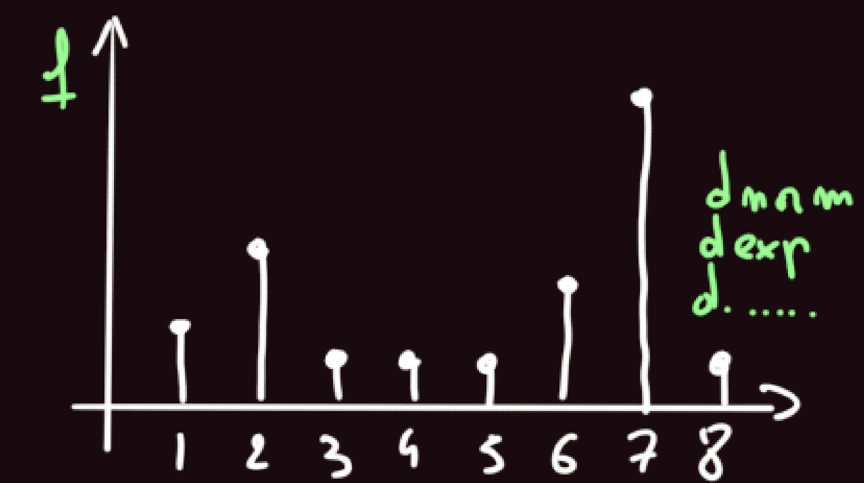
$X \sim f$ density function

i	1	2	3	4	5	6	7	8
p_i	.1	.2	.05	.05	.05	.15	.35	.05

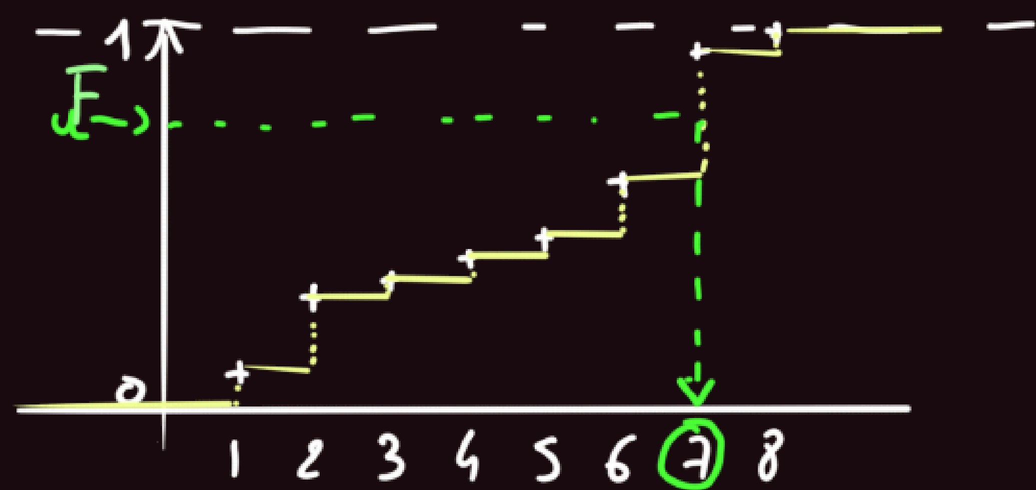


cumulative density function

$u \sim U[0,1]$

$\underbrace{x = F^{-1}(u)}_{\text{inverse de la cdf}}$

i		1	2	3	4	5	6	7	8
P_i	0	.1	.3	.35	.4	.45	.6	.95	1.0

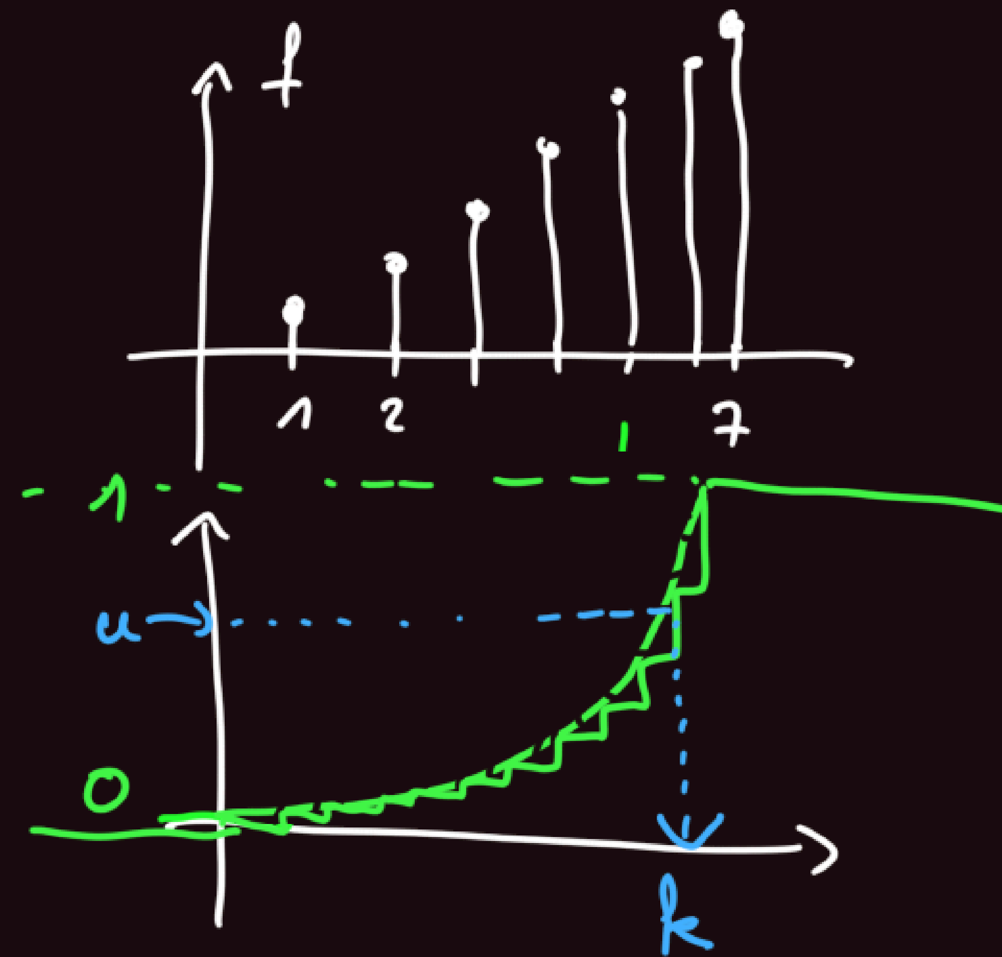


N

$\mathbb{P}[N=i] = \alpha \cdot i$ pour $i \in [\![1, n]\!]$

$$1 = \sum_{i=1}^{n} \alpha i = \alpha \sum_{i=1}^{n} i = \alpha \frac{(n(n+1))}{2}$$

$$F(k) = \sum_{i=1}^{k} \alpha i = \frac{k(k+1)}{n(n+1)}$$

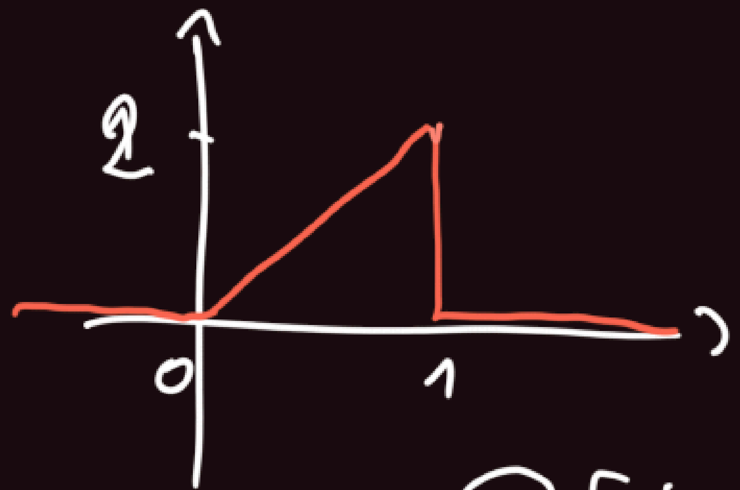


$F(k) = u$ $\qquad k(k+1) = u(n(n+1))$

$$k^2 + k - u(n(n+1)) = 0$$

$\Delta \rightsquigarrow x_1$ $\qquad \lfloor \quad \rfloor$

x_2



* $f(x) = \begin{cases} 2x & \text{si } x \in [0,1] \\ 0 & \text{sinon} \end{cases}$

$f \geqslant 0$

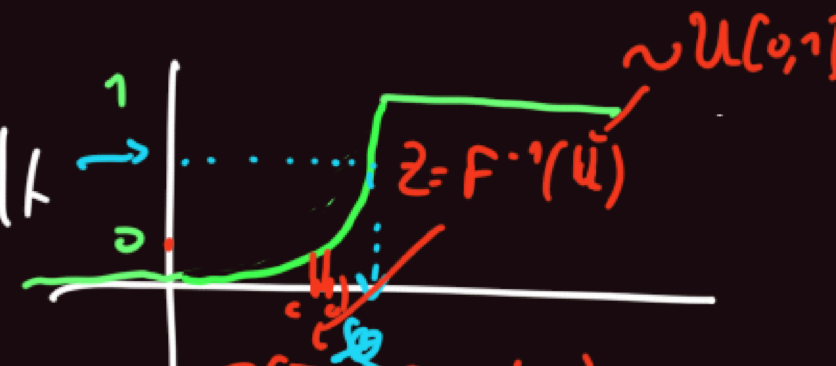
$\int f = 1$

$F : 0 \nearrow 1$

* $F(x) = \mathbb{P}[X \leqslant x] = \int_{-\infty}^{x} f(t)\,dt$

$= \int_0^x 2t\,dt$ pour $x \in [0,1]$

$= \left[t^2 \right]_0^x = x^2$



$\mathbb{P}(Z \in [c,d]) =$

$\mathbb{P}(F^{-1}(U) \in [c,d]) =$

$\mathbb{P}(U \in [F(c), F(d)]) =$

$F(d) - F(c) = \mathbb{P}[X \in [c,d]]$

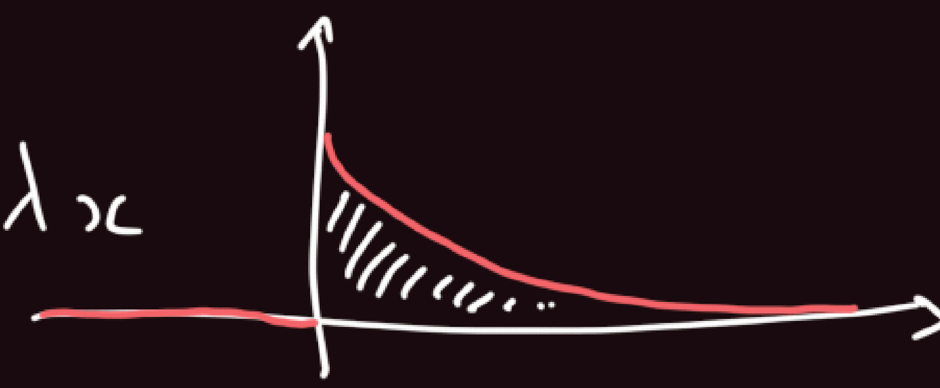
si $X \sim f$

* $F^{-1}(u) = x \iff F(x) = u \iff x^2 = u$

$\iff \boxed{x = \sqrt{u}}$

Loi exponentielle.

$X \sim \mathcal{E}(\lambda)$ si $f(x) = \lambda e^{-\lambda x}$
pour $x \geq 0$



$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_{0}^{x} \lambda e^{-\lambda t}\,dt = \left[-e^{-\lambda t}\right]_0^x = 1 - e^{-\lambda x}$$

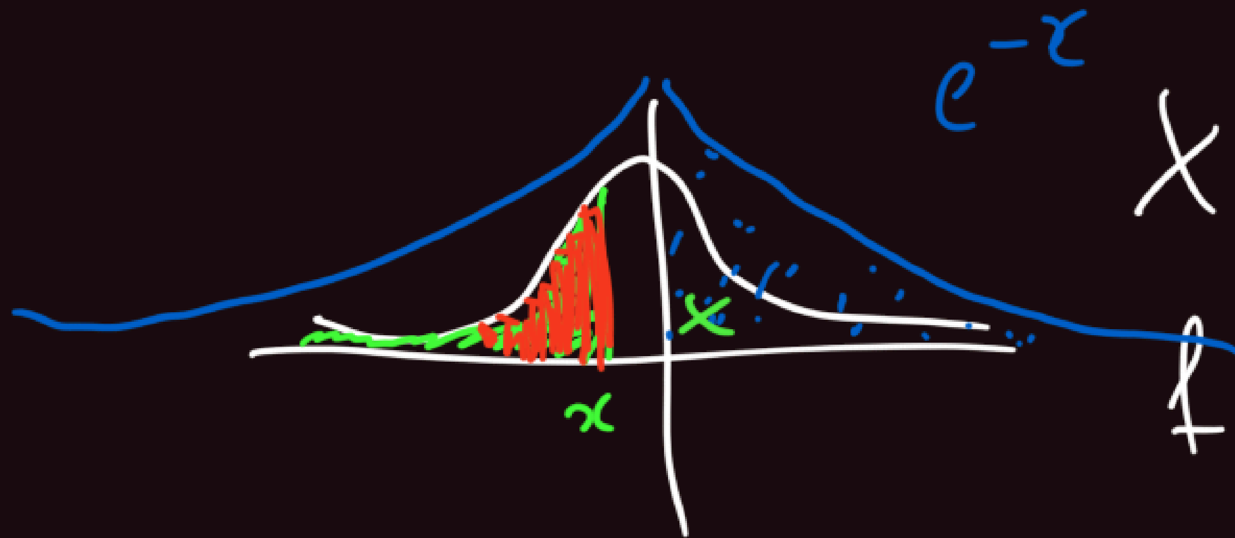
$$F(x) = u \iff 1 - e^{-\lambda x} = u$$

$$\iff e^{-\lambda x} = 1 - u$$

$$\iff -\lambda x = \ln(1-u)$$

$$\iff x = \frac{\ln(1-u)}{-\lambda}$$

-log(runif(N))/lambda



$$X \sim \mathcal{N}(0,1)$$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_{-\infty}^{x} \underbrace{e^{-\frac{x^2}{2}}}_{erf}\,dx$$

while(T)

$$\left[\begin{array}{l} x \sim U[a,b] \\ y \sim U[0,M] \\ \text{if } y \leq f(x) \text{ return}(x) \end{array}\right.$$

$$h e^{-x} \geq \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$