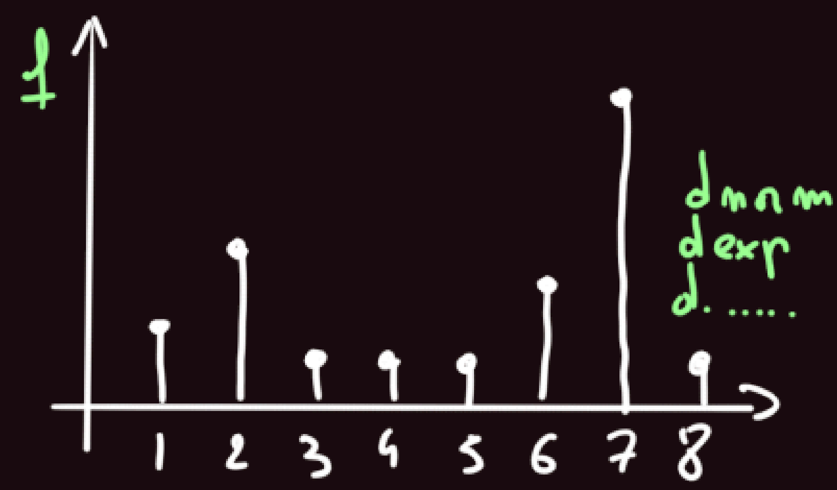
$X \sim f$ density function

i	1	2	3	4	5	6	7	8
p_i	.1	.2	.05	.05	.05	.15	.35	.05



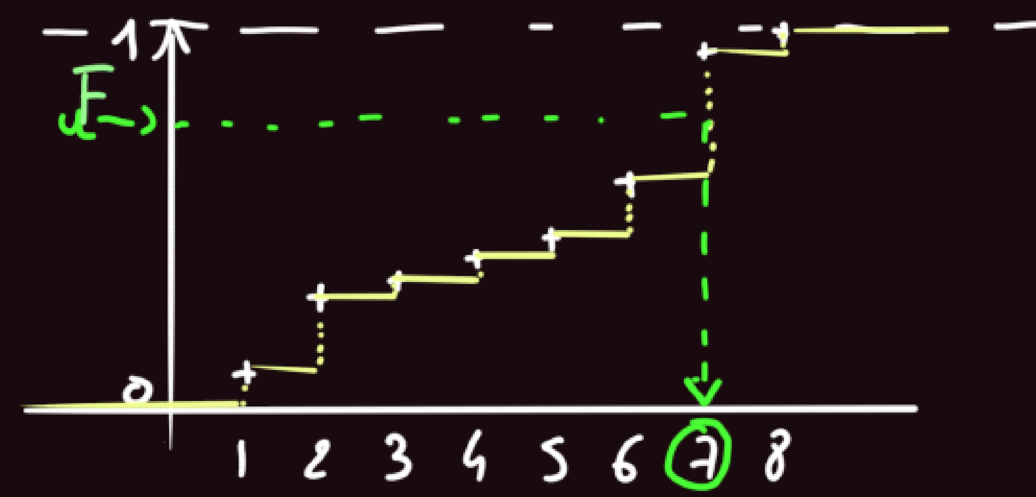
cumulative density function

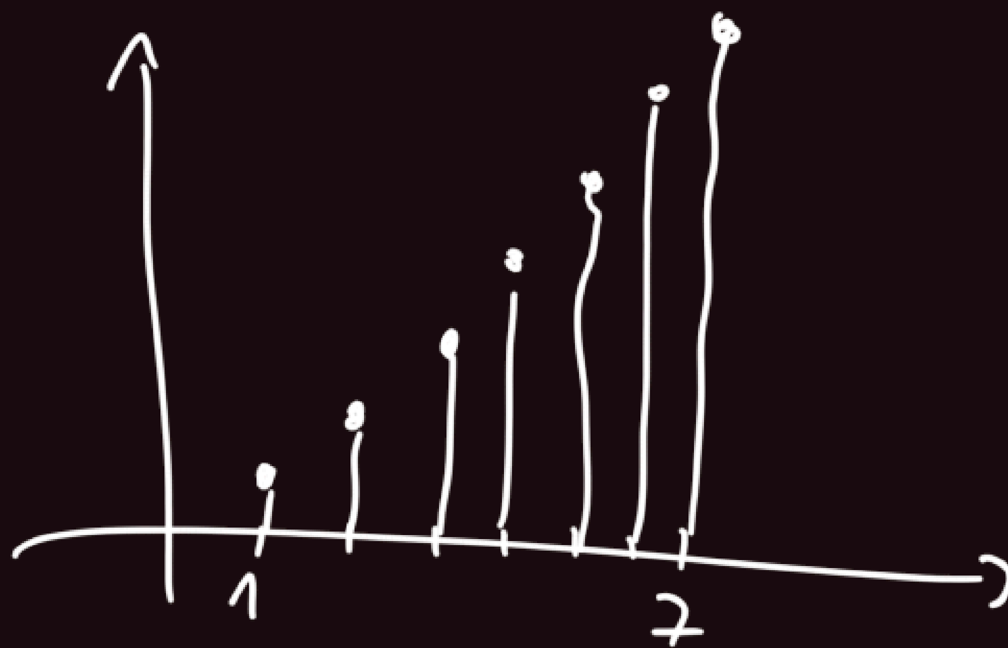
i		1	2	3	4	5	6	7	8
P_i	0	.1	.3	.35	.4	.45	.6	.95	1.0

$u \sim U[0,1]$

$\underbrace{x = F^{-1}(u)}$

inverse de la cdf





$$P(X=i)=\alpha \cdot i$$
$$i \in [\![1;n]\!].$$

$$\alpha \underbrace{\sum_{i=1}^{n} i}_{\frac{n(n+1)}{2}} = 1 \Rightarrow \alpha = \frac{2}{n(n+1)}$$

1) Inverse CDF

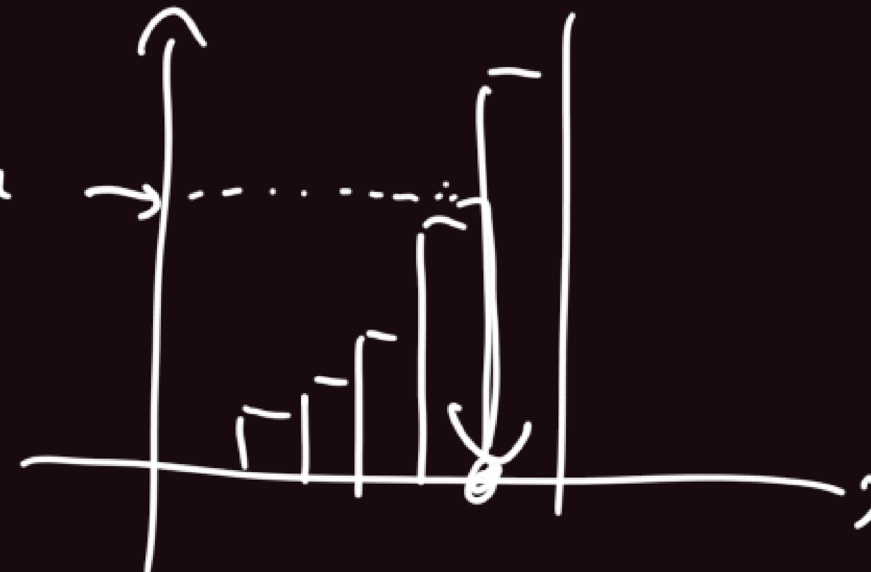
$f \rightsquigarrow F \rightsquigarrow F^{-1}(u)$

2) Reject

3) Adhoc

$$F(1)=\alpha$$
$$F(2)=\alpha+2\alpha$$
$$F(3)=\alpha+2\alpha+3\alpha$$
$$F(k)=\alpha\sum_{i=1}^{k} i = \frac{k(k+1)}{2}\alpha$$
$$F(k)=\frac{k(k+1)}{n(n+1)}$$

$$F(k) = \frac{k(k+1)}{n(n+1)}$$



$$F(k) = u$$

$$k^2 + k = n(n+1)u$$

Δ

$\Rightarrow x_1$

$ x_2$

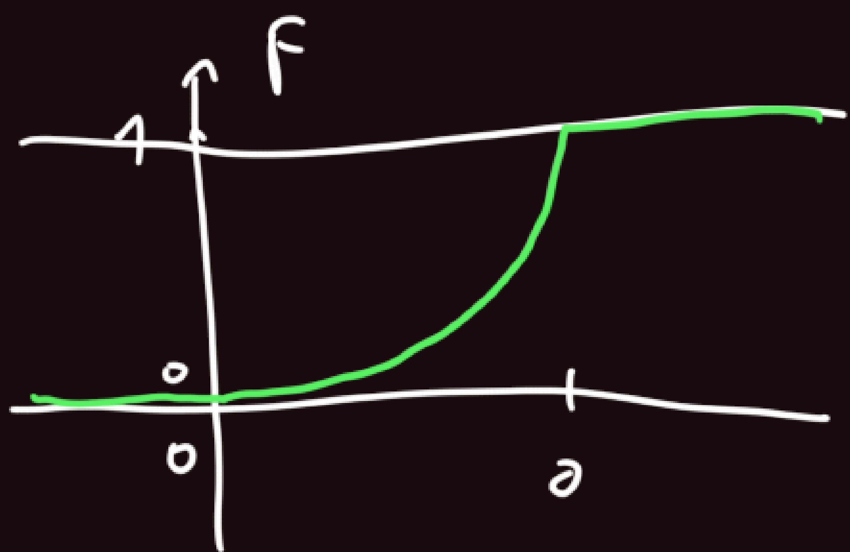
$f(x) = 2x$ pour $x \in [0,1]$

$f : \mathbb{R} \to \mathbb{R}^+$ et $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$F : \mathbb{R} \to [0,1]$ et $\nearrow$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

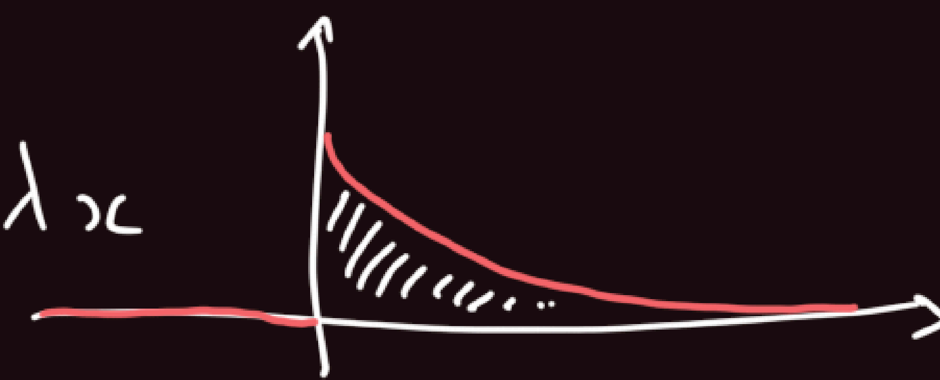
$$= \int_0^x 2t\,dt = \left[t^2 \right]_0^x = x^2$$



$x = \text{sqrt}(\text{runif}(n, 0, 1))$

Loi exponentielle.

$X \sim \mathcal{E}(\lambda)$ si $f(x) = \lambda e^{-\lambda x}$ pour $x \geq 0$



$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_{0}^{x} \lambda e^{-\lambda t}\,dt = \left[-e^{-\lambda t}\right]_{0}^{x} = 1 - e^{-\lambda x}$$

$$F(x) = u \iff 1 - e^{-\lambda x} = u$$

$$\iff e^{-\lambda x} = 1 - u$$

$$\iff -\lambda x = \ln(1-u)$$

$$\iff x = \frac{\ln(1-u)}{-\lambda}$$

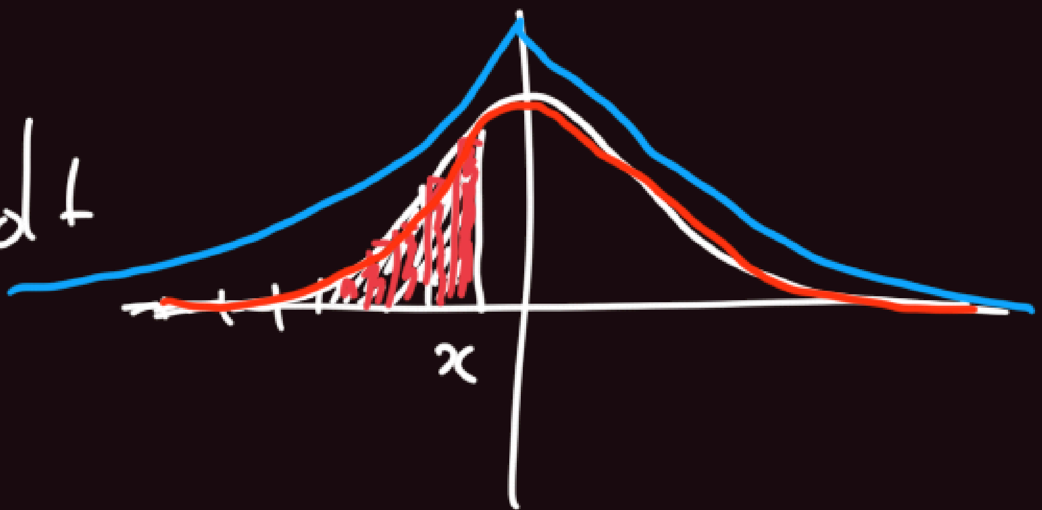
-log(runif(N))/lambda

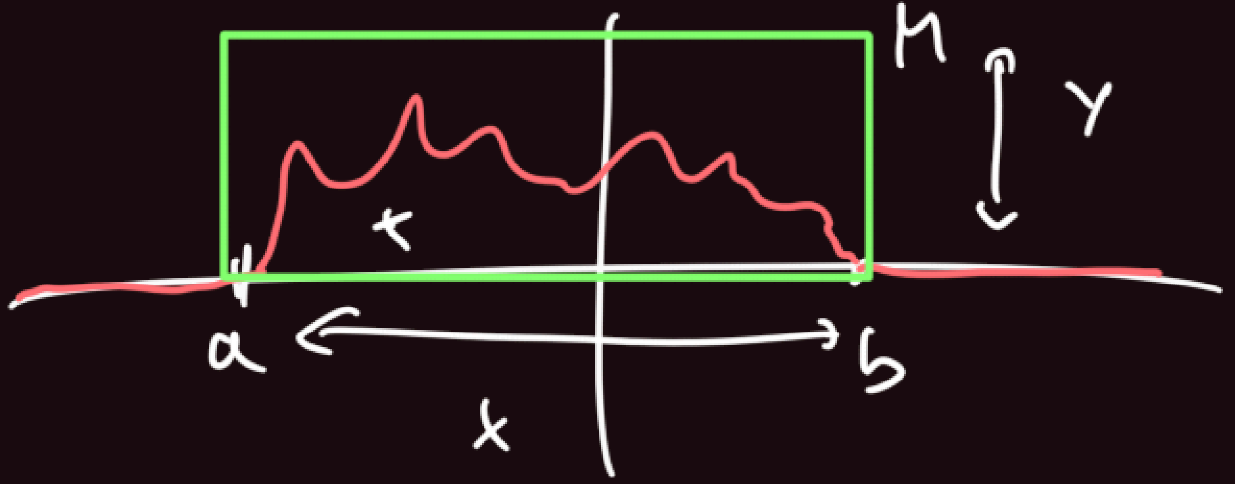
Loi normale : $X \sim \mathcal{N}(0,1)$

$$\boxed{\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad \mathcal{N}(\mu,\sigma^2)}$$

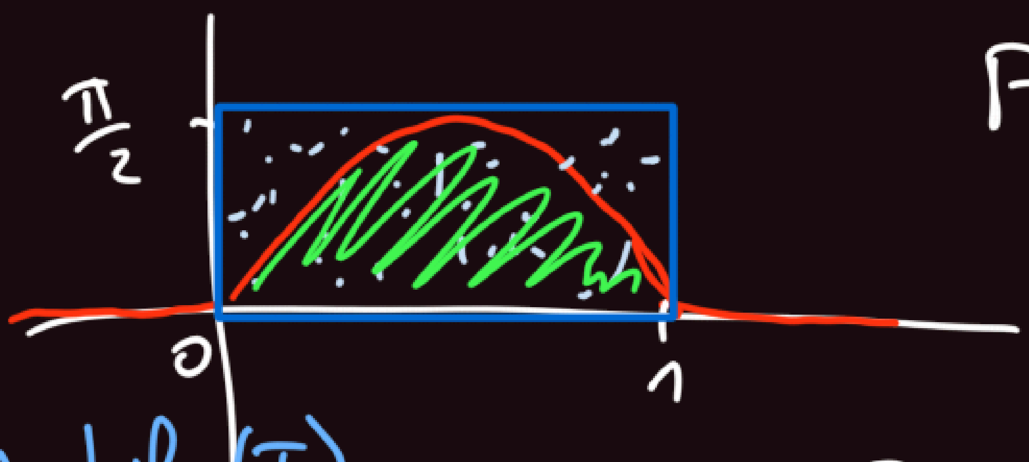
$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}}\,dt$$





$$f(x) = \frac{\pi}{2}\sin(\pi x)\,.\,\mathbb{1}_{[0,1]}(x)$$



$$F(x) = \int_0^x \frac{\pi}{2}\sin(\pi t)\,dt = \frac{1}{2}\Big[-\cos(\pi t)\Big]_0^x$$
$$= \frac{1}{2}\big(1-\cos(\pi x)\big)$$

while(T)

X = runif(1, 0, 1)

Y = runif(1, 0, $\frac{\pi}{2}$)

if $y \leq \frac{\pi}{2}\sin(\pi * x)$ return(x)

$$F(x) = u \iff \frac{1}{2}\big(1-\cos(\pi x)\big) = u$$
$$\iff \cos(\pi x) = 1-2u$$
$$\iff x = \frac{\arccos(1-2u)}{\pi}$$

Proba_accept = $\frac{\text{green}}{\text{blue}} = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$

$$\mathbb{E}(N) = \frac{\pi}{2}$$