

# Computational Metaphysics 1

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## Exercise 1)

a)

"The ship is huge and it is blue."

$$Huge(the\_ship) \wedge Blue(the\_ship) \quad (1)$$

b)

"I'm sad if the sun does not shine."

$$\neg Sun\_is\_shining \rightarrow Sad(I) \quad (2)$$

c)

"Either it's raining or it is not."

$$Is\_raining \vee Is\_not\_raining \quad (3)$$

d)

"I'm only going if she is going!"

$$I\_am\_going \leftrightarrow She\_is\_going \quad (4)$$

e)

"Everyone loves chocolate or ice cream."

$$\forall x Is\_someone(x) \rightarrow Loves\_ice\_cream(x) \vee Loves\_chocolate(x) \quad (5)$$

f)

"There is somebody who loves ice cream and loves chocolate as well."

$$\exists x Is\_someone(x) \wedge Loves\_ice\_cream(x) \wedge Loves\_chocolate(x) \quad (6)$$

g)

"Everyone has got someone to play with."

$$\forall x \exists y Is\_someone(x) \wedge Is\_someone(y) \wedge Can\_play\_with(x, y) \quad (7)$$

h)

"Nobody has somebody to play with if they are all mean."

$$\neg \exists x \exists y Is\_someone(x) \wedge Is\_somenone(y) \wedge Is\_mean(x) \wedge Can\_play\_with(x, y) \quad (8)$$

i)

"Cats have the same annoying properties as dogs."

$$\forall P \forall cat \forall dog (Is\_annoying(P) \wedge Is\_cat(cat) \wedge Is\_dog(dog)) \rightarrow (P(cat) \leftrightarrow P(dog)) \quad (9)$$

## Excercise 2)

a)

propositional

**b)**

higher-order

**c)**

first-order

**d)**

higher-order

### **Excercise 3)**

**a)**

$$A \wedge B \rightarrow C, B \rightarrow A, B \vdash C \quad (10)$$

**b)**

$$A \vdash B \rightarrow A \quad (11)$$

**c)**

$$A \rightarrow (B \rightarrow C) \vdash B \rightarrow (A \rightarrow C) \quad (12)$$

**d)**

$$\neg A \vdash A \rightarrow B \quad (13)$$

**e)**

$$\vdash A \vee \neg A \quad (14)$$

f)

$$A \vee B \vdash A \rightarrow B \quad (15)$$

g)

$$\neg A \vee B \vdash A \rightarrow B \quad (16)$$

1	X	<b>A</b>
2	(A → ¬A) ∧ (¬A → A)	<b>A</b>
3	A → ¬A	2 ∧ <b>E</b>
4	¬A → A	2 ∧ <b>E</b>
5	A	<b>A</b>
6	¬A	3, 5 → <b>E</b>
7	A	5 <b>R</b>
8	¬A	5-7 ¬ <b>I</b>
9	¬A	<b>A</b>
10	A	4, 9 → <b>E</b>
11	¬A	9 <b>R</b>
12	A	9-11 ¬ <b>I</b> <sup>+</sup>
13	¬((A → ¬A) ∧ (¬A → A))	2-12 ¬ <b>I</b> <sup>+</sup>

1	$AAA$	Assumption 1
2	$BBB$	Assumption 2
3	$CCC$	Assumption 3
4	$DDD$	Main proof step
5	$EEE$	Another main proof step
6	$FFF$	Another main proof step

#### Exercise 4)

a)

$$\frac{\frac{[A]}{A} \text{ id}}{A \rightarrow A} \text{ impl} \quad (17)$$

b)

$$\frac{\frac{\frac{[A]}{A} \text{ id}}{B \rightarrow A} \text{ impl}}{A \rightarrow (B \rightarrow A)} \text{ impl} \quad (18)$$

Note that A follows independently of B, so in particular, it follows from B. We can always add arbitrary assumptions, even if our conclusions do not need them.

c)

$$\begin{array}{c}
\frac{[A]^1}{A} \text{ id} \quad \frac{[A \rightarrow B]^3}{B} \text{ mp} \quad \frac{[A]^1}{A} \text{ id} \quad \frac{[A \rightarrow (B \rightarrow C)]^2}{B \rightarrow C} \text{ mp} \\
\hline
\frac{C}{A \rightarrow C} \text{ impI}_1 \\
\frac{(A \rightarrow B) \rightarrow (A \rightarrow C)}{(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))} \text{ impI}_3 \\
\hline
(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C)) \text{ impI}_2
\end{array} \tag{19}$$

d)

$$\begin{array}{c}
\frac{[B]^1}{[B]^1} \quad \frac{[\neg A]^2 \quad [\neg A \rightarrow \neg B]^3}{\neg B} \text{ mp} \\
\hline
\frac{\perp}{A} \text{ ccontr}_2 \\
\frac{B \rightarrow A}{(\neg A \rightarrow \neg B) \rightarrow (B \rightarrow A)} \text{ impI}_1 \\
\hline
(\neg A \rightarrow \neg B) \rightarrow (B \rightarrow A) \text{ impI}_3
\end{array} \tag{20}$$