

Computational Metaphysics 1

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26. April 2016

Exercise 1)

a)

"The ship is huge and it is blue."

$$Huge(the_ship) \wedge Blue(the_ship) \quad (1)$$

b)

"I'm sad if the sun does not shine."

$$\neg Sun_is_shining \rightarrow Sad(I) \quad (2)$$

c)

"Either it's raining or it is not."

$$Is_raining \vee Is_not_raining \quad (3)$$

d)

"I'm only going if she is going!"

$$I_am_going \leftrightarrow She_is_going \quad (4)$$

e)

"Everyone loves chocolate or ice cream."

$$\forall x Is_someone(x) \rightarrow Loves_ice_cream(x) \vee Loves_chocolate(x) \quad (5)$$

f)

"There is somebody who loves ice cream and loves chocolate as well."

$$\exists x Is_someone(x) \wedge Loves_ice_cream(x) \wedge Loves_chocolate(x) \quad (6)$$

g)

"Everyone has got someone to play with."

$$\forall x \exists y Is_someone(x) \wedge Is_someone(y) \wedge Can_play_with(x, y) \quad (7)$$

h)

"Nobody has somebody to play with if they are all mean."

$$\forall x Is_someone(x) \wedge Is_mean(x) \rightarrow \neg \exists y \wedge Is_somenone(y) \wedge Can_play_with(x, y) \quad (8)$$

i)

"Cats have the same annoying properties as dogs."

$$\forall P \forall cat \forall dog (Is_annoying(P) \wedge Is_cat(cat) \wedge Is_dog(dog)) \rightarrow (P(cat) \leftrightarrow P(dog)) \quad (9)$$

Excercise 2)

a)

propositional

b)

higher-order

c)

first-order

d)

higher-order

Excercise 3)

a)

$$A \wedge B \rightarrow C, B \rightarrow A, B \vdash C \quad (10)$$

1	B	$ass.$
2	$B \rightarrow A$	$ass.$
3	A	$mp(1, 2)$
4	$A \wedge B$	$conjI(1, 3)$
5	$A \wedge B \rightarrow C$	$ass.$
6	C	$mp(4, 5)$

b)

$$A \vdash B \rightarrow A \quad (11)$$

1	A	$ass.$
2	B	$hyp.$
3	$B \rightarrow A$	$impI(1, 2)$

c)

$$A \rightarrow (B \rightarrow C) \vdash B \rightarrow (A \rightarrow C) \quad (12)$$

1	B	$hyp.$	
2	<div style="border-left: 1px solid black; padding-left: 10px; border-bottom: 1px solid black;">A</div>	$hyp.$	
3	<div style="border-left: 1px solid black; padding-left: 10px;">$A \rightarrow (B \rightarrow C)$</div>	$ass.$	
4	<div style="border-left: 1px solid black; padding-left: 10px;">$B \rightarrow C$</div>	$mp(2, 3)$	(13)
5	C	$mp(1, 4)$	
6	$A \rightarrow C$	$impI(2, 5)$	
7	$B \rightarrow (A \rightarrow C)$	$impI(1, 6)$	

d)

$$\neg A \vdash A \rightarrow B \quad (14)$$

1	$\neg A$	$ass.$	
2	A	$hyp.$	
3	<div style="border-left: 1px solid black; padding-left: 10px; border-bottom: 1px solid black;">$\neg B$</div>	hyp	
4	<div style="border-left: 1px solid black; padding-left: 10px;">\perp</div>	$notE(1, 2)$	(15)
5	B	$ccontr(2, 4)$	
6	$A \rightarrow B$	$imp(2, 5)$	

e)

$$\vdash A \vee \neg A \quad (16)$$

$$\begin{array}{lcl}
1 & | & \neg(A \vee \neg A) \quad hyp. \\
2 & | & | \quad A \quad hyp. \\
3 & | & | \quad A \vee \neg A \quad disjI(2) \\
4 & | & \perp \quad notE(1,3) \\
5 & & A \vee \neg A \quad ccontr(1,4)
\end{array} \tag{17}$$

Excercise 4)

a)

$$\frac{\frac{[A]}{A} id}{A \rightarrow A} impI \tag{18}$$

b)

$$\frac{\frac{\frac{[A]}{A} id}{B \rightarrow A} impI}{A \rightarrow (B \rightarrow A)} impI \tag{19}$$

Note that A follows independently of B, so in particular, it follows from B. We can always add arbitrary assumptions, even if our conclusions do not need them.

c)

$$\frac{\frac{\frac{[A]^1}{A} id}{B} [A \rightarrow B]^3 mp}{\frac{\frac{\frac{[A]^1}{A} id}{B \rightarrow C} [A \rightarrow (B \rightarrow C)]^2 mp}{\frac{C}{A \rightarrow C} impI_1} impI_3} impI_2 \tag{20}$$

d)

$$\begin{array}{c}
 \frac{[B]^1 \quad \frac{[\neg A]^2 \quad [\neg A \rightarrow \neg B]^3}{\neg B} \text{mp}}{\neg B} \text{notE} \\
 \frac{\frac{\perp}{A} \text{ccontr}_2 \quad \frac{\neg B}{B \rightarrow A} \text{implI}_1}{(\neg A \rightarrow \neg B) \rightarrow (B \rightarrow A)} \text{implI}_3
 \end{array} \tag{21}$$