Logik und Komplexität Übung 7

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Excercise 1)

a)

"The ship is huge and it is blue."

$$Huge(the\_ship) \wedge Blue(the\_ship) \tag{1}$$

b)

"I'm sad if the sun does not shine."

$$\neg Sun\_is\_shining \rightarrow Sad(I) \tag{2}$$

c)

"Either it's raining or it is not."

$$Is\_raining \vee Is\_not\_raining \tag{3}$$

d)

"I'm only going if she is going!"

$$I\_am\_going \leftrightarrow She\_is\_going \tag{4}$$

e)

"Everyone loves chocolate or ice cream."

$$\forall x Is\_someone(x) \rightarrow Loves\_ice\_cream(x) \vee Loves\_chocolate(x) \tag{5}$$

f)

"There is somebody who loves ice cream and loves chocolate as well."

$$\exists x Is\_someone(x) \land Loves\_ice\_cream(x) \land Loves\_chocolate(x) \tag{6}$$

g)

"Everyone has got someone to play with."

$$\forall x \exists y Is\_someone(x) \land Is\_someone(y) \land Can\_play\_with(x, y) \tag{7}$$

h)

"Nobody has somebody to play with if they are all mean."

$$\neg \exists x \exists y Is\_someone(x) \land Is\_somenone(y) \land Is\_mean(x) \land Can\_play\_with(x, y) \tag{8}$$

i)

"Cats have the same annoying properties as dogs."

$$\forall P \forall cat \forall dog(Is\_annoying(P) \land Is\_cat(cat) \land Is\_dog(dog)) \rightarrow (P(cat) \leftrightarrow P(dog))) \tag{9}$$

Excercise 2)

a)

propositional

b)

higher-order

c)

first-order

d)

higher-order

Excercise 3)

a)

$$A \land B \to C, B \to A, B \vdash C \tag{10}$$

1	X	**A**
2	$(A \to \neg A) \land (\neg A \to A)$	**A**
3	$A \to \neg A$	2 $\land$ **E**
4	$\neg A \to A$	2 $\land$ **E**
5	A	**A**
6	$\neg A$	3, 5 $\to$ **E**
7	A	5 **R**
8	$\neg A$	5-7 $\neg$ **I**
9	$\neg A$	**A**
10	A	4, 9 $\to$ **E**
11	$\neg A$	9 **R**
12	A	9-11 $\neg$ $\mathbf{I}^+$
13	$\neg((A \to \neg A) \land (\neg A \to A))$	2-12 $\neg$ $\mathbf{I}^+$

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1	AAA	Assumption 1
2	BBB	Assumption 2
3	CCC	Assumption 3
4	DDD	Main proof step
5	EEE	Another main proof step
6	FFF	Another main proof step

Excercise 4)