Computational Metaphysics 1

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26. April 2016

Excercise 1)

a)

"The ship is huge and it is blue."

$$Huge(the\_ship) \wedge Blue(the\_ship) \tag{1}$$

b)

"I'm sad if the sun does not shine."

$$\neg Sun\_is\_shining \rightarrow Sad(I) \tag{2}$$

c)

"Either it's raining or it is not."

$$Is\_raining \vee Is\_not\_raining \tag{3}$$

d)

"I'm only going if she is going!"

$$I\_am\_going \leftrightarrow She\_is\_going \tag{4}$$

e)

"Everyone loves chocolate or ice cream."

$$\forall x Is\_someone(x) \rightarrow Loves\_ice\_cream(x) \vee Loves\_chocolate(x) \quad (5)$$

f)

"There is somebody who loves ice cream and loves chocolate as well."

$$\exists x Is\_someone(x) \wedge Loves\_ice\_cream(x) \wedge Loves\_chocolate(x) \quad (6)$$

g)

"Everyone has got someone to play with."

$$\forall x \exists y Is\_someone(x) \wedge Is\_someone(y) \wedge Can\_play\_with(x, y) \quad (7)$$

h)

"Nobody has somebody to play with if they are all mean."

$$\forall x Is\_someone(x) \wedge Is\_mean(x) \rightarrow \neg \exists y \wedge Is\_somenone(y) \wedge Can\_play\_with(x, y) \quad (8)$$

i)

"Cats have the same annoying properties as dogs."

$$\forall P \forall cat \forall dog(Is\_annoying(P) \wedge Is\_cat(cat) \wedge Is\_dog(dog)) \rightarrow (P(cat) \leftrightarrow P(dog))) \quad (9)$$

Excercise 2)

a)

propositional

b)

higher-order

c)

first-order

d)

higher-order

Excercise 3)

a)

$$A \wedge B \to C, B \to A, B \vdash C \tag{10}$$

1	B	$ass.$
2	$B \to A$	$ass.$
3	A	$mp(1,2)$
4	$A \wedge B$	$conjI(1,3)$
5	$A \wedge B \to C$	$ass.$
6	C	$mp(4,5)$

b)

$$A \vdash B \to A \tag{11}$$

1	A	$ass.$
2	$\vert\ B$	$hyp.$
3	$\vert\ B \to A$	$impI(1,2)$

c)

$$A \to (B \to C) \vdash B \to (A \to C) \tag{12}$$

1	B	$hyp.$
2	$\quad A$	$hyp.$
3	$\quad A \to (B \to C)$	$ass.$
4	$\quad B \to C$	$mp(2,3)$
5	C	$mp(1,4)$
6	$A \to C$	$impI(2,5)$
7	$B \to (A \to C)$	$impI(1,6)$

(13)

d)

$$\neg A \vdash A \to B \tag{14}$$

1	$\neg A$	$ass.$
2	$\quad A$	$hyp.$
3	$\quad\quad \neg B$	hyp
4	$\quad\quad \bot$	$notE(1,2)$
5	$\quad B$	$ccontr(2,4)$
6	$\quad A \to B$	$imp(2,5)$

(15)

e)

$$\vdash A \vee \neg A \tag{16}$$

1	$\neg(A \vee \neg A)$	$hyp.$
2	$\quad A$	$hyp.$
3	$\quad A \vee \neg A$	$disjI(2)$
4	$\bot$	$notE(1,3)$
5	$A \vee \neg A$	$ccontr(1,4)$

(17)

Excercise 4)

a)

$$\dfrac{\dfrac{[A]}{A}\,\mathrm{id}}{A \to A}\,\mathrm{impI} \tag{18}$$

b)

$$\dfrac{\dfrac{\dfrac{[A]}{A}\,\mathrm{id}}{B \to A}\,\mathrm{impI}}{A \to (B \to A)}\,\mathrm{impI} \tag{19}$$

Note that A follows independently of B, so in particular, it follows from B. We can always add arbitrary assumptions, even if our conclusions do not need them.

c)

$$\dfrac{\dfrac{\dfrac{\dfrac{[A]^1}{A}\,\mathrm{id} \quad \dfrac{[A \to B]^3}{B}\,\mathrm{mp} \qquad \dfrac{\dfrac{[A]^1}{A}\,\mathrm{id} \quad \dfrac{[A \to (B \to C)]^2}{B \to C}\,\mathrm{mp}}{C}}{A \to C}\,\mathrm{impI}_1}{(A \to B) \to (A \to C)}\,\mathrm{impI}_3}{(A \to (B \to C)) \to ((A \to B) \to (A \to C))}\,\mathrm{impI}_2 \tag{20}$$

d)

$$\dfrac{\dfrac{\dfrac{[B]^1 \quad \dfrac{[\neg A]^2 \quad [\neg A \to \neg B]^3}{\neg B}\,\text{mp}}{\bot}\,\text{notE}}{A}\,\text{ccontr}_2}{B \to A}\,\text{impI}_1}{(\neg A \to \neg B) \to (B \to A)}\,\text{impI}_3 \tag{21}$$