

# Computational Metaphysics 1

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## Exercise 1)

a)

"The ship is huge and it is blue."

$$Huge(the\_ship) \wedge Blue(the\_ship) \quad (1)$$

b)

"I'm sad if the sun does not shine."

$$\neg Sun\_is\_shining \rightarrow Sad(I) \quad (2)$$

c)

"Either it's raining or it is not."

$$Is\_raining \vee Is\_not\_raining \quad (3)$$

d)

"I'm only going if she is going!"

$$I\_am\_going \leftrightarrow She\_is\_going \quad (4)$$

e)

"Everyone loves chocolate or ice cream."

$$\forall x Is\_someone(x) \rightarrow Loves\_ice\_cream(x) \vee Loves\_chocolate(x) \quad (5)$$

f)

"There is somebody who loves ice cream and loves chocolate as well."

$$\exists x Is\_someone(x) \wedge Loves\_ice\_cream(x) \wedge Loves\_chocolate(x) \quad (6)$$

g)

"Everyone has got someone to play with."

$$\forall x \exists y Is\_someone(x) \wedge Is\_someone(y) \wedge Can\_play\_with(x, y) \quad (7)$$

h)

"Nobody has somebody to play with if they are all mean."

$$\neg \exists x \exists y Is\_someone(x) \wedge Is\_somenone(y) \wedge Is\_mean(x) \wedge Can\_play\_with(x, y) \quad (8)$$

i)

"Cats have the same annoying properties as dogs."

$$\forall P \forall cat \forall dog (Is\_annoying(P) \wedge Is\_cat(cat) \wedge Is\_dog(dog)) \rightarrow (P(cat) \leftrightarrow P(dog)) \quad (9)$$

## Excercise 2)

a)

propositional

b)

higher-order

c)

first-order

d)

higher-order

### Excercise 3)

a)

$$A \wedge B \rightarrow C, B \rightarrow A, B \vdash C \quad (10)$$

1	$B$	$ass.$
2	$B \rightarrow A$	$ass.$
3	$A$	$mp(1, 2)$
4	$A \wedge B$	$conjI(1, 3)$
5	$A \wedge B \rightarrow C$	$ass.$
6	$C$	$mp(4, 5)$

b)

$$A \vdash B \rightarrow A \quad (11)$$

1	$A$	$ass.$
2	$\left  \begin{array}{l} B \end{array} \right.$	$hyp.$
3	$\left  \begin{array}{l} B \rightarrow A \end{array} \right.$	$impI(1, 2)$

c)

$$A \rightarrow (B \rightarrow C) \vdash B \rightarrow (A \rightarrow C) \quad (12)$$

1	$B$	$hyp.$	
2	<div style="border-left: 1px solid black; padding-left: 10px; border-bottom: 1px solid black;"><math>A</math></div>	$hyp.$	
3	<div style="border-left: 1px solid black; padding-left: 10px;"><math>A \rightarrow (B \rightarrow C)</math></div>	$ass.$	
4	<div style="border-left: 1px solid black; padding-left: 10px;"><math>B \rightarrow C</math></div>	$mp(2, 3)$	(13)
5	$C$	$mp(1, 4)$	
6	$A \rightarrow C$	$impI(2, 5)$	
7	$B \rightarrow (A \rightarrow C)$	$impI(1, 6)$	

d)

$$\neg A \vdash A \rightarrow B \quad (14)$$

1	$\neg A$	$ass.$	
2	$A$	$hyp.$	
3	<div style="border-left: 1px solid black; padding-left: 10px; border-bottom: 1px solid black;"><math>\neg B</math></div>	$hyp$	
4	<div style="border-left: 1px solid black; padding-left: 10px;"><math>\perp</math></div>	$notE(1, 2)$	(15)
5	$B$	$ccontr(2, 4)$	
6	$A \rightarrow B$	$imp(2, 5)$	

e)

$$\vdash A \vee \neg A \quad (16)$$

$$\begin{array}{lcl}
1 & | & \neg(A \vee \neg A) \quad hyp. \\
& | & \hline
2 & | & A \quad hyp. \\
& | & \hline
3 & | & A \vee \neg A \quad disjI(2) \\
4 & | & \perp \quad notE(1,3) \\
5 & A \vee \neg A & ccontr(1,4)
\end{array} \tag{17}$$

f)

$$A \vee B, \neg A \vdash B \tag{18}$$

g)

$$\neg A \vee B \vdash A \rightarrow B \tag{19}$$

#### Excercise 4)

a)

$$\frac{\frac{[A]}{A} id}{A \rightarrow A} impl \tag{20}$$

b)

$$\frac{\frac{\frac{[A]}{A} id}{B \rightarrow A} impl}{A \rightarrow (B \rightarrow A)} impl \tag{21}$$

Note that A follows independently of B, so in particular, it follows from B. We can always add arbitrary assumptions, even if our conclusions do not need them.

c)

$$\begin{array}{c}
\frac{[A]^1}{A} \text{ id} \quad \frac{[A \rightarrow B]^3}{B} \text{ mp} \quad \frac{[A]^1}{A} \text{ id} \quad \frac{[A \rightarrow (B \rightarrow C)]^2}{B \rightarrow C} \text{ mp} \\
\hline
\frac{C}{A \rightarrow C} \text{ impI}_1 \\
\frac{(A \rightarrow B) \rightarrow (A \rightarrow C)}{(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))} \text{ impI}_3 \\
\hline
(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C)) \text{ impI}_2
\end{array} \tag{22}$$

d)

$$\begin{array}{c}
\frac{[B]^1}{[B]^1} \quad \frac{[\neg A]^2 \quad [\neg A \rightarrow \neg B]^3}{\neg B} \text{ mp} \\
\hline
\frac{\perp}{A} \text{ ccontr}_2 \\
\frac{B \rightarrow A}{(\neg A \rightarrow \neg B) \rightarrow (B \rightarrow A)} \text{ impI}_1 \\
\hline
(\neg A \rightarrow \neg B) \rightarrow (B \rightarrow A) \text{ impI}_3
\end{array} \tag{23}$$