Exercise 6

By Martin Lundfall, Denis Erfurt k0025944

May 30, 2016

Contents

theory *LundfallErfurtEx06*
imports *QML Main*

begin

Exercise 1

consts $X :: \sigma$
consts $Y :: \sigma$
theorem K:
  shows $\lfloor\ \Box(X \rightarrow\ Y) \rightarrow (\Box\ X \rightarrow \Box\ Y)\rfloor$
proof *simp*
qed

consts $mayVote :: \mu{\Rightarrow}\sigma$
consts $bornEqual :: \mu{\Rightarrow}\mu{\Rightarrow}\sigma$

theorem *Ex1-a*:
  assumes *1*: $\lfloor\ \neg\ \Diamond\ ((\forall\ \ h1.\ \forall\ \ h2.\ bornEqual\ h1\ h2) \wedge (\exists\ \ h3.\ \neg\ mayVote(h3)))$
$\rfloor$ *AND*
  assumes *2*: $\lfloor\ \Box\ (\forall\ \ h1.\ \forall\ \ h2.\ bornEqual\ h1\ h2)\ \rfloor$
  shows $\lfloor\ (\Box\ (\forall\ \ h.(mayVote(h))))\ \rfloor$
proof $-$
 from *assms* **have** $\lfloor\ \Box\ (\neg\ ((\forall\ \ h1.\ \forall\ \ h2.\ bornEqual\ h1\ h2) \wedge (\exists\ \ h3.\ \neg\ mayVote(h3))))$
$\rfloor$ **by** *simp*
 then have $\lfloor\ \Box\ ((\forall\ \ h1.\ \forall\ \ h2.\ bornEqual\ h1\ h2) \rightarrow (\neg\ (\exists\ \ h3.\ \neg\ mayVote(h3))))$
$\rfloor$ **by** *metis*
 then have *3*: $\lfloor\ (\Box\ (\forall\ \ h1.\ \forall\ \ h2.\ (bornEqual\ h1\ h2))) \rightarrow (\Box\ \neg\ (\exists\ \ h3.\ \neg$
$mayVote(h3)))\ \rfloor$ **using** K **by** *simp*
 from *2 3* **have** $\lfloor(\Box\ \neg\ (\exists\ \ h3.\ \neg\ mayVote(h3)))\rfloor$ **by** *simp*
 then show *?thesis* **by** *simp*
qed

consts $isRaining :: \sigma$
consts $StreetIsWet :: \sigma$

theorem *Ex1-b*:
  assumes *1*: $\lfloor\Box(isRaining \longrightarrow StreetIsWet)\rfloor$ *AND*
  assumes *2*: $\lfloor isRaining\rfloor$
  shows $\lfloor StreetIsWet\rfloor$
nitpick

oops

axiomatization where
  T: $\lfloor\Box A \longrightarrow A\ \rfloor$

theorem *Ex1-c*:
  assumes *1*: $\lfloor\Box(\ isRaining \longrightarrow StreetIsWet)\rfloor$ *AND*
  assumes *2*: $\lfloor isRaining\rfloor$
  shows $\lfloor StreetIsWet\rfloor$
proof −
  from *assms* **have** *3*: $\lfloor(\Box isRaining) \longrightarrow (\Box\ StreetIsWet)\rfloor$ **by** *simp*
  from *2* **have** *4*: $\lfloor\Box isRaining\rfloor$ **by** *simp*
  from *2 3* **have** $\lfloor\Box\ StreetIsWet\rfloor$ **by** *auto*
  then show *?thesis* **using** *T* **by** *auto*
qed

axiomatization where
  5: $\lfloor\Diamond A \longrightarrow \Box\Diamond A\ \rfloor$

The intuition behind this axiom is: If something is possible, then it is necessarily possible

theorem *Ex1-d*:
  assumes $\lfloor isRaining\rfloor$
  shows $\lfloor\Box\Diamond isRaining\rfloor$
proof −
  from *assms* **show** *?thesis* **using** *5* **by** *metis*
qed

Exercise 2

consts *unlimited* :: $\mu\Rightarrow\sigma$

definition *God* **where** *God G* $\equiv$ *unlimited*(G)

axiomatization where
  B: $\lfloor(\Box\ (\exists\ x.\ unlimited(x))) \vee (\neg\Diamond(\exists\ x.\ unlimited(x)))\rfloor$

axiomatization where
  C: $\lfloor\neg\Diamond(\exists\ x.\ unlimited(x))\rfloor$

theorem *d*:
  shows $\lfloor\Box\ (\exists\ x.\ x = G)\rfloor$
proof −
  show *?thesis* **using** *B C* **by** *auto*

qed

To prove this ontological argument we do not require axiom K or 5

end