

# Computational Metaphysics 1

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## Exercise 1)

a)

"The ship is huge and it is blue."

$$Huge(the\_ship) \wedge Blue(the\_ship) \quad (1)$$

b)

"I'm sad if the sun does not shine."

$$\neg Sun\_is\_shining \rightarrow Sad(I) \quad (2)$$

c)

"Either it's raining or it is not."

$$Is\_raining \vee Is\_not\_raining \quad (3)$$

d)

"I'm only going if she is going!"

$$I\_am\_going \leftrightarrow She\_is\_going \quad (4)$$

e)

"Everyone loves chocolate or ice cream."

$$\forall x Is\_someone(x) \rightarrow Loves\_ice\_cream(x) \vee Loves\_chocolate(x) \quad (5)$$

f)

"There is somebody who loves ice cream and loves chocolate as well."

$$\exists x Is\_someone(x) \wedge Loves\_ice\_cream(x) \wedge Loves\_chocolate(x) \quad (6)$$

g)

"Everyone has got someone to play with."

$$\forall x \exists y Is\_someone(x) \wedge Is\_someone(y) \wedge Can\_play\_with(x, y) \quad (7)$$

h)

"Nobody has somebody to play with if they are all mean."

$$\neg \exists x \exists y Is\_someone(x) \wedge Is\_somenone(y) \wedge Is\_mean(x) \wedge Can\_play\_with(x, y) \quad (8)$$

i)

"Cats have the same annoying properties as dogs."

$$\forall P \forall cat \forall dog (Is\_annoying(P) \wedge Is\_cat(cat) \wedge Is\_dog(dog)) \rightarrow (P(cat) \leftrightarrow P(dog)) \quad (9)$$

## Excercise 2)

a)

propositional

b)

higher-order

c)

first-order

d)

higher-order

### Excercise 3)

a)

$$A \wedge B \rightarrow C, B \rightarrow A, B \vdash C \quad (10)$$

|   |                            |               |
|---|----------------------------|---------------|
| 1 | $B$                        | $ass.$        |
| 2 | $B \rightarrow A$          | $ass.$        |
| 3 | $A$                        | $mp(1, 2)$    |
| 4 | $A \wedge B$               | $conjI(1, 3)$ |
| 5 | $A \wedge B \rightarrow C$ | $ass.$        |
| 6 | $C$                        | $mp(4, 5)$    |

b)

$$A \vdash B \rightarrow A \quad (11)$$

|   |  |              |
|---|--|--------------|
| 1 | $A$  | $ass.$       |
| 2 | $\left  \begin{array}{l} B \end{array} \right.$                      | $hyp.$       |
| 3 | $\left  \begin{array}{l} \hline B \rightarrow A \end{array} \right.$ | $impI(1, 2)$ |

c)

$$A \rightarrow (B \rightarrow C) \vdash B \rightarrow (A \rightarrow C) \quad (12)$$

|   |   |              |      |
|---|---|--------------|------|
| 1 | $B$   | $hyp.$       |      |
| 2 | <div style="border-left: 1px solid black; padding-left: 10px; border-bottom: 1px solid black;"><math>A</math></div> | $hyp.$       |      |
| 3 | <div style="border-left: 1px solid black; padding-left: 10px;"><math>A \rightarrow (B \rightarrow C)</math></div>   | $ass.$       |      |
| 4 | <div style="border-left: 1px solid black; padding-left: 10px;"><math>B \rightarrow C</math></div>                   | $mp(2, 3)$   | (13) |
| 5 | $C$   | $mp(1, 4)$   |      |
| 6 | $A \rightarrow C$   | $impI(2, 5)$ |      |
| 7 | $B \rightarrow (A \rightarrow C)$   | $impI(1, 6)$ |      |

d)

$$\neg A \vdash A \rightarrow B \quad (14)$$

|   |  |                |      |
|---|--|----------------|------|
| 1 | $\neg A$   | $ass.$         |      |
| 2 | $A$  | $hyp.$         |      |
| 3 | <div style="border-left: 1px solid black; padding-left: 10px; border-bottom: 1px solid black;"><math>\neg B</math></div> | $hyp$          |      |
| 4 | <div style="border-left: 1px solid black; padding-left: 10px;"><math>\perp</math></div>                                  | $notE(1, 2)$   | (15) |
| 5 | $B$  | $ccontr(2, 4)$ |      |
| 6 | $A \rightarrow B$  | $imp(2, 5)$    |      |

e)

$$\vdash A \vee \neg A \quad (16)$$

$$\begin{array}{lcl}
1 & | & \neg(A \vee \neg A) \quad hyp. \\
2 & | & | \quad A \quad hyp. \\
3 & | & | \quad A \vee \neg A \quad disjI(2) \\
4 & | & \perp \quad notE(1,3) \\
5 & & A \vee \neg A \quad ccontr(1,4)
\end{array} \tag{17}$$

### Excercise 4)

a)

$$\frac{\frac{[A]}{A} id}{A \rightarrow A} impI \tag{18}$$

b)

$$\frac{\frac{\frac{[A]}{A} id}{B \rightarrow A} impI}{A \rightarrow (B \rightarrow A)} impI \tag{19}$$

Note that A follows independently of B, so in particular, it follows from B. We can always add arbitrary assumptions, even if our conclusions do not need them.

c)

$$\frac{\frac{\frac{[A]^1}{A} id}{B} [A \rightarrow B]^3 mp}{\frac{\frac{\frac{[A]^1}{A} id}{B \rightarrow C} [A \rightarrow (B \rightarrow C)]^2 mp}{\frac{C}{A \rightarrow C} impI_1} impI_3} impI_2 \tag{20}$$

d)

$$\begin{array}{c}
 \frac{[B]^1 \quad \frac{[\neg A]^2 \quad [\neg A \rightarrow \neg B]^3}{\neg B} \text{mp}}{\neg B} \text{notE} \\
 \frac{\frac{\perp}{A} \text{ccontr}_2 \quad \frac{\neg B}{B \rightarrow A} \text{implI}_1}{(\neg A \rightarrow \neg B) \rightarrow (B \rightarrow A)} \text{implI}_3
 \end{array} \tag{21}$$