The diagram model of linear dependent type theory

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Abstract

We present a type theory dealing with non-linear, "ordinary" dependent types (which we will call *cartesian*), and *linear types*, where both constructs may depend on terms of the former. In the interplay between these, we find the new type formers $\sqcap_{x:A} B$ and $\sqsubset_{x:A} B$, akin to Π and Σ, but where the dependent type B, (and therefore the resulting construct) is a linear type. These can be seen as internalizing universal and existential quantification over linear propositions, respectively. We also consider two modalities, M and L, transforming linear types into cartesian types and vice versa.

The theory is interpreted in a split comprehension category $\pi : \mathcal{T} \to \mathcal{C}^{\to}$ [2], accompanied by a split monoidal fibration, $q : \mathcal{L} \to \mathcal{C}$. We interpret $\mathcal{C}$ as a category of contexts, which for any $\Gamma \in \mathcal{C}$, determines the fibers $\mathcal{T}_\Gamma$ and $\mathcal{L}_\Gamma$. We interpret $\mathcal{T}_\Gamma$ as category of the cartesian types over Γ, and $\mathcal{L}_\Gamma$ as the monoidal category of linear types in Γ. In this setting, the type formers $\sqcap_{x:A}$ and $\sqsubset_{x:A}$ are understood as right and left adjoints of the monoidal reindexing functor $\pi_A^* : \mathcal{L}_\Gamma \to \mathcal{L}_{\Gamma.A}$ corresponding to the weakening projection $\pi_A : \Gamma.A \to \Gamma$ in $\mathcal{C}$. The operators M and L give rise to a fiberwise adjunction $L \dashv M$ between $\mathcal{L}$ and $\mathcal{T}$, where we understand the traditional exponential modality as the comonad $! = LM$.

We provide a model of this theory called the *Diagram model*, which extends the groupoid model of dependent type theory [1] to accomodate linear types. Here, cartesian types over a context Γ are interpreted as a family of groupoids indexed over the groupoid Γ, while linear types are interpreted as diagrams over groupoids, $A : \Gamma \to \mathcal{V}$ in any symmetric monoidal category $\mathcal{V}$. We show that the *diagrams model* can under certain conditions support a linear analogue of the univalence axiom, and provide some discussion on the higher-dimensional nature of linear dependent types.

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References

1 Martin Hofmann and Thomas Streicher. The groupoid interpretation of type theory. In *Twenty-five years of constructive type theory (Venice, 1995)*, volume 36 of *Oxford Logic Guides*, pages 83–111. Oxford Univ. Press, New York, 1998.

2 Bart Jacobs. Comprehension categories and the semantics of type dependency. *Theoret. Comput. Sci.*, 107(2):169–207, 1993. URL: `https://doi.org/10.1016/0304-3975(93)90169-T`.