REPASO FINAL:

$\delta(x,y) \xrightarrow{DFT} 1$

EX 20172:

PREGUNTA 1:

$$H(u,v) \triangleq H(e^{j\omega x}, e^{j\omega y})\Big|_{\substack{\omega x = \frac{2\pi u}{M} \\ \omega y = \frac{2\pi v}{N}}} \quad ; \quad \begin{matrix} u \in \{0,1,\dots,M-1\} \\ v \in \{0,1,\dots,N-1\} \end{matrix}$$

A)

$$\therefore H(u,v) \begin{cases} 1, & \underbrace{\left|\frac{2\pi u}{M}\right| \le \frac{\pi}{8}}_{|u|\le 1}, \ \underbrace{\left|\frac{2\pi v}{N}\right| \le \frac{\pi}{8}}_{|v|\le 1} \\ 0; & \text{otros casos} \end{cases}$$

$M = N = 16$

$$h(x,y) = \underset{\substack{M=16\\N=16}}{IDFT2D}\left\{ \begin{aligned} &\delta(u,v) + \delta(u-1,v-1) + \delta(u+1,v+1) + \delta(u-1,v+1) \\ &+ \delta(u+1,v-1) + \delta(u,v+1) + \delta(u,v-1) + \delta(u+1,v) + \delta(u-1,v) \end{aligned}\right.$$

$$= \frac{1}{16(16)} \sum_{u=-1}^{1} \sum_{v=-1}^{1} H(u,v) \cdot e^{j2\pi\left(\frac{ux}{16} + \frac{vy}{16}\right)}$$

$$= \frac{1}{16^2}\Big\{ 1 + e^{j\frac{2\pi}{16}x} + e^{-j\frac{2\pi}{16}x} + e^{j\frac{2\pi}{16}y} + e^{-j\frac{2\pi}{16}y} + e^{j\frac{2\pi}{16}(x+y)} + e^{-j\frac{2\pi}{16}(x+y)} + e^{j\frac{2\pi}{16}(x-y)} + e^{-j\frac{2\pi}{16}(-x+y)} \Big\}$$

$$\boxed{\cancel{+e^{-j\frac{2\pi}{16}(x-y)}}_{\text{error}} \ \cancel{+e^{j\frac{2\pi}{16}(-x+y)}}_{\text{error}}}$$

$$= \frac{1}{16^2}\left[1 + e^{j\frac{\pi}{8}x} + e^{-j\frac{\pi}{8}x}\right]\left[1 + e^{j\frac{\pi}{8}y} + e^{-j\frac{\pi}{8}y}\right]$$

$$= \frac{1}{256}\left[1 + 2\cos\left(\frac{\pi}{8}x\right)\right]\left[1 + 2\cos\left(\frac{\pi}{8}y\right)\right]$$

b) $r(x,y) = h(x,y) \cdot \sin\left(\frac{\pi}{2}x\right)$; $f(x,y) = \underbrace{\cos\left(\frac{\pi}{2}x\right)}$

$1 \longleftrightarrow MN\delta(u,v)$

$$\frac{1}{2}e^{j\frac{\pi}{2}x} + \frac{1}{2}e^{-j\frac{\pi}{2}x}$$

$$\frac{2\pi}{16}(4) \qquad \frac{2\pi}{16}(-4)$$

$$F(u,v) = \underset{\substack{M=16\\N=16}}{DFT2D}\left\{\frac{1}{2}e^{j\frac{2\pi}{16}(4)x} + \frac{1}{2}e^{j\frac{2\pi}{16}(-4)x}\right\}$$

$$= \frac{16^2}{2}\delta(u-4,v) + \frac{16^2}{2}\delta(u+4,v)$$

$$R(u,v) = \underset{\substack{M=16\\N=16}}{DFT2D}\left\{-\frac{j}{2}h(x,y)\,e^{j\frac{2\pi}{16}(4)} + \frac{j}{2}h(x,y)\,e^{+j\frac{2\pi}{16}(-4)x}\right\}$$

$$= -\frac{j}{2}H(u-4,v) + \frac{j}{2}H(u+4,v)$$

$$Z(u,v) = F(u,v) \cdot R(u,v) = -\frac{j}{2}\left(\frac{16^2}{2}\right)\delta(u-4,v) + \frac{j}{2}\left(\frac{16^2}{2}\right)\delta(u+4,v)\}$$

$$\therefore z(x,y) = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)$$

Práctica 2:

$$T$$

$$\Delta s = 2.5 \times 10^{-3} \longrightarrow f = 400$$

$$\Delta t = 2 \cdot 10^{-3} \longrightarrow f = 500$$

A)

$$f(x,y) \triangleq f_c(\Delta s \cdot x, \Delta t \cdot y)$$

$$= 2^7 + 3.5 \sin\left(\frac{200\pi}{400} x + \frac{100\pi y}{500}\right)$$

$$= \begin{pmatrix} 125 & 125 & 125 & 127 \\ 126 & \underline{128} & 130 & 131 \\ 131 & 132 & 131 & 129 \\ 130 & 128 & 126 & 125 \end{pmatrix}$$

B)

$$h_i = \{4, 2, 1, 2, 1, 2, 3, 1\}$$

índice 125 126 127 128 129 130 131 132

$$h_N[i] = \frac{h_i}{16} = P_i$$

OTSU - SIMPLIFICADO:

i	125	126	127	128	129	130	131	~~132~~
$P_1(i)$	0.25	0.375	0.4375	0.5625	0.625	0.75	0.9375	
$m_1(i)$	31.5	47.38	55.38	71.5	79.63	96	120.75	
m_g	129.063							
σ_B^2	3.1263	4.469	4.8265	4.8959	4.6065	3.3867	1.0336	

→ No se segmenta

NOTA: se rechaza las probabilidad 1 en cualquier clase.

$$\boxed{4.8959}$$

$$K = 128 \cdot \underset{K}{\arg\max}\ \sigma_B^2(K)$$

$$\therefore = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} \Big\} \ g(x,y) = \begin{cases} 1 ; & f(x,y) \le 128 \\ 0 ; & \text{otros} \end{cases}$$

PROBLEMA 3:

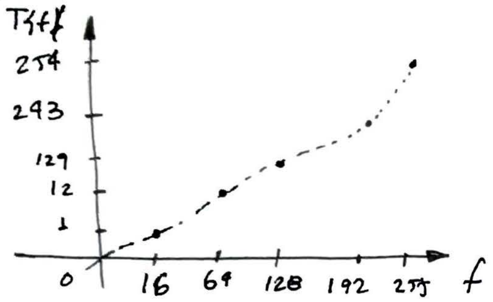
3A)

$$P(x,y) = \begin{pmatrix} 0 & 255 & 192 & 16 \\ 128 & 255 & 16 & 192 \\ 64 & 16 & 0 & 128 \\ 128 & 64 & 16 & 0 \end{pmatrix}$$

$$T\{P(x,y)\} = \begin{pmatrix} 1 & 254 & 243 & 1 \\ 129 & 254 & 1 & 243 \\ 12 & 1 & 1 & 129 \\ 129 & 12 & 1 & 1 \end{pmatrix}$$

función sigmoide.

* contraste:



COMPARAMOS $T\{f\}$ vs. f.

* TRANSFORMACIÓN AUMENTA EL CONTRASTE EN INTENSIDADES MEDIAS ~~{64 – 128}~~ {128 – 192}

B) B.P7 (más significativo)

$$B.P.7 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$H = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

~~$BP \ominus H =$~~

~~1 1 1 1~~
~~1 1~~
~~1 1 1 1 1~~
~~1 1 1 1 1 1 1~~

~~$BP \ominus H =$~~

~~1 0 0 0~~
~~0 0 0 0~~
~~0 0 1 0~~
~~0 0 0 0 0~~
~~0 0 0 0 0 0 0 0~~

$$B.P \ominus H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$(BP \ominus H) \oplus H = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

PREGUNTA 4:

A) $$T\{\alpha(x,y)\} = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \underbrace{\alpha(m,n)}_{\delta(m-1,n-1)} h(x-2m, y-2n)$$

$$\delta(m-1,n-1) \cdot h(x-2, y-2)$$

$$= h(x-2,y-2) \cdot \underbrace{\sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \delta(m-1,n-1)}_{1}$$

1 $$= h(x-2, y-2)$$

$$T\{\beta(x,y)\} = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \underbrace{\beta(m,n)}_{\delta(x-2,y-2)} \cdot h(x-2m, y-2n)$$

$$= h(x-4, y-4) \cdot \underbrace{\sum_{m=\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \delta(m-2,n-2)}_{1}$$

2 $$= h(x-4, y-4)$$

1 $\neq$ 2 $\Rightarrow$ VARIANTE ANTE DESPLASAMIENTO

LSI

B)

$$G_1(\mu,\nu) = \underset{M=N=9}{\text{DFT2D}} \{ g_1(x,y) \}$$

$$f_1\left(\frac{x}{3},\frac{y}{3}\right) e^{j\frac{8\pi}{3}(x+y)}$$

$$= \sum_{x=\langle 9\rangle} \sum_{y=\langle 9\rangle} f_1\left(\tfrac{x}{3},\tfrac{y}{3}\right) \cdot e^{j\frac{8\pi}{3}(x+y)} \cdot e^{-j\frac{2\pi}{9}(\mu x+\nu y)}$$

(UPSAMPLING)

* CAMBIO DE VARIABLE:

$$\omega x = \frac{2\pi\mu}{M} \; ; \; \omega y = \frac{2\pi\nu}{N}$$

$$\hat{x} = \frac{x}{3} \; ; \; \hat{y} = \frac{y}{3} \; ; \; \mu \in \{0,\dots,8\} \; ; \; \nu \in \{0,\dots,8\}$$

$$\sum_{\hat{x}=\langle 3\rangle} \sum_{\hat{y}=\langle 3\rangle} f_1(\hat{x},\hat{y}) \cdot \underbrace{e^{j\frac{8\pi}{3}(3\hat{x}+3\hat{y})}}_{1} \cdot e^{-j\frac{2\pi}{3}(\mu\hat{x}+\nu\hat{y})}$$

$$= F_1(\mu,\nu) : \underset{\substack{M=3\\N=3}}{\text{DFT2D}} \; ; (\mu,\nu) \in \begin{matrix}\{0,\dots,8\}\\ \{0,\dots,8\}\end{matrix}$$

$$\therefore g_{1p}(x,y) = \sum_{l=-\infty}^{+\infty} \sum_{r=-\infty}^{+\infty} f_1(x-3l,\, y-3r)$$

periódica

Nos quedamos con una parte de duración finita:

$$g_1(x,y) = \begin{cases} g_{1p}(x,y) \; ; & x \in \{0,1,\dots,8\} \\ & y \in \{0,1,\dots,8\} \\ 0 & ; \text{otros casos} \end{cases}$$

Metodo 1.

No es necesario

$$\therefore g_1(x,y) = \left(\begin{array}{ccc|ccc|ccc} 1&2&3&1&2&3&1&2&3\\ 2&2&3&2&2&3&2&2&3\\ 3&3&3&3&3&3&3&3&3\\ \hline 1&2&3&1&2&3&1&2&3\\ 2&2&3&2&2&3&2&2&3\\ 3&3&3&3&3&3&3&3&3\\ \hline 1&2&3&1&2&3&1&2&3\\ 2&2&3&2&2&3&2&2&3\\ 3&3&3&3&3&3&3&3&3 \end{array}\right)$$

Metodo 2

5.A) $W(x,y) = W_1(x,y) * W_2(x,y)$

$W_1(x,y) = \frac{1}{4}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$; $W_2(x,y) = \frac{1}{4}\begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$ $\Big\}$ $\alpha(x,y) = \underbrace{W(x,y)}_{} * f(x,y)$

$(W_1(x,y) * W_2(x,y)) * f(x,y)$

- CONVOLUCIONAR ~~MULTIPLICAR~~ ESTAS 2 MÁSCARAS GAUSS. NOS DA LA DE 3x3.

$* \; N^{3\times3}_{(2,2)} = \begin{pmatrix} 75 & 105 & 107 \\ 77 & \underline{93} & 115 \\ 107 & 132 & 175 \end{pmatrix}$

Respuesta $W_1(x,y)$: al vecindario

$(84 \quad \underline{105.75} \quad 128)$

Respuesta $W_2(x,y)$: $\alpha(2,2) = 105.87$

$* \; N^{3\times3}_{(4,5)} = \begin{pmatrix} 168 & 164 & 127 \\ 114 & \underline{148} & 72 \\ 84 & 147 & 72 \end{pmatrix}$

Respuesta a $w_1(x,y)$:

Respuesta a $w_2(x,y)$: $\alpha(4,5) = 127.3125$

B) CANNY : $f_s(x,y) = f(x,y) * \underbrace{\frac{1}{9}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}}_{\text{filtro promedio}}$

$\nabla f_s(x,y) = \left\{ \frac{\partial f_s}{\partial x}, \frac{\partial f_s}{\partial y} \right\}$

$f_s(x+1,y) - f_s(x,y)$

$f(s,y+1) - f_s(x,y)$

$\therefore \nabla f_s(2,3) = \{ f_s(3,3) - f_s(2,3) ; \; f_s(2,4) - f(2,3) \}$

$= \{ \underbrace{134.67 - 121.33}_{13.34}, \; \underbrace{136.22 - 121.33}_{14.89} \}$

$|\nabla f_s(2,3)| = (13.34^2 + 14.89^2)^{1/2} = 19.99$

$\angle f_s(2,3) = tg^{-1}\{ 14.89 / 13.34 \} = 48.15°$

$\rightarrow$ tener presente si es $\frac{(-)}{(-)}$ tercer cuadrante.

Canny : $d(2,3) = d_2$

$\longrightarrow$ Analizamos vecinos: ~~inf. derecha~~

vecinos: $f_s(1,2) - f_s(3,4)$

i) $\nabla f_s(1,2) = \{14.889,\ 6.67\}$

$|\nabla f(1,2)| = 16.313$

ii) $\nabla f_s(3,4) = \{-3.33,\ -7.22\}$

$|\nabla f_s(1,2)| = 7.9043$

$|\nabla f_s(2,3)| >$ magnitud de vecino

$\therefore f_N(2,3) = 19.99$

* $\{(3,5)\}$: $|\nabla f_s(3,5)| =$

$d(3,5) = d_4$

vecinos: $f_s(4,4),\ f_s(2,6)$

$|\nabla f_s(4,4)| = 24.91$

$|\nabla f_s(2.6)| = 9.55$

* $\{(5,6)\}$: $|\nabla f_s(5,6)| = 22.82$

$d(5,6) = d_3$

vecinos: $f_s(5,5),\ f_s(5,7)$

$|\nabla f_s(5,5)| = 8.33$

$|\nabla f_s(5,7)| = 24.56$

$f_N(5,6) = 0$