NE336 Quiz3 Nov 19$^{\text{th}}$

1 Instructions

- The first line in each file should contain a comment with your complete name and student ID.

- All module import statements have been given but you may modify them however you wish.

- A minimal amount of comments (at least a few lines to explain your method of thought) is required and code with no comments will lose 0.5 marks in total.

- Any written answers can be provided in a word document or from a note taking application of your choice.

2 Conceptual question (2.5 points)

A coop student has been hired by a manufacturer of nicotine patches to predict the concentration of nicotine in the tissue near the patch over a 24-h time period.

The concentration of nicotine in the tissue can be described by the equation:

$$\frac{\partial \rho}{\partial t} = D \frac{\partial^2 \rho}{\partial x^2} - k\rho$$

Where ρ is the is the concentration in mg/cm^3, t the time in hours, D=0.1 cm^2/h the diffusivity and k, the first-order decay rate of Nicotine in h^{-1}.

Here are the known parameters:

- The concentration of nicotine is zero initially.

- At the skin adjacent to the patch, the concentration is 1 mg/cm^3.

- The soft tissue beneath the patch is 2cm and beneath this is impermeable bone so

$$\frac{\partial \rho}{\partial x}\big|_{x=L=2cm} = 0$$

- Rate of drug decay, k, varies between individuals so it could be as low as 0 up to as high as $k = 1.0\ h^{-1}$. (High value of k would mean that the individual's body would break down nicotine more rapidly)

A solution to this PDE is attached to the quiz dropbox. Please download the code, *run it* and answer the following questions based on what you observe.

1. Please remark on whether you think an implicit or explicit method is used.

    - Please justify your choice in one or two sentences.

    - State the benefits and the downside of the method briefly.

2. Based on the code, please comment on whether a centered, forward or backward difference approximation has been applied for

    - the generic internal nodes

    - the BCs

3. **BONUS (upto +0.25):** Looking at the results, what do you observe for users with a high rate of drug decay k? what would you recommend for the Nicotine patch manufacturer for these users?

Note : the answer to this question is only written work and no code needs to be returned for it.

3   Programming question (7.5 points)

Consider the following ODE,

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

with the BCs

$$T(r = 5) = 20$$
$$T(r = 10) = 200$$

We would like to solve this using a FD approach. The first few steps of the tasks below are on paper.

1. Setup your grid and identify your nodes, choose a suitable number of interior nodes.

2. Apply FD approximations to the ODE and write the resulting equation for a generic internal node, i. Use second order approximations.

3. Write these equations for your number of nodes. Include your treatment of the BCs.

4. Setup the code to solve for T(r) and plot T vs r (for all values of r) when done.

   Note : You may wish to increase the number of nodes at this point to get a smoother plot!

5. **BONUS (upto +0.75)**: solve the same problem but with $\frac{dT}{dr}|_{r=10} = 10$

Import statements

```
import numpy as np
import matplotlib.pyplot as plt
```