NE336 Quiz2 Oct 31st

1 Instructions

- The first line in each file should contain a comment with your complete name and ID.
- All module import statements have been given but you may modify them however you wish.
- A minimal amount of comments (at least a few lines to explain your method of thought) is required and code with no comments will lose 0.5 marks in total.
- Any written answers can be provided in a word document or from a note taking application of your choice.

2 Programming questions

2.1 Question 1 (5 points)

Solve the following second-order initial value problem from $t = 0$ to $t = 25$

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - \mu\left(1 - x^2\right)\frac{\mathrm{d}x}{\mathrm{d}t} + x = A\sin(\omega t)$$

where

$$x(t = 0) = 1$$
$$\frac{\mathrm{d}x}{\mathrm{d}t}(t = 0) = 1$$

Note : you will need to decompose the equation into two first-order differential equations.

Tasks

1. Is this ODE linear or nonlinear?
   Comment on the linearity when $\mu = 0$ and when $\mu > 0$. (1 point)
2. Solve the IVP using `solve_ivp` for each of the parts below (4 points)

   a) $A = 0$, $\omega = 2$ and $\mu = [0, 1, 2]$.
   Plot x vs $\frac{\mathrm{d}x}{\mathrm{d}t}$ for all three cases on one graph. Use appropriate labels.

   b) $A = 1$, $\omega = 2$ and $\mu = 0.01$. Plot both x and $\frac{\mathrm{d}x}{\mathrm{d}t}$ vs t in the same figure.

   Note : You may wish to define your system of ODEs function such that μ, A, and ω can be passed as parameters to the `solve_ivp` function but this is optional.
3. **BONUS** Imagine that the IVP is a BVP (you will need to be creative to find the second BC) and solve using `solve_bvp`.

Note : there is another question on the next page.

2.2 Question 2 (5 points)

Consider the following BVP,

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 10y = 0$$

with the BCs

$$\frac{dy}{dx}|_{x=0} = 20$$
$$\frac{dy}{dx}|_{x=10} = 10\,[y(x=10) - 5]$$

Solve the ODE to using a shooting method of your choice. Plot y vs x in your results.

BONUS : Solve the problem one more time with a different shooting method approach. Plot the results on the same graph as above with labels to distinguish the methods.

Import statements

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp, solve_bvp
from scipy.optimize import fsolve
```