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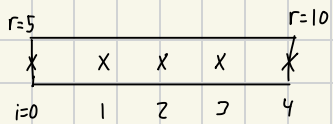
$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$ $\quad T = 20$ @ $r = 5$

$T = 200$ @ $r = 10$

1. Grid



Minimum 3 Interior Nodes Needed

2. Apply Finite Difference

$$\frac{d^2T}{dr^2} = \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta r^2}$$

$$\frac{dT}{dr} = \frac{T_{i+1} - T_{i-1}}{2\Delta r}$$

Plug into Equation

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

$$\frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta r^2} + \frac{1}{r_i}\frac{T_{i+1} - T_{i-1}}{2\Delta r} = 0$$

$$T_{i+1}\left(\frac{1}{\Delta r^2} + \frac{1}{2r_i\Delta r}\right) - T_i\left(\frac{2}{\Delta r^2}\right) + T_{i-1}\left(\frac{1}{\Delta r^2} - \frac{1}{2r_i\Delta r}\right) = 0$$ Generic Node

3. Make Matrix

$i = 0 \quad T = 20$

$i = 1 \quad T_2\left(\frac{1}{\Delta r^2} + \frac{1}{2r_1\Delta r}\right) - T_1\left(\frac{2}{\Delta r^2}\right) + T_0\left(\frac{1}{\Delta r^2} - \frac{1}{2r_1\Delta r}\right) = 0$

$$T_2\left(\frac{1}{\Delta r^2} + \frac{1}{2r_1\Delta r}\right) - T_1\left(\frac{2}{\Delta r^2}\right) = -20\left(\frac{1}{\Delta r^2} - \frac{1}{2r_1\Delta r}\right)$$

$i = 2 \quad T_3\left(\frac{1}{\Delta r^2} + \frac{1}{2r_2\Delta r}\right) - T_2\left(\frac{2}{\Delta r^2}\right) + T_1\left(\frac{1}{\Delta r^2} - \frac{1}{2r_2\Delta r}\right) = 0$

$i=3 \quad T_4\left(\frac{1}{\Delta r^2}+\frac{1}{2r_3\Delta r}\right)-T_3\left(\frac{2}{\Delta r^2}\right)+T_2\left(\frac{1}{\Delta r^2}-\frac{1}{2r_3\Delta r}\right)=0$

$-T_3\left(\frac{2}{\Delta r^2}\right)+T_2\left(\frac{1}{\Delta r^2}-\frac{1}{2r_3\Delta r}\right)=-200\left(\frac{1}{\Delta r^2}+\frac{1}{2r_3\Delta r}\right)$

$i=4 \quad T=200$

Matrix

D

$$\begin{bmatrix} -\frac{2}{\Delta r^2} & \frac{1}{\Delta r^2}+\frac{1}{2r_1\Delta r} & 0 \\ \frac{1}{\Delta r^2}-\frac{1}{2r_2\Delta r} & -\frac{2}{\Delta r^2} & \frac{1}{\Delta r^2}+\frac{1}{2r_2\Delta r} \\ 0 & \frac{1}{\Delta r^2}-\frac{1}{2r_3\Delta r} & -\frac{2}{\Delta r^2} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} -20\left(\frac{1}{\Delta r^2}-\frac{1}{2r_1\Delta r}\right) \\ 0 \\ -200\left(\frac{1}{\Delta r^2}+\frac{1}{2r_3\Delta r}\right) \end{bmatrix}$$

E

Plot is in the .Py File

Bonus Question

Boundary Condition Change

$\frac{dT}{dr}=10$ @ $r=10$ Use Backward Difference so that we can Plug in

$\frac{dT}{dr}=\frac{3T_i-4T_{i-1}+T_{i-2}}{2\Delta r}=10$

$3T_i-4T_{i-1}+T_{i-2}=20\Delta r$ For the Last Equation