Practice Problems

Questions

Question 1

A large well-mixed tank used to allow settling of large particles from brine usually operates at steady-state with constant input (10 kg/min of 0.20 mass fraction salt in water) and output flowrates (10 kg/min, at the salt concentration in the tank). Due to operator error, a valve is opened and an additional 10 kg/min of pure water is introduced to the tank. Material balances on the total mass (M, in kg) and mass of salt (S, in kg) in the tank yield the following pair of equations:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = 10$$
$$\frac{\mathrm{d}S}{\mathrm{d}t} = 10 \times 0.2 - 10\left(\frac{S}{M}\right)$$

Where S/M represents the mass fraction of salt in the tank.

At $t = 0$, the mass of brine $M = 1000$ kg at 0.20 mass fraction, so $S = (0.20/\mathrm{M}) = 200$ kg.

- Using solve_ivp and the initial conditions provided, solve the system above for a time long enough that tank overflow occurs, estimated to be at $M = 2000kg$.
- Plot the total mass and the mass fraction of salt in the tank as a function of time.

Question 2

An irreversible first-order reaction A $\rightarrow$B occurs in a catalyst of thickness L = 0.001 m; assume that the catalyst is wide in the other directions. At steady state, diffusion and reaction in the slab are described by,

$$D\frac{\mathrm{d}^2 C_A}{\mathrm{d}x^2} = kC_A$$

Where the diffusivity $D = 1.2 \times 10^{-9}\ m^2/s$, and the rate constant $k = 0.001\ s^{-1}$.

The reactant at the surface of the catalyst (x = 0) is at concentration C_{A0} = 0.2 mol/L. The catalyst is supported by an impermeable wall, so there is no flux of either species through the lower surface of the catalyst:

$$\left.\frac{\mathrm{d}C_A}{\mathrm{d}x}\right|_{x=L} = 0$$

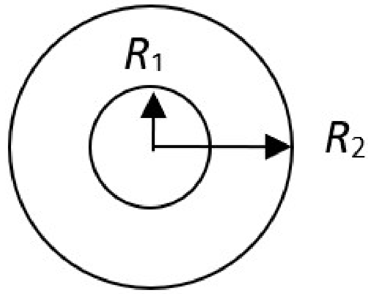
a) Use the shooting method (with solve_ivp) to obtain a solution for the concentration profile $C_A(x)$.

b) Use solve_bvp to obtain the same solution.

c) Present your results.

Question 3

You wish to solve for the steady-state temperature distribution in a long annular cylinder:



With $R_1 = 5$ cm and $R_2 = 10$ cm. In cylindrical coordinates, the governing equation is:

$$\frac{\mathrm{d}^2 T}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}T}{\mathrm{d}r} = 0$$

The temperature of the inside wall is constant, $T(R_1) = 120°\mathrm{C}$. The heat loss through the outside wall has been measured as $\dot{Q} = 1$ kW per m of pipe length (along the axis), so

$$-k\frac{\mathrm{d}T}{\mathrm{d}r}\bigg|_{r=R_2} = \frac{\dot{Q}}{2\pi R_2}$$

The pipe is steel, with $k = 14\ W\ m^{-1} K^{-1}$.

- Write code to implement the finite difference approach for this problem; clearly state the finite difference approximation you are using as a comment within the code. Make n configurable (i.e., there is a line n = ... near the top of the file; you can decide whether n represents internal nodes or all nodes as convenient).
- Note: You need to consider the boundary nodes separately to the interior nodes. You can use in-built methods to solve the resulting matrix equation.
- Make suitable plot(s) to illustrate your results. What is the temperature at R_2 and the temperature gradient at R_1?

Question 4

The governing equation for the steady-state temperature distribution in a pin fin (pictured below) is:

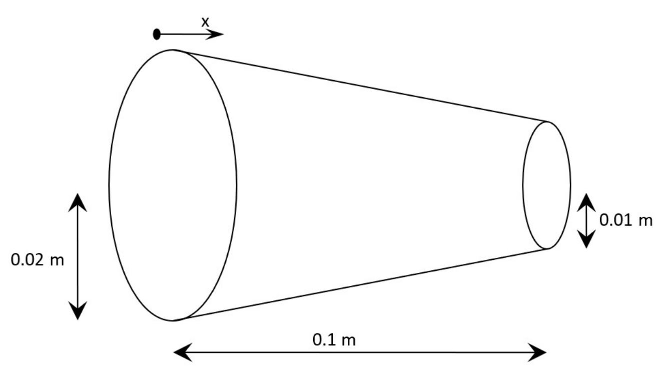
$$\frac{\mathrm{d}}{\mathrm{d}x}\left([r(x)]^2\frac{\mathrm{d}T}{\mathrm{d}x}\right) - \frac{2h}{k}r(x)(T - T_{\text{air}}) = 0$$

Where the boundary conditions are given by:

$$T(0) = 100°C \qquad\qquad -k\frac{\mathrm{d}T}{\mathrm{d}x}\bigg|_{x=L} = h\left(T(L) - T_{\text{air}}\right)$$

The radius r of the fin varies with distance according to $r(x) = 0.02–0.1x$ [in m] and the length $L = 0.10$ m. The air temperature $T_{\text{air}} = 20°C$ and the pin is steel, with $h = 20Wm^{-2}K^{-1}$ and $k = 14Wm^{-1}K^{-1}$.

- Convert the system into a pair of first-order differential equations. Please write your system of first-order equations as a comment in your ode file, together with the boundary conditions you are using.
- Solve the system as a boundary-value problem with solve_bvp. [If you prefer, you can solve the problem via shooting with solve_ivp].
- Make suitable plot(s) to illustrate your results.



Question 5

Carbon dioxide is being absorbed into the surface of an alkaline solution containing a catalyst. The concentration, C(t,x), of CO_2 in solution is described by the following equation,

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} - kC$$

Where t is time, x is the distance from the surface, D is the diffusivity of CO_2 in the alkaline solution and k is a reaction rate constant.

Initially, there is no CO_2 dissolved in the solution. At time 0, the partial pressure of CO_2 in the gas at the solution's surface is suddenly increased to 1 atm, which gives a concentration at the surface of C(t,0) = 0.030 mol/L. Assume that the solution is deep, so that the dissolved CO_2 concentration far from the surface is 0. Take D = 1.5×10^{-9} m^2/s and k = 35 s^{-1}.

a) State suitable boundary and initial conditions for this problem.

b) Solve this PDE at steady state first. So set $\dfrac{\partial C}{\partial t} = 0$ and solve the resulting ODE.

c) Establish a set of equations to solve the PDE using the method of lines; include equations to implement the boundary conditions. Use second-order approximations wherever difference equations are required.

d) Implement the solution; you should obtain a solution that approaches steady-state (start using a 0.01 s timeframe). The boundary layer is quite thin; try using a depth of 10^{-5} m and node spacing of 10^{-6} m. Does the steady state solution match the one you first obtained?

e) Check that your solution is reasonable by halving the step size Δx.

f) Make suitable plots to illustrate the solution.

Question 6

Heat transfer, at unsteady state and in one-dimension, is governed by the equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

You wish to model the temperature T(t,x) at various depths in soil. Assume that the temperature at the soil surface T(t,0) varies over the year according to:

$$T(t,0) = 10 - 14\cos\left[\frac{2\pi}{365}(t-37)\right]$$

Where t is the time in days and T is in Celsius. The temperature gradient is zero at large depth (assume 20 m is "large"),

$$\frac{\partial T}{\partial x} = 0 \ \text{ at } \ \mathrm{x} = 20\,\mathrm{m}$$

The thermal diffusivity of soil $\alpha = 0.081$ m^2/day.

- Solve this problem using the method of lines. Use a node spacing of $\Delta x = 2$ m, model the soil temperature to a depth of 20 m over a 4-year (1460 day) period. Assume that the soil is initially at 10°C.
- Make suitable plot(s) to illustrate your results.
- Why was it necessary to solve over more than one year? [write a brief comment in the file]

Hint: in developing your solution, you may find it convenient to first obtain a solution for a constant surface temperature T(t,0) of, say, 20°C (note: do not choose 10°C, since this will just give T(t,x) = constant!) before substituting the actual boundary condition above.