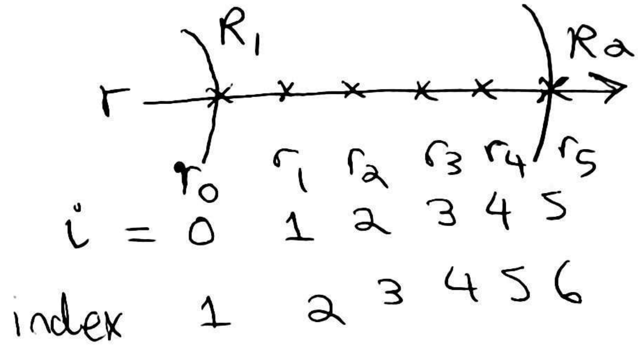

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

$$T(R_1) = 120$$

$$-k \left.\frac{dT}{dr}\right|_{r=R_2} = \frac{\dot{Q}}{2\pi R_2}$$

FD: $\dfrac{d^2T}{dr^2} = \dfrac{T_{i+1} - 2T_i + T_{i-1}}{\Delta r^2}$ $\quad O(\Delta r^2)$ $\quad$ Central.

$\dfrac{dT}{dr} = \dfrac{T_{i+1} - T_{i-1}}{2\Delta r}$ $\quad O(\Delta r^2)$ $\quad$ Central.

Sub in PDE

$$\frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta r^2} + \frac{1}{r_i}\frac{T_{i+1} - T_{i-1}}{2\Delta r} = 0$$

$\times \Delta r^2$ and group

$$\left(1 + \frac{\Delta r}{2 r_i}\right) T_{i+1} - 2T_i + \left(1 - \frac{\Delta r}{2 r_i}\right) T_{i-1} = 0$$

these will be true for the interior nodes. (1,2,3,4)

now for the BCs.

For $i=0$ (inner radius) we have the temp => this will come directly into the first row of our matrix as a const (for $i=1$)

$$i=1 \Rightarrow \left(1+\frac{\Delta r}{2r_1}\right)T_2 - 2T_1 = \underbrace{\left(\frac{\Delta r}{2r_1}-1\right)T_0}_{b}$$

$$\begin{bmatrix} -2 & 1+\frac{\Delta r}{2r_1} & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} T_1 \\ T_2 \\ \\ \end{bmatrix} = \begin{bmatrix} \left(\frac{\Delta r}{2r_1}-1\right)T_0 \\ \\ \\ \end{bmatrix}$$

what about the other BC?

at $i=n+1$, we have the derivative.

$$-k\left.\frac{dT}{dr}\right|_{r=R_2} = \frac{\dot{Q}}{2\pi R_2} \Rightarrow \left.\frac{dT}{dr}\right|_{r=R_2} = -\frac{\dot{Q}}{2\pi k R_2}$$

since we only have the nodes $n+1$ and prior, apply a backward diff (maintains $O(2)$)

$$\Rightarrow \frac{3T_{n+1}-4T_n+T_{n-1}}{\cancel{2}\Delta r} = -\frac{\dot{Q}}{\cancel{2}\pi k R_2}$$

$$\Rightarrow T_{n-1}-4T_n+3T_{n+1} = -\frac{\dot{Q}\Delta r}{k\pi R_2}$$

=> last row of A

$$\begin{bmatrix} & & \\ & & \\ 1 & -4 & 3 \end{bmatrix}\begin{bmatrix} \\ T_{n-1} \\ T_n \\ T_{n+1} \end{bmatrix} = \begin{bmatrix} \\ \\ \vdots \\ -\frac{\dot{Q}\Delta r}{k\pi R_2} \end{bmatrix}$$

Rest of A?

$$
\begin{bmatrix}
-2 & 1+\frac{\Delta r}{2r_i} & & & \\
1-\frac{\Delta r}{2r_i} & -2 & 1+\frac{\Delta r}{2r_i} & & \\
 & 1-\frac{\Delta r}{2r_i} & -2 & 1+\frac{\Delta r}{2r_i} & \\
 & & \ddots & \ddots & \ddots \\
 & & 1 & -4 & 3
\end{bmatrix}
\begin{bmatrix}
T_1 \\ T_2 \\ T_3 \\ \vdots \\ T_{n+1}
\end{bmatrix}
=
\begin{bmatrix}
\left(\frac{\Delta r}{2r_1}-1\right)\overset{120}{T_0} \\ 0 \\ 0 \\ \vdots \\ -\frac{\dot{Q}\,\Delta r}{k\pi R_2}
\end{bmatrix}
$$

or for four interior nodes:

$$
\begin{bmatrix}
-2 & 1+\frac{\Delta r}{2r_1} & & & \\
1-\frac{\Delta r}{2r_2} & -2 & 1+\frac{\Delta r}{2r_2} & & \\
 & 1-\frac{\Delta r}{2r_3} & -2 & 1+\frac{\Delta r}{2r_3} & \\
 & & 1-\frac{\Delta r}{2r_4} & -2 & 1+\frac{\Delta r}{2r_4} \\
 & & 1 & -4 & 3
\end{bmatrix}
\begin{bmatrix}
T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5
\end{bmatrix}
=
\begin{bmatrix}
\left(\frac{\Delta r}{2r_1}-1\right)T_0 \\ 0 \\ 0 \\ 0 \\ -\frac{\dot{Q}\,\Delta r}{k\pi R_2}
\end{bmatrix}
$$

please be careful in entering the index values for r.