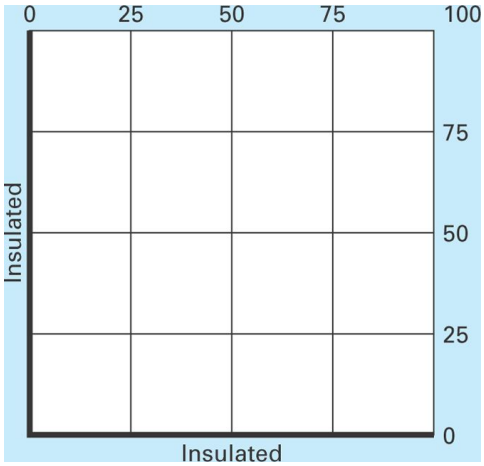
NE 336: Lab 3 Group 1

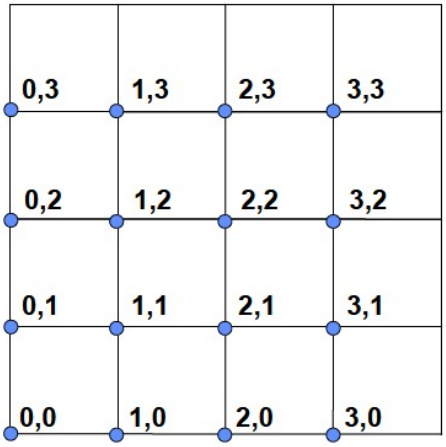
1 Finding the Temperature distribution in a strange system

As we've already seen, the temperature distribution in a simple plate (in the absence of heat generation) is governed by the Laplacian equation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{1}$$

Develop the finite difference approximation and solve for the temperature of the nodes shown on the graph.





Since the bottom and left boundaries are insulated, ($\frac{\partial T}{\partial x} = 0$ on left boundary and $\frac{\partial T}{\partial y} = 0$ at the bottom), we require additional nodes on these faces of the system.

2 Lab tasks

Please note that all written work may be written on paper (digital or not) and submitted as images or a pdf to the dropbox.

2.1 Setting up

2.1.1 Approximating terms in PDE with FD for a generic node

Show (on paper) that by using a centered difference formula, with $\Delta x = \Delta y$, equation 1 becomes :

$$T_{i+1,j} + T_{i-1,j} - 4T_{i,j} + T_{i,j+1} + T_{i,j-1} = 0 \tag{2}$$

2.1.2 Observation

As part of your solution, please comment on the fact that the size of the system (length of each side) is not provided. Is information missing?

2.2 Considering the BCs

The BC conditions on the right and top face are given at each node. Using a centered difference formula, write (on paper) the approximation for the two boundaries that involve derivative terms to show that we get the following equations for the bottom boundary :

$$T_{i,1} = T_{i,-1} \tag{3}$$

And the following for the left boundary:

$$T_{1,j} = T_{-1,j} \tag{4}$$

2.2.1 Observation

How would equations 3 and 4 be incorporated into our general equation 2? Please write out the resulting equations that need to be used for

- a generic interior node,
- nodes at the bottom,
- nodes at the left.

2.3 Solving the system of equations

You have two options for solving the resulting system of equations.

Note: you only need to solve for one of these approaches, the other is optional and if solved correctly, will count as a bonus.

1. Form as a matrix and solve once. Submit as `solve_as_matrix.py`
2. Use Liebmann's method (Gauss Seidel). Submit as `solve_by_iterative.py`

3 How to submit your answers to the lab

Please keep these points in mind as you go on to complete the lab questions.

- When working on these questions, please ask for help from the TAs and myself as needed.
- Discussion with classmates is encouraged but takings somebody else's code and submitting their work as your own is not acceptable. Please ask for clarification on this item if unsure.
- In terms of formatting your submission:
  - If the question is asking for a script or function, please submit as a python file.
  - If asked for analysis, you may include this as comments in your python file, handwritten work or however else you wish.
- Please include your name and student ID in each file that you submit.

4 Checklist of submission items

This is included for your convenience.

1. Written work as pdf or image.
2. One of `solve_as_matrix.py` OR `solve_by_iterative.py`.
   (Both if you'd like a bonus)