NE336 Assignment 4 Due Nov 29^{th}

Instructions

Last assignment of the course so a celebratory dance is mandatory after finishing the assignment.

Parabolic PDEs

The problem of radial heat flow in a circular rod in non-dimensional form is as follows:

$$\frac{\partial^2 u}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial u}{\partial \bar{r}} = \frac{\partial u}{\partial \bar{z}}$$

The axial direction ($\bar{z}$) may be treated like time for additional accuracy and stability in our numerical solution.

The boundary conditions are :

$$u(1, \bar{z}) = 1$$
$$\frac{\partial u}{\partial \bar{r}}(0, \bar{z}) = 0$$

and initial condition (at $\bar{z} = 0$) :

$$u(\bar{r}, 0) = 0 \quad 0 \leq \bar{r} \leq 1$$

Predict the distribution of $u(\bar{r}, \bar{z})$

a) at $\bar{z} = 1.0$ using the Crank-Nicholson method. Assume $\Delta\bar{r} = 0.2$, $\Delta\bar{z} = 0.1$.

b) at $\bar{z} = 1.0$ using the method of lines. Assume $\Delta\bar{r} = 0.2$, $\Delta\bar{z} = 0.01$.

Note: You only need to solve one of the two to get full marks (either CN or MOL) but solving both will get you bonus marks on the assignment. Please note that this question is asking for both setup (paper) and code implementation.

FEA (not optional... :( )

In class, we solved the following ODE using FEA to find the temperature distribution of a heated rod

$$\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} = -f(x)$$

with two Dirichlet BCs. Solve the same problem using FEA for $f(x) = 15$, and $T(x = 0) = 75$ and $T(x = L = 10) = 150$ once again for 4 elements (five nodes).

Please include all the written steps for element equations and assembly, apply the BCs to get your system of equations.

For finding the solution, please include your few lines of code required to solve it in a python file.

Submit both the written work and your python code for this question.