NE336 Quiz3 Nov 21st

1 Instructions

- The first line in each file should contain a comment with your complete name and student ID.
- All module import statements have been given but you may modify them however you wish.
- A minimal amount of comments (at least a few lines to explain your method of thought) is required and code with no comments will lose 0.5 marks in total.
- Any written answers can be provided in a word document or from a note taking application of your choice.

2 Conceptual question (2.5 points)

We have solved the following PDE many times in class using different approaches.

$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$$

The system is well insulated from its surroundings and has thermal conductivity $k = 0.835\ \text{cm}^2/s$ and we had the following IC and BCs.

$$T(t, x = 0) = 100\ °\text{C}$$
$$T(t, x = 10) = 50\ °\text{C}$$
$$T(t = 0, x) = 0\ °\text{C}$$

One of these solutions is attached to the quiz dropbox. Please download the code, *run it* and answer the following questions based on what you observe.

1. Please remark on whether you think an implicit or explicit method is used. Please justify your choice in one or two sentences.
2. Comment on the phenomenon you observe in the plot.
   What is it? What modification can we make to the code to remove this effect ?
3. Once you fix the code, please comment on the following. Has this system reached steady state at $t = 12$ seconds? Explain your reasoning for the answer you provide.

Note : the answer to this question is only written work and no code needs to be returned for it.

3 Programming question (7.5 points)

In your last quiz, you solved the following BVP,

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 10y = 0$$

using a shooting method. In that problem we had Neumann and Robin BCs. We will initially consider a more simple problem with the following Dirichlet BCs :

$$y(x = 0) = 20$$
$$y(x = 10) = 10$$

We would now like to solve this using a FD approach. The first few steps of the tasks below are on paper.

1. Setup your grid and identify your nodes, choose a suitable number of interior nodes.
2. Apply FD approximations to the ODE and write the resulting equation for a generic internal node, i. Use second order approximations.
3. Write these equations for your number of nodes. Include your treatment of the BCs.
4. Setup the code to solve for y(x) and plot y vs x when done. **Note** : You may wish to increase the number of nodes at this point to get a smoother plot!
5. BONUS: solve the problem for the original BCs we had last time, which were

$$\frac{dy}{dx}|_{x=0} = 20$$
$$\frac{dy}{dx}|_{x=10} = 10\left[y(x = 10) - 5\right]$$

   Please maintain second order accuracy when applying FD approximations to the BCs.

Import statements

```python
import numpy as np
import matplotlib.pyplot as plt
```