Euler's method

$$y_{i+1} = y_i + f(t_i, y_i)h$$

Implicit Euler's method

$$y_{i+1} = y_i + f(t_{i+1}, y_{i+1})h$$

Heun's method

$$\begin{aligned} y_{i+1}^0 &= y_i + f(t_i, y_i)h \\ y_{i+1} &= y_i + \frac{f(t_i, y_i) + f(t_{i+1}, y_{i+1}^0)}{2}h \end{aligned}$$

Second-order Runge-Kutta method

$$\begin{aligned} y_{i+1} &= y_i + \frac{1}{2}(k_1 + k_2)h \\ k_1 &= f(t_i, y_i) \\ k_2 &= f(t_i + h, y_i + k_1 h) \end{aligned}$$

Fourth-order Runge-Kutta method

$$\begin{aligned} y_{i+1} &= y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \\ k_1 &= f(t_i, y_i) \\ k_2 &= f(t_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h) \\ k_3 &= f(t_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h) \\ k_4 &= f(t_i + h, y_i + k_3 h) \end{aligned}$$

Method of Weighted Residuals for FEA

$$\int_D R N_i \mathrm{d}D = 0 \quad i = 1, 2, \ldots$$

Finite Difference Approximations to Derivatives

Centered
$\mathbf{O(h^2)}$
$f'(x_i) = \frac{1}{2h}\left[f(x_{i+1}) - f(x_{i-1})\right]$ $f''(x_i) = \frac{1}{h^2}\left[f(x_{i+1}) - 2f(x_i) + f(x_{i-1})\right]$

Backward	**Forward**
$\mathbf{O(h)}$	
$f'(x_i) = \frac{1}{h}\left[f(x_i) - f(x_{i-1})\right]$ $f''(x_i) = \frac{1}{h^2}\left[f(x_i) - 2f(x_{i-1}) + f(x_{i-2})\right]$	$f'(x_i) = \frac{1}{h}\left[f(x_{i+1}) - f(x_i)\right]$ $f''(x_i) = \frac{1}{h^2}\left[f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)\right]$
$\mathbf{O(h^2)}$	
$f'(x_i) = \frac{1}{2h}\left[3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})\right]$ $f''(x_i) = \frac{1}{h^2}\left[2f(x_i) - 5f(x_{i-1}) + 4f(x_{i-2}) - f(x_{i-3})\right]$	$f'(x_i) = \frac{1}{2h}\left[-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)\right]$ $f''(x_i) = \frac{1}{h^2}\left[-f(x_{i+3}) + 4f(x_{i+2}) - 5f(x_{i+1}) + 2f(x_i)\right]$