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Homework #4

Interpolation and Polynomial Approximation / Curve Fitting

DATE: 26th January 2021 DUE : **14nd February 2021**

Name: ______________________ **Scholar ID**: ______________________

1 INDICATIONS

- You **could** fill this sheet with just the answer to each problem and return it to the professor. However, you have to present the process on a separate exam sheet.
- Answers with no process are **not valid**.
- Make all calculations with 5 decimal places of precision.

2 INTERPOLATION AND POLYNOMIAL APPROXIMATION

1. (0.6 points) Find the Taylor polynomial of degree $N = 4$ and $N = 6$ for $f(x) = e^{-x^2/2}$ about $x_0 = 0$

Process:

$\mathbf{P_4(x)} =$
$\mathbf{P_6(x)} =$

2. (0.6 points) Find the Taylor polynomial of degree $N = 5$ for $f(x) = (3 + x)^{1/2}$ about $x_0 = 3$, and use it to find approximation to $f = 4^{1/2}$.

Process:

$\mathbf{P_5(x)} =$
$\mathbf{4^{1/2}} =$

3. (0.6 points) Compute the divided difference table for each tabulated function.

a) $f(x) = (x+1)^{1/2}$

k	x_k	$f(x_k)$
0	8.0	3.00000
1	9.0	3.16227
2	10.0	3.31662
3	11.0	3.46410
4	12.0	3.60555

b) $f(x) = 7.8/x^2$

k	x_k	$f(x_k)$
0	6.0	0.21666
1	7.0	0.15918
2	8.0	0.12187
3	9.0	0.09629
4	10.0	0.07800

$f(x) = (x+1)^{1/2}$

x_k	$f(x_k)$	1st divided difference	2nd divided difference	3th divided difference	4th divided difference
$x_0 =$ $x_1 =$ $x_2 =$ $x_3 =$ $x_4 =$					

$f(x) = 7.8/x^2$

x_k	$f(x_k)$	1st divided difference	2nd divided difference	3th divided difference	4th divided difference
$x_0 =$ $x_1 =$ $x_2 =$ $x_3 =$ $x_4 =$					

4. (0.6 points) Write down the Newton polynomial $P_1(x), P_2(x)$ and $P_3(x)$ for each function in Exercise 3.

a) $f(x) = (x+1)^{1/2}$

$\mathbf{P_1(x)} =$
$\mathbf{P_2(x)} =$
$\mathbf{P_3(x)} =$

b) $f(x) = 7.8/x^2$

$\mathbf{P_1(x)} =$
$\mathbf{P_2(x)} =$
$\mathbf{P_3(x)} =$

5. (0.8 point) Use appropriate Lagrange interpolating polynomials $P_1(x), P_2(x)$ and $P_3(x)$ of degrees $1, 2$ and 3 respectively to approximate each of the following:

   a) $\mathbf{f(8.4)}$, if $f(8.1) = 16.94410$, $f(8.3) = 17.56492$, $f(8.6) = 18.50515$, $f(8.7) = 18.82091$. Specify each Lagrange multiplier.

Process:

$\mathbf{L_{1,0}} =$	$\mathbf{L_{1,1}} =$
$P_1(x) =$	$P_1(8.4) =$

$\mathbf{L_{2,0}} =$	$\mathbf{L_{2,1}} =$
$\mathbf{L_{2,2}} =$	$P_2(8.4) =$
$P_2(x) =$	

$\mathbf{L_{3,0}} =$
$\mathbf{L_{3,1}} =$
$\mathbf{L_{3,2}} =$
$\mathbf{L_{3,3}} =$
$P_3(x) =$
$P_3(8.4) =$

b) $\mathbf{f}(-\frac{\mathbf{1}}{\mathbf{3}})$, if $f(-0.75) = -0.0718125$, $f(-0.5) = -0.02475000$, $f(-0.25) = 0.33493750$, $f(0) = 1.10100000$. Specify each Lagrange multiplier.

Process:

$\mathbf{L_{1,0}} =$	$\mathbf{L_{1,1}} =$
$P_1(x) =$	$P_1(-\frac{1}{3}) =$

$\mathbf{L_{2,0}} =$	$\mathbf{L_{2,1}} =$
$\mathbf{L_{2,2}} =$	$P_2(-\frac{1}{3}) =$
$P_2(x) =$	

$\mathbf{L_{3,0}} =$
$\mathbf{L_{3,1}} =$
$\mathbf{L_{3,2}} =$
$\mathbf{L_{3,3}} =$
$P_3(x) =$
$P_3(-\frac{1}{3}) =$

c) $\mathbf{f(0.25)}$, if $f(0.1) = 0.62049958$, $f(0.2) = -0.28395668$, $f(0.3) = 0.00660095$, $f(0.4) = 0.24842440$. Specify each Lagrange multiplier.

Process:

$\mathbf{L_{1,0}} =$	$\mathbf{L_{1,1}} =$
$P_1(x) =$	$P_1(0.25) =$

$\mathbf{L_{2,0}} =$	$\mathbf{L_{2,1}} =$
$\mathbf{L_{2,2}} =$	$P_2(0.25) =$
$P_2(x) =$	

$\mathbf{L_{3,0}} =$
$\mathbf{L_{3,1}} =$
$\mathbf{L_{3,2}} =$
$\mathbf{L_{3,3}} =$
$P_3(x) =$
$P_3(0.25) =$

d) $\mathbf{f(0.9)}$, if $f(0.6) = -0.17694460$, $f(0.7) = 0.01375227, f(0.8) = 0.22363362, f(1.0) = 0.65809197$. Specify each Lagrange multiplier.

Process:

$\mathbf{L_{1,0}} =$	$\mathbf{L_{1,1}} =$
$P_1(x) =$	$P_1(0.9) =$

$\mathbf{L_{2,0}} =$	$\mathbf{L_{2,1}} =$
$\mathbf{L_{2,2}} =$	$P_2(0.9) =$
$P_2(x) =$	

$\mathbf{L_{3,0}} =$
$\mathbf{L_{3,1}} =$
$\mathbf{L_{3,2}} =$
$\mathbf{L_{3,3}} =$
$P_3(x) =$
$P_3(0.9) =$

6. (0.6 points) Use the Lagrange polynomial error formula to find an error bound for the approximations in Exercise 5.

a) $f(8.4)$

$\mathbf{E_1(x)} =$
$\mathbf{E_2(x)} =$
$\mathbf{E_3(x)} =$

b) $f(-\frac{1}{3})$

$\mathbf{E_1(x)} =$
$\mathbf{E_2(x)} =$
$\mathbf{E_3(x)} =$

c) $f(0.25)$

$\mathbf{E_1(x)} =$
$\mathbf{E_2(x)} =$
$\mathbf{E_3(x)} =$

d) $f(0.9)$

$\mathbf{E_1(x)} =$
$\mathbf{E_2(x)} =$
$\mathbf{E_3(x)} =$

3 Curve Fitting

1. (0.4 points) For the following data, find the least-squares curve.

x_k	-1	0	1	2	3
y_k	6.62	3.94	2.17	1.35	0.89

a) $y = f_1(x) = Ce^{Ax}$, by using the change of variables $X = x$, $Y = \ln(y)$, and $C = e^B$ to linearize the data points.

Process:

$C =$	$A =$

b) $y = f_2(x) = 1/(Ax + B)$, by using the change of variables $X = x$ and $Y = 1/y$ to linearize the data points.

Process:

$A =$	$B =$

c) Use $E_2(f)$ to determine which curve gives the best fit.

Process:

$E_2(f_1) =$	$E_2(f_2) =$

2. (0.4 points) Find the Least-square line for $y = Ax + B$ for the data and calculate $E_2(y)$.

x_k	-6.0	-2.0	0.0	2.0	6.0
y_k	-5.3	-3.5	-1.7	0.2	4.0

Process:

$A =$	$b =$	$E_2(y) =$

3. (0.4 points) Find the least-squares parabola $f(x) = Ax^2 + Bx + C$ for the data and calculate $E_2(f)$.

x_k	-2.0	-1.0	0.0	1.0	2.0
y_k	2.8	2.1	3.25	6.0	11.5

Process:

$A =$	$B =$	$C =$	$E_2(f) =$