

Joint distributions summary

Two discrete random variables

Two continuous random variables

Joint cdf

$$F_{X,Y}(x,y) = \mathbb{P}(X \leq x, Y \leq y)$$

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Joint

$$f_{X,Y}(x,y) = \mathbb{P}(X = x, Y = y)$$

$$f_{X,Y}(x,y) = \frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y)$$

pmf/pdf

$$\bullet f_{X,Y}(x,y) \geq 0$$

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$$\bullet \sum_x \sum_y f_{X,Y}(x,y) = 1$$

$$\bullet \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y) dx dy = 1$$

Marginal

$$f_X(x) = \mathbb{P}(X = x) = \sum_y f_{X,Y}(x,y)$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dy$$

pmf/pdf

$$= \sum_y f_{X|Y}(x|y) f_Y(y)$$

$$= \int_{-\infty}^{\infty} f_{X|Y}(x|y) f_Y(y) dy$$

Conditional

pmf/pdf

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

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Independence

$$\mathbb{P}(X \leq x, Y \leq y) = \mathbb{P}(X \leq x) \mathbb{P}(Y \leq y)$$

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$$F_{X,Y}(x,y) = F_X(x) F_Y(y)$$

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for all x and y

for all x and y

Expectation

of a function

$$\mathbb{E}(g(X,Y)) = \sum_x \sum_y g(x,y) f_{X,Y}(x,y)$$

$$\mathbb{E}(g(X,Y)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y) dx dy$$

Covariance

$$\text{Cov}(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X) \mathbb{E}(Y)$$

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Correlation

$$\rho_{X,Y} = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \text{Var}(Y)}}$$

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Conditional

Expectation

$$\psi_Y(x) = \mathbb{E}(Y|X = x) = \sum_y y f_{Y|X}(y|x)$$

$$\psi_Y(x) = \mathbb{E}(Y|X = x) = \int_{-\infty}^{\infty} y f_{Y|X}(y|x) dy$$