1. For each of the following descriptions, say whether the random variable X is or is not a Binomial random variable. If X is Binomial, give the values of the parameters n and p. If X is not Binomial, give a brief (few words) reason why not.

Example questions:

(E1) X is the number of Heads in 10 tosses of a fair coin.

(E2) X is the number of tosses of a fair coin needed before 10 heads are obtained.

Example answers:

(E1) Yes: $X \sim \text{Binomial}(n = 10; p = 0.5)$.

(E2) No: not a fixed number of trials.

Give similar answers for the questions below.

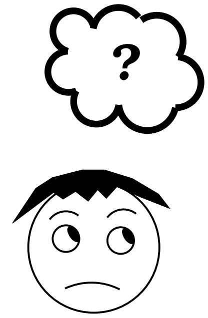
(a) An opinion poll samples 500 people at random and asks which party they plan to vote for at the next election. Nationwide, support for the Green Party is currently at 14%. X is the number of the 500 people who say they plan to vote for the Green Party.

(b) There are 50 lecture theatres on the university campus. Each lecture theatre has a clock, which shows the right time with probability 0.7, independently of all other clocks. Amy has lectures in 8 different lecture theatres during the week. X is the number of Amy's lecture theatres in which the clock is correct.
(Assume that clocks are never fixed.)

(c) Valerie practises throwing the shot-put for the Olympics. She is satisfied with 80% of her throws, and all throws are independent. X is the number of throws Valerie makes before she makes a satisfactory throw.

(d) Suppose that 2% of free-range eggs bought at the supermarket would hatch into chickens if they were incubated in the right conditions. Assume all eggs in a box of 12 are independent of each other. X is the number of chickens that would hatch from incubating a box of 12 eggs.

(e) Jill runs a cafe. 40% of people take sugar in their coffee. Jill only has enough sugar for 15 coffees today. She gets about 60 customers each day. X is the number of coffees that Jill serves before she runs out of sugar.



2. In Stats 101, students have to complete four multi-choice tests. Each test has 10 questions with 5 possible answers to each question. Tom wonders if he can get by on the tests just by guessing. Consider *one* test of 10 questions, with 5 multi-choice answers for each question. Tom guesses each question completely at random. Let X be the number of the 10 questions he gets correct.

   (a) X has a Binomial distribution. State the values of n and p.

   The table shows the probability mass function, $f_X(x)$, and the cumulative distribution function, $F_X(x)$, of X. (You could check your answer to part (a) using some of the entries in the table.) Some of the probabilities are missing.

x	0	1	2	3	4	5	6	7	8	9	10
$f_X(x) = \mathbb{P}(X = x)$	0.11	0.27	p_2	p_3	0.09	0.02	0.01	0.00	0.00	0.00	0.00
$F_X(x) = \mathbb{P}(X \leq x)$	c_0	0.38	0.68	0.88	c_4	c_5	1.00	1.00	1.00	1.00	1.00

   (b) Use the formula for Binomial probabilities to calculate p_2 and p_3, showing your calculation as part of your answer. **Quote your answer to 2 decimal places only.**

   (c) Using values taken from the table, and your answer to (b), calculate c_0, c_4, and c_5. Show your working. You do **not** need to use the formula for Binomial probabilities. Again, quote your answer to 2 decimal places only.

   Tom wants to know the probability that he will get at least 6 questions correct, just by guessing. Unfortunately, he has done no Stats study whatsoever, so is unable to calculate his probability himself. We will have to help him out.

   (d) Write down Tom's probability as $\mathbb{P}(X \ldots)$. Then rewrite it in terms of the cumulative distribution function, $F_X(\cdot)$. Hence read off the table the probability that Tom will get at least 6 out of 10, just by guessing.

   Now remember that there are four tests in total. Tom guesses for all of them. Write Tom's four scores from the four tests as X_1, X_2, X_3, and X_4. Define $T = X_1 + X_2 + X_3 + X_4$ to be Tom's total mark out of 40 for the multi-choice tests. Assume $X_1, \ldots, X_4$ are independent.

   (e) Name the distribution of T, with parameters.

   (f) Find $\mathbb{P}(T = 10)$, the probability that Tom scores 10/40 on the multi-choice tests.

   (g) In order to pass this component of the coursework, Tom needs a score of at least 20/40, in other words T must be at least 20. You are given the information that $F_T(18) = 0.9999148$, where $F_T(t)$ is the cumulative distribution function of T. Using this information, do some extra calculation and find the probability that Tom passes the multi-choice tests just by guessing. Quote your answer to 3 significant figures. Would you recommend this strategy?

   Note: If Tom could exclude TWO wrong answers from each question, and only guess among the other three answers, his chance of passing is still only $\mathbb{P}(\text{pass}) = 0.02$.

3. Answer the following questions. Show your working for each part.

(a) Suppose that X is a random variable that represents the number shown when we roll the die D_1 which has 9 sides. The probability function of X is given by

$$\mathbb{P}(X = x) = \log_{10}\left(\frac{x+1}{x}\right), \quad \text{for } x \in \{1, 2, \ldots, 9\},$$

where $\log_{10}(\cdot)$ denotes the logarithm base 10. Show that the cumulative distribution function of X is:

$$F_X(x) = \begin{cases} 0, & \text{if } x \in (-\infty, 1), \\ \log_{10}\left(\lfloor x \rfloor + 1\right), & \text{if } x \in [1, 9], \\ 1, & \text{if } x \in (9, \infty), \end{cases}$$

where $\lfloor \cdot \rfloor$ is the greatest integer less than or equal to the real number in the argument. For example, $\lfloor 2.0 \rfloor = 2$, $\lfloor 2.5 \rfloor = 2$, $\lfloor 2.999999 \rfloor = 2$.

(b) Let Y be a random variable that represents the number shown when we roll the 9-sided die D_2 which is known to be fair. Hence, we have:

$$\mathbb{P}(Y = y) = \frac{1}{9}, \quad \text{for } y \in \{1, 2, \ldots, 9\}.$$

Calculate $\mathbb{P}(X + Y = 10)$ for the case when the random variables X and Y are assumed to be independent.

4. Suppose that $X \sim \text{Bernoulli}(1/10)$, $Y \sim \text{Bernoulli}(1/2)$, and the random variables X and Y are independent. The random variable Z is given by $Z = X \oplus Y$, where the operator $\oplus$ is defined in Table 1. The operation that corresponds to the symbol $\oplus$ is called *exclusive or* (often abbreviated to XOR), or *addition mod 2*.

Note that for any $x, y \in \{0, 1\}$, $x \oplus y = 0$ if $x = y$ and $x \oplus y = 1$ if $x \neq y$.

x	y	$z = x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

Table 1: Definition of the operator $\oplus$.

Answer the following questions:

(a) Show that $Z \sim \text{Bernoulli}(1/2)$.

(b) Calculate $\mathbb{P}(Y = 0, Z = 0)$.

(c) Decide if the random variables Y and Z are independent (or not).
Explain your answer.