Integration

Ex1

We will consider the function $f(x) = \sin x$ on the interval $[0, \pi/2]$. Because $\int_0^{\pi/2} \sin x \, dx = 1$, this is a good example for comparative purposes.

(a) We use 100 subintervals and choose x_i^* to be the *left* endpoint of each subinterval.

(b) We choose the value of x_i^* to be the *right* endpoint of each subinterval. (This time we expect an over-approximation.)

Ex2

To improve the accuracy of the approximation offered in earlier Example, we can choose the value of x_i.. To be the midpoint of each subinterval. This leads to an approximation method called the *midpoint rule.*

Ex3

Approximate $\int_0^{\pi/2} \sin x \, dx$ using the trapezoidal rule.

Ex4

Compute the Riemann sums of $f(x) = x\, e^x \sqrt{x}$ over the interval $[0, 2]$ using

(a) the left endpoint of each subinterval.

(b) the right endpoint of each subinterval.

(c) the midpoint of each subinterval.

Compare with *Mathematica*'s approximation to the integral $\int_0^2 f(x)\, dx$.

Ex5

Approximate $\int_1^2 x \ln x \, dx$ using the trapezoidal rule with $n = 100$ and compare the result with *Mathematica*'s approximation.

Ex6

Compute the lower and upper Riemann sums for the function $f(x) = x^2$ on the interval $[0, 1]$ for $n = 2, 4, 8, 16, \ldots, 2^{20}$ subintervals. Explain the behavior of the approximations in terms of the integral $\int_0^1 x^2 \, dx$.

Ex7

Compute an approximation of $\int_0^1 e^{x^2}\,dx$ using the trapezoidal rule with 10, 50, and 100 subintervals. Compare with *Mathematica*'s approximation.

Monte Carlo Integration

Ex8

Using the Monte Carlo method calculate the value of integral $\int_0^1 \frac{1}{\sqrt{x}}dx$

Ex9

Using the Monte Carlo method calculate the value of integral $\iint_{-2}^{2} e^{-(x^4+y^4)}dxdy$