

Linear Algebraic Systems with Mathematica

Linear Algebraic Systems

A linear algebraic system is given in scalar form by

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1,n-1}x_{n-1} + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2,n-1}x_{n-1} + a_{2n}x_n = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \dots + a_{3,n-1}x_{n-1} + a_{3n}x_n = b_3$$

.....

$$a_{m-1,1}x_1 + a_{m-1,2}x_2 + a_{m-1,3}x_3 + \dots + a_{m-1,n-1}x_{n-1} + a_{m-1,n}x_n = b_{m-1}$$

$$a_{m,1}x_1 + a_{m,2}x_2 + a_{m,3}x_3 + \dots + a_{m,n-1}x_{n-1} + a_{m,n}x_n = b_m$$

and in vector form by: $Ax = b$.

Here it is assumed that all of the elements in A , b , and x are real. Both A and b are given and x is unknown.

If $m < n$, then this system is said to be **undetermined** (fewer equations than unknowns) and is not uniquely solvable.

If $m = n$, then this system is square. If this system is also nonsingular, then it is said to be determined and hence solvable with **the unique solution x** .

If $m > n$, then this system is **overdetermined** (more equations than unknowns) and its solution is called the least-squares solution. Below is a table of Mathematica functions that can be used to solve problems related to linear systems.

Function	Definition
Det [A]	Gives the determinant of the square matrix A
MatrixRank [A]	Gives the rank of the matrix A
Tr [A]	Finds the trace of the matrix A
Inverse [A]	Produces the matrix inverse of a non – singular matrix A (such that $A A^{-1} = I$)
LinearSolve [A, b]	Solves the linear system in the matrix – vector form of $A x = b$
PseudoInverse [A]	Forms the pseudo – inverse of a rectangular matrix A
Norm [array, pval]	Produces the p – norm of an array for $p = 1, 2, \infty$

Zad1 Dokonać dekompozycji macierzy $m = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 11 & 14 \\ 6 & 29 & 43 \end{bmatrix}$ na dolną i górną macierz trójkątną (LU decomposition) postaci

$$\begin{bmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ x & x & 1 \end{bmatrix} \begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{bmatrix}$$

Zad2

Rozwiązać układ równań z trzema niewiadomymi analitycznie.

Zad3

Rozwiązać układ równań liniowych $A x = b$ o wymiarach macierzy A 800x800.

Elementy macierzy A, B wygeneruj jako liczby losowe rzeczywiste z przedziału [-100,100]..

Zad4

Dla danych z zadania 3 oblicz rząd macierzy A, wyznacznik macierzy A ,ślad macierzy A, normę $A * x = b$. Oszacuj czas wyliczeń używając funkcji Timing.