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Maharashtra State Board 12th Commerce Maths Solutions Chapter 5 Integration Ex 5.1

Question 1.
Evaluate ∫−25x−4√−5x−2√dx
Solution:

$$\int \frac{-2}{\sqrt{5x-4}-\sqrt{5x-2}}\,dx$$

$$=\int \frac{-2}{\sqrt{5x-4}-\sqrt{5x-2}} \times \frac{\sqrt{5x-4}+\sqrt{5x-2}}{\sqrt{5x-4}+\sqrt{5x-2}}\,dx$$

$$=\int \frac{-2(\sqrt{5x-4}+\sqrt{5x-2})}{(5x-4)-(5x-2)}\,dx$$

$$=\int (\sqrt{5x-4}+\sqrt{5x-2})\,dx$$

$$=\int (\sqrt{5x-4})^{\frac{1}{2}}dx+\int (\sqrt{5x-2})^{\frac{1}{2}}\,dx$$

$$=\frac{(5x-4)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} \times \frac{1}{5}+\frac{(5x-2)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} \times \frac{1}{5}+c$$

$$=\frac{2}{15}\left[(5x-4)^{\frac{3}{2}}+(5x-2)^{\frac{3}{2}}\right]+c.$$

Question 2.
Evaluate ∫(1+x+x22!)dx
Solution:

$$\int\left(1+x+\frac{x^2}{2!}\right)dx$$

$$=\int 1dx+\int xdx+\frac{1}{2!}\int x^2dx$$

$$=x+\frac{x^2}{2}+\frac{1}{2!} \times \frac{x^3}{3}+c$$

$$=x+\frac{x^2}{2}+\frac{x^3}{6}+c.$$

Question 3.
Evaluate ∫3x3−2x√xdx
Solution:

$$\int \frac{3x^3-2\sqrt{x}}{x}\,dx$$

$$=\int\left(\frac{3x^3}{x}-\frac{2\sqrt{x}}{x}\right)dx$$

$$=\int\left(3x^2-\frac{2}{\sqrt{x}}\right)dx$$

$$=3\int x^2dx-2\int x^{-\frac{1}{2}}\,dx$$

$$=3\cdot\left(\frac{x^3}{3}\right)-2\cdot\left[\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]+c$$

$$=x^3-4\sqrt{x}+c.$$

Question 4.
Evaluate ∫(3x2 – 5)2 dx

Solution:
$\int(3x^2 - 5)^2\, dx$
$= \int(9x^4 - 30x^2 + 25)\, dx$
$= 9\int x^4\, dx - 30\int x^2\, dx + 25\int 1\, dx$
$= 9\left(\frac{x^5}{5}\right) - 30\left(\frac{x^3}{3}\right) + 25x + c$
$= \frac{9x^5}{5} - 10x^3 + 25x + c.$

Question 5.
Evaluate $\int \frac{1}{x(x-1)} dx$
Solution:

$$\int \frac{1}{x(x-1)} dx = \int \frac{x-(x-1)}{x(x-1)}\, dx$$

$$= \int \left(\frac{1}{x-1} - \frac{1}{x}\right) dx$$

$$= \int \frac{1}{x-1}\, dx - \int \frac{1}{x}\, dx$$

$$= \log|x-1| - \log|x| + c$$

$$= \log\left|\frac{x-1}{x}\right| + c.$$

Question 6.
If $f'(x) = x^2 + 5$ and $f(0) = -1$, then find the value of $f(x)$.
Solution:
By the definition of integral
$f(x) = \int f'(x)\, dx$
$= \int(x^2 + 5)\, dx$
$= \int x^2\, dx + 5\int 1\, dx$
$= \frac{x^3}{3} + 5x + c$
Now, $f(0) = -1$ gives
$f(0) = 0 + 0 + c = -1$
$\therefore c = -1$
$\therefore$ from (1), $f(x) = \frac{x^3}{3} + 5x - 1$.

Question 7.
If $f(x) = 4x^3 - 3x^2 + 2x + k$, $f(0) = -1$ and $f(1) = 4$, find $f(x)$.
Solution:
By the definition of integral
$f(x) = \int f'(x)\, dx$
$= \int(4x^3 - 3x^2 + 2x + k)\, dx$
$= 4\int x^3\, dx - 3\int x^2\, dx + 2\int x\, dx + k\int 1\, dx$
$= 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^3}{3}\right) + 2\left(\frac{x^2}{2}\right) + kx + c$
$\therefore f(x) = x^4 - x^3 + x^2 + kx + c$
Now, $f(0) = 1$ gives
$f(0) = 0 - 0 + 0 + 0 + c = 1$
$\therefore c = 1$
$\therefore$ from (1), $f(x) = x^4 - x^3 + x^2 + kx + 1$
Further $f(1) = 4$ gives
$f(1) = 1 - 1 + 1 + k + 1 = 4$
$\therefore k = 2$
$\therefore$ from (2), $f(x) = x^4 - x^3 + x^2 + 2x + 1$.

Question 8.
If $f(x) = \frac{x^2}{2} - kx + 1$, $f(0) = 2$ and $f(3) = 5$, find $f(x)$.
Solution:

By the definition of integral

$$f(x) = \int f'(x)dx = \int \left(\frac{x^2}{2} - kx + 1\right) dx$$

$$= \frac{1}{2}\int x^2 dx - k\int x dx + \int 1\, dx$$

$$= \frac{1}{2}\left(\frac{x^3}{3}\right) - k\left(\frac{x^2}{2}\right) + x + c$$

$$\therefore\ f(x) = \frac{x^3}{6} - \frac{kx^2}{2} + x + c \quad \text{.........} (1)$$

Now, $f(0) = 2$ gives

$$f(0) = 0 - 0 + 0 + c = 2 \qquad \therefore\ c = 2$$

$$\therefore\ \text{from (1)},\ f(x) = \frac{x^3}{6} - \frac{kx^2}{2} + x + 2 \quad \text{.......} (2)$$

Further $f(3) = 5$ gives

$$f(3) = \frac{27}{6} - \frac{9k}{2} + 3 + 2 = 5$$

$$\therefore\ \frac{9k}{2} = \frac{9}{2} \qquad \therefore\ k = 1$$

$$\therefore\ \text{from (2)},\ f(x) = \frac{x^3}{6} - \frac{x^2}{2} + x + 2.$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 5 Integration Ex 5.2

Evaluate the following.

Question 1.
∫x1+x2−−−−−√dx

Solution:

$$\text{Let } I = \int x\sqrt{1+x^2}\, dx = \int \sqrt{1+x^2}\cdot x\, dx$$

Put $1 + x^2 = t$

$$\therefore\ 2xdx = dt \qquad \therefore\ xdx = \frac{dt}{2}$$

$$\therefore\ I = \int \sqrt{t}\,\frac{dt}{2} = \frac{1}{2}\int t^{\frac{1}{2}} dt$$

$$= \frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{(3/2)} + c$$

$$= \frac{1}{3}(1+x^2)^{\frac{3}{2}} + c.$$

Question 2.
∫x31+x4√dx

Solution:

$$\text{Let } I = \int \frac{x^3}{\sqrt{1+x^4}}\,dx$$

Put $1 + x^4 = t \qquad \therefore\ 4x^3 dx = dt$

$$\therefore\ x^3 dx = \frac{dt}{4}$$

$$\therefore\ I = \int \frac{1}{\sqrt{t}} \cdot \frac{dt}{4} = \frac{1}{4}\int t^{-\frac{1}{2}} dt$$

$$= \frac{1}{4} \cdot \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c$$

$$= \frac{1}{2}\sqrt{1+x^4} + c.$$

Question 3.

$\int (e^x+e^{-x})^2(e^x-e^{-x})\,dx$

Solution:

$$\text{Let } I = \int (e^x + e^{-x})^2 (e^x - e^{-x})\,dx$$

Put $e^x + e^{-x} = t$

$$\therefore\ (e^x - e^{-x})\,dx = dt$$

$$\therefore\ I = \int t^2 dt = \frac{t^3}{3} + c$$

$$= \frac{(e^x + e^{-x})^3}{3} + c.$$

Question 4.

$\int \frac{1+x}{x+e^{-x}}\,dx$

Solution:

$$\text{Let } I = \int \frac{1+x}{x+e^{-x}}\,dx$$

$$= \int \frac{(1+x)e^x}{(x+e^{-x})e^x}\,dx$$

$$= \int \frac{(1+x)e^x}{xe^x+1}\,dx$$

Put $xe^x + 1 = t$

$$\therefore\ (xe^x + e^x \times 1)\,dx = dt$$

$$\therefore\ (1+x)e^x\,dx = dt$$

$$\therefore\ I = \int \frac{1}{t}\,dt = \log|t| + c$$

$$= \log|xe^x + 1| + c.$$

Question 5.

$\int (x + 1)(x + 2)^7(x + 3)\,dx$

Solution:

Let $I = \int (x + 1)(x + 2)^7(x + 3)\,dx$

$= \int (x + 2)^7 (x + 1)(x + 3)\,dx$

$= \int (x + 2)^7 [(x + 2) - 1][(x + 2) + 1]\,dx$

$= \int (x + 2)^7 [(x + 2)^2 - 1]\,dx$

$= \int [(x + 2)^9 - (x + 2)^7]\,dx$

$= \int (x + 2)^9\,dx - \int (x + 2)^7\,dx$

$= \frac{(x+2)^{10}}{10} - \frac{(x+2)^8}{8} + c$

Question 6.

$\int \frac{1}{x \log x} dx$

Solution:

Put log x = t

∴ $\frac{1}{x}$ dx = dt

∴ $\int \frac{dx}{x \cdot \log x} = \int \frac{1}{\log x} \cdot \frac{1}{x} dx$

= $\int \frac{1}{t}$ dt

= log |t| + c

= log|log x| + c.

Question 7.

$\int \frac{x^5}{x^2+1} dx$

Solution:

$$\text{Let } I = \int \frac{x^5}{x^2+1} dx = \int \frac{x^4}{x^2+1} \cdot x \, dx$$
$$= \int \frac{(x^2)^2}{x^2+1} \cdot x \, dx$$

Put $x^2 + 1 = t \quad \therefore \; 2x\,dx = dt$

$\therefore \; x dx = \frac{dt}{2}$ and $x^2 = t - 1$

$$\therefore \; I = \int \frac{(t-1)^2}{t} \cdot \frac{dt}{2} = \frac{1}{2} \int \left(\frac{t^2 - 2t + 1}{t}\right) dt$$
$$= \frac{1}{2} \int \left(t - 2 + \frac{1}{t}\right) dt$$
$$= \frac{1}{2} \int t\,dt - \int 1\,dt + \frac{1}{2} \int \frac{1}{t}\,dt$$
$$= \frac{1}{2} \cdot \frac{t^2}{2} - t + \frac{1}{2} \log |t| + c$$
$$= \frac{1}{4}(x^2+1)^2 - (x^2+1) + \frac{1}{2} \log |x^2+1| + c.$$

Question 8.

$\int \frac{2x+6}{\sqrt{x^2+6x+3}} dx$

Solution:

$$\text{Let } I = \int \frac{2x+6}{\sqrt{x^2+6x+3}} dx$$

Put $x^2 + 6x + 3 = t$

$\therefore \; (2x+6)dx = dt$

$$\therefore \; I = \int \frac{1}{\sqrt{t}} dt = \int t^{-\frac{1}{2}} dt$$
$$= \frac{t^{\frac{1}{2}}}{1/2} + c = 2\sqrt{x^2+6x+3} + c.$$

Question 9.

$\int \frac{1}{\sqrt{x} + x} dx$

Solution:

$$\text{Let } I=\int \frac{1}{\sqrt{x}+x}dx=\int \frac{1}{\sqrt{x}(1+\sqrt{x})}dx$$
$$=\int \frac{1}{1+\sqrt{x}}\cdot\frac{1}{\sqrt{x}}dx$$

Put $1+\sqrt{x}=t \qquad \therefore \frac{1}{2\sqrt{x}}dx=dt$

$\therefore \frac{1}{\sqrt{x}}dx=2\,dt$

$$\therefore I=\int \frac{1}{t}\cdot 2dt=2\int \frac{1}{t}dt$$
$$=2\log|t|+c=2\log|1+\sqrt{x}|+c.$$

Question 10.
$\int \frac{1}{x(x^6+1)}dx$
Solution:

$$\text{Let } I=\int \frac{1}{x(x^6+1)}dx$$
$$=\int \frac{x^5}{x^6(x^6+1)}dx$$

Put $x^6=t \qquad \therefore 6x^5dx=dt$

$\therefore x^5dx=\frac{1}{6}dt$

$$\therefore I=\int \frac{1}{t(t+1)}\cdot\frac{dt}{6}$$
$$=\frac{1}{6}\int \frac{(t+1)-t}{t(t+1)}dt=\frac{1}{6}\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt$$
$$=\frac{1}{6}\left[\int \frac{1}{t}dt-\int \frac{1}{t+1}dt\right]$$
$$=\frac{1}{6}[\log(t)-\log|t+1|]+c$$
$$=\frac{1}{6}\log\left|\frac{t}{t+1}\right|+c=\frac{1}{6}\log\left|\frac{x^6}{x^6+1}\right|+c.$$

Maharashtra Board 12th Commerce Maths Solutions Chapter 5 Integration Ex 5.3

May 8, 2023 by Bhagya

Balbharati Maharashtra State Board **Std 12 Commerce Statistics Part 1 Digest Pdf** Chapter 5 Integration Ex 5.3 Questions and Answers.

Maharashtra State Board 12th Commerce Maths Solutions Chapter 5 Integration Ex 5.3

Evaluate the following:

Question 1.

$\int \frac{3e^{2t}+5}{4e^{2t}-5} dt$

Solution:

Let I = $\int \frac{3e^{2t}+5}{4e^{2t}-5} dt$

Put, Numerator = A(Denominator) + B[$\frac{d}{dx}$(Denominator)]

∴ $3e^{2t} + 5 = A(4e^{2t} - 5) + B[\frac{d}{dt}(4e^{2t} - 5)]$

∴ $3e^{2t} + 5 = A(4e^{2t} - 5) + B[4e^{2t} \times 2 - 0]$

∴ $3e^{2t} + 5 = (4A + 8B) e^{2t} - 5A$

Equating the coefficient of e^{2t} and constant on both sides, we get

4A + 8B = 3

and -5A = 5

∴ A = -1

∴ from (1), 4(-1) + 8B = 3

∴ 8B = 7

∴ B = $\frac{7}{8}$

$$\therefore\ 3e^{2t}+5 = -(4e^{2t}-5)+\frac{7}{8}(8e^{2t})$$

$$\therefore\ I = \int\left[\frac{-(4e^{2t}-5)+\frac{7}{8}(8e^{2t})}{4e^{2t}-5}\right]dt$$

$$= \int\left[-1+\frac{\frac{7}{8}(8e^{2t})}{4e^{2t}-5}\right]dt$$

$$= -\int 1dt + \frac{7}{8}\int\frac{8e^{2t}}{4e^{2t}-5}dt$$

$$= -t+\frac{7}{8}\log|4e^{2t}-5|+c$$

$$\ldots\left[\because \int\frac{f'(x)}{f(x)}dx = \log|f(x)|+c\right].$$

Question 2.

$\int \frac{20-12e^{x}}{3e^{x}-4} dx$

Solution:

Let I = $\int \frac{20-12e^{x}}{3e^{x}-4} dx$

Put, Numerator = A (Denominator) + B[$\frac{d}{dx}$(Denominator)]

∴ $20 - 12e^{x} = A(3e^{x} - 4) + B[\frac{d}{dx}(3e^{x} - 4)]$

∴ $20 - 12e^{x} = A(3e^{x} - 4) + B(3e^{x} - 0)$

∴ $20 - 12e^{x} = (3A + 3B)e^{x} - 4A$

Equating the coefficient of ex and constant on both sides, we get

3A + 3B = -12 ……(1)

and -4A = 20

∴ A = -5

from (1), 3(-5) + 3B = -12

∴ 3B = 3

∴ B = 1

∴ $20 - 12e^{x} = -5(3e^{x} - 4) + (3e^{x})$

$$\therefore\ I = \int\left[\frac{-5(3e^{x}-4)+(3e^{x})}{3e^{x}-4}\right]dx$$

$$= \int\left(-5+\frac{3e^{x}}{3e^{x}-4}\right)dx$$

$$= -5\int 1dx + \int\frac{3e^{x}}{3e^{x}-4}dx$$

$$= -5x+\log|3e^{x}-4|+c$$

$$\ldots\left[\because \int\frac{f'(x)}{f(x)}dx = \log|f(x)|+c\right]$$

Question 3.

$\int \frac{3e^x+4}{2e^x-8} dx$

Solution:

Let I = $\int \frac{3e^x+4}{2e^x-8} dx$

Put, Numerator = A (Denominator) + B[$\frac{d}{dx}$(Denominator)]

∴ $3e^x + 4 = A(2e^x - 8) + B[\frac{d}{dx}(2e^x - 8)]$

∴ $3e^x + 4 = A(2e^x - 8) + B(2e^x - 0)$

∴ $3e^x + 4 = (2A + 2B)e^x - 8A$

Equating the coefficient of ex and constant on both sides, we get

2A + 2B = 3 ……..(1)

and -8A = 4

∴ A = $-\frac{1}{2}$

∴ from (1), $2(-\frac{1}{2}) + 2B = 3$

∴ 2B = 4

∴ B = 2

$$\therefore\ 3e^x + 4 = -\frac{1}{2}(2e^x - 8) + 2(2e^x)$$

$$\therefore\ I = \int \left[\frac{-\frac{1}{2}(2e^x - 8) + 2(2e^x)}{2e^x - 8}\right] dx$$

$$= \int \left[-\frac{1}{2} + \frac{2(2e^x)}{2e^x - 8}\right] dx$$

$$= -\frac{1}{2}\int 1\, dx + 2\int \frac{2e^x}{2e^x - 8}\, dx$$

$$= -\frac{1}{2}x + 2\log|2e^x - 8| + c$$

$$\ldots \left[\because \int \frac{f'(x)}{f(x)}\, dx = \log|f(x)| + c\right]$$

Question 4.

$\int \frac{2e^x+5}{2e^x+1} dx$

Solution:

Let $I = \int \frac{2e^x + 5}{2e^x + 1} dx$

Let $2e^x + 5 = A(2e^x + 1) + B \frac{d}{dx}(2e^x + 1)$

$= 2Ae^x + A + B(2e^x)$

$\therefore 2e^x + 5 = (2A + 2B)e^x + A$

Comparing the coefficients of e^x and constant term on both sides, we get

$2A + 2B = 2$ and $A = 5$

Solving these equations, we get

$B = -4$

$\therefore I = \int \frac{5\left(2e^x + 1\right) - 4\left(2e^x\right)}{2e^x + 1} dx$

$= 5\int dx - 4\int \frac{2e^x}{2e^x + 1} dx$

$\therefore I = 5x - 4 \log \left|2e^x + 1\right| + c \quad \ldots \left[\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c\right]$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 5 Integration Ex 5.4

Evaluate the following.

Question 1.
$\int \frac{1}{4x^2-1} dx$
Solution:

$$\int \frac{1}{4x^2-1} dx = \frac{1}{4}\int \frac{1}{x^2-(1/4)} dx$$

$$= \frac{1}{4}\int \frac{1}{x^2-\left(\frac{1}{2}\right)^2} dx$$

$$= \frac{1}{4} \times \frac{1}{2\left(\frac{1}{2}\right)} \log \left|\frac{x-\frac{1}{2}}{x+\frac{1}{2}}\right| + c$$

$$= \frac{1}{4} \log \left|\frac{2x-1}{2x+1}\right| + c.$$

Question 2.
$\int \frac{1}{x^2+4x-5} dx$

Solution:

$$\int \frac{1}{x^2+4x-5}\,dx$$

$$= \int \frac{1}{(x^2+4x+4)-4-5}\,dx$$

$$= \int \frac{1}{(x+2)^2-(3)^2}\,dx$$

$$= \frac{1}{2\times 3}\log\left|\frac{x+2-3}{x+2+3}\right| + c$$

$$= \frac{1}{6}\log\left|\frac{x-1}{x+5}\right| + c.$$

Question 3.

$\int \frac{1}{4x^2-20x+17}dx$

Solution:

$$\int \frac{1}{4x^2-20x+17}\,dx$$

$$= \frac{1}{4}\int \frac{1}{x^2-5x+\frac{17}{4}}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\left(x^2-5x+\frac{25}{4}\right)-\frac{25}{4}+\frac{17}{4}}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\left(x-\frac{5}{2}\right)^2-(\sqrt{2})^2}\,dx$$

$$= \frac{1}{4}\times\frac{1}{2\sqrt{2}}\log\left|\frac{x-\frac{5}{2}-\sqrt{2}}{x-\frac{5}{2}+\sqrt{2}}\right| + c$$

$$= \frac{1}{8\sqrt{2}}\log\left|\frac{2x-5-2\sqrt{2}}{2x-5+2\sqrt{2}}\right| + c.$$

Question 4.

$\int \frac{x}{4x^4-20x^2-3}dx$

Solution:

$$\text{Let } I=\int \frac{x}{4x^4-20x^2-3}\,dx$$

Put $x^2=t \qquad \therefore\ 2x\,dx=dt$

$$\therefore\ x\,dx=\frac{dt}{2}$$

$$\therefore\ I=\int \frac{1}{4t^2-20t-3}\cdot\frac{dt}{2}$$

$$=\frac{1}{2}\times\frac{1}{4}\int \frac{1}{t^2-5t-\frac{3}{4}}\,dt$$

$$=\frac{1}{8}\int \frac{1}{\left(t^2-5t+\frac{25}{4}\right)-\frac{25}{4}-\frac{3}{4}}\,dt$$

$$=\frac{1}{8}\int \frac{1}{\left(t-\frac{5}{2}\right)^2-(\sqrt{7})^2}\,dt$$

$$=\frac{1}{8}\times\frac{1}{2\sqrt{7}}\log\left|\frac{t-\frac{5}{2}-\sqrt{7}}{t-\frac{5}{2}+\sqrt{7}}\right|+c$$

$$=\frac{1}{16\sqrt{7}}\log\left|\frac{2t-5-2\sqrt{7}}{2t-5+2\sqrt{7}}\right|+c$$

$$=\frac{1}{16\sqrt{7}}\log\left|\frac{2x^2-5-2\sqrt{7}}{2x^2-5+2\sqrt{7}}\right|+c.$$

Question 5.

$\int \frac{x^3}{16x^8-25}dx$

Solution:

$$\text{Let } I=\int \frac{x^3}{16x^8-25}\,dx$$

Put $x^4=t \qquad \therefore\ 4x^3dx=dt$

$$\therefore\ x^3dx=\frac{dt}{4}$$

$$\therefore\ I=\int \frac{1}{16t^2-25}\cdot\frac{dt}{4}$$

$$=\frac{1}{4}\times\frac{1}{16}\int \frac{1}{t^2-\frac{25}{16}}\,dt$$

$$=\frac{1}{64}\int \frac{1}{t^2-\left(\frac{5}{4}\right)^2}\,dt$$

$$=\frac{1}{64}\times\frac{1}{2\times\frac{5}{4}}\log\left|\frac{t-\frac{5}{4}}{t+\frac{5}{4}}\right|+c$$

$$=\frac{1}{160}\log\left|\frac{4t-5}{4t+5}\right|+c$$

$$=\frac{1}{160}\log\left|\frac{4x^4-5}{4x^4+5}\right|+c.$$

Question 6.

∫1a2−b2x2dx

Solution:

$$\int \frac{1}{a^2 - b^2x^2}\,dx = \frac{1}{b^2}\int \frac{1}{\frac{a^2}{b^2} - x^2}\,dx$$

$$= \frac{1}{b^2}\int \frac{1}{\left(\frac{a}{b}\right)^2 - x^2}\,dx$$

$$= \frac{1}{b^2} \times \frac{1}{2\left(\frac{a}{b}\right)} \log\left|\frac{\frac{a}{b} + x}{\frac{a}{b} - x}\right| + c$$

$$= \frac{1}{2ab}\log\left|\frac{a + bx}{a - bx}\right| + c.$$

Question 7.

∫17+6x−x2dx

Solution:

$$\int \frac{1}{7 + 6x - x^2}\,dx$$

$$= \int \frac{1}{7 - (x^2 - 6x + 9) + 9}\,dx$$

$$= \int \frac{1}{(4)^2 - (x - 3)^2}\,dx$$

$$= \frac{1}{2 \times 4}\log\left|\frac{4 + x - 3}{4 - x + 3}\right| + c$$

$$= \frac{1}{8}\log\left|\frac{1 + x}{7 - x}\right| + c.$$

Question 8.

∫13x2+8√dx

Solution:

$$\int \frac{1}{\sqrt{3x^2 + 8}}\,dx = \int \frac{1}{\sqrt{(\sqrt{3}x)^2 + (\sqrt{8})^2}}\,dx$$

$$= \frac{\log\left|\sqrt{3}x + \sqrt{(\sqrt{3}x)^2 + (\sqrt{8})^2}\right|}{\sqrt{3}} + c$$

$$= \frac{1}{\sqrt{3}}\log\left|\sqrt{3}x + \sqrt{3x^2 + 8}\right| + c.$$

Question 9.

∫1x2+4x+29√dx

Solution:

$$\int \frac{1}{\sqrt{x^2+4x+29}} dx$$

$$= \int \frac{1}{\sqrt{(x^2+4x+4)+25}} dx$$

$$= \int \frac{1}{\sqrt{(x+2)^2+(5)^2}} dx$$

$$= \log \left|(x+2)+\sqrt{(x+2)^2+(5)^2}\right|+c$$

$$= \log \left|(x+2)+\sqrt{x^2+4x+29}\right|+c.$$

Question 10.

∫13x2−5√dx

Solution:

$$\int \frac{1}{\sqrt{3x^2-5}} dx = \int \frac{1}{\sqrt{(\sqrt{3}x)^2-(\sqrt{5})^2}} dx$$

$$= \frac{\log \left|\sqrt{3}x+\sqrt{(\sqrt{3}x)^2-(\sqrt{5})^2}\right|}{\sqrt{3}}+c$$

$$= \frac{1}{\sqrt{3}} \log \left|\sqrt{3}x+\sqrt{3x^2-5}\right|+c.$$

Question 11.

∫1x2−8x−20√dx

Solution:

$$\int \frac{1}{\sqrt{x^2-8x-20}} dx$$

$$= \int \frac{1}{\sqrt{(x^2-8x+16)-16-20}} dx$$

$$= \int \frac{1}{\sqrt{(x-4)^2-(6)^2}} dx$$

$$= \log \left|(x-4)+\sqrt{(x-4)^2-(6)^2}\right|+c$$

$$= \log \left|(x-4)+\sqrt{x^2-8x-20}\right|+c.$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 5 Integration Ex 5.5

Evaluate the following.

Question 1.

∫x log x

Solution:

$$\int x \log x \, dx = \int (\log x) \cdot x \, dx$$

$$= (\log x) \int x \, dx - \int \left[\frac{d}{dx} (\log x) \int x \, dx \right] dx$$

$$= (\log x) \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} \, dx$$

$$= \frac{1}{2} x^2 \log x - \frac{1}{2} \int x \, dx$$

$$= \frac{x^2}{2} \log x - \frac{1}{2} \cdot \frac{x^2}{2} + c = \frac{x^2}{2} \log x - \frac{x^2}{4} + c.$$

Question 2.

$\int x^2 e^{4x} \, dx$

Solution:

$$\int x^2 e^{4x} \, dx = x^2 \int e^{4x} \, dx - \int \left[\frac{d}{dx} (x^2) \int e^{4x} \, dx \right] dx$$

$$= x^2 \cdot \frac{e^{4x}}{4} - \int 2x \cdot \frac{e^{4x}}{4} \, dx$$

$$= \frac{1}{4} x^2 e^{4x} - \frac{1}{2} \int x e^{4x} \, dx$$

$$= \frac{1}{4} x^2 e^{4x} - \frac{1}{2} \left[x \int e^{4x} \, dx - \int \left\{ \frac{d}{dx} (x) \int e^{4x} \, dx \right\} dx \right]$$

$$= \frac{1}{4} x^2 e^{4x} - \frac{1}{2} \left[x \cdot \frac{e^{4x}}{4} - \int 1 \cdot \frac{e^{4x}}{4} \, dx \right]$$

$$= \frac{1}{4} x^2 e^{4x} - \frac{1}{8} x \cdot e^{4x} + \frac{1}{8} \int e^{4x} \, dx$$

$$= \frac{1}{4} x^2 e^{4x} - \frac{1}{8} x e^{4x} + \frac{1}{8} \cdot \frac{e^{4x}}{4} + c$$

$$= \frac{1}{4} e^{4x} \left[x^2 - \frac{x}{2} + \frac{1}{8} \right] + c.$$

Question 3.

$\int x^2 e^{3x} \, dx$

Solution:

$$\text{Let } I = \int x^2 e^{3x}\,dx$$

$$= x^2 \int e^{3x}\,dx - \int\left[\frac{d}{dx}\left(x^2\right)\int e^{3x}\,dx\right]dx$$

$$= x^2\cdot\left(\frac{e^{3x}}{3}\right) - \int 2x\cdot\frac{e^{3x}}{3}\,dx$$

$$= \frac{x^2}{3}e^{3x} - \frac{2}{3}\int x\cdot e^{3x}\,dx$$

$$= \frac{x^2}{3}e^{3x} - \frac{2}{3}\left[x\int e^{3x}\,dx - \int\left(\frac{d}{dx}(x)\int e^{3x}\,dx\right)dx\right]$$

$$= \frac{x^2\cdot e^{3x}}{3} - \frac{2}{3}\left[x\cdot\frac{e^{3x}}{3} - \int 1\cdot\frac{e^{3x}}{3}\,dx\right]$$

$$= \frac{x^2\cdot e^{3x}}{3} - \frac{2}{3}\left[\frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}\,dx\right]$$

$$= \frac{x^2\cdot e^{3x}}{3} - \frac{2}{3}\left[\frac{1}{3}xe^{3x} - \frac{1}{3}\cdot\frac{e^{3x}}{3}\right] + c$$

$$\therefore I = \frac{1}{3}x^2\cdot e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + c$$

Question 4.

$\int x^3 e^{x^2} dx$

Solution:

$$\text{Let } I = \int x^3 e^{x^2} dx = \int x^2 e^{x^2}\cdot x\,dx$$

Put $x^2 = t$ $\quad\therefore\ 2x\,dx = dt$

$$\therefore\ x\,dx = \frac{dt}{2}$$

$$\therefore\ I = \int te^t\cdot\frac{dt}{2} = \frac{1}{2}\int te^t dt$$

$$= \frac{1}{2}\left[t\int e^t dt - \int\left\{\frac{d}{dt}(t)\int e^t dt\right\}dt\right]$$

$$= \frac{1}{2}[te^t - \int 1\cdot e^t dt]$$

$$= \frac{1}{2}[te^t - e^t] + c$$

$$= \frac{1}{2}(t-1)e^t + c$$

$$= \frac{1}{2}(x^2-1)e^{x^2} + c.$$

Question 5.

$\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$

Solution:

$$\text{Let } I=\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$$

Put $f(x)=\frac{1}{x}$. Then $f'(x)=-\frac{1}{x^2}$

$$\therefore\ I=\int e^x-[f(x)+f'(x)]\,dx$$

$$=e^x\cdot f(x)+c=e^x\cdot\frac{1}{x}+c.$$

Question 6.

$\int e^x \frac{x}{(x+1)^2} dx$

Solution:

$$\text{Let } I=\int e^x\cdot\frac{x}{(x+1)^2}\,dx$$

$$=\int e^x\left[\frac{(x+1)-1}{(x+1)^2}\right]dx$$

$$=\int e^x\left[\frac{1}{x+1}-\frac{1}{(x+1)^2}\right]dx$$

Put $f(x)=\frac{1}{x+1}$

Then $f'(x)=\frac{d}{dx}(x+1)^{-1}=-1(x+1)^{-2}\cdot\frac{d}{dx}(x+1)$

$$=\frac{-1}{(x+1)^2}\times(1+0)=\frac{-1}{(x+1)^2}$$

$$\therefore\ I=\int e^x[f(x)+f'(x)]\,dx$$

$$=e^x\cdot f(x)+c=e^x\cdot\frac{1}{x+1}+c.$$

Question 7.

$\int e^x \frac{x-1}{(x+1)^3} dx$

Solution:

$$\text{Let } I=\int e^x\cdot\frac{x-1}{(x+1)^3}\,dx$$

$$=\int e^x\left[\frac{(x+1)-2}{(x+1)^3}\right]dx$$

$$=\int e^x\left[\frac{1}{(x+1)^2}-\frac{2}{(x+1)^3}\right]dx$$

Put $f(x)=\frac{1}{(x+1)^2}$

Then $f'(x)=\frac{d}{dx}(x+1)^{-2}=-2(x+1)^{-3}\cdot\frac{d}{dx}(x+1)$

$$=\frac{-2}{(x+1)^3}\times(1+0)=\frac{-2}{(x+1)^3}$$

$$\therefore\ I=\int e^x\,[f(x)+f'(x)]\,dx$$

$$=e^x\cdot f(x)+c=e^x\cdot\frac{1}{(x+1)^2}+c.$$

Question 8.

$\int e^x\left[(\log x)^2+\frac{2\log x}{x}\right]dx$

Solution:

$$\text{Let } I=\int e^x\left[(\log x)^2+\frac{2\log x}{x}\right]dx$$

Put $f(x)=(\log x)^2$

Then $f'(x)=\frac{d}{dx}(\log x)^2=2\log x\cdot\frac{d}{dx}(\log x)$

$$=2\log x\times\frac{1}{x}=\frac{2\log x}{x}$$

$\therefore\ I=\int e^x[f(x)+f'(x)]\,dx$

$=e^x\cdot f(x)+c=e^x\cdot(\log x)^2+c.$

Question 9.

$\int[1\log x-1(\log x)2]dx$

Solution:

$$\text{Let } I=\int\left[\frac{1}{\log x}-\frac{1}{(\log x)^2}\right]dx$$

Put $\log x=t\qquad\therefore\ x=e^t$

$\therefore\ dx=e^t\,dt$

$\therefore\ I=\int\left(\frac{1}{t}-\frac{1}{t^2}\right)e^t\,dt$

Let $f(t)=\frac{1}{t}$. Then $f'(t)=-\frac{1}{t^2}$

$\therefore\ I=\int e^t[f(t)+f'(t)]\,dt$

$=e^t\cdot f(t)+c=e^t\times\frac{1}{t}+c$

$=\frac{x}{\log x}+c.$

Question 10.

$\int\log x(1+\log x)2dx$

Solution:

$$\text{Let } I=\int\frac{\log x}{(1+\log x)^2}\,dx$$

Put $\log x=t\qquad\therefore\ x=e^t$

$\therefore\ dx=e^t\,dt$

$\therefore\ I=\int\frac{t}{(1+t)^2}\cdot e^t\,dt$

$=\int e^t\left[\frac{(1+t)-1}{(1+t)^2}\right]dt$

$=\int e^t\left[\frac{1}{1+t}-\frac{1}{(1+t)^2}\right]dt$

Let $f(t)=\frac{1}{1+t}$.

$\therefore\ f'(t)=\frac{d}{dt}(1+t)^{-1}=-1(1+t)^{-2}(0+1)$

$=\frac{-1}{(1+t)^2}$

$\therefore\ I=\int e^t[f(t)+f'(t)]\,dt$

$=e^t\cdot f(t)+c=e^t\times\frac{1}{1+t}+c=\frac{x}{1+\log x}+c.$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 5 Integration Ex 5.6

Evaluate:

Question 1.
$\int 2x+1(x+1)(x-2)dx$
Solution:
Let I = $\int 2x+1(x+1)(x-2)dx$

Let $2x+1(x+1)(x-2)=Ax+1+Bx-2$
∴ 2x + 1 = A(x – 2) + B(x + 1)
Put x + 1 = 0, i.e. x = -1, we get
2(-1) + 1 = A(-3) + B(0)
∴ A = 13
Put x – 2 = 0, i.e. x = 2, we get
2(2) + 1 = A(0) + B(3)
∴ B = 53

$$\therefore \frac{2x+1}{(x+1)(x-2)}=\frac{(1/3)}{x+1}+\frac{(5/3)}{x-2}$$

$$\therefore I=\int\left[\frac{(1/3)}{x+1}+\frac{(5/3)}{x-2}\right]dx$$

$$=\frac{1}{3}\int\frac{1}{x+1}\,dx+\frac{5}{3}\int\frac{1}{x-2}\,dx$$

$$=\frac{1}{3}\log|x+1|+\frac{5}{3}\log|x-2|+c.$$

Question 2.
$\int 2x+1x(x-1)(x-4)dx$
Solution:
Let I = $\int 2x+1x(x-1)(x-4)dx$

Let $\int 2x+1x(x-1)(x-4)=Ax+Bx-1+Cx-4$
∴ 2x + 1 = A(x – 1)(x – 4) + Bx(x – 4) + Cx(x – 1)
Put x = 0, we get
2(0) + 1 = A(-1)(-4) + B(0)(-4) + C(0)(-1)
∴ 1 = 4A
∴ A = 14
Put x – 1 = 0, i.e. x = 1, we get
2(1) + 1 = A(0)(-3) + B(1)(-3) + C(1)(0)
∴ 3 = -3B
∴ B = -1
Put x – 4 = 0, i.e x = 4, we get
2(4) + 1 = A(3)(0) + B(4)(0) + C(4)(3)
∴ 9 = 12C
∴ C = 34

$$\therefore \frac{2x+1}{x(x-1)(x-4)}=\frac{\left(\frac{1}{4}\right)}{x}+\frac{(-1)}{x-1}+\frac{\left(\frac{3}{4}\right)}{x-4}$$

$$\therefore I=\int\left[\frac{\left(\frac{1}{4}\right)}{x}+\frac{(-1)}{x-1}+\frac{\left(\frac{3}{4}\right)}{x-4}\right]dx$$

$$=\frac{1}{4}\int\frac{1}{x}\,dx-\int\frac{1}{x-1}\,dx+\frac{3}{4}\int\frac{1}{x-4}\,dx$$

$$=\frac{1}{4}\log|x|-\log|x-1|+\frac{3}{4}\log|x-4|+c.$$

Question 3.
$\int \frac{x^2+x-1}{x^2+x-6} dx$
Solution:

$$\text{Let } I = \int \frac{x^2+x-1}{x^2+x-6}\,dx$$
$$= \int \frac{(x^2+x-6)+5}{x^2+x-6}\,dx$$
$$= \int \left[1 + \frac{5}{x^2+x-6}\right] dx$$
$$= \int 1\,dx + 5\int \frac{1}{x^2+x-6}\,dx$$

$$\text{Let } \frac{1}{x^2+x-6} = \frac{1}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$$

$\therefore$ 1 = A(x – 2) + B(x + 3)
Put x + 3 = 0, i.e. x = -3, we get
1 = A(-5) + B (0)
$\therefore$ A = $-\frac{1}{5}$
Put x – 2 = 0, i.e. x = 2, we get
1 = A(0) + B(5)
$\therefore$ B = $\frac{1}{5}$

$$\therefore \frac{1}{x^2+x-6} = \frac{(-1/5)}{x+3} + \frac{(1/5)}{x-2}$$
$$\therefore I = \int 1\,dx + 5\int \left[\frac{(-1/5)}{x+3} + \frac{(1/5)}{x-2}\right] dx$$
$$= \int 1\,dx - \int \frac{1}{x+3}\,dx + \int \frac{1}{x-2}\,dx$$
$$= x - \log|x+3| + \log|x-2| + c.$$

Question 4.
$\int \frac{x}{(x-1)^2(x+2)} dx$
Solution:
Let I = $\int \frac{x}{(x-1)^2(x+2)} dx$
Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$
$\therefore$ x = A(x – 1)(x + 2) + B(x + 2) + C$(x - 1)^2$
Put x – 1 = 0, i.e. x = 1, we get
1 = A(0)(3) + B(3) + C(0)
$\therefore$ B = $\frac{1}{3}$
Put x + 2 = 0, i.e. x = -2, we get
-2 = A (-3)(0) + B(0) + C(9)
$\therefore$ C = $-\frac{2}{9}$
Put x = -1, we get,
-1 = A(-2)(1) + B(1) + C(4)

But B = 13 and C = −29

$$\therefore\ -1=-2A+\frac{1}{3}-\frac{8}{9}$$

$$\therefore\ 2A=-\frac{5}{9}+1=\frac{4}{9} \qquad \therefore\ A=\frac{2}{9}$$

$$\therefore\ \frac{x}{(x-1)^2(x+2)}=\frac{(2/9)}{x-1}+\frac{(1/3)}{(x-1)^2}+\frac{(-2/9)}{x+2}$$

$$\therefore\ I=\int\left[\frac{(2/9)}{x-1}+\frac{(1/3)}{(x-1)^2}+\frac{(-2/9)}{x+2}\right]dx$$

$$=\frac{2}{9}\int\frac{1}{x-1}dx+\frac{1}{3}\int(x-1)^{-2}dx-\frac{2}{9}\int\frac{1}{x+2}dx$$

$$=\frac{2}{9}\log|x-1|+\frac{1}{3}\cdot\frac{(x-1)^{-1}}{-1}-\frac{2}{9}\log|x+2|+c$$

$$=\frac{2}{9}\log\left|\frac{x-1}{x+2}\right|-\frac{1}{3(x-1)}+c.$$

Question 5.

∫3x−2(x+1)2(x+3)dx

Solution:

Let I = ∫3x−2(x+1)2(x+3)dx

Let 3x−2(x+1)2(x+3)=Ax+1+B(x+1)2+Cx+3

∴ 3x – 2 = A(x + 1)(x + 3) + B(x + 3) + C(x + 1)^2

Put x + 1 = 0, i.e. x = -1, we get

3(-1) – 2 = A(0)(2) + B(2) + C(0)

∴ -5 = 2B

∴ B = −52

Put x + 3 = 0, i.e. x = -3, we get

3(-3) – 2 = A(-2)(0) + B(0) + C(4)

∴ -11 = 4C

∴ C = −114

Put x = 0, we get

3(0) – 2 = A(1)(3) + B(3) + C(1)

∴ -2 = 3A + 3B + C

But B = −52 and C = −114

$$\therefore\ -2=3A+3\left(-\frac{5}{2}\right)-\frac{11}{4}$$

$$\therefore\ 3A=-2+\frac{15}{2}+\frac{11}{4}=\frac{-8+30+11}{4}=\frac{33}{4}$$

$$\therefore\ A=\frac{11}{4}$$

$$\therefore\ \frac{3x-2}{(x+1)^2(x+3)}=\frac{\left(\frac{11}{4}\right)}{x+1}+\frac{\left(-\frac{5}{2}\right)}{(x+1)^2}+\frac{\left(-\frac{11}{4}\right)}{(x+3)}$$

$$\therefore\ I=\int\left[\frac{\left(\frac{11}{4}\right)}{x+1}+\frac{\left(-\frac{5}{2}\right)}{(x+1)^2}+\frac{\left(-\frac{11}{4}\right)}{x+3}\right]dx$$

$$=\frac{11}{4}\int\frac{1}{x+1}\,dx-\frac{5}{2}\int(x+1)^{-2}\,dx-\frac{11}{4}\int\frac{1}{x+3}\,dx$$

$$=\frac{11}{4}\log|x+1|-\frac{5}{2}\cdot\frac{(x+1)^{-1}}{-1}-\frac{11}{4}\log|x+3|+c$$

$$=\frac{11}{4}\log\left|\frac{x+1}{x+3}\right|+\frac{5}{2(x+1)}+c.$$

Question 6.

$\int \frac{1}{x(x^5+1)} dx$

Solution:

Let I = $\int \frac{1}{x(x^5+1)} dx$

= $\int \frac{x^4}{x^5(x^5+1)} dx$

Put $x^5 = t$. Then $5x^4\,dx = dt$

$\therefore\ x^4\,dx = \frac{dt}{5}$

$$\therefore\ I = \int \frac{1}{t(t+1)} \cdot \frac{dt}{5}$$

$$= \frac{1}{5}\int \frac{(t+1)-t}{t(t+1)}\,dt$$

$$= \frac{1}{5}\int \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$$

$$= \frac{1}{5}\left[\int \frac{1}{t}dt - \int \frac{1}{t+1}dt\right]$$

$$= \frac{1}{5}[\log|t| - \log|t+1|] + c$$

$$= \frac{1}{5}\log\left|\frac{t}{t+1}\right| + c = \frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right| + c.$$

Question 7.

$\int \frac{1}{x(x^n+1)} dx$

Solution:

$$\text{Let } I = \int \frac{1}{x(x^n+1)}\,dx$$

$$= \int \frac{x^{n-1}}{x^n(x^n+1)}\,dx$$

Put $x^n = t \quad \therefore\ nx^{n-1}\,dx = dt$

$\therefore\ x^{n-1}\,dx = \frac{dt}{n}$

$$\therefore\ I = \int \frac{1}{t(t+1)} \cdot \frac{dt}{n}$$

$$= \frac{1}{n}\int \frac{(t+1)-t}{t(t+1)}\,dt$$

$$= \frac{1}{n}\int \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$$

$$= \frac{1}{n}\left[\int \frac{1}{t}dt - \int \frac{1}{t+1}\,dt\right]$$

$$= \frac{1}{n}[\log|t| - \log|t+1|] + c$$

$$= \frac{1}{n}\log\left|\frac{t}{t+1}\right| + c = \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + c.$$

Question 8.

$\int \frac{5x^2+20x+6}{x^3+2x^2+x} dx$

Solution:

$$\text{Let } I=\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx$$
$$=\int \frac{5x^2+20x+6}{x(x^2+2x+1)}\,dx$$
$$=\int \frac{5x^2+20x+6}{x(x+1)^2}\,dx$$

Let $\frac{5x^2+20x+6}{x(x+1)^2}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$

∴ $5x^2 + 20x + 6 = A(x + 1)^2 + Bx(x + 1) + Cx$

Put x = 0, we get

0 + 0 + 6 = A(1) + B(0)(1) + C(0)

∴ A = 6

Put x + 1 = 0, i.e. x = -1, we get

5(1) + 20(-1) + 6 = A(0) + B(-1)(0) + C(-1)

∴ -9 = -C

∴ C = 9

Put x = 1, we get

5(1) + 20(1) + 6 = A(4) + B(1)(2) + C(1)

But A = 6 and C = 9

∴ 31 = 24 + 2B + 9

∴ B = -1

$$\therefore \frac{5x^2+20x+6}{x(x+1)^2}=\frac{6}{x}-\frac{1}{x+1}+\frac{9}{(x+1)^2}$$

$$\therefore I=\int\left[\frac{6}{x}-\frac{1}{x+1}+\frac{9}{(x+1)^2}\right]dx$$
$$=6\int\frac{1}{x}dx-\int\frac{1}{x+1}dx+9\int (x+1)^{-2}\,dx$$
$$=6\log|x|-\log|x+1|+9.\frac{(x+1)^{-1}}{-1}+c$$
$$=6\log|x|-\log|x+1|-\frac{9}{x+1}+c.$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 5 Integration Miscellaneous Exercise 5

(I) Choose the correct alternative from the following:

Question 1.

The value of $\int\frac{dx}{\sqrt{1-x}}$ is

(a) $2\sqrt{1-x}$ + c

(b) $-2\sqrt{1-x}$ + c

(c) √x + c

(d) x + c

Answer:

(b) $-2\sqrt{1-x}$ + c

Question 2.

$\int\sqrt{1+x^2}\,dx$ =

(a) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log(x+\sqrt{1+x^2})+c$

(b) $\frac{2}{3}(1+x^2)^{3/2}+c$

(c) $\frac{1}{3}(1+x^2)+c$

(d) $\frac{(x)}{\sqrt{1+x^2}}+c$

Answer:

(a) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log(x+\sqrt{1+x^2})+c$

Question 3.

$\int x^2(3)^{x^3}dx$ =

(a) $(3)^{x^3}+c$

(b) $\frac{(3)^{x^3}}{3\cdot\log 3}+c$

(c) $\log 3(3)^{x^3}+c$

(d) $x^2(3)^{x^3}$

Answer:

(b) $\frac{(3)^{x^3}}{3\cdot\log 3}+c$

Hint:
Put x^3 = t

Question 4.

$\int\frac{x+2}{2x^2+6x+5}dx=p\int\frac{4x+6}{2x^2+6x+5}dx + \frac{1}{2}\int\frac{dx}{2x^2+6x+5}$, then p = ________

(a) $\frac{1}{3}$
(b) $\frac{1}{2}$
(c) $\frac{1}{4}$
(d) 2
Answer:
(c) $\frac{1}{4}$
Hint:

$\int\frac{x+2}{2x^2+6x+5}dx=\int\frac{\frac{1}{4}(4x+6)+\frac{1}{2}}{2x^2+6x+5}dx$

Question 5.

$\int\frac{dx}{(x-x^2)}$ = _______

(a) log x – log(1 – x) + c
(b) log(1 – x^2) + c
(c) -log x + log(1 – x) + c
(d) log(x – x^2) + c
Answer:
(a) log x – log(1 – x) + c

Question 6.

$\int\frac{dx}{(x-8)(x+7)}$ = _________

(a) $\frac{1}{15}\log\left|\frac{x+2}{x-1}\right|+c$

(b) $\frac{1}{15}\log\left|\frac{x+8}{x+7}\right|+c$

(c) $\frac{1}{15}\log\left|\frac{x-8}{x+7}\right|+c$

(d) (x – 8)(x – 7) + c
Answer:

(c) $\frac{1}{15}\log\left|\frac{x-8}{x+7}\right|+c$

Question 7.

$\int\left(x+\frac{1}{x}\right)^3dx$ = ________

(a) $\frac{1}{4}\left(x+\frac{1}{x}\right)^4+c$

(b) $\frac{x^4}{4}+\frac{3x^2}{2}+3\log x-\frac{1}{2x^2}+c$

(c) $\frac{x^4}{4}+\frac{3x^2}{2}+3\log x+\frac{1}{x^2}+c$

(d) $(x-x^{-1})^3+c$

Answer:

(b) $\frac{x^4}{4}+\frac{3x^2}{2}+3\log x-\frac{1}{2x^2}+c$

Hint:

$\left(x+\frac{1}{x}\right)^3=x^3+3x+\frac{3}{x}+\frac{1}{x^3}$

Question 8.

$\int\left(\frac{e^{2x}+e^{-2x}}{e^x}\right)dx$

(a) $e^x-\frac{1}{3e^{3x}}+c$

(b) $e^x+\frac{1}{3e^{3x}}+c$

(c) $e^{-x}+\frac{1}{3e^{3x}}+c$

(d) $e^{-x}-\frac{1}{3e^{3x}}+c$

Answer:

(a) $e^x-\frac{1}{3e^{3x}}+c$

Question 9.
$\int(1-x)^{-2}$ dx = __________
(a) $(1 + x)^{-1} + c$
(b) $(1 - x)^{-1} + c$
(c) $(1 - x)^{-1} - 1 + c$
(d) $(1 - x)^{-1} + 1 + c$
Answer:
(b) $(1 - x)^{-1} + c$

Question 10.

$\int\frac{(x^3+3x^2+3x+1)}{(x+1)^5}dx$ = ______

(a) $\frac{-1}{x+1}+c$

(b) $\left(\frac{-1}{x+1}\right)^5+c$

(c) log(x + 1) + c
(d) $\log|x + 1|^5 + c$
Answer:

(a) $\frac{-1}{x+1}+c$

Hint:
$x^3 + 3x^2 + 3x + 1 = (x + 1)^3$

(II) Fill in the blanks.

Question 1.

$\int\frac{5(x^6+1)}{x^2+1}dx = x^4 +$ ___$x^3 + 5x + c$.

Answer:

$-\frac{5}{3}$

Hint:
$x^6 + 1 = (x^2 + 1)(x^4 - x^2 + 1)$

Question 2.

$\int\frac{x^2+x-6}{(x-2)(x-1)}dx = x +$ _____ + c

Answer:
4 log|x – 1|
Hint:
$x^2 + x - 6 = (x + 3)(x - 2)$

Question 3.
If f'(x) = $\frac{1}{x}$ + x and f(1) = $\frac{5}{2}$ then f(x) = log x + $\frac{x^2}{2}$ + ______
Answer:
2

Hint:

$$f(x)=\int\left(\frac{1}{x}+x\right)dx=\log|x|+\frac{x^2}{2}+c$$

$$\therefore\ f(1)=\log 1+\frac{1}{2}+c=\frac{5}{2}\qquad \therefore\ c=2$$

$$\therefore\ f(x)=\log|x|+\frac{x^2}{2}+2$$

Question 4.

To find the value of $\int(1+logx)dxx$ the proper substitution is ________

Answer:

1 + log x = t

Question 5.

$\int 1x_3[logx_x]2dx$ = p(log x)3 + c, then p = ______

Answer:

13

Hint:

$1x_3(logx_x)2=1x_3(xlogx)2=(logx)_2x$

(III) State whether each of the following is True or False:

Question 1.

The proper substitution for $\int x(x_x)_x(2logx+1)dx$ is $(x_x)_x=t$

Answer:

True

Question 2.

If ∫x e_{2x} dx is equal to e_{2x} f(x) + c where c is constant of integration, then f(x) is $(2x-1)2$.

Answer:

False

Question 3.

If ∫x f(x) dx = $f(x)2$, then f(x) = e_{x_2}.

Answer:

True

Question 4.

If $\int(x-1)dx(x+1)(x-2)$ = A log|x + 1| + B log|x – 2|, then A + B = 1.

Answer:

True

Question 5.

For $\int x-1(x+1)_3 e_x dx$ = e_x f(x) + c, f(x) = (x + 1)2.

Answer:

False

(IV) Solve the following:

1. Evaluate:

(i) $\int 5x_2-6x+32x-3dx$

Solution:

$$\text{Let } I = \int \frac{5x^2 - 6x + 3}{2x - 3}\, dx$$

$$
\begin{array}{r|l}
2x-3 & 5x^2 - 6x + 3 \quad \left(\frac{5}{2}x + \frac{3}{4} \right. \\
 & 5x^2 - \frac{15}{2}x \\
 & - \qquad + \\
\hline
 & \frac{3}{2}x + 3 \\
 & \frac{3}{2}x - \frac{9}{4} \\
 & - \qquad + \\
\hline
 & \frac{21}{4}
\end{array}
$$

$$\therefore\ 5x^2 - 6x + 3 = \left(\frac{5}{2}x + \frac{3}{4}\right)(2x-3) + \frac{21}{4}$$

$$\therefore\ I = \int \left[\frac{\left(\frac{5}{2}x + \frac{3}{4}\right)(2x-3) + \frac{21}{4}}{2x-3} \right] dx$$

$$= \int \left[\frac{5}{2}x + \frac{3}{4} + \frac{\left(\frac{21}{4}\right)}{2x-3} \right] dx$$

$$= \frac{5}{2}\int x\, dx + \frac{3}{4}\int 1\, dx + \frac{21}{4}\int \frac{1}{2x-3}\, dx$$

$$= \frac{5}{2}\cdot\frac{x^2}{2} + \frac{3}{4}x + \frac{21}{4}\cdot\frac{\log|2x-3|}{2} + c$$

$$= \frac{5x^2}{4} + \frac{3x}{4} + \frac{21}{8}\log|2x-3| + c.$$

(ii)∫(5x+1)49dx

Solution:

$$\int (5x+1)^{\frac{4}{9}} dx = \frac{(5x+1)^{\frac{4}{9}+1}}{\frac{4}{9}+1} \times \frac{1}{5} + c$$

$$= \frac{9}{65}(5x+1)^{\frac{13}{9}} + c.$$

(iii)∫12x+3dx

Solution:

$$\int \frac{1}{2x+3}\, dx = \frac{\log|2x+3|}{2} + c$$

$$= \frac{1}{2}\log|2x+3| + c.$$

(iv)∫x−1x+4√dx

Solution:

$$\int \frac{x-1}{\sqrt{x+4}}\,dx = \int \frac{(x+4)-5}{\sqrt{x+4}}\,dx$$

$$= \int \left(\frac{x+4}{\sqrt{x+4}} - \frac{5}{\sqrt{x+4}}\right) dx$$

$$= \int \left(\sqrt{x+4} - \frac{5}{\sqrt{x+4}}\right) dx$$

$$= \int (x+4)^{\frac{1}{2}}\,dx - 5\int (x+4)^{-\frac{1}{2}}\,dx$$

$$= \frac{(x+4)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - 5\cdot\frac{(x+4)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c$$

$$= \frac{2}{3}(x+4)^{\frac{3}{2}} - 10\sqrt{x+4} + c.$$

(v) If f'(x) = √x and f(1) = 2, then find the value of f(x).
Solution:
By the definition of integral

$$f(x) = \int f'(x)\,dx = \int \sqrt{x}\,dx$$

$$= \frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{2}{3}x^{\frac{3}{2}} + c \qquad \text{....... (1)}$$

$$\therefore\ f(1) = \frac{2}{3}(1)^{\frac{3}{2}} + c = \frac{2}{3} + c$$

But $f(1) = 2$

$$\therefore\ \frac{2}{3} + c = 2 \qquad \therefore\ c = \frac{4}{3}$$

$$\therefore\ \text{from (1), } f(x) = \frac{2}{3}x^{\frac{3}{2}} + \frac{4}{3}.$$

(vi) ∫|x| dx if x < 0
Solution:
∫|x| dx = ∫-x dx .....[∵ x < 0]
= -∫x dx
= −x22 + c

2. Evaluate:

(i) Find the primitive of 11+ex
Solution:
Let I be the primitive of 11+ex

$$\text{Then } I = \int \frac{1}{1+e^x}\,dx$$

$$= \int \frac{\left(\frac{1}{e^x}\right)}{\left(\frac{1+e^x}{e^x}\right)}\,dx = \int \frac{e^{-x}}{e^{-x}+1}\,dx$$

$$= -\int \frac{-e^{-x}}{e^{-x}+1}\,dx$$

$$= -\log|e^{-x}+1| + c$$

$$\ldots \left[\because \frac{d}{dx}(e^{-x}+1) = -e^{-x} \text{ and } \int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c\right]$$

(ii) $\int \frac{ae^x+be^{-x}}{(ae^x-be^{-x})^2} dx$

Solution:

$$\text{Let } I = \int \frac{ae^x + be^{-x}}{(ae^x - be^{-x})^2} dx$$

Put $ae^x - be^{-x} = t$

$\therefore (ae^x + be^{-x}) dx = dt$

$$\therefore I = \int \frac{1}{t^2} dt = \int t^{-2} dt$$

$$= \frac{t^{-1}}{-1} + c = \frac{-1}{t} + c$$

$$= \frac{-1}{ae^x + be^{-x}} + c.$$

(iii) $\int \frac{1}{2x+3x \log x} dx$

Solution:

$$\text{Let } I = \int \frac{1}{2x + 3x \log x} dx$$

$$= \int \frac{1}{(2 + 3 \log x)} \cdot \frac{1}{x} dx$$

Put $2 + 3 \log x = t$ $\quad \therefore \frac{3}{x} dx = dt$

$$\therefore \frac{1}{x} dx = \frac{dt}{3}$$

$$\therefore I = \int \frac{1}{t} \cdot \frac{dt}{3} = \frac{1}{3} \int \frac{1}{t} dt$$

$$= \frac{1}{3} \log |t| + c$$

$$= \frac{1}{3} \log |2 + 3 \log x| + c.$$

(iv) $\int \frac{1}{\sqrt{x}+x} dx$

Solution:

$$\text{Let I} = \int \frac{1}{\sqrt{x} + x} dx$$

$$= \int \frac{1}{\sqrt{x}\left(1 + \sqrt{x}\right)} dx$$

Put $1 + \sqrt{x} = t$

$$\therefore \frac{1}{2\sqrt{x}} dx = dt$$

$$\therefore \frac{1}{\sqrt{x}} dx = 2\, dt$$

$$\therefore I = \int \frac{2 \cdot dt}{t}$$

$$= 2 \int \frac{1}{t} dt$$

$$= 2 \log |t| + c$$

$$\therefore I = 2 \log \left|1 + \sqrt{x}\right| + c$$

(v) ∫2ex−34ex+1dx

Solution:

$$\text{Let I} = \int \frac{2e^x - 3}{4e^x + 1}\,dx$$

$$\text{Let } 2e^x - 3 = A\left(4e^x + 1\right) + B\frac{d}{dx}\left(4e^x + 1\right)$$

$$\therefore 2e^x - 3 = (4A + 4B)e^x + A$$

Comparing the coefficients of e^x and constant term on both sides, we get

4A + 4B = 2 and A = - 3

Solving these equations, we get

$$B = \frac{7}{2}$$

$$\therefore I = \frac{-3\left(4e^x + 1\right) + \frac{7}{2}\left(4e^x\right)}{4e^x + 1}\,dx$$

$$= -3\int dx + \frac{7}{2}\int \frac{4e^x}{4e^x + 1}dx$$

$$\therefore I = -3x + \frac{7}{2}\log\left|4e^x + 1\right| + c \quad ...\left[\because \int \frac{f\prime(x)}{f(x)}dx = \log|f(x)| + c\right]$$

3. Evaluate:

(i) ∫dx4x2−5√dx

Solution:

$$\int \frac{dx}{\sqrt{4x^2 - 5}} = \frac{1}{2}\int \frac{1}{\sqrt{x^2 - \frac{5}{4}}}\,dx$$

$$= \frac{1}{2}\log\left|x + \sqrt{x^2 - \frac{5}{4}}\right| + c.$$

(ii) ∫dx3−2x−x2dx

Solution:

$$\int \frac{dx}{3 - 2x - x^2} = \int \frac{dx}{3 - (x^2 + 2x + 1) + 1}$$

$$= \int \frac{1}{(2)^2 - (x+1)^2}\,dx$$

$$= \frac{1}{2\times 2}\log\left|\frac{2 + x + 1}{2 - x - 1}\right| + c$$

$$= \frac{1}{4}\log\left|\frac{3 + x}{1 - x}\right| + c.$$

(iii) ∫dx9x2−25

Solution:

$$\int \frac{dx}{9x^2-25} = \frac{1}{9}\int \frac{1}{x^2-\frac{25}{9}}\,dx$$

$$= \frac{1}{9}\int \frac{1}{x^2-\left(\frac{5}{3}\right)^2}\,dx$$

$$= \frac{1}{9} \times \frac{1}{2\times\frac{5}{3}} \log\left|\frac{x-\frac{5}{3}}{x+\frac{5}{3}}\right| + c$$

$$= \frac{1}{30}\log\left|\frac{3x-5}{3x+5}\right| + c.$$

(iv) ∫exe2x+4ex+13√dx

Solution:

$$\text{Let } I = \int \frac{e^x}{\sqrt{e^{2x}+4e^x+13}}\,dx$$

Put $e^x dx \quad \therefore \; e^x dx = dt$

$$\therefore\; I = \int \frac{1}{\sqrt{t^2+4t+13}}\,dt$$

$$= \int \frac{1}{\sqrt{(t^2+4t+4)+9}}\,dt$$

$$= \int \frac{1}{\sqrt{(t+2)^2+(3)^2}}\,dt$$

$$= \log|(t+2)+\sqrt{(t+2)^2+(3)^2}| + c$$

$$= \log|(t+2)+\sqrt{t^2+4t+13}| + c$$

$$= \log|(e^x+2)+\sqrt{e^{2x}+4e^x+13}| + c.$$

(v) ∫dxx[(logx)2+4logx−1]

Solution:

$$\text{Let } I = \int \frac{dx}{x[(\log x)^2+4\log x-1]}$$

$$= \int \frac{1}{(\log x)^2+4\log x-1}\cdot\frac{1}{x}\,dx$$

Put $\log x = t \quad \therefore\; \frac{1}{x}dx = dt$

$$\therefore\; I = \int \frac{1}{t^2+4t-1}\,dt$$

$$= \int \frac{1}{(t^2+4t+4)-5}\,dt$$

$$= \int \frac{1}{(t+2)^2-(\sqrt{5})^2}\,dt$$

$$= \frac{1}{2\sqrt{5}}\log\left|\frac{t+2-\sqrt{5}}{t+2+\sqrt{5}}\right| + c$$

$$= \frac{1}{2\sqrt{5}}\log\left|\frac{\log x+2-\sqrt{5}}{\log x+2+\sqrt{5}}\right| + c.$$

(vi) ∫dx5−16x2

Solution:

$$\int \frac{dx}{5-16x^2} = \frac{1}{16}\int \frac{dx}{\frac{5}{16}-x^2}\,dx$$

$$= \frac{1}{16}\int \frac{1}{\left(\frac{\sqrt{5}}{4}\right)^2 - x^2}\,dx$$

$$= \frac{1}{16} \times \frac{1}{2 \times \frac{\sqrt{5}}{4}} \log\left|\frac{\frac{\sqrt{5}}{4}+x}{\frac{\sqrt{5}}{4}-x}\right| + c$$

$$= \frac{1}{8\sqrt{5}} \log\left|\frac{\sqrt{5}+4x}{\sqrt{5}-4x}\right| + c.$$

(vii) ∫dx25x−x(logx)2

Solution:

$$\text{Let } I = \int \frac{dx}{25x - x(\log\ x)^2}$$

$$= \int \frac{1}{25-(\log\ x)^2} \cdot \frac{1}{x}\,dx$$

$$\text{Put } \log x = t \qquad \therefore\ \frac{1}{x}\,dx = dt$$

$$\therefore\ I = \int \frac{1}{25-t^2}\,dt$$

$$= \frac{1}{2\times 5} \log\left|\frac{5+t}{5-t}\right| + c$$

$$= \frac{1}{10} \log\left|\frac{5+\log\ x}{5-\log\ x}\right| + c.$$

(viii) ∫ex4e2x−1dx

Solution:

$$\text{Let } I = \int \frac{e^x}{4e^{2x}-1}\,dx$$

$$\text{Put } e^x = t \qquad \therefore\ e^x dx = dt$$

$$\therefore\ I = \int \frac{1}{4t^2-1}\,dt = \frac{1}{4}\int \frac{1}{t^2 - \frac{1}{4}}\,dt$$

$$= \frac{1}{4}\int \frac{1}{t^2 - \left(\frac{1}{2}\right)^2}\,dt$$

$$= \frac{1}{4} \times \frac{1}{2\times\frac{1}{2}} \log\left|\frac{t-\frac{1}{2}}{t+\frac{1}{2}}\right| + c$$

$$= \frac{1}{4} \log\left|\frac{2t-1}{2t+1}\right| + c$$

$$= \frac{1}{4} \log\left|\frac{2e^x-1}{2e^x+1}\right| + c.$$

4. Evaluate:

(i) ∫(log x)2 dx

Solution:

$$\int (\log x)^2 dx = \int (\log x)^2 \cdot 1\, dx$$

$$= (\log x)^2 \int 1 dx - \int \left[\frac{d}{dx}(\log x)^2 \cdot \int 1\, dx\right] dx$$

$$= (\log x)^2 \cdot x - \int \left[2 \log x \cdot \frac{d}{dx}(\log x) \times x\right] dx$$

$$= x(\log x)^2 - \int 2 \log x \times \frac{1}{x} \times x\, dx$$

$$= x(\log x)^2 - 2\int (\log x) \cdot 1\, dx$$

$$= x(\log x)^2 - 2\left\{(\log x) \int 1\, dx - \left[\frac{d}{dx}(\log x) \int 1\, dx\right] dx\right\}$$

$$= x(\log x)^2 - 2\left\{(\log x) \cdot x - \int \frac{1}{x} \times x\, dx\right\}$$

$$= x(\log x)^2 - 2x \log x + 2\int 1\, dx$$

$$= x(\log x)^2 - 2x \log x + 2x + c.$$

(ii) ∫ex1+x(2+x)2dx

Solution:

$$\text{Let } I = \int e^x \frac{1+x}{(2+x)^2}\, dx$$

$$= \int e^x \left[\frac{(2+x)-1}{(2+x)^2}\right] dx$$

$$= \int e^x \left[\frac{1}{2+x} - \frac{1}{(2+x)^2}\right] dx$$

$$\text{Let } f(x) = \frac{1}{2+x}$$

$$\therefore f'(x) = \frac{-1}{(2+x)^2}$$

$$\therefore I = \int e^x [f(x) + f\prime(x)]\, dx$$

$$= e^x \cdot f(x) + c$$

$$= e^x \cdot \frac{1}{2+x} + c$$

$$\therefore I = \frac{e^x}{2+x} + c$$

(iii) ∫x e2x dx

Solution:

$$\int xe^{2x}dx = x\int e^{2x}dx - \int\left[\frac{d}{dx}(x)\int e^{2x}dx\right]dx$$

$$= x\cdot\frac{e^{2x}}{2} - \int 1\cdot\frac{e^{2x}}{2}\,dx$$

$$= \frac{1}{2}xe^{2x} = \frac{1}{2}\int e^{2x}\,dx$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{2}\cdot\frac{e^{2x}}{2} + c$$

$$= e^{2x}\left(\frac{x}{2} - \frac{1}{4}\right) + c$$

$$= \left(\frac{2x-1}{4}\right)e^{2x} + c.$$

(iv) ∫log(x2 + x) dx

Solution:

$$\text{Let } I = \int \log(x^2+x)dx = \int[\log(x^2+x)]\cdot 1\,dx$$

$$= [\log(x^2+x)]\int 1\,dx - \int\left[\frac{d}{dx}\{\log(x^2+x)\}\cdot\int dx\right]dx$$

$$= [\log\,(x^2+x)]\cdot x - \int\frac{1}{x^2+x}\cdot\frac{d}{dx}(x^2+x)\times x\,dx$$

$$= x\,\log(x^2+x) - \int\frac{1}{x(x+1)}\cdot(2x+1)\cdot x\,dx$$

$$= x\,\log(x^2+x) - \int\frac{2x+1}{x+1}\,dx$$

$$= x\,\log(x^2+x) - \int\frac{2(x+1)-1}{x+1}\,dx$$

$$= x\,\log(x^2+x) - \int\left(2 - \frac{1}{x+1}\right)dx$$

$$= x\,\log(x^2+x) - 2\int 1\,dx + \int\frac{1}{x+1}\,dx$$

$$= x\,\log(x^2+x) - 2x + \log|x+1| + c.$$

(v) ∫ex√dx

Solution:

$$\text{Let } I = \int e^{\sqrt{x}}dx$$

Put $\sqrt{x} = t \qquad \therefore\ x = t^2$

$\therefore\ dx = 2t\,dt$

$\therefore\ I = \int e^t\cdot 2t\,dt = 2\int te^t dt$

$$= 2[t\int e^t dt - \int\left\{\frac{d}{dt}(t)\int e^t dt\right\}]dt$$

$$= 2[t\cdot e^t - \int 1\cdot e^t dt]$$

$$= 2[t\cdot e^t - e^t] + c$$

$$= 2(t-1)e^t + c$$

$$= 2(\sqrt{x}-1)\,e^{\sqrt{x}} + c.$$

(vi) $\int \sqrt{x^2+2x+5}\, dx$

Solution:

$$\int \sqrt{x^2+2x+5}\, dx$$
$$= \int \sqrt{(x^2+2x+1)+4}\, dx$$
$$= \int \sqrt{(x+1)^2+(2)^2}\, dx$$
$$= \frac{(x+1)}{2}\sqrt{(x+1)^2+(2)^2} + \frac{(2)^2}{2}\log|(x+1)+\sqrt{(x+1)^2+(2)^2}| + c$$
$$= \frac{(x+1)}{2}\sqrt{x^2+2x+5} + 2\log|(x+1)+\sqrt{x^2+2x+5}| + c.$$

(vii) $\int \sqrt{x^2-8x+7}\, dx$

Solution:

$$\int \sqrt{x^2-8x+7}\, dx$$
$$= \int \sqrt{(x^2-8x+16)-9}\, dx$$
$$= \int \sqrt{(x-4)^2-(3)^2}\, dx$$
$$= \frac{(x-4)}{2}\sqrt{(x-4)^2-(3)^2} - \frac{(3)^2}{2}\log|(x-4)+\sqrt{(x-4)^2-(3)^2}| + c$$
$$= \frac{(x-4)}{2}\sqrt{x^2-8x+7} - \frac{9}{2}\log|(x-4)+\sqrt{x^2-8x+7}| + c.$$

5. Evaluate:

(i) $\int \frac{3x-1}{2x^2-x-1}\, dx$

Solution:

$$\text{Let } I = \int \frac{3x-1}{2x^2-x-1}\,dx$$
$$= \int \frac{3x-1}{(x-1)(2x+1)}\,dx$$

Let $\frac{3x-1}{(x-1)(2x+1)} = \frac{A}{x-1} + \frac{B}{2x+1}$

$\therefore\ 3x-1 = A(2x+1) + B(x-1)$

Put $x-1=0$, i.e. $x=1$, we get

$3(1)-1 = A(3)+B(0) \quad \therefore\ 2 = 3A \quad \therefore\ A = \frac{2}{3}$

Put $2x+1=0$, i.e. $x = -\frac{1}{2}$, we get

$3\left(-\frac{1}{2}\right) - 1 = A(0) + B\left(-\frac{3}{2}\right)$

$\therefore\ -\frac{5}{2} = -\frac{3}{2}B \quad \therefore\ B = \frac{5}{3}$

$$\therefore\ \frac{3x-1}{(x-1)(2x+1)} = \frac{\left(\frac{2}{3}\right)}{x-1} + \frac{\left(\frac{5}{3}\right)}{2x+1}$$

$$\therefore\ I = \int \left[\frac{\left(\frac{2}{3}\right)}{x-1} + \frac{\left(\frac{5}{3}\right)}{2x+1}\right] dx$$
$$= \frac{2}{3}\int \frac{1}{x-1}\,dx + \frac{5}{3}\int \frac{1}{2x+1}\,dx$$
$$= \frac{2}{3}\log|x-1| + \frac{5}{3}\cdot\frac{\log|2x+1|}{2} + c$$
$$= \frac{2}{3}\log|x-1| + \frac{5}{6}\log|2x+1| + c.$$

(ii) $\int \frac{2x^3-3x^2-9x+1}{2x^2-x-10}dx$

Solution:

$$\text{Let } I = \int \frac{2x^3-3x^2-9x+1}{2x^2-x-10}\,dx$$

$$\begin{array}{r|l} 2x^2-x-10 & 2x^3-3x^2-9x+1 \quad (x-1 \\ & \underline{\begin{array}{l}2x^3-x^2-10x \\ -\quad +\quad +\end{array}} \\ & \quad -2x^2+x+1 \\ & \quad \underline{\begin{array}{l}-2x^2+x+10 \\ +\quad -\quad -\end{array}} \\ & \qquad\qquad -9 \end{array}$$

$\therefore\ 2x^3-3x^2-9x+1 = (x-1)(2x^2-x-10) - 9$

$$\therefore\ I=\int\left[\frac{(x-1)(2x^2-x-10)-9}{2x^2-x-10}\right]dx$$

$$=\int\left[(x-1)-\frac{9}{2x^2-x-10}\right]dx$$

$$=\int(x-1)\,dx-\frac{9}{2}\int\frac{1}{x^2-\frac{1}{2}x-5}\,dx$$

$$=\int xdx-\int 1\,dx-\frac{9}{2}\int\frac{1}{\left(x^2-\frac{1}{2}x+\frac{1}{16}\right)-\frac{1}{16}-5}\,dx$$

$$=\int xdx-\int 1\,dx-\frac{9}{2}\int\frac{1}{\left(x-\frac{1}{4}\right)^2-\left(\frac{9}{4}\right)^2}\,dx$$

$$=\frac{x^2}{2}-x-\frac{9}{2}\times\frac{1}{2\times\frac{9}{4}}\log\left|\frac{x-\frac{1}{4}-\frac{9}{4}}{x-\frac{1}{4}+\frac{9}{4}}\right|+c_1$$

$$=\frac{x^2}{2}-x-\log\left|\frac{x-\frac{5}{2}}{x+2}\right|+c_1$$

$$=\frac{x^2}{2}-x-\log\left|\frac{2x-5}{2(x+2)}\right|+c_1$$

$$=\frac{x^2}{2}-x+\log\left|\frac{2(x+2)}{2x-5}\right|+c_1$$

$$=\frac{x^2}{2}-x+\log\left|\frac{x+2}{2x-5}\right|+\log 2+c_1$$

$$=\frac{x^2}{2}-x+\log\left|\frac{x+2}{2x-5}\right|+c,\text{ where } c_1=\log 2+c_1$$

(iii) $\int\frac{1+\log x}{x(3+\log x)(2+3\log x)}dx$

Solution:

$$\text{Let } I=\int\frac{1+\log x}{x(3+\log x)(2+3\log x)}\,dx$$

$$=\int\frac{1+\log x}{(3+\log x)(2+3\log x)}\cdot\frac{1}{x}\,dx$$

Put $\log x=t \qquad \therefore\ \frac{1}{x}dx=dt$

$$\therefore\ I=\int\frac{1+t}{(3+t)(2+3t)}\,dt$$

$$\text{Let } \frac{1+t}{(3+t)(2+3t)}=\frac{A}{3+t}+\frac{B}{2+3t}$$

$$\therefore\ 1+t=A(2+3t)+B(3+t)$$

Put $3+t=0$, i.e. $t=-3$, we get

$$1-3=A(-7)+B(0)$$

$$\therefore\ -2=-7A \qquad \therefore\ A=\frac{2}{7}$$

Put $2+3t=0$, i.e. $t=-\frac{2}{3}$, we get

$$1-\frac{2}{3}=A(0)+B\left(\frac{7}{3}\right)$$

$$\therefore\ \frac{1}{3}=\frac{7}{3}B \qquad \therefore\ B=\frac{1}{7}$$

$$\therefore\ \frac{1+t}{(3+t)(2+3t)}=\frac{\left(\frac{2}{7}\right)}{3+t}+\frac{\left(\frac{1}{7}\right)}{2+3t}$$

$$\therefore\ I=\int\left[\frac{\left(\frac{2}{7}\right)}{3+t}+\frac{\left(\frac{1}{7}\right)}{2+3t}\right]dt$$

$$=\frac{2}{7}\int\frac{1}{3+t}\,dt+\frac{1}{7}\int\frac{1}{2+3t}\,dt$$

$$=\frac{2}{7}\log|3+t|+\frac{1}{7}\cdot\frac{\log|2+3t|}{3}+c$$

$$=\frac{2}{7}\log|3+\log x|+\frac{1}{21}\log|2+3\log x|+c.$$