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Maharashtra State Board 12th Commerce Maths Solutions Chapter 6 Definite Integration Ex 6.1

Evaluate the following definite integrals:

Question 1.
∫941x√dx
Solution:

$$\int_4^9 \frac{1}{\sqrt{x}}\,dx = \int_4^9 x^{-\frac{1}{2}}\,dx$$

$$= \left[\frac{x^{\frac{1}{2}}}{1/2}\right]_4^9 = 2\left[\sqrt{x}\right]_4^9$$

$$= 2(\sqrt{9} - \sqrt{4})$$

$$= 2(3 - 2) = 2.$$

Question 2.
∫3−21x+5dx
Solution:

$$\int_{-2}^{3} \frac{1}{x+5}\,dx$$

$$= \left[\log|x+5|\right]_{-2}^{3}$$

$$= \log 8 - \log 3$$

$$= \log\left(\frac{8}{3}\right)$$

Question 3.
∫32xx2−1dx
Solution:

$$\int_2^3 \frac{x}{x^2-1}\,dx$$

$$= \frac{1}{2}\int_2^3 \frac{2x}{x^2-1}\,dx$$

$$= \frac{1}{2}\left[\log|x^2-1|\right]_2^3 \qquad \ldots \left[\because \frac{d}{dx}(x^2-1) = 2x \text{ and } \int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c\right]$$

v

$$= \frac{1}{2}\left[\log(9-1) - \log(4-1)\right] = \frac{1}{2}\log\left(\frac{8}{3}\right).$$

Question 4.
∫10x2+3x+2x√dx

Solution:

$$\int_0^1 \frac{x^2+3x+2}{\sqrt{x}}\,dx$$

$$= \int_0^1 \left(\frac{x^2}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{2}{\sqrt{x}}\right) dx$$

$$= \int_0^1 \left(x^{\frac{3}{2}} + 3x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}\right) dx$$

$$= \left[\frac{x^{\frac{5}{2}}}{5/2} + 3\left(\frac{x^{\frac{3}{2}}}{3/2}\right) + 2\left(\frac{x^{\frac{1}{2}}}{1/2}\right)\right]_0^1$$

$$= \left[\frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 4x^{\frac{1}{2}}\right]_0^1$$

$$= \left[\frac{2}{5}(1)^{\frac{5}{2}} + 2(1)^{\frac{3}{2}} + 4(1)^{\frac{1}{2}}\right] - (0+0+0)$$

$$= \frac{2}{5} + 2 + 4 = \frac{32}{5}.$$

Question 5.

$\int_2^3 \frac{x}{(x+2)(x+3)} dx$

Solution:

Let I = $\int_2^3 \frac{x}{(x+2)(x+3)} dx$

Let $\frac{x}{(x+2)(x+3)} = \frac{A}{x+3} + \frac{B}{x+2}$

∴ x = A(x + 2) + B(x + 3)

Put x + 3 = 0, i.e. x = -3, we get

-3 = A(-1) + B(0)

∴ A = 3

Put x + 2 = 0, i.e. x = -2, we get

-2 = A(0) + B(1)

∴ B = -2

$$\therefore \frac{x}{(x+2)(x+3)} = \frac{3}{x+3} + \frac{(-2)}{x+2}$$

$$\therefore I = \int_2^3 \left[\frac{3}{x+3} + \frac{(-2)}{x+2}\right] dx$$

$$= \left[3\log(x+3) - 2\log(x+2)\right]_2^3$$

$$= [3\log(3+3) - 2\log(3+2)] - [3\log(2+3) - 2\log(2+2)]$$

$$= 3\log 6 - 5\log 5 + 2\log 4$$

$$= \log 6^3 - \log 5^5 + \log 4^2$$

$$= \log 216 - \log 3125 + \log 16$$

$$= \log\left(\frac{216 \times 16}{3125}\right) = \log\left(\frac{3456}{3125}\right).$$

Question 6.

$\int_1^2 \frac{dx}{x^2+6x+5}$

Solution:

$$\int_1^2 \frac{dx}{x^2+6x+5}$$

$$= \int_1^2 \frac{dx}{(x^2+6x+9)-4}$$

$$= \int_1^2 \frac{1}{(x+3)^2-(2)^2}\,dx$$

$$= \frac{1}{2(2)}\left[\log\left|\frac{x+3-2}{x+3+2}\right|\right]_1^2 = \frac{1}{4}\left[\log\left|\frac{x+1}{x+5}\right|\right]_1^2$$

$$= \frac{1}{4}\left[\log\frac{3}{7} - \log\frac{2}{6}\right]$$

$$= \frac{1}{4}\log\left(\frac{3}{7}\times\frac{6}{2}\right) = \frac{1}{4}\log\left(\frac{9}{7}\right).$$

Question 7.

If $\int_0^a (2x+1)dx$ = 2, find the real values of 'a'.

Solution:

Let I = $\int_0^a (2x+1)dx$

= $\left[2\cdot\frac{x^2}{2}+x\right]_0^a$

= $a^2 + a - 0$

= $a^2 + a$

∴ I = 2 gives $a^2 + a = 2$

∴ $a^2 + a - 2 = 0$

∴ $(a + 2)(a - 1) = 0$ 1

∴ $a + 2 = 0$ or $a - 1 = 0$

∴ a = -2 or a = 1.

Question 8.

If $\int_1^a (3x^2+2x+1)dx$ = 11, find 'a'.

Solution:

Let I = $\int_1^a (3x^2+2x+1)dx$

= $\left[3\left(\frac{x^3}{3}\right)+2\left(\frac{x^2}{2}\right)+x\right]_1^a$

= $\left[x^3+x^2+x\right]_1^a$

= $(a^3 + a^2 + a) - (1 + 1 + 1)$

= $a^3 + a^2 + a - 3$

∴ I = 11 gives $a^3 + a^2 + a - 3 = 11$

∴ $a^3 + a^2 + a - 14 = 0$

∴ $(a^3 - 8) + (a^2 + a - 6) = 0$

∴ $(a - 2)(a^2 + 2a + 4) + (a + 3)(a - 2) = 0$

∴ $(a - 2)(a^2 + 2a + 4 + a + 3) = 0$

∴ $(a - 2)(a^2 + 3a + 7) = 0$

∴ $a - 2 = 0$ or $a^2 + 3a + 7 = 0$

∴ a = 2 or a = $\frac{-3\pm\sqrt{9-28}}{2}$

The latter two roots are not real.

∴ they are rejected.

∴ a = 2.

Question 9.

$\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}}dx$

Solution:

$$\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}}\,dx$$

$$= \int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}} \times \frac{\sqrt{1+x}-\sqrt{x}}{\sqrt{1-x}-\sqrt{x}}\,dx$$

$$= \int_0^1 \frac{\sqrt{1+x}-\sqrt{x}}{1+x-x}\,dx$$

$$= \int_0^1 (\sqrt{1+x}-\sqrt{x})\,dx$$

$$= \int_0^1 (1+x)^{\frac{1}{2}}dx - \int_0^1 x^{\frac{1}{2}}dx$$

$$= \left[\frac{(1+x)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_0^1 - \left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_0^1$$

$$= \frac{2}{3}\left[(1+x)^{\frac{3}{2}}\right]_0^1 - \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{3}(2^{\frac{3}{2}}-1) - \frac{2}{3}(1-0)$$

$$= \frac{2}{3}(2^{\frac{3}{2}}-1-1) = \frac{2}{3}(2\sqrt{2}-2)$$

$$= \frac{4}{3}(\sqrt{2}-1).$$

Question 10.

$\int_1^2 \frac{1}{3x\sqrt{9x^2-1}}dx$

Solution:

Let I = $\int_1^2 \frac{1}{3x\sqrt{9x^2-1}}dx = \int_1^2 \frac{1}{3x\sqrt{(3x)^2-1}}dx$

Put 3x = t

∴ 3 dx = dt

∴ dx = $\frac{dt}{3}$

When x = 1, t = 3 × 1 = 3

When x = 2, t = 3 × 2 = 6

$$\therefore\ I=\int_3^6 \frac{t}{t^2-1}\cdot\frac{dt}{3}=\frac{1}{6}\int_3^6 \frac{2t}{t^2-1}\,dt$$

$$=\frac{1}{6}\Big[\log|t^2-1|\Big]_3^6 \qquad \ldots\left[\because \frac{d}{dt}(t^2-1)=2t\right]$$

$$=\frac{1}{6}[\log 35-\log 8]$$

$$=\frac{1}{6}\log\left(\frac{35}{8}\right).$$

Alternative Method :

$$\int_1^2 \frac{3x}{9x^2-1}\,dx$$

$$=\frac{1}{6}\int_1^2 \frac{18x}{9x^2-1}\,dx$$

$$=\frac{1}{6}\Big[\log|9x^2-1|\Big]_1^2$$

$$\ldots\left[\because \frac{d}{dx}(9x^2-1)=18x \text{ and } \int\frac{f'(x)}{f(x)}\,dx=\log|f(x)|\right]$$

$$=\frac{1}{6}[\log 35-\log 8]$$

$$=\frac{1}{6}\log\left(\frac{35}{8}\right).$$

Question 11.

∫31logxdx

Solution:

$$\int_1^3 \log x\,dx=\int_1^3(\log x)\cdot 1\,dx$$

$$=[(\log x)]\int 1dx]_1^3-\int_1^3\left[\frac{d}{dx}(\log x)\int 1\,dx\right]dx$$

$$=[(\log x)x]_1^3-\int_1^3\frac{1}{x}\times x\,dx$$

$$=(3\log 3-\log 1)-\int_1^3 1\,dx$$

$$=3\log 3-[x]_1^3 \qquad \ldots[\because \log 1=0]$$

$$=\log 3^3-(3-1)$$

$$=\log 27-2.$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 6 Definite Integration Ex 6.2

Evaluate the following integrals:

Question 1.
$\int_{-9}^{9} \frac{x^3}{4-x^2} dx$
Solution:

$$\text{Let } I = \int_{-9}^{9} \frac{x^3}{4-x^2}\, dx$$

$$\text{Let } f(x) = \frac{x^3}{4-x^2}$$

$$\therefore\ f(-x) = \frac{(-x)^3}{4-(-x)^2} = \frac{-x^3}{4+x^2} = -f(x)$$

$\therefore$ f is an odd function.

$$\therefore\ \int_{-9}^{9} f(x)\, dx = 0$$

$$\text{i.e. } \int_{-9}^{9} \frac{x^3}{4-x^2}\, dx = 0.$$

Question 2.
$\int_0^a x^2(a-x)^{3/2} dx$
Solution:
We use the property

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$\therefore\ \int_0^a x^2(a-x)^{\frac{3}{2}}dx = \int_0^a (a-x)^2(a-a+x)^{\frac{3}{2}}dx$$

$$= \int_0^a (a^2-2ax+x^2)x^{\frac{3}{2}}\, dx$$

$$= \int_0^a (a^2x^{\frac{3}{2}} - 2ax^{\frac{5}{2}} + x^{\frac{7}{2}})\, dx$$

$$= a^2\int_0^a x^{\frac{3}{2}}\, dx - 2a\int_0^a x^{\frac{5}{2}}\, dx + \int_0^a x^{\frac{7}{2}}\, dx$$

$$= a^2\left[\frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}\right]_0^a - 2a\left[\frac{x^{\frac{7}{2}}}{\left(\frac{7}{2}\right)}\right]_0^a + \left[\frac{x^{\frac{9}{2}}}{\left(\frac{9}{2}\right)}\right]_0^a$$

$$= \frac{2a^2}{5}[a^{\frac{5}{2}}-0] - \frac{4a}{7}[a^{\frac{7}{2}}-0] + \frac{2}{9}[a^{\frac{9}{2}}-0]$$

$$= \frac{2}{5}a^{\frac{9}{2}} - \frac{4}{7}a^{\frac{9}{2}} + \frac{2}{9}a^{\frac{9}{2}} = \left(\frac{2}{5}-\frac{4}{7}+\frac{2}{9}\right)a^{\frac{9}{2}}$$

$$= \left(\frac{126-180+70}{315}\right)a^{\frac{9}{2}} = \frac{16}{315}a^{\frac{9}{2}}.$$

Question 3.
$\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}} dx$

Solution:

$$\text{Let } I=\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx \quad \ldots (1)$$

We use the property, $\int_a^b f(x)dx=\int_a^b f(a+b-x)\,dx$

Hence in I, we replace x by $1+3-x$.

$$\therefore\ I=\int_1^3 \frac{\sqrt[3]{1+3-x+5}}{\sqrt[3]{1+3-x+5}+\sqrt[3]{9-1-3+x}}\,dx$$

$$=\int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\,dx \quad \ldots (2)$$

Adding (1) and (2), we get

$$2I=\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx+\int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\,dx$$

$$=\int_1^3 \frac{\sqrt[3]{x+5}+\sqrt[3]{9-x}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx$$

$$=\int_1^3 1dx=[x]_1^3$$

$$=3-1=2$$

$$\therefore\ I=1$$

Hence, $\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx=1.$

Question 4.

∫52x√x√+7−x√dx

Solution:

$$\text{Let } I=\int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}}\,dx \quad \ldots (1)$$

We use the property, $\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx.$

Hence in I, we change x by $2+5-x$.

$$\therefore\ I=\int_2^5 \frac{\sqrt{2+5-x}}{\sqrt{2+5-x}+\sqrt{7-2-5+x}}\,dx$$

$$=\int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-x}+\sqrt{x}}\,dx \quad \ldots (2)$$

Adding (1) and (2), we get

$$2I=\int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx+\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx$$

$$=\int_2^5 \frac{\sqrt{x}+\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx$$

$$=\int_2^5 1\,dx=[x]_2^5=5-2=3$$

$$\therefore\ I=\frac{3}{2}$$

Hence, $\int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}}\,dx=\frac{3}{2}.$

Question 5.

∫21x√3−x√+x√dx

Solution:

Let I = $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} \cdot dx$ ...(i)

= $\int_1^2 \frac{\sqrt{1+2-x}}{\sqrt{3-(1+2-x)}+\sqrt{1+2-x}} \cdot dx$ ...$\left[\because \int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx\right]$

$\therefore$ I = $\int_1^2 \frac{\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}} \cdot dx$ ...(ii)

Adding (i) and (ii), we get

2I = $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} \cdot dx + \int_1^2 \frac{\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}} \cdot dx$

= $\int_1^2 \frac{\sqrt{x}+\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}} \cdot dx$

= $\int_1^2 1 \cdot dx$

= $[x]_1^2$

$\therefore$ 2I = 2 – 1 = 1

$\therefore$ I = $\frac{1}{2}$.

Question 6.

∫72x√x√+9−x√dx

Solution:

Let I = $\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} \cdot dx$ ...(i)

= $\int_2^7 \frac{\sqrt{2+7-x}}{\sqrt{2+7-x}+\sqrt{9-(2+7-x)}} \cdot dx$ ...$\left[\because \int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx\right]$

$\therefore$ I = $\int_2^7 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}} \cdot dx$ ...(ii)

Adding (i) and (ii), we get

2I = $\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} \cdot dx + \int_2^7 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}} \cdot dx$

= $\int_2^7 \frac{\sqrt{x}+\sqrt{9-x}}{\sqrt{x}+\sqrt{9-x}} \cdot dx$

= $\int_2^7 1 \cdot dx$

= $[x]_2^7$

$\therefore$ 2I = 7 – 2 = 5

$\therefore$ I = $\frac{5}{2}$.

Question 7.

∫10log(1x−1)dx

Solution:

$$\text{Let } I = \int_0^1 \log\left(\frac{1}{x} - 1\right) dx = \int_0^1 \log\left(\frac{1-x}{x}\right) dx$$

We use the property, $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$\therefore\ I = \int_0^1 \log\left[\frac{1-(1-x)}{1-x}\right] dx = \int_0^1 \log\left(\frac{x}{1-x}\right) dx$$

$$= \int_0^1 -\log\left(\frac{1-x}{x}\right) dx = -\int_0^1 \log\left(\frac{1-x}{x}\right) dx$$

$\therefore\ I = -I$

$\therefore\ 2I = 0 \qquad \therefore\ I = 0$

Hence, $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx = 0.$

Question 8.

$\int_0^1 x(1-x)^5 dx$

Solution:

We use the property

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx.$$

$$\therefore\ \int_0^1 x(1-x)^5 dx = \int_0^1 (1-x)(1-1+x)^5\,dx$$

$$= \int_0^1 (1-x)x^5 dx = \int_0^1 (x^5 - x^6)\,dx$$

$$= \int_0^1 x^5 dx - \int_0^1 x^6 dx$$

$$= \left[\frac{x^6}{6}\right]_0^1 - \left[\frac{x^7}{7}\right]_0^1$$

$$= \frac{1}{6}(1-0) - \frac{1}{7}(1-0)$$

$$= \frac{1}{6} - \frac{1}{7} = \frac{1}{42}.$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 6 Definite Integration Miscellaneous Exercise 6

(I) Choose the correct alternative:

Question 1.

$\int_{-9}^{9} \frac{x^3}{4-x^2} dx$ = _______

(a) 0
(b) 3
(c) 9
(d) -9

Answer:

(a) 0

Question 2.
∫3−2dxx+5 = ________
(a) -log(83)
(b) log(83)
(c) log(38)
(d) -log(38)
Answer:
(b) log(83)

Question 3.
∫32xx2−1dx = ________
(a) log(83)
(b) -log(83)
(c) 12 log(83)
(d) −12 log(83)
Answer:
(c) 12 log(83)

Question 4.
∫94dxx√ = __________
(a) 9
(b) 4
(c) 2
(d) 0
Answer:
(c) 2

Question 5.
If ∫a03x2dx=8, then a = _________
(a) 2
(b) 0
(c) 83
(d) a
Answer:
(a) 2

Question 6.
∫32x4 dx = _______
(a) 12
(b) 52
(c) 5211
(d) 2115
Answer:
(d) 2115

Question 7.
∫20ex dx = ______
(a) e – 1
(b) 1 – e
(c) 1 – e2
(d) e2 – 1
Answer:
(d) e2 – 1

Question 8.
∫baf(x)dx = _______
(a) ∫abf(x)dx
(b) −∫baf(x)dx
(c) −∫abf(x)dx
(d) ∫a0f(x)dx

Answer:
(c) $-\int ab f(x) dx$

Question 9.
$\int 7-7 x3x2+7 dx$ = _______
(a) 7
(b) 49
(c) 0
(d) 72
Answer:
(c) 0

Question 10.
$\int 72 x\sqrt{}x\sqrt{}+9-x\sqrt{} dx$ = _______
(a) 72
(b) 52
(c) 7
(d) 2
Answer:
(b) 52

(II) Fill in the blanks:

Question 1.
$\int 20 ex dx$ = ______
Answer:
e2 – 1

Question 2.
$\int 32 x4 dx$ = ________
Answer:
2115

Question 3.
$\int 10 dx2x+5$ = __________
Answer:
12log(75)

Question 4.
If $\int a0 3x2 dx$ = 8, then a = _______
Answer:
2

Question 5.
$\int 94 1x\sqrt{} dx$ = _______
Answer:
2

Question 6.
$\int 32 xx2-1 dx$ = _______
Answer:
12log(83)

Question 7.
$\int 3-2 dxx+5$ = _______
Answer:
log(83)

Question 8.
$\int 9-9x3 4-x2 dx$ = __________
Answer:
0

(III) State whether each of the following is True or False:

Question 1.
$\int ba f(x)dx=\int -a-b f(x)dx$
Answer:
True

Question 2.
$\int ba f(x)dx=\int ba f(t)dt$
Answer:
True

Question 3.
$\int a0 f(x)dx=\int 0a f(a-x)dx$
Answer:
False

Question 4.
$\int ba f(x)dx=\int ba f(x-a-b)dx$
Answer:
False

Question 5.
$\int 5-5 x3 x2+7 dx=0$
Answer:
True

Question 6.
$\int 21 x\sqrt{}3-x\sqrt{}+x\sqrt{} dx=12$
Answer:
True

Question 7.
$\int 72 x\sqrt{}x\sqrt{}+9-x\sqrt{} dx=92$
Answer:
False

Question 8.
$\int 74 (11-x)2 (11-x)2+x2 dx=32$
Answer:
True

(IV) Solve the following:

Question 1.
$\int 32 x(x+2)(x+3) dx$
Solution:

Let I = $\int_2^3 \frac{x}{(x+2)(x+3)} \cdot dx$

Let $\frac{x}{(x+2)x+3} = \frac{A}{x+2} + \frac{B}{x+3}$ ...(i)

∴ x = A(x + 3) + B(x + 2) ...(ii)

Putting x = – 3 in (ii) we get

– 2 = A

∴ B = 3

Putting x = – 2 in (ii),we get

– 2 = A

∴ A = – 2

From (i), we get

$\frac{x}{(x+2(x+3))} = \frac{-2}{x+2} + \frac{3}{x+3}$

$\therefore \mathrm{I} = \int_2^3 \left[\frac{-2}{x+2} + \frac{3}{x+3}\right] \cdot dx$

$= -2\int_2^3 \frac{1}{x+2} \cdot dx + 3\int_2^3 \frac{1}{x+3} \cdot dx$

$= -2[\log|x+2|]_2^3 + 3[\log|x+3|]_2^3$

$= -2\log[\log 5 - \log 4] + 3[\log 6 - \log 5]$

$= -2\left[\log\left(\frac{5}{4}\right)\right] + 3\left[\log\left(\frac{6}{5}\right)\right]$

$= 3\log\left(\frac{6}{5}\right) - 2\log\left(\frac{5}{4}\right)$

$= \log\left(\frac{6}{5}\right)^2 - 2\log\left(\frac{5}{4}\right)^2$

$= \log\left(\frac{216}{125}\right) - \log\left(\frac{25}{16}\right)$

$= \log\left(\frac{216}{125} \times \frac{16}{25}\right)$

$\therefore \mathrm{I} = \log\left(\frac{3456}{3125}\right)$.

Question 2.

∫21x+3x(x+2)dx

Solution:

Let I = ∫21x+3x(x+2)dx

Let x+3x(x+2)=Ax+Bx+2

∴ x + 3 = A(x + 2) + Bx

Put x = 0, we get

3 = A(2) + B(0)

∴ A = 32

Put x + 2 = 0, i.e. x = -2, we get

-2 + 3 = A(0) + B(-2)

∴ 1 = -2B

∴ B = $-\frac{1}{2}$

$$\therefore \frac{x+3}{x(x+2)} = \frac{\left(\frac{3}{2}\right)}{x} + \frac{\left(-\frac{1}{2}\right)}{x+2}$$

$$\therefore I = \int_1^2 \left[\frac{\left(\frac{3}{2}\right)}{x} + \frac{\left(-\frac{1}{2}\right)}{x+2}\right] dx$$

$$= \frac{3}{2}\int_1^2 \frac{1}{x}\, dx - \frac{1}{2}\int_1^2 \frac{1}{x+2}\, dx$$

$$= \frac{3}{2}[\log|x|]_1^2 - \frac{1}{2}[\log|x+2|]_1^2$$

$$= \frac{3}{2}(\log 2 - \log 1) - \frac{1}{2}(\log 4 - \log 3)$$

$$= \frac{3}{2}\log 2 - \frac{1}{2}\log 4 + \frac{1}{2}\log 3 \qquad \ldots [\because \log 1 = 0]$$

$$= \frac{1}{2}(3\log 2 - \log 4 + \log 3)$$

$$= \frac{1}{2}(\log 8 - \log 4 + \log 3)$$

$$= \frac{1}{2}\log\left(\frac{8\times 3}{4}\right) = \frac{1}{2}\log 6.$$

Question 3.
$\int_1^3 x^2 \log x\, dx$
Solution:

$$\int_1^3 x^2 \log x\, dx = \int_1^3 (\log x)\cdot x^2\, dx$$

$$= [(\log x)\int x^2 dx]_1^3 - \int_1^3 \left[\frac{d}{dx}(\log x)\int x^2 dx\right] dx$$

$$= \left[(\log x)\left(\frac{x^3}{3}\right)\right]_1^3 - \int_1^3 \frac{1}{x}\times\frac{x^3}{3}\, dx$$

$$= \frac{1}{3}[x^3 \log x]_1^3 - \frac{1}{3}\int_1^3 x^2 dx$$

$$= \frac{1}{3}[27\log 3 - 0] - \frac{1}{3}\left[\frac{x^3}{3}\right]_1^3 \qquad \ldots [\because \log 1 = 0]$$

$$= 9\log 3 - \frac{1}{9}(27-1)$$

$$= 9\log 3 - \frac{26}{9}.$$

Question 4.
$\int_0^1 e^{x^2} \cdot x^3 dx$
Solution:

$$\text{Let } I=\int_0^1 e^{x^2}\cdot x^3\,dx=\int_0^1 e^{x^2}\cdot x^2\cdot x\,dx$$

Put $x^2=t \qquad \therefore\ 2x\,dx=dt$

$\therefore\ x\,dx=\frac{dt}{2}$

When $x=0,\ t=0$

When $x=1,\ t=1$

$$\therefore\ I=\int_0^1 e^t\cdot t\cdot\frac{dt}{2}=\frac{1}{2}\int_0^1 te^t\,dt$$

$$=\frac{1}{2}\left\{[t\textstyle\int e^t\,dt]_0^1-\displaystyle\int_0^1\left[\frac{d}{dt}(t)\int e^t\,dt\right]dt\right\}$$

$$=\frac{1}{2}[te^t]_0^1-\frac{1}{2}\int_0^1 1\cdot e^t\,dt$$

$$=\frac{1}{2}(e-0)-\frac{1}{2}[e^t]_0^1$$

$$=\frac{e}{2}-\frac{1}{2}(e-1)$$

$$=\frac{e}{2}-\frac{e}{2}+\frac{1}{2}=\frac{1}{2}.$$

Question 5.
$\int_1^2 e^{2x}\left(\frac{1}{x}-\frac{1}{2x^2}\right)dx$
Solution:

$$\text{Let } I=\int_1^2 e^{2x}\left(\frac{1}{x}-\frac{1}{2x^2}\right)dx$$

Put $2x=t \qquad \therefore\ 2dx=dt$

$\therefore\ dx=\frac{dt}{2}$ and $x=\frac{t}{2}$

When $x=1,\ t=2$

When $x=2,\ t=4$

$$\therefore\ I=\int_2^4 e^t\left(\frac{2}{t}-\frac{2}{t^2}\right)\frac{dt}{2}=\frac{1}{2}\int_2^4 e^t\left(\frac{2}{t}-\frac{2}{t^2}\right)dt$$

Let $f(t)=\frac{2}{t}$

Then $f'(t)=2\left(-\frac{1}{t^2}\right)=\frac{-2}{t^2}$

$$\therefore\ I=\frac{1}{2}\int_2^4 e^t[f(t)+f'(t)]\,dt$$

$$=\frac{1}{2}[e^t\cdot f(t)]_2^4=\frac{1}{2}\left[e^t\cdot\frac{2}{t}\right]_2^4$$

$$=\frac{1}{2}\left[e^4\times\frac{2}{4}-e^2\times\frac{2}{2}\right]$$

$$=\frac{e^4}{4}-\frac{e^2}{2}.$$

Question 6.
$\int_4^9 \frac{1}{\sqrt{x}}\,dx$

Solution:

Let I = $\int_{4}^{9} \frac{1}{\sqrt{x}} \cdot dx$

= $\int_{4}^{9} x^{\frac{1}{2}} \cdot dx = \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_{4}^{9}$

= $2\left[\sqrt{x}\right]_{4}^{9}$

= $2\left(\sqrt{9}-\sqrt{4}\right)$

= 2 (3 – 2)

∴ I = 2.

Question 7.

∫3-21x+5dx

Solution:

Let I = $\int_{-2}^{3} \frac{1}{x+5} \cdot dx$

= $[\log|x+5|]_{-2}^{3}$

= [log |3 + 5| – log |–2 + 5|]

= log 8 – log 3

∴ I = $\log\left(\frac{8}{3}\right)$.

Question 8.

∫32xx2-1dx

Solution:

Let I = $\int_{2}^{3} \frac{x}{x^2-1} \cdot dx$

Put $x^2 - 1 = t$

∴ 2x·dx = dt

∴ x·dx = $\frac{1}{2} \cdot dt$

When x = 2, t = $2^2 - 1 = 3$

When x = 3, t = $3^2 - 1 = 8$

∴ I = $\int_{3}^{8} \frac{1}{\mathrm{t}} \cdot \frac{\mathrm{dt}}{2}$

= $\frac{1}{2} \int_{3}^{8} \frac{\mathrm{dt}}{\mathrm{t}}$

= $\frac{1}{2}[\log|\mathrm{t}|]_{3}^{8}$

= $\frac{1}{2}(\log 8 - \log 3)$

∴ I = $\frac{1}{2} \log\left(\frac{8}{3}\right)$.

Question 9.

∫10x2+3x+2x√dx

Solution:

Let I = $\int_0^1 \frac{x^2+3x+2}{\sqrt{x}} \cdot dx$

= $\int_0^1 \left(\frac{x^2+3x+2}{x^{\frac{1}{2}}}\right) \cdot dx$

= $\int_0^1 \left(\frac{x^2}{x^{\frac{1}{2}}} + \frac{3x}{x^{\frac{1}{2}}} + \frac{2}{x^{\frac{1}{2}}}\right) \cdot dx$

= $\int_0^1 \left(x^{\frac{1}{2}} + 3x^{\frac{1}{2}} + 2x^{\frac{1}{2}}\right) \cdot dx$

= $\int_0^1 x^{\frac{3}{2}} \cdot dx + 3\int_0^1 x^{\frac{1}{2}} \cdot dx + 2\int_0^1 x^{\frac{1}{2}} \cdot dx$

= $\left[\frac{\frac{x^5}{2}}{\frac{5}{2}}\right]_0^1 + 3\left[\frac{\frac{x^3}{2}}{\frac{3}{2}}\right]_0^1 + 2\left[\frac{\frac{x^1}{2}}{\frac{1}{2}}\right]_0^1$

= $\frac{2}{5}(1-0)\,3 \times \frac{2}{3}(1-0) + 2 \times 2(1-0)$

= $\frac{2}{5}\ 2+4$

$\therefore$ I = $\frac{32}{5}$.

Question 10.
∫53dxx+4√+x−2√
Solution:

$$\int_3^5 \frac{dx}{\sqrt{x+4}+\sqrt{x-2}}$$

$$=\int_3^5 \frac{1}{\sqrt{x+4}+\sqrt{x-2}} \times \frac{\sqrt{x+4}-\sqrt{x-2}}{\sqrt{x+4}-\sqrt{x-2}}\, dx$$

$$=\int_3^5 \frac{\sqrt{x+4}-\sqrt{x-2}}{x+4-x+2}\, dx$$

$$=\frac{1}{6}\int_3^5 [(x+4)^{\frac{1}{2}}-(x-2)^{\frac{1}{2}}]\, dx$$

$$=\frac{1}{6}\left[\frac{(x+4)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}-\frac{(x-2)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_3^5$$

$$=\frac{1}{9}\left[(x+4)^{\frac{3}{2}}-(x-2)^{\frac{3}{2}}\right]_3^5$$

$$=\frac{1}{9}\left[(9^{\frac{3}{2}}-3^{\frac{3}{2}})-(7^{\frac{3}{2}}-1)\right]$$

$$=\frac{1}{9}(27-3\sqrt{3}-7\sqrt{7}+1)$$

$$=\frac{1}{9}(28-3\sqrt{3}-7\sqrt{7}).$$

Question 11.
∫32xx2+1dx
Solution:

$$\text{Let } I = \int_2^3 \frac{x}{x^2+1}\,dx$$

Put $x^2 + 1 = t$ $\quad \therefore\ 2x\,dx = dt$

$$\therefore\ x\,dx = \frac{dt}{2}$$

When $x = 2,\ t = 4 + 1 = 5$

When $x = 3,\ t = 9 + 1 = 10$

$$\therefore\ I = \int_5^{10} \frac{1}{t}\cdot\frac{dt}{2} = \frac{1}{2}\int_5^{10}\frac{1}{t}\,dt$$

$$= \frac{1}{2}\left[\log|t|\right]_5^{10}$$

$$= \frac{1}{2}(\log 10 - \log 5) = \frac{1}{2}\log\left(\frac{10}{5}\right)$$

$$= \frac{1}{2}\log 2 = \log\sqrt{2}.$$

Question 12.

∫21x2dx

Solution:

$$\int_1^2 x^2 dx = \left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{3}(8-1) = \frac{7}{3}.$$

Question 13.

∫-1−41xdx

Solution:

$$\int_{-4}^{-1}\frac{1}{x}\,dx = \left[\log|x|\right]_{-4}^{-1}$$

$$= \log|-1| - \log|-4|$$

$$= \log 1 - \log 4$$

$$= -\log 4. \qquad \ldots\ [\because \log 1 = 0]$$

Question 14.

∫1011+x√+x√dx

Solution:

Let I = $\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}} \cdot dx$

= $\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}} \times \frac{\sqrt{1+x}-\sqrt{x}}{\sqrt{1+x}-\sqrt{x}} \cdot dx$

= $\int_0^1 \frac{\sqrt{1+x}-\sqrt{x}}{\left(\sqrt{1+x}\right)^2-\left(\sqrt{x}^2\right) \cdot dx}$

= $\int_0^1 \frac{\sqrt{1+x}-\sqrt{x}}{1+x-x} \cdot dx$

= $\int_0^1 \left[(1+x)^{\frac{1}{2}}-x^{\frac{1}{2}}\right] \cdot dx$

= $\int_0^1 (1+x)^{\frac{1}{2}} \cdot dx - \int_0^1 x^{\frac{1}{2}} \cdot dx$

= $\left[\frac{(1+x)^{\frac{1}{2}}}{\frac{3}{2}}\right]_0^1 - \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$

= $\frac{2}{3}\left[(2)^{\frac{3}{2}}-(1)^{\frac{3}{2}}\right]-\frac{2}{3}\left[(1)^{\frac{3}{2}}-0\right]$

= $\frac{2}{3}\left(2\sqrt{2}-1\right)-\frac{2}{3}(1)$

= $\frac{4\sqrt{2}}{3}-\frac{2}{3}-\frac{2}{3}$

$\therefore$ I = $\frac{4}{2}\left(\sqrt{2}-1\right)$.

Question 15.

∫401x2+2x+3√dx

Solution:

$$\int_0^4 \frac{1}{\sqrt{x^2+2x+3}}\, dx$$

$$\int_0^4 \frac{1}{\sqrt{(x^2+2x+1)+2}}\, dx$$

$$\int_0^4 \frac{1}{\sqrt{(x+1)^2+(\sqrt{2})^2}}$$

$$= [\log|(x+1)+\sqrt{(x+1)^2+\sqrt{2})^2}|]_0^4$$

$$= [\log|(x+1)+\sqrt{x^2+2x+3}|]_0^4$$

$$= \log(5+\sqrt{27}) - \log(1+\sqrt{3})$$

$$= \log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right).$$

Question 16.

∫42xx2+1dx

Solution:

Let I = $\int_2^4 \frac{x}{x^2+1} \cdot dx$

Put $x^2 + 1 = t$

$\therefore 2x \cdot dx = dt$

$\therefore x \cdot dx = \frac{dt}{2}$

When x = 2, t = $2^2 + 1 = 5$

When x = 4, t = 42 + 1 = 17

$\therefore I = \int_5^{17} \frac{1}{t} \cdot \frac{dt}{2}$

$= \frac{1}{2} \int_5^{17} \frac{dt}{t}$

$= \frac{1}{2} [\log|t|]_5^{17}$

$= \frac{1}{2} (\log 17 - \log 5)$

$\therefore I = \frac{1}{2} \log\left(\frac{17}{5}\right)$.

Question 17.

∫1012x-3dx

Solution:

$$\int_0^1 \frac{1}{2x-3}\, dx = \left[\frac{\log |2x-3|}{2}\right]_0^1$$

$$= \frac{1}{2}[\log|-1| - \log|-3|]$$

$$= \frac{1}{2}(\log 1 - \log 3)$$

$$= -\frac{1}{2}\log 3. \qquad \ldots [\because \log 1 = 0]$$

Question 18.

∫215x2x2+4x+3dx

Solution:

$$\text{Let } I=\int_1^2 \frac{5x^2}{x^2+4x+3}\,dx$$

$$=\int_1^2\left[\frac{5(x^2+4x+3)-5(4x+3)}{x^2+4x+3}\right]dx$$

$$=\int_1^2\left[5-\frac{20x+15}{x^2+4x+3}\right]dx$$

$$\therefore\ I=\int_1^2 5\,dx-\int_1^2\frac{20x+15}{x^2+4x+3}\,dx \qquad \text{.......... (1)}$$

$$\text{Let } \frac{20x+15}{x^2+4x+3}=\frac{20x+15}{(x+1)(x+3)}=\frac{A}{x+1}+\frac{B}{x+3}$$

$\therefore\ 20x+15=A(x+3)+B(x+1)$

Put $x+1=0$, i.e. $x=-1$, we get

$20(-1)+15=A(2)+B(0)$

$\therefore\ -5=2A \qquad \therefore\ A=-\frac{5}{2}$

Put $x+3=0$, i.e. $x=-3$, we get

$20(-3)+15=A(0)+B(-2)$

$\therefore\ -45=-2B \qquad \therefore\ B=\frac{45}{2}$

$$\therefore\ \frac{20x+15}{x^2+4x+3}=\frac{\left(-\frac{5}{2}\right)}{x+1}+\frac{\left(\frac{45}{2}\right)}{x+3}$$

$\therefore$ from (1),

$$I=\int_1^2 5\,dx-\int_1^2\left[\frac{\left(-\frac{5}{2}\right)}{x+1}+\frac{\left(\frac{45}{2}\right)}{x+3}\right]dx$$

$$=5\int_1^2 1\,dx+\frac{5}{2}\int_1^2\frac{1}{x+1}\,dx-\frac{45}{2}\int_1^2\frac{1}{x+3}\,dx$$

$$=5[x]_1^2+\frac{5}{2}[\log|x+1|]_1^2-\frac{45}{2}[\log|x+3|]_1^2$$

$$=5(2-1)+\frac{5}{2}(\log 3-\log 2)-\frac{45}{2}(\log 5-\log 4)$$

$$=5+\frac{1}{2}(5\log 3-5\log 2-45\log 5+90\log 2)$$

$$=5+\frac{1}{2}(5\log 3+85\log 2-45\log 5).$$

Question 19.

∫21dxx(1+logx)2

Solution:

$$\text{Let } I=\int_1^2 \frac{dx}{x(1+\log x)^2}$$
$$=\int_1^2 \frac{1}{(1+\log x)^2}\cdot\frac{1}{x}\,dx$$

Put $1+\log x=t \qquad \therefore\ \frac{1}{x}\,dx=dt$

When $x=1$, $t=1+\log 1=1+0=1$

When $x=2$, $t=1+\log 2$

$$\therefore\ I=\int_1^{1+\log 2}\frac{1}{t^2}\,dt=\int_1^{1+\log 2} t^{-2}\,dt$$
$$=\left[\frac{t^{-1}}{-1}\right]_1^{1+\log 2}=-\left[\frac{1}{t}\right]_1^{1+\log 2}$$
$$=-\left[\frac{1}{1+\log 2}-1\right]$$
$$=-\left[\frac{1-(1+\log 2)}{1+\log 2}\right]=\frac{\log 2}{1+\log 2}.$$

Question 20.

$\int_0^9 \frac{1}{1+\sqrt{x}}\,dx$

Solution:

$$\text{Let } I=\int_0^9 \frac{1}{1+\sqrt{x}}\,dx$$

Put $\sqrt{x}=t$, i.e. $x=t^2$

$\therefore\ dx=2t\,dt$

When $x=0$, $t=0$

When $x=9$, $t=3$

$$\therefore\ I=\int_0^3 \frac{1}{1+t}\cdot 2t\,dt$$
$$=2\int_0^3\left[\frac{(1+t)-1}{1+t}\right]dt$$
$$=2\int_0^3\left(1-\frac{1}{1+t}\right)dt$$
$$=2\left[t-\log|1+t|\right]_0^3 \qquad \ldots\ [\because \log 1=0]$$
$$=2[(3-\log 4)-0]$$
$$=6-2\log 4=6-4\log 2.$$