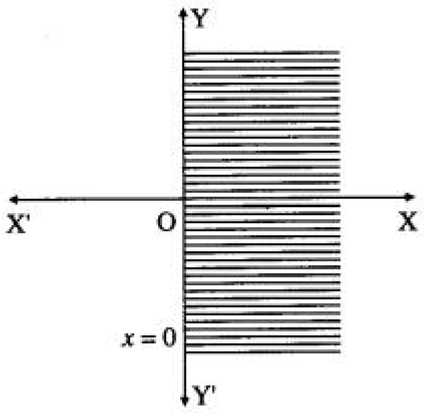
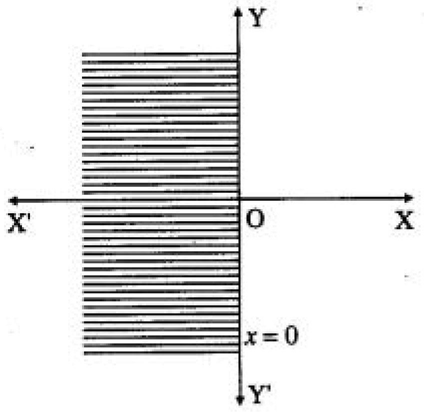
Maharashtra State Board 12th Maths Solutions Chapter 7 Linear Programming Ex 7.1

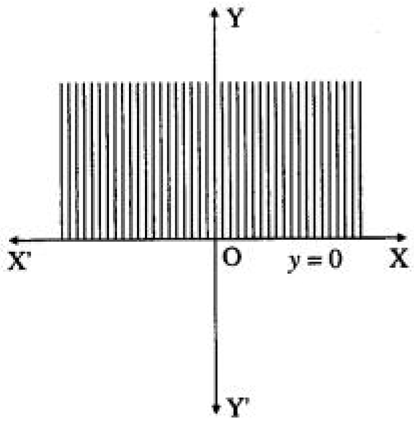
Question 1.
Solve graphically :
(i) $x \geq 0$
Solution:
Consider the line whose equation is $x = 0$. This represents the Y-axis.
To find the solution set, we have to check any point other than origin.
Let us check the point (1, 1)
When $x = 1$, $x \geq 0$
∴ (1, 1) lies in the required region
Therefore, the solution set is the Y-axis and the right side of the Y-axis which is shaded in the graph.



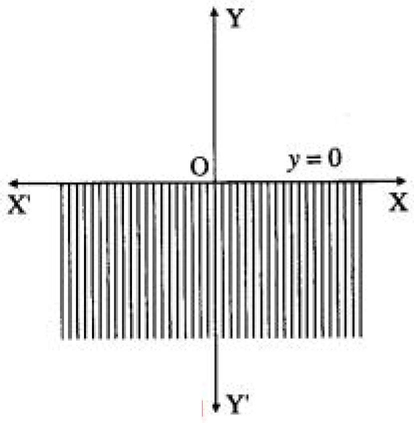
(ii) $x \leq 0$
Solution:
Consider the line whose equation is $x = 0$.
This represents the Y-axis.
To find the solution set, we have to check any point other than origin.
Let us check the point (1, 1).
When $x = 1$, $x \nleq 0$
∴ (1, 1) does not lie in the required region.
Therefore, the solution set is the Y-axis and the left side of the Y-axis which is shaded in the graph.



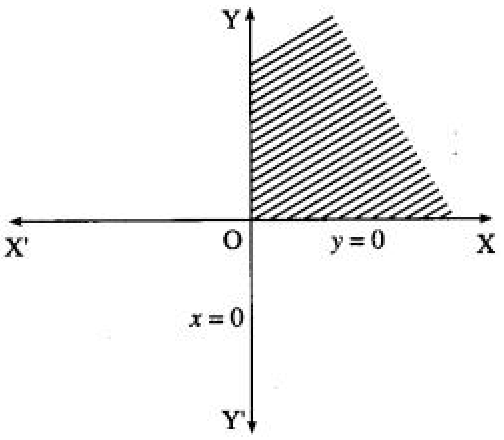
(iii) $y \geq 0$
Solution:
Consider the line whose equation is $y = 0$. This represents the X-axis. To find the solution set, we have to check any point other than origin. Let us check the point (1, 1).
When $y = 1$, $y \geq 0$
∴ (1, 1) lies in the required region.
Therefore, the solution set is the X-axis and above the X-axis which is shaded in the graph.



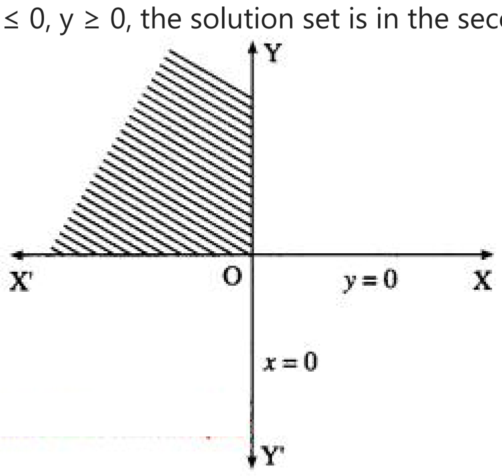
(iv) y ≤ 0
Solution:
(iv) Consider the line whose equation is y = 0. This represents the X-axis.
To find the solution set, we have to check any point other than origin.
Let us check the point (1, 1).
When y = 1, y ≰ 0.
∴ (1, 1) does not lie in the required region.
Therefore, the solution set is the X-axis and below the X-axis which is shaded in the graph.



Question 2.
Solve graphically :
(i) x ≥ 0 and y ≥ 0
Solution:
Consider the lines whose equations are x = 0, y = 0.
These represents the equations of Y-axis and X-axis respectively, which divide the plane into four parts.
(i) Since x ≥ 0, y ≥ 0, the solution set is in the first quadrant which is shaded in the graph.

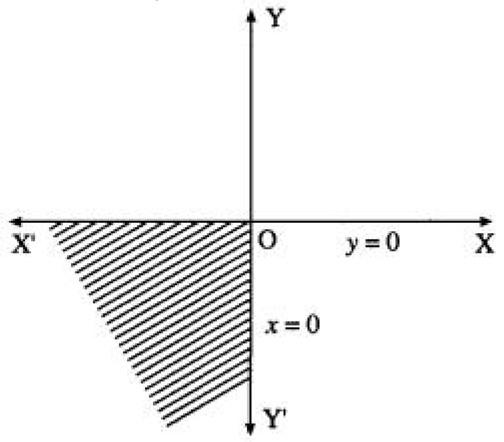


(ii) x ≤ 0 and y ≥ 0
Solution:
Since x ≤ 0, y ≥ 0, the solution set is in the second quadrant which is shaded in the graph.

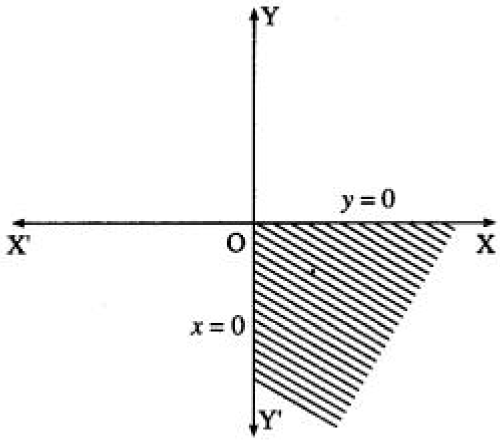


(iii) x ≤ 0 and y ≤ 0
Solution:

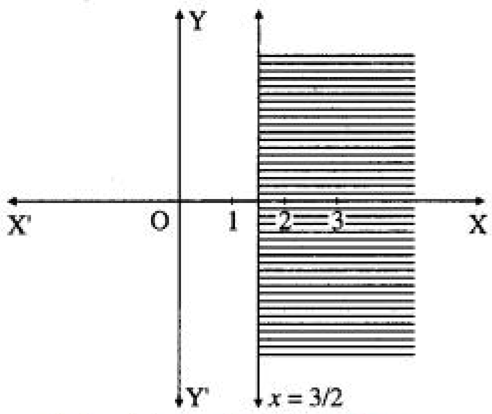
Since x ≤ 0, y ≤ 0, the solution set is in the third quadrant which is shaded in the graph.



(iv) x ≥ 0 and y ≤ 0
Solution:
Since x ≥ 0, y ≤ 0, the solution set is in the fourth ! quadrant which is shaded in the graph.

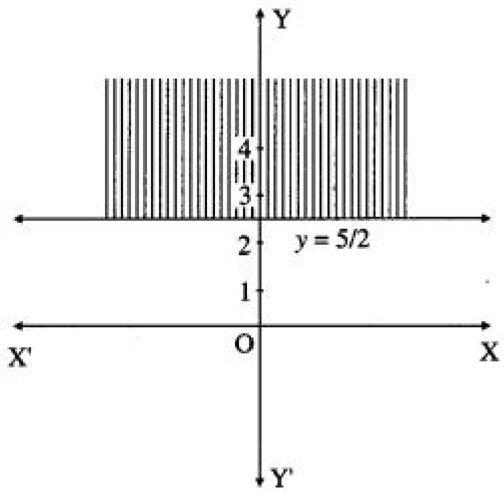


Question 3.
Solve graphically :
(i) 2x – 3 ≥ 0
Solution:
Consider the line whose equation is 2x – 3 = 0,
i.e. x = $\frac{3}{2}$
This represents a line parallel to Y-axis passing through the point ($\frac{3}{2}$, 0)
Draw the line x =$\frac{3}{2}$.
To find the solution set, we have to check the position of the origin (0, 0).
When x = 0, 2x – 3 = 2 × 0 – 3 = -3 ≯ 0
∴ the coordinates of the origin does not satisfy the given inequality.
∴ the solution set consists of the line x = $\frac{3}{2}$ and the non-origin side of the line which is shaded in the graph.



(ii) 2y – 5 ≥ 0
Solution:
Consider the line whose equation is 2y – 5 = 0, i.e. y = $\frac{5}{2}$
This represents a line parallel to X-axis passing through the point (0, $\frac{5}{2}$).
Draw the line y = $\frac{5}{2}$.
To find the solution set, we have to check the position of the origin (0, 0).
When y = 0, 2y – 5 = 2 × 0 – 5 = -5 ≯ 0
∴ the coordinates of the origin does not satisfy the given inequality.

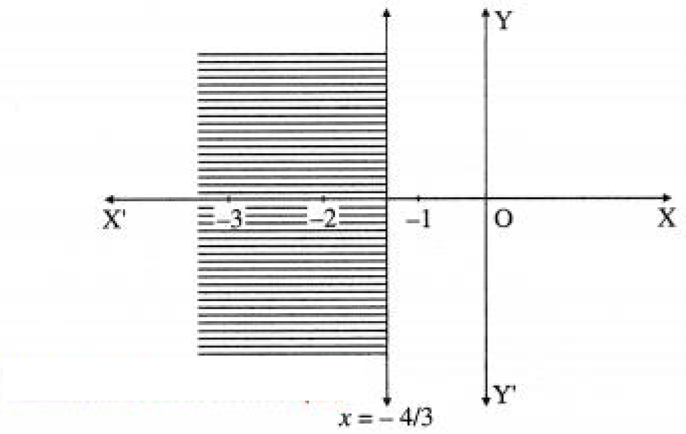
∴ the solution set consists of the line y = $\frac{5}{2}$ and the non-origin side of the line which is shaded in the graph.



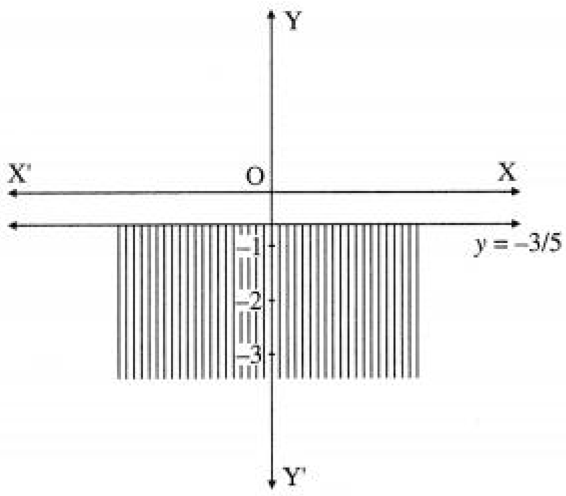
(iii) $3x + 4 \leq 0$
Solution:
(iii) Consider the line whose equation is $3x + 4 = 0$,
i.e. x = $-\frac{4}{3}$

This represents a line parallel to Y-axis passing through the point ($-\frac{4}{3}$, 0).

Draw the line x = $-\frac{4}{3}$.
To find the solution set, we have to check the position of the origin (0, 0).
When x = 0, $3x + 4 = 3 \times 0 + 4 = 4 \nleq 0$
∴ the coordinates of the origin does not satisfy the given inequality.
∴ the solution set consists of the line x = $-\frac{4}{3}$ and the non-origin side of the line which is shaded in the graph.

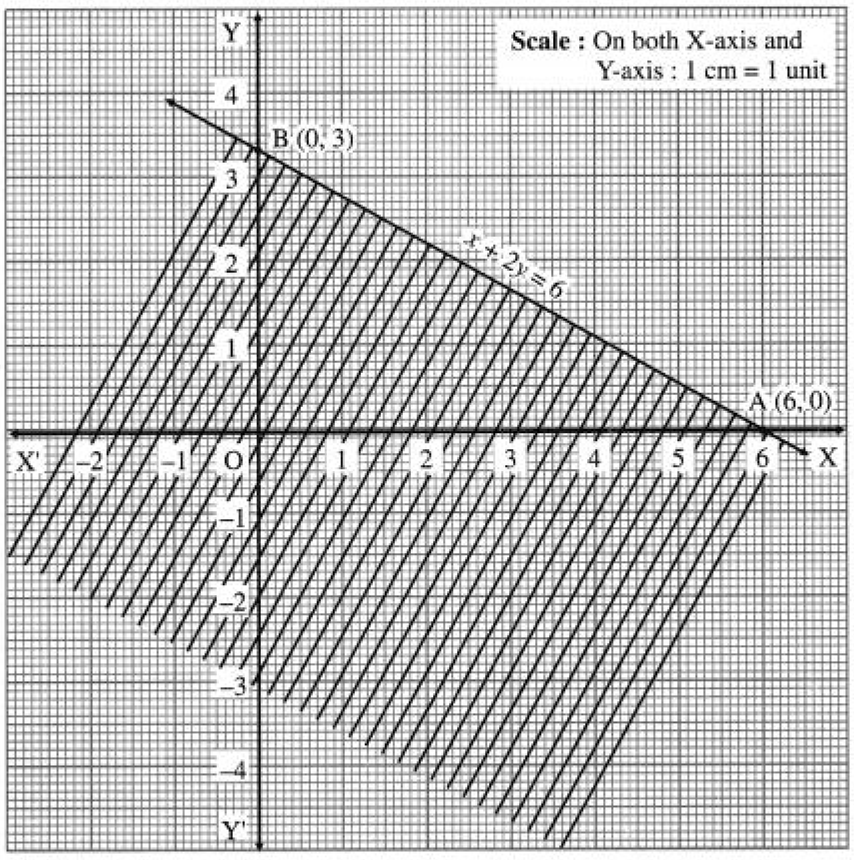


(iv) $5y + 3 \leq 0$
Solution:
(iv) Consider the line whose equation is $5y + 3 = 0$,
i.e. y = $-\frac{3}{5}$
This represents a line parallel to X-axis passing through the point (0, $-\frac{3}{5}$)
Draw the line y = $-\frac{3}{5}$.
To find the solution set, we have to check the position of the origin (0, 0).
When y = 0, $5y + 3 = 5 \times 0 + 3 = 3 \nleq 0$
∴ the coordinates of the origin does not satisfy the given inequality.
∴ the solution set consists of the line y = $-\frac{3}{5}$ and the non-origin side of the line which is shaded in the graph.



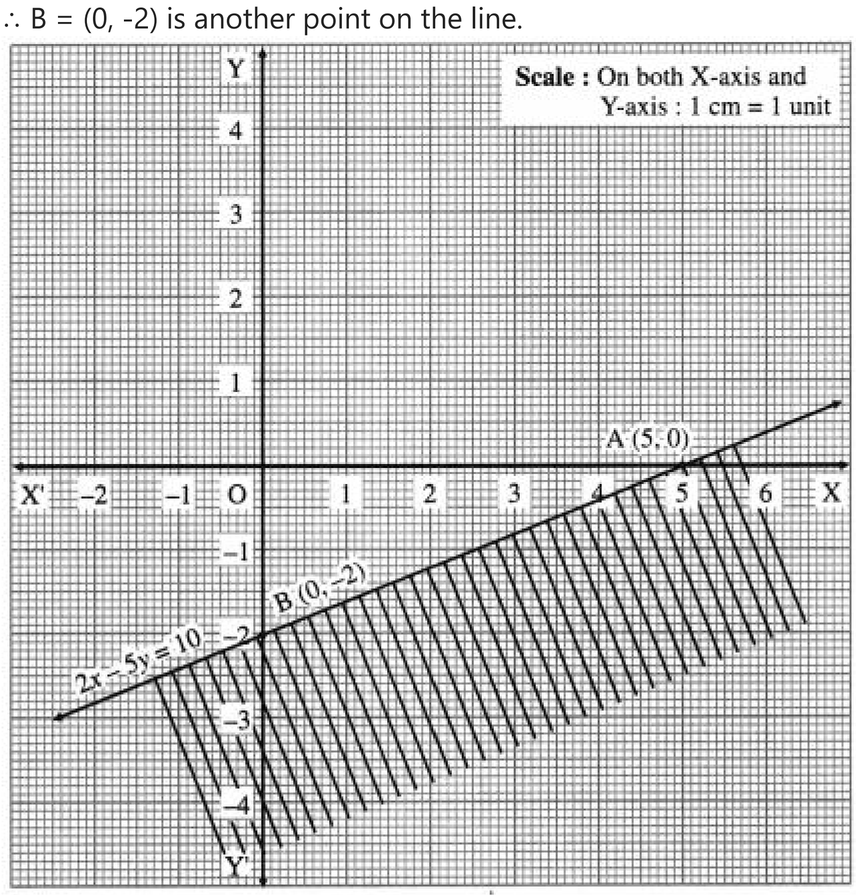
Question 4.
Solve graphically :
(i) $x + 2y \leq 6$
Solution:
Consider the line whose equation is $x + 2y = 6$.

To find the points of intersection of this line with the coordinate axes.
Put y = 0, we get x = 6.
∴ A = (6, 0) is a point on the line.
Put x = 0, we get 2y = 6, i.e. y = 3
∴ B = (0, 3) is another point on the line.



Draw the line AB joining these points. This line divide the line into two parts.
1. Origin side 2. Non-origin side
To find the solution set, we have to check the position of the origin (0, 0) with respect to the line.
When x = 0, y = 0, then x + 2y = 0 which is less than 6.
∴ x + 2y ≤ 6 in this case.
Hence, origin lies in the required region. Therefore, the given inequality is the origin side which is shaded in the graph.
This is the solution set of x + 2y ≤ 6.

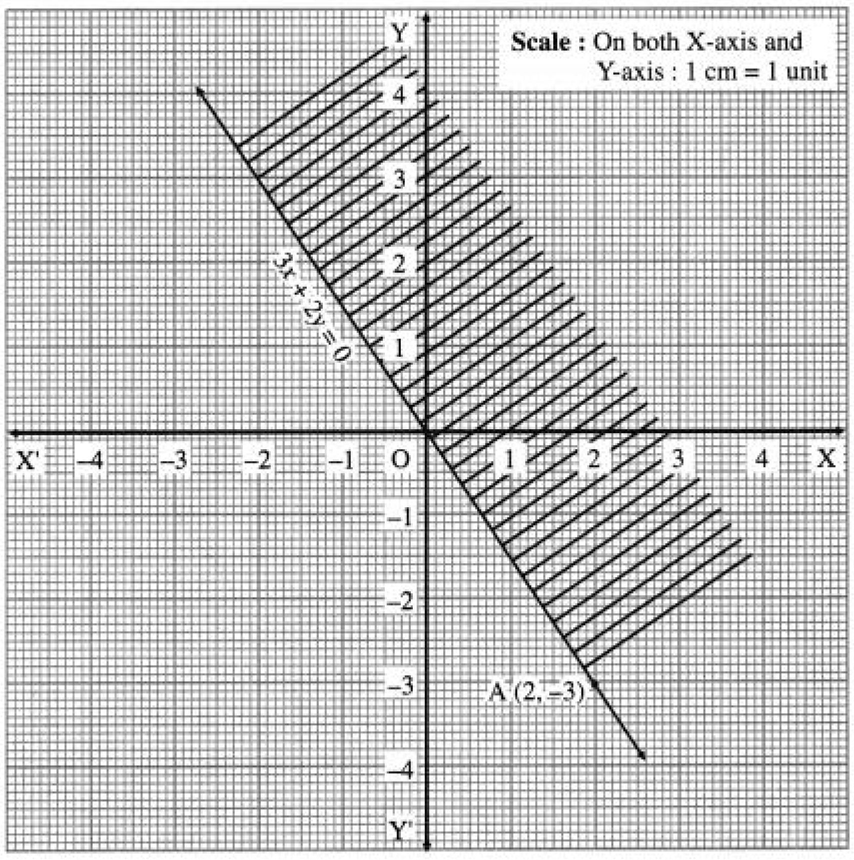
(ii) 2x – 5y ≥ 10
Solution:
Consider the line whose equation is 2x – 5y = 10.
To find the points of intersection of this line with the coordinate axes.
Put y = 0, we get 2x = 10, i.e. x = 5.
∴ A = (5, 0) is a point on the line.
Put x = 0, we get -5y = 10, i.e. y = -2
∴ B = (0, -2) is another point on the line.



Draw the line AB joining these points. This line J divide the plane in two parts.
1. Origin side 2. Non-origin side

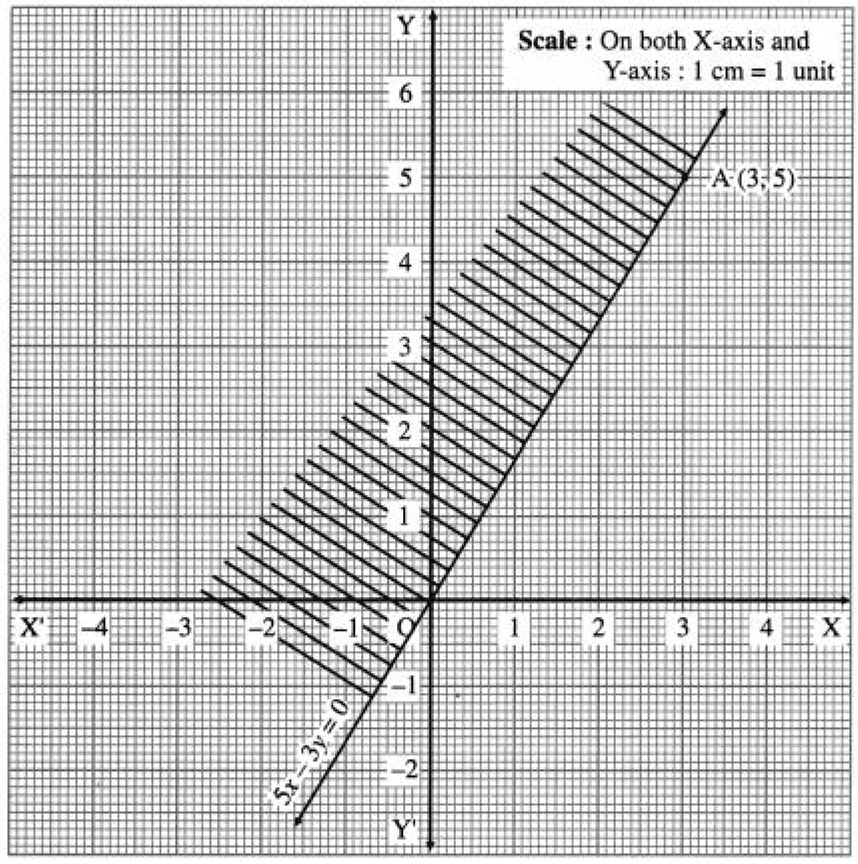
To find the solution set, we have to check the position of the origin (0, 0) with respect to the line. When x = 0, y = 0, then 2x – 5y = 0 which is neither greater nor equal to 10.
∴ $2x - 5y \ngtr 10$ in this case.
Hence (0, 0) will not lie in the required region.
Therefore, the given inequality is the non-origin side, which is shaded in the graph.
This is the solution set of $2x - 5y \geq 10$.

(iii) $3x + 2y \geq 0$
Solution:
Consider the line whose equation is $3x + 2y = 0$.
The constant term is zero, therefore this line is passing through the origin.
∴ one point on the line is O ≡ (0, 0).
To find the another point, we can give any value of x and get the corresponding value of y.
Put x = 2, we get 6 + 2y = 0 i.e. y = – 3
∴ A = (2, -3) is another point on the line.
Draw the line OA.
To find the solution set, we cannot check (0, 0) as it is already on the line.
We can check any other point which is not on the line.
Let us check the point (1, 1)



When x = 1, y = 1, then 3x + 2y = 3 + 2 = 5 which is greater than zero.
∴ $3x + 2y > 0$ in this case.
Hence (1, 1) lies in the required region. Therefore, the required region is the upper side which is shaded in the graph.
This is the solution set of $3x + 2y \geq 0$.

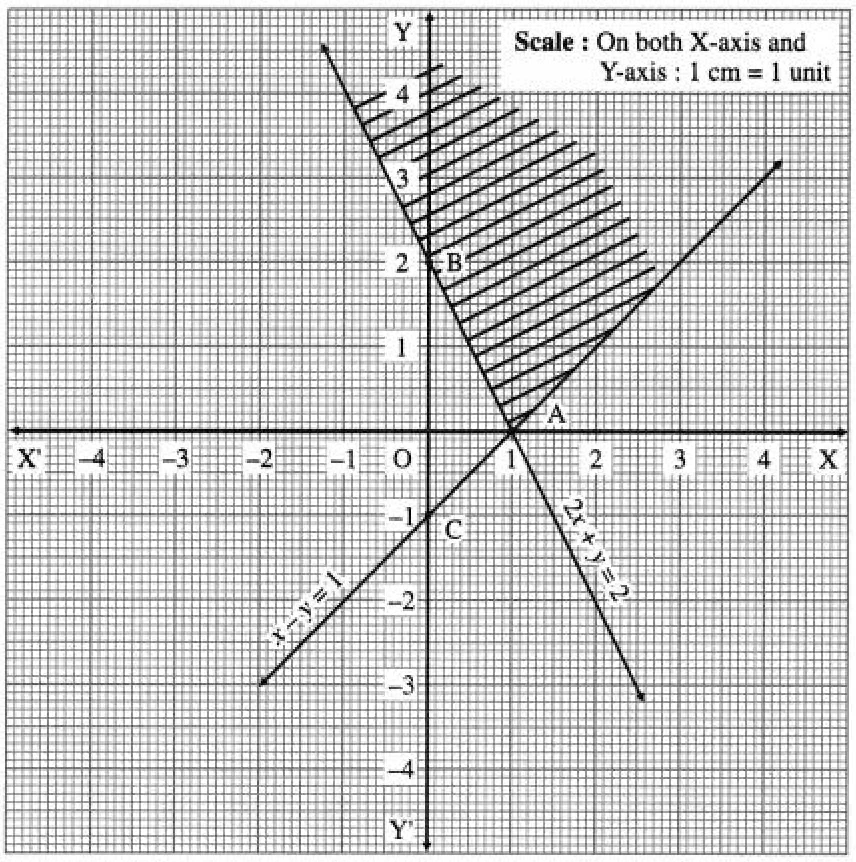
(iv) $5x - 3y \leq 0$
Solution:
Consider the line whose equation is $5x - 3y = 0$. The constant term is zero, therefore this line is passing through the origin.
∴ one point on the line is the origin O = (0, 0).
To find the other point, we can give any value of x and get the corresponding value of y.
Put x = 3, we get 15 – 3y = 0, i.e. y = 5
∴ A ≡ (3, 5) is another point on the line.
Draw the line OA.
To find the solution set, we cannot check 0(0, 0), as it is already on the line. We can check any other point which is not on the line.
Let us check the point (1, -1).
When x = 1, y = -1 then 5x – 3y = 5 + 3 = 8
which is neither less nor equal to zero.
∴ $5x - 3y \nless 0$ in this case.
Hence (1, -1) will not lie in the required region. Therefore, the required region is the upper side which is shaded in the graph.



This is the solution set of 5x – 3y ≤ 0.

Question 5.
Solve graphically :
(i) 2x + y ≥ 2 and x – y ≤ 1
Solution:
First we draw the lines AB and AC whose equations are 2x + y = 2 and x – y = 1 respectively.

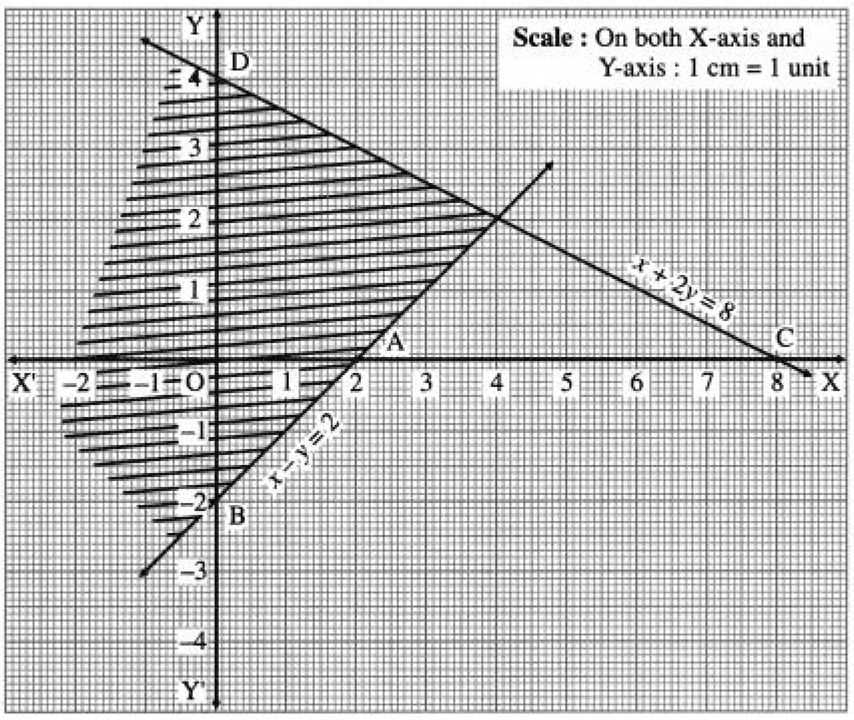
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$2x+y=2$	A (1, 0)	B (0, 2)	$\geq$	non-origin side of line AB
AC	$x-y=1$	A (1, 0)	C (0, −1)	$\leq$	origin side of line AC



The solution set of the given system of inequalities is shaded in the graph.

(ii) x – y ≤ 2 and x + 2y ≤ 8
Solution:
First we draw the lines AB and CD whose equations are x – y = 2 and x + 2y = 8 respectively.

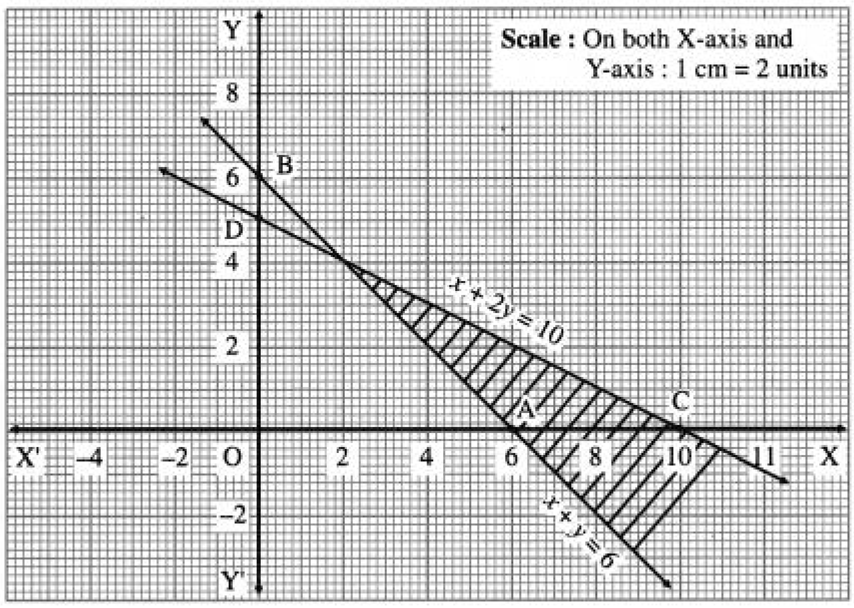
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x - y = 2$	A (2, 0)	B (0, −2)	$\leqslant$	origin side of line AB
CD	$x + 2y = 8$	C (8, 0)	D (0, 4)	$\leqslant$	origin side of line CD



The solution set of the given system of inequalities is shaded in the graph.

(iii) x + y ≥ 6 and x + 2y ≤ 10
Solution:
First we draw the lines AB and CD whose equations are x + y = 6 and x + 2y = 10 respectively.

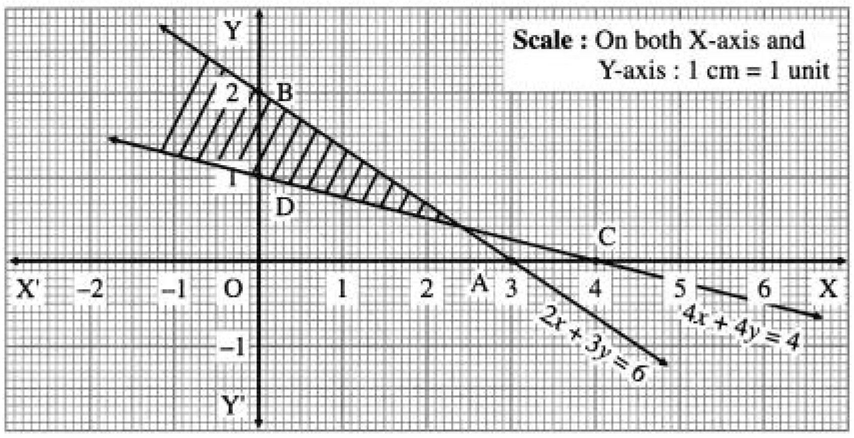
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x + y = 6$	A (6, 0)	B (0, 6)	$\geqslant$	non-origin side of line AB
CD	$x + 2y = 10$	C (10, 0)	D (0, 5)	$\leqslant$	origin side of line CD



The solution set of the given system of inequalities is shaded in the graph.

(iv) 2x + 3y ≤ 6 and x + 4y ≥ 4
Solution:
First we draw the lines AB and CD whose equations are 2x + 3y = 6 and x + 4y = 4 respectively.

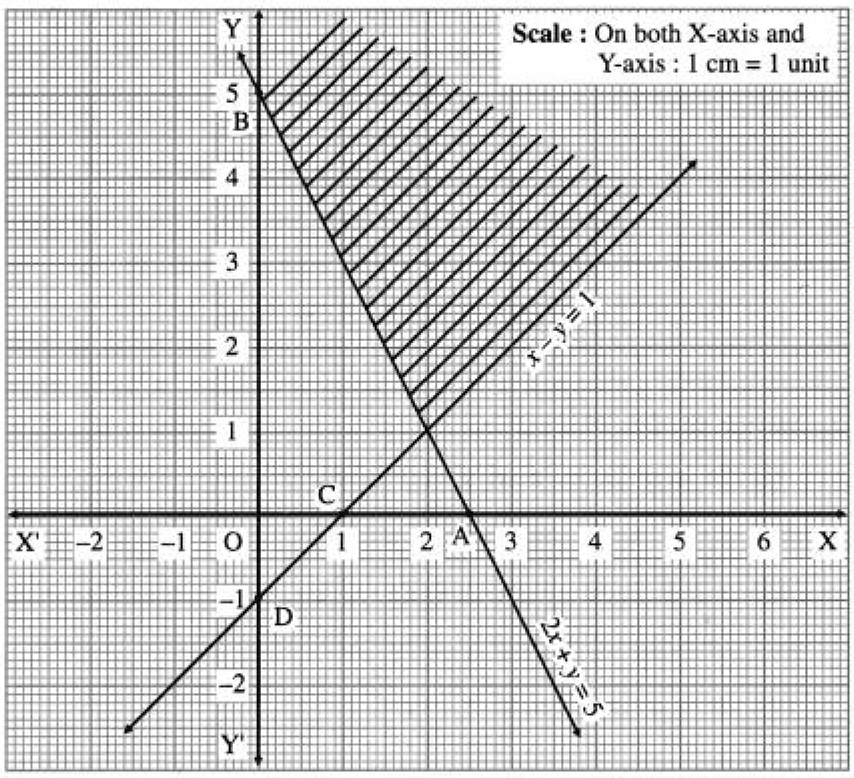
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$2x+3y=6$	A (3, 0)	B (0, 2)	$\leqslant$	origin side of line AB
CD	$x+4y=4$	C (4, 0)	D (0, 1)	$\geqslant$	non-origin side of line CD



The solution set of the given system of inequalities is shaded in the graph.

(v) 2x + y ≥ 5 and x – y ≤ 1
Solution:
First we draw the lines AB and CD whose equations are 2x + y = 5 and x – y = 1 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$2x+y=5$	A (2.5, 0)	B (0, 5)	$\geqslant$	non-origin side of line AB
CD	$x-y=1$	C (1, 0)	D (0, −1)	$\leqslant$	origin side of line CD

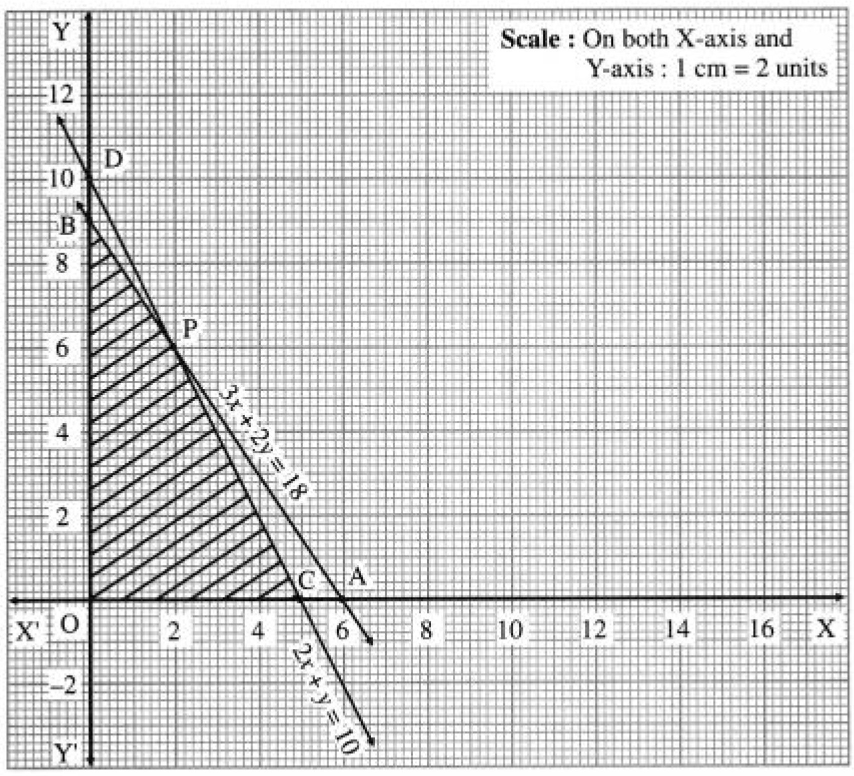


The solution set of the given system of inequations is shaded in the graph.

Maharashtra State Board 12th Maths Solutions Chapter 7 Linear Programming Ex 7.2

I) Find the feasible solution of the following inequations graphically.
Question 1.
3x + 2y ≤ 18, 2x + y ≤ 10, x ≥ 0, y ≥ 0
Solution:
First we draw the lines AB and CD whose equations are 3x + 2y = 18 and 2x + y = 10 respectively.

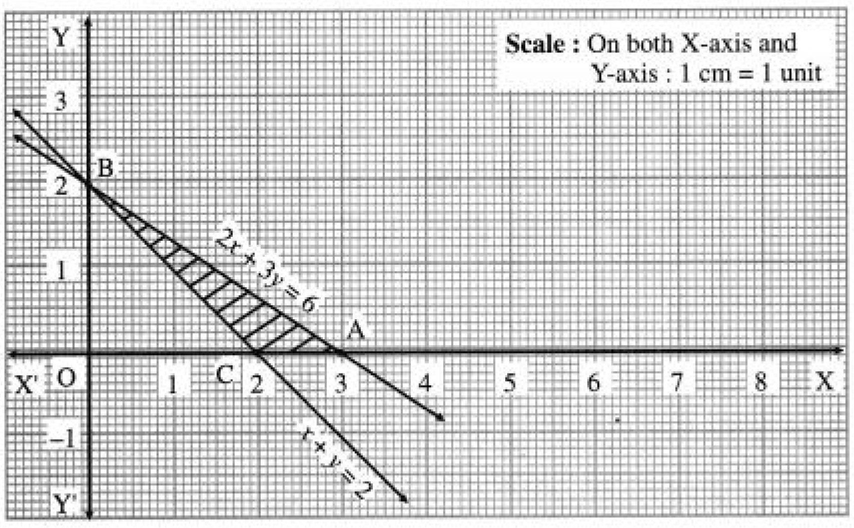
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x+2y=18$	A (6, 0)	B (0, 9)	$\leqslant$	origin side of line AB
CD	$2x+y=10$	C (5, 0)	D (0, 10)	$\leqslant$	origin side of line CD



The feasible solution is OCPBO which is shaded in the graph.

Question 2.
2x + 3y ≤ 6, x + y ≥ 2, x ≥ 0, y ≥ 0
Solution:
First we draw the lines AB and CB whose equations are 2x + 3y = 6 and x + y = 2 respectively.

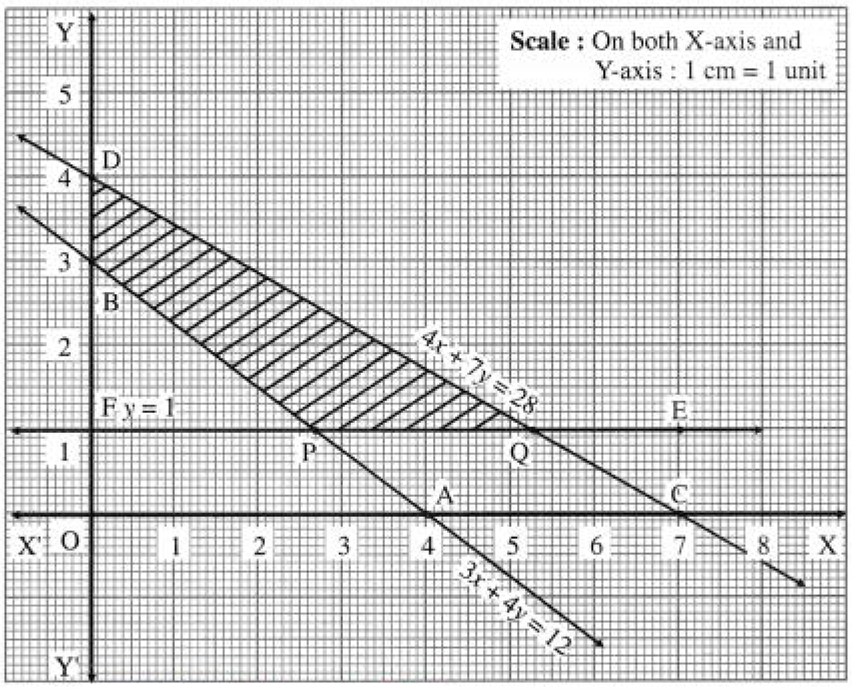
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$2x+3y=6$	A (3, 0)	B (0, 2)	$\leqslant$	origin side of line AB
CB	$x+y=2$	C (2, 0)	B (0, 2)	$\geqslant$	non-origin side of line CB



The feasible solution is ∆ABC which is shaded in the graph.

Question 3.
3x + 4y ≥ 12, 4x + 7y ≤ 28, y ≥ 1, x ≥ 0
Solution:
First we draw the lines AB, CD and EF whose equations are 3x + 4 y = 12, 4x + 7y = 28 and y = 1 respectively.

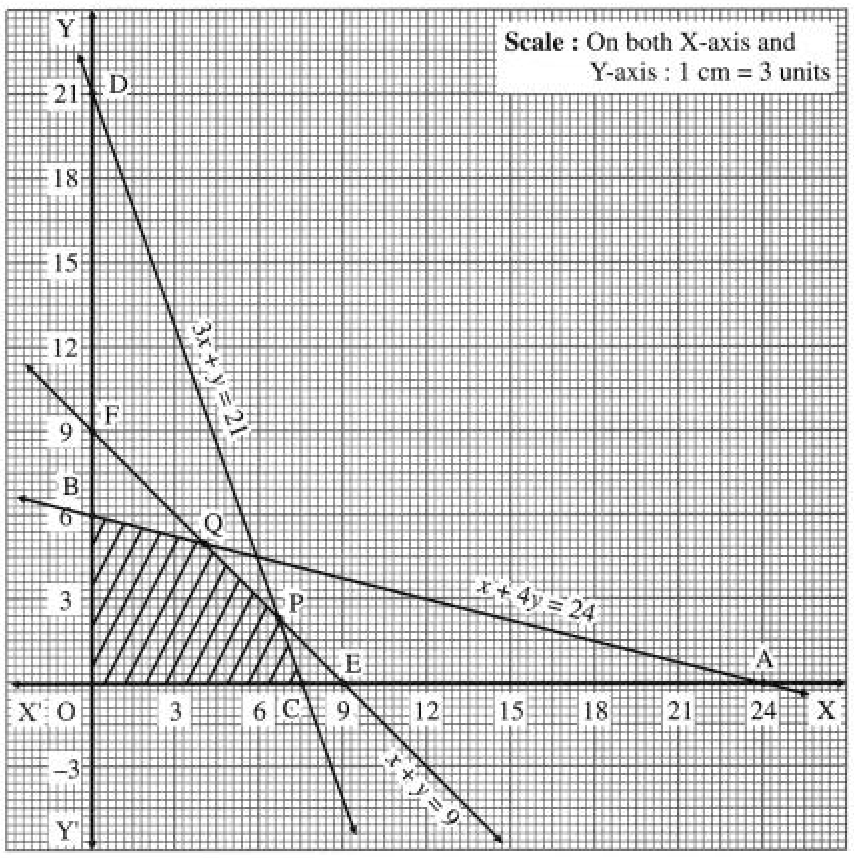
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x + 4y = 12$	A (4, 0)	B (0, 3)	$\geqslant$	non-origin side of line AB
CD	$4x + 7y = 28$	C (7, 0)	D (0, 4)	$\leqslant$	origin side of line CD
EF	$y = 1$	—	F (0, 1)	$\geqslant$	non-origin side of line EF



The feasible solution is PQDBP. which is shaded in the graph.

Question 4.
x + 4y ≤ 24, 3x + y ≤ 21, x + y ≤ 9, x ≥ 0, y ≥ 0.
Solution:
First we draw the lines AB, CD and EF whose equations are x + 4y = 24, 3x + y = 21 and x + y = 9 respectively.

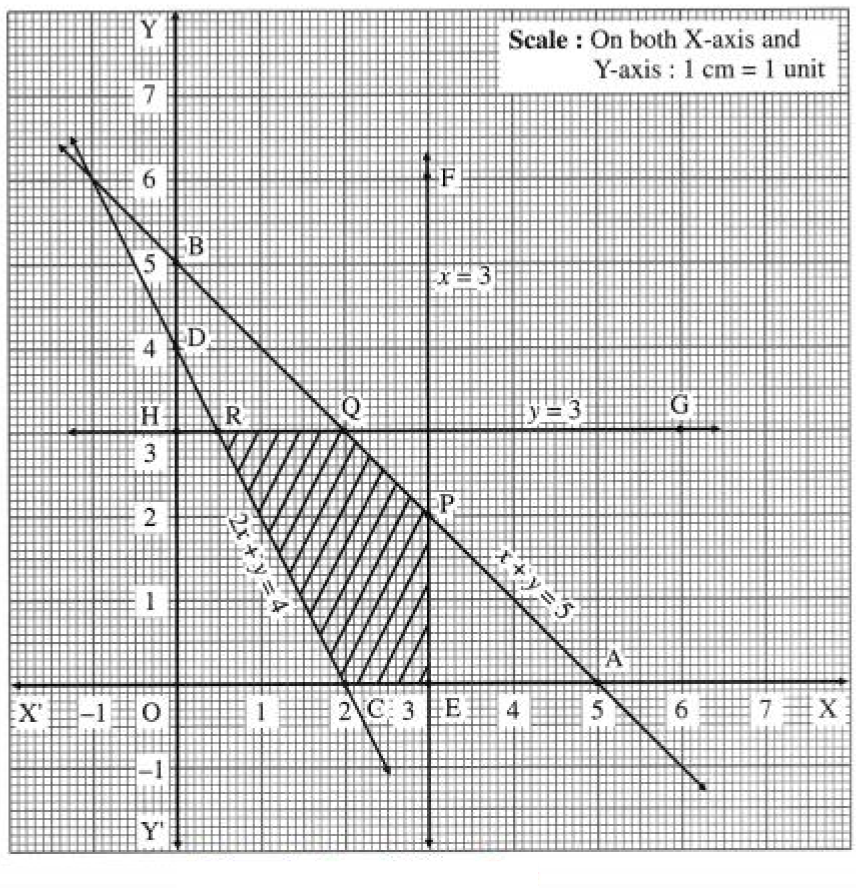
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x + 4y = 24$	A (24, 0)	B (0, 6)	$\leqslant$	origin side of line AB
CD	$3x + y = 21$	C (7, 0)	D (0, 21)	$\leqslant$	origin side of line CD
EF	$x + y = 9$	E (9, 0)	F (0, 9)	$\leqslant$	origin side of line EF



The feasible solution is OCPQBO. which is shaded in the graph.

Question 5.
0 ≤ x ≤ 3, 0 ≤ y ≤ 3, x + y ≤ 5, 2x + y ≥ 4
Solution:
First we draw the lines AB, CD, EF and GH whose equations are x + y = 5, 2x + y = 4, x = 3 and y = 3 respectively.

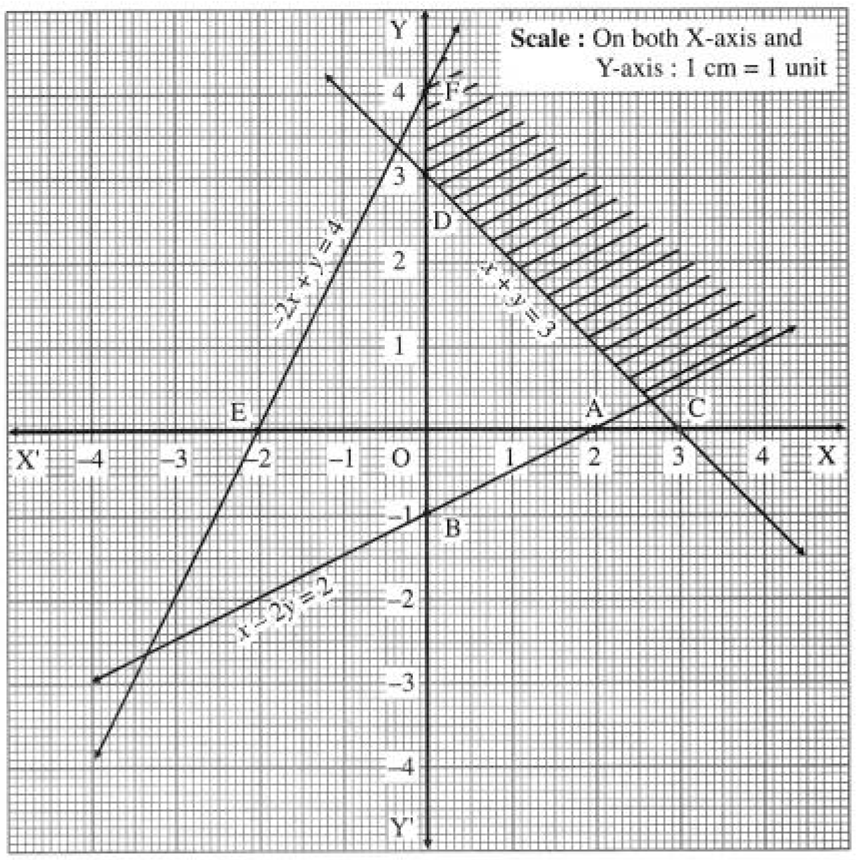
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x + y = 5$	A (5, 0)	B (0, 5)	$\leqslant$	origin side of line AB
CD	$2x + y = 4$	C (2, 0)	D (0, 4)	$\geqslant$	non-origin side of line CD
EF	$x = 3$	E (3, 0)	—	$\leqslant$	origin side of line EF
GH	$y = 3$	—	H (0, 3)	$\leqslant$	origin side of line GH



The feasible solution is CEPQRC. which is shaded in the graph.

Question 6.
x – 2y ≤ 2, x + y ≥ 3, -2x + y ≤ 4, x ≥ 0, y ≥ 0
Solution:
First we draw the lines AB, CD and EF whose equations are x – 2y = 2, x + y = 3 and -2x + y = 4 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x-2y=2$	A (2, 0)	B (0, −1)	≤	origin side of line AB
CD	$x+y=3$	C (3, 0)	D (0, 3)	≥	non-origin side of line CD
EF	$-2x+y=4$	E (−2, 0)	F (0, 4)	≤	origin side of line EF



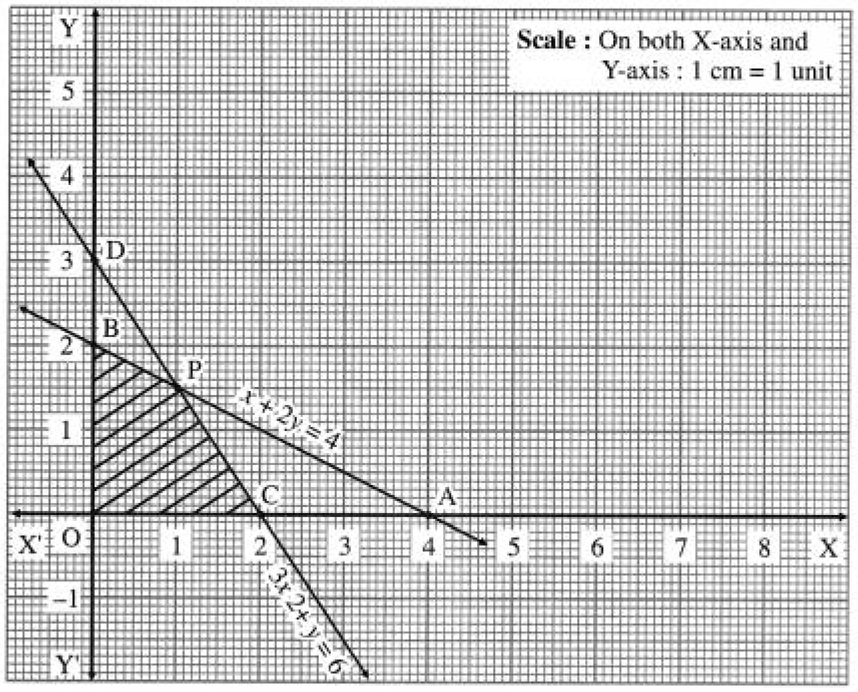
The feasible solution is shaded in the graph.

Question 7.
A company produces two types of articles A and B which requires silver and gold. Each unit of A requires 3 gm of silver and 1 gm of gold, while each unit of B requires 2 gm of silver and 2 gm of gold. The company has 6 gm of silver and 4 gm of gold. Construct the inequations and find the feasible solution graphically.
Solution:
Let the company produces x units of article A and y units of article B.
The given data can be tabulated as:

	Article A (x)	Article B (y)	Availability
Gold	1	2	4
Silver	3	2	6

Inequations are :
$x + 2y \leq 4$ and $3x + 2y \leq 6$
x and y are number of items, $x \geq 0$, $y \geq 0$
First we draw the lines AB and CD whose equations are $x + 2y = 4$ and $3x + 2y = 6$ respectively.

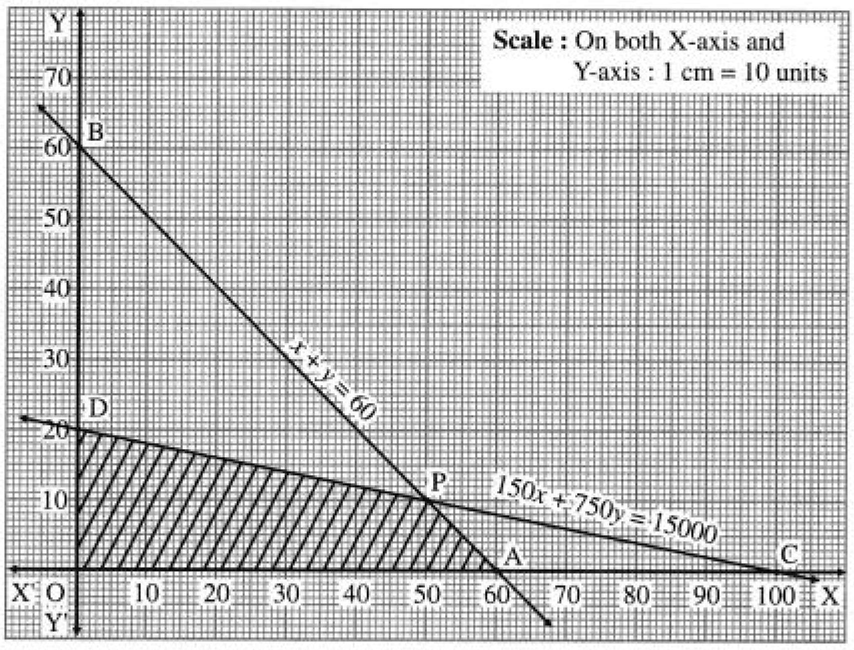
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x+2y=4$	A (4, 0)	B (0, 2)	$\leqslant$	origin side of line AB
CD	$3x+2y=6$	C (2, 0)	D (0, 3)	$\leqslant$	origin side of line CD



The feasible solution is OCPBO. which is shaded in the graph.

Question 8.
A furniture dealer deals in tables and chairs. He has Rs.1,50,000 to invest and a space to store at most 60 pieces. A table costs him Rs.1500 and a chair Rs.750. Construct the inequations and find the feasible solution.
Question is modified
A furniture dealer deals in tables and chairs. He has ₹ 15,000 to invest and a space to store at most 60 pieces. A table costs him ₹ 150 and a chair ₹ 750. Construct the inequations and find the feasible solution.
Solution:
Let x be the number of tables and y be the number of chairs. Then $x \geq 0$, $y \geq 0$.
The dealer has a space to store at most 60 pieces.
$\therefore x + y \leq 60$
Since, the cost of each table is ₹ 150 and that of each chair is ₹ 750, the total cost of x tables and y chairs is $150x + 750y$. Since the dealer has ₹ 15,000 to invest, $150x + 750y \leq 15,000$
Hence the system of inequations are
$x + y \leq 60$, $150x + 750y \leq 15000$, $x \geq 0$, $y \geq 0$.
First we draw the lines AB and CD whose equations are $x + y = 60$ and $150x + 750y = 15,000$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x+y=60$	A (60, 0)	B (0, 60)	$\leqslant$	origin side of line AB
CD	$150x+750y=15000$	C (100, 0)	D (0, 20)	$\leqslant$	origin side of line CD



The feasible solution is OAPDO. which is shaded in the graph.

Maharashtra State Board 12th Maths Solutions Chapter 7 Linear Programming Ex 7.3

Question 1.
A manufacturing firm produces two types of gadgets A and B, which are first processed in the foundry and then sent to machine shop for finishing. The number of man hours of labour required in each shop for production of A and B per unit and the number of man hours available for the firm are as follows :

Gadgets	Foundry	Machine Shop
A	10	5
B	6	4
Time available (hour)	60	35

Profit on the sale of A is ₹ 30 and B is ₹ 20 per units. Formulate the L.P.P. to have maximum profit.
Solution:
Let the number of gadgets A produced by the firm be x and the number of gadgets B produced by the firm be y.
The profit on the sale of A is ₹ 30 per unit and on the sale of B is ₹ 20 per unit.
∴ total profit is z = 30x + 20y.
This is a linear function which is to be maximized. Hence it is the objective function.
The constraints are as per the following table :

	Gadgets A (x)	Gadgets B (y)	Total available Time (in hour)
Foundry	10	6	60
Machine shop	5	4	35

From the table total man hours of labour required for x units of gadget A and y units of gadget B in foundry is (10x + 6y) hours and total man hours of labour required in machine shop is (5x + 4y) hours.
Since, maximum time avilable in foundry and machine shops are 60 hours and 35 hours respectively.
Therefore, the constraints are 10x + 6y ≤ 60, 5x + 4y ≤ 35. Since, x and y cannot be negative, we have x ≥ 0, y ≥ 0. Hence, the given LPP can be formulated as :
Maximize z = 30x + 20y, subject to 10x + 6y ≤ 60, 5x + 4y ≤ 35, x ≥ 0, y ≥ 0.

Question 2.
In a cattle breading firm, it is prescribed that the food ration for one animal must contain 14, 22 and 1 units of nutrients A, B and C respectively. Two different kinds of fodder are available. Each unit of these two contains the following amounts of these three nutrients :

Fodder / Nutrient	Fodder 1	Fodder 2
Nutrients A	2	1
Nutrients B	2	3
Nutrients C	1	1

The cost of fodder 1 is ₹3 per unit and that of fodder ₹ 2, Formulate the L.P.P. to minimize the cost.
Solution:
Let x units of fodder 1 and y units of fodder 2 be prescribed.
The cost of fodder 1 is ₹ 3 per unit and cost of fodder 2 is ₹ 2 per unit.
∴ total cost is z = 3x + 2y
This is the linear function which is to be minimized. Hence it is the objective function. The constraints are as per the following table :

Fodder → / Nutrient ↓	Fodder 1	Fodder 2	Minimum requirements
Nutrients A	2	1	14
Nutrients B	2	3	22
Nutrients C	1	1	1

From table fodder contains (2x + y) units of nutrients A, (2x + 3y) units of nutrients B and (x + y) units of nutrients C. The minimum requirements of these nutrients are 14 units, 22 units and 1 unit respectively.
Therefore, the constraints are
2x + y ≥ 14, 2x + 3y ≥ 22, x + y ≥ 1
Since, number of units (i.e. x and y) cannot be negative, we have, x ≥ 0, y ≥ 0.
Hence, the given LPP can be formulated as
Minimize z = 3x + 2y, subject to
2x + y ≥ 14, 2x + 3y ≥ 22, x + y ≥ 1, x ≥ 0, y ≥ 0.

Question 3.
A company manufactures two types of chemicals A and B. Each chemical requires two types of raw material P and Q. The table below shows number of units of P and Q required to manufacture one unit of A and one unit of B and the total availability of P and Q.

Chemical / Raw Material	A	B	Availability
P	3	2	120
Q	2	5	160

The company gets profits of ₹350 and ₹400 by selling one unit of A and one unit of B respectively. (Assume that the entire production of A and B can be sold). How many units of the chemicals A and B should be manufactured so that the company get maximum profit? Formulate the problem as L.P.P. to maximize the profit.
Solution:
Let the company manufactures x units of chemical A and y units of chemical B. Then the total profit f to the company is p = ₹ (350x + 400y).
This is a linear function which is to be maximized.
Hence, it is the objective function.
The constraints are as per the following table:

Raw Material ↓ \ Chemical →	A (x)	B (y)	Availability
P	3	2	120
Q	2	5	160

The raw material P required for x units of chemical A and y units of chemical B is $3x + 2y$. Since, the maximum availability of P is 120, we have the first constraint as $3x + 2y \leq 120$.
Similarly, considering the raw material Q, we have : $2x + 5y \leq 160$.
Since, x and y cannot be negative, we have, $x \geq 0$, $y \geq 0$.
Hence, the given LPP can be formulated as :
Maximize $p = 350x + 400y$, subject to
$3x + 2y \leq 120$, $2x + 5y \leq 160$, $x \geq 0$, $y \geq 0$.

Question 4.
A printing company prints two types of magazines A and B. The company earns ₹ 10 and ₹ 15 on magazines A and B per copy. These are processed on three machines I, II, III. Magazine A requires 2 hours on Machine I, 5 hours on Machine II and 2 hours on Machine III. Magazine B requires 3 hours on Machine I, 2 hours on Machine II and 6 hours on Machine III. Machines I, II, III are available for 36, 50, 60 hours per week respectively. Formulate the L.P.P. to determine weekly production of A and B, so that the total profit is maximum.
Solution:
Let the company prints x magazine of type A and y magazine of type B.
Profit on sale of magazine A is ₹ 10 per copy and magazine B is ₹ 15 per copy.
Therefore, the total earning z of the company is
$z = ₹\ (10x + 15y)$.
This is a linear function which is to be maximized.
Hence, it is the objective function.
The constraints are as per the following table:

Machine type ↓ \ Magazine type →	Time required per unit		Available time per week (in hours)
	Magazine A (x)	Magazine B (y)	
Machine I	2	3	36
Machine II	5	2	50
Machine III	2	6	60

From the table, the total time required for Machine I is $(2x + 3y)$ hours, for Machine II is $(5x + 2y)$ hours and for Machine III is $(2x + 6y)$ hours. The machines I, II, III are available for 36,50 and 60 hours per week. Therefore, the constraints are $2x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$.
Since x and y cannot be negative. We have, $x \geq 0$, $y \geq 0$. Hence, the given LPP can be formulated as :
Maximize $z = 10x + 15y$, subject to
$2x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$, $x \geq 0$, $y \geq 0$.

Question 5.
A manufacture produces bulbs and tubes. Each of these must be processed through two machines M_1 and M_2. A package of bulbs require 1 hour of work on Machine M_1 and 3 hours of work on M_2. A package of tubes require 2 hours on Machine M_1 and 4 hours on Machine M_2. He earns a profit of ₹ 13.5 per package of bulbs and ₹ 55 per package of tubes. Formulate the LLP to maximize the profit, if he operates the machine M_1, for atmost 10 hours a day and machine M_2 for atmost 12 hours a day.
Solution:
Let the number of packages of bulbs produced by manufacturer be x and packages of tubes be y. The manufacturer earns a profit of ₹ 13.5 per package of bulbs and ₹ 55 per package of tubes.
Therefore, his total profit is $p = ₹\ (13.5x + 55y)$
This is a linear function which is to be maximized.
Hence, it is the objective function.
The constraints are as per the following table :

	Bulbs (x)	Tubes (y)	Available Time
Machine M_1	1	2	10
Machine M_2	3	4	12

From the table, the total time required for Machine M_1 is $(x + 2y)$ hours and for Machine M_2 is $(3x + 4y)$ hours.

Given Machine M_1 and M_2 are available for atmost 10 hours and 12 hours a day respectively.
Therefore, the constraints are $x + 2y \leq 10$, $3x + 4y \leq 12$. Since, x and y cannot be negative, we have, $x \geq 0$, $y \geq 0$. Hence, the given LPP can be formulated as :
Maximize $p = 13.5x + 55y$, subject to $x + 2y \leq 10$, $3x + 4y \leq 12$, $x \geq 0$, $y \geq 0$.

Question 6.
A company manufactures two types of fertilizers F_1 and F_2. Each type of fertilizer requires
two raw materials A and B. The number of units of A and B required to manufacture one unit of fertilizer F_1 and F_2 and availability of the raw materials A and B per day are given in the
table below :

Fertilizers / Raw Material	F1	F2	Availability
A	2	3	40
B	1	4	70

By selling one unit of F_1 and one unit of F_2, company gets a profit of ₹ 500 and ₹ 750
respectively. Formulate the problem as L.P.P. to maximize the profit.
Solution:
Let the company manufactures x units of fertilizers F_1 and y units of fertilizers F_1. Then the total profit to the company is
$z = ₹(500x + 750y)$.
This is a linear function that is to be maximized. Hence, it is an objective function.

Fertilizers→ Raw Material ↓	F_1	F_2	**Availability**
A	2	3	40
B	1	4	70

The raw material A required for x units of Fertilizers F_1 and y units of Fertilizers F_2 is 2x + Since the maximum availability of A is 40, we have the first constraint as $2x + 3y \leq 40$.
Similarly, considering the raw material B, we have $x + 4y \leq 70$.
Since, x and y cannot be negative, we have, $x \geq 0$, $y \geq 0$.
Hence, the given LPP can be formulated as:
Maximize $z = 500x + 750y$, subject to
$2x + 3y \leq 40$, $x + 4y \leq 70$, $x \geq 0$, $y \geq 0$.

Question 7.
A doctor has prescribed two different units of foods A and B to form a weekly diet for a sick person. The minimum requirements of fats, carbohydrates and proteins are 18, 28, 14 units respectively. One unit of food A has 4 units of fats. 14 units of carbohydrates and 8 units of protein. One unit of food B has 6 units of fat, 12 units of carbohydrates and 8 units of protein. The price of food A is ₹ 4.5 per unit and that of food B is ₹ 3.5 per unit. Form the L.P.P. so that the sick person's diet meets the requirements at a minimum cost.
Solution:
Let the diet of sick person include x units of food A and y units of food B.
Then $x \geq 0$, $y \geq 0$.
The prices of food A and B are ₹ 4.5 and ₹ 3.5 per unit respectively.
Therefore, the total cost is $z = ₹ (4.5x + 3.5y)$
This is the linear function which is to be minimized.
Hence, it is objective function.
The constraints are as per the following table :

Food Type → Ingredients ↓	Food A ₹ (x)	Food B ₹ (y)	Minimum requirements
Fats	4	6	18
Carbohydrates	14	12	28
Proteins	8	8	14

From the table, the sick person's diet will include $(4x + 6y)$ units of fats, $(14x + 12y)$ units of carbohydrates and $(8x + 8y)$ units of proteins. The minimum requirements of these ingredients are 18 units, 28 units and 14 units respectively.
Therefore, the constraints are
$4x + 6y \geq 18$, $14x + 12y \geq 28$, $8x + 8y \geq 14$.
Hence, the given LPP can be formulated as
Minimize $z = 4.5x + 3.5y$, subject to
$4x + 6y \geq 18$, $14x + 12y \geq 28$, $8x + 8y \geq 14$, $x \geq 0$, $y \geq 0$.

Question 8.
If John drives a car at a speed of 60 kms/hour he has to spend ₹ 5 per km on petrol. If he drives at a faster speed of 90 kms/hour, the cost of petrol increases to ₹ 8 per km. He has ₹ 600 to spend on petrol and wishes to travel the maximum distance within an hour. Formulate the above problem as L.P.P.
Solution:
Let John travel xl km at a speed of 60 km/ hour and x_1 km at a speed of 90 km/hour.
Therefore, time required to travel a distance of x_1 km is $\frac{x_1}{60}$ hours and the time required to travel a distance of x_2 km is $\frac{x_2}{90}$ hours.

∴ total time required to travel is $\left(\frac{x_1}{60}+\frac{x_2}{90}\right)$ hours.

Since he wishes to travel the maximum distance within an hour,

$\frac{x_1}{60}+\frac{x_2}{90} \leq 1$

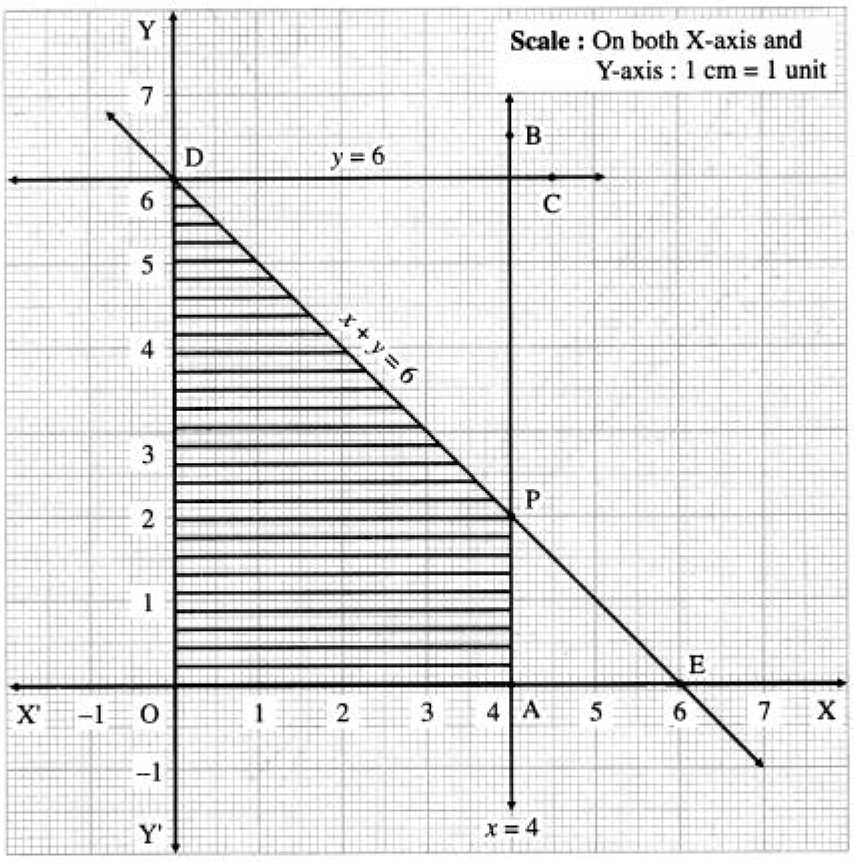
He has to spend ₹ 5 per km on petrol at a speed of 60 km/hour and ₹ 8 per km at a speed of 90 km/hour.
∴ the total cost of travelling is ₹ $(5x_1 + 8x_2)$
Since he has ₹ 600 to spend on petrol,
$5x_1 + 8x_2 \leq 600$
Since distance is never negative, $x_1 \geq 0$, $x_2 \geq 0$.
Total distance travelled by John is z. = $(x_1 + x_2)$ km.
This is the linear function which is to be maximized.
Hence, it is objective function.
Hence, the given LPP can be formulated as :
Maximize $z = x_1 + x_2$, subject to
$\frac{x_1}{60}+\frac{x_2}{90} \leq 1$, $5x_1 + 8x_2 \leq 600$, $x_1 \geq 0$, $x_2 \geq 0$.

Question 9.
The company makes concrete bricks made up of cement and sand. The weight of a concrete brick has to be least 5 kg. Cement costs ₹ 20 per kg. and sand costs of ₹ 6 per kg. strength consideration dictate that a concrete brick should contain minimum 4 kg. of cement and not more than 2 kg. of sand. Form the L.P.P. for the cost to be minimum.
Solution:
Let the company use x_1 kg of cement and x_2 kg of sand to make concrete bricks.
Cement costs ₹ 20 per kg and sand costs ₹ 6 per kg.
∴ the total cost c = ₹ $(20x_1 + 6x_2)$
This is a linear function which is to be minimized.
Hence, it is the objective function.
Total weight of brick = $(x_1 + x_2)$ kg
Since the weight of concrete brick has to be at least 5 kg,
∴ $x_1 + x_2 \geq 5$.
Since concrete brick should contain minimum 4 kg of cement and not more than 2 kg of sand,
$x_1 \geq 4$ and $0 \leq x_2 \leq 2$
Hence, the given LPP can be formulated as :
Minimize $c = 20x_1 + 6x_2$, subject to
$x_1 + x_2 \geq 5$, $x_1 \geq 4$, $0 \leq x_2 \leq 2$.

Maharashtra State Board 12th Maths Solutions Chapter 7 Linear Programming Ex 7.4

Question 1.
Maximize : $z = 11x + 8y$ subject to $x \leq 4$, $y \leq 6$,
$x + y \leq 6$, $x \geq 0$, $y \geq 0$.
Solution:
First we draw the lines AB, CD and ED whose equations are $x = 4$, $y = 6$ and $x + y = 6$ respectively.

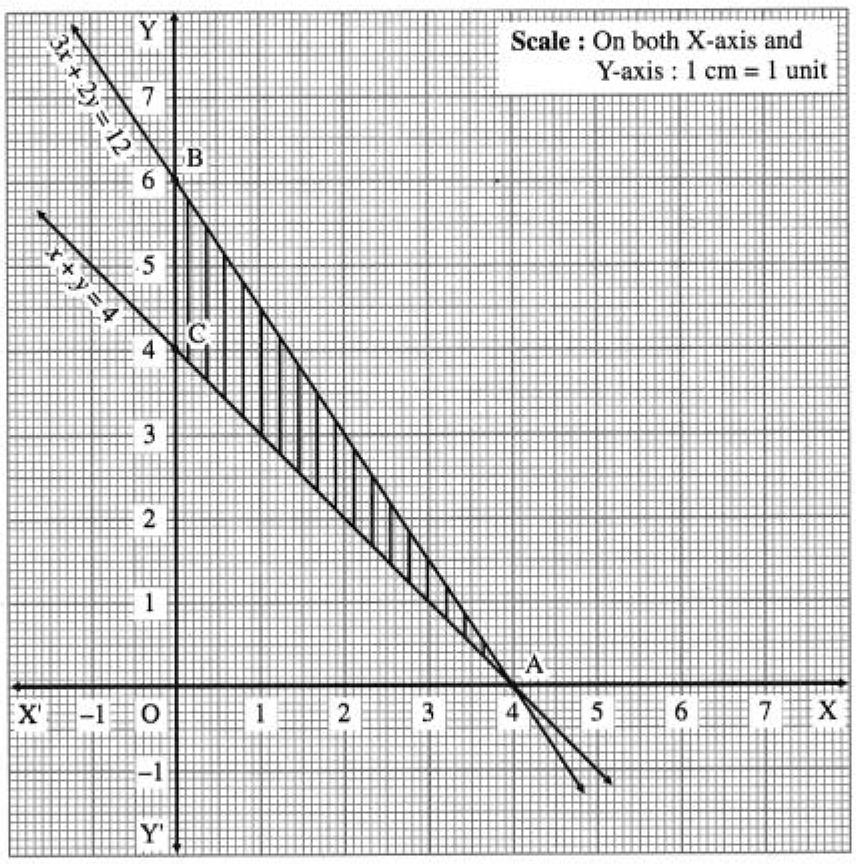
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x = 4$	A (4, 0)	—	$\leqslant$	origin side of the line AB
CD	$y = 6$	—	D (0, 6)	$\leqslant$	origin side of the line CD
ED	$x + y = 6$	E (6, 0)	D (0, 6)	$\leqslant$	origin side of the line ED



The feasible region is shaded portion OAPDO in the graph.
The vertices of the feasible region are O (0, 0), A (4, 0), P and D (0, 6)
P is point of intersection of lines x + y = 6 and x = 4.
Substituting x = 4 in x + y = 6, we get
4 + y = 6 ∴ y = 2 ∴ P is (4, 2).
∴ the corner points of feasible region are O (0, 0), A (4, 0), P(4, 2) and D(0 ,6).
The values of the objective function z = 11x + 8y at these vertices are
z (O) = 11(0) + 8(0) = 0 + 0 = 0
z(a) = 11(4) + 8(0) = 44 + 0 = 44
z (P) = 11(4) + 8(2) = 44 + 16 = 60
z (D) = 11(0) + 8(2) = 0 + 16 = 16
∴ z has maximum value 60, when x = 4 and y = 2.

Question 2.
Maximize : z = 4x + 6y subject to 3x + 2y ≤ 12,
x + y ≥ 4, x, y ≥ 0.
Solution:
First we draw the lines AB and AC whose equations are 3x + 2y = 12 and x + y = 4 respectively.

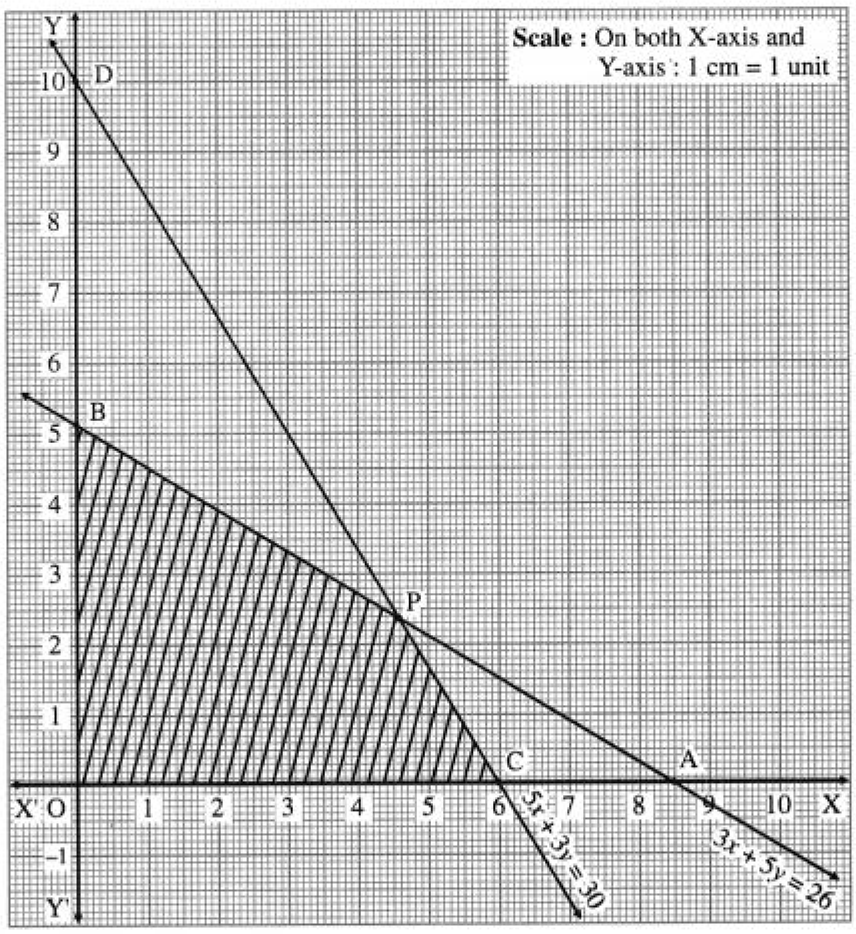
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x + 2y = 12$	A (4, 0)	B (0, 6)	$\leqslant$	origin side of line AB
AC	$x + y = 4$	A (4, 0)	C (0, 4)	$\geqslant$	non-origin side of line AC



The feasible region is the ΔABC which is shaded in the graph.
The vertices of the feasible region (i.e. corner points) are A (4, 0), B (0, 6) and C (0, 4).
The values of the objective function z = 4x + 6y at these vertices are
z(a) = 4(4) + 6(0) = 16 + 0 = 16
z(B) = 4(0)+ 6(6) = 0 + 36 = 36
z(C) = 4(0) + 6(4) = 0 + 24 = 24
∴ has maximum value 36, when x = 0, y = 6.

Question 3.
Maximize : z = 7x + 11y subject to 3x + 5y ≤ 26
5x + 3y ≤ 30, x ≥ 0, y ≥ 0.
Solution:
First we draw the lines AB and CD whose equations are 3x + 5y = 26 and 5x + 3y = 30 respectively.

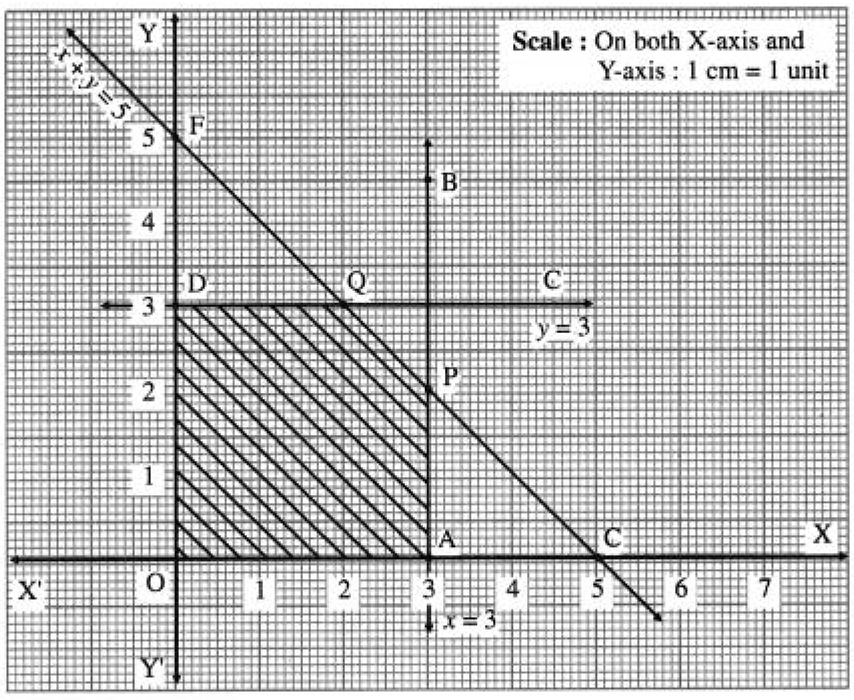
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x + 5y = 26$	$A\left(\frac{26}{3}, 0\right)$	$B\left(0, \frac{26}{5}\right)$	$\leqslant$	origin side of line AB
CD	$5x + 3y = 30$	$C(6, 0)$	$D(0, 10)$	$\leqslant$	origin side of line CD



The feasible region is OCPBO which is shaded in the graph.
The vertices of the feasible region are O (0, 0), C (6, 0), p and B(0, $\frac{26}{5}$)
The vertex P is the point of intersection of the lines
3x + 5y = 26 … (1)
and 5x + 3y = 30 … (2)
Multiplying equation (1) by 3 and equation (2) by 5, we get
9x + 15y = 78
and 25x + 15y = 150
On subtracting, we get
16x = 72 ∴ x = $\frac{72}{16}=\frac{9}{2}$ = 4.5
Substituting x = 4.5 in equation (2), we get
5(4.5) + 3y = 30
22.5 + 3y = 30
∴ 3y = 7.5 ∴ y = 2.5
∴ P is (4.5, 2.5)
The values of the objective function z = 7x + 11y at these corner points are
z (O) = 7(0) + 11(0) = 0 + 0 = 0
z (C) = 7(6) + 11(0) = 42 + 0 = 42
z (P) = 7(4.5) + 11 (2.5) = 31.5 + 27.5 = 59.0 = 59
z(B) = 7(0) + 11$\left(\frac{26}{5}\right)=\frac{286}{5}$ = 57.2
∴ z has maximum value 59, when x = 4.5 and y = 2.5.

Question 4.
Maximize : z = 10x + 25y subject to 0 ≤ x ≤ 3,
0 ≤ y ≤ 3, x + y ≤ 5 also find maximum value of z.
Solution:
First we draw the lines AB, CD and EF whose equations are x = 3, y = 3 and x + y = 5 respectively.

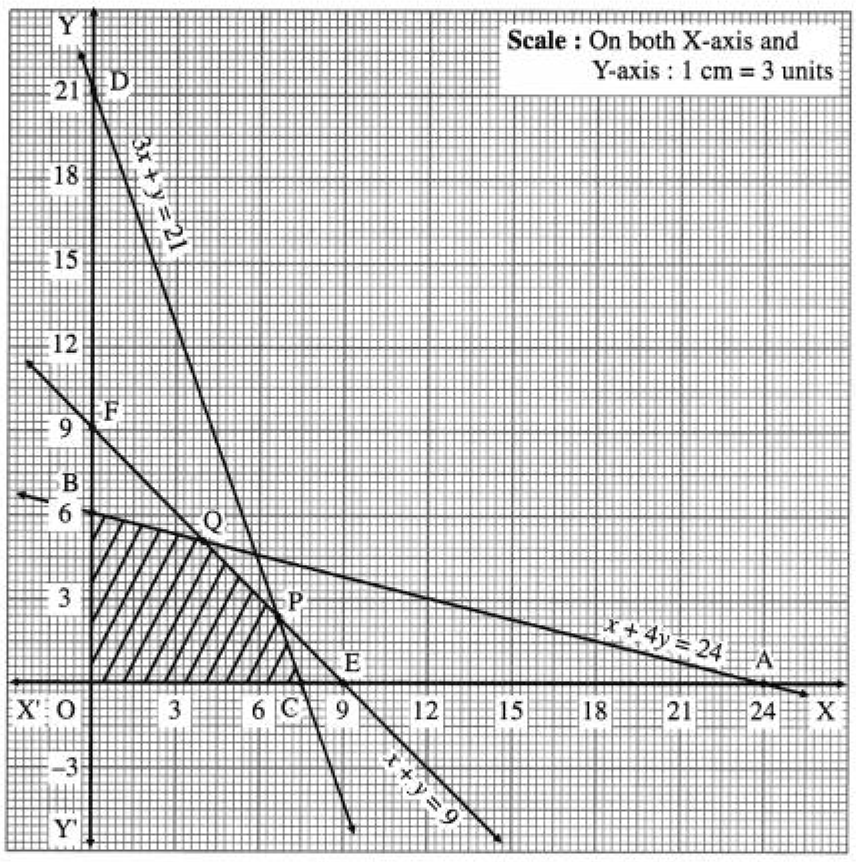
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x=3$	A (3, 0)	—	⩽	origin side of line AB
CD	$y=3$	—	D (0, 3)	⩽	origin side of line CD
EF	$x+y=5$	E (5, 0)	F (0, 5)	⩽	origin side of line EF



The feasible region is OAPQDO which is shaded in the i graph.
The vertices of the feasible region are O (0, 0), A (3, 0), P, Q and D(0, 3).
t P is the point of intersection of the lines x + y = 5 and x = 3.
Substituting x = 3 in x + y = 5, we get
3 + y = 5 ∴ y = 2
∴ P is (3, 2)
Q is the point of intersection of the lines x + y = 5 and y = 3
Substituting y = 3 in x + y = 5, we get
x + 3 = 5 ∴ x = 2
∴ Q is (2, 3)
The values of the objective function z = 10x + 25y at these vertices are
z(O) = 10(0) + 25(0) = 0 + 0 = 0
z(a) = 10(3) + 25(0) = 30 + 0 = 30
z(P) = 10(3) + 25(2) = 30 + 50 = 80
z(Q) = 10(2) + 25(3) = 20 + 75 = 95
z(D) = 10(0)+ 25(3) = 0 + 75 = 75
∴ z has maximum value 95, when x = 2 and y = 3.

Question 5.
Maximize : z = 3x + 5y subject to x + 4y ≤ 24, 3x + y ≤ 21,
x + y ≤ 9, x ≥ 0, y ≥ 0 also find maximum value of z.
Solution:
First we draw the lines AB, CD and EF whose equations are x + 4y = 24, 3x + y = 21 and x + y = 9 respectively.

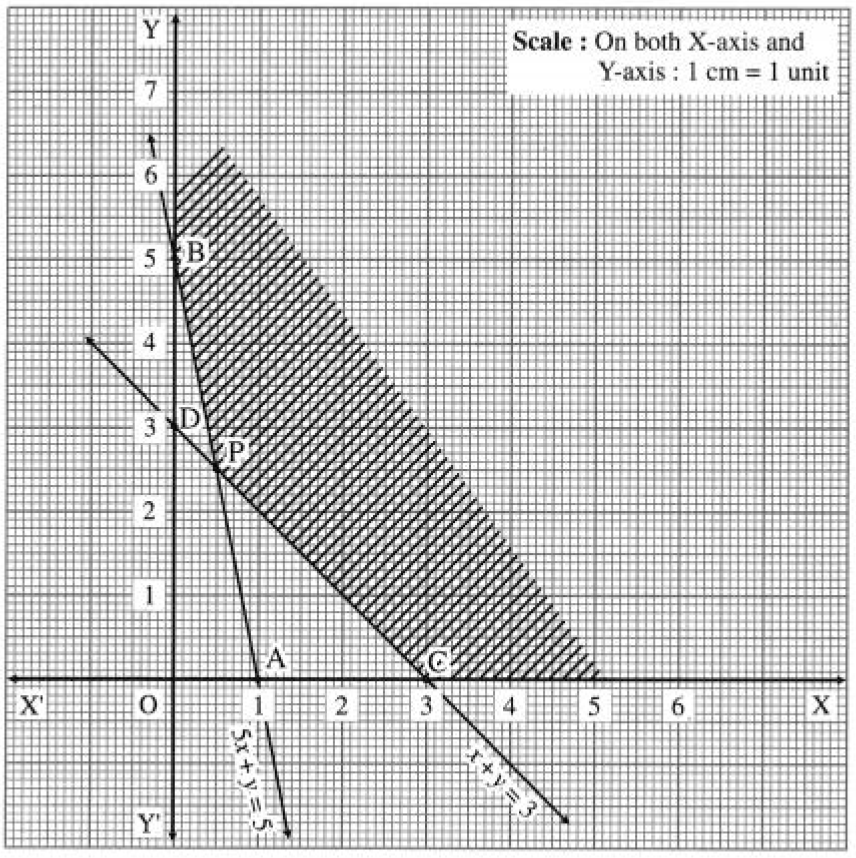
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x + 4y = 24$	A (24, 0)	B (0, 6)	≤	origin side of line AB
CD	$3x + y = 21$	C (7, 0)	D (0, 21)	≤	origin side of line CD
EF	$x + y = 9$	E (9, 0)	F (0, 9)	≤	origin side of line EF



The feasible region is OCPQBO which is shaded in the graph.
The vertices of the feasible region are O (0, 0), C (7, 0), P, Q and B (0, 6).
P is the point of intersection of the lines
$3x + y = 21$ ... (1)
and $x + y = 9$ ... (2)
On subtracting, we get $2x = 12 \therefore x = 6$
Substituting $x = 6$ in equation (2), we get
$6 + y = 9 \therefore y = 3$
$\therefore$ P = (6, 3)
Q is the point of intersection of the lines
$x + 4y = 24$ ... (3)
and $x + y = 9$ ... (2)
On subtracting, we get
$3y = 15 \therefore y = 5$
Substituting $y = 5$ in equation (2), we get
$x + 5= 9 \therefore x = 4$
$\therefore$ Q = (4, 5)
$\therefore$ the corner points of the feasible region are 0(0,0), C(7, 0), P (6, 3), Q (4, 5) and B (0, 6).
The values of the objective function $2 = 3x + 5y$ at these corner points are
z(O) = 3(0)+ 5(0) = 0 + 0 = 0
z(C) = 3(7) + 5(0) = 21 + 0 = 21
z(P) = 3(6) + 5(3) = 18 + 15 = 33
z(Q) = 3(4) + 5(5) = 12 + 25 = 37
z(B) = 3(0)+ 5(6) = 0 + 30 = 30
$\therefore$ z has maximum value 37, when x = 4 and y = 5.

Question 6.
Minimize : $z = 7x + y$ subject to $5x + y \geq 5$, $x + y \geq 3$,
$x \geq 0$, $y \geq 0$.
Solution:
First we draw the lines AB and CD whose equations are $5x + y = 5$ and $x + y = 3$ respectively.

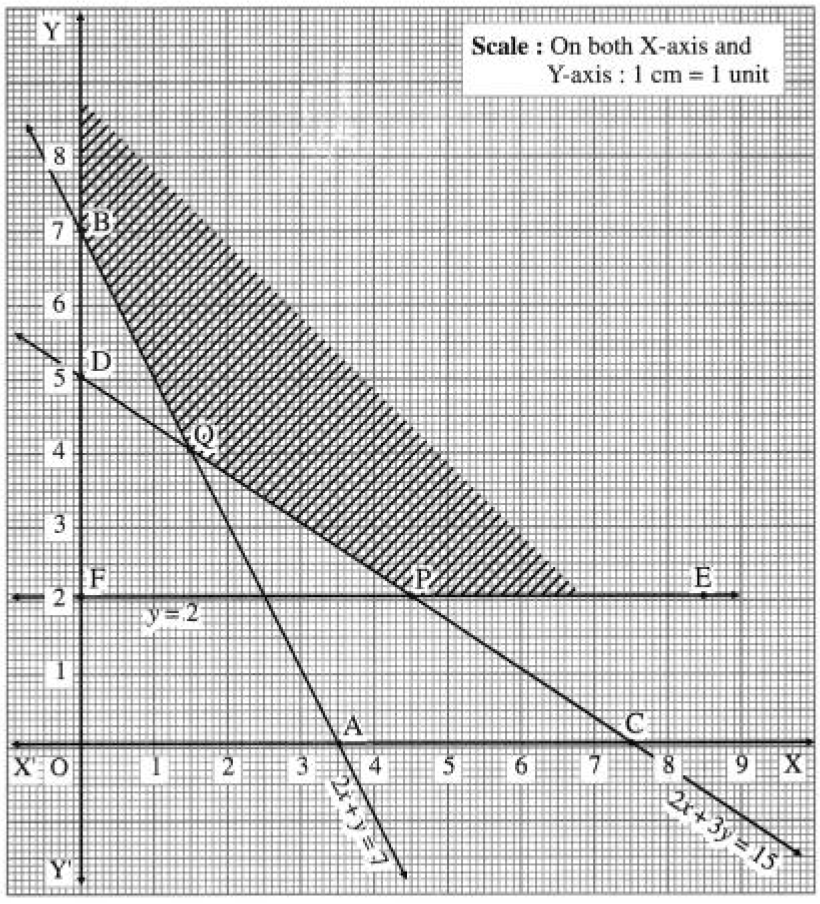
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$5x+y=5$	A (1, 0)	B (0, 5)	$\geqslant$	non-origin side of line AB
CD	$x+y=3$	C (3, 0)	D (0, 3)	$\geqslant$	non-origin side of line CD



The feasible region is XCPBY which is shaded in the graph.
The vertices of the feasible region are C (3, 0), P and B (0, 5).
P is the point of the intersection of the lines
5x + y = 5
and x + y = 3
On subtracting, we get
4x = 2 ∴ x = 12
Substituting x = 12 in x + y = 3, we get
12 + y = 3
∴ y = 52 ∴ P = (12,52)
The values of the objective function z = 7x + y at these vertices are
z(C) = 7(3) + 0 = 21
z(B) = 7(0) + 5 = 5
∴ z has minimum value 5, when x = 0 and y = 5.

Question 7.
Minimize : z = 8x + 10y subject to 2x + y ≥ 7, 2x + 3y ≥ 15,
y ≥ 2, x ≥ 0, y ≥ 0.
Solution:
First we draw the lines AB, CD and EF whose equations are 2x + y = 7, 2x + 3y = 15 and y = 2 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$2x + y = 7$	A (3.5, 0)	B (0, 7)	$\geqslant$	non-origin side of line AB
CD	$2x + 3y = 15$	C (7.5, 0)	D (0, 5)	$\geqslant$	non-origin side of line CD
EF	$y = 2$	—	F (0, 2)	$\geqslant$	non-origin side of line EF



The feasible region is EPQBY which is shaded in the graph. The vertices of the feasible region are P, Q and B(0,7). P is the point of intersection of the lines 2x + 3y = 15 and y = 2.

Substituting y – 2 in 2x + 3y = 15, we get 2x + 3(2) = 15

∴ 2x = 9 ∴ x = 4.5 ∴ P = (4.5, 2)

Q is the point of intersection of the lines

2x + 3y = 15 … (1)

and 2x + y = 7 … (2)

On subtracting, we get

2y = 8 ∴ y = 4

∴ from (2), 2x + 4 = 7

∴ 2x = 3 ∴ x = 1.5

∴ Q = (1.5, 4)

The values of the objective function z = 8x + 10y at these vertices are

z(P) = 8(4.5) + 10(2) = 36 + 20 = 56

z(Q) = 8(1.5) + 10(4) = 12 + 40 = 52

z(B) = 8(0) +10(7) = 70

∴ z has minimum value 52, when x = 1.5 and y = 4

Question 8.

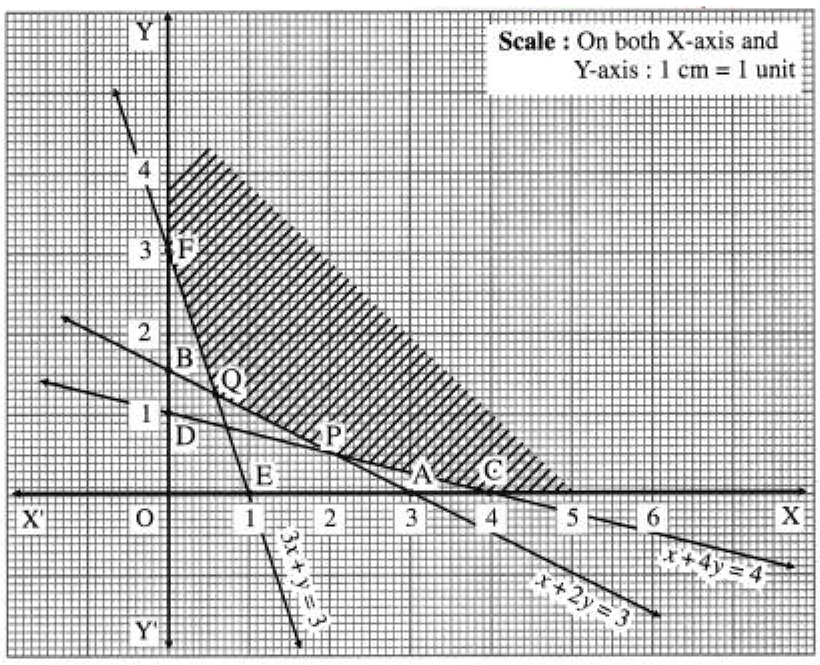
Minimize : z = 6x + 21y subject to x + 2y ≥ 3, x + 4y ≥ 4,

3x + y ≥ 3, x ≥ 0, y ≥ 0.

Solution:

First we draw the lines AB, CD and EF whose equations are x + 2y = 3, x + 4y = 4 and 3x + y = 3 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x + 2y = 3$	A (3, 0)	$B\left(0, \frac{3}{2}\right)$	$\geqslant$	non-origin side of line AB
CD	$x + 4y = 4$	C (4, 0)	D (0, 1)	$\geqslant$	non-origin side of line CD
EF	$3x + y = 3$	E (1, 0)	F (0, 3)	$\geqslant$	non-origin side of line EF



The feasible region is XCPQFY which is shaded in the graph.
The vertices of the feasible region are C (4, 0), P, Q and F(0, 3).
P is the point of intersection of the lines x + 4y = 4 and x + 2y = 3
On subtracting, we get
2y = 1 ∴ y = $\frac{1}{2}$
Substituting y = $\frac{1}{2}$ in x + 2y = 3, we get
x + 2$\left(\frac{1}{2}\right)$ = 3
∴ x = 2
∴ P = (2, $\frac{1}{2}$)
Q is the point of intersection of the lines
x + 2y = 3 ... (1)
and 3x + y = 3 ....(2)
Multiplying equation (1) by 3, we get 3x + 6y = 9
Subtracting equation (2) from this equation, we get
5y = 6
∴ y = $\frac{6}{5}$
∴ from (1), x + 2$\left(\frac{6}{5}\right)$ = 3
∴ x = 3 – $\frac{12}{5}=\frac{3}{5}$
Q ≡ $\left(\frac{3}{5},\frac{6}{5}\right)$
The values of the objective function z = 6x + 21y at these vertices are
z(C) = 6(4) + 21(0) = 24
z(P) = 6(2) + 21$\left(\frac{1}{2}\right)$
= 12 + 10.5 = 22.5
z(Q)= 6$\left(\frac{3}{5}\right)$ + 21$\left(\frac{6}{5}\right)$
= $\frac{18}{5}+\frac{126}{5}=\frac{144}{5}$ = 28.8
2 (F) = 6(0) + 21(3) = 63
∴ z has minimum value 22.5, when x = 2 and y = $\frac{1}{2}$.

Maharashtra State Board 12th Maths Solutions Chapter 7 Linear Programming Miscellaneous Exercise 7

I) Select the appropriate alternatives for each of the following :
Question 1.
The value of objective function is maximum under linear constraints _______.
(A) at the centre of feasible region
(B) at (0, 0)
(C) at a vertex of feasible region
(D) the vertex which is of maximum distance from (0, 0)
Solution:
(C) at a vertex of feasible region

Question 2.
Which of the following is correct _______.
(A) every L.P.P. has an optimal solution
(B) a L.P.P. has unique optimal solution
(C) if L.P.P. has two optimal solutions then it has infinite number of optimal solutions
(D) the set of all feasible solution of L.P.P. may not be convex set
Solution:
(C) if L.P.P. has two optimal solutions then it has infinite number of optimal solutions

Question 3.
Objective function of L.P.P. is _______.
(A) a constraint
(B) a function to be maximized or minimized
(C) a relation between the decision variables
(D) equation of a straight line
Solution:
(B) a function to be maximized or minimized

Question 4.
The maximum value of z = 5x + 3y subjected to the constraints 3x + 5y ≤ 15, 5x + 2y ≤ 10, x, y≥ 0 is _______.
(A) 235
(B) 2359
(C) 23519
(D) 2353
Solution:
(C) 23519

Question 5.
The maximum value of z = 10x + 6y subjected to the constraints 3x + y ≤ 12, 2x + 5y ≤ 34, x ≥ 0, y≥ 0. _______.
(A) 56
(B) 65
(C) 55
(D) 66
Solution:
(A) 56

Question 6.
The point at which the maximum value of x + y subject to the constraints x + 2y ≤ 70, 2x + y ≤ 95, x ≥ 0, y ≥ 0 is obtained at _______.
(A) (30, 25)
(B) (20, 35)
(C) (35, 20)
(D) (40, 15)
Solution:
(D) (40, 15)

Question 7.
Of all the points of the feasible region, the optimal value ofz obtained at the point lies _______.
(A) inside the feasible region
(B) at the boundary of the feasible region
(C) at vertex of feasible region
(D) outside the feasible region

Solution:
(C) at vertex of feasible region

Question 8.
Feasible region is the set of points which satisfy _______.
(A) the objective function
(B) all of the given constraints
(C) some of the given constraints
(D) only one constraint
Solution:
(B) all of the given constraints

Question 9.
Solution of L.P.P. to minimize z = 2x + 3y such that x ≥ 0, y ≥ 0, 1 ≤ x + 2y ≤ 10 is _______.
(A) x = 0, y = 12
(B) x = 12, y = 0
(C) x = 1, y = 2
(D) x = 12, y = 12
Solution:
(A) x = 0, y = 12

Question 10.
The corner points of the feasible solution given by the inequation x + y ≤ 4, 2x + y ≤ 7, x ≥ 0, y ≥ 0 are _______.
(A) (0, 0), (4, 0), (7, 1), (0, 4)
(B) (0, 0), (72, 0), (3, 1), (0, 4)
(C) (0, 0), (72, 0), (3, 1), (0, 7)
(D) (0, 0), (4, 0), (3, 1), (0, 7)
Solution:
(B) (0, 0), (72, 0), (3, 1), (0, 4)

Question 11.
The corner points of the feasible solution are (0, 0), (2, 0), (127, 37), (0, 1). Then z = 7x + y is maximum at _______.
(A) (0, 0)
(B) (2, 0)
(C) (127, 37)
(D) (0, 1)
Solution:
(B) (2, 0)

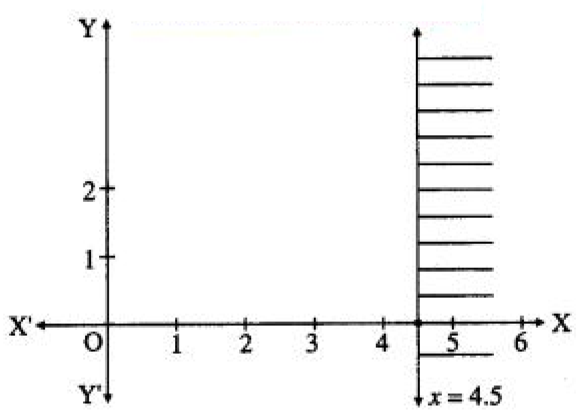
Question 12.
If the corner points of the feasible solution are (0, 0), (3, 0), (2, 1) and (0, 73), the maximum value of z = 4x + 5y is _______.
(A) 12
(B) 13
(C) 35
(D) 0
Solution:
(B) 13

Question 13.
If the corner points of the feasible solution are (0, 10), (2, 2) and (4, 0) then the point of minimum z = 3x + 2y is _______.
(A) (2, 2)
(B) (0, 10)
(C) (4, 0)
(D) (3 ,4)
Solution:
(A) (2, 2)

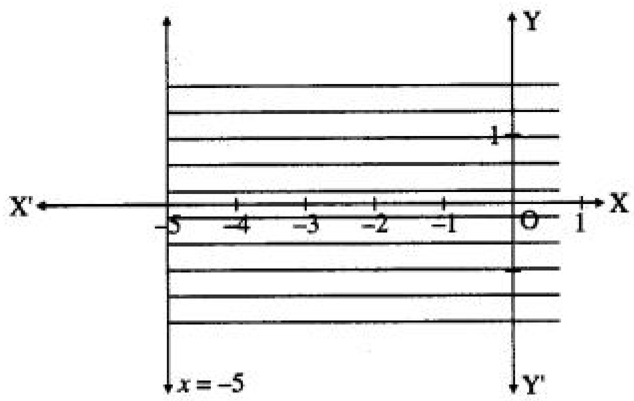
Question 14.
The half plane represented by 3x + 2y < 8 contains the point _______.
(A) (1, 52)
(B) (2, 1)
(C) (0, 0)
(D) (5, 1)
Solution:
(C) (0, 0)

Question 15.
The half plane represented by 4x + 3y > 14 contains the point _______.
(A) (0, 0)
(B) (2, 2)
(C) (3, 4)
(D) (1, 1)
Solution:
(C) (3, 4)

II) Solve the following :
Question 1.
Solve each of the following inequations graphically using X Y plane.
(i) 4x – 18 ≥ 0
Solution:
Consider the line whose equation is 4x – 18 ≥ 0 i.e. x = 184=92 = 4.5
This represents a line parallel to Y-axis passing3through the point (4.5, 0)
Draw the line x = 4.5
To find the solution set we have to check the position of the origin (0, 0).
When x = 0, 4x – 18 = 4 × 0 – 18 = -18 > 0
∴ the coordinates of the origin does not satisfy thegiven inequality.
∴ the solution set consists of the line x = 4.5 and the non-origin side of the line which is shaded in the graph.

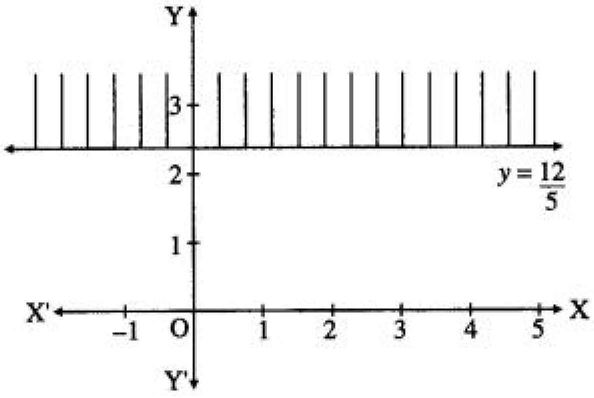


(ii) -11x – 55 ≤ 0
Solution:
Consider the line whose equation is -11x – 55 ≤ 0 i.e. x = -5
This represents a line parallel to Y-axis passing3through the point (-5, 0)
Draw the line x = – 5
To find the solution set we have to check the position of the origin (0, 0).
When x = 0, -11x – 55 = – 11(0) – 55 = -55 > 0
∴ the coordinates of the origin does not satisfy thegiven inequality.
∴ the solution set consists of the line x = -5 and the non-origin side of the line which is shaded in the graph.

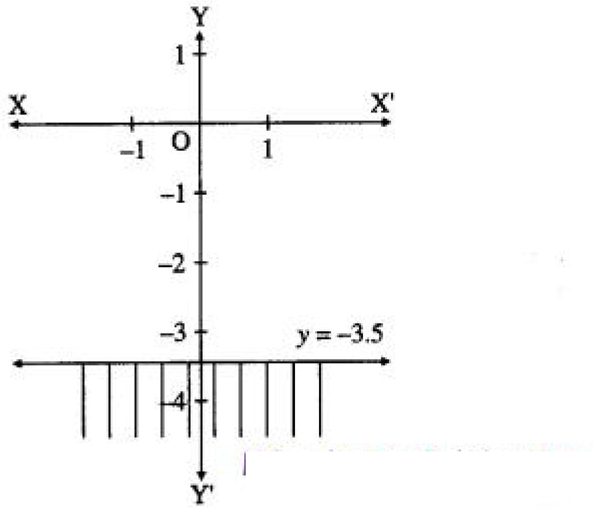


(iii) 5y – 12 ≥ 0
Solution:
Consider the line whose equation is 5y – 12 ≥ 0 i.e. y = 125
This represents a line parallel to X-axis passing through the point (o, 125)
Draw the line y = 125
To find the solution set, we have to check the position of the origin (0, 0).
When y = 0, 5y – 12 = 5(0) – 12 = -12 > 0
∴ the coordinates of the origin does not satisfy the given inequality.

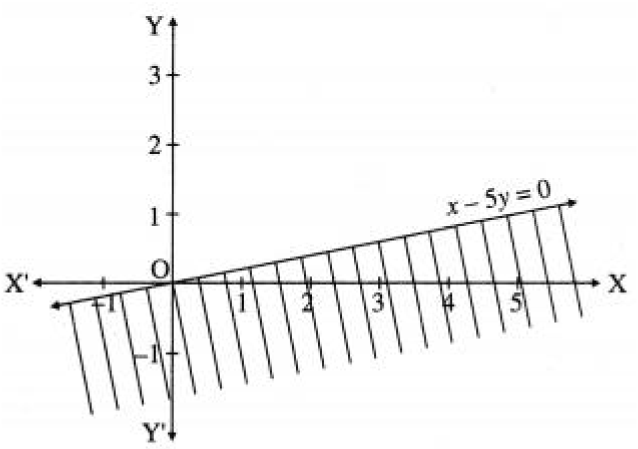
∴ the solution set consists of the line y = 125 and the non-origin side of the line which is shaded in the graph.



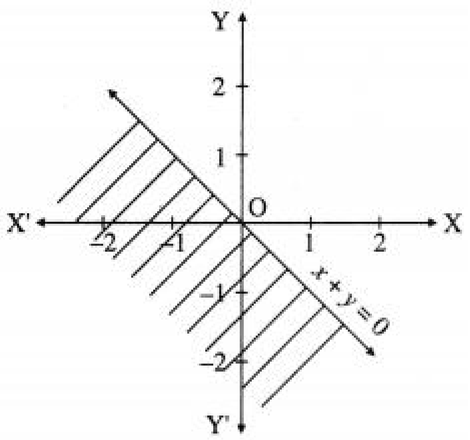
(iv) y ≤ -3.5
Solution:
Consider the line whose equation is y ≤ – 3.5 i.e. y = – 3.5
This represents a line parallel to X-axis passing3through the point (0, -3.5)
Draw the line y = – 3.5
To find the solution set, we have to check the position of the origin (0, 0).
∴ the coordinates of the origin does not satisfy the given inequality.
∴ the solution set consists of the line y = – 3.5 and the non-origin side of the line which is shaded in the graph.



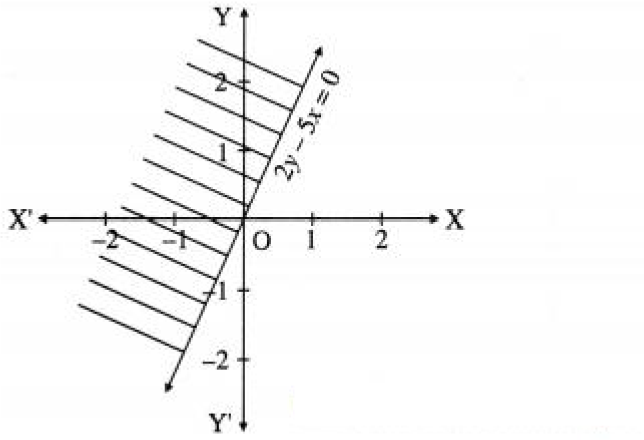
Question 2.
Sketch the graph of each of following inequations in XOY co-ordinate system.
(i) x ≥ 5y
Solution:



(ii) x + y≤ 0
Solution:



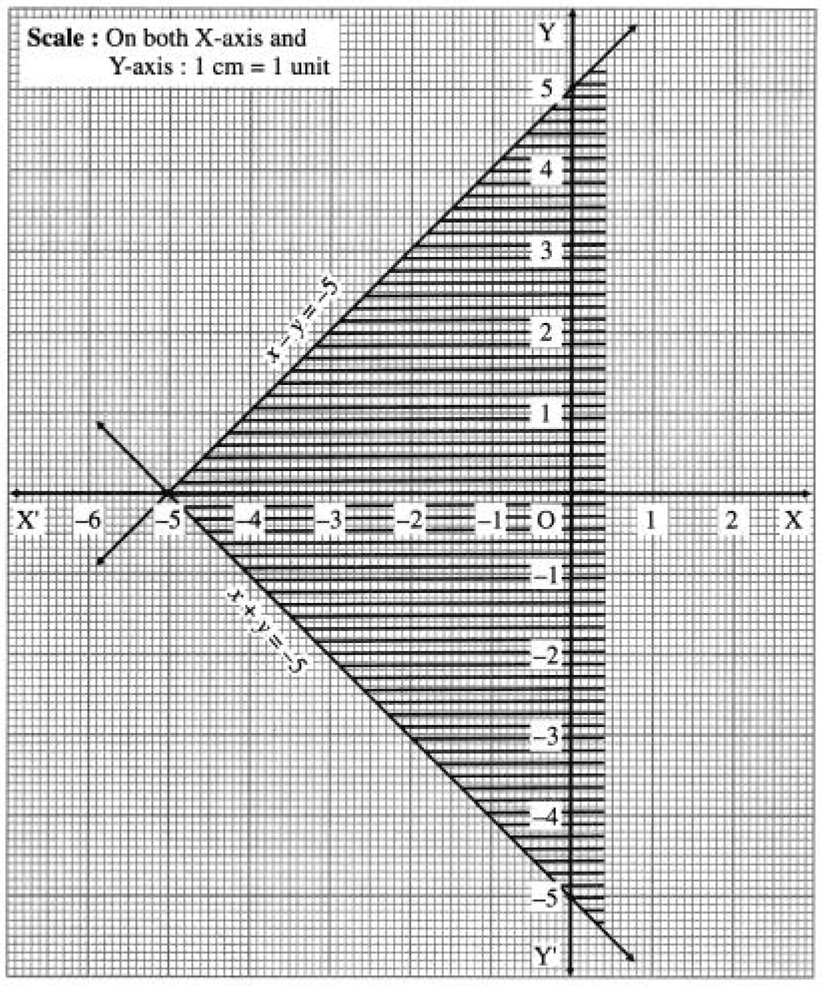
(iii) $2y - 5x \geq 0$
Solution:



(iv) $|x + 5| \leq y$
Solution:
$|x + 5| \leq y$
$\therefore -y \leq x + 5 \leq y$
$\therefore -y \leq x + 5$ and $x + 5 \leq y$
$\therefore x + y \geq -5$ and $x - y \leq -5$
First we draw the lines AB and AC whose equations are
$x + y = -5$ and $x - y = -5$ respectively.

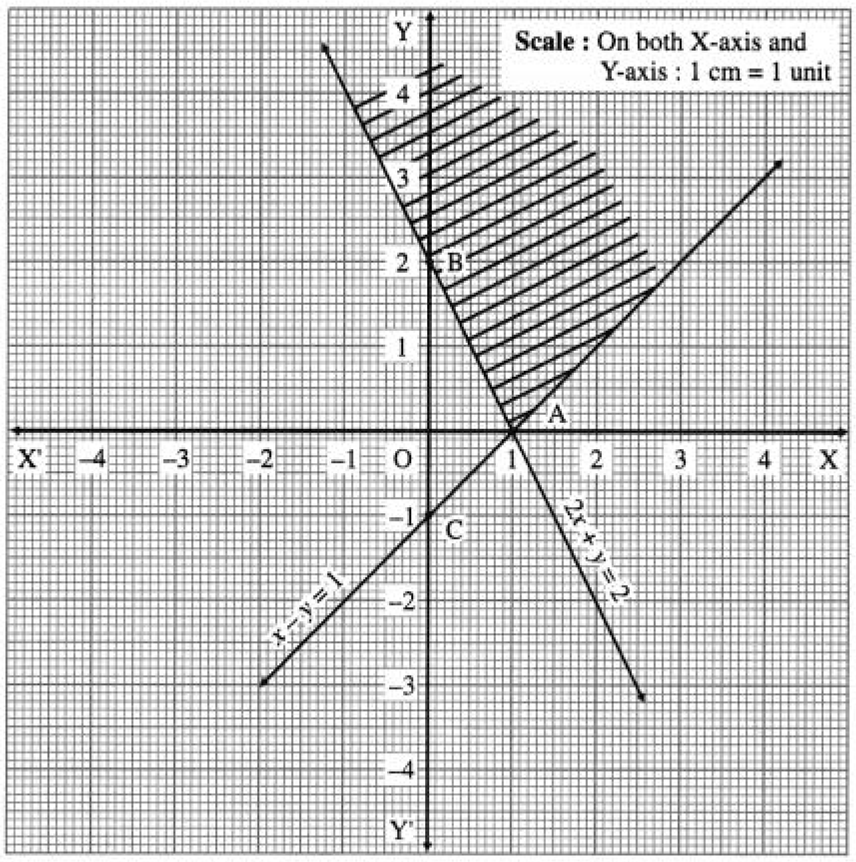
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x+y=-5$	A $(-5, 0)$	B $(0, -5)$	$\geqslant$	origin side of line AB
AC	$x-y=-5$	A $(-5, 0)$	C $(0, 5)$	$\leqslant$	non-origin side of line AC

The graph of $|x + 5| \leq y$ is as below:



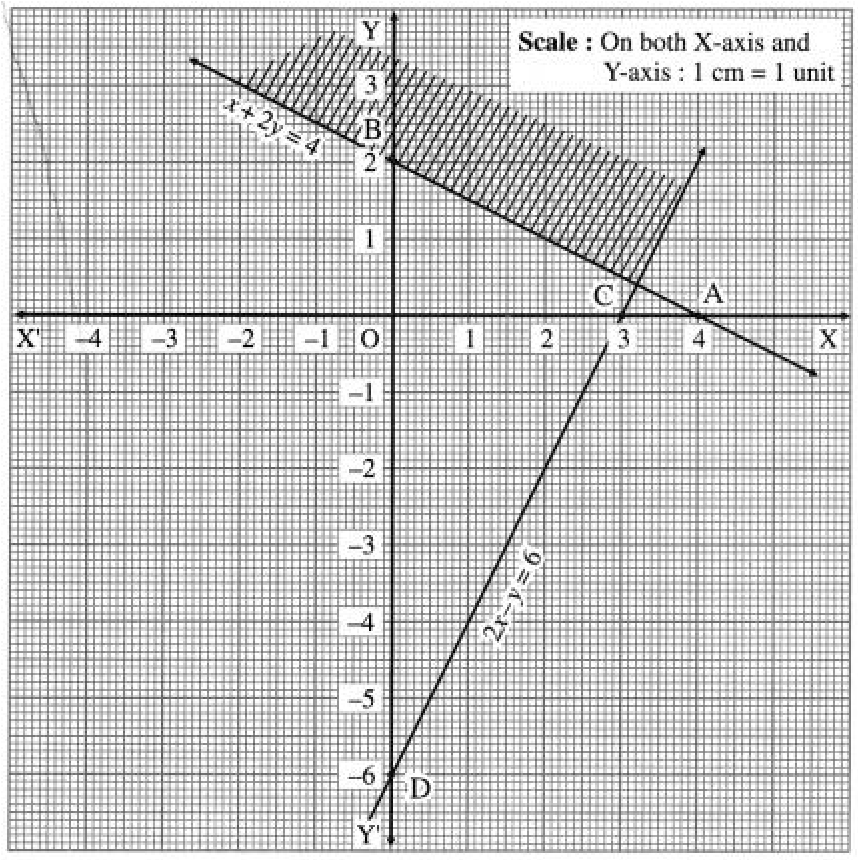
Question 3.
Find graphical solution for each of the following system of linear inequation.
(i) $2x + y \geq 2$, $x - y \leq 1$
Solution:
First we draw the lines AB and AC whose equations are $2x + y = 2$ and $x - y = 1$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$2x + y = 2$	A (1, 0)	B (0, 2)	$\geqslant$	non-origin side of line AB
AC	$x - y = 1$	A (1, 0)	C (0, −1)	$\leqslant$	origin side of line AC



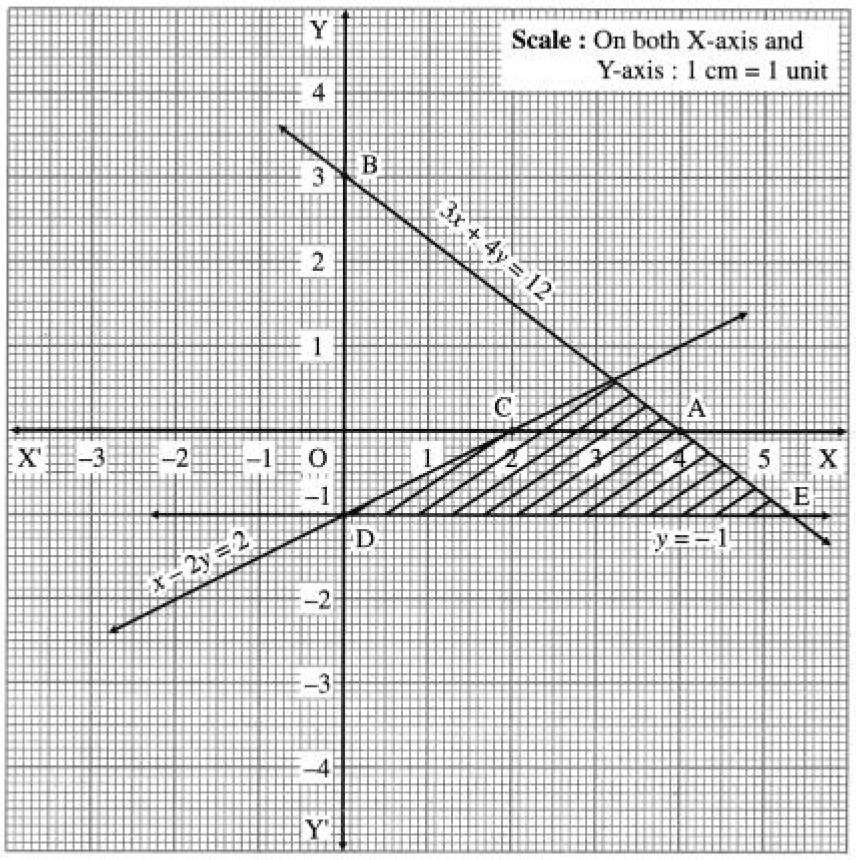
The solution set of the given system of inequalities is shaded in the graph.

(ii) x + 2y ≥ 4, 2x – y ≤ 6
Solution:



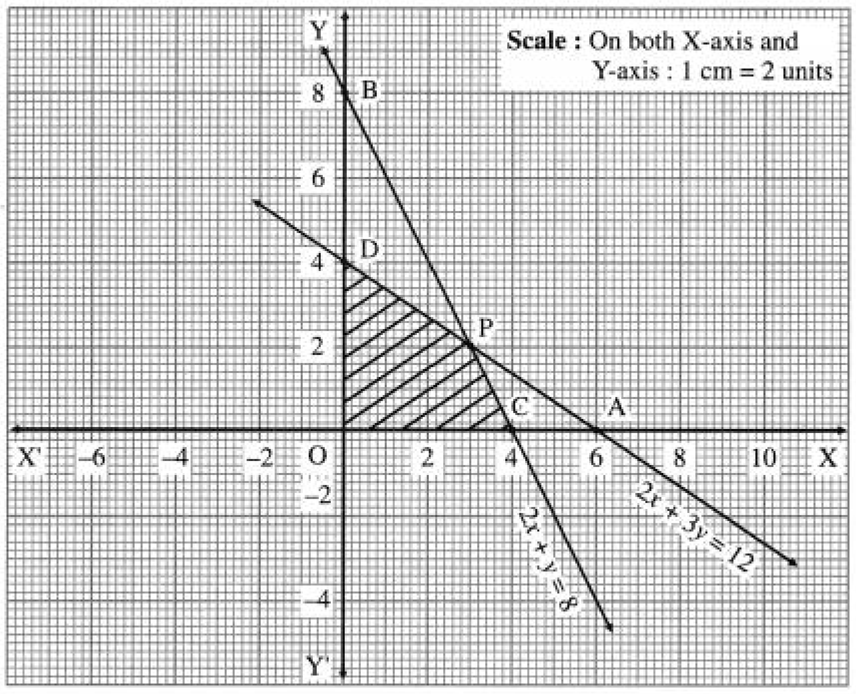
(iii) 3x + 4y ≤ 12, x – 2y ≥ 2, y ≥ -1
Solution:
First we draw the lines AB, CD and ED whose equations are 3x + 4y = 12, x – 2y = 2 and y = -1 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x + 4y = 12$	A (4, 0)	B (0, 3)	$\leqslant$	origin side of line AB
CD	$x - 2y = 2$	C (2, 0)	D (0, −1)	$\geqslant$	non-origin side of line CD
ED	$y = -1$	—	D (0, −1)	$\geqslant$	origin side of line ED



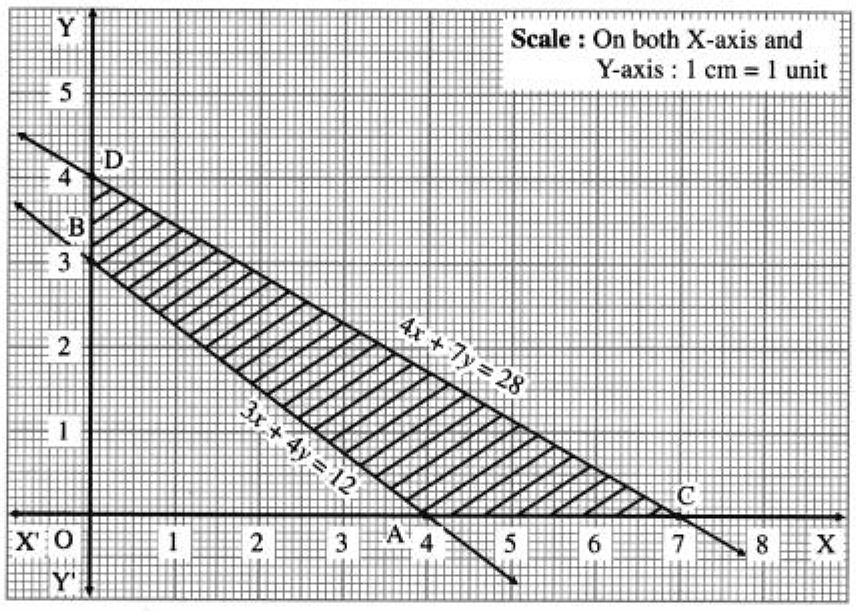
The solution set of given system of inequation is shaded in the graph.

Question 4.
Find feasible solution for each of the following system of linear inequations graphically.
(i) $2x + 3y \leq 12$, $2x + y \leq 8$, $x \geq 0$, $y \geq 0$
Solution:



The feasible solution is OCPBO.

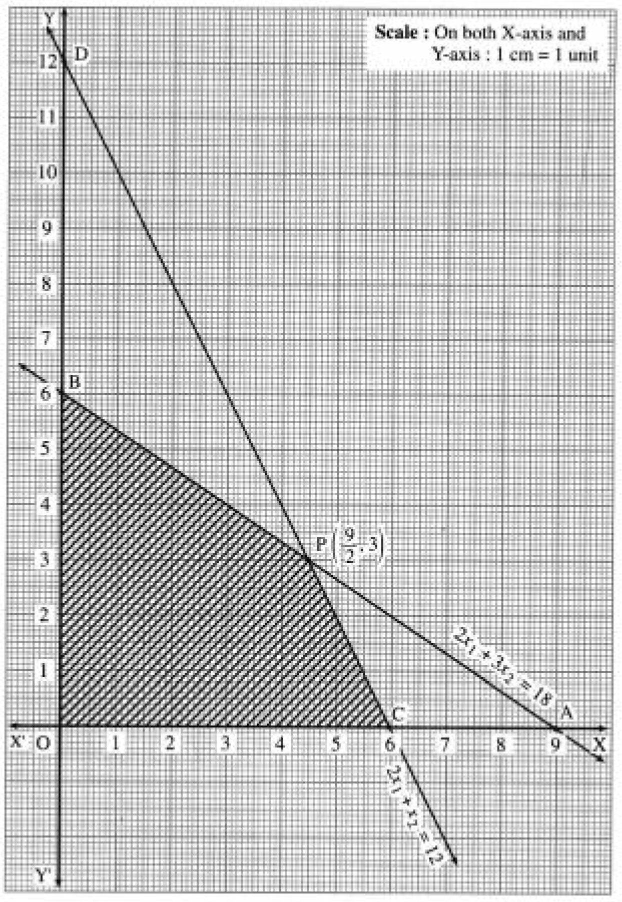
(ii) $3x + 4y \geq 12$, $4x + 7y \leq 28$, $x \geq 0$, $y \geq 0$
Solution:



The feasible solution is ACDBA.

Question 5.
Solve each of the following L.P.P.
(i) Maximize $z = 5x_1 + 6x_2$ subject to $2x_1 + 3x_2 \leq 18$, $2x_1 + x_2 \leq 12$, $x_1 \geq 0$, $x_2 \geq 0$
Solution:
First we draw the lines AB and CD whose equations are $2x_1 + 3x_2 = 18$ and $2x_1 + x_2 = 12$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$2x_1+3x_2=18$	A(9, 0)	B(0, 6)	$\leqslant$	origin side of line AB
CD	$2x_1+x_2=12$	C(6, 0)	O(0, 12)	$\leqslant$	origin side of line CD



The feasible region is OCPBO which is shaded in the graph. The vertices of the feasible region are O(0, 0), C(6, 0), P and B (0,6).
P is the point of intersection of the lines
$2x_1 + 3x_2 = 18$ ....(1)
and $2x_1 + x_2 = 12$
On subtracting, we get
$2x_2 = 6 \therefore x_2 = 3$
Substituting $x_2 = 3$ in (2), we get
$2x_1 + 3 = 12 \therefore x_2 = 9$
$\therefore$ P is (৭2, 3)
The values of objective function $z = 5x_1 + 6x_2$ at these vertices are
$z(O) = 5(0) + 6(0) = 0 + 0 = 0$

z(C) = 5(6) + 6(0) = 30 + 0 = 30

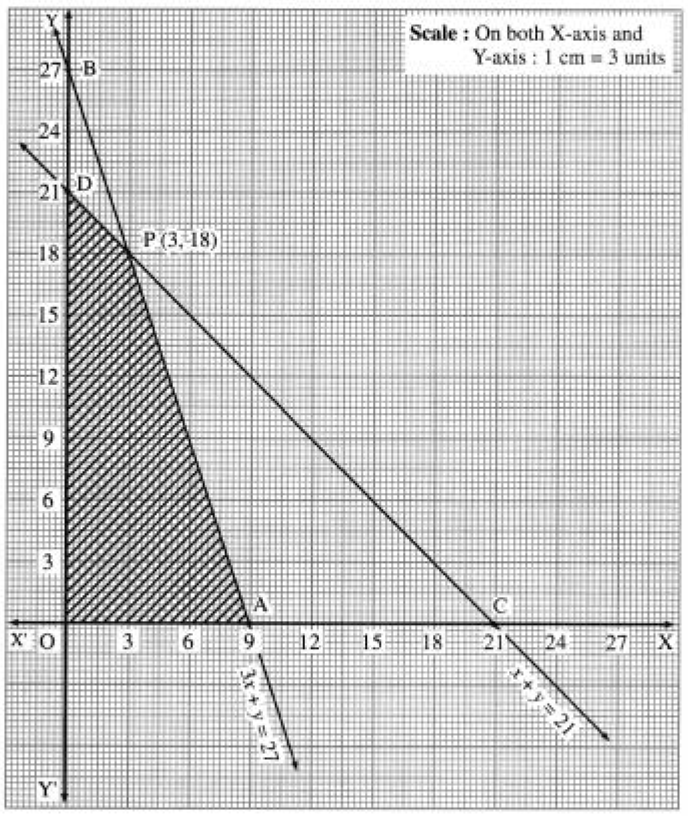
z(P) = 5(92) + 6(3) = 452 + 18 = 45+362=812 = 40.5

z(B) = 5(0) + 6(3) = 0 + 18 = 18

Maximum value of z is 40.5 when x_1 = 9/2, y = 3.

(ii) Maximize z = 4x + 2y subject to 3x + y ≥ 27, x + y ≥ 21
Question is modified.
Maximize z = 4x + 2y subject to 3x + y ≤ 27, x + y ≤ 21, x ≥ 0, y ≥ 0
Solution:
First we draw the lines AB and CD whose equations are 3x + y = 27 and x + y = 21 respectively.

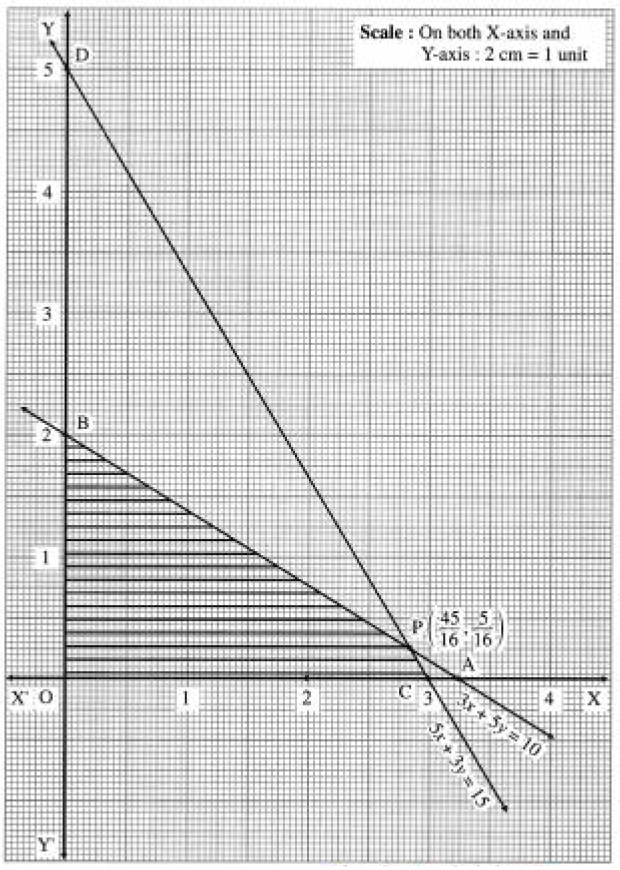
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x + y = 27$	A (9, 0)	B (0, 27)	≤	origin side of line AB
CD	$x + y = 21$	C (21, 0)	O (0, 21)	≤	origin side of line CD



The feasible region is OAPDO which is shaded region in the graph. The vertices of the feasible region are 0(0, 0), A (9, 0), P and D(0, 21).
P is the point of intersection of lines
3x + y = 27 ... (1)
and x + y = 21 ... (2)
On substracting, we get 2x = 6 ∴ x = 3
Substituting x = 3 in equation (1), we get
9 + y = 27 ∴ y = 18
∴ P = (3, 18)
The values of the objective function z = 4x + 2y at these vertices are
z(O) = 4(0) + 2(0) = 0 + 0 = 0
z(a) = 4(9) + 2(0) = 36 + 0 = 36
z(P) = 4(3) + 2(18) = 12 + 36 = 48
z (D) = 4(0) + 2(21) = 0 + 42 = 42
∴ 2 has minimum value 48 when x = 3, y = 18.

(iii) Maximize z = 6x + 10y subject to 3x + 5y ≤ 10, 5x + 3y ≤ 15, x ≥ 0, y ≥ 0
Solution:
First we draw the lines AB and CD whose equations are 3x + 5y = 10 and 5x + 3y = 15 respectively.

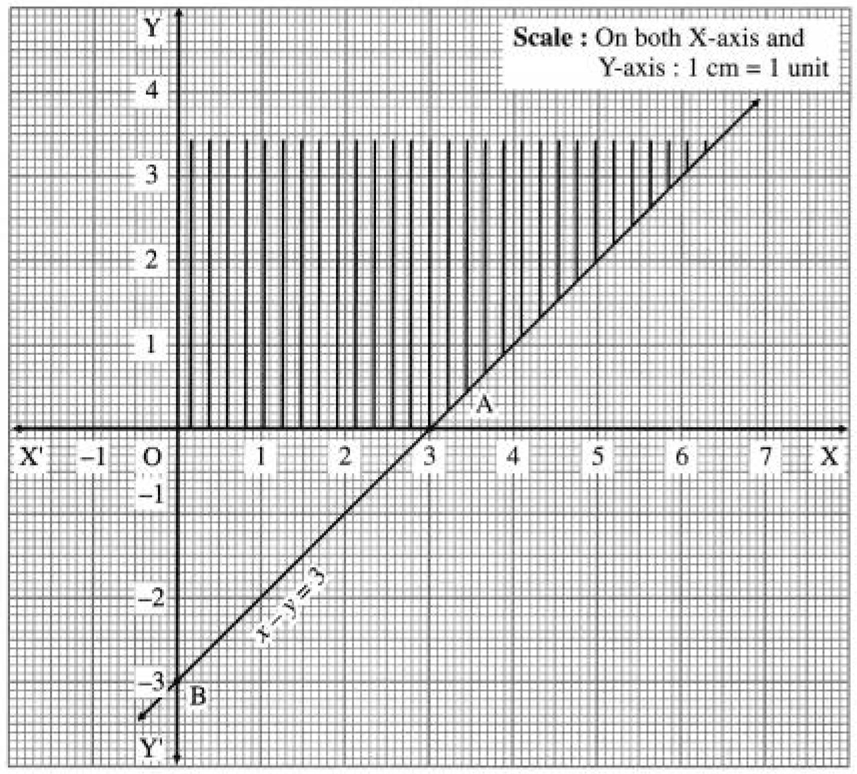
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x + 5y = 10$	$A\left(\frac{10}{3}, 0\right)$	$B(0, 2)$	$\leqslant$	origin side of the line AB
CD	$5x + 3y = 15$	$C(3, 0)$	$D(0, 5)$	$\leqslant$	origin side of the line CD



The feasible region is OCPBD which is shaded in the graph.
The vertices of the feasible region are 0(0, 0), C(3, 0), P and B (0, 2).
P is the point of intersection of the lines
3x + 5y = 10 … (1)
and 5x + 3y = 15 … (2)
Multiplying equation (1) by 5 and equation (2) by 3, we get
15x + 25y = 50
15x + 9y = 45
On subtracting, we get
16y = 5 ∴ y = 516
Substituting y = 516 in equation (1), we get
3x + 2516 = 10 ∴ 3x = 10 – 2516=13516
∴ x = 4516 ∴ P ≡ (4516,516)
The values of objective function z = 6x + 10y at these vertices are
z(O) = 6(0) + 10(0) = 0 + 0 = 0
z(C) = 6(3) + 10(0) = 18 + 0 = 18
z(P) = 6(4516) + 10(510) = 27016+5016=32016 = 20
z(B) = 6(0) + 10(2) = 0 + 20 = 20
The maximum value of z is 20 at P(4516,516) and B (0, 2) two consecutive vertices.
∴ z has maximum value 20 at each point of line segment PB where B is (0, 2) and P is (4516,516).
Hence, there are infinite number of optimum solutions.

(iv) Maximize z = 2x + 3y subject to x – y ≥ 3, x ≥ 0, y ≥ 0
Solution:
First we draw the lines AB whose equation is x – y = 3.

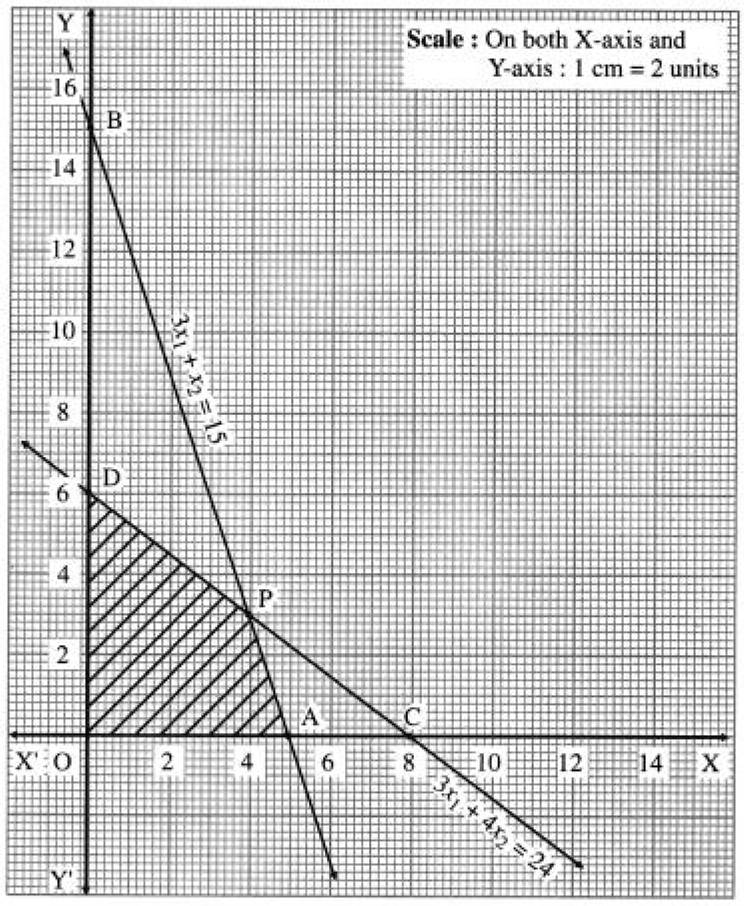
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x - y = 3$	A (3, 0)	B (0, −3)	$\geqslant$	non-origin side of line AB



The feasible region is shaded which is unbounded.
Therefore, the value of objective function can be in- j creased indefinitely. Hence, this LPP has unbounded solution.

Question 6.
Solve each of the following L.P.P.
(i) Maximize $z = 4x_1 + 3x_2$ subject to $3x_1 + x_2 \leq 15$, $3x_1 + 4x_2 \leq 24$, $x_1 \geq 0$, $x_2 \geq 0$
Solution:
We first draw the lines AB and CD whose equations are $3x_1 + x_2 = 15$ and $3x_1 + 4x_2 = 24$ respectively.

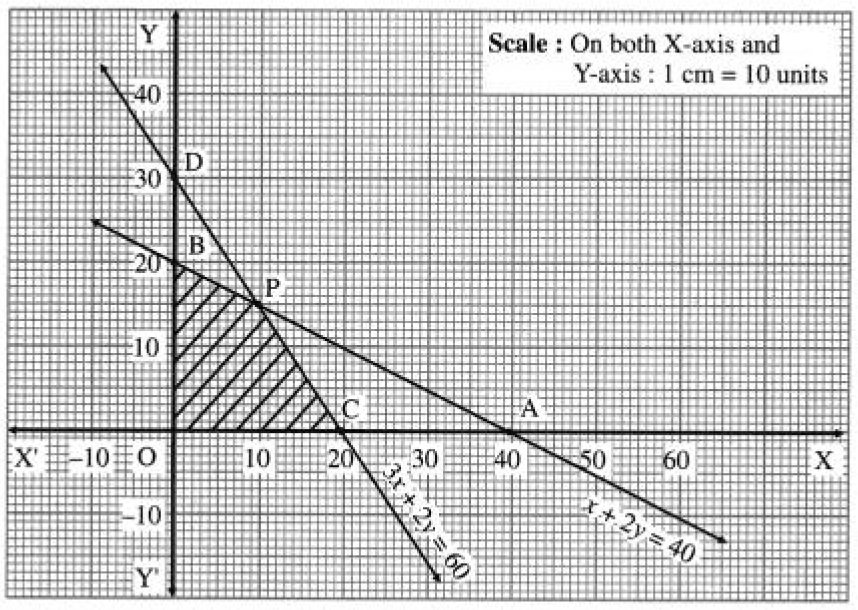
Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x_1 + x_2 = 15$	A (5, 0)	B (0, 15)	$\leqslant$	origin side of the line AB
CD	$3x_1 + 4x_2 = 24$	C (8, 0)	D (0, 6)	$\leqslant$	origin side of the line CD



The feasible region is OAPDO which is shaded in the graph.
The Vertices of the feasible region are 0(0, 0), A(5, 0), P and D(0, 6).
P is the point of intersection of lines.
$3x_1 + 4x_2 = 24$ … (1)
and $3x_1 + x_2 = 15$ … (2)
On subtracting, we get
$3x_2 = 9 \therefore x_2 = 3$
Substituting $x_2 = 3$ in (2), we get
$3x_1 + 3 = 15$
$\therefore 3x_1 = 12 \therefore x_1 = 4 \therefore$ P is (4, 3)
The values of objective function $z = 4x_1 + 3x_2$ at these vertices are
z(O) = 4(0) + 3(0) = 0 + 0 = 0
z(a) = 4(5) + 3(0) = 20 + 0 = 20
z(P) = 4(4) + 3(3) = 16 + 9 = 25
z(D) = 4(0) + 3(6) = 0 + 18 = 18
$\therefore$ z has maximum value 25 when x = 4 and y = 3.

(ii) Maximize z = 60x + 50y subject to x + 2y ≤ 40, 3x + 2y ≤ 60, x ≥ 0, y ≥ 0
Solution:
We first draw the lines AB and CD whose equations are x + 2y = 40 and 3x + 2y = 60 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x+2y=40$	A (40, 0)	B (0, 20)	$\leqslant$	origin side of line AB
CD	$3x+2y=60$	C (20, 0)	D (0, 30)	$\leqslant$	origin side of line CD

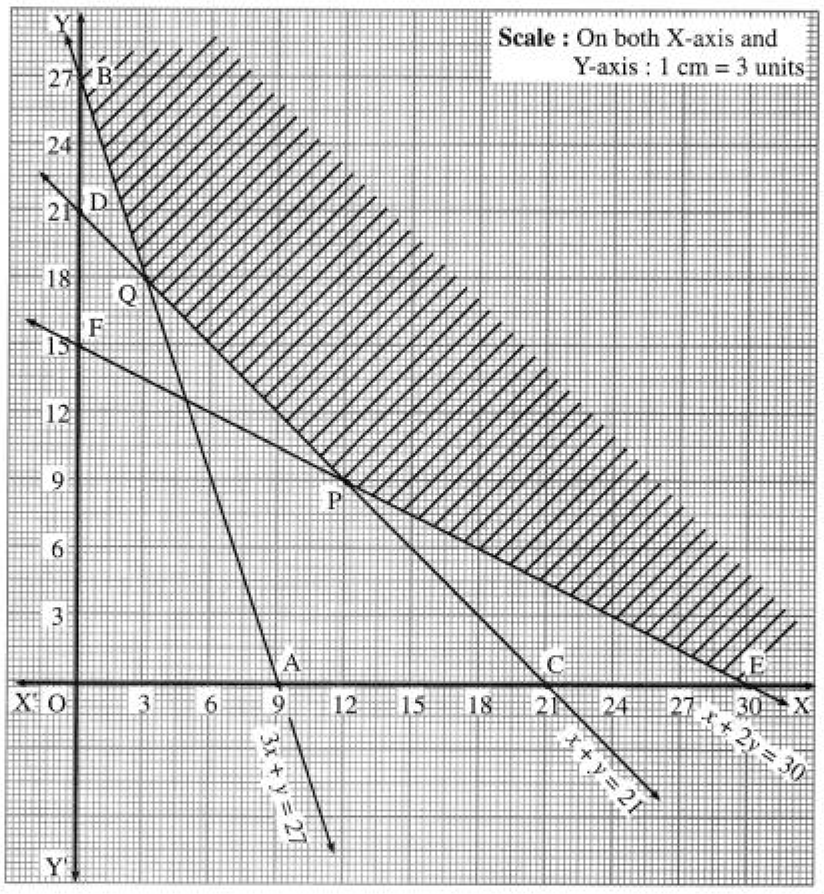


The feasible region is OCPBO which is shaded in the graph.
The vertices of the feasible region are O (0, 0), C (20, 0), P and B (0, 20).
P is the point of intersection of the lines.

3x + 2y = 60 ... (1)
and x + 2y = 40 ... (2)
On subtracting, we get
2x = 20 ∴ x = 10
Substituting x = 10 in (2), we get
10 + 2y = 40
∴ 2y = 30 ∴ y = 15 ∴ P is (10, 15)
The values of the objective function z = 60x + 50y at these vertices are
z(O) = 60(0) + 50(0) = 0 + 0 = 0
z(C) = 60(20) + 50(0) = 1200 + 0 = 1200
z(P) = 60(10) + 50(15) = 600 + 750 = 1350
z(B) = 60(0) + 50(20) = 0 + 1000 = 1000 .
∴ z has maximum value 1350 at x = 10, y = 15.

(iii) Maximize z = 4x + 2y subject to 3x + y ≥ 27, x + y ≥ 21, x + 2y ≥ 30; x ≥ 0, y ≥ 0
Solution:
We first draw the lines AB, CD and EF whose equations are 3x + y = 27, x + y = 21, x + 2y = 30 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x+y=27$	A (9, 0)	B (0, 27)	$\geqslant$	non-origin side of line AB
CD	$x+y=21$	C (21, 0)	D (0, 21)	$\geqslant$	non-origin side of line CD
EF	$x+2y=30$	E (30, 0)	F (0, 15)	$\geqslant$	non-origin side of line EF



The feasible region is XEPQBY which is shaded in the graph.
The vertices of the feasible region are E (30,0), P, Q and B (0,27).
P is the point of intersection of the lines
x + 2y = 30 ... (1)
and x + y = 21 ... (2)
On subtracting, we get
y = 9
Substituting y = 9 in (2), we get
x + 9 = 21 ∴ x = 12
∴ P is (12, 9)
Q is the point of intersection of the lines
x + y = 21 ... (2)

and $3x + y = 27$ ... (3)
On subtracting, we get
$2x = 6 \therefore x = 3$
Substituting $x = 3$ in (2), we get
$3 + y = 21 \therefore y = 18$
$\therefore$ Q is (3, 18).
The values of the objective function $z = 4x + 2y$ at these vertices are
$z(E) = 4(30) + 2(0) = 120 + 0 = 120$
$z(P) = 4(12) + 2(9) = 48 + 18 = 66$
$z(Q) = 4(3) + 2(18) = 12 + 36 = 48$
$z(B) = 4(0) + 2(27) = 0 + 54 = 54$
$\therefore$ z has minimum value 48, when $x = 3$ and $y = 18$.

Question 7.
A carpenter makes chairs and tables. Profits are ₹140/- per chair and ₹ 210/- per table. Both products are processed on three machines : Assembling, Finishing and Polishing. The time required for each product in hours and availability of each machine is given by following table:

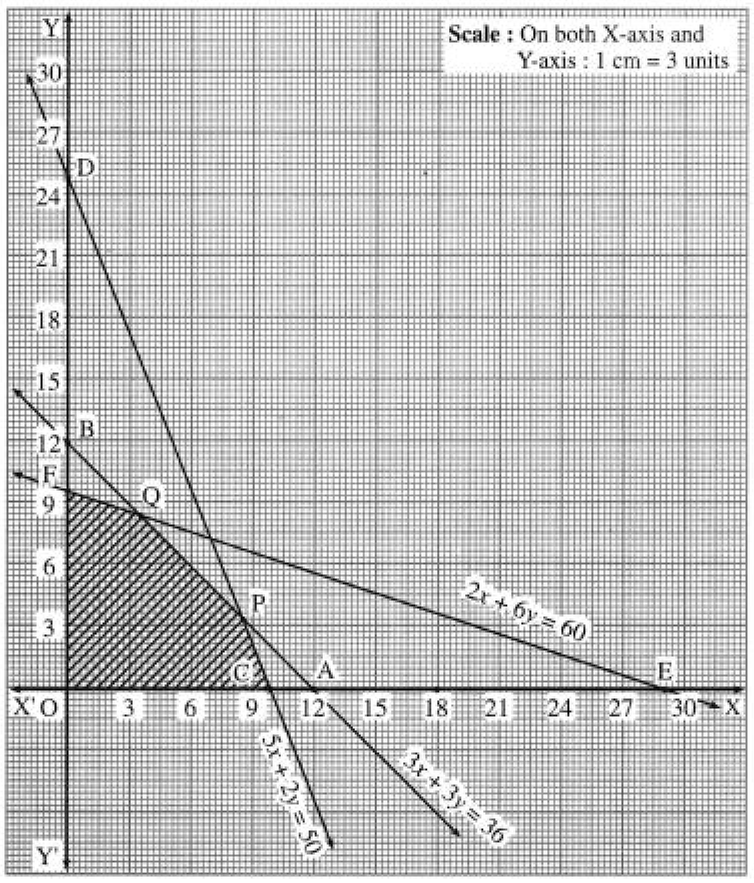
Product Machine	Chair (x)	Table (y)	Available time (hours)
Assembling	3	3	36
Finishing	5	2	50
Polishing	2	6	60

Formulate the above problem as L.P.P. Solve it graphically to get maximum profit.
Solution:
Let the number of chairs and tables made by the carpenter be x and y respectively.
The profits are ₹ 140 per chair and ₹ 210 per table.
$\therefore$ total profit z = ₹ $(140x + 210y)$ This is the objective function which is to be maximized.
The constraints are as per the following table :

	Chair (x)	Table (y)	Available time (hours)
Assembling	3	3	36
Finishing	5	2	50
Polishing	2	6	60

From the table, the constraints are
$3x + 3y \le 36$, $5x + 2y \le 50$, $2x + 6y \le 60$.
The number of chairs and tables cannot be negative.
$\therefore x \ge 0, y \ge 0$
Hence, the mathematical formulation of given LPP is :
Maximize $z = 140x + 210y$, subject to
$3x + 3y \le 36$, $5x + 2y \le 50$, $2x + 6y \le 60$, $x \ge 0$, $y \ge 0$.
We first draw the lines AB, CD and EF whose equations are $3x + 3y = 36$, $5x + 2y = 50$ and $2x + 6y = 60$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x+3y=36$	A (12, 0)	B (0, 12)	$\leqslant$	origin side of line AB
CD	$5x+2y=50$	C (10, 0)	D (0, 25)	$\leqslant$	origin side of line CD
EF	$2x+6y=60$	E (30, 0)	F (0, 10)	$\leqslant$	origin side of line EF



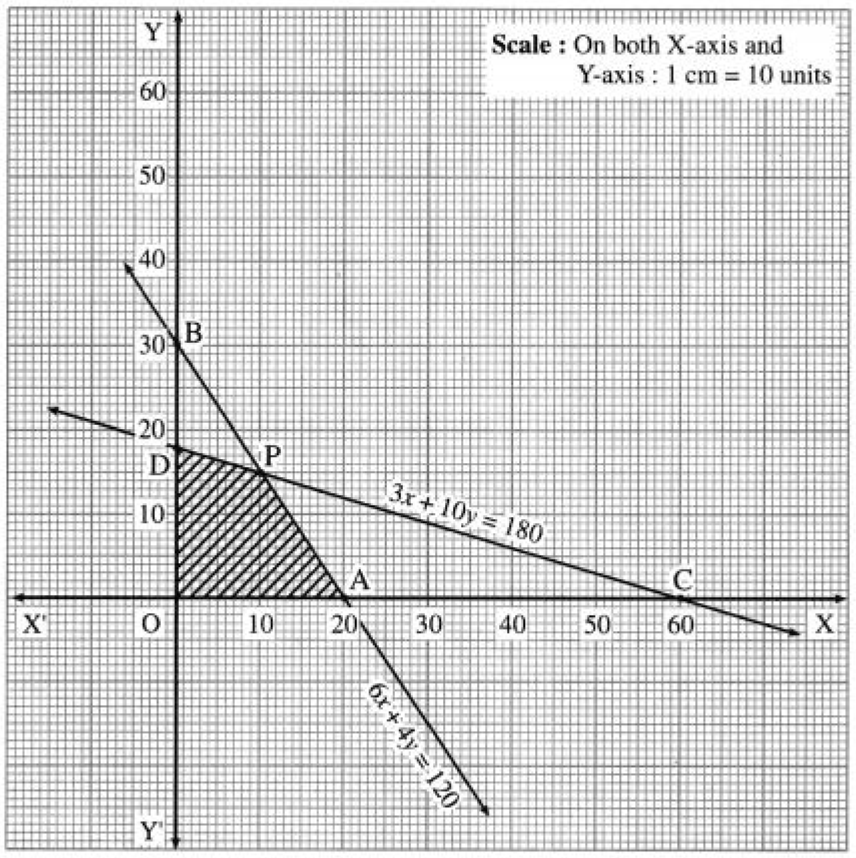
The feasible region is OCPQFO which is shaded in the graph.
The vertices of the feasible region are O (0, 0), C (10, 0), P, Q and F (0, 10).
P is the point of intersection of the lines
5x + 2y = 50 … (1)
and 3x + 3y = 36 … (2)
Multiplying equation (1) by 3 and equation (2) by 2, we get
15x + 6y = 150
6x + 6y = 72
On subtracting, we get 26
9x = 78 ∴ x = 263
Substituting x = 263 in (2), we get
3(263) + 3y = 36
3y = 1o y = 103
Q is the point of intersection of the lines
3x + 3y = 36 … (2)
and 2x + 6y = 60 … (3)
Multiplying equation (2) by 2, we get
6x + 6 y = 72
Subtracting equation (3) from this equation, we get
4x = 12 ∴ x = 3
Substituting x = 3 in (2), we get
3(3) + 3y = 36
∴ 3y = 27 ∴ y = 9
∴ Q is (3, 9).
Hence, the vertices of the feasible region are O (0, 0),
C(10, 0), P(263,103), Q(3, 9) and F(0, 10).
The values of the objective function z = 140x + 210y at these vertices are
z(O) = 140(0) + 210(0) = 0 + 0 = 0
z(C) = 140 (10) + 210(0) = 1400 + 0 = 1400
z(P) = 140(263) + 210(103) = 3640+21003=57403 = 1913.33
z(Q) = 140(3) + 210(9) = 420 + 1890 = 2310
z(F) = 140(0) + 210(10) = 0 + 2100 = 2100
∴ z has maximum value 2310 when x = 3 and y = 9 Hence, the carpenter should make 3 chairs and 9 tables to get the maximum profit of ₹ 2310.

Question 8.
A company manufactures bicycles and tricycles, each of which must be processed through two machines A and B. Maximum availability of Machine A and B is respectively 120 and 180 hours. Manufacturing a bicycle requires 6 hours on Machine A and 3 hours on Machine B. Manufacturing a tricycles requires 4 hours on Machine A and 10 hours on Machine B. If profits are ₹180/- for a bicycle and ₹220/- for a tricycle. Determine the number of bicycles and tricycles that should be manufactured in order to maximize the profit.
Solution:
Let x bicycles and y tricycles are to be manu¬factured. Then the total profit is z = ₹ (180x + 220y)
This is a linear function which is to be maximized. Hence, it is the objective function. The constraints are as per the following table :

Machine	Bicycle (x)	Tricycle (y)	Maximum hours available
A	6	4	120
B	3	10	180

From the table, the constraints are
6x + 4y ≤ 120, 3x +10y ≤ 180
Also, the number of bicycles and tricycles cannot be i negative.
∴ x ≥ 0, y ≥ 0.
Hence, the mathematical formulation of given LPP is :
Maximize z = 180x + 220y, subject to
6x + 4y ≤ 120, 3x + 10y ≤ 180, x ≥ 0, y ≥ 0.
First we draw the lines AB and CD whose equations are 6x + 4y = 120 and 3x + 10y = 180 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$6x+4y=120$	A (20, 0)	B (0, 30)	$\leqslant$	origin side of line AB
CD	$3x+10y=180$	C (60, 0)	D (0, 18)	$\leqslant$	origin side of line CD



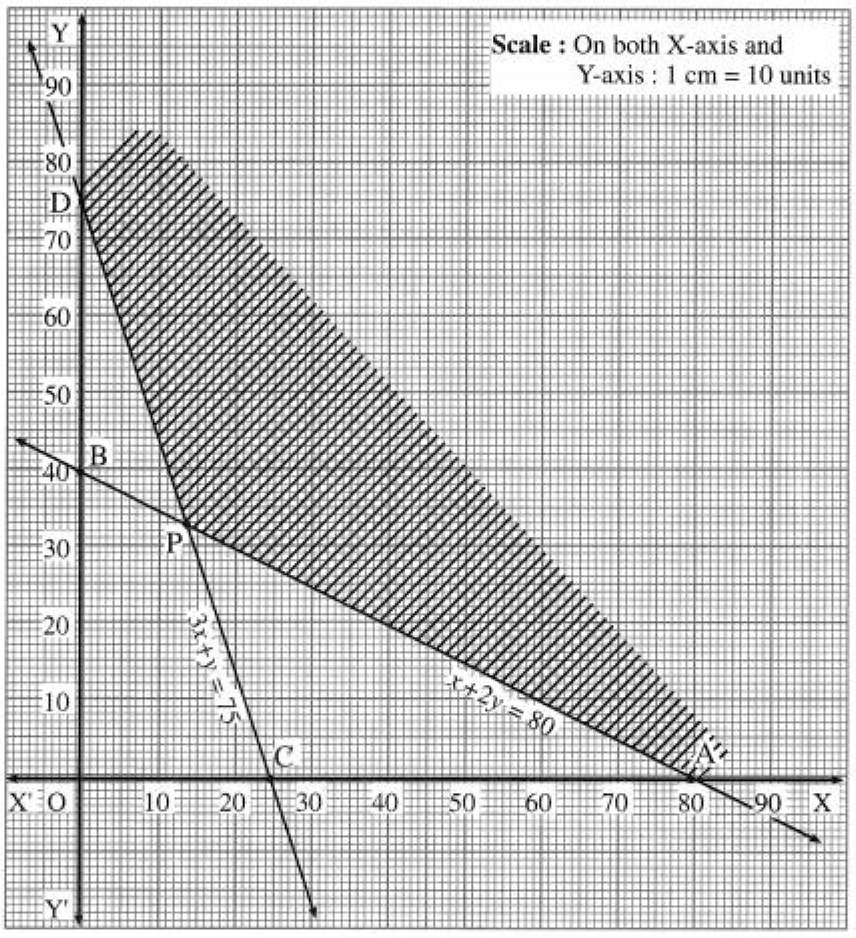
The feasible region is OAPDO which is shaded in the graph.
The vertices of the feasible region are O(0, 0), A(20, 0) P and D(0, 18).
P is the point of intersection of the lines
3x + 10y = 180 … (1)
and 6x + 4y = 120 … (2)
Multiplying equation (1) by 2, we get
6x + 20y = 360
Subtracting equation (2) from this equation, we get
16y = 240 ∴ y = 15
∴ from (1), 3x + 10(15) = 180
∴ 3x = 30 ∴ x = 10
∴ P = (10, 15)
The values of the objective function z = 180x + 220y at these vertices are
z(O) = 180(0) + 220(0) = 0 + 0 = 0
z(a) = 180(20) + 220(0) = 3600 + 0 = 3600
z(P) = 180(10) + 220(15) = 1800 + 3300 = 5100
z(D) = 180(0) +220(18) = 3960
∴ the maximum value of z is 5100 at the point (10, 15).
Hence, 10 bicycles and 15 tricycles should be manufactured in order to have the maximum profit of ₹ 5100.

Question 9.
A factory produced two types of chemicals A and B. The following table gives the units of ingredients P and Q (per kg) of chemicals A and B as well as minimum requirements of P and Q and also cost per kg. chemicals A and B :

Chemicals in units / Ingredients per kg.	A (x)	B (y)	Minimum requirements in units
P	1	2	80
Q	3	1	75
Cost (in Rs.)	4	6	--

Find the number of units of chemicals A and B should be produced so as to minimize the cost.
Solution:
Let the factory produce x units of chemical A and y units of chemical B. Then the total cost is z = ₹ (4x + 6y). This is the objective function which is to be minimized.
From the given table, the constraints are
$x + 2y \geq 80$, $3x + y \geq 75$.
Also, the number of units x and y of chemicals A and B cannot be negative.
$\therefore x \geq 0, y \geq 0$.
∴ the mathematical formulation of given LPP is
Minimize $z = 4x + 6y$, subject to
$x + 2y \geq 80$, $3x + y \geq 75$, $x \geq 0$, $y \geq 0$.
First we draw the lines AB and CD whose equations are $x + 2y = 80$ and $3x + y = 75$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x + 2y = 80$	A (80, 0)	B (0, 40)	$\geqslant$	non-origin side of line AB
CD	$3x + y = 75$	C (25, 0)	D (0, 75)	$\geqslant$	non-origin side of line CD



The feasible region is shaded in the graph.
The vertices of the feasible region are A (80, 0), P and D (0, 75).
P is the point of intersection of the lines
$x + 2y = 80$ ... (1)
and $3x + y = 75$ ... (2)
Multiplying equation (2) by 2, we get
$6x + 2y = 150$
Subtracting equation (1) from this equation, we get
$5x = 70 \therefore x = 14$
∴ from (2), $3(14) + y = 75$
$\therefore 42 + y = 75 \therefore y = 33$
∴ P = (14, 33)
The values of the objective function $z = 4x + 6y$ at these vertices are
$z(a) = 4(80) + 6(0) = 320 + 0 = 320$

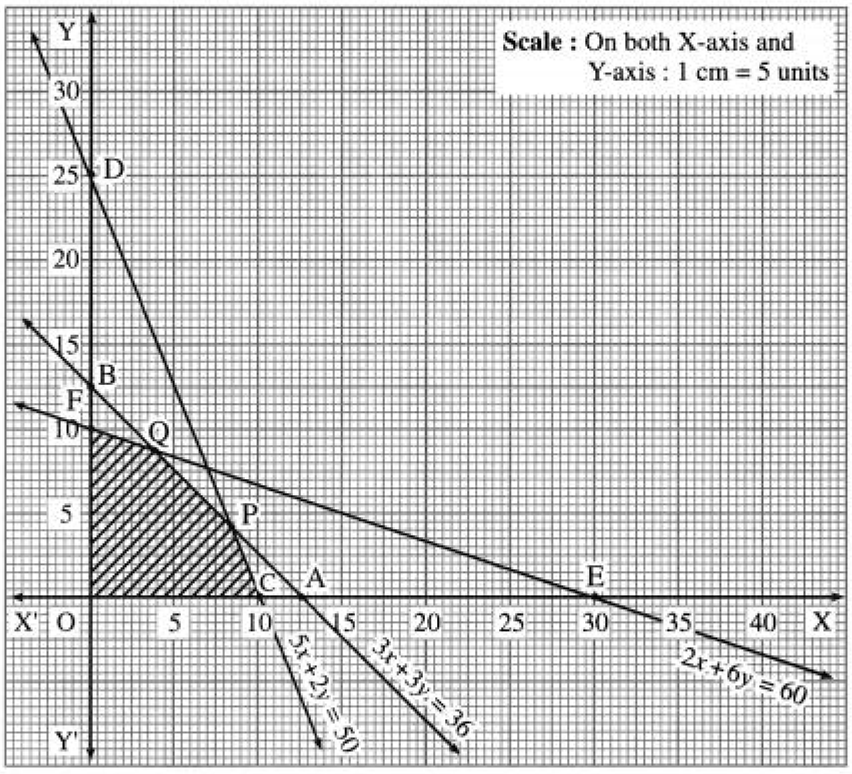
z(P) = 4(14)+ 6(33) = 56+ 198 = 254
z(D) = 4(0) + 6(75) = 0 + 450 = 450
∴ the minimum value of z is 254 at the point (14, 33).
Hence, 14 units of chemical A and 33 units of chemical B are to be produced in order to have the j minimum cost of ₹ 254.

Question 10.
A company produces mixers and food processors. Profit on selling one mixer and one food processor is ₹ 2,000/- and ₹ 3,000/- respectively. Both the products are processed through three Machines A, B, C. The time required in hours by each product and total time available in hours per week on each machine are as follows :

Machine \ Product	Mixer (per unit)	Food Processor (per unit)	Available time
$A \to A$	3	3	36
$B \to B$	5	2	50
$C \to C$	2	6	60

How many mixers and food processors should be produced to maximize the profit?
Solution:
Let the company produce x mixers and y food processors.
Then the total profit is z = ₹ (2000x + 3000y)
This is the objective function which is to be maximized. From the given table in the problem, the constraints are $3x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$
Also, the number of mixers and food processors cannot be negative,
∴ $x \geq 0, y \geq 0$.
∴ the mathematical formulation of given LPP is
Maximize z = 2000x + 3000y, subject to $3x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$, $x \geq 0$, $y \geq 0$.
First we draw the lines AB, CD and EF whose equations are 3x + 3y = 36, 5x + 2y = 50 and 2x + 6y = 60 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x+3y=36$	A(12, 0)	B(0, 12)	$\leqslant$	origin side of line AB
CD	$5x+2y=50$	C(10, 0)	D(0, 25)	$\leqslant$	origin side of line CD
EF	$2x+6y=60$	E(30, 0)	F(0, 10)	$\leqslant$	origin side of line EF



The feasible region is OCPQFO which is shaded in the graph.
The vertices of the feasible region are O(0, 0), C(10, 0), P, Q and F(0,10).
P is the point of intersection of the lines
3x + 3y = 36 … (1)
and 5x + 2y = 50 … (2)
Multiplying equation (1) by 2 and equation (2) by 3, we get

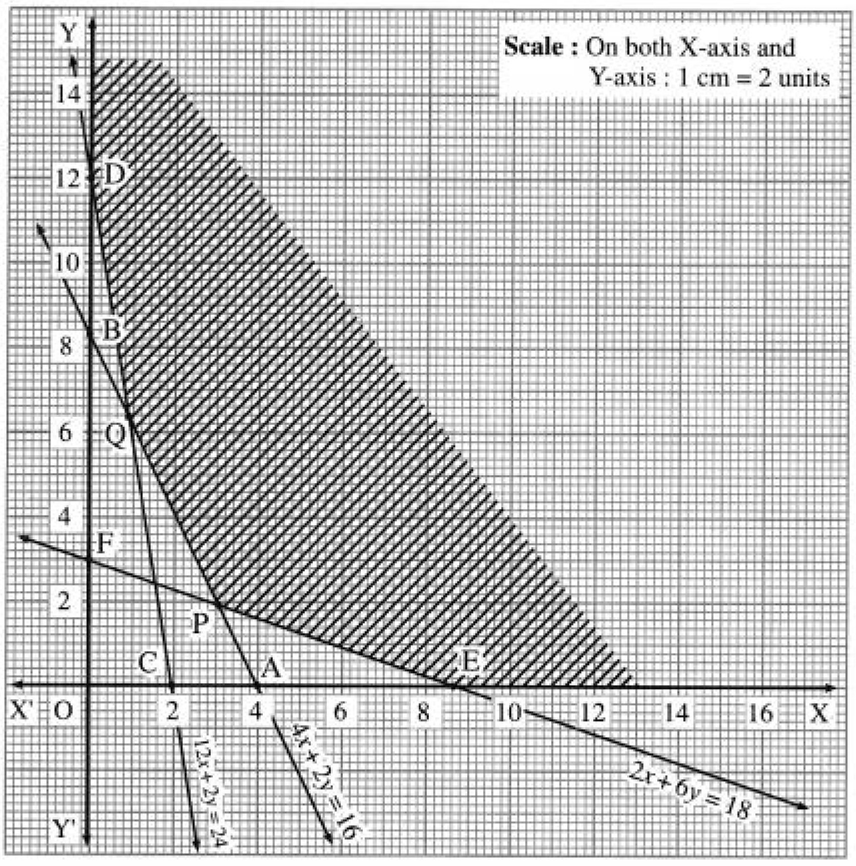
6x + 6y = 72
15x + 6y = 150
On subtracting, we get
9x = 78
∴ x = 263
∴ from (1), 3(263) + 3y = 36
∴ 3y = 10
∴ y = 103
∴ P = (263,103)
Q is the point of intersection of the lines
3x + 3y = 36 … (1)
and 2x + 6y = 60 … (3)
Multiplying equation (1) by 2, we get
6x + 6y = 72
Subtracting equation (3), from this equation, we get
4x = 12
∴ x = 3
∴ from (1), 3(3) + 3y = 36
∴ 3y = 27
∴ y = 9
∴ Q = (3, 9)
The values of the objective function z = 2000x + 3000y at these vertices are
z(O) = 2000(0) + 3000(0) = 0 + 0 = 0
z(C) = 2000(10) + 3000(0) = 20000 + 0 = 20000
z(P) = 2000(263) + 3000(103) = 520003+300003=820003
z(Q) = 2000(3) + 3000(9) = 6000 + 27000 = 33000
z(F) = 2000(0) + 3000(10) = 30000 + 0 = 30000
∴ the maximum value of z is 33000 at the point (3, 9).
Hence, 3 mixers and 9 food processors should be produced in order to get the maximum profit of ₹ 33,000.

Question 11.
A chemical company produces a chemical containing three basic elements A, B, C so that it has at least 16 liters of A, 24 liters of B and 18 liters of C. This chemical is made by mixing two compounds I and II. Each unit of compound I has 4 liters of A, 12 liters of B, 2 liters of C. Each unit of compound II has 2 liters of A, 2 liters of B and 6 liters of C. The cost per unit of compound I is ₹ 800/- and that of compound II is ₹ 640/-. Formulate the problem as L.P.P. and solve it to minimize the cost.
Solution:
Let the company buy x units of compound I and y units of compound II.
Then the total cost is z = ₹(800x + 640y).
This is the objective function which is to be minimized.
The constraints are as per the following table :

	Compound I (x)	Compound II (y)	Minimum requirement
Element A	4	2	16
Element B	12	2	24
Element C	2	6	18

From the table, the constraints are
4x + 2y ≥ 16, 12x + 2y ≥ 24, 2x + 6y ≥ 18.
Also, the number of units of compound I and compound II cannot be negative.
∴ x ≥ 0, y ≥ 0.
∴ the mathematical formulation of given LPP is
Minimize z = 800x + 640y, subject to 4x + 2y ≥ 16, 12x + 2y ≥ 24, 2x + 6y ≥ 18, x ≥ 0, y ≥ 0.
First we draw the lines AB, CD and EF whose equations are 4x + 2y = 16, 12x + 2y = 24 and 2x + 6y = 18

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$4x + 2y = 16$	A (4, 0)	B (0, 8)	$\geqslant$	non-origin side of line AB
CD	$12x + 2y = 24$	C (2, 0)	D (0, 12)	$\geqslant$	non-origin side of line CD
EF	$2x + 6y = 18$	E (9, 0)	F (0, 3)	$\geqslant$	non-origin side of line EF



The feasible region is shaded in the graph.
The vertices of the feasible region are E(9, 0), P, Q, and D(0, 12).
P is the point of intersection of the lines
2x + 6y = 18 ... (1)
and 4x + 2y = 16 ... (2)
Multiplying equation (1) by 2, we get 4x + 12y = 36
Subtracting equation (2) from this equation, we get
10y = 20
∴ y = 2
∴ from (1), 2x + 6(2) = 18
∴ 2x = 6
∴ x = 3
∴ P = (3, 2)
Q is the point of intersection of the lines
12x + 2y = 24 ... (3)
and 4x + 2y = 16 ... (2)
On subtracting, we get
8x = 8 ∴ x = 1
∴ from (2), 4(1) + 2y = 16
∴ 2y = 12 ∴ y = 6
∴ Q = (1, 6)
The values of the objective function z = 800x + 640y at these vertices are
z(E) = 800(9)+ 640(0) =7200 + 0 = 7200
z(P) = 800(3) + 640(2) = 2400 + 1280 = 3680
z(Q) = 800(1) + 640(6) =800 + 3840 =4640
z(D) = 800(0) + 640(12) = 0 + 7680 = 7680
∴ the minimum value of z is 3680 at the point (3, 2).
Hence, the company should buy 3 units of compound I and 2 units of compound II to have the minimum cost of ₹ 3680.

Question 12.
A person makes two types of gift items A and B requires the services of a cutter and a finisher. Gift item A requires 4 hours of cutter's time and 2 hours of finisher's time. B requires 2 hours of cutter's time and 4 hours of finisher's time. The cutter and finisher have 208 hours and 152 hours available times respectively every month. The profit of one gift item of type A is ₹ 75/- and on gift item B is ₹ 125/-. Assuming that the person can sell all the gift items produced, determine how many gift items of each type should he make every month to obtain the best returns?
Solution:
Let x: number of gift item A
y: number of gift item B
As numbers of the items are never negative
$x \geq 0; y \geq 0$

	A (x)	**B (y)**	**Max.time available**
Cutter	4	2	208
Finisher	2	4	152
Profit	75	125	

Total time required for the cutter = $4x + 2y$
Maximum available time 208 hours
$\therefore 4x+ 2y \leq 208$
Total time required for the finisher $2x + 4y$
Maximum available time 152 hours
$2x + 4y \leq 152$
Total Profit is $75x + 125y$
∴ L.P.P. of the above problem is
Minimize $z = 75x + 125y$
Subject to $4x+ 2y \leq 208$
$2x + 4y \leq 152$
$x \geq 0; y \geq 0$
Graphical solution

2x + y = 104		
x	0	52
y	104	0
(0 , 104) (52 , 0)		

x + 2y = 76		
x	0	0
y	38	76
(0 , 38) (76 , 0)		

Corner points
Now, Z at
$x = (75x + 125y)$
$O(0, 0) = 75 \times 0 + 125 \times 0 = 0$
$A(52,0) = 75 \times 52 + 125 \times 0 = 3900$
$B(44, 16) = 75 \times 44 + 125 \times 16 = 5300$
$C(0, 38) = 75 \times 0 + 125 \times 38 = 4750$
A person should make 44 items of type A and 16 Uems of type Band his returns are ₹ 5,300.

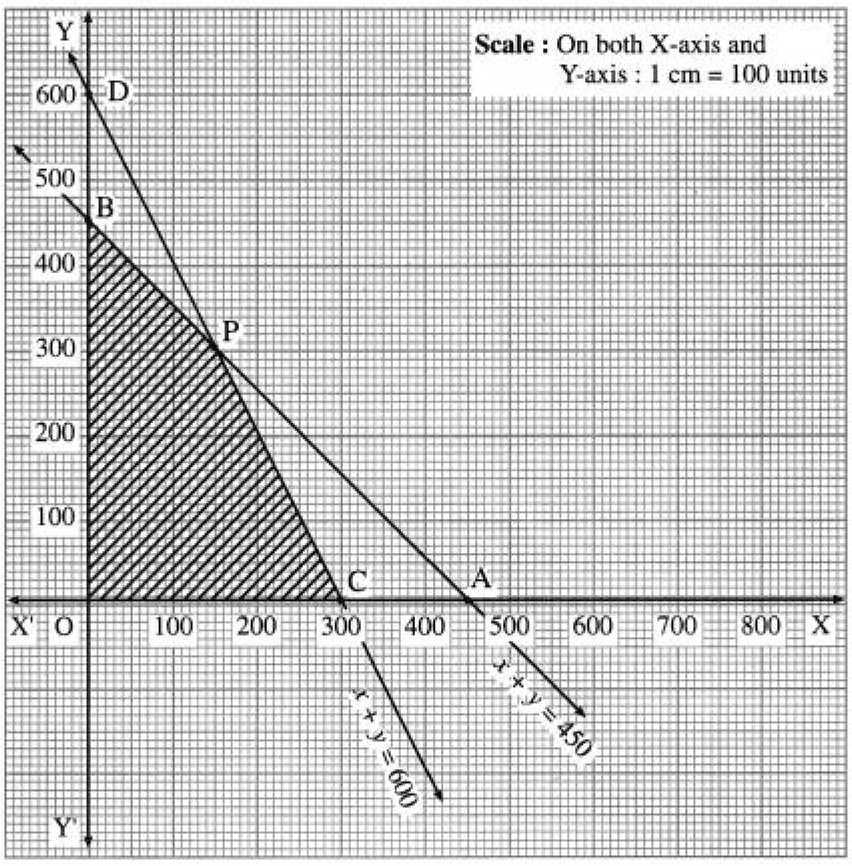
Question 13.
A firm manufactures two products A and B on which profit earned per unit ₹3/- and ₹4/- respectively. Each product is processed on two machines M_1 and M_2. The product A requires one minute of processing time on M_1 and two minute of processing time on M_2, B requires one minute of processing time on M_1 and one minute of processing time on M_2. Machine M_1 is available for use for 450 minutes while M_2 is available for 600 minutes during any working day. Find the number of units of product A and B to be manufactured to get the maximum profit.
Solution:
Let the firm manufactures x units of product
A and y units of product B.
The profit earned per unit of A is ₹3 and B is ₹ 4.
Hence, the total profit is z = ₹ $(3x + 4y)$.
This is the linear function which is to be maximized.
Hence, it is the objective function.

The constraints are as per the following table :

Machine	Product A (x)	Product B (y)	Total availability of time (minutes)
M_1	1	1	450
M_2	2	1	600

From the table, the constraints are
x + y ≤ 450, 2x + y ≤ 600
Since, the number of gift items cannot be negative, x ≥ 0, y ≥ o.
∴ the mathematical formulation of LPP is,
Maximize z = 3x + 4y, subject to x + y ≤ 450, 2x + y ≤ 600, x ≥ 0, y ≥ 0.
Now, we draw the lines AB and CD whose equations are x + y = 450 and 2x + y — 600 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x+y=450$	A (450, 0)	B (0, 450)	$\leqslant$	origin side of line AB
CD	$2x+y=600$	C (300, 0)	D (0, 600)	$\leqslant$	origin side of line CD



The feasible region is OCPBO which is shaded in the graph.
The vertices of the feasible region are O(0, 0), C(300, 0), P and B (0, 450).
P is the point of intersection of the lines
2x + y = 600 ... (1)
and x + y = 450 ... (2)
On subtracting, we get
∴ x = 150
Substituting x = 150 in equation (2), we get
150 + y = 450
∴ y = 300
∴ P = (150, 300)
The values of the objective function z = 3x + 4y at these vertices are
z(O) = 3(0) + 4(0) = 0 + 0 = 0
z(C) = 3(300) + 4(0) = 900 + 0 = 900
z(P) = 3(150) + 4(300) = 450 + 1200 = 1650
z(B) = 3(0) + 4(450) = 0 + 1800 = 1800
∴ z has the maximum value 1800 when x = 0 and y = 450 Hence, the firm gets maximum profit of ₹ 1800 if it manufactures 450 units of product B and no unit product A.

Question 14.

A firm manufacturing two types of electrical items A and B, can make a profit of ₹ 20/- per unit of A and ₹ 30/- per unit of B. Both A and B make use of two essential components a motor and a transformer. Each unit of A requires 3 motors and 2 transformers and each units of B requires 2 motors and 4 transformers. The total supply of components per month is restricted to 210 motors and 300 transformers. How many units of A and B should the manufacture per month to maximize profit? How much is the maximum profit?

Solution:

Let the firm manufactures x units of item A and y units of item B.

Firm can make profit of ₹ 20 per unit of A and ₹ 30 per unit of B.

Hence, the total profit is z = ₹ (20x + 30y).

This is the objective function which is to be maximized. The constraints are as per the following table :

	Item A (x)	Item B (y)	Total supply
Motor	3	2	210
Transformer	2	4	300

From the table, the constraints are

$3x + 2y \leq 210$, $2x + 4y \leq 300$

Since, number of items cannot be negative, $x \geq 0$, $y \geq 0$.

Hence, the mathematical formulation of given LPP is :

Maximize $z = 20x + 30y$, subject to $3x + 2y \leq 210$, $2x + 4y \leq 300$, $x \geq 0$, $y \geq 0$.

We draw the lines AB and CD whose equations are $3x + 2y = 210$ and $2x + 4y = 300$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x+2y=210$	A (70, 0)	B (0, 105)	$\leqslant$	origin side of line AB
CD	$2x+4y=300$	C (150, 0)	D (0, 75)	$\leqslant$	origin side of line CD

The feasible region is OAPDO which is shaded in the graph.

The vertices of the feasible region are O (0, 0), A (70, 0), P and D (0, 75).

P is the point of intersection of the lines

$2x + 4y = 300$ … (1)

and $3x + 2y = 210$ … (2)

Multiplying equation (2) by 2, we get

$6x + 4y = 420$

Subtracting equation (1) from this equation, we get

$\therefore 4x = 120 \therefore x = 30$

Substituting x = 30 in (1), we get

$2(30) + 4y = 300$

$\therefore 4y = 240 \therefore y = 60$

$\therefore$ P is (30, 60)

The values of the objective function $z = 20x + 30y$ at these vertices are

$z(O) = 20(0) + 30(0) = 0 + 0 = 0$

$z(A) = 20(70) + 30(0) = 1400 + 0 = 1400$

$z(P) = 20(30) + 30(60) = 600 + 1800 = 2400$

$z(D) = 20(0) + 30(75) = 0 + 2250 = 2250$

$\therefore$ z has the maximum value 2400 when x = 30 and y = 60. Hence, the firm should manufactured 30 units of item A and 60 units of item B to get the maximum profit of ₹ 2400.