Maharashtra State Board 11th Maths Solutions Chapter 9 Differentiation Ex 9.1

Question 1.
Find the derivatives of the following w.r.t. x by using the method of the first principle.
(a) $x^2 + 3x - 1$
Solution:
Let $f(x) = x^2 + 3x - 1$
$\therefore f(x + h) = (x + h)^2 + 3(x + h) - 1$
$= x^2 + 2xh + h^2 + 3x + 3h - 1$
By first principle, we get

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{\left[x^2 + 2xh + h^2 + 3x + 3h - 1\right] - \left[x^2 + 3x - 1\right]}{h}$$
$$= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h}$$
$$= \lim_{h \to 0} \frac{h(2x + h + 3)}{h}$$
$$= \lim_{h \to 0} (2x + h + 3) \quad \ldots[\because h \to 0, \therefore h \neq 0]$$
$$= 2x + 3$$

(b) sin(3x)
Solution:
Let $f(x) = \sin 3x$
$f(x + h) = \sin 3(x + h) = \sin(3x + 3h)$
By first principle, we get

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{\sin(3x + 3h) - \sin 3x}{h}$$
$$= \lim_{h \to 0} \left\{ \frac{2\cos\left(\frac{3x + 3h + 3x}{2}\right) \sin\left(\frac{3x + 3h - 3x}{2}\right)}{h} \right\}$$
$$\ldots\left[\because \sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$$
$$= \lim_{h \to 0} \left\{ 2\cos\left(\frac{6x + 3h}{2}\right) \left[\frac{\sin\left(\frac{3h}{2}\right)}{h}\right] \right\}$$
$$= \lim_{h \to 0} \left\{ 2\cos\left(\frac{6x + 3h}{2}\right) \left(\frac{\sin\frac{3h}{2}}{\frac{3h}{2}}\right) \times \left(\frac{3}{2}\right) \right\}$$
$$= 2 \times \frac{3}{2} \lim_{h \to 0} \cos\frac{(6x + 3h)}{2} \lim_{h \to 0} \frac{\sin\frac{3h}{2}}{\frac{3h}{2}}$$
$$= 3\cos\left(\frac{6x + 0}{2}\right)(1) \quad \ldots\left[\because \lim_{\theta \to 0} \frac{\sin p\theta}{p\theta} = 1\right]$$
$$= 3\cos 3x$$

(c) e^{2x+1}
Solution:

Let $f(x) = e^{2x+1}$
$f(x+h) = e^{2(x+h)+1}$
$= e^{2x+2h+1}$

By first principle, we get

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{e^{2x+2h+1} - e^{2x+1}}{h}$$
$$= \lim_{h\to 0} e^{2x+1} \frac{(e^{2h}-1)}{h}$$
$$= e^{2x+1}\left(\lim_{h\to 0} \frac{e^{2h}-1}{2h}\right) \times 2$$
$$= 2e^{2x+1}(1) \qquad \ldots\left[\because \lim_{x\to 0} \frac{e^{px}-1}{px} = 1\right]$$
$$= 2e^{2x+1}$$

(d) 3^x
Solution:

Let $f(x) = 3^x$
$f(x+h) = 3^{x+h}$

By first principle, we get

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{3^{x+h}-3^x}{h}$$
$$= \lim_{h\to 0} 3^x\left(\frac{3^h-1}{h}\right)$$
$$= 3^x \lim_{h\to 0}\left(\frac{3^h-1}{h}\right)$$
$$= 3^x \log 3 \qquad \ldots\left[\because \lim_{x\to 0} \frac{a^x-1}{x} = \log a\right]$$

(e) log(2x + 5)
Solution:
Let f(x) = log(2x + 5)
∴ f(x + h) = log[2(x + h) + 5] = log (2x + 2h + 5)
By first principle, we get

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\log(2x+2h+5) - \log(2x+5)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \log\left(\frac{2x+2h+5}{2x+5}\right)$$

$$= \lim_{h \to 0} \frac{1}{h} \log\left(\frac{2x+5+2h}{2x+5}\right)$$

$$= \lim_{h \to 0} \frac{1}{h} \log\left(1 + \frac{2h}{2x+5}\right)$$

$$= \lim_{h \to 0} \log\left(1 + \frac{2h}{2x+5}\right)^{\frac{1}{h}}$$

$$= \lim_{h \to 0} \log\left[\left(1 + \frac{2h}{2x+5}\right)^{\frac{2x+5}{2h}}\right]^{\frac{2}{2x+5}}$$

$$= \log e^{\frac{2}{2x+5}} \quad \ldots\left[\because \lim_{x \to 0} (1+px)^{\frac{1}{px}} = e\right]$$

$$= \frac{2}{2x+5} \log e$$

$$= \frac{2}{2x+5} \quad \ldots[\because \log e = 1]$$

(f) tan(2x + 3)
Solution:
Let f(x) = tan(2x + 3)
∴ f(x + h) = tan[2(x + h) + 3] = tan(2x + 2h + 3)
By first principle, we get

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\tan(2x+2h+3) - \tan(2x+3)}{h}$$

$$= \lim_{h \to 0} \left(\frac{\frac{\sin(2x+2h+3)}{\cos(2x+2h+3)} - \frac{\sin(2x+3)}{\cos(2x+3)}}{h}\right)$$

$$= \lim_{h \to 0} \left[\frac{\sin(2x+2h+3).\cos(2x+3) - \sin(2x+3).\cos(2x+2h+3)}{h\cos(2x+3)\cos(2x+2h+3)}\right]$$

$$= \lim_{h \to 0} \frac{\sin[(2x+2h+3) - (2x+3)]}{h\cos(2x+3)\cos(2x+2h+3)}$$

$$= \lim_{h \to 0} \left[\left(\frac{\sin 2h}{h}\right)\frac{1}{\cos(2x+3)\cos(2x+2h+3)}\right]$$

$$= \frac{2}{\cos(2x+3)} \lim_{h \to 0}\left(\frac{\sin 2h}{2h}\right) \lim_{h \to 0}\left(\frac{1}{\cos(2x+2h+3)}\right)$$

$$= \frac{2}{\cos(2x+3)}(1)\left(\frac{1}{\cos(2x+3)}\right) \quad \ldots\left[\because \lim_{\theta \to 0} \frac{\sin p\theta}{p\theta} = 1\right]$$

$$= 2\sec^2(2x+3)$$

(g) sec(5x – 2)
Solution:
Let f(x) = sec(5x – 2)
f(x + h) = sec[5(x + h) – 2] = sec(5x + 5h – 2)
By first principle, we get

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sec(5x+5h-2)-\sec(5x-2)}{h}$$

$$= \lim_{h\to 0}\left[\frac{\frac{1}{\cos(5x+5h-2)}-\frac{1}{\cos(5x-2)}}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\cos(5x-2)-\cos(5x+5h-2)}{h\cos(5x-2)\cos(5x+5h-2)}\right]$$

$$= \lim_{h\to 0}\left[\frac{2\sin\left(\frac{5x-2+5x+5h-2}{2}\right)\sin\left(\frac{5x+5h-2-5x+2}{2}\right)}{h\cos(5x-2)\cos(5x+5h-2)}\right]$$

$$\ldots\left[\because \cos C-\cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)\right]$$

$$= \frac{1}{\cos(5x-2)}\lim_{h\to 0}\frac{2\sin\left(5x-2+\frac{5h}{2}\right)\sin\frac{5h}{2}}{h\cos(5x+5h-2)}$$

$$= \frac{5}{\cos(5x-2)}\lim_{h\to 0}\left[\frac{\sin\left(5x-2+\frac{5h}{2}\right)}{\cos(5x+5h-2)}\cdot\frac{\sin\left(\frac{5h}{2}\right)}{\frac{5h}{2}}\right]$$

$$= \frac{5}{\cos(5x-2)}\left[\frac{\lim_{h\to 0}\sin\left(5x-2+\frac{5h}{2}\right)}{\lim_{h\to 0}\cos(5x+5h-2)}\lim_{h\to 0}\frac{\sin\left(\frac{5h}{2}\right)}{\frac{5h}{2}}\right]$$

$$= \frac{5}{\cos(5x-2)}\cdot\frac{\sin(5x-2)}{\cos(5x-2)}(1) \quad \ldots\left[\because \lim_{\theta\to 0}\frac{\sin p\theta}{p\theta}=1\right]$$

$$= 5\sec(5x-2)\tan(5x-2)$$

(h) x√x

Solution:

Let $f(x) = x\sqrt{x} = x^{3/2}$

$\therefore \quad f(x+h) = (x+h)^{\frac{3}{2}}$

By first principle, we get

$$f'(x) = \lim_{h\to0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to0} \frac{(x+h)^{3/2} - x^{3/2}}{h}$$

$$= \lim_{h\to0} \frac{\left[(x+h)^{3/2} - x^{3/2}\right]\left[(x+h)^{3/2} + x^{3/2}\right]}{h\left[(x+h)^{3/2} + x^{3/2}\right]}$$

$$= \lim_{h\to0} \frac{(x+h)^3 - x^3}{h\left[(x+h)^{3/2} + x^{3/2}\right]}$$

$$= \lim_{h\to0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h\left[(x+h)^{3/2} + x^{3/2}\right]}$$

$$= \lim_{h\to0} \frac{h\left(3x^2 + 3xh + h^2\right)}{h\left[(x+h)^{3/2} + x^{3/2}\right]}$$

$$= \lim_{h\to0} \frac{3x^2 + 3xh + h^2}{(x+h)^{3/2} + x^{3/2}}$$

$$= \frac{\lim_{h\to0}\left(3x^2 + 3xh + h^2\right)}{\lim_{h\to0}(x+h)^{3/2} + \lim_{h\to0} x^{3/2}}$$

$$= \frac{3x^2 + 3x \times 0 + 0^2}{(x+0)^{3/2} + x^{3/2}} = \frac{3x^2}{2x^{3/2}}$$

$$= \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$$

Question 2.

Find the derivatives of the following w.r.t. x. at the points indicated against them by using the method of the first principle.

(i) $\sqrt{2x+5}$ at x = 2

Solution:

Let $f(x) = \sqrt{2x+5}$

$\therefore \quad f(2) = \sqrt{2(2)+5} = \sqrt{9} = 3$ and

$f(2+h) = \sqrt{2(2+h)+5} = \sqrt{2h+9}$

By first principle, we get

$$f'(a) = \lim_{h\to0} \frac{f(a+h)-f(a)}{h}$$

$$\therefore \quad f'(2) = \lim_{h\to0} \frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h\to0} \frac{\sqrt{2h+9}-3}{h}$$

$$= \lim_{h\to0} \frac{\sqrt{2h+9}-3}{h} \times \frac{\sqrt{2h+9}+3}{\sqrt{2h+9}+3}$$

$$= \lim_{h\to0} \frac{2h+9-9}{h(\sqrt{2h+9}+3)}$$

$$= \lim_{h\to0} \frac{2h}{h\left(\sqrt{2h+9}+3\right)}$$

$$= \lim_{h\to0} \frac{2}{\sqrt{2h+9}+3} \quad \ldots [\because h \to 0, h \neq 0]$$

$$= \frac{2}{\sqrt{0+9}+3}$$

$$= \frac{2}{3+3} = \frac{1}{3}$$

(ii) tan x at x = π4

Solution:

Let f (x) = tan x

$$\therefore \quad f\left(\frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1 \text{ and } f\left(\frac{\pi}{4}+h\right) = \tan\left(\frac{\pi}{4}+h\right)$$

By first principle, we get

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$$\therefore \quad f'\left(\frac{\pi}{4}\right) = \lim_{h\to 0}\frac{f\left(\frac{\pi}{4}+h\right)-f\left(\frac{\pi}{4}\right)}{h}$$

$$= \lim_{h\to 0}\frac{\tan\left(\frac{\pi}{4}+h\right)-1}{h}$$

$$= \lim_{h\to 0}\frac{\frac{\sin\left(\frac{\pi}{4}+h\right)}{\cos\left(\frac{\pi}{4}+h\right)}-1}{h}$$

$$= \lim_{h\to 0}\frac{\sin\left(\frac{\pi}{4}+h\right)-\cos\left(\frac{\pi}{4}+h\right)}{h\cos\left(\frac{\pi}{4}+h\right)}$$

$$= \lim_{h\to 0}\frac{\left[\left(\sin\frac{\pi}{4}\cos h+\cos\frac{\pi}{4}\sin h\right)-\left(\cos\frac{\pi}{4}\cos h-\sin\frac{\pi}{4}\sin h\right)\right]}{h\cos\left(\frac{\pi}{4}+h\right)}$$

$$= \lim_{h\to 0}\frac{\frac{1}{\sqrt{2}}\cos h+\frac{1}{\sqrt{2}}\sin h-\frac{1}{\sqrt{2}}\cos h+\frac{1}{\sqrt{2}}\sin h}{h\cos\left(\frac{\pi}{4}+h\right)}$$

$$= \lim_{h\to 0}\frac{\frac{2}{\sqrt{2}}\sin h}{h\cos\left(\frac{\pi}{4}+h\right)}$$

$$= \frac{2}{\sqrt{2}}\lim_{h\to 0}\left(\frac{\sin h}{h}\right)\cdot\lim_{h\to 0}\frac{1}{\cos\left(\frac{\pi}{4}+h\right)}$$

$$= \sqrt{2}\,(1)\cdot\frac{1}{\cos\left(\frac{\pi}{4}+0\right)} \qquad \ldots\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$= \sqrt{2}\times\frac{1}{\left(\frac{1}{\sqrt{2}}\right)}$$

$$= 2$$

(iii) 2^{3x+1} at x = 2

Solution:

Let $f(x) = 2^{3x+1}$

$\therefore$ $f(2) = 2^{3(2)+1} = 2^7$ and

$f(2+h) = 2^{3(2+h)+1} = 2^{3h+7}$

By first principle, we get

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore \quad f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{2^{3h+7} - 2^7}{h}$$

$$= \lim_{h \to 0} \frac{2^{3h} \cdot 2^7 - 2^7}{h}$$

$$= \lim_{h \to 0} \frac{2^7 (2^{3h} - 1)}{h}$$

$$= 2^7 \lim_{h \to 0} \left(\frac{2^{3h} - 1}{3h}\right) \times 3$$

$$= 2^7 (\log 2) \times 3 \quad \ldots \left[\because \lim_{x \to 0} \left(\frac{a^{px} - 1}{px}\right) = \log a\right]$$

$$= 384 \log 2$$

(iv) log(2x + 1) at x = 2

Solution:

Let f(x) = log(2x + 1)

$\therefore$ f(2) = log [2(2) + 1] = log 5 and

f(2 + h) = log [2(2 + h) + 1] = log(2h + 5)

By first principle, we get

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{\log(2h+5) - \log 5}{h}$$

$$= \lim_{h \to 0} \frac{\log\left(\frac{2h+5}{5}\right)}{h} = \lim_{h \to 0} \frac{\log\left(1 + \frac{2h}{5}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\log\left(1 + \frac{2h}{5}\right)}{\frac{2h}{5} \times \frac{5}{2}}$$

$$= \frac{2}{5} \lim_{h \to 0} \frac{\log\left(1 + \frac{2h}{5}\right)}{\frac{2h}{5}} = \frac{2}{5}\ (1)$$

$$\ldots \left[\because \lim_{x \to 0} \frac{\log(1+px)}{px} = 1\right]$$

$$= \frac{2}{5}$$

(v) e^{3x-4} at x = 2

Solution:

Let $f(x) = e^{3x-4}$

$f(2) = e^{3(2)-4} = e^2$ and

$f(2+h) = e^{3(2+h)-4} = e^{3h+2}$

By first principle, we get

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$$f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h\to 0}\frac{e^{3h+2}-e^2}{h}$$

$$= \lim_{h\to 0}\frac{e^{3h}e^2-e^2}{h}$$

$$= \lim_{h\to 0}\frac{e^2(e^{3h}-1)}{h}$$

$$= e^2 \lim_{h\to 0}\left(\frac{e^{3h}-1}{3h}\right)\times 3$$

$$= 3e^2(1) \qquad \ldots\left[\because \lim_{x\to 0}\frac{e^{px}-1}{px}=1\right]$$

$$= 3e^2$$

(vi) cos x at x = $\frac{5\pi}{4}$

Solution:

Let $f(x) = \cos x$

$$f\left(\frac{5\pi}{4}\right) = \cos\left(\frac{5\pi}{4}\right)$$

and $$f\left(\frac{5\pi}{4}+h\right) = \cos\left(\frac{5\pi}{4}+h\right)$$

By first principle, we get

$$f'(a) = \lim_{h\to0}\frac{f(a+h)-f(a)}{h}$$

$$f'\left(\frac{5\pi}{4}\right) = \lim_{h\to0}\frac{f\left(\frac{5\pi}{4}+h\right)-f\left(\frac{5\pi}{4}\right)}{h}$$

$$= \lim_{h\to0}\frac{\cos\left(\frac{5\pi}{4}+h\right)-\cos\left(\frac{5\pi}{4}\right)}{h}$$

$$= \lim_{h\to0}\frac{-2\sin\left(\frac{\frac{10\pi}{4}+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$$

$$\ldots\left[\because \cos C-\cos D=-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$$

$$= \lim_{h\to0}\frac{-2\sin\left(\frac{5\pi}{4}+\frac{h}{2}\right)\sin\frac{h}{2}}{h}$$

$$= -2\lim_{h\to0}\sin\left(\frac{5\pi}{4}+\frac{h}{2}\right)\cdot\lim_{h\to0}\frac{\sin\frac{h}{2}}{\frac{h}{2}\times2}$$

$$= -2\lim_{h\to0}\sin\left(\frac{5\pi}{4}+\frac{h}{2}\right)\left(\frac{1}{2}\right)\lim_{h\to0}\frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= -2\sin\left(\frac{5\pi}{4}+0\right)\cdot\frac{1}{2}(1)$$

$$\ldots\left[\because \lim_{\theta\to0}\frac{\sin p\theta}{p\theta}=1\right]$$

$$= -\sin\frac{5\pi}{4}$$

$$= -\sin\left(\pi+\frac{\pi}{4}\right)$$

$$= \sin\frac{\pi}{4}\quad\ldots[\because \sin(\pi+\theta)=-\sin\theta]$$

$$= \frac{1}{\sqrt{2}}$$

Question 3.
Show that the function f is not differentiable at x = -3,
where f(x) = x^2 + 2 for x < -3
= 2 – 3x for x ≥ -3
Solution:

$$f(x) = x^2+2 \qquad \text{for } x<-3$$
$$= 2-3x \qquad \text{for } x\geq-3$$

$$L f'(-3) = \lim_{h\to0^-}\frac{f(-3+h)-f(-3)}{h}$$

$$= \lim_{h\to0^-}\frac{\left[(-3+h)^2+2\right]-\left[2-3(-3)\right]}{h}$$

$$= \lim_{h \to 0^-} \frac{9 - 6h + h^2 + 2 - 11}{h}$$

$$= \lim_{h \to 0^-} \frac{h^2 - 6h}{h}$$

$$= \lim_{h \to 0^-} \frac{h(h-6)}{h}$$

$$= \lim_{h \to 0^-} (h - 6) \quad \ldots[\because h \to 0, h \neq 0]$$

$$= -6$$

$$R\ f'(-3) = \lim_{h \to 0^+} \frac{f(-3+h) - f(-3)}{h}$$

$$= \lim_{h \to 0^+} \frac{[2 - 3(-3+h)] - [2 - 3(-3)]}{h}$$

$$= \lim_{h \to 0^+} \frac{(11 - 3h) - 11}{h}$$

$$= \lim_{h \to 0^+} \frac{-3h}{h}$$

$$= \lim_{h \to 0^+} (-3) \quad \ldots[\because h \to 0, h \neq 0]$$

$$= -3$$

∴ L f'(-3) ≠ R f'(-3)
∴ f is not differentiable at x = -3.

Question 4.
Show that f(x) = x^2 is continuous and differentiable at x = 0.
Solution:

Continuity at $x = 0$:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} x^2 = (0)^2 = 0$$

Also, $f(x) = x^2$

∴ $f(0) = (0)^2 = 0$

∴ $\lim_{x \to 0} f(x) = f(0)$

∴ $f(x)$ is continuous at $x = 0$.

Differentiability at $x = 0$:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

∴ $$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{h^2 - 0}{h}$$

$$= \lim_{h \to 0} h \quad \ldots[\because h \to 0, h \neq 0]$$

$$= 0$$

∴ $f'(0)$ exists.

∴ $f(x)$ is differentiable at $x = 0$.

Question 5.
Discuss the continuity and differentiability of
(i) f(x) = x |x| at x = 0

Solution:

$f(x) = x|x|$

$f(x) = x(-x), x < 0$

$= x(x), \quad x \geq 0$

Continuity at $x = 0$:

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(-x^2) = 0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(x^2) = 0$

$f(0) = 0$

$\therefore \quad \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\therefore$ f(x) is continuous at $x = 0$.

Differentiability at $x = 0$:

$$L f'(0) = \lim_{h\to 0^-} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0^-} \frac{-h^2 - 0}{h}$$

$$= \lim_{h\to 0^-} (-h) \quad \ldots[\because h \to 0, h \neq 0]$$

$$= 0$$

$$R f'(0) = \lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0^+} \frac{h^2 - 0}{h}$$

$$= \lim_{h\to 0^+} (h) \quad \ldots[\because h \to 0, h \neq 0]$$

$$= 0$$

$\therefore \quad L f'(0) = R f'(0)$

$\therefore$ f(x) is differentiable at $x = 0$.

(ii) f(x) = (2x + 3) |2x + 3| at x = −32

Solution:

$f(x) = (2x+3)\,|2x+3|$

$\therefore \quad f(x) = -(2x+3)^2, \quad x < -\frac{3}{2}$

$= (2x+3)^2, \quad x \geq -\frac{3}{2}$

Continuity at $x = -\frac{3}{2}$:

$$\lim_{x\to -\frac{3}{2}^-} f(x) = \lim_{x\to -\frac{3}{2}^-} -(2x+3)^2$$

$$= -\left(2\times\frac{-3}{2}+3\right)^2$$

$$= 0$$

$$\lim_{x\to -\frac{3}{2}^+} f(x) = \lim_{x\to -\frac{3}{2}^+} (2x+3)^2$$

$$= \left(2\times\frac{-3}{2}+3\right)^2 = 0$$

$$f\left(-\frac{3}{2}\right) = \left(2\times\frac{-3}{2}+3\right)^2$$

$$\therefore \quad f\left(-\frac{3}{2}\right) = 0$$

$\therefore \quad \lim_{x \to -\frac{3}{2}^-} f(x) = \lim_{x \to -\frac{3}{2}^+} f(x) = f\left(-\frac{3}{2}\right)$

$\therefore \quad$ f(x) is continuous at $x = -\frac{3}{2}$.

Differentiability at $x = -\frac{3}{2}$:

$$Lf'\left(-\frac{3}{2}\right) = \lim_{h \to 0^-} \frac{f\left(-\frac{3}{2}+h\right) - f\left(-\frac{3}{2}\right)}{h}$$

$$= \lim_{h \to 0^-} \frac{-\left[2\left(-\frac{3}{2}+h\right)+3\right]^2 - 0}{h}$$

$$= \lim_{h \to 0^-} -\left(\frac{4h^2}{h}\right)$$

$$= -\lim_{h \to 0^-} 4h \quad \ldots[\because h \to 0, h \neq 0]$$

$$= 0$$

$$Rf'\left(-\frac{3}{2}\right) = \lim_{h \to 0^+} \frac{f\left(-\frac{3}{2}+h\right) - f\left(-\frac{3}{2}\right)}{h}$$

$$= \lim_{h \to 0^+} \frac{\left[2\left(-\frac{3}{2}+h\right)+3\right]^2 - 0}{h}$$

$$= \lim_{h \to 0^+} \frac{4h^2}{h}$$

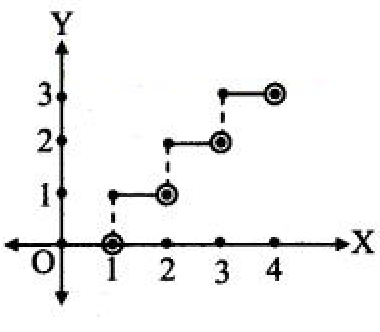
$$= \lim_{h \to 0^+} 4h \quad \ldots[\because h \to 0, \because h \neq 0]$$

$$= 0$$

$\therefore \quad Lf'\left(-\frac{3}{2}\right) = Rf'\left(-\frac{3}{2}\right)$

$\therefore \quad$ f is differentiable at $x = -\frac{3}{2}$.

Question 6.
Discuss the continuity and differentiability of f(x) at x = 2.
f(x) = [x] if x ∈ [0, 4). [where [ ] is a greatest integer (floor) function]
Solution:
Explanation:
x ∈ [0, 4)
∴ 0 ≤ x < 4
We will plot graph for 0 ≤ x < 4
not for x < 0 and upto x = 4 making on X-axis.



f(x) = [x]
∴ Greatest integer function is discontinuous at all integer values of x and hence not differentiable at all integers.
∴ f is not continuous at x = 2.
∵ f(x) = 1, x < 2
= 2, x ≥ 2
x ∈ neighbourhood of x = 2.
∴ L.H.L. = 1, R.H.L. = 2
∴ f is not continuous at x = 2.
∴ f is not differentiable at x = 2.

Question 7.
Test the continuity and differentiability of

f(x) = 3x + 2 if x > 2
= 12 – x2 if x ≤ 2 at x = 2.
Solution:

$$f(x) = 3x + 2,\ x > 2$$
$$= 12 - x^2,\ x \le 2$$

Continuity at $x = 2$:

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (12 - x^2) = 12 - (2)^2 = 8$$
$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (3x + 2) = 3(2) + 2 = 8$$
$$f(2) = 12 - (2)^2 = 12 - 4 = 8$$
$$\therefore \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$

$\therefore$ f(x) is continuous at $x = 2$.

Differentiability at $x = 2$:

$$Lf'(2) = \lim_{h \to 0^-} \frac{f(2+h) - f(2)}{h}$$
$$= \lim_{h \to 0^-} \frac{12 - (2+h)^2 - 8}{h}$$
$$= \lim_{h \to 0^-} \frac{12 - (4 + 4h + h^2) - 8}{h}$$
$$= \lim_{h \to 0^-} - \frac{(4h + h^2)}{h}$$
$$= -\lim_{h \to 0^-} (4 + h) \quad \ldots[\because h \to 0, h \neq 0]$$
$$= -4$$
$$Rf'(2) = \lim_{h \to 0^+} \frac{f(2+h) - f(2)}{h}$$
$$= \lim_{h \to 0^+} \frac{3(2+h) + 2 - 8}{h}$$
$$= \lim_{h \to 0^+} \frac{3h}{h}$$
$$= \lim_{h \to 0^+} (3) \quad \ldots[\because h \to 0, h \neq 0]$$
$$= 3$$

$\therefore$ $Lf'(2) \neq Rf'(2)$

$\therefore$ **f is not differentiable at $x = 2$.**

Question 8.
If f(x) = sin x – cos x if x ≤ π2
= 2x – π + 1 if x > π2
Test the continuity and differentiability of f at x = π2.
Solution:

$$f(x) = \sin x - \cos x, \quad x \le \frac{\pi}{2}$$
$$= 2x - \pi + 1, \quad x > \frac{\pi}{2}$$

Continuity at $x = \frac{\pi}{2}$:

$$\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^-} (\sin x - \cos x)$$
$$= \sin \frac{\pi}{2} - \cos \frac{\pi}{2}$$
$$= 1 - 0 = 1$$
$$\lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{x \to \frac{\pi}{2}^+} (2x - \pi + 1)$$
$$= 2\left(\frac{\pi}{2}\right) - \pi + 1$$
$$= 1$$
$$f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} - \cos \frac{\pi}{2} = 1 - 0 = 1$$

$\therefore \quad \lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^+} f(x) = f\left(\frac{\pi}{2}\right)$

$\therefore \quad f(x)$ is continuous at $x = \frac{\pi}{2}$.

Differentiability at $x = \frac{\pi}{2}$:

$$Lf'\left(\frac{\pi}{2}\right) = \lim_{h \to 0^-} \frac{f\left(\frac{\pi}{2} + h\right) - f\left(\frac{\pi}{2}\right)}{h}$$
$$= \lim_{h \to 0^-} \frac{\sin\left(\frac{\pi}{2} + h\right) - \cos\left(\frac{\pi}{2} + h\right) - 1}{h}$$
$$= \lim_{h \to 0^-} \frac{\cos h + \sin h - 1}{h}$$
$$= \lim_{h \to 0^-} \left(\frac{\sin h}{h} - \frac{1 - \cos h}{h}\right)$$
$$= \lim_{h \to 0^-} \left(\frac{\sin h}{h} - \frac{2 \sin^2 \frac{h}{2}}{h}\right)$$
$$= 1 - \lim_{h \to 0^-} \left(\frac{\sin^2\left(\frac{h}{2}\right)}{\frac{h}{2}}\right)$$
$$= 1 - 0 = 1$$

$$Rf'\left(\frac{\pi}{2}\right) = \lim_{h \to 0^+} \frac{f\left(\frac{\pi}{2} + h\right) - f\left(\frac{\pi}{2}\right)}{h}$$
$$= \lim_{h \to 0^+} \frac{\left[2\left(\frac{\pi}{2} + h\right) - \pi + 1\right] - 1}{h}$$
$$= \lim_{h \to 0^+} \left(\frac{2h}{h}\right)$$
$$= \lim_{h \to 0^+} 2 \qquad \ldots[\because h \to 0, h \neq 0]$$
$$= 2$$

$\therefore \quad Lf'\left(\frac{\pi}{2}\right) \neq Rf'\left(\frac{\pi}{2}\right)$

$\therefore \quad f(x)$ is not differentiable at $x = \frac{\pi}{2}$.

Question 9.
Examine the function
f(x) = $x^2 \cos(\frac{1}{x})$, for x ≠ 0
= 0, for x = 0
for continuity and differentiability at x = 0.
Solution:

$$f(x) = x^2 \cos\left(\frac{1}{x}\right) \quad , x \neq 0$$
$$= 0 \quad , x = 0$$

Continuity at $x = 0$:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} x^2 \cos \frac{1}{x}$$
$$= 0 \quad \ldots\left[-1 \leq \cos \frac{1}{x} \leq 1\right]$$
$$f(0) = 0$$
$$\therefore \quad \lim_{x \to 0} f(x) = f(0)$$

$\therefore$ f(x) is continuous at $x = 0$.

Differentiability at $x = 0$:

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$
$$= \lim_{h \to 0} \left(\frac{h^2 \cos\left(\frac{1}{h}\right) - 0}{h}\right)$$
$$= \lim_{h \to 0} h \cos \frac{1}{h} \quad \ldots[\because h \to 0, h \neq 0]$$
$$= 0$$

$\therefore$ f′(0) exists.
$\therefore$ f(x) is differentiable at $x = 0$.

Maharashtra State Board 11th Maths Solutions Chapter 9 Differentiation Ex 9.2

(I) Differentiate the following w.r.t. x

Question 1.
y = $x^{\frac{4}{3}} + e^x - \sin x$

Solution:

$$y = x^{\frac{4}{3}} + e^x - \sin x$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{4}{3}} + e^x - \sin x\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{4}{3}}\right) + \frac{d}{dx}\left(e^x\right) - \frac{d}{dx}(\sin x)$$

$$= \frac{4}{3}x^{\frac{4}{3}-1} + e^x - \cos x$$

$$= \frac{4}{3}x^{\frac{1}{3}} + e^x - \cos x$$

Question 2.
$y = \sqrt{x} + \tan x - x^3$
Solution:

$$y = \sqrt{x} + \tan x - x^3$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\sqrt{x} + \tan x - x^3)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x}\right) + \frac{d}{dx}(\tan x) - \frac{d}{dx}\left(x^3\right)$$

$$= \frac{1}{2\sqrt{x}} + \sec^2 x - 3x^2$$

Question 3.
$y = \log x - \operatorname{cosec} x + 5^x - \frac{3}{x^{\frac{3}{2}}}$
Solution:

$$y = \log x - \operatorname{cosec} x + 5^x - \frac{3}{x^{\frac{3}{2}}}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\log x - \operatorname{cosec} x + 5^x - \frac{3}{x^{\frac{3}{2}}}\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}(\log x) - \frac{d}{dx}(\operatorname{cosec} x) + \frac{d}{dx}\left(5^x\right) - 3\frac{d}{dx}\left(x^{-\frac{3}{2}}\right)$$

$$= \frac{1}{x} - (-\operatorname{cosec} x \cot x) + 5^x \log 5 - 3\left(-\frac{3}{2}x^{\frac{-3}{2}-1}\right)$$

$$= \frac{1}{x} - (-\operatorname{cosec} x \cot x) + 5^x \log 5 - 3\left(-\frac{3}{2}x^{\frac{-5}{2}}\right)$$

$$= \frac{1}{x} + \operatorname{cosec} x \cot x + 5^x \log 5 + \frac{9}{2x^{\frac{5}{2}}}$$

Question 4.
$y = x^{\frac{7}{3}} + 5x^{\frac{4}{5}} - \frac{5}{x^{\frac{2}{5}}}$
Solution:

$$y = x^{\frac{7}{3}} + 5x^{\frac{4}{5}} - \frac{5}{x^{\frac{2}{5}}}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{7}{3}} + 5x^{\frac{4}{5}} - \frac{5}{x^{\frac{2}{5}}}\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{7}{3}}\right) + 5\frac{d}{dx}\left(x^{\frac{4}{5}}\right) - 5\frac{d}{dx}\left(x^{-\frac{2}{5}}\right)$$

$$= \frac{7}{3}x^{\frac{7}{3}-1} + 5\left(\frac{4}{5}x^{\frac{4}{5}-1}\right) - 5\left(-\frac{2}{5}x^{-\frac{2}{5}-1}\right)$$

$$= \frac{7}{3}x^{\frac{4}{3}} + 4x^{\frac{-1}{5}} + 2x^{\frac{-7}{5}}$$

$$= \frac{7}{3}x^{\frac{4}{3}} + \frac{4}{x^{\frac{1}{5}}} + \frac{2}{x^{\frac{7}{5}}}$$

Question 5.
y = $7^x + x^7 - \frac{2}{3}x\sqrt{x} - \log x + 7^7$
Solution:

$$y = 7^x + x^7 - \frac{2}{3}x\sqrt{x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(7^x + x^7 - \frac{2}{3}x\sqrt{x} - \log x + 7^7\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(7^x\right) + \frac{d}{dx}\left(x^7\right) - \frac{2}{3}\cdot\frac{d}{dx}\left(x^{\frac{3}{2}}\right) - \frac{d}{dx}(\log x) + \frac{d}{dx}(7^7)$$

$$= 7^x \log 7 + 7x^6 - \frac{2}{3}\times\frac{3}{2}x^{\frac{3}{2}-1} - \frac{1}{x} + 0$$

$$= 7^x \log 7 + 7x^6 - x^{\frac{1}{2}} - \frac{1}{x}$$

$$= 7^x \log 7 + 7x^6 - \sqrt{x} - \frac{1}{x}$$

Question 6.
y = 3 cot x – 5e^x + 3 log x – $\frac{4}{x^{\frac{3}{4}}}$
Solution:

$$y = 3\cot x - 5e^x + 3\log x - \frac{4}{x^{\frac{3}{4}}}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(3\cot x - 5e^x + 3\log x - \frac{4}{x^{\frac{3}{4}}}\right)$$

$$\frac{dy}{dx} = 3\frac{d}{dx}(\cot x) - 5\frac{d}{dx}\left(e^x\right) + 3\frac{d}{dx}(\log x) - 4\frac{d}{dx}\left(x^{-\frac{3}{4}}\right)$$

$$= 3(-\operatorname{cosec}^2 x) - 5e^x + 3\left(\frac{1}{x}\right) - 4\left(-\frac{3}{4}x^{-\frac{3}{4}-1}\right)$$

$$= -3\operatorname{cosec}^2 x - 5e^x + \frac{3}{x} + 3x^{\frac{-7}{4}}$$

$$= -3\operatorname{cosec}^2 x - 5e^x + \frac{3}{x} + \frac{3}{x^{\frac{7}{4}}}$$

(II) Diffrentiate the following w.r.t. x

Question 1.
y = x5 tan x
Solution:

$y = x^5 \tan x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^5 \tan x\right)$$

$$= x^5 \frac{d}{dx}\tan x + \tan x \frac{d}{dx}x^5$$

$$= x^5 \sec^2 x + \tan x\,(5x^4)$$

$$= x^4 (x \sec^2 x + 5\tan x)$$

Question 2.
y = x3 log x
Solution:

$y = x^3 \log x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 \log x)$$

$$\frac{dy}{dx} = x^3\left(\frac{d}{dx}\log x\right) + \log x\left(\frac{d}{dx}x^3\right)$$

$$= x^3\left(\frac{1}{x}\right) + \log x\,(3x^2)$$

$$= x^2 + 3x^2 \log x$$

$$= x^2 (1 + 3 \log x)$$

Question 3.
y = (x2 + 2)2 sin x
Solution:

$y = (x^2 + 2)^2 \sin x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[(x^2 + 2)^2 \sin x]$$

$$\frac{dy}{dx} = (x^2 + 2)^2 \frac{d}{dx}\sin x + \sin x\left[\frac{d}{dx}\left(x^2 + 2\right)^2\right]$$

$$= \left(x^2+2\right)^2 \cos x + \sin x \frac{d}{dx}\left(x^4 + 4x^2 + 4\right)$$

$$= \left(x^2+2\right)^2 \cos x + \sin x\left(4x^3 + 8x\right)$$

$$= \left(x^2+2\right)^2 \cos x + \sin x.(4x)\left(x^2 + 2\right)$$

$$= \left(x^2 + 2\right)\left[\left(x^2 + 2\right)\cos x + 4x \sin x\right]$$

Question 4.
y = ex log x
Solution:

$y = e^x \log x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(e^x \log x)$$

$$\frac{dy}{dx} = e^x \frac{d}{dx}\log x + \log x\left(\frac{d}{dx}e^x\right)$$

$$= e^x\left(\frac{1}{x}\right) + \log x\,(e^x)$$

$$= e^x\left(\frac{1}{x} + \log x\right)$$

Question 5.
y = x32exlogx

Solution:

$$y = x^{\frac{3}{2}}\left(e^x \log x\right)$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{3}{2}}\left(e^x \log x\right)\right)$$

$$\frac{dy}{dx} = x^{\frac{3}{2}}\frac{d}{dx}\left(e^x \log x\right) + \left(e^x \log x\right)\frac{d}{dx}\left(x^{\frac{3}{2}}\right)$$

$$= x^{\frac{3}{2}}\left[e^x \frac{d}{dx}\log x + \log x\frac{d}{dx}e^x\right] + \left(e^x \log x\right)\left(\frac{3}{2}x^{\frac{3}{2}-1}\right)$$

$$= x^{\frac{3}{2}}\left[e^x\left(\frac{1}{x}\right) + \log x\left(e^x\right)\right] + \left(e^x \log x\right)\left(\frac{3}{2}x^{\frac{1}{2}}\right)$$

$$= e^x x^{\frac{3}{2}}\left(\frac{1}{x} + \log x\right) + \left(e^x \log x\right)\left(\frac{3}{2}x^{\frac{1}{2}}\right)$$

$$= x^{\frac{1}{2}}e^x\left[x\left(\frac{1}{x} + \log x\right) + \left(\log x\right)\left(\frac{3}{2}\right)\right]$$

$$= \sqrt{x}e^x\left[1 + x\log x + \frac{3}{2}\log x\right]$$

Question 6.

y = $\log e^{x^3} \log x^3$

Solution:

$$y = \log e^{x^3} \log x^3$$

$$= x^3(\log e)\ 3(\log x) \quad \ldots[\because \log m^n = n \log m]$$

$$= 3x^3 \log x \quad \ldots[\because \log e = 1]$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(3x^3 \log x)$$

$$\frac{dy}{dx} = 3\left[x^3\frac{d}{dx}\log x + \log x\left(\frac{d}{dx}x^3\right)\right]$$

$$= 3\left[x^3\left(\frac{1}{x}\right) + \log x\left(3x^2\right)\right]$$

$$= 3\left[x^2 + \log x\left(3x^2\right)\right]$$

$$= 3x^2\left(1 + 3\log x\right)$$

(III) Diffrentiate the following w.r.t. x

Question 1.

y = $x^2\sqrt{x} + x^4 \log x$

Solution:

$$y = x^2\sqrt{x} + x^4 \log x$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2\sqrt{x} + x^4 \log x\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{5}{2}}\right) + \frac{d}{dx}\left(x^4 \log x\right)$$

$$= \frac{5}{2}x^{\frac{5}{2}-1} + \left[x^4\frac{d}{dx}\log x + \log x\left(\frac{d}{dx}x^4\right)\right]$$

$$= \frac{5}{2}x^{\frac{3}{2}} + \left[x^4\left(\frac{1}{x}\right) + \log x\left(4x^3\right)\right]$$

$$= \frac{5}{2}x^{\frac{3}{2}} + x^3\left(1 + 4\log x\right)$$

Question 2.

y = $e^x \sec x - x^{\frac{5}{3}} \log x$

Solution:

$$y = e^x \sec x - x^{\frac{5}{3}} \log x$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x \sec x - x^{\frac{5}{3}} \log x\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x \sec x\right) - \frac{d}{dx}\left(x^{\frac{5}{3}} \log x\right)$$

$$= \left[e^x\left(\frac{d}{dx}\sec x\right) + \sec x\frac{d}{dx}\left(e^x\right)\right] - \left[x^{\frac{5}{3}}\left(\frac{d}{dx}\log x\right) + \log x\left(\frac{d}{dx}x^{\frac{5}{3}}\right)\right]$$

$$= [e^x \sec x \tan x + \sec x\,(e^x)] - \left[x^{\frac{5}{3}}\left(\frac{1}{x}\right) + \log x\left(\frac{5}{3}x^{\frac{5}{3}-1}\right)\right]$$

$$= e^x \sec x\,(\tan x + 1) - \left[x^{\frac{2}{3}} + (\log x)\frac{5}{3}x^{\frac{2}{3}}\right]$$

$$= e^x \sec x\,(\tan x + 1) - x^{\frac{2}{3}}\left(1 + \frac{5}{3}\log x\right)$$

Question 3.

$y = x^4 + x\sqrt{x} \cos x - x^2 e^x$

Solution:

$$y = x^4 + x\sqrt{x}\ \cos x - x^2 e^x$$

$$= x^4 + x^{\frac{3}{2}}\ \cos x - x^2 e^x$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^4 + x^{\frac{3}{2}} \cos x - x^2 e^x\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^4\right) + \frac{d}{dx}\left(x^{\frac{3}{2}} \cos x\right) - \frac{d}{dx}\left(x^2 e^x\right)$$

$$= 4x^3 + x^{\frac{3}{2}}\frac{d}{dx}\cos x + \cos x\frac{d}{dx}\left(x^{\frac{3}{2}}\right) - \left[x^2\left(\frac{d}{dx}e^x\right) + e^x\left(\frac{d}{dx}x^2\right)\right]$$

$$= 4x^3 + x^{\frac{3}{2}}(-\sin x) + \cos x\left(\frac{3}{2}x^{\frac{1}{2}}\right) - \left[x^2 e^x + e^x\,(2x)\right]$$

$$= 4x^3 - x^{\frac{3}{2}}\sin x + \frac{3}{2}\sqrt{x}\cos x - xe^x(x+2)$$

Question 4.

$y = (x^3 - 2) \tan x - x \cos x + 7^x . x^7$

Solution:

$y = (x^3 - 2)\tan x - x\cos x + 7^x x^7$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(x^3-2\right)\tan x\right] - \frac{d}{dx}(x\cos x) + \frac{d}{dx}\left(7^x x^7\right)$$

$$= \left(x^3-2\right)\frac{d}{dx}(\tan x) + \tan x\frac{d}{dx}\left(x^3-2\right)$$

$$-\left(x\frac{d}{dx}\cos x + \cos x\frac{d}{dx}x\right) + 7^x\left(\frac{d}{dx}x^7\right) + x^7\frac{d}{dx}\left(7^x\right)$$

$$\therefore \quad \frac{dy}{dx} = (x^3-2)\sec^2 x + \tan x\,(3x^2)$$

$$-(-x\sin x + \cos x) + 7^x(7x^6) + x^7\,7^x(\log 7)$$

$$= (x^3-2)\sec^2 x + 3x^2\tan x + x\sin x - \cos x + 7^x x^6(7 + x\log 7)$$

Question 5.
y = sin x log x + e^x cos x – e^x √x
Solution:

$y = \sin x\log x + e^x\cos x - e^x\sqrt{x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x\log x) + \frac{d}{dx}\left(e^x\cos x\right) - \frac{d}{dx}\left(e^x\sqrt{x}\right)$$

$$= \sin x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(\sin x)$$

$$+ e^x\frac{d}{dx}(\cos x) + \cos x\frac{d}{dx}\left(e^x\right)$$

$$-\left[e^x\frac{d}{dx}\left(\sqrt{x}\right) + \sqrt{x}\left(\frac{d}{dx}e^x\right)\right]$$

$$= \sin x\left(\frac{1}{x}\right) + \log x(\cos x)$$

$$- e^x\sin x + \cos x\left(e^x\right) - e^x\left(\frac{1}{2\sqrt{x}}\right) - \sqrt{x}\,e^x$$

$$= \frac{1}{x}\sin x + (\log x)(\cos x) + e^x(-\sin x + \cos x)$$

$$- e^x\left(\frac{1}{2\sqrt{x}} + \sqrt{x}\right)$$

$$= \frac{\sin x}{x} + (\cos x)(\log x) + e^x(-\sin x + \cos x)$$

$$- e^x\left(\frac{1+2x}{2\sqrt{x}}\right)$$

Question 6.
y = e^x tan x + cos x log x – √x 5^x
Solution:

$y = e^x \tan x + (\cos x)(\log x) - (\sqrt{x})5^x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(e^x \tan x) + \frac{d}{dx}(\cos x \log x) - \frac{d}{dx}\left[(\sqrt{x})5^x\right]$$

$$= \left(e^x \frac{d}{dx}\tan x + \tan x \frac{d}{dx} e^x\right) + \left[\cos x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(\cos x)\right] - \left[\sqrt{x}\frac{d}{dx}(5^x) + 5^x \frac{d}{dx}(\sqrt{x})\right]$$

$$= e^x \sec^2 x + \tan x\,(e^x) + \cos x\left(\frac{1}{x}\right) + \log x(-\sin x) - \left[\sqrt{x}(5^x \log 5) + 5^x \frac{1}{2\sqrt{x}}\right]$$

$$= e^x(\sec^2 x + \tan x) + \frac{\cos x}{x} - \sin x \log x - \left[\frac{2x\, 5^x \log 5 + 5^x}{2\sqrt{x}}\right]$$

$$= e^x(\sec^2 x + \tan x) + \frac{\cos x}{x} - \sin x \log x - 5^x\left(\frac{2x \log 5 + 1}{2\sqrt{x}}\right)$$

(IV) Diffrentiate the following w.r.t.x.

Question 1.
y = $x2+3x2-5$
Solution:

$$y = \frac{x^2+3}{x^2-5}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+3}{x^2-5}\right)$$

$$= \frac{(x^2-5)\frac{d}{dx}(x^2+3) - (x^2+3)\frac{d}{dx}(x^2-5)}{(x^2-5)^2}$$

$$= \frac{(x^2-5)(2x) - (x^2+3)(2x)}{(x^2-5)^2}$$

$$= \frac{2x(x^2-5-x^2-3)}{(x^2-5)^2}$$

$$= \frac{2x(-8)}{(x^2-5)^2} = \frac{-16x}{(x^2-5)^2}$$

Question 2.
y = $x\sqrt{}+5x\sqrt{}-5$
Solution:

$$y = \frac{\sqrt{x}+5}{\sqrt{x}-5}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sqrt{x}+5}{\sqrt{x}-5}\right)$$

$$= \frac{\left(\sqrt{x}-5\right)\frac{d}{dx}\left(\sqrt{x}+5\right)-\left(\sqrt{x}+5\right)\frac{d}{dx}\left(\sqrt{x}-5\right)}{\left(\sqrt{x}-5\right)^2}$$

$$= \frac{\left(\sqrt{x}-5\right)\left(\frac{1}{2\sqrt{x}}\right)-\left(\sqrt{x}+5\right)\left(\frac{1}{2\sqrt{x}}\right)}{\left(\sqrt{x}-5\right)^2}$$

$$= \frac{\frac{1}{2\sqrt{x}}\left(\sqrt{x}-5-\sqrt{x}-5\right)}{\left(\sqrt{x}-5\right)^2}$$

$$= \frac{\frac{1}{2\sqrt{x}}(-10)}{\left(\sqrt{x}-5\right)^2} = \frac{-5}{\sqrt{x}\left(\sqrt{x}-5\right)^2}$$

Question 3.
y = $xe^x x+e^x$
Solution:

$$y = \frac{xe^x}{x+e^x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{xe^x}{x+e^x}\right)$$

$$= \frac{\left(x+e^x\right)\frac{d}{dx}xe^x - xe^x\frac{d}{dx}\left(x+e^x\right)}{\left(x+e^x\right)^2}$$

$$= \frac{\left(x+e^x\right)\left(x\frac{d}{dx}e^x + e^x\frac{d}{dx}x\right) - xe^x\left(\frac{d}{dx}x + \frac{d}{dx}e^x\right)}{\left(x+e^x\right)^2}$$

$$= \frac{\left(x+e^x\right)\left(x\,e^x + e^x\right) - xe^x\left(1+e^x\right)}{\left(x+e^x\right)^2}$$

$$= \frac{\left(x+e^x\right)e^x(x+1) - xe^x\left(1+e^x\right)}{\left(x+e^x\right)^2}$$

$$= \frac{e^x\left[\left(x+e^x\right)(x+1) - x\left(1+e^x\right)\right]}{\left(x+e^x\right)^2}$$

$$= \frac{e^x\left[x^2+x+x\,e^x+e^x-x-x\,e^x\right]}{\left(x+e^x\right)^2}$$

$$= \frac{e^x\left(x^2+e^x\right)}{\left(x+e^x\right)^2}$$

Question 4.
y = $xlogxx+logx$
Solution:

y = xlogxx+logx

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x\log x}{x+\log x}\right)$$

$$= \frac{(x+\log x)\frac{d}{dx}(x\log x) - x\log x\frac{d}{dx}(x+\log x)}{(x+\log x)^2}$$

$$= \frac{(x+\log x)\left(x\frac{d}{dx}\log x + \log x\frac{d}{dx}x\right) - x\log x\left(\frac{d}{dx}x + \frac{d}{dx}\log x\right)}{(x+\log x)^2}$$

$$= \frac{(x+\log x)\left[x\left(\frac{1}{x}\right)+\log x\,(1)\right] - x\log x\left(1+\frac{1}{x}\right)}{(x+\log x)^2}$$

$$= \frac{(x+\log x)(1+\log x) - x\log x\left(1+\frac{1}{x}\right)}{(x+\log x)^2}$$

$$= \frac{x + x\log x + \log x + (\log x)^2 - x\log x - \log x}{(x+\log x)^2}$$

$$= \frac{x+(\log x)^2}{(x+\log x)^2}$$

Question 5.

y = x2sinxx+cosx

Solution:

$$y = \frac{x^2\sin x}{x+\cos x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2\sin x}{x+\cos x}\right)$$

$$= \frac{(x+\cos x)\frac{d}{dx}(x^2\sin x) - x^2\sin x\frac{d}{dx}(x+\cos x)}{(x+\cos x)^2}$$

$$= \frac{(x+\cos x)\left(x^2\frac{d}{dx}\sin x + \sin x\frac{d}{dx}x^2\right) - x^2\sin x\left(\frac{d}{dx}x + \frac{d}{dx}\cos x\right)}{(x+\cos x)^2}$$

$$= \frac{(x+\cos x)\left[x^2\cos x + \sin x(2x)\right] - x^2\sin x(1-\sin x)}{(x+\cos x)^2}$$

$$= \frac{x^3\cos x + 2x^2\sin x + x^2\cos^2 x + 2x\sin x\cos x - x^2\sin x + x^2\sin^2 x}{(x+\cos x)^2}$$

$$= \frac{x^3\cos x + x^2\sin x + x^2\cos^2 x + x^2\sin^2 x + 2x\sin x\cos x}{(x+\cos x)^2}$$

$$= \frac{x^3\cos x + x^2\sin x + x^2(\sin^2 x + \cos^2 x) + x\sin 2x}{(x+\cos x)^2}$$

$$= \frac{x^3\cos x + x^2\sin x + x^2(1) + x\sin 2x}{(x+\cos x)^2}$$

$$= \frac{x^2 + x^2\sin x + x^3\cos x + x\sin 2x}{(x+\cos x)^2}$$

$$= \frac{x^2(1+\sin x + x\cos x) + x\sin 2x}{(x+\cos x)^2}$$

Question 6.

y = 5ex−43ex−2

Solution:

$$y = \frac{5e^x - 4}{3e^x - 2}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{5e^x - 4}{3e^x - 2}\right)$$

$$= \frac{(3e^x - 2)\frac{d}{dx}(5e^x - 4) - (5e^x - 4)\frac{d}{dx}(3e^x - 2)}{(3e^x - 2)^2}$$

$$= \frac{(3e^x - 2)\left(5\frac{d}{dx}e^x - \frac{d}{dx}4\right) - (5e^x - 4)\left(3\frac{d}{dx}e^x - \frac{d}{dx}2\right)}{(3e^x - 2)^2}$$

$$= \frac{(3e^x - 2)(5e^x - 0) - (5e^x - 4)(3e^x - 0)}{(3e^x - 2)^2}$$

$$= \frac{15(e^x)^2 - 10e^x - 15(e^x)^2 + 12e^x}{(3e^x - 2)^2} = \frac{2e^x}{(3e^x - 2)^2}$$

(V).

Question 1.
If f(x) is a quadratic polynomial such that f(0) = 3, f'(2) = 2 and f'(3) = 12, then find f(x).
Solution:
Let f(x) = ax^2 + bx + c .....(i)
∴ f(0) = $a(0)^2$ + b(0) + c
∴ f(0) = c
But, f(0) = 3 .....(given)
∴ c = 3 .....(ii)
Differentiating (i) w.r.t. x, we get
f'(x) = 2ax + b
∴ f'(2) = 2a(2) + b
∴ f'(2) = 4a + b
But, f'(2) = 2 .....(given)
∴ 4a + b = 2 .....(iii)
Also, f'(3) = 2a(3) + b
∴ f'(3) = 6a + b
But, f'(3) = 12 .....(given)
∴ 6a + b = 12 .....(iv)
equation (iv) – equation (iii), we get
2a = 10
∴ a = 5
Substituting a = 5 in (iii), we get
4(5) + b = 2
∴ b = -18
∴ a = 5, b = -18, c = 3
∴ f(x) = $5x^2$ – 18x + 3

Check:
If f(0) = 3, f'(2) = 2 and f'(3) = 12, then our answer is correct.
f(x) = $5x^2$ – 18x + 3 and f'(x) = 10x – 18
f(0) = $5(0)^2$ – 18(0) + 3 = 3
f'(2) = 10(2) – 18 = 2
f'(3) = 10(3) – 18 = 12
Thus, our answer is correct.

Question 2.
If f(x) = a sin x – b cos x, f'($\frac{\pi}{4}$) = √2 and f'($\frac{\pi}{6}$) = 2, then find f(x).
Solution:
f(x) = a sin x – b cos x
Differentiating w.r.t. x, we get
f'(x) = a cos x – b (- sin x)
∴ f'(x) = a cos x + b sin x

$\therefore \quad f'\left(\frac{\pi}{4}\right) = a\cos\left(\frac{\pi}{4}\right) + b\sin\left(\frac{\pi}{4}\right)$

But, $f'\left(\frac{\pi}{4}\right) = \sqrt{2}$ ...(given)

$\therefore \quad a\cos\frac{\pi}{4} + b\sin\frac{\pi}{4} = \sqrt{2}$

$\therefore \quad a\left(\frac{1}{\sqrt{2}}\right) + b\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2}$

$\therefore \quad \frac{a+b}{\sqrt{2}} = \sqrt{2}$

$\therefore \quad a + b = 2$ ...(i)

Also, $f'\left(\frac{\pi}{6}\right) = a\cos\frac{\pi}{6} + b\sin\frac{\pi}{6}$

But, $f'\left(\frac{\pi}{6}\right) = 2$ ...(given)

$\therefore \quad a\cos\frac{\pi}{6} + b\sin\frac{\pi}{6} = 2$

$\therefore \quad a\left(\frac{\sqrt{3}}{2}\right) + b\left(\frac{1}{2}\right) = 2$

$\therefore \quad \sqrt{3}a + b = 4$ ...(ii)

equation (ii) – equation (i), we get

$\sqrt{3}a - a = 2$

$\therefore \quad a\left(\sqrt{3} - 1\right) = 2$

$\therefore \quad a = \frac{2}{\sqrt{3}-1} = \frac{2\left(\sqrt{3}+1\right)}{3-1}$

$\therefore \quad a = \sqrt{3} + 1$

Substituting $a = \sqrt{3} + 1$ in equation (i), we get

$\sqrt{3} + 1 + b = 2$

$\therefore \quad b = 1 - \sqrt{3}$

Now, f(x) = a sin x – b cos x

∴ f(x) = (√3 + 1) sin x + (√3 – 1) cos x

VI. Fill in the blanks. (Activity Problems)

Question 1.

$y = e^x \cdot \tan x$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}(e^x \tan x)$$

$$= \square \frac{d}{dx}\tan x + \tan x \frac{d}{dx}\square$$

$$= \square\,\square + \tan x \cdot \square$$

$$= e^x[\square + \square]$$

Solution:

$y = e^x.\tan x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(e^x \tan x)$$

$$= \boxed{e^x}\frac{d}{dx}\tan x + \tan x\frac{d}{dx}\boxed{e^x}$$

$$= \boxed{e^x}\,\boxed{\sec^2 x} + e^x \tan x$$

$$= e^x\left[\boxed{\sec^2 x} + \boxed{\tan x}\right]$$

Question 2.

$y = \sin x x^2 + 2$

diff. w.r.t. x

$$\frac{dy}{dx} = \frac{\square \frac{d}{dx}(\sin x) - \sin x \frac{d}{dx}\square}{(x^2+2)^2}$$

$$= \frac{\square\square - \sin x \square}{(x^2+2)^2}$$

$$= \frac{\square - \square}{(x^2+2)^2}$$

Solution:

$$y = \frac{\sin x}{x^2+2}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{\boxed{x^2+2}\frac{d}{dx}(\sin x) - \sin x \frac{d}{dx}\boxed{x^2+2}}{(x^2+2)^2}$$

$$= \frac{\boxed{(x^2+2)}\boxed{(\cos x)} - \sin x \boxed{(2x)}}{(x^2+2)^2}$$

$$= \frac{\boxed{(x^2 \cos x + 2\cos x)} - \boxed{2x \sin x}}{(x^2+2)^2}$$

Question 3.

y = (3x2 + 5) cos x

Diff. w.r.t. x

Diff. $w.r.t. x$

$$\frac{dy}{dx} = \frac{d}{dx}\left[(3x^2+5)\cos x\right]$$

$$= (3x^2+5)\frac{d}{dx}[\ \square\ ] + \cos x \frac{d}{dx}[\ \square\ ]$$

$$= (3x^2+5)[\ \square\ ] + \cos x[\ \square\ ]$$

$$\therefore \quad \frac{dx}{dy} = (3x^2+5)[\ \square\ ] + [\ \square\ ]\cos x$$

Solution:

$y = (3x^2 + 5) \cos x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[(3x^2+5)\cos x\right]$$

$$= (3x^2+5)\frac{d}{dx}\left[\boxed{\cos x}\right] + \cos x \frac{d}{dx}\left[\boxed{3x^2+5}\right]$$

$$= (3x^2+5)\left[\boxed{-\sin x}\right] + \cos x \left[\boxed{6x}\right]$$

$$\frac{dy}{dx} = (3x^2+5)\left[\boxed{-\sin x}\right] + \left[\boxed{6x}\right]\cos x$$

Question 4.

Differentiate tan x and sec x w.r.t. x using the formulae for differentiation of $\frac{u}{v}$ and $\frac{1}{v}$ respectively.

Solution:

Let $y = \tan x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\tan x)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{\cos x \frac{d}{dx}\sin x - \sin x \frac{d}{dx}\cos x}{\cos^2 x}$$

$$= \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\frac{dy}{dx} = \sec^2 x$$

Let $y = \sec x$

$$y = \frac{1}{\cos x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{\cos x \frac{d}{dx}(1) - 1\frac{d}{dx}\cos x}{\cos^2 x}$$

$$= \frac{\cos x(0) - 1(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \sec x \tan x$$

Maharashtra State Board 11th Maths Solutions Chapter 9 Differentiation Miscellaneous Exercise 9

(I) Select the appropriate option from the given alternatives.

Question 1.
If y = x−4x+2√, then dydx is
(A) 1x+4
(B) x√(x+2)2√
(C) 12x√
(D) x(x√+2)2
Answer:
(C) 12x√

Hint:

$$y=\frac{x-4}{\sqrt{x}+2}=\frac{(\sqrt{x})^2-(2)^2}{\sqrt{x}+2}$$

$$=\frac{(\sqrt{x}+2)(\sqrt{x}-2)}{(\sqrt{x}+2)}$$

$$y=\sqrt{x}-2$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1}{2\sqrt{x}}$$

Question 2.
If y = ax+bcx+d,then dydx =
(A) ab−cd(cx+d)2
(B) ax−c(cx+d)2
(C) ac−bd(cx+d)2
(D) ad−bc(cx+d)2
Answer:
(D) ad−bc(cx+d)2
Hint:

$$y=\frac{ax+b}{cx+d}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{(cx+d)\frac{d}{dx}(ax+b)-(ax+b)\frac{d}{dx}(cx+d)}{(cx+d)^2}$$

$$=\frac{a(cx+d)-c(ax+b)}{(cx+d)^2}=\frac{ad-bc}{(cx+d)^2}$$

Question 3.
If y = 3x+54x+5, then dydx =
(A) −15(3x+5)2
(B) −15(4x+5)2
(C) −5(4x+5)2
(D) −13(4x+5)2
Answer:
(C) −5(4x+5)2
Hint:

$$y=\frac{3x+5}{4x+5}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{(4x+5)\frac{d}{dx}(3x+5)-(3x+5)\frac{d}{dx}(4x+5)}{(4x+5)^2}$$

$$=\frac{3(4x+5)-4(3x+5)}{(4x+5)^2}$$

$$=-\frac{5}{(4x+5)^2}$$

Question 4.
If y = 5sinx−24sinx+3, then dydx =
(A) 7cosx(4sinx+3)2
(B) 23cosx(4sinx+3)2
(C) −7cosx(4sinx+3)2
(D) −15cosx(4sinx+3)2
Answer:
(B) 23cosx(4sinx+3)2

Hint:

$$y = \frac{5\sin x - 2}{4\sin x + 3}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{(4\sin x + 3)\frac{d}{dx}(5\sin x - 2) - (5\sin x - 2)\frac{d}{dx}(4\sin x + 3)}{(4\sin x + 3)^2}$$

$$= \frac{(4\sin x + 3)(5\cos x) - (5\sin x - 2)(4\cos x)}{(4\sin x + 3)^2}$$

$$= \frac{20\sin x\cos x + 15\cos x - 20\sin x\cos x + 8\cos x}{(4\sin x + 3)^2}$$

$$= \frac{23\cos x}{(4\sin x + 3)^2}$$

Question 5.
Suppose f(x) is the derivative of g(x) and g(x) is the derivative of h(x).
If h(x) = a sin x + b cos x + c, then f(x) + h(x) =
(A) 0
(B) c
(C) -c
(D) -2(a sin x + b cos x)
Answer:
(B) c
Hint:
h(x) = a sin x + b cos x + c
Differentiating w.r.t. x, we get
h'(x) = a cos x – b sin x = g(x) …..[given]
Differentiating w.r.t. x, we get
g'(x) = -a sin x – b cos x = f(x) …..[given]
∴ f(x) + h(x) = -a sin x – b cos x + a sin x + b cos x + c
∴ f(x) + h(x) = c

Question 6.
If f(x) = 2x + 6, for 0 ≤ x ≤ 2
= ax^2 + bx, for 2 < x ≤ 4
is differentiable at x = 2, then the values of a and b are
(A) a = −32, b = 3
(B) a = 32, b = 8
(C) a = 12, b = 8
(D) a = −32, b = 8
Answer:
(D) a = −32, b = 8
Hint:
f(x) = 2x + 6, 0 ≤ x ≤ 2
= ax^2 + bx, 2 < x ≤ 4
Lf'(2) = 2, Rf'(2) = 4a + b
Since f is differentiable at x = 2,
Lf'(2) = Rf'(2)
∴ 2 = 4a + b …..(i)
f is continuous at x = 2.
∴ $lim_{x\to 2^+} f(x) = f(2) = lim_{x\to 2^-} f(x)$
∴ 4a + 2b = 2(2) + 6
∴ 4a + 2b = 10
∴ 2a + b = 5 …..(ii)
Solving (i) and (ii), we get
a = −32, b = 8

Question 7.
If f(x) = x^2 + sin x + 1, for x ≤ 0
= x^2 – 2x + 1, for x ≤ 0, then
(A) f is continuous at x = 0, but not differentiable at x = 0
(B) f is neither continuous nor differentiable at x = 0
(C) f is not continuous at x = 0, but differentiable at x = 0

(D) f is both continuous and differentiable at x = 0
Answer:
(A) f is continuous at x = 0, but not differentiable at x = 0
Hint:

$$f(x) = x^2 + \sin x + 1,\ x \le 0$$
$$= x^2 - 2x + 1,\quad x > 0$$
$$\lim_{x\to0^-} f(x) = \lim_{x\to0^-} (x^2 + \sin x + 1) = 0 + 0 + 1 = 1$$
$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} (x^2 - 2x + 1) = 0 - 0 + 1 = 1$$
$$f(0) = 1$$

$\therefore$ f is continuous at $x = 0$.

$$Lf'(0) = \lim_{h\to0^-} \frac{f(0+h) - f(0)}{h}$$
$$= \lim_{h\to0^-} \frac{h^2 + \sin h + 1 - 1}{h}$$
$$= \lim_{h\to0^-} \left(h + \frac{\sin h}{h}\right) = 0 + 1 = 1$$
$$Rf'(0) = \lim_{h\to0^+} \frac{f(0+h) - f(0)}{h}$$
$$= \lim_{h\to0^+} \frac{h^2 - 2h + 1 - 1}{h}$$
$$= \lim_{h\to0} (h - 2)$$
$$= -2$$

Here, $Lf'(0) \ne Rf'(0)$

$\therefore$ f is not differentiable at $x = 0$.

Question 8.
If f(x) = $\frac{x^{50}}{50}+\frac{x^{49}}{49}+\frac{x^{48}}{48}+\dots+\frac{x^2}{2}+x+1$, then f'(1) =
(A) 48
(B) 49
(C) 50
(D) 51
Answer:
(C) 50
Hint:

$$f(x) = \frac{x^{50}}{50} + \frac{x^{49}}{49} + \dots + \frac{x^2}{2} + x + 1$$

Differentiating w.r.t. x, we get

$$f'(x) = \frac{50x^{49}}{50} + \frac{49x^{48}}{49} + \dots + \frac{2x}{2} + 1 + 0$$
$$= x^{49} + x^{48} + \dots + x + 1$$
$$f'(1) = \underbrace{1 + 1 + \dots + 1}_{49\text{ times}} + 1 = 50$$

(II).

Question 1.
Determine whether the following function is differentiable at x = 3 where,
f(x) = x^2 + 2, for x ≥ 3
= 6x – 7, for x < 3.
Solution:
f(x) = x^2 + 2, x ≥ 3
= 6x – 7, x < 3
Differentiability at x = 3

$$
\begin{aligned}
Lf'(3) &= \lim_{h\to0^-}\frac{f(3+h)-f(3)}{h}\\
&= \lim_{h\to0^-}\frac{6(3+h)-7-(3^2+2)}{h}\\
&= \lim_{h\to0^-}\frac{18+6h-7-11}{h}\\
&= \lim_{h\to0^-}\frac{6h}{h}\\
&= \lim_{h\to0^-}6 \quad \ldots[\because h\to0, \therefore h\neq0]\\
&= 6
\end{aligned}
$$

$$
\begin{aligned}
Rf'(3) &= \lim_{h\to0^+}\frac{f(3+h)-f(3)}{h}\\
&= \lim_{h\to0^+}\frac{(3+h)^2+2-(3^2+2)}{h}\\
&= \lim_{h\to0^+}\frac{h^2+6h+9+2-11}{h}\\
&= \lim_{h\to0^+}\frac{h^2+6h}{h}\\
&= \lim_{h\to0^+}(h+6) \quad \ldots[\because h\to0, \therefore h\neq0]\\
&= 6
\end{aligned}
$$

Here, Lf'(3) = Rf'(3)
∴ f is differentiable at x = 3.

Question 2.
Find the values of p and q that make function f(x) differentiable everywhere on R.
f(x) = 3 – x, for x < 1
= px2 + qx, for x ≥ 1.
Solution:
f(x) is differentiable everywhere on R.
∴ f(x) is differentiable at x = 1.
∴ f(x) is continuous at x = 1.

Continuity at $x = 1$:

f(x) is continuous at x = 1.

$$\therefore \quad \lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x)$$

$$\therefore \quad \lim_{x\to1^-}(3-x) = \lim_{x\to1^+}(px^2+qx)$$

$$\therefore \quad 2 = p+q \quad \ldots(i)$$

Differentiability at $x = 1$:

$$
\begin{aligned}
Lf'(1) &= \lim_{h\to0^-}\frac{f(1+h)-f(1)}{h}\\
&= \lim_{h\to0^-}\frac{3-(1+h)-(p+q)}{h}\\
&= \lim_{h\to0^-}\left(\frac{2-(p+q)}{h}-1\right)\\
&= -1 \quad \ldots[\because p+q=2]
\end{aligned}
$$

$$
\begin{aligned}
Rf'(1) &= \lim_{h\to0^+}\frac{f(1+h)-f(1)}{h}\\
&= \lim_{h\to0^+}\frac{p(1+h)^2+q(1+h)-(p+q)}{h}\\
&= \lim_{h\to0^+}\frac{p(1+2h+h^2)+q+qh-p-q}{h}\\
&= \lim_{h\to0^+}\frac{h(ph+2p+q)}{h}\\
&= \lim_{h\to0^+}(ph+2p+q)\\
&\qquad \ldots[\because h\to0, \therefore h\neq0]\\
&= 2p+q
\end{aligned}
$$

f(x) is differentiable at x = 1.
∴ Lf'(1) = Rf'(1)
∴ -1 = 2p + q .....(ii)

Subtracting (i) from (ii), we get
p = -3
Substituting p = -3 in (i), we get
p + q = 2
∴ -3 + q = 2
∴ q = 5

Question 3.
Determine the values of p and q that make the function f(x) differentiable on R where
f(x) = px3, for x < 2
= x2 + q, for x ≥ 2
Solution:
f(x) is differentiable on R.
∴ f(x) is differentiable at x = 2.
∴ f(x) is continuous at x = 2.
Continuity at x = 2:
f(x) is continuous at x = 2.

$$\therefore \quad \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$\therefore \quad \lim_{x \to 2^-} px^3 = \lim_{x \to 2^+} \left(x^2 + q\right)$$

$$\therefore \quad 8p = 4 + q$$

$$\therefore \quad 8p - q = 4 \qquad \ldots(i)$$

Differentiability at $x = 2$:

$$Lf'(2) = \lim_{h \to 0^-} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0^-} \frac{p(2+h)^3 - (2^2 + q)}{h}$$

$$= \lim_{h \to 0^-} \left(\frac{ph^3 + 6ph^2 + 12hp + 8p - (4+q)}{h}\right)$$

$$= \lim_{h \to 0^-} \frac{h(ph^2 + 6ph + 12p)}{h}$$

$$\ldots[\because 8p = 4 + q]$$

$$= \lim_{h \to 0^-} (ph^2 + 6ph + 12p)$$

$$\ldots[\because h \to 0, \therefore h \neq 0]$$

$$= 12p$$

$$Rf'(2) = \lim_{h \to 0^+} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0^+} \frac{(2+h)^2 + q - (2^2 + q)}{h}$$

$$= \lim_{h \to 0^+} \frac{h^2 + 4h + 4 + q - (4+q)}{h}$$

$$= \lim_{h \to 0} \left(\frac{h^2 + 4h}{h}\right)$$

$$= \lim_{h \to 0} (h + 4) \qquad \ldots[\because h \to 0, \therefore h \neq 0]$$

$$= 4$$

f(x) is differentiable at x = 2.
∴ Lf'(2) = Rf'(2)
∴ 12p = 4
∴ p = 13
Substituting p = 13 in (i), we get
8(13 – q = 4
∴ q = 83 – 4 = -43

Question 4.
Determine all real values of p and q that ensure the function
f(x) = px + q, for x ≤ 1
= tan(πx4), for 1 < x < 2
is differentiable at x = 1.

Solution:
f(x) is differentiable at x = 1.
∴ f(x) is continuous at x = 1.
Continuity at x= 1:
f(x) is continuous at x = 1.

$$\therefore \quad \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$\therefore \quad \lim_{x \to 1^-} (px + q) = \lim_{x \to 1^+} \tan\left(\frac{\pi x}{4}\right)$$

$$\therefore \quad p + q = \tan\frac{\pi}{4} = 1 \qquad \ldots(i)$$

Differentiability at $x = 1$:

$$Lf'(1) = \lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0^-} \frac{p(1+h) + q - (p+q)}{h}$$

$$= \lim_{h \to 0} \frac{ph}{h}$$

$$= p \qquad \ldots[\because h \to 0, \therefore h \neq 0]$$

$$Rf'(1) = \lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0^+} \frac{\tan\frac{\pi}{4}(1+h) - (p+q)}{h}$$

$$= \lim_{h \to 0^+} \frac{\tan\left(\frac{\pi}{4} + \frac{\pi h}{4}\right) - 1}{h} \qquad \ldots[\because p + q = 1]$$

$$= \lim_{h \to 0^+} \left[\frac{\frac{1 + \tan\frac{\pi h}{4}}{1 - \tan\frac{\pi h}{4}} - 1}{h}\right]$$

$$= \lim_{h \to 0^+} \left[\frac{2\tan\frac{\pi h}{4}}{h\left(1 - \tan\frac{\pi h}{4}\right)}\right]$$

$$= \frac{2\pi}{4} \lim_{h \to 0^+} \frac{\tan\left(\frac{\pi h}{4}\right)}{\left(\frac{\pi h}{4}\right)} \left(\frac{1}{1 - \tan\frac{\pi h}{4}}\right)$$

$$= \frac{\pi}{2}(1)\left(\frac{1}{1-0}\right) = \frac{\pi}{2}$$

f(x) is differentiable at $x = 1$.

$$\therefore \quad Lf'(1) = Rf'(1)$$

$$\therefore \quad p = \frac{\pi}{2}$$

Substituting $p = \frac{\pi}{2}$ in (i), we get

$$\frac{\pi}{2} + q = 1$$

$$\therefore \quad q = 1 - \frac{\pi}{2} = \frac{2 - \pi}{2}$$

Question 5.
Discuss whether the function f(x) = |x + 1| + |x – 1| is differentiable ∀ x ∈ R.
Solution:

$$\begin{aligned} f(x) &= |x+1| + |x-1| \\ &= (1-x)-(1+x), \quad x < -1 \\ &= 1+x+1-x, \quad -1 \le x < 1 \\ &= x+1+x-1, \quad x \ge 1 \end{aligned}$$

$$\begin{aligned} \text{i.e., } f(x) &= -2x, \quad x < -1 \\ &= 2, \quad -1 \le x < 1 \\ &= 2x, \quad x \ge 1 \end{aligned}$$

Differentiability at $x = -1$:

$$Lf'(-1) = \lim_{h \to 0^-} \frac{f(-1+h) - f(-1)}{h}$$

$$= \lim_{h \to 0^-} \frac{-2(-1+h) - (2)}{h}$$

$$= \lim_{h \to 0^-} \left(\frac{-2h}{h}\right)$$

$$= -2 \quad \ldots[\because h \to 0, \therefore h \neq 0]$$

$$Rf'(-1) = \lim_{h \to 0^+} \frac{f(-1+h) - f(-1)}{h}$$

$$= \lim_{h \to 0^+} \frac{2-2}{h} = 0$$

Here, $Lf'(-1) \neq Rf'(-1)$

f is not differentiable at $x = -1$.

Differentiability at $x = 1$:

$$Lf'(1) = \lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0^-} \frac{2-2}{h} = 0$$

$$Rf'(1) = \lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0^+} \frac{2(1+h) - (2)}{h}$$

$$= \lim_{h \to 0^-} \left(\frac{2h}{h}\right)$$

$$= 2 \quad \ldots[\because h \to 0, \therefore h \neq 0]$$

Here, $Lf'(1) \neq Rf'(1)$

$\therefore$ f is not differentiable at $x = 1$.

$\therefore$ f is not differentiable at $x = -1$ and $x = 1$.

$\therefore$ f is not differentiable $\forall\ x \in R$.

Question 6.

Test whether the function

$f(x) = 2x - 3$, for $x \ge 2$

$= x - 1$, for $x < 2$

is differentiable at $x = 2$.

Solution:

$$f(x) = 2x - 3,\ x \geq 2$$
$$= x - 1,\ x < 2$$

$$R\,f'(2) = \lim_{h \to 0^+} \frac{f(2+h) - f(2)}{h}$$
$$= \lim_{h \to 0^+} \frac{2(2+h) - 3 - [2(2) - 3]}{h}$$
$$= \lim_{h \to 0^+} \frac{4 + 2h - 3 - 1}{h}$$
$$= \lim_{h \to 0^+} \left(\frac{2h}{h}\right)$$
$$= 2 \quad \ldots[\because h \to 0,\ \therefore h \neq 0]$$

$$L\,f'(2) = \lim_{h \to 0^-} \frac{f(2+h) - f(2)}{h}$$
$$= \lim_{h \to 0^-} \frac{(2+h) - 1 - [2(2) - 3]}{h}$$
$$= \lim_{h \to 0^-} \left(\frac{2 + h - 1 - 1}{h}\right)$$
$$= \lim_{h \to 0^-} \left(\frac{h}{h}\right)$$
$$= 1 \quad \ldots[\because h \to 0,\ \therefore h \neq 0]$$

Here, $Lf'(2) \neq Rf'(2)$
$f(x)$ is not differentiable at $x = 2$.

Question 7.
Test whether the function
$f(x) = x^2 + 1$, for $x \geq 2$
$= 2x + 1$, for $x < 2$
is differentiable at $x = 2$.
Solution:

$$f(x) = x^2 + 1,\ \ x \geq 2$$
$$= 2x + 1,\ \ x < 2$$

$$R\,f'(2) = \lim_{h \to 0^+} \frac{f(2+h) - f(2)}{h}$$
$$= \lim_{h \to 0^+} \frac{(2+h)^2 + 1 - (2^2 + 1)}{h}$$
$$= \lim_{h \to 0^+} \frac{4 + 4h + h^2 + 1 - 5}{h}$$
$$= \lim_{h \to 0^+} \left(\frac{h^2 + 4h}{h}\right)$$
$$= \lim_{h \to 0^+} \frac{h(h+4)}{h}$$
$$= \lim_{h \to 0} (h + 4) \quad \ldots[\because h \to 0,\ \therefore h \neq 0]$$
$$= 4$$

$$L\,f'(2) = \lim_{h \to 0^-} \frac{f(2+h) - f(2)}{h}$$
$$= \lim_{h \to 0^-} \frac{2(2+h) + 1 - (2^2 + 1)}{h}$$
$$= \lim_{h \to 0^-} \frac{4 + 2h + 1 - 5}{h}$$
$$= \lim_{h \to 0} \left(\frac{2h}{h}\right)$$
$$= 2 \quad \ldots[\because h \to 0,\ \therefore h \neq 0]$$

Here, $Lf'(2) \neq Rf'(2)$
$f(x)$ is not differentiable at $x = 2$.

Question 8.
Test whether the function

$f(x) = 5x - 3x^2$, for $x \geq 1$
$= 3 - x$, for $x < 1$
is differentiable at x = 1.
Solution:

$$f(x) = 5x - 3x^2, \quad x \geq 1$$
$$= 3 - x, \quad x < 1$$
$$Rf'(1) = \lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h \to 0^+} \frac{5(1+h) - 3(1+h)^2 - \left[5(1) - 3(1)^2\right]}{h}$$
$$= \lim_{h \to 0^+} \frac{5 + 5h - 3(1 + 2h + h^2) - 2}{h}$$
$$= \lim_{h \to 0^+} \frac{-h - 3h^2}{h}$$
$$= \lim_{h \to 0^+} \frac{h(-1 - 3h)}{h}$$
$$= \lim_{h \to 0^+} (-1 - 3h) \ldots [\because h \to 0, \therefore h \neq 0]$$
$$= -1$$
$$Lf'(1) = \lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h \to 0^-} \frac{3 - (1+h) - \left[5(1) - 3(1)^2\right]}{h}$$
$$= \lim_{h \to 0^-} \frac{3 - 1 - h - 2}{h}$$
$$= \lim_{h \to 0^-} \left(-\frac{h}{h}\right)$$
$$= -1 \qquad \ldots [\because h \to 0, \therefore h \neq 0]$$

Here, Lf'(1) = Rf'(1)
∴ f(x) is differentiable at x = 1.

Question 9.
If f(2) = 4, f'(2) = 1, then find $lim_{x \to 2}\left[\frac{xf(2) - 2f(x)}{x - 2}\right]$
Solution:

$$\lim_{x \to 2} \frac{xf(2) - 2f(x)}{x - 2}$$
$$= \lim_{x \to 2} \frac{f(2) - 2f'(x)}{1} \qquad \ldots[\text{By L' Hospital Rule}]$$
$$= f(2) - 2f'(2)$$
$$= 4 - 2(1) = 2$$

Question 10.
If y = $e^{\sqrt{x}}$, find $\frac{dy}{dx}$ when x = 1.

Solution:

$$y = \frac{e^x}{\sqrt{x}}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x}{\sqrt{x}}\right)$$

$$= \frac{\sqrt{x}\frac{d}{dx}e^x - e^x\frac{d}{dx}\sqrt{x}}{\left(\sqrt{x}\right)^2}$$

$$= \frac{\sqrt{x}\,e^x - e^x\frac{1}{2\sqrt{x}}}{x}$$

$$\frac{dy}{dx} = \frac{2x.e^x - e^x}{2\sqrt{x}.x}$$

When $x = 1$,

$$\frac{dy}{dx} = \frac{2(1)e^1 - e^1}{2\sqrt{1}.1} = \frac{2e - e}{2} = \frac{e}{2}$$