- *Digvijay*
- *Arjun*

Practice Set 4.1 Algebra 9th Std Maths Part 1 Answers Chapter 4 Ratio and Proportion

Question 1.
From the following pairs of numbers, find the reduced form of ratio of first number to second number.
i. 72,60
ii. 38,57
iii. 52,78
Solution:
i. 72, 60
$\text{Ratio} = 7260 = 12 \times 612 \times 5 = 65 = 6:5$
ii. 38, 57
$\text{Ratio} = 3857 = 19 \times 219 \times 3 = 23 = 2:3$
iii. 52, 78
$\text{Ratio} = 5278 = 26 \times 226 \times 3 = 23 = 2:3$
Question 2.
Find the reduced form of the ratio of the first quantity to second quantity.
i. ₹ 700, ₹ 308
ii. ₹ 14, ₹ 12 and 40 paise
iii. 5 litres, 2500 ml
iv. 3 years 4 months, 5 years 8 months
v. 3.8 kg, 1900 gm
vi. 7 minutes 20 seconds, 5 minutes 6 seconds
Solution:
i. ₹ 700, ₹ 308
$\text{Ratio} = 700308 = 28 \times 2528 \times 11 = 2511 = 25:11$
ii. ₹ 14, ₹12 and 40 paise
₹ 14 = 14 x 100 paise = 1400 paise
₹ 12 and 40 paise = 12 x 100 paise + 40 paise
= (1200 + 40) paise

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= 1240 paise

$$\therefore \quad \text{Ratio} = \frac{₹14}{₹12 \text{ and } 40 \text{ paise}} = \frac{1400 \text{ paise}}{1240 \text{ paise}}$$

$$= \frac{1400}{1240} = \frac{40 \times 35}{40 \times 31} = \frac{35}{31} = \mathbf{35 : 31}$$

iii. 5 litres, 2500 ml

5 litres = 5 x 1000 ml = 5000ml

$$\therefore \quad \text{Ratio} = \frac{5 \text{ litres}}{2500 \text{ ml}} = \frac{5000 \text{ ml}}{2500 \text{ ml}} = \frac{5000}{2500}$$

$$= \frac{2500 \times 2}{2500 \times 1} = \frac{2}{1} = \mathbf{2 : 1}$$

iv. 3 years 4 months, 5 years 8 months

3 years 4 months = 3×12 months + 4 months

= (36 + 4) months

= 40 months

5 years 8 months = 5 x 12 months + 8 months

= (60 + 8) months

= 68 months

$$\therefore \quad \text{Ratio} = \frac{3 \text{ years } 4 \text{ months}}{5 \text{ years } 8 \text{ months}} = \frac{40 \text{ months}}{68 \text{ months}} = \frac{40}{68}$$

$$= \frac{4 \times 10}{4 \times 17} = \frac{10}{17} = \mathbf{10 : 17}$$

v. 3.8 kg, 1900 gm

3.8 kg = 3.8 x 1000 gm = 3800 gm

$$\therefore \quad \text{Ratio} = \frac{3.8 \text{kg}}{1900 \text{gm}} = \frac{3800 \text{gm}}{1900 \text{gm}} = \frac{3800}{1900}$$

$$= \frac{1900 \times 2}{1900 \times 1} = \frac{2}{1} = \mathbf{2 : 1}$$

vi. 7 minutes 20 seconds, 5 minutes 6 seconds

7 minutes 20 seconds = 7 x 60 seconds + 20 seconds

= (420 + 20) seconds

= 440 seconds

5 minutes 6 seconds = 5 x 60 seconds + 6 seconds

= (300 + 6) seconds

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= 306 seconds

$$\therefore \quad \text{Ratio} = \frac{7 \text{ minutes } 20 \text{ seconds}}{5 \text{ minutes } 6 \text{ seconds}} = \frac{440 \text{ seconds}}{306 \text{ seconds}}$$

$$= \frac{440}{306} = \frac{2 \times 220}{2 \times 153} = \frac{220}{153} = \mathbf{220 : 153}$$

Question 3.
Express the following percentages as ratios
i. 75 : 100
ii. 44 : 100
iii. 6.25%
iv. 52: 100
v. 0.64%
Solution:

i. $$\text{Ratio} = 75 : 100 = \frac{75}{100} = \frac{25 \times 3}{25 \times 4}$$

$$= \frac{3}{4} = \mathbf{3 : 4}$$

ii. $$\text{Ratio} = 44 : 100 = \frac{44}{100} = \frac{4 \times 11}{4 \times 25}$$

$$= \frac{11}{25} = \mathbf{11 : 25}$$

iii. $$\text{Ratio} = 6.25\% = \frac{6.25}{100} = \frac{625}{100 \times 100}$$

$$= \frac{25 \times 25}{25 \times 4 \times 25 \times 4} = \frac{1}{16} = \mathbf{1 : 16}$$

iv. $$\text{Ratio} = 52 : 100 = \frac{52}{100} = \frac{4 \times 13}{4 \times 25}$$

$$= \frac{13}{25} = \mathbf{13 : 25}$$

v. $$\text{Ratio} = 0.64\% = \frac{0.64}{100} = \frac{64}{100 \times 100}$$

$$= \frac{4 \times 4 \times 4}{25 \times 4 \times 25 \times 4} = \frac{4}{25 \times 25}$$

$$= \frac{4}{625} = \mathbf{4 : 625}$$

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Question 4.
Three persons can build a small house in 8 days. To build the same house in 6 days, how many persons are required?
Solution:
Let the persons required to build a house in 6 days be x.
Days required to build a house and number of persons are in inverse proportion.
∴ 6 × x = 8 × 3
∴ 6 x = 24
∴ x = 4
∴ 4 persons are required to build the house in 6 days.

Question 5.
Convert the following ratios into percentages.
i. 15 : 25
ii. 47 : 50
iii. 710
iv. 546600
v. 716
Solution:
Let 15 : 25 = x %

$$\therefore \quad \frac{15}{25} = \frac{x}{100}$$

$$\therefore \quad x = \frac{15}{25} \times 100 = 15 \times 4 = 60\%$$

∴ 15 : 25 = 60 %
ii. Let 47 : 50 = x%

$$\therefore \quad \frac{47}{50} = \frac{x}{100}$$

$$\therefore \quad x = \frac{47}{50} \times 100 = 47 \times 2 = 94\ \%$$

∴ 47 : 50 = 94 %

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iii. Let 710 = x %

$$\therefore \quad \frac{7}{10} = \frac{x}{100}$$

$$\therefore \quad x = \frac{7}{10} \times 100 = 7 \times 10 = 70\ \%$$

$$\therefore \quad \mathbf{\frac{7}{10} = 70\ \%}$$

iv. Let 546600 = x %

$$\therefore \quad \frac{546}{600} = \frac{x}{100}$$

$$\therefore \quad x = \frac{546}{600} \times 100 = \frac{546}{6} = 91\%$$

$$\therefore \quad \mathbf{\frac{546}{600} = 91\ \%}$$

v. Let 716 = x %

$$\therefore \quad \frac{7}{16} = \frac{x}{100}$$

$$\therefore \quad x = \frac{7}{16} \times 100 = \frac{7}{4} \times 25 = \frac{175}{4} = 43.75\%$$

$$\therefore \quad \mathbf{\frac{7}{16} = 43.75\ \%}$$

Question 6.
The ratio of ages of Abha and her mother is 2 : 5. At the time of Abha's birth her mothers age was 27 years. Find the present ages of Abha and her mother.
Solution:
The ratio of ages of Abha and her mother is 2 : 5.
Let the common multiple be x.
∴ Present age of Abha = 2x years and
Present age of Abha's mother = 5x years
According to the given condition, the age of Abha's mother at the time of Abha's birth = 27 years
∴ 5x – 2x = 27
∴ 3x = 27
∴ x = 9
∴ Present age of Abha = 2x = 2 x 9 = 18 years
∴ Present age of Abha's mother = 5x =5 x 9 = 45 years

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The present ages of Abha and her mother are 18 years and 45 years respectively.

Question 7.
Present ages of Vatsala and Sara are 14 years and 10 years respectively. After how many years the ratio of their ages will become 5 : 4?
Solution:
Present age of Vatsala = 14 years
Present age of Sara = 10 years
After x years,
Vatsala's age = (14 + x) years
Sara's age = (10 + x) years
According to the given condition,
After x years the ratio of their ages will become 5 : 4
$\therefore \frac{14+x}{10+x} = \frac{5}{4}$
$\therefore 4(14 + x) = 5(10 + x)$
$\therefore 56 + 4x = 50 + 5x$
$\therefore 56 - 50 = 5x - 4x$
$\therefore 6 = x$
$\therefore x = 6$
$\therefore$ After 6 years, the ratio of their ages will become 5 : 4.
Question 8.
The ratio of present ages of Rehana and her mother is 2 : 7. After 2 years, the ratio of their ages will be 1 : 3. What is Rehana's present age ?
Solution:
The ratio of present ages of Rehana and her mother is 2 : 7
Let the common multiple be x.
$\therefore$ Present age of Rehana = 2x years and Present age of Rehana's mother = 7x years
After 2 years,
Rehana's age = (2x + 2) years
Age of Rehana's mother = (7x + 2) years
According to the given condition,
After 2 years, the ratio of their ages will be 1 : 3
$\therefore \frac{2x+2}{7x+2} = \frac{1}{3}$
$\therefore 3(2x + 2) = 1(7x + 2)$
$\therefore 6x + 6 = 7x + 2$

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∴ 6 – 2 = 7x – 6x

∴ 4 = x

∴ x = 4

∴ Rehana's present age = 2x = 2 x 4 = 8 years

∴ Rehana's present age is 8 years.

Practice Set 4.2 Algebra 9th Std Maths Part 1 Answers Chapter 4 Ratio and Proportion

Question 1.

Using the property ab = akbk, fill in the blanks by substituting proper numbers in the following.

i. $\frac{5}{7} = \frac{....}{28} = \frac{35}{....} = \frac{....}{3.5}$

ii. $\frac{9}{14} = \frac{4.5}{....} = \frac{....}{42} = \frac{....}{3.5}$

Solution:

i. $\frac{5}{7} = \frac{....}{28} = \frac{5\times4}{7\times4} = \frac{20}{28}$

$\frac{5}{7} = \frac{35}{....} = \frac{5\times7}{7\times7} = \frac{35}{49}$

$\frac{5}{7} = \frac{....}{3.5} = \frac{5\times0.5}{7\times0.5} = \frac{2.5}{3.5}$

∴ $\frac{5}{7} = \frac{\mathbf{20}}{28} = \frac{35}{\mathbf{49}} = \frac{\mathbf{2.5}}{3.5}$

ii. $\frac{9}{14} = \frac{4.5}{....} = \frac{9\times0.5}{14\times0.5} = \frac{4.5}{7}$

$\frac{9}{14} = \frac{....}{42} = \frac{9\times3}{14\times3} = \frac{27}{42}$

$\frac{9}{14} = \frac{....}{3.5} = \frac{9\times0.25}{14\times0.25} = \frac{2.25}{3.5}$

∴ $\frac{9}{14} = \frac{4.5}{\mathbf{7}} = \frac{\mathbf{27}}{42} = \frac{\mathbf{2.25}}{3.5}$

Question 2.

Find the following ratios.

i. The ratio of radius to circumference of the circle.

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ii. The ratio of circumference of circle with radius r to its area.
iii. The ratio of diagonal of a square to its side, if the length of side is 7 cm.
iv. The lengths of sides of a rectangle are 5 cm and 3.5 cm. Find the ratio of numbers denoting its perimeter to area.
Solution:
i. Let the radius of circle be r.
then, its circumference = 2πr
Ratio of radius to circumference of the circle

$$= \frac{\text{radius}}{\text{circumference}}$$

$$= \frac{r}{2\pi r}$$

$$= \frac{1}{2\pi}$$

$$= 1 : 2\pi$$

The ratio of radius to circumference of the circle is 1 : 2π.

ii. Let the radius of the circle is r.
∴ circumference = 2πr and area = πr^2
Ratio of circumference to the area of circle

$$= \frac{\text{circumference}}{\text{area}}$$

$$= \frac{2\pi r}{\pi r^2}$$

$$= \frac{2}{r}$$

$$= 2 : r$$

∴ The ratio of circumference of circle with radius r to its area is 2 : r.

iii. Length of side of square = 7 cm
∴ Diagonal of square = √2 x side
= √2 x 7
= 7 √2 cm
Ratio of diagonal of a square to its side

$$= \frac{\text{diagonal}}{\text{side}}$$

$$= \frac{7\sqrt{2}}{7}$$

$$= \frac{\sqrt{2}}{1}$$

$$= \sqrt{2} : 1$$

∴ The ratio of diagonal of a square to its side is √2 : 1.

iv. Length of rectangle = (l) = 5 cm,
Breadth of rectangle = (b) = 3.5 cm
Perimeter of the rectangle = 2(l + b)
= 2(5 + 3.5)
= 2 x 8.5
= 17 cm
Area of the rectangle = l x b
= 5 x 3.5
= 17.5 cm^2
Ratio of numbers denoting perimeter to the area of rectangle

$$= \frac{\text{perimeter}}{\text{area}}$$

$$= \frac{17}{17.5}$$

$$= \frac{17 \times 10}{17.5 \times 10}$$

$$= \frac{170}{175}$$

$$= \frac{5 \times 34}{5 \times 35}$$

$$= \frac{34}{35}$$

$$= 34 : 35$$

∴ Ratio of numbers denoting perimeter to the area of rectangle is 34 : 35.

Question 3.
Compare the following

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i. $\frac{\sqrt{5}}{3}, \frac{3}{\sqrt{7}}$

ii. $\frac{3\sqrt{5}}{5\sqrt{7}}, \frac{\sqrt{63}}{\sqrt{125}}$

iii. $\frac{5}{18}, \frac{17}{121}$

iv. $\frac{\sqrt{80}}{\sqrt{48}}, \frac{\sqrt{45}}{\sqrt{27}}$

v. $\frac{9.2}{5.1}, \frac{3.4}{7.1}$

Solution:

i. $\frac{\sqrt{5}}{3}, \frac{3}{\sqrt{7}}$

$\sqrt{5} \times \sqrt{7} = \sqrt{35}$

- *Digvijay*
- *Arjun*

$3 \times 3 = 9 = \sqrt{9^2} = \sqrt{81}$

Here, $35 < 81$

$\therefore \quad \sqrt{35} < \sqrt{81}$

$\therefore \quad \dfrac{\sqrt{5}}{3} < \dfrac{3}{\sqrt{7}}$

ii. $\dfrac{3\sqrt{5}}{5\sqrt{7}}, \dfrac{\sqrt{63}}{\sqrt{125}}$

$3\sqrt{5} \times \sqrt{125} = 3 \times \sqrt{5 \times 125}$

$= 3 \times \sqrt{5 \times 25 \times 5}$

$= 3 \times 5 \times 5$

$= 3 \times 25 = 75$

$5\sqrt{7} \times \sqrt{63} = 5 \times \sqrt{7 \times 63}$

$= 5 \times \sqrt{7 \times 9 \times 7}$

$= 5 \times 3 \times 7 = 105$

Here, $75 < 105$

$\therefore \quad \dfrac{3\sqrt{5}}{5\sqrt{7}} < \dfrac{\sqrt{63}}{\sqrt{125}}$

iii. $\dfrac{5}{18}, \dfrac{17}{121}$

$5 \times 121 = 605$

$18 \times 17 = 306$

Here, $605 > 306$

$\therefore \quad \dfrac{5}{18} > \dfrac{17}{121}$

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iv. $\frac{\sqrt{80}}{\sqrt{48}}, \frac{\sqrt{45}}{\sqrt{27}}$

$\sqrt{80} \times \sqrt{27} = \sqrt{2160}$

$\sqrt{48} \times \sqrt{45} = \sqrt{2160}$

Here, $2160 = 2160$

$\therefore \quad \sqrt{2160} = \sqrt{2160}$

$\therefore \quad \mathbf{\frac{\sqrt{80}}{\sqrt{48}} = \frac{\sqrt{45}}{\sqrt{27}}}$

Alternate method:

$\frac{\sqrt{80}}{\sqrt{48}}, \frac{\sqrt{45}}{\sqrt{27}}$

Consider, $\frac{\sqrt{80}}{\sqrt{48}} = \frac{\sqrt{16 \times 5}}{\sqrt{16 \times 3}} = \frac{4\sqrt{5}}{4\sqrt{3}} = \frac{\sqrt{5}}{\sqrt{3}}$

$\frac{\sqrt{45}}{\sqrt{27}} = \frac{\sqrt{9 \times 5}}{\sqrt{9 \times 3}} = \frac{3\sqrt{5}}{3\sqrt{3}} = \frac{\sqrt{5}}{\sqrt{3}}$

Here, $\frac{\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{5}}{\sqrt{3}}$

$\therefore \quad \mathbf{\frac{\sqrt{80}}{\sqrt{48}} = \frac{\sqrt{45}}{\sqrt{27}}}$

v. $\frac{9.2}{5.1}, \frac{3.4}{7.1}$

$9.2 \times 7.1 = 65.32$

$5.1 \times 3.4 = 17.34$

Here, $65.32 > 17.34$

$\therefore \quad \mathbf{\frac{9.2}{5.1} > \frac{3.4}{7.1}}$

Question 4.
Solve.
ABCD is a parallelogram. The ratio of ∠A and ∠B of this parallelogram is 5 : 4. FInd the measure of ∠B. [2 Marksl
Solution:
Ratio of ∠A and ∠B for given parallelogram is 5 : 4
Let the common multiple be x.

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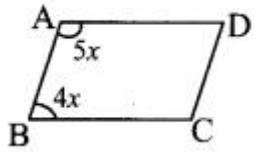

m∠A = 5x°and m∠B=4x°
Now, m∠A + m∠B = 180° ...[Adjacent angles of a parallelogram arc supplementary]
∴ 5x° + 4x°= 180°
∴ 9x° = 180°
∴ x° = 20°
∴ m∠B=4x°= 4 x 20° = 80°
∴ The measure of ∠B is 800.

ii. The ratio of present ages of Albert and Salim is 5 : 9. Five years hence ratio of their ages will be 3 : 5. Find their present ages.
Solution:
The ratio of present ages of Albert and Salim is 5 : 9
Let the common multiple be x.
∴ Present age of Albert = 5x years and
Present age of Salim = 9x years
After 5 years,
Albert's age = (5x + 5) years and
Salim's age = (9x + 5) years
According to the given condition,
Five years hence ratio of their ages will be 3 : 5
$5x+59x+5=35$
∴ 5(5x + 5) = 3(9x + 5)
∴ 25x + 25 = 27x + 15
∴ 25 – 15 = 27 x – 25 x
∴ 10 = 2x
∴ x = 5
∴ Present age of Albert = 5x = 5 x 5 = 25 years
Present age of Salim = 9x = 9 x 5 = 45 years
∴ The present ages of Albert and Salim are 25 years and 45 years respectively.
iii. The ratio of length and breadth of a rectangle is 3 : 1, and its perimeter is 36 cm. Find the length and breadth of the rectangle.
Solution:

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The ratio of length and breadth of a rectangle is 3 : 1
Let the common multiple be x.
Length of the rectangle (l) = 3x cm
and Breadth of the rectangle (b) = x cm
Given, perimeter of the rectangle = 36 cm
Since, Perimeter of the rectangle = 2(l + b)
∴ 36 = 2(3x + x)
∴ 36 = 2(4x)
∴ 36 = 8x
∴ $x=368=92=4.5$
Length of the rectangle = 3x = 3 x 4.5 = 13.5 cm
∴ The length of the rectangle is 13.5 cm and its breadth is 4.5 cm.
iv. The ratio of two numbers is 31 : 23 and their sum is 216. Find these numbers.
Solution:
The ratio of two numbers is 31 : 23
Let the common multiple be x.
∴ First number = 31x and
Second number = 23x
According to the given condition,
Sum of the numbers is 216
∴ 31x + 23x = 216
∴ 54x = 216
∴ x = 4
∴ First number = 31x = 31 x 4 = 124
Second number = 23x = 23 x 4 = 92
∴ The two numbers are 124 and 92.

v. If the product of two numbers is 360 and their ratio is 10 : 9, then find the numbers.
Solution:
Ratio of two numbers is 10 : 9
Let the common multiple be x.
∴ First number = 10x and
Second number = 9x
According to the given condition,
Product of two numbers is 360

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∴ (10x) (9x) = 360
∴ $90x^2 = 360$
∴ $x^2 = 4$
∴ x = 2 .... [Taking positive square root on both sides]
∴ First number = 10x = $10x^2 = 20$
Second number = 9x = $9x^2 = 18$
∴ The two numbers are 20 and 18.

Question 5.
If a : b = 3 : 1 and b : c = 5 : 1, then find the value of [3 Marks each]

i. $\left(\frac{a^3}{15b^2c}\right)^3$ **ii.** $\frac{a^2}{7bc}$

Solution:
Given, a : b = 3 : 1
∴ ab = 31
∴ a = 3b ....(i)
and b : c = 5 : 1
∴ bc = 51
b = 5c .....(ii)
Substituting (ii) in (i),
we get a = 3(5c)
∴ a = 15c ...(iii)

i. $\left(\frac{a^3}{15b^2c}\right)^3 = \left[\frac{(15c)^3}{15(5c)^2c}\right]^3$ ...[From (ii) and (iii)]

$= \left[\frac{15c \times 15c \times 15c}{15 \times 5c \times 5c \times c}\right]^3$

$= (3 \times 3)^3$

$= (9)^3$

∴ $\left(\frac{a^3}{15b^2c}\right)^3 = 729$

ii. $\frac{a^2}{7bc} = \frac{(15c)^2}{7 \times 5c \times c}$ ...[From (ii) and (iii)]

$= \frac{15c \times 15c}{7 \times 5c \times c}$

∴ $\frac{a^2}{7bc} = \frac{45}{7}$

- *Digvijay*
- *Arjun*

Ratio and Proportion 9th Class Practice Set 4.1 Question 6.
If $0.04\times0.4\times a----------\sqrt{}=0.4\times0.04\times b\sqrt{}$, then find the ratio ab.
Solution:
$0.04\times0.4\times a----------\sqrt{}=0.4\times0.04\times b\sqrt{}$ ... [Given]
∴ 0.04 x 0.4 x a = $(0.4)^2$ x $(0.04)^2$ x b ... [Squaring both sides]

$$\therefore \quad \frac{a}{b} = \frac{(0.04)^2 \times (0.4)^2}{0.04 \times 0.4}$$

$$= 0.04 \times 0.4 = 0.016$$

$$= \frac{16}{1000}$$

$$= \frac{8 \times 2}{8 \times 125}$$

$$\therefore \quad \frac{\mathbf{a}}{\mathbf{b}} = \frac{\mathbf{2}}{\mathbf{125}}$$

9th Algebra Practice Set 4.2 Question 7. (x + 3) : (x + 11) = (x – 2) : (x + 1), then find the value of x.
Solution:
(x + 3) : (x + 11) = (x- 2) : (x+ 1)
$x+3x+11=x-2x+1$
∴ (x + 3)(x +1) = (x – 2)(x + 11)
∴ x(x +1) + 3(x + 1) = x(x + 11) – 2(x + 11)
∴ x^2 + x + 3x + 3 = x^2 + 1 lx – 2x – 22
∴ x^2 + 4x + 3 = x^2 + 9x – 22
∴ 4x + 3 = 9x – 22
∴ 3 + 22 = 9x – 4x
∴ 25 = 5x
∴ x = 5

Practice Set 4.3 Algebra 9th Std Maths Part 1 Answers Chapter 4 Ratio and Proportion

Ratio and Proportion Practice Set 4.3 Question 1.
If $ab = 73$, then find the alues of the following ratios.

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i. $\dfrac{5a+3b}{5a-3b}$

ii. $\dfrac{2a^2+3b^2}{2a^2-3b^2}$

iii. $\dfrac{a^3-b^3}{b^3}$

iv. $\dfrac{7a+9b}{7a-9b}$

Solution:

i. $\dfrac{a}{b}=\dfrac{7}{3}$ ...[Given]

$\therefore \quad \dfrac{a}{b}\times\dfrac{5}{3}=\dfrac{7}{3}\times\dfrac{5}{3}$

$\ldots\left[\text{Multiplying both sides by } \dfrac{5}{3}\right]$

$\therefore \quad \dfrac{5a}{3b}=\dfrac{35}{9}$

$\therefore \quad \dfrac{5a+3b}{5a-3b}=\dfrac{35+9}{35-9}$

...[By componendo - dividendo]

$\therefore \quad \dfrac{5a+3b}{5a-3b}=\dfrac{44}{26}$

$=\dfrac{2\times22}{2\times13}=\dfrac{22}{13}$

$\therefore \quad \mathbf{\dfrac{5a+3b}{5a-3b}=22:13}$

Alternate method:

$\dfrac{a}{b}=\dfrac{7}{3}$ ...[Given]

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Let the common multiple be m.

then, a = 7m and b = 3m

$$\therefore \quad \frac{5a+3b}{5a-3b} = \frac{5(7m)+3(3m)}{5(7m)-3(3m)}$$

$$= \frac{35m+9m}{35m-9m} = \frac{44m}{26m} = \frac{44}{26}$$

$$= \frac{2\times 22}{2\times 13} = \frac{22}{13}$$

$$\therefore \quad \mathbf{\frac{5a+3b}{5a-3b} = 22 : 13}$$

ii. $\frac{a}{b} = \frac{7}{3}$ ...[Given]

$\therefore \quad \left(\frac{a}{b}\right)^2 = \left(\frac{7}{3}\right)^2$ ...[Squaring both sides]

$\therefore \quad \frac{a^2}{b^2} = \frac{49}{9}$

$\therefore \quad \frac{a^2}{b^2} \times \frac{2}{3} = \frac{49}{9} \times \frac{2}{3}$

... $\left[\text{Multiplying both sides by } \frac{2}{3}\right]$

$\therefore \quad \frac{2a^2}{3b^2} = \frac{98}{27}$

$\therefore \quad \frac{2a^2+3b^2}{2a^2-3b^2} = \frac{98+27}{98-27}$

...[By componendo - dividendo]

$\therefore \quad \frac{2a^2+3b^2}{2a^2-3b^2} = \frac{125}{71}$

$\therefore \quad \mathbf{\frac{2a^2+3b^2}{2a^2-3b^2} = 125 : 71}$

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iii. $\frac{a}{b} = \frac{7}{3}$ ...[Given]

$\therefore \left(\frac{a}{b}\right)^3 = \left(\frac{7}{3}\right)^3$ ...[Cubing both sides]

$\therefore \frac{a^3}{b^3} = \frac{343}{27}$

$\therefore \frac{a^3 - b^3}{b^3} = \frac{343 - 27}{27}$ ...[By dividendo]

$\therefore \frac{a^3 - b^3}{b^3} = \frac{316}{27}$

$\therefore \mathbf{\frac{a^3 - b^3}{b^3} = 316 : 27}$

iv. $\frac{a}{b} = \frac{7}{3}$ ...[Given]

$\therefore \frac{a}{b} \times \frac{7}{9} = \frac{7}{3} \times \frac{7}{9}$

$\ldots \left[\text{Multiplying both sides by } \frac{7}{9}\right]$

$\therefore \frac{7a}{9b} = \frac{49}{27}$

$\therefore \frac{7a + 9b}{7a - 9b} = \frac{49 + 27}{49 - 27}$

...[By componendo - dividendo]

$\therefore \frac{7a + 9b}{7a - 9b} = \frac{76}{22} = \frac{2 \times 38}{2 \times 11} = \frac{38}{11}$

$\therefore \mathbf{\frac{7a + 9b}{7a - 9b} = 38 : 11}$

Ratio and Proportion Class 9 Practice Set 4.3 Question 2.

If $\frac{15a^2+4b^2}{15a^2-4b^2} = 477$, then find the value of the following ratios.

i. $\frac{a}{b}$

ii. $\frac{7a - 3b}{7a + 3b}$

iii. $\frac{b^2 - 2a^2}{b^2 + 2a^2}$

iv. $\frac{b^3 - 2a^3}{b^3 + 2a^3}$

Solution:

- *Digvijay*
- *Arjun*

i. $\dfrac{15a^2+4b^2}{15a^2-4b^2}=\dfrac{47}{7}$ ...[Given]

$\therefore \dfrac{15a^2+4b^2+\left(15a^2-4b^2\right)}{15a^2+4b^2-\left(15a^2-4b^2\right)}=\dfrac{47+7}{47-7}$

...[By componendo - dividendo]

$\therefore \dfrac{15a^2+4b^2+15a^2-4b^2}{15a^2+4b^2-15a^2+4b^2}=\dfrac{54}{40}$

$\therefore \dfrac{30a^2}{8b^2}=\dfrac{54}{40}$

$\therefore \dfrac{a^2}{b^2}=\dfrac{54\times 8}{40\times 30}$

$\therefore \dfrac{a^2}{b^2}=\dfrac{9}{25}$

$\therefore \dfrac{a}{b}=\dfrac{3}{5}$

...[Taking positive square root of both sides]

$\therefore \dfrac{\mathbf{a}}{\mathbf{b}}=\mathbf{3:5}$

ii. $\dfrac{a}{b}=\dfrac{3}{5}$

$\therefore \dfrac{a}{b}\times\dfrac{7}{3}=\dfrac{3}{5}\times\dfrac{7}{3}$

$\ldots\left[\text{Multiplying both sides by } \dfrac{7}{3}\right]$

$\therefore \dfrac{7a}{3b}=\dfrac{7}{5}$

$\therefore \dfrac{7a+3b}{7a-3b}=\dfrac{7+5}{7-5}$

...[By componendo - dividendo]

$\therefore \quad \frac{7a+3b}{7a-3b} = \frac{12}{2}$

$\therefore \quad \frac{7a+3b}{7a-3b} = \frac{6}{1}$

$\therefore \quad \frac{7a-3b}{7a+3b} = \frac{1}{6}$ …[By invertendo]

$\therefore \quad \mathbf{\frac{7a-3b}{7a+3b} = 1 : 6}$

iii. $\frac{a}{b} = \frac{3}{5}$

$\therefore \quad \left(\frac{a}{b}\right)^2 = \left(\frac{3}{5}\right)^2$ …[Squaring both sides]

$\therefore \quad \frac{a^2}{b^2} = \frac{9}{25}$

$\therefore \quad \frac{b^2}{a^2} = \frac{25}{9}$ …[By invertendo]

$\therefore \quad \frac{b^2}{a^2} \times \frac{1}{2} = \frac{25}{9} \times \frac{1}{2}$

$\ldots \left[\text{Multiplying both sides by } \frac{1}{2}\right]$

$\therefore \quad \frac{b^2}{2a^2} = \frac{25}{18}$

$\therefore \quad \frac{b^2+2a^2}{b^2-2a^2} = \frac{25+18}{25-18}$

…[By componendo - dividendo]

$\therefore \quad \frac{b^2+2a^2}{b^2-2a^2} = \frac{43}{7}$

$\therefore \quad \frac{b^2-2a^2}{b^2+2a^2} = \frac{7}{43}$ …[By invertendo]

$\therefore \quad \mathbf{\frac{b^2-2a^2}{b^2+2a^2} = 7 : 43}$

- *Digvijay*
- *Arjun*

iv. $\frac{a}{b} = \frac{3}{5}$

$\therefore \quad \left(\frac{a}{b}\right)^3 = \left(\frac{3}{5}\right)^3$ …[Cubing both sides]

$\therefore \quad \frac{a^3}{b^3} = \frac{27}{125}$

$\therefore \quad \frac{b^3}{a^3} = \frac{125}{27}$ …[By invertendo]

$\therefore \quad \frac{b^3}{a^3} \times \frac{1}{2} = \frac{125}{27} \times \frac{1}{2}$

$\ldots\left[\text{Multiplying both sides by } \frac{1}{2}\right]$

$\therefore \quad \frac{b^3}{2a^3} = \frac{125}{54}$

$\therefore \quad \frac{b^3 + 2a^3}{b^3 - 2a^3} = \frac{125 + 54}{125 - 54}$

…[By componendo - dividendo]

$\therefore \quad \frac{b^3 + 2a^3}{b^3 - 2a^3} = \frac{179}{71}$

$\therefore \quad \frac{b^3 - 2a^3}{b^3 + 2a^3} = \frac{71}{179}$ …[By invertendo]

$\therefore \quad \mathbf{\frac{b^3 - 2a^3}{b^3 + 2a^3} = 71 : 179}$

Practice Set 4.3 Algebra 9th Question 3.

If $\frac{3a+7b}{3a-7b}=\frac{43}{}$then find the value of the ratio $\frac{3a^2-7b^2}{3a^2+7b^2}$.

Solution:

- *Digvijay*
- *Arjun*

$$\frac{3a+7b}{3a-7b}=\frac{4}{3} \quad \text{...[Given]}$$

$$\therefore \quad \frac{3a+7b+(3a-7b)}{3a+7b-(3a-7b)}=\frac{4+3}{4-3}$$

...[By componendo - dividendo]

$$\therefore \quad \frac{3a+7b+3a-7b}{3a+7b-3a+7b}=\frac{7}{1}$$

$$\therefore \quad \frac{6a}{14b}=\frac{7}{1}$$

$$\therefore \quad \frac{a}{b}=\frac{7\times14}{6}$$

$$\therefore \quad \frac{a}{b}=\frac{49}{3}$$

$$\therefore \quad \left(\frac{a}{b}\right)^2=\left(\frac{49}{3}\right)^2 \quad \text{...[Squaring both sides]}$$

$$\therefore \quad \frac{a^2}{b^2}=\frac{2401}{9}$$

$$\therefore \quad \frac{a^2}{b^2}\times\frac{3}{7}=\frac{2401}{9}\times\frac{3}{7}$$

$$\ldots \left[\text{Multiplying both sides by } \frac{3}{7}\right]$$

$$\therefore \quad \frac{3a^2}{7b^2}=\frac{343}{3}$$

$$\therefore \quad \frac{3a^2+7b^2}{3a^2-7b^2}=\frac{343+3}{343-3}$$

...[By componendo - dividendo]

$$\therefore \quad \frac{3a^2+7b^2}{3a^2-7b^2}=\frac{346}{340}$$

$$\therefore \quad \frac{3a^2+7b^2}{3a^2-7b^2}=\frac{2\times173}{2\times170}=\frac{173}{170}$$

$$\therefore \quad \frac{3a^2-7b^2}{3a^2+7b^2}=\frac{170}{173} \quad \text{...[By invertendo]}$$

$$\therefore \quad \mathbf{\frac{3a^2-7b^2}{3a^2+7b^2}=170:173}$$

Class 9 Maths Chapter 4 Ratio And Proportion Practice Set 4.3 Question 4.

- *Digvijay*
- *Arjun*

Solve the following equations.

i. $\frac{x^2+12x-20}{3x-5} = \frac{x^2+8x+12}{2x+3}$

ii. $\frac{10x^2+15x+63}{5x^2-25x+12} = \frac{2x+3}{x-5}$

iii. $\frac{(2x+1)^2+(2x-1)^2}{(2x+1)^2-(2x-1)^2} = \frac{17}{8}$

iv. $\frac{\sqrt{4x+1}+\sqrt{x+3}}{\sqrt{4x+1}-\sqrt{x+3}} = \frac{4}{1}$

v. $\frac{(4x+1)^2+(2x+3)^2}{4x^2+12x+9} = \frac{61}{36}$

vi. $\frac{(3x-4)^3-(x+1)^3}{(3x-4)^3+(x+1)^3} = \frac{61}{189}$

Solution:

i. $\frac{x^2+12x-20}{3x-5} = \frac{x^2+8x+12}{2x+3}$

$\therefore \frac{x^2+12x-20}{4(3x-5)} = \frac{x^2+8x+12}{4(2x+3)}$

$\ldots \left[\text{Multiplying both sides by } \frac{1}{4}\right]$

$\therefore \frac{x^2+12x-20}{12x-20} = \frac{x^2+8x+12}{8x+12}$

$\therefore \frac{(x^2+12x-20)-(12x-20)}{12x-20} = \frac{(x^2+8x+12)-(8x+12)}{8x+12}$

…[By dividendo]

$\therefore \frac{x^2+12x-20-12x+20}{12x-20} = \frac{x^2+8x+12-8x-12}{8x+12}$

$\therefore \frac{x^2}{12x-20} = \frac{x^2}{8x+12}$

This equation is true for x = 0

∴ x = 0 is one of the solutions.

If $x \neq 0$, then $x^2 \neq 0$

∴ 112x–20 = 18x+12 … [Dividing both sides by x^2]

∴ 8x + 12 = 12x – 20

∴ 12 + 20 = 12x – 8x

∴ 32 = 4x

- *Digvijay*
- *Arjun*

$\therefore x = 8$

$\therefore x = 0$ or $x = 8$ are the solutions of the given equation.

ii. $\dfrac{10x^2+15x+63}{5x^2-25x+12} = \dfrac{2x+3}{x-5}$

If $x = 0$ then

$$\text{L.H.S.} = \frac{10x^2+15x+63}{5x^2-25x+12} = \frac{10(0)^2+15(0)+63}{5(0)^2-25(0)+12}$$

$$= \frac{63}{12} = \frac{21}{4}$$

$$\text{R.H.S.} = \frac{2x+3}{x-5} = \frac{2(0)+3}{-5} = -\frac{3}{5}$$

$\therefore$ $\dfrac{21}{4} = -\dfrac{3}{5}$ which is a contradiction.

$\therefore$ $x \neq 0$

Now, $\dfrac{10x^2+15x+63}{5x^2-25x+12} = \dfrac{2x+3}{x-5}$

$\therefore$ $\dfrac{10x^2+15x+63}{2x+3} = \dfrac{5x^2-25x+12}{x-5}$

…[By alternendo]

$\therefore$ $\dfrac{10x^2+15x+63}{5x(2x+3)} = \dfrac{5x^2-25x+12}{5x(x-5)}$

… $\left[\text{Multiplying both sides by } \dfrac{1}{5x} \text{ as } x \neq 0\right]$

$\therefore$ $\dfrac{10x^2+15x+63}{10x^2+15x} = \dfrac{5x^2-25x+12}{5x^2-25x}$

$\therefore$ $\dfrac{(10x^2+15x+63)-(10x^2+15x)}{10x^2+15x}$

$$= \frac{(5x^2 - 25x + 12) - (5x^2 - 25x)}{5x^2 - 25x}$$

...[By dividendo]

$$\therefore \quad \frac{10x^2 + 15x + 63 - 10x^2 - 15x}{10x^2 + 15x}$$

$$= \frac{5x^2 - 25x + 12 - 5x^2 + 25x}{5x^2 - 25x}$$

$$\therefore \quad \frac{63}{10x^2 + 15x} = \frac{12}{5x^2 - 25x}$$

$$\therefore \quad \frac{63}{5x(2x+3)} = \frac{12}{5x(x-5)}$$

$$\therefore \quad \frac{21}{2x+3} = \frac{4}{x-5}$$

$$\ldots \left[\text{Multiplying both sides by } \frac{5x}{3} \text{ as } x \neq 0\right]$$

∴ 21(x – 5) = 4(2x + 3)
∴ 21x – 105 = 8x + 12
∴ 21x – 8x = 12 + 105
∴ 13x = 117
∴ x = 9
∴ x = 9 ¡s the solution of the given equation.

iii. $\dfrac{(2x+1)^2+(2x-1)^2}{(2x+1)^2-(2x-1)^2}=\dfrac{17}{8}$

$\therefore \quad \dfrac{\left[(2x+1)^2+(2x-1)^2\right]+\left[(2x+1)^2-(2x-1)^2\right]}{\left[(2x+1)^2+(2x-1)^2\right]-\left[(2x+1)^2-(2x-1)^2\right]}$

$$=\frac{17+8}{17-8}$$

…[By componendo - dividendo]

$\therefore \quad \dfrac{(2x+1)^2+(2x-1)^2+(2x+1)^2-(2x-1)^2}{(2x+1)^2+(2x-1)^2-(2x+1)^2+(2x-1)^2}=\dfrac{25}{9}$

$\therefore \quad \dfrac{2(2x+1)^2}{2(2x-1)^2}=\dfrac{25}{9}$

$\therefore \quad \dfrac{(2x+1)^2}{(2x-1)^2}=\dfrac{25}{9}$

$\therefore \quad \dfrac{2x+1}{2x-1}=\pm\dfrac{5}{3}$

…[Taking square root of both sides]

$\therefore \quad \dfrac{2x+1}{2x-1}=\dfrac{5}{3}$ or $\dfrac{2x+1}{2x-1}=-\dfrac{5}{3}$

$\therefore \quad 3(2x+1)=5(2x-1)$ or $3(2x+1)=-5(2x-1)$

$\therefore \quad 6x+3=10x-5$ or $6x+3=-10x+5$

$\therefore \quad 3+5=10x-6x$ or $6x+10x=5-3$

$\therefore \quad 8=4x$ or $16x=2$

$\therefore \quad x=2$ or $x=\dfrac{2}{16}$ $\qquad\qquad$ $\therefore \quad x=2$ or $x=\dfrac{1}{8}$

$\therefore$ **$x=2$ or $x=\dfrac{1}{8}$ is the solution of the given equation.**

iv. $\dfrac{\sqrt{4x+1}+\sqrt{x+3}}{\sqrt{4x+1}-\sqrt{x+3}} = \dfrac{4}{1}$

$\therefore \quad \dfrac{\left(\sqrt{4x+1}+\sqrt{x+3}\right)+\left(\sqrt{4x+1}-\sqrt{x+3}\right)}{\left(\sqrt{4x+1}+\sqrt{x+3}\right)-\left(\sqrt{4x+1}-\sqrt{x+3}\right)} = \dfrac{4+1}{4-1}$

…[By componendo - dividendo]

$\therefore \quad \dfrac{\sqrt{4x+1}+\sqrt{x+3}+\sqrt{4x+1}-\sqrt{x+3}}{\sqrt{4x+1}+\sqrt{x+3}-\sqrt{4x+1}+\sqrt{x+3}} = \dfrac{5}{3}$

$\therefore \quad \dfrac{2\sqrt{4x+1}}{2\sqrt{x+3}} = \dfrac{5}{3}$

$\therefore \quad \dfrac{\sqrt{4x+1}}{\sqrt{x+3}} = \dfrac{5}{3}$

$\therefore \quad \dfrac{4x+1}{x+3} = \dfrac{25}{9}$ …[Squaring both sides]

$\therefore 9(4x + 1) = 25(x + 3)$

$\therefore 36x + 925x + 75$

$\therefore 36x – 25 = 75 – 9$

$\therefore 11x = 66$

$\therefore x = 6$

$\therefore x = 6$ is the solution of the given equation.

v. $\dfrac{(4x+1)^2+(2x+3)^2}{4x^2+12x+9}=\dfrac{61}{36}$

$\therefore \quad \dfrac{(4x+1)^2+(2x+3)^2}{(2x)^2+2\times 2x\times 3+(3)^2}=\dfrac{61}{36}$

$\therefore \quad \dfrac{(4x+1)^2+(2x+3)^2}{(2x+3)^2}=\dfrac{61}{36}$

...[$\because a^2 + 2ab + b^2 = (a + b)^2$]

$\therefore \quad \dfrac{\left[(4x+1)^2+(2x+3)^2\right]-(2x+3)^2}{(2x+3)^2}=\dfrac{61-36}{36}$

…[By dividendo]

$\therefore \quad \dfrac{(4x+1)^2}{(2x+3)^2}=\dfrac{25}{36}$

$\therefore \quad \dfrac{4x+1}{2x+3}=\pm\dfrac{5}{6}$

…[Taking square root of both sides]

$\therefore \quad \dfrac{4x+1}{2x+3}=\dfrac{5}{6}$ or $\dfrac{4x+1}{2x+3}=-\dfrac{5}{6}$

$\therefore \quad 6(4x + 1) = 5(2x + 3)$ or $6(4x + 1) = -5(2x + 3)$

$\therefore \quad 24x + 6 = 10x + 15$ or $24x + 6 = -10x - 15$

$\therefore \quad 24x - 10x = 15 - 6$ or $24x + 10x = -15 - 6$

$\therefore \quad 14x = 9$ or $34x = -21$

$\therefore \quad x = \dfrac{9}{14}$ or $x = -\dfrac{21}{34}$

$\therefore$ **$x = \dfrac{9}{14}$ or $x = -\dfrac{21}{34}$ is the solution of the given equation.**

- *Digvijay*
- *Arjun*

vi. $\dfrac{(3x-4)^3-(x+1)^3}{(3x-4)^3+(x+1)^3}=\dfrac{61}{189}$

$\therefore \dfrac{(3x-4)^3+(x+1)^3}{(3x-4)^3-(x+1)^3}=\dfrac{189}{61}$ …[By invertendo]

$\therefore \dfrac{\left[(3x-4)^3+(x+1)^3\right]+\left[(3x-4)^3-(x+1)^3\right]}{\left[(3x-4)^3+(x+1)^3\right]-\left[(3x-4)^3-(x+1)^3\right]}$

$=\dfrac{189+61}{189-61}$

…[By componendo - dividendo]

$\therefore \dfrac{(3x-4)^3+(x+1)^3+(3x-4)^3-(x+1)^3}{(3x-4)^3+(x+1)^3-(3x-4)^3+(x+1)^3}=\dfrac{250}{128}$

$\therefore \dfrac{2(3x-4)^3}{2(x+1)^3}=\dfrac{2\times 125}{2\times 64}$

$\therefore \dfrac{(3x-4)^3}{(x+1)^3}=\dfrac{125}{64}$

$\therefore \dfrac{3x-4}{x+1}=\dfrac{5}{4}$

…[Taking cube root of both sides]

∴ 4(3x – 4) = 5(x + 1)
∴ 12x – 16 = 5x + 5
∴ 12x – 5x = 5 + 16
∴ 7x = 21
∴ x = 3
∴ x = 3 ¡s the solution of the given equation.

Practice Set 4.4 Algebra 9th Std Maths Part 1 Answers Chapter 4 Ratio and Proportion

Question 1.
Fill in the blanks of the following.

- *Digvijay*
- *Arjun*

i. $\frac{x}{7} = \frac{y}{3} = \frac{3x+5y}{.....} = \frac{7x-9y}{.....}$

ii. $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = \frac{a-2b+3c}{.....} = \frac{.....}{6-8+14}$

Solution:

i. $\frac{x}{7} = \frac{y}{3} = \frac{3x+5y}{.....} = \frac{7x-9y}{.....}$

$\frac{x}{7} = \frac{y}{3} = \frac{3x+5y}{3\times7+5\times3}$

…[Theorem on equal ratios]

$= \frac{3x+5y}{21+15} = \frac{3x+5y}{36}$

$\frac{x}{7} = \frac{y}{3} = \frac{7x-9y}{7\times7-9\times3}$

…[Theorem on equal ratios]

$= \frac{7x-9y}{49-27} = \frac{7x-9y}{22}$

$\therefore \frac{x}{7} = \frac{y}{3} = \frac{3x+5y}{\mathbf{36}} = \frac{7x-9y}{\mathbf{22}}$

ii. $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = \frac{a-2b+3c}{.....} = \frac{.....}{6-8+14}$

$\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = \frac{a-2b+3c}{3-2\times4+3\times7}$

…[Theorem on equal ratios]

$= \frac{a-2b+3c}{3-8+21} = \frac{a-2b+3c}{16}$

$\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = \frac{2a-2b+2c}{2\times3-2\times4+2\times7}$

…[Theorem on equal ratios]

$= \frac{2a-2b+2c}{6-8+14}$

$\therefore \frac{a}{3} = \frac{b}{4} = \frac{c}{7} = \frac{a-2b+3c}{\mathbf{16}} = \frac{\mathbf{2a-2b+2c}}{6-8+14}$

Question 2.

5m – n = 3m + 4n, then find the values of the following expressions.

i. $\frac{m^2+n^2}{m^2-n^2}$ ii. $\frac{3m+4n}{3m-4n}$

- *Digvijay*
- *Arjun*

Solution:

5m – n = 3m + 4n ... [Given]

∴ 5m – 3m = 4n + n

∴ 2m = 5n

i. $\frac{m^2}{n^2} = \frac{25}{4}$ ...[Squaring both sides]

∴ $\frac{m^2+n^2}{m^2-n^2} = \frac{25+4}{25-4}$

...[By componendo - dividendo]

∴ $\frac{m^2+n^2}{m^2-n^2} = \frac{29}{21}$

∴ $\mathbf{\frac{m^2+n^2}{m^2-n^2} = 29 : 21}$

ii. $\frac{m}{n} = \frac{5}{2}$

∴ $\frac{m}{n} \times \frac{3}{4} = \frac{5}{2} \times \frac{3}{4}$

$\ldots\left[\text{Multiplying both sides by } \frac{3}{4}\right]$

∴ $\frac{3m}{4n} = \frac{15}{8}$

∴ $\frac{3m+4n}{3m-4n} = \frac{15+8}{15-8}$

...[By componendo - dividendo]

∴ $\frac{3m+4n}{3m-4n} = \frac{23}{7}$

∴ $\mathbf{\frac{3m+4n}{3m-4n} = 23 : 7}$

Question 3.

Solve:

i. If a(y + z) = b(z + x) = c(x + y) and out of a, b, c no two of them are equal, then show that, $\frac{y-z}{a(b-c)} = \frac{z-x}{b(c-a)} = \frac{x-y}{c(a-b)}$.

Solution:

Here, no two of a, b and c are equal.

∴ values of (b – c), (c – a) and (a – b) are not zero.

a(y + z) = b(z + x) = c(x + y) ... [Given]

- *Digvijay*
- *Arjun*

$\therefore \quad \frac{a(y+z)}{abc} = \frac{b(z+x)}{abc} = \frac{c(x+y)}{abc}$

…[Dividing each term by abc]

$\therefore \quad \frac{y+z}{bc} = \frac{z+x}{ac} = \frac{x+y}{ab}$

Let $\frac{y+z}{bc} = \frac{z+x}{ac} = \frac{x+y}{ab} = k$ …(i)

$k = \frac{x+y}{ab} = \frac{z+x}{ac}$ …[From (i)]

- *Digvijay*
- *Arjun*

$$\therefore \quad k = \frac{(x+y)-(z+x)}{ab-ac}$$

…[Theorem on equal ratios]

$$= \frac{x+y-z-x}{a(b-c)}$$

$$\therefore \quad k = \frac{y-z}{a(b-c)} \qquad \text{...(ii)}$$

$$k = \frac{y+z}{bc} = \frac{x+y}{ab} \qquad \text{…[From (i)]}$$

$$k = \frac{(y+z)-(x+y)}{bc-ab}$$

…[Theorem on equal ratios]

$$= \frac{y+z-x-y}{b(c-a)}$$

$$\therefore \quad k = \frac{z-x}{b(c-a)} \qquad \text{…(iii)}$$

$$k = \frac{z+x}{ac} = \frac{y+z}{bc} \qquad \text{…[From (i)]}$$

$$\therefore \quad k = \frac{(z+x)-(y+z)}{ac-bc}$$

…[Theorem on equal ratios]

$$= \frac{z+x-y-z}{c(a-b)}$$

$$\therefore \quad k = \frac{x-y}{c(a-b)} \qquad \text{...(iv)}$$

$$\therefore \quad \frac{y-z}{a(b-c)} = \frac{z-x}{b(c-a)} = \frac{x-y}{c(a-b)}$$

…[From (ii), (iii) and (iv)]

ii. If $\frac{x}{3x-y-z} = \frac{y}{3y-z-x} = \frac{z}{3z-x-y}$ and $x + y + z \neq 0$, then show that the value of each ratio is equal to 1.

- *Digvijay*
- *Arjun*

Solution:

$$\text{Let } \frac{x}{3x-y-z} = \frac{y}{3y-z-x} = \frac{z}{3z-x-y} = k$$

$$\therefore \quad k = \frac{x+y+z}{(3x-y-z)+(3y-z-x)+(3z-x-y)}$$

…[Theorem on equal ratios]

$$\therefore \quad k = \frac{x+y+z}{3x-x-x+3y-y-y+3z-z-z}$$

$$= \frac{x+y+z}{x+y+z}$$

$$\therefore \quad k = 1 \qquad \ldots[\because x+y+z \neq 0]$$

$$\therefore \quad \textbf{Each ratio} = \mathbf{1}$$

iii. $x3x-y-z=y3y-z-x=z3z-x-y$ and x + y + z ≠ 0,then show that $a+b2$.

Solution:

$$\text{Let } \frac{ax+by}{x+y} = \frac{bx+az}{x+z} = \frac{ay+bz}{y+z} = k$$

$$\therefore \quad k = \frac{(ax+by)+(bx+az)+(ay+bz)}{(x+y)+(x+z)+(y+z)}$$

…[Theorem on equal ratios]

$$= \frac{ax+bx+ay+by+az+bz}{2x+2y+2z}$$

$$= \frac{x(a+b)+y(a+b)+z(a+b)}{2(x+y+z)}$$

$$= \frac{(a+b)(x+y+z)}{2(x+y+z)}$$

$$\therefore \quad k = \frac{a+b}{2} \qquad \ldots[\because x+y+z \neq 0]$$

$$\therefore \quad \textbf{Each ratio} = \frac{\mathbf{a+b}}{\mathbf{2}}$$

iv. If $y+za=z+xb=x+yc$, then show that $xb+c-a=yc+a-b=za+b-c$.

Solution:

- *Digvijay*
- *Arjun*

Let $\frac{y+z}{a} = \frac{z+x}{b} = \frac{x+y}{c} = k$ ...(i)

$\therefore \quad k = \frac{z+x}{b} = \frac{x+y}{c} = \frac{y+z}{a}$

$\therefore \quad k = \frac{(z+x)+(x+y)-(y+z)}{b+c-a}$

...[Theorem on equal ratios]

$= \frac{z+x+x+y-y-z}{b+c-a}$

$\therefore \quad k = \frac{2x}{b+c-a}$ ...(ii)

$k = \frac{x+y}{c} = \frac{y+z}{a} = \frac{z+x}{b}$ ...[From (i)]

$\therefore \quad k = \frac{(x+y)+(y+z)-(z+x)}{c+a-b}$

...[Theorem on equal ratios]

$= \frac{x+y+y+z-z-x}{c+a-b}$

$\therefore \quad k = \frac{2y}{c+a-b}$ ...(iii)

$k = \frac{y+z}{a} = \frac{z+x}{b} = \frac{x+y}{c}$ ...[From (i)]

$\therefore \quad k = \frac{(y+z)+(z+x)-(x+y)}{a+b-c}$

...[Theorem on equal ratios]

$= \frac{y+z+z+x-x-y}{a+b-c}$

$\therefore \quad k = \frac{2z}{a+b-c}$ ...(iv)

$\therefore \quad \frac{2x}{b+c-a} = \frac{2y}{c+a-b} = \frac{2z}{a+b-c}$

...[From (ii), (iii) and (iv)]

$\therefore \quad \frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$

...[Multiplying each ratio by $\frac{1}{2}$]

- *Digvijay*
- *Arjun*

v. If $3x-5y5z+3y=x+5zy-5x=y-zx-z$, then show that every ratio = xy.

Solution:

$$\text{Let } \frac{3x-5y}{5z+3y} = \frac{x+5z}{y-5x} = \frac{y-z}{x-z} = k$$

$$\therefore \quad k = \frac{3x-5y}{5z+3y} = \frac{x+5z}{y-5x} = \frac{5y-5z}{5x-5z}$$

…[Multiplying numerator and denominator of third ratio by 5]

$$\therefore \quad k = \frac{(3x-5y)+(x+5z)+(5y-5z)}{(5z+3y)+(y-5x)+(5x-5z)}$$

…[Theorem on equal ratios]

$$= \frac{4x}{4y}$$

$$= \frac{x}{y}$$

$$\therefore \quad \textbf{Each ratio} = \frac{x}{y}$$

Ratio And Proportion Class 9 Practice Set 4.4 Question 4.

Solve:

i. $\dfrac{16x^2-20x+9}{8x^2+12x+21} = \dfrac{4x-5}{2x+3}$

ii. $\dfrac{5y^2+40y-12}{5y+10y^2-4} = \dfrac{y+8}{1+2y}$

Solution:

i. $\dfrac{16x^2-20x+9}{8x^2+12x+21} = \dfrac{4x-5}{2x+3}$

If $x = 0$ then

$$\text{L.H.S} = \frac{16x^2-20x+9}{8x^2+12x+21} = \frac{16(0)^2-20(0)+9}{8(0)^2+12(0)+21}$$

$$= \frac{9}{21} = \frac{3}{7}$$

- *Digvijay*
- *Arjun*

$$\text{R.H.S.} = \frac{4x-5}{2x+3} = \frac{4(0)-5}{2(0)+3}$$

$$= \frac{-5}{3}$$

$\frac{3}{7} = \frac{-5}{3}$, which is a contradiction.

$\therefore \quad x \neq 0.$

Let $\frac{16x^2-20x+9}{8x^2+12x+21} = \frac{4x-5}{2x+3} = k$ ...(i)

$\therefore \quad k = \frac{16x^2-20x+9}{8x^2+12x+21} = \frac{4x(4x-5)}{4x(2x+3)}$

...[Multiplying numerator and denominator of second ratio by $4x$ as $x \neq 0$]

$\therefore \quad k = \frac{16x^2-20x+9}{8x^2+12x+21} = \frac{16x^2-20x}{8x^2+12x}$

$$= \frac{(16x^2-20x+9)-(16x^2-20x)}{(8x^2+12x+21)-(8x^2+12x)}$$

...[Theorem on equal ratios]

$$= \frac{16x^2-20x+9-16x^2+20x}{8x^2+12x+21-8x^2-12x}$$

$$= \frac{9}{21} = \frac{3\times3}{3\times7}$$

$\therefore \quad k = \frac{3}{7}$

$\frac{4x-5}{2x+3} = k$ ...[From (i)]

$\therefore \quad \frac{4x-5}{2x+3} = \frac{3}{7}$

$\therefore 7(4x - 5)3(2x + 3)$

$\therefore 28x - 35 = 6x + 9$

$\therefore 28x - 6x = 9 + 35$

$\therefore 22x = 44$

$\therefore x = 2$

$\therefore x = 2$ is the solution of the given equation.

- *Digvijay*
- *Arjun*

ii. $5y^2+40y-125y+10y^2-4=y+81+2y$

If $y = 0$ then

$$\text{L.H.S.} = \frac{5y^2+40y-12}{5y+10y^2-4} = \frac{5(0)^2+40(0)-12}{5(0)+10(0)^2-4}$$

$$= \frac{-12}{-4} = \frac{3}{1}$$

$$\text{R.H.S.} = \frac{y+8}{1+2y} = \frac{0+8}{1+2(0)} = \frac{8}{1}$$

$\therefore \quad \frac{3}{1} = \frac{8}{1}$, which is a contradiction

$\therefore \quad y \neq 0$

$$\text{Let } \frac{5y^2+40y-12}{5y+10y^2-4} = \frac{y+8}{1+2y} = k \qquad \ldots(i)$$

$$\therefore \quad k = \frac{5y^2+40y-12}{5y+10y^2-4} = \frac{5y(y+8)}{5y(1+2y)}$$

…[Multiplying numerator and denominator of second ratio by $5y$ as $y \neq 0$]

$$\therefore \quad k = \frac{5y^2+40y-12}{5y+10y^2-4} = \frac{5y^2+40y}{5y+10y^2}$$

$$\therefore \quad k = \frac{(5y^2+40y-12)-(5y^2+40y)}{(5y+10y^2-4)-(5y+10y^2)}$$

…[Theorem on equal ratios]

$$= \frac{5y^2+40y-12-5y^2-40y}{5y+10y^2-4-5y-10y^2}$$

$$= \frac{-12}{-4}$$

$\therefore \quad k = 3$

$$\frac{y+8}{1+2y} = k \qquad \text{...[From (i)]}$$

$$\therefore \quad \frac{y+8}{1+2y} = 3$$

$\therefore y + 8 = 3(1 + 2y)$

$\therefore y + 8 = 3 + 6y$

$\therefore 8 - 3 = 6y - y$

$\therefore 5 = 5y$

$\therefore y = 1$

$\therefore y = 1$ is the solution of the given equation.

- *Digvijay*
- *Arjun*

Practice Set 4.5 Algebra 9th Std Maths Part 1 Answers Chapter 4 Ratio and Proportion

Question 1.
Which number should be subtracted from 12, 16 and 21 so that resultant numbers are in continued proportion?
Solution:
Let the number to be subtracted be x.
∴ (12 – x), (16 – x) and (21 – x) are in continued proportion.

$$\therefore \quad \frac{12-x}{16-x} = \frac{16-x}{21-x}$$

$$\therefore \quad \frac{(12-x)-(16-x)}{16-x} = \frac{(16-x)-(21-x)}{21-x}$$

…[By dividendo]

$$\therefore \quad \frac{12-x-16+x}{16-x} = \frac{16-x-21+x}{21-x}$$

$$\therefore \quad \frac{-4}{16-x} = \frac{-5}{21-x}$$

$$\therefore \quad \frac{4}{16-x} = \frac{5}{21-x}$$

$$\therefore \quad 4(21-x) = 5(16-x)$$

∴ 84 – 4x = 80 – 5x
∴ 5x – 4x = 80 – 84
∴ x = -4
∴ -4 should be subtracted from 12,16 and 21 so that the resultant numbers in continued proportion.

Question 2.
If (28 – x) is the mean proportional of (23 – x) and (19 – x), then find the value ofx.
Solution:
(28 – x) is the mean proportional of (23 – x) and (19-x). …[Given]

$$\therefore \quad \frac{23-x}{28-x} = \frac{28-x}{19-x}$$

$$\therefore \quad \frac{(23-x)-(28-x)}{28-x} = \frac{(28-x)-(19-x)}{19-x}$$

…[By dividendo]

$$\therefore \quad \frac{23-x-28+x}{28-x} = \frac{28-x-19+x}{19-x}$$

$$\therefore \quad \frac{-5}{28-x} = \frac{9}{19-x}$$

∴ -5(19 – x) = 9(28 – x)

- *Digvijay*
- *Arjun*

∴ -95 + 5x = 252 – 9x
∴ 5x + 9x = 252 + 95
∴ 14x = 347
∴ x = 34714

Question 3.
Three numbers are in continued proportion, whose mean proportional is 12 and the sum of the remaining two numbers is 26, then find these numbers.
Solution:
Let the first number be x.
∴ Third number = 26 – x
12 is the mean proportional of x and (26 – x).
∴ x12 = 1226−x
∴ x(26 – x) = 12 x 12
∴ $26x – x^2 = 144$
∴ $x^2 – 26x + 144 = 0$
∴ $x^2 – 18x – 8x + 144 = 0$
∴ x(x – 18) – 8(x – 18) = 0
∴ (x – 18) (x – 8) = 0
∴ x = 18 or x = 8
∴ Third number = 26 – x = 26 – 18 = 8 or 26 – x = 26 – 8 = 18
∴ The numbers are 18, 12, 8 or 8, 12, 18.

Question 4.
If $(a + b + c)(a – b + c) = a^2 + b^2 + c^2$, show that a, b, c are in continued proportion.
Solution:
$(a + b + c)(a – b + c) = a^2 + b^2 + c^2$ …[Given]
∴ a(a – b + c) + b(a – b + c) + c(a – b + c) = a2 + b2 + c2
∴ $a^2 – ab + ac + ab – b^2 + be + ac – be + c^2 = a^2 + b^2 + c^2$
∴ $a^2 + 2ac – b^2 + c^2 = a^2 + b^2 + c^2$
∴ $2ac – b^2 = b^2$
∴ $2ac = 2b^2$
∴ $ac = b^2$
∴ $b^2 = ac$
∴ a, b, c are in continued proportion.

Question 5.
If ab = bc and a, b, c > 0, then show that,
i. $(a + b + c)(b – c) = ab – c^2$
ii. $(a^2 + b^2)(b^2 + c^2) = (ab + be)^2$
iii. a2+b2ab=a+cb
Solution:
Let ab = bc = k
∴ b = ck
∴ a = bk =(ck)k
∴ $a = ck^2$ …(ii)
i. $(a + b + c)(b – c) = ab – c^2$
L.H.S = (a + b + c) (b – c)
= $[ck^2 + ck + c] [ck – c]$ … [From (i) and (ii)]
= $c(k^2 + k + 1)$ c (k – 1)

- *Digvijay*
- *Arjun*

$= c^2 (k^2 + k + 1) (k – 1)$
R.H.S $= ab – c^2$
$= (ck^2) (ck) – c^2$ … [From (i) and (ii)]
$= c^2k^3 – c^2$
$= c^2(k^3 – 1)$
$= c^2 (k – 1) (k^2 + k + 1)$ … $[a^3 – b^3 = (a – b) (a^2 + ab + b^2]$
∴ L.H.S = R.H.S
∴ $(a + b + c) (b – c) = ab – c^2$

ii. $(a^2 + b^2)(b^2 + c^2) = (ab + bc)^2$
$b = ck$; $a = ck^2$
L.H.S $= (a^2 + b^2) (b^2 + c^2)$
$= [(ck^2) + (ck)^2] [(ck)^2 + c^2]$ … [From (i) and (ii)]
$= [c^2k^4 + c^2k^2] [c^2k^2 + c^2]$
$= c^2k^2 (k^2 + 1) c^2 (k^2 + 1)$
$= c4k^2 (k^2 + 1)^2$
R.H.S $= (ab + bc)^2$
$= [(ck^2) (ck) + (ck)c]^2$ …[From (i) and (ii)]
$= [c^2k^3 + c^2k]^2$
$= [c^2k (k^2 + 1)]2 = c^4(k^2 + 1)^2$
∴ L.H.S = R.H.S
∴ $(a^2 + b^2) (b^2 + c^2) = (ab + bc)^2$

- *Digvijay*
- *Arjun*

iii. $a^2+b^2ab=a+cb$

$b = ck;\ a = ck^2$

$$L.H.S = \frac{a^2 + b^2}{ab}$$

$$= \frac{\left(ck^2\right)^2 + (ck)^2}{\left(ck^2\right)(ck)} \quad \ldots[\text{From (i) and (ii)}]$$

$$= \frac{c^2k^4 + c^2k^2}{c^2k^3}$$

$$= \frac{c^2k^2(k^2 + 1)}{c^2k^3}$$

$$= \frac{k^2 + 1}{k}$$

$$R.H.S = \frac{a + c}{b}$$

$$= \frac{ck^2 + c}{ck} \quad \ldots[\text{From (i) and (ii)}]$$

$$= \frac{c\left(k^2 + 1\right)}{ck}$$

$$= \frac{k^2 + 1}{k}$$

$\therefore$ L.H.S = R.H.S

$$\therefore \quad \frac{a^2 + b^2}{ab} = \frac{a + c}{b}$$

9th Standard Algebra Practice Set 4.5 Question 6. Find mean proportional of $x+yx-y, x^2-y^2x^2y^2$.

Solution:

- *Digvijay*
- *Arjun*

Let a be the mean proportional of x+yx−y and x2−y2x2y2

$$\therefore \quad a^2 = \frac{x+y}{x-y} \times \frac{x^2-y^2}{x^2y^2}$$

$$= \frac{x+y}{x-y} \times \frac{(x+y)(x-y)}{x^2y^2}$$

...[$\because a^2 - b^2 = (a + b)(a - b)$]

$$\therefore \quad a^2 = \frac{(x+y)^2}{x^2y^2}$$

$$\therefore \quad a = \frac{x+y}{xy}$$

…[Taking square root of both sides]

$\therefore$ **Mean proportional of** $\frac{x+y}{x-y}$, $\frac{x^2-y^2}{x^2y^2}$ **is**

$$\frac{x+y}{xy}.$$

- *Digvijay*
- *Arjun*

Problem Set 4 Algebra 9th Std Maths Part 1 Answers Chapter 4 Ratio and Proportion

Question 1.
Select the appropriate alternative answer for the following questions.

i . If 6 : 5 = y : 20, then what will be the value of y?
(A) 15
(B) 24
(C) 18
(D) 22.5
Answer:
(B) 24

ii. What is the ratio of 1 mm to 1 cm ?
(A) 1 : 100
(B) 10: 1
(C) 1 : 10
(D) 100: 1
Answer:
(C) 1 : 10

iii. The ages of Jatin, Nitin and Mohasin are 16, 24 and 36 years respectively. What is the ratio of Nitin's age to Mohasin's age ?
(A) 3 : 2
(B) 2 : 3
(C) 4 : 3
(D) 3 : 4
Answer:
(B) 2 : 3

iv. 24 bananas were distributed between Shubham and Anil in the ratio 3 : 5, then how many bananas did Shubham get?
(A) 8
(B) 15

- *Digvijay*
- *Arjun*

(C) 12
(D) 9
Answer:
(D) 9

v. What is the mean proportional of 4 and 25?
(A) 6
(B) 8
(C) 10
(D) 12
Answer:
(C) 10

Hints:
i . $65 = y20$
$\therefore y = 6 \times 205 = 24$
ii. $1mm1cm = 1mm10mm = 110 = 1:10$
iii. $\text{Age of Nitin}\ \ \text{Age of Mohasin} = 2436 = 12 \times 212 \times 3$
23 = 2 : 3
iv. 3x + 5x = 24
∴ 8x = 24
∴ x = 3
∴ Number of bananas with Shubham = 3x = 9

v. x^2 = 4 x 25
∴ x^2 = 100
∴ x = 10

Question 2.
For the following numbers write the ratio of first number to second number in the reduced form. [1 Mark each]
i. 21,48
ii. 36,90
iii. 65,117
iv. 138,161
v. 114,133
Solution:

- *Digvijay*
- *Arjun*

i. 21,48

Ratio $=2148=3\times73\times16=716=7:16$

ii. 36,90

Ratio $=3690=18\times218\times5=25=2:5$

iii. 65,117

Ratio $=65117=13\times513\times9=59=5:9$

iv. 138,161

Ratio $=138161=23\times623\times7=67=6:7$

v. 114,133

Ratio $=114113=19\times619\times7=67=6:7$

Question 3.

Write the following ratios in the reduced form.

i. Radius to the diameter of a circle.

ii. The ratio of diagonal to the length of a rectangle, having length 4 cm and breadth 3 cm.

iii. The ratio of numbers denoting perimeter to area of a square, having side 4 cm.

Solution:

i. Radius to the diameter of a circle.

Let radius of the circle be r

then, diameter = 2 x radius = 2r

Ratio of radius to diameter of circle

$$= \frac{\text{Radius of the circle}}{\text{Diameter of the circle}}$$

$$= \frac{r}{2r}$$

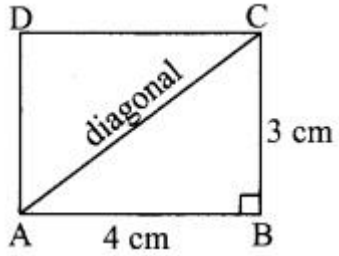
$$= \frac{1}{2}$$

$$= 1:2$$

∴ Ratio of radius to diameter of circle is 1 : 2.

ii. The ratio of diagonal to the length of a rectangle, having length 4 cm and breadth 3 cm.

- *Digvijay*
- *Arjun*



Let □ ABCD be a rectangle.
In ΔABC, ∠B = 90°
$AC^2 = AB^2 + BC^2$ … [Pythagoras theorem]
$= 4^2 + 3^2 = 16 + 9$
∴ $AC^2 = 25$
AC = 5 … [Taking square root on both side]
The ratio of diagonal to the length of a rectangle = ACAB
$= 54$
= 5 : 4
∴ The ratio of diagonal to the length of a rectangle is 5 : 4
iii. The ratio of numbers denoting perimeter to area of a square, having side 4 cm. side of square = 4cm
Perimeter of square = 4 x side = 4 x 4 = 16 cm
Area of square = $(\text{side})^2 = (4)^2 = 16\ cm^2$
Ratio of numbers denoting perimeter to area of square

$$= \frac{\text{Perimeter of the square}}{\text{Area of the square}}$$

$$= \frac{16}{16} = \frac{1}{1} = 1 : 1$$

∴ The ratio of numbers denoting perimeter to area of a square is 1 : 1.

Question 4.
Check whether the following numbers are in continued proportion.
i. 2, 4, 8
ii. 1, 2, 3
iii. 9, 12, 16
iv. 3, 5, 8
Solution:
If a, b, c are in continued proportion then $b^2 = ac$.
i. 2, 4, 8
Let, a = 2, b = 4 and c = 8
Here, $b^2 = 4^2 = 16$

- *Digvijay*
- *Arjun*

ac = 2 x 8 = 16
∴ b^2 = ac
∴ 2, 4,8 are in continued proportion.

ii. 1, 2, 3
Let, a = 1, b = 2 and c = 3
Here, $b^2 = 2^2 = 4$
ac = 1 x 3 = 3
∴ $b^2 \neq$ ac
∴ 1, 2,3 are not in continued proportion.

iii. 9, 12, 16
Let, a = 9, b = 12 and c = 16
Here, $b^2 = 12^2 = 144$
ac = 9 x 16 = 144
∴ b^2 = ac
∴ 9, 12, 16 are in continued proportion.

iv. 3, 5, 8
Let, a = 3, b = 5 and c = 8
Here, $b^2 = 5^2 = 25$
ac = 3 x 8 = 24
∴ $b^2 \neq$ ac
∴ 3, 5, 8 are not in continued proportion.

Question 5.
a, b, c are in continued proportion. If a = 3 and c = 27, then find b.
Solution:
a, b, c are in continued proportion. ...[Given]
∴ b^2 = ac
∴ b^2 = 3 x 27 ...[∵ a = 3 and c = 27]
∴ b^2 = 81
∴ b = 9 ...[Taking square root of both sides]

Question 6.
Convert the following ratios into percentages.

- *Digvijay*
- *Arjun*

i. 37 : 500 ii. $\frac{5}{8}$

iii. $\frac{22}{30}$ iv. $\frac{5}{16}$

v. $\frac{144}{1200}$

Solution:

i. Let 37 : 500 = x%

$\therefore \quad \frac{37}{500} = \frac{x}{100}$

$\therefore \quad x = \frac{37}{500} \times 100 = \frac{37}{5} = 7.4\ \%$

- *Digvijay*
- *Arjun*

$\therefore$ 37 : 500 = 7.4%

ii. Let $\frac{5}{8} = x\%$

$\therefore \quad \frac{5}{8} = \frac{x}{100}$

$\therefore \quad x = \frac{5}{8} \times 100 = \frac{5}{2} \times 25 = 62.5\%$

$\therefore \quad \mathbf{\frac{5}{8} = 62.5\%}$

iii. Let $\frac{22}{30} = x\%$

$\therefore \quad \frac{22}{30} = \frac{x}{100}$

$\therefore \quad x = \frac{22}{30} \times 100 = \frac{22}{3} \times 10 = 73.33\%$

$\therefore \quad \mathbf{\frac{22}{30} = 73.33\%}$

iv. Let $\frac{5}{16} = x\%$

$\therefore \quad \frac{5}{16} = \frac{x}{100}$

$\therefore \quad x = \frac{5}{16} \times 100 = \frac{5}{4} \times 25 = 31.25\%$

$\therefore \quad \mathbf{\frac{5}{16} = 31.25\%}$

v. Let $\frac{144}{1200} = x\%$

$\therefore \quad \frac{144}{1200} = \frac{x}{100}$

$\therefore \quad x = \frac{144}{1200} \times 100 = \frac{144}{12} = 12\%$

$\therefore \quad \mathbf{\frac{144}{1200} = 12\%}$

Question 7.

Write the ratio of first quantity to second quantity in the reduced form.

- *Digvijay*
- *Arjun*

i. 1024 MB, 1.2 GB [(1024 MB = 1GB)]
ii. ₹ 17, ₹ 25 and 60 paise
iii. 5 dozen, 120 units
iv. 4 sq.m, 800 sq.cm
v. 1.5 kg, 2500 gm
Solution:
i. 1024 MB, 1.2 GB
1024 MB = 1 GB

$$\text{Ratio} = \frac{1024\text{ MB}}{1.2\text{ GB}} = \frac{1\text{ GB}}{1.2\text{ GB}} = \frac{1}{1.2}$$

$$= \frac{10}{12} = \frac{2\times5}{2\times6}$$

$$= \frac{5}{6}$$

$$= \mathbf{5:6}$$

ii. ₹ 17, ₹ 25 and 60 paise
₹ 17 = 17 x 100 paise = 1700 paise
₹ 25 and 60 paise = (25x 100) paise + 60 paise
= (2500 + 60) paise
= 2560 paise

$$\text{Ratio} = \frac{₹17}{₹25\text{ and }60\text{paise}}$$

$$= \frac{1700\text{paise}}{2560\text{paise}}$$

$$= \frac{1700}{2560}$$

$$= \frac{20\times85}{20\times128}$$

$$= \frac{85}{128}$$

$$= \mathbf{85:128}$$

- *Digvijay*
- *Arjun*

iii. 5 dozen, 120 units

5 dozen = 5 x 12 units = 60 units

$$\text{Ratio} = \frac{5\,\text{dozen}}{120\,\text{units}} = \frac{60\,\text{units}}{120\,\text{units}}$$

$$= \frac{60}{120}$$

$$= \frac{60\times1}{60\times2} = \frac{1}{2}$$

$$= \mathbf{1 : 2}$$

iv. 4 sq.m, 800 sq.cm

4 sq.m = 4 x 10000 sq.cm = 40000 sq.cm

$$\text{Ratio} = \frac{4\,\text{sq.m}}{800\,\text{sq.cm}} = \frac{40000\,\text{sq.cm}}{800\,\text{sq.cm}}$$

$$= \frac{40000}{800}$$

$$= \frac{400}{8}$$

$$= \frac{8\times50}{8\times1} = \frac{50}{1}$$

$$= \mathbf{50 : 1}$$

v. 1.5 kg, 2500 gm

1.5 kg = 1.5 x 1000 gm = 1500gm

$$\text{Ratio} = \frac{1.5\,\text{kg}}{2500\,\text{gm}} = \frac{1500\,\text{gm}}{2500\,\text{gm}}$$

$$= \frac{1500}{2500}$$

$$= \frac{15}{25}$$

$$= \frac{5\times3}{5\times5} = \frac{3}{5}$$

$$= \mathbf{3 : 5}$$

Question 8.

If $ab = 23$, then find the values of the following expressions.

- *Digvijay*
- *Arjun*

i. $\frac{4a+3b}{3b}$

ii. $\frac{5a^2+2b^2}{5a^2-2b^2}$

iii. $\frac{a^3+b^3}{b^3}$

iv. $\frac{7b-4a}{7b+4a}$

Solution:

i. $\frac{a}{b} = \frac{2}{3}$ …[Given]

$\therefore \quad \frac{a}{b} \times \frac{4}{3} = \frac{2}{3} \times \frac{4}{3}$

…[Multiplying both sides by $\frac{4}{3}$]

$\therefore \quad \frac{4a}{3b} = \frac{8}{9}$

$\therefore \quad \frac{4a+3b}{3b} = \frac{8+9}{9}$ …[By componendo]

$\therefore \quad \mathbf{\frac{4a+3b}{3b} = \frac{17}{9}}$

ii. $\frac{a}{b} = \frac{2}{3}$ …[Given]

$\therefore \quad \left(\frac{a}{b}\right)^2 = \left(\frac{2}{3}\right)^2$ …[Squaring both sides]

$\therefore \quad \frac{a^2}{b^2} = \frac{4}{9}$

$\therefore \quad \frac{a^2}{b^2} \times \frac{5}{2} = \frac{4}{9} \times \frac{5}{2}$

…[Multiplying both sides by $\frac{5}{2}$]

$\therefore \quad \frac{5a^2}{2b^2} = \frac{10}{9}$

$\therefore \quad \frac{5a^2+2b^2}{5a^2-2b^2} = \frac{10+9}{10-9}$

…[By componendo - dividendo]

$= \frac{19}{1}$

$\therefore \quad \mathbf{\frac{5a^2+2b^2}{5a^2-2b^2} = 19}$

iii. $\frac{a}{b} = \frac{2}{3}$ …[Given]

$\therefore \quad \left(\frac{a}{b}\right)^3 = \left(\frac{2}{3}\right)^3$ …[Cubing both sides]

$\therefore \quad \frac{a^3}{b^3} = \frac{8}{27}$

$\therefore \quad \frac{a^3+b^3}{b^3} = \frac{8+27}{27}$ …[By componendo]

$\therefore \quad \mathbf{\frac{a^3+b^3}{b^3} = \frac{35}{27}}$

iv. $\frac{a}{b} = \frac{2}{3}$ ...[Given]

$\therefore \quad \frac{b}{a} = \frac{3}{2}$ …[By invertendo]

$\therefore \quad \frac{b}{a} \times \frac{7}{4} = \frac{3}{2} \times \frac{7}{4}$

…[Multiplying both sides by $\frac{7}{4}$]

$\therefore \quad \frac{7b}{4a} = \frac{21}{8}$

$\therefore \quad \frac{7b+4a}{7b-4a} = \frac{21+8}{21-8}$

…[By componendo - dividendo]

$\therefore \quad \frac{7b+4a}{7b-4a} = \frac{29}{13}$

$\therefore \quad \mathbf{\frac{7b-4a}{7b+4a} = \frac{13}{29}}$ …[By invertendo]

Question 9.

If a, b, c, d are in proportion, then prove that

- *Digvijay*
- *Arjun*

i. $\dfrac{11a^2+9ac}{11b^2+9bd} = \dfrac{a^2+3ac}{b^2+3bd}$

ii. $\sqrt{\dfrac{a^2+5c^2}{b^2+5d^2}} = \dfrac{a}{b}$

iii. $\dfrac{a^2+ab+b^2}{a^2-ab+b^2} = \dfrac{c^2+cd+d^2}{c^2-cd+d^2}$

Solution:

a, b, c are in continued proportion. ...[Given]

$\therefore \quad \dfrac{a}{b} = \dfrac{c}{d}$

Let $\dfrac{a}{b} = \dfrac{c}{d} = k$

$\therefore \quad a = bk$ and $c = dk$ ...(i)

- *Digvijay*
- *Arjun*

i. $\dfrac{11a^2+9ac}{11b^2+9bd} = \dfrac{a^2+3ac}{b^2+3bd}$

$$\text{L.H.S.} = \frac{11a^2+9ac}{11b^2+9bd}$$

$$= \frac{11(bk)^2+9(bk)(dk)}{11b^2+9bd} \quad \ldots[\text{From (i)}]$$

$$= \frac{11b^2k^2+9bdk^2}{11b^2+9bd}$$

$$= \frac{k^2(11b^2+9bd)}{11b^2+9bd}$$

$$= k^2$$

$$\text{R.H.S.} = \frac{a^2+3ac}{b^2+3bd}$$

$$= \frac{(bk)^2+3(bk)(dk)}{b^2+3bd} \quad \ldots[\text{From (i)}]$$

$$= \frac{b^2k^2+3bdk^2}{b^2+3bd}$$

$$= \frac{k^2(b^2+3bd)}{b^2+3bd}$$

$$= k^2$$

$\therefore$ L.H.S. = R.H.S.

$\therefore$ $\boldsymbol{\dfrac{11a^2+9ac}{11b^2+9bd} = \dfrac{a^2+3ac}{b^2+3bd}}$

ii. $\sqrt{\frac{a^2+5c^2}{b^2+5d^2}} = \frac{a}{b}$

$$\text{L.H.S.} = \sqrt{\frac{a^2+5c^2}{b^2+5d^2}}$$

$$= \sqrt{\frac{(bk)^2+5(dk)^2}{b^2+5d^2}} \quad \ldots[\text{From (i)}]$$

$$= \sqrt{\frac{b^2k^2+5d^2k^2}{b^2+5d^2}}$$

$$= \sqrt{\frac{k^2(b^2+5d^2)}{b^2+5d^2}}$$

$$= \sqrt{k^2}$$

$$= k$$

$$\text{R.H.S.} = \frac{a}{b} = \frac{bk}{b} \quad \ldots[\text{From (i)}]$$

$$= k$$

$\therefore$ L.H.S. = R.H.S.

$\therefore$ $\sqrt{\frac{a^2+5c^2}{b^2+5d^2}} = \frac{a}{b}$

- *Digvijay*
- *Arjun*

iii. $\dfrac{a^2+ab+b^2}{a^2-ab+b^2} = \dfrac{c^2+cd+d^2}{c^2-cd+d^2}$

L.H.S. $= \dfrac{a^2+ab+b^2}{a^2-ab+b^2}$

$= \dfrac{(bk)^2+(bk)b+b^2}{(bk)^2-(bk)b+b^2}$ …[From (i)]

$= \dfrac{b^2k^2+b^2k+b^2}{b^2k^2-b^2k+b^2}$

$= \dfrac{b^2(k^2+k+1)}{b^2(k^2-k+1)}$

$= \dfrac{k^2+k+1}{k^2-k+1}$

R.H.S. $= \dfrac{c^2+cd+d^2}{c^2-cd+d^2}$

$= \dfrac{(dk)^2+(dk)d+d^2}{(dk)^2-(dk)d+d^2}$ …[From (i)]

$= \dfrac{d^2k^2+d^2k+d^2}{d^2k^2-d^2k+d^2}$

$= \dfrac{d^2(k^2+k+1)}{d^2(k^2-k+1)}$

$= \dfrac{k^2+k+1}{k^2-k+1}$

∴ L.H.S. = R.H.S.

∴ $\dfrac{a^2+ab+b^2}{a^2-ab+b^2} = \dfrac{c^2+cd+d^2}{c^2-cd+d^2}$

Question 10.

If a, b, c are in continued proportion, then prove that

i. $\dfrac{a}{a+2b} = \dfrac{a-2b}{a-4c}$ ii. $\dfrac{b}{b+c} = \dfrac{a-b}{a-c}$

Solution:

a, b, c are in continued proportion. … [Given]

∴ ab = bc

Let ab = bc = k

∴ b = ck

- *Digvijay*
- *Arjun*

$\therefore a = bk$

$= (ck)k$ . .. [From (j)]

$a = ck^2$ ...(ii)

i. $\frac{a}{a+2b} = \frac{a-2b}{a-4c}$

$L.H.S. = \frac{a}{a+2b}$

$= \frac{ck^2}{ck^2+2ck}$ ...[From (i) and (ii)]

$= \frac{ck^2}{ck(k+2)} = \frac{k}{k+2}$

- *Digvijay*
- *Arjun*

$$\text{R.H.S.} = \frac{a-2b}{a-4c}$$

$$= \frac{ck^2-2ck}{ck^2-4c} \quad \ldots[\text{From (i) and (ii)}]$$

$$= \frac{ck(k-2)}{c(k^2-4)}$$

$$= \frac{k(k-2)}{(k+2)(k-2)}$$

$$\ldots[\because a^2-b^2=(a+b)(a-b)]$$

$$= \frac{k}{k+2}$$

$\therefore$ L.H.S. = R.H.S.

$\therefore$ $\mathbf{\frac{a}{a+2b} = \frac{a-2b}{a-4c}}$

ii. $\frac{b}{b+c} = \frac{a-b}{a-c}$

$$\text{L.H.S.} = \frac{b}{b+c}$$

$$= \frac{ck}{ck+c} \quad \ldots[\text{From (i) and (ii)}]$$

$$= \frac{ck}{c(k+1)} = \frac{k}{k+1}$$

$$\text{R.H.S.} = \frac{a-b}{a-c}$$

$$= \frac{ck^2-ck}{ck^2-c} \quad \ldots[\text{From (i) and (ii)}]$$

$$= \frac{ck(k-1)}{c(k^2-1)}$$

$$= \frac{k(k-1)}{(k+1)(k-1)}$$

$$\ldots[\because a^2-b^2=(a+b)(a-b)]$$

$$= \frac{k}{k+1}$$

$\therefore$ L.H.S. = R.H.S.

$\therefore$ $\mathbf{\frac{b}{b+c} = \frac{a-b}{a-c}}$

- *Digvijay*
- *Arjun*

Question 11.

Solve:

$12x^2+18x+4218x^2+12x+58=2x+33x+2$

Solution:

$$\frac{12x^2+18x+42}{18x^2+12x+58}=\frac{2x+3}{3x+2}$$

If $x=0$ then

$$\text{L.H.S}=\frac{12x^2+18x+42}{18x^2+12x+58}$$

$$=\frac{12(0)^2+18(0)+42}{18(0)^2+12(0)+58}$$

$$=\frac{42}{58}=\frac{21}{29}$$

- *Digvijay*
- *Arjun*

$$\text{R.H.S} = \frac{2x+3}{3x+2} = \frac{2(0)+3}{3(0)+2} = \frac{3}{2}$$

$\therefore \quad \frac{21}{29} = \frac{3}{2}$, which is a contradiction

$\therefore \quad x \neq 0$

$$\text{Let } \frac{12x^2+18x+42}{18x^2+12x+58} = \frac{2x+3}{3x+2} = k \qquad \ldots(i)$$

$$\therefore \quad k = \frac{12x^2+18x+42}{18x^2+12x+58} = \frac{6x(2x+3)}{6x(3x+2)}$$

…[Multiplying numerator and denominator of second ratio by $6x$, as $x \neq 0$]

$$\therefore \quad k = \frac{12x^2+18x+42}{18x^2+12x+58} = \frac{12x^2+18x}{18x^2+12x}$$

$$\therefore \quad k = \frac{(12x^2+18x+42)-(12x^2+18x)}{(18x^2+12x+58)-(18x^2+12x)}$$

…[Theorem on equal ratios]

$$\therefore \quad k = \frac{12x^2+18x+42-12x^2-18x}{18x^2+12x+58-18x^2-12x}$$

$$\therefore \quad k = \frac{42}{58} = \frac{2\times 21}{2\times 29}$$

$$\therefore \quad k = \frac{21}{29}$$

$$\frac{2x+3}{3x+2} = k \qquad \ldots[\text{From (i)}]$$

$$\therefore \quad \frac{2x+3}{3x+2} = \frac{21}{29}$$

∴ 29(2x + 3) = 21 (3x + 2)
∴ 5 + 87= 63x + 42
∴ 87 – 42 = 63x – 58x
∴ 45 = 5x
∴ x = 9
∴ x = 9 is the solution of the given equation.

Question 12.
If $2x-3y3z+y=z-yz-x=x+3z2y-3x$,then prove that every ratio = xy.
Solution:

- *Digvijay*
- *Arjun*

Let $\frac{2x-3y}{3z+y} = \frac{z-y}{z-x} = \frac{x+3z}{2y-3x} = k$

$\therefore \quad k = \frac{2x-3y}{3z+y} = \frac{-3(z-y)}{-3(z-x)} = \frac{x+3z}{2y-3x}$

…[Multiplying numerator and denominator of second ratio by –3]

$\therefore \quad k = \frac{(2x-3y)-3(z-y)+(x+3z)}{(3z+y)-3(z-x)+(2y-3x)}$

…[Theorem on equal ratios]

$= \frac{2x-3y-3z+3y+x+3z}{3z+y-3z+3x+2y-3x} = \frac{3x}{3y}$

$\therefore \quad k = \frac{x}{y}$

$\therefore \quad$ **Each ratio** $= \frac{x}{y}$

Question 13.

If $by+cz b^2+c^2=cz+ax c^2+a^2=ax+by a^2+b^2$, then prove $xa=yb=zc$

- *Digvijay*
- *Arjun*

Solution:

$$\text{Let } \frac{by+cz}{b^2+c^2} = \frac{cz+ax}{c^2+a^2} = \frac{ax+by}{a^2+b^2} = k \quad \ldots(i)$$

$$\therefore \quad k = \frac{(by+cz)-(cz+ax)+(ax+by)}{(b^2+c^2)-(c^2+a^2)+(a^2+b^2)}$$

…[Theorem on equal ratios]

$$= \frac{by+cz-cz-ax+ax+by}{b^2+c^2-c^2-a^2+a^2+b^2}$$

$$= \frac{2by}{2b^2}$$

$$\therefore \quad k = \frac{y}{b} \quad \ldots(ii)$$

$$\frac{by+cz}{b^2+c^2} = \frac{cz+ax}{c^2+a^2} = \frac{ax+by}{a^2+b^2} = k \quad \ldots[\text{From(i)}]$$

$$\therefore \quad k = \frac{-(by+cz)+(cz+ax)+(ax+by)}{-(b^2+c^2)+(c^2+a^2)+(a^2+b^2)}$$

…[Theorem on equal ratios]

$$= \frac{-by-cz+cz+ax+ax+by}{-b^2-c^2+c^2+a^2+a^2+b^2}$$

$$= \frac{2ax}{2a^2}$$

$$\therefore \quad k = \frac{x}{a} \quad \ldots(iii)$$

$$\frac{by+cz}{b^2+c^2} = \frac{cz+ax}{c^2+a^2} = \frac{ax+by}{a^2+b^2} = k \quad \ldots[\text{From(i)}]$$

$$\therefore \quad k = \frac{(by+cz)+(cz+ax)-(ax+by)}{(b^2+c^2)+(c^2+a^2)-(a^2+b^2)}$$

…[Theorem on equal ratios]

$$= \frac{by+cz+cz+ax-ax-by}{b^2+c^2+c^2+a^2-a^2-b^2}$$

$$= \frac{2cz}{2c^2}$$

$$\therefore \quad k = \frac{z}{c} \quad \ldots(iv)$$

$$\therefore \quad \frac{x}{a} = \frac{y}{b} = \frac{z}{c} \quad \ldots[\text{From (ii),(iii) and (iv)}]$$

Question 1.

Take 5 pieces of card paper. Write the following statements, one on each paper.

- *Digvijay*
- *Arjun*

i. $\frac{a+b}{b}=\frac{c+d}{d}$ ii. $\frac{a}{c}=\frac{b}{d}$

iii. $\frac{a}{b}=\frac{ac}{bd}$ iv. $\frac{c}{d}=\frac{c-a}{d-b}$

v. $\frac{a}{b}=\frac{rc}{rd}$

a, b, c, d are positive numbers and ab=cd is given. Which of the above statements are true or false, write at the back of each card, if false explain why. (Textbook pg. no. 70)

Answer:

i. True

ii. True

iii. False

Here, numerator and denominator are multiplied by two different numbers a and b.

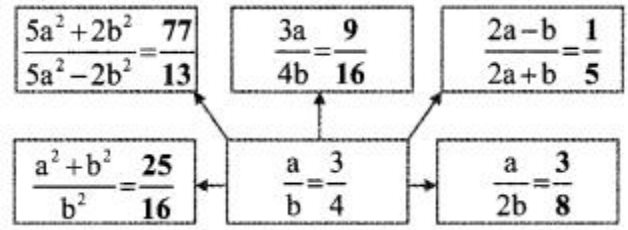
iv. False

Here, different numbers a and b are subtracted from numerator and denominator.

v. True

Question 2.

In the following activity, the values of a and b can be changed. That is by changing a : b we can create many examples. Teachers should give lot of practice to the students and encourage them to construct their own examples. (Textbook pg. no. 70)



Question 3.

Observe the political map of India from a Geography text book. Study the scale of this map.

From the given scale find the straight line distances between various cities like

i. New Delhi to Bengaluru

ii. Mumbai to Kolkata

- *Digvijay*
- *Arjun*

iii. Jaipur to Bhuvaneshvar. (Textbook pg. no. 77)

[Students should attempt the above activity on their own.]