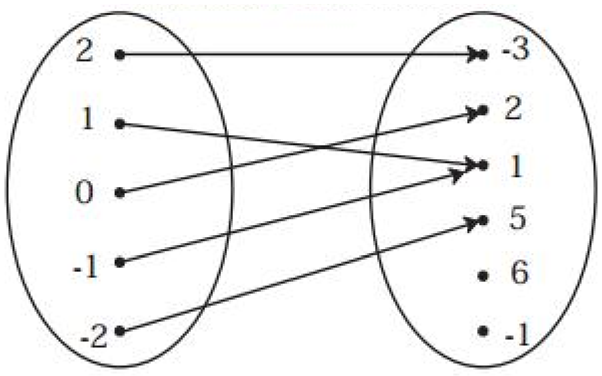
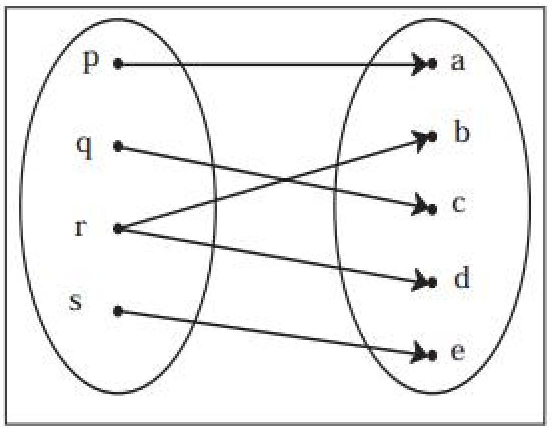
Maharashtra State Board 11th Commerce Maths Solutions Chapter 2 Functions Ex 2.1

Question 1.
Check if the following relations are functions.

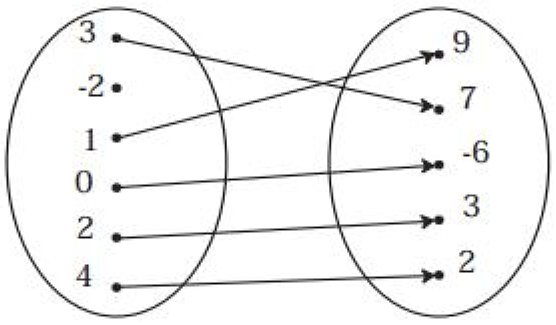
(a)



(b)



(c)



Solution:
(a) Yes
Reason: Every element of set A has been assigned a unique element in set B.

(b) No
Reason: An element of set A has been assigned more than one element from set B.

(c) No
Reason: Not every element of set A has been assigned an image from set B.

Question 2.
Which sets of ordered pairs represent functions from A = {1, 2, 3, 4} to B = {-1, 0, 1, 2, 3}? Justify.
(i) {(1, 0), (3, 3), (2, -1), (4, 1), (2, 2)}
(ii) {(1, 2), (2, -1), (3, 1), (4, 3)}
(iii) {(1, 3), (4, 1), (2, 2)}
(iv) {(1, 1), (2, 1), (3, 1), (4, 1)}
Solution:
(i) {(1, 0), (3, 3), (2, -1), (4, 1), (2, 2)} does not represent a function.
Reason: (2, -1) and (2, 2) show that element 2 ∈ A has been assigned two images -1 and 2 from set B.

(ii) {(1, 2), (2, -1), (3, 1), (4, 3)} represents a function.
Reason: Every element of set A has a unique image in set B.

(iii) {(1, 3), (4, 1), (2, 2)} does not represent a function.
Reason: 3 ∈ A does not have an image in set B.

(iv) {(1, 1), (2, 1), (3, 1), (4, 1)} represents a function
Reason: Every element of set A has been assigned a unique image in set B.

Question 3.
If $f(m) = m^2 - 3m + 1$, find
(i) f(0)
(ii) f(-3)
(iii) f(12)
(iv) f(x + 1)
(v) f(-x)
Solution:
$f(m) = m^2 - 3m + 1$
(i) $f(0) = 0^2 - 3(0) + 1 = 1$

(ii) $f(-3) = (-3)^2 - 3(-3) + 1$
$= 9 + 9 + 1$
$= 19$

(iii) f(12)=(12)2−3(12)+1
= 14−32+1
= 1-6+44
= −14

(iv) $f(x + 1) = (x + 1)^2 - 3(x + 1) + 1$
$= x^2 + 2x + 1 - 3x - 3 + 1$
$= x^2 - x - 1$

(v) $f(-x) = (-x)^2 - 3(-x) + 1 = x^2 + 3x + 1$

Question 4.
Find x, if g(x) = 0 where
(i) g(x) = 5x-67
(ii) g(x) = 18-2x27
(iii) $g(x) = 6x^2 + x - 2$
Solution:
(i) g(x) = 5x-67
g(x) = 0
∴ 5x-67 = 0
∴ 5x – 6 = 0
∴ x = 65

(ii) g(x) = 18-2x27
g(x) = 0
∴ 18-2x27 = 0
∴ $18 - 2x^2 = 0$
∴ $x^2 = 9$
∴ $x = \pm 3$

(iii) $g(x) = 6x^2 + x - 2$
g(x) = 0
∴ $6x^2 + x - 2 = 0$
∴ $6x^2 + 4x - 3x - 2 = 0$
∴ $2x(3x + 2) - 1(3x + 2) = 0$
∴ $(2x - 1)(3x + 2) = 0$
∴ 2x – 1 = 0 or 3x + 2 = 0
∴ x = 12 or x = -23

Question 5.
Find x, if f(x) = g(x) where $f(x) = x^4 + 2x^2$, $g(x) = 11x^2$.
Solution:
$f(x) = x^4 + 2x^2$, $g(x) = 11x^2$
f(x) = g(x)

∴ $x^4 + 2x^2 = 11x^2$
∴ $x^4 - 9x^2 = 0$
∴ $x^2(x^2 - 9) = 0$
∴ $x^2 = 0$ or $x^2 - 9 = 0$
∴ $x = 0$ or $x^2 = 9$
∴ $x = 0$ or $x = \pm 3$

Question 6.
If $f(x) = \begin{cases} x^2+3, & x \le 2 \\ 5x+7, & x > 2 \end{cases}$, then find
(i) f(3)
(ii) f(2)
(iii) f(0)
Solution:
$f(x) = x^2 + 3, x \le 2$
$= 5x + 7, x > 2$
(i) $f(3) = 5(3) + 7 = 15 + 7 = 22$
(ii) $f(2) = 2^2 + 3 = 4 + 3 = 7$
(iii) $f(0) = 0^2 + 3 = 3$

Question 7.
If $f(x) = \begin{cases} 4x-2, & x \le -3 \\ 5, & -3 < x < 3 \\ x^2, & x \ge 3 \end{cases}$, then fmd
(i) f(-4)
(ii) f(-3)
(iii) f(1)
(iv) f(5)
Solution:
$f(x) = 4x - 2, x \le -3$
$= 5, -3 < x < 3$
$= x^2, x \ge 3$
(i) $f(-4) = 4(-4) - 2 = -16 - 2 = -18$
(ii) $f(-3) = 4(-3) - 2 = -12 - 2 = -14$
(iii) $f(1) = 5$
(iv) $f(5) = 5^2 = 25$

Question 8.
If $f(x) = 3x + 5$, $g(x) = 6x - 1$, then find
(i) $(f + g)(x)$
(ii) $(f - g)(2)$
(iii) $(fg)(3)$
(iv) $\left(\frac{f}{g}\right)(x)$ and its domain
Solution:
$f(x) = 3x + 5$, $g(x) = 6x - 1$
(i) $(f + g)(x) = f(x) + g(x)$
$= 3x + 5 + 6x - 1$
$= 9x + 4$

(ii) $(f - g)(2) = f(2) - g(2)$
$= [3(2) + 5] - [6(2) - 1]$
$= 6 + 5 - 12 + 1$
$= 0$

(iii) $(fg)(3) = f(3)\, g(3)$
$= [3(3) + 5]\, [6(3) - 1]$
$= (14)(17)$
$= 238$

(iv) $\left(\frac{f}{g}\right)x = \frac{f(x)}{g(x)} = \frac{3x+5}{6x-1}, x \neq \frac{1}{6}$
Domain = $R - \left\{\frac{1}{6}\right\}$

Question 9.
If $f(x) = 2x^2 + 3$, $g(x) = 5x - 2$, then find
(i) fog
(ii) gof
(iii) fof

(iv) gog
Solution:
$f(x) = 2x^2 + 3$, $g(x) = 5x – 2$
(i) $(fog)(x) = f(g(x))$
$= f(5x – 2)$
$= 2(5x – 2)^2 + 3$
$= 2(25x^2 – 20x + 4) + 3$
$= 50x^2 – 40x + 8 + 3$
$= 50x^2 – 40x + 11$

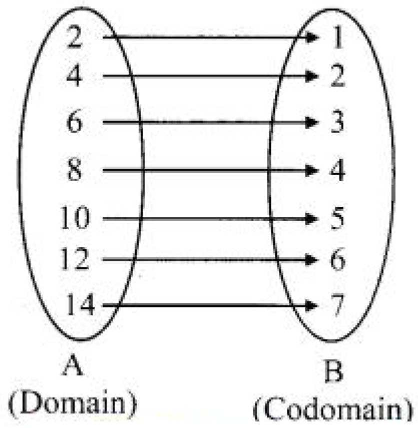
(ii) $(gof)(x) = g(f(x))$
$= g(2x^2 + 3)$
$= 5(2x^2 + 3) – 2$
$= 10x^2 + 15 – 2$
$= 10x^2 + 13$

(iii) $(fof)(x) = f(f(x))$
$= f(2x^2 + 3)$
$= 2(2x^2 + 3)^2 + 3$
$= 2(4x^4 + 12x^2 + 9) + 3$
$= 8x^4 + 24x^2 + 18 + 3$
$= 8x^4 + 24x^2 + 21$

(iv) $(gog)(x) = g(g(x))$
$= g(5x – 2)$
$= 5(5x – 2) – 2$
$= 25x – 10 – 2$
$= 25x – 12$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 2 Functions Miscellaneous Exercise 2

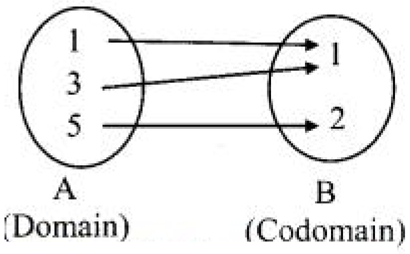
Question 1.
Which of the following relations are functions? If it is a function determine its domain and range.
(i) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5) (12, 6), (14, 7)}
(ii) {(0, 0), (1, 1), (1, -1), (4, 2), (4, -2), (9, 3), (9, -3), (16, 4), (16, -4)}
(iii) {(1, 1), (3, 1), (5, 2)}
Solution:
(i) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5) (12, 6), (14, 7)}



Every element of set A has been assigned a unique element in set B.
∴ Given relation is a function.
Domain = {2, 4, 6, 8, 10, 12, 14},
Range = {1, 2, 3, 4, 5, 6, 7}

(ii) {(0, 0), (1, 1), (1, -1), (4, 2), (4, -2), (9, 3), (9, -3), (16, 4), (16, -4)}
∴ (1, 1), (1, -1) ∈ the relation
∴ Given relation is not a function.
As element 1 of the domain has not been assigned a unique element of co-domain.

(iii) {(1, 1), (3, 1), (5, 2)}



Every element of set A has been assigned a unique element in set B.
∴ Given relation is a function.
Domain = {1, 3, 5}, Range = {1, 2}

Question 2.
A function f: R → R defined by f(x) = $\frac{3x}{5}$ + 2, x ∈ R. Show that f is one-one and onto. Hence, find f^{-1}.
Solution:
f: R → R defined by f(x) = $\frac{3x}{5}$ + 2
First we have to prove that f is one-one function for that we have to prove if
$f(x_1) = f(x_2)$ then $x_1 = x_2$
Here f(x) = $\frac{3x}{5}$ + 2
Let $f(x_1) = f(x_2)$
∴ $\frac{3x_1}{5}+2=\frac{3x_2}{5}+2$
∴ $\frac{3x_1}{5}=\frac{3x_2}{5}$
∴ $x_1 = x_2$
∴ f is a one-one function.
Now, we have to prove that f is an onto function.
Let y ∈ R be such that
y = f(x)
∴ y = $\frac{3x}{5}$ + 2
∴ y – 2 = $\frac{3x}{5}$
∴ x = $\frac{5(y-2)}{3}$ ∈ R
∴ for any y ∈ co-domain R, there exist an element x = $\frac{5(y-2)}{3}$ ∈ domain R such that f(x) = y
∴ f is an onto function.
∴ f is one-one onto function.
∴ f^{-1} exists.
∴ $f^{-1}(y)=\frac{5(y-2)}{3}$
∴ $f^{-1}(x)=\frac{5(x-2)}{3}$

Question 3.
A function f is defined as follows:
f(x) = 4x + 5, for -4 ≤ x < 0. Find the values of f(-1), f(-2), f(0), if they exist.
Solution:
f(x) = 4x + 5, -4 ≤ x < 0
f(-1) = 4(-1) + 5 = -4 + 5 = 1
f(-2) = 4(-2) + 5 = -8 + 5 = -3
x = 0 ∉ domain of f
∴ f(0) does not exist.

Question 4.
A function f is defined as follows:
f(x) = 5 – x for 0 ≤ x ≤ 4. Find the value of x such that f(x) = 3.
Solution:
f(x) = 5 – x

f(x) = 3
∴ 5 – x = 3
∴ x = 5 – 3 = 2

Question 5.
If $f(x) = 3x^2 – 5x + 7$, find f(x – 1).
Solution:
$f(x) = 3x^2 – 5x + 7$
∴ $f(x – 1) = 3(x – 1)^2 – 5(x – 1) + 7$
$= 3(x^2 – 2x + 1) – 5(x – 1) + 7$
$= 3x^2 – 6x + 3 – 5x + 5 + 7$
$= 3x^2 – 11x + 15$

Question 6.
If f(x) = 3x + a and f(1) = 7, find a and f(4).
Solution:
f(x) = 3x + a,
f(1) = 7
∴ 3(1) + a = 7
∴ a = 7 – 3 = 4
∴ f(x) = 3x + 4
∴ f(4) = 3(4) + 4
= 12 + 4
= 16

Question 7.
If $f(x) = ax^2 + bx + 2$ and f(1) = 3, f(4) = 42, find a and b.
Solution:
$f(x) = ax^2 + bx + 2$
f(1) = 3
∴ $a(1)^2 + b(1) + 2 = 3$
∴ a + b = 1 …….(i)
f(4) = 42
∴ $a(4)^2 + b(4) + 2 = 42$
∴ 16a + 4b = 40
Dividing by 4, we get
4a + b = 10 ……….(ii)
Solving (i) and (ii), we get
a = 3, b = -2

Question 8.
If $f(x) = \frac{2x-1}{5x-2}, x \neq \frac{2}{5}$, verify whether (fof)(x) = x
Solution:
(fof)(x) = f(f(x))

$$= f\left(\frac{2x-1}{5x-2}\right)$$

$$= \frac{2\left(\frac{2x-1}{5x-2}\right)-1}{5\left(\frac{2x-1}{5x-2}\right)-2}$$

$$= \frac{4x-2-5x+2}{10x-5-10x+4} = \frac{-x}{-1} = x$$

Question 9.
If $f(x) = \frac{x+3}{4x-5}$, $g(x) = \frac{3+5x}{4x-1}$, then verify that (fog)(x) = x.

Solution:

$$f(x)=\frac{x+3}{4x-5},\ g(x)=\frac{3+5x}{4x-1}$$

$$\begin{aligned}(fog)(x) &= f(g(x))\\ &= f\left(\frac{3+5x}{4x-1}\right)\\ &= \frac{\frac{3+5x}{4x-1}+3}{4\left(\frac{3+5x}{4x-1}\right)-5}\\ &= \frac{3+5x+12x-3}{12+20x-20x+5} = \frac{17x}{17}=x\end{aligned}$$