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Maharashtra State Board 11th Commerce Maths Solutions Chapter 9 Differentiation Ex 9.1

I. Find the derivatives of the following functions w.r.t. x.

Question 1.
x^{12}
Solution:
Let y = x^{12}
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}x^{12} = 12x^{12-1} = 12x^{11}$$

Question 2.
x^{-9}
Solution:
Let y = x^{-9}
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}x^{-9} = -9x^{-9-1} = -9x^{-10}$$

Question 3.
$x^{\frac{3}{2}}$
Solution:
Let y = $x^{\frac{3}{2}}$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}x^{\frac{3}{2}} = \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$$

Question 4.
$7x\sqrt{x}$
Solution:
Let $y = 7x\sqrt{x}$

$= 7x^1x^{\frac{1}{2}}$

$y = 7x^{\frac{3}{2}}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}7x^{\frac{3}{2}} = 7 \times \frac{3}{2}x^{\frac{3}{2}-1} = \frac{21}{2}x^{\frac{1}{2}} = \frac{21}{2}\sqrt{x}$$

Question 5.
3^5
Solution:
Let y = 3^5

Differentiating w.r.t. x, we get

$\frac{dy}{dx}=\frac{d}{dx}\frac{3}{5}=0$ .....[$\frac{3}{5}$ is a constant]

II. Differentiate the following w.r.t. x.

Question 1.
$x^5 + 3x^4$
Solution:
Let $y = x^5 + 3x^4$
Differentiating w.r.t. x, we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(x^5+3x^4\right)\\ &= \frac{d}{dx}x^5+3\frac{d}{dx}x^4\\ &= 5x^4+3(4x^3)\\ \frac{dy}{dx} &= 5x^4+12x^3\end{aligned}$$

Question 2.
$x\sqrt{x} + \log x - e^x$
Solution:
Let $y = x\sqrt{x} + \log x - e^x$
$= x^{\frac{3}{2}}+\log x-e^x$
Differentiating w.r.t. x, we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(x^{\frac{3}{2}}+\log x-e^x\right)\\ &= \frac{d}{dx}x^{\frac{3}{2}}+\frac{d}{dx}\log x-\frac{d}{dx}e^x\\ &= \frac{3}{2}x^{\frac{3}{2}-1}+\frac{1}{x}-e^x\\ &= \frac{3}{2}x^{\frac{1}{2}}+\frac{1}{x}-e^x\\ &= \frac{3}{2}\sqrt{x}+\frac{1}{x}-e^x\end{aligned}$$

Question 3.
$x^{\frac{5}{2}}+5x^{\frac{7}{5}}$
Solution:
Let $y = x^{\frac{5}{2}}+5x^{\frac{7}{5}}$
Differentiating w.r.t. x, we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(x^{\frac{5}{2}}+5x^{\frac{7}{5}}\right)\\ &= \frac{d}{dx}x^{\frac{5}{2}}+5\frac{d}{dx}x^{\frac{7}{5}}\\ &= \frac{5}{2}x^{\frac{5}{2}-1}+5\frac{7}{5}x^{\frac{7}{5}-1}\\ &= \frac{5}{2}x^{\frac{3}{2}}+7x^{\frac{2}{5}}\end{aligned}$$

Question 4.
$\frac{2}{7}x^{\frac{7}{2}}+\frac{5}{2}x^{\frac{2}{5}}$
Solution:
Let $y = \frac{2}{7}x^{\frac{7}{2}}+\frac{5}{2}x^{\frac{2}{5}}$

Differentiating w.r.t. x, we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(\frac{2}{7}x^{\frac{7}{2}}+\frac{5}{2}x^{\frac{2}{5}}\right)\\ &= \frac{2}{7}\frac{d}{dx}x^{\frac{7}{2}}+\frac{5}{2}\frac{d}{dx}x^{\frac{2}{5}}\\ &= \frac{2}{7}\times\frac{7}{2}x^{\frac{7}{2}-1}+\frac{5}{2}\times\frac{2}{5}x^{\frac{2}{5}-1}\\ &= x^{\frac{5}{2}}+x^{\frac{-3}{5}}\end{aligned}$$

Question 5.

$\sqrt{x}(x^2+1)^2$

Solution:

Let y = $\sqrt{x}(x^2+1)^2$

$$y = x^{\frac{1}{2}}\left(x^4+2x^2+1\right)$$

$$y = x^{\frac{9}{2}}+2x^{\frac{5}{2}}+x^{\frac{1}{2}}$$

Differentiating w.r.t. x, we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(x^{\frac{9}{2}}+2x^{\frac{5}{2}}+x^{\frac{1}{2}}\right)\\ &= \frac{d}{dx}x^{\frac{9}{2}}+2\frac{d}{dx}x^{\frac{5}{2}}+\frac{d}{dx}\sqrt{x}\\ &= \frac{9}{2}x^{\frac{9}{2}-1}+2\times\frac{5}{2}x^{\frac{5}{2}-1}+\frac{1}{2\sqrt{x}}\\ &= \frac{9}{2}x^{\frac{7}{2}}+5x^{\frac{3}{2}}+\frac{1}{2\sqrt{x}}\end{aligned}$$

III. Differentiate the following w.r.t. x.

Question 1.

$x^3 \log x$

Solution:

Let y = $x^3 \log x$

Differentiating w.r.t. x, we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}x^3\log x\\ &= x^3\frac{d}{dx}(\log x)+(\log x)\frac{d}{dx}\left(x^3\right)\\ &= x^3\times\frac{1}{x}+(\log x)\left(3x^2\right)\\ &= x^2+3x^2\log x\end{aligned}$$

Question 2.

$x^{\frac{5}{2}}e^x$

Solution:

Let y = $x^{\frac{5}{2}}e^x$

Differentiating w.r.t. x, we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(x^{\frac{5}{2}}e^x\right)\\ &= x^{\frac{5}{2}}\frac{d}{dx}\left(e^x\right)+e^x\frac{d}{dx}\left(x^{\frac{5}{2}}\right)\\ &= x^{\frac{5}{2}}\left(e^x\right)+e^x\left(\frac{5}{2}x^{\frac{3}{2}}\right)\\ &= e^x\left(x^{\frac{5}{2}}+\frac{5}{2}x^{\frac{3}{2}}\right)\end{aligned}$$

Question 3.
$e^x \log x$
Solution:
Let $y = e^x \log x$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x \log x\right)$$
$$= e^x \frac{d}{dx}(\log x) + (\log x)\frac{d}{dx}\left(e^x\right)$$
$$= e^x\left(\frac{1}{x}\right) + (\log x)\left(e^x\right)$$
$$= e^x\left(\frac{1}{x} + \log x\right)$$

Question 4.
$x^3 . 3^x$
Solution:
Let $y = x^3 . 3^x$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3 3^x\right)$$
$$= x^3 \frac{d}{dx}\left(3^x\right) + 3^x \frac{d}{dx}\left(x^3\right)$$
$$= \left(x^3\right)\left(3^x \log 3\right) + 3^x\left(3x^2\right)$$
$$= x^2 3^x\left(x \log 3 + 3\right)$$

IV. Find the derivatives of the following w.r.t. x.

Question 1.
$\frac{x^2+a^2}{x^2-a^2}$
Solution:

Let $y = \frac{x^2+a^2}{x^2-a^2}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+a^2}{x^2-a^2}\right)$$
$$= \frac{\left(x^2-a^2\right)\frac{d}{dx}\left(x^2+a^2\right)-\left(x^2+a^2\right)\frac{d}{dx}\left(x^2-a^2\right)}{\left(x^2-a^2\right)^2}$$
$$= \frac{\left(x^2-a^2\right)\left(\frac{d}{dx}x^2+\frac{d}{dx}a^2\right)-\left(x^2+a^2\right)\left(\frac{d}{dx}x^2-\frac{d}{dx}a^2\right)}{\left(x^2-a^2\right)^2}$$
$$= \frac{\left(x^2-a^2\right)(2x+0)-\left(x^2+a^2\right)(2x-0)}{\left(x^2-a^2\right)^2}$$
$$= \frac{2x\left(x^2-a^2\right)-2x\left(x^2+a^2\right)}{\left(x^2-a^2\right)^2}$$
$$= \frac{2x\left(x^2-a^2-x^2-a^2\right)}{\left(x^2-a^2\right)^2}$$
$$= \frac{2x\left(-2a^2\right)}{\left(x^2-a^2\right)^2}$$
$$= \frac{-4xa^2}{\left(x^2-a^2\right)^2}$$

Question 2.
$\frac{3x^2+5}{2x^2-4}$
Solution:

Let $y=\frac{3x^2+5}{2x^2-4}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{3x^2+5}{2x^2-4}\right)$$

$$=\frac{(2x^2-4)\frac{d}{dx}(3x^2+5)-(3x^2+5)\frac{d}{dx}(2x^2-4)}{(2x^2-4)^2}$$

$$=\frac{(2x^2-4)\left(\frac{d}{dx}(3x^2)+\frac{d}{dx}5\right)-(3x^2+5)\left(\frac{d}{dx}(2x^2)-\frac{d}{dx}4\right)}{(2x^2-4)^2}$$

$$=\frac{(2x^2-4)(6x+0)-(3x^2+5)(4x-0)}{(2x^2-4)^2}$$

$$=\frac{6x(2x^2-4)-4x(3x^2+5)}{(2x^2-4)^2}$$

$$=\frac{2x\left[3(2x^2-4)-2(3x^2+5)\right]}{(2x^2-4)^2}$$

$$=\frac{2x(6x^2-12-6x^2-10)}{(2x^2-4)^2}$$

$$=\frac{2x(-22)}{(2x^2-4)^2}$$

$$=\frac{-44x}{(2x^2-4)^2}$$

Question 3.

logxx3−5

Solution:

Let $y=\frac{\log x}{x^3-5}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{\log x}{x^3-5}\right)$$

$$=\frac{(x^3-5)\frac{d}{dx}(\log x)-(\log x)\frac{d}{dx}(x^3-5)}{(x^3-5)^2}$$

$$=\frac{(x^3-5)\left(\frac{1}{x}\right)-\log x\left(\frac{d}{dx}(x^3)-\frac{d}{dx}(5)\right)}{(x^3-5)^2}$$

$$=\frac{(x^3-5)\frac{1}{x}-\log x(3x^2-0)}{(x^3-5)^2}$$

$$=\frac{(x^3-5)\frac{1}{x}-3x^2\log x}{(x^3-5)^2}$$

Question 4.

3ex−23ex+2

Solution:

$$\text{Let } y = \frac{3e^x - 2}{3e^x + 2}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3e^x - 2}{3e^x + 2}\right)$$

$$= \frac{(3e^x+2)\frac{d}{dx}(3e^x-2)-(3e^x-2)\frac{d}{dx}(3e^x+2)}{(3e^x+2)^2}$$

$$= \frac{(3e^x+2)\left(\frac{d}{dx}(3e^x)-\frac{d}{dx}(2)\right)-(3e^x-2)\left(\frac{d}{dx}(3e^x)+\frac{d}{dx}(2)\right)}{(3e^x+2)^2}$$

$$= \frac{(3e^x+2)(3e^x-0)-(3e^x-2)(3e^x+0)}{(3e^x+2)^2}$$

$$= \frac{3e^x(3e^x+2)-3e^x(3e^x-2)}{(3e^x+2)^2}$$

$$= \frac{3e^x(3e^x+2-3e^x+2)}{(3e^x+2)^2}$$

$$= \frac{3e^x(4)}{(3e^x+2)^2}$$

$$= \frac{12e^x}{(3e^x+2)^2}$$

Question 5.
$\frac{xe^x}{x+e^x}$
Solution:

$$\text{Let } y = \frac{xe^x}{x+e^x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{xe^x}{x+e^x}\right)$$

$$= \frac{(x+e^x)\frac{d}{dx}(xe^x)-(xe^x)\frac{d}{dx}(x+e^x)}{(x+e^x)^2}$$

$$= \frac{(x+e^x)\left[x\frac{d}{dx}(e^x)+e^x\frac{d}{dx}(x)\right]-xe^x\left(\frac{d}{dx}(x)+\frac{d}{dx}(e^x)\right)}{(x+e^x)^2}$$

$$= \frac{(x+e^x)\left[xe^x+e^x(1)\right]-xe^x(1+e^x)}{(x+e^x)^2}$$

$$= \frac{(x+e^x)(xe^x+e^x)-xe^x(1+e^x)}{(x+e^x)^2}$$

$$= \frac{(x+e^x)e^x(x+1)-xe^x(1+e^x)}{(x+e^x)^2}$$

$$= \frac{e^x\left[(x+e^x)(x+1)-x(1+e^x)\right]}{(x+e^x)^2}$$

V. Find the derivatives of the following functions by the first principle:

Question 1.
$3x^2 + 4$
Solution:
Let $f(x) = 3x^2 + 4$
$\therefore f(x + h) = 3(x + h)^2 + 4$
$= 3(x^2 + 2xh + h^2) + 4$
$= 3x^2 + 6xh + 3h^2 + 4$

By first principle, we get

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{(3x^2+6xh+3h^2+4)-(3x^2+4)}{h}$$
$$= \lim_{h\to 0} \frac{3h^2+6xh}{h}$$
$$= \lim_{h\to 0} \frac{h(3h+6x)}{h}$$
$$= \lim_{h\to 0} (6x+3h) \quad \ldots[\because h\to 0, \therefore h \neq 0]$$
$$= 6x + 3(0)$$
$$= 6x$$

Question 2.
x√x
Solution:
Let f(x) = x√x
∴ f(x + h) = $(x+h)^{\frac{3}{2}}$
By first principle, we get

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{(x+h)^{3/2}-x^{3/2}}{h}$$
$$= \lim_{h\to 0} \frac{\left[(x+h)^{3/2}-x^{3/2}\right]\left[(x+h)^{3/2}+x^{3/2}\right]}{h\left[(x+h)^{3/2}+x^{3/2}\right]}$$
$$= \lim_{h\to 0} \frac{(x+h)^3-x^3}{h\left[(x+h)^{3/2}+x^{3/2}\right]}$$
$$= \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h\left[(x+h)^{3/2}+x^{3/2}\right]}$$
$$= \lim_{h\to 0} \frac{h(3x^2+3xh+h^2)}{h\left[(x+h)^{3/2}+x^{3/2}\right]}$$
$$= \lim_{h\to 0} \frac{3x^2+3xh+h^2}{(x+h)^{3/2}+x^{3/2}} \quad \ldots[\because h\to 0, \therefore h \neq 0]$$
$$= \frac{3x^2+3x\times 0+0^2}{(x+0)^{3/2}+x^{3/2}}$$
$$= \frac{3x^2}{2x^{3/2}}$$
$$= \frac{3}{2}x^{\frac{1}{2}}$$
$$= \frac{3}{2}\sqrt{x}$$

Question 3.
$\frac{1}{2x+3}$
Solution:
Let f(x) = $\frac{1}{2x+3}$
∴ f(x + h) = $\frac{1}{2(x+h)+3} = \frac{1}{2x+2h+3}$
By first principle, we get

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2x+2h+3} - \frac{1}{2x+3}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{2x+3-2x-2h-3}{(2x+2h+3)(2x+3)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{-2h}{(2x+2h+3)(2x+3)}\right]$$

$$= \lim_{h\to 0} \frac{-2}{(2x+2h+3)(2x+3)}$$

$$\ldots[\because h \to 0, \therefore h \neq 0]$$

$$= \frac{-2}{(2x+2\times 0+3)(2x+3)}$$

$$= \frac{-2}{(2x+3)^2}$$

Question 4.

x-12x+7

Solution:

Let f(x) = x-12x+7

∴ f(x + h) = x+h-12(x+h)+7=x+h-12x+2 h+7

By first principle, we get

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x+h-1}{2x+2h+7} - \frac{x-1}{2x+7}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{(x+h-1)(2x+7)-(x-1)(2x+2h+7)}{(2x+2h+7)(2x+7)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\begin{pmatrix}2x^2+2xh-2x+7x+7h-7 \\ -2x^2-2xh-7x+2x+2h+7\end{pmatrix}}{(2x+2h+7)(2x+7)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{9h}{(2x+2h+7)(2x+7)}\right]$$

$$= \lim_{h\to 0} \frac{9}{(2x+2h+7)(2x+7)}$$

$$\ldots[\because h \to 0, \therefore h \neq 0]$$

$$= \frac{9}{(2x+2\times 0+7)(2x+7)}$$

$$= \frac{9}{(2x+7)^2}$$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 9 Differentiation Ex 9.2

I. Differentiate the following functions w.r.t. x.

Question 1.
$\frac{x}{x+1}$
Solution:
Let $y = \frac{x}{x+1}$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{x+1}\right)$$

$$= \frac{(x+1)\frac{d}{dx}(x) - x\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{(x+1)(1) - x(1+0)}{(x+1)^2}$$

$$= \frac{x+1-x}{(x+1)^2}$$

$$= \frac{1}{(x+1)^2}$$

Question 2.
$\frac{x^2+1}{x}$
Solution:
Let $y = \frac{x^2+1}{x}$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x}\right)$$

$$= \frac{x\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x(2x+0) - (x^2+1)(1)}{x^2}$$

$$= \frac{2x^2 - x^2 - 1}{x^2}$$

$$\frac{dy}{dx} = \frac{x^2-1}{x^2}$$

Question 3.
$\frac{1}{e^x+1}$

Solution:

Let $y = \frac{1}{e^x+1}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{e^x+1}\right)$$

$$= \frac{\left(e^x+1\right)\frac{d}{dx}(1) - (1)\frac{d}{dx}\left(e^x+1\right)}{\left(e^x+1\right)^2}$$

$$= \frac{\left(e^x+1\right)(0) - (1)\left(e^x+0\right)}{\left(e^x+1\right)^2}$$

$$= \frac{e^x+1-e^x}{\left(e^x+1\right)^2}$$

$$= \frac{1}{\left(e^x+1\right)^2}$$

Question 4.

exex+1

Solution:

$y = \frac{e^x}{e^x+1}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x}{e^x+1}\right)$$

$$= \frac{\left(e^x+1\right)\frac{d}{dx}\left(e^x\right) - e^x\frac{d}{dx}\left(e^x+1\right)}{\left(e^x+1\right)^2}$$

$$= \frac{\left(e^x+1\right)e^x - e^x\left(e^x+0\right)}{\left(e^x+1\right)^2}$$

$$= \frac{e^x\left(e^x+1-e^x\right)}{\left(e^x+1\right)^2}$$

$$= \frac{e^x}{\left(e^x+1\right)^2}$$

Question 5.

xlogx

Solution:

Let $y = \frac{x}{\log x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{\log x}\right)$$

$$= \frac{\log x\frac{d}{dx}(x) - x\frac{d}{dx}(\log x)}{(\log x)^2}$$

$$= \frac{\log x(1) - x\left(\frac{1}{x}\right)}{(\log x)^2}$$

$$= \frac{\log x - 1}{(\log x)^2}$$

Question 6.

2xlogx

Solution:

Let $y = \frac{2^x}{\log x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2^x}{\log x}\right)$$

$$= \frac{\log x \frac{d}{dx}(2^x) - 2^x \frac{u}{dx}(\log x)}{(\log x)^2}$$

$$= \frac{\log x\left(2^x \log 2\right) - 2^x\left(\frac{1}{x}\right)}{(\log x)^2}$$

$$= \frac{\left(2^x \log x \cdot \log 2\right)\left(-\frac{1}{x}\right)}{(\log x)^2}$$

Question 7.

$(2e^x-1)(2e^x+1)$

Solution:

Let $y = \frac{2e^x-1}{2e^x+1}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2e^x-1}{2e^x+1}\right)$$

$$= \frac{(2e^x+1)\frac{d}{dx}(2e^x-1) - (2e^x-1)\frac{d}{dx}(2e^x+1)}{(2e^x+1)^2}$$

$$= \frac{(2e^x+1)(2e^x-0) - (2e^x-1)(2e^x+0)}{(2e^x+1)^2}$$

$$= \frac{(2e^x+1)(2e^x) - (2e^x-1)(2e^x)}{(2e^x+1)^2}$$

$$= \frac{2e^x(2e^x+1-2e^x+1)}{(2e^x+1)^2}$$

$$= \frac{2e^x(2)}{(2e^x+1)^2}$$

$$= \frac{4e^x}{(2e^x+1)^2}$$

Question 8.

$(x+1)(x-1)(e^x+1)$

Solution:

Let $y = \frac{(x+1)(x-1)}{(e^x+1)}$

$y = \frac{x^2-1}{(e^x+1)}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2-1}{e^x+1}\right)$$

$$= \frac{(e^x+1)\frac{d}{dx}(x^2-1)-(x^2-1)\frac{d}{dx}(e^x+1)}{(e^x+1)^2}$$

$$= \frac{(e^x+1)(2x)-(x^2-1)(e^x+0)}{(e^x+1)^2}$$

$$= \frac{2xe^x+2x-x^2e^x+e^x}{(e^x+1)^2}$$

$$= \frac{2xe^x+e^x-x^2e^x+2x}{(e^x+1)^2}$$

$$= \frac{e^x(2x+1-x^2)+2x}{(e^x+1)^2}$$

II. Solve the following examples:

Question 1.
The demand D for a price P is given as D = $\frac{27}{P}$, find the rate of change of demand when the price is 3.
Solution:
Demand, D = $\frac{27}{P}$
Rate of change of demand = $\frac{dD}{dP}$

$$= \frac{d}{dP}\left(\frac{27}{P}\right)$$

$$= 27\frac{d}{dP}\left(\frac{1}{P}\right)$$

$$= 27\frac{d}{dP}(P^{-1})$$

$$= 27((-1)P^{-2})$$

$$= 27\left(\frac{-1}{P^2}\right) = \frac{-27}{P^2}$$

When price P = 3,
Rate of change of demand,
$\left(\frac{dD}{dP}\right)_{P=3} = \frac{-27}{(3)^2} = -3$
∴ When price is 3, Rate of change of demand is -3.

Question 2.
If for a commodity; the price-demand relation is given as D = $\frac{P+5}{P-1}$. Find the marginal demand when the price is 2.

Solution:

Given, $D = \frac{P+5}{P-1}$

Marginal demand $= \frac{dD}{dP} = \frac{d}{dP}\left(\frac{P+5}{P-1}\right)$

$= \frac{(P-1)\frac{d}{dP}(P+5)-(P+5)\frac{d}{dP}(P-1)}{(P-1)^2}$

$= \frac{(P-1)(1+0)-(P+5)(1-0)}{(P-1)^2}$

$= \frac{P-1-P-5}{(P-1)^2} = \frac{-6}{(P-1)^2}$

When P = 2,

Marginal demand, $\left(\frac{dD}{dP}\right)_{P=2} = \frac{-6}{(2-1)^2} = -6$

When price is 2, marginal demand is −6.

Question 3.
The demand function of a commodity is given as $P = 20 + D - D^2$. Find the rate at which price is changing when demand is 3.
Solution:
Given, $P = 20 + D - D^2$
Rate of change of price = $\frac{dP}{dD}$
$= \frac{d}{dD}(20 + D - D^2)$
$= 0 + 1 - 2D$
$= 1 - 2D$
Rate of change of price at D = 3 is
$\left(\frac{dP}{dD}\right)_{D=3} = 1 - 2(3) = -5$
∴ Price is changing at a rate of -5, when demand is 3.

Question 4.
If the total cost function is given by; $C = 5x^3 + 7x^2 + 7$; find the average cost and the marginal cost when x = 4.
Solution:
Total cost function, $C = 5x^3 + 7x^2 + 7$
Average cost = $\frac{C}{x}$

$= \frac{5x^3+2x^2+7}{x}$

$= 5x^2 + 2x + \frac{7}{x}$

When $x = 4$,

Average cost $= 5(4)^2 + 2(4) + \frac{7}{4}$

$= 80 + 8 + \frac{7}{4}$

$= \frac{320+32+7}{4}$

$= \frac{359}{4}$

Marginal cost $= \frac{dC}{dx}$

$= \frac{d}{dx}(5x^3 + 2x^2 + 7)$

$= 5\frac{d}{dx}(x^3) + 2\frac{d}{dx}(x^2) + \frac{d}{dx}(7)$

$= 5(3x^2) + 2(2x) + 0$

$= 15x^2 + 4x$

When x = 4, Marginal cost = $\left(\frac{dC}{dx}\right)_{x=4}$
$= 15(4)^2 + 4(4)$
$= 240 + 16$
$= 256$
∴ the average cost and marginal cost at x = 4 are $\frac{359}{4}$ and 256 respectively.

Question 5.
The total cost function of producing n notebooks is given by
C = 1500 – 75n + 2n2 + n35
Find the marginal cost at n = 10.
Solution:
The total cost function,

$$C = 1500 - 75n + 2n^2 + \frac{n^3}{5}$$

$$\text{Marginal Cost} = \frac{dC}{dn}$$

$$= \frac{d}{dn}\left(1500 - 75n + 2n^2 + \frac{n^3}{5}\right)$$

$$= \frac{d}{dn}(1500) - 75\frac{d}{dn}(n) + 2\frac{d}{dn}(n^2) + \frac{1}{5}\frac{d}{dn}(n^3)$$

$$= 0 - 75(1) + 2(2n) + \frac{1}{5}(3n^2)$$

$$= -75 + 4n + \frac{3n^2}{5}$$

When n = 10,
Marginal cost

$$= \left(\frac{dC}{dn}\right)_{n=10} = -75 + 4(10) + \frac{3}{5}(10)^2$$

$$= -75 + 40 + 60$$

$$= 25$$

∴ Marginal cost at n = 10 is 25.

Question 6.
The total cost of 't' toy cars is given by C = 5(2t) + 17. Find the marginal cost and average cost at t = 3.
Solution:
Total cost of 't' toy cars, C = 5(2t) + 17

$$\text{Marginal Cost} = \frac{dC}{dt}$$

$$= \frac{d}{dt}[5(2^t) + 17]$$

$$= 5\frac{d}{dt}(2^t) + \frac{d}{dt}(17)$$

$$= 5(2^t \cdot \log 2) + 0$$

$$= 5(2^t \cdot \log 2)$$

When t = 3,

$$\text{Marginal cost} = \left(\frac{dC}{dt}\right)_{t=3}$$

$$= 5(2^3 . \log 2) = 40 \log 2$$

$$\text{Average cost} = \frac{C}{t} = \frac{5(2^t) + 17}{t}$$

$$\text{When } t = 3, \text{ average cost} = \frac{5(2^3) + 17}{3}$$

$$= \frac{40 + 17}{3} = 19$$

∴ at t = 3, the Marginal cost is 40 log 2 and the Average cost is 19.

Question 7.
If for a commodity; the demand function is given by, D = 75−3P−−−−−−−√. Find the marginal demand function when P = 5.
Solution:
Demand function, D = 75−3P−−−−−−−√
Now, Marginal demand = dDdP

$= \frac{d}{dP}\left(\sqrt{75-3P}\right)$

$= \frac{1}{2\sqrt{75-3P}} \cdot \frac{d}{dP}(75-3P)$

$= \frac{1}{2\sqrt{75-3P}} \cdot (0 - 3 \times 1)$

$= \frac{-3}{2\sqrt{75-3P}}$

When P = 5,

Marginal demand $= \left(\frac{dD}{dP}\right)_{P=5}$

$= \frac{-3}{2\sqrt{75-3(5)}}$

$= \frac{-3}{2\sqrt{60}} = \frac{-3}{4\sqrt{15}}$

$\therefore$ Marginal demand $= \frac{-3}{4\sqrt{15}}$ at P = 5.

Question 8.
The total cost of producing x units is given by C = $10e^{2x}$, find its marginal cost and average cost when x = 2.
Solution:

Total cost, $C = 10e^{2x}$

Marginal cost $= \frac{dC}{dx}$

$= \frac{d}{dx}(10e^{2x}) = 10\frac{d}{dx}(e^{2x})$

$= 10 \cdot e^{2x} \cdot \frac{d}{dx}(2x) = 10 \cdot e^{2x} \cdot 2(1)$

$= 20e^{2x}$

When $x = 2$,

Marginal cost $= \left(\frac{dC}{dx}\right)_{x=2} = 20e^4$

Average cost $= \frac{C}{x} = \frac{10e^{2x}}{x}$

When $x = 2$ average cost $= \frac{10e^4}{2} = 5e^4$

When $x = 2$, marginal cost is $20e^4$ and average cost is $5e^4$.

Question 9.
The demand function is given as P = 175 + 9D + $25D^2$. Find the revenue, average revenue, and marginal revenue when demand is 10.
Solution:
Given, P = 175 + 9D + $25D^2$
Total revenue, R = P.D
= (175 + 9D + $25D^2$)D
= 175D + $9D^2$ + $25D^3$
Average revenue = P = 175 + 9D + $25D^2$
Marginal revenue = $\frac{dR}{dD}$
= $\frac{d}{dD}$ (175D + $9D^2$ + $25D^3$)
= 175 $\frac{d}{dD}$ (D) + 9 $\frac{d}{dD}$ (D^2) + 25 $\frac{d}{dD}$ (D^3)
= 175(1) + 9(2D) + 25($3D^2$)
= 175 + 18D + $75D^2$
When D = 10,
Total revenue = 175(10) + $9(10)^2$ + $25(10)^3$
= 1750 + 900 + 25000
= 27650
Average revenue = 175 + 9(10) + $25(10)^2$
= 175 + 90 + 2500
= 2765
Marginal revenue = 175 + 18(10) + $75(10)^2$
= 175 + 180 + 7500
= 7855
∴ When Demand = 10,
Total revenue = 27650, Average revenue = 2765, Marginal revenue = 7855.

Question 10.
The supply S for a commodity at price P is given by S = P^2 + 9P – 2. Find the marginal supply when the price is 7.
Solution:
Given, S = P^2 + 9P – 2

$$\text{Marginal supply} = \frac{dS}{dP}$$
$$= \frac{d}{dP}\left(P^2 + 9P - 2\right)$$
$$= \frac{d}{dP}(P^2) + 9\frac{d}{dP}(P) - \frac{d}{dP}(2)$$
$$= 2P + 9(1) - 0$$
$$= 2P + 9$$

When P = 7,

$$\text{Marginal supply} = \left(\frac{dS}{dP}\right)_{P=7} = 2(7) + 9$$
$$= 14 + 9 = 23$$

∴ The marginal supply is 23, at P = 7.

Question 11.
The cost of producing x articles is given by C = x^2 + 15x + 81. Find the average cost and marginal cost functions. Find marginal cost when x = 10. Find x for which the marginal cost equals the average cost.
Solution:
Given, cost C = x^2 + 15x + 81

$$\text{Average cost} = \frac{C}{x} = \frac{x^2 + 15x + 81}{x}$$
$$= x + 15 + \frac{81}{x}$$

$$\text{and Marginal cost} = \frac{dC}{dx}$$
$$= \frac{d}{dx}(x^2 + 15x + 81)$$
$$= \frac{d}{dx}(x^2) + 15\frac{d}{dx}(x) + \frac{d}{dx}(81)$$
$$= 2x + 15(1) + 0 = 2x + 15$$

When $x = 10$,

$$\text{Marginal cost} = \left(\frac{dC}{dx}\right)_{x=10} = 2(10) + 15 = 35$$

If marginal cost = average cost, then
2x + 15 = x + 15 + $\frac{81}{x}$
∴ x = $\frac{81}{x}$
∴ x^2 = 81
∴ x = 9 …..[∵ x > 0]

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Maharashtra State Board 11th Commerce Maths Solutions Chapter 9 Differentiation Miscellaneous Exercise 9

I. Differentiate the following functions w.r.t.x.

Question 1.
x^5
Solution:
Let y = x^5
Differentiating w.r.t. x, we get
$\frac{dy}{dx}=\frac{d}{dx}x^5=5x^4$

Question 2.
x^{-2}
Solution:
Let y = x^{-2}
Differentiating w.r.t. x, we get
$\frac{dy}{dx}=\frac{d}{dx}(x^{-2})=-2x^{-3}=\frac{-2}{x^3}$

Question 3.
$\sqrt{x}$
Solution:
Let y = $\sqrt{x}$
Differentiating w.r.t. x, we get
$\frac{dy}{dx}=\frac{d}{dx}\sqrt{x}=\frac{1}{2\sqrt{x}}$

Question 4.
$x\sqrt{x}$
Solution:
Let y = $x\sqrt{x}$
∴ y = $x^{\frac{3}{2}}$
Differentiating w.r.t. x, we get
$\frac{dy}{dx}=\frac{d}{dx}x^{\frac{3}{2}}=\frac{3}{2}x^{\frac{1}{2}}$

Question 5.
$\frac{1}{\sqrt{x}}$
Solution:
Let y = $\frac{1}{\sqrt{x}}$
∴ y = $x^{-\frac{1}{2}}$
Differentiating w.r.t. x, we get
$\frac{dy}{dx}=-\frac{1}{2}x^{-\frac{3}{2}}=\frac{-1}{2x^{\frac{3}{2}}}$

Question 6.
7^x
Solution:
Let y = 7^x
Differentiating w.r.t. x, we get
$\frac{dy}{dx}=\frac{d}{dx}7^x=7^x\log 7$

II. Find $\frac{dy}{dx}$ if

Question 1.
y = $x^2 + \frac{1}{x^2}$

Solution:

$$y = x^2 + \frac{1}{x^2}$$

$$\therefore \quad y = x^2 + x^{-2}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + x^{-2})$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(x^{-2})$$

$$= 2x - 2x^{-3}$$

$$= 2x - \frac{2}{x^3}$$

Question 2.
y = (√x + 1)2
Solution:

$$y = \left(\sqrt{x} + 1\right)^2$$

$$\therefore \quad y = x + 2\sqrt{x} + 1$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x + 2\sqrt{x} + 1)$$

$$= \frac{d}{dx}(x) + 2\frac{d}{dx}\left(\sqrt{x}\right) + \frac{d}{dx}(1)$$

$$= 1 + 2\left(\frac{1}{2\sqrt{x}}\right) + 0$$

$$\frac{dy}{dx} = 1 + \frac{1}{\sqrt{x}}$$

Question 3.
y = (x−−√+1x√)2
Solution:

$$y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$$

$$\therefore \quad y = x + 2 + \frac{1}{x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x + 2 + \frac{1}{x}\right)$$

$$= \frac{d}{dx}(x) + \frac{d}{dx}(2) + \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$= 1 + 0 + \frac{d}{dx}(x^{-1})$$

$$= 1 + (-1)x^{-2}$$

$$= 1 - \frac{1}{x^2}$$

Question 4.
y = x3 – 2x2 + √x + 1

Solution:

$$y = x^3 - 2x^2 + \sqrt{x} + 1$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 - 2x^2 + \sqrt{x} + 1)$$

$$= \frac{d}{dx}(x^3) - 2\frac{d}{dx}(x^2) + \frac{d}{dx}\left(\sqrt{x}\right) + \frac{d}{dx}(1)$$

$$= 3x^2 - 2(2x) + \frac{d}{dx}\left(x^{\frac{1}{2}}\right) + 0$$

$$= 3x^2 - 4x + \frac{1}{2}x^{\frac{1}{2}-1}$$

$$= 3x^2 - 4x + \frac{1}{2}x^{\frac{-1}{2}}$$

$$\frac{dy}{dx} = 3x^2 - 4x + \frac{1}{2\sqrt{x}}$$

Question 5.

y = x2 + 2x – 1

Solution:

$$y = x^2 + 2^x - 1$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + 2^x - 1)$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(2^x) - \frac{d}{dx}(1)$$

$$= 2x + 2^x \log 2 - 0$$

$$= 2x + 2^x \log 2$$

Question 6.

y = (1 – x)(2 – x)

Solution:

$$y = (1 - x)(2 - x)$$

$$= 2 - 3x + x^2$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(2 - 3x + x^2)$$

$$= \frac{d}{dx}(2) - 3\frac{d}{dx}(x) + \frac{d}{dx}(x^2)$$

$$= 0 - 3(1) + 2x$$

$$= -3 + 2x$$

Question 7.

y = 1+x2+x

Solution:

$$y = \frac{1+x}{2+x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+x}{2+x}\right)$$

$$= \frac{(2+x)\frac{d}{dx}(1+x) - (1+x)\frac{d}{dx}(2+x)}{(2+x)^2}$$

$$= \frac{(2+x)(0+1) - (1+x)(0+1)}{(2+x)^2}$$

$$\frac{dy}{dx} = \frac{(2+x) - (1+x)}{(2+x)^2} = \frac{2+x-1-x}{(2+x)^2} = \frac{1}{(2+x)^2}$$

Question 8.

y = (logx+1)x

Solution:

$$y=\frac{(\log x+1)}{x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left[\frac{\log x+1}{x}\right]$$

$$=\frac{x\frac{d}{dx}(\log x+1)-(\log x+1)\frac{d}{dx}(x)}{x^2}$$

$$=\frac{x\left(\frac{1}{x}+0\right)-(\log x+1)(1)}{x^2}$$

$$=\frac{1-\log x-1}{x^2}$$

$$=\frac{-\log x}{x^2}$$

Question 9.
y = $\frac{e^x}{\log x}$
Solution:

$$y=\frac{e^x}{\log x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{e^x}{\log x}\right)$$

$$=\frac{(\log x)\frac{d}{dx}(e^x)-(e^x)\frac{d}{dx}(\log x)}{(\log x)^2}$$

$$=\frac{(\log x)e^x-e^x\left(\frac{1}{x}\right)}{(\log x)^2}$$

$$=\frac{e^x\left(\log x-\frac{1}{x}\right)}{(\log x)^2}$$

Question 10.
y = x log x (x^2 + 1)
Solution:

$$y=x\log x\,(x^2+1)$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\,(x)(\log x)(x^2+1)$$

$$=(x)(\log x)\frac{d}{dx}\,(x^2+1)$$

$$-(x^2+1)\,\frac{d}{dx}((x)(\log x))$$

$$=(x\log x)(2x+0$$

$$+(x^2+1)\left[x\frac{d}{dx}(\log x)+(\log x)\frac{d}{dx}(x)\right]$$

$$=2x^2\log x+(x^2+1)\left[x\times\frac{1}{x}+(\log x)(1)\right]$$

$$=2x^2\log x+(x^2+1)(1+\log x)$$

$$=2x^2\log x+(x^2+1)+(x^2+1)\log x$$

III. Solve the following:

Question 1.
The relation between price (P) and demand (D) of a cup of Tea is given by D = $\frac{12}{P}$. Find the rate at which the demand changes when the price is ₹ 2/-. Interpret the result.
Solution:
Demand, D = $\frac{12}{P}$
Rate of change of demand

$= \frac{dD}{dP}$

$= \frac{d}{dP}\left(\frac{12}{P}\right)$

$= 12\frac{d}{dP}\left(P^{-1}\right)$

$= 12\,((-1)P^{-2})$

$= 12\left(\frac{-1}{P^2}\right) = \frac{-12}{P^2}$

When price P = 2,
Rate of change of demand,

$\left(\frac{dD}{dP}\right)_{P=2} = \frac{-12}{(2)^2} = -3$

∴ When the price is 2, the rate of change of demand is -3.
∴ Here, the rate of change of demand is negative demand would fall when the price becomes ₹ 2.

Question 2.
The demand (D) of biscuits at price P is given by D = $\frac{64}{P^3}$, find the marginal demand when the price is ₹ 4/-.
Solution:
Given demand D = $\frac{64}{P^3}$
Now, marginal demand

$= \frac{dD}{dP}$

$= \frac{d}{dP}\left(\frac{64}{P^3}\right)$

$= 64\ \frac{d}{dP}\left(P^{-3}\right)$

$= 64\,(-3)\,P^{-4}$

$= \frac{-192}{P^4}$

When P = 4
Marginal demand

$= \left(\frac{dD}{dP}\right)_{P=4}$

$= \frac{-192}{(4)^4}$

$= \frac{-192}{256}$

$= \frac{-3}{4}$

Question 3.
The supply S of electric bulbs at price P is given by S = $2p^3 + 5$. Find the marginal supply when the price is ₹ 5/-. Interpret the result.
Solution:
Given, supply S = $2p^3 + 5$
Now, marginal supply

$= \frac{dS}{dp}$

$= \frac{d}{dp}\ (2p^3 + 5)$

$= 2\ \frac{d}{dp}(p^3) + \frac{d}{dp}(5)$

$= 2(3p^2) + 0$

$= 6p^2$

∴ When p = 5
Marginal supply = $\left(\frac{dS}{dp}\right)_{p=5}$
= $6(5)^2$
= 150
Here, the rate of change of supply with respect to the price is positive which indicates that the supply increases.

Question 4.
The total cost of producing x items is given by C = $x^2 + 4x + 4$. Find the average cost and the marginal cost. What is the marginal cost

when x = 7?
Solution:
Total cost C = $x^2 + 4x + 4$
Now. Average cost = $\frac{C}{x} = \frac{x^2+4x+4}{x}$
= $x + 4 + \frac{4}{x}$
and Marginal cost = $\frac{dC}{dx} = \frac{d}{dx}(x^2 + 4x + 4)$
= $\frac{d}{dx}(x^2) + 4\frac{d}{dx}(x) + \frac{d}{dx}(4)$
= 2x + 4(1) + 0
= 2x + 4
∴ When x = 7,
Marginal cost = $\left(\frac{dC}{dx}\right)_{x=7}$
= 2(7) + 4
= 14 + 4
= 18

Question 5.
The demand D for a price P is given as D = $\frac{27}{P}$, find the rate of change of demand when the price is ₹ 3/-.
Solution:
Demand, D = $\frac{27}{P}$
Rate of change of demand = $\frac{dD}{dP}$

$$= \frac{d}{dP}\left(\frac{27}{P}\right)$$
$$= 27\frac{d}{dP}\left(\frac{1}{P}\right)$$
$$= 27\frac{d}{dP}\left(P^{-1}\right)$$
$$= 27\left((-1)P^{-2}\right)$$
$$= 27\left(\frac{-1}{P^2}\right) = \frac{-27}{P^2}$$

When price P = 3,
Rate of change of demand,
$\left(\frac{dD}{dP}\right)_{P=3} = \frac{-27}{(3)^2} = -3$
∴ When price is 3, Rate of change of demand is -3.

Question 6.
If for a commodity; the price demand relation is given as D = $\left(\frac{P+5}{P-1}\right)$. Find the marginal demand when price is ₹ 2/-
Solution:

Given, $D = \frac{P+5}{P-1}$

Marginal demand = $\frac{dD}{dP} = \frac{d}{dP}\left(\frac{P+5}{P-1}\right)$

$$= \frac{(P-1)\frac{d}{dP}(P+5)-(P+5)\frac{d}{dP}(P-1)}{(P-1)^2}$$
$$= \frac{(P-1)(1+0)-(P+5)(1-0)}{(P-1)^2}$$
$$= \frac{P-1-P-5}{(P-1)^2} = \frac{-6}{(P-1)^2}$$

When P = 2,

Marginal demand, $\left(\frac{dD}{dP}\right)_{P=2} = \frac{-6}{(2-1)^2} = -6$

When price is 2, marginal demand is −6.

Question 7.
The price function P of a commodity is given as P = 20 + D – D^2 where D is demand. Find the rate at which price (P) is changing when demand D = 3.
Solution:
Given, P = 20 + D – D^2
Rate of change of price = $\frac{dP}{dD}$
= $\frac{d}{dD}(20 + D - D^2)$
= 0 + 1 – 2D

$= 1 - 2D$

Rate of change of price at D = 3 is

$\left(\frac{dP}{dD}\right)_{D=3} = 1 - 2(3) = -5$

∴ Price is changing at a rate of -5, when demand is 3.

Question 8.
If the total cost function is given by $C = 5x^3 + 2x^2 + 1$; find the average cost and the marginal cost when x = 4.
Solution:
Total cost function $C = 5x^3 + 2x^2 + 1$

Average cost = $\frac{C}{x}$

$= \frac{5x^3+2x^2+1}{x}$

$= 5x^2 + 2x + \frac{1}{x}$

When x = 4,

Average cost = $5(4)^2 + 2(4) + \frac{1}{4}$

$= 80 + 8 + \frac{1}{4}$

$= \frac{320+32+1}{4}$

$= \frac{353}{4}$

Marginal cost = $\frac{dC}{dx}$

$= \frac{d}{dx}(5x^3 + 2x^2 + 1)$

$= 5\frac{d}{dx}(x^3) + 2\frac{d}{dx}(x^2) + \frac{d}{dx}(1)$

$= 5(3x^2) + 2(2x) + 0$

$= 15x^2 + 4x$

When x = 4, marginal cost = $\left(\frac{dC}{dx}\right)_{x=4}$

$= 15(4)^2 + 4(4)$

$= 240 + 16$

$= 256$

∴ The average cost and marginal cost at x = 4 are $\frac{353}{4}$ and 256 respectively.

Question 9.
The supply S for a commodity at price P is given by $S = P^2 + 9P - 2$. Find the marginal supply when the price is 7/-.
Solution:
Given, $S = P^2 + 9P - 2$

$$\text{Marginal supply} = \frac{dS}{dP}$$

$$= \frac{d}{dP}\left(P^2 + 9P - 2\right)$$

$$= \frac{d}{dP}(P^2) + 9\frac{d}{dP}(P) - \frac{d}{dP}(2)$$

$$= 2P + 9(1) - 0$$

$$= 2P + 9$$

When P = 7,

$$\text{Marginal supply} = \left(\frac{dS}{dP}\right)_{P=7} = 2(7) + 9$$

$$= 14 + 9 = 23$$

∴ The marginal supply is 23, at P = 7.

Question 10.
The cost of producing x articles is given by $C = x^2 + 15x + 81$. Find the average cost and marginal cost functions. Find the marginal cost when x = 10. Find x for which the marginal cost equals the average cost.
Solution:
Given, cost $C = x^2 + 15x + 81$

$$\text{Average cost} = \frac{C}{x} = \frac{x^2 + 15x + 81}{x}$$

$$= x + 15 + \frac{81}{x}$$

$$\text{and Marginal cost} = \frac{dC}{dx}$$

$$= \frac{d}{dx}(x^2 + 15x + 81)$$

$$= \frac{d}{dx}(x^2) + 15\frac{d}{dx}(x) + \frac{d}{dx}(81)$$

$$= 2x + 15(1) + 0 = 2x + 15$$

When $x = 10$,

$$\text{Marginal cost} = \left(\frac{dC}{dx}\right)_{x=10} = 2(10) + 15 = 35$$

If marginal cost = average cost, then

$2x + 15 = x + 15 + \frac{81}{x}$

$\therefore x = \frac{81}{x}$

$\therefore x^2 = 81$

$\therefore x = 9$ .....[$\because x > 0$]