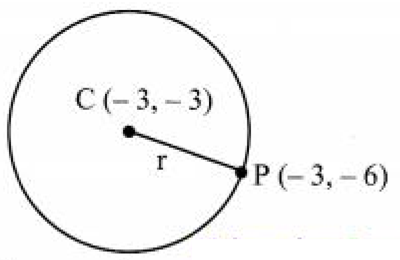
Maharashtra State Board 11th Maths Solutions Chapter 6 Circle Ex 6.1

Question 1.
Find the equation of a circle with
(i) centre at origin and radius 4.
(ii) centre at (-3, -2) and radius 6.
(iii) centre at (2, -3) and radius 5.
(iv) centre at (-3, -3) passing through point (-3, -6).
Solution:
(i) The equation of a circle with centre at origin and radius 'r' is given by
$x^2 + y^2 = r^2$
Here, r = 4
∴ The required equation of the circle is $x^2 + y^2 = 4^2$ i.e., $x^2 + y^2 = 16$.

(ii) The equation of a circle with centre at (h, k) and radius 'r' is given by
$(x - h)^2 + (y - k)^2 = r^2$
Here, h = -3, k = -2 and r = 6
∴ The required equation of the circle is
$[x - (-3)]^2 + [y - (-2)]^2 = 6^2$
⇒ $(x + 3)^2 + (y + 2)^2 = 36$
⇒ $x^2 + 6x + 9 + y^2 + 4y + 4 - 36 = 0$
⇒ $x^2 + y^2 + 6x + 4y - 23 = 0$

(iii) The equation of a circle with centre at (h, k) and radius 'r' is given by
$(x - h)^2 + (y - k)^2 = r^2$
Here, h = 2, k = -3 and r = 5
The required equation of the circle is
$(x - 2)^2 + [y - (-3)]^2 = 5^2$
⇒ $(x - 2)^2 + (y + 3)^2 = 25$
⇒ $x^2 - 4x + 4 + y^2 + 6y + 9 - 25 = 0$
⇒ $x^2 + y^2 - 4x + 6y - 12 = 0$

(iv) Centre of the circle is C (-3, -3) and it passes through the point P (-3, -6).



By distance formula,
$$\text{Radius (r)} = CP = \sqrt{[-3-(-3)]^2 + [-6-(-3)]^2}$$
$$= \sqrt{(-3+3)^2 + (-6+3)^2}$$
$$= \sqrt{0^2 + (-3)^2}$$
$$= \sqrt{9} = 3$$
The equation of a circle with centre at (h, k) and radius 'r' is given by
$(x - h)^2 + (y - k)^2 = r^2$
Here, h = -3, k = -3, r = 3
The required equation of the circle is
$[x - (-3)]^2 + [y - (-3)]^2 = 3^2$
⇒ $(x + 3)^2 + (y + 3)^2 = 9$
⇒ $x^2 + 6x + 9 + y^2 + 6y + 9 - 9 = 0$
⇒ $x^2 + y^2 + 6x + 6y + 9 = 0$

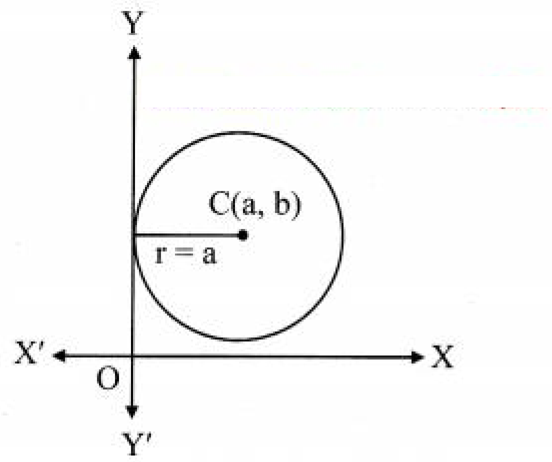
Check:
If the point (-3, -6) satisfies $x^2 + y^2 + 6x + 6y + 9 = 0$, then our answer is correct.
L.H.S. = $x^2 + y^2 + 6x + 6y + 9$
= $(-3)^2 + (-6)^2 + 6(-3) - 6(-6) + 9$
= 9 + 36 – 18 – 36 + 9
= 0
= R.H.S.
Thus, our answer is correct.

Question 2.
Find the centre and radius of the following circles:
(i) $x^2 + y^2 = 25$
(ii) $(x - 5)^2 + (y - 3)^2 = 20$
(iii) $(x-12)^2+(y+13)^2=136$
Solution:
(i) Given equation of the circle is
$x^2 + y^2 = 25$
$\Rightarrow x^2 + y^2 = (5)^2$
Comparing this equation with $x^2 + y^2 = r^2$, we get r = 5
Centre of the circle is (0, 0) and radius of the circle is 5.

(ii) Given equation of the circle is
$(x - 5)^2 + (y - 3)^2 = 20$
$\Rightarrow (x - 5)^2 + (y - 3)^2 = (\sqrt{20})^2$
Comparing this equation with $(x - h)^2 + (y - k)^2 = r^2$, we get
h = 5, k = 3 and r = $\sqrt{20} = 2\sqrt{5}$
Centre of the circle = (h, k) = (5, 3)
and radius of the circle = $2\sqrt{5}$.

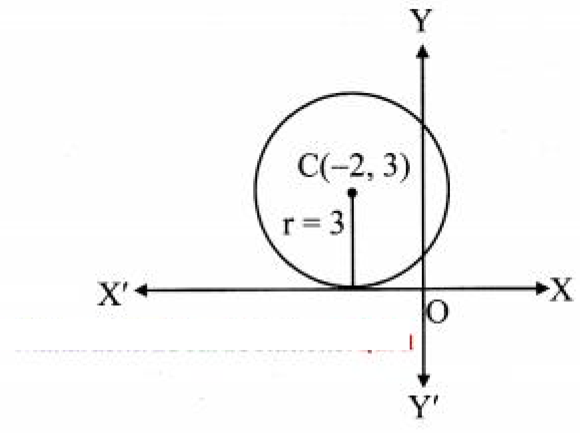
(iii) Given the equation of the circle is
$$\left(x-\frac{1}{2}\right)^2 + \left(y+\frac{1}{3}\right)^2 = \frac{1}{36}$$
$$\left(x-\frac{1}{2}\right)^2 + \left(y-\left(-\frac{1}{3}\right)\right)^2 = \left(\frac{1}{6}\right)^2$$
Comparing this equation with $(x - h)^2 + (y - k)^2 = r^2$, we get
h = 12, k = -13 and r = 16
Centre of the circle = (h, k) = (12, -13) and radius of the circle = 16

Question 3.
Find the equation of the circle with centre
(i) at (a, b) and touching the Y-axis.
(ii) at (-2, 3) and touching the X-axis.
(iii) on the X-axis and passing through the origin having radius 4.
(iv) at (3, 1) and touching the line 8x – 15y + 25 = 0.
Solution:
(i) Since the circle is touching the Y-axis, the radius of the circle is X-co-ordinate of the centre.



∴ r = a
The equation of a circle with centre at (h, k) and radius r is given by
$(x - h)^2 + (y - k)^2 = r^2$
Here, h = a, k = b
The required equation of the circle is
$\Rightarrow (x - a)^2 + (y - b)^2 = a^2$
$\Rightarrow x^2 - 2ax + a^2 + y^2 - 2by + b^2 = a^2$
$\Rightarrow x^2 + y^2 - 2ax - 2by + b^2 = 0$

(ii) Since the circle is touching the X-axis, the radius of the circle is the Y co-ordinate of the centre.



∴ r = 3

The equation of a circle with centre at (h, k) and radius r is given by

$(x - h)^2 + (y - k)^2 = r^2$

Here, h = -2, k = 3

The required equation of the circle is

⇒ $(x + 2)^2 + (y - 3)^2 = 3^2$

⇒ $x^2 + 4x + 4 + y^2 - 6y + 9 = 9$

⇒ $x^2 + y^2 + 4x - 6y + 4 = 0$

(iii) Let the co-ordinates of the centre of the required circle be C (h, 0).

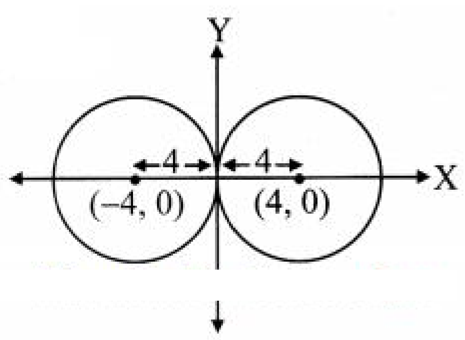
Since the circle passes through the origin i.e., O(0, 0)

OC = radius

⇒ $\sqrt{(h-0)^2+(0-0)^2}=4$

⇒ $h^2 = 16$

⇒ h = ±4



the co-ordinates of the centre are (4, 0) or (-4, 0).

The equation of a circle with centre at (h, k) and radius r is given by

$(x - h)^2 + (y - k)^2 = r^2$

Here, h = ± 4, k = 0, r = 4

The required equation of the circle is

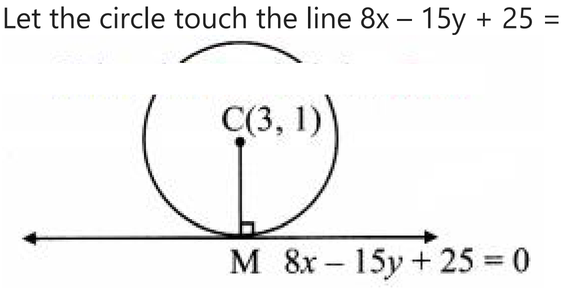
⇒ $(x - 4)^2 + (y - 0)^2 = 4^2$ or $(x + 4)^2 + (y - 0)^2 = 4^2$

⇒ $x^2 - 8x + 16 + y^2 = 16$ or $x^2 + 8x + 16 + y^2 = 16$

⇒ $x^2 + y^2 - 8x = 0$ or $x^2 + y^2 + 8x = 0$

(iv) Centre of the circle is C (3, 1).

Let the circle touch the line 8x – 15y + 25 = 0 at point M.



CM = radius (r)

CM = Length of perpendicular from centre C(3, 1) on the line 8x – 15y + 25 = 0

$$= \left|\frac{8(3) - 15(1) + 25}{\sqrt{8^2 + (-15)^2}}\right|$$

$$= \left|\frac{24 - 15 + 25}{\sqrt{64 + 225}}\right|$$

$$= \left|\frac{34}{\sqrt{289}}\right|$$

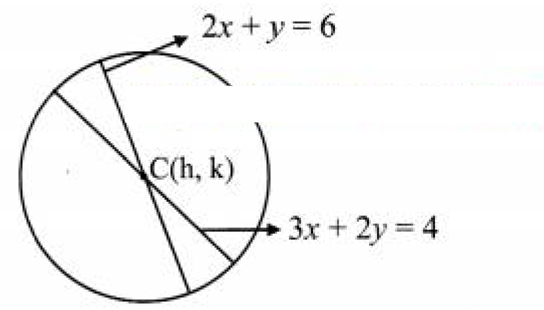
$$r = \left|\frac{34}{17}\right| = 2$$

The equation of a circle with centre at (h, k) and radius r is given by

$(x - h)^2 + (y - k)^2 = r^2$

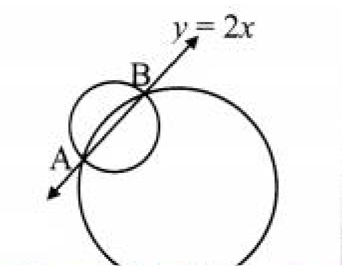
Here, h = 3, k = 1 and r = 2
The required equation of the circle is
⇒ $(x - 3)^2 + (y - 1)^2 = 2^2$
⇒ $x^2 - 6x + 9 + y^2 - 2y + 1 = 4$
⇒ $x^2 + y^2 - 6x - 2y + 10 - 4 = 0$
⇒ $x^2 + y^2 - 6x - 2y + 6 = 0$

Question 4.
Find the equation of the circle, if the equations of two diameters are 2x + y = 6 and 3x + 2y = 4 and radius is 9.
Solution:



Given equations of diameters are 2x + y = 6 and 3x + 2y = 4.
Let C (h, k) be the centre of the required circle.
Since point of intersection of diameters is the centre of the circle,
x = h, y = k
Equations of diameters become
2h + k = 6 .....(i)
and 3h + 2k = 4 ....(ii)
By (ii) – 2 × (i), we get
-h = -8
⇒ h = 8
Substituting h = 8 in (i), we get
2(8) + k = 6
⇒ k = 6 – 16
⇒ k = -10
Centre of the circle is C (8, -10) and radius, r = 9
The equation of a circle with centre at (h, k) and radius r is given by
$(x - h)^2 + (y - k)^2 = r^2$
Here, h = 8, k = -10
The required equation of the circle is
⇒ $(x - 8)^2 + (y + 10)^2 = 9^2$
⇒ $x^2 - 16x + 64 + y^2 + 20y + 100 = 81$
⇒ $x^2 + y^2 - 16x + 20y + 100 + 64 - 81 = 0$
⇒ $x^2 + y^2 - 16x + 20y + 83 = 0$

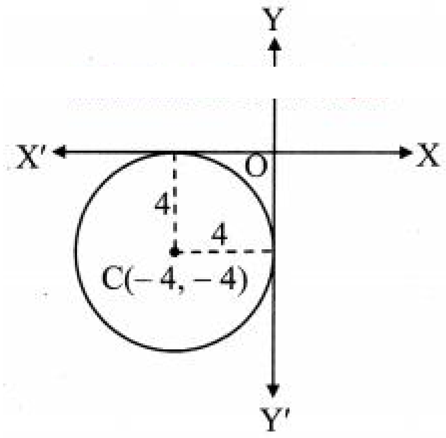
Question 5.
If y = 2x is a chord of the circle $x^2 + y^2 - 10x = 0$, find the equation of the circle with this chord as diameter.
Solution:



y = 2x is the chord of the given circle.
It satisfies the equation of a given circle.
Substituting y = 2x in $x^2 + y^2 - 10x = 0$, we get
⇒ $x^2 + (2x)^2 - 10x = 0$
⇒ $x^2 + 4x^2 - 10x = 0$
⇒ $5x^2 - 10x = 0$
⇒ 5x(x – 2) = 0
⇒ x = 0 or x = 2
When x = 0, y = 2x = 2(0) = 0
∴ A = (0, 0)
When x = 2, y = 2x = 2 (2) = 4
∴ B = (2, 4)
End points of chord AB are A(0, 0) and B(2, 4).
Chord AB is the diameter of the required circle.
The equation of a circle having (x_1, y_1) and (x_2, y_2) as end points of diameter is given by
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

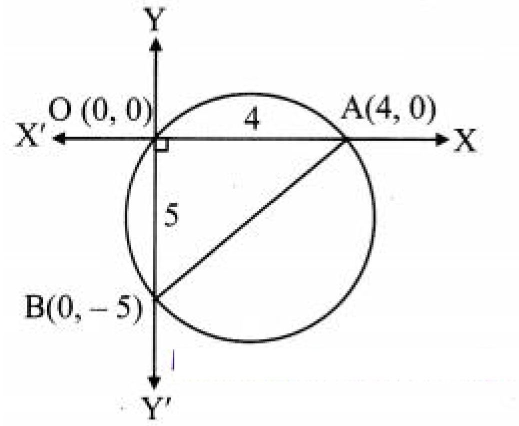
Here, $x_1 = 0$, $y_1 = 0$, $x_2 = 2$, $y_2 = 4$
The required equation of the circle is
⇒ $(x – 0) (x – 2) + (y – 0) (y – 4) = 0$
⇒ $x^2 – 2x + y^2 – 4y = 0$
⇒ $x^2 + y^2 – 2x – 4y = 0$

Question 6.
Find the equation of a circle with a radius of 4 units and touch both the co-ordinate axes having centre in the third quadrant. Solution:
The radius of the circle = 4 units
Since the circle touches both the co-ordinate axes and its centre is in the third quadrant,
the centre of the circle is C(-4, -4).



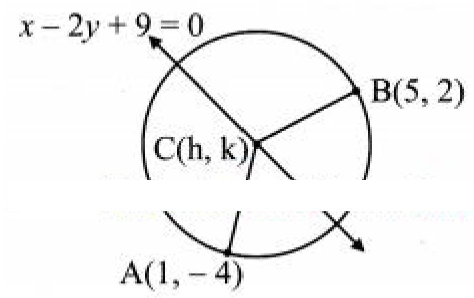
The equation of a circle with centre at (h, k) and radius r is given by $(x – h)^2 + (y – k)^2 = r^2$
Here, h = -4, k = -4, r = 4
the required equation of the circle is
⇒ $[x – (-4)]^2 + [y – (-4)]^2 = 4^2$
⇒ $(x + 4)^2 + (y + 4)^2 = 16$
⇒ $x^2 + 8x + 16 + y^2 + 8y + 16 – 16 = 0$
⇒ $x^2 + y^2 + 8x + 8y + 16 = 0$

Question 7.
Find the equation of the circle passing through the origin and having intercepts 4 and -5 on the co-ordinate axes.
Solution:
Let the circle intersect X-axis at point A and intersect Y-axis at point B.
the co-ordinates of point A are (4, 0) and the co-ordinates of point B are (0, -5).



Since ∠AOB is a right angle,
AB represents the diameter of the circle.
The equation of a circle having (x_1, y_1) and (x_2, y_2) as end points of diameter is given by
$(x – x_1) (x – x_2) + (y – y_1) (y – y_2) = 0$
Here, $x_1 = 4$, $y_1 = 0$, $x_2 = 0$, $y_2 = -5$
The required equation of the circle is
⇒ $(x – 4) (x – 0) + (y – 0) [y – (-5)] = 0$
⇒ $x(x – 4) + y(y + 5) = 0$
⇒ $x^2 – 4x + y^2 + 5y = 0$
⇒ $x^2 + y^2 – 4x + 5y = 0$

Question 8.
Find the equation of a circle passing through the points (1, -4), (5, 2) and having its centre on line x – 2y + 9 = 0.
Solution:



Let C(h, k) be the centre of the required circle which lies on the line x – 2y + 9 = 0.
Equation of line becomes
h – 2k + 9 = 0 .....(i)
Also, the required circle passes through points A(1, -4) and B(5, 2).
CA = CB = radius
CA = CB
By distance formula,
$\sqrt{(h-1)^2+[k-(-4)]^2}=\sqrt{(h-5)^2+(k-2)^2}$
Squaring both the sides, we get
⇒ $(h-1)^2 + (k+4)^2 = (h-5)^2 + (k-2)^2$
⇒ $h^2 - 2h + 1 + k^2 + 8k + 16 = h^2 - 10h + 25 + k^2 - 4k + 4$
⇒ -2h + 8k + 17 = -10h – 4k + 29
⇒ 8h + 12k – 12 = 0
⇒ 2h + 3k – 3 = 0 ......(ii)
By (ii) – (i) × 2, we get
7k = 21
⇒ k = 3
Substituting k = 3 in (i), we get
h – 2(3) + 9 = 0
⇒ h – 6 + 9 = 0
⇒ h = -3
Centre of the circle is C(-3, 3).
radius (r) = CA

$= \sqrt{[1-(-3)]^2 + (-4-3)^2}$

$= \sqrt{4^2 + (-7)^2}$

$= \sqrt{16+49}$

$= \sqrt{65}$

The equation of a circle with centre at (h, k) and radius r is given by $(x-h)^2 + (y-k)^2 = r^2$
Here, h = -3, k = 3, r = √65
The required equation of the circle is
⇒ $[x-(-3)]^2 + (y-3)^2 = (\sqrt{65})^2$
⇒ $(x+3)^2 + (y-3)^2 = 65$
⇒ $x^2 + 6x + 9 + y^2 - 6y + 9 - 65 = 0$
⇒ $x^2 + y^2 + 6x - 6y - 47 = 0$

Maharashtra State Board 11th Maths Solutions Chapter 6 Circle Ex 6.2

Question 1.
Find the centre and radius of each of the following circles:
(i) $x^2 + y^2 - 2x + 4y - 4 = 0$
(ii) $x^2 + y^2 - 6x - 8y - 24 = 0$
(iii) $4x^2 + 4y^2 - 24x - 8y - 24 = 0$
Solution:
(i) Given equation of the circle is $x^2 + y^2 - 2x + 4y - 4 = 0$
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
2g = -2, 2f = 4 and c = -4
⇒ g = -1, f = 2 and c = -4
Centre of the circle = (-g, -f) = (1, -2)
and radius of the circle

$= \sqrt{g^2 + f^2 - c}$

$= \sqrt{(-1)^2 + (2)^2 - (-4)}$

$= \sqrt{1+4+4}$

$= \sqrt{9} = 3$

(ii) Given equation of the circle is $x^2 + y^2 - 6x - 8y - 24 = 0$
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
$2g = -6$, $2f = -8$ and $c = -24$
⇒ $g = -3$, $f = -4$ and $c = -24$
Centre of the circle = $(-g, -f) = (3, 4)$
and radius of the circle

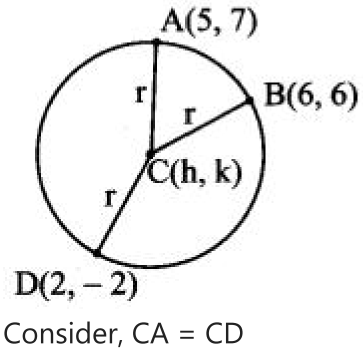
$$= \sqrt{g^2 + f^2 - c}$$
$$= \sqrt{(-3)^2 + (-4)^2 - (-24)}$$
$$= \sqrt{9 + 16 + 24}$$
$$= \sqrt{49} = 7$$

(iii) Given equation of the circle is $4x^2 + 4y^2 - 24x - 8y - 24 = 0$
Dividing throughout by 4, we get $x^2 + y^2 - 6x - 2y - 6 = 0$
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
$2g = -6$, $2f = -2$ and $c = -6$
⇒ $g = -3$, $f = -1$ and $c = -6$
Centre of the circle = $(-g, -f) = (3, 1)$
and radius of the circle

$$= \sqrt{g^2 + f^2 - c}$$
$$= \sqrt{(-3)^2 + (-1)^2 - (-6)}$$
$$= \sqrt{9 + 1 + 6}$$
$$= \sqrt{16} = 4$$

Question 2.
Show that the equation $3x^2 + 3y^2 + 12x + 18y - 11 = 0$ represents a circle.
Solution:
Given equation is $3x^2 + 3y^2 + 12x + 18y - 11 = 0$
Dividing throughout by 3, we get
$x^2 + y^2 + 4x + 6y - \frac{11}{3} = 0$
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
$2g = 4$, $2f = 6$, $c = -\frac{11}{3}$
⇒ $g = 2$, $f = 3$, $c = -\frac{11}{3}$
Now, $g^2 + f^2 - c = (2)^2 + (3)^2 - (-\frac{11}{3})$
$= 4 + 9 + \frac{11}{3}$
$= \frac{50}{3} > 0$
∴ The given equation represents a circle.

Question 3.
Find the equation of the circle passing through the points (5, 7), (6, 6), and (2, -2).
Solution:
Let C(h, k) be the centre of the required circle.
Since the required circle passes through points A(5, 7), B(6, 6), and D(2, -2),
CA = CB = CD = radius



Consider, CA = CD
By distance formula,
$$\sqrt{(h-5)^2+(k-7)^2} = \sqrt{(h-2)^2+[k-(-2)]^2}$$
Squaring both the sides, we get
⇒ $(h - 5)^2 + (k - 7)^2 = (h - 2)^2 + (k + 2)^2$
⇒ $h^2 - 10h + 25 + k^2 - 14k + 49 = h^2 - 4h + 4 + k^2 + 4k + 4$
⇒ $-10h - 14k + 74 = -4h + 4k + 8$
⇒ $6h + 18k - 66 = 0$
⇒ $h + 3k - 11 = 0$ .....(i)

Consider, CB = CD

By distance formula,

$$\sqrt{(h-6)^2+(k-6)^2}=\sqrt{(h-2)^2+[k-(-2)]^2}$$

Squaring both the sides, we get

⇒ $(h-6)^2 + (k-6)^2 = (h-2)^2 + (k+2)^2$

⇒ $h^2 - 12h + 36 + k^2 - 12k + 36 = h^2 - 4h + 4 + k^2 + 4k + 4$

⇒ $-12h - 12k + 72 = -4h + 4k + 8$

⇒ $8h + 16k - 64 = 0$

⇒ $h + 2k - 8 = 0$ ……(ii)

By (i) – (ii), we get k = 3

Substituting k = 3 in (i), we get

$h + 3(3) - 11 = 0$

⇒ $h + 9 - 11 = 0$

⇒ $h = 2$

Centre of the circle is C(2, 3).

radius (r) = CD

$= \sqrt{(2-2)^2+(3+2)^2}$

$= \sqrt{0+5^2}$

$= \sqrt{25}$

= 5

The equation of a circle with centre at (h, k) and radius r is given by $(x-h)^2 + (y-k)^2 = r^2$

Here, h = 2, k = 3

The required equation of the circle is

$(x-2)^2 + (y-3)^2 = 5^2$

⇒ $x^2 - 4x + 4 + y^2 - 6y + 9 = 25$

⇒ $x^2 + y^2 - 4x - 6y + 4 + 9 - 25 = 0$

⇒ $x^2 + y^2 - 4x - 6y - 12 = 0$

Question 4.

Show that the points (3, -2), (1, 0), (-1, -2) and (1, -4) are concyclic.

Solution:

Let the equation of the circle passing through the points (3, -2), (1, 0) and (-1, -2) be

$x^2 + y^2 + 2gx + 2fy + c = 0$ …..(i)

For point (3, -2),

Substituting x = 3 and y = -2 in (i), we get

$9 + 4 + 6g - 4f + c = 0$

⇒ $6g - 4f + c = -13$ ….(ii)

For point (1, 0),

Substituting x = 1 andy = 0 in (i), we get

$1 + 0 + 2g + 0 + c = 0$

⇒ $2g + c = -1$ ……(iii)

For point (-1, -2),

Substituting x = -1 and y = -2, we get

$1 + 4 - 2g - 4f + c = 0$

⇒ $2g + 4f - c = 5$ …….(iv)

Adding (ii) and (iv), we get

$8g = -8$

⇒ $g = -1$

Substituting g = -1 in (iii), we get

$-2 + c = -1$

⇒ $c = 1$

Substituting g = -1 and c = 1 in (iv), we get

$-2 + 4f - 1 = 5$

⇒ $4f = 8$

⇒ $f = 2$

Substituting g = -1, f = 2 and c = 1 in (i), we get

$x^2 + y^2 - 2x + 4y + 1 = 0$ ……….(v)

If (1, -4) satisfies equation (v), the four points are concyclic.

Substituting x = 1, y = -4 in L.H.S of (v), we get

L.H.S. $= (1)^2 + (-4)^2 - 2(1) + 4(-4) + 1$

$= 1 + 16 - 2 - 16 + 1$

= 0

= R.H.S.

Point (1, -4) satisfies equation (v).

∴ The given points are concyclic.

Maharashtra State Board 11th Maths Solutions Chapter 6 Circle Ex 6.3

Question 1.
Write the parametric equations of the circles:
(i) $x^2 + y^2 = 9$
(ii) $x^2 + y^2 + 2x - 4y - 4 = 0$
(iii) $(x - 3)^2 + (y + 4)^2 = 25$
Solution:
(i) Given equation of the circle is
$x^2 + y^2 = 9$
$\Rightarrow x^2 + y^2 = 3^2$
Comparing this equation with $x^2 + y^2 = r^2$, we get $r = 3$
The parametric equations of the circle in terms of θ are
$x = r \cos\theta$ and $y = r \sin\theta$
$\Rightarrow x = 3 \cos\theta$ and $y = 3 \sin\theta$

(ii) Given equation of the circle is
$x^2 + y^2 + 2x - 4y - 4 = 0$
$\Rightarrow x^2 + 2x + y^2 - 4y - 4 = 0$
$\Rightarrow x^2 + 2x + 1 - 1 + y^2 - 4y + 4 - 4 - 4 = 0$
$\Rightarrow (x^2 + 2x + 1) + (y^2 - 4y + 4) - 9 = 0$
$\Rightarrow (x + 1)^2 + (y - 2)^2 = 9$
$\Rightarrow (x + 1)^2 + (y - 2)^2 = 3^2$
Comparing this equation with $(x - h)^2 + (y - k)^2 = r^2$, we get
$h = -1$, $k = 2$ and $r = 3$
The parametric equations of the circle in terms of θ are
$x = h + r \cos\theta$ and $y = k + r \sin\theta$
$\Rightarrow x = -1 + 3 \cos\theta$ and $y = 2 + 3 \sin\theta$

(iii) Given equation of the circle is
$(x - 3)^2 + (y + 4)^2 = 25$
$\Rightarrow (x - 3)^2 + (y + 4)^2 = 5^2$
Comparing this equation with $(x - h)^2 + (y - k)^2 = r^2$, we get
$h = 3$, $k = -4$ and $r = 5$
The parametric equations of the circle in terms of θ are
$x = h + r \cos\theta$ and $y = k + r \sin\theta$
$\Rightarrow x = 3 + 5 \cos\theta$ and $y = -4 + 5 \sin\theta$

Question 2.
Find the parametric representation of the circle $3x^2 + 3y^2 - 4x + 6y - 4 = 0$.
Solution:
Given equation of the circle is $3x^2 + 3y^2 - 4x + 6y - 4 = 0$
Dividing throughout by 3, we get

$$x^2 + y^2 - \frac{4}{3}x + 2y - \frac{4}{3} = 0$$

$$x^2 - \frac{4}{3}x + y^2 + 2y - \frac{4}{3} = 0$$

$$x^2 - \frac{4}{3}x + \frac{4}{9} - \frac{4}{9} + y^2 + 2y + 1 - 1 - \frac{4}{3} = 0$$

$$\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) + \left(y^2 + 2y + 1\right) - \frac{25}{9} = 0$$

$$\left(x - \frac{2}{3}\right)^2 + (y + 1)^2 = \frac{25}{9}$$

$$\left(x - \frac{2}{3}\right)^2 + [y - (-1)]^2 = \left(\frac{5}{3}\right)^2$$

Comparing this equation with $(x - h)^2 + (y - k)^2 = r^2$, we get
$h = \frac{2}{3}$, $k = -1$ and $r = \frac{5}{3}$
The parametric representation of the circle in terms of θ are
$x = h + r \cos\theta$ and $y = k + r \sin\theta$
$\Rightarrow x = \frac{2}{3} + \frac{5}{3} \cos\theta$ and $y = -1 + \frac{5}{3} \sin\theta$

Question 3.
Find the equation of a tangent to the circle $x^2 + y^2 – 3x + 2y = 0$ at the origin.
Solution:
Given equation of the circle is $x^2 + y^2 – 3x + 2y = 0$
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
2g = -3, 2f = 2, c = 0
⇒ g = −32, f = 1, c = 0
The equation of a tangent to the circle
$x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
The equation of the tangent at (0, 0) is
x(0) + y(0) + (−32) (x + 0) + 1(y + 0) + 0 = 0
⇒ −32x + y = 0
⇒ 3x – 2y = 0

Question 4.
Show that the line 7x – 3y – 1 = 0 touches the circle $x^2 + y^2 + 5x – 7y + 4 = 0$ at point (1, 2).
Solution:
Given equation of the circle is $x^2 + y^2 + 5x – 7y + 4 = 0$
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
2g = 5, 2f = -7, c = 4
⇒ g = 52, f = -72, c = 4
The equation of a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is
$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
The equation of the tangent at (1, 2) is

$$x(1) + y(2) + \frac{5}{2}(x+1) - \frac{7}{2}(y+2) + 4 = 0$$
$$x + 2y + \frac{5}{2}x + \frac{5}{2} - \frac{7}{2}y - 7 + 4 = 0$$
$$\frac{7}{2}x - \frac{3}{2}y - \frac{1}{2} = 0$$

7x – 3y – 1 = 0, which is same as the given line.
The line 7x – 3y – 1 = 0 touches the given circle at (1, 2).

Question 5.
Find the equation of tangent to the circle $x^2 + y^2 – 4x + 3y + 2 = 0$ at the point (4, -2).
Solution:
Given equation of the circle is $x^2 + y^2 – 4x + 3y + 2 = 0$
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
2g = -4, 2f = 3, c = 2
g = -2, f = 32, c = 2
The equation of a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is
$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
The equation of the tangent at (4, -2) is
x(4) + y(-2) – 2(x + 4) + 32(y – 2) + 2 = 0
⇒ 4x – 2y – 2x – 8 + 32 y – 3 + 2 = 0
⇒ 2x – 12y – 9 = 0
⇒ 4x – y – 18 = 0

Maharashtra State Board 11th Maths Solutions Chapter 6 Circle Miscellaneous Exercise 6

(I) Choose the correct alternative.

Question 1.
Equation of a circle which passes through (3, 6) and touches the axes is

(A) $x^2 + y^2 + 6x + 6y + 3 = 0$
(B) $x^2 + y^2 – 6x – 6y – 9 = 0$
(C) $x^2 + y^2 – 6x – 6y + 9 = 0$
(D) $x^2 + y^2 – 6x + 6y – 3 = 0$
Answer:
(C) $x^2 + y^2 – 6x – 6y + 9 = 0$

Question 2.
If the lines 2x – 3y = 5 and 3x – 4y = 7 are the diameters of a circle of area 154 sq. units, then find the equation of the circle.
(A) $x^2 + y^2 – 2x + 2y = 40$
(B) $x^2 + y^2 – 2x – 2y = 47$
(C) $x^2 + y^2 – 2x + 2y = 47$
(D) $x^2 + y^2 – 2x – 2y = 40$
Answer:
(C) $x^2 + y^2 – 2x + 2y = 47$
Hint:
Centre of circle = Point of intersection of diameters.
Solving 2x – 3y = 5 and 3x – 4y = 7, we get
x = 1, y = -1
Centre of the circle C(h, k) = C(1, -1)
∴ Area = 154
$\pi r^2 = 154$
$227 \times r^2 = 154$
$r^2 = 154 \times 227 = 49$
∴ r = 7
equation of the circle is
$(x – 1)^2 + (y + 1)^2 = 7^2$
$x^2 + y^2 – 2x + 2y = 47$

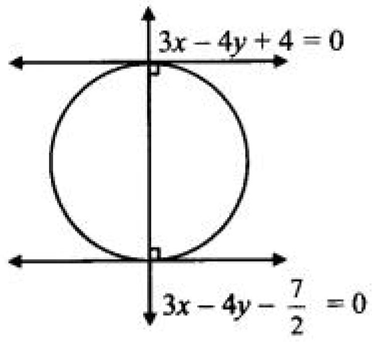
Question 3.
Find the equation of the circle which passes through the points (2, 3) and (4, 5), and the center lies on the straight line y – 4x + 3 = 0.
(A) $x^2 + y^2 – 4x – 10y + 25 = 0$
(B) $x^2 + y^2 – 4x – 10y – 25 = 0$
(C) $x^2 + y^2 – 4x + 10y – 25 = 0$
(D) $x^2 + y^2 + 4x – 10y + 25 = 0$
Answer:
(A) $x^2 + y^2 – 4x – 10y + 25 = 0$

Question 4.
The equation(s) of the tangent(s) to the circle $x^2 + y^2 = 4$ which are parallel to x + 2y + 3 = 0 are
(A) x – 2y = 2
(B) x + 2y = ±2√3
(C) x + 2y = ±2√5
(D) x – 2y = ±2√5
Answer:
(C) x + 2y = ±2√5

Question 5.
If the lines 3x – 4y + 4 = 0 and 6x – 8y – 7 = 0 are tangents to a circle, then find the radius of the circle.
(A) 34
(B) 43
(C) 14
(D) 74
Answer:
(A) 34
Hint:
Tangents are parallel to each other.

The perpendicular distance between tangents = diameter

$$\left|\frac{4-\left(-\frac{7}{2}\right)}{\sqrt{3^2+(-4)^2}}\right| = 2r$$

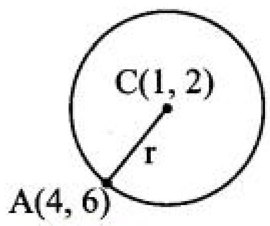


$$\frac{\frac{15}{2}}{5} = 2r$$

$$r = \frac{3}{4}$$

Question 6.
The area of the circle having centre at (1, 2) and passing through (4, 6) is
(A) 5π
(B) 10π
(C) 25π
(D) 100π
Answer:
(C) 25π
Hint:

$$r = CA$$
$$= \sqrt{(4-1)^2+(6-2)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25} = 5$$
$$\text{area} = \pi r^2 = \pi \times 5^2$$
$$= 25\pi$$



Question 7.
If a circle passes through the points (0, 0), (a, 0), and (0, b), then find the co-ordinates of its centre.
(A) (−a2,−b2)
(B) (a2,−b2)
(C) (−a2,b2)
(D) (a2,b2)
Answer:
(D) (a2,b2)

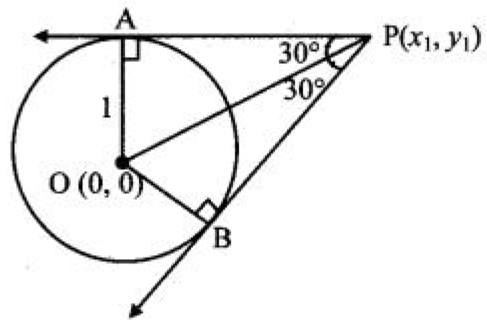
Question 8.
The equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length 3a is
(A) x2 + y2 = 9a2
(B) x2 + y2 = 16a2
(C) x2 + y2 = 4a2
(D) x2 + y2 = a2
Answer:
(C) x2 + y2 = 4a2
Hint:
Since the triangle is equilateral.
The centroid of the triangle is same as the circumcentre
and radius of the circumcircle = 23 (median) = 23(3a) = 2a
Hence, the equation of the circumcircle whose centre is at (0, 0) and radius 2a is x2 + y2 = 4a2

Question 9.
A pair of tangents are drawn to a unit circle with centre at the origin and these tangents intersect at A enclosing an angle of 60. The area enclosed by these tangents and the arc of the circle is
(A) 23√−π6
(B) 3−√−π3
(C) π3−3√6
(D) 3−√(1−π6)
Answer:

(B) $\sqrt{3}-\frac{\pi}{3}$

Hint:

In Δ OAP,



$\sin 30^\circ = \frac{1}{OP}$

$OP = 2$

$\cos 30^\circ = \frac{AP}{OP}$

$\frac{\sqrt{3}}{2} = \frac{AP}{2}$

$AP = \sqrt{3}$

$A(\square\ AOBP) = 2A(\Delta\ OAP)$

$= 2 \times \frac{1}{2} \times 1 \times \sqrt{3}$

$= \sqrt{3}$

$A(\text{sector } AOB) = \frac{1}{2} \times (1)^2 \times \frac{2\pi}{3}$

$= \frac{\pi}{3}$

Required area $= A(\square\ AOBP) - A(\text{sector } AOB)$

$= \sqrt{3} - \frac{\pi}{3}$

Question 10.
The parametric equations of the circle $x^2 + y^2 + mx + my = 0$ are
(A) $x = \frac{-m}{2}+\frac{m}{\sqrt{2}}\cos\theta$, $y = \frac{-m}{2}+\frac{m}{\sqrt{2}}\sin\theta$
(B) $x = \frac{-m}{2}+\frac{m}{\sqrt{2}}\cos\theta$, $y = \frac{+m}{2}+\frac{m}{\sqrt{2}}\sin\theta$
(C) x = 0, y = 0
(D) x = m cos θ, y = m sin θ
Answer:
(A) $x = \frac{-m}{2}+\frac{m}{\sqrt{2}}\cos\theta$, $y = \frac{-m}{2}+\frac{m}{\sqrt{2}}\sin\theta$

(II) Answer the following:

Question 1.
Find the centre and radius of the circle $x^2 + y^2 - x + 2y - 3 = 0$.
Solution:
Given equation of the circle is $x^2 + y^2 - x + 2y - 3 = 0$
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
2g = -1, 2f = 2 and c = -3
$g = \frac{-1}{2}$, f = 1 and c = -3
Centre of the circle = (-g, -f) = $(\frac{1}{2}, -1)$
and radius of the circle

$$= \sqrt{g^2+f^2-c}$$

$$= \sqrt{\left(-\frac{1}{2}\right)^2+(1)^2-(-3)}$$

$$= \sqrt{\frac{1}{4}+1+3}$$

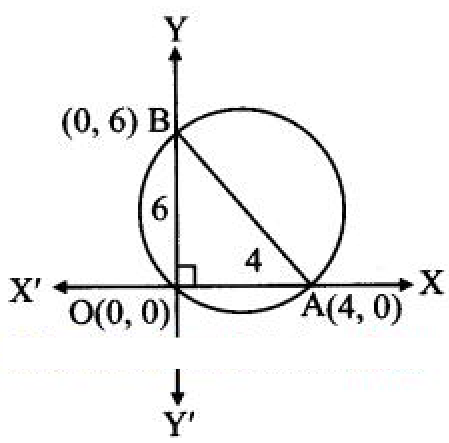
$$= \sqrt{\frac{17}{4}}$$

$$= \frac{\sqrt{17}}{2}$$

Question 2.
Find the centre and radius of the circle x = 3 – 4 sin θ, y = 2 – 4 cos θ.
Solution:
Given, x = 3 – 4 sin θ, y = 2 – 4 cos θ
⇒ x – 3 = -4 sin θ, y – 2 = -4 cos θ
On squaring and adding, we get
⇒ $(x-3)^2 + (y-2)^2 = (-4 \sin\theta)^2 + (-4 \cos\theta)^2$
⇒ $(x-3)^2 + (y-2)^2 = 16 \sin^2\theta + 16 \cos^2\theta$

⇒ $(x - 3)^2 + (y - 2)^2 = 16(\sin^2 \theta + \cos^2 \theta)$
⇒ $(x - 3)^2 + (y - 2)^2 = 16(1)$
⇒ $(x - 3)^2 + (y - 2)^2 = 16$
⇒ $(x - 3)^2 + (y - 2)^2 = 4^2$
Comparing this equation with $(x - h)^2 + (y - k)^2 = r^2$, we get
h = 3, k = 2, r = 4
∴ Centre of the circle is (3, 2) and radius is 4.

Question 3.
Find the equation of circle passing through the point of intersection of the lines x + 3y = 0 and 2x – 7y = 0 and whose centre is the point of intersection of lines x + y + 1 = 0 and x – 2y + 4 = 0.
Solution:
Required circle passes through the point of intersection of the lines x + 3y = 0 and 2x – 7y = 0.
x + 3y = 0
⇒ x = -3y ……..(i)
2x – 7y = 0 ……(ii)
Substituting x = -3y in (ii), we get
⇒ 2(-3y) – 7y = 0
⇒ -6y – 7y = 0
⇒ -13y = 0
⇒ y = 0
Substituting y = 0 in (i), we get
x = -3(0) = 0
Point of intersection is O(0, 0).
This point O(0, 0) lies on the circle.
Let C(h, k) be the centre of the required circle.
Since, point of intersection of lines x + y = -1 and x – 2y = -4 is the centre of circle.
∴ x = h, y = k
∴ Equations of lines become
h + k = -1 ……(iii)
h – 2k = -4 …..(iv)
By (iii) – (iv), we get
3k = 3
⇒ k = 1
Substituting k = 1 in (iii), we get
h + 1 = -1
⇒ h = -2
∴ Centre of the circle is C(-2, 1) and it passes through point O(0, 0).
Radius(r) = OC
$= \sqrt{(0+2)^2+(0-1)^2}$
$= \sqrt{4+1}$
= √5
The equation of a circle with centre at (h, k) and radius r is given by
$(x - h)^2 + (y - k)^2 = r^2$
Here, h = -2, k = 1
the required equation of the circle is
$(x + 2)^2 + (y - 1)^2 = (\sqrt{5})^2$
⇒ $x^2 + 4x + 4 + y^2 - 2y + 1 = 5$
⇒ $x^2 + y^2 + 4x - 2y = 0$

Question 4.
Find the equation of the circle which passes through the origin and cuts off chords of lengths 4 and 6 on the positive side of the X-axis and Y-axis respectively.
Solution:


Let the circle cut the chord of length 4 on X-axis at point A and the chord of length 6 on the Y-axis at point B.
∴ the co-ordinates of point A are (4, 0) and co-ordinates of point B are (0, 6).

Since ∠BOA is a right angle.
AB represents the diameter of the circle.
The equation of a circle having (x_1, y_1) and (x_2, y_2) as endpoints of diameter is given by
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
Here, $x_1 = 4$, $y_1 = 0$, $x_2 = 0$, $y_2 = 6$
∴ the required equation of the circle is
⇒ $(x - 4)(x - 0) + (y - 0)(y - 6) = 0$
⇒ $x^2 - 4x + y^2 - 6y = 0$
⇒ $x^2 + y^2 - 4x - 6y = 0$

Question 5.
Show that the points (9, 1), (7, 9), (-2, 12) and (6, 10) are concyclic.
Solution:
Let the equation of circle passing through the points (9, 1), (7, 9), (-2, 12) be
$x^2 + y^2 + 2gx + 2fy + c = 0$ …….(i)
For point (9, 1),
Substituting x = 9 andy = 1 in (i), we get
$81 + 1 + 18g + 2f + c = 0$
⇒ $18g + 2f + c = -82$ …..(ii)
For point (7, 9),
Substituting x = 7 andy = 9 in (i), we get
$49 + 81 + 14g + 18f + c = 0$
⇒ $14g + 18f + c = -130$ ……(iii)
For point (-2, 12),
Substituting x = -2 and y = 12 in (i), we get
$4 + 144 - 4g + 24f + c = 0$
⇒ $-4g + 24f + c = -148$ …..(iv)
By (ii) – (iii), we get
$4g - 16f = 48$
⇒ $g - 4f = 12$ …..(v)
By (iii) – (iv), we get
$18g - 6f = 18$
⇒ $3g - f = 3$ ……(vi)
By 3 × (v) – (vi), we get
$-11f = 33$
⇒ $f = -3$
Substituting f = -3 in (vi), we get
$3g - (-3) = 3$
⇒ $3g + 3 = 3$
⇒ $g = 0$
Substituting g = 0 and f = -3 in (ii), we get
$18(0) + 2(-3) + c = -82$
⇒ $-6 + c = -82$
⇒ $c = -76$
Equation of the circle becomes
$x^2 + y^2 + 2(0)x + 2(-3)y + (-76) = 0$
⇒ $x^2 + y^2 - 6y - 76 = 0$ ……(vii)
Now for the point (6, 10),
Substituting x = 6 and y = 10 in L.H.S. of (vii), we get
L.H.S $= 6^2 + 10^2 - 6(10) - 76$
$= 36 + 100 - 60 - 76$
$= 0$
= R.H.S.
∴ Point (6,10) satisfies equation (vii).
∴ the given points are concyciic.

Question 6.
The line $2x - y + 6 = 0$ meets the circle $x^2 + y^2 + 10x + 9 = 0$ at A and B. Find the equation of circle with AB as diameter. Solution:
$2x - y + 6 = 0$
⇒ $y = 2x + 6$
Substituting $y = 2x + 6$ in $x^2 + y^2 + 10x + 9 = 0$, we get
⇒ $x^2 + (2x + 6)^2 + 10x + 9 = 0$
⇒ $x^2 + 4x^2 + 24x + 36 + 10x + 9 = 0$
⇒ $5x^2 + 34x + 45 = 0$
⇒ $5x^2 + 25x + 9x + 45 = 0$
⇒ $(5x + 9)(x + 5) = 0$
⇒ $5x = -9$ or $x = -5$
⇒ $x = -\frac{9}{5}$ or $x = -5$

When x = −95,
y = 2 × −95 + 6
= −185 + 6
= −18+305
= 125
∴ Point of intersection is A(−95,125)
When x = -5,
y = -10 + 6 = -4
∴ Point of intersection in B (-5, -4).
By diameter form, equation of circle with AB as diameter is
(x + 95) (x + 5) + (y – 125) (y + 4) = 0
⇒ (5x + 9) (x + 5) + (5y – 12) (y + 4) = 0
⇒ $5x^2$ + 25x + 9x + 45 + $5y^2$ + 20y – 12y – 48 = 0
⇒ $5x^2$ + $5y^2$ + 34x + 8y – 3 = 0

Question 7.
Show that x = -1 is a tangent to circle $x^2 + y^2 – 4x – 2y – 4 = 0$ at (-1, 1).
Solution:
Given equation of circle is $x^2 + y^2 – 4x – 2y – 4 = 0$.
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
2g = -4, 2f = -2, c = -4
⇒ g = -2, f = -1, c = -4
The equation of a tangent to the circle
$x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
the equation of the tangent at (-1, 1) is
⇒ x(-1) + y(1) – 2(x – 1) – 1(y + 1) – 4 = 0
⇒ -3x – 3 = 0
⇒ -x – 1 = 0
⇒ x = -1
∴ x = -1 is the tangent to the given circle at (-1, 1).

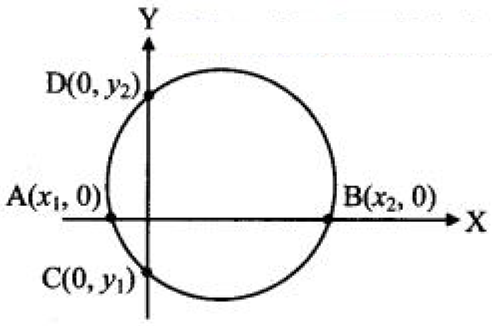
Question 8.
Find the equation of tangent to the circle $x^2 + y^2 = 64$ at the point P(2π3).
Solution:
Given equation of circle is $x^2 + y^2 = 64$
Comparing this equation with $x^2 + y^2 = r^2$, we get r = 8
The equation of a tangent to the circle $x^2 + y^2 = r^2$ at P(θ) is x cos θ + y sin θ = r
∴ the equation of the tangent at P(2π3) is
⇒ x cos 2π3 + y sin 2π3 = 9
⇒ x(−12)+y(3√2)=8
⇒ -x + √3y = 16
⇒ x – √3y + 16 = 0

Question 9.
Find the equation of locus of the point of intersection of perpendicular tangents drawn to the circle x = 5 cos θ and y = 5 sin θ.
Solution:
The locus of the point of intersection of perpendicular tangents is the director circle of the given circle.
x = 5 cos θ and y = 5 sin θ
⇒ $x^2 + y^2 = 25 \cos^2 θ + 25 \sin^2 θ$
⇒ $x^2 + y^2 = 25 (\cos^2 θ + \sin^2 θ)$
⇒ $x^2 + y^2 = 25(1) = 25$
The equation of the director circle of the circle $x^2 + y^2 = a^2$ is $x^2 + y^2 = 2a^2$.
Here, a = 5
∴ the required equation is
$x^2 + y^2 = 2(5)^2 = 2(25)$
∴ $x^2 + y^2 = 50$

Question 10.
Find the equation of the circle concentric with $x^2 + y^2 – 4x + 6y = 1$ and having radius 4 units.
Solution:
Given equation of circle is
$x^2 + y^2 – 4x + 6y = 1$ i.e., $x^2 + y^2 – 4x + 6y – 1 = 0$
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
2g = -4, 2f = 6

⇒ g = -2, f = 3
Centre of the circle = (-g, -f) = (2, -3)
Given circle is concentric with the required circle.
∴ They have same centre.
∴ Centre of the required circle = (2, -3)
The equation of a circle with centre at (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$
Here, h = 2, k = -3 and r = 4
∴ the required equation of the circle is
$(x - 2)^2 + [y - (-3)]^2 = 4^2$
⇒ $(x - 2)^2 + (y + 3)^2 = 16$
⇒ $x^2 - 4x + 4 + y^2 + 6y + 9 - 16 = 0$
⇒ $x^2 + y^2 - 4x + 6y - 3 = 0$

Question 11.
Find the lengths of the intercepts made on the co-ordinate axes, by the circles.
(i) $x^2 + y^2 - 8x + y - 20 = 0$
(ii) $x^2 + y^2 - 5x + 13y - 14 = 0$
Solution:
To find x-intercept made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$,
substitute y = 0 and get a quadratic equation in x, whose roots are, say, x_1 and x_2.



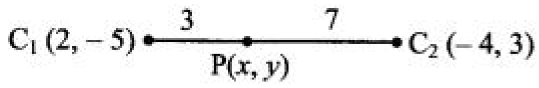
These values represent the abscissae of ends A and B of the x-intercept.
Length of x-intercept = |AB| = $|x_2 - x_1|$
Similarly, substituting x = 0, we get a quadratic equation in y whose roots, say, y_1 and y_2 are ordinates of the ends C and D of the y-intercept.
Length of y-intercept = |CD| = $|y_2 - y_1|$
(i) Given equation of the circle is
$x^2 + y^2 - 8x + y - 20 = 0$ ……(i)
Substituting y = 0 in (i), we get
$x^2 - 8x - 20 = 0$ ……(ii)
Let AB represent the x-intercept, where
A = $(x_1, 0)$, B = $(x_2, 0)$
Then from (ii),
$x_1 + x_2 = 8$ and $x_1x_2 = -20$
$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$
$= (8)^2 - 4(-20)$
= 64 + 80
= 144
∴ $|x_1 - x_2| = \sqrt{(x_1 - x_2)^2} = \sqrt{144} = 12$
∴ Length of x – intercept =12 units
Substituting x = 0 in (i), we get
$y^2 + y - 20 = 0$ …..(iii)
Let CD represent the y – intercept,
where C = $(0, y_1)$ and D = $(0, y_2)$
Then from (iii),
$y_1 + y_2 = -1$ and $y_1y_2 = -20$
$(y_1 - y_2)^2 = (y_1 + y_2)^2 - 4y_1y_2$
$= (-1)^2 - 4(-20)$
= 1 + 80
= 81
∴ $|y_1 - y_2| = \sqrt{(y_1 - y_2)^2} = \sqrt{81} = 9$
∴ Length of y – intercept = 9 units.

Alternate Method:
Given equation of the circle is $x^2 + y^2 - 8x + y - 20 = 0$ ……(i)
x-intercept:
Substituting y = 0 in (i), we get
$x^2 - 8x - 20 = 0$
⇒ $(x - 10)(x + 2) = 0$
⇒ x = 10 or x = -2
length of x-intercept = |10 – (-2)| = 12 units
y-intercept:

Substituting x = 0 in (i), we get
$y^2 + y - 20 = 0$
$\Rightarrow (y + 5)(y - 4) = 0$
$\Rightarrow y = -5$ or $y = 4$
length of y-intercept = $|-5 - 4| = 9$ units

(ii) Given equation of the circle is
$x^2 + y^2 - 5x + 13y - 14 = 0$
Substituting y = 0 in (i), we get
$x^2 - 5x - 14 = 0$ ……(ii)
Let AB represent the x-intercept, where
$A = (x_1, 0)$, $B = (x_2, 0)$
Then from (ii),
$x_1 + x_2 = 5$ and $x_1x_2 = -14$
$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$
$= (5)^2 - 4(-14)$
$= 25 + 56$
$= 81$
$\therefore |x_1 - x_2| = \sqrt{(x_1-x_2)^2} = \sqrt{81} = 9$
$\therefore$ Length of x-intercept = 9 units
Substituting x = 0 in (i), we get
$y^2 + 13y - 14 = 0$ ……(iii)
Let CD represent they-intercept,
where $C = (0, y_1)$, $D = (0, y_2)$.
Then from (iii),
$y_1 + y_2 = -13$ and $y_1y_2 = -14$
$(y_1 - y_2)^2 = (y_1 + y_2)^2 - 4y_1y_2$
$= (-13)^2 - 4(-14)$
$= 169 + 56$
$= 225$
$\therefore |y_1 - y_2| = \sqrt{(y_1-y_2)^2} = \sqrt{225} = 15$
$\therefore$ Length ofy-intercept = 15 units

Question 12.
Show that the circles touch each other externally. Find their point of contact and the equation of their common tangent.
(i) $x^2 + y^2 - 4x + 10y + 20 = 0$
$x^2 + y^2 + 8x - 6y - 24 = 0$
(ii) $x^2 + y^2 - 4x - 10y + 19 = 0$
$x^2 + y^2 + 2x + 8y - 23 = 0$
Solution:
(i) Given equation of the first circle is $x^2 + y^2 - 4x + 10y + 20 = 0$
Here, g = -2, f = 5, c = 20
Centre of the first circle is $C_1 = (2, -5)$
Radius of the first circle is
$r_1 = \sqrt{(-2)^2+5^2-20}$
$= \sqrt{4+25-20}$
$= \sqrt{9}$
$= 3$
Given equation of the second circle is $x^2 + y^2 + 8x - 6y - 24 = 0$
Here, g = 4, f = -3, c = -24
Centre of the second circle is $C_2 = (-4, 3)$
Radius of the second circle is
$r_2 = \sqrt{4^2+(-3)^2+24}$
$= \sqrt{16+9+24}$
$= \sqrt{49}$
$= 7$
By distance formula,
$C_1C_2 = \sqrt{(-4-2)^2+[3-(-5)]^2}$
$= \sqrt{36+64}$
$= \sqrt{1oo}$
$= 10$
$r_1 + r_2 = 3 + 7 = 10$
Since, $C_1C_2 = r_1 + r_2$
$\therefore$ the given circles touch each other externally.



Let P(x, y) be the point of contact.
∴ P divides C_1C_2 internally in the ratio $r_1 : r_2$ i.e. 3 : 7.
∴ By internal division,

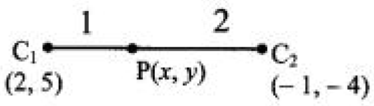
$$x = \frac{3(-4)+7(2)}{3+7} = \frac{-12+14}{10} = \frac{1}{5}$$

$$\text{and } y = \frac{3(3)+7(-5)}{3+7} = \frac{9-35}{10} = -\frac{13}{5}$$

$$\text{Point of contact} = \left(\frac{1}{5}, -\frac{13}{5}\right)$$

Equation of common tangent is
$(x^2 + y^2 - 4x + 10y + 20) - (x^2 + y^2 + 8x - 6y - 24) = 0$
⇒ $-4x + 10y + 20 - 8x + 6y + 24 = 0$
⇒ $-12x + 16y + 44 = 0$
⇒ $3x - 4y - 11 = 0$

(ii) Given equation of the first circle is $x^2 + y^2 - 4x - 10y + 19 = 0$
Here, g = -2, f = -5, c = 19
Centre of the first circle is $C_1 = (2, 5)$
Radius of the first circle is
$r_1 = \sqrt{(-2)^2+(-5)^2-19}$
$= \sqrt{4+25-19}$
$= \sqrt{10}$
Given equation of the second circle is $x^2 + y^2 + 2x + 8y - 23 = 0$
Here, g = 1, f = 4, c = -23
Centre of the second circle is $C_2 = (-1, -4)$
Radius of the second circle is
$r_2 = \sqrt{(-1)^2+4^2+23}$
$= \sqrt{1+16+23}$
$= \sqrt{40}$
$= 2\sqrt{10}$
By distance formula,
$C_1C_2 = \sqrt{(-1-2)^2+(-4-5)^2}$
$= \sqrt{9+81}$
$= \sqrt{90}$
$= 3\sqrt{10}$
$r_1 + r_2 = \sqrt{10} + 2\sqrt{10} = 3\sqrt{10}$
Since, $C_1C_2 = r_1 + r_2$
the given circles touch each other externally.
$r_1 : r_2 = \sqrt{10} : 2\sqrt{10} = 1 : 2$
Let P(x, y) be the point of contact.



∴ P divides C_1 C_2 internally in the ratio $r_1 : r_2$ i.e. 1 : 2
∴ By internal division,

$$x = \frac{1(-1)+2(2)}{1+2} = \frac{-1+4}{3} = 1$$

$$\text{and } \quad y = \frac{1(-4)+2(5)}{1+2} = \frac{-4+10}{3} = 2$$

Point of contact = (1, 2)
Equation of common tangent is
$(x^2 + y^2 - 4x - 10y + 19) - (x^2 + y^2 + 2x + 8y - 23) = 0$
⇒ $-4x - 10y + 19 - 2x - 8y + 23 = 0$
⇒ $-6x - 18y + 42 = 0$
⇒ $x + 3y - 7 = 0$

Question 13.
Show that the circles touch each other internally. Find their point of contact and the equation of their common tangent.
(i) $x^2 + y^2 - 4x - 4y - 28 = 0$,
$x^2 + y^2 - 4x - 12 = 0$

(ii) $x^2 + y^2 + 4x – 12y + 4 = 0$,
$x^2 + y^2 – 2x – 4y + 4 = 0$
Solution:
(i) Given equation of the first circle is $x^2 + y^2 – 4x – 4y – 28 = 0$
Here, g = -2, f = -2, c = -28
Centre of the first circle is $C_1 = (2, 2)$
Radius of the first circle is
$r_1 = \sqrt{(-2)^2+(-2)^2+28}$
$= \sqrt{4+4+28}$
$= \sqrt{36}$
= 6
Given equation of the second circle is $x^2 + y^2 – 4x – 12 = 0$
Here, g = -2, f = 0, c = -12
Centre of the second circle is $C_2 = (2, 0)$
Radius of the second circle is
$r_2 = \sqrt{(-2)^2+0^2+12}$
$= \sqrt{4+12}$
$= \sqrt{16}$
= 4
By distance formula,
$C_1C_2 = \sqrt{(2-2)^2+(0-2)^2}$
$= \sqrt{4}$
= 2
$|r_1 – r_2| = 6 – 4 = 2$
Since, $C_1C_2 = |r_1 – r_2|$
∴ the given circles touch each other internally.
Equation of common tangent is
$(x^2 + y^2 – 4x – 4y – 28) – (x^2 + y^2 – 4x – 12) = 0$
⇒ -4x – 4y – 28 + 4x + 12 = 0
⇒ -4y – 16 = 0
⇒ y + 4 = 0
⇒ y = -4
Substituting y = -4 in $x^2 + y^2 – 4x – 12 = 0$, we get
⇒ $x^2 + (-4)^2 – 4x – 12 = 0$
⇒ $x^2 + 16 – 4x – 12 = 0$
⇒ $x^2 – 4x + 4 = 0$.
⇒ $(x – 2)^2 = 0$
⇒ x = 2
∴ Point of contact is (2, -4) and equation of common tangent is y + 4 = 0.

(ii) Given equation of the first circle is $x^2 + y^2 + 4x – 12y + 4 = 0$
Here, g = 2, f = -6, c = 4
Centre of the first circle is $C_1 = (-2, 6)$
Radius of the first circle is
$r_1 = \sqrt{2^2+(-6)^2-4}$
$= \sqrt{4+36-4}$
$= \sqrt{36}$
= 6
Given equation of the second circle is $x^2 + y^2 – 2x – 4y + 4 = 0$
Here, g = -1, f = -2, c = 4
Centre of the second circle is $C_2 = (1, 2)$
Radius of the second circle is
$r_2 = \sqrt{(-1)^2+(-2)^2-4}$
$= \sqrt{1+4-4}$
$= \sqrt{1}$
= 1
By distance formula,
$C_1C_2 = \sqrt{[1-(-2)]^2+(2-6)^2}$
$= \sqrt{9+16}$
$= \sqrt{25}$
= 5
$|r_1 – r_2| = 6 – 1 = 5$
Since, $C_1C_2 = |r_1 – r_2|$
the given circles touch each other internally.
Equation of common tangent is

$(x^2 + y^2 + 4x – 12y + 4) – (x^2 + y^2 – 2x – 4y + 4) = 0$
⇒ $4x – 12y + 4 + 2x + 4y – 4 = 0$
⇒ $6x – 8y = 0$
⇒ $3x – 4y = 0$
⇒ $y = \frac{3x}{4}$
Substituting $y = \frac{3x}{4}$ in $x^2 + y^2 – 2x – 4y + 4 = 0$, we get

$$x^2 + \left(\frac{3x}{4}\right)^2 - 2x - 4\left(\frac{3x}{4}\right) + 4 = 0$$

$$x^2 + \frac{9x^2}{16} - 2x - 3x + 4 = 0$$

$$\frac{25x^2}{16} - 5x + 4 = 0$$

$$25x^2 - 80x + 64 = 0$$

$$(5x - 8)^2 = 0$$

$$5x - 8 = 0$$

$$x = \frac{8}{5}$$

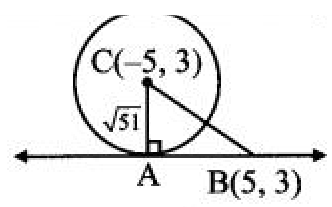
Substituting $x = \frac{8}{5}$ in $y = \frac{3x}{4}$, we get

$$y = \frac{3}{4}\left(\frac{8}{5}\right) = \frac{6}{5}$$

∴ Point of contact is $\left(\frac{8}{5}, \frac{6}{5}\right)$ and equation of common tangent is $3x – 4y = 0$.

Question 14.
Find the length of the tangent segment drawn from the point (5, 3) to the circle $x^2 + y^2 + 10x – 6y – 17 = 0$.
Solution:
Given equation of circle is $x^2 + y^2 + 10x – 6y – 17 = 0$
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
$2g = 10, 2f = -6, c = -17$
⇒ $g = 5, f = -3, c = -17$
Centre of circle = (-g, -f) = (-5, 3)

$$\textbf{Radius of circle} = \sqrt{g^2 + f^2 - c}$$
$$= \sqrt{5^2 + (-3)^2 - (-17)}$$
$$= \sqrt{25 + 9 + 17}$$
$$= \sqrt{51}$$



$$BC = \sqrt{(-5-5)^2 + (3-3)^2}$$
$$= \sqrt{100 + 0} = 10$$

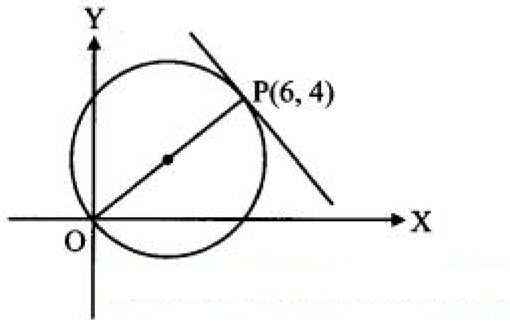
In right angled ΔABC,
$BC^2 = AB^2 + AC^2$ .....[Pythagoras theorem]
⇒ $(10)^2 = AB^2 + (\sqrt{51})^2$
⇒ $AB^2 = 100 – 51 = \sqrt{49}$
⇒ AB = 7
∴ Length of the tangent segment from (5, 3) is 7 units.

Alternate method:
Given equation of circle is $x^2 + y^2 + 10x – 6y – 17 = 0$
Here, g = 5, f = -3, c = -17
Length of the tangent segment to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the point (x_1, y_1)
is $\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$
Length of the tangent segment from (5, 3)
$= \sqrt{(5)^2 + (3)^2 + 10(5) - 6(3) - 17}$
$= \sqrt{25 + 9 + 50 - 18 - 17}$
$= \sqrt{49}$
= 7 units

Question 15.
Find the value of k, if the length of the tangent segment from the point (8, -3) to the circle $x^2 + y^2 - 2x + ky - 23 = 0$ is $\sqrt{10}$.
Solution:
Given equation of the circle is $x^2 + y^2 - 2x + ky - 23 = 0$
Here, $g = -1$, $f = \frac{k}{2}$, $c = -23$
Length of the tangent segment to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the point (x_1, y_1)
is $\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$
Length of the tangent segment from (8, -3) = $\sqrt{10}$
$\Rightarrow \sqrt{8^2 + (-3)^2 - 2(8) + k(-3) - 23} = \sqrt{10}$
$\Rightarrow 64 + 9 - 16 - 3k - 23 = 10$ .....[Squaring both the sides]
$\Rightarrow 34 - 3k = 10$
$\Rightarrow 3k = 24$
$\Rightarrow k = 8$

Question 16.
Find the equation of tangent to circle $x^2 + y^2 - 6x - 4y = 0$, at the point (6, 4) on it.
Solution:
Given equation of the circle is $x^2 + y^2 - 6x - 4y = 0$
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
$2g = -6$, $2f = -4$, $c = 0$
$\Rightarrow g = -3$, $f = -2$, $c = 0$
The equation of a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is
$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
the equation of the tangent at (6, 4) is
$x(6) + y(4) - 3(x + 6) - 2(y + 4) + 0 = 0$
$\Rightarrow 6x + 4y - 3x - 18 - 2y - 8 = 0$
$\Rightarrow 3x + 2y - 26 = 0$

Alternate method:
Given equation of the circle is $x^2 + y^2 - 6x - 4y = 0$
$x(x - 6) + y(y - 4) = 0$, which is in diameter form where (0, 0) and (6, 4) are endpoints of diameter.



Slope of OP = $\frac{4-0}{6-0} = \frac{2}{3}$
Since, OP is perpendicular to the required tangent.
Slope of the required tangent = $\frac{-3}{2}$
the equation of the tangent at (6, 4) is
$y - 4 = \frac{-3}{2}(x - 6)$
$\Rightarrow 2(y - 4) = 3(x - 6)$
$\Rightarrow 2y - 8 = -3x + 18$
$\Rightarrow 3x + 2y - 26 = 0$

Question 17.
Fihd the equation of tangent to circle $x^2 + y^2 = 5$, at the point (1, -2) on it.
Solution:
Given equation of the circle is $x^2 + y^2 = 5$
Comparing this equation with $x^2 + y^2 = r^2$, we get
$r^2 = 5$
The equation of a tangent to the circle $x^2 + y^2 = r^2$ at (x_1, y_1) is $xx_1 + yy_1 = r^2$
the equation of the tangent at (1, -2) is
$x(1) + y(-2) = 5$
$\Rightarrow x - 2y = 5$

Question 18.
Find the equation of tangent to circle $x = 5 \cos \theta$, $y = 5 \sin \theta$, at the point $\theta = \frac{\pi}{3}$ on it.
Solution:
The equation of a tangent to the circle $x^2 + y^2 = r^2$ at $P(\theta)$ is $x \cos \theta + y \sin \theta = r$
Here, $r = 5$, $\theta = \frac{\pi}{3}$
the equation of the tangent at $P(\frac{\pi}{3})$ is
$x \cos \frac{\pi}{3} + y \sin \frac{\pi}{3} = 5$

⇒ $x(12)+y(3\sqrt{2})=5$
⇒ $x + \sqrt{3}y = 10$

Question 19.
Show that $2x + y + 6 = 0$ is a tangent to $x^2 + y^2 + 2x – 2y – 3 = 0$. Find its point of contact.
Solution:
Given equation of circle is
$x^2 + y^2 + 2x – 2y – 3 = 0$ ….(i)
Given equation of line is $2x + y + 6 = 0$
$y = -6 – 2x$ ……(ii)
Substituting $y = -6 – 2x$ in (i), we get
$x + (-6 – 2x)^2 + 2x – 2(-6 – 2x) – 3 = 0$
⇒ $x^2 + 36 + 24x + 4x^2 + 2x + 12 + 4x – 3 = 0$
⇒ $5x^2 + 30x + 45 = 0$
⇒ $x^2 + 6x + 9 = 0$
⇒ $(x + 3)^2 = 0$
⇒ $x = -3$
Since, the roots are equal.
∴ $2x + y + 6 = 0$ is a tangent to $x^2 + y^2 + 2x – 2y – 3 = 0$
Substituting $x = -3$ in (ii), we get
$y = -6 – 2(-3) = -6 + 6 = 0$
Point of contact = $(-3, 0)$

Question 20.
If the tangent at $(3, -4)$ to the circle $x^2 + y^2 = 25$ touches the circle $x^2 + y^2 + 8x – 4y + c = 0$, find c.
Solution:
The equation of a tangent to the circle
$x^2 + y^2 = r^2$ at (x_1, y_1) is $xx_1 + yy_1 = r^2$
Equation of the tangent at $(3, -4)$ is
$x(3) + y(-4) = 25$
⇒ $3x – 4y – 25 = 0$ ……(i)
Given equation of circle is $x^2 + y^2 + 8x – 4y + c = 0$
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
$2g = 8, 2f = -4$
⇒ $g = 4, f = -2$
∴ $C = (-4, 2)$ and r = $42+(−2)2−c−−−−−−−−−−−−√=20−c−−−−−√$
Since line (i) is a tangent to this circle also, the perpendicular distance from C(-4, 2) to line (i) is equal to radius r.

$$\left|\frac{3(-4)+(-4)(2)-25}{\sqrt{3^2+4^2}}\right| = \sqrt{20-c}$$
$$\left|\frac{-45}{\sqrt{25}}\right| = \sqrt{20-c}$$
$$\left|\frac{-45}{5}\right| = \sqrt{20-c}$$
$$|-9| = \sqrt{20-c}$$
$$81 = 20 - c \quad \text{…[Squaring both the sides]}$$
$$c = -61$$

Question 21.
Find the equations of the tangents to the circle $x^2 + y^2 = 16$ with slope -2.
Solution:
Given equation of the circle is $x^2 + y^2 = 16$
Comparing this equation with $x^2 + y^2 = a^2$, we get
$a^2 = 16$
Equations of the tangents to the circle $x^2 + y^2 = a^2$ with slope m are
$y=mx±a2(1+m2)−−−−−−−−−√$
Here, $m = -2$, $a^2 = 16$
the required equations of the tangents are
y = $−2x±16[1+(−2)2]−−−−−−−−−−−√$
⇒ y = $−2x±16(5)−−−−√$
⇒ $y = -2x ± 4\sqrt{5}$
⇒ $2x + y ± 4\sqrt{5} = 0$

Question 22.
Find the equations of the tangents to the circle $x^2 + y^2 = 4$ which are parallel to $3x + 2y + 1 = 0$.
Solution:
Given equation of the circle is $x^2 + y^2 = 4$
Comparing this equation with $x^2 + y^2 = a^2$, we get
$a^2 = 4$
Given equation of the line is $3x + 2y + 1 = 0$
Slope of this line = $-\frac{3}{2}$
Since, the required tangents are parallel to the given line.
Slope of required tangents (m) = $-\frac{3}{2}$
Equations of the tangents to the circle $x^2 + y^2 = a^2$ with slope m are
$y = mx \pm \sqrt{a^2(1+m^2)}$
the required equations of the tangents are

$$y = \frac{-3}{2}x \pm \sqrt{4\left[1+\left(\frac{-3}{2}\right)^2\right]}$$
$$= \frac{-3}{2}x \pm \sqrt{4\left(1+\frac{9}{4}\right)}$$
$$y = \frac{-3}{2}x \pm \sqrt{13}$$
$$2y = -3x \pm 2\sqrt{13}$$
$$3x + 2y \pm 2\sqrt{13} = 0$$

Question 23.
Find the equations of the tangents to the circle $x^2 + y^2 = 36$ which are perpendicular to the line $5x + y = 2$.
Solution:
Given equation of the circle is $x^2 + y^2 = 36$
Comparing this equaiton with $x^2 + y^2 = a^2$, we get
$a^2 = 36$
Given equation of line is $5x + y = 2$
Slope of this line = -5
Since, the required tangents are perpendicular to the given line.
Slope of required tangents (m) = $\frac{1}{5}$
Equations of the tangents to the circle $x^2 + y^2 = a^2$ with slope m are
$y = mx \pm \sqrt{a^2(1+m^2)}$
the required equations of the tangents are

$$y = \frac{1}{5}x \pm \sqrt{36\left[1+\left(\frac{1}{5}\right)^2\right]}$$
$$= \frac{1}{5}x \pm \sqrt{36\left(1+\frac{1}{25}\right)}$$
$$y = \frac{1}{5}x \pm \frac{6}{5}\sqrt{26}$$
$$5y = x \pm 6\sqrt{26}$$
$$x - 5y \pm 6\sqrt{26} = 0$$

Question 24.
Find the equations of the tangents to the circle $x^2 + y^2 - 2x + 8y - 23 = 0$ having slope 3.
Solution:
Let the equation of the tangent with slope 3 be $y = 3x + c$.
$3x - y + c = 0$ ……(i)
Given equation of circle is $x^2 + y^2 - 2x + 8y - 23 = 0$
Comparing this equation with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get
$2g = -2$, $2f = 8$, $c = -23$
$g = -1$, $f = 4$, $c = -23$
The centre of the circle is C(1, -4)
and its radius = $\sqrt{1+16+23}$
= $\sqrt{40}$
= $2\sqrt{10}$
Since line (i) is a tangent to this circle the perpendicular distance from C(1, -4) to line (i) is equal to radius r.
$\left|\frac{3(1)+4+c}{\sqrt{9+1}}\right| = 2\sqrt{10}$
$\Rightarrow \left|\frac{7+c}{\sqrt{10}}\right| = 2\sqrt{10}$
$\Rightarrow (7 + c) = \pm 20$
$\Rightarrow 7 + c = 20$ or $7 + c = -20$

⇒ c = 13 or c = – 27
∴ Equations of the tangents are 3x – y + 13 = 0 and 3x – y – 21 = 0

Question 25.
Find the equation of the locus of a point, the tangents from which to the circle $x^2 + y^2 = 9$ are at right angles.
Solution:
Given equation of the circle is $x^2 + y^2 = 9$
Comparing this equation with $x^2 + y^2 = a^2$, we get
$a^2 = 9$
The locus of the point of intersection of perpendicular tangents is the director circle of the given circle.
The equation of the director circle of the circle $x^2 + y^2 = a^2$ is $x^2 + y^2 = 2a^2$.
the required equation is
$x^2 + y^2 = 2(9)$
$x^2 + y^2 = 18$

Alternate method:
Given equation of the circle is $x^2 + y^2 = 9$
Comparing this equation with $x^2 + y^2 = a^2$, we get $a^2 = 9$
Let P(x_1, y_1) be a point on the required locus.
Equations of the tangents to the circle $x^2 + y^2 = a^2$ with slope m are
$y = mx \pm \sqrt{a^2(1+m^2)}$
∴ Equations of the tangents are
$y = mx \pm \sqrt{9(m^2+1)}$
⇒ $y = mx \pm 3\sqrt{1+m^2}$
Since, these tangents pass through (x_1, y_1).
$y_1 = mx_1 \pm 3\sqrt{1+m^2}$
⇒ $y_1 - mx_1 = \pm 3\sqrt{1+m^2}$
⇒ $(y_1 - mx_1)^2 = 9(1 + m^2)$ ……[Squaring both the sides]
⇒ $y_1^2 - 2mx_1y_1 + m^2x_1^2 = 9 + 9m^2$
⇒ $(x_1^2 - 9)m^2 - 2mx_1y_1 + (y_1^2 - 9) = 0$
This is a quadratic equation which has two roots m_1 and m_2.
$m_1m_2 = \frac{y_1^2-9}{x_1^2-9}$
Since, the tangents are at right angles.
$m_1m_2 = -1$
⇒ $\frac{y_1^2-9}{x_1^2-9} = -1$
⇒ $y_1^2 - 9 = 9 - x_1^2$
⇒ $x_1^2 + y_1^2 = 18$
Equation of the locus of point P is $x^2 + y^2 = 18$.

Question 26.
Tangents to the circle $x^2 + y^2 = a^2$ with inclinations, θ_1 and θ_2 intersect in P. Find the locus of P such that
(i) $\tan \theta_1 + \tan \theta_2 = 0$
(ii) $\cot \theta_1 + \cot \theta_2 = 5$
(iii) $\cot \theta_1 . \cot \theta_2 = c$
Solution:
Let P(x_1, y_1) be a point on the required locus.
Equations of the tangents to the circle $x^2 + y^2 = a^2$ with slope m are
$y = mx \pm \sqrt{a^2(1+m^2)}$
Since, these tangents pass through (x_1, y_1).

$y_1 = mx_1 \pm \sqrt{a^2(1+m^2)}$

$\therefore \quad y_1 - mx_1 = \pm\sqrt{a^2(1+m^2)}$

$\therefore \quad {y_1}^2 - 2mx_1y_1 + m^2{x_1}^2 = a^2 + a^2m^2$

$\therefore \quad ({x_1}^2 - a^2)m^2 - 2mx_1y_1 + ({y_1}^2 - a^2) = 0$

This is a quadratic equation which has two roots m_1 and m_2.

$\therefore \quad m_1 + m_2 = \dfrac{2x_1y_1}{x_1^2 - a^2}$ and $m_1 m_2 = \dfrac{y_1^2 - a^2}{x_1^2 - a^2}$

i. Let $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$

Given, $\tan\theta_1 + \tan\theta_2 = 0$

$\therefore \quad m_1 + m_2 = 0$

$\therefore \quad \dfrac{2x_1y_1}{x_1^2 - a^2} = 0$

$\therefore \quad 2x_1y_1 = 0$

$\therefore \quad x_1y_1 = 0$

$\therefore$ Equation of the locus of point P is $xy = 0$.

ii. Given, $\cot\theta_1 + \cot\theta_2 = 5$

$\therefore \quad \dfrac{1}{\tan\theta_1} + \dfrac{1}{\tan\theta_2} = 5$

$\therefore \quad \dfrac{1}{m_1} + \dfrac{1}{m_2} = 5$

$\therefore \quad \dfrac{m_1+m_2}{m_1m_2} = 5$

$\therefore \quad \dfrac{\frac{2x_1y_1}{x_1^2 - a^2}}{\frac{y_1^2 - a^2}{x_1^2 - a^2}} = 5$

$\therefore \quad \dfrac{2x_1y_1}{{y_1}^2 - a^2} = 5$

$\therefore \quad 2x_1y_1 = 5{y_1}^2 - 5a^2$

$\therefore \quad 5{y_1}^2 - 2x_1y_1 = 5a^2$

$\therefore$ Equation of the locus of point P is $5y^2 - 2xy = 5a^2$

iii. $\cot\theta_1 \cdot \cot\theta_2 = c$

$\therefore \quad \dfrac{1}{\tan\theta_1} \cdot \dfrac{1}{\tan\theta_2} = c$

$\therefore \quad \dfrac{1}{m_1m_2} = c$

$\therefore \quad \dfrac{1}{\frac{y_1^2 - a^2}{x_1^2 - a^2}} = c$

$\therefore \quad \dfrac{{x_1}^2 - a^2}{{y_1}^2 - a^2} = c$

$\therefore \quad {x_1}^2 - a^2 = c({y_1}^2 - a^2)$

$\therefore$ Equation of the locus of point P is $x^2 - a^2 = c(y^2 - a^2)$.