Maharashtra State Board 11th Commerce Maths Solutions Chapter 3 Complex Numbers Ex 3.1

Question 1.
Write the conjugates of the following complex numbers:
(i) 3 + i
(ii) 3 – i
(iii) -√5 – √7i
(iv) -√-5
(v) 5i
(vi) √5 – i
(vii) √2 + √3i
Solution:
(i) Conjugate of (3 + i) is (3 – i)
(ii) Conjugate of (3 – i) is (3 + i)
(iii) Conjugate of (-√5 – √7i) is (-√5 + √7i)
(iv) -√-5 = -√5 × √-1 = -√5i
Conjugate of -√-5 is √5i
(v) Conjugate of 5i is -5i
(vi) Conjugate of √5 – i is √5 + i
(vii) Conjugate of √2 + √3i is √2 – √3i

Question 2.
Express the following in the form of a + ib, a, b ∈ R, i = √-1. State the values of a and b:
(i) (1 + 2i)(-2 + i)
(ii) i(4+3i)(1−i)
(iii) (2+i)(3−i)(1+2i)
(iv) 3+2i2−5i+3−2i2+5i
(v) 2+−3√4+−3√
(vi) (2 + 3i)(2 – 3i)
(vii) $4i_8-3i_9+33i_{11}-4i_{10}-2$
Solution:

i. $(1+2i)(-2+i) = -2+i-4i+2i^2$
$= -2-3i+2(-1)$ ...[∵ $i^2 = -1$]

∴ $(1+2i)(-2+i) = -4-3i$
∴ $a = -4$ and $b = -3$

ii. $\frac{i(4+3i)}{1-i} = \frac{4i+3i^2}{1-i}$
$= \frac{-3+4i}{1-i}$ ...[∵ $i^2 = -1$]
$= \frac{(-3+4i)(1+i)}{(1-i)(1+i)}$
$= \frac{-3-3i+4i+4i^2}{1-i^2}$
$= \frac{-3+i+4(-1)}{1-(-1)}$ ...[∵ $i^2 = -1$]
$= \frac{-7+i}{2}$

∴ $\frac{i(4+3i)}{1-i} = \frac{-7}{2}+\frac{1}{2}i$
∴ $a = \frac{-7}{2}$ and $b = \frac{1}{2}$

iii. $\frac{2+i}{(3-i)(1+2i)} = \frac{2+i}{3+6i-i-2i^2}$
$= \frac{2+i}{3+5i-2(-1)}$ ...[∵ $i^2 = -1$]

$= \frac{2+i}{5+5i}$

$= \frac{2+i}{5(1+i)} = \frac{(2+i)(1-i)}{5(1+i)(1-i)}$

$= \frac{2-2i+i-i^2}{5(1-i^2)}$

$= \frac{2-i-(-1)}{5[1-(-1)]}$ ...[$\because$ $i^2 = -1$]

$= \frac{3-i}{10}$

$\therefore \quad \frac{2+i}{(3-i)(1+2i)} = \frac{3}{10} - \frac{1}{10}i$

$\therefore \quad a = \frac{3}{10}$ and $b = \frac{-1}{10}$

iv. $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$

$= \frac{(3+2i)(2+5i)+(2-5i)(3-2i)}{(2-5i)(2+5i)}$

$= \frac{6+15i+4i+10i^2+6-4i-15i+10i^2}{4-25i^2}$

$= \frac{12+20i^2}{4-25i^2}$

$= \frac{12+20(-1)}{4-25(-1)}$ ...[$\because$ $i^2 = -1$]

$= \frac{-8}{29}$

$\therefore \quad \frac{3+2i}{2-5i} + \frac{3-2i}{2+5i} = \frac{-8}{29} + 0i$

$\therefore \quad a = \frac{-8}{29}$ and $b = 0$

v. $\frac{2+\sqrt{-3}}{4+\sqrt{-3}} = \frac{2+\sqrt{3}i}{4+\sqrt{3}i}$

$$= \frac{\left(2+\sqrt{3}i\right)\left(4-\sqrt{3}i\right)}{\left(4+\sqrt{3}i\right)\left(4-\sqrt{3}i\right)}$$

$$= \frac{8-2\sqrt{3}i+4\sqrt{3}i-3i^2}{16-3i^2}$$

$$= \frac{8+2\sqrt{3}i-3(-1)}{16-3(-1)} \qquad ...[\because i^2 = -1]$$

$$= \frac{11+2\sqrt{3}i}{19}$$

$\therefore \quad \frac{2+\sqrt{-3}}{4+\sqrt{-3}} = \frac{11}{19} + \frac{2\sqrt{3}}{19}i$

$\therefore \quad a = \frac{11}{19}$ and $b = \frac{2\sqrt{3}}{19}$

vi. $(2 + 3i)(2 - 3i) = 4 - 9i^2$

$$= 4 - 9(-1) \qquad ...[\because i^2 = -1]$$

$$= 4 + 9 = 13$$

$\therefore \quad (2 + 3i)(2 - 3i) = 13 + 0i$

$\therefore \quad a = 13$ and $b = 0$

vii. $\frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2} = \frac{4\left(i^4\right)^2 - 3\left(i^4\right)^2.i+3}{3\left(i^4\right)^2.i^3 - 4\left(i^4\right)^2.i^2 - 2}$

Since, $i^2 = -1$, $i^3 = -i$ and $i^4 = 1$

$\therefore \quad \frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2} = \frac{4(1)^2 - 3(1)^2.i+3}{3(1)^2(-i) - 4(1)^2(-1) - 2}$

$$= \frac{4-3i+3}{-3i+4-2}$$

$$= \frac{7-3i}{2-3i}$$

$$= \frac{(7-3i)(2+3i)}{(2-3i)(2+3i)}$$

$$= \frac{14+21i-6i-9i^2}{4-9i^2}$$

$$= \frac{14+15i-9(-1)}{4-9(-1)}$$

$$= \frac{23+15i}{13}$$

$\therefore \quad \frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2} = \frac{23}{13} + \frac{15}{13}i$

$\therefore \quad a = \frac{23}{13}$ and $b = \frac{15}{13}$

Question 3.
Show that $(-1 + \sqrt{3}i)^3$ is a real number.
Solution:
$(-1 + \sqrt{3}i)^3$
$= (-1)^3 + 3(-1)^2 (\sqrt{3}i) + 3(-1)(\sqrt{3}i)^2 + (\sqrt{3}i)^3$ [$\because (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]
$= -1 + 3\sqrt{3}i - 3(3i^2) + 3\sqrt{3}\ i^3$
$= -1 + 3\sqrt{3}i - 3(-3) - 3\sqrt{3}i$ [$\because i^2 = -1, i^3 = -1$]
$= -1 + 9$
$= 8$, which is a real number.

Question 4.
Evaluate the following:
(i) i^{35}
(ii) i^{888}
(iii) i^{93}
(iv) i^{116}
(v) i^{403}
(vi) $1i_{58}$

(vii) $i^{30} + i^{40} + i^{50} + i^{60}$
Solution:
We know that, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$
(i) $i^{35} = (i^4)^8 (i^2)\, i = (1)^8 (-1)\, i = -i$
(ii) $i^{888} = (i^4)^{222} = (1)^{222} = 1$
(iii) $i^{93} = (i^4)^{23} \,.\, i = (1)^{23} \,.\, i = i$
(iv) $i^{116} = (i^4)^{29} = (1)^{29} = 1$
(v) $i^{403} = (i^4)^{100} (i^2)\, i = (1)^{100} (-1)\, i = -i$
(vi) $\frac{1}{i^{88}} = \frac{1}{(i^4)^{14} \cdot i^2} = \frac{1}{(1)^{14}(-1)} = -1$
(vii) $i^{30} + i^{40} + i^{50} + i^{60}$
$= (i^4)^7 i^2 + (i^4)^{10} + (i^4)^{12} i^2 + (i^4)^{15}$
$= (1)^7 (-1) + (1)^{10} + (1)^{12} (-1) + (1)^{15}$
$= -1 + 1 - 1 + 1$
$= 0$

Question 5.
Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number.
Solution:
$1 + i^{10} + i^{20} + i^{30}$
$= 1 + (i^4)^2 \,.\, i^2 + (i^4)^5 + (i^4)^7 \,.\, i^2$
$= 1 + (1)^2 (-1) + (1)^5 + (1)^7 (-1)$ [$\because i^4 = 1, i^2 = -1$]
$= 1 - 1 + 1 - 1$
$= 0$, which is a real number.

Question 6.
Find the value of
(i) $i^{49} + i^{68} + i^{89} + i^{110}$
(ii) $i + i^2 + i^3 + i^4$
Solution:
(i) $i^{49} + i^{68} + i^{89} + i^{110}$
$= (i^4)^{12} \,.\, i + (i^4)^{17} + (i^4)^{22} \,.\, i + (i^4)^{27} \,.\, i^2$
$= (1)^{12} \,.\, i + (1)^{17} + (1)^{22} \,.\, i + (1)^{27}(-1)$ ......[$\because i^4 = 1, i^2 = -1$]
$= i + 1 + i - 1$
$= 2i$

(ii) $i + i^2 + i^3 + i^4$
$= i + i^2 + i^2 \,.\, i + i^4$
$= i - 1 - i + 1$ [$\because i^2 = -1, i^4 = 1$]
$= 0$

Question 7.
Find the value of $1 + i^2 + i^4 + i^6 + i^8 + \ldots\ldots + i^{20}$.
Solution:
$1 + i^2 + i^4 + i^6 + i^8 + \ldots.. + i^{20}$
$= 1 + (i^2 + i^4) + (i^6 + i^8) + (i^{10} + i^{12}) + (i^{14} + i^{16}) + (i^{18} + i^{20})$
$= 1 + [i^2 + (i^2)^2] + [(i^2)^3 + (i^2)^4] + [(i^2)^5 + (i^2)^6] + [(i^2)^7 + (i^2)^8] + [(i^2)^9 + (i^2)^{10}]$
$= 1 + [-1 + (-1)^2] + [(-1)^3 + (-1)^4] + [(-1)^5 + (-1)^6] + [(-1)^7 + (-1)^8] + [(-1)^9 + (-1)^{10}]$ [$\because i^2 = -1$]
$= 1 + (-1 + 1) + (-1 + 1) + (-1 + 1) + (-1 + 1) + (-1 + 1)$
$= 1 + 0 + 0 + 0 + 0 + 0$
$= 1$

Question 8.
Find the values of x and y which satisfy the following equations ($x, y \in R$):
(i) $(x + 2y) + (2x - 3y)i + 4i = 5$
(ii) $\frac{x+1}{1+i} + \frac{y-1}{1-i} = i$
Solution:
(i) $(x + 2y) + (2x - 3y)i + 4i = 5$
$\therefore (x + 2y) + (2x - 3y)i = 5 - 4i$
Equating real and imaginary parts, we get
$x + 2y = 5$ ....(i)
and $2x - 3y = -4$ ....(ii)
Equation (i) × 2 – equation (ii) gives
$7y = 14$
$\therefore y = 2$
Putting y- 2 in (i), we get
$x + 2(2) = 5$
$\therefore x + 4 = 5$

∴ x = 1
∴ x = 1 and y = 2
Check:
If x = 1 and y = 2 satisfy the given condition, then our answer is correct.
L.H.S. = (x + 2y) + (2x – 3y)i + 4i
= (1 + 4) + (2 – 6)i + 4i
= 5 – 4i + 4i
= 5
= R.H.S.
Thus, our answer is correct.

(ii) $\frac{x+1}{1+i}+\frac{y-1}{1-i}=i$

$$\frac{(x+1)(1-i)+(y-1)(1+i)}{(1+i)(1-i)}=i$$

$$\frac{x-xi+1-i+y+yi-1-i}{1-i^2}=i$$

$$\frac{(x+y)+(y-x-2)i}{1-(-1)}=i \qquad \ldots[\because i^2=-1]$$

(x + y) + (y – x – 2)i = 2i
(x + y) + (y – x – 2)i = 0 + 2i
Equating real and imaginary parts, we get
x + y = 0 and y – x – 2 = 2
∴ x + y = 0 ……(i)
and -x + y = 4 ……..(ii)
Adding (i) and (ii), we get
2y = 4
∴ y = 2
Putting y = 2 in (i), we get
x + 2 = 0
∴ x = -2
∴ x = -2 and y = 2

Question 9.
Find the value of:
(i) $x^3 – x^2 + x + 46$, if x = 2 + 3i
(ii) $2x^3 – 11x^2 + 44x + 27$, if x = $\frac{25}{3-4i}$
Solution:
(i) x = 2 + 3i
∴ x – 2 = 3i
∴ $(x – 2)^2 = 9i^2$
∴ $x^2 – 4x + 4 = 9(-1)$ …..[∵ $i^2 = -1$]
∴ $x^2 – 4x + 13 = 0$ ……(i)

$$\begin{array}{r|l} & x+3 \\ \hline x^2-4x+13 & x^3-x^2+x+46 \\ & x^3-4x^2+13x \\ & -\quad +\quad - \\ \hline & 3x^2-12x+46 \\ & 3x^2-12x+39 \\ & -\quad +\quad - \\ \hline & 7 \end{array}$$

∴ $x^3 – x^2 + x + 46 = (x^2 – 4x + 13)(x + 3) + 7$
= 0(x + 3) + 7 ……[From (i)]
= 7

(ii) $x = \frac{25}{3-4i}$

$$x = \frac{25(3+4i)}{(3-4i)(3+4i)}$$

$$= \frac{25(3+4i)}{9-16i^2}$$

$$= \frac{25(3+4i)}{9-16(-1)} \qquad \ldots[\because i^2 = -1]$$

$$= \frac{25(3+4i)}{25}$$

$\therefore$ x = 3 + 4i
$\therefore$ x – 3 = 4i
$\therefore$ $(x – 3)^2 = 16i^2$
$\therefore$ $x^2 – 6x + 9 = 16(-1)$ ....[$\because$ $i^2 = -1$]
$\therefore$ $x^2 – 6x + 25 = 0$ ....(i)

$$\begin{array}{r|l}
 & 2x+1 \\
\hline
x^2-6x+25 & 2x^3-11x^2+44x+27 \\
 & 2x^3-12x^2+50x \\
 & - \quad + \quad - \\
\hline
 & x^2 - 6x + 27 \\
 & x^2 - 6x + 25 \\
 & - \quad + \quad - \\
\hline
 & 2
\end{array}$$

$\therefore$ $2x^3 – 11x^2 + 44x + 27$
$= (x^2 – 6x + 25)(2x + 1) + 2$
= 0 . (2x + 1) + 2 ......[From (i)]
= 0 + 2
= 2

Maharashtra State Board 11th Commerce Maths Solutions Chapter 3 Complex Numbers Ex 3.2

Question 1.
Find the square root of the following complex numbers:
(i) -8 – 6i
Solution:
Let $\sqrt{-8-6i}$ = a + bi, where a, b ∈ R
Squaring on both sides, we get
$-8 – 6i = (a + bi)^2$
$-8 – 6i = a^2 + b^2i^2 + 2abi$
$-8 – 6i = (a^2 – b^2) + 2abi$ .....[$\because$ $i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = -8$ and $2ab = -6$

$\therefore \quad a^2 - b^2 = -8$ and $b = \frac{-3}{a}$

$\therefore \quad a^2 - \left(-\frac{3}{a}\right)^2 = -8$

$\therefore \quad a^2 - \frac{9}{a^2} = -8$

$\therefore \quad a^4 - 9 = -8a^2$

$\therefore \quad a^4 + 8a^2 - 9 = 0$

$\therefore \quad (a^2 + 9)(a^2 - 1) = 0$

$\therefore \quad a^2 = -9$ or $a^2 = 1$

But $a \in R$

$\therefore \quad a^2 \neq -9$

$\therefore \quad a^2 = 1$

$\therefore \quad a = \pm 1$

When $a = 1$, $b = \frac{-3}{1} = -3$

When $a = -1$, $b = \frac{-3}{-1} = 3$

$\therefore \quad \sqrt{-8-6i} = \pm(1 - 3i)$

(ii) 7 + 24i
Solution:
Let $\sqrt{7+24i}$ = a + bi, where a, b ∈ R
Squaring on both sides, we get
$7 + 24i = (a + bi)^2$
$7 + 24i = a^2 + b^2i^2 + 2abi$
$7 + 24i = (a^2 - b^2) + 2abi$ .....[∵ $i^2 = -1$]
Equating real and imaginary parts, we get

$a^2 - b^2 = 7$ and $2ab = 24$

$\therefore \quad a^2 - b^2 = 7$ and $b = \frac{12}{a}$

$\therefore \quad a^2 - \left(\frac{12}{a}\right)^2 = 7$

$\therefore \quad a^2 - \frac{144}{a^2} = 7$

$\therefore \quad a^4 - 144 = 7a^2$

$\therefore \quad a^4 - 7a^2 - 144 = 0$

$\therefore \quad (a^2 - 16)(a^2 + 9) = 0$

$\therefore \quad a^2 = 16$ or $a^2 = -9$

But $a \in R$

$\therefore \quad a^2 \neq -9$

$\therefore \quad a^2 = 16$

$\therefore \quad a = \pm 4$

When $a = 4$, $b = \frac{12}{4} = 3$

When $a = -4$, $b = \frac{12}{-4} = -3$

$\therefore \quad \sqrt{7+24i} = \pm(4 + 3i)$

(iii) 1 + 4√3i
Solution:
Let $\sqrt{1+4\sqrt{3}i}$ = a + bi, where a, b ∈ R
Squaring on both sides, we get
$1 + 4\sqrt{3}i = (a + bi)^2$
$1 + 4\sqrt{3}i = a^2 + b^2i^2 + 2abi$
$1 + 4\sqrt{3}i = (a^2 - b^2) + 2abi$ ......[∵ $i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 1$ and $2ab = 4\sqrt{3}$

$\therefore \quad a^2 - b^2 = 1$ and $b = \frac{2\sqrt{3}}{a}$

$\therefore \quad a^2 - \left(\frac{2\sqrt{3}}{a}\right)^2 = 1$

$\therefore \quad a^2 - \frac{12}{a^2} = 1$

$\therefore \quad a^4 - 12 = a^2$

$\therefore \quad a^4 - a^2 - 12 = 0$

$\therefore \quad (a^2 - 4)(a^2 + 3) = 0$

$\therefore \quad a^2 = 4$ or $a^2 = -3$

But $a \in R$

$\therefore \quad a^2 \neq -3$

$\therefore \quad a^2 = 4$

$\therefore \quad a = \pm 2$

When $a = 2$, $b = \frac{2\sqrt{3}}{2} = \sqrt{3}$

When $a = -2$, $b = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

$\therefore \quad \sqrt{1+4\sqrt{3}i} = \pm(2 + \sqrt{3}i)$

(iv) 3 + 2√10i

Solution:

Let 3+210−−√−−−−−−−−√i = a + bi, where a, b ∈ R

Squaring on both sides, we get

$3 + 2\sqrt{10}i = (a + bi)^2$

$3 + 2\sqrt{10}i = a^2 + b^2i^2 + 2abi$

$3 + 2\sqrt{10}i = (a^2 - b^2) + 2abi$ .....[∵ $i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 3$ and $2ab = 2\sqrt{10}$

$a^2 - b^2 = 3$ and b = 10√a

$\therefore \quad a^2 - \left(\frac{\sqrt{10}}{a}\right)^2 = 3$

$\therefore \quad a^2 - \frac{10}{a^2} = 3$

$\therefore \quad a^4 - 10 = 3a^2$

$\therefore \quad a^4 - 3a^2 - 10 = 0$

$\therefore \quad (a^2 - 5)(a^2 + 2) = 0$

$\therefore \quad a^2 = 5$ or $a^2 = -2$

But $a \in R$

$\therefore \quad a^2 \neq -2$

$\therefore \quad a^2 = 5$

$\therefore \quad a = \pm\sqrt{5}$

When $a = \sqrt{5}$, $b = \frac{\sqrt{10}}{\sqrt{5}} = \sqrt{2}$

When $a = -\sqrt{5}$, $b = \frac{\sqrt{10}}{-\sqrt{5}} = -\sqrt{2}$

$\therefore \quad \sqrt{3+2\sqrt{10}i} = \pm\left(\sqrt{5} + \sqrt{2}i\right)$

(v) 2(1 – √3i)

Solution:

Let 2(1−3−√i)−−−−−−−−−√ = a + bi, where a, b ∈ R

Squaring on both sides, we get

$2(1 - \sqrt{3}i) = (a + bi)^2$

$2(1 - \sqrt{3}i) = a^2 + b^2i^2 + 2abi$

$2 - 2\sqrt{3}i = (a^2 - b^2) + 2abi$ .....[∵ $i^2 = -1$]

Equating real and imaginary parts, we get

$$a^2 - b^2 = 2 \text{ and } 2ab = -2\sqrt{3}$$

$\therefore \quad a^2 - b^2 = 2 \text{ and } b = -\frac{\sqrt{3}}{a}$

$\therefore \quad a^2 - \left(-\frac{\sqrt{3}}{a}\right)^2 = 2$

$\therefore \quad a^2 - \frac{3}{a^2} = 2$

$\therefore \quad a^4 - 3 = 2a^2$

$\therefore \quad a^4 - 2a^2 - 3 = 0$

$\therefore \quad (a^2 - 3)(a^2 + 1) = 0$

$\therefore \quad a^2 = 3 \text{ or } a^2 = -1$

But $a \in R$

$\therefore \quad a^2 \neq -1$

$\therefore \quad a^2 = 3$

$\therefore \quad a = \pm\sqrt{3}$

When $a = \sqrt{3}$, $b = \frac{-\sqrt{3}}{\sqrt{3}} = -1$

When $a = -\sqrt{3}$, $b = \frac{-\sqrt{3}}{-\sqrt{3}} = 1$

$\therefore \quad \sqrt{2\left(1-\sqrt{3}i\right)} = \pm\left(\sqrt{3}-i\right)$

Question 2.
Solve the following quadratic equations.
(i) $8x^2 + 2x + 1 = 0$
Solution:
Given equation is $8x^2 + 2x + 1 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
a = 8, b = 2, c = 1
Discriminant = $b^2 - 4ac$
$= (2)^2 - 4 \times 8 \times 1$
$= 4 - 32$
$= -28 < 0$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{-28}}{2(8)}$$

$$= \frac{-2 \pm 2\sqrt{7}i}{16}$$

$$x = \frac{-1 \pm \sqrt{7}i}{8}$$

∴ the roots of the given equation are $\frac{-1+\sqrt{7}i}{8}$ and $\frac{-1-\sqrt{7}i}{8}$

(ii) $2x^2 - \sqrt{3}x + 1 = 0$
Solution:
Given equation is $2x^2 - \sqrt{3}x + 1 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
a = 2, b = $-\sqrt{3}$, c = 1
Discriminant = $b^2 - 4ac$
$= (-\sqrt{3})^2 - 4 \times 2 \times 1$
$= 3 - 8$
$= -5 < 0$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-\sqrt{3}) \pm \sqrt{-5}}{2(2)}$$

$$x = \frac{\sqrt{3} \pm \sqrt{5}\,i}{4}$$

∴ the roots of the given equation are $\frac{\sqrt{3}+\sqrt{5}i}{4}$ and $\frac{\sqrt{3}-\sqrt{5}i}{4}$

(iii) $3x^2 - 7x + 5 = 0$
Solution:
Given equation is $3x^2 - 7x + 5 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 3$, $b = -7$, $c = 5$
Discriminant = $b^2 - 4ac$
$= (-7)^2 - 4 \times 3 \times 5$
$= 49 - 60$
$= -11 < 0$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-7) \pm \sqrt{-11}}{2(3)}$$

$$x = \frac{7 \pm \sqrt{11}\,i}{6}$$

∴ the roots of the given equation are $\frac{7+\sqrt{11}i}{6}$ and $\frac{7-\sqrt{11}i}{6}$

(iv) $x^2 - 4x + 13 = 0$
Solution:
Given equation is $x^2 - 4x + 13 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = -4$, $c = 13$
Discriminant = $b^2 - 4ac$
$= (-4)^2 - 4 \times 1 \times 13$
$= 16 - 52$
$= -36 < 0$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{-36}}{2(1)}$$

$$= \frac{4 \pm 6i}{2} = 2 \pm 3i$$

∴ the roots of the given equation are 2 + 3i and 2 – 3i.

Question 3.
Solve the following quadratic equations.
(i) $x^2 + 3ix + 10 = 0$
Solution:
Given equation is $x^2 + 3ix + 10 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = 3i$, $c = 10$
Discriminant = $b^2 - 4ac$
$= (3i)^2 - 4 \times 1 \times 10$
$= 9i^2 - 40$
$= -9 - 40$ …..[∵ $i^2 = -1$]
$= -49$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3i \pm \sqrt{-49}}{2(1)}$$
$$x = \frac{-3i \pm 7i}{2}$$
$$x = \frac{-3i + 7i}{2} \text{ or } x = \frac{-3i - 7i}{2}$$

∴ x = 2i or x = -5i
∴ the roots of the given equation are 2i and -5i.
Check:
If x = 2i and x = -5i satisfy the given equation, then our answer is correct.
L.H.S. = $x^2 + 3ix + 10$
= $(2i)^2 + 3i(2i) + 10i$
= $4i^2 + 6i^2 + 10$
= $10i^2 + 10$
= -10 + 10 ……[∵ $i^2 = -1$]
= 0
= R.H.S.
L.H.S. = $x^2 + 3ix + 10$
= $(-5i)^2 + 3i(-5i) + 10$
= $25i^2 – 15i^2 + 10$
= $10i^2 + 10$
= -10 + 10 …..[∵ $i^2 = -1$]
= 0
= R.H.S.
Thus, our answer is correct.

(ii) $2x^2 + 3ix + 2 = 0$
Solution:
Given equation is $2x^2 + 3ix + 2 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
a = 2, b = 3i, c = 2
Discriminant = $b^2 – 4ac$
= $(3i)^2 – 4 × 2 × 2$
= $9i^2 – 16$
= -9 – 16
= -25 < 0
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3i \pm \sqrt{-25}}{2(2)}$$
$$x = \frac{-3i \pm 5i}{4}$$
$$x = \frac{-3i + 5i}{4} \text{ or } x = \frac{-3i - 5i}{4}$$
$$x = \frac{1}{2}i \text{ or } x = -2i$$

∴ the roots of the given equation are $\frac{1}{2}$i and -2i.

(iii) $x^2 + 4ix – 4 = 0$
Solution:
Given equation is $x^2 + 4ix – 4 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
a = 1, b = 4i, c = -4
Discriminant = $b^2 – 4ac$
= $(4i)^2 – 4 × 1 × -4$
= $16i^2 + 16$
= -16 + 16 …..[∵ $i^2 = -1$]
= 0
So, the given equation has equal roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4i \pm \sqrt{0}}{2(1)} = \frac{-4i}{2}$$
$$x = -2i$$

∴ the roots of the given equation are -2i and -2i.

(iv) $ix^2 – 4x – 4i = 0$
Solution:
$ix^2 – 4x – 4i = 0$
Multiplying throughout by i, we get
$i^2x^2 – 4ix – 4i^2 = 0$
∴ $-x^2 – 4ix + 4 = 0$ ……[∵ $i^2 = -1$]
∴ $x^2 + 4ix – 4 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
a = 1, b = 4i, c = -4
Discriminant = $b^2 – 4ac$
= $(4i)^2 – 4 × 1 × -4$
= $16i^2 + 16$
= -16 + 16 …..[∵ $i^2 = -1$]
= 0
So, the given equation has equal roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4i \pm \sqrt{0}}{2(1)} = \frac{-4i}{2}$$
$$x = -2i$$

∴ the roots of the given equation are -2i and -2i.

Question 4.
Solve the following quadratic equations.
(i) $x^2 – (2 + i) x – (1 – 7i) = 0$
Solution:
Given equation is $x^2 – (2 + i)x – (1 – 7i) = 0$
Comparing with $ax^2 + bx + c = 0$, we get
a = 1, b = -(2 + i), c = -(1 – 7i)
Discriminant = $b^2 – 4ac$
= $[-(2 + i)]^2 – 4 × 1 × -(1 – 7i)$
= $4 + 4i + i^2 + 4 – 28i$
= 4 + 4i – 1 + 4 – 28i …….[∵ $i^2 = -1$]
= 7 – 24i
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-[-(2+i)] \pm \sqrt{7-24i}}{2(1)}$$
$$= \frac{(2+i) \pm \sqrt{7-24i}}{2}$$

Let $\sqrt{7-24i} = a + bi$, where $a, b \in R$

Squaring on both sides, we get

$7 - 24i = a^2 + i^2b^2 + 2abi$

$\therefore \quad 7 - 24i = (a^2 - b^2) + 2abi \quad \ldots[\because i^2 = -1]$

Equating real and imaginary parts, we get

$a^2 - b^2 = 7$ and $2ab = -24$

$\therefore \quad a^2 - b^2 = 7$ and $b = \dfrac{-12}{a}$

$\therefore \quad a^2 - \left(\dfrac{-12}{a}\right)^2 = 7$

$\therefore \quad a^2 - \dfrac{144}{a^2} = 7$

$\therefore \quad a^4 - 144 = 7a^2$

$\therefore \quad a^4 - 7a^2 - 144 = 0$

$\therefore \quad (a^2 - 16)(a^2 + 9) = 0$

$\therefore \quad a^2 = 16$ or $a^2 = -9$

But $a \in R$

$\therefore \quad a^2 \neq -9$

$\therefore \quad a^2 = 16$

$\therefore \quad a = \pm 4$

When $a = 4$, $b = \dfrac{-12}{4} = -3$

When $a = -4$, $b = \dfrac{-12}{-4} = 3$

$\therefore \quad \sqrt{7-24i} = \pm(4 - 3i)$

$\therefore \quad x = \dfrac{(2+i) \pm (4-3i)}{2}$

$\therefore \quad x = \dfrac{(2+i)+(4-3i)}{2}$ or $x = \dfrac{(2+i)-(4-3i)}{2}$

$\therefore \quad x = 3 - i$ or $x = -1 + 2i$

(ii) $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

Solution:

Given equation is $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = -(3\sqrt{2} + 2i)$, $c = 6\sqrt{2}i$

Discriminant $= b^2 - 4ac$

$= [-(3\sqrt{2} + 2i)]^2 - 4 \times 1 \times 6\sqrt{2}i$

$= 18 + 12\sqrt{2}i + 4i^2 - 24\sqrt{2}i$

$= 18 - 12\sqrt{2}i - 4$ .....[$\because i^2 = -1$]

$= 14 - 12\sqrt{2}i$

So, the given equation has complex roots.

These roots are given by

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-[-(3\sqrt{2}+2i)] \pm \sqrt{14-12\sqrt{2}i}}{2(1)}$$

$$= \frac{(3\sqrt{2}+2i) \pm \sqrt{14-12\sqrt{2}i}}{2}$$

Let $\sqrt{14-12\sqrt{2}i} = a + bi$, where $a, b \in R$

Squaring on both sides, we get

$14 - 12\sqrt{2}\,i = a^2 + i^2b^2 + 2abi$

$14 - 12\sqrt{2}\,i = (a^2 - b^2) + 2abi \quad \ldots[\because i^2 = -1]$

Equating real and imaginary parts, we get

$a^2 - b^2 = 14$ and $2ab = -12\sqrt{2}$

$\therefore\quad a^2 - b^2 = 14$ and $b = \frac{-6\sqrt{2}}{a}$

$\therefore\quad a^2 - \left(\frac{-6\sqrt{2}}{a}\right)^2 = 14$

$\therefore\quad a^2 - \frac{72}{a^2} = 14$

$\therefore\quad a^4 - 72 = 14a^2$

$\therefore\quad a^4 - 14a^2 - 72 = 0$

$\therefore\quad (a^2 - 18)(a^2 + 4) = 0$

$\therefore\quad a^2 = 18$ or $a^2 = -4$

But $a \in R$

$\therefore\quad a^2 \neq -4$

$\therefore\quad a^2 = 18$

$\therefore\quad a = \pm 3\sqrt{2}$

When $a = 3\sqrt{2}$, $b = \frac{-6\sqrt{2}}{3\sqrt{2}} = -2$

When $a = -3\sqrt{2}$, $b = \frac{-6\sqrt{2}}{-3\sqrt{2}} = 2$

$\therefore\quad \sqrt{14-12\sqrt{2}i} = \pm\left(3\sqrt{2}-2i\right)$

$\therefore\quad x = \frac{(3\sqrt{2}+2i) \pm (3\sqrt{2}-2i)}{2}$

$\therefore\quad x = \frac{(3\sqrt{2}+2i) + (3\sqrt{2}-2i)}{2}$

or $x = \frac{(3\sqrt{2}+2i) - (3\sqrt{2}-2i)}{2}$

$\therefore\quad x = 3\sqrt{2}$ or $x = 2i$

(iii) $x^2 - (5 - i)x + (18 + i) = 0$
Solution:
Given equation is $x^2 - (5 - i)x + (18 + i) = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = -(5 - i)$, $c = 18 + i$
Discriminant $= b^2 - 4ac$
$= [-(5 - i)]^2 - 4 \times 1 \times (18 + i)$
$= 25 - 10i + i^2 - 72 - 4i$
$= 25 - 10i - 1 - 72 - 4i$ .....[$\because i^2 = -1$]
$= -48 - 14i$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-[-(5-i)] \pm \sqrt{-48-14i}}{2(1)} = \frac{(5-i) \pm \sqrt{-48-14i}}{2}$$

Let $\sqrt{-48-14i} = a + bi$, where a, $b \in R$

Squaring on both sides, we get

$-48 - 14i = a^2 + b^2i^2 + 2abi$

$-48 - 14i = (a^2 - b^2) + 2abi \quad \ldots[\because i^2 = -1]$

Equating real and imaginary parts, we get

$a^2 - b^2 = -48$ and $2ab = -14$

$\therefore \quad a^2 - b^2 = -48$ and $b = \frac{-7}{a}$

$\therefore \quad a^2 - \left(\frac{-7}{a}\right)^2 = -48$

$\therefore \quad a^2 - \frac{49}{a^2} = -48$

$\therefore \quad a^4 - 49 = -48a^2$

$\therefore \quad a^4 + 48a^2 - 49 = 0$

$\therefore \quad (a^2 + 49)(a^2 - 1) = 0$

$\therefore \quad a^2 = -49$ or $a^2 = 1$

But $a \in R$

$\therefore \quad a^2 \neq -49$

$\therefore \quad a^2 = 1$

$\therefore \quad a = \pm 1$

When $a = 1$, $b = -\frac{7}{1} = -7$

When $a = -1$, $b = \frac{-7}{-1} = 7$

$\therefore \quad \sqrt{-48-14i} = \pm(1 - 7i)$

$\therefore \quad x = \frac{(5-i) \pm (1-7i)}{2}$

$\therefore \quad x = \frac{5-i+1-7i}{2}$ or $x = \frac{5-i-1+7i}{2}$

$\therefore \quad x = 3 - 4i$ or $x = 2 + 3i$

(iv) $(2 + i)x^2 - (5 - i)x + 2(1 - i) = 0$

Solution:

Given equation is

$(2 + i)x^2 - (5 - i)x + 2(1 - i) = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 2 + i$, $b = -(5 - i)$, $c = 2(1 - i)$

Discriminant = $b^2 - 4ac$

$= [-(5 - i)]^2 - 4 \times (2 + i) \times 2(1 - i)$

$= 25 - 10i + i^2 - 8(2 + i)(1 - i)$

$= 25 - 10i + i^2 - 8(2 - 2i + i - i^2)$

$= 25 - 10i - 1 - 8(2 - i + 1)$ .....[$\because i^2 = -1$]

$= 25 - 10i - 1 - 16 + 8i - 8$

$= -2i$

So, the given equation has complex roots.

These roots are given by

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-[-(5-i)] \pm \sqrt{-2i}}{2(2+i)} = \frac{(5-i) \pm \sqrt{-2i}}{2(2+i)}$

Let $\sqrt{-2i} = a + bi$, where $a, b \in R$

Squaring on both sides, we get

$-2i = a^2 + b^2 i^2 + 2abi$

$0 - 2i = (a^2 - b^2) + 2abi \quad \ldots[\because i^2 = -1]$

Equating real and imaginary parts, we get

$a^2 - b^2 = 0$ and $2ab = -2$

$a^2 - b^2 = 0$ and $b = -\frac{1}{a}$

$\therefore \quad a^2 - \left(\frac{-1}{a}\right)^2 = 0$

$\therefore \quad a^2 - \frac{1}{a^2} = 0$

$\therefore \quad a^4 - 1 = 0$

$\therefore \quad (a^2 - 1)(a^2 + 1) = 0$

$\therefore \quad a^2 = 1$ or $a^2 = -1$

But $a \in R$

$\therefore \quad a^2 \neq -1$

$\therefore \quad a^2 = 1$

$\therefore \quad a = \pm 1$

When $a = 1$, $b = -1$

When $a = -1$, $b = 1$

$\therefore \quad \sqrt{-2i} = \pm(1 - i)$

$\therefore \quad x = \frac{(5-i) \pm (1-i)}{2(2+i)}$

$\therefore \quad x = \frac{5 - i + 1 - i}{2(2+i)}$ or $x = \frac{5 - i - 1 + i}{2(2+i)}$

$\therefore \quad x = \frac{6-2i}{2(2+i)}$ or $x = \frac{4}{2(2+i)}$

$\therefore \quad x = \frac{2(3-i)}{2(2+i)}$ or $x = \frac{2}{2+i}$

$\therefore \quad x = \frac{3-i}{2+i}$ or $x = \frac{2(2-i)}{(2+i)(2-i)}$

$\therefore \quad x = \frac{(3-i)(2-i)}{(2+i)(2-i)}$ or $x = \frac{2(2-i)}{4-i^2}$

$\therefore \quad x = \frac{6 - 5i + i^2}{4 - i^2}$ or $x = \frac{4-2i}{4-i^2}$

$\therefore \quad x = \frac{5-5i}{5}$ or $x = \frac{4-2i}{5} \quad \ldots[\because i^2 = -1]$

$\therefore \quad x = 1 - i$ or $x = \frac{4}{5} - \frac{2i}{5}$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 3 Complex Numbers Ex 3.3

Question 1.
If ω is a complex cube root of unity, show that
(i) $(2 - \omega)(2 - \omega^2) = 7$
(ii) $(2 + \omega + \omega^2)^3 - (1 - 3\omega + \omega^2)^3 = 65$
(iii) $\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = \omega^2$
Solution:
ω is the complex cube root of unity.
$\therefore \omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$
(i) L.H.S. $= (2 - \omega)(2 - \omega^2)$
$= 4 - 2\omega^2 - 2\omega + \omega^3$
$= 4 - 2(\omega^2 + \omega) + 1$
$= 4 - 2(-1) + 1$
$= 4 + 2 + 1$
$= 7$
= R.H.S.

(ii) L.H.S. $= (2 + \omega + \omega^2)^3 - (1 - 3\omega + \omega^2)^3$
$= [2 + (\omega + \omega^2)]^3 - [-3\omega + (1 + \omega^2)]^3$
$= (2 - 1)^3 - (-3\omega - \omega)^3$
$= 1^3 - (-4\omega)^3$
$= 1 + 64\omega^3$
$= 1 + 64(1)$
$= 65$
= R.H.S.

(iii) L.H.S. $= \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$
$= \frac{a\omega^3+b\omega^4+c\omega^2}{c+a\omega+b\omega^2}$ ......[$\because \omega^3 = 1, \omega^4 = \omega$]
$= \frac{\omega^2(c+a\omega+b\omega^2)}{c+a\omega+b\omega^2}$
$= \omega^2$
= R.H.S.

Question 2.
If ω is a complex cube root of unity, find the value of
(i) $\omega + \frac{1}{\omega}$
(ii) $\omega^2 + \omega^3 + \omega^4$
(iii) $(1 + \omega^2)^3$
(iv) $(1 - \omega - \omega^2)^3 + (1 - \omega + \omega^2)^3$
(v) $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$
Solution:
ω is the complex cube root of unity.
$\therefore \omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$
(i) $\omega + \frac{1}{\omega}$
$= \frac{\omega^2+1}{\omega}$
$= \frac{-\omega}{\omega}$
$= -1$

(ii) $\omega^2 + \omega^3 + \omega^4$
$= \omega^2 (1 + \omega + \omega^2)$
$= \omega^2 (0)$
$= 0$

(iii) $(1 + \omega^2)^3$
$= (-\omega)^3$
$= -\omega^3$
$= -1$

(iv) $(1 - \omega - \omega^2)^3 + (1 - \omega + \omega^2)^3$
$= [1 - (\omega + \omega^2)]^3 + [(1 + \omega^2) - \omega]^3$
$= [1 - (-1)]^3 + (-\omega - \omega)^3$
$= 2^3 + (-2\omega)^3$
$= 8 - 8\omega^3$
$= 8 - 8(1)$
$= 0$

(v) $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$
$= (1 + \omega)(1 + \omega^2)(1 + \omega)(1 + \omega^2)$ .....[$\because \omega^3 = 1, \omega^4 = \omega$]
$= (-\omega^2)(-\omega)(-\omega^2)(-\omega)$
$= \omega^6$
$= (\omega^3)^2$
$= (1)^2$
$= 1$

Question 3.
If α and β are the complex cube roots of unity, show that $\alpha^2 + \beta^2 + \alpha\beta = 0$.
Solution:
α and β are the complex cube roots of unity.

$$\therefore \quad \alpha = \frac{-1+i\sqrt{3}}{2} \text{ and } \beta = \frac{-1-i\sqrt{3}}{2}$$

$$\therefore \quad \alpha\beta = \left(\frac{-1+i\sqrt{3}}{2}\right)\left(\frac{-1-i\sqrt{3}}{2}\right)$$

$$= \frac{(-1)^2 - \left(i\sqrt{3}\right)^2}{4} = \frac{1-(-1)(3)}{4} \quad \ldots[\because i^2 = -1]$$

$$= \frac{1+3}{4}$$

$$\therefore \quad \alpha\beta = 1$$

$$\text{Also, } \alpha + \beta = \frac{-1+i\sqrt{3}}{2} + \frac{-1-i\sqrt{3}}{2}$$

$$= \frac{-1+i\sqrt{3}-1-i\sqrt{3}}{2}$$

$$= \frac{-2}{2}$$

$\therefore \alpha - \beta = -1$
L.H.S. $= \alpha^2 + \beta^2 + \alpha\beta$
$= \alpha^2 + 2\alpha\beta + \beta^2 + \alpha\beta - 2\alpha\beta$ ......[Adding and subtracting $2\alpha\beta$]
$= (\alpha^2 + 2\alpha\beta + \beta^2) - \alpha\beta$
$= (\alpha + \beta)^2 - \alpha\beta$
$= (-1)^2 - 1$
$= 1 - 1$
$= 0$
= R.H.S.

Question 4.
If $x = a + b$, $y = \alpha a + \beta b$ and $z = a\beta + b\alpha$, where α and β are the complex cube roots of unity, show that $xyz = a^3 + b^3$.
Solution:
$x = a + b$, $y = \alpha a + \beta b$, $z = a\beta + b\alpha$
α and β are the complex cube roots of unity.

$\therefore \quad \alpha = \frac{-1+i\sqrt{3}}{2}$ and $\beta = \frac{-1-i\sqrt{3}}{2}$

$\therefore \quad \alpha\beta = \left(\frac{-1+i\sqrt{3}}{2}\right)\left(\frac{-1-i\sqrt{3}}{2}\right)$

$= \frac{(-1)^2-(i\sqrt{3})^2}{4}$

$= \frac{1-(-1)(3)}{4} \quad \ldots[\because i^2=-1]$

$= \frac{1+3}{4}$

$\therefore \quad \alpha\beta = 1$

Also, $\alpha + \beta = \frac{-1+i\sqrt{3}}{2} + \frac{-1-i\sqrt{3}}{2}$

$= \frac{-1+i\sqrt{3}-1-i\sqrt{3}}{2}$

$= \frac{-2}{2}$

$\therefore \quad \alpha + \beta = -1$

L.H.S. $= xyz = (a + b)(\alpha a + \beta b)(a\beta + b\alpha)$

$= (a + b)(\alpha\beta a^2 + \alpha^2 ab + \beta^2 ab + \alpha\beta b^2)$

$= (a + b)[1.(a^2) + (\alpha^2 + \beta^2)ab + 1.(b^2)]$

$= (a + b)\{a^2 + [(\alpha + \beta)^2 - 2\alpha\beta]ab + b^2\}$

$= (a + b)\ \{a^2 + [(-1)^2 - 2(1)]ab + b^2\}$

$= (a + b)\ [a^2 + (1 - 2)ab + b^2]$

$= (a + b)(a^2 - ab + b^2)$

$= a^3 + b^3$

= R.H.S.

Question 5.
If ω is a complex cube root of unity, then prove the following:
(i) $(\omega^2 + \omega - 1)^3 = -8$
(ii) $(a + b) + (a\omega + b\omega^2) + (a\omega^2 + b\omega) = 0$
Solution:
ω is the complex cube root of unity.
$\therefore \omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$
(i) L.H.S. $= (\omega^2 + \omega - 1)^3$
$= (-1 - 1)^3$
$= (-2)^3$
$= -8$
= R.H.S.

(ii) L.H.S. $= (a + b) + (a\omega + b\omega^2) + (a\omega^2 + b\omega)$
$= (a + a\omega + a\omega^2) + (b + b\omega + b\omega^2)$
$= a(1 + \omega + \omega^2) + b(1 + \omega + \omega^2)$
$= a(0) + b(0)$
$= 0$
= R.H.S.

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