Maharashtra State Board 12th Maths Solutions Chapter 5 Application of Definite Integration Ex 5.1

1. Find the area of the region bounded by the following curves, X-axis, and the given lines:

(i) y = 2x, x = 0, x = 5.
Solution:
Required area = $\int_0^5 y\,dx$, where y = 2x
= $\int_0^5 2x\,dx$
= $\left[\frac{2x^2}{2}\right]_0^5$
= 25 – 0
= 25 sq units.

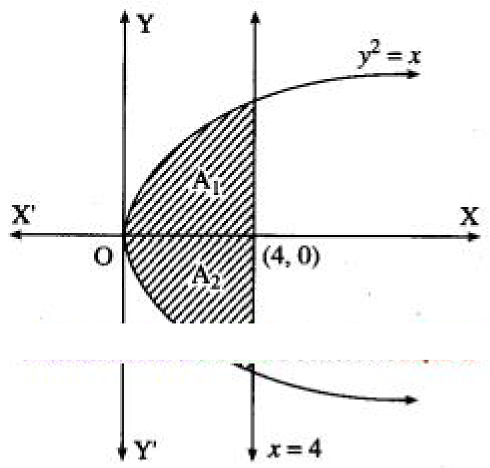
(ii) x = 2y, y = 0, y = 4.
Solution:
Required area = $\int_0^4 x\,dy$, where x = 2y
= $\int_0^4 2y\,dy$
= $\left[\frac{2y^2}{2}\right]_0^4$
= 16 – 0
= 16 sq units.

(iii) x = 0, x = 5, y = 0, y = 4.
Solution:
Required area = $\int_0^5 y\,dx$, where y = 4
= $\int_0^5 4\,dx$
= $[4x]_0^5$
= 20 – 0
= 20 sq units.

(iv) y = sin x, x = 0, x = $\frac{\pi}{2}$
Solution:
Required area = $\int_0^{\pi/2} y\,dx$, where y = sin x
= $\int_0^{\pi/2} \sin x\,dx$
= $[-\cos x]_0^{\pi/2}$
= -cos $\frac{\pi}{2}$ + cos 0
= 0 + 1
= 1 sq unit.

(v) xy = 2, x = 1, x = 4.
Solution:
For xy = 2, y = $\frac{2}{x}$
Required area = $\int_1^4 y\,dx$, where y = $\frac{2}{x}$
= $\int_1^4 \frac{2}{x}\,dx$
= $[2\log|x|]_1^4$
= 2 log 4 – 2 log 1
= 2 log 4 – 0
= 2 log 4 sq units.

(vi) y^2 = x, x = 0, x = 4.
Solution:



The required area consists of two bounded regions A1 and A2 which are equal in areas.
For $y^2 = x$, $y = \sqrt{x}$
Required area = $A_1 + A_2 = 2A_1$

$$= 2\int_0^4 y\, dx, \text{ where } y = \sqrt{x}$$

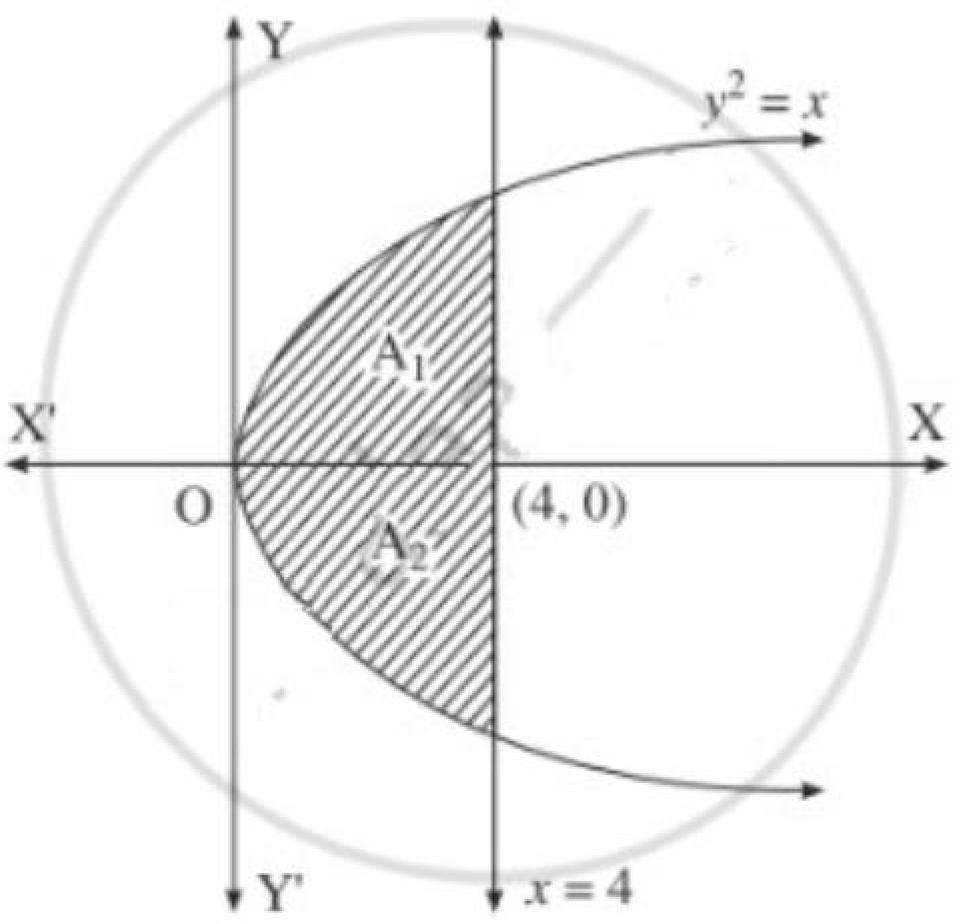
$$= 2\int_0^4 \sqrt{x}\, dx$$

$$= 2\left[\frac{x^{3/2}}{3/2}\right]_0^4$$

$$= 2\left[\frac{2}{3}(4)^{3/2} - 0\right]$$

$$= 2\left[\frac{2}{3}(2^2)^{3/2}\right] = \frac{32}{3} \text{ sq units}$$

(vii) $y^2 = 16x$, $x = 0$, $x = 4$.
Solution:



The required area consists of two bounded regions A1 and A2 which are equal in areas.
For $y^2 = x$, $y = \sqrt{x}$

Required area = $A_1 + A_2 = 2A_1$

$$= 2\int_0^4 y \cdot dx, \text{ where } y = \sqrt{x}$$

$$= 2\int_0^4 \sqrt{x} \cdot dx$$

$$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$$

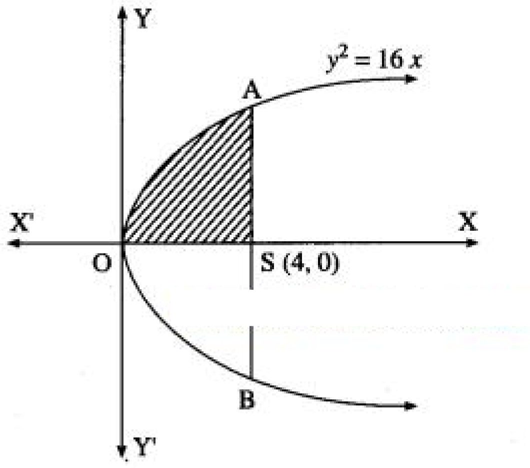
$$= 2\left[\frac{2}{3}(4)^{\frac{3}{2}} - 0\right]$$

$$= 2\left[\frac{2}{3}(2^2)^{\frac{3}{2}}\right]$$

$$= \frac{128}{3} \text{ sq units.}$$

2. Find the area of the region bounded by the parabola:

(i) $y^2 = 16x$ and its latus rectum.
Solution:
Comparing $y^2 = 16x$ with $y^2 = 4ax$, we get
$4a = 16$
$\therefore a = 4$
$\therefore$ focus is $S(a, 0) = (4, 0)$



For $y^2 = 16x$, $y = 4\sqrt{x}$
Required area = area of the region OBSAO
= 2 [area of the region OSAO]

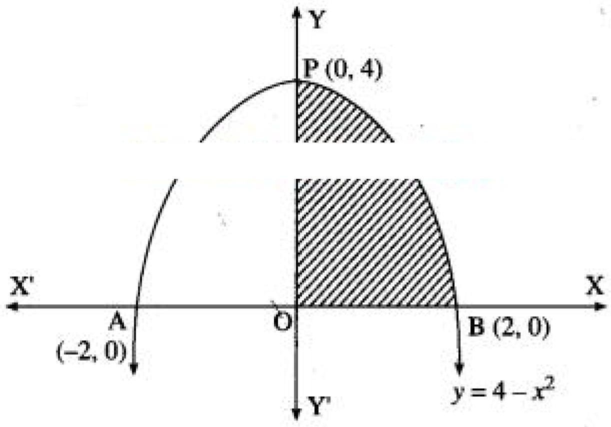
$$= 2\int_0^4 y\, dx, \text{ where } y = 4\sqrt{x}$$

$$= 2\int_0^4 4\sqrt{x}\, dx$$

$$= 8\left[\frac{x^{3/2}}{3/2}\right]_0^4 = 8\left[\frac{2}{3}(4)^{\frac{3}{2}} - 0\right]$$

$$= 8\left[\frac{2}{3}(2^2)^{\frac{3}{2}}\right] = \frac{128}{3} \text{ sq units.}$$

(ii) $y = 4 - x^2$ and the X-axis.
Solution:
The equation of the parabola is $y = 4 - x^2$
$\therefore x^2 = 4 - y$
i.e. $(x - 0)^2 = -(y - 4)$
It has vertex at P(0, 4)
For points of intersection of the parabola with X-axis,
we put y = 0 in its equation.
$\therefore 0 = 4 - x^2$
$\therefore x^2 = 4$
$\therefore x = \pm 2$
$\therefore$ the parabola intersect the X-axis at A(-2, 0) and B(2, 0)



Required area = area of the region APBOA
= 2[area of the region OPBO]

$$= 2\int_0^2 y\,dx, \text{ where } y = 4 - x^2$$

$$= 2\int_0^2 (4 - x^2)\,dx$$

$$= 8\int_0^2 1\,dx - 2\int_0^2 x^2\,dx$$

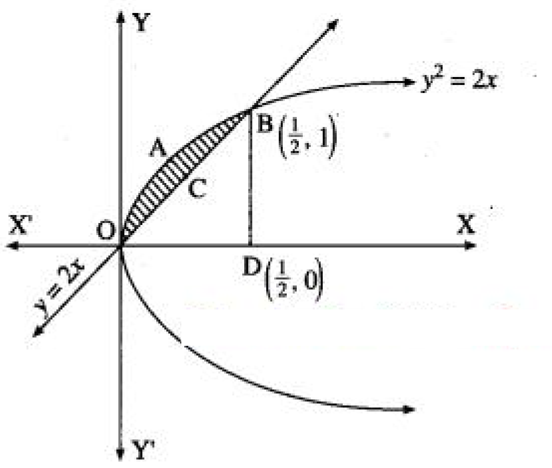
$$= 8[x]_0^2 - 2\left[\frac{x^3}{3}\right]_0^2$$

$$= 8(2-0) - \frac{2}{3}(8-0)$$

$$= 16 - \frac{16}{3} = \frac{32}{3} \text{ sq units.}$$

3. Find the area of the region included between:

(i) $y^2 = 2x$ and $y = 2x$.
Solution:
The vertex of the parabola $y^2 = 2x$ is at the origin O = (0, 0).



To find the points of intersection of the line and the parabola, equaling the values of 2x from both the equations we get,
$y^2 = y$
∴ $y^2 - y = 0$
∴ y = 0 or y = 1
When y = 0, $x = \frac{0}{2} = 0$
When y = 1, $x = \frac{1}{2}$
∴ the points of intersection are 0(0, 0) and B($\frac{1}{2}$, 1)
Required area = area of the region OABCO = area of the region OABDO – area of the region OCBDO
Now, area of the region OABDO = area under the parabola $y^2 = 2x$ between x = 0 and $x = \frac{1}{2}$

$$= \int_0^{1/2} y\,dx, \text{ where } y = \sqrt{2x}$$

$$= \int_0^{1/2} \sqrt{2x}\,dx = \sqrt{2}\left[\frac{x^{\frac{3}{2}}}{3/2}\right]_0^{1/2}$$

$$= \sqrt{2}\left[\frac{2}{3}\left(\frac{1}{2}\right)^{3/2} - 0\right] = \sqrt{2}\left[\frac{2}{3}\cdot\frac{1}{2\sqrt{2}}\right] = \frac{1}{3}$$

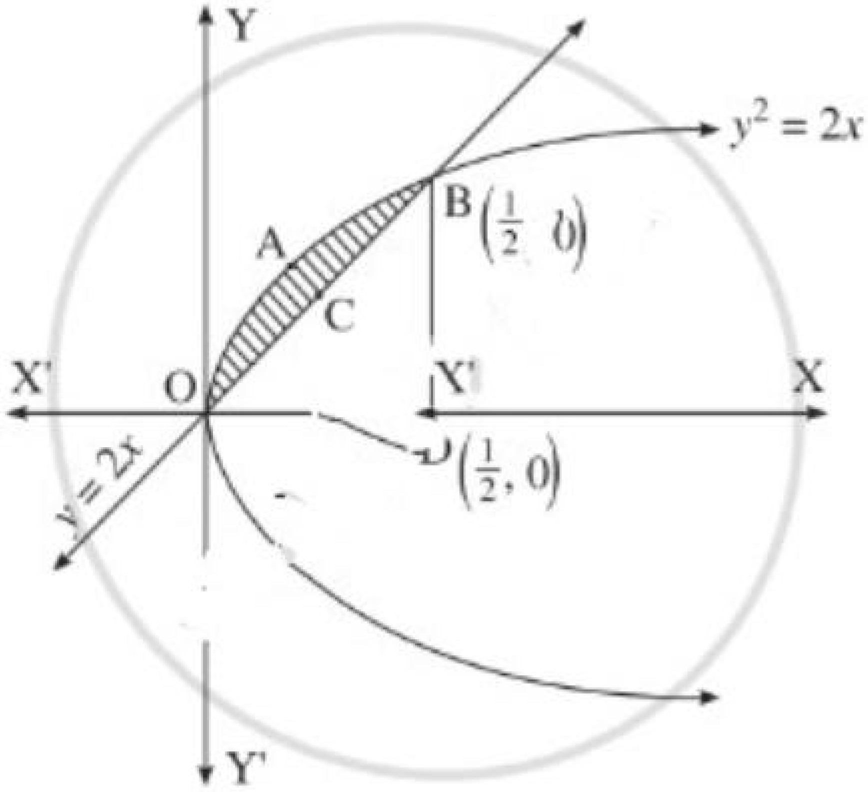
Area of the region OCBDO = area under the line y = 2x between x = 0 and x = 12

$$= \int_0^{1/2} y\,dx, \text{ where } y = 2x$$

$$= \int_0^{1/2} 2x\,dx = \left[\frac{2x^2}{2}\right]_0^{1/2} = \frac{1}{4} - 0 = \frac{1}{4}$$

$$\therefore \text{ required area} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \text{ sq unit.}$$

(ii) $y^2 = 4x$ and y = x.
Solution:
The vertex of the parabola $y^2 = 4x$ is at the origin O = (0, 0).



To find the points of intersection of the line and the parabola, equaling the values of 4x from both the equations we get,
∴ $y^2 = y$
∴ $y^2 - y = 0$
∴ y(y – 1) = 0
∴ y = 0 or y = 1
When y = 0, x = 02 = 0
When y = 1, x = 12
∴ the points of intersection are O(0, 0) and B(12, 1)
Required area = area of the region OABCO = area of the region OABDO – area of the region OCBDO
Now, area of the region OABDO = area under the parabola $y^2 = 4x$ between x = 0 and x = 12

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = \sqrt{x}x$$

$$= \int_0^{\frac{1}{2}} \sqrt{2}x dx$$

$$= \sqrt{2}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{1}{2}}$$

$$= \sqrt{2}\left[\frac{2}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}} - 0\right]$$

$$= \sqrt{2}\left[\frac{2}{3} \cdot \frac{1}{2\sqrt{2}}\right]$$

$$= \frac{1}{3}$$

Area of the region OCBDO = area under the line y = 2x between x = 0 and x = 12

$= \int_0^{\frac{1}{2}} y \cdot dx$, where $y = x$

$= \int_0^{\frac{1}{2}} 2x \cdot dx$

$= \left[\frac{2x^2}{2}\right]_0^{\frac{1}{2}}$

$= \frac{4}{1} - 0$

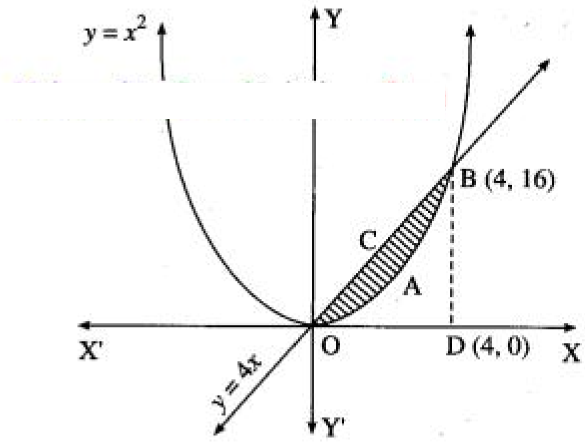
$= \frac{4}{3}$

∴ required area

$= \frac{4}{1} = \frac{4}{3}$

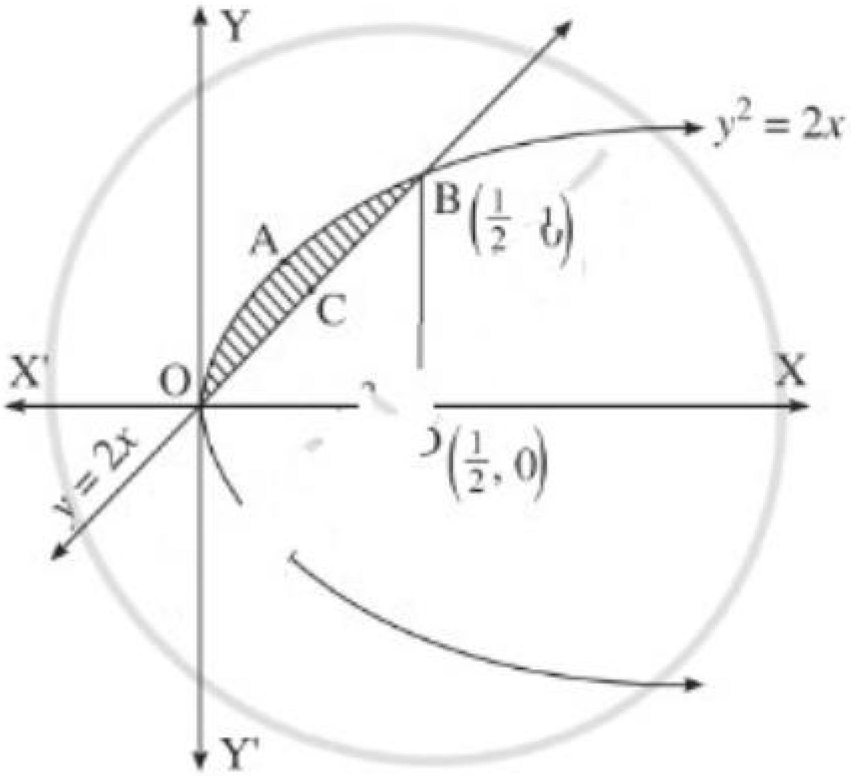
$= \frac{8}{3}$ sq units.

(iii) y = x2 and the line y = 4x.
Solution:
The vertex of the parabola y = x2 is at the origin 0(0, 0)
To find the points of the intersection of a line and the parabola.



Equating the values of y from the two equations, we get
x2 = 4x
∴ x2 – 4x = 0
∴ x(x – 4) = 0
∴ x = 0, x = 4
When x = 0, y = 4(0) = 0
When x = 4, y = 4(4) = 16
∴ the points of intersection are 0(0, 0) and B(4, 16)
Required area = area of the region OABCO = (area of the region ODBCO) – (area of the region ODBAO)
Now, area of the region ODBCO = area under the line y = 4x between x = 0 and x = 4
= ∫40ydx, where y = 4x
= ∫404xdx
= 4∫40xdx
= 4[latex]∫40xdx[/latex]
= 2(16 – 0)
= 32
Area of the region ODBAO = area under the parabola y = x2 between x = 0 and x = 4
= ∫40ydx, where y = x2
= ∫40x2dx
= [x33]40
= 13 (64 – 0)
= 643
∴ required area = 32 – 643 = 323 sq units.

(iv) $y^2 = 4ax$ and $y = x$.
Solution:
The vertex of the parabola $y^2 = 4ax$ is at the origin O = (0, 0).



To find the points of intersection of the line and the parabola, equaling the values of 4ax from both the equations we get,
∴ $y^2 = y$
∴ $y^2 - y = 0$
∴ $y(y - 1) = 0$
∴ y = 0 or y = 1
When y = 0, x = 02 = 0
When y = 1, x = 12
∴ the points of intersection are O(0, 0) and B(12, 1)
Required area = area of the region OABCO = area of the region OABDO – area of the region OCBDO
Now, area of the region OABDO
= area under the parabola $y^2 = 4ax$ between x = 0 and x = 12

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = \sqrt{2}x$$

$$= \int_0^{\frac{1}{2}} \sqrt{2}x dx$$

$$= \sqrt{2}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{1}{2}}$$

$$= \sqrt{2}\left[\frac{2}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}} - 0\right]$$

$$= \sqrt{2}\left[\frac{2}{3} \cdot \frac{1}{2\sqrt{2}}\right]$$

$$= \frac{1}{3}$$

Area of the region OCBDO
= area under the line y

= 4ax between x = 0 and x = $14ax$

$= \int_0^{\frac{1}{2}} y \cdot dx$, where $y = x$

$= \int_0^{\frac{1}{2}} 2x \cdot dx$

$= \left[\frac{2x^2}{2}\right]_0^{\frac{1}{2}}$

$= \frac{4}{3} - 0$

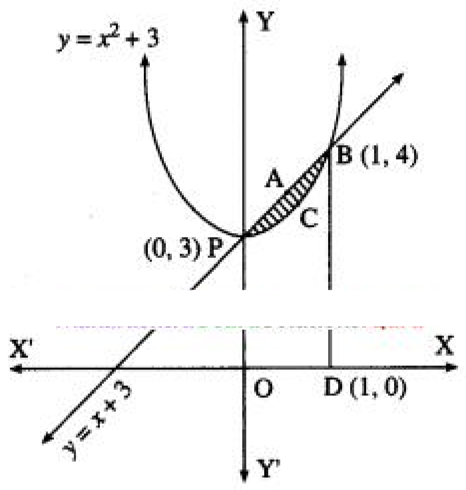
$= \frac{2a^2}{1}$

∴ required area

$= \frac{4}{3} = \frac{2a^2}{1}$

$= \frac{8a^2}{3}$ sq units.

(v) $y = x^2 + 3$ and $y = x + 3$.
Solution:
The given parabola is $y = x^2 + 3$, i.e. $(x - 0)^2 = y - 3$
∴ its vertex is P(0, 3).



To find the points of intersection of the line and the parabola.
Equating the values of y from both the equations, we get
$x^2 + 3 = x + 3$
∴ $x^2 - x = 0$
∴ $x(x - 1) = 0$
∴ x = 0 or x = 1
When x = 0, y = 0 + 3 = 3
When x = 1, y = 1 + 3 = 4
∴ the points of intersection are P(0, 3) and B(1, 4)
Required area = area of the region PABCP = area of the region OPABDO – area of the region OPCBDO
Now, area of the region OPABDO
= area under the line y = x + 3 between x = 0 and x = 1

$= \int_0^1 y\, dx$, where $y = x + 3$

$= \int_0^1 (x + 3)\, dx$

$= \int_0^1 x\, dx + 3\int_0^1 1 dx = \left[\frac{x^2}{2}\right]_0^1 + 3[x]_0^1$

$= \left(\frac{1}{2} - 0\right) + 3(1 - 0) = \frac{7}{2}$

Area of the region OPCBDO = area under the parabola $y = x^2 + 3$ between x = 0 and x = 1

$$= \int_0^1 y\,dx, \quad \text{where } y = x^2 + 3$$

$$= \int_0^1 (x^2+3)\,dx = \int_0^1 x^2\,dx + 3\int_0^1 1dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 + 3[x]_0^1 = \left(\frac{1}{3} - 0\right) + 3(1-0) = \frac{10}{3}$$

$$\therefore \text{ required area} = \frac{7}{2} - \frac{10}{3} = \frac{21-20}{6} = \frac{1}{6} \text{ sq unit.}$$

Maharashtra State Board 12th Maths Solutions Chapter 5 Application of Definite Integration Miscellaneous Exercise 5

I. Choose the correct option from the given alternatives:

Question 1.
The area bounded by the region 1 ≤ x ≤ 5 and 2 ≤ y ≤ 5 is given by
(a) 12 sq units
(b) 8 sq units
(c) 25 sq units
(d) 32 sq units
Answer:
(a) 12 sq units

Question 2.
The area of the region enclosed by the curve y = 1x, and the lines x = e, x = e^2 is given by
(a) 1 sq unit
(b) 12 sq units
(c) 32 sq units
(d) 52 sq units
Answer:
(a) 1 sq unit

Question 3.
The area bounded by the curve $y = x^3$, the X-axis and the lines x = -2 and x = 1 is
(a) -9 sq units
(b) −154 sq units
(c) 154 sq units
(d) 174 sq units
Answer:
(c) 154 sq units

Question 4.
The area enclosed between the parabola $y^2 = 4x$ and line y = 2x is
(a) 23 sq units
(b) 13 sq units
(c) 14 sq units
(d) 34 sq units
Answer:
(b) 13 sq units

Question 5.
The area of the region bounded between the line x = 4 and the parabola $y^2 = 16x$ is
(a) 1283 sq units
(b) 1083 sq units
(c) 1183 sq units
(d) 2183 sq units
Answer:
(a) 1283 sq units

Question 6.
The area of the region bounded by y = cos x, Y-axis and the lines x = 0, x = 2π is
(a) 1 sq unit
(b) 2 sq units
(c) 3 sq units
(d) 4 sq units
Answer:
(d) 4 sq units

Question 7.
The area bounded by the parabola $y^2 = 8x$, the X-axis and the latus rectum is
(a) 313 sq units
(b) 323 sq units
(c) 322√3 sq units
(d) 163 sq units
Answer:
(b) 323 sq units

Question 8.
The area under the curve y = 2√x, enclosed between the lines x = 0 and x = 1 is
(a) 4 sq units
(b) 34 sq units
(c) 23 sq units
(d) 43 sq units
Answer:
(d) 43 sq units

Question 9.
The area of the circle $x^2 + y^2 = 25$ in first quadrant is
(a) 25π3 sq units
(b) 5π sq units
(c) 5 sq units
(d) 3 sq units
Answer:
(a) 25π3 sq units

Question 10.
The area of the region bounded by the ellipse $x2a2+y2b2=1$ is
(a) ab sq units
(b) πab sq units
(c) πab sq units ab
(d) πa^2 sq units
Answer:
(b) πab sq units

Question 11.
The area bounded by the parabola $y^2 = x$ and the line 2y = x is
(a) 43 sq units
(b) 1 sq unit
(c) 23 sq unit
(d) 13 sq unit
Answer:
(a) 43 sq units

Question 12.
The area enclosed between the curve y = cos 3x, 0 ≤ x ≤ $\pi 6$ and the X-axis is
(a) 12 sq unit
(b) 1 sq unit
(c) 23 sq unit
(d) 13 sq unit
Answer:
(d) 13 sq unit

Question 13.
The area bounded by y = √x and line x = 2y + 3, X-axis in first quadrant is
(a) 2√3 sq units
(b) 9 sq units
(c) 343 sq units
(d) 18 sq units
Answer:
(b) 9 sq units

Question 14.
The area bounded by the ellipse $x2a2+y2b2=1$ and the line $xa+yb=1$ is
(a) (πab – 2ab) sq units
(b) $\pi ab4-ab2$ sq units
(c) (πab – ab) sq units
(d) πab sq units
Answer:
(b) $\pi ab4-ab2$ sq units

Question 15.
The area bounded by the parabola y = x^2 and the line y = x is
(a) 12 sq unit
(b) 13 sq unit
(c) 16 sq unit
(d) 112 sq unit
Answer:
(c) 16 sq unit

Question 16.
The area enclosed between the two parabolas y^2 = 4x and y = x is
(a) 83 sq units
(b) 323 sq units
(c) 163 sq units
(d) 43 sq units
Answer:
(c) 163 sq units

Question 17.
The area bounded by the curve y = tan x, X-axis and the line x = $\pi 4$ is
(a) 13 log 2 sq units
(b) log 2 sq units
(c) 2 log 2 sq units
(d) 3 log 2 sq units
Answer:
(a) 13 log 2 sq units

Question 18.
The area of the region bounded by x^2 = 16y, y = 1, y = 4 and x = 0 in the first quadrant, is
(a) 73 sq units
(b) 83 sq units
(c) 643 sq units
(d) 563 sq units
Answer:
(d) 563 sq units

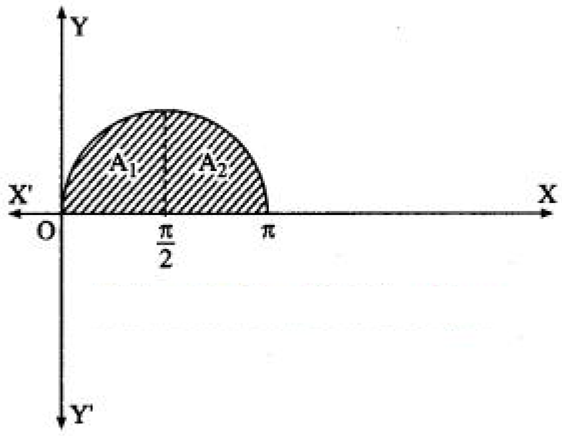
Question 19.
The area of the region included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, $(a > 0)$ is given by
(a) $\frac{16a^2}{3}$ sq units
(b) $\frac{8a^2}{3}$ sq units
(c) $\frac{4a^2}{3}$ sq units
(d) $\frac{32a^2}{3}$ sq units
Answer:
(a) $\frac{16a^2}{3}$ sq units

Question 20.
The area of the region included between the line $x + y = 1$ and the circle $x^2 + y^2 = 1$ is
(a) $\frac{\pi}{2}-1$ sq units
(b) $\pi - 2$ sq units
(c) $\frac{\pi}{4}-\frac{1}{2}$ sq units
(d) $\pi - \frac{1}{2}$ sq units
Answer:
(c) $\frac{\pi}{4}-\frac{1}{2}$ sq units

(II) Solve the following:

Question 1.
Find the area of the region bounded by the following curve, the X-axis and the given lines:
(i) $0 \le x \le 5$, $0 \le y \le 2$
(ii) $y = \sin x$, $x = 0$, $x = \pi$
(iii) $y = \sin x$, $x = 0$, $x = \frac{\pi}{3}$
Solution:
(i) Required area = $\int_0^5 y\,dx$, where $y = 2$
$= \int_0^5 2\,dx$
$= [2x]_0^5$
$= 2 \times 5 - 0$
$= 10$ sq units.

(ii) The curve $y = \sin x$ intersects the X-axis at $x = 0$ and $x = \pi$ between $x = 0$ and $x = \pi$.



Two bounded regions A_1 and A_2 are obtained. Both the regions have equal areas.
∴ required area = $A_1 + A_2 = 2A_1$

$$= 2\int_0^{\pi/2} y\,dx, \text{ where } y = \sin x$$
$$= 2\int_0^{\pi/2} \sin x\,dx$$
$$= 2[-\cos x]_0^{\pi/2}$$
$$= 2\left[-\cos\frac{\pi}{2} + \cos 0\right]$$
$$= 2(-0 + 1) = 2 \text{ sq units.}$$

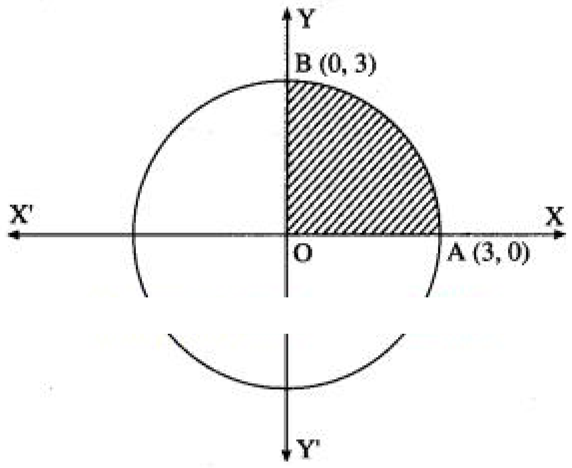
(iii) Required area = $\int_0^{\pi/3} y\,dx$, where y = sin x

$$= \int_0^{\pi/3} \sin x\, dx$$

$$= \Big[-\cos x\Big]_0^{\pi/3} = -\cos\frac{\pi}{3} + \cos 0$$

$$= -\frac{1}{2} + 1 = \frac{1}{2} \text{ sq unit.}$$

Question 2.
Find the area of the circle $x^2 + y^2 = 9$, using integration.
Solution:
By the symmetry of the circle, its area is equal to 4 times the area of the region OABO.
Clearly, for this region, the limits of integration are 0 and 3.



From the equation of the circle, $y^2 = 9 - x^2$.
In the first quadrant, y > 0
∴ y = $\sqrt{9-x^2}$
∴ area of the circle = 4 (area of the region OABO)

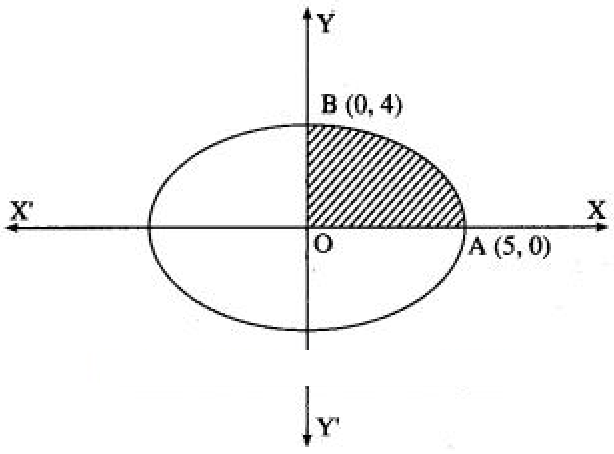
$$= 4\int_0^3 y\, dx = 4\int_0^3 \sqrt{9-x^2}\, dx$$

$$= 4\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$$

$$= 4\left[\frac{3}{2}\sqrt{9-9} + \frac{9}{2}\sin^{-1}\left(\frac{3}{3}\right)\right] - 4\left[\frac{0}{2}\sqrt{9-0} + \frac{9}{2}\sin^{-1}(0)\right]$$

$$= 4\cdot\frac{9}{2}\cdot\frac{\pi}{2} = 9\pi \text{ sq units.}$$

Question 3.
Find the area of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ using integration.
Solution:



By the symmetry of the ellipse, its area is equal to 4 times the area of the region OABO.
Clearly, for this region, the limits of integration are 0 and 5.
From the equation of the ellipse

$\frac{y^2}{16} = 1 - \frac{x^2}{25} = \frac{25-x^2}{25}$

∴ $y^2 = \frac{16}{25}(25 - x^2)$

In the first quadrant y > 0

∴ y = $\frac{4}{5}\sqrt{25-x^2}$

∴ area of the ellipse = 4(area of the region OABO)

$$= 4\int_0^5 y\,dx = 4\int_0^5 \frac{4}{5}\sqrt{25-x^2}\,dx$$

$$= \frac{16}{5}\int_0^5 \sqrt{25-x^2}\,dx$$

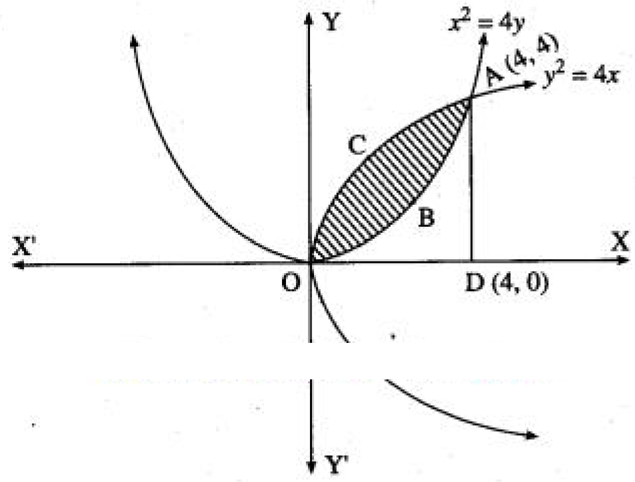
$$= \frac{16}{5}\left[\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5$$

$$= \frac{16}{5}\left(\frac{5}{2}\sqrt{25-25} + \frac{25}{2}\sin^{-1}(1)\right) - \frac{16}{5}\left[\frac{5}{2}\sqrt{25-0} + \frac{25}{2}\sin^{-1}(0)\right]$$

$$= \frac{16}{5} \times \frac{25}{2} \times \frac{\pi}{2}$$

$$= 20\pi \text{ sq units.}$$

Question 4.
Find the area of the region lying between the parabolas:
(i) $y^2 = 4x$ and $x^2 = 4y$
(ii) $4y^2 = 9x$ and $3x^2 = 16y$
(iii) $y^2 = x$ and $x^2 = y$.
Solution:
(i)



For finding the points of intersection of the two parabolas, we equate the values of y^2 from their equations.
From the equation $x^2 = 4y$, $y = \frac{x^2}{4}$
$y = \frac{x^4}{16}$
$\frac{x^4}{16} = 4x$
∴ $x^4 - 64x = 0$
∴ $x(x^3 - 64) = 0$
∴ x = 0 or $x^3 = 64$ i.e. x = 0 or x = 4
When x = 0, y = 0
When x = 4, y = $\frac{4^2}{4}$ = 4
∴ the points of intersection are 0(0, 0) and A(4, 4).
Required area = area of the region OBACO = [area of the region ODACO] – [area of the region ODABO]
Now, area of the region ODACO = area under the parabola $y^2 = 4x$, i.e. y = 2√x between x = 0 and x = 4

$$= \int_0^4 2\sqrt{x}\,dx = \left[2\frac{x^{3/2}}{3/2}\right]_0^4$$

$$= 2 \times \frac{2}{3} \times 4^{3/2} - 0 = \frac{4}{3} \times (2^3) = \frac{32}{3}$$

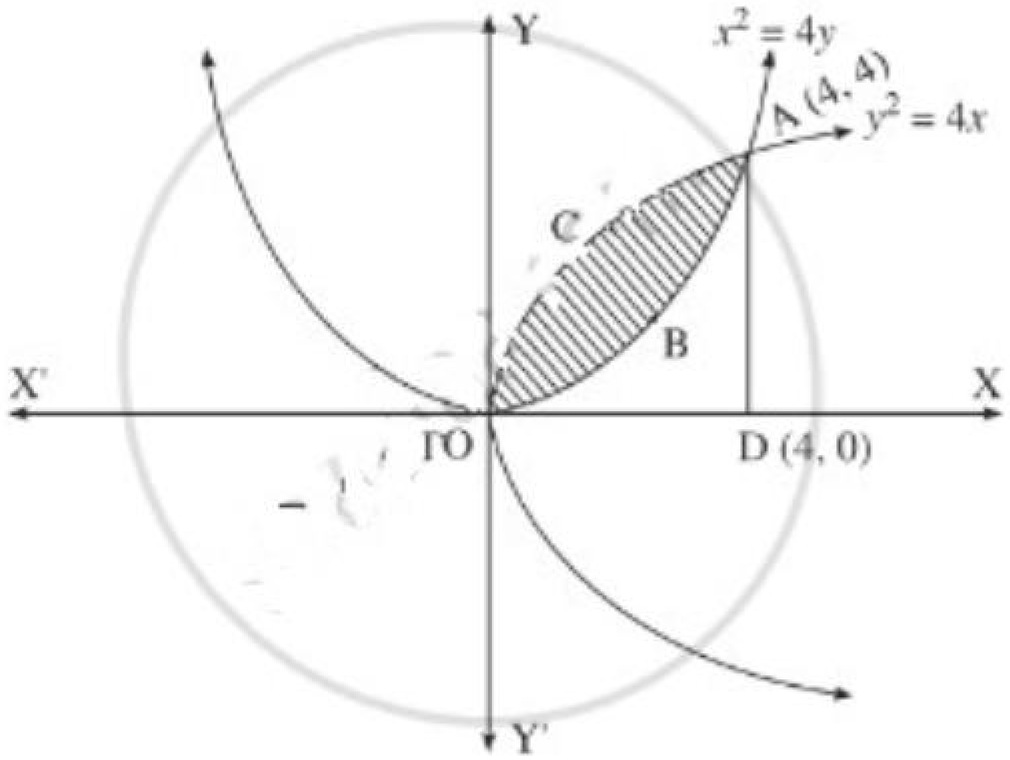
Area of the region ODABO

= area under the parabola $x^2 = 4y$, i.e. $y = \frac{x^2}{4}$ between $x = 0$ and $x = 4$

$$= \int_0^4 \frac{1}{4}x^2\,dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4 = \frac{1}{4}\left(\frac{64}{3} - 0\right) = \frac{16}{3}$$

∴ required area $= \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$ sq units.

(ii)



For finding the points of intersection of the two parabolas, we equate the values of $4y^2$ from their equations.
From the equation $3x^2 = 16y$, $y = \frac{3x^2}{16}$
$\therefore y = \frac{3x^4}{256}$
$\therefore \frac{3x^4}{256} = 9x$
$\therefore 3x^4 - 2304x = 0$
$\therefore x(x^3 - 2304) = 0$
$\therefore x = 0$ or $x^3 = 2304$ i.e. $x = 0$ or $x = 4$
When $x = 0$, $y = 0$
When $x = 4$, $y = \frac{4^2}{4}$
$\therefore$ the points of intersection are O(0, 0) and A(4, 4).
Required area = area of the region OBACO = [area of the region ODACO] – [area of the region ODABO]
Now, area of the region ODACO = area under the parabola $y^2 = 4x$,
i.e. $y = 2\sqrt{x}$ between $x = 0$ and $x = 4$

$$= \int_0^4 2\sqrt{x} \cdot dx$$

$$= \left[2\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$$

$$= 2 \times \frac{2}{3} \times 4^{\frac{3}{2}} - 0$$

$$= \frac{4}{3} \times \left(2^3\right)$$

$$= \frac{32}{3}$$

Area of the region ODABO = area under the rabola $x^2 = 4y$,
i.e. $y = \frac{x^2}{4}$ between $x = 0$ and $x = 4$

$$= \int_0^4 \frac{1}{4}x^2 \cdot dx$$

$$= \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$$

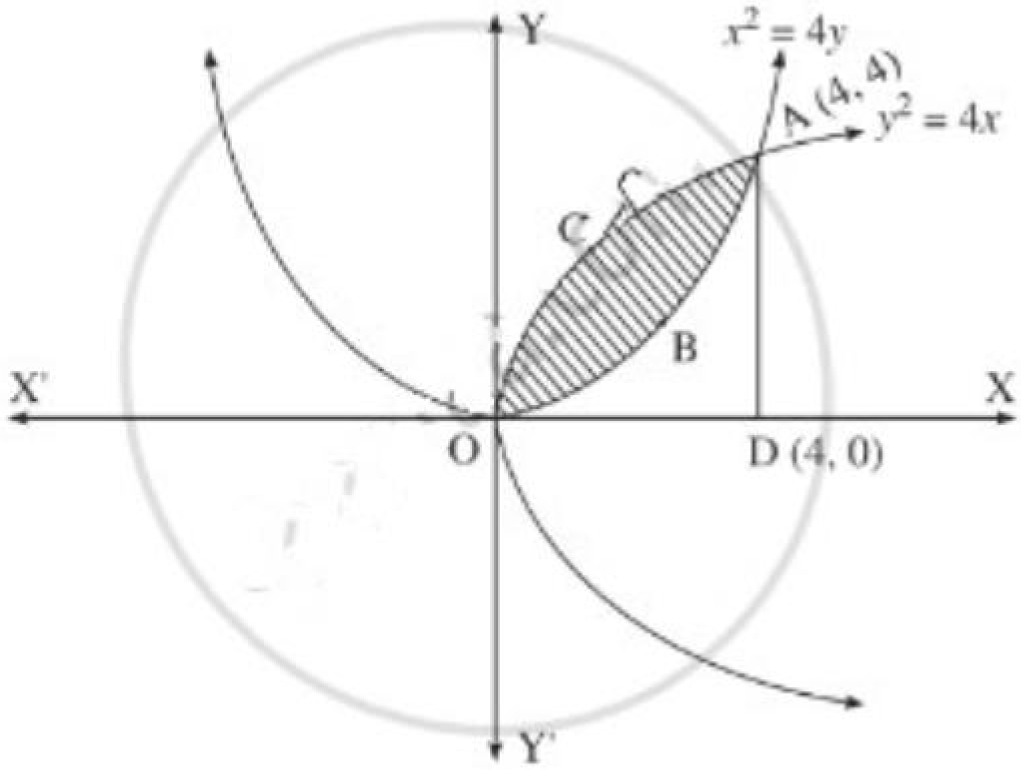
$$= \frac{1}{4}\left(\frac{2304}{3} - 0\right)$$

$$= \frac{16}{3}$$

$\therefore$ required area = $\frac{32}{3} - \frac{16}{3}$

$= \frac{8}{3}$ sq units.

(iii)



For finding the points of intersection of the two parabolas, we equate the values of y^2 from their equations.
From the equation $x^2 = y$, y = *x2y*
∴ y = *x2y*
∴ *x2y* = x
∴ $x^2 - y = 0$
∴ $x(x^3 - y) = 0$
∴ x = 0 or $x^3 = y$
i.e. x = 0 or x = 4
When x = 0, y = 0
When x = 4, y = *424* = 4
∴ the points of intersection are O(0, 0) and A(4, 4).
Required area = area of the region OBACO = [area of the region ODACO] – [area of the region ODABO]
Now, area of the region ODACO = area under the parabola $y^2 = 4x$,
i.e. y = 2√x between x = 0 and x = 4

$$= \int_0^4 2\sqrt{x} \cdot dx$$

$$= \left[2\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$$

$$= 2 \times \frac{2}{3} \times 4^{\frac{3}{2}} - 0$$

$$= \frac{4}{3} \times \left(2^3\right)$$

$$= \frac{32}{3}$$

Area ofthe region ODABO = area under the rabola $x^2 = 4y$, i.e. y = $\frac{x^2}{3}$ between x = 0 and x = 4

$$= \int_0^4 \frac{1}{3}x^2 \cdot dx$$

$$= \frac{1}{3}\left[\frac{x}{3}\right]_0^4$$

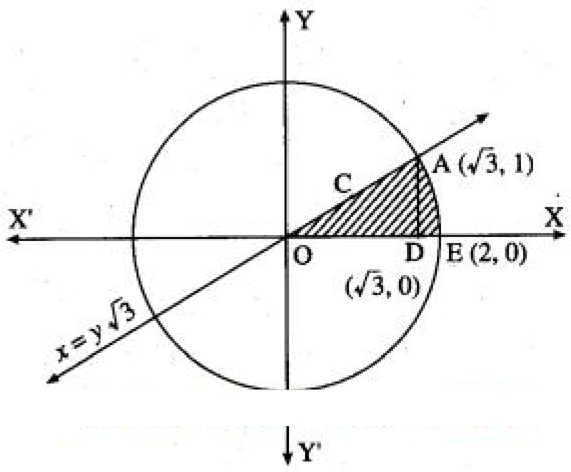
$$= \frac{1}{3}\left(\frac{y}{3} - 0\right)$$

$$= \frac{y}{3}$$

$$\therefore \text{ required area } = \frac{1}{3} - \frac{y}{3}$$

$$= \frac{1}{3} \text{sq units.}$$

Question 5.
Find the area of the region in the first quadrant bounded by the circle $x^2 + y^2 = 4$ and the X-axis and the line x = y√3.
Solution:



For finding the points of intersection of the circle and the line, we solve
$x^2 + y^2 = 4$ ....(1)
and x = y√3 ....(2)
From (2), $x^2 = 3y^2$
From (1), $x^2 = 4 – y^2$
$3y^2 = 4 – y^2$
$4y^2 = 4$
$y^2 = 1$
y = 1 in the first quadrant.
When y = 1, r = 1 × √3 = √3
∴ the circle and the line intersect at A(√3, 1) in the first quadrant
Required area = area of the region OCAEDO = area of the region OCADO + area of the region DAED
Now, area of the region OCADO = area under the line x = y√3, i.e. y = $\frac{x}{\sqrt{3}}$ between x = 0

and x = √3

$$= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}}\,dx = \left[\frac{x^2}{2\sqrt{3}}\right]_0^{\sqrt{3}} = \frac{3}{2\sqrt{3}} - 0 = \frac{\sqrt{3}}{2}$$

Area of the region DAED

$= $ area under the circle $x^2 + y^2 = 4$ i.e. $y = +\sqrt{4 - x^2}$ (in the first quadrant) between $x = \sqrt{3}$ and $x = 2$

$$= \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2}$$

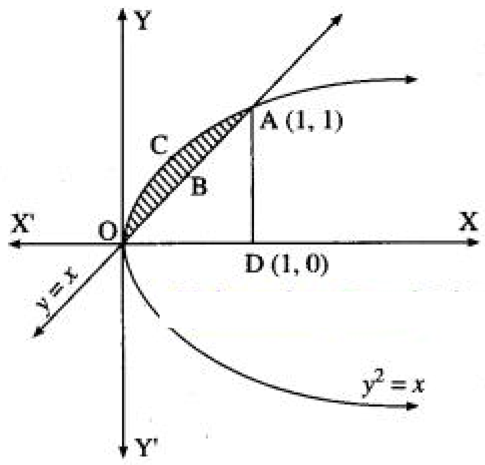
$$= \left[\frac{2}{2}\sqrt{4 - 4} + 2\sin^{-1}(1)\right] - \left[\frac{\sqrt{3}}{2}\sqrt{4 - 3} + 2\sin^{-1}\frac{\sqrt{3}}{2}\right]$$

$$= 0 + 2\left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{2} - 2\left(\frac{\pi}{3}\right)$$

$$= \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$$

$$\therefore \text{ required area} = \frac{\sqrt{3}}{2} + \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} \text{ sq units.}$$

Question 6.
Find the area of the region bounded by the parabola $y^2 = x$ and the line y = x in the first quadrant.
Solution:
To obtain the points of intersection of the line and the parabola, we equate the values of x from both equations.



∴ $y^2 = y$
∴ $y^2 - y = 0$
∴ y(y – 1) = 0
∴ y = 0 or y = 1
When y = 0, x = 0
When y = 1, x = 1
∴ the points of intersection are O(0, 0) and A(1, 1).
Required area = area of the region OCABO = area of the region OCADO – area of the region OBADO
Now, area of the region OCADO = area under the parabola $y^2 = x$ i.e. y = +√x (in the first quadrant) between x = 0 and x = 1

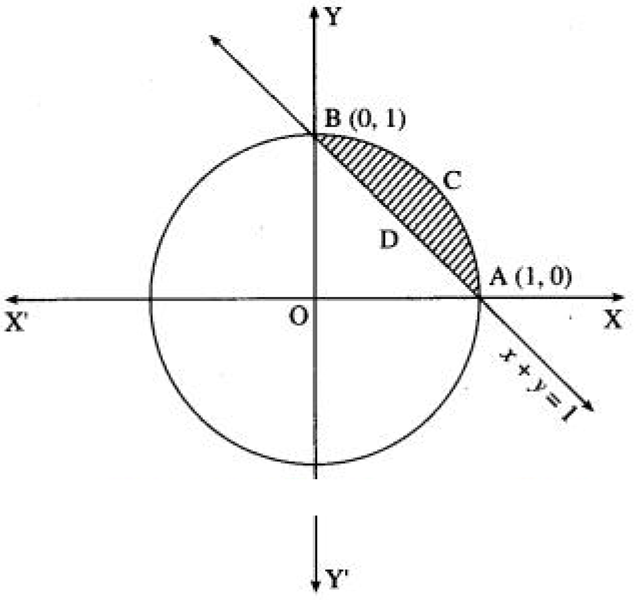
$$= \int_0^1 \sqrt{x}\,dx = \left[\frac{x^{3/2}}{3/2}\right]_0^1 = \frac{2}{3} \times (1 - 0) = \frac{2}{3}$$

Area of the region OBADO = area under the line y = x between x = 0 and x = 1

$$= \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$$

$$\therefore \text{ required area} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \text{ sq unit.}$$

Question 7.
Find the area enclosed between the circle $x^2 + y^2 = 1$ and the line x + y = 1, lying in the first quadrant.
Solution:



Required area = area of the region ACBPA = (area of the region OACBO) – (area of the region OADBO)
Now, area of the region OACBO = area under the circle $x^2 + y^2 = 1$ between x = 0 and x = 1

$$= \int_0^1 y\,dx, \text{ where } y^2 = 1 - x^2, \text{ i.e. } y = \sqrt{1-x^2}, \text{ as } y > 0$$

$$= \int_0^1 \sqrt{1-x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}(x)\right]_0^1$$

$$= \frac{1}{2}\sqrt{1-1} + \frac{1}{2}\sin^{-1}1 - 0$$

$$= \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{4}$$

Area of the region OADBO = area under the line x + y = 1 between x = 0 and x = 1

$$= \int_0^1 y\,dx, \text{ where } y = 1 - x$$

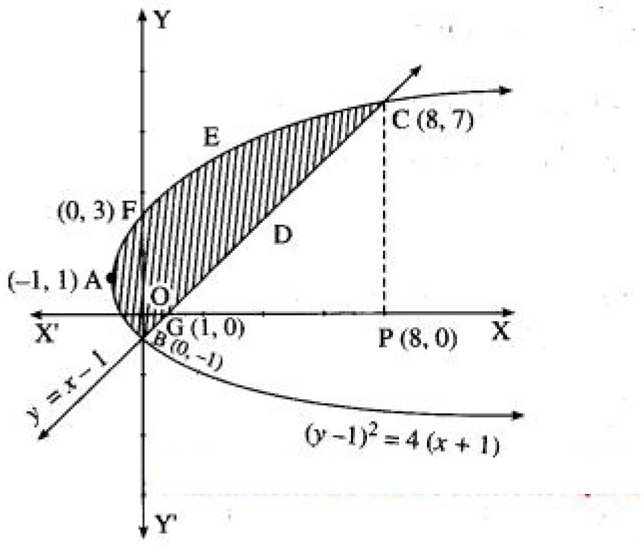
$$= \int_0^1 (1-x)\,dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1$$

$$= 1 - \frac{1}{2} - 0 = \frac{1}{2}$$

∴ required area = $\left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq units.

Question 8.
Find the area of the region bounded by the curve $(y - 1)^2 = 4(x + 1)$ and the line y = (x – 1).
Solution:
The equation of the curve is $(y - 1)^2 = 4(x + 1)$
This is a parabola with vertex at A (-1, 1).
To find the points of intersection of the line y = x – 1 and the parabola.
Put y = x – 1 in the equation of the parabola, we get
$(x - 1 - 1)^2 = 4(x + 1)$
∴ $x^2 - 4x + 4 = 4x + 4$
∴ $x^2 - 8x = 0$
∴ x(x – 8) = 0
∴ x = 0, x = 8
When x = 0, y = 0 – 1 = -1
When x = 8, y = 8 – 1 = 7
∴ the points of intersection are B (0, -1) and C (8, 7).



To find the points where the parabola $(y-1)^2 = 4(x+1)$ cuts the Y-axis.
Put x = 0 in the equation of the parabola, we get
$(y-1)^2 = 4(0+1) = 4$
$\therefore y - 1 = \pm 2$
$\therefore y - 1 = 2$ or $y - 1 = -2$
$\therefore y = 3$ or $y = -1$
$\therefore$ the parabola cuts the Y-axis at the points B(0, -1) and F(0, 3).
To find the point where the line y = x – 1 cuts the X-axis.
Put y = 0 in the equation of the line, we get
x – 1 = 0
$\therefore$ x = 1
$\therefore$ the line cuts the X-axis at the point G (1, 0).
Required area = area of the region BFAB + area of the region OGDCEFO + area of the region OBGO
Now, area of the region BFAB = area under the parabola $(y-1)^2 = 4(x+1)$, Y-axis from y = -1 to y = 3

$$= \int_{-1}^{3} x\,dy, \text{ where } x+1 = \frac{(y-1)^2}{4}, \text{ i.e. } x = \frac{(y-1)^2}{4} - 1$$

$$= \int_{-1}^{3} \left[\frac{(y-1)^2}{4} - 1\right] dy$$

$$= \left[\frac{1}{4} \cdot \frac{(y-1)^3}{3} - y\right]_{-1}^{3}$$

$$= \left[\left\{\frac{1}{12}(3-1)^3 - 3\right\} - \left\{\frac{1}{12}(-1-1)^3 - (-1)\right\}\right]$$

$$= \frac{8}{12} - 3 + \frac{8}{12} - 1 = \frac{16}{12} - 4$$

$$= \frac{4}{3} - 4 = -\frac{8}{3}$$

Since, the area cannot be negative,
Area of the region BFAB = $\left|-\frac{8}{3}\right| = \frac{8}{3}$ sq units.
Area of the region OGDCEFO = area of the region OPCEFO – area of the region GPCDG

$$= \int_0^8 y\,dx, \text{ where } (y-1)^2 = 4(x+1), \text{ i.e. } y = 2\sqrt{x+1}+1$$

$$-\int_1^8 y\,dx, \text{ where } y = x-1$$

$$= \int_0^8 [2\sqrt{x+1}+1]\,dx - \int_1^8 (x-1)\,dx$$

$$= \left[2\cdot\frac{(x+1)^{\frac{3}{2}}}{3/2} + x\right]_0^8 - \left[\frac{x^2}{2} - x\right]_1^8$$

$$= \left[\frac{4}{3}(9)^{\frac{3}{2}} + 8 - \frac{4}{3}(1)^{\frac{3}{2}} - 0\right] - \left[\left(\frac{64}{2} - 8\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= \left(36 + 8 - \frac{4}{3}\right) - \left(24 + \frac{1}{2}\right)$$

$$= 44 - \frac{4}{3} - 24 - \frac{1}{2}$$

$$= 20 - \left(\frac{4}{3} + \frac{1}{2}\right) = 20 - \frac{11}{6} = \frac{109}{6} \text{ sq units.}$$

Area of region OBGO $= \int_0^1 y\,dx$, where $y = x - 1$

$$= \int_0^1 (x-1)\,dx = \left[\frac{x^2}{2} - x\right]_0^1$$

$$= \frac{1}{2} - 1 - 0 = -\frac{1}{2}$$

Since, area cannot be negative,

area of the region = | | −12 | | =12 sq units.

∴ required area = 83+1096+12

= 16+109+36

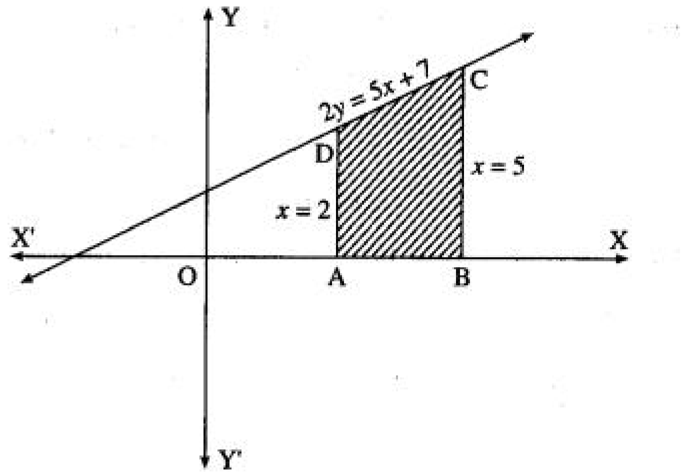
= 1286

= 643 sq units.

Question 9.
Find the area of the region bounded by the straight line 2y = 5x + 7, X-axis and x = 2, x = 5.
Solution:
The equation of the line is

2y = 5x + 7, i.e., y = 52X+72

Required area = area of the region ABCDA = area under the line y = $\frac{5}{2}x+\frac{7}{2}$ between x = 2 and x = 5



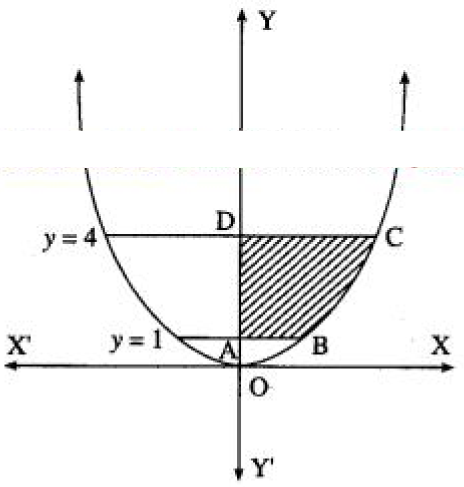
$$= \int_2^5 \left(\frac{5}{2}x+\frac{7}{2}\right)dx = \frac{5}{2}\cdot\int_2^5 x\,dx + \frac{7}{2}\int_2^5 1dx$$

$$= \frac{5}{2}\left[\frac{x^2}{2}\right]_2^5 + \frac{7}{2}[x]_2^5$$

$$= \frac{5}{2}\left[\frac{25}{2}-\frac{4}{2}\right]+\frac{7}{2}[5-2]$$

$$= \frac{5}{2}\times\frac{21}{2}+\frac{21}{2} = \frac{105}{4}+\frac{42}{4} = \frac{147}{4} \text{ sq units.}$$

Question 10.
Find the area of the region bounded by the curve y = $4x^2$, Y-axis and the lines y = 1, y = 4.
Solution:



By symmetry of the parabola, the required area is 2 times the area of the region ABCD.
From the equation of the parabola, $x^2 = \frac{y}{4}$
In the first quadrant, x > 0
∴ x = $\frac{1}{2}\sqrt{y}$
∴ required area = $\int_1^4 x\,dy$

$$= \frac{1}{2}\int_1^4 \sqrt{y}\,dy = \frac{1}{2}\left[\frac{y^{3/2}}{3/2}\right]_1^4$$

$$= \frac{1}{2}\times\frac{2}{3}[4^{3/2}-1^{3/2}]$$

$$= \frac{1}{3}[(2^2)^{3/2}-1] = \frac{1}{3}[8-1] = \frac{7}{3} \text{ sq units.}$$