Allguidesite -
- Arjun
- Digvijay

Maharashtra State Board 11th Maths Solutions Chapter 1 Complex Numbers Ex 1.1

Question 1.
Simplify:
(i) $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$
Solution:
$$\sqrt{-16}+3\sqrt{-25}+\sqrt{-36}-\sqrt{-625}$$
$$=\sqrt{16\times-1}+3\sqrt{25\times-1}+\sqrt{36\times-1}-\sqrt{625\times-1}$$

= 4i + 3(5i) + 6i – 25i
= 25i – 25i
= 0

(ii) $4\sqrt{-4} + 5\sqrt{-9} - 3\sqrt{-16}$
Solution:
$$4\sqrt{-4}+5\sqrt{-9}-3\sqrt{-16}$$
$$= 4\sqrt{4\times-1}+5\sqrt{9\times-1}-3\sqrt{16\times-1}$$
$$= 4(2i) + 5(3i) - 3(4i)$$
$$= 8i + 15i - 12i$$
$$= 11i$$

Question 2.
Write the conjugates of the following complex numbers
(i) 3 + i
Solution:
Conjugate of (3 + i) is (3 – i).

(ii) 3 – i
Solution:
Conjugate of (3 – i) is (3 + i).

(iii) $\sqrt{-5} - \sqrt{7}\, i$
Solution:
Conjugate of $(\sqrt{-5} - \sqrt{7}\, i)$ is $(\sqrt{-5} + \sqrt{7}\, i)$.

(iv) $-\sqrt{-5}$
Solution:
$-\sqrt{-5} = -\sqrt{5} \times \sqrt{-1} = -\sqrt{5}\, i$
Conjugate of $(-\sqrt{-5})$ is $\sqrt{5}\, i$

(v) 5i
Solution:
Conjugate of (5i) is (-5i).

(vi) $\sqrt{5} - i$
Solution:
Conjugate of $(\sqrt{5} - i)$ is $(\sqrt{5} + i)$.

(vii) $\sqrt{2} + \sqrt{3}\, i$
Solution:
Conjugate of $(\sqrt{2} + \sqrt{3}\, i)$ is $(\sqrt{2} - \sqrt{3}\, i)$

(viii) $\cos\theta + i\sin\theta$
Solution:
Conjugate of $(\cos\theta + i\sin\theta)$ is $(\cos\theta - i\sin\theta)$

Question 3.
Find a and b if
(i) a + 2b + 2ai = 4 + 6i
Solution:
a + 2b + 2ai = 4 + 6i
Equating real and imaginary parts, we get
a + 2b = 4 .....(i)
2a = 6 ......(ii)

∴ a = 3
Substituting, a = 3 in (i), we get
3 + 2b = 4
∴ b = 12
∴ a = 3 and b = 12

Check:
For a = 3 and b = 12
Consider, L.H.S. = a + 2b + 2ai
= 3 + 2(12) + 2(3)i
= 4 + 6i
= R.H.S.

(ii) (a – b) + (a + b)i = a + 5i
Solution:
(a – b) + (a + b)i = a + 5i
Equating real and imaginary parts, we get
a – b = a ……(i)
a + b = 5 ……(ii)
From (i), b = 0
Substituting b = 0 in (ii), we get
a + 0 = 5
∴ a = 5
∴ a = 5 and b = 0

(iii) (a + b) (2 + i) = b + 1 + (10 + 2a)i
Solution:
(a + b) (2 + i) = b + 1 + (10 + 2a)i
2(a + b) + (a + b)i = (b + 1) + (10 + 2a)i
Equating real and imaginary parts, we get
2(a + b) = b + 1
∴ 2a + b = 1 ……(i)
and a + b = 10 + 2a
-a + b = 10 …….(ii)
Subtracting equation (ii) from (i), we get
3a = -9
∴ a = -3
Substituting a = – 3 in (ii), we get
-(-3) + b = 10
∴ b = 7
∴ a = -3 and b = 7

(iv) abi = 3a – b + 12i
Solution:
abi = 3a – b + 12i
∴ 0 + abi = (3a – b) + 12i
Equating real and imaginary parts, we get
3a – b = 0
∴ 3a = b …..(i)
and ab = 12
∴ b = 12a ……..(ii)
Substituting b = 12a in (i), we get
3a = 12a
$3a^2 = 12$
$a^2 = 4$
a = ±2
When a = 2, b = 12a = 122 = 6
When a = -2, b = 12a = 12-2 = -6
∴ a = 2 and b = 6 or a = -2 and b = -6

(v) $\frac{1}{a+ib}$ = 3 – 2i

Solution:

$$\frac{1}{a+ib} = 3 - 2i$$

$$\therefore \quad a + ib = \frac{1}{3-2i}$$

$$\therefore \quad a + ib = \frac{1}{3-2i} \times \frac{3+2i}{3+2i}$$

$$\therefore \quad a + ib = \frac{3+2i}{3^2 - 2^2 i^2}$$

$$\therefore \quad a + ib = \frac{3+2i}{9-4(-1)} \qquad \ldots[\because i^2 = -1]$$

$$\therefore \quad a + ib = \frac{3+2i}{13}$$

$$\therefore \quad a + ib = \frac{3}{13} + \frac{2}{13}i$$

Equating real and imaginary parts, we get

$$\therefore \quad a = \frac{3}{13} \text{ and } b = \frac{2}{13}$$

(vi) (a + ib) (1 + i) = 2 + i

Solution:

(a + ib)(1 + i) = 2 + i

a + ai + bi + bi^2 = 2 + i

a + (a + b)i + b(-1) = 2 + i ……($\because i^2 = -1$)

(a – b) + (a + b)i = 2 + i

Equating real and imaginary parts, we get

a – b = 2 ……(i)

a + b = 1 …….(ii)

Adding equations (i) and (ii), we get

2a = 3

∴ a = $\frac{3}{2}$

Substituting a = $\frac{3}{2}$ in (ii), we get

$\frac{3}{2}$ + b = 1

∴ b = 1 – $\frac{3}{2}$ = $-\frac{1}{2}$

∴ a = $\frac{3}{2}$ and b = $-\frac{1}{2}$

Question 4.

Express the following in the form of a + ib, a, b ∈ R, i = √-1. State the values of a and b:

(i) (1 + 2i)(-2 + i)

Solution:

(1 + 2i)(-2 + i) = -2 + i – 4i + $2i^2$

= -2 – 3i + 2(-1) ……[$\because i^2 = -1$]

∴ (1 + 2i)(-2 + i) = -4 – 3i

∴ a = -4 and b = -3

(ii) $(1 + i)(1 – i)^{-1}$

Solution:

$$(1+i)(1-i)^{-1} = \frac{1+i}{1-i}$$

$$= \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+2i+i^2}{1-i^2}$$

$$= \frac{1+2i-1}{1-(-1)} \qquad \ldots[\because i^2 = -1]$$

$$= \frac{2i}{2}$$

$$= i$$

$$\therefore \quad (1+i)(1-i)^{-1} = 0 + i$$

$$\therefore \quad a = 0 \text{ and } b = 1$$

(iii) $\frac{i(4+3i)}{1-i}$

Solution:

iii. $\frac{i(4+3i)}{1-i} = \frac{4i+3i^2}{1-i}$

$= \frac{-3+4i}{1-i}$ ...[$\because i^2 = -1$]

$= \frac{(-3+4i)(1+i)}{(1-i)(1+i)}$

$= \frac{-3-3i+4i+4i^2}{1-i^2}$

$= \frac{-3+i+4(-1)}{1-(-1)}$ ...[$\because i^2 = -1$]

$= \frac{-7+i}{2}$

$\therefore \quad \frac{i(4+3i)}{1-i} = \frac{-7}{2} + \frac{1}{2}i$

$\therefore \quad a = \frac{-7}{2}$ and $b = \frac{1}{2}$

(iv) $\frac{(2+i)}{(3-i)(1+2i)}$

Solution:

$\frac{2+i}{(3-i)(1+2i)} = \frac{2+i}{3+6i-i-2i^2}$

$= \frac{2+i}{3+5i-2(-1)}$ ...[$\because i^2 = -1$]

$= \frac{2+i}{5+5i}$

$= \frac{2+i}{5(1+i)}$

$= \frac{(2+i)(1-i)}{5(1+i)(1-i)}$

$= \frac{2-2i+i-i^2}{5(1-i^2)}$

$= \frac{2-i-(-1)}{5[1-(-1)]}$ ...[$\because i^2 = -1$]

$= \frac{3-i}{10}$

$\therefore \quad \frac{2+i}{(3-i)(1+2i)} = \frac{3}{10} - \frac{1}{10}i$

$\therefore \quad a = \frac{3}{10}$ and $b = \frac{-1}{10}$

(v) $\left(\frac{1+i}{1-i}\right)^2$

Solution:

$\left(\frac{1+i}{1-i}\right)^2 = \frac{1+2i+i^2}{1-2i+i^2}$

$= \frac{1+2i-1}{1-2i-1}$ ...[$\because i^2 = -1$]

$= \frac{2i}{-2i}$

$= -1$

$\therefore \quad \left(\frac{1+i}{1-i}\right)^2 = -1 + 0i$

$\therefore \quad a = -1$ and $b = 0$

(vi) 3+2i2-5i+3-2i2+5i

Solution:

$$\frac{3+2i}{2-5i}+\frac{3-2i}{2+5i}$$

$$=\frac{(3+2i)(2+5i)+(2-5i)(3-2i)}{(2-5i)(2+5i)}$$

$$=\frac{6+15i+4i+10i^2+6-4i-15i+10i^2}{4-25i^2}$$

$$=\frac{12+20i^2}{4-25i^2}$$

$$=\frac{12+20(-1)}{4-25(-1)} \quad \ldots[\because i^2=-1]$$

$$=\frac{-8}{29}$$

$\therefore \quad \frac{3+2i}{2-5i}+\frac{3-2i}{2+5i}=\frac{-8}{29}+0i$

$\therefore \quad a=\frac{-8}{29}$ and $b=0$

(vii) $(1 + i)^{-3}$

Solution:

$$(1+i)^{-3}=\frac{1}{(1+i)^3}$$

$$=\frac{1}{1+3i+3i^2+i^3}$$

$$\ldots[\because (a+b)^3=a^3+3a^2b+3ab^2+b^3]$$

$$=\frac{1}{1+3i-3-i} \quad \ldots[\because i^2=-1, i^3=-i]$$

$$=\frac{1}{-2+2i}$$

$$=\frac{1}{2(-1+i)}$$

$$=\frac{1}{2}\times\frac{-1-i}{(-1+i)(-1-i)}$$

$$=\frac{1}{2}\times\frac{(-1-i)}{1-i^2}$$

$$=\frac{-(1+i)}{2[1-(-1)]} \quad \ldots[\because i^2=-1]$$

$$=\frac{-1}{4}(1+i)$$

$\therefore \quad (1+i)^{-3}=\frac{-1}{4}-\frac{1}{4}i$

$\therefore \quad a=b=\frac{-1}{4}$

(viii) 2+−3√4+−3√

Solution:

$$\frac{2+\sqrt{-3}}{4+\sqrt{-3}}=\frac{2+\sqrt{3}i}{4+\sqrt{3}i}$$

$$=\frac{(2+\sqrt{3}i)(4-\sqrt{3}i)}{(4+\sqrt{3}i)(4-\sqrt{3}i)}$$

$$=\frac{8-2\sqrt{3}i+4\sqrt{3}i-3i^2}{16-3i^2}$$

$$=\frac{8+2\sqrt{3}i-3(-1)}{16-3(-1)} \qquad ...[\because i^2=-1]$$

$$=\frac{11+2\sqrt{3}i}{19}$$

$$\therefore \quad \frac{2+\sqrt{-3}}{4+\sqrt{-3}}=\frac{11}{19}+\frac{2\sqrt{3}}{19}i$$

$$\therefore \quad a=\frac{11}{19} \text{ and } b=\frac{2\sqrt{3}}{19}$$

(ix) (-√5 + 2√-4) + (1 – √-9) + (2 + 3i)(2 – 3i)

Solution:

(-√5 + 2√-4) + (1 – √-9) + (2 + 3i)(2 – 3i)
= (-√5 + 2√4.√-1) + (1 – √9.√-1) + 4 – 9i2
= [-√5 + 2(2)i] + (1 – 3i) + 4 – 9i2
= -√5 + 4i + 1 – 3i + 4 – 9(-1) ……[∵ i2 = -1]
= (14 – √5) + i
∴ a = 14 – √5 and b = 1

(x) (2 + 3i)(2 – 3i)

Solution:

(2 + 3i)(2 – 3i)
= 4 – 9i2
= 4 – 9(-1) …[∵ i2 = -1]
= 4 + 9
= 13
∴ (2 + 3i)(2 – 3i) = 13 + 0i
∴ a = 13 and b = 0

(xi) $4i8-3i9+33i11-4i10-2$

Solution:

$$\frac{4i^8-3i^9+3}{3i^{11}-4i^{10}-2}=\frac{4\left(i^4\right)^2-3\left(i^4\right)^2.i+3}{3\left(i^4\right)^2.i^3-4\left(i^4\right)^2.i^2-2}$$

$\because \quad i^2=-1,\ i^3=-i$ and $i^4=1$

$$\therefore \quad \frac{4i^8-3i^9+3}{3i^{11}-4i^{10}-2}=\frac{4(1)^2-3(1)^2.i+3}{3(1)^2(-i)-4(1)^2(-1)-2}$$

$$=\frac{4-3i+3}{-3i+4-2}$$

$$=\frac{7-3i}{2-3i}$$

$$=\frac{(7-3i)(2+3i)}{(2-3i)(2+3i)}$$

$$=\frac{14+21i-6i-9i^2}{4-9i^2}$$

$$=\frac{14+15i-9(-1)}{4-9(-1)}$$

$$=\frac{23+15i}{13}$$

$$\therefore \quad \frac{4i^8-3i^9+3}{3i^{11}-4i^{10}-2}=\frac{23}{13}+\frac{15}{13}i$$

$$\therefore \quad a=\frac{23}{13} \text{ and } b=\frac{15}{13}$$

Question 5.

Show that $(-1 + \sqrt{3}i)3$ is a real number.

Solution:

$$\left(-1+\sqrt{3}i\right)^3=(-1)^3+3(-1)^2\left(\sqrt{3}i\right)+3(-1)\left(\sqrt{3}i\right)^2+\left(\sqrt{3}i\right)^3$$

$$\ldots[(a+b)^3=a^3+3a^2b+3ab^2+b^3]$$

$$=-1+3\sqrt{3}i-3(3i^2)+3\sqrt{3}i^3$$

$$=-1+3\sqrt{3}i-3(-3)-3\sqrt{3}i$$

$$\ldots.[\because i^2=-1,\ i^3=-i]$$

$$=-1+9$$

$= 8$, which is a real number.

Question 6.

Find the value of $(3 + 2i)\ (i6 - i7)\ (1 + i11)$.

Solution:

$$\left(3+\frac{2}{i}\right)(i^6-i^7)(1+i^{11})$$
$$=\left(3+\frac{2}{i}\right)\left[\left(i^2\right)^3-i^4\cdot i^3\right]\left[1+\left(i^4\right)^2\cdot i^3\right]$$
$$=\left(3+\frac{2}{i}\right)\left[(-1)^3-(1)(-i)\right]\left[1+(1)^2(-i)\right]$$
$$\ldots[\because\ i^2=-1,\ i^3=-i \text{ and } i^4=1]$$
$$=\left(3+\frac{2}{i}\right)(-1+i)(1-i)$$
$$=\left(3+\frac{2i}{i^2}\right)(-1+i+i-i^2)$$
$$=\left(3+\frac{2i}{-1}\right)[-1+2i-(-1)]$$
$$=(3-2i)(2i)$$
$$=6i-4i^2$$
$$=6i-4(-1)$$
$$=4+6i$$

Question 7.
Evaluate the following:
(i) i^{35}
(ii) i^{888}
(iii) i^{93}
(iv) i^{116}
(v) i^{403}
(vi) $\frac{1}{i^{58}}$
(vii) i^{-888}
(viii) $i^{30} + i^{40} + i^{50} + i^{60}$
Solution:

We know that, $i^2=-1,\ i^3=-i,\ i^4=1$

i. $i^{35}=(i^4)^8(i^2)i=(1)^8(-1)i=-i$

ii. $i^{888}=(i^4)^{222}=(1)^{222}=1$

iii. $i^{93}=(i^4)^{23}.i=(1)^{23}.i=i$

iv. $i^{116}=(i^4)^{29}=(1)^{29}=1$

v. $i^{403}=(i^4)^{100}(i^2)i=(1)^{100}(-1)i=-i$

vi. $\frac{1}{i^{58}}=\frac{1}{(i^4)^{14}.i^2}=\frac{1}{(1)^{14}(-1)}=-1$

vii. $i^{-888}=(i^4)^{-222}=(1)^{-222}=\frac{1}{(1)^{222}}=1$

viii. $i^{30}+i^{40}+i^{50}+i^{60}$
$=(i^4)^7 i^2+(i^4)^{10}+(i^4)^{12} i^2+(i^4)^{15}$
$=(1)^7(-1)+(1)^{10}+(1)^{12}(-1)+(1)^{15}$
$=-1+1-1+1=0$

Question 8.
Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number.
Solution:

$$1+i^{10}+i^{20}+i^{30}$$
$$=1+(i^4)^2.i^2+(i^4)^5+(i^4)^7.i^2$$
$$=1+(1)^2(-1)+(1)^5+(1)^7(-1)$$
$$\ldots[\because\ i^4=1,\ i^2=-1]$$

= 1 – 1 + 1 – 1
= 0, which is a real number.

Question 9.
Find the value of
(i) $i^{49} + i^{68} + i^{89} + i^{110}$
(ii) $i + i^2 + i^3 + i^4$

Solution:

i. $i^{49} + i^{68} + i^{89} + i^{110}$

$= (i^4)^{12}.i + (i^4)^{17} + (i^4)^{22}.i + (i^4)^{27}.i^2$

$= (1)^{12}.i + (1)^{17} + (1)^{22}.i + (1)^{27}(-1)$ ...[$\because i^4 = 1, i^2 = -1$]

$= i + 1 + i - 1 = 2i$

ii. $i + i^2 + i^3 + i^4 = i + i^2 + i^2.i + i^4$

$= i - 1 - i + 1$ ...[$\because i^2 = -1, i^4 = 1$]

$= 0$

Question 10.

Simplify: $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$

Solution:

$$\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$$

$$= \frac{i^{10}(i^{582}+i^{580}+i^{578}+i^{576}+i^{574})}{(i^{582}+i^{580}+i^{578}+i^{576}+i^{574})}$$

$= i^{10} = (i^4)^2.i^2$

$= (1)^2(-1)$ ...[$\because i^4 = 1, i^2 = -1$]

$= -1$

Question 11.

Find the value of $1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20}$.

Solution:

$1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20}$

$= 1 + (i^2 + i^4) + (i^6 + i^8) + (i^{10} + i^{12}) + (i^{14} + i^{16}) + (i^{18} + i^{20})$

$= 1 + [i^2 + (i^2)^2] + [(i^2)^3 + (i^2)^4] + [(i^2)^5 + (i^2)^6] + [(i^2)^7 + (i^2)^8] + [(i^2)^9 + (i^2)^{10}]$

$= 1 + [-1 + (-1)^2] + [(-1)^3 + (-1)^4] + [(-1)^5 + (-1)^6] + [(-1)^7 + (-1)^8] + [(-1)^9 + (-1)^{10}]$ ...[$\because i^2 = -1$]

$= 1 + (-1 + 1) + (-1 + 1) + (-1 + 1) + (-1 + 1) + (-1 + 1)$

$= 1 + 0 + 0 + 0 + 0 + 0 = 1$

Question 12.

Show that $1 + i^{10} + i^{100} - i^{1000} = 0$.

Solution:

$1 + i^{10} + i^{100} - i^{1000}$

$= 1 + (i^4)^2.i^2 + (i^4)^{25} - (i^4)^{250}$

$= 1 + (1)^2.(-1) + (1)^{25} - (1)^{250}$ ...[$\because i^2 = -1, i^4 = 1$]

$= 1 - 1 + 1 - 1 = 0$

Question 13.

Is $(1 + i^{14} + i^{18} + i^{22})$ a real number? Justify your answer.

Solution:

$1 + i^{14} + i^{18} + i^{22}$

$= 1 + (i^4)^3.i^2 + (i^4)^4.i^2 + (i^4)^5.i^2$

$= 1 + (1)^3(-1) + (1)^4(-1) + (1)^5(-1)$ ..[$\because i^2 = -1, i^4 = 1$]

$= 1 - 1 - 1 - 1$

$= -2$, which is a real number.

Question 14.
Evaluate: $\left(i^{37}+\frac{1}{i^{67}}\right)$
Solution:

$$i^{37}+\frac{1}{i^{67}} = (i^4)^9 i + \frac{1}{(i^4)^{16} i^3}$$
$$= (1)^9 i + \frac{1}{(1)^{16} \cdot (-i)} \quad \ldots[\because i^3 = -i, i^4 = 1]$$
$$= i - \frac{1}{i}$$
$$= i + \frac{i^2}{i} \quad \ldots[-1 = i^2]$$
$$= i + i$$
$$= 2i$$

Question 15.
Prove that: $(1 + i)^4 \times \left(1+\frac{1}{i}\right)^4 = 16$
Solution:

$$(1+i)^4 \times \left(1+\frac{1}{i}\right)^4$$
$$= \left[(1+i)\left(1+\frac{1}{i}\right)\right]^4 = \left[(1+i)\frac{(1+i)}{i}\right]^4 = \left[\frac{(1+i)^2}{i}\right]^4$$
$$= \frac{(1+2i+i^2)^4}{i^4}$$
$$= \frac{(1+2i-1)^4}{1} \quad \ldots[\because i^2 = -1]$$
$$= 16i^4$$
$$= 16 \quad \ldots[\because i^4 = 1]$$

Question 16.
Find the value of $\frac{i^6+i^7+i^8+i^9}{i^2+i^3}$
Solution:

$$\frac{i^6+i^7+i^8+i^9}{i^2+i^3} = \frac{i^4.i^2+i^4.i^3+(i^4)^2+(i^4)^2.i}{i^2+i^3}$$
$$= \frac{(1).(-1)+(1).(-i)+(1)^2+(1)^2.i}{-1-i} \quad \ldots[\because i^2 = -1, i^3 = -i, i^4 = 1]$$
$$= \frac{-1-i+1+i}{-1-i} = 0$$

Question 17.
If $a = \frac{-1+\sqrt{3}i}{2}$, $b = \frac{-1-\sqrt{3}i}{2}$, then show that $a^2 = b$ and $b^2 = a$.

Solution:

$$a = \frac{-1+\sqrt{3}i}{2},\ b = \frac{-1-\sqrt{3}i}{2}$$

$$a^2 = \left(\frac{-1+\sqrt{3}i}{2}\right)^2 = \frac{1-2\sqrt{3}i+3i^2}{4}$$

$$= \frac{1-2\sqrt{3}i+3(-1)}{4}$$

$$\ldots[\because i^2 = -1]$$

$$= \frac{-2-2\sqrt{3}i}{4}$$

$$= \frac{-1-\sqrt{3}i}{2} = b$$

$$\text{and } b^2 = \left(\frac{-1-\sqrt{3}i}{2}\right)^2 = \frac{1+2\sqrt{3}i+3i^2}{4}$$

$$= \frac{1+2\sqrt{3}i+3(-1)}{4}$$

$$\ldots[\because i^2 = -1]$$

$$= \frac{-2+2\sqrt{3}i}{4}$$

$$= \frac{-1+\sqrt{3}i}{2} = a$$

Question 18.
If $x + iy = (a + ib)^3$, show that $\frac{x}{a}+\frac{y}{b} = 4(a^2 - b^2)$
Solution:
$x + iy = (a + ib)^3$
$x + iy = a^3 + 3a^2bi + 3ab^2i^2 + b^3i^3$
$x + iy = a^3 + 3a^2bi - 3ab^2 - b^3i$ ……$[\because i^2 = -1, i^3 = -i]$
$x + iy = (a^3 - 3ab^2) + (3a^2b - b^3)i$
Equating real and imaginary parts, we get
$x = a^3 - 3ab^2$ and $y = 3a^2b - b^3$
$\frac{x}{a} = a^2 - 3b^2$ and $\frac{y}{b} = 3a^2 - b^2$
$\frac{x}{a}+\frac{y}{b} = a^2 - 3b + 3a^2 - b^2$
$\frac{x}{a}+\frac{y}{b} = 4a^2 - 4b^2$
$\frac{x}{a}+\frac{y}{b} = 4(a^2 - b^2)$

Alternate Method:
$x + iy = (a + ib)^3$
$x + iy = a^3 + 3a^2bi + 3ab^2i^2 + b^3i^3$
$x + iy = a^3 + 3a^2bi - 3ab^2 - b^3i$ …..$[\because i^2 = -1, i^3 = -i]$
$x + iy = (a^3 - 3ab^2) + (3a^2b - b^3)i$
Equating real and imaginary parts, we get
$x = a^3 - 3ab^2$ and $y = 3a^2b - b^3$
Consider

$$\text{L.H.S.} = \frac{x}{a}+\frac{y}{b}$$

$$= \frac{a^3-3ab^2}{a} + \frac{3a^2b-b^3}{b}$$

$$= \frac{a(a^2-3b^2)}{a} + \frac{b(3a^2-b^2)}{b}$$

$$= a^2 - 3b^2 + 3a^2 - b^2$$

$$= 4a^2 - 4b^2 = 4(a^2 - b^2) = \text{R.H.S.}$$

$$\frac{x}{a}+\frac{y}{b} = 4(a^2 - b^2)$$

Question 19.
If $\frac{a+3i}{2+ib} = 1 - i$, show that $(5a - 7b) = 0$.
Solution:

$\frac{a+3i}{2+ib} = 1 - i$

$a + 3i = (1 - i)(2 + ib)$

$= 2 + bi - 2i - bi^2$

$= 2 + (b - 2)i - b(-1)$ ......[$\because i^2 = -1$]

$a + 3i = (2 + b) + (b - 2)i$

Equating real and imaginary parts, we get

$a = 2 + b$ and $3 = b - 2$

$a = 2 + b$ and $b = 5$

$a = 2 + 5 = 7$

$5a - 7b = 5(7) - 7(5) = 35 - 35 = 0$

Question 20.

If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, prove that $(x^2+y^2)^2=\frac{a^2+b^2}{c^2+d^2}$

Solution:

$$x + iy = \sqrt{\frac{a+ib}{c+id}}$$

$$\therefore \quad (x+iy)^2 = \frac{a+bi}{c+di}$$

$$\therefore \quad x^2 + 2xyi + y^2i^2 = \frac{(a+bi)(c-di)}{(c+di)(c-di)}$$

$$\therefore \quad x^2 + 2xyi + y^2(-1) = \frac{ac - adi + bci - bdi^2}{c^2 - d^2i^2}$$

$$\ldots[\because i^2 = -1]$$

$$\therefore \quad (x^2 - y^2) + 2xyi = \frac{(ac+bd) + (bc-ad)i}{c^2+d^2}$$

$$\therefore \quad (x^2 - y^2) + 2xyi = \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i$$

Equating real and imaginary parts, we get

$$x^2 - y^2 = \frac{ac+bd}{c^2+d^2} \quad \text{and} \quad 2xy = \frac{bc-ad}{c^2+d^2}$$

$$\therefore \quad (x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$$

$$= \left(\frac{ac+bd}{c^2+d^2}\right)^2 + \left(\frac{bc-ad}{c^2+d^2}\right)^2$$

$$= \frac{a^2c^2 + b^2d^2 + 2abcd + b^2c^2 + a^2d^2 - 2abcd}{(c^2+d^2)^2}$$

$$= \frac{a^2c^2 + b^2c^2 + a^2d^2 + b^2d^2}{(c^2+d^2)^2}$$

$$= \frac{c^2(a^2+b^2) + d^2(a^2+b^2)}{(c^2+d^2)^2}$$

$$= \frac{(a^2+b^2)(c^2+d^2)}{(c^2+d^2)^2}$$

$$\therefore \quad (x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$$

Question 21.

If $(a + ib) = \frac{1+i}{1-i}$, then prove that $a^2 + b^2 = 1$.

Solution:

$$a + bi = \frac{1+i}{1-i} = \frac{(1+i)^2}{(1-i)(1+i)}$$

$$a + bi = \frac{1+2i+i^2}{1-i^2} = \frac{1+2i-1}{1-(-1)} \quad \ldots[\because i^2 = -1]$$

$$= \frac{2i}{2} = i$$

$\therefore a + bi = 0 + i$

Equating real and imaginary parts, we get

$a = 0$ and $b = 1$

$a^2 + b^2 = 0^2 + 1^2 = 1$

Question 22.

Show that $\left(\frac{\sqrt{7}+i\sqrt{3}}{\sqrt{7}-i\sqrt{3}}+\frac{\sqrt{7}-i\sqrt{3}}{\sqrt{7}+i\sqrt{3}}\right)$ is real.

Solution:

$$\left(\frac{\sqrt{7}+\sqrt{3}i}{\sqrt{7}-\sqrt{3}i}+\frac{\sqrt{7}-\sqrt{3}i}{\sqrt{7}+\sqrt{3}i}\right)$$

$$=\left[\frac{\left(\sqrt{7}+\sqrt{3}i\right)^2+\left(\sqrt{7}-\sqrt{3}i\right)^2}{\left(\sqrt{7}+\sqrt{3}i\right)\left(\sqrt{7}-\sqrt{3}i\right)}\right]$$

$$=\frac{7+2\sqrt{21}i+3i^2+7-2\sqrt{21}i+3i^2}{7-3i^2}=\frac{14+6i^2}{7-3i^2}$$

$$=\frac{14+6(-1)}{7-3(-1)} \qquad \ldots[\because i^2=-1]$$

$$=\frac{8}{10}=\frac{4}{5}\text{, which is real.}$$

Question 23.

If $(x + iy)^3 = y + vi$, then show that $\frac{y}{x}+\frac{v}{y} = 4(x^2 - y^2)$.

Solution:

$$(x+yi)^3=y+vi$$

$\therefore$ $x^3+3x^2yi+3xy^2i^2+y^3i^3=y+vi$

$\therefore$ $x^3+3x^2yi+3xy^2(-1)-y^3i=y+vi$

$\ldots[\because i^2=-1,\ i^3=-i]$

$\therefore$ $(x^3-3xy^2)+(3x^2y-y^3)i=y+vi$

Equating real and imaginary parts, we get

$y=x^3-3xy^2$ and $v=3x^2y-y^3$

$\therefore$ $\frac{y}{x}=x^2-3y^2$ and $\frac{v}{y}=3x^2-y^2$

$\therefore$ $\frac{y}{x}+\frac{v}{y}=x^2-3y^2+3x^2-y^2=4x^2-4y^2$

$\therefore$ $\frac{y}{x}+\frac{v}{y}=4(x^2-y^2)$

Question 24.

Find the values of x and y which satisfy the following equations (x, y ∈ R)

(i) (x + 2y) + (2x – 3y)i + 4i = 5

Solution:

(x + 2y) + (2x – 3y)i + 4i = 5

(x + 2y) + (2x – 3y)i = 5 – 4i

Equating real and imaginary parts, we get

x + 2y = 5 ……(i)

and 2x – 3y = -4 …..(ii)

Equation (i) x 2 – equation (ii) gives

7y = 14

∴ y = 2

Substituting y = 2 in (i), we get

x + 2(2) = 5

x + 4 = 5

∴ x = 1

∴ x = 1 and y = 2

Check:

For x = 1 and y = 2

Consider, L.H.S. = (x + 2y) + (2x – 3y)i + 4i

= (1 + 4) + (2 – 6)i + 4i

= 5 – 4i + 4i

= 5

= R.H.S.

(ii) $\frac{x+1}{1+i}+\frac{y-1}{1-i}=i$

Solution:

$$\frac{x+1}{1+i}+\frac{y-1}{1-i}=i$$

$$\frac{(x+1)(1-i)+(y-1)(1+i)}{(1+i)(1-i)}=i$$

$$\frac{x-xi+1-i+y+yi-1-i}{1-i^2}=i$$

$$\frac{(x+y)+(y-x-2)i}{1-(-1)}=i \quad \ldots[\because i^2=-1]$$

(x + y) + (y – x – 2)i = 2i
(x + y) + (y – x – 2)i = 0 + 2i
Equating real and imaginary parts, we get
x + y = 0 and y – x – 2 = 2
∴ x + y = 0 …….(i)
and -x + y = 4 …….(ii)
Adding (i) and (ii), we get
2y = 4
∴ y = 2
Substituting y = 2 in (i), we get
x + 2 = 0
∴ x = -2
∴ x = -2 and y = 2

(iii) x+iy2+3i+2+i2-3i=913(1+i)
Solution:

$$\frac{x+iy}{2+3i}+\frac{2+i}{2-3i}=\frac{9}{13}(1+i)$$

$$\frac{(x+yi)(2-3i)+(2+i)(2+3i)}{(2+3i)(2-3i)}=\frac{9}{13}(1+i)$$

$$\frac{2x-3xi+2yi-3yi^2+4+6i+2i+3i^2}{4-9i^2}=\frac{9}{13}(1+i)$$

$$\frac{2x-3xi+2yi+3y+4+8i-3}{4+9}=\frac{9+9i}{13}$$

$$\ldots[\because i^2=-1]$$

$$\frac{2x+3y+1+8i-3xi+2yi}{13}=\frac{9+9i}{13}$$

(2x + 3y + 1) + (8 – 3x + 2y)i = 9 + 9i
Equating real and imaginary parts, we get
2x + 3y + 1 = 9 and 8 – 3x + 2y = 9
2x + 3y = 8 ……(i)
and 3x – 2y = – 1 ……(ii)
Equation (i) × 2 + equation (ii) × 3 gives
13x = 13
∴ x = 1
Substituting x = 1 in (i), we get
2(1) + 3y = 8
3y = 6
∴ y = 2
∴ x = 1 and y = 2

(iv) If x(1 + 3i) + y(2 – i) – 5 + i^3 = 0, find x + y
Solution:
x(1 + 3i) + y(2 – i) – 5 + i^3 = 0
x + 3xi + 2y – yi – 5 – i = 0 ……[∵ i^3 = -i]
(x + 2y – 5) + (3x – y – 1)i = 0 + 0i
Equating real and imaginary parts, we get
x + 2y – 5 = 0 …..(i)
and 3x – y – 1 = 0 ……(ii)
Equation (i) + equation (ii) × 2 gives
7x – 7 = 0
7x = 1
∴ x = 1
Substituting x = 1 in (i), we get
1 + 2y – 5 = 0
2y = 4

$y = 2$
$\therefore x = 1$ and $y = 2$
$\therefore x + y = 1 + 2 = 3$

(v) If $x + 2i + 15i^6y = 7x + i^3(y + 4)$, find $x + y$
Solution:
$x + 2i + 15i^6y = 7x + i^3(y + 4)$
$x + 2i + 15(i^2)^3 y = 7x + i^3(y + 4)$
$x + 2i + 15(-1)^3 y = 7x - i(y + 4)$ ……[$\because i^2 = -1, i^3 = -i$]
$x + 2i - 15y - 7x + iy + 4i = 0$
$(-6x - 15y) + i(y + 6) = 0 + 0i$
Equating real and imaginary parts, we get
$-6x - 15y = 0$ and $y + 6 = 0$
$-6x - 15y = 0$ and $y = -6$
$-6x - 15(-6) = 0$
$-6x + 90 = 0$
$\therefore x = 15$
$\therefore x + y = 15 - 6 = 9$

Maharashtra State Board 11th Maths Solutions Chapter 1 Complex Numbers Ex 1.2

Question 1.
Find the square root of the following complex numbers:
(i) $-8 - 6i$
Solution:
Let $\sqrt{-8-6i}$ = a + bi, where a, b ∈ R.
Squaring on both sides, we get
$-8 - 6i = (a + bi)^2$
$-8 - 6i = a^2 + b^2i^2 + 2abi$
$-8 - 6i = (a^2 - b^2) + 2abi$ …..[$\because i^2 = -1$]
Equating real and imaginary parts, we get
$a^2 - b^2 = -8$ and $2ab = -6$

$a^2 - b^2 = -8$ and $b = \frac{-3}{a}$

$a^2 - \left(-\frac{3}{a}\right)^2 = -8$

$a^2 - \frac{9}{a^2} = -8$

$a^4 - 9 = -8a^2$
$a^4 + 8a^2 - 9 = 0$
$(a^2 + 9)(a^2 - 1) = 0$
$a^2 = -9$ or $a^2 = 1$
But $a \in R$
$a^2 \neq -9$
$a^2 = 1$
$a = \pm 1$

When $a = 1$, $b = \frac{-3}{1} = -3$

When $a = -1$, $b = \frac{-3}{-1} = 3$

$\sqrt{-8-6i} = \pm(1 - 3i)$

(ii) $7 + 24i$
Solution:
Let $\sqrt{7+24i}$ = a + bi, where a, b ∈ R.
Squaring on both sides, we get
$7 + 24i = (a + bi)^2$
$7 + 24i = a^2 + b^2i^2 + 2abi$
$7 + 24i = (a^2 - b^2) + 2abi$ …..[∵ $i^2 = -1$]
Equating real and imaginary parts, we get
$a^2 - b^2 = 7$ and $2ab = 24$

$$a^2 - b^2 = 7 \text{ and } b = \frac{12}{a}$$
$$a^2 - \left(\frac{12}{a}\right)^2 = 7$$
$$a^2 - \frac{144}{a^2} = 7$$
$$a^4 - 144 = 7a^2$$
$$a^4 - 7a^2 - 144 = 0$$
$$(a^2 - 16)(a^2 + 9) = 0$$
$$a^2 = 16 \text{ or } a^2 = -9$$
But $a \in R$
$$a^2 \neq -9$$
$$a^2 = 16$$
$$a = \pm 4$$
When $a = 4$, $b = \frac{12}{4} = 3$

When $a = -4$, $b = \frac{12}{-4} = -3$

$$\sqrt{7 + 24i} = \pm(4 + 3i)$$

(iii) $1 + 4\sqrt{3}\, i$
Solution:
Let $\sqrt{1+4\sqrt{3}i}$ = a + bi, where a, b ∈ R.
Squaring on both sides, we get
$1 + 4\sqrt{3}\, i = (a + bi)^2$
$1 + 4\sqrt{3}i = a^2 + b^2i^2 + 2abi$
$1 + 4\sqrt{3}i = (a^2 - b^2) + 2abi$ …..[∵ $i^2 = -1$]
Equating real arid imaginary parts, we get
$a^2 - b^2 = 1$ and $2ab = 4\sqrt{3}$

$$a^2 - b^2 = 1 \text{ and } b = \frac{2\sqrt{3}}{a}$$
$$a^2 - \left(\frac{2\sqrt{3}}{a}\right)^2 = 1$$
$$a^2 - \frac{12}{a^2} = 1$$
$$a^4 - 12 = a^2$$
$$a^4 - a^2 - 12 = 0$$
$$(a^2 - 4)(a^2 + 3) = 0$$
$$a^2 = 4 \text{ or } a^2 = -3$$
But $a \in R$
$$a^2 \neq -3$$
$$a^2 = 4$$
$$a = \pm 2$$
When $a = 2$, $b = \frac{2\sqrt{3}}{2} = \sqrt{3}$

When $a = -2$, $b = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

$$\sqrt{1 + 4\sqrt{3}i} = \pm(2 + \sqrt{3}i)$$

(iv) $3 + 2\sqrt{10}\, i$
Solution:

Let 3+210−−√−−−−−−−−√i = a + bi, where a, b ∈ R.
Squaring on both sides, we get
$3 + 2\sqrt{10}\, i = a^2 + b^2i^2 + 2abi$
$3 + 2\sqrt{10}\, i = (a^2 – b^2) + 2abi$ ……[∵ $i^2 = -1$]
Equating real and imaginary parts, we get
$a^2 – b^2 = 3$ and $2ab = 2\sqrt{10}$

$$a^2 - b^2 = 3 \text{ and } b = \frac{\sqrt{10}}{a}$$
$$a^2 - \left(\frac{\sqrt{10}}{a}\right)^2 = 3$$
$$a^2 - \frac{10}{a^2} = 3$$
$$a^4 - 10 = 3a^2$$
$$a^4 - 3a^2 - 10 = 0$$
$$(a^2 - 5)(a^2 + 2) = 0$$
$$a^2 = 5 \text{ or } a^2 = -2$$
But $a \in R$
$$a^2 \neq -2$$
$$a^2 = 5$$
$$a = \pm\sqrt{5}$$
When $a = \sqrt{5}$, $b = \frac{\sqrt{10}}{\sqrt{5}} = \sqrt{2}$

When $a = -\sqrt{5}$, $b = \frac{\sqrt{10}}{-\sqrt{5}} = -\sqrt{2}$
$$\sqrt{3 + 2\sqrt{10}\,i} = \pm\left(\sqrt{5} + \sqrt{2}\,i\right)$$
$$a^2 - \left(-\frac{\sqrt{3}}{a}\right)^2 = 2$$
$$a^2 - \frac{3}{a^2} = 2$$
$$a^4 - 3 = 2a^2$$
$$a^4 - 2a^2 - 3 = 0$$
$$(a^2 - 3)(a^2 + 1) = 0$$
$$a^2 = 3 \text{ or } a^2 = -1$$
But $a \in R$
$$a^2 \neq -1$$
$$a^2 = 3$$
$$a = \pm\sqrt{3}$$
When $a = \sqrt{3}$, $b = \frac{-\sqrt{3}}{\sqrt{3}} = -1$

When $a = -\sqrt{3}$, $b = \frac{-\sqrt{3}}{-\sqrt{3}} = 1$
$$\sqrt{2\left(1 - \sqrt{3}\,i\right)} = \pm\left(\sqrt{3} - i\right)$$

(v) 2(1 – √3 i)
Solution:
Let 2(1−3–√i)−−−−−−−−−√ = a + bi, where a, b ∈ R.
Squaring on both sides, we get
$2(1 – \sqrt{3}\, i) = a^2 + b^2i^2 + 2abi$
$2 – 2\sqrt{3}\, i = (a^2 – b^2) + 2abi$ ….[∵ $i^2 = -1$]
Equating real and imaginary parts, we get
$a^2 – b^2 = 2$ and $2ab = -2\sqrt{3}$

$a^2 - b^2 = 2$ and $b = -\frac{\sqrt{3}}{a}$

$$a^2 - \left(-\frac{\sqrt{3}}{a}\right)^2 = 2$$

$$a^2 - \frac{3}{a^2} = 2$$

$$a^4 - 3 = 2a^2$$

$$a^4 - 2a^2 - 3 = 0$$

$$(a^2 - 3)(a^2 + 1) = 0$$

$$a^2 = 3 \text{ or } a^2 = -1$$

But $a \in R$

$$a^2 \neq -1$$

$$a^2 = 3$$

$$a = \pm\sqrt{3}$$

When $a = \sqrt{3}$, $b = \frac{-\sqrt{3}}{\sqrt{3}} = -1$

When $a = -\sqrt{3}$, $b = \frac{-\sqrt{3}}{-\sqrt{3}} = 1$

$$\sqrt{2\left(1-\sqrt{3}i\right)} = \pm\left(\sqrt{3} - i\right)$$

Question 2.
Solve the following quadratic equations:
(i) $8x^2 + 2x + 1 = 0$
Solution:
Given equation is $8x^2 + 2x + 1 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 8$, $b = 2$, $c = 1$
Discriminant $= b^2 - 4ac$
$= (2)^2 - 4 \times 8 \times 1$
$= 4 - 32$
$= -28 < 0$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{-28}}{2(8)} = \frac{-2 \pm 2\sqrt{7}i}{16}$$

$$x = \frac{-1 \pm \sqrt{7}i}{8}$$

The roots of the given equation are $\frac{-1+\sqrt{7}i}{8}$ and $\frac{-1-\sqrt{7}i}{8}$.

(ii) $2x^2 - \sqrt{3}x + 1 = 0$
Solution:
Given equation is $2x^2 - \sqrt{3}x + 1 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 2$, $b = -\sqrt{3}$, $c = 1$
Discriminant $= b^2 - 4ac$
$= (-\sqrt{3})^2 - 4 \times 2 \times 1$
$= 3 - 8$
$= -5 < 0$
So, the given equation has complex roots.

These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-\sqrt{3}) \pm \sqrt{-5}}{2(2)}$$
$$x = \frac{\sqrt{3} \pm \sqrt{5}i}{4}$$

The roots of the given equation are $\frac{\sqrt{3}+\sqrt{5}i}{4}$ and $\frac{\sqrt{3}-\sqrt{5}i}{4}$.

(iii) $3x^2 - 7x + 5 = 0$
Solution:
Given equation is $3x^2 - 7x + 5 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 3$, $b = -7$, $c = 5$
Discriminant $= b^2 - 4ac$
$= (-7)^2 - 4 \times 3 \times 5$
$= 49 - 60$
$= -11 < 0$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-7) \pm \sqrt{-11}}{2(3)}$$
$$x = \frac{7 \pm \sqrt{11}i}{6}$$

The roots of the given equation are $7+11\sqrt{i}6$ and $7-11\sqrt{i}6$

(iv) $x^2 - 4x + 13 = 0$
Solution:
Given equation is $x^2 - 4x + 13 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = -4$, $c = 13$
Discriminant $= b^2 - 4ac$
$= (-4)^2 - 4 \times 1 \times 13$
$= 16 - 52$
$= -36 < 0$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{-36}}{2(1)}$$
$$= \frac{4 \pm 6i}{2} = 2 \pm 3i$$

∴ The roots of the given equation are $2 + 3i$ and $2 - 3i$.

Question 3.
Solve the following quadratic equations:
(i) $x^2 + 3ix + 10 = 0$
Solution:
Given equation is $x^2 + 3ix + 10 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = 3i$, $c = 10$
Discriminant $= b^2 - 4ac$
$= (3i)^2 - 4 \times 1 \times 10$
$= 9i^2 - 40$
$= -9 - 40$ ……[∵ $i^2 = -1$]
$= -49 < 0$
So, the given equation has complex roots.

These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3i \pm \sqrt{-49}}{2(1)}$$
$$x = \frac{-3i \pm 7i}{2}$$
$$x = \frac{-3i + 7i}{2} \text{ or } x = \frac{-3i - 7i}{2}$$

∴ x = 2i or x = -5i
∴ The roots of the given equation are 2i and -5i.

(ii) $2x^2 + 3ix + 2 = 0$
Solution:
Given equation is $2x^2 + 3ix + 2 = 0$
Comparing with ax + bx + c = 0, we get
a = 2, b = 3i, c = 2
Discriminant = $b^2 – 4ac$
= $(3i)^2 – 4 × 2 × 2$
= $9i^2 – 16$
= -9 – 16 …..[∵ i^2 = -1]
= -25 < 0
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \frac{-3i \pm \sqrt{-25}}{2(2)}$$
$$x = \frac{-3i \pm 5i}{4}$$
$$x = \frac{-3i + 5i}{4} \text{ or } x = \frac{-3i - 5i}{4}$$
$$x = \frac{1}{2}i \text{ or } x = -2i$$

∴ The roots of the given equation are $\frac{1}{2}$i and -2i.

(iii) $x^2 + 4ix – 4 = 0$
Solution:
Given equation is $x^2 + 4ix – 4 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
a = 1, b = 4i, c = -4
Discriminant = $b^2 – 4ac$
= $(4i)^2 – 4 × 1 × (-4)$
= $16i^2 + 16$
= -16 + 16 …..[∵ i^2 = -1]
= 0
So, the given equation has equal roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4i \pm \sqrt{0}}{2(1)} = \frac{-4i}{2}$$
$$x = -2i$$

∴ x = -2i
∴ The root of the given equation is -2i.

(iv) $ix^2 – 4x – 4i = 0$
Solution:
$ix^2 – 4x – 4i = 0$
Multiplying throughout by i, we get
$i^2x^2 – 4ix – 4i^2 = 0$
$-x^2 – 4ix + 4 = 0$ …[∵ i^2 = -1]
$x^2 + 4ix – 4 = 0$
Comparing with $ax^2 + bx + c = 0$, we get
a = 1, b = 4i, c = -4
Discriminant = $b^2 – 4ac$

$= (4i)^2 - 4 \times 1 \times (-4)$
$= 16i^2 + 16$
$= -16 + 16$
$= 0$
So, the given equation has equal roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4i \pm \sqrt{0}}{2(1)}$$
$$= \frac{-4i}{2}$$
$$x = -2i$$

∴ The root of the given equation is -2i

Question 4.
Solve the following quadratic equations:
(i) $x^2 - (2 + i) x - (1 - 7i) = 0$
Solution:
Given equation is $x^2 - (2 + i)x - (1 - 7i) = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 1, b = -(2 + i), c = -(1 - 7i)$
Discriminant = $b^2 - 4ac$
$= [-(2 + i)]^2 - 4 \times 1 \times -(1 - 7i)$
$= 4 + 4i + i^2 + 4 - 28i$
$= 4 + 4i - 1 + 4 - 28i$ .....[∵ $i^2 = -1$]
$= 7 - 24i$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-[-(2+i)] \pm \sqrt{7-24i}}{2(1)} = \frac{(2+i) \pm \sqrt{7-24i}}{2}$$

Let $\sqrt{7-24i}$ = a + bi, where a, b ∈ R
Squaring on both sides, we get
$7 - 24i = a^2 + i^2b^2 + 2abi$
$7 - 24i = a^2 - b^2 + 2abi$
Equating real and imaginary parts, we get

$a^2 - b^2 = 7$ and $2ab = -24$

$a^2 - b^2 = 7$ and $b = \frac{-12}{a}$

$a^2 - \left(\frac{-12}{a}\right)^2 = 7$

$a^2 - \frac{144}{a^2} = 7$

$a^4 - 144 = 7a^2$

$a^4 - 7a^2 - 144 = 0$

$(a^2 - 16)(a^2 + 9) = 0$

$a^2 = 16$ or $a^2 = -9$

But $a \in R$

$a^2 \neq -9$

$a^2 = 16$

$a = \pm 4$

When $a = 4$, $b = \frac{-12}{4} = -3$

When $a = -4$, $b = \frac{-12}{-4} = 3$

$\sqrt{7-24i} = \pm(4 - 3i)$

$x = \frac{(2+i) \pm (4-3i)}{2}$

$x = \frac{(2+i)+(4-3i)}{2}$ or $x = \frac{(2+i)-(4-3i)}{2}$

$x = 3 - i$ or $x = -1 + 2i$

(ii) $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

Solution:

Given equation is $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = -(3\sqrt{2} + 2i)$, $c = 6\sqrt{2}i$

Discriminant = $b^2 - 4ac$

$= [-(3\sqrt{2} + 2i)]^2 - 4 \times 1 \times 6\sqrt{2}i$

$= 18 + 12\sqrt{2}i + 4i^2 - 24\sqrt{2}i$

$= 18 - 12\sqrt{2}i - 4$ ……[∵ $i^2 = -1$]

$= 14 - 12\sqrt{2}i$

So, the given equation has complex roots.

These roots are given by

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-[-(3\sqrt{2}+2i)] \pm \sqrt{14-12\sqrt{2}i}}{2(1)}$$

$$= \frac{(3\sqrt{2}+2i) \pm \sqrt{14-12\sqrt{2}i}}{2}$$

Let $\sqrt{14-12\sqrt{2}i} = a + bi$, where $a, b \in R$

Squaring on both sides, we get

$14 - 12\sqrt{2}i = a^2 + i^2b^2 + 2abi$

$14 - 12\sqrt{2}i = a^2 - b^2 + 2abi$

Equating real and imaginary parts, we get

$a^2 - b^2 = 14$ and $2ab = -12\sqrt{2}$

$a^2 - b^2 = 14$ and $b = \frac{-6\sqrt{2}}{a}$

$a^2 - \left(\frac{-6\sqrt{2}}{a}\right)^2 = 14$

$a^2 - \frac{72}{a^2} = 14$

$a^4 - 72 = 14a^2$

$a^4 - 14a^2 - 72 = 0$

$(a^2 - 18)(a^2 + 4) = 0$

$a^2 = 18$ or $a^2 = -4$

But $a \in R$

$a^2 \neq -4$

$a^2 = 18$

$a = \pm 3\sqrt{2}$

When $a = 3\sqrt{2}$, $b = \frac{-6\sqrt{2}}{3\sqrt{2}} = -2$

When $a = -3\sqrt{2}$, $b = \frac{-6\sqrt{2}}{-3\sqrt{2}} = 2$

$\sqrt{14 - 12\sqrt{2}i} = \pm\left(3\sqrt{2} - 2i\right)$

$x = \frac{(3\sqrt{2} + 2i) \pm (3\sqrt{2} - 2i)}{2}$

$x = \frac{(3\sqrt{2} + 2i) + (3\sqrt{2} - 2i)}{2}$

or $x = \frac{(3\sqrt{2} + 2i) - (3\sqrt{2} - 2i)}{2}$

$x = 3\sqrt{2}$ or $x = 2i$

(iii) $x^2 - (5 - i)x + (18 + i) = 0$

Solution:

Given equation is $x^2 - (5 - i)x + (18 + i) = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = -(5 - i)$, $c = 18 + i$

Discriminant $= b^2 - 4ac$

$= [-(5 - i)]^2 - 4 \times 1 \times (18 + i)$

$= 25 - 10i + i^2 - 72 - 4i$

$= 25 - 10i - 1 - 72 - 4i$ ......[$\because i^2 = -1$]

$= -48 - 14i$

So, the given equation has complex roots.

These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-[-(5-i)] \pm \sqrt{-48-14i}}{2(1)} = \frac{(5-i) \pm \sqrt{-48-14i}}{2}$$

Let $\sqrt{-48-14i} = a + bi$, where a, b∈R

Squaring on both sides, we get

$-48 - 14i = a^2 + b^2i^2 + 2abi$

$-48 - 14i = a^2 - b^2 + 2abi$

Equating real and imaginary parts, we get

$a^2 - b^2 = -48$ and $2ab = -14$

$\therefore \quad a^2 - b^2 = -48$ and $b = \frac{-7}{a}$

$\therefore \quad a^2 - \left(\frac{-7}{a}\right)^2 = -48$

$\therefore \quad a^2 - \frac{49}{a^2} = -48$

$\therefore \quad a^4 - 49 = -48a^2$

$\therefore \quad a^4 + 48a^2 - 49 = 0$

$\therefore \quad (a^2 + 49)(a^2 - 1) = 0$

$\therefore \quad a^2 = -49$ or $a^2 = 1$

But $a \in R$

$\therefore \quad a^2 \neq -49$

$\therefore \quad a^2 = 1$

$\therefore \quad a = \pm 1$

When $a = 1$, $b = -\frac{7}{1} = -7$

When $a = -1$, $b = \frac{-7}{-1} = 7$

$\therefore \quad \sqrt{-48-14i} = \pm(1 - 7i)$

$\therefore \quad x = \frac{(5-i) \pm (1-7i)}{2}$

$\therefore \quad x = \frac{5-i+1-7i}{2}$ or $x = \frac{5-i-1+7i}{2}$

$\therefore \quad x = 3 - 4i$ or $x = 2 + 3i$

(iv) $(2 + i) x^2 – (5 – i) x + 2(1 – i) = 0$
Solution:
Given equation is $(2 + i) x^2 – (5 – i) x + 2(1 – i) = 0$
Comparing with $ax^2 + bx + c = 0$, we get
$a = 2 + i$, $b = -(5 – i)$, $c = 2(1 – i)$
Discriminant = $b^2 – 4ac$
$= [-(5 – i)]^2 – 4 × (2 + i) × 2(1 – i)$
$= 25 – 10i + i^2 – 8(2 + i) (1 – i)$
$= 25 – 10i + i^2 – 8(2 – 2i + i – i^2)$
$= 25 – 10i – 1 – 8(2 – i + 1)$ .....[∵ $i^2 = -1$]
$= 25 – 10i – 1 – 16 + 8i – 8$
$= -2i$
So, the given equation has complex roots.
These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-[-(5-i)] \pm \sqrt{-2i}}{2(2+i)} = \frac{(5-i) \pm \sqrt{-2i}}{2(2+i)}$$

Let $\sqrt{-2i}$ = a + bi, where a, b ∈ R
Squaring on both sides, we get
$-2i = a^2 + b^2i^2 + 2abi$
$-2i = a^2 – b^2 + 2abi$
Equating real and imaginary parts, we get
$a^2 – b^2 = 0$ and $2ab = -2$
$a^2 – b^2 = 0$ and $b = \frac{-1}{a}$

$a^2 - \left(\frac{-1}{a}\right)^2 = 0$

$a^2 - \frac{1}{a^2} = 0$

$a^4 - 1 = 0$

$(a^2 - 1)(a^2 + 1) = 0$

$a^2 = 1$ or $a^2 = -1$

But $a \in R$

$a^2 \neq -1$

$a^2 = 1$

$a = \pm 1$

When $a = 1, b = -1$

When $a = -1, b = 1$

$\sqrt{-2i} = \pm(1 - i)$

$x = \frac{(5-i) \pm (1-i)}{2(2+i)}$

$x = \frac{5-i+1-i}{2(2+i)}$ or $x = \frac{5-i-1+i}{2(2+i)}$

$x = \frac{6-2i}{2(2+i)}$ or $x = \frac{4}{2(2+i)}$

$x = \frac{2(3-i)}{2(2+i)}$ or $x = \frac{2}{2+i}$

$x = \frac{3-i}{2+i}$ or $x = \frac{2(2-i)}{(2+i)(2-i)}$

$x = \frac{(3-i)(2-i)}{(2+i)(2-i)}$ or $x = \frac{2(2-i)}{4-i^2}$

$x = \frac{6-5i+i^2}{4-i^2}$ or $x = \frac{4-2i}{4-i^2}$

$x = \frac{5-5i}{5}$ or $x = \frac{4-2i}{5}$ ...[$\because i^2 = -1$]

$x = 1 - i$ or $x = \frac{4}{5} - \frac{2i}{5}$

Question 5.
Find the value of
(i) $x^3 - x^2 + x + 46$, if $x = 2 + 3i$
Solution:
$x = 2 + 3i$
$x - 2 = 3i$
$(x - 2)^2 = 9i^2$
$x^2 - 4x + 4 = 9(-1)$ .....[$\because i^2 = -1$]
$x^2 - 4x + 13 = 0$ .....(i)

$$\begin{array}{r|l} & x+3 \\ \hline x^2-4x+13 & x^3 - x^2 + x + 46 \\ & x^3 - 4x^2 + 13x \\ & - \quad + \quad - \\ \hline & 3x^2 - 12x + 46 \\ & 3x^2 - 12x + 39 \\ & - \quad + \quad - \\ \hline & 7 \end{array}$$

Dividend = Divisor × Quotient + Remainder
$\therefore x^3 - x^2 + x + 46 = (x^2 - 4x + 13)(x + 3) + 7$
$= 0(x + 3) + 7$ .....[from(i)]
$= 7$

Alternate Method:
$x = 2 + 3i$

$\alpha = 2 + 3i$, $\bar{\alpha} = 2 - 3i$

$\alpha\bar{\alpha} = (2 + 3i)(2 - 3i)$
$= 4 - 6i + 6i - 9i^2$
$= 4 - 9(-1)$
$= 4 + 9$
$= 13$

$\alpha + \bar{\alpha} = 2 + 3i + 2 - 3i = 4$

∴ Standard form of quadratic equation,
x^2 – (Sum of roots) x + Product of roots = 0
$x^2 - 4x + 13 = 0$

$$\begin{array}{r|l} & x+3 \\ \hline x^2-4x+13 & x^3 - x^2 + x + 46 \\ & x^3 - 4x^2 + 13x \\ & - \quad + \quad - \\ \hline & 3x^2 - 12x + 46 \\ & 3x^2 - 12x + 39 \\ & - \quad + \quad - \\ \hline & 7 \end{array}$$

Dividend = Divisor × Quotient + Remainder
∴ $x^3 - x^2 + x + 46 = (x^2 - 4x + 13).(x + 3) + 7$
$= 0(x + 3) + 7$ .....[From (i)]
$= 7$

(ii) $2x^3 - 11x^2 + 44x + 27$, if $x = \frac{25}{3-4i}$
Solution:

$$x = \frac{25}{3-4i}$$
$$x = \frac{25(3+4i)}{(3-4i)(3+4i)}$$
$$= \frac{25(3+4i)}{9-16i^2}$$
$$= \frac{25(3+4i)}{9-16(-1)} \quad \ldots[\because i^2 = -1]$$
$$= \frac{25(3+4i)}{25}$$
$$x = 3 + 4i$$
$$x - 3 = 4i$$
$$(x-3)^2 = 16i^2$$
$$x^2 - 6x + 9 = 16(-1) \quad \ldots[\because i^2 = -1]$$
$$x^2 - 6x + 25 = 0 \quad \ldots(i)$$

$$\begin{array}{r|l} & 2x+1 \\ \hline x^2-6x+25 & 2x^3 - 11x^2 + 44x + 27 \\ & 2x^3 - 12x^2 + 50x \\ & - \quad + \quad - \\ \hline & x^2 - 6x + 27 \\ & x^2 - 6x + 25 \\ & - \quad + \quad - \\ \hline & 2 \end{array}$$

Dividend = Divisor × Quotient + Remainder
$2x^3 - 11x^2 + 44x + 27 = (x^2 - 6x + 25)(2x + 1) + 2$
$= 0.(2x + 1) + 2$ .....[From (i)]
$= 0 + 2$
$= 2$

(iii) $x^3 + x^2 - x + 22$, if $x = \frac{5}{1-2i}$
Solution:

$$x = \frac{5}{1-2i}$$
$$= \frac{5}{1-2i} \times \frac{1+2i}{1+2i} = \frac{5(1+2i)}{1-4i^2}$$
$$= \frac{5(1+2i)}{1-4(-1)} \quad \ldots[\because i^2 = -1]$$
$$= \frac{5(1+2i)}{5}$$
$$x = 1 + 2i$$
$$x - 1 = 2i$$
$$(x-1)^2 = 4i^2$$
$$x^2 - 2x + 1 = 4(-1) \quad \ldots[\because i^2 = -1]$$
$$x^2 - 2x + 1 = -4$$
$$x^2 - 2x + 5 = 0 \quad \ldots(i)$$

$$\begin{array}{r|l} & x+3 \\ \hline x^2-2x+5 & x^3 + x^2 - x + 22 \\ & x^3 - 2x^2 + 5x \\ & - \quad + \quad - \\ \hline & 3x^2 - 6x + 22 \\ & 3x^2 - 6x + 15 \\ & - \quad + \quad - \\ \hline & 7 \end{array}$$

Dividend = Divisor × Quotient + Remainder
$x^3 + x^2 - x + 22 = (x^2 - 2x + 5)(x + 3) + 7$
$= 0.(x + 3) + 7$ .....[From (i)]
$= 0 + 7$
$= 7$

(iv) $x^4 + 9x^3 + 35x^2 - x + 4$, if $x = -5 + \sqrt{-4}$
Solution:
$x = -5 + \sqrt{-4}$
$x + 5 = \sqrt{-4}$
$x + 5 = \sqrt{4}\ \sqrt{-1}$
$x + 5 = 2i$
$(x + 5)^2 = 4i^2$
$x^2 + 10x + 25 = 4(-1)$ ....[$\because i^2 = -1$]
$x^2 + 10x + 29 = 0$ .....(i)

$$\begin{array}{r|l} & x^2 - x + 16 \\ \hline x^2+10x+29 & x^4 + 9x^3 + 35x^2 - x + 4 \\ & x^4 + 10x^3 + 29x^2 \\ & - \quad - \quad - \\ \hline & -x^3 + 6x^2 - x + 4 \\ & -x^3 - 10x^2 - 29x \\ & + \quad + \quad + \\ \hline & 16x^2 + 28x + 4 \\ & 16x^2 + 160x + 464 \\ & - \quad - \quad - \\ \hline & -132x - 460 \end{array}$$

Dividend = Divisor × Quotient + Remainder
$x^4 + 9x^3 + 35x^2 - x + 4 = (x^2 + 10x + 29)(x^2 - x + 16) - 132x - 460$
$= 0.(x^2 - x + 16) - 132x - 460$ .....[From (i)]
$= -132(-5 + 2i) - 460$
$= 660 - 264i - 460$
$= 200 - 264i$

(v) $2x^4 + 5x^3 + 7x^2 - x + 41$, if $x = -2 - \sqrt{3}i$
Solution:

$x = -2 - \sqrt{3}\, i$

$x + 2 = -\sqrt{3}\, i$

$(x + 2)^2 = 3i^2$

$x^2 + 4x + 4 = 3(-1)$ ...[$\because i^2 = -1$]

$x^2 + 4x + 7 = 0$ ...(i)

$$
\begin{array}{r|l}
 & 2x^2 - 3x + 5 \\
\hline
x^2 + 4x + 7 & 2x^4 + 5x^3 + 7x^2 - x + 41 \\
 & 2x^4 + 8x^3 + 14x^2 \\
 & - \quad - \quad - \\
\hline
 & -3x^3 - 7x^2 - x + 41 \\
 & -3x^3 - 12x^2 - 21x \\
 & + \quad + \quad + \\
\hline
 & 5x^2 + 20x + 41 \\
 & 5x^2 + 20x + 35 \\
 & - \quad - \quad - \\
\hline
 & 6
\end{array}
$$

Dividend = Divisor × Quotient + Remainder

$2x^4 + 5x^3 + 7x^2 - x + 41 = (x^2 + 4x + 7)(2x^2 - 3x + 5) + 6$

$= 0(2x^2 - 3x + 5) + 6$ ......[From (i)]

$= 0 + 6$

$= 6$

Maharashtra State Board 11th Maths Solutions Chapter 1 Complex Numbers Ex 1.3

Question 1.
Find the modulus and amplitude for each of the following complex numbers:
(i) 7 – 5i
Solution:
Let z = 7 – 5i
a = 7, b = -5
i.e. a > 0, b < 0

$|z| = \sqrt{a^2+b^2} = \sqrt{7^2+(-5)^2} = \sqrt{49+25} = \sqrt{74}$

Here, (7, -5) lies in 4th quadrant.

$$
\begin{aligned}
\therefore \quad \text{amp}(z) &= -\tan^{-1}\left|\frac{b}{a}\right| \\
&= -\tan^{-1}\left|\frac{-5}{7}\right| \\
&= -\tan^{-1}\left(\frac{5}{7}\right)
\end{aligned}
$$

(ii) √3 + √2 i
Solution:
Let z = √3 + √2 i
a = √3, b = √2,

i.e. a > 0, b > 0

$$\therefore \quad |z| = \sqrt{a^2+b^2} = \sqrt{\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2} = \sqrt{3+2} = \sqrt{5}$$

Here, $\left(\sqrt{3}, \sqrt{2}\right)$ lies in 1st quadrant.

$$\therefore \quad \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{3}}\right)$$

$$= \tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

(iii) -8 + 15i
Solution:
Let z = -8 + 15i
a = -8, b = 15 , i.e. a < 0, b > 0

$$\therefore \quad |z| = \sqrt{a^2+b^2} = \sqrt{(-8)^2+15^2} = \sqrt{64+225}$$

$$= \sqrt{289} = 17$$

Here, (–8, 15) lies in 2nd quadrant.

$$\therefore \quad \text{amp}(z) = \pi - \tan^{-1}\left|\frac{b}{a}\right|$$

$$= \pi - \tan^{-1}\left|\frac{15}{-8}\right|$$

$$= \pi - \tan^{-1}\left(\frac{15}{8}\right)$$

(iv) -3(1 – i)
Solution:
Let z = -3(1 – i) = -3 + 3i
a = -3, b = 3 , i.e. a < 0, b > 0
$|z| = \sqrt{a^2+b^2} = \sqrt{(-3)^2+3^2} = \sqrt{9+9} = 3\sqrt{2}$
Here, (-3, 3) lies in 2nd quadrant.
$\text{amp}(z) = \pi - \tan^{-1}\left|\frac{b}{a}\right|$

$$= \pi - \tan^{-1}\left|\frac{3}{-3}\right|$$

$$= \pi - \tan^{-1}|-1| = \pi - \tan^{-1}(1)$$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

(v) -4 – 4i
Solution:
Let z = -4 – 4i
a = -4, b = -4 , i.e. a < 0, b < 0

$$\therefore \quad |z| = \sqrt{a^2+b^2} = \sqrt{(-4)^2+(-4)^2} = \sqrt{16+16}$$

$$= \sqrt{32} = 4\sqrt{2}$$

Here, (– 4, – 4) lies in 3rd quadrant.

$$\therefore \quad \text{amp}(z) = \pi + \tan^{-1}\left|\frac{b}{a}\right| = \pi + \tan^{-1}\left|\frac{-4}{-4}\right|$$

$$= \pi + \tan^{-1}(1)$$

$$\therefore \quad \text{amp}(z) = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

(vi) √3 – i
Solution:
Let z = √3 – i

a = √3, b = -1, i.e. a > 0, b < 0

$$\therefore \quad |z| = \sqrt{a^2+b^2} = \sqrt{\left(\sqrt{3}\right)^2+(-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$

Here, $(\sqrt{3}, -1)$ lies in 4^{th} quadrant.

$$\therefore \quad \text{amp}(z) = 2\pi - \tan^{-1}\left|\frac{b}{a}\right|$$

$$= 2\pi - \tan^{-1}\left|\frac{-1}{\sqrt{3}}\right| = 2\pi - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

(vii) 3
Solution:
Let z = 3 + 0i
a = 3, b = 0
z is a real number, it lies on the positive real axis.
|z|= $\sqrt{a^2+b^2} = \sqrt{3^2+0^2} = \sqrt{9+0}$ = 3
and amp (z) = 0

(viii) 1 + i
Solution:
Let z = 1 + i
a = 1, b = 1, i.e. a > 0, b > 0
|z| = $\sqrt{a^2+b^2} = \sqrt{1^2+1^2} = \sqrt{1+1} = \sqrt{2}$
Here, (1, 1) lies in 1st quadrant.
amp (z) = $\tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

(ix) 1 + i√3
Solution:
Let z = 1 + i√3
a = 1, b = √3, i.e. a > 0, b > 0
|z| = $\sqrt{a^2+b^2} = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{1+3} = 2$
Here, (1, √3) lies in 1st quadrant.
amp (z) = $\tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

(x) $(1 + 2i)^2 (1 – i)$
Solution:
Let z = $(1 + 2i)^2 (1 – i)$
= $(1 + 4i + 4i^2) (1 – i)$
= [1 + 4i + 4(-1)] (1 – i) ....[∵ i^2 = -1]
= (-3 + 4i) (1 – i)
= $-3 + 3i + 4i – 4i^2$
= -3 + 7i – 4(-1)
= -3 + 7i + 4
∴ z = 1 + 7i
∴ a = 1, b = 7, i. e. a > 0, b > 0
∴ |z| = $\sqrt{a^2+b^2} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{52}$
Here, (1, 7) lies in 1st quadrant.
∴ amp(z) = $\tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}(7)$

Question 2.
Find real values of θ for which $\left(\frac{4+3i\sin\theta}{1-2i\sin\theta}\right)$ is purely real.

Solution:

$$\frac{4+3\,i\sin\theta}{1-2\,i\sin\theta}=\frac{(4+3\,i\sin\theta)(1+2\,i\sin\theta)}{(1-2\,i\sin\theta)(1+2\,i\sin\theta)}$$

$$=\frac{4+8\,i\sin\theta+3\,i\sin\theta+6\,i^2\sin^2\theta}{1-4\,i^2\sin^2\theta}$$

$$=\frac{4+11(i\sin\theta)-6\sin^2\theta}{1+4\sin^2\theta}\qquad\ldots[\because i^2=-1]$$

$$=\left(\frac{4-6\sin^2\theta}{1+4\sin^2\theta}\right)+\left(\frac{11\sin\theta}{1+4\sin^2\theta}\right)i$$

Since z is purely real, Im(z) = 0

$\therefore\quad \dfrac{11\sin\theta}{1+4\sin^2\theta}=0$

$\therefore\quad \sin\theta=0$

$\therefore\quad \sin\theta=\sin n\pi$, where $n\in Z$

$\therefore\quad \theta=n\pi$, where $n\in Z$

Question 3.

If z = 3 + 5i, then represent the z, $\bar{z}$, -z, $\overline{-z}$ in Argand's diagram.

Solution:

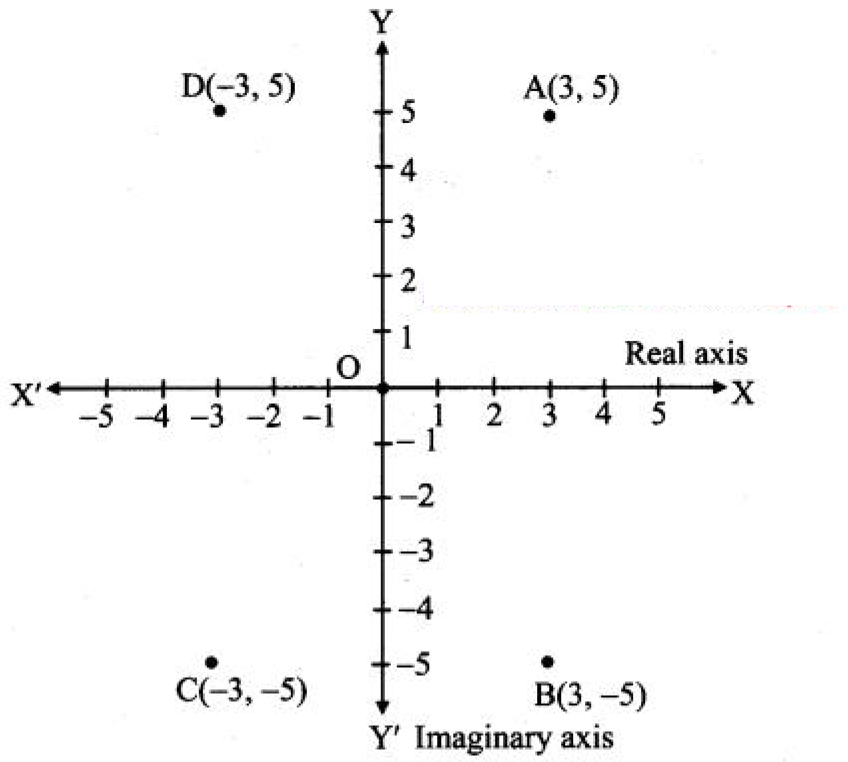
z = 3 + 5i

$\bar{z}$ = 3 – 5i

-z = – 3 – 5i

$\overline{-z}$ = -3 + 5i

The above complex numbers will be represented by the points
A (3, 5), B (3, -5), C (-3, -5) , D (-3, 5) respectively as shown below:



Question 4.

Express the following complex numbers in polar form and exponential form.

(i) -1 + √3 i

Solution:

Let z = – 1 + √3

a = -1, b = √3

$\therefore \quad |z| = r = \sqrt{a^2+b^2} = \sqrt{(-1)^2+(\sqrt{3})^2} = \sqrt{1+3} = 2$

Here, $(-1, \sqrt{3})$ lies in 2^{nd} quadrant.

$$\therefore \quad \theta = \text{amp}(z) = \pi - \tan^{-1}\left|\frac{b}{a}\right|$$
$$= \pi - \tan^{-1}\left|\frac{\sqrt{3}}{-1}\right|$$
$$= \pi - \tan^{-1}(\sqrt{3})$$
$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$\therefore \quad \theta = 120^\circ = \frac{2\pi}{3}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$= 2(\cos 120^\circ + i \sin 120^\circ)$

$= 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

The exponential form of $z = re^{i\theta} = 2e^{\frac{2\pi}{3}i}$

(ii) -i
Solution:
Let z = -i = 0 – i
a = 0, b = -1
z lies on negative imaginary Y-axis.
$|z| = r = \sqrt{a^2+b^2} = \sqrt{0^2+(-1)^2}$ = 1 and
θ = amp z = 270° = $\frac{3\pi}{2}$
The polar form of z = r (cos θ + i sin θ)
= 1 (cos 270° + i sin 270°)
= 1 (cos $\frac{3\pi}{2}$ + i sin $\frac{3\pi}{2}$)
The exponential form of z = $re^{i\theta} = e^{\frac{3\pi}{2}i}$

(iii) -1
Solution:
Let z = -1 = -1 + 0.i
a = -1, b = 0
z lies on negative real X-axis.
$|z| = r = \sqrt{a^2+b^2} = \sqrt{(-1)^2+0^2}$ = 1 and
θ = amp z = 180° = π
The polar form of z = r (cos θ + i sin θ)
= 1 (cos 180° + i sin 180°)
= 1 (cos π + i sin π)
The exponential form of z = $re^{i\theta} = e^{\pi i}$

(iv) $\frac{1}{1+i}$
Solution:

$$\text{Let } z = \frac{1}{1+i} = \frac{1-i}{(1+i)(1-i)}$$

$$= \frac{1-i}{1-i^2} = \frac{1-i}{1-(-1)} \quad \ldots[\because i^2 = -1]$$

$$= \frac{1-i}{2}$$

$\therefore \quad z = \frac{1}{2} - \frac{1}{2}i$

$\therefore \quad a = \frac{1}{2}, b = \frac{-1}{2}$

$\therefore \quad |z| = r = \sqrt{a^2 + b^2}$

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{1}{\sqrt{2}}$$

Here, $\left(\frac{1}{2}, \frac{-1}{2}\right)$ lies in 4^{th} quadrant.

$\therefore \quad \theta = \text{amp }(z) = 2\pi - \tan^{-1}\left|\frac{b}{a}\right|$

$$= 2\pi - \tan^{-1}\left|\frac{\frac{-1}{2}}{\frac{1}{2}}\right|$$

$$= 2\pi - \tan^{-1}|-1|$$

$$= 2\pi - \tan^{-1}(1)$$

$$= 2\pi - \frac{\pi}{4}$$

$$= \frac{7\pi}{4}$$

$\therefore \quad \theta = 315° = \frac{7\pi}{4}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$$= \frac{1}{\sqrt{2}}(\cos 315° + i \sin 315°)$$

$$= \frac{1}{\sqrt{2}}\left[\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right]$$

The exponential form of $z = re^{i\theta} = \frac{1}{\sqrt{2}}e^{\frac{7\pi}{4}i}$

(v) $\frac{1+2i}{1-3i}$

Solution:

$$\text{Let } z = \frac{1+2i}{1-3i} = \frac{(1+2i)(1+3i)}{(1-3i)(1+3i)}$$

$$= \frac{1+2i+3i+6i^2}{1-9i^2}$$

$$= \frac{1+5i+6(-1)}{1-9(-1)} \qquad \ldots[\because i^2 = -1]$$

$$= \frac{-5+5i}{10}$$

$$= \frac{-1}{2} + \frac{1}{2} i$$

$\therefore \quad a = \frac{-1}{2}, b = \frac{1}{2}$

$\therefore \quad |z| = r = \sqrt{a^2+b^2} = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$$= \sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{1}{\sqrt{2}}$$

Here, $\left(\frac{-1}{2}, \frac{1}{2}\right)$ lies in 2nd quadrant.

$\therefore \quad \theta = \text{amp } z = \pi - \tan^{-1}\left|\frac{b}{a}\right|$

$$= \pi - \tan^{-1}\left|\frac{\frac{1}{2}}{\frac{-1}{2}}\right|$$

$$= \pi - \tan^{-1}|-1|$$

$$= \pi - \tan^{-1}(1)$$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

$\therefore \quad \theta = 135° = \frac{3\pi}{4}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$$= \frac{1}{\sqrt{2}}(\cos 135° + i \sin 135°)$$

$$= \frac{1}{\sqrt{2}}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

The exponential form of $z = re^{i\theta} = \frac{1}{\sqrt{2}} e^{\frac{3\pi}{4}i}$

(vi) $\frac{1+7i}{(2-i)^2}$

Solution:

$$\text{Let } z = \frac{1+7i}{(2-i)^2}$$

$$= \frac{1+7i}{4-4i+i^2} = \frac{1+7i}{4-4i-1}$$

$$= \frac{1+7i}{3-4i} = \frac{(1+7i)(3+4i)}{(3-4i)(3+4i)}$$

$$= \frac{3+4i+21i+28i^2}{9-16i^2}$$

$$= \frac{25i+3+28(-1)}{9-16(-1)}$$

$$= \frac{25i-25}{25}$$

$\therefore$ $z = -1 + i$

$\therefore$ $a = -1, b = 1$

$\therefore$ $|z| = r = \sqrt{a^2+b^2} = \sqrt{(-1)^2+1^2} = \sqrt{2}$

Here, $(-1, 1)$ lies in 2^{nd} quadrant

$\therefore$ $\theta = \text{amp } z = \pi - \tan^{-1}\left|\frac{b}{a}\right|$

$$= \pi - \tan^{-1}\left|\frac{1}{-1}\right|$$

$$= \pi - \tan^{-1}|-1| = \pi - \tan^{-1}(1)$$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

$\therefore$ $\theta = 135° = \frac{3\pi}{4}$

$\therefore$ The polar form of $z = r(\cos\theta + i.\sin\theta)$

$$= \sqrt{2}(\cos 135° + i \sin 135°)$$

$$= \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

The exponential form of $z = re^{i\theta} = \sqrt{2}\, e^{\frac{3\pi}{4}i}$

Question 5.
Express the following numbers in the form x + iy:
(i) $\sqrt{3}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$
Solution:

$$\sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \sqrt{3}\left(\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right)$$

$$= \frac{3}{2} + \frac{\sqrt{3}}{2}i$$

(ii) $\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)$
Solution:

$$\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

$$= \sqrt{2}\left[\cos\left(2\pi - \frac{\pi}{4}\right) + i\sin\left(2\pi - \frac{\pi}{4}\right)\right]$$

$$= \sqrt{2}\left[\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right]$$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}} + i\left(-\frac{1}{\sqrt{2}}\right)\right]$$

$$= 1 - i$$

(iii) $7(\cos(-\frac{5\pi}{6})+i\sin(-\frac{5\pi}{6}))$

Solution:

$$7\left[\cos\left(-\frac{5\pi}{6}\right)+i\sin\left(-\frac{5\pi}{6}\right)\right]$$
$$=7\left[\cos\frac{5\pi}{6}-i\sin\frac{5\pi}{6}\right]$$
$$=7\left[\cos\left(\pi-\frac{\pi}{6}\right)-i\sin\left(\pi-\frac{\pi}{6}\right)\right]$$
$$=7\left[-\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right]$$
$$=7\left[-\frac{\sqrt{3}}{2}-i\left(\frac{1}{2}\right)\right]$$
$$=-\frac{7\sqrt{3}}{2}-\frac{7}{2}i$$

(iv) $e^{\frac{\pi}{3}i}$

Solution:

$$z=re^{i\theta}=e^{\frac{\pi}{3}i}$$
$$\therefore \quad r=1,\ \theta=\frac{\pi}{3}$$
$$\text{Polar form of } z=r(\cos\theta+i\sin\theta)$$
$$=1\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$$
$$=\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

(v) $e^{-\frac{4\pi}{3}i}$

Solution:

$$z=re^{i\theta}=e^{\frac{-4\pi}{3}i}$$
$$\therefore \quad r=1,\ \theta=\frac{-4\pi}{3}$$
$$\text{Polar form of } z=r(\cos\theta+i\sin\theta)$$
$$=1\left[\cos\left(-\frac{4\pi}{3}\right)+i\sin\left(-\frac{4\pi}{3}\right)\right]$$
$$=\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}$$
$$=\cos\left(\pi+\frac{\pi}{3}\right)-i\sin\left(\pi+\frac{\pi}{3}\right)$$
$$=-\cos\frac{\pi}{3}-i\left(-\sin\frac{\pi}{3}\right)$$
$$=\frac{-1}{2}-i\left(-\frac{\sqrt{3}}{2}\right)$$
$$=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

(vi) [latex]$e^{\frac{5\pi}{6}i}$[/latex]
Solution:

$$z = re^{i\theta} = e^{\frac{5\pi}{6}i}$$

$$\therefore \quad r = 1, \theta = \frac{5\pi}{6}$$

Polar form of $z = r(\cos\theta + i\sin\theta)$

$$= 1\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$$

$$= \cos\left(\pi - \frac{\pi}{6}\right) + i\sin\left(\pi - \frac{\pi}{6}\right)$$

$$= -\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$$

$$= -\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}$$

$$= -\frac{\sqrt{3}}{2} + \frac{1}{2}i$$

Question 6.
Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$.
Solution:

Let $z = \frac{1+2i}{1-3i}$

$$= \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i}$$

$$= \frac{1+3i+2i+6i^2}{1-9i^2}$$

$$= \frac{1+5i-6}{1+9} \quad ...[\because i^2 = -1]$$

$$\therefore z = \frac{-5+5i}{10}$$

$$= -\frac{1}{2} + \frac{1}{2}i$$

This is of the form a + bi, where $a = -\frac{1}{2}$, $b = \frac{1}{2}$

$\therefore$ modulus = r

$$= \sqrt{a^2 + b^2}$$

$$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{1}{4}}$$

$$= \frac{1}{\sqrt{2}}$$

If θ is the argument, then

$\cos\theta = \frac{a}{r}$

$= \frac{\left(-\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$

$= -\frac{1}{\sqrt{2}}$

and $\sin\theta = \frac{b}{r}$

$= \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$

$= \frac{1}{\sqrt{2}}$

$\therefore \theta = \frac{3\pi}{4} \quad \ldots \left[\because \cos\frac{3\pi}{4} = \cos\left(\pi - \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}} \text{ and } \sin\frac{3\pi}{4} = \sin\left(\pi - \frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right]$

Hence, modulus = $\frac{1}{\sqrt{2}}$ and argurement = $\frac{3\pi}{4}$

Question 7.
Convert the complex number $z=i-1\cos\pi 3+i\sin\pi 3$ in the polar form.
Solution:

$$z = \frac{i-1}{\cos\frac{\pi}{3} + i\ \sin\frac{\pi}{3}}$$

$$= \frac{i-1}{\frac{1}{2}+\frac{\sqrt{3}}{2}i} = \frac{2(i-1)}{1+\sqrt{3}\,i}$$

$$= \frac{2(i-1)(1-\sqrt{3}\,i)}{(1+\sqrt{3}\,i)(1-\sqrt{3}\,i)} = \frac{2(i-\sqrt{3}\,i^2-1+\sqrt{3}\,i)}{1-3i^2}$$

$$= \frac{2[i-\sqrt{3}(-1)-1+\sqrt{3}\,i]}{1-3(-1)} \quad \ldots[\because i^2=-1]$$

$$= \frac{2}{4}(i+\sqrt{3}-1+\sqrt{3}\,i)$$

$$\therefore \quad z = \frac{\sqrt{3}-1}{2} + \frac{(\sqrt{3}+1)}{2}i$$

$$\therefore \quad a = \frac{\sqrt{3}-1}{2},\ b = \frac{\sqrt{3}+1}{2}$$

$$\therefore \quad |z| = r = \sqrt{a^2+b^2} = \sqrt{\left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{\sqrt{3}+1}{2}\right)^2}$$

$$= \sqrt{\frac{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2}{4}}$$

$$= \sqrt{\frac{3-2\sqrt{3}+1+3+2\sqrt{3}+1}{4}}$$

$$= \sqrt{\frac{8}{4}} = \sqrt{2}$$

Here, $\left(\frac{\sqrt{3}-1}{2}, \frac{\sqrt{3}+1}{2}\right)$ lies in 1st quadrant.

$$\therefore \quad \text{amp}(z) = \theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{\frac{\sqrt{3}+1}{2}}{\frac{\sqrt{3}-1}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$$

$\therefore$ The polar form of $z = r(\cos\theta + i.\sin\theta)$

$= \sqrt{2}(\cos\theta + i.\sin\theta)$, where $\theta = \tan^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

Question 8.
For z = 2 + 3i, verify the following:
(i) $\overline{(\bar{z})} = z$
Solution:
z = 2 + 3i
$\therefore \bar{z} = 2 - 3i$
$\therefore \bar{\bar{z}} = 2 + 3i = z$

(ii) $z\bar{z} = |z|^2$
Solution:
$z\bar{z} = (2 + 3i)(2 - 3i)$
$= 4 - 9i^2$
$= 4 - 9(-1)$ .....[$\because i^2 = -1$]
= 13
$|z|^2 = (\sqrt{2^2+3^2})^2$
$= 2^2 + 3^2$
= 4 + 9

= 13

$\therefore z\bar{z} = |z|^2$

(iii) $(z + \bar{z})$ is real

Solution:

$(z + \bar{z}) = (2 + 3i) + (2 - 3i)$

$= 2 + 3i + 2 - 3i$

= 4, which is a real number.

$\therefore z + \bar{z}$ is real.

(iv) $z - \bar{z} = 6i$

Solution:

$z - \bar{z} = (2 + 3i) - (2 - 3i)$

$= 2 + 3i - 2 + 3i$

$= 6i$

Question 9.

$z_1 = 1 + i$, $z_2 = 2 - 3i$, verify the following:

(i) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

Solution:

$$z_1 = 1 + i,\ z_2 = 2 - 3i$$
$$\therefore \quad \bar{z}_1 = 1 - i,\ \bar{z}_2 = 2 + 3i$$
$$z_1 + z_2 = (1 + i) + (2 - 3i) = 3 - 2i$$
$$\therefore \quad \overline{z_1 + z_2} = 3 + 2i$$
$$\bar{z}_1 + \bar{z}_2 = (1 - i) + (2 + 3i)$$
$$= 3 + 2i$$
$$\therefore \quad \overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

(ii) $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$

Solution:

$$z_1 - z_2 = (1 + i) - (2 - 3i)$$
$$= 1 + i - 2 + 3i$$
$$= -1 + 4i$$
$$\therefore \quad \overline{z_1 - z_2} = -1 - 4i$$
$$\bar{z}_1 - \bar{z}_2 = (1 - i) - (2 + 3i)$$
$$= 1 - i - 2 - 3i$$
$$= -1 - 4i$$
$$\therefore \quad \overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$$

(iii) $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$

Solution:

$$(z_1.z_2) = (1 + i)(2 - 3i)$$
$$= 2 - 3i + 2i - 3i^2$$
$$= 2 - i - 3(-1) \qquad \ldots[\because i^2 = -1]$$
$$= 5 - i$$
$$\therefore \quad \overline{z_1.z_2} = 5 + i$$
$$\bar{z}_1 . \bar{z}_2 = (1 - i)(2 + 3i)$$
$$= 2 + 3i - 2i - 3i^2$$
$$= 2 + i - 3(-1) \qquad \ldots[\because i^2 = -1]$$
$$= 5 + i$$
$$\therefore \quad \overline{z_1 z_2} = \bar{z}_1 . \bar{z}_2$$

(iv) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

Solution:

$$\frac{z_1}{z_2} = \frac{1+i}{2-3i} = \frac{(1+i)(2+3i)}{(2-3i)(2+3i)}$$

$$= \frac{2+3i+2i+3i^2}{4-9i^2} = \frac{2+5i+3(-1)}{4-9(-1)} \quad \ldots[\because i^2 = -1]$$

$$= \frac{-1+5i}{13} = \frac{-1}{13} + \frac{5}{13}i$$

$$\therefore \quad \overline{\left(\frac{z_1}{z_2}\right)} = \frac{-1}{13} - \frac{5}{13}i$$

$$\frac{\bar{z}_1}{\bar{z}_2} = \frac{1-i}{2+3i}$$

$$= \frac{(1-i)(2-3i)}{(2+3i)(2-3i)}$$

$$= \frac{2-3i-2i+3i^2}{4-9i^2}$$

$$= \frac{2-5i+3(-1)}{4-9(-1)} \quad \ldots[\because i^2 = -1]$$

$$= \frac{-1-5i}{13} = \frac{-1}{13} - \frac{5}{13}i$$

$$\therefore \quad \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$$

Maharashtra State Board 11th Maths Solutions Chapter 1 Complex Numbers Ex 1.3

Question 1.
Find the modulus and amplitude for each of the following complex numbers:
(i) 7 – 5i
Solution:
Let z = 7 – 5i
a = 7, b = -5
i.e. a > 0, b < 0
$|z| = \sqrt{a^2+b^2} = \sqrt{7^2+(-5)^2} = \sqrt{49+25} = \sqrt{74}$
Here, (7, -5) lies in 4th quadrant.

$$\therefore \quad \text{amp}(z) = -\tan^{-1}\left|\frac{b}{a}\right|$$

$$= -\tan^{-1}\left|\frac{-5}{7}\right|$$

$$= -\tan^{-1}\left(\frac{5}{7}\right)$$

(ii) √3 + √2 i
Solution:
Let z = √3 + √2 i

$a = \sqrt{3}$, $b = \sqrt{2}$,
i.e. $a > 0$, $b > 0$

$\therefore \quad |z| = \sqrt{a^2 + b^2} = \sqrt{\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2} = \sqrt{3+2} = \sqrt{5}$

Here, $\left(\sqrt{3}, \sqrt{2}\right)$ lies in 1st quadrant.

$\therefore \quad \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{3}}\right)$

$= \tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$

(iii) -8 + 15i
Solution:
Let z = -8 + 15i
a = -8, b = 15 , i.e. a < 0, b > 0

$\therefore \quad |z| = \sqrt{a^2 + b^2} = \sqrt{(-8)^2 + 15^2} = \sqrt{64 + 225}$

$= \sqrt{289} = 17$

Here, (−8, 15) lies in 2nd quadrant.

$\therefore \quad \text{amp}(z) = \pi - \tan^{-1}\left|\frac{b}{a}\right|$

$= \pi - \tan^{-1}\left|\frac{15}{-8}\right|$

$= \pi - \tan^{-1}\left(\frac{15}{8}\right)$

(iv) -3(1 – i)
Solution:
Let z = -3(1 – i) = -3 + 3i
a = -3, b = 3 , i.e. a < 0, b > 0

$|z| = \sqrt{a^2+b^2} = \sqrt{(-3)^2+3^2} = \sqrt{9+9} = 3\sqrt{2}$

Here, (-3, 3) lies in 2nd quadrant.

$\text{amp}(z) = \pi - \tan^{-1}\left|\frac{b}{a}\right|$

$= \pi - \tan^{-1}\left|\frac{3}{-3}\right|$

$= \pi - \tan^{-1}|-1| = \pi - \tan^{-1}(1)$

$= \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

(v) -4 – 4i
Solution:
Let z = -4 – 4i
a = -4, b = -4 , i.e. a < 0, b < 0

$\therefore \quad |z| = \sqrt{a^2 + b^2} = \sqrt{(-4)^2 + (-4)^2} = \sqrt{16 + 16}$

$= \sqrt{32} = 4\sqrt{2}$

Here, (– 4, – 4) lies in 3rd quadrant.

$\therefore \quad \text{amp}(z) = \pi + \tan^{-1}\left|\frac{b}{a}\right| = \pi + \tan^{-1}\left|\frac{-4}{-4}\right|$

$= \pi + \tan^{-1}(1)$

$\therefore \quad \text{amp}(z) = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

(vi) √3 – i
Solution:
Let z = √3 – i

a = √3, b = -1, i.e. a > 0, b < 0

$$\therefore \quad |z| = \sqrt{a^2+b^2} = \sqrt{\left(\sqrt{3}\right)^2+(-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$

Here, ($\sqrt{3}$, −1) lies in 4^{th} quadrant.

$$\therefore \quad \text{amp}(z) = 2\pi - \tan^{-1}\left|\frac{b}{a}\right|$$

$$= 2\pi - \tan^{-1}\left|\frac{-1}{\sqrt{3}}\right| = 2\pi - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

(vii) 3
Solution:
Let z = 3 + 0i
a = 3, b = 0
z is a real number, it lies on the positive real axis.
|z|= $\sqrt{a^2+b^2}=\sqrt{3^2+0^2}=\sqrt{9+0}$ = 3
and amp (z) = 0

(viii) 1 + i
Solution:
Let z = 1 + i
a = 1, b = 1, i.e. a > 0, b > 0
|z| = $\sqrt{a^2+b^2}=\sqrt{1^2+1^2}=\sqrt{1+1}=\sqrt{2}$
Here, (1, 1) lies in 1st quadrant.
amp (z) = $\tan^{-1}\left(\frac{b}{a}\right)=\tan^{-1}(1)=\frac{\pi}{4}$

(ix) 1 + i√3
Solution:
Let z = 1 + i√3
a = 1, b = √3, i.e. a > 0, b > 0
|z| = $\sqrt{a^2+b^2}=\sqrt{1^2+(\sqrt{3})^2}=\sqrt{1+3}=2$
Here, (1, √3) lies in 1st quadrant.
amp (z) = $\tan^{-1}\left(\frac{b}{a}\right)=\tan^{-1}(\sqrt{3})=\frac{\pi}{3}$

(x) $(1 + 2i)^2 (1 – i)$
Solution:
Let z = $(1 + 2i)^2 (1 – i)$
= $(1 + 4i + 4i^2) (1 – i)$
= [1 + 4i + 4(-1)] (1 – i) ….[∵ i^2 = -1]
= (-3 + 4i) (1 – i)
= $-3 + 3i + 4i – 4i^2$
= -3 + 7i – 4(-1)
= -3 + 7i + 4
∴ z = 1 + 7i
∴ a = 1, b = 7, i. e. a > 0, b > 0
∴ |z| = $\sqrt{a^2+b^2}=\sqrt{1^2+7^2}=\sqrt{1+49}=\sqrt{52}$
Here, (1, 7) lies in 1st quadrant.
∴ amp(z) = $\tan^{-1}\left(\frac{b}{a}\right)=\tan^{-1}(7)$

Question 2.
Find real values of θ for which $\left(\frac{4+3i\sin\theta}{1-2i\sin\theta}\right)$ is purely real.

Solution:

$$\frac{4+3i\sin\theta}{1-2i\sin\theta} = \frac{(4+3i\sin\theta)(1+2i\sin\theta)}{(1-2i\sin\theta)(1+2i\sin\theta)}$$

$$= \frac{4+8i\sin\theta+3i\sin\theta+6i^2\sin^2\theta}{1-4i^2\sin^2\theta}$$

$$= \frac{4+11(i\sin\theta)-6\sin^2\theta}{1+4\sin^2\theta} \quad \ldots[\because i^2=-1]$$

$$= \left(\frac{4-6\sin^2\theta}{1+4\sin^2\theta}\right)+\left(\frac{11\sin\theta}{1+4\sin^2\theta}\right)i$$

Since z is purely real, Im(z) = 0

$\therefore \quad \dfrac{11\sin\theta}{1+4\sin^2\theta} = 0$

$\therefore \quad \sin\theta = 0$

$\therefore \quad \sin\theta = \sin n\pi$, where $n \in Z$

$\therefore \quad \theta = n\pi$, where $n \in Z$

Question 3.

If z = 3 + 5i, then represent the z, $\bar{z}$, -z, $\overline{-z}$ in Argand's diagram.

Solution:

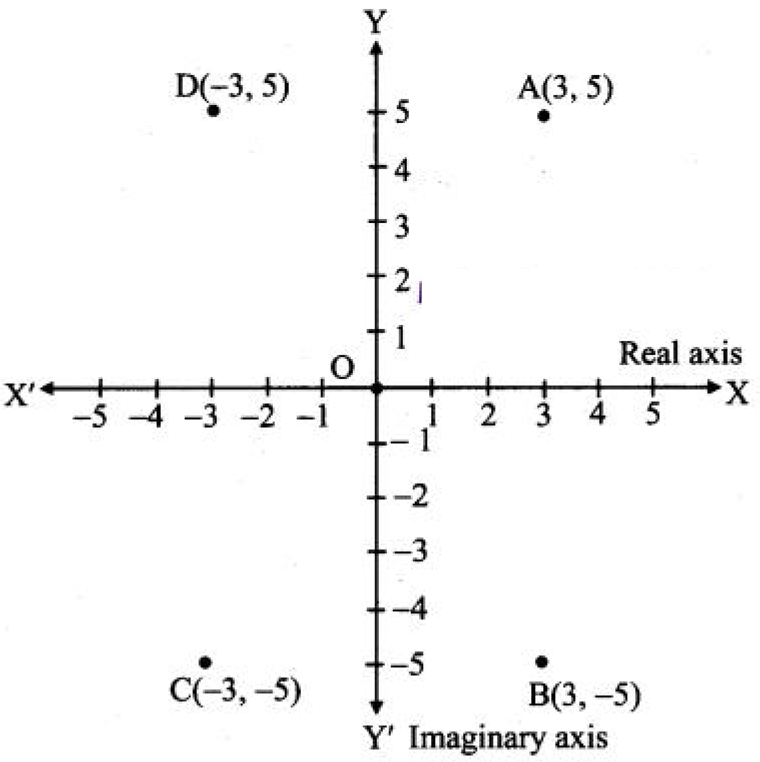
z = 3 + 5i

$\bar{z}$ = 3 – 5i

-z = – 3 – 5i

$\overline{-z}$ = -3 + 5i

The above complex numbers will be represented by the points
A (3, 5), B (3, -5), C (-3, -5) , D (-3, 5) respectively as shown below:



Question 4.

Express the following complex numbers in polar form and exponential form.

(i) -1 + √3 i

Solution:

Let z = – 1 + √3

a = -1, b = √3

$\therefore \quad |z| = r = \sqrt{a^2+b^2} = \sqrt{(-1)^2+(\sqrt{3})^2} = \sqrt{1+3} = 2$

Here, $(-1, \sqrt{3})$ lies in 2[nd] quadrant.

$$\therefore \quad \theta = \text{amp}(z) = \pi - \tan^{-1}\left|\frac{b}{a}\right|$$

$$= \pi - \tan^{-1}\left|\frac{\sqrt{3}}{-1}\right|$$

$$= \pi - \tan^{-1}\left(\sqrt{3}\right)$$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\therefore \quad \theta = 120^\circ = \frac{2\pi}{3}$$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$$= 2(\cos 120^\circ + i\sin 120^\circ)$$

$$= 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

The exponential form of $z = re^{i\theta} = 2e^{\frac{2\pi}{3}i}$

(ii) -i
Solution:
Let z = -i = 0 – i
a = 0, b = -1
z lies on negative imaginary Y-axis.
$|z| = r = \sqrt{a^2+b^2} = \sqrt{0^2+(-1)^2}$ = 1 and
θ = amp z = 270° = $\frac{3\pi}{2}$
The polar form of z = r (cos θ + i sin θ)
= 1 (cos 270° + i sin 270°)
= 1 (cos $\frac{3\pi}{2}$ + i sin $\frac{3\pi}{2}$)
The exponential form of z = $re^{i\theta} = e^{\frac{3\pi}{2}i}$

(iii) -1
Solution:
Let z = -1 = -1 + 0.i
a = -1, b = 0
z lies on negative real X-axis.
$|z| = r = \sqrt{a^2+b^2} = \sqrt{(-1)^2+0^2}$ = 1 and
θ = amp z = 180° = π
The polar form of z = r (cos θ + i sin θ)
= 1 (cos 180° + i sin 180°)
= 1 (cos π + i sin π)
The exponential form of z = $re^{i\theta} = e^{\pi i}$

(iv) $\frac{1}{1+i}$
Solution:

Let $z = \frac{1}{1+i} = \frac{1-i}{(1+i)(1-i)}$

$= \frac{1-i}{1-i^2} = \frac{1-i}{1-(-1)}$ ...[$\because i^2 = -1$]

$= \frac{1-i}{2}$

$\therefore$ $z = \frac{1}{2} - \frac{1}{2}i$

$\therefore$ $a = \frac{1}{2}, b = \frac{-1}{2}$

$\therefore$ $|z| = r = \sqrt{a^2 + b^2}$

$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{1}{\sqrt{2}}$

Here, $\left(\frac{1}{2}, \frac{-1}{2}\right)$ lies in 4^{th} quadrant.

$\therefore$ $\theta = \text{amp}(z) = 2\pi - \tan^{-1}\left|\frac{b}{a}\right|$

$= 2\pi - \tan^{-1}\left|\frac{\frac{-1}{2}}{\frac{1}{2}}\right|$

$= 2\pi - \tan^{-1}|-1|$

$= 2\pi - \tan^{-1}(1)$

$= 2\pi - \frac{\pi}{4}$

$= \frac{7\pi}{4}$

$\therefore$ $\theta = 315° = \frac{7\pi}{4}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$= \frac{1}{\sqrt{2}}(\cos 315° + i \sin 315°)$

$= \frac{1}{\sqrt{2}}\left[\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right]$

The exponential form of $z = re^{i\theta} = \frac{1}{\sqrt{2}}e^{\frac{7\pi}{4}i}$

(v) $\frac{1+2i}{1-3i}$

Solution:

Let $z = \frac{1+2i}{1-3i} = \frac{(1+2i)(1+3i)}{(1-3i)(1+3i)}$

$= \frac{1+2i+3i+6i^2}{1-9i^2}$

$= \frac{1+5i+6(-1)}{1-9(-1)}$ $\quad \ldots[\because i^2 = -1]$

$= \frac{-5+5i}{10}$

$= \frac{-1}{2} + \frac{1}{2}i$

$\therefore \quad a = \frac{-1}{2}, b = \frac{1}{2}$

$\therefore \quad |z| = r = \sqrt{a^2+b^2} = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$= \sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{1}{\sqrt{2}}$

Here, $\left(\frac{-1}{2}, \frac{1}{2}\right)$ lies in 2^{nd} quadrant.

$\therefore \quad \theta = \text{amp } z = \pi - \tan^{-1}\left|\frac{b}{a}\right|$

$= \pi - \tan^{-1}\left|\frac{\frac{1}{2}}{\frac{-1}{2}}\right|$

$= \pi - \tan^{-1}|-1|$

$= \pi - \tan^{-1}(1)$

$= \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

$\therefore \quad \theta = 135° = \frac{3\pi}{4}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$= \frac{1}{\sqrt{2}}(\cos 135° + i \sin 135°)$

$= \frac{1}{\sqrt{2}}\left(\cos\frac{3\pi}{4} + i \sin\frac{3\pi}{4}\right)$

The exponential form of $z = re^{i\theta} = \frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}i}$

(vi) $\frac{1+7i}{(2-i)^2}$

Solution:

$$\text{Let } z = \frac{1+7i}{(2-i)^2}$$

$$= \frac{1+7i}{4-4i+i^2} = \frac{1+7i}{4-4i-1}$$

$$= \frac{1+7i}{3-4i} = \frac{(1+7i)(3+4i)}{(3-4i)(3+4i)}$$

$$= \frac{3+4i+21i+28i^2}{9-16i^2}$$

$$= \frac{25i+3+28(-1)}{9-16(-1)}$$

$$= \frac{25i-25}{25}$$

$\therefore \quad z = -1 + i$

$\therefore \quad a = -1, b = 1$

$\therefore \quad |z| = r = \sqrt{a^2+b^2} = \sqrt{(-1)^2+1^2} = \sqrt{2}$

Here, $(-1, 1)$ lies in 2^{nd} quadrant

$$\therefore \quad \theta = \text{amp } z = \pi - \tan^{-1}\left|\frac{b}{a}\right|$$

$$= \pi - \tan^{-1}\left|\frac{1}{-1}\right|$$

$$= \pi - \tan^{-1}|-1| = \pi - \tan^{-1}(1)$$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

$$\therefore \quad \theta = 135° = \frac{3\pi}{4}$$

$\therefore$ The polar form of $z = r(\cos\theta + i.\sin\theta)$

$$= \sqrt{2}(\cos 135° + i \sin 135°)$$

$$= \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

The exponential form of $z = re^{i\theta} = \sqrt{2}\, e^{\frac{3\pi}{4}i}$

Question 5.
Express the following numbers in the form x + iy:
(i) $\sqrt{3}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$
Solution:

$$\sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \sqrt{3}\left(\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right)$$

$$= \frac{3}{2} + \frac{\sqrt{3}}{2}i$$

(ii) $\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)$
Solution:

$$\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

$$= \sqrt{2}\left[\cos\left(2\pi - \frac{\pi}{4}\right) + i\sin\left(2\pi - \frac{\pi}{4}\right)\right]$$

$$= \sqrt{2}\left[\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right]$$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}} + i\left(-\frac{1}{\sqrt{2}}\right)\right]$$

$$= 1 - i$$

(iii) $7(\cos(-\frac{5\pi}{6})+i\sin(-\frac{5\pi}{6}))$

Solution:

$$7\left[\cos\left(-\frac{5\pi}{6}\right)+i\sin\left(-\frac{5\pi}{6}\right)\right]$$
$$=7\left[\cos\frac{5\pi}{6}-i\sin\frac{5\pi}{6}\right]$$
$$=7\left[\cos\left(\pi-\frac{\pi}{6}\right)-i\sin\left(\pi-\frac{\pi}{6}\right)\right]$$
$$=7\left[-\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right]$$
$$=7\left[-\frac{\sqrt{3}}{2}-i\left(\frac{1}{2}\right)\right]$$
$$=-\frac{7\sqrt{3}}{2}-\frac{7}{2}i$$

(iv) $e^{\frac{\pi}{3}i}$

Solution:

$$z=re^{i\theta}=e^{\frac{\pi}{3}i}$$
$$\therefore\quad r=1,\ \theta=\frac{\pi}{3}$$
Polar form of z $=r(\cos\theta+i\sin\theta)$
$$=1\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$$
$$=\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

(v) $e^{\frac{-4\pi}{3}i}$

Solution:

$$z=re^{i\theta}=e^{\frac{-4\pi}{3}i}$$
$$\therefore\quad r=1,\ \theta=\frac{-4\pi}{3}$$
Polar form of z $=r(\cos\theta+i\sin\theta)$
$$=1\left[\cos\left(-\frac{4\pi}{3}\right)+i\sin\left(-\frac{4\pi}{3}\right)\right]$$
$$=\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}$$
$$=\cos\left(\pi+\frac{\pi}{3}\right)-i\sin\left(\pi+\frac{\pi}{3}\right)$$
$$=-\cos\frac{\pi}{3}-i\left(-\sin\frac{\pi}{3}\right)$$
$$=\frac{-1}{2}-i\left(-\frac{\sqrt{3}}{2}\right)$$
$$=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

(vi) [latex]e5π6i[/latex]
Solution:

$$z = re^{i\theta} = e^{\frac{5\pi}{6}i}$$

$$\therefore \quad r = 1, \theta = \frac{5\pi}{6}$$

Polar form of $z = r(\cos\theta + i\sin\theta)$

$$= 1\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$$

$$= \cos\left(\pi - \frac{\pi}{6}\right) + i\sin\left(\pi - \frac{\pi}{6}\right)$$

$$= -\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$$

$$= -\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}$$

$$= -\frac{\sqrt{3}}{2} + \frac{1}{2}i$$

Question 6.
Find the modulus and argument of the complex number 1+2i1−3i.
Solution:

Let $z = \frac{1+2i}{1-3i}$

$$= \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i}$$

$$= \frac{1+3i+2i+6i^2}{1-9i^2}$$

$$= \frac{1+5i-6}{1+9} \quad \ldots[\because i^2 = -1]$$

$$\therefore z = \frac{-5+5i}{10}$$

$$= -\frac{1}{2} + \frac{1}{2}i$$

This is of the form a + bi, where $a = -\frac{1}{2}$, $b = \frac{1}{2}$

$\therefore$ modulus = r

$$= \sqrt{a^2 + b^2}$$

$$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{1}{4}}$$

$$= \frac{1}{\sqrt{2}}$$

If θ is the argument, then

$\cos \theta = \frac{a}{r}$

$= \frac{\left(-\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$

$= -\frac{1}{\sqrt{2}}$

and $\sin \theta = \frac{b}{r}$

$= \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$

$= \frac{1}{\sqrt{2}}$

$$\therefore \theta = \frac{3\pi}{4} \quad \ldots \left[\begin{array}{c} \because \cos \frac{3\pi}{4} = \cos\left(\pi - \frac{\pi}{4}\right) = -\cos \frac{\pi}{4} = \\ -\frac{1}{\sqrt{2}} \text{ and } \sin \frac{3\pi}{4} = \sin\left(\pi - \frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} \end{array}\right]$$

Hence, modulus = $\frac{1}{\sqrt{2}}$ and argurement = $\frac{3\pi}{4}$

Question 7.

Convert the complex number $z=i-1\cos_{\pi 3}+i\sin_{\pi 3}$ in the polar form.

Solution:

$$z = \frac{i-1}{\cos\frac{\pi}{3} + i\ \sin\frac{\pi}{3}}$$

$$= \frac{i-1}{\frac{1}{2} + \frac{\sqrt{3}}{2}i} = \frac{2(i-1)}{1+\sqrt{3}\,i}$$

$$= \frac{2(i-1)(1-\sqrt{3}\,i)}{(1+\sqrt{3}\,i)(1-\sqrt{3}\,i)} = \frac{2(i-\sqrt{3}\,i^2-1+\sqrt{3}\,i)}{1-3i^2}$$

$$= \frac{2[i-\sqrt{3}(-1)-1+\sqrt{3}\,i]}{1-3(-1)} \quad \ldots[\because i^2 = -1]$$

$$= \frac{2}{4}(i+\sqrt{3}-1+\sqrt{3}\,i)$$

$$\therefore \quad z = \frac{\sqrt{3}-1}{2} + \frac{(\sqrt{3}+1)}{2}i$$

$$\therefore \quad a = \frac{\sqrt{3}-1}{2},\ b = \frac{\sqrt{3}+1}{2}$$

$$\therefore \quad |z| = r = \sqrt{a^2+b^2} = \sqrt{\left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{\sqrt{3}+1}{2}\right)^2}$$

$$= \sqrt{\frac{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2}{4}}$$

$$= \sqrt{\frac{3-2\sqrt{3}+1+3+2\sqrt{3}+1}{4}}$$

$$= \sqrt{\frac{8}{4}} = \sqrt{2}$$

Here, $\left(\frac{\sqrt{3}-1}{2}, \frac{\sqrt{3}+1}{2}\right)$ lies in 1st quadrant.

$$\therefore \quad \text{amp}\,(z) = \theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{\frac{\sqrt{3}+1}{2}}{\frac{\sqrt{3}-1}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$$

$\therefore$ The polar form of $z = r(\cos\theta + i.\sin\theta)$

$= \sqrt{2}(\cos\theta + i.\sin\theta)$, where $\theta = \tan^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

Question 8.
For z = 2 + 3i, verify the following:
(i) $\overline{(\bar{z})} = z$
Solution:
z = 2 + 3i
$\therefore \bar{z} = 2 - 3i$
$\therefore \bar{\bar{z}} = 2 + 3i = z$

(ii) $z\bar{z} = |z|^2$
Solution:
$z\bar{z} = (2 + 3i)(2 - 3i)$
$= 4 - 9i^2$
$= 4 - 9(-1)$ .....[$\because i^2 = -1$]
= 13
$|z|^2 = (\sqrt{2^2+3^2})^2$
$= 2^2 + 3^2$
= 4 + 9

= 13

$\therefore z\bar{z} = |z|^2$

(iii) $(z + \bar{z})$ is real

Solution:

$(z + \bar{z}) = (2 + 3i) + (2 - 3i)$

$= 2 + 3i + 2 - 3i$

= 4, which is a real number.

$\therefore z + \bar{z}$ is real.

(iv) $z - \bar{z} = 6i$

Solution:

$z - \bar{z} = (2 + 3i) - (2 - 3i)$

$= 2 + 3i - 2 + 3i$

$= 6i$

Question 9.

$z_1 = 1 + i$, $z_2 = 2 - 3i$, verify the following:

(i) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

Solution:

$$z_1 = 1 + i,\ z_2 = 2 - 3i$$

$$\therefore \quad \bar{z}_1 = 1 - i,\ \bar{z}_2 = 2 + 3i$$

$$z_1 + z_2 = (1 + i) + (2 - 3i) = 3 - 2i$$

$$\therefore \quad \overline{z_1 + z_2} = 3 + 2i$$

$$\bar{z}_1 + \bar{z}_2 = (1 - i) + (2 + 3i) = 3 + 2i$$

$$\therefore \quad \overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$$

(ii) $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$

Solution:

$$z_1 - z_2 = (1 + i) - (2 - 3i) = 1 + i - 2 + 3i = -1 + 4i$$

$$\therefore \quad \overline{z_1 - z_2} = -1 - 4i$$

$$\bar{z}_1 - \bar{z}_2 = (1 - i) - (2 + 3i) = 1 - i - 2 - 3i = -1 - 4i$$

$$\therefore \quad \overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$$

(iii) $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$

Solution:

$$(z_1.z_2) = (1 + i)(2 - 3i)$$
$$= 2 - 3i + 2i - 3i^2$$
$$= 2 - i - 3(-1) \qquad \ldots[\because i^2 = -1]$$
$$= 5 - i$$

$$\therefore \quad \overline{z_1.z_2} = 5 + i$$

$$\bar{z}_1.\bar{z}_2 = (1 - i)(2 + 3i)$$
$$= 2 + 3i - 2i - 3i^2$$
$$= 2 + i - 3(-1) \qquad \ldots[\because i^2 = -1]$$
$$= 5 + i$$

$$\therefore \quad \overline{z_1 z_2} = \bar{z}_1.\bar{z}_2$$

(iv) $\overline{(z_1 z_2)} = \bar{z}_1 \bar{z}_2$

Solution:

$$\frac{z_1}{z_2} = \frac{1+i}{2-3i} = \frac{(1+i)(2+3i)}{(2-3i)(2+3i)}$$

$$= \frac{2+3i+2i+3i^2}{4-9i^2} = \frac{2+5i+3(-1)}{4-9(-1)} \quad \ldots[\because i^2 = -1]$$

$$= \frac{-1+5i}{13} = \frac{-1}{13} + \frac{5}{13}i$$

$$\therefore \overline{\left(\frac{z_1}{z_2}\right)} = \frac{-1}{13} - \frac{5}{13}i$$

$$\frac{\bar{z}_1}{\bar{z}_2} = \frac{1-i}{2+3i}$$

$$= \frac{(1-i)(2-3i)}{(2+3i)(2-3i)}$$

$$= \frac{2-3i-2i+3i^2}{4-9i^2}$$

$$= \frac{2-5i+3(-1)}{4-9(-1)} \quad \ldots[\because i^2 = -1]$$

$$= \frac{-1-5i}{13} = \frac{-1}{13} - \frac{5}{13}i$$

$$\therefore \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$$

Maharashtra State Board 11th Maths Solutions Chapter 1 Complex Numbers Ex 1.4

Question 1.
Find the value of
(i) ω^{18}
(ii) ω^{21}
(iii) ω^{-30}
(iv) ω^{-105}
Solution:

$\omega^3 = 1$

i. $\omega^{18} = (\omega^3)^6 = (1)^6 = 1$

ii. $\omega^{21} = (\omega^3)^7 = (1)^7 = 1$

iii. $\omega^{-30} = (\omega^3)^{-10} = (1)^{-10} = \frac{1}{(1)^{10}} = 1$

iv. $\omega^{-105} = (\omega^3)^{-35} = (1)^{-35} = \frac{1}{(1)^{35}} = 1$

Question 2.
If ω is the complex cube root of unity, show that

(i) $(2 - \omega)(2 - \omega^2) = 7$
Solution:
ω is the complex cube root of unity.
$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$
L.H.S. $= (2 - \omega)(2 - \omega^2)$
$= 4 - 2\omega^2 - 2\omega + \omega^3$
$= 4 - 2(\omega^2 + \omega) + 1$
$= 4 - 2(-1) + 1$
$= 4 + 2 + 1$
$= 7$
= R.H.S.

(ii) $(1 + \omega - \omega^2)^6 = 64$
Solution:
ω is the complex cube root of unity.
$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$

$$\begin{aligned}\text{L.H.S.} &= (1 + \omega - \omega^2)^6 \\ &= [(1 + \omega) - \omega^2]^6 \\ &= (-\omega^2 - \omega^2)^6 \\ &= (-2\omega^2)^6 \\ &= 64.\omega^{12} \\ &= 64(\omega^3)^4 \\ &= 64(1)^4 \\ &= 64 = \text{R.H.S.}\end{aligned}$$

(iii) $(1 + \omega)^3 - (1 + \omega^2)^3 = 0$
Solution:
ω is the complex cube root of unity.
$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$

$$\begin{aligned}\text{L.H.S.} &= (1 + \omega)^3 - (1 + \omega^2)^3 \\ &= (-\omega^2)^3 - (-\omega)^3 \\ &= -\omega^6 + \omega^3 \\ &= -(\omega^3)^2 + \omega^3 \\ &= -(1)^2 + 1 \\ &= -1 + 1 = 0 = \text{R.H.S.}\end{aligned}$$

(iv) $(2 + \omega + \omega^2)^3 - (1 - 3\omega + \omega^2)^3 = 65$
Solution:
ω is the complex cube root of unity.
$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$

$$\begin{aligned}\text{L.H.S.} &= (2 + \omega + \omega^2)^3 - (1 - 3\omega + \omega^2)^3 \\ &= [2 + (\omega + \omega^2)]^3 - [-3\omega + (1 + \omega^2)]^3 \\ &= (2 - 1)^3 - (-3\omega - \omega)^3 \\ &= 1^3 - (-4\omega)^3 \\ &= 1 + 64\omega^3 \\ &= 1 + 64(1) = 65 \\ &= \text{R.H.S.}\end{aligned}$$

(v) $(3 + 3\omega + 5\omega^2)^6 - (2 + 6\omega + 2\omega^2)^3 = 0$
Solution:
ω is the complex cube root of unity.
$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$

$$\begin{aligned}
\text{L.H.S.} &= (3 + 3\omega + 5\omega^2)^6 - (2 + 6\omega + 2\omega^2)^3 \\
&= [3(1 + \omega) + 5\omega^2]^6 - [2(1 + \omega^2) + 6\omega]^3 \\
&= [3(-\omega^2) + 5\omega^2]^6 - [2(-\omega) + 6\omega]^3 \\
&= (2\omega^2)^6 - (4\omega)^3 \\
&= 64\omega^{12} - 64\omega^3 \\
&= 64(\omega^3)^4 - 64\omega^3 \\
&= 64(1) - 64(1) \\
&= 0 = \text{R.H.S.}
\end{aligned}$$

(vi) $\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = \omega^2$

Solution:

ω is the complex cube root of unity.

$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$

$$\text{L.H.S.} = \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = \frac{a\omega^3+b\omega^4+c\omega^2}{c+a\omega+b\omega^2}$$

$$\ldots[\because \omega^3 = 1, \therefore \omega^4 = \omega]$$

$$= \frac{\omega^2(c+a\omega+b\omega^2)}{c+a\omega+b\omega^2} = \omega^2 = \text{R.H.S.}$$

(vii) $(a + b) + (a\omega + b\omega^2) + (a\omega^2 + b\omega) = 0$

Solution:

ω is the complex cube root of unity.

$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$

$$\begin{aligned}
\text{L.H.S.} &= (a + b) + (a\omega + b\omega^2) + (a\omega^2 + b\omega) \\
&= (a + a\omega + a\omega^2) + (b + b\omega + b\omega^2) \\
&= a(1 + \omega + \omega^2) + b(1 + \omega + \omega^2) \\
&= a(0) + b(0) \\
&= 0 = \text{R.H.S.}
\end{aligned}$$

(viii) $(a - b)(a - b\omega)(a - b\omega^2) = a^3 - b^3$

Solution:

ω is the complex cube root of unity.

$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$

$$\begin{aligned}
\text{L.H.S.} &= (a - b)(a - b\omega)(a - b\omega^2) \\
&= (a - b)(a^2 - ab\omega^2 - ab\omega + b^2\omega^3) \\
&= (a - b)[a^2 - ab(\omega^2 + \omega) + b^2(1)] \\
&= (a - b)[a^2 - ab(-1) + b^2] \\
&= (a - b)(a^2 + ab + b^2) \\
&= a^3 - b^3 = \text{R.H.S.}
\end{aligned}$$

(ix) $(a + b)^2 + (a\omega + b\omega^2)^2 + (a\omega^2 + b\omega)^2 = 6ab$

Solution:

ω is the complex cube root of unity.

$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$

$$\begin{aligned}
\text{L.H.S.} &= (a+b)^2 + (a\omega + b\omega^2)^2 + (a\omega^2 + b\omega)^2 \\
&= (a+b)^2 + \omega^2(a + b\omega)^2 + \omega^2(a\omega + b)^2 \\
&= (a+b)^2 + \omega^2[(a + b\omega)^2 + (a\omega + b)^2] \\
&= a^2 + b^2 + 2ab + \omega^2(a^2 + 2ab\omega + b^2\omega^2 \\
&\qquad + a^2\omega^2 + 2ab\omega + b^2) \\
&= a^2 + b^2 + 2ab + a^2\omega^2 + 2ab\omega^3 + b^2\omega^4 \\
&\qquad + a^2\omega^4 + 2ab\omega^3 + b^2\omega^2 \\
&= a^2 + b^2 + 2ab + a^2\omega^2 + 2ab(1) + b^2(\omega) \\
&\qquad + a^2(\omega) + 2ab(1) + b^2\omega^2 \\
&= a^2(1 + \omega + \omega^2) + b^2(1 + \omega + \omega^2) + 2ab \\
&\qquad + 2ab + 2ab \\
&= a^2(0) + b^2(0) + 6ab \\
&= 6ab = \text{R.H.S.}
\end{aligned}$$

Question 3.
If ω is the complex cube root of unity, find the value of
(i) $\omega + \frac{1}{\omega}$
Solution:
ω is the complex cube root of unity.
$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$

$\omega + \frac{1}{\omega} = \frac{\omega^2 + 1}{\omega} = \frac{-\omega}{\omega} = -1$

(ii) $\omega^2 + \omega^3 + \omega^4$
Solution:
ω is the complex cube root of unity.
$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$
$\omega^2 + \omega^3 + \omega^4$
$= \omega^2(1 + \omega + \omega^2)$
$= \omega^2(0)$
$= 0$

(iii) $(1 + \omega^2)^3$
Solution:
ω is the complex cube root of unity.
$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$
$(1 + \omega^2)^3$
$= (-\omega)^3$
$= -\omega^3$
$= -1$

(iv) $(1 - \omega - \omega^2)^3 + (1 - \omega + \omega^2)^3$
Solution:
ω is the complex cube root of unity.
$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$
$(1 - \omega - \omega^2)^3 + (1 - \omega + \omega^2)^3$
$= [1 - (\omega + \omega^2)]^3 + [(1 + \omega^2) - \omega]^3$
$= [1 - (-1)]^2 + (-\omega - \omega)^3$
$= 2^3 + (-2\omega)^3$
$= 8 - 8\omega^3$
$= 8 - 8(1)$
$= 0$

(v) $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$
Solution:
ω is the complex cube root of unity.
$\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$ and $\omega + \omega^2 = -1$
$(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$
$= (1 + \omega)(1 + \omega^2)(1 + \omega)(1 + \omega^2)$ .....[$\because \omega^3 = 1, \omega^4 = \omega$]
$= (-\omega^2)(-\omega)(-\omega^2)(-\omega)$

$= \omega^6$
$= (\omega^3)^2$
$= (1)^2$
$= 1$

Question 4.
If α and β are the complex cube roots of unity, show that
(i) $\alpha^2 + \beta^2 + \alpha\beta = 0$
(ii) $\alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1} = 0$
Solution:
α and β are the complex cube roots of unity.

$\therefore \quad \alpha = \frac{-1+i\sqrt{3}}{2}$ and $\beta = \frac{-1-i\sqrt{3}}{2}$

$\therefore \quad \alpha\beta = \left(\frac{-1+i\sqrt{3}}{2}\right)\left(\frac{-1-i\sqrt{3}}{2}\right)$

$= \frac{(-1)^2-\left(i\sqrt{3}\right)^2}{4} = \frac{1-(-1)(3)}{4}$

$= \frac{1+3}{4}$

$\therefore \quad \alpha\beta = 1$

Also, $\alpha + \beta = \frac{-1+i\sqrt{3}}{2} + \frac{-1-i\sqrt{3}}{2}$

$= \frac{-1+i\sqrt{3}-1-i\sqrt{3}}{2} = \frac{-2}{2}$

$\therefore \quad \alpha + \beta = -1$

i. $\alpha^2 + \beta^2 + \alpha\beta$
$= \alpha^2 + 2\alpha\beta + \beta^2 + \alpha\beta - 2\alpha\beta$
…[Adding and subtracting $2\alpha\beta$]
$= (\alpha^2 + 2\alpha\beta + \beta^2) - \alpha\beta$
$= (\alpha + \beta)^2 - \alpha\beta$
$= (-1)^2 - 1 = 1 - 1$
$= 0$

ii. $\alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1}$
$= \alpha^4 + \beta^4 + 2\alpha^2\beta^2 - 2\alpha^2\beta^2 + \frac{1}{\alpha\beta}$
…[Adding and subtracting $2\alpha^2\beta^2$]
$= (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 + \frac{1}{\alpha\beta}$
$= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2 + \frac{1}{\alpha\beta}$
$= [(-1)^2 - 2(1)]^2 - 2(1)^2 + \frac{1}{1}$
$= (1 - 2)^2 - 2 + 1$
$= (-1)^2 - 1$
$= 1 - 1$
$= 0$

Question 5.
If x = a + b, y = αa + βb and z = aβ + bα, where α and β are complex cube roots of unity, show that $xyz = a^3 + b^3$.
Solution:
x = a + b, y = αa + βb, z = aβ + bα
α and β are the complex cube roots of unity.

$\therefore \alpha = \frac{-1+i\sqrt{3}}{2}$ and $\beta = \frac{-1-i\sqrt{3}}{2}$

$$\therefore \quad \alpha\beta = \left(\frac{-1+i\sqrt{3}}{2}\right)\left(\frac{-1-i\sqrt{3}}{2}\right)$$

$$= \frac{(-1)^2 - (i\sqrt{3})^2}{4}$$

$$= \frac{1-(-1)(3)}{4} \qquad \ldots[\because i^2 = -1]$$

$$= \frac{1+3}{4}$$

$$\therefore \quad \alpha\beta = 1$$

$$\text{Also, } \alpha + \beta = \frac{-1+i\sqrt{3}}{2} + \frac{-1-i\sqrt{3}}{2}$$

$$= \frac{-1+i\sqrt{3}-1-i\sqrt{3}}{2} = \frac{-2}{2}$$

$$\therefore \quad \alpha + \beta = -1$$

$$\therefore \quad xyz = (a+b)(\alpha a + \beta b)(a\beta + b\alpha)$$

$$= (a+b)(\alpha\beta a^2 + \alpha^2 ab + \beta^2 ab + \alpha\beta b^2)$$

$$= (a+b)[1.(a^2) + (\alpha^2 + \beta^2)ab + 1.(b^2)]$$

$$= (a+b)\{a^2 + [(\alpha+\beta)^2 - 2\alpha\beta]ab + b^2\}$$

$$= (a+b)\ \{a^2 + [(-1)^2 - 2(1)]ab + b^2\}$$

$$= (a+b)\ [a^2 + (1-2)ab + b^2]$$

$$= (a+b)(a^2 - ab + b^2) = a^3 + b^3$$

Question 6.
Find the equation in cartesian coordinates of the locus of z if
(i) |z| = 10
Solution:
Let z = x + iy
|z| = 10
|x + iy| = 10
$\sqrt{x^2+y^2} = 10$
$\therefore x^2 + y^2 = 100$

(ii) |z – 3| = 2
Solution:
Let z = x + iy
|z – 3| = 2
|x + iy – 3| = 2
|(x – 3) + iy| = 2
$\sqrt{(x-3)^2+y^2} = 2$
$\therefore (x-3)^2 + y^2 = 4$

(iii) |z – 5 + 6i| = 5
Solution:
Let z = x + iy
|z – 5 + 6i| = 5
|x + iy – 5 + 6i| = 5
|(x – 5) + i(y + 6)| = 5
$\sqrt{(x-5)^2+(y+6)^2} = 5$
$\therefore (x-5)^2 + (y+6)^2 = 25$

(iv) |z + 8| = |z – 4|
Solution:
Let z = x + iy
|z + 8| = |z – 4|
|x + iy + 8| = |x + iy – 4|
|(x + 8) + iy | = |(x – 4) + iy|
$\sqrt{(x+8)^2+y^2} = \sqrt{(x-4)^2+y^2}$
$(x+8)^2 + y^2 = (x-4)^2 + y^2$
$x^2 + 16x + 64 + y^2 = x^2 - 8x + 16 + y^2$
16x + 64 = -8x + 16

$24x + 48 = 0$
$\therefore x + 2 = 0$

(v) $|z - 2 - 2i| = |z + 2 + 2i|$
Solution:
Let $z = x + iy$
$|z - 2 - 2i| = |z + 2 + 2i|$
$|x + iy - 2 - 2i| = |x + iy + 2 + 2i|$
$|(x - 2) + i(y - 2)| = |(x + 2) + i(y + 2)|$
$\sqrt{(x-2)^2+(y-2)^2}=\sqrt{(x+2)^2+(y+2)^2}$
$(x - 2)^2 + (y - 2)^2 = (x + 2)^2 + (y + 2)^2$
$x^2 - 4x + 4 + y^2 - 4y + 4 = x^2 + 4x + 4 + y^2 + 4y + 4$
$-4x - 4y = 4x + 4y$
$8x + 8y = 0$
$x + y = 0$
$y = -x$

(vi) $\left|\frac{z+3i}{z-6i}\right|=1$
Solution:
Let $z = x + iy$

$$\left|\frac{z+3i}{z-6i}\right|=1$$

$$\frac{|x+iy+3i|}{|x+iy-6i|}=1$$

$$\frac{|x+(y+3)i|}{|x+(y-6)i|}=1$$

$$\frac{\sqrt{x^2+(y+3)^2}}{\sqrt{x^2+(y-6)^2}}=1$$

$$\frac{x^2+(y+3)^2}{x^2+(y-6)^2}=1$$

$x^2 + (y + 3)^2 = x^2 + (y - 6)^2$
$y^2 + 6y + 9 = y^2 - 12y + 36$
$18y - 27 = 0$
$2y - 3 = 0$

Question 7.
Use De Moivre's theorem and simplify the following:
(i) $\frac{(\cos2\theta+i\sin2\theta)^7}{(\cos4\theta+i\sin4\theta)^3}$
Solution:

$$\frac{(\cos 2\theta + i\sin 2\theta)^7}{(\cos 4\theta + i\sin 4\theta)^3}$$

$$= \frac{(\cos\theta + i\sin\theta)^{2\times 7}}{(\cos\theta + i\sin\theta)^{4\times 3}}$$

$\ldots[\because \cos n\theta + i\sin n\theta = (\cos\theta + i\sin\theta)^n]$

$= (\cos\theta + i\sin\theta)^{14}(\cos\theta + i\sin\theta)^{-12}$

$= (\cos\theta + i\sin\theta)^{14-12}$

$= (\cos\theta + i\sin\theta)^2$

$= \cos 2\theta + i\sin 2\theta$

$\ldots[\because (\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)]$

Alternate Method:

$$\frac{(\cos 2\theta + i\sin 2\theta)^7}{(\cos 4\theta + i\sin 4\theta)^3}$$

$$= \frac{\cos 7(2\theta) + i\sin 7(2\theta)}{\cos 3(4\theta) + i\sin 3(4\theta)}$$

$\ldots[\because (\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)]$

$$= \frac{\cos 14\theta + i\sin 14\theta}{\cos 12\theta + i\sin 12\theta}$$

$$= \frac{(\cos 14\theta + i\sin 14\theta)(\cos 12\theta - i\sin 12\theta)}{(\cos 12\theta + i\sin 12\theta)(\cos 12\theta - i\sin 12\theta)}$$

$$= \frac{\cos 14\theta\cos 12\theta - i\cos 14\theta\sin 12\theta + i\sin 14\theta\cos 12\theta - i^2\sin 14\theta\sin 12\theta}{\cos^2 12\theta - i^2\sin^2 12\theta}$$

$$= \frac{(\cos 14\theta\cos 12\theta + \sin 14\theta\sin 12\theta) + i(\sin 14\theta\cos 12\theta - \cos 14\theta\sin 12\theta)}{\cos^2 12\theta + \sin^2 12\theta}$$

$$= \frac{\cos(14\theta - 12\theta) + i\sin(14\theta - 12\theta)}{(1)^2}$$

$= \cos 2\theta + i\sin 2\theta$

(ii) cos5θ+isin5θ(cos3θ−isin3θ)2

Solution:

$$\frac{\cos 5\theta + i\sin 5\theta}{(\cos 3\theta - i\sin 3\theta)^2}$$

$$= \frac{\cos 5\theta + i\sin 5\theta}{[\cos(-3\theta) + i\sin(-3\theta)]^2}$$

$\ldots[\because \sin(-\theta) = -\sin\theta, \cos(-\theta) = \cos\theta]$

$$= \frac{(\cos\theta + i\sin\theta)^5}{(\cos\theta + i\sin\theta)^{-3\times 2}}$$

$\ldots[\because \cos n\theta + i\sin n\theta = (\cos\theta + i\sin\theta)^n]$

$$= \frac{(\cos\theta + i\sin\theta)^5}{(\cos\theta + i\sin\theta)^{-6}}$$

$= (\cos\theta + i\sin\theta)^5(\cos\theta + i\sin\theta)^6$

$= (\cos\theta + i\sin\theta)^{5+6}$

$= (\cos\theta + i\sin\theta)^{11}$

$= \cos 11\theta + i\sin 11\theta$

$\ldots[\because (\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)]$

(iii) $\frac{(\cos\frac{7\pi}{13}+i\sin\frac{7\pi}{13})^4}{(\cos\frac{4\pi}{13}-i\sin\frac{4\pi}{13})^6}$

Solution:

$$\frac{\left(\cos\frac{7\pi}{13}+i\sin\frac{7\pi}{13}\right)^4}{\left(\cos\frac{4\pi}{13}-i\sin\frac{4\pi}{13}\right)^6}$$

$$=\frac{\left(\cos\frac{7\pi}{13}+i\sin\frac{7\pi}{13}\right)^4}{\left[\cos\left(\frac{-4\pi}{13}\right)+i\sin\left(\frac{-4\pi}{13}\right)\right]^6}$$

$\ldots[\because \sin(-\theta)=-\sin\theta, \cos(-\theta)=\cos\theta]$

$$=\frac{(\cos\pi+i\sin\pi)^{\frac{7}{13}\times 4}}{(\cos\pi+i\sin\pi)^{\frac{-4}{13}\times 6}}$$

$\ldots[\because \cos n\theta+i\sin n\theta=(\cos\theta+i\sin\theta)^n]$

$$=(\cos\pi+i\sin\pi)^{\frac{7}{13}\times 4}(\cos\pi+i\sin\pi)^{\frac{4}{13}\times 6}$$

$$=(\cos\pi+i\sin\pi)^{\frac{28}{13}+\frac{24}{13}}$$

$$=(\cos\pi+i\sin\pi)^4$$

$$=\cos 4\pi+i\sin 4\pi$$

$\ldots[\because (\cos\theta+i\sin\theta)^n=(\cos n\theta+i\sin n\theta)]$

$= 1+i(0)$

$= 1$

Question 8.

Express the following in the form a + ib, a, b ∈ R, using De Moivre's theorem.

(i) $(1-i)^5$

Solution:

Let $z=1-i$

$\therefore$ $a=1, b=-1$, i.e. $a>0, b<0$

$\therefore$ $|z|=\sqrt{a^2+b^2}=\sqrt{1^2+(-1)^2}=\sqrt{2}$

Here, $(1,-1)$ lies in 4^{th} quadrant.

$\therefore$ $\text{amp}(z)=2\pi-\tan^{-1}\left|\frac{b}{a}\right|$

$$=2\pi-\tan^{-1}\left|\frac{-1}{1}\right|$$

$$=2\pi-\tan^{-1}|-1|$$

$$=2\pi-\tan^{-1}(1)$$

$$=2\pi-\frac{\pi}{4}=\frac{7\pi}{4}$$

$$z^5=(1-i)^5=\left[\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)\right]^5$$

$$=4\sqrt{2}\left[\cos\left(2\pi-\frac{\pi}{4}\right)+i\sin\left(2\pi-\frac{\pi}{4}\right)\right]^5$$

$$=4\sqrt{2}\left[\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right]^5$$

$$=4\sqrt{2}\left[\cos\frac{5\pi}{4}-i\sin\frac{5\pi}{4}\right]$$

$$=4\sqrt{2}\left[\cos\left(\pi+\frac{\pi}{4}\right)-i\sin\left(\pi+\frac{\pi}{4}\right)\right]$$

$$=4\sqrt{2}\left(\frac{-1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)$$

$= -4+4i$

(ii) $(1 + i)^6$

Solution:

Let $z = 1 + i$

$\therefore$ $a = 1, b = 1$, i.e. $a > 0, b > 0$

$\therefore$ $|z| = \sqrt{a^2 + b^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

Here, (1, 1) lies in 1st quadrant.

$$\therefore \quad \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right)$$

$$= \tan^{-1}\left(\frac{1}{1}\right)$$

$$= \frac{\pi}{4}$$

$$z^6 = (1 + i)^6$$

$$= \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^6$$

$$= 8\left[\cos\frac{6\pi}{4} + i\sin\frac{6\pi}{4}\right]$$

$\ldots[\because (\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)]$

$$= 8\left[\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right]$$

$$= 8[0 + i(-1)]$$

$$= -8i$$

(iii) $(1 - \sqrt{3}\, i)^4$

Solution:

Let $z = 1 - \sqrt{3}\,i$

$\therefore$ $a = 1, b = -\sqrt{3}$, i.e. $a > 0, b < 0$

$\therefore$ $|z| = \sqrt{a^2 + b^2} = \sqrt{1^2 + \left(-\sqrt{3}\right)^2} = 2$

Here, $(1, -\sqrt{3})$ lies in 4th quadrant.

$$\therefore \quad \text{amp}(z) = 2\pi - \tan^{-1}\left|\frac{b}{a}\right|$$

$$= 2\pi - \tan^{-1}\left|\frac{-\sqrt{3}}{1}\right|$$

$$= 2\pi - \tan^{-1}\left(\sqrt{3}\right)$$

$$= 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$z^4 = \left(1 - \sqrt{3}\,i\right)^4$$

$$= \left[2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)\right]^4$$

$$= 16\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)^4$$

$$= 16\left[\cos\left(2\pi - \frac{\pi}{3}\right) + i\sin\left(2\pi - \frac{\pi}{3}\right)\right]^4$$

$$= 16\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^4$$

$$= 16\left(\cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}\right)$$

$\ldots[\because (\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)]$

$$= 16\left[\cos\left(\pi + \frac{\pi}{3}\right) - i\sin\left(\pi + \frac{\pi}{3}\right)\right]$$

$$= 16\left(-\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$= 16\left[-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right]$$

$$= -8 + 8\sqrt{3}\,i$$

(iv) $(-2\sqrt{3} - 2i)^5$

Solution:

Let $z = -2\sqrt{3} - 2i$

$\therefore \quad a = -2\sqrt{3},\ b = -2$, i.e. $a < 0, b < 0$

$\therefore \quad |z| = \sqrt{a^2 + b^2} = \sqrt{\left(-2\sqrt{3}\right)^2 + (-2)^2} = \sqrt{12+4}$

$= 4$

Here, $(-2\sqrt{3}, -2)$ lies in 3^{rd} quadrant.

$\therefore \quad \text{amp}(z) = \pi + \tan^{-1}\left|\frac{b}{a}\right| = \pi + \tan^{-1}\left|\frac{-2}{-2\sqrt{3}}\right|$

$= \pi + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$z^5 = \left(-2\sqrt{3} - 2i\right)^5$

$= \left[4\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)\right]^5$

$= 1024\left(\cos\frac{35\pi}{6} + i\sin\frac{35\pi}{6}\right)$

$\ldots[\because (\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)]$

$= 1024\left[\cos\left(6\pi - \frac{\pi}{6}\right) + i\sin\left(6\pi - \frac{\pi}{6}\right)\right]$

$= 1024\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

$= 1024\left[\frac{\sqrt{3}}{2} - \frac{1}{2}i\right] = 512\sqrt{3} - 512i$

Maharashtra State Board 11th Maths Solutions Chapter 1 Complex Numbers Miscellaneous Exercise 1

(I) Select the correct answer from the given alternatives.

Question 1.
If n is an odd positive integer, then the value of $1 + (i)^{2n} + (i)^{4n} + (i)^{6n}$ is:
(A) -4i
(B) 0
(C) 4i
(D) 4
Answer:
(B) 0
Hint:
$1 + (i^2)^n + (i^4)^n + (i^2)^{3n}$
= 1 – 1 + 1 – 1 .....(n odd positive integer)
= 0

Question 2.
The value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$ is equal to:
(A) -2
(B) 1
(C) 0
(D) -1
Answer:
(D) -1
Hint:

$$\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$$
$$= \frac{i^{584}\left[i^8+i^6+i^4+i^2+1\right]}{i^{574}\left[i^8+i^6+i^4+i^2+1\right]} = i^{10} = (i^2)^5$$
$$= (-1)^5 = -1$$

Question 3.
√-3 √-6 is equal to
(A) -3√2
(B) 3√2
(C) 3√2 i
(D) -3√2 i
Answer:
(A) -3√2
Hint:
√-3 √-6
= (√3 i) (√6 i)
= 3√2 (-1)
= -3√2

Question 4.
If ω is a complex cube root of unity, then the value of $\omega^{99} + \omega^{100} + \omega^{101}$ is:
(A) -1
(B) 1
(C) 0
(D) 3
Answer:
(C) 0
Hint:
$\omega^{99} + \omega^{100} + \omega^{101}$
$= \omega^{99} (1 + \omega + \omega^2)$
$= \omega^{99} (0)$
= 0

Question 5.
If z = r(cos θ + i sin θ), then the value of $\frac{z}{\bar{z}}+\frac{\bar{z}}{z}$ is
(A) cos 2θ
(B) 2cos 2θ
(C) 2cos θ
(D) 2sin θ
Answer:
(B) 2cos 2θ
Hint:

$$\frac{z}{\bar{z}} = \frac{r(\cos\theta + i\sin\theta)}{r(\cos\theta - i\sin\theta)} = \frac{\cos\theta + i\sin\theta}{\cos(-\theta) + i\sin(-\theta)}$$
$$= (\cos\theta + i\sin\theta)(\cos(-\theta) + i\sin(-\theta))^{-1}$$
$$= (\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)$$
...[De Moivre's Theorem]
$$= (\cos\theta + i\sin\theta)^2$$
$$= \cos 2\theta + i\sin 2\theta$$
...[De Moivre's Theorem]

Question 6.
If ω(≠1) is a cube root of unity and $(1 + \omega)^7 = A + B\omega$, then A and B are respectively the numbers
(A) 0, 1

(B) 1, 1
(C) 1, 0
(D) -1, 1
Answer:
(B) 1, 1
Hint:
$(1 + \omega)^7$
$= (-\omega^2)^7$
$= -\omega^{14}$
$= -\omega^2(\omega^3)^4$
$= -\omega^2$
$= 1 + \omega$
A = 1, B = 1

Question 7.
The modulus and argument of $(1 + i\sqrt{3})^8$ are respectively
(A) 2 and $\frac{2\pi}{3}$
(B) 256 and $\frac{8\pi}{3}$
(C) 256 and $\frac{2\pi}{3}$
(D) 64 and $\frac{4\pi}{3}$
Answer:
(C) 256 and $\frac{2\pi}{3}$
Hint:

Let $z = (1 + i\sqrt{3})^8 = [r\ (\cos\theta + i\sin\theta)]^8$
$r\cos\theta = 1,\ r\sin\theta = \sqrt{3},$
$r = \sqrt{1+3} = 2$
$\therefore \quad \cos\theta = \frac{1}{2},\ \sin\theta = \frac{\sqrt{3}}{2}$
$\therefore \quad \arg z = \frac{\pi}{3}$

$$z = \left[2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^8$$
$$= 2^8\left(\cos\frac{8\pi}{3} + i\sin\frac{8\pi}{3}\right)$$
$$= 256\left[\cos\left(2\pi + \frac{2\pi}{3}\right) + i\sin\left(2\pi + \frac{2\pi}{3}\right)\right]$$
$$= 256\left[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right]$$

Question 8.
If arg (z) = θ, then arg $\overline{(z)}$ =
(A) -θ
(B) θ
(C) π – θ
(D) π + θ
Answer:
(A) -θ
Hint:
Let z = $re^{i\theta}$, then $\bar{z} = re^{-i\theta}$
∴ arg $\bar{z}$ = -θ.

Question 9.
If -1 + √3 i = $re^{i\theta}$, then θ =
(A) $-\frac{2\pi}{3}$
(B) $\frac{\pi}{3}$
(C) $-\frac{\pi}{3}$
(D) $\frac{2\pi}{3}$
Answer:
(D) $\frac{2\pi}{3}$

Hint:

$$r e^{i\theta} = -1 + i\sqrt{3}$$
$$= 2\left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right) \quad \ldots\begin{bmatrix} a = \frac{-1}{2}, \\ b = \frac{\sqrt{3}}{2} \end{bmatrix}$$
$$= 2\left[\cos\left(\pi - \frac{\pi}{3}\right) + i\sin\left(\pi - \frac{\pi}{3}\right)\right]$$
$$= 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$
$$\theta = \frac{2\pi}{3}$$

Question 10.
If z = x + iy and |z – zi| = 1, then
(A) z lies on X-axis
(B) z lies on Y-axis
(C) z lies on a rectangle
(D) z lies on a circle
Answer:
(D) z lies on a circle
Hint:
|z – zi | = |z| |1 – i| = 1
∴ |z| = 12√
∴ $x^2 + y^2 = 12$

(II) Answer the following:

Question 1.
Simplify the following and express in the form a + ib.
(i) 3 + √-64
Solution:
3 + √-64
= 3 + √64 √-1
= 3 + 8i

(ii) $(2i^3)^2$
Solution:
$(2i^3)^2$
= $4i^6$
= $4(i^2)^3$
= $4(-1)^3$
= -4 .....[∵ i^2 = -1]
= -4 + 0i

(iii) (2 + 3i) (1 – 4i)
Solution:
(2 + 3i)(1 – 4i)
= 2 – 8i + 3i – $12i^2$
= 2 – 5i – 12(-1) .....[∵ i^2 = -1]
= 14 – 5i

(iv) 52i(-4 – 3i)
Solution:

$$\frac{5}{2} i(-4-3i) = \frac{5}{2}(-4i - 3i^2)$$
$$= \frac{5}{2}[-4i - 3(-1)] \ldots[\because i^2 = -1]$$
$$= \frac{5}{2}(3 - 4i)$$
$$= \frac{15}{2} - 10i$$

(v) $(1 + 3i)^2 (3 + i)$
Solution:
$(1 + 3i)^2 (3 + i)$
$= (1 + 6i + 9i^2)(3 + i)$
$= (1 + 6i - 9)(3 + i)$ ......[$\because i^2 = -1$]
$= (-8 + 6i)(3 + i)$
$= -24 - 8i + 18i + 6i^2$
$= -24 + 10i + 6(-1)$
$= -24 + 10i - 6$
$= -30 + 10i$

(vi) $\frac{4+3i}{1-i}$
Solution:

$$\frac{4+3i}{1-i} = \frac{(4+3i)(1+i)}{(1-i)(1+i)}$$
$$= \frac{4+4i+3i+3i^2}{1-i^2}$$
$$= \frac{4+7i+3(-1)}{1-(-1)} \quad \ldots[\because i^2 = -1]$$
$$= \frac{1+7i}{2} = \frac{1}{2} + \frac{7}{2}i$$

(vii) $\left(1+\frac{2}{i}\right)\left(3+\frac{4}{i}\right)(5+i)^{-1}$
Solution:

$$\left(1+\frac{2}{i}\right)\left(3+\frac{4}{i}\right)(5+i)^{-1}$$
$$= \frac{(i+2)}{i} \cdot \frac{(3i+4)}{i} \cdot \frac{1}{5+i}$$
$$= \frac{3i^2+4i+6i+8}{i^2(5+i)} = \frac{-3+10i+8}{-1(5+i)} \quad \ldots[\because i^2 = -1]$$
$$= \frac{(5+10i)}{-(5+i)} = \frac{(5+10i)(5-i)}{-(5+i)(5-i)}$$
$$= \frac{25-5i+50i-10i^2}{-(25-i^2)}$$
$$= \frac{25+45i-10(-1)}{-[25-(-1)]} = \frac{35+45i}{-26}$$
$$= \frac{-35}{26} - \frac{45}{26}i$$

(viii) $\frac{\sqrt{5}+\sqrt{3}i}{\sqrt{5}-\sqrt{3}i}$
Solution:

$$\frac{\sqrt{5}+\sqrt{3}i}{\sqrt{5}-\sqrt{3}i}$$
$$= \frac{(\sqrt{5}+\sqrt{3}i)(\sqrt{5}+\sqrt{3}i)}{(\sqrt{5}-\sqrt{3}i)(\sqrt{5}+\sqrt{3}i)}$$
$$= \frac{5+2\sqrt{15}i+3i^2}{5-3i^2}$$
$$= \frac{5+2\sqrt{15}i+3(-1)}{5-3(-1)} \quad \ldots[\because i^2 = -1]$$
$$= \frac{2+2\sqrt{15}i}{8} = \frac{1+\sqrt{15}i}{4}$$
$$= \frac{1}{4} + \frac{\sqrt{15}i}{4}$$

(ix) $\frac{3i^5+2i^7+i^9}{i^6+2i^8+3i^{18}}$

Solution:

$$\frac{3i^5+2i^7+i^9}{i^6+2i^8+3i^{18}} = \frac{3(i^4.i)+2(i^4.i^3)+(i^4)^2.i}{i^4.i^2+2(i^4)^2+3(i^2)^9}$$

$$= \frac{3(1).i+2(1)(-i)+(1)^2.i}{(1)(-1)+2(1)^2+3(-1)^9}$$

$$\ldots[\because i^2=-1,\ i^3=-i,\ i^4=1]$$

$$= \frac{3i-2i+i}{-1+2-3}$$

$$= \frac{2i}{-2}$$

$$= -i$$

(x) $\frac{5+7i}{4+3i}+\frac{5+7i}{4-3i}$

Solution:

$$\frac{5+7i}{4+3i}+\frac{5+7i}{4-3i}$$

$$= (5+7i)\left[\frac{1}{4+3i}+\frac{1}{4-3i}\right]$$

$$= (5+7i)\left[\frac{4-3i+4+3i}{(4+3i)(4-3i)}\right]$$

$$= (5+7i)\left[\frac{8}{16-9i^2}\right]$$

$$= (5+7i).\left[\frac{8}{16-9(-1)}\right] \quad \ldots[\because i^2=-1]$$

$$= \frac{8(5+7i)}{25} = \frac{40+56i}{25}$$

$$= \frac{40}{25}+\frac{56}{25}i$$

Question 2.
Solve the following equations for x, y ∈ R
(i) (4 – 5i)x + (2 + 3i)y = 10 – 7i
Solution:
(4 – 5i)x + (2 + 3i)y = 10 – 7i
(4x + 2y) + (3y – 5x) i = 10 – 7i
Equating real and imaginary parts, we get
4x + 2y= 10 i.e., 2x + y = 5 ……(i)
and 3y – 5x = -7 ……(ii)
Equation (i) × 3 – equation (ii) gives
11x = 22
∴ x = 2
Putting x = 2 in (i), we get
2(2) + y = 5
∴ y = 1
∴ x = 2 and y = 1

(ii) $\frac{x+iy}{2+3i}$ = 7 – i
Solution:
$\frac{x+iy}{2+3i}$ = 7 – i
x + iy = (7 – i)(2 + 3i)
x + iy = 14 + 21i – 2i – $3i^2$
x + iy = 14 + 19i – 3(-1)
x + iy = 17 + 19i
Equating real and imaginary parts, we get
∴ x = 17 and y = 19

(iii) $(x + iy)(5 + 6i) = 2 + 3i$

Solution:

$$(x + iy)(5 + 6i) = 2 + 3i$$

$$\therefore \quad x + iy = \frac{2+3i}{5+6i}$$

$$\therefore \quad x + iy = \frac{(2+3i)(5-6i)}{(5+6i)(5-6i)}$$

$$= \frac{10-12i+15i-18i^2}{25-36i^2} = \frac{10+3i-18(-1)}{25-36(-1)}$$

$$\therefore \quad x + iy = \frac{28+3i}{61} = \frac{28}{61} + \frac{3}{61}i$$

Equating real and imaginary parts, we get

$$x = \frac{28}{61} \text{ and } y = \frac{3}{61}$$

(iv) $2x + i^9 y(2 + i) = x i^7 + 10 i^{16}$

Solution:

$2x + i^9 y(2 + i) = x i^7 + 10 i^{16}$

$2x + (i^4)^2 . i . y(2 + i) = x(i^2)^3 . i + 10 . (i^4)^4$

$2x + (1)^2 . iy(2 + i) = x(-1)^3 . i + 10(1)^4$ ....[$\because$ $i^2 = -1$, $i^4 = 1$]

$2x + 2yi + y i^2 = -xi + 10$

$2x + 2yi – y + xi = 10$

$(2x – y) + (x + 2y)i = 10 + 0 . i$

Equating real and imaginary parts, we get

$2x – y = 10$ ......(i)

and $x + 2y = 0$ ....(ii)

Equation (i) × 2 + equation (ii) gives, we get

$5x = 20$

$\therefore x = 4$

Putting $x = 4$ in (i), we get

$2(4) – y = 10$

$y = 8 – 10$

$\therefore y = -2$

$\therefore x = 4$ and $y = -2$

Question 3.

Evaluate

(i) $(1 – i + i^2)^{-15}$

Solution:

$$(1 - i + i^2)^{-15} = (1 - i - 1)^{-15}$$

$$= (-i)^{-15} = \frac{1}{(-i)^{15}}$$

$$= \frac{-1}{(i^4)^3 . i^3} = \frac{-1}{(1)^3(-i)}$$

$$= \frac{1}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$$

(ii) $i^{131} + i^{49}$

Solution:

$i^{131} + i^{49}$

$= (i^4)^{32} . i^3 + (i^4)^{12} . i$

$= (1)^{32} (-i) + (1)^{12} . i$

$= -i + i$

$= 0$

Question 4.

Find the value of

(i) $x^3 + 2x^2 – 3x + 21$, if $x = 1 + 2i$

Solution:

$$x = 1 + 2i$$
$$\therefore \quad x - 1 = 2i$$
$$\therefore \quad (x-1)^2 = 4i^2$$
$$\therefore \quad x^2 - 2x + 1 = -4 \qquad \ldots[\because i^2 = -1]$$
$$\therefore \quad x^2 - 2x + 5 = 0 \qquad \ldots\text{(i)}$$

$$\because \quad \begin{array}{r|l} & x+4 \\ \hline x^2-2x+5 & x^3+2x^2-3x+21 \\ & x^3-2x^2+5x \\ & -\quad+\quad- \\ \hline & 4x^2-8x+21 \\ & 4x^2-8x+20 \\ & -\quad+\quad- \\ \hline & 1 \end{array}$$

$$\therefore \quad x^3 + 2x^2 - 3x + 21$$
$$= (x^2 - 2x + 5)(x + 4) + 1$$
$$= 0.(x + 4) + 1 \qquad \ldots\text{[From (i)]}$$
$$= 0 + 1$$
$$\therefore \quad x^3 + 2x^2 - 3x + 21 = 1$$

(ii) $x^4 + 9x^3 + 35x^2 - x + 164$, if $x = -5 + 4i$

Solution:

$$x = -5 + 4i$$
$$\therefore \quad x + 5 = 4i$$
$$\therefore \quad (x+5)^2 = 16i^2$$
$$\therefore \quad x^2 + 10x + 25 = -16 \qquad \ldots[\because i^2 = -1]$$
$$\therefore \quad x^2 + 10x + 41 = 0 \qquad \ldots\text{(i)}$$

$$\begin{array}{r|l} & x^2-x+4 \\ \hline x^2+10x+41 & x^4+9x^3+35x^2-x+164 \\ & x^4+10x^3+41x^2 \\ & -\quad-\quad- \\ \hline & -x^3-6x^2-x+164 \\ & -x^3-10x^2-41x \\ & +\quad+\quad+ \\ \hline & 4x^2+40x+164 \\ & 4x^2+40x+164 \\ & -\quad-\quad- \\ \hline & 0 \end{array}$$

$$\therefore \quad x^4 + 9x^3 + 35x^2 - x + 164$$
$$= (x^2 + 10x + 41)(x^2 - x + 4)$$
$$= 0(x^2 - x + 4) \qquad \ldots\text{[From (i)]}$$
$$\therefore \quad x^4 + 9x^3 + 35x^2 - x + 164 = 0$$

Question 5.
Find the square roots of
(i) -16 + 30i
Solution:
Let $\sqrt{-16+30i}$ = a + bi, where a, b ∈ R.
Squaring on both sides, we get
$-16 + 30i = a^2 + b^2 i^2 + 2abi$
$-16 + 30i = (a^2 - b^2) + 2abi$ .....[∵ $i^2 = -1$]
Equating real and imaginary parts, we get
$a^2 - b^2 = -16$ and $2ab = 30$

$a^2 - b^2 = -16$ and $b = \frac{15}{a}$

$\therefore \quad a^2 - \left(\frac{15}{a}\right)^2 = -16$

$\therefore \quad a^2 - \frac{225}{a^2} = -16$

$\therefore \quad a^4 - 225 = -16a^2$

$\therefore \quad a^4 + 16a^2 - 225 = 0$

$\therefore \quad (a^2 + 25)(a^2 - 9) = 0$

$\therefore \quad a^2 = -25$ or $a^2 = 9$

But $a \in R$

$\therefore \quad a^2 \neq -25$

$\therefore \quad a^2 = 9$

$\therefore \quad a = \pm 3$

When $a = 3$, $b = \frac{15}{3} = 5$

When $a = -3$, $b = \frac{15}{-3} = -5$

$\therefore \quad \sqrt{-16 + 30i} = \pm (3 + 5i)$

(ii) 15 – 8i
Solution:
Let $\sqrt{15 - 8i}$ = a + bi, where a, b ∈ R.
Squaring on both sides, we get
$15 - 8i = a^2 + b^2 i^2 + 2abi$
$15 - 8i = (a^2 - b^2) + 2abi$ .....[∵ $i^2 = -1$]
Equating real and imaginary parts, we get
$a^2 - b^2 = 15$ and $2ab = -8$
$a^2 - b^2 = 15$ and $b = \frac{-4}{a}$

$\therefore \quad a^2 - \left(\frac{-4}{a}\right)^2 = 15$

$\therefore \quad a^2 - \frac{16}{a^2} = 15$

$\therefore \quad a^4 - 16 = 15a^2$

$\therefore \quad a^4 - 15a^2 - 16 = 0$

$\therefore \quad (a^2 - 16)(a^2 + 1) = 0$

$\therefore \quad a^2 = 16$ or $a^2 = -1$

But $a \in R$

$\therefore \quad a^2 \neq -1$

$\therefore \quad a^2 = 16$

$\therefore \quad a = \pm 4$

When a = 4, $b = \frac{-4}{4} = -1$
When a = -4, $b = \frac{-4}{-4} = 1$
$\therefore \sqrt{15 - 8i} = \pm(4 - i)$

(iii) 2 + 2√3 i
Solution:
Let $\sqrt{2 + 2\sqrt{3}i}$ = a + bi, where a, b ∈ R.
Squaring on both sides, we get
$2 + 2\sqrt{3}i = a^2 + b^2 i^2 + 2abi$
$2 + 2\sqrt{3}i = a^2 - b^2 + 2abi$ .....[∵ $i^2 = -1$]
Equating real and imaginary parts, we get
$a^2 - b^2 = 2$ and $2ab = 2\sqrt{3}$

$a^2 - b^2 = 2$ and $b = \frac{\sqrt{3}}{a}$

$\therefore \quad a^2 - \left(\frac{\sqrt{3}}{a}\right)^2 = 2$

$\therefore \quad a^2 - \frac{3}{a^2} = 2$

$\therefore \quad a^4 - 3 = 2a^2$

$\therefore \quad a^4 - 2a^2 - 3 = 0$

$\therefore \quad (a^2 - 3)(a^2 + 1) = 0$

$\therefore \quad a^2 = 3$ or $a^2 = -1$

But $a \in R$,

$\therefore \quad a^2 \neq -1$

$\therefore \quad a^2 = 3$

$\therefore \quad a = \pm\sqrt{3}$

When $a = \sqrt{3}$, $b = \frac{\sqrt{3}}{\sqrt{3}} = 1$

When $a = -\sqrt{3}$, $b = \frac{\sqrt{3}}{-\sqrt{3}} = -1$

$\therefore \quad \sqrt{2 + 2\sqrt{3}i} = \pm\left(\sqrt{3} + i\right)$

(iv) 18i
Solution:
Let $\sqrt{18i} = a + bi$, where $a, b \in R$.
Squaring on both sides, we get
$18i = a^2 + b^2 i^2 + 2abi$
$0 + 18i = a^2 - b^2 + 2abi$ .....[$\because i^2 = -1$]
Equating real and imaginary parts, we get
$a^2 - b^2 = 0$ and $2ab = 18$
$a^2 - b^2 = 0$ and $b = \frac{9}{a}$

$a^2 - \left(\frac{9}{a}\right)^2 = 0$

$a^2 - \frac{81}{a^2} = 0$

$a^4 - 81 = 0$
$(a^2 - 9)(a^2 + 9) = 0$
$a^2 = 9$ or $a^2 = -9$
But $a \in R$
$\therefore a^2 \neq -9$
$\therefore a^2 = 9$
$\therefore a = \pm 3$
When $a = 3$, $b = \frac{9}{3} = 3$
When $a = -3$, $b = \frac{9}{-3} = -3$
$\therefore \sqrt{18i} = \pm(3 + 3i) = \pm 3(1 + i)$

(v) 3 – 4i
Solution:
Let $\sqrt{3 - 4i} = a + bi$, where $a, b \in R$.
Squaring on both sides, we get
$3 - 4i = a^2 + b^2 i^2 + 2abi$
$3 - 4i = a^2 - b^2 + 2abi$ ......[$\because i^2 = -1$]
Equating real and imaginary parts, we get
$a^2 - b^2 = 3$ and $2ab = -4$

$a^2 - b^2 = 3$ and $b = \frac{-2}{a}$

$\therefore \quad a^2 - \left(-\frac{2}{a}\right)^2 = 3$

$\therefore \quad a^2 - \frac{4}{a^2} = 3$

$\therefore \quad a^4 - 4 = 3a^2$

$\therefore \quad a^4 - 3a^2 - 4 = 0$

$\therefore \quad (a^2 - 4)(a^2 + 1) = 0$

$\therefore \quad a^2 = 4$ or $a^2 = -1$

But, $a \in R$

$\therefore \quad a^2 \neq -1$

$\therefore \quad a^2 = 4$

$\therefore \quad a = \pm 2$

When $a = 2$, $b = \frac{-2}{2} = -1$

When $a = -2$, $b = \frac{-2}{-2} = 1$

$\therefore \quad \sqrt{3-4i} = \pm(2 - i)$

(vi) 6 + 8i
Solution:
Let $\sqrt{6+8i}$ = a + bi, where a, b ∈ R.
Squaring on both sides, we get
$6 + 8i = a^2 + b^2 i^2 + 2abi$
$6 + 8i = a^2 - b^2 + 2abi$ ……[∵ $i^2 = -1$]
Equating real and imaginary parts, we get
$a^2 - b^2 = 6$ and $2ab = 8$

$\therefore \quad a^2 - b^2 = 6$ and $b = \frac{4}{a}$

$\therefore \quad a^2 - \left(\frac{4}{a}\right)^2 = 6$

$\therefore \quad a^2 - \frac{16}{a^2} = 6$

$\therefore \quad a^4 - 16 = 6a^2$

$\therefore \quad a^4 - 6a^2 - 16 = 0$

$\therefore \quad (a^2 - 8)(a^2 + 2) = 0$

$\therefore \quad a^2 = 8$ or $a^2 = -2$

But $a \in R$

$\therefore \quad a^2 \neq -2$

$\therefore \quad a^2 = 8$

$\therefore \quad a = \pm 2\sqrt{2}$

When $a = 2\sqrt{2}$, $b = \frac{4}{2\sqrt{2}} = \sqrt{2}$

When $a = -2\sqrt{2}$, $b = \frac{4}{-2\sqrt{2}} = -\sqrt{2}$

$\therefore \quad \sqrt{6+8i} = \pm(2\sqrt{2} + \sqrt{2}\,i) = \pm\sqrt{2}(2 + i)$

Question 6.
Find the modulus and argument of each complex number and express it in the polar form.
(i) 8 + 15i

Solution:

Let $z = 8 + 15i$

$\therefore \quad a = 8, b = 15, \quad a, b > 0$

$\therefore \quad |z| = r = \sqrt{a^2 + b^2} = \sqrt{(8)^2 + (15)^2}$

$= \sqrt{64 + 225} = \sqrt{289} = 17$

Here, (8, 15) lies in 1^{st} quadrant.

$\therefore \quad \theta = \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{15}{8}\right)$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$= 17(\cos\theta + i\sin\theta)$, where $\theta = \tan^{-1}\left(\frac{15}{8}\right)$

(ii) 6 – i

Solution:

Let $z = 6 - i$

$\therefore \quad a = 6, b = -1, \ a > 0, b < 0$

$\therefore \quad |z| = r = \sqrt{a^2 + b^2} = \sqrt{6^2 + (-1)^2} = \sqrt{36 + 1}$

$= \sqrt{37}$

Here, (6, –1) lies in 4^{th} quadrant.

$\therefore \quad \theta = \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{-1}{6}\right)$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$= \sqrt{37}(\cos\theta + i\sin\theta)$

where $\theta = \tan^{-1}\left(-\frac{1}{6}\right)$

(iii) 1+3√i2

Solution:

Let $z = \frac{1+\sqrt{3}i}{2} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$\therefore \quad a = \frac{1}{2}, b = \frac{\sqrt{3}}{2}, a, b > 0$

$\therefore \quad |z| = r = \sqrt{a^2 + b^2}$

$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

Here, $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ lies in 1st quadrant.

$\therefore \quad \theta = \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)$

$= \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$\therefore \quad \theta = 60° = \frac{\pi}{3}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$= 1(\cos 60° + i\sin 60°)$

$= 1\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(iv) $\frac{-1-i}{\sqrt{2}}$

Solution:

Let $z = \frac{-1-i}{\sqrt{2}} = \frac{-1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$

$\therefore \quad a = \frac{-1}{\sqrt{2}}, b = \frac{-1}{\sqrt{2}}, a, b < 0$

$\therefore \quad |z| = r = \sqrt{a^2 + b^2} = \sqrt{\left(\frac{-1}{\sqrt{2}}\right)^2 + \left(\frac{-1}{\sqrt{2}}\right)^2}$

$= \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$

Here, $\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ lies in 3rd quadrant.

$\therefore \quad \theta = \text{amp}(z) = \pi + \tan^{-1}\left|\frac{b}{a}\right|$

$= \pi + \tan^{-1}\left|\frac{\frac{-1}{\sqrt{2}}}{\frac{-1}{\sqrt{2}}}\right|$

$= \pi + \tan^{-1}(1)$

$= \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$\therefore \quad \theta = 225° = \frac{5\pi}{4}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$= 1(\cos 225° + i\sin 225°)$

$= 1\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

(v) 2i
Solution:

Let $z = 2i = 0 + 2i$

$\therefore \quad a = 0, b = 2$

$\therefore$ z lies on positive imaginary Y-axis.

$\therefore \quad |z| = r = \sqrt{a^2+b^2} = \sqrt{0^2+2^2} = \sqrt{0+4} = 2$

$\therefore \quad \theta = \text{amp}(z) = \frac{\pi}{2}$

$\therefore \quad \theta = 90^\circ = \frac{\pi}{2}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$= 2(\cos 90^\circ + i\sin 90^\circ)$

$= 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

(vi) -3i
Solution:

Let $z = -3i = 0 - 3i$

$\therefore \quad a = 0, b = -3$

z lies on negative imaginary Y-axis.

$\therefore \quad |z| = r = \sqrt{a^2+b^2} = \sqrt{0^2+(-3)^2} = \sqrt{0+9} = 3$

$\therefore \quad \theta = \text{amp}(z) = \frac{3\pi}{2}$

$\therefore \quad \theta = 270^\circ = \frac{3\pi}{2}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$= 3(\cos 270^\circ + i\sin 270^\circ)$

$= 3\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

(vii) 12√+12√i
Solution:

Let $z = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

$\therefore \quad a = \frac{1}{\sqrt{2}}, b = \frac{1}{\sqrt{2}}, a > 0, b > 0$

$\therefore \quad |z| = r = \sqrt{a^2+b^2} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$

$= \sqrt{\frac{1}{2}+\frac{1}{2}} = 1$

Here, $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ lies in 1^{st} quadrant.

$\therefore \quad \theta = \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right)$

$= \tan^{-1}(1) = \frac{\pi}{4}$

$\therefore \quad \theta = 45^\circ = \frac{\pi}{4}$

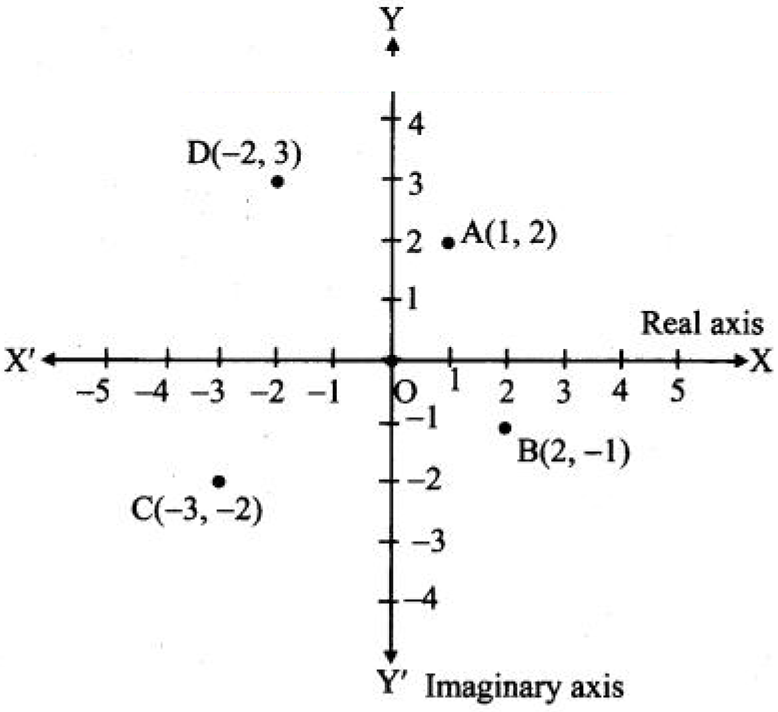
$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$= 1(\cos 45^\circ + i\sin 45^\circ)$

$= 1\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Question 7.
Represent 1 + 21, 2 – i, -3 – 2i, -2 + 3i by points in Argand's diagram.
Solution:

The complex numbers 1 + 2i, 2 – i, -3 – 2i, -2 + 3i will be represented by the points A(1, 2), B(2, -1), C(-3, -2), D(-2, 3) respectively as shown below:



Question 8.
Show that $z = 5(1-i)(2-i)(3-i)$ is purely imaginary number.
Solution:

$$z = \frac{5}{(1-i)(2-i)(3-i)}$$
$$= \frac{5}{(2-i-2i+i^2)(3-i)}$$
$$= \frac{5}{(2-3i-1)(3-i)} \quad \ldots[\because i^2 = -1]$$
$$= \frac{5}{(1-3i)(3-i)}$$
$$= \frac{5}{3-i-9i+3i^2}$$
$$= \frac{5}{3-10i-3} = \frac{5}{-10i}$$
$$= \frac{5i}{-10i^2}$$
$$= \frac{5i}{10}$$
$$= \frac{1}{2}i, \text{ which is a purely imaginary number.}$$

Question 9.
Find the real numbers x and y such that $x1+2i+y3+2i=5+6i-1+8i$
Solution:
$x1+2i+y3+2i=5+6i-1+8i$

$$\therefore \quad \frac{x(3+2i)+y(1+2i)}{(3+2i)(1+2i)} = \frac{5+6i}{-1+8i}$$
$$\therefore \quad \frac{3x+2xi+y+2yi}{3+6i+2i+4i^2} = \frac{5+6i}{-1+8i}$$
$$\therefore \quad \frac{(3x+y)+(2x+2y)i}{3+8i+4(-1)} = \frac{5+6i}{-1+8i} \quad \ldots[i^2 = -1]$$
$$\therefore \quad \frac{(3x+y)+(2x+2y)i}{-1+8i} = \frac{5+6i}{-1+8i}$$

(3x + y) + 2(x + y)i = 5 + 6i
Equating real and imaginary parts, we get
3x + y = 5 ……(i)
and 2(x + y) = 6
i.e., x + y = 3 …….(ii)
Subtracting (ii) from (i), we get
2x = 2
∴ x = 1
Putting x = 1 in (ii), we get

$1 + y = 3$
$\therefore y = 2$
$\therefore x = 1, y = 2$

Question 10.
Show that $\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{10}+\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^{10}=0$
Solution:

$$\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^2 = \frac{1}{2} + 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{i}{\sqrt{2}}\right) + \frac{i^2}{2}$$
$$= \frac{1}{2} + i - \frac{1}{2} = i$$
$$\therefore \quad \left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{10} = \left[\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^2\right]^5$$
$$= i^5 = i^4 . i = i \quad \ldots(i)$$

Also, $$\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^2 = \frac{1}{2} - 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{i}{\sqrt{2}}\right) + \frac{i^2}{2}$$
$$= \frac{1}{2} - i - \frac{1}{2} = -i$$
$$\therefore \quad \left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^{10} = \left[\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^2\right]^5 = (-i)^5$$
$$= i^4(-i) = -i \quad \ldots(ii)$$

Adding (i) and (ii), we get
$$\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{10} + \left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^{10} = i - i = 0$$

Question 11.
Show that $\left(\frac{1+i}{\sqrt{2}}\right)^8+\left(\frac{1-i}{\sqrt{2}}\right)^8=2$
Solution:

$$\left(\frac{1+i}{\sqrt{2}}\right)^2 = \frac{1+2i+i^2}{2} = \frac{1+2i-1}{2} = i$$
$$\left(\frac{1+i}{\sqrt{2}}\right)^8 = \left[\left(\frac{1+i}{\sqrt{2}}\right)^2\right]^4 = i^4 = 1 \quad \ldots(i)$$

Also, $$\left(\frac{1-i}{\sqrt{2}}\right)^2 = \frac{1-2i+i^2}{2} = \frac{1-2i-1}{2} = -i$$
$$\left(\frac{1-i}{\sqrt{2}}\right)^8 = \left[\left(\frac{1-i}{\sqrt{2}}\right)^2\right]^4$$
$$= (-i)^4 = (-1)^4 \times (i)^4$$
$$= 1 \times i^4 = 1 \quad \ldots(ii)$$

Adding (i) and (ii), we get
$$\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8 = 1 + 1 = 2$$

Question 12.
Convert the complex numbers in polar form and also in exponential form.
(i) z = $\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$
Solution:

$$z = \frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$$
$$= \frac{(2+6\sqrt{3}i)(5-\sqrt{3}i)}{(5+\sqrt{3}i)(5-\sqrt{3}i)}$$
$$= \frac{10-2\sqrt{3}i+30\sqrt{3}i-6(3)i^2}{25-3i^2}$$
$$= \frac{10+28\sqrt{3}i+18}{25+3} \quad \ldots[\because i^2 = -1]$$
$$= \frac{28+28\sqrt{3}i}{28} = 1+\sqrt{3}\,i$$

$\therefore$ $a = 1, b = \sqrt{3}$, i.e. $a, b > 0$

$\therefore$ $r = \sqrt{a^2+b^2} = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{1+3} = 2$

Here, $(1, \sqrt{3})$ lies in 1^{st} quadrant.

$\therefore$ $\theta = \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right)$
$$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$
$$= \frac{\pi}{3}$$

$\therefore$ $\theta = 60° = \frac{\pi}{3}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$
$$= 2(\cos 60° + i\sin 60°)$$
$$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right).$$

$\therefore$ The exponential form of $z = re^{i\theta} = 2e^{\frac{\pi}{3}i}$

(ii) z = -6 + √2 i
Solution:
z = -6 + √2 i
∴ a = -6, b = √2
i.e. a < 0, b > 0

$\therefore$ $r = \sqrt{a^2+b^2} = \sqrt{(-6)^2+(\sqrt{2})^2} = \sqrt{36+2} = \sqrt{38}$

Here, $(-6, \sqrt{2})$ lies in 2^{nd} quadrant.

$\therefore$ $\theta = \text{amp}(z) = \pi - \tan^{-1}\left|\frac{b}{a}\right| = \pi - \tan^{-1}\left(\frac{\sqrt{2}}{6}\right)$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$= \sqrt{38}(\cos\theta + i\sin\theta)$,

where $\theta = \pi - \tan^{-1}\left(\frac{\sqrt{2}}{6}\right)$

$\therefore$ The exponential form of $z = re^{i\theta} = \sqrt{38}\,e^{i\theta}$

(iii) -32+33√i2
Solution:

Let $z = \frac{-3}{2} + \frac{3\sqrt{3}}{2} i$

$\therefore \quad a = \frac{-3}{2}, b = \frac{3\sqrt{3}}{2} \quad , a < 0, b > 0$

$\therefore \quad r = \sqrt{a^2 + b^2}$

$= \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{9}{4} + \frac{27}{4}}$

$= 3$

Here, $\left(\frac{-3}{2}, \frac{3\sqrt{3}}{2}\right)$ lies in 2^{nd} quadrant.

$\therefore \quad \theta = \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right) + \pi$

$= \tan^{-1}\left(\frac{\frac{3\sqrt{3}}{2}}{\frac{-3}{2}}\right) + \pi$

$= \tan^{-1}(-\sqrt{3}) + \pi$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

$\therefore \quad \theta = 120° = \frac{2\pi}{3}$

$\therefore$ The polar form of $z = r(\cos\theta + i.\sin\theta)$

$= 3(\cos 120° + i \sin 120°)$

$= 3\left(\cos\frac{2\pi}{3} + i \sin\frac{2\pi}{3}\right)$

$\therefore$ The exponential form of $z = re^{i\theta} = 3e^{\frac{2\pi}{3} i}$

Question 13.
If x + iy = $a+iba-ib$, prove that $x^2 + y^2 = 1$.

Solution:

$$x + iy = \frac{a+ib}{a-ib} = \frac{(a+ib)(a+ib)}{(a-ib)(a+ib)}$$

$$= \frac{a^2 + i^2b^2 + 2abi}{a^2 - i^2b^2}$$

$$= \frac{(a^2 - b^2) + 2abi}{a^2 + b^2} \quad \ldots[\because i^2 = -1]$$

$$\therefore \quad x + iy = \frac{a^2 - b^2}{a^2 + b^2} + \frac{2ab}{a^2 + b^2} i$$

Equating real and imaginary parts, we get

$$x = \frac{a^2 - b^2}{a^2 + b^2} \text{ and } y = \frac{2ab}{a^2 + b^2}$$

$$\therefore \quad x^2 + y^2 = \frac{(a^2 - b^2)^2}{(a^2 + b^2)^2} + \frac{4a^2b^2}{(a^2 + b^2)^2}$$

$$= \frac{a^4 + b^4 - 2a^2b^2 + 4a^2b^2}{(a^2 + b^2)^2}$$

$$= \frac{(a^2 + b^2)^2}{(a^2 + b^2)^2}$$

$$\therefore \quad x^2 + y^2 = 1$$

Question 14.
Show that z = (−1+−3√2)3 is a rational number.
Solution:

$$\left(\frac{-1+\sqrt{-3}}{2}\right)^3 = \frac{(-1+\sqrt{3}.\sqrt{-1})^3}{8}$$

$$= \frac{(-1+\sqrt{3}i)^3}{8}$$

$$= \frac{(-1)^3 + 3(-1)^2(i\sqrt{3}) + 3(-1)(i\sqrt{3})^2 + (i\sqrt{3})^3}{8}$$

$$= \frac{-1 + 3\sqrt{3}i - 3 \times 3(-1) - 3\sqrt{3}i}{8}$$

$$\ldots[\because i^2 = -1, i^3 = -i]$$

$$= \frac{-1+9}{8} = \frac{8}{8}$$

= 1, which is a rational number.

Question 15.
Show that 1−2i3−4i+1+2i3+4i is real.
Solution:

$$\frac{1-2i}{3-4i} + \frac{1+2i}{3+4i}$$

$$= \frac{(1-2i)(3+4i) + (3-4i)(1+2i)}{(3+4i)(3-4i)}$$

$$= \frac{3 + 4i - 6i - 8i^2 + 3 + 6i - 4i - 8i^2}{9 - 16i^2}$$

$$= \frac{6 - 16i^2}{9 - 16(-1)}$$

$$= \frac{6 - 16(-1)}{9 + 16} \quad \ldots[\because i^2 = -1]$$

$$= \frac{22}{25}, \text{ which is a real number.}$$

Question 16.
Simplify

(i) i29+i39+i49i30+i40+i50

Solution:

$$\frac{i^{29}+i^{39}+i^{49}}{i^{30}+i^{40}+i^{50}}$$
$$=\frac{i^{29}+i^{39}+i^{49}}{i\left(i^{29}+i^{39}+i^{49}\right)}=\frac{1}{i}=\frac{i}{i^2}=\frac{i}{-1}$$
$$=-i$$

(ii) (i65+1i145)

Solution:

$$\left(i^{65}+\frac{1}{i^{145}}\right)$$
$$=\left[(i^4)^{16}\cdot\ \ 1\ \ \right]_{\cdot\ \cdot}\ 1\ \_\ i^2+1$$
$$=\frac{-1+1}{i}$$
$$=0$$

(iii) i238+i236+i234+i232+i230i228+i226+i224+i222+i220

Solution:

$$\frac{i^{238}+i^{236}+i^{234}+i^{232}+i^{230}}{i^{228}+i^{226}+i^{224}+i^{222}+i^{220}}$$
$$=\frac{i^{10}(i^{228}+i^{226}+i^{224}+i^{222}+i^{220})}{i^{228}+i^{226}+i^{224}+i^{222}+i^{220}}$$
$$=i^{10}=(i^4)^2.\ i^2=(1)^2(-1)$$
$$=-1$$

Question 17.

Simplify [11-2i+31+i][3+4i2-4i]

Solution:

$$\left[\frac{1}{1-2i}+\frac{3}{1+i}\right]\left[\frac{3+4i}{2-4i}\right]$$
$$=\left[\frac{1+i+3-6i}{(1-2i)(1+i)}\right]\left[\frac{3+4i}{2-4i}\right]$$
$$=\left[\frac{4-5i}{1+i-2i-2i^2}\right]\left[\frac{3+4i}{2-4i}\right]$$
$$=\frac{(4-5i)(3+4i)}{(3-i)(2-4i)}$$
$$=\frac{12+16i-15i-20i^2}{6-12i-2i+4i^2}$$

$$=\frac{12+i+20}{6-14i-4}=\frac{32+i}{2-14i}$$
$$=\frac{(32+i)(2+14i)}{(2-14i)(2+14i)}=\frac{64+448i+2i+14i^2}{4-196i^2}$$
$$=\frac{64+450i-14}{4+196}=\frac{50+450i}{200}=\frac{50}{200}(1+9i)$$
$$=\frac{1}{4}+\frac{9}{4}i$$

Question 18.

If α and β are complex cube roots of unity, prove that (1 – α) (1 – β) (1 – α^2) (1 – β^2) = 9.

Solution:

α and β are the complex cube roots of unity.

$$\therefore \quad \alpha = \frac{-1+i\sqrt{3}}{2} \text{ and } \beta = \frac{-1-i\sqrt{3}}{2}$$

$$\therefore \quad \alpha\beta = \left(\frac{-1+i\sqrt{3}}{2}\right)\left(\frac{-1-i\sqrt{3}}{2}\right)$$

$$= \frac{(-1)^2 - \left(i\sqrt{3}\right)^2}{4}$$

$$= \frac{1-(-1)(3)}{4}$$

$$= \frac{1+3}{4}$$

$$\therefore \quad \alpha\beta = 1$$

$$\text{Also, } \alpha+\beta = \frac{-1+i\sqrt{3}}{2} + \frac{-1-i\sqrt{3}}{2}$$

$$= \frac{-1+i\sqrt{3}-1-i\sqrt{3}}{2} = \frac{-2}{2}$$

$$\therefore \quad \alpha + \beta = -1$$

$$\therefore \quad (1-\alpha)(1-\beta)(1-\alpha^2)(1-\beta^2)$$

$$= (1-\alpha)(1-\beta)(1-\alpha)(1+\alpha)(1-\beta)(1+\beta)$$

$$= (1-\alpha)^2(1-\beta)^2(1+\alpha)(1+\beta)$$

$$= [(1-\alpha)(1-\beta)]^2(1+\alpha)(1+\beta)$$

$$= (1-\beta-\alpha+\alpha\beta)^2(1+\alpha+\beta+\alpha\beta)$$

$$= [1-(\alpha+\beta)+\alpha\beta]^2[1+(\alpha+\beta)+\alpha\beta]$$

$$= [1-(-1)+1]^2(1-1+1)$$

$$= 3^2(1) = 9$$

Question 19.
If ω is a complex cube root of unity, prove that $(1 - \omega + \omega^2)^6 + (1 + \omega - \omega^2)^6 = 128$.
Solution:
ω is the complex cube root of unity.
$\therefore \omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$
$\therefore$ L.H.S. $= (1 - \omega + \omega^2)^6 + (1 + \omega - \omega^2)^6$
$= [(1 + \omega^2) - \omega]^6 + [(1 + \omega) - \omega^2]^6$
$= (-\omega - \omega))^6 + (-\omega^2 - \omega^2)^6$
$= (-2\omega)^6 + (-2\omega^2)^6$
$= 64\omega^6 + 64\omega^{12}$
$= 64(\omega^3)^2 + 64(\omega^3)^4$
$= 64(1)^2 + 64(1)^4$
$= 128$
$=$ R.H.S.

Question 20.
If ω is the cube root of unity, then find the value of $\left(\frac{-1+i\sqrt{3}}{2}\right)^{18}+\left(\frac{-1-i\sqrt{3}}{2}\right)^{18}$
Solution:
If ω is the complex cube root of unity, then

$$\omega^3 = 1,\ \omega = \frac{-1+i\sqrt{3}}{2} \text{ and } \omega^2 = \left(\frac{-1-i\sqrt{3}}{2}\right)^2$$

$$\text{Consider, } \left(\frac{-1+i\sqrt{3}}{2}\right)^{18} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{18}$$

Given Expression $= \omega^{18} + (\omega^2)^{18}$
$= \omega^{18} + \omega^{36}$
$= (\omega^3)^6 + (\omega^3)^{12}$
$= (1)^6 + (1)^{12}$
$= 2$