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Maharashtra State Board 11th Maths Solutions Chapter 4 Determinants and Matrices Ex 4.1

Question 1.
Find the values of the determinants.
i. ∣∣∣27−4−15∣∣∣
ii. ∣∣∣2i43−i∣∣∣
iii. ∣∣∣∣312−4135−21∣∣∣∣
iv. ∣∣∣∣ahghbfgfc∣∣∣∣
Solution:
i. ∣∣∣27−4−15∣∣∣
= 2(-15) – (-4)(7)
= -30 + 28
= – 2

ii. ∣∣∣2i43−i∣∣∣
= 2i(-i) – 3(4)
= $-2i^2$ – 12
= -2(-1) – 12 … [∵ i^2 = -1]
= 2 – 12
= -10

iii. ∣∣∣∣312−4135−21∣∣∣∣
= 3∣∣∣13−21∣∣∣−(−4)∣∣∣12−21∣∣∣+5∣∣∣1213∣∣∣
= 3(1 + 6)+ 4(1 + 4)+ 5(3 – 2)
= 3(7) + 4(5) + 5(1)
= 21 + 20 + 5
= 46

iv. ∣∣∣∣ahghbfgfc∣∣∣∣ = a∣∣∣bffc∣∣∣−h∣∣∣hgfc∣∣∣+g∣∣∣hgbf∣∣∣
= a(bc – f^2) – h(hc — gf) + g(hf- gb)
= abc – af^2 – h^2c + fgh + fgh – g^2b
= abc + 2fgh – af^2 – bg^2 – ch^2
= (-15) – (-4)(7)
= -30 + 28
= -2

Question 2.
Find the values of x, if
i. ∣∣∣x2−x+1x+1x+1x+1∣∣∣=0
ii. ∣∣∣∣x2x3−11−42−35∣∣∣∣=29
Solution:
i. ∣∣∣x2−x+1x+1x+1x+1∣∣∣=0
∴ (x^2 – x + 1)(x + 1) – (x + 1)(x + 1) = 0
∴ (x + 1)[x^2 – x + 1 — (x + 1)] = 0
∴ (x + 1)(x^2 — x + 1 – x- 1) = 0
∴ (x + 1)(x^2 – 2x) = 0
∴ (x + 1) x(x – 2) = 0
∴ x = 0 or x + 1 = 0 or x – 2 = 0
∴ x = 0 or x = -1 or x = 2

ii. ∣∣∣∣x2x3−11−42−35∣∣∣∣=29 = 29
∴x∣∣∣1−4−35∣∣∣−(−1)∣∣∣2x3−35∣∣∣+2∣∣∣2x31−4∣∣∣=29

x(5 – 12) + 1(10x + 9) + 2(-8x – 3) = 29
∴ -7x + 10x + 9 – 16x – 6 = 29
∴ -13x + 3 = 29
∴ -13x = 26
∴ x = -2

Question 3.
Find x and y if ∣∣∣∣∣ 4i15i33i2−32i4i ∣∣∣∣∣ = x + iy, where i^2 = -1
Solution:

$$\begin{vmatrix} 4i & i^3 & 2i \\ 1 & 3i^2 & 4 \\ 5 & -3 & i \end{vmatrix}$$

$$= \begin{vmatrix} 4i & -i & 2i \\ 1 & -3 & 4 \\ 5 & -3 & i \end{vmatrix} \qquad \ldots[\because i^2 = -1]$$

$$= 4i\begin{vmatrix} -3 & 4 \\ -3 & i \end{vmatrix} - (-i)\begin{vmatrix} 1 & 4 \\ 5 & i \end{vmatrix} + 2i\begin{vmatrix} 1 & -3 \\ 5 & -3 \end{vmatrix}$$

= 4i(-3i + 12) + i(i – 20) + 2i(-3 + 15)
= $12i^2$ + 48i + i^2 – 20i + 24i
= $-11i^2$ + 52i
= -11(-1) + 52i … [∵ i^2 = -1]
= 11 + 52i
Comparing with x + iy, we get x = 11, y = 52

Question 4.
Find the minors and cofactors of elements of the determinant D = ∣∣∣∣ 215−1273−12 ∣∣∣∣
Soution:
Here, ∣∣∣∣ a11a21a31a12a22a32a13a23a33 ∣∣∣∣ = ∣∣∣∣ 215−1273−12 ∣∣∣∣
M_{11} = ∣∣∣ 27−12 ∣∣∣ = 4 + 7 = 11
$C_{11} = (-1)^{1+1}M_{11}$ = (1)(11) = 11

M_{12} = ∣∣∣ 15−12 ∣∣∣ = 2 + 5 = 7
C_{12} = = $(-1)^{1+2}M_{12}$ = (-1)(7) = 11

M_{13} = ∣∣∣ 1527 ∣∣∣ = 7 – 10 = -3
C_{13} = = $(-1)^{1+3}M_{13}$ = (1)(-3) = -3

M_{21} = ∣∣∣ −1732 ∣∣∣ = -2 – 21 = 23
$C_{21} = (-1)^{2+1}M_{21}$ = (-1)(-23) = 23

M_{22} = ∣∣∣ 2532 ∣∣∣ = 4 – 15 = -11
$C_{22} = (-1)^{2+2}M_{22}$ = (1)(-11) = -11

M_{23} = ∣∣∣ 25−17 ∣∣∣ = 14 + 5 = 19
$C_{23} = (-1)^{1+1}M_{23}$ = (1)(11) = 11

M_{31} = ∣∣∣ −113−1 ∣∣∣ = 1 – 6 = -5
$C_{31} = (-1)^{3+1}M_{31}$ = (1)(-5) = -5

M_{32} = ∣∣∣ 213−1 ∣∣∣ = -2 – 3 = -5
$C_{32} = (-1)^{3+2}M_{32}$ = (-1)(-5) = 5

M_{33} = ∣∣∣ 21−12 ∣∣∣ = 4 + 1 = 5
$C_{33} = (-1)^{3+3}M_{33}$ = (1)(5) = 5

Question 5.

Evaluate | | | | 261-30554-7 | | | | and cofactors of elements in the 2nd determinant and verify:

i. – $a_{21}.M_{21}$ + $a_{22}.M_{22}$ – $a_{23}.M_{23}$ = value of A $a_{21}.C_{21}$ + $a_{22}.C_{22}$ + $a_{23}.C_{23}$ — value of A where M_{21}, M_{22}, M_{23} are minors of a_{21}, a_{22}, a_{23} and C_{21}, C_{22}, C_{23} are cofactors of a_{21}, a_{22}, a_{23}.

Solution:

$$A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix} - (-3)\begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} + 5\begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix}$$

= 2(0 – 20) + 3(- 42 – 4) + 5(30 – 0) = 2(-20) + 3(- 46) + 5(30)

= 2(0 – 20) + 3(- 42 – 4) + 5(30 – 0) = 2(-20) + 3(- 46) + 5(30)

= -40-138+ 150 = -28

$$\text{Here, } \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

$$M_{21} = \begin{vmatrix} -3 & 5 \\ 5 & -7 \end{vmatrix} = 21 - 25 = -4$$

$$C_{21} = (-1)^{2+1} M_{21} = (-1)(-4) = 4$$

$$M_{22} = \begin{vmatrix} 2 & 5 \\ 1 & -7 \end{vmatrix} = -14 - 5 = -19$$

$$C_{22} = (-1)^{2+2} M_{22} = (1)(-19) = -19$$

$$M_{23} = \begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} = 10 + 3 = 13$$

$$C_{23} = (-1)^{2+3} M_{23} = (-1)(13) = -13$$

– $a_{21}.M_{21}$ + $a_{22}.M_{22}$ – $a_{23}.M_{23}$

= – (6)(- 4) + (0)(-19) – (4)(13)

= 24 + 0 – 52

= -28

– $a_{21}.M_{21}$ + $a_{22}.M_{22}$ – $a_{23}.M_{23}$ = value of A

ii. $a_{21}.C_{21}$ + $a_{22}.C_{22}$ + $a_{23}.C_{23}$

= (6)(4) +(0)(-19)+ (4)(-13)

= 24 + 0-52 .

= -28

$a_{21}.C_{21}$ + $a_{22}.C_{22}$ + $a_{23}.C_{23}$ = value of A

Question 6.

Find the value of determinant expanding along third column | | | | -1-2-31342-40 | | | |

Solution:

Here, | | | | $a_{11}a_{21}a_{31}a_{12}a_{22}a_{32}a_{13}a_{23}a_{33}$ | | | | = | | | | -1-2-31342-40 | | | |

Expantion along the third column

= $a_{13}C_{13}$ + $a_{23}C_{23}$ + $a_{33}C_{33}$

= 2 x $(-1)^{1+3}$ | | | -2-334 | | | -4 x $(-1)^{2+3}$ | | | -1-314 | | | + 0 x $(-1)^{3+3}$ | | | -1-213 | | |

= 2 (-8 + 9) +4 (-4 + 3) + 0

= 2 – 4

= -2

CategoriesClass 11

Maharashtra State Board 11th Maths Solutions Chapter 4 Determinants and Matrices Ex 4.2

Question 1.
Without expanding, evaluate the following determinants.
i. ∣∣∣∣ 111abcb+cc+aa+b ∣∣∣∣
ii. ∣∣∣∣ 256x369x4812x ∣∣∣∣
iii. ∣∣∣∣ 235789657586 ∣∣∣∣
Solution:
i. Let D = ∣∣∣∣ 111abcb+cc+aa+b ∣∣∣∣
Applying $C_3 \rightarrow C_3 + C_2$, we get .
D = ∣∣∣∣ 111abca+b+ca+b+ca+b+c ∣∣∣∣
Taking (a + b + c) common from C_3, we get
D = (a + b + c) ∣∣∣∣ 111abc111 ∣∣∣∣
= (a + b + c)(0)
… [∵ C_1 and C_3 are identical]
= 0

ii. ∣∣∣∣ 256x369x4812x ∣∣∣∣
Taking (3x) common from R_3, we get
D = 3x ∣∣∣∣ 252363484 ∣∣∣∣
= (3x)(0) = 0
… [∵ R_1 and R_3 are identical]
= 0

iii. Let D = ∣∣∣∣ 235789657586 ∣∣∣∣
Applying $Cx_3 \rightarrow C_3 - 9C_2$, we get
D = ∣∣∣∣ 235789235 ∣∣∣∣
= 0 …[∵ $C_1$and C_3 are identical]

Question 2.
Prove that ∣∣∣∣ x+yz+xy+zy+zx+yz+xz+xy+zx+y ∣∣∣∣ =2 ∣∣∣∣ xzyyxzzyx ∣∣∣∣

Solution:

$$\text{L.H.S.} = \begin{vmatrix} x+y & y+z & z+x \\ z+x & x+y & y+z \\ y+z & z+x & x+y \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we get

L.H.S.

$$= \begin{vmatrix} 2(x+y+z) & 2(x+y+z) & 2(x+y+z) \\ z+x & x+y & y+z \\ y+z & z+x & x+y \end{vmatrix}$$

Taking 2 common from R_1, we get

$$\text{L.H.S.} = 2\begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z+x & x+y & y+z \\ y+z & z+x & x+y \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$, we get

$$\text{L.H.S.} = 2\begin{vmatrix} x & y & z \\ z+x & x+y & y+z \\ y+z & z+x & x+y \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, we get

$$\text{L.H.S.} = 2\begin{vmatrix} x & y & z \\ z & x & y \\ y+z & z+x & x+y \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, we get

$$\text{L.H.S.} = 2\begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix} = \text{R.H.S.}$$

Question 3.

Using properties of determinant, show that

i. | | | | a+babaa+ccbcb+c | | | | =4abc

ii. | | | | | 1logyxlog2xlogxy1logxylogxzlogyz1 | | | | | =0

Solution:

i. L.H.S. =

Applying $C_1 \to C_1 - (C_2 + C_3)$, we get

L.H.S. = | | | | 0−2c−2caa+ccbcb+c | | | |

Taking (-2) common from C_1, we get

L.H.S. = -2 | | | | 0ccaa+ccbcb+c | | | |

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get

L.H.S. = -2 | | | | 0ccaa0b0b | | | |

= -2[0(ab – 0) – a(bc – 0) + b(0 – ac)]

= -2(0 – abc – abc)

= -2(-2abc)

= 4abc = R.H.S.

ii.

$$\text{L.H.S.} = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \dfrac{\log_e x}{\log_e x} & \dfrac{\log_e y}{\log_e x} & \dfrac{\log_e z}{\log_e x} \\ \dfrac{\log_e x}{\log_e y} & \dfrac{\log_e y}{\log_e y} & \dfrac{\log_e z}{\log_e y} \\ \dfrac{\log_e x}{\log_e z} & \dfrac{\log_e y}{\log_e z} & \dfrac{\log_e z}{\log_e z} \end{vmatrix}$$

$$\ldots\left[\because \log_c b = \frac{\log_e b}{\log_e c}\right]$$

Taking $\frac{1}{\log_e x}, \frac{1}{\log_e y}, \frac{1}{\log_e z}$ common from R_1, R_2, R_3 respectively, we get

L.H.S.

$$= \frac{1}{\log_e x . \log_e y . \log_e z} \begin{vmatrix} \log_e x & \log_e y & \log_e z \\ \log_e x & \log_e y & \log_e z \\ \log_e x & \log_e y & \log_e z \end{vmatrix}$$

$$= \frac{1}{\log_e x . \log_e y . \log_e z} (0)$$

$$\ldots[\because R_1, R_2, R_3 \text{ are identical}]$$

$= 0 = $ R.H.S.

Question 4.
Solve the following equations.

i. ∣∣∣∣ x+2x+6x−1x+6x−1x+2x−1x+2x+6 ∣∣∣∣ =0

ii. Solution:i.[latex ∣∣∣∣ x+2x+6x−1x+6x−1x+2x−1x+2x+6 ∣∣∣∣ =0

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we get

∣∣∣∣ x+24−3x+6−7−4x−137 ∣∣∣∣ =0

∴ (x + 2)(- 49 + 12) – (x + 6)(28 + 9) + (x- 1)(- 16 – 21) = 0
∴ (x + 2) (-37) – (x + 6) (37) + (x – 1) (-37) = 0
∴ -37(x + 2+ x + 6 + x – 1) = 0
∴ 3x + 7 = 0
∴ x = −73

ii. ∣∣∣∣ x−100xx−20x−2x−3x−3 ∣∣∣∣ =0

Applying $R_2 \rightarrow R_2 - R_3$, we get

∣∣∣∣ x−100xx−20x−20x−3 ∣∣∣∣ =0

∴ (x – 1)(x – 2)(x – 3) – 0] – x(0 – 0) + (x – 2)(0 – 0) =
∴ (x – 1)(x – 2)(x – 3) = 0
∴ x — 1 = 0 or x-2 = 0 or x-3 = 0
∴ x = 1 or x = 2 or x = 3

Question 5.
If ∣∣∣∣ 4+x4−x4−x4−x4+x4−x4−x4−x4+x ∣∣∣∣ = 0, then find the values of x.
Solution:

$$\begin{vmatrix} 4+x & 4-x & 4-x \\ 4-x & 4+x & 4-x \\ 4-x & 4-x & 4+x \end{vmatrix} = 0$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 12-x & 4-x & 4-x \\ 12-x & 4+x & 4-x \\ 12-x & 4-x & 4+x \end{vmatrix} = 0$$

Taking $(12 - x)$ common from C_1, we get

$$(12-x)\begin{vmatrix} 1 & 4-x & 4-x \\ 1 & 4+x & 4-x \\ 1 & 4-x & 4+x \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$(12-x)\begin{vmatrix} 1 & 4-x & 4-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$$

$(12 - x)[1(4x^2 - 0) - (4 - x)(0 - 0) + (4 - x)(0 - 0)] = 0$

$\therefore (12 - x)(4x^2) = 0$

$\therefore x^2(12 - x) = 0$

$\therefore x = 0$ or $12 - x = 0$

$\therefore x = 0$ or $x = 12$

Question 6.

Without expanding determinant, show that

∣∣∣∣ 1633176412 ∣∣∣∣ +4 ∣∣∣∣ 221317326 ∣∣∣∣ =10 ∣∣∣∣ 133212176 ∣∣∣∣

Solution:

L.H.S. = ∣∣∣∣ 1633176412 ∣∣∣∣ +4 ∣∣∣∣ 221317326 ∣∣∣∣

In 1st determinant, taking 2 common from C_3 we get

$$\text{L.H.S.} = \begin{vmatrix} 1 & 3 & 6 \\ 6 & 1 & 4 \\ 3 & 7 & 12 \end{vmatrix} + 4\begin{vmatrix} 2 & 3 & 3 \\ 2 & 1 & 2 \\ 1 & 7 & 6 \end{vmatrix}$$

In 1st determinant, taking 2 common from C_3, we get

$$\text{L.H.S.} = 2\begin{vmatrix} 1 & 3 & 3 \\ 6 & 1 & 2 \\ 3 & 7 & 6 \end{vmatrix} + 4\begin{vmatrix} 2 & 3 & 3 \\ 2 & 1 & 2 \\ 1 & 7 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 & 3 \\ 12 & 1 & 2 \\ 6 & 7 & 6 \end{vmatrix} + \begin{vmatrix} 8 & 3 & 3 \\ 8 & 1 & 2 \\ 4 & 7 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 2+8 & 3 & 3 \\ 12+8 & 1 & 2 \\ 6+4 & 7 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 10 & 3 & 3 \\ 20 & 1 & 2 \\ 10 & 7 & 6 \end{vmatrix}$$

Interchanging rows and columns, we get

L.H.S. = ∣∣∣∣ 103320121076 ∣∣∣∣

Taking 10 common from R_1, we get

L.H.S = 10 ∣∣∣∣ 133212176 ∣∣∣∣ = R.H.S.

Maharashtra State Board 11th Maths Solutions Chapter 4 Determinants and Matrices Ex 4.3

Question 1.
Solve the following linear equations by using Cramer's Rule.
x+y + z = 6, x – y + z = 2,.x + 2y – z = 2
x + y – 2z = -10,
2x +y – 3z = -19, 4x + 6y + z = 2
x + z = 1, y + z = 1, x + y = 4
$-2x-1y-3z$ = 3, $2x-3y+1z$ = -13 and $2x-3z$ = -11
Solution:
Given equations are
x + y + z = 6,
x – y + z = 2,
x + 2y – z = 2.
D = ∣∣∣∣ $1111-1211-1$ ∣∣∣∣
1 2 -1 = 1(1 -2) – 1(-1 – 1) + 1(2 + 1)
= 1 (-1)-1 (-2)+ 1(3)
= -1 + 2 + 3
= 4 ≠ 0

D_x = ∣∣∣∣ $6221-1211-1$ ∣∣∣∣
= 6(1 – 2) – 1(-2 – 2) + 1(4 + 2)
= 6(-1) -1 (-4) + 1(6)
= -6 + 4 + 6
= 4

D_y = ∣∣∣∣ $11162211-1$ ∣∣∣∣
= 1(-2 – 2) – 6(-l – 1) + 1(2 = 1 (- 4) – 6 (- 2) + 1(0)
= -4+12 + 0 = 8

D_z = ∣∣∣∣ $1111-12622$ ∣∣∣∣
= l(-2 – 4) – 1(2 – 2) + 6(2 + 1)
= l(-6)-l(0) + 6(3)
= -6 + 0+18 = 12

By Cramer's Rule,
$$x = \frac{D_x}{D} = \frac{4}{4} = 1, \ y = \frac{D_y}{D} = \frac{8}{4} = 2 \text{ and}$$
$$z = \frac{D_z}{D} = \frac{12}{4} = 3$$
∴ x = 1, y = 2 and z = 3 are the solutions of the given equations.

ii. Given equations are x+y- 2z = -10,
2x + y – 3z = -19,
Ax + 6y + z = 2.
D = ∣∣∣∣ $124116-2-31$ ∣∣∣∣
= 1(1 + 18)- 1(2+ 12)-2(12-4)
= 1(19)-1(14)-2(8)
= 19-14-16 = -11 ≠ 0

D_x = ∣∣∣∣ $-10-192116-2-31$ ∣∣∣∣
= -10(1 + 18) – 1(-19 + 6) – 2(- 114 – 2)
= -10(19)- 1(-13) -2(-l 16)
= -190+ 13 + 232 = 55

D_y = ∣∣∣∣ $124-10-192-2-31$ ∣∣∣∣
= 1(-19 + 6) – (-10)(2 + 12) – 2(4 + 76) = 1(-13) + 10(14) – 2(80)
= -13 + 140-160 = -33

D_z = ∣∣∣∣ 124116-10-192 ∣∣∣∣
= 1(2+ 114)-1(4+ 76)-10(12-4)
= 1(116)-1(80)-10(8)
= 116-80-80 .
= -44
By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{55}{-11} = -5,\ y = \frac{D_y}{D} = \frac{-33}{-11} = 3,$$

$$z = \frac{D_z}{D} = \frac{-44}{-11} = 4$$

∴ x = -5, y = 3 and z = 4 are the solutions of the given equations.
[Note: The question has been modified]

iii. Given equations are
x + z = 1, i.e.,x + 0y + z = 1,
y + z = 1, i.e., 0x + y + z = 1,
x + y = 4, i.e., x + y + 0z = 4.
D = ∣∣∣∣ 101011110 ∣∣∣∣
= 1(0 – 1) – 0 + 1(0 – 1)
= 1(-1)+1(-1)
= -1-1 = -2 ≠ 0
D_x = ∣∣∣∣ 101011110 ∣∣∣∣
= 1(0 – 1) – 0 + 1(1 -4) = l(-l)+l(-3)
= -1 – 3
= -4

D_y = ∣∣∣∣ 101114110 ∣∣∣∣
= 1(0 – 4) – 1(0 – 1) + 1(0 – 1)
= 1(-4) – 1(-1) + 1(-1)
= -4 + 1 – 1
= -4

D_z = ∣∣∣∣ 101011114 ∣∣∣∣
= 1(4 – 1) – 0 + 1(0 – 1)
= 1(3) + 1(-1)
= 3 – 1
= 2

By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{-4}{-2} = 2,\ y = \frac{D_y}{D} = \frac{-4}{-2} = 2,$$

$$z = \frac{D_z}{D} = \frac{2}{-2} = -1$$

∴ x = 2, y = 2 and z = -1 are the solutions of the given equations.

Let $1x$ = p, $1y$ = q, $1z$ = r
∴ The given equations become
-2p – q – 3r = 3, i.e., 2p + q + 3r = -3,
2p-3q + r = -13,
2p – 3r = -11, i.e., 2p + 0q – 3r = -11.
D = ∣∣∣∣ 2221-3031-3 ∣∣∣∣
= 2(9 – 0) – 1(-6 – 2) + 3(0 + 6)
= 2(9) – 1(-8) + 3(6)
= 18 + 8 + 18
= 44 ≠ 0

D_P = ∣∣∣∣ -3-13-111-3031-3 ∣∣∣∣
= -3(9 – 0) – 1(39 + 11) + 3(0 – 33)
= -3(9) – 1(50) + 3(-33)
= -27 – 50 – 99
= -176

D_q = ∣∣∣∣ 222-3-13-1131-3 ∣∣∣∣
= 2(39 + 11)- (-3)(-6 – 2) + 3(-22 + 26)
= 2(50) + 3(-8) + 3(4)
= 100 – 24 + 12
= 88

D_r = ∣∣∣∣ 2221-30-3-13-11 ∣∣∣∣
= 2(33 – 0) – 1(-22 + 26) – 3(0 + 6)
= 2(33) – 1(4) – 3(6)
= 66 – 4 – 18
= 44

By Cramer's Rule,

$$p = \frac{D_p}{D} = \frac{-176}{44} = -4, \; q = \frac{D_q}{D} = \frac{88}{44} = 2,$$

$$r = \frac{D_r}{D} = \frac{44}{44} = 1$$

$$\therefore \quad \frac{1}{x} = -4, \; \frac{1}{y} = 2, \; \frac{1}{z} = 1$$

∴ x = /-14, y = 12 z = 1 are the solutions of the given equations.

Question 2.
The sum of three numbers is 15. If the second number is subtracted from the sum of first and third numbers, then we get 5. When the third number is subtracted from the sum of twice the first number and the second number, we get 4. Find the three numbers.
Solution:
Let the three numbers be x, y and z.
According to the given conditions, x + y + z = 15,
x + z-y = 5, i.e., x – y + z = 5,
2x + y – z = 4.
D = ∣∣∣∣ 1121-1111-1 ∣∣∣∣
= 1(1 – 1) – 1(-1 – 2) + 1(1 + 2)
= 1(0) – 1(-3) + 1(3)
= 0 + 3 + 3
= 6 ≠ 0

D_x = ∣∣∣∣ 15541-1111-1 ∣∣∣∣
= 15(1 – 1) – 1(-5 – 4) + 1(5 + 4)
= 15(0) – 1(-9) + 1(9)
= 0 + 9 + 9
= 18

D_y = ∣∣∣∣ 112155411-1 ∣∣∣∣
= 1(-5 – 4) – 15(-1 – 2) + 1(4 – 10)
= 1(-9) – 15(-3) + 1(-6)
= -9 + 45 – 6 = 30

D_z = ∣∣∣∣ 1121-111554 ∣∣∣∣
= 1(-4 – 5) – 1(4 – 10) + 15(1 + 2)
= 1(-9) – 1(-6) + 15(3)
= -9 + 6 + 45
= 42
By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{18}{6} = 3, \; y = \frac{D_y}{D} = \frac{30}{6} = 5,$$

$$z = \frac{D_z}{D} = \frac{42}{6} = 7$$

∴ The three numbers are 3, 5 and 7.

Question 3.
Examine the consistency of the following equations.
i. 2x – y + 3 = 0, 3x + y – 2 = 0, 11x + 2y – 3 = 0
ii. 2x + 3y – 4 = 0, x + 2y = 3, 3x + 4y + 5 = 0
iii. x + 2y – 3 = 0,7x + 4y – 11 = 0,2x + 4y – 6 = 0

Solution:
i. Given equations are 2x – y + 3 = 0,
3x + y – 2 = 0,
11x + 2y – 3 = 0.
D = ∣∣∣∣ 2311-1123-2-3 ∣∣∣∣
= 2(-3 + 4) – (-l)(-9 + 22) + 3(6-11)
= 2(1)+1(13)+ 3(-5)
= 2 + 13-15 = 0
∴ The given equations are consistent.

ii. Given equations are 2x + 3y – 4 = 0,
x + 2y = 3, i.e., x + 2y – 3 = 0,
3x + 4y + 5 = 0.
∣∣∣∣ 213324-4-35 ∣∣∣∣
= 2(10 + 12) – 3(5 + 9) – 4(4 – 6)
= 2 (22) – 3(14) – 4(-2)
= 44 – 42 + 8
= 10 ≠ 0
∴ The given equations are not consistent.

iii. Given equations are x + 2y – 3 =
7x + 4y – 11 =0,
2x + 4y – 6 = 0.
∣∣∣∣ 172244-3-11-6 ∣∣∣∣
= 1(-24 + 44) – 2(-42 + 22) – 3(28 – 8)
= 1(20) – 2(-20) – 3(20)
= 20 + 40 – 60
= 0
∴ The given equations are consistent.

Question 4.
Find k, if the following equations are consistent.
i. 2x + 3y-2 = 0,2x + 4y-k = 0,x-2j + 3k = 0
ii. kx + 3,y +1 = 0, x + 2y+1 = 0, x + y = 0
Solution:
i. Given equations are 2x + 3y – 2 = 0,
2x + 4y – k = 0,
x – 2y + 3k = 0.
Since these equations are consistent,
∣∣∣∣ 22134-2-2-k3k ∣∣∣∣ = 0
∴ 2(12k – 2k) – 3(6k + k) – 2(- 4 – 4) = 0
∴ 2(10k) – 3(7k) – 2(- 8) = 0
∴ 20k – 21k + 16 = 0
∴ k = 16

Given equations are are
kx + 3y + 1 = 0,
x + 2y +1=0,
x + y = 0, i.e., x + y + 0 = 0.
Since these equations are consistent,
∣∣∣∣ k11321110 ∣∣∣∣ = 0
∴ k(0 – 1) – 3(0 – 1) + 1(1 – 2) = 0
∴ k(-1) – 3(-1) + 1(-1) = 0
∴ -k + 3 – 1 = 0
∴ k = 2.

Question 5.
Find the area of triangle whose vertices are
i. A (5,8), B (5,0), C (1,0)
ii. P(3/2, 1), Q(4,2), R(4, -1/2)
iii. M (0, 5), N (- 2, 3), T (1, – 4)
Solution:
i. Here, A(x_1, y_1) ≡ A(5, 8), B(x_2, y_2) = B(5, 0), C(x_3, y_3) = C(1,0)
Area of a triangle = 12 ∣∣∣∣ $x_1x_2x_3y_1y_2y_3$111 ∣∣∣∣
A(ΔABC) = 12 ∣∣∣∣ 551800111 ∣∣∣∣

= 12[5(0 – 0) – 8(5 – 1) + 1(0 – 0)]
= 12[0 – 8(4) + 0]
= 12(-32)
= -16
Since area cannot be negative,
A(ΔABC) = 16 sq. units

ii. Here, P(x_1, y_1) ≡ P(3/2, 1), Q(x_2, y_2) ≡ Q(4, 2), R(x_3, y_3) ≡ R(4,-12)

$$\text{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore \quad A(\Delta PQR) = \frac{1}{2}\begin{vmatrix} \frac{3}{2} & 1 & 1 \\ 4 & 2 & 1 \\ 4 & \frac{-1}{2} & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[\frac{3}{2}\left(2+\frac{1}{2}\right)-1(4-4)+1(-2-8)\right]$$

$$= \frac{1}{2}\left[\frac{3}{2}\left(\frac{5}{2}\right)-1(0)+1(-10)\right]$$

$$= \frac{1}{2}\left(\frac{15}{4}-10\right)$$

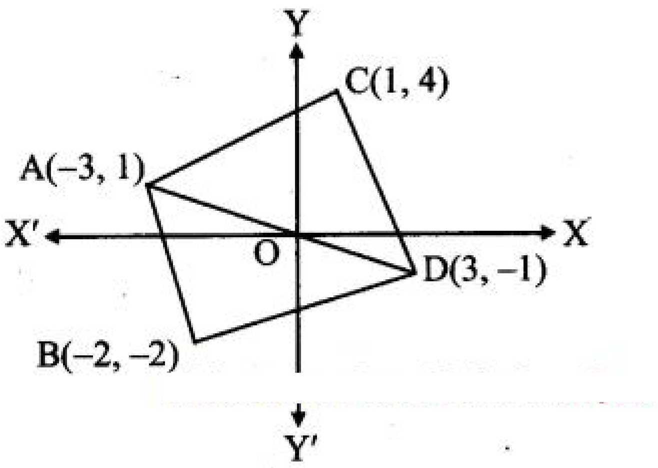
$$= \frac{1}{2}\left(\frac{-25}{4}\right)$$

$$= \frac{-25}{8}$$

Since area cannot be negative
A(ΔPQR) = 25/8 sq. units

iii. Here, M(x_1, y_1) ≡ M(0, 5), N(x_2, y_2) ≡ N(-2, 3)
T(x_3, y_3) ≡ T(1, -4)
Area of a triangle = 12 ∣∣∣∣ x1x2x3y1y2y3111 ∣∣∣∣
∴ A(ΔMNT) = 12 ∣∣∣∣ 0-2153-4111 ∣∣∣∣
= 12 [0 – 5 (-2 -1) + 1 (8 – 3)]
= 12[-5 (-3) + 1(5)]
= 12 (15 + 5)
= 12 (20)
= 10 sq. units

Question 6.
Find the area of quadrilateral whose vertices are A (- 3,1), B (- 2, – 2), C (1,4), D (3, – 1).
Solution:
A(-3, 1), B(-2, -2), C(l, 4), D(3, -1)



A(□ ABDC) = A(ΔABD) + A(ΔADC)
Area of triangle = 12 ∣∣∣∣ x1x2x3y1y2y3111 ∣∣∣∣
A(ΔABD) = 12 ∣∣∣∣ -3-231-2-1111 ∣∣∣∣
= 12 [-3 (-2 + 1) – 1(-2 – 3) + 1(2 + 6)
= 12 [-3(-1) – 1(-5) + 1(8)]

= 12 (3 + 5 + 8)
= 12 (16)
∴ A(ΔABD) = 8 sq. units
A(ΔADC) = 12 ∣∣∣∣ −3311−14111 ∣∣∣∣
= 12 [-3(-1-4) – 1(3 – 1) + 1(12 + 1)]
= 12 [-3(-5) – 1(2) + 1(13)]
= 12 [15 – 2 + 13]
= 12 (26)
∴ A(ΔADC) = 13 sq. units
∴ A(□ ABDC) = A(ΔABD) + A(ΔADC)
= 8 + 13
= 21 sq. units

Question 7.
Find the value of k, if the area of triangle whose vertices are P (k, 0), Q (2,2), R (4,3) is 32 sq. units.
Solution:
Here, P(x_1, y_1) ≡ P(k, 0), Q(x_2, y_2) ≡ Q(2, 2), R(x_3, y_3) ≡ R(4,3)
∴ A(ΔPQR) = 32 sq. units
Area if triangle = 12 ∣∣∣∣ $x_1x_2x_3y_1y_2y_3$111 ∣∣∣∣
∴ ±32=12 ∣∣∣∣ k24023111 ∣∣∣∣
∴ ± [k(2−3)−0+1(6−8)]∴±[latex32=12 (-k -2)
∴ ± 3 = -k – 2
∴ 3 = -k – 2 or -3 = -k – 2
∴ k = -5 or k = 1

Question 8.
Examine the collinearity of the following set of points:
i. A (3, – 1), B (0, – 3), C (12, 5)
ii. P (3, – 5), Q (6,1), R (4, 2)
iii. L(0,1/2), M(2,-1), N(-4, 7/2)
Solution:
i. Here, A(x_1, y_1) ≡ A(3, -1), B(x_2, y_2) ≡ B(0, -3), C(x_3, y_3) ≡ C(12, 5)
If A(ΔABC) = 0, then the points A, B, C are collinear.
∴ A(ΔABC) = 12 ∣∣∣∣ 3012−1−35111 ∣∣∣∣
= 12[3(-3 – 5) – (-1) (0 – 12) + 1(0 + 36)]
= 12[3(-8)+ 1(-12)+ 1(36)]
= 12(-24 – 12 + 36)
= 0
∴ The points A, B, C are collinear.

ii. Here, P(x_1, y_1) ≡ P(3, -5), Q(x_2, y_2) ≡ Q(6, 1), R(x_3, y_3) ≡ R(4,2)
∴ If A(ΔPQR) = 0, then the points P,Q, R are collinear
∴ A(ΔPQR) = 12 ∣∣∣∣ 364−512111 ∣∣∣∣
= 12 [3(1-2) – (-5)(6 – 4) + 1(12 – 4)]
= 12 [3(-1) + 5(2) + 1(8)]
= 12(-3 + 10 + 8)= 152 ≠ 0
∴ The points P, Q, R are non-collinear.

iii. Here, L(x_1, y_1) ≡ L(0,1/2), M(x_2, y_2) ≡ M(2, -1), N(x_3, y_3) ≡ N(-4, 7/2)
If A(ΔLMN) = 0, then the points L, M, N are collinear.
∴ A(ΔLMN) = 12 ∣∣∣∣∣ 02−412−172111 ∣∣∣∣∣
= 12 [0 –]12 (2 + 4) + 1(7 – 4)]
= 12[−12 (6) + 1(3)]
= 12 (-3 + 3) = 0
∴ The points L, M, N are collinear.

Maharashtra State Board 11th Maths Solutions Chapter 4 Determinants and Matrices Miscellaneous Exercise 4(A)

I. Select the correct option from the given alternatives.

Question 1.
The determinant D = $\begin{vmatrix} a & b & a+b \\ b & c & b+c \\ a+b & b+c & 0 \end{vmatrix}$ = 0, if
(a) a, b, c are in A.P.
(b) a, b, c are in G.P.
(c) a, b, c are in H.P.
(d) α is a root of $ax^2 + 2bx + c = 0$
Answer:
(b) a, b, c are in G.P.
Hint:
Applying $R_3 \to R_3 - (R_1 + R_2)$, we get
$\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ a+b & b+c & -(a+2b+c) \end{vmatrix} = 0$
∴ a[-c(a + 2b + c) – 0] – b[-b(a + 2b + c) – 0] + (a + b) (0 – 0) = 0
∴ $(-ac + b^2)$ (a + 2b + c) = 0
∴ $-ac + b^2 = 0$ or a + 2b + c = 0
∴ $b^2 = ac$
∴ a, b, c are in G.P.

Question 2.
If $\begin{vmatrix} x^k & x^{k+2} & x^{k+3} \\ y^k & y^{k+2} & y^{k+3} \\ z^k & z^{k+2} & z^{k+3} \end{vmatrix}$ = (x – y) (y – z) (z – x) $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ then
(a) k = -3
(b) k = -1
(c) k = 1
(d) k = 3
Answer:
(b) k = -1
Hint:

$$\begin{vmatrix} x^k & x^{k+2} & x^{k+3} \\ y^k & y^{k+2} & y^{k+3} \\ z^k & z^{k+2} & z^{k+3} \end{vmatrix} = x^k y^k z^k \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$$
$$= (xyz)^k (x-y)(y-z)(z-x)(xy+yz+zx)$$
$$= (xyz)^{k+1} (x-y)(y-z)(z-x)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$$
$$k + 1 = 0, \qquad \therefore \quad k = -1$$

Question 3.
Let D = $\begin{vmatrix} \sin\theta\cdot\cos\phi & \cos\theta\cdot\cos\phi & -\sin\theta\cdot\sin\phi \\ \sin\theta\cdot\sin\phi & \cos\theta\cdot\sin\phi & \sin\theta\cdot\cos\phi \\ \cos\theta & -\sin\theta & 0 \end{vmatrix}$ then
(a) D is independent of θ
(b) D is independent of φ
(c) D is a constant
(d) $\frac{dD}{d\theta}$ at θ = $\frac{\pi}{2}$ is equal to 0
Answer:
(b) D is independent of φ

Question 4.
The value of a for which the system of equations $a^3x + (a + 1)y + (a + 2)^3 z = 0$, ax + (a + 1)y + (a + 2)z = 0 and x + y + z = 0 has a non zero solution is
(a) 0
(b) -1
(c) 1
(d) 2
Answer:
(b) -1
Hint:

The given system of equations will have a non-zero solution, if

$$\begin{vmatrix} a^3 & (a+1)^3 & (a+2)^3 \\ a & a+1 & a+2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_2$, we get

$$\begin{vmatrix} a^3 & 3a^2+3a+1 & 3a^2+9a+7 \\ a & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$a^3(0-0) - (3a^2+3a+1)(0-1) + (3a^2+9a+7)(0-1) = 0$$

$$-6a - 6 = 0, \quad \therefore \quad a = -1$$

Question 5.
∣∣∣∣ b+cq+ry+zc+ar+pz+xa+bp+qx+y ∣∣∣∣ =
(a) 2 ∣∣∣∣ crzbqyapx ∣∣∣∣
(b) 2 ∣∣∣∣ bqyapxcrz ∣∣∣∣
(c) 2 ∣∣∣∣ apxbqycrz ∣∣∣∣
(d) 2 ∣∣∣∣ apxcrzbqy ∣∣∣∣
Answer:
(c) 2 ∣∣∣∣ apxbqycrz ∣∣∣∣
Hint:

$$\text{Let } D = \begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$D = \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(p+q+r) & r+p & p+q \\ 2(x+y+z) & z+x & x+y \end{vmatrix}$$

Taking 2 common from C_1, we get

$$D = 2\begin{vmatrix} a+b+c & c+a & a+b \\ p+q+r & r+p & p+q \\ x+y+z & z+x & x+y \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get

$$D = 2\begin{vmatrix} a+b+c & -b & -c \\ p+q+r & -q & -r \\ x+y+z & -y & -z \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$D = 2\begin{vmatrix} a & -b & -c \\ p & -q & -r \\ x & -y & -z \end{vmatrix}$$

Taking (−1) common from C_2 and C_3, we get

$$D = 2(-1)(-1)\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

Question 6.
The system $3x - y + 4z = 3$, $x + 2y - 3z = -2$ and $6x + 5y + \lambda z = -3$ has atleast one solution when
(a) $\lambda = -5$
(b) $\lambda = 5$
(c) $\lambda = 3$
(d) $\lambda = -13$
Answer:
(a) $\lambda = -5$
Hint:

The given system of equations will have more than one solution if

$$\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & \lambda \end{vmatrix} = 0$$

Applying $R_3 \to R_3 - (R_1 + 3R_2)$, we get

$$\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 0 & 0 & \lambda + 5 \end{vmatrix} = 0$$

$\therefore \quad 3[2(\lambda + 5) - 0] - (-1)(\lambda + 5 - 0) + 4(0 - 0) = 0$

$\therefore \quad 7\lambda + 35 = 0$

$\therefore \quad \lambda = -\frac{35}{7} = -5$

Question 7.
If x = -9 is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, has other two roots are
(a) 2, -7
(b) -2, 7
(c) 2, 7
(d) -2, -7
Answer:
(c) 2, 7
Hint:

$$\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

Applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\begin{vmatrix} x+9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

Taking $(x + 9)$ common from R_1, we get

$$(x+9)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

$(x + 9)[1(x^2 - 12) - 1(2x - 14) + 1(12 - 7x)] = 0$
$(x + 9)(x^2 - 9x + 14) = 0$
$(x + 9)(x - 7)(x - 2) = 0$
The other two roots are $x = 2$ and $x = 7$.

Question 8.
If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$ = x + iy, then
(a) x = 3, y = 1
(b) x = 1, y = 3
(c) x = 0, y = 3
(d) x = 0, y = 0
Answer:
(d) x = 0, y = 0

Question 9.
If A(0, 0), B(1, 3) and C(k, 0) are vertices of triangle ABC whose area is 3 sq.units, then the value of k is
(a) 2
(b) -3
(c) 3 or -3
(d) -2 or 2
Answer:
(d) -2 or 2

Question 10.
Which of the following is correct?
(a) Determinant is a square matrix
(b) Determinant is number associated to matrix
(c) Determinant is a number associated with a square matrix
(d) None of these

Answer:
(c) Determinant is a number associated with a square matrix

II. Answer the following questions.

Question 1.
Evaluate:
(i) ∣∣∣∣ 259−520712 ∣∣∣∣
(ii) ∣∣∣∣ 109−32712−42 ∣∣∣∣
Solution:
(i) ∣∣∣∣ 259−520712 ∣∣∣∣

$$\begin{vmatrix} 2 & -5 & 7 \\ 5 & 2 & 1 \\ 9 & 0 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} - (-5)\begin{vmatrix} 5 & 1 \\ 9 & 2 \end{vmatrix} + 7\begin{vmatrix} 5 & 2 \\ 9 & 0 \end{vmatrix}$$

= 2(4 – 0) + 5(10 – 9) + 7(0 – 18)
= 2(4) + 5(1) + 7(-18)
= 8 + 5 – 126
= -113

(ii) ∣∣∣∣ 109−32712−42 ∣∣∣∣

$$\begin{vmatrix} 1 & -3 & 12 \\ 0 & 2 & -4 \\ 9 & 7 & 2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & -4 \\ 7 & 2 \end{vmatrix} - (-3)\begin{vmatrix} 0 & -4 \\ 9 & 2 \end{vmatrix} + 12\begin{vmatrix} 0 & 2 \\ 9 & 7 \end{vmatrix}$$

= 1(4 + 28) + 3(0 + 36) + 12(0 – 18)
= 1(32) + 3(36) + 12(-18)
= 32 + 108 – 216
= -76

Question 2.
Evaluate determinant along second column ∣∣∣∣ 130−1212−2−2 ∣∣∣∣
Solution:

$$\text{Here, } \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 2 & -2 \\ 0 & 1 & -2 \end{vmatrix}$$

Expansion along the second column

$$= a_{12} C_{12} + a_{22} C_{22} + a_{32} C_{32}$$

$$= (-1) \times (-1)^{1+2} \begin{vmatrix} 3 & -2 \\ 0 & -2 \end{vmatrix}$$

$$+ (2) \times (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} + (1) \times (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix}$$

$$= 1(-6-0) + 2(-2-0) - 1(-2-6)$$
$$= 1(-6) + 2(-2) - 1(-8)$$
$$= -6 - 4 + 8$$
$$= -2$$

Question 3.
Evaluate:
(i) ∣∣∣∣ 240048360047510018 ∣∣∣∣
(ii) ∣∣∣∣ 101106110210721031083 ∣∣∣∣
by using properties.
Solution:

i. Let D = $\begin{vmatrix} 2 & 3 & 5 \\ 400 & 600 & 1000 \\ 48 & 47 & 18 \end{vmatrix}$

Taking 200 common from R_2, we get

$D = 200 \begin{vmatrix} 2 & 3 & 5 \\ 2 & 3 & 5 \\ 48 & 47 & 18 \end{vmatrix}$

$= 200(0)$ …[$\because R_1$ and R_2 are identical]

$= 0$

ii. Let D = $\begin{vmatrix} 101 & 102 & 103 \\ 106 & 107 & 108 \\ 1 & 2 & 3 \end{vmatrix}$

Applying $C_3 \to C_3 - C_2$, we get

$D = \begin{vmatrix} 101 & 102 & 1 \\ 106 & 107 & 1 \\ 1 & 2 & 1 \end{vmatrix}$

Applying $C_2 \to C_2 - C_1$, we get

$D = \begin{vmatrix} 101 & 1 & 1 \\ 106 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$

$= 0$ …[$\because C_2$ and C_3 are identical]

Question 4.
Find the minors and cofactors of elements of the determinants.

(i) ∣∣∣∣−1−2001−4432∣∣∣∣

(ii) ∣∣∣∣131−1002−23∣∣∣∣

Solution:

i. Here, $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} -1 & 0 & 4 \\ -2 & 1 & 3 \\ 0 & -4 & 2 \end{vmatrix}$

$M_{11} = \begin{vmatrix} 1 & 3 \\ -4 & 2 \end{vmatrix} = 2 + 12 = 14$

$\therefore$ $C_{11} = (-1)^{1+1} M_{11} = 1(14) = 14$

$M_{12} = \begin{vmatrix} -2 & 3 \\ 0 & 2 \end{vmatrix} = -4 - 0 = -4$

$\therefore$ $C_{12} = (-1)^{1+2} M_{12} = (-1)(-4) = 4$

$M_{13} = \begin{vmatrix} -2 & 1 \\ 0 & -4 \end{vmatrix} = 8 - 0 = 8$

$\therefore$ $C_{13} = (-1)^{1+3} M_{13} = 1(8) = 8$

$M_{21} = \begin{vmatrix} 0 & 4 \\ -4 & 2 \end{vmatrix} = 0 + 16 = 16$

$\therefore$ $C_{21} = (-1)^{2+1} M_{21} = (-1)(16) = -16$

$M_{22} = \begin{vmatrix} -1 & 4 \\ 0 & 2 \end{vmatrix} = -2 - 0 = -2$

$\therefore \quad C_{22} = (-1)^{2+2} M_{22} = 1(-2) = -2$

$M_{23} = \begin{vmatrix} -1 & 0 \\ 0 & -4 \end{vmatrix} = 4 - 0 = 4$

$\therefore \quad C_{23} = (-1)^{2+3} M_{23} = (-1)(4) = -4$

$M_{31} = \begin{vmatrix} 0 & 4 \\ 1 & 3 \end{vmatrix} = 0 - 4 = -4$

$\therefore \quad C_{31} = (-1)^{3+1} M_{31} = 1(-4) = -4$

$M_{32} = \begin{vmatrix} -1 & 4 \\ -2 & 3 \end{vmatrix} = -3 + 8 = 5$

$\therefore \quad C_{32} = (-1)^{3+2} M_{32} = (-1)(5) = -5$

$M_{33} = \begin{vmatrix} -1 & 0 \\ -2 & 1 \end{vmatrix} = -1 + 0 = -1$

$\therefore \quad C_{33} = (-1)^{3+3} M_{33} = 1(-1) = -1$

ii. Here, $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$

$M_{11} = \begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = 0 - 0 = 0$

$\therefore \quad C_{11} = (-1)^{1+1} M_{11} = 1(0) = 0$

$M_{12} = \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = 9 + 2 = 11$

$\therefore \quad C_{12} = (-1)^{1+2} M_{12} = (-1)(11) = -11$

$M_{13} = \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0$

$\therefore \quad C_{13} = (-1)^{1+3} M_{13} = 1(0) = 0$

$M_{21} = \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -3 - 0 = -3$

$\therefore \quad C_{21} = (-1)^{2+1} M_{21} = (-1)(-3) = 3$

$M_{22} = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 3 - 2 = 1$

$\therefore \quad C_{22} = (-1)^{2+2} M_{22} = 1(1) = 1$

$M_{23} = \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 0 + 1 = 1$

$\therefore \quad C_{23} = (-1)^{2+3} M_{23} = (-1)(1) = -1$

$M_{31} = \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 2 - 0 = 2$

$\therefore \quad C_{31} = (-1)^{3+1} M_{31} = 1(2) = 2$

$M_{32} = \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -2 - 6 = -8$

$\therefore \quad C_{32} = (-1)^{3+2} M_{32} = (-1)(-8) = 8$

$M_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 0 + 3 = 3$

$\therefore \quad C_{33} = (-1)^{3+3} M_{33} = 1(3) = 3$

Question 5.
Find the values of x, if
(i) $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$
(ii) $\begin{vmatrix} 1 & 2x & 4x \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix} = 0$
Solution:
(i) $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$
⇒ $1(-10x^2 + 10x) – 4(5x^2 + 5) + 20(2x + 2) = 0$
⇒ $-10x^2 + 10x – 20x^2 – 20 + 40x + 40 = 0$
⇒ $-30x^2 + 50x + 20 = 0$
⇒ $3x^2 – 5x – 2 = 0$ …..[Dividing throughout by (-10)]
⇒ $3x^2 – 6x + x – 2 = 0$
⇒ $3x(x – 2) + 1(x – 2) = 0$

⇒ (x – 2) (3x + 1) = 0
⇒ x – 2 = 0 or 3x + 1 = 0
⇒ x = 2 or x = $-\frac{1}{3}$

(ii) $\begin{vmatrix} 1 & 2x & 4x \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix} = 0$
⇒ 1(4 – 16) – 2x(1 – 16) + 4x(1 – 4) = 0
⇒ 1(-12) – 2x(-15) + 4x(-3) = 0
⇒ -12 + 30x – 12x = 0
⇒ 18x = 12
⇒ x = $\frac{12}{18} = \frac{2}{3}$

Question 6.
By using properties of determinant, prove that $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0$
Solution:

$$\text{L.H.S.} = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2$, we get

$$\text{L.H.S.} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

Taking $(x+y+z)$ common from R_1, we get

$$\text{L.H.S.} = (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$= (x+y+z)(0)$
…[∵ R_1 and R_3 are identical]
$= 0 = \text{R.H.S.}$

Question 7.
Without expanding the determinants, show that

i) $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix} = 0$ ii) $\begin{vmatrix} xa & yb & zc \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix}$

$= \begin{vmatrix} x & y & z \\ a & b & c \\ bc & ca & ab \end{vmatrix}$

iii) $\begin{vmatrix} l & m & n \\ e & d & f \\ u & v & w \end{vmatrix} = \begin{vmatrix} n & f & w \\ l & e & u \\ m & d & v \end{vmatrix}$

iv) $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0$

Solution:

i. $\text{L.H.S.} = \begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix}$

Taking bc, ca, ab common from R_1, R_2, R_3 respectively, we get

$$\text{L.H.S.} = (bc)(ca)(ab)\begin{vmatrix} \frac{b+c}{bc} & 1 & bc \\ \frac{c+a}{ca} & 1 & ca \\ \frac{a+b}{ab} & 1 & ab \end{vmatrix}$$

Taking abc common from C_3, we get

$$\text{L.H.S.} = (a^2b^2c^2)\,(abc)\begin{vmatrix} \frac{1}{c}+\frac{1}{b} & 1 & \frac{1}{a} \\ \frac{1}{a}+\frac{1}{c} & 1 & \frac{1}{b} \\ \frac{1}{b}+\frac{1}{a} & 1 & \frac{1}{c} \end{vmatrix}$$

Applying $C_1 \to C_1 + C_3$, we get

$$\text{L.H.S.} = a^3b^3c^3\begin{vmatrix} \frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1 & \frac{1}{a} \\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1 & \frac{1}{b} \\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1 & \frac{1}{c} \end{vmatrix}$$

Taking $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ common from C_1, we get

$$\text{L.H.S.} = a^3b^3c^3\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & \frac{1}{a} \\ 1 & 1 & \frac{1}{b} \\ 1 & 1 & \frac{1}{c} \end{vmatrix}$$

$$= a^3b^3c^3\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)(0)$$

…[$\because$ C_1 and C_2 are identical]

$= 0 = \text{R.H.S.}$

ii. $\text{L.H.S.} = \begin{vmatrix} xa & yb & zc \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix}$

Taking a, b, c common from C_1, C_2, C_3 respectively, we get

$$\text{L.H.S.} = abc\begin{vmatrix} x & y & z \\ a & b & c \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \end{vmatrix}$$

$$= \begin{vmatrix} x & y & z \\ a & b & c \\ \frac{abc}{a} & \frac{abc}{b} & \frac{abc}{c} \end{vmatrix}$$

$$= \begin{vmatrix} x & y & z \\ a & b & c \\ bc & ca & ab \end{vmatrix} = \text{R.H.S.}$$

iii. $\text{L.H.S.} = \begin{vmatrix} l & m & n \\ e & d & f \\ u & v & w \end{vmatrix}$

Interchanging rows and columns, we get

$\text{L.H.S.} = \begin{vmatrix} l & e & u \\ m & d & v \\ n & f & w \end{vmatrix}$

Applying $R_2 \leftrightarrow R_3$, we get

$\text{L.H.S.} = -\begin{vmatrix} l & e & u \\ n & f & w \\ m & d & v \end{vmatrix}$

Applying $R_1 \leftrightarrow R_2$, we get

$\text{L.H.S.} = \begin{vmatrix} n & f & w \\ l & e & u \\ m & d & v \end{vmatrix}$

$= \text{R.H.S.}$

iv. Let $D = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$

Taking (-1) common from R_1, R_2, R_3, we get

$D = (-1)^3 \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$

Interchanging rows and columns, we get

$D = -1 \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$

$\therefore$ $D = -1(D)$

$\therefore$ $2D = 0$

$\therefore$ $D = 0$

$\therefore$ $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0$

Question 8.

If $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$ then show that $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$

Solution:

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix} = 0$$

$\therefore \quad a[(b-1)(c-1)-0] - 1[(1-a)(c-1)-0] + 1[0-(b-1)(1-a)] = 0$

$\therefore \quad a(1-b)(1-c) + (1-a)(1-c) + (1-b)(1-a) = 0$

Dividing throughout by $(1-a)(1-b)(1-c)$, we get

$$\frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

Adding 1 on both the sides, we get

$$1 + \frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

$$\therefore \quad \frac{1-a+a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

$$\therefore \quad \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

Question 9.
Solve the following linear equations by Cramer's Rule.
(i) 2x – y + z = 1, x + 2y + 3z = 8, 3x + y – 4z = 1
(ii) 1x+1y=32,1y+1z=56,1z+1x=43
(iii) 2x + 3y + 3z = 5, x – 2y + z = -4, 3x – y – 2z = 3
(iv) x + y + 2z = 7, 3x + 4y – 5z = 5, 2x – y + 3z = 12
Solution:
(i) Given equations are
2x – y + z = 1
x + 2y + 3z = 8
3x + y – 4z = 1
D = ∣∣∣∣213-12113-4∣∣∣∣
= 2(-8 – 3) – (-1)(-4 – 9) + 1(1 – 6)
= 2(-11) + 1(-13) + 1(-5)
= -22 – 13 – 5
= -40 ≠ 0

Dx = ∣∣∣∣181-12113-4∣∣∣∣
= 1(-8 – 3) – (-1)(-32 – 3) + 1(8 – 2)
= 1(-11) + 1(-35) + 1(6)
= -11 – 35 + 6
= -40

Dy = ∣∣∣∣21318113-4∣∣∣∣
= 2(-32 – 3) -1(-4 – 9) + 1(1 – 24)
= 2(-35) – 1(-13) + 1(-23)
= -70 + 13 – 23
= -80

Dz = ∣∣∣∣213-121181∣∣∣∣
= 2(2 – 8) – (-1)(1 – 24) + 1(1 – 6)
= 2(-6) + 1(-23) + 1(-5)
= -12 – 23 – 5
= -40
By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{-40}{-40} = 1, \; y = \frac{D_y}{D} = \frac{-80}{-40} = 2,$$

$$z = \frac{D_z}{D} = \frac{-40}{-40} = 1$$

∴ x = 1, y = 2 and z = 1 are the solutions of the given equations.

(ii) Let 1x = p, 1y = q, 1z = r
∴ The given equations become
p + q = 32
i.e., 2p + 2q = 3
i.e., 2p + 2q + 0 = 3
q + r = 56
i.e., 6q + 6r = 5,
i.e., 0p + 6q + 6r = 5
r + p = 43
i.e., 3r + 3p = 4,
i.e., 3p + 0q + 3r = 4
D = ∣∣∣∣203260063∣∣∣∣
= 2(18 – 0) -2(0 – 18) + 0
= 2(18) – 2(-18)
= 36 + 36
= 72 ≠ 0

Dp = ∣∣∣∣354260063∣∣∣∣
= 3(18 – 0) – 2(15 – 24) + 0
= 3(18) – 2(-9)
= 54 + 18
= 72

Dq = ∣∣∣∣203354063∣∣∣∣
= 2(15 – 24) – 3(0 – 18) + 0
= 2(-9) – 3(-18)
= -18 + 54
= 36

Dr = ∣∣∣∣203260354∣∣∣∣
= 2(24 – 0) – 2(0 – 15) + 3(0 – 18)
= 2(24) – 2(-15) + 3(-18)
= 48 + 30 – 54
= 24
By Cramer’s Rule,

$$p = \frac{D_p}{D} = \frac{72}{72} = 1, \; q = \frac{D_q}{D} = \frac{36}{72} = \frac{1}{2},$$

$$r = \frac{D_r}{D} = \frac{24}{72} = \frac{1}{3}$$

$$\frac{1}{x} = 1, \frac{1}{y} = \frac{1}{2}, \frac{1}{z} = \frac{1}{3}$$

∴ x = 1, y = 2 and z = 3 are the solutions of the given equations.

(iii) Given equations are
2x + 3y + 3z = 5
x – 2y + z = -4
3x – y – 2z = 3
D = ∣∣∣∣2133-2-131-2∣∣∣∣
= 2(4 + 1) – 3(-2 – 3) + 3(-1 + 6)
= 2(5) – 3(-5) + 3(5)
= 10 + 15 + 15
= 40 ≠ 0

Dx = ∣∣∣∣5-433-2-131-2∣∣∣∣
= 5(4 + 1) – 3(8 – 3) + 3(4 + 6)
= 5(5) – 3(5) + 3(10)
= 25 – 15 + 30
= 40

Dy = ∣∣∣∣2135-4331-2∣∣∣∣
= 2(8 – 3) – 5(-2 – 3) + 3(3 + 12)
= 2(5) – 5(-5) + 3(15)

= 10 + 25 + 45
= 80

Dz = ∣∣∣∣ 2133-2-15-43 ∣∣∣∣
= 2(-6 – 4) – 3(3 + 12) + 5(-1 + 6)
= 2(-10) – 3(15) + 5(5)
= -20 -45 + 25
= -40
By Cramer’s Rule,

$$x = \frac{D_x}{D} = \frac{40}{40} = 1,\ y = \frac{D_y}{D} = \frac{80}{40} = 2,$$

$$z = \frac{D_z}{D} = \frac{-40}{40} = -1$$

∴ x = 1, y = 2 and z = -1 are the solutions of the given equations.

(iv) Given equations are
x – y + 2z = 7
3x + 4y – 5z = 5
2x – y + 3z = 12
D = ∣∣∣∣ 132-14-12-53 ∣∣∣∣
= 1(12 – 5) – (-1)(9 + 10) + 2(-3 – 8)
= 1(7) + 1(19) + 2(-11)
= 7 + 19 – 22
= 4 ≠ 0

Dx = ∣∣∣∣ 7512-14-12-53 ∣∣∣∣
= 7(12 – 5) – (-1)(15 + 60) + 2(-5 – 48)
= 7(7) + 1(75) + 2(-53)
= 49 + 75 – 106
= 18

Dy = ∣∣∣∣ 13275122-53 ∣∣∣∣
= 1(15 + 60) – 7(9 + 10) + 2(36 – 10)
= 1(75) – 7(19) + 2(26)
= 75 – 133 + 52
= -6

Dz = ∣∣∣∣ 132-14-17512 ∣∣∣∣
= 1(48 + 5) – (-1)(36 – 10) + 7(-3 – 8)
= 1(53) + 1(26) + 7(-11)
= 53 + 26 – 77
= 2
By Cramer’s Rule,

$$x = \frac{D_x}{D} = \frac{18}{4} = \frac{9}{2},\ y = \frac{D_y}{D} = \frac{-6}{4} = \frac{-3}{2},$$

$$z = \frac{D_z}{D} = \frac{2}{4} = \frac{1}{2}$$

∴ x = 92, y = -32 and z = 12 are the solutions of the given equations.

Question 10.
Find the value of k, if the following equations are consistent.
(i) (k + 1)x + (k – 1)y + (k – 1) = 0
(k – 1)x + (k + 1)y + (k – 1) = 0
(k – 1)x + (k – 1)y + (k + 1) = 0
(ii) 3x + y – 2 = 0, kx + 2y – 3 = 0 and 2x – y = 3
(iii) (k – 2)x + (k – 1)y = 17, (k – 1)x +(k – 2)y = 18 and x + y = 5
Solution:
(i) Given equations are
(k + 1)x + (k – 1)y + (k – 1) = 0
(k – 1)x + (k + 1)y + (k – 1) = 0
(k – 1)x + (k – 1)y + (k + 1) = 0
Since these equations are consistent,

$$\begin{vmatrix} k+1 & k-1 & k-1 \\ k-1 & k+1 & k-1 \\ k-1 & k-1 & k+1 \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_2$, we get

$$\begin{vmatrix} 2 & k-1 & k-1 \\ -2 & k+1 & k-1 \\ 0 & k-1 & k+1 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_3$, we get

$$\begin{vmatrix} 2 & 0 & k-1 \\ -2 & 2 & k-1 \\ 0 & -2 & k+1 \end{vmatrix} = 0$$

⇒ 2(2k + 2 + 2k – 2) – 0 + (k – 1) (4 – 0) = 0
⇒ 2(4k) + (k – 1)4 = 0
⇒ 8k + 4k – 4 = 0
⇒ 12k – 4 = 0
⇒ k = 412=13

(ii) Given equations are
3x + y – 2 = 0
kx + 2y – 3 = 0
2x – y = 3, i.e., 2x – y – 3 = 0.
Since these equations are consistent,
∣∣∣∣3k212-1-2-3-3∣∣∣∣=0
⇒ 3(-6 – 3) – 1(-3k + 6) – 2(-k – 4) = 0
⇒ 3(-9) – 1(-3k + 6) – 2(-k – 4) = 0
⇒ -27 + 3k – 6 + 2k + 8 = 0
⇒ 5k – 25 = 0
⇒ k = 5

(iii) Given equations are
(k – 2)x + (k – 1)y = 17
⇒ (k – 2)x + (k – 1)y – 17 = 0
(k – 1)x + (k – 2)y = 18
⇒ (k – 1)x + (k – 2)y – 18 = 0
x + y = 5
⇒ x + y – 5 = 0
Since these equations are consistent,

$$\begin{vmatrix} k-2 & k-1 & -17 \\ k-1 & k-2 & -18 \\ 1 & 1 & -5 \end{vmatrix} = 0$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{vmatrix} -1 & 1 & 1 \\ k-1 & k-2 & -18 \\ 1 & 1 & -5 \end{vmatrix} = 0$$

⇒ -1(-5k + 10 + 18) – 1(-5k + 5 + 18) + 1(k – 1 – k + 2) = 0
⇒ -1(-5k + 28) – 1(-5k + 23) + 1(1) = 0
⇒ 5k – 28 + 5k – 23 + 1 = 0
⇒ 10k – 50 = 0
⇒ k = 5

Question 11.
Find the area of triangle whose vertices are
(i) A(-1, 2), B(2, 4), C(0, 0)
(ii) P(3, 6), Q(-1, 3), R(2, -1)
(iii) L(1, 1), M(-2, 2), N(5, 4)
Solution:
(i) Here, A(x_1, y_1) = A(-1, 2)
B(x_2, y_2) = B(2, 4)
C(x_3, y_3) = C(0, 0)

$$\text{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$A(\Delta ABC) = \frac{1}{2}\begin{vmatrix} -1 & 2 & 1 \\ 2 & 4 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[-1(4-0) - 2(2-0) + 1(0-0)]$$

$$= \frac{1}{2}(-4-4) = \frac{1}{2}(-8) = -4$$

Since area cannot be negative,
A(ΔABC) = 4 sq.units

(ii) Here, $P(x_1, y_1) = P(3, 6)$
$Q(x_2, y_2) = Q(-1, 3)$
$R(x_3, y_3) = R(2, -1)$

$$\text{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$A(\Delta PQR) = \frac{1}{2}\begin{vmatrix} 3 & 6 & 1 \\ -1 & 3 & 1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[3(3+1) - 6(-1-2) + 1(1-6)]$$

$$= \frac{1}{2}[3(4) - 6(-3) + 1(-5)]$$

$$= \frac{1}{2}(12 + 18 - 5)$$

A(ΔPQR) = 252 sq.units

(iii) Here, $L(x_1, y_1) = L(1, 1)$
$M(x_2, y_2) = M(-2, 2)$
$N(x_3, y_3) = N(5, 4)$

$$\text{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$A(\Delta LMN) = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ -2 & 2 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[1(2-4) - 1(-2-5) + 1(-8-10)]$$

$$= \frac{1}{2}[1(-2) - 1(-7) + 1(-18)]$$

$$= \frac{1}{2}(-2 + 7 - 18)$$

$$= -\frac{13}{2}$$

Since area cannot be negative,
A(ΔLMN) = 132 sq.units

Question 12.
Find the value of k,
(i) if the area of a triangle is 4 square units and vertices are P(k, 0), Q(4, 0), R(0, 2).
(ii) if area of triangle is 332 square units and vertices are L(3, -5), M(-2, k), N(1, 4).
Solution:
(i) Here, $P(x_1, y_1) = P(k, 0)$
$Q(x_2, y_2) = Q(4, 0)$
$R(x_3, y_3) = R(0, 2)$

A(ΔPQR) = 4 sq.units

$$\text{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore \quad \pm 4 = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$\therefore \quad \pm 4 = \frac{1}{2}[k(0-2) - 0 + 1\,(8-0)]$$

$\therefore \quad \pm 8 = -2k + 8$

$\therefore \quad 8 = -2k + 8 \quad \text{or} \quad -8 = -2k + 8$

$\therefore \quad -2k = 0 \quad \text{or} \quad 2k = 16$

$\therefore \quad k = 0 \quad \text{or} \quad k = 8$

(ii) Here, L(x_1, y_1) = L(3, -5), M(x_2, y_2) = M(-2, k), N(x_3, y_3) = N(1, 4)

A(ΔLMN) = 332 sq. units

Area of a triangle = 12 ∣∣∣∣ x1x2x3y1y2y3111 ∣∣∣∣

±332=12 ∣∣∣∣ 3-21-5k4111 ∣∣∣∣

⇒ ±332=12 [3(k – 4) – (-5) (-2 – 1) + 1 (-8 – k)]

⇒ ±33 = 3k – 12 – 15 – 8 – k

⇒ ±33 = 2k – 35

⇒ 2k – 35 = 33 or 2k – 35 = -33

⇒ 2k = 68 or 2k = 2

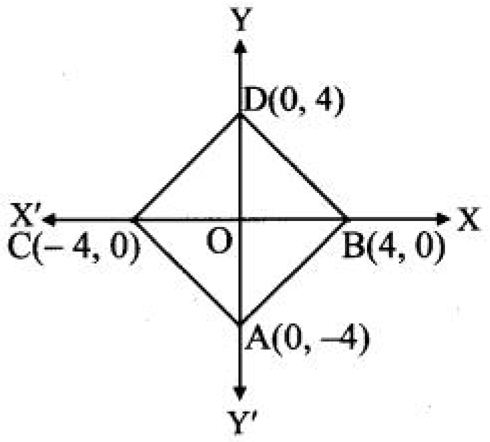
⇒ k = 34 or k = 1

Question 13.

Find the area of quadrilateral whose vertices are A(0, -4), B(4, 0), C(-4,0), D (0, 4).

Solution:

A(0, -4), B(4, 0), C(-4, 0), D(0, 4)



A(□ABDC) = A(ΔABC) + A(ΔBDC)

$$\text{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$A(\Delta ABC) = \frac{1}{2}\begin{vmatrix} 0 & -4 & 1 \\ 4 & 0 & 1 \\ -4 & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\,[0 - (-4)(4+4) + 1(0-0)]$$

$$= \frac{1}{2}(32) = 16 \text{ sq. units}$$

$$A(\Delta BDC) = \frac{1}{2}\begin{vmatrix} 4 & 0 & 1 \\ 0 & 4 & 1 \\ -4 & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[4(4-0) - 0 + 1(0+16)]$$

$$= \frac{1}{2}[4(4) + 1(16)]$$

$$= \frac{1}{2}(16+16)$$

$$= \frac{1}{2}(32)$$

$$= 16 \text{ sq. units}$$

∴ A(ABDC) = A(ΔABC) + A(ΔBDC)
= 16 + 16
= 32 sq.units

Question 14.
An amount of ₹ 5000 is put into three investments at the rate of interest of 6%, 7%, and 8% per annum respectively. The total annual income is ₹ 350. If the combined income from the first two investments is ₹ 70 more than the income from the third, find the amount of each investment.
Solution:
Let the amount of each investment be ₹ x, ₹ y and ₹ z.
According to the given conditions,
x + y + z = 5000,
6% x + 7% y + 8% z = 350

$$\therefore \quad \frac{6}{100}x + \frac{7}{100}y = \frac{8}{100}z + 70$$

$$\therefore \quad 6x + 7y = 8z + 7000$$

$$\therefore \quad 6x + 7y - 8z = 7000$$

$$\therefore \quad D = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{vmatrix}$$

$$= 1(-56-56) - 1(-48-48) + 1(42-42)$$

$$= -112 + 96 + 0$$

$$= -16 \neq 0$$

$$D_x = \begin{vmatrix} 5000 & 1 & 1 \\ 35000 & 7 & 8 \\ 7000 & 7 & -8 \end{vmatrix}$$

Taking 1000 common from C_1, we get

$$D_x = 1000\begin{vmatrix} 5 & 1 & 1 \\ 35 & 7 & 8 \\ 7 & 7 & -8 \end{vmatrix}$$

Applying $C_1 \to C_1 - 5C_3$ and $C_2 \to C_2 - C_3$, we get

$$D_x = 1000\begin{vmatrix} 0 & 0 & 1 \\ -5 & -1 & 8 \\ 47 & 15 & -8 \end{vmatrix}$$

$$= 1000[0 - 0 + 1(-75+47)]$$

$$= 1000 \times (-28)$$

$$= -28000$$

$$D_y = \begin{vmatrix} 1 & 5000 & 1 \\ 6 & 35000 & 8 \\ 6 & 7000 & -8 \end{vmatrix}$$

Taking 1000 common from C_2, we get

$$D_y = 1000\begin{vmatrix} 1 & 5 & 1 \\ 6 & 35 & 8 \\ 6 & 7 & -8 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - 5C_3$, we get

$$D_y = 1000 \begin{vmatrix} 0 & 0 & 1 \\ -2 & -5 & 8 \\ 14 & 47 & -8 \end{vmatrix}$$

$= 1000 [0 - 0 + 1(-94 + 70)$
$= 1000 (-24)$
$= -24000$

$$D_z = \begin{vmatrix} 1 & 1 & 5000 \\ 6 & 7 & 35000 \\ 6 & 7 & 7000 \end{vmatrix}$$

Taking 1000 common from C_3, we get

$$D_z = 1000 \begin{vmatrix} 1 & 1 & 5 \\ 6 & 7 & 35 \\ 6 & 7 & 7 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$ and $C_3 \to C_3 - 5C_2$, we get

$$D_z = 1000 \begin{vmatrix} 0 & 1 & 0 \\ -1 & 7 & 0 \\ -1 & 7 & -28 \end{vmatrix}$$

$= 1000 [0 - 1 (28 - 0) + 0]$
$= 1000 \times (-28)$
$= -28000$

By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{-28000}{-16} = 1750,$$

$$y = \frac{D_y}{D} = \frac{-24000}{-16} = 1500,$$

$$z = \frac{D_z}{D} = \frac{-28000}{-16} = 1750,$$

∴ The amounts of investments are ₹ 1750, ₹ 1500, and ₹ 1750.

Question 15.
Show that the lines x – y = 6, 4x – 3y = 20 and 6x + 5y + 8 = 0 are concurrent. Also, find the point of concurrence.
Solution:
Given equations of the lines are
x – y = 6, i.e., x – y – 6 = 0 ……(i)
4x – 3y = 20, i.e., 4x – 3y – 20 = 0 …..(ii)
6x + 5y + 8 = 0 ……(iii)
The given lines will be concurrent, if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -1 & -6 \\ 4 & -3 & -20 \\ 6 & 5 & 8 \end{vmatrix}$$

= 1(-24 + 100) – (-1) (32 + 120) – 6(20 + 18)
= 1(76) + 1(152) – 6(38)
= 76 + 152 – 228
= 0
∴ The given lines are concurrent.
To find the point of concurrence, solve any two equations.
Multiplying (i) by 5, we get
5x – 5y – 30 = 0 …….(iv)
Adding (iii) and (iv), we get
11x – 22 = 0
∴ x = 2
Substituting x = 2 in (i), we get
2 – y – 6 = 0
∴ y = -4
∴ The point of concurrence is (2, -4).

Question 16.
Show that the following points are collinear using determinants:
(i) L(2, 5), M(5, 7), N(8, 9)
(ii) P(5,1), Q(1, -1), R(11, 4)
Solution:
(i) Here, $L(x_1, y_1)$ = L(2, 5)
$M(x_2, y_2)$ = M(5, 7)
$N(X_3\ y_3)$ = N(8, 9)
If A(ΔLMN) = 0, then the points L, M, N are collinear.

$$A(\Delta LMN) = \frac{1}{2}\begin{vmatrix} 2 & 5 & 1 \\ 5 & 7 & 1 \\ 8 & 9 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\ [2(7-9) - 5\,(5-8) + 1(45-56)]$$

$$= \frac{1}{2}\ [2(-2) - 5(-3) + 1(-11)]$$

$$= \frac{1}{2}\ (-4 + 15 - 11) = 0$$

∴ The points L, M, N are collinear.

(ii) Here, $P(x_1, y_1)$ = P(5, 1)
$Q(x_2, y_2)$ = Q(1, -1)
$R(x_3, y_3)$ = R(11, 4)
If A(ΔPQR) = 0, then the points P, Q, R are collinear.

$$A(\Delta PQR) = \frac{1}{2}\begin{vmatrix} 5 & 1 & 1 \\ 1 & -1 & 1 \\ 11 & 4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\ [5(-1-4) - 1\,(1-11) + 1(4+11)]$$

$$= \frac{1}{2}\ [5(-5) - 1(-10) + 1(15)]$$

$$= \frac{1}{2}\ (-25 + 10 + 15) = 0$$

∴ The points P, Q, R are collinear.

Maharashtra State Board 11th Maths Solutions Chapter 4 Determinants and Matrices Ex 4.4

Question 1.
Construct a matrix A = $[a_{ij}]_{3 \times 2}$ whose elements ay are given by
i. $a_{ij} = (i-j)_2 5-i$
ii. a_{ij} = i – 3j
iii. a_{ij} $(i+j)_3 5$
Solution:

$$A = \left[a_{ij}\right]_{3\times 2}$$

$$\therefore \quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

i. $$a_{ij} = \frac{(i-j)^2}{5-i}$$

$$\therefore \quad a_{11} = \frac{(1-1)^2}{5-1} = 0, \; a_{12} = \frac{(1-2)^2}{5-1} = \frac{(-1)^2}{4} = \frac{1}{4},$$

$$a_{21} = \frac{(2-1)^2}{5-2} = \frac{1}{3}, \; a_{22} = \frac{(2-2)^2}{5-2} = 0,$$

$$a_{31} = \frac{(3-1)^2}{5-3} = \frac{2^2}{2} = 2, \; a_{32} = \frac{(3-2)^2}{5-3} = \frac{1}{2}$$

$$\therefore \quad A = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \\ 2 & \frac{1}{2} \end{bmatrix}$$

[Note: Answer given in the textbook is A = ⎡ ⎣ ⎢ ⎢ 0122140 12⎤ ⎦ ⎥ ⎥

However, as per our calculation it is ⎡ ⎣ ⎢ ⎢ 0132140 12⎤ ⎦ ⎥ ⎥].

ii. $a_{ij} = i - 3j$

$\therefore a_{11} = 1 - 3(1) = 1 - 3 = -2,$

$a_{12} = 1 - 3(2) = 1 - 6 = -5,$

$a_{21} = 2 - 3(1) = 2 - 3 = -1,$

$a_{22} = 2 - 3(2) = 2 - 6 = -4$

$a_{31} = 3 - 3(1) = 3 - 3 = 0,$

$a_{32} = 3 - 3(2) = 3 - 6 = -3$

∴ A = ⎡ ⎣ ⎢ -2-10-5-4-3⎤ ⎦ ⎥

iii. $a_{ij} = \frac{(i+j)^3}{5}$

$$\therefore \quad a_{11} = \frac{(1+1)^3}{5} = \frac{2^3}{5} = \frac{8}{5}, \; a_{12} = \frac{(1+2)^3}{5} = \frac{3^3}{5} = \frac{27}{5}$$

$$a_{21} = \frac{(2+1)^3}{5} = \frac{3^3}{5} = \frac{27}{5}, \; a_{22} = \frac{(2+2)^3}{5} = \frac{4^3}{5} = \frac{64}{5}$$

$$a_{31} = \frac{(3+1)^3}{5} = \frac{4^3}{5} = \frac{64}{5}, \; a_{32} = \frac{(3+2)^3}{5} = \frac{5^3}{5} = \frac{125}{5}$$

$$\therefore \quad A = \begin{bmatrix} \frac{8}{5} & \frac{27}{5} \\ \frac{27}{5} & \frac{64}{5} \\ \frac{64}{5} & \frac{125}{5} \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 8 & 27 \\ 27 & 64 \\ 64 & 125 \end{bmatrix}$$

Question 2.
Classify the following matrices as a row, a column, a square, a diagonal, a scalar, a unit, an upper triangular, a lower triangular, a symmetric or a skew- symmetric matrix.

i. ⎡ ⎣ ⎢ 300-2004-50⎤ ⎦ ⎥

Solution:

Let A = ⎡ ⎣ ⎢ 300-2004-50⎤ ⎦ ⎥

As every element below the diagonal is zero in matrix A.

∴ A is an upper triangular matrix.

ii. ⎡ ⎣ ⎢ 0-4-74037-30⎤ ⎦ ⎥

Solution:

Let A = ⎡⎣⎢⎢ 0−4−74037−30⎤⎦⎥⎥

∴ A^T = ⎡⎣⎢⎢ 047−40−3−730⎤⎦⎥⎥

∴ A^T = −⎡⎣⎢⎢ 0−4−74037−30⎤⎦⎥⎥

∴ A^T = -A, i.e., A = $-A^T$

∴ A is a skew-symmetric matrix.

iii. ⎡⎣⎢⎢ 54−3⎤⎦⎥⎥

Solution:

Let A = ⎡⎣⎢⎢ 54−3⎤⎦⎥⎥

∴ As matrix A has only one column.

∴ A is a column matrix.

iv. [92−√−3]

Solution:

Let A = [92−√−3]

As matrix A has only one row.

∴ A is a row matrix.

v. [6006]

Solution:

Let A = [6006]

As matrix A has all its non-diagonal elements zero and diagonal elements same.

∴ A is a scalar matrix.

vi. ⎡⎣⎢⎢ 23−70−13001⎤⎦⎥⎥

Solution:

Let A = ⎡⎣⎢⎢ 23−70−13001⎤⎦⎥⎥

As every element above the diagonal is zero in matrix A.

∴ A is a lower triangular matrix.

vii. ⎡⎣⎢⎢ 30005000$_{13}$⎤⎦⎥⎥

Solution:

Let A = ⎡⎣⎢⎢ 30005000$_{13}$⎤⎦⎥⎥

As matrix A has all its non-diagonal elements zero.

∴ A is a diagonal matrix.

viii. ⎡⎣⎢⎢ 10−1527−15034−−√2734−−√$_{53}$⎤⎦⎥⎥

Solution:

$$\text{Let } A = \begin{bmatrix} 10 & -15 & 27 \\ -15 & 0 & \sqrt{34} \\ 27 & \sqrt{34} & \frac{5}{3} \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} 10 & -15 & 27 \\ -15 & 0 & \sqrt{34} \\ 27 & \sqrt{34} & \frac{5}{3} \end{bmatrix}$$

∴ A^T = A, i/e., A = A^T

∴ A is a symmetric matrix.

ix. ⎡⎣⎢⎢ 100010001⎤⎦⎥⎥

Solution:

A = ⎡⎣⎢⎢ 100010001⎤⎦⎥⎥

In matrix A, all the non-diagonal elements are zero and diagonal elements are one.
∴ A is a unit (identity) matrix.

x. ⎡ ⎣ ⎢ 001010100⎤ ⎦ ⎥
Solution:
Let A = ⎡ ⎣ ⎢ 001010100⎤ ⎦ ⎥
∴ A^T = A, i/e., A = A^T
∴ A is a symmetric matrix.
∴ A is a symmetric matrix.

Question 3.
Which of the following matrices are singular or non-singular?
i. ⎡ ⎣ ⎢ ap2a−pbq2b−qcr2c−r⎤ ⎦ ⎥
ii. ⎡ ⎣ ⎢ 516099995100105⎤ ⎦ ⎥
iii. ⎡ ⎣ ⎢ 3−23512745⎤ ⎦ ⎥
iv. [7−457]
Solution:

$$\text{Let } A = \begin{bmatrix} a & b & c \\ p & q & r \\ 2a-p & 2b-q & 2c-r \end{bmatrix}$$

$$\therefore \quad |A| = \begin{vmatrix} a & b & c \\ p & q & r \\ 2a-p & 2b-q & 2c-r \end{vmatrix}$$

Applying $R_3 \to R_3 + R_2$, we get

$$|A| = \begin{vmatrix} a & b & c \\ p & q & r \\ 2a & 2b & 2c \end{vmatrix}$$

Taking 2 common from R_3, we get

$$|A| = 2\begin{vmatrix} a & b & c \\ p & q & r \\ a & b & c \end{vmatrix}$$

$= 2(0) \quad \ldots[\because R_1 \text{ and } R_3 \text{ are identical}]$

$= 0$

∴ **A is a singular matrix.**

ii. Let A = ⎡ ⎣ ⎢ 516099995100105⎤ ⎦ ⎥
∴ |A| = ∣ ∣ ∣ ∣ 516099995100105∣ ∣ ∣ ∣
Applying $C_2 \to C_2 + C_1$
|A| = ∣ ∣ ∣ ∣ 516510010555100105∣ ∣ ∣ ∣
= 0 … [∵ C_2 and C_3 are identical]
∴ A is a singular matrix.

iii. Let A = ⎡ ⎣ ⎢ 3−23512745⎤ ⎦ ⎥
∴ |A| = ∣ ∣ ∣ ∣ 3−23512745∣ ∣ ∣ ∣
= 3(5 – 8) – 5(-10 – 12) + 7(-4 – 3)
= -9 + 110 – 49 = 52 ≠ 0
∴ A is a non-singular matrix.

iv. Let A = [7−457]
∴ |A| = [7−457] = 49 + 20 = 69 ≠ 0

Question 4.
Find k, if the following matrices are singular.
i. [7−23k]

ii. ⎡ ⎣ ⎢ 47103k9111⎤ ⎦ ⎥

iii. ⎡ ⎣ ⎢ k−13121−2324⎤ ⎦ ⎥

Solution:

Let A = [7−23k]

Since A is a singular matrix,
|A|=0

∴ ∣ ∣ ∣ 7−23k ∣ ∣ ∣ = o

∴ 7k + 6 = 0
∴ 7k = -6
k = -6/7

ii. Let A = ⎡ ⎣ ⎢ 47103k9111⎤ ⎦ ⎥

Since A is a singular matrix,
|A|= 0

∴ ∣ ∣ ∣ ∣ 47103k9111 ∣ ∣ ∣ ∣ = 0

∴ 4(k – 9) – 3(7 – 10) + 1(63 – 10k) = 0
∴ 4k – 36 + 9 + 63 – 10k = 0
∴ -6k + 36 = 0
∴ 6k = 36
∴ k = 6

iii. Let A = ⎡ ⎣ ⎢ k−13121−2324⎤ ⎦ ⎥

Since A is a singular matrix
|A| = 0

∴ ∣ ∣ ∣ ∣ k−13121−2324 ∣ ∣ ∣ ∣

∴ (k – 1)(4 + 4) – 2(12 – 2) + 3 (-6 – 1) = 0
∴ 8k-8-20-21 =0
∴ 8k = 49
∴ k = 49/8

Question 5.

If A = [5312−10], find (AT)T.

Solution:

$$A = \begin{bmatrix} 5 & 1 & -1 \\ 3 & 2 & 0 \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} 5 & 3 \\ 1 & 2 \\ -1 & 0 \end{bmatrix}$$

$$\therefore \quad (A^T)^T = \begin{bmatrix} 5 & 1 & -1 \\ 3 & 2 & 0 \end{bmatrix}$$

Question 6.

If A = , find (AT)T.

Solution:

$$A = \begin{bmatrix} 7 & 3 & 1 \\ -2 & -4 & 1 \\ 5 & 9 & 1 \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} 7 & -2 & 5 \\ 3 & -4 & 9 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad (A^T)^T = \begin{bmatrix} 7 & 3 & 1 \\ -2 & -4 & 1 \\ 5 & 9 & 1 \end{bmatrix}$$

Question 7.

Find a, b, c, if ⎡ ⎣ ⎢ 1b−435−5ca−70⎤ ⎦ ⎥ is a symmetric matrix.

Solution:

Let $A = \begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix}$

$\therefore \quad A^T = \begin{bmatrix} 1 & b & -4 \\ \frac{3}{5} & -5 & c \\ a & -7 & 0 \end{bmatrix}$

Since A is a symmetric matrix,

$A = A^T$

$\therefore \quad \begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix} = \begin{bmatrix} 1 & b & -4 \\ \frac{3}{5} & -5 & c \\ a & -7 & 0 \end{bmatrix}$

$\therefore$ By equality of matrices, we get

$a = -4, b = \frac{3}{5}, c = -7$

Question 8.
Find x, y, z, if is a symmetric matrix.
Solution:

Let $A = \begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix}$

$\therefore \quad A^T = \begin{bmatrix} 0 & y & \frac{3}{2} \\ -5i & 0 & -\sqrt{2} \\ x & z & 0 \end{bmatrix}$

Since A is a skew–svmmetric matrix.

$A = -A^T$

$\therefore \quad \begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix} = -\begin{bmatrix} 0 & y & \frac{3}{2} \\ -5i & 0 & -\sqrt{2} \\ x & z & 0 \end{bmatrix}$

$\therefore \quad \begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -y & \frac{-3}{2} \\ 5i & 0 & \sqrt{2} \\ -x & -z & 0 \end{bmatrix}$

$\therefore$ By equality of matrices, we get

$x = \frac{-3}{2}, y = 5i, z = \sqrt{2}$

Question 9.
For each of the following matrices, using its transpose, state whether it is symmetric, skew-symmetric or neither.

i. ⎡ ⎣ ⎢ 12−52−34−549⎤ ⎦ ⎥

ii. ⎡ ⎣ ⎢ 2−5−154−6163⎤ ⎦ ⎥

iii. ⎡ ⎣ ⎢ 0−1−2i2−i1+2i07i−2−70⎤ ⎦ ⎥

Solution:

i. Let A = ⌈ ⌊ ∣ 12−52−34−549⌉ ⌋ ∣

∴ A^T =⌈ ⌊ ∣ 12−52−34−549⌉ ⌋ ∣

∴ $A^T = A$, i.e., $A = A^T$

∴ A is a symmetric matrix.

ii.

$$\text{Let } A = \begin{bmatrix} 2 & 5 & 1 \\ -5 & 4 & 6 \\ -1 & -6 & 3 \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} 2 & -5 & -1 \\ 5 & 4 & -6 \\ 1 & 6 & 3 \end{bmatrix}$$

$$\therefore \quad A^T = -\begin{bmatrix} -2 & 5 & 1 \\ -5 & -4 & 6 \\ -1 & -6 & -3 \end{bmatrix}$$

∴ $A \neq A^T$, i.e., $A \neq -A^T$

∴ A is neither a symmetric nor skew-symmetric matrix.

iii.

$$\text{Let } A = \begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} 0 & -1-2i & 2-i \\ 1+2i & 0 & 7 \\ i-2 & -7 & 0 \end{bmatrix}$$

$$\therefore \quad A^T = -\begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$$

∴ $A^T = -A$, i.e., $A = -A^T$

∴ A is a skew-symmetric matrix.

Question 10.

Construct the matrix $A = [a_{ij}]_{3 \times 3}$, where $a_{ij} = i - j$. State whether A is symmetric or skew-symmetric.

Solution:

$A = [a_{ij}]_{3 \times 3}$

∴ A = ⌈ ⌊ ∣ a11a21a31a12a22a32a13a23a33⌉ ⌋ ∣

Given, $a_{ij} = i - j$

a_{11} = 1-1 = 0, a_{12} = 1-2 = – 1, a_{13} = 1 – 3 = – 2,

a_{21} – 2 – 1 = 1, a_{22} = 2 – 2 = 0, a_{23} =2 – 3 = – 1,

a_{31} = 3 – 1 = 2, a_{32} = 3 – 2 = 1, a_{33} = 3 – 3 = 0

$$\therefore \quad A = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

∴ $A^T = -A$, i.e., $A = -A^T$

∴ A is a skew-symmetric matrix.

Maharashtra State Board 11th Maths Solutions Chapter 4 Determinants and Matrices Ex 4.5

Question 1.
If A = ⌈ ⌊ ∣ 25−6−3−41⌉ ⌋ ∣ , B = ⌈ ⌊ ∣ −120223⌉ ⌋ ∣ and C = ⌈ ⌊ ∣ 4−1−2341⌉ ⌋ ∣
Show that
i. A+B=B+A
ii. (A + B) + C = A + (B + C)
Solution:

i. $$A+B=\begin{bmatrix}2 & -3\\ 5 & -4\\ -6 & 1\end{bmatrix}+\begin{bmatrix}-1 & 2\\ 2 & 2\\ 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2-1 & -3+2\\ 5+2 & -4+2\\ -6+0 & 1+3\end{bmatrix}$$

$$\therefore \quad A+B=\begin{bmatrix}1 & -1\\ 7 & -2\\ -6 & 4\end{bmatrix} \quad \ldots\text{(i)}$$

$$B+A=\begin{bmatrix}-1 & 2\\ 2 & 2\\ 0 & 3\end{bmatrix}+\begin{bmatrix}2 & -3\\ 5 & -4\\ -6 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-1+2 & 2-3\\ 2+5 & 2-4\\ 0-6 & 3+1\end{bmatrix}$$

$$\therefore \quad B+A=\begin{bmatrix}1 & -1\\ 7 & -2\\ -6 & 4\end{bmatrix} \quad \ldots\text{(ii)}$$

From (i) and (ii), we get
A + B = B + A

ii. $$(A+B)+C = \left\{\begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix}\right\} + \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & -3+2 \\ 5+2 & -4+2 \\ -6+0 & 1+3 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 7 & -2 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & -1+3 \\ 7-1 & -2+4 \\ -6-2 & 4+1 \end{bmatrix}$$

$$\therefore \quad (A+B)+C = \begin{bmatrix} 5 & 2 \\ 6 & 2 \\ -8 & 5 \end{bmatrix} \qquad \ldots\text{(i)}$$

$$A+(B+C) = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \left\{\begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix}\right\}$$

$$= \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} -1+4 & 2+3 \\ 2-1 & 2+4 \\ 0-2 & 3+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ 1 & 6 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3 & -3+5 \\ 5+1 & -4+6 \\ -6-2 & 1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 6 & 2 \\ -8 & 5 \end{bmatrix} \qquad \ldots\text{(ii)}$$

$$(A+B)+C = A+(B+C)$$

Question 2.
If A = [15−23], B = [14−3−7] then find the matrix A – 2B + 6I, where I is the unit matrix of order 2.
Solution:

$$A - 2B + 6I = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix} - 2\begin{bmatrix} 1 & -3 \\ 4 & -7 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 8 & -14 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+6 & -2+6+0 \\ 5-8+0 & 3+14+6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ -3 & 23 \end{bmatrix}$$

Question 3.
If A = , B = then find the matrix C such that A + B + C is a zero matrix.
Solution:
A+ B + C is a zero matrix.
∴ A + B + C = O
C = -(A + B)

$$= -\left\{\begin{bmatrix} 1 & 2 & -3 \\ -3 & 7 & -8 \\ 0 & -6 & 1 \end{bmatrix} + \begin{bmatrix} 9 & -1 & 2 \\ -4 & 2 & 5 \\ 4 & 0 & -3 \end{bmatrix}\right\}$$

$$= -\begin{bmatrix} 1+9 & 2-1 & -3+2 \\ -3-4 & 7+2 & -8+5 \\ 0+4 & -6+0 & 1-3 \end{bmatrix}$$

$$= -\begin{bmatrix} 10 & 1 & -1 \\ -7 & 9 & -3 \\ 4 & -6 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -1 & 1 \\ 7 & -9 & 3 \\ -4 & 6 & 2 \end{bmatrix}$$

Question 4.
If A = ⎡ ⎣ ⎢ 13−6−2−50⎤ ⎦ ⎥ B = ⎡ ⎣ ⎢ −141−225⎤ ⎦ ⎥ and C = ⎡ ⎣ ⎢ 2−1−34−46⎤ ⎦ ⎥ , find the matrix X such that 3A – 4B + 5X = C.
Solution:
3A-4B + 5X = C
∴ 5X = C + 4B – 3A

$$= \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix} + 4\begin{bmatrix} -1 & -2 \\ 4 & 2 \\ 1 & 5 \end{bmatrix} - 3\begin{bmatrix} 1 & -2 \\ 3 & -5 \\ -6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ 16 & 8 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 3 & -6 \\ 9 & -15 \\ -18 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2-4-3 & 4-8+6 \\ -1+16-9 & -4+8+15 \\ -3+4+18 & 6+20-0 \end{bmatrix}$$

$$\therefore \quad 5X = \begin{bmatrix} -5 & 2 \\ 6 & 19 \\ 19 & 26 \end{bmatrix}$$

$$\therefore \quad X = \frac{1}{5}\begin{bmatrix} -5 & 2 \\ 6 & 19 \\ 19 & 26 \end{bmatrix} = \begin{bmatrix} -1 & \frac{2}{5} \\ \frac{6}{5} & \frac{19}{5} \\ \frac{19}{5} & \frac{26}{5} \end{bmatrix}$$

Question 5.
Solve the following equations for X and Y, if 3X – Y = =[1−1−11] and X – 3Y = [00−1−1]
Solution:
Given equations are
=[1−1−11]………………. (i)
and X – 3Y = [00−1−1] ……………..(ii)

By (i) x 3 – (ii) we get

$$8X = 3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-0 & -3+1 \\ -3-0 & 3+1 \end{bmatrix}$$

$$\therefore \quad 8X = \begin{bmatrix} 3 & -2 \\ -3 & 4 \end{bmatrix}$$

$$\therefore \quad X = \frac{1}{8}\begin{bmatrix} 3 & -2 \\ -3 & 4 \end{bmatrix}$$

$$\therefore \quad X = \begin{bmatrix} \frac{3}{8} & \frac{-2}{8} \\ \frac{-3}{8} & \frac{4}{8} \end{bmatrix} = \begin{bmatrix} \frac{3}{8} & \frac{-1}{4} \\ \frac{-3}{8} & \frac{1}{2} \end{bmatrix}$$

By (i) – (ii) × 3, we get

$$8Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-0 & -1+3 \\ -1-0 & 1+3 \end{bmatrix}$$

$$\therefore \quad 8Y = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$$

$$\therefore \quad Y = \frac{1}{8}\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{2}{8} \\ \frac{-1}{8} & \frac{4}{8} \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ \frac{-1}{8} & \frac{1}{2} \end{bmatrix}$$

Question 6.

Find the matrices A and B, if 2A – B = =[6−4−6201] and A – 2B = [3−2218−7]

Solution:

Given equations are

2A – B = =[6−4−6201] .... (i)

and A – 2B = [3−2218−7] ....(ii)

By (i) – (ii) x 2, we get

$$3B = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - 2\begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 4 & 16 \\ -4 & 2 & -14 \end{bmatrix}$$

$$= \begin{bmatrix} 6-6 & -6-4 & 0-16 \\ -4+4 & 2-2 & 1+14 \end{bmatrix}$$

$$\therefore \quad 3B = \begin{bmatrix} 0 & -10 & -16 \\ 0 & 0 & 15 \end{bmatrix}$$

$$\therefore \quad B = \frac{1}{3}\begin{bmatrix} 0 & -10 & -16 \\ 0 & 0 & 15 \end{bmatrix}$$

$$\therefore \quad B = \begin{bmatrix} 0 & \frac{-10}{3} & \frac{-16}{3} \\ 0 & 0 & 5 \end{bmatrix}$$

By (i) × 2 – (ii), we get

$$3A = 2\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 12-3 & -12-2 & 0-8 \\ -8+2 & 4-1 & 2+7 \end{bmatrix}$$

$$\therefore \quad 3A = \begin{bmatrix} 9 & -14 & -8 \\ -6 & 3 & 9 \end{bmatrix}$$

$$\therefore \quad A = \frac{1}{3}\begin{bmatrix} 9 & -14 & -8 \\ -6 & 3 & 9 \end{bmatrix}$$

$$\therefore \quad A = \begin{bmatrix} 3 & \frac{-14}{3} & \frac{-8}{3} \\ -2 & 1 & 3 \end{bmatrix}$$

Question 7.
Simplify $cos\theta[cos\theta-sin\theta sin\theta cos\theta]+sin\theta[sin\theta cos\theta-cos\theta sin\theta]$
Solution:

$$\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\cos\theta\sin\theta \\ -\cos\theta\sin\theta+\cos\theta\sin\theta & \cos^2\theta+\sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Quesiton 8.
If A = $[1-32i2]$ and B = $[2i21-3]$ where i =$-1---\sqrt{}$, find A + B and A – B. Show that A + B is singular. Is A – B singular? Justify your answer.

Solution:

$$A + B = \begin{bmatrix} i & 2i \\ -3 & 2 \end{bmatrix} + \begin{bmatrix} 2i & i \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} i+2i & 2i+i \\ -3+2 & 2-3 \end{bmatrix} = \begin{bmatrix} 3i & 3i \\ -1 & -1 \end{bmatrix}$$

$$\therefore \quad |A+B| = \begin{vmatrix} 3i & 3i \\ -1 & -1 \end{vmatrix}$$

$$= 3i(-1) - (-1)3i = 0$$

$\therefore$ A + B is a singular matrix.

$$A - B = \begin{bmatrix} i & 2i \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 2i & i \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} i-2i & 2i-i \\ -3-2 & 2+3 \end{bmatrix} = \begin{bmatrix} -i & i \\ -5 & 5 \end{bmatrix}$$

$$\therefore \quad |A-B| = \begin{vmatrix} -i & i \\ -5 & 5 \end{vmatrix} = (-i)5 - (-5)i = 0$$

$\therefore$ A – B is also a singular matrix.

Question 9.
Find x and y, if [2x+y3−14y14]+[−136043]=[3651857]
Solution:

$$\begin{bmatrix} 2x+y & -1 & 1 \\ 3 & 4y & 4 \end{bmatrix} + \begin{bmatrix} -1 & 6 & 4 \\ 3 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 5 \\ 6 & 18 & 7 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} 2x+y-1 & -1+6 & 1+4 \\ 3+3 & 4y+0 & 4+3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 5 \\ 6 & 18 & 7 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} 2x+y-1 & 5 & 5 \\ 6 & 4y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 5 \\ 6 & 18 & 7 \end{bmatrix}$$

∴ By equality of matrices, we get
2x + y – 1 = 3 and 4y = 18
∴ 2x + y = 4 and y = 184=92
∴ 2x + 92 = 4
∴ 2x = 4 – 92
∴ 2x = 12=
∴ x = −14= and y = 92=

Question 10.
If [2a+bc+2d3a−b2c−d]=[243−1], find a, b, c and d.
Solution:
[2a+bc+2d3a−b2c−d]=[243−1]

∴ By equality of matrices, we get
2a + b = 2 ….(i)
3a – b = 3 ….(ii)
c + 2d = 4 ….(iii)
2c – d = -1 ….(iv)
Adding (i) and (ii), we get
5a = 5
∴ a = 1
Substituting a = 1 in (i), we get
2(1) + b = 2
∴ b = 0
By (iii) + (iv) x 2, we get
5c = 2
∴ c = 25
Substituting c = 25 in (iii), we get
25 + 2d = 4
∴ 2d = 4 – 25
∴ 2d = 185
∴ d = 95

[Note: Answer given in the textbook is d = 35.
However, as per our calculation it is d = 95.]

Question 11.
There are two book shops owned by Suresh and Ganesh. Their sales (in Rupees) for books in three subjects – Physics, Chemistry and Mathematics for two months, July and August 2017 are given by two matrices A and B.

July Sales (in Rupees):

$$\begin{array}{cc} & \begin{array}{ccc} \text{Physics} & \text{Chemistry} & \text{Mathematics} \end{array} \\ A = & \begin{bmatrix} 5600 & 6750 & 8500 \\ 6650 & 7055 & 8905 \end{bmatrix} \begin{array}{l} \text{Suresh} \\ \text{Ganesh} \end{array} \end{array}$$

August Sales (in Rupees):

$$\begin{array}{cc} & \begin{array}{ccc} \text{Physics} & \text{Chemistry} & \text{Mathematics} \end{array} \\ B = & \begin{bmatrix} 6650 & 7055 & 8905 \\ 7000 & 7500 & 10200 \end{bmatrix} \begin{array}{l} \text{Suresh} \\ \text{Ganesh} \end{array} \end{array}$$

i. Find the increase in sales in Rupees from July to August 2017.
ii. If both book shops got 10% profit in the month of August 2017, find the profit for each bookseller in each subject in that month.
Solution:
i. Increase in sales in rupees from July to August 2017
For Suresh:
Increase in sales for Physics books
= 6650 – 5600= ₹ 1050
Increase in sales for Chemistry books
= 7055 – 6750 = ₹ 305
Increase in sales of Mathematics books
= 8905 – 8500 = ₹ 405
For Ganesh:
Increase in sales for Physics books
= 7000 – 6650 = ₹ 350
Increase in sales for Chemistry books
= 7500 – 7055 = ₹ 445
Increase in sales for Mathematics books
= 10200 – 8905 = ₹ 1295
[Note: Answers given in the textbook are 1760, 2090. However, as per our calculation, they are 1050, 305, 405, 350, 445, 1295.]

ii. Both book shops got 10% profit in the month of August 2017.
For Suresh:
Profit for Physics books = $\frac{6650\times10}{100}$ = ₹ 665
Profit for Chemistry books = $\frac{7055\times10}{100}$ = ₹ 705.50
Profit for Mathematics books = $\frac{8905\times10}{100}$ = ₹ 890.50

For Ganesh:
Profit for Physics books = $\frac{7000\times10}{100}$ = ₹ 700
Profit for Chemistry books = $\frac{7500\times10}{100}$ = ₹ 750
Profit for Mathematics books = $\frac{10200\times10}{100}$ = ₹ 1020
[Note: Answers given in the textbook for Suresh’s profit in Chemistry and Mathematics books are ? 675 and ?850 respectively. However, as per our calculation profit amounts are ₹ 705.50 and ₹ 890.50 respectively.]

Maharashtra State Board 11th Maths Solutions Chapter 4 Determinants and Matrices Ex 4.6

Question 1.
Evaluate:
i. $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 2 & -4 & 3 \end{bmatrix}$
ii. $\begin{bmatrix} 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$
Solution:
i. $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 2 & -4 & 3 \end{bmatrix}=\begin{bmatrix} 3(2) & 3(-4) & 3(3) \\ 2(2) & 2(-4) & 2(3) \\ 1(2) & 1(-4) & 1(3) \end{bmatrix}=\begin{bmatrix} 6 & -12 & 9 \\ 4 & -8 & 6 \\ 2 & -4 & 3 \end{bmatrix}$

ii. $\begin{bmatrix} 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$
= [2(4)-1(3)+ 3(1)]
= [8 – 3 + 3] = [8]

Question 2.
If A = $\begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 4 & 1 \\ 3 & -2 \end{bmatrix}$, = show that AB ≠ BA.
Solution:
$$AB = \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 3 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 4-9 & 1+6 \\ 16+6 & 4-4 \end{bmatrix}$$
$$= \begin{bmatrix} -5 & 7 \\ 22 & 0 \end{bmatrix} \quad \ldots(i)$$
$$BA = \begin{bmatrix} 4 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 4+4 & -12+2 \\ 3-8 & -9-4 \end{bmatrix}$$
$$= \begin{bmatrix} 8 & -10 \\ -5 & -13 \end{bmatrix} \quad \ldots(ii)$$
From (i) and (ii), we get
AB ≠ BA

Question 3.
If A = $\begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix}$,B = $\begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ state whether AB = BA? Justify your answer.
Solution:
$$AB = \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} -2+3+1 & -1+0+2 & -4+2+1 \\ 4+9+0 & 2+0+0 & 8+6+0 \\ 2-9+1 & 1+0+2 & 4-6+1 \end{bmatrix}$$

$$\therefore \quad AB = \begin{bmatrix} 2 & 1 & -1 \\ 13 & 2 & 14 \\ -6 & 3 & -1 \end{bmatrix} \quad \ldots\text{(i)}$$

$$BA = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+2+4 & 2+3-12 & 2+0+4 \\ -3+0+2 & 3+0-6 & 3+0+2 \\ -1+4+1 & 1+6-3 & 1+0+1 \end{bmatrix}$$

$$\therefore \quad BA = \begin{bmatrix} 4 & -7 & 6 \\ -1 & -3 & 5 \\ 4 & 4 & 2 \end{bmatrix} \quad \ldots\text{(ii)}$$

From (i) and (ii), we get
AB ≠ BA

Question 4.
Show that AB = BA, where
i. A = ⎡⎣⎢⎢ −2−1−6329−1−1−4 ⎤⎦⎥⎥ , B = ⎡⎣⎢⎢ 123320−1−1−1 ⎤⎦⎥⎥
ii. A = [cosθ−sinθsinθcosθ], B = [cosθsinθ−sinθcosθ]
Solution:

$$\text{i.} \quad AB = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+6-3 & -6+6-0 & 2-3+1 \\ -1+4-3 & -3+4-0 & 1-2+1 \\ -6+18-12 & -18+18+0 & 6-9+4 \end{bmatrix}$$

$$\therefore \quad AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \ldots\text{(i)}$$

$$BA = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4-2+6 & 6+4-9 & -2-2+4 \\ -6+0+6 & 9+0-9 & -3+0+4 \end{bmatrix}$$

$$\therefore \quad BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \ldots\text{(ii)}$$

From (i) and (ii), we get
AB = BA

ii. $$AB = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & -\sin\theta\cos\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \ldots\text{(i)}$$

$$BA = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \ldots\text{(ii)}$$

From (i) and (ii), we get
AB = BA
[Note: The question has been modified.]

Question 5.
If A = [4−28−4], prove that A^2 = 0.
Solution:
A^2 = A.A
= [4−28−4][4−28−4]
= [16−16−8+832−32−16+16]
= [0000] = 0

Question 6.
Verify A(BC) = (AB)C in each of the following cases:
i. A = =[42−23], B = [−131−2] and C = [421−1]
ii. A = [12−1332], B = ⌈ ⌊ | 1−24033⌉ ⌋ | and C = ⌈ ⌊ | 1−2420−3⌉ ⌋ |
Solution:

i. $$BC = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-4 & 4-0 & -2+4 \\ -3+2 & -2+0 & 1-2 \\ 0+6 & 0+0 & 0-6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 2 \\ -1 & -2 & -1 \\ 6 & 0 & -6 \end{bmatrix}$$

$$\therefore \quad A(BC) = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 & 2 \\ -1 & -2 & -1 \\ 6 & 0 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0+6 & 4-0+0 & 2-0-6 \\ 4-3+0 & 8-6+0 & 4-3-0 \\ 0-4+30 & 0-8+0 & 0-4-30 \end{bmatrix}$$

$$\therefore \quad A(BC) = \begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & -34 \end{bmatrix} \qquad \ldots(i)$$

$$AB = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+0 & -2+0+3 \\ 4-3+0 & -4+3+0 \\ 0-4+0 & 0+4+15 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix}$$

$$\therefore \quad (AB)C = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6+2 & 4+0 & -2-2 \\ 3-2 & 2-0 & -1+2 \\ -12+38 & -8+0 & 4-38 \end{bmatrix}$$

$$\therefore \quad (AB)C = \begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & -34 \end{bmatrix} \qquad \ldots(ii)$$

$$\therefore \quad A(BC) = \begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & -34 \end{bmatrix} \qquad \ldots(i)$$

$$AB = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+0 & -2+0+3 \\ 4-3+0 & -4+3+0 \\ 0-4+0 & 0+4+15 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix}$$

$$\therefore \quad (AB)C = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6+2 & 4+0 & -2-2 \\ 3-2 & 2-0 & -1+2 \\ -12+38 & -8+0 & 4-38 \end{bmatrix}$$

$$\therefore \quad (AB)C = \begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & -34 \end{bmatrix} \qquad \ldots(ii)$$

From (i) and (ii), we get
A(BC) = (AB)C.

ii. $BC = \begin{bmatrix} 2 & -2 \\ 3 & 3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 6-2 & 2-6 \\ 9+3 & 3+9 \\ -3+1 & -1+3 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 12 & 12 \\ -2 & 2 \end{bmatrix}$

$\therefore \quad A(BC) = \begin{bmatrix} 2 & 4 & 3 \\ -1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 4 & -4 \\ 12 & 12 \\ -2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 8+48-6 & -8+48+6 \\ -4+36-4 & 4+36+4 \end{bmatrix}$

$\therefore \quad A(BC) = \begin{bmatrix} 50 & 46 \\ 28 & 44 \end{bmatrix}$ ...(i)

$AB = \begin{bmatrix} 2 & 4 & 3 \\ -1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 3 & 3 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4+12-3 & -4+12+3 \\ -2+9-2 & 2+9+2 \end{bmatrix}$

$\therefore \quad AB = \begin{bmatrix} 13 & 11 \\ 5 & 13 \end{bmatrix}$

$\therefore \quad (AB)C = \begin{bmatrix} 13 & 11 \\ 5 & 13 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 39+11 & 13+33 \\ 15+13 & 5+39 \end{bmatrix}$

$\therefore \quad (AB)C = \begin{bmatrix} 50 & 46 \\ 28 & 44 \end{bmatrix}$ ...(ii)

From (i) and (ii), we get
A(BC) = (AB)C

Question 7.
Verify that A(B + C) = AB + AC in each of the following matrices:
i. A = [42−23], B = [−131−2] and C = =[421−1]
ii. A = [12−1332], B = [⌊ ⎢ 1−24033 ⌋ ⎥ and C = [⌊ ⎢ 1−2420−3 ⌋ ⎥
Solution:

i. $$A(B+C) = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \left\{ \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1+4 & 1+1 \\ 3+2 & -2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 12-10 & 8+6 \\ 6+15 & 4-9 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 14 \\ 21 & -5 \end{bmatrix} \quad \ldots(i)$$

AB + AC

$$= \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -4-6 & 4+4 \\ -2+9 & 2-6 \end{bmatrix} + \begin{bmatrix} 16-4 & 4+2 \\ 8+6 & 2-3 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 8 \\ 7 & -4 \end{bmatrix} + \begin{bmatrix} 12 & 6 \\ 14 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -10+12 & 8+6 \\ 7+14 & -4-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 14 \\ 21 & -5 \end{bmatrix} \quad \ldots(ii)$$

From (i) and (ii), we get
A(B + C) = AB + AC.
[Note: The question has been modified.]

ii. $$A(B+C) = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 0 \\ -2 & 3 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -2 & 0 \\ 4 & -3 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1+1 & 0+2 \\ -2-2 & 3+0 \\ 4+4 & 3-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -4 & 3 \\ 8 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4+24 & 2-3+0 \\ 4-12+16 & 4+9+0 \end{bmatrix}$$

$$= \begin{bmatrix} 30 & -1 \\ 8 & 13 \end{bmatrix} \quad \ldots(i)$$

AB + AC

$$= \begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 3 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & 0 \\ 4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+12 & 0-3+9 \\ 2-6+8 & 0+9+6 \end{bmatrix} + \begin{bmatrix} 1+2+12 & 2+0-9 \\ 2-6+8 & 4+0-6 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 6 \\ 4 & 15 \end{bmatrix} + \begin{bmatrix} 15 & -7 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 15+15 & 6-7 \\ 4+4 & 15-2 \end{bmatrix}$$

$$= \begin{bmatrix} 30 & -1 \\ 8 & 13 \end{bmatrix} \quad \ldots(ii)$$

Question 8.
If A = $[15-26]$, B = $[33-17]$, find AB – 2I, where I is unit matrix of order 2.

Solution:

$$\mathbf{AB-2I} = \begin{bmatrix} 1 & -2 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 3 & 7 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-6 & -1-14 \\ 15+18 & -5+42 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -15 \\ 33 & 37 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3-2 & -15-0 \\ 33-0 & 37-2 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -15 \\ 33 & 35 \end{bmatrix}$$

Question 9.

If A = $[4−13220]$, B = ⌈ ⌊ | $1−1120−2$⌉ ⌋ | , show that matrix AB is non singular.

Solution:

im

∴ AB is non-singular matrix.

Question 10.

If A = , find the product (A + I)(A – I).

Solution:

$$\mathbf{A+I} = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+0 & 0+0 \\ 5+0 & 4+1 & 2+0 \\ 0+0 & 7+0 & -3+1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 \\ 5 & 5 & 2 \\ 0 & 7 & -2 \end{bmatrix}$$

$$\mathbf{A-I} = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-0 & 0-0 \\ 5-0 & 4-1 & 2-0 \\ 0-0 & 7-0 & -3-1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 5 & 3 & 2 \\ 0 & 7 & -4 \end{bmatrix}$$

$$\therefore \quad \mathbf{(A+I)(A-I)}$$

$$= \begin{bmatrix} 2 & 2 & 0 \\ 5 & 5 & 2 \\ 0 & 7 & -2 \end{bmatrix}\begin{bmatrix} 0 & 2 & 0 \\ 5 & 3 & 2 \\ 0 & 7 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0+10+0 & 4+6+0 & 0+4-0 \\ 0+25+0 & 10+15+14 & 0+10-8 \\ 0+35+0 & 0+21-14 & 0+14+8 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 10 & 4 \\ 25 & 39 & 2 \\ 35 & 7 & 22 \end{bmatrix}$$

[Note : Answer given in the textbook is ⌈ ⌊ | $91535632−74−229$⌉ ⌋ |
However, as per our calculation it is ⌈ ⌊ | 102535103974222⌉ ⌋ | .]

Question 11.

If A = $[\alpha 101]$, B = $[1201]$, find α, if A^2 = B.

Solution:

A^2 = B

$$\therefore \quad \begin{bmatrix}\alpha & 0\\1 & 1\end{bmatrix}\begin{bmatrix}\alpha & 0\\1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}\alpha^2+0 & 0+0\\\alpha+1 & 0+1\end{bmatrix}=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}\alpha^2 & 0\\\alpha+1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

∴ By equality of matrices, we get
$\alpha^2 = 1$ and $\alpha + 1 = 2$
∴ $\alpha = \pm 1$ and $\alpha = 1$
∴ $\alpha = 1$

Question 12.
If A = $\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix}$, show that $A^2 – 4A$ is scalar matrix.
Solution:
$A^2 – 4A = A.A – 4A$

$$=\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix}-4\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+4+4 & 2+2+4 & 2+4+2\\2+2+4 & 4+1+4 & 4+2+2\\2+4+2 & 4+2+2 & 4+4+1\end{bmatrix}-\begin{bmatrix}4 & 8 & 8\\8 & 4 & 8\\8 & 8 & 4\end{bmatrix}$$

$$=\begin{bmatrix}9 & 8 & 8\\8 & 9 & 8\\8 & 8 & 9\end{bmatrix}-\begin{bmatrix}4 & 8 & 8\\8 & 4 & 8\\8 & 8 & 4\end{bmatrix}$$

$$=\begin{bmatrix}9-4 & 8-8 & 8-8\\8-8 & 9-4 & 8-8\\8-8 & 8-8 & 9-4\end{bmatrix}$$

$$=\begin{bmatrix}5 & 0 & 0\\0 & 5 & 0\\0 & 0 & 5\end{bmatrix}, \text{ which is a scalar matrix.}$$

Question 13.
If A = $\begin{bmatrix}1 & 0\\-1 & 7\end{bmatrix}$, find k so that $A^2 – 8A – kI = O$, where I is a unit matrix and O is a null matrix of order 2.
Solution:
$A^2 – 8A – kI = O$
∴ $A.A – 8A – kI = O$

$$\therefore \quad \begin{bmatrix}1 & 0\\-1 & 7\end{bmatrix}\begin{bmatrix}1 & 0\\-1 & 7\end{bmatrix}-8\begin{bmatrix}1 & 0\\-1 & 7\end{bmatrix}-k\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}1+0 & 0+0\\-1-7 & 0+49\end{bmatrix}-\begin{bmatrix}8 & 0\\-8 & 56\end{bmatrix}-\begin{bmatrix}k & 0\\0 & k\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}1 & 0\\-8 & 49\end{bmatrix}-\begin{bmatrix}8 & 0\\-8 & 56\end{bmatrix}-\begin{bmatrix}k & 0\\0 & k\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}1-8-k & 0-0-0\\-8+8-0 & 49-56-k\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

∴ by equality of matrices, we get
$1 – 8 – k = 0$
∴ $k = -7$

Question 14.
If A = $\begin{bmatrix}8 & 10\\4 & 5\end{bmatrix}$, B = $\begin{bmatrix}5 & 10\\-4 & -8\end{bmatrix}$, show that $(A+B)^2 = A^2 + AB + B^2$.
Solution:
We have to prove that $(A + B)^2 = A^2 + AB + B^2$,

i.e., to prove $A^2 + AB + BA + B^2 = A^2 + AB + B^2$,
i.e., to prove BA = 0.
BA = [510–4–8][81045]
[40–4080–8020–2040–40]=[0000]

Question 15.
If A = [3–112], prove that $A^2 - 5A + 7I = 0$, where I is unit matrix of order 2.
Solution:
$A^2 - 5A + 7I = 0 = A.A - 5A + 7I = 0$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$
$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$
$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Question 16.
If A = [3–443] and B = [2–112], show that $(A + B)(A - B) = A^2 - B^2$.
Solution:
We have to prove that $(A + B)(A - B) = A^2 - B^2$,
i.e., to prove $A^2 - AB + BA - B^2 = A^2 - B^2$,
i.e., to prove – AB + BA = 0,
i.e., to prove AB – BA.

$$AB = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 6-4 & 3+8 \\ -8-3 & -4+6 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 11 \\ -11 & 2 \end{bmatrix} \quad \ldots(1)$$
$$BA = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 6-4 & 8+3 \\ -3-8 & -4+6 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 11 \\ -11 & 2 \end{bmatrix} \quad \ldots(ii)$$

From (i) and (ii), we get AB = BA

Question 17.
If A = [1–12–2], B = [2–1ab] and $(A + B)^2 = A^2 + B^2$, find the values of a and b.
Solution:
Given, $(A + B)^2 = A^2 + B^2$
∴ $A^2 + AB + BA + B^2 = A^2 + B^2$
∴ AB + BA = 0
∴ AB = -BA

$$\therefore \quad \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix} = -\begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$
$$\therefore \quad \begin{bmatrix} 2-2 & a+2b \\ -2+2 & -a-2b \end{bmatrix} = -\begin{bmatrix} 2-a & 4-2a \\ -1-b & -2-2b \end{bmatrix}$$
$$\therefore \quad \begin{bmatrix} 0 & a+2b \\ 0 & -a-2b \end{bmatrix} = \begin{bmatrix} -2+a & -4+2a \\ 1+b & 2+2b \end{bmatrix}$$

∴ by equality of matrices, we get
– 2 + a = 0 and 1 + b = 0
a = 2 and b = -1
[Note: The question has been modified.]

Question 18.
Find matrix X such that AX = B,
where A = $[1-2-21]$ and B = $[-3-1]$
Solution:
Let X = $[ab]$
But AX = B
∴ $[1-2-21][ab]=[-3-1]$
∴ $[a-2b-2a+b]=[-3-1]$
By equality of matrices, we get
a – 2b = -3 …(i)
-2a + b = -l …(ii)
By (i) x 2 + (ii), we get
-3b =-7
∴ b = 73
Substituting b = 73 in (i), we get
a – 2 (73) = -3
∴ a = -3 + $143=53$
∴ X = $[5373]$

Question 19.
Find k, if A = $[34-2-2]$ and A_2 = KA – 2I
Solution:
A_2 = kA – 2I
∴ AA + 2I = kA

$$\therefore \quad \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}+2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}.$$

$$\therefore \quad \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix}+\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}=\begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}+\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}=\begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$$

∴ $[34-2-2]=[3k4k-2k-2k]$
∴ By equality of matrices, we get
3k = 3
∴ k = 1

Question 20.
Find x, if $[1x1]$⎡ ⎣ ⎢ 143252365⎤ ⎦ ⎥ ⎡ ⎣ ⎢ $1-23$⎤ ⎦ ⎥ = 0
Solution:

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}=0$$

$$\therefore \quad \begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1-4+9 \\ 4-10+18 \\ 3-4+15 \end{bmatrix}=0$$

$$\therefore \quad \begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 12 \\ 14 \end{bmatrix}=0$$

∴ [6 + 12x + 14] =[0]
∴ By equality of matrices, we get
∴ 12x + 20 = 0
∴ 12x =-20
∴ x = -53

Question 21.
Find x and y, if $\{4[21-1032]-[32-3141]\}$⎡ ⎣ ⎢ $2-11$⎤ ⎦ ⎥ $=[xy]$
Solution:

$$\left\{4\begin{bmatrix}2 & -1 & 3\\1 & 0 & 2\end{bmatrix}-\begin{bmatrix}3 & -3 & 4\\2 & 1 & 1\end{bmatrix}\right\}\begin{bmatrix}2\\-1\\1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\therefore \quad \left\{\begin{bmatrix}8 & -4 & 12\\4 & 0 & 8\end{bmatrix}-\begin{bmatrix}3 & -3 & 4\\2 & 1 & 1\end{bmatrix}\right\}\begin{bmatrix}2\\-1\\1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}8-3 & -4+3 & 12-4\\4-2 & 0-1 & 8-1\end{bmatrix}\begin{bmatrix}2\\-1\\1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}5 & -1 & 8\\2 & -1 & 7\end{bmatrix}\begin{bmatrix}2\\-1\\1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}10+1+8\\4+1+7\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}19\\12\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

∴ By equality of matrices, we get
x = 19 andy = 12

Question 22.
Find x, y, z if
⌈ ⌊ ⎨ | | 3⌈ ⌊ | 202022⌉ | | −4⌈ ⌊ | 1−13121⌉ ⌋ | | ⌉ ⌋ ⎬ | | | [12]=⌈ ⌊ | x−3y−12z⌉ ⌋ |
Solution:

$$\left\{3\begin{bmatrix}2 & 0\\0 & 2\\2 & 2\end{bmatrix}-4\begin{bmatrix}1 & 1\\-1 & 2\\3 & 1\end{bmatrix}\right\}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \quad \left\{\begin{bmatrix}6 & 0\\0 & 6\\6 & 6\end{bmatrix}-\begin{bmatrix}4 & 4\\-4 & 8\\12 & 4\end{bmatrix}\right\}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}6-4 & 0-4\\0+4 & 6-8\\6-12 & 6-4\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}2 & -4\\4 & -2\\-6 & 2\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}2-8\\4-4\\-6+4\end{bmatrix}=\begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}-6\\0\\-2\end{bmatrix}=\begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

∴ By equality of matrices, we get
x – 3 = -6,y – 1 = 0, 2z = -2
∴ x = – 3, y = 1, z = – 1

Question 23.
If A = [cosα−sinαsinαcosα] show that A_2 = =[cos2α−sin2αsin2αcos2α]

Solution:

$$A^2 = A.A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & \cos\alpha\sin\alpha + \cos\alpha\sin\alpha \\ -\cos\alpha\sin\alpha - \cos\alpha\sin\alpha & -\sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & 2\sin\alpha\cos\alpha \\ -2\sin\alpha\cos\alpha & \cos^2\alpha - \sin^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

Question 24.
If A = $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, B = $\begin{bmatrix} 0 & 4 \\ 2 & -1 \end{bmatrix}$
show that AB ≠ BA, but |AB| = |A| . |B|.
Solution:
AB = $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 2 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 0+4 & 4-2 \\ 0+10 & 12-5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 \\ 10 & 7 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0+12 & 0+20 \\ 2-3 & 4-5 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 20 \\ -1 & -1 \end{bmatrix} \neq AB$$

Now, |AB| = $\begin{vmatrix} 4 & 2 \\ 10 & 7 \end{vmatrix}$ = 28 – 20 = 8

|A| = $\begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix}$ = 5 – 6 = -1

|B| = $\begin{vmatrix} 0 & 4 \\ 2 & -1 \end{vmatrix}$ = 0 – 8 = -8

∴ |A| . |B| = (-1).(-8) = 8 = |AB|
∴ AB ≠ BA, but |AB| = |A|.|B|

Question 25.
Jay and Ram are two friends in a class. Jay wanted to buy 4 pens and 8 notebooks, Ram wanted to buy 5 pens and 12 notebooks. Both of them went to a shop. The price of a pen and a notebook which they have selected was 6 and ₹ 10. Using matrix multiplication, find the amount required from each one of them.
Solution:
Let A be the matrix of pens and notebooks and B be the matrix of prices of one pen and one notebook.

$$\therefore \quad A = \begin{array}{c} \begin{array}{cc} \text{Pens} & \text{Notebooks} \end{array} \\ \begin{bmatrix} 4 & 8 \\ 5 & 12 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Jay} \\ \text{Ram} \end{array}$$

$$\text{and } B = \begin{bmatrix} 6 \\ 10 \end{bmatrix} \begin{array}{l} \text{Pen} \\ \text{Notebook} \end{array}$$

The total amount required for each one of them is obtained by matrix AB.

$$\therefore \quad AB = \begin{bmatrix} 4 & 8 \\ 5 & 12 \end{bmatrix} \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 24+80 \\ 30+120 \end{bmatrix}$$

$$= \begin{bmatrix} 104 \\ 150 \end{bmatrix}$$

∴ Jay needs ₹ 104 and Ram needs ₹ 150.

Maharashtra State Board 11th Maths Solutions Chapter 4 Determinants and Matrices Ex 4.7

Question 1.
Find A^T, if
i. A = [1-435]
ii. A = [2-4-6015]
Solution:
i. A = [1-435]
∴ A^T = [13-45]

ii. A = [-4052-61]
∴ A^T = ⎡ ⎣ ⎢ 2-61-405⎤ ⎦ ⎥

[Note: Answer given in the textbook is A^T = ⎡ ⎣ ⎢ 261-405⎤ ⎦ ⎥ . However, as per our calculation it is A^T = ⎡ ⎣ ⎢ 2-61-405⎤ ⎦ ⎥ .]

Question 2.
If $[a_{ij}]_{3\times3}$ where $a_{ij} = 2(i – j)$, find A and A^T. State whether A and A^T are symmetric or skew-symmetric matrices?
Solution:
A = $[a_{ij}]_{3\times3}$ = ⎡ ⎣ ⎢ a11a21a31a12a22a32a13a23a33⎤ ⎦ ⎥
Given a_{ij} = 2 (i — j)
∴ a_{11} = 2(1-1) = 0,
a_{12} = 2(1-2) = -2,
a_{13} = 2(1-3) = -4,
a_{21} = 2(2-1) = 2,
a_{22} = 2(2-2) = 0,
a_{23}=2(2-3) = -2,
a_{31} = 2(3-1) = 4,
a_{32} = 2(3-2) = 2,
a_{33}=2(3-3) = 0

$$\therefore \quad A = \begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -2 \\ 4 & 2 & 0 \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & 2 \\ -4 & -2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -2 \\ 4 & 2 & 0 \end{bmatrix} = -A$$

∴ A^T = -A and A = $-A^T$
∴ A and A^T both are skew-symmetric matrices.

Questionn 3.
If A = ⎡ ⎣ ⎢ 54-2-3-31⎤ ⎦ ⎥ , prove that $(2A)^T = 2A^T$.
Solution:

$$A = \begin{bmatrix} 5 & -3 \\ 4 & -3 \\ -2 & 1 \end{bmatrix}$$

$$\therefore \quad 2A = 2\begin{bmatrix} 5 & -3 \\ 4 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 10 & -6 \\ 8 & -6 \\ -4 & 2 \end{bmatrix}$$

$$\therefore \quad (2A)^T = \begin{bmatrix} 10 & 8 & -4 \\ -6 & -6 & 2 \end{bmatrix} \qquad \ldots(i)$$

$$A^T = \begin{bmatrix} 5 & 4 & -2 \\ -3 & -3 & 1 \end{bmatrix}$$

$$\therefore \quad 2A^T = 2\begin{bmatrix} 5 & 4 & -2 \\ -3 & -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 8 & -4 \\ -6 & -6 & 2 \end{bmatrix} \qquad \ldots(ii)$$

From (i) and (ii), we get
$(2A)^T = 2A^T$

Question 4.
If A = ⌈ ⌊ | 12−52−34−549⌉ ⌋ | , prove that $(3A)^T = 3A^T$.
Solution:

$$A = \begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$$

$$\therefore \quad 3A = 3\begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 3 & 6 & -15 \\ 6 & -9 & 12 \\ -15 & 12 & 27 \end{bmatrix}$$

$$\therefore \quad (3A)^T = \begin{bmatrix} 3 & 6 & -15 \\ 6 & -9 & 12 \\ -15 & 12 & 27 \end{bmatrix} \qquad \ldots(i)$$

$$A^T = \begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$$

$$\therefore \quad 3A^T = 3\begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 3 & 6 & -15 \\ 6 & -9 & 12 \\ -15 & 12 & 27 \end{bmatrix} \ldots(ii)$$

From (i) and (ii), we get
$(3A)^T = 3A^T$

Question 5.
If A = ⌈ ⌊ | 0−1−2i2−i1+2i071−2−70⌉ ⌋ | ,
where i = −1−−−√, prove that $A^T = -A$.

Solution:

$$A = \begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} 0 & -1-2i & 2-i \\ 1+2i & 0 & 7 \\ i-2 & -7 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$$

$$\therefore \quad A^T = -A$$

Question 6.

If $A = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 4 & -1 \\ -3 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ -2 & 3 \end{bmatrix}$ then show that

i. $(A + B)^T = A^T + B^T$

ii. $(A - C)^T = A^T - C^T$

Solution:

$$\text{i.} \quad A + B = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 4 & -1 \\ -3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & -3+1 \\ 5+4 & -4-1 \\ -6-3 & 1+3 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 9 & -5 \\ -9 & 4 \end{bmatrix}$$

$$\therefore \quad (A+B)^T = \begin{bmatrix} 4 & 9 & -9 \\ -2 & -5 & 4 \end{bmatrix} \qquad \ldots\text{(i)}$$

$$\text{Now, } A^T = \begin{bmatrix} 2 & 5 & -6 \\ -3 & -4 & 1 \end{bmatrix} \text{ and } B^T = \begin{bmatrix} 2 & 4 & -3 \\ 1 & -1 & 3 \end{bmatrix}$$

$$\therefore \quad A^T + B^T = \begin{bmatrix} 2 & 5 & -6 \\ -3 & -4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 4 & -3 \\ 1 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 5+4 & -6-3 \\ -3+1 & -4-1 & 1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 9 & -9 \\ -2 & -5 & 4 \end{bmatrix} \qquad \ldots\text{(ii)}$$

From (i) and (ii), we get

$(A + B)^T = A^T + B^T$

[Note: The question has been modified.]

ii. $$A - C = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & -3-2 \\ 5+1 & -4-4 \\ -6+2 & 1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -5 \\ 6 & -8 \\ -4 & -2 \end{bmatrix}$$

$$\therefore \quad (A - C)^T = \begin{bmatrix} 1 & 6 & -4 \\ -5 & -8 & -2 \end{bmatrix} \quad \ldots\text{(i)}$$

Now, $A^T = \begin{bmatrix} 2 & 5 & -6 \\ -3 & -4 & 1 \end{bmatrix}$ and

$$C^T = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 4 & 3 \end{bmatrix}$$

$$\therefore \quad A^T - C^T = \begin{bmatrix} 2 & 5 & -6 \\ -3 & -4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 & -2 \\ 2 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 5+1 & -6+2 \\ -3-2 & -4-4 & 1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 & -4 \\ -5 & -8 & -2 \end{bmatrix} \quad \ldots\text{(ii)}$$

From (i) and (ii), we get
$(A – C)^T = A^T – C^T$</sup

Question 7.
If A = [5−243] and [−143−1] then find C^T, such that 3A – 2B + C = I, where I is the unit matrix of order 2.
Solution:
3A – 2B + C = I
∴ C = I + 2B – 3A

$$\therefore \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 2\begin{bmatrix} -1 & 3 \\ 4 & -1 \end{bmatrix} - 3\begin{bmatrix} 5 & 4 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 6 \\ 8 & -2 \end{bmatrix} - \begin{bmatrix} 15 & 12 \\ -6 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2-15 & 0+6-12 \\ 0+8+6 & 1-2-9 \end{bmatrix}$$

$$\therefore \quad C = \begin{bmatrix} -16 & -6 \\ 14 & -10 \end{bmatrix}$$

$$\therefore \quad C^T = \begin{bmatrix} -16 & 14 \\ -6 & -10 \end{bmatrix}$$

Question 8.
If A = [70340−2], B = [02−213−4], then find
i. $A^T + 4B^T$
ii. $5A^T – 5B^T$

Solution:

$$A = \begin{bmatrix} 7 & 3 & 0 \\ 0 & 4 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 1 & -4 \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} 7 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix} \text{ and } B^T = \begin{bmatrix} 0 & 2 \\ -2 & 1 \\ 3 & -4 \end{bmatrix}$$

i.

$$A^T + 4B^T = \begin{bmatrix} 7 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix} + 4\begin{bmatrix} 0 & 2 \\ -2 & 1 \\ 3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 8 \\ -8 & 4 \\ 12 & -16 \end{bmatrix}$$

$$= \begin{bmatrix} 7+0 & 0+8 \\ 3-8 & 4+4 \\ 0+12 & -2-16 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ -5 & 8 \\ 12 & -18 \end{bmatrix}$$

ii. ii. $5A^T - 5B^T = 5(A^T - B^T)$

$$= 5\left(\begin{bmatrix} 7 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ -2 & 1 \\ 3 & -4 \end{bmatrix}\right)$$

$$= 5\begin{bmatrix} 7-0 & 0-2 \\ 3+2 & 4-1 \\ 0-3 & -2+4 \end{bmatrix} = 5\begin{bmatrix} 7 & -2 \\ 5 & 3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 35 & -10 \\ 25 & 15 \\ -15 & 10 \end{bmatrix}$$

Question 9.

If A = [130112], B = [2315−4−2] and C = [0−12−130], verify that $(A + 2B + 3C)^T = A^T + 2B^T + 3C^T$.

Solution:

A + 2B + 3C

$$= \begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 2 & 1 & -4 \\ 3 & 5 & -2 \end{bmatrix} + 3\begin{bmatrix} 0 & 2 & 3 \\ -1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 2 & -8 \\ 6 & 10 & -4 \end{bmatrix} + \begin{bmatrix} 0 & 6 & 9 \\ -3 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+0 & 0+2+6 & 1-8+9 \\ 3+6-3 & 1+10-3 & 2-4+0 \end{bmatrix}$$

$$\therefore \quad A + 2B + 3C = \begin{bmatrix} 5 & 8 & 2 \\ 6 & 8 & -2 \end{bmatrix}$$

$$\therefore \quad [A+2B+3C]^T = \begin{bmatrix} 5 & 6 \\ 8 & 8 \\ 2 & -2 \end{bmatrix} \qquad \ldots(i)$$

$$\text{Now, } A^T = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}, B^T = \begin{bmatrix} 2 & 3 \\ 1 & 5 \\ -4 & -2 \end{bmatrix}$$

$$\text{and } C^T = \begin{bmatrix} 0 & -1 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$$

$$\therefore \quad A^T + 2B^T + 3C^T$$

$$= \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 2 & 3 \\ 1 & 5 \\ -4 & -2 \end{bmatrix} + 3\begin{bmatrix} 0 & -1 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ 2 & 10 \\ -8 & -4 \end{bmatrix} + \begin{bmatrix} 0 & -3 \\ 6 & -3 \\ 9 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+0 & 3+6-3 \\ 0+2+6 & 1+10-3 \\ 1-8+9 & 2-4+0 \end{bmatrix}$$

∴ $A^T + 2B^T + 3C^T$ = ⎡ ⎣ ⎢ 58268−2⎤ ⎦ ⎥

From (i) and (ii), we get

$(A + 2B + 3C)^T = A^T + 2B^T + 3C^T$

Question 10.

If A = [−1−3221−3] and B = ⎡ ⎣ ⎢ 2−3−1123⎤ ⎦ ⎥ , prove that $(A + B^T)^T = A^T + B$.

prove that $(A + B^T)^T = A^T + B$

Solution:

$$A = \begin{bmatrix} -1 & 2 & 1 \\ -3 & 2 & -3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ -3 & 2 \\ -1 & 3 \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} -1 & -3 \\ 2 & 2 \\ 1 & -3 \end{bmatrix} \text{ and } B^T = \begin{bmatrix} 2 & -3 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\therefore \quad A+B^T = \begin{bmatrix} -1 & 2 & 1 \\ -3 & 2 & -3 \end{bmatrix} + \begin{bmatrix} 2 & -3 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 2-3 & 1-1 \\ -3+1 & 2+2 & -3+3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & 0 \end{bmatrix}$$

$$\therefore \quad (A+B^T)^T = \begin{bmatrix} 1 & -2 \\ -1 & 4 \\ 0 & 0 \end{bmatrix} \quad \ldots(i)$$

$$\text{Now, } A^T+B = \begin{bmatrix} -1 & -3 \\ 2 & 2 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ -3 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & -3+1 \\ 2-3 & 2+2 \\ 1-1 & -3+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ -1 & 4 \\ 0 & 0 \end{bmatrix} \quad \ldots(ii)$$

From (i) and (ii), we get
$(A + B^T)^T = A^T + B$

Question 11.
Prove that $A + A^T$ is a symmetric and $A – A^T$ is a skew symmetric matrix, where
i. A = ⎡ ⎣ ⎢ 13-222-3412⎤ ⎦ ⎥
ii. A = ⎡ ⎣ ⎢ 5342-7-5-42-3⎤ ⎦ ⎥
Solution:

$$\text{i.} \quad A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & -3 & 2 \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}$$

$$\therefore \quad A + A^T = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & -3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 & -2 \\ 2 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+3 & 4-2 \\ 3+2 & 2+2 & 1-3 \\ -2+4 & -3+1 & 2+2 \end{bmatrix}$$

$$\therefore \quad A + A^T = \begin{bmatrix} 2 & 5 & 2 \\ 5 & 4 & -2 \\ 2 & -2 & 4 \end{bmatrix}$$

$$\therefore \quad (A + A^T)^T = \begin{bmatrix} 2 & 5 & 2 \\ 5 & 4 & -2 \\ 2 & -2 & 4 \end{bmatrix}$$

∴ $(A + A^T)^T = A + A^T$, i.e., $A + A^T = (A + A^T)^T$
∴ $A + A^T$ is a symmetric matrix.

$$A - A^T = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & -3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -2 \\ 2 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-3 & 4+2 \\ 3-2 & 2-2 & 1+3 \\ -2-4 & -3-1 & 2-2 \end{bmatrix}$$

$$\therefore \quad A - A^T = \begin{bmatrix} 0 & -1 & 6 \\ 1 & 0 & 4 \\ -6 & -4 & 0 \end{bmatrix}$$

$$\therefore \quad (A - A^T)^T = \begin{bmatrix} 0 & 1 & -6 \\ -1 & 0 & -4 \\ 6 & 4 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & 6 \\ 1 & 0 & 4 \\ -6 & -4 & 0 \end{bmatrix}$$

∴ $(A – A^T)^T = – (A – A^T)$,
i.e., $A – A^T = -(A – A^T)^T$
∴ $A – A^T$ is skew symmetric matrix.

ii. $A = \begin{bmatrix} 5 & 2 & -4 \\ 3 & -7 & 2 \\ 4 & -5 & -3 \end{bmatrix}$

$$\therefore \quad A^T = \begin{bmatrix} 5 & 3 & 4 \\ 2 & -7 & -5 \\ -4 & 2 & -3 \end{bmatrix}$$

$$\therefore \quad A + A^T = \begin{bmatrix} 5 & 2 & -4 \\ 3 & -7 & 2 \\ 4 & -5 & -3 \end{bmatrix} + \begin{bmatrix} 5 & 3 & 4 \\ 2 & -7 & -5 \\ -4 & 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+5 & 2+3 & -4+4 \\ 3+2 & -7-7 & 2-5 \\ 4-4 & -5+2 & -3-3 \end{bmatrix}$$

$$\therefore \quad A + A^T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & -14 & -3 \\ 0 & -3 & -6 \end{bmatrix}$$

$$\therefore \quad (A + A^T)^T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & -14 & -3 \\ 0 & -3 & -6 \end{bmatrix}$$

∴ $(A + A^T)^T = A + A^T$, i.e., $A + A^T = (A + A^T)^T$
∴ $A + A^T$ is a symmetric matrix.

$$A - A^T = \begin{bmatrix} 5 & 2 & -4 \\ 3 & -7 & 2 \\ 4 & -5 & -3 \end{bmatrix} - \begin{bmatrix} 5 & 3 & 4 \\ 2 & -7 & -5 \\ -4 & 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 5-5 & 2-3 & -4-4 \\ 3-2 & -7+7 & 2+5 \\ 4+4 & -5-2 & -3+3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & -8 \\ 1 & 0 & 7 \\ 8 & -7 & 0 \end{bmatrix}$$

$$\therefore \quad (A - A^T)^T = \begin{bmatrix} 0 & 1 & 8 \\ -1 & 0 & -7 \\ -8 & 7 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & -8 \\ 1 & 0 & 7 \\ 8 & -7 & 0 \end{bmatrix}$$

∴ $(A – A^T)^T = – (A – A^T)$,
i.e., $A – A^T = -(A – A^T)^T$
∴ $A – A^T$ is skew symmetric matrix.

Question 12.
Express the following matrices as the sum of a symmetric and a skew symmetric matrix.
i. [43-2-5]
ii. ⎡ ⎣ ⎢ 3-2-43-2-5-112⎤ ⎦ ⎥
Solution:

A square matrix A can be expressed as the sum of a symmetric and a skew symmetric matrix as
$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

i. Let $A = \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix}$

$\therefore \quad A^T = \begin{bmatrix} 4 & 3 \\ -2 & -5 \end{bmatrix}$

$\therefore \quad A + A^T = \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -2 & -5 \end{bmatrix}$

$= \begin{bmatrix} 8 & 1 \\ 1 & -10 \end{bmatrix}$

Also, $A - A^T = \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ -2 & -5 \end{bmatrix}$

$= \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A^T) = \begin{bmatrix} 4 & \frac{1}{2} \\ \frac{1}{2} & -5 \end{bmatrix}$

and $Q = \frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$

P is symmetric matrix ...[∵ $a_{ij} = a_{ji}$]
and Q is a skew symmetric matrix [∵ $-a_{ij} = -a_{ji}$]
A = P + Q
$A = \begin{bmatrix} 4 & \frac{1}{2} \\ \frac{1}{2} & -5 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} \\ -\frac{5}{2} & 0 \end{bmatrix}$

ii. Let $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$\therefore \quad A^T = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

$\therefore \quad A + A^T = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$\text{Also, } A - A^T = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A^T) = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}$$

$$\text{and } Q = \frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ \frac{-3}{2} & -3 & 0 \end{bmatrix}$$

∴ P is symmetric matrix ...[∵ $a_{ij} = a_{ji}$]
and Q is a skew symmetric matrix [∵ $-a_{ij} = -a_{ji}$]
∴ A = P + Q
∴ A = [41212-5]+[052-520]

Question 13.
If A = [⌊ | 234-1-21] | | and B = [023-1-41], verify that
i. $(AB)^T = B^T A^T$
ii. $(BA)^T = A^T B^T$
Solution:

$$A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 3 & -4 \\ 2 & -1 & 1 \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} 2 & 3 & 4 \\ -1 & -2 & 1 \end{bmatrix} \text{ and } B^T = \begin{bmatrix} 0 & 2 \\ 3 & -1 \\ -4 & 1 \end{bmatrix}$$

$$\text{i.} \quad AB = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 0 & 3 & -4 \\ 2 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0-2 & 6+1 & -8-1 \\ 0-4 & 9+2 & -12-2 \\ 0+2 & 12-1 & -16+1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 7 & -9 \\ -4 & 11 & -14 \\ 2 & 11 & -15 \end{bmatrix}$$

$$\therefore \quad (AB)^T = \begin{bmatrix} -2 & -4 & 2 \\ 7 & 11 & 11 \\ -9 & -14 & -15 \end{bmatrix} \quad \ldots(i)$$

$$B^T A^T = \begin{bmatrix} 0 & 2 \\ 3 & -1 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ -1 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0-2 & 0-4 & 0+2 \\ 6+1 & 9+2 & 12-1 \\ -8-1 & -12-2 & -16+1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -4 & 2 \\ 7 & 11 & 11 \\ -9 & -14 & -15 \end{bmatrix} \quad \ldots(ii)$$

From (i) and (ii), we get
$(AB)^T = B^T A^T$

$$\text{ii.} \quad BA = \begin{bmatrix} 0 & 3 & -4 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9-16 & 0-6-4 \\ 4-3+4 & -2+2+1 \end{bmatrix}$$

$$\therefore \quad BA = \begin{bmatrix} -7 & -10 \\ 5 & 1 \end{bmatrix}$$

$$\therefore \quad (BA)^T = \begin{bmatrix} -7 & 5 \\ -10 & 1 \end{bmatrix} \quad \ldots(i)$$

$$A^T B^T = \begin{bmatrix} 2 & 3 & 4 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 3 & -1 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9-16 & 4-3+4 \\ 0-6-4 & -2+2+1 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 5 \\ -10 & 1 \end{bmatrix} \quad \ldots(ii)$$

From (i) and (ii) we get
$(BA)^T = A^T B^T$

Question 14.
If A = $[\cos\alpha - \sin\alpha \sin\alpha \cos\alpha]$, show that $A^T A$ = I, where I is the unit matrix of order 2.
Solution:

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\therefore \quad A^T A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ $A^T A$ = I, where I is the unit matrix of order 2.

Maharashtra State Board 11th Maths Solutions Chapter 4 Determinants and Matrices Miscellaneous Exercise 4(B)

(I) Select the correct option from the given alternatives.

Question 1.
Given A = [1232], I = [1001] if A – λI is a singular matrix, then _________
(a) $\lambda = 0$
(b) $\lambda^2 - 3\lambda - 4 = 0$
(c) $\lambda^2 + 3\lambda - 4 = 0$
(d) $\lambda^2 - 3\lambda - 6 = 0$
Answer:
(b) $\lambda^2 - 3\lambda - 4 = 0$
Hint:

$$A - \lambda I = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\lambda & 3 \\ 2 & 2-\lambda \end{bmatrix}$$

Since A – λI is a singular matrix,

$$\begin{vmatrix} 1-\lambda & 3 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(2-\lambda) - 6 = 0$$
$$2 - 3\lambda + \lambda^2 - 6 = 0$$
$$\lambda^2 - 3\lambda - 4 = 0$$

Question 2.
Consider the matrices A = ⌈ ⌊ ∣ 43160-2-125⌉ ⌋ ∣ , B = ⌈ ⌊ ∣ 20-1412⌉ ⌋ ∣ , C = ⌈ ⌊ ∣ 312⌉ ⌋ ∣ . Out of the given matrix products, _________
(i) $(AB)^TC$
(ii) $C^TC(AB)^T$
(iii) C^TAB
(iv) A^TABB^TC
(a) Exactly one is defined
(b) Exactly two are defined
(c) Exactly three are defined
(d) all four are defined
Answer:
(c) Exactly three are defined
Hint:
A is of order 3 × 3, B is of order 3 × 2 and C is of order 3 × 1.
$(AB)^TC$ is of order 2 × 1.
C^TC and $(AB)^T$ are of different orders.
$C^TC\ (AB)^T$ is not defined.
C^TAB is of order 1 × 2.
A^TABB^TC is of order 3 × 1.

Question 3.
If A and B are square matrices of equal order, then which one is correct among the following?
(a) A + B = B + A
(b) A + B = A – B
(c) A – B = B – A
(d) AB = BA
Answer:
(a) A + B = B + A
Hint:
Matrix addition is commutative.
∴ A + B = B + A

Question 4.
If A = ⌈ ⌊ ∣ 12a2122-2b⌉ ⌋ ∣ is a matrix satisfying the equation $AA^T = 9I$, where I is the identity matrix of order 3, then the ordered pair (a, b) is equal to _________
(a) (2, -1)
(b) (-2, 1)
(c) (2, 1)

(d) (-2, -1)
Answer:
(d) (-2, -1)

Question 5.
If A = [α22α] and $|A^3|$ = 125, then α = __________
(a) ±3
(b) ±2
(c) ±5
(d) 0
Answer:
(a) ±3
Hint:
$|A^3| = 125$
$|A|^3 = 5^3$ …….[∵ $|A^n| = |A|^n$, n ∈ N]
∴ |A| = 5
| α22α | = 5
$\alpha^2 - 4 = 5$
$\alpha^2 = 9$
∴ α = ± 3

Question 6.
If [5x2716] − [1−3225y] = [4405−44], then __________
(a) x = 1, y = -2
(b) x = -1, y = 2
(c) x = 1, y = 2
(d) x = -1, y = -2
Answer:
(c) x = 1, y = 2

Question 7.
If A + B = [7849] and A – B = [1023], then the value of A is __________
(a) [3413]
(b) [4436]
(c) [6826]
(d) [78612]
Answer:
(b) [4436]

Question 8.
If [xzx+z3x−y3y−w]=[3427], then __________
(a) x = 3, y = 7, z = 1, w = 14
(b) x = 3, y = -5, z = -1, w = -4
(c) x = 3, y = 6, z = 2, w = 7
(d) x = -3, y = -7, z = -1, w = -14
Answer:
(a) x = 3, y = 7, z = 1, w = 14

Question 9.
For suitable matrices A, B, the false statement is __________
(a) $(AB)^T = A^TB^T$
(B) $(A^T)^T = A$
(C) $(A - B)^T = A^T - B^T$
(D) $(A + B)^T = A^T + B^T$
Answer:
(a) $(AB)^T = A^TB^T$
Hint:
$(AB)^T = B^TA^T$

Question 10.
If A = [−2013] and $f(x) = 2x^2 - 3x$, then f(A) = __________

(a) [1401−9]

(b) [−14019]

(c) [140−19]

(d) [−140−1−9]

Answer:

(c) [140−19]

Hint:

$$f(A) = 2A^2 - 3A$$

$$= 2\begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix} - 3\begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$$

$$= 2\begin{bmatrix} 4 & 1 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} -6 & 3 \\ 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 8+6 & 2-3 \\ 0-0 & 18-9 \end{bmatrix} = \begin{bmatrix} 14 & -1 \\ 0 & 9 \end{bmatrix}$$

(II) Answer the following questions.

Question 1.
If A = diag[2, -3, -5], B = diag[4, -6, -3] and C = diag [-3, 4, 1], then find
i. B + C – A
ii. 2A + B – 5C.
Solution:

$$A = \text{diag } [2, -3, -5]$$

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$B = \text{diag } [4, -6, -3]$$

$$B = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$C = \text{diag } [-3, 4, 1]$$

$$C = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i. $B + C - A$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -3 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3-2 & 0 & 0 \\ 0 & -6+4+3 & 0 \\ 0 & 0 & -3+1+5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$= \text{diag}\,[-1, 1, 3]$

ii. $2A + B - 5C$

$$= 2\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -5 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -3 \end{bmatrix} - 5\begin{bmatrix} -3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -10 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -3 \end{bmatrix} - \begin{bmatrix} -15 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4+15 & 0 & 0 \\ 0 & -6-6-20 & 0 \\ 0 & 0 & -10-3-5 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & 0 & 0 \\ 0 & -32 & 0 \\ 0 & 0 & -18 \end{bmatrix}$$

$= \text{diag}\,[23, -32, -18]$

Question 2.
If f(α) = A = [⌊ | cosαsinα0−sinαcosα0001⌉ | | , find
i. f(-α)
ii. f(-α) + f(α)
Solution:

i. $$f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad f(-\alpha) = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad f(-\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ii. $f(-\alpha) + f(\alpha)$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \cos\alpha & \sin\alpha - \sin\alpha & 0+0 \\ -\sin\alpha + \sin\alpha & \cos\alpha + \cos\alpha & 0+0 \\ 0+0 & 0+0 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2\cos\alpha & 0 & 0 \\ 0 & 2\cos\alpha & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Question 3.
Find matrices A and B, where
(i) 2A – B = [10−11] and A + 3B = [10−11]

(ii) 3A – B = $[-112015]$ and A + 5B = $[0-10010]$

Solution:

i. Given equations are

2A – B = $[10-11]$ …….(i)

$$\text{and} \quad A + 3B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad \ldots\text{(ii)}$$

By (i) × 3 + (ii), we get

$$7A = 3\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad 7A = \begin{bmatrix} 3 & -3 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad 7A = \begin{bmatrix} 4 & -4 \\ 0 & 4 \end{bmatrix}$$

$$\therefore \quad A = \frac{1}{7}\begin{bmatrix} 4 & -4 \\ 0 & 4 \end{bmatrix}$$

By (i) – (ii) × 2, we get

$$-7B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ 0 & 2 \end{bmatrix}$$

$$\therefore \quad -7B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\therefore \quad B = -\frac{1}{7}\begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\therefore \quad B = \frac{1}{7}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

Answers given in the textbook are $A = \frac{1}{7}\begin{bmatrix} 3 & -4 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{-1}{7} & \frac{-1}{7} \\ \frac{2}{7} & \frac{-1}{7} \end{bmatrix}$. However, as per our calculation we found that $A = \frac{1}{7}\begin{bmatrix} 4 & -4 \\ 0 & 4 \end{bmatrix}$ and $B = \frac{1}{7}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.]

ii. Given equations are

$$3A - B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 5 \end{bmatrix} \quad \ldots(i)$$

and $$A + 5B = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \quad \ldots(ii)$$

By (i) × 5 + (ii), we get

$$16A = 5\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 10 & 5 \\ 5 & 0 & 25 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\therefore \quad 16A = \begin{bmatrix} -5 & 10 & 6 \\ 4 & 0 & 25 \end{bmatrix}$$

$$\therefore \quad A = \frac{1}{16}\begin{bmatrix} -5 & 10 & 6 \\ 4 & 0 & 25 \end{bmatrix}$$

By (i) – (ii) × 3, we get

$$-16B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 5 \end{bmatrix} - 3\begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 3 \\ -3 & 0 & 0 \end{bmatrix}$$

$$\therefore \quad -16B = \begin{bmatrix} -1 & 2 & -2 \\ 4 & 0 & 5 \end{bmatrix}$$

$$\therefore \quad B = \frac{-1}{16}\begin{bmatrix} -1 & 2 & -2 \\ 4 & 0 & 5 \end{bmatrix}$$

$$\therefore \quad B = \frac{1}{16}\begin{bmatrix} 1 & -2 & 2 \\ -4 & 0 & -5 \end{bmatrix}$$

Question 4.
If A = [⌊ ∣ 23−1−3−24⌋ ∣ ∣ , B = [−324−11−3], verify
i. $(A + 2B^T)^T = A^T + 2B$
ii. $(3A - 5B^T)^T = 3A^T - 5B$.
Solution:

i. $$A = \begin{bmatrix} 2 & -3 \\ 3 & -2 \\ -1 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 4 & 1 \\ 2 & -1 & -3 \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} 2 & 3 & -1 \\ -3 & -2 & 4 \end{bmatrix} \text{ and } B^T = \begin{bmatrix} -3 & 2 \\ 4 & -1 \\ 1 & -3 \end{bmatrix}$$

$$\therefore \quad A + 2B^T = \begin{bmatrix} 2 & -3 \\ 3 & -2 \\ -1 & 4 \end{bmatrix} + 2\begin{bmatrix} -3 & 2 \\ 4 & -1 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ 3 & -2 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} -6 & 4 \\ 8 & -2 \\ 2 & -6 \end{bmatrix}$$

$$\therefore \quad A + 2B^T = \begin{bmatrix} -4 & 1 \\ 11 & -4 \\ 1 & -2 \end{bmatrix}$$

$$\therefore \quad (A + 2B^T)^T = \begin{bmatrix} -4 & 11 & 1 \\ 1 & -4 & -2 \end{bmatrix} \quad \ldots(i)$$

$$A^T + 2B = \begin{bmatrix} 2 & 3 & -1 \\ -3 & -2 & 4 \end{bmatrix} + 2\begin{bmatrix} -3 & 4 & 1 \\ 2 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & -1 \\ -3 & -2 & 4 \end{bmatrix} + \begin{bmatrix} -6 & 8 & 2 \\ 4 & -2 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 11 & 1 \\ 1 & -4 & -2 \end{bmatrix} \quad \ldots(ii)$$

From (i) and (ii), we get

$(A + 2B^T)^T = A^T + 2B$

ii. $$3A - 5B^T = 3\begin{bmatrix} 2 & -3 \\ 3 & -2 \\ -1 & 4 \end{bmatrix} - 5\begin{bmatrix} -3 & 2 \\ 4 & -1 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -9 \\ 9 & -6 \\ -3 & 12 \end{bmatrix} - \begin{bmatrix} -15 & 10 \\ 20 & -5 \\ 5 & -15 \end{bmatrix}$$

$$\therefore \quad 3A - 5B^T = \begin{bmatrix} 21 & -19 \\ -11 & -1 \\ -8 & 27 \end{bmatrix}$$

$$\therefore \quad (3A - 5B^T)^T = \begin{bmatrix} 21 & -11 & -8 \\ -19 & -1 & 27 \end{bmatrix} \quad \ldots(i)$$

$$3A^T - 5B = 3\begin{bmatrix} 2 & 3 & -1 \\ -3 & -2 & 4 \end{bmatrix} - 5\begin{bmatrix} -3 & 4 & 1 \\ 2 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 9 & -3 \\ -9 & -6 & 12 \end{bmatrix} - \begin{bmatrix} -15 & 20 & 5 \\ 10 & -5 & -15 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & -11 & -8 \\ -19 & -1 & 27 \end{bmatrix} \quad \ldots(ii)$$

From (i) and (ii), we get

$(3A - 5B^T)^T = 3A^T - 5B.$

Question 5.

If A = [cosαsinα−sinαcosα] and A + A^T = I, where I is a unit matrix of order 2 × 2, then find the value of α.

Solution:

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\therefore \quad A^T = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$A + A^T = I$$

$$\therefore \quad \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

$2\cos\alpha = 1$

$$\therefore \quad \cos\alpha = \frac{1}{2}$$

$$\therefore \quad \cos\alpha = \cos\frac{\pi}{3}$$

$$\therefore \quad \alpha = \frac{\pi}{3} \text{ or } 60^\circ$$

Question 6.

If A = ⎡ ⎣ ⎢ 13−1220⎤ ⎦ ⎥ and B = [143−12−3], show that AB is singular.

Solution:

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 4 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 3-2 & 2-6 \\ 3+8 & 9-2 & 6-6 \\ -1+0 & -3+0 & -2+0 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 1 & -4 \\ 11 & 7 & 0 \\ -1 & -3 & -2 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 9 & 1 & -4 \\ 11 & 7 & 0 \\ -1 & -3 & -2 \end{vmatrix}$$

$$= 9(-14+0) - 1(-22+0) - 4(-33+7)$$
$$= -126 + 22 + 104$$
$$= 0$$

AB is a singular matrix.

Question 7.
If A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$, show that AB and BA are both singular matrices.
Solution:

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6-6 & -1+4+3 & 1-2+0 \\ 2-12-12 & -2+8+6 & 2-4+0 \\ 1-6-6 & -1+4+3 & 1-2+0 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{vmatrix}$$

$$= 0 \quad \ldots[\because R_1 \text{ an } R_3 \text{ are identical}]$$

AB is a singular matrix

$$BA = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+1 & 2-4+2 & 3-6+3 \\ -3+4-1 & -6+8-2 & -9+12-3 \\ -2+2+0 & -4+4+0 & -6+6+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$|BA| = 0$$

BA is a singular matrix.

Question 8.
If A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, show that BA = 6I.

Solution:

$$\mathbf{BA} = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4-0 & -2+6-4 & 0+8-8 \\ -4+4+0 & 4+6-4 & 0+8-8 \\ 2-2+0 & -2-3+5 & 0-4+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{BA} = 6\mathbf{I}$$

Question 9.
If A = [2013], B = [132−2], verify that |AB| = |A|.|B|.
Solution:

$$\mathbf{AB} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3 & 4-2 \\ 0+9 & 0-6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 9 & -6 \end{bmatrix}$$

$$|\mathbf{AB}| = \begin{vmatrix} 5 & 2 \\ 9 & -6 \end{vmatrix}$$

$$|\mathbf{AB}| = 5(-6) - 9(2) = -30 - 18 = -48$$

$$|\mathbf{A}| = \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = 2(3) - 0(1) = 6 \blacklozenge 0 = 6$$

$$|\mathbf{B}| = \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = 1(-2) - 2(3) = -2 - 6 = -8$$

$$|\mathbf{A}|.|\mathbf{B}|. = 6(-8) = -48 = |\mathbf{AB}|$$

$$|\mathbf{AB}| = |\mathbf{A}|.|\mathbf{B}|$$

Question 10.
If Aα = [cosα−sinαsinαcosα], show that Aα . Aβ = Aα+β
Solution:

$$\mathbf{A}_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\therefore \quad \mathbf{A}_\beta = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$\therefore \quad \mathbf{A}_\alpha.\mathbf{A}_\beta = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & \cos\alpha\sin\beta + \sin\alpha\cos\beta \\ -\sin\alpha\cos\beta - \cos\alpha\sin\beta & -\sin\alpha\sin\beta + \cos\alpha\cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & \cos\alpha\sin\beta + \sin\alpha\cos\beta \\ -[\sin\alpha\cos\beta + \cos\alpha\sin\beta] & \cos\alpha\sin\beta - \sin\alpha\cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$$

$$\therefore \quad \mathbf{A}_\alpha\,\mathbf{A}_\beta = \mathbf{A}_{(\alpha+\beta)}$$

Question 11.
If A = [1ω2ω1], B = [ω211ω], where ω is a complex cube root of unity, then show that AB + BA + A – 2B is a null matrix.
Solution:
ω is the complex cube root of unity.
$\omega^3 = 1$
$\omega^3 - 1 = 0$
$(\omega - 1)(\omega^2 + \omega + 1) = 0$
$\omega = 1$ or $\omega^2 + \omega + 1 = 0$
But, ω is a complex number.
$1 + \omega + \omega^2 = 0$ …….(i)

AB + BA + A – 2B

$$= \begin{bmatrix} 1 & \omega \\ \omega^2 & 1 \end{bmatrix}\begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix} + \begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix}\begin{bmatrix} 1 & \omega \\ \omega^2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & \omega \\ \omega^2 & 1 \end{bmatrix} - 2\begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix}$$

$$= \begin{bmatrix} \omega^2+\omega & 1+\omega^2 \\ \omega^4+1 & \omega^2+\omega \end{bmatrix} + \begin{bmatrix} \omega^2+\omega^2 & \omega^3+1 \\ 1+\omega^3 & \omega+\omega \end{bmatrix} + \begin{bmatrix} 1 & \omega \\ \omega^2 & 1 \end{bmatrix} - \begin{bmatrix} 2\omega^2 & 2 \\ 2 & 2\omega \end{bmatrix}$$

$$= \begin{bmatrix} \omega^2+\omega+2\omega^2+1-2\omega^2 & 1+\omega^2+\omega^3+1+\omega-2 \\ \omega^4+1+1+\omega^3+\omega^2-2 & \omega^2+\omega+2\omega+1-2\omega \end{bmatrix}$$

$$= \begin{bmatrix} \omega^2+\omega+1 & \omega^2+\omega+1 \\ \omega^2+\omega+1 & \omega^2+\omega+1 \end{bmatrix} \quad \ldots[\because \omega^3 = 1 \therefore \omega^4 = \omega]$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \ldots[\text{From (i)}]$$

which is a null matrix.

Question 12.
If A = [⌊ | 2-11-23-2-44-3] ⌋ | , show that $A^2 = A$.
Solution:

$$A^2 = A.A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2-4 & -4-6+8 & -8-8+12 \\ -2-3+4 & 2+9-8 & 4+12-12 \\ 2+2-3 & -2-6+6 & -4-8+9 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A^2 = A$$

Question 13.
If A = [⌊ | 433-10-1-4-4-3] ⌋ | , show that $A^2 = I$.
Solution:

$$A^2 = A.A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}\begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 16-3-12 & -4+0+4 & -16+4+12 \\ 12+0-12 & -3+0+4 & -12+0+12 \\ 12-3-9 & -3-0+3 & -12+4+9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Question 14.
If A = [3-4-52], show that $A^2 - 5A - 14I = 0$.

Solution:

$$A^2 - 5A - 14I$$
$$= A.A - 5A - 14I$$
$$= \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$
$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$
$$= \begin{bmatrix} 29-15-14 & -25+25-0 \\ -20+20-0 & 24-10-14 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$= 0$$

Question 15.
If A = [2−1−12], show that A – 4A + 3I = 0.
Solution:

$$A^2 - 4A + 3I$$
$$= A.A - 4A + 3I$$
$$= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix} - \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} - \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 5-8+3 & -4+4+0 \\ -4+4+0 & 5-8+3 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$= 0$$

Question 16.
If A = [−322−4], B = [1yx0] and (A + B)(A – B) = $A^2 - B^2$, find x and y.
Solution:
(A + B)(A – B) = $A^2 - B^2$
$A^2 - AB + BA - B^2 = A^2 - B^2$
-AB + BA = 0
AB = BA

$$\begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} 1 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 1 & x \\ y & 0 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix}$$
$$\begin{bmatrix} -3+2y & -3x+0 \\ 2-4y & 2x+0 \end{bmatrix} = \begin{bmatrix} -3+2x & 2-4x \\ -3y+0 & 2y+0 \end{bmatrix}$$

By equality of matrices, we get
2 – 4x = -3x
∴ x = 2 and 2y = 2x
y = x
∴ y = 2
∴ x = 2, y = 2

Question 17.
If A = [0110] and B = [01−10], show that (A + B)(A – B) ≠ $A^2 - B^2$.
Solution:
We have to prove that
(A + B) . (A – B) ≠ $A^2 - B^2$
i.e., to prove that A(A – B) + B(A – B) ≠ $A^2 - B^2$
i.e., to prove that $A^2 - AB + BA - B^2 \neq A^2 - B^2$
i.e., to prove that AB ≠ BA.

$$AB = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
$$BA = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0-1 & 0-0 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore AB \neq BA$

Question 18.
If A = $[23-1-2]$, find A^3.
Solution:

$$A^2 = A.A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix}$$
$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore A^2 = I$
Multiplying throughout by A, we get
$A^3 = A . I$
$\therefore A^3 = A$

Question 19.
Find x, y if,

$$\text{i) } [0 \ \ -1 \ \ 4]\left\{2\begin{bmatrix} 4 & 5 \\ 3 & 6 \\ 2 & -1 \end{bmatrix} + 3\begin{bmatrix} 4 & 3 \\ 1 & 4 \\ 0 & -1 \end{bmatrix}\right\}$$
$$= [x \quad y].$$

ii)

$$\left\{-1\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 3\begin{bmatrix} 2 & -3 & 7 \\ 1 & -1 & 3 \end{bmatrix}\right\}\begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Solution:

i. $\begin{bmatrix}0 & -1 & 4\end{bmatrix}\left\{2\begin{bmatrix}4 & 5\\3 & 6\\2 & -1\end{bmatrix}+3\begin{bmatrix}4 & 3\\1 & 4\\0 & -1\end{bmatrix}\right\}=[x\ y]$

$\therefore$ $\begin{bmatrix}0 & -1 & 4\end{bmatrix}\left\{\begin{bmatrix}8 & 10\\6 & 12\\4 & -2\end{bmatrix}+\begin{bmatrix}12 & 9\\3 & 12\\0 & -3\end{bmatrix}\right\}=[x\ y]$

$\therefore$ $\begin{bmatrix}0 & -1 & 4\end{bmatrix}\begin{bmatrix}20 & 19\\9 & 24\\4 & -5\end{bmatrix}=[x\ y]$

$\therefore$ $[0-9+16 \quad 0-24-20]=[x\ y]$

$\therefore$ $[7 \quad -44]=[x, y]$

$\therefore$ By equality of matrices, we get

$x=7, y=-44$

ii. $\left\{-1\begin{bmatrix}1 & 2 & 1\\2 & 0 & 3\end{bmatrix}+3\begin{bmatrix}2 & -3 & 7\\1 & -1 & 3\end{bmatrix}\right\}\begin{bmatrix}5\\0\\-1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$

$\therefore$ $\left\{\begin{bmatrix}-1 & -2 & -1\\-2 & 0 & -3\end{bmatrix}+\begin{bmatrix}6 & -9 & 21\\3 & -3 & 9\end{bmatrix}\right\}\begin{bmatrix}5\\0\\-1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$

$\therefore$ $\begin{bmatrix}5 & -11 & 20\\1 & -3 & 6\end{bmatrix}\begin{bmatrix}5\\0\\-1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$

$\therefore$ $\begin{bmatrix}25-0-20\\5-0-6\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$

$\therefore$ $\begin{bmatrix}5\\-1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$

$\therefore$ By equality of matrices, we get

$x=5, y=-1$

Question 20.
Find x, y, z if

i) $\left\{5\begin{bmatrix}0 & 1\\1 & 0\\1 & 1\end{bmatrix}-3\begin{bmatrix}2 & 1\\3 & -2\\1 & 3\end{bmatrix}\right\}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$

ii) $\left\{\begin{bmatrix}1 & 3 & 2\\2 & 0 & 1\\3 & 1 & 2\end{bmatrix}+2\begin{bmatrix}3 & 0 & 2\\1 & 4 & 5\\2 & 1 & 0\end{bmatrix}\right\}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$

Solution:

i. $\left\{5\begin{bmatrix}0 & 1\\1 & 0\\1 & 1\end{bmatrix}-3\begin{bmatrix}2 & 1\\3 & -2\\1 & 3\end{bmatrix}\right\}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$

$\therefore \quad \left\{\begin{bmatrix}0 & 5\\5 & 0\\5 & 5\end{bmatrix}-\begin{bmatrix}6 & 3\\9 & -6\\3 & 9\end{bmatrix}\right\}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$

$\therefore \quad \begin{bmatrix}-6 & 2\\-4 & 6\\2 & -4\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$

$\therefore \quad \begin{bmatrix}-12+2\\-8+6\\4-4\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$

$\therefore \quad \begin{bmatrix}-10\\-2\\0\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$

$\therefore$ By equality of matrices, we get

$x-1=-10 \quad \therefore \quad x=-9$

$y+1=-2 \quad \therefore \quad y=-3$

$2z=0 \quad \therefore \quad z=0$

ii. $\left\{\begin{bmatrix}1 & 3 & 2\\2 & 0 & 1\\3 & 1 & 2\end{bmatrix}+2\begin{bmatrix}3 & 0 & 2\\1 & 4 & 5\\2 & 1 & 0\end{bmatrix}\right\}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$

$\therefore \quad \left\{\begin{bmatrix}1 & 3 & 2\\2 & 0 & 1\\3 & 1 & 2\end{bmatrix}+\begin{bmatrix}6 & 0 & 4\\2 & 8 & 10\\4 & 2 & 0\end{bmatrix}\right\}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$

$\therefore \quad \begin{bmatrix}7 & 3 & 6\\4 & 8 & 11\\7 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$

$\therefore \quad \begin{bmatrix}7+6+18\\4+16+33\\7+6+6\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$

$\therefore \quad \begin{bmatrix}31\\53\\19\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$

$\therefore$ By equality of matrices, we get

$x=31, y=53, z=19$

Question 21.
If A = [2012−36], B = [130−1−24], find AB^T and A^TB.
Solution:

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 2 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 0 \\ 1 & 2 \\ -3 & 6 \end{bmatrix} \text{ and } B^T = \begin{bmatrix} 1 & 3 \\ 0 & -1 \\ -2 & 4 \end{bmatrix}$$

$$AB^T = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 2 & 6 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & -1 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+6 & 6-1-12 \\ 0+0-12 & 0-2+24 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -7 \\ -12 & 22 \end{bmatrix}$$

$$\text{and } A^TB = \begin{bmatrix} 2 & 0 \\ 1 & 2 \\ -3 & 6 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & 0+0 & -4+0 \\ 1+6 & 0-2 & -2+8 \\ -3+18 & 0-6 & 6+24 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -4 \\ 7 & -2 & 6 \\ 15 & -6 & 30 \end{bmatrix}$$

Question 22.
If A = $\begin{bmatrix} 2 & -4 \\ 3 & -2 \\ 0 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 0 \end{bmatrix}$, show that $(AB)^T = B^TA^T$.
Solution:

$$AB = \begin{bmatrix} 2 & -4 \\ 3 & -2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2+8 & -2-4 & 4+0 \\ 3+4 & -3-2 & 6-0 \\ 0-2 & 0+1 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -6 & 4 \\ 7 & -5 & 6 \\ -2 & 1 & 0 \end{bmatrix}$$

$$\therefore \quad (AB)^T = \begin{bmatrix} 10 & 7 & -2 \\ -6 & -5 & 1 \\ 4 & 6 & 0 \end{bmatrix} \quad \ldots\text{(i)}$$

$$\text{Now, } A^T = \begin{bmatrix} 2 & 3 & 0 \\ -4 & -2 & 1 \end{bmatrix} \text{ and } B^T = \begin{bmatrix} 1 & -2 \\ -1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$\therefore \quad B^TA^T = \begin{bmatrix} 1 & -2 \\ -1 & 1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 & 0 \\ -4 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+8 & 3+4 & 0-2 \\ -2-4 & -3-2 & 0+1 \\ 4-0 & 6-0 & 0+0 \end{bmatrix}$$

$$\therefore \quad B^TA^T = \begin{bmatrix} 10 & 7 & -2 \\ -6 & -5 & 1 \\ 4 & 6 & 0 \end{bmatrix} \quad \ldots\text{(ii)}$$

From (i) and (ii), we get
$(AB)^T = B^TA^T$

Question 23.
If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, for all n ∈ N.

Solution:

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

Let $P(n) \equiv A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, for all $n \in N$.

Step 1: Put n = 1

$$\text{R.H.S.} = \begin{bmatrix} 1+2(1) & -4(1) \\ 1 & 1-2(1) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= A = \text{L.H.S.}$$

$\therefore$ P(n) is true for n = 1.

Step 2: Let us consider that P(n) is true for n = k.

$\therefore$ $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$ ...(i)

Step 3: We have to prove that P(n) is true for n = k + 1,

i.e., to prove that

$$A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix}$$

$$\text{R.H.S.} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+2k+2 & -4k-4 \\ k+1 & 1-2k-2 \end{bmatrix}$$

$$= \begin{bmatrix} 3+2k & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}$$

$$\text{L.H.S.} = A^{k+1}$$

$$= A^k.A$$

$$= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \text{...[From (i)]}$$

$$= \begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix}$$

$$= \begin{bmatrix} 3+2k & -4k-4 \\ k+1 & -2k-1 \end{bmatrix} = \text{R.H.S.}$$

$\therefore$ P(n) is true for n = k + 1.

Step 4: From all steps above, by the principle of Mathematical induction P(n) is true for all $n \in N$.

$\therefore$ $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for all $n \in N$.

Question 24.

If A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, prove that A^n = $\begin{bmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix}$, for all n ∈ N.

Solution:

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Let $P(n) \equiv A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$, for all $n \in N$.

Step 1: Put $n = 1$

$\therefore$ R.H.S. $= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = A =$ L.H.S.

$\therefore$ P(n) is true for n = 1.

Step 2: Let us consider that P(n) is true for n = k.

$\therefore \quad A^k = \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix} \quad \ldots$(i)

Step 3: We have to prove that P(n) is true for $n = k + 1$,
i.e., to prove that

$$A^{k+1} = \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

R.H.S. $= \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$

L.H.S. $= A^{k+1} = A^k.A$

$= \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad \ldots$[From (i)]

$= \begin{bmatrix} \cos k\theta\cos\theta - \sin k\theta\sin\theta & \cos k\theta\sin\theta + \sin k\theta\cos\theta \\ -\sin k\theta\cos\theta - \cos k\theta\sin\theta & -\sin k\theta\sin\theta + \cos k\theta\cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos k\theta\cos\theta - \sin k\theta\sin\theta & \sin k\theta\cos\theta + \cos k\theta\sin\theta \\ -(\sin k\theta\cos\theta + \cos k\theta\sin\theta) & \cos k\theta\cos\theta - \sin k\theta\sin\theta \end{bmatrix}$

$= \begin{bmatrix} \cos(k\theta+\theta) & \sin(k\theta+\theta) \\ -\sin(k\theta+\theta) & \cos(k\theta+\theta) \end{bmatrix}$

$= \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$

= R.H.S.

$\therefore$ P(n) is true for n = k + 1.

Step 4: From all steps above, by the principle of Mathematical induction, P(n) is true for all $n \in N$.

$\therefore \quad A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$ for all $n \in N$.

Question 25.
Two farmers Shantaram and Kantaram cultivate three crops rice, wheat, and groundnut. The sale (in Rupees) of these crops by both the farmers for the month of April and may 2008 is given below,

April sale (In Rs.)

	Rice	Wheat	Groundnut
Shantaram	15000	13000	12000
Kantaram	18000	15000	8000

May sale (In Rs.)

	Rice	Wheat	Groundnut
Shantaram	18000	15000	12000
Kantaram	21000	16500	16000

Find
(i) the total sale in rupees for two months of each farmer for each crop.
(ii) the increase in sales from April to May for every crop of each farmer.
Solution:
(i) Total sale for Shantaram:
For rice = 15000 + 18000 = ₹ 33000.
For wheat = 13000 + 15000 = ₹ 28000.

For groundnut = 12000 + 12000 = ₹ 24000.
Total sale for Kantaram:
For rice = 18000 + 21000 = ₹ 39000
For wheat = 15000 + 16500 = ₹ 31500
For groundnut = 8000 + 16000 = ₹ 24000

Alternate method:
Matrix form

$$= \begin{bmatrix} 15000 & 13000 & 12000 \\ 18000 & 15000 & 8000 \end{bmatrix} + \begin{bmatrix} 18000 & 15000 & 12000 \\ 21000 & 16500 & 16000 \end{bmatrix}$$

$$= \begin{bmatrix} 33000 & 28000 & 24000 \\ 39000 & 31500 & 24000 \end{bmatrix}$$

∴ The total sale of April and May of Shantaram in ₹ is ₹ 33000 (rice), ₹ 28000 (wheat), ₹ 24000 (groundnut), and that of Kantaram in ₹ is ₹ 39000(rice), ₹ 31500(wheat), and ₹ 24000 (groundnut).

(ii) Increase in sale from April to May for Shantaram:
For rice = 18000 – 15000 = ₹ 3000
For wheat = 15000 – 13000 = ₹ 2000
For groundnut = 12000 – 12000 = ₹ 0
Increase in sale from April to May for Kantaram:
For rice = 21000 – 18000 = ₹ 3000
For wheat = 16500 – 15000 = ₹ 1500
For groundnut = 16000 – 8000 = ₹ 8000

Alternate method:
Matrix form

$$= \begin{bmatrix} 18000 & 15000 & 12000 \\ 21000 & 16500 & 16000 \end{bmatrix} - \begin{bmatrix} 15000 & 13000 & 12000 \\ 18000 & 15000 & 8000 \end{bmatrix}$$

$$= \begin{bmatrix} 3000 & 2000 & 0 \\ 3000 & 1500 & 8000 \end{bmatrix}$$

∴ The increase in sales for Shantaram from April to May in each crop is ₹ 3000 (rice), ₹ 2000(wheat), 0 (groundnut), and that for Kantaram is ₹ 3000 (rice), ₹ 1500 (wheat), and ₹ 8000 (groundnut).