Maharashtra State Board 12th Commerce Maths Solutions Chapter 2 Matrices Ex 2.1

Question 1.
Construct a matrix A = $[a_{ij}]_{3×2}$ whose elements aij isgiven by
(i) a_{ij} = (i–j)2 5–i
Solution:

$$A = [a_{ij}]_{3\times2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\text{Now, } a_{ij} = \frac{(i-j)^2}{5-i}$$

$$\therefore\ a_{11} = \frac{(1-1)^2}{5-1} = \frac{0}{4} = 0$$

$$a_{12} = \frac{(1-2)^2}{5-1} = \frac{1}{4}$$

$$a_{21} = \frac{(2-1)^2}{5-2} = \frac{1}{3}$$

$$a_{22} = \frac{(2-2)^2}{5-2} = \frac{0}{3} = 0$$

$$a_{31} = \frac{(3-1)^2}{5-3} = \frac{4}{2} = 2$$

$$a_{32} = \frac{(3-2)^2}{5-3} = \frac{1}{2}$$

$$\therefore\ A = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \\ 2 & \frac{1}{2} \end{bmatrix}$$

(ii) a_{ij} = i – 3j
Solution:
a_{ij} = i – 3j
∴ a_{11} = 1 – 3(1) = 1 – 3 = -2
a_{12} = 1 – 3(2) = 1 – 6 = -5
a_{21} = 2 – 3(1) = 2 – 3 = -1
a_{22} = 2 – 3(2) = 2 – 6 = -4
a_{31} = 3 – 3(1) = 3 – 3 = 0
a_{32} = 3 – 3(2) = 3 – 6 = -3
∴ A = ⎡ ⎣ ⎢ -2-10-5-4-3⎤ ⎦ ⎥

(iii) aij = (i+j)35

Solution:

$$a_{ij} = \frac{(i+j)^3}{5}$$

$$\therefore a11 = \frac{(1+1)^3}{5} = \frac{2^3}{5} = \frac{8}{5}, a_{12} = \frac{(1+2)^3}{5} = \frac{3^3}{5} = \frac{27}{5}$$

$$a_{21} = \frac{(2+1)^3}{5} = \frac{3^3}{5} = \frac{27}{5}, a_{22} = \frac{(2+2)^2}{5} = \frac{4^3}{5} = \frac{64}{5}$$

$$a_{31} = \frac{(3+1)^3}{5} = \frac{4^3}{5} = \frac{64}{5}, a_{32} = \frac{(3+2)^2}{5} = \frac{5^3}{5} = \frac{125}{5}$$

$$\therefore A = \begin{bmatrix} \frac{8}{5} & \frac{27}{5} \\ \frac{27}{5} & \frac{64}{5} \\ \frac{64}{5} & \frac{125}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 27 \\ 27 & 64 \\ 64 & 125 \end{bmatrix}.$$

Question 2.

Classify each of the following matrices as a row, a column, a square, a diagonal, a scalar, a unit, an upper triangular, a lower triangular matrix:

(i) $\begin{bmatrix} 3 & -2 & 4 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 5 \\ 4 \\ -3 \end{bmatrix}$ (iii) $\begin{bmatrix} 9 & \sqrt{2} & -3 \end{bmatrix}$ (iv) $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

(v) $\begin{bmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ -7 & 3 & 1 \end{bmatrix}$ (vi) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$ (vii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution:

(i) Since, all the elements below the diagonal are zero, it is an upper triangular matrix.
(ii) This matrix has only one column, it is a column matrix.
(iii) This matrix has only one row, it is a row matrix.
(iv) Since, diagonal elements are equal and non-diagonal elements are zero, it is a scalar matrix.
(v) Since, all the elements above the diagonal are zero, it is a lower triangular matrix.
(vi) Since, all the non-diagonal elements are zero, it is a diagonal matrix.
(vii) Since, diagonal elements are 1 and non-diagonal elements are 0, it is an identity (or unit) matrix.

Question 3.

Which of the following matrices are singular or non-singular:

(i) ⎡ ⎣ ⎢ ap2a−pbq2b−qcr2c−r⎤ ⎦ ⎥

Solution:

$$\text{Let } A = \begin{bmatrix} a & b & c \\ p & q & r \\ 2a-p & 2b-q & 2c-r \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} a & b & c \\ p & q & r \\ 2a-p & 2b-q & 2c-r \end{vmatrix}$$

By $R_3 + R_2$, we get,

$$|A| = \begin{vmatrix} a & b & c \\ p & q & r \\ 2a & 2b & 2c \end{vmatrix}$$

$$= 2\begin{vmatrix} a & b & c \\ p & q & r \\ a & b & c \end{vmatrix}$$

$$= 2 \times 0 \qquad \ldots [\because R_1 \equiv R_3]$$

$$= 0$$

$\therefore$ A is a **singular** matrix.

(ii) $\begin{bmatrix} 5 & 0 & 5 \\ 1 & 99 & 100 \\ 6 & 99 & 105 \end{bmatrix}$

Solution:

Let $B = \begin{bmatrix} 5 & 0 & 5 \\ 1 & 99 & 100 \\ 6 & 99 & 105 \end{bmatrix}$

$\therefore \ |B| = \begin{vmatrix} 5 & 0 & 5 \\ 1 & 99 & 100 \\ 6 & 99 & 105 \end{vmatrix}$

By $R_3 - R_2$, we get

$|B| = \begin{vmatrix} 5 & 0 & 5 \\ 1 & 99 & 100 \\ 5 & 0 & 5 \end{vmatrix}$

$= 0$. $[\because R_1 \equiv R_3]$

$\therefore$ B is a **singular** matrix.

(iii) $\begin{bmatrix} 3 & 5 & 7 \\ -2 & 1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$

Solution:

Let C = $\begin{bmatrix} 3 & 5 & 7 \\ -2 & 1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$

$\therefore$ |C| = $\begin{vmatrix} 3 & 5 & 7 \\ -2 & 1 & 4 \\ 3 & 2 & 5 \end{vmatrix}$

= 3(5 – 8) – 5(-10 – 12) + 7(-4 – 3)

= -9 + 110 – 49

= 52 ≠ 0

$\therefore$ C is a non-singular matrix.

(iv) $\begin{bmatrix} 7 & 5 \\ -4 & 7 \end{bmatrix}$

Solution:

Let D = $\begin{bmatrix} 7 & 5 \\ -4 & 7 \end{bmatrix}$

$\therefore$ |D| = $\begin{vmatrix} 7 & 5 \\ -4 & 7 \end{vmatrix}$

= 49 – (-20)

= 69 ≠ 0

$\therefore$ D is a non-singular matrix.

Question 4.

Find k, if the following matrices are singular:

(i) $\begin{bmatrix} 7 & 3 \\ -2 & K \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 7 & 3 \\ -2 & K \end{bmatrix}$

Since, A is a singular matrix, |A| = 0

$\therefore$ $\begin{vmatrix} 7 & 3 \\ -2 & k \end{vmatrix}$ = 0

$\therefore$ 7k – (-6) = 0

$\therefore$ 7k = -6

$\therefore$ k = $\frac{-6}{7}$

(ii) $\begin{bmatrix} 4 & 3 & 1 \\ 7 & K & 1 \\ 10 & 9 & 1 \end{bmatrix}$

Solution:

Let B = $\begin{bmatrix} 4 & 3 & 1 \\ 7 & K & 1 \\ 10 & 9 & 1 \end{bmatrix}$

Since, B is a singular matrix, |B| = 0

$\therefore$ $\begin{vmatrix} 4 & 3 & 1 \\ 7 & k & 1 \\ 10 & 9 & 1 \end{vmatrix}$ = 0

$\therefore$ 4(k – 9) – 3(7 – 10) + 1(63 – 10k) = 0

$\therefore$ 4k – 36 + 9 + 63 – 10k = 0

$\therefore$ -6k + 36 = 0

$\therefore$ 6k = 36

$\therefore$ k = 6.

(iii) ⎡ ⎣ ⎢ K-13121-2324 ⎤ ⎦ ⎥

Solution:

Let C = ⎡ ⎣ ⎢ K-13121-2324 ⎤ ⎦ ⎥

Since, C is a singular matrix, |C| = 0

∴ ∣ ∣ ∣ ∣ k-13121-2324 ∣ ∣ ∣ ∣ = 0

∴ (k – 1)(4 + 4) – 2(12 – 2) + 3(-6 – 1) = 0

∴ 8k – 8 – 20 – 21 = 0

∴ 8k = 49

∴ k = $\frac{49}{8}$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 2 Matrices Ex 2.2

Question 1.

If A = ⎡ ⎣ ⎢ 25-6-3-41 ⎤ ⎦ ⎥ , B = ⎡ ⎣ ⎢ -120223 ⎤ ⎦ ⎥ and C = ⎡ ⎣ ⎢ 4-1-2341 ⎤ ⎦ ⎥ show that

(i) A + B = B + A

(ii) (A + B) + C = A + (B + C)

Solution:

$$A + B = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & -3+2 \\ 5+2 & -4+2 \\ -6+0 & 1+3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 7 & -2 \\ -6 & 4 \end{bmatrix} \quad \ldots (1)$$

$$B + A = \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 2-3 \\ 2+5 & 2-4 \\ 0-6 & 3+1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 7 & -2 \\ -6 & 4 \end{bmatrix} \quad \ldots (2)$$

$$A+B=\begin{bmatrix}2 & -3\\ 5 & -4\\ -6 & 1\end{bmatrix}+\begin{bmatrix}-1 & 2\\ 2 & 2\\ 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2-1 & -3+2\\ 5+2 & -4+2\\ -6+0 & 1+3\end{bmatrix}=\begin{bmatrix}1 & -1\\ 7 & -2\\ -6 & 4\end{bmatrix}$$

$$\therefore\ (A+B)+C=\begin{bmatrix}1 & -1\\ 7 & -2\\ -6 & 4\end{bmatrix}+\begin{bmatrix}4 & 3\\ -1 & 4\\ -2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+4 & -1+3\\ 7-1 & -2+4\\ -6-2 & 4+1\end{bmatrix}=\begin{bmatrix}5 & 2\\ 6 & 2\\ -8 & 5\end{bmatrix}\qquad \ldots (1)$$

$$\text{Also, } B+C=\begin{bmatrix}-1 & 2\\ 2 & 2\\ 0 & 3\end{bmatrix}+\begin{bmatrix}4 & 3\\ -1 & 4\\ -2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-1+4 & 2+3\\ 2-1 & 2+4\\ 0-2 & 3+1\end{bmatrix}=\begin{bmatrix}3 & 5\\ 1 & 6\\ -2 & 4\end{bmatrix}$$

$$\therefore\ A+(B+C)=\begin{bmatrix}2 & -3\\ 5 & -4\\ -6 & 1\end{bmatrix}+\begin{bmatrix}3 & 5\\ 1 & 6\\ -2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}2+3 & -3+5\\ 5+1 & -4+6\\ -6-2 & 1+4\end{bmatrix}=\begin{bmatrix}5 & 2\\ 6 & 2\\ -8 & 5\end{bmatrix}\qquad \ldots (2)$$

From (1) and (2), we get
(A + B) + C = A + (B + C).

Question 2.
If A = [15−23], B = [14−3−7], then find the matrix A – 2B + 6I, where I is the unit matrix of order 2.
Solution:

$$A-2B+6I=\begin{bmatrix}1 & -2\\ 5 & 3\end{bmatrix}-2\begin{bmatrix}1 & -3\\ 4 & -7\end{bmatrix}+6\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & -2\\ 5 & 3\end{bmatrix}-\begin{bmatrix}2 & -6\\ 8 & -14\end{bmatrix}+\begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}$$

$$=\begin{bmatrix}1-2+6 & -2-(-6)+0\\ 5-8+0 & 3-(-14)+6\end{bmatrix}$$

$$=\begin{bmatrix}5 & 4\\ -3 & 23\end{bmatrix}$$

Question 3.
If A = ⎡ ⎣ ⎢ 1−3027−6−3−81⎤ ⎦ ⎥ , B = ⎡ ⎣ ⎢ 9−44−12025−3⎤ ⎦ ⎥ , then find the matrix C such that A + B + C is a zero matrix.
Solution:
A + B + C = 0

∴ C = -A – B

$$= -\begin{bmatrix} 1 & 2 & -3 \\ -3 & 7 & -8 \\ 0 & -6 & 1 \end{bmatrix} - \begin{bmatrix} 9 & -1 & 2 \\ -4 & 2 & 5 \\ 4 & 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 & 3 \\ 3 & -7 & 8 \\ 0 & 6 & -1 \end{bmatrix} - \begin{bmatrix} 9 & -1 & 2 \\ -4 & 2 & 5 \\ 4 & 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-(-1) & 3-2 \\ 3-(-4) & -7-2 & 8-5 \\ 0-4 & 6-0 & -1-(-3) \end{bmatrix}$$

$$\therefore\ C = \begin{bmatrix} -10 & -1 & 1 \\ 7 & -9 & 3 \\ -4 & 6 & 2 \end{bmatrix}$$

Question 4.

If A = ⎡⎣⎢ 13−6−2−50 ⎤⎦⎥ , B = ⎡⎣⎢ −141−225 ⎤⎦⎥ and C = ⎡⎣⎢ 2−1−34−46 ⎤⎦⎥ , find the matrix X such that 3A – 4B + 5X = C.

Solution:

3A – 4B + 5X = C

∴ 5X = C – 3A + 4B

$$= \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix} - 3\begin{bmatrix} 1 & -2 \\ 3 & -5 \\ -6 & 0 \end{bmatrix} + 4\begin{bmatrix} -1 & -2 \\ 4 & 2 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix} - \begin{bmatrix} 3 & -6 \\ 9 & -15 \\ -18 & 0 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ 16 & 8 \\ 4 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3+(-4) & 4-(-6)-8 \\ -1-9+16 & -4-(-15)+8 \\ -3-(-18)+4 & 6-0+20 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 2 \\ 6 & 19 \\ 19 & 26 \end{bmatrix}$$

$$\therefore\ X = \frac{1}{5}\begin{bmatrix} -5 & 2 \\ 6 & 19 \\ 19 & 26 \end{bmatrix} = \begin{bmatrix} -1 & \frac{2}{5} \\ \frac{6}{5} & \frac{19}{5} \\ \frac{19}{5} & \frac{26}{5} \end{bmatrix}$$

Question 5.

If A = [5312−40], find $(A^T)^T$.

Solution:

A = [5312−40]

∴ A^T = ⎡⎣⎢ 51−4320 ⎤⎦⎥

∴ $(A^T)^T$ = [5312−40] = A

Question 6.

If A = ⎡⎣⎢ 7−253−49111 ⎤⎦⎥ , find $(A^T)^T$.

Solution:

A = ⎡⎣⎢ 7−253−49111 ⎤⎦⎥

∴ A^T = ⎡⎣⎢ 731−2−41591 ⎤⎦⎥

∴ $(A^T)^T$ = ⎡⎣⎢ 7−253−49111 ⎤⎦⎥ = A

Question 7.

Find a, b, c if ⎡⎣⎢ 1b−435−5ca−70 ⎤⎦⎥ is a symetric matrix.

Solution:

Let A = ⌈ ⌊ ∣ 1b−435−5ca−70⌉ ⌋ ∣

Since, A is a symmetric matrix, $a_{ij} = a_{ji}$ for all i and j

$\therefore\ a_{13} = a_{31}, a_{12} = a_{21}$ and $a_{23} = a_{32}$

$\therefore\ a = -4, \frac{3}{5} = b$ and $-7 = c$

$\therefore\ a = -4, b = \frac{3}{5}$ and $c = -7.$

Alternative Method :

Let $A = \begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix}$

Then $A^T = \begin{bmatrix} 1 & b & -4 \\ \frac{3}{5} & -5 & -7 \\ a & -7 & 0 \end{bmatrix}$

Since, A is symmetric matrix, $A = A^T$

$$\therefore \begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix} = \begin{bmatrix} 1 & b & -4 \\ \frac{3}{5} & -5 & -7 \\ a & -7 & 0 \end{bmatrix}$$

By equality of matrices

$a = -4, b = \frac{3}{5}$ and $c = -7.$

Question 8.

Find x, y, z if ⌈ ⌊ ∣ 0y32−5i0−2−√xz0⌉ ⌋ ∣ is a skew symmetric matrix.

Solution:

Let A = ⌈ ⌊ ∣ 0y32−5i0−2−√xz0⌉ ⌋ ∣

Since, A is skew-symmetric matrix,

$a_{ij} = -a_{ji}$ for all i and j.

$\therefore\ a_{13} = -a_{31},\ a_{12} = -a_{21}$ and $a_{23} = -a_{32}$

$\therefore\ x = -\frac{3}{2},\ -5i = -y$ and $z = -(-\sqrt{2})$

$\therefore\ x = -\frac{3}{2},\ y = 5i$ and $z = \sqrt{2}.$

Alternative Method :

Let $A = \begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix}$

Then $A^T = \begin{bmatrix} 0 & y & \frac{3}{2} \\ -5i & 0 & -\sqrt{2} \\ x & z & 0 \end{bmatrix}$

Since, A is skew-symmetric matrix, $A = -A^T$

$$\therefore \begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix} = -\begin{bmatrix} 0 & y & \frac{3}{2} \\ -5i & 0 & -\sqrt{2} \\ x & z & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -y & -\frac{3}{2} \\ 5i & 0 & \sqrt{2} \\ -x & -z & 0 \end{bmatrix}$$

By equality of matrices

$x = -\frac{3}{2},\ y = 5i$ and $z = \sqrt{2}.$

Question 9.
For each of the following matrices, find its transpose and state whether it is symmetric, skew-symmetric or neither:
(i) ⎡ ⎣ ⎢ 12−52−34−549⎤ ⎦ ⎥
Solution:
Let A = ⎡ ⎣ ⎢ 12−52−34−549⎤ ⎦ ⎥
Then AT = ⎡ ⎣ ⎢ 12−52−34−549⎤ ⎦ ⎥
Since, A = AT, A is a symmetric matrix.

(ii) ⎡ ⎣ ⎢ 2−5−154−6163⎤ ⎦ ⎥
Solution:
Let B = ⎡ ⎣ ⎢ 2−5−154−6163⎤ ⎦ ⎥
Then BT = ⎛ ⎝ ⎜ 251−546−1−63⎞ ⎠ ⎟
∴ B ≠ BT
Also,
-BT = ⎛ ⎝ ⎜ 251−546−1−63⎞ ⎠ ⎟ =⎛ ⎝ ⎜ −2−5−15−4−616−3⎞ ⎠ ⎟
∴ B ≠ -BT
Hence, B is neither symmetric nor skew-symmetric matrix.

(iii) ⎡ ⎣ ⎢ 0−1−2i2−i1+2i07i−2−70⎤ ⎦ ⎥
Solution:

$$\text{Let } C = \begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$$

$$\text{Then } C^T = \begin{bmatrix} 0 & -1-2i & 2-i \\ 1+2i & 0 & 7 \\ i-2 & -7 & 0 \end{bmatrix}$$

$$\therefore \; -C^T = \begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$$

$$\therefore \; C = -C^T$$

Hence, C is a skew-symmetric matrix.

Question 10.
Construct the matrix A = $[a_{ij}]_{3\times3}$, where a_{ij} = i – j. State whether A is symmetric or skew-symmetric.
Solution:

$$A = [a_{ij}]_{3\times3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Now, $a_{ij} = i - j$ for all i and j

$$\therefore \; a_{11} = 1-1 = 0, \; a_{12} = 1-2 = -1$$

$$a_{13} = 1-3 = -2, \; a_{21} = 2-1 = 1$$

$$a_{22} = 2-2 = 0, \; a_{23} = 2-3 = -1$$

$$a_{31} = 3-1 = 2, \; a_{32} = 3-2 = 1, \; a_{33} = 3-3 = 0$$

$$\therefore \; A = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

Since, $a_{ij} = i - j = -(j-i) = -a_{ji}$ for all i and j,

A is skew-symmetric matrix.

Question 11.
Solve the following equations for X and Y, if 3X – Y = $[1 -1 -1 1]$ and X – 3Y = $[0 0 -1 -1]$
Solution:

$$3X - Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \ldots (1)$$

$$X - 3Y = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix} \quad \ldots (2)$$

Multiplying (1) by 3, we get

$$9X - 3y = 3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \quad \ldots (3)$$

Subtracting (2) from (3), we get

$$8X = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-0 & -3-(-1) \\ -3-0 & 3-(-1) \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -3 & 4 \end{bmatrix}$$

$$\therefore X = \frac{1}{8}\begin{bmatrix} 3 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} \frac{3}{8} & -\frac{1}{4} \\ -\frac{3}{8} & \frac{1}{2} \end{bmatrix}$$

$$3\begin{bmatrix} \frac{3}{8} & -\frac{1}{4} \\ -\frac{3}{8} & \frac{1}{2} \end{bmatrix} - Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} \frac{9}{8} & -\frac{3}{4} \\ -\frac{9}{8} & \frac{3}{2} \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{8}-1 & -\frac{3}{4} & -(-1) \\ -\frac{9}{8}-(-1) & \frac{3}{2} & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{2} \end{bmatrix}$$

$$\text{Hence, } X = \begin{bmatrix} \frac{3}{8} & -\frac{1}{4} \\ -\frac{3}{8} & \frac{1}{2} \end{bmatrix} \text{ and } Y = \begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{2} \end{bmatrix}.$$

Question 12.

Find matrices A and B, if 2A – B = [6−4−6201] and A – 2B = [3−2218−7]

Solution:

Given equations are

$$2A - B = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} \quad \ldots(i)$$

$$\text{and } A - 2B = \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix} \quad \ldots(ii)$$

By (i) – (ii) x 2, we get

$$3B = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - 2\begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 4 & 16 \\ -4 & 2 & -14 \end{bmatrix}$$

$$= \begin{bmatrix} 6-6 & -6-4 & 0-16 \\ -4+4 & 2-2 & 1+14 \end{bmatrix}$$

$\therefore 3B = \begin{bmatrix} 0 & -10 & -16 \\ 0 & 0 & 15 \end{bmatrix}$

$\therefore B = \frac{1}{3}\begin{bmatrix} 0 & -10 & -16 \\ 0 & 0 & 15 \end{bmatrix}$

$\therefore B = \begin{bmatrix} 0 & \frac{-10}{3} & \frac{-16}{3} \\ 0 & 0 & 5 \end{bmatrix}$

By (i) x 2 – (ii), we get

$3A = 2\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$

$= \begin{bmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$

$= \begin{bmatrix} 12-3 & -12-2 & 0-8 \\ -8+2 & 4-1 & 2+7 \end{bmatrix}$

$\therefore 3A = \begin{bmatrix} 9 & -14 & -8 \\ -6 & 3 & 9 \end{bmatrix}$

$\therefore A = \frac{1}{3}\begin{bmatrix} 9 & 14 & -8 \\ -6 & 3 & 9 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 3 & \frac{-14}{3} & \frac{-8}{3} \\ -2 & 1 & 3 \end{bmatrix}$

Question 13.
Find x and y, if [2x+y3−14y14]+[−136043]=[3651857]
Solution:

$\begin{bmatrix} 2x+y & -1 & 1 \\ 3 & 4y & 4 \end{bmatrix} + \begin{bmatrix} -1 & 6 & 4 \\ 3 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 5 \\ 6 & 18 & 7 \end{bmatrix}$

$\therefore \begin{bmatrix} 2x+y-1 & -1+6 & 1+4 \\ 3+3 & 4y+0 & 4+3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 5 \\ 6 & 18 & 7 \end{bmatrix}$

$\therefore \begin{bmatrix} 2x+y-1 & 5 & 5 \\ 6 & 4y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 5 \\ 6 & 18 & 7 \end{bmatrix}$

By equality of matrices, we get
2x + y – 1 = 3 ……..(1)
and 4y = 18 ……….(2)
From (2), y = 92
Substituting y = 92 in (1), we get
2x + 92 – 1 = 3
∴ 2x = 3 – 72 = -12
∴ x = -14
Hence, x = -14 and y = 92.

Question 14.
If [2a+bc+2d3a−b2c−d]=[243−1], find a, b, c and d.
Solution:
[2a+bc+2d3a−b2c−d]=[243−1]
By equality of matrices,
2a + b = 2 ….. (1)
3a – b = 3 …… (2)
c + 2d = 4 …… (3)
2c – d = -1 …… (4)
Adding (1) and (2), we get
5a = 5
∴ a = 1
Substituting a = 1 in (1), we get

2(1) + b = 2
∴ b = 0
Multiplying equation (4) by 2, we get
4c – 2d = -2 …… (5)
Adding (3) and (5), we get
5c = 2
∴ c = 25
Substituting c = 25 in (4), we get
2(25) – d = -1
∴ d = 45 + 1 = 95
Hence, a = 1, b = 0, c = 25 and d = 95.

Question 15.
There are two book shops owned by Suresh and Ganesh. Their sales (in Rupees) for books in three subjects – Physics, Chemistry and Mathematics for two months, July and August 2017 are given by two matrices A and B:
July sales (in Rupees), Physics, Chemistry, Mathematics
A = [560066506750705585008905] First Row Suresh / Second Row Ganesh
August Sales (in Rupees), Physics, Chemistry, Mathematics
B = [66507000705575008905510200] First Row Suresh / Second Row Ganesh
(i) Find the increase in sales in Z from July to August 2017.
(ii) If both book shops get 10% profit in the month of August 2017,
find the profit for each bookseller in each subject in that month.
Solution:
Th d Ganesh are given by the matrices A and B as:

July Sales (in ₹)

$$\begin{array}{cccc} & \text{Physics} & \text{Chemistry} & \text{Mathematics} & \\ A = & \left(\begin{matrix} 5600 \\ 6650 \end{matrix}\right. & \begin{matrix} 6750 \\ 7055 \end{matrix} & \left.\begin{matrix} 8500 \\ 8905 \end{matrix}\right) & \begin{matrix} \textbf{Suresh} \\ \textbf{Ganesh} \end{matrix} \end{array}$$

August Sales (in ₹)

$$\begin{array}{cccc} & \text{Physics} & \text{Chemistry} & \text{Mathematics} & \\ B = & \left(\begin{matrix} 6650 \\ 7000 \end{matrix}\right. & \begin{matrix} 7055 \\ 7500 \end{matrix} & \left.\begin{matrix} 8905 \\ 10200 \end{matrix}\right) & \begin{matrix} \textbf{Suresh} \\ \textbf{Ganesh} \end{matrix} \end{array}$$

(i) The increase in sales (in ₹) from July to August 2017 is obtained by subtracting the matrix A from B.

$$\text{Now, } B - A = \begin{pmatrix} 6650 & 7055 & 8905 \\ 7000 & 7500 & 10200 \end{pmatrix} - \begin{pmatrix} 5600 & 6750 & 8500 \\ 6650 & 7055 & 8905 \end{pmatrix}$$

$$= \begin{pmatrix} 6650-5600 & 7055-6750 & 8905-8500 \\ 7000-6650 & 7500-7055 & 10200-8905 \end{pmatrix}$$

$$\begin{array}{cccc} & \text{Physics} & \text{Chemistry} & \text{Mathematics} & \\ = & \left(\begin{matrix} 1050 \\ 350 \end{matrix}\right. & \begin{matrix} 305 \\ 445 \end{matrix} & \left.\begin{matrix} 405 \\ 1295 \end{matrix}\right) & \begin{matrix} \textbf{Suresh} \\ \textbf{Ganesh} \end{matrix} \end{array}$$

Hence, the increase in sales (in ₹) from July to August 2017 for:
Suresh book shop: ₹ 1050 in Physics, ₹ 305 in Chemistry, and ₹ 405 in Mathematics.
Ganesh book shop: ₹ 350 in Physics, ₹ 445 in Chemistry, and ₹ 1295 in Mathematics.
(ii) Both the book shops get 10% profit in August 2017,
the profit for each bookseller in each subject in August 2017 is obtained by the scalar multiplication of matrix B by 10%,
i.e. 10100=110

$$\text{Now, } \frac{1}{10}B = \frac{1}{10}\begin{pmatrix} 6650 & 7055 & 8905 \\ 7000 & 7500 & 10200 \end{pmatrix}$$

$$\begin{array}{cccc} & \text{Physics} & \text{Chemistry} & \text{Mathematics} & \\ = & \left(\begin{matrix} 665 \\ 700 \end{matrix}\right. & \begin{matrix} 705.5 \\ 750 \end{matrix} & \left.\begin{matrix} 890.5 \\ 1020 \end{matrix}\right) & \begin{matrix} \textbf{Suresh} \\ \textbf{Ganesh} \end{matrix} \end{array}$$

Hence, the profit for Suresh book shop are ₹ 665 in Physics, ₹ 705.50 in Chemistry and ₹ 890.50 in Mathematics and for Ganesh book shop are ₹ 700 in Physics, ₹ 750 in Chemistry and ₹ 1020 in Mathematics.

Maharashtra State Board 12th Commerce Maths Solutions Chapter 2 Matrices Ex 2.3

Question 1.
Evaluate:
(i) $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 3 \end{bmatrix}$
Solution:
$\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -12 & 9 \\ 4 & -8 & 6 \\ 2 & -4 & 3 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$
Solution:
$\begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$ = [8 – 3 + 3] = [8]

Question 2.
If A = $\begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$. State whether AB = BA? Justify your answer.
Solution:

$$AB = \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} -2+3+1 & -1+0+2 & -4+2+1 \\ 4+9+0 & 2+0+0 & 8+6+0 \\ 2-9+1 & 1-0+2 & 4-6+1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 1 & -1 \\ 13 & 2 & 14 \\ -6 & 3 & -1 \end{bmatrix} \quad \ldots (1)$$
$$BA = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} -2+2+4 & 2+3-12 & 2+0+4 \\ -3+0+2 & 3+0-6 & 3+0+2 \\ -1+4+1 & 1+6-3 & 1+0+1 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & -7 & 6 \\ -1 & -3 & 5 \\ 4 & 4 & 2 \end{bmatrix} \quad \ldots (2)$$

From (1) and (2), AB ≠ BA.

Question 3.
Show that AB = BA, where A = $\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$
Solution:

$$AB = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+6-3 & -6+6-0 & 2-3+1 \\ -1+4-3 & -3+4-0 & 1-2+1 \\ -6+18-12 & -18+18-0 & 6-9+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \ldots (1)$$

$$B = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4-2+6 & 6+4-9 & -2-2+4 \\ -6-0+6 & 9+0-9 & -3-0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \ldots (2)$$

From (1) and (2), AB = BA.

Question 4.
Verify A(BC) = (AB)C, if A = ⌈ ⌊ ∣ 120034105⌉ ⌋ ∣ , B = ⌈ ⌊ ∣ 2-10-213⌉ ⌋ ∣ , and C = [3220-1-2]
Solution:

$$BC = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-4 & 4-0 & -2+4 \\ -3+2 & -2+0 & 1-2 \\ 0+6 & 0+0 & 0-6 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 \\ -1 & -2 & -1 \\ 6 & 0 & -6 \end{bmatrix}$$

$$\therefore\ A(BC) = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 & 2 \\ -1 & -2 & -1 \\ 6 & 0 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0+6 & 4-0+0 & 2-0-6 \\ 4-3+0 & 8-6+0 & 4-3-0 \\ 0-4+30 & 0-8+0 & 0-4-30 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & -34 \end{bmatrix} \qquad \ldots (1)$$

$$\text{Also, } AB = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0+0 & -2+0+3 \\ 4-3+0 & -4+3+0 \\ 0-4+0 & 0+4+15 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix}$$

$$\therefore\ (AB)C = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6+2 & 4+0 & -2-2 \\ 3-2 & 2-0 & -1+2 \\ -12+38 & -8+0 & 4-38 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & -34 \end{bmatrix} \qquad \ldots (2)$$

From (1) and (2), A(BC) = (AB)C.

Question 5.

Verify that A(B + C) = AB + AC, if A = [42−23], B = [−131−2] and C = [421−1]

Solution:

$$B+C=\begin{bmatrix}-1 & 1\\ 3 & -2\end{bmatrix}+\begin{bmatrix}4 & 1\\ 2 & -1\end{bmatrix}$$

$$=\begin{bmatrix}-1+4 & 1+1\\ 3+2 & -2-1\end{bmatrix}=\begin{bmatrix}3 & 2\\ 5 & -3\end{bmatrix}$$

$$\therefore\ A(B+C)=\begin{bmatrix}4 & -2\\ 2 & 3\end{bmatrix}\begin{bmatrix}3 & 2\\ 5 & -3\end{bmatrix}$$

$$=\begin{bmatrix}12-10 & 8+6\\ 6+15 & 4-9\end{bmatrix}=\begin{bmatrix}2 & 14\\ 21 & -5\end{bmatrix}\quad \ldots (1)$$

$$\text{Also, } AB=\begin{bmatrix}4 & -2\\ 2 & 3\end{bmatrix}\begin{bmatrix}-1 & 1\\ 3 & -2\end{bmatrix}$$

$$=\begin{bmatrix}-4-6 & 4+4\\ -2+9 & 2-6\end{bmatrix}=\begin{bmatrix}-10 & 8\\ 7 & -4\end{bmatrix}$$

$$AC=\begin{bmatrix}4 & -2\\ 2 & 3\end{bmatrix}\begin{bmatrix}4 & 1\\ 2 & -1\end{bmatrix}$$

$$=\begin{bmatrix}16-4 & 4+2\\ 8+6 & 2-3\end{bmatrix}=\begin{bmatrix}12 & 6\\ 14 & -1\end{bmatrix}$$

$$\therefore\ AB+AC=\begin{bmatrix}-10 & 8\\ 7 & -4\end{bmatrix}+\begin{bmatrix}12 & 6\\ 14 & -1\end{bmatrix}$$

$$=\begin{bmatrix}-10+12 & 8+6\\ 7+14 & -4-1\end{bmatrix}=\begin{bmatrix}2 & 14\\ 21 & -5\end{bmatrix}\ldots (2)$$

From (1) and (2), A(B + C) = AB + AC.

Question 6.

If A = [4−13220], B = ⌈ ⌊ ∣ 1−1120−2⌉ ⌋ ∣ , show that matrix AB is non-singular.

Solution:

$$AB=\begin{bmatrix}4 & 3 & 2\\ -1 & 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & 0\\ 1 & -2\end{bmatrix}$$

$$=\begin{bmatrix}4-3+2 & 8+0-4\\ -1-2+0 & -2+0-0\end{bmatrix}=\begin{bmatrix}3 & 4\\ -3 & -2\end{bmatrix}$$

$$\therefore\ |AB|=\begin{vmatrix}3 & 4\\ -3 & -2\end{vmatrix}$$

$$=-6-(-12)=6\neq 0$$

Hence, AB is a non-singular matrix.

Question 7.

If A + I = ⌈ ⌊ ∣ 15024702−3⌉ ⌋ ∣ , find the product (A + I)(A – I).

Solution:

$$A - I = (A + I) - 2I$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 0 \\ 5 & 2 & 2 \\ 0 & 7 & -5 \end{bmatrix}$$

$$\therefore\ (A + I)(A - I) = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 0 \\ 5 & 2 & 2 \\ 0 & 7 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -1+10+0 & 2+4+0 & 0+4-0 \\ -5+20+0 & 10+8+14 & 0+8-10 \\ 0+35-0 & 0+14-21 & 0+14+15 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 6 & 4 \\ 15 & 32 & -2 \\ 35 & -7 & 29 \end{bmatrix}$$

Question 8.

If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that $A^2 - 4A$ is a scalar matrix.

Solution:

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$\therefore\ A^2 - 4A = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

which is a scalar matrix.

Question 9.

If A = $\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find k so that $A^2 - 8A - kI = O$, where I is a 2 × 2 unit matrix and O is null matrix of order 2.

Solution:

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1-0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$$

$$\therefore \ A^2 - 8A - kI = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - 8 \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} - \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$= \begin{bmatrix} 1-8-k & 0-0-0 \\ -8+8-0 & 49-56-k \end{bmatrix}$$

$$= \begin{bmatrix} -k-7 & 0 \\ 0 & -k-7 \end{bmatrix}$$

But, $A^2 - 8A - kI = 0$

$$\therefore \ \begin{bmatrix} -k-7 & 0 \\ 0 & -k-7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

By equality of matrices,
-k – 7 = 0
∴ k = -7.

Question 10.
If A = [3−112], prove that A^2 – 5A + 7I = 0, where I is a 2 × 2 unit matrix.
Solution:

$$A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore \ A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \ A^2 - 5A + 7I = 0.$$

Question 11.
If A = [1−12−2], B = [2−1ab] and if$(A + B)^2 = A^2 + B^2$, find values of a and b.
Solution:
$(A + B)^2 = A^2 + B^2$
∴ (A + B)(A + B) = $A^2 + B^2$
∴ A^2 + AB + BA + B^2 = $A^2 + B^2$
∴ AB + BA = 0
∴ AB = -BA

$$\therefore \ \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix} = - \begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$\therefore \ \begin{bmatrix} 2-2 & a+2b \\ -2+2 & -a-2b \end{bmatrix} = - \begin{bmatrix} 2-a & 4-2a \\ -1-b & -2-2b \end{bmatrix}$$

$$\therefore \ \begin{bmatrix} 0 & a+2b \\ 0 & -a-2b \end{bmatrix} = \begin{bmatrix} a-2 & 2a-4 \\ 1+b & 2+2b \end{bmatrix}$$

By the equality of matrices, we get
0 = a – 2 ……..(1)
0 = 1 + b ……..(2)
a + 2b = 2a – 4 ……..(3)

-a – 2b = 2 + 2b ……..(4)
From equations (1) and (2), we get
a = 2 and b = -1
The values of a and b satisfy equations (3) and (4) also.
Hence, a = 2 and b = -1.

Question 12.
Find k, if A = [34-2-2] and A2 = kA – 2I.
Solution:

$$A^2 = A \cdot A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

$$kA - 2I = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

But, $A^2 = kA - 2I$

$$\therefore \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

By equality of matrices,
1 = 3k – 2 ……..(1)
-2 = -2k ……..(2)
4 = 4k ……..(3)
-4 = -2k – 2 ……..(4)
From (2), k = 1.
k = 1 also satisfies equation (1), (3) and (4).
Hence, k = 1.

Question 13.
Find x and y, if {4[21-1032]-[32-3141]}⎡⎣⎢⎢2-11⎤⎦⎥⎥ =[xy]
Solution:

$$\left\{4\begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -3 & 4 \\ 2 & 1 & 1 \end{bmatrix}\right\}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \left\{\begin{bmatrix} 8 & -4 & 12 \\ 4 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 3 & -3 & 4 \\ 2 & 1 & 1 \end{bmatrix}\right\}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \begin{bmatrix} 5 & -1 & 8 \\ 2 & -1 & 7 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \begin{bmatrix} 10+1+8 \\ 4+1+7 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \begin{bmatrix} 19 \\ 12 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

By equality of matrices,
x = 19 and y = 12.

Question 14.
Find x, y, z, if ⎧⎩⎨⎪⎪3⎡⎣⎢202022⎤⎦⎥-4⎡⎣⎢1-13121⎤⎦⎥⎫⎭⎬⎪⎪[12]=⎡⎣⎢x-3y-12z⎤⎦⎥
Solution:

$$\left\{3\begin{bmatrix}2 & 0\\0 & 2\\2 & 2\end{bmatrix}-4\begin{bmatrix}1 & 1\\-1 & 2\\3 & 1\end{bmatrix}\right\}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore\ \left\{\begin{bmatrix}6 & 0\\0 & 6\\6 & 6\end{bmatrix}-\begin{bmatrix}4 & 4\\-4 & 8\\12 & 4\end{bmatrix}\right\}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore\ \begin{bmatrix}2 & -4\\4 & -2\\-6 & 2\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore\ \begin{bmatrix}2-8\\4-4\\-6+4\end{bmatrix}=\begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore\ \begin{bmatrix}-6\\0\\-2\end{bmatrix}=\begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

By equality of matrices,
-6 = x – 3, 0 = y – 1 and -2 = 2z
∴ x = -3, y = 1 and z = -1.

Question 15.
Jay and Ram are two friends. Jay wants to buy 4 pens and 8 notebooks. Ram wants to buy 5 pens and 12 notebooks. The price of one pen and one notebook was ₹ 6 and ₹ 10 respectively. Using matrix multiplication, find the amount each one of them requires for buying the pens and notebooks.
Solution:
The given data can be written in matrix form as:
Number of Pens and Notebooks

$$\begin{array}{cc} & \begin{array}{cc}\textbf{Pens} & \textbf{Notebooks}\end{array}\\ \mathbf{A}= & \begin{bmatrix}4 & 8\\5 & 12\end{bmatrix}\begin{array}{l}\textbf{Jay}\\\textbf{Ram}\end{array}\end{array}$$

Price in ₹

$$B=\begin{bmatrix}6\\10\end{bmatrix}\begin{array}{l}\textbf{Pen}\\\textbf{Notebook}\end{array}$$

For finding the amount each one of them requires to buy the pens and notebook, we require the multiplication of the two matrices A and B.

$$\text{Consider } AB=\begin{bmatrix}4 & 8\\5 & 12\end{bmatrix}\begin{bmatrix}6\\10\end{bmatrix}$$

$$=\begin{bmatrix}24+80\\30+120\end{bmatrix}=\begin{bmatrix}104\\150\end{bmatrix}$$

Hence, Jay requires ₹ 104 and Ram requires ₹ 150 to buy the pens and notebooks.

Maharashtra State Board 12th Commerce Maths Solutions Chapter 2 Matrices Ex 2.3

Question 1.
Evaluate:
(i) ⌈ ⌊ ∣ 321⌉ ⌋ ∣ [2-43]
Solution:
⌈ ⌊ ∣ 321⌉ ⌋ ∣ [2-43] = ⌈ ⌊ ∣ 642-12-8-4963⌉ ⌋ ∣

(ii) $\begin{bmatrix} 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$

Solution:

$\begin{bmatrix} 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$ = [8 – 3 + 3] = [8]

Question 2.

If A = $\begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \\ 4 & 2 & 1 \end{bmatrix}$. State whether AB = BA? Justify your answer.

Solution:

$$\mathbf{AB} = \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+3+1 & -1+0+2 & -4+2+1 \\ 4+9+0 & 2+0+0 & 8+6+0 \\ 2-9+1 & 1-0+2 & 4-6+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & -1 \\ 13 & 2 & 14 \\ -6 & 3 & -1 \end{bmatrix} \qquad \ldots (1)$$

$$\mathbf{BA} = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+2+4 & 2+3-12 & 2+0+4 \\ -3+0+2 & 3+0-6 & 3+0+2 \\ -1+4+1 & 1+6-3 & 1+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -7 & 6 \\ -1 & -3 & 5 \\ 4 & 4 & 2 \end{bmatrix} \qquad \ldots (2)$$

From (1) and (2), AB ≠ BA.

Question 3.

Show that AB = BA, where A = $\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$

Solution:

$$\mathbf{AB} = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}\begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+6-3 & -6+6-0 & 2-3+1 \\ -1+4-3 & -3+4-0 & 1-2+1 \\ -6+18-12 & -18+18-0 & 6-9+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \ldots (1)$$

$$\mathbf{B} = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4-2+6 & 6+4-9 & -2-2+4 \\ -6-0+6 & 9+0-9 & -3-0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \ldots (2)$$

From (1) and (2), AB = BA.

Question 4.

Verify A(BC) = (AB)C, if A = $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 1 & 0 & 5 \end{bmatrix}$, B = $\begin{bmatrix} 2 & -1 \\ 0 & -2 \\ 1 & 3 \end{bmatrix}$, and C = $\begin{bmatrix} 3 & 2 & 2 \\ 0 & -1 & -2 \end{bmatrix}$

Solution:

$$BC = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-4 & 4-0 & -2+4 \\ -3+2 & -2+0 & 1-2 \\ 0+6 & 0+0 & 0-6 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 \\ -1 & -2 & -1 \\ 6 & 0 & -6 \end{bmatrix}$$

$$\therefore\ A(BC) = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 & 2 \\ -1 & -2 & -1 \\ 6 & 0 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0+6 & 4-0+0 & 2-0-6 \\ 4-3+0 & 8-6+0 & 4-3-0 \\ 0-4+30 & 0-8+0 & 0-4-30 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & -34 \end{bmatrix} \qquad \ldots (1)$$

$$\text{Also, } AB = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0+0 & -2+0+3 \\ 4-3+0 & -4+3+0 \\ 0-4+0 & 0+4+15 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix}$$

$$\therefore\ (AB)C = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6+2 & 4+0 & -2-2 \\ 3-2 & 2-0 & -1+2 \\ -12+38 & -8+0 & 4-38 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & -34 \end{bmatrix} \qquad \ldots (2)$$

From (1) and (2), A(BC) = (AB)C.

Question 5.

Verify that A(B + C) = AB + AC, if A = [42−23], B = [−131−2] and C = [421−1]

Solution:

$$B + C = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+4 & 1+1 \\ 3+2 & -2-1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}$$

$$\therefore\ A(B+C) = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 12-10 & 8+6 \\ 6+15 & 4-9 \end{bmatrix} = \begin{bmatrix} 2 & 14 \\ 21 & -5 \end{bmatrix} \quad \ldots (1)$$

$$\text{Also, } AB = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -4-6 & 4+4 \\ -2+9 & 2-6 \end{bmatrix} = \begin{bmatrix} -10 & 8 \\ 7 & -4 \end{bmatrix}$$

$$AC = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 16-4 & 4+2 \\ 8+6 & 2-3 \end{bmatrix} = \begin{bmatrix} 12 & 6 \\ 14 & -1 \end{bmatrix}$$

$$\therefore\ AB + AC = \begin{bmatrix} -10 & 8 \\ 7 & -4 \end{bmatrix} + \begin{bmatrix} 12 & 6 \\ 14 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -10+12 & 8+6 \\ 7+14 & -4-1 \end{bmatrix} = \begin{bmatrix} 2 & 14 \\ 21 & -5 \end{bmatrix} \ldots (2)$$

From (1) and (2), A(B + C) = AB + AC.

Question 6.
If A = [4−13220], B = ⎡⎣⎢⎢1−1120−2⎤⎦⎥⎥, show that matrix AB is non-singular.
Solution:

$$AB = \begin{bmatrix} 4 & 3 & 2 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3+2 & 8+0-4 \\ -1-2+0 & -2+0-0 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -3 & -2 \end{bmatrix}$$

$$\therefore\ |AB| = \begin{vmatrix} 3 & 4 \\ -3 & -2 \end{vmatrix}$$

$$= -6-(-12) = 6 \neq 0$$

Hence, AB is a non-singular matrix.

Question 7.
If A + I = ⎡⎣⎢⎢15024702−3⎤⎦⎥⎥, find the product (A + I)(A – I).

Solution:

$$A - I = (A + I) - 2I$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 0 \\ 5 & 2 & 2 \\ 0 & 7 & -5 \end{bmatrix}$$

$$\therefore \ (A + I)(A - I) = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix}\begin{bmatrix} -1 & 2 & 0 \\ 5 & 2 & 2 \\ 0 & 7 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -1+10+0 & 2+4+0 & 0+4-0 \\ -5+20+0 & 10+8+14 & 0+8-10 \\ 0+35-0 & 0+14-21 & 0+14+15 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 6 & 4 \\ 15 & 32 & -2 \\ 35 & -7 & 29 \end{bmatrix}$$

Question 8.

If A = ⌈ ⌊ ∣ 122212221⌉ ⌋ ∣ , show that A2 – 4A is a scalar matrix.

Solution:

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$\therefore \ A^2 - 4A = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

which is a scalar matrix.

Question 9.

If A = [1−107], find k so that A2 – 8A – kI = O, where I is a 2 × 2 unit matrix and O is null matrix of order 2.

Solution:

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1-0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$$

$$\therefore\ A^2 - 8A - kI = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} - \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$= \begin{bmatrix} 1-8-k & 0-0-0 \\ -8+8-0 & 49-56-k \end{bmatrix}$$

$$= \begin{bmatrix} -k-7 & 0 \\ 0 & -k-7 \end{bmatrix}$$

But, $A^2 - 8A - kI = 0$

$$\therefore\ \begin{bmatrix} -k-7 & 0 \\ 0 & -k-7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

By equality of matrices,
$-k - 7 = 0$
$\therefore k = -7$.

Question 10.
If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, prove that $A^2 - 5A + 7I = 0$, where I is a 2 × 2 unit matrix.
Solution:

$$A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore\ A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore\ A^2 - 5A + 7I = 0.$$

Question 11.
If A = $\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$, B = $\begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix}$ and if $(A + B)^2 = A^2 + B^2$, find values of a and b.
Solution:
$(A + B)^2 = A^2 + B^2$
$\therefore (A + B)(A + B) = A^2 + B^2$
$\therefore A^2 + AB + BA + B^2 = A^2 + B^2$
$\therefore AB + BA = 0$
$\therefore AB = -BA$

$$\therefore\ \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix} = -\begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$\therefore\ \begin{bmatrix} 2-2 & a+2b \\ -2+2 & -a-2b \end{bmatrix} = -\begin{bmatrix} 2-a & 4-2a \\ -1-b & -2-2b \end{bmatrix}$$

$$\therefore\ \begin{bmatrix} 0 & a+2b \\ 0 & -a-2b \end{bmatrix} = \begin{bmatrix} a-2 & 2a-4 \\ 1+b & 2+2b \end{bmatrix}$$

By the equality of matrices, we get
$0 = a - 2$ ……..(1)
$0 = 1 + b$ ……..(2)
$a + 2b = 2a - 4$ ……..(3)

-a – 2b = 2 + 2b ……..(4)
From equations (1) and (2), we get
a = 2 and b = -1
The values of a and b satisfy equations (3) and (4) also.
Hence, a = 2 and b = -1.

Question 12.
Find k, if A = [34−2−2] and A2 = kA – 2I.
Solution:

$$A^2 = A \cdot A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

$$kA - 2I = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

But, $A^2 = kA - 2I$

$$\therefore \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

By equality of matrices,
1 = 3k – 2 ……..(1)
-2 = -2k ……..(2)
4 = 4k ……..(3)
-4 = -2k – 2 ……..(4)
From (2), k = 1.
k = 1 also satisfies equation (1), (3) and (4).
Hence, k = 1.

Question 13.
Find x and y, if {4[21−1032]−[32−3141]}⎡⎣⎢2−11⎤⎦⎥=[xy]
Solution:

$$\left\{4\begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -3 & 4 \\ 2 & 1 & 1 \end{bmatrix}\right\}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \left\{\begin{bmatrix} 8 & -4 & 12 \\ 4 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 3 & -3 & 4 \\ 2 & 1 & 1 \end{bmatrix}\right\}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \begin{bmatrix} 5 & -1 & 8 \\ 2 & -1 & 7 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \begin{bmatrix} 10+1+8 \\ 4+1+7 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \begin{bmatrix} 19 \\ 12 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

By equality of matrices,
x = 19 and y = 12.

Question 14.
Find x, y, z, if ⎧⎩⎨⎪⎪3⎡⎣⎢202022⎤⎦⎥−4⎡⎣⎢1−13121⎤⎦⎥⎫⎭⎬⎪⎪[12]=⎡⎣⎢x−3y−12z⎤⎦⎥
Solution:

$$\left\{3\begin{bmatrix}2 & 0\\0 & 2\\2 & 2\end{bmatrix} - 4\begin{bmatrix}1 & 1\\-1 & 2\\3 & 1\end{bmatrix}\right\}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \left\{\begin{bmatrix}6 & 0\\0 & 6\\6 & 6\end{bmatrix} - \begin{bmatrix}4 & 4\\-4 & 8\\12 & 4\end{bmatrix}\right\}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \begin{bmatrix}2 & -4\\4 & -2\\-6 & 2\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \begin{bmatrix}2-8\\4-4\\-6+4\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \begin{bmatrix}-6\\0\\-2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

By equality of matrices,
-6 = x – 3, 0 = y – 1 and -2 = 2z
∴ x = -3, y = 1 and z = -1.

Question 15.
Jay and Ram are two friends. Jay wants to buy 4 pens and 8 notebooks. Ram wants to buy 5 pens and 12 notebooks. The price of one pen and one notebook was ₹ 6 and ₹ 10 respectively. Using matrix multiplication, find the amount each one of them requires for buying the pens and notebooks.
Solution:
The given data can be written in matrix form as:
Number of Pens and Notebooks

$$\mathbf{A} = \begin{array}{c} \begin{array}{cc}\textbf{Pens} & \textbf{Notebooks}\end{array} \\ \begin{bmatrix}4 & 8\\5 & 12\end{bmatrix} \end{array}\begin{array}{l}\textbf{Jay}\\\textbf{Ram}\end{array}$$

Price in ₹

$$B = \begin{bmatrix}6\\10\end{bmatrix}\begin{array}{l}\textbf{Pen}\\\textbf{Notebook}\end{array}$$

For finding the amount each one of them requires to buy the pens and notebook, we require the multiplication of the two matrices A and B.

$$\text{Consider AB} = \begin{bmatrix}4 & 8\\5 & 12\end{bmatrix}\begin{bmatrix}6\\10\end{bmatrix}$$

$$= \begin{bmatrix}24+80\\30+120\end{bmatrix} = \begin{bmatrix}104\\150\end{bmatrix}$$

Hence, Jay requires ₹ 104 and Ram requires ₹ 150 to buy the pens and notebooks.

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Question 1.
Find A^T, if
(i) A = $[1 -4 3 5]$

(ii) A = [2-4-6015]

Solution:

(i) $A = \begin{bmatrix} 1 & 3 \\ -4 & 5 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 1 & -4 \\ 3 & 5 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 2 & -6 & 1 \\ -4 & 0 & 5 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 2 & -4 \\ -6 & 0 \\ 1 & 5 \end{bmatrix}$

Question 2.

If A = $[a_{ij}]_{3\times3}$ where a_{ij} = 2(i – j). Find A and A^T. State whether A and A^T both are symmetric or skew-symmetric matrices.

Solution:

$$A = [a_{ij}]_{3\times3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Given : $a_{ij} = 2(i - j)$

$\therefore a_{11} = 2(1-1) = 0,\ a_{12} = 2(1-2) = -2,$

$a_{13} = 2(1-3) = -4,\ a_{21} = 2(2-1) = 2,$

$a_{22} = 2(2-2) = 0,\ a_{23} = 2(2-3) = -2,$

$a_{31} = 2(3-1) = 4,\ a_{32} = 2(3-2) = 2,$

$a_{33} = 2(3-3) = 0$

$$\therefore A = \begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -2 \\ 4 & 2 & 0 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & 2 \\ -4 & -2 & 0 \end{bmatrix}$$

$$\therefore -A^T = -\begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & 2 \\ -4 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -2 \\ 4 & 2 & 0 \end{bmatrix}$$

$\therefore A = -A^T$ and $A^T = -A$

Hence, A and A^T are both skew-symmetric matrices.

Question 3.

If A = ⎡ ⎣ ⎢ 54-2-3-31⎤ ⎦ ⎥ , prove that $(A^T)^T$ = A.

Solution:

$$A = \begin{bmatrix} 5 & -3 \\ 4 & -3 \\ -2 & 1 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 5 & 4 & -2 \\ -3 & -3 & 1 \end{bmatrix}$$

$$\therefore (A^T)^T = \begin{bmatrix} 5 & -3 \\ 4 & -3 \\ -2 & 1 \end{bmatrix} = A.$$

Question 4.

If A = ⎡ ⎣ ⎢ 12-52-34-549⎤ ⎦ ⎥ , prove that A^T = A.

Solution:

$$A = \begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix} \quad \ldots (1)$$

$$\therefore\ A^T = \begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix} \quad \ldots (2)$$

From (1) and (2), $A^T = A$.

Question 5.
If A = ⎡ ⎣ ⎢ 25−6−3−41⎤ ⎦ ⎥ , B = ⎡ ⎣ ⎢ 24−31−13⎤ ⎦ ⎥ , C = ⎡ ⎣ ⎢ 1−1−2243⎤ ⎦ ⎥ , then show that
(i) $(A + B)^T = A^T + B^T$
(ii) $(A - C)^T = A^T - C^T$
Solution:

$$\textbf{(i)}\ A + B = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 4 & -1 \\ -3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2+2 & -3+1 \\ 5+4 & -4-1 \\ -6-3 & 1+3 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 9 & -5 \\ -9 & 4 \end{bmatrix}$$

$$\therefore\ (A+B)^T = \begin{bmatrix} 4 & 9 & -9 \\ -2 & -5 & 4 \end{bmatrix} \quad \ldots (1)$$

$$A^T = \begin{bmatrix} 2 & 5 & -6 \\ -3 & -4 & 1 \end{bmatrix},\ B^T = \begin{bmatrix} 2 & 4 & -3 \\ 1 & -1 & 3 \end{bmatrix}$$

$$\therefore\ A^T + B^T = \begin{bmatrix} 2 & 5 & -6 \\ -3 & -4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 4 & -3 \\ 1 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 5+4 & -6-3 \\ -3+1 & -4-1 & 1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 9 & -9 \\ -2 & -5 & 4 \end{bmatrix} \quad \ldots (2)$$

From (1) and (2),

$(A+B)^T = A^T + B^T$.

$$\textbf{(ii)}\ A - C = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & -3-2 \\ 5-(-1) & -4-4 \\ -6-(-2) & 1-3 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ 6 & -8 \\ -4 & -2 \end{bmatrix}$$

$$\therefore\ (A-C)^T = \begin{bmatrix} 1 & 6 & -4 \\ -5 & -8 & -2 \end{bmatrix} \quad \ldots (1)$$

$$A^T = \begin{bmatrix} 2 & 5 & -6 \\ -3 & -4 & 1 \end{bmatrix},\ C^T = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 4 & 3 \end{bmatrix}$$

$$\therefore\ A^T - C^T = \begin{bmatrix} 2 & 5 & -6 \\ -3 & -4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 & -2 \\ 2 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 5-(-1) & -6-(-2) \\ -3-2 & -4-4 & 1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 & -4 \\ -5 & -8 & -2 \end{bmatrix} \quad \ldots (2)$$

From (1) and (2),

$(A-C)^T = A^T - C^T$.

Question 6.

If A = [5−243] and B = [−143−1], then find C^T, such that 3A – 2B + C = I, where I is the unit matrix of order 2.

Solution:

$$3A - 2B + C = I$$

$$\therefore\ C = I - 3A + 2B$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 3\begin{bmatrix} 5 & 4 \\ -2 & 3 \end{bmatrix} + 2\begin{bmatrix} -1 & 3 \\ 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 12 \\ -6 & 9 \end{bmatrix} + \begin{bmatrix} -2 & 6 \\ 8 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-15+(-2) & 0-12+6 \\ 0-(-6)+8 & 1-9-2 \end{bmatrix}$$

$$\therefore\ C = \begin{bmatrix} -16 & -6 \\ 14 & -10 \end{bmatrix}$$

$$\therefore\ C^T = \begin{bmatrix} -16 & 14 \\ -6 & -10 \end{bmatrix}.$$

Question 7.

If A = [70340−2], B = [02−213−4], then find

(i) $A^T + 4B^T$

(ii) $5A^T - 5B^T$

Solution:

$$A = \begin{bmatrix} 7 & 3 & 0 \\ 0 & 4 & -2 \end{bmatrix},\ B = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 1 & -4 \end{bmatrix}$$

$$\therefore\ A^T = \begin{bmatrix} 7 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix},\ B = \begin{bmatrix} 0 & 2 \\ -2 & 1 \\ 3 & -4 \end{bmatrix}$$

$$\textbf{(i)}\ A^T + 4B^T = \begin{bmatrix} 7 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix} + 4\begin{bmatrix} 0 & 2 \\ -2 & 1 \\ 3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 8 \\ -8 & 4 \\ 12 & -16 \end{bmatrix}$$

$$= \begin{bmatrix} 7+0 & 0+8 \\ 3-8 & 4+4 \\ 0+12 & -2-16 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ -5 & 8 \\ 12 & -18 \end{bmatrix}.$$

$$\textbf{(ii)}\ 5A^T - 5B^T = 5\begin{bmatrix} 7 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix} - 5\begin{bmatrix} 0 & 2 \\ -2 & 1 \\ 3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 35 & 0 \\ 15 & 20 \\ 0 & -10 \end{bmatrix} - \begin{bmatrix} 0 & 10 \\ -10 & 5 \\ 15 & -20 \end{bmatrix}$$

$$= \begin{bmatrix} 35-0 & 0-10 \\ 15-(-10) & 20-5 \\ 0-15 & -10-(-20) \end{bmatrix}$$

$$= \begin{bmatrix} 35 & -10 \\ 25 & 15 \\ -15 & 10 \end{bmatrix}$$

Question 8.

If A = [130112], B = [2315−4−2] and C = [0−12−130], verify that $(A + 2B + 3C)^T = A^T + 2B^T + 3C^T$

Solution:

$A + 2B + 3C$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 2 & 1 & -4 \\ 3 & 5 & -2 \end{bmatrix} + 3\begin{bmatrix} 0 & 2 & 3 \\ -1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 2 & -8 \\ 6 & 10 & -4 \end{bmatrix} + \begin{bmatrix} 0 & 6 & 9 \\ -3 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+0 & 0+2+6 & 1-8+9 \\ 3+6-3 & 1+10-3 & 2-4+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 8 & 2 \\ 6 & 8 & -2 \end{bmatrix}$$

$$\therefore\ (A + 2B + 3C)^T = \begin{bmatrix} 5 & 6 \\ 8 & 8 \\ 2 & -2 \end{bmatrix} \qquad \ldots (1)$$

$$A^T = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix},\ B^T = \begin{bmatrix} 2 & 3 \\ 1 & 5 \\ -4 & -2 \end{bmatrix},\ C^T = \begin{bmatrix} 0 & -1 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$$

$\therefore\ A^T + 2B^T + 3C^T$

$$= \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 2 & 3 \\ 1 & 5 \\ -4 & -2 \end{bmatrix} + 3\begin{bmatrix} 0 & -1 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ 2 & 10 \\ -8 & -4 \end{bmatrix} + \begin{bmatrix} 0 & -3 \\ 6 & -3 \\ 9 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+0 & 3+6-3 \\ 0+2+6 & 1+10-3 \\ 1-8+9 & 2-4+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 6 \\ 8 & 8 \\ 2 & -2 \end{bmatrix} \qquad \ldots (2)$$

From (1) and (2),

$(A + 2B + 3C)^T = A^T + 2B^T + 3C^T$.

Question 9.
If A = [−1−3221−3] and B = [⌊ ∣ 2−3−1123⌋ ∣ ∣ , prove that $(A + B^T)^T = A^T + B$.
Solution:

$$A = \begin{bmatrix} -1 & 2 & 1 \\ -3 & 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ -3 & 2 \\ -1 & 3 \end{bmatrix}$$

$$\therefore\ A^T = \begin{bmatrix} -1 & -3 \\ 2 & 2 \\ 1 & -3 \end{bmatrix}, B^T = \begin{bmatrix} 2 & -3 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\therefore\ A + B^T = \begin{bmatrix} -1 & 2 & 1 \\ -3 & 2 & -3 \end{bmatrix} + \begin{bmatrix} 2 & -3 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 2-3 & 1-1 \\ -3+1 & 2+2 & -3+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & 0 \end{bmatrix}$$

$$\therefore\ (A + B^T)^T = \begin{bmatrix} 1 & -2 \\ -1 & 4 \\ 0 & 0 \end{bmatrix} \quad \ldots (1)$$

$$A^T + B = \begin{bmatrix} -1 & -3 \\ 2 & 2 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ -3 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & -3+1 \\ 2-3 & 2+2 \\ 1-1 & -3+3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1 & 4 \\ 0 & 0 \end{bmatrix} \quad \ldots (2)$$

From (1) and (2),
$(A + B^T)^T = A^T + B$.

Question 10.
Prove that $A + A^T$ is symmetric and $A - A^T$ is a skew-symmetric matrix, where
(i) A = ⎡ ⎣ ⎢ 13−222−3412⎤ ⎦ ⎥
(ii) A = ⎡ ⎣ ⎢ 5342−7−5−42−3⎤ ⎦ ⎥
Solution:

$$\textbf{(i)}\ A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & -3 & 2 \end{bmatrix} \quad \therefore\ A^T = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}$$

$$\therefore\ A + A^T = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & -3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 & -2 \\ 2 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+3 & 4-2 \\ 3+2 & 2+2 & 1-3 \\ -2+4 & -3+1 & 2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & 2 \\ 5 & 4 & -2 \\ 2 & -2 & 4 \end{bmatrix}$$

This is a symmetric matrix (by definition).

$$\text{Also, } A - A^T = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & -3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -2 \\ 2 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-3 & 4-(-2) \\ 3-2 & 2-2 & 1-(-3) \\ -2-4 & -3-1 & 2-2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 6 \\ 1 & 0 & 4 \\ -6 & -4 & 0 \end{bmatrix}$$

This is a skew-symmetric matrix (by definition).

A = $\begin{bmatrix} 5 & 2 & -4 \\ 3 & -7 & 2 \\ 4 & -5 & -3 \end{bmatrix}$

∴ A^T = $\begin{bmatrix} 5 & 3 & 4 \\ 2 & -7 & -5 \\ -4 & 2 & -3 \end{bmatrix}$

∴ $A + A^T$ = $\begin{bmatrix} 5 & 2 & -4 \\ 3 & -7 & 2 \\ 4 & -5 & -3 \end{bmatrix} + \begin{bmatrix} 5 & 3 & 4 \\ 2 & -7 & -5 \\ -4 & 2 & -3 \end{bmatrix}$

= $\begin{bmatrix} 5+5 & 2+3 & -4+4 \\ 3+2 & -7-7 & 2-5 \\ 4-4 & -5+2 & -3-3 \end{bmatrix}$

∴ $A + A^T$ = $\begin{bmatrix} 10 & 5 & 0 \\ 5 & -14 & -3 \\ 0 & -3 & -6 \end{bmatrix}$

∴ $(A + AT)^T = A + A^T$ i.e., $A + A^T = (A + AT)^T$

∴ $A + A^T$ = is a symmetric matrix.

$A - A^T$ = $\begin{bmatrix} 5 & 2 & -4 \\ 3 & -7 & 2 \\ 4 & -5 & -3 \end{bmatrix} - \begin{bmatrix} 5 & 3 & 4 \\ 2 & -7 & -5 \\ -4 & 2 & -3 \end{bmatrix}$

= $\begin{bmatrix} 5-5 & 2-3 & -4-4 \\ 3-2 & -7+7 & 2+5 \\ 4+4 & -5-2 & -3+3 \end{bmatrix}$

= $\begin{bmatrix} 0 & -1 & -8 \\ 1 & 0 & 7 \\ 8 & -7 & 0 \end{bmatrix}$

∴ $(A - A^T)^T$ = $\begin{bmatrix} 0 & 1 & 8 \\ -1 & 0 & -7 \\ -8 & 7 & 0 \end{bmatrix}$

= $\begin{bmatrix} 0 & -1 & -8 \\ 1 & 0 & 7 \\ 8 & -7 & 0 \end{bmatrix}$

∴ $(A - A^T)^T = -(A - A^T)$

i.e., $A - A^T = -(A - A^T)^T$

∴ $A - A^T$ is a skew symmetric matrix.

Question 11.
Express each of the following matrix as the sum of a symmetric and a skew-symmetric matrix:
(i) [43−2−5]
(ii) [⌊ ∣ 3−2−43−2−5−112⌋ ∣ ∣
Solution:

(i) Let $A = \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix}$

Then $A^T = \begin{bmatrix} 4 & 3 \\ -2 & -5 \end{bmatrix}$

$\therefore\ A + A^T = \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -2 & -5 \end{bmatrix}$

$= \begin{bmatrix} 4+4 & -2+3 \\ 3-2 & -5-5 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 1 & -10 \end{bmatrix}$

This is a symmetric matrix.

Also, $A - A^T = \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ -2 & -5 \end{bmatrix}$

$= \begin{bmatrix} 4-4 & -2-3 \\ 3-(-2) & -5-(-5) \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

This is a skew-symmetric matrix.

Now, $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

$= \frac{1}{2}\begin{bmatrix} 8 & 1 \\ 1 & -10 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

$\therefore\ A = \begin{bmatrix} 4 & \frac{1}{2} \\ \frac{1}{2} & -5 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}.$

(ii) Let $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$$\text{Then } A^T = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\therefore\ A + A^T = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3+3 & 3-2 & -1-4 \\ -2+3 & -2-2 & 1-5 \\ -4-1 & -5+1 & 2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

This is a symmetric matrix.

Also, $A - A^T$

$$= \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-3 & 3-(-2) & -1-(-4) \\ -2-3 & -2-(-2) & 1-(-5) \\ -4-(-1) & -5-1 & 2-2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

This is a skew-symmtric matrix.

$$\text{Now, } A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

$$\therefore\ A = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}.$$

Question 12.
If A = [⌊ ∣ 234−1−21] ∣ ∣ and B = [023−1−41], verify that
(i) $(AB)^T = B^TA^T$
(ii) $(BA)^T = A^TB^T$
Solution:

$$A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0 & 3 & -4 \\ 2 & -1 & 1 \end{bmatrix}$$

$$\therefore\ A^T = \begin{bmatrix} 2 & 3 & 4 \\ -1 & -2 & 1 \end{bmatrix},\ B^T = \begin{bmatrix} 0 & 2 \\ 3 & -1 \\ -4 & 1 \end{bmatrix}$$

$$\textbf{(i)}\ AB = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 0 & 3 & -4 \\ 2 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0-2 & 6+1 & -8-1 \\ 0-4 & 9+2 & -12-2 \\ 0+2 & 12-1 & -16+1 \end{bmatrix} = \begin{bmatrix} -2 & 7 & -9 \\ -4 & 11 & -14 \\ 2 & 11 & -15 \end{bmatrix}$$

$$\therefore\ (AB)^T = \begin{bmatrix} -2 & -4 & 2 \\ 7 & 11 & 11 \\ -9 & -14 & -15 \end{bmatrix} \quad \ldots (1)$$

$$B^T A^T = \begin{bmatrix} 0 & 2 \\ 3 & -1 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ -1 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0-2 & 0-4 & 0+2 \\ 6+1 & 9+2 & 12-1 \\ -8-1 & -12-2 & -16+1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -4 & 2 \\ 7 & 11 & 11 \\ -9 & -14 & -15 \end{bmatrix} \quad \ldots (2)$$

From (1) and (2),
$(AB)^T = B^T A^T$.

(ii) $BA = \begin{bmatrix} 0 & 3 & -4 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0+9-16 & 0-6-4 \\ 4-3+4 & -2+2+1 \end{bmatrix} = \begin{bmatrix} -7 & -10 \\ 5 & 1 \end{bmatrix}$$

$$\therefore\ (BA)^T = \begin{bmatrix} -7 & 5 \\ -10 & 1 \end{bmatrix} \quad \ldots (1)$$

$$A^T B^T = \begin{bmatrix} 2 & 3 & 4 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 3 & -1 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9-16 & 4-3+4 \\ 0-6-4 & -2+2+1 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 5 \\ -10 & 1 \end{bmatrix} \quad \ldots (2)$$

From (1) and (2), $(BA)^T = A^T B^T$.

Maharashtra State Board 12th Commerce Maths Solutions Chapter 2 Matrices Ex 2.5

Question 1.
Apply the given elementary transformation on each of the following matrices:
(i) [32−42], $R_1 \leftrightarrow R_2$
(ii) [214−5], $C_1 \leftrightarrow C_2$
(iii) ⎡ ⎣ ⎢ 31−1131−113⎤ ⎦ ⎥ 3R_2 and $C_2 \rightarrow C_2 - 4C_1$

Solution:

(i) Let $A = \begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}$

By $R_1 \leftrightarrow R_2$, we get

$A \sim \begin{bmatrix} 2 & 2 \\ 3 & -4 \end{bmatrix}$

(ii) Let $B = \begin{bmatrix} 2 & 4 \\ 1 & -5 \end{bmatrix}$

By $C_1 \leftrightarrow C_2$, we get

$B \sim \begin{bmatrix} 4 & 2 \\ -5 & 1 \end{bmatrix}$.

(iii) Let $C = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

By $3R_2$, we get

$C \sim \begin{bmatrix} 3 & 1 & -1 \\ 3 & 9 & 3 \\ -1 & 1 & 3 \end{bmatrix}$

By $C_2 - 4C_1$ on C, we get

$C \sim \begin{bmatrix} 3 & -11 & -1 \\ 1 & -1 & 1 \\ -1 & 5 & 3 \end{bmatrix}$.

Question 2.

Transform $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$ into an upper triangularmatrix by suitable row transformations.

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 3R_1$, we get

$A \sim \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 5 & -2 \end{bmatrix}$

By $\left(\frac{1}{3}\right) R_2$, we get

$A \sim \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 5 & -2 \end{bmatrix}$

By $R_3 - 5R_2$, we get

$A \sim \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & -\frac{1}{3} \end{bmatrix}$

This is an upper triangular matrix.

Question 3.

Find the cofactor matrix of the following matrices:

(i) $\begin{bmatrix} 1 & 2 \\ 5 & -8 \end{bmatrix}$

(ii) ⌈ ⌊ | 5-1-28-21711⌉ ⌋ |

Solution:

(i) Let $A = \begin{bmatrix} 1 & 2 \\ 5 & -8 \end{bmatrix}$

Here, $a_{11} = 1$, $M_{11} = -8$

$\therefore A_{11} = (-1)^{1+1} M_{11} = -8$

$a_{12} = 2$, $M_{12} = 5$

$\therefore A_{12} = (-1)^{1+2} M_{12} = -1(5) = -5$

$a_{21} = 5$, $M_{21} = 2$

$\therefore A_{21} = (-1)^{2+1} M_{21} = -1(2) = -2$

$a_{22} = -8$, $M_{22} = 1$

$\therefore A_{22} = (-1)^{2+2} M_{22} = 1.$

$$\therefore \text{cofactor matrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -5 \\ -2 & 1 \end{bmatrix}$$

(ii) Let $A = \begin{bmatrix} 5 & 8 & 7 \\ -1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix}$

The cofactor of a_{ij} is given by $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $M_{11} = \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} = -2 - 1 = -3$

$\therefore A_{11} = (-1)^{1+1} M_{11} = 1(-3) = -3$

$M_{12} = \begin{vmatrix} -1 & 1 \\ -2 & 1 \end{vmatrix} = -1 - (-2) = 1$

$\therefore A_{12} = (-1)^{1+2} M_{12} = -1(1) = -1$

$M_{13} = \begin{vmatrix} -1 & -2 \\ -2 & 1 \end{vmatrix} = -1 - 4 = -5$

$\therefore A_{13} = (-1)^{1+3} M_{13} = 1(-5) = -5$

$$M_{21} = \begin{vmatrix} 8 & 7 \\ 1 & 1 \end{vmatrix} = 8 - 7 = 1$$

$$\therefore\ A_{21} = (-1)^{2+1} M_{21} = -1(1) = -1$$

$$M_{22} = \begin{vmatrix} 5 & 7 \\ -2 & 1 \end{vmatrix} = 5 + 14 = 19$$

$$\therefore\ A_{22} = (-1)^{2+2} M_{22} = 1(19) = 19$$

$$M_{23} = \begin{vmatrix} 5 & 8 \\ -2 & 1 \end{vmatrix} = 5 - (-16) = 21$$

$$\therefore\ A_{23} = (-1)^{2+3} M_{23} = -1(21) = -21$$

$$M_{31} = \begin{vmatrix} 8 & 7 \\ -2 & 1 \end{vmatrix} = 8 - (-14) = 22$$

$$\therefore\ A_{31} = (-1)^{3+1} M_{31} = 1(22) = 22$$

$$M_{32} = \begin{vmatrix} 5 & 7 \\ -1 & 1 \end{vmatrix} = 5 - (-7) = 12$$

$$\therefore\ A_{32} = (-1)^{3+2} M_{32} = -1(12) = -12$$

$$M_{33} = \begin{vmatrix} 5 & 8 \\ -1 & -2 \end{vmatrix} = -10 - (-8) = -2$$

$$\therefore\ A_{33} = (-1)^{3+3} M_{33} = 1(-2) = -2$$

$$\therefore\ \text{cofactor matrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 & -5 \\ -1 & 19 & -21 \\ 22 & -12 & -2 \end{bmatrix}$$

Question 4.
Find the adjoint of the following matrices:
(i) [23-35]
(ii) [⌊ | 1-2-2-13025-1] ⌋ |
Solution:

$$\text{(i)}\ A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$$

Here, $a_{11} = 2$, $M_{11} = 5$

$$\therefore\ A_{11} = (-1)^{1+1}(5) = 5$$

$a_{12} = -3$, $M_{12} = 3$

$$\therefore\ A_{12} = (-1)^{1+2}(3) = -3$$

$a_{21} = 3$, $M_{21} = -3$

$$\therefore\ A_{21} = (-1)^{2+1}(-3) = 3$$

$a_{22} = 5$, $M_{22} = 2$

$$\therefore\ A_{22} = (-1)^{2+2} = 2$$

$\therefore$ the cofactor matrix $= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ 3 & 2 \end{bmatrix}$

$\therefore$ adj A $= \begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$

(ii) Let A $= \begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

The cofactor of a_{ij} is given by $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $M_{11} = \begin{vmatrix} 3 & 5 \\ 0 & -1 \end{vmatrix} = -3 - 0 = -3$

$\therefore A_{11} = (-1)^{1+1}(-3) = -3$

$M_{12} = \begin{vmatrix} -2 & 5 \\ -2 & -1 \end{vmatrix} = 2 + 10 = 12$

$\therefore A_{12} = (-1)^{1+2}(12) = -12$

$M_{13} = \begin{vmatrix} -2 & 3 \\ -2 & 0 \end{vmatrix} = 0 + 6 = 6$

$\therefore A_{13} = (-1)^{1+3}(6) = 6$

$M_{21} = \begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix} = 1 - 0 = 1$

$\therefore A_{21} = (-1)^{2+1}(1) = -1$

$M_{22} = \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = -1 + 4 = 3$

$\therefore A_{22} = (-1)^{2+2}(3) = 3$

$M_{23} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = 0 - 2 = -2$

$\therefore A_{23} = (-1)^{2+3}(-2) = 2$

$M_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5 - 6 = -11$

$\therefore A_{31} = (-1)^{3+1}(-11) = -11$

$M_{32} = \begin{vmatrix} 1 & 2 \\ -2 & 5 \end{vmatrix} = 5 + 4 = 9$

$\therefore A_{32} = (-1)^{3+2}(9) = -9$

$M_{33} = \begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix} = 3 - 2 = 1$

$\therefore A_{33} = (-1)^{3+3}(1) = 1.$

$\therefore$ the cofactor matrix $= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

$= \begin{bmatrix} -3 & -12 & 6 \\ -1 & 3 & 2 \\ -11 & -9 & 1 \end{bmatrix}$

$\therefore$ adj A $= \begin{bmatrix} -3 & -1 & -11 \\ -12 & 3 & -9 \\ 6 & 2 & 1 \end{bmatrix}$

Question 5.
Find the inverses of the following matrices by the adjoint mathod:

(i) [32-1-1]

(ii) [24-25]

(iii) ⎡ ⎣ ⎢ 100220345⎤ ⎦ ⎥

Solution:

(i) Let $A = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$

Then $|A| = \begin{vmatrix} 3 & -1 \\ 2 & -1 \end{vmatrix} = -3-(-2) = -1 \neq 0$

$\therefore$ A^{-1} exists.

First we have to find the cofactor matrix

$= [A_{ij}]_{2 \times 2}$, where $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $A_{11} = (-1)^{1+1} M_{11} = -1$

$A_{12} = (-1)^{1+2} M_{12} = -2$

$A_{21} = (-1)^{2+1} M_{21} = -(-1) = 1$

$A_{22} = (-1)^{2+2} M_{22} = 3$

$\therefore$ the cofactor matrix

$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 1 & 3 \end{bmatrix}$

$\therefore$ adj $A = \begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix}$

$\therefore$ $A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-1}\begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix}$

$\therefore$ $A^{-1} = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$.

Let A = $\begin{bmatrix} 2 & -2 \\ 4 & 5 \end{bmatrix}$

$\therefore$ |A| = $\begin{bmatrix} 2 & -2 \\ 4 & 5 \end{bmatrix}$ = 10 + 8 = 18 ≠ 0

$\therefore$ A^{-1} exists.

$A_{11} = (-1)^{1+1} M_{11} = (1)(5) = 5$

$A_{12} = (-1)^{1+2} M_{12} = (-1)(4) = -4$

$A_{21} = (-1)^{2+1} M_{21} = (-1)(-2) = 2$

$A_{22} = (-1)^{2+2} M_{22} = (1)(2) = 2$

$\therefore$ The matrix of the co-factors is

$[A_{ij}]_{2x2} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ 2 & 2 \end{bmatrix}$

Now adj A = $[A_{ij}]^T_{2\times 2} = \begin{bmatrix} 5 & 2 \\ -4 & 2 \end{bmatrix}$

$\therefore$ A–1 = $\frac{1}{|A|}$(adj A)

$= \frac{1}{18}\begin{bmatrix} 5 & 2 \\ -4 & 2 \end{bmatrix}$.

(iii) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Then $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{vmatrix}$

$= 1(10-0) - 2(0-0) + 3(0-0)$

$= 10 \neq 0$

$\therefore$ A^{-1} exist.

First we have to find the cofactor matrix

$= [A_{ij}]_{3 \times 3}$, where $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} = 10 - 0 = 10$

$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix} = -(0-0) = 0$

$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$

$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = -(10-0) = -10$

$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = 5 - 0 = 5$

$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = -(0-0) = 0$

$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = 8 - 6 = 2$

$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -(4-0) = -4$

$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$

$\therefore$ the cofactor matrix

$$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix}$$

$$\therefore \text{ adj } A = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$\therefore A^{-1} = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

Question 6.
Find the inverses of the following matrices by the transformation method:
(i) [122−1]
(ii) [⌊ | 250011−103] ⌋ |
Solution:

(i) Let $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Then $|A| = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1-4 = -5 \neq 0$

$\therefore$ A^{-1} exists.

A^{-1} by Row transformations :

We write $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

By $\left(-\frac{1}{5}\right) R_2$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$$

By $R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \quad \text{......... (1)}$$

A^{-1} by Column transformations :

We write $A^{-1}A = I$

$$\therefore A^{-1}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $C_2 - 2C_1$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 \\ 2 & -5 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

By $\left(-\frac{1}{5}\right) C_2$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{2}{5} \\ 0 & -\frac{1}{5} \end{bmatrix}$$

By $C_1 - 2C_2$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \quad \text{......... (2)}$$

From (1) and (2),

$A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, which is unique.

(ii) Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Then $|A| = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$

$= 2(3-0) - 0(15-0) - 1(5-0)$

$= 6 - 0 - 5 = 1 \neq 0$

$\therefore$ A^{-1} exists.

We write $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $3R_1$, we get

$$\begin{bmatrix} 6 & 0 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$, we get

$$\begin{bmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - 5R_1$, we get

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & 6 & 15 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - 5R_3$, we get

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 + R_2$ and $R_3 - R_2$, we get

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -12 & 5 & -5 \\ -15 & 6 & -5 \\ 15 & -6 & 6 \end{bmatrix}$$

By $\left(\frac{1}{3}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -12 & 5 & -5 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

By $R_1 + 3R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Question 7.

Find the inverse of A = ⎡ ⎣ ⎢ 101022131⎤ ⎦ ⎥ by elementary column transformations.

Solution:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\therefore\ |A| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= 1(2-6) - 0 + 1(0-2)$$

$$= -4 - 2 = -6 \neq 0$$

$\therefore\ A^{-1}$ exists.

We write $A^{-1}A = I$

$$\therefore\ A^{-1}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $C_3 - C_1$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $\left(\frac{1}{2}\right)C_2$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $C_3 - 3C_2$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & \frac{1}{2} & -\frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

By $\left(-\frac{1}{3}\right)C_3$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -\frac{1}{3} \end{bmatrix}$$

By $C_1 - C_3$ and $C_2 - C_3$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{6}\begin{bmatrix} 4 & -2 & 2 \\ -3 & 0 & 3 \\ 2 & 2 & -2 \end{bmatrix}$$

Question 8.

Find the inverse of ⎡ ⎣ ⎢ 112214357⎤ ⎦ ⎥ by the elementary row transformations.

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$$

$$\text{Then } |A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{vmatrix}$$

$$= 1(7-20) - 2(7-10) + 3(4-2)$$

$$= -13 + 6 + 6 = -1 \neq 0$$

$\therefore$ A^{-1} exists.

We write $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - R_1$ and $R_3 - 2R_1$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $(-1)R_2$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $R_1 - 7R_3$ and $R_3 + 2R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

Question 9.

If A = ⌈ ⌊ ∣ 101022131⌉ ⌋ ∣ and B = ⌈ ⌊ ∣ 112214357⌉ ⌋ ∣ , then find matrix X such that XA = B.

Solution:

$$XA = B$$

$$\therefore\ X\begin{bmatrix}1 & 0 & 1\\0 & 2 & 3\\1 & 2 & 1\end{bmatrix} = \begin{bmatrix}1 & 2 & 3\\1 & 1 & 5\\2 & 4 & 7\end{bmatrix}$$

By $C_3 - C_1$, we get

$$X\begin{bmatrix}1 & 0 & 0\\0 & 2 & 3\\1 & 2 & 0\end{bmatrix} = \begin{bmatrix}1 & 2 & 2\\1 & 1 & 4\\2 & 4 & 5\end{bmatrix}$$

By $\left(\frac{1}{2}\right)C_2$, we get

$$X\begin{bmatrix}1 & 0 & 0\\0 & 1 & 3\\1 & 1 & 0\end{bmatrix} = \begin{bmatrix}1 & 1 & 2\\1 & \frac{1}{2} & 4\\2 & 2 & 5\end{bmatrix}$$

By $C_3 - 3C_2$, we get

$$X\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\1 & 1 & -3\end{bmatrix} = \begin{bmatrix}1 & 1 & -1\\1 & \frac{1}{2} & \frac{5}{2}\\2 & 2 & -1\end{bmatrix}$$

By $\left(-\frac{1}{3}\right)C_3$, we get

$$X\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\1 & 1 & 1\end{bmatrix} = \begin{bmatrix}1 & 1 & \frac{1}{3}\\1 & \frac{1}{2} & -\frac{5}{6}\\2 & 2 & \frac{1}{3}\end{bmatrix}$$

By $C_1 - C_3$ and $C_2 - C_3$, we get

$$X\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\frac{2}{3} & \frac{2}{3} & \frac{1}{3}\\\frac{11}{6} & \frac{4}{3} & -\frac{5}{6}\\\frac{5}{3} & \frac{5}{3} & \frac{1}{3}\end{bmatrix}$$

$$\therefore\ X = \frac{1}{6}\begin{bmatrix}4 & 4 & 2\\11 & 8 & -5\\10 & 10 & 2\end{bmatrix}.$$

Question 10.
Find matrix X, if AX = B, where A = ⎡ ⎣ ⎢ 1-11212324⎤ ⎦ ⎥ and B = ⎡ ⎣ ⎢ 123⎤ ⎦ ⎥
Solution:

$$AX = B$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

By $R_2 + R_1$ and $R_3 - R_1$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

By $\left(\frac{1}{3}\right)R_2$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

By $R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

By $R_1 + \frac{1}{3}R_3$ and $R_2 - \frac{5}{3}R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} -\frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -\frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix}.$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 2 Matrices Ex 2.6

Question 1.
Solve the following equations by the method of inversion:
(i) x + 2y = 2, 2x + 3y = 3
Solution:
The given equations can be written in the matrix form as:

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

This is of the form AX = B, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1 \neq 0$$

$\therefore$ A^{-1} exists.

We write $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

By $(-1)R_2$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$$

By $R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

Now, premultiply AX = B by A^{-1}, we get

$$A^{-1}(AX) = A^{-1}B$$

$$\therefore (A^{-1}A)X = A^{-1}B$$

$$\therefore IX = A^{-1}B$$

$$\therefore X = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6+6 \\ 4-3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

By equality of matrices,
x = 0, y = 1 is the required solution.

(ii) 2x + y = 5, 3x + 5y = -3
Solution:

Matrix form of the given system of equations is

$$\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

This is of the form AX = B,

where $A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

To deterrmine X, we have to find A^{-1}.

$$|A| = \begin{vmatrix} 2 & 1 \\ 3 & 5 \end{vmatrix}$$

$= 10 - 3$

$= 7 \neq 0$

$\therefore$ A^{-1} exissts.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_2$, we get

$$\begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & 4 \\ 0 & -7 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

Applying $R_2 \to \left(-\frac{1}{7}\right) R_2$, we get

$$\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 1 \\ -\frac{3}{7} & \frac{2}{7} \end{bmatrix}$$

Applying $R_1 \to R_1 - 4R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{5}{7} & -\frac{1}{7} \\ -\frac{3}{7} & \frac{2}{7} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{5}{7} & -\frac{1}{7} \\ -\frac{3}{7} & \frac{2}{7} \end{bmatrix}$$

Pre-multiplying $AX = B$ by A^{-1}, we get

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1} A)X = A^{-1} B$

$\therefore Ix = A^{-1} B$

$\therefore X = A^{-1} B$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5}{7} & -\frac{1}{7} \\ -\frac{3}{7} & \frac{2}{7} \end{bmatrix} \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{25}{7} & \frac{+3}{7} \\ -\frac{15}{7} & -\frac{6}{7} \end{bmatrix} = \begin{bmatrix} \frac{28}{7} \\ -\frac{21}{7} \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

x = 4 and y = – 3.

(iii) $2x - y + z = 1$, $x + 2y + 3z = 8$ and $3x + y - 4z = 1$

Solution:

The given equations can be written in the matrix form as:

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$$

This is of the form $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{vmatrix}$$

$= 2(-8-3) + 1(-4-9) + 1(1-6)$

$= -22 - 13 - 5 = -40 \neq 0$

$\therefore$ A^{-1} exists.

We write $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & -4 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 3R_1$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 0 & -5 & -13 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

By $\left(-\frac{1}{5}\right) R_2$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -5 & -13 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{5} & \frac{2}{5} & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$ and $R_3 + 5R_2$, we get

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -8 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{2}{5} & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

By $\left(-\frac{1}{8}\right)R_3$, we get

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{2}{5} & 0 \\ \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \end{bmatrix}$$

By $R_1 - R_3$ and $R_2 - R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{11}{40} & \frac{3}{40} & \frac{1}{8} \\ -\frac{13}{40} & \frac{11}{40} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{40}\begin{bmatrix} 11 & 3 & 5 \\ -13 & 11 & 5 \\ 5 & 5 & -5 \end{bmatrix}$$

Now, premultiply $AX = B$ by A^{-1}, we get

$A^{-1}(AX) = A^{-1}B$

$\therefore\ (A^{-1}A)X = A^{-1}B$

$\therefore\ IX = A^{-1}B$

$$\therefore\ X = \frac{1}{40}\begin{bmatrix} 11 & 3 & 5 \\ -13 & 11 & 5 \\ 5 & 5 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 11+24+5 \\ -13+88+5 \\ 5+40-5 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ 40 \end{bmatrix}$$

$$\therefore\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

By equality of matrices,
x = 1, y = 2, z = 1 is the required solution.

(iv) x + y + z = 1, x – y + z = 2 and x + y – z = 3
Solution:

Matrix form of the given system of equations is

$$\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

This is of the form AX = B,

where A = $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$, X = $\begin{bmatrix}x\\y\\z\end{bmatrix}$ and B = $\begin{bmatrix}1\\2\\3\end{bmatrix}$

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}, A^{-1}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Appying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we get

$$\begin{bmatrix}1 & 1 & 1\\0 & -2 & 0\\0 & 0 & -2\end{bmatrix}A^{-1}=\begin{bmatrix}1 & 0 & 0\\-1 & 1 & 0\\-1 & 0 & 1\end{bmatrix}$$

Appying $R_2 \rightarrow \left(-\frac{1}{2}\right) R_2$, we get

$$\begin{bmatrix}1 & 0 & 1\\0 & 1 & 0\\0 & 0 & -2\end{bmatrix}A^{-1}=\begin{bmatrix}1 & 0 & 0\\\frac{1}{2} & -\frac{1}{2} & 0\\-1 & 0 & 1\end{bmatrix}$$

Appying $R_1 \rightarrow R_1 - R_2$, we get

$$\begin{bmatrix}1 & 0 & 1\\0 & 1 & 0\\0 & 0 & -2\end{bmatrix}A^{-1}=\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & 0\\\frac{1}{2} & -\frac{1}{2} & 0\\-1 & 0 & 1\end{bmatrix}$$

Appying $R_3 \rightarrow \left(-\frac{1}{2}\right) R_3$, we get

$$\begin{bmatrix}1 & 0 & 1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A^{-1}=\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & 0\\-\frac{1}{2} & -\frac{1}{2} & 0\\-\frac{1}{2} & 0 & -\frac{1}{2}\end{bmatrix}$$

Appying $R_1 \to R_1 - R_3$, we get

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

pre-multiplying AX = B by A^{-1}, we get

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)X = A^{-1}B$

$\therefore IX = A^{-1}B$

$\therefore X = A^{-1}B$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 + 1 + \frac{3}{2} \\ \frac{1}{2} - 1 + 0 \\ \frac{1}{2} + 0 - \frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{5}{2} \\ -\frac{1}{2} \\ -1 \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

$x = \dfrac{5}{2}, y = -\dfrac{1}{2}$ and $z = -1$.

Question 2.
Express the following equations in matrix form and solve them by method of reduction:
(i) x + 3y = 2, 3x + 5y = 4.
Solution:
The given equations can be written in the matrix form as:

$$\begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

By $R_2 - 3R_1$, we get

$$\begin{pmatrix} 1 & 3 \\ 0 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x + 3y \\ 0 - 4y \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

By equality of matrices,

$$x + 3y = 2 \quad \text{........ (1)}$$

$$-4y = -2 \quad \text{......... (2)}$$

From (2), $y = \frac{1}{2}$

Substituting $y = \frac{1}{2}$ in (1), we get

$$x + \frac{3}{2} = 2$$

$$\therefore\ x = 2 - \frac{3}{2} = \frac{1}{2}$$

Hence, x = 12, y = 12 is the required solution.

(ii) 3x – y = 1, 4x + y = 6.
Solution:
The given equations can be written in the matrix form as:

$$\begin{bmatrix} 3 & -1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

By $4R_1$ and $3R_2$, we get

$$\begin{bmatrix} 12 & -4 \\ 12 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 18 \end{bmatrix}$$

By $R_2 - R_1$, we get

$$\begin{bmatrix} 12 & -4 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 12x - 4y \\ 0 + 7y \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$$

By equality of matrices,
12x – 4y = 4 .....(1)
7y = 14 .....(2)
From (2), y = 2
Substituting y = 2 in (1), we get
12x – 8 = 4
∴ 12x = 12
∴ x = 1
Hence, x = 1, y = 2 is the required solution.

(iii) x + 2y + z = 8, 2x + 3y – z = 11 and 3x – y – 2z = 5.
Solution:
The given equations can be written in the matrix form as:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \\ 5 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 3R_1$, we get

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & -7 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ -19 \end{bmatrix}$$

By $R_3 - 7R_2$, we get

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & 16 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 16 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + 2y + z \\ 0 - y - 3z \\ 0 + 0 + 16z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 16 \end{bmatrix}$$

By equality of matrices,
x + 2y + z = 8 ……(1)
-y – 3z = -5 …….(2)
16z = 16 ……….(3)
From (3), z = 1
Substituting z = 1 in (2), we get
-y – 3= -5
∴ y = 2
Substituting y = 2, z = 1 in (1), we get
x + 4 + 1 = 8
∴ x = 3
Hence, x = 3, y = 2, z = 1 is the required solution.

(iv) x + y + z = 1, 2x + 3y + 2z = 2 and x + y + 2z = 4.
Solution:
The given equations can be written in the matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - R_1$, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + y + z \\ 0 + y + 0 \\ 0 + 0 + z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

By equality of matrices,
x + y + z = 1 ……(1)
y = 0
z = 3
Substituting y = 0, z = 3 in (1), we get
x + 0 + 3 = 1
∴ x = -2
Hence, x = -2, y = 0, z = 3 is the required solution.

Question 3.
The total cost of 3 T.V. and 2 V.C.R. is ₹ 35000. The shopkeeper wants a profit of ₹ 1000 per T.V. and ₹ 500 per V.C.R. He sells 2 T.V. and 1 V.C.R. and he gets total revenue of ₹ 21500. Find the cost and selling price of T.V. and V.C.R.
Solution:
Let the cost of each T.V. be ₹ x and each V.C.R. be ₹ y.
Then the total cost of 3 T.V. and 2 V.C.R. is ₹ (3x + 2y) which is given to be ₹ 35000.
∴ 3x + 2y = 35000
The shopkeeper wants a profit of ₹ 1000 per T.V. and ₹ 500 per V.C.R.
The selling price of each T.V. is ₹ (x + 1000) and of each V.C.R. is ₹ (y + 500).
∴ selling price of 2 T.V. and 1 V.C.R is
₹ [2(x + 1000) + (y + 500)] which is given to be ₹ 21500.
∴ 2(x + 1000) + (y + 500) = 21500
∴ 2x + 2000 + y + 500 = 21500
∴ 2x + y = 19000

Hence, the system of linear equations is
3x + 2y = 35000
2x + y = 19000
The equations can be written in matrix form as:

$$\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35000 \\ 19000 \end{bmatrix}$$

By $R_1 - 2R_2$, we get

$$\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3000 \\ 19000 \end{bmatrix}$$

$$\therefore \begin{bmatrix} -x+0 \\ 2x+y \end{bmatrix} = \begin{bmatrix} -3000 \\ 19000 \end{bmatrix}$$

By equality of matrices,
-x = -3000 ....(1)
2x + y = 19000 ....(2)
From (1), x = 3000
Substituting x = 3000 in (2), we get
2(3000) + y = 19000
∴ y = 19000 – 6000 = 13000
Hence, the cost price of one T.V. is ₹ 3000 and of one V.C.R. is ₹ 13000 and the selling price of one T.V. is ₹ 4000 and of one V.C.R. is ₹ 13500.

Question 4.
The sum of the cost of one Economics book, one Cooperation book, and one Account book is ₹ 420. The total cost of an Economic book, 2 Cooperation books, and an Account book is ₹ 480. Also, the total cost of an Economic book, 3 Cooperation books, and 2 Account books is ₹ 600. Find the cost of each book.
Solution:
Let the cost of 1 Economic book, 1 Cooperation book and 1 Account book be ₹ x, ₹ y and ₹ z respectively.
Then, from the given information
x + y + z = 420
x + 2y + z = 480
x + 3y + 2z = 600
These equations can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 420 \\ 480 \\ 600 \end{bmatrix}$$

By $R_2 - R_1$ and $R_3 - R_1$, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 420 \\ 60 \\ 180 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x+y+z \\ 0+y+0 \\ 0+2y+z \end{bmatrix} = \begin{bmatrix} 420 \\ 60 \\ 180 \end{bmatrix}$$

By equality of matrices,
x + y + z = 420 ....(1)
y = 60
2y + z = 180 ....(2)
Substituting y = 60 in (2), we get
2(60) + z = 180
∴ z = 180 – 120 = 60
Substituting y = 60, z = 60 in (1), we get
x + 60 + 60 = 420
∴ x = 420 – 120 = 300
Hence, the cost of each Economic book is ₹ 300, each Cooperation book is ₹ 60 and each Account book is ₹ 60.

Maharashtra State Board 12th Commerce Maths Solutions Chapter 2 Matrices Miscellaneous Exercise 2

(I) Choose the correct alternative.

Question 1.
If AX = B, where A = [−122−1], B = [11] then X = _________

(a) [3537]

(b) [7353]

(c) [11]

(d) [12]

Answer:

(c) [11]

Question 2.
The matrix ⎡⎣⎢800080008⎤⎦⎥ is _________
(a) identity matrix
(b) scalar matrix
(c) null matrix
(d) diagonal matrix
Answer:
(b) scalar matrix

Question 3.
The matrix [000000] is _________
(a) identity matrix
(b) diagonal matrix
(c) scalar matix
(d) null matrix
Answer:
(d) null matrix

Question 4.
If A = ⎡⎣⎢a000a000a⎤⎦⎥, then |adj A| = _________
(a) a^{12}
(b) a^{9}
(c) a^{6}
(d) a^{-3}
Answer:
(c) a6
Hint:
adj A = ⎡⎣⎢a2000a2000a2⎤⎦⎥
∴ |adj A| = $a^2(a^4 - 0) = a^6$

Question 5.
Adjoint of [24−3−6] is _________

(a) [−6−432]

(b) [6−432]

(c) [−64−32]

(d) [−643−2]

Answer:

(a) [−6−432]

Question 6.
If A = diag. [d_1, d_2, d_3, ..., d_n], where $d_1 \neq 0$, for i = 1, 2, 3, ..., n, then A^{-1} = _________
(a) diag [1/d_1, 1/d_2, 1/d_3, ..., 1/d_n]
(b) D
(c) I
(d) O

Answer:
(a) diag [$1/d_1$, $1/d_2$, $1/d_3$, ..., $1/d_n$]

Question 7.
If $A^2 + mA + nI = O$ and $n \neq 0$, $|A| \neq 0$, then A^{-1} = __________
(a) $-\frac{1}{m}(A + nI)$
(b) $-\frac{1}{n}(A + mI)$
(c) $-\frac{1}{m}(I + mA)$
(d) $(A + mnI)$
Answer:
(b) $-\frac{1}{n}(A + mI)$
Hint:
$A^2 + mA + nI = 0$
$\therefore (A^2 + mA + nI) . A^{-1} = 0 . A^{-1}$
$\therefore A(AA^{-1}) + m(AA^{-1}) + nIA^{-1} = 0$
$\therefore AI + mI + nA^{-1} = 0$
$\therefore nA^{-1} = -A - mI$
$\therefore A^{-1} = -\frac{1}{n}(A + mI)$

Question 8.
If a 3 × 3 matrix B has its inverse equal to B, then B^2 = __________
(a) $[0011110101]$
(b) $\begin{bmatrix}1&1&1\\1&1&0\\1&1&1\end{bmatrix}$
(c) $\begin{bmatrix}1&0&0\\0&1&0\\1&0&0\end{bmatrix}$
(d) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$
Answer:
(d) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$
Hint:
$B^{-1} = B$
$\therefore B^2 = B.B^{-1} = I$

Question 9.
If A = $\begin{bmatrix}\alpha&4\\4&\alpha\end{bmatrix}$ and $|A^3| = 729$ then α = __________
(a) ±3
(b) ±4
(c) ±5
(d) ±6
Answer:
(c) ±5
Hint:
$|A| = \begin{vmatrix}\alpha&4\\4&\alpha\end{vmatrix} = \alpha^2 - 16$
$\therefore |A^3| = |A|^3 = (\alpha^2 - 16)^3 = 729$
$\therefore \alpha^2 - 16 = 9$
$\therefore \alpha^2 = 25$
$\therefore \alpha = \pm 5$

Question 10.
If A and B square matrices of order n × n such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true?
(a) AB = BA
(b) either A or B is a zero matrix
(c) either of A and B is an identity matrix
(d) A = B
Answer:
(a) AB = BA
Hint:
$A^2 - B^2 = (A - B)(A + B)$
$\therefore A^2 - B^2 = A^2 + AB - BA - B^2$
$\therefore 0 = AB - BA$
$\therefore AB = BA$

Question 11.
If A = $\begin{bmatrix}2&1\\5&3\end{bmatrix}$ then A^{-1} = __________
(a) $\begin{bmatrix}3&1\\-5&2\end{bmatrix}$

(b) [3−1−52]

(c) [3−152]

(d) [31−5−2]

Answer:

(b) [3−1−52]

Question 12.

If A is a 2 × 2 matrix such that A(adj A) = [5005], then |A| = __________

(a) 0
(b) 5
(c) 10
(d) 25
Answer:
(b) 5
Hint:
A(adj A) = |A|.I

Question 13.
If A is a non-singular matrix, then det(A^{-1}) = __________
(a) 1
(b) 0
(c) det(A)
(d) 1/det(A)
Answer:
(d) 1/det(A)
Hint:
AA^{-1} = I
∴ |A|.$|A^{-1}|$ = 1
∴ $|A^{-1}|$ = 1| A|

Question 14.

If A = [1−32−1], B = [−1105] then AB = __________

(a) [11−1020]

(b) [1−11020]

(c) [111020]

(d) [1−110−20]

Answer:

(c) [111020]

Question 15.
If x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6, then (y, z) = __________
(a) (-1, 0)
(b) (1, 0)
(c) (1, -1)
(d) (-1, 1)
Answer:
(b) (1, 0)

(II) Fill in the blanks:

Question 1.

A = [31] is __________ matrix.

Answer:
column

Question 2.

Order of matrix [251118] is __________

Answer:
2 × 3

Question 3.

If A = [46x3] is a singular matrix, then x is __________

Answer:
2

Question 4.
Matrix B = $\begin{bmatrix} 0 & 3 & 1 \\ -3 & 0 & -4 \\ p & 4 & 0 \end{bmatrix}$ is a skew-symmetric, then value of p is _________

Answer:
-1

Question 5.
If A = $[a_{ij}]_{2\times3}$ and B = $[b_{ij}]_{m\times1}$, and AB is defined, then m = _________

Answer:
3

Question 6.
If A = $\begin{bmatrix} 3 & -5 \\ 2 & 5 \end{bmatrix}$, then cofactor of a_{12} is _________

Answer:
-2

Question 7.
If A = $[a_{ij}]_{m\times m}$ is non-singular matrix, then $A^{-1} = \frac{1}{\ldots\ldots}$ adj (A).

Answer:
|A|

Question 8.
$(A^T)^T$ = _________

Answer:
A

Question 9.
If A = $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ and A^{-1} = $\begin{bmatrix} 1 & -1 \\ x & 2 \end{bmatrix}$, then x = _________

Answer:
-1

Question 10.
If $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, then matrix form is $\begin{bmatrix} \cdots & \cdots \\ \ldots & \ldots \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \ldots \\ \cdots \end{bmatrix}$

Answer:
$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

(III) State whether each of the following is True or False:

Question 1.
Single element matrix is row as well as a column matrix.

Answer:
True

Question 2.
Every scalar matrix is unit matrix.

Answer:
False

Question 3.
A = $\begin{bmatrix} 4 & 5 \\ 6 & 1 \end{bmatrix}$ is non-singular matrix.

Answer:
True

Question 4.
If A is symmetric, then $A = -A^T$.

Answer:
False

Question 5.
If AB and BA both exist, then AB = BA.

Answer:
False

Question 6.
If A and B are square matrices of same order, then $(A + B)^2 = A^2 + 2AB + B^2$.
Answer:
False

Question 7.
If A and B are conformable for the product AB, then $(AB)^T = A^T B^T$.
Answer:
False

Question 8.
Singleton matrix is only row matrix.
Answer:
False

Question 9.
A = [21015] is invertible matrix.
Answer:
False

Question 10.
A(adj A) = |A| I, where I is the unit matrix.
Answer:
True.

(IV) Solve the following:

Question 1.
Find k, if [753k] is a singular matrix.
Solution:
Let A = [753k]
Since, A is singular matrix, |A| = 0
∴ ∣ ∣ ∣ 753k ∣ ∣ ∣ = 0
∴ 7k – 15 = 0
∴ k = 157

Question 2.
Find x, y, z if ⎡ ⎣ ⎢ 23yx155z8⎤ ⎦ ⎥ is a symmetric matrix.
Solution:

$$\text{Let } A = \begin{bmatrix} 2 & x & 5 \\ 3 & 1 & z \\ y & 5 & 8 \end{bmatrix}$$

$$\text{Then } A^T = \begin{bmatrix} 2 & 3 & y \\ x & 1 & 5 \\ 5 & z & 8 \end{bmatrix}$$

Since, A is symmetric matrix, $A = A^T$

$$\therefore \begin{bmatrix} 2 & x & 5 \\ 3 & 1 & z \\ y & 5 & 8 \end{bmatrix} = \begin{bmatrix} 2 & 3 & y \\ x & 1 & 5 \\ 5 & z & 8 \end{bmatrix}$$

By equality of matrices,
x = 3, y = 5 and z = 5.

Question 3.
If A = ⎡ ⎣ ⎢ 179585⎤ ⎦ ⎥ , B = ⎡ ⎣ ⎢ 21-8456⎤ ⎦ ⎥ , C = ⎡ ⎣ ⎢ -2173-58⎤ ⎦ ⎥ then show that (A + B) + C = A + (B + C).
Solution:

$$A+B=\begin{bmatrix}1&5\\7&8\\9&5\end{bmatrix}+\begin{bmatrix}2&4\\1&5\\-8&6\end{bmatrix}$$

$$=\begin{bmatrix}1+2&5+4\\7+1&8+5\\9-8&5+6\end{bmatrix}=\begin{bmatrix}3&9\\8&13\\1&11\end{bmatrix}$$

$$\therefore\ (A+B)+C=\begin{bmatrix}3&9\\8&13\\1&11\end{bmatrix}+\begin{bmatrix}-2&3\\1&-5\\7&8\end{bmatrix}$$

$$=\begin{bmatrix}3-2&9+3\\8+1&13-5\\1+7&11+8\end{bmatrix}=\begin{bmatrix}1&12\\9&8\\8&19\end{bmatrix} \quad \text{....... (1)}$$

$$\text{Also, } B+C=\begin{bmatrix}2&4\\1&5\\-8&6\end{bmatrix}+\begin{bmatrix}-2&3\\1&-5\\7&8\end{bmatrix}$$

$$=\begin{bmatrix}2-2&4+3\\1+1&5-5\\-8+7&6+8\end{bmatrix}=\begin{bmatrix}0&7\\2&0\\-1&14\end{bmatrix}$$

$$\therefore\ A+(B+C)=\begin{bmatrix}1&5\\7&8\\9&5\end{bmatrix}+\begin{bmatrix}0&7\\2&0\\-1&14\end{bmatrix}$$

$$=\begin{bmatrix}1+0&5+7\\7+2&8+0\\9-1&5+14\end{bmatrix}=\begin{bmatrix}1&12\\9&8\\8&19\end{bmatrix} \quad \text{.......... (2)}$$

From (1) and (2),
(A + B) + C = A + (B + C).

Question 4.
If A = [2357], B = [1−370], find the matrix A – 4B + 7I, where I is the unit matrix of order 2.
Solution:

$$A-4B+7I=\begin{bmatrix}2&5\\3&7\end{bmatrix}-4\begin{bmatrix}1&7\\-3&0\end{bmatrix}+7\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$=\begin{bmatrix}2&5\\3&7\end{bmatrix}-\begin{bmatrix}4&28\\-12&0\end{bmatrix}+\begin{bmatrix}7&0\\0&7\end{bmatrix}$$

$$=\begin{bmatrix}2-4+7&5-28+0\\3-(-12)+0&7-0+7\end{bmatrix}$$

$$=\begin{bmatrix}5&-23\\15&14\end{bmatrix}.$$

Question 5.
If A = [23−1−3−24], B = [−324−11−3] verify
(i) $(A + 2B^T)^T = A^T + 2B$
(ii) $(3A - 5B^T)^T = 3A^T - 5B$
Solution:

$$A=\begin{bmatrix}2&-3\\3&-2\\-1&4\end{bmatrix}, B=\begin{bmatrix}-3&4&1\\2&-1&-3\end{bmatrix}$$

$$\therefore\ A^T = \begin{bmatrix} 2 & 3 & -1 \\ -3 & -2 & 4 \end{bmatrix},\ B^T = \begin{bmatrix} -3 & 2 \\ 4 & -1 \\ 1 & -3 \end{bmatrix}$$

(i) $$A + 2B^T = \begin{bmatrix} 2 & -3 \\ 3 & -2 \\ -1 & 4 \end{bmatrix} + 2\begin{bmatrix} -3 & 2 \\ 4 & -1 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ 3 & -2 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} -6 & 4 \\ 8 & -2 \\ 2 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 2-6 & -3+4 \\ 3+8 & -2-2 \\ -1+2 & 4-6 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 11 & -4 \\ 1 & -2 \end{bmatrix}$$

$$\therefore\ (A + 2B^T)^T = \begin{bmatrix} -4 & 11 & 1 \\ 1 & -4 & -2 \end{bmatrix} \qquad \dots (1)$$

Also, $A^T + 2B$

$$= \begin{bmatrix} 2 & 3 & -1 \\ -3 & -2 & 4 \end{bmatrix} + 2\begin{bmatrix} -3 & 4 & 1 \\ 2 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & -1 \\ -3 & -2 & 4 \end{bmatrix} + \begin{bmatrix} -6 & 8 & 2 \\ 4 & -2 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 2-6 & 3+8 & -1+2 \\ -3+4 & -2-2 & 4-6 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 11 & 1 \\ 1 & -4 & -2 \end{bmatrix} \qquad \dots\dots\dots (2)$$

From (1) and (2),

$(A + 2B^T)^T = A^T + 2B.$

(ii) $$3A - 5B^T = 3\begin{bmatrix} 2 & -3 \\ 3 & -2 \\ -1 & 4 \end{bmatrix} - 5\begin{bmatrix} -3 & 2 \\ 4 & -1 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -9 \\ 9 & -6 \\ -3 & 12 \end{bmatrix} - \begin{bmatrix} -15 & 10 \\ 20 & -5 \\ 5 & -15 \end{bmatrix}$$

$$= \begin{bmatrix} 6-(-15) & -9-10 \\ 9-20 & -6-(-5) \\ -3-5 & 12-(-15) \end{bmatrix}$$

$$= \begin{bmatrix} 21 & -19 \\ -11 & -1 \\ -8 & 27 \end{bmatrix}$$

$$\therefore\ (3A - 5B^T)^T = \begin{bmatrix} 21 & -11 & -8 \\ -19 & -1 & 27 \end{bmatrix} \qquad \dots (1)$$

Also, $3A^T - 5B$

$$= 3\begin{bmatrix} 2 & 3 & -1 \\ -3 & -2 & 4 \end{bmatrix} - 5\begin{bmatrix} -3 & 4 & 1 \\ 2 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 9 & -3 \\ -9 & -6 & 12 \end{bmatrix} - \begin{bmatrix} -15 & 20 & 5 \\ 10 & -5 & -15 \end{bmatrix}$$

$$= \begin{bmatrix} 6-(-15) & 9-20 & -3-5 \\ -9-10 & -6-(-5) & 12-(-15) \end{bmatrix}$$

$$= \begin{bmatrix} 21 & -11 & -8 \\ -19 & -1 & 27 \end{bmatrix} \qquad \dots (2)$$

From (1) and (2),

$(3A - 5B^T)^T = 3A^T = 3A^T - 5B.$

Question 6.
If A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$ thenshow that AB and BA are both singular matrices.
Solution:

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6-6 & -1+4+3 & 1-2+0 \\ 2-12-12 & -2+8+6 & 2-4+0 \\ 1-6-6 & -1+4+3 & 1-2+0 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{bmatrix}$$

$$\therefore\ |AB| = \begin{bmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{bmatrix}$$

By taking -6 common from C_2, we get

$$\therefore\ |AB| = -6\begin{vmatrix} -11 & -1 & -1 \\ -22 & -2 & -2 \\ -11 & -1 & -1 \end{vmatrix}$$

$$= -6 \times 0 \qquad \ldots [\because C_2 \equiv C_3]$$

$$= 0$$

$\therefore$ AB is a singular matrix.

$$\text{Also, } BA = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+1 & 2-4+2 & 3-6+3 \\ -3+4-1 & -6+8-2 & -9+12-3 \\ -2+2+0 & -4+4+0 & -6+6+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 \qquad \therefore\ |BA| = 0$$

∴ BA is also a singular matrix.
Hence, AB and BA are both singular matrices.

Question 7.
If A = $\begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 2 \\ 5 & -2 \end{bmatrix}$, verify |AB| = |A| |B|.
Solution:

$$AB = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3+5 & 6-2 \\ 1+25 & 2-10 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 26 & -8 \end{bmatrix}$$

$$\therefore\ |AB| = \begin{vmatrix} 8 & 4 \\ 26 & -8 \end{vmatrix}$$

$$= -64 - 104 = -168 \qquad \ldots (1)$$

$$|A| = \begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} = 15 - 1 = 14$$

$$|B| = \begin{vmatrix} 1 & 2 \\ 5 & -2 \end{vmatrix} = -2 - 10 = -12$$

$$\therefore\ |A||B| = 14(-12) = -168 \qquad \ldots (2)$$

From (1) and (2), $|AB| = |A||B|$.

Question 8.
If A = $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 3I = 0$.

Solution:

$$A^2 = A \cdot A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

$$\therefore \; A^2 - 4A + 3I$$

$$= \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} - 4\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} - \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5-8+3 & -4-(-4)+0 \\ -4-(-4)+0 & 5-8+3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \; A^2 - 4A + 3I = 0.$$

Question 9.
If A = $[-3224]$, B = $[1ba0]$ and (A + B)(A – B) = A2 – B2, find a and b.
Solution:
(A + B)(A – B) = A2 – B2
∴ A2 – AB + BA – B2 = A2 – B2
∴ -AB + BA = 0
∴ AB = BA

$$\therefore \; \begin{bmatrix} -3 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & a \\ b & 0 \end{bmatrix} = \begin{bmatrix} 1 & a \\ b & 0 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 2 & 4 \end{bmatrix}$$

$$\therefore \; \begin{bmatrix} -3+2b & -3a+0 \\ 2+4b & 2a+0 \end{bmatrix} = \begin{bmatrix} -3+2a & 2+4a \\ -3b+0 & 2b+0 \end{bmatrix}$$

$$\therefore \; \begin{bmatrix} -3+2b & -3a \\ 2+4b & 2a \end{bmatrix} = \begin{bmatrix} -3+2a & 2+4a \\ -3b & 2b \end{bmatrix}$$

By equality of matrices,
-3 + 2b = -3 + 2a ……..(1)
-3a = 2 + 4a ……..(2)
2 + 4b = -3b ……..(3)
2a = 2b ……..(4)
From (2), 7a = -2
∴ a = −27
From (3), 7b = -2
∴ b = −27
These values of a and b also satisfy equations (1) and (4).
Hence, a = −27 and b = −27

Question 10.
If A = $[1-123]$, then find A3.
Solution:

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & 2+6 \\ -1-3 & -2+9 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ -4 & 7 \end{bmatrix}$$

$$\therefore \; A^3 = A^2 \cdot A = \begin{bmatrix} -1 & 8 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1-8 & -2+24 \\ -4-7 & -8+21 \end{bmatrix}$$

$$\therefore \; A^3 = \begin{bmatrix} -9 & 22 \\ -11 & 13 \end{bmatrix}$$

Question 11.
Find x, y, z if ⎡ ⎣ ⎨ ⎪ ⎪ 5⎡ ⎣ ⎢ 011101⎤ ⎦ ⎥ −⎡ ⎣ ⎢ 2311−23⎤ ⎦ ⎥ ⎫ ⎭ ⎬ ⎪ ⎪ [21]=⎡ ⎣ ⎢ x−1y+12z⎤ ⎦ ⎥

Solution:

$$\left\{5\begin{bmatrix}0 & 1\\1 & 0\\1 & 1\end{bmatrix}-\begin{bmatrix}2 & 1\\3 & -2\\1 & 3\end{bmatrix}\right\}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$$

$$\therefore\left\{\begin{bmatrix}0 & 5\\5 & 0\\5 & 5\end{bmatrix}-\begin{bmatrix}2 & 1\\3 & -2\\1 & 3\end{bmatrix}\right\}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$$

$$\therefore\begin{bmatrix}0-2 & -1\\5-3 & 0+2\\5-1 & -3\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$$

$$\therefore\begin{bmatrix}-2 & 4\\2 & 2\\4 & 2\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$$

$$\therefore\begin{bmatrix}-4+4\\4+2\\8+2\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$$

$$\therefore\begin{bmatrix}0\\6\\10\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$$

∴ By equality of matrices, we get
x – 1 = 0 ∴ x = 1
y + 1 = 6 ∴ y = 5
2z = 10 ∴ z = 5

Question 12.

If A = ⌈ ⌊ | 230–4–21⌉ ⌋ | , B = [1–2–1120] then showthat$(AB)^T = B^TA^T$.

Solution:

$$AB = \begin{bmatrix} 2 & -4 \\ 3 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2+8 & -2-4 & 4+0 \\ 3+4 & -3-2 & 6-0 \\ 0-2 & 0+1 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -6 & 4 \\ 7 & -5 & 6 \\ -2 & 1 & 0 \end{bmatrix}$$

$$\therefore (AB)^T = \begin{bmatrix} 10 & 7 & -2 \\ -6 & -5 & 1 \\ 4 & 6 & 0 \end{bmatrix} \quad \ldots(i)$$

Now, $A^T = \begin{bmatrix} 2 & 3 & 0 \\ -4 & -2 & 1 \end{bmatrix}$ and $B^T = \begin{bmatrix} 1 & -2 \\ -1 & 1 \\ 2 & 0 \end{bmatrix}$

$$\therefore B^T A^T = \begin{bmatrix} 1 & -2 \\ -1 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 & 0 \\ -4 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+8 & 3+4 & 0-2 \\ -2-4 & -3-2 & 0+1 \\ 4-0 & 6-0 & 0+0 \end{bmatrix}$$

$$\therefore B^T A^T = \begin{bmatrix} 10 & 7 & -2 \\ -6 & -5 & 1 \\ 4 & 6 & 0 \end{bmatrix} \quad \ldots(ii)$$

From (i) and (ii), we get

$(AB)^T = B^T A^T$.

Question 13.

If A = ⎡ ⎣ ⎢ 123013001⎤ ⎦ ⎥ , then reduce it to unit matrix by row tranformation.

Solution:

AB = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$

$\therefore$ |A| = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$

= 1(1 – 0) –0(2 –0) + 0(6 – 3)

= 1 – 0 + 0

= 1 ≠ 0

$\therefore$ A is non-singular matrix.

Hence, row transformations are possible.

Now, A = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$, we get

A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$

Applying $R_3 \to R_3 - 3R_2$, we get

A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Question 14.
Two farmers Shantaram and Kantaram cultivate three crops rice, wheat, and groundnut. The sale (in ₹) of these crops by both the farmers for the month of April and May 2016 is given below:

April 2016 (In Rs.)

	Rice	Wheat	Groundnut
Shantaram	15000	13000	12000
Kantaram	18000	15000	8000

May 2016 (In Rs.)

	Rice	Wheat	Groundnut
Shantaram	18000	15000	12000
Kantaram	21000	16500	16000

Find (i) the total sale in rupees for two months of each farmer for each crop.
(ii) the increase in sales from April to May for every crop of each farmer.
Solution:
The given information can be written in matrix form as:

April 2016 (in ₹)

$$A = \begin{array}{c} \begin{array}{ccc} \text{Rice} & \text{Wheat} & \text{Groundnut} \end{array} \\ \begin{bmatrix} 15000 & 13000 & 12000 \\ 18000 & 15000 & 8000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shantaram} \\ \text{Kantaram} \end{array}$$

May 2016 (in ₹)

$$B = \begin{array}{c} \begin{array}{ccc} \text{Rice} & \text{Wheat} & \text{Groundnut} \end{array} \\ \begin{bmatrix} 18000 & 15000 & 12000 \\ 21000 & 16500 & 16000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shantaram} \\ \text{Kantaram} \end{array}$$

(i) The total sale in ₹ for two months of each farmer for each crop can be obtained by the addition A + B.
Now, A + B

$$= \begin{bmatrix} 15000 & 13000 & 12000 \\ 18000 & 15000 & 8000 \end{bmatrix} + \begin{bmatrix} 18000 & 15000 & 12000 \\ 21000 & 16500 & 16000 \end{bmatrix}$$

$$= \begin{bmatrix} 15000+18000 & 13000+15000 & 12000+12000 \\ 18000+21000 & 15000+16500 & 8000+16000 \end{bmatrix}$$

$$= \begin{bmatrix} 33000 & 28000 & 24000 \\ 39000 & 31500 & 24000 \end{bmatrix}$$

∴ total sale in ₹ for two months of each farmer for each crop is given by

$$\begin{array}{cc} \begin{array}{ccc} \textbf{Rice} & \textbf{Wheat} & \textbf{Groundnut} \end{array} & \\ \begin{bmatrix} 33000 & 28000 & 24000 \\ 39000 & 31500 & 24000 \end{bmatrix} & \begin{array}{l} \textbf{Shantaram} \\ \textbf{Kantaram} \end{array} \end{array}$$

Hence, the total sale for Shantaram are ₹ 33000 for Rice, ₹ 28000 for Wheat, ₹ 24000 for Groundnut, and for Kantaram are ₹ 39000 for Rice, ₹ 31500 for Wheat, ₹ 24000 for Groundnut.
(ii) The increase in sales from April to May for every crop of each farmer can be obtained by the subtraction of A from B.
Now, B – A

$$= \begin{bmatrix} 18000 & 15000 & 12000 \\ 21000 & 16500 & 16000 \end{bmatrix} - \begin{bmatrix} 15000 & 13000 & 12000 \\ 18000 & 15000 & 8000 \end{bmatrix}$$

$$= \begin{bmatrix} 18000-15000 & 15000-13000 & 12000-12000 \\ 21000-18000 & 16500-15000 & 16000-8000 \end{bmatrix}$$

$$= \begin{bmatrix} 3000 & 2000 & 0 \\ 3000 & 1500 & 8000 \end{bmatrix}$$

∴ the increase in sale from April to May is given by

$$\begin{array}{cc} \begin{array}{ccc} \textbf{Rice} & \textbf{Wheat} & \textbf{Groundnut} \end{array} & \\ \begin{bmatrix} 3000 & 2000 & 0 \\ 3000 & 1500 & 8000 \end{bmatrix} & \begin{array}{l} \textbf{Shantaram} \\ \textbf{Kantaram} \end{array} \end{array}$$

Hence, the increase in sales from April to May of Shantam is ₹ 3000 in Rice, ₹ 2000 in Wheat, nothing in Groundnut and of Kantaram are ₹ 3000 in Rice, ₹ 1500 in Wheat, ₹ 8000 in Groundnut.

Question 15.
Check whether following matrices are invertible or not:
(i) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Solution:
Let A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Then |A| = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$
= 1 – 0
= 1 ≠ 0
∴ A is a non-singular matrix.
Hence, A^{-1} exists.

(ii) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
Solution:
Let A = $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
Then |A| = $\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$
= 1 – 1
= 0
∴ A is a singular matrix.
Hence, A^{-1} does not exist.

(iii) $\begin{bmatrix} 3 & 1 & 1 \\ 4 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$
Solution:
Let A = $\begin{bmatrix} 3 & 1 & 1 \\ 4 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$
Then |A| = $\begin{vmatrix} 3 & 1 & 1 \\ 4 & 1 & 4 \\ 3 & 0 & 5 \end{vmatrix}$
= 3(5 – 0) – 4(5 – 0) + 3(4 – 1)
= 15 – 20 + 9
= 4 ≠ 0
∴ A is a non-singular matrix.
Hence, A^{-1} exists.

(iv) ⎡ ⎣ ⎢ 122244356⎤ ⎦ ⎥

Solution:

Let A = ⎡ ⎣ ⎢ 122244356⎤ ⎦ ⎥

Then |A| = ∣ ∣ ∣ ∣ 122244356 ∣ ∣ ∣ ∣

= 1(24 – 20) – 2(12 – 10) + 3(8 – 8)
= 4 – 4 + 0
= 0
∴ A is a singular matrix.
Hence, A^{-1} does not exist.

Question 16.
Find inverse of the following matrices (if they exist) by elementary transformation:
(i) [12-13]
Solution:

$$\text{Let } A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$$

Then $|A| = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 - (-2) = 5 \neq 0$

∴ A^{-1} exists.

We write, $AA^{-1} = I$

$$\therefore \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

By $R_2 - 2R_1$, we get

$$\begin{pmatrix} 1 & -1 \\ 0 & 5 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

By $\left(\frac{1}{5}\right) R_2$, we get

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

By $R_1 + R_2$, we get

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

(ii) [2714]

Solution:

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

$$\text{Then } |A| = \begin{vmatrix} 2 & 1 \\ 7 & 4 \end{vmatrix} = 8 - 7 = 1 \neq 0$$

$\therefore$ A^{-1} exist.

We write $A^{-1}A = I$

$$\therefore A^{-1}\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $C_1 - C_2$, we get

$$A^{-1}\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

By $C_2 - C_1$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

By $C_1 - 3C_2$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}.$$

(iii) ⎡ ⎣ ⎢ ⎢ 223-32-2332⎤ ⎦ ⎥ ⎥

Solution:

$$\text{Let } A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

$$\text{Then } |A| = \begin{vmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= 2(4+6) + 3(4-9) + 3(-4-6)$$

$$= 20 - 15 - 30 = -25 \neq 0$$

$\therefore$ A^{-1} exists.

We write $A^{-1}A = I$

$$\therefore A^{-1}\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $C_1 \leftrightarrow C_3$, we get

$$A^{-1}\begin{bmatrix} 3 & -3 & 2 \\ 3 & 2 & 2 \\ 2 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

By $C_1 - C_3$, we get

$$A^{-1}\begin{bmatrix} 1 & -3 & 2 \\ 1 & 2 & 2 \\ -1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

By $C_2 + 3C_1$ and $C_3 - 2C_1$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 5 & 0 \\ -1 & -5 & 5 \end{bmatrix} = \begin{bmatrix} -1 & -3 & 3 \\ 0 & 1 & 0 \\ 1 & 3 & -2 \end{bmatrix}$$

By $\left(\frac{1}{5}\right)C_2$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 5 \end{bmatrix} = \begin{bmatrix} -1 & -\frac{3}{5} & 3 \\ 0 & \frac{1}{5} & 0 \\ 1 & \frac{3}{5} & -2 \end{bmatrix}$$

By $C_1 - C_2$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 5 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & -\frac{3}{5} & 3 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{3}{5} & -2 \end{bmatrix}$$

By $\left(\frac{1}{5}\right)C_3$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & -\frac{3}{5} & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{3}{5} & -\frac{2}{5} \end{bmatrix}$$

By $C_2 + C_3$, we get

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{5}\begin{bmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix}$$

(iv) ⎡ ⎣ ⎢ 250011-103⎤ ⎦ ⎥

Solution:

$$\text{Let } A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\therefore \ |A| = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$$

$$= 2(3-0) - 0 - 1(5-0)$$

$$= 6 - 0 - 5$$

$$= 1 \neq 0$$

$\therefore$ A^{-1} exists

We write $AA^{-1} = 1$

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get

$$\begin{bmatrix} 5 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 + 3R_3$, we get

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 4 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$ and $R_3 - R_2$, we get

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ -5 & 2 & -2 \end{bmatrix}$$

By $(-1)R_3$, we get

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ 5 & -2 & 2 \end{bmatrix}$$

By $R_1 + 2R_3$ and $R_2 - 4R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\therefore \ A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}.$$

Question 17.

Find the inverse of ⎡ ⎣ ⎢ 321172585⎤ ⎦ ⎥ by adjoint method.

Solution:

$$\text{Let } A = \begin{bmatrix} 3 & 1 & 5 \\ 2 & 7 & 8 \\ 1 & 2 & 5 \end{bmatrix}$$

$$\text{Then } |A| = \begin{vmatrix} 3 & 1 & 5 \\ 2 & 7 & 8 \\ 1 & 2 & 5 \end{vmatrix}$$

$$= 3(35-16) - 1(10-8) + 5(4-7)$$

$$= 57 - 2 - 15 = 40 \neq 0$$

$\therefore$ A^{-1} exists

First, we have to find the cofactor matrix

$= [A_{ij}]_{3\times 3}$, where $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 7 & 8 \\ 2 & 5 \end{vmatrix}$

$$= 35 - 16 = 19$$

$$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 2 & 8 \\ 1 & 5 \end{vmatrix}$$

$$= -(10-8) = -2$$

$$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 2 & 7 \\ 1 & 2 \end{vmatrix}$$

$$= 4 - 7 = -3$$

$$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} 1 & 5 \\ 2 & 5 \end{vmatrix}$$

$$= -(5-10) = 5$$

$$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 3 & 5 \\ 1 & 5 \end{vmatrix}$$

$$= 15 - 5 = 10$$

$$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= -(6-1) = -5$$

$$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} 1 & 5 \\ 7 & 8 \end{vmatrix}$$

$$= 8 - 35 = -27$$

$$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix}$$

$$= -(24-10) = -14$$

$$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 3 & 1 \\ 2 & 7 \end{vmatrix}$$

$$= 21 - 2 = 19$$

$\therefore$ the cofactor matrix

$$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 19 & -2 & -3 \\ 5 & 10 & -5 \\ -27 & -14 & 19 \end{bmatrix}$$

$$\therefore \text{ adj } A = \begin{bmatrix} 19 & 5 & -27 \\ -2 & 10 & -14 \\ -3 & -5 & 19 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$\therefore A^{-1} = \frac{1}{40} \begin{bmatrix} 19 & 5 & -27 \\ -2 & 10 & -14 \\ -3 & -5 & 19 \end{bmatrix}.$$

Question 18.
Solve the following equations by method of inversion:
(i) $4x - 3y - 2 = 0$, $3x - 4y + 6 = 0$
Solution:
The given equations are
$4x - 3y = 2$
$3x - 4y = -6$
These equations can be written in matrix form as:

$$\begin{bmatrix} 4 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

This is of the form AX = B, where

$$A = \begin{bmatrix} 4 & -3 \\ 3 & -4 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix} 4 & -3 \\ 3 & -4 \end{vmatrix} = -16 - (-9) = -7 \neq 0$$

$\therefore$ A^{-1} exists.

We write $AA^{-1} = I$

$$\therefore \begin{bmatrix} 4 & -3 \\ 3 & -4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 1 \\ 3 & -4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 3R_1$, we get

$$\begin{bmatrix} 1 & 1 \\ 0 & -7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$$

By $\left(-\frac{1}{7}\right)R_2$, we get

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ \frac{3}{7} & -\frac{4}{7} \end{bmatrix}$$

By $R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{4}{7} & -\frac{3}{7} \\ \frac{3}{7} & -\frac{4}{7} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{4}{7} & -\frac{3}{7} \\ \frac{3}{7} & -\frac{4}{7} \end{bmatrix}$$

Now, premultiply AX = B by A^{-1}, we get

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)X = A^{-1}B$

$\therefore IX = A^{-1}B$

$$\therefore X = \begin{bmatrix} \frac{4}{7} & -\frac{3}{7} \\ \frac{3}{7} & -\frac{4}{7} \end{bmatrix}\begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{8}{7} + \frac{18}{7} \\ \frac{6}{7} + \frac{24}{7} \end{bmatrix} = \begin{bmatrix} \frac{26}{7} \\ \frac{30}{7} \end{bmatrix}$$

By equality of matrices,
x = 267, y = 307 is the required solution.

(ii) x + y – z = 2, x – 2y + z = 3 and 2x – y – 3z = -1
Solution:

The given equations can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 2 & -1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

This is of the form AX = B, where

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 2 & -1 & -3 \end{bmatrix}\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 2 & -1 & -3 \end{vmatrix}$$

$$= 1(6+1) - 1(-3-2) - 1(-1+4)$$

$$= 7 + 5 - 3 = 9 \neq 0$$

$\therefore$ A^{-1} exists.

We write $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 2 & -1 & -3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - R_1$ and $R_3 - 2R_1$, we get

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -3 & 2 \\ 0 & -3 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $\left(-\frac{1}{3}\right) R_2$, we get

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -\frac{2}{3} \\ 0 & -3 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & -\frac{1}{3} & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$ and $R_3 + 3R_2$, we get

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & -3 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & -\frac{1}{3} & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

By $\left(-\frac{1}{3}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$$

By $R_1 + \frac{1}{3}R_3$ and $R_2 + \frac{2}{3}R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{7}{9} & \frac{4}{9} & -\frac{1}{9} \\ \frac{5}{9} & -\frac{1}{9} & -\frac{2}{9} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{9}\begin{bmatrix} 7 & 4 & -1 \\ 5 & -1 & -2 \\ 3 & 3 & -3 \end{bmatrix}$$

Now, premultiply $AX = B$ by A^{-1}, we get

$A^{-1}(AX) = A^{-1}B$

$\therefore\ (A^{-1}A)X = A^{-1}B$

$\therefore\ IX = A^{-1}B$

$$\therefore\ X = \frac{1}{9}\begin{bmatrix} 7 & 4 & -1 \\ 5 & -1 & -2 \\ 3 & 3 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

$$\therefore\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 14+12+1 \\ 10-3+2 \\ 6+9+3 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 27 \\ 9 \\ 18 \end{bmatrix}$$

$$\therefore\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

By equality of matrices,
$x = 3$, $y = 1$, $z = 2$ is the required solution.

(iii) $x - y + z = 4$, $2x + y - 3z = 0$ and $x + y + z = 2$
Solution:

Matrix form of the given system of equations is

$$\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$$

This is of the form AX = B, where

A = $\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$, X = $\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and B = $\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$

To determine X, we have to find A^{-1}.

|A| = $\begin{vmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{vmatrix}$

= 1(1 + 3) + 1(2 + 3) + 1(2 – 1)

= 1(4) + 1(5) + 1(1)

= 4 + 5 + 1

= 10 ≠ 0

∴ A^{-1} exists.

Consider $AA^{-1} = I$

$$\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}A - 1 = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{bmatrix}1 & -1 & 1\\ 0 & 3 & -5\\ 0 & 2 & 0\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\ -2 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$$

Applying $R_2 \to \left(\frac{1}{3}\right) R_2$, we get

$$\begin{bmatrix}1 & -1 & 1\\ 0 & 1 & \frac{-5}{3}\\ 0 & 2 & 0\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\ \frac{-2}{3} & \frac{1}{3} & 0\\ -1 & 0 & 1\end{bmatrix}$$

Applying $R_1 \to R_1 + R_2$ and $R_3 \to R_3 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & \frac{-2}{3} \\ 0 & 1 & \frac{-5}{3} \\ 0 & 0 & \frac{10}{3} \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{-2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{-2}{3} & 1 \end{bmatrix}$$

Applying $R_3 \to \left(\frac{3}{10}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & \frac{-2}{3} \\ 0 & 1 & \frac{-5}{3} \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{-2}{3} & \frac{1}{3} & 0 \\ \frac{1}{10} & \frac{-1}{3} & \frac{3}{10} \end{bmatrix}$$

Applying $R_1 \to R_1 + \left(\frac{2}{3}\right) R_3$ and $R_2 \to R_2 + \left(\frac{5}{3}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{-1}{2} & 0 & \frac{1}{2} \\ \frac{1}{10} & \frac{-1}{5} & \frac{3}{10} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

Pre-multiplying $AX = B$ by A^{-1}, we get

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A) X = A^{-1}B$

$\therefore IX = A^{-1}B$

$\therefore X = A^{-1}B$

$$\therefore X = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4-0+6 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\therefore$ By equality of martices, we get

$x = 2$, $y = -1$ and $z = 1$.

Question 19.
Solve the following equations by method of reduction:
(i) $2x + y = 5$, $3x - 5y = -3$
Solution:
The given equation can be written in matrix form as:
[2315][xy]=[5−3]
By $R_2 - 5R_1$, we get

$$\begin{bmatrix} 2 & 1 \\ -7 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -28 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2x+y \\ -7x+0 \end{bmatrix} = \begin{bmatrix} 5 \\ -28 \end{bmatrix}$$

By equality of matrices,
2x + y = 5 …….(1)
-7x = -28 ……(2)
From (2), x = 4
Substituting x = 4 in (1), we get
2(4) + y = 5
∴ y = -3
Hence, x = 4 and y = -3 is the required solution.

(ii) x + 2y + z = 3, 3x – y + 2z = 1 and 2x – 3y + 3z = 2
Solution:
The given equations can be written in matrix form as:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & -1 & 2 \\ 2 & -3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

By $R_2 - 3R_1$ and $R_3 - 2R_1$, we get

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -7 & -1 \\ 0 & -7 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -8 \\ -4 \end{bmatrix}$$

By $R_3 - R_2$, we get

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -7 & -1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -8 \\ 4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x+2y+z \\ 0-7y-z \\ 0+0+2z \end{bmatrix} = \begin{bmatrix} 3 \\ -8 \\ 4 \end{bmatrix}$$

By equality of matrices,
x + 2y + z = 3 …….(1)
-7y – z = -8 …….(2)
2z = 4 …….(3)
From (3), z = 2
Substituting z = 2 in (2), we get
-7y – 2 = -8
∴ -7y = -6
∴ y = 67
Substituting y = 67, z = 2 in (1), we get
x + 2(67) + 2 = 3
x = 3 – 2 – 127 = -57
Hence, x = -57, y = 67 and z = 2 is the required solution.

(iii) x – 3y + z = 2, 3x + y + z = 1 and 5x + y + 3z = 3.
Solution:

Matrix form of the given system of equations is

$$\begin{bmatrix}1 & -3 & 1\\3 & 1 & 1\\5 & 1 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\1\\3\end{bmatrix}$$

This is of the form AX = B, where

A = $\begin{bmatrix}1 & -3 & 1\\3 & 1 & 1\\5 & 1 & 3\end{bmatrix}$, X = $\begin{bmatrix}x\\y\\z\end{bmatrix}$ and B = $\begin{bmatrix}2\\1\\3\end{bmatrix}$

Applying $R_2 \to R_2 - 3R_1$ and $R_3 \to R_3 - 5R_1$, we get

$$\begin{bmatrix}1 & -3 & 1\\0 & 10 & -2\\0 & 16 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-5\\-7\end{bmatrix}$$

Applying $R_3 \to R_3 - \left(\frac{8}{5}\right) R_2$, we get

$$\begin{bmatrix}1 & -3 & 1\\0 & 10 & -2\\0 & 0 & \frac{6}{5}\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-5\\1\end{bmatrix}$$

Hence, the original matrix A is reduced to an upper triangular matrix.

$$\therefore \begin{bmatrix}x-3y+z\\0+10y-2z\\0+0+\frac{6}{5}z\end{bmatrix}=\begin{bmatrix}2\\-5\\1\end{bmatrix}$$

∴ By equality of martices, we get

$x - 3y + z = 2$ ...(i)

$10y - 2z = -5$ ...(ii)

$\frac{6}{5}z = 1$

$\therefore z = \frac{5}{6}$

Substituting z = $\frac{5}{6}$ in equation (ii), we get

$$10y - 2\left(\frac{5}{6}\right) = -5$$

$$\therefore 10y - \frac{10}{6} = -5$$

$$\therefore 10y = -5 + \frac{10}{6} = \frac{-20}{6}$$

$$\therefore 10y = \frac{-10}{3}$$

$$\therefore y = \frac{-1}{3}$$

Substituting y = $\frac{-1}{3}$ and z = $\frac{5}{6}$ in equation (i), we get

$$x - 3\left(\frac{-1}{3}\right) + \frac{5}{6} = 2$$

$$\therefore x + 1 + \frac{5}{6} = 2$$

$$\therefore x = 2 - 1 - \frac{5}{6} = \frac{1}{6}$$

$\therefore$ x = $\frac{1}{6}$, y = $\frac{-1}{3}$ and z = $\frac{5}{6}$ is the required solution.

Question 20.
The sum of three numbers is 6. If we multiply the third number by 3 and add it to the second number, we get 11. By adding first and third numbers we get a number that is double the second number. Use this information and find a system of linear equations. Find the three numbers using matrices.
Solution:
Let the three numbers be x, y, and z.
According to the given condition,
x + y + z = 6
3z + y = 11, i.e. y + 3z = 11
and x + z = 2y, i.e. x – 2y + z = 0
Hence, the system of linear equations is
x + y + z = 6
y + 3z = 11
x – 2y + z = 0
These equations can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

By $R_3 - R_1$, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ -6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x+y+z \\ 0+y+3z \\ 0-3y+0 \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ -6 \end{bmatrix}$$

By equality of matrices,
x + y + z = 6 …….(1)
y + 3z = 11 ………(2)
-3y = -6 ………(3)
From (3), y = 2
Substituting y = 2 in (2), we get
2 + 3z = 11
∴ 3z = 9

∴ $z = 3$

Substituting $y = 2$, $z = 3$ in (1), we get

$x + 2 + 3 = 6$

∴ $x = 1$

∴ $x = 1$, $y = 2$, $z = 3$

Hence, the required numbers are 1, 2 and 3.