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Practice Set 6.1 Geometry 10th Std Maths Part 2 Answers Chapter 6 Trigonometry

Question 1.
If $\sin \theta = \frac{7}{25}$, find the values of $\cos \theta$ and $\tan \theta$.
Solution:
$\sin \theta = \frac{7}{25}$ … [Given]
We know that,
$\sin^2 \theta + \cos^2 \theta = 1$

$$\therefore \quad \left(\frac{7}{25}\right)^2 + \cos^2 \theta = 1$$

$$\therefore \quad \frac{49}{625} + \cos^2 \theta = 1$$

$$\therefore \quad \cos^2 \theta = 1 - \frac{49}{625}$$

$$\therefore \quad \cos^2 \theta = \frac{625 - 49}{625}$$

$$\therefore \quad \cos^2 \theta = \frac{576}{625}$$

$$\therefore \quad \cos \theta = \frac{24}{25}$$

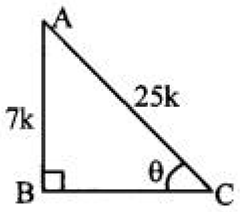
…[Taking square root of both sides] Now, $\tan \theta = \frac{\sin\theta}{\cos\theta}$

$$\text{Now, } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\left(\frac{7}{25}\right)}{\left(\frac{24}{25}\right)} = \frac{7}{25} \div \frac{24}{25} = \frac{7}{25} \times \frac{25}{24}$$

$$\therefore \quad \tan \theta = \frac{7}{24}$$

$$\therefore \quad \mathbf{\cos \theta = \frac{24}{25} \text{ and } \tan \theta = \frac{7}{24}}$$

Alternate Method:
$\sin \theta = \frac{7}{25}$ …(i) [Given]
Consider ΔABC, where ∠ABC 90° and ∠ACB = θ.
$\sin \theta = \frac{AB}{AC}$ … (ii) [By definition]
$\therefore \frac{AB}{AC} = \frac{7}{25}$ … [From (i) and (ii)]



LetAB = 7k and AC = 25k
In ΔABC, ∠B = 90°
$\therefore AB^2 + BC^2 = AC^2$ … [Pythagoras theorem]
$\therefore (7k)^2 + BC^2 = (25k)^2$
$\therefore 49k^2 + BC^2 = 625k^2$
$\therefore BC^2 = 625k^2 - 49k^2$
$\therefore BC^2 = 576k^2$

∴ BC = 24k ...[Taking square root of both sides]

$$\text{Now, } \cos\theta = \frac{BC}{AC} \quad \text{...[By definition]}$$

$$= \frac{24k}{25k}$$

$$\therefore \quad \cos\theta = \frac{24}{25}$$

$$\text{Also, } \tan\theta = \frac{AB}{BC} \quad \text{...[By definition]}$$

$$= \frac{7k}{24k}$$

$$\therefore \quad \tan\theta = \frac{7}{24}$$

$$\therefore \quad \mathbf{\cos\theta = \frac{24}{25} \text{ and } \tan\theta = \frac{7}{24}}$$

Question 2.
If tan θ = 34, find the values of sec θ and cos θ.
Solution:

$$\tan\theta = \frac{3}{4} \quad \text{...[Given]}$$

We know that,

$$1 + \tan^2\theta = \sec^2\theta$$

$$\therefore \quad 1 + \left(\frac{3}{4}\right)^2 = \sec^2\theta$$

$$\therefore \quad 1 + \frac{9}{16} = \sec^2\theta$$

$$\therefore \quad \frac{16+9}{16} = \sec^2\theta$$

$$\therefore \quad \sec^2\theta = \frac{25}{16}$$

$$\therefore \quad \sec\theta = \frac{5}{4}$$

...[Taking square root of both sides]

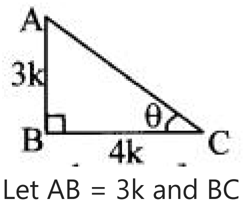
$$\text{Now, } \cos\theta = \frac{1}{\sec\theta}$$

$$= \frac{1}{\left(\frac{5}{4}\right)}$$

$$\therefore \quad \cos\theta = \frac{4}{5}$$

$$\therefore \quad \mathbf{\sec\theta = \frac{5}{4} \text{ and } \cos\theta = \frac{4}{5}}$$

Alternate Method:
tan θ = 34 ...(i)[Given]
Consider ΔABC, where ∠ABC 90° and ∠ACB = θ.
tan θ = $ABBC$ ... (ii) [By definition]
∴ $ABBC$ = 34 ... [From (i) and (ii)]



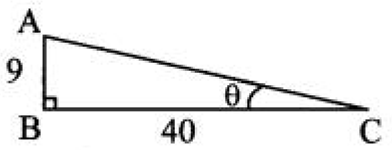
Let AB = 3k and BC 4k
In ΔABC,∠B = 90°
∴ $AB^2 + BC^2 = AC^2$ ...[Pythagoras theorem]
∴ $(3k)^2 + (4k)^2 = AC^2$
∴ $9k^2 + 16k^2 = AC^2$
∴ $AC^2 = 25k^2$

∴ AC = 5k …[Taking square root of both sides]

$$\text{Now, } \sec\theta = \frac{AC}{BC} \qquad \ldots\text{[By definition]}$$
$$= \frac{5k}{4k}$$
$$\therefore \quad \sec\theta = \frac{5}{4}$$
$$\text{Also, } \cos\theta = \frac{BC}{AC} \qquad \ldots\text{[By definition]}$$
$$= \frac{4k}{5k}$$
$$\therefore \quad \cos\theta = \frac{4}{5}$$
$$\therefore \quad \mathbf{\sec\theta = \frac{5}{4} \text{ and } \cos\theta = \frac{4}{5}}$$

Question 3.
If cot θ = $\frac{40}{9}$, find the values of cosec θ and sin θ
Solution:

$$\cot\theta = \frac{40}{9} \qquad \ldots\text{[Given]}$$
We know that,
$$1 + \cot^2\theta = \text{cosec}^2\theta$$
$$\therefore \quad 1 + \left(\frac{40}{9}\right)^2 = \text{cosec}^2\theta$$
$$\therefore \quad 1 + \frac{1600}{81} = \text{cosec}^2\theta$$
$$\therefore \quad \frac{81 + 1600}{81} = \text{cosec}^2\theta$$
$$\therefore \quad \text{cosec}^2\theta = \frac{1681}{81}$$
$$\therefore \quad \text{cosec}\,\theta = \frac{41}{9}$$
..[Taking square root of both sides]
$$\text{Now, } \sin\theta = \frac{1}{\text{cosec}\,\theta}$$
$$= \frac{1}{\left(\frac{41}{9}\right)}$$
$$\therefore \quad \sin\theta = \frac{9}{41}$$
$$\therefore \quad \mathbf{\text{cosec}\,\theta = \frac{41}{9} \text{ and } \sin\theta = \frac{9}{41}}$$

Alternate Method:
cot θ = $\frac{40}{9}$ ….(i) [Given]
Consider ΔABC, where ∠ABC = 90° and
∠ACB = θ
cot θ = $\frac{BC}{AB}$ …(ii) [By defnition]
∴ $\frac{BC}{AB}$ = $\frac{40}{9}$ ….. [From (i) and (ii)]
Let BC = 40k and AB = 9k


In ΔABC, ∠B = 90°
∴ $AB^2 + BC^2 = AC^2$ … [Pythagoras theorem]
∴ $(9k)^2 + (40k)^2 = AC^2$
∴ $81k^2 + 1600k^2 = AC^2$
∴ $AC^2 = 1681k^2$

∴ AC = 41k … [Taking square root of both sides]

Now, $\text{cosec}\,\theta = \frac{AC}{AB}$ …[By definition]

$= \frac{41k}{9k}$

∴ $\text{cosec}\,\theta = \frac{41}{9}$

Also, $\sin\theta = \frac{AB}{AC}$ …[By definition]

$= \frac{9k}{41k}$

∴ $\sin\theta = \frac{9}{41}$

∴ $\mathbf{cosec\,\theta = \frac{41}{9}}$ **and** $\mathbf{\sin\theta = \frac{9}{41}}$

Question 4.
If 5 sec θ – 12 cosec θ = θ, find the values of sec θ, cos θ and sin θ.
Solution:
5 sec θ – 12 cosec θ = 0 …[Given]
∴ 5 sec θ = 12 cosec θ

∴ $\frac{5}{\cos\theta} = \frac{12}{\sin\theta}$ … $\left[\because \sec\theta = \frac{1}{\cos\theta}, \text{cosec}\theta = \frac{1}{\sin\theta}\right]$

∴ $\frac{\sin\theta}{\cos\theta} = \frac{12}{5}$

∴ $\tan\theta = \frac{12}{5}$

We know that,
$1 + \tan^2\theta = \sec^2\theta$

∴ $1 + \left(\frac{12}{5}\right)^2 = \sec^2\theta$

∴ $1 + \frac{144}{25} = \sec^2\theta$

∴ $\frac{25 + 144}{25} = \sec^2\theta$

∴ $\sec^2\theta = \frac{169}{25}$

∴ $\sec\theta = \frac{13}{5}$ …[Taking square root of both sides]

Now, $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\left(\frac{13}{5}\right)}$

∴ $\cos\theta = \frac{5}{13}$

We know that, $\sin^2\theta + \cos^2\theta = 1$

∴ $\sin^2\theta + \left(\frac{5}{13}\right)^2 = 1$

∴ $\sin^2\theta + \frac{25}{169} = 1$

∴ $\sin^2\theta = 1 - \frac{25}{169}$

∴ $\sin^2\theta = \frac{169 - 25}{169}$

∴ $\sin^2\theta = \frac{144}{169}$

∴ $\sin\theta = \frac{12}{13}$

…[Taking square root of both sides]

∴ $\mathbf{\sec\theta = \frac{13}{5}, \cos\theta = \frac{5}{13}, \sin\theta = \frac{12}{13}}$

Question 5.
If tan θ = 1, then find the value of

$\frac{\sin\theta + \cos\theta}{\sec\theta + \text{cosec}\,\theta}$.

Solution:
$\tan\theta = 1$ … [Given]
We know that, $\tan 45° = 1$
$\therefore \tan\theta = \tan 45°$
$\therefore \theta = 45°$

$$\text{Now, } \sin\theta = \sin 45° = \frac{1}{\sqrt{2}}$$

$$\cos\theta = \cos 45° = \frac{1}{\sqrt{2}}$$

$$\sec\theta = \sec 45° = \sqrt{2}$$

$$\operatorname{cosec}\theta = \operatorname{cosec} 45° = \sqrt{2}$$

$$\frac{\sin\theta + \cos\theta}{\sec\theta + \operatorname{cosec}\theta} = \frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{\left(\frac{2}{\sqrt{2}}\right)}{2\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} \div 2\sqrt{2}$$

$$= \frac{2}{\sqrt{2}} \times \frac{1}{2\sqrt{2}}$$

$$\therefore \quad \frac{\sin\theta + \cos\theta}{\sec\theta + \operatorname{cosec}\theta} = \frac{1}{2}$$

Question 6.
Prove that:

i. $\frac{\sin^2\theta}{\cos\theta}+\cos\theta=\sec\theta$

ii. $\cos^2\theta\,(1+\tan^2\theta) = 1$

iii. $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta$

iv. $(\sec\theta - \cos\theta)(\cot\theta + \tan\theta)\tan\theta.\ \sec\theta$

v. $\cot\theta + \tan\theta\ \operatorname{cosec}\theta.\ \sec\theta$

vi. $\frac{1}{\sec\theta-\tan\theta}=\sec\theta+\tan\theta$

vii. $\sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$

viii. $\sec\theta+\tan\theta=\frac{\cos\theta}{1-\sin\theta}$

ix. If $\tan\theta + \frac{1}{\tan\theta} = 2$, then show that

$$\tan^2\theta + \frac{1}{\tan^2\theta} = 2$$

x. $$\frac{\tan A}{(1+\tan^2 A)^2} + \frac{\cot A}{(1+\cot^2 A)^2} = \sin A \cos A$$

xi. $$\sec^4 A\,(1-\sin^4 A) - 2\tan^2 A = 1$$

xii. $$\frac{\tan\theta}{\sec\theta - 1} = \frac{\tan\theta + \sec\theta + 1}{\tan\theta + \sec\theta - 1}$$

Proof:

i. L.H.S. = $\frac{\sin^2\theta}{\cos\theta}+\cos\theta$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} \qquad \text{…}[\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \sec\theta$$

= R.H.S.

$$\therefore \quad \frac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta$$

ii. L.H.S. = $\cos^2\theta(1 + \tan^2\theta)$
= $\cos^2\theta\sec^2\theta$ …[$\because 1 + \tan^2\theta = \sec^2\theta$]

$= \cos^2\theta \cdot \sec^2\theta$

$\quad ...[\because 1 + \tan^2\theta = \sec^2\theta]$

$= \cos^2\theta \cdot \frac{1}{\cos^2\theta} \quad ...\left[\because \sec\theta = \frac{1}{\cos\theta}\right]$

= 1
= R.H.S.
$\therefore \cos^2\theta\,(1 + \tan^2\theta) = 1$

iii. L.H.S. $= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}}$

$\quad \dots\left[\begin{array}{l}\text{On rationalising}\\ \text{the denominator}\end{array}\right]$

$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$

$\quad \dots\left[\begin{array}{l}\because \sin^2\theta + \cos^2\theta = 1\\ \therefore 1 - \sin^2\theta = \cos^2\theta\end{array}\right]$

$= \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$= \sec\theta - \tan\theta$
= R.H.S.

$\therefore \quad \boldsymbol{\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta}$

iv. L.H.S. = $(\sec\theta - \cos\theta)(\cot\theta + \tan\theta)$

$= \left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)$

$= \left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}\right)$

$= \frac{\sin^2\theta}{\cos\theta} \times \frac{1}{\sin\theta\cos\theta}$

$\quad \dots\left[\begin{array}{l}\because \sin^2\theta + \cos^2\theta = 1\\ \therefore \sin^2\theta = 1 - \cos^2\theta\end{array}\right]$

$= \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$

$= \tan\theta . \sec\theta$
= R.H.S.

$\therefore (\sec\theta - \cos\theta)(\cot\theta + \tan\theta) = \tan\theta . \sec\theta$

v. L.H.S. = $\cot\theta + \tan\theta$

$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta} \quad \dots[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{1}{\sin\theta} . \frac{1}{\cos\theta}$

$= \operatorname{cosec}\theta . \sec\theta$
= R.H.S.

$\therefore \cot\theta + \tan\theta = \operatorname{cosec}\theta . \sec\theta$

vi. $\text{L.H.S.} = \dfrac{1}{\sec\theta - \tan\theta}$

$= \dfrac{1}{\sec\theta - \tan\theta} \times \dfrac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$ ...[On rationalising the denominator]

$= \dfrac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$

$= \dfrac{\sec\theta + \tan\theta}{1}$ ...$\left[\because 1 + \tan^2\theta = \sec^2\theta \;\; \therefore \sec^2\theta - \tan^2\theta = 1\right]$

$= \sec\theta + \tan\theta$

= R.H.S.

$\therefore \quad \mathbf{\dfrac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta}$

vii. L.H.S. = $\sin^4\theta - \cos^4\theta$
= $(\sin^2\theta)^2 - (\cos^2\theta)^2$
= $(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$
= $(1)(\sin^2\theta - \cos^2\theta)$ ....[$\because \sin^2\theta + \cos^2\theta = 1$]
= $\sin^2\theta - \cos^2\theta$
= $(1 - \cos^2\theta) - \cos^2\theta$ ...[θ $\sin^2\theta = 1 - \cos^2\theta$]
= $1 - 2\cos^2\theta$
= R.H.S.
$\therefore \sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$

viii. L.H.S. = $\sec\theta + \tan\theta$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1 + \sin\theta}{\cos\theta}$

$= \dfrac{1 + \sin\theta}{\cos\theta} \times \dfrac{1 - \sin\theta}{1 - \sin\theta}$

$= \dfrac{1^2 - \sin^2\theta}{\cos\theta(1 - \sin\theta)}$

$= \dfrac{1 - \sin^2\theta}{\cos\theta(1 - \sin\theta)}$

$= \dfrac{\cos^2\theta}{\cos\theta(1 - \sin\theta)}$ ...$\left[\because \sin^2\theta + \cos^2\theta = 1 \;\; \therefore 1 - \sin^2\theta = \cos^2\theta\right]$

$= \dfrac{\cos\theta}{1 - \sin\theta}$ = R.H.S.

$\therefore \quad \mathbf{\sec\theta + \tan\theta = \dfrac{\cos\theta}{1 - \sin\theta}}$

ix. $\tan\theta + \dfrac{1}{\tan\theta} = 2$ …[Given]

$\therefore \left(\tan\theta + \dfrac{1}{\tan\theta}\right)^2 = 4$ ...[Squaring both sides]

$\therefore \tan^2\theta + 2(\tan\theta)\left(\dfrac{1}{\tan\theta}\right) + \dfrac{1}{\tan^2\theta} = 4$

$\therefore \tan^2\theta + 2 + \dfrac{1}{\tan^2\theta} = 4$

$\therefore \tan^2\theta + \dfrac{1}{\tan^2\theta} = 4 - 2$

$\therefore \mathbf{\tan^2\theta + \dfrac{1}{\tan^2\theta} = 2}$

x. L.H.S. $= \dfrac{\tan A}{(1+\tan^2 A)^2} + \dfrac{\cot A}{(1+\cot^2 A)^2}$

$= \dfrac{\tan A}{(\sec^2 A)^2} + \dfrac{\cot A}{(\text{cosec}^2 A)^2}$

$\ldots\left[\begin{array}{l}\because 1+\tan^2\theta = \sec^2\theta, \\ 1+\cot^2\theta = \text{cosec}^2\theta\end{array}\right]$

$= \dfrac{\tan A}{\sec^4 A} + \dfrac{\cot A}{\text{cosec}^4 A}$

$= \dfrac{\sin A}{\cos A} \times \cos^4 A + \dfrac{\cos A}{\sin A} \times \sin^4 A$

$= \sin A \cos^3 A + \cos A \sin^3 A$

$= \sin A \cos A (\cos^2 A + \sin^2 A)$

$= \sin A \cos A (1)$ …$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \sin A \cos A$

= R.H.S.

$\therefore \mathbf{\dfrac{\tan A}{(1+\tan^2 A)^2} + \dfrac{\cot A}{(1+\cot^2 A)^2} = \sin A \cos A}$

xi. L.H.S. $= \sec^4 A (1 - \sin^4 A) - 2\tan^2 A$

$= \sec^4 A [1^2 - (\sin^2 A)^2] - 2\tan^2 A$

$= \sec^4 A (1 - \sin^2 A)(1 + \sin^2 A) - 2\tan^2 A$

$= \sec^4 A \cos^2 A (1 + \sin^2 A) - 2\tan^2 A$

$[\because \sin^2\theta + \cos^2\theta = 1, \because 1 - \sin^2\theta = \cos^2\theta]$

$= \frac{1}{\cos^4 A} \cdot \cos^2 A (1 + \sin^2 A) - 2 \tan^2 A$

$= \frac{1}{\cos^2 A} (1 + \sin^2 A) - 2 \tan^2 A$

$= \frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} - 2 \tan^2 A$

$= \sec^2 A + \tan^2 A - 2 \tan^2 A$

$= \sec^2 A - \tan^2 A$

$= 1 \qquad \ldots[\because \sec^2\theta - \tan^2\theta = 1]$

= R.H.S.

$\therefore \quad \mathbf{\sec^4 A(1 - \sin^4 A) - 2\tan^2 A = 1}$

xii. L.H.S. $= \frac{\tan\theta}{\sec\theta - 1}$

$= \frac{\tan\theta}{\sec\theta-1} \times \frac{\sec\theta+1}{\sec\theta+1} \qquad \ldots\left[\begin{array}{l}\text{On rationalising} \\ \text{the denominator}\end{array}\right]$

$= \frac{\tan\theta(\sec\theta+1)}{\sec^2\theta - 1}$

$= \frac{\tan\theta(\sec\theta+1)}{\tan^2\theta} \qquad \ldots\left[\begin{array}{l}\because 1 + \tan^2\theta = \sec^2\theta \\ \therefore \sec^2\theta - 1 = \tan^2\theta\end{array}\right]$

$= \frac{\sec\theta+1}{\tan\theta}$

$\therefore \quad \frac{\tan\theta}{\sec\theta-1} = \frac{\sec\theta+1}{\tan\theta}$

$\therefore$ By theorem on equal ratios,

$\frac{\tan\theta}{\sec\theta-1} = \frac{\sec\theta+1}{\tan\theta} = \frac{\tan\theta+(\sec\theta+1)}{\sec\theta-1+(\tan\theta)}$

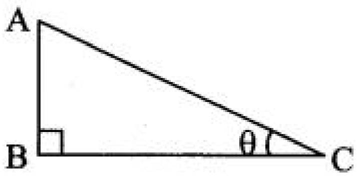
$= \frac{\tan\theta+\sec\theta+1}{\tan\theta+\sec\theta-1}$

= R.H.S.

$\therefore \quad \mathbf{\frac{\tan\theta}{\sec\theta-1} = \frac{\tan\theta+\sec\theta+1}{\tan\theta+\sec\theta-1}}$

Maharashtra Board Class 10 Maths Chapter 6 Trigonometry Intext Questions and Activities

Question 1.
Fill in the blanks with reference to the figure given below. (Textbook pg. no. 124)



Solution:

i. $\sin\theta = \frac{\boxed{AB}}{\boxed{AC}}$

ii. $\cos\theta = \frac{\boxed{BC}}{\boxed{AC}}$

iii. $\tan\theta = \frac{\boxed{AB}}{\boxed{BC}}$

Question 2.
Complete the relations in ratios given below. (Textbook pg, no. 124)

i. $\frac{\sin\theta}{\cos\theta} = \boxed{}$

ii. $\sin\theta = \cos\left(90 - \boxed{}\right)$

iii. $\cos\theta = \sin\left(90 - \boxed{}\right)$

iv. $\tan\theta \times \tan(90 - \theta) = \boxed{}$

Solution:
i. $\frac{\sin\theta}{\cos\theta}$ = [tan θ]
ii. sin θ = cos (90 – θ)
iii. cos θ = (90 – θ)
iv. tan θ × tan (90 – θ) = 1

Question 3.
Complete the equation. (Textbook pg. no, 124)
$\sin^2\theta + \cos^2\theta$ = [______]
Solution:
$\sin^2\theta + \cos^2\theta$ = [1]

Question 4.
Write the values of the following trigonometric ratios. (Textbook pg. no. 124)

i. $\sin 30° = \frac{1}{\boxed{}}$ ii. $\cos 30° = \frac{\boxed{}}{\boxed{}}$

iii. $\tan 30° = \frac{\boxed{}}{\boxed{}}$ iv. $\sin 60° = \frac{\boxed{}}{\boxed{}}$

v. $\cos 45° = \frac{\boxed{}}{\boxed{}}$ vi. $\tan 45° = \boxed{}$

Solution:

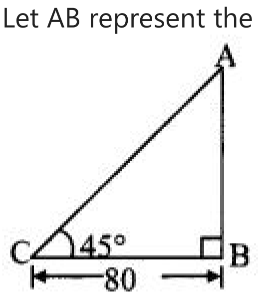
i. $\sin 30° = \frac{1}{\boxed{2}}$ ii. $\cos 30° = \frac{\boxed{\sqrt{3}}}{\boxed{2}}$

iii. $\tan 30° = \frac{\boxed{1}}{\boxed{\sqrt{3}}}$ iv. $\sin 60° = \frac{\boxed{\sqrt{3}}}{\boxed{2}}$

v. $\cos 45° = \frac{\boxed{1}}{\boxed{\sqrt{2}}}$ vi. $\tan 45° = \boxed{1}$

Practice Set 6.2 Geometry 10th Std Maths Part 2 Answers Chapter 6 Trigonometry

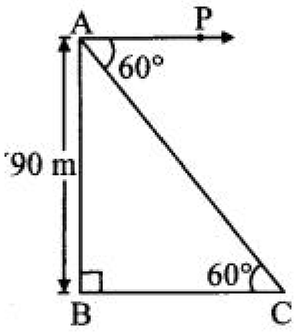
Question 1.
A person is standing at a distance of 80 m from a church looking at its top. The angle of elevation is of 45°. Find the height of the church.
Solution:
Let AB represent the height of the church and point C represent the position of the person.



BC = 80 m
Angle of elevation = ∠ACB = 45°
In right angled ΔABC,
tan 45° = $\frac{AB}{BC}$ ... [By definition]
∴ 1 = $\frac{AB}{80}$

∴ AB = 80m
∴ The height of the church is 80 m.

Question 2.
From the top of a lighthouse, an observer looking at a ship makes angle of depression of 60°. If the height of the lighthouse is 90 metre, then find how far the ship is from the lighthouse. ($\sqrt{3}$ = 1.73)
Solution:
Let AB represent the height of lighthouse and point C represent the position of the ship.
AB = 90 m



Angle of depression = ∠PAC = 60°
Now, ray AP || seg BC
∴ ∠ACB = ∠PAC ... [Alternate angles]
∴ ∠ACB = 60°
In right angled ΔABC,
tan 60° = $\frac{AB}{BC}$ ... [By definition]

$$\therefore \quad \sqrt{3} = \frac{90}{BC}$$

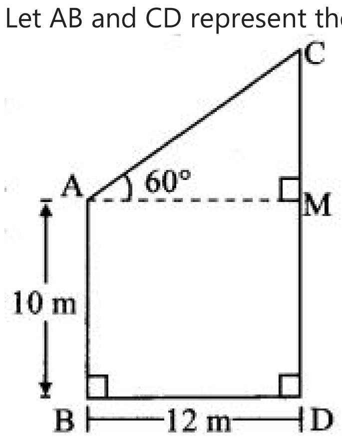
$$\therefore \quad BC = \frac{90}{\sqrt{3}}$$

$$= \frac{90}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \quad \ldots \left[\begin{array}{l}\text{On rationalising} \\ \text{the denominator}\end{array}\right]$$

$$= \frac{90\sqrt{3}}{3} = 30\sqrt{3} = 30 \times 1.73 = 51.90 \text{ m}$$

∴ The ship is 51.90 m away from the lighthouse.

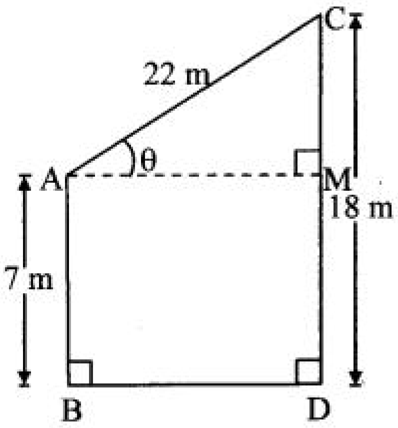
Question 3.
Two buildings are facing each other on a road of width 12 metre. From the top of the first building, which is 10 metre high, the angle of elevation of the top of the second is found to be 60°. What is the height of the second building?
Solution:
Let AB and CD represent the heights of the two buildings, and BD represent the width of the road.



Draw seg AM ⊥ seg CD.
Angle of elevation = ∠CAM = 60°
AB = 10 m
BD= 12 m
In ⌡ ABDM,
∠B = ∠D = 90°
∠M = 90° ... [seg AM ⊥ seg CD]
∴ ∠A = 90° ... [Remaining angle of ⌡ ABDM]
∴ ⌡ ABDM is a rectangle .... [Each angle is 90°]
∴ AM = BD = 12 m opposite sides
DM = AB = 10 m of a rectangle
In right angled ΔAMC,
tan 60° = $\frac{CM}{AM}$ ...[By definition]
∴ $\sqrt{3} = \frac{CM}{12}$
∴ CM = $12\sqrt{3}$ m
Now, CD = DM + CM ... [C – M – D]
∴ CD = $(10 + 12\sqrt{3})$m
= 10 + 12 × 1.73

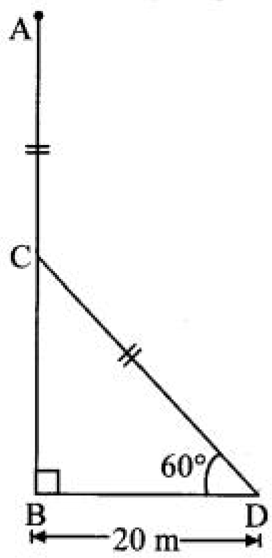
= 10 + 20.76 = 30.76
∴ The height of the second building is 30.76 m.

Question 4.
Two poles of heights 18 metre and 7 metre are erected on a ground. The length of the wire fastened at their tops is 22 metre. Find the angle made by the wire with the horizontal.
Solution:
Let AB and CD represent the heights of two poles, and AC represent the length of the wire.



Draw seg AM ⊥ seg CD.
Angle of elevation = ∠CAM = θ
AB = 7 m
CD = 18 m
AC = 22 m
In □ ABDM,
∠B = ∠D = 90°
∠M = 90° ...[seg AM ⊥ seg CD]
∴ ∠A = 90° ... [Remaining angle of □ ABDM]
∴ □ABDM is a rectangle. ... [Each angle is 90°]
∴ DM = AB = 7 m ... [Opposite sides of a rectangle]
Now, CD = CM + DM ... [C – M – D]
∴ 18 = CM + 7
∴ CM = 18 – 7 = 11 m
In right angled ΔAMC,
sin θ = $\frac{CM}{AC}$ .....[By definition]
∴ sin θ = $\frac{11}{22} = \frac{1}{2}$
But, sin 30° = $\frac{1}{2}$
∴ θ = 30°
∴ The angle made by the wire with the horizontal is 30°.

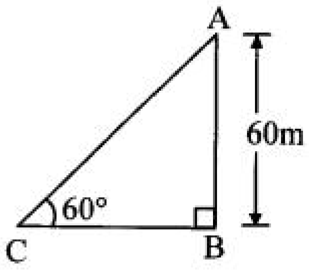
Question 5.
A storm broke a tree and the treetop rested 20 m from the base of the tree, making an angle of 60° with the horizontal. Find the height of the tree.
Solution:
Let AB represent the height of the tree.
Suppose the tree broke at point C and its top touches the ground at D.
AC is the broken part of the tree which takes position CD such that ∠CDB = 60°
∴ AC = CD ...(i)
BD = 20 m
In right angled ΔCBD,



tan 60° = $\frac{BC}{BD}$ ... [By definition]
∴ $\sqrt{3} = \frac{BC}{20}$
∴ BC = $20\sqrt{3}$ m
Also, cos 60° = $\frac{BC}{CD}$ ... [By definition]
∴ $\frac{1}{2} = \frac{20}{CD}$
∴ CD = 20 × 2 = 40 m

∴ AC = 40 m …[From(i)]
Now, AB = AC + BC ….[A – C – B]
= 40 + 20$\sqrt{3}$
= 40 + 20 × 1.73
= 40 + 34.6
= 74.6
∴ The height of the tree is 74.6 m.

Question 6.
A kite is flying at a height of 60 m above the ground. The string attached to the kite is tied at the ground. It makes an angle of 60° with the ground. Assuming that the string is straight, find the length of the string. ($\sqrt{3}$ = 1.73)
Solution:
Let AB represent the height at which kite is flying and point C represent the point where the string is tied at the ground.
∠ACB is the angle made by the string with the ground.



∠ACB = 60°
AB = 60 m
In right angled ΔABC,
sin 60° = $\frac{AB}{AC}$ … [By definition]

$$\therefore \quad \frac{\sqrt{3}}{2} = \frac{60}{AC}$$

$$\therefore \quad AC = \frac{60 \times 2}{\sqrt{3}}$$

$$= \frac{120}{\sqrt{3}}$$

$$= \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \quad \ldots \left[\begin{array}{l}\text{On rationalising}\\ \text{the denominator}\end{array}\right]$$

$$= \frac{120\sqrt{3}}{3}$$

∴ AC = 40 $\sqrt{3}$ = 40 × 1.73 = 69.20 m
∴ The length of the string is 69.20 m.

Problem Set 6 Geometry 10th Std Maths Part 2 Answers Chapter 6 Trigonometry

Question 1.
Choose the correct alternative answer for the following questions.

i. sin θ.cosec θ = ?
(A) 1
(B) 0
(C) $\frac{1}{2}$
(D) $\sqrt{2}$
Answer:
(A)

ii. cosec 45° = ?
(A) $\frac{1}{\sqrt{2}}$
(B) $\sqrt{2}$

(C) 3√2

(D) 13√

Answer:
(B)

iii. $1 + \tan^2 \theta$ = ?

(A) $\cot^2 \theta$

(B) $\operatorname{cosec}^2 \theta$

(C) $\sec^2 \theta$

(D) $\tan^2 \theta$

Answer:
(C)

iv. When we see at a higher level, from the horizontal line, angle formed is ______

(A) angle of elevation.

(B) angle of depression.

(C) 0

(D) straight angle.

Answer:
(A)

Question 2.
If sin θ = 1161, find the value of cos θ using trigonometric identity.

Solution:
sin θ = 1161 ... [Given]

We know that,
$\sin^2 \theta + \cos^2 \theta = 1$

$$\therefore \quad \left(\frac{11}{61}\right)^2 + \cos^2 \theta = 1$$

$$\therefore \quad \frac{121}{3721} + \cos^2 \theta = 1$$

$$\therefore \quad \cos^2 \theta = 1 - \frac{121}{3721}$$

$$\therefore \quad \cos^2 \theta = \frac{3721 - 121}{3721}$$

$$\therefore \quad \cos^2 \theta = \frac{3600}{3721}$$

$$\therefore \quad \mathbf{\cos \theta = \frac{60}{61}}$$

...[Taking square root of both sides]

Question 3.
If tan θ = 2, find the values of other trigonometric ratios.

Solution:
tan θ = 2 ...[Given]

We know that,
$1 + \tan^2 \theta = \sec^7 \theta$

$\therefore 1 + (2)^7 = \sec^7 \theta$

$\therefore 1 + 4 = \sec^7 \theta$

$\therefore \sec^7 \theta = 5$

∴ sec θ = $\sqrt{5}$ ...[Taking square root of both sides]

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{5}}$$

We know that, $\sin^2\theta + \cos^2\theta = 1$

$$\therefore \quad \sin^2\theta + \left(\frac{1}{\sqrt{5}}\right)^2 = 1$$

$$\therefore \quad \sin^2\theta + \frac{1}{5} = 1$$

$$\therefore \quad \sin^2\theta = 1 - \frac{1}{5}$$

$$\therefore \quad \sin^2\theta = \frac{5-1}{5}$$

$$\therefore \quad \sin^2\theta = \frac{4}{5}$$

$$\therefore \quad \sin\theta = \frac{2}{\sqrt{5}}$$ ...[Taking square root of both sides]

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{2}{\sqrt{5}}} = \frac{\sqrt{5}}{2}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{2}$$

$$\therefore \quad \mathbf{\sin\theta = \frac{2}{\sqrt{5}}, \ \cos\theta = \frac{1}{\sqrt{5}}, \ \cot\theta = \frac{1}{2},}$$

$$\mathbf{\sec\theta = \sqrt{5}, \ \operatorname{cosec}\theta = \frac{\sqrt{5}}{2}}$$

Question 4.
If sec θ = $\frac{13}{12}$, find the values of other trigonometric ratios.
Solution:
sec θ = $\frac{13}{12}$ ... [Given]
We know that,
$1 + \tan^2\theta = \sec^2\theta$

$$\therefore \quad 1 + \tan^2\theta = \left(\frac{13}{12}\right)^2$$

$$\therefore \quad 1 + \tan^2\theta = \frac{169}{144}$$

$$\therefore \quad \tan^2\theta = \frac{169}{144} - 1$$

$$\therefore \quad \tan^2\theta = \frac{169-144}{144}$$

$$\therefore \quad \tan^2\theta = \frac{25}{144}$$

$$\therefore \quad \tan\theta = \frac{5}{12}$$

...[Taking square root of both sides]

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{5}{12}} = \frac{12}{5}$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13}$$

We know that, $\sin^2\theta + \cos^2\theta = 1$

$\therefore \quad \sin^2\theta + \left(\frac{12}{13}\right)^2 = 1$

$\therefore \quad \sin^2\theta + \frac{144}{169} = 1$

$\therefore \quad \sin^2\theta = 1 - \frac{144}{169}$

$\therefore \quad \sin^2\theta = \frac{169 - 144}{169}$

$\therefore \quad \sin^2\theta = \frac{25}{169}$

$\therefore \quad \sin\theta = \frac{5}{13}$

...[Taking square root of both sides]

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{5}{13}} = \frac{13}{5}$

$\therefore$ $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$, $\cot\theta = \frac{12}{5}$, $\operatorname{cosec}\theta = \frac{13}{5}$

Question 5.
Prove the following:
i. $\sec\theta (1 - \sin\theta)(\sec\theta + \tan\theta) = 1$
ii. $(\sec\theta + \tan\theta)(1 - \sin\theta) = \cos\theta$
iii. $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \times \operatorname{cosec}^2\theta$
iv. $\cot^2\theta - \tan^2\theta = \operatorname{cosec}^2\theta - \sec^2\theta$
v. $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$
vi. $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$
vii. $\sec^6 x - \tan^6 x = 1 + 3\sec^2 x \times \tan^2 x$
viii. $\frac{\tan\theta}{\sec\theta + 1} = \frac{\sec\theta - 1}{\tan\theta}$
ix. $\frac{\tan^3\theta - 1}{\tan\theta - 1} = \sec^2\theta + \tan\theta$
x. $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$

Proof:
i. L.H.S. = $\sec\theta (1 - \sin\theta)(\sec\theta + \tan\theta)$

$= \frac{1}{\cos\theta}(1 - \sin\theta)\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$

$= \frac{1 - \sin\theta}{\cos\theta}\left(\frac{1 + \sin\theta}{\cos\theta}\right)$

$= \frac{1 - \sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta}{\cos^2\theta} \quad \ldots\left[\begin{array}{l}\because \sin^2\theta + \cos^2\theta = 1 \\ \therefore 1 - \sin^2\theta = \cos^2\theta\end{array}\right]$

$= 1$

= R.H.S.

$\therefore$ $\sec\theta (1 - \sin\theta)(\sec\theta + \tan\theta) = 1$

ii. L.H.S. = $(\sec\theta + \tan\theta)(1 - \sin\theta)$

$= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta)$

$= \left(\frac{1 + \sin\theta}{\cos\theta}\right)(1 - \sin\theta)$

$= \frac{1 - \sin^2\theta}{\cos\theta}$

$= \frac{\cos^2\theta}{\cos\theta} \quad \ldots\left[\begin{array}{l}\because \sin^2\theta + \cos^2\theta = 1 \\ \therefore 1 - \sin^2\theta = \cos^2\theta\end{array}\right]$

$= \cos\theta$

= R.H.S.

$\therefore$ $(\sec\theta + \tan\theta)(1 - \sin\theta) = \cos\theta$

iii. L.H.S. = $\sec^2\theta + \csc^2\theta$

$$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta.\sin^2\theta}$$

$$= \frac{1}{\cos^2\theta.\sin^2\theta} \quad ...[\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{\cos^2\theta} \times \frac{1}{\sin^2\theta}$$

$$= \sec^2\theta \times \operatorname{cosec}^2\theta$$

= R.H.S.

∴ $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \times \operatorname{cosec}^2\theta$

iv. L.H.S. = $\cot^2\theta - \tan^2\theta$
= $(\operatorname{cosec}^2\theta - 1) - (\sec^2\theta - 1)$
[∵ $\tan^2\theta = \sec^2\theta - 1$,
$\cot^2\theta = \operatorname{cosec}^2\theta - 1$]
= $\operatorname{cosec}^2\theta - 1 - \sec^2\theta + 1$
$\operatorname{cosec}^2\theta - \sec^2\theta$
= R.H.S.
∴ $\cot^2\theta - \tan^2\theta = \operatorname{cosec}^2\theta - \sec^2\theta$

v. L.H.S. = $\tan^4\theta + \tan^2\theta$
= $\tan^2\theta(\tan^2\theta + 1)$
= $\tan^2\theta.\sec^2\theta$
...[∵ $1 + \tan^2\theta = \sec^2\theta$]
= $(\sec^2\theta - 1)\sec^2\theta$
...[∵ $\tan^2\theta = \sec^2\theta - 1$]
= $\sec^4\theta - \sec^2\theta$
= R.H.S.
∴ $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

vi. L.H.S. = $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}$

$$= \frac{(1+\sin\theta)+(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta}$$

$$= \frac{2}{\cos^2\theta} \qquad ...[\because 1-\sin^2\theta = \cos^2\theta]$$

$$= 2 \times \frac{1}{\cos^2\theta}$$

$$= 2\sec^2\theta$$

= R.H.S.

$$\therefore \quad \frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$$

vii. L.H.S. = $\sec^6 x - \tan^6 x$
= $(\sec^2 x)^3 - \tan^6 x$
= $(1 + \tan^2 x)^3 - \tan^6 x$ ...[∵ $1 + \tan^2\theta = \sec^2\theta$]
= $1 + 3\tan^2 x + 3(\tan^2 x)^2 + (\tan^2 x)^3 - \tan^6 x$ ...[∵ $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]
= $1 + 3\tan^2 x(1 + \tan^2 x) + \tan^6 x - \tan^6 x$
= $1 + 3\tan^2 x\sec^2 x$ ...[∵ $1 + \tan^2\theta = \sec^2\theta$]
= R.H.S.
∴ $\sec^3 x - \tan^6 x = 1 + 3\sec^2 x.\tan^2 x$

viii. L.H.S. $= \dfrac{\tan\theta}{\sec\theta+1}$

$= \dfrac{\tan\theta}{\sec\theta+1} \times \dfrac{\sec\theta-1}{\sec\theta-1}$

$\ldots\begin{bmatrix}\text{On rationalising}\\ \text{the denominator}\end{bmatrix}$

$= \dfrac{\tan\theta(\sec\theta-1)}{\sec^2\theta-1}$

$= \dfrac{\tan\theta(\sec\theta-1)}{\tan^2\theta}$

$\ldots\begin{bmatrix}\because 1+\tan^2\theta=\sec^2\theta\\ \therefore \sec^2\theta-1=\tan^2\theta\end{bmatrix}$

$= \dfrac{\sec\theta-1}{\tan\theta}$

= R.H.S.

$\therefore \quad \mathbf{\dfrac{\tan\theta}{\sec\theta+1} = \dfrac{\sec\theta-1}{\tan\theta}}$

ix. L.H.S. $= \dfrac{\tan^3\theta-1}{\tan\theta-1} = \dfrac{\tan^3\theta-1^3}{\tan\theta-1}$

$= \dfrac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{(\tan\theta-1)}$

$\ldots[\because a^3-b^3=(a-b)(a^2+ab+b^2)]$

$= \tan^2\theta+\tan\theta+1$

$= (1+\tan^2\theta)+\tan\theta$

$= \sec^2\theta+\tan\theta$

$\ldots[\because 1+\tan^2\theta=\sec^2\theta]$

= R.H.S.

$\therefore \quad \mathbf{\dfrac{\tan^3\theta-1}{\tan\theta-1} = \sec^2\theta+\tan\theta}$

x. We know that,

$\sin^2\theta + \cos^2\theta = 1$

$\therefore 1 - \sin^2\theta = \cos^2\theta$

$\therefore (1 - \sin\theta)(1 + \sin\theta) = \cos\theta.\cos\theta$

$\therefore \quad \dfrac{1+\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1-\sin\theta}$

$\therefore$ By theorem on equal ratios,

$\dfrac{1+\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1-\sin\theta} = \dfrac{1+\sin\theta-\cos\theta}{\cos\theta-(1-\sin\theta)}$

$\dfrac{1+\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1-\sin\theta} = \dfrac{\sin\theta-\cos\theta+1}{\cos\theta-1+\sin\theta}$

$\therefore \quad \dfrac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1} = \dfrac{1+\sin\theta}{\cos\theta} \quad \ldots(i)$

Consider,

$$\frac{1}{\sec\theta - \tan\theta}$$

$$= \frac{1}{\sec\theta - \tan\theta} \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta} \quad \ldots\begin{bmatrix}\text{On rationalising} \\ \text{the denominator}\end{bmatrix}$$

$$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$$= \frac{\sec\theta + \tan\theta}{1} \quad \ldots\begin{bmatrix}\because 1 + \tan^2\theta = \sec^2\theta \\ \therefore \sec^2\theta - \tan^2\theta = 1\end{bmatrix}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$\therefore \quad \frac{1}{\sec\theta - \tan\theta} = \frac{1 + \sin\theta}{\cos\theta} \quad \ldots\text{(ii)}$$

From (i) and (ii), we get

$$\mathbf{\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}}$$

Alternate method:

$$\text{L.H.S.} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

$$= \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}}$$

$$\ldots[\text{Dividing numerator and denominator by } \cos\theta]$$

$$= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + \left(\sec^2\theta - \tan^2\theta\right)}$$

$$\ldots[\because \sec^2\theta - \tan^2\theta = 1]$$

$$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}$$

$$= \frac{\tan\theta + \sec\theta - 1}{-(\sec\theta - \tan\theta) + (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}$$

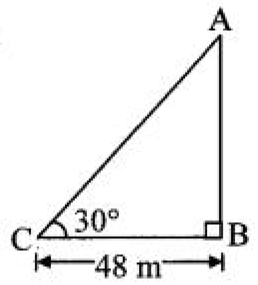
$$= \frac{\tan\theta + \sec\theta - 1}{(\sec\theta - \tan\theta)(-1 + \sec\theta + \tan\theta)}$$

$$= \frac{1}{\sec\theta - \tan\theta}$$

$$= \text{R.H.S.}$$

$$\therefore \quad \mathbf{\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}}$$

Question 6.
A boy standing at a distance of 48 metres from a building observes the top of the building and makes an angle of elevation of 30°. Find the height of the building.
Solution:
Let AB represent the height of the building and point C represent the position of the boy.
Angle of elevation = $\angle ACB = 30°$
BC = 48 m



In right angled ΔABC,
tan 30° = $\frac{AB}{BC}$ ... [By definition]

$$\therefore \quad \frac{1}{\sqrt{3}} = \frac{AB}{48}$$

$$\therefore \quad AB = \frac{48}{\sqrt{3}}$$

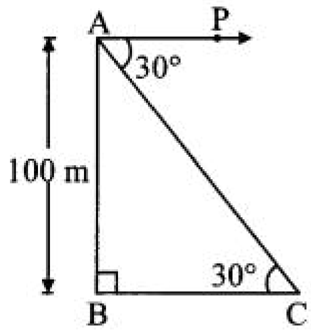
$$\therefore \quad AB = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \qquad \cdots \left[\begin{array}{l}\text{On rationalising} \\ \text{the denominator}\end{array}\right]$$

$$\therefore \quad AB = \frac{48\sqrt{3}}{3}$$

$$\therefore \quad AB = 16\sqrt{3} \text{ m}$$

∴ The height of the building is $16\sqrt{3}$ m.

Question 7.
From the top of the lighthouse, an observer looks at a ship and finds the angle of depression to be 30°. If the height of the lighthouse is 100 metres, then find how far the ship is from the lighthouse.
Solution:
Let AB represent the height of lighthouse and point C represent the position of the ship.



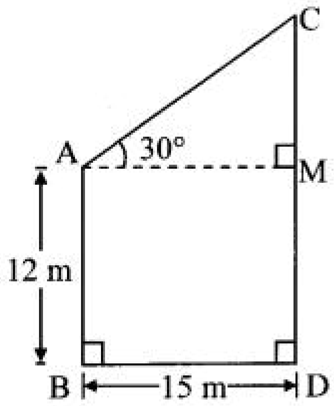
Angle of depression ∠PAC 30°
AB = 100m.
Now, ray AP || seg BC
∴ ∠ACB = ∠PAC ... [Alternate angles]
∴ ∠ACB = 30°
AB = 100m
In right angled ΔABC,
tan 30° = $\frac{AB}{BC}$ ...[By definition]

∴ $\frac{1}{\sqrt{3}} = \frac{100}{BC}$

∴ BC = $100\sqrt{3}$m

∴ The ship is 1oo$\sqrt{3}$m far from the lighthouse.

Question 8.
Two buildings are in front of each other on a road of width 15 metres. From the top of the first building, having a height of 12 metre, the angle of elevation of the top of the second building is 30°. What is the height of the second building?
Solution:
Let AB and CD represent the heights of the two buildings, and BD represent the width of the road.



Draw seg AM ⊥ seg CD
Angle of elevation = ∠CAM = 30°
AB = 12m
BD = 15m
In □ ABDM,

∠B = ∠D = 90°
∠M 90° …[segAM ⊥ segCD]
∠A 90° …[Remaining angle of ⅃ ABDM]
⅃ ABDM is a rectangle …[Each angle is 90°]

$\therefore$ $\left.\begin{array}{l} AM = BD = 15 \text{ m} \\ DM = AB = 12 \text{ m} \end{array}\right\}$ … $\left[\begin{array}{l}\text{Opposite sides} \\ \text{of a rectangle}\end{array}\right]$

In right angled ΔAMC,

$\tan 30° = \frac{CM}{AM}$ …[By definition]

$\therefore \frac{1}{\sqrt{3}} = \frac{CM}{15}$

$\therefore CM = \frac{15}{\sqrt{3}}$

$\therefore CM = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ … $\left[\begin{array}{l}\text{On rationalising} \\ \text{the denominator}\end{array}\right]$

$\therefore CM = \frac{15\sqrt{3}}{3} = 5\sqrt{3}$ m

Now, CD = DM + CM …[C–M–D]

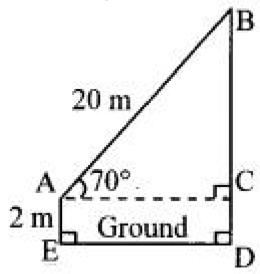
$= 12 + 5\sqrt{3}$

$= 12 + 5 \times 1.73$

$= 12 + 8.65 = 20.65$ m

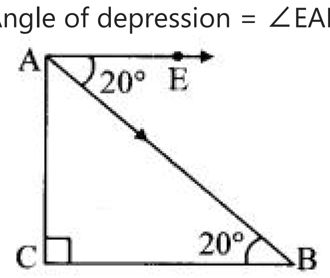
∴ The height of the second building is 20.65 m.

Question 9.
A ladder on the platform of a fire brigade van can be elevated at an angle of 70° to the maximum. The length of the ladder can be extended upto 20 m. If the platform is 2 m above the ground, find the maximum height from the ground upto which the ladder can reach. (sin 70° = 0.94)
Solution:
Let AB represent the length of the ladder and AE represent the height of the platform.



Draw seg AC ⊥ seg BD.
Angle of elevation = ∠BAC = 70°
AB = 20 m
AE = 2m
In right angled ΔABC,
sin 70° = $BCAB$ …..[By definition]
∴ 0.94 = $BC20$
∴ BC = 0.94 × 20
= 18.80 m
In ⅃ ACDE,
∠E = ∠D = 90°
∠C = 90° … [seg AC ⊥ seg BD]
∴ ∠A = 90° … [Remaining angle of ⅃ ACDE]
∴ ⅃ ACDE is a rectangle. … [Each angle is 90°]
∴ CD = AE = 2 m … [Opposite sides of a rectangle]
Now, BD = BC + CD … [B – C – D]
= 18.80 + 2
= 20.80 m
∴ The maximum height from the ground upto which the ladder can reach is 20.80 metres.

Question 10.
While landing at an airport, a pilot made an angle of depression of 20°. Average speed of the plane was 200 km/hr. The plane reached the ground after 54 seconds. Find the height at which the plane was when it started landing, (sin 20° = 0.342)
Solution:
Let AC represent the initial height and point A represent the initial position of the plane.
Let point B represent the position where plane lands.
Angle of depression = ∠EAB = 20°



Now, seg AE || seg BC
∴ ∠ABC = ∠EAB … [Alternate angles]
∴ ∠ABC = 20°
Speed of the plane = 200 km/hr
= 200 × $\frac{1000}{3600}$ m/sec
= $\frac{500}{9}$ m/sec
∴ Distance travelled in 54 sec = speed × time
= $\frac{500}{9}$ × 54
= 3000 m
∴ AB = 3000 m
In right angled ΔABC,
sin 20° = $\frac{AC}{AB}$ ….[By definition]
∴ 0.342 = $\frac{AC}{3000}$
∴ AC = 0.342 × 3000
= 1026 m
∴ The plane was at a height of 1026 m when it started landing.