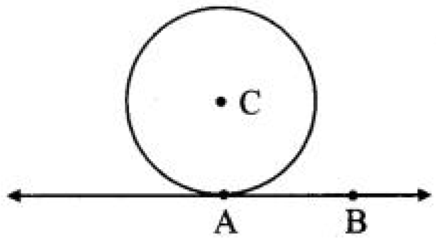
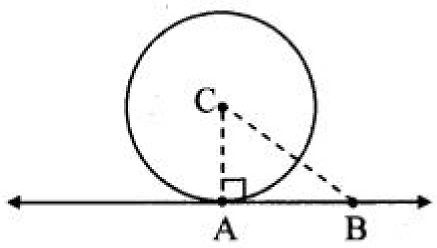
Practice Set 3.1 Geometry 10th Std Maths Part 2 Answers Chapter 3 Circle

Question 1.
In the adjoining figure, the radius of a circle with centre C is 6 cm, line AB is a tangent at A. Answer the following questions.
i. What is the measure of ∠CAB? Why?
ii. What is the distance of point C from line AB? Why?
iii. d(A, B) = 6 cm, find d(B, C).
iv. What is the measure of ∠ABC? Why?

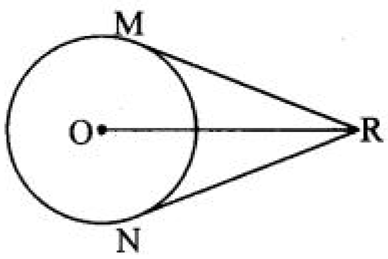


Solution:
i. line AB is the tangent to the circle with centre C and radius AC. [Given]
∴ ∠CAB = 90° (i) [Tangent theorem]
ii. seg CA ⊥ line AB [From (i)]
radius = l(AC) = 6 cm
∴ The distance of point C from line AB is 6 cm.
iii. In ΔCAB, ∠CAB = 90° [From (i)]
∴ $BC^2 = AB^2 + AC^2$. [Pythagoras theorem]

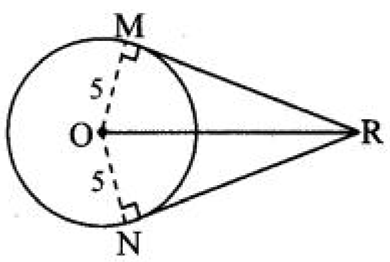


$= 6^2 + 6^2$
$= 2 \times 6^2$
∴ BC = $\sqrt{2 \times 6^2}$ [Taking square root of both sides]
= $6\sqrt{2}$ cm
∴ d(B, C) = 6 cm
iv. In ΔABC,
AC = AB = 6cm
∴ ∠ABC = ∠ACB [Isosceles triangle theorem]
Let ∠ABC = ∠ACB =x
In ΔABC,
∠CAB + ∠ABC + ∠ACB = 180° [Sum of the measures of angles of a triangle is 180°]
∴ 90° + x + x = 180°
∴ 90 + 2x = 180°
∴ 2x = 180°- 90°
∴ x = $\frac{90^\circ}{2}$
∴ x = 45°
∴ ∠ABC = 45°

Question 2.
In the adjoining figure, O is the centre of the circle. From point R, seg RM and seg RN are tangent segments touching the circle at M and N. If (OR) = 10 cm and radius of the circle = 5 cm, then
i. What is the length of each tangent segment?
ii. What is the measure of ∠MRO?
iii. What is the measure of ∠MRN?



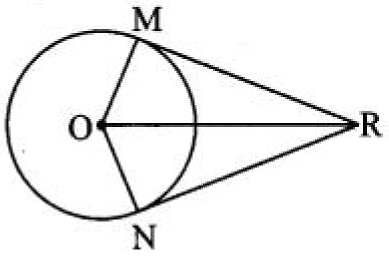
Solution:
seg RM and seg RN are tangents to the circle with centre O. [Given]
∴ ∠OMR = ∠ONR = 90° [Tangent theorem]



i. In ΔOMR, ∠OMR = 90°

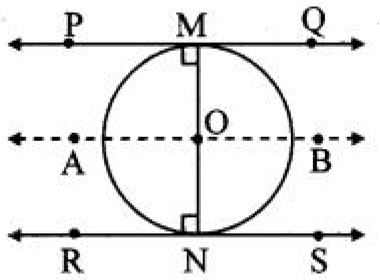
∴ $OR^2 = OM^2 + RM^2$ [Pythagoras theorem]
∴ $10^2 = 5^2 + RM^2$
∴ $100 = 25 + RM^2$
∴ $RM^2 = 75$
∴ RM = $\sqrt{75}$ [Taking square root of both sides]
∴ RM = RN [Tangent segment theorem]
Length of each tangent segment is $5\sqrt{3}$ cm.
ii. In ΔRMO,
∠OMR = 90° [Tangent theorem]
OM = 5 cm and OR = 10 cm
∴ OM = $\frac{1}{2}$ OR
∴ ∠MRO = 30° (i) [Converse of 30° – 60° – 90° theorem]
Similarly, ∠NRO = 30°
iii. But, ∠MRN = ∠MRO + ∠NRO [Angle addition property]
= 30° + 30° [From (i)]
∴ ∠MRN = 60°

Question 3.
Seg RM and seg RN are tangent segments of a circle with centre O. Prove that seg OR bisects ∠MRN as well as ∠MON.



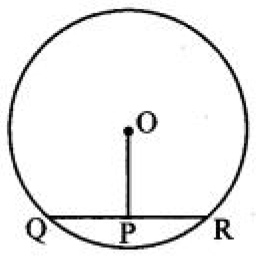
Solution:
Proof:
In ΔOMR and ΔONR,
seg RM ≅ seg RN [Tangent segment theorem]
seg OM ≅ seg ON [Radii of the same circle]
seg OR ≅ seg OR [Common side]
∴ ΔOMR ≅ ΔONR [SSS test of congruency]
{∴ ∠MRO ≅ ∠NRO
∠MOR ≅ ∠NOR } [c.a.c.t.]
∴ seg OR bisects ∠MRN and ∠MON.

Question 4.
What is the distance between two parallel tangents of a circle having radius 4.5 cm? Justify your answer.
Solution:
Let the lines PQ and RS be the two parallel tangents to circle at M and N respectively. Through centre O, draw line AB || line RS.

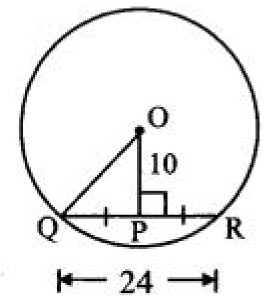


OM = ON = 4.5 [Given]
line AB || line RS [Construction]
line PQ || line RS [Given]
∴ line AB || line PQ || line RS
Now, ∠OMP = ∠ONR = 90° (i) [Tangent theorem]
For line PQ || line AB,
∠OMP = ∠AON = 90° (ii) [Corresponding angles and from (i)]
For line RS || line AB,
∠ONR = ∠AOM = 90° (iii) [Corresponding angles and from (i)]
∠AON + ∠AOM = 90° + 90° [From (ii) and (iii)]
∴ ∠AON + ∠AOM = 180°
∴ ∠AON and ∠AOM form a linear pair.
∴ ray OM and ray ON are opposite rays.
∴ Points M, O, N are collinear. (iv)
∴ MN = OM + ON [M – O – N, From (iv)]
∴ MN = 4.5 + 4.5
∴ MN = 9 cm
∴ Distance between two parallel tangents PQ and RS is 9 cm.

Question 1.
In the adjoining figure, seg QR is a chord of the circle with centre O. P is the midpoint of the chord QR. If QR = 24, OP = 10, find radius of the circle. To find solution of the problem, write the theorems that are useful. Using them, solve the problem. (Textbook pg. no. 48)



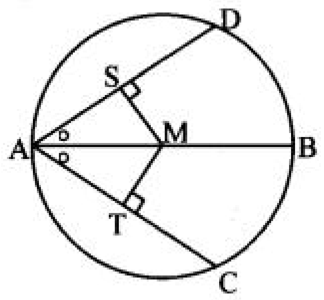
Solution:
Theorems which are useful to find solution:
i. The segment joining the centre of a circle and the midpoint of a chord is perpendicular to the chord.
ii. In a right angled triangle, sum of the squares of the perpendicular sides is equal to square of its hypotenuse.
QP = 12 (QR) [P is the midpoint of chord QR]



12 × 24 = 12 units
Also, seg OP ⊥ chord QR [The segment joining centre of a circle and midpoint of a chord is perpendicular to the chord]
In ΔOPQ, ∠OPQ = 90°
∴ $OQ^2 = OP^2 + QP^2$ [Pythagoras theorem]
$= 10^2 + 12^2$
= 100 + 144
= 244
∴ OQ = 244−−−√ = 261−−√ units.

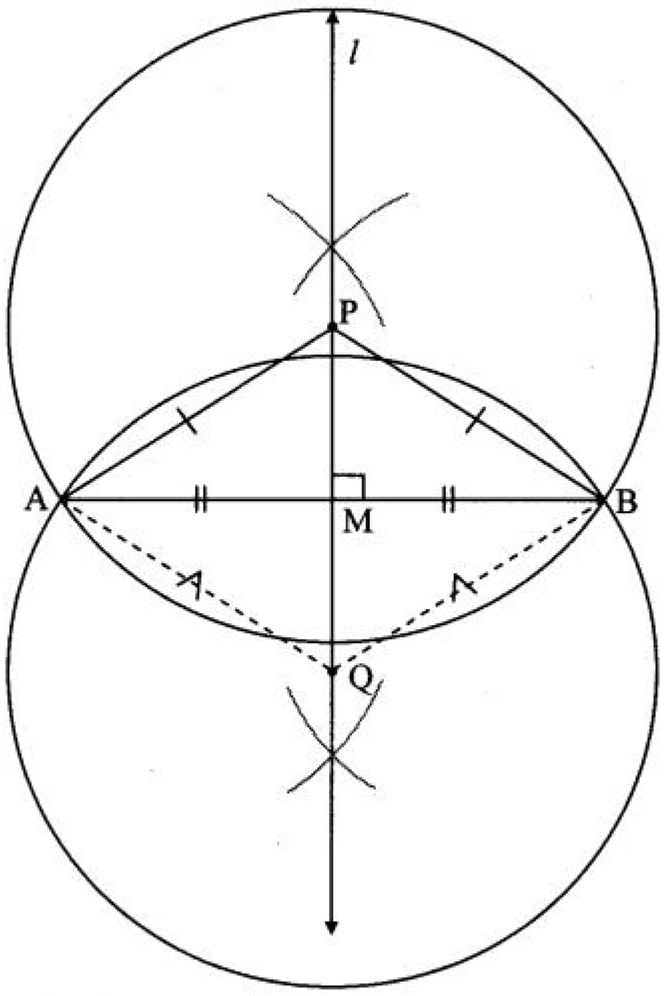
∴ The radius of the circle is 261−−√ units.

Question 2.
In the adjoining figure, M is the centre of the circle and seg AB is a diameter, seg MS ⊥ chord AD, seg MT ⊥ chord AC, ∠DAB ≅ ∠CAB.
i. Prove that: chord AD ≅ chord AC.
ii. To solve this problem which theorems will you use?
a. The chords which are equidistant from the centre are equal in length.
b. Congruent chords of a circle are equidistant from the centre.



iii. Which of the following tests of congruence of triangles will be useful?
a. SAS b. ASA c. SSS d. AAS e. Hypotenuse-side test.
Using appropriate test and theorem write the proof of the above example. (Textbook pg. no, 48)
Solution:
Proof:
i. ∠DAB ≅ ∠CAB [Given]
∴ ∠SAM ≅ ∠TAM (i) [A – S – D, A – M – B, A -T – C]
In ΔSAM and ΔTAM,
∠SAM ≅ ∠TAM [From (i)]
∠ASM ≅ ∠ATM [Each angle is of measure 90°]
seg AM ≅ seg AM [Common side]
∴ ΔSAM ≅ ΔTAM [AAS [SAA] test of congruency]
∴ side MS ≅ side MT [c.s.c.t]
But, seg MS ⊥ chord AD [Given]
seg MT ⊥chord AC
∴ chord AD ≅ chord AC [Chords of a circle equidistant from the centre are congruent]
ii. Theorem used for solving the problem:
The chords which are equidistant from the centre are equal in length.
iii. Test of congruency useful in solving the above problem is AAS ISAAI test of congruency.

Question 3.
i. Draw segment AB. Draw perpendicular bisector l of the segment AB. Take point P on the line l as centre,
PA as radius and draw a circle. Observe that the circle passes through point B also. Find the reason.
ii. Taking any other point Q on the line l, if a circle is drawn with centre Q and radius QA, will it pass through B? Think.
iii. How many such circles can be drawn, passing through A and B? Where will their centres lie? (Textbook pg. no. 49)
Solution:

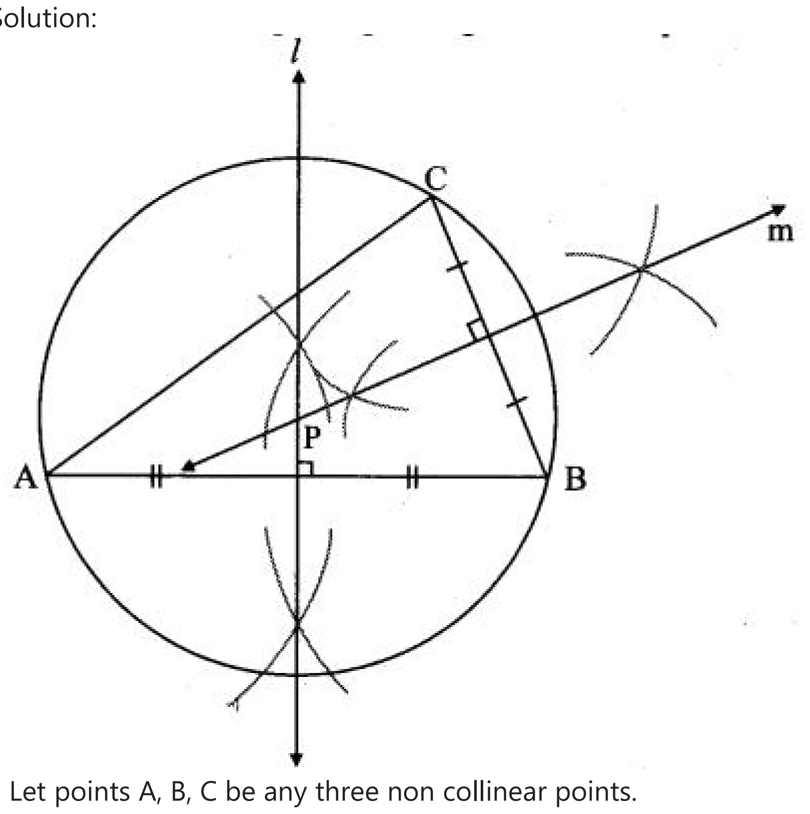


i. Draw the circle with centre P and radius PA.
line l is the perpendicular bisector of seg AB.
Every point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.
∴ PA = PB … [Perpendicular bisector theorem]
∴ PA = PB = radius
∴ The circle with centre P and radius PA passes through point B.

ii. The circle with any other point Q and radius QA is drawn.
QA = QB = radius … [Perpendicular bisector theorem]
∴ The circle with centre Q and radius QA passes through point B.

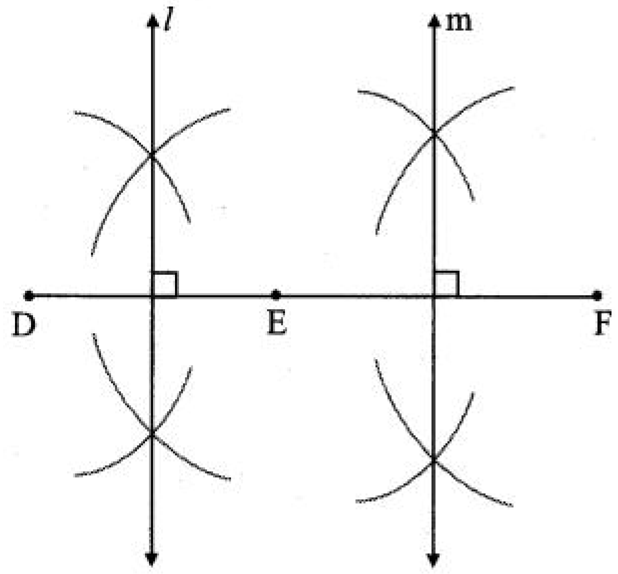
iii. We can draw infinite number of circles passing through A and B.
All their centres will lie on the perpendicular bisector of AB (i.e., line l)

Question 4.
i. Take any three non-collinear points. What should be done to draw a circle passing through all these points? Draw a circle through these points.
ii. Is it possible to draw one more circle passing through these three points? Think of it. (Textbook pg. no. 49)
Solution:

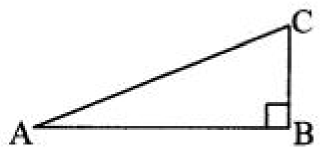


i. Let points A, B, C be any three non collinear points.
Draw the perpendicular bisector of seg AB (line l).
∴ Points A and B are equidistant from any point of line l ….(i)[Perpendicular bisector theorem]
Draw the perpendicular bisector of seg BC (line m) to intersect line l at point P.
∴ Points B and C are equidistant from any point of line m ….(ii) [Perpendicular bisector theorem]
∴ PA = PB …[From (i)]
PB = PC … [From (ii)]
∴ PA = PB = PC = radius

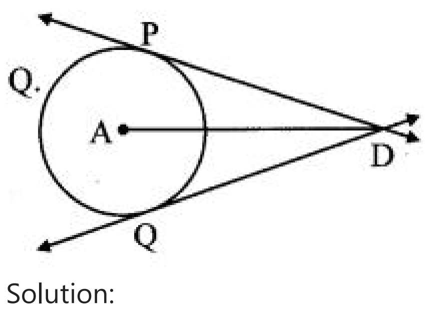
∴ With PA as radius the required circle is drawn through points A, B, C.
ii. It is not possible to draw more than one circle passing through these three points.

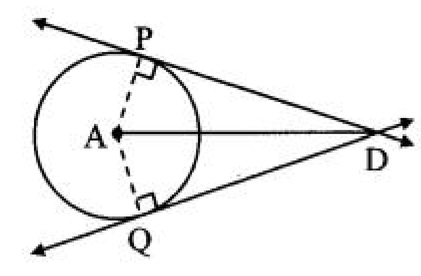
Question 5.
Take 3 collinear points D, E, F. Try to draw a circle passing through these points. If you are not able to draw a circle, think of the reason. (Textbook pg. no. 49)
Solution:


Let D, E, F be the collinear points.
The perpendicular bisector of DE and EF drawn (i.e., line l and line m) do not intersect at a common point.
∴ There is no single common point (centre of circle) from which a circle can be drawn passing through points D, E and F.
Hence, we cannot draw a circle passing through points D, E and F.

Question 6.
Which theorem do we use in proving that hypotenuse is the longest side of a right angled triangle? (Textbook pg. no. 52)


Solution:
In ΔABC,
∠ABC = 90°
∴ ∠BAC < 90° and ∠ACB < 90° [Given]
∴ ∠ABC > ∠BAC and ∠ABC > ∠ACB
∴ AC > BC and AC > AB [Side opposite to greater angle is greater]
∴ Hypotenuse is the longest side in right angled triangle.
We use theorem, If two angles of a triangle are not equal, then the side opposite to the greater angle is greater than the side opposite to the smaller angle.

Question 7.
Theorem: Tangent segments drawn from an external point to a circle are congruent
Draw radius AP and radius AQ and complete the following proof of the theorem.
Given: A is the centre of the circle.
Tangents through external point D touch the circle at the points P and Q.
To prove: seg DP ≅ seg DQ
Construction: Draw seg AP and seg AQ.


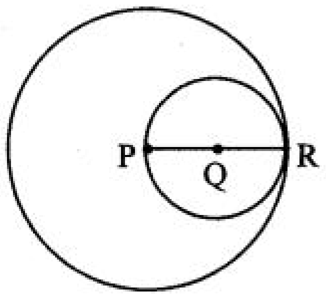
Solution:
Proof:
In ΔPAD and ΔQAD,
seg PA ≅ [segQA] [Radii of the same circle]
seg AD ≅ seg AD [Common side]


∠APD = ∠AQD = 90° [Tangent theorem]

∴ ΔPAD = ΔQAD [By Hypotenuse side test]
∴ seg DP = seg DQ [c.s.c.t]

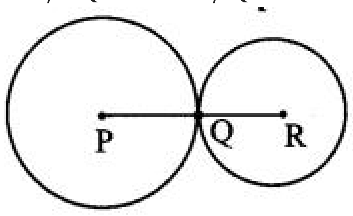
Practice Set 3.2 Geometry 10th Std Maths Part 2 Answers Chapter 3 Circle

Question 1.
Two circles having radii 3.5 cm and 4.8 cm touch each other internally. Find the distance between their centres.
Solution:
Let the two circles having centres P and Q touch each other internally at point R.
Here, QR = 3.5 cm, PR = 4.8 cm



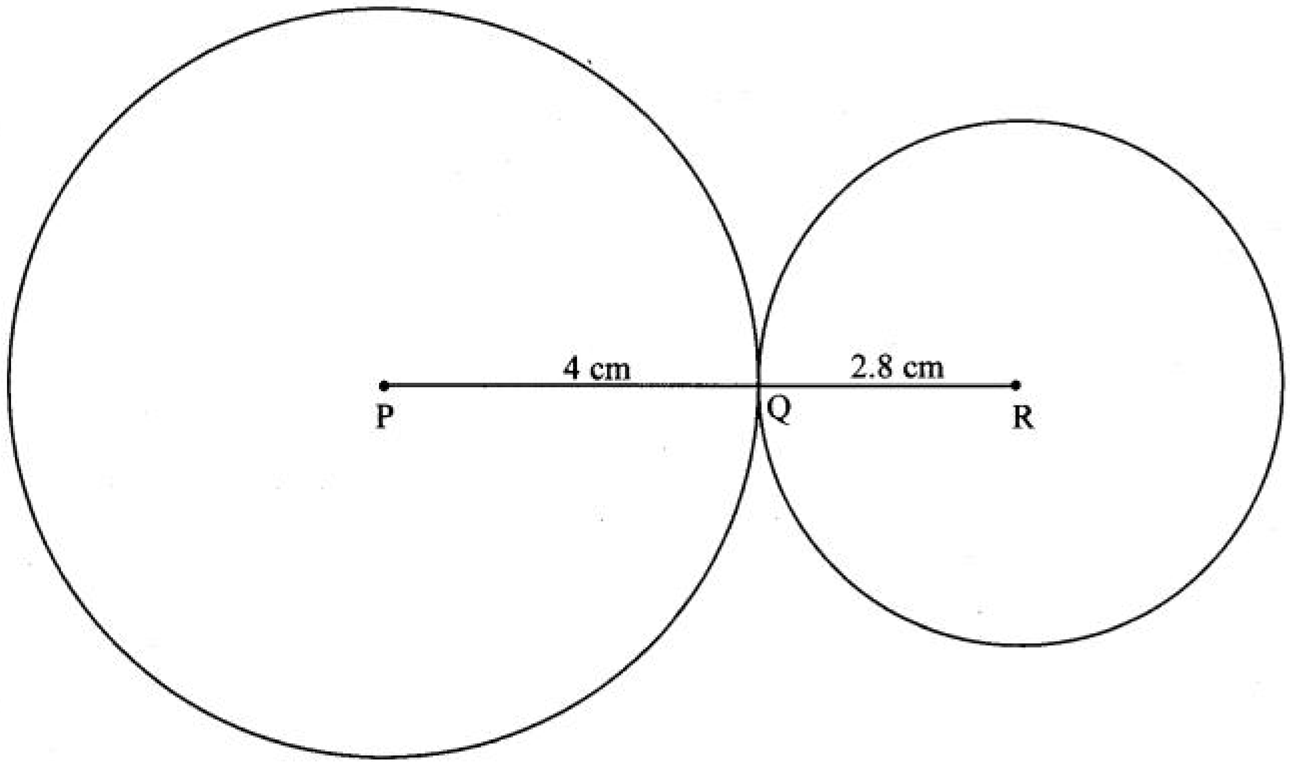
The two circles touch each other internally.
∴ By theorem of touching circles,
P – Q – R
PQ = PR – QR
= 4.8 – 3.5
= 1.3 cm
[The distance between the centres of circles touching internally is equal to the difference in their radii]

Question 2.
Two circles of radii 5.5 cm and 4.2 cm touch each other externally. Find the distance between their centres.
Solution:
Let the two circles having centres P and R touch each other externally at point Q.
Here, PQ = 5.5 cm, QR = 4.2 cm

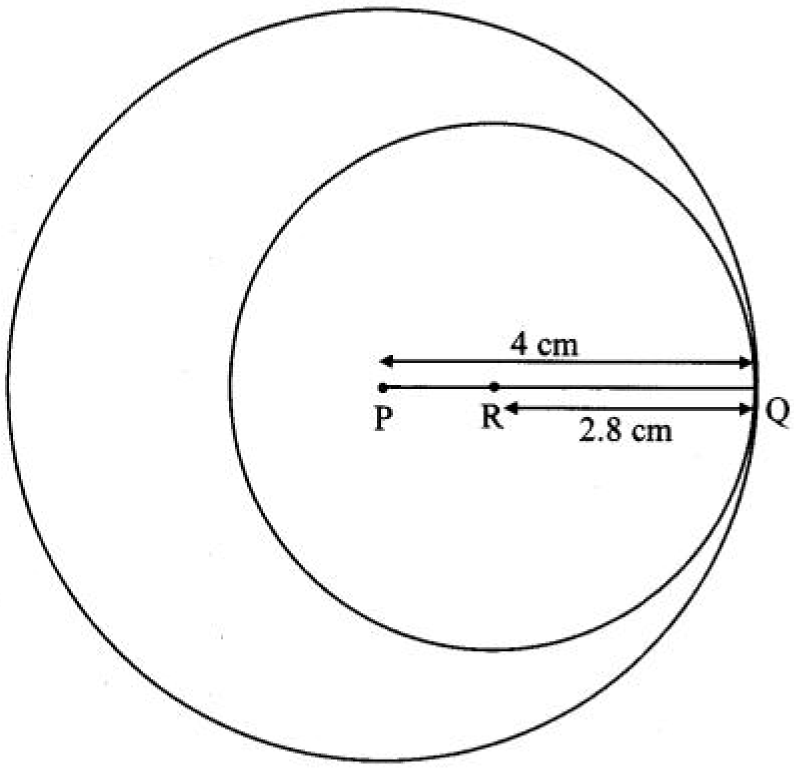


The two circles touch each other externally.
∴ By theorem of touching circles,
P – Q – R
PR = PQ + QR
= 5.5 + 4.2
= 9.7 cm
[The distance between the centres of the circles touching externally is equal to the sum of their radii]

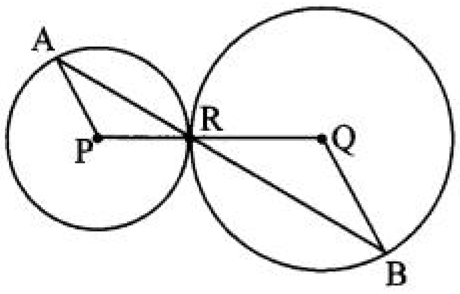
Question 3.
If radii of two circles are 4 cm and 2.8 cm. Draw figure of these circles touching each other
i. externally
ii. internally.
Solution:
i. Circles touching externally:



ii. Circles touching internally:



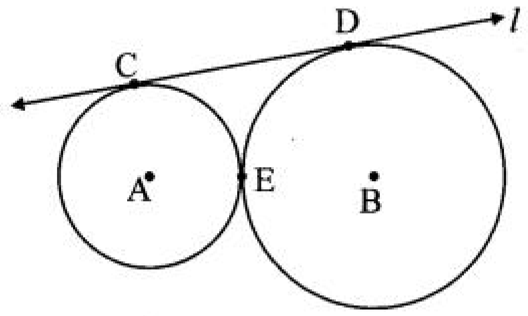
Question 4.
In the adjoining figure, the circles with centres P and Q touch each other at R A line passing through R meets the circles at A and B respectively.
Prove that –



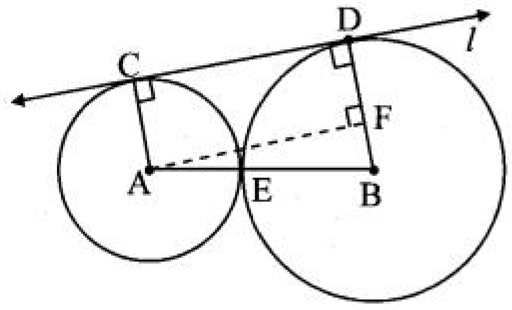
i. seg AP || seg BQ,
ii. ΔAPR ~ ΔRQB, and
iii. Find ∠RQB if ∠PAR = 35°.
Solution:
The circles with centres P and Q touch each other at R.
∴ By theorem of touching circles,
P – R – Q
i. In ΔPAR,
seg PA = seg PR [Radii of the same circle]
∴ ∠PRA ≅ ∠PAR (i) [Isosceles triangle theorem]
Similarly, in ΔQBR,
seg QR = seg QB [Radii of the same circle]
∴ ∠RBQ ≅ ∠QRB (ii) [Isosceles triangle theorem]
But, ∠PRA ≅ ∠QRB (iii) [Vertically opposite angles]
∴ ∠PAR ≅ ∠RBQ (iv) [From (i) and (ii)]
But, they are a pair of alternate angles formed by transversal AB on seg AP and seg BQ.
∴ seg AP || seg BQ [Alternate angles test]
ii. In ΔAPR and ΔRQB,
∠PAR ≅ ∠QRB [From (i) and (iii)]

∠APR ≅ ∠RQB [Alternate angles]
∴ ΔAPR – ΔRQB [AA test of similarity]
iii. ∠PAR = 35° [Given]
∴ ∠RBQ = ∠PAR= 35° [From (iv)]
In ΔRQB,
∠RQB + ∠RBQ + ∠QRB = 180° [Sum of the measures of angles of a triangle is 180°]
∴ ∠RQB + ∠RBQ + ∠RBQ = 180° [From (ii)]
∴ ∠RQB + 2 ∠RBQ = 180°
∴ ∠RQB + 2 × 35° = 180°
∴ ∠RQB + 70° = 180°
∴ ∠RQB = 110°

Question 5.
In the adjoining figure, the circles with centres A and B touch each other at E. Line l is a common tangent which touches the circles at C and D respectively. Find the length of seg CD if the radii of the circles are 4 cm, 6 cm.



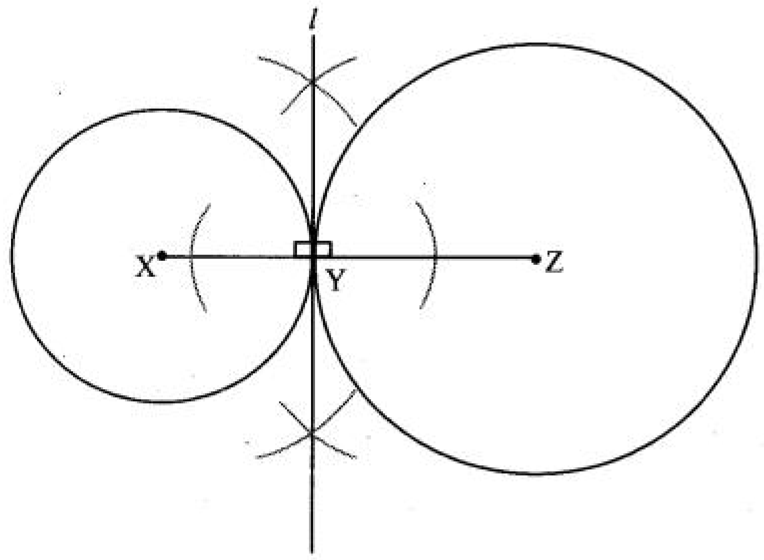
Construction : Draw seg AF ⊥ seg BD.
Solution:
i. The circles with centres A and B touch each other at E. [Given]
∴ By theorem of touching circles,
A – E – B



∴ ∠ACD = ∠BDC = 90° [Tangent theorem]
∠AFD = 90° [Construction]
∴ ∠CAF = 90° [Remaining angle of □ AFDC]
∴ □ AFDC is a rectangle. [Each angle is of measure 900]
∴ AC = DF = 4 cm [Opposite sides of a rectangle]
Now, BD = BF + DF [B – F – C]
∴ 6 = BF + 4 BF = 2 cm
Also, AB = AE + EB
= 4 + 6 = 10 cm
[The distance between the centres of circles touching externally is equal to the sum of their radii]

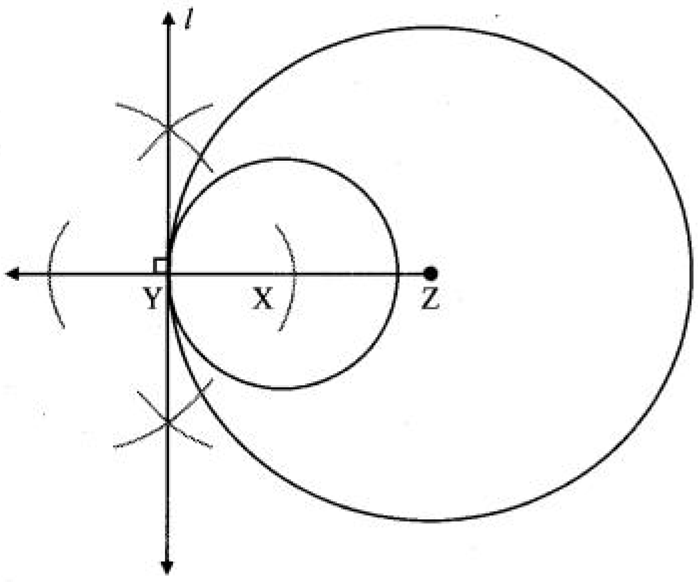
ii. Now, in ΔAFB, ∠AFB = 90° [Construction]
∴ $AB^2 = AF^2 + BF^2$ [Pythagoras theorem]
∴ $10^2 = AF^2 + 2^2$
∴ $100 = AF^2 + 4$
∴ $AF^2 = 96$
∴ AF = $\sqrt{96}$ [Taking square root of both sides]
= $\sqrt{16 \times 6}$
= $4\sqrt{6}$ cm
But, CD = AF [Opposite sides of a rectangle]
∴ CD = $4\sqrt{6}$ cm

Question 1.
Take three collinear points X – Y – Z as shown in figure.
Draw a circle with centre X and radius XY. Draw another circle with centre Z and radius YZ.
Note that both the circles intersect each other at the single point Y. Draw a line / through point Y and perpendicular to seg XZ. What is line l
(Textbook pg. no. 56)
Solution:



Line l is a common tangent of the two circles.

Question 2.
Take points Y – X – Z as shown in the figure. Draw a circle with centre Z and radius ZY.
Also draw a circle with centre X and radius XY. Note that both the circles intersect each other at the point Y.
Draw a line l perpendicular to seg YZ through point Y. What is line l? (Textbook pg. no. 56)
Solution:

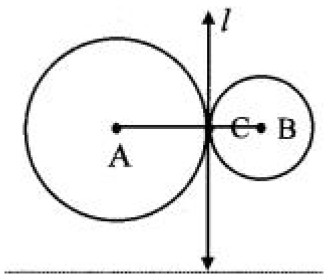


Line l is a common tangent of the two circles.

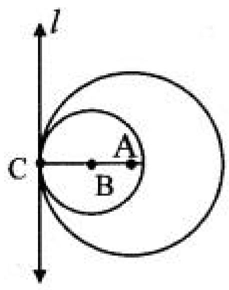
If two circles in the same plane intersect with a line in the plane in only one point, they are said to be touching circles and the line is their common tangent.

The point common to the circles and the line is called their common point of contact.

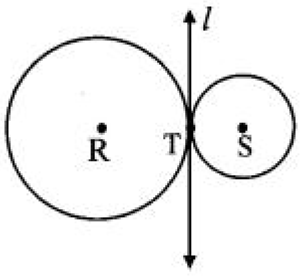
1. Circles touching externally:
For circles touching externally, the distance between their centres is equal to sum of their radii, i.e. AB = AC + BC



2. Circles touching internally:
For circles touching internally, the distance between their centres is equal to difference of their radii,
i. e. AB = AC – BC

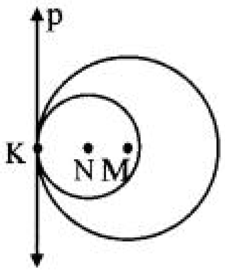


Question 3.
The circles shown in the given figure are called externally touching circles. Why? (iexthook pg. no. 57)



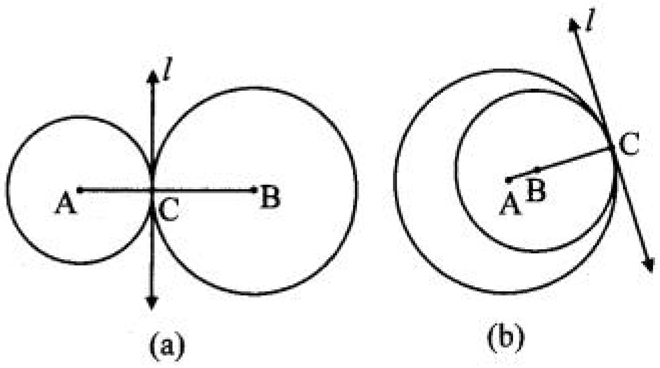
Answer:
Circles with centres R and S lie in the same plane and intersect with a line l in the plane in one and only one point T [R – T – S].
Hence the given circles are externally touching circles.

Question 4.
The circles shown in the given figure are called internally touching circles, why? (Textbook pg. no. 57)



Answer:
Circles with centres N and M lie in the same plane and intersect with a line p in the plane in one and only one point T [K – N – M].
Hence, the given circles are internally touching circles.

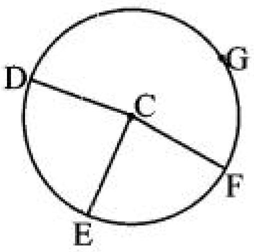
Question 5.
In the given figure, the radii of the circles with centres A and B are 3 cm and 4 cm respectively. Find
i. d(A,B) in figure (a)
ii. d(A,B) in figure (b) (Textbook pg. no. 57)



Solution:
i. Here, circle with centres A and B touch each other externally at point C.
∴ d(A, B) = d(A, C) + d(B ,C)
= 3 + 4
∴ d(A,B) = 7 cm
[The distance between the centres of circles touching externally is equal to the sum of their radii]
ii. Here, circle with centres A and 13 touch each other internally at point C.
∴ d(A, B) = d(A, C) – d(B, C)
= 4 – 3
∴ d(A,B) = 1 cm
[The distance between the centres of circles touching internally is equal to the difference in their radii]

Practice Set 3.3 Geometry 10th Std Maths Part 2 Answers Chapter 3 Circle

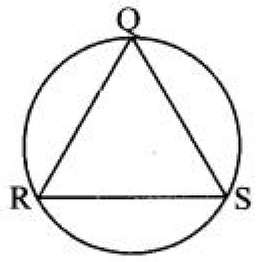
Question 1.
In the adjoining figure, points G, D, E, F are concyclic points of a circle with centre C.
∠ECF = 70°, m(arc DGF) = 200°. Find m(arc DE) and m(arc DEF).



Solution:

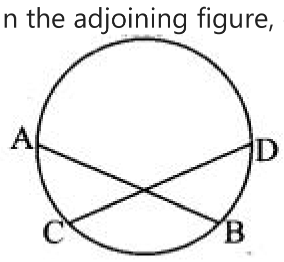
m(arc EF) = m∠ECF [Definition of measure of minor arc]
∴ m(arc EF) = 70°
i. m(arc DE) + m(arc DGF)
+ m(arc EF) = 360° [Measure of a circle is 360°]
∴ m(arc DE) = 360° – m(arc DGF) – m(arc EF)
= 360° – 200° – 70°
∴ m(arc DE) = 90°
ii. m(arc DEF) = m(arc DE) + m(arc EF) [Arc addition property]
= 90° + 70°
∴ m(arc DEF) = 160°

Question 2.
In the adjoining figure, AQRS is an equilateral triangle. Prove that,
i. arc RS ≅ arc QS ≅ arc QR
ii. m(arc QRS) = 240°.



Solution:
Proof:
i. ΔQRS is an equilateral triangle, [Given]
∴ seg RS ≅ seg QS ≅ seg QR [Sides of an equilateral triangle]
∴ arc RS ≅ arc QS ≅ arc QR [Corresponding arcs of congruents chords of a circle are congruent]
ii. Let m(arc RS) = m(arc QS)= m(arc QR) = x
m(arc RS) + m(arc QS) + m(arc QR) = 360° [Measure of a circle is 360°, arc addition property]
∴ x + x + x = 360°
∴ 3x = 360°
∴ x = $360° ÷ 3 = 120°$
∴ m(arc RS) = m(arc QS) = m(arc QR) = 120° (i)
Now, m(arc QRS) = m(arc QR) + m(arc RS) [Arc addition property]
= 120° + 120° [From (i)]
∴ m(arc QRS) = 240°

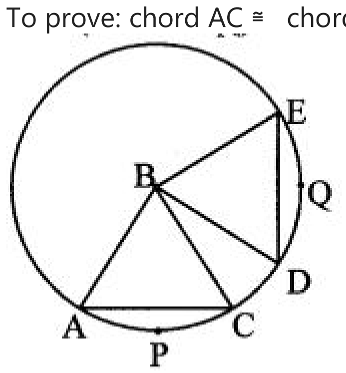
Question 3.
In the adjoining figure, chord AB ≅ chord CD. Prove that, arc AC = arc BD.



Solution:
Proof:
chord AB ≅ chord CD [Given]
∴ arc AB ≅ arc CD [Corresponding arcs of congruents chords of a circle are congruent]
∴ m(arc AB) = m(arc CD)
∴ m(arc AC) + m(arc BC) = m(arc BC) + m(arc BD) [Arc addition property]
∴ m(arc AC) = m(arc BD)
∴ arc AC ≅ arc BD

Maharashtra Board Class 10 Maths Chapter 3 Circle Intext Questions and Activities

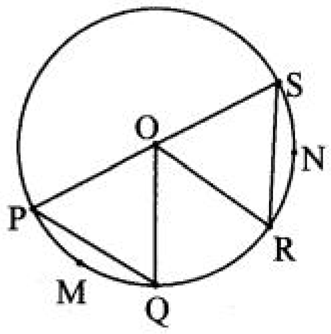
Question 1.
Theorem : The chords corresponding to congruent arcs of a circle (or congruent circles) are congruent. (Textbook pg. no. 61)
Given: B is the centre of circle.
arc APC ≅ arc DQE
To prove: chord AC ≅ chord DE



Proof:
[m(arc APC) = ∠ABC (i) [Definition of measure of
m(arc DQE) = ∠DBE] (ii) minor arc]

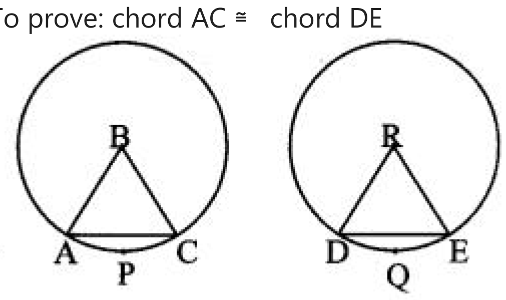
arc APC ≅ arc ∠DQE (iii) [Given]
∴ ∠ABC ≅ ∠DBE [From (i), (ii) and (iii)]
In ΔABC and ΔDBE,
side AB ≅ side DB [Radil of the same circle]
side [CB] side [EB] [Radii of the same circle]
∠ABC ≅ ∠DBE [From (iii), Measures of congruent arcs]
∴ ΔABC ≅ ΔDBE [SAS test of congruency]
∴ chord AC ≅ chord DE [c.s.c.t]

Question 2.
Theorem: Corresponding arcs of congruent chords of a circle (or congruent circles) are congruent (Textbook pg. no. 61)
Given: O is the centre of circle, chord PQ = chord RS
To prove: arc PMQ = arc RNS



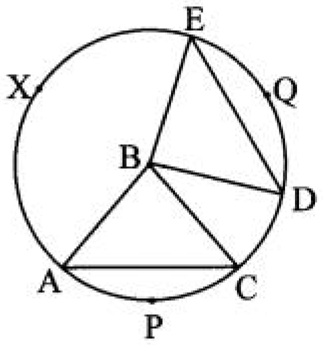
Proof:
In ΔPOQ and ΔROS,
[side PO ≅ side RO
side OQ ≅ side OS] [Radii of the same circle]
chord PQ ≅ chord RS [Given]
∴ ΔPOQ ≅ ΔROS [SSS test of congruency]
∴ ∠POQ ≅ ∠ROS (i) [c.a.c.t.]
m(arc PMQ) = ∠POQ (ii)
m(arc RNS) = ∠ROS (iii) [Definition of measure of minor arc]
∴ arc PMQ ≅ arc RNS [From (i), (ii) and (iii)]

Question 3.
Prove the two theorems on textbook pg.no.61 for congruent circles. (Textbook pg. no. 62)
Theorem : The chords corresponding to congruent arcs of congruent circles are congruent
Given: In congruent circles with centres B and R,
arc APC ≅ arc DQE
To prove: chord AC ≅ chord DE



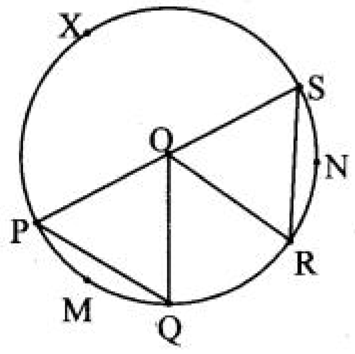
Proof:
[m(arc APC) = ∠ABC (i)
m(arc DQE) = ∠DRE] (ii) [Definition of measure of minor arc]
arc APC ≅ arc DQE (iii) [Given]
∴ ∠ABC = ∠DRE (iv) [From (i), (ii) and (iii)]
In ΔABC and ΔDRE,
[side AB ≅ side DR [Radii of congruent circles]
side CB ≅ side ER] [From (iv)]
∠ABC ≅ ∠DRE
∴ ΔABC ≅ ΔDRE [SAS test of congruency]

Question 4.
While proving the first theorem of the two, we assume that the minor arc APC and minor arc DQE are congruent. Can you prove the same theorem by assuming that corresponding major arcs congruent? (Textbook pg. no. 62)
Statement:
The chords corresponding to congruent major arcs of a circle are congruent.
Given: B is the centre of circle.
arc AXC ≅ arc DXE
To prove: chord AC ≅ chord DE



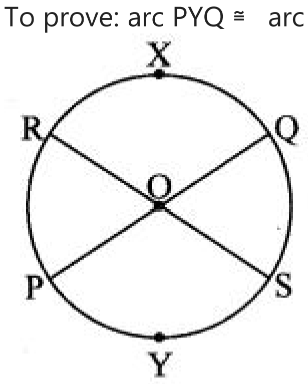
Proof:
m(major arc) = 360° – m(minor arc)
∴ m(arc AXC) = 360° – m(arc APC) (i)
m(arc DXE) = 360° – m(arc DQE) (ii)
m(arc AXC) = m(arc DXE) (iii) [Given]
∴ 360° – m(arc APC) = 360°- m(arc DQE) [From (i), (ii) and (iii)]
∴ m(arc APC) = m(arc DQE) (iv)
∴ m(arc APC) = ∠ABC (v) [Definition of measure of minor arc]
m(arc DQE) = ∠DBE (vi)
∴ ∠ABC = ∠DBE (vii) [From (iv), (v) and (vi)]
In ΔABC and ΔDBE,
[side AB ≅ side DB
Side CB ≅ side EB] [Radii of the same circle]
∠ABC ≅ ∠DBE [From (vii)]
∴ ΔABC ≅ ΔDBE [SAS test of congruency]
∴ chord AC ≅ chord DE [c.s.c.t.]

Question 5.
i. In the second theorem, are the major arcs corresponding to congruent chords congruent?
ii. Is the theorem true, when the chord PQ and chord RS are diameters of the circle? (Textbook pg. no. 62)
Solution:
i. Yes, the major arcs corresponding to congruent chords are congruent.



Proof:
In ΔPOQ and ΔROS,
seg OP ≅ seg OR [Radii of the same circle]
seg OQ ≅ seg OS [Radii of the same circle]
seg PQ ≅ seg RS [Given]
∴ ΔPOQ ≅ ΔROS [SSS test of congruence]
∴ ∠POQ ≅ ∠SOR (i) [c.a.c.t]
[m(arc PMQ) = ∠POQ (ii)
m(arc RNS) = ∠SOR] (iii) [Definition of measure of minor arc]
∴ m(arc PMQ) = m(arc RNS)
m(minor arc) = 360° – m(major arc) (iv) [From (i), (ii) and (iii)]
m(arc PMQ) = 360° – m(arc PXQ) (v)
and m(arc RNS) = 360° – m(arc RXS) (vi)
∴ 360°- m(arc PXQ) = 360°- m(arc RXS) [From (iv), (v) and (vi)]
∴ m(arc PXQ) = m(arc RXS)

ii. Yes, the major arcs corresponding to congruent chords (diameters) are congruent.
Given: O is the centre of circle.
seg PQ and seg RS are the diameters.
To prove: arc PYQ ≅ arc RYS

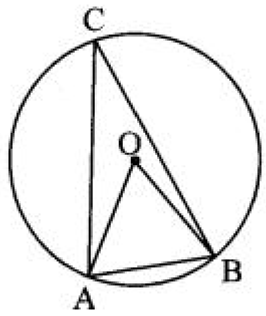


Proof:
seg PQ and seg RS are the diameters of the same circle. [Given]
∴ arc PYQ and arc RYS are semicircular arcs.

∴ m(arc PYQ) = m(arc RYS) = 180° [Measure of a semicircular arc is 180°]
∴ arc PYQ ≅ arc RYS

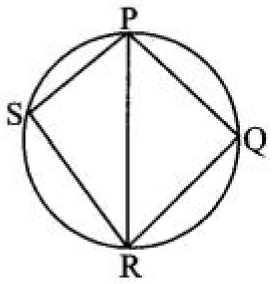
Practice Set 3.4 Geometry 10th Std Maths Part 2 Answers Chapter 3 Circle

Question 1.
In the adjoining figure, in a circle with centre O, length of chord AB is equal to the radius of the circle. Find measure of each of the following.
i. ∠AOB
ii. ∠ACB
iii. arc AB
iv. arc ACB.



Solution:
i. seg OA = seg OB = radius…… (i) [Radii of the same circle]
seg AB = radius…… (ii) [Given]
∴ seg OA = seg OB = seg AB [From (i) and (ii)]
∴ ΔOAB is an equilateral triangle.
∴ m∠AOB = 60° [Angle of an equilateral triangle]
ii. m ∠ACB = 12 m ∠AOB [Measure of an angle subtended by an arc at a point on the circle is half of the measure of the angle subtended by the arc at the centre]
= 12 × 60°
∴ m ∠ACB = 30°
iii. m(arc AB) = m ∠AOB [Definition of measure of minor arc]
∴ m(arc AB) = 60°
iv. m(arc ACB) + m(arc AB) = 360° [Measure of a circle is 360°]
∴ m(arc ACB) = 360° – m(arc AB)
= 360° – 60°
∴ m(arc ACB) = 300°

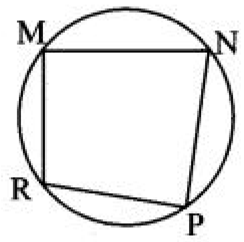
Question 2.
In the adjoining figure, ⌋ PQRS is cyclic, side PQ ≅ side RQ, ∠PSR = 110°. Find
i. measure of ∠PQR
ii. m (arc PQR)
iii. m (arc QR)
iv. measure of ∠PRQ



Solution:
i. ⌋ PQRS is a cyclic quadrilateral. [Given]
∴ ∠PSR + ∠PQR = 180° [Opposite angles of a cyclic quadrilateral are supplementary]
∴ 110° + ∠PQR = 180°
∴ ∠PQR = 180° – 110°
∴ m ∠PQR = 70°
ii. ∠PSR= 12 m (arcPQR) [Inscribed angle theorem]
110°= 12 m (arcPQR)
∴ m(arc PQR) = 220°
iii. In ΔPQR,
side PQ ≅ side RQ [Given]
∴ ∠PRQ = ∠QPR [Isosceles triangle theorem]
Let ∠PRQ = ∠QPR = x
Now, ∠PQR + ∠QPR + ∠PRQ = 180° [Sum of the measures of angles of a triangle is 180°]
∴ ∠PQR + x + x= 180°
∴ 70° + 2x = 180°

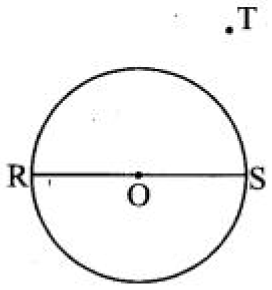
∴ 2x = 180° – 70°
∴ 2x = 110°
∴ x=110∘ 2=55∘
∴ ∠PRQ = ∠QPR = 55°….. (i)
But, ∠QPR = 12 m(arc QR) [Inscribed angle theorem]
∴ 55° = 12 m(arc QR)
∴ m(arc QR) = 110°
iv. ∠PRQ = ∠QPR =55° [From (i)]
∴ m ∠PRQ = 55°

Question 3.
□ MRPN is cyclic, ∠R = (5x -13)°, ∠N = (Ax + 4)°. Find measures of ∠R and ∠N.
Solution:
□ MRPN is a cyclic quadrilateral. [Given]
∴ ∠R + ∠N = 180° [Opposite angles of a cyclic quadrilateral are supplementary]

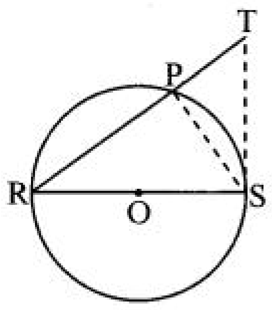


∴ 5x – 13 + 4x + 4 = 180
∴ 9x – 9 = 180
∴ 9x = 189
∴ x = 1899
∴ x = 21
∴ ∠R = 5x – 13
= 5 × 21 – 13
= 105 – 13
= 92°
∠N = 4x + 4
= 4 × 21 +4
= 84 +4
= 88°
∴ m∠R = 92° and m ∠N = 88°

Question 4.
In the adjoining figure, seg RS is a diameter of the circle with centre O. Point T lies in the exterior of the circle. Prove that ∠RTS is an acute angle.
Given: O is the centre of the circle, seg RS is the diameter of the circle.
To prove: ∠RTS is an acute angle.
Construction: Let seg RT intersect the circle at point P. Join PS and PT.



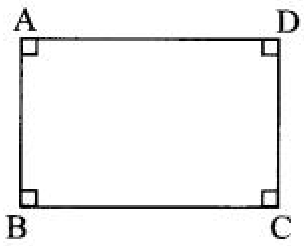
Proof:
seg RS is the diameter. [Given]
∴ ∠RPS = 90° [Angle inscribed in a semicircle]
Now, ∠RPS is the exterior angle of ΔPTS.
∴ ∠RPS > ∠PTS [Exterior angle is greater than the remote interior angles]



∴ 90° > ∠PTS
i.e. ∠PTS < 90°
i.e, ∠RTS < 90° [R – P -T]
∠RTS is an acute angle.

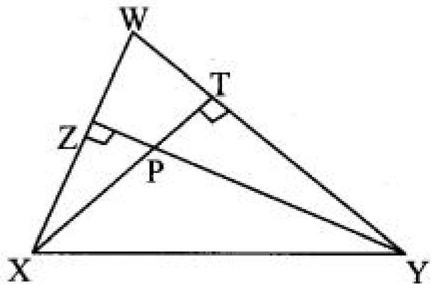
Question 5.
Prove that, any rectangle is a cyclic quadrilateral.
Given: ɿ ABCD is a rectangle.

To prove: □ ABCD is a cyclic quadrilateral.



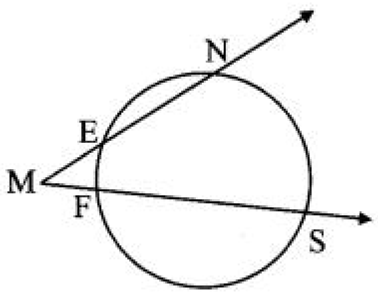
Proof:
□ XBCD is a rectangle. [Given]
∴ ∠A = ∠B = ∠C = ∠D = 90° [Angles of a rectangle]
Now, ∠A + ∠C = 90° + 90°
∴ ∠A + ∠C = 180°
∴ □ ABCD is a cyclic quadrilateral. [Converse of cyclic quadrilateral theorem]

Question 6.
In the adjoining figure, altitudes YZ and XT of ΔWXY intersect at P. Prove that,
i. □ WZPT is cyclic.
ii. Points X, Z, T, Y are concyclic.
Given: seg YZ ⊥ side XW
seg XT ⊥ side WY
To prove: i. □WZPT is cyclic.
ii. Points X, Z, T, Y are concyclic.



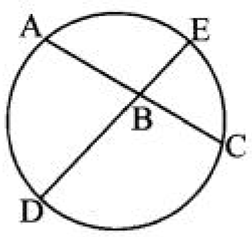
Proof:
i. segYZ ⊥ side XW [Given]
∴∠PZW = 90°...... (i)
seg XT I side WY [Given]
∴ ∠PTW = 90° ......(ii)
∠PZW + ∠PTW = 90° + 90° [Adding (i) and (ii)]
∴∠PZW + ∠PTW = 180°
∴□WZPT Ls a cyclic quadrilateral. [Converse of cyclic quadrilateral theorem]
ii. ∠XZY = ∠YTX = 90° [Given]
∴ Points X and Y on line XY subtend equal angles on the same side of line XY.
∴ Points X, Z, T and Y are concydic. [If two points on a given line subtend equal angles at two distinct points which lie on the same side of the line, then the four points are concyclic]

Question 7.
In the adjoining figure, m (arc NS) = 125°, m(arc EF) = 37°, find the measure of ∠NMS.
Solution:



Chords EN and FS intersect externally at point M.
m∠NMS = 12 [m (arc NS) – m(arc EF)]
= 12 (125° – 37°) = 12 × 88°
∴ m∠NMS = 44°

Question 8.
In the adjoining figure, chords AC and DE intersect at B. If ∠ABE = 108°, m(arc AE) = 95°, find m (arc DC).

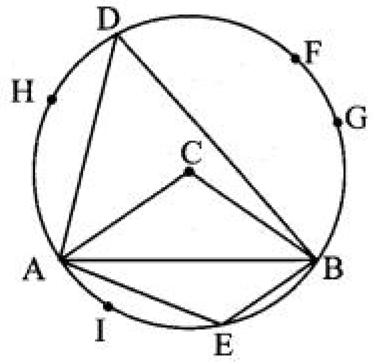


Solution:
Chords AC and DE intersect internally at point B.
∴ ∠ABE = 12 [m(arc AE) + m(arc DC)]
∴ 108° = 12 [95° + m(arc DC)]

∴ 108° × 2 = 95° + m(arc DC)
∴ 95° + m(arc DC) = 216°
∴ m(arc DC) = 216° – 95°
∴ m(arc DC) = 121°

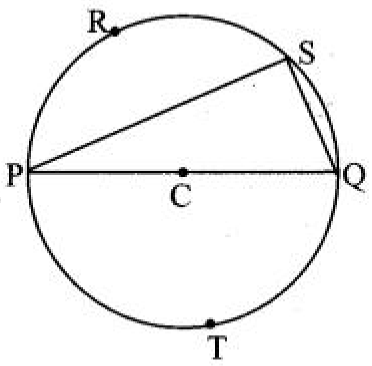
Maharashtra Board Class 10 Maths Chapter 3 Circle Intext Questions and Activities

Question 1.
Draw a sufficiently large circle of any radius as shown in the figure below. Draw a chord AB and central ∠ACB. Take any point D on the major arc and point E on the minor arc.



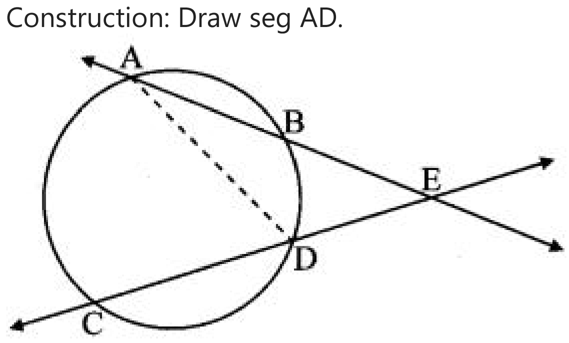
i. Measure ∠ADB and ∠ACB and compare the measures.
ii. Measure ∠ADB and ∠AEB. Add the measures.
iii. Take points F, G, H on the arc ADB. Measure ∠AFB, ∠AGB, ∠AHB. Compare these measures with each other as well as with measure of ∠ADB.
iv. Take any point I on the arc AEB. Measure ∠AIB and compare it with ∠AEB. (Textbook pg, no. 64)
Answer:
i. ∠ACB = 2 ∠ADB.
ii. ∠ADB + ∠AEB = 180°.
iii. ∠AHB = ∠ADB = ∠AFB = ∠AGB
iv. ∠AEB = ∠AIB

Question 2.
Draw a sufficiently large circle with centre C as shown in the figure. Draw any diameter PQ. Now take points R, S, T on both the semicircles. Measure ∠PRQ, ∠PSQ, ∠PTQ. What do you observe? (Textbook pg. no.65)



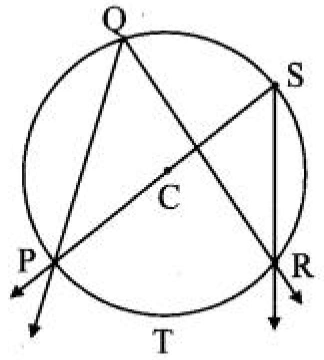
Answer:
∠PRQ = ∠PSQ = ∠PTQ = 90°
[Student should draw and verily the above answers.]

Question 3.
Prove that, if two lines containing chords of a circle intersect each other outside the circle, then the measure of angle between them is half the difference in measures of the arcs intercepted by the angle. (Textbook pg. no. 72)
Given: Chord AB and chord CD intersect at E in the exterior of the circle.
To prove: ∠AEC = 12 [m(arc AC) – m(arc BD)]
Construction: Draw seg AD.



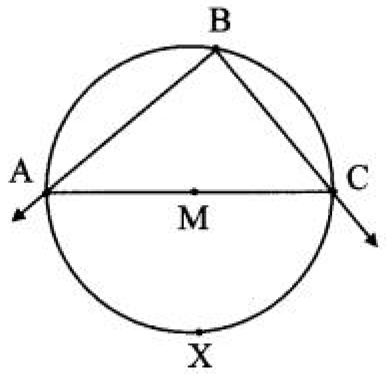
Proof:
∠ADC is the exterior angle of ΔADE.
∴ ∠ADC = ∠DAE + ∠AED [Remote interior angle theorem]
∴ ∠ADC = ∠DAE + ∠AEC [C – D – E]
∴ ∠AEC = ∠ADC – ∠DAE ……(i)
∠ADC = 12 m(arc AC) (ii) [Inscribed angle theorem]
∠DAE = 12 m(arc BD) (iii) [A – B – E, Inscribed angle theorem]
∴ ∠AEC = 12 m(arc AC) – 12 m (arc BD) [From (i), (ii) and (iii)]
∴ ∠AEC = 12 m(arc AC) – m (arc BD)

Question 4.
Angles inscribed in the same arc are congruent.
Write 'given' and 'to prove' with the help of the given figure.
Think of the answers of the following questions and write the proof.
i. Which arc is intercepted by ∠PQR ?
ii. Which arc is intercepted by ∠PSR ?
iii. What is the relation between an inscribed angle and the arc intercepted by it? (Textbook: pg. no. 68)



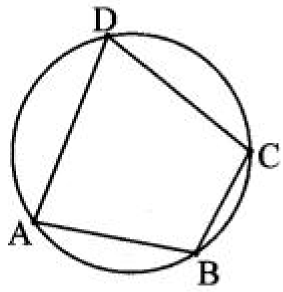
Given: C is the centre of circle. ∠PQR and ∠PSR are inscribed in same arc PTR.
To prove: ∠PQR ≅ ∠PSR
Proof:
i. arc PTR is intercepted by ∠PQR.
ii. arc PTR is intercepted by ∠PSR.
iii. ∠PQR = 12 m(arc PTR), and (i) [inscribed angle theorem]
∠PSR = 12 m(arcPTR) (ii) [Inscribed angle theorem]
∴ ∠PQR ≅ ∠PSR [From (i) and (ii)]

Question 5.
Angle inscribed in a semicircle is a right angle. With the help of given figure write 'given', 'to prove' and 'the proof. (Textbook pg. no. 68)



Given: M is the centre of circle. ∠ABC is inscribed in arc ABC.
Arcs ABC and AXC are semicircles.
To prove: ∠ABC = 90°
Proof:
∠ABC = 12 m(arc AXC) (i) [Inscribed angle theorem]
arc AXC is a semicircle.
∴ m(arc AXC) = 180° (ii) [Measure of semicircular arc is 1800]
∴ ∠ABC = 12 × 180°
∴ ∠ABC = 90° [From (i) and (ii)]

Question 6.
Theorem: Opposite angles of a cyclic quadrilateral are supplementry.
Fill in the blanks and complete the following proof. (Textbook pg. no. 68)
Given: □ ABCD is cyclic.
To prove: ∠B + ∠D = 180°
∠A + ∠C = 180°

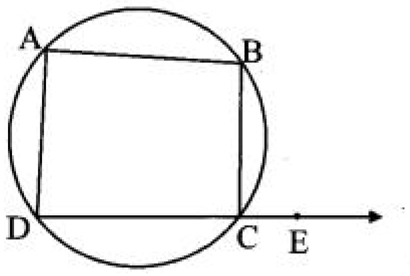


Proof:
arc ABC is intercepted by the inscribed angle ∠ADC.
∴ ∠ADC m(arcABC) (i) [Inscribed angle theorem]
Similarly, ∠ABC is an inscribed angle. It intercepts arc ADC.
∴ ABC = 12 m(arc ADC) (ii) [Inscribed angle theorem]
∴ ∠ADC + ∠ABC
= 12 m(arcABC) + 12 m(arc ADC) [Adding (i) and (ii)]
∴ ∠D + ∠B = 12 m(areABC) + m(arc ADC)]
∴ ∠B + ∠D = 12 × 360° [arc ABC and arc ADC constitute a complete circle]
= 180°

∴ ∠B + ∠D = 180°
Similarly we can prove,
∠A + ∠C = 180°

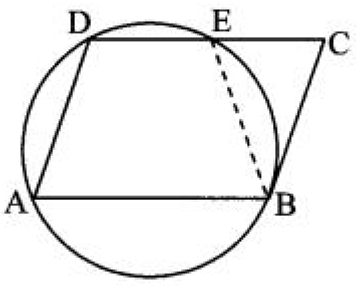
Question 7.
In the above theorem, after proving ∠B + ∠D = 180°, can you use another way to prove ∠A + ∠C = 180°? (Textbook pg. no. 69)
Proof:
Yes, we can prove ∠A + ∠C = 180° by another way.
∠B + ∠D = 180°
In ⌋ ABCD,
∠A + ∠B + ∠C + ∠D = 360° [Sum of the measures of all angles of a quadrilateral is 360°.]
∴ ∠A + ∠C + 180° = 360°
∴ ∠A + ∠C = 360° – 180°
∴ ∠A + ∠C = 180°

Question 8.
An exterior angle of a cyclic quadrilateral is congruent to the angle opposite to its adjacent interior angle. (Textbook pg. no. 69)
Given: ⌋ ABCD is a cyclic quadrilateral.
∠BCE is the exterior angle of ⌋ ABCD.
To prove: ∠BCE ≅ ∠BAD

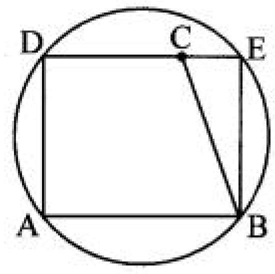


Proof:
∠BCE + ∠BCD = 180°...... (i) [Angles in a linear pair]
⌋ ABCD is a cyclic quadrilateral. [Given]
∠BAD + ∠BCD = 180°.......... (ii) [Opposite angles of a cyclic quadrilateral are supplementary]
∴ ∠BCE + ∠BCD = ∠BAD + ∠BCD [From (i) and (ii)]
∴ ∠BCE = ∠BAD

Question 9.
Theorem : If a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic. (Textbook pg. no. 69)
Given: In ⌋ ABCD, ∠A + ∠C = 180°
To prove: ⌋ ABCD is a cyclic quadrilateral.
Proof:
(Indirect method)
Suppose ⌋ ABCD is not a cyclic quadrilateral.
We can still draw a circle passing through three non collinear points A, B, D.



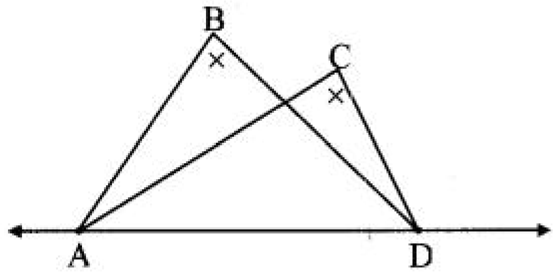
Case I: Point C lies outside the circle.
Then, take point E on the circle
such that D – E – C.
∴ ⌋ ABED is a cyclic quadrilateral.
∠DAB + ∠DEB = 180° (i) [Opposite angles of a cyclic quadrilateral are supplementary]
∠DAB + ∠DCB = 180° (ii) [Given]
∴ ∠DAB + ∠DEB = ∠DAB + ∠DCB [From (i) and (ii)]
∴ ∠DEB = ∠DCB



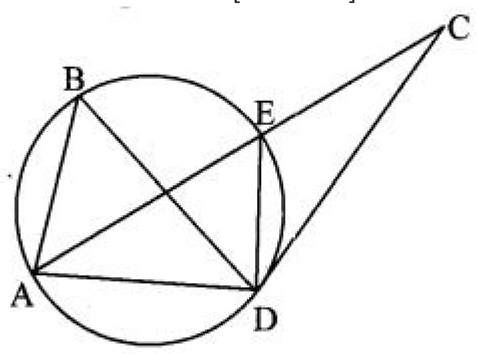
But, ∠DEB ≠ ∠DCB as ∠DEB is the exterior angle of ΔBEC.
∴ Our supposition is wrong.
∴ ⌋ ABCD is a cyclic quadrilateral.
Case II: Point C lies inside the circle.
Then, take point E on the circle such that
D – C – E
∴ □ABED is a cyclic quadrilateral.

∠DAB + ∠DEB = 180° (iii) [Opposite angles of a cyclic quadrilateral are supplementary]
∠DAB + ∠DCB = 180° (iv) [Given]
∴ ∠DAB + ∠DEB = ∠DAB + ∠DCB [From (iii) and (iv)]
∴ ∠DEB = ∠DCB
But ∠DEB ≠ ∠DCB as ∠DCB is the exterior angle of ΔBCE.
∴ Our supposition is wrong.
∴ □ABCD is a cyclic quadrilateral.

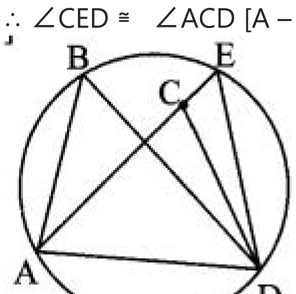
Question 10.
Theorem: If two points on a given line subtend equal angles at two distinct points which lie on the same side of the line, then the four points are concyclic. (Textbook pg. no. 70)
Given: Points B and C lie on the same side of the line AD.
∠ABD = ∠ACD
To prove: Points A, B, C, D are concyclic. i.e., □ABCD is a cyclic quadrilateral.



Proof:
Suppose points A, B, C, D are not concyclic points.
We can still draw a circle passing through three non collinear points A, B, D.
Case I: Point C lies outside the circle.
Then, take point E on the circle such that
A – E – C.
∠ABD ≅ ∠AED (i) [Angles inscribed in the same arc]
∠ABD ≅ ∠ACD (ii) [Given]
∴ ∠AED ≅ ∠ACD [From (i) and (ii)]
∴ ∠AED ≅ ∠ECD [A – E – C]



But, ∠AED ≅ ∠ECD as ∠AED is the exterior angle of ΔECD.
∴ Our supposition is wrong.
∴ Points A, B, C, D are concyclic points.
Case II: Point C lies inside the circle. Then, take point E on the circle such that A – C – E.
∠ABD ≅ ∠AED (iii) [Angles inscribed in the same arc]
∠ABD ≅ ∠ACD (iv) [Given]
∴ ∠AED ≅ ∠ACD [From (iii) and (iv)]
∴ ∠CED ≅ ∠ACD [A – C – E]

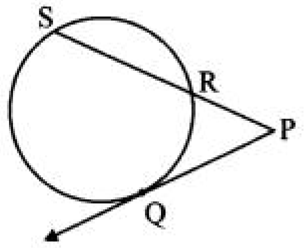


But, ∠CED ≅ ∠ACD as ∠ACD is the exterior angle of ΔECD.
∴ Our supposition is wrong.
∴ Points A, B, C, D are concyclic points.

Question 11.
The above theorem is converse of a certain theorem. State it. (Textbook pg. no. 70)
Answer:
If four points are concyclic, then the line joining any two points subtend equal angles at the other two points which are on the same side of that line.

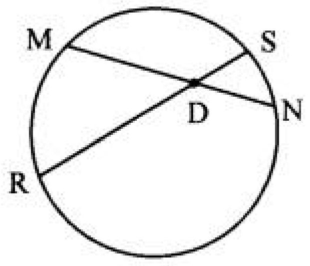
Practice Set 3.5 Geometry 10th Std Maths Part 2 Answers Chapter 3 Circle

Question 1.
In the adjoining figure, ray PQ touches the circle at point Q. PQ = 12, PR = 8, find PS and RS.
Solution:
i. Ray PQ is a tangent to the circle at point Q and seg PS is the secant. [Given]
∴ PR × PS = PQ^2 [Tangent secant segments theorem]



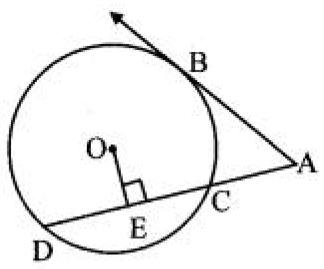
∴ 8 × PS = 12^2
∴ 8 × PS = 144
∴ PS = $\frac{144}{8}$
∴ PS = 18 units
ii. Now, PS = PR + RS [P – R – S]
∴ 18 = 8 + RS
∴ RS = 18 – 8
∴ RS = 10 units

Question 2.
In the adjoining figure, chord MN and chord RS intersect at point D.
i. If RD = 15, DS = 4, MD = 8 find DN
ii. If RS = 18, MD = 9, DN = 8 find DS



Solution:
i. Chords MN and RS intersect internally at point D. [Given]
∴ MD × DN = RD × DS [Theorem of internal division of chords]
∴ 8 × DN = 15 × 4
∴ DN = $\frac{15 \times 4}{8}$
∴ DN = 7.5 units
ii. Let the value of RD be x.
RS = RD + DS [R – D – S]
∴ 18 = x + DS
∴ DS = 18 – x
Now, MD × DN = RD × DS [Theorem of internal division of chords]
∴ 9 × 8 = x(18 – x)
∴ 72 = 18x – x^2
∴ x^2 – 18x + 72 = 0
∴ x^2 – 12x – 6x + 72 = 0
∴ x (x – 12) – 6 (x – 12) = 0
∴ (x – 12) (x – 6) = 0
∴ x – 12 = 0 or x – 6 = 0
∴ x = 12 or x = 6
∴ DS = 18 – 12 or DS = 18 – 6
∴ DS = 6 units or DS = 12 units

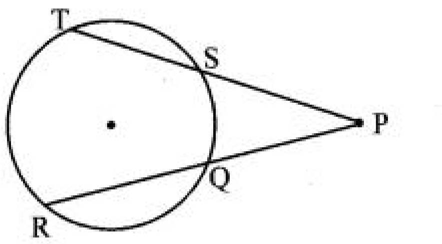
Question 3.
In the adjoining figure, O is the centre of the circle and B is a point of contact. Seg OE ⊥ seg AD, AB = 12, AC = 8, find
i. AD
ii. DC
iii. DE.



Solution:
i. Line AB is the tangent at point B and seg AD is the secant. [Given]
∴ AC × AD = AB^2 [Tangent secant segments theorem]

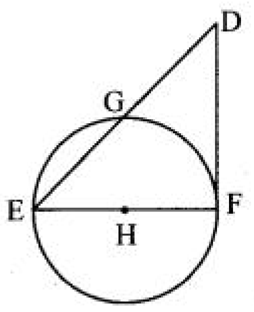
∴ 8 × AD = 12^2
∴ 8 × AD = 144
∴ AD = $\frac{144}{8}$
∴ AD = 18 units
ii. AD = AC + DC [A – C – D]
∴ 18 = 8 + DC
∴ DC = 18 – 8
∴ DC = 10 units
iii. seg OE ⊥ seg AD [Given]
i.e. seg OE ⊥ seg CD [A – C – D]
∴ DE = $\frac{1}{2}$ DC [Perpendicular drawn from the centre of the circle to the chord bisects the chord]
= $\frac{1}{2}$ × 10
∴ DE = 5 units

Question 4.
In the adjoining figure, if PQ = 6, QR = 10, PS = 8, find TS.
Solution:
PR = PQ + QR [P-Q-R]
∴ PR = 6 + 10 = 16 units

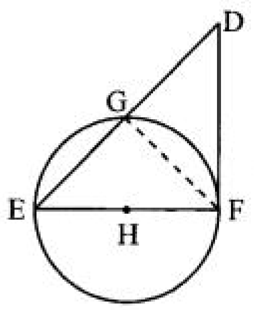


Chords TS and RQ intersect externally at point P.
PQ × PR = PS × PT [Theorem of external division of chords]
∴ 6 × 16 = 8 × PT
∴ PT = $\frac{6\times16}{8}$ = 12 units
But, PT = PS + TS [P – S – T]
∴ 12 = 8 + TS
∴ TS = 12 – 8
∴ TS = 4 units

Question 5.
In the adjoining figure, seg EF is a diameter and seg DF is a tangent segment. The radius of the circle is r. Prove that, DE × GE = $4r^2$.
Given: seg EF is the diameter.
seg DF is a tangent to the circle,
radius = r



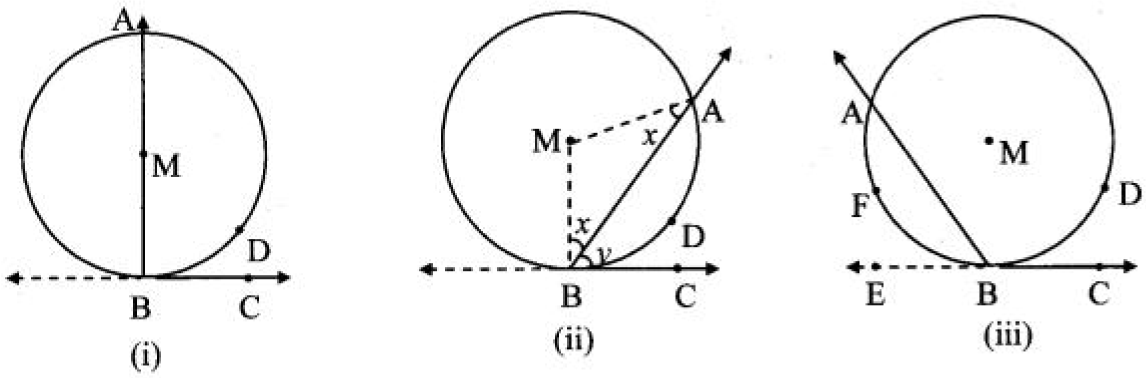
To prove: DE × GE = $4r^2$
Construction: Join seg GF.
Proof:
seg EF is the diameter. [Given]
∴ ∠EGF = 90° (i) [Angle inscribed in a semicircle]
seg DF is a tangent to the circle at F. [Given]



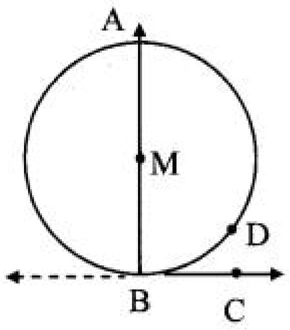
∴ ∠EFD = 90° (ii) [Tangent theorem]
In ΔDFE,
∠EFD = 90 ° [From (ii)]
seg FG ⊥ side DE [From (i)]
∴ ΔEFD ~ ΔEGF [Similarity of right angled triangles]
∴ $\frac{EF}{GE} = \frac{DE}{EF}$ [Corresponding sides of similar triangles]
∴ DE × GE = EF^2
∴ DE × GE = $(2r)^2$ [diameter = 2r]
∴ DE × GE = $4r^2$

Maharashtra Board Class 10 Maths Chapter 3 Circle Intext Questions and Activities

Question 1.
Theorem: If an angle has its vertex on the circle, its one side touches the circle and the other intersects the circle in one more point, then the measure of the angle is half the measure of its intercepted arc. (Textbook pg.no. 75 and 76)

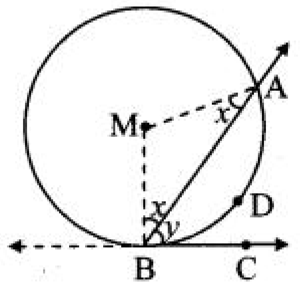


Given: ∠ABC is any angle, whose vertex B lies on the circle with centre M.
Line BC is tangent at B and line BA is secant intersecting the circle at point A.
Arc ADB is intercepted by ∠ABC.
To prove: ∠ABC = 12 m(arc ADB)
Proof:
Case I: Centre M lies on arm BA of ∠ABC.
∠MBC = 90° [Trangnet theorem]



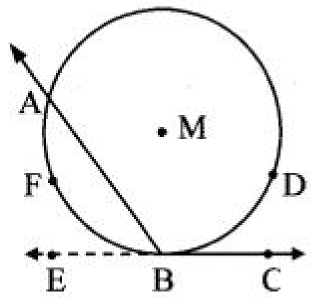
i.e. ∠ABC 90° (i) [A – M – B]
arc ADB is a semicircular arc.
∴ m(arc ADB) = 180° (ii) [Measure ofa semicircle is 180°]
∴ ∠ABC = 12 m(arc ADB) [(From (i) and (ii)]

Case II: Centre M lies in the exterior of ∠ABC.
Draw radii MA and MB.
∴ ∠MBA = ∠MAB [Isosceles triangle theorem]



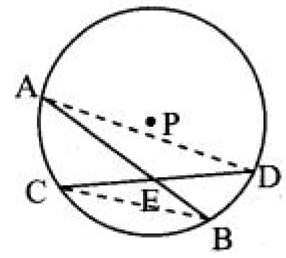
Let, ∠MHA = ∠MAB =x, ∠ABC = y In ΔABM,
∠AMB + ∠MBA + ∠MAB = 180° [Sum of the measures of all the angles of a triangle is 1800]
∴ ∠AMB + x + x = 180°
∴ ∠AMB = 180° – 2x …… (i)
Now, ∠MBC = ∠MBA + ∠ABC [Angle addition property]
∴ 90° = x + y [Tangent theorem]
∴ x = 90° – y ……(ii)
∠AMB = 180° – 2 (90° – y) [From (i) and (ii)]
∴ ∠AMB = 180° – 180° + 2y
∴ 2y = ∠AMB
∴ y = 12 ∠AMB
∴ ∠ABC = 12 ∠AMB
∴ ∠ABC = 12 m(arc ADB) [Definition of measure of minor arc]

Case III: Centre M lies in the interior of ∠ABC.
Ray BE is the opposite ray of ray BC.
Now, ∠ABE = 12 m (arc AFB) (i) [Proved in case II]



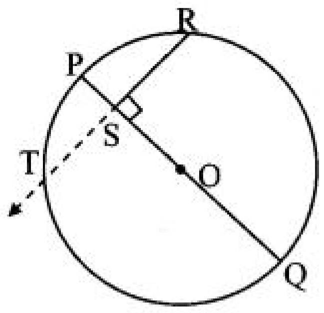
∠ABC + ∠ABE = 180° [Angles in a linear pair]
∴ 180 – ∠ABC = ∠ABE
∴ 180 – ∠ABC = 12 m(arc AFB) [From (i)]
= 12 [360 – m (arc ADB)]
∴ 180 – ∠ABC = 180 – 12 m(arc ADB)
∴ -∠ABC = – 12 m(arc ADB)
∴ ∠ABC = 12 m(arc ADB)

Question 2.
We have proved the above theorem by drawing seg AC and seg DB. Can the theorem be proved by drawing seg AD and seg CB, instead of seg AC and seg DB? (Textbook pg. no. 77)
Solution:
Yes, the theorem can be proved by drawing seg AD and seg CB.
Given: P is the centre of circle, chords AB and CD intersect internally at point E.
To prove: AE × EB = CE × ED
Construction: Draw seg AD and seg CB.



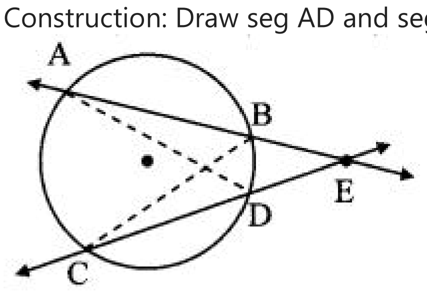
Proof:
In ΔCEB and ΔAED,
∠CEB = ∠DEA [Vertically opposite angles]
∠CBE = ∠ADE [Angles inscribed in the same arc]
∴ ΔCEB ~ ΔAED [by AA test of similarity]
∴ CEAE = EBED [Corresponding sides of similar triangles]
∴ AE × EB = CE × ED

Question 3.
In figure, seg PQ is a diameter of a circle with centre O. R is any point on the circle, seg RS ⊥ seg PQ. Prove that, SR is the geometric mean of PS and SQ. [That is, SR^2 = PS × SQ] (Textbook pg. no. 81)
Given: seg PQ is the diameter.
seg RS ⊥ seg PQ
To prove: SR^2 = PS × SQ
Construction: Extend ray RS, let it intersect the circle at point T.



Proof:
seg PQ ⊥ seg RS [Given]
∴ seg OS ⊥ chord RT [R – S – T, P – S – O]
∴ segSR = segTS (i) [Perpendicular drawn from the centre of the circle to the chord bisects the chord]
Chords PQ and RT intersect internally at point S.
∴ SR × TS = PS × SQ [Theorem of internal division of chords]
∴ SR × SR = PS × SQ [From (i)]
∴ SR^2 = PS × SQ

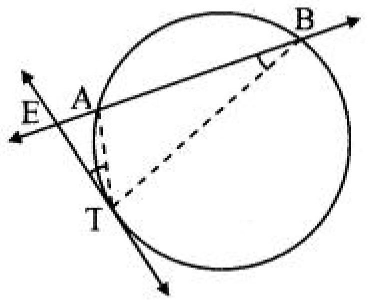
Question 4.
Theorem: If secants containing chords AB and CD of a circle intersect outside the circle in point E, then
AE × EB = CE × ED. (Textbook pg. no. 78)
Given: Chords AB and CD of a circle intersect outside the circle in point E.
To prove: AE × EB = CE × ED
Construction: Draw seg AD and seg BC.



Proof:
In ΔADE and ΔCBE,
∠AED = ∠CEB [Common angle]
∠DAE ≅ ∠BCE [Angles inscribed in the same arc]

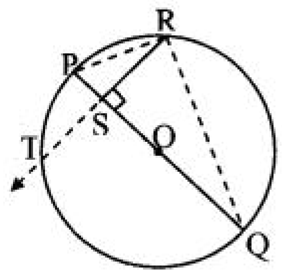
∴ ΔADE ~ ΔCBE [AA testof similaritv]
∴ $AECE = EDEB$ [Corresponding sides of similar triangles]
∴ AE × EB = CE × ED

Question 5.
Theorem: Point E is in the exterior of a circle. A secant through E intersects the circle at points A and B, and a tangent through E touches the circle at point T, then EA × EB = ET^2.
Given: Secant through point E intersects the circle in points A and B.
Tangent drawn through point E touches the circle in point T.
To prove: EA × EB = ET^2
Construction: Draw seg TA and seg TB.



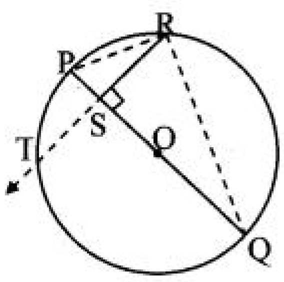
Proof:
In ΔEAT and ΔETB,
∠AET ≅ ∠TEB [Common angle]
∠ETA ≅ ∠EBT [Theorem of angle between tangent and secant, E – A – B]
∴ ΔEAT ~ ΔETB [AA test of similarity]
∴ $EAET = ETEB$ [Corresponding sides of similar triangles]
∴ EA × EB = ET^2

Question 6.
In the figure in the above example, if seg PR and seg RQ are drawn, what is the nature of ΔPRQ. (Textbook pg. no, 81)
Answer:
seg PQ is the diameter of the circle.
∴ ∠PRQ = 90°



∴ ΔPRQ is a right angled triangle. [Angle inscribed in a semicircle]

Question 7.
Have you previously proved the property proved in the above example? (Textbook pg. no. 81)
Answer:
Yes. It is the theorem of geometric mean.
ΔPSR ~ ΔRSQ [Similarity of right angled triangles]



∴ $PSSR = SRSQ$ [Corresponding sides of similar triangles]
∴ SR^2 = PS × SQ

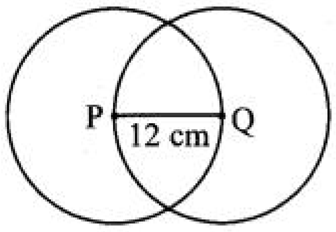
Problem Set 3 Geometry 10th Std Maths Part 2 Answers Chapter 3 Circle

Problem Set 3 Geometry Class 10 Question 1.
Four alternative answers for each of the following questions are given. Choose the correct alternative.

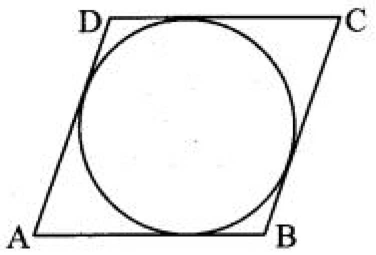
i. Two circles of radii 5.5 cm and 3.3 cm respectively touch each other. What is the distance between their centres?
(A) 4.4 cm
(B) 8.8 cm

(C) 2.2 cm
(D) 8.8 or 2.2 cm
Answer: (D)
Two circles can touch each other internally or externally.
∴ Distance between centres = 5.5 + 3.3 or 5.5 – 3.3 = 8.8 or 2.2

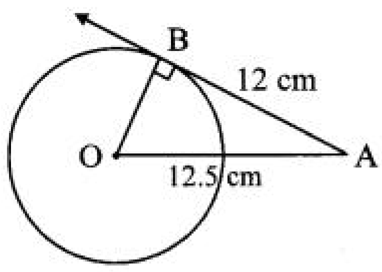
ii. Two circles intersect each other such that each circle passes through the centre of the other. If the distance between their centres is 12, what is the radius of each circle?
(A) 6 cm
(B) 12 cm
(C) 24 cm
(D) can't say
Answer: (B)
PQ is the radius = 12 cm



iii. A circle touches all sides of a parallelogram. So the parallelogram must be a _________.
(A) rectangle
(B) rhombus
(C) square
(D) trapezium
Answer: (B)
J ABCD is a rhombus.
Note: It cannot be square as the angles are not mentioned as 90°.



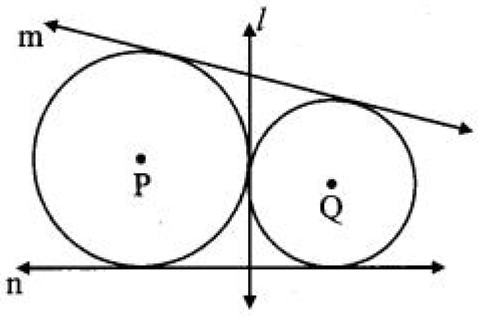
iv. Length of a tangent segment drawn from a point which is at a distance 12.5 cm from the centre of a circle is 12 cm, find the diameter of the circle.
(A) 25 cm
(B) 24 cm
(C) 7 cm
(D) 14 cm
Answer: (C)



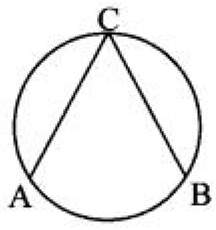
In ΔOAB, ∠B = 90° [Tangent theorem]
∴ $OA^2 = OB^2 + AB^2$ [Pythagoras theorem]
∴ $12.5^2 = OB^2 + 12^2$
∴ $OB^2 = 156.25 - 144$
∴ $OB = \sqrt{12.25} = 3.5$ cm
∴ Diameter = 2 × OB = 2 × 3.5 = 7 cm

v. If two circles are touching externally, how many common tangents of them can be drawn?
(A) One
(B) Two
(C) Three
(D) Four
Answer: (C)

line l, line m and line n are the tangents.

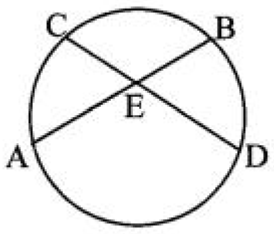


vi. ∠ACB is inscribed in arc ACB of a circle with centre O. If ∠ACB = 65°, find m(arc ACB).
(A) 65°
(B) 130°
(C) 295°
(D) 230°
Answer: (D)
m∠ACB = 12 m(arc AB) [Inscribed angle theorem]



∴ m(arc AB) = 2 m∠ACB =
= 2 × 65
= 130°
m(arc ACB) = 360° – m(arc AB) [Measure of a circle is 360°]
= 360° – 130°
= 230°

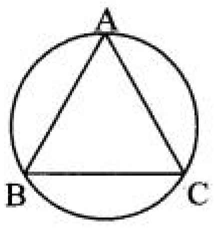
vii. Chords AB and CD of a circle intersect inside the circle at point E. If AE = 5.6, EB = 10, CE = 8, find ED.
(A) 7
(B) 8
(C) 11.2
(D) 9
Answer: (A)
Chords AB and CD intersect
internally at E.
AE × EB = CE × ED [Theorem of internal division of chords]



∴ 5.6 × 10 = 8 × ED
∴ ED = 7 units

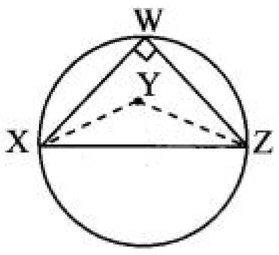
viii. In a cyclic ⌋ ABCD, twice the measure of ∠A is thrice the measure of ∠C. Find the measure of ∠C?
(A) 36°
(B) 72°
(C) 90°
(D) 108°
Answer: (B)
∠A + ∠C = 180° [Theorem of cyclic quadrilateral]
∴ 2∠A + 2∠C = 2 × 180° [Multiplying both sides by 2]
∴ 3∠C + 2∠C = 360° [∵ 2∠A = 3∠C]
∴ 5∠C = 360°
∴ ∠C = 72°

ix. Points A, B, C are on a circle, such that m(arc AB) = m(arc BC) = 120°. No point, except point B, is common to the arcs. Which is the type of ΔABC?
(A) Equilateral triangle
(B) Scalene triangle
(C) Right angled triangle
(D) Isosceles triangle
Answer: (A)
m(arc AB) + m(arc BC) + m(arc AC) = 360° [Measure of a circle is 360°]
∴ 120° + 120° + m (arc AC) = 360°



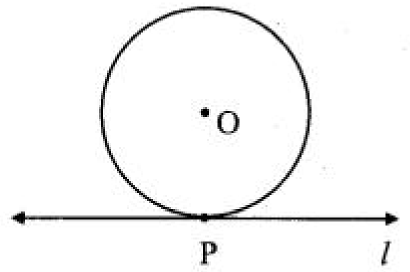
∴ m(arc AC) = 120°
∴ arc AB = arc BC = arc AC
∴ seg AB ≅ seg BC ≅ seg AC [Corresponding chords of congruents arcs of a circle are congruent]
∴ ΔABC is an equilateral triangle.

x. Seg XZ is a diameter of a circle. Point Y lies in its interior. How many of the following statements are true?
(i) It is not possible that ∠XYZ is an acute angle.
(ii) ∠XYZ can't be a right angle.
(iii) ∠XYZ is an obtuse angle.
(iv) Can't make a definite statement for measure of ∠XYZ.
(A) Only one
(B) Only two
(C) Only three
(D) All
Answer: (C)
x. seg XZ is the diameter.
∴ ∠XWZ is a right angle. [Angle inscribed in a semicircle]

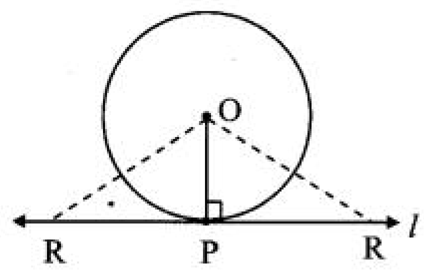


Since, Y lies in the interior of ΔXWZ,
∴ ∠XYZ > 90°
i.e., ∠XYZ is an obtuse angle.

Problem Set 3 Question 2.
Line l touches a circle with centre O at point P. If radius of the circle is 9 cm, answer the following.
i. What is d(O, P) = ? Why?
ii. If d(O, Q) = 8 cm, where does the point Q lie?
iii. If d(O, R) = 15 cm, how many locations of point R are on line l ?
At what distance will each of them be from point P?



Solution:
i. seg OP is the radius of the circle.
∴ d(0, P) = 9 cm
ii. Here, 8 cm < 9 cm
∴ d(0, Q) < d(0, P)
∴ d(0, Q) < radius
Point Q lies in the interior of the circle.
iii. There can be two locations of point R on line l.

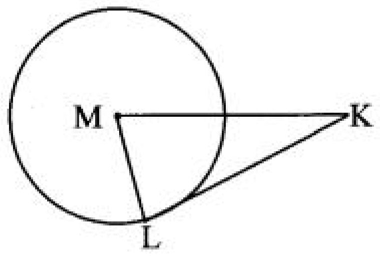


d(0, R) = 15 cm
Now, in ΔOPR, ∠OPR = 90° [Tangent theorem]
∴ $OR^2 = OP^2 + PR^2$ [Pythagoras theorem]
∴ $15^2 = 9^2 + PR^2$
∴ $225 = 81 + PR^2$
∴ $PR^2 = 225 - 81 = 144$ [Taking square root of both sides]
∴ PR = $\sqrt{144}$
= 12 cm

Standard 10th Geometry Problem Set 3 Question 3.

In the adjoining figure, M is the centre of the circle and seg KL is a tangent segment. If MK = 12, KL = $6\sqrt{3}$,then find

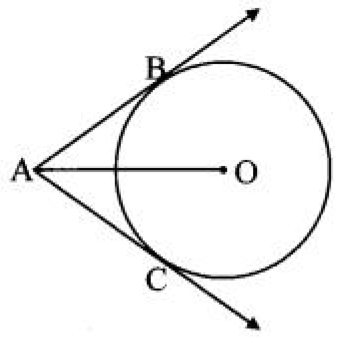
i. Radius of the circle.
ii. Measures of ∠K and ∠M.



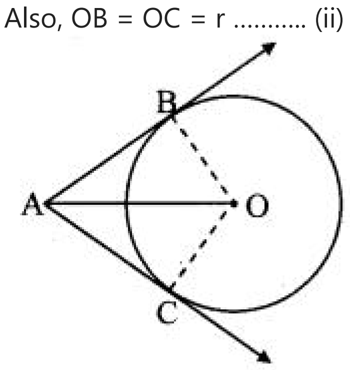
Solution:
i. Line KL is the tangent to the circle at point L and seg ML is the radius. [Given]
∴ ∠MLK = 90°............ (i) [Tangent theorem]
In ΔMLK, ∠MLK = 90°
∴ $MK^2 = ML^2 + KL^2$ [Pythagoras theorem]
∴ $12^2 = ML^2 + (6\sqrt{3})^2$
∴ $144 = ML^2 + 108$
∴ $ML^2 = 144 – 108$
∴ $ML^2 = 36$
∴ ML = $\sqrt{36}$ = 6 units. [Taking square root of both sides]
∴ Radius of the circle is 6 units.

ii. We know that,
ML = $\frac{1}{2}$ MK
∴ ∠K = 30° .... (ii) [Converse of 30° – 60° – 90° theorem]
In ΔMLK ,
∠L = 90° [From (i)]
∠K = 30° [From (ii)]
∴ ∠M = 60° [Remaining angle of ΔMLK]

10th Class Geometry Problem Set 3 Question 4.
In the adjoining figure, O is the centre of the circle. Seg AB, seg AC are tangent segments. Radius of the circle is r and l(AB) = r. Prove that, □ABOC is a square.

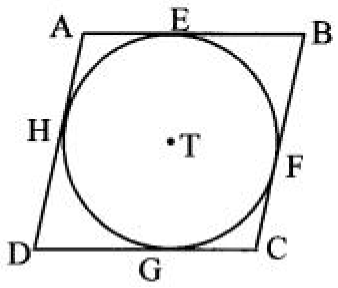


Given: O is the centre of circle.
seg AB and seg AC are the tangents, radius = r, /(AB) = r.
To prove: □ABOC is a square.
Construction: Draw seg OB and seg OC.
Proof:
seg AB and seg AC are the tangents to the circle. [Given]
∴ AB = AC [Tangent segment theorem]
But, AB = r [Given]
∴ AB = AC = r .... (i)
Also, OB = OC = r .... (ii) [Radii of the same circle]

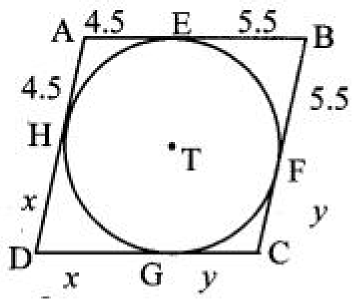


∴ AB = AC = OB = OC [From (i) and (ii)]
∴ □ABOC is a rhombus.
∠OBA = 90° [Tangent theorem]
∴ □ABOC is a square [A rhombus is a square, if one of its angles is a right angle]

Question 5.
In the adjoining figure, ⌡ ABCD is a parallelogram. It circumscribes the circle with centre T. Points E, F, G, H are touching points. If AE = 4.5, EB = 5.5, find AD.

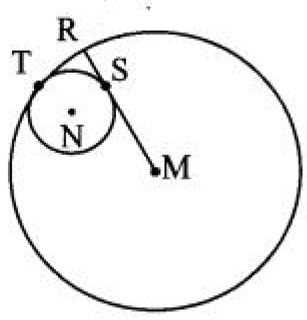


Solution:
Let the values of DH and CF be x and y respectively.
[AE = AH = 4.5
BE = BF = 5.5
DH = DG = x
CF = CG = y] [Tangent segment theorem]
□ABCD is a parallelogram. [Given]

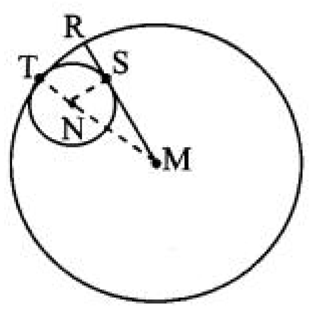


∴ AB = CD [Opposite sides of a parallelogram]
∴ AE + BE = DG + CG [A – E – B, D – G – C]
∴ 4.5 + 5.5 = x + y
∴ x + y = 10 .... (i)
Also, AD = BC [Opposite sides of a parallelogram]
∴ AH + DH = BF + CF [A – H – D, B – F – C]
∴ 4.5 + x = 5.5 + y
∴ x – y = 1 .... (ii)
Adding equations (i) and (ii), we get
2x = 11
∴ x = $\frac{11}{2}$ = 5.5
∴ AD = AH + DH [A – H – D]
= 4.5 + 5.5
∴ AD = 10 units

Question 6.
In the adjoining figure, circle with centre M touches the circle with centre N at point T. Radius RM touches the smaller circle at S. Radii of circles are 9 cm and 2.5 cm. Find the answers to the following questions, hence find the ratio MS : SR.



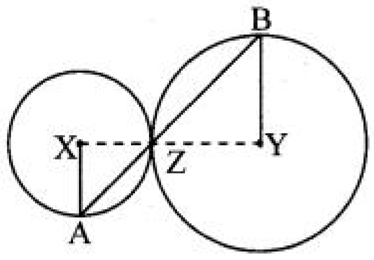
i. Find the length of segment MT.
ii. Find the length of seg MN.
iii. Find the measure of ∠NSM.
Solution:
i. MT = 9 cm [Radius of the bigger circle]
ii. MT = MN + NT [M – N – T]
∴ 9 = MN + 2.5
∴ MN = 9 – 2.5
∴ MN = 6.5 cm
iii. seg MR is a tangent to the smaller circle and NS is its radius.



∴ ∠NSM = 90° [Tangent theorem]
iv. In ΔNSM, ∠NSM = 90°
∴ $MN^2 = NS^2 + MS^2$ [Pythagoras theorem]
∴ $6.5^2 = 2.5^2 + MS^2$
∴ $MS^2 = 6.5^2 – 2.5^2$

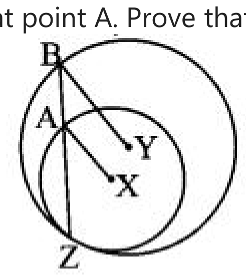
= (6.5 + 2.5) (6.5 – 2.5) [∵ $a^2 - b^2 = (a + b)(a - b)$]
= 9 × 4=36
∴ MS = $\sqrt{36}$ [Taking square root of both sides]
= 6 cm
But, MR = MS + SR [M – S – R]
∴ 9 = 6 + SR
∴ SR = 9 – 6
∴ SR = 3cm
Now, $\frac{MS}{SR} = \frac{6}{3} = \frac{2}{1}$
∴ $\frac{MS}{SR}$ = 2 : 1

10th Std Geometry Circle Problem Set Question 7.
In the adjoining figure, circles with centres X and Y touch each other at point Z. A secant passing through Z intersects the circles at points A and B respectively. Prove that, radius XA || radius YB. Fill in the blanks and complete the proof.

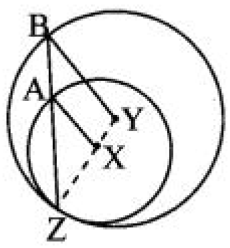


Given: X and Y are the centres of circle.
To prove: radius XA || radius YB
Construction: Draw segments XZ and YZ.
Proof:
By theorem of touching circles, points X, Z,
Y are collinear.
∴ ∠XZA ≅ ∠BZY [Vertically opposite angles]
Let ∠XZA = ∠BZY = a …………….. (i)
Now, seg XA seg XZ [Radii of the same circle]
∴ ∠XAZ ≅ ∠XZA = a …………….. (ii) [Isosceles triangle theorem]
Similarly, seg YB ≅ seg YZ [Radii of the same circie]
∴ ∠BZY = ∠ZBY = a …………….. (iii) [Isosceles triangle theorem]
∴ ∠XAZ = ∠ZBY [From (i), (ii) and (iii)]
∴ radius XA || radius YB [Alternate angles test]

Circle Problem Set 3 Question 8.
In the adjoining figure, circles with centres X and Y touch internally at point Z. Seg BZ is a chord of bigger circle and it intersects smaller circle at point A. Prove that, seg AX || seg BY.

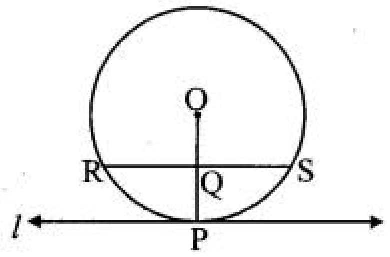


Given: X and Y are the centres of the circle.
To prove: seg AX || seg BY
Proof:
In ΔXAZ,
seg XA ≅ seg XZ [Radii of the same circle]
∴ ∠XZA ≅ ∠XAZ ………… (i) [Isosceles triangle theorem]
Also, in ΔYBZ,
seg YB ≅ seg YZ [Radii of the sanie circle]
∴ ∠YZB ≅ ∠YBZ [Isosceles triangle theorem]

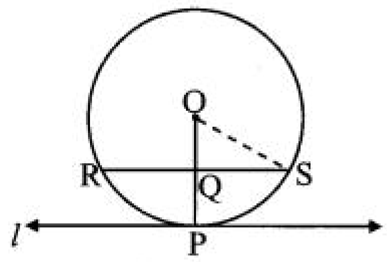


∴ ∠XZA ≅ ∠YBZ …………. (ii) [Y – X – Z,B – A – Z]
∴ ∠XAZ ≅ ∠YBZ [From (i) and (ii)]
∴ seg AX || seg BY [Corresponding angles test]

Question 9.
In the adjoining figure, line l touches the circle with centre O at point P, Q is the midpoint of radius OP. RS is a chord through Q such that chords RS || line l. If RS = 12, find the radius of the circle.

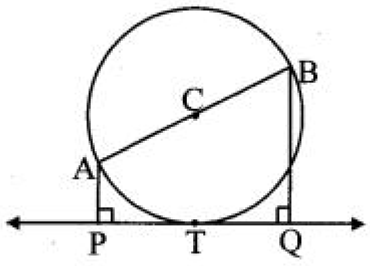


Solution:
Let the radius of the circle be r.
line l is the tangent to the circle and [Given]
seg OP is the radius.
∴ seg OP ⊥ line l [Tangent theorem]
chord RS || line l [Given]
∴ seg OP ⊥ chord RS
∴ QS = 12 RS [Perpendicular drawn from the centre of the circle to the chord bisects the chord]
= 12 × 12 = 6 cm
Also, OQ = 12 OP [Q is the midpoint of OP]
= 12 r
In ΔOQS, ∠OQS = 90° [seg OP ⊥ chord RS]

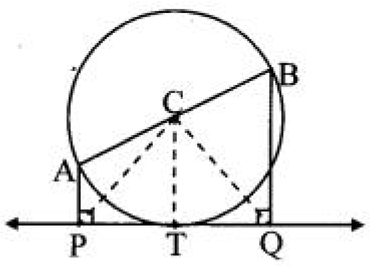


∴ OS2 = OQ2 + QS2 [Pythagoras theorem]
∴ r2 = (12r)2 + 62
∴ r2 = 14 r2 + 36
∴ r2 – 14 r2 = 36
∴ 34 r2 = 36
∴ r2 = 36×43
∴ r2 = 48
∴ r = 48−−√ [Taking square root of both sides]
= 4 3–√
∴ The radius of the given circle is 43–√ cm.

Question 10.
In the adjoining figure, seg AB is a diameter of a circle with centre C. Line PQ is a tangent, which touches the circle at point T. Seg AP ⊥ line PQ and seg BQ ⊥ line PQ. Prove that seg CP ≅ seg CQ.



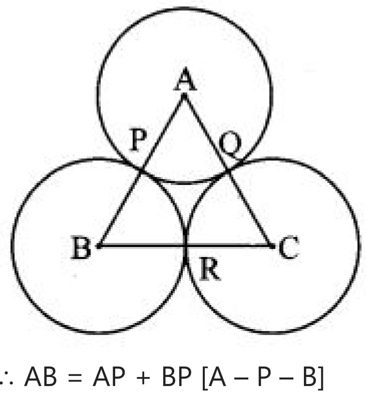
Given: C is the centre of circle.
seg AB is the diameter of circle.
line PQ is a tangent, seg AP ⊥ line PQ and seg BQ ⊥ line PQ.
To prove: seg CP ≅ seg CQ
Construction: Draw seg CT, seg CP and seg CQ.
Proof:
Line PQ is the tangent to the circle at point T. [Given]
∴ seg CT ⊥ line PQ (i) [Tangent theorem]
Also, seg AP ⊥ line PQ,
seg BQ ⊥ line PQ [Given]



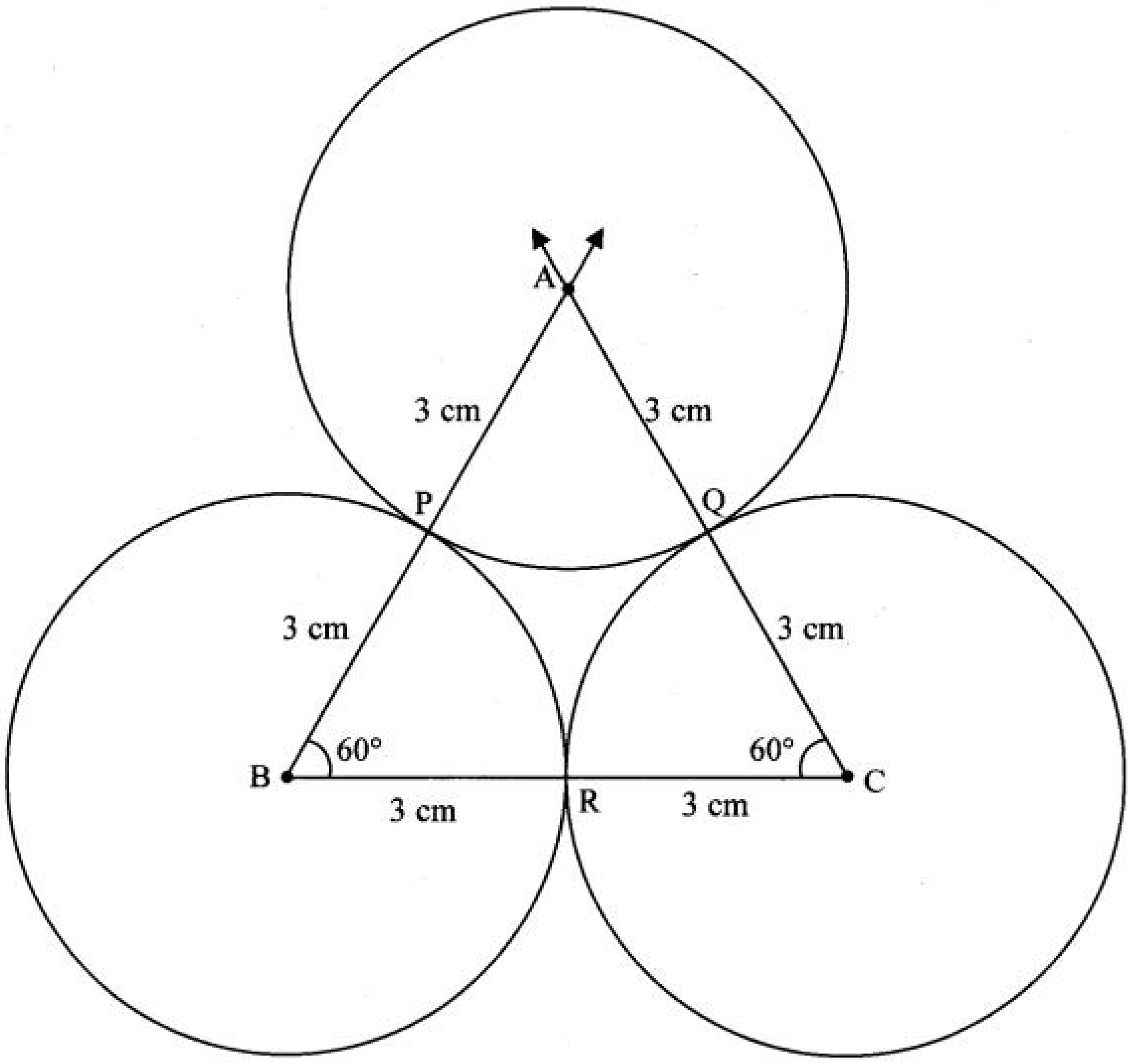
∴ seg AP || seg CT || seg BQ [Lines perpendicular to the same line are parallel to each other]
∴ ACCB = PTTQ [Property of intercepts made by three parallel lines and their transversals]
But, AC = CB [Radii of the same circle]
∴ ACAC = PTTQ
∴ PTTQ = 1

∴ PT = TQ .... (ii)
∴ In ΔCTP and ΔCTQ,
seg PT ≅ seg QT [From (ii)]
∠CTP ≅ ∠CTQ [From (i), each angle is of measure 90°]
seg CT ≅ seg CT [Common side]
∴ ΔCTP ≅ ΔCTQ [SAS test of congruence]
∴ seg CP ≅ seg CQ [c.s.c.t]

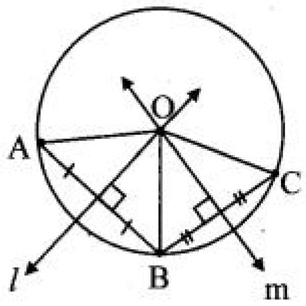
Question 11.
Draw circles with centres A, B and C each of radius 3 cm, such that each circle touches the other two circles.
Analysis:
Let the circles with centres A, B, C touch each other at points P, Q, R.
[∴ A – P – B
A – Q – C
B – R – C] [Theorem of touching circles]



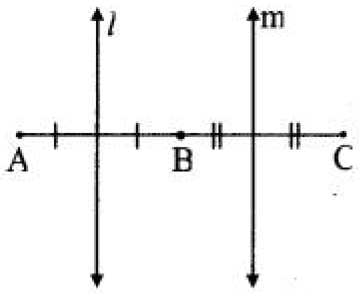
∴ AB = AP + BP [A – P – B]
∴ AB = 3 + 3 = 6 cm
Similarly, BC = 6 cm, AC = 6 cm
So, if we construct triangle ΔABC of side 6 cm each, then with A, B, C as the centres and radius 3 cm, the touching circles can be drawn.



Question 12.
Prove that any three points on a circle cannot be collinear
Given: A circle with centre O.
Points A, B and C lie on the circle.
To prove: Points A, B and C are not collinear.
Proof:
OA = OB [Radii of the same circle]

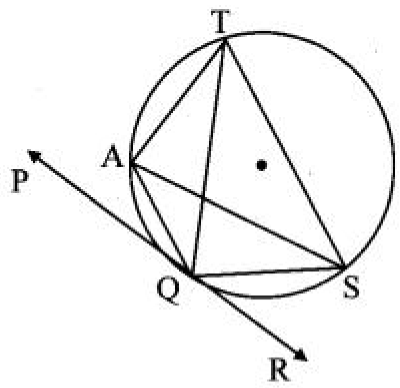


∴ Point O is equidistant from the endpoints A and B of seg AB.
∴ Point O lies on the perpendicular bisector of AB. [Perpendicular bisector theorem]
Similarly, we can prove that,
Point O lies on the perpendicular bisector of BC.



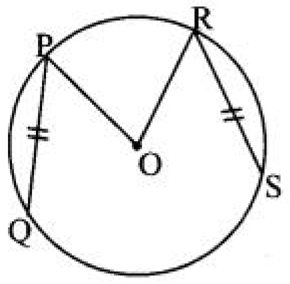
∴ Point O is the point of intersection of perpendicular bisectors of AB and BC (i.e., circumcentre of ΔABC) ……… (i)
Now, suppose that the points A, B, C are collinear.
Then, the perpendicular bisector of AB and BC will be parallel. [Perpendiculars to the same line are parallel]
∴ The perpendicular bisector do not intersect at O.
This contradicts statement (i) that the perpendicular bisectors intersect each other at O.
∴ Our supposition that A, B, C are collinear is false.
∴ Points A, B and C are non collinear points.

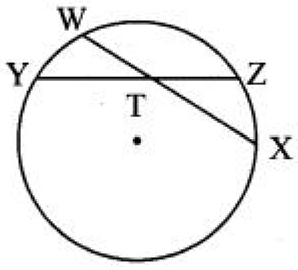
Question 13.
In the adjoining figure, line PR touches the circle at point Q. Answer the following questions with the help of the figure.
i. What is the sum of ∠TAQ and ∠TSQ?
ii. Find the angles which are congruent to ∠AQP.
iii. Which angles are congruent to ∠QTS?
iv. ∠TAS = 65°, find the measures of ∠TQS and arc TS.
v. If ∠AQP = 42° and ∠SQR = 58° find measure of ∠ATS.



Solution:
i. □ AQST is a cyclic quadrilateral. [Given]
∴ ∠TAQ + ∠TSQ = 180° [Opposite angles of a cyclic quadrilateral are supplementary]
ii. line PR is the tangent and seg AQ is the secant. [Given]
∴∠AQP = 12 m(arc AQ) [Theorem of angle between tangent and secant]
But,∠ASQ = 12 m(arc AQ) [Inscribed angle theorem]
∴∠AQP ≅ ∠ZASQ
Similarly, we can prove that,
∠AQP ≅ ∠ATQ
iii. ∠QTS = 12 m(arc QS) [Inscribed angle theorem]
But, ∠SQR = 12 m(arc QS) [Theorem of angle between tangent and secant]
∴ ∠QTS ≅ ∠SQR
Also, ∠QTS = ∠QAS [Angles inscribed in the same arc]
iv. ∠TQS = ∠TAS [Angles inscribed in the same arc]
∴ ∠TQS = 65°
Now, ∠TQS = 12 m(arc TS) [Inscribed angle theorem]
∴ 65°= 12 m(arcTS)
∴ m(arc TS) = 65° × 2
∴ m(arc TS) = 130°
v. ∠AQP + ∠AQS + ∠SQR = 180° [Angles in a linear pair]
∴ 42° + ∠AQS + 58° = 180°
∴∠AQS + 100° = 180° ………… (i)
But, □ AQST is a cyclic quadrilateral.
∴ ∠AQS + ∠ATS = 180° ………… (ii) [Theorem of cyclic quadrilateral]
∴ ∠ATS = 100° [From (i) and (ii)]

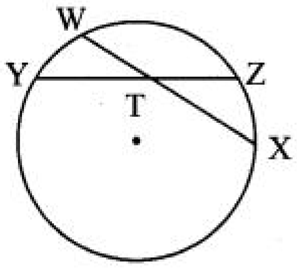
Question 14.
In the adjoining figure, O is the centre of a circle, chord PQ ≅ chord RS.
If ∠POR = 70° and (arc RS) = 80°, find
i. m (arc PR)
ii. m (arc QS)
iii. m (arc QSR).





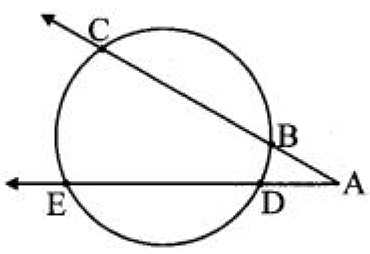
Solution:
i. m(arc PR) = m∠POR [Definition of measure of arc]
∴ m(arc PR) = 70°
ii. chord PQ chord RS [Given]
∴ m(arc PQ) = m(arc RS) = 80° [Corresponding arcs of congruents chords of a circle are congruent]
Now, m(arc QS) + m(arc PQ) + m(arc PR) + m(arcRS) = 360°
∴ m(arc QS) + 80° + 70° + 80° = 360° [Measure of a circle is 360°]
∴ m(arc QS) + 230° = 360°
∴ m(arc QS) = 130°
iii. m(arc QSR) = m(arc QS) + m(arc SR) [Arc addition property]
= 130° + 80°
∴ m(arc QSR) = 210°

Question 15.
In the adjoining figure, m(arc WY) = 44°, m(arc ZX) = 68°, then
i. Find the measure of ∠ZTX.
ii. If WT = 4.8, TX = 8.0, YT = 6.4, find TZ.
iii. If WX = 25, YT = 8, YZ = 26, find WT.



Solution:
i. Chords WX and YZ intersect internally at point T.
∴ ∠ZTX = 12 m(arc WY) + m(arc ZX)]
= 12 (44° + 68°)
= 12 × 112°
∴ m ∠ZTX = 56°
ii. WT × TX = YT × TZ [Theorem of internal division of chords]
∴ 4.8 × 8.0 = 6.4 × TZ
∴ TZ = 4.8×8.06.4
∴ l(TZ) = 6.0 units
iii. Let the value of WT be x. [W – T – X]
WT + TX = WX
∴ x + TX = 25
∴ TX = 25 – x
Also, YT + TZ = YZ [Y – T – Z]
∴ 8 + TZ = 26
∴ TZ = 26 – 8
= 18 units
But, WT × TX = YT × TZ [Theorem of internal division of chords]
∴ x × (25 – x) = 8 × 18
∴ 25x – x^2 = 144
∴ x^2 – 25x + 144 = 0
∴ (x – 16)(x – 9) = 0
∴ x = 16 or x = 9
∴ WT = 16 units or WT = 9units

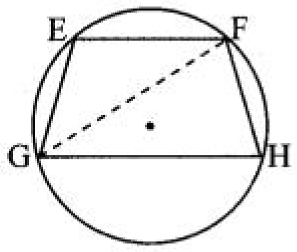
Question 16.
In the adjoining figure,
i. Mn (arc CE) = 54°, m (arc BD) = 23°, find measure of ∠CAE.
ii. If AB = 4.2,BC = 5.4, AE = 12.0, find AD.
iii. If AB = 3.6, AC = 9.0, AD = 5.4, find AE.



Solution:
i. Chords BC and ED intersect each other externally at point A.

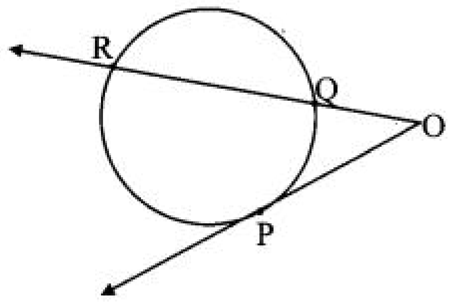
∴ ∠CAE = 12 [m(arc CE) – m(arc BD)]
= 12 (54° – 23°)
= 12 × 31°
∴ m∠CAE = 15.5°
ii. AC = AB + BC [A – B – C]
= 4.2 + 5.4
= 9.6 units
Now, AB × AC = AD × AE [Theorem of external division of chords]
∴ 4.2 × 9.6 = AD × 12.0
∴ AD = 4.2×9.612.0
∴ AD = 3.36 units
iii. AB × AC = AD × AE [Theorem of external division of chords]
∴ 3.6 × 9.0 = 5.4 × AE
∴ AE = 3.6×9.05.4
∴ AE = 6 units

Geometry Problem Set 3 Question 17.
In the adjoining figure, chord EF || chord GH. Prove that, chord EG ≅ chord FH.
Fill in the blanks and write the proof.
Given: chord EF || chord GH
To prove: chord EG = chord FH
Construction: Draw seg GF.

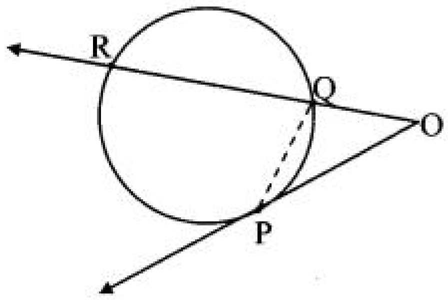


Proof:
∠EFG = ∠FGH (i) Alternate angles
∠EFG = 12 m (arcEG)] (ii) [Inscribed angle theorem]
∠FGH = 12m(arcFH) (iii) [Inscribed angle theorem]
∴ m(arcEG) = m (are FH) [From (i), (ii) and (iii)]
∴ chord EG ≅ chord FH The chords corresponding to congruent arcs of a circle are congruent

SSC Geometry Circle Chapter Solutions Pdf Question 18.
In the adjoining figure, P is the point of contact.
i. If m (arc PR) = 140°, L POR = 36°, find m (arc PQ)
ii. If OP = 7.2, OQ = 3.2, find OR and QR
iii. If OP = 7.2, OR = 16.2, find QR.



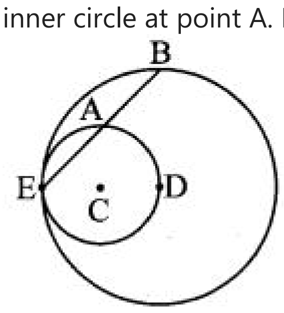
Solution:
i. ∠PQR m(arc PR) [Inscribed angle theorem]
= 12 × 140° = 70°
∠PQR is the exterior angle of ΔPOQ. [Remote interior angle theorem]



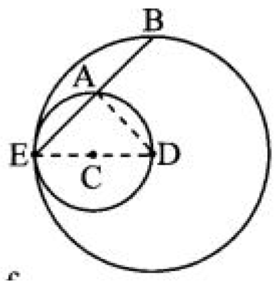
∴ ∠PQR = ∠POQ + ∠QPO [R – Q – O]
∴ 70° = ∠POR + ∠QPO
∴ 70 = 36° + ∠QPO
∴ ∠QPO = 70° – 36° = 340
Now, ray OP is tangent at point P and segment PQ is a secant.
∴ ∠QPO = 12 m(arcPQ) [Theorem of angle between tangent and secant]
∴ 34° = 12 m(arc PQ)
∴ m(arc PQ) = 68°
ii. Here, OP = 7.2, OQ = 3.2
Line OP is the tangent at point P [Given]
and seg OR is the secant.

∴ $OP^2 = OQ \times OR$ [Tangent secant segments theorem]
∴ $7.2^2 = 3.2 \times OR$
∴ $51.84 = 3.2 \times OR$
∴ OR $\frac{51.84}{3.2}$
∴ OR = 16.2 units
Now, OR = OQ + QR [O – Q – R]
∴ 16.2,= 3.2 + QR
∴ QR = 16.2 – 3.2
∴ QR = 13 units
iii. Here, OP = 7.2, OR = 16.2
$OP^2 = OQ \times OR$ [Tangent secant segments theorem]
∴ $7.2^2 = OQ \times 16.2$
∴ OQ = $\frac{51.84}{16.2}$
∴ OQ = 3.2 units
Now, OR = OQ + QR [O – Q – R]
∴ 16.2 = 3.2 + QR
∴ QR = 16.2 – 3.2
∴ QR = 13 units

Question 19.
In the adjoining figure, circles with centres C and D touch internally at point E. D lies on the inner circle. Chord EB of the outer circle intersects inner circle at point A. Prove that, seg EA ≅ seg AB.

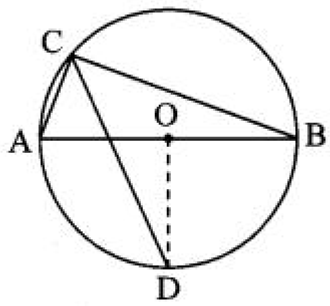


Given: Circles with centres C and D touch each other internally.
To prove: seg EA ≅ seg AB
Construction: Join seg ED and seg DA.
Proof:
E – C – D [Theorem of touching circles]
seg ED is the diameter of smaller circle.
∴∠EAD = 90° [Angle inscribed in a semicircle]



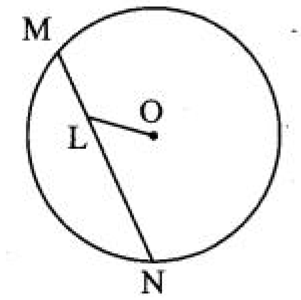
∴ seg AD ⊥ chord EB
∴ seg EA ≅ seg AB [Perpendicular drawn from the centre of the circle to the chord bisects the chord]

Question 20.
In the adjoining figure, seg AB is a diameter of a circle with centre O. The bisector of ∠ACB intersects the circle at point D. Prove that, seg AD ≅ seg BD. Complete the following proof by filling the blanks.

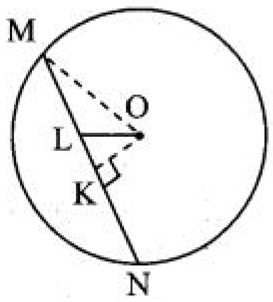


Given: seg AB is a diameter, seg CD bisects ∠ACB.
To prove: seg AD ≅ seg BD
Construction: Draw seg OD.
Proof:
∠ACB = 90° [Angle inscribed in a semicircle]
∠DCB = ∠DCA = 45° [CD is the bisector of ∠C]
m(arcDB) = 2∠DCA = 90° [Inscribed angle theorem]
∠DOB = m(arc DB) = 90° ………… (i) [Definition of measure of arc]
segOA ≅ segOB …………. (ii) [[Radii of the same circle]
∴ line OD is the perpendicular biscctor of [From (i) and (ii)]
seg AB.
∴ seg AD ≅ seg BD

10th Geometry Circle Question 21.
In the adjoining figure, seg MN is a chord of a circle with centre O. MN = 25, L is a point on chord MN such that ML = 9 and d(0, L) = 5. Find the radius of the circle.

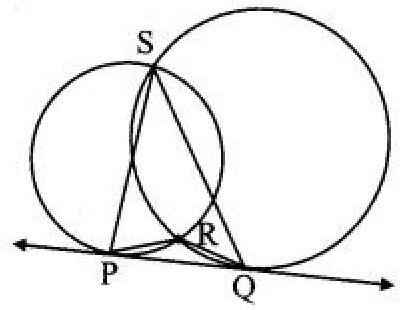


Construction: Draw seg OK ⊥ chord MN. Join OM.
Solution:
seg OK ⊥chord MN [Construction]
∴ MK = $\frac{1}{2}$ MN [Perpendicular drawn from the centre of the circle to the chord bisects the chord]
= $\frac{1}{2}$ × 25
= 12.5 units
MK = ML + LK [M – L – K]

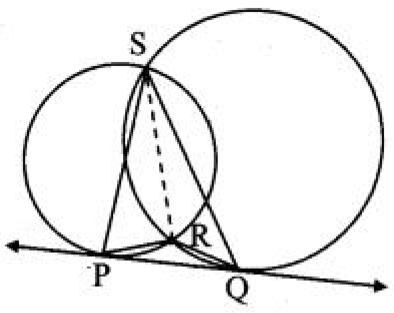


∴ 12.5 = 9 + LK
∴ LK= 12.5 – 9 = 3.5 units
In ΔOKL, ∠OKL = 90°
∴ $OL^2 = KL^2 + OK^2$ [Pythagoras theorem]
∴ $5^2 = 3.5^2 + OK^2$
∴ OK^2 = 25 – 12.25 = 12.75
Now, in ΔOKM, ∠OKM = 90°
∴ $OM^2 = OK^2 + MK^2$
= $12.75 + 12.5^2$
= 12.75 + 156.25
= 169
∴ OM = $\sqrt{169}$
= 13 units [Taking square root of both sides]
∴ The radius of the given circle is 13 units.

Question 22.
In the adjoining figure, two circles intersect each other at points S and R. Their common tangent PQ touches the circle at points P, Q.
Prove that, ∠PRQ + ∠PSQ = 180°.



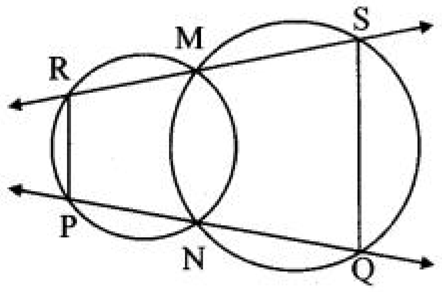
Given: Two circles intersect each other at points S and R.
line PQ is a common tangent.
To prove: ∠PRQ + ∠PSQ = 180°
Proof:
Line PQ is the tangent at point P and seg PR is a secant.
∴ [∠RPQ = ∠PSR .... (i)
and ∠PQR = ∠QSR] .... (ii) [Tangent secant theorem]
In Δ PQR,
∠PQR + ∠PRQ + ∠RPQ = 180° [Sum of the measures of angles of a triangle is 180°]



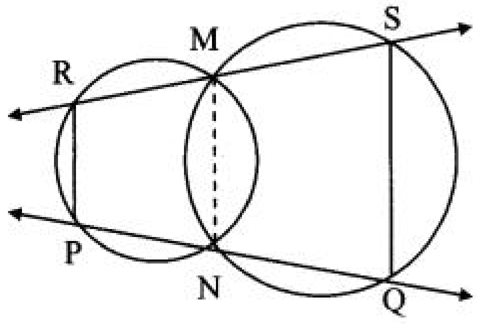
∴ ∠QSR + ∠PRQ + ∠PSR = 180° [From (i) and (ii)]

∴ ∠PRQ + ∠QSR + ∠PSR = 180°
∴ ∠PRQ + ∠PSQ = 180° [Angle addition property]

Question 23.
In the adjoining figure, two circles intersect at points M and N. Secants drawn through M and N intersect the circles at points R, S and P, Q respectively. Prove that: seg SQ || seg RP.

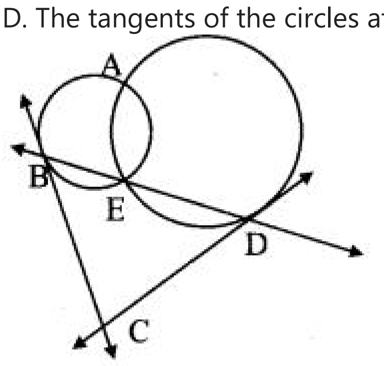


Given: Two circles intersect each other at points M and N.
To prove: seg SQ || seg RP
Construction: Join seg MN.
Proof:
□RMNP is a cyclic quadrilateral.
∴ ∠MRP = ∠MNQ ………… (i) [Corollary of cyclic quadrilateral theorem]
Also, □MNQS is a cyclic quadrilateral.
∴ ∠MNQ+ ∠MSQ = 180° [Theorem of cyclic quadrilateral]
∴ ∠MRP + ∠MSQ = 180° [From (i)]

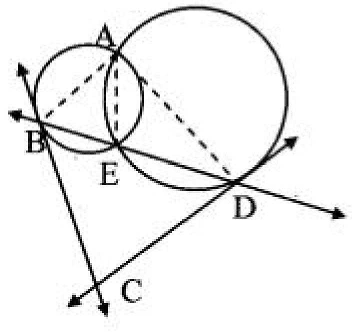


But, they are a pair of interior angles on the sarpe side of transversal RS on lines SQ and RP.
∴ seg SQ || seg RP [Interior angles test]

Question 24.
In the adjoining figure, two circles intersect each other at points A and E. Their common secant through E intersects the circles at points B and D. The tangents of the circles at points B and D intersect each other at point C. Prove that □ABCD is cyclic.

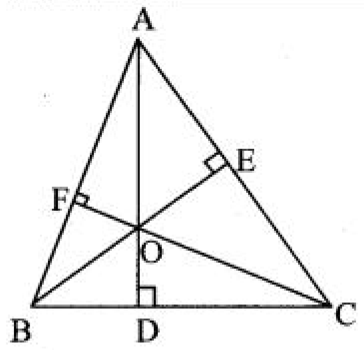


Given: Two circles intersect each other at A and E. seg BC and seg CD are the tangents to the circles.
To prove: □ABCD is cyclic.
Construction: Draw AB, AE and AD.
Proof:
[∠EBC = ∠BAE (i)
∠EDC = ∠DAE] (ii) [Tangent secant theorem]
In ΔBCD,

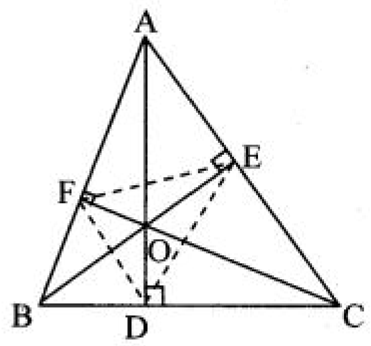


∠DBC + ∠BDC + ∠BCD = 180° [Sum of the measures of angles of a triangle is 180°]
∴ ∠EBC + ∠EDC + ∠BCD = 180° (iii) [B – E – D]
∴ ∠BAE + ∠DAE + ∠BCD = 180° [From (i), (ii) and (iii)]
∴ ∠BAD + ∠BCD = 180° [Angle addition property]
∴ □ABCD is cyclic. [Converse of cyclic quadrilateral theorem]

Question 25.
In the adjoining figure, seg AD ⊥ side BC, seg BE ⊥ side AC, seg CF ⊥ side AB. Point O is the orthocentre. Prove that, point O is the incentre of ΔDEF.



Given: seg AD ⊥ side BC,
seg BE ⊥ side AC,
seg CF ⊥ side AB.
To prove: Point O is the incentre of ΔDEF.
Construction: Draw DE, EF and DF.
Proof:
∠OFA = ∠OEA = 90° [Given]



Now, ∠OFA + ∠OEA = 90° + 90°
∴ ∠OFA + ∠OEA = 180°
∴ □OFAE is a cyclic quadrilateral. [Converse of cyclic quadrilateral theorem]
∴ Points O, F, A, E are concyclic points.
∴ seg 0E subtends equal angles ∠OFE and ∠OAE on the same side of OE.
∴ ∠OFE = ∠OAE .... (i)
∠OFB ∠ODB = 90° [Given]
Now, ∠OFB + ∠ODB = 90° + 90°
∴ ∠OFB + ∠ODB = 180°
∴ ⌡ OFBD is a cyclic quadrilateral. [Converse of cyclic quadrilateral theorem]
∴ Points O, F, B, D are concyclic points.
∴ seg OD subtends equal angles ∠OFD and
∠OBD on the same side of OD.
∠OFD = ∠OBD .... (ii)
In ΔAEO and ΔBDO,
∠AEO = ∠BDO [Each angle is 90°]
∠AOE = ∠BOD [Vertically opposite angles]
∴ ΔAEO ~ ΔBDO [AA test of similarity]
∴ ∠OAE = ∠OBD .... (iii) [Corresponding angles of similar triangles]
∴ ∠OFE = ∠OFD [From (i), (ii) and (iii)]
∴ ray FO bisects ∠EFD.
Similarly, we can prove ray EO and ray DO bisects ∠FED and ∠FDE respectively.
∴ Point O is the intersection of angle bisectors of ∠D, ∠E and ∠F of ΔDEF.
∴ Point O is the incentre of ΔDEF.