

Maharashtra State Board 11th Physics Solutions Chapter 5 Gravitation

1. Choose the correct option.

Question 1.

The value of acceleration due to gravity is maximum at

- (A) the equator of the Earth .
- (B) the centre of the Earth.
- (C) the pole of the Earth.
- (D) slightly above the surface of the Earth.

Answer:

- (C) the pole of the Earth.

Question 2.

The weight of a particle at the centre of the Earth is _____

- (A) infinite.
- (B) zero.
- (C) same as that at other places.
- (D) greater than at the poles.

Answer:

- (B) zero.

Question 3.

The gravitational potential due to the Earth is minimum at

- (A) the centre of the Earth.
- (B) the surface of the Earth.
- (C) a points inside the Earth but not at its centre.
- (D) infinite distance.

Answer:

- (A) the centre of the Earth.

Question 4.

The binding energy of a satellite revolving around planet in a circular orbit is 3×10^9 J. Its kinetic energy is _____

- (A) 6×10^9 J
- (B) -3×10^9 J
- (C) $-6 \times 10^{+9}$ J
- (D) $3 \times 10^{+9}$ J

Answer:

- (D) $3 \times 10^{+9}$ J

2. Answer the following questions.

Question 1.

State Kepler's law equal of area.

Answer:

The line that joins a planet and the Sun sweeps equal areas in equal intervals of time.

Question 2.

State Kepler's law of period.

Answer:

The square of the time period of revolution of a planet around the Sun is proportional to the cube of the semimajor axis of the ellipse traced by the planet.

Question 3.

What are the dimensions of the universal gravitational constant?

Answer:

The dimensions of universal gravitational constant are: $[L^3M^{-1}T^{-2}]$.

Question 4.

Define binding energy of a satellite.

Answer:

The minimum energy required by a satellite to escape from Earth 's gravitational influence is the binding energy of the satellite.

Question 5.

What do you mean by geostationary satellite?

Answer:

Some satellites that revolve around the Earth in equatorial plane have same sense of rotation as that of the Earth. The also have the same period of rotation as that of the Earth i.e.. 24 hours. Due to this, these satellites appear stationary from the Earth's surface and are known as geostationary satellites.

Question 6.

State Newton's law of gravitation.

Answer:

Statement:

Every particle of matter attracts every other particle of matter with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Question 7.

Define escape velocity of a satellite.

Answer:

The minimum velocity with which a body should be thrown vertically upwards from the surface of the Earth so that it escapes the Earth's gravitational field, is called the escape velocity (v_e) of the body.

Question 8.

What is the variation in acceleration due to gravity with altitude?

Answer:

Variation in acceleration due to gravity due to altitude is given by, $g_h = g \left(\frac{R}{R+h} \right)^2$

where,

g_h = acceleration due to gravity of an object placed at h altitude

g = acceleration due to gravity on surface of the Earth

R = radius of the Earth

h = altitude height of the object from the surface of the Earth.

Hence, acceleration due to gravity decreases with increase in altitude.

Question 9.

On which factors does the escape speed of a body from the surface of Earth depend?

Answer:

The escape speed depends only on the mass and radius of the planet.

[Note: Escape velocity does not depend upon the mass of the body]

Question 10.

As we go from one planet to another planet, how will the mass and weight of a body change?

Answer:

1. As we go from one planet to another, mass of a body remains unaffected.
2. However, due to change in mass and radius of planet, acceleration due to gravity acting on the body changes as, $g \propto \frac{M}{R^2}$.
Hence, weight of the body also changes as, $W \propto MR^2$.

Question 11.

What is periodic time of a geostationary satellite?

Answer:

The periodic time of a geostationary satellite is same as that of the Earth i.e., one day or 24 hours.

Question 12.

State Newton's law of gravitation and express it in vector form.

Answer:

1. Statement:
Every particle of matter attracts every other particle of matter with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
2. In vector form, it can be expressed as,
$$\vec{F}_{21} = G \frac{m_1 m_2}{r^2} \hat{r}_{21}$$

where, \hat{r}_{21} is the unit vector from m_1 to m_2 .
The force \vec{F}_{21} is directed from m_2 to m_1 .

Question 13.

What do you mean by gravitational constant? State its SI units.

Answer:

1. From Newton's law of gravitation,
$$F = G \frac{m_1 m_2}{r^2}$$

where, G = constant called universal gravitational constant Its value is $667 \times 10^{-11} \text{ N m}^2/\text{kg}^2$.
2. $G = \frac{F r^2}{m_1 m_2}$
If $m_1 = m_2 = 1 \text{ kg}$, $r = 1 \text{ m}$ then $F = G$.
Hence, the universal gravitational constant is the force of gravitation between two particles of unit mass separated by unit distance.
3. Unit: $\text{N m}^2/\text{kg}^2$ in SI system.

Question 14.

Why is a minimum two stage rocket necessary for launching of a satellite?

Answer:

1. For the projection of an artificial satellite, it is necessary for the satellite to have a certain velocity.
2. In a single stage rocket, when the fuel in first stage of rocket is ignited on the surface of the Earth, it raises the satellite vertically.
3. The velocity of projection of satellite normal to the surface of the Earth is the vertical velocity.
4. If this vertical velocity is less than the escape velocity (v_e), the satellite returns to the Earth's surface. While, if the vertical velocity is greater than or equal to the escape velocity, the satellite will escape from Earth's gravitational influence and go to infinity.
5. Hence, minimum two stage rocket, one to raise the satellite to desired height and another to provide required horizontal velocity, is necessary for launching of a satellite.

Question 15.

State the conditions for various possible orbits of a satellite depending upon the horizontal speed of projection

Answer:

The path of the satellite depends upon the value of horizontal speed of projection v_h relative to critical velocity v_c and escape velocity v_e .

Case (I) $v_h < v_c$:

The orbit of satellite is an ellipse with point of projection as apogee and Earth at one of the foci. During this elliptical path, if the satellite passes through the Earth's atmosphere, it experiences a nonconservative force of air resistance. As a result it loses energy and spirals down to the Earth.

Case (II) $v_h = v_c$:

The satellite moves in a stable circular orbit around the Earth.

Case (III) $v_c < v_h < v_e$:

The satellite moves in an elliptical orbit round the Earth with the point of projection as perigee.

Case (IV) $v_h = v_e$

The satellite travels along parabolic path and never returns to the point of projection. Its speed will be zero at infinity.

Case (V) $v_h > v_e$:

The satellite escapes from gravitational influence of Earth traversing a hyperbolic path.

3. Answer the following questions in detail.

Question 1.

Derive an expression for critical velocity of a satellite.

Answer:

Expression for critical velocity:

1. Consider a satellite of mass m revolving round the Earth at height h above its surface. Let M be the mass of the Earth and R be its radius.
2. If the satellite is moving in a circular orbit of radius $(R + h) = r$, its speed must be equal to the magnitude of critical velocity v_c .
3. The centripetal force necessary for circular motion of satellite is provided by gravitational force exerted by the satellite on the Earth.

\therefore Centripetal force = Gravitational force

$$\therefore \frac{mv_c^2}{r} = \frac{GMm}{r^2}$$

$$\therefore v_c^2 = \frac{GM}{r}$$

$$\therefore v_c = \sqrt{\frac{GM}{r}}$$

$$\therefore v_c = \sqrt{\frac{GM}{(R+h)}} = \sqrt{g_h(R+h)}$$

This is the expression for critical speed at the orbit of radius $(R + h)$.

4. The critical speed of a satellite is independent of the mass of the satellite. It depends upon the mass of the Earth and the height at which the satellite is revolving or gravitational acceleration at that altitude.

Question 2.

State any four applications of a communication satellite.

Answer:

Applications of communication satellite:

1. For the transmission of television and radiowave signals over large areas of Earth's surface.
2. For broadcasting telecommunication.
3. For military purposes.
4. For navigation surveillance.

Question 3.

Show that acceleration due to gravity at height h above the Earth's surface is $g_h = \frac{gR^2}{(R+h)^2}$

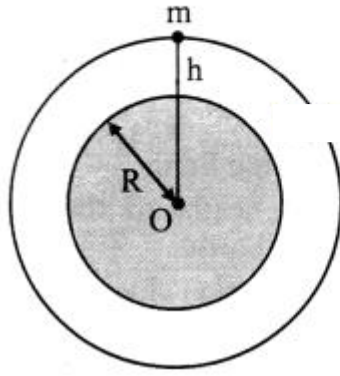
Answer:

Variation of g due to altitude:

1. Let,
 R = radius of the Earth,
 M = mass of the Earth.
 g = acceleration due to gravity at the surface of the Earth.

2. Consider a body of mass m on the surface of the Earth. The acceleration due to gravity on the Earth's surface is given by,

$$g = \frac{GM}{R^2} \quad \dots(1)$$



3. The body is taken at height h above the surface of the Earth as shown in figure. The acceleration due to gravity now changes to,
 $g_h = \frac{GM}{(R+h)^2} \dots\dots\dots (2)$

4. Dividing equation (2) by equation (1), we get,

$$\frac{g_h}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}}$$

$$\therefore \frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

$$\therefore g_h = \frac{gR^2}{(R+h)^2}$$

We can rewrite,

$$\therefore g_h = \frac{gR^2}{R^2 \left(1 + \frac{h}{R}\right)^2}$$

$$\therefore g_h = g \left(1 + \frac{h}{R}\right)^{-2}$$

- v. For small altitude h , i.e., for $\frac{h}{R} \ll 1$, by

neglecting higher power terms of $\frac{h}{R}$,

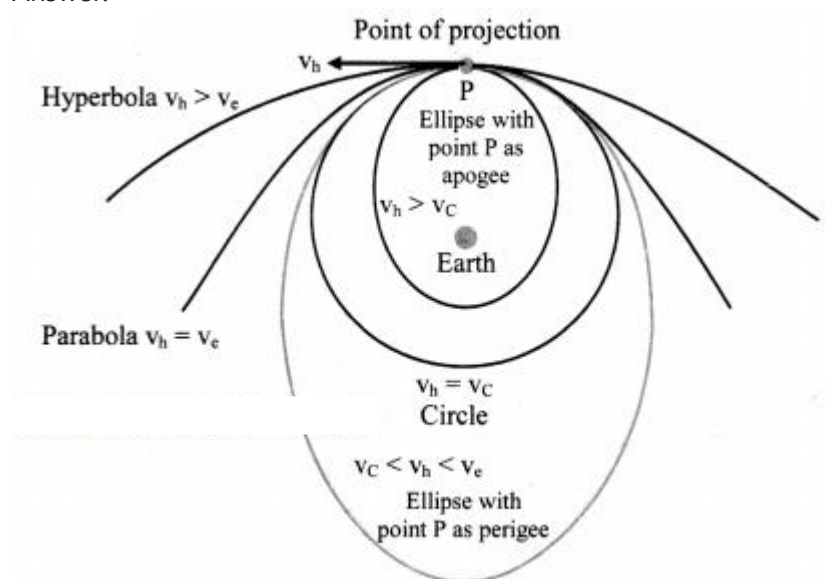
$$g_h = g \left(1 - \frac{2h}{R}\right)$$

This expression can be used to calculate the value of g at height h above the surface of the Earth as long as $h \ll R$.

Question 4.

Draw a labelled diagram to show different trajectories of a satellite depending upon the tangential projection speed.

Answer:



v_h = horizontal speed of projection

v_c = critical velocity

v_e = escape velocity

Question 5.

Derive an expression for binding energy of a body at rest on the Earth's surface.

Answer:

1. Let,

M = mass of the Earth

m = mass of the satellite

R = radius of the Earth.

- Since the satellite is at rest on the Earth, $v = 0$
 \therefore Kinetic energy of satellite.
 $K.E = \frac{1}{2}mv^2 = 0$
- Gravitational potential at the Earth's surface
 $= -\frac{GM}{R}$
 \therefore Potential energy of satellite = Gravitational potential \times mass of satellite
 $= -GMm/R$
- Total energy of satellite = T.E = P.E + K.E
 \therefore T.E. = $-\frac{GMm}{R} + 0 = -\frac{GMm}{R}$
- Negative sign in the energy indicates that the satellite is bound to the Earth, due to gravitational force of attraction.
- For the satellite to be free from Earth's gravitational influence, its total energy should become positive. That energy is the binding energy of the satellite at rest on the surface of the Earth.
 \therefore B.E. = $\frac{GMm}{R}$

Question 6.

Why do astronauts in an orbiting satellite have a feeling of weightlessness?

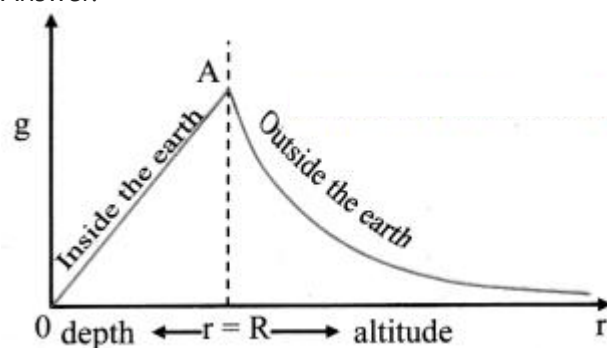
Answer:

- For an astronaut, in a satellite, the net force towards the centre of the Earth will always be, $F = mg - N$, where, N is the normal reaction.
- In the case of a revolving satellite, the satellite is performing a circular motion. The acceleration for this motion is centripetal, which is provided by the gravitational acceleration g at the location of the satellite.
- In this case, the downward acceleration, $a_d = g$, or the satellite (along with the astronaut) is in the state of free fall.
- Thus, the net force acting on astronaut will be, $F = mg - ma_d$ i.e., the apparent weight will be zero, giving the feeling of total weightlessness.

Question 7.

Draw a graph showing the variation of gravitational acceleration due to the depth and altitude from the Earth's surface.

Answer:



Question 8.

At which place on the Earth's surface is the gravitational acceleration maximum? Why?

Answer:

- Gravitational acceleration on the surface of the Earth depends on latitude of the place as well as rotation and shape of the Earth.
- At poles, latitude $\theta = 90^\circ$.
 $\therefore g' = g$
 i.e., there is no reduction in acceleration due to gravity at poles.
- Also, shape of the Earth is actually an ellipsoid, bulged at equator. The polar radius of the Earth is 6356 km which is minimum. As $g \propto \frac{1}{R^2}$, acceleration due to gravity is maximum at poles i.e., 9.8322 m/s^2 .

Question 9.

At which place on the Earth surface the gravitational acceleration minimum? Why?

Answer:

- Gravitational acceleration on the surface of the Earth depends on latitude of the place as well as rotation and shape of the Earth.
- At equator, latitude $\theta = 0^\circ$.
 $\therefore g' = g - R\omega^2$
 i.e., the acceleration due to gravity is reduced by amount $R\omega^2 (\approx 0.034 \text{ m/s}^2)$ at equator.
- Also, shape of the Earth is actually an ellipsoid, bulged at equator. The equatorial radius of the Earth is 6378 km, which is maximum. As $g \propto \frac{1}{R^2}$, acceleration due to gravity is minimum on equator i.e., 9.7804 m/s^2 .

Question 10.

Define the binding energy of a satellite. Obtain an expression for binding energy of a satellite revolving around the Earth at certain altitude.

Answer:

The minimum energy required by a satellite to escape from Earth's gravitational influence is the binding energy of the satellite.

Expression for binding energy of satellite revolving in circular orbit round the Earth:

- Consider a satellite of mass m revolving at height h above the surface of the Earth in a circular orbit. It possesses potential energy as well as kinetic energy.
- Let M be the mass of the Earth, R be the Radius of the Earth, v_c be critical velocity of satellite, $r = (R + h)$ be the radius of the orbit.
- Kinetic energy of satellite = $\frac{1}{2}mv_c^2 = \frac{1}{2}\frac{GMm}{r}$

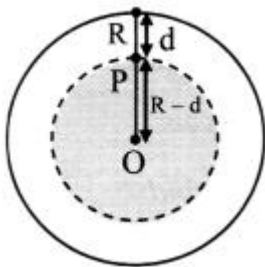
- The gravitational potential at a distance r from the centre of the Earth is $-GM/r$
 \therefore Potential energy of satellite = Gravitational potential \times mass of satellite
 $= -GMmr$
- The total energy of satellite is given as T.E. = K.E. + P.E.
 $= \frac{1}{2}GMmr - GMmr = -\frac{1}{2}GMmr$
- Total energy of a circularly orbiting satellite is negative. Negative sign indicates that the satellite is bound to the Earth, due to gravitational force of attraction. For the satellite to be free from the Earth's gravitational influence its total energy should become zero or positive.
- Hence the minimum energy to be supplied to unbind the satellite is $+\frac{1}{2}GMmr$. This is the binding energy of a satellite.

Question 11.

Obtain the formula for acceleration due to gravity at the depth 'd' below the Earth's surface.

Answer:

- The Earth can be considered to be a sphere made of large number of concentric uniform spherical shells.
- When an object is on the surface of the Earth it experiences the gravitational force as if the entire mass of the Earth is concentrated at its centre.
- The acceleration due to gravity on the surface of the Earth is, $g = GM/R^2$
- Assuming that the density of the Earth is uniform, mass of the Earth is given by
 $M = \text{volume} \times \text{density} = \frac{4}{3}\pi R^3 \rho$
 $\therefore g = G \times \frac{4}{3}\pi R^3 \rho / R^2 = \frac{4}{3}\pi R \rho G$ (1)
- Consider a body at a point P at the depth d below the surface of the Earth as shown in figure.



Here the force on a body at P due to outer spherical shell shown by shaded region, cancel out due to symmetry. The net force on P is only due to the inner sphere of radius $OP = R - d$.

- Acceleration due to gravity because of this sphere is,

$$g_d = \frac{GM'}{(R-d)^2}$$

where,

$M' = \text{volume of the inner sphere} \times \text{density}$

$$\therefore M' = \frac{4}{3}\pi (R-d)^3 \times \rho$$

$$\therefore g_d = \frac{G \times \frac{4}{3}\pi (R-d)^3 \rho}{(R-d)^2}$$

$$\therefore g_d = G \times \frac{4}{3}\pi (R-d) \rho \quad \dots (2)$$

- Dividing equation (2) by equation (1) we get,

$$\frac{g_d}{g} = \frac{R-d}{R}$$

$$\therefore \frac{g_d}{g} = 1 - \frac{d}{R}$$

$$\therefore g_d = g \left(1 - \frac{d}{R} \right)$$

This equation gives acceleration due to gravity at depth d below the Earth's surface.

Question 12.

State Kepler's three laws of planetary motion.

Answer:

- All planets move in elliptical orbits around the Sun with the Sun at one of the foci of the ellipse.
- The line that joins a planet and the Sun sweeps equal areas in equal intervals of time.
- The square of the time period of revolution of a planet around the Sun is proportional to the cube of the semimajor axis of the ellipse traced by the planet.

Question 13.

State the formula for acceleration due to gravity at depth 'd' and altitude 'h'. Hence show that their ratio is equal to $(R-d)/R(1+2h/R)$ by assuming that the altitude is very small as compared to the radius of the Earth.

Answer:

- For an object at depth d, acceleration due to gravity of the Earth is given by,
 $g_d = g(1 - d/R)$ (1)

- Also, the acceleration due to gravity at smaller altitude h is given by,

$$g_h = g \left(1 - \frac{2h}{R} \right) \dots\dots\dots (2)$$

- Hence, dividing equation (1) by equation (2), we get,

$$\frac{g_d}{g_h} = \frac{g \left(1 - \frac{d}{R} \right)}{g \left(1 - \frac{2h}{R} \right)} = \frac{R-d}{R} \times \frac{R}{R-2h}$$

$$\therefore \frac{g_d}{g_h} = \frac{R-d}{R-2h}$$

Question 14.

What is critical velocity? Obtain an expression for critical velocity of an orbiting satellite. On what factors does it depend?

Answer:

The exact horizontal velocity of projection that must be given to a satellite at a certain height so that it can revolve in a circular orbit round the Earth is called the critical velocity or orbital velocity (v_c).

Expression for critical velocity:

- Consider a satellite of mass m revolving round the Earth at height h above its surface. Let M be the mass of the Earth and R be its radius.
- If the satellite is moving in a circular orbit of radius $(R + h) = r$, its speed must be equal to the magnitude of critical velocity v_c .
- The centripetal force necessary for circular motion of satellite is provided by gravitational force exerted by the satellite on the Earth.

\therefore Centripetal force = Gravitational force

$$\therefore \frac{mv_c^2}{r} = \frac{GMm}{r^2}$$

$$\therefore v_c^2 = \frac{GM}{r}$$

$$\therefore v_c = \sqrt{\frac{GM}{r}}$$

$$\therefore v_c = \sqrt{\frac{GM}{(R+h)}} = \sqrt{g_h (R+h)}$$

This is the expression for critical speed at the orbit of radius $(R + h)$.

- The critical speed of a satellite is independent of the mass of the satellite. It depends upon the mass of the Earth and the height at which the satellite is revolving or gravitational acceleration at that altitude.

Question 15.

Define escape speed. Derive an expression for the escape speed of an object from the surface of the earth.

Answer:

- The minimum velocity with which a body should be thrown vertically upwards from the surface of the Earth so that it escapes the Earth's gravitational field, is called the escape velocity (v_e) of the body.
- As the gravitational force due to Earth becomes zero at infinite distance, the object has to reach infinite distance in order to escape.
- Let us consider the kinetic and potential energies of an object thrown vertically upwards with escape velocity v_e .
- On the surface of the Earth,

$$\text{K.E.} = \frac{1}{2}mv_e^2$$

$$\text{P.E.} = -GMm/R$$

$$\text{Total energy} = \text{P.E.} + \text{K.E.}$$

$$\therefore \text{T.E.} = \frac{1}{2}mv_e^2 - GMm/R \dots\dots\dots (1)$$

- The kinetic energy of the object will go on decreasing with time as it is pulled back by Earth's gravitational force. It will become zero when it reaches infinity. Thus, at infinite distance from the Earth,

$$\text{K.E.} = 0$$

Also,

$$\text{P.E.} = -GMm/\infty = 0$$

$$\therefore \text{Total energy} = \text{P.E.} + \text{K.E.} = 0$$

- As energy is conserved

$$\frac{1}{2}mv_e^2 - GMm/R = 0 \dots\dots[\text{From (1)}]$$

$$\text{or, } v_e = \sqrt{2GM/R}$$

Question 16.

Describe how an artificial satellite using two stage rocket is launched in an orbit around the Earth.

Answer:

- Launching of a satellite in an orbit around the Earth cannot take place by use of single stage rocket. It requires minimum two stage rocket.
- With the help of first stage of rocket, satellite can be taken to a desired height above the surface of the Earth.
- Then the launcher is rotated in horizontal direction i.e., through 90° using remote control and the first stage of the rocket is detached.
- With the help of second stage of rocket, a specific horizontal velocity (v_h) is given to satellite so that it can revolve in a circular path around the Earth.
- The satellite follows different paths depending upon the horizontal velocity provided to it.

4. Solve the following problems.

Question 1.

At what distance below the surface of the Earth, the acceleration due to gravity decreases by 10% of its value at the surface, given radius of Earth is 6400 km.

Solution:

Given: $g_d = 90\%$ of g i.e., $g_d/g = 0.9$,

$R = 6400\text{ km} = 6.4 \times 10^6\text{ m}$

To find: Distance below the Earth's surface (d)

Formula: $g_d = g \left(1 - \frac{d}{R}\right)$

Calculation: From formula,

$$\frac{g_d}{g} = \left(1 - \frac{d}{R}\right)$$

$$\therefore \frac{d}{R} = 1 - \frac{g_d}{g}$$

$$\begin{aligned}\therefore d &= R \left(1 - \frac{g_d}{g}\right) \\ &= 6.4 \times 10^6 \times 0.1 \\ &= 640 \times 10^3\text{ m} \\ &= \mathbf{640\text{ km}}\end{aligned}$$

At distance 640 km below the surface of the Earth, value of acceleration due to gravity decreases by 10%.

Question 2.

If the Earth were made of wood, the mass of wooden Earth would have been 10% as much as it is now (without change in its diameter).

Calculate escape speed from the surface of this Earth.

Solution:

Given: $M_w = 10\%$ of $M = \frac{M}{10}$, $D_w = D$ or $R_w = R$,

To find: Escape speed (v_e)

Formula: $v_e = \sqrt{\frac{2GM}{R}}$

Calculation: From formula,

$$v_{e_w} = \sqrt{\frac{2GM_w}{R_w}}$$

$$\therefore \frac{v_{e_w}}{v_e} = \sqrt{\frac{2GM_w}{R_w} \times \frac{R}{2GM}}$$

$$\therefore \frac{v_{e_w}}{v_e} = \sqrt{\left(\frac{R}{R_w}\right) \times \left(\frac{M_w}{M}\right)} = \sqrt{1 \times \frac{1}{10}} = \sqrt{\frac{1}{10}}$$

$$\therefore v_{e_w} = v_e \times \frac{1}{\sqrt{10}}$$

As, we know that the escape speed from surface of the Earth is 11.2 km/s, Substituting value of $v_e = 11.2\text{ km/s}$

$$v_{e_w} = 11.2 \times \frac{1}{\sqrt{10}} = 11.2 \times 0.3162$$

$$= 11.2 \times 0.3162$$

..... [Taking square root value]

$$= \text{antilog} \{\log(11.2) - \log(3.162)\}$$

$$= \text{antilog} \{1.0492 - 0.5000\}$$

$$= \text{antilog} \{0.5492\} = 3.542$$

$$\therefore v_{e_w} = 3.54\text{ km/s}$$

The escape velocity from the surface of wooden Earth is 3.54 km/s.

Question 3.

Calculate the kinetic energy, potential energy, total energy and binding energy of an artificial satellite of mass 2000 kg orbiting at a height of 3600 km above the surface of the Earth.

Given:- $G = 6.67 \times 10^{-11}\text{ Nm}^2/\text{kg}^2$

$R = 6400\text{ km}$

$M = 6 \times 10^{24}\text{ kg}$

Solution:

Given:- $m = 2000\text{ kg}$, $h = 3600\text{ km} = 3.6 \times 10^6\text{ m}$,

$G = 6.67 \times 10^{-11}\text{ Nm}^2/\text{kg}^2$

$R = 6400\text{ km}$

$M = 6 \times 10^{24}\text{ kg}$

To find: i) Kinetic energy (K.E.)

ii) Potential Energy (P.E.)

iii) Total Energy (T.E.)

iv) Binding Energy (B.E.)

$$\text{Formulae: i. K.E.} = \frac{GMm}{2(R+h)}$$

$$\text{ii. P.E.} = -\frac{GMm}{(R+h)} = -2(\text{K.E.})$$

$$\text{iii. T.E.} = \text{K.E.} + \text{P.E.}$$

$$\text{iv. B.E.} = -\text{T.E.}$$

Calculation: From formula (i),

$$\begin{aligned}\text{K.E.} &= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 2 \times 10^3}{2 \times \left[(6.4 \times 10^6) + (3.6 \times 10^6) \right]} \\ &= \frac{6.67 \times 6 \times 10^{16}}{10^7} \\ &= \mathbf{40.02 \times 10^9 \text{ J}}\end{aligned}$$

From formula (ii),

$$\text{P.E.} = -2 \times 40.02 \times 10^9$$

$$= -80.04 \times 10^9 \text{ J}$$

From formula (iii),

$$\text{T.E.} = (40.02 \times 10^9) + (-80.02 \times 10^9)$$

$$= -40.02 \times 10^9 \text{ J}$$

From formula (iv),

$$\text{B.E.} = -(-40.02 \times 10^9)$$

$$= 40.02 \times 10^9 \text{ J}$$

Kinetic energy of the satellite is $40.02 \times 10^9 \text{ J}$, potential energy is $-80.04 \times 10^9 \text{ J}$, total energy is $-40.02 \times 10^9 \text{ J}$ and binding energy is $40.02 \times 10^9 \text{ J}$.

[Note: Total energy of orbiting satellite is negative.]

Question 4.

Two satellites A and B are revolving around a planet. Their periods of revolution are 1 hour and 8 hours respectively. The radius of orbit of satellite B is $4 \times 10^4 \text{ km}$. find radius of orbit of satellite A .

Solution:

$$\text{Given: } T_A = 1 \text{ hour, } T_B = 8 \text{ hour,}$$

$$r_B = 4 \times 10^4 \text{ km}$$

To find: Radius of orbit of satellite A (r_A)

$$\text{Formula: } T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Calculation: From formula,

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\therefore T^2 \propto r^3$$

$$\dots (\because \frac{4\pi^2 r^3}{GM} = \text{constant for a planet})$$

$$\therefore \left(\frac{T_A}{T_B} \right)^2 = \left(\frac{r_A}{r_B} \right)^3$$

$$\therefore \left(\frac{1}{8} \right)^2 = \left(\frac{r_A}{4 \times 10^4} \right)^3$$

$$\therefore r_A^3 = \frac{1}{(8)^2} \times (4 \times 10^4)^3$$

$$\therefore r_A^3 = 10^{12}$$

$$\therefore r_A = \mathbf{1 \times 10^4 \text{ km}}$$

Radius of orbit of satellite A will be $1 \times 10^4 \text{ km}$.

Question 5.

Find the gravitational force between the Sun and the Earth.

$$\text{Given Mass of the Sun} = 1.99 \times 10^{30} \text{ kg}$$

$$\text{Mass of the Earth} = 5.98 \times 10^{24} \text{ kg}$$

$$\text{The average distance between the Earth and the Sun} = 1.5 \times 10^{11} \text{ m.}$$

Solution:

$$\text{Given: } M_s = 1.99 \times 10^{30} \text{ kg}$$

$$M_E = 5.98 \times 10^{24} \text{ kg, } R = 1.5 \times 10^{11} \text{ m.}$$

To find: Gravitational force between the Sun and the Earth (F)

$$\text{Formula: } F = G \frac{m_1 m_2}{r^2}$$

$$\text{Calculation: As, we know, } G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

From formula,

$$F = \frac{GM_s M_E}{R^2}$$

$$= \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 5.98 \times 10^{24}}{(1.5 \times 10^{11})^2}$$

$$\therefore F = \frac{6.67 \times 1.99 \times 5.98}{2.25} \times 10^{21}$$

$$= \text{antilog} \{(\log(6.67) + \log(1.99) + \log(5.98) - \log(2.25)) \times 10^{21}\}$$

$$= \text{antilog} \{(0.8241) + (0.2989) + (0.7767) - (0.3522)\} \times 10^{21}$$

$$= \text{antilog} \{1.5475\} \times 10^{21}$$

$$= 35.28 \times 10^{21}$$

$$= 3.5 \times 10^{22} \text{ N}$$

The gravitational force between the Sun and the Earth is $3.5 \times 10^{22} \text{ N}$.

Question 6.

Calculate the acceleration due to gravity at a height of 300 km from the surface of the Earth. ($M = 5.98 \times 10^{24} \text{ kg}$, $R = 6400 \text{ km}$).

Solution:

Given: $h = 300 \text{ km} = 0.3 \times 10^6 \text{ m}$,

$M = 5.98 \times 10^{24} \text{ kg}$,

$R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$

$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

To find: Acceleration due to gravity at height (g_h)

Formula: $g_h = \frac{GM}{(R+h)^2}$

Calculation: From formula,

$$g_h = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{[(6.4 \times 10^6) + (0.3 \times 10^6)]^2}$$

$$= \frac{6.67 \times 5.98 \times 10^{13}}{(6.7)^2 \times 10^{12}}$$

$$6.67 \times 10^{-11} \times 5.98 \times 10^{24}$$

$$= \text{antilog} \{\log(6.67) + \log(5.98) - 2\log(6.7)\} \times 10$$

$$= \text{antilog}\{0.8241 + 0.7767 - 2(0.8261)\} \times 10$$

$$= \text{antilog} \{1.6008 - 1.6522\} \times 10$$

$$= \text{antilog} \{1^{-0.9486}\} \times 10$$

$$= 0.8884 \times 10 = 8.884 \text{ m/s}^2$$

Acceleration due to gravity at 300 km will be 8.884 m/s^2 .

Question 7.

Calculate the speed of a satellite in an orbit at a height of 1000 km from the Earth's surface. $M_E = 5.98 \times 10^{24} \text{ kg}$, $R = 6.4 \times 10^6 \text{ m}$.

Solution:

Given: $h = 1000 \text{ km} = 1 \times 10^6 \text{ m}$,

$M_E = 5.98 \times 10^{24} \text{ kg}$, $R = 6.4 \times 10^6 \text{ m}$,

$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$

To find: Speed of satellite (v_c)

Formula: $v_c = \sqrt{\frac{GM}{r}}$

Calculation:

From formula,

$$v_c = \sqrt{\frac{GM}{(R+h)}}$$

$$= \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{[(6.4 \times 10^6) + (1 \times 10^6)]}}$$

$$= \sqrt{\frac{6.67 \times 5.98 \times 10^7}{7.4}}$$

$$= \sqrt{\text{antilog}\{\log(6.67) + \log(5.98) - \log(7.4) \times 10^7\}}$$

$$= \sqrt{\text{antilog}\{0.8241 + 0.7767 - 0.8692\} \times 10^7}$$

$$= \sqrt{\text{antilog}\{0.7316\} \times 10^7}$$

$$= \sqrt{5.391 \times 10^7}$$

$$= \sqrt{53.91 \times 10^6}$$

$$= 7.343 \times 10^3 \text{[Taking square root value]}$$

$$= 7.34 \times 10^3 \text{ m/s}$$

Speed of the satellite at height 1000 km is $7.34 \times 10^3 \text{ m/s}$.

Question 8.

Calculate the value of acceleration due to gravity on the surface of Mars if the radius of Mars = 3.4×10^3 km and its mass is 6.4×10^{23} kg.

Solution:

Given:

$$M = 6.4 \times 10^{23} \text{ kg}$$

$$R = 3.4 \times 10^3 = 3.4 \times 10^6 \text{ m,}$$

To find: Acceleration due to gravity on the surface of the Mars (g_M)

$$\text{Formula: } g = \frac{GM}{R^2}$$

$$\text{Calculation: As, } G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

From formula,

$$g_M = \frac{6.67 \times 10^{-11} \times 6.4 \times 10^{23}}{(3.4 \times 10^6)^2} = \frac{6.67 \times 6.4 \times 3.4 \times 3.4}{10^{-11+23-12-12}}$$

$$= \text{antilog} \{ \log(6.67) + \log(6.4) - \log(3.4) - \log(3.4) \}$$

$$= \text{antilog} \{ (0.8241) + (0.8062) - (0.5315) - (0.5315) \}$$

$$= \text{antilog} \{ 0.5673 \}$$

$$= 3.693 \text{ m/s}^2$$

Acceleration due to gravity on the surface of Mars is 3.693 m/s².

Question 9.

A planet has mass 6.4×10^{24} kg and radius 3.4×10^6 m. Calculate energy required to remove an object of mass 800 kg from the surface of the planet to infinity.

Solution:

$$\text{Given: } M = 6.4 \times 10^{24} \text{ kg, } R = 3.4 \times 10^6 \text{ m, } m = 800 \text{ kg}$$

To find: Energy required to remove the object from surface of planet to infinity = B.E.

$$\text{Formula: } \text{B.E.} = \frac{GMm}{R}$$

Calculation: We know that,

$$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

From formula,

$$\text{B.E.} = \frac{6.67 \times 10^{-11} \times 6.4 \times 10^{24} \times 800}{3.4 \times 10^6}$$

$$\therefore \text{B.E.} = \frac{6.67 \times 6.4 \times 8}{3.4} \times 10^9$$

$$= \frac{6.67 \times 51.2}{3.4} \times 10^9$$

$$= \text{antilog} \{ \log(6.67) + \log(51.2) - \log(3.4) \} \times 10^9$$

$$= \text{antilog} \{ 0.8241 + 1.7093 - 0.5315 \} \times 10^9$$

$$= \text{antilog} \{ 2.0019 \} \times 10^9$$

$$= 1.004 \times 10^2 \times 10^9$$

$$= 1.004 \times 10^{11} \text{ J}$$

Energy required to remove the object from the surface of the planet is $1.004 \times 10^{11} \text{ J}$.

[Note: Answer calculated above is in accordance with retual methods of calculation.]

Question 10.

Calculate the value of the universal gravitational constant from the given data. Mass of the Earth = 6×10^{24} kg, Radius of the Earth = 6400 km and the acceleration due to gravity on the surface = 9.8 m/s²

Solution:

$$\text{Given: } M = 6 \times 10^{24} \text{ kg,}$$

$$R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m,}$$

$$g = 9.8 \text{ m/s}^2$$

To find: Gravitational constant (G)

$$\text{Formula. } g = \frac{GM}{R^2}$$

Calculation: From formula,

$$G = \frac{gR^2}{M}$$

$$G = \frac{9.8 \times (6.4 \times 10^6)^2}{6 \times 10^{24}} = \frac{401.4 \times 10^{12}}{6 \times 10^{24}}$$

$$\therefore G = 6.69 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

The value of gravitational constant is $6.69 \times 10^{-11} \text{ N m}^2/\text{kg}^2$.

Question 11.

A body weighs 5.6 kg wt on the surface of the Earth. How much will be its weight on a planet whose mass is 1/7 times the mass of the Earth and radius twice that of the Earth's radius.

Solution:

$$\text{Given: } W_E = 5.6 \text{ kg-wt,}$$

$$M_p/M_E = 1/7, R_p/R_E = 2$$

To find: Weight of the body on the surface of planet (W_p)

$$\text{Formula: } W = mg = \frac{GMm}{R^2}$$

Calculation: From formula,

$$\frac{W_p}{W_E} = \frac{M_p}{M_E} \times \frac{R_E^2}{R_p^2}$$

$$\frac{W_p}{5.6} = \frac{1}{7} \times \left(\frac{1}{2}\right)^2 = \frac{1}{28}$$

$$\therefore W_p = \frac{1}{28} \times 5.6 = \mathbf{0.2 \text{ kg-wt.}}$$

Weight of the body on the surface of a planet will be 0.2 kg-wt.

[Note: The answer given above is calculated in accordance with textual method considering the given data].

Question 12.

What is the gravitational potential due to the Earth at a point which is at a height of $2R_E$ above the surface of the Earth, Mass of the Earth is 6×10^{24} kg, radius of the Earth = 6400 km and $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$.

Solution:

Given: $M = 6 \times 10^{24} \text{ kg}$,

$R_E = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$,

$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$,

$h = 2R_E$

To find: Gravitational potential (V)

Formula: $V = -\frac{GM}{r}$

Calculation: From formula,

$$\begin{aligned} V &= -\frac{GM}{(R_E + 2R_E)} \\ &= -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{3 \times 6.4 \times 10^6} \\ &= -\frac{6.67 \times 2}{6.4} \times 10^7 \end{aligned}$$

$$= -2.08 \times 10^7 \text{ J kg}^{-1}$$

Negative sign indicates the attractive nature of gravitational potential.

Gravitational potential due to Earth will be $2.08 \times 10^7 \text{ J kg}^{-1}$ towards the centre of the Earth.

[Note: According to definition of gravitational potential its SI unit is J/kg.]

11th Physics Digest Chapter 5 Gravitation Intext Questions and Answers

[Can you recall? \(Textbook Page No. 78\)](#)

Question 1.

i) What are Kepler's laws?

ii) What is the shape of the orbits of planets?

Answer:

1. The Kepler's laws are:

- Kepler's first law: The orbit of a planet is an ellipse with the Sun at one of the foci.
- Kepler's second law: The line joining the planet and the Sun sweeps equal areas in equal intervals of time.
- Kepler's third law: The square of orbital period of revolution of a planet around the Sun is directly proportional to the cube of the mean distance of the planet from the Sun.

2. The orbits of the planet are elliptical in shape.

Question 2.

When released from certain height why do objects tend to fall vertically downwards?

Answer:

When released from certain height, objects tend to fall vertically downwards because of the gravitational force exerted by the Earth.