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Maharashtra State Board 12th Commerce Maths Solutions Chapter 8 Probability Distributions Ex 8.1

Question 1.
Let X represent the difference between a number of heads and the number of tails obtained when a coin is tossed 6 times. What are the possible values of X?
Solution:
∵ A coin is tossed 6 times
S = {6H and 0T, 5H and 1T, 4H and 2T, 3H and 3T, 2H and 4T, 1H and 5T, 0H and 6T}
X: Difference between no. of heads and no. of tails.
X = 6 – 0 = 6
X = 5 – 1 = 4
X = 4 – 2 = 2
X = 3 – 3 = 0
X = 2 – 4 = -2
X = 1 – 5 = -4
X = 0 – 6 = -6
X = {-6, -4, -2, 0, 2, 4, 6}

Question 2.
An urn contains 5 red and 2 black balls. Two balls are drawn at random. X denotes the number of black balls drawn. What are the possible values of X?
Solution:
S : Two bolts are drawn from the Urn
S = {RR, RB, BR, BB}
X : No. of black balls
∴ X = {0, 1, 2}

Question 3.
Determine whether each of the following is a probability distribution. Give reasons for your answer.
(i)

x	0	1	2
$P(x)$	0.4	0.4	0.2

Solution:
Here, P(X = x) ≥ 0, ∀ x and
$\sum_{x=0}^{2} P(X=x)$ = p(0) + p(1) + p(2)
= 0.4 + 0.4 + 0.2
= 1
∴ The function is a p.m.f.

(ii)

x	0	1	2	3	4
$P(x)$	0.1	0.5	0.2	–0.1	0.3

Solution:
Here, p(3) = -0.1 < 0
∴ P(X = x) $\ngeq$ 0, ∀ x
∴ The function is not a p.m.f.

(iii)

x	0	1	2
$P(x)$	0.1	0.6	0.3

Solution:
Here, P(X = x) ≥ 0, ∀ x and
$\sum_{x=0}^{2} P(X=x)$ = p(0) + p(1) + p(2)
= 0.1 + 0.6 + 0.3
= 1
∴ The function is a p.m.f.

(iv)

z	3	2	1	0	−1
$P(z)$	0.3	0.2	0.4	0.05	0.05

Solution:
Here, P(Z = z) ≥ 0, ∀ z and
$\sum_{x=-1}^{3} P(Z=z)$ = p(-1) + p(0) + p(1) + p(2) + p(3)
= 0.05 + 0 + 0.4 + 0.2 + 0.3
= 0.95
≠ 1
∴ The function is not a p.m.f.

(v)

y	−1	0	1
$P(y)$	0.6	0.1	0.2

Solution:
Here, P(Y = y) ≥ 0, ∀ y and
$\sum_{x=-1}^{2} P(Y=y)$ = p(-1) + p(0) + p(1)
= 0.1 + 0.6 + 0.2
= 0.9
≠ 1
∴ The function is not a p.m.f.

(vi)

x	0	1	2
$P(x)$	0.3	0.4	0.2

Solution:
Here, P(X = x) ≥ 0, ∀ x and
$\sum_{x=0}^{0} P(X=x)$ = p(-2) + p(-1) + p(0)
= 0.3 + 0.4 + 0.2
= 0.9
≠ 1
∴ The function is not a p.m.f.

Question 4.
Find the probability distribution of
(i) number of heads in two tosses of a coin,
(ii) number of trails in three tosses of a coin,
(iii) number of heads in four tosses of a coin.
Solution:
(i) S: Coin is tossed two times
S = {HH, HT, TH, TT}
n(S) = 4
X: No. of heads
Range of X = {0, 1, 2}
p.m.f. Table

x	0	1	2
p	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

(ii) S: 3 coin are tossed
S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
n(S) = 8
X: No. of heads
Range of X = {0, 1, 2, 3}
p.m.f. Table

x	0	1	2	3
p	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii) S: Four coin are tossed
S = {HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}
n(S) = 16
X: No. of heads
Range of X = {0, 1, 2, 3, 4}
p.m.f. Table

x	0	1	2	3	4
p	$\frac{1}{16}$	$\frac{4}{16}=\frac{1}{4}$	$\frac{6}{16}=\frac{3}{8}$	$\frac{4}{16}=\frac{1}{4}$	$\frac{1}{16}$

Question 5.
Find the probability distribution of the number of successes in two tosses of a die if successes are defined as getting a number greater than 4.
Solution:
S = A die is tossed 2 times
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
n(S) = 36
X = No. getting greater than 4
Range of X = {0, 1, 2}
p(0) = 1636=49
p(1) = 1636=49
p(2) = 436=19

x	0	1	2
p	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

Question 6.
A sample of 4 bulbs is drawn at random with replacement from a lot of 30 bulbs which includes 6 defective bulbs. Find the probability distribution of the number of defective bulbs.
Solution:
Total no. of bulbs = 30
No. of defective bulbs = 6
A sample of 4 bulbs are drawn from 30 bulbs.
∴ n(S) = 30C4
∴ No. of non-defective bulbs = 24
Let X = No. of defective bulbs drawn in sample of 4 bulbs.

$\therefore X = \{0, 1, 2, 3, 4\}$.

For $X = 0$, $p(0) = \dfrac{{}^6C_0 \times {}^{24}C_4}{{}^{30}C_4}$

$= 1 \times \dfrac{24 \times 23 \times 22 \times 21}{1 \times 2 \times 3 \times 4} \times \dfrac{1 \times 2 \times 3 \times 4}{30 \times 29 \times 28 \times 27}$

$= \dfrac{3542}{9135} = \dfrac{506}{1305}$

For $X = 1$, $p(1) = \dfrac{{}^6C_1 \times {}^{24}C_3}{{}^{30}C_4}$

$= 6 \times \dfrac{24 \times 23 \times 22}{1 \times 2 \times 3} \times \dfrac{1 \times 2 \times 3 \times 4}{30 \times 29 \times 28 \times 27}$

$= \dfrac{4048}{9135}$

For $X = 2$, $p(2) = \dfrac{{}^6C_2 \times {}^{24}C_2}{{}^{30}C_4}$

$= \dfrac{6 \times 5}{1 \times 2} \times \dfrac{24 \times 23}{1 \times 2} \times \dfrac{1 \times 2 \times 3 \times 4}{30 \times 29 \times 28 \times 27}$

$= \dfrac{1380}{9135} = \dfrac{92}{609}$

For $X = 3$, $p(3) = \dfrac{{}^6C_3 \times {}^{24}C_1}{{}^{30}C_4}$

$= \dfrac{6 \times 5 \times 4}{1 \times 2 \times 3} \times 24 \times \dfrac{1 \times 2 \times 3 \times 4}{30 \times 29 \times 28 \times 27}$

$= \dfrac{160}{9135} = \dfrac{32}{1827}$

For $X = 4$, $p(4)$

$= \dfrac{{}^6C_4 \times {}^{24}C_0}{{}^{30}C_4}$

$= \dfrac{6 \times 5 \times 4 \times 3}{1 \times 2 \times 3 \times 4} \times 1 \times \dfrac{1 \times 2 \times 3 \times 4}{30 \times 29 \times 28 \times 27}$

$= \dfrac{5}{9135} = \dfrac{1}{1827}$

$\therefore$ Probability distribution of no. of defective bulbs is

x	0	1	2	3	4
$p(x)$	$\dfrac{506}{1305}$	$\dfrac{4048}{9135}$	$\dfrac{92}{609}$	$\dfrac{32}{1827}$	$\dfrac{1}{1827}$

Question 7.
A coin is biased so that the head is 3 times as likely to occur as the tail. Find the probability distribution of a number of tails in two tosses.
Solution:
Here, the head is 3 times as likely to occur as the tail.
i.e., If 4 times coin is tossed, 3 times there will be a head and 1 time there will be the tail.
∴ p(H) = 34 and p(T) = 14
Let X : No. of tails in two tosses.
And coin is tossed twice.
∴ X = {0, 1, 2}
For X = 0,
p(0) = p(both heads)
= p(H) × p(H)
= 34×34
= 916
For X = 1,
p(1) = p(HT or TH)
= p(HT) + p(TH)
= p(H) × p(T) + p(T) × p(H)
= 34×14+14×34
= 616

For X = 2,
p(2) = p(both tails)
= p(T) × p(T)
= 14×14
= 116
The probability distribution of the number of tails in two tosses is

x	0	1	2
p	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

Question 8.
A random variable X has the following probability distribution:

x	1	2	3	4	5	6	7
$P(x)$	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Determine (i) k, (ii) P(X < 3), (iii) P(0 < X < 3), (iv) P(X > 4).
Solution:
(i) It is a p.m.f. of r.v. X
∴ Σp(x) = 1
∴ p(1) + p(2) + p(3) + p(4) + p(5) + p(6) + p(7) = 1
∴ k + 2k + 2k + 3k + k2 + 2k2 + (7k2 + k) = 1
∴ 10k2 + 9k = 1
∴ 10k2 + 9k – 1 = 0
∴ 10k2 + 10k – k – 1 = 0
∴ 10k(k + 1) – (k + 1) = 0
∴ (10k – 1)(k + 1) = 0
∴ 10k – 1 = 0 or k + 1 = 0
∴ k = 110 or k = -1
but k = -1 is not accepted
∴ k = 12 is accepted

(ii) P(X < 3) = p(1) + p(2)
= k + 2k
= 3k
= 3 × 110
= 310

(iii) P(0 < X < 3) = p(1) + p(2)
= k + 2k
= 3k
= 3 × 110
= 310

(iv) P(X > 4) = p(5) + p(6) + p(7)
= k2 + 2k2 + (7k2 + k)
= 10k2 + k
= 10(110)2+110
= 210
= 15

Question 9.
Find expected value and variance of X using the following p.m.f.

x	–2	–1	0	1	2
$P(x)$	0.2	0.3	0.1	0.15	0.25

Solution:

x	p	px	x^2	x^2p
–2	0.2	–0.4	4	0.8
–1	0.3	–0.3	1	0.3
0	0.1	0	0	0
1	0.15	0.15	1	0.15
2	0.25	0.5	4	1
Total		**–0.05**		**2.25**

$E(X) = \Sigma xp =$

E(X) = Σxp = -0.05
V(X) = $\Sigma x^2p - (\Sigma xp)^2$
= $2.25 - (-0.05)^2$
= 2.25 – 0.0025
= 2.2475

Question 10.
Find expected value and variance of X, the number on the uppermost face of a fair die.
Solution:
S : A fair die is thrown
S = {1, 2, 3, 4, 5, 6}
n(S) = 6
X: No obtained on uppermost face of die
Range of X = {1, 2, 3, 4, 5, 6}

x	p	px	x^2	x^2p
1	$\frac{1}{6}$	$\frac{1}{6}$	1	$\frac{1}{6}$
2	$\frac{1}{6}$	$\frac{2}{6}$	4	$\frac{4}{6}$
3	$\frac{1}{6}$	$\frac{3}{6}$	9	$\frac{9}{6}$
4	$\frac{1}{6}$	$\frac{4}{6}$	16	$\frac{16}{6}$
5	$\frac{1}{6}$	$\frac{5}{6}$	25	$\frac{25}{6}$
6	$\frac{1}{6}$	$\frac{6}{6}$	36	$\frac{36}{6}$
Total		$\frac{21}{6}$		$\frac{91}{6}$

E(X) = Σxp = $\frac{21}{6} = \frac{7}{2}$ = 3.5
V(X) = $\Sigma x^2p - (\Sigma xp)^2$
= $\frac{91}{6} - (3.5)^2$
= 15.17 – 12.25
= 2.92

Question 11.
Find the mean of the number of heads in three tosses of a fair coin.
Solution:
S : A coin is tossed 3 times
S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
n(S) = 8
Range of X = {0, 1, 2, 3}

x	p	px
0	$\frac{1}{8}$	0
1	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$
Total		$\frac{12}{8}$

∴ Mean = E(X) = Σxp = 128=32 = 1.5

Question 12.
Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.
Solution:
S : Two dice are thrown
S = {(1, 1), (1, 2), (1, 3), ……, (6, 6)}
n(S) = 36
Range of X = {0, 1, 2}
First 6 positive integers are 1, 2, 3, 4, 5, 6
X = Larger two numbers selected
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
n(S) = 36

x	p	px
0	$\frac{25}{36}$	0
1	$\frac{10}{36}$	$\frac{10}{36}$
2	$\frac{1}{36}$	$\frac{2}{36}$
Total		$\frac{12}{36}$

∴ E(X) = Σxp = 1236=13

Question 13.
Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers. Find E(X).
Solution:
First 6 positive integers are 1, 2, 3, 4, 5, 6
X : The larger of the selected two numbers
S = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)}
n(S) = 30

x	p	px
2	$\frac{2}{30}$	$\frac{4}{30}$
3	$\frac{4}{30}$	$\frac{12}{30}$
4	$\frac{6}{30}$	$\frac{24}{30}$
5	$\frac{8}{30}$	$\frac{40}{30}$
6	$\frac{10}{30}$	$\frac{60}{30}$
Total		$\frac{140}{30}$

E(X) = Σxp = 14030=143 = 4.67

Question 14.
Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance of X.
Solution:
S : Two fair dice are rolled
S = {(1, 1), (1, 2), (1, 4), ……, (6, 6)}
n(S) = 36
X : Sum of the two numbers.
Range of X = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

x	p	xp	x^2p
2	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$
3	$\frac{2}{36}$	$\frac{6}{36}$	$\frac{18}{36}$
4	$\frac{3}{36}$	$\frac{12}{36}$	$\frac{48}{36}$
5	$\frac{4}{36}$	$\frac{20}{36}$	$\frac{100}{36}$
6	$\frac{5}{36}$	$\frac{30}{36}$	$\frac{180}{36}$
7	$\frac{6}{36}$	$\frac{42}{36}$	$\frac{294}{36}$
8	$\frac{5}{36}$	$\frac{40}{36}$	$\frac{320}{36}$
9	$\frac{4}{36}$	$\frac{30}{36}$	$\frac{324}{36}$
10	$\frac{3}{36}$	$\frac{30}{36}$	$\frac{300}{36}$
11	$\frac{2}{36}$	$\frac{22}{36}$	$\frac{220}{36}$
12	$\frac{1}{36}$	$\frac{12}{36}$	$\frac{144}{36}$
Total		$\frac{252}{36}$	$\frac{1952}{36}$

V(X) = $\Sigma x^2p - (\Sigma xp)^2$
= 195236−(25236)2
= $54.22 - (7)^2$
= 5.22
SD(X) = √V(X) = √5.22 = 2.28

Question 15.
A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. If X denotes the age of a randomly selected student, find the probability distribution of X. Find the mean and variance of X.

Solution:

x (Age)	No. of students	$p = \frac{\text{No. of students}}{\text{Total students}}$	xp	x^2p
14	2	$\frac{2}{15}$	$\frac{28}{15}$	$\frac{392}{15}$
15	1	$\frac{1}{15}$	$\frac{15}{15}$	$\frac{225}{15}$
16	2	$\frac{2}{15}$	$\frac{32}{15}$	$\frac{512}{15}$
17	3	$\frac{3}{15}$	$\frac{51}{15}$	$\frac{867}{15}$
18	1	$\frac{1}{15}$	$\frac{18}{15}$	$\frac{324}{15}$
19	2	$\frac{2}{15}$	$\frac{38}{15}$	$\frac{722}{15}$
20	3	$\frac{3}{15}$	$\frac{60}{15}$	$\frac{1200}{15}$
21	1	$\frac{1}{15}$	$\frac{21}{15}$	$\frac{441}{15}$
Total			$\frac{263}{15}$	$\frac{4683}{15}$

$$E(X) = \Sigma\, xp = \frac{263}{15} = 17.53$$

$$V(X) = \Sigma x^2p - (\Sigma xp)^2$$
$$= \frac{4683}{15} - \left(\frac{263}{15}\right)^2$$
$$= \frac{4683}{15} - \frac{69169}{225}$$
$$= \frac{70245 - 69169}{225}$$
$$= \frac{1076}{225} = 4.78$$
$$SD(X) = \sqrt{V(X)} = \sqrt{4.78} = 2.18$$

Question 16.
70% of the member's favour and 30% oppose a proposal in a meeting. The random variable X takes the value 0 if a member opposes the proposal and the value 1 if a member is in favour. Find E(X) and V(X).
Solution:

x	p	xp	x^2	x^2p
0	0.3	0	0	0
1	0.7	0.7	1	0.7
		0.7		0.7

E(X) = Σxp = 0.7
V(X) = $\Sigma x^2p - (\Sigma xp)^2$
= $0.7 - (0.7)^2$
= 0.7 – 0.49
= 0.21

Maharashtra State Board 12th Commerce Maths Solutions Chapter 8 Probability Distributions Ex 8.2

Question 1.
Check whether each of the following is p.d.f.
(i) f(x)={x2−x for 0≤x≤1 for 1<x≤2
Solution:
Given function is
f(x) = x, 0 ≤ x ≤ 1
Each f(x) ≥ 0, as x ≥ 0.

$$\text{and} \int_0^1 f(x)dx = \int_0^1 x dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$$

Also, $f(x) = 2 - x$, $1 \le x \le 2$
$\Rightarrow$ Each $f(x) \ge 0$, as $x \le 2$.

$$\text{and} \int_1^2 f(x)dx = \int_1^2 (2-x)dx = 2\int_1^2 1dx - \int_1^2 xdx = 2[x]_1^2 - \left(\frac{x^2}{2}\right)_1^2 = 2(2-1) - \frac{1}{2}(4-1) = 1 - \frac{3}{2} = \frac{1}{2}$$

Now, for the total range of $0 \le x \le 2$.

$$\int_0^2 f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx = \frac{1}{2} + \frac{1}{2} = 1$$

∴ The given function is a p.d.f. of x.

(ii) f(x) = 2 for 0 < x < 1
Solution:
Given function is
f(x) = 2 for 0 < x < 1 Each f(x) > 0,

$$\text{but} \int_0^1 f(x)dx = \int_0^1 2dx = 2[x]_0^1 = 2(1) = 2 > 1.$$

∴ The given function is not a p.d.f.

Question 2.
The following is the p.d.f. of a r.v. X.
f(x)={x8 0 for 0<x<4 otherwise
Find (i) P(X < 1.5), (ii) P(1 < X < 2), (iii) P(X > 2)
Solution:

$$f(x) = \begin{cases} \frac{x}{8} & \text{for } 0 < x < 4, \\ 0 & \text{otherwise} \end{cases}$$

(i) $P(X < 1.5) = \int_0^{1.5} \frac{x}{8}\, dx$

$$= \frac{1}{8}\int_0^{1.5} x\, dx$$

$$= \frac{1}{8}\left[\frac{x^2}{2}\right]_0^{1.5}$$

$$= \frac{1}{8}\left[(1.5)^2 - 0^2\right]$$

$$= \frac{2.25}{16} = 0.14$$

(ii) $P(1 < X < 2) = \int_1^2 \frac{x}{8}\, dx = \frac{1}{8}\int_1^2 x\, dx$

$$= \frac{1}{8}\left[\frac{x^2}{2}\right]_1^2 = \frac{1}{16}[2^2 - 1^2]$$

$$= \frac{1}{16}[4-1] = \frac{3}{16} = 0.1875$$

(iii) $P(X > 2) = \int_2^4 \frac{x}{8}\, dx = \frac{1}{8}\int_2^4 x\, dx$

$$= \frac{1}{8}\left[\frac{x^2}{2}\right]_2^4 = \frac{1}{16}[4^2 - 2^2]$$

$$= \frac{1}{16}[16-4]$$

$$= \frac{12}{16} \qquad = \frac{3}{4}$$

$$= 0.75$$

Question 3.
It is felt that error in measurement of reaction temperature (in Celsius) in an experiment is a continuous r.v. with p.d.f.

$$f(x) = \begin{cases} \frac{x^3}{64} & \text{for } 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

(i) Verify whether f(x) is a p.d.f.
(ii) Find $P(0 < X \le 1)$.
(iii) Find the probability that X is between 1 and 3.
Solution:
(i) f(x) is p.d.f. of r.v. X if
(a) $f(x) \ge 0,\ \forall\ x \in R$
(b) $\int_0^4 f(x)\,dx = 1$

i.e.(a) $f(x) = \frac{x^3}{64}, f(x) \geq 0, 0 \leq x \leq 4$

(b) $\int_0^4 f(x)dx = \int_0^4 \frac{x^3}{64} dx$

$$= \frac{1}{64}\left[\frac{x^4}{4}\right]_0^4$$

$$= \frac{1}{256}[4^4 - 0^4]$$

$$= \frac{256}{256}$$

$$= 1$$

Hence, $f(x)$ is a p.d.f. of r.v. X

(ii) $P(0 < X \leq 1) = \int_0^1 \frac{x^3}{64} dx$

$$= \frac{1}{64}\left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{1}{256}[1^4 - 0]$$

$$= \frac{1}{256}$$

(iii) $P(1 < X < 3) = \int_1^3 \frac{x^3}{64} dx = \frac{1}{64}\left[\frac{x^4}{4}\right]_1^3$

$$= \frac{1}{256}[3^4 - 1^4]$$

$$= \frac{80}{256}$$

$$= \frac{5}{16}$$

Question 4.
Find k, if the following function represents the p.d.f. of a r.v. X.
(i) f(x)={kx0 for 0<x<2 otherwise
Also find P[14 < X < 12]
Solution:

$$f(x) = \begin{cases} kx & \text{for } 0 < x < 2, \\ 0 & \text{otherwise} \end{cases}$$

$$\because f(x) = kx,\ 0 < x < 2$$

$$\therefore \int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\therefore \int_0^2 kx\, dx = 1$$

$$\therefore k\int_0^2 x\, dx = 1$$

$$\therefore k\left[\frac{x^2}{2}\right]_0^2 = 1$$

$$\therefore k\left(\frac{2^2 - 0^2}{2}\right) = 1$$

$$\therefore 2k = 1$$

$$\therefore k = \frac{1}{2}$$

$$\therefore f(x) = kx = \frac{x}{2}$$

$$P\left(\frac{1}{4} < X < \frac{3}{2}\right) = \int_{1/4}^{3/2} \frac{x}{2}\, dx$$

$$= \frac{1}{2}\int_{1/4}^{3/2} x dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2}\right]_{\frac{1}{4}}^{\frac{3}{2}}$$

$$= \frac{1}{4}\left[\left(\frac{3}{2}\right)^2 - \left(\frac{1}{4}\right)^2\right]$$

$$= \frac{1}{4}\left[\frac{9}{4} - \frac{1}{16}\right]$$

$$= \frac{1}{4}\left[\frac{36-1}{16}\right]$$

$$= \frac{35}{64} \qquad = 0.55$$

(ii) $f(x)=\{kx(1-x)0$ for $0<x<1$ otherwise

Also find (a) P[14 < X < 12], (b) P[X < 12]

Solution:

We know that

$$\int_0^1 f(x)\, dx \quad = 1$$

$$\therefore \int_0^1 kx(1-x)\, dx = 1$$

$$\therefore k\int_0^0 \left(x - x^2\right) dx = 1$$

$$\therefore k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 \quad = 1$$

$$\therefore k\left[\frac{1}{2} - \frac{1}{3}\right] \quad = 1$$

$$\therefore k \cdot \frac{1}{6} \quad = 1$$

$$\therefore k \quad = 6$$

$$\therefore f(x) = 6x(1-x)$$

(a)

$$P\left(\frac{1}{4}<X<\frac{1}{2}\right)=\int_{1/4}^{1/2}6x(1-x)dx=6\int_{1/4}^{1/2}(x-x^2)dx$$

$$=6\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_{1/4}^{1/2}$$

$$=6\left[\frac{1}{2}\left(\frac{1}{4}-\frac{1}{16}\right)-\frac{1}{3}\left(\frac{1}{8}-\frac{1}{64}\right)\right]$$

$$=3\left(\frac{3}{16}\right)-2\left(\frac{7}{64}\right)$$

$$=\frac{9}{16}-\frac{7}{32}$$

$$=\frac{18-7}{32}$$

$$=\frac{11}{32}$$

$$=0.34375$$

(b) $$P\left(X<\frac{1}{2}\right)=\int_{0}^{1/2}6x(1-x)dx=6\int_{0}^{1/2}(x-x^2)dx$$

$$=6\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_{0}^{1/2}$$

$$=6\left[\frac{1}{2}\left(\frac{1}{4}-0\right)-\frac{1}{3}\left(\frac{1}{8}-0\right)\right]$$

$$=3\left(\frac{1}{4}\right)-2\left(\frac{1}{8}\right)$$

$$=\frac{3}{4}-\frac{1}{4}$$

$$=\frac{2}{4}$$

$$=0.5$$

Question 5.
Let X be the amount of time for which a book is taken out of the library by a randomly selected student and suppose that X has p.d.f.
f(x)={0.5x0 for 0≤x≤2 otherwise
Calculate (i) P(X ≤ 1), (ii) P(0.5 ≤ X ≤ 1.5), (iii) P(X ≥ 1.5).
Solution:
Given p.d.f. of X is f(x) = 0.5x for 0 ≤ x ≤ 2
∴ Its c.d.f. F(x) is given by

$$f(x)=\int_0^x f(y)dy, \text{ where } f(y)=0.5y$$

$$=\int_0^x 0.5y\,dy=0.5\left[\frac{y^2}{2}\right]_0^x=0.25x^2$$

(i) P(X < 1) = F(1)
= $0.25(1)^2$
= 0.25

(ii) P(0.5 < X < 1.5) = F(1.5) – F(0.5)
= $0.25(1.5)^2 - 0.25(0.5)^2$
= 0.25[2.25 – 0.25]
= 0.25(2)
= 0.5

(iii) P(X ≥ 1.5) = 1 – P(X ≤ 1.5)
= 1 – F(1.5)
= $1 - 0.25(1.5)^2$
= 1 – 0.25(2.25)

= 1 – 0.5625
= 0.4375

Question 6.
Suppose X is the waiting time (in minutes) for a bus and its p.d.f. is given by
$f(x)=\begin{cases}\frac{1}{5} & \text{for } 0 \le x \le 5\\ 0 & \text{otherwise}\end{cases}$
Find the probability that (i) waiting time is between 1 and 3 minutes, (ii) waiting time is more than 4 minutes.
Solution:
p.d.f. of r.v. X is given by
f(x) = $\frac{1}{5}$ for 0 ≤ x ≤ 5
This is a constant function.
(i) Probability that waiting time X is between 1 and 3 minutes

$$\text{i.e. } P(1 < X < 3) = \int_1^3 f(x)\,dx = \int_1^3 \frac{1}{5}\,dx = \frac{1}{5}[x]_1^3 = \frac{1}{5}[3-1] = \frac{2}{5} = 0.4$$

(ii) Probability that waiting time X is more than 4 minutes

$$\text{i.e. } P(X > 4) = \int_4^5 f(x)\,dx = \int_4^5 \frac{1}{5}\,dx = \frac{1}{5}[x]_4^5 = \frac{1}{5}(5-4) = \frac{1}{5} = 0.2$$

Question 7.
Suppose error involved in making a certain measurement is a continuous r.v. X with p.d.f.
$f(x)=\begin{cases}k(4-x^2) & \text{for } -2 \le x \le 2\\ 0 & \text{otherwise}\end{cases}$
Compute (i) P(X > 0), (ii) P(-1 < X < 1), (iii) P(X < -0.5 or X > 0.5)
Solution:
Since given f(x) is a p.d.f. of r.v. X
Since -2 ≤ x ≤ 2
∴ $x^2 \le 4$
∴ $4 - x^2 \ge 0$
∴ $k(4 - x^2) \ge 0$
∴ k ≥ 0 [∵ f(x) ≥ 0]

Also $\int_{-2}^{2} f(x)\, dx = 1$

$\therefore \int_{-2}^{2} k(4 - x^2)\, dx = 1$

$\therefore 2\int_{0}^{2} k(4 - x^2)\, dx = 1$ ($\because 4 - x^2$ is even function)

$\therefore 2k\int_{0}^{2} (4 - x^2)\, dx = 1$

$\therefore 2k\left[4x - \frac{x^3}{3}\right]_0^2 = 1$

$\therefore 2k\left(8 - \frac{8}{3}\right) = 1$

$\therefore 2k \cdot \frac{16}{3} = 1$

$\therefore k = \frac{3}{32}$

$\therefore$ p.d.f. of X is

$f(x) = \frac{3}{32}\ (4 - x^2)\ for\ -2 \le x \le 2$

(i) P(X > 0)

$= \int_{0}^{2} f(x)\, dx$

$= \int_{0}^{2} \frac{3}{32}(4 - x^2)\, dx$

$= \frac{3}{32}\int_{0}^{2} (4 - x^2)\, dx$

$= \frac{3}{16}\left[4x - \frac{x^3}{3}\right]_0^2$

$= \frac{3}{32}\left[8 - \frac{8}{3}\right]$

$= \frac{3}{32} \cdot \frac{16}{3}$

$= \frac{1}{2}$
$= 0.5$

(ii) P(–1 < X < 1)

$$= \int_{-1}^{1} f(x)\,dx$$
$$= \int_{-1}^{1} \frac{3}{32}(4-x^2)\,dx$$
$$= 2\cdot\frac{3}{32}\int_{0}^{1}(4-x^2)\,dx$$
$$= \frac{3}{16}\left[4x-\frac{x^3}{3}\right]_0^1$$
$$= \frac{3}{16}\left[4-\frac{1}{3}\right] \qquad = \frac{3}{16}\cdot\frac{11}{3}$$
$$= \frac{11}{16} \qquad = 0.6875$$

(iii) P(X < –0.5 or X > 0.5)

$$= \int_{-2}^{-0.5} f(x)\,dx + \int_{0.5}^{2} f(x)\,dx$$
$$= \int_{-2}^{-0.5} \frac{3}{32}(4-x^2)\,dx + \int_{0.5}^{2}\frac{3}{32}(4-x^2)\,dx$$

Since $4 - x^2$ is a symmetric function
Area under the curve between –2 and –0.5 and 0.5 and 2 are equal.

$$= 2\int_{0.5}^{2}\frac{3}{32}(4-x^2)\,dx = \frac{3}{16}\int_{0.5}^{2}(4-x^2)\,dx$$
$$= \frac{3}{16}\left[4x-\frac{x^3}{3}\right]_{1/2}^{2} = \frac{3}{16}\left[8-\frac{8}{3}-2+\frac{1}{24}\right]$$
$$= \frac{3}{16}\cdot\frac{192-64-48+1}{24} = \frac{3}{16}\cdot\frac{81}{24} = \frac{81}{128}$$
$$= 0.6328$$

Question 8.
Following is the p.d.f. of a continuous r.v. X.

$f(x)=\begin{cases}\frac{x}{8} & \text{for } 0<x<4\\ 0 & \text{otherwise}\end{cases}$

(i) Find an expression for the c.d.f. of X.
(ii) Find F(x) at x = 0.5, 1.7, and 5.

Solution:
The p.d.f. of a continuous r.v. X is

$f(x)=\begin{cases}\frac{x}{8} & \text{for } 0<x<4\\ 0 & \text{otherwise}\end{cases}$

(i) c.d.f. of continuous r.v. X is given by

$$f(x) = \int_{-\infty}^{x} f(y)dy$$
$$\therefore f(x) = \int_{0}^{x}\frac{y}{8}dy = \frac{1}{8}\int_{0}^{x} ydy$$
$$= \frac{1}{8}\left[\frac{y^2}{2}\right]_0^x$$
$$= \frac{1}{16}[x^2 - 0^2]$$
$$= \frac{x^2}{16}$$

(ii) F(0.5) = $\frac{(0.5)^2}{16} = \frac{0.25}{16} = \frac{1}{64}$ = 0.015

F(1.7) = $\frac{(1.7)^2}{16} = \frac{2.89}{16}$ = 0.18

For any of x greater than or equal to 4, F(x) = 1
∴ F(5) = 1

Question 9.
The p.d.f. of a continuous r.v. X is

$f(x) = \begin{cases} \frac{3x^2}{8} & \text{for } 0<x<2 \\ 0 & \text{otherwise} \end{cases}$

Determine the c.d.f. of X and hence find (i) P(X < 1), (ii) P(X < -2), (iii) P(X > 0), (iv) P(1 < X < 2).
Solution:
The p.d.f. of a continuous r.v. X is

$$f(x) = \begin{cases} \frac{3x^2}{8} & \text{for } 0 < x < 2, \\ 0 & \text{otherwise} \end{cases}$$

c.d.f. of X is given by

$$F(x) = \int_0^x f(y)dy$$

$$= \int_0^x \frac{3y^2}{8}dy$$

$$= \frac{3}{8}[\frac{y^3}{3}]_0^x$$

$$= \frac{1}{8}[x^3 - 0^3]$$

$$= \frac{x^3}{8}$$

(i) $P(X < 1) = F(1) = \frac{1^3}{8} = \frac{1}{8}$

(ii) $P(X < -2) = 0 \quad \because \quad$ Range of X is (0, 2)

(iii) $P(X > 0) = 1 - P(X \leq 0)$

$$= 1 - F(0)$$

$$= 1 - [\frac{0^3}{8}]$$

$$= 1 - 0$$

$$= 1$$

(iv) $P(1 < X < 2) = F(2) - F(1)$

$$= \left[\frac{2^3}{8} - \frac{1^3}{8}\right]$$

$$= \frac{8-1}{8}$$

$$= \frac{7}{8}$$

Question 10.
If a r.v. X has p.d.f.

$f(x) = \begin{cases} \frac{c}{x} & \text{for } 1<x<3, c>0 \\ 0 & \text{otherwise} \end{cases}$

Find c, E(X) and V(X). Also find f(x).
Solution:
The p.d.f. of r.v. X is
f(x) = $\frac{c}{x}$, 1 < x < 3, c > 0
For p.d.f. of X, we have

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\therefore \int_1^3 \frac{c}{x}dx = 1$$

$$\therefore c[\log x]_1^3 = 1$$

$$\therefore c[\log 3 - \log 1] = 1$$

$$\therefore c\log\left(\frac{3}{1}\right) = 1$$

$$\therefore c\log 3 = 1$$

$$\therefore c = \frac{1}{\log 3}$$

$$\therefore f(x) = \frac{1}{x\log 3}$$

$$\begin{aligned} E(X) &= \int_1^3 xf(x)dx \\ &= \int_1^3 x\cdot\frac{1}{x\log 3}dx \\ &= \int_1^3 \frac{1}{\log 3}dx \\ &= \frac{1}{\log 3}[x]_1^3 \\ &= \frac{1}{\log 3}(3-1) \end{aligned}$$

Hence, $E(X) = \dfrac{2}{\log 3}$

$$\begin{aligned} E(X^2) &= \int_1^3 x^2\cdot f(x)dx \\ &= \int_1^3 x^2\cdot\frac{1}{x\log 3}dx \\ &= \frac{1}{\log 3}\int_1^3 xdx \\ &= \frac{1}{\log 3}[\frac{x^2}{2}]_1^3 \end{aligned}$$

$$= \frac{1}{\log 3}[\frac{9}{2} - \frac{1}{2}]$$
$$= \frac{1}{\log 3} \cdot 4$$
$$= \frac{4}{\log 3}$$

$$\therefore V(X) = E(X^2) - [E(X)]^2$$
$$= \frac{4}{\log 3} - (\frac{2}{\log 3})^2$$
$$= \frac{4}{\log 3} - \frac{4}{(\log 3)^2}$$

c.d.f. of $F(x)$ is given by

$$F(x) = \int_1^x \frac{1}{y \log 3} dy$$
$$= \frac{1}{\log 3} \int_1^x \frac{1}{y} dy$$
$$= \frac{1}{\log 3} [\log y]_1^x$$
$$= \frac{1}{\log 3} [\log x - \log 1]$$
$$= \frac{\log x}{\log 3}$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 8 Probability Distributions Ex 8.3

Question 1.
A die is thrown 4 times. If 'getting an odd number' is a success, find the probability of (i) 2 successes (ii) at least 3 successes (iii) at most 2 successes.
Solution:
X: Getting an odd no.
p: Probability of getting an odd no.
A die is thrown 4 times
$\therefore n = 4$
$\because p = \frac{3}{6} = \frac{1}{2}$
$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$
$\because X \sim B(3, \frac{1}{2})$
$\therefore p(x) = {}^nC_x p^x q^{n-x}$
(i) P(Two Successes)

$$P(X=2) = {}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{4-2}$$
$$= 6\times\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$$
$$= 6\times\left(\frac{1}{2}\right)^4$$
$$= 6\times\frac{1}{16}$$
$$= \frac{3}{8}$$

(ii) P(Atleast 3 Successes)

$$P(X\geq 3) = p(3)+p(4)$$
$$= {}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{4-3} + {}^4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{4-4}$$
$$= 4\times\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) + 1\times\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0$$
$$= 4\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4$$
$$= \left(\frac{1}{2}\right)^4(4+1)$$
$$= \left(\frac{5}{16}\right)$$

(iii) P(Atmost 2 Successes)

$$P(X\leq 2) = p(0)+p(1)+p(2)$$
$$= {}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{4-0} + {}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{4-1}$$
$$+ {}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{4-2}$$
$$= 1\times1\times\left(\frac{1}{2}\right)^4 + 4\times\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3 + 6$$
$$\times\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$$
$$= \left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^4 + 6\left(\frac{1}{2}\right)^4$$
$$= \left(\frac{1}{2}\right)^4(1+4+6)$$
$$= \frac{11}{16}$$

Question 2.
A pair of dice is thrown 3 times. If getting a doublet is considered a success, find the probability of two successes.
Solution:
n: No. of times die is thrown = 3
X: No. of doublets
p: Probability of getting doublets
Getting a doublet means, same no. is obtained on 2 throws of a die
There are 36 outcomes

No. of doublets are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

$$\therefore \quad p = \frac{6}{36} = \frac{1}{6}$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

$$\because \quad X \sim B\left(3, \frac{1}{6}\right)$$

$\therefore$ $p(x) = {}^nC_x\, p^x\, q^{n-x}$

P(getting two successes)

$$= P(X = 2)$$

$$= {}^3C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{3-2}$$

$$= 3 \times \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

$$= 3 \times \frac{25}{36 \times 36}$$

$$= \frac{25}{432}$$

Question 3.
There are 10% defective items in a large bulk of items. What is the probability that a sample of 4 items will include not more than one defective item?
Solution:
n: No of sample items = 4
X: No of defective items
p: Probability of getting defective items
$\therefore$ p = 0.1
$\therefore$ q = 1 – p = 1 – 0.1 = 0.9
X ~ B(4, 0.1)
$\therefore$ p(x) = ${}^nC_x p^x q^{n-x}$
P(Not include more than 1 defective)
P(X ≤ 1) = p(0) + p(1)
= ${}^4C_0 (0.1)^0 (0.9)^4 + {}^4C_1 (0.1)^1 (0.9)^{4-1}$
= $1 \times 1 \times (0.9)^4 + 4 \times 0.1 \times (0.9)^3$
= $(0.9)^3 [0.9 + 0.4]$
= $(0.9)^3 \times 1.3$
= 0.977

Question 4.
Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability that (i) all the five cards are spades, (ii) only 3 cards are spades, (iii) none is a spade.
Solution:
X: No. of spade cards
Number of cards drawn
$\therefore$ n = 5
p: Probability of getting spade card

$$\therefore p = \frac{13}{52} = \frac{1}{4}$$

$$\therefore q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\because X \sim B\left(5, \frac{1}{4}\right)$$

$\therefore$ $p(x) = {}^nC_x\, p^x\, q^{n-x}$

(i) P(All five cards are spades)

$$P(X = 5) = {}^5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^{5-5}$$
$$= 1 \times \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0$$
$$= 1 \times \frac{1}{1024} \times 1$$
$$= \frac{1}{1024}$$

(ii) P(Only 3 cards are spades)

$$P(X = 3) = {}^5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{5-3}$$
$$= 10 \times \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2$$
$$= 10 \times \frac{1}{64} \times \frac{9}{16}$$
$$= \frac{90}{1024} = \frac{45}{512}$$

(iii) P(None is a spade)

$$P(X = 0) = {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{5-0}$$
$$= 1 \times 1 \times \frac{3^5}{4^5}$$
$$= \frac{243}{1024}$$

Question 5.
The probability that a bulb produced by a factory will use fuse after 200 days of use is 0.2. Let X denote the number of bulbs (out of 5) that fuse after 200 days of use. Find the probability of (i) X = 0, (ii) X ≤ 1, (iii) X > 1, (iv) X ≥ 1.
Solution:
X : No. of bulbs fuse after 200 days of use
p : Probability of getting fuse bulbs
No. of bulbs in a sample
∴ n = 5
∴ p = 0.2
∴ q = 1 – p = 1 – 0.2 = 0.8
∵ X ~ B(5, 0.2)
∴ $p(x) = {}^nC_x p^x q^{n-x}$
(i) $P(X = 0) = {}^5C_0 (0.2)^0 (0.8)^{5-0}$
$= 1 \times 1 \times (0.8)^5$
$= (0.8)^5$

(ii) $P(X \le 1) = p(0) + p(1)$
$= {}^5C_0 (0.2)^0 (0.8)^{5-0} + {}^5C_1 (0.2)^1 (0.8)^{5-1}$
$= 1 \times 1 \times (0.8)^5 + 5 \times 0.2 \times (0.8)^4$
$= (0.8)^4 [0.8 + 1]$
$= 1.8 \times (0.8)^4$

(iii) $P(X > 1) = 1 - [p(0) + p(1)]$
$= 1 - 1.8 \times (0.8)^4$

(iv) $P(X \ge 1) = 1 - p(0)$
$= 1 - (0.8)^5$

Question 6.
10 balls are marked with digits 0 to 9. If four balls are selected with replacement. What is the probability that none is marked 0?
Solution:
X : No. of balls drawn marked with the digit 0
n : No. of balls drawn
∴ n = 4
p : Probability of balls marked with 0.

∴ p = 110
∴ q = 1 – p = 1 – 110 = 910
p(x) = nCxpxqn−x
P(None of the ball is marked with digit 0)

$$P(X = 0) = {}^4C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{4-0}$$
$$= 1\times1\times\left(\frac{9}{10}\right)^4$$
$$= \left(\frac{9}{10}\right)^4$$

Question 7.
In a multiple-choice test with three possible answers for each of the five questions, what is the probability of a candidate getting four or more correct answers by random choice?
Solution:
n: No. of Questions
∴ n = 5
X: No. of correct answers by guessing
p: Probability of getting correct answers

$$\therefore p = \frac{1}{3}$$
$$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$
$$\because X \sim B\left(5, \frac{1}{3}\right)$$
$$\therefore p(x) = {}^nC_x\ p^x\ q^{n-x}$$

P(Four or more correct answers)

$$P(X \geq 4) = p(4) + p(5)$$
$$= {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^{5-4} + {}^5C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^{5-5}$$
$$= 5\times\frac{1}{3^4}\times\frac{2}{3} + 1\times\frac{1}{3^5}\times\left(\frac{2}{3}\right)^0$$
$$= \frac{10}{3^5} + \frac{1}{3^5}$$
$$= \frac{11}{243}$$

Question 8.
Find the probability of throwing at most 2 sixes in 6 throws of a single die.
Solution:
X : No. of sixes in 6 throws
n : No. of times dice thrown
∴ n = 6
p : Probability of getting six
∴ p = 16
∴ q = 1 – p = 1 – 16 = 56
∵ X ~ B(6, 16)
∴ p(x) = nCxpxqn−x
P(At most 2 sixes)

$P(X \le 2) = p(0) + p(1) + p(2)$

$$= {}^6C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{6-0} + {}^6C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{6-1}$$

$$+ {}^6C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{6-2}$$

$$= 1\times1\times\left(\frac{5}{6}\right)^6 + 6\times\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^5$$

$$+ 15\times\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^6 + \left(\frac{5}{6}\right)^5 + 15\times\frac{1}{30}\times\left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^6 + \left(\frac{5}{6}\right)^5 + \frac{5}{12}\left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^4\left[\left(\frac{5}{6}\right)^2 + \frac{5}{6} + \frac{5}{12}\right]$$

$$= \left(\frac{5}{6}\right)^4\left[\frac{25}{36} + \frac{30}{36} + \frac{15}{36}\right]$$

$$= \left(\frac{5}{6}\right)^4\left(\frac{70}{36}\right)$$

$$= \left(\frac{5}{6}\right)^4\left(\frac{35}{18}\right)$$

Question 9.
Given that $X \sim B(n, p)$,
(i) if n = 10 and p = 0.4, find E(X) and Var(X).
(ii) if p = 0.6 and E(X) = 6, find n and Var(X).
(iii) if n = 25, E(X) = 10, find p and Var(X).
(iv) if n = 10, E(X) = 8, find Var(X).
Solution:
$\because X \sim B(n, p)$, $E(X) = np$, $V(X) = npq$, $q = 1 - p$
(i) $E(X) = np = 10 \times 0.4 = 4$
$\because q = 1 - p = 1 - 0.4 = 0.6$
$V(X) = npq = 10 \times 0.4 \times 0.6 = 2.4$

(ii) $\because p = 0.6$
$\therefore q = 1 - p = 1 - 0.6 = 0.4$
$E(X) = np$
$\therefore 6 = n \times 0.6$
$\therefore n = 10$
$\therefore V(X) = npq = 10 \times 0.6 \times 0.4 = 2.4$

(iii) $E(X) = np$
$\therefore 10 = 25 \times p$
$\therefore p = 0.4$
$\therefore q = 1, p = 1 - 0.4 = 0.6$
$\therefore S.D.(X) = \sqrt{V(X)}$
= npq−−−√
= 25×0.4×0.6−−−−−−−−−−−√
$= \sqrt{6}$
$= 2.4494$

(iv) $\because E(X) = np$
$\therefore 8 = 10p$
$\therefore p = 0.8$
$\therefore q = 1 - p = 1 - 0.8 = 0.2$
$\because V(X) = npq = 10 \times 0.8 \times 0.2 = 1.6$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 8 Probability Distributions Ex 8.4

Question 1.
If X has Poisson distribution with m = 1, then find $P(X \leq 1)$ given $e^{-1} = 0.3678$.
Solution:
∵ m = 1
∵ X follows Poisson Distribution

$$\therefore \quad p(x) = \frac{e^{-m} \cdot m^x}{x!}$$

$$P(X \leq 1) = p(0) + p(1)$$

$$= \frac{e^{-1} \times 1^0}{0!} + \frac{e^{-1} \times 1^1}{1!}$$

$= e^{-m} \times 1 + e^{-m} \times 1$
$= e^{-1} + e^{-1}$
$= 2 \times e^{-1}$
$= 2 \times 0.3678$
$= 0.7356$

Question 2.
If X ~ P(12), then find P(X = 3) given $e^{-0.5} = 0.6065$.
Solution:

$$\because \quad X \sim P(m = \frac{1}{2})$$

$$\therefore \quad p(x) = \frac{e^{-m} \cdot m^x}{x!}$$

$$\because \quad P(X = 3) = \frac{e^{-0.5} \times (0.5)^3}{3!}$$

$$= \frac{0.6065 \times 0.125}{6}$$

$$= \frac{0.076}{6}$$

$$= 0.013$$

Question 3.
If X has Poisson distribution with parameter m and P(X = 2) = P(X = 3), then find $P(X \geq 2)$. Use $e^{-3} = 0.0497$
Solution:

∵ X follows Poisson Distribution

$$\therefore \quad p(x) = \frac{e^{-m} \cdot m^x}{x!}$$

$$\because \quad P(X = 2) = P(X = 3)$$

$$\therefore \quad \frac{e^{-m} \cdot m^2}{2!} = \frac{e^{-m} \cdot m^3}{3!}$$

$$\therefore \quad \frac{m^2}{2} = \frac{m^3}{6}$$

$$\therefore \quad \frac{6}{2} = m$$

$$\therefore \quad m = 3$$

$$\because \quad P(X \geq 2) = 1 - p(X < 2) = 1 - [p(0) + p(1)]$$

$$= 1 - [\frac{e^{-3} \times 3^0}{0!} + \frac{e^{-3} \times 3^1}{1!}]$$

$$= 1 - (e^{-3} + e^{-3} \times 3)$$

$$= 1 - e^{-3}[1 + 3]$$

$$= 1 - 0.497 \times 4$$

$$= 1 - 0.1988$$

$$= 0.8012$$

Question 4.
The number of complaints which a bank manager receives per day follows a Poisson distribution with parameter m = 4. Find the probability that the manager receives (i) only two complaints on a given day, (ii) at most two complaints on a given day. Use $e^{-4} = 0.0183$.
Solution:
∵ m = 1
∵ X ~ P(m = 4)
∴ $p(x) = \frac{e^{-m} \cdot m^x}{x!}$
X = No. of complaints recieved
(i) P(Only two complaints on a given day)

$$P(X = 2) = \frac{e^{-4} \times 4^2}{2!}$$

$$= \frac{0.0183 \times 16}{2}$$

$$= \frac{0.2928}{2}$$

$$= 0.1464$$

(ii) P(Atmost two complaints on a given day)
$P(X \leq 2) = p(0) + p(1) + p(2)$
$= \frac{e^{-4} \times 4^0}{0!} + \frac{e^{-4} \times 4^1}{1!} + 0.1464$
$= e^{-4} + e^{-4} \times 4 + 0.1464$
$= e^{-4} [1 + 4] + 0.1464$
$= 0.0183 \times 5 + 0.1464$
$= 0.0915 + 0.1464$
$= 0.2379$

Question 5.
A car firm has 2 cars, which are hired out day by day. The number of cars hired on a day follows a Poisson distribution with a mean of 1.5. Find the probability that
(i) no car is used on a given day.
(ii) some demand is refused on a given day, given $e^{-1.5} = 0.2231$.
Solution:
Let X = No. of demands for a car on any day
∴ No. of cars hired
n = 2
m = 1.5

∵ X ~ P(m = 1.5)

$$\therefore \quad p(x) = \frac{e^{-m} \cdot m^x}{x!}$$

(i) P(No car is used)

$$P(X = 0) = \frac{e^{-1.5} \cdot (1.5)^0}{0!} = e^{-1.5} = 0.2231$$

(ii) P(Some demand is refused on a given day)

$$\begin{aligned} P(X \geq 3) &= 1 - [p(0) + p(1) + p(2)] \\ &= 1 - \left[\frac{e^{-1.5} \times 1.5^0}{0!} + \frac{e^{-1.5} \times (1.5)^1}{1!} + \frac{e^{-1.5} \times (1.5)^2}{2!}\right] \\ &= 1 - [e^{-1.5} + 1.5 \times e^{-1.5} + 2.25 \times e^{-1.5}] \\ &= 1 - [0.2231 + 0.3346 - 0.2510] \\ &= 0.1913 \end{aligned}$$

Question 6.
Defects on plywood sheets occur at random with an average of one defect per 50 sq. ft. Find the probability that such a sheet has (i) no defect, (ii) at least one defect. Use e^{-1} = 0.3678.
Solution:
∵ X = No. of defects on a plywood sheet
∵ m = -1
∵ X ~ P(m = -1)
∴ p(x) = $\frac{e^{-m} \cdot m^x}{x!}$
(i) P(No defect)
P(X = 0) = $\frac{e^{-1} \times 1^0}{0!}$
= e^{-1}
= 0.3678

(ii) P(At least one defect)
P(X ≥ 1) = 1 – P(X < 1)
= 1 – p(0)
= 1 – 0.3678
= 0.6322

Question 7.
It is known that, in a certain area of a large city, the average number of rats per bungalow is five. Assuming that the number of rats follows Poisson distribution, find the probability that a randomly selected bungalow has
(i) exactly 5 rats
(ii) more than 5 rats
(iii) between 5 and 7 rats, inclusive. Given e^{-5} = 0.0067.
Solution:
X = No. of rats
∵ m = 5
∴ X ~ P(m = 5)
∴ p(x) = $\frac{e^{-m} \cdot m^x}{x!}$
(i) P(Exactly five rats)

$$\begin{aligned} P(X = 5) &= \frac{e^{-5} \times 5^5}{5!} \\ &= \frac{0.0067 \times 3125}{120} \\ &= \frac{20.9375}{20} \\ &= 0.17 \end{aligned}$$

(ii) P(More than five rats)
P(X > 5) = 1 – P(X ≤ 5)

$$= 1 - \left[\frac{e^{-5} \times 5^0}{0!} + \frac{e^{-5} \times 5^1}{1!} + \frac{e^{-5} \times 5^2}{2!} + \frac{e^{-5} \times 5^3}{3!} \right.$$
$$\left. \frac{e^{-5} \times 5^4}{4!} + \frac{e^{-5} \times 5^5}{5!}\right]$$

$$= 1 - e^{-5}\left[1 + 5 + \frac{25}{2} + \frac{125}{6} + \frac{625}{24} + \frac{3125}{120}\right]$$

$$= 1 - e^{-5}\left[6 + \frac{1500 + 2500 + 3125 + 3125}{120}\right]$$

$= 1 - e^{-5}$ [6 + 85.42]
= 1 – 0.0067 × 91.42
= 1 – 0.61
= 0.39

(iii) P(between 5 and 7 rats, inclusive)
P(5 ≤ x ≤ 7) = p(5) + p(6) + p(7)

$$= \frac{e^{-5} \times 5^5}{5!} + \frac{e^{-5} \times 5^6}{6!} + \frac{e^{-5} \times 5^7}{7!}$$

$$= e^{-5} \times 5^5\left[\frac{1}{5!} + \frac{5}{6!} + \frac{5^2}{7!}\right]$$

$$= e^{-5} \times 3125\left[\frac{1}{120} + \frac{5}{720} + \frac{25}{5040}\right]$$

$$= e^{-5} \times 3125\left[\frac{42 + 35 + 25}{5040}\right]$$

= 0.0067 × 3125 × 0.02
= 0.0067 × 62.5
= 0.42

Maharashtra State Board 12th Commerce Maths Solutions Chapter 8 Probability Distributions Miscellaneous Exercise 8

(I) Choose the correct alternative.

Question 1.
F(x) is c.d.f. of discreter r.v. X whose p.m.f. is given by P(x) = k(4x), for x = 0, 1, 2, 3, 4 & P(x) = 0 otherwise then F(5) = _________
(a) 116
(b) 18
(c) 14
(d) 1
Answer:
(d) 1

Question 2.
F(x) is c.d.f. of discrete r.v. X whose distribution is

X_i	−2	−1	0	1	2
P_i	0.2	0.3	0.15	0.25	0.1

then F(-3) = ________
(a) 0
(b) 1
(c) 0.2
(d) 0.15
Answer:
(a) 0

Question 3.
X : number obtained on uppermost face when a fair die is thrown then E(X) = ________
(a) 3.0
(b) 3.5
(c) 4.0
(d) 4.5
Answer:
(b) 3.5

Question 4.
If p.m.f. of r.v. X is given below.

x	0	1	2
$P(x)$	q^2	$2pq$	p^2

then Var(X) = ________
(a) p2
(b) q2
(c) pq
(d) 2pq
Answer:
(d) 2pq

Question 5.
The expected value of the sum of two numbers obtained when two fair dice are rolled is ________
(a) 5
(b) 6
(c) 7
(d) 8
Answer:
(c) 7

Question 6.
Given p.d.f. of a continuous r.v. X as
f(x) = x23 for -1 < x < 2
= 0 otherwise then F(1) =
(a) 19
(b) 29
(c) 39
(d) 49
Answer:
(b) 29

Question 7.
X is r.v. with p.d.f.
f(x) = kx√, 0 < x < 4
= 0 otherwise then E(X) = ________
(a) 13
(b) 43
(c) 23
(d) 1
Answer:
(b) 43

Question 8.
If X follows B(20, $\frac{1}{10}$) then E(X) = _________
(a) 2
(b) 5
(c) 4
(d) 3
Answer:
(a) 2

Question 9.
If E(X) = m and Var(X) = m then X follows _________
(a) Binomial distribution
(b) Possion distribution
(c) Normal distribution
(d) none of the above
Answer:
(b) Possion distribution

Question 10.
If E(X) > Var(X) then X follows _________
(a) Binomial distribution
(b) Possion distribution
(c) Normal distribution
(d) none of the above
Answer:
(a) Binomial distribution

(II) Fill in the blanks.

Question 1.
The values of discrete r.v. are generally obtained by _________
Answer:
counting

Question 2.
The values of continuous r.v. are generally obtained by _________
Answer:
measurement

Question 3.
If X is dicrete random variable takes the values x_1, x_2, x_3, …… x_n then $\sum_{i=1}^{n} p(x_i)$ = _________
Answer:
1

Question 4.
If f(x) is distribution function of discrete r.v. X with p.m.f. p(x) = $\frac{x-1}{3}$ for x = 1, 2, 3, and p(x) = 0 otherwise then F(4) = _________
Answer:
1

Question 5.
If f(x) is distribution function of discrete r.v. X with p.m.f. p(x) = $k\binom{4}{x}$ for x = 0, 1, 2, 3, 4, and p(x) = 0 otherwise then F(-1) = _________
Answer:
0

Question 6.
E(X) is considered to be _________ of the probability distribution of X.
Answer:
centre of gravity

Question 7.

If X is continuous r.v. and f(x_i) = P(X ≤ x_i) = $\int x_i - \infty f(x) dx$ then f(x) is called _________

Answer:

Cumulative Distribution Function

Question 8.

In Binomial distribution probability of success _______ from trial to trial.

Answer:

remains constant/independent

Question 9.

In Binomial distribution, if n is very large and probability success of p is very small such that np = m (constant) then _______ distribution is applied.

Answer:

Possion

(III) State whether each of the following is True or False.

Question 1.

If P(X = x) = $k(4x)$ for x = 0, 1, 2, 3, 4, then F(5) = 14 when f(x) is c.d.f.

Answer:

False

Question 2.

X	–2	–1	0	1	2
$P(X = x)$	0.2	0.3	0.15	0.25	0.1

If F(x) is c.d.f. of discrete r.v. X then F(-3) = 0.

Answer:

True

Question 3.

X is the number obtained on the uppermost face when a die is thrown the E(X) = 3.5.

Answer:

True

Question 4.

If p.m.f. of discrete r.v.X is

X	0	1	2
$P(X = x)$	q^2	$2pq$	p^2

then E(X) = 2p.

Answer:

True

Question 5.

The p.m.f. of a r.v. X is p(x) = $2xn(n+1)$, x = 1, 2,......n

= 0 otherwise,

Then E(X) = $2n+13$

Answer:

True

Question 6.

If f(x) = kx (1 – x) for 0 < x < 1

= 0 otherwise then k = 12

Answer:

False

Question 7.

If X ~ B(n, p) and n = 6 and P(X = 4) = P(X = 2) then p = 12.

Answer:

True

Question 8.
If r.v. X assumes values 1, 2, 3,………, n with equal probabilities then E(X) = $\frac{n+1}{2}$
Answer:
True

Question 9.
If r.v. X assumes the values 1, 2, 3,………, 9 with equal probabilities, E(X) = 5.
Answer:
True

(IV) Solve the following problems.

Part – I

Question 1.
Identify the random variable as discrete or continuous in each of the following. Identify its range if it is discrete.
(i) An economist is interested in knowing the number of unemployed graduates in the town with a population of 1 lakh.
Solution:
X = No. of unemployed graduates in a town.
∵ The population of the town is 1 lakh
∴ X takes finite values
∴ X is a Discrete Random Variable
∴ Range of = {0, 1, 2, 4, …. 1,00,000}

(ii) Amount of syrup prescribed by a physician.
Solution:
X : Amount of syrup prescribed.
∴ X Takes infinite values
∴ X is a Continuous Random Variable.

(iii) A person on a high protein diet is interested in the weight gained in a week.
Solution:
X : Gain in weight in a week.
X takes infinite values
∴ X is a Continuous Random Variable.

(iv) Twelve of 20 white rats available for an experiment are male. A scientist randomly selects 5 rats and counts the number of female rats among them.
Solution:
X : No. of female rats selected
X takes finite values.
∴ X is a Discrete Random Variable.
Range of X = {0, 1, 2, 3, 4, 5}

(v) A highway safety group is interested in the speed (km/hrs) of a car at a checkpoint.
Solution:
X : Speed of car in km/hr
X takes infinite values
∴ X is a Continuous Random Variable.

Question 2.
The probability distribution of a discrete r.v. X is as follows.

x	1	2	3	4	5	6
$P(X=x)$	k	$2k$	$3k$	$4k$	$5k$	$6k$

(i) Determine the value of k.
(ii) Find P(X ≤ 4), P(2 < X < 4), P(X ≥ 3).
Solution:
(i) Assuming that the given distribution is a p.m.f. of X
∴ Each P(X = x) ≥ 0 for x = 1, 2, 3, 4, 5, 6
k ≥ 0
ΣP(X = x) = 1 and
k + 2k + 3k + 4k + 5k + 6k = 1
∴ 21k = 1 ∴ k = $\frac{1}{21}$

(ii) P(X ≤ 4) = 1 – P(X > 4)
= 1 – [P(X = 5) + P(X = 6)]
= 1 – [latex]\frac{5}{21}+\frac{6}{21}[/latex]
= 1 – 1121
= 1021
P(2 < X < 6) = p(3) + p(4) + p(5)
= 3k + 4k + 5k
= 321+421+521
= 1221
= 47

(iii) P(X ≥ 3) = p(3) + p(4) + p(5) + p(6)
= 3k + 4k + 5k + 6k

$$= 3 \times \frac{1}{21} + 4 \times \frac{1}{21} + 5 \times \frac{1}{21} + 6 \times \frac{1}{21}$$

$$= \frac{3}{21} + \frac{4}{21} + \frac{5}{21} + \frac{6}{21}$$

$$= \frac{18}{21}$$

$$= \frac{6}{7}$$

Question 3.
Following is the probability distribution of an r.v. X.

x	−3	−2	−1	0	1	2	3
$P(X=x)$	0.05	0.1	0.15	0.20	0.25	0.15	0.1

Find the probability that
(i) X is positive.
(ii) X is non-negative.
(iii) X is odd.
(iv) X is even.
Solution:
(i) P(X is positive)
P(X = 0) = p(1) + p(2) + p(3)
= 0.25 + 0.15 + 0.10
= 0.50

(ii) P(X is non-negative)
P(X ≥ 0) = p(0) + p(1) + p(2) + p(3)
= 0.20 + 0.25 + 0.15 + 0.10
= 0.70

(iii) P(X is odd)
P(X = -3, -1, 1, 3)
= p(- 3) +p(-1) + p(1) + p(3)
= 0.05 + 0.15 + 0.25 + 0.10
= 0.55

(iv) P(X is even)
= 1 – P(X is odd)
= 1 – 0.55
= 0.45

Question 4.
The p.m.f of a r.v. X is given by
P(X=x)=⎧ ⎨ ⎩ (5x)125,0x=0,1,2,3,4,5. otherwise
Show that P(X ≤ 2) = P(X ≥ 3).
Solution:

For x = 0, 1, 2, 3, 4, 5

$$P(X = x) = \frac{^5C_x}{2^5}$$

$$\therefore P(X = 0) = \frac{^5C_0}{2^5} = \frac{1}{32}$$

$$P(X = 1) = \frac{^5C_1}{2^5} = \frac{5}{32}$$

$$P(X = 2) = \frac{^5C_2}{2^5} = \frac{10}{32}$$

$$P(X = 3) = \frac{^5C_3}{2^5} = \frac{10}{32}$$

$$P(X = 4) = \frac{^5C_4}{2^5} = \frac{5}{32}$$

$$P(X = 5) = \frac{^5C_5}{2^5} = \frac{1}{32}$$

$\therefore$ The given p.m.f. of X is

$X = x$	0	1	2	3	4	5
$P(X = x)$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

$$P(X \le 2) = p(0) + p(1) + p(2)$$
$$= \frac{1}{32} + \frac{5}{32} + \frac{10}{32} = \frac{16}{32} = \frac{1}{2}$$
$$P(X \ge 3) = p(3) + p(4) + p(5)$$
$$= \frac{10}{32} + \frac{5}{32} + \frac{1}{32} = \frac{16}{32} = \frac{1}{2}$$
$$P(X \le 2) = P(X \ge 3)$$

Question 5.
In the following probability distribution of an r.v. X

x	1	2	3	4	5
$P(x)$	1/20	3/20	a	$2a$	1/20

Find a and obtain the c.d.f. of X.
Solution:
Given distribution is p.m.f. of r.v. X
$\Sigma P(X = x) = 1$
$\therefore p(1) + p(2) + p(3) + p(4) + p(5) = 1$

$\therefore \quad \frac{1}{20}+\frac{3}{20}+a+2a+\frac{1}{20}=1$

$\therefore \quad \frac{1}{20}+\frac{3}{20}+a+2a+\frac{1}{20}=1 \ldots(\because b=2a)$

$\therefore \quad \frac{5}{20}+3a=1$

$\therefore \quad 3a=1-\frac{5}{20}=\frac{15}{20}$

$\therefore \quad a=\frac{5}{20}=\frac{1}{4} \Rightarrow 2a=\frac{10}{20}=\frac{1}{2}$

$\therefore$ The given p.m.f. is

$X = x$	1	2	3	4	5
$P(X = x)$	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{10}{20}$	$\frac{1}{20}$

Let $F(x)$ be the c.d.f. of X

$F(1) = p(1) = \frac{1}{20}$

$F(2) = p(1) + p(2) = \frac{1}{20} + \frac{3}{20} = \frac{4}{20} = \frac{1}{5}$

$F(3) = p(1) + p(2) + p(3)$

$= \frac{1}{20} + \frac{3}{20} + \frac{5}{20} = \frac{9}{20}$

$F(4) = p(1) + p(2) + p(3) + p(4)$

$= \frac{1}{20} + \frac{3}{20} + \frac{5}{20} + \frac{10}{20} = \frac{19}{20}$

$F(5) = p(1) + ... + p(5)$

$\therefore$ The c.d.f. is given as

$X = x$	1	2	3	4	5
$P(X = x)$	[illegible]/20	[illegible]/5	[illegible]/20	[illegible]/20	1

Question 6.
A fair coin is tossed 4 times. Let X denote the number of heads obtained. Identify the probability distribution of X and state the formula for p.m.f. of X.
Solution:
A fair coin is tossed 4 times
$\therefore$ Sample space contains 16 outcomes
Let X = Number of heads obtained
$\therefore$ X takes the values x = 0, 1, 2, 3, 4.
$\therefore$ The number of heads obtained in a toss is an even

$= \ {}^4C_x \text{ for } x = 0,1,2,3,4$

$\therefore \quad {}^4C_0 = 1, {}^4C_1 = 4, {}^4C_2 = 6, {}^4C_3 = 4, {}^4C_4 = 1$

$\therefore$ The p.m.f. of X is given as

$X = x$	0	1	2	3	4
$P(X = x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

Question 7.
Find the probability of the number of successes in two tosses of a die, where success is defined as (i) number greater than 4 (ii) six appearing in at least one toss.
Solution:
S : A die is tossed two times
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
n(S) = 36
(i) X : No. is greater than 4

Range of X = {0, 1, 2}

x	p
0	$\frac{16}{36} = \frac{4}{9}$
1	$\frac{16}{36} = \frac{4}{9}$
2	$\frac{4}{36} = \frac{1}{9}$

(ii) X : Six appears on aleast one die.
Range of X = {0, 1, 2}

x	p
0	$\frac{25}{36}$
1	$\frac{10}{36} = \frac{5}{18}$
2	$\frac{1}{36}$

Question 8.
A random variable X has the following probability distribution.

x	1	2	3	4	5	6	7
$P(x)$	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Determine (i) k, (ii) P(X < 3), (iii) P(X > 6), (iv) P(0 < X < 3).
Solution:
(i) It is a p.m.f. of r.v. X
Σp(x) = 1
p(1) + p(2) + p(3) + p(4) + p(5) + p(6) + p(7) = 1
$k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$
$9k + 10k^2 = 1$
$10k^2 + 9k – 1 = 0$
$10k^2 + 10k – k – 1 = 0$
∴ 10k(k + 1) – 1(k + 1) = 0
∴ (10k – 1) (k + 1) = 0
∴ 10k – 1 = 0r k + 1 = 0
∴ k = 110 or k = -1
k = -1 is not accepted, p(x) ≥ 0, ∀ x ∈ R
∴ k = 110

(ii) P(X < 3) = p(1) + p(2)
= k + 2k
= 3k
= 3 × 110
= 310

(iii) P(X > 6) = p(7)
$= 7k^2 + k$
= 7(110)2+110
= 7100+110
= 17100

(iv) P(0 < X < 3) = p(1) + p(2)
= k + 2k
= 3k
= 3 × 110
= 310

Question 9.
The following is the c.d.f. of a r.v. X.

x	-3	-2	-1	0	1	2	3	4
$F(x)$	0.1	0.3	0.5	0.65	0.75	0.85	0.9	1

Find the probability distribution of X and P(-1 ≤ X ≤ 2).
Solution:

x	F(x)	$p(x)$
-3	0.1	0.1
-2	0.3	0.2
-1	0.5	0.2
0	0.65	0.15
1	0.75	0.10
2	0.85	0.10
3	0.9	0.05
4	1	0.10

P(-1 ≤ X ≤ 2) = p(-1) + p(0) + p(1) + p(2)
= 0.2 + 0.15 + 0.10 + 0.10
= 0.55

Question 10.
Find the expected value and variance of the r.v. X if its probability distribution is as follows.
(i)

x	1	2	3
$P(X = x)$	1/5	2/5	2/5

Solution:

$$\begin{aligned} E(X) &= \Sigma x \cdot p(x) \\ &= 1\left(\frac{1}{5}\right) + 2\left(\frac{2}{5}\right) + 3\left(\frac{2}{5}\right) \\ &= \frac{1}{5} + \frac{4}{5} + \frac{6}{5} = \frac{11}{5} = 2.2 \end{aligned}$$

$$\begin{aligned} E(X^2) &= \Sigma x^2 \cdot p(x) \\ &= 1\left(\frac{1}{5}\right) + 4\left(\frac{2}{5}\right) + 9\left(\frac{2}{5}\right) \\ &= \frac{1}{5} + \frac{8}{5} + \frac{18}{5} = \frac{27}{5} = 5.4 \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= (5.4) - (2.2)^2 = 5.4 - 4.48 = 0.56 \end{aligned}$$

$$\text{S.D. of } X = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{0.56} = 0.7483$$

(ii)

x	-1	0	1
$P(X = x)$	1/5	2/5	2/5

Solution:

E(X) = Σx . p(x)

$$= -1\left(\frac{1}{5}\right) + 0\left(\frac{2}{5}\right) + 1\left(\frac{2}{5}\right)$$

$$= -\frac{1}{5} + 0 + \frac{2}{5} = \frac{1}{5} = 0.2$$

$$E(X)^2 = \Sigma x^2 \cdot p(x)$$

$$= (-1)^2\left(\frac{1}{5}\right) + 0\left(\frac{2}{5}\right) + (1)^2\left(\frac{2}{5}\right)$$

$$= \frac{1}{5} + 0 + \frac{2}{5} = \frac{3}{5} = 0.6$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$= 0.6 - (0.2)^2 = 0.6 - 0.04$$

$$= 0.56$$

$$\text{S.D. of X} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{0.56}$$

$$= 0.7483$$

(iii)

x	1	2	3	...	n
$P(X = x)$	$1/n$	$1/n$	$1/n$	...	$1/n$

Solution:

$$\begin{aligned} E(X) &= \Sigma x \cdot p(x) \\ &= 1\left(\frac{1}{n}\right) + 2\left(\frac{1}{n}\right) + 3\left(\frac{1}{n}\right) + \ldots + n\left(\frac{1}{n}\right) \\ &= \frac{1}{n}(1 + 2 + 3 + \ldots + n) \\ &= \frac{1}{n} \cdot \frac{n(n+1)}{2} \\ & \qquad \ldots\ldots\left(\because \Sigma r = \frac{n(n+1)}{2}\right) \\ &= \frac{n+1}{2} \end{aligned}$$

$$\begin{aligned} E(X)^2 &= \Sigma x^2 \cdot p(x) \\ &= 1\left(\frac{1}{n}\right) + 4\left(\frac{1}{n}\right) + 9\left(\frac{1}{n}\right) + \ldots + n^2\left(\frac{1}{n}\right) \\ &= \frac{1}{n}(1 + 2 + 9 + \ldots + n^2) \\ &= \frac{1}{n} \frac{n(n+1)(2n+1)}{2} \\ & \qquad \ldots\ldots\left(\because \Sigma r^2 = \frac{n(n+1)(2n+1)}{6}\right) \\ &= \frac{(n+1)(2n+1)}{6} \end{aligned}$$

$$\begin{aligned} \mathrm{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 \\ &= \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right] \\ &= \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right] \\ &= \frac{n+1}{2}\left[\frac{n-1}{6}\right] \\ &= \frac{n^2-1}{12} \end{aligned}$$

$$\text{S.D. of } X = \sigma_x = \sqrt{\mathrm{Var}(X)} = \sqrt{\frac{n^2-1}{12}}$$

(iv)

x	0	1	2	3	4	5
$P(X = x)$	1/32	5/32	10/32	10/32	5/32	1/32

Solution:

$$
\begin{aligned}
E(X) &= \Sigma x \cdot p(x) \\
&= 0\left(\frac{1}{32}\right)+1\left(\frac{5}{32}\right)+2\left(\frac{10}{32}\right) \\
&\quad +3\left(\frac{10}{32}\right)+4\left(\frac{5}{32}\right)+5\left(\frac{1}{32}\right) \\
&= 0+\frac{5}{32}+\frac{20}{32}+\frac{30}{32}+\frac{20}{32}+\frac{5}{32} \\
&= \frac{80}{32}=\frac{5}{2} \\
&= 2.5 \\
E(X)^2 &= \Sigma x^2 \cdot p(x) \\
&= 0\left(\frac{1}{32}\right)+1\left(\frac{5}{32}\right)+4\left(\frac{10}{32}\right) \\
&\quad +9\left(\frac{10}{32}\right)+16\left(\frac{5}{32}\right)+25\left(\frac{1}{32}\right) \\
&= 0+\frac{5}{32}+\frac{40}{32}+\frac{90}{32}+\frac{80}{32}+\frac{25}{32} \\
&= \frac{240}{32}=\frac{15}{2} \\
&= 7.5 \\
\text{Var}(X) &= E(X^2)-[E(X)]^2 \\
&= 7.5-(2.5)^2 \\
&= 7.5-6.25
\end{aligned}
$$

= 1.25
S.D. of X = σ_x = √Var(X)
= √1.25
= 1.118

Question 11.
A player tosses two coins. He wins ₹ 10 if 2 heads appear, ₹ 5 if 1 head appears, and ₹ 2 if no head appears. Find the expected value and variance of the winning amount.
Solution:
S : Two fair coin are tossed
S = {HH, HT, TT, TH}
n(S) = 4
∴ Range of X = {0, 1, 2}
∴ Let Y = amount received corresponds to values of X

0	2	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{4}{4}$
1	5	$\frac{2}{4}$	$\frac{10}{4}$	$\frac{50}{4}$
2	10	$\frac{1}{4}$	$\frac{10}{4}$	$\frac{100}{4}$

Expected winning amount
E(Y) = Σpy = 224 = ₹ 5.5
V(Y) = $\Sigma py^2 - (\Sigma py)^2$
= 1544 – $(5.5)^2$
= 38.5 – 30.25
= ₹ 8.25

Question 12.
Let the p.m.f. of the r.v. X be
p(x)={3-x10O for x=-1,0,1,2 otherwise
Calculate E(X) and Var(X).
Solution:

For $x = -1$,

$$p(-1) = \frac{3-(-1)}{10} = \frac{4}{10}$$

For $x = 0$,

$$p(0) = \frac{3-0}{10} = \frac{3}{10}$$

For $x = 1$,

$$p(1) = \frac{3-1}{10} = \frac{2}{10}$$

For $x = 2$,

$$p(2) = \frac{3-2}{10} = \frac{1}{10}$$

x	p	x^2	xp	x^2
-1	$\frac{4}{10}$	1	$\frac{-4}{10}$	$\frac{4}{10}$
0	$\frac{3}{10}$	0	0	0
1	$\frac{2}{10}$	1	$\frac{2}{10}$	$\frac{2}{10}$
2	$\frac{1}{10}$	4	$\frac{2}{10}$	$\frac{4}{10}$
Total			**0**	$\frac{10}{10}$

$E(X) = \Sigma xp = 0$

$V(X) = \Sigma x^2p - (\Sigma xp)^2$

$= \frac{10}{10} - (0)^2$

$= 1 - 0$

$= 1$

Question 13.
Suppose error involved in making a certain measurement is a continuous r.v. X with p.d.f.

$f(x)=\{k(4-x2)0$ for $-2≤x≤2$ otherwise

Compute (i) P(X > 0), (ii) P(-1 < X < 1), (iii) P(X < -0.5 or X > 0.5)
Solution:
We know that

$$\int_{-2}^{2} k(4-x^2)dx = 1$$

$$\therefore k\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 1$$

$$\therefore k\left[\left(8-\frac{8}{3}\right)-\left(-8+\frac{8}{3}\right)\right] = 1$$

$$\therefore 2k\left(8-\frac{8}{3}\right)=1$$

$$\therefore 2k\times 8\left(1-\frac{1}{3}\right)=1$$

$$\therefore \frac{32k}{3}=1 \quad \therefore k=\frac{3}{32}$$

(i) $P(X > 0) \quad = \int_0^2 \frac{3}{32}(4-x^2)dx$

$$=\frac{3}{32}\left[4x-\frac{x^3}{3}\right]_0^2$$

$$=\frac{3}{32}\left(8-\frac{8}{3}\right)=\frac{3\times 8}{32}\left(\frac{2}{3}\right)=\frac{1}{2}$$

(ii) $P(-1 < X < 1) = \int_{-1}^{1} \frac{3}{32}(4-x^2)dx$

$$=\frac{3}{32}\left[4x-\frac{x^3}{3}\right]_{-1}^{1}$$

$$=\frac{3}{32}\left[\left(4-\frac{1}{3}\right)-\left(-4+\frac{1}{3}\right)\right]$$

$$=\frac{3}{32}\left[\frac{22}{3}\right]$$

$$=\frac{32}{3} = \frac{11}{16}$$

(iii) $P(X < -0.5 \text{ or } X > 0.5)$

$$=\int_{-2}^{\frac{-1}{2}} \frac{3}{32}(4-x^2)dx+\int_{\frac{1}{2}}^{2} \frac{3}{32}(4-x^2)dx$$

$$=\frac{3}{32}\left[4x-\frac{x^3}{3}\right]_{-2}^{-\frac{1}{2}}+\frac{3}{32}\left[4x-\frac{x^3}{3}\right]_{\frac{1}{2}}^{2}$$

$$=\frac{3}{32}\left\{\left[\left(-2+\frac{1}{24}\right)-\left(-8+\frac{8}{3}\right)\right] +\left[\left(8-\frac{8}{3}\right)-\left(2-\frac{1}{24}\right)\right]\right\}$$

$$=\frac{3}{32}\left[-2+\frac{1}{24}+8-\frac{8}{3}+8-\frac{8}{3}-2+\frac{1}{24}\right]$$

$$=\frac{3}{32}\left[12-\frac{63}{12}\right]$$

$$=\frac{3}{32}\left[\frac{144-63}{12}\right] = \frac{81}{128}$$

$$= 0.633$$

Question 14.
The p.d.f. of the r.v. X is given by
$f(x)=\begin{cases}\frac{1}{2a} & \text{for } 0<x<2a\\ 0 & \text{otherwise}\end{cases}$
Show that $P(X < \frac{a}{2}) = P(X > \frac{3a}{2})$

Solution:

$$P\left(X<\frac{a}{2}\right)=\int_0^{\frac{a}{2}}\frac{1}{2a}dx=\left[\frac{x}{2a}\right]_0^{\frac{a}{2}}$$
$$=\frac{1}{2a}[x]_0^{a/2}=\frac{1}{2a}\cdot\frac{a}{2}$$
$$=\frac{1}{4}$$
$$P\left(X>\frac{3a}{2}\right)=\int_{\frac{3a}{2}}^{2a}\frac{1}{2a}dx$$
$$=\frac{1}{2a}[x]_{3a/2}^{2a}$$
$$=\frac{1}{2a}\left[2a-\frac{3a}{2}\right]$$
$$=\frac{1}{2a}\cdot\frac{a}{2}=\frac{1}{4}$$
$$\therefore P\left(X<\frac{a}{2}\right)=P\left(X>\frac{3a}{2}\right)$$

Question 15.
Determine k if
$f(x)=\{ke^{-\theta x} 0$ for $0\le x<\infty, \theta>0$ otherwise
is the p.d.f. of the r.v. X. Also find P(X > $\frac{1}{\theta}$). Find M if P(0 < X < M) = $\frac{1}{2}$
Solution:
We know that

$$\int_0^\infty ke^{-\theta x}dx = 1$$

$$\therefore k[\frac{e^{-\theta x}}{-\theta}]_0^\infty = 1$$

$$\therefore -k(0-\frac{1}{\theta}) = 1$$

$$\therefore k = \theta$$

$$\therefore f(x) = \theta e^{-\theta x}$$

$$P(X > \frac{1}{\theta}) = \int_{\frac{1}{\theta}}^\infty \theta e^{-\theta x}dx = \theta\int_{\frac{1}{\theta}}^\infty e^{-\theta x}dx$$

$$= \theta\left[\frac{e^{-\theta x}}{-\theta}\right]_{\frac{1}{\theta}}^\infty = \left[-\left(e^{-\theta x}\right)\right]_{\frac{1}{\theta}}^\infty$$

$$= -[e^{-\infty} - e^{-\theta\cdot\frac{1}{\theta}}]$$

$$= e^{-1}$$

$$P(0 < X < M) = \frac{1}{2}$$

$$\therefore \int_0^M \theta e^{-\theta x}dx = \frac{1}{2}$$

$$\therefore \theta\int_0^M e^{-\theta x}dx = \frac{1}{2}$$

$$\therefore \theta\left[\frac{e^{-\theta x}}{-\theta}\right]_0^M = \frac{1}{2}$$

$$\therefore -\left[e^{-M\theta} - 1\right] = \frac{1}{2}$$

$$\therefore e^{-M\theta} = \frac{1}{2}$$

$$\therefore e^{M\theta} = 2$$

$$\therefore M\theta = \log 2$$

$$\therefore M = \frac{1}{\theta}\log 2$$

Question 16.
The p.d.f. of the r.v. X is given by
$f_X(x)=\{kx\surd,0<x<40, \text{ otherwise}$
Determine k, c.d.f. of X and hence find P(X ≤ 2) and P(X ≥ 1).
Solution:
We know that

$$\int_0^4 \frac{k}{\sqrt{x}}dx = 1$$

$$\therefore k\int_0^4 x^{\frac{-1}{2}}dx = 1$$

$$\therefore k[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}]_0^4 = 1$$

$$\therefore 2k[\sqrt{4}-\sqrt{0}] = 1$$
$$\therefore 4k = 1$$
$$\therefore k = \frac{1}{4}$$

c.d.f. of $f(x)$ is

$$F(x) = \int_0^x \frac{k}{\sqrt{x}}dx$$

$$\therefore F(x) = \int_0^x \frac{1}{4}x^{\frac{-1}{2}}dx = \frac{1}{4}[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}]_0^x = \frac{\sqrt{x}}{2}$$

$$P(X \le 2) = F(2)$$
$$= \frac{\sqrt{2}}{2} = 0.7071$$
$$P(X \ge 1) = 1 - P(X < 1) = 1 - F(1)$$
$$= 1 - \frac{1}{2} = \frac{1}{2} = 0.5$$

Question 17.
Let X denote the reaction temperature (in °C) of a certain chemical process. Let X be a continuous r.v. with p.d.f.
$f(x)=\{\frac{1}{10}, 0, -5 \le x \le 5 \text{ otherwise}$
Compute P(X < 0).
Solution:
Given p.d.f. is f(x) = $\frac{1}{10}$, for -5 ≤ x ≤ 5
Let its c.d.f. F(x) be given by

$$F(x) = \int_{-5}^x f(y)dy, \qquad \text{where } f(y) = \frac{1}{10}$$
$$= \int_{-5}^x \frac{1}{10}dy = \frac{1}{10}[y]_{-5}^x$$
$$= \frac{1}{10}[x+5] = \frac{x}{10} + \frac{1}{2}$$

$$P(X < 0) = F(0) = \frac{0}{10} + \frac{1}{2} = \frac{1}{2}$$

Part – II

Question 1.
Let X ~ B(10, 0.2). Find (i) P(X = 1) (ii) P(X ≥ 1) (iii) P(X ≤ 8)
Solution:
X ~ B(10, 0.2)
n = 10, p = 0.2
∴ q = 1 – p = 1 – 0.2 = 0.8
(i) P(X = 1) = ${}^{10}C_1 (0.2)^1 (0.8)^9$ = 0.2684

(ii) P(X ≥ 1) = 1 – P(X < 1)
= 1 – P(X = 0)
= 1 – ${}^{10}C_0 (0.2)^0 (0.8)^{10}$
= 1 – 0.1074
= 0.8926

(iii) P(X ≤ 8) = 1 – P(x > 1)
= 1 – [p(9) + p(10)]
= 1 – [$^{10}C_9 (0.2)^9 (0.8)^1 + {}^{10}C_{10} (0.2)^{10}$]
= 1 – 0.00000041984
= 0.9999

Question 2.
Let X ~ B(n, p) (i) If n = 10 and E(X) = 5, find p and Var(X), (ii) If E(X) = 5 and Var(X) = 2.5, find n and p.
Solution:
X ~ B(n, p)
(i) n = 10, E(X) = 5
∴ np = 5
∴ 10p = 5
∴ p = 12
∴ q = 1 – p = 1 – 12 = 12
V(X) = npq
= 10 × 12 × 12
= 2.5

(ii) E(X) = 5, V(X) = 2.5
∴ np = 5, ∴ npq = 2.5
∴ 5q = 2.5
∴ q = 2.55 = 0.5, p = 1 – 0.5 = 0.5
But np = 5
∴ n(0.5) = 5
∴ n = 10

Question 3.
If a fair coin is tossed 4 times, find the probability that it shows (i) 3 heads, (ii) head in the first 2 tosses, and tail in the last 2 tosses.
Solution:
n : No. of times a coin is tossed
∴ n = 4
X : No. of heads
P : Probability of getting heads

$$\therefore \quad p = \frac{1}{2}$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\because \quad X \sim B(4, \frac{1}{2})$$

$$\therefore \quad p(x) = {}^nC_x \ p^x \ q^{n-x}$$

(i) P(3 heads)

$$P(X=3) = {}^4C_3 \ (\frac{1}{2})^3 (\frac{1}{2})^{4-3}$$

$$= 4 \times (\frac{1}{2})^3 (\frac{1}{2})$$

$$= 4 \times (\frac{1}{2})^4$$

$$= 4 \times \frac{1}{16} = \frac{1}{4}$$

(ii) P(Head in 1st two tosses and tail in last 2 tosses)

$$P(X=2) = {}^4C_2 \ (\frac{1}{2})^2 (\frac{1}{2})^{4-2}$$

$$= 6 \times (\frac{1}{2})^2 (\frac{1}{2})^2$$

$$= 6 \times \frac{1}{2^4}$$

$$= 6 \times \frac{1}{16}$$

$$= \frac{3}{8}$$

Question 4.
The probability that a bomb will hit the target is 0.8. Find the probability that, out of 5 bombs, exactly 2 will miss the target.
Solution:
X : No. of bombs miss the target
p : Probability that bomb miss the target
$\therefore$ q = 0.8
$\therefore$ p = 1 – q = 1 – 0.8 = 0.2
n = No. of bombs = 5
$\therefore$ X ~ B(5, 0.2)
$\therefore p(x) = {}^nC_x p^x q^{n-x}$
$P(X = 2) = {}^5C_2 (0.2)^2 (0.8)^{5-2}$
$= 10 \times 0.04 \times (0.8)^3$
$= 10 \times 0.04 \times 0.512$
$= 0.4 \times 0.512$
$= 0.2048$

Question 5.
The probability that a lamp in the classroom will burn is 0.3. 3 lamps are fitted in the classroom. The classroom is unusable if the number of lamps burning in it is less than 2. Find the probability that the classroom can not be used on a random occasion.
Solution:
X : No. of lamps not burning
p : Probability that the lamp is not burning
$\therefore$ q = 0.3
$\therefore$ p = 1 – q = 1 – 0.3 = 0.7
n = No. of lamps fitted = 3
$\therefore$ X ~ B(3, 0.7)
$\therefore p(x) = {}^nC_x p^x q^{n-x}$
P(classroom cannot be used)
P(X < 2) = p(0) + p(1)
$= {}^3C_0 (0.7)^0 (0.3)^{3-0} + {}^3C_1 (0.7)^1 (0.3)^{3-1}$
$= 1 \times 1 \times (0.3)^3 + 3 \times 0.7 \times (0.3)^2$
$= (0.3)2 [0.3 + 3 \times 0.7]$
$= 0.09 [0.3 + 2.1]$
$= 0.09 [2.4]$
$= 0.216$

Question 6.
A large chain retailer purchases an electric device from the manufacturer. The manufacturer indicates that the defective rate of the device is 10%. The inspector of the retailer randomly selects 4 items from a shipment. Find the probability that the inspector finds at most one defective item in the 4 selected items.
Solution:
X : No. of defective items
n : No. of items selected = 4
p : Probability of getting defective items
$\therefore$ p = 0.1
$\therefore$ q = 1 – p = 1 – 0.1 = 0.9
P(At most one defective item)
$P(X \le 1) = p(0) + p(1)$
$= {}^4C_0 (0.1)^0 (0.9)^{4-0} + {}^4C_1 (0.1)^1 (0.9)^{4-1}$
$= 1 \times 1 \times (0.9)^4 + 4 \times 0.1 \times (0.9)^3$
$= (0.9)3 [0.9 + 4 \times 0.1]$
$= (0.9)3 \times [0.9 + 0.4]$
$= 0.729 \times 1.3$
$= 0.9477$

Question 7.
The probability that a component will survive a check test is 0.6. Find the probability that exactly 2 of the next 4 components tested survive.
Solution:
p = 0.6, q = 1 – 0.6 = 0.4, n = 4
x = 2
$\therefore p(x) = {}^nC_x p^x q^{n-x}$
$P(X = 2) = {}^4C_2 (0.6)^2 (0.4)^2 = 0.3456$

Question 8.
An examination consists of 5 multiple choice questions, in each of which the candidate has to decide which one of 4 suggested answers is correct. A completely unprepared student guesses each answer randomly. Find the probability that this student gets 4 or more correct answers.
Solution:
n : No. of multiple-choice questions
∴ n = 5
X : No. of correct answers
p : Probability of getting correct answer
∵ There are 4 options out of which one is correct
∴ p = $\frac{1}{4}$
∴ q = 1 – p = 1 – $\frac{1}{4}$ = $\frac{3}{4}$
∵ X ~ B(5, $\frac{1}{4}$)
∴ p(x) = $^nC_x\, p^x\, q^{n-x}$
P(Four or more correct answers)
P(X ≥ 4) = p(4) + p(5)

$$= {}^5C_4(\frac{1}{4})^4(\frac{3}{4})^{5-4} + {}^5C_5(\frac{1}{4})^5(\frac{3}{4})^{5-5}$$

$$= 5 \times (\frac{1}{4})^4(\frac{3}{4}) + 1 \times (\frac{1}{4})^5(\frac{3}{4})^0$$

$$= 5 \times \frac{1}{4^4} \times \frac{3}{4} + \frac{1}{4^5} \times 1$$

$$= \frac{15}{4^5} + \frac{1}{4^5}$$

$$= \frac{1}{4^5}(15+1) = \frac{16}{1024} = \frac{1}{64}$$

Question 9.
The probability that a machine will produce all bolts in a production run with in the specification is 0.9. A sample of 3 machines is taken at random. Calculate the probability that all machines will produce all bolts in a production run within the specification.
Solution:
n : No. of samples selected
∴ n = 3
X : No. of bolts produce by machines
p : Probability of getting bolts
∴ p = 0.9
∴ q = 1 – p = 1 – 0.9 = 0.1
∴ X ~ B(3, 0.9)
∴ p(x) = $^nC_x\, p^x\, q^{n-x}$
P(Machine will produce all bolts)
P(X = 3) = $^3C_3\,(0.9)^3\,(0.1)^{3-3}$
= 1 × $(0.9)^3$ × $(0.1)^0$
= 1 × $(0.9)^3$ × 1
= $(0.9)^3$
= 0.729

Question 10.
A computer installation has 3 terminals. The probability that anyone terminal requires attention during a week is 0.1, independent of other terminals. Find the probabilities that (i) 0 (ii) 1 terminal requires attention during a week.
Solution:
n : No. of terminals
∴ n = 3
X : No. of terminals need attention
p : Probability of getting terminals need attention
∴ p = 0.1
∴ q = 1 – p = 1 – 0.1 = 0.9
∵ X ~ B(3, 0.1)
∴ p(x) = $^nC_x\, p^x\, q^{n-x}$
(i) P(No attention)
∴ P(X = 0) = $^3C_0$ × $(0.1)^0$ $(0.9)^{3-1}$
= 1 × 1 × $(0.9)^3$
= 0.729

(ii) P(One terminal need attention)
∴ P(X = 1) = $^3C_1\,(0.1)^1\,(0.9)^{3-1}$
= 3 × 0.1 × $(0.9)^2$

$= 0.3 \times 0.81$
$= 0.243$

Question 11.
In a large school, 80% of the students like mathematics. A visitor asks each of 4 students, selected at random, whether they like mathematics, (i) Calculate the probabilities of obtaining an answer yes from all of the selected students, (ii) Find the probability that the visitor obtains the answer yes from at least 3 students.
Solution:
X : No. of students like mathematics
p: Probability that students like mathematics
$\therefore p = 0.8$
$\therefore q = 1 - p = 1 - 0.8 = 0.2$
n : No. of students selected
$\therefore n = 4$
$\because X \sim B(4, 0.8)$
$\therefore p(x) = {}^nC_x\, p^x\, q^{n-x}$
(i) P(All students like mathematics)
$\therefore P(X = 4) = {}^4C_4\, (0.8)^4\, (0.2)^{4-4}$
$= 1 \times (0.8)^4 \times (0.2)^0$
$= 1 \times (0.8)^4 \times 1$
$= 0.4096$

(ii) P(Atleast 3 students like mathematics)
$\therefore P(X \geq 3) = p(3) + p(4)$
$= {}^4C_3\, (0.8)^3\, (0.2)^{4-3} + 0.4096$
$= 4 \times (0.8)^3\, (0.2)^1 + 0.4096$
$= 0.8 \times (0.8)^3 + 0.4096$
$= (0.8)^4 \times 0.4096$
$= 0.4096 + 0.4096$
$= 0.8192$

Question 12.
It is observed that it rains on 10 days out of 30 days. Find the probability that
(i) it rains on exactly 3 days of a week.
(ii) it rains at most 2 days a week.
Solution:
X : No. of days it rains in a week
p : Probability that it rains
$\therefore p = \frac{10}{30} = \frac{1}{3}$
$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$
n : No. of days in a week
$\therefore n = 7$
$\therefore X \sim B(7, \frac{1}{3})$
(i) P(Rains on Exactly 3 days of a week)

$$\therefore P(X = 3) = {}^7C_3 (\frac{1}{3})^3 (\frac{2}{3})^{7-3}$$
$$= 35 \times (\frac{1}{3})^3 (\frac{2}{3})^4$$
$$= 35 \times \frac{16}{3^7}$$
$$= \frac{560}{2187}$$

(ii) P(Rains on at most 2 days of a week)

$\therefore$ P(X ≤ 2) = p(0) + p(1) + p(2)

$$= {}^7C_0(\frac{1}{3})^0(\frac{2}{3})^{7-0} + {}^7C_1(\frac{1}{3})^1(\frac{2}{3})^{7-1}$$
$$+ {}^7C_2(\frac{1}{3})^2(\frac{2}{3})^{7-2}$$
$$= 1 \times 1 \times (\frac{2}{3})^7 + 7 \times (\frac{1}{3})(\frac{2}{3})^6 +$$
$$21 \times (\frac{1}{3})^2(\frac{2}{3})^5$$
$$= \left(\frac{2}{3}\right)^5 \left[\left(\frac{2}{3}\right)^2 + \frac{14}{9} + \frac{21}{9}\right]$$
$$= \frac{32}{243} \times \left[\frac{4}{9} + \frac{14}{9} + \frac{21}{9}\right]$$
$$= \frac{32}{243} \times \frac{39}{9}$$
$$= \frac{416}{729}$$

Question 13.

If X follows Poisson distribution such that P(X = 1) = 0.4 and P(X = 2) = 0.2, find variance of X.

Solution:

X : Follows Possion Distribution

$$\therefore \quad p(x) = \frac{e^{-m} \cdot m^x}{x!}$$
$$\because P(X = 1) = 0.4$$
$$\text{and } P(X = 2) = 0.2$$
$$\therefore P(X = 1) = 2 \; P(X = 2)$$
$$\therefore \quad \frac{e^{-m} \cdot m^1}{1!} = 2 \times \frac{e^{-m} \cdot m^2}{2!}$$
$$\therefore \quad e^{-m} \times m = \frac{2 \times e^{-m} \times m^2}{2}$$
$$\therefore m = m^2$$

$\therefore$ m = 1

$\therefore$ Mean = m = Variance of X = 1

Question 14.

If X has Poisson distribution with parameter m, such that

$P(X=x+1)P(X=x)=mx+1$

find probabilities P(X = 1) and P(X = 2), when X follows Poisson distribution with m = 2 and P(X = 0) = 0.1353.

Solution:

Given that the random variable X follows the Poisson distribution with parameter m = 2

i.e. X ~ P(2)

Its p.m.f. is satisfying the given equation.

$P(X=x+1)P(X=x)=mx+1$

When x = 0,

$P(X=1)P(X=0)=20+1$

P(X = 1) = 2P(X = 0)

= 2(0.1353)

= 0.2706

When x = 1,

$P(X=2)P(X=1)=21+1$

P(X = 2) = P(X = 1) = 0.2706

CategoriesClass 12