Maharashtra State Board 11th Maths Solutions Chapter 9 Probability Ex 9.1

Question 1.
There are four pens: Red, Green, Blue, and Purple in a desk drawer of which two pens are selected at random one after the other with replacement. State the sample space and the following events.
(a) A : Select at least one red pen.
(b) B : Two pens of the same colour are not selected.
Solution:
The drawer contains 4 pens out of which one is red (R), one is green (G), one is blue (B) and the other one is purple (P).
From this drawer, two pens are selected one after the other with replacement.
∴ The sample space S is given by
S = {RR, RG, RB, RP, GR, GG, GB, GP, BR, BG, BB, BP, PR, PG, PB, PP}
(a) A : Select at least one red pen.
At least one means one or more than one.
∴ A = {RR, RG, RB, RP, GR, BR, PR}

(b) B : Two pens of the same colour are not selected.
B = {RG, RB, RP, GR, GB, GP, BR, BG, BP, PR, PG, PB}

Question 2.
A coin and a die are tossed simultaneously. Enumerate the sample space and the following events.
(a) A : Getting a tail and an odd number.
(b) B : Getting a prime number.
(c) C : Getting head and a perfect square.
Solution:
When a coin and a die are tossed simultaneously, the sample space S is given by
S = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}
(a) A : Getting a tail and an odd number.
∴ A = {(T, 1), (T, 3), (T, 5)}

(b) B : Getting a prime number.
∴ B = {(H, 2), (H, 3), (H, 5), (T, 2), (T, 3), (T, 5)}

(c) C : Getting a head and a perfect square.
∴ C = {(H, 1), (H, 4)}

Question 3.
Find n(S) for each of the following random experiments.
(a) From an urn containing 5 gold and 3 silver coins, 3 coins are drawn at random.
(b) 5 letters are to be placed into 5 envelopes such that no envelope is empty.
(c) 6 books of different subjects are arranged on a shelf.
(d) 3 tickets are drawn from a box containing 20 lottery tickets.
Solution:
(a) There are 5 gold and 3 silver coins, i.e., 8 coins.
3 coins can be drawn from these 8 coins in ${}^8C_3$ ways.
∴ n(s) = ${}^8C_3 = \frac{8!}{5!3!} = \frac{8\times7\times6\times5!}{5!\times3\times2\times1} = 56$

(b) 5 letters have to be placed in 5 envelopes in such a way that no envelope is empty.
∴ The first letter can be placed into 5 envelopes in 5 different ways, the second letter in 4 ways.
Similarly, the third, fourth and fifth letters can be placed in 3 ways, 2 ways and 1 way, respectively.
∴ Total number of ways = 5!
= 5 × 4 × 3 × 2 × 1
= 120
∴ n(S) = 120

(c) 6 books can be arranged on a shelf in ${}^6P_6$ = 6! ways.
∴ n(S) = 6! = 6 × 5 × 4 × 3 × 2 × 1 = 720

(d) 3 tickets are drawn at random from 20 tickets.
∴ 3 tickets can be selected in ${}^{20}C_3$ ways.
∴ n(S) = ${}^{20}C_3 = \frac{20!}{17!3!} = \frac{20\times19\times18\times17!}{17!\times3\times2\times1}$ = 1140

Question 4.
Two fair dice are thrown. State the sample space and write the favourable outcomes for the following events.
(a) A : Sum of numbers on two dice is divisible by 3 or 4.
(b) B : The sum of numbers on two dice is 7.
(c) C : Odd number on the first die.
(d) D : Even number on the first die.
(e) Check whether events A and B are mutually exclusive and exhaustive.
(f) Check whether events C and D are mutually exclusive and exhaustive.
Solution:
When two dice are thrown, the sample space is
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6,4), (6, 5), (6, 6)}
∴ n(S) = 36
(a) A: Sum of the numbers on two dice is divisible by 3 or 4.
∴ A = {(1, 2), (1, 3), (1, 5), (2, 1), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (3, 6), (4, 2), (4, 4), (4, 5), (5, 1), (5, 3), (5, 4), (6, 2), (6, 3), (6, 6)}

(b) B: Sum of the numbers on two dice is 7.
∴ B = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

(c) C: Odd number on the first die.
∴ C = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}

(d) D: Even number on the first die.
∴ D = {(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

(e) A and B are mutually exclusive events as A ∩ B = Φ.
A ∪ B = {(1, 2), (1, 3), (1, 5), (1, 6), (2, 1), (2, 2), (2, 4), (2, 5), (2, 6), (3, 1), (3, 3), (3, 4), (3, 5), (3, 6), (4, 2), (4, 3), (4, 4), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 6)} ≠ S
∴ A and B are not exhaustive events as A ∪ B ≠ S.

(f) C and D are mutually exclusive events as C ∩ D = Φ.
C ∪ D = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} = S
∴ C and D are exhaustive events.

Question 5.
A bag contains four cards marked as 5, 6, 7, and 8. Find the sample space if two cards are drawn at random
(a) with replacement.
(b) without replacement.
Solution:
The bag contains 4 cards marked 5, 6, 7, and 8. Two cards are to be drawn from this bag.
(a) If the two cards are drawn with replacement, then the sample space is
S = {(5, 5), (5, 6), (5, 7), (5, 8), (6, 5), (6, 6), (6, 7), (6, 8), (7, 5), (7, 6), (7, 7), (7, 8), (8, 5), (8, 6), (8, 7), (8, 8)}

(b) If the two cards are drawn without replacement, then the sample space is
S = {(5, 6), (5, 7), (5, 8), (6, 5), (6, 7), (6, 8), (7, 5), (7, 6), (7, 8), (8, 5), (8, 6), (8, 7)}

Question 6.
A fair die is thrown two times. Find the probability that
(a) the sum of the numbers on them is 5.
(b) the sum of the numbers on them is at least 8.
(c) the first throw gives a multiple of 2 and the second throw gives a multiple of 3.
(d) product of numbers on them is 12.
Solution:
When two dice are thrown, the sample space is
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6,4), (6, 5), (6, 6)}
∴ n(S) = 36
(a) Let event A: Sum of the numbers on uppermost face is 5.
∴ A = {(1, 4), (2, 3), (3, 2), (4, 1)}
∴ n(A) = 4
∴ P(A) = $\frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

(b) Let event B: Sum of the numbers on uppermost face is at least 8 (i.e., 8 or more than 8)
∴ B = {(2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(B) = 15
∴ P(B) = $\frac{n(B)}{n(S)}=\frac{15}{36}=\frac{5}{12}$

(c) Let event C: First throw gives a multiple of 2 and second throw gives a multiple of 3.
∴ C = {(2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6)}
∴ n(C) = 6
∴ P(C) = $\frac{n(C)}{n(S)}=\frac{6}{36}=\frac{1}{6}$

(d) Let event D: The product of the numbers on uppermost face is 12.
∴ D = {(2, 6), (3, 4), (4, 3), (6, 2)}
∴ n(D) = 4
∴ P(D) = $\frac{n(D)}{n(S)}=\frac{4}{36}=\frac{1}{9}$

Question 7.
Two cards are drawn from a pack of 52 cards. Find the probability that
(a) one is a face card and the other is an ace card.
(b) one is a club and the other is a diamond.
(c) both are from the same suit.
(d) both are red cards.
(e) one is a heart card and the other is a non-heart card.
Solution:
Two cards can be drawn from a pack of 52 cards in ${}^{52}C_2$ ways.
∴ n(S) = ${}^{52}C_2$

(a) Let event A: Out of the two cards drawn, one is a face card and the other is an ace card.
There are 12 face cards and 4 ace cards in a pack of 52 cards.
∴ One face card can be drawn from 12 face cards in ${}^{12}C_1$ ways and one ace card can be drawn from 4 ace cards in ${}^{4}C_1$ ways.
∴ n(A) = ${}^{12}C_1 \times {}^{4}C_1$
∴ P(A) = $\frac{n(A)}{n(S)}=\frac{{}^{12}C_1 \times {}^{4}C_1}{{}^{52}C_2}=\frac{12\times 4}{\frac{52\times 51}{2\times 1}}=\frac{8}{221}$

(b) Let event B: Out of the two cards drawn, one is club and the other is a diamond card.
There are 13 club cards and 13 diamond cards.
∴ One club card can be drawn from 13 club cards in ${}^{13}C_1$ ways and one diamond card can be drawn from 13 diamond cards in ${}^{13}C_1$ ways.
∴ n(B) = ${}^{13}C_1 \times {}^{13}C_1$
∴ P(B) = $\frac{n(B)}{n(S)}=\frac{{}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_2}=\frac{13\times 13}{\frac{52\times 51}{2\times 1}}=\frac{13}{102}$

(c) Let event C: Both the cards drawn are of the same suit.
A pack of 52 cards consists of 4 suits each containing 13 cards.
∴ 2 cards can be drawn from the same suit in ${}^{13}C_2$ ways.
∴ n(C) = ${}^{13}C_2 \times 4$
∴ P(C) = $\frac{n(C)}{n(S)}=\frac{4\times {}^{13}C_2}{{}^{52}C_2}=\frac{4\times 13\times 12}{52\times 51}=\frac{4}{17}$

(d) Let event D: Both the cards drawn are red.
There are 26 red cards in the pack of 52 cards.
∴ 2 cards can be drawn from them in ${}^{26}C_2$ ways.
∴ n(D) = ${}^{26}C_2$
∴ P(D) = $\frac{n(D)}{n(S)}=\frac{{}^{26}C_2}{{}^{52}C_2}=\frac{26\times 25}{52\times 51}=\frac{25}{102}$

(e) Let event E: Out of the two cards drawn, one is heart and other is non-heart.
There are 13 heart cards in a pack of 52 cards, i.e., 39 cards are non-heart.
∴ One heart card can be drawn from 13 hdart cards in ${}^{13}C_1$ ways and one non-heart card can be drawn from 39 cards in ${}^{39}C_1$ ways.
∴ n(E) = ${}^{13}C_1 \times {}^{39}C_1$
∴ P(E) = $\frac{n(E)}{n(S)}=\frac{{}^{13}C_1 \times {}^{39}C_1}{{}^{52}C_2}=\frac{13\times 39}{\frac{52\times 51}{2\times 1}}=\frac{13}{34}$

Question 8.
Three cards are drawn from a pack of 52 cards. Find the chance that
(a) two are queen cards and one is an ace card.
(b) at least one is a diamond card.
(c) all are from the same suit.
(d) they are a king, a queen, and a jack.
Solution:
3 cards can be drawn from a pack of 52 cards in $52C3$ ways.

$\therefore$ n(S) = $52C3$

(a) Let event A: Out of the three cards drawn, 2 are queens and 1 is an ace card.
There are 4 queens and 4 aces in a pack of 52 cards.
$\therefore$ 2 queens can be drawn from 4 queens in $4C2$ ways and 1 ace can be drawn out of 4 aces in $4C1$ ways.

$$n(A) = {}^4C_2 \times {}^4C_1$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{{}^4C_2 \times {}^4C_1}{{}^{52}C_3}$$
$$= \frac{\frac{4\times3}{2}\times4}{\frac{52\times51\times50}{3\times2\times1}} = \frac{6}{5525}$$

(b) Let event B: Out of the three cards drawn, at least one is a diamond.
$\therefore$ B′ is the event that all 3 cards drawn are non-diamond cards.
In a pack of 52 cards, there are 39 non-diamond cards.
$\therefore$ 3 non-diamond cards can be drawn in $39C3$ ways.

$$n(B') = {}^{39}C_3$$
$$P(B') = \frac{n(B')}{n(S)} = \frac{{}^{39}C_3}{{}^{52}C_3} = \frac{39\times38\times37}{52\times51\times50} = \frac{703}{1700}$$
$$P(B) = 1 - P(B') = 1 - \frac{703}{1700} = \frac{997}{1700}$$

(c) Let event C: All the cards drawn are from the same suit.
A pack of 52 cards consists of 4 suits each containing 13 cards.
$\therefore$ 3 cards can be drawn from the same suit in $13C3$ ways.

$$n(C) = {}^{13}C_3 \times 4$$
$$P(C) = \frac{n(C)}{n(S)} = \frac{4\times {}^{13}C_3}{{}^{52}C_3}$$
$$= \frac{4\times13\times12\times11}{52\times51\times50} = \frac{22}{425}$$

(d) Let event D: The cards drawn are a king, a queen, and a jack.
There are 4 kings, 4 queens and 4 jacks in a pack of 52 cards.
$\therefore$ 1 king can be drawn from 4 kings in $4C1$ ways,

1 queen can be drawn from 4 queens in $4C1$ ways and

1 jack can be drawn from 4 jacks in $4C1$ ways.

$$n(D) = {}^4C_1 \times {}^4C_1 \times {}^4C_1 = 4\times4\times4$$
$$P(D) = \frac{n(D)}{n(S)} = \frac{4\times4\times4}{{}^{52}C_3}$$
$$= \frac{4\times4\times4}{\frac{52\times51\times50}{3\times2\times1}} = \frac{16}{5525}$$

Question 9.
From a bag containing 10 red, 4 blue, and 6 black balls, a ball is drawn at random. Find the probability of drawing
(a) a red bail.
(b) a blue or black ball.
(c) not a black ball.
Solution:
The bag contains 10 red, 4 blue, and 6 black balls,

i.e., 10 + 4 + 6 = 20 balls.

One ball can be drawn from 20 balls in ${}^{20}C_1$ ways.

∴ n(S) = ${}^{20}C_1$ = 20

(a) Let event A: Ball drawn is red.
There are total 10 red balls.

∴ 1 red ball can be drawn from 10 red balls in ${}^{10}C_1$ ways.

∴ n(A) = ${}^{10}C_1$ = 10

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{10}{20} = \frac{1}{2}$

(b) Let event B: The ball drawn is blue or black.
There are 4 blue and 6 black balls.

∴ 1 blue ball can be drawn from 4 blue balls in ${}^{4}C_1$ ways

or 1 black ball can be drawn from 6 black balls in ${}^{6}C_1$ ways.

∴ n(B) = ${}^{4}C_1$ + ${}^{6}C_1$ = 4 + 6 = 10

∴ P(B) = $\frac{n(B)}{n(S)} = \frac{10}{20} = \frac{1}{2}$

(c) Let event C: Ball drawn is not black,
i.e., ball drawn is red or blue.
There are total 14 red and blue balls.

∴ 1 ball can be drawn from 14 balls in ${}^{14}C_1$ ways.

∴ n(C) = ${}^{14}C_1$ = 14

∴ P(C) = $\frac{n(C)}{n(S)} = \frac{14}{20} = \frac{7}{10}$

Question 10.
A box contains 75 tickets numbered 1 to 75. A ticket is drawn at random from the box. Find the probability that,
(a) number on the ticket is divisible by 6.
(b) the number on the ticket is a perfect square.
(c) the number on the ticket is prime.
(d) the number on the ticket is divisible by 3 and 5.
Solution:
The box contains 75 tickets numbered 1 to 75.

∴ 1 ticket can be drawn from the box in ${}^{75}C_1$ = 75 ways.

∴ n(S) = 75

(a) Let event A: Number on the ticket is divisible by 6.
∴ A = {6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72}
∴ n(A) = 12

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{12}{75} = \frac{4}{25}$

(b) Let event B: Number on the ticket is a perfect square.
∴ B = (1, 4, 9, 16, 25, 36, 49, 64}
∴ n(B) = 8

∴ P(B) = $\frac{n(B)}{n(S)} = \frac{8}{75}$

(c) Let event C: Number on the ticket is a prime number.
∴C = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73}

(d) Let event D: Number on the ticket is divisible by 3 and 5,
i.e., divisible by L.C.M. of 3 and 5,
i.e., 15.
∴D = {15, 30, 45, 60, 75}
∴ n(D) = 5

∴ P(D) = $\frac{n(D)}{n(S)} = \frac{5}{75} = \frac{1}{15}$

Question 11.
What is the chance that a leap year, selected at random, will contain 53 Sundays?
Solution:
A leap year consists of 366 days.
It has 52 complete weeks and two more days.

These two days can be {(Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thur), (Thur, Fri), (Fri, Sat), (Sat, Sun)}.
∴ n(S) = 7
Let event E : There are 53 Sundays.
∴ E = {(Sun, Mon), (Sat, Sun)}
∴ n(E) = 2
∴ P(E) = $\frac{n(E)}{n(S)} = \frac{2}{7}$

Question 12.
Find the probability of getting both red balls, when from a bag containing 5 red and 4 black balls, two balls are drawn,
(i) with replacement
(ii) without replacement
Solution:
The bag contains 5 red and 4 black balls,
i.e., 5 + 4 = 9 balls.
(i) 2 balls can be drawn from 9 balls with replacement in ${}^9C_1 \times {}^9C_1$ ways.
∴ n(S) = ${}^9C_1 \times {}^9C_1$ = 9 × 9 = 81
Let event A: Balls drawn are red.
2 red balls can be drawn from 5 red balls with replacement in ${}^5C_1 \times {}^5C_1$ ways.
∴ n(A) = ${}^5C_1 \times {}^5C_1$ = 5 × 5 = 25
∴ P(A) = $\frac{n(A)}{n(S)} = \frac{25}{81}$

(ii) 2 balls can be drawn from 9 balls without replacement in ${}^9C_1 \times {}^8C_1$ ways.
∴ n(S) = ${}^9C_1 \times {}^8C_1$ = 9 × 8 = 72
2 red balls can be drawn from 5 red balls without replacement in ${}^5C_1 \times {}^4C_1$ ways.
∴ n(B) = ${}^5C_1 \times {}^4C_1$ = 5 × 4 = 20
∴ P(B) = $\frac{n(B)}{n(S)} = \frac{20}{72} = \frac{5}{18}$

Question 13.
A room has three sockets for lamps. From a collection of 10 bulbs of which 6 are defective. At night a person selects 3 bulbs, at random and puts them in sockets. What is the probability that
(i) room is still dark.
(ii) the room is lit.
Solution:
Total number of bulbs = 10
Number of defective bulbs = 6
∴ Number of non-defective bulbs = 4
3 bulbs can be selected out of 10 bulbs in ${}^{10}C_3$ ways.
∴ n(S) = ${}^{10}C_3$

(i) Let event A: The room is dark.
For event A to happen the bulbs should be selected from the 6 defective bulbs. This can be done in ${}^6C_3$ ways.
∴ n(A) = ${}^6C_3$
∴ P(A) = $\frac{n(A)}{n(S)} = \frac{{}^6C_3}{{}^{10}C_3} = \frac{6 \times 5 \times 4}{10 \times 9 \times 8} = \frac{1}{6}$

(ii) Let event A′: The room is lit.
∴ P(Room is lit) = 1 – P(Room is not lit)
∴ P(A′) = 1 – P(A) = 1 – $\frac{1}{6}$ = $\frac{5}{6}$

Question 14.
Letters of the word MOTHER are arranged at random. Find the probability that in the arrangement
(a) vowels are always together.
(b) vowels are never together.
(c) O is at the beginning and end with T.
(d) starting with a vowel and ending with a consonant.
Solution:
There are 6 letters in the word MOTHER.
These letters can be arranged among themselves in ${}^6P_6$ = 6! ways.
∴ n(S) = 6!
(a) Let event A: Vowels are always together.
The word MOTHER consists of 2 vowels (O, E) and 4 consonants (M, T, H, R).

2 vowels can be arranged among themselves in $^2P_2$ = 2! ways.

Let us consider 2 vowels as one group.

This one group with 4 consonants can be arranged in $^5P_5$ = 5! ways.

∴ n(A) = 2! × 5!

∴ P(A) = $\frac{n(A)}{n(S)}=\frac{2!\times5!}{6!}=\frac{1}{3}$

(b) Let event B: Vowels are never together.
4 consonants create 5 gaps, in which vowels are arranged.
Consider the following arrangement of consonants
_C_C_C_C_

2 vowels can be arranged in 5 gaps in $^5P_2$ ways.

Also 4 consonants can be arranged among themselves in $^4P_4$ = 4! ways.

∴ n(B) = 4! × $^5P_2$

∴ P(B) = $\frac{n(B)}{n(S)}=\frac{4!\times{}^5P_2}{6!}=\frac{4!\times5\times4}{6\times5\times4!}=\frac{2}{3}$

(iii) Let event C: Word begin with O and end with T.

Thus first and last letters can be arranged in one way each and the remaining 4 letters can be arranged in $^4P_4$ = 4! ways

∴ n(C) = 4! × 1 × 1 = 4!

∴ P(C) = $\frac{n(C)}{n(S)}=\frac{4!}{6!}=\frac{1}{30}$

(d) Let event D: Word starts with a vowel and ends with a consonant.
There are 2 vowels and 4 consonants in the word MOTHER.
∴ The first place can be arranged in 2 different ways and the last place can be arranged in 4 different ways.

Now, the remaining 4 letters (3 consonants and 1 vowel) can be arranged in $^4P_4$ = 4! ways.

∴ n(D) = 2 × 4 × 4!

∴ P(D) = $\frac{n(D)}{n(S)}=\frac{2\times4\times4!}{6!}=\frac{4}{15}$

Question 15.
4 letters are to be posted in 4 post boxes. If any number of letters can be posted in any of the 4 post boxes, what is the probability that each box contains only one letter?
Solution:
There are 4 letters and 4 post boxes.
Since any number of letters can be posted in all 4 post boxes,
so each letter can be posted in different ways.
∴ n(S) = 4 × 4 × 4 × 4
Let event A: Each box contains only one letter.
∴ 1st letter can be posted in 4 different ways.
Since each box contains only one letter, 2nd letter can be posted in 3 different ways.
Similarly, 3rd and 4th letters can be posted in 2 different ways and 1 way respectively.
∴ n(A) = 4 × 3 × 2 × 1

∴ P(A) = $\frac{n(A)}{n(S)}=\frac{4\times3\times2\times1}{4\times4\times4\times4}=\frac{3}{32}$

Question 16.
15 professors have been invited for a round table conference by the Vice-chancellor of a university. What is the probability that two particular professors occupy the seats on either side of the Vice-chancellor during the conference?
Solution:
Since a Vice-chancellor invited 15 professors for a round table conference, there were all 16 persons in the conference.
These 16 persons can be arranged among themselves around a round table in (16 – 1)! = 15! ways.
∴ n(S) = 15!
Let event A: Two particular professors be seated on either side of the Vice-chancellor.

Those two particular persons sit on either side of a Vice chancellor in $^2P_2$ = 2! ways.

Thus the remaining 13 persons can be arranged in $^{13}P_{13}$ = 13! ways.

∴ n(A) = 13! 2!

∴ P(A) = $\frac{n(A)}{n(S)}=\frac{13!\times2!}{15!}=\frac{13!\times2\times1}{15\times14\times13!}=\frac{1}{105}$

Question 17.
A bag contains 7 black and 4 red balls. If 3 balls are drawn at random, find the probability that
(i) all are black.
(ii) one is black and two are red.
Solution:
The bag contains 7 black and 4 red balls,

i.e., 7 + 4 = 11 balls.

∴ 3 balls can be drawn out of 11 balls in $^{11}C_3$ ways.

∴ n(S) = $^{11}C_3$

(i) Let event A: All 3 balls drawn are black.
There are 7 black balls.

∴ 3 black balls can be drawn from 7 black balls in $^7C_3$ ways.

∴ n(A) = $^7C_3$

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{^7C_3}{^{11}C_3} = \frac{7\times6\times5}{11\times10\times9} = \frac{7}{33}$

(ii) Let event B: Out of 3 balls drawn, one is black and two are red.
There are 7 black and 4 red balls.

∴ One black ball can be drawn from 7 black balls in $^7C_1$ ways and 2 red balls can be drawn from 4 red balls in $^4C_2$ ways.

∴ n(A) = $^7C_1 \times {^4C_2}$

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{^7C_1\times{^4C_2}}{^{11}C_3} = \frac{7\times\frac{4\times3}{2}}{\frac{11\times10\times9}{3\times2\times1}} = \frac{14}{55}$

Maharashtra State Board 11th Maths Solutions Chapter 9 Probability Ex 9.2

Question 1.
First, 6 faced die which is numbered 1 to 6 is thrown, then a 5 faced die which is numbered 1 to 5 is thrown. What is the probability that sum of the numbers on the upper faces of the dice is divisible by 2 or 3?
Solution:
When a 6 faced die and a 5 faced die are thrown, the sample space is
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (4,4), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)}
∴ n(S) = 30
Let event A: The sum of the numbers on the upper faces of the dice is divisible by 2.
A = {(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4)}
∴ n(A) = 15

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{15}{30}$

Let event B: Sum of the numbers on the upper faces of the dice is divisible by 3.
B = {(1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3)}
∴ n(B) = 10

∴ P(B) = $\frac{n(B)}{n(S)} = \frac{10}{30}$

Now,
A ∩ B = {(1, 5), (2,4), (3, 3), (4, 2), (5, 1)}
∴ n(A ∩ B) = 5

∴ P(A ∩ B) = $\frac{n(A\cap B)}{n(S)} = \frac{5}{30}$

∴ Required probability
P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

= $\frac{15}{30}+\frac{10}{30}-\frac{5}{30}$

= $\frac{20}{30}$

= $\frac{2}{3}$

Question 2.
A card is drawn from a pack of 52 cards. What is the probability that,
(i) card is either red or black?

(ii) card is either black or a face card?
Solution:
One card can be drawn from the pack of 52 cards in 52C1 = 52 ways.
∴ n(S) = 52
The pack of 52 cards consists of 26 red and 26 black cards.
(i) Let event A: A red card is drawn.
∴ Red card can be drawn in 26C1 = 26ways
∴ n(A) = 26
∴ P(A) = n(A)n(S)=2652
Let event B: A black card is drawn.
∴ Black card can be drawn in 26C1 = 26 ways.
∴ n(B) = 26
∴ P(B) = n(B)n(S)=2652
Since A and B are mutually exclusive events,
P(A ∩ B) = 0
∴ Required probability
P(A ∪ B) = P(A) + P(B)
= 2652+2652
= 1

(ii) Let event A: A black card is drawn.
∴ Black card can be drawn in 26C1 = 26 ways.
n(A) = 26
n(A) 26 n(S) ~ 52
Let event B: A face card is drawn.
There are 12 face cards in the pack of 52 cards.
∴ 1 face card can be drawn in 12C1 = 12 ways.
∴ n(B) = 12
∴ P(B) = n(B)n(S)=1252
There are 6 black face cards.
∴ n(A ∩ B) = 6
∴ P(A ∩ B) = n(A∩B)n(S)=652
∴ Required probability
P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
= 2652+1252−652
= 3252
= 813

Question 3.
A girl is preparing for National Level Entrance exam and State Level Entrance exam for professional courses. The chances of her cracking National Level exam is 0.42 and that of State Level exam is 0.54. The probability that she clears both the exams is 0.11. Find the probability that
(i) she cracks at least one of the two exams.
(ii) she cracks only one of the two.
(iii) she cracks none.
Solution:
Let event A: The girl cracks the National Level exam.
∴ P(A) = 0.42
Let event B: The girl cracks the State Level exam.
∴ P(B) = 0.54
Also, P(A ∩ B) = 0.11
(i) P(the girl cracks at least one of the two exams)
P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
= 0.42 + 0.54 – 0.11
= 0.85

(ii) P(the girl cracks only one of the two exams)
= P(A) – P(B) – 2P(A ∩ B)
= 0.42 + 0.54 – 2(0.11)
= 0.74

(iii) P(the girl cracks none of the exams)
= P(A′ ∩ B′)

= P(A ∪ B)′
= 1 – P(A ∪ B)
= 1 – 0.85
= 0.15

Question 4.
A bag contains 75 tickets numbered from 1 to 75. One ticket is drawn at random. Find the probability that,
(i) number on the ticket is a perfect square or divisible by 4.
(ii) number on the ticket is a prime number or greater than 40.
Solution:
Out of the 75 tickets, one ticket can be drawn in $75C1$ = 75 ways.
∴ n(S) = 75
(i) Let event A: The number on the ticket is a perfect square.
∴ A = {1, 4, 9, 16, 25, 36, 49, 64}
∴ n(A) = 8
∴ P(A) = $n(A)n(S)=875$
Let event B: The number on the ticket is divisible by 4.
∴ B = {4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72}
∴ n(B) = 18
∴ P(B) = $n(B)n(S)=1875$
Now, A ∩ B = {4, 16, 36, 64}
∴ n(A ∩ B) = 4
∴ P(A ∩ B) = $n(A∩B)n(S)=475$
∴ Required probability
P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
= $875+1875−475$
= 2275

(ii) Let event A: The number on the ticket is a prime number.
∴ A = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73}
∴ n(A) = 21
∴ P(A) = $n(A)n(S)=2175$
Let event B: The number is greater than 40.
∴ B = {41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75}
∴ n(B) = 35
∴ P(B) = $n(B)n(S)=3575$
Now,
A ∩ B = {41, 43, 47, 53, 59, 61, 67, 71, 73}
∴ n(A ∩ B) = 9
∴ n(A ∩ B) = $n(A∩B)n(S)=975$
∴ Required probability
P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
= $2175+3575−975$
= 4775

Question 5.
The probability that a student will pass in French is 0.64, will pass in Sociology is 0.45 and will pass in both is 0.40. What is the probability that the student will pass in at least one of the two subjects?
Solution:
Let event A: The student will pass in French.
∴ P(A) = 0.64
Let event B: The student will pass in Sociology.
∴ P(B) = 0.45
Also, P(A ∩ B) = 0.40
∴ Required probability
P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
= 0.64 + 0.45 – 0.40
= 0.69

Question 6.
Two fair dice are thrown. Find the probability that the number on the upper face of the first die is 3 or sum of the numbers on their upper faces is 6.
Solution:
When two dice are thrown, the sample space is

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(S) = 36
Let event A: The number on the upper face of the first die is 3.
∴ A = {(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)}
∴ n(A) = 6
∴ P(A) = n(A)n(S)=636
Let event B: Sum of the numbers on their upper faces is 6.
∴ B = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)}
∴ n(B) = 5
∴ P(B) = n(B)n(S)=536
Now, A ∩ B = {(3, 3)}
∴ n(A ∩ B) = 1
∴ P(A ∩ B) = n(A∩B)n(S)=136
∴ Required probability
P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
= 636+536−136
= 1036
= 518

Question 7.
For two events A and B of a sample space S, if P(A) = 38, P(B) = 12 and P(A ∪ B) = 58. Find the value of the following.
(a) P(A ∩ B)
(b) P(A′ ∩ B′)
(c) P(A′ ∪ B′)
Solution:
Here, P(A) = 38, P(B) = 12, P(A ∪ B) = 58
(a) P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
∴ P(A ∩ B) = P(A) + P(B) – P(A ∪ B)
= 38+12−58
= 14

(b) P(A′ ∩ B′) = P(A ∪ B)′
= 1 – P(A ∪ B)
= 1 – 58
= 38

(c) P(A′ ∪ B′) = P(A ∩ B)′
= 1 – P(A ∩ B)
= 1 – 14
= 34

Question 8.
For two events A and B of a sample space S, if P(A ∪ B) = 56, P(A ∩ B) = 13 and P(B′) = 13, then find P(A).
Solution:
Here, P(A ∪ B) = 56, P(A ∩ B) = 13, P(B′) = 13
P(B) = 1 – P(B′)
= 1 – 13
= 23
Since P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
56 = P(A) + 23−13
∴ 56 = P(A) + 13
∴ P(A) = 56−13 = 12

Question 9.
A bag contains 5 red, 4 blue and an unknown number m of green balls. If the probability of getting both the balls green, when two balls are selected at random is 17, find m.
Solution:
Total number of balls in the bag = 5 + 4 + m = 9 + m
Two balls are selected from (9 + m) balls in 9+mC2 ways.
∴ n(S) = 9+mC2
Let event A: The two balls selected are green.

∴ 2 balls can be selected from m balls in $^mC_2$ ways.

∴ n(A) = $^mC_2$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\frac{1}{7} = \frac{^mC_2}{^{9+m}C_2}$$

$$= \frac{\frac{m!}{(m-2)!\,2!}}{\frac{(9+m)!}{(9+m-2)!\,2!}}$$

$$= \frac{\frac{m(m-1)(m-2)!}{(m-2)! \times 2 \times 1}}{\frac{(9+m)(8+m)(7+m)!}{(7+m)! \times 2 \times 1}}$$

$$\frac{1}{7} = \frac{m(m-1)}{(9+m)(8+m)}$$

(9 + m)(8 + m) = 7m(m – 1)
72 + 9m + 8m + m2 = 7m2 – 7m
6m2 – 24m – 72 = 0
m2 – 4m – 12 = 0
(m – 6)(m + 2) = 0
m = 6 or m = -2
Since number of balls cannot be negative, m ≠ -2
∴ m = 6

Question 10.
From a group of 4 men, 4 women and 3 children, 4 persons are selected at random. Find the probability that,
(i) no child is selected.
(ii) exactly 2 men are selected.
Solution:
The group consists of 4 men, 4 women and 3 children, i.e., 4 + 4 + 3 = 11 persons.
4 persons are to be selected from this group.
∴ 4 persons can be selected from 11 persons in $^{11}C_4$ ways.

∴ n(S) = $^{11}C_4$

(i) Let event A: No child is selected.

∴ 4 persons can be selected from 4 men and 4 women, i.e., from 8 persons in $^8C_4$ ways.

∴ n(A) = $^8C_4$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{^8C_4}{^{11}C_4}$$

$$= \frac{8 \times 7 \times 6 \times 5}{11 \times 10 \times 9 \times 8}$$

$$= \frac{1680}{7920}$$

$$= \frac{7}{33}$$

(ii) Let event B: Exactly 2 men are selected.

∴ 2 men are selected from 4 men in $^4C_2$ ways, and remaining 2 persons are selected from 7 persons (i.e., 4 women and 3 children) in $^7C_2$ ways.

$$n(A) = {}^4C_2 \times {}^7C_2$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{{}^4C_2 \times {}^7C_2}{{}^{11}C_4}$$

$$= \frac{\frac{4 \times 3}{2} \times \frac{7 \times 6}{2}}{\frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1}}$$

$$= \frac{126}{330}$$

$$= \frac{21}{55}$$

Question 11.
A number is drawn at random from the numbers 1 to 50. Find the probability that it is divisible by 2 or 3 or 10.
Solution:
One number can be drawn at random from the numbers 1 to 50 in $^{50}C_1$ = 50 ways.
∴ n(S) = 50
Let event A: The number drawn is divisible by 2.
∴ A = {2, 4, 6, 8, 10, ..., 48, 50}
∴ n(A) = 25
∴ P(A) = $\frac{n(A)}{n(S)} = \frac{25}{50}$
Let event B: The number drawn is divisible by 3.
B = {3, 6, 9, 12, ..., 48}
∴ n(B) = 16
∴ P(B) = $\frac{n(B)}{n(S)} = \frac{16}{50}$
Let event C: The number drawn is divisible by 10.
C = {10, 20, 30, 40, 50}
∴ n(C) = 5
∴ P(C) = $\frac{n(C)}{n(S)} = \frac{5}{50}$
Now, A ∩ B = {6, 12, 18, 24, 30, 36, 42, 48}
∴ n(A ∩ B) = 8
∴ P(A ∩ B) = $\frac{n(A \cap B)}{n(S)} = \frac{8}{50}$
B ∩ C = {30}
∴ n(B ∩ C) = 1
∴ P(B ∩ C) = $\frac{n(B \cap C)}{n(S)} = \frac{1}{50}$
A ∩ C = {10, 20, 30, 40, 50}
∴ n(A ∩ C) = 5
∴ P(A ∩ C) = $\frac{n(A \cap C)}{n(S)} = \frac{5}{50}$
A ∩ B ∩ C = {30}
∴ n(A ∩ B ∩ C) = 1
∴ P(A ∩ B ∩C) = $\frac{n(A \cap B \cap C)}{n(S)} = \frac{1}{50}$
∴ P(the number is divisible by 2 or 3 or 10)
P(A ∪ B ∪ C) = P(A) + P(B) + P(C) – P(A ∩ B) – P(B ∩ C) – P(A ∩ C) + P(A ∩ B ∩ C)
= $\frac{25}{50} + \frac{16}{50} + \frac{5}{50} - \frac{8}{50} - \frac{1}{50} - \frac{5}{50} + \frac{1}{50}$
= $\frac{33}{50}$

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Maharashtra State Board 11th Maths Solutions Chapter 9 Probability Ex 9.3

Question 1.
A bag contains 3 red marbles and 4 blue marbles. Two marbles are drawn at random without replacement. If the first marble drawn is red, what is the probability that the second marble is blue?
Solution:
Total number of marbles = 3 + 4 = 7
Let event A: The first marble drawn is red.
∴ P(A) = $\frac{^3C_1}{^7C_1}=\frac{3}{7}$
Let event B: The second marble drawn is blue.
Since the first red marble is not replaced in the bag, we now have 6 marbles out of which 4 are blue.
∴ Probability that the second marble is blue under the condition that the first red marble is not replaced in the bag = P(B/A)
= $\frac{^4C_1}{^6C_1}=\frac{4}{6}=\frac{2}{3}$
∴ Required probability = P(A ∩ B) = P(B/A) . P(A)
= $\frac{2}{3}\times\frac{3}{7}$
= $\frac{2}{7}$

Alternate Method:
Total number of marbles = 3 + 4 = 7
Two marbles are drawn at random without replacement.
∴ n(S) = $^7C_1\times{}^6C_1$ = 7 × 6 = 42
Let event A: The first marble is red and second marble is blue.
First red marble can be drawn from 3 red marbles in $^3C_1$ ways and second blue marble can be drawn from 4 blue marbles in $^4C_1$ ways.
∴ n(A) = $^3C_1\times{}^4C_1$ = 3 × 4 = 12
∴ P(A) = $\frac{n(A)}{n(S)}=\frac{12}{42}=\frac{2}{7}$

Question 2.
A box contains 5 green pencils and 7 yellow pencils. Two pencils are chosen at random from the box without replacement. What is the probability that both are yellow?
Solution:
Total number of pencils = 5 + 7 = 12
Let event A: The first pencil chosen is yellow.
∴ P(A) = $\frac{^7C_1}{^{12}C_1}=\frac{7}{12}$
Let event B: The second pencil chosen is yellow.
Since the first yellow pencil is not replaced in the box, we now have 11 pencils, out of which 6 are yellow.
∴ Probability that the second pencil is yellow under the condition that the first yellow pencil is not replaced in the box = P(B/A)
= $\frac{^6C_1}{^{11}C_1}$
= $\frac{6}{11}$
Required probability = P(A ∩ B)
= P(B/A) . P(A)
= $\frac{6}{11}\times\frac{7}{12}$
= $\frac{7}{22}$

Question 3.
In a sample of 40 vehicles, 18 are red, 6 are trucks, of which 2 are red. Suppose that a randomly selected vehicle is red. What is the probability it is a truck?
Solution:
One vehicle is selected from 40 vehicles.
Let event A: The selected vehicle is red.
There are total of 18 red vehicles.
∴ P(A) = $\frac{^{18}C_1}{^{40}C_1}=\frac{18}{40}=\frac{9}{20}$
Let event B: The selected vehicle is a truck.
There are total of 6 trucks.
Since 2 trucks are red, they are common between A and B.
∴ P(A ∩ B) = $\frac{^2C_1}{^{40}C_1}=\frac{2}{40}=\frac{1}{20}$
∴ Probability that the selected vehicle is a truck under the condition that it is red = P(B/A)
= $\frac{P(A\cap B)}{P(A)}$
= $\frac{\frac{1}{20}}{\frac{9}{20}}$
= $\frac{1}{9}$

Question 4.
From a pack of well-shuffled cards, two cards are drawn at random. Find the probability that both the cards are diamonds when
(i) the first card drawn is kept aside.
(ii) the first card drawn is replaced in the pack.
Solution:
In a pack of 52 cards, there are 13 diamond cards.
Let event A: The first card drawn is a diamond card.

∴ P(A) = $\frac{^{13}C_1}{^{52}C_1} = \frac{13}{52} = \frac{1}{4}$

(i) Let event B: The second card drawn is a diamond card.
Since the first diamond card is kept aside, we now have 51 cards, out of which 12 are diamond cards.
Probability that the second card is a diamond card under the condition that the first diamond card is kept aside in the pack = P(B/A)

= $\frac{^{12}C_1}{^{51}C_1} = \frac{12}{51} = \frac{4}{17}$

∴ Required probability = P(A ∩ B)
= P(B/A) . P(A)

= $\frac{1}{4} \times \frac{4}{17}$

= $\frac{1}{17}$

(ii) Let event B: The second card drawn is a diamond card.
Since the first diamond card is replaced in the pack, we now again have 52 cards, out of which 13 are diamond cards.
∴ Probability that the second card is a diamond card under the condition that the first diamond card is replaced in the pack = P(B/A)

= $\frac{^{13}C_1}{^{52}C_1} = \frac{13}{52} = \frac{1}{4}$

Required probability = P(A ∩ B)
= P(B/A) . P(A)

= $\frac{1}{4} \times \frac{1}{4}$

= $\frac{1}{16}$

Question 5.
A, B, and C try to hit a target simultaneously but independently. Their respective probabilities of hitting the target are $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{5}{8}$.
Find the probability that the target
(a) is hit exactly by one of them.
(b) is not hit by any one of them.
(c) is hit.
(d) is exactly hit by two of them.
Solution:
Let event A: A can hit the target,
event B: B can hit the target,
event C: C can hit the target.

$$\therefore \quad P(A) = \frac{3}{4},\ P(B) = \frac{1}{2},\ P(C) = \frac{5}{8}$$

$$\therefore \quad P(A') = 1 - P(A) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$P(B') = 1 - P(B) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(C') = 1 - P(C) = 1 - \frac{5}{8} = \frac{3}{8}$$

Since A, B, C are independent events,
A′, B′, C′ are also independent events.
(a) Let event W: Target is hit exactly by one of them.

$$P(W) = P(A \cap B' \cap C') \cup P(A' \cap B \cap C') \cup P(A' \cap B' \cap C)$$

$$= P(A) \cdot P(B') \cdot P(C') + P(A') \cdot P(B) \cdot P(C') + P(A') \cdot P(B') \cdot P(C)$$

$$= \left(\frac{3}{4} \times \frac{1}{2} \times \frac{3}{8}\right) + \left(\frac{1}{4} \times \frac{1}{2} \times \frac{3}{8}\right) + \left(\frac{1}{4} \times \frac{1}{2} \times \frac{5}{8}\right)$$

$$= \frac{9}{64} + \frac{3}{64} + \frac{5}{64} = \frac{17}{64}$$

(b) Let event X: Target is not hit by any one of them.
∴ P(X) = P(A′ ∩ B′ ∩ C′)
= P(A′) P(B′) P(C′)

= $\frac{1}{4} \times \frac{1}{2} \times \frac{3}{8}$

= $\frac{3}{64}$

(c) Let event Y: Target is hit.
∴ P(Y) = 1 – P(target is not hit by any one of them)
= 1 – 364
= 6164

(d) Let event Z: Target is hit by exactly two of them.

$$P(Z) = P(A \cap B \cap C') \cup P(A \cap B' \cap C) \cup P(A' \cap B \cap C)$$

$$= P(A) \cdot P(B) \cdot P(C') + P(A) \cdot P(B') \cdot P(C) + P(A') \cdot P(B) \cdot P(C)$$

$$= \left(\frac{3}{4} \times \frac{1}{2} \times \frac{3}{8}\right) + \left(\frac{3}{4} \times \frac{1}{2} \times \frac{5}{8}\right) + \left(\frac{1}{4} \times \frac{1}{2} \times \frac{5}{8}\right)$$

$$= \frac{9}{64} + \frac{15}{64} + \frac{5}{64} = \frac{29}{64}$$

Question 6.
The probability that a student X solves a problem in dynamics is 25 and the probability that student Y solves the same problem is 14.
What is the probability that
(i) the problem is not solved?
(ii) the problem is solved?
(iii) the problem is solved exactly by one of them?
Solution:
Let event A: Student X solves the problem in dynamics,
event B: Student Y solves the problem in dynamics.
∴ P(A) = 25, P(B) = 14
∴ P(A′) = 1 – P(A) = 1 – 25 = 35
P(B′) = 1 – P(B) = 1 – 14 = 34
Since A and B are independent events,
A′ and B′ are also independent events.
(i) Let event C: Problem is not solved.
∴ P(C) = P(A′ ∩ B′)
= P(A′) . P(B′)
= 35×34
= 920

(ii) Let event D: Problem is solved.
Problem can be solved if at least one of the two students solves the problem.
∴ P(D) = P(at least one student solves the problem)
= 1 – P(no student solves the problem)
= 1 – P(A′ ∩ B′)
= 1 – P(A′) P(B′)
= 1 – 35×34
= 1 – 920
= 1120

(iii) Let event E: The problem is solved exactly by one of them.
∴ P(E) = P(A′ ∩ B) ∪ P(A ∩ B′)
= P(A′) . P(B) + P(A) . P(B′)
= (35×14)+(25×34)
= 320+620
= 920

Question 7.
A speaks truth in 80% of the cases and B speaks truth in 60% of the cases. Find the probability that they contradict each other in narrating an incident.
Solution:
Let event A : A speaks the truth,
event B : B speaks the truth.
∴ P(A) = 80100=45
and P(B) = 60100=35
P(A′) = 1 – P(A) = 1 – 45 = 15
and P(B′) = 1 – P(B) = 1 – 35 = 25
∴ P(A and B contradict each other) = P(A speaks the truth and B lies) + P (A lies and B speaks the truth)

= P(A ∩ B′) + P(A′ ∩ B)
= P(A) P(B′) + P(A′) P(B)
= (45×25)+(15×35)
= 825+325
= 1125

Question 8.
Two hundred patients who had either Eye surgery or Throat surgery were asked whether they were satisfied or unsatisfied regarding the result of their surgery. The following table summarizes their response.

Surgery	Satisfied	Unsatisfied	Total
Throat	70	25	95
Eye	90	15	105
Total	160	40	200

If one person from the 200 patients is selected at random, determine the probability
(a) that the person was satisfied given that the person had Throat surgery.
(b) that person was unsatisfied given that the person had eye surgery.
(c) the person had Throat surgery given that the person was unsatisfied.
Solution:
(a) Let event A: The patient was satisfied,
event B: The patient had throat surgery.
Given, n(S) = 200
n(A ∩ B) = 70
∴ P(A ∩ B) = n(A∩B)n(S)=70200
n(B) = 95
∴ P(B) = n(B)n(S)=95200
∴ Required probability = P(A / B)
= P(A∩B)P(B)
= (70200)(95200)
= 7095
= 1419

Check:
Reduce the sample space to the set of throat patients only.
n(S) = 95
Let E : Patient had satisfactory throat surgery.
n(E) = 70
∴ P(E) = n(E)n(S)=7095=1419

(b) Let event C : The patient was unsatisfied,
event D : The patient had a eye surgery.
Given, n(S) = 200
n(C ∩ D) = 15
∴ P(C ∩ D) = n(C∩D)n(S)=15200
n(D) = 105
∴ P(D) = 105200
Required probability = P(C / D)
= P(C∩D)P(D)
= (15200)(105200)
= 17

(c) Let event F : The patient had a throat surgery,
event G : The patient was unsatisfied.
Given, n(S) = 200
n(F ∩ G) = 25
∴ P(F ∩ G) = n(F∩G)n(S)=25200
n(G) = 40
∴ P(G) = n(G)n(S)=40200
∴ Required probability = P(F / G)
= P(F∩G)P(G)

= (25200)(40200)

= 58

Question 9.

Two dice are thrown together. Let A be the event ‘getting 6 on the first die’ and B be the event ‘getting 2 on the second die’. Are events A and B independent?

Solution:

When two dice are thrown, the sample space is

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

∴ n(S) = 36

Let event A: Getting 6 on the first die.

∴ A = {(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

∴ n(A) = 6

∴ P(A) = n(A)n(S)=636=16

Let event B : Gettting 2 on the second die.

∴ B = {(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)}

∴ n(B) = 6

∴ P(B) = n(B)n(S)=636=16

Now, A ∩ B = {(6, 2)}

∴ n(A ∩ B) = 1

∴ P(A ∩ B) = n(A∩B)n(S)=136 …..(i)

P(A) × P(B) = 16×16=136 ……..(ii)

From (i) and (ii), we get

P(A ∩ B) = P(A) × P(B)

∴ A and B are independent events.

Question 10.

The probability that a man who is 45 years old will be alive till he becomes 70 is 512. The probability that his wife who is 40 years old will be alive till she becomes 65 is 38. What is the probability that, 25 years hence,

(a) the couple will be alive?

(b) exactly one of them will be alive?

(c) none of them will be alive?

(d) at least one of them will be alive?

Solution:

Let event A: The man will be alive till 70.

∴ P(A) = 512

Let event B: The wife will be alive till 65.

∴ P(B) = 38

∴ P(A’) = 1 – P(A) = 1 – 512 = 712

P(B’) = 1 – P(B) = 1 – 38 = 58

Since A and B are independent events,

A’ and B’ are also independent events.

(a) Let event C : Both man and his wife will be alive.

∴ P(C) = P(A ∩ B) = P(A) . P(B)

= 512×38

= 532

(b) Let event D: Exactly one of them will be alive.

∴ P(D) = P(A’ ∩ B) + P(A ∩ B’)

= P(A’) . P(B) + P(A) . P(B’)

= (712×38)+(512×58)

= 2196+2596

= 2348

(c) Let event E: None of them will be alive.

∴ P(E) = P(A’ ∩ B’) = P(A’) . P(B’)

= 712×58

= 3596

(d) Let event F: At least one of them will be alive.

∴ P(F) = 1 – P(none of them will be alive)

= 1 – 3596
= 6196

Question 11.
A box contains 10 red balls and 15 green balls. Two balls are drawn in succession without replacement. What is the probability that,
(a) the first is red and the second is green?
(b) one is red and the other is green?
Solution:
Total number of balls = 10 + 15 = 25
(a) Let event A: First ball drawn is red.
∴ P(A) = $_{10}C_{1}{}_{25}C_{1}$=1025=25
Let event B: Second ball drawn is green.
Since the first red ball is not replaced in the box, we now have 24 balls, out of which 15 are green.
∴ Probability that the second ball is green under the condition that the first red ball is not replaced in the box = P(B/A)
= $_{15}C_{1}{}_{24}C_{1}$=1524=58
∴ Required probability = P(A ∩ B) = P(B/A) . P(A)
= 25×58
= 14

(b) To find the probability that one ball is red and the other is green, there are two possibilities:
First ball is red and second ball is green.
OR
The first ball is the green and the second ball is red.
From above, we get
P(First ball is red and second ball is green) = 14
Similarly,
P(First ball is green and second ball is red) = $_{15}C_{1}{}_{25}C_{1}$×$_{10}C_{1}{}_{24}C_{1}$=1525×1024=14
∴ Required probability = P(First ball is red and second ball is green) + P(First ball is green and second ball is red)
= 14 + 14
= 12

Question 12.
A bag contains 3 yellow and 5 brown balls. Another bag contains 4 yellow and 6 brown balls. If one ball is drawn from each bag, what is the probability that,
(a) both the balls are of the same colour?
(b) the balls are of a different colours?
Solution:
(a) Let event A: A yellow ball is drawn from each bag.
Probability of drawing one yellow ball from total 8 balls of first bag and that of drawing one yellow ball out of total 10 balls of second bag is
P(A) = $_{3}C_{1}{}_{8}C_{1}$×$_{4}C_{1}{}_{10}C_{1}$ = 38×410=320
Let event B: A brown ball is drawn from each bag.
Probability of drawing one brown ball out of total 8 balls of first bag and that of drawing one brown ball out of total 10 balls of second bag is
P(B) = $_{5}C_{1}{}_{8}C_{1}$×$_{6}C_{1}{}_{10}C_{1}$ = 58×610=38
Since both the events are mutually exclusive events,
P(A ∩ B) = 0
∴ P(both the balls are of the same colour) = P(both are of yellow colour) or P(both are of brown colour)
= P(A) + P(B)
= 320+38
= 2140

(b) P(both the balls are of different colour) = 1 – P(both the balls are of the same colour)
= 1 – 2140
= 1940

Question 13.
An urn contains 4 black, 5 white, and 6 red balls. Two balls are drawn one after the other without replacement. What is the probability that at least one of them is black?
Solution:
Total number of balls in the um = 4 + 5 + 6 = 15
Two balls are drawn from 15 balls without replacement.

∴ n(S) = 15C1×14C1 = 15 × 14 = 210

Let event A: At least one ball is black.
i.e., the first ball is black, and the second ball is non-black or the first ball is non-black and the second ball is black, or both the first and second balls are black.

∴ n(A) = 4C1×11C1+11C1×4C1+4C1×3C1
= 4 × 11 + 11 × 4 + 4 × 3
= 100

∴ P(A) = n(A)n(S) = 100210=1021

Check:
Required probability = 1 – P(no black ball in two balls)
= 1 – 11C215C2=1−11×1015×14=1−1121=1021

Question 14.
Three fair coins are tossed. What is the probability of getting three heads given that at least two coins show heads?
Solution:
When three fair coins are tossed, the sample space is
S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
∴ n(S) = 8
Let event A: Getting three heads.
∴ A = {HHH}
Let event B: Getting at least two heads.
∴ B = {HHT, HTH, THH, HHH}
∴ n(B) = 4
∴ P(B) = n(B)n(S)=48
Now, A ∩ B = {HHH}
∴ n(A ∩ B) = 1
∴ P(A ∩ B) = n(A∩B)n(S)=18
∴ Probability of getting three heads, given that at least two coins show heads, is given by
P(A/B) = P(A∩B)P(B)
= 1848
= 14

Question 15.
Two cards are drawn one after the other from a pack of 52 cards without replacement. What is the probability that both the cards are drawn are face cards?
Solution:
In a pack of52 cards, there are 12 face cards.
Let event A: The first card drawn is a face card.
∴ P(A) = 12C152C1=1252=313
Let event B: The second card drawn is a face card.
Since the first card is not replaced in the pack, we now have 51 cards, out of which 11 are face cards.
∴ Probability that the second card is a face card under the condition that the first card is not replaced in the pack = P(B/A)
= 11C151C1=1151
∴ Required probability = P(A ∩ B) = P(B/A) . P(A)
= 1151×313
= 11221

Question 16.
Bag A contains 3 red and 2 white balls and bag B contains 2 red and 5 white balls. A bag is selected at random, a ball is drawn and put into the other bag, and then a ball is drawn from that bag. Find the probability that both the balls are drawn are of the same colour.
Solution:
Let event C_1: The first ball drawn is red and from bag A,
event D_1: The first ball drawn is white and from bag A,
event E_1: The first ball drawn is red and from bag B,
event F_1: The first ball drawn is white and from bag B,
event C_2: Second ball drawn is red and from bag B,
event D_2: Second ball drawn is white and from bag B,
event E_2: Second ball drawn is red and from bag A,
event F_2: Second ball drawn is white and from bag A,
event G: Selecting bag A in the first place,
event H: Selecting bag B in the first place.
P(G) = P(H) = 12
Let event X: Both the balls drawn are of same colour.

∴ P(X) = P(G) × P (X/G) + P(H) × P(X/H) …….(i)
If bag A is selected in first place, then In bag A, we have 5 balls, out of which 3 are red.
Probability of getting first red ball from bag A = P(C1) = ${}_{3}C_{1}{}_{5}C_{1}=35$
Since first red ball is put into the bag B, we now have 8 balls in bag B, out of which 3 are red.
∴ Probability of getting second red ball from bag B.
$P(C_2/C_1)$ = ${}_{3}C_{1}{}_{8}C_{1}=38$
Similarly, probability of getting first white ball from bag A = $P(D_1)$ = ${}_{2}C_{1}{}_{5}C_{1}=25$
and probability of getting second white ball form bag B = $P(D_2/D_1)$ = ${}_{6}C_{1}{}_{8}C_{1}=68$
∴ $P(X/G) = P(C_1)\ P(C_2/C_1) + P(D_1)\ P(D_2/D_1)$
= $35×38+25×68$
= 2140 …..(ii)
Similarly, $P(X/H) = P(E_1)\ P(E_2/E_1) + P(F_1)\ P(F_2/F_1)$
= $27×46+57×36$
= 2342 ………(iii)
From (i), (ii), (iii),
Required probability = $12×2140+12×2342$
= 36046720
= 9011680

Question 17.
Activity: A bag contains 3 red and 5 white balls. Two balls are drawn at random one after the other without replacement. Find the probability that both the balls are white.
Solution:
Let, event A: The first ball drawn is white
event B: Second ball drawn is white.
P(A) = 58
After drawing the first ball, without replacing it into the bag a second ball is drawn from the remaining 7 balls.
∴ P(B/A) = 47
∴ P(Both balls are white) = P(A ∩ B)
= P(A) . P(B/A)
= $58 × 47$
= 514

Question 18.
A family has two children. Find the probability that both the children are girls, given that at least one of them is a girl.
Solution:
A family has two children.
∴ Sample space S = {BB, BG, GB, GG}
∴ n(S) = 4
Let event A: At least one of the children is a girl.
∴ A = {GG, GB, BG}
∴ n(A) = 3
∴ P(A) = $n(A)n(S)=34$
Let event B: Both children are girls.
∴ B = {GG}
∴ n(B) = 1
∴ P(B) = $n(B)n(S)=14$
Also, A ∩ B = B
∴ P(A ∩ B) = P(B) = 14
∴ Required probability = P(B/A)
= $P(B∩A)P(A)$
= 1434
= 13

Maharashtra State Board 11th Maths Solutions Chapter 9 Probability Ex 9.4

Question 1.
There are three bags, each containing 100 marbles. Bag 1 has 75 red and 25 blue marbles. Bag 2 has 60 red and 40 blue marbles and Bag 3 has 45 red and 55 blue marbles. One of the bags is chosen at random and marble is picked from the chosen bag. What is the probability that the chosen marble is red?
Solution:
Let event R: Chosen marble is red.
Let event B_i: ith bag is chosen.
∴ $P(B_i) = \frac{1}{3}$
If Bag 1 is chosen, it has 75 red and 25 blue marbles.
∴ Probability that the chosen marble is red under the condition that it is from Bag 1 = $P(R/B_1)$
$= \frac{^{75}C_1}{^{100}C_1}$
$= \frac{75}{100}$
= 0.75
Similarly we get,
$P(R/B_2) = \frac{60}{100} = 0.60$
$P(R/B_3) = \frac{45}{100} = 0.45$
∴ Required probability
$P(R) = P(B_1)\ P(R/B_1) + P(B_2)\ P(R/B_2) + P(B_3)\ P(R/B_3)$
$= \frac{1}{3}(0.75) + \frac{1}{3}(0.60) + \frac{1}{3}(0.45)$
$= \frac{1}{3}(1.8)$
= 0.60

Question 2.
A box contains 2 blue and 3 pink balls and another box contains 4 blue and 5 pink balls. One ball is drawn at random from one of the two boxes and it is found to be pink. Find the probability that it was drawn from
(i) first box
(ii) second box
Solution:
Let event A_1: The ball is drawn from 1st box and
event A_2: The ball is drawn from the 2nd box.
∴ $P(A_1) = \frac{1}{2}$, $P(A_2) = \frac{1}{2}$
Let event B: The ball drawn is pink.
There are 5 balls in the 1st box, of which 3 are pink.
∴ $P(B/A_1) = \frac{3}{5}$
There are 9 balls in the 2nd box, of which 5 are pink.
∴ $P(B/A_2) = \frac{5}{9}$
(i) By Bayes' theorem,
the probability that a pink ball is drawn from the first box, is given by

$$P(A_1/B) = \frac{P(A_1)\cdot P(B/A_1)}{P(A_1)\cdot P(B/A_1) + P(A_2)\cdot P(B/A_2)}$$

$$= \frac{\frac{1}{2}\times\frac{3}{5}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{5}{9}}$$

$$= \frac{\frac{3}{10}}{\frac{3}{10}+\frac{5}{18}} = \frac{\frac{3}{10}}{\frac{27+25}{90}} = \frac{\frac{3}{10}}{\frac{52}{90}} = \frac{27}{52}$$

(ii) By Bayes' theorem,
the probability that a pink ball is drawn from the second box, is given by

$$P(A_2/B)$$
$$= \frac{P(A_2)\cdot P(B/A_2)}{P(A_1)\cdot P(B/A_1)+P(A_2)\cdot P(B/A_2)}$$
$$= \frac{\frac{1}{2}\times\frac{5}{9}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{5}{9}}$$
$$= \frac{\frac{5}{18}}{\frac{3}{10}+\frac{5}{18}} = \frac{\frac{5}{18}}{\frac{27+25}{90}} = \frac{\frac{5}{18}}{\frac{52}{90}} = \frac{25}{52}$$

Question 3.
There is a working women's hostel in a town, where 75% are from neighbouring town. The rest all are from the same town. 48% of women who hail from the same town are graduates and 83% of the women who have come from the neighbouring town are also graduates. Find the probability that a woman selected at random is a graduate from the same town.
Solution:
Let the total number of women be 100.
∴ n(S) = 100
Let event N: Women are from neighbouring town,
event W: Women are from same town and
event G: Women are graduates.
Number of women from neighbouring town,
n(N) = 75
Number of women from same town,
n(W) = 25
∴ P(N) = $\frac{n(N)}{n(S)}=\frac{75}{100}$ and

P(W) = $\frac{n(W)}{n(S)}=\frac{25}{100}$

P(G/N), P(G/W) represent probabilities that woman is graduate given that she is from neighbouring town or same town respectively.
∴ P(G/N) = $\frac{n(G/N)}{n(S)}=\frac{83}{100}$ and

P(G/W) = $\frac{n(G/W)}{n(S)}=\frac{48}{100}$

By Bayes' theorem, the probability that a women selected at random is a graduate from the same town, is given by

$$P(W/G) = \frac{P(W)P(G/W)}{P(W).P(G/W)+P(N).P(G/N)}$$
$$= \frac{\left(\frac{25}{100}\right)\cdot\left(\frac{48}{100}\right)}{\left(\frac{25}{100}\right)\cdot\left(\frac{48}{100}\right)+\left(\frac{75}{100}\right)\cdot\left(\frac{83}{100}\right)}$$
$$= \frac{25\times 48}{(25\times 48)+(75\times 83)}$$
$$= \frac{48}{48+249}$$
$$= \frac{16}{99}$$

Question 4.
If E_1 and E_2 are equally likely, mutually exclusive and exhaustive events and $P(A/E_1) = 0.2$, $P(A/E_2) = 0.3$. Find $P(E_1/A)$.
Solution:
E_1 and E_2 are equally likely, mutually exclusive and exhaustive events.

$\therefore P(E_1) = P(E_2) = 12$

$$P(A/E_1) = \frac{P(A \cap E_1)}{P(E_1)} = 0.2 = \frac{2}{10}$$

$$P(A/E_2) = \frac{P(A \cap E_2)}{P(E_2)} = 0.3 = \frac{3}{10}$$

By Bayes' theorem,

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

$$= \frac{\frac{1}{2}\cdot\left(\frac{2}{10}\right)}{\frac{1}{2}\left(\frac{2}{10}\right)+\frac{1}{2}\left(\frac{3}{10}\right)} \quad \ldots[\because P(E_1) = P(E_2)]$$

$$= \frac{2}{2+3}$$

$$= \frac{2}{5} = 0.4$$

Question 5.
Jar I contains 5 white and 7 black balls. Jar II contains 3 white and 12 black balls. A fair coin is flipped; if it is Head, a ball is drawn from Jar I, and if it is Tail, a ball is drawn from Jar II. Suppose that this experiment is done and a white ball was drawn. What is the probability that this ball was in fact taken from Jar II?
Solution:
Let event J_1: Ball drawn from jar I,
event J_2: Ball drawn from jar II.
$P(J_1) = P(\text{head}) = 12$
$P(J_2) = P(\text{tail}) = 12$
Let event W: Ball drawn is white.
In Jar I, there are total 12 balls, out of which 5 balls are white.
∴ Probability that the ball drawn is white under the condtion that it is drawn from Jar I.
$P(W/J_1) = {}_5C_1{}_{12}C_1 = 512$
Similarly, $P(W/J_2) = {}_3C_1{}_{15}C_1 = 315 = 15$
Required probability = $P(J_2/W)$
By Bayes' theorem,

$$P(J_2/W) = \frac{P(J_2)\,P(W/J_2)}{P(J_1)\,P(W/J_1) + P(J_2)\,P(W/J_2)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{5}}{\frac{1}{2}\times\frac{5}{12}+\frac{1}{2}\times\frac{1}{5}}$$

$$= \frac{\frac{1}{5}}{\frac{25+12}{60}} = \frac{12}{37}$$

Question 6.
A diagnostic test has a probability 0.95 of giving a positive result when applied to a person suffering from a certain disease, and a probability 0.10 of giving a (false) positive result when applied to a non-sufferer. It is estimated that 0.5% of the population are sufferers. Suppose that the test is now administered to a person about whom we have no relevant information relating to the disease (apart from the fact that he/she comes from this population). Calculate the probability that:
(i) given a positive result, the person is a sufferer.
(ii) given a negative result, the person is a non-sufferer.
Solution:
Let event T: Test positive
event S: Sufferer
$P(S) = 0.5100 = 0.005$
$\therefore P(S') = 1 - P(S) = 1 - 0.005 = 0.995$
Since a probability of getting a positive result when applied to a person suffering from a disease is 0.95 and probability of getting positive result when applied to a non sufferer is 0.10.
$\therefore P(T/S) = 0.95$ and $P(T/S') = 0.10$
$\therefore P(T) = P(S)\,P(T/S) + P(S')\,P(T/S')$
$= 0.005 \times 0.95 + 0.995 \times 0.10$
$= 0.10425$
$\therefore P(T') = 1 - P(T) = 1 - 0.10425 = 0.8958$
(i) Required probability = $P(S/T)$

By Bayes' theorem,

$$P(S/T)=\frac{P(S)\,P(T/S)}{P(T)}$$

$$=\frac{0.005\times 0.95}{0.10425}=\frac{0.00475}{0.10425}$$

(ii) P(T'/S') = 1 – 0.1 = 0.9
Required probability = P(S'/T')
By Bayes' theorem

$$P(S'/T')=\frac{P(S')\,P(T'/S')}{P(T')}$$

$$=\frac{0.995\times 0.9}{0.8958}$$

$$=\frac{0.8955}{0.8958}$$

Question 7.
A doctor is called to see a sick child. The doctor has prior information that 80% of the sick children in that area have the flu, while the other 20% are sick with measles. Assume that there is no other disease in that area. A well-known symptom of measles is rash. From the past records, it is known that, chances of having rashes given that sick child is suffering from measles is 0.95. However occasionally children with flu also develop rash, whose chance are 0.08. Upon examining the child, the doctor finds a rash. What is the probability that child is suffering from measles?
Solution:
Let the total number of sick children be 100.
∴ n(S) = 100.
Let event A: The child is sick with flu,
event B: The child is sick with measles,
event C: The child is sick with rash.
∴ n(A) = 80 and n(B) = 20
∴ P(A) = $\frac{n(A)}{n(S)}=\frac{80}{100}=\frac{4}{5}$
P(B) = $\frac{n(B)}{n(S)}=\frac{20}{100}=\frac{1}{5}$
Since the chances of having rashes, if the child is suffering from measles is 0.95 and the chances of having rashes if the child has flu is 0.08,
P(C/B) = 0.95 = $\frac{95}{100}$ and
P(C/A) = 0.08 = $\frac{8}{100}$
Required probability = P(B/C)
By Bayes' theorem,

$$P(B/C)=\frac{P(B)\cdot P(C/B)}{P(A)\cdot P(C/A)+P(B)\cdot P(C/B)}$$

$$=\frac{\left(\frac{1}{5}\right)\left(\frac{95}{100}\right)}{\left(\frac{4}{5}\right)\left(\frac{8}{100}\right)+\left(\frac{1}{5}\right)\left(\frac{95}{100}\right)}$$

$$=\frac{95}{32+95}$$

$$=\frac{95}{127}=0.748$$

Question 8.
2% of the population have a certain blood disease of a serious form: 10% have it in a mild form; and 88% don't have it at all. A new blood test is developed; the probability of testing positive is $\frac{9}{10}$ if the subject has the serious form, $\frac{6}{10}$ if the subject has the mild form, and $\frac{1}{10}$ if the subject doesn't have the disease. A subject is tested positive. What is the probability that the subject has serious form of the disease?
Solution:
Let event A_1: Disease in serious form,
event A_2: Disease in mild form,
event A_3: Subject does not have disease,
event B: Subject tests positive.
$P(A_1) = 0.02$, $P(A_2) = 0.1$, $P(A_3) = 0.88$
The probability of testing positive is $\frac{9}{10}$ if the subject has the serious form, $\frac{6}{10}$ if the subject has the mild form, and $\frac{1}{10}$ if the subject doesn't have the disease.
∴ $P(B/A_1) = 0.9$, $P(B/A_2) = 0.6$, $P(B/A_3) = 0.1$
$P(B) = P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + P(A_3)\,P(B/A_3)$

= 0.02 × 0.9 + 0.1 × 0.6 + 0.88 × 0.1
= 0.166
Required probability = $P(A_1/B)$
By Baye's theorem

$$P(A_1 / B) = \frac{P(A_1)\,P(B / A_1)}{P(B)}$$
$$= \frac{0.9 \times 0.02}{0.166}$$
$$= 0.108$$

Question 9.
A box contains three coins: two fair coins and one fake two-headed coin. A coin is picked randomly from the box and tossed.
(i) What is the probability that it lands head up?
(ii) If happens to be head, what is the probability that it is the two-headed coin?
Solution:
Let event A: Fair coin is tossed,
event B: Fake coin is tossed
and event H: Head occur.
Clearly, a fair coin has one head.
∴ Probability that head occur under the condition that the fair coin is tossed = P(H/A) = 12
Fake coin has two heads.
∴ Probability that head occur under the condition that the fake coin is tossed = P(H/B) = 1
n(A) = 2, n(B) = 1, n(S) = 3
∴ P(A) = n(A)n(S)=23
P(B) = n(B)n(S)=13
(i) Required probability
P(H) = P(A) P(H/A) + P(B) P(H/B)
= 23×12+13×1
= 13+13
= 23

(ii) Required probability = P(B/H)
By Baye's theorem

$$P(B / H) = \frac{P(B)\ P(H / B)}{P(H)}$$
$$= \frac{\frac{1}{3} \times 1}{\frac{2}{3}}$$
$$= \frac{1}{2}$$

Question 10.
There are three social media groups on a mobile: Group I, Group II and Group III. The probabilities that Group I, Group II and Group III sending the messages on sports are 25, 12 and 23 respectively. The probability of opening the messages by Group I, Group II and Group III are 12, 14 and 14 respectively. Randomly one of the messages is opened and found a message on sports. What is the probability that the message was from Group III.
Solution:
Let event A: Message sent on sports by group I,
event B: Message sent on sports by group II,
event C: Message sent on sports by group III,
event E: Message is opened.
Given that the probabilities that Group I, Group II and Group III sending the messages on sports are 25, 12 and 23 respectively and the probability of opening the messages by Group I, Group II and Group III are 12, 14 and 14 respectively.
∴ P(A) = 25
P(B) = 12
P(C) = 23
P(E/A) = 12
P(E/B) = 14
P(E/C) = 14
Required probability = P(C/E)

By Baye's theorem

$$P(C / E)$$
$$= \frac{P(C)P(E/C)}{P(A)P(E/A)+P(B)P(E/B)+P(C)P(E/C)}$$
$$= \frac{\frac{2}{3}\times\frac{1}{4}}{\frac{2}{5}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{4}+\frac{2}{3}\times\frac{1}{4}}$$
$$= \frac{\frac{1}{6}}{\frac{1}{5}+\frac{1}{8}+\frac{1}{6}}$$
$$= \frac{\frac{1}{6}}{\frac{59}{120}} = \frac{20}{59}$$

Question 11.

Mr. X goes to office by Auto, Car and train. The probabilities of him travelling by these modes are 27, 37, 27 respectively. The chances of him being late to the office are 12, 14, 14 respectively by Auto, Car and train. On one particular day he was late to the office. Find the probability that he travelled by car.

Solution:

Let A, C and T be the events that Mr. X goes to office by Auto, Car and Train respectively.

Let L be event that he is late.

Given that $P(A) = \boxed{\frac{2}{7}}$, $P(C) = \boxed{\frac{3}{7}}$, $P(T) = \boxed{\frac{2}{7}}$

$P(L/A) = \frac{1}{2}$, $P(L/C) = \boxed{\frac{1}{4}}$, $P(L/T) = \frac{1}{4}$.

$$P(L) = P(A \cap L) + P(C \cap L) + P(T \cap L)$$
$$= P(A).P(L/A) + P(C).P(L/C) + P(T).P(L/T)$$
$$= \boxed{\frac{2}{7}}\times\boxed{\frac{1}{2}}+\boxed{\frac{3}{7}}\times\boxed{\frac{1}{4}}+\boxed{\frac{2}{7}}\times\boxed{\frac{1}{4}}$$
$$= \boxed{\frac{1}{7}}+\boxed{\frac{3}{28}}+\boxed{\frac{2}{28}} = \boxed{\frac{9}{28}}$$
$$P(C/L) = \frac{P(L\cap C)}{P(L)}$$
$$= \frac{P(C)P(L/C)}{P(L)}$$
$$= \frac{\boxed{\frac{3}{7}}\times\boxed{\frac{1}{4}}}{\boxed{\frac{9}{28}}} = \boxed{\frac{1}{3}}$$

Maharashtra State Board 11th Maths Solutions Chapter 9 Probability Ex 9.5

Question 1.
If odds in favour of X solving a problem are 4 : 3 and odds against Y solving the same problem are 2 : 3. Find the probability of:
(i) X solving the problem
(ii) Y solving the problem
Solution:
(i) Odds in favour of X solving a problem are 4 : 3.
∴ The probability of X solving the problem is
P(X) = 44+3=47

(ii) Odds against Y solving the problem are 2 : 3.
∴ The probability of Y solving the problem is
P(Y) = 1 – P(Y')
= 1 – 22+3
= 1 – 25
= 35

Question 2.
The odds against John solving a problem are 4 to 3 and the odds in favour of Rafi solving the same problem are 7 to 5. What is the chance that the problem is solved when both of them try it?
Solution:
The odds against John solving a problem are 4 to 3.
Let event P(A') = P (John does not solve the problem)
= 44+3
= 47
So, the probability that John solves the problem
P(A) = 1 – P(A') = 1 – 47 = 37
Similarly, Let P(B) = P(Rafi solves the problem)
Since the odds in favour of Rafi solving the problem are 7 to 5,
P(B) = 77+5 = 712
Required probability
P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
Since A, B are independent events,
P(A ∩ B) = P(A) . P(B)
∴ Required probability = P(A) + P(B) – P(A) . P(B)

$$= \frac{3}{7} + \frac{7}{12} - \frac{3}{7} \times \frac{7}{12}$$

$$= \frac{85}{84} - \frac{1}{4}$$

$$= \frac{85-21}{84}$$

$$= \frac{64}{84} = \frac{16}{21}$$

Question 3.
The odds against student X solving a statistics problem are 8 : 6 and odds in favour of student Y solving the same problem are 14 : 16.
Find the chance that
(i) the problem will be solved if they try it independently.
(ii) neither of them solves the problem.
Solution:
The odds against X solving a problem are 8 : 6.
Let P(X') = P(X does not solve the problem) = 88+6 = 814
So, the probability that X solves the problem
P(X) = 1 – P(X') = 1 – 814 = 614
Similarly, let P(Y) = P(Y solves the problem)
Since odds in favour of Y solving the problem are 14 : 16,
P(Y) = 1414+16=1430
So, the probability that Y does not solve the problem
P(Y') = 1 – P(Y)
= 1 – 1430
= 1630

(i) Required probability
P(X ∪ Y) = P(X) + P(Y) – P(X ∩ Y)
Since X and Y are independent events,
P(X ∩ Y) = P(X) . P(Y)
∴ Required probability = P(X) + P(Y) – P(X) . P(Y)
= 614+1430−614×1430
= 73105

(ii) Required probability = P(X′ ∩ Y′)
Since X and Y are independent events, X′ and Y′ are also independent events.
∴ Required probability = P(X′) . P(Y′)
= 814×1630
= 32105

Question 4.
The odds against a husband who is 60 years old, living till he is 85 are 7 : 5. The odds against his wife who is now 56, living till she is 81 are 5 : 3. Find the probability that
(i) at least one of them will be alive 25 years hence.
(ii) exactly one of them will be alive 25 years hence.
Solution:
The odds against her husband living till he is 85 are 7 : 5.
Let P(H′) = P(husband dies before he is 85) = 77+5=712
So, the probability that the husband would be alive till age 85
P(H) = 1 – P(H′) = 1 – 712 = 512
Similarly, P(W′) = P(Wife dies before she is 81)
Since the odds against wife will be alive till she is 81 are 5 : 3.
∴ P(W′) = 55+3=58
So, the probability that the wife would be alive till age 81
P(W) = 1 – P(W′) = 1 – 58 = 38
(i) Required probability
P(H ∪ W) = P(H) + P(W) – P(H ∩ W)
Since H and W are independent events,
P(H ∩ W) = P(H) . P(W)
∴ Required probability = P(H) + P(W) – P(H) . P(W)
= 512+38−512×38
= 40+36-1596
= 6196

(ii) Required probability = P(H ∩ W′) + P(H′ ∩ W)
Since H and W are independent events, H′ and W′ are also independent events.
∴ Required probability = P(H) . P(W′) + P(H′) . P(W)
= 512×58+712×38
= 25+2196
= 4696
= 2348

Question 5.
There are three events A, B, and C, one of which must, and only one can happen. The odds against event A are 7 : 4 and odds against event B are 5 : 3. Find the odds against event C.
Solution:
Since odds against A are 7 : 4,
P(A) = 47+4=411
Since odds against B are 5 : 3,
P(B) = 35+3=38
Since only one of the events A, B and C can happen,
P(A) + P(B) + P(C) = 1
411 + 38 + P(C) = 1
∴ P(C) = 1 – (411 + 38)
= 1 – (32+3388)
= 2388
∴ P(C′) = 1 – P(C)

$= 1 - \frac{23}{88}$
$= \frac{65}{88}$
∴ Odds against the event C are P(C′) : P(C)
$= \frac{65}{88} : \frac{23}{88}$
= 65 : 23

Question 6.
In a single toss of a fair die, what are the odds against the event that number 3 or 4 turns up?
Solution:
When a fair die is tossed, the sample space is
S = {1, 2, 3, 4, 5, 6}
∴ n(S) = 6
Let event A: 3 or 4 turns up.
∴ A = {3, 4}
∴ n(A) = 2
∴ P(A) = $\frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$
P(A′) = 1 – P(A) = $1 - \frac{1}{3} = \frac{2}{3}$
∴ Odds against the event A are P(A′) : P(A)
$= \frac{2}{3} : \frac{1}{3}$
= 2 : 1

Question 7.
The odds in favour of A winning a game of chess against B are 3 : 2. If three games are to be played, what are the odds in favour of A's winning at least two games out of the three?
Solution:
Let event A: A wins the game and event B: B wins the game.
Since the odds in favour of A winning a game against B are 3 : 2,
the probability of occurrence of event A and B is given by
P(A) = $\frac{3}{3+2} = \frac{3}{5}$ and P(B) = $\frac{2}{3+2} = \frac{2}{5}$
Let event E: A wins at least two games out of three games.
∴ P(E) = P(A) . P(A) . P(B) + P(A) . P(B) . P(A) + P(B) . P(A) . P(A) + P(A) . P(A) . P(A)

$$= \frac{3}{5} \times \frac{3}{5} \times \frac{2}{5} + \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5} + \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$$

$$= \frac{18}{125} + \frac{18}{125} + \frac{18}{125} + \frac{27}{125}$$

$$= \frac{81}{125}$$

$$P(E') = 1 - P(E)$$

$$= 1 - \frac{81}{125}$$

$$= \frac{44}{125}$$

∴ Odds in favour of A's winning at least two games out of three are P(E) : P(E′)
$= \frac{81}{125} : \frac{44}{125}$
= 81 : 44

Maharashtra State Board 11th Maths Solutions Chapter 9 Probability Miscellaneous Exercise 9

(I) Select the correct answer from the given four alternatives.

Question 1.
There are 5 girls and 2 boys, then the probability that no two boys are sitting together for a photograph is
(A) 121
(B) 47
(C) 27
(D) 57
Answer:
(D) 57
Hint:
There are 5 girls and 2 boys.
They can be arranged among themselves in $7P7$ = 7! ways.
∴ Girls can be arranged among themselves in $5P5$ = 5! ways.
No two boys should sit together.
Let girls be denoted by the letter G.
– G – G – G – G – G –
There are 6 places, marked by '-' where boys can sit.
∴ Boys can be arranged in
$6P2=6!(6-2)!$
= $6\times5\times4!4!$
= 30 ways.
∴ Required probability = $5!\times307!=5!\times307\times6\times5!=57$

Question 2.
In a jar, there are 5 black marbles and 3 green marbles. Two marbles are picked randomly one after the other without replacement. What is the possibility that both the marbles are black?
(A) 514
(B) 58
(C) 57
(D) 516
Answer:
(A) 514

Question 3.
Two dice are thrown simultaneously. Then the probability of getting two numbers whose product is even is
(A) 34
(B) 14
(C) 57
(D) 12
Answer:
(A) 34
Hint:
Two dice are thrown.
∴ n(S) = 36
Getting two numbers whose product is even, i.e., one of the two numbers must be even.
Let event A: Getting even number on first dice,
event B: Getting even number on second dice.
n(A) = 18, n(B) = 18, n(A ∩ B) = 9
Required probability = P(A ∩ B)
= $n(A)+n(B)-n(A\cap B)n(S)$
= $18+18-936$
= 34

Question 4.
In a set of 30 shirts, 17 are white and the rest are black. 4 white and 5 black shirts are tagged as 'PARTY WEAR'. If a shirt is chosen at random from this set, the possibility of choosing a black shirt or a 'PARTY WEAR' shirt is
(A) 1115

(B) 1330
(C) 913
(D) 1730
Answer:
(D) 1730
Hint:
17 white + 13 black = 30 shirts
4 white and 5 black are 'PARTY WEAR'
A: Choosing a black shirt
∴ P(A) = ${}_{13}C_{1}{}_{30}C_{1}=1330$
B: Choosing a 'PARTY WEAR' shirt.
∴ P(B) = ${}_{9}C_{1}{}_{30}C_{1}=930$
There are 5 black 'PARTY WEAR' shirts.
∴ P(A ∩ B) = ${}_{5}C_{1}{}_{30}C_{1}=530$
∴ Required probability
P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
= 1330 + 930 – 530
= 1730

Question 5.
There are 2 shelves. One shelf has 5 Physics and 3 Biology books and the other has 4 Physics and 2 Biology books. The probability of drawing a Physics book is
(A) 914
(B) 3148
(C) 938
(D) 12
Answer:
(B) 3148
Hint:
Let event S_1: First shelve is selected,
event S_2: Second shelve is selected,
event P: Drawing a physics book.
∴ P(S_1) = 12 and P(S_2) = 12
First shelve has 5 physics and 3 biology books, i.e., total 8 books.
∴ P(P/S_1) = ${}_{5}C_{1}{}_{8}C_{1}=58$
Similarly, P(P/S_2) = ${}_{4}C_{1}{}_{6}C_{1}=46=23$
∴ P(P) = P(S_1) . P(P/S_1) + P(S_2) . P(P/S_2)
= 12×58+12×23
= 3148

Question 6.
Two friends A and B apply for a job in the same company. The chances of A getting selected is 2/5 and that of B is 4/7. The probability that both of them get selected is
(A) 3435
(B) 135
(C) 835
(D) 2735
Answer:
(C) 835

Question 7.
The probability that a student knows the correct answer to a multiple-choice question is 23. If the student does not know the answer, then the student guesses the answer. The probability of the guessed answer being correct is 14. Given that the student has answered the question correctly, the probability that the student knows the correct answer is
(A) 56
(B) 67
(C) 78
(D) 89
Answer:
(D) 89
Hint:
Let event A: Student knows the correct answer,
event A': Student guesses the answer,

event B: Answer is correct.
∴ P(A) = 23, P(A’) = 13, P(B/A’) = 14
Clearly, P(B/A) = 1
Required probability = P(A/B)
= P(A)· P(B/A)P(A)· P(B/A)+P(A′)P(B/A′)
= 23×123×1+13×14
= 89

Question 8.
The bag I contain 3 red and 4 black balls while Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. The probability that it was drawn from Bag II is
(A) 3368
(B) 3569
(C) 3467
(D) 3568
Answer:
(D) 3568

Question 9.
A fair die is tossed twice. What are the odds in favour of getting 4, 5, or 6 on the first toss and 1, 2, 3, or 4 on the second toss?
(A) 1 : 3
(B) 3 : 1
(C) 1 : 2
(D) 2 : 1
Answer:
(C) 1 : 2
Hint:
A fair dice is tossed twice.
∴ n(S) = 36
A: Getting 4, 5, or 6 on the first toss and Getting 1, 2, 3, or 4 on the second toss.
∴ A = {(4, 1), (4, 2), (4, 3), (4, 4), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4)}
∴ P(A) = n(A)n(S)=1236=13
∴ Required answer = P(A) : P(A’) = 1 : 2

Question 10.
The odds against an event are 5 : 3 and the odds in favour of another independent event are 7 : 5. The probability that at least one of the two events will occur is
(A) 5296
(B) 7196
(C) 6996
(D) 1396
Answer:
(B) 7196

(II) Solve the following.

Question 1.
The letters of the word ‘EQUATION’ are arranged in a row. Find the probability that
(i) all the vowels are together
(ii) arrangement starts with a vowel and ends with a consonant.
Solution:
The letters of the word EQUATION can be arranged in 8! ways.
∴ n(S) = 8!
There are 5 vowels and 3 consonants.
(i) A: all vowels are together we need to arrange (E, U, A, I, O), Q, T, N
Let us consider all vowels as one unit.
So, there are 4 units, which can be arranged in 4! ways.
Also, 5 vowels can be arranged among themselves in 5! ways.
∴ n(A) = 4! × 5!
Required probability = P(A)
= n(A)n(S)
= 4!×5!8!
= 114

(ii) B: arrangement start with a vowel and ends with a consonant.
First and last places can be filled in 5 and 3 ways respectively.

Remaining 6 letters are arranged in 6! Ways.
∴ n(B) = 5 × 3 × 6!
Required probability = P(B)
= n(B)n(S)
= 5×3×6!8!
= 1556

Question 2.
There are 6 positive and 8 negative numbers. Four numbers are chosen at random, without replacement, and multiplied. Find the probability that the product is a positive number.
Solution:
Let event A: Four positive numbers are chosen,
event B: Four negative numbers are chosen,
event C: Two positive and two negative numbers are chosen.
Since four numbers are chosen without replacement,
n(A) = 6 × 5 × 4 × 3 = 360
n(B) = 8 × 7 × 6 × 5 = 1680
In event C, four numbers are to be chosen without replacement such that two numbers are positive and two numbers ate negative. This can be done in following ways:
+ + – – OR + – + – OR + – – + OR – + – + OR – – + + OR – + + –
∴ n(C) = 6 × 5 × 8 × 7 + 6 × 8 × 5 × 7 + 6 × 8 × 7 × 5 + 8 × 6 × 7 × 5 + 6 × 5 × 8 × 7 + 8 × 6 × 5 × 7
= 6 × (8 × 7 × 6 × 5)
=10080
Here, total number of numbers = 14
∴ n(S) = 14 × 13 × 12 × 11 = 24024
Since A, B, C are mutually exclusive events,
Required probability = P(A) + P(B) + P(C)

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} + \frac{n(C)}{n(S)}$$

$$= \frac{360 + 1680 + 10080}{24024}$$

$$= \frac{505}{1001}$$

Question 3.
Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly, and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?
Solution:
S = {1, 2,…., 10}
∴ n(S) = 10
A: Number is more than 3.
A = {4, 5, 6, 7, 8, 9, 10}
∴ n(A) = 7
∴ P(A) = n(A)n(S)=710
B: Number is even.
B = {2, 4, 6, 8, 10}
∴ A ∩ B = {4, 6, 8, 10}
∴ n(A ∩ B) = 4
∴ P(A ∩ B) = n(A∩B)n(S)=410
Required probability = P(B/A)
= P(A∩B)P(A)
= (410)(710)
= 47

Question 4.
If A, B and C are independent events, P(A ∩ B) = 12, P(B ∩ C) = 13, P(C ∩ A) = 16, then find P(A), P(B) and P(C).
Solution:
Since A and B are independent events,
P(A ∩ B) = P(A) . P(B)
∴ P(A) . P(B) = 12 ……(i)
B and C are independent events.
∴ P(B ∩ C) = P(B) . P(C)

∴ P(B) . P(C) = $\frac{1}{3}$ ……(ii)
A and C are independent events.
∴ P(A ∩ C) = P(A) . P(C)
∴ P(A) . P(C) = $\frac{1}{6}$ ……(iii)
Dividing (i) by (ii), we get
$\frac{P(A)P(B)}{P(B)P(C)} = \frac{\frac{1}{2}}{\frac{1}{3}}$
P(A) = $\frac{3}{2}$ P(C) ……(iv)
Substituting equation (iv) in (iii), we get
$\frac{3}{2}$ P(C) . P(C) = $\frac{1}{6}$
$[P(C)]^2 = \frac{1}{9}$
∴ P(C) = $\frac{1}{3}$
Substituting P(C) = $\frac{1}{3}$ in equation (ii), we get P(B) = 1
Substituting P(B) = 1 in equation (i), we get P(A) = $\frac{1}{2}$

Question 5.
If the letters of the word 'REGULATIONS' be arranged at random, what is the probability that there will be exactly 4 letters between R and E?
Solution:
There are 11 letters in the word 'REGULATIONS' which can be arranged among themselves in 11! ways.
∴ n(S) = 11!
Let event A: There will be exactly 4 letters between R and E.
R, E can occur at (1, 6), (2, 7), ….,(6, 11) positions. So, there are 6 possibilities.
Also, R and E can interchange their positions.
So, R, E can be arranged in 2 × 6 = 12 ways.
Remaining 9 letters can be arranged in 9! ways.
∴ n(A) = 12 × 9!
∴ P(A) = $\frac{n(A)}{n(S)} = \frac{12 \times 9!}{11!} = \frac{12 \times 9!}{11 \times 10 \times 9!} = \frac{6}{55}$

Question 6.
In how many ways can the letters of the word ARRANGEMENTS be arranged?
(i) Find the chance that an arrangement chosen at random begins with the letters EE.
(ii) Find the probability that the consonants are together.
Solution:
The word 'ARRANGEMENTS' has 12 letters in which 2A, 2E, 2N, 2R, G, M, T, S are there.
n(S) = Total number of arrangements = $\frac{12!}{2!2!2!2!} = \frac{12!}{(2!)^4}$
(i) A: Arrangement chosen at random begins with the letters EE.
If the first and second places are filled with EE, there are 10 letters left in which 2A, 2N, 2R, G, M, T, S are there.
∴ n(A) = $\frac{10!}{2!2!2!} = \frac{10!}{(2!)^3}$

(ii) B: Consonants (G, M, T, S, 2N, 2R) are together.
2A, 2E, and the group containing consonants form total 5 units. Which can be arranged in $\frac{5!}{2!2!}$ ways.
Also, 8 consonants can be arranged among themselves in $\frac{8!}{2!2!}$ ways.

$$n(B) = \frac{5!}{2!\,2!} \times \frac{8!}{2!\,2!}$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{5!}{2! \times 2!} \times \frac{8!}{2! \times 2!} \times \frac{(2!)^4}{(12)!}$$

$$= \frac{5!}{12 \times 11 \times 10 \times 9}$$

$$= \frac{1}{99}$$

Question 7.
A letter is taken at random from the letters of the word 'ASSISTANT' and another letter is taken at random from the letters of the word 'STATISTICS'. Find the probability that the selected letters are the same.
Solution:
Word ASSISTANT has 2A, I, N, 3S, 2T, and word STATISTICS has A, C, 2I, 3S, 3T.
C and N are uncommon letters.
In the words ASSISTANT, there are 9 letters out of which 2 letters are 'A', and in the word STATISTICS, there are 10 letters, out of which 1 letter is A.
∴ Probability of choosing A from both the letters = $\frac{^2C_1}{^9C_1} \times \frac{^1C_1}{^{10}C_1} = \frac{2}{9} \times \frac{1}{10} = \frac{1}{45}$

Similarly,

Probability of choosing I from both the letters = 1C19C1×2C110C1=19×210=145

Probability of choosing S from both the letters = 3C19C1×3C110C1=39×310=110

Probability of choosing T from both the letters = 2C19C1×3C110C1=29×310=115

Required probability = 145+145+110+115 = 1990

Question 8.
A die is loaded in such a way that the probability of the face with j dots turning up is proportional to j for j = 1, 2,….., 6. What is the probability, in one roil of the die, that an odd number of dots will turn up?
Solution:
According to the given condition, the probability of the face with 1, 2, 3, 4, 5, 6 dots turning up is proportional to 1, 2, 3, 4, 5, 6.
Let k be the common ration of proportionality.
∴ The probabilities of the faces with 1, 2, 3, 4, 5, 6 dots turning up are 1k , 2k, 3k, 4k, 5k, 6k respectively.
Since sum of the probabilities = 1,
k(1 + 2+ ….. + 6) = 1
k(6×72) = 1
k = 121
Required probability = P(1) + P(3) + P(5)
= 121+321+521
= 921
= 37

Question 9.
An urn contains 5 red balls and 2 green balls. A ball is drawn. If it's green, a red ball is added to the urn, and if it's red, a green ball is added to the urn. (The original ball is not returned to the urn). Then a second ball is drawn. What is the probability that the second ball is red?
Solution:
A: Event of drawing a red ball and placing a green ball in the urn
B: Event of drawing a green ball and placing a red ball
C: Event of drawing a red ball in the second draw
P(A) = 57
P(B) = 27
P(C/A) = 47
P(C/B) = 67
Required probability
P(C) = P(A) P(C/A) + P(B) P(C/B)
= 57×47+27×67
= 3249

Question 10.
The odds against A solving a certain problem are 4 to 3 and the odds in favour of B solving the same problem are 7 to 5, find the probability that the problem will be solved.
Solution:
The odds against A solving the problems are 4 : 3.
Let P(A') = P(A does not solve the problem) = 44+3=47
So, the probability that A solves the problem = P(A) = 1 – P(A')
= 1 – 47
= 37
Similarly, let P(B) = P(B solves the problem)
Since odds in favour of B solving the problem are 7 : 5.
∴ P(B) = 77+5=712
Required probability
P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
Since A and B are independent events.
∴ P(A ∩ B) = P(A) . P(B)
∴ Required probability = P(A) + P(B) – P(A) . P(B)
= 37+712−37×712
= 1621

Question 11.
If P(A) = P(A/B) = 15, P(B/A) = 13, then find
(i) P(A'/B)
(ii) P(B'/A')

Solution:
Since P(A) = P(A/B) = 15
P(A) = 15
and P(A∩B)P(B)=15
∴ P(A) = 15 ……(i)
P(B) = 5 P(A ∩ B) ……..(ii)
Since P(B/A) = 13
P(A∩B)P(A)=13
∴ P(A) = 3 P(A ∩ B) ………(iii)

i. $$P(A'/B) = \frac{P(A' \cap B)}{P(B)}$$
$$= \frac{P(B) - P(A \cap B)}{P(B)}$$
$$= 1 - \frac{P(A \cap B)}{P(B)}$$
$$= 1 - \frac{1}{5} \quad \ldots\text{[From (ii)]}$$
$$= \frac{4}{5}$$

ii. $$3P(A \cap B) = P(A) \quad \ldots\text{[From (iii)]}$$
$$\therefore \quad P(A \cap B) = \frac{1}{3}\left(\frac{1}{5}\right) = \frac{1}{15}$$
$$\therefore \quad P(B) = 5P(A \cap B) \quad \ldots\text{[From (ii)]}$$
$$= \frac{5}{15} = \frac{1}{3}$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{1}{5} + \frac{1}{3} - \frac{1}{15}$$
$$= \frac{3 + 5 - 1}{15} = \frac{7}{15}$$
$$P(A' \cap B') = P(A \cup B)'$$
$$= 1 - P(A \cup B)$$
$$= 1 - \frac{7}{15} = \frac{8}{15}$$
$$P(B' / A') = \frac{P(A' \cap B')}{P(A')} = \frac{\frac{8}{15}}{1 - \frac{1}{5}}$$
$$= \frac{\frac{8}{15}}{\frac{4}{5}} = \frac{2}{3}$$

Question 12.
Let A and B be independent events with P(A) = 14 and P(A ∪ B) = 2P(B) – P(A). Find
(i) P(B)
(ii) P(A/B)
(iii) P(B′/A)
Solution:
A and B are independent events. .
∴ P(A ∩ B) = P(A) × P(B)
(i) P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
∴ P(A ∪ B) = P(A) + P(B) – P(A) × P(B)
∴ 2P(B) – P(A) = P(A) + P(B) – P(A) × P(B) ……[∵ P(A ∪ B) = 2P(B) – P(A)]
∴ 2P(B) – 14 = 14 + P(B) – 14 × P(B)

∴ 2P(B) – P(B) + 14 P(B) = 14 + 14

ii. $P(A/B) = \frac{P(A \cap B)}{P(B)}$

∴ $P(A/B) = \frac{P(A) \times P(B)}{P(B)}$

$= P(A) = \frac{1}{4}$

iii. $P(B'/A) = \frac{P(B' \cap A)}{P(A)}$

$= \frac{P(B') \times P(A)}{P(A)}$

$= P(B')$

$= 1 - P(B)$

$= 1 - \frac{2}{5} = \frac{3}{5}$

Question 13.
Find the probability that a year selected will have 53 Wednesdays.
Solution:
A leap year comes after 3 years.
∴ The probability of a year being a leap year = 14
∴ Probability of a year being a non-leap year = 1 – 14 = 34
In a non-leap year, there are 52 weeks and one extra day, whereas a leap year has 52 weeks and 2 extra days.
∴ 53rd Wednesday's chance in a non-leap year = 17
Two extra days of a leap year can be
(Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun), (Sun, Mon)
∴ There are 2 possibilities of 53rd Wednesday in a leap year.
∴ 53rd Wednesday's chance in a leap year = 27
Required probability = P(a non-leap year and Wednesday) + P(a leap year and Wednesday)
= 34×17+14×27
= 528

Question 14.
The chances of P, Q and R, getting selected as principal of a college are 25, 25, 15 respectively. Their chances of introducing IT in the college are 12, 13, 14 respectively. Find the probability that
(a) IT is introduced in the college after one of them is selected as a principal.
(b) IT is introduced by Q.
Solution:
Let event P: P become principal,
event Q: Q become principal,
event R: R become principal,
event E: Subject IT is introduced.
Given, P(P) = 25
P(Q) = 25
P(R) = 15
P(E/P) = 12
P(E/Q) = 13
P(E/R) = 14
(a) Required probability
P(E) = P(P) P(E/P) + P(Q) P(E/Q) + P(R) P(E/R)
= 25×12+25×13+15×14
= 15+215+120
= 12+8+360
= 2360

(b) Required probability = P(Q/E)
By Bayes' theorem,
P(Q/E) = P(Q)P(E/Q)P(E)

= 25×132360

= 823

Question 15.
Suppose that five good fuses and two defective ones have been mixed up. To find the defective fuses, we test them one-by-one, at random and without replacement What is the probability that we are lucky and find both of the defective fuses in the first two tests?
Solution:
Number of fuses = 5 + 2 = 7

Testing two fuses one-by-one at random, without replacement from 7 can be done in $7C1 \times 6C1$ ways.

∴ n(S) = $7C1 \times 6C1$ = 7 × 6 = 42

Let event A: Getting defective fuses in the first two tests without replacement.
There are two defective fuses.

∴ n(A) = $2C1 \times 1C1$ = 2 × 1 = 2

∴ P(A) = n(A)n(S)=242=121

Question 16.
For three events A, B and C, we know that A and C are independent, B and C are independent, A and B are disjoint, P(A ∪ C) = 23, P(B ∪ C) = 34, P(A ∪ B ∪ C) = 1112. Find P(A), P(B) and P(C).
Solution:
Let P(A) = x, P(B) = y, P(C) = z
Since A, B are disjoint,
A ∩ B = Φ and A ∩ B ∩ C = Φ
∴ P(A ∩ B) = 0, P(A ∩ B ∩ C) = 0 ……(i)
Since A and C are independent,
P(A ∩ C) = P(A) P(C) = xz
Since B and C are independent,
P(B ∩ C) = P(B) P(C) = yz
P(A ∪ C) = P(A) + P(C) – P(A ∩ C)
∴ 23 = x + z – xz ……..(ii)
P(B ∪ C) = P(B) + P(C) – P(B ∩ C)
∴ 34 = y + z – yz ………(iii)
P(A ∪ B ∪ C) = P(A) + P(B) + P(C) – P(A ∩ B) – P(B ∩ C) – P(C ∩ A) + P(A ∩ B ∩ C)
1112 = x + y + z – 0 – yz – zx + 0 …… [From(i)]
= (x + z – xz) + (y + z – yz) – z
= 23+34 – z ……. [From (ii) and (iii)]

∴ $z = \frac{2}{3} + \frac{3}{4} - \frac{11}{12} = \frac{17-11}{12} = \frac{1}{2}$

Substituting z = $\frac{1}{2}$ in (ii), we get

$\frac{2}{3} = x + \frac{1}{2} - \frac{1}{2}x$

∴ $x = 2\left(\frac{2}{3} - \frac{1}{2}\right) = \frac{2}{6}$

∴ $x = \frac{1}{3}$

Substituting z = $\frac{1}{2}$ in (iii), we get

$\frac{3}{4} = y + \frac{1}{2} - \frac{1}{2}y$

∴ $y = 2\left(\frac{3}{4} - \frac{1}{2}\right) = 2\left(\frac{1}{4}\right) = \frac{1}{2}$

∴ **P(A) = $\frac{1}{3}$, P(B) = $\frac{1}{2}$, P(C) = $\frac{1}{2}$**

Question 17.
The ratio of boys to girls in a college is 3 : 2 and 3 girls out of 500 and 2 boys out of 50 of that college are good singers. A good singer is chosen. What is the probability that the chosen singer is a girl?
Solution:
Let event S: The student is a good singer,
event B: The student is a boy,
event G: The student is a girl.

Since the ratio of boys to girls is 3 : 2 and 3 girls out of 500 and 2 boys out of 50 are good singers.

$$P(B) = \frac{3}{5},\ P(G) = \frac{2}{5},\ P(S/G) = \frac{3}{500},$$

$$P(S/B) = \frac{2}{50}.$$

$$P(S) = P(G) \times P(S/G) + P(B) \times P(S/B)$$

$$= \frac{2}{5} \times \frac{3}{500} + \frac{3}{5} \times \frac{2}{50}$$

$$= \frac{2\times3}{5}\left(\frac{1}{500} + \frac{1}{50}\right)$$

$$= \frac{6}{5} \times \frac{11}{500}$$

$$= \frac{33}{1250}$$

Required probability = P(G / S)

By Bayes' theorem,

$$P(G/S) = \frac{P(G)\,P(S/G)}{P(S)}$$

$$= \frac{\frac{2}{5} \times \frac{3}{500}}{\frac{33}{1250}} = \frac{1}{11}$$

Question 18.

A and B throw a die alternatively till one of them gets a 3 and wins the game. Find the respective probabilities of winning. (Assuming A begins the game).

Solution:

Since P(getting 3) = 16,

P(not getting 3) = 1 – 16 = 56

In 1st throw if A gets 3, A wins

∴ P(A win) = 16

In 2nd throw by B (i.e., A does not get 3),

∴ P(B wins) = 56×16

In 3rd throw by A, P(A wins) = 56×56×16

(3rd throw by A shows that B has lost in 2nd throw) and so on.

$$P(\text{A winning}) = \frac{1}{6} + \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \ldots$$

$$= \frac{1}{6}\left[\underbrace{1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \ldots}_{\text{infinite geometric series}}\right]$$

$$= \frac{1}{6}\left(\frac{1}{1-\left(\frac{5}{6}\right)^2}\right)$$

$$\ldots\left[\begin{array}{l}\text{Sum of infinite}\\ \text{geometric series} = \dfrac{a}{1-r}\end{array}\right]$$

$$= \frac{1}{6} \times \frac{36}{36-25} = \frac{6}{11}$$

$$P(\text{B winning}) = \frac{5}{6}\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) + \ldots$$

$$= \left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \ldots\right]$$

$$= \frac{5}{36} \times \frac{1}{1-\left(\frac{5}{6}\right)^2}$$

$$= \frac{5}{36} \times \frac{36}{36-25} = \frac{5}{11}$$

Question 19.
Consider independent trials consisting of rolling a pair of fair dice, over and over. What is the probability that a sum of 5 appears before a sum of 7?
Solution:
When two dice are thrown, the sample space is
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(S) = 36
Let event A: The sum is 5 in a trial.
A = {(2, 3), (3, 2), (1, 4), (4, 1)}
∴ P(A) = $\frac{4}{36}=\frac{1}{9}$
Let event B: The sum is 7 in a trial.
B = {(2, 5), (5, 2), (3, 4), (4, 3), (1, 6), (6, 1)}
∴ P(B) = $\frac{6}{36}=\frac{1}{6}$
Let event C: Neither sum is 5 nor 7.
P(C) = 1 – P(A) – P(B)
= 1 – $\frac{1}{9}$ – $\frac{1}{6}$
= $\frac{26}{36}$
Let the sum of 5 appear in the nth trial for the first time and the sum of 7 has not occurred in the first (n – 1) trials.

Probability of this event = $[P(C)]^{n-1}\,P(A)$

$$\text{Required probability} = \sum_{n=1}^{\infty}\left(\frac{26}{36}\right)^{n-1}\left(\frac{1}{9}\right)$$

$$= \frac{1}{9}\left(\frac{1}{1-\frac{26}{36}}\right)$$

$$= \frac{1}{9}\left(\frac{18}{5}\right) = \frac{2}{5}$$

Question 20.
A machine produces parts that are either good (90%), slightly defective (2%), or obviously defective (8%). Produced parts get passed through an automatic inspection machine, which is able to detect any part that is obviously defective and discard it. What is the probability that the quality of the parts that make it through the inspection machine and get shipped?
Solution:
Let event G: The event that machine produces a good part,
event S: The event that machine produces a slightly defective part,
event D: The event that machine produces an obviously defective part.

$$P(G) = \frac{90}{100} = 0.90,\quad P(S) = \frac{2}{100} = 0.02,$$

$$P(D) = \frac{8}{100} = 0.08$$

Let $D^c = G \cup S$. Then

$$P(G / D^c) = \frac{P(G \cap D^c)}{P(D^c)}$$

$$= \frac{P(G)}{P(G \cup S)} \quad \ldots[\because G \cap (G \cup S) = G]$$

$$= \frac{P(G)}{P(G) + P(S)}$$

$\ldots[\because$ G and S are disjoint sets]

$$= \frac{0.90}{0.90 + 0.02}$$

$$= \frac{0.90}{0.92} = \frac{90}{92} = \frac{45}{46}$$

Question 21.
Given three identical boxes, I, II, and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?
Solution:
Let event B_1: Select box I having two gold coins.
event B_2: Selecting box II having two silver coins,
event B_3: Selecting box III having one silver and one gold coin,

event G: Coin is gold.

$$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$$

$$P(G / B_1) = 1, P(G / B_2) = 0, P(G / B_3) = \frac{1}{2}$$

$$P(G) = P(B_1)\,P(G / B_1) + P(B_2)\,P(G / B_2) + P(B_3)\,P(G / B_3)$$

$$= \frac{1}{3}\left[1 + 0 + \frac{1}{2}\right] = \frac{1}{3}\left(\frac{3}{2}\right) = \frac{1}{2}$$

To find the probability that the other can in the box is also gold. Which is possible only when it is drawn from the box I.
∴ Required probability = $P(B_1/G)$
By Bayes' theorem,

$$P(B_1 / G) = \frac{P(B_1)P(G/B_1)}{P(G)}$$

$$= \frac{\frac{1}{3}(1)}{\frac{1}{2}} = \frac{2}{3}$$

Question 22.
In a factory which manufactures bulbs, machines A, B, and C manufacture respectively 25%, 35% and 40% of the bulbs. Of their outputs, 5, 4, and 2 percent are respectively defective bulbs. A bulb is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by machine B?
Solution:
Let event A: Bulb manufactured by machine A
event B: Bulb manufactured by machine B
event C: Bulb manufactured by machine C
event D: Bulb defective
∴ P(A) = 25100
P(B) = 35100
P(C) = 40100
Machines A, B and C manufacture respectively 25%, 35% and 40% of the bulbs.
Of their outputs, 5, 4, and 2 percent are respectively defective bulbs.

$$P(D / A) = \frac{5}{100},\ P(D / B) = \frac{4}{100},$$

$$P(D / C) = \frac{2}{100}$$

$$P(D) = P(A)\,P(D / A) + P(B)\,P(D / B) + P(C)\,P(D / C)$$

$$= \frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100}$$

$$= \frac{1}{100 \times 100}(125 + 140 + 80) = \frac{345}{100 \times 100}$$

Required probability = P(B/D)
By Bayes' theorem,

$$P(B / D) = \frac{P(B)P(D/B)}{P(D)} = \frac{\frac{35}{100} \times \frac{4}{100}}{\frac{345}{100 \times 100}} = \frac{28}{69}$$

Question 23.
A family has two children. One of them is chosen at random and found that the child is a girl. Find the probability that
(i) both the children are girls.
(ii) both the children are girls given that at least one of them is a girl.
Solution:
A family has two children.
∴ Sample space S = {BB, BG, GB, GG}
(i) A: First child is a girl.
∴ A = {GB, GG}
∴ P(A) = 24=12
B: Second child is a girl.
∴ B = {BG, GG}
∴ A ∩ B = {GG}
∴ P(A ∩ B) = 14

Required probability

$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

(ii) A: At least one of the children is a girl.
∴ A = {GG, GB, BG}
∴ $P(A) = \frac{3}{4}$
B: both children are girls.
B = {GG}
∴ $P(B) = \frac{1}{4}$
Also, $A \cap B = B$

$$P(B / A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B)}{P(A)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$