Maharashtra State Board 11th Commerce Maths Solutions Chapter 3 Skewness Ex 3.1

Question 1.
For a distribution, mean = 100, mode = 127 and S.D. = 60. Find the Pearson coefficient of skewness Sk_p.
Solution:
Given, Mean = 100, Mode = 127, S.D. = 60

$$Sk_p = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$$
$$= \frac{100 - 127}{60}$$
$$= \frac{-27}{60}$$
$$= -0.45$$

Question 2.
The mean and variance of a distribution are 60 and 100 respectively. Find the mode and the median of the distribution if Sk_p = -0.3.
Solution:
Given, Mean = 60, Variance = 100, Sk_p = -0.3
∴ S.D. = √Variance = √100 = 10
Sk_p = Mean − Mode S.D.
∴ -0.3 = 60− Mode 10
∴ -3 = 60 – Mode
∴ Mode = 60 + 3 = 63
Mean – Mode = 3 (Mean – Median)
∴ 60 – 63 = 3(60 – Median)
∴ -3 = 180 – 3Median
∴ 3Median = 180 + 3 = 183
∴ Median = 1833
∴ Median = 61

Question 3.
For a data set, sum of upper and lower quartiles is 100, difference between upper and lower quartiles is 40 and the median is 30. Find the coefficient of skewness.
Solution:
Given, $Q_3 + Q_1$ = 100 ……(i)
$Q_3 - Q_1$ = 40 …..(ii)
Median = Q_2 = 30
Adding (i) and (ii), we get
$2Q_3$ = 140
∴ Q_3 = 70

Substituting the value of Q_3 in (i), we get
$70 + Q_1 = 100$
$\therefore Q_1 = 100 - 70 = 30$

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$
$$= \frac{70 + 30 - 2(30)}{40}$$
$$= \frac{70 + 30 - 60}{40}$$
$$\therefore \quad Sk_b = \frac{40}{40}$$
$$\therefore \quad Sk_b = 1$$

Question 4.
For a data set with an upper quartile equal to 55 and median equal to 42, if the distribution is symmetric, find the value of the lower quartile.
Solution:
Upper quartile = $Q_3 = 55$
Median = $Q_2 = 42$
Since, the distribution is symmetric.
$\therefore Sk_b = 0$
$Sk_b = Q_3+Q_1-2Q_2Q_3-Q_1$
$\therefore 0 = Q_3+Q_1-2Q_2Q_3-Q_1$
$\therefore 0 = Q_3 + Q_1 - 2Q_2$
$\therefore Q_1 = 2Q_2 - Q_3$
$\therefore Q_1 = 2(42) - 55$
$\therefore Q_1 = 84 - 55$
$\therefore Q_1 = 29$

Obtain coefficient of skewness by formula and comment on the nature of the distribution.

Height in inches	No. of Females
Less than 60	10
60-64	20
64-68	40
68-72	10
72-76	2

Solution:
We construct the less than cumulative frequency table as given below.

Height in inches	No. of Females (f)	Less than cumulative frequency (c.f.)
Less than 60	10	10
60-64	20	$30 \leftarrow Q_1$
64-68	40	$70 \leftarrow Q_2, Q_3$
68-72	10	80
72-76	2	82
Total	N = 82	

Q_1 class = class containing (N4)th observation

∴ N4=824 = 20.5

Cumulative frequency which is just greater than (or equal) to 20.5 is 30.

∴ Q_1 lies in the class 60 – 64.

∴ L = 60, h = 4, f = 20, c.f. = 10

$$\therefore \quad Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - \text{c.f.}\right)$$

$$= 60 + \frac{4}{20}(20.5 - 10)$$

$$= 60 + \frac{1}{5}(10.5)$$

$$= 60 + 2.1$$

$$\therefore \quad Q_1 = 62.1$$

Q_2 class = class containing (N2)th observation

∴ N2=822 = 41

Cumulative frequency which is just greater than (or equal) to 41 is 70.

∴ Q_2 lies in the class 64 – 68.

∴ L = 64, h = 4, f = 40, c.f. = 30

$$\therefore \quad Q_2 = L + \frac{h}{f}\left(\frac{N}{2} - \text{c.f.}\right)$$

$$= 64 + \frac{4}{40}(41 - 30)$$

$$= 64 + \frac{1}{10}(11)$$

$$= 64 + 1.1$$

$$\therefore \quad Q_2 = 65.1$$

Q_3 class = class containing (3 N4)th observation

∴ 3 N4=3×824 = 61.5

Cumulative frequency which is just greater than (or equal) to 61.5 is 70.

∴ Q_3 lies in the class 64 – 68.

∴ L = 64, h = 4, f = 40, c.f. = 30

$$f\left(4 \qquad\right)$$

$$= 64 + \frac{4}{40}(61.5 - 30)$$

$$= 64 + \frac{\quad}{10}(31.5)$$

$$= 64 + 3.15$$

$$\therefore \quad Q_3 = 67.15$$

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{67.15 + 62.1 - 2(65.1)}{67.15 - 62.1}$$

$$= \frac{129.25 - 130.2}{5.05}$$

$$= \frac{-0.95}{5.05}$$

$\therefore Sk_b = -0.1881$

Since, $Sk_b < 0$, the distribution is negatively skewed.

Question 6.

Find Sk_p for the following set of observations.

17, 17, 21, 14, 15, 20, 19, 16, 13, 17, 18

Solution:

$\Sigma x_i = 17 + 17 + 21 + 14 + 15 + 20 + 19 + 16 + 13 + 17 + 18 = 187$

Mean $= \frac{\sum x_i}{n} = \frac{187}{11} = 17$

Mode = Observation that occurs most frequently in the data = 17

$$Sk_p = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$$

$$= \frac{17 - 17}{\text{S.D.}}$$

$$= \frac{0}{\text{S.D.}}$$

$$= 0$$

Question 7.

Calculate Sk_b for the following set of observations of the yield of wheat in kg from 13 plots:

5, 4.2, 3.5, 3.6, 5.2

Solution:

The given data can be arranged in ascending order as follows:

3.5, 3.5, 3.5, 3.6, 3.6, 4.2, 4.6, 4.7, 4.7, 4.8, 5.1, 5.2, 5.5

Here, n = 13
Q_1 = value of (n+14)th observation
= value of (13+14)th observation
= value of (3.50)th observation
= value of 3rd observation + 0.50(value of 4th observation – value of 3rd observation)
= 3.5 + 0.50(3.6 – 3.5)
= 3.5 + 0.50(0.1)
= 3.5 + 0.05
∴ Q_1 = 3.55
Q_2 = value of 2(n+14)th observation
= value of 2(13+14)th observation
= value of (2 × 3.50)th observation
= value of 7th observation
∴ Q_2 = 4.6
Q_3 = value of 3(n+14)th observation
= value of 3(13+14)th observation
= value of (3 × 3.50)th observation
= value of (10.50)th observation
= value of 10th observation + 0.50 (value of 11th obseration – value of 10th observation)
= 4.8 + 0.50(5.1 – 4.8)
= 4.8 + 0.50(0.3)
∴ Q_3 = 4.95

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{4.95 + 3.55 - 2(4.6)}{4.95 - 3.55}$$

$$= \frac{8.5 - 9.2}{1.4}$$

$$= \frac{-0.7}{1.4}$$

∴ Sk_b = -0.5

Question 8.
For a frequency distribution $Q_3 – Q_2$ = 90 and $Q_2 – Q_1$ = 120. Find Sk_b.
Solution:
Given, $Q_2 – Q_1$ = 90, $Q_2 – Q_1$ = 120

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{Q_3 - Q_2 - Q_2 + Q_1}{Q_3 - Q_2 + Q_2 - Q_1}$$

$$= \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{90 - 120}{90 + 120}$$

$$= \frac{-30}{210}$$

$$= \frac{-1}{7}$$

$\therefore Sk_b = -0.1429$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 3 Skewness Miscellaneous Exercise 3

Question 1.
For u distribution, mean = 100, mode = 80 and S.D. = 20. Find Pearsonian coefficient of skewness Sk_p.
Solution:
Given, Mean = 100, Mode = 80, S.D. = 20

$$Sk_p = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$$

$$= \frac{100 - 80}{20} = \frac{20}{20} = 1$$

$\therefore \quad Sk_p = 1$

Question 2.
For a distribution, mean = 60, median = 75 and variance = 900. Find Pearsonian coefficient of skewness Sk_p.
Solution:
Given. Mean = 60, Median = 75, Variance = 900

$\therefore$ S.D. = √Variance = √900 = 30

$$Sk_p = \frac{3(\text{Mean} - \text{Median})}{\text{S.D.}}$$

$$= \frac{3(60-75)}{30} = \frac{3(-15)}{30} = \frac{-15}{10}$$

$$\therefore \quad Sk_p = -1.5$$

Question 3.
For a distribution, $Q_1 = 25$, $Q_2 = 35$ and $Q_3 = 50$. Find Bowley's coefficient of skewness Sk_b.
Solution:
Given $Q_1 = 25$, $Q_2 = 35$, $Q_3 = 50$

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{50 + 25 - 2(35)}{50 - 25} = \frac{75 - 70}{25} = \frac{5}{25} = \frac{1}{5}$$

$$\therefore \quad Sk_b = 0.2$$

Question 4.
For a distribution $Q_3 - Q_2 = 40$, $Q_2 - Q_1 = 60$. Find Bowlev's coefficient of skewness Sk_b.
Solution:
Given, $Q_3 - Q_2 = 40$, $Q_2 - Q_1 = 60$

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{Q_3 - Q_2 - Q_2 + Q_1}{Q_3 - Q_2 + Q_2 - Q_1}$$

$$= \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{40 - 60}{40 + 60} = -\frac{20}{100} = -\frac{1}{5}$$

$$Sk_b = -0.2$$

Question 5.
For a distribution, Bowley's coefficient of skewness is 0.6. The sum of upper and lower quartiles is 100 and median is 38. Find the upper and lower quartiles.

Solution:
Given, $Sk_b = 0.6$, $Q_3 + Q_1 = 100$,
Median = $Q_2 = 38$
Skb = $\frac{Q_3+Q_1-2Q_2}{Q_3-Q_1}$
$\therefore 0.6 = \frac{100-2(38)}{Q_3-Q_1}$
$\therefore 0.6(Q_3 - Q_1) = 100 - 76 = 24$
$\therefore Q_3 - Q_1 = 40$ ....(i)
$Q_3 + Q_1 = 100$ .....(ii) (given)
Adding (i) and (ii), we get
$2Q_3 = 140$
$\therefore Q_3 = 70$
Substituting the value of Q_3 in (ii), we get
$70 + Q_1 = 100$
$\therefore Q_1 = 100 - 70 = 30$
∴ upper quartile = 70 and lower quartile = 30

Question 6.
For a frequency distribution, the mean is 200, the coefficient of variation is 8% and Karl Pearsonian's coefficient of skewness is 0.3. Find the mode and median of the distribution.
Solution:
Mean = $\bar{x} = 200$
Coefficient of variation, C.V. = 8%, Skp = 0.3
C.V. = $\frac{\sigma}{\bar{x}} \times 100$, where σ = standard deviation
$\therefore 8 = \frac{\sigma}{200} \times 100$
$\therefore \sigma = \frac{8 \times 200}{100} = 16$
Now, $Sk_p = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$
$\therefore 0.3 = \frac{200 - \text{Mode}}{16}$
∴ 0.3 × 16 = 200 – Mode
∴ Mode = 200 – 4.8 = 195.2
Since, Mean – Mode = 3(Mean – Median)
∴ 200 – 195.2 = 3(200 – Median)
∴ 4.8 = 600 – 3Median
∴ 3Median = 600 – 4.8 = 595.2
∴ Median = 198.4

Question 7.
Calculate Karl Pearsonian's coefficient of skewness Skp from the follow ing data:

Marks above	0	10	20	30	40	50	60	70	80
No of students	120	115	108	98	85	60	18	5	0

Solution:
The given table is the cumulative frequency table of more than type.
From this table, we have to prepare the frequency distribution table and then calculate the value of Sk_p.
Construct the following table:

Marks above	No. of students 'more than' (c.f.)	Class-interval	Frequency f_i	Mid value x_i	f_ix_i	$f_ix_i^2$
0	120	0-10	5	5	25	125
10	115	10-20	7	15	105	1575
20	108	20-30	10	25	250	6250
30	98	30-40	13	35	455	15925
40	85	40-50	25	45	1125	50625
50	60	50-60	42	55	2310	127050
60	18	60-70	13	65	845	54925
70	5	70-80	5	75	375	28125
80	0	80-90	0	85	0	0
		Total	**120**	**–**	**5490**	**284600**

From the table, N = 120, Σf_ix_i = 5490 and $\sum f_ix_i^2$ = 284600
Mean = $\bar{x} = \frac{\sum f_ix_i}{N} = \frac{5490}{120}$ = 45.75
Maximum frequency 42 is of the class 50 – 60
∴ Mode lies in the class 50 – 60

$\therefore$ L = 50, f_1 = 42, f_0 = 25, f_2 = 13, h = 10

$$\therefore \quad \text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 50 + \frac{42 - 25}{2(42) - 25 - 13} \times 10$$

$$= 50 + \frac{17}{84 - 38} \times 10$$

$$= 50 + \frac{17}{46} \times 10$$

$$= 50 + 3.6957$$

$$= 53.6957$$

$$\text{S. D.} = \sqrt{\frac{\sum f_i x_i^2}{N} - (\bar{x})^2}$$

$$= \sqrt{\frac{284600}{120} - (45.75)^2}$$

$$= \sqrt{2371.6667 - 2093.0625}$$

$$= \sqrt{278.6042}$$

$$= 16.6914$$

Pearsonian's coefficient of skewness:

$$Sk_p = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$$

$$= \frac{45.75 - 53.6957}{16.6914}$$

$$= -\frac{7.9457}{16.6914}$$

$$\therefore \quad Sk_p = -0.4760$$

Alternate Method:

Let u = x−4510

Marks above	No. of students 'more than' (c.f.)	Class	Frequency (f_i)	Mid value x_i	u_i	f_iu_i	$f_iu_i^2$
0	120	0 – 10	5	5	– 4	– 20	80
10	115	10 – 20	7	15	– 3	– 21	63
20	108	20 – 30	10	25	– 2	– 20	40
30	98	30 – 40	13	35	– 1	– 13	13
40	85	40 – 50	25	45	0	0	0
50	60	50 – 60	42	55	1	42	42
60	18	60 – 70	13	65	2	26	52
70	5	70 – 80	5	75	3	15	45
80	0	80 – 90	0	85	4	0	0
		Total	120			9	335

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$\bar{u} = \frac{\sum f_i u_i}{N} = \frac{9}{120}$ = 0.075
$\therefore \bar{x}$ = 45 + 10($\bar{u}$)
= 45 + 10(0.075)
= 45 + 0.75
= 45.75
Var(u) = $\sigma^2_u = \frac{\sum f_i u_i^2}{N} - (\bar{u})^2$
= $\frac{335}{120}$ – $(0.075)^2$
= 2.7917 – 0.0056
= 2.7861
Var(X) = h^2 × Var(u)
= 100 × 2.7861
= 278.61
S.D. = √278.61 = 16.6916
Maximum frequency 42 is of the class 50 – 60.
∴ Mode lies in the class 50 – 60.
∴ L = 50, f_1 = 42, f_0 = 25, f_2 = 13, h = 10

$$\therefore \quad \text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$
$$= 50 + \frac{42-25}{2(42)-25-13} \times 10$$
$$= 50 + \frac{17}{84-38} \times 10$$
$$= 50 + \frac{17}{46} \times 10$$
$$= 50 + 3.6957$$
$$= 53.6957$$
$$\therefore \quad Sk_p = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}} = \frac{45.75 - 53.6957}{16.6916}$$
$$= \frac{-7.9457}{16.6916}$$
$$= -0.4760$$

Question 8.
Calculate Bowley’s coefficient of skewness Skb from the following data.

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Marks above	0	10	20	30	40	50	60	70	80
No of students	120	115	108	98	85	60	18	5	0

Solution:
To calculate Bowley's coefficient of skewness Skb, we construct the following table:

Marks above	No. of students 'more than' (c.f.)	Marks	Frequency (f_i)	Less than cumulative frequency (c.f.)
0	120	0-10	5	5
10	115	10-20	7	12
20	108	20-30	10	22
30	98	30-40	13	35 $\leftarrow Q_1$
40	85	40-50	25	60 $\leftarrow Q_2$
50	60	50-60	42	102 $\leftarrow Q_3$
60	18	60-70	13	115
70	5	70-80	5	120
80	0	80-90	0	120
		Total	**120**	–

Here, N = 120
Q_1 class = class containing the (N4)th observation
∴ N4=1204 = 30
Cumulative frequency which is just greater than (or equal to) 30 is 35.
∴ Q_1 lies in the class 30-40.
∴ L = 30, h = 10, f = 13, c.f. = 22

$$\therefore \quad Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f.\right)$$

$$= 30 + \frac{10}{13}(30 - 22)$$

$$= 30 + \frac{10}{13}(8)$$

$$= 30 + 6.1538$$

$$\therefore \quad Q_1 = 36.1538$$

Q_2 class = class containing the (N2)th observation
∴ N2=1202 = 60
Cumulative frequency which is just greater than (or equal to) 60 is 60.
∴ Q_2 lies in the class 40-50.
∴ L = 40, h = 10, f = 25, c.f. = 35

$$\therefore \quad Q_2 = L + \frac{h}{f}\left(\frac{N}{2} - c.f.\right)$$

$$= 40 + \frac{10}{25}(60 - 35)$$

$$= 40 + \frac{10}{25}(25)$$

$$\therefore \quad Q_2 = 50$$

Q_3 class = class containing the $\left(\frac{3N}{4}\right)^{th}$ observation

$\therefore \frac{3N}{4} = \frac{3 \times 120}{4} = 90$

Cumulative frequency which is just greater than (or equal to) 90 is 102.

$\therefore$ Q_3 lies in the class 50 – 60

$\therefore$ L = 50, h = 10, f = 42, c.f. = 60

$$\therefore \quad Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$$

$$= 50 + \frac{10}{42}(90 - 60)$$

$$= 50 + \frac{10}{42}(30)$$

$$= 50 + 7.1429$$

$$\therefore \quad Q_3 = 57.1429$$

Bowley's coefficient of skewness:

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{57.1429 + 36.1538 - 2(50)}{57.1429 - 36.1538}$$

$$= \frac{93.2967 - 100}{20.9891}$$

$$= \frac{-6.7033}{20.9891}$$

$$\therefore \quad Sk_b = -0.3194$$

Question 9.

Find Sk_p for the following set of observations:

18, 27, 10, 25, 31, 13, 28

Solution:

The given data can be arranged in ascending order as follows:

10, 13, 18, 25, 27, 28, 31

Here, n = 7

∴ Median = value of (n+12)th observation
= value of (7+12)th observation
= value of 4th observation
= 25
For finding standard deviation, we construct the following table:

	10	100
	13	169
	18	324
	25	625
	27	729
	28	784
	31	961
Total	152	3692

From the table, $\sum x_i = 152$, $\sum x_i^2 = 3692$

$$\text{Mean} = \bar{x} = \frac{\sum x_i}{n} = \frac{152}{7} = 21.7143$$

$$\therefore \quad \text{S.D.} = \sqrt{\frac{\sum x_i^2}{n} - (\bar{x})^2}$$

$$= \sqrt{\frac{3692}{7} - (21.7143)^2}$$

$$= \sqrt{527.4286 - 471.5108}$$

$$= \sqrt{55.9178}$$

$$= 7.4778$$

Coefficient of skewness,

$$Sk_p = \frac{3(\text{Mean} - \text{Median})}{\text{S.D.}}$$

$$= \frac{3(21.7143 - 25)}{7.4778}$$

$$= \frac{3(-3.2857)}{7.4778}$$

$$= \frac{-9.8571}{7.4778}$$

$$\therefore \quad Sk_p = -1.3182$$

Question 10.
Find Skb for the following set of observations:
18, 27, 10, 25, 31, 13, 28
Solution:
The given data can be arranged in ascending order as follows:
10, 13, 18, 25, 27, 28, 31
Here, n = 7
∴ Q_1 = value of (n+14)th observation
= value of (7+14)th observation
= value of 2nd observation
∴ Q_1 = 13
Q_2 = value of 2(n+14)th observation
= value of 2(7+14)th observation
= value of (2 × 2)th observation
= value of 4th observation
∴ Q_2 = 25
Q_3 = value of 3(n+14)th observation
= value of 3(7+14)th observation
= value of (3 × 2)th observation
= value of 6th observation
∴ Q_3 = 28
Coefficient of skewness,

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{28 + 13 - 2(25)}{28 - 13}$$

$$= \frac{41 - 50}{15}$$

$$= -\frac{9}{15}$$

∴ Sk_b = -0.6
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