Maharashtra State Board 12th Commerce Maths Solutions Chapter 3 Differentiation Ex 3.1

1. Find $\frac{dy}{dx}$ if,

Question 1.
y = $\sqrt{x+\frac{1}{x}}$
Solution:

$$\text{Given : } y = \sqrt{x+\frac{1}{x}}$$

$$\text{Let } u = x+\frac{1}{x}$$

$$\text{Then } y = \sqrt{u}$$

$$\therefore \frac{dy}{du} = \frac{d}{du}(u^{\frac{1}{2}}) = \frac{1}{2}u^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{u}} = \frac{1}{2\sqrt{x+\frac{1}{x}}}$$

$$\text{and } \frac{du}{dx} = \frac{d}{dx}\left(x+\frac{1}{x}\right)$$

$$= \frac{d}{dx}(x) + \frac{d}{dx}(x^{-1})$$

$$= 1+(-1)x^{-2} = 1-\frac{1}{x^2}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{1}{2\sqrt{x+\frac{1}{x}}}\cdot\left(1-\frac{1}{x^2}\right)$$

$$= \frac{1}{2}\left(x+\frac{1}{x}\right)^{-\frac{1}{2}}\left(1-\frac{1}{x^2}\right).$$

Question 2.
y = $\sqrt[3]{a^2+x^2}$
Solution:

$$\text{Given : } y = \sqrt[3]{a^2+x^2}$$

$$\text{Let } u = a^2+x^2$$

$$\text{Then } y = \sqrt[3]{u}$$

$$\therefore \frac{dy}{du} = \frac{d}{du}(u^{\frac{1}{3}}) = \frac{1}{3}u^{-\frac{2}{3}}$$

$$= \frac{1}{3}(a^2+x^2)^{-\frac{2}{3}}$$

$$\text{and } \frac{du}{dx} = \frac{d}{dx}(a^2+x^2)$$

$$= 0+2x = 2x$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$= \frac{1}{3}(a^2+x^2)^{-\frac{2}{3}}\cdot 2x$$

$$= \frac{2x}{3}(a^2+x^2)^{-\frac{2}{3}}$$

Question 3.
y = $(5x^3 - 4x^2 - 8x)^9$

Solution:

Given : $y = (5x^3 - 4x^2 - 8x)^9$

Let $u = 5x^3 - 4x^2 - 8x$

Then $y = u^9$

$$\therefore \frac{dy}{du} = \frac{d}{du}(u^9) = 9u^8$$
$$= 9(5x^3 - 4x^2 - 8x)^8$$

and $\frac{du}{dx} = \frac{d}{dx}(5x^3 - 4x^2 - 8x)$

$$= 5\frac{d}{dx}(x^3) - 4\frac{d}{dx}(x^2) - 8\frac{d}{dx}(x)$$
$$= 5 \times 3x^2 - 4 \times 2x - 8 \times 1$$
$$= 15x^2 - 8x - 8$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= 9(5x^3 - 4x^2 - 8x)^8 (15x^2 - 8x - 8).$$

2. Find $\frac{dy}{dx}$ if:

Question 1.
y = log(log x)
Solution:
Given y = log(log x)
Let u = log x
Then y = log u

$$\therefore \frac{dy}{du} = \frac{d}{du}(\log u)$$
$$= \frac{1}{u} = \frac{1}{\log x}$$

and $\frac{du}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{\log x} \times \frac{1}{x}$$
$$= \frac{1}{x \log x}.$$

Question 2.
$y = \log(10x^4 + 5x^3 - 3x^2 + 2)$
Solution:
Given $y = \log(10x^4 + 5x^3 - 3x^2 + 2)$
Let $u = 10x^4 + 5x^3 - 3x^2 + 2$

Then y = log u

$$\therefore \frac{dy}{du} = \frac{d}{du}(\log u) = \frac{1}{u}$$

$$= \frac{1}{10x^4 + 5x^3 - 3x^2 + 2}$$

$$\text{and } \frac{du}{dx} = \frac{d}{dx}(10x^4 + 5x^3 - 3x^2 + 2)$$

$$= 10\frac{d}{dx}(x^4) + 5\frac{d}{dx}(x^3) - 3\frac{d}{dx}(x^2) + \frac{d}{dx}(2)$$

$$= 10 \times 4x^3 + 5 \times 3x^2 - 3 \times 2x + 0$$

$$= 40x^3 + 15x^2 - 6x$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{10x^4 + 5x^3 - 3x^2 + 2} \times (40x^3 + 15x^2 - 6x)$$

$$= \frac{40x^3 + 15x^2 - 6x}{10x^4 + 5x^3 - 3x^2 + 2}.$$

Question 3.
y = log(ax^2 + bx + c)
Solution:
Given y = log(ax^2 + bx + c)
Let u = ax^2 + bx + c
Then y = log u

$$\therefore \frac{dy}{du} = \frac{d}{du}(\log u) = \frac{1}{u}$$

$$= \frac{1}{ax^2 + bx + c}$$

$$\text{and } \frac{du}{dx} = \frac{d}{dx}(ax + bx + c)$$

$$= a\frac{d}{dx}(x^2) + b\frac{d}{dx}(x) + \frac{d}{dx}(c)$$

$$= a \times 2x + b \times 1 \times 0 = 2ax + b$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{ax^2 + bx + c} \times (2ax + b)$$

$$= \frac{2ax + b}{ax^2 + bx + c}.$$

3. Find $\frac{dy}{dx}$ if:

Question 1.
y = e^{5x^2-2x+4}

Solution:

Given : $y = e^{5x^2 - 2x + 4}$

Let $u = 5x^2 - 2x + 4$

Then $y = e^u$

$$\therefore \frac{dy}{du} = \frac{d}{du}(e^u) = e^u$$
$$= e^{5x^2 - 2x + 4}$$

and $\frac{du}{dx} = \frac{d}{dx}(5x^2 - 2x + 4)$

$$= 5\frac{d}{dx}(x^2) - 2\frac{d}{dx}(x) + \frac{d}{dx}(4)$$
$$= 5 \times 2x - 2 \times 1 + 0 = 10x - 2$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= e^{5x^2 - 2x + 4} \times (10x - 2)$$
$$= (10x - 2)e^{5x^2 - 2x + 4}.$$

Question 2.

y = $a^{(1+\log x)}$

Solution:

Given : $y = a^{(1 + \log x)}$

Let $u = 1 + \log x$

Then $y = a^u$

$$\therefore \frac{dy}{du} = \frac{d}{du}(a^u) = a^u \cdot \log a$$
$$= a^{(1 + \log x)} \cdot \log a$$

and $\frac{du}{dx} = \frac{d}{dx}(1 + \log x)$

$$= 0 + \frac{1}{x} = \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= a^{(1 + \log x)} \cdot \log a \cdot \frac{1}{x}.$$

Question 3.

y = $5^{(x+\log x)}$

Solution:

Given : $y = 5^{(x+\log x)}$

Let $u = x + \log x$

Then $y = 5^u$

$$\therefore \frac{dy}{du} = \frac{d}{du}(5^u) = 5^u \cdot \log 5$$

$$= 5^{(x+\log x)} \cdot \log 5$$

and $\frac{du}{dx} = \frac{d}{dx}(x + \log x)$

$$= 1 + \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 5^{(x+\log x)} \cdot \log 5 \cdot \left(1 + \frac{1}{x}\right).$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 3 Differentiation Ex 3.2

1. Find the rate of change of demand (x) of a commodity with respect to price (y) if:

Question 1.
y = 12 + 10x + 25x2
Solution:
Given y = 12 + 10x + 25x2

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(12 + 10x + 25x^2)$$

$$= \frac{d}{dx}(12) + 10\frac{d}{dx}(x) + 25\frac{d}{dx}(x^2)$$

$$= 0 + 10 \times 1 + 25 \times 2x = 10 + 50x$$

By derivative of inverse function,

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{1}{10 + 50x}$$

Hence, the rate of change of demand (x) with respect to price (y) =dxdy=110+50x

Question 2.
y = 18x + log(x – 4)
Solution:
Given y = 18x + log (x – 4)

$$\therefore \frac{dy}{dx} = \frac{d}{dx}[18x + \log(x-4)]$$
$$= 18\frac{d}{dx}(x) + \frac{d}{dx}[\log(x-4)]$$
$$= 18 \times 1 + \frac{1}{x-4} \cdot \frac{d}{dx}(x-4)$$
$$= 18 + \frac{1}{x-4} \times (1-0)$$
$$= 18 + \frac{1}{x-4} = \frac{18x - 72 + 1}{x-4}$$
$$= \frac{18x-71}{x-4}$$

By derivative of inverse function

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{x-4}{18x-71}$$

Hence, the rate of change of demand (x) with respect to price (y) $= \frac{dx}{dy} = \frac{x-4}{18x-71}$

Question 3.
$y = 25x + \log(1 + x^2)$
Solution:
Given $y = 25x + \log(1 + x^2)$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}[25x + \log(1+x^2)]$$
$$= 25\frac{d}{dx}(x) + \frac{d}{dx}[\log(1+x^2)]$$
$$= 25 \times 1 + \frac{1}{1+x^2} \cdot \frac{d}{dx}(1+x^2)$$
$$= 25 + \frac{1}{1+x^2} \times (0+2x)$$
$$= 25 + \frac{2x}{1+x^2} = \frac{25 + 25x^2 + 2x}{1+x^2}$$
$$= \frac{25x^2 + 2x + 25}{1+x^2}$$

By derivative of inverse function,

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{1+x^2}{25x^2+2x+25}$$

Hence, the rate of change of demand (x) with respect to price (y) $\frac{dx}{dy} = \frac{1+x^2}{25x^2+2x+25}$

2. Find the marginal demand of a commodity where demand is x and price is y.

Question 1.
$y = xe^{-x} + 7$
Solution:

Given : $y = xe^{-x} + 7$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(xe^{-x} + 7)$$

$$= \frac{d}{dx}(xe^{-x}) + \frac{d}{dx}(7)$$

$$= x \cdot \frac{d}{dx}(e^{-x}) + e^{-x} \cdot \frac{d}{dx}(x) + 0$$

$$= x \times e^{-x} \cdot \frac{d}{dx}(-x) + e^{-x} \times 1$$

$$= xe^{-x}(-1) + e^{-x}$$

$$= e^{-x}(-x+1) = \frac{1-x}{e^x}$$

By the derivative of inverse function,

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{e^x}{1-x}$$

Hence, marginal demand $= \frac{dx}{dy} = \frac{e^x}{1-x}$.

Question 2.
y = $\frac{x+2}{x^2+1}$
Solution:

Given : $y = \frac{x+2}{x^2+1}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x+2}{x^2+1}\right)$$

$$= \frac{(x^2+1) \cdot \frac{d}{dx}(x+2) - (x+2) \cdot \frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(1+0) - (x+2)(2x+0)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2-4x}{(x^2+1)^2} = \frac{1-4x-x^2}{(x^2+1)^2}$$

By the derivative of inverse function,

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{(x^2+1)^2}{1-4x-x^2}$$

Hence, marginal demand $= \frac{dx}{dy}$

$$= \frac{(x^2+1)^2}{1-4x-x^2}$$

Question 3.
y = $\frac{5x+9}{2x-10}$

Solution:

Given : $y = \frac{5x+9}{2x-10}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{5x+9}{2x-10}\right)$$

$$= \frac{(2x-10)\cdot\frac{d}{dx}(5x+9) - (5x+9)\cdot\frac{d}{dx}(2x-10)}{(2x-10)^2}$$

$$= \frac{(2x-10)(5\times1+0) - (5x+9)(2\times1-0)}{(2x-10)^2}$$

$$= \frac{10x-50-10x-18}{(2x-10)^2} = \frac{-68}{(2x-10)^2}$$

By the derivative of inverse function,

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = -\frac{(2x-10)^2}{68}$$

Hence, marginal demand $= \frac{dx}{dy} = \frac{-(2x-10)^2}{68}$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 3 Differentiation Ex 3.3

1. Find $\frac{dy}{dx}$ if:

Question 1.

$y = x^{x^{2x}}$

Solution:

$$y = x^{x^{2x}}$$

$$\therefore \log y = \log x^{x^{2x}} = x^{2x}\cdot\log x$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{d}{dx}(x^{2x}\cdot\log x)$$

$$= x^{2x}\cdot\frac{d}{dx}(\log x) + (\log x)\cdot\frac{d}{dx}(x^{2x})$$

$$= x^{2x}\times\frac{1}{x} + (\log x)\cdot\frac{d}{dx}(x^{2x})$$

$$\therefore \frac{dy}{dx} = y\left[\frac{x^{2x}}{x} + (\log x)\cdot\frac{d}{dx}(x^{2x})\right]$$

$$= x^{x^{2x}}\left[\frac{x^{2x}}{x} + (\log x)\cdot\frac{d}{dx}(x^{2x})\right] \qquad \dots (1)$$

Let $u = x^{2x}$

Then $\log u = \log x^{2x} = 2x \log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{u}\cdot\frac{du}{dx} = 2\frac{d}{dx}(x \log x)$$

$$= 2\left[x\frac{d}{dx}(\log x) + (\log x)\cdot\frac{d}{dx}(x)\right]$$

$$= 2\left[x \times \frac{1}{x} + (\log x) \times 1\right]$$

$$\therefore \frac{du}{dx} = 2u(1 + \log x)$$

$$\therefore \frac{d}{dx}(x^{2x}) = 2x^{2x}(1 + \log x)$$

$\therefore$ from (1),

$$\frac{dy}{dx} = x^{x^{2x}}\left[\frac{x^{2x}}{x} + (\log x) \times 2x^{2x}(1 + \log x)\right]$$

$$= x^{x^{2x}} \cdot x^{2x}\cdot\log x\left[2(1 + \log x) + \frac{1}{x \log x}\right].$$

Question 2.

y = x^{e^x}

Solution:

$$y = x^{e^x}$$

$$\therefore \log y = \log x^{e^x} = e^x\cdot\log x$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{d}{dx}(e^x\cdot\log x)$$

$$= e^x\cdot\frac{d}{dx}(\log x) + (\log x)\cdot\frac{d}{dx}(e^x)$$

$$= e^x\cdot\frac{1}{x} + (\log x)(e^x)$$

$$\therefore \frac{dy}{dx} = y\left[\frac{e^x}{x} + e^x\cdot\log x\right]$$

$$= x^{e^x}\cdot e^x\left[\frac{1}{x} + \log x\right].$$

Question 3.

y = e^{x^x}

Solution:

$$y = e^{x^x}$$

$$\therefore \log y = \log e^{x^x} = x^x \log e$$

$$\therefore \log y = x^x \quad \ldots [\because \log e = 1]$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}(x^x)$$

$$\therefore \frac{dy}{dx} = y \cdot \frac{d}{dx}(x^x) = e^{x^x} \cdot \frac{d}{dx}(x^x) \quad \ldots (1)$$

Let $u = x^x$

Then $\log u = \log x^x = x \log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx}(x \log x)$$

$$= x \cdot \frac{d}{dx}(\log x) + (\log x) \cdot \frac{d}{dx}(x)$$

$$= x \times \frac{1}{x} + (\log x) \times 1$$

$$\therefore \frac{du}{dx} = u(1 + \log x)$$

$$\therefore \frac{d}{dx}(x^x) = x^x(1 + \log x)$$

$\therefore$ from (1),

$$\frac{dy}{dx} = e^{x^x} \cdot x^x(1 + \log x).$$

2. Find $\frac{dy}{dx}$ if:

Question 1.
y = $\left(1+\frac{1}{x}\right)^x$
Solution:

$$y = \left(1 + \frac{1}{x}\right)^x$$

$$\therefore \log y = \log\left(1 + \frac{1}{x}\right)^x = x \log\left(1 + \frac{1}{x}\right)$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}\left[x \log\left(1 + \frac{1}{x}\right)\right]$$

$$= x \frac{d}{dx}\left[\log\left(1 + \frac{1}{x}\right)\right] + \left[\log\left(1 + \frac{1}{x}\right)\right] \cdot \frac{d}{dx}(x)$$

$$= x \times \frac{1}{1 + \frac{1}{x}} \cdot \frac{d}{dx}\left(1 + \frac{1}{x}\right) + \left[\log\left(1 + \frac{1}{x}\right)\right] \times 1$$

$$= x \times \frac{x}{x+1} \times \left(0 - \frac{1}{x^2}\right) + \log\left(1 + \frac{1}{x}\right)$$

$$\therefore \frac{dy}{dx} = y\left[\frac{-1}{x+1} + \log\left(1 + \frac{1}{x}\right)\right]$$

$$= \left(1 + \frac{1}{x}\right)^x \left[\log\left(1 + \frac{1}{x}\right) - \frac{1}{1+x}\right].$$

Question 2.
y = $(2x + 5)^x$

Solution:

$$y=(2x+5)^x$$

$$\therefore\ \log y=\log(2x+5)^x=x\log(2x+5)$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{d}{dx}[x\log(2x+5)]$$

$$=x\frac{d}{dx}[\log(2x+5)]+[\log(2x+5)]\cdot\frac{d}{dx}(x)$$

$$=x\times\frac{1}{2x+5}\cdot\frac{d}{dx}(2x+5)+[\log(2x+5)]\times 1$$

$$=\frac{x}{2x+5}\times(2\times 1+0)+\log(2x+5)$$

$$\therefore\ \frac{dy}{dx}=y\left[\frac{2x}{2x+5}+\log(2x+5)\right]$$

$$=(2x+5)^x\left[\log(2x+5)+\frac{2x}{2x+5}\right].$$

Question 3.

y = (3x−1)(2x+3)(5−x)2−−−−−−−−−√3

Solution:

$$y=\sqrt[3]{\frac{3x-1}{(2x+3)(5-x)^2}}$$

$$\therefore\ \log y=\log\left[\frac{3x-1}{(2x+3)(5-x)^2}\right]^{\frac{1}{3}}$$

$$=\frac{1}{3}\log\left[\frac{3x-1}{(2x+3)(5-x)^2}\right]$$

$$=\frac{1}{3}[\log(3x-1)-\log(2x+3)-\log(5-x)^2]$$

$$=\frac{1}{3}\log(3x-1)-\frac{1}{3}\log(2x+3)-\frac{2}{3}\log(5-x)$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{1}{3}\frac{d}{dx}[\log(3x-1)]-\frac{1}{3}\frac{d}{dx}[\log(2x+3)]-\frac{2}{3}\frac{d}{dx}[\log(5-x)]$$

$$=\frac{1}{3}\times\frac{1}{3x-1}\cdot\frac{d}{dx}(3x-1)-\frac{1}{3}\times\frac{1}{2x+3}\cdot\frac{d}{dx}(2x+3)-\frac{2}{3}\times\frac{1}{5-x}\cdot\frac{d}{dx}(5-x)$$

$$=\frac{1}{3(3x-1)}\times(3\times 1-0)-\frac{1}{3(2x+3)}\times(2\times 1+0)-\frac{2}{3(5-x)}\times(0-1)$$

$$\therefore\ \frac{dy}{dx}=y\left[\frac{3}{3(3x-1)}-\frac{2}{3(2x+3)}+\frac{2}{3(5-x)}\right]$$

$$=\frac{1}{3}\cdot\sqrt[3]{\frac{3x-1}{(2x+3)(5-x)^2}}\left[\frac{3}{3x-1}-\frac{2}{2x+3}+\frac{2}{5-x}\right].$$

3. Find dydx if:

Question 1.

y = (log x)x + xlog x

Solution:

$y = (\log x)^x + x^{\log x}$

Let $u = (\log x)^x$ and $v = x^{\log x}$

Then $y = u + v$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \ldots (1)$$

Take $u = (\log x)^x$

$\therefore \log u = \log(\log x)^x = x \log(\log x)$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{u}\cdot\frac{du}{dx} = \frac{d}{dx}[x \log(\log x)]$$

$$= x\cdot\frac{d}{dx}[\log(\log x)] + [\log(\log x)]\cdot\frac{d}{dx}(x)$$

$$= x \times \frac{1}{\log x}\cdot\frac{d}{dx}(\log x) + [\log(\log x)] \times 1$$

$$= x \times \frac{1}{\log x} \times \frac{1}{x} + \log(\log x)$$

$$\therefore \frac{du}{dx} = u\left[\frac{1}{\log x} + \log(\log x)\right]$$

$$= (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] \quad \ldots (2)$$

Also, $v = x^{\log x}$

$\therefore \log v = \log x^{\log x} = \log x \cdot \log x = (\log x)^2$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{v}\cdot\frac{dv}{dx} = \frac{d}{dx}(\log x)^2$$

$$= 2(\log x)\cdot\frac{d}{dx}(\log x)$$

$$= 2 \log x \times \frac{1}{x}$$

$$\therefore \frac{dv}{dx} = v\left[\frac{2 \log x}{x}\right]$$

$$= x^{\log x}\left[\frac{2 \log x}{x}\right] \quad \ldots (3)$$

From (1), (2) and (3), we get

$$\frac{dy}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] + x^{\log x}\left[\frac{2 \log x}{x}\right].$$

Question 2.

$y = x^x + a^x$

Solution:

$$y = x^x + a^x$$

Let $u = x^x$

Then $\log u = \log x^x = x \cdot \log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx}(x \cdot \log x)$$

$$= x \cdot \frac{d}{dx}(\log x) + (\log x) \cdot \frac{d}{dx}(x)$$

$$= x \times \frac{1}{x} + (\log x) \times 1$$

$$\therefore \frac{du}{dx} = u(1 + \log x) = x^x(1 + \log x) \quad \dots (1)$$

Now, $y = u + a^x$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{d}{dx}(a^x)$$

$$= x^x(1 + \log x) + a^x \cdot \log a \quad \dots \text{[By (1)]}$$

Question 3.

$y = 10^{x^x} + 10^{x^{10}} + 10^{10^x}$

Solution:

$$y = 10^{x^x} + 10^{x^{10}} + 10^{10^x}$$

Let $u = x^x$

Then $\log u = \log x^x = x \log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx}(x \log x)$$

$$= x\frac{d}{dx}(\log x) + (\log x) \cdot \frac{d}{dx}(x)$$

$$= x \times \frac{1}{x} + (\log x) \times 1$$

$$\therefore \frac{du}{dx} = u(1 + \log x) = x^x(1 + \log x) \quad \dots (1)$$

Now, $y = 10^u + 10^{x^{10}} + 10^{10^x}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(10^u) + \frac{d}{dx}(10^{x^{10}}) + \frac{d}{dx}(10^{10^x})$$

$$= 10^u \cdot \log 10 \cdot \frac{du}{dx} + 10^{x^{10}} \cdot \log 10 \cdot \frac{d}{dx}(x^{10}) + 10^{10^x} \cdot \log 10 \cdot \frac{d}{dx}(10^x)$$

$$= 10^{x^x} \cdot \log 10 \cdot x^x(1 + \log x) + 10^{x^{10}} \cdot \log 10 \times 10x^9 + 10^{10^x} \cdot \log 10 \times 10^x \cdot \log 10 \quad \dots \text{[By (1)]}$$

$$\therefore \frac{dy}{dx} = 10^{x^x} \cdot x^x \cdot (\log 10)(1 + \log x) + 10^{x^{10}}(10x^9) \log 10 + 10^{10^x} \cdot 10^x \cdot (\log 10)^2.$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 3 Differentiation Ex 3.4

1. Find $dydx$ if:

Question 1.
$\sqrt{x} + \sqrt{y} = \sqrt{a}$
Solution:
$\sqrt{x} + \sqrt{y} = \sqrt{a}$
Differentiating both sides w.r.t. x, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$$

$$\therefore \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = -\frac{2\sqrt{y}}{2\sqrt{x}} = -\sqrt{\frac{y}{x}}.$$

Question 2.
$x^3 + y^3 + 4x^3y = 0$
Solution:
$x^3 + y^3 + 4x^3y = 0$
Differentiating both sides w.r.t. x, we get

$$3x^2 + 3y^2\frac{dy}{dx} + 4\left[x^3\frac{dy}{dx} + y\frac{d}{dx}(x^3)\right] = 0$$

$$\therefore 3x^2 + 3y^2\frac{dy}{dx} + 4x^3\frac{dy}{dx} + 4y \times 3x^2 = 0$$

$$\therefore 3y^2\frac{dy}{dx} + 4x^3\frac{dy}{dx} = -3x^2 - 12x^2y$$

$$\therefore (3y^2 + 4x^3)\frac{dy}{dx} = -3x^2(1 + 4y)$$

$$\therefore \frac{dy}{dx} = -\frac{3x^2(1 + 4y)}{3y^2 + 4x^3}.$$

Question 3.
$x^3 + x^2y + xy^2 + y^3 = 81$
Solution:
$x^3 + x^2y + xy^2 + y^3 = 81$
Differentiating both sides w.r.t. x, we get

$$3x^2 + \left[x^2\frac{dy}{dx} + y \cdot \frac{d}{dy}(x^2)\right] + \left[x \cdot \frac{d}{dx}(y^2) + y^2 \cdot \frac{d}{dx}(x)\right] + 3y^2\frac{dy}{dx} = 0$$

$$\therefore 3x^2 + x^2\frac{dy}{dx} + y \times 2x + x \times 2y \cdot \frac{dy}{dx} + y^2 \times 1 + 3y^2\frac{dy}{dx} = 0$$

$$\therefore x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -3x^2 - 2xy - y^2$$

$$\therefore (x^2 + 2xy + 3y^2)\frac{dy}{dx} = -(3x^2 + 2xy + y^2)$$

$$\therefore \frac{dy}{dx} = -\left(\frac{3x^2 + 2xy + y^2}{x^2 + 2xy + 3y^2}\right).$$

2. Find $\frac{dy}{dx}$ if:

Question 1.
$y.e^x + x.e^y = 1$
Solution:
$y.e^x + x.e^y = 1$
Differentiating both sides w.r.t. x, we get

$$\frac{d}{dx}(ye^x)+\frac{d}{dx}(xe^y)=0$$

$$\therefore\ y\cdot\frac{d}{dx}(e^x)+e^x\cdot\frac{dy}{dx}+x\cdot\frac{d}{dx}(e^y)+e^y\cdot\frac{d}{dx}(x)=0$$

$$\therefore\ y\cdot e^x+e^x\cdot\frac{dy}{dx}+x\cdot e^y\cdot\frac{dy}{dx}+e^y\times 1=0$$

$$\therefore\ (e^x+xe^y)\frac{dy}{dx}=-e^y-ye^x$$

$$\therefore\ \frac{dy}{dx}=-\left(\frac{e^y+ye^x}{e^x+xe^y}\right).$$

Question 2.
$x^y = e^{(x-y)}$
Solution:
$x^y = e^{(x-y)}$
∴ $\log x^y = \log e^{(x-y)}$
∴ y log x = (x – y) log e
∴ y log x = x – y .....[∵ log e = 1]
∴ y + y log x = x
∴ y(1 + log x) = x
∴ $y = \frac{x}{1+\log x}$

$$\therefore\ \frac{dy}{dx}=\frac{d}{dx}\left(\frac{x}{1+\log x}\right)$$

$$\frac{(1+\log x)\cdot\frac{d}{dx}(x)-x\frac{d}{dx}\ (1+\log x)}{(1+\log x)^2}$$

$$=\frac{(1+\log x)\cdot 1-x\left(0+\frac{1}{x}\right)}{(1+\log x)^2}$$

$$=\frac{1+\log x-1}{(1+\log x)^2}$$

$$=\frac{\log x}{(1+\log x)^2}.$$

Question 3.
xy = log(xy)
Solution:
xy = log (xy)
∴ xy = log x + log y

Differentiating both sides w.r.t. x, we get

$$xy = \log(xy)$$

$$\therefore\ xy = \log x + \log y$$

Differentiating both sides w.r.t. x, we get

$$x \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}(x) = \frac{1}{x} + \frac{1}{y} \cdot \frac{dy}{dx}$$

$$\therefore\ x \cdot \frac{dy}{dx} + y \times 1 = \frac{1}{x} + \frac{1}{y} \cdot \frac{dy}{dx}$$

$$\therefore\ \left(x - \frac{1}{y}\right)\frac{dy}{dx} = \frac{1}{x} - y$$

$$\therefore\ \left(\frac{xy-1}{y}\right)\frac{dy}{dx} = \frac{1-xy}{x} = \frac{-(xy-1)}{x}$$

$$\therefore\ \frac{1}{y}\frac{dy}{dx} = -\frac{1}{x}$$

$$\therefore\ \frac{dy}{dx} = -\frac{y}{x}.$$

3. Solve the following:

Question 1.

If $x^5 \cdot y^7 = (x + y)^{12}$, then show that $\frac{dy}{dx} = \frac{y}{x}$

Solution:

$x^5 \cdot y^7 = (x + y)^{12}$

$\therefore \log(x^5 \cdot y^7) = \log(x + y)^{12}$

$\therefore \log x^5 + \log y^7 = \log(x + y)^{12}$

$\therefore 5 \log x + 7 \log y = 12 \log(x + y)$

Differentiating both sides w.r.t. x, we get

$$5 \times \frac{1}{x} + 7 \times \frac{1}{y} \cdot \frac{dy}{dx} = 12 \times \frac{1}{x+y} \cdot \frac{d}{dx}(x+y)$$

$$\therefore\ \frac{5}{x} + \frac{7}{y} \cdot \frac{dy}{dx} = \frac{12}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\therefore\ \frac{5}{x} + \frac{7}{y}\frac{dy}{dx} = \frac{12}{x+y} + \frac{12}{x+y} \cdot \frac{dy}{dx}$$

$$\therefore\ \left(\frac{7}{y} - \frac{12}{x+y}\right)\frac{dy}{dx} = \frac{12}{x+y} - \frac{5}{x}$$

$$\therefore\ \left[\frac{7x + 7y - 12y}{y(x+y)}\right]\frac{dy}{dx} = \frac{12x - 5x - 5y}{x(x+y)}$$

$$\therefore\ \left[\frac{7x - 5y}{y(x+y)}\right]\frac{dy}{dx} = \frac{7x - 5y}{x(x+y)}$$

$$\therefore\ \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x}$$

$$\therefore\ \frac{dy}{dx} = \frac{y}{x}$$

Question 2.

If $\log(x + y) = \log(xy) + a$, then show that $\frac{dy}{dx} = -\frac{y^2}{x^2}$

Solution:

$\log(x + y) = \log(xy) + a$

$\therefore \log(x + y) = \log x + \log y + a$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{x+y}\cdot\frac{d}{dx}(x+y)=\frac{1}{x}+\frac{1}{y}\cdot\frac{dy}{dx}+0$$

$$\therefore\ \frac{1}{x+y}\cdot\left(1+\frac{dy}{dx}\right)=\frac{1}{x}+\frac{1}{y}\cdot\frac{dy}{dx}$$

$$\therefore\ \frac{1}{x+y}+\frac{1}{x+y}\cdot\frac{dy}{dx}=\frac{1}{x}+\frac{1}{y}\frac{dy}{dx}$$

$$\therefore\ \left(\frac{1}{x+y}-\frac{1}{y}\right)\frac{dy}{dx}=\frac{1}{x}-\frac{1}{x+y}$$

$$\therefore\ \left[\frac{y-x-y}{y(x+y)}\right]\frac{dy}{dx}=\frac{x+y-x}{x(x+y)}$$

$$\therefore\ \left[\frac{-x}{y(x+y)}\right]\frac{dy}{dx}=\frac{y}{x(x+y)}$$

$$\therefore\ -\frac{x}{y}\cdot\frac{dy}{dx}=\frac{y}{x}$$

$$\therefore\ \frac{dy}{dx}=-\frac{y^2}{x^2}.$$

Question 3.

If $e^x + e^y = e^{(x+y)}$, then show that $\frac{dy}{dx}=-e^{y-x}$.

Solution:

$e^x + e^y = e^{(x+y)}$ ……….(1)

Differentiating both sides w.r.t. x, we get

$$e^x+e^y\cdot\frac{dy}{dx}=e^{(x+y)}\cdot\frac{d}{dx}(x+y)$$

$$\therefore\ e^x+e^y\cdot\frac{dy}{dx}=e^{(x+y)}\cdot\left(1+\frac{dy}{dx}\right)$$

$$\therefore\ e^x+e^y\cdot\frac{dy}{dx}=e^{(x+y)}+e^{(x+y)}\cdot\frac{dy}{dx}$$

$$\therefore\ [e^y-e^{(x+y)}]\frac{dy}{dx}=e^{(x+y)}-e^x$$

$$\therefore\ (e^y-e^x-e^y)\frac{dy}{dx}=e^x+e^y-e^x \qquad \dots \text{[By (1)]}$$

$$\therefore\ -e^x\cdot\frac{dy}{dx}=e^y$$

$$\therefore\ \frac{dy}{dx}=-\frac{e^y}{e^x}=-e^{y-x}.$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 3 Differentiation Ex 3.4

1. Find $\frac{dy}{dx}$ if:

Question 1.

√x + √y = √a

Solution:

√x + √y = √a

Differentiating both sides w.r.t. x, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$$

$$\therefore \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = -\frac{2\sqrt{y}}{2\sqrt{x}} = -\sqrt{\frac{y}{x}}.$$

Question 2.
x3 + y3 + 4x3y = 0
Solution:
x3 + y3 + 4x3y = 0
Differentiating both sides w.r.t. x, we get

$$3x^2 + 3y^2\frac{dy}{dx} + 4\left[x^3\frac{dy}{dx} + y\frac{d}{dx}(x^3)\right] = 0$$

$$\therefore 3x^2 + 3y^2\frac{dy}{dx} + 4x^3\frac{dy}{dx} + 4y \times 3x^2 = 0$$

$$\therefore 3y^2\frac{dy}{dx} + 4x^3\frac{dy}{dx} = -3x^2 - 12x^2y$$

$$\therefore (3y^2 + 4x^3)\frac{dy}{dx} = -3x^2(1 + 4y)$$

$$\therefore \frac{dy}{dx} = -\frac{3x^2(1+4y)}{3y^2 + 4x^3}.$$

Question 3.
x3 + x2y + xy2 + y3 = 81
Solution:
x3 + x2y + xy2 + y3 = 81
Differentiating both sides w.r.t. x, we get

$$3x^2 + \left[x^2\frac{dy}{dx} + y\cdot\frac{d}{dy}(x^2)\right] + \left[x\cdot\frac{d}{dx}(y^2) + y^2\cdot\frac{d}{dx}(x)\right] + 3y^2\frac{dy}{dx} = 0$$

$$\therefore 3x^2 + x^2\frac{dy}{dx} + y \times 2x + x \times 2y\cdot\frac{dy}{dx} + y^2 \times 1 + 3y^2\frac{dy}{dx} = 0$$

$$\therefore x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -3x^2 - 2xy - y^2$$

$$\therefore (x^2 + 2xy + 3y^2)\frac{dy}{dx} = -(3x^2 + 2xy + y^2)$$

$$\therefore \frac{dy}{dx} = -\left(\frac{3x^2 + 2xy + y^2}{x^2 + 2xy + 3y^2}\right).$$

2. Find $dydx$ if:

Question 1.
y.ex + x.ey = 1
Solution:
y.ex + x.ey = 1

Differentiating both sides w.r.t. x, we get

$$\frac{d}{dx}(ye^x)+\frac{d}{dx}(xe^y)=0$$

$$\therefore\ y\cdot\frac{d}{dx}(e^x)+e^x\cdot\frac{dy}{dx}+x\cdot\frac{d}{dx}(e^y)+e^y\cdot\frac{d}{dx}(x)=0$$

$$\therefore\ y\cdot e^x+e^x\cdot\frac{dy}{dx}+x\cdot e^y\cdot\frac{dy}{dx}+e^y\times 1=0$$

$$\therefore\ (e^x+xe^y)\frac{dy}{dx}=-e^y-ye^x$$

$$\therefore\ \frac{dy}{dx}=-\left(\frac{e^y+ye^x}{e^x+xe^y}\right).$$

Question 2.
$x^y = e^{(x-y)}$
Solution:
$x^y = e^{(x-y)}$
∴ log x^y = log $e^{(x-y)}$
∴ y log x = (x – y) log e
∴ y log x = x – y .....[∵ log e = 1]
∴ y + y log x = x
∴ y(1 + log x) = x
∴ y = $\frac{x}{1+\log x}$

$$\therefore\ \frac{dy}{dx}=\frac{d}{dx}\left(\frac{x}{1+\log x}\right)$$

$$\frac{(1+\log x)\cdot\frac{d}{dx}(x)-x\frac{d}{dx}\ (1+\log x)}{(1+\log x)^2}$$

$$=\frac{(1+\log x)\cdot 1-x\left(0+\frac{1}{x}\right)}{(1+\log x)^2}$$

$$=\frac{1+\log x-1}{(1+\log x)^2}$$

$$=\frac{\log x}{(1+\log x)^2}.$$

Question 3.
xy = log(xy)
Solution:
xy = log (xy)
∴ xy = log x + log y

Differentiating both sides w.r.t. x, we get

$$xy = \log(xy)$$

$$\therefore \ xy = \log x + \log y$$

Differentiating both sides w.r.t. x, we get

$$x \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}(x) = \frac{1}{x} + \frac{1}{y} \cdot \frac{dy}{dx}$$

$$\therefore \ x \cdot \frac{dy}{dx} + y \times 1 = \frac{1}{x} + \frac{1}{y} \cdot \frac{dy}{dx}$$

$$\therefore \ \left(x - \frac{1}{y}\right)\frac{dy}{dx} = \frac{1}{x} - y$$

$$\therefore \ \left(\frac{xy-1}{y}\right)\frac{dy}{dx} = \frac{1-xy}{x} = \frac{-(xy-1)}{x}$$

$$\therefore \ \frac{1}{y}\frac{dy}{dx} = -\frac{1}{x}$$

$$\therefore \ \frac{dy}{dx} = -\frac{y}{x}.$$

3. Solve the following:

Question 1.

If $x^5 \cdot y^7 = (x + y)^{12}$, then show that $\frac{dy}{dx} = \frac{y}{x}$

Solution:

$x^5 \cdot y^7 = (x + y)^{12}$

$\therefore \log(x^5 \cdot y^7) = \log(x + y)^{12}$

$\therefore \log x^5 + \log y^7 = \log(x + y)^{12}$

$\therefore$ 5 log x + 7 log y = 12 log (x + y)

Differentiating both sides w.r.t. x, we get

$$5 \times \frac{1}{x} + 7 \times \frac{1}{y} \cdot \frac{dy}{dx} = 12 \times \frac{1}{x+y} \cdot \frac{d}{dx}(x+y)$$

$$\therefore \ \frac{5}{x} + \frac{7}{y} \cdot \frac{dy}{dx} = \frac{12}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\therefore \ \frac{5}{x} + \frac{7}{y}\frac{dy}{dx} = \frac{12}{x+y} + \frac{12}{x+y} \cdot \frac{dy}{dx}$$

$$\therefore \ \left(\frac{7}{y} - \frac{12}{x+y}\right)\frac{dy}{dx} = \frac{12}{x+y} - \frac{5}{x}$$

$$\therefore \ \left[\frac{7x+7y-12y}{y(x+y)}\right]\frac{dy}{dx} = \frac{12x-5x-5y}{x(x+y)}$$

$$\therefore \ \left[\frac{7x-5y}{y(x+y)}\right]\frac{dy}{dx} = \frac{7x-5y}{x(x+y)}$$

$$\therefore \ \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x}$$

$$\therefore \ \frac{dy}{dx} = \frac{y}{x}$$

Question 2.

If log(x + y) = log(xy) + a, then show that $\frac{dy}{dx} = \frac{-y^2}{x^2}$

Solution:

log (x + y) = log (xy) + a

$\therefore$ log(x + y) = log x + log y + a

Differentiating both sides w.r.t. x, we get

$$\frac{1}{x+y}\cdot\frac{d}{dx}(x+y)=\frac{1}{x}+\frac{1}{y}\cdot\frac{dy}{dx}+0$$

$$\therefore \frac{1}{x+y}\cdot\left(1+\frac{dy}{dx}\right)=\frac{1}{x}+\frac{1}{y}\cdot\frac{dy}{dx}$$

$$\therefore \frac{1}{x+y}+\frac{1}{x+y}\cdot\frac{dy}{dx}=\frac{1}{x}+\frac{1}{y}\frac{dy}{dx}$$

$$\therefore \left(\frac{1}{x+y}-\frac{1}{y}\right)\frac{dy}{dx}=\frac{1}{x}-\frac{1}{x+y}$$

$$\therefore \left[\frac{y-x-y}{y(x+y)}\right]\frac{dy}{dx}=\frac{x+y-x}{x(x+y)}$$

$$\therefore \left[\frac{-x}{y(x+y)}\right]\frac{dy}{dx}=\frac{y}{x(x+y)}$$

$$\therefore -\frac{x}{y}\cdot\frac{dy}{dx}=\frac{y}{x}$$

$$\therefore \frac{dy}{dx}=-\frac{y^2}{x^2}.$$

Question 3.
If $e^x + e^y = e^{(x+y)}$, then show that $\frac{dy}{dx}=-e^{y-x}$.
Solution:
$e^x + e^y = e^{(x+y)}$ ....(1)
Differentiating both sides w.r.t. x, we get

$$e^x+e^y\cdot\frac{dy}{dx}=e^{(x+y)}\cdot\frac{d}{dx}(x+y)$$

$$\therefore e^x+e^y\cdot\frac{dy}{dx}=e^{(x+y)}\cdot\left(1+\frac{dy}{dx}\right)$$

$$\therefore e^x+e^y\cdot\frac{dy}{dx}=e^{(x+y)}+e^{(x+y)}\cdot\frac{dy}{dx}$$

$$\therefore [e^y-e^{(x+y)}]\frac{dy}{dx}=e^{(x+y)}-e^x$$

$$\therefore (e^y-e^x-e^y)\frac{dy}{dx}=e^x+e^y-e^x \quad \text{... [By (1)]}$$

$$\therefore -e^x\cdot\frac{dy}{dx}=e^y$$

$$\therefore \frac{dy}{dx}=-\frac{e^y}{e^x}=-e^{y-x}.$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 3 Differentiation Ex 3.5

1. Find $\frac{dy}{dx}$ if:

Question 1.
$x = at^2$, $y = 2at$
Solution:
$x = at^2$, $y = 2at$

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt} = a\frac{d}{dt}(t^2) = a \times 2t = 2at$$

$$\text{and } \frac{dy}{dt} = 2a\frac{d}{dt}(t) = 2a \times 1 = 2a$$

$$\therefore \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{2a}{2at} = \frac{1}{t}.$$

Question 2.
$x = 2at^2$, $y = at^4$
Solution:

$x = 2at^2$

Differentiating both sides w.r.t. t, we get

$$\frac{dx}{dt} = 4at$$

$y = at^4$

Differentiating both sides w.r.t. t, we get

$$\frac{dy}{dt} = 4at^3$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dy}{dt}\right)} = \frac{4at^3}{4at} = t^2$$

Question 3.
$x = e^{3t}$, $y = e^{(4t+5)}$
Solution:
$x = e^{3t}$, $y = e^{(4t+5)}$
Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt} = \frac{d}{dt}(e^{3t}) = e^{3t} \cdot \frac{d}{dt}(3t)$$

$$= e^{3t} \times 3 \times 1 = 3e^{3t}$$

$$\text{and } \frac{dy}{dt} = \frac{d}{dt}[e^{(4t+5)}] = e^{(4t+5)} \cdot \frac{d}{dt}(4t+5)$$

$$= e^{(4t+5)} \times (4 \times 1 + 0) = 4e^{(4t+5)}$$

$$\therefore \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{4e^{(4t+5)}}{3e^{3t}}$$

$$= \frac{4}{3}e^{4t+5-3t} = \frac{4}{3}e^{t+5}.$$

2. Find $\frac{dy}{dx}$ if:

Question 1.
$x = \left(u+\frac{1}{u}\right)^2$, $y = (2)^{\left(u+\frac{1}{u}\right)}$
Solution:
$x = \left(u+\frac{1}{u}\right)^2$, $y = (2)^{\left(u+\frac{1}{u}\right)}$ ……(1)
Differentiating x and y w.r.t. u, we get,

$$\frac{dx}{du}=\frac{d}{du}\left(u+\frac{1}{u}\right)^2=2\left(u+\frac{1}{u}\right)\cdot\frac{d}{du}\left(u+\frac{1}{u}\right)$$

$$=2\left(u+\frac{1}{u}\right)\left(1-\frac{1}{u^2}\right)$$

and $\frac{dy}{du}=\frac{d}{du}\left[2^{\left(u+\frac{1}{u}\right)}\right]$

$$=2^{\left(u+\frac{1}{u}\right)}\cdot\log 2\cdot\frac{d}{du}\left(u+\frac{1}{u}\right)$$

$$=2^{\left(u+\frac{1}{u}\right)}\cdot\log 2\cdot\left(1-\frac{1}{u^2}\right)$$

$$\therefore\ \frac{dy}{dx}=\frac{(dy/du)}{(dx/du)}=\frac{2^{\left(u+\frac{1}{u}\right)}\cdot\log 2\cdot\left(1-\frac{1}{u^2}\right)}{2\left(u+\frac{1}{u}\right)\left(1-\frac{1}{u^2}\right)}$$

$$=\frac{2^{\left(u+\frac{1}{u}\right)}\cdot\log 2}{2\left(u+\frac{1}{u}\right)}$$

$$=\frac{y\log 2}{2\sqrt{x}}\qquad\dots\text{ [By (1)]}$$

Question 2.
$x = \sqrt{1+u^2}$, $y = \log(1 + u^2)$
Solution:
$x = \sqrt{1+u^2}$, $y = \log(1 + u^2)$ ......(1)
Differentiating x and y w.r.t. u, we get,

$$\frac{dx}{du}=\frac{d}{du}(\sqrt{1+u^2})=\frac{1}{2\sqrt{1+u^2}}\cdot\frac{d}{du}(1+u^2)$$

$$=\frac{1}{2\sqrt{1+u^2}}\times(0+2u)=\frac{u}{\sqrt{1+u^2}}$$

and $\frac{dy}{du}=\frac{d}{du}[\log(1+u^2)]$

$$=\frac{1}{1+u^2}\cdot\frac{d}{du}(1+u^2)$$

$$=\frac{1}{1+u^2}\times(0+2u)=\frac{2u}{1+u^2}$$

$$\therefore\ \frac{dy}{dx}=\frac{(dy/du)}{(dx/du)}=\frac{\left(\frac{2u}{1+u^2}\right)}{\left(\frac{u}{\sqrt{1+u^2}}\right)}$$

$$=\frac{2u}{1+u^2}\times\frac{\sqrt{1+u^2}}{u}=\frac{2}{\sqrt{1+u^2}}.$$

Question 3.
Differentiate 5^x with respect to log x.
Solution:
Let $u = 5^x$ and $v = \log x$
Then we want to find $\frac{du}{dv}$
Differentiating u and v w.r.t. x, we get

$\frac{du}{dx}=\frac{d}{dx}(5^x)=5^x \cdot \log 5$

$$\text{and } \frac{dv}{dx}=\frac{d}{dx}(\log x)=\frac{1}{x}$$

$$\therefore \frac{du}{dv}=\frac{(du/dx)}{(dv/dx)}=\frac{5^x\cdot\log 5}{\left(\frac{1}{x}\right)}$$

$$=x\cdot 5^x\cdot\log 5.$$

3. Solve the following:

Question 1.
If x = $a\left(1-\frac{1}{t}\right)$, y = $a\left(1+\frac{1}{t}\right)$, then show that $\frac{dy}{dx}$ = -1
Solution:

$$x=a\left(1-\frac{1}{t}\right),\ y=a\left(1+\frac{1}{t}\right)$$

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt}=a\frac{d}{dt}\left(1-\frac{1}{t}\right)=a[0-(-1)t^{-2}]=\frac{a}{t^2}$$

$$\text{and } \frac{dy}{dt}=a\frac{d}{dt}\left(1+\frac{1}{t}\right)=a[0+(-1)t^{-2}]=-\frac{a}{t^2}$$

$$\therefore \frac{dy}{dx}=\frac{(dy/dt)}{(dx/dt)}=\frac{\left(-\frac{a}{t^2}\right)}{\left(\frac{a}{t^2}\right)}=-1.$$

Question 2.
If x = $\frac{4t}{1+t^2}$, y = $3\left(\frac{1-t^2}{1+t^2}\right)$, then show that $\frac{dy}{dx}=-\frac{9x}{4y}$
Solution:

$$x=\frac{4t}{1+t^2},\quad y=3\left(\frac{1-t^2}{1+t^2}\right)$$

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt}=\frac{d}{dt}\left(\frac{4t}{1+t^2}\right)=\frac{(1+t^2)\cdot\frac{d}{dt}(4t)-4t\cdot\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$=\frac{(1+t^2)(4)-4t(0+2t)}{(1+t^2)^2}$$

$$=\frac{4+4t^2-8t^2}{(1+t^2)^2}=\frac{4-4t^2}{(1+t^2)^2}$$

$$=\frac{4(1-t^2)}{(1+t^2)^2}$$

and $\frac{dy}{dt}=3\frac{d}{dt}\left(\frac{1-t^2}{1+t^2}\right)$

$$=3\left[\frac{(1+t^2)\cdot\frac{d}{dt}(1-t^2)-(1-t^2)\cdot\frac{d}{dt}(1+t^2)}{(1+t^2)^2}\right]$$

$$=3\left[\frac{(1+t^2)(0-2t)-(1-t^2)(0+2t)}{(1+t^2)^2}\right]$$

$$=3\left[\frac{-2t-2t^3-2t+2t^3}{(1+t^2)^2}\right]$$

$$=\frac{-12t}{(1+t^2)^2}$$

$$\therefore\ \frac{dy}{dx}=\frac{(dy/dt)}{(dx/dt)}=\frac{\left[\frac{-12t}{(1+t^2)^2}\right]}{\left[\frac{4(1-t^2)}{(1+t^2)^2}\right]}$$

$$\therefore\ \frac{dy}{dx}=\frac{-3t}{1-t^2} \qquad \text{........ (1)}$$

$$\frac{-9x}{4y}=\frac{-9}{4}\cdot\frac{\left(\frac{4t}{1+t^2}\right)}{3\left(\frac{1-t^2}{1+t^2}\right)}=\frac{-3t}{1-t^2} \qquad \text{...... (2)}$$

From (1) and (2)

$$\frac{dy}{dx}=-\frac{9x}{4y}.$$

Question 3.
If x = t . log t, y = t^t, then show that $\frac{dy}{dx}$ – y = 0.
Solution:
x = t log t
Differentiating w.r.t. t, we get

$$\frac{dx}{dt}=\frac{d}{dt}(t\cdot\log t)$$

$$=t\frac{d}{dt}(\log t)+(\log t)\cdot\frac{d}{dt}(t)$$

$$=t\times\frac{1}{t}+(\log t)\times 1=1+\log t.$$

Also, $y=t^t$

$\therefore\ \log y=\log t^t=t\log t.$

Differentiating both sides w.r.t. t, we get

$$\frac{1}{y}\cdot\frac{dy}{dt}=\frac{d}{dt}(t\log t)$$

$$=t\cdot\frac{d}{dt}(\log t)+(\log t)\cdot\frac{d}{dt}(t)$$

$$=t\times\frac{1}{t}+(\log t)\times 1$$

$$\therefore\ \frac{dy}{dt}=y(1+\log t)$$

$$\therefore\ \frac{dy}{dx}=\frac{(dy/dt)}{(dx/dt)}=\frac{y(1+\log t)}{1+\log t}=y$$

$$\therefore\ \frac{dy}{dx}-y=0.$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 3 Differentiation Ex 3.6

1. Find $\frac{d^2y}{dx^2}$ if,

Question 1.
y = √x
Solution:

$y=\sqrt{x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}(\sqrt{x})=\frac{1}{2\sqrt{x}}$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{1}{2\sqrt{x}}\right)=\frac{1}{2}\frac{d}{dx}(x^{-\frac{1}{2}})$$

$$=\frac{1}{2}\cdot\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}=-\frac{1}{4}x^{-\frac{3}{2}}.$$

Question 2.
y = x^5

Solution:

$y = x^5$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x^5) = 5x^4$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(5x^4) = 5\frac{d}{dx}(x^4)$$

$$= 5 \times 4x^3 = 20x^3.$$

Question 3.
y = x-7
Solution:

$y = x^{-7}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x^{-7}) = -7x^{-8}$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-7x^{-8}) = -7\frac{d}{dx}(x^{-8})$$

$$= (-7)(-8)x^{-9} = 56x^{-9}.$$

2. Find d2ydx2 if,

Question 1.
y = ex
Solution:

$y = e^x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(e^x) = e^x$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(e^x) = e^x.$$

Question 2.
y = e(2x+1)
Solution:

$y = e^{(2x+1)}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[e^{(2x+1)}] = e^{(2x+1)}\cdot\frac{d}{dx}(2x+1)$$

$$= e^{(2x+1)} \times (2 \times 1 + 0) = 2e^{(2x+1)}$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{d}{dx}[2e^{(2x+1)}] = 2\frac{d}{dx}[e^{(2x+1)}]$$

$$= 2e^{(2x+1)}\cdot\frac{d}{dx}(2x+1) = 2e^{(2x+1)} \times (2 \times 1 + 0)$$

$$= 4e^{(2x+1)}.$$

Question 3.
$y = e^{\log x}$
Solution:

$$y = e^{\log x} = x \qquad \ldots [\because a^{\log_a x} = x]$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x) = 1$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(1) = 0.$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 3 Differentiation Miscellaneous Exercise 3

(I) Choose the correct alternative:

Question 1.
If $y = (5x^3 - 4x^2 - 8x)^9$, then $\frac{dy}{dx}$ = __________
(a) $9(5x^3 - 4x^2 - 8x)^8 (15x^2 - 8x - 8)$
(b) $9(5x^3 - 4x^2 - 8x)^9 (15x^2 - 8x - 8)$
(c) $9(5x^3 - 4x^2 - 8x)^8 (5x^2 - 8x - 8)$
(d) $9(5x^3 - 4x^2 - 8x)^9 (5x^2 - 8x - 8)$
Answer:
(a) $9(5x^3 - 4x^2 - 8x)^8 (15x^2 - 8x - 8)$

Question 2.
If $y = \sqrt{x+\frac{1}{x}}$, then $\frac{dy}{dx}$ = ?
(a) $\frac{x^2-1}{2x^2\sqrt{x^2+1}}$
(b) $\frac{1-x^2}{2x^2\sqrt{x^2+1}}$
(c) $\frac{x^2-1}{2x\sqrt{x}\sqrt{x^2+1}}$
(d) $\frac{1-x^2}{2x\sqrt{x}\sqrt{x^2+1}}$
Answer:
(c) $\frac{x^2-1}{2x\sqrt{x}\sqrt{x^2+1}}$
Hint:

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+\frac{1}{x}}} \cdot \frac{d}{dx}\left(x+\frac{1}{x}\right)$$

$$= \frac{\sqrt{x}}{2\sqrt{x^2+1}}\left(1-\frac{1}{x^2}\right) = \frac{x^2-1}{2x\sqrt{x}\sqrt{x^2+1}}$$

Question 3.
If $y = e^{\log x}$ then $\frac{dy}{dx}$ = ?
(a) $\frac{e^{\log x}}{x}$
(b) $\frac{1}{x}$

(c) 0
(d) $\frac{1}{2}$
Answer:
(a) $e^{\log x} x$

Question 4.
If y = $2x^2 + 2^2 + a^2$, then $\frac{dy}{dx}$ = ?
(a) x
(b) 4x
(c) 2x
(d) -2x
Answer:
(b) 4x

Question 5.
If y = $5^x . x^5$, then $\frac{dy}{dx}$ = ?
(a) $5^x . x^4(5 + \log 5)$
(b) $5^x . x^5(5 + \log 5)$
(c) $5^x . x^4(5 + x \log 5)$
(d) $5^x . x^5(5 + x \log 5)$
Answer:
(c) $5^x . x^4(5 + x \log 5)$

Question 6.
If y = $\log\left(\frac{e^x}{x^2}\right)$ then $\frac{dy}{dx}$ = ?
(a) $\frac{2-x}{x}$
(b) $\frac{x-2}{x}$
(c) $\frac{e-x}{ex}$
(d) $\frac{x-e}{ex}$
Answer:
(b) $\frac{x-2}{x}$
Hint:

$$y = \log\left(\frac{e^x}{x^2}\right) = \log e^x = \log x^2$$
$$= x - 2\log x \qquad \ldots [\because \log e = 1]$$
$$\therefore \frac{dy}{dx} = 1 - \frac{2}{x} = \frac{x-2}{x}$$

Question 7.
If $ax^2 + 2hxy + by^2 = 0$, then $\frac{dy}{dx}$ = ?
(a) $\frac{(ax+hy)}{(hx+by)}$
(b) $\frac{-(ax+hy)}{(hx+by)}$
(c) $\frac{(ax-hy)}{(hx+by)}$
(d) $\frac{(2ax+hy)}{(hx+3by)}$
Answer:
(b) $\frac{-(ax+hy)}{(hx+by)}$

Question 8.
If $x^4 . y^5 = (x + y)^{(m+1)}$ and $\frac{dy}{dx} = \frac{y}{x}$ then m = ?
(a) 8
(b) 4
(c) 5
(d) 20
Answer:
(a) 8
Hint:
If $x^p . y^q = (x + y)^{p+q}$, then $\frac{dy}{dx} = \frac{y}{x}$
∴ m + 1 = 4 + 5 = 9
∴ m = 8.

Question 9.
If x = $\frac{e^t+e^{-t}}{2}$, y = $\frac{e^t-e^{-t}}{2}$ then $\frac{dy}{dx}$ = ?
(a) $\frac{-y}{x}$
(b) $\frac{y}{x}$

(c) $-\frac{x}{y}$
(d) $\frac{x}{y}$
Answer:
(d) $\frac{x}{y}$
Hint:

$$\frac{dx}{dt}=\frac{1}{2}(e^t-e^{-t}),\ \frac{dy}{dt}=\frac{1}{2}(e^t+e^{-t})$$

$$\therefore\ \frac{dy}{dx}=\frac{(dy/dt)}{(dx/dt)}=\left(\frac{e^t+e^{-t}}{2}\right)\Big/\left(\frac{e^t-e^{-t}}{2}\right)=\frac{x}{y}$$

Question 10.
If $x = 2at^2$, $y = 4at$, then $\frac{dy}{dx}$ = ?
(a) $-\frac{1}{2at^2}$
(b) $\frac{1}{2at^3}$
(c) $\frac{1}{t}$
(d) $\frac{1}{4at^3}$
Answer:
(c) $\frac{1}{t}$

(II) Fill in the blanks:

Question 1.
If $3x^2y + 3xy^2 = 0$ then $\frac{dy}{dx}$ = …………
Answer:
-1
Hint:
$3x^2y + 3xy^2 = 0$
$\therefore$ $3xy(x + y) = 0$
$\therefore$ $x + y = 0$
$\therefore$ $y = -x$
$\therefore$ $\frac{dy}{dx} = -1$

Question 2.
If $x^m \cdot y^n = (x+y)^{(m+n)}$ then $\frac{dy}{dx}=\frac{\ldots\ldots}{x}$
Answer:
y

Question 3.
If $0 = \log(xy) + a$ then $\frac{dy}{dx}=\frac{-y}{\ldots\ldots}$
Answer:
x

Question 4.
If $x = t \log t$ and $y = t^t$ then $\frac{dy}{dx}$ = …………
Answer:
y
Hint:
$x = t \log t = \log t^t = \log y$
$\therefore$ $1 = \frac{1}{y} \cdot \frac{dy}{dx}$
$\therefore$ $\frac{dy}{dx} = y$

Question 5.
If $y = x \cdot \log x$ then $\frac{d^2y}{dx^2}$ = ………….
Answer:
$\frac{1}{x}$

Question 6.
If $y = [\log(x)]^2$ then $\frac{d^2y}{dx^2}$ = ………….
Answer:
$\frac{2(1-\log x)}{x^2}$

Hint:

$$y=(\log x)^2 \quad \therefore \frac{dy}{dx}=2\log x\cdot\frac{d}{dx}(\log x)$$
$$=2\log x\times\frac{1}{x}=\frac{2\log x}{x}$$
and $$\frac{d^2y}{dx^2}=2\frac{d}{dx}\left(\frac{\log x}{x}\right)$$
$$=2\left[\frac{x\frac{d}{dx}(\log x)-(\log x)\cdot\frac{d}{dx}(x)}{x^2}\right]$$
$$=2\left[\frac{x\times\frac{1}{x}-(\log x)\times 1}{x^2}\right]$$
$$=\frac{2(1-\log x)}{x^2}$$

Question 7.
If x = y + $\frac{1}{y}$ then $\frac{dy}{dx}$ = ...
Answer:
$\frac{y^2}{y^2-1}$
Hint:

$$\frac{dx}{dy}=\frac{d}{dy}\left(y+\frac{1}{y}\right)=1-\frac{1}{y^2}=\frac{y^2-1}{y^2}$$
$$\therefore \frac{dy}{dx}=\frac{1}{\left(\frac{dx}{dy}\right)}=\frac{y^2}{y^2-1}$$

Question 8.
If y = e^{ax}, then x.$\frac{dy}{dx}$ = ...
Answer:
axy

Question 9.
If x = t . log t, y = t^t then $\frac{dy}{dx}$ = ...
Answer:
y

Question 10.
If y = $(x+\sqrt{x^2-1})^m$ then $(\sqrt{x^2-1})\frac{dy}{dx}$ = ...
Answer:
my
Hint:

$$y=(x+\sqrt{x^2-1})^m$$
$$\therefore \frac{dy}{dx}=m(x+\sqrt{x^2-1})^{m-1}\cdot\frac{d}{dx}(x+\sqrt{x^2-1})$$
$$=m(x+\sqrt{x^2-1})^{m-1}\cdot\left[1+\frac{1}{2\sqrt{x^2-1}}\cdot\frac{d}{dx}(x^2-1)\right]$$
$$=m(x+\sqrt{x^2-1})^{m-1}\cdot\left[1+\frac{1}{2\sqrt{x^2-1}}\times 2x\right]$$
$$=m(x+\sqrt{x^2-1})^{m-1}\cdot\left[\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}\right]$$
$$\therefore \frac{dy}{dx}=\frac{m(x+\sqrt{x^2-1})^m}{\sqrt{x^2-1}}=\frac{my}{\sqrt{x^2-1}}$$
$$\therefore \sqrt{x^2-1}\cdot\frac{dy}{dx}=my$$

(III) State whether each of the following is True or False:

Question 1.
If f' is the derivative of f, then the derivative of the inverse of f is the inverse of f'.
Answer:
False

Question 2.
The derivative of $\log_a x$, where a is constant is $\frac{1}{x \cdot \log a}$.
Answer:
True

Question 3.
The derivative of $f(x) = a^x$, where a is constant is $x \cdot a^{x-1}$
Answer:
False

Question 4.
The derivative of a polynomial is polynomial.
Answer:
True

Question 5.
$\frac{d}{dx}(10^x) = x \cdot 10^{x-1}$
Answer:
False

Question 6.
If y = log x, then $\frac{dy}{dx} = \frac{1}{x}$.
Answer:
True

Question 7.
If $y = e^2$, then $\frac{dy}{dx} = 2e$.
Answer:
False

Question 8.
The derivative of a^x is $a^x \cdot \log a$.
Answer:
True

Question 9.
The derivative of $x^m \cdot y^n = (x + y)^{(m+n)}$ is $\frac{x}{y}$
Answer:
False

(IV) Solve the following:

Question 1.
If $y = (6x^3 - 3x^2 - 9x)^{10}$, find $\frac{dy}{dx}$
Solution:

Given y = (6x3 – 3x2 – 9x)10

Let $u = 6x^3 - 3x^2 - 9x$

Then $y = u^{10}$

$$\therefore \frac{dy}{du} = \frac{d}{du}(u^{10}) = 10u^9$$

$$= 10(6x^3 - 3x^2 - 9x)^9$$

and $$\frac{du}{dx} = \frac{d}{dx}(6x^3 - 3x^2 - 9x)$$

$$= 6\frac{d}{dx}(x^3) - 3\frac{d}{dx}(x^2) - 9\frac{d}{dx}(x)$$

$$= 6 \times 3x^2 - 3 \times 2x - 9 \times 1$$

$$= 18x^2 - 6x - 9$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 10(6x^2 - 3x^2 - 9x)^9 \cdot (18x^2 - 6x - 9).$$

Question 2.
If y = (3X2+8X+5)4−−−−−−−−−−−−−√5, find dydx.
Solution:

$$\text{Given : } y = \sqrt[5]{(3x^2 + 8x + 5)^4}$$

Let $u = 3x^2 + 8x + 5$

Then $y = \sqrt[5]{u^4} = u^{\frac{4}{5}}$

$$\therefore \frac{dy}{du} = \frac{d}{du}(u^{\frac{4}{5}}) = \frac{4}{5}u^{\frac{4}{5}-1}$$

$$= \frac{4}{5}(3x^2 + 8x + 5)^{-\frac{1}{5}}$$

and $$\frac{du}{dx} = \frac{d}{dx}(3x^2 + 8x + 5)$$

$$= 3\frac{d}{dx}(x^2) + 8\frac{d}{dx}(x) + \frac{d}{dx}(5)$$

$$= 3 \times 2x + 8 \times 1 + 0 = 6x + 8$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{4}{5}(3x^2 + 8x + 5)^{-\frac{1}{5}} \cdot (6x + 8).$$

Question 3.
If y = [log(log(log x))]2, find dydx.
Solution:

$$y = [\log(\log(\log x))]^2$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} \cdot [\log(\log(\log x))]^2$$

$$= 2[\log(\log(\log x))] \cdot \frac{d}{dx}[\log(\log(\log x))]$$

$$= 2\log[\log(\log x)] \times \frac{1}{\log(\log x)} \cdot \frac{d}{dx}[\log(\log x)]$$

$$= \frac{2\log[\log(\log x)]}{\log(\log x)} \times \frac{1}{\log x} \cdot \frac{d}{dx}(\log x)$$

$$= \frac{2\log[\log(\log x)]}{\log(\log x)} \times \frac{1}{\log x} \times \frac{1}{x}$$

$$= \frac{2\log[\log(\log x)]}{x \cdot \log x \cdot \log(\log x)}.$$

Question 4.
Find the rate of change of demand (x) of a commodity with respect to its price (y) if y = 25 + 30x – x^2.
Solution:

$$y = 25 + 30x - x^2$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(25 + 30x - x^2)$$

$$= \frac{d}{dx}(25) + 30\frac{d}{dx}(x) - \frac{d}{dx}(x^2)$$

$$= 0 + 30 \times 1 - 2x$$

$$= 30 - 2x$$

Hence, the rate of change of demand (x) w.r.t. price (y)

$$= \frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{1}{30 - 2x}.$$

Question 5.
Find the rate of change of demand (x) of a commodity with respect to its price (y) if y = $\frac{5x+7}{2x-13}$
Solution:

$$y = \frac{5x + 7}{2x - 13}$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{5x + 7}{2x - 13}\right)$$

$$= \frac{(2x - 13) \cdot \frac{d}{dx}(5x + 7) - (5x + 7) \cdot \frac{d}{dx}(2x - 13)}{(2x - 13)^2}$$

$$= \frac{(2x - 13) \cdot (5 \times 1 + 0) - (5x + 7) \cdot (2 \times 1 - 0)}{(2x - 13)^2}$$

$$= \frac{10x - 65 - 10x - 14}{(2x - 13)^2} = \frac{-79}{(2x - 13)^2}$$

Hence, rate of change of demand (x) w.r.t. price (y)

$$= \frac{dx}{dy} = \frac{1}{\left(\frac{xy}{dx}\right)} = -\frac{(2x - 13)^2}{79}.$$

Question 6.
Find $\frac{dy}{dx}$ if $y = x^x$.
Solution:
$y = x^x$
$\therefore \log y = \log x^x = x \log x$
Differentiating both sides w.r.t. x, we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}(x \log x)$$

$$= x \cdot \frac{d}{dx}(\log x) + (\log x) \cdot \frac{d}{dx}(x)$$

$$= x \times \frac{1}{x} + (\log x) \times 1$$

$$\therefore \frac{dy}{dx} = y(1 + \log x)$$

$$= x^x(1 + \log x).$$

Question 7.
Find $\frac{dy}{dx}$ if $y = 2^{x^x}$.
Solution:

$$\text{Given : } y = 2^{x^x}$$

Let $u = x^x$

Then $y = 2^u$

$$\therefore \frac{dy}{du} = \frac{d}{du}(2^u) = 2^u \cdot \log 2$$

$$= 2^{x^x} \cdot \log 2 \quad \ldots (1)$$

Now, $u = x^x$

$\therefore \log u = \log x^x = x \log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx}(x \log x)$$

$$= x \frac{d}{dx}(\log x) + (\log x) \cdot \frac{d}{dx}(x)$$

$$= x \times \frac{1}{x} + (\log x) \times 1$$

$$\therefore \frac{du}{dx} = u(1 + \log x) = x^x(1 + \log x) \quad \ldots (2)$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 2^{x^x} \cdot \log 2 \cdot x^x(1 + \log x) \quad \ldots \text{[By (1) and (2)]}$$

$$= 2^{x^x} \cdot x^x (\log 2)(1 + \log x).$$

Question 8.
Find $\frac{dy}{dx}$, if $y = \sqrt{\frac{(3x-4)^3}{(x+1)^4(x+2)}}$
Solution:

$$y = \sqrt{\frac{(3x-4)^3}{(x+1)^4(x+2)}}$$

$$= \frac{(3x-4)^{\frac{3}{2}}}{(x+1)^{\frac{4}{2}} \cdot (x+2)^{\frac{1}{2}}}$$

Taking logarithm of both sides, we get

$$\log y = \log\left[\frac{(3x-4)^{\frac{3}{2}}}{(x+1)^{\frac{4}{2}} \cdot (x+2)^{\frac{1}{2}}}\right]$$

$$= \log(3x-4)^{\frac{3}{2}} - \left[\log(x+1)^2 + \log(x+2)^{\frac{1}{2}}\right]$$

$$= \frac{3}{2}\log(3x-4) - 2\log(x+1) - \frac{1}{2}\log(x+2)$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{3}{2} \cdot \frac{d}{dx}[\log(3x-4)] - 2\frac{d}{dx}[\log(x+1)] - \frac{1}{2} \cdot \frac{d}{dx}[\log(x+2)]$$

$$= \frac{3}{2} \cdot \frac{1}{3x-4} \cdot \frac{d}{dx}(3x-4) - 2 \cdot \frac{1}{x+1} \cdot \frac{d}{dx}(x+1) - \frac{1}{2} \cdot \frac{1}{x+2} \cdot \frac{d}{dx}(x+2)$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = \frac{3}{2(3x-4)} \times 3 - \frac{2}{x+1} \times 1 - \frac{1}{2(x+2)} \times 1$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = \frac{9}{2(3x-4)} - \frac{2}{x+1} - \frac{1}{2(x+2)}$$

$$\therefore \frac{dy}{dx} = \frac{y}{2}\left[\frac{9}{3x-4} - \frac{4}{x+1} - \frac{1}{x+2}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(3x-4)^3}{(x+1)^4(x+2)}}\left[\frac{9}{3x-4} - \frac{4}{x+1} - \frac{1}{x+2}\right]$$

Question 9.
Find $dydx$ if y = x_x + $(7x - 1)_x$
Solution:

$$y = x^x + (7x-1)^x$$

Let $u = x^x$ and $v = (7x-1)^x$

Then $y = u + v$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \ldots (1)$$

Take $u = x^x$

$\therefore \log u = \log x^x = x \log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx}(x \log x)$$

$$= x \frac{d}{dx}(\log x) + (\log x) \cdot \frac{d}{dx}(x)$$

$$= x \times \frac{1}{x} + (\log x) \times 1$$

$$\therefore \frac{du}{dx} = u(1 + \log x) = x^x(1 + \log x) \quad \ldots (2)$$

Also, $v = (7x-1)^x$

$\therefore \log v = \log (7x-1)^x = x \log (7x-1)$

$$\frac{1}{v} \cdot \frac{dv}{dx} = \frac{d}{dx}[x \log (7x-1)]$$

$$= x \frac{d}{dx}[\log (7x-1)] + [\log (7x-1)] \cdot \frac{d}{dx}(x)$$

$$= x \times \frac{1}{7x-1} \cdot \frac{d}{dx}(7x-1) + [\log (7x-1)] \times 1$$

$$= \frac{x}{7x-1} \times (7 \times 1 - 0) + \log (7x-1)$$

$$\therefore \frac{dv}{dx} = v\left[\frac{7x}{7x-1} + \log (7x-1)\right]$$

$$= (7x-1)^x \left[\frac{7x}{7x-1} + \log (7x-1)\right] \quad \ldots (3)$$

From (1), (2) and (3), we get

$$\frac{dy}{dx} = x^x(1 + \log x) + (7x-1)^x \left[\frac{7x}{7x-1} + \log (7x-1)\right].$$

Question 10.
If y = $x^3 + 3xy^2 + 3x^2y$, find $\frac{dy}{dx}$.
Solution:
y = $x^3 + 3xy^2 + 3x^2y$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = 3x^2 + 3\left[x\cdot\frac{d}{dx}(y^2) + y^2\cdot\frac{d}{dx}(x)\right] + 3\left[x^2\cdot\frac{dy}{dx} + y\cdot\frac{d}{dx}(x^2)\right]$$

$$\therefore\ \frac{dy}{dx} = 3x^2 + 3\left[x \times 2y\frac{dy}{dx} + y^2 \times 1\right] + 3\left[x^2\cdot\frac{dy}{dx} + y \times 2x\right]$$

$$\therefore\ \frac{dy}{dx} = 3x^2 + 6xy\frac{dy}{dx} + 3y^2 + 3x^2\frac{dy}{dx} + 6xy$$

$$\therefore\ (1 - 6xy - 3x^2)\frac{dy}{dx} = 3x^2 + 3y^2 + 6xy$$

$$\therefore\ \frac{dy}{dx} = \frac{3x^2 + 3y^2 + 6xy}{1 - 6xy - 3x^2} = \frac{-3(x^2 + y^2 + 2xy)}{6xy + 3x^2 - 1}.$$

Question 11.
If $x^3 + y^2 + xy = 7$, find $\frac{dy}{dx}$.
Solution:
$x^3 + y^2 + xy = 7$
Differentiating both sides w.r.t. x, we get

$$3x^2 + 2y\cdot\frac{dy}{dx} + x\frac{dy}{dx} + y\cdot\frac{d}{dx}(x) = 0$$

$$\therefore\ 3x^2 + 2y\frac{dy}{dx} + x\frac{dy}{dx} + y \times 1 = 0$$

$$\therefore\ (2y + x)\frac{dy}{dx} = -3x^2 - y$$

$$\therefore\ \frac{dy}{dx} = \frac{-(y + 3x^2)}{2y + x}.$$

Question 12.
If $x^3y^3 = x^2 - y^2$, find $\frac{dy}{dx}$.
Solution:
$x^3y^3 = x^2 - y^2$
Differentiating both sides w.r.t. x, we get

$$x^3\cdot\frac{d}{dx}(y^3) + y^3\cdot\frac{d}{dx}(x^3) = 2x - 2y\frac{dy}{dx}$$

$$\therefore\ x^3 \times 3y^2\frac{dy}{dx} + y^3 \times 3x^2 = 2x - 2y\frac{dy}{dx}$$

$$\therefore\ (3x^2y^2 + 2y)\frac{dy}{dx} = 2x - 3x^2y^3$$

$$\therefore\ y(2 + 3x^3y)\frac{dy}{dx} = x(2 - 3xy^3)$$

$$\therefore\ \frac{dy}{dx} = \frac{x(2 - 3xy^3)}{y(2 + 3x^3y)}.$$

Question 13.
If $x^7 \cdot y^9 = (x + y)^{16}$, then show that $\frac{dy}{dx} = \frac{y}{x}$.
Solution:

$x^7 \cdot y^9 = (x + y)^{16}$

Taking logarithm of both sides, we get

$\log x^7 \cdot y^9 = \log (x + y)^{16}$

$\therefore \log x^7 + \log y^9 = 16 \log(x + y)$

$\therefore 7 \log x + 9 \log y = 16 \log (x + y)$

Differentiating both sides w.r.t. x, we get

$$7\left(\frac{1}{x}\right) + 9\left(\frac{1}{y}\right)\frac{dy}{dx} = 16\left(\frac{1}{x+y}\right)\frac{d}{dx}(x + y)$$

$$\therefore \frac{7}{x} + \frac{9}{y}\frac{dy}{dx} = \frac{16}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\therefore \frac{7}{x} + \frac{9}{y}\frac{dy}{dx} = \frac{16}{x+y} + \frac{16}{x+y}\frac{dy}{dx}$$

$$\therefore \frac{9}{y}\frac{dy}{dx} - \frac{16}{x+y}\frac{dy}{dx} = \frac{16}{x+y} - \frac{7}{x}$$

$$\therefore \left(\frac{9}{y} - \frac{16}{x+y}\right)\frac{dy}{dx} = \frac{16}{x+y} - \frac{7}{x}$$

$$\therefore \left[\frac{9x + 9y - 16y}{y(x + y)}\right]\frac{dy}{dx} = \frac{16x - 7x - 7y}{x(x + y)}$$

$$\therefore \left[\frac{9x - 7y}{y(x + y)}\right]\frac{dy}{dx} = \frac{9x - 7y}{x(x + y)}$$

$$\therefore \frac{dy}{dx} = \frac{9x - 7y}{x(x + y)} \times \frac{y(x + y)}{9x - 7y}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}$$

Question 14.

If $x^a \cdot y^b = (x + y)^{a+b}$, then show that $\frac{dy}{dx} = \frac{y}{x}$.

Solution:

$$x^a \cdot y^b = (x+y)^{a+b}$$

$$\therefore \log(x^a \cdot y^b) = \log(x+y)^{a+b}$$

$$\therefore \log x^a + \log y^b = \log(x+y)^{a+b}$$

$$\therefore a \log x + b \log y = (a+b)\log(x+y)$$

Differentiating both sides w.r.t. x, we get

$$a \times \frac{1}{x} + b \times \frac{1}{y}\frac{dy}{dx} = (a+b) \times \frac{1}{x+y} \cdot \frac{d}{dx}(x+y)$$

$$\therefore \frac{a}{x} + \frac{b}{y}\frac{dy}{dx} = \frac{a+b}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\therefore \frac{a}{x} + \frac{b}{y}\frac{dy}{dx} = \frac{a+b}{x+y} + \frac{a+b}{x+y} \cdot \frac{dy}{dx}$$

$$\therefore \left(\frac{b}{y} - \frac{a+b}{x+y}\right)\frac{dy}{dx} = \frac{a+b}{x+y} - \frac{a}{x}$$

$$\therefore \left[\frac{bx + by - ay - by}{y(x+y)}\right]\frac{dy}{dx} = \frac{ax + bx - ax - ay}{a(x+y)}$$

$$\therefore \left[\frac{bx - ay}{y(x+y)}\right]\frac{dy}{dx} = \frac{bx - ay}{x(x+y)}$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}.$$

Question 15.
Find $\frac{dy}{dx}$ if $x = 5t^2$, $y = 10t$.
Solution:

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt} = 5\frac{d}{dt}(t^2) = 5 \times 2t = 10t$$

$$\text{and } \frac{dy}{dt} = 10\frac{d}{dt}(t) = 10 \times 1 = 10$$

$$\therefore \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{10}{10t} = \frac{1}{t}.$$

Question 16.

Find $\frac{dy}{dx}$ if $x = e^{3t}$, $y = e^{\sqrt{t}}$.

Solution:

$x = e^{3t}$, $y = e^{\sqrt{t}}$

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt} = \frac{d}{dt}(e^{3t}) = e^{3t} \cdot \frac{d}{dt}(3t)$$

$$= e^{3t} \times 3 = 3e^{3t}$$

$$\text{and } \frac{dy}{dt} = \frac{d}{dt}(e^{\sqrt{t}}) = e^{\sqrt{t}} \cdot \frac{d}{dt}(\sqrt{t})$$

$$= e^{\sqrt{t}} \times \frac{1}{2\sqrt{t}} = \frac{e^{\sqrt{t}}}{2\sqrt{t}}$$

$$\therefore \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{\left(\frac{e^{\sqrt{t}}}{2\sqrt{t}}\right)}{3e^{3t}}$$

$$= \frac{1}{6\sqrt{t}} \cdot e^{(\sqrt{t} - 3t)}.$$

Question 17.
Differentiate $\log(1 + x^2)$ with respect to a^x.
Solution:
Let $u = \log(1 + x^2)$ and $v = a^x$
Then we want to find $\frac{du}{dv}$
Differentiating u and v w.r.t. x, we get

$$\frac{du}{dx} = \frac{d}{dx}[\log(1+x^2)] = \frac{1}{1+x^2} \cdot \frac{d}{dx}(1+x^2)$$

$$= \frac{1}{1+x^2} \times (0+2x) = \frac{2x}{1+x^2}$$

$$\text{and } \frac{dv}{dx} = \frac{d}{dx}(a^x) = a^x \cdot \log a$$

$$\therefore \frac{du}{dv} = \frac{(du/dx)}{(dv/dx)} = \frac{\left(\frac{2x}{1+x^2}\right)}{a^x \cdot \log a} = \frac{2x}{(1+x^2) \cdot a^x \log a}.$$

Question 18.
Differentiate $e^{(4x+5)}$ with resepct to 10^{4x}.
Solution:
Let $u = e^{(4x+5)}$ and $v = 10^{4x}$
Then we want to find $\frac{du}{dv}$
Differentiating u and v w.r.t. x, we get

$$\frac{du}{dx} = \frac{d}{dx}[e^{(4x+5)}] = e^{(4x+5)} \cdot \frac{d}{dx}(4x+5)$$

$$= e^{(4x+5)} \times (4 \times 1 + 0) = 4e^{(4x+5)}$$

$$\text{and } \frac{dv}{dx} = \frac{d}{dx}(10^{4x}) = 10^{4x} \cdot \log 10 \cdot \frac{d}{dx}(4x)$$

$$= 10^{4x} \cdot (\log 10) \times 4 = 4 \cdot 10^{4x} \cdot \log 10$$

$$\therefore \frac{du}{dv} = \frac{(du/dx)}{(dv/dx)} = \frac{4e^{(4x+5)}}{4 \cdot 10^{4x} \cdot \log 10} = \frac{e^{(4x+5)}}{10^{4x} \cdot \log 10}.$$

Question 19.
Find $\frac{d^2y}{dx^2}$, if y = log x.
Solution:
y = log x
Differentiating w.r.t. x, we get
$\frac{dy}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$
Differentiating again w.r.t. x, we get
$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

Question 20.
Find $\frac{d^2y}{dx^2}$, if y = 2at, $x = at^2$.
Solution:
$x = at^2$, y = 2at

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt}=\frac{d}{dt}(at^2)=a\frac{d}{dt}(t^2)$$

$$=a\times 2t=2at \qquad \ldots (1)$$

and $$\frac{dy}{dt}=\frac{d}{dt}(2at)=2a\frac{d}{dt}(t)$$

$$=2a\times 1=2a$$

$$\therefore \frac{dy}{dx}=\frac{(dy/dt)}{(dx/dt)}=\frac{2a}{2at}=\frac{1}{t}$$

$$\therefore \frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{1}{t}\right)=\frac{d}{dt}\left(\frac{1}{t}\right)\cdot\frac{dt}{dx}$$

$$=-\frac{1}{t^2}\times\frac{1}{\left(\frac{dx}{dt}\right)}=-\frac{1}{t^2}\times\frac{1}{2at} \qquad \ldots [\text{By } (1)]$$

$$=-\frac{1}{2at^3}.$$

Question 21.
Find $\frac{d^2y}{dx^2}$, if y = $x^2 \cdot e^x$
Solution:
y = $x^2 \cdot e^x$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}(x^2e^x)=x^2\frac{d}{dx}(e^x)+e^x\cdot\frac{d}{dx}(x^2)$$

$$=x^2\cdot e^x+e^x\times 2x=e^x(x^2+2x)$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2}=\frac{d}{dx}[e^x(x^2+2x)]$$

$$=e^x\cdot\frac{d}{dx}(x^2+2x)+(x^2+2x)\cdot\frac{d}{dx}(e^x)$$

$$=e^x(2x+2)+(x^2+2x)\cdot e^x$$

= e^x (2x + 2 + x^2 + 2x)
= e^x (x^2 + 4x + 2).

Question 22.
If x^2 + 6xy + y^2 = 10, then show that $\frac{d^2y}{dx^2}=\frac{80}{(3x+y)^3}$.
Solution:
x^2 + 6xy + y^2 = 10 ……..(1)
Differentiating both sides w.r.t. a, we get

$$2x+6\left[x\frac{dy}{dx}+y\cdot\frac{d}{dx}(x)\right]+2y\frac{dy}{dx}=$$

$$\therefore 2x+6x\frac{dy}{dx}+6y\times 1+2y\frac{dy}{dx}=0$$

$$\therefore (6x+2y)\frac{dy}{dx}=-2x-6y$$

$$\therefore \frac{dy}{dx}=\frac{-2(x+3y)}{2(3x+y)}=-\left(\frac{x+3y}{3x+y}\right) \qquad \ldots (2)$$

$$\therefore \frac{d^2y}{dx^2}=-\frac{d}{dx}\left(\frac{x+3y}{3x+y}\right)$$

$$= -\left[\frac{(3x+y)\cdot\frac{d}{dx}(x+3y)-(x+3y)\cdot\frac{d}{dx}(3x+y)}{(3x+y)^2}\right]$$

$$= -\left[\frac{(3x+y)\left(1+3\frac{dy}{dx}\right)-(x+3y)\left(3+\frac{dy}{dx}\right)}{(3x+y)^2}\right]$$

$$=\frac{1}{(3x+y)^2}\left[-(3x+y)\left\{1-\frac{3(x+3y)}{3x+y}\right\}+\right.$$
$$\left.(x+3y)\left(3-\frac{x+3y}{3x+y}\right)\right] \quad \ldots \text{[By (2)]}$$

$$=\frac{1}{(3x+y)^2}\left[-(3x+y)\left(\frac{3x+y-3x-9y}{3x+y}\right)+\right.$$
$$\left.(x+3y)\left(\frac{9x+3y-x-3y}{3x+y}\right)\right]$$

$$=\frac{1}{(3x+y)^2}\left[8y+\frac{(x+3y)(8x)}{3x+y}\right]$$

$$=\frac{1}{(3x+y)^2}\left[\frac{8y(3x+y)+(x+3y)8x}{(3x+y)}\right]$$

$$=\frac{24xy+8y^2+8x^2+24xy}{(3x+y)^3}$$

$$=\frac{8x^2+48xy+8y^2}{(3x+y)^3}=\frac{8(x^2+6xy+y^2)}{(3x+y)^3}$$

$$=\frac{8(10)}{(3x+y)^3} \quad \ldots \text{[By (1)]}$$

$$\therefore \frac{d^2y}{dx^2}=\frac{80}{(3x+y)^3}.$$

Question 23.
If $ax^2 + 2hxy + by^2 = 0$, then show that $\frac{d^2y}{dx^2} = 0$.
Solution:
$ax^2 + 2hxy + by^2 = 0$ ……..(1)
$\therefore ax^2 + hxy + hxy + by^2 = 0$
$\therefore x(ax + hy) + y(hx + by) = 0$
$\therefore x(ax + hy) = -y(hx + by)$
$\therefore \frac{ax+hy}{hx+by} = -\frac{y}{x}$ …….(2)

Differentiating (1) w.r.t. x, we get

$$a \times 2x + 2h\left[x\frac{dy}{dx} + y\cdot\frac{d}{dx}(x)\right] + b \times 2y\frac{dy}{dx} = 0$$

$$\therefore\ 2ax + 2hx\frac{dy}{dx} + 2hy \times 1 + 2by\frac{dy}{dx} = 0$$

$$\therefore\ (2hx + 2by)\frac{dy}{dx} = -2ax - 2hy$$

$$\therefore\ \frac{dy}{dx} = \frac{-2(ax + hy)}{2(hx + by)} = -\left(\frac{ax + hy}{hx + by}\right)$$

$$\therefore\ \frac{dy}{dx} = \frac{y}{x} \qquad \ldots\ [\text{By (1)}]$$

$$\therefore\ \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{y}{x}\right)$$

$$= \frac{x\ \frac{dy}{dx} - y\cdot\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x\left(\frac{y}{x}\right) - y \times 1}{x^2} \qquad \ldots\ \left[\because \frac{dy}{dx} = \frac{y}{x}\right]$$

$$= \frac{y - y}{x^2} = \frac{0}{x^2}$$

$$\therefore\ \frac{d^2y}{dx^2} = 0.$$