Maharashtra State Board 11th Maths Solutions Chapter 2 Sequences and Series Ex 2.1

Question 1.
Check whether the following sequences are G.P. If so, write t_n.
(i) 2, 6, 18, 54, ……
Solution:
2, 6, 18, 54, …
$t_1 = 2, t_2 = 6, t_3 = 18, t_4 = 54, \ldots$
Here, $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = 3$
$\because$ the ratio of any two consecutive terms is a constant, the given sequence is a Geometric progression.
Here, a = 2, r = 3
$t_n = ar^{n-1}$
$\therefore$ $t_n = 2(3^{n-1})$

(ii) 1, -5, 25, -125, ………
Solution:
1, –5, 25, –125, …
$t_1 = 1, t_2 = -5, t_3 = 25, t_4 = -125$
Here, $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = -5$
$\because$ the ratio of any two consecutive terms is a constant, the given sequence is a Geometric progression.
Here, a = 1, r = – 5
$t_n = ar^{n-1}$
$\therefore$ $t_n = (-5)^{n-1}$

(iii) $\sqrt{5}, \frac{1}{\sqrt{5}}, \frac{1}{5\sqrt{5}}, \frac{1}{25\sqrt{5}}, \ldots$
Solution:
$\sqrt{5}, \frac{1}{\sqrt{5}}, \frac{1}{5\sqrt{5}}, \frac{1}{25\sqrt{5}}, \ldots$
$t_1 = \sqrt{5}, t_2 = \frac{1}{\sqrt{5}}, t_3 = \frac{1}{5\sqrt{5}}, t_4 = \frac{1}{25\sqrt{5}}, \ldots$
Here, $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \frac{1}{5}$
$\because$ the ratio of any two consecutive terms is a constant, the given sequence is a Geometric progression.
Here, $a = \sqrt{5}$, $r = \frac{1}{5}$
$t_n = ar^{n-1}$
$\therefore$ $t_n = \sqrt{5}\left(\frac{1}{5}\right)^{n-1}$

(iv) 3, 4, 5, 6, ……
Solution:
3, 4, 5, 6, …
$t_1 = 3, t_2 = 4, t_3 = 5, t_4 = 6$
Here, $\frac{t_2}{t_1} = \frac{4}{3}, \frac{t_3}{t_2} = \frac{5}{4}, \frac{t_4}{t_3} = \frac{6}{5}$
$\because$ $\frac{t_2}{t_1} \neq \frac{t_3}{t_2} \neq \frac{t_4}{t_3}$, the given sequence is not a Geometric progression.

(v) 7, 14, 21, 28, ...
Solution:

7, 14, 21, 28, ...
$t_1 = 7, t_2 = 14, t_3 = 21, t_4 = 28$
Here, $\frac{t_2}{t_1} = 2, \frac{t_3}{t_2} = \frac{3}{2}, \frac{t_4}{t_3} = \frac{4}{3}$
$\because$ $\frac{t_2}{t_1} \neq \frac{t_3}{t_2} \neq \frac{t_4}{t_3}$, the given sequence is not a Geometric progression.

Question 2.
For the G.P.
(i) If r = 13, a = 9, find t_7.
Solution:

Given, $r = \frac{1}{3}$, a = 9
$t_n = ar^{n-1}$
$\therefore$ $t_7 = 9 \times \left(\frac{1}{3}\right)^{7-1}$
$= \frac{9}{3^6}$
$= \frac{1}{81}$

(ii) If a = 7243, r = 3, find t_6.
Solution:

Given, $a = \frac{7}{243}$, r = 3
$t_n = ar^{n-1}$
$\therefore$ $t_6 = \frac{7}{243} \times (3)^{6-1}$
$= \frac{7}{243} \times 3^5$
$= 7$

(iii) If r = -3 and t_6 = 1701, find a.
Solution:

(iv) If a = 23, t_6 = 162, find r.
Solution:

Given, $a = \frac{2}{3}$, $t_6 = 162$
$t_n = ar^{n-1}$
$\therefore$ $t_6 = \left(\frac{2}{3}\right)(r^{6-1})$
$\therefore$ $162 = \frac{2}{3} r^5$
$\therefore$ $r^5 = 162 \times \frac{3}{2}$
$\therefore$ $r^5 = 3^5$
$\therefore$ $r = 3$

Question 3.
Which term of the G. P. 5, 25, 125, 625, ... is 5^{10}?

Solution:

Here, $t_1 = a = 5$, $r = \frac{t_2}{t_1} = \frac{25}{5} = 5$, $t_n = 5^{10}$

$t_n = ar^{n-1}$

$\therefore \quad 5^{10} = 5 \times 5^{(n-1)}$

$\therefore \quad 5^{10} = 5^{(1+n-1)}$

$\therefore \quad 5^{10} = 5^n$

$\therefore \quad n = 10$

$\therefore \quad 5^{10}$ is the 10^{th} term of the G.P.

Alternate Method:

$t_1 = 5$, $t_2 = 25 = 5^2$, $t_3 = 125 = 5^3$, $t_4 = 625 = 5^4$,

$\therefore \quad t_{10} = 5^{10}$

Question 4.

For what values of x, the terms 43, x, 427 are in G. P.?

Solution:

$\frac{4}{3}, x, \frac{4}{27}$ are in Geometric progression.

$\therefore \quad \frac{t_2}{t_1} = \frac{t_3}{t_2}$

$\therefore \quad \frac{x}{\frac{4}{3}} = \frac{\frac{4}{27}}{x}$

$\therefore \quad x^2 = \frac{4}{3} \times \frac{4}{27}$

$\therefore \quad x^2 = \frac{16}{81}$

$\therefore \quad x = \pm \frac{4}{9}$

Question 5.

If for a sequence, $t_n=5_{n-3}2_{n-3}$, show that the sequence is a G. P. Find its first term and the common ratio.

Solution:

The sequence (t_n) is a G.P. if $\frac{t_{n+1}}{t_n}$ = constant for all $n \in N$.

Now, $t_n = \frac{5^{n-3}}{2^{n-3}}$

$t_{n+1} = \frac{5^{n+1-3}}{2^{n+1-3}} = \frac{5^{n-2}}{2^{n-2}}$

$\therefore \quad \frac{t_{n+1}}{t_n} = \frac{5^{n-2}}{2^{n-2}} \times \frac{2^{n-3}}{5^{n-3}}$

$= (5)^{(n-2)-(n-3)} \times (2)^{(n-3)-(n-2)}$

$= (5)^1 \times (2)^{-1}$

$= \frac{5}{2}$, which is a constant, for all $n \in N$.

$\therefore \quad r = \frac{5}{2}$

$\therefore$ The sequence is a G.P. with common ratio $\frac{5}{2}$.

First term $t_1 = \frac{5^{1-3}}{2^{1-3}} = \frac{5^{-2}}{2^{-2}} = \frac{2^2}{5^2} = \frac{4}{25}$

Question 6.

Find three numbers in G. P. such that their sum is 21 and the sum of their squares is 189.

Solution:

Let the three numbers in G. P. be ar, a, ar.

According to the given conditions,

$$\frac{a}{r} + a + ar = 21$$

$$\therefore \quad \frac{1}{r} + 1 + r = \frac{21}{a}$$

$$\therefore \quad \frac{1}{r} + r = \frac{21}{a} - 1 \qquad \ldots(i)$$

$$\text{and } \frac{a^2}{r^2} + a^2 + a^2r^2 = 189$$

$$\therefore \quad \frac{1}{r^2} + 1 + r^2 = \frac{189}{a^2}$$

$$\therefore \quad \frac{1}{r^2} + r^2 = \frac{189}{a^2} - 1 \qquad \ldots(ii)$$

On squaring equation (i), we get

$$\frac{1}{r^2} + r^2 + 2 = \frac{441}{a^2} - \frac{42}{a} + 1$$

$$\therefore \quad \left(\frac{189}{a^2} - 1\right) + 2 = \frac{441}{a^2} - \frac{42}{a} + 1 \qquad \ldots\text{[From (ii)]}$$

$$\therefore \quad \frac{189}{a^2} + 1 = \frac{441}{a^2} - \frac{42}{a} + 1$$

$$\therefore \quad \frac{441}{a^2} - \frac{189}{a^2} = \frac{42}{a}$$

$$\therefore \quad \frac{252}{a^2} = \frac{42}{a}$$

$$\therefore \quad 252 = 42a$$

$$\therefore \quad a = 6$$

Substituting the value of a in (i), we get

$$\frac{1}{r} + r = \frac{21}{6} - 1$$

$$\therefore \quad \frac{1+r^2}{r} = \frac{15}{6}$$

$$\therefore \quad \frac{1+r^2}{r} = \frac{5}{2}$$

$$\therefore \quad 2r^2 - 5r + 2 = 0$$

$$\therefore \quad 2r^2 - 4r - r + 2 = 0$$

$$\therefore \quad (2r-1)(r-2) = 0$$

$$\therefore \quad r = \frac{1}{2} \text{ or } 2$$

When $a = 6$, $r = \frac{1}{2}$,

$$\frac{a}{r} = 12, a = 6, ar = 3$$

When a = 6, r = 2,
ar = 3, a = 6, ar = 12
Hence, the three numbers in G.P. are 12, 6, 3 or 3, 6, 12.

Check:
If sum of the three numbers is 21 and sum of their squares is 189, then our answer is correct.
Sum of the numbers = 12 + 6 + 3 = 21
Sum of the squares of the numbers = $12^2 + 6^2 + 3^2$
= 144 + 36 + 9
= 189
Thus, our answer is correct.

Question 7.
Find four numbers in G. P. such that the sum of the middle two numbers is 103 and their product is 1.
Solution:
Let the four numbers in G.P. be ar3,ar,ar,ar3
According to the given conditions,

$$\frac{a}{r^3}\left(\frac{a}{r}\right)(ar)(ar^3) = 1$$

$\therefore \quad a^4 = 1$

$\therefore \quad a = 1 \qquad \ldots(i)$

and $\frac{a}{r} + ar = \frac{10}{3}$

$\therefore \quad \frac{1}{r} + (1)r = \frac{10}{3} \qquad \ldots[\text{From (i)}]$

$\therefore \quad \frac{1+r^2}{r} = \frac{10}{3}$

$\therefore \quad 3 + 3r^2 = 10r$

$\therefore \quad 3r^2 - 10r + 3 = 0$

$\therefore \quad (r-3)(3r-1) = 0$

$\therefore \quad r = 3 \text{ or } r = \frac{1}{3}$

When $a = 1, r = 3$,

$\frac{a}{r^3} = \frac{1}{(3)^3} = \frac{1}{27}, \ \frac{a}{r} = \frac{1}{3}, \ ar = 1(3) = 3$ and

$ar^3 = 1(3)^3 = 27$

When $a = 1, r = \frac{1}{3}$,

$\frac{a}{r^3} = \frac{1}{\left(\frac{1}{3}\right)^3} = 27, \ \frac{a}{r} = \frac{1}{\left(\frac{1}{3}\right)} = 3,$

$ar = 1\left(\frac{1}{3}\right) = \frac{1}{3}$ and $ar^3 = 1\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

Hence, the four numbers in G.P.are

$\frac{1}{27}, \frac{1}{3}, 3, 27$ or $27, 3, \frac{1}{3}, \frac{1}{27}$.

Question 8.
Find five numbers in G. P. such that their product is 1024 and the fifth term is square of the third term.
Solution:
Let the five numbers in G. P. be
ar2,ar,a,ar,ar2
According to the given conditions,

$$\frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 = 1024$$

$\therefore \quad a^5 = 4^5$

$\therefore \quad a = 4 \qquad \ldots(i)$

and $ar^2 = a^2$

$\therefore \quad r^2 = a$

$\therefore \quad r^2 = 4 \qquad \ldots[\text{From (i)}]$

$\therefore \quad r = \pm 2$

When $a = 4, r = 2$,

$\frac{a}{r^2} = 1, \ \frac{a}{r} = 2, \ a = 4, \ ar = 8, \ ar^2 = 16$

When $a = 4, r = -2$,

$\frac{a}{r^2} = 1, \ \frac{a}{r} = -2, \ a = 4, \ ar = -8, \ ar^2 = 16$

Hence, the five numbers in G.P. are
1, 2, 4, 8, 16 or 1, -2, 4, -8, 16.

Question 9.
The fifth term of a G. P. is x, the eighth term of a G.P. is y and the eleventh term of a G.P. is z, verify whether y_2 = xz.

Solution:

Given, $t_5 = x$, $t_8 = y$, $t_{11} = z$
Since $t_n = ar^{n-1}$,
$t_5 = ar^4$, $t_8 = ar^7$, $t_{11} = ar^{10}$
Consider,
L.H.S. $= y^2$
$= (t_8)^2 = (ar^7)^2 = a^2r^{14}$
R.H.S. $= xz$
$= t_5.t_{11} = ar^4.ar^{10} = a^2r^{14}$
$\therefore$ L.H.S. = R.H.S.
$\therefore$ $y^2 = xz$

Question 10.
If p, q, r, s are in G.P., show that p + q, q + r, r + s are also in G. P.
Solution:
p, q, r, s are in G.P.
$\therefore$ $qp=rq=sr$
Let $qp=rq=sr$ = k
$\therefore$ q = pk, r = qk, s = rk
We have to prove that p + q, q + r, r + s are in G.P.
i.e., to prove that $q+rp+q=r+sq+r$

$$\text{L.H.S.} = \frac{q+r}{p+q} = \frac{q+qk}{p+pk} = \frac{q(1+k)}{p(1+k)} = \frac{q}{p} = k$$

$$\text{R.H.S.} = \frac{r+s}{q+r} = \frac{r+rk}{q+qk} = \frac{r(1+k)}{q(1+k)} = \frac{r}{q} = k$$

$$\therefore \quad \frac{q+r}{p+q} = \frac{r+s}{q+r}$$

$\therefore$ p + q, q + r, r + s are in G.P.

Question 11.
The number of bacteria in a culture doubles every hour. If there were 50 bacteria originally in the culture, how many bacteria will be there at the end of the 5th hour?
Solution:
Since the number of bacteria in culture doubles every hour, increase in number of bacteria after every hour is in G.P.
$\therefore$ a = 50, r = 10050 = 2
$t_n = ar^{n-1}$
To find the number of bacteria at the end of the 5th hour.
(i.e., to find the number of bacteria at the beginning of the 6th hour, i.e., to find t_6.)
$\therefore$ $t_6 = ar^5$
= 50 × (2^5)
= 50 × 32
= 1600

Question 12.
A ball is dropped from a height of 80 ft. The ball is such that it rebounds $(34)th$ of the height it has fallen. How high does the ball rebound on the 6th bounce? How high does the ball rebound on the nth bounce?
Solution:
Since the ball rebounds $(34)th$ of the height it has fallen, the height in successive bounce is in G.P.
1st height in the bounce = 80 × 34

$$a = 80 \times \frac{3}{4}, \ r = \frac{3}{4}$$
$$t_n = ar^{n-1}$$

$$\text{Height in the } 6^{\text{th}} \text{ bounce} = 80 \times \frac{3}{4} \times \left(\frac{3}{4}\right)^5$$
$$= 80 \times \left(\frac{3}{4}\right)^6$$

$$\text{Height in the } n^{\text{th}} \text{ bounce} = 80\left(\frac{3}{4}\right)^n$$

Question 13.
The numbers 3, x and x + 6 are in G. P. Find
(i) x
(ii) 20th term
(iii) nth term.
Solution:
(i) 3, x and x + 6 are in G. P.

$\frac{x}{3}=\frac{x+6}{x}$

$x^2 = 3x + 18$
$x^2 – 3x – 18 = 0$
$(x – 6) (x + 3) = 0$
$x = 6, -3$

ii. $r = \frac{6}{3} = 2$ or $r = \frac{-3}{3} = -1$

$t_n = ar^{n-1}$

$\therefore$ $t_{20} = 3(2^{19})$ or $t_{20} = 3(-1)^{19}$

$\therefore$ $t_{20} = 3(2^{19})$ or $t_{20} = -3$

iii. $t_n = 3(2)^{n-1}$ or $t_n = 3(-1)^{n-1}$

Question 14.
Mosquitoes are growing at a rate of 10% a year. If there were 200 mosquitoes in the beginning, write down the number of mosquitoes after
(i) 3 years
(ii) 10 years
(iii) n years
Solution:
a = 200, r = 1 + $\frac{10}{100} = \frac{11}{10}$
Mosquitoes at the end of 1st year = 200 × $\frac{11}{10}$
(i) Number of mosquitoes after 3 years
= 200 × $\frac{11}{10}\times\left(\frac{11}{10}\right)^2$
= 200 $\left(\frac{11}{10}\right)^3$
= 200 $(1.1)^3$

(ii) Number of mosquitoes after 10 years = 200 $(1.1)^{10}$

(iii) Number of mosquitoes after n years = 200 $(1.1)^n$

Question 15.
The numbers x – 6, 2x and x^2 are in G. P. Find
(i) x
(ii) 1st term
(iii) nth term
Solution:
(i) x – 6, 2x and x are in Geometric progression.
$\therefore \frac{2x}{x-6}=\frac{x^2}{2x}$
$4x^2 = x^2(x – 6)$
$4 = x – 6$
$x = 10$

(ii) $t_1 = x – 6 = 10 – 6 = 4$

iii. $a = 4, r = \frac{2x}{x-6} = \frac{2(10)}{4} = 5$

$t_n = ar^{n-1}$

$\therefore$ $t_n = 4(5^{n-1})$

Maharashtra State Board 11th Maths Solutions Chapter 2 Sequences and Series Ex 2.2

Question 1.
For the following G.P.s, find S_n.
(i) 3, 6, 12, 24, ……..
Solution:

3, 6, 12, 24, …

Here, $a = 3$, $r = \frac{6}{3} = 2 > 1$

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{ for } r > 1$$

$$\therefore \quad S_n = \frac{3(2^n - 1)}{2 - 1}$$

$$\therefore \quad S_n = 3(2^n - 1)$$

(ii) p, q, $\frac{q^2}{p}, \frac{q^3}{p^2}, \ldots$
Solution:

$$p, q, \frac{q^2}{p}, \frac{q^3}{p^2}, \ldots$$

Here, $a = p$, $r = \frac{q}{p}$

Let $\frac{q}{p} < 1$

$$S_n = \frac{a(1 - r^n)}{1 - r}, \text{ for } r < 1$$

$$\therefore \quad S_n = \frac{p\left[1 - \left(\frac{q}{p}\right)^n\right]}{1 - \frac{q}{p}}$$

$$\therefore \quad S_n = \frac{p^2}{p - q}\left[1 - \left(\frac{q}{p}\right)^n\right]$$

Let $\frac{q}{p} > 1$

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{ for } r > 1$$

$$\therefore \quad S_n = \frac{p\left[\left(\frac{q}{p}\right)^n - 1\right]}{\frac{q}{p} - 1} = \frac{p^2}{q - p}\left[\left(\frac{q}{p}\right)^n - 1\right]$$

(iii) 0.7, 0.07, 0.007, …….
Solution:

$$a = 0.7 = \frac{7}{10}, r = \frac{0.07}{0.7} = \frac{1}{10} < 1$$

$$S_n = \frac{a(1 - r^n)}{1 - r}, \text{ for } r < 1$$

$$= \frac{\frac{7}{10}\left[1 - \left(\frac{1}{10}\right)^n\right]}{1 - \frac{1}{10}} = \frac{7}{9}\left(1 - \frac{1}{10^n}\right)$$

(iv) √5, -5, 5√5, -25, …….
Solution:

$$a = \sqrt{5},\ r = \frac{-5}{\sqrt{5}} = -\sqrt{5} < 1$$

$$S_n = \frac{a\left(1-r^n\right)}{1-r},\ \text{for } r < 1$$

$$= \frac{\sqrt{5}\left[1-\left(-\sqrt{5}\right)^n\right]}{1-\left(-\sqrt{5}\right)}$$

$$= \frac{\sqrt{5}}{1+\sqrt{5}}\left[1-\left(-\sqrt{5}\right)^n\right]$$

$$= \frac{-\sqrt{5}}{\left(\sqrt{5}+1\right)}\left[\left(-\sqrt{5}\right)^n-1\right]$$

Question 2.
For a G.P.
(i) a = 2, r = −23, find S_6.
Solution:

$$a = 2,\ r = -\frac{2}{3}$$

$$S_n = \frac{a\left(1-r^n\right)}{1-r},\ \text{for } r < 1$$

$$S_6 = \frac{2\left[1-\left(-\frac{2}{3}\right)^6\right]}{1-\left(-\frac{2}{3}\right)}$$

$$= \frac{2\left[1-\left(\frac{2}{3}\right)^6\right]}{\frac{5}{3}}$$

$$= \frac{6}{5}\left[\frac{729-64}{3^6}\right]$$

$$= \frac{6}{5}\left[\frac{665}{729}\right]$$

$$S_6 = \frac{266}{243}$$

(ii) If S_5 = 1023, r = 4, find a.
Solution:

$$r = 4,\ S_5 = 1023$$

$$S_n = a\left(\frac{r^n-1}{r-1}\right),\ \text{for } r > 1$$

$$\therefore\quad S_5 = a\left(\frac{4^5-1}{4-1}\right)$$

$$\therefore\quad 1023 = a\left(\frac{1024-1}{3}\right)$$

$$\therefore\quad 1023 = \frac{a}{3}(1023)$$

$$\therefore\quad a = 3$$

Question 3.
For a G.P.
(i) If a = 2, r = 3, S_n = 242, find n.

Solution:

$a = 2, r = 3, S_n = 242$

$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$, for $r > 1$

$\therefore \quad 242 = 2\left(\frac{3^n - 1}{3 - 1}\right)$

$\therefore \quad 242 = 3^n - 1$

$\therefore \quad 3^n = 243$

$\therefore \quad 3^n = 3^5$

$\therefore \quad n = 5$

(ii) For a G.P. sum of the first 3 terms is 125 and the sum of the next 3 terms is 27, find the value of r.

Solution:

$S_3 = 125, S_6 = 125 + 27 = 152$

$S_n = a\left(\frac{1 - r^n}{1 - r}\right)$

$\therefore \quad S_3 = a\left(\frac{1 - r^3}{1 - r}\right)$

$\therefore \quad 125 = a\left(\frac{1 - r^3}{1 - r}\right) \quad \ldots(i)$

Also, $S_6 = a\left(\frac{1 - r^6}{1 - r}\right)$

$\therefore \quad 152 = a\left(\frac{1 - r^6}{1 - r}\right) \quad \ldots(ii)$

Dividing (ii) by (i), we get

$\frac{152}{125} = \frac{1 - r^6}{1 - r^3}$

$\therefore \quad \frac{152}{125} = \frac{(1 + r^3)(1 - r^3)}{(1 - r^3)}$

$\therefore \quad 1 + r^3 = \frac{152}{125}$

$\therefore \quad r^3 = \frac{152}{125} - 1$

$\therefore \quad r^3 = \frac{27}{125} \quad \therefore \quad r^3 = \left(\frac{3}{5}\right)^3$

$\therefore \quad r = \frac{3}{5}$

Question 4.

For a G.P.

(i) If $t_3 = 20$, $t_6 = 160$, find S_7.

Solution:

$t_3 = 20, t_6 = 160$

$t_n = ar^{n-1}$

$\therefore \quad t_3 = ar^{3-1} = ar^2$

$\therefore \quad ar^2 = 20$

$\therefore \quad a = \frac{20}{r^2} \quad \ldots(i)$

Also, $t_6 = ar^5$

$\therefore \quad ar^5 = 160$

$\therefore \quad \left(\frac{20}{r^2}\right)r^5 = 160 \quad \ldots[\text{From (i)}]$

$\therefore \quad r^3 = \frac{160}{20} = 8$

$\therefore \quad r = 2$

Substituting the value of r in (i) we get

$a = \frac{20}{2^2} = 5$

Now, $S_n = \frac{a(r^n - 1)}{r - 1}$, for $r > 1$

$\therefore \quad S_7 = \frac{5(2^7 - 1)}{2 - 1} = 5(128 - 1) = 635$

(ii) If $t_4 = 16$, $t_9 = 512$, find S_{10}.
Solution:

$t_4 = 16, t_9 = 512$

$t_n = ar^{n-1}$

$\therefore \quad t_4 = ar^{4-1} = ar^3$

$\therefore \quad ar^3 = 16$

$\therefore \quad a = \frac{16}{r^3} \quad \ldots(i)$

Also, $t_9 = ar^8$

$\therefore \quad ar^8 = 512$

$\therefore \quad \frac{16}{r^3} \times r^8 = 512$

$\therefore \quad r^5 = 32$

$\therefore \quad r = 2$

Substituting r = 2 in (i), we get

$a = \frac{16}{2^3} = \frac{16}{8} = 2$

Now, $S_n = \frac{a\left(r^n - 1\right)}{r - 1}$, for $r > 1$

$\therefore \quad S_{10} = \frac{2(2^{10} - 1)}{2 - 1} = 2(1024 - 1) = 2046$

Question 5.
Find the sum to n terms
(i) 3 + 33 + 333 + 3333 + …..
Solution:
S_n = 3 + 33 + 333 +….. upto n terms
= 3(1 + 11 + 111 +….. upto n terms)
= $\frac{3}{9}$(9 + 99 + 999 + ….. upto n terms)
= $\frac{3}{9}$[(10 – 1) + (100 – 1) + (1000 – 1) +… upto n terms]
= $\frac{3}{9}$[(10 + 100 + 1000 + … upto nterms) – (1 + 1 + 1 + ….. n times)]
But 10, 100, 1000, ….. n terms are in G.P. with
a = 10, r = $\frac{100}{10}$ = 10

$$\therefore \quad S_n = \frac{3}{9}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right] = \frac{3}{9}\left[\frac{10}{9}\left(10^n - 1\right) - n\right]$$

$$\therefore \quad S_n = \frac{3}{81}[10(10^n - 1) - 9n]$$

(ii) 8 + 88 + 888 + 8888 + ...
Solution:
S_n = 8 + 88 + 888 + ... upto n terms
= 8(1 + 11 + 111 + ... upto n terms)
= $\frac{8}{9}$(9 + 99 + 999 + ... upto n terms)
= $\frac{8}{9}$[(10 – 1) + (100 – 1) + (1000 – 1) +... upto n terms]
= $\frac{8}{9}$[(10 + 100 + 1000 + ... upto n terms) – (1 + 1 + 1 + ... n times)]
But 10, 100, 1000, ... n terms are in G.P. with
a = 10, r = $\frac{100}{10}$ = 10

$$\therefore \quad S_n = \frac{8}{9}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right]$$
$$= \frac{8}{9}\left[\frac{10}{9}\left(10^n - 1\right) - n\right]$$
$$\therefore \quad S_n = \frac{8}{81}\left[10(10^n - 1) - 9n\right]$$

Question 6.
Find the sum to n terms
(i) 0.4 + 0.44 + 0.444 + ...
Solution:
S_n = 0.4 + 0.44 + 0.444 + ... upto n terms
= 4(0.1 + 0.11 +0.111 + ... upto n terms)
= $\frac{4}{9}$(0.9 + 0.99 + 0.999 + ... upto n terms)
= $\frac{4}{9}$[(1 – 0.1) + (1 – 0.01) + (1 – 0.001) ... upto n terms]
= $\frac{4}{9}$[(1 + 1 + 1 + ...n times) – (0.1 + 0.01 + 0.001 +... upto n terms)]
But 0.1, 0.01, 0.001, ... n terms are in G.P. with
a = 0.1, r = $\frac{0.01}{0.1}$ = 0.1

$$\therefore \quad S_n = \frac{4}{9}\left\{n - 0.1\left[\frac{1-(0.1)^n}{1-0.1}\right]\right\}$$
$$\therefore \quad S_n = \frac{4}{9}\left\{n - \frac{0.1}{0.9}\left[1-(0.1)^n\right]\right\}$$
$$\therefore \quad S_n = \frac{4}{9}\left[n - \frac{1}{9}\left(1-(0.1)^n\right)\right]$$
$$\therefore \quad S_n = \frac{4}{81}\left[9n - \left(1 - \frac{1}{10^n}\right)\right]$$

(ii) 0.7 + 0.77 + 0.777 + ...
Solution:
S_n = 0.7 + 0.77 + 0.777 + ... upto n terms
= 7(0.1 + 0.11 + 0.111 + ... upton terms)
= $\frac{7}{9}$(0.9 + 0.99 + 0.999 + ... upto n terms)
= $\frac{7}{9}$[(1 – 0.1) + (1 – 0.01) + (1 – 0.001) +... upto n terms]
= $\frac{7}{9}$[(1 + 1 + 1 +... n times) – (0.1 + 0.01 + 0.001 +... upto n terms)]
But 0.1, 0.01, 0.001, ... n terms are in G.P. with
a = 0.1, r = $\frac{0.01}{0.1}$ = 0.1

$$\therefore \quad S_n = \frac{7}{9}\left\{n - 0.1\left[\frac{1-(0.1)^n}{1-0.1}\right]\right\}$$
$$= \frac{7}{9}\left\{n - \frac{0.1}{0.9}\left[1-(0.1)^n\right]\right\}$$
$$= \frac{7}{9}\left\{n - \frac{1}{9}\left[1-(0.1)^n\right]\right\}$$
$$\therefore \quad S_n = \frac{7}{81}\left[9n - \left(1 - \frac{1}{10^n}\right)\right]$$

Question 7.
Find the sum to n terms of the sequence
(i) 0.5, 0.05, 0.005, ...

Solution:

Here, $t_1 = 0.5, t_2 = 0.05, t_3 = 0.005$

$\therefore \quad \frac{t_2}{t_1} = \frac{0.05}{0.5} = 0.1$ and $\frac{t_3}{t_2} = \frac{0.005}{0.05} = 0.1$

$\therefore$ The given sequence is a G.P.

$\therefore \quad a = 0.5$ and $r = 0.1$

$$\therefore \quad S_n = \frac{a(1-r^n)}{1-r}, \text{ for } r < 1$$

$$= \frac{0.5\left[1-(0.1)^n\right]}{1-0.1}$$

$$= \frac{0.5}{0.9}\left[1-(0.1)^n\right]$$

$$= \frac{5}{9}\left[1-\left(\frac{1}{10}\right)^n\right]$$

(ii) 0.2, 0.02, 0.002, ……

Solution:

Here, $t_1 = 0.2, t_2 = 0.02, t_3 = 0.002$

$\therefore \quad \frac{t_2}{t_1} = \frac{0.02}{0.2} = 0.1$ and $\frac{t_3}{t_2} = \frac{0.002}{0.02} = 0.1$

$\therefore$ The given sequence is a G.P.

$\therefore \quad a = 0.2$ and $r = 0.1$

$$\therefore \quad S_n = \frac{a(1-r^n)}{1-r}, \text{ for } r < 1$$

$$= \frac{0.2\left[1-(0.1)^n\right]}{1-0.1}$$

$$= \frac{0.2}{0.9}\left[1-(0.1)^n\right]$$

$$= \frac{2}{9}\left[1-\left(\frac{1}{10}\right)^n\right]$$

Question 8.

For a sequence, if $S_n = 2(3n - 1)$, find the nth term, hence showing that the sequence is a G.P.

Solution:

$S_n = 2(3^n - 1)$

$\therefore \quad S_{n-1} = 2(3^{n-1} - 1)$

But $t_n = S_n - S_{n-1}$

$= 2(3^n - 1) - 2(3^{n-1} - 1)$

$= 2(3^n - 1 - 3^{n-1} + 1)$

$= 2(3^n - 3^{n-1}) = 2(3^{n-1+1} - 3^{n-1})$

$\therefore \quad t_n = 2.3^{n-1}(3-1) = 4.3^{n-1}$

$\therefore \quad t_{n+1} = 4.3^{(n+1-1)}$

$= 4(3)^n$

The sequence (t_n) is a G.P., if $\frac{t_{n+1}}{t_n}$ = constant for $n \in N$.

$\therefore \quad \frac{t_{n+1}}{t_n} = \frac{4(3)^n}{4(3)^{n-1}} = 3$ = constant, for $n \in N$

$\therefore \quad r = 3$

$\therefore$ the sequences is a G.P. with

$t_n = 4(3)^{n-1}$

Question 9.

If S, P, R are the sum, product, and sum of the reciprocals of n terms of a G.P, respectively, then verify that $\left[\frac{S}{R}\right]^n = P^2$.

Solution:

Let a be the 1st term and r be the common ratio of the G.P.

$\therefore$ the G.P. is $a, ar, ar^2, ar^3, \ldots, ar^{n-1}$

$\therefore \quad S = a + ar + ar^2 + \ldots + ar^{n-1} = a\left(\frac{r^n - 1}{r - 1}\right)$

$P = a(ar)(ar^2) \ldots (ar^{n-1})$
$= a^n . r^{1+2+3+\ldots+(n-1)}$
$= a^n . r^{\frac{n(n-1)}{2}}$

$\therefore \quad P^2 = a^{2n} . r^{n(n-1)} \qquad \ldots$(i)

$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \ldots + \frac{1}{ar^{n-1}}$

$= \frac{r^{n-1} + r^{n-2} + r^{n-3} + \ldots + r^2 + r + 1}{a . r^{n-1}}$

$= \frac{1 + r + r^2 + \ldots + r^{n-2} + r^{n-1}}{a . r^{n-1}}$

$1, r, r^2, \ldots, r^{n-1}$ are in G.P., with $a = 1, r = r$

$\therefore \quad 1 + r + r^2 + \ldots + r^{n-1} = 1.\left(\frac{r^n - 1}{r - 1}\right)$

$\therefore \quad R = \frac{1}{ar^{n-1}}\left(\frac{r^n - 1}{r - 1}\right) = \frac{1}{a^2 . r^{n-1}} \times a \times \left(\frac{r^n - 1}{r - 1}\right)$

$\therefore \quad R = \frac{1}{a^2 . r^{n-1}} S$

$\therefore \quad a^2 . r^{n-1} = \frac{S}{R}$

$\therefore \quad (a^2 . r^{n-1})^n = \left(\frac{S}{R}\right)^n$

$\therefore \quad a^{2n} . r^{n(n-1)} = \left(\frac{S}{R}\right)^n$

$\therefore \quad P^2 = \left(\frac{S}{R}\right)^n \qquad \ldots$[From (i)]

Question 10.
If S_n, S_{2n}, S_{3n} are the sum of n, 2n, 3n terms of a G.P. respectively, then verify that $S_n(S_{3n} – S_{2n}) = (S_{2n} – S_n)^2$.
Solution:
Let a and r be the 1st term and common ratio of the G.P. respectively.

$\therefore \quad S_n = a\left(\frac{r^n-1}{r-1}\right), S_{2n} = a\left(\frac{r^{2n}-1}{r-1}\right), S_{3n} = a\left(\frac{r^{3n}-1}{r-1}\right)$

$\therefore \quad S_{2n} - S_n = a\left(\frac{r^{2n}-1}{r-1}\right) - a\left(\frac{r^n-1}{r-1}\right)$

$= \frac{a}{r-1}(r^{2n} - 1 - r^n + 1)$

$= \frac{a}{r-1}(r^{2n} - r^n)$

$= \frac{a\,r^n}{r-1}(r^n - 1)$

$\therefore \quad S_{2n} - S_n = r^n.\frac{a(r^n-1)}{r-1} \quad \ldots(i)$

$S_{3n} - S_{2n} = a\left(\frac{r^{3n}-1}{r-1}\right) - a\left(\frac{r^{2n}-1}{r-1}\right)$

$= \frac{a}{r-1}(r^{3n} - 1 - r^{2n} + 1)$

$= \frac{a}{r-1}(r^{3n} - r^{2n})$

$= \frac{a}{r-1}.r^{2n}(r^n - 1)$

$= a.\left(\frac{r^n-1}{r-1}\right).r^{2n}$

$\therefore \quad S_n(S_{3n} - S_{2n}) = \left[a.\left(\frac{r^n-1}{r-1}\right)\right]\left[a.\left(\frac{r^n-1}{r-1}\right)r^{2n}\right]$

$= \left[r^n.\frac{a(r^n-1)}{r-1}\right]^2$

$\therefore \quad S_n(S_{3n} - S_{2n}) = (S_{2n} - S_n)^2 \quad \ldots[\text{From (i)}]$

Question 11.
Find
(i) $\Sigma_{10r=1}(3\times2r)$
Solution:

$\sum_{r=1}^{10} 3\times2^r$
$= 3\times2 + 3\times2^2 + 3\times2^3 + \ldots + 3\times2^{10}$
$= 3[2 + 2^2 + 2^3 + \ldots + 2^{10}]$
Here, $2, 2^2, 2^3, \ldots 2^{10}$ are the terms in G.P. with $a = 2, r = 2$

$\sum_{r=1}^{10} 3\times2^r = 3.\frac{2[2^{10}-1]}{2-1}$
$= 6(1024 - 1)$
$= 6(1023) = 6138$

(ii) $\Sigma_{10r=1}5\times3r$
Solution:

$\sum_{r=1}^{10} 5\times3^r$
$= 5\times3 + 5\times3^2 + 5\times3^3 + \ldots + 5\times3^{10}$
$= 5[3 + 3^2 + 3^3 + \ldots + 3^{10}]$
Here, $3, 3^2, 3^3, \ldots, 3^{10}$ are the terms in G.P. with $a = 3, r = 3$

$\sum_{r=1}^{10} 5\times3^r = 5.\frac{3[3^{10}-1]}{3-1} = \frac{15}{2}(3^{10} - 1)$

Question 12.
The value of a house appreciates 5% per year. How much is the house worth after 6 years if its current worth is Rs. 15 Lac. [Given: $(1.05)^5 = 1.28$, $(1.05)^6 = 1.34$]
Solution:
The value of a house is Rs. 15 Lac.
Appreciation rate = 5% = $\frac{5}{100}$ = 0.05
Value of house after 1st year = 15(1 + 0.05) = 15(1.05)
Value of house after 6 years = $15(1.05)(1.05)^5$
$= 15(1.05)^6$
= 15(1.34)
= 20.1 lac.

Question 13.
If one invests Rs. 10,000 in a bank at a rate of interest 8% per annum, how long does it take to double the money by compound interest? [$(1.08)^5 = 1.47$]
Solution:
Amount invested = Rs. 10000
Interest rate = $\frac{8}{100}$ = 0.08
amount after 1st year = 10000(1 + 0.08) = 10000(1.08)
Value of the amount after n years
$= 10000(1.08) \times (1.08)^{n-1}$
$= 10000(1.08)^n$
= 20000
$\therefore (1.08)^n = 2$
$\therefore (1.08)^5 = 1.47$ .....[Given]
$\therefore$ n = 10 years, (approximately)

Maharashtra State Board 11th Maths Solutions Chapter 2 Sequences and Series Ex 2.3

Question 1.
Determine whether the sum to infinity of the following G.P.s exist, if exists find them.
(i) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$
Solution:
$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

Here, $a = \frac{1}{2}$, $r = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$, $|r| < 1$

$\therefore$ Sum to infinity exists.

$\therefore$ Sum to infinity $= \frac{a}{1-r}$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

(ii) $2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \ldots$
Solution:

$2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \ldots$

Here, $a = 2$, $r = \frac{\frac{4}{3}}{2} = \frac{2}{3}$, $|r| < 1$

$\therefore$ Sum to infinity exists.

$\therefore$ Sum to infinity $= \frac{a}{1-r}$

$= \frac{2}{1-\frac{2}{3}} = 6$

(iii) $-3, 1, \frac{-1}{3}, \frac{1}{9}, \ldots$
Solution:

$-3, 1, \frac{-1}{3}, \frac{1}{9}, \ldots$

Here, $a = -3$, $r = -\frac{1}{3}$, $|r| < 1$

$\therefore$ Sum to infinity exists.

$\therefore$ Sum to infinity $= \frac{a}{1-r}$

$= \frac{-3}{1-\left(-\frac{1}{3}\right)}$

$= -\frac{9}{4}$

(iv) $\frac{1}{5}, \frac{-2}{5}, \frac{4}{5}, \frac{-8}{5}, \frac{16}{5}, \ldots$
Solution:

$\frac{1}{5}, \frac{-2}{5}, \frac{4}{5}, \frac{-8}{5}, \frac{16}{5}, \ldots$

Here, $a = \frac{1}{5}$, $r = \frac{\frac{-2}{5}}{\frac{1}{5}} = -2$, $|r| = |-2| > 1$

$\therefore$ Sum to infinity does not exist.

(v) 9, 8.1, 7.29, ...
Solution:
9, 8.1, 7.29, ...
Here, $a = 9$, $r = \frac{8.1}{9} = 0.9$, $|r| < 1$
∴ Sum to infinity exists.
∴ Sum to infinity $= \frac{a}{1-r}$
$= \frac{9}{1-0.9}$
$= \frac{9}{0.1}$
= 90

Question 2.
Express the following recurring decimals as rational numbers.
(i) $0.\overline{7}$
(ii) $2.\overline{4}$
(iii) $2.\overline{35}$
(iv) $51.0\overline{2}$
Solution:
(i) $0.\overline{7}$ = 0.7777... = 0.7 + 0.07 + 0.007 + ...
The terms 0.7, 0.07, 0.007,... are in G.P.
∴ $a = 0.7$, $r = \frac{0.07}{0.7} = 0.1$, $|r| = |0.1| < 1$
∴ Sum to infinity exists.

∴ Sum to infinity

$$= \frac{a}{1-r} = \frac{0.7}{1-0.1}$$
$$= \frac{0.7}{0.9}$$
$$= \frac{7}{9}$$

(ii) 2.4¯ = 2.444 … = 2 + 0.4 + 0.04 + 0.004 + …

The terms 0.4, 0.04, 0.004,… are in G.P.

∴ a = 0.4, r = 0.070.7 = 0.1, |r| = 10.11 < 1

∴ Sum to infinity exists.

∴ Sum to infinity

$$= 2 + \frac{a}{1-r} = 2 + \frac{0.4}{1-0.1}$$
$$= 2 + \frac{0.4}{0.9}$$
$$= 2 + \frac{4}{9} = \frac{22}{9}$$

(iii) 2.35¯ = 2.3555… = 2.3 + 0.05 + 0.005 + 0.0005 + …

The terms 0.05,0.005,0.0005,… are in G.P.

∴ a = 0.05, r = 0.0050.05 = 0.1, |r| = |0.1| < 1

∴ Sum to infinity exists.

∴ Sum to infinity

$$= 2.3 + \frac{a}{1-r}$$
$$= 2.3 + \frac{0.05}{1-0.1}$$
$$= 2.3 + \frac{0.05}{0.9}$$
$$= \frac{23}{10} + \frac{5}{90}$$
$$= \frac{212}{90} = \frac{106}{45}$$

(iv) 51.02¯ = 51.0222 …. = 51 + 0.02 + 0.002 + 0.0002 + …..

The terms 0.02, 0.002, 0.0002,… are in G.P.

∴ a = 0.02, r = 0.0020.02 = 0.1, |r| = |0.1| < 1

∴ Sum to infinity exists.

∴ Sum to infinity

$$= 51 + \frac{a}{1-r} = 51 + \frac{0.02}{1-0.1}$$
$$= 51 + \frac{0.02}{0.9}$$
$$= 51 + \frac{2}{90}$$
$$= 51 + \frac{1}{45}$$
$$= \frac{2296}{45}$$

Question 3.

If the common ratio of a G.P. is 23 and the sum to infinity is 12, find the first term.

Solution:

r = 23, sum to infinity = 12 ….. [Given]

Sum to infinity = a1−r

12 = a1−23

a = 12 × 13

∴ a = 4

Question 4.
If the first term of the G.P. is 6 and its sum to infinity is $\frac{96}{17}$, find the common ratio.
Solution:
a = 6, sum to infinity = $\frac{96}{17}$ .....[Given]
Sum to infinity = $\frac{a}{1-r}$

$\therefore \quad \frac{96}{17} = \frac{6}{1-r}$

$\therefore \quad \frac{16}{17} = \frac{1}{1-r}$

$\therefore \quad 16 - 16r = 17$

$\therefore \quad 16r = 16 - 17$

$\therefore \quad r = \frac{-1}{16}$

Question 5.
The sum of an infinite G.P. is 5 and the sum of the squares of these terms is 15, find the G.P.
Solution:
Let the required G.P. be a, ar, ar^2, ar^3, ...
Sum to infinity of this G.P. = 5

$\therefore \quad 5 = \frac{a}{1-r}$

$\therefore \quad a = 5(1-r) \quad \ldots(i)$

Also, the sum of the squares of the terms is 15.

$\therefore \quad (a^2 + a^2r^2 + a^2r^4 + \ldots) = 15$

$\therefore \quad 15 = \frac{a^2}{1-r^2}$

$\therefore \quad 15(1-r^2) = a^2$

$\therefore \quad 15(1-r)(1+r) = 25(1-r)^2 \quad \ldots$[From (i)]

$\therefore \quad 3(1+r) = 5(1-r)$

$\therefore \quad 3 + 3r = 5 - 5r$

$\therefore \quad 8r = 2$

$\therefore \quad r = \frac{1}{4}$

$\therefore \quad a = 5\left(1 - \frac{1}{4}\right) = 5\left(\frac{3}{4}\right) = \frac{15}{4}$

$\therefore$ Required G.P. is a, ar, ar^2, ar^3, ...

i.e. $\frac{15}{4}, \frac{15}{16}, \frac{15}{64}, \ldots$

Question 6.
Find
(i) $\sum_{r=1}^{\infty} 4(0.5)^r$
Solution:

$$\sum_{r=1}^{\infty} 4(0.5)^r$$

$$= 4(0.5) + 4(0.5)^2 + 4(0.5)^3 + \ldots$$

$$= 4\left\{\frac{5}{10} + \left(\frac{5}{10}\right)^2 + \left(\frac{5}{10}\right)^3 + \ldots\right\}$$

$$= 4\left\{\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots\right\}$$

The terms $\frac{1}{2}, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3, \ldots$ are in G.P.

$\therefore \quad a = \frac{1}{2}, r = \frac{1}{2}, |r| = \left|\frac{1}{2}\right| < 1$

$\therefore$ Sum to infinity exists.

$\therefore \quad \sum_{r=1}^{\infty} 4(0.5)^r = 4 \times \frac{a}{1-r} = 4 \times \frac{\frac{1}{2}}{1-\frac{1}{2}} = 4$

(ii) $\sum_{r=1}^{\infty}\left(-\frac{1}{3}\right)^r$
Solution:

$$\sum_{r=1}^{\infty}\left(-\frac{1}{3}\right)^r$$

$$= \left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^3 + \dots$$

The terms $\left(-\frac{1}{3}\right), \left(-\frac{1}{3}\right)^2, \left(-\frac{1}{3}\right)^3, \dots$ are in G.P.

$\therefore$ $a = -\frac{1}{3}, r = -\frac{1}{3}, |r| = \left|-\frac{1}{3}\right| < 1$

$\therefore$ Sum to infinity exists.

$\therefore$ $\sum_{r=1}^{\infty}\left(-\frac{1}{3}\right)^r = \frac{a}{1-r} = \frac{-\frac{1}{3}}{1-\left(-\frac{1}{3}\right)} = \frac{-\frac{1}{3}}{\frac{4}{3}} = -\frac{1}{4}$

(iii) $\sum_{r=0}^{\infty}(-8)\left(-\frac{1}{2}\right)^r$
Solution:

$$\sum_{r=1}^{\infty}(-8)\left(-\frac{1}{2}\right)^r$$

$$= (-8)\left[\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^3 + \dots\right]$$

The terms $-\frac{1}{2}, \left(-\frac{1}{2}\right)^2, \left(-\frac{1}{2}\right)^3, \dots$ are in G.P.

$\therefore$ $a = -\frac{1}{2}, r = -\frac{1}{2}, |r| = \left|-\frac{1}{2}\right| < 1$

$\therefore$ Sum to infinity exists.

$\therefore$ $\sum_{r=1}^{\infty}(-8)\left(-\frac{1}{2}\right)^r = (-8)\frac{a}{1-r}$

$$= (-8)\left[\frac{-\frac{1}{2}}{1-\left(-\frac{1}{2}\right)}\right]$$

$$= (-8)\left(\frac{-\frac{1}{2}}{\frac{3}{2}}\right) = \frac{8}{3}$$

(iv) $\sum_{n=1}^{\infty}0.4^n$
Solution:

$$\sum_{n=1}^{\infty}(0.4)^n$$

$= 0.4 + (0.4)^2 + (0.4)^3 + \dots$

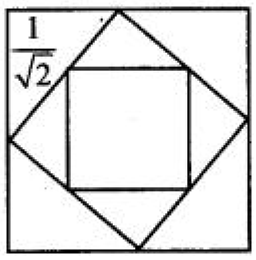
The terms $0.4, (0.4)^2, (0.4)^3, \dots$ are in G.P.

$\therefore$ $a = 0.4, r = 0.4, |r| = |0.4| < 1$

$\therefore$ Sum to infinity exists.

$\therefore$ $\sum_{n=1}^{\infty}(0.4)^n = \frac{a}{1-r} = \frac{0.4}{1-0.4} = \frac{0.4}{0.6} = \frac{2}{3}$

Question 7.
The midpoints of the sides of a square of side 1 are joined to form a new square. This procedure is repeated indefinitely. Find the sum of
(i) the areas of all the squares.
(ii) the perimeters of all the squares.
Solution:



(i) Area of the 1st square = 12

Area of the 2nd square = (12√)2

Area of the 3rd square = (12)2

and so on.

∴ Sum of the areas of all the squares

$$= 1^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \dots$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \dots$$

The terms $1, \frac{1}{2}, \frac{1}{4}, \dots$ are in G.P.

$a = 1, r = \frac{1}{2}, |r| = \left|\frac{1}{2}\right| < 1$

∴ Sum to infinity exists.

∴ Sum of the areas of all the squares = 11−12 = 2

(ii) Perimeter of 1st square = 4

Perimeter of 2nd square = 4(12√)

Perimeter of 3rd square = 4(12)

and so on.

Sum of the perimeters of all the squares

$$= 4 + 4\left(\frac{1}{\sqrt{2}}\right) + 4\left(\frac{1}{2}\right) + \dots$$

$$= 4\left(1 + \frac{1}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}}\right)^2 + \dots\right)$$

The terms $1, \frac{1}{\sqrt{2}}, \left(\frac{1}{\sqrt{2}}\right)^2, \dots$ are in G.P.

∴ $a = 1, r = \frac{1}{\sqrt{2}}, |r| = \left|\frac{1}{\sqrt{2}}\right| < 1$

∴ Sum to infinity exists.

∴ Sum of the perimeters of all the squares

$$= 4.\left(\frac{1}{1 - \frac{1}{\sqrt{2}}}\right)$$

$$= \frac{4\sqrt{2}}{\sqrt{2} - 1}$$

Question 8.

A ball is dropped from a height of 10 m. It bounces to a height of 6m, then 3.6 m, and so on. Find the total distance travelled by the ball.

Solution:

Here, on the first bounce, the ball will go 6 m and it will return 6 m.

On the second bounce, the ball will go 3.6 m and it will return 3.6 m, and so on.

Given that, a ball is dropped from a height of 10 m.

∴ Total distance travelled by the ball is = 10 + 2[6 + 3.6 + …]

The terms 6, 3.6 … are in G.P.

$a = 6, r = 0.6, |r| = |0.6| < 1$

∴ Sum to infinity exists.

∴ Total distance travelled by the ball

$$= 10 + 2\left[\frac{6}{1-\frac{6}{10}}\right] = 10 + 2\left[\frac{6}{\frac{10-6}{10}}\right]$$

$$= 10 + 2\left[\frac{6\times10}{4}\right]$$

$$= 10 + 30$$

$$= 40 \text{ m}$$

Maharashtra State Board 11th Maths Solutions Chapter 2 Sequences and Series Ex 2.3

Question 1.
Determine whether the sum to infinity of the following G.P.s exist, if exists find them.

(i) 12,14,18,116,…

Solution:

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$$

Here, $a = \frac{1}{2}$, $r = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$, $|r| < 1$

∴ Sum to infinity exists.

∴ Sum to infinity $= \frac{a}{1-r}$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

(ii) 2,43,89,1627,…

Solution:

$$2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \ldots$$

Here, $a = 2$, $r = \frac{\frac{4}{3}}{2} = \frac{2}{3}$, $|r| < 1$

∴ Sum to infinity exists.

∴ Sum to infinity $= \frac{a}{1-r}$

$$= \frac{2}{1-\frac{2}{3}} = 6$$

(iii) −3,1,−13,19,...

Solution:

$$-3, 1, \frac{-1}{3}, \frac{1}{9}, \ldots$$

Here, a = −3, r = $-\frac{1}{3}$, $|r| < 1$

∴ Sum to infinity exists.

∴ Sum to infinity = $\frac{a}{1-r}$

$$= \frac{-3}{1-\left(-\frac{1}{3}\right)}$$

$$= -\frac{9}{4}$$

(iv) 15,−25,45,−85,165,...

Solution:

$$\frac{1}{5}, \frac{-2}{5}, \frac{4}{5}, \frac{-8}{5}, \frac{16}{5}, \ldots$$

Here, a = $\frac{1}{5}$, r = $\frac{\frac{-2}{5}}{\frac{1}{5}} = -2$, $|r| = |-2| > 1$

∴ Sum to infinity does not exist.

(v) 9, 8.1, 7.29, ……

Solution:

9, 8.1, 7.29, …..

Here, a = 9, r = 8.19 = 0.9, |r| < 1

∴ Sum to infinity exists.

∴ Sum to infinity = a1−r

= 91−0.9

= 90.1

= 90

Question 2.

Express the following recurring decimals as rational numbers.

(i) $0.\overline{7}$

(ii) $2.\overline{4}$

(iii) $2.3\overline{5}$

(iv) $51.0\overline{2}$

Solution:

(i) $0.\overline{7}$ = 0.7777… = 0.7 + 0.07 + 0.007 + ….

The terms 0.7, 0.07, 0.007,… are in G.P.

∴ a = 0.7, r = 0.070.7 = 0.1, |r| = |0.1| < 1

∴ Sum to infinity exists.

∴ Sum to infinity

$$= \frac{a}{1-r} = \frac{0.7}{1-0.1}$$

$$= \frac{0.7}{0.9}$$

$$= \frac{7}{9}$$

(ii) $2.\overline{4}$ = 2.444 … = 2 + 0.4 + 0.04 + 0.004 + …

The terms 0.4, 0.04, 0.004,… are in G.P.

∴ a = 0.4, r = 0.070.7 = 0.1, |r| = 10.11 < 1

∴ Sum to infinity exists.

∴ Sum to infinity

$$= 2 + \frac{a}{1-r} = 2 + \frac{0.4}{1-0.1}$$

$$= 2 + \frac{0.4}{0.9}$$

$$= 2 + \frac{4}{9} = \frac{22}{9}$$

(iii) $2.3\overline{5}$ = 2.3555... = 2.3 + 0.05 + 0.005 + 0.0005 + ...

The terms 0.05,0.005,0.0005,... are in G.P.

∴ a = 0.05, r = $\frac{0.005}{0.05}$ = 0.1, |r| = |0.1| < 1

∴ Sum to infinity exists.

∴ Sum to infinity

$$= 2.3 + \frac{a}{1-r}$$

$$= 2.3 + \frac{0.05}{1-0.1}$$

$$= 2.3 + \frac{0.05}{0.9}$$

$$= \frac{23}{10} + \frac{5}{90}$$

$$= \frac{212}{90} = \frac{106}{45}$$

(iv) $51.0\overline{2}$ = 51.0222 .... = 51 + 0.02 + 0.002 + 0.0002 + ...

The terms 0.02, 0.002, 0.0002,... are in G.P.

∴ a = 0.02, r = $\frac{0.002}{0.02}$ = 0.1, |r| = |0.1| < 1

∴ Sum to infinity exists.

∴ Sum to infinity

$$= 51 + \frac{a}{1-r} = 51 + \frac{0.02}{1-0.1}$$

$$= 51 + \frac{0.02}{0.9}$$

$$= 51 + \frac{2}{90}$$

$$= 51 + \frac{1}{45}$$

$$= \frac{2296}{45}$$

Question 3.
If the common ratio of a G.P. is $\frac{2}{3}$ and the sum to infinity is 12, find the first term.
Solution:
r = $\frac{2}{3}$, sum to infinity = 12 ..... [Given]
Sum to infinity = $\frac{a}{1-r}$
12 = $\frac{a}{1-\frac{2}{3}}$
a = 12 × $\frac{1}{3}$
∴ a = 4

Question 4.
If the first term of the G.P. is 6 and its sum to infinity is $\frac{96}{17}$, find the common ratio.
Solution:
a = 6, sum to infinity = $\frac{96}{17}$ .....[Given]

Sum to infinity = $\frac{a}{1-r}$

$\therefore \quad \frac{96}{17} = \frac{6}{1-r}$

$\therefore \quad \frac{16}{17} = \frac{1}{1-r}$

$\therefore \quad 16 - 16r = 17$

$\therefore \quad 16r = 16 - 17$

$\therefore \quad r = \frac{-1}{16}$

Question 5.
The sum of an infinite G.P. is 5 and the sum of the squares of these terms is 15, find the G.P.
Solution:
Let the required G.P. be a, ar, ar^2, ar^3, …..
Sum to infinity of this G.P. = 5

$\therefore \quad 5 = \frac{a}{1-r}$

$\therefore \quad a = 5(1-r) \qquad \ldots(i)$

Also, the sum of the squares of the terms is 15.

$\therefore \quad (a^2 + a^2r^2 + a^2r^4 + \ldots) = 15$

$\therefore \quad 15 = \frac{a^2}{1-r^2}$

$\therefore \quad 15(1-r^2) = a^2$

$\therefore \quad 15(1-r)(1+r) = 25(1-r)^2 \qquad \ldots$[From (i)]

$\therefore \quad 3(1+r) = 5(1-r)$

$\therefore \quad 3 + 3r = 5 - 5r$

$\therefore \quad 8r = 2$

$\therefore \quad r = \frac{1}{4}$

$\therefore \quad a = 5\left(1 - \frac{1}{4}\right) = 5\left(\frac{3}{4}\right) = \frac{15}{4}$

$\therefore$ Required G.P. is a, ar, ar^2, ar^3, …

i.e. $\frac{15}{4}, \frac{15}{16}, \frac{15}{64}, \ldots$

Question 6.
Find
(i) $\sum_{r=1}^{\infty} 4(0.5)^r$
Solution:

$\sum_{r=1}^{\infty} 4(0.5)^r$

$= 4(0.5) + 4(0.5)^2 + 4(0.5)^3 + \ldots$

$= 4\left\{\frac{5}{10} + \left(\frac{5}{10}\right)^2 + \left(\frac{5}{10}\right)^3 + \ldots\right\}$

$= 4\left\{\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots\right\}$

The terms $\frac{1}{2}, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3, \ldots$ are in G.P.

$\therefore \quad a = \frac{1}{2}, r = \frac{1}{2}, |r| = \left|\frac{1}{2}\right| < 1$

$\therefore$ Sum to infinity exists.

$\therefore \quad \sum_{r=1}^{\infty} 4(0.5)^r = 4 \times \frac{a}{1-r} = 4 \times \frac{\frac{1}{2}}{1-\frac{1}{2}} = 4$

(ii) $\sum_{r=1}^{\infty} \left(\frac{-1}{3}\right)^r$
Solution:

$$\sum_{r=1}^{\infty}\left(-\frac{1}{3}\right)^{r}$$
$$=\left(-\frac{1}{3}\right)+\left(-\frac{1}{3}\right)^{2}+\left(-\frac{1}{3}\right)^{3}+\ldots$$

The terms $\left(-\frac{1}{3}\right),\left(-\frac{1}{3}\right)^{2},\left(-\frac{1}{3}\right)^{3},\ldots$ are in G.P.

$\therefore \quad a=-\frac{1}{3}, r=-\frac{1}{3}, |r|=\left|-\frac{1}{3}\right|<1$

$\therefore$ Sum to infinity exists.

$$\therefore \quad \sum_{r=1}^{\infty}\left(-\frac{1}{3}\right)^{r}=\frac{a}{1-r}=\frac{-\frac{1}{3}}{1-\left(-\frac{1}{3}\right)}=\frac{-\frac{1}{3}}{\frac{4}{3}}=-\frac{1}{4}$$

(iii) $\sum_{r=0}^{\infty}(-8)\left(-\frac{1}{2}\right)^{r}$

Solution:

$$\sum_{r=1}^{\infty}(-8)\left(-\frac{1}{2}\right)^{r}$$
$$=(-8)\left[\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)^{2}+\left(-\frac{1}{2}\right)^{3}+\ldots\right]$$

The terms $-\frac{1}{2},\left(-\frac{1}{2}\right)^{2},\left(-\frac{1}{2}\right)^{3},\ldots$ are in G.P.

$\therefore \quad a=-\frac{1}{2}, r=-\frac{1}{2}, |r|=\left|-\frac{1}{2}\right|<1$

$\therefore$ Sum to infinity exists.

$$\therefore \quad \sum_{r=1}^{\infty}(-8)\left(-\frac{1}{2}\right)^{r}=(-8)\frac{a}{1-r}$$
$$=(-8)\left[\frac{-\frac{1}{2}}{1-\left(-\frac{1}{2}\right)}\right]$$
$$=(-8)\left(\frac{-\frac{1}{2}}{\frac{3}{2}}\right)=\frac{8}{3}$$

(iv) $\sum_{n=1}^{\infty} 0.4^{n}$

Solution:

$$\sum_{n=1}^{\infty}(0.4)^{n}$$
$$=0.4+(0.4)^{2}+(0.4)^{3}+\ldots$$

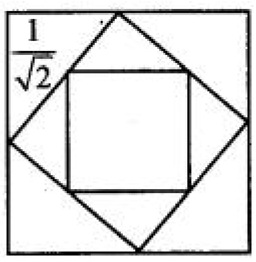
The terms $0.4,(0.4)^{2},(0.4)^{3},\ldots$ are in G.P.

$\therefore \quad a=0.4, r=0.4, |r|=|0.4|<1$

$\therefore$ Sum to infinity exists.

$$\therefore \quad \sum_{n=1}^{\infty}(0.4)^{n}=\frac{a}{1-r}=\frac{0.4}{1-0.4}=\frac{0.4}{0.6}=\frac{2}{3}$$

Question 7.
The midpoints of the sides of a square of side 1 are joined to form a new square. This procedure is repeated indefinitely. Find the sum of
(i) the areas of all the squares.
(ii) the perimeters of all the squares.
Solution:



(i) Area of the 1st square = 12

Area of the 2nd square = (12√)2

Area of the 3rd square = (12)2

and so on.

∴ Sum of the areas of all the squares

$$= 1^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \ldots$$

$$= 1 + \frac{1}{2} + \frac{1}{4} +$$

The terms $1, \frac{1}{2}, \frac{1}{4}, \ldots$ are in G.P.

$$a = 1, r = \frac{1}{2}, |r| = \left|\frac{1}{2}\right| < 1$$

∴ Sum to infinity exists.

∴ Sum of the areas of all the squares = 11−12 = 2

(ii) Perimeter of 1st square = 4

Perimeter of 2nd square = 4(12√)

Perimeter of 3rd square = 4(12)

and so on.

Sum of the perimeters of all the squares

$$= 4 + 4\left(\frac{1}{\sqrt{2}}\right) + 4\left(\frac{1}{2}\right) + \ldots$$

$$= 4\left(1 + \frac{1}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}}\right)^2 + \ldots\right)$$

The terms $1, \frac{1}{\sqrt{2}}, \left(\frac{1}{\sqrt{2}}\right)^2, \ldots$ are in G.P.

∴ $a = 1, r = \frac{1}{\sqrt{2}}, |r| = \left|\frac{1}{\sqrt{2}}\right| < 1$

∴ Sum to infinity exists.

∴ Sum of the perimeters of all the squares

$$= 4.\left(\frac{1}{1 - \frac{1}{\sqrt{2}}}\right)$$

$$= \frac{4\sqrt{2}}{\sqrt{2} - 1}$$

Question 8.

A ball is dropped from a height of 10 m. It bounces to a height of 6m, then 3.6 m, and so on. Find the total distance travelled by the ball.

Solution:

Here, on the first bounce, the ball will go 6 m and it will return 6 m.

On the second bounce, the ball will go 3.6 m and it will return 3.6 m, and so on.

Given that, a ball is dropped from a height of 10 m.

∴ Total distance travelled by the ball is = 10 + 2[6 + 3.6 + …]

The terms 6, 3.6 … are in G.P.

a = 6, r = 0.6, |r| = |0.6| < 1

∴ Sum to infinity exists.

∴ Total distance travelled by the ball

$$= 10 + 2\left[\frac{6}{1 - \frac{6}{10}}\right] = 10 + 2\left[\frac{6}{\frac{10-6}{10}}\right]$$

$$= 10 + 2\left[\frac{6 \times 10}{4}\right]$$

$$= 10 + 30$$

$$= 40 \text{ m}$$

Maharashtra State Board 11th Maths Solutions Chapter 2 Sequences and Series Ex 2.4

Question 1.
Verify whether the following sequences are H.P.

(i) 13,15,17,19,…

Solution:

13,15,17,19,…

Here, the reciprocal sequence is 3, 5, 7, 9,…
$t_1 = 3, t_2 = 5, t_3 = 7, t_4 = 9$, …..
$t_2 – t_1 = t_3 – t_2 = t_4 – t_3 = 2$ = constant
∴ The reciprocal sequence is an A.P.
∴ The given sequence is a H.P.

(ii) 13,16,112,124,…

Solution:

13,16,112,124,…

Here, the reciprocal sequence is 3, 6, 12, 24,…
$t_1 = 3, t_2 = 6, t_3 = 12$, ……
$t_2 – t_1 = 3, t_3 – t_2 = 6$
$t_2 – t_1 \neq t_3 – t_2$
∴ The reciprocal sequence is not an A.P.
∴ The given sequence is not a H.P.

(iii) 5,1017,1032,1047,…

Solution:

$5, \frac{10}{17}, \frac{10}{32}, \frac{10}{47}, \ldots$

Here, the reciprocal sequence is

$\frac{1}{5}, \frac{17}{10}, \frac{32}{10}, \frac{47}{10}, \ldots$

$t_1 = \frac{1}{5}, t_2 = \frac{17}{10}, t_3 = \frac{32}{10}, t_4 = \frac{47}{10}, \ldots$

$t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \frac{3}{2}$ = constant

∴ The reciprocal sequence is an A.P.
∴ The given sequence is a H.P.

Question 2.
Find the nth term and hence find the 8th term of the following HPs.

(i) 12,15,18,111,…

Solution:

12,15,18,111,… are in H.P.

∴ 2, 5, 8, 11,… are in A.P.
∴ a = 2, d = 3
$t_n = a + (n – 1)d$
= 2 + (n – 1)(3)
= 3n – 1
∴ nth term of H.P. = 13n−1
∴ 8th term of H.P. = 13(8)−1 = 123

(ii) 14,16,18,110,…

Solution:

14,16,18,110,… are in H.P.

∴ 4, 6, 8, 10, … are in A.P.
∴ a = 4, d = 2
$t_n = a + (n – 1)d$
= 4 + (n – 1) (2)
= 2n + 2

∴ nth term of H.P. = 12n+2
∴ 8th term of H.P. = 12(8)+2 = 118

(iii) 15,110,115,120,…
Solution:
15,110,115,120,… are in H.P.
∴ 5, 10, 15, 20, … are in A.P.
∴ a = 5, d = 5
t_n = a + (n – 1)d
= 5 + (n – 1) (5)
= 5n
∴ nth term of H.P. = 15n
∴ 8th term of H.P. = 15(8) = 140

Question 3.
Find A.M. of two positive numbers whose G.M. and H.M. are 4 and 165 respectively.
Solution:
G.M. = 4, H.M. = 165
Now, $(G.M.)^2$ = (A.M.) (H.M.)
∴ 4^2 = A.M. × 165
∴ A.M. = 16 × 516
∴ A.M. = 5

Question 4.
Find H.M. of two positive numbers whose A.M. and G.M. are 152 and 6.
Solution:
A.M. = 152, G.M. = 6
Now, $(G.M.)^2$ = (A.M.) (H.M.)
∴ 6^2 = 152 × H.M.
∴ H.M. = 36 × 215
∴ H.M. = 245

Question 5.
Find G.M. of two positive numbers whose A.M. and H.M. are 75 and 48.
Solution:
A.M. = 75, H.M. = 48
Now, $(G.M.)^2$ = (A.M.) (H.M.)
∴ $(G.M.)^2$ = 75 × 48
∴ $(G.M.)^2$ = 25 × 3 × 16 × 3
∴ $(G.M.)^2$ = $5^2 \times 4^2 \times 3^2$
∴ G.M. = 5 × 4 × 3
∴ G.M. = 60

Question 6.
Insert two numbers between 14 and 13 so that the resulting sequence is a H.P.
Solution:
Let the required numbers be $1H_1$ and $1H_2$.
∴ 14,$1H_1$,$1H_2$,13 are in H.P.
∴ 4, H_1, H_2, 3 are in A.P.
$t_1 = 4$, $t_2 = H_1$, $t_3 = H_2$, $t_4 = 3$
∴ $t_1 = a = 4$, $t_4 = 3$
t_n = a + (n – 1)d
t_4 = 4 + (4 – 1)d
3 = 4 + 3d
3d = -1
∴ d = –13
$H_1 = t_2$ = a + d = 4 – 13 = 113
$H_2 = t_3$ = a + 2d = 4 – 23 = 103
∴ For resulting sequence to be H.P. we need to insert numbers 311 and 310.

Question 7.
Insert two numbers between 1 and -27 so that the resulting sequence is a G.P.
Solution:

Let the required numbers be G_1 and G_2.
$\therefore$ 1, G_1, G_2, -27 are in G.P.
$t_1 = 1$, $t_2 = G_1$, $t_3 = G_2$, $t_4 = -27$
$\therefore t_1 = a = 1$
$t_n = ar^{n-1}$
$t_4 = (1)\, r^{4-1}$
$-27 = r^3$
$r^3 = (-3)^3$
$\therefore r = -3$
$\therefore G_1 = t_2 = ar = 1(-3) = -3$
$G_2 = t_3 = ar^2 = 1(-3)^2 = 9$
$\therefore$ For resulting sequence to be G.P. we need to insert numbers -3 and 9.

Question 8.
If the A.M. of two numbers exceeds their G.M. by 2 and their H.M. by $\frac{18}{5}$, find the numbers.
Solution:
Let a and b be the two numbers.
A = $\frac{a+b}{2}$, G = $\sqrt{ab}$, H = $\frac{2ab}{a+b}$
According to the given conditions,

$$A = G + 2,\ A = H + \frac{18}{5}$$
$$\therefore \quad G = A - 2, \quad H = A - \frac{18}{5}$$
$$\text{Now, } G^2 = AH$$
$$\therefore \quad (A-2)^2 = A\left(A - \frac{18}{5}\right)$$
$$\therefore \quad A^2 - 4A + 4 = A^2 - \frac{18}{5}A$$
$$\therefore \quad \frac{18}{5}A - 4A = -4$$
$$\therefore \quad \frac{-2A}{5} = -4$$
$$\therefore \quad A = 10$$
$$\therefore \quad \frac{a+b}{2} = 10$$
$$\therefore \quad a + b = 20$$
$$\therefore \quad b = 20 - a \qquad \ldots(i)$$

Consider, G = A – 2 = 10 – 2 = 8
$\sqrt{ab} = 8$
ab = 64
a(20 – a) = 64 .....[From (i)]
$a^2 - 20a + 64 = 0$
(a – 4)(a – 16) = 0
$\therefore$ a = 4 or a = 16
When a = 4, b = 20 – 4 = 16
When a = 16, b = 20 – 16 = 4
$\therefore$ The two numbers are 4 and 16.

Question 9.
Find two numbers whose A.M. exceeds their G.M. by $\frac{1}{2}$ and their H.M. by $\frac{25}{26}$.
Solution:
Let a and b be the two numbers.
A = $\frac{a+b}{2}$, G = $\sqrt{ab}$, H = $\frac{2ab}{a+b}$
According to the given conditions,

$$A = G + \frac{1}{2}, A = H + \frac{25}{26}$$

$\therefore \quad G = A - \frac{1}{2}, H = A - \frac{25}{26}$

Now, $G^2 = AH$

$\therefore \quad \left(A - \frac{1}{2}\right)^2 = A\left(A - \frac{25}{26}\right)$

$\therefore \quad A^2 - A + \frac{1}{4} = A^2 - \frac{25}{26}A$

$\therefore \quad A - \frac{25}{26}A = \frac{1}{4}$

$\therefore \quad \frac{1}{26}A = \frac{1}{4}$

$\therefore \quad A = \frac{13}{2}$

$\therefore \quad \frac{a+b}{2} = \frac{13}{2}$

$\therefore \quad a + b = 13$

$\therefore \quad b = 13 - a \qquad \ldots(i)$

Consider, $G = A - \frac{1}{2} = \frac{13}{2} - \frac{1}{2} = 6$

$\sqrt{ab} = 6$
ab = 36
a(13 – a) = 36 ……[From (i)]
$a^2 - 13a + 36 = 0$
(a – 4)(a – 9) = 0
∴ a = 4 or a = 9
When a = 4, b = 13 – 4 = 9
When a = 9, b = 13 – 9 = 4
∴ The two numbers are 4 and 9.

Maharashtra State Board 11th Maths Solutions Chapter 2 Sequences and Series Ex 2.5

Question 1.
Find Sn of the following arithmetico-geometric sequences.
(i) $2, 4x, 6x^2, 8x^3, 10x^4, \ldots\ldots$
Solution:
$2, 4x, 6x^2, 8x^3, 10x^4, \ldots\ldots$
Here, 2, 4, 6, 8, 10,… are in A.P.
∴ a = 2, d = 2
∴ nth term = a + (n – 1)d
= 2 + (n – 1)(2)

= 2n

Also, 1, x, x^2, x^3, … are in G.P.

$\therefore$ a = 1, r = x

$\therefore$ n^{th} term = $a.r^{n-1}$

$= x^{n-1}$

$\therefore$ n^{th} term of arithmetico-geometric sequence is

$t_n = 2n.x^{n-1}$

$\therefore$ $S_n = 2 + 4x + 6x^2 + 8x^3 + 10x^4 + \ldots + 2n.x^{n-1}$ …(i)

Multiplying throughout by x, we get

$x S_n = 2x + 4x^2 + 6x^3 + 8x^4 + 10x^5 + \ldots + 2n.x^n$ …(ii)

Equation (i) – equation (ii), we get

$S_n - xS_n$

$= 2 + 2x + 2x^2 + 2x^3 + \ldots + 2x^{n-1} - 2nx^n$

$\therefore$ $(1-x)S_n = 2 + 2x\left(\frac{1-x^{n-1}}{1-x}\right) - 2nx^n$

$= 2 - 2nx^n + \frac{2x(1-x^{n-1})}{1-x}$

$\therefore$ $(1-x)S_n = 2(1-nx^n) + \frac{2x(1-x^{n-1})}{1-x}$

$\therefore$ $S_n = \frac{2(1-nx^n)}{1-x} + \frac{2x(1-x^{n-1})}{(1-x)^2}$

(ii) 1, 4x, $7x^2$, $10x^3$, $13x^4$, ……
Solution:
1, 4x, $7x^2$, $10x^3$, $13x^4$, ……
Here, 1, 4, 7, 10, 13,… are in A.P.
a = 1, d = 3
∴ nth term = a + (n – 1)d
= 1 + (n – 1)(3)
= 3n – 2
Also, 1, x, x^2, x^3,… are in G.P.
∴ a = 1, r = x,
nth term = $ar^{n-1} = x^{n-1}$
nth term of arithmetico-geometric sequence is

$t_n = (3n-2)x^{n-1}$

$\therefore$ $S_n = 1 + 4x + 7x^2 + 10x^3 + \ldots + (3n-2)x^{n-1}$ …(i)

Multiplying throughout by x, we get

$xS_n = x + 4x^2 + 7x^3 + 10x^4 + \ldots + (3n-2)x^n$ …(ii)

Equation (i) – equation (ii), we get

$S_n - xS_n$

$= 1 + 3x + 3x^2 + 3x^3 + \ldots + 3x^{n-1} - (3n-2).x^n$

$\therefore$ $(1-x)S_n = 1 + 3x\left(\frac{1-x^{n-1}}{1-x}\right) - (3n-2).x^n$

$\therefore$ $(1-x)S_n = 1 - (3n-2).x^n + \frac{3x(1-x^{n-1})}{1-x}$

$\therefore$ $S_n = \frac{1-(3n-2)x^n}{1-x} + \frac{3x(1-x^{n-1})}{(1-x)^2}$

(iii) 1, 2 × 3, 3 × 9, 4 × 27, 5 × 81, ……
Solution:
1, 2 × 3, 3 × 9, 4 × 27, 5 × 81, …..
Here, 1, 2, 3, 4, 5, … are in A.P.
∴ a = 1, d = 1
∴ nth term = a + (n – 1)d
= 1 + (n – 1)1

= n

Also, 1, 3, 3^2, 3^3, 3^4 ,… are in G.P.

∴ a = 1, r = 3

∴ n^{th} term = ar^{n-1}
$= 1.(3^{n-1})$
$= 3^{n-1}$

∴ n^{th} term of arithmetico-geometric sequence is
$t_n = n.3^{n-1}$

∴ $S_n = 1 + 2 \times 3 + 3 \times 3^2 + 4 \times 3^3 + 5 \times 3^4 + \ldots + n.3^{n-1}$ …(i)

Multiplying throughout by 3, we get
$3S_n = 1 \times 3 + 2 \times 3^2 + 3 \times 3^3 + 4 \times 3^4 + 5 \times 3^5 + \ldots + n.3^n$ …(ii)

Equation (i) – equation (ii), we get
$S_n - 3S_n = [1 - n.(3^n)] + [3 + 3^2 + 3^3 + \ldots + 3^{n-1}]$

∴ $-2S_n = [1 - n.(3^n)] + \frac{3\left(3^{n-1}-1\right)}{3-1}$

∴ $-2S_n = [1 - n.(3^n)] + \frac{3}{2}\,(3^{n-1} - 1)$

∴ $S_n = \frac{n\left(3^n\right)-1}{2} - \frac{3}{4}(3^{n-1} - 1)$

∴ $S_n = \frac{n\left(3^n\right)-1}{2} + \frac{3-3^n}{4}$

(iv) 3, 12, 36, 96, 240, ……
Solution:
3, 12, 36, 96, 240, ……
i.e., 1 × 3, 2 × 6, 3 × 12, 4 × 24, 5 × 48, …….
Here, 1, 2, 3, 4, 5, ….. are in A.P.
∴ nth term = n
Also, 3, 6, 12, 24, 48, ….. are in G.P.
∴ a = 3, r = 2
∴ nth term = ar^{n-1} = 3 . (2^{n-1})
∴ nth term of arithmetico-geometric sequence is
$t_n = n.3(2^{n-1})$

∴ $S_n = 1 \times 3 + 2 \times 6 + 3 \times 12 + 4 \times 24 + 5 \times 48 + \ldots + n.3(2^{n-1})$ …(i)

Multiplying throughout by 2, we get
$2S_n = 1 \times 6 + 2 \times 12 + 3 \times 24 + 4 \times 48 + 5 \times 96 + \ldots + n.3(2^n)$ …(ii)

Equation (i) – equation (ii), we get
$S_n - 2S_n = [3 - 3n(2^n)] + [6 + 12 + 24 + \ldots + 3(2^{n-1})]$

∴ $-S_n = [3 - 3n(2^n)] + \frac{6\left(2^{n-1}-1\right)}{2-1}$
$= 3 - 3n(2^n) + 6(2^{n-1} - 1)$

∴ $S_n = 3n(2^n) - 3 - 6(2^{n-1}) + 6$
$= 3n(2^n) - 3(2^n) + 3$

∴ $S_n = (n - 1)3(2^n) + 3 = 3[(n - 1)2^n + 1]$

Question 2.
Find the sum to infinity of the following arithmetico-geometric sequence.
(i) 1,24,316,464,…

Solution:

$$S = 1 + \frac{2}{4} + \frac{3}{4^2} + \frac{4}{4^3} + \dots \quad \dots\text{(i)}$$

Multiplying (i) by $\frac{1}{4}$, we get

$$\frac{1}{4}S = \frac{1}{4} + \frac{2}{4^2} + \frac{3}{4^3} + \frac{4}{4^4} + \dots \quad \dots\text{(ii)}$$

Equation (i) – equation (ii), we get

$$\frac{3}{4}S = 1 + \left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots\right)$$

The terms $\frac{1}{4}, \frac{1}{4^2}, \frac{1}{4^3}, \dots$ are in G.P.

$\therefore \quad a = \frac{1}{4}, r = \frac{1}{4}, |r| = \left|\frac{1}{4}\right| < 1$

$\therefore$ Sum to infinity exists.

$$\therefore \quad \frac{3}{4}S = 1 + \frac{\frac{1}{4}}{1 - \frac{1}{4}}$$

$$= 1 + \frac{1}{3}$$

$$\therefore \quad \frac{3}{4}S = \frac{4}{3}$$

$$\therefore \quad S = \frac{16}{9}$$

(ii) 3,65,925,12125,15625,…

Solution:

$$S = 3 + \frac{6}{5} + \frac{9}{5^2} + \frac{12}{5^3} + \frac{15}{5^4} + \dots \quad \dots\text{(i)}$$

Multiplying (i) by $\frac{1}{5}$, we get

$$\frac{1}{5}S = \frac{3}{5} + \frac{6}{5^2} + \frac{9}{5^3} + \frac{12}{5^4} + \frac{15}{5^5} + \dots \quad \dots\text{(ii)}$$

Equation (i) – equation (ii), we get

$$\frac{4}{5}S = 3 + \left(\frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + \dots\right)$$

The terms $\frac{3}{5}, \frac{3}{5^2}, \frac{3}{5^3}, \dots$ are in G.P.

$\therefore \quad a = \frac{3}{5}, r = \frac{1}{5}, |r| = \left|\frac{1}{5}\right| < 1$

$\therefore$ Sum to infinity exists.

$$\therefore \quad \frac{4}{5}S = 3 + \frac{\frac{3}{5}}{1 - \frac{1}{5}}$$

$$= 3 + \frac{3}{4}$$

∴ 45S=154

∴ S = 7516

(iii) 1,-43,79,-1027…

Solution:

$$S = 1 + \left(\frac{-4}{3}\right) + \frac{7}{(-3)^2} + \frac{10}{(-3)^3} + \ldots \quad \ldots(i)$$

Multiplying (i) by $-\frac{1}{3}$, we get

$$\frac{-1}{3} S = \frac{1}{(-3)} + \frac{4}{(-3)^2} + \frac{7}{(-3)^3} + \frac{10}{(-3)^4} + \ldots \quad \ldots(ii)$$

Equation (i) – equation (ii), we get

$$\frac{4}{3} S = 1 + \frac{3}{(-3)} + \frac{3}{(-3)^2} + \frac{3}{(-3)^3} + \ldots$$

The terms $\frac{3}{(-3)}, \frac{3}{(-3)^2}, \frac{3}{(-3)^3}, \ldots$ are in G.P.

$\therefore$ $a = \frac{-3}{3}$, $r = \frac{-1}{3}$, $|r| = \left|\frac{-1}{3}\right| < 1$

$\therefore$ Sum to infinity exists.

$$\therefore \quad \frac{4}{3} S = 1 + \frac{\left(\frac{3}{-3}\right)}{1 - \left(-\frac{1}{3}\right)}$$

$$= 1 + \frac{-1}{\frac{4}{3}} = 1 - \frac{3}{4}$$

$$\therefore \quad \frac{4}{3} S = \frac{1}{4}$$

$$\therefore \quad S = \frac{3}{16}$$

Maharashtra State Board 11th Maths Solutions Chapter 2 Sequences and Series Ex 2.6

Question 1.

Find the sum $\sum_{r=1}^{n}(r+1)(2r-1)$.

Solution:

$$\sum_{r=1}^{n}(r+1)(2r-1)$$

$$= \sum_{r=1}^{n}(2r^2 + r - 1)$$

$$= 2\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r - \sum_{r=1}^{n} 1$$

$$= 2 . \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} - n$$

$$= \frac{n}{6}[2(2n^2 + 3n + 1) + 3(n + 1) - 6]$$

$$= \frac{n}{6}(4n^2 + 9n - 1)$$

Question 2.

Find $\sum_{r=1}^{n}(3r^2 - 2r + 1)$

Solution:

$$\sum_{r=1}^{n}\left(3r^2-2r+1\right)$$

$$=3\sum_{r=1}^{n}r^2-2\sum_{r=1}^{n}r+\sum_{r=1}^{n}1$$

$$=3.\frac{n(n+1)(2n+1)}{6}-2\frac{n(n+1)}{2}+n$$

$$=\frac{n}{2}[(2n^2+3n+1)-2(n+1)+2]$$

$$=\frac{n}{2}(2n^2+n+1)$$

Question 3.

Find $\sum_{r=1}^{n}\left(\frac{1+2+3....+r}{r}\right)$

Solution:

$$\sum_{r=1}^{n}\left(\frac{1+2+3+...+r}{r}\right)$$

$$=\sum_{r=1}^{n}\frac{r(r+1)}{2r}$$

$$=\frac{1}{2}\sum_{r=1}^{n}(r+1)$$

$$=\frac{1}{2}\left[\sum_{r=1}^{n}r+\sum_{r=1}^{n}1\right]$$

$$=\frac{1}{2}\left[\frac{n(n+1)}{2}+n\right]$$

$=\frac{n}{4}$ [(n + 1) + 2]

$=\frac{n}{4}$ (n + 3)

Question 4.

Find $\sum_{r=1}^{n}\left(\frac{1^3+2^3+.....+r^3}{r(r+1)}\right)$

Solution:

$$\sum_{r=1}^{n}\frac{1^3+2^3+...+r^3}{r(r+1)}$$

$$=\sum_{r=1}^{n}\frac{r^2(r+1)^2}{4}\times\frac{1}{r(r+1)}$$

$$=\frac{1}{4}\sum_{r=1}^{n}r(r+1)$$

$$=\frac{1}{4}\sum_{r=1}^{n}\left(r^2+r\right)$$

$$=\frac{1}{4}\left[\sum_{r=1}^{n}r^2+\sum_{r=1}^{n}r\right]$$

$$=\frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right]$$

$$=\frac{1}{4}.\frac{n(n+1)}{6}\left[(2n+1)+3\right]$$

$$=\frac{n(n+1)}{24}2(n+2)$$

$$=\frac{n(n+1)(n+2)}{12}$$

Question 5.

Find the sum 5 × 7 + 9 × 11 + 13 × 15 + ….. upto n terms.

Solution:

5 × 7 + 9 × 11 + 13 × 15 + ….. upto n terms

Now, 5, 9, 13, … are in A.P. with

rth term = 5 + (r – 1) (4) = 4r + 1

7, 11, 15, ….. are in A.P. with

rth term = 7 + (r – 1) (4) = 4r + 3

∴ 5 × 7 + 9 × 11 + 13 × 15 + …… upto n terms

$$= \sum_{r=1}^{n}(4r+1)(4r+3)$$

$$= \sum_{r=1}^{n}(16r^2+16r+3)$$

$$= 16\sum_{r=1}^{n}r^2 + 16\sum_{r=1}^{n}r + 3\sum_{r=1}^{n}1$$

$$= 16\frac{n(n+1)(2n+1)}{6} + 16\frac{n(n+1)}{2} + 3n$$

$$= \frac{n}{3}[8(2n^2+3n+1)+24(n+1)+9]$$

$$= \frac{n}{3}(16n^2+48n+41)$$

Question 6.

Find the sum $2^2 + 4^2 + 6^2 + 8^2 + \ldots$ upto n terms.

Solution:

$$2^2 + 4^2 + 6^2 + 8^2 + \ldots \text{ upto n terms}$$

$$= (2\times1)^2 + (2\times2)^2 + (2\times3)^2 + (2\times4)^2 + \ldots$$

$$= \sum_{r=1}^{n}(2r)^2$$

$$= 4\sum_{r=1}^{n}r^2$$

$$= \frac{4.n(n+1)(2n+1)}{6}$$

$$= \frac{2n(n+1)(2n+1)}{3}$$

Question 7.

Find $(70^2 - 69^2) + (68^2 - 67^2) + (66^2 - 65^2) + \ldots + (2^2 - 1^2)$

Solution:

Let $S = (70^2 - 69^2) + (68^2 - 67^2) + \ldots + (2^2 - 1^2)$

∴ $S = (2^2 - 1^2) + (4^2 - 3^2) + \ldots + (70^2 - 69^2)$

Here, 2, 4, 6,…, 70 are in A.P. with rth term = 2r

and 1, 3, 5, …,69 are in A.P. with rth term = 2r – 1

$$S = \sum_{r=1}^{35}\left[(2r)^2-(2r-1)^2\right]$$

$$= \sum_{r=1}^{35}\left[4r^2-(4r^2-4r+1)\right]$$

$$= \sum_{r=1}^{35}(4r-1)$$

$$= 4\sum_{r=1}^{35}r - \sum_{r=1}^{35}1$$

$$= 4.\frac{35\times36}{2} - 35$$

$$= (72-1)(35)$$

$$= 71\times35$$

$$= 2485$$

Question 8.

Find the sum 1 × 3 × 5 + 3 × 5 × 7 + 5 × 7 × 9 + …… + (2n – 1) (2n + 1) (2n + 3)

Solution:

Let S = 1 × 3 × 5 + 3 × 5 × 7 + ….. upto n terms

Here, 1, 3, 5, 7 … are in A.P. with rth term = 2r – 1,

3, 5, 7, 9,… are in A.P. with rth term = 2r + 1,

5, 7, 9, 11,… are in A.P. with rth term = 2r + 3

$$S = \sum_{r=1}^{n}(2r-1)(2r+1)(2r+3)$$
$$= \sum_{r=1}^{n}(4r^2-1)(2r+3)$$
$$= \sum_{r=1}^{n}(8r^3+12r^2-2r-3)$$
$$= 8\sum_{r=1}^{n}r^3+12\sum_{r=1}^{n}r^2-2\sum_{r=1}^{n}r-3\sum_{r=1}^{n}1$$
$$= 8.\frac{n^2(n+1)^2}{4} + 12\frac{n(n+1)(2n+1)}{6} - 2\frac{n(n+1)}{2} - 3n$$
$$= 2n^2(n+1)^2 + 2n(n+1)(2n+1) - n(n+1) - 3n$$
$$= n(n+1)\,[2n(n+1) + 4n + 2 - 1] - 3n$$
$$= n(n+1)\,(2n^2 + 6n + 1) - 3n$$
$$= n(2n^3 + 8n^2 + 7n + 1 - 3)$$
$$= n(2n^3 + 8n^2 + 7n - 2)$$
$$= n(n+2)\,(2n^2 + 4n - 1)$$

Question 9.
If 1×2+2×3+3×4+4×5+... upto n terms 1+2+3+4+... upto n terms = 1003, find n.
Solution:

$$\frac{1\times2+2\times3+3\times4+\text{...upto n terms}}{1+2+3+4+\text{...upto n terms}} = \frac{100}{3}$$
$$\therefore \quad \frac{\sum_{r=1}^{n}r(r+1)}{\sum_{r=1}^{n}r} = \frac{100}{3}$$
$$\therefore \quad \frac{\sum_{r=1}^{n}r^2+\sum_{r=1}^{n}r}{\sum_{r=1}^{n}r} = \frac{100}{3}$$
$$\therefore \quad \frac{\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}}{\frac{n(n+1)}{2}} = \frac{100}{3}$$
$$\therefore \quad \frac{\frac{n(n+1)}{6}[(2n+1)+3]}{\frac{n(n+1)}{2}} = \frac{100}{3}$$
$$\therefore \quad \frac{2(n+2)}{3} = \frac{100}{3}$$
$$\therefore \quad n+2 = 50$$
$$\therefore \quad n = 48$$

Question 10.
If S_1, S_2 and S_3 are the sums of first n natural numbers, their squares and their cubes respectively, then show that $9S_2^2 = S_3(1 + 8S_1)$.
Solution:

$$S_1 = 1 + 2 + 3 + \ldots + n = \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

$$S_2 = 1^2 + 2^2 + 3^2 + \ldots + n^2$$
$$= \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

$$S_3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

R.H.S. $= S_3(1 + 8S_1)$

$$= \frac{n^2(n+1)^2}{4}\left[1 + 8.\frac{n(n+1)}{2}\right]$$

$$= \frac{n^2(n+1)^2}{4}(1 + 4n^2 + 4n)$$

$$= \frac{n^2(n+1)^2}{4}(2n + 1)^2$$

$$= \frac{9.n^2(n+1)^2(2n+1)^2}{36}$$

$$= 9\left[\frac{n(n+1)(2n+1)}{6}\right]^2$$

$$= 9{S_2}^2$$

$= $ L.H.S.

Maharashtra State Board 11th Maths Solutions Chapter 2 Sequences and Series Miscellaneous Exercise 2

(I) Select the correct answer from the given alternative:

Question 1.
The common ratio for the G.P. 0.12, 0.24, 0.48, is
(A) 0.12
(B) 0.2
(C) 0.02
(D) 2
Answer:
(D) 2

Question 2.
The tenth term of the geometric sequence is 14,-12,1,-2,... is
(A) 1024
(B) 11024
(C) -128
(D) -1128
Answer:
(C) -128

Hint:

$$\text{Here, } a = \frac{1}{4},\ r = \frac{\frac{-1}{2}}{\frac{1}{4}} = \frac{-1}{2} \times 4 = -2$$

n^{th} term of geometric sequence = ar^{n-1}

10^{th} term of geometric sequence = $\frac{1}{4}(-2)^{10-1}$

$$= \frac{(-2)^9}{4}$$

$$= -128$$

Question 3.

If for a G.P. $t_6t_3=145854$ then r = ?

(A) 3
(B) 2
(C) 1
(D) -1

Answer:
(A) 3

Hint:

3. $t_n = ar^{n-1}$

$\therefore$ $t_6 = ar^5$ and $t_3 = ar^2$

Given, $\frac{t_6}{t_3} = \frac{1458}{54}$

$\therefore$ $\frac{ar^5}{ar^2} = 27$

$\therefore$ $r^3 = 27 = 3^3$

$\therefore$ $r = 3$

Question 4.
Which term of the geometric progression 1, 2, 4, 8, ….. is 2048.
(A) 10th
(B) 11th
(C) 12th
(D) 13th
Answer:
(C) 12th
Hint:
Here, a = 1, r = 2
nth term of geometric progression = ar^{n-1}
$\therefore ar^{n-1} = 2048$
$2^{n-1} = 2^{11}$
n – 1 = 11
$\therefore$ n = 12

Question 5.
If the common ratio of the G.P. is 5, the 5th term is 1875, the first term is
(A) 3
(B) 5
(C) 15
(D) -5
Answer:
(A) 3

Question 6.
The sum of 3 terms of a G.P. is 214 and their product is 1, then the common ratio is
(A) 1
(B) 2
(C) 4
(D) 8
Answer:
(C) 4
Hint:
Let three terms be ar, a, ar
According to the given conditions,
ar + a + ar = 214 …..(i)

and ar × a × ar = 1,
i.e., a^3 = 1
∴ a = 1
∴ from equation (i), we get
1r + 1 + r = 214
By solving this, we get r = 4.

Question 7.
Sum to infinity of a G.P. 5, −52,54,−58,516,… is
(A) 5
(B) −12
(C) 103
(D) 310
Answer:
(C) 103
Hint:
Here, a = 5, r = −12, |r| < 1
∴ Sum to the infinity = a1−r=51+12=103

Question 8.
The tenth term of H.P. 29,17,219,112,… is
(A) 127
(B) 92
(C) 52
(D) 27
Answer:
(A) 127
Hint:

Sequence $\frac{2}{9}, \frac{1}{7}, \frac{2}{19}, \frac{1}{12}, \ldots$ is in H.P.

$\frac{9}{2}, 7, \frac{19}{2}, 12, \ldots$ is in A.P.

where, a = $\frac{9}{2}$ and d = $7 - \frac{9}{2} = \frac{5}{2}$

10^{th} term in A.P. = a + (10 − 1)d

$= \frac{9}{2} + 9 \times \frac{5}{2} = 27$

10^{th} term in H.P. = $\frac{1}{27}$

Question 9.
Which of the following is not true, where A, G, H are the AM, GM, HM of a and b respectively, (a, b > 0)
(A) A = a+b2
(B) G = ab−−√
(C) H = 2aba+b
(D) A = GH
Answer:
(D) A = GH

Question 10.
The G.M. of two numbers exceeds their H.M. by 65, the A.M. exceeds G.M. by 32 the two numbers are
(A) 6, 152
(B) 15, 25
(C) 3, 12
(D) 65, 32
Answer:
(C) 3, 12
Hint:

Let two numbers be a and b.

$A = \frac{a+b}{2}, G = \sqrt{ab}, H = \frac{2ab}{a+b}$

According to the given condition,

$G - H = \frac{6}{5}$ ...(i)

and $A - G = \frac{3}{2}$ ...(ii)

Multiply (i) by G, we get

$G^2 - GH = \frac{6}{5}G$

$AH - GH = \frac{6}{5}G$ ...$[G^2 = AH]$

$H(A - G) = \frac{6}{5}G$

$\therefore$ $H \times \frac{3}{2} = \frac{6}{5}G$ ...[From (ii)]

$\therefore$ $H = \frac{4}{5}G$

$\therefore$ From (i), we get $G - \frac{4}{5}G = \frac{6}{5}$

$\therefore$ $G = 6$, i.e., $ab = 36$

Option (C) satisfies the condition.

(II) Answer the following:

Question 1.
In a G.P., the fourth term is 48 and the eighth term is 768. Find the tenth term.
Solution:

Given, $t_4 = 48$, $t_8 = 768$

$t_n = ar^{n-1}$

$\therefore$ $t_4 = ar^3$

$\therefore$ $ar^3 = 48$ ...(i)

and $ar^7 = 768$ ...(ii)

Equation (ii) ÷ equation (i), we get

$\therefore$ $\frac{ar^7}{ar^3} = \frac{768}{48}$

$\therefore$ $r^4 = 16$

$\therefore$ $r = 2$

Substituting r = 2 (i), we get

$a.(2^3) = 48$

$\therefore$ $a = 6$

$\because$ $t_{10} = ar^9$

$\therefore$ $t_{10} = ar^9 = 6(2^9) = 3072$

Question 2.
Find the sum of the first 5 terms of the G.P. whose first term is 1 and the common ratio is 23.

Solution:

$$\text{Given, } a = 1,\ r = \frac{2}{3}$$

$$S_n = \frac{a(1-r^n)}{1-r},\ \text{for } r < 1$$

$$\therefore \quad S_5 = \frac{1\left[1-\left(\frac{2}{3}\right)^5\right]}{1-\frac{2}{3}}$$

$$= 3\left[1-\frac{32}{243}\right]$$

$$= 3 \times \frac{211}{243}$$

$$= \frac{211}{81}$$

Question 3.
For a G.P. a = 43 and t_7 = 2431024, find the value of r.
Solution:

$$\text{Given, } a = \frac{4}{3},\ t_7 = \frac{243}{1024}$$

$$t_n = ar^{n-1}$$

$$\therefore \quad t_7 = ar^6$$

$$\therefore \quad \frac{243}{1024} = ar^6$$

$$\therefore \quad \frac{243}{1024} = \frac{4}{3}r^6$$

$$\therefore \quad r^6 = \frac{3^6}{4^6}$$

$$\therefore \quad r = \frac{3}{4}$$

Question 4.
For a sequence, if t_n=5n-27n-3, verify whether the sequence is a G.P. If it is a G.P., find its first term and the common ratio.
Solution:
The sequence (t_n) is a G.P.
if tn+1tn = constant for all n ∈ N.

$$\text{Now, } t_n = \frac{5^{n-2}}{7^{n-3}}$$

$$\therefore \quad t_{n+1} = \frac{5^{n+1-2}}{7^{n+1-3}} = \frac{5^{n-1}}{7^{n-2}}$$

$$\therefore \quad \frac{t_{n+1}}{t_n} = \frac{5^{n-1}}{7^{n-2}} \times \frac{7^{n-3}}{5^{n-2}}$$

$$= 5^{(n-1)-(n-2)}\, 7^{(n-3)-(n-2)}$$

$$= 5^{(1)}.7^{-1}$$

$$= \frac{5}{7} = \text{constant, for all } n \in N.$$

$$\therefore \quad \text{the sequence is a G.P. with common ratio} = \frac{5}{7}$$

$$\therefore \quad \text{first term} = t_1 = \frac{5^{1-2}}{7^{1-3}} = \frac{5^{-1}}{7^{-2}} = \frac{7^2}{5} = \frac{49}{5}$$

Question 5.
Find three numbers in G.P. such that their sum is 35 and their product is 1000.
Solution:
Let the three numbers in G.P. be ar, a, ar.
According to the given conditions,
ar + a + ar = 35
a(1r + 1 + r) = 35 …..(i)
Also, (ar)(a)(ar) = 1000

$a^3 = 1000$
$\therefore$ a = 10
Substituting the value of a in (i), we get

$$10\left(\frac{1}{r}+1+r\right) = 35$$

$\therefore \quad \frac{1}{r} + r + 1 = \frac{35}{10}$

$\therefore \quad \frac{1}{r} + r = \frac{35}{10} - 1$

$\therefore \quad \frac{1}{r} + r = \frac{25}{10}$

$\therefore \quad \frac{1}{r} + r = \frac{5}{2}$

$\therefore \quad 2r^2 - 5r + 2 = 0$

$\therefore \quad (2r - 1)(r - 2) = 0$

$\therefore \quad r = \frac{1}{2}$ or $r = 2$

When $a = 10, r = \frac{1}{2}$,

$\frac{a}{r} = \frac{10}{\left(\frac{1}{2}\right)} = 20$, a = 10 and $ar = 10\left(\frac{1}{2}\right) = 5$

When a = 10, r = 2,

$\frac{a}{r} = \frac{10}{2} = 5$, a = 10 and $ar = 10(2) = 20$

Hence, the three numbers in G.P. are 20, 10, 5, or 5, 10, 20.

Question 6.
Find five numbers in G.P. such that their product is 243 and the sum of the second and fourth numbers is 10.
Solution:
Let the five numbers in G.P. be $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$.
According to the given condition,

$$\frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 = 243$$

$\therefore \quad a^5 = 243$

$\therefore \quad a = 3$

Also, $\frac{a}{r} + ar = 10$

$\therefore \quad \frac{1}{r} + r = \frac{10}{a}$

$\therefore \quad \frac{1+r^2}{r} = \frac{10}{3}$

$\therefore \quad 3r^2 - 10r + 3 = 0$

$\therefore \quad 3r^2 - 9r - r + 3 = 0$

$\therefore \quad (3r - 1)(r - 3) = 0$

$\therefore \quad r = \frac{1}{3}, 3$

When $a = 3, r = \frac{1}{3}$,

$\frac{a}{r^2} = 27, \frac{a}{r} = 9, a = 3, ar = 1, ar^2 = \frac{1}{3}$

When a = 3, r = 3,

$\frac{a}{r^2} = \frac{1}{3}, \frac{a}{r} = 1, a = 3, ar = 9, ar^2 = 27$

Hence, the five numbers in G.P. are

$27, 9, 3, 1, \frac{1}{3}$ or $\frac{1}{3}, 1, 3, 9, 27$

Question 7.
For a sequence, $S_n = 4(7^n - 1)$, verify that the sequence is a G.P.
Solution:

$S_n = 4(7^n - 1)$

$\therefore \quad S_{n-1} = 4(7^{n-1} - 1)$

But, $t_n = S_n - S_{n-1}$

$= 4(7^n - 1) - 4(7^{n-1} - 1)$

$= 4(7^n - 1 - 7^{n-1} + 1)$

$= 4(7^n - 7^{n-1})$

$= 4(7^{n-1+1} - 7^{n-1})$

$= 4.7^{n-1}(7 - 1)$

$\therefore \quad t_n = 24.7^{n-1}$

$\therefore \quad t_{n+1} = 24(7)^{n+1-1} = 24(7)^n$

The sequence is a G.P., if $\frac{t_{n+1}}{t_n}$ = constant for all $n \in N$.

$$\therefore \quad \frac{t_{n+1}}{t_n} = \frac{24(7)^n}{24(7)^{n-1}} = 7 = \text{constant, for all } n \in N$$

∴ The given sequence is a G.P.

Question 8.
Find 2 + 22 + 222 + 2222 + … upto n terms.
Solution:
S_n = 2 + 22 + 222 +… upto n terms
= 2(1 + 11 + 111 + ….. upto n terms)
= $\frac{2}{9}$ (9 + 99 + 999 + … upto n terms)
= $\frac{2}{9}$ [(10 – 1) + (100 – 1) + (1000 – 1) + …… upto n terms]
= $\frac{2}{9}$ [(10 + 100 + 1000 + … upto n terms) – (1 + 1 + 1 + ….. n times)]
Since 10, 100, 1000, ….. n terms are in G.P. with
a = 10, r = $\frac{100}{10}$ = 10,

$$S_n = \frac{2}{9}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right]$$

$$= \frac{2}{9}\left[\frac{10}{9}\left(10^n - 1\right) - n\right]$$

$$S_n = \frac{2}{81}[10(10^n - 1) - 9n]$$

Question 9.
Find the nth term of the sequence 0.6, 0.66, 0.666, 0.6666,…
Solution:
0.6, 0.66, 0.666, 0.6666, …
∴ t_1 = 0.6
t_2 = 0.66 = 0.6 + 0.06
t_3 = 0.666 = 0.6 + 0.06 + 0.006
Hence, in general
t_n = 0.6 + 0.06 + 0.006 + …..upto n terms.
The terms are in G.P. with
a = 0.6, r = $\frac{0.06}{0.6}$ = 0.1
∴ t_n = the sum of first n terms of the G.P.

$$\therefore \quad t_n = 0.6\left[\frac{1 - (0.1)^n}{1 - 0.1}\right] = \frac{0.6}{0.9}[1 - (0.1)^n]$$

$$\therefore \quad t_n = \frac{6}{9}[1 - (0.1)^n] = \frac{2}{3}\left[1 - \left(\frac{1}{10}\right)^n\right]$$

Question 10.
Find $\sum_{r=1}^{n}(5r^2 + 4r - 3)$

Solution:

$$\sum_{r=1}^{n}\left(5r^2+4r-3\right)$$

$$= 5\sum_{r=1}^{n}r^2+4\sum_{r=1}^{n}r-3\sum_{r=1}^{n}1$$

$$= 5.\ \frac{n(n+1)(2n+1)}{6}+4.\frac{n(n+1)}{2}-3n$$

$$= \frac{n}{6}\ [5(2n^2+3n+1)+12(n+1)-18]$$

$$= \frac{n}{6}(10n^2+27n-1)$$

Question 11.

Find $\sum_{r=1}^{n} r(r-3)(r-2)$

Solution:

$$\sum_{r=1}^{n}r(r-3)(r-2)$$

$$= \sum_{r=1}^{n}\left(r^3-5r^2+6r\right)$$

$$= \sum_{r=1}^{n}r^3-5\sum_{r=1}^{n}r^2+6\sum_{r=1}^{n}r$$

$$= \frac{n^2(n+1)^2}{4}-5\frac{n(n+1)(2n+1)}{6}+6.\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{12}\left[3n(n+1)-10(2n+1)+36\right]$$

$$= \frac{n(n+1)\left(3n^2-17n+26\right)}{12}$$

Question 12.

Find $\sum_{r=1}^{n} \frac{1^2+2^2+3^2+\ldots+r^2}{2r+1}$

Solution:

We know that

$$1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$$

$$\therefore \quad 1^2+2^2+3^2+\ldots+r^2=\frac{r(r+1)(2r+1)}{6}$$

$$\therefore \quad \frac{1^2+2^2+3^2+\ldots+r^2}{2r+1}=\frac{r(r+1)}{6}$$

$$\therefore \quad \sum_{r=1}^{n}\left(\frac{1^2+2^2+3^2+\ldots+r^2}{2r+1}\right)$$

$$= \sum_{r=1}^{n}\frac{r(r+1)}{6}=\frac{1}{6}\sum_{r=1}^{n}\left(r^2+r\right)$$

$$= \frac{1}{6}\left(\sum_{r=1}^{n}r^2+\sum_{r=1}^{n}r\right)$$

$$= \frac{1}{6}\left[\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right]$$

$$= \frac{1}{6}\times\frac{n(n+1)}{2}\left(\frac{2n+1}{3}+1\right)$$

$$= \frac{n(n+1)}{12}\left(\frac{2n+1+3}{3}\right)$$

$$= \frac{n(n+1)(2n+4)}{36}$$

$$= \frac{2n(n+1)(n+2)}{36}$$

$$= \frac{n(n+1)(n+2)}{18}$$

Question 13.

Find $\sum_{r=1}^{n} \frac{1^3+2^3+3^3+\ldots r^3}{(r+1)^2}$

Solution:

$$\sum_{r=1}^{n} \frac{1^3+2^3+3^3+\ldots+r^3}{(r+1)^2}$$
$$= \sum_{r=1}^{n} \frac{r^2(r+1)^2}{4} \times \frac{1}{(r+1)^2}$$
$$= \frac{1}{4} \sum_{r=1}^{n} r^2$$
$$= \frac{1}{4} \cdot \frac{n(n+1)(2n+1)}{6}$$
$$= \frac{n(n+1)(2n+1)}{24}$$

Question 14.

Find 2 × 6 + 4 × 9 + 6 × 12 + ….. upto n terms.

Solution:

2, 4, 6, ….. are in A.P.

∴ rth term = 2 + (r – 1) 2 = 2r

6, 9, 12, ….. are in A.P.

∴ rth term = 6 + (r – 1)(3) = (3r + 3)

∴ 2 × 6 + 4 × 9 + 6 × 12 + ….. to n terms

$$= \sum_{r=1}^{n} 2r \times (3r+3)$$
$$= 6\sum_{r=1}^{n} r^2 + 6\sum_{r=1}^{n} r$$
$$= 6 . \frac{n(n+1)(2n+1)}{6} + 6\frac{n(n+1)}{2}$$

= n(n + 1) [2n + 1 + 3]

= 2n(n + 1)(n + 2)

Question 15.

Find 2 × 5 × 8 + 4 × 7 × 10 + 6 × 9 × 12 + …… upto n terms.

Solution:

2, 4, 6,… are in A.P.

∴ rth term = 2 + (r – 1) 2 = 2r

5, 7, 9, … are in A.P.

∴ rth term = 5 + (r – 1) (2) = (2r + 3)

8, 10, 12, … are in A.P.

∴ rth term = 8 + (r – 1) (2) = (2r + 6)

2 × 5 × 8 + 4 × 7 × 10 + 6 × 9 × 12 + ….. to n terms

$$= \sum_{r=1}^{n} 2r(2r+3)(2r+6)$$
$$= \sum_{r=1}^{n}\left(8r^3+36r^2+36r\right)$$
$$= 8\sum_{r=1}^{n} r^3 + 36\sum_{r=1}^{n} r^2 + 36\sum_{r=1}^{n} r$$
$$= 8 . \frac{n^2(n+1)^2}{4} + 36 . \frac{n(n+1)(2n+1)}{6} + 36 . \frac{n(n+1)}{2}$$

= 2n (n + 1) [n(n + 1) + 3(2n + 1) + 9]

= 2n (n + 1)(n^2 + 7n + 12)

= 2n (n + 1) (n + 3) (n + 4)

Question 16.

Find $\frac{1^2}{1}+\frac{1^2+2^2}{2}+\frac{1^2+2^2+3^2}{3}+\ldots$ upto n terms.

Solution:

$$\frac{1^2}{1}+\frac{1^2+2^2}{2}+\frac{1^2+2^2+3^2}{3}+\ldots \text{ upto n terms}$$

$$=\sum_{r=1}^{n}\frac{1^2+2^2+3^2+\ldots.+r^2}{r}$$

$$=\sum_{r=1}^{n}\frac{r(r+1)(2r+1)}{6r}$$

$$=\frac{1}{6}\sum_{r=1}^{n}\left(2r^2+3r+1\right)$$

$$=\frac{2}{6}\sum_{r=1}^{n}r^2+\frac{3}{6}\sum_{r=1}^{n}r+\frac{1}{6}\sum_{r=1}^{n}1$$

$$=2.\ \frac{n(n+1)(2n+1)}{36}+\frac{3n(n+1)}{12}+\frac{n}{6}$$

$$=\frac{n}{36}[2(2n^2+3n+1)+9(n+1)+6]$$

$$=\frac{n}{36}(4n^2+15n+17)$$

Question 17.
Find $12^2 + 13^2 + 14^2 + 15^2 + \ldots.. 20^2$
Solution:

$$12^2+13^2+14^2+15^2+\ldots 20^2$$

$$=(1^2+2^2+3^2+4^2+\ldots+20^2)-(1^2+2^2+3^2+4^2+\ldots+11^2)$$

$$=\sum_{r=1}^{20}r^2-\sum_{r=1}^{11}r^2$$

$$=\frac{20(20+1)(2\times 20+1)}{6}-\frac{11(11+1)(2\times 11+1)}{6}$$

$$=\frac{20\times 21\times 41}{6}-\frac{11\times 12\times 23}{6}$$

$$=2870-506=2364$$

Question 18.
If $1+2+3+4+5+\ldots$ *upto n terms* $1\times2+2\times3+3\times4+4\times5+\ldots$ *upto n terms* $=322$, Find the value of n.

Solution:

$$\frac{1+2+3+4+5+\ldots \text{ upto n terms}}{1\times 2+2\times 3+3\times 4+4\times 5+\ldots\text{upto n terms}}=\frac{3}{22}$$

$$\therefore \quad \frac{\sum_{r=1}^{n} r}{\sum_{r=1}^{n} r(r+1)}=\frac{3}{22}$$

$$\therefore \quad \frac{\sum_{r=1}^{n} r}{\sum_{r=1}^{n} r^2+\sum_{r=1}^{n} r}=\frac{3}{22}$$

$$\therefore \quad \frac{\frac{n(n+1)}{2}}{\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}}=\frac{3}{22}$$

$$\therefore \quad \frac{1}{\frac{2n+1}{3}+1}=\frac{3}{22}$$

$$\therefore \quad \frac{3}{2(n+2)}=\frac{3}{22}$$

$$\therefore \quad \frac{1}{n+2}=\frac{1}{11}$$

$$\therefore \quad n=9$$

Question 19.
Find $(50^2 – 49^2) + (48^2 – 47^2) + (46^2 – 45^2) +\ldots + (2^2 – 1^2)$.
Solution:

$$(50^2-49^2)+(48^2-47^2)+(46^2-45^2)+\ldots+(2^2-1^2)$$
$$=(50^2+48^2+46^2+\ldots+2^2)-(49^2+47^2+45^2+\ldots+1^2)$$
$$=\sum_{r=1}^{25}(2r)^2-\sum_{r=1}^{25}(2r-1)^2$$
$$=\sum_{r=1}^{25}4r^2-\sum_{r=1}^{25}(4r^2-4r+1)$$
$$=\sum_{r=1}^{25}\left[4r^2-(4r^2-4r+1)\right]$$
$$=\sum_{r=1}^{25}(4r-1)$$
$$=4\sum_{r=1}^{25}r-\sum_{r=1}^{25}1$$
$$=4\times\frac{25(25+1)}{2}-25$$
$$=\frac{4(25)(26)}{2}-25$$
$$=1300-25=1275$$

Question 20.
If $1\times3+2\times5+3\times7+\ldots$ *upto n terms* $1^3+2^3+3^3+\ldots$ *upto n terms* $=59$, find the value of n.
Solution:

$$\frac{1\times 3+2\times 5+3\times 7+\ldots\text{upto n terms}}{1^3+2^3+3^3+\ldots\text{ upto n terms}}=\frac{5}{9}$$

Here, 1, 2, 3, … is an A.P. with r^{th} term = r.

3, 5, 7, …, is an A.P. with r^{th} term = 2r + 1

$\therefore \quad \dfrac{\sum_{r=1}^{n} r(2r+1)}{\sum_{r=1}^{n} r^3}=\dfrac{5}{9}$

$\therefore \quad \dfrac{2\sum_{r=1}^{n} r^2+\sum_{r=1}^{n} r}{\sum_{r=1}^{n} r^3}=\dfrac{5}{9}$

$\therefore \quad \dfrac{\dfrac{2n(n+1)(2n+1)}{6}+\dfrac{n(n+1)}{2}}{\dfrac{n^2(n+1)^2}{4}}=\dfrac{5}{9}$

$\therefore \quad \dfrac{n(n+1)\left[\dfrac{2n+1}{3}+\dfrac{1}{2}\right]}{\dfrac{n^2(n+1)^2}{4}}=\dfrac{5}{9}$

$\therefore \quad \dfrac{\dfrac{4n+2+3}{6}}{\dfrac{n(n+1)}{4}}=\dfrac{5}{9}$

$\therefore \quad \dfrac{4n+5}{3}\times\dfrac{2}{n^2+n}=\dfrac{5}{9}$

$\therefore \quad \dfrac{8n+10}{n^2+n}=\dfrac{5}{3}$

$\therefore \quad 5n^2+5n=24n+30$

$\therefore \quad 5n^2-19n-30=0$

$\therefore \quad 5n^2-25n+6n-30=0$

$\therefore \quad 5n(n-5)+6(n-5)=0$

$\therefore \quad (n-5)(5n+6)=0$

$\therefore \quad n=5, -\dfrac{6}{5}$

$\because \quad n\in N$

$\therefore \quad n=5$

Question 21.
For a G.P. if t_2 = 7, t_4 = 1575, find a.
Solution:

Given, $t_2 = 7$, $t_4 = 1575$

$t_n = ar^{n-1}$

$\therefore \quad t_2 = ar$

$\therefore \quad ar = 7$

$\therefore \quad r = \dfrac{7}{a}$ …(i)

Also, $t_4 = ar^3$

$\therefore \quad ar^3 = 1575$

$\therefore \quad a\times\left(\dfrac{7}{a}\right)^3 = 1575$ …[From (i)]

$\therefore \quad a^2 = \dfrac{7^3}{1575}$

$\therefore \quad a^2 = \dfrac{49}{225}$

$\therefore \quad a = \dfrac{7}{15}$

Question 22.
If for a G.P. t_3 = 13, t_6 = 181 find r.

Solution:

Given, $t_3 = \frac{1}{3}$, $t_6 = \frac{1}{81}$

$t_n = ar^{n-1}$

$\therefore \quad t_3 = ar^2$

$\therefore \quad ar^2 = \frac{1}{3}$

$\therefore \quad a = \frac{1}{3r^2} \qquad \ldots(i)$

Also, $t_6 = ar^5$

$\therefore \quad ar^5 = \frac{1}{81}$

$\therefore \quad \frac{1}{3r^2} \times r^5 = \frac{1}{81} \qquad \ldots[\text{From (i)}]$

$\therefore \quad r^3 = \frac{1}{27}$

$\therefore \quad r = \frac{1}{3}$

Question 23.
Find $\sum_{r=1}^{n} \left(\frac{2}{3}\right)^r$.
Solution:

$$\sum_{r=1}^{n} \left(\frac{2}{3}\right)^r = \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \ldots + \left(\frac{2}{3}\right)^n$$

The terms $\frac{2}{3}, \left(\frac{2}{3}\right)^2, \left(\frac{2}{3}\right)^3$ are in G.P.

$\therefore \quad a = \frac{2}{3}, r = \frac{2}{3}$

$$\therefore \quad \sum_{r=1}^{n} \left(\frac{2}{3}\right)^r = \frac{\frac{2}{3}\left[1 - \left(\frac{2}{3}\right)^n\right]}{1 - \frac{2}{3}}$$

$$\therefore \quad \sum_{r=1}^{n} \left(\frac{2}{3}\right)^r = 2\left[1 - \left(\frac{2}{3}\right)^n\right]$$

Question 24.
Find k so that k – 1, k, k + 2 are consecutive terms of a G.P.
Solution:
Since k – 1, k, k + 2 are consecutive terms of a G.P.,
$\frac{k}{k-1} = \frac{k+2}{k}$
$k^2 = k^2 + k - 2$
$k - 2 = 0$
$\therefore k = 2$

Question 25.
If for a G.P. first term is $(27)^2$ and the seventh term is $(8)^2$, find S_8.

Solution:

$a = (27)^2,$

$t_n = ar^{n-1}$

$\therefore \quad t_7 = ar^6$

$\therefore \quad ar^6 = 8^2$

$\therefore \quad r^6 = \frac{8^2}{27^2} = \frac{2^6}{3^6}$

$\therefore \quad r = \frac{2}{3}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{for } r < 1$$

$$\therefore \quad S_8 = \frac{(27)^2\left[1-\left(\frac{2}{3}\right)^8\right]}{1-\frac{2}{3}}$$

$$= 3 \times 729\left[1-\left(\frac{2}{3}\right)^8\right]$$

$$= 2187\left[1-\left(\frac{2}{3}\right)^8\right]$$

Question 26.

If pth, qth and rth terms of a G.P. are x, y, z respectively. Find the value of $x^{q-r} \cdot y^{r-p} \cdot z^{p-q}$.

Solution:

Let a be the first term and R be the common ratio of the G.P.

$\therefore \quad t_n = a.R^{n-1}$

$\therefore \quad x = a.R^{p-1}, y = a.R^{q-1}, z = a.R^{r-1}$

$\therefore \quad x^{q-r}.y^{r-p}.z^{p-q}$

$= (a.R^{p-1})^{q-r}.(a.R^{q-1})^{r-p}.(a.R^{r-1})^{p-q}$

$= a^{q-r}R^{(p-1)(q-r)}.a^{r-p}R^{(q-1)(r-p)}.a^{p-q}R^{(r-1)(p-q)}$

$= a^{(q-r+r-p+p-q)}.R^{[(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)]}$

$= a^0.R^{(pq-pr-q+r+qr-pq-r+p+pr-qr-p+q)}$

$= (1).R^0 = 1$

Question 27.

Which 2 terms are inserted between 5 and 40 so that the resulting sequence is G.P.

Solution:

Let the required numbers be G_1 and G_2.

$\therefore \quad t_1 = 5, t_2 = G_1, t_3 = G_2, t_4 = 40$

$\therefore \quad t_1 = a = 5, t_4 = 40$

$t_n = ar^{n-1}$

$\therefore \quad t_4 = 5(r)^{4-1}$

$\therefore \quad 40 = 5r^3$

$\therefore \quad r^3 = 8 = 2^3$

$\therefore \quad r = 2$

$G_1 = t_2 = ar = 5(2) = 10$

$G_2 = t_3 = ar^2 = 5(2)^2 = 20$

$\therefore$ For the resulting sequence to be in G.P. we need to insert numbers 10 and 20.

Question 28.

If p, q, r are in G.P. and $p^{1/x} = q^{1/y} = r^{1/z}$, verify whether x, y, z are in A.P. or G.P. or neither.

Solution:

p, q, r are in G.P.

$\therefore \quad \frac{q}{p} = \frac{r}{q}$

$\therefore \quad q^2 = pr \qquad \ldots(i)$

Also, $p^{\frac{1}{x}} = q^{\frac{1}{y}} = r^{\frac{1}{z}}$

$\therefore \quad p = q^{\frac{x}{y}}, r = q^{\frac{z}{y}}$

Substituting value of p and r in (i), we get

$q^2 = q^{\frac{x}{y}} . q^{\frac{z}{y}}$

$q^2 = q^{\frac{x}{y}+\frac{z}{y}}$

$q^2 = q^{\frac{x+z}{y}}$

$\therefore \quad 2 = \frac{x+z}{y}$

$\therefore \quad 2y = x + z$

$\therefore \quad y - x = z - y$

$\therefore \quad x, y, z$ are in A.P.

Question 29.
If a, b, c are in G.P. and $ax^2 + 2bx + c = 0$ and $px^2 + 2qx + r = 0$ have common roots, then verify that $pb^2 – 2qba + ra^2 = 0$.
Solution:
a, b, c are in G.P.
$\therefore b^2 = ac$
$ax^2 + 2bx + c = 0$ becomes

$ax^2 + 2\sqrt{ac}\, x + c = 0$

$\left(\sqrt{a}x + \sqrt{c}\right)^2 = 0$

$\therefore \quad x = \frac{-\sqrt{c}}{\sqrt{a}}$

$\therefore \quad ax^2 + 2bx + c = 0$ and $px^2 + 2qx + r = 0$ have a common root, $x = \frac{-\sqrt{c}}{\sqrt{a}}$

Satisfying $px^2 + 2qx + r = 0$

$\therefore \quad p.\frac{c}{a} + 2q.\left(\frac{-\sqrt{c}}{\sqrt{a}}\right) + r = 0$

$pc - 2q\sqrt{ac} + ra = 0$

$p.\frac{b^2}{a} - 2qb + ra = 0$

$\ldots\left[\because b^2 = ac,\ c = \frac{b^2}{a},\ \sqrt{c} = \frac{b}{\sqrt{a}},\ \sqrt{ac} = b\right]$

$\therefore \quad pb^2 - 2qba + ra^2 = 0$

Question 30.
If p, q, r, s are in G.P., show that $(p^2 + q^2 + r^2)(q^2 + r^2 + s^2) = (pq + qr + rs)^2$.
Solution:

p, q, r, s are in G.P.

$$\frac{q}{p} = \frac{r}{q} = \frac{s}{r}$$

$$\therefore \quad q^2 = pr, \qquad r^2 = qs, \qquad qr = ps$$

$$\begin{aligned} \text{L.H.S.} &= (p^2 + q^2 + r^2)(q^2 + r^2 + s^2) \\ &= p^2q^2 + p^2r^2 + p^2s^2 + (q^2)^2 + q^2r^2 \\ &\qquad + q^2s^2 + q^2r^2 + (r^2)^2 + r^2s^2 \\ &= (pq)^2 + (qr)^2 + (rs)^2 + pq^2r + pq^2r \\ &\qquad + psqr + psqr + qr^2s + qr^2s \\ &\qquad \ldots[\because q^2 = pr, ps = qr, r^2 = qs] \\ &= (pq)^2 + (qr)^2 + (rs)^2 + 2pq.qr \\ &\qquad + 2pq.rs + 2qr.rs \\ &= (pq + qr + rs)^2 \\ &= \text{R.H.S.} \end{aligned}$$

Question 31.
If p, q, r, s are in G.P., show that $(p_n + q_n)$, $(q_n + r_n)$, $(r_n + s_n)$ are also in G.P.
Solution:
p, q, r, s are in G.P.
Let the common ratio be R
∴ let p = aR_3, q = aR, r = aR and s = aR_3
To show that $(p_n + q_n)$, $(q_n + r_n)$, $(r_n + s_n)$ are in G.P,
i.e., we have to show

$$\frac{q^n + r^n}{p^n + q^n} = \frac{r^n + s^n}{q^n + r^n}$$

$$\text{L.H.S.} = \frac{q^n + r^n}{p^n + q^n} = \frac{\left(\frac{a}{R}\right)^n + (aR)^n}{\left(\frac{a}{R^3}\right)^n + \left(\frac{a}{R}\right)^n} = \frac{\frac{1}{R^n} + R^n}{\frac{1}{R^{3n}} + \frac{1}{R^n}}$$

$$= \frac{\frac{1 + R^{2n}}{R^n}}{\frac{1 + R^{2n}}{R^{3n}}} = \frac{R^{3n}}{R^n} = R^{2n}$$

$$\text{R.H.S.} = \frac{r^n + s^n}{q^n + r^n} = \frac{(aR)^n + (aR^3)^n}{\left(\frac{a}{R}\right)^n + (aR)^n} = \frac{R^n + R^{3n}}{\frac{1}{R^n} + R^n}$$

$$= \frac{R^n(1 + R^{2n})}{\frac{1}{R^n}(1 + R^{2n})} = R^{2n}$$

$$\therefore \quad \text{we get } \frac{q^n + r^n}{p^n + q^n} = \frac{r^n + s^n}{q^n + r^n}$$

$$\therefore \quad p^n + q^n, q^n + r^n, r^n + s^n \text{ are in G.P.}$$

Question 32.
Find the coefficient x_6 in the expression of e_{2x} using series expansion.
Solution:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$e^{2x} = 1 + \frac{2x}{1!} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \frac{(2x)^5}{5!} + \frac{(2x)^6}{6!} + \ldots$$

$$\therefore \quad \text{Coefficient of } x^6 = \frac{2^6}{6!} = \frac{64}{720} = \frac{8}{90} = \frac{4}{45}$$

Question 33.
Find the sum of infinite terms of 1+45+725+10125+13625+…
Solution:

$S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \dots$ ...(i)

Multiplying (i) by $\frac{1}{5}$, we get

$\frac{1}{5}S = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \frac{10}{5^4} + \dots$ ...(ii)

Equation (i) – (ii), we get

$\frac{4}{5}S = 1 + \frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + \dots$

The terms $\frac{3}{5}, \frac{3}{5^2}, \frac{3}{5^3}, \dots$ are in G.P.

$\therefore$ $a = \frac{3}{5}, r = \frac{1}{5}, |r| < 1$

$\therefore$ Sum to the infinity exists.

$\therefore$ $\frac{4}{5}S = 1 + \frac{a}{1-r}$

$= 1 + \frac{\frac{3}{5}}{1-\frac{1}{5}}$

$= 1 + \frac{3}{4}$

$\therefore$ $\frac{4}{5}S = \frac{7}{4}$

$\therefore$ $S = \frac{35}{16}$