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Maharashtra State Board 11th Commerce Maths Solutions Chapter 6 Permutations and Combinations Ex 6.1

Question 1.
A teacher wants to select the class monitor in a class of 30 boys and 20 girls. In how many ways can he select a student if the monitor can be a boy or a girl?
Solution:
There are 30 boys and 20 girls in a class.
The teacher wants to select a class monitor from these boys and girls.
A boy can be selected in 30 ways and a girl can be selected in 20 ways.
∴ By using the fundamental principle of addition,
in a number of ways either a boy or a girl is selected as a class monitor = 30 + 20 = 50.

Question 2.
In question 1, in how many ways can the monitor be selected if the monitor must be a boy? What is the answer if the monitor must be a girl?
Solution:
(i) Since there are 30 boys in the class
∴ A boy monitor can be selected in 30 ways.
(ii) Since there are 20 girls in the class
∴ A girl monitor can be selected in 20 ways.

Question 3.
A Signal is generated from 2 flags by putting one flag above the other. If 4 flags of different colours are available, how many different signals can be generated?
Solution:
A signal is generated from 2 flags and there are 4 flags of different colours available.
∴ 1st flag can be any one of the available 4 flags.
∴ It can be selected in 4 ways.
Now, 2nd flag is to be selected for which 3 flags are available for a different

signal.
∴ 2nd flag can be anyone from these 3 flags.
∴ It can be selected in 3 ways.
∴ By using the fundamental principle of multiplication,
Total number of ways in which a signal can be generated = $4 \times 3 = 12$
∴ 12 different signals can be generated.

Question 4.
How many two-letter words can be formed using letters from the word SPACE when repetition of letters
(i) is allowed
(ii) is not allowed
Solution:
A two-letter word is to be formed out of the letters of the word SPACE.
(i) When repetition of the letters is allowed
1st letter can be selected in 5 ways
2nd letter can be selected in 5 ways
∴ By using the fundamental principle of multiplication,
total number of 2-letter words = $5 \times 5 = 25$

(ii) When repetition of the letters is not allowed
1st letter can be selected in 5 ways
2nd letter can be selected in 4 ways
∴ By using the fundamental principle of multiplication,
total number of 2-letter words = $5 \times 4 = 20$

Question 5.
How many three-digit numbers can be formed from the digits 0, 1, 3, 5, 6 if repetitions of digits
(i) are allowed
(ii) are not allowed
Solution:
The three-digit number is to be formed from the digits 0, 1, 3, 5, 6
(i) When repetition of digits is allowed:
100's place digit should be a non-zero number.
Hence, it can be anyone from digits 1, 3, 5, 6

∴ 100's place digit can be selected in 4 ways.
0 can appear in 10's and unit's place and digits can be repeated.
∴ 10's place digit can be selected in 5 ways and the unit's place digit can be selected in 5 ways.
∴ By using the fundamental principle of multiplication,
the total number of three-digit numbers = $4 \times 5 \times 5 = 100$

(ii) When repetition of digits is not allowed:
100's place digit should be a non-zero number.
Hence, it can be anyone from digits 1, 3, 5, 6
∴ 100's place digit can be selected in 4 ways
0 can appear in 10's and unit's place and digits can't be repeated.
∴ 10's place digit can be selected in 4 ways and the unit's place digit can be selected in 3 ways
∴ By using the fundamental principle of multiplication,
total number of three-digit numbers = $4 \times 4 \times 3 = 48$

Question 6.
How many three-digit numbers can be formed using the digits 2, 3, 4, 5, 6 if digits can be repeated?
Solution:
A 3-digit number is to be formed from the digits 2, 3, 4, 5, 6 where digits can be repeated.
∴ The unit's place digit can be selected in 5 ways.
10's place digit can be selected in 5 ways.
100's place digit can be selected in 5 ways.
∴ By using fundamental principle of multiplication,
the total number of 3-digit numbers = $5 \times 5 \times 5 = 125$

Question 7.
A letter lock has 3 rings and each ring has 5 letters. Determine the maximum number of trials that may be required to open the lock.
Solution:
A letter lock has 3 rings, each ring containing 5 different letters.
∴ A letter from each ring can be selected in 5 ways.
∴ By using fundamental principle of multiplication,

the total number of trials that can be made = 5 × 5 × 5 = 125
Out of these 124 wrong attempts are made and in the 125th attempt, the lock gets opened, for a maximum number of trials.
∴ A maximum number of trials required to open the lock is 125.

Question 8.
In a test that has 5 true/false questions, no student has got all correct answers and no sequence of answers is repeated. What is the maximum number of students for this to be possible?
Solution:
For a set of 5 true/false questions, each question can be answered in 2 ways.
∴ By using fundamental principle of multiplication,
the total number of possible sequences of answers = 2 × 2 × 2 × 2 × 2 = 32
Since no student has written all the correct answers.
∴ Total number of sequences of answers given by the students in the class = 32 – 1 = 31
Also, no student has given the same sequence of answers.
∴ Maximum number of students in the class = Number of sequences of answers given by the students = 31

Question 9.
How many numbers between 100 and 1000 have 4 in the unit's place?
Solution:
Numbers between 100 and 1000 are 3-digit numbers.
A 3-digit number is to be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 where the unit place digit is 4.
Since Unit's place digit is 4.
∴ it can be selected in 1 way only.
10's place digit can be selected in 10 ways.
For 3-digit number 100's place digit should be a non-zero number.
∴ 100's place digit can be selected in 9 ways.
∴ By using fundamental principle of multiplication,
total number of numbers between 100 and 1000 which have 4 in the units place = 1 × 10 × 9 = 90

Question 10.
How many numbers between 100 and 1000 have the digit 7 exactly once?
Solution:
Numbers between 100 and 1000 are 3-digit numbers.
A 3-digit number is to be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, where exactly one of the digits is 7.
When 7 is in the unit's place
The unit's place digit is 7.
∴ it can be selected in 1 way only.
10's place digit can be selected in 9 ways.
100's place digit can be selected in 8 ways.
∴ total number of numbers which have 7 in the unit's place = 1 × 9 × 8 = 72
When 7 is in 10's place
The unit's place digit can be selected in 9 ways.
10's place digit is 7
∴ it can be selected in 1 way only.
100's place digit can be selected in 8 ways.
∴ total number of numbers which have 7 in 10's place = 9 × 1 × 8 = 72
When 7 is in 100's place
The unit's place digit can be selected in 9 ways.
10's place digit can be selected in 9 ways.
100's place digit is 7
∴ it can be selected in 1 way.
∴ total numbers which have 7 in 100's place = 9 × 9 × 1 = 81
∴ total number of numbers between 100 and 1000 having digit 7 exactly once = 72 + 72 + 81 = 225.

Question 11.
How many four-digit numbers will not exceed 7432 if they are formed using the digits 2, 3, 4, 7 without repetition?
Solution:
Among many set's of digits, the greatest number is possible when digits are arranged in descending order.
∴ 7432 is the greatest number, formed from the digits 2, 3, 4, 7.
∴ Since a 4-digit number is to be formed from the digits 2, 3, 4, 7, where repetition of the digit is not allowed.
∴ 1000's place digit can be selected in 4 ways.

100's place digit can be selected in 3 ways.
10's place digit can be selected in 2 ways.
The unit's place digit can be selected in 1 way.
∴ Total number of numbers not exceeding 7432 that can be formed from the digits 2, 3, 4, 7
= Total number of four-digit numbers formed from the digits 2, 3, 4, 7
= 4 × 3 × 2 × 1
= 24

Question 12.
If numbers are formed using digits 2, 3, 4, 5, 6 without repetition, how many of them will exceed 400?
Solution:
Case I: Three-digit numbers with 4 occurring in hundred's place:
100's place digit can be selected in 1 way.
Ten's place can be filled by any one of the numbers 2, 3, 5, 6.
∴ 10's place digit can be selected in 4 ways.
The unit's place digit can be selected in 3 ways.
∴ total number of numbers which have 4 in 100's place = 1 × 4 × 3 = 12

Case II: Three-digit numbers more than 500
100's place digit can be selected in 2 ways.
10's place digit can be selected in 4 ways.
Unit's place digit can be selected in 3 ways.
∴ total number of three digit numbers more than 500 = 2 × 4 × 3 = 24

Case III: Number of four digit numbers formed from 2, 3, 4, 5, 6
Since, repetition of digits is not allowed
∴ total four digit numbers formed = 5 × 4 × 3 × 2 = 120

Case IV: Number of five digit numbers formed from 2, 3, 4, 5, 6
Since, repetition of digits is not allowed
∴ total five digit numbers formed = 5 × 4 × 3 × 2 × 1 = 120
∴ total number of numbers that exceed 400 = 12 + 24 + 120 + 120 = 276

Question 13.
How many numbers formed with the digits 0, 1, 2, 5, 7, 8 will fall between

13 and 1000 if digits can be repeated?
Solution:
Case I: 2-digit numbers more than 13, less than 20, formed from the digits 0, 1, 2, 5, 7, 8
Number of such numbers = 3

Case II: 2-digit numbers more than 20 formed from 0, 1, 2, 5, 7, 8
Ten’s place digit is selected from 2, 5, 7, 8.
∴ Ten’s place digit can be selected in 4 ways.
Unit’s place digit is anyone from 0, 1, 2, 5, 7, 8
∴ The unit’s place digit can be selected in 6 ways.
Using the multiplication principle,
the number of such numbers (repetition allowed) = $4 \times 6 = 24$

Case III: 3-digit numbers formed from 0, 1, 2, 5, 7, 8
100’s place digit is anyone from 1, 2, 5, 7, 8.
∴ 100’s place digit can be selected in 5 ways.
As digits can be repeated, the 10’s place and unit’s place digits are selected from 0, 1, 2, 5, 7, 8
∴ 10’s place and unit’s place digits can be selected in 6 ways each.
Using multiplication principle,
the number of such numbers (repetition allowed) = $5 \times 6 \times 6 = 180$
All cases are mutually exclusive and exhaustive.
∴ Required number = $3 + 24 + 180 = 207$

Question 14.
A school has three gates and four staircases from the first floor to the second floor. How many ways does a student have to go from outside the school to his classroom on the second floor?
Solution:
A student can go inside the school from outside in 3 ways and from the first floor to the second floor in 4 ways.
∴ Number of ways to choose gates = 3
Number of ways to choose staircase = 4
∴ By using fundamental principle of multiplication,
number of ways in which a student has to go from outside the school to his classroom = $4 \times 3 = 12$

Question 15.
How many five-digit numbers formed using the digit 0, 1, 2, 3, 4, 5 are divisible by 3 if digits are not repeated?
Solution:
For a number to be divisible by 3.
The sum of digits must be divisible by 3.
Given 6 digits are 0, 1,2, 3, 4, 5.
Sum of 1, 2, 3, 4, 5 = 15, which is divisible by 3.
∴ There are two cases of 5 digit numbers formed from 0, 1, 2, 3, 4, 5 and divisible by 3.
Either 3 is selected in 5 digits (and 0 not selected) or 3 is not selected in 5 digits (and 0 is selected)
Case I:
3 is not selected (and 0 is selected) i.e., the digits are 0, 1, 2, 4, 5.
10000's place digit can be selected in 4 ways (as 0 cannot appear).
As digits are not repeated, 1000's place digit can be selected in 4 ways.
100's place digit can be selected in 3 ways.
10's place digit can be selected in 2 ways.
The unit's place digit can be selected in 1 way.
∴ Using multiplication theorem,
Number of 5-digit number formed from 0, 1, 2, 4, 5 (with no repetition of digits) = $4 \times 4 \times 3 \times 2 \times 1 = 96$

Case II:
3 is selected (and 0 is not selected) i.e., 1, 2, 3, 4, 5
10000's place digit can be selected in 5 ways.
1000's place digit can be selected in 4 ways.
100's place digit can be selected in 3 ways.
10's place digit can be selected in 2 ways.
The unit's place digit can be selected in 1 way.
Using multiplication theorem,
Number of 5-digit numbers formed from 1, 2, 3, 4, 5 = $5 \times 4 \times 3 \times 2 \times 1 = 120$
Both the cases are mutually exclusive and exhaustive.
∴ Required number = 96 + 120 = 216

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Question 1.
Evaluate:
(i) 8!
Solution:
8!
= 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1
= 40320

(ii) 6!
Solution:
6!
= 6 × 5 × 4 × 3 × 2 × 1
= 720

(iii) 8! – 6!
Solution:
8! – 6!
= 8 × 7 × 6! – 6!
= 6! (8 × 7 – 1)
= 6! (56 – 1)
= 6 × 5 × 4 × 3 × 2 × 1 × 55
= 39,600

(iv) (8 – 6)!
Solution:
(8 – 6)!
= 2!
= 2 × 1
= 2

Question 2.
Compute:
(i) 12!6!
Solution:
12!6!
= 12×11×10×9×8×7×6!6!
= 12 × 11 × 10 × 9 × 8 × 7
= 665280

(ii) (126)!
Solution:
(126)!
= 2!
= 2 × 1
= 2

(iii) (3 × 2)!
Solution:
(3 × 2)!
= 6!
= 6 × 5 × 4 × 3 × 2 × 1
= 720

(iv) 3! × 2!
Solution:
3! × 2!
= 3 × 2 × 1 × 2 × 1
= 12

Question 3.
Compute:
(i) 9!3!6!
Solution:

$$\frac{9!}{3!\,6!} = \frac{9 \times 8 \times 7 \times 6!}{(3 \times 2 \times 1) \times 6!} = 84$$

(ii) $\frac{6!-4!}{4!}$
Solution:
$$\frac{6!-4!}{4!} = \frac{6\times5\times4!-4!}{4!} = \frac{4!(6\times5-1)}{4!} = 29$$

(iii) $\frac{8!}{6!-4!}$
Solution:
$$\frac{8!}{6!-4!} = \frac{8\times7\times6\times5\times4!}{6\times5\times4!-4!}$$
$$= \frac{8\times7\times6\times5\times4!}{4!(6\times5-1)} = \frac{1680}{29}$$

(iv) $\frac{8!}{(6-4)!}$
Solution:
$$\frac{8!}{(6-4)!} = \frac{8!}{2!} = \frac{8\times7\times6\times5\times4\times3\times2!}{2!} = 20160$$

Question 4.
Write in terms of factorials
(i) 5 × 6 × 7 × 8 × 9 × 10
Solution:
5 × 6 × 7 × 8 × 9 × 10
= 10 × 9 × 8 × 7 × 6 × 5
Multiplying and dividing by 4!, we get
$$= \frac{10\times9\times8\times7\times6\times5\times4!}{4!}$$
$$= \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{4!} = \frac{10!}{4!}$$

(ii) 3 × 6 × 9 × 12 × 15
Solution:
3 × 6 × 9 × 12 × 15
= 3 × (3 × 2) × (3 × 3) × (3 × 4) × (3 × 5)
= (3^5) (5 × 4 × 3 × 2 × 1)
= 3^5 (5!)

(iii) 6 × 7 × 8 × 9
Solution:

6 × 7 × 8 × 9
= 9 × 8 × 7 × 6
Multiplying and dividing by 5!, we get

$$= \frac{9\times8\times7\times6\times5!}{5!}$$

$$= \frac{9\times8\times7\times6\times5\times4\times3\times2\times1}{5!} = \frac{9!}{5!}$$

(iv) 5 × 10 × 15 × 20 × 25
Solution:
5 × 10 × 15 × 20 × 25
= (5 × 1) × (5 × 2) × (5 × 3) × (5 × 4) × (5 × 5)
= (5^5) (5 × 4 × 3 × 2 × 1)
= (5^5) (5!)

Question 5.
Evaluate: n!r!(n−r)! for
(i) n = 8, r = 6
Solution:
n = 8, r = 6

$$\therefore \quad \frac{n!}{r!(n-r)!} = \frac{8!}{6!(8-6)!}$$

$$= \frac{8\times7\times6!}{6!\,2!}$$

$$= \frac{8\times7}{2!}$$

$$= \frac{8\times7}{1\times2} = 28$$

(ii) n = 12, r = 12
Solution:
n = 12, r = 12

$$\therefore \quad \frac{n!}{r!(n-r)!} = \frac{12!}{12!(12-12)!}$$

$$= \frac{12!}{12!\,0!}$$

$$= 1 \qquad \ldots[\because 0! = 1]$$

Question 6.
Find n, if
(i) $\frac{n}{8!}=\frac{3}{6!}+\frac{1}{4!}$
Solution:

$$\frac{n}{8!}=\frac{3}{6!}+\frac{1}{4!}$$

$$\therefore \quad \frac{n}{8!}=\frac{3}{6!}+\frac{6\times5}{6\times5\times4!}$$

$$\therefore \quad \frac{n}{8!}=\frac{3}{6!}+\frac{30}{6!}$$

$$\therefore \quad \frac{n}{8\times7\times6!}=\frac{33}{6!}$$

$$\therefore \quad \frac{n}{56}=33$$

$$\therefore \quad n=56\times33=1848$$

(ii) $\frac{n}{6!}=\frac{4}{8!}+\frac{3}{6!}$
Solution:

$$\frac{n}{6!}=\frac{4}{8!}+\frac{3}{6!}$$

$$\therefore \quad \frac{n}{6!}-\frac{3}{6!}=\frac{4}{8!}$$

$$\therefore \quad \frac{n-3}{6!}=\frac{4}{8\times7\times6!}$$

$$\therefore \quad n-3=\frac{4}{8\times7}$$

$$\therefore \quad n-3=\frac{1}{14}$$

$$\therefore \quad n=\frac{1}{14}+3=\frac{43}{14}$$

(iii) $\frac{1}{n!} = \frac{1}{4!} - \frac{4}{5!}$

Solution:

$$\frac{1}{n!} = \frac{1}{4!} - \frac{4}{5!}$$

$$\therefore \quad \frac{1}{n!} = \frac{1}{4!} - \frac{4}{5!}$$

$$\therefore \quad \frac{1}{n!} = \frac{5}{5 \times 4!} - \frac{4}{5!}$$

$$\therefore \quad \frac{1}{n!} = \frac{5}{5!} - \frac{4}{5!}$$

$$\therefore \quad \frac{1}{n!} = \frac{1}{5!}$$

$$\therefore \quad n! = 5!$$

$$\therefore \quad n = 5$$

Question 7.

Find n, if

(i) (n + 1)! = 42 × (n – 1)!

Solution:

(n + 1)! = 42(n – 1)!

∴ (n + 1) n (n – 1)! = 42(n – 1)!

∴ $n^2 + n = 42$

∴ n(n + 1) = 6 × 7

Comparing on both sides, we get

∴ n = 6

(ii) (n + 3)! = 110 × (n + 1)!

Solution:

(n + 3)! = 110 × (n + 1)!

∴ (n + 3) (n + 2) (n + 1)! = 110 (n + 1)!

∴ (n + 3) (n + 2) = (11) (10)

Comparing on both sides, we get

n + 3 = 11

∴ n = 8

Question 8.

Find n, if:

(i) $\frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-5)!} = 5:3$
Solution:

$$\frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-5)!} = 5 : 3$$

$$\therefore \quad \frac{n!}{3!(n-3)!} \times \frac{5!(n-5)!}{n!} = \frac{5}{3}$$

$$\therefore \quad \frac{n!}{3!(n-3)(n-4)(n-5)!} \times \frac{5 \times 4 \times 3!(n-5)!}{n!} = \frac{5}{3}$$

$$\therefore \quad \frac{5 \times 4}{(n-3)(n-4)} = \frac{5}{3}$$

∴ 12 = (n – 3)(n – 4)
∴ (n – 3)(n – 4) = 4 × 3
Comparing on both sides, we get
n – 3 = 4
∴ n = 7

(ii) $\frac{n!}{3!(n-5)!} : \frac{n!}{5!(n-7)!} = 10:3$
Solution:

$$\frac{n!}{3!(n-5)!} : \frac{n!}{5!(n-7)!} = 10:3$$

$$\therefore \quad \frac{n!}{3!(n-5)!} \times \frac{5!(n-7)!}{n!} = \frac{10}{3}$$

$$\therefore \quad \frac{n!}{3!(n-5)(n-6)(n-7)!} \times \frac{5 \times 4 \times 3!(n-7)!}{n!} = \frac{10}{3}$$

$$\therefore \quad \frac{5 \times 4}{(n-5)(n-6)} = \frac{10}{3}$$

∴ (n – 5) (n – 6) = 3 × 2
Comparing on both sides, we get
n – 5 = 3
∴ n = 8

Question 9.
Find n, if:
(i) $\frac{(17-n)!}{(14-n)!} = 5!$
Solution:
$\frac{(17-n)!}{(14-n)!} = 5!$
∴ $\frac{(17-n)(16-n)(15-n)(14-n)!}{(14-n)!} = 5 \times 4 \times 3 \times 2 \times 1$
∴ (17 – n) (16 – n) (15 – n) = 6 × 5 × 4

Comparing on both sides, we get
17 – n = 6
∴ n = 11

(ii) $(15-n)!(13-n)!$ = 12
Solution:
$(15-n)!(13-n)!$ = 12
∴ $(15-n)(14-n)(13-n)!(13-n)!$ = 12
∴ (15 – n) (14 – n) = 4 × 3
Comparing on both sides, we get
15 – n = 4
∴ n = 11

Question 10.
Find n if $(2n)!7!(2n-7)!:n!4!(n-4)!$ = 24 : 1
Solution:

$$\frac{(2n)!}{7!(2n-7)!} : \frac{n!}{4!(n-4)!} = 24 : 1$$

$$\therefore \quad \frac{(2n)!}{7!(2n-7)!} \times \frac{4!(n-4)!}{n!} = 24$$

$$\therefore \quad \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)(2n-5)(2n-6)(2n-7)!}{7\times6\times5\times4!(2n-7)!} \times \frac{4!(n-4)!}{n(n-1)(n-2)(n-3)(n-4)!} = 24$$

$$\therefore \quad \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)(2n-5)(2n-6)}{7\times6\times5} \times \frac{1}{n(n-1)(n-2)(n-3)} = 24$$

$$\therefore \quad \frac{(2n)(2n-1)2(n-1)(2n-3)2(n-2)(2n-5)2(n-3)}{7\times6\times5} \times \frac{1}{n(n-1)(n-2)(n-3)} = 24$$

$$\therefore \quad \frac{16(2n-1)(2n-3)(2n-5)}{7\times6\times5} = 24$$

$$\therefore \quad (2n-1)(2n-3)(2n-5) = \frac{24\times7\times6\times5}{16}$$

∴ (2n – 1) (2n – 3) (2n – 5) = 9 × 7 × 5
Comparing on both sides, we get

2n – 1 = 9
∴ n = 5

Question 11.
Show that $n!r!(n-r)!+n!(r-1)!(n-r+1)!=(n+1)!r!(n-r+1)$
Solution:

$$
\begin{aligned}
\text{L.H.S.} &= \frac{n!}{r!(n-r)!}+\frac{n!}{(r-1)!(n-r+1)!}\\
&= \frac{n!}{r(r-1)!(n-r)!}+\frac{n!}{(r-1)!(n-r+1)(n-r)!}\\
&= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r}+\frac{1}{n-r+1}\right]\\
&= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(n+1).n!}{r.(r-1)!(n-r+1)(n-r)!}\\
&= \frac{(n+1)!}{r!(n-r+1)} = \text{R.H.S}
\end{aligned}
$$

Question 12.
Show that $9!3!6!+9!4!5!= 10!4!6!$
Solution:

$$
\begin{aligned}
\text{L.H.S.} &= \frac{9!}{3!\,6!}+\frac{9!}{4!\,5!} = \frac{9!}{3!\times 6\times 5!}+\frac{9!}{4\times 3!\times 5!}\\
&= \frac{9!}{3!\,5!}\left[\frac{1}{6}+\frac{.}{4}\right]\\
&= \frac{9!}{3!\,5!}\left[\frac{4+6}{6\times 4}\right] = \frac{10\times 9!}{6\times 5!\times 4\times 3!}\\
&= \frac{10!}{6!\,4!} = \frac{10!}{4!\,6!} = \text{R.H.S.}
\end{aligned}
$$

Question 13.
Find the value of:
(i) $8!+5(4!)4!-12$

Solution:

$$\begin{aligned}\frac{8!+5(4!)}{4!-12} &= \frac{8!+5!}{4\times3\times2-12}\\ &= \frac{8\times7\times6\times5!+5!}{4\times3\times(2-1)}\\ &= \frac{5!(8\times7\times6+1)}{4\times3}\\ &= \frac{5\times4\times3\times2\times1(336+1)}{4\times3}\\ &= 5\times2\times337 = 3370\end{aligned}$$

(ii) $5(26!)+(27!)4(27!)-8(26!)$

Solution:

$$\begin{aligned}\frac{5(26!)+27!}{4(27!)-8(26!)} &= \frac{5(26!)+27(26!)}{4(27\times26!)-8(26!)}\\ &= \frac{26!(5+27)}{4(26!)(27-2)}\\ &= \frac{32}{(4)(25)} = \frac{8}{25}\end{aligned}$$

Question 14.

Show that

$(2n)!n! = 2^n (2n-1)(2n-3)\ldots5.3.1$

Solution:

$$\text{L.H.S.} = \frac{(2n)!}{n!}$$

$$= \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)\ldots 6\times5\times4\times3\times2\times1}{n!}$$

$$= \frac{(2n)(2n-1)[2(n-1)](2n-3)[2(n-2)]\ldots(2\times3)\times5 \times(2\times2)\times3\times(2\times1)\times1}{n!}$$

$$= \frac{2^n[n(n-1)(n-2)\ldots3.2.1][(2n-1)(2n-3)\ldots5.3.1]}{n!}$$

$$= \frac{2^n(n!)(2n-1)(2n-3)\ldots5.3.1}{n!}$$

$$= 2^n(2n-1)(2n-3)\ldots5.3.1 = \text{R.H.S.}$$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 6 Permutations and Combinations Ex 6.3

Question 1.
Find n if ${}^nP_6 : {}^nP_3 = 120 : 1$
Solution:
${}^nP_6 : {}^nP_3 = 120 : 1$
$\therefore n!(n-6)! \div n!(n-3)! = 1201$
$\therefore n!(n-6)! \times (n-3)!n! = 120$
$\therefore n!(n-6)! \times (n-3)(n-4)(n-5)(n-6)!n! = 120$
$\therefore (n-3)(n-4)(n-5) = 120$
$\therefore (n-3)(n-4)(n-5) = 6 \times 5 \times 4$
Comparing on both sides, we get
$n - 3 = 6$
$\therefore n = 9$

Question 2.
Find m and n if ${}^{(m+n)}P_2 = 56$ and ${}^{(m-n)}P_2 = 12$
Solution:

${}^{m+n}P_2 = 56$

$\therefore (m+n)!(m+n-2)! = 56$

$\therefore (m+n)(m+n-1)(m+n-2)!(m+n-2)! = 56$

$\therefore (m + n)(m + n - 1) = 8 \times 7$

Comparing on both sides, we get

$m + n = 8$ .....(i)

Also ${}^{m-n}P_2 = 12$

$\therefore (m-n)!(m-n-2)! = 12$

$\therefore (m-n)(m-n-1)(m-n-2)!(m-n-2)! = 12$

$\therefore (m - n)(m - n - 1) = 4 \times 3$

Comparing on both sides, we get

$m - n = 4$ .....(ii)

Adding (i) and (ii), we get

$2m = 12$

$\therefore m = 6$

Substituting $m = 6$ in (ii), we get

$6 - n = 4$

$\therefore n = 2$

Question 3.

Find r if ${}^{12}P_{r-2} : {}^{11}P_{r-1} = 3 : 14$

Solution:

$${}^{12}P_{r-2} : {}^{11}P_{r-1} = 3 : 14$$

$$\therefore \quad \frac{12!}{(12-r+2)!} \div \frac{11!}{(11-r+1)!} = \frac{3}{14}$$

$$\therefore \quad \frac{12!}{(14-r)!} \times \frac{(12-r)!}{11!} = \frac{3}{14}$$

$$\therefore \quad \frac{12 \times 11!}{(14-r)(13-r)(12-r)!} \times \frac{(12-r)!}{11!} = \frac{3}{14}$$

$$\therefore \quad \frac{12}{(14-r)(13-r)} = \frac{}{14}$$

$\therefore (14 - r)(13 - r) = 8 \times 7$

Comparing on both sides, we get

$14 - r = 8$

$\therefore r = 6$

Question 4.

Show that $(n + 1)\,{}^{n}P_r = (n - r + 1)\,{}^{(n+1)}P_r$.

Solution:

$$\text{L.H.S.} = (n + 1)\ {}^{n}P_r = (n + 1)\frac{n!}{(n-r)!} = \frac{(n+1)!}{(n-r)!}$$

$$\text{R.H.S.} = (n - r + 1)\ {}^{(n+1)}P_r = (n - r + 1)\ \frac{(n+1)!}{(n-r+1)!}$$

$$= \frac{(n-r+1)(n+1)!}{(n-r+1)(n-r)!}$$

$$= \frac{(n+1)!}{(n-r)!}$$

$\therefore$ L.H.S. = R.H.S.

Question 5.
How many 4 letter words can be formed using letters in the word MADHURI if
(i) letters can be repeated?
(ii) letters cannot be repeated?
Solution:
There are 7 letters in the word MADHURI.
(i) A 4 letter word is to be formed from the letters of the word MADHURI and repetition of letters is allowed.
$\therefore$ 1st letter can be filled in 7 ways.
2nd letter can be filled in 7 ways.
3rd letter can be filled in 7 ways.
4th letter can be filled in 7 ways.
$\therefore$ Total no. of ways a 4-letter word can be formed = $7 \times 7 \times 7 \times 7 = 2401$
$\therefore$ 2401 four-lettered words can be formed when repetition of letters is allowed.

(ii) When repetition of letters is not allowed, the number of 4-letter words formed from the letters of the word MADHURI is
${}^7P_4 = \frac{7!}{(7-4)!} = \frac{7\times6\times5\times4\times3!}{3!} = 840$
$\therefore$ 840 four-letter words can be formed when repetition of letters is not allowed.

Alternate method:
There are 7 letters in the word MADHURI.

(i) Since letters can be repeated
∴ In all places of a four-letter word, any one of seven letters M, A, D, H, U, R, I can appear.
∴ Using the Multiplication theorem, we get
Number of four-letter words with repetition of letters M, A, D, H, U, R, I = 7 × 7 × 7 × 7 = 2401

(ii) Since the letters cannot be repeated therefore 1st, 2nd, 3rd, 4th places can be filled in 7, 6, 5, 4 ways respectively
∴ Using the multiplication theorem, we get
Number of four-letter words, with no repetition of letters M, A, D, H, U, R, I = 7 × 6 × 5 × 4 = 840

Question 6.
Determine the number of arrangements of letters of the word ALGORITHM if
(i) vowels are always together.
(ii) no two vowels are together.
(iii) Consonants are at even positions
(iv) O is first and T is last.
Solution:
A word is to be formed using the letters of the word ALGORITHM.
There are 9 letters in the word ALGORITHM.
(i) When vowels are always together:
There are 3 vowels in the word ALGORITHM. (i.e, A, I, O)
Let us consider these 3 vowels as one unit.
This unit with 6 other letters is to be arranged.
∴ It becomes an arrangement of 7 things which can be done in ${}^7P_7$ i.e., 7! ways and 3 vowels can be arranged among themselves in ${}^3P_3$ i.e., 3! ways.
∴ the total number of ways in which the word can be formed = 7! × 3!
= 5040 × 6
= 30240
∴ 30240 words can be formed if vowels are always together.

(ii) When no two vowels are together:
There are 6 consonants in the word ALGORITHM.
They can be arranged among themselves in ${}^6P_6$ i.e., 6! ways.
Let consonants be denoted by C.

_C_C_ C_C_C_C_
6 consonants create 7 gaps in which 3 vowels are to arranged.
∴ 3 vowels can be filled in $^7P_3$
$= 7!(7-3)!$
$= 7\times6\times5\times4!4!$
= 210 ways
∴ total number of ways in which the word can be formed = 6! × 210
= 720 × 210
= 151200
∴ 151200 words can be formed if no two vowels are together.

(iii) When consonants are at even positions:
There are 4 even places and 6 consonants in the word ALGORITHM.
1st, 2nd, 3rd, 4th even places are filled in 6, 5, 4, 3 way respectively.
∴ The number of ways to fill four even places by consonants = 6 × 5 × 4 × 3 = 360
The remaining 5 letters (3 vowels and 2 consonants) can be arranged among themselves in $^5P_5$ i.e., 5! ways.
∴ Total number of ways the words can be formed
In which even places are occupied by consonants = 360 × 5!
= 360 × 120
= 43200
∴ 43200 words can be formed if even positions are occupied by consonants.

(iv) When beginning with O and ends with T:
All the letters of the word ALGORITHM are to be arranged among themselves such that arrangement begins with O and ends with T.
7 letters other than O and T can be filled between O and T in $^7P_7$ i.e., 7! ways = 5040 ways.
∴ 5040 words beginning with O and ending with T can be formed.

Question 7.
In a group photograph, 6 teachers and principals are in the first row and 18 students are in the second row. There are 12 boys and 6 girls among the students. If the middle position is reserved for the principal and if no two girls are together, find the number of arrangements.
Solution:

In 1st row middle seat is fixed for the principal.
Also 1st row, 6 teachers can be arranged among themselves in ${}^6P_6$ i.e., 6! ways.
In the 2nd row, 12 boys can be arranged among themselves in ${}^{12}P_{12}$ i.e., 12! ways.
13 gaps are created by 12 boys, in which 6 girls are to be arranged.
together which can be done in ${}^{13}P_6$ ways.
∴ total number of arrangements = 6! × 12! × ${}^{13}P_6$ …..[using Multiplications Principle]
= 6! × 12! × $\frac{13!}{(13-6)!}$
= 6! × 12! × $\frac{13!}{7!}$
= $\frac{6! \times 12! \times 13!}{7 \times 6!}$
= $\frac{12!\,13!}{7}$

Question 8.
Find the number of ways letters of the word HISTORY can be arranged if
(i) Y and T are together
(ii) Y is next to T.
Solution:
There are 7 letters in the word HISTORY
(i) When 'Y' and 'T' are together.
Let us consider ' Y' and 'T' as one unit
This unit with the other 5 letters is to be arranged.
∴ The number of arrangements of one unit and 5 letters = ${}^6P_6$ = 6!
Also, 'Y' and 'T' can be arranged among themselves in ${}^2P_2$ i.e., 2! ways.
∴ a total number of arrangements when Y and T are always together = 6! × 2!
= 720 × 2
= 1440
∴ 1440 words can be formed if Y and T are together.

(ii) When 'Y' is next to 'T'
Let us take this ('Y' next to 'T') as one unit.
This unit with 5 other letters is to be arranged.
∴ The number of arrangements of 6 letters and one unit = ${}^6P_6$ = 6!
Also 'Y' has to be always next to 'T'.

So they can be arranged in 1 way.
∴ total number of arrangements possible when Y is next to T = 6! × 1 = 720
∴ 720 words can be formed if Y is next to T.

Question 9.
Find the number of arrangements of the letters in the word BERMUDA so that consonants and vowels are in the same relative positions.
Solution:
There are 7 letters in the word "BERMUDA" out of which 3 are vowels and 4 are consonants.
If relative positions of consonants and vowels are not changed.
3 vowels can be arranged among themselves in ${}^3P_3$ i.e., 3! ways.
4 consonants can be arranged among themselves in ${}^4P_4$ i.e., 4! ways.
∴ total no. of arrangements possible if relative positions of vowels and consonants are not changed = 3! × 4!
= 6 × 24
= 144

Question 10.
Find the number of 4-digit numbers that can be formed using the digits 1, 2, 4, 5, 6, 8 if
(i) digits can be repeated
(ii) digits cannot be repeated
Solution:
(i) A 4 digit number is to be made from the digits 1, 2, 4, 5, 6, 8 such that digits can be repeated.
∴ The unit's place digit can be filled in 6 ways.
10's place digit can be filled in 6 ways.
100's place digit can be filled in 6 ways.
1000's place digit can be filled in 6 ways.
∴ total number of numbers = 6 × 6 × 6 × 6 = 6^4 = 1296
∴ 1296 four-digit numbers can be formed if repetition of digits is allowed.

(ii) A 4 different digit number is to be made from the digits 1, 2, 4, 5, 6, 8 without repetition of digits.
∴ 4 different digits are to be arranged from 6 given digits which can be done in ${}^6P_4$
= $6!(6-4)!$

= $6\times5\times4\times3\times2!2!$
= 360 ways
∴ 360 four-digit numbers can be formed, if repetition of digits is not allowed.

Question 11.
How many numbers can be formed using the digits 0, 1, 2, 3, 4, 5 without repetition so that the resulting numbers are between 100 and 1000?
Solution:
A number between 100 and 1000 is a 3 digit number and is to be formed from the digits 0, 1, 2, 3, 4, 5, without repetition of digits.
∴ 100's place digit must be a non-zero number which can be filled in 5 ways.
10's place digits can be filled in 5 ways.
Unit's place digit can be filled in 4 ways.
∴ total number of ways the number can be formed = 5 × 5 × 4 = 100
∴ 100 numbers between 100 and 1000 can be formed.

Question 12.
Find the number of 6-digit numbers using the digits 3, 4, 5, 6, 7, 8 without repetition. How many of these numbers are
(i) divisible by 5?
(ii) not divisible by 5?
Solution:
A number of 6 different digits is to be formed from the digits 3, 4, 5, 6, 7, 8 which can be done in $^6P_6$ i.e., 6! = 720 ways
(i) If the number is divisible by 5, then
The unit's place digit must be 5, and hence unit's place can be filled in 1 way
Other 5 digits can be arranged among themselves in $^5P_5$ i.e., 5! ways
∴ Total number of ways in which numbers divisible by 5 can be formed = 1 × 5! = 120

(ii) If the number is not divisible by 5, then
Unit's place can be any digit from 3, 4, 6, 7, 8 which can be selected in 5 ways.

Other 5 digits can be arranged in $^5P_5$ i.e., 5! ways
∴ The total number of ways in which numbers not divisible by 5 can be formed = 5 × 5!
= 5 × 120
= 600

Question 13.
A code word is formed by two distinct English letters followed by two non-zero distinct digits. Find the number of such code words. Also, find the number of such code words that end with an even digit.
Solution:
(i) There is a total of 26 alphabets.
A code word contains 2 English alphabets.
∴ 2 alphabets can be filled in $^{26}P_2$
= $26!(26-2)!$
= $26\times25\times24!24!$
= 650 ways
Also, alphabets to be followed by two distinct non-zero digits from 1 to 9 which can be filled in $^9P_2$
= $9!(9-2)!$
= $9\times8\times7!7!$
= 72 ways
∴ Total number of a code words = 650 × 72 = 46800

(ii) There are total 26 alphabets.
A code word contains 2 English alphabets.
∴ 2 alphabets can be filled in $^{26}P_2$
= $26(26-2)!$
= $26\times25\times24!24!$
= 650 ways
For a code word to end with an even integer, the digit in the unit's place should be an even number between 1 to 9 which can be filled in 4 ways.
Also, 10's place can be filled in 8 ways.
∴ Total number of codewords = 650 × 4 × 8 = 20800 ways
∴ 20800 codewords end with an even integer.

Question 14.
Find the number of ways in which 5 letters can be posted in 3 post boxes if any number of letters can be posted in a post box.
Solution:
There are 5 letters and 3 post boxes and any number of letters can be posted in all three post boxes.
∴ Each letter can be posted in 3 ways.
∴ Total number of ways in which 5 letters can be posted = 3 × 3 × 3 × 3 × 3 = 243

Question 15.
Find the number of arranging 11 distinct objects taken 4 at a time so that a specified object
(i) always occurs
(ii) never occurs
Solution:
There are 11 distinct objects and 4 are to be taken at a time.
(i) The number of permutations of n distinct objects, taken r at a time, when one specified object will always occur is $r \times {}^{(n-1)}P_{(r-1)}$
Here, r = 4, n = 11
∴ The number of permutations of 4 out of 11 objects when a specified object occurs.
$= 4 \times {}^{(11-1)}P_{(4-1)}$
$= 4 \times {}^{10}P_3$
$= 4 \times 10!(10-3)!$
$= 4 \times 10!7!$
$= 4 \times 10\times9\times8\times7!7!$
= 2880
∴ There are 2880 permutations of 11 distinct objects, taken 4 at a time, in which one specified object always occurs.

(ii) When one specified object does not occur then 4 things are to be arranged from the remaining 10 things, which can be done in ${}^{10}P_4$ ways
= 10 × 9 × 8 × 7 ways
= 5040 ways

∴ There are 5040 permutations of 11 distinct objects, taken 4 at a time, in which one specified object never occurs.

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Question 1.
Find the number of permutations of letters in each of the following words:
(i) DIVYA
(ii) SHANTARAM
(iii) REPRESENT
(iv) COMBINE
Solution:
(i) There are 5 letters in the word DIVYA which can be arranged in 5! Way = 120 ways

(ii) There are 9 letters in the word SHANTARAM in which 'A' repeats 3 times.
∴ Number of permutations of the letters of the word SHANTARAM = 9!3!
$= 9 \times 8 \times 7 \times 6 \times 5 \times 4$
$= 60480$

(iii) There are 9 letters in the word REPRESENT in which 'E' repeats 3 times and 'R' repeats 2 times.
∴ Number of permutations of the letters of the word REPRESENT = 9!3!2!
$= 9×8×7×6×5×42$
$= 30240$

(iv) There are 7 distinct letters in the word COMBINE which can be arranged among themselves in 7! = 5040 ways

Question 2.
You have 2 identical books on English, 3 identical books on Hindi and 4

identical books on Mathematics. Find the number of distinct ways of arranging them on a shelf.
Solution:
There are total 9 books to be arranged on a shelf.
Out of these 9 books, 2 books on English, 3 books on Hindi and 4 books on Mathematics are identical.
∴ Total number of arrangements = $9!2!3!4!$
= $9×8×7×6×5×4!2×3×2×4!$
= 9 × 4 × 7 × 5
= 1260
∴ In 1260 distinct ways the books can be arranged on a shelf.

Question 3.
A coin is tossed 8 times. In how many ways can we obtain
(i) 4 heads and 4 tails?
(ii) at least 6 heads?
Solution:
A coin is tossed 8 times. All heads are identical and all tails are identical.
(i) We can obtain 4 heads and 4 tails in $8!4!4!$
= $8×7×6×54×3×2$
= 70 ways
∴ In 70 different ways we can obtain 4 heads and 4 tails.

(ii) When at least 6 heads are to be obtained
∴ Outcome can be (6 heads and 2 tails) or (7 heads and 1 tail) or (8 heads)
∴ Number of ways in which it can be obtained = $8!6!2!+8!7!1!+8!8!$
= $8×72$ + 8 + 1
= 28 + 8 + 1
= 37
∴ In 37 different ways we can obtain at least 6 heads.

Question 4.
A bag has 5 red, 4 blue, and 4 green marbles. If all are drawn one by one and their colours are recorded, how many different arrangements can be found?
Solution:
There is a total of 13 marbles in a bag.
Out of these 5 are Red, 4 Blue, and 4 are Green marbles.

All balls of the same colour are taken to be identical.
∴ Required number of arrangements = 13!5!4!4!

Question 5.
Find the number of ways of arranging letters of the word MATHEMATICAL. How many of these arrangements have all vowels together?
Solution:
There are 12 letters in the word MATHEMATICAL in which 'M' repeats 2 times, 'A' repeats 3 times, and 'T' repeats 2 times.
∴ Total number of arrangements = 12!2!3!2!
When all the vowels
i.e., 'A', 'A', 'A', 'E', T are to be kept together
Number of arrangements of these vowels = 5!3! ways.
Let us consider these vowels together as one unit.
This unit is to be arranged with 7 other letters in which 'M' and 'T' repeated 2 times each.
∴ Number of arrangements = 8!2!2!
∴ Total number of arrangements = 8!×5!2!2!3!

Question 6.
Find the number of different arrangements of letters in the word MAHARASHTRA. How many of these arrangements have
(i) letters M and T never together?
(ii) all vowels together?
Solution:
There are 11 letters in the word MAHARASHTRA in which 'A' is repeated 4 times, 'H' repeated 2 times, and 'R' repeated 2 times.
∴ Total number of arrangements is 11!4!2!2!
∴ 11!4!2!2! different words can be formed from the letters of the word MAHARASHTRA.
(i) Other than M and T. there are 9 letters in which A repeats 4 times, H repeats twice, R repeats twice
The number of arrangements of the a letter = 9!4!2!2!
These 9 letters create 10 gaps in which M and T are to be arranged
The number of arrangements of M and T = $^{10}P_2$
∴ Total number arrangement having M and T never together = 9!×10P24!2!2!

(ii) When all vowels are together.
There are 4 vowels in the word MAHARASHTRA i.e., A, A, A, A
Let us consider these 4 vowels as one unit, they themselves can be arranged in $\frac{4!}{4!}$ = 1 way.
This unit is to be arranged with 7 other letters which can be done in 8! ways
∴ Total number of arrangements = $\frac{8!}{2!2!}$
∴ $\frac{8!}{2!2!}$ different words can be formed if vowels are always together.

Question 7.
How many different words are formed if the letter R is used thrice and letters S and T are used twice each?
Solution:
When ‘R’ is used thrice, ‘S’ is used twice and ‘T’ is used twice,
∴ Total number of letters available = 7, of which ‘S’ and ‘T’ repeat 2 times each, ‘R’ repeats 3 times.
∴ Required number of arrangements = $\frac{7!}{2!2!3!}$
= $\frac{7\times6\times5\times4\times3!}{2\times1\times2\times1\times3!}$
= 7 × 6 × 5
= 210
∴ 210 different words can be formed with the letter R is used thrice and letters S and T are used twice each.

Question 8.
Find the number of arrangements of letters in the word MUMBAI so that the letter B is always next to A.
Solution:
There are 6 letters in the word MUMBAI.
These letters are to be arranged in such a way that ‘B’ is always next to ‘A’.
Let us consider AB as one unit. This unit with other 4 letters in which ‘M’ repeats twice, is to be arranged.
∴ Total number of arrangements when B is always next to A = $\frac{5!}{2!}$
= $\frac{5\times4\times3\times2!}{2!}$
= 60

Question 9.
Find the number of arrangements of letters in the word CONSTITUTION

that begin and end with N.
Solution:
There are 12 letters in the word CONSTITUTION, in which ‘O’, ‘N’, T repeat two times each, ‘T’ repeats 3 times.
The arrangement starts and ends with ‘N’, 10 letters other than N can be arranged between two N, in which ‘O’ and ‘I’ repeat twice each and ‘T’ repeats 3 times.
∴ Total number of arrangements with the letter N at the beginning and at the end = 10!2!2!3!

Question 10.
Find the number of different ways of arranging letters in the word ARRANGE. How many of these arrangements have two R’s and two A’s not together?
Solution:
(i) There are 7 letters in the word ARRANGE in which A is repeated 2 times and R is repeated 2 times
∴ The number of arrangements = 7!2!2! = 1260

(ii) A: set of words having 2A together
B: set of words having 2R together
Number of words having both A and both R not together
= 1260 – $n(A \cup B)$
= 1260 – [n(a) + n(B) – $n(A \cap B)$] ……(i)
n(A) = number of ways in which (AA) R, R, N, G, E are to be arranged
∴ n(A) = 6!2! = 360
n(B) = number of ways in which (RR), A, A, N, G, E are to be arranged
∴ n(B) = 6!2! = 360
$n(A \cap B)$ = number of ways in which (AA), (RR), N, G, E are to be arranged
∴ $n(A \cap B)$ = 5! = 120
Substituting n(A), n(B), $n(A \cap B)$ in (i), we get
Number of words having both A and both R not together
= 1260 – [360 + 360 – 120]
= 1260 – 600
= 660

Question 11.
How many distinct 5 digit numbers can be formed using the digits 3, 2, 3, 2,

4, 5.
Solution:
5 digit numbers are to be formed from 2, 3, 2, 3, 4, 5.
Case I: Numbers formed from 2, 2, 3, 4, 5 OR 2, 3, 3, 4, 5
Number of such numbers = $\frac{5!}{2!} \times 2$
= 5!
= 120

Case II: Numbers formed from 2, 2, 3, 3 and any one of 4 or 5
Number of such numbers = $\frac{5!}{2!2!} \times 2 = 60$
Required number = 120 + 60 = 180
∴ 180 distinct 5 digit numbers can be formed using the digit 3, 2, 3, 2, 4, 5.

Question 12.
Find the number of distinct numbers formed using the digits 3, 4, 5, 6, 7, 8, 9, so that odd positions are occupied by odd digits.
Solution:
A number is to be formed with digits 3, 4, 5, 6, 7, 8, 9 such that odd digits always occupy the odd places.
There are 4 odd digits i.e. 3, 5, 7, 9.
They can be arranged at 4 odd places among themselves in 4! ways = 24 ways
3 even places of the number are occupied by even digits (i.e. 4, 6, 8).
∴ They can be arranged in 3! ways = 6 ways
∴ Total number of arrangements = 24 × 6 = 144
∴ 144 numbers can be formed so that odd digits always occupy the odd positions.

Question 13.
How many different 6-digit numbers can be formed using digits in the number 659942? How many of them are divisible by 2?
Solution:
A 6-digit number is to be formed using digits of 659942, in which 9 repeats twice.
∴ Total number of arrangements = $\frac{6!}{2!}$
= $\frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$

= 360
∴ 360 different 6-digit numbers can be formed.
For a number to be divisible by 2,
Last digits can be selected in 3 ways
Remaining 5 digit in which, 9 appears twice are arranged in $5!2!$ ways
∴ Total number of arrangements = $5!2!$ × 3 = 180
∴ 180 numbers are divisible by 2.

Question 14.
Find the number of distinct words formed from letters in the word INDIAN. How many of them have the two N's together?
Solution:
There are 6 letters in the word INDIAN in which I and N repeat twice.
Number of different words that can be formed using the letters of the word INDIAN = $6!2!2!$
= $6\times5\times4\times3\times2!2\times2!$
= 180
∴ 180 different words can be formed with the letters of the word INDIAN.
When two N's are together.
Let us consider the two N's as one unit.
They can be arranged with 4 other letters in $5!2!$
= $5\times4\times3\times2!2!$
= 60 ways.
∴ 2N can be arranged in 1 way
∴ Total number of arrangements = 60 × 1 = 60 ways
∴ 60 words are such that two N's are together.

Question 15.
Find the number of different ways of arranging letters in the word PLATOON if
(i) the two O's are never together.
(ii) consonants and vowels occupy alternate positions.
Solution:
(i) When the two O's are never together:
Let us arrange the other 5 letters first, which can be done in 5! = 120 ways.
The letters P, L, A, T, N create 6 gaps, in which O's are arranged.

∴ Two O's in 6 gaps can be arranged in $\frac{^6P_2}{2!}$ ways
= $\frac{6!}{(6-2)!2!}$ ways
= $\frac{6 \times 5 \times 4!}{4! \times 2 \times 1}$ ways
= 3 × 5 ways
= 15 ways
∴ Total number of arrangements if the two O's are never together = 120 × 15 = 1800

(ii) When consonants and vowels occupy alternate positions:
There are 4 consonants and 3 vowels in the word PLATOON.
∴ At odd places consonants occur and at even places vowels occur.
4 consonants can be arranged among themselves in 4! ways
3 vowels in which O occurs twice and A occurs once.
∴ They can be arranged in $\frac{3!}{2!}$ ways
∴ Required number of arrangements if the consonants and vowels occupy alternate positions = 4! × $\frac{3!}{2!}$
= 4 × 3 × 2 × $\frac{3 \times 2!}{2!}$
= 72

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Question 1.
In how many different ways can 8 friends sit around a table?
Solution:
We know that 'n' persons can sit around a table in (n – 1)! ways
∴ 8 friends can sit around a table in 7! ways
= 7 × 6 × 5 × 4 × 3 × 2 × 1
= 5040 ways.
∴ 8 friends can sit around a table in 5040 ways.

Question 2.
A party has 20 participants and a host. Find the number of distinct ways for the host to sit with them around a circular table. How many of these ways have two specified persons on either side of the host?

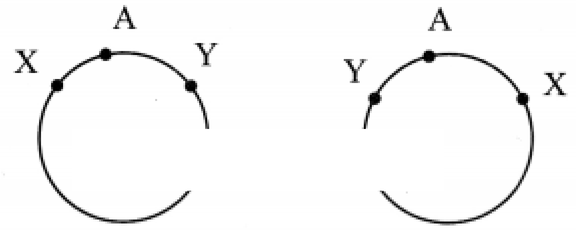
Solution:
A party has 20 participants.
All of them and the host (i.e., 21 persons) can be seated at a circular table in (21 – 1)! = 20! ways.
When two particular participants be seated on either side of the host.
Host takes chair in 1 way.
These 2 persons can sit on either side of host in 2! ways
Once host occupies his chair, it is not circular permutation any more.
Remaining 18 people occupy their chairs in 18! ways.
∴ Total number of arrangement possible if two particular participants be seated on either side of the host = 2! × 18!

Question 3.
Delegates from 24 countries participate in a round table discussion. Find the number of seating arrangements where two specified delegates are
(i) always together.
(ii) never together.
Solution:
(i) Delegates of 24 countries are to participate in a round table discussion such that two specified delegates are always together.
Let us consider these 2 delegates as one unit.
They can be arranged among themselves in 2! ways.
Also, these two delegates are to be seated with 22 other delegates (i.e. total 23) which can be done in (23 – 1)! = 22! ways.
∴ The total number of arrangements if two specified delegates are always together = 22! × 2!

(ii) When 2 specified delegates are never together then, the other 22 delegates can participate in a round table discussion in (22 – 1)! = 21! ways.
∴ There are 22 places of which any 2 places can be filled by those 2 delegates who are never together.
∴ Two specified delegates can be arranged in $^{22}P_2$ ways.
∴ Total number of arrangements if two specified delegates are never together = $^{22}P_2$ × 21!
= $\frac{22!}{(22-2)!}$ × 21!
= $\frac{22!}{20!}$ × 21!

= 22 × 21 × 21!
= 21 × 22 × 21!
= 21 × 22!

Question 4.
Find the number of ways for 15 people to sit around the table so that no two arrangements have the same neighbours.
Solution:
There are 15 people to sit around a table.
∴ They can be arranged in (15 – 1)! = 14! ways.
But, they should not have the same neighbour in any two arrangements.
Around the table, arrangements (i.e. clockwise and anticlockwise) coincide.



∴ Number of arrangements possible for not to have same neighbours
= 14!2

Question 5.
A committee of 20 members sits around a table. Find the number of arrangements that have the president and the vice president together.
Solution:
A committee of 20 members sits around a table.
But, President and Vice-president sit together.
Let us consider President and Vice-president as one unit.
They can be arranged among themselves in 2! ways.
Now, this unit with the other 18 members of the committee is to be arranged around a table, which can be done in (19 – 1)! = 18! ways.
∴ The total number of arrangements possible if President and Vice-president sit together = 18! × 2!

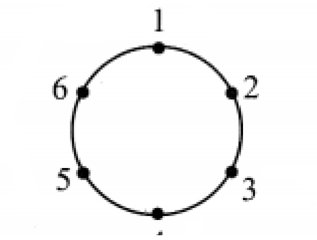
Question 6.
Five men, two women, and a child sit around a table. Find the number of arrangements where the child is seated

(i) between the two women.
(ii) between two men.
Solution:
(i) 5 men, 2 women, and a child sit around a table
When a child is seated between two women
∴ The two women can be seated on either side of the child in 2! ways.
Let us consider these 3 (two women and a child) as one unit.
Also, these 3 are to be seated with 5 men, (i.e. a total of 6 units) which can be done in (6 – 1)! = 5! ways.
∴ The total number of arrangements if the child is seated between two women = 5! × 2!

(ii) Two men out of 5 men can sit on either side of the child in 5P2 ways.
Let us take two men and a child as one unit.
Now these are to be arranged with the remaining 3 men and 2 women i.e., a total of 6 events (3 + 2 + 1) is to be arranged around a round table which can be done in (6 – 1)! = 5! ways.
∴ The total number of arrangements, if the child is seated between two men = ${}^5P_2 \times 5!$

Question 7.
Eight men and six women sit around a table. How many sitting arrangements will have no two women together?
Solution:
8 men can be seated around a table in (8 – 1)! = 7! ways.
There are 8 gaps created by 8 men's seats.
∴ 6 Women can be seated in 8 gaps in ${}^8P_6$ ways
∴ Total number of arrangements so that no two women are together = $7! \times {}^8P_6$

Question 8.
Find the number of seating arrangements for 3 men and 3 women to sit around a table so that exactly two women are together.
Solution:



Two women sit together and one woman sits separately.
Women sitting separately can be selected in 3 ways.
The other two women occupy two chairs in one way (as it is a circular arrangement).
They can be seated on those two chairs in 2 ways. Suppose two chairs are chairs 1 and 2 shown in the figure.
Then the third woman has only two options viz chairs 4 or 5.
∴ The third woman can be seated in 2 ways. 3 men are seated in 3! ways
∴ Required number = 3 × 2 × 2 × 3!
= 12 × 6
= 72

Question 9.
Four objects in a set of ten objects are alike. Find the number of ways of arranging them in a circular order.
Solution:
Ten things can be arranged in a circular order of which 4 are alike
in 9!4! ways.
∴ Required total number of arrangements = 9!4!

Question 10.
Fifteen persons sit around a table. Find the number of arrangements that have two specified persons not sitting side by side.
Solution:
Since 2 particular persons can't be sitting side by side.
The other 13 persons can be arranged around the table in (13 – 1)! = 12!
13 people around a table create 13 gaps in which 2 people are to be seated
Number of arrangements of 2 people = $^{13}P_2$
∴ The total number of arrangements in which two specified persons not sitting side by side = 12! × $^{13}P_2$

= 12! × 13 × 12
= 13 × 12! × 12
= 12 × 13!

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Question 1.
Find the value of
(i) $^{15}C_4$
Solution:

$$^{15}C_4 = \frac{15!}{4!(15-4)!} = \frac{15!}{4!11!}$$
$$= \frac{15 \times 14 \times 13 \times 12 \times 11!}{4 \times 3 \times 2 \times 1 \times 11!}$$
$$= \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}$$
$$= 1365$$

(ii) $^{80}C_2$
Solution:

$$^{80}C_2 = \frac{80!}{2!(80-2)!} = \frac{80!}{2!\ 78!}$$
$$= \frac{80 \times 79 \times 78!}{2 \times 78!}$$
$$= 40 \times 79 = 3160$$

(iii) $^{15}C_4 + {}^{15}C_5$
Solution:

$$^{15}C_4 + {}^{15}C_5 = {}^{15}C_5 + {}^{15}C_4 = {}^{15}C_5 + {}^{15}C_{5-1}$$
$$= {}^{16}C_5 \quad \ldots[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$$
$$= 4368$$

(iv) $^{20}C_{16} - {}^{19}C_{16}$

Solution:

$$^{20}C_{16} - {}^{19}C_{16} = \frac{20!}{16!\,4!} - \frac{19!}{16!\,3!}$$

$$= \frac{20 \times 19!}{16! \times 4 \times 3!} - \frac{19!}{16!\,3!}$$

$$= \frac{19!}{3!\,16!}\left[\frac{20}{4} - 1\right]$$

$$= \frac{19!}{3!\,16!}\ (4)$$

$$= \frac{19!}{3!\,(16)(15!)}\ 4$$

$$= \frac{19!}{4\,(3!)(15!)}$$

$$= \frac{19!}{4!\,15!}$$

$$= {}^{19}C_{15} \text{ or } {}^{19}C_4 = 3876$$

Question 2.

Find n if

(i) $^{6}P_2 = n\ {}^{6}C_2$

Solution:

$$^{6}P_2 = n\left({}^{6}C_2\right)$$

$$\therefore \quad \frac{6!}{(6-2)!} = n\ \frac{6!}{2!(6-2)!}$$

$$\therefore \quad \frac{6!}{4!} = n\ \frac{6!}{2!\,4!}$$

$$\therefore \quad n = 2! = 2 \times 1 = 2$$

(ii) ${}^{2n}C_3 : {}^{n}C_2 = 52 : 3$
Solution:

$${}^{2n}C_3 : {}^{n}C_2 = 52 : 3$$

$$\therefore \quad \frac{(2n)!}{3!(2n-3)!} \div \frac{n!}{2!(n-2)!} = \frac{52}{3}$$

$$\therefore \quad \frac{(2n)!}{3!(2n-3)!} \times \frac{2!(n-2)!}{n!} = \frac{52}{3}$$

$$\therefore \quad \frac{(2n)(2n-1)(2n-2)(2n-3)!}{3\times 2!(2n-3)!} \times \frac{2!(n-2)!}{n(n-1)(n-2)!} = \frac{52}{3}$$

$$\therefore \quad \frac{2n(2n-1).2(n-1)}{3} \times \frac{1}{n(n-1)} = \frac{52}{3}$$

$$\therefore \quad \frac{4(2n-1)}{3} = \frac{52}{3}$$

$$\therefore \quad 2n - 1 = 13$$

$$\therefore \quad 2n = 14$$

$$\therefore \quad n = 7$$

(iii) ${}^{n}C_{n-3} = 84$
Solution:
${}^{n}C_{n-3} = 84$
$\therefore$ n!(n−3)![n−(n−3)]! = 84
$\therefore$ n(n−1)(n−2)(n−3)!(n−3)!×3! = 84
$\therefore n(n-1)(n-2) = 84 \times 6$
$\therefore n(n-1)(n-2) = 9 \times 8 \times 7$
Comparing on both sides, we get
$\therefore n = 9$

Question 3.
Find r if ${}^{14}C_{2r} : {}^{10}C_{2r-4} = 143 : 10$
Solution:

$^{14}C_{2r} : {}^{10}C_{2r-4} = 143 : 10$

$$\therefore \quad \frac{14!}{2r!(14-2r)!} \div \frac{10!}{(2r-4)!(14-2r)!} = \frac{143}{10}$$

$$\therefore \quad \frac{14!}{2r!(14-2r)!} \times \frac{(2r-4)!(14-2r)!}{10!} = \frac{143}{10}$$

$$\therefore \quad \frac{14\times13\times12\times11\times10!}{2r(2r-1)(2r-2)(2r-3)(2r-4)!(14-2r)!} \times \frac{(2r-4)!(14-2r)!}{10!} = \frac{143}{10}$$

$\therefore \frac{14\times13\times12\times11}{2r(2r-1)\times(2r-2)(2r-3)}=\frac{143}{10}$

$\therefore 2r(2r-1)(2r-2)(2r-3) = 14 \times 12 \times 10$

$\therefore 2r(2r-1)(2r-2)(2r-3) = 8 \times 7 \times 6 \times 5$

Comparing on both sides, we get

$\therefore r = 4$

Question 4.

Find n and r if.

(i) $^nP_r = 720$ and $^nC_{n-r} = 120$

Solution:

$^nP_r = 720$

$\therefore \quad \frac{n!}{(n-r)!} = 720 \quad \ldots(i)$

Also, $^nC_{n-r} = 120$

$\therefore \quad \frac{n!}{(n-r)!(n-n+r)!} = 120$

$\therefore \quad \frac{n!}{r!(n-r)!} = 120 \quad \ldots(ii)$

Dividing (i) by (ii), we get

$$\frac{\frac{n!}{(n-r)!}}{\frac{n!}{r!(n-r)!}} = \frac{720}{120}$$

$\therefore \quad r! = 6$

$\therefore \quad r = 3$

Substituting r = 3 in (i), we get

$\frac{n!}{(n-3)!} = 720$

$\therefore \quad \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 720$

$\therefore \quad n(n-1)(n-2) = 10 \times 9 \times 8$

$\therefore \quad n = 10$

(ii) $^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 20 : 35 : 42$

Solution:

$${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 20 : 35 : 42$$

$\therefore \quad {}^nC_{r-1} : {}^nC_r = \frac{20}{35}$

$\therefore \quad \frac{n!}{(r-1)![n-(r-1)]!} \div \frac{n!}{r!(n-r)!} = \frac{4}{7}$

$\therefore \quad \frac{n!}{(r-1)!(n+1-r)!} \times \frac{r!(n-r)!}{n!} = \frac{4}{7}$

$\therefore \quad \frac{n!}{(r-1)!(n+1-r)!} \times \frac{r(r-1)!(n-r)!}{n!} = \frac{4}{7}$

$\therefore \quad \frac{r(n-r)!}{(n+1-r)(n-r)!} = \frac{4}{7}$

$\therefore \quad 7r = 4(n + 1 - r)$

$\therefore \quad 7r = 4n + 4 - 4r$

$\therefore \quad 11r = 4n + 4 \quad \ldots(i)$

Also, ${}^nC_r : {}^nC_{r+1} = \frac{35}{42}$

$\therefore \quad \frac{n!}{r!(n-r)!} \div \frac{n!}{(r+1)!(n-r-1)!} = \frac{5}{6}$

$\therefore \quad \frac{n!}{r!(n-r)!} \times \frac{(r+1)!(n-r-1)!}{n!} = \frac{5}{6}$

$\therefore \quad \frac{(r+1)r!(n-r-1)!}{r!(n-r)(n-r-1)!} = \frac{5}{6}$

$\therefore \quad \frac{r+1}{n-r} = \frac{5}{6}$

$\therefore \quad 6r + 6 = 5n - 5r$

$\therefore \quad 11r = 5n - 6$

$\therefore \quad 4n + 4 = 5n - 6 \quad \ldots$[From (i)]

$\therefore \quad n = 10$

$\therefore \quad 11r = 4(10) + 4 \quad \ldots$[From (i)]

$= 44$

$\therefore \quad r = 4$

Question 5.
If ${}^nP_r$ = 1814400 and ${}^nC_r$ = 45, find r.
Solution:

$^nP_r = 1814400,\ ^nC_r = 45$

$\therefore \quad \frac{^nP_r}{^nC_r} = \frac{1814400}{45}$

$\therefore \quad \frac{n!}{(n-r)!} \times \frac{r!(n-r)!}{n!} = \frac{1814400}{45}$

∴ r! = 40320

∴ r! = 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1

∴ r! = 8!

∴ r = 8

Question 6.

If $^nC_{r-1} = 6435$, $^nC_r = 5005$, $^nC_{r+1} = 3003$, find $^rC_5$.

Solution:

$^nC_{r-1} = 6435, {}^nC_r = 5005, {}^nC_{r+1} = 3003$

$\therefore \quad \frac{^nC_{r-1}}{^nC_r} = \frac{6435}{5005}$

$\therefore \quad \frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{9}{7}$

$\therefore \quad \frac{n!}{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{9}{7}$

$\therefore \quad \frac{n!}{(r-1)!(n-r+1)(n-r)!} \times \frac{r(r-1)!(n-r)!}{n!} = \frac{9}{7}$

$\therefore \quad \frac{r}{n-r+1} = \frac{9}{7}$

$\therefore \quad 7r = 9n - 9r + 9$

$\therefore \quad 16r - 9n = 9 \quad \ldots(i)$

Now, $\frac{^nC_r}{^nC_{r+1}} = \frac{5005}{3003}$

$\therefore \quad \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{5}{3}$

$\therefore \quad \frac{n!}{r!(n-r)!} \times \frac{(r+1)!(n-r-1)!}{n!} = \frac{5}{3}$

$\therefore \quad \frac{n!}{r!(n-r)(n-r-1)!} \times \frac{(r+1)r!(n-r-1)!}{n!} = \frac{5}{3}$

$\therefore \quad \frac{r+1}{n-r} = \frac{5}{3}$

$\therefore \quad 3r + 3 = 5n - 5r$

$\therefore \quad 8r - 5n = -3 \quad \ldots(ii)$

Solving (i) and (ii), we get

$n = 15, r = 9$

$^rC_5 = {}^9C_5 = \frac{9!}{4!5!}$

$= \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 1 \times 5!}$

$= 126$

Question 7.

Find the number of ways of drawing 9 balls from a bag that has 6 red balls,

5 green balls and 7 blue balls so that 3 balls of every colour are drawn.
Solution:
9 balls are to be selected from 6 red, 5 green, 7 blue balls such that the selection consists of 3 balls of each colour.
∴ 3 red balls can be selected from 6 red balls in $^6C_3$ ways.
3 green balls can be selected from 5 green balls in $^5C_3$ ways.
3 blue balls can be selected from 7 blue balls in $^7C_3$ ways.
∴ Number of ways selection can be done if the selection consists of 3 balls of each colour

$= {}^6C_3 . {}^5C_3 . {}^7C_3$

$= \frac{6!}{3!3!} \times \frac{5!}{3!2!} \times \frac{7!}{3!4!}$

$= \frac{6\times5\times4\times3!}{3\times2\times1\times3!} \times \frac{5\times4\times3!}{3!\times2} \times \frac{7\times6\times5\times4!}{3\times2\times1\times4!}$

$= 20 \times 10 \times 35$

$= 7000$

Question 8.
Find the number of ways of selecting a team of 3 boys and 2 girls from 6 boys and 4 girls.
Solution:
There are 6 boys and 4 girls.
A team of 3 boys and 2 girls is to be selected.
∴ 3 boys can be selected from 6 boys in $^6C_3$ ways.
2 girls can be selected from 4 girls in $^4C_2$ ways.
∴ Number of ways the team can be selected = $^6C_3 \times {}^4C_2$

= 6!3!3!×4!2!2!

= 6×5×4×3!3×2×1×3!×4×3×2!2×2!

= 20 × 6

= 120

∴ The team of 3 boys and 2 girls can be selected in 120 ways.

Question 9.
After a meeting, every participant shakes hands with every other participants. If the number of handshakes is 66, find the number of participants in the meeting.
Solution:

Let there be n participants present in the meeting.
A handshake occurs between 2 persons.
∴ Number of handshakes = ${}^nC_2$
Given 66 handshakes were exchanged.
∴ 66 = ${}^nC_2$
∴ 66 = $\frac{n!}{2!(n-2)!}$
∴ 66 × 2 = $\frac{n(n-1)(n-2)!}{(n-2)!}$
∴ 132 = n(n – 1)
∴ n(n – 1) = 12 × 11
Comparing on both sides, we get n = 12
∴ 12 participants were present at the meeting.

Question 10.
If 20 points are marked on a circle, how many chords can be drawn?
Solution:
To draw a chord we need to join two points on the circle.
There are 20 points on a circle.
∴ Total number of chords possible from these points = ${}^{20}C_2$
= $\frac{20!}{2!18!}$
= $\frac{20\times19\times18!}{2\times1\times18!}$
= 190

Question 11.
Find the number of diagonals of an n-sided polygon. In particular, find the number of diagonals when
(i) n = 10
(ii) n = 15
(iii) n = 12
Solution:
In n-sided polygon, there are 'n' points and 'n' sides. .
∴ Through 'n' points we can draw ${}^nC_2$ lines including sides.
∴ Number of diagonals in n sided polygon = ${}^nC_2$ – n (∴ n = number of

sides)

i. $n = 10$,

$$^{n}C_2 - n = {}^{10}C_2 - 10$$
$$= \frac{10 \times 9}{1 \times 2} - 10$$
$$= 45 - 10 = 35$$

ii. $n = 15$, $^{n}C_2 - n = {}^{15}C_2 - 15$

$$= \frac{15 \times 14}{1 \times 2} - 15$$
$$= 105 - 15 = 90$$

iii. $n = 12$, $^{n}C_2 - n = {}^{12}C_2 - 12$

$$= \frac{12 \times 11}{1 \times 2} - 12$$
$$= 66 - 12 = 54$$

Question 12.
There are 20 straight lines in a plane so that no two lines are parallel and no three lines are concurrent. Determine the number of points of intersection.
Solution:
There are 20 lines such that no two of them are parallel and no three of them are concurrent.
Since no two lines are parallel
∴ they intersect at a point
∴ Number of points of intersection if no two lines are parallel and no three lines are concurrent = $^{20}C_2$

$= \frac{20!}{2!18!}$

$= \frac{20 \times 19 \times 18!}{2 \times 1 \times 18!}$

$= 190$

Question 13.
Ten points are plotted on a plane. Find the number of straight lines obtained by joining these points if
(i) no three points are collinear
(ii) four points are collinear
Solution:

There are 10 points on a plane.
(i) No three of them are collinear:
Since a line is obtained by joining 2 points,
number of lines passing through these points if no three points are collinear = ${}^{10}C_2$
= 10!2!8!
= 10×9×8!2×1×8!
= 5 × 9
= 45

(ii) When 4 of them arc collinear:
∴ Number of lines passing through these points if 4 points are collinear
= ${}^{10}C_2 - {}^{4}C_2 + 1$
= 45 – 4!2!2! + 1
= 45 – 4×3×2!2×2! + 1
= 45 – 6 + 1
= 40

Question 14.
Find the number of triangles formed by joining 12 points if
(i) no three points are collinear
(ii) four points are collinear
Solution:
There are 12 points on the plane
(i) When no three of them are collinear:
Since a triangle can be drawn by joining any three non-collinear points.
∴ Number of triangles that can be obtained from these points = ${}^{12}C_3$
= 12!3!9!
= 12×11×10×9!3×2×1×9!
= 220

(ii) When 4 of these points are collinear:
∴ Number of triangles that can be obtained from these points = ${}^{12}C_3 - {}^{4}C_3$
= 220 – 4!3!×1!
= 220 – 4×3!3!
= 220 – 4
= 216

Question 15.
A word has 8 consonants and 3 vowels. How many distinct words can be formed if 4 consonants and 2 vowels are chosen?
Solution:
Out of 8 consonants, 4 can be selected in ${}^{8}C_{4}$
= 8!4!4!
= 8×7×6×5×4!4×3×2×1×4!
= 70 ways
From 3 vowels, 2 can be selected in ${}^{3}C_{2}$
= 3!2!1!
= 3×2!2!
= 3 ways
Now, to form a word, these 6 letters (i.e., 4 consonants and 2 vowels) can be arranged in ${}^{6}P_{6}$ i.e., 6! ways.
∴ Total number of words that can be formed = 70 × 3 × 6!
= 70 × 3 × 720
= 151200
∴ 151200 words of 4 consonants and 2 vowels can be formed.

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Question 1.
Find n if ${}^{n}C_{8} = {}^{n}C_{12}$
Solution:
${}^{n}C_{8} = {}^{n}C_{12}$
If ${}^{n}C_{x} = {}^{n}C_{y}$, then either x = y or x = n – y
∴ 8 = 12 or 8 = n – 12
But 8 = 12 is not possible
∴ 8 = n – 12
∴ n = 20

Question 2.
Find n if ${}^{23}C_{3n} = {}^{23}C_{2n+3}$
Solution:
${}^{23}C_{3n} = {}^{23}C_{2n+3}$
If ${}^{n}C_{x} = {}^{n}C_{y}$, then either x = y or x = n – y
∴ 3n = 2n + 3 or 3n = 23 – 2n – 3
∴ n = 3 or n = 4

Question 3.
Find n if ${}^{21}C_{6n} = {}^{21}C_{n^2+5}$
Solution:
${}^{21}C_{6n} = {}^{21}C_{n^2+5}$
If ${}^{n}C_{x} = {}^{n}C_{y}$, then either x = y or x = n – y
∴ $6n = n^2 + 5$ or $6n = 21 - (n^2 + 5)$
∴ $n^2 - 6n + 5 = 0$ or $6n = 21 - n^2 - 5$
∴ $n^2 - 6n + 5 = 0$ or $n^2 + 6n - 16 = 0$
If $n^2 - 6n + 5 = 0$ then $(n - 1)(n - 5) = 0$
∴ n = 1 or n = 5
If n = 5 then $n^2 + 5 = 30 > 21$
∴ $n \neq 5$
∴ n = 1
If $n^2 + 6n - 16 = 0$ then $(n + 8)(n - 2) = 0$
n = -8 or n = 2
$n \neq -8$
∴ n = 2

Question 4.
Find n if ${}^{2n}C_{r-1} = {}^{2n}C_{r+1}$
Solution:
${}^{2n}C_{r-1} = {}^{2n}C_{r+1}$
If ${}^{n}C_{x} = {}^{n}C_{y}$, then either x = y or x = n – y
∴ r – 1 = r + 1 or r – 1 = 2n – (r + 1)
But r – 1 = r + 1 is not possible
∴ r – 1 = 2n – (r + 1)
∴ r + r = 2n
∴ r = n

Question 5.
Find n if ${}^{n}C_{n-2} = 15$
Solution:
${}^{n}C_{n-2} = 15$
$\therefore {}^{n}C_{2} = 15$ ....[$\because {}^{n}C_{r} = {}^{n}C_{n-r}$]
$\therefore n!(n-2)!2! = 15$
$\therefore n(n-1)(n-2)!(n-2)!2\times1 = 15$
$\therefore n(n-1) = 30$
$\therefore n(n-1) = 6 \times 5$
Comparing both sides, we get
$\therefore n = 6$

Question 6.
Find x if ${}^{n}P_{r} = x\ {}^{n}C_{r}$
Solution:
$$^{n}P_{r} = x(^{n}C_{r})$$
$$x = \frac{^{n}P_{r}}{^{n}C_{r}}$$
$$= \frac{\frac{n!}{(n-r)!}}{\frac{n!}{(n-r!)r!}}$$
$$= r!$$

Question 7.
Find r if ${}^{11}C_{4} + {}^{11}C_{5} + {}^{12}C_{6} + {}^{13}C_{7} = {}^{14}C_{r}$
Solution:
$${}^{11}C_{4} + {}^{11}C_{5} + {}^{12}C_{6} + {}^{13}C_{7} = {}^{14}C_{r}$$
$$\therefore \left({}^{11}C_{4} + {}^{11}C_{5}\right) + {}^{12}C_{6} + {}^{13}C_{7} = {}^{14}C_{r}$$
$$\ldots\left[{}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}\right]$$
$$\therefore ({}^{12}C_{5} + {}^{12}C_{6}) + {}^{13}C_{7} = {}^{14}C_{r}$$
$$\therefore ({}^{13}C_{6} + {}^{13}C_{7}) = {}^{14}C_{r}$$
$$\therefore {}^{14}C_{7} = {}^{14}C_{r}$$
If ${}^{n}C_{x} = {}^{n}C_{y}$, then either $x = y$ or $x = n - y$
$\therefore r = 7$ or $r = 14 - 7 = 7$

Question 8.
Find the value of $\sum_{r=1}^{4} {}^{21-r}C_4 + {}^{17}C_5$
Solution:

$$\sum_{r=1}^{4} {}^{(21-r)}C_4 + {}^{17}C_5 = ({}^{20}C_4 + {}^{19}C_4 + {}^{18}C_4 + {}^{17}C_4) + {}^{17}C_5$$

$$= ({}^{20}C_4 + {}^{19}C_4 + {}^{18}C_4 + {}^{18}C_5 - {}^{17}C_5) + {}^{17}C_5 \quad \ldots \left[{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right]$$

$$= {}^{20}C_4 + {}^{19}C_4 + ({}^{18}C_4 + {}^{18}C_5)$$

$$= {}^{20}C_4 + {}^{19}C_4 + {}^{19}C_5$$

$$= {}^{20}C_4 + {}^{20}C_5$$

$$= {}^{21}C_5$$

$$\sum_{r=1}^{4} {}^{(21-r)}C_4 + {}^{17}C_5 = {}^{21}C_5$$

Question 9.
Find the differences between the largest values in the following:
(i) ${}^{14}C_r - {}^{12}C_r$
Solution:
Greatest value of ${}^{14}C_r$
Here n = 14, which is even
Greatest value of ${}^{n}C_r$ occurs at r = $\frac{n}{2}$ if n is even

$\therefore \quad r = \frac{n}{2}$

$\therefore \quad r = \frac{14}{2} = 7$

$\therefore \quad$ Greatest value of $^{14}C_r = {}^{14}C_7 = \frac{14!}{7!7!}$

$= \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7!}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 7!}$

$= 3432$

Also, for greatest value of $^{12}C_r$

$n = 12$, which is even

$\therefore \quad r = \frac{12}{2} = 6$

$\therefore \quad {}^{12}C_6 = \frac{12!}{6!6!}$

$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6!}{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 6!}$

$= 924$

$\therefore$ Difference between the greatest values of $^{14}C_r$ and $^{12}C_r = 3432 - 924 = 2508$

(ii) $^{13}C_r - {}^{8}C_r$

Solution:

Greatest value of $^{13}C_r$

Here n = 13, which is odd

Greatest value of $^{n}C_r$ occurs at $r = n-12$ if n is odd

$\therefore \quad r = \frac{n-1}{2}$

$\therefore \quad r = \frac{13-1}{2} = 6$

$\therefore$ Greatest value of ${}^{13}C_r = {}^{13}C_6 = \frac{13!}{6!7!}$

$= \frac{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7!}{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 7!}$

$= 1716$

Also, for greatest value of ${}^{8}C_r$

$n = 8$, which is even

$\therefore \quad r = \frac{8}{2} = 4$

$\therefore \quad {}^{8}C_4 = \frac{8!}{4!4!}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4!}{4 \times 3 \times 2 \times 1 \times 4!}$

$= 70$

$\therefore$ Difference between the greatest values of ${}^{13}C_r$ and ${}^{8}C_r$ = 1716 – 70 = 1646

(iii) ${}^{15}C_r - {}^{11}C_r$

Solution:

Greatest value of ${}^{15}C_r$

Here n = 15, which is odd

Greatest value of ${}^{n}C_r$ occurs at r = $n-12$ if n is odd

$\therefore \quad r = \frac{n-1}{2}$

$\therefore \quad r = \frac{15-1}{2} = 7$

$\therefore \quad$ Greatest value of ${}^{15}C_r = {}^{15}C_7 = \frac{15!}{7!8!}$

$= \frac{15\times14\times13\times12\times11\times10\times9\times8!}{7\times6\times5\times4\times3\times2\times1\times8!}$

$= 6435$

Also, for greatest value of ${}^{11}C_r$

$n = 11$, which is odd

$\therefore \quad r = \frac{11-1}{2} = 5$

$\therefore \quad {}^{11}C_5 = \frac{11!}{5!6!}$

$= \frac{11\times10\times9\times8\times7\times6!}{5\times4\times3\times2\times1\times6!}$

$= 462$

Difference between the greatest values of ${}^{15}C_r$ and ${}^{11}C_r$ = 6435 – 462 = 5973

Question 10.
In how many ways can a boy invite his 5 friends to a party so that at least three join the party?
Solution:
Boy can invite = (3 or 4 or 5 friends)
Consider the following table:

Case I	Case II	Case III
3 friends are invited	4 friends are invited	All 5 friends are invited
Number of ways = ${}^5C_3$ = 10	Number of ways = ${}^5C_4$ = 5	Number of ways = ${}^5C_5$ = 1

$\therefore$ Number of ways a boy can invite his friends to a party so that three or more join the party = 10 + 5 + 1 = 16

Question 11.
A group consists of 9 men and 6 women. A team of 6 is to be selected. How

many possible selections will have at least 3 women?
Solution:
There are 9 men and 6 women.
A team of 6 persons is to be formed such that it consists of at least 3 women.

	Case I	Case II	Case III	Case IV
	3W 3M	4W 2M	5W 1M	6W –
Number of ways	${}^6C_3 \times {}^9C_3$ $= 20 \times 84$ $= 1680$	${}^6C_4 \times {}^9C_2$ $= 15 \times 36$ $= 540$	${}^6C_5 \times {}^9C_1$ $= 6 \times 9$ $= 54$	1

∴ Number of ways this can be done = 1680 + 540 + 54 + 1 = 2275
∴ 2275 teams can be formed if team consists of at least 3 women.

Question 12.
A committee of 10 persons is to be formed from a group of 10 women and 8 men. How many possible committees will have at least 5 women? How many possible committees will have men in the majority?
Solution:
(i) A committee of 10 persons is to be formed from 10 women and 8 men such that the committee contains at least 5 women
Consider the following table:

		Number of ways
Case I	5W, 5M	${}^{10}C_5 \times {}^8C_5 = 252 \times 26$ $= 14112$
Case II	6W, 4M	${}^{10}C_6 \times {}^8C_4 = 210 \times 70$ $= 14700$
Case III	7W, 3M	${}^{10}C_7 \times {}^8C_3 = 120 \times 56$ $= 6720$
Case IV	8W, 2M	${}^{10}C_8 \times {}^8C_2 = 45 \times 28$ $= 1260$
Case V	9W, 1M	${}^{10}C_9 \times {}^8C_1 = 10 \times 8$ $= 80$
Case VI	10 W	1

∴ Number of committees = 14112 + 14700 + 6720 + 1260 + 81 = 36873
∴ At least 5 women are there in 36873 committees.

(ii) Number of committees with men in majority = Total number of committees – (Number of committees with women in majority + women and men equal in number)
= $^{18}C_{10}$ – 36873
= $^{18}C_{8}$ – 36873
= 43758 – 36873
= 6885

Question 13.
A question paper has two sections. Section I has 5 questions and section II has 6 questions. A student must answer at least two questions from each section among 6 questions he answers. How many different choices does the student have in choosing questions?
Solution:
There are 11 questions, out of which 5 questions are from section I and 6 questions are from section II.
The student has to select 6 questions taking at least 2 questions from each section.
Consider the following table:

	Case I	Case II	Case III
No. of questions	Sec I (2Q) Sec II (4Q)	Sec I (3Q) Sec II (3Q)	Sec I (4Q) Sec II (2Q)
Number of ways	$^5C_2 \times {^6C_4}$ $= 10 \times 15$ $= 150$	$^5C_3 \times {^6C_3}$ $= 10 \times 20$ $= 200$	$^5C_4 \times {^6C_2}$ $= 5 \times 15$ $= 75$

∴ Number of choices = 150 + 200 + 75 = 425
∴ In 425 ways students can select 6 questions, taking at least 2 questions from each section.

Question 14.
There are 3 wicketkeepers and 5 bowlers among 22 cricket players. A team of 11 players is to be selected so that there is exactly one wicketkeeper and at least 4 bowlers in the team. How many different teams can be formed?
Solution:
There are 22 cricket players, of which 3 are wicketkeepers and 5 are bowlers.

A team of 11 players is to be chosen such that exactly one wicketkeeper and at least 4 bowlers are to be included in the team.
Consider the following table:

	Case I	Case II
	1 wicketkeeper + 4 bowlers + 6 players	1 wicketkeeper + 5 bowlers + 5 players
Number of ways	${}^3C_1 \times {}^5C_4 \times {}^{14}C_6$ $= 3 \times 5 \times 3003$ $= 45045$	${}^3C_1 \times {}^5C_5 \times {}^{14}C_5$ $= 3 \times 1 \times 2002$ $= 6006$

∴ Number of ways a team of 11 players can be selected = 45045 + 6006 = 51051

Question 15.
Five students are selected from 11. How many ways can these students be selected if
(i) two specified students are selected?
(ii) two specified students are not selected?
Solution:
5 students are to be selected from 11 students
(i) When 2 specified students are included
then remaining 3 students can be selected from (11 – 2) = 9 students.
∴ Number of ways of selecting 3 students from 9 students = ${}^9C_3$
= 9!3!×6!
= 9×8×7×6!3×2×1×6!
= 84
∴ Selection of students is done in 126 ways when 2 specified students are not selected.

(ii) When 2 specified students are not included then 5 students can be selected from the remaining (11 – 2) = 9 students
∴ Number of ways of selecting 5 students from 9 students = ${}^9C_5$
= 9!5!4!
= 9×8×7×6×5!5!×4×3×2×1
= 126

∴ Selection of students is done in 126 ways when 2 specified students are not selected.

Maharashtra State Board 11th Commerce Maths Solutions Chapter 6 Permutations and Combinations Miscellaneous Exercise 6

Question 1.
Find the value of r if $^{56}C_{r+6} : {}^{54}P_{r-1} = 30800 : 1$
Solution:

$^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$

$\therefore \quad \frac{^{56}P_{r+6}}{^{54}P_{r+3}} = \frac{30800}{1}$

$\therefore \quad \frac{\frac{56!}{(56-r-6)!}}{\frac{54!}{(54-r-3}} = 30800$

$\therefore \quad \frac{56!}{(56-r-6)!} \times \frac{(54-r-3)!}{54!} = 30800$

$\therefore \quad \frac{56!}{(50-r)!} \times \frac{(51-r)!}{54!} = 30800$

$\therefore \quad \frac{56 \times 55 \times 54!}{(50-r)!} \times \frac{(51-r)(50-r)!}{54!} = 30800$

$\therefore \quad 51 - r = \frac{30800}{56 \times 55}$

$\therefore \quad 51 - r = 10$

$\therefore \quad r = 51 - 10$

$\therefore \quad r = 41$

Question 2.
How many words can be formed by writing letters in the word CROWN in a different order?
Solution:
Five Letters of the word CROWN are to be permuted.
Number of different words = 5! = 120

Question 3.
Find the number of words that can be formed by using all the letters in the word REMAIN. If these words are written in dictionary order, what will be the 40th word?
Solution:
There are 6 letters A, E, I, M, N, R
The number of words starting with A = 5!
The number of words starting with E = 5!
The number of words starting with I = 5!
The number of words starting with M = 5!
The number of words starting with N = 5!
The number of words starting with R = 5!
Total number of words = 6 × 5! = 720
Number of words starting with AE = 4! = 24
Number of words starting with AIE = 3! = 6
Number of words starting with AIM = 3! = 6
The number of words starting with AINE = 2!
Total words = 24 + 6 + 6 + 2 = 38
39th word is AINMER
40th word is AINMRE

Question 4.
Find the number of ways of distributing n balls in n cells. What will be the number of ways if each cell must be occupied?
Solution:
There are n balls and n cells
(i) Every ball can be put in any of the n cells.
Number of distributions = $n \times n \times \ldots\ldots \times n = (n)^n$
(ii) For filling the first cell, n balls are available.
The first cell is filled in n ways.
The second cell is filled in (n – 1) ways
The third cell is filled in (n – 2) ways and so on.
the nth cell is tilled in one way.
Required number = n(n – 1)(n – 2) …… 1 = n!

Question 5.
Thane is the 20th station from C.S.T. If a passenger can purchase a ticket from any station to any other station, how many different tickets must be

available at the booking window?
Solution:
Taking CST as the first station and Thane as 20th,
Let us name CST as A_0 next station as A_1 and so on, Thane is A_{20}
From station A_0, 20 different journeys are possible
From station A_1, 20 different journeys are possible.
From station A_{20}, 20 different journeys are possible.
Total number of different tickets of different journeys = 21 × 20 = 420

Question 6.
English alphabet has 11 symmetric letters that appear the same when looked at in a mirror. These letters are A, H, I, M, O, T, U, V, W, X, and Y. How many symmetric three letters passwords can be formed using these letters?
Solution:
Number of 3 Letter passwords = $^{11}P_3$
= 11 × 10 × 9
= 990

Question 7.
How many numbers formed using the digits 3, 2, 0, 4, 3, 2, 3 exceed one million?
Solution:
A number that exceeds one million is to be formed from the digits 3, 2, 0, 4, 3, 2, 3.
Then the numbers should be any number of 7 digits which can be formed from these digits.
Also among the given numbers 2 repeats twice and 3 repeats thrice.
∴ Required number of numbers = Total number of arrangements possible among these digits – number of arrangements of 7 digits which begin with 0.
= $7!2!3! - 6!2!3!$
= $7\times6\times5\times4\times3!2\times3! - 6\times5\times4\times3!2\times3!$
= 7 × 6 × 5 × 2 – 6 × 5 × 2
= 6 × 5 × 2(7 – 1)
= 60 × 6
= 360

∴ 360 numbers that exceed one million can be formed with the digits 3, 2, 0, 4, 3, 2, 3.

Question 8.
Ten students are to be selected for a project from a class of 30 students. There are 4 students who want to be together either in the project or not in the project. Find the number of possible selections.
Solution:

Case I	Case II
4 Students are in	4 Students are out
Number of ways = $^{26}C_6$	Number of ways = $^{26}C_{10}$

Required number = $^{26}C_6 + {}^{26}C_{10}$

Question 9.
A student finds 7 books of his interest but can borrow only three books. He wants to borrow the Chemistry part II book only if Chemistry Part I can also be borrowed. Find the number of ways he can choose three books that he wants to borrow.
Solution:

Chemistry Part I borrowed	Chemistry Part I not borrowed
Only one book from remaining 5 books borrowed	All three books borrowed from remaining 5 books
Number of selections = $^5C_1 = 5$	Number of selections = $^5C_3 = 10$

Required Number = 5 + 10 = 15

Question 10.
30 objects are to be divided into three groups containing 7, 10, 13 objects. Find the number of distinct ways of doing so. Solution:
Required number = $^{30}C_7 \times {}^{23}C_{10} \times {}^{13}C_{13}$

Question 11.
A student passes an examination if he secures a minimum in each of the 7 subjects. Find the number of ways a student can fail.
Solution:

Every subject a student may pass or fail.
∴ Total number of outcomes = 2^7 = 128
This number includes one case when the student passes in all subjects.
∴ Required number = 128 – 1 = 127

Question 12.
Nine friends decide to go for a picnic in two groups. One group decides to go by car and the other group decides to go by train. Find the number of different ways of doing so if there must be at least 3 friends in each group.
Solution:

	Train	**Car**	**Number of outcomes**
No. of friends	3	6	${}^9C_3$
	4	5	${}^9C_4$
	5	4	${}^9C_5$
	6	3	${}^9C_6$

Required number = ${}^9C_3 + {}^9C_4 + {}^9C_5 + {}^9C_6$
$= ({}^9C_4 + {}^9C_3) + ({}^9C_6 + {}^9C_5) = {}^{10}C_4 + {}^{10}C_6$
$= \frac{10!}{6!4!} + \frac{10!}{4!6!}$
$= 210 + 210 = 420$

Question 13.
Five balls are to be placed in three boxes, where each box can contain upto five balls. Find the number of ways if no box is to remain empty.
Solution:
Let boxes be named as I, II, III
Let sets A, B, C represent cases in which boxes I, II, III remain empty
Then A ∪ B ∪ C represent the cases in which at least one box remains empty.
Then we use method of indirect counting
Required number = Total number of distributions – n(A ∪ B ∪ C) …..(i)
n(A ∪ B ∪ C) represent the number of undesirable cases
Total number of distributions = 3 × 3 × 3 × 3 × 3 = 3^5 = 243 …….(ii)
n(A ∪ B ∪ C) = n(A) + n(B) + n(C) – n(A ∩ B) – n(B ∩ C) – n(C ∩ A) + n(A ∩ B ∩ C) …..(iii)
In box I is empty then every ball has two places (boxes) to go.

Similarly for box II and III.
∴ $n(A) + n(B) + n(C) = 3 \times 2^5$ ......(iv)
If boxes I and II remain empty then all balls go to box III
Similarly we would have two more cases.
∴ $n(A \cap B) + n(B \cap C) + n(C \cap A) = 3 \times 1^5$ ......(v)
∴ $n(A \cap B \cap C) = 0$ ....(vi) [as all boxes cannot be empty]
Substitute from (iv), (v), (vi) to (iii) to get
$n(A \cup B \cup C) = 3 \times 2^5 - 3 \times 1^5$
$= 96 - 3$
$= 93$
Substitute $n(A \cup B \cup C)$ and from (ii) to (i), we get
Required number = 243 – 93 = 150

Question 14.
A hall has 12 lamps and every lamp can be switched on independently. Find the number of ways of illuminating the hall.
Solution:
Every lamp is either ON or OFF.
There are 12 lamps
Number of instances = 2^{12}
This number includes the case in which all 12 lamps are OFF.
∴ Required Number = $2^{12} - 1 = 4095$

Question 15.
How many quadratic equations can be formed using numbers from 0, 2, 4, 5 as coefficients if a coefficient can be repeated in an equation?
Solution:
Let the quadratic equation be $ax^2 + bx + c = 0$, $a \neq 0$

Coefficient	Values	Number of ways
a	2, 4, 5	3
b	0, 2, 4, 5	4
c	0, 2, 4, 5	4

∴ Required number = 3 × 4 × 4 = 48

Question 16.
How many six-digit telephone numbers can be formed if the first two digits are 45 and no digit can appear more than once?

Solution:
Let the telephone number be 45abcd

	Number of ways to fill
a	8
b	7
c	6
d	5

∴ Required number = ${}^8P_4$ = 1680

Question 17.
A question paper has 6 questions. How many ways does a student have if he wants to solve at least one question?
Solution:
Every question is 'SOLVED' or 'NOT SOLVED'.
There are 6 question.
Number of outcomes = 2^6
This number includes the case when the student solves NONE of the question.
Required number = $2^6 - 1$
= 64 – 1
= 63

Question 18.
Find the number of ways of dividing 20 objects in three groups of sizes 8, 7 and 5.
Solution:
Select 8 objects out of 20 in ${}^{20}C_8$ ways
Select 7 objects from the remaining 12 in ${}^{12}C_7$ ways and 5 objects from the remaining 5 in ${}^5C_5$ ways
Required number is = ${}^{20}C_8 \times {}^{12}C_7 \times {}^5C_5$

Question 19.
There are 8 doctors and 4 lawyers in a panel. Find the number of ways for selecting a team of 6 if at least one doctor must be in the team.
Solution:
There are 8 doctors and 4 lawyers.
We need to select a team of 6 which contains at least one doctor.
Since there are only 4 lawyers any team of 6 will contain at least two

doctors.
Required number = ${}^{12}C_6 = 924$

Question 20.
Four parallel lines intersect another set of five parallel lines. Find the number of distinct parallelograms that can be formed.
Solution:
We need 2 lines from each set.
Required number = ${}^{4}C_2 \times {}^{5}C_2$
$= 6 \times 10$
$= 60$