Maharashtra State Board 12th Maths Solutions Chapter 1 Mathematical Logic Ex 1.1

Question 1.
State which of the following sentences are statements. Justify your answer. In case of the statement, write down the truth value :
(i) $5 + 4 = 13$.
Solution:
It is a statement which is false, hence its truth value is 'F'.

(ii) $x - 3 = 14$.
Solution:
It is an open sentence, hence it is not a statement.

(iii) Close the door.
Solution:
It is an imperative sentence, hence it is not a statement.

(iv) Zero is a complex number.
Solution:
It is a statement which is true, hence its truth value is 'T'.

(v) Please get me breakfast.
Solution:
It is an imperative sentence, hence it is not a statement.

(vi) Congruent triangles are also similar.
Solution:
It is a statement which is true, hence its truth value is 'T'.

(vii) $x^2 = x$.
Solution:
It is an open sentence, hence it is not a statement,

(viii) A quadratic equation cannot have more than two roots.
Solution:
It is a statement which is true, hence its truth value is 'T'.

(ix) Do you like Mathematics ?
Solution:
It is an interrogative sentence, hence it is not a statement.

(x) The sun sets in the west.
Solution:
It is a statement which is true, hence its truth value is 'T'.

(xi) All real numbers are whole numbers.
Solution:
It is a statement which is false, hence its truth value is 'F'.

(xii) Can you speak in Marathi ?
Solution:
It is an interrogative sentence, hence it is not a statement.

(xiii) $x^2 - 6x - 7 = 0$, when $x = 7$.
Solution:
It is a statement which is true, hence its truth value is 'T'.

(xiv) The sum of cuberoots of unity is zero.
Solution:
It is a statement which is true, hence its truth value is 'T'.

(xv) It rains heavily.
Solution :
It is an open sentence, hence it is not a statement.

Question 2.
Write the following compound statements symbolically:
(i) Nagpur is in Maharashtra and Chennai is in Tamil Nadu.
Solution:
Let p : Nagpur is in Maharashtra.
q : Chennai is in Tamil Nadu.
Then the symbolic form of the given statement is $P \wedge q$.

(ii) Triangle is equilateral or isosceles,
Solution:
Let p : Triangle is equilateral.
q : Triangle is isosceles.
Then the symbolic form of the given statement is $P \vee q$.

(iii) The angle is right angle if and only if it is of measure 90°.
Solution:
Let p : The angle is right angle.
q : It is of measure 90°.
Then the symbolic form of the given statement is $p \leftrightarrow q$

(iv) Angle is neither acute nor obtuse.
Solution:
Let p : Angle is acute.
q : Angle is obtuse.
Then the symbolic form of the given statement is
$\sim p \wedge \sim q$.

(v) If Δ ABC is right angled at B, then $m\angle A + m\angle C = 90°$.
Solution:
Let p : Δ ABC is right angled at B.
q : $m\angle A + m\angle C = 90°$.
Then the symbolic form of the given statement is $p \rightarrow q$

(vi) Hima Das wins gold medal if and only if she runs fast.
Solution:
Let p : Hima Das wins gold medal
q : She runs fast.
Then the symbolic form of the given statement is $p \leftrightarrow q$.

(vii) x is not irrational number but it is a square of an integer.
Solution:
Let p : x is not irrational number
q : It is a square of an integer
Then the symbolic form of the given statement is $p \wedge q$
Note : If p : x is irrational number, then the symbolic form of the given statement is $\sim p \wedge q$.

Question 3.
Write the truth values of the following :
(i) 4 is odd or 1 is prime.
Solution:
Let p : 4 is odd.
q : 1 is prime.
Then the symbolic form of the given statement is $p \vee q$.
The truth values of both p and q are F.
$\therefore$ the truth value of p v q is F. ... $[F \vee F = F]$

(ii) 64 is a perfect square and 46 is a prime number.
Solution:
Let p : 64 is a perfect square.
q : 46 is a prime number.
Then the symbolic form of the given statement is $p \wedge q$.
The truth values of p and q are T and F respectively.
$\therefore$ the truth value of $p \wedge q$ is F. ... $[T \wedge F \equiv F]$

(iii) 5 is a prime number and 7 divides 94.
Solution:
Let p : 5 is a prime number.
q : 7 divides 94.
Then the symbolic form of the given statement is $p \wedge q$.

The truth values of p and q are T and F respectively.
∴ the truth value of $p \wedge q$ is F. … $[T \wedge F \equiv F]$

(iv) It is not true that 5 – 3i is a real number.
Solution:
Let p : 5 – 3i is a real number.
Then the symbolic form of the given statement is $\sim p$.
The truth values of p is F.
∴ the truth values of $\sim p$ is T. … $[\sim F \equiv T]$

(v) If 3 × 5 = 8, then 3 + 5 = 15.
Solution:
Let p : 3 × 5 = 8.
q : 3 + 5 = 15.
Then the symbolic form of the given statement is $p \rightarrow q$.
The truth values of both p and q are F.
∴ the truth value of $p \rightarrow q$ is T. … $[F \rightarrow F \equiv T]$

(vi) Milk is white if and only if sky is blue.
Solution:
Let p : Milk is white.
q : Sky is blue
Then the symbolic form of the given statement is $p \leftrightarrow q$.
The truth values of both p and q are T.
∴ the truth value of $p \leftrightarrow q$ is T. … $[T \leftrightarrow T \equiv T]$

(vii) 24 is a composite number or 17 is a prime number.
Solution :
Let p : 24 is a composite number.
q : 17 is a prime number.
Then the symbolic form of the given statement is $p \vee q$.
The truth values of both p and q are T.
∴ the truth value of $p \vee q$ is T. … $[T \vee T \equiv T]$

Question 4.
If the statements p, q are true statements and r, s are false statements, then determine the truth values of the following:
(i) $p \vee (q \wedge r)$
Solution:
Truth values of p and q are T and truth values of r and s are F.
$p \vee (q \wedge r) \equiv T \vee (T \wedge F)$
$\equiv T \wedge F \equiv T$
Hence the truth value of the given statement is true.

(ii) $(p \rightarrow q) \vee (r \rightarrow s)$
Solution:
$(p \rightarrow q) \vee (r \rightarrow s) \equiv (T \rightarrow T) \vee (F \rightarrow F)$
$\equiv T \vee T \equiv T$
Hence the truth value of the given statement is true.

(iii) $(q \wedge r) \vee (\sim p \wedge s)$
Solution:
$(q \wedge r) \vee (\sim p \wedge s) \equiv (T \wedge F) \vee (\sim T \wedge F)$
$\equiv F \vee (F \wedge F)$
$\equiv F \vee F \equiv F$
Hence the truth value of the given statement is false.

(iv) $(p \rightarrow q) \wedge (\sim r)$
Solution:
$(p \rightarrow q) \wedge (\sim r) \equiv (T \rightarrow T) \wedge (\sim F)$
$\equiv T \wedge T \equiv T$
Hence the truth value of the given statement is true.

(v) $(\sim r \leftrightarrow p) \rightarrow (\sim q)$
Solution:
$(\sim r \leftrightarrow p) \rightarrow (\sim q) \equiv (\sim F \leftrightarrow T) \rightarrow (\sim T)$
$\equiv (T \leftrightarrow T) \rightarrow F$

$\equiv T \to F \equiv F$

Hence the truth value of the given statement is false.

(vi) $[\sim p \wedge (\sim q \wedge r) \vee (q \wedge r) \vee (p \wedge r)]$

Solution:

$[\sim p \wedge (\sim q \wedge r) \vee (q \wedge r) \vee (p \wedge r)]$

$\equiv [\sim T \wedge (\sim T \wedge F)] \vee [(T \wedge F) \vee (T \wedge F)]$

$\equiv [F \wedge (F \wedge F)] \vee [F \vee F]$

$\equiv (F \wedge F) \vee F$

$\equiv F \vee F \equiv F$

Hence the truth value of the given statement is false.

(vii) $[(\sim p \wedge q) \wedge (\sim r)] \vee [(q \to p) \to (\sim s \vee r)]$

Solution:

$[(\sim p \wedge q) \wedge (\sim r)] \vee [(q \to p) \to (\sim s \vee r)]$

$\equiv [(\sim T \wedge T) \wedge (\sim F)] \vee [(T \to T) \to (\sim F \vee F)]$

$\equiv [(F \wedge T) \wedge T] \vee [T \to (T \vee F)]$

$\equiv (F \wedge T) \vee (T \to T)$

$\equiv F \vee T \equiv T$

Hence the truth value of the given statement is true.

(viii) $\sim [(\sim p \wedge r) \vee (s \to \sim q)] \leftrightarrow (p \wedge r)$

Solution :

$\sim [(\sim p \wedge r) \vee (s \to \sim q)] \leftrightarrow (p \wedge r)$

$\equiv \sim [(\sim T \wedge F) \vee (F \to \sim T)] \leftrightarrow (T \wedge F)$

$\equiv \sim [(F \wedge F) \vee (F \to F)] \leftrightarrow F$

$\equiv \sim (F \vee T) \leftrightarrow F$

$\equiv \sim T \leftrightarrow F$

$\equiv F \leftrightarrow F \equiv T$

Hence the truth value of the given statement is true.

Question 5.

Write the negations of the following :

(i) Tirupati is in Andhra Pradesh.

Solution:

The negations of the given statements are :

Tirupati is not in Andhra Pradesh.

(ii) 3 is not a root of the equation $x^2 + 3x - 18 = 0$.

Solution:

3 is a root of the equation $x^2 + 3x - 18 = 0$.

(iii) $2 - \sqrt{}$ is a rational number.

Solution:

$2 - \sqrt{}$ is not a rational number.

(iv) Polygon ABCDE is a pentagon.

Solution:

Polygon ABCDE is not a pentagon.

(v) 7 + 3 > 5.

Solution :

7 + 3 > 5.

Maharashtra State Board 12th Maths Solutions Chapter 1 Mathematical Logic Ex 1.2

Question 1.
Construct the truth table for each of the following statement patterns:
(i) $[(p \rightarrow q) \wedge q] \rightarrow p$
Solution :
Here are two statements and three connectives.
∴ there are 2 × 2 = 4 rows and 2 + 3 = 5 columns in the truth table.

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	$[(p \rightarrow q) \wedge q] \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

(ii) $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$
Solution:

p	q	$\sim q$	$p \wedge \sim q$	$p \rightarrow q$	$(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

(iii) $(p \wedge q) \leftrightarrow (q \vee r)$
Solution:

p	q	r	$p \wedge q$	$q \vee r$	$(p \wedge q) \leftrightarrow (q \vee r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	F	F	T
F	T	T	F	T	F
F	T	F	F	T	F
F	F	T	F	T	F
F	F	F	F	F	T

(iv) p → [~(q ∧ r)]
Solution:

p	q	r	$q \wedge r$	$\sim(q \wedge r)$	$p \rightarrow [\sim(q \wedge r)]$
T	T	T	T	F	F
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T

(v) ~p ∧ [(p ∨ ~q) ∧ q]
Solution:

p	q	$\sim p$	$\sim q$	$p \vee \sim q$	$(p \vee \sim q) \wedge q$	$\sim p \wedge [(p \vee \sim q) \wedge q]$
T	T	F	F	T	T	F
T	F	F	T	T	F	F
F	T	T	F	F	F	F
F	F	T	T	T	F	F

(vi) (~p → ~q) ∧ (~q → ~p)
Solution:

p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$	$(\sim p \rightarrow \sim q) \wedge (\sim q \rightarrow \sim p)$
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

(vii) (q → p) ∨ (~p ↔ q)
Solution:

p	q	$\sim p$	$q \rightarrow p$	$\sim p \leftrightarrow q$	$(q \rightarrow p) \vee (\sim p \leftrightarrow q)$
T	T	F	T	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	T

(viii) [p → (q → r)] ↔ [(p ∧ q) → r]
Solution:

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$	$[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

(ix) p → [~(q ∧ r)]
Solution:

p	q	r	$q \wedge r$	$\sim(q \wedge r)$	$p \to [\sim(q \wedge r)]$
T	T	T	T	F	F
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T

(x) (p ∨ ~q) → (r ∧ p)
Solution:

p	q	r	$\sim q$	$p \vee \sim q$	$r \wedge p$	$(p \vee \sim q) \to (r \wedge p)$
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	T	F	F	F	F	T
F	F	T	T	T	F	F
F	F	F	T	T	F	F

Question 2.
Using truth tables prove the following logical equivalences.
(i) ~p ∧ q ≡ (p ∨ q) ∧ ~p
Solution:

1	2	3	4	5	6
p	q	$\sim p$	$\sim p \wedge q$	$p \vee q$	$(p \vee q) \wedge \sim p$
T	T	F	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	F	F	F

The entries in the columns 4 and 6 are identical.
∴ ~p ∧ q ≡ (p ∨ q) ∧ ~p.

(ii) ~(p ∨ q) ∨ (~p ∧ q) ≡ ~p
Solution:

1	2	3	4	5	6	7
p	q	$\sim p$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge q$	$\sim(p \vee q) \vee (\sim p \wedge q)$
T	T	F	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	T
F	F	T	F	T	F	T

The entries in the columns 3 and 7 are identical.
∴ ~(p ∨ q) ∧ (~p ∧ q) = ~p.

(iii) p ↔ q ≡ ~[(p ∨ q) ∧ ~(p ∧ q)]
Solution:

1	2	3	4	5	6	7	8
p	q	$p \leftrightarrow q$	$p \vee q$	$p \wedge q$	$\sim(p \wedge q)$	$(p \vee q) \wedge \sim(p \wedge q)$	$\sim[(p \vee q) \wedge \sim(p \wedge q)]$
T	T	T	T	T	F	F	T
T	F	F	T	F	T	T	F
F	T	F	T	F	T	T	F
F	F	T	F	F	T	F	T

The entries in the columns 3 and 8 are identical.
∴ p ↔ q ≡ ~[(p ∨ q) ∧ ~(p ∧ q)].

(iv) p → (q → p) ≡ ~p → (p → q)
Solution:

1	2	3	4	5	6	7
p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$	$\sim p$	$p \rightarrow q$	$\sim p \rightarrow (p \rightarrow q)$
T	T	T	T	F	T	T
T	F	T	T	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T

The entries in the columns 4 and 7 are identical.
∴ p → (q → p) ≡ ~p → (p → q).

(v) (p ∨ q) → r ≡ (p → r) ∧ (q → r)
Solution:

1	2	3	4	5	6	7	8
p	q	r	$p \vee q$	$(p \vee q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The entries in the columns 5 and 8 are identical.
∴ $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$.

(vi) $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$
Solution:

1	2	3	4	5	6	7	8
p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \wedge (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The entries in the columns 5 and 8 are identical.
∴ $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$.

(vii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Solution:

1	2	3	4	5	6	7	8
p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

The entries in the columns 5 and 8 are identical.
∴ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

(viii) $[\sim(p \vee q) \vee (p \vee q)] \wedge r \equiv r$
Solution:

1	2	3	4	5	6	7
p	q	r	$p \vee q$	$\sim(p \vee q)$	$\sim(p \vee q) \vee (p \vee q)$	$[\sim(p \vee q) \vee (p \vee q)] \wedge r$
T	T	T	T	F	T	T
T	T	F	T	F	T	F
T	F	T	T	F	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	F	T	F	T	T	T
F	F	F	F	T	T	F

The entries in the columns 3 and 7 are identical.
∴ $[\sim(p \vee q) \vee (p \vee q)] \wedge r \equiv r$.

(ix) $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$
Solution:

1	2	3	4	5	6	7	8	9
p	q	$\sim p$	$\sim q$	$p \leftrightarrow q$	$\sim(p \leftrightarrow q)$	$p \wedge \sim q$	$q \wedge \sim p$	$(p \wedge \sim q) \vee (q \wedge \sim p)$
T	T	F	F	T	F	F	F	F
T	F	F	T	F	T	T	F	T
F	T	T	F	F	T	F	T	T
F	F	T	T	T	F	F	F	F

The entries in the columns 6 and 9 are identical.
∴ $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$.

Question 3.
Examine whether each of the following statement patterns is a tautology or a contradiction or a contingency.
(i) $(p \wedge q) \rightarrow (q \vee p)$
Solution:

p	q	$p \wedge q$	$q \vee p$	$(p \wedge q) \rightarrow (q \vee p)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

All the entries in the last column of the above truth table are T.
∴ $(p \wedge q) \rightarrow (q \vee p)$ is a tautology.

(ii) $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$
Solution:

p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$	$(p \rightarrow q) \leftrightarrow (\sim p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

All the entries in the last column of the above truth table are T.
∴ $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$ p is a tautology.

(iii) $[\sim(\sim p \wedge \sim q)] \vee q$
Solution:

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$\sim(\sim p \wedge \sim q)$	$[\sim(\sim p \wedge \sim q)] \vee q$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	F

The entries in the last column of the above truth table are neither all T nor all F.
∴ [~(~p ∧ ~q)] ∨ q is a contingency.

(iv) [(p → q) ∧ q)] → p
Solution:

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	$[(p \rightarrow q) \wedge q] \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

The entries in the last column of the above truth table are neither all T nor all F.
∴ [(p → q) ∧ q)] → p is a contingency

(v) [(p → q) ∧ ~q] → ~p
Solution:

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \sim q$	$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

All the entries in the last column of the above truth table are T.
∴ [(p → q) ∧ ~q] → ~p is a tautology.

(vi) (p ↔ q) ∧ (p → ~q)
Solution:

p	q	$\sim q$	$p \leftrightarrow q$	$p \rightarrow \sim q$	$(p \leftrightarrow q) \wedge (p \rightarrow \sim q)$
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	T	T

The entries in the last column of the above truth table are neither all T nor all F.
∴ (p ↔ q) ∧ (p → ~q) is a contingency.

(vii) ~(~q ∧ p) ∧ q
Solution:

p	q	$\sim q$	$\sim q \wedge p$	$\sim(\sim q \wedge p)$	$\sim(\sim q \wedge p) \wedge q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	F

The entries in the last column of the above truth table are neither all T nor all F.
∴ ~(~q ∧ p) ∧ q is a contingency.

(viii) (p ∧ ~q) ↔ (p → q)
Solution:

p	q	$\sim q$	$p \wedge \sim q$	$p \rightarrow q$	$(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

All the entries in the last column of the above truth table are F.
∴ (p ∧ ~q) ↔ (p → q) is a contradiction.

(ix) (~p → q) ∧ (p ∧ r)
Solution:

p	q	r	$\sim p$	$\sim p \rightarrow q$	$p \wedge r$	$(\sim p \rightarrow q) \wedge (p \wedge r)$
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	T	F	F
F	F	T	T	F	F	F
F	F	F	T	F	F	F

The entries in the last column of the above truth table are neither all T nor all F.
∴ (~p → q) ∧ (p ∧ r) is a contingency.

(x) [p → (~q ∨ r)] ↔ ~[p → (q → r)]
Solution:

p	q	r	$\sim q$	$\sim q \vee r$	$p \rightarrow (\sim q \vee r)$	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$\sim [p \rightarrow (q \rightarrow r)]$	$[p \rightarrow (\sim q \vee r)] \leftrightarrow \sim [p \rightarrow (q \rightarrow r)]$
T	T	T	F	T	T	T	T	F	F
T	T	F	F	F	F	F	F	T	F
T	F	T	T	T	T	T	T	F	F
T	F	F	T	T	T	T	T	F	F
F	T	T	F	T	T	T	T	F	F
F	T	F	F	F	T	F	T	F	F
F	F	T	T	T	T	T	T	F	F
F	F	F	T	T	T	T	T	F	F

All the entries in the last column of the above truth table are F.
∴ [p → (~q ∨ r)] ↔ ~[p → (q → r)] is a contradiction

Maharashtra State Board 12th Maths Solutions Chapter 1 Mathematical Logic Ex 1.3

Question 1.
If A = {3, 5, 7, 9, 11, 12}, determine the truth value of each of the following.
(i) $\exists$ x $\in$ A such that x – 8 = 1
Solution:
Clearly x = 9 $\in$ A satisfies x – 8 = 1. So the given statement is true, hence its truth value is T.

(ii) $\forall$ x $\in$ A, x^2 + x is an even number
Solution:
For each x $\in$ A, x^2 + x is an even number. So the given statement is true, hence its truth value is T.

(iii) $\exists$ x $\in$ A such that $x^2 < 0$
Solution:
There is no x $\in$ A which satisfies $x^2 < 0$. So the given statement is false, hence its truth value is F.

(iv) $\forall$ x $\in$ A, x is an even number
Solution:
x = 3 $\in$ A, x = 5 $\in$ A, x = 7 $\in$ A, x = 9 $\in$ A, x = 11 $\in$ A do not satisfy x is an even number. So the given statement is false, hence its truth value is F.

(v) $\exists$ x $\in$ A such that 3x + 8 > 40
Solution:
Clearly x = 11 $\in$ A and x = 12 $\in$ A satisfies 3x + 8 > 40. So the given statement is true, hence its truth value is T.

(vi) $\forall$ x $\in$ A, 2x + 9 > 14
Solution:
For each x $\in$ A, 2x + 9 > 14. So the given statement is true, hence its truth value is T.

Question 2.
Write the duals of each of the following.
(i) $p \vee (q \wedge r)$
Solution:
The duals of the given statement patterns are :
$p \wedge (q \vee r)$

(ii) $p \wedge (q \wedge r)$
Solution:
$p \vee (q \vee r)$

(iii) $(p \vee q) \wedge (r \vee s)$
Solution:
$(p \wedge q) \vee (r \wedge s)$

(iv) $p \wedge \sim q$
Solution:
$p \vee \sim q$

(v) $(\sim p \vee q) \wedge (\sim r \wedge s)$
Solution:
$(\sim p \wedge q) \vee (\sim r \vee s)$

(vi) $\sim p \wedge (\sim q \wedge (p \vee q) \wedge \sim r)$
Solution:
$\sim p \vee (\sim q \vee (p \wedge q) \vee \sim r)$

(vii) [~(p ∨ q)] ∧ [p ∨ ~(q ∧ ~s)]
Solution:
[~(p ∧ q)] ∨ [p ∧ ~(q ∨ ~s)]

(viii) c ∨ {p ∧ (q ∨ r)}
Solution:
t ∧ {p ∧ (q Ar)}

(ix) ~p ∨ (q ∧ r) ∧ t
Solution:
~p ∧ (q ∨ r) ∨ c

(x) (p ∨ q) ∨ c
Solution:
(p ∧ q) ∧ t

Question 3.
Write the negations of the following.
(i) x + 8 > 11 or y – 3 = 6
Solution:
Let p : x + 8 > 11, q : y — 3 = 6.
Then the symbolic form of the given statement is p ∨ q.
Since ~(p ∨ q) ≡ ~p ∧ ~q, the negation of given statement is :
'x + 8 > 11 and y – 3 ≠ 6' OR
'x + 8 ≯ 11 and y – 3 ≠ 6'

(ii) 11 < 15 and 25 > 20
Solution:
Let p: 11 < 15, q : 25 > 20.
Then the symbolic form of the given statement is p ∧ q.
Since ~(p ∧ q) ≡ ~p ∨ ~q, the negation of given statement is :
'11 ≮ 15 or 25 > 20.' OR
'11 ≯ 15 or 25 ≮ 20.'

(iii) Qudrilateral is a square if and only if it is a rhombus.
Solution:
Let p : Quadrilateral is a square.
q : It is a rhombus.
Then the symbolic form of the given statement is p ↔ q.
Since ~(p ↔ q) ≡ (p ∧ ~q) ∨ (q ∧ ~p), the negation of given statement is :
' Quadrilateral is a square but it is not a rhombus or quadrilateral is a rhombus but it is not a square.'

(iv) It is cold and raining.
Solution:
Let p : It is cold.
q : It is raining.
Then the symbolic form of the given statement is p ∧ q.
Since ~(p ∧ q) ≡ ~p ∨ ~q, the negation of the given statement is :
'It is not cold or not raining.'

(v) If it is raining then we will go and play football.
Solution:
Let p : It is raining.
q : We will go.
r : We play football.
Then the symbolic form of the given statement is p → (q ∧ r).
Since ~[p → (q ∧ r)] ≡ p ∧ ~(q ∧ r) ≡ p ∧ (q ∨ ~r), the negation of the given statement is :
'It is raining and we will not go or not play football.'

(vi) 2–√ is a rational number.
Solution:
Let p : 2–√ is a rational number.

The negation of the given statement is
‘ ~p : $2 - \surd$ is not a rational number.’

(vii) All natural numbers are whole numers.
Solution:
The negation of the given statement is :
‘Some natural numbers are not whole numbers.’

(viii) $\forall$ n $\in$ N, $n^2 + n + 2$ is divisible by 4.
Solution:
The negation of the given statement is :
‘$\exists$ n $\in$ N, such that $n^2 + n + 2$ is not divisible by 4.’

(ix) $\exists$ x $\in$ N such that $x – 17 < 20$
Solution:
The negation of the given statement is :
‘$\forall$ x $\in$ N, $x – 17 \nsucc 20$.’

Question 4.
Write converse, inverse and contrapositive of the following statements.
(i) If $x < y$ then $x^2 < y^2$ (x, y $\in$ R)
Solution:
Let p : $x < y$, q : $x^2 < y^2$.
Then the symbolic form of the given statement is $p \to q$.
Converse : $q \to p$ is the converse of $p \to q$.
i.e. If $x^2 < y^2$, then $x < y$.
Inverse : $\sim p \to \sim q$ is the inverse of $p \to q$.
i.e. If $x \nsucc y$, then $x^2 \nsucc y^2$. OR
If $x \nprec y$, then $x^2 \nprec y^2$.
Contrapositive : $\sim q \to p$ is the contrapositive of
$p \to q$ i.e. If $x^2 \nsucc y^2$, then $x \nsucc y$. OR
If $x^2 \nprec y^2$, then $x \nprec y$.

(ii) A family becomes literate if the woman in it is literate.
Solution:
Let p : The woman in the family is literate.
q : A family become literate.
Then the symbolic form of the given statement is $p \to q$
Converse : $q \to p$ is the converse of $p \to q$.
i.e. If a family become literate, then the woman in it is literate.
Inverse : $\sim p \to \sim q$ is the inverse of $p \to q$.
i.e. If the woman in the family is not literate, then the family does not become literate.
Contrapositive : $\sim q \to \sim p$ is the contrapositive of $p \to q$. i e. If a family does not become literate, then the woman in it is not literate.

(iii) If surface area decreases then pressure increases.
Solution:
Let p : The surface area decreases.
q : The pressure increases.
Then the symbolic form of the given statement is $p \to q$.
Converse : $q \to p$ is the converse of $p \to q$.
i.e. If the pressure increases, then the surface area decreases.
Inverse : $\sim p \to \sim q$ is the inverse of $p \to q$.
i.e. If the surface area does not decrease, then the pressure does not increase.
Contrapositive : $\sim q \to \sim p$ is the contrapositive of $p \to q$.
i.e. If the pressure does not increase, then the surface area does not decrease.

(iv) If voltage increases then current decreases.
Solution:
Let p : Voltage increases.
q : Current decreases.
Then the symbolic form of the given statement is $p \to q$.
Converse : $q \to p$ is the converse of $p \to q$.
i.e. If current decreases, then voltage increases.
Inverse : $\sim p \to \sim q$ is the inverse of $p \to q$.
i.e. If voltage does not increase, then current does not decrease.
Contrapositive : $\sim q \to \sim p$, is the contrapositive of $p \to q$.
i.e. If current does not decrease, then voltage doesnot increase.

Maharashtra State Board 12th Maths Solutions Chapter 1 Mathematical Logic Ex 1.4

Question 1.
Using rules of negation write the negations of the following with justification.
(i) $\sim q \rightarrow p$
Solution:
The negation of $\sim q \rightarrow p$ is
$\sim(\sim q \rightarrow p) \equiv \sim q \wedge \sim p$.... (Negation of implication)

(ii) $p \wedge \sim q$
Solution:
The negation of $p \wedge \sim q$ is
$\sim(p \wedge \sim q) \equiv \sim p \vee \sim(\sim q)$ ... (Negation of conjunction)
$\equiv \sim p \vee q$ ... (Negation of negation)

(iii) $p \vee \sim q$
Solution:
The negation of $p \vee \sim p$ is
$\sim (p \vee \sim(q) \equiv \sim p \wedge \sim(\sim(q)$ ... (Negation of disjunction)
$\equiv \sim p \wedge q$ ... (Negation of negation)

(iv) $(p \vee \sim q) \wedge r$
Solution:
The negation of $(p \vee \sim q) \wedge r$ is
$\sim[(p \vee \sim q) \wedge r] \equiv \sim(p \vee \sim q) \vee \sim r$ ... (Negation of conjunction)
$\equiv [\sim p \wedge \sim(\sim q)] \vee \sim r$... (Negation of disjunction)
$\equiv (\sim p \wedge q) \wedge \sim r$ ... (Negation of negation)

(v) $p \rightarrow (p \vee \sim q)$
Solution:
The negation of $p \rightarrow (p \vee \sim q)$ is
$\sim [p \rightarrow (p \vee \sim q)] \equiv p \wedge \sim (p \wedge \sim p)$ ... (Negation of implication)
$\equiv p \wedge [\sim p \wedge \sim (\sim(q)]$ ... (Negation of disjunction)
$\equiv p \wedge (\sim p \wedge q)$ (Negation of negation)

(vi) $\sim(p \wedge q) \vee (p \vee \sim q)$
Solution:
The negation of $\sim(p \wedge q) \vee (p \vee \sim q)$ is
$\sim[\sim(p \wedge q) \vee (p \vee \sim q)] \equiv \sim[\sim(p \wedge q)] \wedge \sim(p \vee \sim q)$ ... (Negation of disjunction)
$\equiv \sim[\sim(p \wedge q)] \wedge [p \wedge \sim(\sim q)]$ ... (Negation of disjunction)
$\equiv (p \wedge q) \wedge (\sim p \wedge q)$ ... (Negation of negation)

(vii) $(p \vee \sim q) \rightarrow (p \wedge \sim q)$
Solution:
The negation of $(p \vee \sim q) \rightarrow (p \wedge \sim q)$ is
$\sim[(p \vee \sim q) \rightarrow (p \wedge \sim q)]$
$\equiv (p \vee \sim q) \wedge \sim(p \wedge \sim q)$ ... (Negation of implication)
$\equiv (p \vee \sim q) \wedge [\sim p \vee \sim(\sim q)]$ ... (Negation of conjunction)
$\equiv (p \vee \sim q) \wedge (\sim p \vee q)$ ... (Negation of negation)

(viii) $(\sim p \vee \sim q) \vee (p \wedge \sim q)$
Solution:
The negation of $(\sim p \vee \sim q) \vee (p \wedge \sim q)$ is
$\sim [(\sim p \vee \sim q) \vee (p \wedge \sim q)]$
$\equiv \sim(\sim p \vee \sim q) \wedge \sim(p \wedge \sim q)$ ... (Negation of disjunction)
$\equiv [\sim(\sim p) \wedge \sim(\sim q)] \wedge [\sim p \vee \sim(\sim q)]$ ... (Negation of disjunction and conjunction)
$\equiv (p \wedge q) \wedge (\sim p \vee q)$ ... (Negation of negation)

Question 2.
Rewrite the following statements without using if .. then.

(i) If a man is a judge then he is honest.
Solution:
Since $p \rightarrow \equiv \sim p \vee q$, the given statements can be written as :
A man is not a judge or he is honest.

(ii) It 2 is a rational number then $2 - \surd$ is irrational number.
Solution:
2 is not a rational number or $2 - \surd$ is irrational number.

(iii) It f(2) = 0 then f(x) is divisible by (x – 2).
Solution:
$f(2) \neq 0$ or f(x) is divisible by (x – 2).

Question 3.
Without using truth table prove that :
(i) $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$
Solution:
LHS = $p \leftrightarrow q$
$\equiv (p \leftrightarrow q) \wedge (q \leftrightarrow p)$ … (Biconditional Law)
$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$ … (Conditional Law)
$\equiv [\sim p \wedge (\sim q \vee p)] \vee [q \wedge (\sim q \vee p)]$ … (Distributive Law)
$\equiv [(\sim p \wedge \sim q) \vee (\sim p \wedge p)] \vee [(q \wedge \sim q) \vee (q \wedge p)]$ … (Distributive Law)
$\equiv [(\sim p \wedge \sim q) \vee F] \vee [F \vee (q \wedge p)]$ … (ComplementLaw)
$\equiv (\sim p \wedge \sim q) \vee (q \wedge p)$ … (Identity Law)
$\equiv (\sim p \wedge \sim q) \vee (p \wedge q)$ … (Commutative Law)
$\equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ … (Commutative Law)
≡ RHS.

(ii) $(p \vee q) \wedge (p \vee \sim q) \equiv p$
Solution:
LHS = $(p \vee q) \wedge (p \vee \sim q)$
$\equiv p \vee (q \wedge \sim q)$ … (Distributive Law)
$\equiv p \vee F$ … (Complement Law)
$\equiv p$ … (Identity Law)
≡ RHS.

(iii) $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$
Solution:
LHS = $(p \wedge q) v (\sim p \wedge q) \vee (p \wedge \sim q)$
$\equiv [(p \vee \sim p) \wedge q] \vee (p \wedge \sim q)$ … (Distributive Law)
$\equiv (T \wedge q) \vee (p \wedge \sim q)$ … (Complement Law)
$\equiv q \vee (p \wedge \sim q)$ … (Identity Law)
$\equiv (q \vee p) \wedge (q \vee \sim q)$ … (Distributive Law)
$\equiv (q \vee p) \wedge T$ .. (Complement Law)
$\equiv q \vee p$ … (Identity Law)
$\equiv p \vee q$ … (Commutative Law)
≡ RHS.

(iv) $\sim[(p \vee \sim q) \rightarrow (p \wedge \sim q)] \equiv (p \vee \sim q) \wedge (\sim p \vee q)$
Solution:
LHS = $\sim[(p \vee \sim q) \rightarrow (p \wedge \sim q)]$
$\equiv (p \vee \sim q) \wedge \sim(p \wedge \sim q)$ … (Negation of implication)
$\equiv (p \vee \sim q) \wedge [\sim p \vee \sim(\sim q)]$ … (Negation of conjunction)
$\equiv (p \vee \sim q) \wedge (\sim p \vee q)$… (Negation of negation)
≡ RHS.

Maharashtra State Board 12th Maths Solutions Chapter 1 Mathematical Logic Ex 1.5

Question 1.
Express the following circuits in the symbolic form of logic and write the input-output table.
(i)

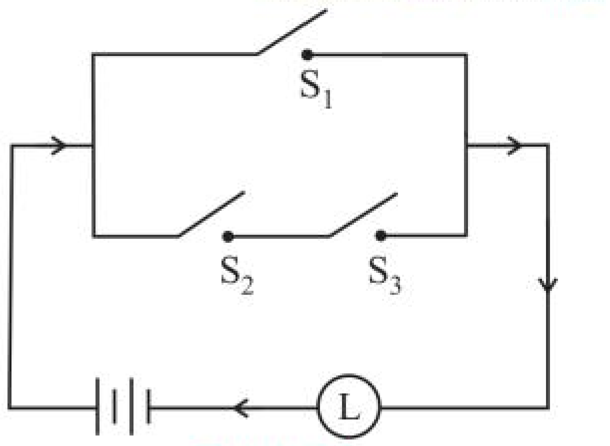


Fig. 1.17

Solution:
Let p : the switch S_1 is closed
q : the switch S_2 is closed
r : the switch S_3 is closed
~p : the switch S_1' is closed or the switch $S_1$is open
~q : the switch S_2' is closed or the switch S_2 is open
~r : the switch S_3' is closed or the switch S_3 is open
l : the lamp L is on
(i) The symbolic form of the given circuit is : $p \vee (q \wedge r) = l$
l is generally dropped and it can be expressed as : $p \vee (q \wedge r)$.

Input–Output Table

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
1	1	1	1	1
1	1	0	0	1
1	0	1	0	1
1	0	0	0	1
0	1	1	1	1
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

(ii)

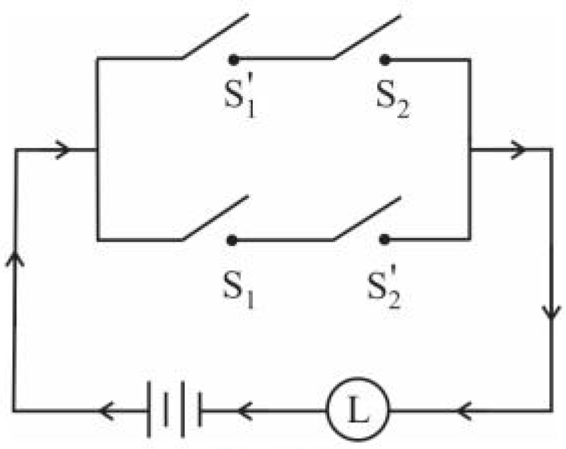


Fig. 1.18

Solution:

The symbolic form of the given circuit is : (~ p ∧ q) ∨ (p ∧ ~ q).

Input–Output Table

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$p \wedge \sim q$	$(\sim p \wedge q) \vee (p \wedge \sim q)$
1	1	0	0	0	0	0
1	0	0	1	0	1	1
0	1	1	0	1	0	1
0	0	1	1	0	0	0

(iii)

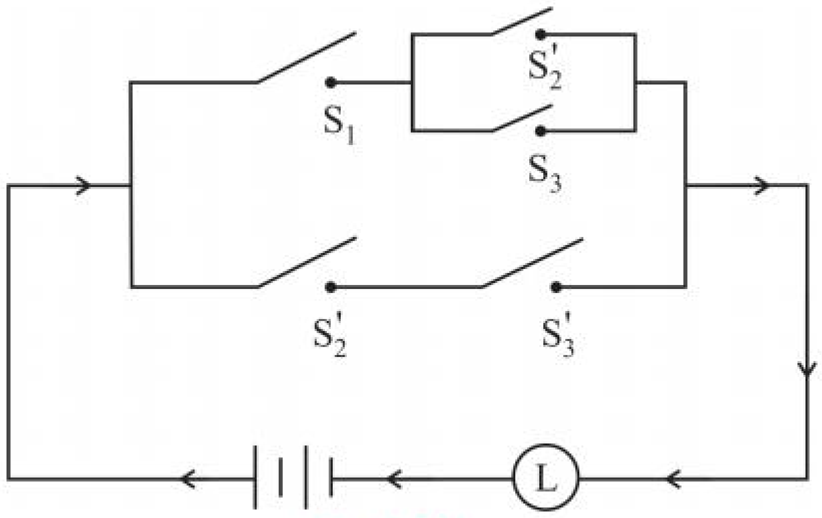


Fig. 1.19

Solution:
The symbolic form of the given circuit is : [p ∧ (~q ∨ r)] ∨ (~q ∧ ~ r).

Input–Output Table

p	q	r	$\sim q$	$\sim r$	$\sim q \vee r$	$p \wedge (\sim q \vee r)$	$\sim q \wedge \sim r$	$[p \wedge (\sim q \vee r)] \vee (\sim q \wedge \sim r)$
1	1	1	0	0	1	1	0	1
1	1	0	0	1	0	0	0	0
1	0	1	1	0	1	1	0	1
1	0	0	1	1	1	1	1	1
0	1	1	0	0	1	0	0	0
0	1	0	0	1	0	0	0	0
0	0	1	1	0	1	0	0	0
0	0	0	1	1	1	0	1	1

(iv)

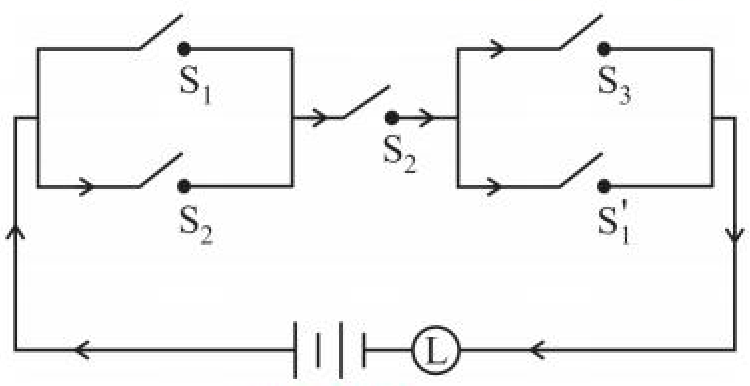


Fig. 1.20

Solution:

The symbolic form of the given circuit is : (p ∨ q) ∧ q ∧ (r ∨ ~p).

Input-Output Table

p	q	r	$\sim p$	$p \vee q$	$r \vee \sim p$	$(p \vee q) \wedge q \wedge (r \vee \sim p)$
1	1	1	0	1	1	1
1	1	0	0	1	0	0
1	0	1	0	1	1	0
1	0	0	0	1	0	0
0	1	1	1	1	1	1
0	1	0	1	1	1	1
0	0	1	1	0	1	0
0	0	0	1	0	1	0

(v)

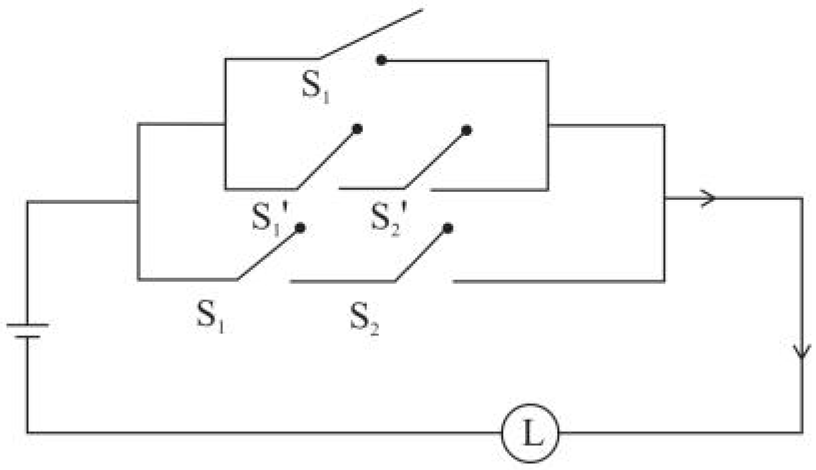


Fig. 1.21

Solution:
The symbolic form of the given circuit is : [p ∨ (~p ∧ ~q)] ∨ (p ∧ q).

Input–Output Table

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \vee (\sim p \wedge \sim q)$	$p \wedge q$	$[p \vee (\sim p \wedge \sim q)] \vee (p \wedge q)$
1	1	0	0	0	1	1	1
1	0	0	1	0	1	0	1
0	1	1	0	0	0	0	0
0	0	1	1	1	1	0	1

(vi)

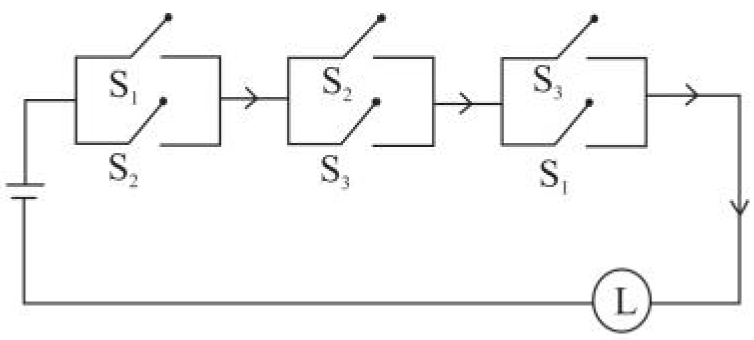


Fig. 1.22

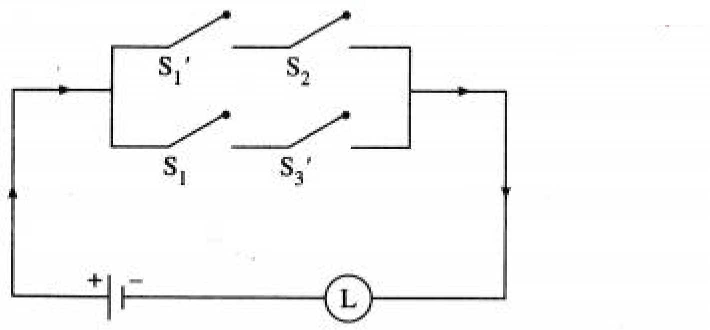
Solution:

The symbolic form of the given circuit is : (p ∨ q) ∧ (q ∨ r) ∧ (r ∨ p)

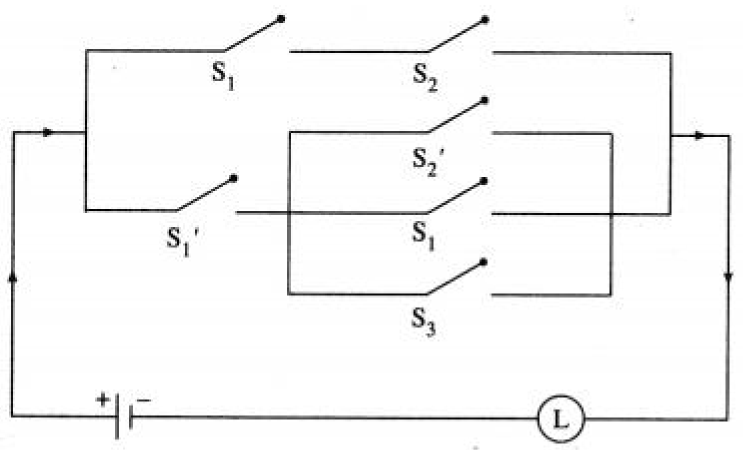
Input–Output Table

p	q	r	$p \vee q$	$q \vee r$	$r \vee p$	$(p \vee q) \wedge (q \vee r) \wedge (r \vee p)$
1	1	1	1	1	1	1
1	1	0	1	1	1	1
1	0	1	1	1	1	1
1	0	0	1	0	1	0
0	1	1	1	1	1	1
0	1	0	1	1	0	0
0	0	1	0	1	1	0
0	0	0	0	0	0	0

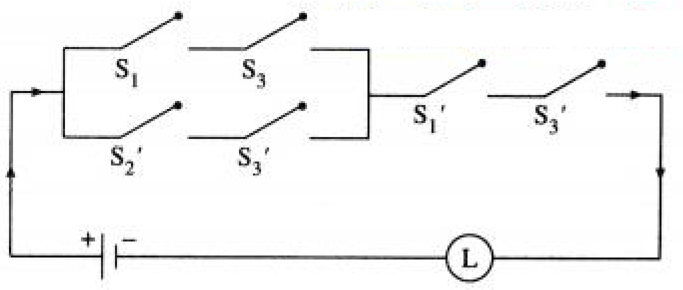
Question 2.
Construct the switching circuit of the following :
(i) (~p∧ q) ∨ (p∧ ~r)
Solution:
Let p : the switch S_1 is closed
q : the switch S_2 is closed
r : the switch S_3 is closed
~p : the switch S_1' is closed or the switch S_1 is open
~ q : the switch S_2' is closed or the switch S_2 is open
~ r : the switch S_3' is closed or the switch S_3 is open.
Then the switching circuits corresponding to the given statement patterns are :



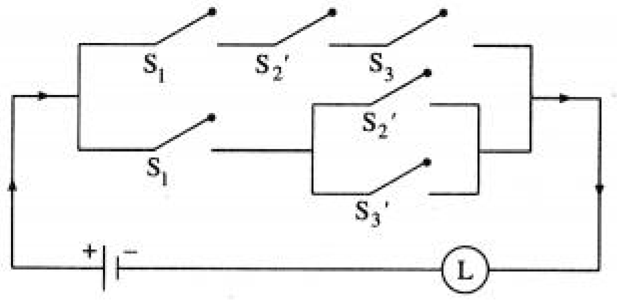
(ii) (p∧ q) ∨ [~p ∧ (~q ∨ p ∨ r)]
Solution:



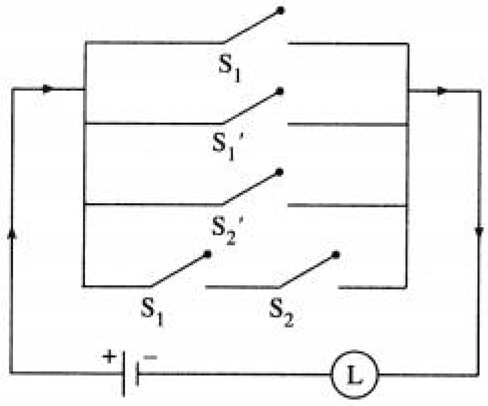
(iii) [(p ∧ r) ∨ (~q ∧ ~r)] ∧ (~p ∧ ~r)
Solution:



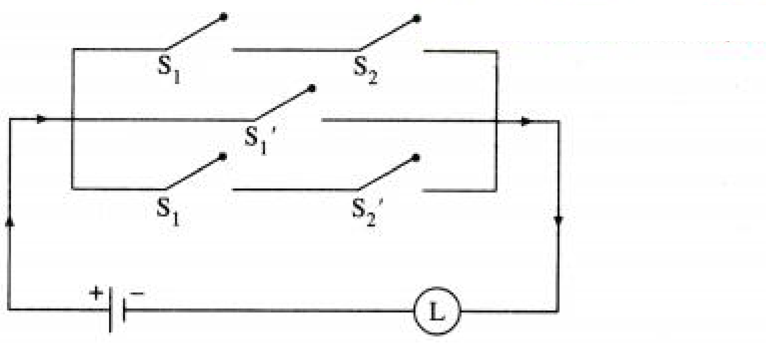
(iv) $(p \land \sim q \land r) \lor [p \land (\sim q \lor \sim r)]$
Solution:



(v) $p \lor (\sim p) \lor (\sim q) \lor (p \land q)$
Solution:



(vi) $(p \land q) \lor (\sim p) \lor (p \land \sim q)$
Solution:



Question 3.
Give an alternative equivalent simple circuits for the following circuits :
(i)

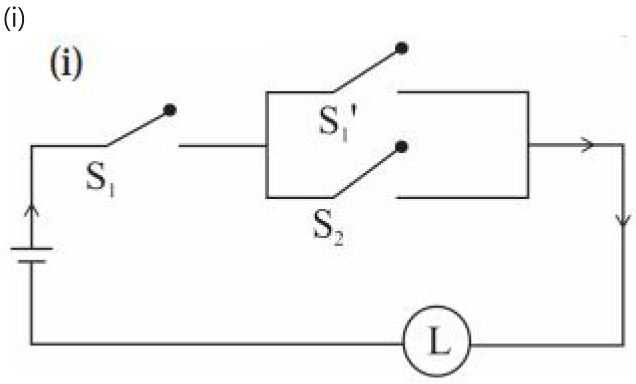
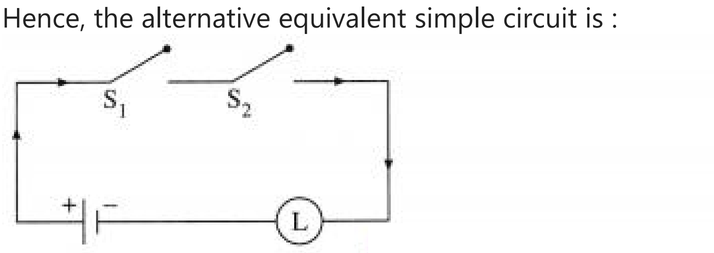


Fig. 1.23

Solution:
(i) Let p : the switch S_1 is closed
q : the switch S_2 is closed
~ p : the switch S_1' is closed or the switch Si is open Then the symbolic form of the given circuit is :
$p \land (\sim p \lor q)$.
Using the laws of logic, we have,
$p \land (\sim p \lor q)$
$= (p \land \sim p) \lor (p \land q)$ ...(By Distributive Law)
$= F \lor (p \land q)$ ... (By Complement Law)
$= p \land q$... (By Identity Law)
Hence, the alternative equivalent simple circuit is :



(ii)

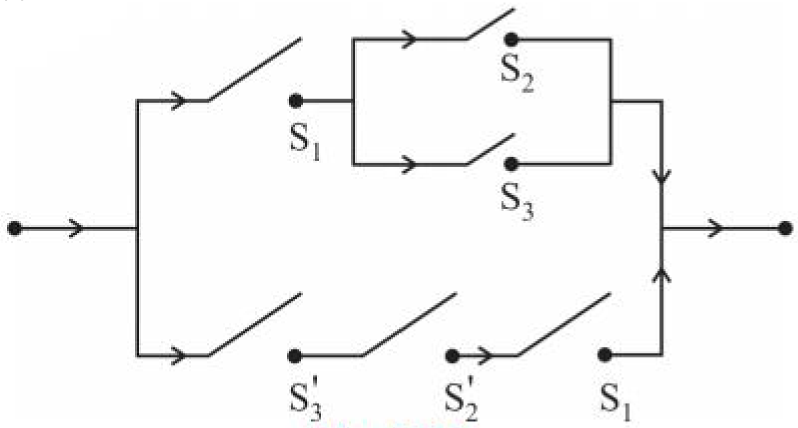
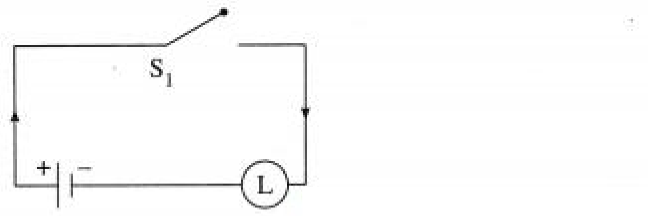


Fig. 1.24

Let p : the switch S_1 is closed
q : the switch S_2 is closed
r : the switch S_3 is closed
~q : the switch S_2' is closed or the switch S_2 is open
~r : the switch S_3' is closed or the switch S_3 is open.
Then the symbolic form of the given circuit is :
[p ∧ (q ∨ r)] ∨ (~r ∧ ~q ∧ p).
Using the laws of logic, we have
[p ∧ (q ∨ r)] ∨ (~r ∧ ~q ∧ p)
≡ [p ∧ (q ∨ r)] ∨ [~(r ∨ q) ∧ p] .... (By De Morgan's Law)
≡ [p ∧ (q ∨ r)] ∨ [p ∧ ~(q ∨ r)] ... (By Commutative Law)
≡ p ∧ [(q ∨ r) ∨ ~(q ∨ r)) ... (By Distributive Law)
≡ p ∧ T ... (By Complement Law)
≡ p ... (By Identity Law)
Hence, the alternative equivalent simple circuit is :



Question 4.
Write the symbolic form of the following switching circuits construct its switching table and interpret it.
i)

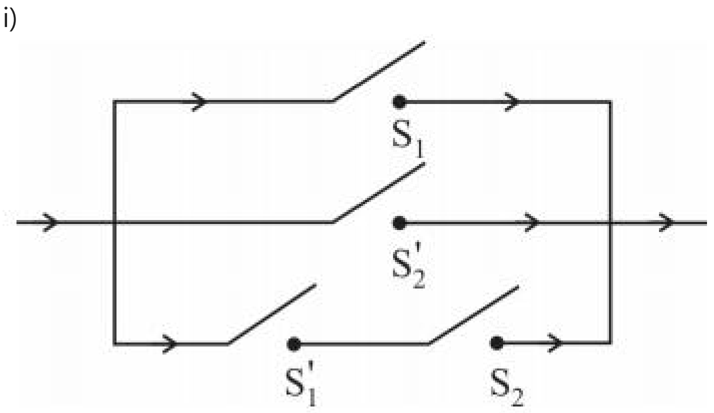


Fig. 1.25

Solution:
Let p : the switch S_1 is closed
q : the switch S_2 is closed
~p : the switch S_1' is closed or the switch S_1 is open
~ q : the switch S_2' is closed or the switch S_2 is open.
Then the symbolic form of the given circuit is :
(p ∨ ~q) ∨ (~p ∧ q)

Switching Table

p	q	$\sim p$	$\sim q$	$p \vee \sim q$	$\sim p \wedge q$	$(p \vee \sim q) \vee (\sim p \wedge q)$
1	1	0	0	1	0	1
1	0	0	1	1	0	1
0	1	1	0	0	1	1
0	0	1	1	1	0	1

Since the final column contains all' 1', the lamp will always glow irrespective of the status of switches.

ii)

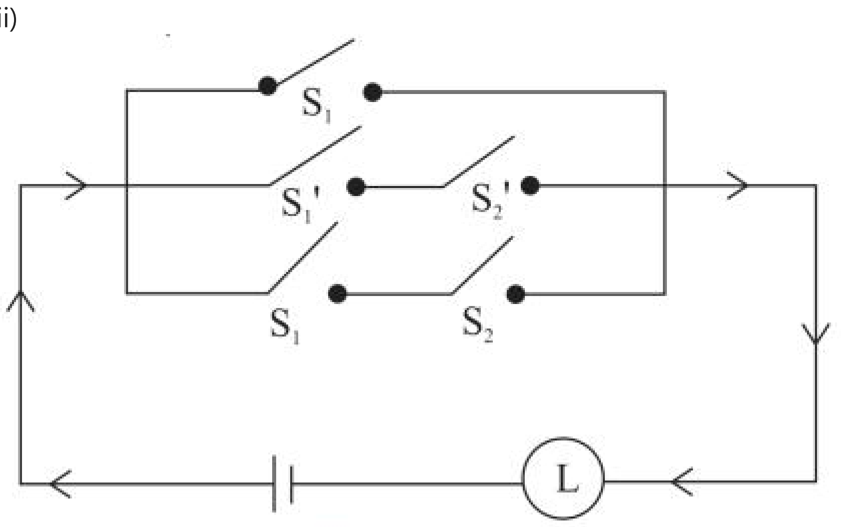


Fig. 1.26

Solution:
Let p : the switch S_1 is closed
q : the switch S_2 is closed
~p : the switch S_1 is closed or the switch S_1 is open.
~q : the switch S_2' is closed or the switch S_2 is open.
Then the symbolic form of the given circuit is : $p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$

Switching Table

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \wedge q$	$p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$
1	1	0	0	0	1	1
1	0	0	1	0	0	1
0	1	1	0	0	0	0
0	0	1	1	1	0	1

Since the final column contains '0' when p is 0 and q is '1', otherwise it contains '1'.
Hence, the lamp will not glow when S_1 is OFF and S_2 is ON, otherwise the lamp will glow.

iii)

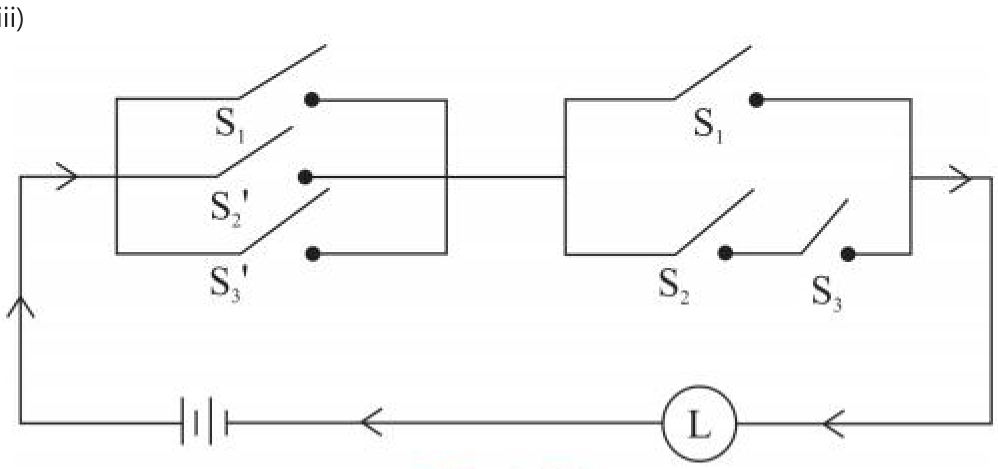


Fig. 1.27

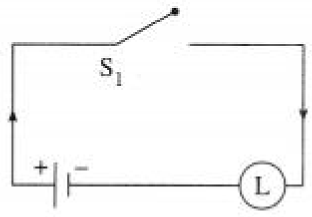
Solution:
Let p : the switch S_1 is closed
q : the switch S_2 is closed
r : the switch S_3 is closed
~q : the switch S_2' is closed or the switch S_2 is open
~r: the switch S_3' is closed or the switch S_3 is open.
Then the symbolic form of the given circuit is : $[p \vee (\sim q) \vee r)] \wedge [p \vee (q \wedge r)]$

Switching Table

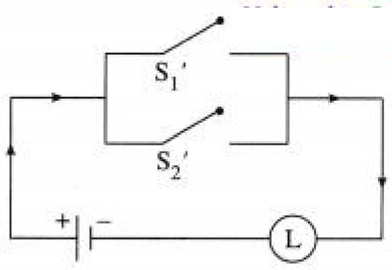
p	q	r	$\sim q$	$\sim r$	$p\vee(\sim q)\vee(\sim r)$ (I)	$q\wedge r$	$p\vee(q\wedge r)$ (II)	Final column (I) $\wedge$ (II)
1	1	1	0	0	1	1	1	1
1	1	0	0	1	1	0	1	1
1	0	1	1	0	1	0	1	1
1	0	0	1	1	1	0	1	1
0	1	1	0	0	0	1	1	0
0	1	0	0	1	1	0	0	0
0	0	1	1	0	1	0	0	0
0	0	0	1	1	1	0	0	0

From the switching table, the 'final column' and the column of p are identical. Hence, the lamp will glow which S_1 is 'ON'.

Question 5.
Obtain the simple logical expression of the following. Draw the corresponding switching circuit.
(i) $p \vee (q \wedge \sim q)$
Solution:
Using the laws of logic, we have, $p \vee (q \wedge \sim q)$
$\equiv p \vee F$ … (By Complement Law)
$\equiv p$ … (By Identity Law)
Hence, the simple logical expression of the given expression is p.
Let p : the switch S_1 is closed
Then the corresponding switching circuit is :



(ii) $(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)]$
Solution:
Using the laws of logic, we have,
$(\sim p \wedge q) \vee (\sim p \vee \sim q) \vee (p \wedge \sim q)$
$\equiv [\sim p \wedge (q \vee \sim q)] \vee (p \wedge \sim q)$… (By Distributive Law)
$\equiv (\sim p \wedge T) \vee (p \wedge \sim q)$ … (By Complement Law)
$\equiv \sim p \vee (p \wedge \sim q)$ … (By Identity Law)
$\equiv (\sim p \vee p) \wedge (\sim p \wedge \sim q)$ … (By Distributive Law)
$\equiv T \wedge (\sim p \wedge \sim q)$ … (By Complement Law)
$\equiv \sim p \vee \sim q$ … (By Identity Law)
Hence, the simple logical expression of the given expression is $\sim p \vee \sim q$.
Let p : the switch S_1 is closed
q : the switch S_2 is closed
$\sim p$: the switch S_1' is closed or the switch S_1 is open
$\sim q$: the switch S_2' is closed or the switch S_2 is open,
Then the corresponding switching circuit is :



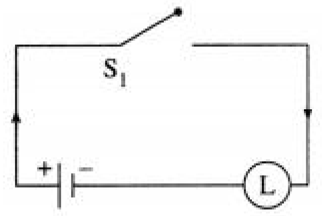
(iii) $[p (\vee (\sim q) \vee \sim r)] \wedge (p \vee (q \wedge r)$
Solution:
Using the laws of logic, we have,
$[p \vee (\sim (q) \vee (\sim r)] \wedge [p \vee (q \wedge r)]$
$= [p \vee \{ \sim(q \wedge r)\}] \wedge [p \vee (q \wedge r)]$ … (By De Morgan's Law)
$= p \vee [\sim(q \wedge r) \wedge (q \wedge r)]$ … (By Distributive Law)
$= p \vee F$ … (By Complement Law)

= p ... (By Identity Law)

Hence, the simple logical expression of the given expression is p.

Let p : the switch S_1 is closed

Then the corresponding switching circuit is :



(iv) (p ∧ q ∧ ~p) ∨ (~p ∧ q ∧ r) ∨ (p ∧ ~q ∧ r) ∨ (p ∧ q ∧ r)

Question is Modified

(p ∧ q ∧ ~p) ∨ (~p ∧ q ∧ r)∨ (p ∧ q ∧ r)

Solution:

Using the laws of logic, we have,

(p ∧ q ∧ ~p) ∨ (~p ∧ q ∧ r) ∨ (p ∧ q ∧ r)

= (p ∧ ~p ∧ q) ∨ (~p ∧ q ∧ r) ∨ (p ∧ q ∧ r) ... (By Commutative Law)

= (F ∧ q) ∨ (~p ∧ q ∧ r) ∨ (p ∧ q ∧ r) ... (By Complement Law)

= F ∨ (~p ∧ q ∧ r) ∨ (p ∧ q ∧ r) ... (By Identity Law)

= (~p ∧ q ∧ r) ∨ (p ∧ q ∧ r) ... (By Identity Law)

= (~ p ∨ p) ∧ (q ∧ r) ... (By Distributive Law)

= T ∧ (q ∧ r) ... (By Complement Law)

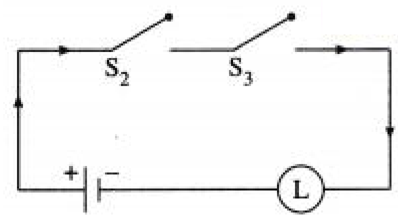
= q ∧ r ... (By Identity Law)

Hence, the simple logical expression of the given expression is q ∧ r.

Let q : the switch S_2 is closed

r : the switch S_3 is closed.

Then the corresponding switching circuit is :



Maharashtra State Board 12th Maths Solutions Chapter 1 Mathematical Logic Miscellaneous Exercise 1

Question 1.
Select and write the correct answer from the given alternatives in each of the following questions:
i) If $p \wedge q$ is false and $p \vee q$ is true, the _______ is not true.
(A) $p \vee q$
(B) $p \leftrightarrow q$
(C) $\sim p \vee \sim q$
(D) $q \vee \sim p$
Solution:
(b) $p \leftrightarrow q$.

(ii) $(p \wedge q) \rightarrow r$ is logically equivalent to _______.
(A) $p \rightarrow (q \rightarrow r)$
(B) $(p \wedge q) \rightarrow \sim r$
(C) $(\sim p \vee \sim q) \rightarrow \sim r$
(D) $(p \vee q) \rightarrow r$
Solution:
(a) $p \rightarrow (q \rightarrow r)$ [Hint: Use truth table.]

(iii) Inverse of statement pattern $(p \vee q) \rightarrow (p \wedge q)$ is _______.
(A) $(p \wedge q) \rightarrow (p \vee q)$
(B) $\sim(p \vee q) \rightarrow (p \wedge q)$
(C) $(\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$
(D) $(\sim p \vee \sim q) \rightarrow (\sim p \wedge \sim q)$
Solution:
(c) $(\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$

(iv) If $p \wedge q$ is F, $p \rightarrow q$ is F then the truth values of p and q are _______.
(A) T, T
(B) T, F
(C) F, T
(D) F, F
Solution:
(b) T, F

(v) The negation of inverse of $\sim p \rightarrow q$ is _______.
(A) $q \wedge p$
(B) $\sim p \wedge \sim q$
(C) $p \wedge q$
(D) $\sim q \rightarrow \sim p$
Solution:
(a) $q \wedge p$

(vi) The negation of $p \wedge (q \rightarrow r)$ is _______.
(A) $\sim p \wedge (\sim q \rightarrow \sim r)$
(B) $p \vee (\sim q \vee r)$
(C) $\sim p \wedge (\sim q \rightarrow \sim r)$
(D) $\sim p \vee (\sim q \wedge \sim r)$
Solution:
(d) $\sim p \vee (q \wedge \sim r)$

(vii) If $A = \{1, 2, 3, 4, 5\}$ then which of the following is not true?
(A) $\exists x \in A$ such that $x + 3 = 8$
(B) $\exists x \in A$ such that $x + 2 < 9$
(C) $\forall x \in A, x + 6 \geq 9$
(D) $\exists x \in A$ such that $x + 6 < 10$
Solution:
(c) $\exists x \in A, x + 6 \geq 9$.

Question 2.
Which of the following sentences are statements in logic? Justify. Write down the truth value of the statements :
(i) $4! = 24$.

Solution:
It is a statement which is true, hence its truth value is 'T'.

(ii) π is an irrational number.
Solution:
It is a statement which is true, hence its truth value is 'T'.

(iii) India is a country and Himalayas is a river.
Solution:
It is a statement which is false, hence its truth value is 'F'. ….[T ∧ F ≡ F]

(iv) Please get me a glass of water.
Solution:
It is an imperative sentence, hence it is not a statement.

(v) $\cos^2\theta - \sin^2\theta = \cos 2\theta$ for all $\theta \in R$.
Solution:
It is a statement which is true, hence its truth value is 'T'.

(vi) If x is a whole number the x + 6 = 0.
Solution:
It is a statement which is false, hence its truth value is 'F'.

Question 3.
Write the truth values of the following statements :
(i) $\sqrt{5}$ is an irrational but $3\sqrt{5}$ is a complex number.
Solution:
Let p : $\sqrt{5}$ is an irrational.
q : $3\sqrt{5}$ is a complex number.
Then the symbolic form of the given statement is p ∧ q.
The truth values of p and q are T and F respectively.
∴ the truth value of p ∧ q is F. … [T ∧ F ≡ F]

(ii) $\forall n \in N$, $n^2 + n$ is even number while $n^2 - n$ is an odd number.
Solution:
Let p : $\forall n \in N$, $n^2 + n$ is an even number.
q : $\forall n \in N$, $n^2 - n$ is an odd number.
Then the symbolic form of the given statement is p ∧ q.
The truth values of p and q are T and F respectively.
∴ the truth value of p ∧ q is F. … [T ∧ F ≡ F].

(iii) $\exists n \in N$ such that $n + 5 > 10$.
Solution:
$\exists n \in N$, such that $n + 5 > 10$ is a true statement, hence its truth value is T.
(All $n \geq 6$, where $n \in N$, satisfy $n + 5 > 10$).

(iv) The square of any even number is odd or the cube of any odd number is odd.
Solution:
Let p : The square of any even number is odd.
q : The cube of any odd number is odd.
Then the symbolic form of the given statement is p ∨ q.
The truth values of p and q are F and T respectively.
∴ the truth value of p ∨ q is T. … [F ∨ T ≡ T].

(v) In Δ ABC if all sides are equal then its all angles are equal.
Solution:
Let p : ABC is a triangle and all its sides are equal.
q : Its all angles are equal.
Then the symbolic form of the given statement is p → q
If the truth value of p is T, then the truth value of q is T.
∴ the truth value of p → q is T. … [T → T ≡ T].

(vi) $\forall n \in N$, $n + 6 > 8$.
Solution:

$\forall n \in N$, $11 + 6 > 8$ is a false statement, hence its truth value is F.
{$n = 1 \in N$, $n = 2 \in N$ do not satisfy $n + 6 > 8$).

Question 4.
If A = {1, 2, 3, 4, 5, 6, 7, 8, 9}, determine the truth value of each of the following statement :
(i) $\exists x \in A$ such that $x + 8 = 15$.
Solution:
True

(ii) $\forall x \in A$, $x + 5 < 12$.
Solution:
False

(iii) $\exists x \in A$, such that $x + 7 \geq 11$.
Solution:
True

(iv) $\forall x \in A$, $3x \leq 25$.
Solution:
False

Question 5.
Write the negations of the following :
(i) $\forall n \in A$, $n + 7 > 6$.
Solution:
The negation of the given statements are :
$\exists n \in A$, such that $n + 7 \leq 6$.
OR $\exists n \in A$, such that $n + 7 \not> 6$.

(ii) $\exists x \in A$, such that $x + 9 \leq 15$.
Solution:
$\forall x \in A$, $x + 9 > 15$.

(iii) Some triangles are equilateral triangle.
Solution:
All triangles are not equilateral triangles.

Question 6.
Construct the truth table for each of the following :
(i) $p \rightarrow (q \rightarrow p)$
Solution:

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

(ii) $(\sim p \vee \sim q) \leftrightarrow [\sim(p \wedge q)]$
Solution:

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \wedge q$	$\sim(p \wedge q)$	$(\sim p \vee \sim q) \leftrightarrow [\sim(p \wedge q)]$
T	T	F	F	F	T	F	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	T	T

(iii) ~(~p ∧ ~q) ∨ q
Solution:

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$\sim(\sim p \wedge \sim q)$	$\sim(\sim p \wedge \sim q) \vee q$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	F

(iv) [(p ∧ q) ∨ r] ∧ [~r ∨ (p ∧ q)]
Solution:

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$	$\sim r$	$\sim r \vee (p \wedge q)$	$[(p \wedge q) \vee r] \wedge [\sim r \vee (p \wedge q)]$
T	T	T	T	T	F	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	F	F	F
T	F	F	F	F	T	T	F
F	T	T	F	T	F	F	F
F	T	F	F	F	T	T	F
F	F	T	F	T	F	F	F
F	F	F	F	F	T	T	F

(v) [(~p ∨ q) ∧ (q → r)] → (p → r)
Solution:

p	q	r	$\sim p$	$\sim p \vee q$	$q \rightarrow r$	$(\sim p \vee q) \wedge (q \rightarrow r)$	$p \rightarrow r$	$[(\sim p \vee q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	F	F	T	F	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

Question 7.
Determine whether the following statement patterns are tautologies contradictions or contingencies :
(i) [(p → q) ∧ ~q)] → ~p
Solution:

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \sim q$	$[(p \rightarrow q) \wedge \sim q] \rightarrow (\sim p)$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

All the entries in the last column of the above truth table are T.
∴ [(p → q) ∧ ~q)] → ~p is a tautology.

(ii) [(p ∨ q) ∧ ~p] ∧ ~q
Solution:

p	q	$\sim p$	$\sim q$	$p \vee q$	$(p \vee q) \wedge \sim p$	$[(p \vee q) \wedge \sim p] \wedge (\sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	F
F	F	T	T	F	F	F

All the entries in the last column of the above truth table are F.
∴ [(p ∨ q) ∧ ~p] ∧ ~q is a contradiction.

(iii) (p → q) ∧ (p ∧ ~q)
Solution:

p	q	$\sim q$	$p \to q$	$p \wedge \sim q$	$(p \to q) \wedge (p \wedge \sim q)$
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	T	F	F
F	F	T	T	F	F

All the entries in the last column of the above truth table are F.
∴ (p → q) ∧ (p ∧ ~q) is a contradiction.

(iv) [p → (q → r)] ↔ [(p ∧ q) → r]
Solution:

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$	$[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

All the entries in the last column of the above truth table are T.
∴ [p → (q → r)] ↔ [(p ∧ q) → r] is a tautology.

(v) [(p ∧ (p → q)] → q
Solution:

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

All the entries in the last column of the above truth table are T.
∴ [(p ∧ (p → q)] → q is a tautology.

(vi) (p ∧ q) ∨ (~p ∧ q) ∨ (p ∨ ~q) ∨ (~p ∧ ~q)
Solution:

p	q	$\sim p$	$\sim q$	$p \wedge q$ (I)	$\sim p \wedge q$ (II)	$p \vee \sim q$ (III)	$\sim p \wedge \sim q$ (IV)	(I) $\vee$ (II) $\vee$ (III) $\vee$ (IV)
T	T	F	F	T	F	T	F	T
T	F	F	T	F	F	T	F	T
F	T	T	F	F	T	F	F	T
F	F	T	T	F	F	T	T	T

All the entries in the last column of the above truth table are T.
∴ (p ∧ q) ∨ (~p ∧ q) ∨ (p ∨ ~q) ∨ (~p ∧ ~q) is a tautology.

(vii) [(p ∨ ~q) ∨ (~p ∧ q)] ∧ r
Solution:

p	q	r	$\sim p$	$\sim q$	$p \vee \sim q$	$\sim p \wedge q$	$(p \vee \sim q) \vee (\sim p \wedge q)$ (I)	(I) $\wedge r$
T	T	T	F	F	T	F	T	T
T	T	F	F	F	T	F	T	F
T	F	T	F	T	T	F	T	T
T	F	F	F	T	T	F	T	F
F	T	T	T	F	F	T	T	T
F	T	F	T	F	F	T	T	F
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	F

The entries in the last column are neither T nor all F.
∴ [(p ∨ ~q) ∨ (~p ∧ q)] ∧ r is a contingency.

(viii) (p → q) ∨ (q → p)
Solution:

p	q	$p \to q$	$q \to p$	$(p \to q) \vee (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

All the entries in the last column of the above truth table are T.
∴ (p → q) ∨ (q → p) is a tautology.

Question 8.
Determine the truth values ofp and q in the following cases :
(i) (p ∨ q) is T and (p ∧ q) is T
Solution:

p	q	$p \vee q$	$p \wedge q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

Since p ∨ q and p ∧ q both are T, from the table the truth values of both p and q are T.

(ii) (p ∨ q) is T and (p ∨ q) → q is F
Solution:

p	q	$p \vee q$	$(p \vee q) \rightarrow q$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	T

Since the truth values of (p ∨ q) is T and (p ∨ q) → q is F, from the table, the truth values of p and q are T and F respectively.

(iii) (p ∧ q) is F and (p ∧ q) → q is T
Solution:

p	q	$p \wedge q$	$(p \wedge q) \rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Since the truth values of (p ∧ q) is F and (p ∧ q) → q is T, from the table, the truth values of p and q are either T and F respectively or F and T respectively or both F.

Question 9.
Using truth tables prove the following logical equivalences :
(i) p ↔ q ≡ (p ∧ q) ∨ (~p ∧ ~q)
Solution:

1	2	3	4	5	6	7	8
p	q	$p \leftrightarrow q$	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
T	T	T	F	F	T	F	T
T	F	F	F	T	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	T	F	T	T

The entries in the columns 3 and 8 are identical.
∴ p ↔ q ≡ (p ∧ q) ∨ (~p ∧ ~q).

(ii) (p ∧ q) → r ≡ p → (q → r)
Solution:

1	2	3	4	5	6	7
p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow r$	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	T	T	T
F	T	F	F	T	F	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

The entries in the columns 5 and 7 are identical.
∴ (p ∧ q) → r ≡ p → (q → r).

Question 10.
Using rules in logic, prove the following :
(i) $p \leftrightarrow q \equiv \sim(p \land \sim q) \land \sim(q \land \sim p)$
Solution:
By the rules of negation of biconditional,
$\sim(p \leftrightarrow q) \equiv (p \land \sim q) \lor (q \land \sim p)$
$\therefore \sim[(p \land \sim q) \lor (q \land \sim p)] \equiv p \leftrightarrow q$
$\therefore \sim(p \land \sim q) \land \sim(q \land \sim p) \equiv p \leftrightarrow q$ … (Negation of disjunction)
$\equiv p \leftrightarrow q \equiv \sim(p \land \sim q) \land \sim(q \land \sim p)$.

(ii) $\sim p \land q \equiv (p \lor q) \land \sim p$
Solution:
$(p \lor q) \land \sim p$
$\equiv (p \land \sim p) \lor (q \land \sim p)$ … (Distributive Law)
$\equiv F \lor (q \land \sim p)$ … (Complement Law)
$\equiv q \land \sim p$ … (Identity Law)
$\equiv \sim p \land q$ …(Commutative Law)
$\therefore \sim p \land q \equiv (p \lor q) \land \sim p$.

(iii) $\sim(p \lor q) \lor (\sim p \land q) \equiv \sim p$
Solution:
$\sim(p \lor q) \lor (\sim p \land q)$
$\equiv (\sim p \land \sim q) \lor (\sim p \land q)$ … (Negation of disjunction)
$\equiv \sim p \land (\sim q \lor q)$ … (Distributive Law)
$\equiv \sim p \land T$ … (Complement Law)
$\equiv \sim p$ … (Identity Law)
$\therefore \sim(p \lor q) \lor (\sim p \land q) \equiv \sim p$.

Question 11.
Using the rules in logic, write the negations of the following :
(i) $(p \lor q) \land (q \lor \sim r)$
Solution:
The negation of $(p \lor q) \land (q \lor \sim r)$ is
$\sim[(p \lor q) \land (q \lor \sim r)]$
$\equiv \sim(p \lor q) \lor \sim(q \lor \sim r)$ … (Negation of conjunction)
$\equiv (\sim p \land \sim q) \lor [\sim q \land \sim(\sim r)]$ … (Negation of disjunction)
$\equiv \{\sim p \land \sim q) \lor (\sim q \land r)$ … (Negation of negation)
$\equiv (\sim q \land \sim p) \lor (\sim q \land r)$ … (Commutative law)
$\equiv (\sim q) \land (\sim p \lor r)$ … (Distributive Law)

(ii) $p \land (q \lor r)$
Solution:
The negation of $p \land (q \lor r)$ is
$\sim[p \land (q \lor r)]$
$\equiv \sim p \lor \sim(q \lor r)$ … (Negation of conjunction)
$\equiv \sim p \lor (\sim q \land \sim r)$ … (Negation of disjunction)

(iii) $(p \rightarrow q) \land r$
Solution:
The negation of $(p \rightarrow q) \land r$ is
$\sim[(p \rightarrow q) \land r]$
$\equiv \sim(p \rightarrow q) \lor (\sim r)$ … (Negation of conjunction)
$\equiv (p \land \sim q) \lor (\sim r)$ … (Negation of implication)

(iv) $(\sim p \land q) \lor (p \land \sim q)$
Solution:
The negation of $(\sim p \land q) \lor (p \land \sim q)$ is
$\sim[(\sim p \land q) \lor (p \land \sim q)]$
$\equiv \sim(\sim p \land q) \land \sim(p \land \sim q)$ … (Negation of disjunction)
$\equiv [\sim(\sim p) \lor \sim q] \land [\sim p \lor \sim(q)]$ … (Negation of conjunction)
$\equiv (p \lor \sim q) \land (\sim p \lor q)$ … (Negation of negation)

Question 12.
Express the following circuits in the symbolic form. Prepare the switching table :
(i)

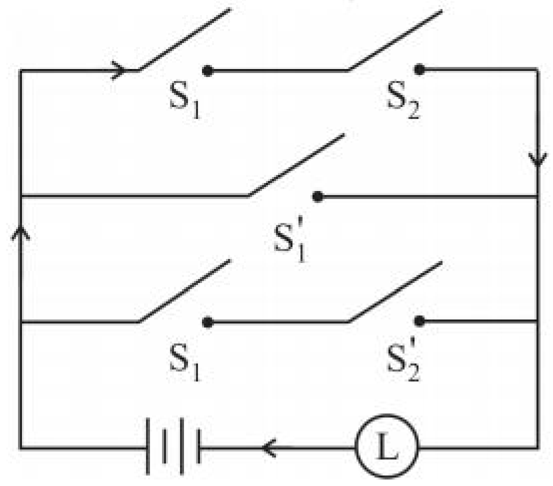


Fig. 1.30

Solution:
Let p : the switch S_1 is closed
q : the switch S_2 is closed
~ p : the switch S_1' is closed or the switch S_1 is open
~ q: the switch S_2' is closed or the switch S_2 is open.
Then the symbolic form of the given circuit is :
$(p \wedge q) \vee (\sim p) \vee (p \wedge \sim q)$.

Switching Table

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$(p\wedge q)\vee(\sim p)\vee(p\wedge \sim q)$
1	1	0	0	1	0	1
1	0	0	1	0	1	1
0	1	1	0	0	0	1
0	0	1	1	0	0	1

(ii)

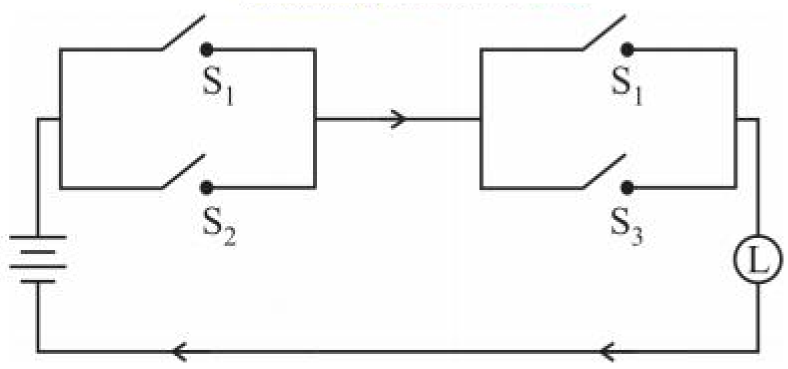


Fig. 1.31

Solution:
Let p : the switch S_1 is closed
q : the switch S_2 is closed
r : the switch S_3 is closed.

Then the symbolic form of the given statement is : (p ∨ q) ∧ (p ∨ r).

Switching Table

p	q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
1	1	1	1	1	1
1	1	0	1	1	1
1	0	1	1	1	1
1	0	0	1	1	1
0	1	1	1	1	1
0	1	0	1	0	0
0	0	1	0	1	0
0	0	0	0	0	0

Question 13.
Simplify the following so that the new circuit has minimum number of switches. Also, draw the simplified circuit.

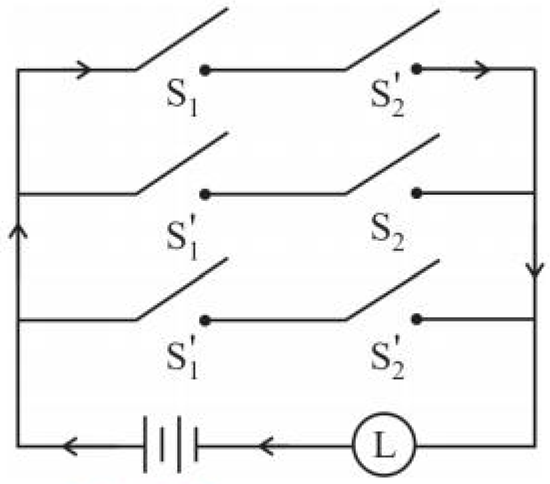
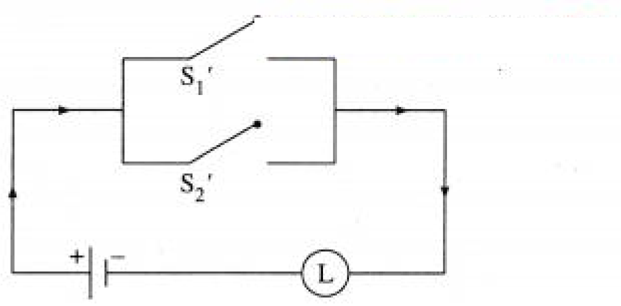


Fig. 1.32

Solution:
Let p : the switch S_1 is closed
q : the switch S_2 is closed
~ p: the switch S_1' is closed or the switch S_1 is open
~ q: the switch S_2' is closed or the switch S_2 is open.
Then the given circuit in symbolic form is :
(p ∧ ~q) ∨ (~p ∧ q) ∨ (~p ∧ ~q)
Using the laws of logic, we have,
(p ∧ ~q) ∨ (~p ∧ q) ∨ (~p ∧ ~ q)
= (p ∧ ~q) ∨ [(~p ∧ q) ∨ (~p ∧ ~q) …(By Complement Law)
= (p ∧ ~q) ∨ [~p ∧ (q ∨ ~q)} (By Distributive Law)
= (p ∧ ~q) ∨ (~p ∧ T) …(By Complement Law)
= (p ∧ ~q) ∨ ~ p …(By Identity Law)
= (p ∨ ~p) ∧ (~q ∨ ~p) …(By Distributive Law)
= ~q ∨ ~p …(By Identity Law)
= ~p ∨ ~p …(By Commutative Law)
Hence, the simplified circuit for the given circuit is :



(ii)

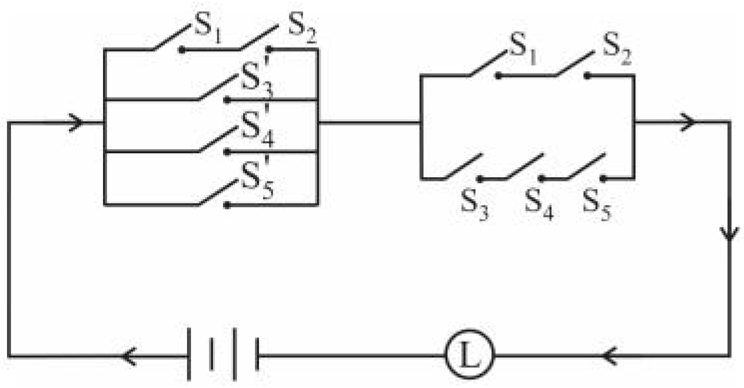
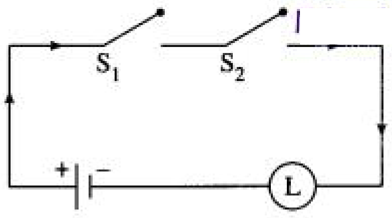


Fig. 1.33

Solution:
(ii) Let p : the switch S_1 is closed
q : the switch S_2 is closed
r : the switch S_3 is closed
s : the switch S_4 is closed
t : the switch S_5 is closed
~ p : the switch S_1' is closed or the switch S_1 is open
~ q : the switch S_2' is closed or the switch S_2 is open
~ r : the switch S_3' is closed or the switch S_3 is open
~ s : the switch S_4' is closed or the switch S_4 is open
~ t : the switch S_5' is closed or the switch S_5 is open.
Then the given circuit in symbolic form is
[(p ∧ q) ∨ ~r ∨ ~s ∨ ~t] ∧ [(p ∧ q) ∨ (r ∧ s ∧ t)]
Using the laws of logic, we have,
[(p ∧ q) ∨ ~r ∨ ~s ∨ ~ t] ∧ [(p A q) ∨ (r ∧ s ∧ t)]
= [(p∧ q) ∨ ~(r ∧ s ∧ t)] ∧ [(p ∧ q) ∨ (r ∧ s ∧ t)] … (By De Morgan's Law)
= (p ∧ q) ∨ [~(r ∧ s ∧ t) ∧ (r ∧ s ∧ t)] … (By Distributive Law)
= (p ∧ q) ∨ F … (By Complement Law)
= p ∧ q … (By Identity Law)
Hence, the alternative simplified circuit is :



Question 14.
Check whether the following switching circuits are logically equivalent – Justify.
(A)

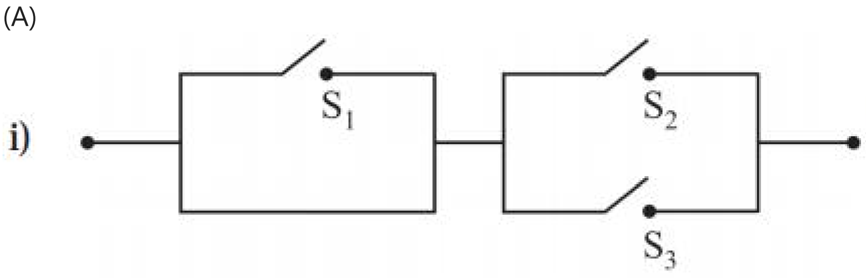


Fig. 1.34

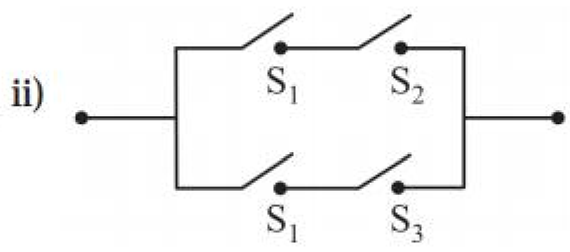


Fig. 1.35

Solution:
Let p : the switch S_1 is closed
q : the switch S_2 is closed
r : the switch S_3 is closed
(A) The symbolic form of the given switching circuits are
p ∧ (q ∨ r) and (p ∧ q) ∨ (p ∧ r) respectively.
By Distributive Law, p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r)
Hence, the given switching circuits are logically equivalent.

(B)

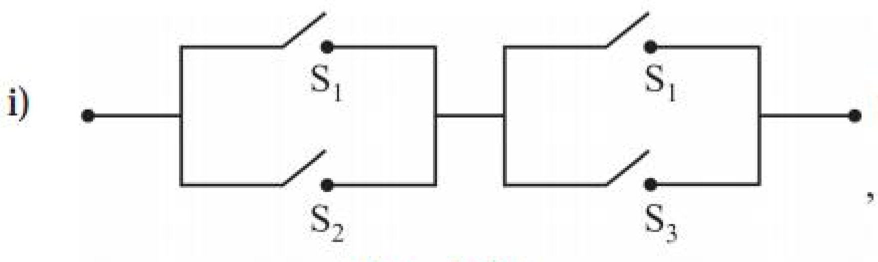


Fig. 1.36

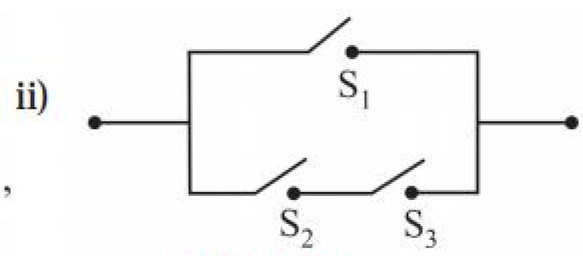


Fig. 1.37

Solution:
The symbolic form of the given switching circuits are

(p ∨ q) ∧ (p ∨ r) and p ∨ (q ∧ r)
By Distributive Law,
p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)
Hence, the given switching circuits are logically equivalent.

Question 15.
Give alternative arrangement of the switching following circuit, has minimum switches.

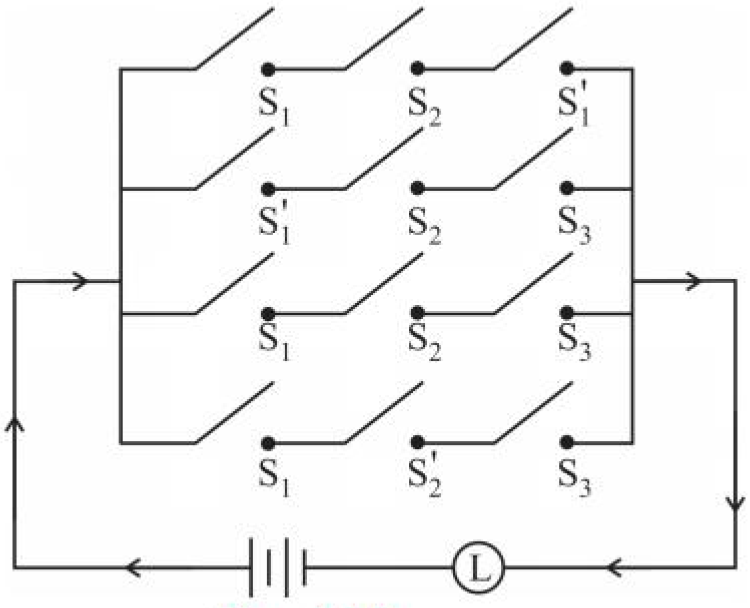
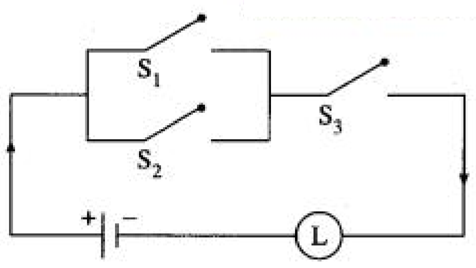


Fig. 1.38

Solution:
Let p : the switch S_1 is closed
q : the switch S_2 is closed
r : the switch S_3 is closed
~p : the switch S_1' is closed, or the switch S_1 is open
~q : the switch S_2' is closed or the switch S_2 is open.
Then the symbolic form Of the given circuit is :
(p ∧ q ∧ ~p) ∨ (~p ∧ q ∧ r) ∨ (p ∧ q ∧ r) ∨ (p ∧ ~q ∧ r)
Using the laws of logic, we have,
(p ∧ q ∧ ~p) ∨ (~p ∧ q ∧ r) ∨ (p ∧ q ∧ r) ∨ (p ∧ ~q ∧ r)
≡ (p ∧ ~p ∧ q) ∨ (~p ∧ q ∧ r) ∨ (p ∧ q ∧ r) y (p ∧ ~q ∧ r) ...(By Commutative Law)
≡ (F ∧ q) ∨ (~p ∧ q ∧ r) ∨ (p ∧ q ∧ r) ∨ (p ∧ ~q ∧ r) ... (By Complement Law)
≡ F ∨ (~p ∧ q ∧ r) ∨ (p ∧ q ∧ r) ∨ (p ∧ ~q ∧ r) ... (By Identity Law)
≡ (~p ∧ q ∧ r) ∨ (p ∧ q ∧ r) ∨ (p ∧ ~q ∧ r) ... (By Identity Law)
≡ [(~p ∨ p) ∧ (q ∧ r)] ∨ (p ∧ ~q ∧ r) ... (By Distributive Law)
≡ [T ∧ (q ∧ r)] ∨ (p ∧ ~q ∧ r) = (q ∧ r) ∨ (p ∧ ~q ∧ r) ...(By Complement Law)
≡ (q ∧ r) ∨ (p ∧ ~q ∧ r) ... (By Identity Law)
≡ [q ∨ (p ∧ ~q)] ∧ r ... (By Distributive Law)
≡ [q ∨ p) ∧ ((q ∨ ~q)] ∧ r ... (By Distributive Law)
≡ [(q ∨ p) ∧ T] ∧ r ...(By Complement Law)
≡ (q ∨ p) ∧ r ... (By Identity Law)
≡ (p ∨ q) ∧ r ...(By Commutative Law)
∴ the alternative arrangement of the new circuit with minimum switches is :



Question 16.
Simplify the following so that the new circuit circuit.

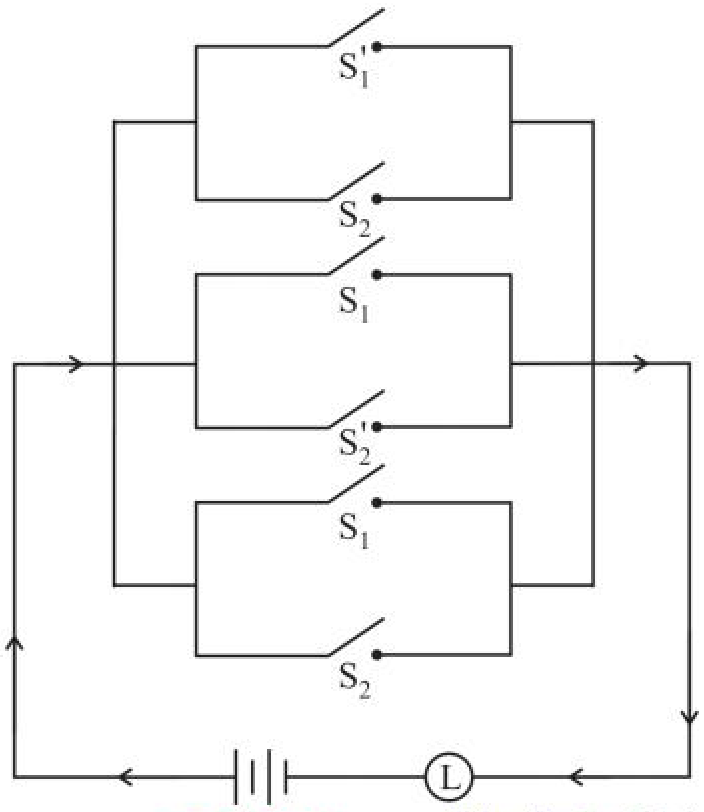
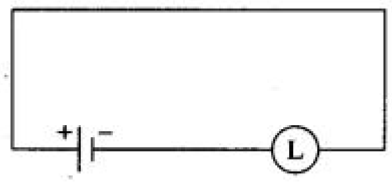


Fig. 1.39

Solution:
Let p : the switch S_1 is closed
q : the switch S_2 is closed
~ p : the switch S_1' is closed or the switch S_1 is open
~ q : the switch S_2' is closed or the switch S_2 is open.
Then the symbolic form of the given switching circuit is :
(~p ∨ q) ∨ (p ∨ ~q) ∨ (p ∨ q)
Using the laws of logic, we have,
(~p ∨ q) ∨ (p ∨ ~q) ∨ (p ∨ q)
≡ (~p ∨ q ∨ p ∨ ~q) ∨ (p ∨ q)
≡ [(~p ∨ p) ∨ (q ∨ ~q)] ∨ (p ∨ q) … (By Commutative Law)
≡ (T ∨ T) ∨ (p ∨ q) … (By Complement Law)
≡ T ∨ (p ∨ q) … (By Identity Law)
≡ T … (By Identity Law)
∴ the current always flows whether the switches are open or closed. So, it is not necessary to use any switch in the circuit.
∴ the simplified form of given circuit is :



Question 17.
Represent the following switching circuit in symbolic form and construct its switching table. Write your conclusion from the switching table.

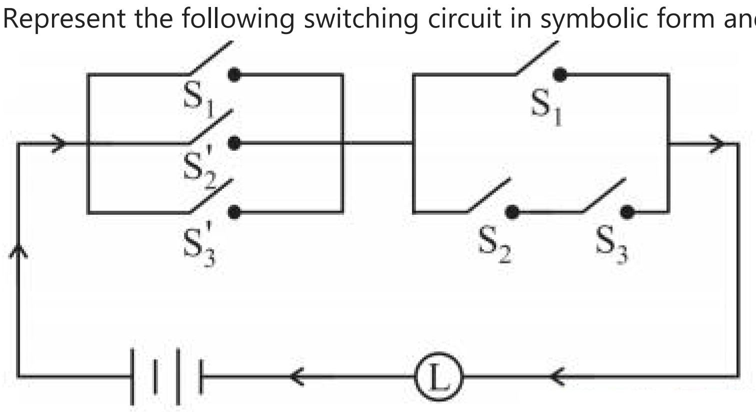


Fig. 1.40

Solution:
Let p : the switch S_1 is closed
q : the switch S_2 is closed
r : the switch S_3 is closed
~ q : the switch S_2' is closed or the switch S_2 is open
~ r : the switch S_3' is closed or the switch S_3 is open.
Then, the symbolic form of the given switching circuit is : [p ∨ (~ q) ∨ (~ r)] ∧ [p ∨ (q ∧ r)]

Switching Table

p	q	r	$\sim q$	$\sim r$	$p \vee (\sim q) \vee (\sim r)$ (I)	$q \wedge r$	$p \vee (q \wedge r)$ (II)	Final column (I) $\wedge$ (II)
1	1	1	0	0	1	1	1	1
1	1	0	0	1	1	0	1	1
1	0	1	1	0	1	0	1	1
1	0	0	1	1	1	0	1	1
0	1	1	0	0	0	1	1	0
0	1	0	0	1	1	0	0	0
0	0	1	1	0	1	0	0	0
0	0	0	1	1	1	0	0	0

From the table, the' final column' and the column of p are identical. Hence, the given circuit is equivalent to the simple circuit with only one switch S_1.
the simplified form of the given circuit is :