Maharashtra State Board 12th Commerce Maths Solutions Chapter 7 Application of Definite Integration Ex 7.1

Question 1.
Find the area of the region bounded by the following curves, the X-axis, and the given lines:
(i) y = x4, x = 1, x = 5
Solution:

$$\text{Required area} = \int_1^5 y\,dx, \text{ where } y = x^4$$

$$= \int_1^5 x^4\,dx = \left[\frac{x^5}{5}\right]_1^5$$

$$= \frac{1}{5}[3125 - 1] = \frac{3124}{5} \text{ sq units.}$$

(ii) y = 6x+4−−−−−√, x = 0, x = 2
Solution:

$$\text{Required area} = \int_0^2 y\,dx, \text{ where } y = \sqrt{6x+4}$$

$$= \int_0^2 \sqrt{6x+4}\,dx = \int_0^2 (6x+4)^{\frac{1}{2}}\,dx$$

$$= \left[\frac{(6x+4)^{\frac{3}{2}}}{3/2} \times \frac{1}{6}\right]_0^2$$

$$= \frac{1}{9}\left[(6x+4)^{\frac{3}{2}}\right]_0^2$$

$$= \frac{1}{9}[64 - 8]$$

$$= \frac{56}{9} \text{ sq units.}$$

(iii) 16−x2−−−−−−√, x = 0, x = 4
Solution:

$$\text{Required area} = \int_0^4 y\,dx, \text{ where } y = \sqrt{16 - x^2}$$

$$= \int_0^4 \sqrt{16 - x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$$

$$\ldots \left[\because \int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]$$

$$= 0 + 8\sin^{-1}(1) - 0 - 0 \qquad \ldots [\because \sin^{-1}(0) = 0]$$

$$= 8 \times \frac{\pi}{2} = 4\pi \text{ sq units.} \qquad \ldots \left[\because \sin^{-1}(1) = \frac{\pi}{2}\right]$$

(iv) 2y = 5x + 7, x = 2, x = 8
Solution:

$$\text{Required area} = \int_2^8 y\,dx, \text{ where } 2y = 5x + 7$$

$$\text{i.e. } y = \frac{5x+7}{2}$$

$$= \int_2^8 \left(\frac{5x+7}{2}\right)dx = \frac{1}{2}\int_2^8 (5x+7)\,dx$$

$$= \frac{1}{2}\left[5 \cdot \frac{x^2}{2} + 7x\right]_2^8$$

$$= \frac{1}{2}[160 + 56 - 10 - 14]$$

$$= \frac{1}{2}(192) = 96 \text{ sq units.}$$

(v) 2y + x = 8, x = 2, x = 4
Solution:

$$\text{Required area} = \int_2^4 y\,dx, \text{ where } 2y + x = 8$$

$$\text{i.e. } y = \frac{8-x}{2}$$

$$= \int_2^4 \left(\frac{8-x}{2}\right)dx = \frac{1}{2}\int_2^4 (8-x)\,dx$$

$$= \frac{1}{2}\left[8x - \frac{x^2}{2}\right]_2^4$$

$$= \frac{1}{2}[(32-8)-(16-2)]$$

$$= \frac{1}{2}(24-14) = 5 \text{ sq units.}$$

(vi) y = x^2 + 1, x = 0, x = 3
Solution:

$$\text{Required area} = \int_0^3 y\,dx, \text{ where } y = x^2 + 1$$

$$= \int_0^3 (x^2+1)\,dx$$

$$= \left[\frac{x^3}{3} + x\right]_0^3$$

$$= 9 + 3 - 0 = 12 \text{ sq units.}$$

(vii) $y = 2 - x^2$, $x = -1$, $x = 1$
Solution:

Required area $= \int_{-1}^{1} y\,dx$, where $y = 2 - x^2$

$= \int_{-1}^{1} (2 - x^2)\,dx$

$= 2\int_{0}^{1} (2 - x^2)\,dx$

... [$\because$ $f(x) = 2 - x^2$ is an even function]

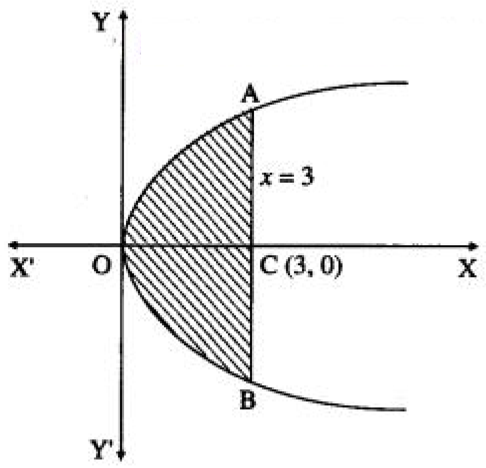
$= 2\left[2x - \frac{x^3}{3}\right]_0^1$

$= 2\left[2 - \frac{1}{3} - 0\right]$

$= 2\left(\frac{5}{3}\right)$

$= \frac{10}{3}$ sq units.

Question 2.
Find the area of the region bounded by the parabola $y^2 = 4x$ and the line $x = 3$.
Solution:



Required area = area of the region OABO
= 2(area of the region OACO)

$= 2\int_{0}^{3} y\,dx$, where $y^2 = 4x$, i.e. $y = 2\sqrt{x}$

$= 2\int_{0}^{3} 2\sqrt{x}\,dx$

$= 4\int_{0}^{3} x^{\frac{1}{2}}\,dx$

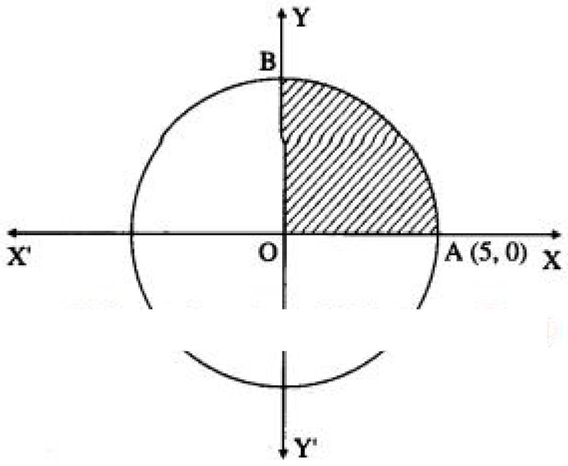
$= 4\cdot\left[\frac{x^{\frac{3}{2}}}{3/2}\right]_0^3$

$= \frac{8}{3}[x^{\frac{3}{2}}]_0^3$

$= \frac{8}{3}(3\sqrt{3} - 0)$

$= 8\sqrt{3}$ sq units.

Question 3.
Find the area of the circle $x^2 + y^2 = 25$.
Solution:



By the symmetry of the circle, its area is equal to 4 times the area of the region OABO.
Clearly, for this region, the limits of integration are 0 and 5.
From the equation of the circle, $y^2 = 25 - x^2$.
In the first quadrant y > 0

∴ y = $\sqrt{25-x^2}$

∴ area of the circle = 4(area of region OABO)

$$= 4\int_0^5 y\,dx = 4\int_0^5 \sqrt{25-x^2}\,dx$$

$$= 4\left[\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5$$

$$\ldots\left[\because \int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]$$

$$= 4\left[\left\{\frac{5}{2}\sqrt{25-25} + \frac{25}{2}\sin^{-1}(1)\right\} - \left\{\frac{0}{2}\sqrt{25-0} + \frac{25}{2}\sin^{-1}(0)\right\}\right]$$

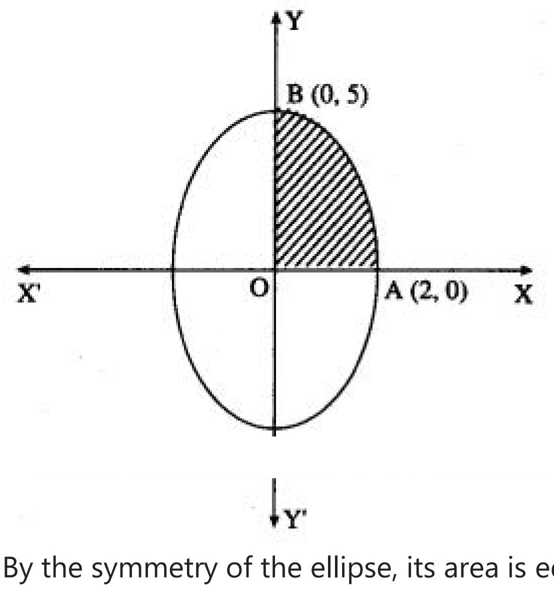
$$= 4\cdot\frac{25}{2}\cdot\frac{\pi}{2} = 25\pi \text{ sq units.}$$

$$\ldots\left[\because \sin^{-1}(1) = \frac{\pi}{2}, \sin^{-1}(0) = 0.\right]$$

Question 4.

Find the area of the ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$

Solution:



By the symmetry of the ellipse, its area is equal to 4 times the area of the region OABO.
Clearly, for this region, the limits of integration are 0 and 2.
From the equation of the ellipse,

$$\frac{y^2}{25}=1-\frac{x^2}{4}=\frac{4-x^2}{4}$$

$$\therefore\ y^2=\frac{25}{4}(4-x^2)$$

In the first quadrant, $y>0$

$$\therefore\ y=\frac{5}{2}\sqrt{4-x^2}$$

$\therefore$ area of ellipse = 4(area of the region OABO)

$$=4\int_0^2 y\,dx$$

$$=4\int_0^2 \frac{5}{2}\sqrt{4-x^2}\,dx$$

$$=10\int_0^2 \sqrt{4-x^2}\,dx$$

$$=10\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$\ldots\left[\because \int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]$$

$$=10\left[\left\{\frac{2}{2}\sqrt{4-4}+2\sin^{-1}(1)\right\}-\left\{\frac{0}{2}\sqrt{4-0}+2\sin^{-1}(0)\right\}\right]$$

$$=10\times 2\times\frac{\pi}{2}=10\pi \text{ sq units.}$$

$$\ldots\left[\because\ \sin^{-1}(1)=\frac{\pi}{2},\ \sin^{-1}(0)=0.\right]$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 7 Application of Definite Integration Miscellaneous Exercise 7

(I) Choose the correct alternatives:

Question 1.
Area of the region bounded by the curve $x^2 = y$, the X-axis and the lines x = 1 and x = 3 is
(a) 263 sq units
(b) 326 sq units
(c) 26 sq units
(d) 3 sq units
Answer:
(a) 263 sq units

Question 2.
The area of the region bounded by $y^2 = 4x$, the X-axis and the lines x = 1 and x = 4 is
(a) 28 sq units
(b) 3 sq unit
(c) 283 sq units
(d) 328 sq units

Answer:
(c) 283 sq units

Question 3.
Area of the region bounded by $x^2 = 16y$, y = 1 and y = 4 and the Y-axis, lying in the first quadrant is
(a) 63 sq units
(b) 356 sq units
(c) 563 sq units
(d) 637 sq units
Answer:
(c) 563 sq units

Question 4.
Area of the region bounded by $y = x^4$, x = 1, x = 5 and the X-axis is
(a) 31425 sq units
(b) 31245 sq units
(c) 31423 sq units
(d) 31243 sq units
Answer:
(b) 31245 sq units

Question 5.
Using definite integration area of circle $x^2 + y^2 = 25$ is
(a) 5π sq units
(b) 4π sq units
(c) 25π sq units
(d) 25 sq units
Answer:
(c) 25π sq units

(II) Fill in the blanks:

Question 1.
Area of the region bounded by $y = x^4$, x = 1, x = 5 and the X-axis is ________
Answer:
31245 sq units

Question 2.
Using definite integration area of the circle $x^2 + y^2 = 49$ is __________
Answer:
49π sq units

Question 3.
Area of the region bounded by $x^2 = 16y$, y = 1, y = 4 and the Y-axis lying in the first quadrant is ________
Answer:
563 sq units

Question 4.
The area of the region bounded by the curve $x^2 = y$, the X-axis and the lines x = 3 and x = 9 is ________
Answer:
234 sq units

Question 5.
The area of the region bounded by $y^2 = 4x$, the X-axis and the lines x = 1 and x = 4 is _________
Answer:
283 sq units

(III) State whether each of the following is True or False.

Question 1.
The area bounded by the curve x = g(y), Y-axis and bounded between the lines y = c and y = d is given by $\int_{dc} x\,dy = \int_{y=dy=c} g(y)\,dy$
Answer:
True

Question 2.
The area bounded by two curves y = f(x), y = g(x) and X-axis is $\left|\int_a^b f(x)dx - \int_a^b g(x)dx\right|$
Answer:
False

Question 3.
The area bounded by the curve y = f(x), X-axis and lines x = a and x = b is $\left|\int_a^b f(x)dx\right|$
Answer:
True

Question 4.
If the curve, under consideration, is below the X-axis, then the area bounded by curve, X-axis, and lines x = a, x = b is positive.
Answer:
False

Question 5.
The area of the portion lying above the X-axis is positive.
Answer:
True

(IV) Solve the following:

Question 1.
Find the area of the region bounded by the curve $xy = c^2$, the X-axis, and the lines x = c, x = 2c.
Solution:

$$\text{Required area} = \int_c^{2c} y\,dx, \text{ where } xy = c^2, \text{ i.e. } y = \frac{c^2}{x}$$

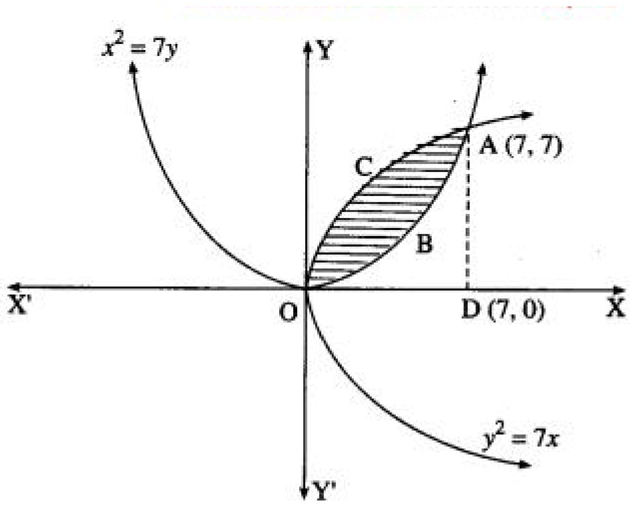
$$= \int_c^{2c} \frac{c^2}{x} dx = c^2 \int_c^{2c} \frac{1}{x} dx$$

$$= c^2 [\log x]_c^{2c}$$

$$= c^2 (\log 2c - \log c)$$

$= c^2 \log\left(\frac{2c}{c}\right)$
$= c^2 . \log 2$ sq units.

Question 2.
Find the area between the parabolas $y^2 = 7x$ and $x^2 = 7y$.
Solution:



For finding the points of intersection of the two parabolas,
we equate the values of y^2 from their equations.
From the equation $x^2 = 7y$, $y^2 = \frac{x^4}{49}$
$\therefore \frac{x^4}{49} = 7x$
$\therefore x^4 = 343x$
$\therefore x^4 - 343x = 0$
$\therefore x(x^3 - 343) = 0$
$\therefore x = 0$ or $x^3 = 343$, i.e. x = 7
When x = 0, y = 0
When x = 7, 7y = 49

∴ y = 7
∴ the points of intersection are O(0, 0) and A(7, 7)
Required area = area of the region OBACO
= (area of the region ODACO) – (area of the region ODABO)
Now, area of the region ODACO = area under the parabola $y^2 = 7x$
i.e. y = √7 √x

$$= \int_0^7 \sqrt{7}\sqrt{x}\,dx = \sqrt{7}\left[\frac{x^{\frac{3}{2}}}{3/2}\right]_0^7$$

$$= \sqrt{7} \times \frac{2}{3}\left[7^{\frac{3}{2}} - 0\right] = \frac{2\sqrt{7}}{3}[7\sqrt{7} - 0]$$

$$= \frac{98}{3}$$

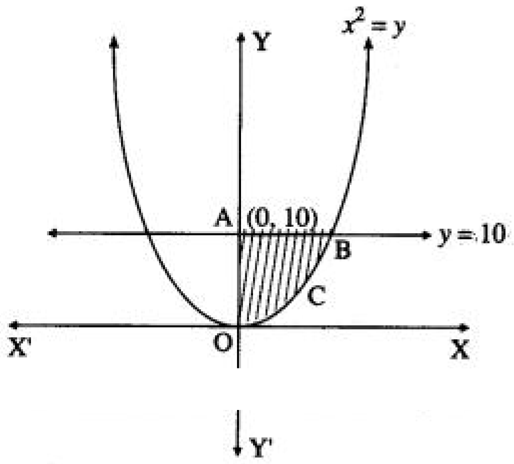
Area of the region ODABO = Area under the parabola
$x^2 = 7y$
i.e. y = $x27$

$$= \int_0^7 \frac{x^2}{7}dx = \frac{1}{7}\left[\frac{x^3}{3}\right]_0^7 = \frac{1}{7}\left[\frac{7^3}{3} - 0\right]$$

$$= \frac{7^2}{3} = \frac{49}{3}$$

∴ required area = 983−493=493 sq units.

Question 3.
Find the area of the region bounded by the curve $y = x^2$ and the line y = 10.
Solution:



By the symmetry of the parabola,
the required area is twice the area of the region OABCO
Now, the area of the region OABCO

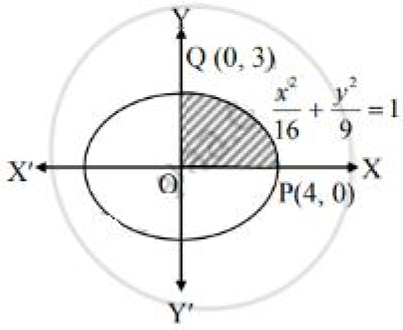
$$= \int_0^{10} x\,dy, \text{ where } x^2 = y \text{ i.e. } x = \sqrt{y}$$

$$= \int_0^{10} \sqrt{y}\,dy = \left[\frac{y^{\frac{3}{2}}}{3/2}\right]_0^{10} = \frac{10^{\frac{3}{2}}}{3/2} - 0 = \frac{2 \times 10\sqrt{10}}{3} = \frac{20\sqrt{10}}{3}$$

$$\therefore \text{ required area} = 2\left[\frac{20\sqrt{10}}{3}\right]$$

$$= \frac{40\sqrt{10}}{3} \text{ sq units.}$$

Question 4.
Find the area of the ellipse $x216+y29$ = 1.
Solution:
By the symmetry of the ellipse, the required area of the ellipse is 4 times the area of the region OPQO.
For the region OPQO, the limits of integration are x = 0 and x = 4.



Given equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$\therefore \frac{y^2}{9} = 1 - \frac{x^2}{16}$

$\therefore y^2 = 9\left(1 - \frac{x^2}{16}\right)$

$= \frac{9}{16}(16 - x^2)$

$\therefore y = \pm\frac{3}{4}\sqrt{16 - x^2}$

$\therefore y = \frac{3}{4}\sqrt{16 - x^2}$ ...[$\because$ In first quadrant, $y > 0$]

$\therefore$ Required area = 4(area of the region OPQO)

$= 4\int_0^4 y \cdot dx$

$= 4\int_0^4 \frac{3}{4}\sqrt{16 - x^2} \cdot dx$

$= 3\int_0^4 \sqrt{(4)^2 - x^2} \cdot dx$

$= 3\left[\frac{x}{2}\sqrt{(4)^2 - x^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$

$= 3\left\{\left[\frac{4}{2}\sqrt{(4)^2 - (4)^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{4}{4}\right)\right] - \left[\frac{0}{2}\sqrt{(4)^2 - (0)^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{0}{4}\right)\right]\right\}$

$= 3\{[0 + 8\sin^{-1}(1)] - [0 + 0]\}$

$= 3\left(8 \times \frac{\pi}{2}\right)$

$= 12\pi$ sq. units.

Question 5.
Find the area of the region bounded by $y = x^2$, the X-axis and $x = 1$, $x = 4$.
Solution:
Required area $= \int_1^4 y dx$, where $y = x^2$

$= \int_1^4 x^2 dx$

$= \left[\frac{x^3}{3}\right]_1^4 = \frac{4^3}{3} - \frac{1^3}{3} = \frac{64}{3} - \frac{1}{3}$

= 21 sq units.

Question 6.
Find the area of the region bounded by the curve $x^2 = 25y$, $y = 1$, $y = 4$, and the Y-axis.

Solution:

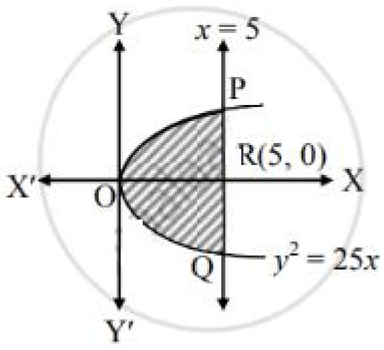
Required area $= \int_1^4 x\,dy$, where $x^2 = 25y$, i.e. $x = 5\sqrt{y}$

$$= \int_1^4 5\sqrt{y}\,dy = 5\left[\frac{y^{\frac{3}{2}}}{3/2}\right]_1^4$$

$$= 5 \times \frac{2}{3}[4^{\frac{3}{2}} - 1] = \frac{10}{3}[(2^2)^{\frac{3}{2}} - 1]$$

$$= \frac{10}{3}[8-1] = \frac{70}{3} \text{ sq units.}$$

Question 7.
Find the area of the region bounded by the parabola $y^2 = 25x$ and the line $x = 5$.
Solution:



Given the equation of the parabola is $y^2 = 25x$
$\therefore y = 5\sqrt{x}$ ...... [∵ IIn first quadrant, $y > 0$]
Required area = area of the region OQRPO
= 2(area of the region ORPO)

$$= 2\int_0^5 y \cdot dx$$

$$= 2\int_0^5 5\sqrt{x} \cdot dx$$

$$= 10\int_0^5 x^{\frac{1}{2}} \cdot dx$$

$$= 10\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^5$$

$$= \frac{20}{3}\left[(5)^{\frac{3}{2}} - 0\right]$$

$$= \frac{20}{3}\left(5\sqrt{(5)}\right.$$

$$= \frac{100\sqrt{5}}{3} \text{sq.units.}$$