Maharashtra State Board 11th Maths Solutions Chapter 7 Limits Ex 7.1

I. Evaluate the following limits:

Question 1.
$\lim_{z \to -3}\left[\frac{\sqrt{z+6}}{z}\right]$
Solution:

$$\lim_{z \to -3}\left[\frac{\sqrt{z+6}}{z}\right] = \frac{\lim_{z \to -3}\sqrt{z+6}}{\lim_{z \to -3} z}$$
$$= \frac{\sqrt{-3+6}}{-3}$$
$$= \frac{\sqrt{3}}{-3}$$
$$= \frac{-1}{\sqrt{3}}$$

Question 2.
$\lim_{y \to -3}\left[\frac{y^5+243}{y^3+27}\right]$
Solution:

$$\lim_{y \to -3}\frac{y^5+243}{y^3+27}$$
$$= \lim_{y \to -3}\frac{\left(\frac{y^5+243}{y+3}\right)}{\left(\frac{y^3+27}{y+3}\right)} \quad \ldots\left[\begin{array}{l}\because y \to -3, y \neq -3,\\ \therefore y+3 \neq 0\end{array}\right]$$
$$= \frac{\lim_{y \to -3}\left[\frac{y^5-(-3)^5}{y-(-3)}\right]}{\lim_{y \to -3}\left[\frac{y^3-(-3)^3}{y-(-3)}\right]}$$
$$= \frac{5(-3)^4}{3(-3)^2} \quad \ldots\left[\because \lim_{x \to a}\frac{x^n-a^n}{x-a} = n.a^{n-1}\right]$$
$$= \frac{5}{3} \times 9$$
$$= 15$$

Question 3.
$\lim_{z \to -5}\left[\frac{\left(\frac{1}{z}+\frac{1}{5}\right)}{z+5}\right]$
Solution:

$$\lim_{z \to -5}\frac{\frac{1}{z}+\frac{1}{5}}{z+5}$$
$$= \lim_{z \to -5}\frac{\frac{5+z}{5z}}{z+5}$$
$$= \lim_{z \to -5}\left[\frac{z+5}{5z(z+5)}\right]$$
$$= \lim_{z \to -5}\frac{1}{5z} \quad \ldots\left[\begin{array}{l}\because z \to -5, \; z \neq -5,\\ \therefore z+5 \neq 0\end{array}\right]$$
$$= \frac{1}{5(-5)}$$
$$= \frac{-1}{25}$$

II. Evaluate the following limits:

Question 1.
$lim_{x\to3}[2x+6\sqrt{x}]$
Solution:

$$\lim_{x\to3}\frac{\sqrt{2x+6}}{x}=\frac{\lim_{x\to3}\sqrt{2x+6}}{\lim_{x\to3}x}$$
$$=\frac{\sqrt{2(3)+6}}{3}$$
$$=\frac{\sqrt{12}}{3}=\frac{2\sqrt{3}}{3}$$

Question 2.
$lim_{x\to2}[x-3-2-3x-2]$
Solution:

$$\lim_{x\to2}\frac{x^{-3}-2^{-3}}{x-2}=(-3).(2)^{-4}$$
$$\ldots\left[\because\lim_{x\to a}\frac{x^n-a^n}{x-a}=n.a^{n-1}\right]$$
$$=-3\times\frac{1}{2^4}$$
$$=\frac{-3}{16}$$

Question 3.
$lim_{x\to5}[x3-125x5-3125]$
Solution:

$$\lim_{x\to5}\frac{x^3-125}{x^5-3125}$$
$$=\lim_{x\to5}\frac{\left(\frac{x^3-5^3}{x-5}\right)}{\left(\frac{x^5-5^5}{x-5}\right)}\qquad\ldots\left[\begin{array}{l}\because x\to5,\ x\neq5,\\ \therefore x-5\neq0\end{array}\right]$$
$$=\frac{\lim_{x\to5}\frac{x^3-5^3}{x-5}}{\lim_{x\to5}\frac{x^5-5^5}{x-5}}$$
$$=\frac{3(5)^2}{5(5)^4}\qquad\ldots\left[\because\lim_{x\to a}\frac{x^n-a^n}{x-a}=n.a^{n-1}\right]$$
$$=\frac{3}{(5)^3}$$
$$=\frac{3}{125}$$

Question 4.
If $lim_{x\to1}[x4-1x-1]=lim_{x\to a}[x3-a3x-a]$, find all possible values of a.

Solution:

$$\lim_{x \to 1} \frac{x^4-1}{x-1} = \lim_{x \to a} \frac{x^3-a^3}{x-a}$$

$$\therefore \quad \lim_{x \to 1} \frac{x^4-(1)^4}{x-1} = \lim_{x \to a} \frac{x^3-a^3}{x-a}$$

$$\therefore \quad 4(1)^3 = 3a^2 \qquad \ldots\left[\because \lim_{x \to a} \frac{x^n-a^n}{x-a} = n.a^{n-1}\right]$$

$$\therefore \quad 3a^2 = 4$$

$$\therefore \quad a^2 = \frac{4}{3}$$

$$\therefore \quad a = \pm\frac{2}{\sqrt{3}}$$

III. Evaluate the following limits:

Question 1.
$\lim_{x \to 1}\left[\frac{x+x^2+x^3+\ldots\ldots\ldots+x^n-n}{x-1}\right]$
Solution:

$$\lim_{x \to 1}\left[\frac{x+x^2+x^3+\ldots+x^n-n}{x-1}\right]$$

$$= \lim_{x \to 1}\left[\frac{x+x^2+x^3+\ldots+x^n-(1+1+1+\ldots \text{n times})}{x-1}\right]$$

$$= \lim_{x \to 1} \frac{(x-1)+(x^2-1)+(x^3-1)+\ldots+(x^n-1)}{x-1}$$

$$= \lim_{x \to 1}\left[\frac{x^1-1^1}{x-1}+\frac{x^2-1^2}{x-1}+\frac{x^3-1^3}{x-1}+\ldots+\frac{x^n-1^n}{x-1}\right]$$

$$= 1\,(1)^0 + 2\,(1)^1 + 3\,(1)^2 + 4\,(1)^3 + \ldots + n\,(1)^{n-1}$$

$$\ldots\left[\because \lim_{x \to a} \frac{x^n-a^n}{x-a} = n.a^{n-1}\right]$$

$$= 1+2+3+4+\ldots+n$$

$$= \frac{n(n+1)}{2}$$

Question 2.
$\lim_{x \to 7}\left[\frac{(\sqrt[3]{x}-\sqrt[3]{7})(\sqrt[3]{x}+\sqrt[3]{7})}{x-7}\right]$
Solution:

$$\lim_{x \to 7}\left[\frac{\left(\sqrt[3]{x}-\sqrt[3]{7}\right)\left(\sqrt[3]{x}+\sqrt[3]{7}\right)}{x-7}\right]$$

$$= \lim_{x \to 7}\left[\frac{(x^{\frac{1}{3}}-7^{\frac{1}{3}})(x^{\frac{1}{3}}+7^{\frac{1}{3}})}{x-7}\right]$$

$$= \lim_{x \to 7}\left[\frac{x^{\frac{2}{3}}-7^{\frac{2}{3}}}{x-7}\right] \quad \ldots[\because (a-b)(a+b) = a^2-b^2]$$

$$= \frac{2}{3}(7)^{\frac{-1}{3}} \qquad \ldots\left[\because \lim_{x \to a} \frac{x^n-a^n}{x-a} = n.a^{n-1}\right]$$

$$= \frac{2}{3}\cdot\frac{1}{7^{\frac{1}{3}}}$$

$$= \frac{2}{3\sqrt[3]{7}}$$

Question 3.
If $\lim_{x \to 5}\left[\frac{x^k-5^k}{x-5}\right]$ = 500, find all possible values of k.

Solution:

$$\lim_{x\to 5}\frac{x^k-5^k}{x-5}=500$$

$\therefore\quad k(5)^{k-1}=500 \qquad \ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=n.a^{n-1}\right]$

$\therefore\quad k(5)^{k-1}=4\times 125$

$\therefore\quad k(5)^{k-1}=4\times (5)^3$

$\therefore\quad k(5)^{k-1}=4\times (5)^{4-1}$

Comparing both sides, we get

$k=4$

Question 4.

$\lim_{x\to 0}\left[\frac{(1-x)^8-1}{(1-x)^2-1}\right]$

Solution:

Put $1-x=y$

As $x\to 0, y\to 1$

$$\therefore\quad \lim_{x\to 0}\frac{(1-x)^8-1}{(1-x)^2-1}$$

$$=\lim_{y\to 1}\frac{y^8-1^8}{y^2-1^2}$$

$$=\lim_{y\to 1}\frac{\frac{y^8-1^8}{y-1}}{\frac{y^2-1^2}{y-1}} \qquad \ldots\left[\begin{array}{l}\because y\to 1,\ y\neq 1,\\ \therefore y-1\neq 0\end{array}\right]$$

$$=\frac{\lim_{y\to 1}\frac{y^8-1^8}{y-1}}{\lim_{y\to 1}\frac{y^2-1^2}{y-1}}$$

$$=\frac{8(1)^7}{2(1)^1} \qquad \ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=n.a^{n-1}\right]$$

$$=4$$

Question 5.

$\lim_{x\to 0}\left[\frac{\sqrt[3]{1+x}-\sqrt{1+x}}{x}\right]$

Solution:

$$\lim_{x \to 0} \frac{\sqrt[3]{1+x}-\sqrt{1+x}}{x}$$

$$= \lim_{x \to 0} \frac{(1+x)^{\frac{1}{3}}-(1+x)^{\frac{1}{2}}}{x}$$

Put $1 + x = y$

As $x \to 0, y \to 1$

$$\lim_{x \to 0} \frac{(1+x)^{\frac{1}{3}}-(1+x)^{\frac{1}{2}}}{x}$$

$$= \lim_{y \to 1} \frac{y^{\frac{1}{3}}-y^{\frac{1}{2}}}{y-1}$$

$$= \lim_{y \to 1} \frac{\left(y^{\frac{1}{3}}-1\right)-\left(y^{\frac{1}{2}}-1\right)}{y-1}$$

$$= \lim_{y \to 1} \left(\frac{y^{\frac{1}{3}}-1}{y-1}-\frac{y^{\frac{1}{2}}-1}{y-1}\right)$$

$$= \lim_{y \to 1} \frac{y^{\frac{1}{3}}-1^{\frac{1}{3}}}{y-1}-\lim_{y \to 1} \frac{y^{\frac{1}{2}}-1^{\frac{1}{2}}}{y-1}$$

$$= \frac{1}{3}(1)^{\frac{-2}{3}}-\frac{1}{2}(1)^{\frac{-1}{2}} \qquad \ldots\left[\because \lim_{x \to a} \frac{x^{n}-a^{n}}{x-a}=n.a^{n-1}\right]$$

$$= \frac{1}{3}-\frac{1}{2}$$

$$= \frac{2-3}{6}$$

$$= -\frac{1}{6}$$

Question 6.

limy→1[2y−27+y√3−2]

Solution:

$$\lim_{y \to 1} \frac{2y-2}{\sqrt[3]{7+y}-2}$$

$$= \lim_{y \to 1} \frac{2(y-1)}{(7+y)^{\frac{1}{3}}-8^{\frac{1}{3}}} \qquad \ldots\left[\because 2=(2^{3})^{\frac{1}{3}}=8^{\frac{1}{3}}\right]$$

$$= \lim_{y \to 1} \frac{2}{\frac{(y+7)^{\frac{1}{3}}-8^{\frac{1}{3}}}{y-1}}$$

$$= \frac{\lim\limits_{y \to 1} 2}{\lim\limits_{y \to 1} \frac{(y+7)^{\frac{1}{3}}-8^{\frac{1}{3}}}{(y+7)-8}}$$

$$= \frac{2}{\frac{1}{3}(8)^{\frac{-2}{3}}} \qquad \ldots\left[\begin{array}{l}\because y \to 1, y+7 \to 8 \\ \text{and } \lim\limits_{x \to a} \frac{x^{n}-a^{n}}{x-a}=n.a^{n-1}\end{array}\right]$$

$$= 2(3).(8)^{\frac{2}{3}}$$

$$= 6(2^{3})^{\frac{2}{3}}$$

$$= 6 \times (2)^{2}$$

$$= 24$$

Question 7.

limz→a[(z+2)32−(a+2)32z−a]

Solution:

$$\lim_{z\to a}\frac{(z+2)^{\frac{3}{2}}-(a+2)^{\frac{3}{2}}}{z-a}$$
$$=\lim_{z\to a}\frac{(z+2)^{\frac{3}{2}}-(a+2)^{\frac{3}{2}}}{(z+2)-(a+2)}$$

Put $z + 2 = y$ and $a + 2 = b$

As $z \to a$, $z + 2 \to a + 2$, i.e., $y \to b$

$$\therefore \quad \lim_{z\to a}\frac{(z+2)^{\frac{3}{2}}-(a+2)^{\frac{3}{2}}}{z-a}$$
$$=\lim_{y\to b}\frac{y^{\frac{3}{2}}-b^{\frac{3}{2}}}{y-b}$$
$$=\frac{3}{2}.b^{\frac{1}{2}} \qquad \ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=n.a^{n-1}\right]$$
$$=\frac{3}{2}\sqrt{a+2} \qquad \ldots[\because b=a+2]$$

Question 8.

$\lim_{x\to 7}\left[\frac{x^3-343}{\sqrt{x}-\sqrt{7}}\right]$

Solution:

$$\lim_{x\to 7}\frac{x^3-343}{\sqrt{x}-\sqrt{7}}$$
$$=\lim_{x\to 7}\frac{x^3-7^3}{x^{\frac{1}{2}}-7^{\frac{1}{2}}}$$
$$=\frac{\lim_{x\to 7}\frac{x^3-7^3}{x-7}}{\lim_{x\to 7}\frac{x^{\frac{1}{2}}-7^{\frac{1}{2}}}{x-7}} \qquad \ldots\left[\begin{array}{l}\because x\to 7, x\neq 7,\\ \therefore x-7\neq 0\end{array}\right]$$
$$=\frac{3(7)^2}{\frac{1}{2}(7)^{-\frac{1}{2}}}$$
$$=6\,(49)\times 7^{\frac{1}{2}}$$
$$=294\sqrt{7}$$

Question 9.

$\lim_{x\to 1}\left(\frac{x+x^3+x^5+\ldots+x^{2n-1}-n}{x-1}\right)$

Solution:

$$\lim_{x\to1}\left[\frac{x+x^3+x^5+\ldots+x^{2n-1}-n}{x-1}\right]$$

$$= \lim_{x\to1}\left[\frac{x+x^3+x^5+\ldots+x^{2n-1}-(1+1+1+\ldots n\text{ times})}{x-1}\right]$$

$$= \lim_{x\to1}\left[\frac{(x-1)+(x^3-1)+(x^5-1)+\ldots+(x^{2n-1}-1)\ldots(\text{n brackets})}{x-1}\right]$$

$\ldots[\because 1, 3, 5, \ldots, 2n - 1$ are the first n odd numbers]

$$= \lim_{x\to1}\left[\frac{x^1-1^1}{x-1}+\frac{x^3-1^3}{x-1}+\frac{x^5-1^5}{x-1}+\ldots+\frac{x^{2n-1}-1^{2n-1}}{x-1}\right]$$

$= 1(1)^0 + 3(1)^2 + 5(1)^4 + \ldots + (2n-1)(1)^{2n-2}$

$= 1 + 3 + 5 + \ldots + (2n-1)$

$$= \sum_{r=1}^{n}(2r-1)$$

$$= 2\sum_{r=1}^{n} r - \sum_{r=1}^{n} 1$$

$$= 2\cdot\frac{n(n+1)}{2} - n$$

$= n(n+1) - n$

$= n^2 + n - n$

$= n^2$

IV. In the following examples, given ∈ > 0, find a δ > 0 such that whenever, |x – a| < δ, we must have |f(x) – l| < ∈.

Question 1.

$\lim_{x\to2}(2x+3)=7$

Solution:

We have to find some δ so that

$\lim_{x\to2}(2x+3)=7$

Here a = 2, l = 1 and f(x) = 2x + 3

Consider ∈ > 0 and |f(x) – l| < ∈

∴ |(2x + 3) – 7| < ∈

∴ |2x + 4| < ∈

∴ 2(x – 2)|< ∈

∴ |x – 2| < $\frac{\epsilon}{2}$

∴ δ ≤ $\frac{\epsilon}{2}$ such that

|2x + 4| < δ ⇒ |f(x) – 7| < ∈

Question 2.

$\lim_{x\to-3}(3x+2)=-7$

Solution:

We have to find some δ so that

$\lim_{x\to-3}(3x+2)=-7$

Here a = -3, l = -7 and f(x) = 3x + 2

Consider ∈ > 0 and |f(x) – l| < ∈

∴ |3x + 2 – (-7)| < ∈

∴ |3x + 9| < ∈

∴ |3(x + 3)| < ∈

∴ |x + 3| < $\frac{\epsilon}{3}$

∴ δ < $\frac{\epsilon}{3}$ such that

|x + 3| ≤ δ ⇒ |f(x) + 7| < ∈

Question 3.

$\lim_{x\to2}(x^2-1)=3$

Solution:

We have to find some δ > 0 such that

$\lim_{x\to2}(x^2-1)=3$

Here, a = 2, l = 3 and f(x) = x^2 – 1

Consider ∈ > 0 and |f(x) – l| < ∈

∴ |(x^2 – 1) – 3| < ∈

∴ $|x^2 - 4| < \in$
∴ $|(x + 2)(x - 2)| < \in$ .....(i)
We have to get rid of the factor $|x + 2|$
As $|x - 2| < \delta$
$-\delta < x - 2 < \delta$
∴ $2 - \delta < x < 2 + \delta$
Since δ can be assumed as very small, let us choose $\delta < 1$
∴ $1 < x < 3$
∴ $3 < x + 2 < 5$ .....(Adding 2 throughout)
∴ $|x + 2| < 5$
∴ $|(x + 2)(x - 2)| < 5|x - 2|$ ......(ii)
From (i) and (ii), we get
$5|x - 2| < \in$
∴ $x - 2 < \frac{\epsilon}{5}$
If $\delta = \frac{\epsilon}{5}$, $|x - 2| < \delta \Rightarrow |x^2 - 4| < \in$
∴ We choose $\delta = \min\{\frac{\epsilon}{5}, 1\}$ then
$|x - 2| < \delta \Rightarrow |f(x) - 3| < \in$

Question 4.
$\lim_{x\to 1}(x^2+x+1)=3$
Solution:
We have to find some $\delta > 0$ such that
$\lim_{x\to 1}(x^2+x+1)=3$
Here a = 1, l = 3 and $f(x) = x^2 + x + 1$
Consider $\in > 0$ and $|f(x) - l| < \in$
∴ $|x^2 + x + 1 - 3| < \in$
∴ $|x^2 + x - 2| < \in$
∴ $|(x + 2)(x - 1)| < \in$ .....(i)
We have to get rid of the factor $|x + 2|$
As $|x - 1| < \delta$
$-\delta < x - 1 < \delta$
∴ $1 - \delta < x < 1 + \delta$
Since δ can be assumed as very small, let us choose $\delta < 1$
∴ $0 < x < 2$
∴ $2 < x + 2 < 4$
∴ $|x + 2| < 4$
∴ $|(x + 2)(x - 1)| < 4|x - 1|$ .....(ii)
From (i) and (ii), we get
$4|x - 1| < \in$
∴ $|x - 1| < \frac{\epsilon}{4}$
If $\delta = \frac{\epsilon}{4}$,
$|x - 1| < \delta \Rightarrow x^2 + x - 2 < \in$
∴ We choose $\delta = \min\{\frac{\epsilon}{4}, 1\}$ then
$|x - 1| < \delta \Rightarrow |f(x) - 3| < \in$

Maharashtra State Board 11th Maths Solutions Chapter 7 Limits Ex 7.2

I. Evaluate the following limits:

Question 1.
$\lim_{z\to 2}\left[\frac{z^2-5z+6}{z^2-4}\right]$

Solution:

$$\lim_{z\to2}\frac{z^2-5z+6}{z^2-4}$$

$$=\lim_{z\to2}\frac{(z-3)(z-2)}{(z+2)(z-2)}$$

$$=\lim_{z\to2}\frac{z-3}{z+2} \qquad \cdots\left[\begin{array}{l}\because z\to2,\ z\neq2,\\ \therefore z-2\neq0\end{array}\right]$$

$$=\frac{\lim_{z\to2}(z-3)}{\lim_{z\to2}(z+2)}$$

$$=\frac{2-3}{2+2}$$

$$=-\frac{1}{4}$$

Question 2.

$\lim_{x\to-3}\left[\frac{x+3}{x^2+4x+3}\right]$

Solution:

$$\lim_{x\to-3}\left[\frac{x+3}{x^2+4x+3}\right]$$

$$=\lim_{x\to-3}\frac{x+3}{(x+3)(x+1)}$$

$$=\lim_{x\to-3}\frac{1}{x+1} \qquad \cdots\left[\begin{array}{l}\because x\to-3,\ x\neq-3,\\ \therefore x+3\neq0\end{array}\right]$$

$$=\frac{1}{-3+1}$$

$$=-\frac{1}{2}$$

Question 3.

$\lim_{y\to0}\left[\frac{5y^3+8y^2}{3y^4-16y^2}\right]$

Solution:

$$\lim_{y\to0}\left[\frac{5y^3+8y^2}{3y^4-16y^2}\right]$$

$$=\lim_{y\to0}\frac{y^2(5y+8)}{y^2(3y^2-16)}$$

$$=\lim_{y\to0}\frac{5y+8}{3y^2-16} \qquad \cdots\left[\begin{array}{l}\because y\to0,\ y\neq0,\\ \therefore y^2\neq0\end{array}\right]$$

$$=\lim_{y\to0}\frac{(5y+8)}{(3y^2-16)}$$

$$=\frac{5(0)+8}{3(0)^2-16}$$

$$=\frac{8}{-16}$$

$$=-\frac{1}{2}$$

Question 4.

$\lim_{x\to-2}\left[\frac{-2x-4}{x^3+2x^2}\right]$

Solution:

$$\lim_{x\to -2}\left[\frac{-2x-4}{x^3+2x^2}\right]$$

$$= \lim_{x\to -2}\frac{-2(x+2)}{x^2(x+2)}$$

$$= \lim_{x\to -2}\frac{-2}{x^2} \qquad \ldots\left[\begin{array}{l}\because x\to -2,\ x\neq -2,\\ \therefore x+2\neq 0\end{array}\right]$$

$$= \frac{\lim\limits_{x\to -2}(-2)}{\lim\limits_{x\to -2}(x^2)}$$

$$= \frac{(-2)}{(-2)^2}$$

$$= \frac{-2}{4}$$

$$= \frac{-1}{2}$$

Question 5.
$\lim_{x\to 3}\left[\frac{x^2+2x-15}{x^2-5x+6}\right]$
Solution:

$$\lim_{x\to 3}\left[\frac{x^2+2x-15}{x^2-5x+6}\right]$$

$$= \lim_{x\to 3}\frac{(x+5)(x-3)}{(x-2)(x-3)}$$

$$= \lim_{x\to 3}\frac{x+5}{x-2} \qquad \ldots\left[\begin{array}{l}\because x\to 3,\ x\neq 3,\\ \therefore x-3\neq 0\end{array}\right]$$

$$= \frac{\lim\limits_{x\to 3}(x+5)}{\lim\limits_{x\to 3}(x-2)} = \frac{3+5}{3-2}$$

$$= 8$$

II. Evaluate the following limits:

Question 1.
$\lim_{u\to 1}\left[\frac{u^4-1}{u^3-1}\right]$
Solution:

$$\lim_{u\to 1}\left[\frac{u^4-1}{u^3-1}\right]$$

$$= \lim_{u\to 1}\frac{\dfrac{u^4-1^4}{u-1}}{\dfrac{u^3-1^3}{u-1}} \qquad \ldots\left[\begin{array}{l}\because u\to 1,\ u\neq 1,\\ \therefore u-1\neq 0\end{array}\right]$$

$$= \frac{\lim\limits_{u\to 1}\left(\dfrac{u^4-1^4}{u-1}\right)}{\lim\limits_{u\to 1}\left(\dfrac{u^3-1^3}{u-1}\right)}$$

$$= \frac{4(1)^3}{3(1)^2} \qquad \ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}\right]$$

$$= \frac{4}{3}$$

Question 2.
$\lim_{x\to 3}\left[\frac{1}{x-3}-\frac{9x}{x^3-27}\right]$

Solution:

$$\lim_{x\to 3}\left[\frac{1}{x-3}-\frac{9x}{x^3-27}\right]$$

$$=\lim_{x\to 3}\left[\frac{1}{x-3}-\frac{9x}{x^3-3^3}\right]$$

$$=\lim_{x\to 3}\left[\frac{1}{x-3}-\frac{9x}{(x-3)(x^2+3x+9)}\right]$$

$$=\lim_{x\to 3}\left[\frac{x^2+3x+9-9x}{(x-3)(x^2+3x+9)}\right]$$

$$=\lim_{x\to 3}\left[\frac{x^2-6x+9}{(x-3)(x^2+3x+9)}\right]$$

$$=\lim_{x\to 3}\left[\frac{(x-3)^2}{(x-3)(x^2+3x+9)}\right]$$

$$=\lim_{x\to 3}\left[\frac{x-3}{x^2+3x+9}\right] \qquad \ldots\left[\begin{array}{l}\because x\to 3, x\neq 3,\\ \therefore x-3\neq 0\end{array}\right]$$

$$=\frac{\lim_{x\to 3}(x-3)}{\lim_{x\to 3}(x^2+3x+9)}$$

$$=\frac{3-3}{(3)^2+3(3)+9}$$

$$=\frac{0}{27}=0$$

Question 3.

$lim_{x\to 2}[x3-4x2+4xx2-1]$

Solution:

$$\lim_{x\to 2}\left[\frac{x^3-4x^2+4x}{x^2-1}\right]$$

$$=\frac{\lim_{x\to 2}(x^3-4x^2+4x)}{\lim_{x\to 2}(x^2-1)}$$

$$=\frac{2^3-4(2)^2+4(2)}{2^2-1}$$

$$=\frac{0}{3}=0$$

Question 4.

$lim_{\Delta x\to 0}[(x+\Delta x)2-2(x+\Delta x)+1-(x2-2x+1)\Delta x]$

Solution:

$$\lim_{\Delta x\to 0}\left[\frac{(x+\Delta x)^2-2(x+\Delta x)+1-(x^2-2x+1)}{\Delta x}\right]$$

$$=\lim_{\Delta x\to 0}\frac{x^2+2x.\Delta x+(\Delta x)^2-2x-2\Delta x+1-x^2+2x-1}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{2x.\Delta x+(\Delta x)^2-2\Delta x}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{\Delta x(2x+\Delta x-2)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}(2x+\Delta x-2) \qquad \ldots[\because \Delta x\to 0, \Delta x\neq 0]$$

$$=2x+0-2$$

$$=2x-2$$

Question 5.

$lim_{x\to\sqrt{2}}\left[\frac{x^2+x\sqrt{2}-4}{x^2-3x\sqrt{2}+4}\right]$

Solution:

$$\lim_{x\to\sqrt{2}}\left[\frac{x^2+x\sqrt{2}-4}{x^2-3x\sqrt{2}+4}\right]$$

Consider, $x^2+x\sqrt{2}-4 = x^2+\sqrt{2}x-4$

$= x^2+2\sqrt{2}x-\sqrt{2}x-4$

$= x\left(x+2\sqrt{2}\right)-\sqrt{2}\left(x+2\sqrt{2}\right)$

$= \left(x+2\sqrt{2}\right)\left(x-\sqrt{2}\right)$

$x^2-3x\sqrt{2}+4 = x^2-3\sqrt{2}x+4$

$= x^2-2\sqrt{2}x-\sqrt{2}x+4$

$= x\left(x-2\sqrt{2}\right)-\sqrt{2}\left(x-2\sqrt{2}\right)$

$= \left(x-2\sqrt{2}\right)\left(x-\sqrt{2}\right)$

Now, $\lim_{x\to\sqrt{2}}\left[\frac{x^2+x\sqrt{2}-4}{x^2-3x\sqrt{2}+4}\right]$

$$= \lim_{x\to\sqrt{2}}\frac{\left(x+2\sqrt{2}\right)\left(x-\sqrt{2}\right)}{\left(x-2\sqrt{2}\right)\left(x-\sqrt{2}\right)}$$

$$= \lim_{x\to\sqrt{2}}\frac{x+2\sqrt{2}}{x-2\sqrt{2}} \quad \ldots\left[\begin{array}{l}\because x\to\sqrt{2}, x\neq\sqrt{2},\\ \therefore x-\sqrt{2}\neq 0\end{array}\right]$$

$$= \frac{\lim_{x\to\sqrt{2}}\left(x+2\sqrt{2}\right)}{\lim_{x\to\sqrt{2}}\left(x-2\sqrt{2}\right)}$$

$$= \frac{\sqrt{2}+2\sqrt{2}}{\sqrt{2}-2\sqrt{2}} = \frac{3\sqrt{2}}{-\sqrt{2}} = -3$$

Question 6.

$lim_{x\to 2}\left[\frac{x^3-7x+6}{x^3-7x^2+16x-12}\right]$

Solution:

$lim_{x\to 2}\left[\frac{x^3-7x+6}{x^3-7x^2+16x-12}\right]$

As x → 2, numerator and denominator both tend to zero

∴ x – 2 is a factor of both.

To find the other factor for both of them, by synthetic division

Consider, Numerator = $x^3 + 0x^2 - 7x + 6$

$$\begin{array}{c|cccc} 2 & 1 & 0 & -7 & 6 \\ & \downarrow & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & \boxed{0} \end{array}$$

∴ Numerator = $(x-2)(x^2+2x-3)$

Now, denominator = $x^3-7x^2+16x-12$

$$\begin{array}{c|cccc} 2 & 1 & -7 & 16 & -12 \\ & \downarrow & 2 & -10 & 12 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

∴ denominator = $(x-2)(x^2-5x+6)$

$$\lim_{x\to 2}\left[\frac{x^3-7x+6}{x^3-7x^2+16x-12}\right]$$

$$= \lim_{x\to 2}\frac{(x-2)\left(x^2+2x-3\right)}{(x-2)\left(x^2-5x+6\right)}$$

$$= \lim_{x \to 2} \frac{x^2+2x-3}{x^2-5x+6} \qquad \ldots \begin{bmatrix} \because x \to 2, x \neq 2 \\ \therefore x-2 \neq 0 \end{bmatrix}$$

$$= \frac{\lim_{x \to 2}\left(x^2+2x-3\right)}{\lim_{x \to 2}\left(x^2-5x+6\right)}$$

$$= \frac{(2)^2+2(2)-3}{(2)^2-5(2)+6}$$

$$= \frac{4+4-3}{4-10+6} = \frac{5}{0}$$

∴ The limit does not exist

III. Evaluate the following limits:

Question 1.
limy→12[1-8y3y-4y3]
Solution:

$$\lim_{y \to \frac{1}{2}} \left[\frac{1-8y^3}{y-4y^3}\right]$$

$$= \lim_{y \to \frac{1}{2}} \frac{1-8y^3}{y\left(1-4y^2\right)}$$

$$= \lim_{y \to \frac{1}{2}} \frac{(1)^3-(2y)^3}{y\left[(1)^2-(2y)^2\right]}$$

$$= \lim_{y \to \frac{1}{2}} \frac{(1-2y)\left(1+2y+4y^2\right)}{y(1-2y)(1+2y)}$$

$$= \lim_{y \to \frac{1}{2}} \frac{1+2y+4y^2}{y(1+2y)} \qquad \ldots \begin{bmatrix} \because y \to \frac{1}{2}, y \neq \frac{1}{2} \\ \therefore 2y \neq 1, \therefore 2y-1 \neq 0 \\ \therefore 1-2y \neq 0 \end{bmatrix}$$

$$= \frac{\lim_{y \to \frac{1}{2}}\left(1+2y+4y^2\right)}{\lim_{y \to \frac{1}{2}}\left[y(1+2y)\right]}$$

$$= \frac{1+2\left(\frac{1}{2}\right)+4\left(\frac{1}{2}\right)^2}{\frac{1}{2}\left[1+2\left(\frac{1}{2}\right)\right]}$$

$$= \frac{1+1+1}{\frac{1}{2}(2)} = 3$$

Question 2.
limx→1[x-2x2-x-1x3-3x2+2x]

Solution:

$$\lim_{x\to1}\left[\frac{x-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right]$$

$$=\lim_{x\to1}\left[\frac{x-2}{x^2-x}-\frac{1}{x\left(x^2-3x+2\right)}\right]$$

$$=\lim_{x\to1}\left[\frac{x-2}{x(x-1)}-\frac{1}{x(x-2)(x-1)}\right]$$

$$=\lim_{x\to1}\left[\frac{(x-2)^2-1}{x(x-2)(x-1)}\right]$$

$$=\lim_{x\to1}\frac{x^2-4x+4-1}{x(x-2)(x-1)}$$

$$=\lim_{x\to1}\frac{x^2-4x+3}{x(x-2)(x-1)}$$

$$=\lim_{x\to1}\frac{(x-3)(x-1)}{x(x-2)(x-1)}$$

$$=\lim_{x\to1}\frac{x-3}{x(x-2)}\qquad\ldots\left[\begin{array}{l}\because x\to1,\ x\neq1,\\ \therefore x-1\neq0\end{array}\right]$$

$$=\frac{\lim_{x\to1}(x-3)}{\lim_{x\to1}x(x-2)}$$

$$=\frac{1-3}{1(1-2)}$$

$$=\frac{-2}{-1}=2$$

Question 3.

$\lim_{x\to1}\left[\frac{x^4-3x^2+2}{x^3-5x^2+3x+1}\right]$

Solution:

$\lim_{x\to1}\left[\frac{x^4-3x^2+2}{x^3-5x^2+3x+1}\right]$

As $x \to 1$, numerator and denominator both tend to zero

$\therefore$ $x-1$ is a factor of both.

To find the factor of numerator and denominator by synthetic division

Consider, numerator = $x^4+0x^3-3x^2+0x+2$

1	1	0	−3	0	2
		1	1	−2	−2
	1	1	−2	−2	$\boxed{0}$

$\therefore$ numerator = $(x-1)(x^3+x^2-2x-2)$

Now, denominator = x^3-5x^2+3x+1

1	1	−5	3	1
		1	−4	−1
	1	−4	−1	$\boxed{0}$

$\therefore$ denominator = $(x-1)(x^2-4x-1)$

$\therefore \lim\limits_{x\to1} \dfrac{x^4-3x^2+2}{x^3-5x^2+3x+1}$

$= \lim\limits_{x\to1} \dfrac{(x-1)(x^3+x^2-2x-2)}{(x-1)(x^2-4x-1)}$

$= \lim\limits_{x\to1} \dfrac{x^3+x^2-2x-2}{x^2-4x-1} \quad \ldots\left[\begin{matrix}\because x\to1,\ x\neq1\\ \because x-1\neq0\end{matrix}\right]$

$= \dfrac{\lim\limits_{x\to1}(x^3+x^2-2x-2)}{\lim\limits_{x\to1}(x^2-4x-1)}$

$= \dfrac{1^3+1^2-2(1)-2}{1^2-4(1)-1}$

$= \dfrac{1+1-2-2}{1-4-1} = \dfrac{-2}{-4}$

$= \dfrac{1}{2}$

Question 4.

$\lim_{x\to1}\left[\frac{x+2}{x^2-5x+4}+\frac{x-4}{3(x^2-3x+2)}\right]$

Solution:

$\lim\limits_{x\to1}\left[\dfrac{x+2}{x^2-5x+4}+\dfrac{x-4}{3(x^2-3x+2)}\right]$

$= \lim\limits_{x\to1}\left[\dfrac{x+2}{(x-4)(x-1)}+\dfrac{x-4}{3(x-2)(x-1)}\right]$

$= \lim\limits_{x\to1} \dfrac{3(x-2)(x+2)+(x-4)^2}{3(x-4)(x-2)(x-1)}$

$= \lim\limits_{x\to1} \dfrac{3(x^2-4)+(x-4)^2}{3(x-4)(x-2)(x-1)}$

$= \lim\limits_{x\to1} \dfrac{3x^2-12+x^2-8x+16}{3(x-4)(x-2)(x-1)}$

$= \lim\limits_{x\to1} \dfrac{4x^2-8x+4}{3(x-4)(x-2)(x-1)}$

$= \lim\limits_{x\to1} \dfrac{4(x^2-2x+1)}{3(x-4)(x-2)(x-1)}$

$= \lim\limits_{x\to1} \dfrac{4(x-1)^2}{3(x-4)(x-2)(x-1)}$

$\ldots[\because a^2-2ab+b^2=(a-b)^2]$

$= \lim\limits_{x\to1} \dfrac{4(x-1)}{3(x-4)(x-2)} \quad \ldots\left[\begin{matrix}\because x\to1,\ x\neq1\\ \therefore x-1\neq0\end{matrix}\right]$

$= \dfrac{\lim\limits_{x\to1} 4(x-1)}{\lim\limits_{x\to1} 3(x-4)(x-2)}$

$= \dfrac{4(1-1)}{3(1-4)(1-2)} = 0$

Question 5.
$\lim_{x \to a}\left[\frac{1}{x^2-3ax+2a^2}+\frac{1}{2x^2-3ax+a^2}\right]$
Solution:

$$\lim_{x \to a}\left[\frac{1}{x^2-3ax+2a^2}+\frac{1}{2x^2-3ax+a^2}\right]$$

Consider,

$$\begin{aligned} x^2-3ax+2a^2 &= x^2-2ax-ax+2a^2 \\ &= x(x-2a)-a(x-2a) \\ &= (x-2a)(x-a) \end{aligned}$$

$$\begin{aligned} 2x^2-3ax+a^2 &= 2x^2-2ax-ax+a^2 \\ &= 2x(x-a)-a(x-a) \\ &= (x-a)(2x-a) \end{aligned}$$

$$\therefore \quad \lim_{x \to a}\left[\frac{1}{x^2-3ax+2a^2}+\frac{1}{2x^2-3ax+a^2}\right]$$

$$= \lim_{x \to a}\left[\frac{1}{(x-2a)(x-a)}+\frac{1}{(x-a)(2x-a)}\right]$$

$$= \lim_{x \to a}\frac{(2x-a)+(x-2a)}{(x-2a)(x-a)(2x-a)}$$

$$= \lim_{x \to a}\frac{3x-3a}{(x-2a)(x-a)(2x-a)}$$

$$= \lim_{x \to a}\frac{3(x-a)}{(x-2a)(x-a)(2x-a)}$$

$$= \lim_{x \to a}\frac{3}{(x-2a)(2x-a)} \quad \ldots\left[\begin{array}{l}\because x \to a, x \neq a \\ \therefore x-a \neq 0\end{array}\right]$$

$$= \frac{3}{(a-2a)(2a-a)}$$

$$= \frac{3}{(-a)(a)} = \frac{-3}{a^2}$$

Maharashtra State Board 11th Maths Solutions Chapter 7 Limits Ex 7.3

I. Evaluate the following limits:

Question 1.
$\lim_{x \to 0}\left[\frac{\sqrt{6+x+x^2}-\sqrt{6}}{x}\right]$
Solution:

$$\lim_{x\to 0}\left[\frac{\sqrt{6+x+x^2}-\sqrt{6}}{x}\right]$$

$$= \lim_{x\to 0}\left[\frac{\sqrt{6+x+x^2}-\sqrt{6}}{x}\times\frac{\sqrt{6+x+x^2}+\sqrt{6}}{\sqrt{6+x+x^2}+\sqrt{6}}\right]$$

...[By rationalization]

$$= \lim_{x\to 0}\frac{\left(6+x+x^2\right)-6}{x\left(\sqrt{6+x+x^2}+\sqrt{6}\right)}$$

$$= \lim_{x\to 0}\frac{x+x^2}{x\left(\sqrt{6+x+x^2}+\sqrt{6}\right)}$$

$$= \lim_{x\to 0}\frac{x(1+x)}{x\left(\sqrt{6+x+x^2}+\sqrt{6}\right)}$$

$$= \lim_{x\to 0}\frac{1+x}{\sqrt{6+x+x^2}+\sqrt{6}} \quad \ldots[\because x\to 0, x\neq 0]$$

$$= \frac{\lim_{x\to 0}(1+x)}{\lim_{x\to 0}\left(\sqrt{6+x+x^2}+\sqrt{6}\right)}$$

$$= \frac{(1+0)}{\sqrt{6}+\sqrt{6}}$$

$$= \frac{1}{2\sqrt{6}}$$

Question 2.
limx→3[2x+3√−4x−3√x2−9]
Solution:

$$\lim_{x\to 3}\left[\frac{\sqrt{2x+3}-\sqrt{4x-3}}{x^2-9}\right]$$

$$= \lim_{x\to 3}\left[\frac{\sqrt{2x+3}-\sqrt{4x-3}}{x^2-9}\times\frac{\sqrt{2x+3}+\sqrt{4x-3}}{\sqrt{2x+3}+\sqrt{4x-3}}\right]$$

...[By rationalization]

$$= \lim_{x\to 3}\left[\frac{(2x+3)-(4x-3)}{\left(x^2-9\right)\left(\sqrt{2x+3}+\sqrt{4x-3}\right)}\right]$$

$$= \lim_{x\to 3}\left[\frac{-2x+6}{\left(x^2-9\right)\left(\sqrt{2x+3}+\sqrt{4x-3}\right)}\right]$$

$$= \lim_{x\to 3}\left[\frac{-2(x-3)}{(x+3)(x-3)\left(\sqrt{2x+3}+\sqrt{4x-3}\right)}\right]$$

$$= \lim_{x\to 3}\left[\frac{-2}{(x+3)\left(\sqrt{2x+3}+\sqrt{4x-3}\right)}\right]$$

$$\ldots\left[\begin{array}{l}\because x\to 3, x\neq 3\\ \therefore x-3\neq 0\end{array}\right]$$

$$= \frac{-2}{(3+3)\left(\sqrt{2(3)+3}+\sqrt{4(3)-3}\right)}$$

$$= \frac{-2}{6(3+3)}$$

$$= \frac{-1}{18}$$

Question 3.
limy→0[1−y2√−1+y2√y2]

Solution:

$$\lim_{y \to 0}\left[\frac{\sqrt{1-y^2}-\sqrt{1+y^2}}{y^2}\right]$$

$$= \lim_{y \to 0}\left[\frac{\sqrt{1-y^2}-\sqrt{1+y^2}}{v^2} \times \frac{\sqrt{1-y^2}+\sqrt{1+y^2}}{\sqrt{1-y^2}+\sqrt{1+y^2}}\right]$$

…[By rationalization]

$$= \lim_{y \to 0}\frac{\left(1-y^2\right)-\left(1+y^2\right)}{y^2\left(\sqrt{1-y^2}+\sqrt{1+y^2}\right)}$$

$$= \lim_{y \to 0}\frac{1-y^2-1-y^2}{y^2\left(\sqrt{1-y^2}+\sqrt{1+y^2}\right)}$$

$$= \lim_{y \to 0}\frac{-2y^2}{y^2\left(\sqrt{1-y^2}+\sqrt{1+y^2}\right)}$$

$$= \lim_{y \to 0}\frac{-2}{\sqrt{1-y^2}+\sqrt{1+y^2}}$$

$$\ldots\left[\begin{array}{l}\because y \to 0,\ y \neq 0,\\ \therefore y^2 \neq 0\end{array}\right]$$

$$= \frac{-2}{\sqrt{1-0^2}+\sqrt{1+0^2}}$$

$$= \frac{-2}{1+1}$$

$$= -1$$

Question 4.

limx→2[2+x√−6−x√x√−2√]

Solution:

$$\lim_{x \to 2}\left(\frac{\sqrt{2+x}-\sqrt{6-x}}{\sqrt{x}-\sqrt{2}}\right)$$

$$= \lim_{x \to 2}\left[\frac{\sqrt{2+x}-\sqrt{6-x}}{\sqrt{x}-\sqrt{2}} \times \frac{\sqrt{2+x}+\sqrt{6-x}}{\sqrt{2+x}+\sqrt{6-x}} \times \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}\right]$$

$$\ldots\left[\begin{array}{l}\text{By taking conjugates of both, the}\\ \text{numerator as well as the denominator.}\end{array}\right]$$

$$= \lim_{x \to 2}\left[\frac{(2+x)-(6-x)}{(x-2)} \times \frac{\sqrt{x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{6-x}}\right]$$

$$= \lim_{x \to 2}\left[\frac{-4+2x}{x-2} \times \frac{\sqrt{x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{6-x}}\right]$$

$$= \lim_{x \to 2}\left[\frac{2(x-2)}{x-2} \times \frac{\sqrt{x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{6-x}}\right]$$

$$= \lim_{x \to 2}\left[\frac{2\left(\sqrt{x}+\sqrt{2}\right)}{\sqrt{2+x}+\sqrt{6-x}}\right]$$

$$\ldots\left[\begin{array}{l}\because x \to 2, x \neq 2,\\ \therefore x-2 \neq 0\end{array}\right]$$

$$= \frac{\lim_{x \to 2} 2\left(\sqrt{x}+\sqrt{2}\right)}{\lim_{x \to 2}\left(\sqrt{2+x}+\sqrt{6-x}\right)}$$

$$= \frac{2\left(\sqrt{2}+\sqrt{2}\right)}{\sqrt{2+2}+\sqrt{6-2}}$$

$$= \frac{2\left(2\sqrt{2}\right)}{2+2}$$

$$= \frac{4\sqrt{2}}{4}$$

$$= \sqrt{2}$$

II. Evaluate the following limits:

Question 1.
$lim_{x \to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$
Solution:

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$=\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$$

$$\ldots\left[\begin{array}{l}\text{By taking conjugates of both, the}\\ \text{numerator as well as the denominator.}\end{array}\right]$$

$$=\lim_{x\to a}\left[\frac{(a+2x)-3x}{(3a+x)-4x}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}\right]$$

$$=\lim_{x\to a}\left[\frac{a-x}{3a-3x}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}\right]$$

$$=\lim_{x\to a}\left[\frac{-(x-a)}{-3(x-a)}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}\right]$$

$$=\lim_{x\to a}\left[\frac{\sqrt{3a+x}+2\sqrt{x}}{3\left(\sqrt{a+2x}+\sqrt{3x}\right)}\right]\qquad\ldots\left[\begin{array}{l}\because x\to a,\ x\neq a\\ \therefore x-a\neq 0\end{array}\right]$$

$$=\frac{\sqrt{3a+a}+2\sqrt{a}}{3\left(\sqrt{a+2a}+\sqrt{3a}\right)}$$

$$=\frac{\sqrt{4a}+2\sqrt{a}}{3\left(\sqrt{3a}+\sqrt{3a}\right)}$$

$$=\frac{2\sqrt{a}+2\sqrt{a}}{3\left(2\sqrt{3a}\right)}$$

$$=\frac{4\sqrt{a}}{6\sqrt{3}\sqrt{a}}$$

$$=\frac{2}{3\sqrt{3}}$$

Question 2.

$lim_{x\to2}[x_2-4x+2\sqrt{}-3x-2\sqrt{}]$

Solution:

$$\lim_{x\to 2}\left[\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right]$$

$$=\lim_{x\to 2}\left[\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\times\frac{\sqrt{x+2}+\sqrt{3x-2}}{\sqrt{x+2}+\sqrt{3x-2}}\right]$$

$$\ldots[\text{By rationalization}]$$

$$=\lim_{x\to 2}\frac{(x^2-4)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{(x+2)-(3x-2)}$$

$$=\lim_{x\to 2}\frac{(x^2-4)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2x+4}$$

$$=\lim_{x\to 2}\frac{(x+2)(x-2)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2(x-2)}$$

$$=\lim_{x\to 2}\frac{(x+2)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2}$$

$$\ldots\left[\begin{array}{l}\because x\to 2,\ x\neq 2\\ \therefore x-2\neq 0\end{array}\right]$$

$$=\frac{(2+2)\left(\sqrt{2+2}+\sqrt{3(2)-2}\right)}{-2}$$

$$=\frac{4(2+2)}{-2}$$

$$=\frac{16}{-2}$$

$$=-8$$

Question 3.

$lim_{x\to2}[1+2+x\sqrt{}\sqrt{}-3\sqrt{}x-2]$

Solution:

$$\lim_{x\to 2}\frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$$

$$=\lim_{x\to 2}\left[\frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}\times\frac{\sqrt{1+\sqrt{2+x}}+\sqrt{3}}{\sqrt{1+\sqrt{2+x}}+\sqrt{3}}\right]$$

...[By rationalization]

$$=\lim_{x\to 2}\frac{1+\sqrt{2+x}-3}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)}$$

$$=\lim_{x\to 2}\frac{\sqrt{2+x}-2}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)}\times\frac{\sqrt{2+x}+2}{\sqrt{2+x}+2}$$

...[By rationalization]

$$=\lim_{x\to 2}\frac{2+x-4}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{2+x}+2\right)}$$

$$=\lim_{x\to 2}\frac{x-2}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{2+x}+2\right)}$$

$$=\lim_{x\to 2}\frac{1}{\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{2+x}+2\right)}$$

...[$\because x\to 2, x\neq 2, \therefore x-2\neq 0$]

$$=\frac{1}{\left(\sqrt{1+\sqrt{2+2}}+\sqrt{3}\right)\left(\sqrt{2+2}+2\right)}$$

$$=\frac{1}{\left(\sqrt{1+2}+\sqrt{3}\right)(2+2)}$$

$$=\frac{1}{\left(2\sqrt{3}\right)(4)}$$

$$=\frac{1}{8\sqrt{3}}$$

Question 4.

$lim_{y\to0}[a+y\sqrt{}-a\sqrt{}ya+y\sqrt{}]$

Solution:

$$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$$

$$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$$

...[By rationalization]

$$=\lim_{y\to 0}\frac{a+y-a}{y\sqrt{a+y}\left(\sqrt{a+y}+\sqrt{a}\right)}$$

$$=\lim_{y\to 0}\frac{y}{y\sqrt{a+y}\left(\sqrt{a+y}+\sqrt{a}\right)}$$

$$=\lim_{y\to 0}\frac{1}{\sqrt{a+y}\left(\sqrt{a+y}+\sqrt{a}\right)}\quad\ldots\left[\begin{array}{l}\because y\to 0,\\ \therefore y\neq 0\end{array}\right]$$

$$=\frac{1}{\sqrt{a+0}\left(\sqrt{a+0}+\sqrt{a}\right)}$$

$$=\frac{1}{\sqrt{a}\left(2\sqrt{a}\right)}$$

$$=\frac{1}{2a}$$

Question 5.
$\lim_{x \to 0}\left(\frac{\sqrt{x^2+9}-\sqrt{2x^2+9}}{\sqrt{3x^2+4}-\sqrt{2x^2+4}}\right)$
Solution:

$$\lim_{x \to 0}\left[\frac{\sqrt{x^2+9}-\sqrt{2x^2+9}}{\sqrt{3x^2+4}-\sqrt{2x^2+4}}\right]$$

$$= \lim_{x \to 0}\left[\frac{\sqrt{x^2+9}-\sqrt{2x^2+9}}{\sqrt{3x^2+4}-\sqrt{2x^2+4}} \times \frac{\sqrt{x^2+9}+\sqrt{2x^2+9}}{\sqrt{x^2+9}+\sqrt{2x^2+9}} \times \frac{\sqrt{3x^2+4}+\sqrt{2x^2+4}}{\sqrt{3x^2+4}+\sqrt{2x^2+4}}\right]$$

$\ldots$ [By taking conjugates of both, the numerator as well as the denominator.]

$$= \lim_{x \to 0}\left[\frac{(x^2+9)-(2x^2+9)}{(3x^2+4)-(2x^2+4)} \times \frac{\sqrt{3x^2+4}+\sqrt{2x^2+4}}{\sqrt{x^2+9}+\sqrt{2x^2+9}}\right]$$

$$= \lim_{x \to 0}\left[\frac{-x^2\left(\sqrt{3x^2+4}+\sqrt{2x^2+4}\right)}{x^2\left(\sqrt{x^2+9}+\sqrt{2x^2+9}\right)}\right]$$

$$= -\lim_{x \to 0}\left[\frac{\sqrt{3x^2+4}+\sqrt{2x^2+4}}{\sqrt{x^2+9}+\sqrt{2x^2+9}}\right]$$

$$\ldots \left[\begin{array}{l} \because x \to 0, x \neq 0 \\ \therefore x^2 \neq 0 \end{array}\right]$$

$$= \frac{-\lim_{x \to 0}\left(\sqrt{3x^2+4}+\sqrt{2x^2+4}\right)}{\lim_{x \to 0}\left(\sqrt{x^2+9}+\sqrt{2x^2+9}\right)}$$

$$= \frac{-\left(\sqrt{0+4}+\sqrt{0+4}\right)}{\sqrt{0+9}+\sqrt{0+9}}$$

$$= \frac{-(2+2)}{3+3}$$

$$= \frac{-2}{3}$$

III. Evaluate the following limits:

Question 1.
$\lim_{x \to 1}\left[\frac{x^2+x\sqrt{x}-2}{x-1}\right]$
Solution:

$$\lim_{x \to 1}\left[\frac{x^2+x\sqrt{x}-2}{x-1}\right]$$

$$= \lim_{x \to 1}\left[\frac{(x^2-1)+(x\sqrt{x}-1)}{x-1}\right]$$

$$= \lim_{x \to 1}\left[\frac{x^2-1}{x-1}+\frac{x^{\frac{3}{2}}-1}{x-1}\right] \quad \ldots \left[\begin{array}{l} \because x\sqrt{x} = x^1 \cdot x^{\frac{1}{2}} \\ \qquad = x^{1+\frac{1}{2}} = x^{\frac{3}{2}} \end{array}\right]$$

$$= \lim_{x \to 1}\left(\frac{x^2-1^2}{x-1}\right) + \lim_{x \to 1}\left(\frac{x^{\frac{3}{2}}-1^{\frac{3}{2}}}{x-1}\right)$$

$$= 2(1)^1 + \frac{3}{2}(1)^{\frac{1}{2}} \qquad \ldots \left[\because \lim_{x \to a}\frac{x^n-a^n}{x-a} = na^{n-1}\right]$$

$$= 2 + \frac{3}{2} = \frac{7}{2}$$

Question 2.
$\lim_{x \to 0}\left[\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}\right]$
Solution:

$$\lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}$$

$$= \lim_{x\to 0} \frac{\left(\sqrt{1+x^2}-\sqrt{1+x}\right)}{\left(\sqrt{1+x^3}-\sqrt{1+x}\right)} \times \frac{\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}{\left(\sqrt{1+x^3}+\sqrt{1+x}\right)} \times \frac{\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}$$

$\ldots$ [By taking conjugates of both, the numerator as well as the denominator.]

$$= \lim_{x\to 0} \frac{\left[1+x^2-(1+x)\right]\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left[1+x^3-(1+x)\right]\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}$$

$$= \lim_{x\to 0} \frac{\left(x^2-x\right)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left(x^3-x\right)\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}$$

$$= \lim_{x\to 0} \frac{x(x-1)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{x\left(x^2-1\right)\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}$$

$$= \lim_{x\to 0} \frac{x(x-1)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{x(x-1)(x+1)\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}$$

$$= \lim_{x\to 0} \frac{\sqrt{1+x^3}+\sqrt{1+x}}{(x+1)\left(\sqrt{1+x^2}+\sqrt{1+x}\right)} \ldots [\because x \to 0, x \neq 0]$$

$$= \frac{\lim\limits_{x\to 0}\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\lim\limits_{x\to 0}(x+1).\lim\limits_{x\to 0}\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}$$

$$= \frac{\sqrt{1+0^3}+\sqrt{1+0}}{(0+1)\left(\sqrt{1+0^2}+\sqrt{1+0}\right)}$$

$$= \frac{1+1}{1(1+1)} = 1$$

Question 3.
limx→4[x2+x−203x+4√−4]

Solution:

$$\lim_{x\to 4}\left[\frac{x^2+x-20}{\sqrt{3x+4}-4}\right]$$

$$=\lim_{x\to 4}\left[\frac{(x+5)(x-4)}{\sqrt{3x+4}-4}\times\frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4}\right]$$

...[By rationalization]

$$=\lim_{x\to 4}\frac{(x+5)(x-4)\left(\sqrt{3x+4}+4\right)}{(3x+4)-16}$$

$$=\lim_{x\to 4}\frac{(x+5)(x-4)\left(\sqrt{3x+4}+4\right)}{3x-12}$$

$$=\lim_{x\to 4}\frac{(x+5)(x-4)\left(\sqrt{3x+4}+4\right)}{3(x-4)}$$

$$=\lim_{x\to 4}\frac{(x+5)\left(\sqrt{3x+4}+4\right)}{3}\quad\ldots\left[\begin{array}{l}\because x\to 4, x\neq 4\\ \therefore x-4\neq 0\end{array}\right]$$

$$=\frac{(4+5)\left(\sqrt{3(4)+4}+4\right)}{3}$$

$$=\frac{9(4+4)}{3}$$

$$=24$$

Question 4.

$lim_{z\to 4}[3-5+z\sqrt{}1-5-z\sqrt{}]$

Solution:

$$\lim_{z\to 4}\left[\frac{3-\sqrt{5+z}}{1-\sqrt{5-z}}\right]$$

$$=\lim_{z\to 4}\left[\frac{3-\sqrt{5+z}}{1-\sqrt{5-z}}\times\frac{3+\sqrt{5+z}}{3+\sqrt{5+z}}\times\frac{1+\sqrt{5-z}}{1+\sqrt{5-z}}\right]$$

...[By taking conjugates of both, the numerator as well as the denominator.]

$$=\lim_{z\to 4}\left[\frac{9-(5+z)}{1-(5-z)}\times\frac{1+\sqrt{5-z}}{3+\sqrt{5+z}}\right]$$

$$=\lim_{z\to 4}\left[\frac{4-z}{-4+z}\times\frac{1+\sqrt{5-z}}{3+\sqrt{5+z}}\right]$$

$$=\lim_{z\to 4}\left[\frac{-(z-4)}{z-4}\times\frac{1+\sqrt{5-z}}{3+\sqrt{5+z}}\right]$$

$$=\lim_{z\to 4}\left[\frac{-\left(1+\sqrt{5-z}\right)}{3+\sqrt{5+z}}\right]\quad\ldots\left[\begin{array}{l}\because z\to 4, z\neq 4,\\ \therefore z-4\neq 0\end{array}\right]$$

$$=\frac{-\lim_{z\to 4}\left(1+\sqrt{5-z}\right)}{\lim_{z\to 4}\left(3+\sqrt{5+z}\right)}$$

$$=\frac{-\left(1+\sqrt{5-4}\right)}{3+\sqrt{5+4}}$$

$$=\frac{-(1+1)}{3+3}$$

$$=\frac{-2}{6}$$

$$=-\frac{1}{3}$$

Question 5.

$lim_{x\to 0}(3x9-x\sqrt{}-1x)$

Solution:

$$\lim_{x \to 0}\left(\frac{3}{x\sqrt{9-x}} - \frac{1}{x}\right)$$

$$= \lim_{x \to 0}\left[\frac{3-\sqrt{9-x}}{x\sqrt{9-x}}\right]$$

$$= \lim_{x \to 0}\left[\frac{3-\sqrt{9-x}}{x\sqrt{9-x}} \times \frac{3+\sqrt{9-x}}{3+\sqrt{9-x}}\right]$$

...[By rationalization]

$$= \lim_{x \to 0}\left[\frac{9-(9-x)}{x\sqrt{9-x}\left(3+\sqrt{9-x}\right)}\right]$$

$$= \lim_{x \to 0}\left[\frac{x}{x\sqrt{9-x}\left(3+\sqrt{9-x}\right)}\right]$$

$$= \lim_{x \to 0}\left[\frac{1}{\sqrt{9-x}\left(3+\sqrt{9-x}\right)}\right] \ldots[\because x \to 0, x \neq 0]$$

$$= \frac{1}{\sqrt{9-0}\left(3+\sqrt{9-0}\right)}$$

$$= \frac{1}{3(3+3)}$$

$$= \frac{1}{18}$$

Maharashtra State Board 11th Maths Solutions Chapter 7 Limits Ex 7.4

I. Evaluate the following limits:

Question 1.
$\lim_{\theta \to 0}\left[\frac{\sin(m\theta)}{\tan(n\theta)}\right]$

Solution:

$$\lim_{\theta \to 0}\left[\frac{\sin(m\theta)}{\tan(n\theta)}\right]$$

$$= \lim_{\theta \to 0}\left[\frac{\frac{\sin(m\theta)}{\theta}}{\frac{\tan(n\theta)}{\theta}}\right]$$

$$\ldots \left[\begin{array}{l}\text{Divide Numerator and Denominator by } \theta \\ \because \theta \to 0, \quad \theta \neq 0\end{array}\right]$$

$$= \frac{\lim_{\theta \to 0}\left[\frac{\sin(m\theta)}{\theta}\right]}{\lim_{\theta \to 0}\left[\frac{\tan(n\theta)}{\theta}\right]}$$

$$= \frac{\lim_{\theta \to 0}\left[\frac{\sin(m\theta)}{m\theta} \times m\right]}{\lim_{\theta \to 0}\left[\frac{\tan(n\theta)}{n\theta} \times n\right]}$$

$$= \frac{m}{n} \frac{\lim_{\theta \to 0} \frac{\sin(m\theta)}{m\theta}}{\lim_{\theta \to 0} \frac{\tan(n\theta)}{n\theta}}$$

$$= \frac{m}{n}\frac{(1)}{(1)} \quad \ldots \left[\because \lim_{\theta \to 0} \frac{\sin p\theta}{p\theta} = 1, \lim_{\theta \to 0} \frac{\tan p\theta}{p\theta} = 1\right]$$

$$= \frac{m}{n}$$

Question 2.

$\lim_{\theta \to 0}\left[\frac{1-\cos 2\theta}{\theta^2}\right]$

Solution:

$$\lim_{\theta \to 0} \frac{1-\cos 2\theta}{\theta^2} = \lim_{\theta \to 0} \frac{2\sin^2\theta}{\theta^2}$$

$$= 2\lim_{\theta \to 0}\left(\frac{\sin\theta}{\theta}\right)^2 = 2(1)^2 = 2$$

Question 3.

$\lim_{x \to 0}\left[\frac{x \cdot \tan x}{1-\cos x}\right]$

Solution:

$$\lim_{x \to 0} \frac{x.\tan x}{1-\cos x}$$

$$= \lim_{x \to 0} \left(\frac{x.\tan x}{1-\cos x} \times \frac{1+\cos x}{1+\cos x}\right)$$

$$= \lim_{x \to 0} \frac{x.\tan x\,(1+\cos x)}{1-\cos^2 x}$$

$$= \lim_{x \to 0} \frac{x.\tan x\,(1+\cos x)}{\sin^2 x}$$

$$= \lim_{x \to 0} \frac{\frac{x.\tan x}{x^2}(1+\cos x)}{\frac{\sin^2 x}{x^2}}$$

$$\ldots \left[\begin{array}{l}\text{Divide numerator and} \\ \text{denominator by } x^2 \\ \because x \to 0, x \neq 0 \therefore x^2 \neq 0\end{array}\right]$$

$$= \frac{\lim_{x \to 0}\left[\frac{\tan x}{x}(1+\cos x)\right]}{\lim_{x \to 0}\left(\frac{\sin x}{x}\right)^2}$$

$$= \frac{\lim_{x \to 0}\frac{\tan x}{x} \times \lim_{x \to 0}(1+\cos x)}{\lim_{x \to 0}\left(\frac{\sin x}{x}\right)^2}$$

$$= \frac{1 \times (1+1)}{(1)^2} \ldots \left[\because \lim_{\theta \to 0}\frac{\sin\theta}{\theta} = 1, \lim_{\theta \to 0}\frac{\tan\theta}{\theta} = 1\right]$$

$$= 2$$

Question 4.

$\lim_{x\to 0}\left(\frac{\sec x-1}{x^2}\right)$

Solution:

$$\lim_{x \to 0} \frac{\sec x - 1}{x^2}$$

$$= \lim_{x \to 0} \frac{(\sec x - 1)(\sec x + 1)}{x^2(\sec x + 1)}$$

$$= \lim_{x \to 0} \frac{\sec^2 x - 1}{x^2(\sec x + 1)} = \lim_{x \to 0} \frac{\tan^2 x}{x^2(\sec x + 1)}$$

$$= \lim_{x \to 0} \left[\left(\frac{\tan x}{x}\right)^2 \times \frac{1}{\sec x + 1}\right]$$

$$= \lim_{x \to 0} \left(\frac{\tan x}{x}\right)^2 \times \lim_{x \to 0} \frac{1}{\sec x + 1}$$

$$= (1)^2 \times \frac{1}{1+1} \qquad \ldots \left[\because \lim_{\theta \to 0}\frac{\tan\theta}{\theta} = 1\right]$$

$$= \frac{1}{2}$$

II. Evaluate the following limits:

Question 1.

$\lim_{x\to 0}\left[\frac{1-\cos(nx)}{1-\cos(mx)}\right]$

Solution:

$$\lim_{x \to 0}\left[\frac{1-\cos(nx)}{1-\cos(mx)}\right] = \lim_{x \to 0}\left[\frac{2\sin^2\left(\frac{nx}{2}\right)}{2\sin^2\left(\frac{mx}{2}\right)}\right]$$

$$= \lim_{x \to 0}\left[\frac{\frac{\left(\sin\frac{nx}{2}\right)^2}{x^2}}{\frac{\left(\sin\frac{mx}{2}\right)^2}{x^2}}\right]$$

$$\ldots\left[\begin{array}{l}\text{Divide numerator and denominator by } x^2\\ \because x \to 0,\ x \neq 0 \therefore x^2 \neq 0\end{array}\right]$$

$$= \frac{\lim_{x \to 0}\left(\frac{\sin\frac{nx}{2}}{x}\right)^2}{\lim_{x \to 0}\left(\frac{\sin\frac{mx}{2}}{x}\right)^2}$$

$$= \frac{\lim_{x \to 0}\left(\frac{\sin\frac{nx}{2}}{\frac{nx}{2}}\right)^2 \times \left(\frac{n}{2}\right)^2}{\lim_{x \to 0}\left(\frac{\sin\frac{mx}{2}}{\frac{mx}{2}}\right)^2 \times \left(\frac{m}{2}\right)^2}$$

$$= \frac{(1)^2 \times \frac{n^2}{4}}{(1)^2 \times \frac{m^2}{4}} \qquad \ldots\left[\because \lim_{\theta \to 0}\frac{\sin p\theta}{p\theta} = 1\right]$$

$$= \frac{n^2}{m^2}$$

Question 2.

$\lim_{x \to \frac{\pi}{6}}\left[\frac{2-\operatorname{cosec} x}{\cot^2 x-3}\right]$

Solution:

$$\lim_{x \to \frac{\pi}{6}}\left[\frac{2-\operatorname{cosec} x}{\cot^2 x-3}\right] = \lim_{x \to \frac{\pi}{6}}\frac{2-\operatorname{cosec} x}{\operatorname{cosec}^2 x-1-3}$$

$$= \lim_{x \to \frac{\pi}{6}}\frac{2-\operatorname{cosec} x}{\operatorname{cosec}^2 x-4}$$

$$= \lim_{x \to \frac{\pi}{6}}\frac{-(\operatorname{cosec} x-2)}{(\operatorname{cosec} x-2)(\operatorname{cosec} x+2)}$$

$$= \lim_{x \to \frac{\pi}{6}}\frac{-1}{(\operatorname{cosec} x+2)}$$

$$\ldots\left[\begin{array}{l}\because x \to \frac{\pi}{6},\ x \neq \frac{\pi}{6}, \therefore \operatorname{cosec} x \neq \operatorname{cosec}\frac{\pi}{6},\\ \therefore \operatorname{cosec} x \neq 2, \therefore \operatorname{cosec} x - 2 \neq 0\end{array}\right]$$

$$= \frac{-1}{\operatorname{cosec}\left(\frac{\pi}{6}\right)+2}$$

$$= \frac{-1}{2+2} = \frac{-1}{4}$$

Question 3.

$\lim_{x \to \frac{\pi}{4}}\left[\frac{\cos x-\sin x}{\cos 2x}\right]$

Solution:

$$\lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos 2x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos^2 x - \sin^2 x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{(\cos x + \sin x)(\cos x - \sin x)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{1}{\cos x + \sin x}$$

$$\ldots \left[\begin{array}{l} \because x \to \frac{\pi}{4},\ \cos x \to \frac{1}{\sqrt{2}} \text{ and } \sin x \to \frac{1}{\sqrt{2}} \\ \therefore \cos x - \sin x \to 0, \therefore \cos x - \sin x \neq 0 \end{array}\right]$$

$$= \frac{1}{\cos\frac{\pi}{4} + \sin\frac{\pi}{4}}$$

$$= \frac{1}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = \frac{1}{\frac{2}{\sqrt{2}}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

III. Evaluate the following limits:

Question 1.

$lim_{x \to 0}\left[\frac{cos(ax) - cos(bx)}{cos(cx) - 1}\right]$

Solution:

$$\lim_{x \to 0} \frac{\cos(ax) - \cos(bx)}{\cos(cx) - 1}$$

$$= \lim_{x \to 0} \frac{-\cos(ax) + \cos(bx)}{1 - \cos(cx)}$$

$$= \lim_{x \to 0} \frac{1 - \cos ax - 1 + \cos bx}{1 - \cos cx}$$

$$= \lim_{x \to 0} \frac{(1 - \cos ax) - (1 - \cos bx)}{1 - \cos cx}$$

$$= \lim_{x \to 0} \frac{2\sin^2\left(\frac{ax}{2}\right) - 2\sin^2\frac{bx}{2}}{2\sin^2\frac{cx}{2}}$$

$$= \lim_{x \to 0} \frac{\frac{\sin^2\left(\frac{ax}{2}\right) - \sin^2\left(\frac{bx}{2}\right)}{x^2}}{\sin^2\left(\frac{cx}{2}\right)}$$

$$\ldots \left[\begin{array}{l} \text{Divide numerator and denominator by } x^2 \\ \because x \to 0, x \neq 0, \therefore x^2 \neq 0 \end{array}\right]$$

$$= \frac{\lim_{x \to 0}\left[\frac{\sin^2\left(\frac{ax}{2}\right)}{x^2} - \frac{\sin^2\left(\frac{bx}{2}\right)}{x^2}\right]}{\lim_{x \to 0}\frac{\sin^2\left(\frac{cx}{2}\right)}{x^2}}$$

$$= \frac{\lim_{x \to 0}\left[\frac{\sin\frac{ax}{2}}{x}\right]^2 - \lim_{x \to 0}\left[\frac{\sin\frac{bx}{2}}{x}\right]^2}{\lim_{x \to 0}\left(\frac{\sin\frac{cx}{2}}{x}\right)^2}$$

$$= \frac{\lim_{x \to 0}\left(\frac{\sin\frac{ax}{2}}{\frac{ax}{2}}\right)^2 \cdot \left(\frac{a}{2}\right)^2 - \lim_{x \to 0}\left(\frac{\sin\frac{bx}{2}}{\frac{bx}{2}}\right)^2 \cdot \left(\frac{b}{2}\right)^2}{\lim_{x \to 0}\left(\frac{\sin\frac{cx}{2}}{\frac{cx}{2}}\right)^2 \cdot \left(\frac{c}{2}\right)^2}$$

$$= \frac{(1)^2 \cdot \frac{a^2}{4} - (1)^2 \cdot \frac{b^2}{4}}{(1)^2 \cdot \frac{c^2}{4}} = \frac{\frac{a^2}{4} - \frac{b^2}{4}}{\frac{c^2}{4}}$$

$$= \frac{a^2 - b^2}{c^2}$$

Question 2.

$lim_{x\to\pi}[1-cosx\surd-2\surd sin_2x]$

Solution:

$$\lim_{x \to \pi} \frac{\sqrt{1-\cos x} - \sqrt{2}}{\sin^2 x}$$

$$= \lim_{x \to \pi} \frac{\sqrt{1-\cos x} - \sqrt{2}}{\sin^2 x} \times \frac{\sqrt{1-\cos x} + \sqrt{2}}{\sqrt{1-\cos x} + \sqrt{2}}$$

...[By rationalization]

$$= \lim_{x \to \pi} \frac{1-\cos x - 2}{\sin^2 x} \times \frac{1}{\sqrt{1-\cos x} + \sqrt{2}}$$

$$= \lim_{x \to \pi} \frac{-\cos x - 1}{1-\cos^2 x} \times \frac{1}{\sqrt{1-\cos x} + \sqrt{2}}$$

$$= \lim_{x \to \pi} \frac{-(1+\cos x)}{(1-\cos x)(1+\cos x)} \times \frac{1}{\sqrt{1-\cos x} + \sqrt{2}}$$

$$= \lim_{x \to \pi} \frac{-1}{(1-\cos x)\left[\sqrt{1-\cos x} + \sqrt{2}\right]}$$

$$\ldots\left[\begin{array}{l}\because x \to \pi,\ \cos x \neq \cos\pi \\ \therefore \cos x \neq -1, \therefore 1+\cos x \neq 0\end{array}\right]$$

$$= \frac{-1}{\lim_{x \to \pi}(1-\cos x) \cdot \lim_{x \to \pi}\left(\sqrt{1-\cos x} + \sqrt{2}\right)}$$

$$= \frac{-1}{(1-\cos\pi)\left[\sqrt{1-\cos\pi} + \sqrt{2}\right]}$$

$$= \frac{-1}{[1-(-1)]\left[\sqrt{1-(-1)} + \sqrt{2}\right]}$$

$$= \frac{-1}{(2)(2\sqrt{2})} = \frac{-1}{4\sqrt{2}}$$

Question 3.

$lim_{x\to\pi4}[tan_2x-cot_2xsecx-cosecx]$

Solution:

$$\lim_{x \to \frac{\pi}{4}} \frac{\tan^2 x - \cot^2 x}{\sec x - \operatorname{cosec} x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(\sec^2 x - 1) - (\operatorname{cosec}^2 x - 1)}{\sec x - \operatorname{cosec} x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x - \operatorname{cosec}^2 x}{\sec x - \operatorname{cosec} x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(\sec x - \operatorname{cosec} x)(\sec x + \operatorname{cosec} x)}{(\sec x - \operatorname{cosec} x)}$$

$$= \lim_{x \to \frac{\pi}{4}} (\sec x + \operatorname{cosec} x)$$

$$\ldots \left[\begin{array}{l} \because x \to \frac{\pi}{4},\ \sec x \to \sqrt{2} \text{ and } \operatorname{cosec} x \to \sqrt{2} \\ \therefore \sec x - \operatorname{cosec} x \to 0, \therefore \sec x - \operatorname{cosec} x \neq 0 \end{array}\right]$$

$$= \lim_{x \to \frac{\pi}{4}} (\sec x) + \lim_{x \to \frac{\pi}{4}} (\operatorname{cosec} x)$$

$$= \sec \frac{\pi}{4} + \operatorname{cosec} \frac{\pi}{4}$$

$$= \sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

Question 4.

limx→π6[2sin2x+sinx−12sin2x−3sinx+1]

Solution:

$$\lim_{x \to \frac{\pi}{6}} \left[\frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}\right]$$

Consider, $2\sin^2 x + \sin x - 1$

$= 2\sin^2 x + 2\sin x - \sin x - 1$

$= 2\sin x(\sin x + 1) - 1(\sin x + 1)$

$= (\sin x + 1)(2\sin x - 1)$

Now,

$2\sin^2 x - 3\sin x + 1 = 2\sin^2 x - 2\sin x - \sin x + 1$

$= 2\sin x(\sin x - 1) - 1(\sin x - 1)$

$= (\sin x - 1)(2\sin x - 1)$

$$\lim_{x \to \frac{\pi}{6}} \left[\frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}\right]$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{(\sin x + 1)(2\sin x - 1)}{(\sin x - 1)(2\sin x - 1)}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{\sin x + 1}{\sin x - 1} \quad \ldots \left[\begin{array}{l} \because x \to \frac{\pi}{6}, \sin x \to \frac{1}{2} \\ \therefore \sin x \neq \frac{1}{2} \therefore 2\sin x \neq 1 \\ \therefore 2\sin x - 1 \neq 0 \end{array}\right]$$

$$= \frac{\lim_{x \to \frac{\pi}{6}} (\sin x + 1)}{\lim_{x \to \frac{\pi}{6}} (\sin x - 1)}$$

$$= \frac{\sin \frac{\pi}{6} + 1}{\sin \frac{\pi}{6} - 1} = \frac{\frac{1}{2} + 1}{\frac{1}{2} - 1} = \frac{\frac{3}{2}}{\frac{-1}{2}} = -3$$

Maharashtra State Board 11th Maths Solutions Chapter 7 Limits Ex 7.5

I. Evaluate the following:

Question 1.
$\lim_{x \to \frac{\pi}{2}} \left[\frac{\operatorname{cosec} x - 1}{\left(\frac{\pi}{2}-x\right)^2}\right]$
Solution:

$$\text{Put } \frac{\pi}{2} - x = h,$$

$$\therefore \quad x = \frac{\pi}{2} - h$$

$$\text{As } x \to \frac{\pi}{2}, h \to 0$$

$$\therefore \quad \lim_{x \to \frac{\pi}{2}} \frac{\operatorname{cosec} x - 1}{\left(\frac{\pi}{2}-x\right)^2} = \lim_{h \to 0} \frac{\operatorname{cosec}\left(\frac{\pi}{2}-h\right)-1}{(h)^2}$$

$$= \lim_{h \to 0} \frac{\sec h - 1}{h^2} \qquad \ldots\left[\because \operatorname{cosec}\left(\frac{\pi}{2}-\theta\right) = \sec\theta\right]$$

$$= \lim_{h \to 0} \frac{\frac{1}{\cos h} - 1}{h^2}$$

$$= \lim_{h \to 0} \frac{1-\cos h}{h^2 . \cos h}$$

$$= \lim_{h \to 0} \frac{2\sin^2 \frac{h}{2}}{h^2} . \frac{1}{\cos h}$$

$$= 2 \lim_{h \to 0} \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 . \frac{1}{4} . \frac{1}{\cos h}$$

$$= \frac{1}{2} \lim_{h \to 0} \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 \lim_{h \to 0} \frac{1}{\cos h}$$

$$= \frac{1}{2}(1)^2 \times \frac{1}{\cos 0} \qquad \ldots\left[\because \lim_{\theta \to 0} \frac{\sin p\theta}{p\theta} = 1\right]$$

$$= \frac{1}{2} \times 1$$

$$= \frac{1}{2}$$

Question 2.
$\lim_{x \to a} \frac{\sin x - \sin a}{\sqrt[5]{x} - \sqrt[5]{a}}$
Solution:

$$\lim_{x \to a} \frac{\sin x - \sin a}{\sqrt[5]{x} - \sqrt[5]{a}}$$

$$= \lim_{x \to a} \frac{2\cos\left(\frac{x+a}{2}\right) \cdot \sin\left(\frac{x-a}{2}\right)}{x^{\frac{1}{5}} - a^{\frac{1}{5}}}$$

$$= \lim_{x \to a} \frac{2\cos\left(\frac{x+a}{2}\right) \cdot \frac{\sin\left(\frac{x-a}{2}\right)}{x-a}}{\frac{x^{\frac{1}{5}} - a^{\frac{1}{5}}}{x-a}}$$

$$\ldots \left[\begin{array}{l}\text{Divide numerator and denominator by } x-a. \\ \because x \to a,\ x \neq a,\ \therefore\ x-a \neq 0\end{array}\right]$$

$$= \frac{\lim_{x \to a} \cos\left(\frac{x+a}{2}\right) \cdot \lim_{x \to a} \left[\frac{\sin\left(\frac{x-a}{2}\right)}{\frac{x-a}{2}}\right]}{\lim_{x \to a} \frac{x^{\frac{1}{5}} - a^{\frac{1}{5}}}{x-a}}$$

$$= \frac{\cos\left(\frac{a+a}{2}\right) \times 1}{\frac{1}{5} \cdot a^{\frac{-4}{5}}}$$

$$\ldots \left[\begin{array}{l}\because x \to a,\ x-a \to 0 \\ \therefore\ \frac{x-a}{2} \to 0;\ \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \\ \text{and } \lim_{x \to a} \frac{x^n - a^n}{x-a} = na^{n-1}\end{array}\right]$$

$$= 5a^{\frac{4}{5}} \cdot \cos a$$

Question 3.

$\lim_{x \to \pi}\left[\frac{\sqrt{5+\cos x}-2}{(\pi-x)^2}\right]$

Solution:

$$\lim_{x \to \pi} \left[\frac{\sqrt{5+\cos x} - 2}{(\pi - x)^2}\right]$$

Put $\pi - x = h$

$\therefore$ $x = \pi - h$

As $x \to \pi$, $h \to 0$

$$\therefore \quad \lim_{x \to \pi} \left[\frac{\sqrt{5+\cos x} - 2}{(\pi - x)^2}\right]$$

$$= \lim_{h \to 0} \left[\frac{\sqrt{5+\cos(\pi-h)}-2}{(h)^2}\right]$$

$$= \lim_{h \to 0} \left[\frac{\sqrt{5-\cos h}-2}{h^2}\right]$$

$$= \lim_{h \to 0} \left[\frac{\sqrt{5-\cos h}-2}{h^2} \times \frac{\sqrt{5-\cos h}+2}{\sqrt{5-\cos h}+2}\right]$$

...[By rationalization]

$$= \lim_{h \to 0} \frac{5-\cos h-4}{h^2\left(\sqrt{5-\cos h}+2\right)}$$

$$= \lim_{h \to 0} \frac{1-\cos h}{h^2\left(\sqrt{5-\cos h}+2\right)}$$

$$= \lim_{h \to 0} \frac{1-\cos h}{h^2\left(\sqrt{5-\cos h}+2\right)} \times \frac{1+\cos h}{1+\cos h}$$

$$= \lim_{h \to 0} \frac{\sin^2 h}{h^2\left(\sqrt{5-\cos h}+2\right)(1+\cos h)}$$

$$= \lim_{h \to 0} \left(\frac{\sin h}{h}\right)^2 \times \frac{1}{\left(\sqrt{5-\cos h}+2\right)(1+\cos h)}$$

$$= \lim_{h \to 0} \left(\frac{\sin h}{h}\right)^2 \times \lim_{h \to 0} \frac{1}{\left(\sqrt{5-\cos h}+2\right)(1+\cos h)}$$

$$= (1)^2 \times \frac{1}{\left(\sqrt{5-1}+2\right)(1+1)} \quad \ldots\left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{1}{\left(\sqrt{4}+2\right)(2)} = \frac{1}{(2+2)(2)}$$

$$= \frac{1}{8}$$

Question 4.
limx→π6[cosx−3√sinxπ−6x]
Solution:

$$\lim_{x \to \frac{\pi}{6}} \left[\frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}\right]$$

Put $\frac{\pi}{6} - x = \text{h}$

$\therefore \quad x = \frac{\pi}{6} - \text{h}$

As $x \to \frac{\pi}{6}$, $\text{h} \to 0$

$$\therefore \quad \lim_{x \to \frac{\pi}{6}} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$$

$$= \lim_{\text{h} \to 0} \frac{\cos\left(\frac{\pi}{6} - \text{h}\right) - \sqrt{3}\sin\left(\frac{\pi}{6} - \text{h}\right)}{\pi - 6\left(\frac{\pi}{6} - \text{h}\right)}$$

$$= \lim_{\text{h} \to 0} \frac{\left[\left(\cos\frac{\pi}{6}\cdot\cos\text{h} + \sin\frac{\pi}{6}\cdot\sin\text{h}\right) - \sqrt{3}\left(\sin\frac{\pi}{6}\cdot\cos\text{h} - \cos\frac{\pi}{6}\cdot\sin\text{h}\right)\right]}{\pi - \pi + 6\text{h}}$$

$$= \lim_{\text{h} \to 0} \frac{\left(\frac{\sqrt{3}}{2}\cos\text{h} + \frac{1}{2}\sin\text{h}\right) - \sqrt{3}\left(\frac{1}{2}\cos\text{h} - \frac{\sqrt{3}}{2}\sin\text{h}\right)}{6\text{h}}$$

$$= \lim_{\text{h} \to 0} \frac{\frac{\sqrt{3}}{2}\cos\text{h} + \frac{1}{2}\sin\text{h} - \frac{\sqrt{3}}{2}\cos\text{h} + \frac{3}{2}\sin\text{h}}{6\text{h}}$$

$$= \lim_{\text{h} \to 0} \frac{2\sin\text{h}}{6\text{h}}$$

$$= \frac{1}{3}\lim_{\text{h} \to 0} \frac{\sin\text{h}}{\text{h}}$$

$$= \frac{1}{3}(1) \qquad \ldots\left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{1}{3}$$

Question 5.

$lim_{x\to 1}[1-x_2 sin\pi x]$

Solution:

$$\lim_{x\to 1}\frac{1-x^2}{\sin \pi x}=\lim_{x\to 1}\frac{(1-x)(1+x)}{\sin \pi x}$$

Put $1-x=h$,

$\therefore$ $x=1-h$

As $x\to 1, h\to 0$

$$\therefore \lim_{x\to 1}\frac{(1-x)(1+x)}{\sin \pi x}$$

$$=\lim_{h\to 0}\frac{(h)(1+1-h)}{\sin \pi(1-h)}$$

$$=\lim_{h\to 0}\frac{(h)(2-h)}{\sin(\pi-\pi h)}$$

$$=\lim_{h\to 0}\frac{(h)(2-h)}{\sin \pi h}\quad \ldots[\because \sin(\pi-\theta)=\sin\theta]$$

$$=\lim_{h\to 0}\frac{(2-h)}{\left(\frac{\sin \pi h}{\pi h}\right).\pi}$$

$$=\frac{1}{\pi}\cdot\frac{\lim_{h\to 0}(2-h)}{\lim_{h\to 0}\left(\frac{\sin \pi h}{\pi h}\right)}$$

$$=\frac{1}{\pi}\cdot\frac{(2-0)}{1}\quad \ldots\left[\because \lim_{\theta\to 0}\frac{\sin p\theta}{p\theta}=1\right]$$

$$=\frac{2}{\pi}$$

II. Evaluate the following:

Question 1.

$\lim_{x\to\frac{\pi}{6}}\left[\frac{2\sin x-1}{\pi-6x}\right]$

Solution:

$$\lim_{x \to \frac{\pi}{6}} \frac{2\sin x - 1}{\pi - 6x}$$

Put $\frac{\pi}{6} - h = x$

$\therefore \quad h = \frac{\pi}{6} - x$

As $x \to \frac{\pi}{6}$, $h \to 0$

$$\therefore \quad \lim_{x \to \frac{\pi}{6}} \frac{2\sin x - 1}{\pi - 6x}$$

$$= \lim_{h \to 0} \frac{2\sin\left(\frac{\pi}{6} - h\right) - 1}{\pi - 6\left(\frac{\pi}{6} - h\right)}$$

$$= \lim_{h \to 0} \frac{2\left[\sin\left(\frac{\pi}{6} - h\right) - \frac{1}{2}\right]}{6h}$$

$$= \frac{1}{3} \lim_{h \to 0} \frac{\sin\left(\frac{\pi}{6} - h\right) - \sin\frac{\pi}{6}}{h}$$

$$= \frac{1}{3} \lim_{h \to 0} \frac{2\cos\left(\frac{\frac{\pi}{6} - h + \frac{\pi}{6}}{2}\right) . \sin\left(\frac{\frac{\pi}{6} - h - \frac{\pi}{6}}{2}\right)}{h}$$

$$= \frac{1}{3} \lim_{h \to 0} \frac{2\cos\left(\frac{\pi}{6} - \frac{h}{2}\right) . \sin\left(\frac{-h}{2}\right)}{h}$$

$$= \frac{-1}{3} \lim_{h \to 0} \cos\left(\frac{\pi}{6} - \frac{h}{2}\right) . \frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= \frac{-1}{3} \lim_{h \to 0} \cos\left(\frac{\pi}{6} - \frac{h}{2}\right) . \lim_{h \to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= \frac{-1}{3} . \cos\left(\frac{\pi}{6} - 0\right) . (1) \qquad \ldots\left[\because \lim_{\theta \to 0} \frac{\sin p\theta}{p\theta} = 1\right]$$

$$= \frac{-1}{3} \times \frac{\sqrt{3}}{2} = \frac{-1}{2\sqrt{3}}$$

Question 2.
$\lim_{x \to \frac{\pi}{4}} \left[\frac{\sqrt{2} - \cos x - \sin x}{(4x - \pi)^2}\right]$
Solution:

$$\lim_{x \to \frac{\pi}{4}} \left[\frac{\sqrt{2} - \cos x - \sin x}{(4x - \pi)^2}\right]$$

$$= \lim_{x \to \frac{\pi}{4}} \left[\frac{\sqrt{2} - (\cos x + \sin x)}{\left[4\left(x - \frac{\pi}{4}\right)\right]^2}\right]$$

Put $x - \frac{\pi}{4} = h$

$\therefore \quad x = \frac{\pi}{4} + h$

As $x \to \frac{\pi}{4}$, $h \to 0$

Also,

$$\cos x + \sin x = \cos x + \cos\left(\frac{\pi}{2} - x\right)$$

$$= \cos\left(\frac{\pi}{4} + h\right) + \cos\left[\frac{\pi}{2} - \left(\frac{\pi}{4} + h\right)\right]$$

$$= \cos\left(\frac{\pi}{4} + h\right) + \cos\left(\frac{\pi}{4} - h\right)$$

$$= 2\cos\frac{\pi}{4}.\cosh$$

...(By defactorisation)

$$= 2\left(\frac{1}{\sqrt{2}}\right)\cosh$$

$$= \sqrt{2}\cosh$$

$\therefore$ Required limit

$$= \lim_{h \to 0} \frac{\sqrt{2} - \sqrt{2}\cos h}{(4h)^2}$$

$$= \lim_{h \to 0} \frac{\sqrt{2}(1 - \cos h)}{16h^2}$$

$$= \frac{\sqrt{2}}{16} \lim_{h \to 0} \left(\frac{1 - \cos h}{h^2} \times \frac{1 + \cos h}{1 + \cos h}\right)$$

$$= \frac{1}{8\sqrt{2}} \lim_{h \to 0} \frac{\sin^2 h}{h^2(1 + \cos h)}$$

$$= \frac{1}{8\sqrt{2}} \lim_{h \to 0} \left(\frac{\sin h}{h}\right)^2 \times \frac{1}{1 + \cos h}$$

$$= \frac{1}{8\sqrt{2}} \times \lim_{h \to 0} \left(\frac{\sin h}{h}\right)^2 \times \lim_{h \to 0} \frac{1}{1 + \cos h}$$

$$= \frac{1}{8\sqrt{2}} \times (1)^2 \times \frac{1}{1+1} \quad \ldots\left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{1}{8\sqrt{2}} \times \frac{1}{2} = \frac{1}{16\sqrt{2}}$$

Question 3.

$\lim_{x\to\frac{\pi}{6}}\left[\frac{2-\sqrt{3}\cos x-\sin x}{(6x-\pi)^2}\right]$

Solution:

$$\lim_{x \to \frac{\pi}{6}} \frac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{2 - \sqrt{3}\cos x - \sin x}{36\left(x - \frac{\pi}{6}\right)^2}$$

Put $x - \frac{\pi}{6} = h$,

$x = \frac{\pi}{6} + h$

As $x \to \frac{\pi}{6}$, $h \to 0$

$$\therefore \quad \lim_{x \to \frac{\pi}{6}} \frac{2-\sqrt{3}\cos x - \sin x}{36\left(x-\frac{\pi}{6}\right)^2}$$

$$= \lim_{h \to 0} \frac{2-\sqrt{3}\cos\left(\frac{\pi}{6}+h\right)-\sin\left(\frac{\pi}{6}+h\right)}{36h^2}$$

$$= \lim_{h \to 0} \frac{2-\sqrt{3}\left(\cos\frac{\pi}{6}.\cosh - \sin\frac{\pi}{6}.\sinh\right)-\left(\sin\frac{\pi}{6}.\cosh + \cos\frac{\pi}{6}.\sinh\right)}{36h^2}$$

$$= \lim_{h \to 0} \frac{2-\sqrt{3}\left(\frac{\sqrt{3}}{2}\cdot\cos h - \frac{1}{2}\sin h\right)-\left(\frac{1}{2}\cos h + \frac{\sqrt{3}}{2}\cdot\sin h\right)}{36h^2}$$

$$= \lim_{h \to 0} \frac{\left[2-\frac{3}{2}.\cos h + \frac{\sqrt{3}}{2}.\ \sin h - \frac{1}{2}\cos h - \frac{\sqrt{3}}{2}\sin h\right]}{36h^2}$$

$$= \lim_{h \to 0} \frac{2-2\cos h}{36h^2}$$

$$= \lim_{h \to 0} \frac{2(1-\cos h)}{36h^2}$$

$$= \frac{1}{18} \lim_{h \to 0} \frac{1-\cos h}{h^2} \times \frac{1+\cos h}{1+\cos h}$$

$$= \frac{1}{18} \lim_{h \to 0} \frac{1-\cos^2 h}{h^2(1+\cos h)}$$

$$= \frac{1}{18} \lim_{h \to 0} \frac{\sin^2 h}{h^2(1+\cos h)}$$

$$= \frac{1}{18}\left(\lim_{h \to 0}\frac{\sin h}{h}\right)^2 \lim_{h \to 0} \frac{1}{(1+\cos h)}$$

$$= \frac{1}{18}(1)^2 \frac{1}{(1+\cos 0)} \qquad \ldots\left[\because \lim_{\theta \to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$= \frac{1}{18(1+1)}$$

$$= \frac{1}{36}$$

Question 4.
$lim_{x\to a}[sin(x\sqrt{})-sin(a\sqrt{})x-a]$
Solution:

$$\lim_{x \to a}\left[\frac{\sin\left(\sqrt{x}\right)-\sin\left(\sqrt{a}\right)}{x-a}\right]$$

Put $\sqrt{x} = y$, $\sqrt{a} = b$ and $y = b + h$.
As $x \to a$, $y \to b$ and $h \to 0$.

$$\lim_{x \to a} \frac{\sin\sqrt{x}-\sin\sqrt{a}}{x-a}$$

$$= \lim_{y \to b} \frac{\sin y - \sin b}{y^2 - b^2}$$

$$= \lim_{y \to b} \frac{\sin y - \sin b}{(y-b)(y+b)}$$

$$= \lim_{h \to 0} \frac{\sin(b+h)-\sin b}{h(b+h+b)}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{b+h+b}{2}\right)\sin\left(\frac{b+h-b}{2}\right)}{h(2b+h)}$$

$$= \lim_{h \to 0} \frac{2\cos\left(b+\frac{h}{2}\right).\sin\left(\frac{h}{2}\right)}{h(2b+h)}$$

$$= \lim_{h \to 0} \frac{\cos\left[b+\frac{h}{2}\right]}{2b+h} \cdot \frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= \lim_{h \to 0} \frac{\cos\left[b+\frac{h}{2}\right]}{2b+h} \cdot \left[\lim_{h \to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}}\right]$$

$$= \frac{\cos(b+0)}{2b+0} \cdot 1 \qquad \ldots\left[\because \lim_{\theta \to 0} \frac{\sin p\theta}{p\theta} = 1\right]$$

$$= \frac{\cos b}{2b}$$

$$= \frac{\cos\sqrt{a}}{2\sqrt{a}}$$

Question 5.
$\lim_{x \to \pi 2}\left[\cos 3x + 3\cos x (2x-\pi) 3\right]$
Solution:

$$\lim_{x \to \frac{\pi}{2}} \frac{3\cos x + \cos 3x}{(2x-\pi)^3}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{3\cos x + 4\cos^3 x - 3\cos x}{(2x-\pi)^3}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{4\cos^3 x}{(2x-\pi)^3} = \lim_{x \to \frac{\pi}{2}} \frac{4\cos^3 x}{8\left(x-\frac{\pi}{2}\right)^3}$$

Put $x - \frac{\pi}{2} = h$,

$\therefore \quad x = \frac{\pi}{2} + h$

As $x \to \frac{\pi}{2}$, $h \to 0$

$$\therefore \quad \lim_{x \to \frac{\pi}{2}} \frac{4\cos^3 x}{8\left(x-\frac{\pi}{2}\right)^3}$$

$$= \lim_{h \to 0} \frac{4\cos^3\left(\frac{\pi}{2}+h\right)}{8h^3}$$

$$= \lim_{h \to 0} \frac{4(-\sin h)^3}{8h^3} \quad \ldots\left[\because \cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta\right]$$

$$= -\frac{1}{2} \cdot \lim_{h \to 0} \left(\frac{\sin h}{h}\right)^3$$

$$= -\frac{1}{2} \cdot \left(\lim_{h \to 0} \frac{\sin h}{h}\right)^3$$

$$= \frac{-1}{2} \cdot (1)^3 \quad \ldots\left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{-1}{2}$$

Maharashtra State Board 11th Maths Solutions Chapter 7 Limits Ex 7.6

I. Evaluate the following limits:

Question 1.
$\lim_{x\to0}\left[\frac{9^x-5^x}{4^x-1}\right]$

Solution:

$$\lim_{x\to 0}\frac{9^x-5^x}{4^x-1} = \lim_{x\to 0}\frac{9^x-1+1-5^x}{4^x-1}$$

$$= \lim_{x\to 0}\frac{(9^x-1)-(5^x-1)}{4^x-1}$$

$$= \lim_{x\to 0}\frac{\frac{(9^x-1)-(5^x-1)}{x}}{\frac{(4^x-1)}{x}}$$

$$\ldots[\because x\to 0,\ x\neq 0]$$

$$= \lim_{x\to 0}\frac{\left(\frac{9^x-1}{x}\right)-\left(\frac{5^x-1}{x}\right)}{\left(\frac{4^x-1}{x}\right)}$$

$$= \frac{\lim\limits_{x\to 0}\frac{9^x-1}{x}-\lim\limits_{x\to 0}\frac{5^x-1}{x}}{\lim\limits_{x\to 0}\frac{4^x-1}{x}}$$

$$= \frac{\log 9-\log 5}{\log 4}\quad \ldots\left[\because \lim_{x\to 0}\frac{a^x-1}{x}=\log a\right]$$

$$= \frac{1}{(\log 4)}\log\left(\frac{9}{5}\right)$$

Question 2.

$\lim_{x\to 0}\left[\frac{5^x+3^x-2^x-1}{x}\right]$

Solution:

$$\lim_{x\to 0}\frac{5^x+3^x-2^x-1}{x}$$

$$= \lim_{x\to 0}\frac{(5^x-1)+(3^x-2^x)}{x}$$

$$= \lim_{x\to 0}\frac{(5^x-1)+(3^x-1)-(2^x-1)}{x}$$

$$= \lim_{x\to 0}\left(\frac{5^x-1}{x}+\frac{3^x-1}{x}-\frac{2^x-1}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{5^x-1}{x}\right)+\lim_{x\to 0}\left(\frac{3^x-1}{x}\right)-\lim_{x\to 0}\left(\frac{2^x-1}{x}\right)$$

$$= \log 5+\log 3-\log 2\quad \ldots\left[\lim_{x\to 0}\frac{a^x-1}{x}=\log a\right]$$

$$= \log\frac{5\times 3}{2}$$

$$= \log\frac{15}{2}$$

Question 3.

$\lim_{x\to 0}\left(\frac{a^x+b^x+c^x-3}{\sin x}\right)$

Solution:

$$\lim_{x\to 0} \frac{a^x + b^x + c^x - 3}{\sin x}$$

$$= \lim_{x\to 0} \frac{(a^x - 1) + (b^x - 1) + (c^x - 1)}{\sin x}$$

$$= \lim_{x\to 0} \frac{\dfrac{(a^x - 1) + (b^x - 1) + (c^x - 1)}{x}}{\dfrac{\sin x}{x}}$$

$$\ldots \left[\begin{array}{l}\text{Divide numerator and} \\ \text{denominator by } x. \\ \because x \to 0,\ x \neq 0\end{array}\right]$$

$$= \lim_{x\to 0} \frac{\left(\dfrac{a^x - 1}{x}\right) + \left(\dfrac{b^x - 1}{x}\right) + \left(\dfrac{c^x - 1}{x}\right)}{\dfrac{\sin x}{x}}$$

$$= \frac{\left(\lim\limits_{x\to 0} \dfrac{a^x - 1}{x}\right) + \left(\lim\limits_{x\to 0} \dfrac{b^x - 1}{x}\right) + \left(\lim\limits_{x\to 0} \dfrac{c^x - 1}{x}\right)}{\left(\lim\limits_{x\to 0} \dfrac{\sin x}{x}\right)}$$

$$= \frac{\log a + \log b + \log c}{1} \qquad \ldots\left[\because \lim_{x\to 0} \frac{a^x - 1}{x} = \log a\right]$$

$$= \log(abc)$$

Question 4.

$\lim_{x\to 0}\left(\frac{6^x+5^x+4^x-3^{x+1}}{\sin x}\right)$

Solution:

$$\lim_{x\to 0} \frac{6^x + 5^x + 4^x - 3^{x+1}}{\sin x}$$

$$= \lim_{x\to 0} \frac{6^x + 5^x + 4^x - 3^x \cdot 3^1}{\sin x}$$

$$= \lim_{x\to 0} \frac{6^x - 1 + 5^x - 1 + 4^x - 1 - 3^x(3) + 3}{\sin x}$$

$$= \lim_{x\to 0} \frac{(6^x - 1) + (5^x - 1) + (4^x - 1) - 3(3^x - 1)}{\sin x}$$

$$= \lim_{x\to 0} \frac{\dfrac{(6^x - 1) + (5^x - 1) + (4^x - 1) - 3(3^x - 1)}{x}}{\dfrac{\sin x}{x}}$$

$$\ldots \left[\begin{array}{l}\text{Divide Numerator and} \\ \text{Denominator by } x. \\ \because x \to 0,\ x \neq 0\end{array}\right]$$

$$= \frac{\lim\limits_{x\to 0}\left[\dfrac{6^x - 1}{x} + \dfrac{5^x - 1}{x} + \dfrac{4^x - 1}{x} - 3\left(\dfrac{3^x - 1}{x}\right)\right]}{\lim\limits_{x\to 0} \dfrac{\sin x}{x}}$$

$$= \frac{\lim\limits_{x\to 0}\left(\dfrac{6^x - 1}{x}\right) + \lim\limits_{x\to 0}\left(\dfrac{5^x - 1}{x}\right) + \lim\limits_{x\to 0}\left(\dfrac{4^x - 1}{x}\right) - 3\lim\limits_{x\to 0}\left(\dfrac{3^x - 1}{x}\right)}{\lim\limits_{x\to 0} \dfrac{\sin x}{x}}$$

$$= \frac{\log 6 + \log 5 + \log 4 - 3\log 3}{1} \quad \ldots \left[\lim_{x\to 0} \frac{a^x - 1}{x} = \log a\right]$$

$$= \log(6 \times 5 \times 4) - \log(3)^3$$

$$= \log \frac{120}{27}$$

$$= \log \frac{40}{9}$$

Question 5.

$lim_{x\to 0}\left(\frac{8^{\sin x}-2^{\tan x}}{e^{2x}-1}\right)$

Solution:

$$\lim_{x\to 0}\frac{8^{\sin x}-2^{\tan x}}{e^{2x}-1}$$

$$=\lim_{x\to 0}\frac{\left(8^{\sin x}-1\right)-\left(2^{\tan x}-1\right)}{e^{2x}-1}$$

$$=\lim_{x\to 0}\frac{\frac{\left(8^{\sin x}-1\right)-\left(2^{\tan x}-1\right)}{x}}{\frac{e^{2x}-1}{x}}$$

$$\ldots\left[\begin{array}{l}\text{Divide Numerator and}\\ \text{Denominator by } x.\\ \because x\to 0,\ x\neq 0\end{array}\right]$$

$$=\frac{\lim_{x\to 0}\left(\frac{8^{\sin x}-1}{x}-\frac{2^{\tan x}-1}{x}\right)}{\lim_{x\to 0}\frac{e^{2x}-1}{x}}$$

$$=\frac{\lim_{x\to 0}\left(\frac{8^{\sin x}-1}{\sin x}\cdot\frac{\sin x}{x}-\frac{2^{\tan x}-1}{\tan x}\cdot\frac{\tan x}{x}\right)}{\lim_{x\to 0}\frac{e^{2x}-1}{x}}$$

$$=\frac{\left(\lim_{x\to 0}\frac{8^{\sin x}-1}{\sin x}\right)\left(\lim_{x\to 0}\frac{\sin x}{x}\right)-\left(\lim_{x\to 0}\frac{2^{\tan x}-1}{\tan x}\right)\left(\lim_{x\to 0}\frac{\tan x}{x}\right)}{\left(\lim_{x\to 0}\frac{e^{2x}-1}{2x}\right)\times 2}=$$

$$\frac{(\log 8)(1)-(\log 2)(1)}{(1)\times 2}\ \ldots\left[\begin{array}{l}\because x\to 0,\ 2x\to 0,\\ \sin x\to 0,\ \tan x\to 0\\ \lim_{x\to 0}\frac{a^x-1}{x}=\log a\end{array}\right]$$

$$=\frac{\log\frac{8}{2}}{2}$$

$$=\frac{\log 4}{2}$$

$$=\frac{\log(2)^2}{2}$$

$$=\frac{2\log 2}{2}=\log 2$$

II. Evaluate the following limits:

Question 1.

$lim_{x\to 0}\left[\frac{3^x+3^{-x}-2}{x\cdot\tan x}\right]$

Solution:

$$\lim_{x\to 0}\frac{3^x+3^{-x}-2}{x\cdot\tan x}$$

$$= \lim_{x\to 0}\frac{3^x+\frac{1}{3^x}-2}{x\cdot\tan x}$$

$$= \lim_{x\to 0}\frac{3^x+\frac{1}{3^x}-2}{x\cdot\tan x}$$

$$= \lim_{x\to 0}\frac{\left(3^x\right)^2+1-2\left(3^x\right)}{3^x\cdot x\tan x}$$

$$= \lim_{x\to 0}\frac{\left(3^x-1\right)^2}{3^x\cdot x\cdot\tan x} \quad \ldots[\because a^2-2ab+b^2=(a-b)^2]$$

$$= \lim_{x\to 0}\frac{\frac{\left(3^x-1\right)^2}{x^2}}{\frac{3^x\cdot x\cdot\tan x}{x^2}} \quad \ldots\left[\begin{array}{l}\text{Divide numerator and}\\ \text{denominator by } x^2.\\ \because x\to 0,\ x\neq 0\\ \therefore x^2\neq 0\end{array}\right]$$

$$= \frac{\lim\limits_{x\to 0}\left(\frac{3^x-1}{x}\right)^2}{\lim\limits_{x\to 0}3^x\cdot\left(\frac{\tan x}{x}\right)}$$

$$= \frac{\left(\lim\limits_{x\to 0}\frac{3^x-1}{x}\right)^2}{\left(\lim\limits_{x\to 0}3^x\right)\left(\lim\limits_{x\to 0}\frac{\tan x}{x}\right)}$$

$$= \frac{(\log 3)^2}{3^0\times 1} \ldots\left[\because \lim_{x\to 0}\frac{a^x-1}{x}=\log a,\ \lim_{\theta\to 0}\frac{\tan\theta}{\theta}=1\right]$$

$$= (\log 3)^2$$

Question 2.

$\lim_{x\to 0}\left[\frac{3+x}{3-x}\right]^{\frac{1}{x}}$

Solution:

$$\lim_{x\to0}\left(\frac{3+x}{3-x}\right)^{\frac{1}{x}}$$

$$= \lim_{x\to0}\left(\frac{1+\frac{x}{3}}{1-\frac{x}{3}}\right)^{\frac{1}{x}} \qquad \dots \left[\begin{array}{l}\text{Divide numerator and}\\ \text{denominator by } 3\end{array}\right]$$

$$= \lim_{x\to0}\frac{\left(1+\frac{x}{3}\right)^{\frac{1}{x}}}{\left(1-\frac{x}{3}\right)^{\frac{1}{x}}}$$

$$= \lim_{x\to0}\frac{\left(1+\frac{x}{3}\right)^{\frac{3}{x}\times\frac{1}{3}}}{\left(1-\frac{x}{3}\right)^{\frac{-3}{x}\times\frac{1}{3}}}$$

$$= \frac{\lim_{x\to0}\left[\left(1+\frac{x}{3}\right)^{\frac{3}{x}}\right]^{\frac{1}{3}}}{\lim_{x\to0}\left[\left(1-\frac{x}{3}\right)^{\frac{-3}{x}}\right]^{-\frac{1}{3}}}$$

$$= \frac{e^{1/3}}{e^{-1/3}} \qquad \dots \left[\begin{array}{l}\because x\to0,\ \frac{x}{3}\to0, \frac{-x}{3}\to0 \text{ and}\\ \lim_{x\to0}(1+x)^{\frac{1}{x}} = e\end{array}\right]$$

$$= e^{\frac{1}{3}+\frac{1}{3}}$$

$$= e^{\frac{2}{3}}$$

Question 3.
$lim_{x\to0}[5x+33-2x]_{2x}$
Solution:

$$\lim_{x\to0}\left(\frac{5x+3}{3-2x}\right)^{\frac{2}{x}}$$

$$= \lim_{x\to0}\left(\frac{3+5x}{3-2x}\right)^{\frac{2}{x}}$$

$$= \lim_{x\to0}\left(\frac{1+\frac{5}{3}x}{1-\frac{2}{3}x}\right)^{\frac{2}{x}} \qquad \dots \left[\begin{array}{l}\text{divide numerator and}\\ \text{denominator by } 3\end{array}\right]$$

$$= \lim_{x\to0}\frac{\left(1+\frac{5x}{3}\right)^{\frac{2}{x}}}{\left(1-\frac{2x}{3}\right)^{\frac{2}{x}}} = \frac{\lim_{x\to0}\left[\left(1+\frac{5x}{3}\right)^{\frac{3}{5x}}\right]^{\frac{5}{3}\times2}}{\lim_{x\to0}\left[\left(1-\frac{2x}{3}\right)^{\frac{-3}{2x}}\right]^{\frac{-2}{3}\times2}}$$

$$= \frac{e^{\frac{10}{3}}}{e^{\frac{-4}{3}}} \qquad \dots \left[\begin{array}{l}\because x\to0,\ \frac{5x}{3}\to0, \frac{-2x}{3}\to0 \text{ and}\\ \lim_{x\to0}(1+x)^{\frac{1}{x}} = e\end{array}\right]$$

$$= e^{\frac{14}{3}}$$

Question 4.
$lim_{x\to0}[log(3-x)-log(3+x)x]$

Solution:

$$\lim_{x\to 0}\frac{\log(3-x)-\log(3+x)}{x}$$

$$=\lim_{x\to 0}\frac{1}{x}\ \log\left(\frac{3-x}{3+x}\right)$$

$$=\lim_{x\to 0}\ \log\left(\frac{3-x}{3+x}\right)^{\frac{1}{x}}$$

$$=\lim_{x\to 0}\log\left(\frac{1-\frac{x}{3}}{1+\frac{x}{3}}\right)^{\frac{1}{x}}=\log\left[\lim_{x\to 0}\frac{\left(1-\frac{x}{3}\right)^{\frac{1}{x}}}{\left(1+\frac{x}{3}\right)^{\frac{1}{x}}}\right]$$

$$=\log\left[\frac{\left\{\lim_{x\to 0}\left(1-\frac{x}{3}\right)^{\frac{-3}{x}}\right\}^{\frac{-1}{3}}}{\left\{\lim_{x\to 0}\left(1+\frac{x}{3}\right)^{\frac{3}{x}}\right\}^{\frac{1}{3}}}\right]$$

$$=\log\left(\frac{e^{-\frac{1}{3}}}{e^{\frac{1}{3}}}\right)$$

$$\ldots\left[\because\ x\to 0,\pm\frac{x}{3}\to 0\text{ and}\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e\right]$$

$$=\log e^{-2/3}$$

$$=-\frac{2}{3}.\log e=-\frac{2}{3}(1)=-\frac{2}{3}$$

Question 5.

$\lim_{x\to 0}\left[\frac{4x+1}{1-4x}\right]^{\frac{1}{x}}$

Solution:

$$\lim_{x\to 0}\left(\frac{4x+1}{1-4x}\right)^{\frac{1}{x}}$$

$$=\lim_{x\to 0}\frac{(1+4x)^{\frac{1}{x}}}{(1-4x)^{\frac{1}{x}}}=\frac{\lim_{x\to 0}\left[(1+4x)^{\frac{1}{4x}}\right]^4}{\lim_{x\to 0}\left[(1-4x)^{\frac{-1}{4x}}\right]^{-4}}$$

$$=\frac{e^4}{e^{-4}}\qquad\ldots\left[\begin{array}{l}\because x\to 0,\ 4x\to 0,-4x\to 0\\ \text{and}\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e\end{array}\right]$$

$$=e^8$$

Question 6.

$\lim_{x\to 0}\left[\frac{5+7x}{5-3x}\right]^{\frac{1}{3x}}$

Solution:

$$\lim_{x\to 0}\left(\frac{5+7x}{5-3x}\right)^{\frac{1}{3x}}$$

$$= \lim_{x\to 0}\left(\frac{1+\frac{7}{5}x}{1-\frac{3}{5}x}\right)^{\frac{1}{3x}} \quad \ldots\left[\begin{array}{l}\text{Divide numerator and}\\ \text{denominator by } 5\end{array}\right]$$

$$= \lim_{x\to 0}\frac{\left(1+\frac{7x}{5}\right)^{\frac{1}{3x}}}{\left(1-\frac{3x}{5}\right)^{\frac{1}{3x}}} = \frac{\lim_{x\to 0}\left[\left(1+\frac{7x}{5}\right)^{\frac{5}{7x}}\right]^{\frac{7}{5}\times\frac{1}{3}}}{\lim_{x\to 0}\left[\left(1-\frac{3x}{5}\right)^{\frac{-5}{3x}}\right]^{\frac{-3}{5}\times\frac{1}{3}}}$$

$$= \frac{e^{\frac{7}{15}}}{e^{\frac{-3}{15}}} \quad \ldots\left[\begin{array}{l}\because x\to 0,\ \frac{7x}{5}\to 0, \frac{-3x}{5}\to 0\\ \text{and } \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e\end{array}\right]$$

$$= e^{\frac{7}{15}+\frac{3}{15}} = e^{\frac{10}{15}}$$

$$= e^{\frac{2}{3}}$$

III. Evaluate the following limits:

Question 1.

$\lim_{x\to 0}\left[\frac{a^x-b^x}{\sin(4x)-\sin(2x)}\right]$

Solution:

$$\lim_{x\to 0}\frac{a^x-b^x}{\sin 4x-\sin 2x}$$

$$= \lim_{x\to 0}\frac{\left(a^x-1\right)-\left(b^x-1\right)}{2\cos\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right)}$$

$$= \lim_{x\to 0}\frac{\left(a^x-1\right)-\left(b^x-1\right)}{2\cos 3x\cdot\sin x}$$

$$= \lim_{x\to 0}\frac{\frac{\left(a^x-1\right)-\left(b^x-1\right)}{x}}{\frac{2\cos 3x\cdot\sin x}{x}} \quad \ldots\left[\begin{array}{l}\text{Divide numerator and}\\ \text{denominator by } x.\\ \because x\to 0,\ x\neq 0\end{array}\right]$$

$$= \frac{\lim_{x\to 0}\left(\frac{a^x-1}{x}-\frac{b^x-1}{x}\right)}{\lim_{x\to 0} 2\cos 3x\cdot\left(\frac{\sin x}{x}\right)}$$

$$= \frac{\left(\lim_{x\to 0}\frac{a^x-1}{x}\right)-\left(\lim_{x\to 0}\frac{b^x-1}{x}\right)}{2\lim_{x\to 0}(\cos 3x)\cdot\lim_{x\to 0}\left(\frac{\sin x}{x}\right)}$$

$$= \frac{\log a-\log b}{2(1)(1)}$$

$$\ldots\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1,\ \lim_{x\to 0}\frac{a^x-1}{x} = \log a\right]$$

$$= \frac{1}{2}\log\left(\frac{a}{b}\right)$$

Question 2.

$\lim_{x\to 0}\left[\frac{(2^x-1)^3}{(3^x-1)\cdot\sin x\cdot\log(1+x)}\right]$

Solution:

$$\lim_{x\to 0}\frac{\left(2^x-1\right)^3}{\left(3^x-1\right)\cdot\sin x\cdot\log(1+x)}$$

$$=\lim_{x\to 0}\frac{\frac{\left(2^x-1\right)^3}{x^3}}{\frac{\left(3^x-1\right)\cdot\sin x\cdot\log(1+x)}{x^3}}$$

$$\ldots\left[\begin{array}{l}\text{Divide numerator and}\\ \text{denominator by } x^3.\\ \because x\to 0,\ x\neq 0\\ \therefore x^3\neq 0\end{array}\right]$$

$$=\frac{\lim_{x\to 0}\left(\frac{2^x-1}{x}\right)^3}{\lim_{x\to 0}\left(\frac{3^x-1}{x}\right)\cdot\frac{\sin x}{x}\cdot\frac{\log(1+x)}{x}}$$

$$=\frac{\left(\lim_{x\to 0}\frac{2^x-1}{x}\right)^3}{\left(\lim_{x\to 0}\frac{3^x-1}{x}\right)\cdot\left(\lim_{x\to 0}\frac{\sin x}{x}\right)\left(\lim_{x\to 0}\frac{\log(1+x)}{x}\right)}$$

$$=\frac{(\log 2)^3}{(\log 3)(1)(1)}$$

$$\ldots\left[\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1,\ \lim_{x\to 0}\frac{a^x-1}{x}=\log a,\ \lim_{x\to 0}\frac{\log(1+x)}{x}=1\right]$$

$$=\frac{(\log 2)^3}{\log 3}$$

Question 3.

$\lim_{x\to 0}\left[\frac{15^x-5^x-3^x+1}{x\cdot\sin x}\right]$

Solution:

$$\lim_{x\to 0}\frac{15^x-5^x-3^x+1}{x\sin x}=\lim_{x\to 0}\frac{5^x.3^x-5^x-3^x+1}{x\sin x}$$

$$=\lim_{x\to 0}\frac{5^x(3^x-1)-1(3^x-1)}{x\sin x}$$

$$=\lim_{x\to 0}\frac{(3^x-1)(5^x-1)}{x\sin x}$$

$$=\lim_{x\to 0}\frac{\frac{(3^x-1)(5^x-1)}{x^2}}{\frac{(x\sin x)}{x^2}}\qquad\ldots[\because x\to 0,\ x\neq 0]$$

$$=\lim_{x\to 0}\frac{\left(\frac{3^x-1}{x}\right)\left(\frac{5^x-1}{x}\right)}{\left(\frac{\sin x}{x}\right)}$$

$$=\frac{\lim_{x\to 0}\left(\frac{3^x-1}{x}\right).\lim_{x\to 0}\left(\frac{5^x-1}{x}\right)}{\left(\lim_{x\to 0}\frac{\sin x}{x}\right)}$$

$$=\frac{(\log 3).(\log 5)}{1}$$

$$\ldots\left[\because\lim_{x\to 0}\frac{a^x-1}{x}=\log a,\ \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$=(\log 3).(\log 5)$$

Question 4.

$\lim_{x\to 0}\left[\frac{(25)^x-2(5)^x+1}{x\cdot\sin x}\right]$

Solution:

$$\lim_{x\to 0}\frac{(25)^x-2(5)^x+1}{x.\sin x}$$

$$=\lim_{x\to 0}\frac{(5^x)^2-2(5^x)+1}{x\cdot\sin x}\quad\ldots\left[(25)^x=(5^2)^x=(5^x)^2\right]$$

$$=\lim_{x\to 0}\frac{(5^x-1)^2}{x\cdot\sin x}$$

$$=\lim_{x\to 0}\frac{\frac{(5^x-1)^2}{x^2}}{\frac{x\cdot\sin x}{x^2}}\quad\ldots\left[\begin{array}{l}\text{Divide numerator and}\\ \text{denominator by } x^2.\\ \because x\to 0,\ x\neq 0\\ \therefore x^2\neq 0\end{array}\right]$$

$$=\frac{\lim_{x\to 0}\left(\frac{5^x-1}{x}\right)^2}{\lim_{x\to 0}\frac{\sin x}{x}}$$

$$=\frac{(\log 5)^2}{1}\quad\ldots\left[\lim_{x\to 0}\frac{a^x-1}{x}=\log a,\ \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$=(\log 5)^2$$

Question 5.

$\lim_{x\to 0}\left[\frac{(49)^x-2(35)^x+(25)^x}{\sin x\cdot\log(1+2x)}\right]$

Solution:

$$\lim_{x\to 0}\frac{(49)^x-2(35)^x+(25)^x}{\sin x\cdot\log(1+2x)}$$

$$=\lim_{x\to 0}\frac{(7^x)^2-2(7^x)(5^x)+(5^x)^2}{\sin x.\log(1+2x)}$$

$$=\lim_{x\to 0}\frac{(7^x-5^x)^2}{\sin x\cdot\log(1+2x)}$$

$$=\lim_{x\to 0}\frac{\frac{\left[(7^x-1)-(5^x-1)\right]^2}{x^2}}{\frac{\sin x\cdot\log(1+2x)}{x^2}}\quad\ldots\left[\begin{array}{l}\text{Divide Numerator and}\\ \text{Denominator by } x^2.\\ \because x\to 0,\ x\neq 0\\ \therefore x^2\neq 0\end{array}\right]$$

$$=\frac{\lim_{x\to 0}\left[\frac{(7^x-1)-(5^x-1)}{x}\right]^2}{\lim_{x\to 0}\frac{\sin x\ \ \log(1+2x)}{}}$$

$$=\frac{\lim_{x\to 0}\left[\frac{7^x-1}{x}-\frac{5^x-1}{x}\right]^2}{\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\log(1+2x)}{x}}$$

$$=\frac{\left[\lim_{x\to 0}\frac{7^x-1}{x}-\lim_{x\to 0}\frac{5^x-1}{x}\right]^2}{\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\log(1+2x)}{2x}\times 2}$$

$$=\frac{(\log 7-\log 5)^2}{1\times 1\times 2}$$

$$\ldots\left[\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1,\ \lim_{x\to 0}\frac{a^x-1}{x}=\log a,\ \lim_{x\to 0}\frac{\log(1+x)}{x}=1\right]$$

$$=\frac{1}{2}\left(\log\frac{7}{5}\right)^2$$

Maharashtra State Board 11th Maths Solutions Chapter 7 Limits Ex 7.7

I. Evaluate the following:

Question 1.
$\lim_{x\to\infty}\left[\frac{ax^3+bx^2+cx+d}{ex^3+fx^2+gx+h}\right]$
Solution:

$$\lim_{x\to\infty}\left[\frac{ax^3+bx^2+cx+d}{ex^3+fx^2+gx+h}\right]$$

$$=\lim_{x\to\infty}\left[\frac{\frac{ax^3+bx^2+cx+d}{x^3}}{\frac{ex^3+fx^2+gx+h}{x^3}}\right]$$

$$\ldots\left[\begin{array}{l}\text{Divide numerator and}\\ \text{denominator by } x^3\end{array}\right]$$

$$=\frac{\lim_{x\to\infty}\left(a+\frac{b}{x}+\frac{c}{x^2}+\frac{d}{x^3}\right)}{\lim_{x\to\infty}\left(e+\frac{f}{x}+\frac{g}{x^2}+\frac{h}{x^3}\right)}$$

$$=\frac{\lim_{x\to\infty}a+\lim_{x\to\infty}\frac{b}{x}+\lim_{x\to\infty}\frac{c}{x^2}+\lim_{x\to\infty}\frac{d}{x^3}}{\lim_{x\to\infty}e+\lim_{x\to\infty}\frac{f}{x}+\lim_{x\to\infty}\frac{g}{x^2}+\lim_{x\to\infty}\frac{h}{x^3}}$$

$$=\frac{a+0+0+0}{e+0+0+0}\ldots\left[\lim_{x\to\infty}\frac{1}{x^k}=0,k>0\right]$$

$$=\frac{a}{e}$$

Question 2.
$\lim_{x\to\infty}\left[\frac{x^3+3x+2}{(x+4)(x-6)(x-3)}\right]$
Solution:

$$\lim_{x\to\infty}\left[\frac{x^3+3x+2}{(x+4)(x-6)(x-3)}\right]$$

$$=\lim_{x\to\infty}\frac{\frac{x^3+3x+2}{x^3}}{\frac{(x+4)(x-6)(x-3)}{x^3}}$$

$$\ldots\left[\begin{array}{l}\text{Divide Numerator and}\\ \text{Denominator by } x^3\end{array}\right]$$

$$=\frac{\lim_{x\to\infty}\left(1+\frac{3}{x^2}+\frac{2}{x^3}\right)}{\lim_{x\to\infty}\left(\frac{x+4}{x}\right)\left(\frac{x-6}{x}\right)\left(\frac{x-3}{x}\right)}$$

$$=\frac{\lim_{x\to\infty}1+\lim_{x\to\infty}\frac{3}{x^2}+\lim_{x\to\infty}\frac{2}{x^3}}{\lim_{x\to\infty}\left(1+\frac{4}{x}\right)\cdot\lim_{x\to\infty}\left(1-\frac{6}{x}\right)\cdot\lim_{x\to\infty}\left(1-\frac{3}{x}\right)}$$

$$=\frac{1+0+0}{(1+0)\,(1-0)\,(1-0)}\quad\ldots\left[\lim_{x\to\infty}\frac{1}{x^k}=0,k>0\right]$$

$$=1$$

Question 3.
$\lim_{x\to\infty}\left[\frac{7x^2+5x-3}{8x^2-2x+7}\right]$

Solution:

$$\lim_{x\to\infty}\frac{7x^2+5x-3}{8x^2-2x+7}$$

$$=\lim_{x\to\infty}\frac{\left(\frac{7x^2+5x-3}{x^2}\right)}{\left(\frac{8x^2-2x+7}{x^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(7+\frac{5}{x}-\frac{3}{x^2}\right)}{\left(8-\frac{2}{x}+\frac{7}{x^2}\right)}$$

$$=\frac{\lim\limits_{x\to\infty}7+\lim\limits_{x\to\infty}\frac{5}{x^2}-\lim\limits_{x\to\infty}\frac{3}{x^2}}{\lim\limits_{x\to\infty}8-\lim\limits_{x\to\infty}\frac{2}{x}+\lim\limits_{x\to\infty}\frac{7}{x^2}}$$

$$=\frac{7+0-0}{8-0+0} \qquad \ldots\left[\because \lim_{x\to\infty}\frac{1}{x^k}=0, k>0\right]$$

$$=\frac{7}{8}$$

II. Evaluate the following:

Question 1.
$\lim_{x\to\infty}\left[\frac{7x^2+2x-3}{\sqrt{x^4+x+2}}\right]$
Solution:

$$\lim_{x\to\infty}\left[\frac{7x^2+2x-3}{\sqrt{x^4+x+2}}\right]$$

$$=\lim_{x\to\infty}\frac{\frac{7x^2+2x-3}{x^2}}{\frac{\sqrt{x^4+x+2}}{x^2}} \qquad \ldots\left[\begin{array}{l}\text{Divide numerator and}\\ \text{denominator by } x^2\end{array}\right]$$

$$=\frac{\lim\limits_{x\to\infty}\left(7+\frac{2}{x}-\frac{3}{x^2}\right)}{\lim\limits_{x\to\infty}\sqrt{\frac{x^4+x+2}{x^4}}}$$

$$=\frac{\lim\limits_{x\to\infty}7+\lim\limits_{x\to\infty}\frac{2}{x}-\lim\limits_{x\to\infty}\frac{3}{x^2}}{\lim\limits_{x\to\infty}\sqrt{1+\frac{1}{x^3}+\frac{2}{x^4}}}$$

$$=\frac{7+0-0}{\sqrt{1+0+0}} \qquad \ldots\left[\lim_{x\to\infty}\frac{1}{x^k}=0, k>0\right]$$

$$=7$$

Question 2.
$\lim_{x\to\infty}\left[\sqrt{x^2+4x+16}-\sqrt{x^2+16}\right]$
Solution:

$$\lim_{x\to\infty} \sqrt{x^2+4x+16} - \sqrt{x^2+16}$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2+4x+16}-\sqrt{x^2+16}\right)\left(\sqrt{x^2+4x+16}+\sqrt{x^2+16}\right)}{\left(\sqrt{x^2+4x+16}+\sqrt{x^2+16}\right)}$$

[By rationalization]

$$= \lim_{x\to\infty} \frac{(x^2+4x+16-x^2-16)}{\sqrt{x^2+4x+16}+\sqrt{x^2+16}}$$

$$= \lim_{x\to\infty} \frac{4x}{\sqrt{x^2+4x+16}+\sqrt{x^2+16}}$$

$$= \lim_{x\to\infty} \frac{\frac{4x}{x}}{\frac{\sqrt{x^2+4x+16}+\sqrt{x^2+16}}{x}}$$

$$= \lim_{x\to\infty} \frac{\frac{4x}{x}}{\sqrt{\frac{x^2+4x+16}{x^2}}+\sqrt{\frac{x^2+16}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{\frac{4x}{x}}{\sqrt{\frac{x^2+4x+16}{x^2}}+\sqrt{\frac{x^2+16}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{4}{\sqrt{1+\frac{4}{x}+\frac{16}{x^2}}+\sqrt{1+\frac{16}{x^2}}}$$

$$= \frac{4}{\lim_{x\to\infty}\sqrt{1+\frac{4}{x}+\frac{16}{x^2}}+\lim_{x\to\infty}\sqrt{1+\frac{16}{x^2}}}$$

$$= \frac{4}{\sqrt{1+0+0}+\sqrt{1+0}} \quad \ldots\left[\because \lim_{x\to\infty}\frac{1}{x^k}=0, k>0\right]$$

$$= \frac{4}{2}$$

$$= 2$$

Question 3.

$lim_{x\to\infty}[x4+4x2−−−−−−−√−x2]$

Solution:

$$\lim_{x\to\infty}\left[\sqrt{x^4+4x^2}-x^2\right]$$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{x^4+4x^2}-x^2\right)\left(\sqrt{x^4+4x^2}+x^2\right)}{\sqrt{x^4+4x^2}+x^2}$$

...[By rationalization]

$$=\lim_{x\to\infty}\frac{x^4+4x^2-x^4}{\sqrt{x^4+4x^2}+x^2}$$

$$=\lim_{x\to\infty}\frac{4x^2}{\sqrt{x^4+4x^2}+x^2}$$

$$=\lim_{x\to\infty}\frac{\frac{4x^2}{x^2}}{\frac{\sqrt{x^4+4x^2}+x^2}{x^2}}\quad\ldots\left[\begin{array}{l}\text{Divide numerator and}\\\text{denominator by } x^2\end{array}\right]$$

$$=\lim_{x\to\infty}\frac{\frac{4x^2}{x^2}}{\sqrt{\frac{x^4+4x^2}{x^4}}+1}$$

$$=\frac{\lim_{x\to\infty}4}{\lim_{x\to\infty}\left(\sqrt{1+\frac{4}{x^2}}+1\right)}$$

$$=\frac{4}{\sqrt{1+0}+1}\quad\ldots\left[\lim_{x\to\infty}\frac{1}{x^k}=0, k>0\right]$$

$$=\frac{4}{2}=2$$

III. Evaluate the following:

Question 1.

$\lim_{x\to\infty}\left[\frac{(3x^2+4)(4x^2-6)(5x^2+2)}{4x^6+2x^4-1}\right]$

Solution:

$$\lim_{x\to\infty}\frac{(3x^2+4)(4x^2-6)(5x^2+2)}{4x^6+2x^4-1}$$

$$=\lim_{x\to\infty}\frac{\frac{(3x^2+4)(4x^2-6)(5x^2+2)}{x^6}}{\frac{4x^6+2x^4-1}{x^6}}$$

$$\ldots\left[\begin{array}{l}\text{Divide numerator and}\\\text{denominator by } x^6\end{array}\right]$$

$$=\lim_{x\to\infty}\frac{\left(\frac{3x^2+4}{x^2}\right)\left(\frac{4x^2-6}{x^2}\right)\left(\frac{5x^2+2}{x^2}\right)}{4+\frac{2}{x^2}-\frac{1}{x^6}}$$

$$=\frac{\lim_{x\to\infty}\left(3+\frac{4}{x^2}\right)\left(4-\frac{6}{x^2}\right)\left(5+\frac{2}{x^2}\right)}{\lim_{x\to\infty}\left(4+\frac{2}{x^2}-\frac{1}{x^6}\right)}$$

$$=\frac{\lim_{x\to\infty}\left(3+\frac{4}{x^2}\right)\cdot\lim_{x\to\infty}\left(4-\frac{6}{x^2}\right)\cdot\lim_{x\to\infty}\left(5+\frac{2}{x^2}\right)}{\lim_{x\to\infty}4+\lim_{x\to\infty}\frac{2}{x^2}-\lim_{x\to\infty}\frac{1}{x^6}}$$

$$=\frac{(3+0)(4-0)(5+0)}{4+0-0}\quad\ldots..\left[\lim_{x\to\infty}\frac{1}{x^k}=0, k>0\right]$$

$$=\frac{3\times4\times5}{4}$$

$$=15$$

Question 2.

$\lim_{x\to\infty}\left[\frac{(3x-4)^3(4x+3)^4}{(3x+2)^7}\right]$

Solution:

$$\lim_{x\to\infty}\left[\frac{(3x-4)^3(4x+3)^4}{(3x+2)^7}\right]$$

$$= \lim_{x\to\infty}\frac{\frac{(3x-4)^3}{x^3}\frac{(4x+3)^4}{x^4}}{\frac{(3x+2)^7}{x^7}} \quad \ldots\left[\begin{array}{l}\text{Divide numerator and}\\ \text{denominator by } x^7\end{array}\right]$$

$$= \frac{\lim_{x\to\infty}\left(\frac{3x-4}{x}\right)^3\left(\frac{4x+3}{x}\right)^4}{\lim_{x\to\infty}\left(\frac{3x+2}{x}\right)^7}$$

$$= \frac{\lim_{x\to\infty}\left(3-\frac{4}{x}\right)^3 \times \lim_{x\to\infty}\left(4+\frac{3}{x}\right)^4}{\lim_{x\to\infty}\left(3+\frac{2}{x}\right)^7}$$

$$= \frac{(3-0)^3\times(4+0)^4}{(3+0)^7} \quad \ldots\left[\lim_{x\to\infty}\frac{1}{x^k}=0, k>0\right]$$

$$= \frac{(3)^3\times(4)^4}{(3)^7}$$

$$= \frac{(4)^4}{(3)^4}$$

$$= \frac{256}{81}$$

Question 3.

$\lim_{x\to\infty}\left[\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)\right]$

Solution:

$$\lim_{x\to 0}\left[\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)\right]$$

$$= \lim_{x\to 0}\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)\times\frac{\left(\sqrt{x+1}+\sqrt{x}\right)}{\sqrt{x+1}+\sqrt{x}}$$

$\ldots$[By rationalization]

$$= \lim_{x\to 0}\frac{\sqrt{x}\,(x+1-x)}{\sqrt{x+1}+\sqrt{x}}$$

$$= \lim_{x\to 0}\frac{\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}$$

$$= \lim_{x\to\infty}\frac{1}{\frac{\sqrt{x+1}}{\sqrt{x}}+1} \quad \ldots\left[\begin{array}{l}\text{Divide numerator and}\\ \text{denominator by } \sqrt{x}\end{array}\right]$$

$$= \lim_{x\to\infty}\frac{1}{\sqrt{\frac{x+1}{x}}+1}$$

$$= \frac{1}{\lim_{x\to\infty}\left(\sqrt{1+\frac{1}{x}}+1\right)}$$

$$= \frac{1}{\sqrt{1+0}+1} \quad \ldots\left[\lim_{x\to\infty}\frac{1}{x^k}=0, k>0\right]$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2}$$

Question 4.

$\lim_{x\to\infty}\left[\frac{(2x-1)^{20}(3x-1)^{30}}{(2x+1)^{50}}\right]$

Solution:

$$\lim_{x\to\infty} \frac{(2x-1)^{20}(3x-1)^{30}}{(2x+1)^{50}}$$

$$= \lim_{x\to\infty} \frac{\frac{(2x-1)^{20}}{x^{20}} \cdot \frac{(3x-1)^{30}}{x^{30}}}{\frac{(2x+1)^{50}}{x^{50}}}$$

$$\ldots \left[\begin{array}{l}\text{Divide numerator and} \\ \text{denominator by } x^{50}\end{array}\right]$$

$$= \frac{\lim_{x\to\infty}\left(\frac{2x-1}{x}\right)^{20} \cdot \left(\frac{3x-1}{x}\right)^{30}}{\lim_{x\to\infty}\left(\frac{2x+1}{x}\right)^{50}}$$

$$= \frac{\lim_{x\to\infty}\left(2-\frac{1}{x}\right)^{20} \cdot \lim_{x\to\infty}\left(3-\frac{1}{x}\right)^{30}}{\lim_{x\to\infty}\left(2+\frac{1}{x}\right)^{50}}$$

$$= \frac{(2-0)^{20} \times (3-0)^{30}}{(2+0)^{50}} \quad \ldots\left[\lim_{x\to\infty}\frac{1}{x^k}=0, k>0\right]$$

$$= \frac{(2)^{20} \times (3)^{30}}{(2)^{50}}$$

$$= \frac{(3)^{30}}{(2)^{30}}$$

$$= \left(\frac{3}{2}\right)^{30}$$

Question 5.

$lim_{x\to\infty}[x_2+5√-x_2-3√x_2+3√-x_2+1√]$

Solution:

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$$

$$= \lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+3}+\sqrt{x^2+1}} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\right]$$

$$\ldots \left[\begin{array}{l}\text{By taking conjugates of both, the} \\ \text{numerator as well as the denominator.}\end{array}\right]$$

$$= \lim_{x\to\infty}\left[\frac{(x^2+5)-(x^2-3)}{(x^2+3)-(x^2+1)} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\right]$$

$$= \lim_{x\to\infty} \frac{8\left(\sqrt{x^2+3}+\sqrt{x^2+1}\right)}{2\left(\sqrt{x^2+5}+\sqrt{x^2-3}\right)}$$

$$= 4\lim_{x\to\infty}\left(\frac{\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{x}}{\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{x}}\right)$$

... [Divide numerator and denominator by x]

$$= 4\lim_{x\to\infty}\left(\frac{\frac{\sqrt{x^2+3}}{x}+\frac{\sqrt{x^2+1}}{x}}{\frac{\sqrt{x^2+5}}{x}+\frac{\sqrt{x^2-3}}{x}}\right)$$

$$= \frac{4\lim\limits_{x\to\infty}\left(\sqrt{\frac{x^2+3}{x^2}}+\sqrt{\frac{x^2+1}{x^2}}\right)}{\lim\limits_{x\to\infty}\left(\sqrt{\frac{x^2+5}{x^2}}+\sqrt{\frac{x^2-3}{x^2}}\right)}$$

$$= \frac{4\left(\lim\limits_{x\to\infty}\sqrt{1+\frac{3}{x^2}}+\lim\limits_{x\to\infty}\sqrt{1+\frac{1}{x^2}}\right)}{\lim\limits_{x\to\infty}\sqrt{1+\frac{5}{x^2}}+\lim\limits_{x\to\infty}\sqrt{1-\frac{3}{x^2}}}$$

$$= \frac{4\left(\sqrt{1+0}+\sqrt{1+0}\right)}{\sqrt{1+0}+\sqrt{1-0}} \quad \ldots\left[\lim_{x\to\infty}\frac{1}{x^k}=0, k>0\right]$$

$$= \frac{4(1+1)}{1+1}$$

$$= 4$$

Maharashtra State Board 11th Maths Solutions Chapter 7 Limits Miscellaneous Exercise 7

I. Select the correct answer from the given alternatives.

Question 1.
$\lim_{x\to 2}\left(\frac{x^4-16}{x^2-5x+6}\right)=$
(A) 23
(B) 32
(C) -32
(D) -16
Answer:
(C) -32
Hint:

$$\lim_{x\to 2}\frac{x^4-16}{x^2-5x+6}$$

$$= \lim_{x\to 2}\frac{\left(x^2+4\right)(x+2)}{x-3}$$

$$= \frac{\left[(2)^2+4\right](2+2)}{2-3} = -32$$

Question 2.

limx→−2(x7+128x3+8)=

(A) 563

(B) 1123

(C) 1213

(D) 283

Answer:

(B) 1123

Hint:

$$\lim_{x\to-2}\frac{x^7+128}{x^3+8}$$

$$=\frac{\lim_{x\to-2}\frac{x^7-(-2)^7}{x-(-2)}}{\lim_{x\to-2}\frac{x^3-(-2)^3}{x-(-2)}}$$

$$=\frac{7(-2)^6}{3(-2)^2}=\frac{112}{3} \qquad \ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

Question 3.

limx→3(1x2−11x+24+1x2−x−6)=

(A) −225

(B) 225

(C) 725

(D) −725

Answer:

(A) −225

Hint:

$$\lim_{x\to3}\left(\frac{1}{x^2-11x+24}+\frac{1}{x^2-x-6}\right)$$

$$=\lim_{x\to3}\left[\frac{1}{(x-8)(x-3)}+\frac{1}{(x-3)(x+2)}\right]$$

$$=\lim_{x\to3}\frac{2x-6}{(x-8)(x-3)(x+2)}$$

$$=\lim_{x\to3}\frac{2}{(x-8)(x+2)}=\frac{-2}{25}$$

Question 4.

limx→5(x+4√−33x−11−2√)=

(A) -29

(B) 27

(C) 59

(D) 29

Answer:

(D) 29

Hint:

$$\lim_{x\to 5}\left(\frac{\sqrt{x+4}-3}{\sqrt{3x-11}-2}\right)$$

$$= \lim_{x\to 5}\left[\frac{\sqrt{x+4}-3}{\sqrt{3x-11}-2}\times\frac{\sqrt{x+4}+3}{\sqrt{3x-11}+2}\times\frac{\sqrt{3x-11}+2}{\sqrt{x+4}+3}\right]$$

$$= \lim_{x\to 5}\frac{(x-5)\left(\sqrt{3x-11}+2\right)}{(3x-15)\left(\sqrt{x+4}+3\right)}$$

$$= \lim_{x\to 5}\frac{\sqrt{3x-11}+2}{3\left(\sqrt{x+4}+3\right)}$$

$$= \frac{\sqrt{4}+2}{3\left(\sqrt{9}+3\right)} = \frac{2}{9}$$

Question 5.

$lim_{x\to\pi 3}(tan2x-3sec3x-8)=$

(A) 1
(B) 12
(C) 13
(D) 14

Answer:
(C) 13

Hint:

$$\lim_{x\to\frac{\pi}{3}}\left(\frac{\tan^2 x-3}{\sec^3 x-8}\right)$$

$$= \lim_{x\to\frac{\pi}{3}}\frac{\sec^2 x-1-3}{\sec^3 x-8}$$

$$= \lim_{x\to\frac{\pi}{3}}\frac{(\sec x-2)(\sec x+2)}{(\sec x-2)(\sec^2 x+2\sec x+4)}$$

$$= \lim_{x\to\frac{\pi}{3}}\frac{\sec x+2}{\sec^2 x+2\sec x+4}$$

$$= \frac{\sec\frac{\pi}{3}+2}{\left(\sec\frac{\pi}{3}\right)^2+2\sec\frac{\pi}{3}+4}$$

$$= \frac{2+2}{(2)^2+2(2)+4} = \frac{1}{3}$$

Question 6.

$lim_{x\to 0}(5$

(A) 0
(B) 1
(C) 2
(D) 3

Answer:
(C) 2

Hint:

$$\lim_{x\to 0}\left(\frac{5\sin x - x\cos x}{2\tan x-3x^2}\right)$$

$$= \frac{\lim_{x\to 0}\left(\frac{5\sin x}{x}-\cos x\right)}{\lim_{x\to 0}\left(\frac{2\tan x}{x}-3x\right)}$$

$$= \frac{5(1)-\cos 0}{2(1)-3(0)}$$

$$= 2$$

Question 7.

$lim_{x\to\pi 2}[3cosx+cos3x(2x-\pi)3]=$

(A) 32

(B) 12

(C) −12

(D) 14

Answer:

(C) −12

Hint:

$$\lim_{x\to\frac{\pi}{2}} \frac{3\cos x+\cos 3x}{(2x-\pi)^3}$$

$$=\lim_{x\to\frac{\pi}{2}} \frac{3\cos x+4\cos^3 x-3\cos x}{(2x-\pi)^3}$$

$$=\lim_{x\to\frac{\pi}{2}} \frac{4\cos^3 x}{8\left(x-\frac{\pi}{2}\right)^3}$$

Put $x=\frac{\pi}{2}+\mathrm{h}$,

$$x-\frac{\pi}{2}=\mathrm{h}$$

As $x\to\frac{\pi}{2}$, $\mathrm{h}\to 0$

$$\lim_{x\to\frac{\pi}{2}} \frac{4\cos^3 x}{8\left(x-\frac{\pi}{2}\right)^3}$$

$$=\lim_{\mathrm{h}\to 0} \frac{4\cos^3\left(\frac{\pi}{2}+\mathrm{h}\right)}{8\mathrm{h}^3}$$

$$=\lim_{\mathrm{h}\to 0} \frac{4(-\sin \mathrm{h})^3}{8\mathrm{h}^3} \qquad \ldots\left[\because \cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta\right]$$

$$=-\frac{1}{2}\cdot\left(\lim_{\mathrm{h}\to 0}\frac{\sin \mathrm{h}}{\mathrm{h}}\right)^3=\frac{-1}{2}$$

Question 8.

$lim_{x\to 0}(15_x-3_x-5_x+1 sin_2 x)$=

(A) log 15

(B) log 3 + log 5

(C) log 3 . log 5

(D) 3 log 5

Answer:

(C) log 3 . log 5

Hint:

$$\lim_{x\to 0}\left(\frac{15^x-3^x-5^x+1}{\sin^2 x}\right)$$

$$=\lim_{x\to 0}\left[\frac{(5^x-1)(3^x-1)}{x^2}\times\frac{x^2}{\sin^2 x}\right]$$

$$\ldots\left[\begin{array}{c}\because x\to 0,\ x\neq 0\\ \therefore\ x^2\neq 0\end{array}\right]$$

$$=\log 3.\ \log 5$$

Question 9.

$lim_{x\to 0}(3+5x3-4x)_{1x}$=

(A) e3

(B) e6

(C) e9

(D) e-3

Answer:

(A) e3

Hint:

$$\lim_{x\to 0}\left(\frac{3+5x}{3-4x}\right)^{\frac{1}{x}}$$

$$=\lim_{x\to 0}\left(\frac{1+\frac{5x}{3}}{1-\frac{4x}{3}}\right)^{\frac{1}{x}} \qquad \ldots\left[\begin{array}{l}\text{Divide numerator and}\\ \text{denominator by 3}\end{array}\right]$$

$$=\frac{\lim_{x\to 0}\left[\left(1+\frac{5x}{3}\right)^{\frac{3}{5x}}\right]^{\frac{5}{3}}}{\lim_{x\to 0}\left[\left(1-\frac{4x}{3}\right)^{\frac{-3}{4x}}\right]^{\frac{-4}{3}}}$$

$$=\frac{e^{\frac{5}{3}}}{e^{\frac{-4}{3}}}=e^3$$

Question 10.

$lim_{x\to 0}[log(5+x)-log(5-x)sinx]=$

(A) 32

(B) -52

(C) -12

(D) 25

Answer:

(D) 25

Hint:

$$\lim_{x\to 0}\left[\frac{\log(5+x)-\log(5-x)}{\sin x}\right]$$

$$=\lim_{x\to 0}\frac{\log\left[5\left(1+\frac{x}{5}\right)\right]-\log\left[5\left(1-\frac{x}{5}\right)\right]}{\sin x}$$

$$=\lim_{x\to 0}\frac{\log 5+\log\left(1+\frac{x}{5}\right)-\left[\log 5+\log\left(1-\frac{x}{5}\right)\right]}{\sin x}$$

$$=\lim_{x\to 0}\left[\frac{\log\left(1+\frac{x}{5}\right)-\log\left(1-\frac{x}{5}\right)}{x}\times\frac{x}{\sin x}\right]$$

$$=\lim_{x\to 0}\left[\frac{\log\left(1+\frac{x}{5}\right)}{5\left(\frac{x}{5}\right)}-\frac{\log\left(1-\frac{x}{5}\right)}{(-5)\left(\frac{-x}{5}\right)}\right]\times\lim_{x\to 0}\frac{x}{\sin x}$$

$$=\left[\frac{1}{5}(1)+\frac{1}{5}(1)\right]\times 1=\frac{2}{5}$$

Question 11.

$lim_{x\to\pi 2}(3_{cosx}-1_{\pi 2}-x)=$

(A) 1

(B) log 3

(C) $3_{\pi 2}$

(D) 3 log 3

Answer:

(B) log 3

Hint:

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{3^{\cos x}-1}{\frac{\pi}{2}-x}\right)$$

Put $\frac{\pi}{2}-x=\mathrm{h}$

$\therefore\quad x=\frac{\pi}{2}-\mathrm{h}$

As $x\to\frac{\pi}{2}$, $\mathrm{h}\to 0$

Required limit

$$=\lim_{\mathrm{h}\to 0}\frac{3^{\cos\left(\frac{\pi}{2}-\mathrm{h}\right)}-1}{\mathrm{h}}$$

$$=\lim_{\mathrm{h}\to 0}\left(\frac{3^{\sin\mathrm{h}}-1}{\sin\mathrm{h}}\times\frac{\sin\mathrm{h}}{\mathrm{h}}\right)=\log 3$$

Question 12.

$\lim_{x\to0}\left[\frac{x\cdot\log(1+3x)}{(e^{3x}-1)^2}\right]=$

(A) $\frac{1}{e^9}$

(B) $\frac{1}{e^3}$

(C) $\frac{1}{9}$

(D) $\frac{1}{3}$

Answer:

(D) $\frac{1}{3}$

Hint:

$$\lim_{x\to0}\frac{x.\log(1+3x)}{\left(e^{3x}-1\right)^2}$$

$$=\frac{\lim_{x\to0}\frac{\log(1+3x)}{x}}{\lim_{x\to0}\left(\frac{e^{3x}-1}{x}\right)^2}$$

$$=\frac{\lim_{x\to0}\left[\frac{\log(1+3x)}{3x}\times3\right]}{\lim_{x\to0}\left[\left(\frac{e^{3x}-1}{3x}\right)^2\times(3)^2\right]}=\frac{1}{3}$$

Question 13.

$\lim_{x\to0}\left[\frac{(3^{\sin x}-1)^3}{(3^x-1)\cdot\tan x\cdot\log(1+x)}\right]=$

(A) 3 log 3

(B) 2 log 3

(C) $(\log 3)^2$

(D) $(\log 3)^3$

Answer:

(C) $(\log 3)^2$

Hint:

$$\lim_{x\to0}\frac{\left(3^{\sin x}-1\right)^3}{\left(3^x-1\right).\tan x.\log(1+x)}$$

$$=\frac{\lim_{x\to0}\frac{\left(3^{\sin x}-1\right)^3}{\sin^3 x}\cdot\frac{\sin^3 x}{x^3}}{\lim_{x\to0}\left(\frac{3^x-1}{x}\right)\left(\frac{\tan x}{x}\right)\cdot\frac{\log(1+x)}{x}}$$

$$=\frac{\lim_{x\to0}\left(\frac{3^{\sin x}-1}{\sin x}\right)^3\cdot\lim_{x\to0}\left(\frac{\sin x}{x}\right)^3}{\lim_{x\to0}\left(\frac{3^x-1}{x}\right)\cdot\lim_{x\to0}\left(\frac{\tan x}{x}\right)\cdot\lim_{x\to0}\frac{\log(1+x)}{x}}$$

$$=(\log 3)^2$$

Question 14.
$\lim_{x\to 3}\left[\frac{5^{x-3}-4^{x-3}}{\sin(x-3)}\right]=$
(A) log 5 – 4
(B) log $\frac{5}{4}$
(C) $\frac{\log 5}{\log 4}$
(D) $\log\frac{5}{4}$
Answer:
(B) log $\frac{5}{4}$
Hint:

$$\lim_{x\to 3}\frac{5^{x-3}-4^{x-3}}{\sin(x-3)}$$

put $x-3=h$

$\therefore$ $x=3+h$

As $x\to 3, h\to 0$

$\therefore$ Required limit

$$=\lim_{h\to 0}\frac{5^h-4^h}{\sin h}$$

$$=\lim_{h\to 0}\frac{(5^h-1)-(4^h-1)}{\sin h}$$

$$=\frac{\lim_{h\to 0}\frac{(5^h-1)}{h}-\frac{(4^h-1)}{h}}{\lim_{h\to 0}\frac{\sin h}{h}}$$

$$=\frac{\log 5-\log 4}{1} \qquad \ldots\left[\lim_{x\to 0}\frac{a^x-1}{x}=\log a\right]$$

$$=\log\left(\frac{5}{4}\right)$$

Question 15.
$\lim_{x\to\infty}\left[\frac{(2x+3)^7(x-5)^3}{(2x-5)^{10}}\right]=$
(A) $\frac{3}{8}$
(B) $\frac{1}{8}$
(C) $\frac{1}{6}$
(D) $\frac{1}{4}$
Answer:
(B) $\frac{1}{8}$
Hint:

$$\lim_{x\to\infty}\frac{(2x+3)^7\cdot(x-5)^3}{(2x-5)^{10}}$$

$$=\frac{\lim_{x\to\infty}\left(\frac{2x+3}{x}\right)^7\cdot\left(\frac{x-5}{x}\right)^3}{\lim_{x\to\infty}\left(\frac{2x-5}{x}\right)^{10}}$$

$$=\frac{\lim_{x\to\infty}\left(2+\frac{3}{x}\right)^7\times\lim_{x\to\infty}\left(1-\frac{5}{x}\right)^3}{\lim_{x\to\infty}\left(2-\frac{5}{x}\right)^{10}}$$

$$=\frac{(2+0)^7\times(1-0)^3}{(2-0)^{10}} \qquad \ldots\left[\lim_{x\to\infty}\frac{1}{x^k}=0, k>0\right]$$

$$=\frac{1}{8}$$

(II) Evaluate the following.

Question 1.
$\lim_{x\to 0}\left[\frac{(1-x)^5-1}{(1-x)^3-1}\right]$

Solution:

$$\lim_{x\to 0}\left[\frac{(1-x)^5-1}{(1-x)^3-1}\right]$$

Put $1-x=y$

As $x\to 0, y\to 1$

$$\lim_{x\to 0}\left[\frac{(1-x)^5-1}{(1-x)^3-1}\right]$$

$$=\lim_{y\to 1}\frac{y^5-1}{y^3-1}$$

$$=\frac{\lim\limits_{y\to 1}\frac{y^5-1}{y-1}}{\lim\limits_{y\to 1}\frac{y^3-1}{y-1}} \ldots[\because y\to 1,\ y\neq 1,\ \therefore y-1\neq 0]$$

$$=\frac{\lim\limits_{y\to 1}\frac{y^5-1^5}{y-1}}{\lim\limits_{y\to 1}\frac{y^3-1^3}{y-1}}$$

$$=\frac{5(1)^4}{3(1)^2} \qquad \ldots\left[\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$=\frac{5}{3}$$

Question 2.

$\lim_{x\to 0}[x]$ ([*] is a greatest integer function.)

Solution:

$\lim\limits_{x\to 0}[x]$

$[x]=-1 \quad ; -1\leq x<0$

$[x]=0 \quad ; 0\leq x<1$

$\lim\limits_{x\to 0^-}[x]=-1$

$\lim\limits_{x\to 0^+}[x]=0$

$\therefore \quad \lim\limits_{x\to 0^-}[x]\neq \lim\limits_{x\to 0^+}[x]$

$\therefore \quad \lim\limits_{x\to 0}[x]$ does not exist.

Question 3.

If $f(r)=\pi r^2$ then find $\lim_{h\to 0}\left[\frac{f(r+h)-f(r)}{h}\right]$

Solution:

$f(r)=\pi r^2$

$f(r+h)=\pi(r+h)^2$

$=\pi(r^2+2rh+h^2)$

$=\pi r^2+2\pi rh+\pi h^2$

$$\therefore \quad \lim_{h\to 0}\frac{f(r+h)-f(r)}{h}$$

$$=\lim_{h\to 0}\frac{\pi r^2+2\pi rh+\pi h^2-\pi r^2}{h}$$

$$=\lim_{h\to 0}\frac{2\pi rh+\pi h^2}{h}$$

$$=\lim_{h\to 0}\frac{h(2\pi r+\pi h)}{h}$$

$$=\lim_{h\to 0}(2\pi r+\pi h) \qquad \ldots[\because h\to 0, h\neq 0]$$

$$=2\pi r+\pi(0)$$

$$=2\pi r$$

Question 4.

$\lim_{x\to 0}\left[\frac{x}{|x|+x^2}\right]$

Solution:

$$\lim_{x\to 0}\left[\frac{x}{|x|+x^2}\right]$$

$|x| = x;\ \ x \geq 0$

$\ \ = -x;\ x < 0$

$$\lim_{x\to 0^-}\left[\frac{x}{|x|+x^2}\right] = \lim_{x\to 0}\frac{x}{-x+x^2}$$

$$= \lim_{x\to 0}\frac{x}{x(-1+x)}$$

$$= \lim_{x\to 0}\frac{x}{-1+x} \ \ldots(\text{As } x \to 0, x \neq 0)$$

$$= \frac{1}{-1+0}$$

$$= -1$$

$$\lim_{x\to 0^+}\left[\frac{x}{|x|+x^2}\right] = \lim_{x\to 0}\frac{x}{x+x^2}$$

$$= \lim_{x\to 0}\frac{x}{x(1+x)}$$

$$= \lim_{x\to 0}\frac{1}{1+x} \ \ldots(\text{As } x \to 0, x \neq 0)$$

$$= \frac{1}{1+0}$$

$$= 1$$

$\therefore\ \ \lim_{x\to 0^-}\left[\frac{x}{|x|+x^2}\right] \neq \lim_{x\to 0}\left[\frac{x}{|x|+x^2}\right]$

$\therefore\ \ \lim_{x\to 0}\frac{x}{|x|+x^2}$ does not exist.

Question 5.
Find the limit of the function, if it exists, at x = 1

$f(x)=\{7-4x x^2+2 \text{ for } \text{ for } x<1 x\geq 1$

Solution:

$f(x) = 7 - 4x;\ x < 1$

$\ \ = x^2 + 2;\ x \geq 1$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1}(7-4x)$$

$$= 7 - 4(1)$$

$$= 3$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(x^2+2)$$

$$= (1)^2 + 2$$

$$= 3$$

$\therefore\ \ \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$\therefore\ \ \lim_{x\to 1^+} f(x)$ exists and is equal to 3.

Question 6.
Given that 7x ≤ f(x) ≤ $3x^2$ – 6 for all x. Determine the value of $\lim_{x\to 3} f(x)$

Solution:

$7x \leq f(x) \leq 3x^2 - 6$

Taking limits as $x \to 3$

$$\lim_{x\to 3}(7x) \leq \lim_{x\to 3} f(x) \leq \lim_{x\to 3}(3x^2-6)$$

$\therefore\ \ 7(3) \leq \lim_{x\to 3} f(x) \leq 3(3)^2 - 6$

$\therefore\ \ 21 \leq \lim_{x\to 3} f(x) \leq 21$

Using squeeze theorem,

$\lim_{x\to 3} f(x) = 21$

Question 7.

$lim_{x \to 0}[\frac{sec x^2 - 1}{x^4}]$

Solution:

$$\lim_{x \to 0} \frac{\sec x^2 - 1}{x^4}$$

Put $x^2 = y$

As $x \to 0, x^2 \to 0$

$\therefore \quad y \to 0$

$\therefore \quad$ Required limit

$$= \lim_{y \to 0} \frac{\sec y - 1}{y^2}$$

$$= \lim_{y \to 0} \frac{\sec y - 1}{y^2} \times \frac{\sec y + 1}{\sec y + 1}$$

$$= \lim_{y \to 0} \frac{\sec^2 y - 1}{y^2(\sec y + 1)}$$

$$= \lim_{y \to 0} \frac{\tan^2 y}{y^2(\sec y + 1)}$$

$$= \lim_{y \to 0} \frac{\tan^2 y}{y^2} \times \frac{1}{\sec y + 1}$$

$$= \lim_{y \to 0} \left(\frac{\tan^2 y}{y^2}\right) \times \lim_{y \to 0} \frac{1}{\sec y + 1}$$

$$= (1)^2 \times \frac{1}{1+1} \qquad \ldots\left[\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1\right]$$

$$= \frac{1}{2}$$

Question 8.

$lim_{x \to 0}[\frac{e^x + e^{-x} - 2}{x \cdot \tan x}]$

Solution:

$$\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x \tan x}$$

$$= \lim_{x \to 0} \frac{e^x + \frac{1}{e^x} - 2}{x \tan x}$$

$$= \lim_{x \to 0} \frac{(e^x)^2 + 1 - 2(e^x)}{e^x . x \tan x}$$

$$= \lim_{x \to 0} \frac{(e^x - 1)^2}{e^x . x \tan x}$$

$$= \lim_{x \to 0} \frac{\left[\frac{(e^x - 1)^2}{x^2}\right]}{\left(\frac{e^x . x \tan x}{x^2}\right)} \qquad \ldots[\because x \to 0;\ x \neq 0]$$

$$= \lim_{x \to 0} \frac{\left(\frac{e^x - 1}{x}\right)^2}{e^x . \left(\frac{\tan x}{x}\right)}$$

$$= \frac{\left(\lim_{x \to 0} \frac{e^x - 1}{x}\right)^2}{\lim_{x \to 0} e^x . \lim_{x \to 0}\left(\frac{\tan x}{x}\right)}$$

$$= \frac{(1)^2}{e^0 . 1}$$

$$= 1$$

Question 9.

$lim_{x \to 0}[\frac{x(6^x - 3^x)}{\cos(6x) - \cos(4x)}]$

Solution:

$$\lim_{x\to 0} \frac{x\left(6^x - 3^x\right)}{\cos 6x - \cos 4x}$$

$$= \lim_{x\to 0} \frac{x \cdot 3^x \left(2^x - 1\right)}{-2\sin\left(\frac{6x+4x}{2}\right)\sin\left(\frac{6x-4x}{2}\right)}$$

$$= \frac{1}{-2} \lim_{x\to 0} \frac{x \cdot 3^x \left(2^x - 1\right)}{\sin 5x \cdot \sin x}$$

$$= \frac{1}{-2} \lim_{x\to 0} \frac{\frac{x \cdot 3^x \left(2^x - 1\right)}{x^2}}{\frac{\sin 5x \cdot \sin x}{x^2}}$$

...[Divide Numerator and Denominator by x^2.
$\because x \to 0,\ x \neq 0 \therefore x^2 \neq 0$]

$$= \frac{1}{-2} \frac{\lim_{x\to 0}\left[3^x \frac{\left(2^x - 1\right)}{x}\right]}{\lim_{x\to 0}\left(\frac{\sin 5x}{x} \cdot \frac{\sin x}{x}\right)}$$

$$= \frac{1}{-2} \frac{\lim_{x\to 0}\left(3^x\right) \cdot \lim_{x\to 0}\left(\frac{2^x - 1}{x}\right)}{\lim_{x\to 0}\left(\frac{\sin x}{5x} \times 5\right) \cdot \lim_{x\to 0}\left(\frac{\sin x}{x}\right)}$$

$$= \frac{1}{-2} \times \frac{3^0 \times \log 2}{1 \times 5 \times 1} \quad \ldots \left(\begin{array}{l} \because x \to 0, 5x \to 0 \\ \lim_{x\to 0} \frac{a^x - 1}{x} = \log a \\ \lim_{x\to 0} \frac{\sin x}{x} = 1 \end{array}\right)$$

$$= \frac{-1}{10} \log 2$$

Question 10.
$\lim_{x\to 0}\left[\frac{a^{3x} - a^{2x} - a^x + 1}{x \cdot \tan x}\right]$
Solution:

$$\lim_{x\to 0} \frac{a^{3x} - a^{2x} - a^x + 1}{x \tan x}$$

$$= \lim_{x\to 0} \frac{a^{2x} \cdot a^x - a^{2x} - a^x + 1}{x \tan x}$$

$$= \lim_{x\to 0} \frac{a^{2x}(a^x - 1) - 1(a^x - 1)}{x \tan x}$$

$$= \lim_{x\to 0} \frac{(a}{x \tan x}$$

$$= \lim_{x\to 0} \frac{\frac{(a^x - 1) \cdot (a^{2x} - 1)}{x^2}}{\frac{x \tan x}{x^2}} \quad \ldots[\because x \to 0,\ x \neq 0]$$

$$= \lim_{x\to 0} \frac{\left(\frac{a^x - 1}{x}\right) \cdot \left(\frac{a^{2x} - 1}{2x}\right) .2}{\frac{\tan x}{x}}$$

$$= \frac{2\lim_{x\to 0}\left(\frac{a^x - 1}{x}\right) . \lim_{x\to 0}\left(\frac{a^{2x} - 1}{2x}\right)}{\lim_{x\to 0} \frac{\tan x}{x}}$$

$$= \frac{2(\log a)(\log a)}{1}$$

$$\ldots\left[\because x \to 0, 2x \to 0 \text{ and } \lim_{x\to 0} \frac{a^x - 1}{x} = \log a\right]$$

$$= 2\ (\log a)^2$$

Question 11.

$lim_{x \to a}[\frac{\sin x - \sin a}{x - a}]$

Solution:

$$\lim_{x \to a} \frac{\sin x - \sin a}{x - a}$$

Put $x = a + h$,

$\therefore \quad x - a = h$

As $x \to a$, $h \to 0$

$$\therefore \quad \lim_{x \to a} \frac{\sin x - \sin a}{x - a}$$

$$= \lim_{h \to 0} \frac{\sin(a + h) - \sin a}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a + h + a}{2}\right)\sin\left(\frac{a + h - a}{2}\right)}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\left(a + \frac{h}{2}\right)\sin\frac{h}{2}}{h}$$

$$= \lim_{h \to 0} \cos\left(a + \frac{h}{2}\right) \cdot \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$$

$$= \cos\ (a + 0)(1) \qquad \ldots\left[\because \lim_{\theta \to 0} \frac{\sin p\theta}{p\theta} = 1\right]$$

$$= \cos a$$

Question 12.

$lim_{x \to 2}[\frac{\log x - \log 2}{x - 2}]$

Solution:

$$\lim_{x \to 2} \frac{\log x - \log 2}{x - 2}$$

Put $x - 2 = h$

$\therefore \quad x = 2 + h$

Required limit

$$= \lim_{h \to 0} \frac{\log(2 + h) - \log 2}{h}$$

$$= \lim_{h \to 0} \frac{\log\left(\frac{2 + h}{2}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\log\left(1 + \frac{h}{2}\right)}{\frac{h}{2} \times 2}$$

$$= 1 \times \frac{1}{2}$$

$$\ldots\left[\because h \to 0, \frac{h}{2} \to 0 \text{ and } \lim_{x \to 0} \frac{\log(1 + x)}{x} = 1\right]$$

$$= \frac{1}{2}$$

Question 13.

$lim_{x \to 1}[\frac{ab^x - a^x b}{x^2 - 1}]$

Solution:

$$\lim_{x\to1}\frac{ab^x-a^xb}{x^2-1} = \lim_{x\to1}\frac{ab\left(b^{x-1}-a^{x-1}\right)}{x^2-1^2}$$

$$= \lim_{x\to1}\frac{ab\left(b^{x-1}-a^{x-1}\right)}{(x-1)(x+1)}$$

Put $x = 1 + h$.

$\therefore \quad x - 1 = h$

As $x \to 1, h \to 0$

$$\therefore \quad \lim_{x\to1}\frac{ab^x-a^xb}{x^2-1} = \lim_{h\to0}\frac{ab\left(b^h-a^h\right)}{h(1+h+1)}$$

$$= ab \lim_{h\to0}\frac{b^h-1+1-a^h}{h(2+h)}$$

$$= ab \lim_{h\to0}\frac{\left(b^h-1\right)-\left(a^h-1\right)}{h(2+h)}$$

$$= ab \lim_{h\to0}\frac{1}{2+h}\left(\frac{b^h-1}{h}-\frac{a^h-1}{h}\right)$$

$$= ab.\frac{1}{\lim\limits_{h\to0}(2+h)}\left(\lim_{h\to0}\frac{b^h-1}{h}-\lim_{h\to0}\frac{a^h-1}{h}\right)$$

$$= ab.\ \frac{1}{2+0}.(\log b-\log a)\ldots\left[\because \lim_{x\to0}\frac{a^x-1}{x}=\log a\right]$$

$$= \frac{ab}{2}\log\left(\frac{b}{a}\right)$$

Question 14.

$\lim_{x\to0}\left[\frac{(5^x-1)^2}{(2^x-1)\log(1+x)}\right]$

Solution:

$$\lim_{x\to0}\frac{\left(5^x-1\right)^2}{\left(2^x-1\right)\cdot\log(1+x)}$$

$$= \lim_{x\to0}\frac{\frac{\left(5^x-1\right)^2}{x^2}}{\frac{\left(2^x-1\right)\cdot\log(1+x)}{x^2}}$$

…[Divide Numerator and Denominator by x^2 $\because$ $x \to 0, x \neq 0 \therefore x^2 \neq 0$]

$$= \frac{\lim\limits_{x\to0}\left(\frac{5^x-1}{x}\right)^2}{\lim\limits_{x\to0}\left(\frac{\quad}{x}\right)\cdot\frac{\quad}{x}}$$

$$= \frac{\left(\lim\limits_{x\to0}\frac{5^x-1}{x}\right)^2}{\lim\limits_{x\to0}\left(\frac{2^x-1}{x}\right)\cdot\lim\limits_{x\to0}\frac{\log(1+x)}{x}}$$

$$= \frac{(\log 5)^2}{\log 2\times1}$$

$$\ldots\left[\lim_{x\to0}\frac{a^x-1}{x}=\log a,\ \lim_{x\to0}\frac{\log(1+x)}{x}=1\right]$$

$$= \frac{(\log 5)^2}{\log 2}$$

Question 15.

$\lim_{x\to\infty}\left[\frac{(2x+1)^2(7x-3)^3}{(5x+2)^5}\right]$

Solution:

$$\lim_{x \to \infty}\left[\frac{(2x+1)^2 \cdot (7x-3)^3}{(5x+2)^5}\right]$$

$$= \lim_{x \to \infty} \frac{\frac{(2x+1)^2}{x^2} \cdot \frac{(7x-3)^3}{x^3}}{\frac{(5x+2)^5}{x^5}}$$

...[Divide Numerator and Denominator by x^5]

$$= \frac{\lim_{x \to \infty}\left(\frac{2x+1}{x}\right)^2 \cdot \left(\frac{7x-3}{x}\right)^3}{\lim_{x \to \infty}\left(\frac{5x+2}{x}\right)^5}$$

$$= \frac{\lim_{x \to \infty}\left(2+\frac{1}{x}\right)^2 \cdot \lim_{x \to \infty}\left(7-\frac{3}{x}\right)^3}{\lim_{x \to \infty}\left(5+\frac{2}{x}\right)^5}$$

$$= \frac{(2+0)^2 \cdot (7-0)^3}{(5+0)^5} \qquad \ldots\left[\lim_{x \to \infty}\frac{1}{x^k} = 0, k > 0\right]$$

$$= \frac{(2)^2 \times (7)^3}{(5)^5} = \frac{1372}{3125}$$

Question 16.

$lim_{x \to a}[xcosa-acosxx-a]$

Solution:

$$\lim_{x \to a} \frac{x \cos a - a \cos x}{x - a}$$

$$= \lim_{x \to a} \frac{x \cos a - a \cos a - a \cos x + a \cos a}{x - a}$$

$$= \lim_{x \to a} \frac{(x - a) \cos a - a (\cos x - \cos a)}{x - a}$$

$$= \lim_{x \to a} \left[\frac{(x - a) \cos a}{x - a} - \frac{a (\cos x - \cos a)}{x - a} \right]$$

$$= \lim_{x \to a} (\cos a) - a \lim_{x \to a} \left(\frac{\cos x - \cos a}{x - a} \right)$$

$$\ldots [\because x \to a,\ x \neq a,\ \therefore x - a \neq 0]$$

Consider, $\lim_{x \to a} \frac{\cos x - \cos a}{x - a}$

$$= \lim_{x \to a} \frac{-2 \sin\left(\frac{x + a}{2}\right) \cdot \sin\left(\frac{x - a}{2}\right)}{x - a}$$

$$= -\lim_{x \to a} \sin\left(\frac{x + a}{2}\right) \cdot \frac{\sin\left(\frac{x - a}{2}\right)}{\frac{x - a}{2}}$$

$$= -\lim_{x \to a} \sin\left(\frac{x + a}{2}\right) \cdot \lim_{x \to a} \frac{\sin\left(\frac{x - a}{2}\right)}{\frac{x - a}{2}}$$

$$= -1 \times \sin\left(\frac{a + a}{2}\right) \times 1$$

$$\ldots \left[\begin{array}{l} \because x \to a, (x - a) \to 0 \\ \therefore \frac{x - a}{2} \to 0 \text{ and } \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \end{array} \right]$$

$= -\sin a$

Required limit

$= \cos a - a(-\sin a)$

$= \cos a + a \sin a$

Question 17.

$\lim_{x \to \frac{\pi}{4}} \left[\frac{(\sin x - \cos x)^2}{\sqrt{2} - \sin x - \cos x} \right]$

Solution:

$$\lim_{x \to \frac{\pi}{4}} \left[\frac{(\sin x - \cos x)^2}{\sqrt{2} - \sin x - \cos x} \right]$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{1 - \sin 2x}{\sqrt{2} - (\sin x + \cos x)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{1 - \sin 2x}{\sqrt{2} - \sqrt{1 + \sin 2x}}$$

Put $1 + \sin 2x = t$

$\therefore \quad \sin 2x = t - 1$

As $x \to \frac{\pi}{4}$, $t \to 1 + \sin 2\left(\frac{\pi}{4}\right)$

$\therefore \quad t \to 1 + \sin \frac{\pi}{2}$

$\therefore \quad t \to 1 + 1$

$\therefore \quad t \to 2$

$\therefore$ Required limit

$$= \lim_{t \to 2} \frac{1 - (t - 1)}{\sqrt{2} - \sqrt{t}}$$

$$= \lim_{t \to 2} \frac{2 - t}{2^{\frac{1}{2}} - t^{\frac{1}{2}}}$$

$$= \lim_{t \to 2} \frac{t - 2}{t^{\frac{1}{2}} - 2^{\frac{1}{2}}}$$

$$= \lim_{t \to 2} \frac{1}{\dfrac{t^{\frac{1}{2}} - 2^{\frac{1}{2}}}{t - 2}}$$

$\cdots \left[\begin{array}{l}\text{Divide Numerator and Denominator by } t-2 \\ \text{As } t \to 2, t \neq 2, \therefore\ t - 2 \neq 0\end{array}\right]$

$$= \frac{1}{\frac{1}{2}(2)^{\frac{-1}{2}}}$$

$$= 2(2)^{\frac{1}{2}} = 2\sqrt{2}$$

Question 18.

$\lim_{x \to 1} \left[\frac{2^{2x-2} - 2^{x} + 1}{\sin^2(x-1)} \right]$

Solution:

$$\lim_{x\to 1}\frac{2^{2x-2}-2^x+1}{\sin^2(x-1)}$$

$$=\lim_{x\to 1}\frac{2^{2(x-1)}-2^x+1}{\sin^2(x-1)}$$

Put $x = 1 + h$, $\therefore x - 1 = h$

As $x \to 1$, $h \to 0$

$$\lim_{x\to 1}\frac{2^{2x-2}-2^x+1}{\sin^2(x-1)}=\lim_{h\to 0}\frac{2^{2h}-2^{1+h}+1}{\sin^2 h}$$

$$=\lim_{h\to 0}\frac{2^{2h}-2.2^h+1}{\sin^2 h}$$

$$=\lim_{h\to 0}\frac{\left(2^h-1\right)^2}{\sin^2 h}$$

$$=\lim_{h\to 0}\left[\frac{\frac{\left(2^h-1\right)^2}{h^2}}{\frac{\sin^2 h}{h^2}}\right]\qquad ...[\because h\to 0,\ h\neq 0]$$

$$=\frac{\left(\lim_{h\to 0}\frac{2^h-1}{h}\right)^2}{\left(\lim_{h\to 0}\frac{\sin h}{h}\right)^2}$$

$$=\frac{(\log 2)^2}{(1)^2}\qquad \ldots\left[\because \lim_{x\to 0}\frac{a^x-1}{x}=\log a\right]$$

$$=(\log 2)^2$$

Question 19.

$\lim_{x\to 1}\left[\frac{4^{x-1}-2^x+1}{(x-1)^2}\right]$

Solution:

$$\lim_{x\to 1}\left[\frac{4^{x-1}-2^x+1}{(x-1)^2}\right]$$

put $x - 1 = h$

$\therefore$ $x = 1 + h$

As $x \to 1$, $h \to 0$

Required limit

$$=\lim_{h\to 0}\frac{4^h-2^{1+h}+1}{h^2}$$

$$=\lim_{h\to 0}\frac{\left(2^2\right)^h-2^1\cdot 2^h+1}{h^2}$$

$$=\lim_{h\to 0}\frac{\left(2^h\right)^2-2\left(2^h\right)+1}{h^2}$$

$$=\lim_{h\to 0}\frac{\left(2^h-1\right)^2}{h^2}$$

$$=\lim_{h\to 0}\left(\frac{2^h-1}{h}\right)^2$$

$$=(\log 2)^2\qquad \ldots\left[\because \lim_{x\to 0}\frac{a^x-1}{x}=\log a\right]$$

Question 20.

$\lim_{x\to 1}\left[\frac{x-1}{\log x}\right]$

Solution:

$$\lim_{x \to 1} \frac{\sqrt{x}-1}{\log x}$$

put $x - 1 = h$

$\therefore \quad x = 1 + h$

As $x \to 1, h \to 0$

$\therefore$ Required limit

$$= \lim_{h \to 0} \frac{\sqrt{1+h}-1}{\log(1+h)}$$

$$= \lim_{h \to 0} \frac{\sqrt{1+h}-1}{\log(1+h)} \times \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1} \quad \ldots\text{[By rationalization]}$$

$$= \lim_{h \to 0} \frac{(1+h)-1}{\log(1+h)} \times \frac{1}{\sqrt{1+h}+1}$$

$$= \lim_{h \to 0} \frac{h}{\log(1+h)} \times \frac{1}{\sqrt{1+h}+1}$$

$$= \lim_{h \to 0} \frac{1}{\frac{\log(1+h)}{h}} \times \frac{1}{\sqrt{1+h}+1}$$

$$\ldots \begin{bmatrix} \text{Divide Numerator and} \\ \text{Denominator by h} \\ \text{As } h \to 0, h \neq 0 \end{bmatrix}$$

$$= \lim_{h \to 0} \frac{1}{\frac{\log(1+h)}{h}} \times \frac{1}{\lim_{h \to 0}\left(\sqrt{1+h}+1\right)}$$

$$= \frac{1}{1} \times \frac{1}{\sqrt{1+0}+1} = \frac{1}{2}$$

Question 21.

$\lim_{x \to 0}(1-\cos x\sqrt{x})$

Solution:

$$\lim_{x \to 0} \frac{\sqrt{1-\cos x}}{x}$$

$$= \lim_{x \to 0} \frac{\sqrt{2\sin^2 \frac{x}{2}}}{x} = \lim_{x \to 0} \frac{\sqrt{2}\left|\sin \frac{x}{2}\right|}{x}$$

$|x| = x \quad ; \ x \geq 0$

$\quad = -x \quad ; \ x < 0$

Left hand limit $= \lim\limits_{x \to 0^-} \dfrac{\sqrt{2}\left|\sin\frac{x}{2}\right|}{x}$

$= \sqrt{2} \lim\limits_{x \to 0} \dfrac{\sin\left(\frac{-x}{2}\right)}{x}$

$= \sqrt{2} \lim\limits_{x \to 0} \dfrac{-\sin\frac{x}{2}}{\frac{x}{2}} \times \dfrac{1}{2}$

$= \sqrt{2} \times (-1) \times \dfrac{1}{2}$

$\ldots \left[\because \lim\limits_{\theta \to 0} \dfrac{\sin p\theta}{p\theta} = 1\right]$

$= \dfrac{-1}{\sqrt{2}}$

Right hand limit $= \lim\limits_{x \to 0^+} \dfrac{\sqrt{2}\left|\sin\frac{x}{2}\right|}{x}$

$= \sqrt{2} \lim\limits_{x \to 0} \dfrac{\sin\left(\frac{x}{2}\right)}{x}$

$= \sqrt{2} \lim\limits_{x \to 0} \dfrac{\sin\frac{x}{2}}{\frac{x}{2}} \times \dfrac{1}{2}$

$= \sqrt{2} \times (1) \times \dfrac{1}{2}$

$\ldots \left[\because \lim\limits_{\theta \to 0} \dfrac{\sin p\theta}{p\theta} = 1\right]$

$= \dfrac{1}{\sqrt{2}}$

$\therefore$ **Left hand limit is not equal to the right hand limit**

$\therefore$ $\lim\limits_{x \to 0}\left(\dfrac{\sqrt{1-\cos x}}{x}\right)$ **does not exist.**

Question 22.

$lim_{x \to 1}\left(\frac{x+3x^2+5x^3+\cdots\cdots\cdots\cdots+(2n-1)x^n-n^2}{x-1}\right)$

Solution:

$\lim\limits_{x \to 1}\left(\dfrac{x+3x^2+5x^3+\cdots+(2n-1)x^n-n^2}{x-1}\right)$

Consider

$1 + 3 + 5 + \ldots + (2n-1)$

$= \sum\limits_{r=1}^{n}(2r-1)$

$= 2\sum\limits_{r=1}^{n} r - \sum\limits_{r=1}^{n} 1$

$$= 2\frac{n(n+1)}{2} - n$$
$$= n(n + 1) - n$$
$$= n^2 + n - n$$
$$= n^2$$

$\therefore \quad n^2 = 1 + 3 + 5 + \ldots + (2n - 1).$

$\therefore$ Required limit

$$= \lim_{x\to 1} \frac{\left[x+3x^2+5x^3+\cdots+(2n-1)x^n\right]-\left[1+3+5+\cdots+(2n-1)\right]}{x-1}$$

$$= \lim_{x\to 1} \frac{(x-1)+(3x^2-3)+(5x^2-5)+\cdots+(2n-1)x^n-(2n-1)}{x-1}$$

$$= \lim_{x\to 1}\left[\frac{x-1}{x-1}+\frac{3(x^2-1)}{x-1}+\frac{5(x^3-1)}{x-1}+\cdots+\frac{(2n-1)(x^n-1)}{x-1}\right]$$

$$= \lim_{x\to 1}\left(\frac{x^1-1^1}{x-1}\right)+3\lim_{x\to 1}\left(\frac{x^2-1^2}{x-1}\right)+5\lim_{x\to 1}\left(\frac{x^3-1^3}{x-1}\right)$$
$$+\cdots+(2n-1)\lim_{x\to 1}\left(\frac{x^n-1^n}{x-1}\right)$$

$$= 1(1)^0 + 3(2)(1)^1 + 5(3)(1)^2 + \ldots + (2n - 1)\, n(1)^{n-1}$$
$$= 1(1) + 3(2) + 5(3) + \ldots + (2n - 1)n$$
$$= \sum_{r=1}^{n}(2r-1)r$$
$$= \sum_{r=1}^{n}(2r^2-r)$$
$$= 2\sum_{r=1}^{n}r^2-\sum_{r=1}^{n}r$$
$$= 2.\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}$$
$$= n(n+1)\left(\frac{2n+1}{3}-\frac{1}{2}\right)$$
$$= n(n+1)\left(\frac{4n+2-3}{6}\right) = \frac{n(n+1)(4n-1)}{6}$$

Question 23.

$\lim_{x\to 0}\frac{1}{x^{12}}\left[1-\cos\left(\frac{x^2}{2}\right)-\cos\left(\frac{x^4}{4}\right)+\cos\left(\frac{x^2}{2}\right)\cdot\cos\left(\frac{x^4}{4}\right)\right]$

Solution:

$$\lim_{x\to 0}\frac{1}{x^{12}}\left[1-\cos\left(\frac{x^2}{2}\right)-\cos\left(\frac{x^4}{4}\right)+\cos\left(\frac{x^2}{2}\right).\cos\left(\frac{x^4}{4}\right)\right]$$

consider

$$1-\cos\left(\frac{x^2}{2}\right)-\cos\left(\frac{x^4}{4}\right)+\cos\left(\frac{x^2}{2}\right)\cdot\cos\left(\frac{x^4}{4}\right)$$

$$=\left[1-\cos\left(\frac{x^2}{2}\right)\right]-\cos\left(\frac{x^4}{4}\right)\left[1-\cos\left(\frac{x^2}{2}\right)\right]$$

$$=\left[1-\cos\left(\frac{x^2}{2}\right)\right]\left[1-\cos\left(\frac{x^4}{2}\right)\right]$$

$$=2\sin^2\left(\frac{x^2}{4}\right)\times 2\sin^2\left(\frac{x^4}{8}\right)$$

Required limit

$$=\lim_{x\to 0}\frac{4\left[\sin\left(\frac{x^2}{4}\right)\right]^2.\left[\sin\left(\frac{x^4}{8}\right)\right]^2}{x^{12}}$$

$$=4\lim_{x\to 0}\frac{\left[\sin\left(\frac{x^2}{4}\right)\right]^2}{x^4}\times\frac{\left[\sin\left(\frac{x^4}{8}\right)\right]^2}{x^8}$$

$$=4\lim_{x\to 0}\left[\frac{\sin\left(\frac{x^2}{4}\right)}{x^2}\right]^2\times\left[\frac{\sin\left(\frac{x^4}{8}\right)}{x^4}\right]^2$$

$$=4\lim_{x\to 0}\left[\frac{\sin\left(\frac{x^2}{4}\right)}{\frac{x^2}{4}}\right]^2\times\frac{1}{16}\times\lim_{x\to 0}\left[\frac{\sin\left(\frac{x^4}{8}\right)}{\frac{x^4}{8}}\right]^2\times\frac{1}{64}$$

$$=4\times(1)^2\times\frac{1}{16}\times(1)^2\times\frac{1}{64}\qquad\ldots\left[\because\lim_{\theta\to 0}\frac{\sin p\theta}{p\theta}=1\right]$$

$$=\frac{1}{256}$$

Question 24.

$\lim_{x\to\infty}\left(\frac{8x^2+5x+3}{2x^2-7x-5}\right)^{\frac{4x+3}{8x-1}}$

Solution:

$$\lim_{x\to\infty}\left(\frac{8x^2+5x+3}{2x^2-7x-5}\right)^{\frac{4x+3}{8x-1}}$$

$$=\left[\lim_{x\to\infty}\left(\frac{8x^2+5x+3}{2x^2-7x-5}\right)\right]^{\lim\limits_{x\to\infty}\frac{4x+3}{8x-1}}$$

Consider

$$\lim_{x\to\infty}\frac{8x^2+5x+3}{2x^2-7x-5}$$

$$=\lim_{x\to\infty}\left(\frac{\frac{8x^2+5x+3}{x^2}}{\frac{2x^2-7x-5}{x^2}}\right)\quad\ldots\left[\begin{array}{l}\text{Divide Numerator and}\\ \text{Denominator by } x^2\end{array}\right]$$

$$=\frac{\lim\limits_{x\to\infty}\left(8+\frac{5}{x}+\frac{3}{x^2}\right)}{\lim\limits_{x\to\infty}\left(2-\frac{7}{x}-\frac{5}{x^2}\right)}$$

$$=\frac{8+0+0}{2-7-0}\qquad\ldots\left[\lim_{x\to\infty}\frac{1}{x^{k}}=0,\ k>0\right]$$

$$=4$$

Consider

$$\lim_{x\to\infty}\frac{4x+3}{8x-1}$$

$$=\lim_{x\to\infty}\frac{\frac{4x+3}{x}}{\frac{8x-1}{x}}$$

…[Divide Numerator and Denominator by x]

$$=\frac{\lim\limits_{x\to\infty}\left(4+\frac{3}{x}\right)}{\lim\limits_{x\to\infty}\left(8-\frac{1}{x}\right)}$$

$$=\frac{4+0}{8-0}\qquad\ldots\left[\lim_{x\to\infty}\frac{1}{x}=0\right]$$

$$=\frac{1}{2}$$

Required limit $=(4)^{\frac{1}{2}}=2$