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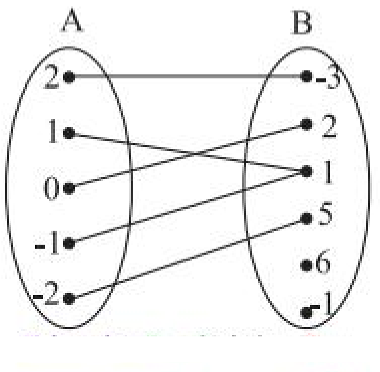
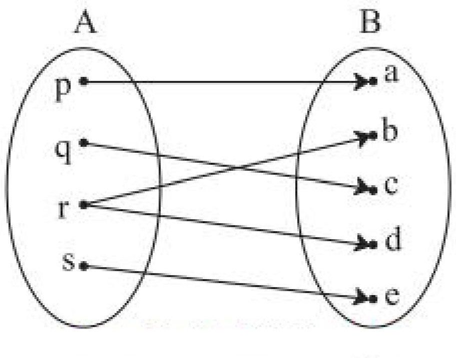
Maharashtra State Board 11th Maths Solutions Chapter 6 Functions Ex 6.1

Question 1.
Check if the following relations are functions.
(a)



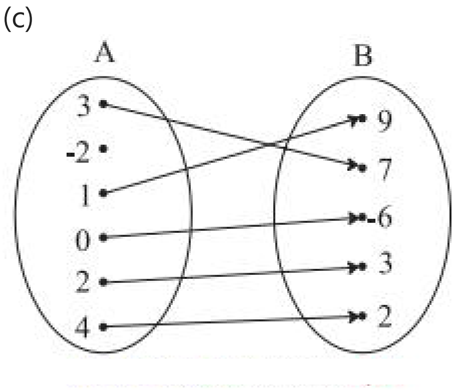
Solution:
Yes.
Reason: Every element of set A has been assigned a unique element in set B.

(b)



Solution:
No.
Reason: An element of set A has been assigned more than one element from set B.

(c)



Solution:
No.
Reason:
Not every element of set A has been assigned an image from set B.

Question 2.
Which sets of ordered pairs represent functions from A = {1, 2, 3, 4} to B = {-1, 0, 1, 2, 3}? Justify.
(i) {(1, 0), (3, 3), (2, -1), (4, 1), (2, 2)}
(ii) {(1, 2), (2, -1), (3, 1), (4, 3)}
(iii) {(1, 3), (4, 1), (2, 2)}
(iv) {(1, 1), (2, 1), (3, 1), (4, 1)}
Solution:
(i) {(1, 0), (3, 3), (2, -1), (4, 1), (2, 2)} does not represent a function.
Reason: (2, -1), (2, 2), show that element 2 ∈ A has been assigned two images -1 and 2 from set B.

(ii) {(1, 2), (2, -1), (3, 1), (4, 3)} represents a function.
Reason: Every element of set A has been assigned a unique image in set B.

(iii) {(1, 3), (4, 1), (2, 2)} does not represent a function.
Reason:
3 ∈ A does not have an image in set B.

(iv) {(1, 1), (2, 1), (3, 1), (4, 1)} represents a function
Reason: Every element of set A has been assigned a unique image in set B.

Question 3.
Check if the relation given by the equation represents y as function of x.
(i) $2x + 3y = 12$
(ii) $x + y^2 = 9$
(iii) $x^2 - y = 25$
(iv) $2y + 10 = 0$
(v) $3x - 6 = 21$
Solution:
(i) $2x + 3y = 12$
∴ $y = 12-2x3$
∴ For every value of x, there is a unique value of y.
∴ y is a function of x.

(ii) $x + y^2 = 9$
∴ $y^2 = 9 - x$
∴ $y = \pm 9-x-----\surd$
∴ For one value of x, there are two values of y.
∴ y is not a function of x.

(iii) $x^2 - y = 25$
∴ $y = x^2 - 25$
∴ For every value of x, there is a unique value of y.
∴ y is a function of x.

(iv) $2y + 10 = 0$
∴ $y = -5$
∴ For every value of x, there is a unique value of y.
∴ y is a function of x.

(v) $3x - 6 = 21$
∴ $x = 9$
∴ $x = 9$ represents a point on the X-axis.
There is no y involved in the equation.
So the given equation does not represent a function.

Question 4.
If $f(m) = m^2 - 3m + 1$, find
(i) f(0)
(ii) f(-3)
(iii) f(12)
(iv) f(x + 1)
(v) f(-x)
(vi) $(f(2+h)-f(2)h)$, $h \neq 0$.
Solution:
$f(m) = m^2 - 3m + 1$
(i) $f(0) = 0^2 - 3(0) + 1 = 1$

(ii) $f(-3) = (-3)^2 - 3(-3) + 1$
$= 9 + 9 + 1$
$= 19$

(iii) $f(12) = (12)^2-3(12)+1$
$= 14-32+1$
$= 1-6+44$
$= -14$

(iv) $f(x + 1) = (x + 1)^2 - 3(x + 1) + 1$
$= x^2 + 2x + 1 - 3x - 3 + 1$
$= x^2 - x - 1$

(v) f(-x) = (-x)2 – 3(-x) + 1 = x2 + 3x + 1

(vi) (f(2+h)-f(2)h)
= $(2+h)_2$-3(2+h)+1-(2_2-3(2)+1)h
= h_2+hh
= h + 1

Question 5.
Find x, if g(x) = 0 where
(i) g(x) = 5x-67
(ii) g(x) = 18-2x27
(iii) g(x) = $6x^2 + x – 2$
(iv) g(x) = $x^3 – 2x^2 – 5x + 6$
Solution:
(i) g(x) = 5x-67
g(x) = 0
∴ 5x-67 = 0
∴ x = 65

(ii) g(x) = 18-2x27
g(x) = 0
18-2x27 = 0
∴ $18 – 2x^2 = 0$
∴ $x^2 = 9$
∴ x = ±3

(iii) g(x) = $6x^2 + x – 2$
g(x) = 0
∴ $6x^2 + x – 2 = 0$
∴ (2x – 1) (3x + 2) = 0
∴ 2x – 1 = 0 or 3x + 2 = 0
∴ x = 12 or x = -23

(iv) g(x) = $x^3 – 2x^2 – 5x + 6$
= (x- 1) (x^2 – x – 6)
= (x – 1) (x + 2) (x – 3)
g(x) = 0
∴ (x – 1) (x + 2) (x – 3) = 0
∴ x – 1 = 0 or x + 2 = 0 or x – 3 = 0
∴ x = 1, -2, 3

Question 6.
Find x, if f(x) = g(x) where
(i) f(x) = $x^4 + 2x^2$, g(x) = $11x^2$
(ii) f(x) = √x – 3, g(x) = 5 – x
Solution:
(i) f(x) = $x^4 + 2x^2$, g(x) = $11x^2$
f(x) = g(x)
∴ $x^4 + 2x^2 = 11x^2$
∴ $x^4 – 9x^2 = 0$
∴ $x^2 (x^2 – 9) = 0$
∴ $x^2 = 0$ or $x^2 – 9 = 0$
∴ x = 0 or $x^2 = 9$
∴ x = 0, ±3

(ii) f(x) = √x – 3, g(x) = 5 – x
f(x) = g(x)
∴ √x – 3 = 5 – x
∴ √x = 5 – x + 3
∴ √x = 8 – x
on squaring, we get
x = 64 + x^2 – 16x
∴ x^2 – 17x + 64 = 0
∴ x = 17±$(-17)_2$-4(64)√2
∴ x = 17±289-256√2
∴ x = 17±33√2

Question 7.
If f(x) = $a-xb-x$, f(2) is undefined, and f(3) = 5, find a and b.
Solution:
f(x) = $a-xb-x$
Given that,
f(2) is undefined
b – 2 = 0
∴ b = 2 …..(i)
f(3) = 5
∴ $a-3b-3$ = 5
∴ $a-32-3$ = 5 ….. [From (i)]
∴ a – 3 = -5
∴ a = -2
∴ a = -2, b = 2

Question 8.
Find the domain and range of the following functions.
(i) f(x) = $7x^2$ + 4x – 1
Solution:
f(x) = $7x^2$ + 4x – 1
f is defined for all x.
∴ Domain of f = R (i.e., the set of real numbers)

$$\begin{aligned}7x^2+4x-1&=7\left(x^2+\frac{4}{7}x\right)-1\\&=7\left(x^2+\frac{4}{7}x\right)+\frac{4}{7}-1-\frac{4}{7}\\&=7\left(x^2+\frac{4}{7}x+\frac{4}{49}\right)-1-\frac{4}{7}\\&=7\left(x+\frac{2}{7}\right)^2-\frac{11}{7}\geq-\frac{11}{7}\end{aligned}$$

∴ Range of f = [-117, ∞)

(ii) g(x) = $x+4x-2$
Solution:
g(x) = $x+4x-2$
Function g is defined everywhere except at x = 2.
∴ Domain of g = R – {2}
Let y = g(x) = $x+4x-2$
∴ (x – 2) y = x + 4
∴ x(y – 1) = 4 + 2y
∴ For every y, we can find x, except for y = 1.
∴ y = 1 ∉ range of function g
∴ Range of g = R – {1}

(iii) h(x) = $x+5\sqrt{}5+x$
Solution:
h(x) = $x+5\sqrt{}5+x=1x+5\sqrt{}$, x ≠ -5
For x = -5, function h is not defined.
∴ x + 5 > 0 for function h to be well defined.
∴ x > -5
∴ Domain of h = (-5, ∞)
Let y = $1x+5\sqrt{}$
∴ y > 0
Range of h = (0, ∞) or R+

(iv) f(x) = $x+1$−−−−−$\sqrt{}_3$
Solution:
f(x) = $x+1$−−−−−$\sqrt{}_3$
f is defined for all real x and the values of f(x) ∈ R
∴ Domain of f = R, Range of f = R

(v) f(x) = $(x-2)(5-x)$−−−−−−−−−−−$\sqrt{}$
Solution:

f(x) = (x−2)(5−x)−−−−−−−−−−−√
For f to be defined,
(x – 2)(5 – x) ≥ 0
∴ (x – 2)(x – 5) ≤ 0
∴ 2 ≤ x ≤ 5 ……[∵ The solution of (x – a) (x – b) ≤ 0 is a ≤ x ≤ b, for a < b]
∴ Domain of f = [2, 5]
(x – 2) (5 – x) = -x2 + 7x – 10
= −(x−72)2+494−10
= 94−(x−72)2≤94
∴ (x−2)(5−x)−−−−−−−−−−−√≤94−−√≤32
Range of f = [0, 32]

(vi) f(x) = x−37−x−−−√
Solution:
f(x) = x−37−x−−−√
For f to be defined,
x−37−x−−−√ ≥ 0, 7 – x ≠ 0
∴ x−37−x−−−√ ≤ 0 and x ≠ 7
∴ 3 ≤ x < 7
Let a < b, x−ax−b ≤ 0 ⇒ a ≤ x < b
∴ Domain of f = [3, 7)
f(x) ≥ 0 … [∵ The value of square root function is non-negative]
∴ Range of f = [0, ∞)

(vii) f(x) = 16−x2−−−−−−√
Solution:
f(x) = 16−x2−−−−−−√
For f to be defined,
16 – x2 ≥ 0
∴ x2 ≤ 16
∴ -4 ≤ x ≤ 4
∴ Domain of f = [-4, 4]
Clearly, f(x) ≥ 0 and the value of f(x) would be maximum when the quantity subtracted from 16 is minimum i.e. x = 0
∴ Maximum value of f(x) = √16 = 4
∴ Range of f = [0, 4]

Question 9.
Express the area A of a square as a function of its
(a) side s
(b) perimeter P
Solution:
(a) area (A) = s2
(b) perimeter (P) = 4s
∴ s = P4
Area (A) = s2 = (P4)2
∴ A = P216

Question 10.
Express the area A of a circle as a function of its
(i) radius r
(ii) diameter d
(iii) circumference C
Solution:
(i) Area (A) = πr2

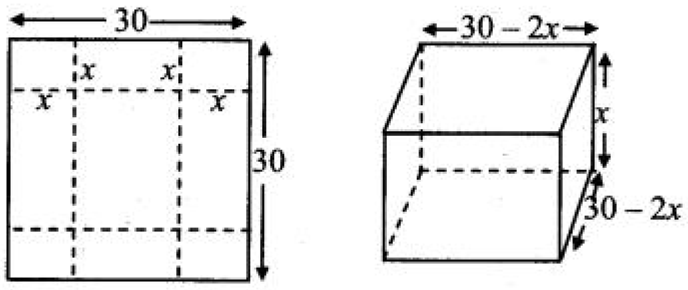
(ii) Diameter (d) = 2r
∴ r = d2
∴ Area (A) = πr2 = πd24

(iii) Circumference (C) = 2πr
∴ r = C2π

Area (A) = $\pi r^2 = \pi\left(\frac{C}{2\pi}\right)^2$

$\therefore$ A = $\frac{C^2}{4\pi}$

Question 11.
An open box is made from a square of cardboard of 30 cms side, by cutting squares of length x centimeters from each corner and folding the sides up. Express the volume of the box as a function of x. Also, find its domain.
Solution:



Length of the box = 30 – 2x
Breadth of the box = 30 – 2x
Height of the box = x
Volume = $(30 - 2x)^2 x$, x < 15, x ≠ 15, x > 0
= $4x(15 - x)^2$, x ≠ 15, x > 0
Domain (0, 15)

Question 12.
Let f be a subset of Z × Z defined by f = {(ab, a + b): a, b ∈ Z}. Is f a function from Z to Z? Justify?
Solution:
f = {(ab, a + b): a, b ∈ Z}
Let a = 1, b = 1. Then, ab = 1, a + b = 2
∴ (1, 2) ∈ f
Let a = -1, b = -1. Then, ab = 1, a + b = -2
∴ (1, -2) ∈ f
Since (1, 2) ∈ f and (1, -2) ∈ f,
f is not a function as element 1 does not have a unique image.

Question 13.
Check the injectivity and surjectivity of the following functions.
(i) f : N → N given by f(x) = x^2
Solution:
f: N → N given by f(x) = x^2

$$\text{Let } f(x_1) = f(x_2),\ x_1, x_2 \in N$$
$$\therefore \quad x_1^2 = x_2^2$$
$$\therefore \quad x_1^2 - x_2^2 = 0$$
$$\therefore \quad (x_1 - x_2)\underbrace{(x_1 + x_2)}_{\text{for } x_1,\, x_2 \,\in\, N} = 0$$
$$\therefore \quad x_1 = x_2$$

∴ f is injective.
For every y = x^2 ∈ N, there does not exist x ∈ N.
Example: 7 ∈ N (codomain) for which there is no x in domain N such that x^2 = 7
∴ f is not surjective.

(ii) f : Z → Z given by f(x) = x^2
Solution:
f: Z → Z given by f(x) = x2

$$\text{Let } f(x_1) = f(x_2), \quad x_1,\ x_2 \in Z$$
$$\therefore \quad x_1^2 = x_2^2$$
$$\therefore \quad x_1^2 - x_2^2 = 0$$
$$\therefore \quad (x_1 - x_2)(x_1 + x_2) = 0$$
$$\therefore \quad x_1 = x_2 \text{ or } x_1 = -x_2$$

∴ f is not injective.
(Example: f(-2) = 4 = f(2). So, -2, 2 have the same image. So, f is not injective.)
Since x^2 ≥ 0,
f(x) ≥ 0
Therefore all negative integers of codomain are not images under f.
∴ f is not surjective.

(iii) f : R → R given by f(x) = x^2
Solution:
f : R → R given by f(x) = x^2

Let $f(x_1) = f(x_2), x_1, x_2 \in R$

$\therefore \quad x_1^2 = x_2^2$

$\therefore \quad x_1^2 - x_2^2 = 0$

$\therefore \quad (x_1 - x_2)(x_1 + x_2) = 0$

$\therefore \quad x_1 = x_2 \quad \text{or} \quad x_1 = -x_2$

∴ f is not injective.
f(x) = $x^2 \geq 0$
Therefore all negative integers of codomain are not images under f.
∴ f is not surjective.

(iv) f : N → N given by f(x) = x^3
Solution:
f: N → N given by f(x) = x^3

Let $f(x_1) = f(x_2), x_1, x_2 \in N$

$\therefore \quad x_1^3 = x_2^3$

$\therefore \quad x_1^3 - x_2^3 = 0$

$$\therefore \quad (x_1 - x_2) \underbrace{(x_1^2 + x_1 x_2 + x_2^2)}_{> 0 \text{ for all } x_1,\, x_2 \text{ as it's discriminant} < 0} = 0$$

$\therefore \quad x_1 = x_2$

∴ f is injective.
Numbers from codomain which are not cubes of natural numbers are not images under f.
∴ f is not surjective.

(v) f : R → R given by f(x) = x^3
Solution:
f: R → R given by f(x) = x^3

Let $x_1^3 = x_2^3, x_1, x_2 \in R$

$\therefore \quad x_1^3 - x_2^3 = 0$

$$\therefore \quad (x_1 - x_2) \underbrace{(x_1^2 + x_1 x_2 + x_2^2)}_{> 0 \text{ for all } x_1,\, x_2 \text{ as it's discriminant} < 0} = 0$$

$\therefore \quad x_1 = x_2$

$\therefore$ **f is injective.**

Let $y = x^3$

$\therefore \quad x = y^{\frac{1}{3}}$

∴ For every y ∈ R, there is some x ∈ R.
∴ f is surjective.

Question 14.
Show that if f : A → B and g : B → C are one-one, then gof is also one-one.
Solution:
f is a one-one function.
Let $f(x_1) = f(x_2)$
Then, $x_1 = x_2$ for all x_1, x_2 …..(i)
g is a one-one function.
Let $g(y_1) = g(y_2)$
Then, $y_1 = y_2$ for all y_1, y_2 …..(ii)
Let (gof) (x_1) = (gof) (x_2)
∴ $g(f(x_1)) = g(f(x_2))$
∴ $g(y_1) = g(y_2)$,
where $y_1 = f(x_1)$, $y_2 = f(x_2) \in B$
∴ $y_1 = y_2$ …..[From (ii)]
i.e., $f(x_1) = f(x_2)$
∴ $x_1 = x_2$ ….[From (i)]
∴ gof is one-one.

Question 15.
Show that if f : A → B and g : B → C are onto, then gof is also onto.
Solution:
Since g is surjective (onto),

there exists y ∈ B for every z ∈ C such that
g(y) = z …….(i)
Since f is surjective,
there exists x ∈ A for every y ∈ B such that
f(x) = y …….(ii)
(gof) x = g(f(x))
= g(y) ……[From (ii)]
= z …..[From(i)]
i.e., for every z ∈ C, there is x in A such that (gof) x = z
∴ gof is surjective (onto).

Question 16.
If $f(x) = 3(4^{x+1})$, find f(-3).
Solution:
$f(x) = 3(4^{x+1})$
∴ $f(-3) = 3(4^{-3+1})$
$= 3(4^{-2})$
$= \frac{3}{16}$

Question 17.
Express the following exponential equations in logarithmic form:
(i) $2^5 = 32$
(ii) $54^0 = 1$
(iii) $23^1 = 23$
(iv) $9^{\frac{3}{2}} = 27$
(v) $3^{-4} = \frac{1}{81}$
(vi) $10^{-2} = 0.01$
(vii) $e^2 = 7.3890$
(viii) $e^{\frac{1}{2}} = 1.6487$
(ix) $e^{-x} = 6$
Solution:

i. $2^5 = 32$
∴ $5 = \log_2 32$ …[By definition of logarithm]
i.e., $\log_2 32 = 5$

ii. $54^0 = 1$

$\therefore$ $0 = \log_{54} 1$ …[By definition of logarithm]

i.e., $\log_{54} 1 = 0$

iii. $23^1 = 23$

$\therefore$ $1 = \log_{23} 23$ …[By definition of logarithm]

i.e., $\log_{23} 23 = 1$

iv. $9^{\frac{3}{2}} = 27$

$\therefore$ $\frac{3}{2} = \log_9 27$ …[By definition of logarithm]

i.e., $\log_9 27 = \frac{3}{2}$

v. $3^{-4} = \frac{1}{81}$

$\therefore$ $-4 = \log_3 \left(\frac{1}{81}\right)$ …[By definition of logarithm]

i.e., $\log_3 \left(\frac{1}{81}\right) = -4$

vi. $10^{-2} = 0.01$

$\therefore$ $-2 = \log_{10}(0.01)$…[By definition of logarithm]

i.e., $\log_{10}(0.01) = -2$

vii. $e^2 = 7.3890$

$\therefore$ $2 = \log_e(7.3890)$ …[By definition of logarithm]

i.e., $\log_e(7.3890) = 2$

(e is a mathematical constant, whose value is approximately 2.71828)

viii. $e^{\frac{1}{2}} = 1.6487$

$\therefore$ $\frac{1}{2} = \log_e(1.6487)$…[By definition of logarithm]

i.e., $\log_e(1.6487) = \frac{1}{2}$

ix. $e^{-x} = 6$

$\therefore$ $-x = \log_e 6$ …[By definition of logarithm]

i.e., $\log_e 6 = -x$

Question 18.

Express the following logarithmic equations in exponential form:

(i) $\log_2 64 = 6$

(ii) $log_5 125$ = -2

(iii) $\log_{10} 0.001 = -3$

(iv) $log_{12}(8)$ = -3

(v) ln 1 = 0

(vi) ln e = 1

(vii) ln 12 = -0.693

Solution:

(i) $\log_2 64 = 6$

$\therefore 64 = 2^6$, i.e., $2^6 = 64$

ii. $\log_5 \left(\frac{1}{25}\right) = -2$

$\therefore \quad \frac{1}{25} = 5^{-2}$, i.e., $5^{-2} = \frac{1}{25}$

iii. $\log_{10}(0.001) = -3$

$\therefore \quad 0.001 = 10^{-3}$, i.e., $10^{-3} = 0.001$

iv. $\log_{\frac{1}{2}}(8) = -3$

$\therefore \quad 8 = \left(\frac{1}{2}\right)^{-3}$, i.e., $\left(\frac{1}{2}\right)^{-3} = 8$

v. $\ln 1 = 0$

$\therefore \quad 1 = e^0$, i.e., $e^0 = 1$

vi. $\ln e = 1$

$\therefore \quad e = e^1$, i.e., $e^1 = e$

vii. $\ln\left(\frac{1}{2}\right) = -0.693$

$\therefore \quad \frac{1}{2} = e^{-0.693}$, i.e., $e^{-0.693} = \frac{1}{2}$

Question 19.
Find the domain of
(i) $f(x) = \ln(x - 5)$
(ii) $f(x) = \log 10 (x^2 - 5x + 6)$
Solution:
(i) $f(x) = \ln(x - 5)$
f is defined, when $x - 5 > 0$
$\therefore x > 5$
$\therefore$ Domain of f = $(5, \infty)$

(ii) $f(x) = \log_{10}(x^2 - 5x + 6)$
$x^2 - 5x + 6 = (x - 2)(x - 3)$
f is defined, when $(x - 2)(x - 3) > 0$
$\therefore x < 2$ or $x > 3$
Solution of $(x - a)(x - b) > 0$ is $x < a$ or $x > b$ where $a < b$
$\therefore$ Domain of f = $(-\infty, 2) \cup (3, \infty)$

Question 20.
Write the following expressions as sum or difference of logarithms:
(a) *log(pqrs)*
Solution:

$$\log\left(\frac{pq}{rs}\right) = \log(pq) - \log(rs)$$

$$\ldots \left[\log\frac{m}{n} = \log m - \log n\right]$$

$$= \log p + \log q - (\log r + \log s)$$

$$\ldots \left[\log mn = \log m + \log n\right]$$

$$= \log p + \log q - \log r - \log s$$

(b) $log(x--\sqrt{y}\sqrt{3})$

Solution:

$$\log\left(\sqrt{x}\,\sqrt[3]{y}\right) = \log\left(\sqrt{x}\right) + \log\left(\sqrt[3]{y}\right) \quad \ldots [\log mn = \log m + \log n]$$

$$= \log x^{\frac{1}{2}} + \log y^{\frac{1}{3}}$$

$$= \frac{1}{2}\log x + \frac{1}{3}\log y \quad \ldots [\log m^n = n\log m]$$

(c) $ln(a3(a-2)2b2+5\sqrt{})$

Solution:

$$\ln\left(\frac{a^3(a-2)^2}{\sqrt{b^2+5}}\right)$$

$$= \ln(a^3(a-2)^2) - \ln\sqrt{b^2+5} \quad \ldots \left[\log\frac{m}{n} = \log m - \log n\right]$$

$$= \ln a^3 + \ln(a-2)^2 - \ln\left(b^2+5\right)^{\frac{1}{2}} \quad \ldots [\log mn = \log m + \log n]$$

$$= 3\ln a + 2\ln(a-2) - \frac{1}{2}\ln(b^2+5) \quad \ldots [\log m^n = n\log m]$$

(d) $ln[x-2\sqrt{3}(2x+1)4(x+4)2x+4\sqrt{}]2$

Solution:

$$\ln\left[\frac{\sqrt[3]{x-2}\,(2x+1)^4}{(x+4)\sqrt{2x+4}}\right]^2$$

$$= 2\ln\left[\frac{\sqrt[3]{x-2}\,(2x+1)^4}{(x+4)\sqrt{2x+4}}\right] \quad \ldots [\log m^n = n\log m]$$

$$= 2\left[\ln\sqrt[3]{x-2}(2x+1)^4 - \ln(x+4)\sqrt{2x+4}\right]$$

$$= 2\left[\ln\sqrt[3]{x-2} + \ln(2x+1)^4 - \left(\ln(x+4) + \ln\sqrt{2x+4}\right)\right]$$

$$= 2\left[\frac{1}{3}\ln(x-2) + 4\ln(2x+1) - \ln(x+4) - \frac{1}{2}\ln(2x+4)\right]$$

Question 21.

Write the following expressions as a single logarithm.

(i) 5 log x + 7 log y – log z

Solution:

$$5\log x + 7\log y - \log z$$

$$= \log(x^5) + \log(y^7) - \log z \quad \ldots [n\log m = \log m^n]$$

$$= \log(x^5 y^7) - \log z \quad \ldots [\log m + \log n = \log mn]$$

$$= \log\left(\frac{x^5 y^7}{z}\right) \quad \ldots \left[\log m - \log n = \log\frac{m}{n}\right]$$

(ii) $\frac{1}{3}$ log(x – 1) + $\frac{1}{2}$ log(x)

Solution:

$$\frac{1}{3}\log(x-1)+\frac{1}{2}\log x$$

$$=\log\left((x-1)^{\frac{1}{3}}\right)+\log\left(x^{\frac{1}{2}}\right)$$

$$\ldots [\text{n log m} = \text{log m}^{\text{n}}]$$

$$=\log\left(\sqrt[3]{x-1}\,\sqrt{x}\right)$$

$$\ldots [\text{log m} + \text{log n} = \text{log mn}]$$

(iii) ln (x + 2) + ln (x – 2) – 3 ln (x + 5)

Solution:

$$\ln(x+2)+\ln(x-2)-3\ln(x+5)$$

$$=\ln[(x+2)(x-2)]-\ln(x+5)^3$$

$$\ldots\begin{bmatrix}\log m+\log n=\log mn\\ n\log m=\log m^n\end{bmatrix}$$

$$=\ln(x^2-4)-\ln(x+5)^3$$

$$=\ln\left(\frac{x^2-4}{(x+5)^3}\right)\qquad\ldots\left[\log m-\log n=\log\frac{m}{n}\right]$$

Question 22.

Given that log 2 = a and log 3 = b, write log √96 terms of a and b.

Solution:

log 2 = a and log 3 = b

log √96 = $\frac{1}{2}$ log (96)

= $\frac{1}{2}$ log (2^5 x 3)

= $\frac{1}{2}$ (log 2^5 + log 3) .....[∵ log mn = log m + log n]

= $\frac{1}{2}$ (5 log 2 + log 3) ......[∵ log m^n = n log m]

= $\frac{5a+b}{2}$

Question 23.

Prove that:

(a) $b^{\log_b a}=a$

Solution:

We have to prove that $b^{\log_b a}=a$

i.e., to prove that (logb a) ($\log_b$ b) = $\log_b$ a

(Taking log on both sides with base b)

L.H.S. = ($\log_b$ a) ($\log_b$ b)

= $\log_b$ a .....[∵ $\log_b$ b = 1]

= R.H.S.

(b) $\log_{b^m}a=\frac{1}{m}\log_b a$

Solution:

$$\log_{b^m}a=\frac{1}{m}\log_b a$$

$$\text{L.H.S.}=\log_{b^m}a$$

$$=\frac{\log a}{\log b^m}\qquad\ldots\left[\log_y x=\frac{\log x}{\log y}\right]$$

$$=\frac{\log a}{m\log b}=\frac{1}{m}\log_b a=\text{R.H.S.}$$

(c) $a^{\log_c b} = b^{\log_c a}$
Solution:

$$a^{\log_c b} = b^{\log_c a}$$

$$\begin{aligned} \text{L.H.S.} &= a^{\log_c b} \\ &= \left(e^{\log a}\right)^{\log_c b} \quad \ldots[x = e^{\log x}] \\ &= \left(e^{\log a}\right)^{\frac{\log b}{\log c}} = \left(e^{\log b}\right)^{\frac{\log a}{\log c}} \\ &= \left(e^{\log b}\right)^{\log_c a} = b^{\log_c a} = \text{R.H.S.} \end{aligned}$$

Question 24.
If $f(x) = ax^2 - bx + 6$ and $f(2) = 3$ and $f(4) = 30$, find a and b.
Solulion:
$f(x) = ax^2 - bx + 6$
$f(2) = 3$
$\therefore a(2)^2 - b(2) + 6 = 3$
$\therefore 4a - 2b + 6 = 3$
$\therefore 4a - 2b + 3 = 0$ …..(i)
$f(4) = 30$
$\therefore a(4)^2 - b(4) + 6 = 30$
$\therefore 16a - 4b + 6 = 30$
$\therefore 16a - 4b - 24 = 0$ …..(ii)
By (ii) – 2 × (i), we get
$8a - 30 = 0$
$\therefore a = \frac{30}{8} = \frac{15}{4}$
Substiting $a = \frac{15}{4}$ in (i), we get
$4\left(\frac{15}{4}\right) - 2b + 3 = 0$
$\therefore 2b = 18$
$\therefore b = 9$
$\therefore a = \frac{15}{4}$, $b = 9$

Question 25.
Solve for x:
(i) $\log 2 + \log (x + 3) - \log (3x - 5) = \log 3$
Solution:
$\log 2 + \log (x + 3) - \log (3x - 5) = \log 3$
$\therefore \log 2(x + 3) - \log(3x - 5) = \log 3$ …..[$\because \log m + \log n = \log mn$]
$\therefore \log \frac{2(x+3)}{3x-5} = \log 3$ …..[$\because \log m - \log n = \log \frac{m}{n}$]
$\therefore \frac{2(x+3)}{3x-5} = 3$
$\therefore 2x + 6 = 9x - 15$
$\therefore 7x = 21$
$\therefore x = 3$

Check:
If x = 3 satisfies the given condition, then our answer is correct.
L.H.S. = $\log 2 + \log (x + 3) - \log (3x - 5)$
$= \log 2 + \log (3 + 3) - \log (9 - 5)$
$= \log 2 + \log 6 - \log 4$
$= \log (2 \times 6) - \log 4$
$= \log \frac{12}{4}$
$= \log 3$
= R.H.S.
Thus, our answer is correct.

(ii) $2\log_{10} x = 1 + \log_{10}\left(x + \frac{11}{10}\right)$
Solution:

$$2 \log_{10} x = 1 + \log_{10}\left(x + \frac{11}{10}\right)$$

$$\log_{10} x^2 - \log_{10}\left(x + \frac{11}{10}\right) = 1$$

$$\ldots[n\log m = \log m^n]$$

$\therefore \quad \log_{10} x^2 - \log_{10}\left(\frac{10x+11}{10}\right) = 1$

$\therefore \quad \log_{10}\left(\frac{x^2}{\frac{10x+11}{10}}\right) = 1 \quad \ldots\left[\log m - \log n = \log \frac{m}{n}\right]$

$\therefore \quad \log_{10}\left(\frac{10x^2}{10x+11}\right) = \log_{10} 10 \quad \ldots[\log_a a = 1]$

$\therefore \quad \frac{10x^2}{10x+11} = 10$

$\therefore \quad \frac{x^2}{10x+11} = 1$

∴ $x^2 = 10x + 11$
∴ $x^2 - 10x - 11 = 0$
∴ $(x - 11)(x + 1) = 0$
∴ $x = 11$ or $x = -1$
But log of a negative numbers does not exist
∴ $x \neq -1$
∴ $x = 11$

(iii) $\log_2 x + \log_4 x + \log_{16} x = 214$
Solution:

$\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$

$\therefore \quad \frac{\log x}{\log 2} + \frac{\log x}{\log 4} + \frac{\log x}{\log 16} = \frac{21}{4} \quad \ldots\left[\log_y x = \frac{\log x}{\log y}\right]$

$\therefore \quad \frac{\log x}{\log 2} + \frac{\log x}{\log (2)^2} + \frac{\log x}{\log (2)^4} = \frac{21}{4}$

$\therefore \quad \frac{\log x}{\log 2} + \frac{\log x}{2.\log 2} + \frac{\log x}{4.\log 2} = \frac{21}{4}$

$\ldots[\log m^n = n\log m]$

$\therefore \quad \frac{\log x}{\log 2}\left(1 + \frac{1}{2} + \frac{1}{4}\right) = \frac{21}{4}$

$\therefore \quad \frac{\log x}{\log 2} \times \left(\frac{7}{4}\right) = \frac{21}{4}$

$\therefore \quad \frac{\log x}{\log 2} = 3$

$\therefore \quad \log x = 3 \log 2$

$\therefore \quad \log x = \log 2^3$

$\therefore \quad x = 2^3$

$\therefore \quad x = 8$

(iv) $x + \log_{10}(1 + 2^x) = x \log_{10} 5 + \log_{10} 6$
Solution:

$x + \log_{10}(1 + 2^x) = x \log_{10} 5 + \log_{10} 6$

$\therefore \quad x \log_{10} 10 + \log_{10}(1 + 2^x) = x \log_{10} 5 + \log_{10} 6$

$\ldots[\log_a a = 1]$

$\therefore \quad \log_{10} 10^x + \log_{10}(1 + 2^x) = \log_{10} 5^x + \log_{10} 6$

$\ldots[n\log m = \log m^n]$

$\therefore \quad \log_{10}[10^x.(1 + 2^x)] = \log_{10}(6 \times 5^x)$

$\ldots[\log m + \log n = \log mn]$

$\therefore \quad 10^x(1 + 2^x) = 6 \times 5^x$

$\therefore \quad 2^x \times 5^x(1 + 2^x) = 6 \times 5^x$

$\therefore \quad 2^x(1 + 2^x) = 6$

Let $2^x = a$

$\therefore \quad a.(1 + a) = 6$

∴ $a + a^2 = 6$
∴ $a^2 + a - 6 = 0$
∴ $(a + 3)(a - 2) = 0$
∴ $a + 3 = 0$ or $a - 2 = 0$
∴ $a = -3$ or $a = 2$
Since $2^x = -3$ is not possible,

2x = 2 = 21
∴ x = 1

Question 26.
If log (x+y3) = 12 log x + 12 log y, show that xy+yx = 7.
Solution:

$\log\left(\frac{x+y}{3}\right) = \frac{1}{2}\log x + \frac{1}{2}\log y$

Multiplying throughout by 2, we get

$2\log\left(\frac{x+y}{3}\right) = \log x + \log y$

∴ $2\log\left(\frac{x+y}{3}\right) = \log xy$ ...[log m + log n = log mn]

∴ $\log\left(\frac{x+y}{3}\right)^2 = \log xy$ ...[nlog m = log m^n]

∴ $\frac{(x+y)^2}{9} = xy$

∴ $x^2 + 2xy + y^2 = 9xy$

∴ $x^2 + y^2 = 7xy$

Dividing throughout by xy, we get

$\frac{x}{y} + \frac{y}{x} = 7$

Question 27.
If log(x−y4) = log√x + log√y, show that (x + y)2 = 20xy.
Solution:

$\log\left(\frac{x-y}{4}\right) = \log\sqrt{x} + \log\sqrt{y}$

∴ $\log\left(\frac{x-y}{4}\right) = \log\left(\sqrt{x}.\sqrt{y}\right)$ ...[log m + log n = log mn]

∴ $\log\left(\frac{x-y}{4}\right) = \log\sqrt{xy}$

∴ $\frac{x-y}{4} = \sqrt{xy}$

Squaring on both sides, we get

$\frac{(x-y)^2}{16} = xy$

∴ $x^2 - 2xy + y^2 = 16xy$

Adding $4xy$ on both sides, we get

$x^2 + 2xy + y^2 = 20xy$

∴ $(x+y)^2 = 20xy$

Question 28.
If x = $\log_a bc$, y = $\log_b ca$, z = $\log_c ab$, then prove that 11+x+11+y+11+z = 1.

Solution:

$x = \log_a (bc), y = \log_b (ca), z = \log_c (ab)$

$$\text{L.H.S.} = \frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$$

$$= \frac{1}{1+\log_a(bc)} + \frac{1}{1+\log_b(ca)} + \frac{1}{1+\log_c(ab)}$$

$$= \frac{1}{\log_a a+\log_a(bc)} + \frac{1}{\log_b b+\log_b(ca)} + \frac{1}{\log_c c+\log_c(ab)} \quad \ldots[\log_a a = 1]$$

$$= \frac{1}{\log_a(abc)} + \frac{1}{\log_b(abc)} + \frac{1}{\log_c(abc)} \quad \ldots[\log_a m + \log_a n = \log_a mn]$$

$$= \frac{\log a}{\log(abc)} + \frac{\log b}{\log(abc)} + \frac{\log c}{\log(abc)} \quad \ldots\left[\log_y x = \frac{\log x}{\log y}\right]$$

$$= \frac{\log a+\log b+\log c}{\log abc}$$

$$= \frac{\log abc}{\log abc}$$

$= 1$

$= \text{R.H.S.}$

Maharashtra State Board 11th Maths Solutions Chapter 6 Functions Ex 6.2

Question 1.
If f(x) = 3x + 5, g(x) = 6x – 1, then find
(i) (f + g) (x)
(ii) (f – g) (2)
(iii) (fg) (3)
(iv) (f/g) (x) and its domain
Solution:
f(x) = 3x + 5, g (x) = 6x – 1
(i) (f + g) (x) = f (x) + g (x)
= 3x + 5 + 6x – 1
= 9x + 4

(ii) (f – g) (2) = f(2) – g(2)
= [3(2) + 5] – [6(2) – 1]
= 6 + 5 – 12 + 1
= 0

(iii) (fg) (3) = f (3) g(3)
= [3(3) + 5] [6(3) – 1]
= (14) (17)
= 238

(iv) $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{3x+5}{6x-1}, x\neq\frac{1}{6}$
Domain = R – $\left\{\frac{1}{6}\right\}$

Question 2.
Let f: (2, 4, 5} → {2, 3, 6} and g: {2, 3, 6} → {2, 4} be given by f = {(2, 3), (4, 6), (5, 2)} and g = {(2, 4), (3, 4), (6, 2)}. Write down gof.
Solution:
f = {(2, 3), (4, 6), (5, 2)}
∴ f(2) = 3, f(4) = 6, f(5) = 2
g ={(2, 4), (3, 4), (6, 2)}
∴ g(2) = 4, g(3) = 4, g(6) = 2
gof: {2, 4, 5} → {2, 4}
(gof) (2) = g(f(2)) = g(3) = 4
(gof) (4) = g(f(4)) = g(6) = 2
(gof) (5) = g(f(5)) = g(2) = 4
∴ gof = {(2, 4), (4, 2), (5, 4)}

Question 3.
If $f(x) = 2x^2 + 3$, $g(x) = 5x – 2$, then find
(i) fog
(ii) gof
(iii) fof
(iv) gog
Solution:
$f(x) = 2x^2 + 3$, $g(x) = 5x – 2$
(i) (fog) (x) = f(g(x))
= f(5x – 2)
$= 2(5x – 2)^2 + 3$
$= 2(25x^2 – 20x + 4) + 3$
$= 50x^2 – 40x + 11$

(ii) (gof) (x) = g(f(x))
$= g(2x^2 + 3)$
= 5(2x + 3) – 2
$= 10x^2 + 13$

(iii) (fof) (x) = f(f(x))
$= f(2x^2 + 3)$
$= 2(2x^2 + 3)^2 + 3$
$= 2(4x^4 + 12x^2 + 9) + 3$
$= 8x^4 + 24x^2 + 21$

(iv) (gog) (x) = g(g(x))
= g(5x – 2)
= 5(5x – 2) – 2
= 25x – 12

Question 4.
Verify that f and g are inverse functions of each other, where
(i) f(x) = x−74, g(x) = 4x + 7
(ii) $f(x) = x^3 + 4$, g(x) = X−4−−−−−√3
(iii) f(x) = x+3x−2, g(x) = 2x+3x−1
Solution:
(i) f(x) = x−74
Replacing x by g(x), we get
f[g(x)] = g(x)−74=4x+7−74 = x
g(x) = 4x + 7
Replacing x by f(x), we get
g[f(x)] = 4f(x) + 7 = 4(x−74) + 7 = x
Here, f[g(x)] = x and g[f(x)] = x.
∴ f and g are inverse functions of each other.

(ii) $f(x) = x^3 + 4$
Replacing x by g(x), we get
$f[g(x)] = [g(x)]^3 + 4$
= (X−4−−−−−√3)3+4
= x – 4 + 4
= x

g(x) = x−4−−−−−√$_3$

Replacing x by f(x), we get

g[f(x)] = f(x)−4−−−−−−−√$_3$=x$_3$+4−4−−−−−−−−√$_3$=x$_3$−−√$_3$ = x

Here, f[g(x)] = x and g[f(x)] = x

∴ f and g are inverse functions of each other.

(iii) f(x) = x+3x−2

Replacing x by g(x), we get

$$f[g(x)] = \frac{g(x)+3}{g(x)-2} = \frac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2}$$

$$= \frac{2x+3+3x-3}{2x+3-2x+2} = \frac{5x}{5} = x$$

$$g(x) = \frac{2x+3}{x-1}$$

Replacing x by f(x), we get

$$g[f(x)] = \frac{2f(x)+3}{f(x)-1} = \frac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1}$$

$$= \frac{2x+6+3x-6}{x+3-x+2} = \frac{5x}{5} = x$$

Here, f[g(x)] = x and g[f(x)] = x.

∴ f and g are inverse functions of each other.

Question 5.

Check if the following functions have an inverse function. If yes, find the inverse function.

(i) f(x) = 5x$_2$

(ii) f(x) = 8

(iii) f(x) = 6x−73

(iv) f(x) = 4x+5−−−−−√

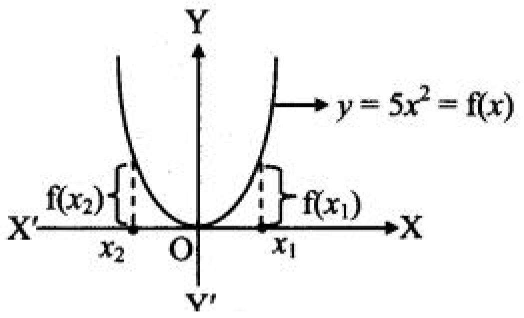
(v) f(x) = 9x$_3$ + 8

$$\begin{cases} x+7 & x<0 \\ 8-x & x\geq 0 \end{cases}$$

(vi) f(x) =

Solution:

(i) f(x) = 5x$_2$ = y (say)



For two values (x$_1$ and x$_2$) of x, values of the function are equal.

∴ f is not one-one.

∴ f does not have an inverse.

(ii) f(x) = 8 = y (say)

For every value of x, the value of the function f is the same.

∴ f is not one-one i.e. (many-one) function.

∴ f does not have the inverse.

(iii) f(x) = 6x−73

Let f(x$_1$) = f(x$_2$)

∴ 6x$_1$−73=6x$_2$−73

∴ x$_1$ = x$_2$

∴ f is a one-one function.

f(x) = 6x−73 = y (say)

∴ x = 3y+76

∴ For every y, we can get x

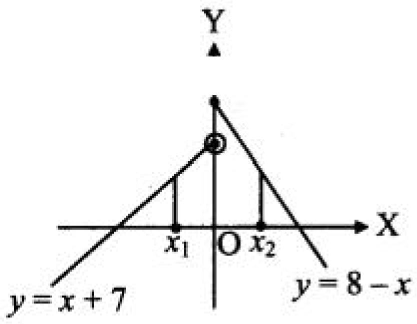
∴ f is an onto function.

∴ x = 3y+76 = f-1 (y)
Replacing y by x, we get
f-1 (x) = 3x+76

(iv) f(x) = 4x+5−−−−−√,x≥−54
Let $f(x_1) = f(x_2)$
∴ $4x_1+5$−−−−−−√=$4x_2+5$−−−−−−√
∴ $x_1 = x_2$
∴ f is a one-one function.
f(x) = 4x+5−−−−−√ = y, (say) y ≥ 0
Squaring on both sides, we get
$y^2 = 4x + 5$
∴ x = y^2−54
∴ For every y we can get x.
∴ f is an onto function.
∴ x = y^2−54 = f^{-1} (y)
Replacing y by x, we get
f^{-1} (x) = x^2−54

(v) f(x) $9x^3 + 8$
Let $f(x_1) = f(x_2)$
∴ $9x_1^3+8=9x_2^3+8$
∴ $x_1 = x_2$
∴ f is a one-one function.
∴ f(x) = $9x^3 + 8$ = y, (say)
∴ x = y−89−−−√3
∴ For every y we can get x.
∴ f is an onto function.
∴ x = y−89−−−√3 = f^{-1} (y)
Replacing y by x, we get
f^{-1} (x) = x−89−−−√3

(vi) f(x) = x + 7, x < 0
= 8 – x, x ≥ 0



We observe from the graph that for two values of x, say x_1, x_2 the values of the function are equal.
i.e. $f(x_1) = f(x_2)$
∴ f is not one-one (i.e. many-one) function.
∴ f does not have inverse.

Question 6.

If f(x) = $\begin{cases} x^2+3, & x \leq 2 \\ 5x+7, & x > 2 \end{cases}$, then find
(i) f(3)
(ii) f(2)
(iii) f(0)
Solution:
f(x) = $x^2 + 3$, x ≤ 2
= 5x + 7, x > 2
(i) f(3) = 5(3) + 7
= 15 + 7
= 22

(ii) $f(2) = 2^2 + 3$
= 4 + 3
= 7

(iii) $f(0) = 0^2 + 3 = 3$

Question 7.
If f(x) = $\begin{cases} 4x-2, & x \le -3 \\ 5, & -3 < x < 3 \\ x^2, & x \ge 3 \end{cases}$, then find
(i) f(-4)
(ii) f(-3)
(iii) f(1)
(iv) f(5)
Solution:
f(x) = 4x – 2, x ≤ -3
= 5, -3 < x < 3
= x^2, x ≥ 3
(i) f(-4) = 4(-4) – 2
= -16 – 2
= -18

(ii) f(-3) = 4(-3) – 2
= -12 – 2
= -14

(iii) f(1) = 5

(iv) $f(5) = 5^2 = 25$

Question 8.
If f(x) = 2 |x| + 3x, then find
(i) f(2)
(ii) f(-5)
Solution:
f(x) = 2 |x| + 3x
(i) f(2) = 2|2| + 3(2)
= 2 (2) + 6 ….. [∵ |x| = x, x > 0]
= 10

(ii) f(-5) = 2 |-5| + 3(-5)
= 2(5) – 15 …..[∵ |x| = -x, x < 0]
= 10 – 15
= -5

Question 9.
If f(x) = 4[x] – 3, where [x] is greatest integer function of x, then find
(i) f(7.2)
(ii) f(0.5)
(iii) $f(-\frac{5}{2})$
(iv) f(2π), where π = 3.14
Solution:
f(x) = 4[x] – 3
(i) f(7.2) = 4 [7.2] – 3
= 4(7) – 3 ………[∵ 7 ≤ 7.2 < 8, [7.2] = 7]
= 25

(ii) f(0.5) = 4[0.5] – 3
= 4(0) – 3 ………[∵ 0 ≤ 0.5 < 1, [0.5] = 0]
= -3

(iii) $f(-\frac{5}{2})$ = f(-2.5)
= 4[-2.5] – 3
= 4(-3) – 3 …….[∵-3 ≤ -2.5 ≤ -2, [-2.5] = -3]
= -15

(iv) f(2π) = 4[2π] – 3
= 4[6.28] – 3 …..[∵ π = 3.14]
= 4(6) – 3 …….[∵ 6 ≤ 6.28 < 7, [6.28] = 6]
= 21

Question 10.
If f(x) = 2{x} + 5x, where {x} is fractional part function of x, then find
(i) f(-1)
(ii) f(14)
(iii) f(-1.2)
(iv) f(-6)
Solution:
f(x) = 2{x} + 5x
(i) {-1} = -1 – [-1] = -1 + 1 = 0
∴ f(-1) = 2 {-1} + 5(-1)
= 2(0) – 5
= -5

(ii) {14} = 14 – [latex]\frac{1}{4}[/latex] = 14 – 0 = 14
f(14) = 2{14} + 5(14)
= 2(14) + 54
= 74
= 1.75

(iii) {-1.2} = -1.2 – [-1.2] = -1.2 + 2 = 0.8
f(-1.2) = 2{-1.2} + 5(-1.2)
= 2(0.8) + (-6)
= -4.4

(iv) {-6} = -6 – [-6] = -6 + 6 = 0
f(-6) = 2{-6} + 5(-6)
= 2(0) – 30
= -30

Question 11.
Solve the following for x, where |x| is modulus function, [x] is the greatest integer function, {x} is a fractional part function.
(i) |x + 4| ≥ 5
(ii) |x – 4| + |x – 2| = 3
(iii) x^2 + 7|x| + 12 = 0
(iv) |x| ≤ 3
(v) 2|x| = 5
(vi) [x + [x + [x]]] = 9
(vii) {x} > 4
(viii) {x} = o
(ix) {x} = 0.5
(x) 2{x} = x + [x]
Solution:
(i) |x + 4| ≥ 5
The solution of |x| ≥ a is x ≤ -a or x ≥ a
∴ |x + 4| ≥ 5 gives
∴ x + 4 ≤ -5 or x + 4 ≥ 5
∴ x ≤ -5 – 4 or x ≥ 5 – 4
∴ x ≤ -9 or x ≥ 1
∴ The solution set = (-∞, – 9] ∪ [1, ∞)

(ii) |x – 4| + |x – 2| = 3 …..(i)
Case I: x < 2
Equation (i) reduces to
4 – x + 2 – x = 3 …….[x < 2 < 4, x – 4 < 0, x – 2 < 0]
∴ 6 – 3 = 2x
∴ x = 32

Case II: 2 ≤ x < 4
Equation (i) reduces to
4 – x + x – 2 = 3
∴ 2 = 3 (absurd)
There is no solution in [2, 4)

Case III: $x \geq 4$
Equation (i) reduces to
$x - 4 + x - 2 = 3$
$\therefore 2x = 6 + 3 = 9$
$\therefore x = \frac{9}{2}$
$\therefore x = \frac{3}{2}, \frac{9}{2}$ are solutions.
The solution set = $\{\frac{3}{2}, \frac{9}{2}\}$

(iii) $x^2 + 7|x| + 12 = 0$
$\therefore (|x|)^2 + 7|x| + 12 = 0$
$\therefore (|x| + 3)(|x| + 4) = 0$
∴ There is no x that satisfies the equation.
The solution set = { } or Φ

(iv) $|x| \leq 3$ The solution set of $|x| \leq a$ is $-a \leq x \leq a$
∴ The required solution is $-3 \leq x \leq 3$
∴ The solution set is $[-3, 3]$

(v) $2|x| = 5$
$\therefore |x| = \frac{5}{2}$
$\therefore x = \pm\frac{5}{2}$

(vi) $[x + [x + [x]]] = 9$
$\therefore [x + [x] + [x]] = 9$ …….$[[x + n] = [x] + n$, if n is an integer]
$\therefore [x + 2[x]] = 9$
$\therefore [x] + 2[x] = 9$ …..[[2[x] is an integer]]
$\therefore [x] = 3$
$\therefore x \in [3, 4)$

(vii) $\{x\} > 4$
This is a meaningless statement as $0 \leq \{x\} < 1$
∴ The solution set = { } or Φ

(viii) $\{x\} = 0$
∴ x is an integer
∴ The solution set is Z.

(ix) $\{x\} = 0.5$
$\therefore x = \ldots, -2.5, -1.5, -0.5, 0.5, 1.5, \ldots$
∴ The solution set = $\{x : x = n + 0.5, n \in Z\}$

(x) $2\{x\} = x + [x]$
$= [x] + \{x\} + [x]$ ……$[x = [x] + \{x\}]$
$\therefore \{x\} = 2[x]$
R.H.S. is an integer
∴ L.H.S. is an integer
$\therefore \{x\} = 0$
$\therefore [x] = 0$
$\therefore x = 0$

Maharashtra State Board 11th Maths Solutions Chapter 6 Functions Miscellaneous Exercise 6

(I) Select the correct answer from the given alternatives.

Question 1.
If log (5x – 9) – log (x + 3) = log 2, then x = _______
(A) 3
(B) 5
(C) 2
(D) 7
Answer:
(B) 5
Hint:
log (5x – 9) – log (x + 3) = log 2
∴ $\frac{5x-9}{x+3}$ = 2
∴ 3x = 9 + 6
∴ x = 5

Question 2.
If $\log_{10}$ ($\log_{10}$ ($\log_{10}$ x)) = 0, then x = _______
(A) 1000
(B) 10^{10}
(C) 10
(D) 0
Answer:
(B) 10^{10}
Hint:
$\log_{10} \log_{10} \log_{10}$ x = 0
∴ $\log_{10}$ ($\log_{10}$ (x)) = 10^0 = 1
∴ $\log_{10}$ x = 10^1 = 10
∴ x = 10^{10}

Question 3.
Find x, if 2 $\log_2$ x = 4
(A) 4, -4
(B) 4
(C) -4
(D) not defined
Answer:
(B) 4
Hint:
2 $\log_2$ x = 4, x > 0
∴ $\log_2$ (x^2) = 4
∴ x^2 = 16
∴ x = ±4
∴ x = 4

Question 4.
The equation $\log_{x^2}16+\log_{2x}64=3$ has,
(A) one irrational solution
(B) no prime solution
(C) two real solutions
(D) one integral solution
Answer:
(A), (B), (C), (D)
Hint:
$\log_{x^2}16+\log_{2x}64=3$
∴ $\frac{\log 16}{\log x^2}+\frac{\log 64}{\log 2x}=3$
∴ 4 log 2 [log x + log 2] + (6 log 2) (2 log x) = 3 (2 log x) (log 2 + log x)
Let log 2 = a, log x = t. Then
∴ 4at + 4a^2 + 12at = 6at + 6t^2
∴ 6t^2 – 10at – 4a^2 = 0
∴ 3t^2 – 5at – 2a^2 = 0
∴ (3t + a) (t – 2a) = 0

∴ t = −13a, 2a

∴ log x = $log(2)_{-13}$, log (2_2)

∴ x = 2_{-13}, 4

∴ x = $12\sqrt{_3}$, 4

Question 5.
If f(x) = 11−x, then f(f(f(x))) is
(A) x – 1
(B) 1 – x
(C) x
(D) -x
Answer:
(C) x
Hint:

$$f(x) = \frac{1}{1-x}$$

$$f(f(x)) = f\left(\frac{1}{1-x}\right) = \frac{1}{1-\left(\frac{1}{1-x}\right)}$$

$$= \frac{1-x}{1-x-1} = \frac{x-1}{x}$$

$$f(f(f(x))) = f\left(\frac{x-1}{x}\right) = \frac{1}{1-\left(\frac{x-1}{x}\right)} = \frac{x}{x-x+1} = x$$

Question 6.
If f: R → R is defined by f(x) = x_3, then f_{-1} (8) is equal to:
(A) {2}
(B) {-2.2}
(C) {-2}
(D) (-2.2)
Answer:
(A) {2}
Hint:

$$f(x) = x^3 = y, \text{ say}$$

$$\therefore \quad x = y^{\frac{1}{3}} = f^{-1}(y)$$

$$\therefore \quad f^{-1}(8) = (8)^{\frac{1}{3}} = \left(2^3\right)^{\frac{1}{3}}$$

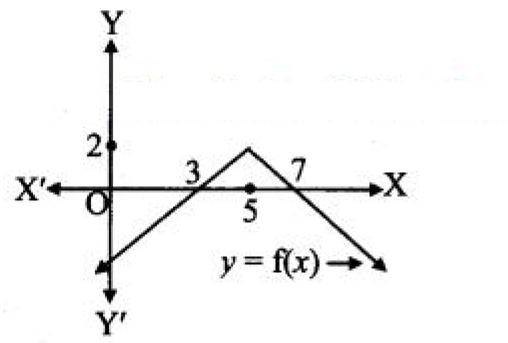
$$\therefore \quad f^{-1}(8) = 2$$

Question 7.
Let the function f be defined by f(x) = 2x+11−3x then f_{-1} (x) is:
(A) x−13x+2
(B) x+13x−2
(C) 2x+11−3x
(D) 3x+2x−1
Answer:
(A) x−13x+2
Hint:
f(x) = 2x+11−3x = y, say. then
2x + 1 = y(1 – 3x)
∴ y – 1 = x(2 + 3y)
∴ x = y−12+3y = f_{-1} (y)
∴ f_{-1} (x) = x−12+3x

Question 8.
If f(x) = $2x_2$ + bx + c and f(0) = 3 and f(2) = 1, then f(1) is equal to
(A) -2
(B) 0
(C) 1
(D) 2
Answer:

(B) 0
Hint:
$f(x) = 2x^2 + bx + c$
$f(0) = 3$
$\therefore 2(0) + b(0) + c = 3$
$\therefore c = 3$ ....(i)
$\therefore f(2) = 1$
$\therefore 2(4) + 2b + c = 1$
$\therefore 2b + c = -7$
$\therefore 2b + 3 = -7$ .....[From (i)]
$\therefore b = -5$
$\therefore f(x) = 2x^2 - 5x + 3$
$\therefore f(1) = 2(1)^2 - 5(1) + 3 = 0$

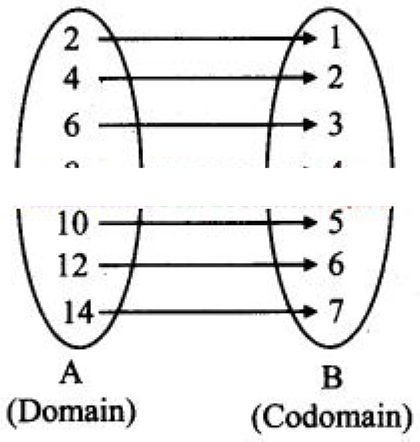
Question 9.
The domain of $\frac{1}{[x]-x}$, where [x] is greatest integer function is
(A) R
(B) Z
(C) R – Z
(D) Q – {0}
Answer:
(C) R – Z
Hint:
$f(x) = \frac{1}{[x]-x} = \frac{1}{-\{x\}}$
For f to be defined, $\{x\} \neq 0$
$\therefore$ x cannot be integer.
$\therefore$ Domain = R – Z

Question 10.
The domain and range of $f(x) = 2 - |x - 5|$ are
(A) R+, (-∞, 1]
(B) R, (-∞, 2]
(C) R, (-∞, 2)
(D) R+, (-∞, 2]
Answer:
(B) R, (-∞, 2]
Hint:


$f(x) = 2 - |x - 5|$
$= 2 - (5 - x), x < 5$
$= 2 - (x - 5), x \geq 5$
$\therefore f(x) = x - 3, x < 5$
$= 7 - x, x \geq 5$
Domain = R,
Range (from graph) = (-∞, 2]

(II) Answer the following:

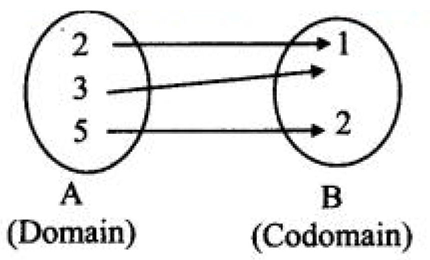
Question 1.
Which of the following relations are functions? If it is a function determine its domain and range.
(i) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5) (12, 6), (14, 7)}
(ii) {(0, 0), (1, 1), (1, -1), (4, 2), (4, -2) (9, 3), (9, -3), (16, 4), (16, -4)}
(iii) {(2, 1), (3, 1), (5, 2)}
Solution:
(i) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5) (12, 6), (14, 7)}



Every element of set A has been assigned a unique element in set B
∴ Given relation is a function
Domain = {2, 4, 6, 8, 10, 12, 14}, Range = {1, 2, 3, 4, 5, 6, 7}

(ii) {(0, 0), (1, 1), (1, -1), (4, 2), (4, -2) (9, 3), (9, -3) (16, 4), (16, -4)}
∵ (1, 1), (1, -1) ∈ the relation
∴ Given relation is not a function.
As element 1 of the domain has not been assigned a unique element of co-domain.

(iii) {(2, 1), (3, 1), (5, 2)}



Every element of set A has been assigned a unique element in set B.
∴ Given relation is a function.
Domain = {2, 3, 5}, Range = {1, 2}

Question 2.
Find whether the following functions are one-one.
(i) f: R → R defined by $f(x) = x^2 + 5$
(ii) f: R – {3} → R defined by $f(x) = \frac{5x+7}{x-3}$ for x ∈ R – {3}
Solution:
(i) f: R → R, defined by $f(x) = x^2 + 5$
Note that $f(-x) = f(x) = x^2 + 5$
∴ f is not one-one (i.e., many-one) function.

(ii) f: R – {3} → R, defined by $f(x) = \frac{5x+7}{x-3}$
Let $f(x_1) = f(x_2)$
∴ $\frac{5x_1+7}{x_1-3}=\frac{5x_2+7}{x_2-3}$
∴ $5x_1 x_2 – 15x_1 + 7x_2 – 21 = 5x_1 x_2 – 15x_2 + 7x_1 – 21$
∴ $22(x_1 – x_2) = 0$
∴ $x_1 = x_2$
∴ f is a one-one function.

Question 3.
Find whether the following functions are onto or not.
(i) f: Z → Z defined by f(x) = 6x – 7 for all x ∈ Z
(ii) f: R → R defined by $f(x) = x^2 + 3$ for all x ∈ R
Solution:
(i) f(x) = 6x – 7 = y (say)
(x, y ∈ Z)
∴ $x = \frac{7+y}{6}$
Since every integer y does not give integer x, f is not onto.

(ii) $f(x) = x^2 + 3 = y$ (say)
(x, y ∈ R)
Clearly y ≥ 3 …..[$x^2 \geq 0$]
∴ All the real numbers less than 3 from codomain R, have not been pre-assigned any element from the domain R.
∴ f is not onto.

Question 4.
Let f: R → R be a function defined by f(x) = $5x^3 - 8$ for all x ∈ R. Show that f is one-one and onto. Hence find f^{-1}.
Solution:

$f(x) = 5x^3 - 8, x \in R$
Let $f(x_1) = f(x_2)$
$\therefore \quad 5x_1^3 - 8 = 5x_2^3 - 8$
$\therefore \quad x_1^3 - x_2^3 = 0$
$\therefore \quad (x_1 - x_2)\underbrace{\left(x_1^2 + x_1 x_2 + x_2^2\right)}_{> 0 \text{ for all } x_1, x_2 \text{ as discriminant} < 0} = 0$
$\therefore \quad x_1 = x_2$
$\therefore$ f is a one-one function.
Let $f(x) = 5x^3 - 8 = y$ (say), $\quad y \in R$
$\therefore \quad x = \sqrt[3]{\frac{y+8}{5}}$
$\therefore$ For every $y \in R$, there is some $x \in R$
$\therefore$ f is an onto function.
$x = \sqrt[3]{\frac{y+8}{5}} = f^{-1}(y)$
$\therefore \quad f^{-1}(x) = \sqrt[3]{\frac{x+8}{5}}$

Question 5.
A function f: R → R defined by f(x) = $\frac{3x}{5}$ + 2, x ∈ R. Show that f is one-one and onto. Hence find f^{-1}.
Solution:

$f(x) = \frac{3x}{5} + 2, \ x \in R$
Let $f(x_1) = f(x_2)$
$\therefore \quad \frac{3x_1}{5} + 2 = \frac{3x_2}{5} + 2$
$\therefore \quad x_1 = x_2$
$\therefore$ f is a one-one function.
Let $f(x) = \frac{3x}{5} + 2 = y$ (say), $\quad y \in R$
$\therefore \quad x = \frac{5(y-2)}{3}$
$\therefore$ for every $y \in R$, there is some $x \in R$
$\therefore$ f is an onto function.
$x = \frac{5(y-2)}{3} = f^{-1}(y)$
$\therefore \quad f^{-1}(x) = \frac{5(x-2)}{3}$

Question 6.
A function f is defined as f(x) = 4x + 5, for -4 ≤ x < 0. Find the values of f(-1), f(-2), f(0), if they exist.
Solution:
f(x) = 4x + 5, -4 ≤ x < 0
f(-1) = 4(-1) + 5 = -4 + 5 = 1
f(-2) = 4(-2) + 5 = -8 + 5 = -3
x = 0 ∉ domain of f
∴ f(0) does not exist.

Question 7.
A function f is defined as f(x) = 5 – x for 0 ≤ x ≤ 4. Find the values of x such that
(i) f(x) = 3
(ii) f(x) = 5
Solution:
(i) f(x) = 3
∴ 5 – x = 3
∴ x = 5 – 3 = 2

(ii) f(x) = 5
∴ 5 – x = 5
∴ x = 0

Question 8.
If $f(x) = 3x^4 - 5x^2 + 7$, find $f(x - 1)$.
Solution:
$f(x) = 3x^4 - 5x^2 + 7$
$\therefore f(x - 1) = 3(x - 1)^4 - 5(x - 1)^2 + 7$
$= 3(x^4 - {}^4C_1 x^3 + {}^4C_2 x^2 - {}^4C_3 x + {}^4C_4) - 5(x^2 - 2x + 1) + 7$
$= 3(x^4 - 4x^3 + 6x^2 - 4x + 1) - 5(x^2 - 2x + 1) + 7$
$= 3x^4 - 12x^3 + 18x^2 - 12x + 3 - 5x^2 + 10x - 5 + 7$
$= 3x^4 - 12x^3 + 13x^2 - 2x + 5$

Question 9.
If $f(x) = 3x + a$ and $f(1) = 7$, find a and $f(4)$.
Solution:
$f(x) = 3x + a$, $f(1) = 7$
$\therefore 3(1) + a = 7$
$\therefore a = 7 - 3 = 4$
$\therefore f(x) = 3x + 4$
$\therefore f(4) = 3(4) + 4 = 12 + 4 = 16$

Question 10.
If $f(x) = ax^2 + bx + 2$ and $f(1) = 3$, $f(4) = 42$, find a and b.
Solution:
$f(x) = ax^2 + bx + 2$
$f(1) = 3$
$\therefore a(1)^2 + b(1) + 2 = 3$
$\therefore a + b = 1$ ....(i)
$f(4) = 42$
$\therefore a(4)^2 + b(4) + 2 = 42$
$\therefore 16a + 4b = 40$
Dividing by 4, we get
$4a + b = 10$ .....(ii)
Solving (i) and (ii), we get
$a = 3$, $b = -2$

Question 11.
Find composite of f and g:
(i) f = {(1, 3), (2, 4), (3, 5), (4, 6)}
g = {(3, 6), (4, 8), (5, 10), (6, 12)}
(ii) f = {(1, 1), (2, 4), (3, 4), (4, 3)}
g = {(1, 1), (3, 27), (4, 64)}
Solution:
(i) f = {(1, 3), (2, 4), (3, 5), (4, 6)}
g = {(3, 6), (4, 8), (5, 10), (6, 12)}
$\therefore f(1) = 3$, $g(3) = 6$
$f(2) = 4$, $g(4) = 8$
$f(3) = 5$, $g(5)=10$
$f(4) = 6$, $g(6) = 12$
$(gof)(x) = g(f(x))$
$(gof)(1) = g(f(1)) = g(3) = 6$
$(gof)(2) - g(f(2)) = g(4) = 8$
$(gof)(3) = g(f(3)) = g(5) = 10$
$(gof)(4) = g(f(4)) = g(6) = 12$
$\therefore$ gof = {(1, 6), (2, 8), (3, 10), (4, 12)}

(ii) f = {(1, 1), (2, 4), (3, 4), (4, 3)}
g = {(1, 1), (3, 27), (4, 64)}
$f(1) = 1$, $g(1) = 1$
$f(2) = 4$, $g(3) = 27$
$f(3) = 4$, $g(4) = 64$
$f(4) = 3$
$(gof)(x) = g(f(x))$
$(gof)(1) = g(f(1)) = g(1) = 1$
$(gof)(2) = g(f(2)) = g(4) = 64$
$(gof)(3) = g(f(3)) = g(4) = 64$
$(gof)(4) = g(f(4)) = g(3) = 27$
$\therefore$ gof = {(1, 1), (2, 64), (3, 64), (4, 27)}

Question 12.
Find fog and gof:

(i) $f(x) = x^2 + 5$, $g(x) = x - 8$
(ii) $f(x) = 3x - 2$, $g(x) = x^2$
(iii) $f(x) = 256x^4$, $g(x) = \sqrt{x}$
Solution:
(i) $f(x) = x^2 + 5$, $g(x) = x - 8$
$(fog)(x) = f(g(x))$
$= f(x - 8)$
$= (x - 8)^2 + 5$
$= x^2 - 16x + 64 + 5$
$= x^2 - 16x + 69$
$(gof)(x) = g(f(x))$
$= g(x^2 + 5)$
$= x^2 + 5 - 8$
$= x - 3$

(ii) $f(x) = 3x - 2$, $g(x) = x^2$
$(fog)(x) = f(g(x)) = f(x^2) = 3x^2 - 2$
$(gof)(x) = g(f(x))$
$= g(3x - 2)$
$= (3x - 2)^2$
$= 9x^2 - 12x + 4$

(iii) $f(x) = 256x^4$, $g(x) = \sqrt{x}$
$(fog)(x) = f(g(x)) = f(\sqrt{x}) = 256(\sqrt{x})^4 = 256x^2$
$(gof)(x) = g(f(x)) = g(256x^4) = \sqrt{256x^4} = 16x^2$

Question 13.
If $f(x) = \frac{2x-1}{5x-2}, x \neq \frac{2}{5}$, show that $(fof)(x) = x$.
Solution:

$$(fof)(x) = f(f(x))$$
$$= f\left(\frac{2x-1}{5x-2}\right)$$
$$= \frac{2\left(\frac{2x-1}{5x-2}\right)-1}{5\left(\frac{2x-1}{5x-2}\right)-2}$$
$$= \frac{4x-2-5x+2}{10x-5-10x+4} = \frac{-x}{-1} = x$$

Question 14.
If $f(x) = \frac{x+3}{4x-5}$, $g(x) = \frac{3+5x}{4x-1}$, then show that $(fog)(x) = x$.
Solution:
$f(x) = \frac{x+3}{4x-5}$, $g(x) = \frac{3+5x}{4x-1}$
$(fog)(x) = f(g(x))$
$= f\left(\frac{3+5x}{4x-1}\right)$

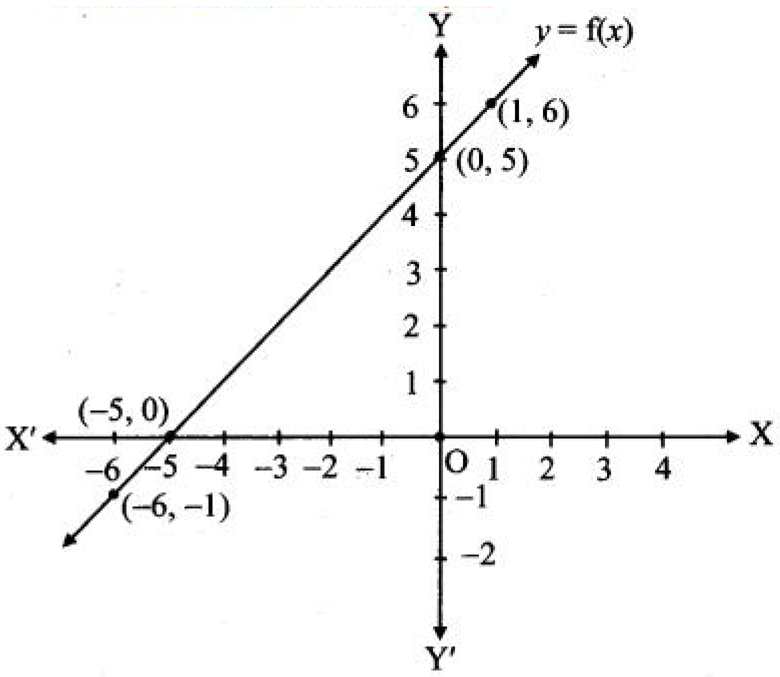
$$= \frac{\frac{3+5x}{4x-1}+3}{4\left(\frac{3+5x}{4x-1}\right)-5}$$
$$= \frac{3+5x+12x-3}{12+20x-20x+5} = \frac{17x}{17} = x$$

Question 15.
Let $f: R - \{2\} \to R$ be defined by $f(x) = \frac{x^2-4}{x-2}$ and $g: R \to R$ be defined by $g(x) = x + 2$. Examine whether $f = g$ or not.
Solution:
$f(x) = \frac{x^2-4}{x-2}$, $x \neq 2$
$\therefore f(x) = x + 2$, $x \neq 2$ and $g(x) = x + 2$,
The domain of $f = R - \{2\}$
The domain of $g = R$
Here, f and g have different domains.
$\therefore f \neq g$

Question 16.
Let $f: R \to R$ be given by $f(x) = x + 5$ for all $x \in R$. Draw its graph.

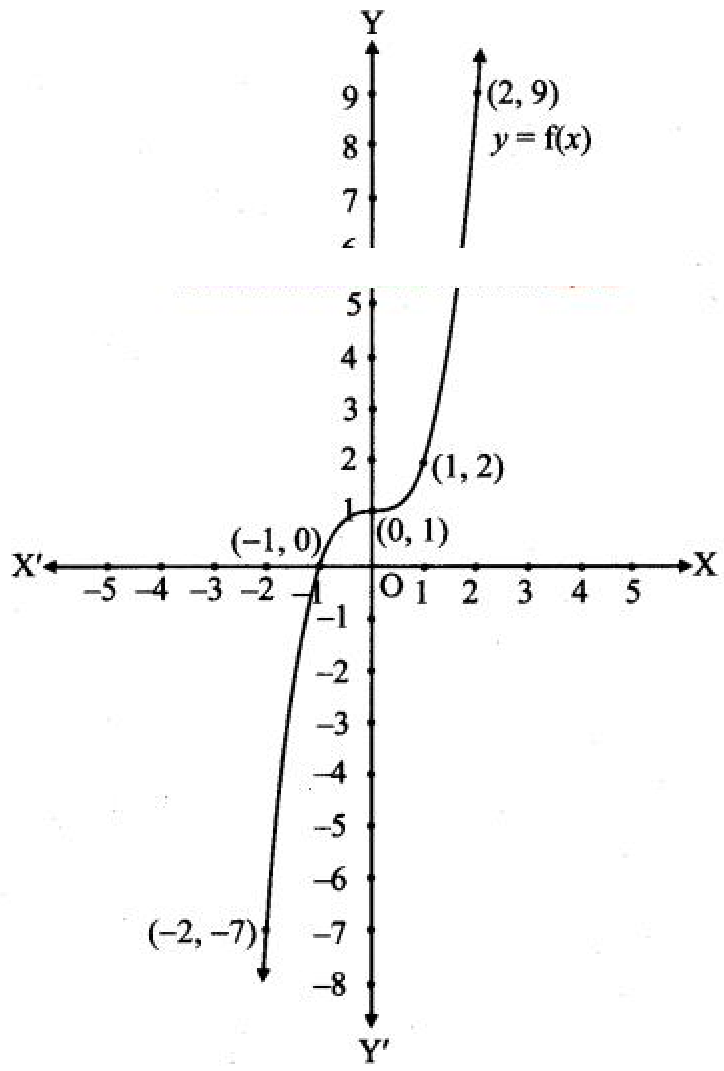
Solution:
f(x) = x + 5

x	1	0	–5	–6
$y = x + 5$	6	5	0	–1



Question 17.
Let f: R → R be given by f(x) = x^3 + 1 for all x ∈ R. Draw its graph.
Solution:
Let y = f(x) = x^3 + 1

x	–2	–1	0	1	2
y	–7	0	1	2	9



Question 18.
For any base show that log(1 + 2 + 3) = log 1 + log 2 + log 3
Solution:
L.H.S. = log(1 + 2 + 3) = log 6
R.H.S. = log 1 + log 2 + log 3
= 0 + log (2 × 3)
= log 6
∴ L.H.S. = R.H.S.

Question 19.
Find x, if $x = 3^{3\log_3 2}$.
Solution:
$x = 3^{3\log_3 2}$
$= 3^{\log_3(2^3)}$
$= 2^3$ ....$[a^{\log_a b} = b]$
= 8

Question 20.
Show that, $\log|\sqrt{x^2+1} + x| + \log|\sqrt{x^2+1} - x| = 0$.
Solution:
L.H.S. = $\log|\sqrt{x^2+1} + x| + \log|\sqrt{x^2+1} - x|$
$= \log\left|(\sqrt{x^2+1}+x)(\sqrt{x^2+1}-x)\right|$
$= \log|x^2 + 1 - x^2|$
= log 1
= 0
= R.H.S.

Question 21.
Show that $\log\frac{a^2}{bc}+\log\frac{b^2}{ca}+\log\frac{c^2}{ab}=0$.
Solution:

$$\text{L.H.S.} = \log\frac{a^2}{bc}+\log\frac{b^2}{ca}+\log\frac{c^2}{ab}$$
$$= \log\left(\frac{a^2}{bc}\times\frac{b^2}{ca}\times\frac{c^2}{ab}\right)$$
$$= \log\left(\frac{a^2\,b^2\,c^2}{a^2\,b^2\,c^2}\right) = \log 1 = 0 = \text{R.H.S.}$$

Question 22.
Simplify $\log(\log x^4) - \log(\log x)$.
Solution:
$\log(\log x^4) - \log(\log x)$
$= \log(4\log x) - \log(\log x)$ .....$[\log m^n = n\log m]$
$= \log 4 + \log(\log x) - \log(\log x)$ .....$[\log(mn) = \log m + \log n]$
= log 4

Question 23.
Simplify $\log_{10}\frac{28}{45}-\log_{10}\frac{35}{324}+\log_{10}\frac{325}{432}-\log_{10}\frac{13}{15}$
Solution:

$$\log_{10}\left(\frac{28}{45}\right)-\log_{10}\left(\frac{35}{324}\right)+\log_{10}\left(\frac{325}{432}\right)-\log_{10}\left(\frac{13}{15}\right)$$
$$= \log_{10}\left(\frac{28}{45}\right)+\log_{10}\left(\frac{325}{432}\right)-\left[\log_{10}\left(\frac{35}{324}\right)+\log_{10}\left(\frac{13}{15}\right)\right]$$
$$= \log_{10}\left(\frac{28}{45}\times\frac{325}{432}\right)-\log_{10}\left(\frac{35}{324}\times\frac{13}{15}\right)$$
$$= \log_{10}\left(\frac{7\times65}{9\times108}\right)-\log_{10}\left(\frac{7\times13}{3\times324}\right)$$
$$= \log_{10}\left(\frac{\frac{7\times65}{9\times108}}{\frac{7\times13}{3\times324}}\right)$$
$$= \log_{10}\left(\frac{7\times65}{9\times108}\times\frac{3\times324}{7\times13}\right) = \log_{10}5$$

Question 24.
If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$, then show that a = b.
Solution:
$\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$\therefore 2 \log \left(\frac{a+b}{2}\right) = \log a + \log b$

$\therefore \log \left(\frac{a+b}{2}\right)^2 = \log ab$

$\therefore \frac{(a+b)^2}{4} = ab$

$\therefore a^2 + 2ab + b^2 = 4ab$

$\therefore a^2 + 2ab - 4ab + b^2 = 0$

$\therefore a^2 - 2ab + b^2 = 0$

$\therefore (a - b)^2 = 0$

$\therefore a - b = 0$

$\therefore a = b$

Question 25.
If $b^2 = ac$. Prove that, $\log a + \log c = 2 \log b$.
Solution:
$b^2 = ac$
Taking log on both sides, we get
$\log b^2 = \log ac$
$\therefore 2 \log b = \log a + \log c$
$\therefore \log a + \log c = 2 \log b$

Question 26.
Solve for x, $\log_x (8x - 3) - \log_x 4 = 2$.
Solution:
$\log_x (8x - 3) - \log_x 4 = 2$

$\therefore \log_x\left(\frac{8x-3}{4}\right) = 2$

$\therefore x^2 = \frac{8x-3}{4}$

$\therefore 4x^2 = 8x - 3$

$\therefore 4x^2 - 8x + 3 = 0$

$\therefore 4x^2 - 2x - 6x + 3 = 0$

$\therefore 2x(2x - 1) - 3(2x - 1) = 0$

$\therefore (2x - 1)(2x - 3) = 0$

$\therefore 2x - 1 = 0$ or $2x - 3 = 0$

$\therefore x = \frac{1}{2}$ or $x = \frac{3}{2}$

Question 27.
If $a^2 + b^2 = 7ab$, show that $\log\left(\frac{a+b}{3}\right)=\frac{1}{2}\log a+\frac{1}{2}\log b$
Solution:
$a^2 + b^2 = 7ab$
$a^2 + 2ab + b^2 = 7ab + 2ab$
$(a + b)^2 = 9ab$
$\frac{(a+b)^2}{9} = ab$
$\left(\frac{a+b}{3}\right)^2 = ab$
Taking log on both sides, we get
$\log \left(\frac{a+b}{3}\right)^2 = \log (ab)$
$2 \log \left(\frac{a+b}{3}\right) = \log a + \log b$
Dividing throughout by 2, we get
$\log\left(\frac{a+b}{3}\right)=\frac{1}{2}\log a+\frac{1}{2}\log b$

Question 28.
If $\log\left(\frac{x-y}{5}\right)=\frac{1}{2}\log x+\frac{1}{2}\log y$, show that $x^2 + y^2 = 27xy$.
Solution:

$$\log\left(\frac{x-y}{5}\right) = \frac{1}{2}(\log x) + \frac{1}{2}(\log y)$$

Multiplying throughout by 2, we get

$$2 \log \left(\frac{x-y}{5}\right) = \log x + \log y$$

$$\therefore \quad \log\left(\frac{x-y}{5}\right)^2 = \log xy$$

$$\therefore \quad \frac{(x-y)^2}{25} = xy$$

$$\therefore \quad x^2 - 2xy + y^2 = 25\,xy$$

$$\therefore \quad x^2 + y^2 = 27xy$$

Question 29.
If $\log_3 [\log_2 (\log_3 x)] = 1$, show that $x = 6561$.
Solution:
$\log_3 [\log_2 (\log_3 x)] = 1$
$\therefore \log_2 (\log_3 x) = 3^1$
$\therefore \log_3 x = 2^3$
$\therefore \log_3 x = 8$
$\therefore x = 3^8$
$\therefore x = 6561$

Question 30.
If $f(x) = \log(1 - x)$, $0 \le x < 1$, show that $f\left(\frac{1}{1+x}\right) = f(1 - x) - f(-x)$.
Solution:

$$f(x) = \log(1-x)$$

Replacing x by $\left(\frac{1}{1+x}\right)$, we get

$$f\left(\frac{1}{1+x}\right) = \log\left(1 - \frac{1}{1+x}\right) = \log\left(\frac{1+x-1}{1+x}\right) = \log\left(\frac{x}{1+x}\right)$$

$$\therefore \quad f\left(\frac{1}{1+x}\right) = \log x - \log(1+x)$$

$$\therefore \quad f\left(\frac{1}{1+x}\right) = \log(1 - 1 + x) - \log(1+x)$$

$$\therefore \quad f\left(\frac{1}{1+x}\right) = \log[1 - (1-x)] - \log[1 - (-x)]$$

$$\therefore \quad f\left(\frac{1}{1+x}\right) = f(1-x) - f(-x)$$

Question 31.
Without using log tables, prove that $\frac{2}{5} < \log_{10} 3 < \frac{1}{2}$.
Solution:
We have to prove that, $\frac{2}{5} < \log_{10} 3 < \frac{1}{2}$
i.e., to prove that $\frac{2}{5} < \log_{10} 3$ and $\log_{10} 3 < \frac{1}{2}$
i.e., to prove that $2 < 5 \log_{10} 3$ and $2 \log_{10} 3 < 1$
i.e., to prove that $2 \log_{10} 10 < 5 \log_{10} 3$ and $2 \log_{10} 3 < \log_{10} 10$ ......[$\because \log_a a = 1$]
i.e., to prove that $\log_{10} 10^2 < \log_{10} 3^5$ and $\log_{10} 3^2 < \log_{10} 10$
i.e., to prove that $10^2 < 3^5$ and $3^2 < 10$
i.e., to prove that $100 < 243$ and $9 < 10$ which is true
$\therefore \frac{2}{5} < \log_{10} 3 < \frac{1}{2}$

Question 32.
Show that $7\log\left(\frac{15}{16}\right)+6\log\left(\frac{8}{3}\right)+5\log\left(\frac{2}{5}\right)+\log\left(\frac{32}{25}\right) = \log 3$
Solution:

$$\text{L.H.S.} = 7\log\left(\frac{15}{16}\right) + 6\log\left(\frac{8}{3}\right) + 5\log\left(\frac{2}{5}\right) + \log\left(\frac{32}{25}\right)$$

$$= \log\left(\frac{15}{16}\right)^7 + \log\left(\frac{8}{3}\right)^6 + \log\left(\frac{2}{5}\right)^5 + \log\left(\frac{32}{25}\right)$$

$$= \log\left(\frac{3\times5}{2^4}\right)^7 + \log\left(\frac{2^3}{3}\right)^6 + \log\left(\frac{2}{5}\right)^5 + \log\left(\frac{2^5}{5^2}\right)$$

$$= \log\left(\frac{3^7\times5^7}{2^{28}}\right) + \log\left(\frac{2^{18}}{3^6}\right) + \log\left(\frac{2^5}{5^5}\right) + \log\left(\frac{2^5}{5^2}\right)$$

$$= \log\left[\frac{3^7\times5^7}{2^{28}} \times \frac{2^{18}}{3^6} \times \frac{2^5}{5^5} \times \frac{2^5}{5^2}\right]$$

$$= \log 3 = \text{R.H.S.}$$

Question 33.
Solve : $\log_2\sqrt{x^4}+4\log_4\sqrt{\frac{2}{x}}=2$
Solution:

$$\sqrt{\log_2 x^4} + 4\log_4\sqrt{\frac{2}{x}} = 2$$

$$\therefore \quad \sqrt{\log_2 x^4} + 4\log_4\left(\frac{2}{x}\right)^{\frac{1}{2}} = 2$$

$$\therefore \quad \sqrt{4\log_2 x} + \frac{4}{2}\log_4\left(\frac{2}{x}\right) = 2$$

$$\therefore \quad 2\sqrt{\log_2 x} + 2\log_4\left(\frac{2}{x}\right) = 2$$

$$\therefore \quad \sqrt{\log_2 x} + \log_4\left(\frac{2}{x}\right) = 1$$

$$\therefore \quad \sqrt{\log_2 x} + \frac{\log_2\left(\frac{2}{x}\right)}{\log_2 4} = 1$$

$$\therefore \quad \sqrt{\log_2 x} + \frac{\log_2\left(\frac{2}{x}\right)}{\log_2 (2)^2} = 1$$

$$\therefore \quad \sqrt{\log_2 x} + \frac{\log_2 2 - \log_2 x}{2\log_2 2} = 1$$

$$\therefore \quad \sqrt{\log_2 x} + \frac{1 - \log_2 x}{2(1)} = 1 \quad \ldots[\because \log_a a = 1]$$

Let $\log_2 x = a$

$$\therefore \quad \sqrt{a} + \frac{1-a}{2} = 1$$

Multiplying throughout by 2, we get

$2\sqrt{a} + 1 - a = 2$

$\therefore \quad 2\sqrt{a} = a + 1$

Squaring on both sides, we get

$4a = (a+1)^2$

$\therefore \quad 4a = a^2 + 2a + 1$

$\therefore \quad a^2 - 2a + 1 = 0$

$\therefore \quad (a-1)^2 = 0$

$\therefore \quad a - 1 = 0$

$\therefore \quad a = 1$

Since $\log_2 x = a$,

$\log_2 x = 1$

$\therefore \quad x = 2^1$

$\therefore \quad x = 2$

Question 34.
Find the value of $3+log_{10}3432+12log_{10}(494)+12log_{10}(125)$

Solution:

$$\frac{3+\log_{10}343}{2+\frac{1}{2}\log_{10}\left(\frac{49}{4}\right)+\frac{1}{2}\log_{10}\left(\frac{1}{25}\right)}$$

$$= \frac{3+\log_{10}7^3}{2+\log_{10}\left(\frac{49}{4}\right)^{\frac{1}{2}}+\log_{10}\left(\frac{1}{25}\right)^{\frac{1}{2}}}$$

$$= \frac{3+3.\log_{10}7}{2+\log_{10}\frac{7}{2}+\log_{10}\frac{1}{5}}$$

$$= \frac{3(1+\log_{10}7)}{2+\log_{10}\left(\frac{7}{2}\times\frac{1}{5}\right)}$$

$$= \frac{3(1+\log_{10}7)}{2+\log_{10}\left(\frac{7}{10}\right)}$$

$$= \frac{3(1+\log_{10}7)}{2+\log_{10}7-\log_{10}10}$$

$$= \frac{3(1+\log_{10}7)}{2+\log_{10}7-1} \qquad \ldots[\because \log_a a = 1]$$

$$= \frac{3(1+\log_{10}7)}{1+\log_{10}7} = 3$$

Question 35.

If $\frac{\log a}{x+y-2z}=\frac{\log b}{y+z-2x}=\frac{\log c}{z+x-2y}$, show that abc = 1.

Solution:

Let $\frac{\log a}{x+y-2z}=\frac{\log b}{y+z-2x}=\frac{\log c}{z+x-2y}$ = k

∴ log a = k(x + y – 2z), log b = k(y + z – 2x), log c = k(z + x – 2y)
log a + log b + log c = k(x + y – 2z) + k(y + z – 2x) + k(z + x – 2y)
= k(x + y – 2z + y + z – 2x + z + x – 2y)
= k(0)
= 0
∴ log (abc) = log 1 …….[∵ log 1 = 0]
∴ abc = 1

Question 36.
Show that, $\log_y x^3 . \log_z y^4 . \log_x z^5 = 60$.
Solution:

L.H.S. $= \log_y (x^3) \log_z (y^4) \log_x (z^5)$
$= (3 \log_y x) (4 \log_z y) (5 \log_x z)$
$= 60 \left(\frac{\log x}{\log y}\right)\left(\frac{\log y}{\log z}\right)\left(\frac{\log z}{\log x}\right)$
$= 60 =$ R.H.S.

Question 37.

If $\frac{\log_2 a}{4}=\frac{\log_2 b}{6}=\frac{\log_2 c}{3k}$ and $a^3b^2c = 1$, find the value of k.

Solution:

Let $\frac{\log_2 a}{4}=\frac{\log_2 b}{6}=\frac{\log_2 c}{3k}=x$

∴ $\log_2 a = 4x, \log_2 b = 6x, \log_2 c = 3k.x$ …(i)
Also, $a^3b^2c = 1$
Taking log to the base 2 throughout, we get
$\log_2 (a^3b^2c) = \log_2 1$
∴ $\log_2 a^3 + \log_2 b^2 + \log_2 c = 0$
∴ $3 \log_2 a + 2 \log_2 b + \log_2 c = 0$
∴ $3(4x) + 2(6x) + 3kx = 0$ …[From (i)]
∴ $12x + 12x + 3kx = 0$
∴ $3kx = -24x$
∴ $k = -8$

Question 38.
If $a^2 = b^3 = c^4 = d^5$, show that $\log_a bcd = 4730$.
Solution:

$a^2 = b^3 = c^4 = d^5$
Taking log to the base a throughout, we get
$\log_a a^2 = \log_a b^3 = \log_a c^4 = \log_a d^5$
$\therefore$ $2 \log_a a = 3 \log_a b = 4 \log_a c = 5 \log_a d$
$\therefore$ $2(1) = 3 \log_a b = 4 \log_a c = 5 \log_a d$
$\therefore$ $\log_a b = \frac{2}{3}$, $\log_a c = \frac{2}{4} = \frac{1}{2}$ and $\log_a d = \frac{2}{5}$
$\therefore$ $\log_a b + \log_a c + \log_a d = \frac{2}{3} + \frac{1}{2} + \frac{2}{5}$
$\therefore$ $\log_a bcd = \frac{47}{30}$

Question 39.
Solve the following for x, where |x| is modulus function, [x] is the greatest integer function, {x} is a fractional part function.
(i) $1 < |x – 1| < 4$
(ii) $|x^2 – x – 6| = x + 2$
(iii) $|x^2 – 9| + |x^2 – 4| = 5$
(iv) $-2 < [x] \le 7$
(v) $2[2x – 5] – 1 = 7$
(vi) $[x]^2 – 5 [x] + 6 = 0$
(vii) $[x – 2] + [x + 2] + \{x\} = 0$
(viii) $[x2]+[x3]=5x6$
Solution:
(i) $1 < |x – 1| < 4$
$\therefore$ $-4 < x – 1 < -1$ or $1 < x – 1 < 4$
$\therefore$ $-3 < x < 0$ or $2 < x < 5$
$\therefore$ Solution set = $(-3, 0) \cup (2, 5)$

(ii) $|x^2 – x – 6| = x + 2$ …..(i)
R.H.S. must be non-negative
$\therefore$ $x \ge -2$ …..(ii)
$|(x – 3) (x + 2)| = x + 2$
$\therefore$ $(x + 2) |x – 3| = x + 2$ as $x + 2 \ge 0$
$\therefore$ $|x – 3| = 1$ if $x \ne -2$
$\therefore$ $x – 3 = \pm 1$
$\therefore$ x = 4 or 2
$\therefore$ x = -2 also satisfies the equation
$\therefore$ Solution set = {-2, 2, 4}

(iii) $|x^2 – 9| + |x^2 – 4| = 5$
$\therefore$ $|(x – 3) (x + 3)| + |(x – 2) (x + 2)| = 5$ ………(i)
Case I: $x < -3$
Also, $x < -2$, $x < 2$, $x < 3$
$\therefore$ $(x – 3) (x + 3) > 0$ and $(x – 2) (x + 2) > 0$
Equation (i) reduces to
$x^2 – 9 + x^2 – 4 = 5$
$\therefore$ $2x^2 = 18$
$\therefore$ x = -3 or 3 (both rejected as $x < -3$)

Case II: $-3 \le x < -2$
As $x < -2$, $x < 3$
$\therefore$ $(x – 3) (x + 3) < 0$, $(x – 2) (x + 2) > 0$
Equation (i) reduces to
$-(x^2 – 9) + x^2 – 4 = 5$
$\therefore$ 5 = 5 (true)
$-3 \le x < -2$ is a solution ….(ii)

Case III: $-2 \le x < 2$ As $x > -3$, $x < 3$
$\therefore$ $(x – 3) (x + 3) < 0$,
$(x – 2) (x + 2) < 0$
Equation (i) reduces to
$9 – x^2 + 4 – x^2 = 5$
$\therefore$ $2x^2 = 13 – 5$
$\therefore$ $x^2 = 4$
$\therefore$ x = -2 is a solution …..(iii)

Case IV: $2 \le x < 3$ As $x > -3$, $x > -2$
$\therefore (x - 3)(x + 3) < 0$, $(x - 2)(x + 2) > 0$
Equation (i) reduces to
$9 - x^2 + x^2 - 4 = 5$
$\therefore 5 = 5$ (true)
$\therefore 2 \le x < 3$ is a solution ……(iv)

Case V: $3 \le x$ As $x > -3$, $x > -2$, $x > 2$
$\therefore (x + 3)(x - 3) > 0$,
$(x - 2)(x + 2) > 0$
Equation (i) reduces to
$x^2 - 9 + x^2 - 4 = 5$
$\therefore 2x^2 = 18$
$\therefore x^2 = 9$
$\therefore x = 3$ …..(v)
($x = -3$ rejected as $x \ge 3$)
From (ii), (iii), (iv), (v), we get
$\therefore$ Solution set = $[-3, -2] \cup [2, 3]$

(iv) $-2 < [x] \le 7$
$\therefore -2 < x < 8$
$\therefore$ Solution set = $(-2, 8)$

(v) $2[2x - 5] - 1 = 7$
$\therefore [2x - 5] = \frac{7+1}{2} = 4$
$\therefore [2x] - 5 = 4$
$\therefore [2x] = 9$
$\therefore 9 \le 2x < 10$
$\therefore \frac{9}{2} \le x < 5$
$\therefore$ Solution set = $[\frac{9}{2}, 5)$

(vi) $[x]^2 - 5[x] + 6 = 0$
$\therefore ([x] - 3)([x] - 2) = 0$
$\therefore [x] = 3$ or 2
If $[x] = 2$, then $2 \le x < 3$
If $[x] = 3$, then $3 \le x < 4$
$\therefore$ Solution set = $[2, 4)$

(vii) $[x - 2] + [x + 2] + \{x\} = 0$
$\therefore [x] - 2 + [x] + 2 + \{x\} = 0$
$\therefore [x] + x = 0$ …..$[\{x\} + [x] = x]$
$\therefore x = 0$

(viii) $\left[\frac{x}{2}\right] + \left[\frac{x}{3}\right] = \frac{5x}{6}$
L.H.S. = an integer
R.H.S. = an integer
$\therefore x = 6k$, where k is an integer

Question 40.
Find the domain of the following functions.
(i) $f(x) = \frac{x^2+4x+4}{x^2+x-6}$
(ii) $f(x) = \sqrt{x-3} + \frac{1}{\log(5-x)}$
(iii) $f(x) = \sqrt{1-\sqrt{1-\sqrt{1-x^2}}}$
(iv) $f(x) = x!$
(v) $f(x) = {}^{5-x}P_{x-1}$
(vi) $f(x) = \sqrt{x-x^2} + \sqrt{5-x}$
(vii) $f(x) = \sqrt{\log(x^2-6x+6)}$
Solution:
(i) $f(x) = \frac{x^2+4x+4}{x^2+x-6} = \frac{x^2+4x+4}{(x+3)(x-2)}$
For f to be defined, $x \ne -3, 2$
$\therefore$ Domain of f = $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

(ii) $f(x) = \sqrt{x-3} + \frac{1}{\log(5-x)}$
For f to be defined,

x – 3 ≥ 0, 5 – x > 0 and 5 – x ≠ 1
x ≥ 3, x < 5 and x ≠ 4
∴ Domain of f = [3, 4) ∪ (4, 5)

(iii) f(x) = $\sqrt{1-\sqrt{1-\sqrt{1-x^2}}}$

$1-x^2 \geq 0$ and $1-\sqrt{1-x^2} \geq 0$
and $1-\sqrt{1-\sqrt{1-x^2}} \geq 0$
Consider $1-x^2 \geq 0$
∴ $x^2 \leq 1$
∴ $-1 \leq x \leq 1$ ...(i)
Consider $1-\sqrt{1-x^2} \geq 0$
∴ $1 \geq \sqrt{1-x^2}$
∴ $1 \geq 1-x^2$
∴ $x^2 \geq 0$ (true)
Consider $1-\sqrt{1-\sqrt{1-x^2}} \geq 0$
∴ $1 \geq \sqrt{1-\sqrt{1-x^2}}$
∴ $1 \geq 1-\sqrt{1-x^2}$ ∴ $\sqrt{1-x^2} \geq 0$ (true)

Equation (i) gives solution set = [-1, 1]
∴ Domain of f = [-1, 1]

(iv) f(x) = x!
∴ Domain of f = set of whole numbers (W)

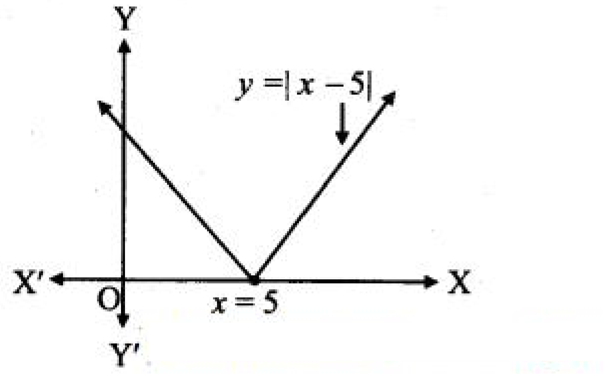
(v) f(x) = ${}^{5-x}P_{x-1}$
5 – x > 0, x – 1 ≥ 0, x – 1 ≤ 5 – x
∴ x < 5, x ≥ 1 and 2x ≤ 6
∴ x ≤ 3
∴ Domain of f = {1, 2, 3}

(vi) f(x) = $\sqrt{x-x^2}+\sqrt{5-x}$
$x - x^2 \geq 0$
∴ $x^2 - x \leq 0$
∴ x(x – 1) ≤ 0
∴ 0 ≤ x ≤ 1 .....(i)
5 – x ≥ 0
∴ x ≤ 5 .....(ii)
Intersection of intervals given in (i) and (ii) gives
Solution set = [0, 1]
∴ Domain of f = [0, 1]

(vii) f(x) = $\sqrt{\log(x^2-6x+6)}$
For f to be defined,
$\log(x^2 - 6x + 6) \geq 0$
∴ $x^2 - 6x + 6 \geq 1$
∴ $x^2 - 6x + 5 \geq 0$
∴ (x – 5)(x – 1) ≥ 0
∴ x ≤ 1 or x ≥ 5 .....(i)
[∵ The solution of (x – a) (x – b) ≥ 0 is x ≤ a or x ≥ b, for a < b]
and $x^2 - 6x + 6 > 0$
∴ $(x - 3)^2 > -6 + 9$
∴ $(x - 3)^2 > 3$
∴ x < 3 – √3 0r x > 3 + √3 ....(ii)
From (i) and (ii), we get
x ≤ 1 or x ≥ 5
Solution set = (-∞, 1] ∪ [5, ∞)
∴ Domain of f = (-∞, 1] ∪ [5, ∞)

Question 41.
(i) f(x) = |x – 5|
(ii) f(x) = $\frac{x}{9+x^2}$
(iii) f(x) = $\frac{1}{\sqrt{1+x}}$

(iv) f(x) = [x] – x
(v) f(x) = 1 + 2x + 4x
Solution:
(i) f(x) = |x – 5|



∴ Range of f = [0, ∞)

(ii) f(x) = $\frac{x}{9+x^2}$ = y (say)
∴ x^2y – x + 9y = 0
For real x, Discriminant > 0
∴ 1 – 4(y)(9y) ≥ 0
∴ $y^2 \leq \frac{1}{36}$
∴ $-\frac{1}{6} \leq y \leq \frac{1}{6}$
∴ Range of f = $[-\frac{1}{6}, \frac{1}{6}]$

(iii) f(x) = $\frac{1}{1+\sqrt{x}}$ = y, (say)
∴ √x y + y = 1
∴ √x = $\frac{1-y}{y}$ ≥ 0
∴ $\frac{y-1}{y}$ ≤ 0
∴ o < y ≤ 1
∴ Range of f = (0, 1]

(iv) f(x) = [x] – x = -{x}
∴ Range of f = (-1, 0] …..[0 ≤ {x} < 1]

(v) f(x) = 1 + 2x + 4x
Since, 2x > 0, 4x > 0
∴ f(x) > 1
∴ Range of f = (1, ∞)

Question 42.
Find (fog) (x) and (gof) (x)
(i) f(x) = e^x, g(x) = log x
(ii) f(x) = $\frac{x}{x+1}$, g(x) = $\frac{x}{1-x}$
Solution:
(i) f(x) = e^x, g(x) = log x
(fog) (x) = f(g(x))
= f(log x)
= $e^{\log x}$
= x
(gof) (x) = g(f(x))
= g(e^x)
= log (e^x)
= x log e
= x …..[∵ log e = 1]

(ii) f(x) = $\frac{x}{x+1}$, g(x) = $\frac{x}{1-x}$

$$(fog)(x) = f(g(x)) = f\left(\frac{x}{1-x}\right)$$

$$= \frac{\frac{x}{1-x}}{\frac{x}{1-x}+1} = \frac{x}{x+1-x} = x$$

$$(gof)\,x = g\,(f(x))$$

$$= g\left(\frac{x}{x+1}\right) = \frac{\frac{x}{x+1}}{1-\frac{x}{x+1}}$$

$$= \frac{x}{x+1-x} = x$$

Question 43.
Find f(x), if
(i) g(x) = x^2 + x – 2 and (gof) (x) = $4x^2$ – 10x + 4
(ii) g(x) = 1 + √x and f [g(x)] = 3 + 2√x + x.
Solution:
(i) g(x) = x^2 + x – 2
(gof) (x) = $4x^2$ – 10x + 4
= $(2x – 3)^2$ + (2x – 3) – 2
= g(2x – 3)
= g(f(x))
∴ f(x) = 2x – 3
(gof) (x) = $4x^2$ – 10x + 4
= $(-2x + 2)^2$ + (-2x + 2) – 2
= g(-2x + 2)
= g(f(x))
∴ f(x) = -2x + 2

(ii) g(x) = 1 + √x
f(g(x)) = 3 + 2√x + x
= x + 2√x + 1 + 2
= $(\sqrt{x} + 1)^2$ + 2
f(√x + 1) = $(\sqrt{x} + 1)^2$ + 2
∴ f(x) = x^2 + 2

Question 44.
Find (fof) (x) if
(i) f(x) = $\frac{x}{\sqrt{1+x^2}}$
(ii) f(x) = $\frac{2x+1}{3x-2}$

Solution:

i. $f(x) = \dfrac{x}{\sqrt{1+x^2}}$

$$f(f(x)) = f\left(\frac{x}{\sqrt{1+x^2}}\right) = \frac{\dfrac{x}{\sqrt{1+x^2}}}{\sqrt{1+\left(\dfrac{x}{\sqrt{1+x^2}}\right)^2}}$$

$$= \frac{\dfrac{x}{\sqrt{1+x^2}}}{\sqrt{\dfrac{1+2x^2}{1+x^2}}} = \frac{x}{\sqrt{1+2x^2}}$$

ii. $f(x) = \dfrac{2x+1}{3x-2}$

$$f(f(x)) = f\left(\frac{2x+1}{3x-2}\right) = \frac{2\left(\dfrac{2x+1}{3x-2}\right)+1}{3\left(\dfrac{2x+1}{3x-2}\right)-2}$$

$$= \frac{4x+2+3x-2}{6x+3-6x+4} = \frac{7x}{7}$$

$$= x$$