Maharashtra State Board 11th Maths Solutions Chapter 5 Straight Line Ex 5.1

Question 1.
If A(1, 3) and B(2, 1) are points, find the equation of the locus of point P such that PA = PB.
Solution:
Let P(x, y) be any point on the required locus.
Given, A(1, 3), B(2, 1) and
PA = PB
$\therefore PA^2 = PB^2$
$\therefore (x - 1)^2 + (y - 3)^2 = (x - 2)^2 + (y - 1)^2$
$\therefore x^2 - 2x + 1 + y^2 - 6y + 9 = x^2 - 4x + 4 + y^2 - 2y + 1$
$-2x - 6y + 10 = -4x - 2y + 5$
$\therefore 2x - 4y + 5 = 0$
$\therefore$ The required equation of locus is $2x - 4y + 5 = 0$.

Question 2.
A(- 5,2) and B(4,1). Find the equation of the locus of point P, which is equidistant from A and B.
Solution:
Let P(x, y) be any point on the required locus.
P is equidistant from A(- 5, 2) and B(4, 1).
$\therefore$ PA = PB
$\therefore PA^2 = PB^2$
$\therefore (x + 5)^2 + (y - 2)^2 = (x - 4)^2 + (y - 1)^2$
$\therefore x^2 + 10x + 25 + y^2 - 4y + 4$
$= x^2 - 8x + 16 + y^2 - 2y + 1$
$\therefore 10x - 4y + 29 = -8x - 2y + 17$
$\therefore 18x - 2y + 12 = 0$
$\therefore 9x - y + 6 = 0$
The required equation of locus is $9x - y + 6 = 0$.

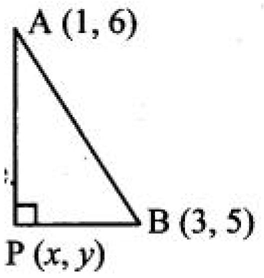
Question 3.
If A(2, 0) and B(0, 3) are two points, find the equation of the locus of point P such that AP = 2BP.
Solution:
Let P(x, y) be any point on the required locus.
Given, A(2, 0), B(0, 3) and AP = 2BP
$\therefore AP^2 = 4BP^2$
$\therefore (x - 2)^2 + (y - 0)^2 = 4[(x - 0)^2 + (y - 3)^2]$
$\therefore x^2 - 4x + 4 + y^2 = 4(x^2 + y^2 - 6y + 9)$
$x^2 - 4x + 4 + y^2 = 4x^2 + 4y^2 - 24y + 36$
$\therefore 3x^2 + 3y^2 + 4x - 24y + 32 = 0$
$\therefore$ The required equation of locus is
$3x^2 + 3y^2 + 4x - 24y + 32 = 0$.
[Note: Answer given in the textbook , is
'$3x^2 + 3y^2 + 4x + 24y + 32 = O$'.
However, as per our calculation it is '$3x^2 + 3y^2 + 4x - 24y + 32 = 0$'.]

Question 4.
If A(4,1) and B(5,4), find the equation of the locus of point P such that $PA^2 = 3PB^2$.
Solution:
Let P(x, y) be any point on the required locus. Given, A(4,1), B(5,4) and PA2 = 3PB2
$\therefore (x - 4)2 + (y - 1)2 = 3[(x - 5)^2 + (y - 4)^2]$
$\therefore x^2 - 8x + 16 + y^2 - 2y + 1 = 3(x^2 - 10x + 25 + y^2 - 8y + 16)$
$\therefore x^2 - 8x + y^2 - 2y + 17 = 3x^2 -30x + 75 + 3y^2 - 24y + 48$
$\therefore 2x^2 + 2y^2 - 22x - 22y + 106 = 0$
$\therefore x^2 + y^2 - 11x - 11y + 53 = 0$
$\therefore$ The required equation of locus is
$x^2 + y^2 - 11x - 11y + 53 = 0$.

Question 5.
A(2, 4) and B(5, 8), find the equation of the
locus of point P such that $PA^2 - PB^2 = 13$.
Solution:
Let P(x, y) be any point on the required locus. Given, A(2,4), B(5, 8) and $PA^2-PB^2 = 13$
$\therefore [(x -2)^2 + (y - 4)^2] - [(x -5)^2 + (y- 8)^2] = 13$

$\therefore$ $(x^2 – 4x + 4 + y^2 – 8y + 16) – (x^2 – 10x + 25 + y^2 – 16y + 64) = 13$
$\therefore$ $x^2 – 4x + y^2 – 8y + 20 – x^2 + 10x – y^2 + 16y – 89 = 13$
$\therefore$ $6x + 8y - 69 = 13$
$\therefore$ $6x + 8y – 82 = 0$
$\therefore$ $3x + 4y – 41 = 0$
$\therefore$ The required equation of locus is $3x + 4y - 41 = 0$.

Question 6.
A(1, 6) and B(3, 5), find the equation of the locus of point P such that segment AB subtends right angle at P. ($\angle APB = 90°$)
Solution:
Let P(x, y) be any point on the required locus. Given,
A(l, 6) and B(3, 5),
$\angle APB = 90°$
$\therefore$ ΔAPB is a right angled triangle,
By Pythagoras theorem,



$AP^2 + PB^2 = AB^2$ P (x,y)
$\therefore$ $[(x – 1)^2 + (y – 6)^2] + [(x – 3)^2 + (y – 5)^2] = (1 – 3)^2 + (6 - 5)^2$
$\therefore$ $x^2 — 2x + 1 + y^2 — 12y + 36 + x^2 – 6x + 9 + y^2 – 10y + 25 = 4 + 1$
$\therefore$ $2x^2 + 2y^2 – 8x – 22y + 66 = 0$
$\therefore$ $x^2 + y^2 – 4x – 11y + 33 = 0$
$\therefore$ The required equation of locus is $x^2 + y^2 – 4x – 11y + 33 = 0$.
[Note: Answer given in the textbook is
'$3x^2 + 4y^2 – 4x – 11y + 33 = 0$'.
However, as per our calculation it is '$x^2 + y^2 – 4x – 11y + 33 = O$

Question 7.
If the origin is shifted to the point 0'(2, 3), the axes remaining parallel to the original axes, find the new co-ordinates of the points
i. A(1, 3) ii. B(2,5)
Solution:
Origin is shifted to (2, 3) = (h, k)
Let the new co-ordinates be (X, Y).
x = X + handy = Y + k
x = X + 2 andy = Y + 3 ...(i)

i. Given, A(x, y) = A(1, 3)
x = X + 2 andy = Y + 3 ...[From(i)]
$\therefore$ 1 = X + 2 and 3 = Y + 3 X = – 1 and Y = 0
$\therefore$ The new co-ordinates of point A are (- 1,0).

ii. Given, B(x, y) = B(2, 5)
x = X + 2 and y = Y + 3 ...[From(i)]
$\therefore$ 2 = X + 2 and 5 = Y + 3
$\therefore$ X = 0 and Y = 2
$\therefore$ The new co-ordinates of point B are (0, 2).

Question 8.
If the origin is shifted to the point O'(1, 3), the axes remaining parallel to the original axes, find the old co-ordinates of the points
i. C(5,4) ii. D(3,3)
Solution:
Origin is shifted to (1,3) = (h, k)
Let the new co-ordinates be (X, Y).
x = X + h andy = Y + k
$\therefore$ x = X+1 andy = Y + 3 ...(i)

i. Given, C(X, Y) = C(5, 4)
x = X +1 andy = Y + 3 ...[From(i)]
$\therefore$ x = 5 + 1 = 6 andy = 4 + 3 = 7
$\therefore$ The old co-ordinates of point C are (6, 7).

ii. Given, D(X, Y) = D(3, 3)
x = X + 1 andy = Y + 3 ...[From(i)]

∴ $x = 3 + 1 = 4$ and $y = 3 + 3 = 6$
∴ The old co-ordinates of point D are (4, 6).

Question 9.
If the co-ordinates A(5, 14) change to B(8, 3) by shift of origin, find the co-ordinates of the point, where the origin is shifted.
Solution:
Let the origin be shifted to (h, k).
Given, $A(x, y) = A(5,14)$, $B(X, Y) = B(8, 3)$
Since $x = X + h$ and $y = Y + k$,
$5 = 8 + h$ and $14 = 3 + k$,
∴ $h = -3$ and $k = 11$
The co-ordinates of the point, where the origin is shifted are (- 3, 11).

Question 10.
Obtain the new equations of the following loci if the origin is shifted to the point 0′(2,2), the direction of axes remaining the same:
i. $3x-y + 2 = 0$
11. $x^2+y^2-3x = 7$
iii. $xy - 2x - 2y + 4 = 0$
iv. $y^2 - 4x - 4y + 12 = 0$
Solution:
Given, $(h,k) = (2,2)$
Let (X, Y) be the new co-ordinates of the point (x,y).
∴ $x = X + h$ and $y = Y + k$
∴ $x = X + 2$ and $y = Y + 2$
i. Substituting the values of x and y in the equation $3x -y + 2 = 0$, we get
$3(X + 2) - (Y + 2) + 2 = 0$
∴ $3X + 6-Y-2 + 2 = 0$
∴ $3 X - Y + 6 = 0$, which is the new equation of locus.

ii. Substituting the values of x and y in the equation
$x^2 + y^2 - 3x = 7$, we get
$(X + 2)^2 + (Y + 2)^2 - 3(X + 2) = 7$
∴ $X^2 + 4X + 4 + Y^2 + 4Y + 4 - 3X - 6 = 7$
∴ $X^2 + Y^2 + X + 4Y - 5 = 0$, which is the new
equation of locus.

iii; Substituting the values of x and y in the equation $xy - 2x - 2y + 4 = 0$, we get
$(X + 2) (Y + 2) - 2(X + 2) - 2(Y + 2) + 4 = 0$
∴ $XY + 2X + 2Y + 4 - 2X - 4-2Y- 4 + 4 = 0$
∴ $XY = 0$, which is the new equation of locus.

iv. Substituting the values of x and y in the equation $y^2 - 4x - 4y + 12 = 0$, we get
$(Y + 2)^2 - 4(X + 2) - 4(Y + 2) + 12 = 0$
∴ $Y^2 + 4Y + 4 - 4X - 8 - 4Y -8 + 12 = 0$
∴ $Y^2 - 4X = 0$, which is the new equation of locus.

Maharashtra State Board 11th Maths Solutions Chapter 5 Straight Line Ex 5.2

Question 1.
Find the slope of each of the lines which passes through the following points:
i. A(2, -1), B(4,3)
ii. C(- 2,3), D(5, 7)
iii. E(2,3), F(2, – 1)
iv. G(7,1), H(- 3,1)
Solution:
i. Here, A = (2, -1) and B = (4, 3)
Slope of line AB = $\frac{y_2-y_1}{x_2-x_1}=\frac{3-(-1)}{4-2}=\frac{4}{2}$ = 2

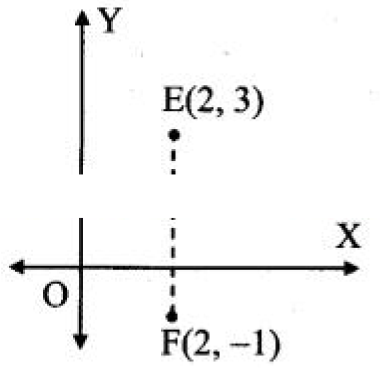
ii. Here, C = (-2, 3) and D = (5, 7)

Slope of line CD = $\frac{y_2-y_1}{x_2-x_1}=\frac{7-3}{5-(-2)}=\frac{4}{7}$

iii. Here, E s (2, 3) and F = (2, -1)

Slope of line EF = $\frac{y_2-y_1}{x_2-x_1}=\frac{-1-3}{2-2}=\frac{-4}{0}$, which ix not defined.

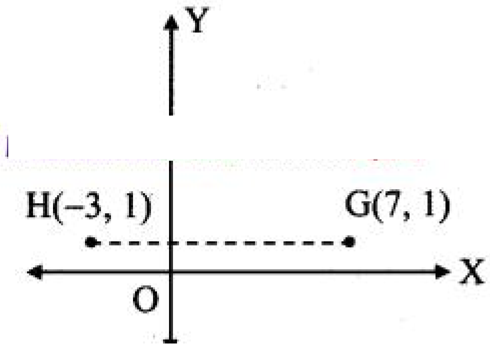
Alternate Method:
Points E and F have same x co-ordinates i.e. 2.
Points E and F lie on a line parallel to Y-axis.
∴ The slope of EF is not defined.



iv. Here, G = (7, 1) and H = (-3, 1)

Slope of line GH = $\frac{y_2-y_1}{x_2-x_1}=\frac{1-1}{-3-7}$ = 0

Alternate Method:
Points G and H have same y co-ordinate i.e. 1.
∴ Points G and H lie on a line parallel to the
X-axis.
∴ The slope of GH is 0.



Question 2.
If the x and x-intercepts of line L are 2 and 3 respectively, then find the slope of line L.
Solution:
Given, x-intercept of line L is 2 and y-intercept of line L is 3
∴ The line L intersects X-axis at (2, 0) and Y-axis at (0,3).
∴ The line L passes through (2, 0) and (0, 3).

Slope of line L = $\frac{y_2-y_1}{x_2-x_1}=\frac{3-0}{0-2}=\frac{-3}{2}$

Question 3.
Find the slope of the line whose inclination is 30°.
Solution:
Given, inclination (θ) = 30°
∴ Slope of the line = tanθ = tan30° = $\frac{1}{\sqrt{3}}$

Question 4.
Find the slope of the line whose inclination is $\frac{\pi}{4}$
Solution:
Given, inclination (0) = $\frac{\pi}{4}$
∴ Slope of the line = tan θ = tan$\frac{\pi}{4}$ = 1

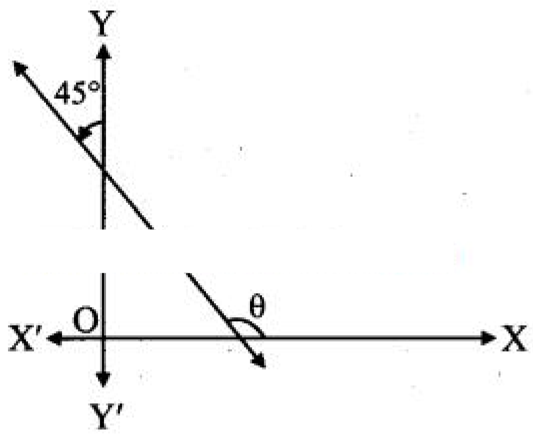
Question 5.
A line makes intercepts 3 and 3 on the co-ordinate axes. Find the inclination of the line.
Solution:
Given, x-intercept of line is 3 and y-intercept of line is 3
∴ The line intersects X-axis at (3, 0) and Y-axis at (0, 3).
∴ The line passes through (3, 0) and (0,3).

∴ Slope of line = $\frac{y_2-y_1}{x_2-x_1}=\frac{3-0}{0-3}$ = -1

But, slope of a line = tan θ
∴ tan θ = – 1
= – tan $\frac{\pi}{4}$
= tan(π-$\frac{\pi}{4}$) ...[v tan(π – θ) = -tan θ]
tan θ = tan $\frac{3\pi}{4}$
θ = $\frac{3\pi}{4}$
The inclination of the line is $\frac{3\pi}{4}$.
[Note: Answer given in the textbook is '-1 However, as per our calculation it is $\frac{3\pi}{4}$]

Question 6.
Without using Pythagoras theorem, show that points A (4, 4), B (3, 5) and C (- 1, – 1) are the vertices of a right-angled triangle.
Solution:
Given, A(4,4), B(3, 5), C (-1, -1).
Slope of AB = $\frac{y_2-y_1}{x_2-x_1}=\frac{5-4}{3-4}$ = – 1
Slope of BC = $\frac{-1-5}{-1-3}=\frac{-6}{-4}=\frac{3}{2}$
Slope of AC = $\frac{-1-4}{-1-4}$ = 1
Slope of AB x slope of AC = – 1 x 1 = – 1
∴ side AB ⊥ side AC
∴ ΔABC is a right angled triangle right angled at A.
∴ The given points are the vertices of a right angled triangle.

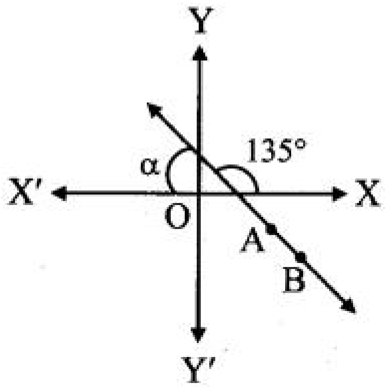
Question 7.
Find the slope of the line which makes angle of 45° with the positive direction of the Y-axis measured anticlockwise.
Solution:



Since the line makes an angle of 45° with positive direction of Y-axis in anticlockwise direction,
Inclination of the line (0) = (90° + 45°)
∴ Slope of the line = tan(90° + 45°)
= – cot 45°
= -1

Question 8.
Find the value of k for which the points P(k, -1), Q(2,1) and R(4,5) are collinear.
Solution:
Given, points P(k, – 1), Q (2, 1) and R(4, 5) are collinear.
∴ Slope of PQ = Slope of QR .
∴ $\frac{1-(-1)}{2-k}=\frac{5-1}{4-2}$
∴ $\frac{2}{2-k}=\frac{4}{2}$
∴ 4 = 4 (2 – k)
∴ 1 = 2 – k
∴ k = 2 – 1 = 1

Question 9.
Find the acute angle between the X-axis and the line joining the points A(3, -1) and B(4, – 2).
Solution:
Given, A (3, – 1) and B (4, – 2)
Slope of AB = $\frac{y_2-y_1}{x_2-x_1}=\frac{-2-(-1)}{4-3}$ = – 1
But, slope of a line = tan θ
∴ tan θ = – 1
= – tan 45°
= tan (180° -45°)
... [∵ tan (180° – θ) = -tan θ]
= tan 135°
∴ θ = 135°



Let α be the acute angle that line AB makes with X-axis.
Then, α + 0 = 180°
α = 180°- 135° = 45°
∴ The acute angle between the X-axis and the line joining the points A and B is 45°.

Question 10.
A line passes through points A(xi, y0 and B(h, k). If the slope of the line is m, then show that $k - y_1 = m(h - x_1)$.
Solution:
Given, $A(x_1, y_1)$, B(h, k) and
slope of line AB = m
Slope of line AB = $\frac{y_2-y_1}{x_2-x_1}$
∴ m = $\frac{k-y_1}{h-x_1}$
∴ $k - y_1 = m(h - x_1)$

Question 11.
If the points A(h, 0), B(0, k) and C(a, b) lie on a line, then show that $\frac{a}{h}+\frac{b}{k} = 1$. '
Solution:
Given, A(h, 0), B(0, k) and C(a, b)
Since the points A, B and C lie on a line, they are collinear.
∴ Slope of AB = slope of BC

$$\therefore \quad \frac{k-0}{0-h} = \frac{b-k}{a-0}$$

$$\therefore \quad ak = -h(b-k)$$

$$\therefore \quad \frac{a}{h} = \frac{k-b}{k}$$

$$\therefore \quad \frac{a}{h} = 1 - \frac{b}{k}$$

$$\therefore \quad \frac{a}{h} + \frac{b}{k} = 1$$

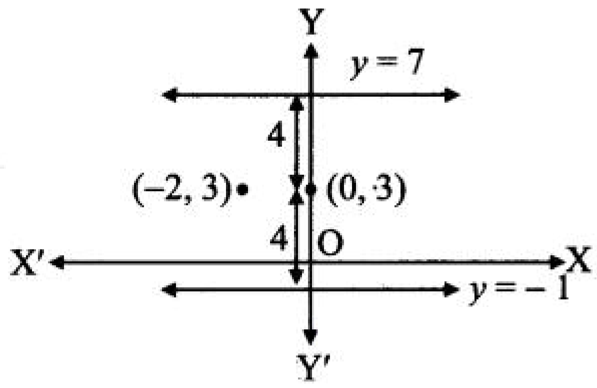
Maharashtra State Board 11th Maths Solutions Chapter 5 Straight Line Ex 5.3

Question 1.
Write the equation of the line:
i. parallel to the X-axis and at a distance of 5 units from it and above it.
ii. parallel to the Y-axis and at a distance of 5 units from it and to the left of it.
iii. parallel to the X-axis and at a distance of 4 units from the point (- 2,3).
Solution:
i. Equation of a line parallel to X-axis is y = k. Since the line is at a distance of 5 units above the X-axis, k = 5
∴ The equation of the required line is y = 5.

ii. Equation of a line parallel to the Y-axis is x = h. Since the line is at a distance of 5 units to the left of the Y-axis, h = -5
∴ The equation of the required line is x = -5.

[Note: Answer given in the textbook is 'y = -5
However, we found that 'x = – 5'.]

iii. Equation of a line parallel to the X-axis is of the form y = k (k > 0 or k < 0).
Since the line is at a distance of 4 units from the point (- 2, 3),
k = 4 + 3 = 7 or k = 3- 4 = -1
∴ The equation of the required line is y = 1 or y = – 1.



Question 2.
Obtain the equation of the line:
i. parallel to the X-axis and making an intercept of 3 units on the Y-axis.
ii. parallel to the Y-axis and making an intercept of 4 units on the X-axis.
Solution:
i. Equation of a line parallel to X-axis with y-intercept 'k' isy = k.
Here, y-intercept = 3
∴ The equation of the required line is y = 3.

ii. Equation of a line parallel to Y-axis with x-intercept 'h' is x = h.
Here, x-intercept = 4
∴ The equation of the required line is x = 4.

Question 3.
Obtain the equation of the line containing the point:
i. A(2, – 3) and parallel to the Y-axis.
ii. B(4, – 3) and parallel to the X-axis.
Solution:
i. Equation of a line parallel to Y-axis is of the form x = h.
Since the line passes through A(2, – 3), h = 2
∴ The equation of the required line is x = 2.

ii. Equation of a line parallel to X-axis is of the formy = k.
Since the line passes through B(4, – 3), k = -3
∴ The equation of the required line is y = – 3.

Question 4.
Find the equation of the line:
i. passing through the points A(2, 0) and B(3,4)
ii. passing through the points P(2, 1) and Q(2,-1)
Solution:
i. The required line passes through the points A(2, 0) and B(3,4).
Equation of the line in two point form is $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$
Here, $(x_1 y_1)$ = (2,0) and (x_1, y_2) = (3,4)
∴ The equation of the required line is
∴ $\frac{y-0}{4-0}=\frac{x-2}{3-2}$
∴ $\frac{y}{4}=\frac{x-2}{1}$
∴ y = 4(x – 2)
∴ y = 4x – 8
∴ 4x – y – 8 = 0

ii. The required line passes through the points P(2, 1) and Q(2,-1).
Since both the given points have same
x co-ordinates i.e. 2,
the given points lie on the line x = 2.
∴ The equation of the required line is x = 2.

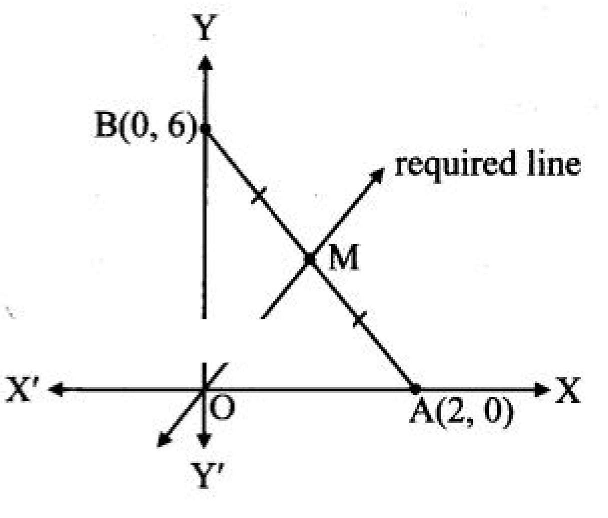
Question 5.
Find the equation of the line:
i. containing the origin and having inclination 60°.
ii. passing through the origin and parallel to AB, where A is (2,4) and B is (1,7).
iii. having slope 1/2 and containing the point (3, -2)
iv. containing the point A(3, 5) and having slope 2/3
v. containing the point A(4, 3) and having inclination 120°.
vi. passing through the origin and which bisects the portion of the line 3JC + y = 6 intercepted between the co-ordinate axes.
Solution:
i. Given, Inclination of line = θ = 60°
Slope of the line (m) = tan θ = tan 60°
= $\sqrt{3}$
Equation of the line having slope m and passing through origin (0, 0) is y = mx.
∴ The equation of the required line is y = $\sqrt{3}$ x

ii. Given, A (2, 4) and B (1, 7)
Slope of AB = $\frac{7-4}{1-2}$ = -3 1-2
Since the required line is parallel to line AB, slope of required line (m) = slope of AB
∴ m = – 3 and the required line passes through the origin.
Equation of the line having slope m and passing through origin (0, 0) is y = mx.
∴ The equation of the required line is y = – 3x

iii. Given, slope(m) = $=\frac{1}{2}$ and the line passes through (3, – 2).
Equation of the line in slope point form is
$y-y_1 = m(x-x_1)$
∴ The equation of the required line is
[y-(- 2)]=$\frac{1}{2}$(x-3)
∴ 2(y + 2)=x – 3
∴ 2y + 4 = x – 3
∴ x – 2y – 7 = 0

iv. Given, slope(m) = $\frac{2}{3}$ and the line passes through (3, 5).
Equation of the line in slope point form is $y-y_1 = m(x-x_1)$
∴ The equation of the required line is y – 5 = $\frac{2}{3}$(x-3)
∴ 3 (y – 5) = 2 (x – 3)
∴ 3y – 15 = 2x – 6
∴ 2x – 3y + 9 = 0

v. Given, Inclination of line = θ = 120°
Slope of the line (m) = tan θ = tan 120°
= tan (90° + 30°)
= – cot 30°
= – $\sqrt{3}$
and the line passes through A(4, 3).
Equation of the line in slope point form is $y-y_1 = m(x-x_1)$
∴ The equation of the required line is
y- 3 = –$\sqrt{3}$(x-4)
∴ y – 3 = –$\sqrt{3}$ x + 4$\sqrt{3}$
∴ $\sqrt{3}$x + y – 3 -4$\sqrt{3}$ = 0

vi.


Given equation of the line is 3x +y = 6.
∴ $\frac{x}{2}+\frac{y}{6}=1$

This equation is of the form $\frac{x}{a}+\frac{y}{b} = 1$,
where a = 2, b = 6
∴ The line 3x + y = 6 intersects the X-axis and Y-axis at A(2, 0) and B(0, 6) respectively. Required line is passing through the midpoint of AB.
∴ Midpoint of AB = $\left(\frac{2+0}{2},\frac{0+6}{2}\right)$ = (1,3)
∴ Required line passes through (0, 0) and (1,3).
Equation of the line in two point form is

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

∴ The equation of the required line is

$$\frac{y-0}{3-0}=\frac{x-0}{1-0}$$

$$\frac{y}{3}=\frac{x}{1}$$

∴ y = 3x
∴ 3x – y = 0

Alternate Method:
Given equation of the line is 3x + y = 6 ...(i)
Substitute y = 0 in (i) to get a point on X-axis.
∴ 3x + 0 = 6
∴ x = 2
Substitute x = 0 in (i) to get a point on Y-axis.
∴ 3(0) + 7 = 6
∴ y = 6
∴ The line 3x + y = 6 intersects the X-axis and Y-axis at A(2,0) and B(0,6) respectively.
Let M be the midpoint of AB.
M = $\left(\frac{2+0}{2},\frac{0+6}{2}\right)$ = (1,3)
Slope of OM (m) = $\frac{3-0}{1-0}$ = 3
Equation of OM is of the formy = mx.
∴ The equation of the required line is y = 3x
∴ 3x – y = 0

Question 6.
Line y = mx + c passes through the points A(2,1) and B(3,2). Determine m and c.
Solution:
Given, A(2, 1) and B(3,2)
Equation of the line in two point form is $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$
∴ The equation of the required line is

$$\frac{y-1}{2-1}=\frac{x-2}{3-2}$$

∴ $\frac{y-1}{1}=\frac{x-2}{1}$
∴ y – 1 = x – 2
∴ y = x – 1
Comparing this equation with y = mx + c, we get
m = 1 and c = – 1

Alternate Method:
Points A(2, 1) and B(3, 2) lie on the line y = mx + c.
∴ They must satisfy the equation.
∴ 2m + c = 1 ...(i)
and 3m + c = 2 ...(ii)
equation (ii) – equation (i) gives m = 1
Substituting m = 1 in (i), we get 2(1) + c = 1
∴ c = 1 – 2 = – 1

Question 7.
Find the equation of the line having inclination 135° and making x-intercept 7.
Solution:
Given, Inclination of line = 0 = 135°
∴ Slope of the line (m) = tan 0 = tan 135°
= tan (90° + 45°)
= – cot 45° = – 1 x-intercept of the required line is 7.
∴ The line passes through (7, 0).
Equation of the line in slope point form is $y – y_1 = m(x – x_1)$
∴ The equation of the required line is y — 0 = – 1 (x – 7)
∴ y = -x + 7
∴ x + y – 7 = 0

Question 8.
The vertices of a triangle are A(3, 4), B(2, 0) and C(- 1, 6). Find the equations of the lines containing
i. side BC
ii. the median AD
iii. the midpoints of sides AB and BC.
Solution:
Vertices of AABC are A(3, 4), B(2, 0) and C(- 1, 6).
i. Equation of the line in two point form is

$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$

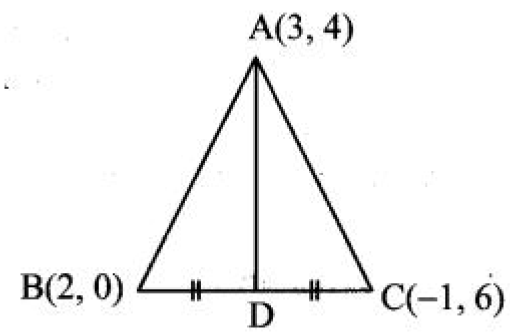
∴ The equation of the side BC is

$\frac{y-0}{6-0}=\frac{x-2}{-1-2}$

$\frac{y}{6}=\frac{x-2}{-3}$

∴ – 3y = 6x – 12
∴ 6x + 3y – 12 = 0
∴ 2x + y – 4 = 0

ii. Let D be the midpoint of side BC.
Then, AD is the median through A.
∴ D = $\left(\frac{2-1}{2},\frac{0+6}{2}\right)=\left(\frac{1}{2},3\right)$
The median AD passes through the points
A(3,4) and D($\frac{1}{2}$, 3)



∴ The equation of the median AD is

$\frac{y-4}{3-4}=\frac{x-3}{\frac{1}{2}-3}$

$\frac{y-4}{-1}=\frac{x-3}{-\frac{5}{2}}$

$\frac{5}{2}$(y-4) = x – 3
∴ 5y – 20 = 2x – 6
∴ 2x – 5y + 14 = 0

iii. Let D and E be the midpoints of side AB and side BC respectively.
The equation of the line DE is

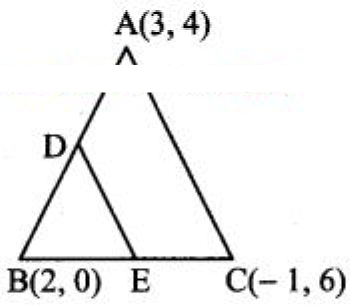
$$\therefore \quad D = \left(\frac{3+2}{2},\frac{4+0}{2}\right) = \left(\frac{5}{2},2\right) \text{ and}$$

$$E = \left(\frac{2-1}{2},\frac{0+6}{2}\right) = \left(\frac{1}{2},3\right)$$

∴ **The equation of the line DE is**

$$\frac{y-2}{3-2} = \frac{x-\frac{5}{2}}{\frac{1}{2}-\frac{5}{2}}$$

$$\therefore \quad \frac{y-2}{1} = \frac{2x-5}{-4}$$



∴ -4(y-2) = 2x-5
∴ 2x + 4y – 13 = 0

Question 9.
Find the x and y-intercepts of the following lines:
i. $\frac{x}{3}+\frac{y}{2}=1$
ii. $\frac{3x}{2}+\frac{2y}{3}=1$
iii. 2x – 3y + 12 = 0
Solution:
i. Given equation of the line is latex]\frac{x}{3}+\frac{y}{2}=1[/latex]
This is of the form $\frac{x}{a}+\frac{y}{b}$ = 1,
where x-intercept = a, y-intercept = b
∴ x-intercept = 3, y-intercept = 2

ii. Given equation of the line is $3x2+2y3$ = 1

∴ $x(23)+y(32)$ = 1

This is of the form = $xa+yb$ = 1,
where x-intercept = a, y-intercept = b
∴ x-intercept = 23 and y-intercept = 32

iii. Given equation of the line is 2x – 3y + 12 = 0
∴ 2x – 3y = – 12
∴ $2x(-12)-3y(-12)=1$

∴ $x-6+y4=1$

This is of the form = 1,
where x-intercept = a, y-intercept = b
∴ x-intercept = – 6 and y-intercept = 4

Question 10.
Find equations of the line which contains the point A(l, 3) and the sum of whose intercepts on the co-ordinate axes is zero.
Solution:
Case I: Line not passing through origin.
Let the equation of the line be

$xa+yb=1$ ....(i)

Since, the sum of the intercepts of the line is zero.
∴ a + b = 0
∴ b = – a
Substituting b = – a in (i), we get

$xa+y(-a)=1$

x – y = a .. .(ii)
Since, the line passes through A(1, 3).
∴ 1 – 3 = a
∴ a = – 2
Substituting the value of a in (ii), equation of the required line is
∴ x – y = – 2,
∴ x – y + 2 = 0

Case II: Line passing through origin.
Slope of line passing through origin and
A(1, 3) is m = $3-01-0$ = 3
∴ Equation of the line having slope m and passing through origin (0, 0) is / = mx.
∴ The equation of the required line is y = 3x
∴ 3x – y = 0

Question 11.
Find equations of the line containing the point A(3, 4) and making equal intercepts on the co-ordinate axes.
Solution:
Case I: Line not passing through origin.

Let the equation of the line be $xa+yb=1$ ....(i)

This line passes through A(3, 4).

∴ $3a+4b=1$.................(ii)

Since, the required line make equal intercepts on the co-ordinate axes.
∴ a = b ...(iii)
Substituting the value of b in (ii), we get

$3a+4a=1$

∴ $7a=1$

∴ a = 7
∴ b = 7 ...[From (iii)]
Substituting the values of a and b in (i), equation of the required line is

$x7+y7=1$ = 1

∴ x + y = 7

Case II: Line passing through origin.

Slope of line passing through origin and A(3,4) is m = $=4-03-0=43$

∴ Equation of the line having slope m and passing through origin (0, 0) is y = mx.
∴ The equation of the required line is 4

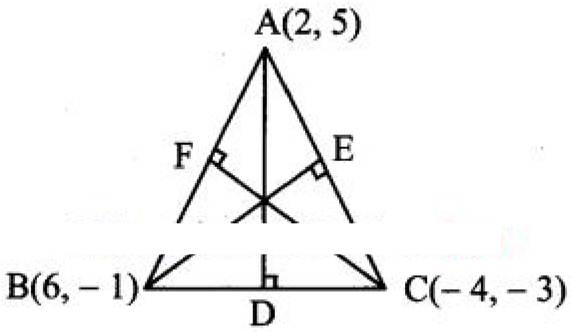
y = 43x

∴ 4x – 3y = 0

Question 12.

Find the equations of the altitudes of the triangle whose vertices are A(2, 5), B(6, – 1) and C(- 4, – 3).

Solution:



A(2, 5), B(6, – 1), C(- 4, – 3) are the vertices of ΔABC.

Let AD, BE and CF be the altitudes through the vertices A, B and C respectively of ΔABC.

∴ Slope of AD = -5 …[∵AD ⊥ BC]

Since altitude AD passes through (2, 5) and has slope – 5,

equation of the altitude AD is y – 5 = -5 (x – 2)

∴ y – 5 = – 5x + 10

∴ 5x +y -15 = 0

Now, slope of AC = -3-5-4-2=-8-6=43

Slope of BE = -34

…[∵ BE ⊥ AC]

Since altitude BE passes through (6,-1) and has slope -34,

equation of the altitude BE is

y-(-1) = -34 (x – 6)

∴ 4 (y + 1) = – 3 (x – 6)

∴ 4y + 4 =-3x+ 18

∴ 3x + 4y – 14 = 0

Also, slope of AB = -1-56-2=-64=-32

∴ Slope of CF = 23 ….[∵ CF ⊥ AB]

Since altitude CF passes through (- 4, – 3) and has slope , 23

equation of the altitude CF is

y-(-3) = 23[x-(-4)]

∴ 3 (y + 3) = 2 (x + 4)

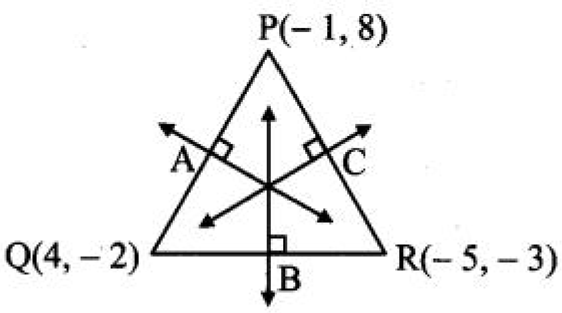
∴ 3y + 9 = 2x + 8

∴ 2x – 3y – 1 = 0

Question 13.

Find the equations of perpendicular bisectors of sides of the triangle whose vertices are P(-1, 8), Q(4, – 2) and R(- 5, – 3).

Solution:



Let A, B and C be the midpoints of sides PQ, QR and PR respectively of APQR.

A is the midpoint of side PQ.

$$\therefore \quad A \equiv \left(\frac{-1+4}{2}, \frac{8-2}{2}\right) = \left(\frac{3}{2}, 3\right)$$

$$\text{Slope of side PQ} = \frac{-2-8}{4-(-1)} = \frac{-10}{5} = -2$$

Slope of perpendicular bisector of PQ is 12 and it passes through (32), 3).

Equation of the perpendicular bisector of side PQ is

y – 3 = 12(x – 32)

y – 3 = (12(2x-32))

∴ 4(y – 3) = 2x – 3

∴ 4y – 12 = 2x – 3

∴ 2x – 4y + 9 = 0

B is the midpoint of side QR

∴ B = (4-52,-2-32)=(-12,-52)

Slope of side QR = -3-(-2)-5-4=-1-9=19

∴ Slope of perpendicular bisector of QR is -9 and it passes through (−12,−52)

∴ Equation of the perpendicular bisector of side QR is

$$y-\left(-\frac{5}{2}\right)=-9\left[x-\left(-\frac{1}{2}\right)\right]$$

$$\therefore \quad \frac{2y+5}{2}=-9\left(\frac{2x+1}{2}\right)$$

∴ 2y + 5 = -18x – 9
∴ 18x + 2y + 14 = 0
∴ 9x + y + 7 = 0
C is the midpoint of side PR.

$$\therefore \quad C \equiv \left(\frac{-1-5}{2}, \frac{8-3}{2}\right)=\left(-3, \frac{5}{2}\right)$$

$$\text{Slope of side PR} = \frac{-3-8}{-5-(-1)}=\frac{-11}{-4}=\frac{11}{4}$$

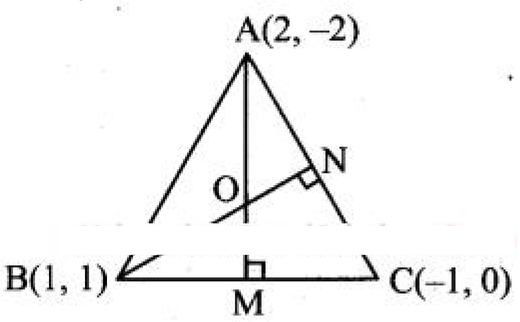
∴ **Slope of perpendicular bisector of PR is** $-\frac{4}{11}$

and it passes through $\left(-3, \frac{5}{2}\right)$.

Equation of the perpendicular bisector of PR is y−52=−411(x+3)

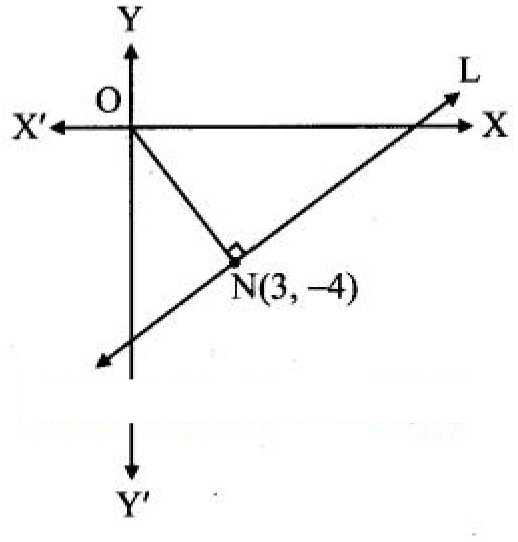
∴ 11(2y-52) =-4(x + 3)
∴ 11(2y – 5) = – 8 (x + 3)
∴ 22y – 55 = – 8x – 24
∴ 8x + 22y -31 = 0

Question 14.
Find the co-ordinates of the orthocentre of the triangle whose vertices are A(2, – 2), B(l, 1) and C(-1,0).
Solution:
Let O be the orthocentre of AABC.
Let AM and BN be the altitudes of sides BC and AC respectively.
Now, slope of BC = 0-1-1-1=-1-2=12
Slope of AM = -2 ,..[∵ AM ⊥ BC]
Since AM passes through (2, – 2) and has slope -2,
equation of the altitude AM is y – (- 2) = – 2 (x – 2)
∴ y + 2 = -2x + 4
∴ 2x + y – 2 = 0 …(i)



Also, slope of AC = 0-(-2)-1-2=2-3
∴ Slope of BN = 32 …[∵ BN ⊥ AC]
Since BN passes through (1,1) and has slope 32, equation of the altitude BN is
y – 1 = 32(x-1)
∴ 2y – 2 = 3x – 3
∴ 3x – 2y – 1 = 0 …(ii)
To find co-ordinates of orthocentre, we have to solve equations (i) and (ii).
By (i) x 2 + (ii), we get
7x – 5 = 0
∴ x = 57
substituting x = 57 in eq (i), we get
2(57) + y – 2 = 0
∴ y = -2(57) + 2
∴ y = -10+147=47
∴ Coordinates of orthocentre O = (57,47)

Question 15.
N(3, – 4) is the foot of the perpendicular drawn from the origin to line L. Find the equation of line L.
Solution:



Slope of ON = $\frac{-4-0}{3-0} = \frac{-4}{3}$
Since line L ⊥ ON,
slope of the line L is $\frac{3}{4}$ and it passes through point N(3, -4).
Equation of the line in slope point form is $y - y_1 = m(x - x_1)$
Equation of line L is
$y-(-4) = \frac{3}{4}(x-3)$
∴ 4(y + 4) = 3(x – 3)
∴ 4y + 16 = 3x – 9
∴ 3x – 4y – 25 = 0

Maharashtra State Board 11th Maths Solutions Chapter 5 Straight Line Ex 5.4

Question 1.
Find the slope, x-intercept, y-intercept of each of the following lines, i. 2x + 3y-6 = 0 ii. 3x-y-9 = 0 iii. x + 2y = 0
Solution:
i. Given equation of the line is 2x + 3y – 6 = 0.
Comparing this equation with ax + by + c = 0,
we get
a = 2, b = 3, c = -6

$$\therefore \quad \text{Slope of the line} = \frac{-a}{b} = \frac{-2}{3}$$

$$x\text{-intercept} = \frac{-c}{a} = \frac{-(-6)}{2} = 3$$

$$y\text{-intercept} = \frac{-c}{b} = \frac{-(-6)}{3} = 2$$

ii. Given equation of the line is 3x – y – 9 = 0.
Comparing this equation with ax + by + c = 0,
we get

a = 3, b = – 1, c = – 9

$$\therefore \quad \text{Slope of the line} = \frac{-a}{b} = \frac{-3}{-1} = 3$$

$$x\text{-intercept} = \frac{-c}{a} = \frac{-(-9)}{3} = 3$$

$$y\text{-intercept} = \frac{-c}{b} = \frac{-(-9)}{-1} = -9$$

iii. Given equation of the line is x + 2y = 0.
Comparing this equation with ax + by + c = 0,
we get
a = 1, b = 2, c = 0

$$\therefore \quad \text{Slope of the line} = \frac{-a}{b} = \frac{-1}{2}$$

$$x\text{-intercept} = \frac{-c}{a} = \frac{-0}{1} = 0$$

$$y\text{-intercept} = \frac{-c}{b} = \frac{-0}{2} = 0$$

Question 2.
Write each of the following equations in ax + by + c = 0 form.
i. y = 2x – 4
ii. y = 4
iii. x2+y4=1
iv. x3−y2=0
i. y = 2x – 4
∴ 2x – y – 4 = 0 is the equation in ax + by + c = 0 form.

ii. y = 4
∴ 0x + 1y – 4 = 0 is the equation in ax + by + c = 0 form.

iii. x2+y4=1
∴ 2x+y4
∴ 2x + y – 4 = 0 is the equation in ax + by + c = 0 form.

iv. x3−y2=0
∴ 2x – 3y = 0
∴ 2x – 3y + 0 = 0 is the equation in ax + by + c = 0 form.
[Note: Answer given in the textbook is '2x – 3y – 6 = 0'. However, as per our calculation it is '2x-3y + 0 = 0'.]

Question 3.
Show that the lines x – 2y – 7 = 0 and 2x – 4y + 15 = 0 are parallel to each other.
Solution:
Let m_1 be the slope of the line x – 2y – 7 = 0.
∴ m_1 = – coefficient of x coefficient of y=-1-2=12
Let m_2 be the slope of the line 2x – 4y + 15 = 0.
∴ m_2 = – coefficient of x coefficient of y=-2-4=12
Since m_1 = m_2
the given lines are parallel to each other.

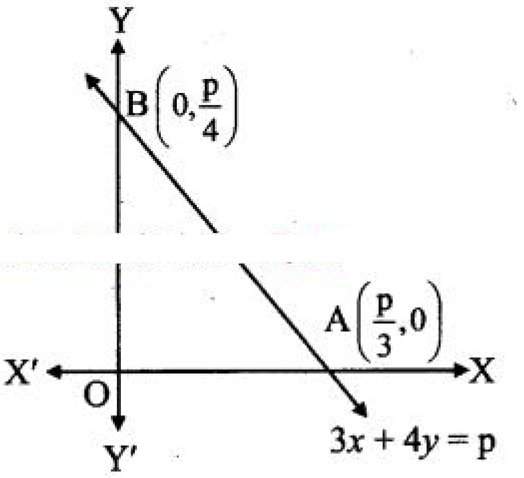
Question 4.
Show that the lines x – 2y – 7 = 0 and 2x + y + 1 = 0 are perpendicular to each other. Find their point of intersection.
Solution:
Let m_1 be the slope of the line x – 2y – 7 = 0.
∴ m_1 = – coefficient of x coefficient of y=-1-2=12
Let m_2 be the slope of the line 2x + y + 1 = 0.
∴ m_2 = – coefficient of x coefficient of y=-21=−2
Since m_1 x m_1 = 12 x (- 2) = -1,
the given lines are perpendicular to each other. Consider,
x – 2y – 7 = 0 ...(i)
2x + y + 1 =0 ...(ii)

Multiplying equation (ii) by 2, we get
4x + 2y + 2 = 0 ...(iii)
Adding equations (i) and (iii), we get
5x – 5 = 0
∴ x = 1
Substituting x = 1 in equation (ii), we get
2 + y + 1 = 0
∴ y = – 3
∴ The point of intersection of the given lines is (1,-3).

Question 5.
If the line 3x + 4y = p makes a triangle of area 24 square units with the co-ordinate axes, then find the value of p.
Solution:
Let the line 3x + 4y = p cuts the X and Y axes at points A and B respectively.
3x + 4y = p

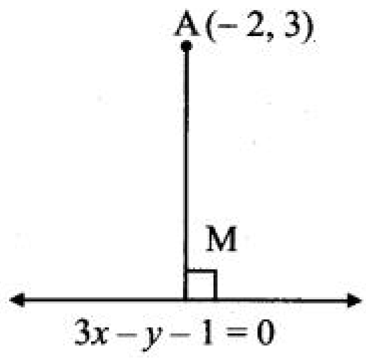
∴ $\frac{3x}{p} + \frac{4y}{p} = 1$

∴ $\frac{x}{\frac{p}{3}} + \frac{y}{\frac{p}{4}} = 1$



This equation is of the form $\frac{x}{a}+\frac{y}{b}=1$,
where a = $\frac{p}{3}$ and b = $\frac{p}{4}$
∴ A (a, 0) ≡ ($\frac{p}{3}$, 0) and B ≡ (0, b) = (0, $\frac{p}{4}$)
∴ OA = $\frac{p}{3}$ and OB = $\frac{p}{4}$
Given, A (ΔOAB) = 24 sq. units
∴ $\left|\frac{1}{2}\times OA\times OB\right| = 24$
∴ $\left|\frac{1}{2}\times\frac{p}{3}\times\frac{p}{4}\right| = 24$
∴ p^2 = 576
∴ p = ± 24

Question 6.
Find the co-ordinates of the foot of the perpendicular drawn from the point A(- 2,3) to the line 3x-y -1 = 0.
Solution:
Let M be the foot of perpendicular drawn from
point A(- 2,3) to the line
3x-y- 1 = 0 ...(i)
Slope of the line 3x-y – 1 = 0 is $\frac{-3}{-1}$ =3.
Since AM ⊥ to line (i),
slope of AM = $\frac{-1}{3}$
∴ Equation of AM is
y – 3 = $\frac{-1}{3}$(x + 2)



∴ 3(y – 3) = – 1(x + 2)
∴ 3y – 9 = -x – 2
∴ x + 3y – 7 = 0 ....(ii)
The foot of perpendicular i.e., point M, is the point of intersection of equations (i) and (ii).
By (i) x 3 + (ii), we get 10x -10 = 0
∴ x = 1
Substituting x = 1 in (ii), we get
1 + 3y – 7 = 0
∴ 3y = 6

∴ y = 2
∴ The co-ordinates of the foot of the perpendicular Mare (1,2).

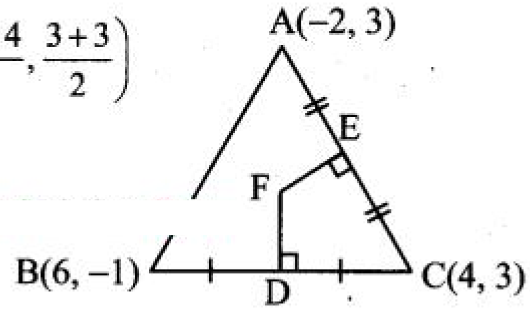
Question 7.
Find the co-ordinates of the circumcentre of the triangle whose vertices are A(- 2, 3), B(6, -1), C(4,3),e
Solution:
Here, A(-2, 3), B(6, -1), C(4, 3) are the vertices of ΔABC.
Let F be the circumcentre of AABC.
Let FD and FE be the perpendicular bisectors of the sides BC and AC respectively.
∴ D and E are the midpoints of side BC and AC respectively.

$\therefore \quad D \equiv \left(\frac{6+4}{2}, \frac{-1+3}{2}\right)$

$\therefore \quad D = (5, 1)$

and $E \equiv \left(\frac{-2+4}{2}, \frac{3+3}{2}\right)$

$\therefore \quad E = (1, 3)$

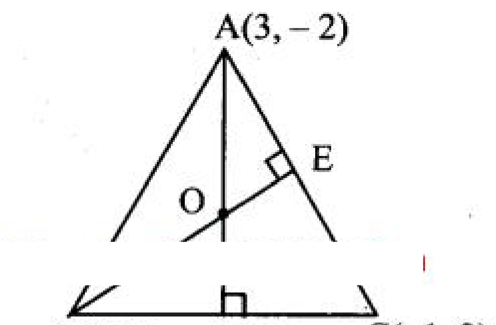


Now, slope of BC $= \frac{3-(-1)}{4-6} = \frac{4}{-2} = -2$

$\therefore \quad$ Slope of FD $= \frac{1}{2}$ $\quad \ldots[\because FD \perp BC]$

Since FD passes through (5, 1) and has slope 1/2 equation of FD is
y – 1 = $\frac{1}{2}$(x-5)
∴ 2 (y – 1) = x – 5
∴ 2y – 2 = x – 5
∴ x – 2y – 3 = 0 …(i)
Since both the points A and C have same y co-ordinates i.e. 3,
the given points lie on the line y = 3.
Since the equation FE passes through E(1, 3),
the equation of FE is x = 1. .. .(ii)
To find co-ordinates of circumcentre, we have to solve equations (i) and (ii).
Substituting the value of x in (i), we get
1 – 2y -3 = 0
∴ y = -1
∴ Co-ordinates of circumcentre F ≡ (1, – 1).

Question 8.
Find the co-ordinates of the orthocentre of the triangle whose vertices are A(3, – 2), B(7,6), C (-1,2).
Solution:
Let O be the orthocentre of ΔABC.
Let AD and BE be the altitudes on the sides BC and AC respectively.



Slope of side BC = $\frac{2-6}{-1-7}=\frac{-4}{-8}=\frac{1}{2}$
∴ Slope of AD = – 2 [∵ AD ⊥ BC]
∴ Equation of line AD is
y – (-2) = (- 2) (x – 3)
∴ y + 2 = -2x + 6
∴ 2x + y -4 = 0 …(i)
Slope of side AC = $\frac{-2-2}{3-(-1)}=\frac{-4}{4}$ = -1
∴ Slope of BE = 1 …[∵ BE ⊥ AC]
∴ Equation of line BE is
y – 6 = 1(x – 7)
∴ y – 6 = x – 1
∴ x = y + 1 …(ii)
Substituting x = y + 1 in (i), we get

2(y + 1) + y – 4 = 0
∴ 2y + 2 + y – 4 = 0
∴ 3y – 2 = 0
∴ y = 23in(ii),wegetSubstitutingy=[latex]23 in (ii), we get
x = 23+1=53
∴ Co-ordinates of orthocentre, O = (53,23)

Question 9.
Show that the lines 3 – 4y + 5 = 0, lx – 8y + 5 = 0 and 4JC + 5y – 45 = 0 are concurrent. Find their point of concurrence.
Solution:
The number of lines intersecting at a point are called concurrent lines and their point of intersection is called the point of concurrence.
Equations of the given lines are
3x – 4y + 5 = 0 …(i)
7x-8y + 5 = 0 …(ii)
4x + 5y – 45 = 0 …(iii)
By (i) x 2 – (ii), we get
– x + 5 = 0
∴ x = 5
Substituting x = 5 in (i), we get
3(5) – 4y + 5 = 0
∴ -4y = – 20
∴ y = 5
∴ The point of intersection of lines (i) and (ii) is given by (5, 5).
Substituting x = 5 and y = 5 in L.H.S. of (iii), we get
L.H.S. = 4(5) + 5(5) – 45
= 20 + 25 – 45
= 0
= R.H.S.
∴ Line (iii) also passes through (5, 5).
Hence, the given three lines are concurrent and the point of concurrence is (5, 5).

Question 10.
Find the equation of the line whose x-intercept is 3 and which ¡s perpendicular to the line 3x – y + 23 = 0.
Solution:
Slope of the line 3x – y + 23 = 0 is 3.
∴ Slope of the required line perpendicular to
3x – y + 23 = 0 is -13
Since the x-intercept of the required line is 3, it passes through (3, 0).
∴ The equation of the required line is ‘
y – 0 = -13(x – 3)
∴ 3y = x + 3
∴ x + 3y = 3

Question 11.
Find the distance of the origin from the line 7x + 24y – 50 = 0.
Solution:
Let p be the perpendicular distance of origin
fromtheline7x + 24y – 50 = 0
Here, a = 7, b = 24, c = -50

$$\therefore \quad p = \left|\frac{c}{\sqrt{a^2+b^2}}\right|$$

$$\therefore \quad p = \left|\frac{-50}{\sqrt{7^2+24^2}}\right| = \left|\frac{-50}{\sqrt{49+576}}\right| = \left|\frac{-50}{25}\right| = 2 \text{ units}$$

Question 12.
Find the distance of the point A(- 2, 3) from the line 12x – 5y – 13 = 0.
Solution:
Let p be the perpendicular distance of the point A(- 2, 3) from the line 12x – 5y – 13 = 0
Here, a = 12, b = – 5, c = – 13, x_1 = -2, y_1 = 3

$$\therefore \quad p = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right| = \left|\frac{12(-2)-5(3)-13}{\sqrt{12^2+(-5)^2}}\right|$$

$$= \left|\frac{-24-15-13}{\sqrt{144+25}}\right| = \left|\frac{-52}{13}\right| = 4 \text{ units}$$

Question 13.
Find the distance between parallel lines 4x – 3y + 5 = 0 and 4xr – 3y + 7 = 0.
Solution:
Equations of the given parallel lines are 4x – 3y + 5 = 0 and 4x – 3y + 1 = 0
Here, a = 4, b = – 3, c_1 = 5 and c_2 = 7
∴ Distance between the parallel lines

$$= \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right| = \left|\frac{5-7}{\sqrt{4^2 + (-3)^2}}\right|$$

$$= \left|\frac{-2}{\sqrt{16+9}}\right| = \left|\frac{-2}{5}\right| = \frac{2}{5} \text{ units}$$

Question 14.
Find the distance between the parallel lines 9x + 6y – 7 = 0 and 3x + 2y + 6 = 0.
Solution:
Equations of the given parallel lines are 3x + 2y + 6 = 0 and
9x + 6y – 1 = 0 i.e., 3x + 2y – 73 =0
Here, a = 3, b = 2, c_1 = 6 and c_2 = –73
∴ Distance between the parallel lines

$$= \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right| = \left|\frac{6 - \left(-\frac{7}{3}\right)}{\sqrt{3^2 + 2^2}}\right|$$

$$= \left|\frac{6 + \frac{7}{3}}{\sqrt{9+4}}\right| = \frac{25}{3\sqrt{13}} \text{ units}$$

Question 15.
Find the points on the line x + y – 4 = 0 which are at a unit distance from the line 4JC + 3y = 10.
Solution:
Let P(x_1, y_1) be a point on the line x + y – 4 = o.
∴ x_1 + y_1 – 4 = 0
∴ y_1 = 4 – x_1 ...(i)
Also, distance of P from the line 4x + 3y- 10 = 0 is 1

$$\therefore \quad 1 = \left|\frac{4x_1 + 3y_1 - 10}{\sqrt{4^2 + 3^2}}\right|$$

$$\therefore \quad 1 = \left|\frac{4x_1 + 3(4 - x_1) - 10}{\sqrt{25}}\right| \quad \ldots\text{[From (i)]}$$

$$\therefore \quad 1 = \left|\frac{4x_1 + 12 - 3x_1 - 10}{5}\right|$$

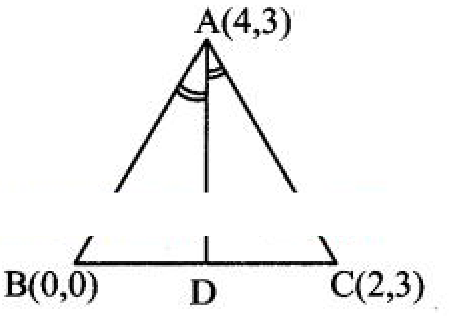
∴ 5 = | x_1 + 2 |
∴ x_1 + 2 = ± 5
∴ x_1 + 2 = 5 or x_1 + 2 = – 5
∴ x_1 = 3 or x_1 = – 7
From (i), when x_1 = 3, y_1 = 1
and when x_1 = -7, y_1 = 11
∴ The required points are (3, 1) and (-7, 11).
[Note: The question has been modified]

Question 16.
Find the equation of the line parallel to the X-axis and passing through the point of intersection of lines x + y – 2 = 0 and 4x + 3y = 10.
Solution:
Let u = x + y – 2 = 0 and v = 4x + 3y – 10 = 0
Equation of the line passing through the point of intersection of lines u = 0 and v = 0 is given by u + kv = 0.
∴ (x + y – 2) + k(4x + 3y – 10) = 0 ...(i)
∴ x + y – 2 + 4kx + 3ky – 10k = 0
∴ x + 4kx + y + 3ky – 2 – 10k = 0
∴ (1+ 4k)x + (1 + 3k)y – 2 – 10k = 0
But, this line is parallel to X-axis.
∴ Its slope = 0
∴ –(1+4k)1+3k = 0
∴ 1 + 4k = 0

∴ k = -14
Substituting the value of k in (i), we get
(x + y – 2) + (4x + 3y – 10) = 0
∴ 4(x +y – 2) – (4x + 3y -10) = 0
∴ 4x + 4y – 8 – 4x – 3y + 10 = 0
∴ y + 2 = 0, which is the equation of the required line.
[Note: Answer given in the textbook is 5y – 8= 0. However, as per our calculation it is y + 2 = 0.]

Question 17.
Find the equation of the line passing through the point of intersection of lines x + y – 2 = 0 and 2xr – 3y + 4 = 0 and making intercept 3 on the X-axis.
Solution:
Let u ≡ x + y – 2 = 0 and v ≡ 2x – 3y + 4 = 0
Equation of the line passing through the point of intersection of lines u = 0 and v = 0 is given by u + kv = 0.
∴ (x +y – 2) + k(2x – 3y + 4) = 0 …(i)
But, x-intercept of line is 3.
∴ It passes through (3, 0).
Substituting x = 3 and y = 0 in (i), we get
(3 + 0 – 2) + k(6 – 0 + 4) = 0
∴ 1 + 10k = 0
k = -110
Substituting the value of k in (i), we get (x + y – 2) + (-110) (2x – 3y + 4) = 0
∴ 10(x + y – 2) – (2x – 3y + 4) = 0
∴ 10x + 10y -20 — 2x + 3y-4 = 0
∴ 8x + 13y – 24 = 0, which is the equation of the required line.

Question 18.
If A(4, 3), B(0, 0) and C(2, 3) are the vertices of ΔABC, then find the equation of bisector of angle BAC.
Solution:
Let the bisector of ∠ BAC meets BC at point D.
∴ Point D divides seg BC in the ratio l(AB) : l(AC)



By distance formula,

$$l(AB) = \sqrt{(4-0)^2 + (3-0)^2} = \sqrt{25} = 5$$

$$l(AC) = \sqrt{(4-2)^2 + (3-3)^2} = \sqrt{4} = 2$$

∴ **Point D divides BC internally in the ratio 5 : 2**

$$\therefore \quad D \equiv \left(\frac{5(2)+2(0)}{5+2}, \frac{5(3)+2(0)}{5+2} \right) \equiv \left(\frac{10}{7}, \frac{15}{7} \right)$$

∴ **Equation of AD is**

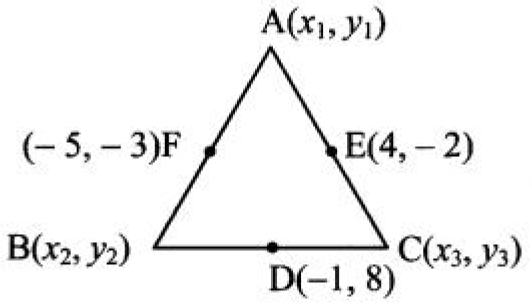
$$\frac{y-3}{\frac{15}{7}-3} = \frac{x-4}{\frac{10}{7}-4}$$

$$\therefore \quad \frac{y-3}{\frac{-6}{7}} = \frac{x-4}{\frac{-18}{7}}$$

∴ 18 (y – 3) = 6 (x – 4)
∴ 3(y – 3) = x – 4
∴ 3y – 9 = x – 4
∴ x – 3y + 5 = 0

Question 19.
D(- 1, 8), E(4, – 2), F(- 5, – 3) are midpoints of sides BC, CA and AB of AABC. Find
i. equations of sides of ΔABC.
ii. co-ordinates of the circumcentre of ΔABC.
Solution:
Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC.

Given, points D, E and F are midpoints of sides BC, CA and AB respectively of ΔABC.



$$D = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

$$\therefore \quad (-1, 8) = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

$$\therefore \quad x_2 + x_3 = -2 \qquad \ldots\text{(i)}$$

$$\text{and } y_2 + y_3 = 16 \qquad \ldots\text{(ii)}$$

$$\text{Also, } E = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$$

$$\therefore \quad (4, -2) = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$$

$$\therefore \quad x_1 + x_3 = 8 \qquad \ldots\text{(iii)}$$

$$\text{and } y_1 + y_3 = -4 \qquad \ldots\text{(iv)}$$

$$\text{Similarly, } F = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\therefore \quad (-5, -3) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

∴ $x_1 + x_2 = -10$ .... (v)
and $y_1 + y_2 = -6$ ....(vi)
For x-coordinates:
Adding (i), (iii) and (v), we get
$2x_1 + 2x_2 + 2x_3 = -4$
∴ $x_1 + x_2 + x_3 = -2$ ....(vii)
Solving (i) and (vii), we get $x_1 = 0$
Solving (iii) and (vii), we get $x_2 = -10$
Solving (v) and (vii), we get $x_3 = 8$

For y-coordinates:
Adding (ii), (iv) and (vi), we get $2y_1 + 2y_2 + 2y_3 = 6$
$y_1 + y_2 + y_3 = 3$ ....(viii)
Solving (ii) and (viii), we get $y_1 = -13$
Solving (iv) and (viii), we get $y_2 = 7$
Solving (vi) and (viii), we get $y_3 = 9$
∴ Vertices of AABC are A(0, – 13), B(- 10, 7), C(8, 9)

a. Equation of side AB is

$$\frac{y+13}{7+13} = \frac{x-0}{-10-0}$$

$$\therefore \quad \frac{y+13}{20} = \frac{x}{-10}$$

$$\therefore \quad \frac{y+13}{2} = -x$$

$$\therefore \quad 2x + y + 13 = 0$$

b. Equation of side BC is

$$\frac{y-7}{9-7} = \frac{x+10}{8+10}$$

$$\therefore \quad \frac{y-7}{2} = \frac{x+10}{18}$$

$$\therefore \quad y - 7 = \frac{x+10}{9}$$

$$\therefore \quad x - 9y + 73 = 0$$

c. Equation of side AC is

$$\frac{y+13}{9+13} = \frac{x-0}{8-0}$$

$$\therefore \quad \frac{y+13}{22} = \frac{x}{8}$$

∴ 8(y + 13) = 22x
∴ 4(y + 13) = 11x
∴ 11x – 4y – 52 = 0

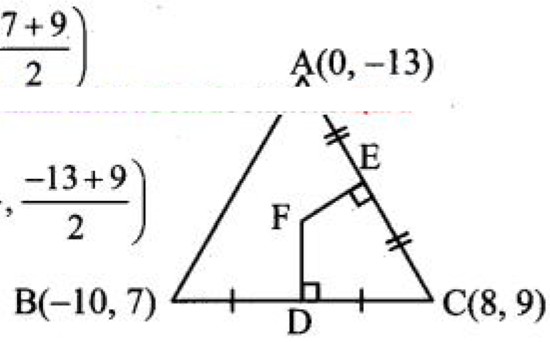
ii. Here, A(0, – 13), B(- 10, 7), C(8, 9) are the vertices of ΔABC.
Let F be the circumcentre of AABC.
Let FD and FE be perpendicular bisectors of the sides BC and AC respectively.
D and E are the midpoints of side BC and AC.

$$\therefore \quad D \equiv \left(\frac{-10+8}{2}, \frac{7+9}{2}\right)$$

$$\therefore \quad D = (-1, 8)$$

$$\text{and } E \equiv \left(\frac{0+8}{2}, \frac{-13+9}{2}\right)$$

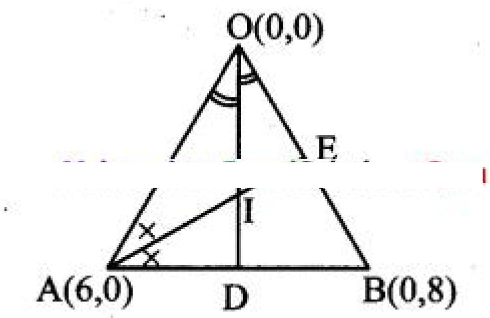
$$\therefore \quad E = (4, -2)$$



$$\text{Now, slope of BC} = \frac{7-9}{-10-8} = \frac{1}{9}$$

∴ Slope of FD = -9 ... [∵ FD ⊥ BC]
Since FD passes through (-1, 8) and has slope -9, equation of FD is
y – 8 = -9 (x +1)
∴ y – 8 = -9x – 9
∴ y = -9x – 1
Also, slope of AC = -13-90-8=114
∴ Slope of FE = -411 [∵ FE ⊥ AC]
Since FE passes through (4, -2) and has slope -4
-411, equation of FE is
(y + 2) = -411 (x – 4)
∴ 11(y + 2) = -4(x – 4)
∴ 11y + 22 = – 4x + 16
∴ 4x + 11y = -6 …………(ii)
To find co-ordinates of circumcentre, we have to solve equations (i) and (ii).
Substituting the value ofy in (ii), we get
4x + 11(-9x- 1) = – 6
∴ 4x – 99x -11 = – 6
∴ -95x = 5
∴ x = -119
Substituting the value of x in (i), we get
y = -9(-119) – 1 = -1019
∴ Co-ordinates of circumcentre F ≡ (-119,-1019)

Question 20.
0(0, 0), A(6, 0) and B(0, 8) are vertices of a triangle. Find the co-ordinates of the incentre of ΔOAB.
Solution:
Let bisector of ∠O meet AB at point D and bisector of ∠A meet BO at point E
∴ Point D divides seg AB in the ratio l(OA): l(OB)
and point E divides seg BO in the ratio l(AB): l(AO)
Let I be the incentre of ∠OAB.



By distance formula,

$$l(OA) = \sqrt{(0-6)^2 + (0-0)^2} = 6$$

$$l(OB) = \sqrt{(0-0)^2 + (0-8)^2} = 8$$

∴ Point D divides AB internally in 6 : 8
i.e. 3 :4

$$\therefore \quad D \equiv \left(\frac{3(0)+4(6)}{3+4}, \frac{3(8)+4(0)}{3+4}\right) \equiv \left(\frac{24}{7}, \frac{24}{7}\right)$$

$$\therefore \quad \text{Equation of OD is } \frac{y-0}{\frac{24}{7}-0} = \frac{x-0}{\frac{24}{7}-0}$$

∴ y = x …(i)
Now, by distance formula,
l(AB) = $=\sqrt{(6-0)^2+(0-8)^2}=\sqrt{36+64}=10$
l(AO) = $\sqrt{(6-0)^2+(0-0)^2}$ = 6
∴ Point E divides BO internally in 10 : 6 i.e. 5:3

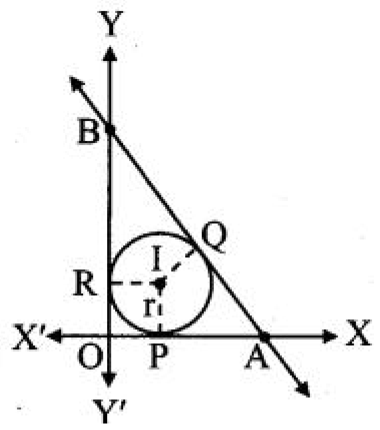
$$\therefore \quad E \equiv \left(\frac{5(0)+3(0)}{5+3}, \frac{5(0)+3(8)}{5+3}\right) \equiv (0, 3)$$

$$\therefore \quad \text{Equation of AE is } \frac{y-0}{3-0} = \frac{x-6}{0-6}$$

$$\therefore \quad \frac{y}{3} = \frac{x-6}{-6}$$

∴ -2y = x – 6
∴ x + 2y = 6 …(ii)
To find co-ordinates of incentre, we have to solve equations (i) and (ii).
Substituting y = x in (ii), we get
x + 2x = 6
∴ x = 2
Substituting the value of x in (i), we get
y = 2
∴ Co-ordinates of incentre I ≡ (2, 2).

Alternate Method:
Let I be the incentre.
I lies in the 1st quadrant.
OPIR is a square having side length r.
Since OA = 6, OP = r,
PA = 6 – r
Since PA = AQ,
AQ = 6 – r …(i)
Since OB = 8, OR = r,
BR = 8 – r
∴ BR = BQ
∴ BQ = 8 – r …(ii)



AB = BQ + AQ

Also, AB = $=\sqrt{OA^2+OB^2}=\sqrt{6^2+8^2}=\sqrt{100}=10$

∴ BQ + AQ= 10
∴ (8 – r) + (6 – r) = 10
∴ 2r = 14- 10 = 4
∴ r = 2
∴ I = (2,2)

Maharashtra State Board 11th Maths Solutions Chapter 5 Straight Line Miscellaneous Exercise 5

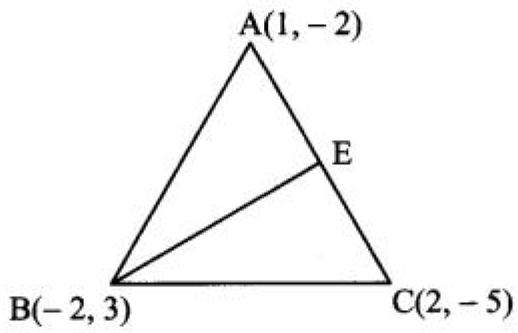
(I) Select the correct option from the given alternatives.

Question 1.
If A is (5, -3) and B is a point on the X-axis such that the slope of line AB is -2, then B ≡
(a) (7, 2)
(b) ($\frac{7}{2}$, 0)
(c) (0, $\frac{7}{2}$)
(d) ($\frac{2}{7}$, 0)
Answer:
(b) ($\frac{7}{2}$, 0)
Hint:
Let B(x, 0) be the point on X-axis.
We have A = (5, -3)
slope of AB = -2
⇒ $\frac{0-(-3)}{x-5}$ = -2
⇒ 3 = -2(x – 5)
⇒ 3 = -2x + 10
⇒ x = $\frac{7}{2}$
Co-ordinates of point B = ($\frac{7}{2}$, 0)

Question 2.
If the point (1, 1) lies on the line passing through the points (a, 0) and (0, b), then $\frac{1}{a}+\frac{1}{b}$=
(a) -1
(b) 0
(c) 1
(d) $\frac{1}{ab}$
Answer:
(c) 1

Hint:
Line passes through (a, 0), (0, b).
x-intercept = a, y-intercept = b
∴ Equation of line is $\frac{x}{a}+\frac{y}{b}=1$ …….(i)
Since line (i) passes through (1, 1), (1, 1) satisfies (i)
∴ $\frac{1}{a}+\frac{1}{b}=1$

Question 3.
If A(1, -2), B(-2, 3) and C(2, -5) are the vertices of ΔABC, then the equation of median BE is
(a) 7x + 13y + 47 = 0
(b) 13x + 7y + 5 = 0
(c) 7x – 13y + 5 = 0
(d) 13x – 7y – 5 = 0
Answer:
(b) 13x + 7y + 5 = 0
Hint:



$$E = \left(\frac{1+2}{2}, \frac{-2-5}{2}\right) = \left(\frac{3}{2}, \frac{-7}{2}\right)$$

Equation of median BE is

$$\frac{y-3}{\frac{-7}{2}-3} = \frac{x+2}{\frac{3}{2}+2}$$

$$\frac{y-3}{\frac{-13}{2}} = \frac{x+2}{\frac{7}{2}}$$

$$7(y-3) = -13(x+2)$$

$$7y - 21 = -13x - 26$$

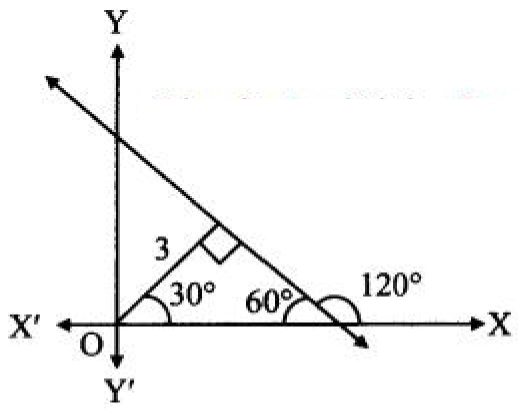
$$13x + 7y + 5 = 0$$

Question 4.
The equation of the line through (1, 2), which makes equal intercepts on the axes, is
(a) x + y = 1
(b) x + y = 2
(c) x + y = 4
(d) x + y = 3
Answer:
(d) x + y = 3
Hint:
Let the equation of required line be
$\frac{x}{a}+\frac{y}{b}=1$ ……..(i)
Since the line makes equal intercepts on the axes, a = b
$\frac{x}{a}+\frac{y}{a}=1$
∴ x + y = a ……(ii)
But, equation (ii) passes through (1, 2).
1 + 2 = a
∴ a = 3
Substituting a = 3 in equation (ii), we get
x + y = 3

Question 5.
If the line kx + 4y = 6 passes through the point of intersection of the two lines 2x + 3y = 4 and 3x + 4y = 5, then k =
(a) 1
(b) 2
(c) 3
(d) 4
Answer:
(b) 2
Hint:

Given two lines are
2x + 3y = 4 ……(i)
3x + 4y = 5 …….(ii)
Multiplying (i) by 3 and (ii) by 2 and then subtracting, we get
y = 2
Substituting y = 2 in (i), we get
x = -1
∴ Point of intersection of lines (i) and (ii) is (-1, 2).
Given that the line kx + 4y = 6 passes through (-1, 2).
k(-1) + 4(2) = 6
∴ k = 2

Question 6.
The equation of a line, having inclination 120° with positive direction of X-axis, which is at a distance of 3 units from the origin is
(a) √3x ± y + 6 = 0
(b) √3x + y ± 6 = 0
(c) x + y = 6
(d) x + y = -6
Answer:
(b) √3x + y ± 6 = 0
Hint:



Here, α = 30° and p = 3 units
Equation of line with inclination a and distance from origin as p is
x cos α + y sin α = p
∴ x cos 30° + y sin 30° = ±3
∴ $\frac{\sqrt{3}x}{2}+\frac{y}{2}=\pm 3$
∴ √3x + y ± 6 = 0

Question 7.
A line passes through (2, 2) and is perpendicular to the line 3x + y = 3. Its y-intercept is
(a) $\frac{1}{3}$
(b) $\frac{2}{3}$
(c) 1
(d) $\frac{4}{3}$
Answer:
(d) $\frac{4}{3}$
Hint:
Slope of line 3x + y = 3 is -3
∴ Slope of line perpendicular to given line = $\frac{1}{3}$
Equation of required line passing through (2, 2) and having slope $\frac{1}{3}$ is
y – 2 = $\frac{1}{3}$(x – 2)
3y – 6 = x – 2
∴ x – 3y + 4 = 0
∴ y-intercept = $\frac{-4}{-3}=\frac{4}{3}$

Question 8.
The angle between the line √3x – y – 2 = 0 and x – √3y + 1 = 0 is
(a) 15°
(b) 30°
(c) 45°
(d) 60°
Answer:
(b) 30°

Hint:

$$\text{Here, } m_1 = \frac{-\sqrt{3}}{-1} = \sqrt{3},$$

$$m_2 = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\text{Now, } \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\frac{1}{\sqrt{3}}}\right| = \left|\frac{3-1}{2\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$$

$$\theta = 30°$$

Question 9.
If kx + 2y – 1 = 0 and 6x – 4y + 2 = 0 are identical lines, then determine k.
(a) -3
(b) −13
(c) 13
(d) 3
Answer:
(a) -3
Hint:
Lines kx + 2y – 1 = 0 and 6x – 4y + 2 = 0 are identical.
∴ k6=2-4=-12
∴ k = -3

Question 10.
Distance between the two parallel lines y = 2x + 7 and y = 2x + 5 is
(a) 2√5√
(b) 15√
(c) 5√2
(d) 25√
Answer:
(d) 25√
Hint:
Here, c_1 = 7, c_2 = 5, a = 2 and b = -1
Distance between parallel lines

$$= \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$$

$$= \frac{|7-5|}{\sqrt{2^2 + (-1)^2}}$$

$$= \frac{2}{\sqrt{5}} \text{ units}$$

II. Answer the following questions.

Question 1.
Find the value of k:
(a) if the slope of the line passing through the points P(3, 4), Q(5, k) is 9.
(b) the points A(1, 3), B(4, 1), C(3, k) are collinear.
(c) the point P(1, k) lies on the line passing through the points A(2, 2) and B(3, 3).
Solution:
(a) Given, P(3, 4), Q(5, k) and
Slope of PQ = 9
k-45-3 = 9
k-42 = 9
k – 4 = 18
k = 22

(b) Given, points A(1, 3), B(4, 1) and C(3, k) are collinear.
Slope of AB = Slope of BC
1-34-1=k-13-4

-23=k-1-1
2 = 3k – 3
k = 53

(c) Given, point P(1, k) lies on the line joining A(2, 2) and B(3, 3).
Slope of AB = Slope of BP
3-23-2=3-k3-1
1 = 3-k2
2 = 3 – k
k = 1

Question 2.
Reduce the equation 6x + 3y + 8 = 0 into slope-intercept form. Hence, find its slope.
Solution:
Given equation is 6x + 3y + 8 = 0, which can be written as
3y = – 6x – 8
y = -6x3−83
y = -2x – 83
This is of the form y = mx + c with m = -2
y = -2x – 83 is in slope-intercept form with slope = -2

Question 3.
Find the distance of the origin from the line x = -2.
Solution:
Given equation of line is x = -2
This equation represents a line parallel to Y-axis and at a distance of 2 units to the left of Y-axis.
∴ Distance of the origin from the line is 2 units.

Question 4.
Does point A(2, 3) lie on the line 3x + 2y – 6 = 0? Give reason.
Solution:
Given equation is 3x + 2y – 6 = 0.
Substituting x = 2 and y = 3 in L.H.S. of given equation, we get
L.H.S. = 3x + 2y – 6
= 3(2) + 2(3) – 6
= 6
≠ R.H.S.
∴ Point A does not lie on the given line.

Question 5.
Which of the following lines passes through the origin?
(a) x = 2
(b) y = 3
(c) y = x + 2
(d) 2x – y = 0
Answer:
(d) 2x – y = 0
Hint:
Any line passing through origin is of the form y = mx or ax + by = 0.
Here in the given option, 2x – y = 0 is in the form ax + by = 0.
∴ Option (d) is the correct answer.

Question 6.
Obtain the equation of the line which is:
(a) parallel to the X-axis and 3 units below it.
(b) parallel to the Y-axis and 2 units to the left of it.
(c) parallel to the X-axis and making an intercept of 5 on the Y-axis.
(d) parallel to the Y-axis and making an intercept of 3 on the X-axis.
Solution:
(a) Equation of a line parallel to X-axis is y = k.
Since the line is at a distance of 3 units below X-axis, k = -3
∴ The equation of the required line is y = -3.

(b) Equation of a line parallel to Y-axis is x = h.
Since the line is at a distance of 2 units to the left of Y-axis, h = -2
∴ The equation of the required line is x = -2.

(c) Equation of a line parallel to X-axis with y-intercept ‘k’ is y = k.
Here, y-intercept = 5
∴ The equation of the required line is y = 5.

(d) Equation of a line parallel to Y-axis with x-intercept ‘h’ is x = h.
Here, x-intercept = 3
∴ The equation of the required line is x = 3.

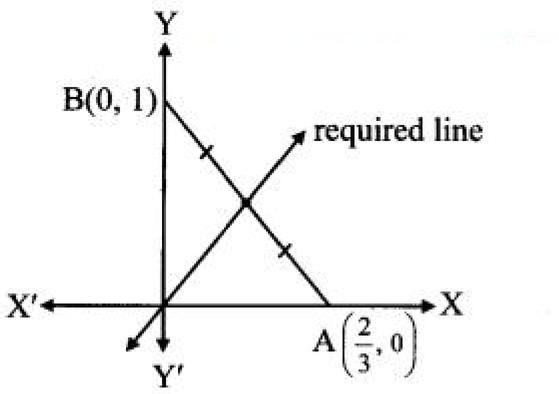
Question 7.
Obtain the equation of the line containing the point:
(i) (2, 3) and parallel to the X-axis.
(ii) (2, 4) and perpendicular to the Y-axis.
Solution:
(i) Equation of a line parallel to X-axis is of the form y = k.
Since the line passes through (2, 3), k = 3
∴ The equation of the required line is y = 3.

(ii) Equation of a line perpendicular to Y-axis
i.e., parallel to X-axis, is of the form y = k.
Since the line passes through (2, 4), k = 4
∴ The equation of the required line is y = 4.

Question 8.
Find the equation of the line:
(a) having slope 5 and containing point A(-1, 2).
(b) containing the point T(7, 3) and having inclination 90°.
(c) through the origin which bisects the portion of the line 3x + 2y = 2 intercepted between the co-ordinate axes.
Solution:
(a) Given, slope(m) = 5 and the line passes through A(-1, 2).
Equation of the line in slope point form is $y - y_1 = m(x - x_1)$
The equation of the required line is
y – 2 = 5(x + 1)
y – 2 = 5x + 5
∴ 5x – y + 7 = 0

(b) Given, Inclination of line = θ = 90°
the required line is parallel to Y-axis.
Equation of a line parallel to Y-axis is of the form x = h.
Since the line passes through (7, 3), h = 7
∴ The equation of the required line is x = 7.

(c) Given equation of the line is 3x + 2y = 2.



$\frac{3x}{2}+\frac{2y}{2}=1$

$\frac{x}{\frac{2}{3}}+\frac{y}{1}=1$

This equation is of the form $\frac{x}{a}+\frac{y}{b}=1$, with a = $\frac{2}{3}$, b= 1.

The line 3x + 2y = 2 intersects the X-axis at A($\frac{2}{3}$, 0) and Y-axis at B(0, 1).

Required line is passing through the midpoint of AB.

Midpoint of AB = $\left(\frac{\frac{2}{3}+0}{2},\frac{0+1}{2}\right)=\left(\frac{1}{3},\frac{1}{2}\right)$

∴ Required line passes through (0, 0) and $\left(\frac{1}{3},\frac{1}{2}\right)$.

Equation of the line in two point form is

$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$

∴ The equation of the required line is

$\frac{y-0}{2-0}=\frac{x-0}{3-0}$

2y = 3x

∴ 3x – 2y = 0

Question 9.
Find the equation of the line passing through the points S(2, 1) and T(2, 3).
Solution:
The required line passes through the points S(2, 1) and T(2, 3).
Since both the given points have same x co-ordinates i.e. 2
the given points lie on a line parallel to Y-axis.
∴ The equation of the required line is x = 2.

Question 10.
Find the distance of the origin from the line 12x + 5y + 78 = 0.
Solution:
Let p be the perpendicular distance of origin from the line 12x + 5y + 78 = 0.
Here, a = 12, b = 5, c = 78

$$p=\left|\frac{c}{\sqrt{a^2+b^2}}\right|$$

$$p=\left|\frac{78}{\sqrt{12^2+5^2}}\right|=\left|\frac{78}{\sqrt{144+25}}\right|$$

$$=\frac{78}{13}$$

$$=6\text{ units}$$

Question 11.
Find the distance between the parallel lines 3x + 4y + 3 = 0 and 3x + 4y + 15 = 0.
Solution:
Equations of the given parallel lines are 3x + 4y + 3 = 0 and 3x + 4y + 15 = 0
Here, a = 3, b = 4, c_1 = 3 and c_2 = 15
∴ Distance between the parallel lines

$$=\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$$

$$=\left|\frac{3-15}{\sqrt{3^2+4^2}}\right|=\left|\frac{-12}{\sqrt{9+16}}\right|$$

$$=\frac{12}{5}\text{ units}$$

Question 12.
Find the equation of the line which contains the point A(3, 5) and makes equal intercepts on the co-ordinates axes.
Solution:
Case I: Line not passing through origin.

Let the equation of the line be $\frac{x}{a}+\frac{y}{b}=1$ …….(i)

This line passes through A(3, 5).

∴ $\frac{3}{a}+\frac{5}{b}=1$ ……..(ii)

Since the required line makes equal intercepts on the co-ordinates axes,
a = b …….(iii)
Substituting the value of b in (ii), we get

$\frac{3}{a}+\frac{5}{a}=1$

∴ a = 8
∴ b = 8 …… [From (iii)]
Substituting the values of a and b in equation (i), the equation of the required line is

$\frac{x}{8}+\frac{y}{8}=1$

∴ x + y = 8

Case II: Line passing through origin.
Slope of line passing through origin and A(3, 5) is

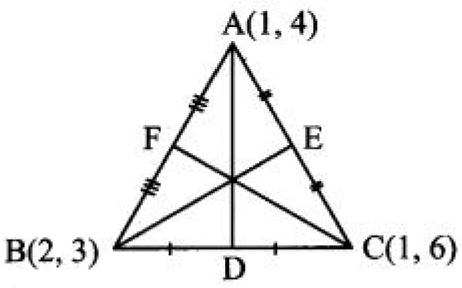
m = $\frac{5-0}{3-0}=\frac{5}{3}$

∴ Equation of the line having slope m and passing through origin (0, 0) is y = mx.
∴ The equation of the required line is

y = 53x
∴ 5x – 3y = 0

Question 13.
The vertices of a triangle are A(1, 4), B(2, 3) and C(1, 6). Find equations of
(a) the sides
(b) the medians
(c) perpendicular bisectors of sides
(d) altitudes of ?ABC
Solution:
Vertices of ΔABC are A(1, 4), B(2, 3) and C(1, 6)
(a) Equation of the line in two point form is y−y1y2−y1 = x−x1x2−x1
Equation of side AB is
y−43−4=x−12−1
y – 4 = -1(x – 1)
y – 4 = -x + 1
x + y = 5
Equation of side BC is
y−36−3=x−21−2
-1(y – 3) = 3(x – 2)
-y + 3 = 3x – 6
∴ 3x + y = 9
Since both the points A and C have same x co-ordinates i.e. 1
the points A and C lie on a line parallel to Y-axis.
∴ The equation of side AC is x = 1.

(b) Let D, E and F be the midpoints of sides AC and AB respectively of ΔABC.



Then $D \equiv \left(\frac{2+1}{2}, \frac{3+6}{2}\right) = \left(\frac{3}{2}, \frac{9}{2}\right)$

$E \equiv \left(\frac{1+1}{2}, \frac{6+4}{2}\right) = (1, 5)$

$F \equiv \left(\frac{1+2}{2}, \frac{4+3}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

Equation of median AD is

$$\frac{y-4}{\frac{9}{2}-4} = \frac{x-1}{\frac{3}{2}-1}$$

$$\frac{y-4}{\frac{1}{2}} = \frac{x-1}{\frac{1}{2}}$$

$x - y + 3 = 0$

Equation of median BE is

$$\frac{y-3}{5-3} = \frac{x-2}{1-2}$$

$\therefore \quad -1(y-3) = 2(x-2)$

$\therefore \quad -y + 3 = 2x - 4$

$\therefore \quad 2x + y = 7$

Equation of median CF is

$$\frac{y-6}{\frac{7}{2}-6} = \frac{x-1}{\frac{3}{2}-1}$$

$$\therefore \quad \frac{y-6}{-\frac{5}{2}} = \frac{x-1}{\frac{1}{2}}$$

$\therefore \quad y - 6 = -5(x-1)$

$\therefore \quad 5x + v - 11 = 0$

(c) Slope of side BC = (6–31–2)=(3–1) = -3

Slope of perpendicular bisector of BC is 13 and the line passes through (32,92).

Equation of the perpendicular bisector of side BC is

(y–92)=13(x–32)

3(2y – 9) = (2x – 3)
6y – 27 = 2x – 3
2x – 6y + 24 = 0
∴ x – 3y + 12 = 0
Since both the points A and C have same x co-ordinates i.e. 1
the points A and C lie on the line x = 1.
AC is parallel to Y-axis and therefore, perpendicular bisector of side AC is parallel to X-axis.
Since, the perpendicular bisector of side AC passes through E(1, 5).
The equation of perpendicular bisector of side AC is y = 5.

Slope of side AB = (3–42–1) = -1

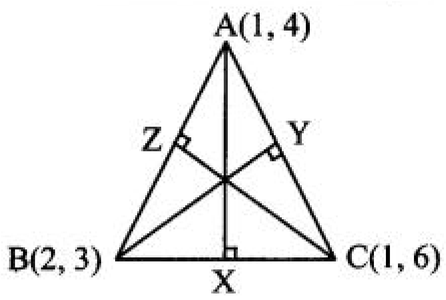
Slope of perpendicular bisector of AB is 1 and the line passes through (32,72).

Equation of the perpendicular bisector of side AB is

(y–72)=1(x–32)

2y – 7 = 2x – 3
2x – 2y + 4 = 0
∴ x – y + 2 = 0

(d) Let AX, BY, and CZ be the altitudes through the vertices A, B and C respectively of ΔABC.



Slope of BC = -3
Slope of AX = 13 ……[∵ AX ⊥ BC]
Since altitude AX passes through (1, 4) and has slope 13,
equation of altitude AX is
y – 4 = 13(x – 1)
3y – 12 = x – 1
∴ x – 3y + 11 = 0
Since both the points A and C have same x co-ordinates i.e. 1
the points A and C lie on the line x = 1.
AC is parallel to Y-axis and therefore, altitude BY is parallel to X-axis.
Since the altitude BY passes through B(2, 3), the equation of altitude BY is y = 3.
Also, slope of AB = -1
Slope of CZ = 1
Since altitude CZ passes through (1, 6) and has slope 1,
equation of altitude CZ is
y – 6 = 1(x – 1)
∴ x – y + 5 = 0

Question 14.
Find the equation of the line which passes through the point of intersection of lines x + y – 3 = 0, 2x – y + 1 = 0 and which is parallel to X-axis.
Solution:
Let u ≡ x + y – 3 = 0 and v ≡ 2x – y + 1 = 0
Equation of the line passing through the point of intersection of lines u = 0 and v = 0 is given by u + kv = 0.
(x + y – 3) + k(2x – y + 1) = 0 …..(i)
x + y – 3 + 2kx – ky + k = 0
x + 2kx + y – ky – 3 + k = 0
(1 + 2k)x + (1 – k)y – 3 + k = 0
But, this line is parallel to X-axis
Its slope = 0
⇒ –(1+2k)1–k=0
⇒ 1 + 2k = 0
⇒ k = –12
Substituting the value of k in (i), we get

$(x + y - 3) + \frac{-1}{2}(2x - y + 1) = 0$
⇒ $2(x + y - 3) - (2x - y + 1) = 0$
⇒ $2x + 2y - 6 - 2x + y - 1 = 0$
⇒ $3y - 7 = 0$, which is the equation of the required line.

Question 15.
Find the equation of the line which passes through the point of intersection of lines $x + y + 9 = 0$, $2x + 3y + 1 = 0$ and which makes x-intercept 1.
Solution:
Let $u \equiv x + y + 9 = 0$ and $v \equiv 2x + 3y + 1 = 0$
Equation of the line passing through the point of intersection of lines u = 0 and v = 0 is given by u + kv = 0.
$(x + y + 9) + k(2x + 3y + 1) = 0$ ……(i)
⇒ $x + y + 9 + 2kx + 3ky + k = 0$
⇒ $(1 + 2k)x + (1 + 3k)y + 9 + k = 0$
But, x-intercept of this line is 1.
⇒ $\frac{-(9+k)}{1+2k}$
⇒ $-9 - k = 1 + 2k$
⇒ $k = \frac{-10}{3}$
Substituting the value of k in (i), we get
$(x + y + 9) + \left(\frac{-10}{3}\right)(2x + 3y + 1) = 0$
⇒ $3(x + y + 9) - 10(2x + 3y + 1) = 0$
⇒ $3x + 3y + 27 - 20x - 30y - 10 = 0$
⇒ $-17x - 27y + 17 = 0$
⇒ $17x + 27y - 17 = 0$, which is the equation of the required line.

Question 16.
Find the equation of the line through A(-2, 3) and perpendicular to the line through S(1, 2) and T(2, 5).
Solution:
Slope of ST = $\frac{5-2}{2-1} = 3$
Since the required line is perpendicular to ST,
slope of required line = $\frac{-1}{3}$ and line passes through A(-2, 3)
Equation of the line in slope point form is $y - y_1 = m(x - x_1)$
The equation of the required line is
$y - 3 = \frac{-1}{3}(x + 2)$
⇒ $3(y - 3) = -(x + 2)$
⇒ $3y - 9 = -x - 2$
⇒ $x + 3y = 7$

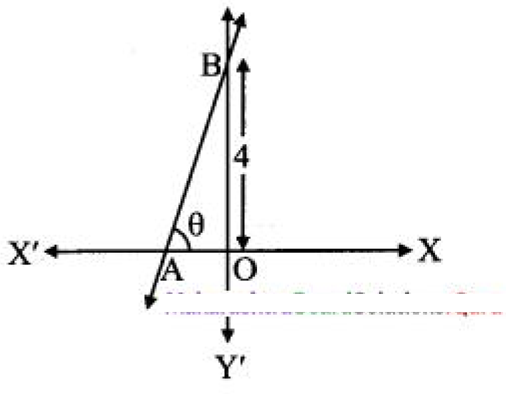
Question 17.
Find the x-intercept of the line whose slope is 3 and which makes intercept 4 on the Y-axis.
Solution:
Equation of a line having slope 'm' and y-intercept 'c' is $y = mx + c$
Given, m = 3, c = 4
The equation of the line is $y = 3x + 4$
$3x - y = -4$

$\frac{3x}{(-4)} - \frac{y}{(-4)} = 1$

$\frac{x}{\left(\frac{-4}{3}\right)} + \frac{y}{4} = 1$

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where
x-intercept = a
x-intercept = $\frac{-4}{3}$

Alternate Method:



Let θ be the inclination of the line.
Then tan θ = 3 …..[∵ slope = 3 (given)]

OBOA=3

4OA=3

OA = 43

x-intercept = −43 as point A is to the left side of Y-axis.

Question 18.
Find the distance of P(-1, 1) from the line 12(x + 6) = 5(y – 2).
Solution:
Given equation of the line is
12(x + 6) = 5(y – 2)
12x + 72 = 5y – 10
12x – 5y + 82 = 0
Let p be the perpendicular distance of the point (-1, 1) from the line 12x – 5y + 82 = 0.

$$p = \left|\frac{12(-1) - 5(1) + 82}{\sqrt{12^2 + (-5)^2}}\right|$$

$$= \left|\frac{-12 - 5 + 82}{\sqrt{144 + 25}}\right| = \left|\frac{65}{13}\right| = 5 \text{ units.}$$

Question 19.
Line through A(h, 3) and B(4,1) intersect the line lx – 9y -19 = 0 at right angle. Find the value of h.
Solution:
Given, A(h, 3) and B(4, 1)
Slope of AB (m_1) = 1-34-h
m_1 = 2h-4
Slope of line 7x – 9y – 19 = 0 is m_2 = 79
Since line AB and line 7x – 9y – 19 = 0 are perpendicular to each other,
$m_1 \times m_2$ = -1
2h-4×79=−1
14 = 9(4 – h)
14 = 36 – 9h
9h = 22
h = 229

Question 20.
Two lines passing through M(2, 3) intersect each other at an angle of 45°. If slope of one line is 2, find the equation of the other line.
Solution:
Let m be the slope of the required line which make an angle of 45° with the other line.
Slope of one of the lines is 2.
tan 45° = ∣ ∣ m-21+m(2) ∣ ∣
1 = ∣ ∣ m-21+2m ∣ ∣
m-21+2m=±1
m-21+2 m = 1 or m-21+2 m = -1
m – 2 = 1 + 2m or m – 2 = -1 – 2m
m = -3 or 3m = 1
m = -3 or m = 13
Required line passes through M(2, 3)
When m = -3, equation of the line is
y – 3 = -3(x – 2)
y – 3 = -3x + 6
∴ 3x + y = 9
When m = 13, equation of the line is
y – 3 = 13(x – 2)
3y – 9 = x – 2
∴ x – 3y + 7 = 0

Question 21.
Find the y-intercept of the line whose slope is 4 and which has x-intercept 5.
Solution:
Given, slope = 4, x-intercept = 5
Since the x-intercept of the line is 5, it passes through (5, 0).
Equation of the line in slope point form is $y - y_1 = m(x - x_1)$
Equation of the required line is
y – 0 = 4(x – 5)

y = 4x – 20
4x – y = 20

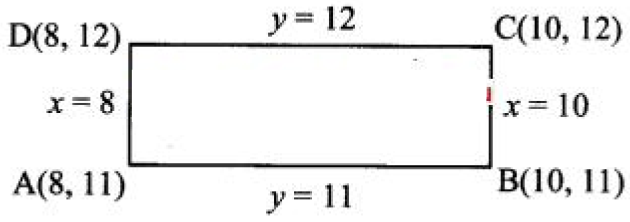
$4x20-y20=1$

$x5+y(-20)=1$

This equation is of the form $xa+yb=1$, where

x-intercept = b, y-intercept = -20

Question 22.
Find the equations of the diagonals of the rectangle whose sides are contained in the lines x = 8, x = 10, y = 11 and y = 12.
Solution:
Given, equations of sides of rectangle are x = 8, x = 10, y = 11 and y = 12



From the above diagram,
Vertices of rectangle are A(8, 11), B(10, 11), C(10, 12) and D(8, 12).
Equation of diagonal AC is

$y-1112-11=x-810-8$

$y-111=x-82$

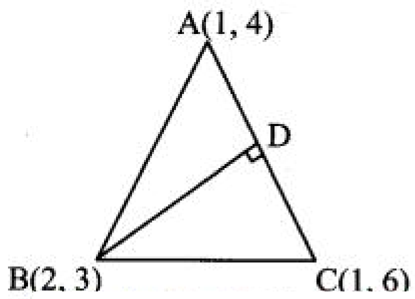
2y – 22 = x – 8
x – 2y + 14 = 0
Equation of diagonal BD is

$y-1112-11=x-108-10$

$y-111=x-10-2$

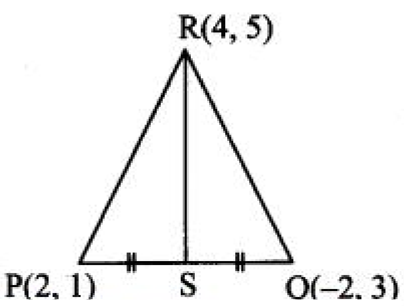
-2y + 22 = x – 10
x + 2y = 32

Question 23.
A(1, 4), B(2, 3) and C(1, 6) are vertices of AABC. Find the equation of the altitude through B and hence find the co-ordinates of the point where this altitude cuts the side AC of ΔABC.
Solution:
Vertices of triangle are A(1, 4), B(2, 3) and C(1, 6).
Let BD be the altitude through the vertex B.



Since both the points A and C have same x co-ordinates i.e. 1
the given points lie on a line parallel to Y-axis.
The equation of the line AC is x = 1 …..(i)
AC is parallel to Y-axis and therefore, altitude BD is parallel to X-axis.
Since the altitude BD passes through B(2, 3), the equation of altitude BD is y = 3 ……(ii)
From (i) and (ii),
Point of intersection of AC and altitude BD is (1, 3).

Question 24.
The vertices of ΔPQR are P(2, 1), Q(-2, 3) and R(4, 5). Find the equation of the median through R.
Solution:



Let S be the midpoint of side PQ.
Then RS is the median through R.

S = $\left(\frac{2-2}{2}, \frac{3+1}{2}\right)$ = (0, 2)

The median RS passes through the points R(4, 5) and S(0, 2).
∴ Equation of median RS is

$\frac{y-5}{2-5}=\frac{x-4}{0-4}$

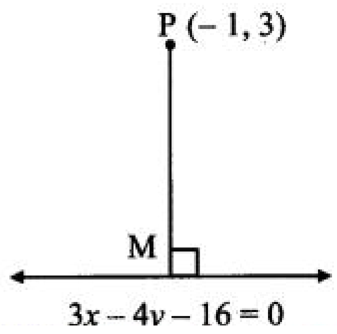
⇒ $\frac{y-5}{-3}=\frac{x-4}{-4}$
⇒ 4(y – 5) = 3(x – 4)
⇒ 4y – 20 = 3x – 12
∴ 3x – 4y + 8 = 0

Question 25.
A line perpendicular to segment joining A(1, 0) and B(2, 3) divides it internally in the ratio 1 : 2. Find the equation of the line. Solution:
Given, A(1, 0), B(2, 3)
Slope of AB = $\frac{3-0}{2-1}$ = 3
Required line is perpendicular to AB.
Slope of required line = $\frac{-1}{3}$
Let point C divide AB in the ratio 1 : 2.

$$C \equiv \left(\frac{1(2)+2(1)}{1+2}, \frac{1(3)+2(0)}{1+2}\right)$$
$$= \left(\frac{4}{3}, \frac{3}{3}\right)$$
$$= \left(\frac{4}{3}, 1\right)$$

Required line passes through $\left(\frac{4}{3}, 1\right)$ and has slope = $\frac{-1}{3}$
Equation of the line in slope point form is y – y_1 = m(x – x_1)
The equation of the required line is
y – 1 = $\frac{-1}{3}\left(x-\frac{4}{3}\right)$
⇒ 3(y – 1) = $-1\left(x-\frac{4}{3}\right)$
⇒ 3y – 3 = -x + $\frac{4}{3}$
⇒ 9y – 9 = -3x + 4
⇒ 3x + 9y = 13

Question 26.
Find the co-ordinates of the foot of the perpendicular drawn from the point P(-1, 3) to the line 3x – 4y – 16 = 0.
Solution:



Let M be the foot of perpendicular drawn from P(-1, 3) to the line 3x – 4y – 16 = 0
Slope of the line 3x – 4y – 16 = 0 is $\frac{-3}{-4}=\frac{3}{4}$
Since PM ⊥ to line (i),
slope of PM = $\frac{-4}{3}$
Equation of PM is
y – 3 = $\frac{-4}{3}$ (x + 1)
⇒ 3(y – 3) = -4(x + 1)
⇒ 3y – 9 = -4x – 4
∴ 4x + 3y – 5 = 0 ……(ii)
The foot of perpendicular i.e., point M, is the point of intersection of equation (i) and (ii).
By (i) × 3 + (ii) × 4, we get
25x = 68
x = $\frac{68}{25}$
Substituting x = $\frac{68}{25}$ in (ii), we get

$$4\left(\frac{68}{25}\right) + 3y - 5 = 0$$

$$3y = 5 - 4\left(\frac{68}{25}\right) = \frac{125-272}{25} = \frac{-147}{25}$$

$$y = \frac{-49}{25}$$

The co-ordinates of the foot of perpendicular M are $(6825,-4925)$

Question 27.

Find points on the X-axis whose distance from the line $x3+y4=1$ is 4 units.

Solution:

The equation of line is $x3+y4=1$

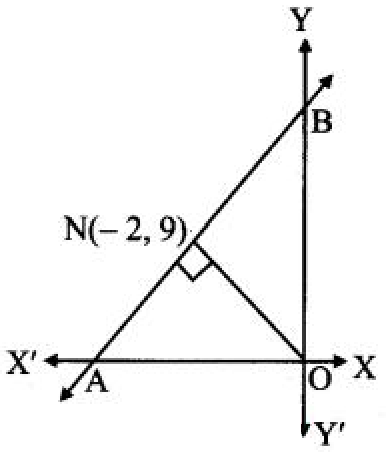
i.e. 4x + 3y – 12 = 0 …..(i)
Let (h, 0) be a point on the X-axis.
The distance of this point from line (i) is 4.

⇒ $|4h+3(0)-12|42+32\sqrt{}=4$

⇒ $|4h-12|5=4$

⇒ |4h – 12| = 20
⇒ 4h – 12 = 20 or 4h – 12 = -20
⇒ 4h = 32 or 4h = -8
⇒ h = 8 or h = -2
∴ The required points are (8, 0) and (-2, 0).

Question 28.
The perpendicular from the origin to a line meets it at (-2, 9). Find the equation of the line.
Solution:



Slope of ON = $9-0-2-0=-92$

Since line AB ⊥ ON,
slope of the line AB perpendicular to ON is 29 and it passes through point N(-2, 9).
Equation of the line in slope point form is $y – y_1 = m(x – x_1)$
Equation of line AB is
y – 9 = 29(x + 2)
⇒ 9(y – 9) = 2(x + 2)
⇒ 9y – 81 = 2x + 4
⇒ 2x – 9y + 85 = 0

Question 29.

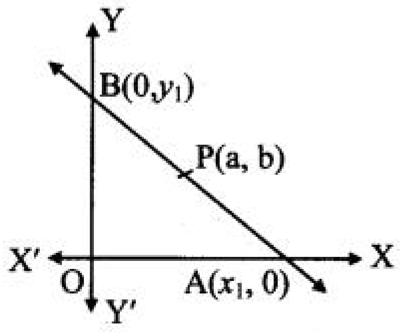
P(a, b) is the midpoint of a line segment intercepted between the axes. Show that the equation of the line is $xa+yb=2$.

Solution:
Let the intercepts of a line AB be x_1 and y_1 on the X and Y-axes respectively.
A ≡ $(x_1, 0)$, B = $(0, y_1)$

P(a, b) is the midpoint of a line segment AB intercepted between the axes.

$\therefore \quad P = \left(\frac{x_1+0}{2}, \frac{0+y_1}{2}\right)$

$\therefore \quad (a, b) = \left(\frac{x_1}{2}, \frac{y_1}{2}\right)$



$\therefore \quad a = \frac{x_1}{2}$

$\therefore \quad x_1 = 2a$

and $b = \frac{y_1}{2}$

$\therefore \quad y_1 = 2b$

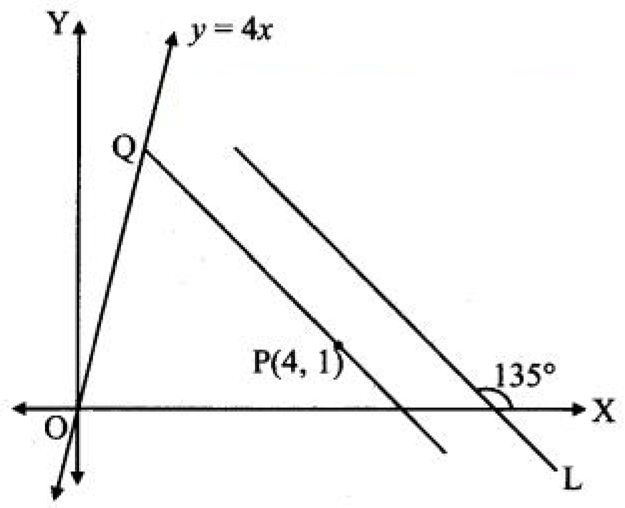
$\therefore$ Equation of the required line AB is

$\frac{x}{x_1} + \frac{y}{y_1} = 1$

$\therefore \quad \frac{x}{2a} + \frac{y}{2b} = 1$

$\therefore \quad \frac{x}{a} + \frac{y}{b} = 2$

Question 30.
Find the distance of the line 4x – y = 0 from the point P(4, 1) measured along the line making an angle of 135° with the positive X-axis.
Solution:



Let a line L make angle 135° with positive X-axis.
Required distance = PQ, where PQ || line L
Slope of PQ = tan 135°
= tan (180° – 45°)
= -tan 45°
= -1
Equation of PQ is
y – 1 = (-1)(x – 4)
y – 1 = -x + 4
x + y = 5 .....(i)
To get point Q we solve the equation 4x – y = 0 with (i)
Substituting y = 4x in (i), we get
5x = 5
x = 1
Substituting x = 1 in (i), we get
1 + y = 5
y = 4
∴ Q = (1, 4)
PQ = $\sqrt{(4-1)^2+(1-4)^2}$
= $\sqrt{9+9}$
= 3√2

Question 31.
Show that there are two lines which pass through A(3, 4) and the sum of whose intercepts is zero.
Solution:
Case I: Line not passing through origin.
Let the equation of the line be $\frac{x}{a}+\frac{y}{b}=1$ ......(1)
This line passes through (3, 4)

$\frac{3}{a}+\frac{4}{b}=1$ .....(ii)

Since the sum of the intercepts of the line is zero,
a + b = 0
a = -b ......(iii)
Substituting the value of a in (ii), we get

$\frac{3}{-b}+\frac{4}{b}=1$

$\frac{1}{b}$ = 1

b = 1
a = -1 ......[From (iii)]
Substituting the values of a and b in (i),
the equation of the required line is

$\frac{x}{-1}+\frac{y}{1}=1$

x – y = -1
∴ x – y + 1 = 0

Case II: Line passing through origin.
Slope of line passing through origin and A(3, 4) is

m = $\frac{4-0}{3-0}=\frac{4}{3}$

Equation of the line having slope m and passing through origin (0, 0) is y = mx.
The equation of the required line is
y = $\frac{4}{3}$x
∴ 4x – 3y = 0
∴ There are two lines which pass through A(3, 4) and the sum of whose intercepts is zero.

Question 32.
Show that there is only one line which passes through B(5, 5) and the sum of whose intercepts is zero.
Solution:
When line is passing through origin, the sum of intercepts made by the line is zero.
Slope of line passing through origin and B(5, 5) is
m = $\frac{5-0}{5-0}$ = 1
Equation of the line having slope m and passing through origin (0, 0) is y = mx.
The equation of the required line is y = x
∴ x – y = 0
∴ There is only one line which passes through B(5, 5) and the sum of whose intercepts is zero.