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- Digvijay

Maharashtra State Board 12th Maths Solutions Chapter 2 Matrices Ex 2.1

Question 1.
Apply the given elementary transformation on each of the following matrices.
A = [1-103], $R_1 \leftrightarrow R_2$
Solution:
A = [1-103]
By $R_1 \leftrightarrow R_2$, we get,
A ~ [-1130]

Question 2.
B = [12-1534], $R_1 \to R_1 \to R_2$
Solution:
B = [12-1534],
$R_1 \to R_1 \to R_2$ gives,
B ~ [-12-65-14]

Question 3.
A = [5143], $C_1 \leftrightarrow C_2$; B = [3415], $R_1 \leftrightarrow R_2$. What do you observe?
Solution:
A = [5143]
By $C_1 \leftrightarrow C_2$, we get,
A ~ [4351] ...(1)
B = [3415]
By $R_1 \leftrightarrow R_2$, we get,
B ~ [4351] ...(2)
From (1) and (2), we observe that the new matrices are equal.

Question 4.
A = [1021-13], $2C_2$
B = [120425], $-3R_1$
Find the addition of the two new matrices.
Solution:
A = [1021-13]
By $2C_2$, we get,
A ~ [1042-13]
B = [120425]
By $-3R_1$, we get,
B ~ [-3204-65]
Now, addition of the two new matrices

$$= \begin{bmatrix} 1 & 4 & -1 \\ 0 & 2 & 3 \end{bmatrix} + \begin{bmatrix} -3 & 0 & -6 \\ 2 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 4 & -7 \\ 2 & 6 & 8 \end{bmatrix}$$

Question 5.
A = ⎡⎣⎢ 123-113301 ⎤⎦⎥ , $3R_3$ and then $C_3 + 2C_2$.
Solution:
A = ⎡⎣⎢ 123-113301 ⎤⎦⎥
By $3R_3$, we get
A ~ ⎡⎣⎢ 129-119303 ⎤⎦⎥
By $C_3 + 2C_2$, we get,
A ~ ⎛⎝⎜ 129-1193+2(-1)0+2(1)3+2(9) ⎞⎠⎟
∴ A ~ ⎛⎝⎜ 129-1191221 ⎞⎠⎟

Question 6.

A = ⎛⎝⎜⎜ 123-113301⎞⎠⎟⎟ , $C_3 + 2C_2$ and then $3R_3$. What do you conclude from Ex. 5 and Ex. 6 ?

Solution:

A = ⎛⎝⎜⎜ 123-113301⎞⎠⎟⎟

By $C_3 + 2C_2$, we get,

A ~ ⎛⎝⎜⎜ 123-1133+2(-1)0+2(1)1+2(3)⎞⎠⎟⎟

∴ A ~ ⎛⎝⎜⎜ 123-113127⎞⎠⎟⎟

By $3R_3$, we get

A ~ ⎛⎝⎜⎜ 129-1191221⎞⎠⎟⎟

We conclude from Ex. 5 and Ex. 6 that the matrix remains same by interchanging the order of the elementary transformations. Hence, the transformations are commutative.

Question 7.

Use suitable transformation on [1324] into an upper triangular matrix.

Solution:

Let A = [1324]

By $R_2 – 3R_1$, we get,

A ~ [102-2]

This is an upper triangular matrix.

Question 8.

Convert [12-13] into an identity matrix by suitable row transformations.

Solution:

Let A = [12-13]

By $R_2 – 2R_1$, we get,

A ~ [10-15]

By (15)R_2, we get,

A ~ [10-11]

By $R_1 + R_2$, we get,

A ~ [1001]

This is an identity matrix.

Question 9.

Transform ⎡⎣⎢⎢ 123-112234⎤⎦⎥⎥ into an upper triangular matrix by suitable row transformations.

Solution:

Let A = ⎡⎣⎢⎢ 123-112234⎤⎦⎥⎥

By $R_2 – 2R_1$ and $R_3 – 3R_1$, we get

A ~ ⎡⎣⎢⎢ 100-1352-1-2⎤⎦⎥⎥

By R_3 – (53)R_2, we get,

A ~ ⎛⎝⎜⎜ 100-1302-1-13⎞⎠⎟⎟

This is an upper triangular matrix.

Maharashtra State Board 12th Maths Solutions Chapter 2 Matrices Ex 2.2

Question 1.
Find the co-factors of the elements of the following matrices
(i) [−1−324]
Solution:
Let A = [−1−324]
Here, a_{11} = -11, M_{11} = 4
∴ $A_{11} = (-1)^{1+1}(4) = 4$
a_{12} = 2, M_{12} = -3
∴ $A_{12} = (-1)^{1+2}(- 3) = 3$
a_{21} = – 3, M_{21} = -2
∴ $A_{21} = (- 1)^{2+1}(2) = -2$
a_{22} = 4, M_{22} = -1
∴ $A_{22} = (-1)^{2+2}(-1) = -1$.

(ii) ⎡⎣⎢⎢1−2−2−13025−1⎤⎦⎥⎥
Solution:
Let A = ⎡⎣⎢⎢1−2−2−13025−1⎤⎦⎥⎥
The co-factor of a_{ij} is given by $A_{ij} = (-1)^{i+j}M_{ij}$

$$\text{Now, } M_{11} = \begin{vmatrix} 3 & 5 \\ 0 & -1 \end{vmatrix} = -3 - 0 = -3$$

$$\therefore A_{11} = (-1)^{1+1}(-3) = -3$$

$$M_{12} = \begin{vmatrix} -2 & 5 \\ -2 & -1 \end{vmatrix} = 2 + 10 = 12$$

$$\therefore A_{12} = (-1)^{1+2}(12) = -12$$

$$M_{13} = \begin{vmatrix} -2 & 3 \\ -2 & 0 \end{vmatrix} = 0 + 6 = 6$$

$$\therefore A_{13} = (-1)^{1+3}(6) = 6$$

$$M_{21} = \begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix} = 1 - 0 = 1$$

$$\therefore A_{21} = (-1)^{2+1}(1) = -1$$

$$M_{22} = \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = -1 + 4 = 3$$

$$\therefore A_{22} = (-1)^{2+2}(3) = 3$$

$$M_{23} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = 0 - 2 = -2$$

$$\therefore A_{23} = (-1)^{2+3}(-2) = 2$$

$$M_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5 - 6 = -11$$

$$\therefore A_{31} = (-1)^{3+1}(-11) = -11$$

$$M_{32} = \begin{vmatrix} 1 & 2 \\ -2 & 5 \end{vmatrix} = 5 + 4 = 9$$

$$\therefore A_{32} = (-1)^{3+2}(9) = -9$$

$$M_{33} = \begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix} = 3 - 2 = 1$$

$$\therefore A_{33} = (-1)^{3+3}(1) = 1.$$

Question 2.
Find the matrix of co-factors for the following matrices
(i) [143−1]
Solution:

Let A = [143-1]

Here, a_{11} = 1, M_{11} = -1

∴ $A_{11} = (-1)^{1+1}(-1) = -1$

a_{12} = 3, M_{12} = 4

∴ $A_{12} = (-1)^{1+2}(4) = -4$

a_{21} = 4, M_{21} = 3

∴ $A_{21} = (-1)^{2+1}(3) = -3$

a_{22} = -1, M_{22} = 1

∴ $A_{22} = (-1)^{2+1}(1) = 1$

∴ the co-factor matrix = $[A_{11}A_{21}A_{12}A_{22}]$

= (-1-3-41)

(ii) ⎡ ⎣ ⎢ 1-2001323-5⎤ ⎦ ⎥

Solution:

Let A = ⎡ ⎣ ⎢ 1-2001323-5⎤ ⎦ ⎥

Here, a_{11} = 1,

$$A_{11} = \begin{vmatrix} 1 & 3 \\ 3 & -5 \end{vmatrix} = -14$$

a_{12} = 0,

$$A_{12} = \begin{vmatrix} -2 & 0 \\ 3 & -5 \end{vmatrix} = -10$$

a_{13} = 2,

$$A_{13} = \begin{vmatrix} -2 & 1 \\ 0 & 3 \end{vmatrix} = -6$$

a_{21} = − 2

$$A_{21} = \begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix} = 6$$

a_{22} = 1

$$A_{22} = \begin{vmatrix} 1 & 0 \\ 2 & -5 \end{vmatrix} = -5$$

a_{23} = 3

$$A_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} = -3$$

a_{31} = 0

$$A_{31} = \begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix} = -2$$

a_{32} = 3

$$A_{32} = \begin{vmatrix} 1 & 2 \\ -2 & 3 \end{vmatrix} = -7$$

a_{33} = − 5

$$A_{33} = \begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix}$$

A_{11} = -14, A_{12} = -10, A_{13} = -6,

A_{21} = 6, A_{22} = -5, A_{23} = -3,

A_{31} = -2, A_{32} = -7, A_{33} = 1.

∴ the co-factor matrix

= ⎡ ⎣ ⎢ $A_{11}A_{21}A_{31}A_{12}A_{22}A_{32}A_{13}A_{23}A_{33}$⎤ ⎦ ⎥ = ⎡ ⎣ ⎢ -146-2-10-5-7-6-31⎤ ⎦ ⎥

Question 3.
Find the adjoint of the following matrices.
(i) [23−35]
Solution:
Let A = [23−35]
Here, a_{11} = 2, M_{11}= 5
∴ $A_{11} = (-1)^{1+1}(5) = 5$
a_{12} = -3, M_{12} = 3
∴ $A_{12} = (-1)^{1+2}(3) = -3$
a_{21} = 3, M_{21} = -3
∴ A $A_{21} = (-1)^{2+1}(-3) = 3$
a_{22} = 5, M_{22} = 2
∴ $A_{22} = (-1)^{2+1} = 2$
∴ the co-factor matrix = [A11A21A12A22]
= [53−32]
∴ adj A = (5−332)

(ii) ⎡⎣⎢ 1−2−2−13025−1⎤⎦⎥
Solution:

$$\text{Now, } M_{11} = \begin{vmatrix} 3 & 5 \\ 0 & -1 \end{vmatrix} = -3 - 0 = -3$$

$$\therefore \; A_{11} = (-1)^{1+1}(-3) = -3$$

$$M_{12} = \begin{vmatrix} -2 & 5 \\ -2 & -1 \end{vmatrix} = 2 + 10 = 12$$

$$\therefore \; A_{12} = (-1)^{1+2}(12) = -12$$

$$M_{13} = \begin{vmatrix} -2 & 3 \\ -2 & 0 \end{vmatrix} = 0 + 6 = 6$$

$$\therefore \; A_{13} = (-1)^{1+3}(6) = 6$$

$$M_{21} = \begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix} = 1 - 0 = 1$$

$$\therefore \; A_{21} = (-1)^{2+1}(1) = -1$$

$$M_{22} = \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = -1 + 4 = 3$$

$$\therefore \; A_{22} = (-1)^{2+2}(3) = 3$$

$$M_{23} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = 0 - 2 = -2$$

$$\therefore \; A_{23} = (-1)^{2+3}(-2) = 2$$

$$M_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5 - 6 = -11$$

$$\therefore \; A_{31} = (-1)^{3+1}(-11) = -11$$

$$M_{32} = \begin{vmatrix} 1 & 2 \\ -2 & 5 \end{vmatrix} = 5 + 4 = 9$$

$$\therefore \; A_{32} = (-1)^{3+2}(9) = -9$$

$$M_{33} = \begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix} = 3 - 2 = 1$$

$$\therefore \; A_{33} = (-1)^{3+3}(1) = 1.$$

A_{11} = -3, A_{12} = -12, A_{13} = 6,
A_{21} = -1, A_{22} = 3, A_{23} = 2,
A_{31} = -11, A_{32} = -9, A_{33} = 1
∴ the co-factor matrix = ⎡⎣⎢ A11 A21 A31 A12 A22 A32 A15 A23 A33⎤⎦⎥

= ⌈ ⌊ ∣ -3-1-11-123-9621⌉ ⌋ ∣

∴ adj A = ⌈ ⌊ ∣ -3-126-132-11-91⌉ ⌋ ∣

Question 4.

If A = ⌈ ⌊ ∣ 131-1002-23⌉ ⌋ ∣ , verify that A (adj A) = (adj A) A = | A | · I

Solution:

A = ⌈ ⌊ ∣ 131-1002-23⌉ ⌋ ∣

$$\therefore\ |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$$

$$= 1(0+0) + 1(9+2) + 2(0-0)$$

$$= 0 + 11 + 0 = 11$$

First we have to find the co-factor matrix $= [A_{ij}]_{3\times 3}$, where $A_{ij} = (-1)^{i+j} M_{ij}$

$$\text{Now, } A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = 0 + 0 = 0$$

$$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -(9+2) = -11$$

$$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0$$

$$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -(-3-0) = 3$$

$$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 3 - 2 = 1$$

$$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -(0+1) = -1$$

$$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 2 - 0 = 2$$

$$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -(-2-6) = 8$$

$$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 0 + 3 = 3$$

Hence the co-factor matrix

$$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{bmatrix}$$

$$\therefore\ \text{adj } A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0-0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \qquad \ldots (1)$$

(adj A)A

$$= \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0-0+0 & 0+2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \quad \ldots (2)$$

$$|A|\cdot I = 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \quad \ldots (3)$$

From (1), (2) and (3), we get,
A(adj A) = (adj A)A = |A|·I.
Note: This relation is valid for any non-singular matrix A.

Question 5.
Find the inverse of the following matrices by the adjoint method
(i) [−1−352]
Solution:
Let A = [−1−352]
∴ |A| = ∣∣∣−1−352∣∣∣ = -2 + 15 = 13 ≠ 0
∴ A^{-1} exists.
First we have to find the co-factor matrix
= $[A_{ij}]_{2\times2}$, where $A_{ij} = (-1)^{i+j}M_{ij}$
Now, $A_{11} = (-1)^{1+1}M_{11} = 2$
$A_{12} = (-1)^{1+2}M_{12} = -(-3) = 3$
$A_{21} = (-1)^{2+1}M_{21} = -5$
$A_{22} = (-1)^{2+2}M_{22} = -1$
Hence, the co-factor matrix

$$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}$$

$$\therefore \text{ adj } A = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

(ii) [24−23]
Solution:
Let A = [24−23]
|A| = = 6 + 8 = 14 ≠ 0
∴ A^{-1} exist
First we have to find the co-factor matrix
= $[A_{ij}]_{2\times2}$ where $A_{ij} = (-1)^{i+j}M_{ij}$
Now, $A_{11} = (-1)^{1+1}M_{11} = 3$
$A_{12} = (-1)^{1+2}M = -4$
$A_{21} = (-2)^{2+1}M_{21} = (-2) = 2$
$A_{22} = (-1)^{2+2}M_{22} = 2$
Hence the co-factor matrix
= [A11A21A12A22] = [32−42]
∴ adj A = [3−422]
∴ A^{-1} = 1∣ A∣ (adj A) = 114(3−422)

(iii) ⎡ ⎣ ⎢ 13503200−1⎤ ⎦ ⎥
Solution:
Let A = ⎡ ⎣ ⎢ 13503200−1⎤ ⎦ ⎥

$$\therefore\ |A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix}$$

$$= 1(-3-0) - 0 + 0$$

$$= -3 \neq 0$$

$\therefore\ A^{-1}$ exists.

First we have to find the co-factor matrix

$= [A_{ij}]_{3\times 3}$, where $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = -3 - 0 = -3$

$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} = -(-3-0) = 3$

$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix} = 6 - 15 = -9$

$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} = -(0-0) = 0$

$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 1 & 0 \\ 5 & -1 \end{vmatrix} = -1 - 0 = -1$

$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} 1 & 0 \\ 5 & 2 \end{vmatrix} = -(2-0) = -2$

$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix} = 0 - 0 = 0$

$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = -(0-0) = 0$

$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} = 3 - 0 = 3$

$\therefore$ the co-factor matrix

$$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\therefore\ \text{adj } A = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$= \frac{1}{-3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

∴ A-1 = 13⎡ ⎣ ⎢ 3−3901200−3⎤ ⎦ ⎥

(iv) ⎡ ⎣ ⎢ 100220345⎤ ⎦ ⎥

Solution:

Let A = ⎡ ⎣ ⎢ 100220345⎤ ⎦ ⎥

∴ |A| = ⎡ ⎣ ⎢ 100220345⎤ ⎦ ⎥

= 1(10 – 0) – 0 + 0

= 1(10) – 0 + 0

= 10 ≠ 0

∴ A-1 exists.

First we have to find the co-factor matrix

$= [A_{ij}]_{3\times3}$, where $A_{ij} = (-1)^{i+j}M_{ij}$

Now, $A_{11} = (-1)^{1+1}M_{11} = \begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} = 10 - 0 = 10$

$A_{12} = (-1)^{1+2}M_{12} = -\begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix} = -0 - 0 = 0$

$A_{13} = (-1)^{1+3}M_{13} = \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$

$A_{21} = (-1)^{2+1}M_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = -10 - 0 = -10$

$A_{22} = (-1)^{2+2}M_{22} = \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = 5 - 0 = 5$

$A_{23} = (-1)^{2+3}M_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = -0 - 0 = 0$

$A_{31} = (-1)^{3+1}M_{31} = \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = 8 - 6 = 2$

$A_{32} = (-1)^{3+2}M_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -4 - 0 = -4$

$A_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$

∴ the co-factor matrix

$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix}$

∴ adj A = $\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$

∴ A-1 = 1| A| (adj A)

= 110((| 1000−10502−42)) |

∴ A-1 = 110((| 1000−10502−42)) |

Question 6.
Find the inverse of the following matrices
(i) [122−1]
Solution:
Let A = [122−1]

$\therefore\ |A| = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1-4 = -5 \neq 0$

$\therefore\ A^{-1}$ exists.

Consider $AA^{-1} = I$

$\therefore\ \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By $R_2 - 2R_1$, we get,

$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

By $\left(-\frac{1}{5}\right)R_2$, we get,

$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 2/5 & -1/5 \end{bmatrix}$

By $R_1 - 2R_2$, we get,

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$

$\therefore\ A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

The answer can be checked by finding the product AA^{-1}.

$$AA^{-1} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$$

$$= \begin{bmatrix} 1\left(\frac{1}{5}\right)+2\left(\frac{2}{5}\right) & 1\left(\frac{2}{5}\right)+2\left(-\frac{1}{5}\right) \\ 2\left(\frac{1}{5}\right)-1\left(\frac{2}{5}\right) & 2\left(\frac{2}{5}\right)-1\left(-\frac{1}{5}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{5}+\frac{4}{5} & \frac{2}{5}-\frac{2}{5} \\ \frac{2}{5}-\frac{2}{5} & \frac{4}{5}+\frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence, A^{-1} is the required answer.

(ii) [2-1-32]

Solution:

Let A = [2-1-32]

$$\therefore\ |A| = \begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix} = 4 - 3 = 1 \neq 0$$

$\therefore\ A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\therefore\ \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_1 + R_2$, we get,

$$\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

By $R_2 + R_1$, we get,

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

By $R_1 + R_2$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

∴ A-1 = (2132)

(iii) ⌈ ⌊ ∣ 013121231⌉ ⌋ ∣

Solution:

Let A = ⌈ ⌊ ∣ 013121231⌉ ⌋ ∣

$$\therefore\ |A| = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$$

$$= 0(2-3) - 1(1-9) + 2(1-6)$$

$$= 0 + 8 - 10$$

$$= -2 \neq 0.$$

$\therefore\ A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\therefore\ \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_3 - 3R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$ and $R_3 + 5R_2$, we get,

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix}$$

By $\left(\frac{1}{2}\right)R_3$, we get,

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$$

By $R_1 + R_3$ and $R_2 - 2R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$$

(iv) ⌈ ⌊ ∣ 250011-103⌉ ⌋ ∣

Solution:

Let A = ⌈ ⌊ ∣ 250011-103⌉ ⌋ ∣

$$\therefore |A| = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$$

$$= 2(3-0) - 0(15-0) - 1(5-0)$$

$$= 6 - 0 - 5 = 1 \neq 0.$$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $3R_1$, we get,

$$\begin{bmatrix} 6 & 0 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - 5R_1$, we get,

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & 6 & 15 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - 5R_3$, we get,

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 + R_2$ and $R_3 - R_2$, we get,

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -12 & 5 & -5 \\ -15 & 6 & -5 \\ 15 & -6 & 6 \end{bmatrix}$$

By $\left(\frac{1}{3}\right)R_3$, we get,

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -12 & 5 & -5 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

By $R_1 + 3R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}.$$

Maharashtra State Board 12th Maths Solutions Chapter 2 Matrices Ex 2.3

Question 1.
Solve the following equations by the inversion method.
(i) x + 2y = 2, 2x + 3y = 3
Solution:
The given equations can be written in the matrix form as :
$[1223][xy] = [23]$
This is of the form AX = B, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1 \neq 0$$

$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

By $(-1)R_2$, we get,

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$\therefore A^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

Now, premultiply AX = B by A^{-1}, we get,

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)X = A^{-1}B$

$\therefore IX = A^{-1}B$

$\therefore X = = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = = \begin{bmatrix} -6+6 \\ 4-3 \end{bmatrix} = = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

By equality of matrices,

x = 0, y = 1 is the required solution.

(ii) x + y = 4, 2x – y = 5

Solution:

x + y = 4, 2x – y = 5

The given equations can be written in the matrix form as:

$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

This is of the form AX = B ⇒ X ⇒ $A^{-1}B$

$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$

|A| = -1 – 2 = -3 ≠ 0

$$\text{Adj } A = \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$$

$$= \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ 8 & -5 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

By equality of matrices.

x = 3, y = 1

(iii) 2x + 6y = 8, x + 3y = 5

Solution:

The given equations can be written in the matrix form as :

[2163][xy]=[85]

This is of the form AX = B, where

A = [2163], X = [xy] and B = [85]

Let us find A^{-1}.

|A| = ∣∣∣2163∣∣∣ = 6 – 6 = 0

∴ A^{-1} does not exist.
Hence, x and y do not exist.

Question 2.
Solve the following equations by reduction method.
(i) 2x + y = 5, 3x + 5y = -3
Solution:
The given equations can be written in the matrix form as :

$$\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

By $2R_2$, we get,

$$\begin{bmatrix} 2 & 1 \\ 6 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$$

By $R_2 - 3R_1$, we get,

$$\begin{bmatrix} 2 & 1 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -21 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2x+y \\ 0+7y \end{bmatrix} = \begin{bmatrix} 5 \\ -21 \end{bmatrix}$$

By equality of matrices,
2x + y = 5 …(1)
7y = -21 …(2)
From (2), y = -3
Substituting y = -3 in (1), we get,
2x – 3 = 5
∴ 2x = 8 ∴ x = 4
Hence, x = 4, y = -3 is the required solution.

(ii) x + 3y = 2, 3x + 5y = 4.
Solution:
The given equations can be written in the matrix form as :

[1335][xy] = [24]

By $R_2 - 3R_1$, we get

[103−4][xy] = (2−2)

∴ [x+30−4y] = [2−2]

By equality of matrices,
x + 3y = 2 …(1)
-4y = -2
From (2), y = 12
Substituting y = 12 in (1), we get,
x + 32 = 2
∴ x = 2 – 32=12
Hence, x = 12, y = 12 is the required solution.

(iii) 3x – y = 1, 4x + y = 6
Solution:
The given equations can be written in the matrix form as :

$$\begin{bmatrix} 3 & -1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

By $4R_1$ and $3R_2$, we get,

$$\begin{bmatrix} 12 & -4 \\ 12 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 18 \end{bmatrix}$$

By $R_2 - R_1$, we get,

$$\begin{bmatrix} 12 & -4 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 12x - 4y \\ 0 + 7y \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$$

By equality of matrices,
12x – 4y = 4 … (1)
7y = 14 … (2)
From (2), y = 2
Substituting y = 2 in (1), we get,
12x – 8 = 4
∴ 12x = 12 ∴ x = 1
Hence, x = 1, y = 2 is the required solution.

(iv) 5x + 2y = 4, 7x + 3y = 5
Solution:
5x + 2y = 4 ………..(1)
7x + 3y = 5 …………(2)
Multiplying Eq. (1) with 7 and Eq. (2) with 5

$$\begin{array}{r} 35x + 14y = 28 \\ 35x + 15y = 25 \\ - \quad\;\; - \quad\;\;\;\; \\ \hline -1y = 3 \\ y = -3 \end{array}$$

Put y = -3 into Eq. (1)
5x + 2y = 4
5x + 2(-3) = 4
5x – 6 = 4
5x = 4 + 6
5x = 10
x = 105
x = 2
Hence, x = 2, y = -3 is the required solution.

Question 3.
The cost of 4 pencils, 3 pens and 2 erasers is ₹ 60. The cost of 2 pencils, 4 pens and 6 erasers is ₹ 90, whereas the cost of 6 pencils, 2 pens and 3 erasers is ₹ 70. Find the cost of each item by using matrices.
Solution:
Let the cost of 1 pencil, 1 pen and 1 eraser be ₹ x, ₹ y and ₹ z respectively.
Then, from the given conditions,
4x + 3y + 2z = 60
2x + 4y + 6z = 90, i.e., x + 2y + 3z = 45
6x + 2y + 3z = 70
These equations can be written in the matrix form as :

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 60 \\ 70 \end{bmatrix}$$

By $R_2 - 4R_1$ and $R_3 - 6R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & -10 & -15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ -120 \\ -200 \end{bmatrix}$$

By $R_3 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ -120 \\ 40 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + 2y + 3z \\ 0 - 5y - 10z \\ 0 + 0 + 5z \end{bmatrix} = \begin{bmatrix} 45 \\ -120 \\ 40 \end{bmatrix}$$

By equality of matrices,
x + 2y + 3z = 45 …….(1)
– 5y – 10z = – 120 …….(2)
5z = 40
From (3), z = 8
Substituting z = 8 in (2), we get,
– 5y – 80 = -120
∴ – 5y = -40 ∴ y = 8
Substituting y = 8, z = 8 in (1), we get,
x + 16 + 24 = 45
∴ x + 40 = 45 ∴ x = 5
∴ x = 5, y = 8, z = 8
Hence, the cost is ₹ 5 for a pencil, ₹ 8 for a pen and ₹ 8 for an eraser.

Question 4.
If three numbers are added, their sum is 2. If 2 times the second number is subtracted from the sum of first and third numbers, we get 8 and if three times the first number is added to the sum of second and third numbers, we get 4. Find the numbers using matrices.
Solution:
Let the three numbers be x, y and z. According to the given conditions,
x + y + z = 2
x + z – 2y = 8, i.e., x – 2y + 2 = 8
and y + z + 3x = 4, i.e., 3x + y + z = 4
Hence, the system of linear equations is
x + y + z = 2
x – 2y + z = 8
3x + y + z = 4
These equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix}$$

By $R_2 - R_1$ and $R_3 - 3R_1$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + y + z \\ 0 - 3y + 0 \\ 0 - 2y - 2z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -2 \end{bmatrix}$$

By equality of matrices,
x + y + z = 2 ……(1)
-3y = 6 ……(2)
– 2y – 2z = -2 ……..(3)

From (2), y = -2
Substituting y = -2 in (3), we get,
-2(-2) – 2z = -2
∴ -2z = -6 ∴ z = 3
Substituting y = -2, z = 3 in (1), we get,
x – 2 + 3 = 2 ∴ x = 1
Hence, the required numbers are 1, -2 and 3.

Question 5.
The total cost of 3 T.V. sets and 2 V.C.R.s is ₹ 35000. The shop-keeper wants profit of ₹ 1000 per television and ₹ 500 per V.C.R. He can sell 2 T. V. sets and 1 V.C.R. and get the total revenue as ₹ 21,500. Find the cost price and the selling price of a T.V. sets and a V.C.R.
Solution:
Let the cost of each T.V. set be ₹ x and each V.C.R. be ₹ y. Then the total cost of 3 T.V. sets and 2 V.C.R.'s is ₹ (3x + 2y) which is given to be ₹ 35,000.
∴ 3x + 2y = 35000
The shopkeeper wants profit of ₹ 1000 per T.V. set and of ₹ 500 per V.C.R.
∴ the selling price of each T.V. set is ₹ (x + 1000) and of each V.C.R. is ₹ (y + 500).
∴ selling price of 2 T.V. set and 1 V.C.R. is
₹ [2(x + 1000) + (y + 500)] which is given to be ₹ 21,500.
∴ 2(x + 1000) + (y + 500) = 21500
∴ 2x + 2000 + y + 500 = 21500
∴ 2x + y = 19000
Hence, the system of linear equations is
3x + 2y = 35000
2x + y = 19000
These equations can be written in the matrix form as :

$$\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35000 \\ 19000 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19000 \\ 35000 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19000 \\ -3000 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2x + y \\ -x + 0 \end{bmatrix} = \begin{bmatrix} 19000 \\ -3000 \end{bmatrix}$$

By equality of matrices,
2x + y = 19000 ……….(1)
-x = -3000 ……….(2)
From (2), x = 3000
Substituting x = 3000 in (1), we get,
2(3000) + y = 19000
∴ y = 13000
∴ the cost price of one T.V. set is ₹ 3000 and of one V.C.R. is ₹ 13000 and the selling price of one T.V. set is ₹ 4000 and of one V.C.R. is ₹ 13500.

Maharashtra State Board 12th Maths Solutions Chapter 2 Matrices Miscellaneous Exercise 2A

Question 1.
If A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ then reduce it to I_3 by using column transformations.
Solution:
|A| = $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{vmatrix}$
= 1(1 – 0) – 0 + 0 = 1 ≠ 0
∴ A is a non-singular matrix.
Hence, the required transformation is possible.
Now, A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$
By $C_1 - 2C_2$, we get, A ~ $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$
By $C_1 + 3C_3$ and $C_2 - 3C_3$, we get,
A ~ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = I_3.

Question 2.
If A = $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 3 & 1 & 1 \end{bmatrix}$, then reduce it to I_3 by using row transformations.
Solution:
|A| = $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 3 & 1 & 1 \end{vmatrix}$
= 2 (0 – 1) – 1(1 – 1) + 3 (1 – 0)
= -2 – 0 + 3 = 1 ≠ 0
∴ A is a non-singular matrix.
Hence, the required transformation is possible.
Now, A = $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 3 & 1 & 1 \end{bmatrix}$
By $R_1 - R_2$, we get,

$$A \sim \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

By $R_2 - R_1$ and $R_3 - R_1$, we get,

$$A \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

By $(-1)R_2$ and $(-1)R_3$, we get,

$$A \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$, we get,

$$A \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - R_3$ and By $R_2 - R_3$, we get
A ~ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = I_3.

Question 3.
Check whether the following matrices are invertible or not:
(i) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Solution:
Let A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Then, |A| = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ = 1 – 0 = 1 ≠ 0.
∴ A is a non-singular matrix.
Hence, A^{-1} exists.

(ii) [1111]
Solution:
Let A = [1111]
Then, |A| = ∣∣∣1111∣∣∣ = 1 – 1 = 0.
∴ A is a singular matrix.
Hence, A^{-1} does not exist.

(iii) [1323]
Solution:
Let A = [1323]
Then, |A| = ∣∣∣1323∣∣∣ = 3 – 6 = -3 ≠ 0.
∴ A is a non-singular matrix.
Hence, A^{-1} exist.

(iv) [210315]
Solution:
Let A = [210315]
Then, |A| = ∣∣∣210315∣∣∣ = 30 – 30 = 0.
∴ A is a singular matrix.
Hence, A^{-1} does not exist.

(v) [cosθ−sinθsinθcosθ]
Solution:
Let A = [cosθ−sinθsinθcosθ]
Then, |A| = ∣∣∣secθtanθtanθsecθ∣∣∣
= $\sec^2\theta - \tan^2\theta = 1 \neq 0$.
∴ A is a non-singular matrix.
Hence, A^{-1} exist.

(vii) ⎡⎣⎢⎢311414305⎤⎦⎥⎥
Solution:
let A = ⎡⎣⎢⎢311414305⎤⎦⎥⎥
Then, |A| = ∣∣∣∣311414305∣∣∣∣
= 3(5 – 0) – 4(5 – 0) + 3(4 – 1)
= 15 – 20 + 9 = 4 ≠ 0
∴ A is a non-singular matrix.
Hence, A^{-1} exist.

(viii) ⎡⎣⎢⎢1212−12333⎤⎦⎥⎥
Solution:
Let A = ⎡⎣⎢⎢1212−12333⎤⎦⎥⎥
Then, |A| = ∣∣∣∣1212−12333∣∣∣∣
= 1 (-3 -6) – 2 (6 – 3) + 3 (4 + 1)
= -9 – 6 + 15 = 0
∴ A is a singular matrix.
Hence, A^{-1} does not exist.

(ix) ⎡⎣⎢⎢134246358⎤⎦⎥⎥
Solution:
Let A = ⎡⎣⎢⎢134246358⎤⎦⎥⎥
Then, |A| = ∣∣∣∣134246358∣∣∣∣
= 1(32 – 30) – 2(24 – 20) + 3(18 – 16)
= 2 – 8 + 6 = 0
∴ A is a singular matrix.
Hence, A^{-1} does not exist.

Question 4.

Find AB, if A = [112-23-3] and B = ⌈ ⌊ | 111-12-2⌉ ⌋ | Examine whether AB has inverse or not.

Solution:

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 1 & 2 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+2(1)+3(1) & 1(-1)+2(2)+3(-2) \\ 1(1)+(-2)(1)+(-3)(1) & 1(-1)+(-2)(2)+(-3)(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+3 & -1+4-6 \\ 1-2-3 & -1-4+6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -3 \\ -4 & 1 \end{bmatrix}$$

$$\therefore\ |AB| = \begin{vmatrix} 6 & -3 \\ -4 & 1 \end{vmatrix} = 6 - 12 = -6 \neq 0$$

∴ A is a non-singular matrix.
Hence, $(AB)^{-1}$ exist.

Question 5.

If A = ⌈ ⌊ | x000y000z⌉ ⌋ | is a nonsingular matrix then find A^{-1} by elementary row transformations.

Hence, find the inverse of ⌈ ⌊ | 20001000-1⌉ ⌋ |

Solution:
Since A is a non-singular matrix, then find A^{-1} by using elementary row transformations.
We write $AA^{-1} = I$

$$\therefore \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $\left(\frac{1}{x}\right)R_1$, $\left(\frac{1}{y}\right)R_2$ and $\left(\frac{1}{z}\right)R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix}$$

$$\therefore\ A^{-1} = \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix}.$$

Comparing ⌈ ⌊ | 20001000-1⌉ ⌋ | with ⌈ ⌊ | x000y000z⌉ ⌋ | ,
we get, x = 2, y = 1, z = -1
∴ 1x = 12, 1y = 11 = 1, 1z = 1-1 = -1
⌈ ⌊ | 20001000-1⌉ ⌋ | is (⌊ | 120001000-1) ⌋ | .

Question 6.

if A = [1324] and X is a 2 × 2 matrix such that AX = I , then find X.

Solution:
We will reduce the matrix A to the identity matrix by using row transformations. During this pro¬cess, I will be converted to the matrix X.

We have AX = I.

$$\therefore \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 3R_1$, we get,

$$\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} X = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

By $\left(-\frac{1}{2}\right)R_2$, we get,

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

Question 7.
Find the inverse of each of the following matrices (if they exist).
(i) $[12-13]$
Solution:
Let A = $[12-13]$
$\therefore$ |A| = $\left|\begin{array}{cc} 1 & 2 \\ -1 & 3 \end{array}\right|$ = 3 + 2 = 5 ≠ 0
$\therefore$ A^{-1} exists.
Consider AA^{-1} = I

$$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

By $\left(\frac{1}{5}\right)R_2$, we get,

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

By $R_1 + R_2$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$$

(ii) $[211-1]$
Solution:
Let A = $[211-1]$
$\therefore$ |A| = $\left|\begin{array}{cc} 2 & 1 \\ 1 & -1 \end{array}\right|$ = -2 – 1 = -3 ≠ 0
$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

By $\left(\frac{1}{3}\right)R_2$, we get,

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ 1/3 & -2/3 \end{bmatrix}$$

By $R_1 + R_2$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1/3 & 1/3 \\ 1/3 & -2/3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}.$$

(iii) [1237]

Solution:

Let A = [1237]

∴ |A| = | | | 1237 | | | = 7 – 6 = 1 ≠ 0

∴ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By R_2 - $2R_1$, we get,

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

By R_1 - $3R_2$ we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

(iv) [25-37]

Solution:

Let A = [25-37]

∴ |A| = | | | 25-37 | | | = 14 + 15 = 29 ≠ 0

∴ A^{-1} exists.

Consider $AA^{-1} = I$

$\therefore \begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By $3R_1$, we get,

$\begin{bmatrix} 6 & -9 \\ 5 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

By $R_1 - R_2$, we get,

$\begin{bmatrix} 1 & -16 \\ 5 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}$

By $R_2 - 5R_1$, we get,

$\begin{bmatrix} 1 & -16 \\ 0 & 87 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 \\ -15 & 6 \end{bmatrix}$

By $\left(\frac{1}{87}\right)R_2$, we get,

$\begin{bmatrix} 1 & -16 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 \\ -5/29 & 2/29 \end{bmatrix}$

By $R_1 + 16R_2$, we get,

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 7/29 & 3/29 \\ -5/29 & 2/29 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{29}\begin{bmatrix} 7 & 3 \\ -5 & 2 \end{bmatrix}$.

(v) $[2714]$

Solution:

Let A = $[2714]$

∴ |A| = $\left|\begin{matrix}2714\end{matrix}\right|$ = 8 – 7 = 1 ≠ 0

∴ A^{-1} exists.

Consider $AA^{-1} = I$

$\therefore \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By $R_1 \to R_1 - \frac{1}{7}R_2$ we get,

$\begin{bmatrix} 1 & \frac{3}{7} \\ 7 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & \frac{-1}{7} \\ 0 & 1 \end{bmatrix}$

By $R_2 \to R_2 - 7R_1$ we get,

$\begin{bmatrix} 1 & \frac{3}{7} \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & \frac{-1}{7} \\ -7 & 2 \end{bmatrix}$

By $R_1 \to R_1 - \frac{3}{7}R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

(vi) $[32-10-7]$

Solution:

Let A = $[32-10-7]$

∴ |A| = $\left|\begin{matrix}32-10-7\end{matrix}\right|$ = -21 + 20 = -1 ≠ 0

∴ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 3 & -10 \\ 2 & -7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix} 1 & -3 \\ 2 & -7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 1 & -3 \\ 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

By $(-1)R_2$, we get,

$$\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$$

By $R_1 + 3R_2$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 7 & -10 \\ 2 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -10 \\ 2 & -3 \end{bmatrix}$$

(vii) $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{vmatrix}$

= 2(4 + 6) +3(4 – 9) + 3(-4 – 6)

= 20 – 15 – 30 = -25 ≠ 0

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_3$, we get,

$$\begin{bmatrix} 3 & -2 & 2 \\ 2 & 2 & 3 \\ 2 & -3 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix} 1 & -4 & -1 \\ 2 & 2 & 3 \\ 2 & -3 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 2R_1$, we get,

$$\begin{bmatrix} 1 & -4 & -1 \\ 0 & 10 & 5 \\ 0 & 5 & 5 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 3 & -2 \\ 1 & 2 & -2 \end{bmatrix}$$

By $\left(\frac{1}{10}\right)R_2$, we get,

$$\begin{bmatrix} 1 & -4 & -1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 5 & 5 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & \frac{3}{10} & -\frac{1}{5} \\ 1 & 2 & -2 \end{bmatrix}$$

By $R_1 + 4R_2$ and $R_3 - 5R_2$, we get,

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & \frac{5}{2} \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & \frac{1}{5} & \frac{1}{5} \\ 0 & \frac{3}{10} & -\frac{1}{5} \\ 1 & \frac{1}{2} & -1 \end{bmatrix}$$

By $\left(\frac{2}{5}\right) R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & \frac{1}{5} & \frac{1}{5} \\ 0 & \frac{3}{10} & -\frac{1}{5} \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

By $R_1 - R_3$ and $R_2 - \frac{1}{2}R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{5}\begin{bmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix}$$

(viii) ⎡ ⎣ ⎢ 1−32305−2−50⎤ ⎦ ⎥
Solution:
Let A = ⎡ ⎣ ⎢ 1−32305−2−50⎤ ⎦ ⎥
∴ |A| = ∣ ∣ ∣ ∣ 1−32305−2−50 ∣ ∣ ∣ ∣
= 1(0 + 25) + 3(0 + 10) + 2(-15 – 0)
= 25 + 30 -30
= 25 ≠ 0
∴ A^{-1} exists.
Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 \to R_2 + 3R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 2 & 5 & 0 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $R \to \frac{1}{9}R_2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -\frac{11}{9} \\ 0 & -1 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -\frac{11}{9} \\ 0 & 0 & \frac{25}{9} \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{5}{3} & \frac{1}{9} & 1 \end{bmatrix}$$

By $R_1 \to R_1 + 3R_2$

$$\begin{bmatrix} 1 & 0 & \frac{5}{3} \\ 0 & 1 & -\frac{11}{9} \\ 0 & 0 & \frac{25}{9} \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{5}{3} & \frac{1}{9} & 1 \end{bmatrix}$$

By $R_3 \to \frac{9}{25}R_3$

$$\begin{bmatrix} 1 & 0 & \frac{5}{3} \\ 0 & 1 & -\frac{11}{9} \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

By $R_2 \to R_2 + \frac{11}{9}R_3$

$$\begin{bmatrix} 1 & 0 & \frac{5}{3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

By $R_1 \to R_1 - \frac{5}{3}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{5}{5} & -\frac{6}{15} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

$$A^{-1} = \frac{1}{25}\begin{bmatrix} 25 & -10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$$

(ix) ⎡ ⎣ ⎢ 250011-103⎤ ⎦ ⎥

Solution:

Let A =⎡ ⎣ ⎢ 250011-103⎤ ⎦ ⎥

∴ |A| = ∣ ∣ ∣ ∣ 250011-103 ∣ ∣ ∣ ∣

= 2(3 – 0) – 0 – 1(5 – 0)
= 6 – 0 – 5 = 1 ≠ 0
∴ A^{-1} exists.
Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 5 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 + 3R_3$, we get,

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 4 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$ and $R_3 - R_2$, we get,

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ -5 & 2 & -2 \end{bmatrix}$$

By $(-1)\,R_3$, we get,

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ 5 & -2 & 2 \end{bmatrix}$$

By $R_1 + 2R_3$ and $R_2 - 4R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

∴ A^{-1} = ⎡ ⎣ ⎢ 3−155−16−21−52⎤ ⎦ ⎥

(x) ⎡ ⎣ ⎢ 10−12−23−210⎤ ⎦ ⎥

Solution:

Let A = ⎡ ⎣ ⎢ 10−12−23−210⎤ ⎦ ⎥

∴ A^{-1} = ⎡ ⎣ ⎢ 10−12−23−210⎤ ⎦ ⎥

= 1 ∣ ∣ ∣ −2310 ∣ ∣ ∣ − 2 ∣ ∣ ∣ 0−111 ∣ ∣ ∣ − 2 ∣ ∣ ∣ 0−1−23 ∣ ∣ ∣

$|A| = 1(0 - 3) - 2(0 + 1) - 2(0 - 2)$

$= -3 - 2 + 4$

$= -1 \neq 0$

∴ A^{-1} exists.

We have

$AA^{-1} = I$

$$\begin{bmatrix}1 & 2 & -2\\0 & -2 & 1\\-1 & 3 & 0\end{bmatrix}A^{-1}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$R_3 \to R_3 + R_1$

$$\begin{bmatrix}1 & 2 & -2\\0 & -2 & 1\\0 & 5 & -2\end{bmatrix}A^{-1}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\1 & 0 & 1\end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix}1 & 2 & -2\\0 & -2 & 1\\0 & 1 & -0\end{bmatrix}A^{-1}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\1 & 2 & 1\end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix}1 & 2 & -2\\0 & 1 & 0\\0 & -2 & 1\end{bmatrix}A^{-1}=\begin{bmatrix}1 & 0 & 0\\1 & 2 & 1\\0 & 1 & 0\end{bmatrix}$$

$R_1 \to R_1 - 2R_2 \quad R3 \to R_3 + 2R_2$

$$\begin{bmatrix}1 & 0 & -2\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A^{-1}=\begin{bmatrix}-1 & -4 & -2\\1 & 2 & 1\\2 & 5 & 2\end{bmatrix}$$

$R_1 \to R_1 + 2R_3$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A^{-1}=\begin{bmatrix}3 & 6 & 2\\1 & 2 & 1\\2 & 5 & 2\end{bmatrix}$$

∴ A^{-1} = ⌈ ⌊ ∣ 312625212⌉ ⌋ ∣

Question 8.
Find the inverse of A = ⌈ ⌊ ∣ cosθsinθ0−sinθcosθ0001⌉ ⌋ ∣ by
(i) elementary row transformations
Solution:
|A| = ∣ ∣ ∣ ∣ ∣ cosθsinθ0−sinθcosθ0001 ∣ ∣ ∣ ∣
= cosθ (cosθ – 0) + sinθ (sinθ – 0) + 0
= $\cos^2\theta + \sin^2\theta = 1 \neq 0$
∴ A^{-1} exists.
(i) Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $\cos\theta \times R_1$, we get,

$$\begin{bmatrix} \cos^2\theta & -\sin\theta\cos\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \cos\theta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 + \sin\theta \times R_2$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - \sin\theta \times R_1$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta\cos\theta & \cos^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $\left(\frac{1}{\cos\theta}\right) \times R_2$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(ii) elementary column transformations
Solution:
Consider $A^{-1}A = I$

$$\therefore A^{-1} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $(\cos\theta) \times C_1$, we get,

$$A^{-1} \begin{bmatrix} \cos^2\theta & -\sin\theta & 0 \\ \sin\theta\cos\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $C_1 - \sin\theta \times C_2$, we get,

$$A^{-1} \begin{bmatrix} 1 & -\sin\theta & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & 0 \\ -\sin\theta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $C_2 + \sin\theta \times C_1$, we get,

$$A^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta\cos\theta & 0 \\ -\sin\theta & \cos^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $\left(\frac{1}{\cos\theta}\right) C_2$, we get,

$$A^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Question 9.

If A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ find AB and $(AB)^{-1}$. Verify that $(AB)^{-1} = B^{-1}A^{-1}$

Solution:

AB = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2+9 & 0+3 \\ 1+6 & 0+2 \end{bmatrix} = \begin{bmatrix} 11 & 3 \\ 7 & 2 \end{bmatrix}$$

$\therefore\ |AB| = \begin{vmatrix} 11 & 3 \\ 7 & 2 \end{vmatrix} = 22 - 21 = 1 \neq 0$

$\therefore\ (AB)^{-1}$ exists.

Now, $(AB)(AB)^{-1} = I$

$\therefore\ \begin{bmatrix} 11 & 3 \\ 7 & 2 \end{bmatrix} (AB)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By $2R_1$, we get, $\begin{bmatrix} 22 & 6 \\ 7 & 2 \end{bmatrix} (AB)^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

By $R_1 - 3R_2$, we get, $\begin{bmatrix} 1 & 0 \\ 7 & 2 \end{bmatrix} (AB)^{-1} = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$

By $R_2 - 7R_1$, we get, $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} (AB)^{-1} = \begin{bmatrix} 2 & -3 \\ -14 & 22 \end{bmatrix}$

By $\left(\frac{1}{2}\right)R_2$, we get, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (AB)^{-1} = \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$

$\therefore\ (AB)^{-1} = \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$ ... (1)

$|A| = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1 \neq 0$

$\therefore\ A^{-1}$ exists.

Consider, $AA^{-1} = I$

$\therefore\ \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By $R_1 \leftrightarrow R_2$, we get, $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

By $R_2 - 2R_1$, we get, $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$

By $(-1)R_2$, we get, $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

By $R_1 - 2R_2$, we get, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$\therefore\ A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$|B| = \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} = 1 - 0 = 1 \neq 0$

$\therefore\ B^{-1}$ exists.

Consider, $BB^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 3R_1$, we get, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} B^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

$$\therefore B^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

$$\therefore B^{-1} \cdot A^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & -3+0 \\ -6-1 & 9+2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix} \quad \dots (2)$$

From (1) and (2), $(AB)^{-1} = B^{-1} \cdot A^{-1}$.

Question 10.
If A = $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$, then show that $A^{-1} = \frac{1}{6}(A - 5I)$
Solution:
$|A| = \begin{vmatrix} 4 & 5 \\ 2 & 1 \end{vmatrix} = 4 - 10 = -6 \neq 0$
$\therefore A^{-1}$ exists.
Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $\left(\frac{1}{4}\right)R_1$, we get, $\begin{bmatrix} 1 & \frac{5}{4} \\ 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & 1 \end{bmatrix}$

By $R_2 - 2R_1$, we get, $\begin{bmatrix} 1 & \frac{5}{4} \\ 0 & -\frac{3}{2} \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix}$

By $\left(-\frac{2}{3}\right)R_2$, we get, $\begin{bmatrix} 1 & \frac{5}{4} \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$

By $R_1 - \frac{5}{4}R_2$, we get, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -\frac{1}{6} & \frac{5}{6} \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$

$$\therefore A^{-1} = \frac{1}{6}\begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix} \quad \dots (1)$$

$$\frac{1}{6}(A - 5I) = \frac{1}{6}\left\{\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

$$= \frac{1}{6}\left\{\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\right\}$$

$$= \frac{1}{6}\begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix} \quad \dots (2)$$

From (1) and (2), $A^{-1} = \frac{1}{6}(A - 5I)$.

Question 11.

Find matrix X such that AX = B, where A = [1−123] and B = [0214]

Solution:

AX = B

$$\therefore \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$$

By $R_2 + R_1$, we get, $\begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix}$

By $\left(\frac{1}{5}\right)R_2$, we get, $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ \frac{2}{5} & 1 \end{bmatrix}$

By $R_1 - 2R_2$, we get, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} -\frac{4}{5} & -1 \\ \frac{2}{5} & 1 \end{bmatrix}$

$$\therefore X = \begin{bmatrix} -\frac{4}{5} & -1 \\ \frac{2}{5} & 1 \end{bmatrix}$$

Question 12.

Find X, if AX = B where A = ⌈ ⌊ ∣ 1−11212324⌉ ⌋ ∣ and B = ⌈ ⌊ ∣ 123⌉ ⌋ ∣ .

Solution:

AX = B

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

By $R_2 + R_1$ and $R_3 - R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

By $\left(\frac{1}{3}\right)R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

By $R_1 + \frac{1}{3}R_3$ and $R_2 - \frac{5}{3}R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} -\frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -\frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix}$$

Question 13.
If A = [1112], B = [4311] and C = [243179] then find matrix X such that AXB = C.
Solution:
AXB = C
∴ (1112)(XB) =[243179]
First we perform the row transformations.

By $R_2 - R_1$, we get, $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}(XB) = \begin{bmatrix} 24 & 7 \\ 7 & 2 \end{bmatrix}$

By $R_1 - R_2$, we get, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}(XB) = \begin{bmatrix} 17 & 5 \\ 7 & 2 \end{bmatrix}$

$\therefore\ XB = \begin{bmatrix} 17 & 5 \\ 7 & 2 \end{bmatrix}$

$\therefore\ X\begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 17 & 5 \\ 7 & 2 \end{bmatrix}$

Now, we perform the column transformations.

By $C_1 \leftrightarrow C_3$, we get, $X\begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 17 \\ 2 & 7 \end{bmatrix}$

By $C_2 - 4C_1$, we get, $X\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix}$

By $(-1)C_2$, we get, $X\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$

By $C_1 - C_2$, we get, $X\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

$\therefore\ X = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$.

Question 14.
Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ by adjoint method.
Solution:
Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{vmatrix}$
= 1(7 – 20) – 2(7 – 10) + 3(4 – 2)
= -13 + 6 + 6 = -1 ≠ 0
$\therefore A^{-1}$ exists.
First we have to find the cofactor matrix
$= [A_{ij}]_{3\times3}$ where $A_{ij} = (-1)^{i+j}M_{ij}$

Now, $A_{11} = (-1)^{1+1}M_{11} = \begin{vmatrix} 1 & 5 \\ 4 & 7 \end{vmatrix}$ = 7 – 20 = -13

$A_{12} = (-1)^{1+2}M_{12} = \begin{vmatrix} 1 & 5 \\ 2 & 7 \end{vmatrix}$ = -(7 – 10) = 3

$$A_{13} = (-1)^{1+3}M_{13} = \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} = 4 - 2 = 2$$

$$A_{21} = (-1)^{2+1}M_{21} = -\begin{vmatrix} 2 & 3 \\ 4 & 7 \end{vmatrix} = -(14 - 12) = -2$$

$$A_{22} = (-1)^{2+2}M_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} = 7 - 6 = 1$$

$$A_{23} = (-1)^{2+3}M_{23} = -\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = -(4 - 4) = 0$$

$$A_{31} = (-1)^{3+1}M_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = 10 - 3 = 7$$

$$A_{32} = (-1)^{3+2}M_{32} = -\begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix} = -(5 - 3) = -2$$

$$A_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1 - 2 = -1$$

$\therefore$ the co-factor matrix =

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -13 & 3 & 2 \\ -2 & 1 & 0 \\ 7 & -2 & -1 \end{bmatrix}$$

$$\therefore \text{ adj } A = \begin{bmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$= \frac{1}{-1}\begin{bmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

Question 15.

Find the inverse of ⎡ ⎣ ⎢ 101022131⎤ ⎦ ⎥ by adjoint method.

Solution:

where A = ⎡ ⎣ ⎢ 101022131⎤ ⎦ ⎥

|A| = 1(2 – 6) – 0(0 – 3) + 1(0 – 2)

|A| = -4 – 2

|A| = -6 ≠ 0

∴ A^{-1} exists.

First we have to find the cofactor matrix

= $[A_{ij}]_{3\times3}$, where $A_{ij} = (-1)^{i+j}M_{ij}$

Now $A_{11} = (-1)^{1+1}M_{11} = \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = 2 - 6 = -4$

$A_{12} = (-1)^{1+2}M_{12} = -\begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} = 0 - 3 = -3$

$A_{13} = (-1)^{1+3}M_{13} = \begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} = 0 - 2 = -2$

$A_{21} = (-1)^{2+1}M_{21} = -\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = -0 - 2 = -2$

$A_{22} = (-1)^{2+2}M_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 - 1 = 0$

$A_{23} = (-1)^{2+3}M_{23} = -\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} = -2 - 0 = -2$

$A_{31} = (-1)^{3+1}M_{31} = \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = 0 - 2 = -2$

$A_{32} = (-1)^{3+2}M_{32} = -\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} = -3 - 0 = -3$

$A_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$

adj (A) = $\begin{bmatrix} -4 & -3 & -2 \\ -2 & 0 & -2 \\ -2 & -3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} . \text{adj(A)}$

$A^{-0} = \frac{1}{6}\begin{bmatrix} -4 & -3 & -2 \\ -2 & 0 & -2 \\ -2 & -3 & 2 \end{bmatrix}$

Question 16.

Find A-1 by adjoint method and by elementary transformations if A = ⎡ ⎣ ∣ 1−11212324⎤ ⎦ ∣

Solution:

|A| = ∣ ∣ ∣ ∣ 1−11212324 ∣ ∣ ∣ ∣

= 1(4 – 4) – 2(-4 – 2) + 3(-2 – 1)

= 0 + 12 – 9 = 3 ≠ 0

∴ A-1 exists.

A-1by adjoint method :

We have to find the cofactor matrix

= $[A_{ij}]_{3\times3}$, where $A_{ij} = (-1)^{i+j} M_{ij}$

$$\text{Now, } A_{11} = (-1)^{1+1}M_{11} = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 4 - 4 = 0$$

$$A_{12} = (-1)^{1+2}M_{12} = -\begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} = -(-4-2) = 6$$

$$A_{13} = (-1)^{1+3}M_{13} = \begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} = -2 - 1 = -3$$

$$A_{21} = (-1)^{2+1}M_{21} = -\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = -(8-6) = -2$$

$$A_{22} = (-1)^{2+2}M_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 4 - 3 = 1$$

$$A_{23} = (-1)^{2+3}M_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = -(2-2) = 0$$

$$A_{31} = (-1)^{3+1}M_{31} = \begin{vmatrix} \text{[illegible]} & \text{[illegible]} \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1$$

$$A_{32} = (-1)^{3+2}M_{32} = -\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} = -(2+3) = -5$$

$$A_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = 1 + 2 = 3$$

$\therefore$ the cofactor matrix =

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 0 & 6 & -3 \\ -2 & 1 & 0 \\ 1 & -5 & 3 \end{bmatrix}$$

$$\therefore \text{ adj } A = \begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$= \frac{1}{3}\begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}$$

A^{-1} by elementary transformations :

Consider $AA^{-1} = I$

By $R_2 + R_1$ and $R_3 - R_1$, we get,

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

By $\left(\frac{1}{3}\right) R_2$, we get,

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1/3 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\therefore \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1/3 & -2/3 & 0 \\ 1/3 & 1/3 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

By $R_1 + \frac{1}{3} R_3$ and $R_2 - \frac{5}{3} R_3$, we get,

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -2/3 & 1/3 \\ 2 & 1/3 & -5/3 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{3} \begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}.$$

Question 17.

Find the inverse of A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 3 & 1 \end{bmatrix}$ by elementary column transformations.

Solution:

|A| = $\begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 3 & 1 \end{vmatrix}$

= 1 (2 – 6) – 0 + 1 (0 – 2)

= -4 – 2= -6 ≠ 0

∴ A^{-1} exists.

Consider $A^{-1}A = I$

∴ $A^{-1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

By $C_3 - C_1$, we get,

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 3\\ 1 & 2 & 0\end{bmatrix}=\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

By $C_2 \leftrightarrow C_3$, we get,

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 3 & 2\\ 1 & 0 & 2\end{bmatrix}=\begin{bmatrix}1 & -1 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$$

By $C_2 - C_3$, we get,

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 2\\ 1 & -2 & 2\end{bmatrix}=\begin{bmatrix}1 & -1 & 0\\ 0 & -1 & 1\\ 0 & 1 & 0\end{bmatrix}$$

By $C_3 - 2C_2$, we get,

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & -2 & 6\end{bmatrix}=\begin{bmatrix}1 & -1 & 2\\ 0 & -1 & 3\\ 0 & 1 & -2\end{bmatrix}$$

By $\left(\frac{1}{6}\right)C_3$, we get,

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & -2 & 1\end{bmatrix}=\begin{bmatrix}1 & -1 & \frac{1}{3}\\ 0 & -1 & \frac{1}{2}\\ 0 & 1 & -\frac{1}{3}\end{bmatrix}$$

By $C_1 - C_3$ and $C_2 + 2C_3$, we get,

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{2}{3} & -\frac{1}{3} & \frac{1}{3}\\ -\frac{1}{2} & 0 & \frac{1}{2}\\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3}\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{6}\begin{bmatrix}4 & -2 & 2\\ -3 & 0 & 3\\ 2 & 2 & -2\end{bmatrix}.$$

Question 18.

Find the inverse of ⎡ ⎣ ⎢ 112214357⎤ ⎦ ⎥ by elementary row transformations.

Solution:

Let A = ⎡ ⎣ ⎢ 112214357⎤ ⎦ ⎥

∴ |A| = ∣ ∣ ∣ ∣ 112214357 ∣ ∣ ∣ ∣

= 1(7 – 20) – 2(7 – 10) + 3(4 – 2)

= -13 + 6 + 6 = -1 ≠ 0

∴ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - R_1$ and $R_3 - 2R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $(-1)R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $R_1 - 7R_3$ and $R_2 + 2R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

Question 19.
Show with usual notations that for any matrix $A = [a_{ij}]_{3\times3}$
(i) $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$
Solution:
$A = [a_{ij}]_{3\times3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$
(i) $A_{21} = (-1)^{2+1}M_{21} = -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$
$= -(a_{12}a_{33} - a_{13}a_{32})$
$= -a_{12}a_{33} + a_{13}a_{32}$
$A_{22} = (-1)^{2+2}M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$
$= a_{11}a_{33} - a_{13}a_{31}$
$A_{23} = (-1)^{2+3}M_{23} = -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$
$= -(a_{11}a_{32} - a_{12}a_{31})$
$= -a_{11}a_{32} + a_{12}a_{31}$
$\therefore a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$
$= a_{11}(-a_{1233} + a_{13}a_{32}) + a_{12}(a_{11}a_{33} - a_{13}a_{31}) + a_{13}(-a_{11}a_{32} + a_{12}a_{31})$
$= -a_{11}a_{12}a_{33} + a_{11}a_{13}a_{32} + a_{11}a_{12}a_{33} - a_{12}a_{13}a_{31} - a_{11}a_{13}a_{32} + a_{12}a_{13}a_{31}$
$= 0$

(ii) $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = |A|$

Solution:

$$A_{11} = (-1)^{1+1}M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$A_{12} = (-1)^{1+2}M_{12} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$A_{13} = (-1)^{1+3}M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\therefore\ a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = |A|.$$

Question 20.

If A = ⎡ ⎣ ⎢ 101022131⎤ ⎦ ⎥ and B = ⎡ ⎣ ⎢ 112214357⎤ ⎦ ⎥ , then find a matrix X such that XA= B.

Solution:

Consider XA = B

$$\therefore\ X\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$$

By $C_3 - C_1$, we get,

$$X\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 1 & 4 \\ 2 & 4 & 5 \end{bmatrix}$$

By $\left(\frac{1}{2}\right)C_2$, we get,

$$X\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1/2 & 4 \\ 2 & 2 & 5 \end{bmatrix}$$

By $C_3 - 3C_2$, we get,

$$X\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1/2 & 5/2 \\ 2 & 2 & -1 \end{bmatrix}$$

By $\left(-\frac{1}{3}\right)C_3$, we get,

$$X\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1/3 \\ 1 & 1/2 & -5/6 \\ 2 & 2 & 1/3 \end{bmatrix}$$

By $C_1 - C_3$ and $C_2 - C_3$, we get,

$$X\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2/3 & 2/3 & 1/3 \\ 11/6 & 4/3 & -5/6 \\ 5/3 & 5/3 & 1/3 \end{bmatrix}$$

$$\therefore\ X = \frac{1}{6}\begin{bmatrix} 4 & 4 & 2 \\ 11 & 8 & -5 \\ 10 & 10 & 2 \end{bmatrix}.$$

Maharashtra State Board 12th Maths Solutions Chapter 2 Matrices Miscellaneous Exercise 2B

I. Choose the correct answer from the given alternatives in each of the following questions:

Question 1.
If A = (1324), adj = (4−3ab) then the values of a and b are,
(a) a = – 2, b = 1
(b) a = 2, b = 4
(c) a = 2, b = –1
(d) a = 1, b = –2
Solution:
(a) a = – 2, b = 1

Question 2.
The inverse of (0110) is

A) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ D) None of these

Solution:
(0110)

Question 3.
If A = (1221) and A(adj A) = k 1, then the value of k is
(a) 1
(b) -1
(c) 0
(d) -3
Solution:
(d) -3 [Hint : A(adj A) = |A| · I]

Question 4.
If A = (23−41), then the adjoint of matrix A is

(a) $\begin{bmatrix} -1 & 3 \\ -4 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$

Solution:(1−342)

Question 5.
If A = (1324) and A(adj A) = kI, then the value of k is
(a) 2
(b) -2
(c) 10
(d) -10
Solution:
(b) -2

Question 6.
If A = (λ−11−λ), then A^{-1} does not exist if λ = ……….
(a) 0
(b) ± 1
(c) 2
(d) 3
Solution:
(b) ± 1

Question 7.
If A = [cosαsinα−sinαcosα] then A^{-1} = ….

(a) $\begin{bmatrix} 1/\cos\alpha & -1/\sin\alpha \\ 1/\sin\alpha & 1/\cos\alpha \end{bmatrix}$

(b) $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

(c) $\begin{bmatrix} -\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

(d) $\begin{bmatrix} -\cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{bmatrix}$

Solution:
[cosα−sinα−sinαcosα]

Question 8.
If F (∝) = ⎡ ⎣ ⎢ cosαsinα0−sinαcosα0001⎤ ⎦ ⎥ where ∝ ∈ R then $[F(\propto)]^{-1}$ is =
(a) F(-∝)
(b) $F(\propto^{-1})$
(c) F(2∝)
(d) None of these
Solution:
(a) F(-∝)

Question 9.
The inverse of A = ⎡ ⎣ ⎢ 010100001⎤ ⎦ ⎥
(a) I
(b) A
(c) A′
(d) -I
Solution:
(b) A

Question 10.
The inverse of a symmetric matrix is
(a) Symmetric
(b) Non-symmetric
(c) Null matrix
(d) Diagonal matrix
Solution:
(a) Symmetric

Question 11.
For a 2 × 2 matrix A, if A(adjA) = (100010) then determinant A equals
(a) 20
(b) 10
(c) 30
(d) 40
Solution:
(b) 10

Question 12.

If $A^2 = -\frac{1}{2}\begin{bmatrix}1 & -1 \\ -4 & 2\end{bmatrix}$ then A =

(a) $\begin{bmatrix}2 & 4 \\ -1 & 1\end{bmatrix}$ (b) $\begin{bmatrix}2 & 4 \\ 1 & -1\end{bmatrix}$

(c) $\begin{bmatrix}2 & -4 \\ 1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}2 & 4 \\ 1 & 1\end{bmatrix}$

Solution:

$-\frac{1}{2}\begin{bmatrix}2 & 1 \\ 4 & 1\end{bmatrix}$

II. Solve the following equations by the methods of inversion.

(i) $2x - y = -2$, $3x + 4y = 5$

Solution:

The given equations can be written in the matrix form as :

$$\begin{bmatrix}2 & -1 \\ 3 & 4\end{bmatrix}\begin{bmatrix}x \\ y\end{bmatrix} = \begin{bmatrix}-2 \\ 3\end{bmatrix}$$

This is of the form AX = B, where

$$A = \begin{bmatrix}2 & -1 \\ 3 & 4\end{bmatrix}, X = \begin{bmatrix}x \\ y\end{bmatrix} \text{ and } B = \begin{bmatrix}-2 \\ 3\end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix}2 & -1 \\ 3 & 4\end{vmatrix} = 8 + 3 = 11 \neq 0$$

$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix}2 & -1 \\ 3 & 4\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix}3 & 4 \\ 2 & -1\end{bmatrix}A^{-1} = \begin{bmatrix}0 & 1 \\ 1 & 0\end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix}1 & 5 \\ 2 & -1\end{bmatrix}A^{-1} = \begin{bmatrix}-1 & 1 \\ 1 & 0\end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix}1 & 5 \\ 0 & -11\end{bmatrix}A^{-1} = \begin{bmatrix}-1 & 1 \\ 3 & -2\end{bmatrix}$$

By $\left(-\frac{1}{11}\right)R_2$, we get,

$$\begin{bmatrix}1 & 5 \\ 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}-1 & 1 \\ -\frac{3}{11} & \frac{2}{11}\end{bmatrix}$$

By $R_1 - 5R_2$, we get,

$$\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}\frac{4}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{2}{11}\end{bmatrix}$$

$$\therefore\ A^{-1} = \begin{bmatrix} \frac{4}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix}$$

Now, premultiply $AX = B$ by A^{-1}, we get,

$$A^{-1}(AX) = A^{-1}B$$

$$\therefore\ (A^{-1}A)X = A^{-1}B \quad \therefore\ IX = A^{-1}B$$

$$\therefore\ X = \begin{bmatrix} 4/11 & 1/11 \\ -3/11 & 2/11 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\therefore\ \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{8}{11}+\frac{3}{11} \\ \frac{6}{11}+\frac{6}{11} \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix}$$

By equality of matrices,

x = $-\frac{5}{11}$, y = $\frac{12}{11}$ is the required solution.

(ii) x + y + z = 1, 2x + 3y + 2z = 2 and ax + ay + 2az = 4, a ≠ 0.
Solution:
The given equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

This is of the form $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}$$

= 1(6a – 2a) – 1(4a – 2a) + 1(2a – 3a)
= 4a – 2a – a = a ≠ 0 ∴ A^{-1} exists.
Consider $AA^{-1} = I$

$$\therefore\ \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - aR_1$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & a \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & a \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$$

By $\left(\frac{1}{a}\right)R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & \frac{1}{a} \end{bmatrix}$$

By $R_1 - R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 4 & -1 & -\frac{1}{a} \\ -2 & 1 & 0 \\ -1 & 0 & \frac{1}{a} \end{bmatrix}$$

$$\therefore\ A^{-1} = \begin{bmatrix} 4 & -1 & -\frac{1}{a} \\ -2 & 1 & 0 \\ -1 & 0 & \frac{1}{a} \end{bmatrix}$$

Now, premultiply $AX = B$ by A^{-1}, we get,

$A^{-1}(AX) = A^{-1}B$

$\therefore\ (A^{-1}A)X = A^{-1}B \qquad \therefore\ IX = A^{-1}B$

$$\therefore\ X = \begin{bmatrix} 4 & -1 & -\frac{1}{a} \\ -2 & 1 & 0 \\ -1 & 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$\therefore\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4-2-\frac{4}{a} \\ -2+2+0 \\ -1+0+\frac{4}{a} \end{bmatrix} = \begin{bmatrix} 2-\frac{4}{a} \\ 0 \\ \frac{4}{a}-1 \end{bmatrix}$$

By equality of matrices,

$x = 2 - \frac{4}{a}$, $y = 0$, $z = \frac{4}{a} - 1$ is the required solution.

(iii) $5x – y + 4z = 5$, $2x + 3y + 5z = 2$ and $5x – 2y + 6z = -1$

Solution:

The given equations can be written in the matrix form as :

$$\begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

This is of the form $AX = B$, where

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix}$$

$= 5(18 + 10) + 1 (12 – 25) + 4(-4 – 15)$

$= 140 – 13 – 76 = 51$ #0

$\therefore$ A^{-1} exists.

Now, we have to find the cofactor matrix

$= [A_{ij}]_{3 \times 3}$, where $A_{ij} = (-1)^{i+j} M_{ij}$

$$A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 3 & 5 \\ -2 & 6 \end{vmatrix} = 18 + 10 = 28$$

$$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 2 & 5 \\ 5 & 6 \end{vmatrix} = -(12 - 25) = 13$$

$$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4 - 15 = -19$$

$$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} -1 & 4 \\ -2 & 6 \end{vmatrix} = -(-6 + 8) = -2$$

$$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 5 & 4 \\ 5 & 6 \end{vmatrix} = (30 - 20) = 10$$

$$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} 5 & -1 \\ 5 & -2 \end{vmatrix} = -(-10 + 5) = 5$$

$$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} -1 & 4 \\ 3 & 5 \end{vmatrix} = -5 - 12 = -17$$

$$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} 5 & 4 \\ 2 & 5 \end{vmatrix} = -(25 - 8) = -17$$

$$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 5 & -1 \\ 2 & 3 \end{vmatrix} = 15 + 2 = 17$$

$\therefore$ the cofactor matrix =

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 28 & 13 & -19 \\ -2 & 10 & 5 \\ -17 & -17 & 17 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$= \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$

Now, premultiply AX = B by A^{-1}, we get,

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)X = A^{-1}B$

$\therefore IX = A^{-1}B$

$$\therefore X = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$= \frac{1}{51}\begin{bmatrix} 140 - 4 + 17 \\ 65 + 20 + 17 \\ -95 + 10 - 17 \end{bmatrix} = \frac{1}{51}\begin{bmatrix} 153 \\ 102 \\ -102 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$$

By equality of matrices,

x = 3, y = 2, z = -2 is the required solution.

(iv) 2x + 3y = -5, 3x + y = 3

Solution:

(v) x + y + z = -1, y + z = 2 and x + y – z = 3
Solution:
The given equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

This is of the form AX = B, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

= 1(-1 – 1) – 1 (0 – 1) + 1(0 – 1)
= -2 + 1 – 1 = -2 ≠ 0 ∴ A^{-1} exists.
Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_3 - R_1$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

By $\left(-\frac{1}{2}\right)R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 1/2 & 0 & -1/2 \end{bmatrix}$$

By $R_2 - R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -1/2 & 1 & 1/2 \\ 1/2 & 0 & -1/2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -1/2 & 0 & 1/2 \\ 1/2 & 0 & -1/2 \end{bmatrix}$$

Now, premultiply AX = B by A^{-1}, we get,
$A^{-1}(AX) = A^{-1}B$
∴ $(A^{-1}A)X = A^{-1}B$
∴ $IX = A^{-1}B$

$$\therefore X = \begin{bmatrix} 1 & -1 & 0 \\ -1/2 & 1 & 1/2 \\ 1/2 & 0 & -1/2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1-2+0 \\ 1/2+2+3/2 \\ -1/2+0-3/2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ -2 \end{bmatrix}$$

∴ by equality of the matrices, x= -3, y = 4, z = -2 is the required solution.

Question 2.
Express the following equation in matrix from and solve them by the method of reduction.
(i) x – y + z = 1, 2x – y = 1, 3x + 3y – 4z = 2
Solution:
The given equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 3 & 3 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 3R_1$, we get,

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 6 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

By $R_3 - 6R_2$, we get,

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x-y+z \\ 0+y-2z \\ 0+0+5z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$$

By equality of matrices,
x – y + z = 1 ……(1)
y – 2z = -1 …..(2)
5z = 5 ….(3)
From (3), z = 1
Substituting z = 1 in (2), we get,
y – 2 = -1 ∴ y = 1
Substituting y = 1, z = 1 in (1), we get,
x – 1 + 1 = 1
∴ x = 1
Hence, x = 1, y = 1, z = 1 is the required solution.

(ii) x + y = 1, y + z = 53, z + x = 43.
Solution:
The given equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5/3 \\ 4/3 \end{bmatrix}$$

By $R_3 - R_1$, we get,

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5/3 \\ 1/3 \end{bmatrix}$$

By $R_3 + R_2$, we get,

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5/3 \\ 2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + y + 0 \\ 0 + y + z \\ 0 + 0 + 2z \end{bmatrix} = \begin{bmatrix} 1 \\ 5/3 \\ 2 \end{bmatrix}$$

By equality of matrices,
x + y = 1 ……(1)
y + z = 53 …(2)
2z = 2 ……..(3)
From (3), z = 1
Substituting z = 1 in (2), we get,
y + 1 = 53 ∴ y = 23
Substituting y = 23 in (1), we get,
x + 23 = 1 ∴ x = 13
Hence, x = 13, y = 23, z = 1 is the required solution.

(iii) 2x – y + z = 1, x + 2y + 3z = 8 and 3x + y – 4z = 1
Solution:
The given equations can be written in the matrix form as :

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 1 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 3R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 0 & -5 & -13 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -15 \\ -23 \end{bmatrix}$$

By $R_3 - R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 0 & 0 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -15 \\ -8 \end{bmatrix}$$

∴ ⎡ ⎣ ⎢ x+2y+3z0−5y−5z0+0−8z ⎤ ⎦ ⎥ = ⎡ ⎣ ⎢ 8−15−8 ⎤ ⎦ ⎥
By equality of matrices,
x + 2y + 3z = 8 …..(1)
-5y – 5z = -15 ….(2)
-8z = -8 …..(3)
From (3), z = 1
Substituting z = 1 in (2), we get,
-5y – 5 = -15
-5y = -10
∴ y = 2
Substituting y = 2, z = 1 in (1), we get,
x + 4 + 3 = 8 ∴ x = 1
Hence, x = 1, y = 2, z = 1 is the required solution.

(iv) x + y + z = 6, 3x – y + 3z =10 and 5x + 5y – 4z = 3.
Solution:

(v) x + 2y + z = 8, 2x + 3y – z =11 and 3x – y – 2z = 5
Solution:
The given equations can be written in the matrix form as :

$$\begin{bmatrix}1 & 2 & 1\\2 & 3 & -1\\3 & -1 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}8\\11\\5\end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 3R_1$, we get,

$$\begin{bmatrix}1 & 2 & 1\\0 & -1 & -3\\0 & -7 & -5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}8\\-5\\-19\end{bmatrix}$$

By $R_3 - 7R_2$, we get,

$$\begin{bmatrix}1 & 2 & 1\\0 & -1 & -3\\0 & 0 & 16\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}8\\-5\\16\end{bmatrix}$$

$$\therefore \begin{bmatrix}x+2y+z\\0-y-3z\\0+0+16z\end{bmatrix}=\begin{bmatrix}8\\-5\\16\end{bmatrix}$$

By equality of matrices,
x + 2y + z = 8 … (1)
-y – 3z = -5 … (2)
16z = 16 … (3)
From (3), z = 1
Substituting z = 1 in (2), we get,
-y – 3 = -5, ∴ y = 2
Substituting y = 2, z = 1 in (1), we get,
x + 4 + 1 = 8 ∴ x = 3
Hence, x = 3, y = 2, z = 1 is the required solution.

(vi) x + 3y + 2z = 6, 3x – 2y + 5z =5 and 2x – 3y + 6z = 7.
Solution:
The given equations can be written in the matrix form as :

$$\begin{bmatrix}1 & 3 & 2\\3 & -2 & 5\\2 & -3 & 6\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\5\\7\end{bmatrix}$$

By R_3 - $2R_1$, R_2 - $3R_1$ we get

$$\begin{bmatrix}1 & 3 & 2\\0 & 11 & 1\\0 & -9 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\13\\-5\end{bmatrix}$$

By $R_2 + R_3$, we get,

$$\begin{bmatrix}1 & 3 & 2\\0 & 2 & 3\\0 & -9 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\8\\-5\end{bmatrix}$$

By $\frac{R_2}{2}$ we get

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & \frac{3}{2} \\ 0 & -9 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ -5 \end{bmatrix}$$

By $R_3 + 9R_2$ we get,

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & \frac{31}{2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 31 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + 3y + 2z \\ 0 + y + \frac{3}{2}z \\ 0 + 0 + \frac{31}{2}z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 31 \end{bmatrix}$$

By equality of matrices,
x + 3y + 2z = 6 ...(1)
y + $\frac{3}{2}$ z = 4 ...(2)
$\frac{31}{2}$z = 31 .....(3)
From (3), z = 2
Substituting z = 2 in (2), we get,
y + $\frac{3}{2}$z = 4
y + $\frac{3}{2}$(2) = 4
y + 3 = 4
y = 1
Substituting y = 1, z = 2 in (2), we get,
x + 3y + 2z = 6
x + 3(1) + 2(2) = 6
x + 3 + 4 = 6
x = -1
Hence, x = -1, y = 1, z = 2 is the required solution.

Question 3.
The sum of three numbers is 6. If we multiply third number by 3 and add it to the second number we get 11. By adding first and the third numbers we get a number which is double the second number. Use this information and find a system of linear equations. Find the three numbers using matrices.
Solution:
Let the three numbers be x, y and z. According to the given conditions,
x + y + z = 6.
3z + y = 11, i.e., y + 3z = 11 and x + z = 2y,
i.e., x – 2y + z = 0
Hence, the system of the linear equations is
x + y + z = 6
y + 3z = 11
x – 2y + z = 0
These equations can be written in the matrix form as :

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \\ 0 \end{pmatrix}$$

By $R_3 - R_1$, we get,

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & -3 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \\ -6 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x + y + z \\ 0 + y + 3z \\ 0 - 3y + 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \\ -6 \end{pmatrix}$$

By equality of matrices,
x + y + z = 6 ...(1)
y + 3z = 11 ...(2)
-3y = -6 ...(3)
From (3), y = 2

Substituting y = 2 in (2), we get,
2 + 3z = 11
∴ 3z = 9 ∴ z = 3
Put y = 2, z — 3 in (1), we get,
x + 2 + 3 = 6 ∴ x = 1
∴ x = 1, y = 2, z = 3
Hence, the required numbers are 1, 2 and 3.

Question 4.
The cost of 4 pencils, 3 pens and 2 books is ₹ 150. The cost of 1 pencil, 2 pens and 3 books is ₹ 125. The cos of 6 pencils, 2 pens and 3 books is ₹ 175. Fild the cost of each item by using Matrices.
Solution:
Let the cost of 1 pencil, 1 pen and 1 book be ₹x, ₹ y, ₹ z respectively.
According to the given conditions,
4x + 3y + 2z = 150
x + 2y + 3z = 125
6x + 2y + 3z = 175
The equations can be written in matrix form as :

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 150 \\ 125 \\ 175 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 125 \\ 150 \\ 175 \end{bmatrix}$$

By $R_2 - 4R_1$ and $R_3 - 6R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & -10 & -15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 125 \\ -350 \\ -575 \end{bmatrix}$$

By $R_3 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 125 \\ -350 \\ 125 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + 2y + 3z \\ 0 - 5y - 10z \\ 0 + 0 + 5z \end{bmatrix} = \begin{bmatrix} 125 \\ -350 \\ 125 \end{bmatrix}$$

By equality of matrices,
x + 2y + 3z = 125 …(1)
-5y – 10z = -350 …(2)
5z = 125 …(3)
From (3), z = 25
Substituting z = 25 in (2), we get
-5y – 10(25) = -350
∴ -5y = -350 + 250 = -100
∴ y = 20
Substituting y = 20, z = 25 in (1), we get
x + 2(20) + 3(25) = 125
∴ x = 125 – 40 – 75 = 10
∴ x = 10, y = 20, z = 25
Hence, the cost of 1 pencil is ₹ 10, 1 pen is ₹ 20 and 1 book is ₹ 25.

Question 5.
The sum of three numbers is 6. Thrice the third number when added to the first number, gives 7. On adding three times first number to the sum of second and third number, we get 12. Find the three numbers by using Matrices.
Solution:
Let the numbers be x, y and z.
According to the given conditions,
x + y + z = 6
3z + x = 7, i.e., x + 3z = 7
and 3x + y + z = 12
Hence, the system of linear equations is
x + y + z = 6
x + 3z = 7

3x + y + z = 12
These equations can be written in matrix form as :

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

By $R_2 - R_1$ and $R_3 - 3R_1$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -6 \end{bmatrix}$$

By $R_3 + R_2$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -5 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x+y+z \\ 0-y+2z \\ 0-3y+0 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -5 \end{bmatrix}$$

By equality of matrices,
x + y + z = 6 ...(1)
-y + 2z = 1 ...(2)
-3y = -5 ...(3)
From (3), y = 53
Substituting y = 53 in (2), we get,
−53 + 2z = 1
∴ 2z = 1 + 53 = 83
∴ z = 43
Substituting y =53, z = 53 in (1), we get,
x + 53+43 = 6
∴ x = 3
∴ x = 3, y = 53, z = 43
Hence, the required numbers are 3, 53 and 43.

Question 6.
The sum of three numbers is 2. If twice the second number is added to the sum of first and third number, we get 1 adding five times the first number to the sum of second and third we get 6. Find the three numbers by using matrices.
Solution:
Let the three numbers be x, y and z.
According to the question,
x + y + 2
x + 2y + z = 1
5x + y + z = 6
The given system of equations can be written in matrix form as follows:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

$AX = B$

Here,

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$

$|A| = 1(2-1) - 1(1-5) + 1(1-10)$

$= 1 + 4 - 9$

$= -4$

Let C_{ij} be the cofactors of t he elements a $_{ij}$ in $A = [a_{ij}]$. Then,

$$C_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1, C_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} = 4, C_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} = -9$$

$$C_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0, C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} = -4, C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} = 4$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1, C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0, C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1$$

$$adjA = \begin{bmatrix} 1 & 4 & -9 \\ 0 & -4 & 4 \\ -1 & 0 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|} adjA$$

$$= \frac{1}{-4}\begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\Rightarrow X = \frac{1}{-4}\begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{-4}\begin{bmatrix} 2+0-6 \\ 8-4+0 \\ -18+4+6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} -4 \\ 4 \\ -8 \end{bmatrix}$$

$\therefore x = 1, y = -1$ and $z = 2$

Question 7.
An amount of ₹ 5000 is invested in three types of investments, at interest rates 6%, 7%, 8% per annum respectively. The total annual income from this investment is ₹ 350. If the total annual income from the first two investments is ₹ 70 more than the income from the third, find the amount of each investment using the matrix method.
Solution:
Let the amounts in three investments by ₹ x, ₹ y, and ₹ z respectively.
Then x + y + z = 5000
Since the rate of interest in these investments are 6%, 7% and 8% respectively, the annual income of the three investments are 6x100, 7y100 and 8z100 respectively.
According to the given conditions,

6x100+7y100+8z100 = 350

i.e. 6x + 7y + 8z = 35000

Also, 6x100+7y100 = 8z100 + 70

i.e. 6x + 7y – 8z = 7000
Hence, the system of linear equation is
x + y + z = 5000
6x + 7y + 8z = 35000
6x + 7y – 8z = 7000
These equations can be written in matrix form as :

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 35000 \\ 7000 \end{bmatrix}$$

By $R_3 - R_2$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 0 & 0 & -16 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 35000 \\ -28000 \end{bmatrix}$$

By $R_2 - 6R_1$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -16 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 5000 \\ -28000 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + y + z \\ 0 + y + 2z \\ 0 + 0 - 16z \end{bmatrix} = \begin{bmatrix} 5000 \\ 5000 \\ -28000 \end{bmatrix}$$

By equality of matrices,
x + y + z = 5000 …(1)
y + 2z = 5000 …(2)
-16z = -28000 ….(3)
From (3), z = 1750
Substituting z = 1750 in (2), we get,
y + 2(1750) = 5000
∴ y = 5000 – 3500 = 1500
Substituting y = 1500, z = 1750 in (1), we get,
x + 1500 + 1750 = 5000
∴ x = 5000 – 3250 = 1750
∴ x = 1750, y = 1500, z = 1750
Hence, the amounts of the three investments are ₹ 1750, ₹ 1500 and ₹ 1750 respectively.

Question 8.
The sum of the costs of one ook each of Mathematics, Physics and Chemistry is ₹ 210. Total cost of a mathematics book, 2 physics books, and a chemistry book is ₹ 240 Also the total cost of a Mathematics book, 3 physics book and chemistry books is Rs. 300/-. Find the cost of each book, using Matrices.