Maharashtra State Board 11th Commerce Maths Solutions Chapter 8 Continuity Ex 8.1

Question 1.
Examine the continuity of
(i) f(x) = $x^3 + 2x^2 - x - 2$ at x = -2
Solution:
f(x) = $x^3 + 2x^2 - x - 2$
Here f(x) is a polynomial function and hence it is continuous for all x ∈ R.
∴ f(x) is continuous at x = -2

(ii) f(x) = $x2-9x-3$ on R
Solution:
f(x) = $x2-9x-3$; x ∈ R
f(x) is a rational function and is continuous for all x ∈ R, except at the points where denominator becomes zero.
Here, denominator x – 3 = 0 when x = 3.
∴ Function f is continuous for all x ∈ R, except at x = 3, where it is not defined.

Question 2.
Examine whether the function is continuous at the points indicated against them.
(i) f(x) = $x^3 - 2x + 1$, for x ≤ 2
= 3x – 2, for x > 2, at x = 2
Solution:

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x^3 - 2x + 1)$$
$$= (2)^3 - 2(2) + 1 = 5$$
$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (3x - 2)$$
$$= 3(2) - 2 = 4$$
$$\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$$

∴ Function f is discontinuous at x = 2

(ii) f(x) = $x2+18x-19x-1$ for x ≠ 1
= 20, for x = 1, at x = 1
Solution:

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^2 + 18x - 19}{x - 1}$$
$$= \lim_{x \to 1} \frac{x^2 + 19x - x - 19}{x - 1}$$
$$= \lim_{x \to 1} \frac{x(x+19) - 1(x+19)}{(x-1)}$$
$$= \lim_{x \to 1} \frac{(x-1)(x+19)}{(x-1)}$$
$$= \lim_{x \to 1} (x + 19)$$
$$\ldots[\because x \to 1, \therefore x \neq 1, \therefore x - 1 \neq 0]$$
$$= 1 + 19 = 20$$

Also, f(1) = 20

$$\lim_{x \to 1} f(x) = f(1)$$

∴ f(x) is continuous at x = 1

Question 3.
Test the continuity of the following functions at the points indicated against them.
(i) f(x) = $x-1\sqrt{}-(x-1)_{13}x-2$ for x ≠ 2
= 15 for x = 2, at x = 2
Solution:

$$f(2) = \frac{1}{5} \qquad \ldots\text{(given)}$$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{\sqrt{x-1} - (x-1)^{\frac{1}{3}}}{x-2}$$

Put $x - 1 = y$

$\therefore \quad x = 1 + y$

As $x \to 2, y \to 1$

$$\therefore \quad \lim_{x \to 2} f(x) = \lim_{y \to 1} \frac{\sqrt{y} - y^{\frac{1}{3}}}{1 + y - 2}$$

$$= \lim_{y \to 1} \frac{y^{\frac{1}{2}} - 1 - y^{\frac{1}{3}} + 1}{y - 1}$$

$$= \lim_{y \to 1} \frac{\left(y^{\frac{1}{2}} - 1\right) - \left(y^{\frac{1}{3}} - 1\right)}{y - 1}$$

$$= \lim_{y \to 1} \left(\frac{y^{\frac{1}{2}} - 1}{y - 1} - \frac{y^{\frac{1}{3}} - 1}{y - 1} \right)$$

$$= \lim_{y \to 1} \frac{y^{\frac{1}{2}} - 1^{\frac{1}{2}}}{y - 1} - \lim_{y \to 1} \frac{y^{\frac{1}{3}} - 1^{\frac{1}{3}}}{y - 1}$$

$$= \frac{1}{2}(1)^{\frac{-1}{2}} - \frac{1}{3}(1)^{\frac{-2}{3}} \qquad \ldots\left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = n.a^{n-1}\right]$$

$$= \frac{1}{2} - \frac{1}{3}$$

$$= \frac{1}{6}$$

$$\therefore \quad \lim_{x \to 2} f(x) \neq f(2)$$

$\therefore \quad f(x)$ is discontinuous at $x = 2$

(ii) f(x) = x3−8x+2√−3x−2√ for x ≠ 2
= -24 for x = 2, at x = 2
Solution:

$f(2) = -24 \quad \ldots(\text{given})$

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x^3-8}{\sqrt{x+2}-\sqrt{3x-2}}$$

$$= \lim_{x\to 2} \frac{x^3-8}{\sqrt{x+2}-\sqrt{3x-2}} \times \frac{\sqrt{x+2}+\sqrt{3x-2}}{\sqrt{x+2}+\sqrt{3x-2}}$$

$$= \lim_{x\to 2} \frac{\left(x^3-8\right)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{(x+2)-(3x-2)}$$

$$= \lim_{x\to 2} \frac{\left(x^3-2^3\right)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2x+4}$$

$$= \lim_{x\to 2} \frac{(x-2)\left(x^2+2x+4\right)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2(x-2)}$$

$$= \lim_{x\to 2} \frac{\left(x^2+2x+4\right)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2}$$

$$\ldots\begin{bmatrix} \because x \to 2, x \neq 2 \\ \therefore x-2 \neq 0 \end{bmatrix}$$

$$= \frac{-1}{2} \lim_{x\to 2} \left(x^2+2x+4\right)\left(\sqrt{x+2}+\sqrt{3x-2}\right)$$

$$= \frac{-1}{2} \lim_{x\to 2} \left(x^2+2x+4\right) \lim_{x\to 2}\left(\sqrt{x+2}+\sqrt{3x-2}\right)$$

$$= \frac{-1}{2} \times \left[2^2+2(2)+4\right] \times \left(\sqrt{2+2}+\sqrt{3(2)-2}\right)$$

$$= \frac{-1}{2} \times 12 \times (2+2)$$

$$= -24$$

$\therefore \quad \lim_{x\to 2} f(x) = f(2)$

$\therefore \quad f(x)$ is continuous at $x = 2$

(iii) f(x) = 4x + 1 for x ≤ 83
= 59-9x3, for x > 83, at x = 83
Solution:

$$\lim_{x \to \left(\frac{8}{3}\right)^-} f(x) = \lim_{x \to \left(\frac{8}{3}\right)^-} (4x+1)$$

$$= 4\left(\frac{8}{3}\right) + 1$$

$$= \frac{32}{3} + 1$$

$$= \frac{35}{3}$$

$$\lim_{x \to \left(\frac{8}{3}\right)^+} f(x) = \lim_{x \to \left(\frac{8}{3}\right)^+} \frac{59-9x}{3}$$

$$= \frac{59 - 9\left(\frac{8}{3}\right)}{3}$$

$$= \frac{59-24}{3}$$

$$= \frac{35}{3}$$

$$f(x) = 4x + 1, \qquad x \leq \left(\frac{8}{3}\right)$$

$$\therefore \quad f\left(\frac{8}{3}\right) = 4\left(\frac{8}{3}\right) + 1$$

$$= \frac{32}{3} + 1$$

$$= \frac{35}{3}$$

$$\therefore \quad \lim_{x \to \left(\frac{8}{3}\right)^-} f(x) = \lim_{x \to \left(\frac{8}{3}\right)^+} f(x) = f\left(\frac{8}{3}\right)$$

$\therefore$ f(x) is continuous at $x = \frac{8}{3}$

(iv) f(x) = x3−27x2−9 for 0 ≤ x < 3
= 92, for 3 ≤ x ≤ 6, at x = 3
Solution:

$$f(3) = \frac{9}{2} \qquad \ldots\text{(given)}$$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9}$$

$$= \lim_{x \to 3} \frac{(x-3)(x^2+3x+9)}{(x-3)(x+3)}$$

$$= \lim_{x \to 3} \frac{x^2+3x+9}{x+3} \qquad \ldots\left[\begin{array}{l}\text{As } x \to 3,\ x \neq 3 \\ \therefore x - 3 \neq 0\end{array}\right]$$

$$= \frac{(3)^2 + 3(3) + 9}{3+3}$$

$$= \frac{9+9+9}{6}$$

$$= \frac{27}{6}$$

$$= \frac{9}{2}$$

$$\therefore \quad \lim_{x \to 3} f(x) = f(3)$$

$\therefore$ Function f is continuous at $x = 3$

Question 4.
(i) If f(x) = 24x−8x−3x+112x−4x−3x+1, for x ≠ 0
= k, for x = 0
is continuous at x = 0, find k.
Solution:

Function f is continuous at x = 0

$$\therefore \quad f(0) = \lim_{x\to 0} f(x)$$

$$\therefore \quad k = \lim_{x\to 0} \frac{24^x - 8^x - 3^x + 1}{12^x - 4^x - 3^x + 1}$$

$$= \lim_{x\to 0} \frac{8^x \cdot 3^x - 8^x - 3^x + 1}{4^x \cdot 3^x - 4^x - 3^x + 1}$$

$$= \lim_{x\to 0} \frac{8^x(3^x - 1) - 1(3^x - 1)}{4^x(3^x - 1) - 1(3^x - 1)}$$

$$= \lim_{x\to 0} \frac{(3^x - 1)(8^x - 1)}{(3^x - 1)(4^x - 1)}$$

$$= \lim_{x\to 0} \frac{8^x - 1}{4^x - 1} \qquad \ldots\begin{bmatrix} \because x \to 0,\ 3^x \to 3^0 \\ \therefore 3^x \to 1 \therefore 3^x \neq 1 \\ \therefore 3^x - 1 \neq 0 \end{bmatrix}$$

$$= \lim_{x\to 0} \left(\frac{\frac{8^x - 1}{x}}{\frac{4^x - 1}{x}} \right) \qquad \ldots[\because x \to 0,\ \therefore x \neq 0]$$

$$= \frac{\log 8}{\log 4} \qquad \ldots\left[\because \lim_{x\to 0}\left(\frac{a^x - 1}{x}\right) = \log a\right]$$

$$= \frac{\log (2)^3}{\log (2)^2}$$

$$= \frac{3 \log 2}{2 \log 2}$$

$$\therefore \quad f(0) = \frac{3}{2}$$

(ii) If f(x) = $\frac{5^x+5^{-x}-2}{x^2}$, for x ≠ 0
= k for x = 0
is continuous at x = 0, find k.
Solution:
Function f is continuous at x = 0

$$\therefore \quad f(0) = \lim_{x\to 0} f(x)$$

$$\therefore \quad k = \lim_{x\to 0} \frac{5^x + 5^{-x} - 2}{x^2}$$

$$= \lim_{x\to 0} \frac{5^x + \frac{1}{5^x} - 2}{x^2}$$

$$= \lim_{x\to 0} \frac{(5^x)^2 + 1 - 2(5^x)}{5^x \cdot x^2}$$

$$= \lim_{x\to 0} \frac{(5^x - 1)^2}{5^x \cdot x^2}$$

$$\ldots[\because a^2 - 2ab + b^2 = (a - b)^2]$$

$$= \lim_{x\to 0} \left(\frac{5^x - 1}{x}\right)^2 \cdot \frac{1}{5^x}$$

$$= \lim_{x\to 0} \left(\frac{5^x - 1}{x}\right)^2 \times \lim_{x\to 0} \frac{1}{5^x}$$

$$= (\log 5)^2 \times \frac{1}{5^0} \qquad \ldots\left[\because \lim_{x\to 0}\left(\frac{a^x - 1}{x}\right) = \log a\right]$$

$$\therefore \quad k = (\log 5)^2$$

(iii) For what values of a and b is the function
f(x) = ax + 2b + 18 for x ≤ 0
= x^2 + 3a – b for 0 < x ≤ 2 = 8x – 2 for x > 2,
continuous for every x?
Solution:
Function f is continuous for every x.
∴ Function f is continuous at x = 0 and x = 2

As f is continuous at x = 0.

∴ $lim_{x\to0^-}f(x)=lim_{x\to0^+}f(x)$

∴ $lim_{x\to0^-}(ax+2b+18)=lim_{x\to0^+}(x^2+3a-b)$

∴ a(0) + 2b + 18 = (0)2 + 3a – b

∴ 3a – 3b = 18

∴ a – b = 6 …..(i)

Also, Function f is continous at x = 2

∴ $lim_{x\to2^-}f(x)=lim_{x\to2^-}f(x)$

∴ $lim_{x\to2^-}(x^2+3a-b)=lim_{x\to2^-}(8x-2)$

∴ $(2)^2$ + 3a – b = 8(2) – 2

∴ 4 + 3a – b = 14

∴ 3a – b = 10 …..(ii)

Subtracting (i) from (ii), we get

2a = 4

∴ a = 2

Substituting a = 2 in (i), we get

2 – b = 6

∴ b = -4

∴ a = 2 and b = -4

(iv) For what values of a and b is the function

f(x) = $\frac{x^2-4}{x-2}$ for x < 2

= ax^2 – bx + 3 for 2 ≤ x < 3

= 2x – a + b for x ≥ 3

continuous in its domain.

Solution:

Function f is continuous for every x on R.

∴ Function f is continuous at x = 2 and x = 3.

As f is continuous at x = 2.

∴ $lim_{x\to2^-}f(x)=lim_{x\to2^+}f(x)$

$$\therefore \quad \lim_{x\to2^-}\frac{x^2-4}{x-2}=\lim_{x\to2^+}(ax^2-bx+3)$$

$$\therefore \quad \lim_{x\to2^-}\frac{(x-2)(x+2)}{x-2}=\lim_{x\to2^+}(ax^2-bx+3)$$

$$\therefore \quad \lim_{x\to2^-}(x+2)=\lim_{x\to2^+}(ax^2-bx+3)$$

$$\cdots\left[\begin{array}{l}\because x\to2\ \therefore x\neq2\\ \therefore\ x-2\neq0\end{array}\right]$$

∴ 2 + 2 = $a(2)^2$ – b(2) + 3

∴ 4 = 4a – 2b + 3

∴ 4a – 2b = 1 …..(i)

Also function f is continuous at x = 3

∴ $lim_{x\to3^-}f(x)=lim_{x\to3^+}f(x)$

∴ $lim_{x\to3^-}(ax^2-bx+3)=lim_{x\to3^+}(2x-a+b)$

∴ $a(3)^2$ – b(3) + 3 = 2(3) – a + b

∴ 9a – 3b + 3 = 6 – a + b

∴ 10a – 4b = 3 …..(ii)

Multiplying (i) by 2, we get

8a – 4b = 2 …..(iii)

Subtracting (iii) from (ii), we get

2a = 1

∴ a = $\frac{1}{2}$

Substituting a = $\frac{1}{2}$ in (i), we get

4($\frac{1}{2}$) – 2b = 1

∴ 2 – 2b = 1

∴ 1 = 2b

∴ b = $\frac{1}{2}$

∴ a = $\frac{1}{2}$ and b = $\frac{1}{2}$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 8 Continuity Miscellaneous Exercise 8

I. Discuss the continuity of the following functions at the point(s) or in the interval indicated against them.

Question 1.
If f(x) = $2x^2 - 2x + 5$ for $0 \leq x < 2$
= $\frac{1-3x-x^2}{1-x}$ for $2 \leq x < 4$
= $\frac{7-x^2}{x-5}$ for $4 \leq x \leq 7$ on its domain.
Solution:
The domain of f is [0, 5) ∪ (5, 7]
We observe that x = 5 is not included in the domain as f is not defined at x = 5
a. For $0 \leq x < 2$
f(x) = $2x^2 - 2x + 5$
It is a polynomial function and is continuous at all point in [0, 2)

b. For $2 < x < 4$
f(x) = $\frac{1-3x-x^2}{1-x}$
It is a rational function and is continuous everwhere except at points where its denominator becomes zero.
Denominator becomes zero at x = 1
But x = 1 does not lie in the interval.
f(x) is continuous at all points in (2, 4)

c. For $4 < x \leq 7$, $x \neq 5$
i.e. for x ∈ [4, 5) ∪ (5, 7]
∴ f(x) = $\frac{7-x^2}{x-5}$
It is a rational function and is continuous everywhere except possibly at points where its denominator becomes zero.
Denominator becomes zero at x = 5
But x = 5 ∉ [4, 5) ∪ (5, 7]
∴ f is continuous at all points in (4, 7] – {5}.

d. Since the definition of function changes around x = 2, x = 4 and x = 7
∴ there is disturbance in behaviour of the function.
So we examine continuity at x = 2, 4, 7 separately.
Continuity at x = 2:

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x^2 - 2x + 5)$$

= $2(2)^2 - 2(2) + 5$
= 8 – 4 + 5
= 9

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{1-3x-x^2}{1-x}$$

$$= \frac{1-3(2)-(2)^2}{1-2}$$

$$= \frac{1-6-4}{-1}$$

$$= \frac{-9}{-1}$$

$$= 9$$

Also, $f(x) = \frac{1-3x-x^2}{1-x}$, at $x = 2$

∴ $f(2) = \frac{1-3(2)-(2)^2}{1-2}$

$f(2) = 9$

∴ $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

∴ f is continuous at x = 2

e. Continuity at x = 4:

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \frac{1-3x-x^2}{1-x}$$

$$= \frac{1-3(4)-(4)^2}{1-4}$$

$$= \frac{1-12-16}{1-4}$$

$$= \frac{-27}{-3}$$

$$= 9$$

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \frac{7-x^2}{x-5}$$

$$= \frac{7-(4)^2}{4-5}$$

$$= \frac{7-16}{-1}$$

$$= 9$$

$$f(x) = \frac{7-x^2}{x-5}, \text{ at } x = 4$$

$$\therefore \quad f(4) = \frac{7-(4)^2}{4-5}$$

$$= 9$$

$$\therefore \quad \lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = f(4)$$

∴ f is continuous at x = 4

Question 2.

f(x) = $\frac{3^x+3^{-x}-2}{x^2}$ for x ≠ 0

= $(\log 3)^2$ for x = 0 at x = 0

Solution:

$$f(0) = (\log 3)^2 \qquad \ldots\text{(given)}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{3^x+3^{-x}-2}{x^2}$$

$$= \lim_{x \to 0} \frac{3^x + \frac{1}{3^x} - 2}{x^2}$$

$$= \lim_{x \to 0} \frac{(3^x)^2 + 1 - 2(3^x)}{x^2 \cdot (3^x)}$$

$$= \lim_{x \to 0} \frac{(3^x-1)^2}{x^2 \cdot (3^x)}$$

$$\ldots[\because a^2 - 2ab + b^2 = (a-b)^2]$$

$$= \lim_{x \to 0} \left[\left(\frac{3^x-1}{x}\right)^2 \times \frac{1}{3^x}\right]$$

$$= \lim_{x \to 0} \left(\frac{3^x-1}{x}\right)^2 \times \frac{1}{\lim_{x \to 0} 3^x}$$

$$= (\log 3)^2 \times \frac{1}{3^0}$$

$$\ldots\left[\because \lim_{x \to 0}\left(\frac{a^n-1}{x}\right) = \log a\right]$$

$$= (\log 3)^2 \times \frac{1}{1}$$

$$= (\log 3)^2$$

∴ $\lim_{x \to 0} f(x) = f(0)$

∴ f is continuous at x = 0

Question 3.
f(x) = $\frac{5^x-e^x}{2x}$ for x ≠ 0
= $\frac{1}{2}$ (log 5 – 1) for x = 0 at x = 0
Solution:

$$f(0) = \frac{1}{2}(\log 5 - 1) \quad \ldots[\text{given}]$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{5^x - e^x}{2x}$$

$$= \lim_{x \to 0} \frac{5^x - 1 - e^x + 1}{2x}$$

$$= \frac{1}{2} \lim_{x \to 0} \frac{(5^x - 1) - (e^x - 1)}{x}$$

$$= \frac{1}{2} \lim_{x \to 0} \left[\frac{(5^x - 1)}{x} - \frac{(e^x - 1)}{x}\right]$$

$$= \frac{1}{2}\left(\lim_{x \to 0} \frac{5^x - 1}{x} - \lim_{x \to 0} \frac{e^x - 1}{x}\right)$$

$$= \frac{1}{2}(\log 5 - \log e)$$

$$\ldots\left[\lim_{x \to 0} \frac{a^x - 1}{x} = \log a\right]$$

$$= \frac{1}{2}(\log 5 - 1) \quad \ldots[\because \log e = 1]$$

∴ $\lim_{x\to 0} f(x) = f(0)$
∴ f is continuous at x = 0

Question 4.
f(x) = $\frac{\sqrt{x+3}-2}{x^3-1}$ for x ≠ 1
= 2 for x = 1, at x = 1
Solution:

$$f(1) = 2 \quad \ldots[\text{given}]$$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{\sqrt{x+3} - 2}{x^3 - 1}$$

$$= \lim_{x \to 1}\left(\frac{\sqrt{x+3} - 2}{x^3 - 1} \times \frac{\sqrt{x+3} + 2}{\sqrt{x+3} + 2}\right)$$

$$= \lim_{x \to 1}\left(\frac{x + 3 - 4}{(x^3 - 1)(\sqrt{x+3} + 2)}\right)$$

$$= \lim_{x \to 1} \frac{x - 1}{(x - 1)(x^2 + x + 1)(\sqrt{x+3} + 2)}$$

$$= \lim_{x \to 1} \frac{1}{(x^2 + x + 1)(\sqrt{x+3} + 2)}$$

$$\ldots\left[\begin{array}{l}\text{As } x \to 1, x \neq 1 \\ \therefore x - 1 \neq 0\end{array}\right]$$

$$= \frac{1}{\lim_{x \to 1}(x^2 + x + 1) \times \lim_{x \to 1}(\sqrt{x+3} + 2)}$$

$$= \frac{1}{(1^2 + 1 + 1) \times (\sqrt{1+3} + 2)}$$

$$= \frac{1}{3(2 + 2)}$$

$$= \frac{1}{12}$$

∴ $\lim_{x\to 1} f(x) \neq f(1)$
∴ f is discontinuous at x = 1

Question 5.
f(x) = $\frac{\log x - \log 3}{x-3}$ for x ≠ 3

= 3 for x = 3, at x = 3

Solution:

$$f(3) = 3 \quad \ldots\text{[given]}$$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{\log x - \log 3}{x - 3}$$

Substitute $x - 3 = h$

$\therefore \quad x = 3 + h,$

as $x \to 3$, $h \to 0$

$$\therefore \quad \lim_{x \to 3} f(x) = \lim_{h \to 0} \frac{\log(h + 3) - \log 3}{3 + h - 3}$$

$$= \lim_{h \to 0} \frac{\log\left(\frac{h+3}{3}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\log\left(1 + \frac{h}{3}\right)}{\left(\frac{h}{3}\right)} \times \frac{1}{3}$$

$$= \frac{1}{3} \lim_{h \to 0} \frac{\log\left(1 + \frac{h}{3}\right)}{\left(\frac{h}{3}\right)}$$

$$= \frac{1}{3}(1) \quad \ldots\left[\because \lim_{x \to 0} \frac{\log(1 + x)}{x} = 1\right]$$

$$= \frac{1}{3}$$

$$\therefore \quad \lim_{x \to 3} f(x) \neq f(3)$$

$\therefore$ f is discontinuous at $x = 3$

(II) Find k if following functions are continuous at the points indicated against them.

Question 1.

$f(x) = \left(\frac{5x-8}{8-3x}\right)^{\frac{3}{2x-4}}$ for $x \neq 2$

= k for x = 2 at x = 2

Solution:

f is continuous at $x = 2$

$$\therefore \quad f(2) = \lim_{x \to 2} f(x)$$

$$\therefore \quad k = \lim_{x \to 2} \left(\frac{5x-8}{8-3x}\right)^{\frac{3}{2x-4}}$$

Substitute $x - 2 = h$

$\therefore \quad x = 2 + h$

As $x \to 2$, $h \to 0$

$$\therefore \quad k = \lim_{h \to 0} \left[\frac{5(2+h)-8}{8-3(2+h)}\right]^{\frac{3}{2(2+h)-4}}$$

$$= \lim_{h \to 0} \left(\frac{10 + 5h - 8}{8 - 6 - 3h}\right)^{\frac{3}{2h}}$$

$$= \lim_{h \to 0} \left(\frac{2 + 5h}{2 - 3h}\right)^{\frac{3}{2h}}$$

$$= \lim_{h \to 0} \left[\frac{2\left(1 + \frac{5h}{2}\right)}{2\left(1 - \frac{3h}{2}\right)} \right]^{\frac{3}{2h}}$$

$$= \lim_{h \to 0} \frac{\left(1 + \frac{5h}{2}\right)^{\frac{3}{2h}}}{\left(1 - \frac{3h}{2}\right)^{\frac{3}{2h}}}$$

$$= \frac{\lim_{h \to 0} \left[\left(1 + \frac{5h}{2}\right)^{\frac{2}{5h}} \right]^{\frac{5}{2} \times \frac{3}{2}}}{\lim_{h \to 0} \left[\left(1 - \frac{3h}{2}\right)^{\frac{-2}{3h}} \right]^{\frac{-3}{2} \times \frac{3}{2}}}$$

$$= \frac{e^{\frac{15}{4}}}{e^{\frac{-9}{4}}} \qquad \ldots \left[\begin{array}{l} \because h \to 0, \frac{5h}{2} \to 0, \frac{-3h}{2} \to 0 \\ \text{and } \lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e \end{array} \right]$$

$$= e^{\frac{24}{4}}$$

$$\therefore \quad k = e^6$$

Question 2.
f(x) = $45x−9x−5x+1(kx−1)(3x−1)$ for x ≠ 0
= 23 for x = 0, at x = 0
Solution:

f is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0)$

$\therefore \lim_{x\to 0} \frac{(45)^x - 9^x - 5^x + 1}{(k^x - 1)(3^x - 1)} = \frac{2}{3}$

$\therefore \lim_{x\to 0} \frac{9^x \cdot 5^x - 9^x - 5^x + 1}{(k^x - 1)(3^x - 1)} = \frac{2}{3}$

$\therefore \lim_{x\to 0} \frac{9^x (5^x - 1) - 1(5^x - 1)}{(k^x - 1)(3^x - 1)} = \frac{2}{3}$

$\therefore \lim_{x\to 0} \frac{(5^x - 1)(9^x - 1)}{(k^x - 1)(3^x - 1)} = \frac{2}{3}$

$\therefore \lim_{x\to 0} \frac{\frac{(5^x - 1)(9^x - 1)}{x^2}}{\frac{(k^x - 1)(3^x - 1)}{x^2}} = \frac{2}{3}$

$\ldots \left[\begin{array}{l} \because x \to 0, \therefore x \neq 0 \quad \therefore x^2 \neq 0 \\ \text{Divide Numerator and Denominator} \\ \text{by } x^2 \end{array}\right]$

$\therefore \frac{\lim_{x\to 0} \left(\frac{5^x - 1}{x}\right)\left(\frac{9^x - 1}{x}\right)}{\lim_{x\to 0} \left(\frac{k^x - 1}{x}\right)\left(\frac{3^x - 1}{x}\right)} = \frac{2}{3}$

$\therefore \frac{\log 5 \cdot \log 9}{\log k \cdot \log 3} = \frac{2}{3} \quad \ldots \left[\because \lim_{x\to 0} \frac{a^x - 1}{x} = \log a\right]$

$\therefore \frac{\log 5 \cdot \log (3)^2}{\log k \cdot \log 3} = \frac{2}{3}$

$\therefore \frac{2 \times \log 5 \times \log 3}{\log k \times \log 3} = \frac{2}{3}$

$\therefore \frac{\log 5}{\log k} = \frac{1}{3}$

$\therefore 3 \log 5 = \log k$

$\therefore \log (5)^3 = \log k$

$\therefore (5)^3 = k$

$\therefore k = 125$

Question 3.
f(x) = $(1+kx)^{\frac{1}{x}}$, for x ≠ 0

= $e^{\frac{3}{2}}$, for x = 0, at x = 0

Solution:

f is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0)$

$\therefore \lim_{x\to 0} (1 + kx)^{\frac{1}{x}} = e^{\frac{3}{2}}$

$\therefore \lim_{x\to 0} \left[(1 + kx)^{\frac{1}{kx}}\right]^k = e^{\frac{3}{2}}$

$\therefore e^k = e^{\frac{3}{2}} \quad \ldots \left[\lim_{x\to 0} (1 + x)^{\frac{1}{x}} = e\right]$

$\therefore k = \frac{3}{2}$

III. Find a and b if following functions are continuous at the point indicated against them.

Question 1.
f(x) = x^2 + a, for x ≥ 0

= $2\sqrt{x^2+1}$ + b, for x < 0 and

f(1) = 2, is continuous at x = 0

Solution:

Since, $f(x) = x^2 + a, \qquad x \geq 0$

$\therefore \quad f(1) = (1)^2 + a$

$\therefore \quad 2 = 1 + a \qquad \ldots[\because f(1) = 2]$

$\therefore \quad a = 1$

Also f is continuous at $x = 0$

$\therefore \quad \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$\therefore \quad \lim_{x \to 0^-} \left(2\sqrt{x^2 + 1} + b\right) = \lim_{x \to 0^+} \left(x^2 + a\right)$

$\therefore \quad 2\sqrt{0^2 + 1} + b = 0^2 + 1$

$\therefore \quad 2(1) + b = 1$

$\therefore \quad b = -1$

$\therefore \quad a = 1$ and $b = -1$

Question 2.
f(x) = x2−9x−3 + a, for x > 3
= 5, for x = 3
= 2x2 + 3x + b, for x < 3
is continuous at x = 3
Solution:

f is continuous at $x = 3$

$\therefore \quad f(3) = \lim_{x \to 3^-} f(x)$

$= \lim_{x \to 3^-} (2x^2 + 3x + b)$

$\therefore \quad 5 = 2(3)^2 + 3(3) + b$

$\therefore \quad 5 = 18 + 9 + b$

$\therefore \quad b = -22$

Also, $f(3) = \lim_{x \to 3^+} f(x)$

$\therefore \quad 5 = \lim_{x \to 3^+} \frac{x^2 - 9}{x - 3} + a$

$= \lim_{x \to 3^+} \frac{(x+3)(x-3)}{(x-3)} + a$

$= \lim_{x \to 3^+} (x + 3) + a \qquad \ldots\begin{bmatrix} \because x \to 3; x \neq 3 \\ \therefore x - 3 \neq 0 \end{bmatrix}$

$= (3 + 3) + a$

$\therefore \quad 5 = 6 + a$

$\therefore \quad a = -1$

$\therefore \quad a = -1, b = -22$

Question 3.
f(x) = 32x−18x−1 + a, for x > 0
= 2, for x = 0
= x + 5 – 2b, for x < 0
is continuous at x = 0
Solution:

f is continuous at $x = 0$

$\therefore \quad \lim_{x \to 0^-} f(x) = f(0)$

$\therefore \quad \lim_{x \to 0^-} (x + 5 - 2b) = 2$

$\therefore \quad 0 + 5 - 2b = 2$

$\therefore \quad 5 - 2 = 2b$

$\therefore \quad 2b = 3$

$\therefore \quad b = \dfrac{3}{2}$

Also $\lim_{x \to 0^-} f(x) = f(0)$

$$\therefore \quad \lim_{x \to 0^+} \left(\frac{32^x - 1}{8^x - 1} + a \right) = 2$$

$$\therefore \quad \lim_{x \to 0^+} \left(\frac{\frac{32^x - 1}{x}}{\frac{8^x - 1}{x}} \right) + \lim_{x \to 0^+} a = 2$$

$$\therefore \quad \frac{\log 32}{\log 8} + a = 2 \qquad \ldots \left[\because \lim_{x \to 0} \frac{a^x - 1}{x} = \log a \right]$$

$$\therefore \quad \frac{\log (2)^5}{\log (2)^3} + a = 2$$

$$\therefore \quad \frac{5 \log 2}{3 \log 2} + a = 2$$

$$\therefore \quad \frac{5}{3} + a = 2$$

$$\therefore \quad a = 2 - \frac{5}{3}$$

$$\therefore \quad a = \frac{1}{3}$$

$$\therefore \quad a = \frac{1}{3} \text{ and } b = \frac{3}{2}$$