Maharashtra State Board 11th Commerce Maths Solutions Chapter 5 Locus and Straight Line Ex 5.1

Question 1.
If A(1, 3) and B(2, 1) are points, find the equation of the locus of point P such that PA = PB.
Solution:
Let P(x, y) be any point on the required locus.
Given, A(1, 3) and B(2, 1).
PA = PB
$\therefore PA^2 = PB^2$
$\therefore (x - 1)^2 + (y - 3)^2 = (x - 2)^2 + (y - 1)^2$
$\therefore x^2 - 2x + 1 + y^2 - 6y + 9 = x^2 - 4x + 4 + y^2 - 2y + 1$
$\therefore -2x - 6y + 10 = -4x - 2y + 5$
$\therefore 2x - 4y + 5 = 0$
$\therefore$ The required equation of locus is $2x - 4y + 5 = 0$.

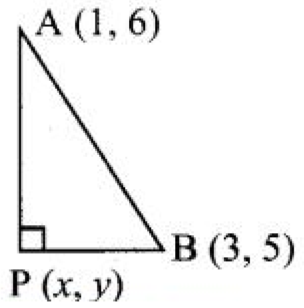
Question 2.
A(-5, 2) and B(4, 1). Find the equation of the locus of point P, which is equidistant from A and B.
Solution:
Let P(x, y) be any point on the required locus.
P is equidistant from A(-5, 2) and B(4, 1).
$\therefore$ PA = PB
$\therefore PA^2 = PB^2$
$\therefore (x + 5)^2 + (y - 2)^2 = (x - 4)^2 + (y - 1)^2$
$\therefore x^2 + 10x + 25 + y^2 - 4y + 4 = x^2 - 8x + 16 + y^2 - 2y + 1$
$\therefore 10x - 4y + 29 = -8x - 2y + 17$
$\therefore 18x - 2y + 12 = 0$
$\therefore 9x - y + 6 = 0$
$\therefore$ The required equation of locus is $9x - y - 6 = 0$

Question 3.
If A(2, 0) and B(0, 3) are two points, find the equation of the locus of point P such that AP = 2BP.
Solution:
Let P(x, y) be any point on the required locus.
Given, A(2, 0), B(0, 3) and AP = 2BP
$\therefore AP^2 = 4BP^2$
$\therefore (x - 2)^2 + (y - 0)^2 = 4[(x - 0)^2 + (y - 3)^2]$
$\therefore x^2 - 4x + 4 + y^2 = 4(x^2 + y^2 - 6y + 9)$
$\therefore x^2 - 4x + 4 + y^2 = 4x^2 + 4y^2 - 24y + 36$
$\therefore 3x^2 + 3y^2 + 4x - 24y + 32 = 0$
$\therefore$ The required equation of locus is $3x^2 + 3y^2 + 4x - 24y + 32 = 0$

Question 4.
If A(4, 1) and B(5, 4), find the equation of the locus of point P if $PA^2 = 3PB^2$.
Solution:
Let P(x, y) be any point on the required locus.
Given, A(4, 1), B(5, 4) and $PA^2 = 3PB^2$
$\therefore (x - 4)^2 + (y - 1)^2 = 3[(x - 5)^2 + (y - 4)^2]$
$\therefore x^2 - 8x + 16 + y^2 - 2y + 1 = 3(x^2 - 10x + 25 + y^2 - 8y + 16)$
$\therefore x^2 - 8x + y^2 - 2y + 17 = 3x^2 - 30x + 75 + 3y^2 - 24y + 48$
$\therefore 2x^2 + 2y^2 - 22x - 22y + 106 = 0$
$\therefore x^2 + y^2 - 11x - 11y + 53 = 0$
$\therefore$ The required equation of locus is $x^2 + y^2 - 11x - 11y + 53 = 0$.

Question 5.
A(2, 4) and B(5, 8), find the equation of the locus of point P such that $PA^2 - PB^2 = 13$.
Solution:
Let P(x, y) be any point on the required locus.
Given, A(2, 4), B(5, 8) and $PA^2 - PB^2 = 13$
$\therefore [(x - 2)^2 + (y - 4)^2] - [(x - 5)^2 + (y - 8)^2] = 13$
$\therefore (x^2 - 4x + 4 + y^2 - 8y + 16) - (x^2 - 10x + 25 + y^2 - 16y + 64) = 13$
$\therefore 6x + 8y - 69 = 13$
$\therefore 6x + 8y - 82 = 0$
$\therefore 3x + 4y - 41 = 0$
$\therefore$ The required equation of locus is $3x + 4y - 41 = 0$

Question 6.
A(1, 6) and B(3, 5), find the equation of the locus of point P such that segment AB subtends a right angle at P. ($\angle APB = 90°$)
Solution:
Let P(x. y) be any point on the required locus.
Given, A(1, 6) and B(3, 5), $\angle APB = 90°$
$\therefore$ ΔAPB is a right-angled triangle.



By Pythagoras theorem,
$AP^2 + PB^2 = AB^2$
$\therefore [(x-1)^2 + (y-6)^2] + [(x-3)^2 + (y-5)^2] = (1-3)^2 + (6-5)^2$
$\therefore x^2 - 2x + 1 + y^2 - 12y + 36 + x^2 - 6x + 9 + y^2 - 10y + 25 = 4 + 1$
$\therefore 2x^2 + 2y^2 - 8x - 22y + 66 = 0$
$\therefore x^2 + y^2 - 4x - 11y + 33 = 0$
$\therefore$ The required equation of locus is $x^2 + y^2 - 4x - 11y + 33 = 0$

Question 7.
If the origin is shifted to the point O'(2, 3), the axes remaining parallel to the original axes, find the new co-ordinates of the points (a) A(1, 3) (b) B(2, 5)
Solution:
Origin is shifted to (2, 3) = (h, k)
Let the new co-ordinates be (X, Y).
$\therefore x = X + h$ and $y = Y + k$
$\therefore x = X + 2$ and $y = Y + 3$ .....(i)
(a) Given, A(x, y) = A(1, 3)
$x = X + 2$ and $y = Y + 3$ .....[From (i)]
$\therefore 1 = X + 2$ and $3 = Y + 3$
$\therefore X = -1$ and $Y = 0$
$\therefore$ the new co-ordinates of point A are (-1, 0).

(b) Given, B(x, y) = B(2, 5)
$x = X + 2$ and$y = Y + 3$ ......[From (i)]
$\therefore 2 = X + 2$ and $5 = Y + 3$
$\therefore X = 0$ and $Y = 2$
$\therefore$ the new co-ordinates of point B are (0, 2).

Question 8.
If the origin is shifted to the point O'(1, 3), the axes remaining parallel to the original axes, find the old co-ordinates of the points (a) C(5, 4) (b) D(3, 3)
Solution:
Origin is shifted to (1, 3) = (h, k)
Let the new co-ordinates be (X, Y)
$x = X + h$ and $y = Y + k$
$\therefore x = X + 1$ and $7 = Y + 3$ .....(i)
(a) Given, C(X, Y) = C(5, 4)
$\therefore x = X + 1$ and$y = Y + 3$ .....[From(i)]
$\therefore x = 5 + 1 = 6$ and $y = 4 + 3 = 7$
$\therefore$ the old co-ordinates of point C are (6, 7).

(b) Given, D(X, Y) = D(3, 3)
$\therefore x = X + 1$ and $y = Y + 3$ .....[From (i)]
$\therefore x = 3 + 1 = 4$ and $y = 3 + 3 = 6$
$\therefore$ the old co-ordinates of point D are (4, 6).

Question 9.
If the co-ordinates (5, 14) change to (8, 3) by the shift of origin, find the co-ordinates of the point, where the origin is shifted.
Solution:
Let the origin be shifted to (h, k).
Given, (x,y) = (5, 14), (X, Y) = (8, 3)
Since, $x = X + h$ and $y = Y + k$
$\therefore 5 = 8 + h$ and $14 = 3 + k$

∴ h = -3 and k = 11
∴ the co-ordinates of the point, where the origin is shifted are (-3, 11).

Question 10.
Obtain the new equations of the following loci if the origin is shifted to the point O'(2, 2), the direction of axes remaining the same:
(a) $3x - y + 2 = 0$
(b) $x^2 + y^2 - 3x = 7$
(c) $xy - 2x - 2y + 4 = 0$
Solution:
Given, (h, k) = (2, 2)
Let (X, Y) be the new co-ordinates of the point (x, y).
∴ x = X + h and y = Y + k
∴ x = X + 2 and y = Y + 2
(a) Substituting the values of x and y in the equation $3x - y + 2 = 0$, we get
$3(X + 2) - (Y + 2) + 2 = 0$
∴ $3X + 6 - Y - 2 + 2 = 0$
∴ $3X - Y + 6 = 0$, which is the new equation of locus.

(b) Substituting the values of x and y in the equation $x^2 + y^2 - 3x = 7$, we get
$(X + 2)^2 + (Y + 2)^2 - 3(X + 2) = 7$
∴ $X^2 + 4X + 4 + Y^2 + 4Y + 4 - 3X - 6 = 7$
∴ $X^2 + Y^2 + X + 4Y - 5 = 0$, which is the new equation of locus.

(c) Substituting the values of x and y in the equation $xy - 2x - 2y + 4 = 0$, we get
$(X + 2)(Y + 2) - 2(X + 2) - 2(Y + 2) + 4 = 0$
∴ $XY + 2X + 2Y + 4 - 2X - 4 - 2Y - 4 + 4 = 0$
∴ $XY = 0$, which is the new equation of locus.

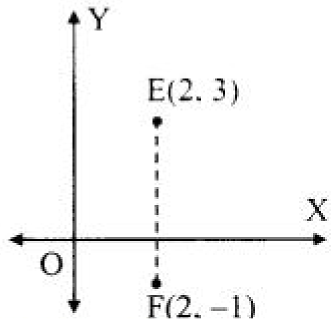
Maharashtra State Board 11th Commerce Maths Solutions Chapter 5 Locus and Straight Line Ex 5.2

Question 1.
Find the slope of each of the following lines which pass through the points:
(a) (2, -1), (4, 3)
(b) (-2, 3), (5, 7)
(c) (2, 3), (2, -1)
(d) (7, 1), (-3, 1)
Solution:
(a) Let A = (x_1, y_1) = (2, -1) and B = (x_2, y_2) = (4, 3).
Slope of line AB = $\frac{y_2-y_1}{x_2-x_1}$
= $\frac{3-(-1)}{4-2}$
= $\frac{4}{2}$
= 2

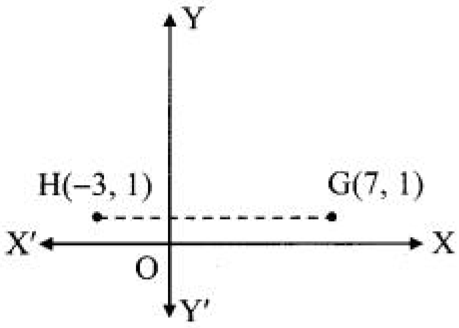
(b) Let C = (x_1, y_1) = (-2, 3) and D = (x_2, y_2) = (5, 7)
Slope of line CD = $\frac{y_2-y_1}{x_2-x_1}$
= $\frac{7-3}{5-(-2)}$
= $\frac{4}{7}$

(c) Let E = (2, 3) = (x_1, y_1) and F = (2, -1) = (x_2, y_2)
Since $x_1 = x_2 = 2$

∴ The slope of EF is not defined. ……[EF || y-axis]



(d) Let G = (7, 1) = (x_1, y_1) and H = (-3, 1) = (x_2, y_2) say.
Since $y_1 = y_2$
∴ The slope of GH = 0 …..[GH || x-axis]



Question 2.
If the X and Y-intercepts of line L are 2 and 3 respectively, then find the slope of line L.
Solution:
Given, x-intercept of line L is 2 and y-intercept of line L is 3
∴ the line L intersects X-axis at (2, 0) and Y-axis at (0, 3).
i.e. the line L passes through (2, 0) = (x_1, y_1) and (0, 3) = (x_2, y_2) say.
Slope of line L = $\frac{y_2-y_1}{x_2-x_1}$
= $\frac{3-0}{0-2}$
= $-\frac{3}{2}$

Question 3.
Find the slope of the line whose inclination is 30°.
Solution:
Given, inclination (θ) = 30°
Slope of the line = tan θ = tan 30° = $\frac{1}{\sqrt{3}}$

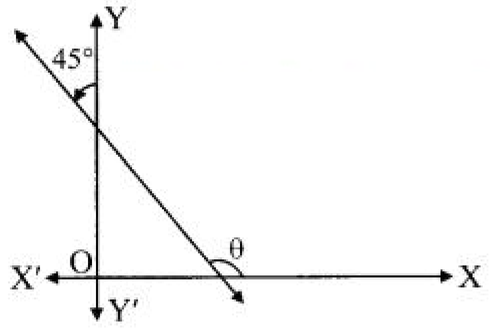
Question 4.
Find the slope of the line whose inclination is 45°.
Solution:
Given, inclination (θ) = 45°
Slope of the line = tan θ = tan 45° = 1

Question 5.
A line makes intercepts 3 and 3 on the co-ordinate axes. Find the slope of the line.
Solution:
Given, x-intercept of line is 3 and y-intercept of line is 3
∴ The line intersects X-axis at (3, 0) and Y-axis at (0, 3).
i.e. the line passes through (3, 0) = (x_1, y_1) and (0, 3) = (x_2, y_2) say.
Slope of line = $\frac{y_2-y_1}{x_2-x_1}$
= $\frac{3-0}{0-3}$
= -1

Question 6.
Without using Pythagoras theorem, show that points A(4, 4), B(3, 5) and C(-1, -1) are the vertices of a right-angled triangle.
Solution:
Given, A(4, 4) = (x_1, y_1), B(3, 5) = (x_2, y_2), C(-1, -1) = (x_3, y_3)
Slope of AB = $\frac{y_2-y_1}{x_2-x_1}=\frac{5-4}{3-4}=-1$
Slope of BC = $\frac{y_3-y_2}{x_3-x_2}=\frac{-1-5}{-1-3}=\frac{-6}{-4}=\frac{3}{2}$
Slope of AC = $\frac{y_3-y_1}{x_3-x_1}=\frac{-1-4}{-1-4}=\frac{-5}{-5}=1$
Slope of AB × slope of AC = -1 × 1 = -1
∴ side AB ⊥ side AC

∴ ΔABC is a right angled triangle, right angled at A.
∴ The given points are the vertices of a right angled triangle.

Question 7.
Find the slope of the line which makes angle of 45° with the positive direction of the Y-axis measured clockwise.
Solution:



Since, the line makes an angle of 45° with positive direction of Y-axis in anticlockwise direction.
∴ Inclination of the line (θ) = (90° + 45°)
∴ Slope of the line = tan(90° + 45°)
= -cot 45° …….[tan(90 + θ°) = -cot θ]
= -1

Question 8.
Find the value of k for which the points P(k, -1), Q(2, 1) and R(4, 5) are collinear.
Solution:
Given, points P(k, -1), Q(2, 1), and R(4, 5) are collinear.
∴ Slope of PQ = Slope of QR
∴ $\frac{1-(-1)}{2-k}=\frac{5-1}{4-2}$
∴ $\frac{2}{2-k}=\frac{4}{2}$
∴ 1 = 2 – k
∴ k = 2 – 1 = 1

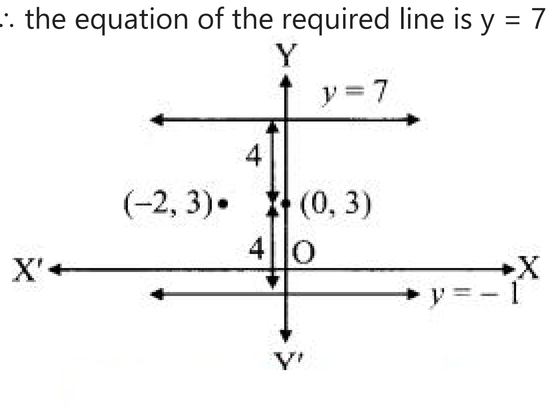
Check:
For collinear points P, Q, R,
Slope of PQ = Slope of QR = Slop of PR
For k = 1, if the given points are collinear, then our answer is correct.
P(1, -1), Q(2, 1) and R(4, 5)
Slope of PQ = $\frac{1-(-1)}{2-1}=\frac{2}{1}=2$
Slope of QR = $\frac{5-1}{4-2}=\frac{4}{2}=2$
Slope of PQ = Slope of QR
∴ The given points are collinear.
Thus, our answer is correct.

Maharashtra State Board 11th Commerce Maths Solutions Chapter 5 Locus and Straight Line Ex 5.3

Question 1.
Write the equation of the line:
(a) parallel to the X-axis and at a distance of 5 units from it and above it.
(b) parallel to the Y-axis and at a distance of 5 units from it and to the left of it.
(c) parallel to the X-axis and at a distance of 4 units from the point (-2, 3).
Solution:
(a) Equation of a line parallel to the X-axis is y = k.
Since the line is at a distance of 5 units above the X-axis.
∴ k = 5
∴ the equation of the required line is y = 5.

(b) Equation of a line parallel to the Y-axis is x = h.
Since the line is at a distance of 5 units to the left of the Y-axis.
∴ h = -5
∴ the equation of the required line is x = -5.

(c) Equation of a line parallel to the X-axis is of the form y = k (k > 0 or k < 0).
Since, the line is at a distance of 4 units from the point (-2, 3).
∴ k = 3 + 4 = 7 or k = 3 – 4 = -1
∴ the equation of the required line is y = 7 or y = -1.



Question 2.
Obtain the equation of the line:
(a) parallel to the X-axis and making an intercept of 3 units on the Y-axis.
(b) parallel to the Y-axis and making an intercept of 4 units on the X-axis.
Solution:
(a) Equation of a line parallel to X-axis with y-intercept 'k' is y = k.
Here, y-intercept = 3
∴ the equation of the required line is y = 3.

(b) Equation of a line parallel to Y-axis with x-intercept 'h' is x = h.
Here, x-intercept = 4
∴ the equation of the required line is x = 4.

Question 3.
Obtain the equation of the line containing the point:
(a) A(2, -3) and parallel to the Y-axis.
(b) B(4, -3) and parallel to the X-axis.
Solution:
(a) Equation of a line parallel to the Y-axis is of the form x = h.
Since, the line passes through A(2, -3).
∴ h = 2
∴ the equation of the required line is x = 2.

(b) Equation of a line parallel to the X-axis is of the form y = k.
Since, the line passes through B(4, -3)
∴ k = -3
∴ the equation of the required line is y = -3.

Question 4.
Find the equation of the line passing through the points A(2, 0) and B(3, 4).
Solution:
The required line passes through the points A(2, 0) = (x_1, y_1) and B(3, 4) = (x_2, y_2) say.
Equation of the line in two-point form is

$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$

∴ the equation of the required line is

∴ $\frac{y-0}{4-0}=\frac{x-2}{3-2}$

∴ $\frac{y}{4}=\frac{x-2}{1}$

∴ y = 4(x – 2)
∴ y = 4x – 8
∴ 4x – y – 8 = 0

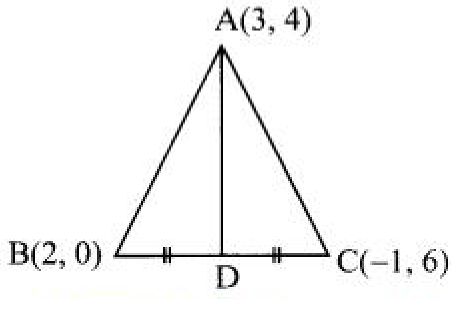
Check:
If the points A(2, 0) and B(3, 4) satisfy 4x – y – 8 = 0, then our answer is correct.
For point A(2, 0),
L.H.S. = 4x – y – 8
= 4(2) – 0 – 8
= 8 – 8

= 0
= R.H.S.
For point B(3, 4),
L.H.S. = 4x – y – 8
= 4(3) – 4 – 8
= 12 – 12
= 0
= R.H.S.
Thus, our answer is correct.

Question 5.
Line y = mx + c passes through the points A(2, 1) and B(3, 2). Determine m and c.
Solution:
Given, A(2, 1) and B(3, 2).
Equation of a line in two-point form is
$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$
∴ the equation of the passing through A and B line is
∴ $\frac{y-1}{2-1}=\frac{x-2}{3-2}$
∴ $\frac{y-1}{1}=\frac{x-2}{1}$
∴ y – 1 = x – 2
∴ y = x – 1
Comparing this equation with y = mx + c, we get
m = 1 and c = -1

Alternate method:
Points A(2, 1) and B(3, 2) lie on the line y = mx + c.
∴ They must satisfy the equation.
∴ 2m + c = 1 ……..(i)
and 3m + c = 2 ……(ii)
equation (ii) – equation (i) gives m = 1
Substituting m = 1 in (i), we get
2(1) – c = 1
∴ c = 1 – 2 = -1

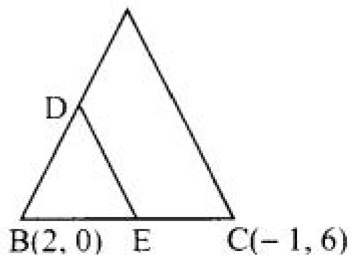
Question 6.
The vertices of a triangle are A(3, 4), B(2, 0), and C(-1, 6). Find the equations of
(a) side BC
(b) the median AD
(c) the midpoints of sides AB and BC.
Solution:
Vertices of ∆ABC are A(3, 4), B(2, 0) and C(-1, 6).
(a) Equation of a line in two-point form is
$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$
∴ the equation of the side BC is
$\frac{y-0}{6-0}=\frac{x-2}{-1-2}$ ……[B = (x_1, y_1) = (2, 0), C = (x_2, y_2) = (-1, 6)]
∴ $\frac{y}{6}=\frac{x-2}{-3}$
∴ y = -2(x – 2)
∴ 2x + y – 4 = 0

(b) Let D be the midpoint of side BC.
Then, AD is the median through A.
∴ D = $\left(\frac{2-1}{2},\frac{0+6}{2}\right)=\left(\frac{1}{2},3\right)$
The median AD passes through the points A(3, 4) and D($\frac{1}{2}$, 3)


∴ the equation of the median AD is
$\frac{y-4}{3-4}=\frac{x-3}{\frac{1}{2}-3}$

$\therefore \frac{y-4}{-1}=\frac{x-3}{-\frac{5}{2}}$
$\therefore \frac{1}{2}(y-4) = x-3$
$\therefore 5y-20 = 2x-6$
$\therefore 2x-5y+14 = 0$

(c) Let D and E be the midpoints of side AB and side BC respectively.
$\therefore$ D = $\left(\frac{3+2}{2}, \frac{4+0}{2}\right)=\left(\frac{5}{2}, 2\right)$ and
E = $\left(\frac{2-1}{2}, \frac{0+6}{2}\right)=\left(\frac{1}{2}, 3\right)$



the equation of the line DE is
$\frac{y-2}{3-2}=\frac{x-\frac{5}{2}}{\frac{1}{2}-\frac{5}{2}}$
$\therefore \frac{y-2}{1}=\frac{2x-5}{-4}$
$\therefore -4(y-2) = 2x-5$
$\therefore -4y+8 = 2x-5$
$\therefore 2x+4y-13 = 0$

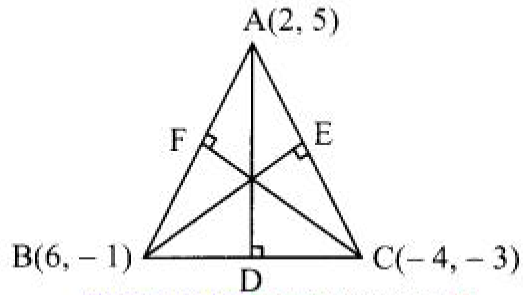
Question 7.
Find the x and y-intercepts of the following lines:
(a) $\frac{x}{3}+\frac{y}{2}=1$
(b) $\frac{3x}{2}+\frac{2y}{3}=1$
(c) $2x-3y+12 = 0$
Solution:
(a) Given equation of the line is $\frac{x}{3}+\frac{y}{2}=1$
This is of the form $\frac{x}{a}+\frac{y}{b}=1$,
where x-intercept = a, y-intercept = b
$\therefore$ x-intercept = 3, y-intercept = 2

(b) Given equation of the line is $\frac{3x}{2}+\frac{2y}{3}=1$
$\therefore \frac{x}{\left(\frac{2}{3}\right)}+\frac{y}{\left(\frac{3}{2}\right)}=1$
This is of the form $\frac{x}{a}+\frac{y}{b}=1$,
where x-intercept = a, y-intercept = b
$\therefore$ x-intercept = $\frac{2}{3}$ and y-intercept = $\frac{3}{2}$

(c) Given equation of the line is $2x-3y+12 = 0$
$\therefore 2x-3y = -12$
$\therefore \frac{2x}{(-12)}-\frac{3y}{(-12)}=1$
$\therefore \frac{x}{-6}+\frac{y}{4}=1$
This is of the form $\frac{x}{a}+\frac{y}{b}=1$,
where x-intercept = a, y-intercept = b
$\therefore$ x-intercept = -6 and y-intercept = 4

Question 8.
Find the equations of a line containing the point A(3, 4) and make equal intercepts on the co-ordinate axes.
Solution:
Let the equation of the line be
$\frac{x}{a}+\frac{y}{b}=1$ .....(i)
Since, the required line make equal intercepts on the co-ordinate axes.
$\therefore$ a = b
$\therefore$ (i) reduces to x + y = a .....(ii)
Since the line passes through A(3, 4).
$\therefore$ 3 + 4 = a
i.e. a = 7
Substituting a = 7 in (ii) to get
x + y = 7

Question 9.
Find the equations of the altitudes of the triangle whose vertices are A(2, 5), B(6, -1) and C(-4, -3).
Solution:



A(2, 5), B(6, -1), C(-4, -3) are the vertices of ΔABC.
Let AD, BE and CF be the altitudes through the vertices A, B and C respectively of ΔABC.
Slope of BC = $\frac{-3-(-1)}{-4-6}=\frac{-2}{-10}=\frac{1}{5}$
∴ slope of AD = -5 ……..[∵ AD ⊥ BC]
Since, altitude AD passes through (2, 5) and has slope -5.
∴ the equation of the altitude AD is
y – 5 = -5(x – 2)
∴ y – 5 = – 5x + 10
∴ 5x + y – 15 = 0
Now, slope of AC = $\frac{-3-5}{-4-2}=\frac{-8}{-6}=\frac{4}{3}$
∴ slope of BE = $\frac{-3}{4}$ …..[∵ BE ⊥ AC]
Since, altitude BE passes through (6, -1) and has slope $\frac{-3}{4}$.
∴ the equation of the altitude BE is
y – (-1) = $\frac{-3}{4}$(x – 6)
∴ 4(y + 1) = -3(x – 6)
∴ 3x + 4y – 14 = 0
Also, slope of AB = $\frac{-1-5}{6-2}=\frac{-6}{4}=\frac{-3}{2}$
∴ slope of CF = $\frac{2}{3}$ ………[∵ CF ⊥ AB]
Since, altitude CF passes through (-4, -3) and has slope $\frac{2}{3}$.
∴ the equation of the altitude CF is
y – (-3) = $\frac{2}{3}$ [x – (-4)]
∴ 3(y + 3) = 2(x + 4)
∴ 2x – 3y – 1 = 0

Maharashtra State Board 11th Commerce Maths Solutions Chapter 5 Locus and Straight Line Ex 5.4

Question 1.
Find the slope, x-intercept, y-intercept of each of the following lines.
(a) 2x + 3y – 6 = 0
(b) x + 2y = 0
Solution:
(a) Given equation of the line is 2x + 3y – 6 = 0
Comparing this equation with ax + by + c = 0, we get
a = 2, b = 3, c = -6
∴ Slope of the line = $\frac{-a}{b}=\frac{-2}{3}$

x-intercept = $\frac{-c}{a}=\frac{-(-6)}{2}$ = 3

y-intercept = $\frac{-c}{b}=\frac{-(-6)}{3}$ = 2

(b) Given equation of the line is $x + 2y = 0$
Comparing this equation with $ax + by + c = 0$, we get
$a = 1, b = 2, c = 0$

$\therefore$ Slope of the line = $-\frac{a}{b} = -\frac{1}{2}$

x-intercept = $-\frac{c}{a} = -\frac{0}{1} = 0$

y-intercept = $-\frac{c}{b} = -\frac{0}{2} = 0$

Question 2.
Write each of the following equations in $ax + by + c = 0$ form.
(a) $y = 2x – 4$
(b) $y = 4$
(c) $\frac{x}{2} + \frac{y}{4} = 1$
(d) $\frac{x}{3} = \frac{y}{2}$
Solution:
(a) $y = 2x – 4$
$\therefore$ $2x – y – 4 = 0$ is the equation in $ax + by + c = 0$ form.

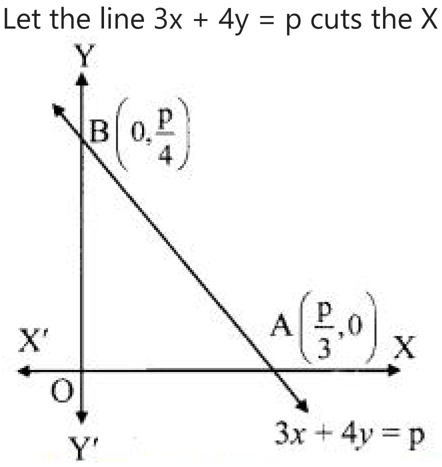
(b) $y = 4$
$\therefore$ $0x + 1y – 4 = 0$ is the equation in $ax + by + c = 0$ form.

(c) $\frac{x}{2} + \frac{y}{4} = 1$
$\therefore$ $\frac{2x + y}{4} = 1$
$\therefore$ $2x + y = 4$
$\therefore$ $2x + y – 4 = 0$ is the equation in $ax + by + c = 0$ form.

(d) $\frac{x}{3} = \frac{y}{2}$
$\therefore$ $2x = 3y$
$\therefore$ $2x – 3y + 0 = 0$ is the equation in $ax + by + c = 0$ form.

Question 3.
Show that the lines $x – 2y – 7 = 0$ and $2x – 4y + 5 = 0$ are parallel to each other.
Solution:
Let m_1 be the slope of the line $x – 2y – 7 = 0$.
$\therefore$ $m_1 = \frac{-1}{-2} = \frac{1}{2}$
Let m_2 be the slope of the line $2x – 4y + 5 = 0$.
$\therefore$ $m_2 = \frac{-2}{-4} = \frac{1}{2}$
Since, $m_1 = m_2$
$\therefore$ The given lines are parallel to each other.

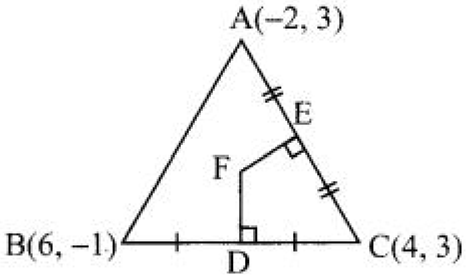
Question 4.
If the line $3x + 4y = p$ makes a triangle of area 24 square units with the co-ordinate axes, then find the value of p.
Solution:
Let the line $3x + 4y = p$ cuts the X and Y-axes at points A and B respectively.



$3x + 4y = p$
$\therefore$ $\frac{3x}{p} + \frac{4y}{p} = 1$
$\therefore$ $\frac{x}{\frac{p}{3}} + \frac{y}{\frac{p}{4}} = 1$
The equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, with $a = \frac{p}{3}$ and $b = \frac{p}{4}$
$\therefore$ A = (a, 0) = $(\frac{p}{3}, 0)$ and B = (0, b) = $(0, \frac{p}{4})$
$\therefore$ OA = $\frac{p}{3}$ and OB = $\frac{p}{4}$

Given, A(ΔOAB) = 24 sq. units
∴ 12 × OA × OB = 24
∴ 12 × p3 × p4 = 24
∴ p^2 = 576
∴ p = ±24

Question 5.
Find the co-ordinates of the circumcentre of the triangle whose vertices are A(-2, 3), B(6, -1), C(4, 3).
Solution:
Here, A(-2, 3), B(6, -1), C(4, 3) are the vertices of ΔABC.
Let F be the circumcentre of ΔABC.
Let FD and FE be the perpendicular bisectors of the sides BC and AC respectively.



∴ D and E are the midpoints of side BC and AC respectively.
∴ D = (6+42,-1+32) = (5, 1)
and E = (-2+42,3+32) = (1, 3)
Now, slope of BC = -1-36-4 = -2
∴ slope of FD = 12 …..[∵ FD ⊥ BC]
Since, FD passes through (5, 1) and has slope 12
∴ Equation of FD is y – 1 = 12(x – 5)
∴ 2(y – 1) = x – 5
∴ x – 2y – 3 = 0 ……(i)
Since, both the points A and C have same y co-ordinates i.e. 3
∴ the points A and C lie on the liney = 3.
Since, FE passes through E(1, 3).
∴ the equation of FE is x = 1. …….(ii)
To find co-ordinates of circumcentre, we have to solve equations (i) and (ii).
Substituting the value of x in (i), we get
1 – 2y – 3 = 0
∴ y = -1
∴ Co-ordinates of circumcentre F = (1, -1).

Question 6.
Find the equation of the line whose x-intercept is 3 and which is perpendicular to the line 3x – y + 23 = 0.
Solution:
Slope of the line 3x – y + 23 = 0 is 3.
∴ slope of the required line which is perpendicular to 3x – y + 23 = 0 is -13.
Since, the x-intercept of the required line is 3.
∴ it passes through (3, 0).
∴ the equation of the required line is
y – 0 = -13 (x – 3)
∴ 3y = -x – 3
∴ x – 3y = 3

Question 7.
Find the distance of the point A(-2, 3) from the line 12x – 5y – 13 = 0.
Solution:
Let p be the perpendicular distance of the point A(-2, 3) from the line 12x – 5y – 13 = 0
Here, a = 12, b = -5, c = -13, x_1 = -2, y_1 = 3

$$\therefore \quad p = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right| = \left|\frac{12(-2) - 5(3) - 13}{\sqrt{12^2 + (-5)^2}}\right|$$

$$= \left|\frac{-24 - 15 - 13}{\sqrt{144 + 25}}\right| = \left|\frac{-52}{13}\right| = 4 \text{ units}$$

Question 8.
Find the distance between parallel lines 9x + 6y – 1 = 0 and 9x + 6y – 32 = 0.
Solution:
Equations of the given parallel lines are 9x + 6y – 7 = 0 and 9x + 6y – 32 = 0.
Here, a = 9, b = 6, C_1 = -7 and C_2 = -32
∴ Distance between the parallel lines

$$= \left|\frac{C_1 - C_2}{\sqrt{a^2+b^2}}\right| = \left|\frac{-7-(-32)}{\sqrt{9^2+6^2}}\right| = \left|\frac{-7+32}{\sqrt{81+36}}\right|$$

$$= \left|\frac{25}{\sqrt{117}}\right| = \frac{25}{\sqrt{117}} \text{ units}$$

Question 9.
Find the equation of the line passing through the point of intersection of lines x + y – 2 = 0 and 2x – 3y + 4 = 0 and making intercept 3 on the X-axis.
Solution:
Given equations of lines are
x + y – 2 = 0 ……(i)
and 2x – 3y – 4 = 0 ……(ii)
Multiplying equation (i) by 3, we get
3x – 3y – 6 = 0 …..(iii)
Adding equation (ii) and (iii), we get
5x – 2 = 0
∴ x = 25
Substituting x = 25 in equation (i), we get
25 + y – 2 = 0
∴ y = 2 – 25 = 85
∴ The required line passes through point (25, 85).
Also, the line makes intercept of 3 on X-axis
∴ it also passes through point (3, 0).
∴ required equation of line passing through points (25, 85) and (3, 0) is

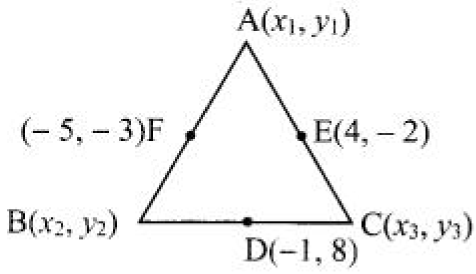
$$\frac{y-\frac{8}{5}}{0-\frac{8}{5}} = \frac{x-\frac{2}{5}}{3-\frac{2}{5}}$$

$$\frac{\frac{5y-8}{5}}{-\frac{8}{5}} = \frac{\frac{5x-2}{5}}{\frac{13}{5}}$$

$$\frac{5y-8}{-8} = \frac{5x-2}{13}$$

∴ 13(5y – 8) = -8(5x – 2)
∴ 65y – 104 = -40x + 16
∴ 40x + 65y – 120 = 0
∴ 8x + 13y – 24 = 0 which is the equation of the required line.

Question 10.
D(-1, 8), E(4, -2), F(-5, -3) are midpoints of sides BC, CA and AB of ΔABC. Find
(i) equations of sides of ΔABC.
(ii) co-ordinates of the circumcentre of ΔABC.
Solution:
(i) Let A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) be the vertices of ΔABC.
Given, points D, E and F are midpoints of sides BC, CA and AB respectively of ΔABC.



$$D = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

$\therefore \quad (-1, 8) = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$

$\therefore \quad x_2 + x_3 = -2 \quad \ldots(i)$

and $y_2 + y_3 = 16 \quad \ldots(ii)$

Also, $E = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$

$\therefore \quad (4, -2) = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$

$\therefore \quad x_1 + x_3 = 8 \quad \ldots(iii)$

and $y_1 + y_3 = -4 \quad \ldots(iv)$

Similarly, $F = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\therefore \quad (-5, -3) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\therefore \quad x_1 + x_2 = -10 \quad \ldots(v)$

and $y_1 + y_2 = -6 \quad \ldots(vi)$

For x-coordinates:
Adding (i), (iii) and (v), we get
$2x_1 + 2x_2 + 2x_3 = -4$
$\therefore x_1 + x_2 + x_3 = -2$ .....(vii)
Solving (i) and (vii), we get $x_1 = 0$
Solving (iii) and (vii), we get $x_2 = -10$
Solving (v) and (vii), we get $x_3 = 8$
For y-coordinates:
Adding (ii), (iv) and (vi), we get
$2y_1 + 2y_2 + 2y_3 = 6$
$\therefore y_1 + y_2 + y_3 = 3$ .....(viii)
Solving (ii) and (viii), we get $y_1 = -13$
Solving (iv) and (viii), we get $y_2 = 7$
Solving (vi) and (viii), we get $y_3 = 9$

∴ Vertices of ΔABC are A(0, -13), B(-10, 7), C(8, 9)

a. Equation of side AB is

$$\frac{y+13}{7+13} = \frac{x-0}{-10-0}$$

∴ $\frac{y+13}{20} = \frac{x}{-10}$

∴ $\frac{y+13}{2} = -x$

∴ $2x + y + 13 = 0$

b. Equation of side BC is

$$\frac{y-7}{9-7} = \frac{x+10}{8+10}$$

∴ $\frac{y-7}{2} = \frac{x+10}{18}$

∴ $y - 7 = \frac{x+10}{9}$

c. Equation of side AC is

$$\frac{y+13}{9+13} = \frac{x-0}{8-0}$$

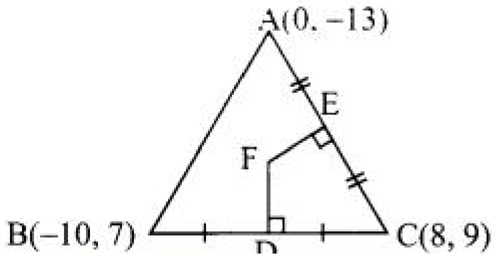
∴ $\frac{y+13}{22} = \frac{x}{8}$

∴ $8(y + 13) = 22x$

∴ $4(y + 13) = 11x$

∴ $11x - 4y - 52 = 0$

(ii) Here, A(0, -13), B(-10, 7), C(8, 9) are the vertices of ΔABC.
Let F be the circumcentre of ΔABC.
Let FD and FE be perpendicular bisectors of the sides BC and AC respectively.



∴ D and E are the midpoints of side BC and AC.
∴ D = (−10+82 · 7+92) = (-1, 8)
and E = (0+82,−13+92) = (4, -2)
Now, slope of BC = 7−9−10−8=19
∴ slope of FD = -9 ……[∵ FD ⊥ BC]
Since, FD passes through (-1, 8) and has slope -9
∴ Equation of FD is y – 8 = -9(x + 1)
∴ y – 8 = -9x – 9
∴ y = -9x – 1 …..(i)
Also, slope of AC = −13−90−8=114
∴ Slope of FE = −411 ….[∵ FE ⊥ AC]
Since, FE passes through (4, -2) and has slope −411
∴ Equation of FE is y + 2 = −411(x – 4)
∴ 11(y + 2) = -4(x – 4)
∴ 11y + 22 = -4x + 16
∴ 4x + 11y = -6 ….(ii)
To find co-ordinates of circumcentre. we have to solve equations (i) and (ii).
S
∴ 4x + 11(-9x – 1) = -6
∴ 4x – 99x – 11 = -6
∴ -95x = 5
∴ x = −119
Substituting the value of x in (i), we get
y = -9(−119) – 1 = −1019
∴ Co-ordinates of circumcentre F = (−119,−1019)

Maharashtra State Board 11th Commerce Maths Solutions Chapter 5 Locus and Straight Line Miscellaneous Exercise 5

Question 1.
Find the slopes of the lines passing through the following points:
(i) (1, 2), (3, -5)
(ii) (1, 3), (5, 2)
(iii) (-1, 3), (3, -1)
(iv) (2, -5), (3, -1)
Solution:
(i) Let A = (1, 2) = (x_1, y_1) and B = (3, -5) = (x_2, y_2) say.
Slope of line AB = $\frac{y_2-y_1}{x_2-x_1}=\frac{-5-2}{3-1}=\frac{-7}{2}$

(ii) Let C = (1, 3) = (x_1, y_1) and D = (5, 2) = (x_2, y_2) say.
Slope of line CD = $\frac{y_2-y_1}{x_2-x_1}=\frac{2-3}{5-1}=\frac{-1}{4}$

(iii) Let E = (-1, 3) = (x_1, y_1) and F = (3, -1) = (x_2, y_2) say.
Slope of line EF = $\frac{y_2-y_1}{x_2-x_1}=\frac{-1-3}{3-(-1)}=\frac{-4}{4}$ = -1

(iv) Let P = (2, -5) = (x_1, y_1) and Q = (3, -1) = (x_2, y_2) say.
Slope of line PQ = $\frac{y_2-y_1}{x_2-x_1}=\frac{-1-(-5)}{3-2}$ = $\frac{-1+5}{1}$ = 4

Question 2.
Find the slope of the line which
(i) makes an angle of 120° with the positive X-axis.
(ii) makes intercepts 3 and -4 on the axes.
(iii) passes through the points A(-2, 1) and the origin.
Solution:
(i) θ = 120°
Slope of the line = tan 120°
= tan (180° – 60°)
= -tan 60° …..[tan(180° – θ) = -tan θ]
= -√3

(ii) Given, x-intercept of line is 3 and y-intercept of line is -4
∴ The line intersects X-axis at (3, 0) and Y-axis at (0, -4).
∴ The line passes through (3, 0) = (x_1, y_1) and (0, -4) = (x_2, y_2) say.
∴ Slope of line = $\frac{y_2-y_1}{x_2-x_1}=\frac{-4-0}{0-3}=\frac{-4}{-3}=\frac{4}{3}$

(iii) Required line passes through O(0, 0) = (x_1, y_1) and A(-2, 1) = (x_2, y_2) say.
Slope of line OA = $\frac{y_2-y_1}{x_2-x_1}=\frac{1-0}{-2-0}=\frac{1}{-2}$ = $-\frac{1}{2}$

Question 3.
Find the value of k:
(i) if the slope of the line passing through the points (3, 4), (5, k) is 9.
(ii) the points (1, 3), (4, 1), (3, k) are collinear.
(iii) the point P(1, k) lies on the line passing through the points A(2, 2) and B(3, 3).
Solution:
(i) Let P(3, 4), Q(5, k).
Slope of PQ = 9 …….[Given]
∴ $\frac{k-4}{5-3}$ = 9
∴ $\frac{k-4}{2}$ = 9
∴ k – 4 = 18
∴ k = 22

(ii) The points A(1, 3), B(4, 1) and C(3, k) are collinear.
∴ Slope of AB = Slope of BC
∴ $\frac{1-3}{4-1}=\frac{k-1}{3-4}$
∴ $\frac{-2}{3}=\frac{k-1}{-1}$

∴ 2 = 3k – 3
∴ k = $\frac{5}{3}$

(iii) Given, point P(1, k) lies on the line joining A(2, 2) and B(3, 3).
∴ Slope of AB = Slope of BP
∴ $\frac{3-2}{3-2}=\frac{3-k}{3-1}$
∴ 1 = $\frac{3-k}{2}$
∴ 2 = 3 – k
∴ k = 1

Question 4.
Reduce the equation 6x + 3y + 8 = 0 into slope-intercept form. Hence, find its slope.
Solution:
Given equation is 6x + 3y + 8 = 0, which can be written as
3y = -6x – 8
∴ y = $\frac{-6x}{3}-\frac{8}{3}$
∴ y = -2x – $\frac{8}{3}$
This is of the form y = mx + c with m = -2
∴ y = -2x – $\frac{8}{3}$ is in slope-intercept form with slope = -2

Question 5.
Verify that A(2, 7) is not a point on the line x + 2y + 2 = 0.
Solution:
Given equation is x + 2y + 2 = 0.
Substituting x = 2 and y = 7 in L.H.S. of given equation, we get
L.H.S. = x + 2y + 2
= 2 + 2(7) + 2
= 2 + 14 + 2
= 18
≠ R.H.S.
∴ Point A does not lie on the given line.

Question 6.
Find the X-intercept of the line x + 2y – 1 = 0.
Solution:
Given equation of the line is x + 2y – 1 = 0
To find the x-intercept, put y = 0 in given equation of the line
∴ x + 2(0) – 1 = 0
∴ x + 0 – 1 = 0
∴ x = 1
∴ X-intercept of the given line is 1.
Alternate method:
Given equation of the line is x + 2y – 1 = 0
i.e. x + 2y = 1
∴ $\frac{x}{1}+\frac{y}{\frac{1}{2}}=1$
Comparing with $\frac{x}{a}+\frac{y}{b}=1$, we get a = 1
X-intercept of the line is 1.

Question 7.
Find the slope of the line y – x + 3 = 0.
Solution:
Equation of given line is y – x + 3 = 0
i.e. y = x – 3
Comparing with y = mx + c, we get
m = Slope = 1

Question 8.
Does point A(2, 3) lie on the line 3x + 2y – 6 = 0? Give reason.
Solution:
Given equation is 3x + 2y – 6 = 0.
Substituting x = 2 and y = 3 in L.H.S. of given equation, we get
L.H.S. = 3x + 2y – 6
= 3(2)+ 2(3) – 6
= 6

≠ R.H.S.
∴ Point A does not lie on the given line.

Question 9.
Which of the following lines passes through the origin?
(a) $x = 2$
(b) $y = 3$
(c) $y = x + 2$
(d) $2x – y = 0$
Solution:
Any line passing through origin is of the form $y = mx$ or $ax + by = 0$.
Here in the given option, $2x – y = 0$ is in the form $ax + by = 0$.

Question 10.
Obtain the equation of the line which is:
(i) parallel to the X-axis and 3 units below it.
(ii) parallel to the Y-axis and 2 units to the left of it.
(iii) parallel to the X-axis and making an intercept of 5 on the Y-axis.
(iv) parallel to the Y-axis and making an intercept of 3 on the X-axis.
Solution:
(i) Equation of a line parallel to X-axis is $y = k$.
Since, the line is at a distance of 3 units below X-axis.
∴ $k = -3$
∴ the equation of the required line is $y = -3$
i.e., $y + 3 = 0$.

(ii) Equation of a line parallel to Y-axis is $x = h$.
Since, the line is at a distance of 2 units to the left of Y-axis.
∴ $h = -2$
∴ the equation of the required line is $x = -2$
i.e., $x + 2 = 0$.

(iii) Equation of a line parallel to X-axis with y-intercept ‘k’ is $y = k$.
Here, y-intercept = 5
∴ the equation of the required line is $y = 5$.

(iv) Equation of a line parallel to Y-axis with x-intercept ‘h’ is $x = h$.
Here, x-intercept = 3
∴ the equation of the required line is $x = 3$.

Question 11.
Obtain the equation of the line containing the point:
(i) (2, 3) and parallel to the X-axis.
(ii) (2, 4) and perpendicular to the Y-axis.
(iii) (2, 5) and perpendicular to the X-axis.
Solution:
(i) Equation of a line parallel to X-axis is of the form $y = k$.
Since, the line passes through (2, 3).
∴ $k = 3$
∴ the equation of the required line is $y = 3$.

(ii) Equation of a line perpendicular to Y-axis
i.e., parallel to X-axis, is of the form $y = k$.
Since, the line passes through (2, 4).
∴ $k = 4$
∴ the equation of the required line is $y = 4$.

(iii) Equation of a line perpendicular to X-axis
i.e., parallel to Y-axis, is of the form $x = h$.
Since, the line passes through (2, 5).
∴ $h = 2$
∴ the equation of the required line is $x = 2$.

Question 12.
Find the equation of the line:
(i) having slope 5 and containing point A(-1, 2).
(ii) containing the point (2, 1) and having slope 13.
(iii) containing the point T(7, 3) and having inclination 90°.

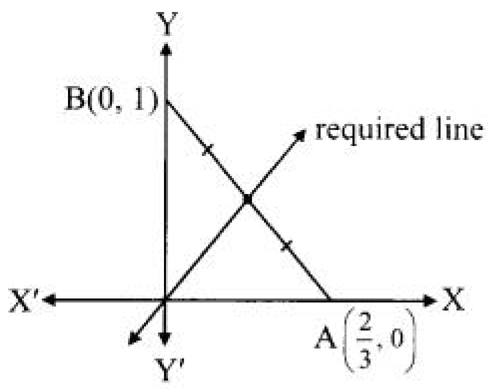
(iv) containing the origin and having inclination 90°.
(v) through the origin which bisects the portion of the line 3x + 2y = 2 intercepted between the co-ordinate axes.
Solution:
(i) Given, slope (m) = 5 and the line passes through A(-1, 2).
Equation of the line in slope point form is $y - y_1 = m(x - x_1)$
∴ the equation of the required line is y – 2 = 5(x + 1)
∴ y – 2 = 5x + 5
∴ 5x – y + 7 = 0

(ii) Given, slope (m) = 13 and the line passes through (2, 1).
Equation of the line in slope point form is $y - y_1 = m(x - x_1)$
∴ the equation of the required line is y – 1 = 13(x – 2)
∴ y – 1 = 13x – 26
∴ 13x – y = 25.

(iii) Given, Inclination of line = θ = 90°
∴ the required line is parallel to Y-axis (or lies on the Y-axis.)
Equation of a line parallel to Y-axis is of the form x = h.
Since, the line passes through (7, 3).
∴ h = 7
∴ the equation of the required line is x = 7.

(iv) Given, Inclination of line = θ = 90°
∴ the required line is parallel to Y-axis (or lies on the Y-axis.)
Equation of a line parallel to Y-axis is of the form x = h.
Since, the line passes through origin (0, 0).
∴ h = 0
∴ the equation of the required line is x = 0.

(v) Given equation of the line is 3x + 2y = 2.
∴ 3x2+2y2=1
∴ x23+y1=1



This equation is of the form xa+y b=1, with
a = 23, b = 1
∴ the line 3x + 2y = 2 intersects the X-axis at A(23, 0) and Y-axis at B(0, 1).
Required line is passing through the midpoint of AB.
∴ Midpoint of AB = (23+02,0+12)=(13,12)
∴ Required line passes through (0, 0) and (13,12).
Equation of the line in two point form is
y−y1y2−y1=x−x1x2−x1
∴ the equation of the required line is
y−012−0=x−013−0
∴ 2y = 3x
∴ 3x – 2y = 0

Question 13.
Find the equation of the line passing through the points A(-3, 0) and B(0, 4).
Solution:
Since, the required line passes through the points A(-3, 0) and B(0, 4).
Equation of the line in two point form is
y−y1y2−y1=x−x1x2−x1
Here, (x_1, y_1) = (-3, 0) and (x_2, y_2) = (0, 4)
∴ the equation of the required line is
∴ y−04−0=x−(−3)0−(−3)
∴ y4=x+33

$\therefore 4x + 12 = 3y$
$\therefore 4x - 3y + 12 = 0$

Question 14.
Find the equation of the line:
(i) having slope 5 and making intercept 5 on the X-axis.
(ii) having an inclination 60° and making intercept 4 on the Y-axis.
Solution:
(i) Since, the x-intercept of the required line is 5.
$\therefore$ it passes through (5, 0).
Also, slope(m) of the line is 5
Equation of the line in slope point form is $y - y_1 = m(x - x_1)$
$\therefore$ the equation of the required line is $y - 0 = 5(x - 5)$
$\therefore y = 5x - 25$
$\therefore 5x - y - 25 = 0$

(ii) Given, Inclination of line = $\theta = 60°$
$\therefore$ Slope of the line (m) = $\tan\theta$
$= \tan 60°$
$= \sqrt{3}$
and the y-intercept of the required line is 4.
$\therefore$ it passes through (0, 4).
Equation of the line in slope point form is $y - y_1 = m(x - x_1)$
$\therefore$ the equation of the required line is $y - 4 = \sqrt{3}(x - 0)$
$\therefore y - 4 = \sqrt{3}x$
$\therefore \sqrt{3}x - y + 4 = 0$

Question 15.
The vertices of a triangle are A(1, 4), B(2, 3), and C(1, 6). Find equations of
(i) the sides
(ii) the medians
(iii) Perpendicular bisectors of sides
(iv) altitudes of ΔABC
Solution:
Vertices of ΔABC are A(1, 4), B(2, 3), and C(1, 6)
(i) Equation of the line in two-point form is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Equation of side AB is

$$\frac{y-4}{3-4} = \frac{x-1}{2-1}$$

$$\therefore \quad \frac{y-4}{-1} = \frac{x-1}{1}$$

$$\therefore \quad y - 4 = -1(x - 1)$$

$$\therefore \quad x + y = 5$$

Equation of side BC is

$$\frac{y-3}{6-3} = \frac{x-2}{1-2}$$

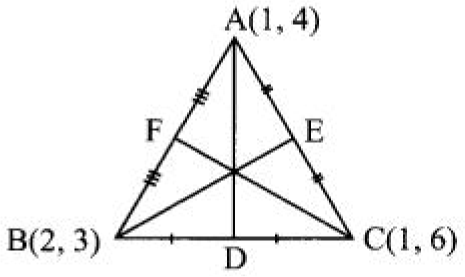
$$\therefore \quad \frac{y-3}{3} = \frac{x-2}{-1}$$

$$\therefore \quad -1(y - 3) = 3(x - 2)$$

$$\therefore \quad 3x + y = 9$$

Since, both the points A and C have same x co-ordinates i.e. 1
$\therefore$ the points A and C lie on a line parallel to Y-axis.
$\therefore$ the equation of side AC is $x = 1$.

(ii) Let D, E, and F be the midpoints of sides BC, AC, and AB respectively of ΔABC.



Then $D = \left(\frac{2+1}{2}, \frac{3+6}{2}\right) = \left(\frac{3}{2}, \frac{9}{2}\right)$

$E = \left(\frac{1+1}{2}, \frac{6+4}{2}\right) = (1, 5)$

$F = \left(\frac{1+2}{2}, \frac{4+3}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

Equation of median AD is

$$\frac{y-4}{\frac{9}{2}-4} = \frac{x-1}{\frac{3}{2}-1}$$

$\therefore \quad \frac{y-4}{\frac{1}{2}} = \frac{x-1}{\frac{1}{2}}$

$\therefore \quad y-4 = x-1$

$\therefore \quad x-y+3=0$

Equation of median BE is

$$\frac{y-3}{5-3} = \frac{x-2}{1-2}$$

$\therefore \quad \frac{y-3}{2} = \frac{x-2}{-1}$

$\therefore \quad -1(y-3) = 2(x-2)$

$\therefore \quad -y+3 = 2x-4$

$\therefore \quad 2x+y=7$

Equation of median CF is

$$\frac{y-6}{\frac{7}{2}-6} = \frac{x-1}{\frac{3}{2}-1}$$

$\therefore \quad \frac{y-6}{-\frac{5}{2}} = \frac{x-1}{\frac{1}{2}}$

$\therefore \quad \frac{y-6}{-5} = \frac{x-1}{1}$

$\therefore \quad y-6 = -5(x-1)$

$\therefore \quad y-6 = -5x+5$

$\therefore \quad 5x+y-11=0$

(iii) Slope of side BC = (6-31-2)=(3-1) = -3

∴ Slope of perpendicular bisector of BC is 13 and the line passes through (32,92)

∴ Equation of the perpendicular bisector of side BC is (y−92)=13(x−32)

∴ 2y-92=13(2x-32)

∴ 3(2y – 9) = (2x – 3)

∴ 2x – 6y + 24 = 0

∴ x – 3y + 12 = 0

Since, both the points A and C have the same x co-ordinates i.e. 1

∴ the points A and C lie on the line x = 1.

AC is parallel to Y-axis and therefore, the perpendicular bisector of side AC is parallel to X-axis.

Since, the perpendicular bisector of side AC passes through E(1, 5).

∴ the equation of the perpendicular bisector of side AC is y = 5.

Slope of side AB = (3-42-1) = -1

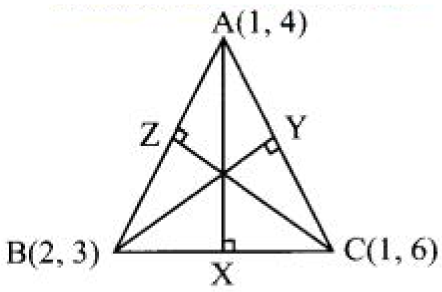
∴ Slope of perpendicular bisector of AB is 1 and the line passes through (32,72).

∴ Equation of the perpendicular bisector of side AB is (y−72)=1(x−32)

∴ 2y-72=2x-32

∴ 2y – 7 = 2x – 3
∴ 2x – 2y + 4 = 0
∴ x – y + 2 = 0

(iv) Let AX, BY and CZ be the altitudes through the vertices A, B, and C respectively of ΔABC.



Slope of BC = -3
∴ Slope of AX = $\frac{1}{3}$ .....[∵ AX ⊥ BC]
Since, altitude AX passes through (1, 4) and has slope $\frac{1}{3}$
∴ equation of altitude AX is y – 4 = $\frac{1}{3}$(x – 1)
∴ 3y – 12 = x – 1
∴ x – 3y + 11 = 0
Since, both the points A and C have the same x co-ordinates i.e. 1
∴ the points A and C lie on the line x = 1.
AC is parallel to Y-axis and therefore, altitude BY is parallel to X-axis.
Since, the altitude BY passes through B(2, 3).
∴ the equation of altitude BY is y = 3.
Also, slope of AB = -1
∴ Slope of CZ = 1 .....[∵ CZ ⊥ AB]
Since, altitude CZ passes through (1, 6) and has slope 1
∴ equation of altitude CZ is y – 6 = 1(x – 1)
∴ y – 6 = x – 1
∴ x – y + 5 = 0