Maharashtra State Board 12th Maths Solutions Chapter 6 Line and Plane Ex 6.1

Question 1.
Find the vector equation of the line passing through the point having position vector $-2\hat{i}+\hat{j}+\hat{k}$ and parallel to vector $4\hat{i}-\hat{j}+2\hat{k}$.
Solution:
The vector equation of the line passing through A ($\bar{a}$) and parallel to the vector $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$, where λ is a scalar.
∴ the vector equation of the line passing through the point having position vector $-2\hat{i}+\hat{j}+\hat{k}$ and parallel to the vector $4\hat{i}-\hat{j}+2\hat{k}$ is
$\bar{r}=(-2\hat{i}+\hat{j}+\hat{k})+\lambda(4\hat{i}-\hat{j}+2\hat{k})$.

Question 2.
Find the vector equation of the line passing through points having position vectors $3\hat{i}+4\hat{j}-7\hat{k}$ and $6\hat{i}-\hat{j}+\hat{k}$.
Solution:
The vector equation of the line passing through the A ($\bar{a}$) and B($\bar{b}$) is $\bar{r}=\bar{a}+\lambda(\bar{b}-\bar{a})$, λ is a scalar
∴ the vector equation of the line passing through the points having position vectors $3\hat{i}+4\hat{j}-7\hat{k}$ and $6\hat{i}-\hat{j}+\hat{k}$ is
is $\bar{r}=(3\hat{i}+4\hat{j}-7\hat{k})+\lambda[(6\hat{i}-\hat{j}+\hat{k})-(3\hat{i}+4\hat{j}-7\hat{k})]$
i.e. $\bar{r}=(3\hat{i}+4\hat{j}-7\hat{k})+\lambda(3\hat{i}-5\hat{j}+8\hat{k})$.

Question 3.
Find the vector equation of line passing through the point having position vector $5\hat{i}+4\hat{j}+3\hat{k}$ and having direction ratios -3, 4, 2.
Solution:
Let A be the point whose position vector is $\bar{a}=5\hat{i}+4\hat{j}+3\hat{k}$.
Let $\bar{b}$ be the vector parallel to the line having direction ratios -3, 4, 2
Then, $\bar{b} = -3\hat{i}+4\hat{j}+2\hat{k}$
The vector equation of the line passing through A ($\bar{a}$) and parallel to $\bar{b}$ is $\bar{r}=\bar{a}+\lambda\bar{b}$, where λ is a scalar.
∴ the required vector equation of the line is
$\bar{r}=5\hat{i}+4\hat{j}+3\hat{k}+\lambda(-3\hat{i}+4\hat{j}+2\hat{k})$.

Question 4.
Find the vector equation of the line passing through the point having position vector $\hat{i}+2\hat{j}+3\hat{k}$ and perpendicular to vectors $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-\hat{j}+\hat{k}$.
Solution:
Let $\bar{b}=\hat{i}+\hat{j}+\hat{k}$ and $\bar{c}=2\hat{i}-\hat{j}+\hat{k}$
The vector perpendicular to the vectors $\bar{b}$ and $\bar{c}$ is given by

$$\bar{b}\times\bar{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 2 & -1 & 1\end{vmatrix}$$

$$=\hat{i}(1+1)-\hat{j}(1-2)+\hat{k}(-1-2)$$
$$=2\hat{i}+\hat{j}-3\hat{k}$$

Since the line is perpendicular to the vector $\bar{b}$ and $\bar{c}$, it is parallel to $\bar{b}\times\bar{c}$. The vector equation of the line passing through A ($\bar{a}$) and parallel to $\bar{b}\times\bar{c}$ is
$\bar{r}=\bar{a}+\lambda(\bar{b}\times\bar{c})$, where λ is a scalar.
Here, $\bar{a} = \hat{i}+2\hat{j}+3\hat{k}$
Hence, the vector equation of the required line is
$\bar{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+\hat{j}-3\hat{k})$

Question 5.
Find the vector equation of the line passing through the point having position vector $-\hat{i}-\hat{j}+2\hat{k}$ and parallel to the

line $\bar{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(3\hat{i}+2\hat{j}+\hat{k})$.

Solution:

Let A be point having position vector $\bar{a} = -\hat{i}-\hat{j}+2\hat{k}$

The required line is parallel to the line

$$\bar{r} = (\hat{i}+2\hat{j}+3\hat{k} + \lambda(3\hat{i}+2\hat{j}+\hat{k})$$

$\therefore$ it is parallel to the vector

$$\bar{b} = 3\hat{i}+2\hat{j}+\hat{k}$$

The vector equation of the line passing through A($\bar{a}$) and parallel to $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$ where λ is a scalar.

∴ the required vector equation of the line is

$\bar{r} = (-\hat{i}-\hat{j}+2\hat{k}) + \lambda(3\hat{i}+2\hat{j}+\hat{k})$.

Question 6.

Find the Cartesian equations of the line passing through A(-1, 2, 1) and having direction ratios 2, 3, 1.

Solution:

The cartesian equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

∴ the cartesian equations of the line passing through the point (-1, 2, 1) and having direction ratios 2, 3, 1 are

$$\frac{x-(-1)}{2} = \frac{y-2}{3} = \frac{z-1}{1}$$

i.e. $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{1}$.

Question 7.

Find the Cartesian equations of the line passing through A(2, 2, 1) and B(1, 3, 0).

Solution:

The cartesian equations of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Here, (x_1, y_1, z_1) = (2, 2, 1) and (x_2, y_2, z_2) = (1, 3, 0)

∴ the required cartesian equations are

$$\frac{x-2}{1-2} = \frac{y-2}{3-2} = \frac{z-1}{0-1}$$

i.e. $\frac{x-2}{-1} = \frac{y-2}{1} = \frac{z-1}{-1}$.

Question 8.

A(-2, 3, 4), B(1, 1, 2) and C(4, -1, 0) are three points. Find the Cartesian equations of the line AB and show that points A, B, C are collinear.

Solution:

We find the cartesian equations of the line AB. The cartesian equations of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

Here, (x_1, y_1, z_1) = (-2, 3, 4) and (x_2, y_2, z_2) = (4, -1, 0)

∴ the required cartesian equations of the line AB are

$$\frac{x-(-2)}{4-(-2)} = \frac{y-3}{-1-3} = \frac{z-4}{0-4}$$

$$\therefore \frac{x+2}{6} = \frac{y-3}{-4} = \frac{z-4}{-4}$$

$$\therefore \frac{x+2}{3} = \frac{y-3}{-2} = \frac{z-4}{-2}$$

$$C = (4, -1, 0)$$

For $x = 4, \frac{x+2}{3} = \frac{4+2}{3} = 2$

For $y = -1, \frac{y-3}{-2} = \frac{-1-3}{-2} = 2$

For $z = 0, \frac{z-4}{-2} = \frac{0-4}{-2} = 2$

∴ coordinates of C satisfy the equations of the line AB.
∴ C lies on the line passing through A and B.
Hence, A, B, C are collinear.

Question 9.

Show that lines $x+1-10=y+3-1=z-41$ and $x+10-1=y+1-3=z-14$ intersect each other. Find the co-ordinates of their point of intersection.

Solution:
The equations of the lines are

$$\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1} = \lambda \qquad \text{... (Say) ... (1)}$$

$$\text{and } \frac{x+10}{-1} = \frac{y+1}{-3} = \frac{z-1}{4} = \mu \qquad \text{... (Say) ... (2)}$$

From (1), x = -1 -10λ, y = -3 – 2, z = 4 + λ
∴ the coordinates of any point on the line (1) are
(-1 – 10λ, – 3 – λ, 4 + λ)
From (2), x = -10 – u, y = -1 – 3u, z = 1 + 4u
∴ the coordinates of any point on the line (2) are
(-10 – u, -1 – 3u, 1 + 4u)
Lines (1) and (2) intersect, if
(- 1 – 10λ, – 3 – λ, 4 + 2) = (- 10 – u, -1 – 3u, 1 + 4u)
∴ the equations -1 – 10λ = -10 – u, -3 – 2= – 1 – 3u
and 4 + λ = 1 + 4u are simultaneously true.
Solving the first two equations, we get, λ = 1 and u = 1. These values of λ and u satisfy the third equation also.
∴ the lines intersect.
Putting λ = 1 in (-1 – 10λ, -3 – 2, 4 + 2) or u = 1 in (-10 – u, -1 – 3u, 1 + 4u), we get
the point of intersection (-11, -4, 5).

Question 10.
A line passes through (3, -1, 2) and is perpendicular to
lines $\bar{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\bar{r}=(2\hat{i}+\hat{j}-3\hat{k})+\mu(\hat{i}-2\hat{j}+2\hat{k})$. Find its equation.
Solution:

The line $\bar{r} = (\hat{i}+\hat{j}-\hat{k}) + \lambda(2\hat{i}-2\hat{j}+\hat{k})$ is parallel to the vector $\bar{b} = 2\hat{i}-2\hat{j}+\hat{k}$ and the line $\bar{r} = (2\hat{i}+\hat{j}-3\hat{k}) + \mu(\hat{i}-2\hat{j}+2\hat{k})$ is parallel to the vector $\bar{c} = \hat{i}-2\hat{j}+2\hat{k}$.

The vector perpendicular to the vectors $\bar{b}$ and $\bar{c}$ is given by

$$\bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -2 & 2 \end{vmatrix}$$

$$= \hat{i}(-4+2) - \hat{j}(4-1) + \hat{k}(-4+2)$$

$$= -2\hat{i} - 3\hat{j} - 2\hat{k}$$

Since the required line is perpendicular to the given lines, it is perpendicular to both $\bar{b}$ and $\bar{c}$.

∴ it is parallel to $\bar{b}\times\bar{c}$

The equation of the line passing through A($\bar{a}$) and parallel to $\bar{b}\times\bar{c}$ is

$\bar{r}=\bar{a}+\lambda(\bar{b}\times\bar{c})$, where λ is a scalar.

Here, $\bar{a} = 3\hat{i}-\hat{j}+2\hat{k}$

∴ the equation of the required line is

$$\bar{r} = (3\hat{i} - \hat{j} + 2\hat{k}) + \lambda(-2\hat{i} - 3\hat{j} - 2\hat{k}) \quad \text{or}$$

$$\bar{r} = (3\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 2\hat{k}), \text{ where } \mu = -\lambda.$$

Question 11.

Show that the line $x-21=y-42=z+4-2$ passes through the origin.

Solution:

The equation of the line is

$x-21=y-42=z+4-2$

The coordinates of the origin O are (0, 0, 0)

$$\text{For } x = 0, \frac{x-2}{1} = \frac{0-2}{1} = -2$$

$$\text{For } y = 0, \frac{y-4}{2} = \frac{0-4}{2} = -2$$

$$\text{For } z = 0, \frac{z+4}{-2} = \frac{0+4}{-2} = -2$$

∴ coordinates of the origin O satisfy the equation of the line.
Hence, the line passes through the origin.

Maharashtra State Board 12th Maths Solutions Chapter 6 Line and Plane Ex 6.2

Question 1.

Find the length of the perpendicular from (2, -3, 1) to the line $x+12=y-33=z+1-1$

Solution:

Let PM be the perpendicular drawn from the point P (2, -3, 1) to the line $x+12=y-33=z+1-1$ = λ ...(Say)

The coordinates of any point on the line are given by x = -1 + 2λ, y = 3 + 3λ, z = -1 – λ
Let the coordinates of M be
(-1 + 2λ, 3 + 3λ, -1 – λ) … (1)
The direction ratios of PM are
-1 + 2λ – 2, 3 + 3λ + 3, -1 – λ – 1
i.e. 2λ – 3, 3λ + 6, -λ – 2
The direction ratios of the given line are 2, 3, -1.
Since PM is perpendicular to the given line, we get
2(2λ – 3) + 3(3λ + 6) – 1(-λ – 2) = 0
∴ 4λ – 6 + 9λ + 18 + λ + 2 = 0
∴ 14λ + 14 = 0 ∴ λ = -1.
Put λ = -1 in (1), the coordinates of M are
(-1 – 2, 3 – 3, -1 + 1) i.e. (-3, 0,0).
∴ length of perpendicular from P to the given line

$$= PM = \sqrt{(-3-2)^2 + (0+3)^2 + (0-1)^2}$$
$$= \sqrt{25+9+1}$$
$$= \sqrt{35} \text{ units.}$$

Alternative Method:
We know that the perpendicular distance from the point P$|\bar{\alpha}|$ to the line $\bar{r} = \bar{a} + \lambda\bar{b}$ is given by

$$\sqrt{|\bar{\alpha}-\bar{a}|^2 - \left[\frac{(\bar{\alpha}-\bar{a})\cdot\bar{b}}{|\bar{b}|}\right]^2} \quad \ldots (1)$$

Here, $\bar{\alpha} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\bar{a} = -\hat{i} + 3\hat{j} - \hat{k}$, $\bar{b} = 2\hat{i} + 3\hat{j} - \hat{k}$

$$\therefore \quad \bar{\alpha} - \bar{a} = (2\hat{i} - 3\hat{j} + \hat{k}) - (-\hat{i} + 3\hat{j} - \hat{k})$$
$$= 3\hat{i} - 6\hat{j} + 2\hat{k}$$

$$\therefore \quad |\bar{\alpha} - \bar{a}|^2 = 3^2 + (-6)^2 + 2^2 = 9 + 36 + 4 = 49$$

Also, $(\bar{\alpha} - \bar{a})\cdot\bar{b} = (3\hat{i} - 6\hat{j} + 2\hat{k})\cdot(2\hat{i} + 3\hat{j} - \hat{k})$
$$= (3)(2) + (-6)(3) + (2)(-1)$$
$$= 6 - 18 - 2 = -14$$

$$|\bar{b}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$$

Substituting these values in (1), we get
length of perpendicular from P to given line

$$= PM = \sqrt{49 - \left(\frac{-14}{\sqrt{14}}\right)^2}$$

$$= \sqrt{49-14} = \sqrt{35} \text{ units.}$$

Question 2.
Find the co-ordinates of the foot of the perpendicular drawn from the point $2\hat{i} - \hat{j} + 5\hat{k}$ to the line $\bar{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$. Also find the length of the perpendicular.
Solution:
Let M be the foot of perpendicular drawn from the point P ($2\hat{i} - \hat{j} + 5\hat{k}$) on the line
$\bar{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$.
Let the position vector of the point M be

$$(11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$$
$$= (11 + 10\lambda)\hat{i} + (-2 - 4\lambda)\hat{j} + (-8 - 11\lambda)\hat{k}.$$

Then $\overline{PM}$ = Position vector of M – Position vector of P
= $[(11 + 10\lambda)\hat{i} + (-2 - 4\lambda)\hat{j} + -8 - 11\lambda)\hat{k}] - (2\hat{i} - \hat{j} + 5\hat{k})$
= $(9 + 10\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (-13 - 11\lambda)\hat{k}$

Since PM is perpendicular to the given line which is parallel to $\bar{b} = 10\hat{i} - 4\hat{j} - 11\hat{k}$,
$\overline{PM} \perp_r \bar{b}$ $\therefore$ $\overline{PM} \cdot \bar{b} = 0$
$\therefore$ $[(9 + 10\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (-13 - 11\lambda)\hat{k}]\cdot(10\hat{i} - 4\hat{j} - 11\hat{k}) = 0$
$\therefore$ $10(9 + 10\lambda) - 4(-1 - 4\lambda) - 11(-13 - 11\lambda) = 0$
$\therefore$ $90 + 100\lambda + 4 + 16\lambda + 143 + 121\lambda = 0$
$\therefore$ $237\lambda + 237 = 0$
$\therefore$ $\lambda = -1$
Putting this value of λ, we get the position vector of M as $\hat{i} + 2\hat{j} + 3\hat{k}$.
$\therefore$ coordinates of the foot of perpendicular M are (1, 2, 3).

Now, $\overline{PM} = (\hat{i} + 2\hat{j} + 3\hat{k}) - (2\hat{i} - \hat{j} + 5\hat{k})$
$$= -\hat{i} + 3\hat{j} - 2\hat{k}$$
$$\therefore \quad |\overline{PM}| = \sqrt{(-1)^2 + (3)^2 + (-2)^2}$$
$$= \sqrt{1+9+4} = \sqrt{14}$$

Hence, the coordinates of the foot of perpendicular are (1,2, 3) and length of perpendicular = $\sqrt{14}$ units.

Question 3.
Find the shortest distance between the lines $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(\hat{i} + 4\hat{j} - 5\hat{k})$
Solution:
We know that the shortest distance between the skew lines $\vec{r} = \overline{a_1} + \lambda\overline{b_1}$ and $\vec{r} = \overline{a_2} + \mu\overline{b_2}$ is given by

$$d = \left|\frac{(\overline{a_2} - \overline{a_1}) \cdot (\overline{b_1} \times \overline{b_2})}{|\overline{b_1} \times \overline{b_2}|}\right|.$$

Here, $\overline{a_1} = 4\hat{i} - \hat{j}$, $\overline{a_2} = \hat{i} - \hat{j} + 2\hat{k}$,

$\overline{b_1} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\overline{b_2} = \hat{i} + 4\hat{j} - 5\hat{k}$.

$$\therefore\ \overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}$$

$$= (-10 + 12)\hat{i} - (-5 + 3)\hat{j} + (4 - 2)\hat{k}$$

$$= 2\hat{i} + 2\hat{j} + 2\hat{k}$$

and $\overline{a_2} - \overline{a_1} = (\hat{i} - \hat{j} + 2\hat{k}) - (4\hat{i} - \hat{j})$

$$= -3\hat{i} + 2\hat{k}$$

$$\therefore\ (\overline{a_2} - \overline{a_1}) \cdot (\overline{b_1} \times \overline{b_2}) = (-3\hat{i} + 2\hat{k}) \cdot (2\hat{i} + 2\hat{j} + 2\hat{k})$$

$$= -3(2) + 0(2) + 2(2)$$

$$= -6 + 0 + 4 = -2$$

and $|\overline{b_1} \times \overline{b_2}| = \sqrt{2^2 + 2^2 + 2^2}$

$$= \sqrt{4 + 4 + 4} = 2\sqrt{3}$$

$\therefore$ required shortest distance between the given lines

$$= \left|\frac{-2}{2\sqrt{3}}\right| = \frac{1}{\sqrt{3}} \text{ units.}$$

Question 4.
Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$
Solution:
The shortest distance between the lines

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1} \quad \text{and} \quad \frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2} \quad \text{is}$$

given by

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2 - m_2n_1)^2 + (l_2n_1 - l_1n_2)^2 + (l_1m_2 - l_2m_1)^2}}$$

The equations of the given lines are

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \quad \text{and} \quad \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

$\therefore\ x_1 = -1, y_1 = -1, z_1 = -1, x_2 = 3, y_2 = 5, z_2 = 7,$

$l_1 = 7, m_1 = -6, n_1 = 1, l_2 = 1, m_2 = -2, n_2 = 1$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

= 4(-6 + 2) – 6(7 – 1) + 8(-14 + 6)
= -16 – 36 – 64 = -116
and $(m_1n_2 - m_2n_1)^2 + (l_2n_1 - l_1n_2)^2 + (l_1m_2 - l_2m_1)^2$
$= (-6 + 2)^2 + (1 - 7)^2 + (-14 + 6)^2$
= 16 + 36 + 64 = 116
Hence, the required shortest distance between the given lines = $\left|\frac{-116}{\sqrt{116}}\right| = \sqrt{116} = 2\sqrt{29}$ units

Question 5.
Find the perpendicular distance of the point (1, 0, 0) from the line x−12=y+1−3=z+108 Also find the co-ordinates of the foot of the perpendicular.
Solution:
Let PM be the perpendicular drawn from the point (1, 0, 0) to the line x−12=y+1−3=z+108 = λ ...(Say)
The coordinates of any point on the line are given by x = -1 + 2λ, y = 3 + 2λ, z = 8 – λ
Let the coordinates of M be
(-1 + 2λ, 3 + 3λ, -1 – λ) .....(1)
The direction ratios of PM are
-1 + 2λ – 2, 3 + 3λ + 3, -1 – λ – 1
i.e. 2λ – 3, 3λ = 6, -λ – 2
The direction ratios of the given line are 2, 3, 8.
Since PM is perpendicular to the given line, we get
2(2λ – 3) + 3(3λ + 6) – 1(-λ – 2) = O
∴ 4λ – 6 + 9λ + 18 + λ + 2 = 0
∴ 14λ + 14 = 0
∴ λ = -1
Put λ in (1), the coordinates of M are
(-1 – 2, 3 – 3, -1 + 1) i.e. (-3, 0, 0).
∴ length of perpendicular from P to the given line
= PM
= (−3−2)2+(0+3)2+(0−1)2−−−−−−−−−−−−−−−−−−−−−−−−−−√
= (25+9+1)−−−−−−−−−−√
= 35−−√units.
Alternative Method :
We know that the perpendicular distance from the point

$$\sqrt{\left|\overline{\infty}-\bar{a}\right|^2-\left[\frac{\left(\overline{oo}-\bar{a}\right).\bar{b}}{\left|\bar{b}\right|}\right]^2} \quad ...(1)$$

Here, $\overline{\infty} = 2\hat{i} - 3\hat{j} + \hat{k}, \bar{a} = -\hat{i} + 3\hat{j} - \hat{k}, \bar{b} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\therefore \overline{\infty} - \bar{a} = \left(2\hat{i} - 3\hat{j} + \hat{k}\right) - \left(-\hat{i} + 3\hat{j} - \hat{k}\right)$

$= 3\hat{i} - 6\hat{j} + 2\hat{k}$

$\therefore \left|\overline{\infty} - \bar{a}\right|^2 = 3^2 + (-6)2 + 2^2 = 9 + 36 + 4 = 49$

Also, $\left(\overline{\infty} - \bar{a}\right).\bar{b} = \left(3\hat{i} - 6\hat{j} + 2\hat{k}\right).\left(2\hat{i} + 3\hat{j} - \hat{k}\right)$

= (3)(2) + (– 6)(3) + (2)(– 1)
= 6 – 18 – 2
= –14

$\left|\bar{b}\right| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$

Substitutng tese values in (1), w get
length of perpendicular from P to given line
= PM

$= \sqrt{49 - \left(-\frac{14}{\sqrt{14}}\right)^2}$

$= \sqrt{49 - 14}$

$= \sqrt{35}$units

or

$2\sqrt{6}$units, $(3, -4, 2)$.

Question 6.
A(1, 0, 4), B(0, -11, 13), C(2, -3, 1) are three points and D is the foot of the perpendicular from A to BC. Find the co-ordinates of D.
Solution:
Equation of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$\therefore$ the equation of the line BC passing through the points

B (0, − 11, 13) and C (2, − 3, 1) is

$$\frac{x-0}{2-0}=\frac{y+11}{-3+11}=\frac{z-13}{1-13}$$

i.e. $\frac{x}{2}=\frac{y+11}{8}=\frac{z-13}{-12}=\lambda$ ... (Say)

AD is the perpendicular from the point A (1, 0, 4) to the line BC.
The coordinates of any point on the line BC are given by x = 2λ, y = -11 + 8λ, z = 13 – 12λ
Let the coordinates of D be (2λ, -11 + 8λ, 13 – 12λ) … (1)
∴ the direction ratios of AD are
2λ – 1, -1λ + 8λ – 0, 13 – 12λ – 4 i.e.
2λ – 1, -11 + 8λ, 9 – 12λ
The direction ratios of the line BC are 2, 8, -12.
Since AD is perpendicular to BC, we get
2(2λ – 1) + 8(-11 + 8λ) – 12(9 – 12λ) = 0
∴ 42λ – 2 – 88 + 64λ – 108 + 144λ = 0
∴ 212λ – 198 = 0

$$\therefore \lambda=\frac{198}{212}=\frac{99}{106}$$

Putting $\lambda=\frac{99}{106}$ in (1), the coordinates of D are

$$\left(\frac{198}{106}, -11+\frac{792}{106}, 13-\frac{1188}{106}\right)$$

i.e. $\left(\frac{198}{106}, \frac{-374}{106}, \frac{190}{106}\right)$, i.e. $\left(\frac{99}{53}, \frac{-187}{53}, \frac{95}{53}\right)$.

Question 7.
By computing the shortest distance, determine whether following lines intersect each other.
(i) $\bar{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+\hat{k})$ and $\bar{r}=(2\hat{i}-\hat{j})+\mu(\hat{i}+\hat{j}-\hat{k})$
Solution:
The shortest distance between the lines

$\bar{r}=\overline{a_1}+\lambda\overline{b_1}$ and $\bar{r}=\overline{a_2}+\mu\overline{b_2}$ is given by

$$d=\left|\frac{(\overline{a_2}-\overline{a_1})\cdot(\overline{b_1}\times\overline{b_2})}{|\overline{b_1}\times\overline{b_2}|}\right|.$$

Here, $\overline{a_1}=\hat{i}-\hat{j}$, $\overline{a_2}=2\hat{i}-\hat{j}$, $\overline{b_1}=2\hat{i}+\hat{k}$, $\overline{b_2}=\hat{i}+\hat{j}-\hat{k}$.

$$\therefore \overline{b_1}\times\overline{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 1\\ 1 & 1 & -1\end{vmatrix}$$

$$=(0-1)\hat{i}-(-2-1)\hat{j}+(2-0)\hat{k}$$

$$=-\hat{i}+3\hat{j}+2\hat{k}$$

and $\overline{a_2}-\overline{a_1}=(2\hat{i}-\hat{j})-(\hat{i}-\hat{j})=\hat{i}$

$$\therefore (\overline{a_2}-\overline{a_1})\cdot(\overline{b_1}\times\overline{b_2})=\hat{i}\cdot(-\hat{i}+3\hat{j}+2\hat{k})$$

$$=1(-1)+0(3)+0(2)=-1$$

and $|\overline{b_1}\times\overline{b_2}|=\sqrt{(-1)^2+3^2+2^2}$

$$=\sqrt{1+9+4}=\sqrt{14}$$

$\therefore$ the shortest distance between the given lines

$$=\left|\frac{-1}{\sqrt{14}}\right|=\frac{1}{\sqrt{14}}\text{ unit}$$

Hence, the given lines do not intersect.

(ii) x-54=y-7-5=z+3-5 and x-87=y-71=z-53

Solution:

The shortest distance between the lines

$$\frac{x-5}{4}=\frac{y-7}{-5}=\frac{z+3}{-5} \text{ and } \frac{x-8}{7}=\frac{y-7}{1}=\frac{z-5}{3} \text{ is give n by}$$

$$d=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2-m_2n_1)^2+(l_2n_1-1_1n_2)^2+(l_1m_2-l_2m_1)^2}}$$

The equation of the given lines are

$$\frac{x-5}{4}=\frac{y-7}{-5}=\frac{z+3}{-5} \text{ and } \frac{x-8}{7}=\frac{y-7}{1}=\frac{z-5}{3}$$

∴ $x_1 = -1$, $y_1 = -1$, $z_1 = -1$, $x_2 = 3$, $y_2 = 5$, $z_2 = 7$,
$l_1 = 7$, $m_1 = -6$, $n_1 = 1$, $l_2 = 1$, $m_2 = -2$, $n_2 = 1$

∣∣∣∣ x2−x1l1l2y2−y1m1m2z2−z1n1n2 ∣∣∣∣ = ∣∣∣∣ 4476−518−53 ∣∣∣∣

= 4(- 6 + 2) – 6(7 – 1) + 8(-14 + 6)
= -16 – 36 – 64
= -116
and
$(m_1n_2 – m_2n_1)^2 + (l_2n_1 – l_1n_2)^2 + (l_1m_2 – l_2m_1)^2$
= $(-6 + 2)^2 + (1 – 7)^2 + (-14 + 6)^2$
= 16 + 36 + 64
= 116
Hence, the required shortest distance between the given lines
= ∣∣ -116116√ ∣∣

= 116---√

=229--√ units

or

The shortest distance between the lines
= 2823830√units
Hence, the gives lines do not intersect.

Question 8.

If lines x-12=y+13=z-14 and x-31=y-k2=z1 intersect each other then find k.

Solution:

The lines

$$\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1} \text{ and } \frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$$

intersect, if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$

The equations of the given lines are

$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4} \text{ and } \frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$$

∴ $x_1 = 1$, $y_1 = -1$, $z_1 = 1$, $x_2 = 3$, $y_2 = k$, $z_2 = 0$,
$l_1 = 2$, $m_1 = 3$, $n_1 = 4$, $l_2 = 1$, $m_2 = 2$, $n_2 = 1$.
Since these lines intersect, we get

∣∣∣∣ 221k+132−141 ∣∣∣∣ = 0

∴ 2 (3 – 8) – (k + 1)(2 – 4) – 1 (4 – 3) = 0
∴ -10 + 2(k + 1) – 1 = 0
∴ 2(k + 1) = 11
∴ k + 1 = 112
∴ k = 92

Maharashtra State Board 12th Maths Solutions Chapter 6 Line and Plane Ex 6.3

Question 1.
Find the vector equation of a plane which is at 42 unit distance from the origin and which is normal to the vector $2\hat{i}+\hat{j}-2\hat{k}$.
Solution:
If $\hat{n}$ is a unit vector along the normal and p is the length of the perpendicular from origin to the plane, then the vector equation of the plane is $\bar{r} \cdot \hat{n} = p$

Here, $\bar{n} = 2\hat{i}+\hat{j}-2\hat{k}$ and $p = 42$

$\therefore\ |\bar{n}| = \sqrt{2^2+1^2+(-2)^2} = \sqrt{9} = 3$

$\hat{n} = \frac{\bar{n}}{|\bar{n}|} = \frac{1}{3}(2\hat{i}+\hat{j}-2\hat{k})$

$\therefore$ the vector equation of the required plane is

$\bar{r}\cdot\left[\frac{1}{3}(2\hat{i}+\hat{j}-2\hat{k})\right] = 42$

i.e. $\bar{r}\cdot(2\hat{i}+\hat{j}-2\hat{k}) = 126.$

Question 2.
Find the perpendicular distance of the origin from the plane 6x – 2y + 3z – 7 = 0.
Solution:
The equation of the plane is
6x – 2y + 3z – 7 = 0
∴ its vector equation is
$\bar{r} \cdot (6\hat{i}-2\hat{j}+3\hat{k})$ = 7 ....(1)
where $\bar{r} = x\hat{i}+y\hat{j}+z\hat{k}$
∴ $\bar{n} = 6\hat{i}-2\hat{j}+3\hat{k}$ is normal to the plane
$|\bar{n}| = \sqrt{6^2+(-2)^2+3^2} = \sqrt{49}$ = 7
Unit vector along $\bar{n}$ is

$\hat{n} = \frac{\bar{n}}{|\bar{n}|} = \frac{6\hat{i}-2\hat{j}+3\hat{k}}{7}$

Dividing both sides of (1) by 7, we get

$\bar{r}\cdot\left(\frac{6\hat{i}-2\hat{j}+3\hat{k}}{7}\right) = \frac{7}{7}$

$\therefore\ \bar{r}\cdot\hat{n} = 1$

Comparing with normal form of equation of the plane $\bar{r} \cdot \hat{n}$ = p, it follows that length of perpendicular from origin is 1 unit.
Alternative Method:
The equation of the plane is 6x – 2y + 3z – 7 = 0 i.e. 6x – 2y + 3z = 7

$$\text{i.e. } \left(\frac{6}{6^2+(-2)^2+3^2}\right)x - \left(\frac{2}{\sqrt{6^2+(-2)^2+3^2}}\right)y + \left(\frac{3}{\sqrt{6^2+(-2)^2+3^2}}\right)z = \frac{7}{\sqrt{6^2+(-2)^2+3^2}}$$

$$\text{i.e. } \frac{6}{7}x - \frac{2}{7}y + \frac{3}{7}z = \frac{7}{7} = 1$$

This is the normal form of the equation of plane.
∴ perpendicular distance of the origin from the plane is p = 1 unit.

Question 3.
Find the coordinates of the foot of the perpendicular drawn from the origin to the plane 2x + 6y – 3z = 63 .
Solution:
The equation of the plane is 2x + 6y – 3z = 63. Dividing each term by $\sqrt{2^2+6^2+(-3)^2} = \sqrt{49}$ = 7, we get

$\frac{2}{7}x + \frac{6}{7}y - \frac{3}{7}z = \frac{63}{7}$ = 9

This is the normal form of the equation of plane.
∴ the direction cosines of the perpendicular drawn from the origin to the plane are

l = 27, m = 67, n = −37

and length of perpendicular from origin to the plane is p = 9.

∴ the coordinates of the foot of the perpendicular from the origin to the plane are (lp, mp, np) i.e. (187,547,−277).

Question 4.
Reduce the equation $\bar{r} \cdot (3\hat{i}+4\hat{j}+12\hat{k})$ = 78 to normal form and hence find
(i) the length of the perpendicular from the origin to the plane
(ii) direction cosines of the normal.
Solution:
The normal form of equation of a plane is $\bar{r} \cdot \hat{n}$ = p where $\hat{n}$ is unit vector along the normal and p is the length of perpendicular drawn from origin to the plane.

Given plane is $\bar{r}\cdot(3\hat{i}+4\hat{j}+12\hat{k}) = 78$ ... (1)

$\bar{n} = 3\hat{i}+4\hat{j}+12\hat{k}$ is normal to the plane

$\therefore\ |\bar{n}| = \sqrt{3^2+4^2+12^2} = \sqrt{169} = 13$

Dividing both sides of (1) by 13, we get

$$\bar{r}\cdot\left(\frac{3\hat{i}+4\hat{j}+12\hat{k}}{13}\right) = \frac{78}{13}$$

$$\text{i.e. } \bar{r}\cdot\left(\frac{3}{13}\hat{i}+\frac{4}{13}\hat{j}+\frac{12}{13}\hat{k}\right) = 6$$

This is the normal form of the equation of plane. Comparing with $\bar{r} \cdot \hat{n}$ = p,
(i) the length of the perpendicular from the origin to plane is 6.
(ii) direction cosines of the normal are 313,413,1213.

Question 5.
Find the vector equation of the plane passing through the point having position vector $\hat{i}+\hat{j}+\hat{k}$ and perpendicular to the vector $4\hat{i}+5\hat{j}+6\hat{k}$.
Solution:
The vector equation of the plane passing through the point A ($\bar{a}$) and perpendicular to the vector $\bar{n}$ is $\bar{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$
Here, $\bar{a} = \hat{i}+\hat{j}+\hat{k}$, $\bar{n} = 4\hat{i}+5\hat{j}+6\hat{k}$
$\therefore \bar{a} \cdot \bar{n} = (\hat{i}+\hat{j}+\hat{k}) \cdot (4\hat{i}+5\hat{j}+6\hat{k})$
= (1)(4) + (1)(5) + (1)(6)
= 4 + 5 + 6 = 15
∴ the vector equation of the required plane is $\bar{r} \cdot (4\hat{i}+5\hat{j}+6\hat{k})$ = 15.

Question 6.
Find the Cartesian equation of the plane passing through A(-1, 2, 3), the direction ratios of whose normal are 0, 2, 5.
Solution:
The cartesian equation of the plane passing ; through (x_1, y_1, z_1), the direction ratios of whose normal are a, b, c, is
$a(x – x_1) + b(y – y_1) + c(z – z_1) = 0$
∴ the cartesian equation of the required plane is
0(x +1) + 2(y – 2) + 5(z – 3) = 0
i.e. 0 + 2y – 4 + 5z – 15 = 0
i.e. 2y + 5z = 19.

Question 7.
Find the Cartesian equation of the plane passing through A(7, 8, 6) and parallel to the XY plane.
Solution:
The cartesian equation of the plane passing through (x_1, y_1, z_1), the direction ratios of whose normal are a, b, c, is
$a(x – x_1) + b(y – y_1) + c(z – z_1) = 0$
The required plane is parallel to XY-plane.
∴ it is perpendicular to Z-axis i.e. Z-axis is normal to the plane. Z-axis has direction ratios 0, 0, 1.
The plane passes through (7, 8, 6).
∴ the cartesian equation of the required plane is
0(x – 7) + 0(y – 8) + 1 (z – 6) = 0
i.e. z = 6.

Question 8.
The foot of the perpendicular drawn from the origin to a plane is M(1, 0, 0). Find the vector equation of the plane.
Solution:
The vector equation of the plane passing ; through A($\bar{a}$) and perpendicular to $\bar{n}$ is $\bar{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$.
M(1, 0, 0) is the foot of the perpendicular drawn from ; origin to the plane. Then the plane is passing through M : and is perpendicular to OM.
If $\bar{m}$ is the position vector of M, then $\bar{m} = \hat{i}$
Normal to the plane is
$\bar{n} = \overline{OM} = \hat{i}$
$\bar{m} \cdot \bar{n} = \hat{i} \cdot \hat{i} = 1$
∴ the vector equation of the required plane is $\bar{r} \cdot \hat{i} = 1$

Question 9.
Find the vector equation of the plane passing through the point A(-2, 7, 5) and parallel to vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$.
Solution:
The vector equation of the plane passing through the point A($\bar{a}$) and parallel to the vectors $\bar{b}$ and $\bar{c}$ is
$\bar{r} \cdot (\bar{b} \times \bar{c}) = \bar{a} \cdot (\bar{b} \times \bar{c})$ ....(1)

Here, $\bar{a} = -2\hat{i} + 7\hat{j} + 5\hat{k}$

$\bar{b} = 4\hat{i} - \hat{j} + 3\hat{k}$, $\bar{c} = \hat{i} + \hat{j} + \hat{k}$

$$\therefore \bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (-1-3)\hat{i} - (4-3)\hat{j} + (4+1)\hat{k}$$

$$= -4\hat{i} - \hat{j} + 5\hat{k}$$

$$\therefore \bar{a} \cdot (\bar{b} \times \bar{c}) = (-2\hat{i} + 7\hat{j} + 5\hat{k}) \cdot (-4\hat{i} - \hat{j} + 5\hat{k})$$

$$= (-2)(-4) + (7)(-1) + (5)(5)$$

$$= 8 - 7 + 25 = 26$$

∴ from (1), the vector equation of the required plane is

$\bar{r} \cdot (-4\hat{i} - \hat{j} + 5\hat{k}) = 26.$

Question 10.
Find the Cartesian equation of the plane $\bar{r} = (5\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$
Solution:
The equation represents a plane passing through a point having position vector $\bar{a}$ and parallel to vectors $\bar{b}$ and $\bar{c}$.

Here, $\bar{a} = 5\hat{i} - 2\hat{j} - 3\hat{k}$, $\bar{b} = \hat{i} + \hat{j} + \hat{k}$, $\bar{c} = \hat{i} - 2\hat{j} + 3\hat{k}$

$$\therefore \bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

$$= (3+2)\hat{i} - (3-1)\hat{j} + (-2-1)\hat{k}$$

$$= 5\hat{i} - 2\hat{j} - 3\hat{k} = \bar{a}$$

Also, $\bar{a} \cdot (\bar{b} \times \bar{c}) = \bar{a} \cdot \bar{a} = |\bar{a}|^2$

$$= (5)^2 + (-2)^2 + (3)^2 = 38$$

The vector equation of the plane passing through A($\bar{a}$)

and parallel to $\bar{b}$ and $\bar{c}$ is

$\bar{r} \cdot (\bar{b} \times \bar{c}) = \bar{a} \cdot (\bar{b} \times \bar{c})$

∴ the vector equation of the given plane is

$\bar{r} \cdot (5\hat{i} - 2\hat{j} - 3\hat{k}) = 38$

If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then this equation becomes

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (5\hat{i} - 2\hat{j} - 3\hat{k}) = 38$

∴ 5x – 2y – 3z = 38.
This is the cartesian equation of the required plane.

Question 11.
Find the vector equation of the plane which makes intercepts 1, 1, 1 on the co-ordinates axes.
Solution:
The vector equation of the plane passing through A($\bar{a}$), B($\bar{b}$). C($\bar{c}$), where A, B, C are non-collinear
is $\bar{r} \cdot (\overline{AB} \times \overline{AC}) = \bar{a} \cdot (\overline{AB} \times \overline{AC})$ ... (1)
The required plane makes intercepts 1, 1, 1 on the coordinate axes.
∴ it passes through the three non-collinear points A (1, 0, 0), B = (0, 1, 0), C = (0, 0, 1)

$\therefore \ \bar{a} = \hat{i}, \bar{b} = \hat{j}, \bar{c} = \hat{k}$

$\overline{AB} = \bar{b} - \bar{a} = \hat{j} - \hat{i} = -\hat{i} + \hat{j}$

$\therefore \ \overline{AC} = \bar{c} - \bar{a} = \hat{k} - \hat{i} = -\hat{i} + \hat{k}$

$$\therefore \ \overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= (1-0)\hat{i} - (-1-0)\hat{j} + (0+1)\hat{k}$$

$$= \hat{i} + \hat{j} + \hat{k}$$

Also, $\bar{a} \cdot (\overline{AB} \times \overline{AC}) = \hat{i} \cdot (\hat{i} + \hat{j} + \hat{k})$

$$= 1 \times 1 + 0 \times 1 + 0 \times 1$$

$$= 1$$

∴ from (1), the vector equation of the required plane is $\bar{r} \cdot (\hat{i}+\hat{j}+\hat{k}) = 1$.

Maharashtra State Board 12th Maths Solutions Chapter 6 Line and Plane Ex 6.4

Question 1.
Find the angle between planes $\bar{r} \cdot (\hat{i}+\hat{j}+2\hat{k}) = 13$ and $\bar{r} \cdot (2\hat{i}-\hat{j}+\hat{k}) = 31$.
Solution:

The acute angle θ between the planes $\bar{r} \cdot \bar{n}_1 = d_1$ and $\bar{r} \cdot \bar{n}_2 = d_2$ is given by

$$\cos\theta = \left|\frac{\bar{n}_1 \cdot \bar{n}_2}{|\bar{n}_1||\bar{n}_2|}\right| \qquad \ldots (1)$$

Here, $\bar{n}_1 = \hat{i} + \hat{j} + 2\hat{k}$, $\bar{n}_2 = 2\hat{i} - \hat{j} + \hat{k}$

$$\therefore\ \bar{n}_1 \cdot \bar{n}_2 = (\hat{i} + \hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j} + \hat{k})$$
$$= (1)(2) + (1)(-1) + (2)(1)$$
$$= 2 - 1 + 2 = 3$$

Also, $|\bar{n}_1| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$

$|\bar{n}_2| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$

$\therefore$ from (1), we have

$$\cos\theta = \left|\frac{3}{\sqrt{6}\cdot\sqrt{6}}\right| = \frac{3}{6} = \frac{1}{2} \cos 60^\circ$$

$\therefore\ \theta = 60^\circ$.

Question 2.
Find the acute angle between the line $\bar{r} \cdot (\hat{i}+2\hat{j}+2\hat{k})+\lambda(2\hat{i}+3\hat{j}-6\hat{k})$ and the plane $\bar{r} \cdot (2\hat{i}-\hat{j}+\hat{k}) = 0$
Solution:
The acute angle θ between the line $\bar{r} = \bar{a} + \lambda\bar{b}$ and the plane $\bar{r} \cdot \bar{n} = d$ is given by

$$\sin\theta = \left|\frac{\bar{b} \cdot \bar{n}}{|\bar{b}||\bar{n}|}\right| \qquad \ldots (1)$$

Here, $\bar{b} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, $\bar{n} = 2\hat{i} - \hat{j} + \hat{k}$

$$\therefore\ \bar{b} \cdot \bar{n} = (2\hat{i} + 3\hat{j} - 6\hat{k}) \cdot (2\hat{i} - \hat{j} + \hat{k})$$
$$= (2)(2) + (3)(-1) + (-6)(1)$$
$$= 4 - 3 - 6 = -5$$

Also, $|\bar{b}| = \sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{49} = 7$

$|\bar{n}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$

$\therefore$ from (1), we have

$$\sin\theta = \left|\frac{-5}{7\sqrt{6}}\right| = \frac{5}{7\sqrt{6}}$$

$$\therefore\ \theta = \sin^{-1}\left(\frac{5}{7\sqrt{6}}\right).$$

Question 3.
Show that lines $\bar{r} = (2\hat{j}-3\hat{k})+\lambda(\hat{i}+2\hat{j}+3\hat{k})$ and $\bar{r} = (2\hat{i}+6\hat{j}+3\hat{k})+\mu(2\hat{i}+3\hat{j}+4\hat{k})$ are coplanar. Find the equation of the plane determined by them.
Solution:

The lines $\bar{r} = \bar{a}_1 + \lambda_1 \bar{b}_1$ and $\bar{r} = \bar{a}_2 + \lambda_2 \bar{b}_2$ are coplanar if $\bar{a}_1 \cdot (\bar{b}_1 \times \bar{b}_2) = \bar{a}_2 \cdot (\bar{b}_1 \times \bar{b}_2)$

Here $\bar{a}_1 = 2\hat{j} - 3\hat{k}$, $\bar{a}_2 = 2\hat{i} + 6\hat{j} + 3\hat{k}$,

$\bar{b}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$, $\bar{b}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\therefore \bar{a}_2 - \bar{a}_1 = (2\hat{i} + 6\hat{j} + 3\hat{k}) - (2\hat{j} - 3\hat{k})$

$= 2\hat{i} + 4\hat{j} + 6\hat{k}$

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$= (8-9)\hat{i} - (4-6)\hat{j} + (3-4)\hat{k}$

$= -\hat{i} + 2\hat{j} - \hat{k}$

$\therefore \bar{a}_1 \cdot (\bar{b}_1 \times \bar{b}_2) = (2\hat{j} - 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k})$

$= 0(-1) + 2(2) + (-3)(-1)$

$= 0 + 4 + 3 = 7$

and $\bar{a}_2 \cdot (\bar{b}_1 \times \bar{b}_2) = (2\hat{i} + 6\hat{j} + 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k})$

= 2(-1) + 6(2) + 3(-1)

= -2 + 12 – 3 = 7

$\therefore \bar{a}_1 \cdot (\bar{b}_1 \times \bar{b}_2) = \bar{a}_2 \cdot (\bar{b}_1 \times \bar{b}_2)$

Hence, the given lines are coplanar.

The plane determined by these lines is given by

$\therefore \bar{r} \cdot (\bar{b}_1 \times \bar{b}_2) = \bar{a}_1 \cdot (\bar{b}_1 \times \bar{b}_2)$

i.e. $\bar{r} \cdot (-\hat{i} + 2\hat{j} - \hat{k})$

Hence, the given lines are coplanar and the equation of the plane determined by these lines is

$\bar{r} \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 7$

Question 4.

Find the distance of the point $4\hat{i} - 3\hat{j} + \hat{k}$ from the plane $\bar{r} \cdot (2\hat{i} + 3\hat{j} - 6\hat{k}) = 21$.

Solution:

The distance of the point A($\bar{a}$) from the plane $\bar{r} \cdot \bar{n} = p$ is given by d = $\frac{|\bar{a} \cdot \bar{n} - p|}{|\bar{n}|}$ ...(1)

Here, $\bar{a} = 4\hat{i} - 3\hat{j} + \hat{k}$, $\bar{n} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, p = 21

$\therefore \bar{a} \cdot \bar{n} = (4\hat{i} - 3\hat{j} + \hat{k}) \cdot (2\hat{i} + 3\hat{j} - 6\hat{k})$

= (4)(2) + (-3)(3) + (1)(-6)

= 8 – 9 – 6 = -7

Also, $\sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{49}$ = 7

∴ from (1), the required distance

= $\frac{|-7-21|}{7}$ = 4units

Question 5.

Find the distance of the point (1, 1, -1) from the plane 3x + 4y – 12z + 20 = 0.

Solution:

The distance of the point (x_1, y_1, z_1) from the plane ax + by + cz + d = 0 is $\left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$

∴ the distance of the point (1, 1, -1) from the plane 3x + 4y – 12z + 20 = 0 is $\left| \frac{3(1) + 4(1) - 12(-1) + 20}{\sqrt{3^2 + 4^2 + (-12)^2}} \right|$

= $\left| \frac{3 + 4 + 12 + 20}{\sqrt{9 + 16 + 144}} \right| = \frac{39}{\sqrt{169}}$

= $\frac{39}{13}$ = 3units

Maharashtra State Board 12th Maths Solutions Chapter 6 Line and Plane Miscellaneous Exercise 6A

Question 1.
Find the vector equation of the line passing through the point having position vector $3\hat{i}+4\hat{j}-7\hat{k}$ and parallel to $6\hat{i}-\hat{j}+\hat{k}$.
Solution:
The vector equation of the line passing through A($\bar{a}$) and parallel to the vector $\bar{b}$ is $\bar{r}=\bar{a}+\lambda\bar{b}$, where λ is a scalar.
∴ the vector equation of the line passing through the point having position vector
$3\hat{i}+4\hat{j}-7\hat{k}$ and parallel to the vector $6\hat{i}-\hat{j}+\hat{k}$ is
$\bar{r}=(3\hat{i}+4\hat{j}-7\hat{k})+\lambda(6\hat{i}^{\wedge}-\hat{j}+\hat{k})$.

Question 2.
Find the vector equation of the line which passes through the point (3, 2, 1) and is parallel to the vector $2\hat{i}+2\hat{j}-3\hat{k}$.
Solution:
The vector equation of the line passing through A($\bar{a}$) and parallel to the vector $\bar{b}$ is $\bar{r}=\bar{a}+\lambda\bar{b}$, where λ is a scalar.
∴ the vector equation of the line passing through the point having position vector $3\hat{i}+2\hat{j}+\hat{k}$ and parallel to the vector
$2\hat{i}+2\hat{j}-3\hat{k}$ *is* $\bar{r}=(3\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}+2\hat{j}-3\hat{k})$

Question 3.
Find the Cartesian equations of the line which passes through the point (-2, 4, -5) and parallel to the line $x+23=y-35=z+56$
Solution:
The line $x+23=y-35=z+56$ has direction ratios 3, 5, 6. The required line has direction ratios 3, 5, 6 as it is parallel to the given line.
It passes through the point (-2, 4, -5).
The cartesian equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

∴ the required cartesian equations of the line are

$$\frac{x-(-2)}{3}=\frac{y-4}{5}=\frac{z-(-5)}{6}$$

i.e. $\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$.

Question 4.
Obtain the vector equation of the line $x+53=y+45=z+56$.
Solution:
The cartesian equations of the line are $x+53=y+45=z+56$.
This line is passing through the point A(-5, -4, -5) and having direction ratios 3, 5, 6.
Let $\bar{a}$ be the position vector of the point A w.r.t. the origin and $\bar{b}$ be the vector parallel to the line.
Then $\bar{a}=-5\hat{i}-4\hat{j}-5\hat{k}$ and $\bar{b}=3\hat{i}+5\hat{j}+6\hat{k}$.
The vector equation of the line passing through A($\bar{a}$) and parallel to $\bar{b}$ is $\bar{r}=\bar{a}+\lambda\bar{b}$ where λ is a scalar.
∴ the vector equation of the required line is $\bar{r}=(-5\hat{i}-4\hat{j}-6\hat{k})+\lambda(3\hat{i}+5\hat{j}+6\hat{k})$

Question 5.
Find the vector equation of the line which passes through the origin and the point (5, -2, 3).
Solution:
Let $\bar{b}$ be the position vector of the point B(5, -2, 3).
Then $\bar{b}=5\hat{i}-2\hat{j}+3\hat{k}$
Origin has position vector $\bar{O}=0\hat{i}+0\hat{j}+0\hat{k}$.
The vector equation the line passing through A($\bar{a}$) and B($\bar{b}$) is $\bar{r}=\bar{a}+\lambda(\bar{b}-\bar{a})$ where λ is a scalar.
∴ the vector equation of the required line is $\bar{r}=\bar{O}+\lambda(\bar{b}-\bar{O})=\lambda(5\hat{i}-2\hat{j}+3\hat{k})$

Question 6.
Find the Cartesian equations of the line which passes through points (3, -2, -5) and (3, -2, 6).
Solution:
Let A = (3, -2, -5), B = (3, -2, 6)
The direction ratios of the line AB are
3 – 3, -2 – (-2), 6 – (-5) i.e. 0, 0, 11.
The parametric equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are
$x = x_1 + a\lambda$, $y = y_1 + b\lambda$, $z = z_1 + c\lambda$
∴ the parametric equattions of the line passing through (3, -2, -5) and having direction ratios are 0, 0, 11 are
$x = 3 + (0)\lambda$, $y = -2 + 0(\lambda)$, $z = -5 + 11\lambda$
i.e. x = 3, y = -2, z = 11λ – 5
∴ the cartesian equations of the line are
x = 3, y = -2, z = 11λ – 5, λ is a scalar.

Question 7.
Find the Cartesian equations of the line passing through A(3, 2, 1) and B(1, 3, 1).
Solution:
The direction ratios of the line AB are 3 – 1, 2 – 3, 1 – 1 i.e. 2, -1, 0.
The parametric equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are
$x = x_1 + a\lambda$, $y = y_1 + b\lambda$, $z = z_1 + c\lambda$
∴ the parametric equattions of the line passing through (3, 2, 1) and having direction ratios 2, -1, 0 are
x = 3 + 2λ, y = 2 – λ, z = 1 + 0(λ)
x – 3 = 2λ, y – 2 = -λ, z = 1
∴ $x-32=y-2-1$ = λ, z = 1
∴ the cartesian equations of the line are
$x-32=y-2-1$, z = 1.

Question 8.
Find the Cartesian equations of the line passing through the point A(1, 1, 2) and perpendicular to
vectors $b^{-}=i^{\wedge}+2j^{\wedge}+k^{\wedge}$ and $c^{-}=3i^{\wedge}+2j^{\wedge}-k^{\wedge}$.
Solution:
Let the required line have direction ratios p, q, r. ,
It is perpendicular to the vectors $b^{-}=i^{\wedge}+2j^{\wedge}+k^{\wedge}$ and $c^{-}=3i^{\wedge}+2j^{\wedge}-k^{\wedge}$.
∴ it is perpendicular to lines whose direction ratios are 1, 2, 1 and 3, 2, -1.
∴ p + 2q + r = 0, 3p + 2q – r = 0

$$\therefore \frac{p}{\begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}} = \frac{q}{\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix}} = \frac{r}{\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}}$$

$$\therefore \frac{p}{-4} = \frac{q}{4} = \frac{r}{-4}$$

$$\therefore \frac{p}{-1} = \frac{q}{1} = \frac{r}{-1}$$

∴ the required line has direction ratios -1, 1, -1.
The cartesian equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are
$x-x_1a=y-y_1b=z-z_1c$
∴ the cartesian equations of the line passing through the point (1, 1, 2) and having direction ratios -1, 1, -1 are
$x-1-1=y-11=z-2-1$

Question 9.
Find the Cartesian equations of the line which passes through the point (2, 1, 3) and perpendicular
to lines $x-11=y-22=z-33$ and $x-3=y2=z5$.
Solution:
Let the required line have direction ratios p, q, r.
It is perpendicular to the vector $b^{-}=i^{\wedge}+2j^{\wedge}+k^{\wedge}$ and $c^{-}=3i^{\wedge}+2j^{\wedge}-k^{\wedge}$.
∴ it is perpendicular to lines whose direction ratios are 1, 2, 1 and 3, 2, -1.
∴ p + 2q + r = 0, 3 + 2q – r = 0

$$\therefore \frac{p}{\begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}} = \frac{q}{\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix}} = \frac{r}{\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}}$$

$$\therefore \frac{p}{-4} = \frac{q}{4} = \frac{r}{-1}$$

$$\therefore \frac{p}{2} = \frac{q}{-7} = \frac{r}{4}$$

∴ the required line has direction ratios 2, -7, 4.
The cartesian equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are

x=x1a=y−y1b=z−z1c

∴ the cartesian equation of the line passing through the point (2, -7, 4) and having directions ratios 2, -7, 4 are

x−22=y−1−7=z−24

Question 10.
Find the vector equation of the line which passes through the origin and intersect the line x – 1 = y – 2 = z – 3 at right angle.
Solution:

The given line is x−11=y−21=z−31 = λ ... (Say)

∴ coordinates of any point on the line are
x = λ + 1, y = λ + 2, z = λ + 3
∴ position vector of any point on the line is

(λ + 1)i^ + (λ + 2)j^ + (λ + 3)k^ ... (1)

If b¯ is parallel to the given line whose direction ratios are 1, 1, 1, then b¯=i^+j^+k^.

Let the required line passing through O meet the given line at M.
∴ position vector of M

= m¯ = (λ + 1)i^ + (λ + 2)j^ + (λ + 3)k^ ... [By (1)]

The required line is perpendicular to given line

$$\therefore \overline{OM} \cdot \bar{b} = 0$$

$$\therefore \bar{m} \cdot \bar{b} = 0$$

$$\therefore [(\lambda+1)\hat{i} + (\lambda+2)\hat{j} + (\lambda+3)\hat{k}] \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$$

$$\therefore (\lambda+1) \times 1 + (\lambda+2) \times 1 + (\lambda+3) \times 1 = 0$$

$$\therefore 3\lambda + 6 = 0 \qquad \therefore \lambda = -2$$

$$\therefore \bar{m} = (-2+1)\hat{i} + (-2+2)\hat{j} + (-2+3)\hat{k}$$
$$= -\hat{i} + \hat{k}$$

The vector equation of the line passing through A(a¯) and B(b¯) is r¯=a¯+λ(b¯−a¯), λ is a scalar.

∴ the vector equation of the line passing through o(o¯) and M(m¯) is

r¯=O¯¯+λ(m¯−O¯¯)=λm¯=λ(−i^+k^) where λ is a scalar.

Hence, vector equation of the required line is .

Question 11.

Find the value of λ so that lines 1−x3=7y−142λ=z−32 and 7−7x3λ=y−51=6−z5 are at right angle.

Solution:
The equations of the given lines are

$$\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{z-3}{2} \qquad \dots (1)$$

and $$\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5} \qquad \dots (2)$$

Equation (1) can be written as :

$$\frac{-(x-1)}{3}=\frac{7(y-2)}{2\lambda}=\frac{z-3}{2}$$

i.e. $$\frac{x-1}{-3}=\frac{y-2}{(2\lambda/7)}=\frac{z-3}{2}$$

The direction ratios of this line are

$$a_1=-3,\ b_1=\frac{2\lambda}{7},\ c_1=2$$

Equation (2) can be written as :

$$\frac{-7(x-1)}{3\lambda}=\frac{y-5}{1}=\frac{-(z-6)}{5}$$

i.e. $$\frac{x-1}{-(3\lambda/7)}=\frac{y-5}{1}=\frac{z-6}{-5}$$

The direction ratios of this line are

$$a_2=\frac{-3\lambda}{7},\ b_2=1,\ c_2=-5$$

Since the lines (1) and (2) are at right angles,

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$\therefore\ (-3)\left(\frac{-3\lambda}{7}\right)+\left(\frac{2\lambda}{7}\right)(1)+2(-5)=0$$

$$\therefore\ \frac{9\lambda}{7}+\frac{2\lambda}{7}-10=0$$

$$\therefore\ \frac{11\lambda}{7}=10 \qquad \therefore\ \lambda=\frac{70}{11}.$$

Question 12.
Find the acute angle between lines x-11=y-2-1=z-32 and x-12=y-21=z-31.
Solution:

Let $\bar{a}$ and $\bar{b}$ be the vectors in the direction of the lines

$\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-3}{2}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{1}$ respectively.

Then $\bar{a}=\hat{i}-\hat{j}+2\hat{k},\ \bar{b}=2\hat{i}+\hat{j}+\hat{k}$

$$\therefore\ \bar{a}\cdot\bar{b}=(\hat{i}-\hat{j}+2\hat{k})\cdot(2\hat{i}+\hat{j}+\hat{k})$$
$$=(1)(2)+(-1)(1)+(2)(1)$$
$$=2-1+2=3$$

$$|\bar{a}|=\sqrt{1^2+(-1)^2+2^2}=\sqrt{6}$$
$$|\bar{b}|=\sqrt{2^2+1^2+1^2}=\sqrt{6}$$

If θ is the angle between the lines, then

$$\cos\theta=\frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}=\frac{3}{\sqrt{6}\sqrt{6}}=\frac{1}{2}=\cos 60^\circ$$

$$\therefore\ \theta=60^\circ.$$

Question 13.
Find the acute angle between lines x = y, z = 0 and x = 0, z = 0.
Solution:
The equations x = y, z = 0 can be written as $\frac{x}{1}=\frac{y}{1}$, z = 0
∴ the direction ratios of the line are 1, 1, 0.
The direction ratios of the line x = 0, z = 0, i.e., Y-axis J are 0, 1, 0.
∴ its directiton ratios are 0, 1, 0.
Let $\bar{a}$ and $\bar{b}$ be the vectors in the direction of the lines x = y, z = 0 and x = 0, z = 0.

$$\text{Then } \bar{a}=\hat{i}+\hat{j}, \quad \bar{b}=\hat{j}$$

$$\therefore \ \bar{a}\cdot\bar{b}=(\hat{i}+\hat{j})\cdot\hat{j}=(1)(0)+(1)(1)+(0)(0)=1$$

$$|\bar{a}|=\sqrt{1^2+1^2}=\sqrt{2}$$

$$|\bar{b}|=|\hat{j}|=1$$

If θ is the acute angle between the lines, then

$$\cos\theta=\left|\frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}\right|=\left|\frac{1}{\sqrt{2}\times 1}\right|=\frac{1}{\sqrt{2}}=\cos 45^\circ.$$

$$\therefore \ \theta=45^\circ.$$

Question 14.
Find the acute angle between lines x = -y, z = 0 and x = 0, z = 0.
Solution:
The equations x = -y, z = 0 can be written as $\frac{x}{1}=\frac{y}{1}$, z = 0.
∴ the direction ratios of the line are 1, 1, 0.
The direction ratios of the line x = 0, z = 0, i.e., Y-axis are 0, 1, 0.
∴ its direction ratios are 0, 1, 0.
Let $\bar{a}$ and $\bar{b}$ be the vectors in the direction of the lines x = y, z = 0 and x = 0, z = 0

$$\text{Then } \bar{a}=\hat{i}+\hat{j}, \bar{b}=\hat{j}$$

$$\therefore \bar{a}.\bar{b}=\left(\hat{i}+\hat{j}\right).\hat{j}$$

$$= (1)(0) + (1)(1) + (0)(0)$$
$$= 1$$

$$\left|\bar{a}\right|=\sqrt{1^2+1^2}=\sqrt{2}$$

$$\left|\bar{b}\right|=\left|\hat{j}\right| = 1$$

If θ is the acute angle between the lines, then

$$\cos\theta=\left|\frac{\bar{a}.\bar{b}}{\left|\bar{a}\right|\bar{b}\ |}\right|=\left|\frac{1}{\sqrt{2}\times 1}\right|=\frac{1}{\sqrt{2}}=\cos 45^\circ.$$

$$\therefore \theta = 45^\circ.$$

Question 15.
Find the co-ordinates of the foot of the perpendicular drawn from the point (0, 2, 3) to the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$.
Solution:
Let P = (0, 2, 3)
Let M be the foot of the perpendicular drawn from P to the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$ = λ ……(Say)
The coordinates of any point on the line are given by
x = 5λ – 3, y = 2λ + 1, z = 3λ – 4
Let M = (5λ – 3, 2λ + 1, 3λ – 4) …(1)
The direction ratios of PM are
5λ – 3 – 0, 2λ + 1 – 2, 3λ – 4 – 3 i.e. 5λ – 3, 2λ – 1, 3λ – 7
Since, PM is perpendicular to the line whose direcction ratios are 5, 2, 3,
5(5λ – 3) + 2(2λ – 1) + 3(3λ – 7) = 0
25λ – 15 + 4λ – 2 + 9λ – 21 =0
38λ – 38 = 0 ∴ λ = 1
Substituting λ = 1 in (1), we get.
M = (5 – 3, 2 + 1, 3 – 4) = (2, 3, -1).
Hence, the coordinates of the foot of perpendicular are (2, 3, – 1).

Question 16.
By computing the shortest distance determine whether following lines intersect each other.
(i) $\bar{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and $\bar{r}=(2\hat{i}+2\hat{j}-3\hat{k})+\mu(\hat{i}+\hat{j}-2\hat{k})$
Solution:
The shortest distance between the lines
$\bar{r}=\bar{a}_1+\lambda\bar{b}_1$ and $\bar{r}=\bar{a}_2+\mu\bar{b}_2$ is given by

$$d=\left|\frac{\left(\bar{a}_2-\bar{a}_1\right).\left(\bar{b}_1\times\bar{b}_2\right)}{\left|\bar{b}_1\times\bar{b}_2\right|}\right|.$$

Here, $\bar{a}_1=\hat{i}+\hat{j}-\hat{k}, \bar{a}_2=2\hat{i}+2\hat{j}-3\hat{k}$,
$\bar{b}_1=2\hat{i}-\hat{j}+\hat{k}, \bar{b}_2=\hat{i}+\hat{j}-2\hat{k}$.

$$\therefore \bar{b}_1\times\bar{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 1 & -2\end{vmatrix}$$

$= (2-1)\hat{i}-(-4-1)\hat{j}+(4+1)\hat{k}$

$= \hat{i}-5\hat{j}+5\hat{k}$

and

$\bar{a}_2-\bar{a}_1=\left(2\hat{i}+2\hat{j}-3\hat{k}\right)-\left(\hat{i}+\hat{j}-\hat{k}\right)$

$\therefore \left(\bar{a}_2-\bar{a}_1\right).\left(\bar{b}_1\times\bar{b}_2\right)=\hat{i}.\left(2\hat{i}+2\hat{j}-3\hat{k}\right)$

= 1(– 1) + 0(3) + 0(2)

= – 1

and

$\left|\bar{b}_1\times\bar{b}_2\right|=\sqrt{(-1)^2+3^2+2^2}$

$= \sqrt{1+9+4}$

$= \sqrt{14}$

Shortest distance between the lines is 0.
∴ the lines intersect each other.

(ii) $\frac{x-5}{4}=\frac{y-7}{5}=\frac{z+3}{5}$ and x – 6 = y – 8 = z + 2.
Solution:
The shortest distance between the lines
$\frac{x-5}{4}=\frac{y-7}{5}=\frac{z+3}{5}$ and x – 6 = y – 8 = z + 2 is given by

$$d=\frac{\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\end{vmatrix}}{\sqrt{(m_1n_2-m_2n_1)^2+(l_2n_1-1_1n_2)^2+(l_1m_2-l_2m_1)^2}}$$

The equation of the given lines are
$\frac{x-5}{4}=\frac{y-7}{5}=\frac{z+3}{5}$ and x – 6 = y – 8 = z + 2
∴ $x_1 = 5, y_1 = 7, z_1 = 3, x_2 = 6, y_2 = 8, z_2 = 2,$
$l_1 = 4, m_1 = 5, n_1 = 1, l_2 = 1, m_2 = -2, n_2 = 1$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 4 & 5 & 5 \\ -6 & -8 & 2 \end{vmatrix}$$

= 4(-6 + 2) – 6(7 – 1) + 8(-14 + 6)
= -16 – 36 – 64
= -116
and
$(m_1n_2 - m_2n_1)^2 + (l_2n_1 - l_1n_2)^2 + (l_1m_2 - l_2m_1)^2$
$= (-6 + 2)^2 + (1 - 7)^2 + (1 - 7)^2 + (-14 + 6)^2$
= 16 + 36 + 64
= 116
Hence, the required shortest distance between the given lines

$= \left|\frac{-116}{\sqrt{116}}\right|$

$= \sqrt{116}$

$= 2\sqrt{29}$ units

or
Shortest distance between the lines is 0.
∴ the lines intersect each other.

Question 17.
If lines x-12=y+13=z-14 and x-21=y+m2=z-21 intersect each other then find m.
Solution:

The lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_2}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ intersect, if

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0 \qquad \ldots (1)$$

Here, $(x_1, y_1, z_1) \equiv (1, -1, 1)$,
$(x_2, y_2, z_2) \equiv (2, -m, 2)$,
$a_1 = 2, b_1 = 3, c_1 = 4$,
$a_2 = 1, b_2 = 2, c_2 = 1$
Substituting these values in (1), we get

$$\begin{vmatrix} 2-1 & -m+1 & 2-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} 1 & 1-m & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

∴ 1(3 – 8) – (1 – m)(2 – 4) + 1 (4 – 3) = 0
∴ -5 + 2 – 2m + 1 = 0
∴ -2m = 2
∴ m = -1.

Question 18.
Find the vector and Cartesian equations of the line passing through the point (-1, -1, 2) and parallel to the line 2x – 2 = 3y + 1 = 6z – 2.
Solution:
Let $\bar{a}$ be the position vector of the point A (-1, -1, 2) w.r.t. the origin.
Then $\bar{a} = -\hat{i} - \hat{j} + 2\hat{k}$
The equation of given line is
x – 2 = 3y + 1 = 6z – 2.

$$\therefore\ 2(x-1)=3\left(y+\frac{1}{3}\right)=6\left(z-\frac{1}{3}\right)$$

$$\therefore\ \frac{x-1}{\left(\frac{1}{2}\right)}=\frac{y+\frac{1}{3}}{\left(\frac{1}{3}\right)}=\frac{z-\frac{1}{3}}{\left(\frac{1}{6}\right)}$$

The direction ratios of this line are

$\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ i.e. 3, 2, 1

Let $\bar{b}$ be the vector parallel to this line.

Then $\bar{b}=3\hat{i}+2\hat{j}+\hat{k}$

The vector equation of the line passing through A $(\bar{a})$ and parallel to $\bar{b}$ is

$\bar{r}=\bar{a}+\lambda\bar{b}$, where λ is a scalar

$\therefore$ the vector equation of the required line is

$\bar{r}=(-\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+2\hat{j}+\hat{k})$.

The line passes through $(-1,\ -1,\ 2)$ and has direction ratios 3, 2, 1

$\therefore$ the cartesian equations of the line are

$$\frac{x-(-1)}{3}=\frac{y-(-1)}{2}=\frac{z-2}{1}$$

i.e. $\frac{x+1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$.

Question 19.
Find the direction cosines of the line $\bar{r}=(-2\hat{i}+\frac{5}{2}\hat{j}-\hat{k})+\lambda(2\hat{i}+3\hat{j})$.
Solution:

The line $\bar{r}=\left(-2\hat{i}+\frac{5}{2}\hat{j}-\hat{k}\right)+\lambda(2\hat{i}+3\hat{j})$ is parallel to $\bar{b}=2\hat{i}+3\hat{j}$.

$\therefore$ direction ratios of the line are 2, 3, 0.

$\therefore$ direction cosines of the line are

$$\frac{2}{\sqrt{2^2+3^2+0}}, \frac{3}{\sqrt{2^2+3^2+0}}, 0$$

i.e. $\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}, 0$.

Question 20.
Find the Cartesian equation of the line passing through the origin which is perpendicular to x – 1 = y – 2 = z – 1 and intersects the $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$.
Solution:
Let the required line have direction ratios a, b, c
Since the line passes through the origin, its cartesian equations are
$\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$ …(1)
This line is perpendicular to the line
x – 1 = y – 2 = z – 1 whose direction ratios are 1, 1, 1.

$$\therefore\ a+b+c=0 \qquad \dots (2)$$

The lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_1}$ intersect, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

Applying this condition for the lines $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$ and

$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$, we get

$$\begin{vmatrix} 1-0 & -1-0 & 1-0 \\ a & b & c \\ 2 & 3 & 4 \end{vmatrix}=0$$

$\therefore$ 1(4b – 3c) + 1(4a – 2c) + 1(3a – 2b) = 0
$\therefore$ 4b – 3c + 4a – 2c + 3a – 2b = 0
$\therefore$ 7a + 2b – 5c = 0
From (2) and (3), we get

$$\frac{a}{\begin{vmatrix} 1 & 1 \\ 2 & -5 \end{vmatrix}}=\frac{b}{\begin{vmatrix} 1 & 1 \\ -5 & 7 \end{vmatrix}}=\frac{a}{\begin{vmatrix} 1 & 1 \\ 7 & 2 \end{vmatrix}}$$

$$\therefore\ \frac{a}{-7}=\frac{b}{12}=\frac{c}{-5}$$

$\therefore$ the required line has direction ratios $-7, 12, -5$.

From (1), cartesian equation of required line are

$$\frac{x}{-7}=\frac{y}{12}=\frac{z}{-5}$$

i.e. $\frac{x}{7}=\frac{y}{-12}=\frac{z}{5}$.

Question 21.
Write the vector equation of the line whose Cartesian equations are y = 2 and 4x – 3z + 5 = 0.
Solution:
4x – 3z + 5 = 0 can be written as

$$4x=3z-5=3\left(z-\frac{5}{3}\right)$$

$$\therefore\ \frac{4x}{12}=\frac{3\left(z-\frac{5}{3}\right)}{12}$$

$$\therefore\ \frac{x}{3}=\frac{z-\frac{5}{3}}{4}$$

$\therefore$ the cartesian equations of the line are

$$\frac{x}{3}=\frac{z-\frac{5}{3}}{4},\ y=2.$$

This line passes through the point A(0, 2, 53) position vector is $\bar{a}=2\hat{j}+53\hat{k}$
Also the line has direction ratio 3, 0, 4.
If $\bar{b}$ is a vector parallel to the line, then $\bar{b}=3\hat{i}+4\hat{k}$
The vector equation of the line passing through A($\bar{a}$) and parallel to $\bar{b}$ is $\bar{r}=\bar{a}+\lambda\bar{b}$ where λ is $\bar{a}$ scalar,
$\therefore$ the vector equation of the required line is
$\bar{r}=(2\hat{j}+53\hat{k})+\lambda(3\hat{i}+4\hat{k})$.

Question 22.
Find the co-ordinates of points on the line $\frac{x-1}{1}=\frac{y-2}{-2}=\frac{z-3}{2}$ which are at the distance 3 unit from the base point A(1, 2, 3).
Solution:
The cartesian equations of the line are $\frac{x-1}{1}=\frac{y-2}{-2}=\frac{z-3}{2}$ = λ
The coordinates of any point on this line are given by
x = λ + 1, y = -2λ + 2, z = 2λ + 3
Let M(λ + 1, -2λ + 2, 2λ + 3) … (1)
be the point on the line whose distance from A(1, 2, 3) is 3 units.

$$\therefore \sqrt{(\lambda+1-1)^2+(-2\lambda+2-2)^2+(2\lambda+3-3)^2}=3$$

$$\therefore \sqrt{\lambda^2+4\lambda^2+4\lambda^2}=3$$

$$\therefore \sqrt{9\lambda^2}=3$$

$$\therefore 9\lambda^2=9$$

$$\therefore \lambda^2=1 \qquad \therefore \lambda=\pm 1$$

When λ = 1, M = (1 + 1, -2 + 2, 2 + 3) … [By (1)]
i. e. M = (2, 0, 5)
When λ = -1, M = (1 – 1, 2 + 2, -2 + 3) … [By (1)]
i. e. M = (0, 4, 1)
Hence, the coordinates of the required points are (2, 0, 5) and (0, 4, 1).

Maharashtra State Board 12th Maths Solutions Chapter 6 Line and Plane Miscellaneous Exercise 6B

Question 1.
If the line $\frac{x}{3}=\frac{y}{4}$ = z is perpendicular to the line $\frac{x-1}{k}=\frac{y+2}{3}=\frac{z-3}{k-1}$ then the value of k is:

A) $\frac{11}{4}$ B) $-\frac{11}{4}$ C) $\frac{11}{2}$ D) $\frac{4}{11}$

Solution:
(b) $-\frac{11}{4}$

Question 2.
The vector equation of line 2x – 1 = 3y + 2 = z – 2 is

A) $\bar{r}=\left(\frac{1}{2}\bar{i}-\frac{2}{3}\bar{j}+2\bar{k}\right)+\lambda\left(3\bar{i}+2\bar{j}+6\bar{k}\right)$

B) $\bar{r}=\bar{i}-\bar{j}+\left(2\bar{i}+\bar{j}+\bar{k}\right)$

C) $\bar{r}=\left(\frac{1}{2}\bar{i}-\bar{j}\right)+\lambda\left(\bar{i}-2\bar{j}+6\bar{k}\right)$

D) $\bar{r}=\left(\bar{i}+\bar{j}\right)+\lambda\left(\bar{i}-2\bar{j}+6\bar{k}\right)$

Solution:
(a) r¯=(12i^−23j^+2k^)+λ(3i^+2j^+6k^)

Question 3.
The direction ratios of the line which is perpendicular to the two lines x−72=y+17−3=z−61 and x+51=y+32=z−6−2 are
(A) 4, 5, 7
(B) 4, -5, 7
(C) 4, -5, -7
(D) -4, 5, 8
Solution:
(A) 4, 5, 7

Question 4.
The length of the perpendicular from (1, 6, 3) to the line x1=y−12=z−23
(A) 3
(B) 11−−√
(C) 13−−√
(D) 5
Solution:
(C) 13−−√

Question 5.
The shortest distance between the lines r¯=(i^+2j^+k^)+λ(i^−j^−k^) and r¯=(2i^−j^−k^)+μ(2i^+j^+2k^) is
Question is modified.
The shortest distance between the lines r¯=(i^+2j^+k^)+λ(i^−j^+k^) and r¯=(2i^−j^−k^)+μ(2i^+j^+2k^) is

A) $\frac{1}{\sqrt{3}}$ B) $\frac{1}{\sqrt{2}}$ C) $\frac{3}{\sqrt{2}}$ D) $\frac{\sqrt{3}}{2}$

Solution:
(c) 32√

Question 6.
The lines x−21=y−31=z−4−k and x−1k=y−42=z−51. and coplanar if
(A) k = 1 or -1
(B) k = 0 or -3
(C) k = + 3
(D) k = 0 or -1
Solution:
(B) k = 0 or -3

Question 7.
The lines x1=y2=z3 and x−1−2=y−2−4=z−36 and are
(A) perpendicular
(B) inrersecting
(C) skew
(D) coincident
Solution:
(B) inrersecting

Question 8.
Equation of X-axis is
(A) x = y = z
(B) y = z
(C) y = 0, z = 0
(D) x = 0, y = 0
Solution:
(C) y = 0, z = 0

Question 9.
The angle between the lines 2x = 3y = -z and 6x = -y = -4z is

(A) 45°
(B) 30°
(C) 0°
(D) 90°
Solution:
(D) 90°

Question 10.
The direction ratios of the line 3x + 1 = 6y – 2 = 1 – z are
(A) 2, 1, 6
(B) 2, 1, -6
(C) 2, -1, 6
(D) -2, 1, 6
Solution:
(B) 2, 1, -6

Question 11.
The perpendicular distance of the plane 2x + 3y – z = k from the origin is $\sqrt{14}$ units, the value of k is
(A) 14
(B) 196
(C) $2\sqrt{14}$
(D) $14\sqrt{2}$
Solution:
(A) 14

Question 12.
The angle between the planes and $\bar{r} \cdot (\bar{i} - 2\bar{j} + 3\bar{k}) + 4 = 0$ and $\bar{r} \cdot (2\bar{i} + \bar{j} - 3\bar{k}) + 7 = 0$ is

A) $\frac{\pi}{2}$ B) $\frac{\pi}{3}$ C) $\cos^{-1}\left(\frac{3}{4}\right)$ D) $\cos^{1}\left(\frac{9}{14}\right)$

Solution:
(d) $\cos^{-1}\left(\frac{9}{14}\right)$

Question 13.
If the planes $\bar{r} \cdot (2\bar{i} - \lambda\bar{j} + \bar{k}) = 3$ and $\bar{r} \cdot (4\bar{i} - \bar{j} + \mu\bar{k}) = 5$ are parallel, then the values of λ and μ are respectively.

A) $\frac{1}{2}, -2$ B) $-\frac{1}{2}, 2$ C) $-\frac{1}{2}, -2$ D) $\frac{1}{2}, 2$

Solution:
(d) $\frac{1}{2}$, 2

Question 14.
The equation of the plane passing through (2, -1, 3) and making equal intercepts on the coordinate axes is
(A) x + y + z =1
(B) x + y + z = 2
(C) x + y + z = 3
(D) x + y + z = 4
Solution:
(D) x + y + z = 4

Question 15.
Measure of angle between the planes 5x – 2y + 3z – 7 = 0 and 15x – 6y + 9z + 5 = 0 is
(A) 0°
(B) 30°
(C) 45°
(D) 90°
Solution:
(A) 0°

Question 16.
The direction cosines of the normal to the plane 2x – y + 2z = 3 are

A) $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$ B) $\frac{-2}{3}, \frac{1}{3}, \frac{-2}{3}$

C) $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$ D) $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$

Solution:

(a) 23,-13,23

Question 17.
The equation of the plane passing through the points (1, -1, 1), (3, 2, 4) and parallel to Y-axis is :
(A) 3x + 2z – 1 = 0
(B) 3x – 2z = 1
(C) 3x + 2z + 1 = 0
(D) 3x + 2z = 2
Solution:
(B) 3x – 2z = 1

Question 18.

The equation of the plane in which the line x-54=y-74=z+3-5 and x-87=y-41=z+53 lie, is

(A) 17x – 47y – 24z + 172 = 0
(B) 17x + 47y – 24z + 172 = 0
(C) 17x + 47y + 24z +172 = 0
(D) 17x – 47y + 24z + 172 = 0
Solution:
(A) 17x – 47y – 24z + 172 = 0

Question 19.

If the line x+12=y-m3=z-46 lies in the plane 3x – 14y + 6z + 49 = 0, then the value of m is:

(A) 5
(B) 3
(C) 2
(D) -5
Solution:
(A) 5

Question 20.
The foot of perpendicular drawn from the point (0,0,0) to the plane is (4, -2, -5) then the equation of the plane is
(A) 4x + y + 5z = 14
(B) 4x – 2y – 5z = 45
(C) x – 2y – 5z = 10
(D) 4x + y + 6z = 11
Solution:
(B) 4x – 2y – 5z = 45

II. Solve the following :
Question 1.

Find the vector equation of the plane which is at a distance of 5 unit from the origin and which is normal to the vector 2i^+j^+2k^

Solution:

If n^ is a unit vector along the normal and p i the length of the perpendicular from origin to the plane, then the vector equation of the plane $\bar{r} \cdot$ n^ = p

Here, $\bar{n} = 2\hat{i}+\hat{j}+2\hat{k}$ and p = 5

$$\therefore \left|\bar{n}\right| = \sqrt{2^2+1^2+(2)^2}$$
$$= \sqrt{9}$$
$$= 3$$
$$\hat{n} = \frac{\bar{n}}{\left|\bar{n}\right|}$$
$$= \frac{1}{3}\left(2\hat{i}+\hat{j}+2\hat{k}\right)$$

$\therefore$ the vector equation of the required plane is

$$\bar{r}.\left[\frac{1}{3}\left(2\hat{i}+\hat{j}+2\hat{k}\right)\right] = 5$$

i.e. $\bar{r}.\left(2\hat{i}+\hat{j}+2\hat{k}\right) = 15.$

Question 2.
Find the perpendicular distance of the origin from the plane 6x + 2y + 3z – 7 = 0
Solution:
The distance of the point (x_1, y_1, z_1) from the plane ax + by + cz + d is $\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$
$\therefore$ the distance of the point (1, 1, -1) from the plane 6x + 2y + 3z – 7 = 0 is

$$\left|\frac{6(1)+2(1-3(-1)+7)}{\sqrt{3^2+4^2+(-12)^2}}\right|$$
$$= \left|\frac{6+4+6+7}{\sqrt{9+16+144}}\right|$$
$$= \frac{23}{\sqrt{169}}$$
$$= \frac{23}{13}$$

= 1units.

Question 3.
Find the coordinates of the foot of the perpendicular drawn from the origin to the plane 2x + 3y + 6z = 49.
Solution:
The equation of the plane is 2x + 3y + 6z = 49
Dividing each term by
$\sqrt{2^2+3^2+(-6)^2}$
= $\sqrt{49}$
= 7
we get
$\frac{2}{7}x + \frac{3}{7}y - \frac{6}{7}z = \frac{49}{7} = 7$
This is the normal form of the equation of plane.
$\therefore$ the direction cosines of the perpendicular drawn from the origin to the plane are
l = $\frac{2}{7}$, m = $\frac{3}{7}$, n = $\frac{6}{7}$
and length of perpendicular from origin to the plane is p = 7.
the coordinates of the foot of the perpendicular from the origin to the plane are
(lp, ∓ , np)i.e.(2, 3, 6)

Question 4.
Reduce the equation $\bar{r}\cdot(\hat{i}+8\hat{j}+24\hat{k})=13$ to normal form and hence find
(i) the length of the perpendicular from the origin to the plane
(ii) direction cosines of the normal.
Solution:
The normal form of equation of a plane is $\bar{r}\cdot\hat{n}$ = p where $\hat{n}$ is unit vector along the normal and p is the length of perpendicular drawn from origin to the plane.

Given pane is $r.\ (6i\hat{}+8j\hat{}+24k\hat{})=13$ ...(1)

$\bar{n} = 6\hat{i} + 8\hat{j} + 24\hat{k}$ is normal to the plane

$\therefore |\bar{n}| = \sqrt{6^2 + 8^2 + 24^2} = \sqrt{76} = 13$

Dividing both sides of (1) by 13, get

$$\bar{r}.\left(\frac{3\hat{i} + 4\hat{j} + 12\hat{k}}{13}\right) = \frac{76}{13}$$

$$\text{i.e. } \bar{r}.\left(\frac{3}{13}\hat{i} + \frac{4}{13}\hat{j} + \frac{12}{13}\hat{k}\right) = \frac{1}{2}$$

This is the normal form of the equation of plane.

Comparing with $\bar{r} \cdot \hat{n} = p$,

(i) the length of the perpendicular from the origin to plane is 12.

(ii) direction cosines of the normal are 313,413,1213

Question 5.
Find the vector equation of the plane passing through the points A(1, -2, 1), B (2, -1, -3) and C (0, 1, 5).
Solution:
The vector equation of the plane passing through three non-collinear points A($\bar{a}$), B($\bar{b}$) and C($\bar{c}$)
is $\bar{r} \cdot (\overline{AB} \times \overline{AC}) = \bar{a} \cdot (\overline{AB} \times \overline{AC})$ ... (1)

Here, $\bar{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\bar{b} = 2\hat{i} - \hat{j} - 3\hat{k}$, $\bar{c} = \hat{j} + 5\hat{k}$

$$\therefore \overline{AB} = \bar{b} - \bar{a} = (2\hat{i} - \hat{j} - 3\hat{k}) - (\hat{i} - 2\hat{j} + \hat{k}) = \hat{i} + \hat{j} - 4\hat{k}$$

$$\overline{AC} = \bar{c} - \bar{a} = (\hat{j} + 5\hat{k}) - (\hat{i} - 2\hat{j} + \hat{k}) = -\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\therefore \overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -4 \\ -1 & 3 & 4 \end{vmatrix}$$

$$= (4+12)\hat{i} - (4-4)\hat{j} + (3+1)\hat{k}$$

$$= 16\hat{i} + 4\hat{k}$$

Now, $\bar{a} \cdot (\overline{AB} \times \overline{AC}) = (\hat{i} - 2\hat{j} + \hat{k}) \cdot (16\hat{i} + 4\hat{k})$

$$= (1)(16) + (-2)(0) + (1)(4) = 20$$

$\therefore$ from (1), the vector equation of the required plane is

$\bar{r} \cdot (16\hat{i} + 4\hat{k}) = 20.$

Question 6.
Find the Cartesian equation of the plane passing through A(1, -2, 3) and the direction ratios of whose normal are 0, 2, 0.
Solution:
The Cartesian equation of the plane passing through (x_1, y_1, z_1), the direction ratios of whose normal are a, b, c, is
$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$
∴ the cartesian equation of the required plane is
o(x + 1) + 2(y + 2) + 5(z – 3) = 0
i.e. 0 + 2y – 4 + 10z – 15 = 0
i.e. y + 2 = 0.

Question 7.
Find the Cartesian equation of the plane passing through A(7, 8, 6) and parallel to the plane $\bar{r} \cdot (6i\hat{}+8j\hat{}+7k\hat{})=0$
Solution:
The cartesian equation of the plane $\bar{r} \cdot (6i\hat{}+8j\hat{}+7k\hat{})=0$ is 6x + 8y + 7z = 0 The required plane is parallel to it
∴ its cartesian equation is
6x + 8y + 7z = p ...(1)
A (7, 8, 6) lies on it and hence satisfies its equation
∴ (6)(7) + (8)(8) + (7)(6) = p

i.e., p = 42 + 64 + 42 = 148.
∴ from (1), the cartesian equation of the required plane is 6x + 8y + 7z = 148.

Question 8.
The foot of the perpendicular drawn from the origin to a plane is M(1, 2,0). Find the vector equation of the plane.
Solution:
The vector equation of the plane passing through A($\bar{a}$) and perpendicular to $\bar{n}$ is $\bar{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$.
M(1, 2, 0) is the foot of the perpendicular drawn from origin to the plane. Then the plane is passing through M and is perpendicular to OM.
If $\bar{m}$ is the position vector of M, then $\bar{m}$ = $\hat{i}$.
Normal to the plane is
$\bar{n} = \overline{OM} = \hat{i}$
$\bar{m} \cdot \bar{n} = \hat{i}, \hat{i} = 5$
∴ the vector equation of the required plane is
$\bar{r} \cdot (\hat{i} + 2\hat{j}) = 5$

Question 9.
A plane makes non zero intercepts a, b, c on the co-ordinates axes. Show that the vector equation of the plane is $\bar{r} \cdot (bc\hat{i} + ca\hat{j} + ab\hat{k})$ = abc
Solution:
The vector equation of the plane passing through A($\bar{a}$), B($\bar{b}$).. C($\bar{c}$), where A, B, C are non collinear is
$\bar{r} \cdot (\overline{AB} \times \overline{AC}) = \bar{a} \cdot (\overline{AB} \times \overline{AC})$ ...(1)
The required plane makes intercepts 1, 1, 1 on the coordinate axes.
∴ it passes through the three non collinear points A = (1, 0, 0), B = (0, 1, 0), C = (0, , 1)

$\therefore \bar{a} = \hat{i}, \bar{b} = \hat{j}, \bar{c} = \hat{k}$

$\overline{AB} = \bar{b} - \bar{a} = \hat{j} - \hat{i} = -\hat{i} + \hat{j}$

$\therefore \overline{AC} = \bar{c} - \bar{a} = \hat{k} - \hat{i} = -\hat{i} + \hat{k}$

$$\therefore \overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & & \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$= (1 - 0)\hat{i} - (-1 - 0)\hat{j} + (0 + 1)\hat{k}$

$= \hat{i} + \hat{j} + \hat{k}$

Also,

$\bar{a}. \left(\overline{AB} \times \overline{AC}\right)$

$= \hat{i}. \left(\hat{i} + \hat{j} + \hat{k}\right)$

= 1 x 1 + 0 x 1 + 0 x 1

= 1

∴ from(1)the vector equation of the required plane is $\bar{r}. \left(\hat{i} + \hat{j} + \hat{k}\right) = 1$.

Question 10.
Find the vector equation of the plane passing through the pointA(-2, 3, 5) and parallel to vectors $4\hat{i} + 3\hat{k}$ and $\hat{i} + \hat{j}$
Solution:
The vector equation of the plane passing through the point A($\bar{a}$) and parallel to the vectors $\bar{b}$ and $\bar{c}$ is

$\bar{r}.\left(\bar{b}\times\bar{c}\right)=\bar{a}.\left(\bar{b}\times\bar{c}\right)$ ...(1)

Here, $\bar{a}=-2\hat{i}+3\hat{j}+5\hat{k}$

$\bar{b}=4\hat{i}+3\hat{k},$

$\bar{c}=\hat{i}+\hat{j}$

$\therefore\ \bar{b}\times\bar{c}=\begin{vmatrix}\hat{i} & & \hat{k}\\ 4 & 0 & 3\\ 1 & 1 & 0\end{vmatrix}$

$=(-1-3)\hat{i}-(4-3)\hat{j}+(3+1)\hat{k}$

$=-4\hat{i}-\hat{j}+4\hat{k}$

$\therefore\ \bar{a}.\left(\bar{b}\times\bar{c}\right)=\left(-2\hat{i}+7\hat{j}+5\hat{k}\right).\left(-4\hat{i}-\hat{j}+4\hat{k}\right)$

= (-2)(-4) + (7)(-1) + (5)(4)
= 8 – 7 + 8
= 35

∴ From (1), the vector equation of the required plane is $\bar{r}\cdot(-3\hat{i}-3atj+4\hat{k})$ = 35.

Question 11.
Find the Cartesian equation of the plane $\bar{r}=\lambda(\hat{i}+\hat{j}-\hat{k})+\mu(\hat{i}+2\hat{j}+3\hat{k})$
Solution:
The equation $\bar{r}=\bar{a}+\lambda\bar{b}+\mu\bar{c}$ represents a plane passing through a point having position vector $\bar{a}$ and parallel to vectors $\bar{b}$ and $\bar{c}$.
Here,

$\bar{b} = \hat{i} + \hat{j} - \hat{k}$,
$\bar{c} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$\therefore \bar{b} \times \bar{c} = \begin{vmatrix} & & \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$= (3+2)\hat{i} - (3-1)\hat{j} + (-2-1)\hat{k}$
$= 5\hat{i} - 4\hat{j} - \hat{k}$
$= \bar{a}$

Also,
$\bar{a}.\left(\bar{b} \times \bar{c}\right)$
$= \bar{a}.\bar{a} = \left|\bar{a}\right|^2$
$= (5)^2 + (4)^2 + (0)^2$
$= 0$

The vector equation of the plane passing through $A(\bar{a})$ and parallel to $\bar{b}$ and $\bar{c}$ is

$\bar{r}.\left(\bar{b} \times \bar{c}\right) = \bar{a}.\left(\bar{b} \times \bar{c}\right)$

$\therefore$ the vector equation of the given plane is

$\bar{r}.\left(5\hat{i} - 4\hat{j} - \hat{k}\right) = 0$

If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then this equation becomes

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(5\hat{i} - 4\hat{j} - \hat{k}\right) = 0$

$\therefore$ 5x – 4y + z = 0.

This is the cartesian equation of the required plane.

Question 12.
Find the vector equations of planes which pass through A(1, 2, 3), B (3, 2, 1) and make equal intercepts on the co-ordinates axes.
Question is modified
Find the cartesian equations of the planes which pass through A(1, 2, 3), B(3, 2, 1) and make equal intercepts on the coordinate axes.
Solution:
Case 1 : Let all the intercepts be 0.
Then the plane passes through the origin.
Then the cartesian equation of the plane is
ax + by + cz = 0 .....(1)
(1, 2, 3) and (3, 2, 1) lie on the plane.
$\therefore$ a + 2b + 3c = 0 and 3a + 2b + c = 0

$$\therefore \frac{a}{\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}}$$

$$\therefore \frac{a}{-4} = \frac{b}{8} = \frac{c}{-4}$$

i.e. $\frac{a}{1} = \frac{b}{-2} = \frac{c}{1}$

$\therefore$ a, b, c are proportional to 1, -2, 1
$\therefore$ from (1), the required cartesian equation is x – 2y + z = 0.
Case 2 : Let the plane make non zero intercept p on each axis.
then its equation is $xp+yp+zp$ = 1
i.e. x + y + z = p ...(2)
Since this plane pass through (1, 2, 3) and (3, 2, 1)
$\therefore$ 1 + 2 + 3 = p and 3 + 2 + 1 = p
$\therefore$ p = 6
$\therefore$ from (2), the required cartesian equation is
x + y + z = 6
Hence, the cartesian equations of required planes are x + y + z = 6 and x – 2y + z = 0.

Question 13.
Find the vector equation of the plane which makes equal non-zero intercepts on the co-ordinates axes and passes through (1, 1, 1).
Solution:
Case 1 : Let all the intercepts be 0.
Then the plane passes through the origin.
Then the vector equation of the plane is ax + by + cz ...(1)
(1, 1, 1) lie on the plane.
∴ 1a + 1b + 1c = 0

$$\therefore \frac{\hat{i}}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{\hat{j}}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{\hat{k}}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}}$$

$$\therefore \frac{\hat{i}}{1} = \frac{\hat{j}}{1} = \frac{\hat{k}}{1}$$

i.e. $\frac{\hat{i}}{1} = \frac{\hat{j}}{1} = \frac{\hat{k}}{1}$

∴ $\hat{i}, \hat{j}, \hat{k}$ are proprtional to 1, 1, 1

∴ from (1), the required cartesian equation is x – y + z = 0
Case 2 : Let he plane make non zero intercept p on each axis.
then its equation is $i\text{^}p+j\text{^}p+k\text{^}p=1$ = 1
i.e. $i\text{^}+j\text{^}+k\text{^}=p$ = p ....(2)
Since this plane pass through (1, 1, 1)
∴ 1 + 1 + 1 = p
∴ p = 3
∴ from (2), the required cartesian equation is $i\text{^}+j\text{^}+k\text{^}$ = 3
Hence, the cartesian equations of required planes are $\bar{r}\cdot(i\text{^}+j\text{^}+k\text{^})=3$

Question 14.
Find the angle between planes $\bar{r}\cdot(-2i\text{^}+j\text{^}+2k\text{^})=17$ and $\bar{r}\cdot(2i\text{^}+2j\text{^}+k\text{^})=71$.
Solution:
The acute angle between the planes

$\bar{r}.\bar{n}_1 = d_1$ and $\bar{r}.\bar{n}_2 = d_2$ is given by

$$\cos\theta = \left|\frac{\bar{n}_1.\bar{n}_2}{|\bar{n}_1||\bar{n}_2|}\right| \quad ...(1)$$

Here,

$\bar{n}_1 = -2\hat{i} + \hat{j} + 2\hat{k}$,

$\bar{n}_2 = 2\hat{i} + 2\hat{j} + \hat{k}$

$\therefore \bar{n}_1.\bar{n}_2$

$= \left(2\hat{i} + \hat{j} + 2\hat{k}\right).\left(2\hat{i} + \hat{j} + \hat{k}\right)$

= (1)(2) + (1)(1) + (2)(1)
= 2 + 1 + 2
= 5

Also,

$$\left|\overline{n}_1\right| = \sqrt{1^2+1^2+2^2} = \sqrt{6}$$

$$\left|\overline{n}_2\right| = \sqrt{2^2+(-1)^2+1^2} = \sqrt{6}$$

$\therefore$ from (1), we have

$$\cos\theta = \left|\frac{3}{\sqrt{6}\sqrt{6}}\right|$$

$= \frac{3}{6}$

$= \frac{1}{2}\cos 90^\circ$

$\therefore \theta = 90^\circ$.

Question 15.
Find the acute angle between the line $\bar{r} = \lambda(\hat{i}-\hat{j}+\hat{k})$ and the plane $\bar{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=23$
Solution:
The acute angle θ between the line $\bar{r}=\bar{a}+\lambda\bar{b}$ and the plane $\bar{r}\cdot\bar{n}$ = d is given by

$$\sin\theta = \left|\frac{\overline{b}.\overline{n}}{\left|\overline{b}\right|\left|\overline{n}\right|}\right| \quad ...(1)$$

Here, $\overline{b} = \hat{i}-\hat{j}+\hat{k}, \overline{n} = 2\hat{i}-\hat{j}+\hat{k}$

$\therefore \overline{b}.\overline{n} = \left(\hat{i}-\hat{j}+\hat{k}\right).\left(2\hat{i}-\hat{j}+\hat{k}\right)$

= (2)(2) + (3)(-1) + (-6)(1)
= 4 – 3 – 6
= -5

Also, $\left|\overline{b}\right| = \sqrt{1^2+1^2+(-1)^2} = \sqrt{2}$ = 1

$$\left|\overline{n}\right| = \sqrt{2^2+(-1)^2+1^2} = \sqrt{4}$$

$\therefore$ from (1), we have

$\sin\theta = \left|\frac{2\sqrt{2}}{-3}\right| = \frac{2\sqrt{2}}{3}$

$\therefore \theta = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$.

Question 16.
Show that lines $\bar{r}=(\hat{i}+4\hat{j})+\lambda(\hat{i}+2\hat{j}+3\hat{k})$ and $\bar{r}=(3\hat{j}-\hat{k})+\mu(2\hat{i}+3\hat{j}+4\hat{k})$
Solution:

Question 17.
Find the distance of the point $3\hat{i}+3\hat{j}+\hat{k}$ from the plane $\bar{r}\cdot(2\hat{i}+3\hat{j}+6\hat{k})=21$
Solution:
The distance of the point A($\bar{a}$) from the plane $\bar{r}\cdot\bar{n}$ = p is given by d = $\left|\frac{\bar{a}\cdot\bar{n}-p}{|\bar{n}|}\right|$ ......(1)

Here, $\overline{a} = 3\hat{i}+3\hat{j}+\hat{k}, \overline{n} = 2\hat{i}+3\hat{j}+6\hat{k}$, p = 21

$\therefore \overline{a}.\overline{n} = \left(3\hat{i}+3\hat{j}+\hat{k}\right).\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$

= (3)(2) + (3)(3) + (1)(-6)
= 6 + 9 – 6

= 9

Also, $\left|\bar{n}\right| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{-12} = 0$

∴ from (1), the required distance

= $\frac{|-12 - 21|}{12}$

= 0 units.

Question 18.
Find the distance of the point (13, 13, -13) from the plane 3x + 4y – 12z = 0.
Solution:

The distance of the point (x_1, y_1, z_1) from the plane ax + by + cz + d = 0 is $\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$

∴ the distance of the point (1, 1, – 1) from the plane 3x + 4y – 12z = 0 is $\left|\frac{3(1) + 4(1 - 12(-1))}{\sqrt{3^2 + 4^2 + (-12)^2}}\right|$

= $\left|\frac{3 + 4 + 12}{\sqrt{9 + 16 + 144}}\right|$

= $\frac{19}{\sqrt{169}}$

= $\frac{19}{13}$

= 19units.

Question 19.
Find the vector equation of the plane passing through the origIn and containing the line $\bar{r} = (\hat{i} + 4\hat{j} + \hat{k}) + \lambda(\hat{i} + 2\hat{j} + \hat{k})$.
Solution:
The vector equation of the plane passing through A($\bar{a}$) and perpendicular to the vector $\bar{n}$ is $\bar{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$ ... (1)
We can take $\bar{a} = \bar{O}$ since the plane passes through the origin.
The point M with position vector $\bar{m} = \hat{i} + 4\hat{j} + \hat{k}$ lies on the line and hence it lies on the plane.
∴ $\overline{OM} = \bar{m} = \hat{i} + 4\hat{j} + \hat{k}$ lies on the plane.
The plane contains the given line which is parallel to $\bar{b} = \hat{i} + 2\hat{j} + \hat{k}$
Let $\bar{n}$ be normal to the plane. Then $\bar{n}$ is perpendicular to $\overline{OM}$ as well as $\bar{b}$

$$\therefore \bar{n} = \overline{OM} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$
$$= (4 - 2)\hat{i} - (1 - 1)\hat{j} + (2 - 4)\hat{k}$$
$$= 2\hat{i} - 2\hat{k}$$

∴ from (1), the vector equation of the required plane is

$\bar{r} \cdot (2\hat{i} - 2\hat{k}) = \bar{0} \cdot \bar{n} = 0$

i.e. $\bar{r} \cdot (\hat{i} - \hat{k}) = 0$.

Question 20.
Find the vector equation of the plane which bisects the segment joining A(2, 3, 6) and B(4, 3, -2) at right angle.
Solution:
The vector equation of the plane passing through A($\bar{a}$) and perpendicular to the vector $\bar{n}$ is $\bar{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$ ....(1)
The position vectors $\bar{a}$ and $\bar{b}$ of the given points A and B are $\bar{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ and $\bar{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}$
If M is the midpoint of segment AB, the position vector $\bar{m}$ of M is given by

$$\overline{m} = \frac{\overline{a}+\overline{b}}{2} = \frac{(2\hat{i}+3\hat{j}+6\hat{k})+(4\hat{i}+3\hat{j}-2\hat{k})}{2}$$

$$= \frac{6\hat{i}+6\hat{j}+4\hat{k}}{2} = 3\hat{i}+3\hat{j}+2\hat{k}$$

The plane passes through $M(\overline{m})$.

AB is perpendicular to the plane

If $\overline{n}$ is normal to the plane, then $\overline{n} = \overline{AB}$

$\therefore\ \overline{n} = \overline{b} - \overline{a} = (4\hat{i}+3\hat{j}-2\hat{k}) - (2\hat{i}+3\hat{j}+6\hat{k})$

$= 2\hat{i} - 8\hat{k}$

$\therefore\ \overline{m}\cdot\overline{n} = (3\hat{i}+3\hat{j}+2\hat{k})\cdot(2\hat{i}-8\hat{k})$

$= (3)(2)+(3)(0)+(2)(-8)$

$= 6+0-16 = -10.$

$\therefore$ from (1), the vector equation of the required plane is

$\overline{r}\cdot\overline{n} = \overline{m}\cdot\overline{n}$

i.e. $\overline{r}\cdot(2\hat{i}-8\hat{k}) = -10$

i.e. $\overline{r}\cdot(\hat{i}-4\hat{k}) = -5.$

Question 21.
Show thatlines x = y, z = 0 and x + y = 0, z = 0 intersect each other. Find the vector equation of the plane determined by them.
Solution:
Given lines are x = y, z = 0 and x + y = 0, z = 0.
It is clear that (0, 0, 0) satisfies both the equations.
$\therefore$ the lines intersect at O whose position vector is $\overline{O}$
Since z = 0 for both the lines, both the lines lie in XY- plane.
Hence, we have to find equation of XY-plane.
Z-axis is perpendicular to XY-plane.
$\therefore$ normal to XY plane is $\hat{k}$.
0($\overline{O}$) lies on the plane.
By using $\overline{r}\cdot\overline{n} = \overline{a}\cdot\overline{n}$, the vector equation of the required plane is $\overline{r}\cdot\hat{k} = \overline{O}\cdot\overline{k}$
i.e. $\overline{r}\cdot\hat{k} = 0$.
Hence, the given lines intersect each other and the vector equation of the plane determine by them is $\overline{r}\cdot\hat{k} = 0$.