Practice Set 3.1 Algebra 10th Std Maths Part 1 Answers Chapter 3 Arithmetic Progression

Question 1.
Which of the following sequences are A.P.? If they are A.P. find the common difference.

i. $2, 4, 6, 8, \ldots$

ii. $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

iii. $-10, -6, -2, 2, \ldots$

iv. $0.3, 0.33, 0.333, \ldots$

v. $0, -4, -8, -12, \ldots$

vi. $-\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, \ldots$

vii. $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

viii. $127, 132, 137, \ldots$

Solution:
i. The given sequence is 2, 4, 6, 8,…
Here, $t_1 = 2$, $t_2 = 4$, $t_3 = 6$, $t_4 = 8$
$\therefore t_2 - t_1 = 4 - 2 = 2$
$t_3 - t_2 = 6 - 4 = 2$
$t_4 - t_3 = 8 - 6 = 2$
$\therefore t_2 - t_1 = t_3 - t_2 = \ldots = 2 = d = \text{constant}$
The difference between two consecutive terms is constant.
∴ The given sequence is an A.P. and common difference (d) = 2.

ii. The given sequence is $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

Here, $t_1 = 2, t_2 = \frac{5}{2}, t_3 = 3, t_4 = \frac{7}{2}$

$\therefore \quad t_2 - t_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$

$t_3 - t_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$

$t_4 - t_3 = \frac{7}{2} - 3 = \frac{7-6}{2} = \frac{1}{2}$

$\therefore \quad t_2 - t_1 = t_3 - t_2 = \ldots = \frac{1}{2} = d = \text{constant}$

The difference between two consecutive terms is constant.
∴ The given sequence is an A.P. and common difference (d) = 12.

iii. The given sequence is -10, -6, -2, 2,…
Here, $t_1 = -10$, $t_2 = -6$, $t_3 = -2$, $t_4 = 2$
$\therefore t_2 - t_1 = -6 - (-10) = -6 + 10 = 4$
$t_3 - t_2 = -2 - (-6) = -2 + 6 = 4$
$t_4 - t_3 = 2 - (-2) = 2 + 2 = 4$
$\therefore t_2 - t_1 = t_3 - t_2 = \ldots = 4 = d = \text{constant}$
The difference between two consecutive terms is constant.
∴ The given sequence is an A.P. and common difference (d) = 4.

iv. The given sequence is 0.3, 0.33, 0.333,…
Here, $t_1 = 0.3$, $t_2 = 0.33$, $t_3 = 0.333$
$\therefore t_2 - t_1 = 0.33 - 0.3 = 0.03$
$t_3 - t_2 = 0.333 - 0.33 = 0.003$
$\therefore t_2 - t_1 \neq t_3 - t_2$
The difference between two consecutive terms is not constant.
∴ The given sequence is not an A.P.

v. The given sequence is 0, -4, -8, -12,…
Here, $t_1 = 0$, $t_2 = -4$, $t_3 = -8$, $t_4 = -12$
$\therefore t_2 - t_1 = -4 - 0 = -4$
$t_3 - t_2 = -8 - (-4) = -8 + 4 = -4$
$t_4 - t_3 = -12 - (-8) = -12 + 8 = -4$
$\therefore t_2 - t_1 = t_3 - t_2 = \ldots = -4 = d = \text{constant}$
The difference between two consecutive terms is constant.
∴ The given sequence is an A.P. and common difference (d) = -4.

vi. The given sequence is $-\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, \ldots$

Here, $t_1 = -\frac{1}{5}, t_2 = -\frac{1}{5}, t_3 = -\frac{1}{5}$

$\therefore \quad t_2 - t_1 = -\frac{1}{5} - \left(-\frac{1}{5}\right) = -\frac{1}{5} + \frac{1}{5} = 0$

$t_3 - t_2 = -\frac{1}{5} - \left(-\frac{1}{5}\right) = -\frac{1}{5} + \frac{1}{5} = 0$

$\therefore \quad t_2 - t_1 = t_3 - t_2 = \ldots = 0 = d = \text{constant}$

The difference between two consecutive terms is constant.
∴ The given sequence is an A.P. and common difference (d) = 0.

vii. The given sequence is
$3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$
Here, $t_1 = 3, t_2 = 3+\sqrt{2}, t_3 = 3+2\sqrt{2},$
$t_4 = 3+3\sqrt{2}$

$\therefore \quad t_2 - t_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$

$t_3 - t_2 = 3 + 2\sqrt{2} - \left(3+\sqrt{2}\right) = \sqrt{2}$

$t_4 - t_3 = 3 + 3\sqrt{2} - \left(3+2\sqrt{2}\right) = \sqrt{2}$

$\therefore \quad t_2 - t_1 = t_3 - t_2 = \ldots = \sqrt{2} = d = \text{constant}$

The difference between two consecutive terms is constant.
∴ The given sequence is an A.P. and common difference (d) = √2.

viii. The given sequence is 127, 132, 137,...
Here, $t_1 = 127$, $t_2 = 132$, $t_3 = 137$
∴ $t_2 - t_1 = 132 - 127 = 5$
$t_3 - t_2 = 137 - 132 = 5$
∴ $t_2 - t_1 = t_3 - t_2 = \ldots = 5 = d$ = constant
The difference between two consecutive terms is constant.
∴ The given sequence is an A.P. and common difference (d) = 5.

Question 2.
Write an A.P. whose first term is a and common difference is d in each of the following.
i. a = 10, d = 5
ii. a = -3, d = 0
iii. a = -7, d = 12
iv. a = -1.25, d = 3
v. a = 6, d = -3
vi. a = -19, d = -4
Solution:
i. a = 10, d = 5 ...[Given]
∴ $t_1 = a = 10$
$t_2 = t_1 + d = 10 + 5 = 15$
$t_3 = t_2 + d = 15 + 5 = 20$
$t_4 = t_3 + d = 20 + 5 = 25$
∴ The required A.P. is 10,15, 20, 25,...

ii. a = -3, d = 0 ...[Given]
∴ $t_1 = a = -3$
$t_2 = t_1 + d = -3 + 0 = -3$
$t_3 = t_2 + d = -3 + 0 = -3$
$t_4 = t_3 + d = -3 + 0 = -3$
∴ The required A.P. is -3, -3, -3, -3,...

iii. $a = -7, d = \frac{1}{2}$ ...[Given]

$\therefore \quad t_1 = a = -7$

$t_2 = t_1 + d = -7 + \frac{1}{2} = \frac{-14+1}{2} = \frac{-13}{2} = -6.5$

$t_3 = t_2 + d = -6.5 + \frac{1}{2} = -6.5 + 0.5 = -6$

$t_4 = t_3 + d = -6 + \frac{1}{2} = \frac{-12+1}{2} = \frac{-11}{2} = -5.5$

∴ The required A.P. is -7, – 6.5, – 6, – 5.5,

iv. a = -1.25, d = 3 ...[Given]
$t_1 = a = -1.25$
$t_2 = t_1 + d = -1.25 + 3 = 1.75$
$t_3 = t_2 + d = 1.75 + 3 = .4.75$
$t_4 = t_3 + d = 4.75 + 3 = 7.75$
∴ The required A.P. is -1.25, 1.75, 4.75, 7.75,...

v. a = 6, d = -3 ...[Given]
∴ $t_1 = a = 6$
$t_2 = t_1 + d = 6 - 3 = 3$
$t_3 = t_2 + d = 3 - 3 = 0$
$t_4 = t_3 + d = 0 - 3 = -3$
∴ The required A.P. is 6, 3, 0, -3,...

vi. a = -19, d = -4 ...[Given]
$t_1 = a = -19$
$t_2 = t_1 + d = -19 - 4 = -23$
$t_3 = t_2 + d = -23 - 4 = -27$
$t_4 = t_3 + d = -27 - 4 = -31$
∴ The required A.P. is -19, -23, -27, -31,...

Question 3.
Find the first term and common difference for each of the A.P.

i. 5, 1, −3, −7, ...
ii. 0.6, 0.9, 1.2, 1.5, ...
iii. 127, 135, 143, 151, ...
iv. $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots$

Solution:
i. The given A.P. is 5, 1,-3,-7,...
Here, $t_1 = 5$, $t_2 = 1$
∴ $a = t_1 = 5$ and
$d = t_2 - t_1 = 1 - 5 = -4$
∴ first term (a) = 5,
common difference (d) = -4

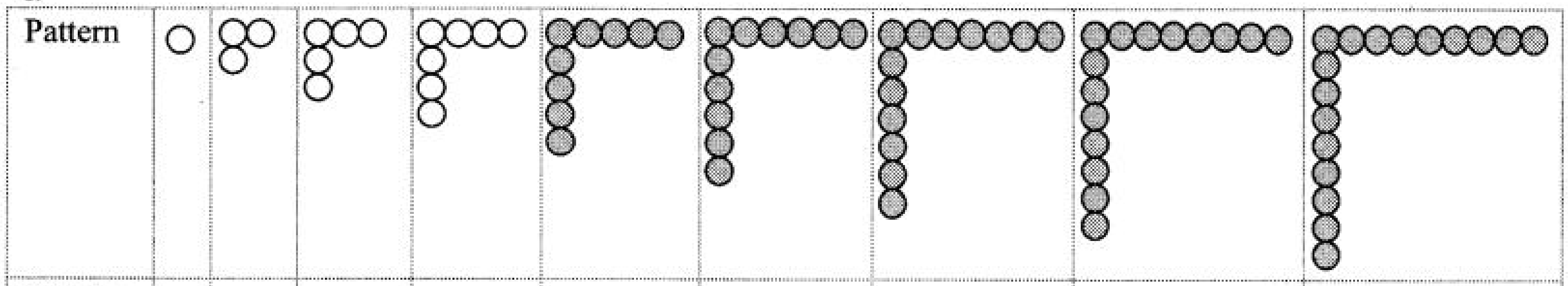
ii. The given A.P. is 0.6, 0.9, 1.2, 1.5,...
Here, $t_1 = 0.6$, $t_2 = 0.9$
∴ $a = t_1 = 0.6$ and
$d = t_2 - t_1 = 0.9 - 0.6 = 0.3$
∴ first term (a) = 0.6,
common difference (d) = 0.3

iii. The given A.P. is 127, 135, 143, 151,...
Here, $t_1 = 127$, $t_2 = 135$
∴ $a = t_1 = 127$ and
$d = t_2 - t_1 = 135 - 127 = 8$
∴ first term (a) = 127,
common difference (d) = 8

iv. The given A.P. is $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots$
Here, $t_1 = \frac{1}{4}$, $t_2 = \frac{3}{4}$
∴ $a = t_1 = \frac{1}{4}$ and
$d = t_2 - t_1 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$
∴ **first term (a) =** $\mathbf{\frac{1}{4}}$**,**
common difference (d) = $\mathbf{\frac{1}{2}}$

Question 1.
Complete the given pattern. Look at the pattern of the numbers. Try to find a rule to obtain the next number from its preceding number. Write the next numbers. (Textbook pg, no. 55 and 56)
Answer:

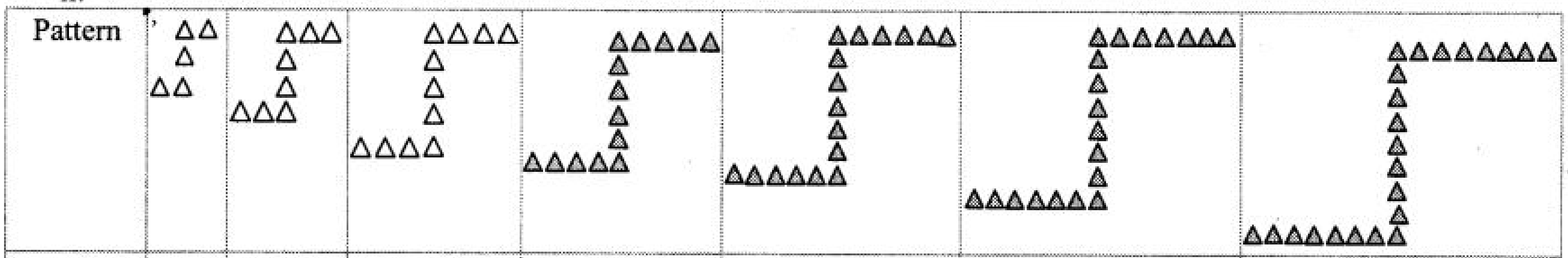
i.



Pattern									
Number of circles	1	3	5	7	9	11	13	15	17

Every pattern is formed by adding a circle in horizontal and vertical rows to the preceding pattern.
∴ The sequence for the above pattern is 1,3, 5, 7, 9,11,13,15,17,....

ii.



Pattern							
Number of triangles	5	8	11	14	17	20	23

Every pattern is formed by adding 2 triangles horizontally and 1 triangle vertically to the preceding pattern.
∴ The sequence for the above pattern is 5,8,11,14,17,20,23,...

Question 2.
Some sequences are given below. Show the positions of the terms by t_1, t_2, t_3,...
Answer:

i. 7, 7, 7, 7, ... Here $t_1 = 7$, $t_2 = \boxed{7}$, $t_3 = \boxed{7}$, ...

ii. –2, –6, –10, –14, ... Here $t_1 = -2$, $t_2 = \boxed{-6}$, $t_3 = \boxed{-10}$, ...

Question 3.
Some sequences are given below. Check whether there is any rule among the terms. Find the similarity between two sequences. To check the rule for the terms of the sequence look at the arrangements and fill the empty boxes suitably. (Textbook pg. no. 56 and 57)
i. .1,4,7,10,13,...
ii. 6,12,18,24,...
iii. 3,3,3,3,...
iv. 4, 16, 64,...
v. -1, -1.5, -2, -2.5,...
vi. 1^3, 2^3, 3^3, 4^3
Answer:

i. 1, 4, 7, 10, $\boxed{13}$, $\boxed{16}$, ...

$1 (+3)$, $4 (+3)$, $7 (+3)$, $10 (+3)$, $\boxed{13} (+3)$

By adding 3 in each term, we get the next term of the sequence.

ii. 6, 12, 18, 24, $\boxed{30}$, $\boxed{36}$, ...

$6+6$, $12+6$, $18+6$, $\boxed{24+6}$, $\boxed{30+6}$

By adding 6 in each term, we get the next term of the sequence.

iii. 3, 3, 3, 3, $\boxed{3}$, $\boxed{3}$, ...

$3+0$, $3+0$, $3+0$, $\boxed{3+0}$, $\boxed{3+0}$

By adding 0 in each term, we get the next term of the sequence.

iv. 4, 16, 64, $\boxed{256}$, $\boxed{1024}$, $\boxed{4096}$, ...

4×4, 16×4, 64×4, $\boxed{256} \times 4$, $\boxed{1024} \times 4$

By multiplying each term by 4, we get the next term of the sequence.

v. -1, -1.5, -2, -2.5, $\boxed{-3}$,....

$(-1) + (-0.5)$, $-1.5 + (-0.5)$, $-2 + (-0.5)$, $-2.5 + (-0.5)$

By adding -0.5 in each term, we get the next term of the sequence.

vi. $1^3, 2^3, 3^3, ...$

$(\text{next term})^3 = (\text{previous term} + 1)^3$

The similarity in the sequences i., ii., iii. and v. is that the next term is obtained by adding a particular fixed number to the previous term.

Note : A Geometric Progression is a sequence in which the ratio of any two consecutive terms is a constant,

i.e. in a G.P., $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \ldots = \frac{t_n}{t_{n-1}} = \text{constant}$

Sequence iv. is a geometric progression.

Question 4.
Write one example of finite and infinite A.P. each. (Textbook pg. no. 59)
Answer:
Finite A.P.:
Even natural numbers from 4 to 50:
4, 6, 8, ... 50.
Infinite A. P.:
Positive multiples of 5:
5, 10, 15, ...

Practice Set 3.2 Algebra 10th Std Maths Part 1 Answers Chapter 3 Arithmetic Progression

Question 1.
Write the correct number in the given boxes from the following A.P.

i. 1, 8, 15, 22, ...

Here $a = \boxed{}$, $t_1 = \boxed{}$, $t_2 = \boxed{}$, $t_3 = \boxed{}$, ...

$t_2 - t_1 = \boxed{} - \boxed{} = \boxed{}$

$t_3 - t_2 = \boxed{} - \boxed{} = \boxed{}$ $\therefore d = \boxed{}$

ii. 3, 6, 9, 12, ...

Here $t_1 = \boxed{}$, $t_2 = \boxed{}$, $t_3 = \boxed{}$, $t_4 = \boxed{}$, ...

$t_2 - t_1 = \boxed{}$, $t_3 - t_2 = \boxed{}$ $\therefore d = \boxed{}$

iii. –3, –8, –13, –18, ...

Here $t_1 = \boxed{}$, $t_2 = \boxed{}$, $t_3 = \boxed{}$, $t_4 = \boxed{}$, ...

$t_2 - t_1 = \boxed{}$, $t_3 - t_2 = \boxed{}$ $\therefore a = \boxed{}$, $d = \boxed{}$

iv. 70, 60, 50, 40, ...

Here $t_1 = \boxed{}$, $t_2 = \boxed{}$, $t_3 = \boxed{}$, ...

$\therefore$ $a = \boxed{}$, $d = \boxed{}$

Solution:

i. 1, 8, 15, 22, ...

Here $a = \boxed{1}$, $t_1 = \boxed{1}$, $t_2 = \boxed{8}$, $t_3 = \boxed{15}$, ...

$t_2 - t_1 = \boxed{8} - \boxed{1} = \boxed{7}$

$t_3 - t_2 = \boxed{15} - \boxed{8} = \boxed{7}$ $\therefore d = \boxed{7}$

ii. 3, 6, 9, 12, ...

Here $t_1 = \boxed{3}$, $t_2 = \boxed{6}$, $t_3 = \boxed{9}$, $t_4 = \boxed{12}$, ...

$t_2 - t_1 = \boxed{3}$, $t_3 - t_2 = \boxed{3}$ $\therefore d = \boxed{3}$

iii. –3, –8, –13, –18, ...

Here $t_1 = \boxed{-3}$, $t_2 = \boxed{-8}$, $t_3 = \boxed{-13}$, $t_4 = \boxed{-18}$, ...

$t_2 - t_1 = \boxed{-5}$, $t_3 - t_2 = \boxed{-5}$ $\therefore a = \boxed{-3}$, $d = \boxed{-5}$

iv. 70, 60, 50, 40, ...

Here $t_1 = \boxed{70}$, $t_2 = \boxed{60}$, $t_3 = \boxed{50}$, ...

$\therefore$ $a = \boxed{70}$, $d = \boxed{-10}$

Question 2.
Decide whether following sequence is an A.P., if so find the 20th term of the progression.
-12, -5, 2, 9,16, 23,30,…
Solution:
i. The given sequence is
-12, -5,2, 9, 16, 23,30,…
Here, $t_1 = -12$, $t_2 = -5$, $t_3 = 2$, $t_4 = 9$
$\therefore t_2 – t_1 – 5 – (-12) – 5 + 12 = 7$
$t_3 – t_2 = 2 – (-5) = 2 + 5 = 7$
$\therefore t_4 – t_3 – 9 – 2 = 7$
$\therefore t_2 – t_1 = t_3 – t_2 = … = 7 = d$ = constant
The difference between two consecutive terms is constant.
∴ The given sequence is an A.P.

ii. $t_n = a + (n – 1)d$
$\therefore t_{20} = -12 + (20 – 1)7$ …[∵ $a = -12$, $d = 7$]
$= -12 + 19 \times 7$
$= -12 + 133$
$\therefore t_{20} = 121$
∴ 20th term of the given A.P. is 121.

Question 3.
Given Arithmetic Progression is 12, 16, 20, 24, … Find the 24_{th} term of this progression.
Solution:
The given A.P. is 12, 16, 20, 24,…
Here, a = 12, d = 16 – 12 = 4 Since,
$t_n = a + (n – 1)d$
∴ $t_{24} = 12 + (24 – 1)4$
$= 12 + 23 \times 4$
$= 12 + 92$
∴ $t_{24} = 104$
∴ 24th term of the given A.P. is 104.

Question 4.
Find the 19th term of the following A.P. 7,13,19,25…..
Solution:
The given A.P. is 7, 13, 19, 25,…
Here, a = 7, d = 13 – 7 = 6
Since, $t_n = a + (n – 1)d$
∴ $t_{19} = 7 + (19 – 1)6$
$= 7 + 18 \times 6$
$= 7 + 108$
∴ $t_{19} = 115$
∴ 19_{th} term of the given A.P. is 115.

Question 5.
Find the 27_{th} term of the following A.P. 9,4,-1,-6,-11,…
Solution:
The given A.P. is 9, 4, -1, -6, -11,…
Here, a = 9, d = 4- 9 = -5
Since, $t_n = a + (n – 1)d$
∴ $t_{27} = 9 + (27 – 1)(-5)$
$= 9 + 26 \times (-5)$
$= 9 – 130$
∴ $t_{27} = -121$
∴ 27_{th} term of the given A.P. is -121.

Question 6.
Find how many three digit natural numbers are divisible by 5.
Solution:
The three digit natural numbers divisible by
5 are 100, 105, 110, …,995
The above sequence is an A.P.
∴ a = 100, d = 105 – 100 = 5
Let the number of terms in the A.P. be n.
Then, $t_n = 995$
Since, $t_n = a + (n – 1)d$
∴ $995 = 100 +(n – 1)5$
∴ $995 – 100 = (n – 1)5$
∴ $895 = (n – 1)5$
∴ n – 1 = 8955
∴ $n – 1 = 179$
∴ $n = 179 + 1 = 180$
∴ There are 180 three digit natural numbers which are divisible by 5.

Question 7.
The 11th term and the 21st term of an A.P. are 16 and 29 respectively, then find the 41st term of that A.P.
Solution:
Bor an A.P., let a be the first term and d be the common difference,
$t_{11} = 16$, $t_{21} = 29$ …[Given]
$t_n = a + (n – 1)d$
∴ $t_{11}, = a + (11 – 1)d$
∴ $16 = a + 10d$
i.e. $a + 10d = 16$ …(i)
Also, $t_{21} = a + (21 – 1)d$
∴ $29 = a + 20d$
i.e. $a + 20d = 29$ …(ii)

Subtracting equation (i) from (ii), we get a

$$\begin{array}{r} a + 20d = 29 \\ a + 10d = 16 \\ - \quad - \quad - \\ \hline 10d = 13 \end{array}$$

$\therefore \quad d = \frac{13}{10}$

Substituting $d = \frac{13}{10}$ in equation (i), we get

$$a + 10\left(\frac{13}{10}\right) = 16$$

$\therefore \quad a + 13 = 16$

$\therefore \quad a = 16 - 13 = 3$

$$t_{41} = 3 + (41 - 1)\left(\frac{13}{10}\right)$$

$$= 3 + 40 \times \frac{13}{10}$$

$$= 3 + 52$$

$\therefore \quad t_{41} = 55$

$\therefore$ **41^{st} term of the A.P. is 55.**

Question 8.
8. 11, 8, 5, 2, … In this A.P. which term is number-151?
Solution:
The given A.P. is 11, 8, 5, 2,…
Here, a = 11, d = 8 – 11 = -3
Let the n_{th} term of the given A.P. be -151.
Then, t_n = – 151
Since, t_n = a + (n – 1)d
∴ -151= 11 + (n – 1)(-3)
∴ -151 – 11 =(n – 1)(-3)
∴ -162 = (n – 1)(-3)
∴ n – 1 = -162-3
∴ n – 1 = 54
∴ n = 54 + 1 = 55
∴ 55th term of the given A.P. is -151.

Question 9.
In the natural numbers from 10 to 250, how many are divisible by 4?
Solution:
The natural numbers from 10 to 250 divisible
by 4 are 12, 16, 20, …,248
The above sequence is an A.P.
∴ a = 12, d = 16 – 12 = 4
Let the number of terms in the A.P. be n.
Then, t_n = 248
Since, t_n = a + (n – 1)d
∴ 248 = 12 + (n – 1)4
∴ 248 – 12 = (n – 1)4
∴ 236 = (n – 1)4
∴ n – 1 = 2364
∴ n – 1 = 59
∴ n = 59 + 1 = 60
∴ There are 60 natural numbers from 10 to 250 which are divisible by 4.

Question 10.
In an A.P. 17_{th} term is 7 more than its 10th term. Find the common difference.
Solution:
For an A.P., let a be the first term and d be the common difference.
According to the given condition,
$t_{17} = t_{10} + 7$
∴ a + (17 – 1)d = a + (10 – 1)d + 7 …[∵ t_n = a + (n – 1)d]
∴ a + 16d = a + 9d + 7
∴ a + 16d – a – 9d = 7
∴ 7d = 7
∴ d = 77 = 1
∴ The common difference is 1.

Question 1.
Kabir's mother keeps a record of his height on each birthday. When he was one year old, his height was 70 cm, at 2 years he was 80 cm tall and 3 years he was 90 cm tall. His aunt Meera was studying in the 10th class. She said, "it seems like Kabir's height grows in Arithmetic Progression". Assuming this, she calculated how tall Kabir will be at the age of 15 years when he is in 10th! She was shocked to find it. You too assume that Kabir grows in A.P. and find out his height at the age of 15 years. (Textbook pg. no. 63)
Solution:
Height of Kabir when he was 1 year old = 70 cm Height of Kabir when he was 2 years old = 80 cm
Height of Kabir when he was 3 years old = 90 cm The heights of Kabir form an A.P.
Here, a = 70, d = 80 – 70 = 10
We have to find height of Kabir at the age of 15years i.e. t_{15}.
Now, tn = a + (n – 1)d
∴ t_{15} = 70 + (15 – 1)10
= 70 + 14 × 10 = 70 + 140
∴ t_{15} = 210
∴ The height of Kabir at the age of 15 years will be 210 cm.

Question 2.
Is 5, 8, 11, 14, …. an A.P.? If so then what will be the 100_{th} term? Check whether 92 is in this A.P.? Is number 61 in this A.P.? (Textbook pg. no, 62)
Solution:
i. The given sequence is
5, 8,11,14,…
Here, t_1 = 5, t_2 = 8, t_3 = 11, t_4 = 14
∴ $t_2 – t_1$ = 8 – 5 = 3
$t_3 – t_2$ = 11 – 8 = 3
$t_4 – t_3$ = 14 – 11 = 3
∴ $t_2 – t_1 = t_3 – t_2 = t_4 – t_3$ = 3 = d = constant
The difference between two consecutive terms is constant
∴ The given sequence is an A.P.

ii. t_n = a + (n – 1)d
∴ t_{100} = 5 + (100 – 1)3 …[∵ a = 5, d = 3]
= 5 + 99 × 3
= 5 + 297
∴ t_{100} = 302
∴ 100_{th} term of the given A.P. is 302.

iii. To check whether 92 is in given A.P., let t_n = 92
∴ t_n = a + (n – 1)d
∴ 92 = 5 + (n – 1)3
∴ 92 = 5 + 3n – 3
∴ 92 = 2 + 3n
∴ 90 = 3n
∴ n = 903 = 30
∴ 92 is the 30th term of given A.P.

iv. To check whether 61 is in given A.P., let t_n = 61
61 = 5 + (n – 1)3
∴ 61 = 5 + 3n – 3
∴ 61 = 2 + 3n
∴ 61 – 2 = 3n
∴ 59 = 3n
∴ n = 593
But, n is natural number 59
∴ n ≠ 593
∴ 61 is not in given A.P.

Practice Set 3.3 Algebra 10th Std Maths Part 1 Answers Chapter 3 Arithmetic Progression

Arithmetic Progression Practice Set 3.3 Question 1.
First term and common difference of an A.P. are 6 and 3 respectively; find S_{27}.

Solution:

$a = 6,\ d = 3,\ S_{27} = ?$

$$S_n = \frac{n}{2}\left[\boxed{2a} + (n-1)d\right]$$

$$\therefore \quad S_{27} = \frac{27}{2}\left[12 + (27-1)\boxed{3}\right]$$

$$= \frac{27}{2} \times \boxed{90}$$

$$= 27 \times 45 = \boxed{1215}$$

Arithmetic Progression Class 10 Practice Set 3.3 Question 2.
Find the sum of first 123 even natural numbers.
Solution:
The even natural numbers are 2, 4, 6, 8,...
The above sequence is an A.P.
∴ a = 2, d = 4 – 2 = 2, n = 123

$$\text{Now, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_n = \frac{123}{2}[2(2) + (123-1)(2)]$$

$$= \frac{123}{2}[2(2) + 122(2)]$$

$$= \frac{123}{2} \times 2[2 + 122]$$

$$= 123 \times 124$$

$$= 15252$$

∴ The sum of first 123 even natural numbers is 15252.

Practice Set 3.3 Question 3.
Find the sum of all even numbers between 1 and 350.
Solution:
The even numbers between 1 and 350 are 2, 4, 6,..., 348.
The above sequence is an A.P.
∴ a = 2, d = 4 – 2 = 2, t_n = 348
Since, t_n = a + (n – 1)d
∴ 348 = 2 + (n – 1)2
∴ 348 – 2 = (n – 1)2
∴ 346 = (n – 1)2
∴ n – 1 = $\frac{346}{2}$
∴ n – 1 = 173
∴ n = 173 + 1 = 174
Now, S_n = $\frac{n}{2}$ [2a + (n – 1)d]
∴ S_{174} = $\frac{174}{2}$ [2 (2) + (174 – 1)2]
= 87(4 + 173 × 2)
= 87(4 + 346)
= 87 × 350
∴ S_{174} = 30450
∴ The sum of all even numbers between 1 and 350 is 30450.

Arithmetic Progression 3.3 Question 4.
In an A.P. 19^{th} term is 52 and 38^{th} term is 128, find sum of first 56 terms.
Solution:
For an A.P., let a be the first term and d be the common difference.
t_{19} = 52, t_{38} = 128 ...[Given]
Since, t_n = a + (n – 1)d
∴ t_{19} = a + (19 – 1)d
∴ 52 = a + 18d
i.e. a + 18d = 52 ...(i)
Also, t_{38} = a + (38 – 1)d
∴ 128 = a + 37d
i.e. a + 37d = 128 ...(ii)

Adding equations (i) and (ii), we get

$$\begin{array}{rl} & a + 18d = 52 \\ + & a + 37d = 128 \\ \hline & 2a + 55d = 180 \end{array} \quad \ldots\text{(iii)}$$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{56} = \frac{56}{2}[2a + (56-1)d]$

$= 28(2a + 55d)$

$= 28 \times 180 \quad \ldots$[From (iii)]

$\therefore \quad S_{56} = 5040$

$\therefore$ **The sum of first 56 terms is 5040.**

3 Arithmetic Progression Question 5.
Complete the following activity to find the sum of natural numbers between 1 to 140 which are divisible by 4.
Solution:

From 1 to 140, natural numbers divisible by 4

↓

4, 8, ..., 136

↓

How many numbers? $\therefore$ n = $\boxed{34}$

↓

n = $\boxed{34}$, a = $\boxed{4}$, d = $\boxed{4}$

↓

$t_n = a + (n-1)d$

↓

$136 = \boxed{4} + (n-1) \times \boxed{4}$

→ n = $\boxed{34}$ → $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{\boxed{34}} = \frac{\boxed{34}}{2}[\,2 \times 4 + (34-1)\,4\,] = \boxed{2380}$

Sum of numbers from 1 to 140, which are divisible by 4 = $\boxed{2380}$

10th Algebra Practice Set 3.3 Question 6.
Sum of first 55 terms in an A.P. is 3300, find its 28^{th} term.
Solution:
For an A.P., let a be the first term and d be the common difference.
$S_{55} = 3300$ ...[Given]
Since, $S_n = \frac{n}{2}[2a + (n-1)d]$
$\therefore S_{55} = \frac{55}{2}[2a + (55-1)d]$

$\therefore \quad 3300 = \frac{55}{2}(2a + 54d)$

$\therefore \quad 3300 = \frac{55}{2} \times 2(a + 27d)$

$\therefore \quad 3300 = 55(a + 27d)$

$\therefore \quad a + 27d = \frac{3300}{55}$

$\therefore \quad a + 27d = 60 \quad \ldots$(i)

Now, $t_n = a + (n-1)d$

$\therefore \quad t_{28} = a + (28-1)d = a + 27d$

$\therefore \quad t_{28} = 60 \quad \ldots$[From (i)]

$\therefore$ **28^{th} term of the A.P. is 60.**

Arithmetic Practice Set Question 7.
In an A.P. sum of three consecutive terms is 27 and their product is 504, find the terms. (Assume that three consecutive terms in A.P. are a – d, a, a + d.)
Solution:
Let the three consecutive terms in an A.P. be

a – d, a and a + d.
According to the first condition,
a – d + a + a + d = 27
∴ 3a = 27
∴ a = $\frac{27}{3}$
∴ a = 9 ….(i)
According to the second condition,
(a – d) a (a + d) = 504
∴ $a(a^2 - d^2) = 504$
∴ $9(a^2 - d^2) = 504$ …[From (i)]
∴ $9(81 - d^2) = 504$
∴ $81 - d^2 = \frac{504}{9}$
∴ $81 - d^2 = 56$
∴ $d^2 = 81 - 56$
∴ $d^2 = 25$
Taking square root of both sides, we get
d = ± 5
When d = 5 and a =9,
a – d 9 – 5 = 4
a = 9
a + d 9 + 5 = 14
When d = -5 and a = 9,
a – d = 9 – (-5) = 9 + 5 = 14
a = 9
a + d = 9 – 5 = 4
∴ The three consecutive terms are 4, 9 and 14 or 14, 9 and 4.

10th Maths 1 Practice Set 3.3 Question 8.
Find four consecutive terms in an A.P. whose sum is 12 and sum of 3rd and 4th term is 14. (Assume the four consecutive terms in A.P. are a – d, a, a + d, a + 2d.)
Solution:
Let the four consecutive terms in an A.P. be
a – d, a, a + d and a + 2d.
According to the first condition,
a – d + a + a + d + a + 2d = 12
∴ 4a + 2d =12
∴ 2(2a + d) = 12
∴ 2a + d = $\frac{12}{2}$
∴ 2a + d = 6 …(i)
According to the second condition,
a + d + a + 2d = 14
∴ 2a + 3d = 14 …(ii)
Subtracting equation (i) from (ii), we get

$$\begin{array}{r} 2a + 3d = 14 \\ 2a + \ \ d = 6 \\ - \quad - \quad - \\ \hline 2d = 8 \end{array}$$

∴ $d = \frac{8}{2} = 4$

Substituting d = 4 in equation (i), we get

$2a + 4 = 6$

∴ $2a = 6 - 4 = 2$

∴ $a = \frac{2}{2} = 1$

∴ $a - d = 1 - 4 = -3$

$a = 1$

$a + d = 1 + 4 = 5$

$a + 2d = 1 + 2(4)$

$= 1 + 8 = 9$

∴ The four consecutive terms are -3,1,5 and 9.

Math 1 Practice Set 3.3 Question 9.
If the 9th term of an A.P. is zero, then show that the 29th term is twice the 19th term.
To prove: $t_{29} = 2t_{19}$
Proof:
For an A.P., let a be the first term and d be the common difference.
$t_9 = 0$ …[Given]
Since, tn = a + (n – 1)d
∴ $t_9 = a + (9 - 1)d$
∴ 0 = a + 8d
∴ a = -8d …(i)

Also, t_{19} = a + (19 – 1)d
= a + 18d
= -8d + 18d … [From (i)]
∴ t_{19} = 10d …(ii)
and t_{29} = a + (29 – 1)d
= a + 28d
= -8d + 28d …[From (i)]
∴ t_{29} = 20d = 2(10d)
∴ t_{29} = 2(t_{19}) … [From (ii)]
∴ The 29^{th} term is twice the 19th term.

10 Class Math Part 1 Practice Set 3.3 Question 1.
Find the sum of all odd numbers from 1 to 150. (Textbook pg, no. 71)
Solution:
Odd numbers from 1 to 150 are 1,3, 5, 7,…, 149
Here, difference between any two consecutive terms is 2.
∴ It is an A.P.
∴ a = 1, d = 2
Let us find how many odd numbers are there from 1 to 150, i.e. find the value of n if
t_n = 149
t_n = a + (n – 1)d
∴ 149 = 1 + (n – 1)2
∴ 149 – 1 = (n – 1)2
∴ 1482 = n – 1
∴ 74 = n – 1
∴ n = 74 + 1 = 75

ii. Now, let’s find the sum of 75 numbers
i. e. 1 + 3 + 5 + 7 + … + 149

Method I:

Here, a = 1, d = 2, n = 75

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_{75} = \boxed{\frac{75}{2}\left[2(1) + (75-1)2\right]}$$

$$= \frac{75}{2}(2 + 74 \times 2)$$

$$= \frac{75}{2}(2 + 148)$$

$$= \frac{75}{2}(150)$$

$$= \boxed{75} \times \boxed{75}$$

$$\therefore \quad S_{75} = \boxed{5625}$$

Method II:

Here, $t_1 = a = 1$, $t_n = 149$, n = 75

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$\therefore \quad S_{75} = \frac{75}{2}(1 + 149)$$

$$= \frac{75}{2} \times 150$$

$$= \boxed{75} \times \boxed{75}$$

$$\therefore \quad S_{75} = \boxed{5625}$$

Practice Set 3.4 Algebra 10th Std Maths Part 1 Answers Chapter 3 Arithmetic Progression

Question 1.
On 1^{st} Jan 2016, Sanika decides to save ₹ 10, ₹ 11 on second day, ₹ 12 on third day. If she decides to save like this, then on 31st Dec 2016 what would be her total saving?
Solution:

i. Sanika's daily savings of 2016 are as follows:
10, 11, 12, ...
The above sequence is an A.P.
$\therefore$ $a = 10$, $d = 11 - 10 = 1$,
$n = 366$ ...[$\because$ 2016 is a leap year]

ii. $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{366} = \frac{366}{2}[2(10) + (366-1)1]$
$= 183(20 + 365 \times 1)$
$= 183(20 + 365)$
$= 183 \times 385$
$\therefore$ $S_{366} = 70455$

$\therefore$ Sanika's total saving on 31st December 2016 would be ₹ 70455.

Question 2.
A man borrows ₹ 8000 and agrees to repay with a total interest of ₹ 1360 in 12 monthly instalments. Each instalment being less than the preceding one by ₹ 40. Find the amount of the first and last instalment.
Solution:
i. The instalments are in A.P.
Amount repaid in 12 instalments (S_{12})
= Amount borrowed + total interest
= 8000 + 1360
$\therefore$ $S_{12} = 9360$
Number of instalments (n) = 12
Each instalment is less than the preceding one by ₹ 40.
$\therefore$ d = -40

ii. $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{12} = \frac{12}{2}[2a + (12-1)(-40)]$
$\therefore$ $9360 = 6[2a + (11)(-40)]$
$\therefore$ $9360 = 6(2a - 440)$
$\therefore$ $\frac{9360}{6} = 2a - 440$
$\therefore$ $1560 = 2a - 440$
$\therefore$ $1560 + 440 = 2a$
$\therefore$ $2000 = 2a$
$\therefore$ $a = \frac{2000}{2}$
$\therefore$ $a = 1000$

iii. $t_n = a + (n-1)d$
$\therefore$ $t_{12} = 1000 + (12-1)(-40)$
$= 1000 + 11(-40)$
$= 1000 - 440$
$\therefore$ $t_{12} = 560$

$\therefore$ Amount of the first instalment is ₹ 1000 and that of the last instalment is ₹ 560.

Question 3.
Sachin invested in a national saving certificate scheme. In the first year he invested ₹ 5000, in the second year ₹ 7000, in the third year ₹ 9000 and so on. Find the total amount that he invested in 12 years.
Solution:
i. Amount invested by Sachin in each year are as follows:
5000, 7000, 9000, …
The above sequence is an A.P.
$\therefore$ a = 5000, d = 7000 – 5000 = 2000, n = 12

ii. $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{12} = \frac{12}{2}[2(5000) + (12-1)2000]$
$= 6(10000 + 11 \times 2000)$
$= 6(10000 + 22000)$
$= 6(32000)$
$\therefore$ $S_{12} = 192000$

$\therefore$ The total amount invested by Sachin in 12 years is ₹ 1,92,000.

Question 4.
There is an auditorium with 27 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row and so on. Find the number of seats in the 15th row and also find how many total seats are there in the auditorium?
Solution:
i. The number of seats arranged row-wise are as follows:
20, 22, 24,
The above sequence is an A.P.
∴ a = 20, d = 22 – 20 = 2, n = 27

ii. $t_n = a + (n – 1)d$
∴ $t_{15} = 20 + (15 – 1)2$
= 20 + 14 × 2
= 20 + 28
∴ $t_{15} = 48$
∴ The number of seats in the 15th row is 48.

iii. $S_n = \frac{n}{2}[2a + (n-1)d]$

∴ $S_{27} = \frac{27}{2}[2(20) + (27-1)2]$

$= \frac{27}{2}(40 + 26 \times 2)$

$= \frac{27}{2}(40 + 52)$

$= \frac{27}{2} \times 92$

$= 27 \times 46$

∴ $S_{27} = 1242$

∴ Total seats in the auditorium are 1242.

Question 5.
Kargil's temperature was recorded in a week from Monday to Saturday. All readings were in A.P. The sum of temperatures of Monday and Saturday was 5°C more than sum of temperatures of Tuesday and Saturday. If temperature of Wednesday was -30° Celsius then find the temperature on the other five days.
Solution:
Let the temperatures from Monday to Saturday in A.P. be
a, a + d, a + 2d, a + 3d, a + 4d, a + 5d.
According to the first condition,
(a) + (a + 5d) = (a + d) + (a + 5d) + 5°
∴ d = -5°
According to the second condition,
a + 2d = -30°
∴ a + 2(-5°) = -30°
∴ a – 10° = -30°
∴ a = -30° + 10° = -20°
∴ a + d = -20° – 5° = – 25°
a + 3d = -20° + 3(- 5°) = -20° – 15° = -35°
a + 4d = -20° + 4(-5°) = -20° – 20° = -40°
a + 5d = -20° + 5(-5°) = -20° – 25° = -45°
∴ The temperatures on the other five days are
-20°C, -25° C, -35° C, -40° C and -45° C.

Question 6.
On the world environment day tree plantation programme was arranged on a land which is triangular in shape. Trees are planted such that in the first row there is one tree, in the second row there are two trees, in the third row three trees and so on. Find the total number of trees in the 25 rows.
Solution:
i. The number of frees planted row-wise are as follows:
1,2,3,...
The above sequence is an A.P.
∴ a = 1, d = 2 – 1 = 1,n = 25

ii. $S_n = \frac{n}{2}[2a + (n-1)d]$

∴ $S_{25} = \frac{25}{2}[2(1) + (25-1)1]$

$= \frac{25}{2}(2 + 24) = \frac{25}{2} \times 26 = 25 \times 13 = 325$

∴ The total number of trees in 25 rows are 325.

Problem Set 3 Algebra 10th Std Maths Part 1 Answers Chapter 3 Arithmetic Progression

Question 1.
Choose the correct alternative answer for each of the following sub questions.

i. The sequence – 10,- 6,- 2, 2, ...
(A) is an A.P. Reason d = – 16
(B) is an A.P. Reason d = 4
(C) is an A.P. Reason d = – 4
(D) is not an A.P.
Answer:
(B)

ii. First four terms of an A.P. are ..., whose first term is -2 and common difference is -2.
(A) -2, 0, 2, 4
(B) -2, 4,- 8, 16
(C) -2, -4, -6, -8
(D) -2,-4, -8, -16
Answer:
(C)

iii. What is the sum of the first 30 natural numbers?
(A) 464
(B) 465
(C) 462
(D) 461
Answer:
(B)

iv. For an given A.P. t_7 = 4, d = – 4, then a = ...
(A) 6
(B) 7
(C) 20
(D) 28
Answer:
(D)

v. For an given A.P. a = 3.5, d = 0, n = 101, then t_n = ...
(A) 0
(B) 3.5
(c) 103.5
(D) 104.5
Answer:
(B)

vi. In an A.P. first two terms are – 3, 4, then 21st term is ....
(A) -143
(B) 143
(C) 137
(D) 17
Answer:
(C)

vii. If for any A.P. d = 5, then $t_{18} – t_{13}$ = ...
(A) 5
(B) 20
(C) 25
(D) 30
Answer:
(C)

viii. Sum of first five multiples of 3 is ...
(A) 45
(B) 55
(C) 15
(D) 75
Answer:
(A)

ix. 15, 10, 5, … In this A.P. sum of first 10 terms is…
(A) -75
(B) -125
(C) 75
(D) 125
Answer:
(A)

x. In an A.P. 1st term is 1 and the last term is 20. The sum of all terms is 399, then n = ….
(A) 42
(B) 38
(C) 21
(D) 19
Answer:
(B)

Hints:

iii. First 30 natural numbers are
1, 2, 3, …, 30
The above sequence is an A.P.

$\therefore\ t_1 = 1, t_{30} = 30$

$\therefore\ S_{30} = \frac{30}{2}(1 + 30)$

$= 15 \times 31 = 465$

vii. $t_{18} - t_{13} = a + (18 - 1)d - [a + (13 - 1)d]$

$= a + 17d - a - 12d$

$= 5d = 5 \times 5 = 25$

x. $S_n = \frac{n}{2}$ (first term + last term)

$\therefore\ 399 = \frac{n}{2}(1 + 20)$

$\therefore\ 399 \times 2 = 21n$

$\therefore\ n = \frac{798}{21} = 38$

Question 2.
Find the fourth term from the end in an
A.P.: -11, -8, -5, …, 49.
Solution:
The given A.P. is
-11,-8,-5, ……. 49
Reversing the A.P., we get 49, …, -5, -8, -11
Here, a = 49, d = -11 -(-8) = -11 + 8 = -3
Since, $t_n = a + (n – 1)d$
∴ $t_4 = 49 + (4 – 1)(-3)$
= 49 + (3) (-3)
= 49 – 9
= 40
∴ Fourth term from the end in the given A.P. is 40.
[Note: If an AY. is reversed, then the resulting sequence is also an A.P.]

Question 3.
In an A.P. the 10th term is 46, sum of the 5th and 7th term is 52. Find the A.P.
Solution:
For an A.P., let a be the first term and d be the common difference.
$t_{10} = 46$, $t_5 + t_7 = 52$ …[Given]
Since, $t_n = a + (n – 1)d$
∴ $t_{10} = a + (10 – 1)d$
∴ 46 = a + 9d
i. e. a + 9d = 46 …(i)
Also, $t_5 + t_7 = 52$
∴ a + (5 – 1)d + a + (7 – 1)d = 52
∴ a + 4d + a + 6d = 52
∴ 2a + 10d = 52
∴ 2 (a + 5d) = 52
∴ a + 5d = 522
∴ a + 5d = 26 …(ii)
Subtracting equation (ii) from (i), we get

$$\begin{array}{r} a + 9d = 46 \\ a + 5d = 26 \\ - \quad - \quad - \\ \hline 4d = 20 \end{array}$$

$\therefore \quad d = \frac{20}{4} = 5$

Substituting d = 5 in equation (ii), we get
a + 5(5) = 26
∴ a + 25 = 26
∴ a = 26 – 25 = 1
t_1 = a = 1
t_2 = t_1 + d = 1 + 5 = 6
t_3 = t_2 + d = 6 + 5 = 11
t_4 = t_3 + d = 11 + 5 = 16
The required A.P. is 1,6,11,16,....

Question 4.
The A.P. in which 4th term is -15 and 9th term is -30. Find the sum of the first 10 numbers.
Solution:
t_4 = -15, t9 = – 30 ...[Given]
Since, t_n = a + (n – 1)d
∴ t_4 = a + (4 – 1)d
∴ – 15 = a + 3d
i. e. a + 3d = -15 ...(i)
Also, t_9 = a + (9 – 1)d
∴ -30 = a + 8d
i.e. a + 8d = -30 ...(ii)

Subtracting equation (i) from (ii), we get

$$\begin{array}{r} a + 8d = -30 \\ a + 3d = -15 \\ - \quad - \quad + \\ \hline 5d = -15 \end{array}$$

$\therefore \quad d = \frac{-15}{5} = -3$

Substituting d = –3 in equation (i), we get

$a + 3(-3) = -15$

$\therefore \quad a - 9 = -15$

$\therefore \quad a = -15 + 9 = -6$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{10} = \frac{10}{2}[2(-6) + (10-1)(-3)]$

$= 5(-12 + 9 \times -3)$

$= 5(-12 - 27)$

$= 5 \times (-39)$

$\therefore \quad S_{10} = -195$

∴ The sum of the first 10 numbers is -195.

Question 5.
Two given A.P.'s are 9, 7, 5, ... and 24, 21, 18, ... If nth term of both the .progressions are equal then find the value of n and n,h term.
Solution:
The first A.P. is 9, 7, 5,...
Here, a = 9, d = 7- 9 = -2
∴nth term = a + (n – 1)d
= 9 + (n – 1) (-2)
= 9 – 2n + 2
= 11 – 2n
The second A.P. is 24, 21, 18, ...
Here, a = 24, d = 21 – 24 = – 3
∴ nth term = a + (n – 1)d
= 24 + (n – 1) (-3)
= 24 – 3n + 3
= 27 – 3n
Since, the nth terms of the two A.P.'s are equal.
∴ 11 – 2n = 27 – 3n
∴ 3n – 2n = 27 – 11
∴ n = 16
∴ t_{16} = 9 + (16 – 1) (-2)
= 9 + 15 × (-2)
= 9 – 30

$\therefore t_{16} = -21$
$\therefore$ The values of n and n_{th} term are 16 and -21 respectively.

Question 6.
If sum of 3_{rd} and 8_{th} terms of an A.P. is 7 and sum of 7th and 14th terms is -3, then find the 10th term.
Solution:
for an A.P., let a be the first term and d be the common difference.
According to the first condition,
$t_3 + tg = 7$
$\therefore a + (3 – 1) d + a + (8 – 1)d = 7$ ...[$\because t_n = a + (n – 1)d$]
$\therefore a + 2d + a + 7d = 7$
$\therefore 2a + 9d = 7$ ...(i)
According to the second condition,
$t_7 + t_{14} = -3$
$\therefore a + (7 – 1)d + a + (14 – 1)d = -3$
$\therefore a + 6d + a + 13d = -3$
$\therefore 2a + 19 d = – 3$ ...(ii)
Subtracting equation (i) from (ii), we get

$$\begin{array}{l} 2a + 19d = -3 \\ 2a + 9d = 7 \\ - \quad - \quad - \\ \hline 10d = -10 \end{array}$$

$\therefore \quad d = \frac{-10}{10} = -1$

Substituting d = –1 in equation (i), we get

$2a + 9(-1) = 7$

$\therefore \quad 2a - 9 = 7$

$\therefore \quad 2a = 7 + 9 = 16$

$\therefore \quad a = \frac{16}{2} = 8$

$\therefore \quad t_{10} = 8 + (10 - 1)(-1)$
$= 8 + 9 \times (-1)$
$= 8 - 9$

$\therefore \quad t_{10} = -1$

$\therefore$ **10^{th} term of the A.P. is –1.**

Question 7.
In an A.P. the first term is -5 and last term is 45. If sum of all numbers in the A.P. is 120, then how many terms are there? What is the common difference?
Solution:
Let the number of terms in the A.P. be n and the common difference be d.
Then, $a = -5, t_n = 45, S_n = 120$
Since, $t_n = a + (n – 1)d$
$\therefore 45 = -5 + (n – 1)d$
$\therefore 45 + 5 = (n – 1)d$
$\therefore (n – 1)d = 50$ ...(i)

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\therefore \quad 120 = \frac{n}{2}[2(-5) + (n-1)d]$

$\therefore \quad 120 = \frac{n}{2}(-10 + 50)$ ...[From (i)]

$\therefore \quad 120 = \frac{n}{2} \times 40$

$\therefore \quad 120 = 20n$

$\therefore \quad n = \frac{120}{20} = 6$

Substituting n = 6 in equation (i), we get
$(6 – 1)d = 50$
$\therefore 5d = 50$
$\therefore d = 505 = 10$
$\therefore$ There are 6 terms in the A.P. and the common difference is 10.

Alternate Method:
Let the number of terms in the A.P. be n.
Then, $t_1 = a = -5, t_n = 45, S_n = 120$

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$\therefore \quad 120 = \frac{n}{2}(-5 + 45)$

$\therefore \quad 120 = \frac{n}{2} \times 40$

$\therefore \quad 120 = 20n$

$\therefore \quad n = \frac{120}{20} = 6$

Since, $t_n = a + (n - 1)d$

$\therefore \quad 45 = -5 + (6 - 1)d$

$\therefore \quad 45 + 5 = 5d$

$\therefore \quad 50 = 5d$

$\therefore \quad d = \frac{50}{5} = 10$

∴ There are 6 terms in the A.P. and the common difference is 10.

Question 8.
Sum of 1 to n natural numbers is 36, then find the value of n.
Solution:
The natural numbers from 1 to n are
1,2, 3, ……, n.
The above sequence is an A.P.
∴ a = 1, d = 2 – 1 = 1
S_n = 36 …[Given]
Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$
∴ $36 = \frac{n}{2}[2(1) + (n - 1)(1)]$
∴ $36 = \frac{n}{2}(2 + n - 1)$
∴ 36 × 2 = n (n + 1)
∴ 72 = n (n + 1)
∴ $72 = n^2 + n$
∴ $n^2 + n - 72 = 0$
∴ $n^2 + 9n - 8n - 72 = 0$
∴ n(n + 9) – 8 (n + 9) = 0
∴ (n + 9) (n – 8) = 0
∴ n + 9 = 0 or n – 8 = 0
∴ n = -9 or n = 8
But, n cannot be negative.
∴ n = 8
∴ The value of n is 8.

Question 9.
Divide 207 in three parts, such that all parts are in A.P. and product of two smaller parts will be 4623.
Solution:
Let the three parts of 207 that are in A.P. be
a – d, a, a + d
According to the first condition,
(a – d) + a + (a + d) = 207
∴ 3a = 207
∴ $a = \frac{207}{3}$
∴ a = 69 …(i)
According to the second condition,
(a – d) × a = 4623
∴ (69 – d) × 69 = 4623 …[From (i)]
∴ $69 - d = \frac{4623}{69}$
∴ d = 69 – 67
∴ d = 2
∴ a – d = 69 – 2 = 67
a = 69
a + d = 69 + 2 = 71
∴ The three parts of 207 that are in A.P. are 67, 69 and 71.

Question 10.
There are 37 terms in an A.P., the sum of three terms placed exactly at the middle is 225 and the sum of last three terms is 429. Write the A.P.
Solution:
Since, there are 37 terms in the A.P.

$\therefore$ The middle term $= \left(\frac{37+1}{2}\right)^{th}$ term

$= 19^{th}$ term

$\therefore$ 18^{th}, 19^{th} and 20^{th} terms are placed exactly in the middle of the sequence.

According to the first condition,

$t_{18} + t_{19} + t_{20} = 225$

$\therefore$ $a + (18-1)d + a + (19-1)d + a + (20-1)d = 225$ ...[$\because t_n = a + (n-1)d$]

$\therefore$ $(a + 17d) + (a + 18d) + (a + 19d) = 225$

$\therefore$ $3a + 54d = 225$ ...(i)

According to the second condition,

$t_{35} + t_{36} + t_{37} = 429$

$\therefore$ $a + (35-1)d + a + (36-1)d + a + (37-1)d = 429$

$\therefore$ $(a + 34d) + (a + 35d) + (a + 36d) = 429$

$\therefore$ $3a + 105d = 429$ ...(ii)

Subtracting equation (i) from (ii), we get

$$\begin{array}{rcl} 3a + 105d & = & 429 \\ 3a + 54d & = & 225 \\ - \quad - & & - \\ \hline 51d & = & 204 \end{array}$$

$\therefore$ $d = \frac{204}{51} = 4$

Substituting d = 4 in equation (i), we get

3a + 54(4) = 225

∴ 3a + 216 = 225

∴ 3a = 225 – 216

∴ 3a = 9

∴ a = 93 = 3

∴The required A. P. is

a, a + d, a + 2d, a + 3d, ..., a + (n – 1)d

i.e. 3, 3 + 4,3 + 2 × 4, 3 + 3 × 4,..., 3 + (37 – 1)4

i.e. 3, 7,11,15, ...,147

Question 11.

If first term of an A.P. is a, second term is b and last term is c, then show that sum of all **terms is** $\frac{(a+c)(b+c-2a)}{2(b-a)}$.

Solution:

i. $t_1 = a, t_2 = b, t_n = c$ ...[Given]

$\therefore$ $d = t_2 - t_1 = b - a$

ii. $t_n = a + (n-1)d$

$\therefore$ $c = a + (n-1)(b-a)$

$\therefore$ $c - a = (n-1)(b-a)$

$\therefore$ $\frac{c-a}{b-a} = n - 1$

$\therefore$ $\frac{c-a}{b-a} + 1 = n$

$\therefore$ $\frac{c-a+b-a}{b-a} = n$

$\therefore$ $n = \frac{b+c-2a}{b-a}$ ...(i)

iii. $S_n = \frac{n}{2}(t_1 + t_n)$

$= \frac{\frac{b+c-2a}{b-a}}{2}(a+c)$ ...[From (i)]

$\therefore$ $S_n = \frac{(a+c)(b+c-2a)}{2(b-a)}$

Question 12.

If the sum of first p terms of an A.P. is equal to the sum of first q terms then show that the sum of its first (p + q) terms is zero, (p ≠ q)

Solution:

For an A.P., let a be the first term and d be the common difference.

The sum of first n terms of an A.P. is given by

Sn = [2a + (n – 1)d]

According to the given condition,
Sp = Sq

$\therefore \quad \frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$

$\therefore \quad p[2a + (p-1)d] = q[2a + (q-1)d]$

$\therefore \quad 2ap + p(p-1)d = 2aq + q(q-1)d$

$\therefore \quad 2ap + p^2d - pd = 2aq + q^2d - qd$

$\therefore \quad 2ap + p^2d - pd - 2aq - q^2d + qd = 0$

$\therefore \quad (2ap - 2aq) + (p^2d - q^2d) - (pd - qd) = 0$

$\therefore \quad 2a(p-q) + d(p^2 - q^2) - d(p-q) = 0$

$\therefore \quad 2a(p-q) + d(p+q)(p-q) - d(p-q) = 0$

$\therefore \quad (p-q)[2a + d(p+q) - d] = 0$

$\therefore \quad (p-q)[2a + (p+q-1)d] = 0$

But $p \neq q$

$\therefore \quad 2a + (p+q-1)d = 0 \quad ...(i)$

Sum of first (p + q) terms,

$S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$

$= \frac{p+q}{2}(0) \qquad ...[\text{From (i)}]$

$\therefore \quad S_{p+q} = 0$

∴ The sum of the first (p + q) terms is zero

Question 13.
If m times the m^{th} term of an A.P. is equal to n times n^{th} term, then show that the $(m + n)^{th}$ term of the A.P. is zero.
Solution:
According to the given condition,
$mt_m = nt_n$
∴ m[a + (m – 1)d] = n[a + (n – 1)d]
∴ ma + md(m – 1) = na + nd(n- 1)
∴ $ma + m^2d - md = na + n^2d - nd$
∴ $ma + m^2d - md - na - n^2d + nd = 0$
∴ $(ma - na) + (m^2d - n^2d) - (md - nd) = 0$
∴ $a(m - n) + d(m^2 - n^2) - d(m - n) = 0$
∴ a(m – n) + d(m + n) (m – n) – d(m – n) = 0
∴ (m – n)[a + (m + n – 1) d] = 0
∴ [a+ (m + n – 1)d] = 0 …[Dividing both sides by (m – n)]
∴ $t_{(m+n)} = 0$
∴ The $(m + n)^{th}$ term of the A.P. is zero.

Question 14.
₹ 1000 is invested at 10 percent simple interest. Check at the end of every year if the total interest amount is in A.P. If this is an A.P. then find interest amount after 20 years. For this complete the following activity.
Solution:

Simple interest = $\frac{P \times R \times N}{100}$

Simple interest after 1 year = $\frac{1000 \times 10 \times 1}{100}$

$= \boxed{₹100}$

Simple interest after 2 years = $\frac{1000 \times 10 \times 2}{100}$

$= \boxed{₹200}$

Simple interest after 3 years = $\frac{\boxed{1000} \times \boxed{10} \times \boxed{3}}{100}$

= ₹ 300

According to this the simple interest for 4, 5, 6 years will be ₹400, $\boxed{₹ 500}$, $\boxed{₹600}$ respectively.

From this d = $\boxed{200 - 100 = 100}$, and a = $\boxed{100}$

Amount of simple interest after 20 years

$t_n = a + (n - 1) d$

$\therefore \quad t_{20} = \boxed{100} + (20 - 1) \boxed{100}$

$= 100 + 19 \times 100 = 100 + 1900$

$\therefore \quad t_{20} = \boxed{2000}$

Amount of simple interest after 20 years

$= \boxed{₹ 2000}$