

Maharashtra State Board 12th Commerce Maths Solutions Chapter 6 Linear Programming Ex 6.1

Question 1.

A manufacturing firm produces two types of gadgets A and B, which are first processed in the foundry and then sent to a machine shop for finishing. The number of man-hours of labour required in each shop for production of A and B and the number of man-hours available for the firm is as follows.

Gadgets	Foundry	Machine Shops
A	10	5
B	6	4
Time available (hours)	60	35

Profit on the sale of A is ₹ 30 and B ₹ 20 Per unit Formulate the LPP to have maximum profit.

Solution:

Let the manufacturing firm produce x gadgets of type A and y gadgets of type B.

On selling x gadgets of type A the firm gets ₹ 30 and that on type B is ₹ 20.

∴ Total profit is $z = ₹ 30x + 20y$.

Since x and y are the numbers of gadgets, $x \geq 0, y \geq 0$

From the given table, the availability of man-hours of labour required in each shop and for the firm is given as 60 and 35.

∴ The inequation are $10x + 6y \leq 60$ and $5x + 4y \leq 35$.

Hence the given LPP can be formulated as Maximize $z = 30x + 20y$

Subject to $10x + 6y \leq 60, 5x + 4y \leq 35, x \geq 0, y \geq 0$.

Question 2.

In a cattle breeding farm, it is prescribed that the food ratio for one animal must contain 14, 22, and 1 unit of nutrients A, B, and C respectively. Two different kinds of fodder are available. Each unit weight of these two contains the following:

	Fodder 1	Fodder 2
Nutrients		
Nutrients A	2	1
Nutrients B	2	3
Nutrients C	1	1

The cost of fodder 1 is ₹ 3 per unit and that of fodder 2 is ₹ 2 per unit. Formulate the LPP to minimize the cost.

Solution:

Let x unit of fodder 1 and y unit of fodder 2 be included in the ration of an animal

The cost of 1 unit of fodder 1 is ₹ 3 and the cost of 1 unit of fodder 2 is ₹ 2.

∴ The total cost is ₹ $3x + 2y$.

The minimum requirement of the nutrients A, B, and C is given as 14 units, 22 units, and 1 unit.

∴ From the given table, the daily food ration will include $(2x + 2y)$ unit of Nutrient A, $(2x + 3y)$ unit of Nutrient B, and $(x + y)$ of Nutrient C.

The total cost is $z = ₹ 3x + 2y$

Hence the given LPP can be formulated as Minimize $z = 3x + 2y$

subject to $2x + y \geq 14, 2x + 3y \geq 22, x + y \geq 1, x \geq 0, y \geq 0$.

Question 3.

A Company manufactures two types of chemicals A and B. Each chemical requires two types of raw materials P and Q. The table below shows a number of units of P and Q required to manufacture one unit of A and one unit of B.

Chemical	A	B	Availability
Raw Material			
P	3	2	120
Q	2	5	160

The company gets profits of ₹ 350/- and ₹ 400/- by selling one unit of A and one unit of B respectively. Formulate the problem as LPP to maximize the profit.

Solution:

∴ Let the company manufactures x unit of chemical A and y unit of chemical B.

The availability of the raw materials for the production of chemicals A and B are given as 120 and 160 units.

The company gets ₹ 350 as profit on selling one unit of chemical A and ₹ 400 as profit on selling one unit of chemical B.

∴ Total profit is ₹ $(350x + 400y)$.

The inequation can be written as.

$3x + 2y \leq 120$

$$2x + 5y \leq 160$$

and x & y cannot be negative

Hence the LPP can be formulated as follows,

$$\text{Maximize } z = 350x + 400y$$

$$\text{Subject to } 3x + 2y \leq 120, 2x + 5y \leq 160, x \geq 0, y \geq 0.$$

Question 4.

A printing company prints two types of magazines A and B. The company earns ₹ 10 and ₹ 15 on magazines A and B per copy. These are processed on three machines I, II, III. Magazine A requires 2 hours on the machine I, 5 hours on machine II, and 2 hours on machine III. Magazine B requires 3 hours on machine I, 2 hours on machine II and 6 hours on machine III. Machines I, II, III are available for 36, 50, 60 hours per week respectively. Formulate the linear programming problem to maximize the profit.

Solution:

Let the company print x magazine of type A and y magazines of type B.

Then the total earnings of the company are ₹ $10x + 15y$.

The given problem can be tabulated as follows.

Magazine type Machine	Time required per unit		Available time per week
	Magazine A (x)	Magazine B (y)	
Machine I	2	3	36
Machine II	5	2	50
Machine III	2	6	60

From the table, the total time required for Machine I is $(2x + 3y)$ hours, for machine II is $(5x + 2y)$ hours, and for machine III is $(2x + 6y)$ hours.

The machine I, II, and III are available for 36, 50, and 60 hours per work.

∴ The constraints are $2x + 3y \leq 36$, $5x + 2y \leq 50$ and $2x + 6y \leq 60$.

Since x and y cannot be negative, we have $x \geq 0$, $y \geq 0$.

Hence the given LPP can be formulated as

$$\text{Maximize } z = 10x + 15y$$

$$\text{Subject to } 2x + 3y \leq 36, 5x + 2y \leq 50, 2x + 6y \leq 60, x \geq 0, y \geq 0.$$

Question 5.

Manufacture produces bulbs and tubes. Each of these must be processed through two machines M_1 and M_2 . A package of bulbs requires 1 hour of work on machine M_1 and 3 hours of work on M_2 . A package of tubes requires 2 hours on machine M_1 and 4 hours on machine M_2 . He earns a profit of ₹ 13.5 per package of bulbs and ₹ 55 per package of tubes. Formulate the LPP to maximize the profit. He operates M_1 for at most 10 hours and M_2 for at most 12 hours a day.

Solution:

Let the manufacturer produce x packages of bulbs and y packages of tubes.

He earns a profit of ₹ 13.5 per packages of bulbs and ₹ 55 per package of tubes.

∴ His total profit = ₹ $(13.5x + 55y)$.

The given problem can be tabulated as follows.

Machines	Time required in hours		Availability
	Bulbs (x)	Tubes (y)	
M_1	1	2	10
M_2	3	4	12

From the above table, the total time required for M_1 is $(x + 2y)$, and that of M_2 is $(3x + 4y)$.

M_1 and M_2 are available for at most 10 hrs per day and 12 hours per day.

∴ The constraint for the objective function is $x + 2y \leq 10$, $3x + 4y \leq 12$

Hence the give LPP can be formulated as

$$\text{Maximize } z = 13.5x + 55y$$

$$\text{Subject to } x + 2y \leq 10, 3x + 4y \leq 12, x \geq 0, y \geq 0.$$

Question 6.

A Company manufactures two types of fertilizers F_1 and F_2 . Each type of fertilizer requires two raw materials A and B. The number of units of A and B required to manufacture one unit of fertilizer F_1 and F_2 and availability of the raw materials A and B per day are given in the table below

Fertilizer Raw Materials	F_1	F_2	Availability
A	2	3	40
B	1	4	70

By selling one unit of F₁ and one unit of F₂, the company gets a profit of ₹ 500 and ₹ 750 respectively. Formulate the problem as LPP to maximize the profit.

Solution:

Let the company manufacture x units of Fertilizers F₁ and y units of fertilizer F₂.

The company gets a profit of ₹ 500 and ₹ 750 by selling a unit of F₁ and F₂.

∴ Total profit = ₹ (500x + 750y)

The availability of raw materials A and B per day is given as 40 and 70.

∴ From the given table the constraints can be written as $2x + 3y \leq 40$ and $x + 4y \leq 70$.

Since x & y cannot be negative, $x \geq 0$, $y \geq 0$

Hence the given LPP can be formulated as

Maximize $z = 500x + 750y$

Subject to $2x + 3y \leq 40$, $x + 4y \leq 70$, $x \geq 0$, $y \geq 0$.

Question 7.

A doctor has prescribed two different kinds of feeds A and B to form a weekly diet for a sick person. The minimum requirement of fats, carbohydrates, and proteins are 18, 28, 14 units respectively. One unit of food A has 4 units of fat, 14 units of carbohydrates, and 8 units of protein. One unit of food B has 6 units of fat, 12 units of carbohydrates and 8 units of protein. The price of food A is ₹ 4.5 per unit and that of food B is ₹ 3.5 per unit. Form the LPP so that the sick person's diet meets the requirements at minimum cost.

Solution:

Let x unit of food A and y unit of food B be consumed by a sick person.

The cost of food A is ₹ 4.5 per unit and food B is ₹ 3.5 per unit.

∴ Total cost = ₹ (4.5x + 3.5y)

The given conditions can be tabulated as follows.

Food	A(x)	B(y)	Minimum requirement
Fats	4	6	18
Carbohydrates	14	12	28
Protein	8	8	14

∴ The given LPP can be formulated as

Minimise $z = 4.5x + 3.5y$

Subject to $4x + 6y \geq 18$, $14x + 12y \geq 28$, $8x + 8y \geq 14$, $x \geq 0$, $y \geq 0$.

Question 8.

If John drives a car at a speed of 60 kms/hour he has to spend ₹ 5 per km on petrol. If he drives at a faster speed of 90 km/ hour, the cost of petrol increases to ₹ 8 per km. He has ₹ 600 to spend on petrol and wishes to travel the maximum distance within an hour.

Formulate the above problem as LPP.

Solution:

Let John drive x km at a speed of 60 km/hr and y km at a speed of 90 km/hr.

∴ Time required to drive a distance of x km is $\frac{x}{60}$ hours and the time required to drive at a distance of y km is $\frac{y}{90}$ hours.

∴ Total time required $(\frac{x}{60} + \frac{y}{90})$ hours.

Since he wishes to drive maximum distance within an hour,

$$\frac{x}{60} + \frac{y}{90} \leq 1$$

He has to spend ₹ 5 per km at a speed of 60 km/hr and ₹ 8 per km at a speed of 90 km/hr.

He has ₹ 600 on petrol to spend, $5x + 8y \leq 600$

The total distance he wishes to travel is (x + y) hours.

∴ The given LPP can be formulated as

Maximize $z = x + y$

Subject to $\frac{x}{60} + \frac{y}{90} \leq 1$, $5x + 8y \leq 600$, $x \geq 0$, $y \geq 0$.

Question 9.

The company makes concrete bricks made up of cement and sand. The weight of a concrete brick has to be at least 5 kg. Cement costs ₹ 20 per kg. and sand costs ₹ 6 per kg. Strength considerations dictate that a concrete brick should contain a minimum of 4 kg of cement and not more than 2 kg of sand. Formulate the LPP for the cost to be minimum.

Solution:

Let the concrete brick contain x kg of cement and y kg of sand.

The cost of cement is ₹ 20 per kg and sand is ₹ 6 per kg.

∴ The total cost = ₹ (20x + 6y)

Since the weight of the concrete brick has to be at least 5 kg, therefore, $x + y \geq 5$

Also, the concrete brick should contain a minimum of 4 kg of cement, i.e. $x \geq 4$, and not more than 2 kg of sand, i.e. $y \leq 2$.

∴ The LPP can be formulated as

Minimize $z = 20x + 6y$

Subject to $x + y \geq 5$, $x \geq 4$, $y \leq 2$, $x \geq 0$, $y \geq 0$.

Maharashtra State Board 12th Commerce Maths Solutions Chapter 6 Linear Programming Ex 6.2

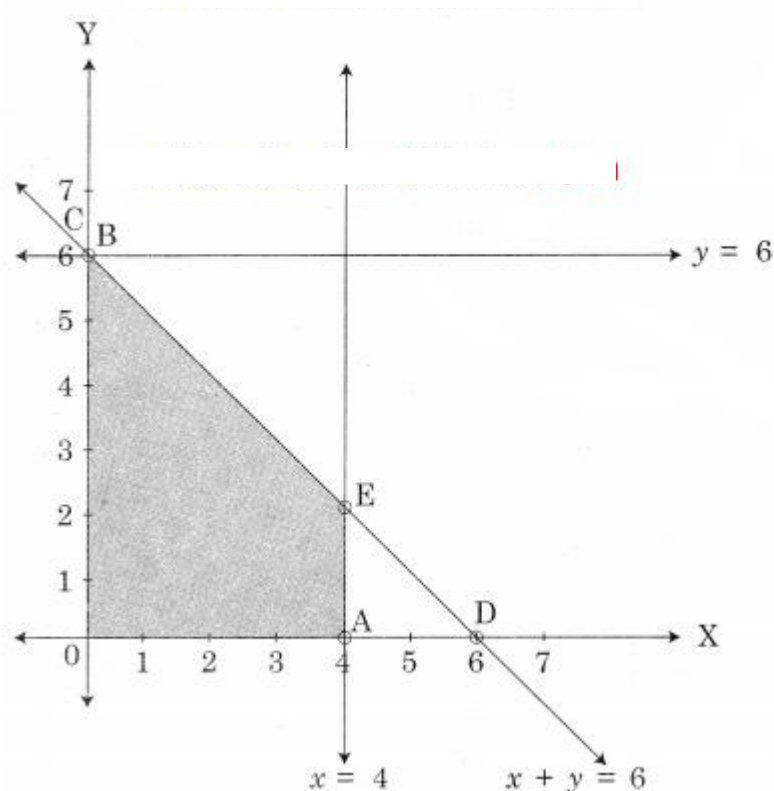
Solve the following LPP by graphical method.

Question 1.

Maximize $z = 11x + 8y$, Subject to $x \leq 4, y \leq 6, x + y \leq 6, x \geq 0, y \geq 0$.

Solution:

Inequation	Corresponding equation	x	y	Points	Region
$x \leq 4$	$x = 4$	4	0	A (4, 0)	Origin side
$y \leq 6$	$y = 6$	0	6	B (0, 6)	Origin side
$x + y \leq 6$	$x + y = 6$	0	6	C (0, 6)	Origin side
		6	0	D (6, 0)	Origin side



The feasible solution is AOB E

Where A(4, 0) O(0, 0) B(0, 6)

E is the point of intersection of $x + y = 6$ and $x = 4$.

$$\therefore 4 + y = 6$$

$$\therefore y = 2$$

$$\therefore E = (4, 2)$$

Corner Points	Value of $z = 11x + 8y$
A (4, 0)	$11 \times 4 + 8 \times 0 = 44$
O (0, 0)	$11 \times 0 + 8 \times 0 = 0$
B (0, 6)	$11 \times 0 + 8 \times 6 = 48$
E (4, 2)	$11 \times 4 + 8 \times 2 = 60$

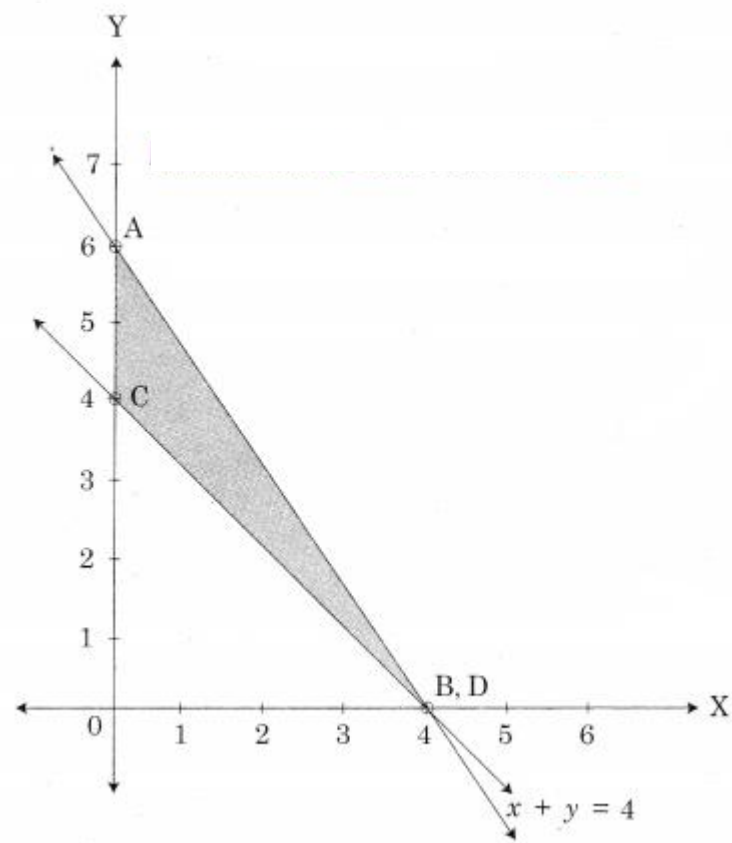
$\therefore z$ is maximum at (4, 2) and the maximum value of $z = 60$

Question 2.

Maximize $z = 4x + 6y$, Subject to $3x + 2y \leq 12, x + y \geq 4, x \geq 0, y \geq 0$.

Solution:

Inequation	Corresponding equation	x	y	Points	Region
$3x + 2y \leq 12$	$3x + 2y = 12$	0	6	A (0, 6)	Origin side
		4	0	B (4, 0)	
$x + y \geq 4$	$x + y = 4$	0	4	C (0, 4)	Non-Origin side
		4	0	D (4, 0)	



From figure, ABC is the feasible region
Where A(0, 6) B(4, 0) C(0, 4)

Corner Points	Value of $z = 4x + 6y$
A (0, 6)	$4 \times 0 + 6 \times 6 = 36$
B (4, 0)	$4 \times 4 + 6 \times 0 = 16$
C (0, 4)	$4 \times 0 + 6 \times 4 = 24$

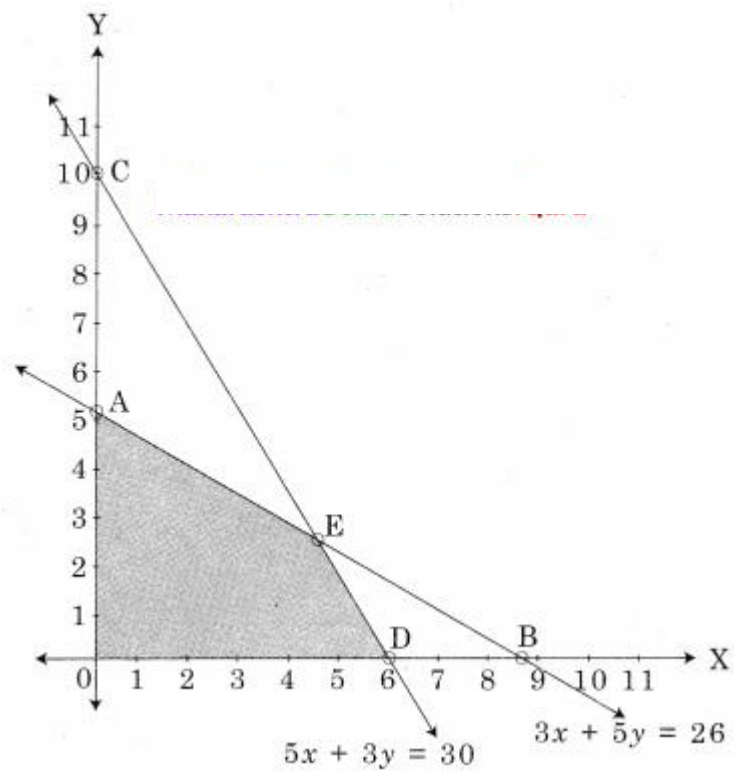
Maximum value of $z = 36$ at A(0, 6)

Question 3.

Maximize $z = 7x + 11y$, Subject to $3x + 5y \leq 26$, $5x + 3y \leq 30$, $x \geq 0$, $y \geq 0$.

Solution:

Inequation	Corresponding equation	x	y	Points	Region
$3x + 5y \leq 26$	$3x + 5y = 26$	0	5.2	A (0, 5.2)	Origin side
		8.6	0	B (8.6, 0)	
$5x + 3y \leq 30$	$5x + 3y = 30$	0	10	C (0, 10)	Origin side
		6	0	D (6, 0)	



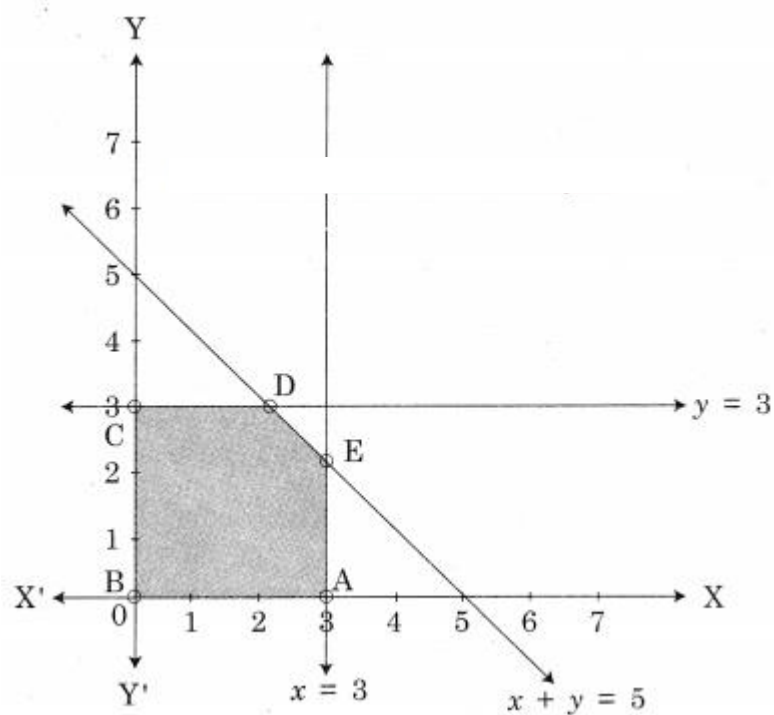
∴ AODE is the feasible region where
A(0, 5.2) O(0, 0) D(6, 0) and E is the intersection of $3x + 5y = 26$ and $5x + 3y = 30$
For E,
Solving $3x + 5y = 26$ (i)
 $5x + 3y = 30$ (ii)
We get, $x = 4.5, y = 2.5$
∴ E = (4.5, 2.5)

Corner Points	Value of $z = 7x + 11y$
A (0, 5.2)	$7 \times 0 + 11 \times 5.2 = 57.2$
O (0, 0)	$7 \times 0 + 11 \times 0 = 0$
D (6, 0)	$7 \times 6 + 11 \times 0 = 42$
E (4.5, 2.5)	$7 \times 4.5 + 11 \times 2.5 = 59$

∴ Maximum value of $z = 59$ at E(4.5, 2.5)

Question 4.
Maximize $z = 10x + 25y$, Subject to $0 \leq x \leq 3, 0 \leq y \leq 3, x + y \leq 5$.
Solution:
The constraints can be written as, $x \leq 3, x \geq 0, y \geq 0, y \leq 3, x + y \leq 5$

Inequation	Corres-ponding equation	x	y	Points	Region
$x \leq 3$	$x = 3$	3	0	(3,0)	Origin side
$y \leq 3$	$y = 3$	0	3	(0,3)	Origin side
$x + y \leq 5$	$x + y = 5$	0	5	(0,5)	Origin side
		5	0	(5,0)	



ABCDE is the feasible region where A(3, 0) B(0, 0) and C(0, 3) D is the intersection of $y = 3$ and $x + 5y = 5$ and E is the intersection of $x = 3$ and $x + 7 = 5$

For D,

Solving $y = 3$ (i)

$x + y = 5$ (ii)

We get $x = 2, y = 3$

$\therefore D = (2, 3)$

For E,

Solving $x = 3$ (i)

$x + y = 5$ (ii)

We get $x = 3, y = 2$

$\therefore E = (3, 2)$

Corner Points	Value of $z = 10x + 25y$
A (3, 0)	$10 \times 3 + 25 \times 0 = 30$
B (0, 0)	$10 \times 0 + 25 \times 0 = 0$
C (0, 3)	$10 \times 0 + 25 \times 3 = 75$
D (2, 3)	$10 \times 2 + 25 \times 3 = 90$
E (3, 2)	$10 \times 3 + 25 \times 2 = 80$

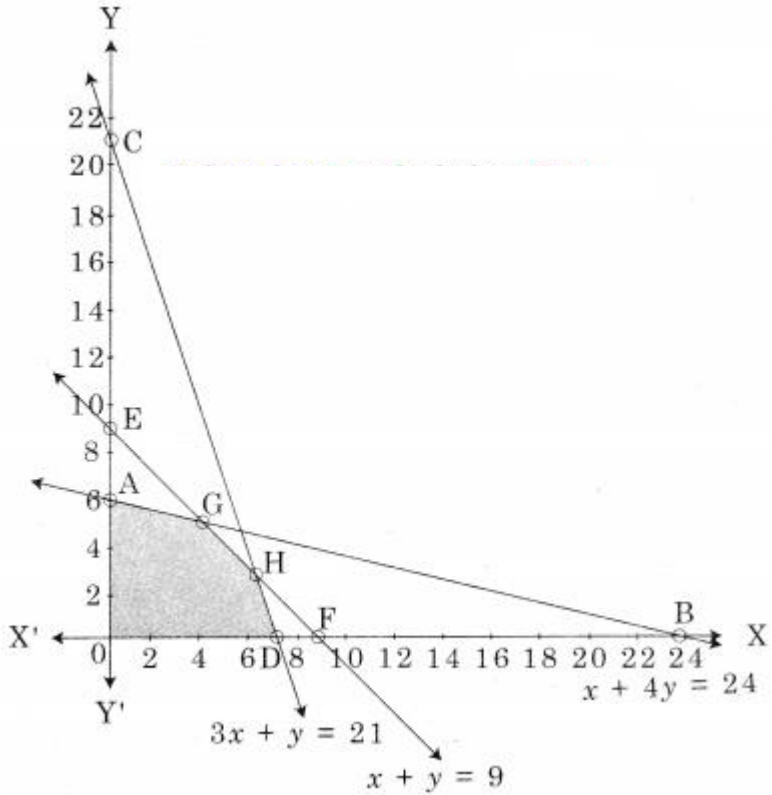
\therefore Maximum value of $z = 90$ at D(2, 3)

Question 5.

Maximize $z = 3x + 5y$, Subject to $x + 4y \leq 24, 3x + y \leq 21, x + y \leq 9, x \geq 0, y \geq 0$.

Solution:

Inequation	Corresponding equation	x	y	Points	Region
$x + 4y \leq 24$	$x + 4y = 24$	0	6	A (0, 6)	Origin side
		24	0	B (24, 0)	
$3x + y \leq 21$	$3x + y = 21$	0	21	C (0, 21)	Origin side
		7	0	D (7, 0)	
$x + y \leq 9$	$x + y = 9$	0	9	E (0, 9)	Origin side
		9	0	F (9, 0)	



OAGHD is the feasible region where O(0, 0), A(0, 6), D(7, 0) G is the intersecting point of $x + 4y = 24$ and $x + y = 9$
H is the intersecting points of $3x + y = 21$ and $x + y = 9$.

For G, Solving $x + 4y = 24$ (i)

$x + y = 9$ (ii)

We get, $x = 4, y = 5$

$\therefore G(4, 5)$

For H, Solving $x + y = 9$ (i)

$3x + y = 21$ (ii)

We get $x = 6, y = 3$

$\therefore H(6, 3)$

Corner Points	Value of $z = 3x + 5y$
O (0, 0)	$3 \times 0 + 5 \times 0 = 0$
A (0, 6)	$3 \times 0 + 5 \times 6 = 30$
G (4, 5)	$3 \times 4 + 5 \times 5 = 37$
H (6, 3)	$3 \times 6 + 5 \times 3 = 33$
D (7, 0)	$3 \times 7 + 5 \times 0 = 21$

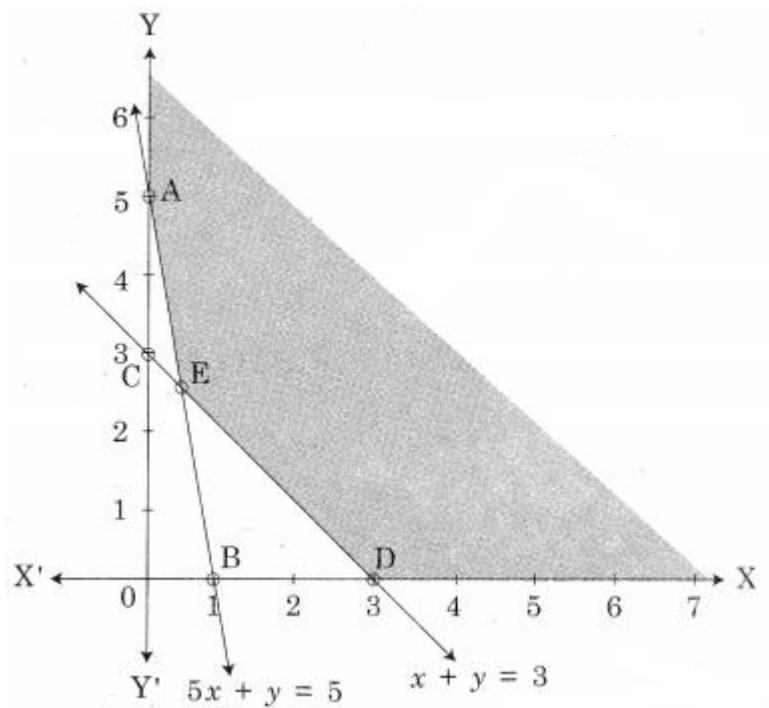
\therefore Maximum value of $z = 37$ at the point G(4, 5)

Question 6.

Minimize $z = 7x + y$ Subject to $5x + y \geq 5, x + y \geq 3, x \geq 0, y \geq 0$

Solution:

Inequation	Corres-ponding equation	x	y	Points	Region
$5x + y \geq 5$	$5x + y = 5$	0	5	A (0, 5)	Non-origin side
		1	0	B (1, 0)	
$x + y \geq 3$	$x + y = 3$	0	3	C (0, 3)	Non-origin side
		3	0	D (3, 0)	



AED is the feasible region where A(0, 5) D(3, 0) and E is the point of intersection of $5x + y = 5$ and $x + y = 3$.

For E, Solving $5x + y = 5$ (i)

$x + y = 3$ (ii)

We get, $x = 12$, $y = 52$

$\therefore E(12, 52)$

Corner Points	Value $z = 7x + y$
A (0, 5)	$7 \times 0 + 5 = 5$
B $(\frac{1}{2}, \frac{5}{2})$	$7 \times \frac{1}{2} + \frac{5}{2} = 6$
D (3, 0)	$7 \times 3 + 0 = 21$

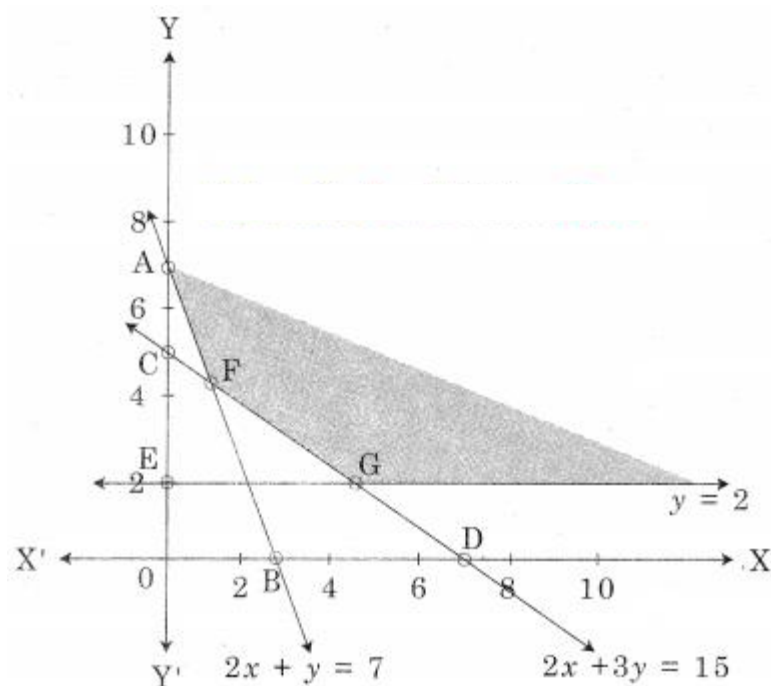
\therefore Minimum value of $z = 5$ at A(0, 5)

Question 7.

Minimize $z = 8x + 10y$, Subject to $2x + y \geq 7$, $2x + 3y \geq 15$, $y \geq 2$, $x \geq 0$, $y \geq 0$

Solution:

Inequation	Corresponding equation	x	y	Points	Region
$2x + y \geq 7$	$2x + y = 7$	0	7	A (0, 7)	Non-origin side
		3.5	0	B (3.5, 0)	
$2x + 3y \geq 15$	$2x + 3y = 15$	0	5	C (0, 5)	Non-origin side
		7.5	0	D (7.5, 0)	
$y \geq 2$	$y = 2$			E (0, 2)	Non-origin



AEG is the feasible solution where A(0, 7)

F is the point of intersection of $2x + y = 7$ and $2x + 3y = 15$

G is the point of intersection of $y = 2$ and $2x + 3y = 15$

For F, Solving $2x + y = 7$ (i)

$2x + 3y = 15$ (ii)

We get $x = 3/2, y = 4$

$\therefore F = (3/2, 4)$

For G, Solving $2x + 3y = 15$ (i)

$y = 2$ (ii)

We get $x = 4.5, y = 2$

$\therefore G = (4.5, 2)$

Corner Points	Value of $z = 8x + 10y$
A (0, 7)	$8 \times 0 + 10 \times 7 = 70$
F ($\frac{3}{2}$, 4)	$8 \times \frac{3}{2} + 10 \times 4 = 52$
G = (4.5, 2)	$8 \times 4.5 + 10 \times 2 = 56$

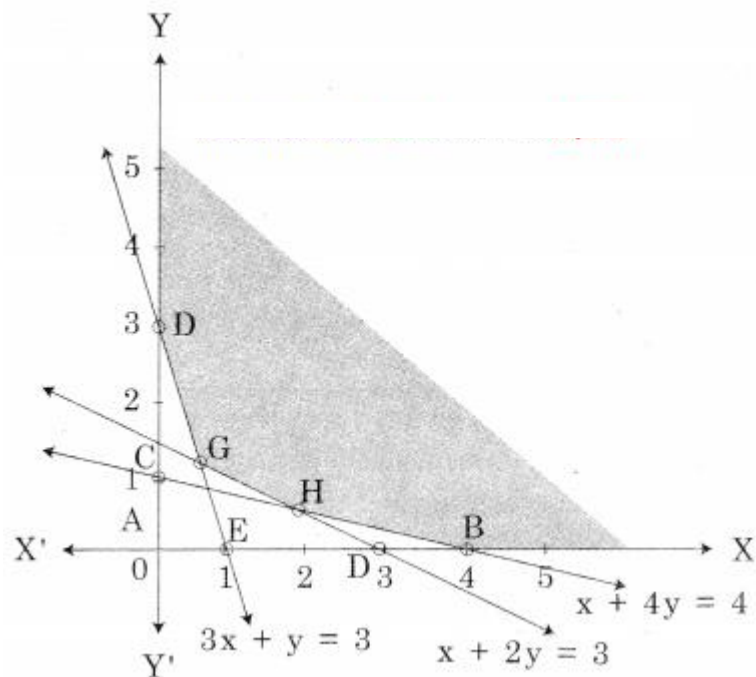
\therefore Minimum value of $z = 52$ at F($3/2, 4$)

Question 8.

Minimize $z = 6x + 2y$, Subject to $x + 2y \geq 3, x + 4y \geq 4, 3x + y \geq 3, x \geq 0, y \geq 0$.

Solution:

Inequation	Corresponding equation	x	y	Points	Region
$x + 4y \geq 4$	$x + 4y = 4$	0	1	A (0, 1)	Non-origin side
		4	0	B (4, 0)	
$x + 2y \geq 3$	$x + 2y = 3$	0	1.5	C (0, 1.5)	Non-origin side
		3	0	D (3, 0)	
$3x + y \geq 3$	$3x + y = 3$	0	3	E (0, 3)	Non-origin side
		1	0	F (1, 0)	



DGHB is the feasible region where D(0, 3), B(4, 0)

G is the point of intersection of $3x + y = 3$ and $x + 2y = 3$ and H is the point of intersection of $x + 2y = 3$ and $x + 4y = 4$

For G, Solving $3x + y = 3$ (i)

$x + 2y = 3$ (ii)

We get $x = 3/5, y = 6/5$

$\therefore G(3/5, 6/5)$

For H, Solving $x + 2y = 3$ (i)

$x + 4y = 4$ (ii)

We get, $x = 2, y = 1/2$

$\therefore H(2, 1/2)$

Corner Points	Value of $z = 6x + 21y$
D (0, 3)	$6 \times 0 + 21 \times 3 = 63$
G ($\frac{3}{5}, \frac{6}{5}$)	$6 \times \frac{3}{5} + 21 \times \frac{6}{5} = 28.8$
H (2, $\frac{1}{2}$)	$6 \times 2 + 21 \times \frac{1}{2} = 22.5$
B (4, 0)	$6 \times 4 + 21 \times 0 = 24$

\therefore Minimum value of $z = 22.5$ at H(2, 1/2)

Maharashtra State Board 12th Commerce Maths Solutions Chapter 6 Linear Programming Miscellaneous Exercise 6

(I) Choose the correct alternative.

Question 1.

The value of objective function is maximized under linear constraints.

- (a) at the centre of feasible region
- (b) at (0, 0)
- (c) at any vertex of feasible region.
- (d) The vertex which is at maximum distance from (0, 0).

Answer:

- (a) at the centre of feasible region

Question 2.

Which of the following is correct?

- (a) Every LPP has on optional solution
- (b) Every LPP has unique optional solution
- (c) If LPP has two optional solutions then it has infinitely many solutions
- (d) The set of all feasible solutions LPP may not be a convex set.

Answer:

(c) If LPP has two optional solutions then it has infinitely many solutions

Question 3.

Objective function of LPP is

- (a) a constraint
- (b) a function to be maximized or minimized
- (c) a relation between the decision variables
- (d) a feasible region.

Answer:

(b) a function to be maximized or minimized

Question 4.

The maximum value of $z = 5x + 3y$. subject to the constraints $3x + 5y = 15$; $5x + 2y \leq 10$, $x, y \geq 0$ is

- (a) 235
- (b) 2359
- (c) 23519
- (d) 2353

Answer:

(c) 23519

Question 5.

The maximum value of $z = 10x + 6y$. subject to the constraints $3x + y \leq 12$, $2x + 5y \leq 34$, $x \geq 0$, $y \geq 0$ is.

- (a) 56
- (b) 65
- (c) 55
- (d) 66

Answer:

(a) 56

Question 6.

The point at which the maximum value of $z = x + y$ subject to the constraint $x + 2y \leq 70$, $2x + y \leq 15$, $x \geq 0$, $y \geq 0$ is

- (a) (36, 25)
- (b) (20, 35)
- (c) (35, 20)
- (d) (40, 15)

Answer:

(d) (40, 15)

Question 7.

Of all the points of the feasible region the optimal value of z is obtained at a point

- (a) Inside the feasible region
- (b) at the boundary of the feasible region
- (c) at vertex of feasible region
- (d) on x -axis

Answer:

(c) at vertex of feasible region

Question 8.

Feasible region; the set of points which satisfy

- (a) The objective function
- (b) All of the given function
- (c) Some of the given constraints
- (d) Only non-negative constraints

Answer:

(b) All of the given function

Question 9.

Solution of LPP to minimize $z = 2x + 3y$ subjected to $x \geq 0$, $y \geq 0$, $1 \leq x + 2y \leq 10$ is

- (a) $x = 0$, $y = 12$
- (b) $x = 12$, $y = 0$
- (c) $x = 1$, $y = -2$
- (d) $x = y = 12$

Answer:

(a) $x = 0$, $y = 12$

Question 10.

The corner points of the feasible region given by the inequation $x + y \leq 4$, $2x + y \leq 7$, $x \geq 0$, $y \geq 0$, are

- (a) (0, 0), (4, 0), (3, 1), (0, 4)
- (b) (0, 0), (7/2, 0), (3, 1), (0, 4)
- (c) (0, 0), (7/2, 0), (3, 1), (5, 7)
- (d) (6, 0), (4, 0), (3, 1), (0, 7)

Answer:

- (b) (0, 0), (7/2, 0), (3, 1), (0, 4)

Question 11.

The corner point of the feasible region are (0, 0), (2, 0), (1/2, 3/2) and (0, 1) then the point of maximum $z = 6.5x + y = 13$

- (a) (0, 0)
- (b) (2, 0)
- (c) (1/2, 3/2)
- (d) (0, 1)

Answer:

- (b) (2, 0)

Question 12.

If the corner points of the feasible region are (0, 0), (3, 0), (2, 1) and (0, 7/3) the maximum value of $z = 4x + 5y$ is

- (a) 12
- (b) 13
- (c) 35/2
- (d) 0

Answer:

- (b) 13

Question 13.

If the corner points of the feasible region are (0, 10), (2, 2), and (4, 0) then the point of minimum $z = 3x + 2y$ is.

- (a) (2, 2)
- (b) (0, 10)
- (c) (4, 0)
- (d) (2, 4)

Answer:

- (a) (2, 2)

Question 14.

The half plane represented by $3x + 2y \leq 0$ contains the point.

- (a) (1, 5/2)
- (b) (2, 1)
- (c) (0, 0)
- (d) (5, 1)

Answer:

- (c) (0, 0)

Question 15.

The half plane represented by $4x + 3y \geq 14$ contains the point

- (a) (0, 0)
- (b) (2, 2)
- (c) (3, 4)
- (d) (1, 1)

Answer:

- (c) (3, 4)

(II) Fill in the blanks.

Question 1.

Graphical solution set of the in equations $x \geq 0$, $y \geq 0$ is in _____ quadrant.

Answer:

First

Question 2.

The region represented by the in equations $x \geq 0$, $y \geq 0$ lines in _____ quadrants.

Answer:

First

Question 3.

The optimal value of the objective function is attained at the _____ points of feasible region.

Answer:

End

Question 4.

The region represented by the inequality $y \leq 0$ lies in _____ quadrants

Answer:

Third and Fourth

Question 5.

The constraint that a factory has to employ more women (y) than men (x) is given by _____

Answer:

$y > x$

Question 6.

A garage employs eight men to work in its showroom and repair shop. The constraints that there must be not less than 3 men in showroom and repair shop. The constraints that there must be at least 3 men in showroom and at least 2 men in repair shop are _____ and _____ respectively.

Answer:

$x \geq 3$ and $y \geq 2$

Question 7.

A train carries at least twice as many first class passengers (y) as second class passengers (x). The constraint is given by _____

Answer:

$x \geq 2y$

Question 8.

A dishwashing machine holds up to 40 pieces of large crockery (x) this constraint is given by _____

Answer:

$x \leq 40$

(III) State whether each of the following is True or False.

Question 1.

The region represented by the inequalities $x \geq 0, y \geq 0$ lies in first quadrant.

Answer:

True

Question 2.

The region represented by the inequalities $x \leq 0, y \leq 0$ lies in first quadrant.

Answer:

False

Question 3.

The optimum value of the objective function of LPP occurs at the center of the feasible region.

Answer:

False

Question 4.

Graphical solution set of $x \leq 0, y \geq 0$ in xy system lies in second quadrant.

Answer:

True

Question 5.

Saina wants to invest at most ₹ 24000 in bonds and fixed deposits. Mathematically this constraint is written as $x + y \leq 24000$ where x is investment in bond and y is in fixed deposits.

Answer:

True

Question 6.

The point (1, 2) is not a vertex of the feasible region bounded by $2x + 3y \leq 6, 5x + 3y \leq 15, x \geq 0, y \geq 0$.

Answer:

True

Question 7.

The feasible solution of LPP belongs to only quadrant I. The Feasible region of graph $x + y \leq 1$ and $2x + 2y \geq 6$ exists.

Answer:

True

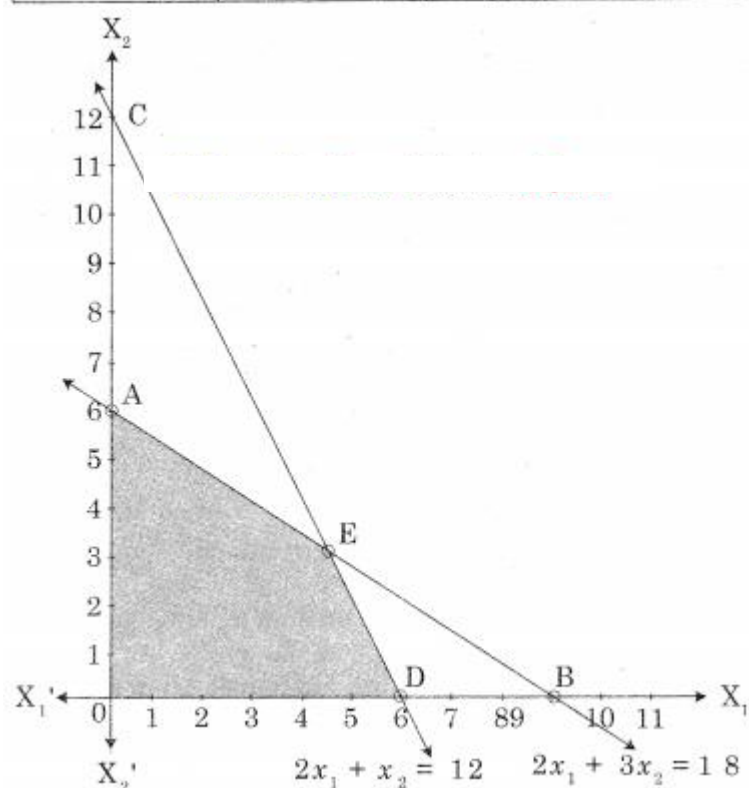
(IV) Solve the following problems.

Question 1.

Maximize $z = 5x_1 + 6x_2$, Subject to $2x_1 + 3x_2 \leq 18$, $2x_1 + x_2 \leq 12$, $x \geq 0$, $y \geq 0$

Solution:

Inequation	Corresponding equation	x_1	x_2	Points	Region
$2x_1 + 3x_2 \leq 18$	$2x_1 + 3x_2 = 18$	0	6	A (0, 6)	Origin side
		9	0	B (9, 0)	
$2x_1 + x_2 \leq 12$	$2x_1 + x_2 = 12$	0	12	C (0, 12)	Origin side
		6	0	D (6, 0)	



OAED is the feasible region $O(0, 0)$ $A(0, 6)$ $D(6, 0)$

and E is the intersection of $2x_1 + 3x_2 = 18$ and $2x_1 + x_2 = 12$

For E, Solving, $2x_1 + 3x_2 = 18$ (i)

$2x_1 + x_2 = 12$ (ii)

We get $x_1 = 4.5$, $x_2 = 3$

$\therefore E = (4.5, 3)$

Corner Points	Value of $z = 5x_1 + 6x_2$
O (0, 0)	$5 \times 0 + 6 \times 0 = 0$
A (0, 6)	$5 \times 0 + 6 \times 6 = 36$
E (4.5, 3)	$5 \times 4.5 + 6 \times 3 = 40.5$
D (6, 0)	$5 \times 6 + 6 \times 0 = 30$

\therefore Maximum value of $z = 40.5$ at $E(4.5, 3)$

Question 2.

Minimize $z = 4x + 2y$, Subject to $3x + y \geq 27$, $x + y \geq 21$, $x \geq 0$, $y \geq 0$

Solution:

Corner Points	Value of $z = 4x + 2y$
A (0, 27)	$4 \times 0 + 2 \times 27 = 54$
E (3, 18)	$4 \times 3 + 2 \times 18 = 48$
D (21, 0)	$4 \times 21 + 2 \times 0 = 84$

AED is the feasible region $A(0, 27)$, $D(21, 0)$

and E is the point of intersection of $3x + y = 27$ and $x + y = 21$

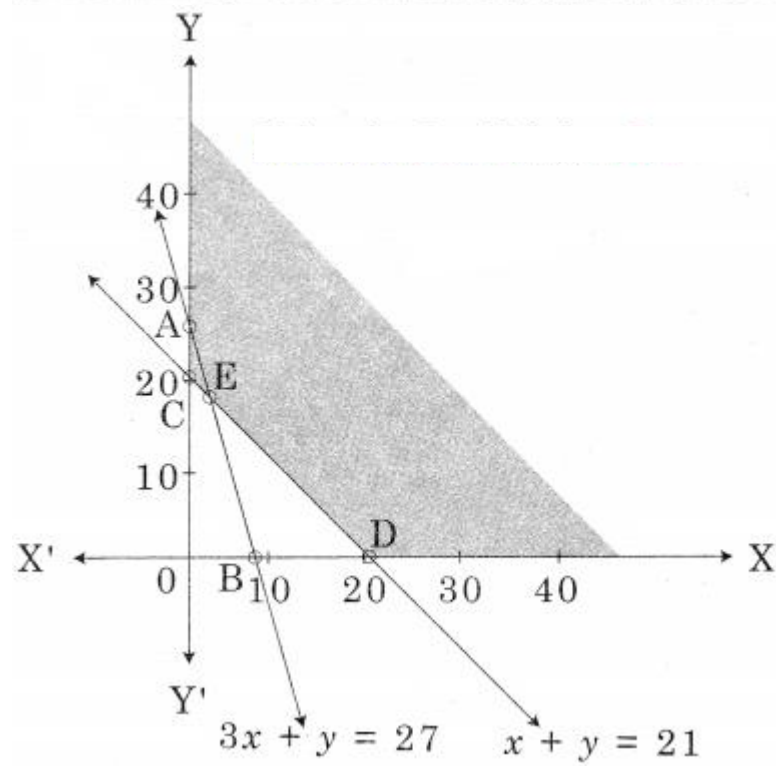
For E, Solving $3x + y = 27$ (i)

$x + y = 21$ (ii)

We get $x = 3, y = 18$

$\therefore E(3, 18)$

Inequation	Corres-ponding equation	x	y	Points	Region
$3x + y \geq 27$	$3x + y = 27$	0	27	A (0, 27)	Non-origin side
		9	0	B (9, 0)	
$x + y \geq 21$	$x + y = 21$	0	21	C (0, 21)	Non-origin side
		21	0	D (21, 0)	



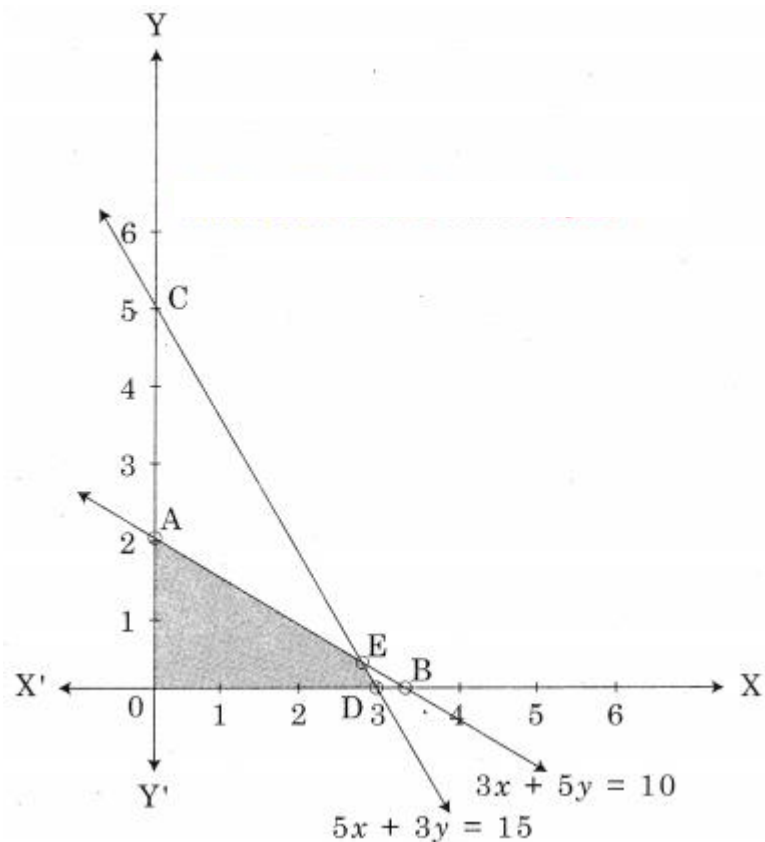
\therefore Minimum value of $z = 48$ at $(3, 18)$

Question 3.

Maximize $z = 6x + 10y$, subject to $3x + 5y \leq 10$, $5x + 3y \leq 15$, $x \geq 0$, $y \geq 0$

Solution:

Inequation	Corres-ponding equation	x	y	Points	Region
$3x + 5y \leq 10$	$3x + 5y = 10$	0	2	A (0, 2)	
		$\frac{10}{3}$	0	B ($\frac{10}{3}$, 0)	Origin side
$5x + 3y \leq 15$	$5x + 3y = 15$	0	5	C (0, 5)	
		3	0	D (3, 0)	Origin side



OAED is the feasible region;

O(0, 0), A (0, 2) D (3, 0) and E is the point of intersection of $3x + 5y = 10$ and $5x + 3y = 15$

For E, Solving $3x + 5y = 10$

$5x + 3y = 15$

We get, $x = \frac{45}{16}$, $y = \frac{5}{16}$

$\therefore E(\frac{45}{16}, \frac{5}{16})$

Corner Points	Value of $z = 6x + 10y$
O (0, 0)	$6 \times 0 + 10 \times 0 = 0$
A (0, 2)	$6 \times 0 + 10 \times 2 = 20$
E ($\frac{45}{16}, \frac{5}{16}$)	$6 \times \frac{45}{16} + 10 \times \frac{5}{16} = 20$
D (3, 0)	$6 \times 3 + 10 \times 0 = 18$

Since the maximum value of $z = 20$ at two points i.e, at A(0, 2) and E($\frac{45}{16}, \frac{5}{16}$).

z is maximum at all point on segment AE.

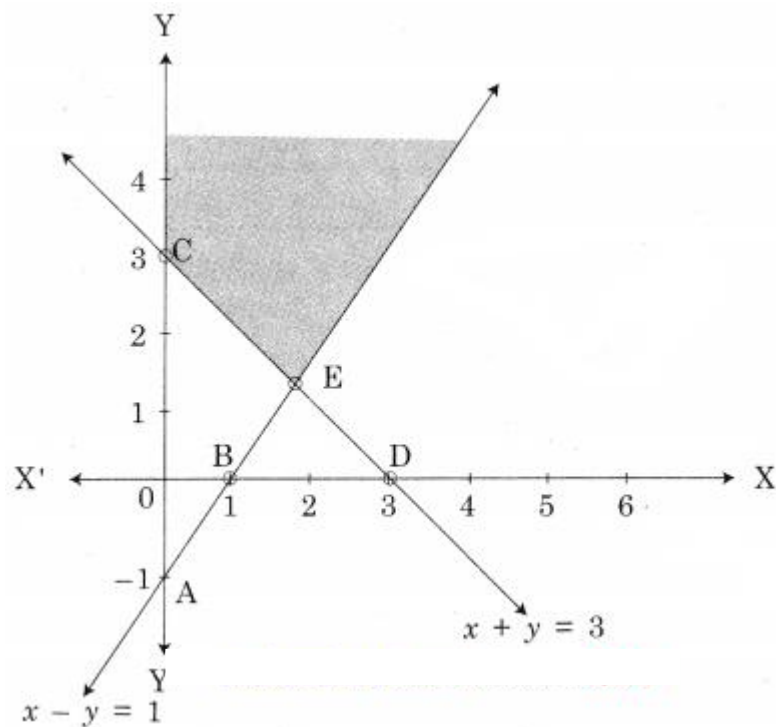
Hence it has infinite number of solutions.

Question 4.

Minimize $z = 2x + 3y$, Subject to $x - y \leq 1$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$

Solution:

Inequation	Corresponding equation	x	y	Points	Region
$x - y \leq 1$	$x - y = 1$	0	-1	A (0, -1)	Origin side
		1	0	B (1, 0)	
$x + y \geq 3$	$x + y = 3$	0	3	C (0, 3)	Non-origin side
		3	0	D (3, 0)	



Shaded portion CE is the feasible region

Where $C = (0, 3)$ and E is the point of intersection of $x - y = 1$ and $x + y = 3$

For E, Solving $x - y = 1$ (i)

$x + y = 3$ (ii)

We get, $x = 2, y = 1$

$\therefore E(2, 1)$

Corner Points	Value of $z = 2x + 3y$
C (0, 3)	$2 \times 0 + 3 \times 3 = 9$
E (2, 1)	$2 \times 2 + 3 \times 1 = 7$

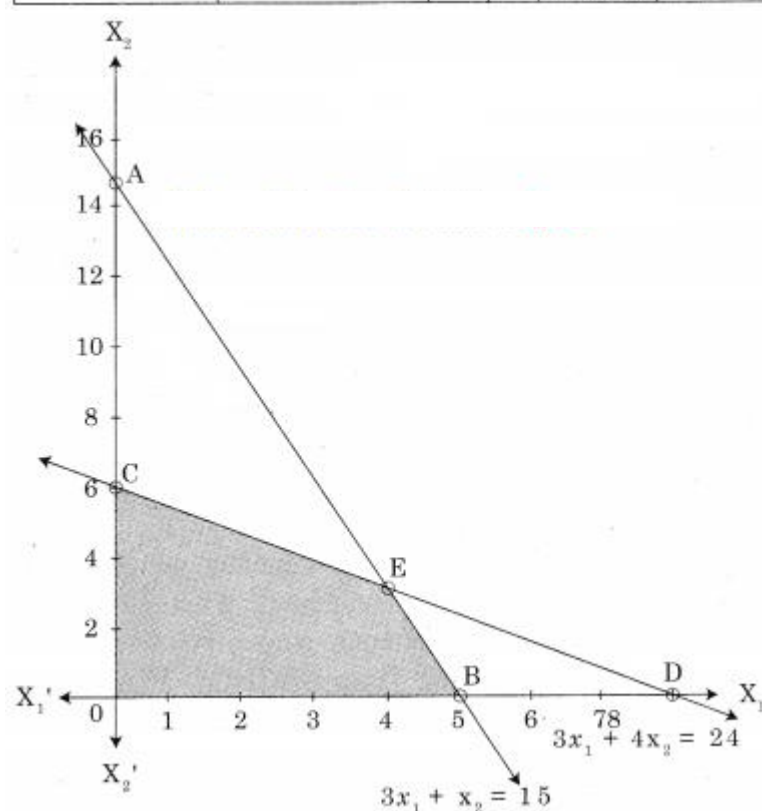
\therefore Minimum value $z = 7$ at $E(2, 1)$

Question 5.

Maximize $z = 4x_1 + 3x_2$, Subject to $3x_1 + x_2 \leq 15$, $3x_1 + 4x_2 \leq 24$, $x \geq 0, y \geq 0$

Solution:

Inequation	Corres-ponding equation	x_1	x_2	Points	Region
$3x_1 + x_2 \leq 15$	$3x_1 + x_2 = 15$	0	15	A (0, 15)	Origin side
		5	0	B (5, 0)	
$3x_1 + 4x_2 \leq 24$	$3x_1 + 4x_2 = 24$	0	6	C (0, 6)	Origin side
		8	0	D (8, 0)	



OCEB is the feasible region where $O(0, 0)$, $C(0, 6)$, $B(5, 0)$

E is the point of intersection of $3x_1 + x_2 = 15$ and $3x_1 + 4x_2 = 24$

For E, Solving $3x_1 + x_2 = 15$ (i)

$3x_1 + 4x_2 = 24$ (ii)

We get, $x_1 = 4, x_2 = 3$

$\therefore E(4, 3)$

Corner Points	Value of $z = 4x_1 + 3x_2$
O (0, 0)	$4 \times 0 + 3 \times 0 = 0$
C (0, 6)	$4 \times 0 + 3 \times 6 = 18$
B (5, 0)	$4 \times 5 + 3 \times 0 = 20$
E (4, 3)	$4 \times 4 + 3 \times 3 = \textcircled{25}$

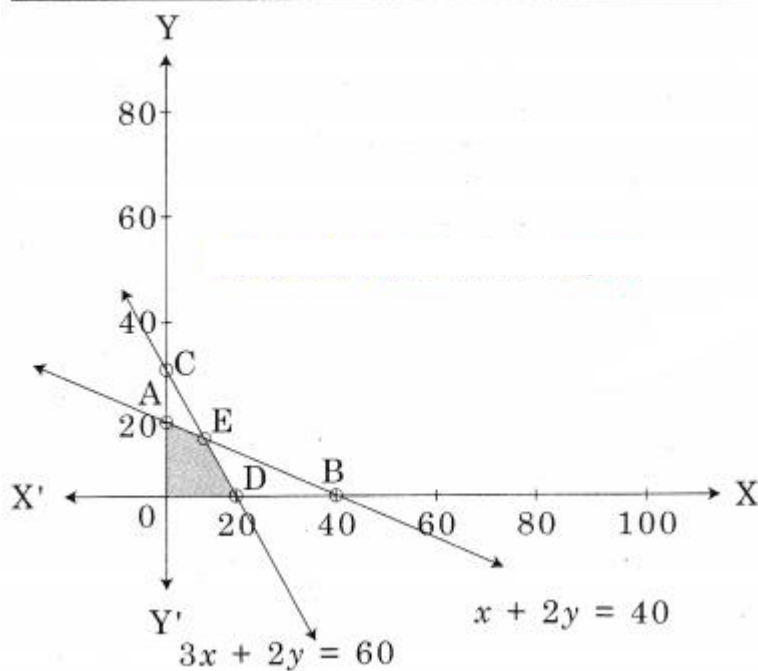
\therefore Maximum value of $z = 25$ at (4, 3)

Question 6.

Maximize $z = 60x + 50y$, Subject to $x + 2y \leq 40, 3x + 2y \leq 60, x \geq 0, y \geq 0$

Solution:

Inequation	Corres-ponding equation	x	y	Points	Region
$x + 2y \leq 40$	$x + 2y = 40$	0	20	A (0, 20)	Origin side
		40	0	B (40, 0)	
$3x + 2y \leq 60$	$3x + 2y = 60$	0	30	C (0, 30)	Origin side
		20	0	D (20, 0)	



OAED is the feasible region O(0, 0), A(0, 20), D(20, 0)

E is $x + 2y = 40$ and $3x + 2y = 60$

For E, Solving $x + 2y = 40$

$3x + 2y = 60$

We get, $x = 10, y = 15$

$\therefore E = (10, 15)$

Corner Points	Value of $z = 60x + 50y$
O (0, 0)	$60 \times 0 + 50 \times 0 = 0$
A (0, 20)	$60 \times 0 + 50 \times 20 = 1000$
D (20, 0)	$60 \times 20 + 50 \times 0 = 1200$
E (10, 15)	$60 \times 10 + 50 \times 15 = \textcircled{1350}$

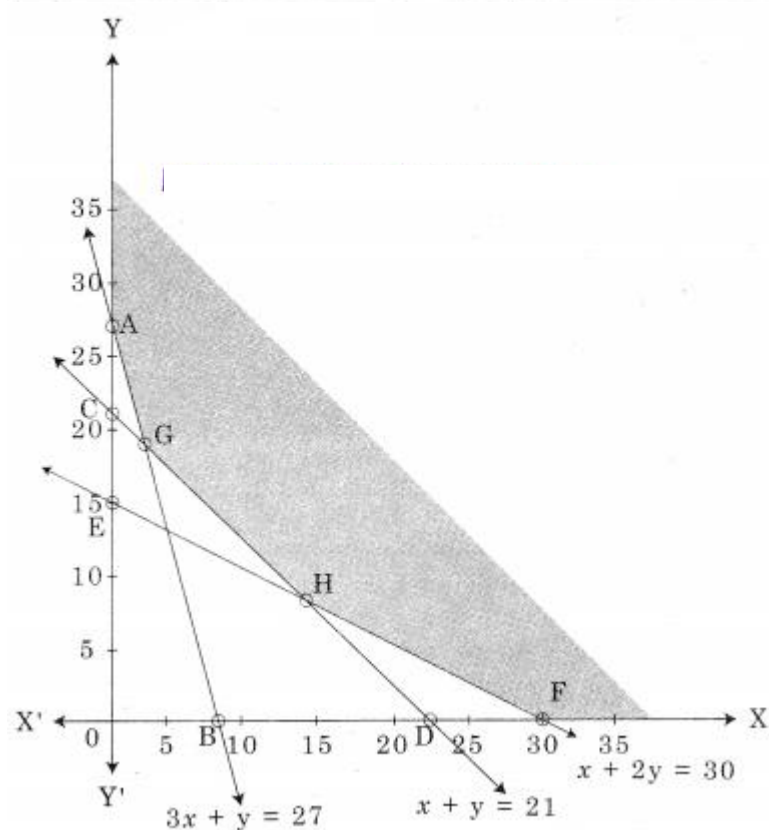
\therefore Maximum value of $z = 1350$ at E(10, 15).

Question 7.

Minimize $z = 4x + 2y$, Subject to $3x + y \geq 27, x + y \geq 21, x + 2y \geq 30, x \geq 0, y \geq 0$

Solution:

Inequation	Corresponding equation	x	y	Points	Region
$3x + y \geq 27$	$3x + y = 27$	0	27	A (0, 27)	Non-origin side
		9	0	B (9, 0)	
$x + y \geq 21$	$x + y = 21$	0	21	C (0, 21)	Non-origin side
		21	0	D (21, 0)	
$x + 2y \geq 30$	$x + 2y = 30$	0	15	E (0, 15)	Non-origin side
		30	0	F (30, 0)	



AGHF is the feasible region where A(0, 27) F(30, 0)
 G is the point of intersection of $3x + y = 27$ and $x + y = 21$
 H is the point of intersection of $x + y = 21$ and $x + 2y = 30$
 For G, Solving $3x + y = 27$ (i)
 $x + y = 21$ (ii)
 We get, $x = 3, y = 18$
 $\therefore G(3, 18)$
 For H, Solving $x + y = 21$ (i)
 $x + 2y = 30$ (ii)
 We get, $x = 12, y = 9$
 $\therefore H = (12, 9)$

Corner Points	Value of $z = 4x + 2y$
A (0, 27)	$4 \times 0 + 2 \times 27 = 54$
G (3, 18)	$4 \times 3 + 2 \times 18 = 48$
H (12, 9)	$4 \times 12 + 2 \times 9 = 66$
F (30, 0)	$4 \times 30 + 2 \times 0 = 120$

\therefore Maximum value of $z = 48$ at G(3, 18)

Question 8.

A carpenter makes chairs and table profit are ₹ 140 per chair and ₹ 210 per table Both products are processed on three machines, Assembling, Finishing and Polishing the time required for each product in hours and availability of each machine is given by the following table.

Product / Machines	Chair (x)	Table (y)	Available time (hours)
Assembling	3	3	36
Finishing	5	2	50
Polishing	2	6	60

Formulate and solve the following Linear programming problem using graphical method.

Solution:

Let z be the profit which can be made by selling x chair and y table.

$$\therefore x \geq 0, y \geq 0$$

$$\text{Total profit} = 140x + 210y$$

According to the table, the constraints can be written as

$$3x + 3y \leq 36$$

$$5x + 2y \leq 50$$

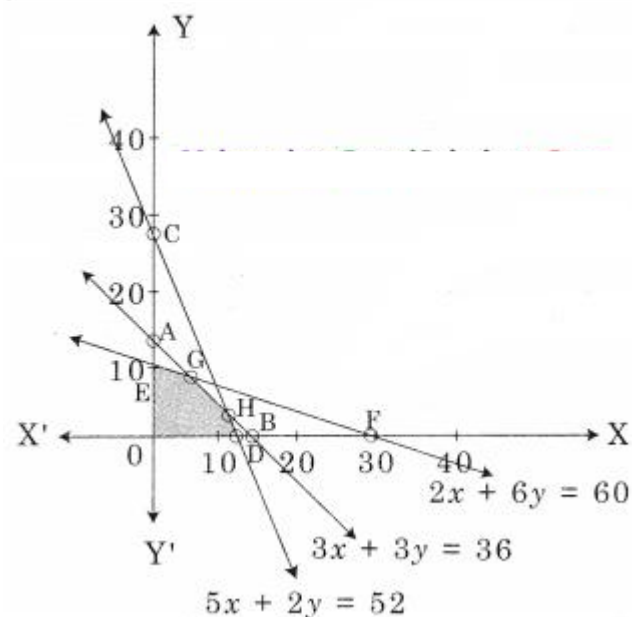
$$2x + 6y \leq 60$$

\therefore The given LPP can be formulated as.

$$\text{Maximize } z = 140x + 210y$$

Subject to $3x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$, $x \geq 0$, $y \geq 0$.

Inequation	Corresponding equation	x	y	Points	Region
$3x + 3y \leq 36$	$3x + 3y = 36$	0	12	A (0, 12)	Origin side
		12	0	B (12, 0)	
$5x + 2y \leq 52$	$5x + 2y = 52$	0	26	C (0, 26)	Origin side
		10.4	0	D (10.4, 0)	
$2x + 6y \leq 60$	$2x + 6y = 60$	0	10	E (0, 10)	Origin side
		30	0	F (30, 0)	



OEGH is the feasible region

O(0, 0), D(10.4, 0), E(0, 10)

For G, Solving $3x + 3y = 36$ (i)

$2x + 6y = 60$ (ii)

$$\therefore G = (3, 9)$$

For H, Solving $5x + 2y = 52$ (i)

$3x + 3y = 36$ (ii)

We get, $x = 283$, $y = 83$

$$\therefore H(283, 83)$$

Corner Points	Value of $z = 140x + 210y$
O (0,0)	$140 \times 0 + 210 \times 0 = 0$
E (0, 10)	$140 \times 0 + 210 \times 10 = 2100$
G (3, 9)	$140 \times 3 + 210 \times 9 = 2310$
H($\frac{28}{3}, \frac{8}{3}$)	$140 \times \frac{28}{3} + 210 \times \frac{8}{3} = 1866.3$
D (10.4, 0)	$140 \times 16.4 + 210 \times 0 = 1456$

$\therefore z$ is maximum at (3, 9) and maximum profit = ₹ 2310.

Question 9.

A company manufactures bicycles and tricycles, each of which must be processed through two machines A and B. Maximum availability of machines A and B are respectively 120 and 180 hours. Manufacturing a bicycle requires 6 hours on machine A and 3 hours on machine B. Manufacturing a tricycle requires 4 hours on machine A and 10 hours on machine B. If profits are ₹ 180 for a bicycle and ₹ 220 on a tricycle, determine the number of bicycles and tricycles that should be manufactured in order to maximize the profit.

Solution:

Let x number of bicycle and y number of tricycle has to be manufactured and to be sold to get the profit (z)

$\therefore x \geq 0, y \geq 0$

Total profit = $180x + 220y$.

The given LPP can be tabulated as follows:

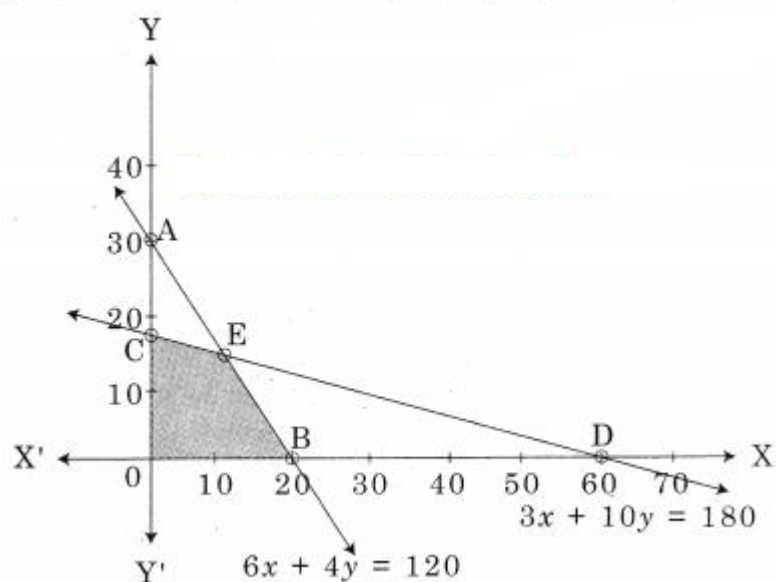
Machines	Bicycle	Tricycle	Available line
A (Hrs)	6	4	120
B (Hrs)	3	10	180

\therefore The given LPP can be formulated as

Maximize $z = 180x + 220y$

Subject to $6x + 4y \leq 120, 3x + 10y \leq 180, x \geq 0, y \geq 0$

Inequation	Corresponding equation	x	y	Points	Region
$6x + 4y \leq 120$	$6x + 4y = 120$	0	30	A (0, 30)	Origin side
		20	0	B (20, 0)	
$3x + 10y \leq 180$	$3x + 10y = 180$	0	18	C (0, 18)	Origin side
		60	0	D (60, 0)	



OCEB is the feasible region where O(0, 0) C(0, 18) B(20, 0)

E is the point of intersection of $6x + 4y = 120$ and $3x + 10y = 180$

For E, Solving $6x + 4y = 120$ (i)

$3x + 10y = 180$ (ii)

We get, $x = 10, y = 15$

$\therefore E(10, 15)$

Corner Points	Value of $z = 180x + 220y$
O (0, 0)	$180 \times 0 + 220 \times 0 = 0$
C (0, 18)	$180 \times 0 + 220 \times 18 = 3960$
B (20, 0)	$180 \times 20 + 220 \times 0 = 3600$
E (10, 15)	$180 \times 10 + 220 \times 15 = 5100$

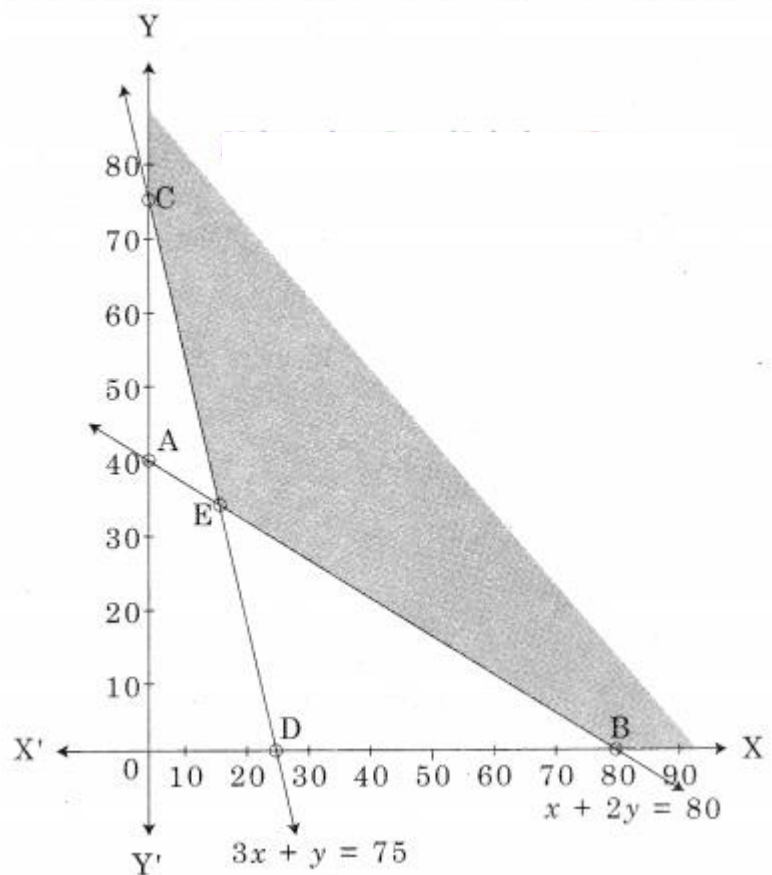
∴ Maximum value of z is 5100 at E(10, 15)
Hence 10 bicycles and 15 tricycles should be produced to get maximum profit.

Question 10.
A factory produced two types of chemicals A and B. The following table gives the units of ingredients P and Q (per kg) of chemicals A and B as well as minimum requirements of P and Q and also cost per kg. Chemicals A and B:

Chemicals Units / Ingredients per kg.	A (x)	B (y)	Minimum requirements in
P	1	2	80
Q	3	1	75
Cost (in Rs.)	4	6	

Find the number of units of chemicals A and B should be produced sp as to minimize the cost.
Solution:
Let x be the no. of units of chemicals, A produced and y be the no. of units of chemical B produced.
Total cost is $4x + 6y$
The LPP is. Minimise $z = 4x + 6y$
Subject to $x + 2y \geq 80, 3x + y \geq 75, x \geq 0, y \geq 0$.

Inequation	Corres- ponding equation	x	y	Points	Region
$x + 2y \geq 80$	$x + 2y = 80$	0	40	A (0, 40)	Non- origin side
		80	0	B (80, 0)	
$3x + y \geq 75$	$3x + y = 75$	0	75	C (0, 75)	Non- origin side
		25	0	D (25, 0)	



The shaded region BEC in the feasible region B(80, 0) C(0, 75)
and E is the point of intersection of $x + 2y = 80$ and $3x + y = 75$
For E, Solving $x + 2y = 80$ (i)
 $3x + y = 75$ (ii)
We get, $x = 14, y = 33$

∴ E(14, 33)

Corner Points	Value of $z = 4x + 6y$
B (80, 0)	$4 \times 80 + 6 \times 0 = 320$
E (14,33)	$4 \times 14 + 6 \times 33 = \underline{254}$
C (0, 75)	$4 \times 0 + 6 \times 75 = 320$

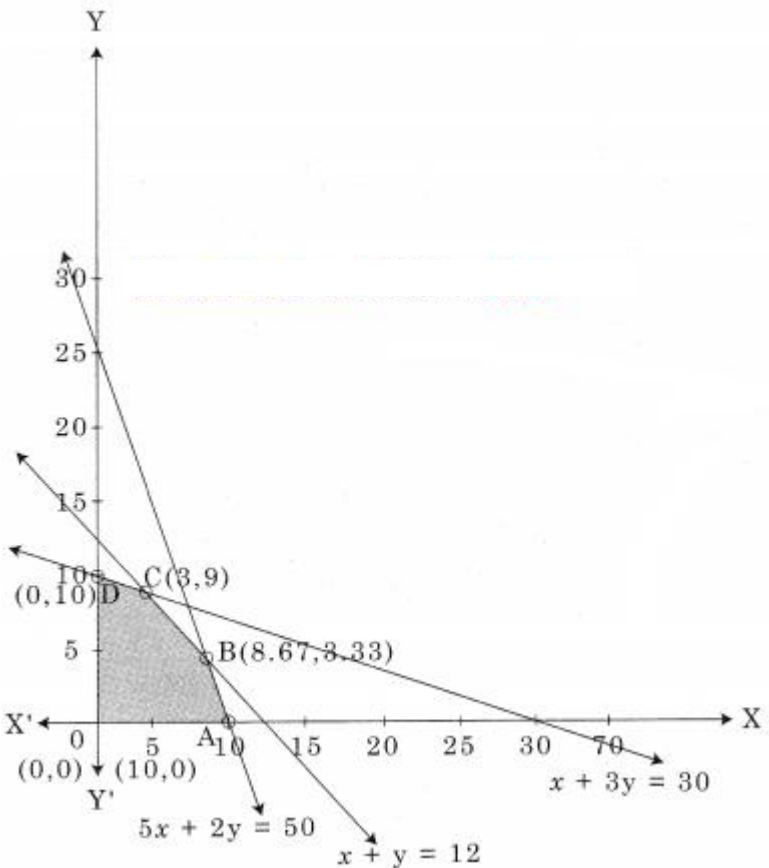
∴ z is minimum at E(14, 33) and the minimum value of z = 254.
Hence 14 units of chemical A and 33 units of chemical B are produced to get a minimum cost of ₹ 254.

Question 11.
A company produces mixers and food processors. Profit on selling one mixer and one food processor is ₹ 2,000/- and ₹ 3,000/- respectively. Both the products are processed through three Machines A, B, C. The time required in hours by each product and total time available in hours per week on each machine are as follows:

Product	Mixer per unit	Food processor per unit	Available time
Machine			
A	3	3	36
B	5	2	50
C	2	6	60

How many mixes and food processors should be produced to maximize profit?

Solution:
Let x be the no. of mixers produced and y be the no.of food processors produced.
The profit is $2000x + 3000y$
The LPP is Maximize $Z = 2000x + 3000y$
Subject to $3x + 3y \leq 36, 5x + 2y \leq 50, 2x + 6y \leq 60, x, y \geq 0$.
Let $3x + 3y = 36$,
i.e. $x + y = 12$
 $x = 0, y = 12, (0, 12)$
 $y = 0, x = 12, (12, 0)$
 $5x + 2y = 50$
 $x = 0, y = 25, (0, 25)$
 $y = 0, x = 10, (10, 0)$
Let $2x + 6y = 60, x + 3y = 30$
 $x = 0, y = 10, (0, 10)$
 $y = 0, x = 30, (30, 0)$



The Shaded region OABCD is the feasible region.
O(0, 0) A (10, 0) D(0, 10)
B is the intersection of $x + y = 12$ and $5x + 2y = 50$
Solving we get $x = 8.67, y = 3.33$
∴ B(8.67, 3.33)
C is the intersection of $x + y = 12$ and $x + 3y = 30$
Solving we get $x = 3, y = 9$
∴ C(3, 9)

Corner Points	Value of $z = 2000x + 3000y$
O (0, 0)	0
A (10, 0)	20,000
B (8, 67, 3.33)	$17340 + 9990 = 27330$
C (3, 9)	$6000 + 27000 = 33000$
D (0, 10)	30000

∴ Maximum value of z is 33000 at C(3, 9)

Hence 3 mixers and 9 food processors should be produced to get a maximum profit of ₹ 33,000.

Question 12.

A chemical company produces a chemical containing three basic elements A, B, C so that it has at least 16 liters of A, 24 liters of B, and 18 liters of C. This chemical is made by mixing two compounds I and II. Each unit of compound I has 4 liters of A, 12 liters of B, 2 liters of C. Each unit of compound II has 2 liters of A, 2 liters of B and 6 liters of C. The cost per unit of compound is ₹ 800/- and that of compound II is ₹ 640/- Formulate the problem as L.P.P and solve it to minimize the cost.

Solution:

Let x be the no. of units of compound I used and y be the no of units of compound II used.

The data can be tabulated as

	A	B	C	Cost
I (x)	4	12	2	800
II (y)	2	2	6	640
Min. Requirement	16	24	18	

The LPP is minimize $z = 800x + 640y$

Subject to $4x + 2y \geq 16$, $12x + 2y \geq 24$, $2x + 6y \geq 18$, $x, y \geq 0$.

Let $4x + 2y = 16$,

$2x + y = 8$

$x = 0, y = 8, (0, 8)$

$y = 0, x = 4, (4, 0)$

$12x + 2y = 24$,

$6x + y = 12$

$x = 0, y = 12, (0, 12)$

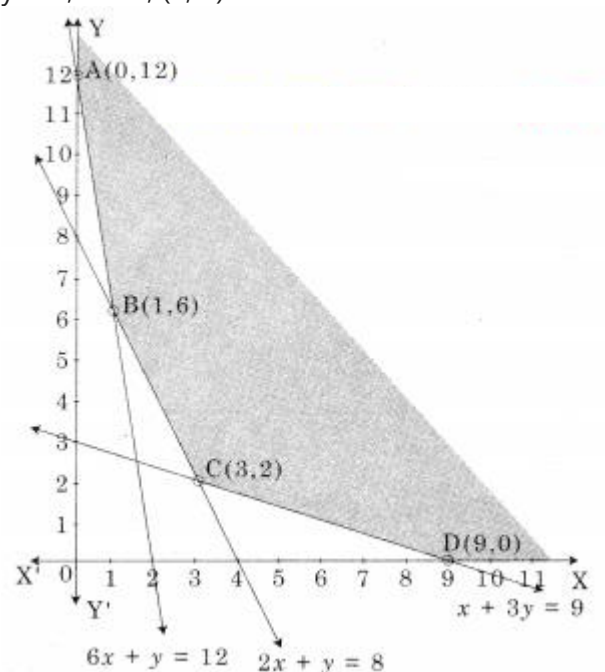
$y = 0, x = 2, (2, 0)$

$2x + 6y = 18$

$x + 3y = 9$

$x = 0, y = 3, (0, 3)$

$y = 0, x = 9, (9, 0)$



The Shaded region ABCD is the feasible region.

A(0, 12) D(9, 0)

B is the intersection of $6x + y = 12$ and $2x + y = 8$

Solving we get $x = 1, y = 6$

∴ B(1, 6)

C is the intersection of $2x + y = 8$ and $x + 3y = 9$

Solving we get $x = 3, y = 2$

∴ C(3, 2)

Corner Points	Value of $z = 800x + 640y$
A (0, 12)	$0 + 7680 = 7680$
B (1, 6)	$800 + 3840 = 4640$
C (3, 2)	$2400 + 1280 = 3680$
D (9, 0)	$7200 + 0 = 7200$

∴ The minimum value of $z = 3680$ at (3, 2)

Hence 3 unit of compound I and 2 units of compound II should be used to get the minimum cost of ₹ 3680.

Question 13.

A person who makes two types of gift items A and B requires the services of a cutter and a finisher. Gift item A requires 4 hours of the cutter's time and 2 hours of the finisher's time. B required 2 hours of the cutter's time and 4 hours of finisher's time. The cutter and finisher have 208 hours and 152 hours available times respectively every month. The profit of one gift item of type A is ₹ 75/- and on gift item B is ₹ 125/-. Assuming that the person can sell all the gift items produced, determine how many gift items of each type should he make every month to obtain the best returns?

Solution:

Let x be the no. of gift A produced and y be the no. of gift B produced.

The data can be tabulated as

	Cutter Hrs.	Finisher Hrs.	Profit (₹)
A (x)	4	2	75
B (y)	2	14	125
Available	208	152	

The LLP is maximize $z = 75x + 125y$

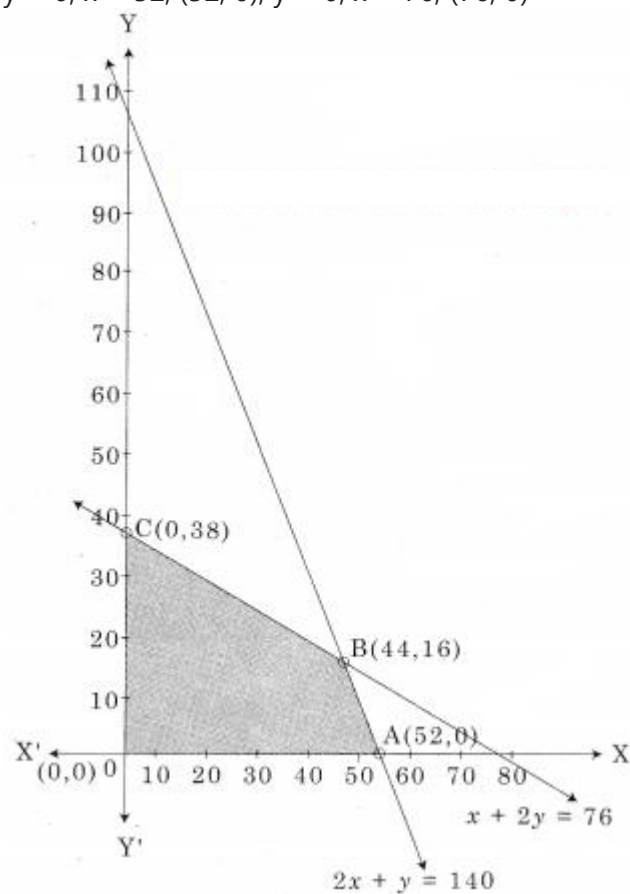
Subject to $4x + 2y \leq 208$, $2x + 4y \leq 152$, $x, y \geq 0$

Let $4x + 2y = 208$, $2x + 4y = 152$,

$2x + y = 104$, $x + 2y = 76$,

$x = 0$, $y = 104$; $(0, 104)$, $x = 0$, $y = 38$; $(0, 38)$

$y = 0$, $x = 52$; $(52, 0)$, $y = 0$, $x = 76$; $(76, 0)$



The shaded region OABC is the feasible region.

O(0,0) A (52, 0) C(0, 38)

B is the intersection of $2x + y = 104$ and $x + 2y = 76$

Solving we get $x = 44$, $y = 16$

∴ B(44, 16)

Corner Points	Value of $z = 75x + 125y$
O (0, 0)	0
A (52, 0)	3900
B (44, 16)	$3300 + 2000 = 5300$
C (0, 38)	$0 + 4750 = 4750$

∴ Maximum value of $z = 5300$ at B(44, 16)

Hence he should produce 44 gifts of type A & 16 of type B to get a maximum profit of ₹ 5300.

Question 14.

A firm manufactures two products A and B on which profit is earned per unit ₹ 3/- and ₹ 4/- respectively. Each product is processed on two machines M₁ and M₂. Product A requires one minute of processing time on M₁ and two minutes of processing time on M₂. B requires one minute of processing time on M₁ and one minute processing time on M₂. Machine M₁ is available for use for 450 minutes while M₂ is available for 600 minutes during any working day. Find the number of units of products A and B to be manufactured to get the maximum profit.

Solution:

Let x denote the number of units of product A and y denote the number of units of product B.

The total profit in ₹ $3x + 4y$,

The given statements can be tabulated as

	A (x)	B (y)	Availability
M ₁	1	1	450
M ₂	2	1	600

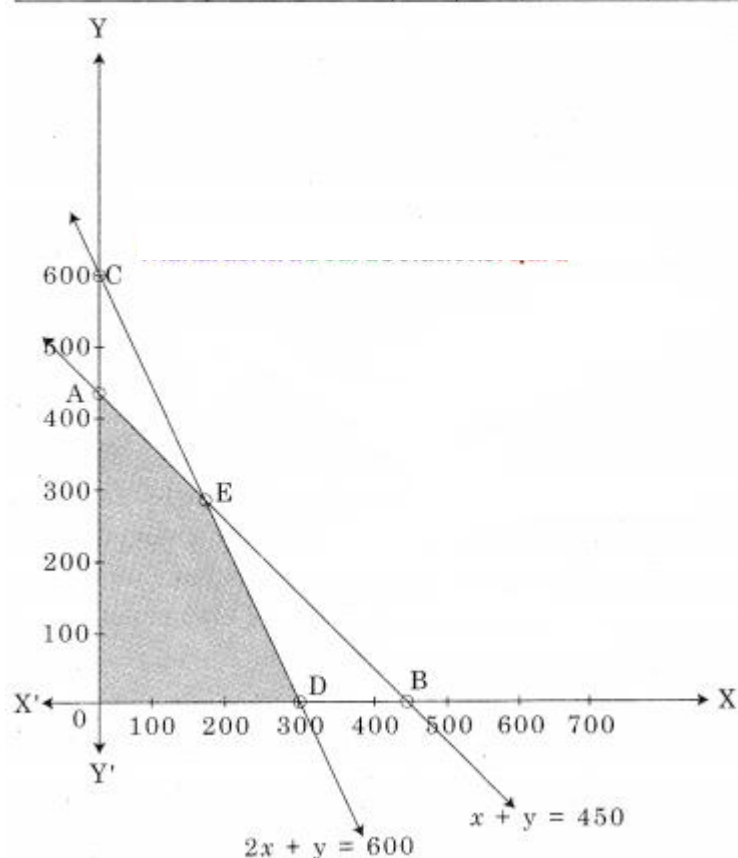
The constraints are $x + y \leq 450$, $2x + y \leq 600$, $x \geq 0$, $y \geq 0$

∴ The LPP can be formulated as

Maximize $z = 3x + 4y$

Subject to $x + y \leq 450$, $2x + y \leq 600$, $x \geq 0$, $y \geq 0$.

Inequation	Corresponding equation	x	y	Points	Region
$x + y \leq 450$	$x + y = 450$	0	450	A (0, 450)	Origin side
		450	0	B (450, 0)	
$2x + y \leq 600$	$2x + y = 600$	0	600	C (0, 600)	Origin side
		300	0	D (300, 0)	



OAED in the feasible region O(0, 0) A(0, 450) D(300, 0)

E is the intersection of $x + y = 450$ and $2x + y = 600$

For E, Solving $x + y = 450$ (i)

$2x + y = 600$ (ii)

Solving $x = 150$, $y = 300$

∴ E(150, 300)

Corner Points	Value of $z = 3x + 4y$
O (0, 0)	$3 \times 0 + 4 \times 0 = 0$
A (0, 450)	$3 \times 0 + 4 \times 450 = 1800$
D (300, 0)	$3 \times 300 + 4 \times 0 = 900$
E (150, 300)	$3 \times 150 + 4 \times 300 = 1650$

\therefore Maximum value of $z = 1800$ at (0, 450)

Question 15.

A firm manufacturing two types of electrical items A and B, can make a profit of ₹ 20/- per unit of A and ₹ 30/- per unit of B. Both A and B make use of two essential components a motor and a transformer. Each unit of A requires 3 motors and 2 transformers and each unit of B required 2 motors and 4 transformers. The total supply of components per month is restricted to 210 motors and 300 transformers. How many units of A and B should they manufacture per month to maximize profit? How much is the maximum profit?

Solution:

Let x be the no. of electrical item A and y be the no. of electrical items to be manufactured per month to maximize the profit.

Total profit is ₹ $20x + 30y$

The given condition can be tabulated as.

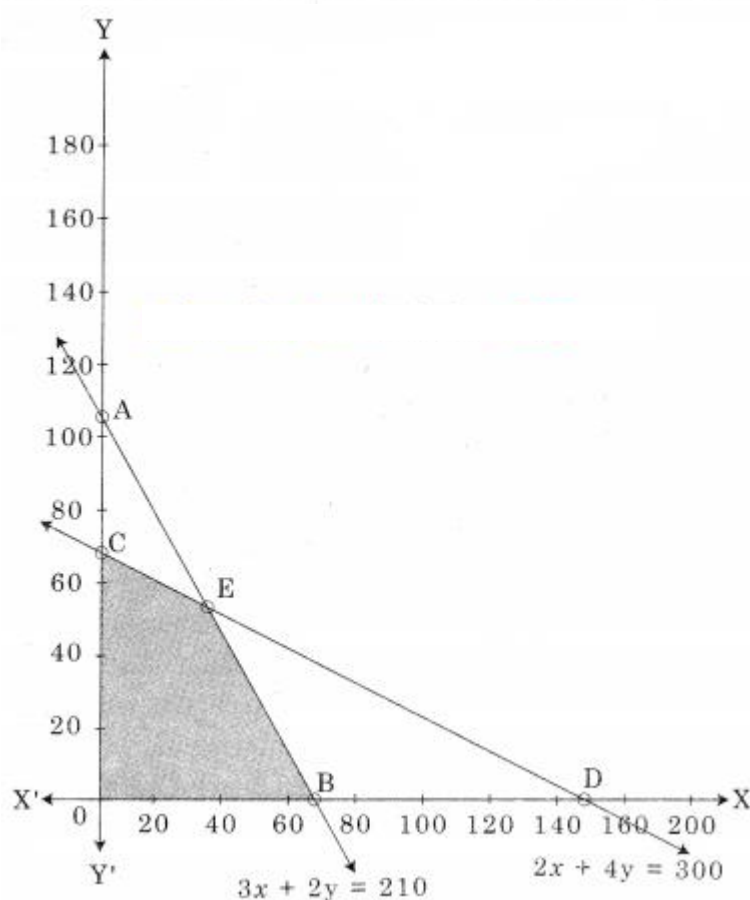
	A (x)	B (y)	Availability
Motor	3	2	210
Transformer	2	4	300

The given LPP can be formulated as

Maximize $z = 20x + 30y$

Subject of $3x + 2y \leq 210$, $2x + 4y \leq 300$, $x \geq 0$, $y \geq 0$.

Inequation	Corresponding equation	x	y	Points	Region
$3x + 2y \leq 210$	$3x + 2y = 210$	0	105	A (0, 105)	Origin side
		70	0	B (70, 0)	
$2x + 4y \leq 300$	$2x + 4y = 300$	0	75	C (0, 75)	Origin Side
		150	0	D (150, 0)	



BOCE is the feasible region

B(70, 0) O(0, 0) C(0, 75)

E is the intersection of $3x + 2y = 210$ and $2x + 4y = 300$

For E, Solving $3x + 2y = 210$ (i)

$2x + 4y = 300$ (ii)

We get $x = 30$, $y = 60$

$\therefore E(30, 60)$

Corner Points	Value of $z = 20x + 30y$
B (70, 0)	$20 \times 70 + 30 \times 0 = 1400$
O (0, 0)	$20 \times 0 + 30 \times 0 = 0$
C (0, 75)	$20 \times 0 + 30 \times 75 = 2250$
E (30, 60)	$20 \times 30 + 30 \times 60 = 2400$

\therefore Maximum value of $z = ₹ 2400$ at (30, 60)

Hence 30 units of A and 60 units of B should be manufactured per month to get the maximum profit of ₹ 2400