Allguidesite -
- Arjun
- Digvijay

Practice Set 5.1 Geometry 10th Std Maths Part 2 Answers Chapter 5 Co-ordinate Geometry

Practice Set 5.1 Geometry Class 10 Question 1. Find the distance between each of the following pairs of points.
i. A (2, 3), B (4,1)
ii. P (-5, 7), Q (-1, 3)
iii. R (0, -3), S (0,$\frac{5}{2}$)
iv. L (5, -8), M (-7, -3)
v. T (-3, 6), R (9, -10)
vi. W($-\frac{7}{2}$,4), X(11, 4)
Solution:
i. Let A (x_1, y_1) and B (x_2, y_2) be the given points.
∴ x_1 = 2, y_1 = 3, x_2 = 4, y_2 = 1
By distance formula,

$$\begin{aligned} \mathbf{d(A, B)} &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(4-2)^2+(1-3)^2} = \sqrt{2^2+(-2)^2} \\ &= \sqrt{4+4} = \sqrt{8} \end{aligned}$$

∴ d(A, B) = $2\sqrt{2}$ units

∴ The distance between the points A and B is $2\sqrt{2}$ units.

ii. Let P (x_1, y_1) and Q (x_2, y_2) be the given points.
∴ x_1 = -5, y_1 = 7, x_2 = -1, y_2 = 3
By distance formula,

$$\begin{aligned} \mathbf{d(P, Q)} &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{[-1-(-5)]^2+(3-7)^2} \\ &= \sqrt{(-1+5)^2+(3-7)^2} \\ &= \sqrt{4^2+(-4)^2} \\ &= \sqrt{16+16} \\ &= \sqrt{32} \end{aligned}$$

∴ d(P, Q) = $4\sqrt{2}$ units

∴ The distance between the points P and Q is $4\sqrt{2}$ units.

iii. Let R (x_1, y_1) and S (x_2, y_2) be the given points.
∴ x_1 = 0, y_1 = -3, x_2 = 0, y_2 = $\frac{5}{2}$
By distance formula,

$$\begin{aligned} \mathbf{d(R, S)} &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(0-0)^2+\left(\frac{5}{2}-(-3)\right)^2} \\ &= \sqrt{\left(\frac{5}{2}+3\right)^2} \\ &= \sqrt{\left(\frac{5+6}{2}\right)^2} \\ &= \sqrt{\left(\frac{11}{2}\right)^2} \end{aligned}$$

$$\mathbf{d(R, S)} = \frac{11}{2}\ \textbf{units}$$

∴ d(R, S) = $\frac{11}{2}$ units
∴ The distance between the points R and S is $\frac{11}{2}$ units.

iv. Let L (x_1, y_1) and M (x_2, y_2) be the given points.
∴ x_1 = 5, y_1 = -8, x_2 = -7, y_2 = -3
By distance formula,

$$\begin{aligned} d(L, M) &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-7-5)^2+[-3-(-8)]^2} \\ &= \sqrt{(-7-5)^2+(-3+8)^2} \\ &= \sqrt{(-12)^2+(5)^2} \\ &= \sqrt{144+25} \\ &= \sqrt{169} \end{aligned}$$

∴ d(L, M) = 13 units
∴ The distance between the points L and M is 13 units.

v. Let T (x_1,y_1) and R (x_2, y_2) be the given points.
∴ x_1 = -3, y_1 = 6,x_2 = 9,y_2 = -10
By distance formula,

$$\begin{aligned} d(T, R) &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{[9-(-3)]^2+(-10-6)^2} \\ &= \sqrt{(9+3)^2+(-10-6)^2} \\ &= \sqrt{12^2+(-16)^2} \\ &= \sqrt{144+256} \\ &= \sqrt{400} \end{aligned}$$

∴ d(T, R) = 20 units
∴ The distance between the points T and R 20 units.

vi. Let W (x_1, y_1) and X (x_2, y_2) be the given points.

$$x_1 = -\frac{7}{2},\ y_1 = 4, x_2 = 11, y_2 = 4$$

By distance formula,

$$\begin{aligned} d(W, X) &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{\left[11-\left(-\frac{7}{2}\right)\right]^2+(4-4)^2} \\ &= \sqrt{\left(11+\frac{7}{2}\right)^2} \\ &= \sqrt{\left(\frac{22+7}{2}\right)^2} \\ &= \sqrt{\left(\frac{29}{2}\right)^2} \end{aligned}$$

∴ d(W, X) = 292 units
∴ The distance between the points W and X is 292 units.

Practice Set 5.1 Geometry 10th Question 2. Determine whether the points are collinear.
i. A (1, -3), B (2, -5), C (-4, 7)
ii. L (-2, 3), M (1, -3), N (5, 4)
iii. R (0, 3), D (2, 1), S (3, -1)
iv. P (-2, 3), Q (1, 2), R (4, 1)
Solution:
i. By distance formula,

$$\begin{aligned} d(A, B) &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(2-1)^2+[-5-(-3)]^2} \\ &= \sqrt{(2-1)^2+(-5+3)^2} \\ &= \sqrt{1^2+(-2)^2} \\ &= \sqrt{1+4} \end{aligned}$$

$$d(A, B) = \sqrt{5} \qquad \ldots(i)$$

$$\begin{aligned}
d(B, C) &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\
&= \sqrt{(-4-2)^2+[7-(-5)]^2} \\
&= \sqrt{(-4-2)^2+(7+5)^2} \\
&= \sqrt{(-6)^2+12^2} \\
&= \sqrt{36+144} \\
&= \sqrt{180}
\end{aligned}$$

$$\therefore \quad d(B, C) = 6\sqrt{5} \qquad \ldots(ii)$$

$$\begin{aligned}
d(A, C) &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\
&= \sqrt{(-4-1)^2+[7-(-3)]^2} \\
&= \sqrt{(-4-1)^2+(7+3)^2} \\
&= \sqrt{(-5)^2+10^2} = \sqrt{25+100} \\
&= \sqrt{125}
\end{aligned}$$

∴ d(A, B) = $\sqrt{5}$ ...(i)

On adding (i) and (iii),

d(A, B) + d(A, C)= $\sqrt{5}$ + 5$\sqrt{5}$ = 6$\sqrt{5}$

∴ d(A, B) + d(A, C) = d(B, C) ... [From (ii)]

∴ Points A, B and C are collinear.

ii. By distance formula,

$$\begin{aligned}
d(L, M) &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\
&= \sqrt{[1-(-2)]^2+(-3-3)^2} \\
&= \sqrt{(1+2)^2+(-3-3)^2} \\
&= \sqrt{3^2+(-6)^2} \\
&= \sqrt{9+36}
\end{aligned}$$

$$\therefore \quad d(L, M) = \sqrt{45} = 3\sqrt{5} \qquad \ldots(i)$$

$$\begin{aligned}
d(M, N) &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\
&= \sqrt{(5-1)^2+[4-(-3)]^2} \\
&= \sqrt{(5-1)^2+(4+3)^2} \\
&= \sqrt{4^2+7^2} \\
&= \sqrt{16+49}
\end{aligned}$$

$$\therefore \quad d(M, N) = \sqrt{65} \qquad \ldots(ii)$$

$$\begin{aligned}
d(L, N) &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\
&= \sqrt{[5-(-2)]^2+(4-3)^2} \\
&= \sqrt{(5+2)^2+(4-3)^2} \\
&= \sqrt{7^2+1^2} \\
&= \sqrt{49+1}
\end{aligned}$$

$$\therefore \quad d(L, N) = \sqrt{50} = 5\sqrt{2} \qquad \ldots(iii)$$

On adding (i) and (iii),

d(L, M) + d(L, N) = 3$\sqrt{5}$ + 5$\sqrt{2}$ ≠ $\sqrt{65}$

∴ d(L, M) + d(L, N) ≠ d(M, N) ... [From (ii)]

∴ Points L, M and N are not collinear.

iii. By distance formula,

$$d(R, D) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2-0)^2 + (1-3)^2}$$
$$= \sqrt{2^2 + (-2)^2}$$
$$= \sqrt{4+4}$$
$$\therefore \quad d(R, D) = \sqrt{8} \quad \ldots(i)$$
$$d(D, S) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3-2)^2 + (-1-1)^2}$$
$$= \sqrt{1^2 + (-2)^2}$$
$$= \sqrt{1+4}$$
$$\therefore \quad d(D, S) = \sqrt{5} \quad \ldots(ii)$$
$$d(R, S) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3-0)^2 + (-1-3)^2}$$
$$= \sqrt{3^2 + (-4)^2}$$
$$= \sqrt{9+16}$$
$$\therefore \quad d(R, S) = \sqrt{25} = 5 \quad \ldots(iii)$$

On adding (i) and (ii),

∴ d(R, D) + d(D, S) = $\sqrt{8} + \sqrt{5} \neq 5$

∴ d(R, D) + d(D, S) ≠ d(R, S) ... [From (iii)]

∴ Points R, D and S are not collinear.

iv. By distance formula,

$$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[1-(-2)]^2 + (2-3)^2}$$
$$= \sqrt{(1+2)^2 + (2-3)^2}$$
$$= \sqrt{3^2 + (-1)^2}$$
$$= \sqrt{9+1}$$
$$\therefore \quad d(P, Q) = \sqrt{10} \quad \ldots(i)$$
$$d(Q, R) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-1)^2 + (1-2)^2}$$
$$= \sqrt{3^2 + (-1)^2}$$
$$= \sqrt{9+1}$$
$$\therefore \quad d(Q, R) = \sqrt{10} \quad \ldots(ii)$$
$$d(P, R) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[4-(-2)]^2 + (1-3)^2}$$
$$= \sqrt{(4+2)^2 + (1-3)^2}$$
$$= \sqrt{6^2 + (-2)^2}$$
$$= \sqrt{36+4}$$
$$= \sqrt{40}$$
$$\therefore \quad d(P, R) = 2\sqrt{10} \quad \ldots(iii)$$

On adding (i) and (ii),

d(P, Q) + d(Q, R) = $\sqrt{10} + \sqrt{10} = 2\sqrt{10}$

∴ d(P, Q) + d(Q, R) = d(P, R) ... [From (iii)]

∴ Points P, Q and R are collinear.

Coordinate Geometry Class 10 Practice Set 5.1 Question 3. Find the point on the X-axis which is equidistant from A (-3,4) and B (1, -4).

Solution:

Let point C be on the X-axis which is equidistant from points A and B.

Point C lies on X-axis.

∴ its y co-ordinate is 0.

Let C = (x, 0)

C is equidistant from points A and B.

∴ AC = BC

$$\sqrt{[x-(-3)]^2+(0-4)^2} = \sqrt{(x-1)^2+[0-(-4)]^2}$$
...[By distance formula]

$$[x-(-3)]^2+(0-4)^2 = (x-1)^2+[0-(-4)]^2$$
...[Squaring both sides]

∴ $(x + 3)^2 + (-4)^2 = (x- 1)^2 + 4^2$
∴ $x^2 + 6x + 9 + 16 = x^2 – 2x + 1 + 16$
∴ 8x = – 8
∴ $x = -\frac{8}{8} = -1$
∴ The point on X-axis which is equidistant from points A and B is (-1,0).

10th Geometry Practice Set 5.1 Question 4. Verify that points P (-2, 2), Q (2, 2) and R (2, 7) are vertices of a right angled triangle.
Solution:
Distance between two points

$$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

By distance formula,

$$PQ = \sqrt{[2-(-2)]^2+(2-2)^2}$$
$$= \sqrt{(2+2)^2+(0)^2} = \sqrt{(4)^2} = 4 \quad \ldots(i)$$
$$QR = \sqrt{(2-2)^2+(7-2)^2}$$
$$= \sqrt{(0)^2+(5)^2} = \sqrt{(5)^2} = 5 \quad \ldots(ii)$$
$$PR = \sqrt{[2-(-2)]^2+(7-2)^2}$$
$$= \sqrt{(2+2)^2+(5)^2} = \sqrt{(4)^2+(5)^2}$$
$$= \sqrt{16+25} = \sqrt{41}$$
$$\text{Now, } PR^2 = \left(\sqrt{41}\right)^2 = 41 \quad \ldots(iii)$$

Consider, $PQ^2 + QR^2 = 4^2 + 5^2 = 16 + 25 = 41$ ... [From (i) and (ii)]
∴ $PR^2 = PQ^2 + QR^2$ ... [From (iii)]
∴ ∆PQR is a right angled triangle. ... [Converse of Pythagoras theorem]
∴ Points P, Q and R are the vertices of a right angled triangle.

Question 5.
Show that points P (2, -2), Q (7, 3), R (11, -1) and S (6, -6) are vertices of a parallelogram.
Proof:
Distance between two points

$$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

By distance formula,

$$PQ = \sqrt{(7-2)^2+[3-(-2)]^2}$$
$$= \sqrt{(7-2)^2+(3+2)^2}$$
$$= \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50} \quad \ldots(i)$$
$$QR = \sqrt{(11-7)^2+(-1-3)^2}$$
$$= \sqrt{4^2+(-4)^2} = \sqrt{16+16} = \sqrt{32} \quad \ldots(ii)$$
$$RS = \sqrt{(6-11)^2+[-6-(-1)]^2}$$
$$= \sqrt{(6-11)^2+(-6+1)^2}$$
$$= \sqrt{(-5)^2+(-5)^2} = \sqrt{25+25} = \sqrt{50} \quad \ldots(iii)$$
$$PS = \sqrt{(6-2)^2+[-6-(-2)]^2}$$
$$= \sqrt{(6-2)^2+(-6+2)^2}$$
$$= \sqrt{4^2+(-4)^2} = \sqrt{16+16} = \sqrt{32} \quad \ldots(iv)$$

PQ = RS ... [From (i) and (iii)]
QR = PS ... [From (ii) and (iv)]
A quadrilateral is a parallelogram, if both the pairs of its opposite sides are congruent.
∴ □ PQRS is a parallelogram.
∴ Points P, Q, R and S are the vertices of a parallelogram.

Question 6.
Show that points A (-4, -7), B (-1, 2), C (8, 5) and D (5, -4) are vertices of rhombus ABCD.
Proof:

Distance between two points

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

By distance formula,

$$AB = \sqrt{[-1-(-4)]^2 + [2-(-7)]^2}$$
$$= \sqrt{(-1+4)^2 + (2+7)^2}$$
$$= \sqrt{3^2 + 9^2} = \sqrt{9+81} = \sqrt{90} \quad \ldots(i)$$
$$BC = \sqrt{[8-(-1)]^2 + (5-2)^2}$$
$$= \sqrt{(8+1)^2 + (5-2)^2}$$
$$= \sqrt{9^2 + 3^2} = \sqrt{81+9} = \sqrt{90} \quad \ldots(ii)$$
$$CD = \sqrt{(5-8)^2 + (-4-5)^2}$$
$$= \sqrt{(-3)^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90} \quad \ldots(iii)$$
$$AD = \sqrt{[5-(-4)]^2 + [-4-(-7)]^2}$$
$$= \sqrt{(5+4)^2 + (-4+7)^2}$$
$$= \sqrt{9^2 + 3^2} = \sqrt{81+9} = \sqrt{90} \quad \ldots(iv)$$

∴ AB = BC = CD = AD …[From (i), (ii), (iii) and (iv)]
In a quadrilateral, if all the sides are equal, then it is a rhombus.
∴ □ ABCD is a rhombus.
∴ Points A, B, C and D are the vertices of rhombus ABCD.

Practice Set 5.1 Question 7. Find x if distance between points L (x, 7) and M (1,15) is 10.
Solution:
$X_1 = x$, $y_1 = 7$, $x_2 = 1$, $y_2 = 15$
By distance formula,

$$d(L, M) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\therefore \quad d(L, M) = \sqrt{(1-x)^2 + (15-7)^2}$$
$$\therefore \quad 10 = \sqrt{(1-x)^2 + 8^2}$$
$$\therefore \quad 100 = (1-x)^2 + 64 \quad \ldots\text{[Squaring both sides]}$$
$$\therefore \quad (1-x)^2 = 100 - 64$$
$$\therefore \quad (1-x)^2 = 36$$
$$\therefore \quad 1 - x = \pm\sqrt{36}$$
$$\ldots\text{[Taking square root of both sides]}$$

∴ 1 – x = ± 6
∴ 1 – x = 6 or l – x = -6
∴ x = – 5 or x = 7
∴ The value of x is – 5 or 7.

Geometry 5.1 Question 8. Show that the points A (1, 2), B (1, 6), C (1 + 23–√, 4) are vertices of an equilateral triangle.
Proof:
Distance between two points

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

By distance formula,

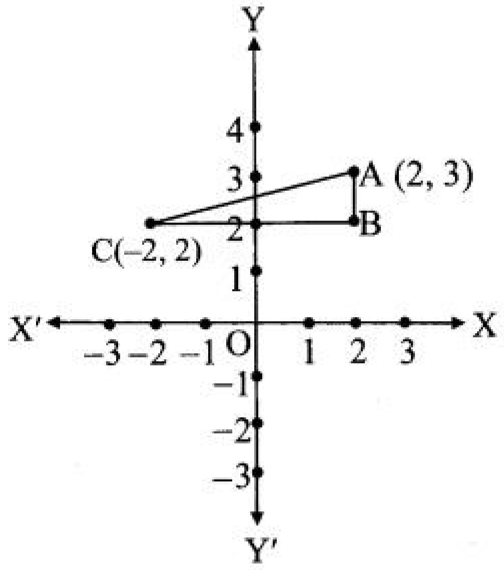
$$AB = \sqrt{(1-1)^2 + (6-2)^2} = \sqrt{0^2 + 4^2}$$
$$= \sqrt{4^2} = 4 \quad \ldots(i)$$
$$BC = \sqrt{\left(1 + 2\sqrt{3} - 1\right)^2 + (4-6)^2}$$
$$= \sqrt{\left(2\sqrt{3}\right)^2 + (-2)^2}$$
$$= \sqrt{12+4} = \sqrt{16} = 4 \quad \ldots(ii)$$
$$AC = \sqrt{\left(1 + 2\sqrt{3} - 1\right)^2 + (4-2)^2}$$
$$= \sqrt{\left(2\sqrt{3}\right)^2 + 2^2}$$
$$= \sqrt{12+4} = \sqrt{16} = 4 \quad \ldots(iii)$$

∴ AB = BC = AC … [From (i), (ii) and (iii)]

∴ ΔABC is an equilateral triangle.
∴ Points A, B and C are the vertices of an equilateral triangle.

Maharashtra Board Class 10 Maths Chapter 5 Coordinate Geometry Intext Questions and Activities

Question 1.
In the figure, seg AB || Y-axis and seg CB || X-axis. Co-ordinates of points A and C are given. To find AC, fill in the boxes given below. (Textbook pa. no. 102)



Solution:
In ΔABC, ∠B = 900
∴ $(AB)_2$ + $(BC)_2$ = [$(Ac)_2$ …(i) … [Pythagoras theorem]
seg CB || X-axis
∴ y co-ordinate of B = 2
seg BA || Y-axis
∴ x co-ordinate of B = 2
∴ co-ordinate of B is (2, 2) = (x_1,y_1)
co-ordinate of A is (2, 3) = (x_2, Y_2)
Since, AB || to Y-axis,
d(A, B) = Y_2 – Y_1
d(A,B) = 3 – 2 = 1
co-ordinate of C is (-2,2) = (x_1,y_1)
co-ordinate of B is (2, 2) = (x_2, y_2)
Since, BC || to X-axis,
d(B, C) = x_2 – x_1
d(B,C) = 2 – -2 = 4
∴ AC_2 = 12 + 42 …[From (i)]
= 1 + 16 = 17
∴ AC = 17−−√ units …[Taking square root of both sides]

Practice Set 5.2 Geometry 10th Std Maths Part 2 Answers Chapter 5 Co-ordinate Geometry

Question 1.
Find the co-ordinates of point P if P divides the line segment joining the points A (-1, 7) and B (4, -3) in the ratio 2:3.
Solution:
Let the co-ordinates of point P be (x, y) and A (x_1, y_1) B (x_2, y_2) be the given points.
Here, x_1 = -1, y_1 = 7, x_2 = 4, y_2 = -3, m = 2, n = 3
∴ By section formula,

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{2(4)+3(-1)}{2+3}$$

$$= \frac{8-3}{5} = \frac{5}{5} = 1$$

$$y = \frac{my_2 + ny_1}{m+n} = \frac{2(-3)+3(7)}{2+3}$$

$$= \frac{-6+21}{5} = \frac{15}{5} = 3$$

The co-ordinates of point P are (1, 3).
∴ The co-ordinates of point P are (1,3).

Question 2.
In each of the following examples find the co-ordinates of point A which divides segment PQ in the ratio a : b.
i. P (-3, 7), Q (1, -4), a : b = 2 : 1
ii. P (-2, -5), Q (4, 3), a : b = 3 : 4
iii. P (2, 6), Q (-4, 1), a : b = 1 : 2
Solution:
Let the co-ordinates of point A be (x, y).
i. Let P (x_1, y_1), Q (x_2, y_2) be the given points.
Here, x_1 = -3, y_1 = 7, x_2 = 1, y_2 = -4, a = 2, b = 1
∴ By section formula,

$$x = \frac{ax_2 + bx_1}{a+b} = \frac{2(1)+1(-3)}{2+1}$$
$$= \frac{2-3}{3} = \frac{-1}{3}$$
$$y = \frac{ay_2 + by_1}{a+b} = \frac{2(-4)+1(7)}{2+1}$$
$$= \frac{-8+7}{3} = \frac{-1}{3}$$

∴ The co-ordinates of point A are (−13,−13).

ii. Let P (x_1,y_1), Q (x_2, y_2) be the given points.
Here, x_1 = -2, y_1 = -5, x_2 = 4, y_2 = 3, a = 3, b = 4
By section formula,

$$x = \frac{ax_2 + bx_1}{a+b} = \frac{3(4)+4(-2)}{3+4}$$
$$= \frac{12-8}{7} = \frac{4}{7}$$
$$y = \frac{ay_2 + by_1}{a+b} = \frac{3(3)+4(-5)}{3+4}$$
$$= \frac{9-20}{7}$$
$$= \frac{-11}{7}$$

∴ The co-ordinates of point A are (47,−117)

iii. Let P (x_1, y_1), Q (x_2, y_2) be the given points.
Here,x_1 = 2,y_1 = 6, x_2 = -4, y_2 = 1, a = 1,b = 2
∴ By section formula,

$$x = \frac{ax_2 + bx_1}{a+b} = \frac{1(-4)+2(2)}{1+2}$$
$$= \frac{-4+4}{3}$$
$$= 0$$
$$y = \frac{ay_2 + by_1}{a+b} = \frac{1(1)+2(6)}{1+2}$$
$$= \frac{1+12}{3}$$
$$= \frac{13}{3}$$

∴ The co-ordinates of point A are (0,133)

Question 3.
Find the ratio in which point T (-1, 6) divides the line segment joining the points P (-3,10) and Q (6, -8).
Solution:
Let P (x_1, y_1), Q (x_2, y_2) and T (x, y) be the given points.
Here, x_1 = -3, y_1 = 10, x_2 = 6, y_2 = -8, x = -1, y = 6
∴ By section formula,

$$x = \frac{mx_2 + nx_1}{m + n}$$

$\therefore \quad -1 = \frac{m(6) + n(-3)}{m + n}$

$\therefore \quad -m - n = 6m - 3n$

$\therefore \quad -n + 3n = 6m + m$

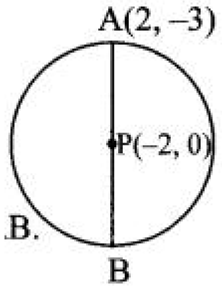
$\therefore \quad 2n = 7m$

$\therefore \quad \frac{m}{n} = \frac{2}{7}$

$\therefore \quad m : n = 2 : 7$

∴ Point T divides seg PQ in the ratio 2 : 7.

Question 4.
Point P is the centre of the circle and AB is a diameter. Find the co-ordinates of point B if co-ordinates of point A and P are (2, -3) and (-2,0) respectively.
Solution:
Let A (x_1, y_1), B (x_2, y_2) and P (x, y) be the given points.
Here, x_1 = 2, y_1 =-3,
x = -2, y = 0



Point P is the midpoint of seg AB.
∴ By midpoint formula,

$$x = \frac{x_1 + x_2}{2}$$

$\therefore \quad -2 = \frac{2 + x_2}{2}$

$\therefore \quad -4 = 2 + x_2$

$\therefore \quad x_2 = -4 - 2 = -6$

$$y = \frac{y_1 + y_2}{2}$$

$\therefore \quad 0 = \frac{-3 + y_2}{2}$

$\therefore \quad -3 + y_2 = 0$

$\therefore \quad y_2 = 3$

∴ The co-ordinates of point B are (-6,3).

Question 5.
Find the ratio in which point P (k, 7) divides the segment joining A (8, 9) and B (1,2). Also find k.
Solution:
Let A (x_1, y_1), B (x_2, y_2) and P (x, y) be the given points.
Here, x_1 = 8, y_1 = 9, x_2 = 1, y_2 = 2, x = k, y = 7
∴ By section formula,

$$y = \frac{my_2 + ny_1}{m+n}$$

$\therefore \quad 7 = \frac{2m+9n}{m+n}$

$\therefore \quad 7m + 7n = 2m + 9n$

$\therefore \quad 5m = 2n$

$\therefore \quad \frac{m}{n} = \frac{2}{5}$

$\therefore \quad m : n = 2 : 5$

$$x = \frac{mx_2 + nx_1}{m+n}$$

$\therefore \quad k = \frac{2(1)+5(8)}{2+5}$

$= \frac{2+40}{7}$

$= \frac{42}{7}$

$= 6$

∴ Point P divides seg AB in the ratio 2 : 5, and the value of k is 6.

Question 6.
Find the co-ordinates of midpoint of the segment joining the points (22, 20) and (0,16).
Solution:
Let A (x_1, y_1) = A (22, 20),
B (x_2,y_2) = B (0, 16)
Let the co-ordinates of the midpoint be P (x,y).
∴ By midpoint formula,

$$x = \frac{x_1 + x_2}{2} = \frac{22+0}{2} = 11$$

$$y = \frac{y_1 + y_2}{2} = \frac{20+16}{2} = \frac{36}{2} = 18$$

The co-ordinates of the midpoint of the segment joining (22, 20) and (0, 16) are (11,18).

Question 7.
Find the centroids of the triangles whose vertices are given below.
i. (-7, 6), (2,-2), (8, 5)
ii. (3, -5), (4, 3), (11,-4)
iii. (4, 7), (8, 4), (7, 11)
Solution:
i. Let A (x_1, y_1) = A (-7, 6),
B (x_2, y_2) = B (2, -2),
C (x_3, y_3) = C(8, 5)
∴ By centroid formula,

$$x = \frac{x_1 + x_2 + x_3}{3}$$

$$= \frac{-7+2+8}{3} = \frac{3}{3} = 1$$

$$y = \frac{y_1 + y_2 + y_3}{3}$$

$$= \frac{6-2+5}{3} = \frac{9}{3} = 3$$

∴ The co-ordinates of the centroid are (1,3).

ii. Let A (x_1 y_1) = A (3, -5),
B (x_2, y_2) = B (4, 3),
C(x_3, y_3) = C(11,-4)
∴ By centroid formula,

$$x = \frac{x_1 + x_2 + x_3}{3}$$

$$= \frac{3+4+11}{3} = \frac{18}{3} = 6$$

$$y = \frac{y_1 + y_2 + y_3}{3}$$

$$= \frac{-5+3-4}{3} = \frac{-6}{3} = -2$$

∴ The co-ordinates of the centroid are (6, -2).

iii. Let A (x_1, y_1) = A (4, 7),
B (x_2, y_2) = B (8,4),
C (x_3, y_3) = C(7,11)
∴ By centroid formula,

$$x = \frac{x_1 + x_2 + x_3}{3}$$
$$= \frac{4+8+7}{3} = \frac{19}{3}$$
$$y = \frac{y_1 + y_2 + y_3}{3}$$
$$= \frac{7+4+11}{3} = \frac{22}{3}$$

∴ The co-ordinates of the centroid are (193,223)

Question 8.
In ΔABC, G (-4, -7) is the centroid. If A (-14, -19) and B (3, 5), then find the co-ordinates of C.
Solution:
G (x, y) = G (-4, -7),
A (x_1, y_1) = A (-14, -19),
B(x_2, y_2) = B(3,5)
Let the co-ordinates of point C be (x_3, y_3).
G is the centroid.
By centroid formula,

$$x = \frac{x_1 + x_2 + x_3}{3}$$
$$\therefore \quad -4 = \frac{-14 + 3 + x_3}{3}$$
$$\therefore \quad -12 = -11 + x_3$$
$$\therefore \quad x_3 = -12 + 11$$
$$\therefore \quad x_3 = -1$$
$$y = \frac{y_1 + y_2 + y_3}{3}$$
$$\therefore \quad -7 = \frac{-19 + 5 + y_3}{3}$$
$$\therefore \quad -21 = -14 + y_3$$
$$\therefore \quad y_3 = -21 + 14$$
$$\therefore \quad y_3 = -7$$

∴ The co-ordinates of point C are (-1, – 7).

Question 9.
A (h, -6), B (2, 3) and C (-6, k) are the co-ordinates of vertices of a triangle whose centroid is G (1,5). Find h and k.
Solution:
A(x_1,y_1) = A(h, -6),
B (x_2, y_2) = B(2, 3),
C (x_3, y_3) = C (-6, k)
∴ centroid G (x, y) = G (1, 5)
G is the centroid.
By centroid formula,

$$x = \frac{x_1 + x_2 + x_3}{3}$$
$$\therefore \quad 1 = \frac{h + 2 + (-6)}{3}$$
$$\therefore \quad 3 = h + 2 - 6$$

∴ 3 = h – 4
∴ h = 3 + 4
∴ h = 7

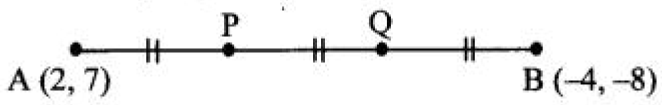
$$y = \frac{y_1 + y_2 + y_3}{3}$$
$$\therefore \quad 5 = \frac{-6 + 3 + k}{3}$$

∴ 15 = -3 + k
∴ k = 15 + 3
∴ k = 18
∴ h = 7 and k = 18

Question 10.
Find the co-ordinates of the points of trisection of the line segment AB with A (2,7) and B (-4, -8).
Solution:
A (2, 7), B H,-8)

Suppose the points P and Q trisect seg AB.

∴ AP = PQ = QB



Let AP = PQ = QB = x

$$\frac{AP}{PB} = \frac{AP}{PQ + QB} \quad \ldots[P - Q - B]$$

$$\therefore \quad \frac{AP}{PB} = \frac{x}{x + x} = \frac{x}{2x} = \frac{1}{2}$$

∴ Point P divides seg AB in the ratio 1:2.

∴ By section formula,

$$x \text{ co-ordinate of P} = \frac{mx_2 + nx_1}{m + n}$$

$$= \frac{1(-4) + 2(2)}{1 + 2}$$

$$= \frac{-4 + 4}{3} = 0$$

$$y \text{ co-ordinate of P} = \frac{my_2 + ny_1}{m + n}$$

$$= \frac{1(-8) + 2(7)}{1 + 2}$$

$$= \frac{-8 + 14}{3} = \frac{6}{3} = 2$$

Co-ordinates of P are (0, 2).

Point Q is the midpoint of PB.

By midpoint formula,

$$x \text{ co-ordinate of Q} = \frac{x_1 + x_2}{2}$$

$$= \frac{0 + (-4)}{2} = \frac{-4}{2} = -2$$

$$y \text{ co-ordinate of Q} = \frac{y_1 + y_2}{2}$$

$$= \frac{2 + (-8)}{2}$$

$$= \frac{2 - 8}{2} = \frac{-6}{2} = -3$$

Co-ordinates of Q are (-2, -3).

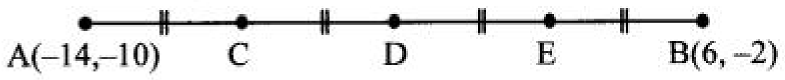
∴ The co-ordinates of the points of trisection seg AB are (0,2) and (-2, -3).

Question 11.

If A (-14, -10), B (6, -2) are given, find the co-ordinates of the points which divide segment AB into four equal parts.

Solution:

Let the points C, D and E divide seg AB in four equal parts.



Point D is the midpoint of seg AB.

∴ By midpoint formula,

$$x \text{ co-ordinate of D} = \frac{x_1 + x_2}{2}$$

$$= \frac{-14 + 6}{2}$$

$$= \frac{-8}{2} = -4$$

$$y \text{ co-ordinate of D} = \frac{y_1 + y_2}{2}$$

$$= \frac{-10 - 2}{2}$$

$$= \frac{-12}{2} = -6$$

∴ Co-ordinates of D are (-4, -6).

Point C is the midpoint of seg AD.

∴ By midpoint formula,

$$x \text{ co-ordinate of C} = \frac{x_1 + x_2}{2} = \frac{-14-4}{2} = \frac{-18}{2} = -9$$

$$y \text{ co-ordinate of C} = \frac{y_1 + y_2}{2} = \frac{-10-6}{2} = \frac{-16}{2} = -8$$

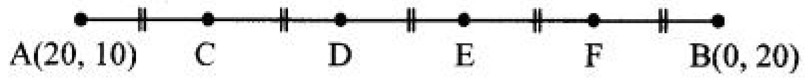
∴ Co-ordinates of C are (-9, -8).
Point E is the midpoint of seg DB.
∴ By midpoint formula,

$$x \text{ co-ordinate of E} = \frac{x_1 + x_2}{2} = \frac{-4+6}{2} = \frac{2}{2} = 1$$

$$y \text{ co-ordinate of E} = \frac{y_1 + y_2}{2} = \frac{-6-2}{2} = \frac{-8}{2} = -4$$

∴ Co-ordinates of E are (1,-4).
∴ The co-ordinates of the points dividing seg AB in four equal parts are C(-9, -8), D(-4, -6) and E(1, – 4).

Question 12.
If A (20, 10), B (0, 20) are given, find the co-ordinates of the points which divide segment AB into five congruent parts.
Solution:
Suppose the points C, D, E and F divide seg AB in five congruent parts.
∴ AC = CD = DE = EF = FB



$$\frac{AC}{CB} = \frac{AC}{CD + DE + EF + FB} \quad \ldots \begin{bmatrix} C - D - E \\ D - E - F \\ E - F - B \end{bmatrix}$$

$$\therefore \quad \frac{AC}{CB} = \frac{x}{x + x + x + x} = \frac{x}{4x} = \frac{1}{4}$$

∴ **Point C divides seg AB in the ratio 1 : 4.**
By section formula,

$$x \text{ co-ordinate of C} = \frac{mx_2 + nx_1}{m+n} = \frac{1(0) + 4(20)}{1+4} = \frac{80}{5} = 16$$

$$y \text{ co-ordinate of C} = \frac{my_2 + ny_1}{m+n} = \frac{1(20) + 4(10)}{1+4} = \frac{60}{5} = 12$$

∴ co-ordinates of C are (16, 12).
E is the midpoint of seg CB.
By midpoint formula,

$$x \text{ co-ordinate of E} = \frac{x_1 + x_2}{2}$$
$$= \frac{16+0}{2} = \frac{16}{2} = 8$$
$$y \text{ co-ordinate of E} = \frac{y_1 + y_2}{2}$$
$$= \frac{12+20}{2} = \frac{32}{2} = 16$$

∴ co-ordinates of E are (8, 16).
D is the midpoint of seg CE.

$$x \text{ co-ordinate of D} = \frac{x_1 + x_2}{2}$$
$$= \frac{16+8}{2} = \frac{24}{2} = 12$$
$$y \text{ co-ordinate of D} = \frac{y_1 + y_2}{2}$$
$$= \frac{12+16}{2} = \frac{28}{2} = 14$$

∴ co-ordinates of D are (12, 14).
F is the midpoint of seg EB.

$$x \text{ co-ordinate of F} = \frac{x_1 + x_2}{2}$$
$$= \frac{8+0}{2} = 4$$
$$y \text{ co-ordinate of F} = \frac{y_1 + y_2}{2}$$
$$= \frac{16+20}{2} = \frac{36}{2} = 18$$

∴ co-ordinates of F are (4, 18).
∴ The co-ordinates of the points dividing seg AB in five congruent parts are C (16, 12), D (12, 14), E (8, 16) and F (4, 18).

Maharashtra Board Class 10 Maths Chapter 5 Co-ordinate Geometry Intext Questions and Activities

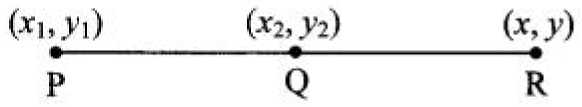
Question 1.
A (15, 5), B (9, 20) and A-P-B. Find the ratio in which point P (11, 15) divides segment AB. Find the ratio using x and y co-ordinates. Write the conclusion. (Textbook pg. no. 113)
Solution:
Suppose point P (11,15) divides segment AB in the ratio m : n.
By section formula,

$$x = \frac{mx_2 + nx_1}{m+n}$$
$$\therefore \quad 11 = \frac{9m + 15n}{m+n}$$
$$\therefore \quad 11m + 11n = 9m + 15n$$
$$\therefore \quad 2m = 4n$$
$$\therefore \quad \frac{m}{n} = \frac{4}{2} = \frac{2}{1} = 2 : 1$$

$$y = \frac{my_2 + ny_1}{m+n}$$
$$\therefore \quad 15 = \frac{20m + 5n}{m+n}$$
$$\therefore \quad 15m + 15n = 20m + 5n$$
$$\therefore \quad 5m = 10n$$
$$\therefore \quad \frac{m}{n} = \frac{10}{5} = \frac{2}{1} = 2 : 1$$

∴ Point P divides seg AB in the ratio 2 : 1.
The ratio obtained by using x and y co-ordinates is the same.

Question 2.
External division: (Textbook pg. no. 115)
Suppose point R divides seg PQ externally in the ratio 3:1.



$\therefore \quad \frac{PR}{QR} = \frac{3}{1}$

Let the common multiple be k.
Let PR = 3k and QR = k
Now, PR = PQ + QR … [P – Q – R]
∴ 3k = PQ + k
∴ PQQR = 2kk = 21
∴ Point Q divides seg PR in the ratio 2 : 1 internally.
Thus, we can find the co-ordinates of point R, when co-ordinates of points P and Q are given.

Practice Set 5.3 Geometry 10th Std Maths Part 2 Answers Chapter 5 Co-ordinate Geometry

Practice Set 5.3 Geometry Class 10 Question 1. Angles made by the line with the positive direction of X-axis are given. Find the slope of these lines.
i. 45°
ii. 60°
iii. 90°
Solution:
i. Angle made with the positive direction of
X-axis (θ) = 45°
Slope of the line (m) = tan θ
∴ m = tan 45° = 1
∴ The slope of the line is 1.

ii. Angle made with the positive direction of X-axis (θ) = 60°
Slope of the line (m) = tan θ
∴ m = tan 60° = 3–√
∴ The slope of the line is 3–√.

iii. Angle made with the positive direction of
X-axis (θ) = 90°
Slope of the line (m) = tan θ
∴ m = tan 90°
But, the value of tan 90° is not defined.
∴ The slope of the line cannot be determined.

Practice Set 5.3 Geometry Question 2. Find the slopes of the lines passing through the given points.
i. A (2, 3), B (4, 7)
ii. P(-3, 1), Q (5, -2)
iii. C (5, -2), D (7, 3)
iv. L (-2, -3), M (-6, -8)
v. E (-4, -2), F (6, 3)
vi. T (0, -3), s (0,4)
Solution:
i. A (x_1, y_1) = A (2, 3) and B (x_2, y_2) = B (4, 7)
Here, x_1 = 2, x_2 = 4, y_1 = 3, y_2 = 7

$$\text{Slope of line AB} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{7-3}{4-2} = \frac{4}{2} = 2$$

∴ The slope of line AB is 2.

ii. P (x_1, y_1) = P (-3, 1) and Q (x_2, y_2) = Q (5, -2)
Here, x_1 = -3, x_2 = 5, y_1 = 1, y_2 = -2

$$\text{Slope of line PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-2-1}{5-(-3)} = \frac{-3}{5+3} = \frac{-3}{8}$$

∴ The slope of line PQ is -38

iii. C (x_1, y_1) = C (5, -2) and D (x_2, y_2) = D (7, 3)
Here, x_1 = 5, x_2 = 7, y_1 = -2, y_2 = 3

$$\text{Slope of line CD} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3-(-2)}{7-5} = \frac{3+2}{2} = \frac{5}{2}$$

∴ The slope of line CD is $\frac{5}{2}$

iv. L (x_1, y_1) = L (-2, -3) and M (x_2,y_2) = M (-6, -8)
Here, x_1 = -2, x_2 = – 6, y_1 = – 3, y_2 = – 8

$$\text{Slope of line LM} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-8-(-3)}{-6-(-2)} = \frac{-8+3}{-6+2}$$
$$= \frac{-5}{-4} = \frac{5}{4}$$

∴ The slope of line LM is $\frac{5}{4}$

v. E (x_1, y_1) = E (-4, -2) and F (x_2, y_2) = F (6, 3)
Here,x_1 = -4, x_2 = 6, y_1 = -2, y_2 = 3

$$\text{Slope of line EF} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3-(-2)}{6-(-4)} = \frac{3+2}{6+4}$$
$$= \frac{5}{10} = \frac{1}{2}$$

∴ The slope of line EF is $\frac{1}{2}$.

vi. T (x_1, y_1) = T (0, -3) and S (x_2, y_2) = S (0, 4)
Here, x_1 = 0, x_2 = 0, y_1 = -3, y_2 = 4

$$\text{Slope of line TS} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4-(-3)}{0-0} = \frac{4+3}{0} = \frac{7}{0}$$
$$= \text{Not defined}$$

∴ The slope of line TS cannot be determined.

5.3.5 Practice Question 3. Determine whether the following points are collinear.
i. A (-1, -1), B (0, 1), C (1, 3)
ii. D (- 2, -3), E (1, 0), F (2, 1)
iii. L (2, 5), M (3, 3), N (5, 1)
iv. P (2, -5), Q (1, -3), R (-2, 3)
v. R (1, -4), S (-2, 2), T (-3,4)
vi. A(-4,4),K[-2,$\frac{5}{2}$], N (4,-2)
Solution:

$$\text{i.} \quad \text{Slope of line AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-1)}{0-(-1)} = \frac{1+1}{0+1}$$
$$= 2$$

$$\text{Slope of line BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{1-0} = 2$$

∴ slope of line AB = slope of line BC
∴ line AB || line BC
Also, point B is common to both the lines.
∴ Both lines are the same.
∴ Points A, B and C are collinear.

$$\text{ii.} \quad \text{Slope of line DE} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-3)}{1-(-2)}$$
$$= \frac{0+3}{1+2} = \frac{3}{3} = 1$$

$$\text{Slope of line EF} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{2-1} = 1$$

∴ slope of line DE = slope of line EF
∴ line DE || line EF
Also, point E is common to both the lines.

∴ Both lines are the same.
∴ Points D, E and F are collinear.

iii. Slope of line LM = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{3-2} = -2$

Slope of line MN = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{5-3} = -\frac{2}{2} = -1$

∴ slope of line LM ≠ slope of line MN
∴ Points L, M and N are not collinear.

iv. Slope of line PQ = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-5)}{1-2}$

$= \frac{-3+5}{-1} = -2$

Slope of line QR = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-3)}{-2-1}$

$= \frac{3+3}{-3} = \frac{6}{-3} = -2$

∴ slope of line PQ = slope of line QR
∴ line PQ || line QR
Also, point Q is common to both the lines.
∴ Both lines are the same.
∴ Points P, Q and R are collinear.

v. Slope of line RS = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-4)}{-2-1}$

$= \frac{2+4}{-3} = \frac{6}{-3} = -2$

Slope of line ST = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{-3-(-2)}$

$= \frac{2}{-3+2} = -2$

∴ slope of line RS = slope of line ST
∴ line RS || line ST
Also, point S is common to both the lines.
∴ Both lines are the same.
∴ Points R, S and T are collinear.

vi. Slope of line AK = $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{\frac{5}{2}-4}{-2-(-4)} = \frac{\frac{5-8}{2}}{-2+4} = \frac{5-8}{2(2)}$

$= \frac{-3}{4}$

Slope of line KN = $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-\frac{5}{2}}{4-(-2)} = \frac{\frac{-4-5}{2}}{4+2}$

$= \frac{-4-5}{2(6)} = \frac{-9}{12} = \frac{-3}{4}$

∴ slope of line AK = slope of line KN
∴ line AK || line KN
Also, point K is common to both the lines.
∴ Both lines are the same.
∴ Points A, K and N are collinear.

Practice Set 5.3 Geometry 9th Standard Question 4. If A (1, -1), B (0,4), C (-5,3) are vertices of a triangle, then find the slope of each side.
Solution:

We know that, slope of line = $\frac{y_2 - y_1}{x_2 - x_1}$

Slope of side AB = $\frac{4-(-1)}{0-1} = \frac{4+1}{-1} = -5$

Slope of side BC = $\frac{3-4}{-5-0} = \frac{-1}{-5} = \frac{1}{5}$

Slope of side AC = $\frac{3-(-1)}{-5-1}$

$= \frac{3+1}{-6} = \frac{-4}{6} = \frac{-2}{3}$

∴ The slopes of the sides AB, BC and AC are -5, 15 and -23 respectively.

Geometry 5.3 Question 5. Show that A (-4, -7), B (-1, 2), C (8, 5) and D (5, -4) are the vertices of a parallelogram.
Proof:

We know that, slope of line = $\frac{y_2 - y_1}{x_2 - x_1}$

Slope of side AB = $\frac{2-(-7)}{-1-(-4)}$

$= \frac{2+7}{-1+4} = \frac{9}{3} = 3 \quad \ldots(i)$

Slope of side BC = $\frac{5-2}{8-(-1)} = \frac{3}{8+1}$

$= \frac{3}{9} = \frac{1}{3} \quad \ldots(ii)$

Slope of side CD = $\frac{-4-5}{5-8} = \frac{-9}{-3} = 3 \quad \ldots(iii)$

Slope of side AD = $\frac{-4-(-7)}{5-(-4)}$

$= \frac{-4+7}{5+4} = \frac{3}{9} = \frac{1}{3} \quad \ldots(iv)$

∴ Slope of side AB = Slope of side CD … [From (i) and (iii)]
∴ side AB || side CD
Slope of side BC = Slope of side AD … [From (ii) and (iv)]
∴ side BC || side AD
Both the pairs of opposite sides of □ ABCD are parallel.
□ ABCD is a parallelogram.
Points A(-4, -7), B(-1, 2), C(8, 5) and D(5, -4) are the vertices of a parallelogram.

Question 6.
Find k, if R (1, -1), S (-2, k) and slope of line RS is -2.
Solution:
R(x_1, y_1) = R (1, -1), S (x_2, y_2) = S (-2, k)
Here, x_1 = 1, x_2 = -2, y_1 = -1, y_2 = k

Slope of line RS = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{k-(-1)}{-2-1} = \frac{k+1}{-3}$

But, slope of line RS is -2. … [Given]
∴ -2 = k+1-3
∴ k + 1 = 6
∴ k = 6 – 1
∴ k = 5

5.3 Class 10 Question 7. Find k, if B (k, -5), C (1, 2) and slope of the line is 7.
Solution:
B(x_1, y_1) = B (k, -5), C (x_2, y_2) = C (1, 2)
Here, x_1 = k, x_2 = 1, y_1 = -5, y_2 = 2

Slope of line BC = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-5)}{1-k}$

$= \frac{2+5}{1-k} = \frac{7}{1-k}$

But, slope of line BC is 7. …[Given]
∴ 7 = 71-k
∴ 7(1 – k) = 7
∴ 1 – k = 77
∴ 1 – k = 1
∴ k = 0

Question 8.
Find k, if PQ || RS and P (2, 4), Q (3, 6), R (3,1), S (5, k).
Solution:

$$\text{Slope of line PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-4}{3-2} = 2$$

$$\text{Slope of line RS} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{k-1}{5-3} = \frac{k-1}{2}$$

But, line PQ || line RS … [Given]
∴ Slope of line PQ = Slope of line RS
∴ 2 = k-12
∴ 4 = k – 1
∴ k = 4 + 1
∴ k = 5

Problem Set 5 Geometry 10th Std Maths Part 2 Answers Chapter 5 Co-ordinate Geometry

Question 1.
Fill in the blanks using correct alternatives.

i. Seg AB is parallel to Y-axis and co-ordinates of point A are (1, 3), then co-ordinates of point B can be _______.
(A) (3,1)
(B) (5,3)
(C) (3,0)
(D) (1,-3)
Answer: (D)
Since, seg AB || Y-axis.
∴ x co-ordinate of all points on seg AB
will be the same,
x co-ordinate of A (1, 3) = 1
x co-ordinate of B (1, – 3) = 1
∴ Option (D) is correct.

ii. Out of the following, point lies to the right of the origin on X-axis.
(A) (-2,0)
(B) (0,2)
(C) (2,3)
(D) (2,0)
Answer: (D)

iii. Distance of point (-3, 4) from the origin is _________.
(A) 7
(B) 1
(C) 5
(D) -5
Answer: (C)
Distance of (-3, 4) from origin
=(−3)2+(4)2−−−−−−−−−−√=9+16−−−−−√=25−−√=5

iv. A line makes an angle of 30° with the positive direction of X-axis. So the slope of the line is _______.
(A) 12
(B) 3√2
(C) 13√
(D) 3–√
Answer: (C)

Question 2.
Determine whether the given points are collinear.
i. A (0, 2), B (1, -0.5), C (2, -3)
ii. P(1,2), Q(2,85),R(3,65)
iii L (1, 2), M (5, 3), N (8, 6)
Solution:

i. Slope of line AB $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-0.5 - 2}{1 - 0} = -2.5$

Slope of line BC $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-3 - (-0.5)}{2 - 1}$

$= \frac{-3 + 0.5}{1} = -2.5$

∴ slope of line AB = slope of line BC
∴ line AB || line BC
Also, point B is common to both the lines.
∴ Both lines are the same.
∴ Points A, B and C are collinear.

ii. Slope of line PQ $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{\frac{8}{5} - 2}{2 - 1} = \frac{8 - 10}{5} = \frac{-2}{5}$

Slope of line QR $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{\frac{6}{5} - \frac{8}{5}}{3 - 2} = \frac{-2}{5}$

∴ slope of line PQ = slope of line QR
∴ line PQ || line QR
Also, point Q is common to both the lines.
∴ Both lines are the same.
∴ Points P, Q and R are collinear.

iii. Slope of line LM $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3 - 2}{5 - 1} = \frac{1}{4}$

Slope of line MN $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{6 - 3}{8 - 5} = \frac{3}{3} = 1$

∴ slope of line LM ≠ slope of line MN
∴ Points L, M and N are not collinear.
[Note: Students can solve the above problems by using distance formula.]

Question 3.
Find the co-ordinates of the midpoint of the line segment joining P (0,6) and Q (12,20).
Solution:
$P(x_1,y_1)$ = P (0, 6), $Q(x_2, y_2)$ = Q (12, 20)
Here, $x_1 = 0$, $y_1 = 6$, $x_2 = 12$, $y_2 = 20$
∴ Co-ordinates of the midpoint of seg PQ

$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{0 + 12}{2}, \frac{6 + 20}{2}\right)$

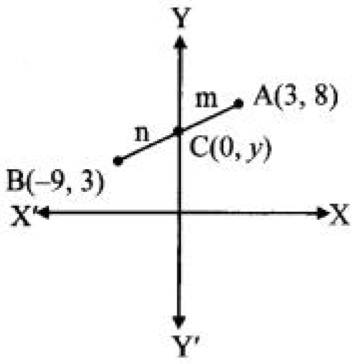
$= \left(\frac{12}{2}, \frac{26}{2}\right)$

$= (6, 13)$

∴ The co-ordinates of the midpoint of seg PQ are (6,13).

Question 4.
Find the ratio in which the line segment joining the points A (3, 8) and B (-9, 3) is divided by the Y-axis.
Solution:
Let C be a point on Y-axis which divides seg AB in the ratio m : n.
Point C lies on the Y-axis
∴ its x co-ordinate is 0.
Let C = (0, y)

Here A (x_1,y_1) = A(3, 8)
B (x_2, y_2) = B (-9, 3)
∴ By section formula,



$$x = \frac{mx_2 + nx_1}{m+n}$$
$$\therefore \quad 0 = \frac{-9m+3n}{m+n}$$
$$\therefore \quad -9m + 3n = 0$$
$$\therefore \quad 9m = 3n$$
$$\therefore \quad \frac{m}{n} = \frac{3}{9}$$
$$\therefore \quad \frac{m}{n} = \frac{1}{3}$$
$$\therefore \quad m : n = 1 : 3$$
∴ Y-axis divides the seg AB in the ratio 1 : 3.

Question 5.
Find the point on X-axis which is equidistant from P (2, -5) and Q (-2,9).
Solution:
Let point R be on the X-axis which is equidistant from points P and Q.
Point R lies on X-axis.
∴ its y co-ordinate is 0.
Let R = (x, 0)
R is equidistant from points P and Q.
∴ PR = QR
$$\therefore \quad \sqrt{(x-2)^2 + [0-(-5)]^2} = \sqrt{[x-(-2)]^2 + (0-9)^2}$$
...[By distance formula]

∴ $(x – 2)^2 + [0 – (-5)]^2 = [x – (- 2)]^2 + (0 – 9)^2$ ...[Squaring both sides]
∴ $(x – 2)^2 + (5)^2 = (x + 2)^2 + (-9)^2$
∴ $4 – 4x + x^2 + 25 = 4 + 4x + x^2 + 81$
∴ – 8x = 56
∴ x = -7
∴ The point on X-axis which is equidistant from points P and Q is (-7,0).

Question 6.
Find the distances between the following points.
i. A (a, 0), B (0, a)
ii. P (-6, -3), Q (-1, 9)
iii. R (-3a, a), S (a, -2a)
Solution:
i. Let A (x_1, y_1) and B (x_2, y_2) be the given points.
∴ x_1 = a, y_1 = 0, x_2 = 0, y_2 = a
By distance formula,
$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(0-a)^2 + (a-0)^2}$$
$$= \sqrt{(-a)^2 + a^2}$$
$$= \sqrt{a^2 + a^2}$$
$$= \sqrt{2a^2}$$
∴ d(A, B) = a2–√ units

ii. Let P (x_1, y_1) and Q (x_2, y_2) be the given points.
∴ x_1 = -6, y_1 = -3, x_2 = -1, y_2 = 9
By distance formula,

$$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[-1-(-6)]^2 + [9-(-3)]^2}$$
$$= \sqrt{(-1+6)^2 + (9+3)^2}$$
$$= \sqrt{5^2 + 12^2}$$
$$= \sqrt{25+144}$$
$$= \sqrt{169}$$

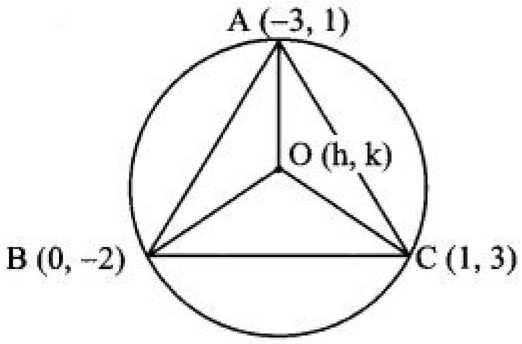
∴ d(P, Q) = 13 units

iii. Let R (x_1, y_1) and S (x_2, y_2) be the given points.
∴ x_1 = -3a, y_1 = a, x_2 = a, y_2 = -2a
By distance formula,

$$d(R, S) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[a-(-3a)]^2 + (-2a-a)^2}$$
$$= \sqrt{(a+3a)^2 + (-2a-a)^2}$$
$$= \sqrt{(4a)^2 + (-3a)^2}$$
$$= \sqrt{16a^2 + 9a^2}$$
$$= \sqrt{25a^2}$$

∴ d(R, S) = 5a units

Question 7.
Find the co-ordinates of the circumcentre of a triangle whose vertices are (-3,1), (0, -2) and (1,3).
Solution:
Let A (-3, 1), B (0, -2) and C (1, 3) be the vertices of the triangle.
Suppose O (h, k) is the circumcentre of ΔABC.



OA = OB ...[Radii of the same circle]

$$\therefore \quad \sqrt{[h-(-3)]^2 + (k-1)^2} = \sqrt{(h-0)^2 + [k-(-2)]^2}$$ ...[By distance formula]

$$\therefore \quad [h-(-3)]^2 + (k-1)^2 = (h-0)^2 + [k-(-2)]^2$$ ...[Squaring both sides]

∴ $(h + 3)^2 + (k – 1)^2 = h^2 + (k + 2)^2$
∴ $h^2 + 6h + 9 + k^2 – 2k + 1 = h^2 + k^2 + 4k + 4$
∴ 6h – 2k + 10 = 4k + 4
∴ 6h – 2k – 4k = 4 – 10
∴ 6h – 6k = – 6
∴ h – k = -1 ,..(i)[Dividing both sides by 6]
OB = OC ...[Radii of the same circle]

$$\therefore \quad \sqrt{(h-0)^2 + [k-(-2)]^2} = \sqrt{(h-1)^2 + (k-3)^2}$$ ...[By distance formula]

$$\therefore \quad (h-0)^2 + [k-(-2)]^2 = (h-1)^2 + (k-3)^2$$ ...[Squaring both sides]

∴ $h^2 + (k + 2)^2 = (h – 1)^2 + (k – 3)^2$
∴ $h^2 + k^2 + 4k + 4 = h^2 – 2h + 1 + k^2 – 6k + 9$
∴ 4k + 4 = -2h + 1 – 6k + 9
∴ 2h+ 10k = 6
∴ h + 5k = 3 ...(ii)
Subtracting equation (ii) from (i), we get

$$\begin{array}{l} h - k = -1 \\ h + 5k = 3 \\ \underline{- \quad - \quad -} \\ -6k = -4 \end{array}$$

$\therefore \quad k = \frac{-4}{-6} = \frac{2}{3}$

Substituting the value of k in equation (i), we get

$h - \frac{2}{3} = -1$

$\therefore \quad h = -1 + \frac{2}{3}$

$\therefore \quad h = \frac{-1}{3}$

∴ The co-ordinates of the circumcentre of the triangle are (-13,23)

Question 8.
In the following examples, can the segment joining the given points form a triangle? If triangle is formed, state the type of the triangle considering sides of the triangle.
i. L (6, 4), M (-5, -3), N (-6, 8)
ii. P (-2, -6), Q (-4, -2), R (-5, 0)
iii. A(2–√,2–√),B(-2–√,-2–√),C(6–√,6–√)
Solution:
i. By distance formula,

$$\begin{aligned} d(L, M) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-5-6)^2 + (-3-4)^2} \\ &= \sqrt{(-11)^2 + (-7)^2} \\ &= \sqrt{121+49} \end{aligned}$$

$\therefore \quad d(L, M) = \sqrt{170} \quad \text{...(i)}$

$$\begin{aligned} d(M, N) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-6-(-5)]^2 + [8-(-3)]^2} \\ &= \sqrt{(-6+5)^2 + (8+3)^2} \\ &= \sqrt{(-1)^2 + 11^2} \\ &= \sqrt{1+121} \end{aligned}$$

$\therefore \quad d(M, N) = \sqrt{122} \quad \text{...(ii)}$

$$\begin{aligned} d(L, N) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-6-6)^2 + (8-4)^2} \\ &= \sqrt{(-12)^2 + (4)^2} \\ &= \sqrt{144+16} \end{aligned}$$

$\therefore \quad d(L, N) = \sqrt{160} \quad \text{...(iii)}$

On adding (ii) and (iii),

∴ d(M, N) + d (L, N) > d (L, M)
∴ Points L, M, N are non collinear points.
We can construct a triangle through 3 non collinear points.
∴ The segment joining the given points form a triangle.
Since MN ≠ LN ≠ LM
∴ ΔLMN is a scalene triangle.
∴ The segments joining the points L, M and N will form a scalene triangle.

ii. By distance formula,

$$\begin{aligned} d(P, Q) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-4-(-2)]^2 + [-2-(-6)]^2} \\ &= \sqrt{(-4+2)^2 + (-2+6)^2} \\ &= \sqrt{(-2)^2 + 4^2} = \sqrt{4+16} \end{aligned}$$

$\therefore \quad d(P, Q) = \sqrt{20} = 2\sqrt{5} \quad ....(i)$

$$d(Q, R) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[-5-(-4)]^2 + [0-(-2)]^2}$$
$$= \sqrt{(-5+4)^2 + (0+2)^2}$$
$$= \sqrt{(-1)^2 + 2^2}$$
$$= \sqrt{1+4}$$

$\therefore \quad d(Q, R) = \sqrt{5} \quad ....(ii)$

$$d(P, R) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[-5-(-2)]^2 + [0-(-6)]^2}$$
$$= \sqrt{(-5+2)^2 + (0+6)^2}$$
$$= \sqrt{(-3)^2 + 6^2}$$
$$= \sqrt{9+36}$$
$$= \sqrt{45} = 3\sqrt{5} \quad ...(iii)$$

On adding (i) and (ii),

$$d(P, Q) + d(Q, R) = 2\sqrt{5} + \sqrt{5}$$
$$= 3\sqrt{5}$$

∴ d(P, Q) + d(Q, R) = d (P, R) ...[From (iii)]
∴ Points P, Q, R are collinear points.
We cannot construct a triangle through 3 collinear points.
∴ The segments joining the points P, Q and R will not form a triangle.

iii. By distance formula,

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{\left(-\sqrt{2}-\sqrt{2}\right)^2 + \left(-\sqrt{2}-\sqrt{2}\right)^2}$$
$$= \sqrt{\left(-2\sqrt{2}\right)^2 + \left(-2\sqrt{2}\right)^2}$$
$$= \sqrt{8+8}$$

$\therefore \quad d(A, B) = \sqrt{16} = 4 \quad ...(i)$

$$d(B, C) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{\left[-\sqrt{6}-\left(-\sqrt{2}\right)\right]^2 + \left[\sqrt{6}-\left(-\sqrt{2}\right)\right]^2}$$
$$= \sqrt{\left(-\sqrt{6}+\sqrt{2}\right)^2 + \left(\sqrt{6}+\sqrt{2}\right)^2}$$
$$= \sqrt{6-2\sqrt{12}+2+6+2\sqrt{12}+2}$$

$\therefore \quad d(B, C) = \sqrt{16} = 4 \quad ...(ii)$

$$d(A, C) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{\left(-\sqrt{6}-\sqrt{2}\right)^2 + \left(\sqrt{6}-\sqrt{2}\right)^2}$$
$$= \sqrt{6+2\sqrt{12}+2+6-2\sqrt{12}+2}$$

$\therefore \quad d(A, C) = \sqrt{16} = 4 \quad ...(iii)$

On adding (i) and (ii),

$$d(A, B) + d(B, C) = 4 + 4$$
$$= 8$$

∴ d(A, B) + d(B, C) + d(A, C) ... [From (iii)]
∴ Points A, B, C are non collinear points.
We can construct a triangle through 3 non collinear points.
∴ The segment joining the given points form a triangle.
Since, AB = BC = AC
∴ ΔABC is an equilateral triangle.
∴ The segments joining the points A, B and C will form an equilateral triangle.

Question 9.
Find k, if the line passing through points P (-12, -3) and Q (4, k) has slope 12.
Solution:
$P(x_1,y_1) = P(-12,-3)$,
$Q(X_2,T_2) = Q(4, k)$
Here, $x_1 = -12$, $x_2 = 4$, $y_1 = -3$, $y_2 = k$

$$\text{Slope of line PQ (m)} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore \quad m = \frac{k-(-3)}{4-(-12)}$$

$$\therefore \quad m = \frac{k+3}{4+12}$$

$$\therefore \quad m = \frac{k+3}{16}$$

But, slope of line PQ (m) is 12 ....[Given]
∴ 12 = k+316
∴ 162 = k + 3
∴ 8 = k + 3
∴ k = 5
The value of k is 5.

Question 10.
Show that the line joining the points A (4,8) and B (5, 5) is parallel to the line joining the points C (2, 4) and D (1 ,7).
Proof:

$$\text{Slope of line AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-8}{5-4} = -3$$

$$\text{Slope of line CD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{1-2} = -3$$

∴ Slope of line AB = Slope of line CD
Parallel lines have equal slope.
∴ line AB || line CD

Question 11.
Show that points P (1, -2), Q (5, 2), R (3, -1), S (-1, -5) are the vertices of a parallelogram.
Proof:
By distance formula,

$$PQ = \sqrt{(5-1)^2 + [2-(-2)]^2}$$
$$= \sqrt{4^2 + 4^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32} = \sqrt{16 \times 2}$$
$$PQ = 4\sqrt{2} \quad \ldots\text{(i)}$$
$$QR = \sqrt{(3-5)^2 + (-1-2)^2}$$
$$= \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9}$$
$$QR = \sqrt{13} \quad \ldots\text{(ii)}$$
$$RS = \sqrt{(-1-3)^2 + [-5-(-1)]^2}$$
$$= \sqrt{(-4)^2 + (-4)^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32} = \sqrt{16 \times 2}$$
$$RS = 4\sqrt{2} \quad \ldots\text{(iii)}$$
$$PS = \sqrt{(-1-1)^2 + [-5-(-2)]^2}$$
$$= \sqrt{(-2)^2 + (-5+2)^2}$$
$$= \sqrt{(-2)^2 + (-3)^2}$$
$$= \sqrt{4+9}$$
$$PS = \sqrt{13} \quad \ldots\text{(iv)}$$

In ⌋ PQRS,
PQ = RS … [From (i) and (iii)]
QR = PS … [From (ii) and (iv)]
∴ ⌋ PQRS is a parallelogram.
[A quadrilateral is a parallelogram, if both the pairs of its opposite sides are congruent]
∴ Points P, Q, R and S are the vertices of a parallelogram.

Question 12.
Show that the ⌋ PQRS formed by P (2, 1), Q (-1, 3), R (-5, -3) and S (-2, -5) is a rectangle.
Proof:
By distance formula,

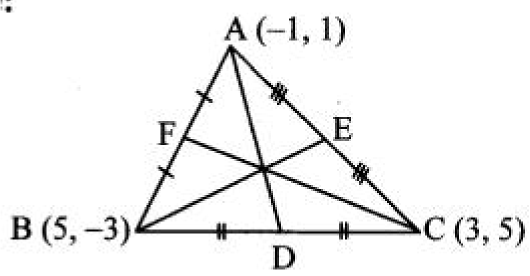
$$d(P, Q) = \sqrt{(-1-2)^2 + (3-1)^2}$$
$$= \sqrt{(-3)^2 + 2^2}$$
$$= \sqrt{9+4} = \sqrt{13} \quad \text{...(i)}$$
$$d(Q, R) = \sqrt{[-5-(-1)]^2 + (-3-3)^2}$$
$$= \sqrt{(-5+1)^2 + (-6)^2}$$
$$= \sqrt{(-4)^2 + (-6)^2}$$
$$= \sqrt{16+36} = \sqrt{52} \quad \text{...(ii)}$$
$$d(R, S) = \sqrt{[-2-(-5)]^2 + [-5-(-3)]^2}$$
$$= \sqrt{(-2+5)^2 + (-5+3)^2}$$
$$= \sqrt{3^2 + (-2)^2}$$
$$= \sqrt{9+4} = \sqrt{13} \quad \text{...(iii)}$$
$$d(P, S) = \sqrt{(-2-2)^2 + (-5-1)^2}$$
$$= \sqrt{(-4)^2 + (-6)^2}$$
$$= \sqrt{16+36} = \sqrt{52} \quad \text{...(iv)}$$

In □ PQRS,
PQ = RS ...[From (i) and (iii)]
QR = PS ...[From (ii) and (iv)]
□ PQRS is a parallelogram.
[A quadrilateral is a parallelogram, if both the pairs of its opposite sides are congruent]

$$d(P, R) = \sqrt{(-5-2)^2 + (-3-1)^2}$$
$$= \sqrt{(-7)^2 + (-4)^2}$$
$$= \sqrt{49+16} = \sqrt{65} \quad \text{...(v)}$$
$$d(Q, S) = \sqrt{[-2-(-1)]^2 + (-5-3)^2}$$
$$= \sqrt{(-2+1)^2 + (-8)^2}$$
$$= \sqrt{(-1)^2 + (-8)^2}$$
$$= \sqrt{1+64} = \sqrt{65} \quad \text{...(vi)}$$

In parallelogram PQRS,
PR = QS ... [From (v) and (vi)]
∴ □ PQRS is a rectangle.
[A parallelogram is a rectangle if its diagonals are equal]

Question 13.
Find the lengths of the medians of a triangle whose vertices are A (-1, 1), B (5, -3) and C (3,5).
Solution:



Suppose AD, BE and CF are the medians.
∴ Points D, E and F are the midpoints of sides BC, AC and AB respectively.
∴ By midpoint formula,

$$\text{Co-ordinates of D} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{5+3}{2}, \frac{-3+5}{2}\right) = \left(\frac{8}{2}, \frac{2}{2}\right)$$
$$\text{Co-ordinates of D} = (4, 1)$$
$$\text{Co-ordinates of E} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-1+3}{2}, \frac{1+5}{2}\right) = \left(\frac{2}{2}, \frac{6}{2}\right)$$

Co-ordinates of E = (1, 3)

$$\text{Co-ordinates of F} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-1+5}{2}, \frac{1-3}{2}\right) = \left(\frac{4}{2}, \frac{-2}{2}\right)$$

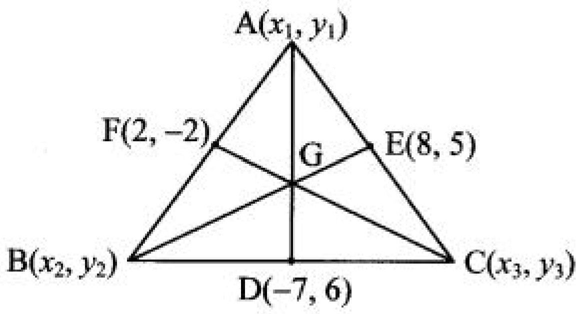
Co-ordinates of F = (2, –1)

By distance formula,

$$d(A, D) = \sqrt{(-1-4)^2 + (1-1)^2}$$
$$= \sqrt{(-5)^2 + 0^2}$$
$$= \sqrt{25} = 5$$
$$d(B, E) = \sqrt{(5-1)^2 + (-3-3)^2}$$
$$= \sqrt{4^2 + (-6)^2}$$
$$= \sqrt{16+36}$$
$$= \sqrt{52} = 2\sqrt{13}$$
$$d(C, F) = \sqrt{(3-2)^2 + [5-(-1)]^2}$$
$$= \sqrt{1^2 + (5+1)^2}$$
$$= \sqrt{1+36} = \sqrt{37}$$

∴ The lengths of the medians of the triangle 5 units, $2\sqrt{13}$ units and $\sqrt{37}$ units.

Question 14.
Find the co-ordinates of centroid of the triangle if points D (-7, 6), E (8, 5) and F (2, -2) are the mid points of the sides of that triangle.
Solution:



Suppose A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of the triangle.
D (-7, 6), E (8, 5) and F (2, -2) are the midpoints of sides BC, AC and AB respectively.
Let G be the centroid of ΔABC.
D is the midpoint of seg BC.
By midpoint formula,

$$\text{Co-ordinates of D} = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

$$\therefore \quad (-7, 6) = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

$$\therefore \quad \frac{x_2+x_3}{2} = -7 \text{ and } \frac{y_2+y_3}{2} = 6$$

$$\therefore \quad x_2 + x_3 = -14 \qquad \ldots\text{(i) and}$$
$$y_2 + y_3 = 12 \qquad \ldots\text{(ii)}$$

E is the midpoint of seg AC.
By midpoint formula,

Co-ordinates of E = $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

$\therefore \quad (8, 5) = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

$\therefore \quad \frac{x_1+x_3}{2} = 8$ and $\frac{y_1+y_3}{2} = 5$

$\therefore \quad x_1 + x_3 = 16$ …(iii) and

$y_1 + y_3 = 10$ …(iv)

F is the midpoint of seg AB.

By midpoint formula,

Co-ordinates of F = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\therefore \quad (2, -2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\therefore \quad \frac{x_1+x_2}{2} = 2$ and $\frac{y_1+y_2}{2} = -2$

$\therefore \quad x_1 + x_2 = 4$ …(v) and

$y_1 + y_2 = -4$ …(vi)

Adding (i), (iii) and (v),

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = -14 + 16 + 4$

$\therefore 2x_1 + 2x_2 + 2x_3 = 6$

$\therefore x_1 + x_2 + x_3 = 3$ …(vii)

Adding (ii), (iv) and (vi),

$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 12 + 10 - 4$

$\therefore 2y_1 + 2y_2 + 2y_3 = 18$

$\therefore y_1 + y_2 + y_3 = 9$ …(viii)

G is the centroid of ΔABC.

By centroid formula,

Co-ordinates of G = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$= \left(\frac{3}{3}, \frac{9}{3}\right)$

…[From (vii) and (viii)]

$= (1, 3)$

∴ The co-ordinates of the centroid of the triangle are (1,3).

Question 15.

Show that A (4, -1), B (6, 0), C (7, -2) and D (5, -3) are vertices of a square.

Proof:

By distance formula,

$$d(A, B) = \sqrt{(6-4)^2 + [0-(-1)]^2}$$

$$= \sqrt{2^2 + (0+1)^2}$$

$$= \sqrt{4+1} = \sqrt{5} \quad \text{...(i)}$$

$$d(B, C) = \sqrt{(7-6)^2 + (-2-0)^2}$$

$$= \sqrt{1^2 + (-2)^2}$$

$$= \sqrt{1+4} = \sqrt{5} \quad \text{...(ii)}$$

$$d(C, D) = \sqrt{(5-7)^2 + [-3-(-2)]^2}$$
$$= \sqrt{(-2)^2 + (-3+2)^2}$$
$$= \sqrt{(-2)^2 + (-1)^2}$$
$$= \sqrt{4+1} = \sqrt{5} \quad \text{...(iii)}$$
$$d(A, D) = \sqrt{(5-4)^2 + [-3-(-1)]^2}$$
$$= \sqrt{1^2 + (-3+1)^2}$$
$$= \sqrt{1 + (-2)^2}$$
$$= \sqrt{1+4} = \sqrt{5} \quad \text{...(iv)}$$

$\therefore$ AB = BC = CD = AD ...[From (i), (ii), (iii) and (iv)]

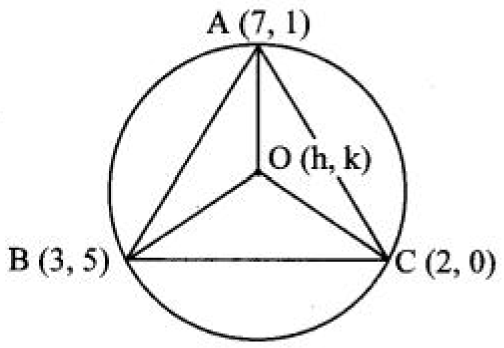
$\therefore$ □ABCD is a rhombus.

$$d(A, C) = \sqrt{(7-4)^2 + [-2-(-1)]^2}$$
$$= \sqrt{3^2 + (-2+1)^2}$$
$$= \sqrt{3^2 + (-1)^2}$$
$$= \sqrt{9+1} = \sqrt{10} \quad \text{...(v)}$$
$$d(B, D) = \sqrt{(5-6)^2 + (-3-0)^2}$$
$$= \sqrt{(-1)^2 + (-3)^2}$$
$$= \sqrt{1+9} = \sqrt{10} \quad \text{...(vi)}$$

In □ABCD,
AC = BD ...[From (v) and (vi)]

$\therefore$ □ABCD is a square.
[A rhombus is a square if its diagonals are equal]

Question 16.
Find the co-ordinates of circumcentre and radius of circumcircle of AABC if A (7, 1), B (3,5) and C (2,0) are given.
Solution:
Suppose, O (h, k) is the circumcentre of ΔABC



OA = OC ...[Radii of the same circle]

$\therefore \sqrt{(h-7)^2 + (k-1)^2} = \sqrt{(h-2)^2 + (k-0)^2}$ ...[By distance formula]

$\therefore (h-7)^2 + (k-1)^2 = (h-2)^2 + (k-0)^2$ ...[Squaring both sides]

$\therefore h^2 - 14h + 49 + k^2 - 2k + 1 = h^2 - 4h + 4 + k^2$

$\therefore 10h + 2k = 46$

$\therefore 5h + k = 23$ ...(i)[Dividing both sides by 2]

OB = OC ...[Radii of the same circle]

$\therefore \sqrt{(h-3)^2 + (k-5)^2} = \sqrt{(h-2)^2 + (k-0)^2}$ ...[By distance formula]

$\therefore (h-3)^2 + (k-5)^2 = (h-2)^2 + (k-0)^2$ ...[Squaring both sides]

$\therefore h^2 - 6h + 9 + k^2 - 10k + 25 = h^2 - 4h + 4 + k^2$
$\therefore 2h + 10k = 30$
$\therefore h + 5k = 15$ ... (ii)[Dividing both sides by 2]
Multiplying equation (i) by 5, we get
$25h + 5k = 115$ ...(iii)
Subtracting equation (ii) from (iii), we get

$$\begin{array}{r} 25h + 5k = 115 \\ h + 5k = \ \ 15 \\ - \quad - \quad \ \ - \\ \hline 24h \qquad = 100 \end{array}$$

$$\therefore \quad h = \frac{100}{24} = \frac{25}{6}$$

Substituting the value of h in equation (i), we get

$$5\left(\frac{25}{6}\right) + k = 23$$

$$\therefore \quad k = 23 - \frac{125}{6}$$

$$\therefore \quad k = \frac{13}{6}$$

$$\therefore \quad O(h, k) = \left(\frac{25}{6}, \frac{13}{6}\right)$$

By distance formula,

$$\text{radius} = d(O, C) = \sqrt{\left(\frac{25}{6} - 2\right)^2 + \left(\frac{13}{6} - 0\right)^2}$$

$$= \sqrt{\left(\frac{25-12}{6}\right)^2 + \left(\frac{13}{6}\right)^2}$$

$$= \sqrt{\left(\frac{13}{6}\right)^2 + \left(\frac{13}{6}\right)^2}$$

$$= \sqrt{2\left(\frac{13}{6}\right)^2}$$

$$= \frac{13\sqrt{2}}{6} \text{ units}$$

∴ The co-ordinates of the circumcentre of the triangle are (256,136) and radius of circumcircle is 132√6 units.

Question 17.
Given A (4, -3), B (8, 5). Find the co-ordinates of the point that divides segment AB in the ratio 3:1.
Solution:
Suppose point C divides seg AB in the ratio 3:1.
Here; $A(x_1, y_1)$ = A (4, -3)
B (x_2, y_2) = B (8, 5)
By section formula,

$$x \text{ co-ordinate of C} = \frac{mx_2 + nx_1}{m+n}$$

$$= \frac{3(8) + 1(4)}{3+1}$$

$$= \frac{24+4}{4} = \frac{28}{4}$$

$$\therefore \quad x \text{ co-ordinate of C} = 7$$

$$y \text{ co-ordinate of C} = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{3(5) + 1(-3)}{3+1}$$

$$\therefore \quad y \text{ co-ordinate of C} = \frac{15-3}{4} = \frac{12}{4} = 3$$

∴ The co-ordinates of point dividing seg AB in ratio 3 : 1 are (7, 3).

Question 18.
Find the type of the quadrilateral if points A (-4, -2), B (-3, -7), C (3, -2) and D (2, 3) are joined serially.
Solution:

$$\text{Slope of AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-(-2)}{-3-(-4)}$$
$$= \frac{-7+2}{-3+4}$$
$$= \frac{-5}{1} = -5$$

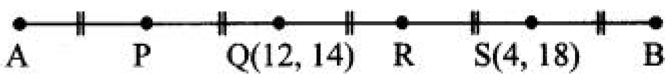
$$\text{Slope of BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-7)}{3-(-3)}$$
$$= \frac{-2+7}{3+3} = \frac{5}{6}$$

$$\text{Slope of CD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{2-3}$$
$$= \frac{3+2}{-1} = -5$$

$$\text{Slope of AD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{2-(-4)}$$
$$= \frac{3+2}{2+4} = \frac{5}{6}$$

Slope of AB = slope of CD
∴ line AB || line CD
slope of BC = slope of AD
∴ line BC || line AD
Both the pairs of opposite sides of ΔABCD are parallel.
∴ ⅃ ABCD is a parallelogram.
∴ The quadrilateral formed by joining the points A, B, C and D is a parallelogram.

Question 19.
The line segment AB is divided into five congruent parts at P, Q, R and S such that A-P-Q-R-S-B. If point Q (12, 14) and S (4, 18) are given, find the co-ordinates of A, P, R, B.
Solution:



Points P, Q, R and S divide seg AB in five congruent parts.
Let A (x_1, y_1), B (x_2, y_2), P (x_3, y_3) and
R (x_4, y_4) be the given points.
Point R is the midpoint of seg QS.
By midpoint formula,
x co-ordinate of R = $\frac{12+4}{2} = \frac{16}{2} = 8$
y co-ordinate of R = $\frac{14+18}{2} = \frac{32}{2} = 16$
∴ co-ordinates of R are (8, 16).
Point Q is the midpoint of seg PR.
By midpoint formula,

$$x \text{ co-ordinate of Q} = \frac{x_3 + 8}{2}$$
$$\therefore \quad 12 = \frac{x_3 + 8}{2}$$
$$\therefore \quad 24 = x_3 + 8$$
$$\therefore \quad x_3 = 16$$
$$y \text{ co-ordinate of Q} = \frac{y_3 + 16}{2}$$
$$\therefore \quad 14 = \frac{y_3 + 16}{2}$$

∴ 28 = y_3 + 16
∴ y_3 = 12
∴ P(x_3,y_3) = (16, 12)
∴ co-ordinates of P are (16, 12).
Point P is the midpoint of seg AQ.
By midpoint formula,

x co-ordinate of P = $\frac{x_1 + 12}{2}$

$\therefore \quad 16 = \frac{x_1 + 12}{2}$

$\therefore \quad 32 = x_1 + 12$

$\therefore \quad x_1 = 20$

y co-ordinate of P = $\frac{y_1 + 14}{2}$

$\therefore \quad 12 = \frac{y_1 + 14}{2}$

$\therefore \quad 24 = y_1 + 14$

$\therefore \quad y_1 = 10$

$\therefore \quad A(x_1, y_1) = (20, 10)$

∴ co-ordinates of A are (20, 10).
Point S is the midpoint of seg RB.
By midpoint formula,

x co-ordinate of S = $\frac{x_2 + 8}{2}$

$\therefore \quad 4 = \frac{x_2 + 8}{2}$

$\therefore \quad 8 = x_2 + 8$

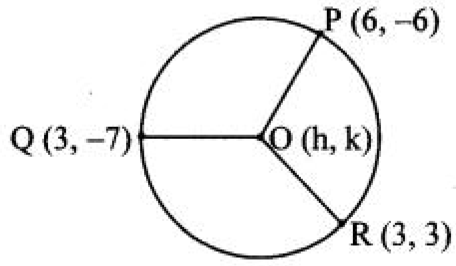
$\therefore \quad x_2 = 0$

y co-ordinate of S = $\frac{y_2 + 16}{2}$

$\therefore \quad 18 = \frac{y_2 + 16}{2}$

∴ $36 = y_2 + 16$
∴ $y_2 = 20$
∴ $B(x_2, y_2) = (0, 20)$
∴ co-ordinates of B are (0, 20).
∴ The co-ordinates of points A, P, R and B are (20, 10), (16, 12), (8, 16) and (0, 20) respectively.

Question 20.
Find the co-ordinates of the centre of the circle passing through the points P (6, -6), Q (3, -7) and R (3,3).
Solution:
Suppose O (h, k) is the centre of the circle passing through the points P, Q and R.



OP = OQ ...[Radii of the same circle]

$\therefore \quad \sqrt{(h-6)^2 + [k-(-6)]^2} = \sqrt{(h-3)^2 + [k-(-7)]^2}$ ...[By distance formula]

∴ $(h - 6)^2 + (k + 6)^2 = (h - 3)^2 + (k + 7)^2$
∴ $h^2 - 12h + 36 + k^2 + 12k + 36$
$= h^2 - 6h + 9 + k^2 + 14k + 49$
∴ $6h + 2k = 14$
∴ $3h + k = 7$ ...(i)[Dividing both sides by 2]
OP = OR ...[Radii of the same circle]

$\therefore \quad \sqrt{(h-6)^2 + [k-(-6)]^2} = \sqrt{(h-3)^2 + (k-3)^2}$ ...[By distance formula]

$\therefore \quad (h-6)^2 + [k-(-6)]^2 = (h-3)^2 + (k-3)^2$ ...[Squaring both sides]

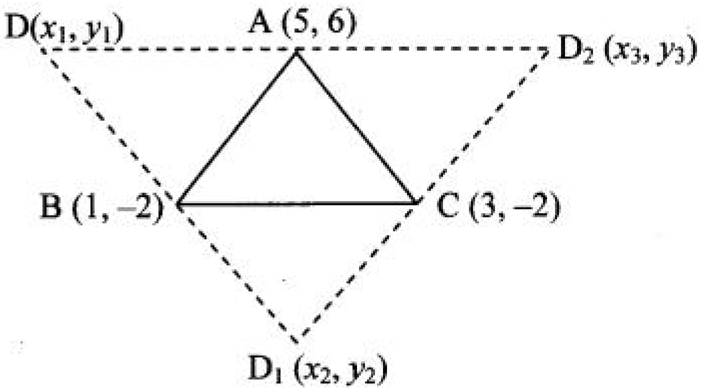
∴ $(h - 6)^2 + (k + 6)^2 = (h - 3)^2 + (k - 3)^2$
∴ $h^2 - 12h + 36 + k^2 + 12k + 36$
$= h^2 - 6h + 9 + k^2 - 6k + 9$
∴ $6h - 18k = 54$
∴ $3h - 9k = 27$ ...(ii)[Dividing both sides by 2]
Subtracting equation (ii) from (i), we get

$$\begin{array}{rl} 3h + k &= 7 \\ 3h - 9k &= 27 \\ - \quad + \quad & - \\ \hline 10k &= -20 \end{array}$$

$\therefore \quad k = \frac{-20}{10} = -2$

Substituting the value of k in equation (i), we get
3h – 2 = 7
∴ 3h = 9
∴ h = $\frac{9}{3}$ = 3
∴ The co-ordinates of the centre of the circle are (3, -2).

Question 21.
Find the possible pairs of co-ordinates of the fourth vertex D of the parallelogram, if three of its vertices are A (5, 6), B (1, -2) and C (3, -2).
Solution:



Let the points A (5, 6), B (1, -2) and C (3, -2) be the three vertices of a parallelogram.
The fourth vertex can be point D or point Di or point D_2 as shown in the figure.
Let $D(x_1,y_1)$, D, (x_2, y_2) and D_2 (x_3,y_3).
Consider the parallelogram ACBD.
The diagonals of a parallelogram bisect each other.
∴ midpoint of DC = midpoint of AB

$\therefore \quad \left(\frac{x_1+3}{2}, \frac{y_1-2}{2}\right) = \left(\frac{5+1}{2}, \frac{6-2}{2}\right)$

$\therefore \quad \left(\frac{x_1+3}{2}, \frac{y_1-2}{2}\right) = \left(\frac{6}{2}, \frac{4}{2}\right)$

$\therefore \quad \frac{x_1+3}{2} = \frac{6}{2}$ and $\frac{y_1-2}{2} = \frac{4}{2}$

$\therefore \quad x_1 + 3 = 6$ and $y_1 - 2 = 4$

$\therefore \quad x_1 = 3$ and $y_1 = 6$

Co-ordinates of point $D(x_1, y_1)$ are (3, 6).
Consider the parallelogram ABD_1C.
The diagonals of a parallelogram bisect each other.
∴ midpoint of AD_1 = midpoint of BC

$\therefore \quad \left(\frac{x_2+5}{2}, \frac{y_2+6}{2}\right) = \left(\frac{3+1}{2}, \frac{-2-2}{2}\right)$

$\therefore \quad \left(\frac{x_2+5}{2}, \frac{y_2+6}{2}\right) = \left(\frac{4}{2}, \frac{-4}{2}\right)$

$\therefore \quad \frac{x_2+5}{2} = \frac{4}{2}$ and $\frac{y_2+6}{2} = \frac{-4}{2}$

$\therefore \quad x_2 + 5 = 4$ and $y_2 + 6 = -4$

$\therefore \quad x_2 = -1$ and $y_2 = -10$

∴ Co-ordinates of $D_1(x_2,y_2)$ are (-1,-10).
Consider the parallelogram $ABCD_2$.
The diagonals of a parallelogram bisect each other.
∴ midpoint of BD_2 = midpoint of AC

$\therefore \quad \left(\frac{x_3+1}{2}, \frac{y_3-2}{2}\right) = \left(\frac{5+3}{2}, \frac{6-2}{2}\right)$

$\therefore \quad \left(\frac{x_3+1}{2}, \frac{y_3-2}{2}\right) = \left(\frac{8}{2}, \frac{4}{2}\right)$

$\therefore \quad \frac{x_3+1}{2} = \frac{8}{2}$ and $\frac{y_3-2}{2} = \frac{4}{2}$

$\therefore \quad x_3 + 1 = 8$ and $y_3 - 2 = 4$

$\therefore \quad x_3 = 7$ and $y_3 = 6$

∴ co-ordinates of point D_2 (x_3, y_3) are (7, 6).
∴ The possible pairs of co-ordinates of the fourth vertex D of the parallelogram are (3, 6), (-1,-10) and (7,6).

Question 22.
Find the slope of the diagonals of a quadrilateral with vertices A (1, 7), B (6,3), C (0, -3) and D (-3,3).
Solution:
Suppose ABCD is the given quadrilateral.

$$\therefore \quad \text{Slope of line AC} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Slope of diagonal AC} = \frac{-3-7}{0-1} = \frac{-10}{-1} = 10$$

$$\text{Slope of diagonal BD} = \frac{3-3}{-3-6} = \frac{0}{-9} = 0$$

∴ The slopes of the diagonals of the quadrilateral are 10 and 0.