# Maharashtra State Board 12th Maths Solutions Chapter 8 Binomial Distribution Ex 8.1

Question 1.
A die is thrown 6 times. If 'getting an odd number' is a success, find the probability of
(i) 5 successes
(ii) at least 5 successes
(iii) at most 5 successes.
Solution:
Let X = number of successes, i.e. number of odd numbers.
p = probability of getting an odd number in a single throw of a die
∴ p = 36=12 and q = 1 – p = 1 – 12 = 12
Given: n = 6
∴ X ~ B(6, 12)
The p.m.f. of X is given by

$$p(X=x) = {}^nC_x\, p^x q^{n-x}$$

$$\text{i.e. } p(x) = {}^6C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{6-x} = {}^6C_x \left(\frac{1}{2}\right)^6,\ x = 0, 1, 2, \ldots, 6$$

**(i)** $P(5 \text{ successes}) = P[X=5]$

$$= p(5) = {}^6C_5 \left(\frac{1}{2}\right)^6$$

$$= {}^6C_1 \times \frac{1}{64} \qquad \ldots [\because {}^nC_x = {}^nC_{n-x}]$$

$$= \frac{6}{64} = \frac{3}{32}$$

Hence, the probability of 5 successes is 332.

(ii) P(at least 5 successes) = P[X ≥ 5]
= p(5) + p(6)

$$= {}^6C_5 \left(\frac{1}{2}\right)^6 + {}^6C_6 \left(\frac{1}{2}\right)^6$$

$$= ({}^6C_5 + {}^6C_6)\left(\frac{1}{2}\right)^6 = (6+1)\frac{1}{64} = \frac{7}{64}$$

Hence, the probability of at least 5 successes is 764.

(iii) P(at most 5 successes) = P[X ≤ 5]
= 1 – P[X > 5]

$$= 1 - p(6) = 1 - {}^6C_6 \left(\frac{1}{2}\right)^6$$

$$= 1 - 1 \times \frac{1}{64} = \frac{63}{64}$$

Hence, the probability of at most 5 successes is 6364.

Question 2.
A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.
Solution:
Let X = number of doublets.
p = probability of getting a doublet when a pair of dice is thrown
∴ p = 636=16 and
q = 1 – p = 1 – 16 = 56
Given: n = 4
∴ X ~ B(4, 16)
The p.m.f. of X is given by

$$P(X=x)={}^{n}C_{x}p^{x}q^{n-x}$$

$$\text{i.e. } p(x)={}^{n}C_{x}\left(\frac{1}{6}\right)^{x}\left(\frac{5}{6}\right)^{4-x},\ x=0,1,2,3,4$$

$$\therefore\ P(2\text{ successes})=P(X=2)$$

$$=p(2)={}^{4}C_{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{4-2}$$

$$\frac{4!}{2!\cdot 2!}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{2}$$

$$=\frac{4\cdot 3\cdot 2!}{2!\cdot 2\cdot 1}\times\frac{1}{36}\times\frac{25}{36}$$

$$=\frac{25}{216}$$

Hence, the probability of two successes is $\frac{25}{216}$.

Question 3.
There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?
Solution:
Let X = number of defective items.
p = probability of defective item
∴ p = 5% = $\frac{5}{100}=\frac{1}{20}$
and q = 1 – p = 1 – $\frac{1}{20}$ = $\frac{19}{20}$
∴ X ~ B(10, $\frac{1}{20}$)
The p.m.f. of X is given by

$$P(X=x)={}^{n}C_{x}p^{x}q^{n-x}$$

$$\text{i.e. } p(x)={}^{10}C_{x}\left(\frac{1}{20}\right)^{x}\left(\frac{19}{20}\right)^{10-x},\ x=0,1,2,\ldots,10.$$

P(sample of 10 items will include not more than one defective item) = P[X ≤ 1]

$$=P(x=0)+P(x=1)$$

$$={}^{10}C_{0}\left(\frac{1}{20}\right)^{0}\left(\frac{19}{20}\right)^{10-0}+{}^{10}C_{1}\left(\frac{1}{20}\right)^{1}\left(\frac{19}{20}\right)^{10-1}$$

$$=1\cdot 1\cdot\left(\frac{19}{20}\right)^{10}+10\times\left(\frac{1}{20}\right)\times\left(\frac{19}{20}\right)^{9}$$

$$=\left(\frac{19}{20}\right)^{9}\left[\frac{19}{20}+\frac{10}{20}\right]$$

$$=\left(\frac{19}{20}\right)^{9}\left(\frac{29}{20}\right)=29\left(\frac{19^{9}}{20^{10}}\right)$$

Hence, the probability that a sample of 10 items will include not more than one defective item = $29\left(\frac{19^{9}}{20^{10}}\right)$.

Question 4.
Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards, find the probability that
(i) all the five cards are spades
(ii) only 3 cards are spades
(iii) none is a spade.
Solution:
Let X = number of spade cards.
p = probability of drawing a spade card from a pack of 52 cards.
Since there are 13 spade cards in the pack of 52 cards.
∴ p = $\frac{13}{52}=\frac{1}{4}$ and
q = 1 – p = 1 – $\frac{1}{4}$ = $\frac{3}{4}$
Given: n = 5
∴ X ~ B(5, $\frac{1}{4}$)
The p.m.f. of X is given by

$$P(X=x)={}^{n}C_{x}\,p^{x}q^{n-x}$$

$$\text{i.e. } p(x)={}^{5}C_{x}\left(\frac{1}{4}\right)^{x}\left(\frac{3}{5}\right)^{5-x},\ x=0,1,2,\ldots,5$$

(i) P(all five cards are spade)

$$= P(X = 5) = p(5) = {}^5C_5\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^{5-5}$$

$$= 1\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0$$

$$= 1\cdot\frac{1}{1024}\cdot 1 = \frac{1}{1024}$$

Hence, the probability of all the five cards are spades = $\frac{1}{1024}$

(ii) P(only 3 cards are spade) = P[X = 3]

$$p(3) = {}^5C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^{5-3}$$

$$= \frac{5!}{3!\ 2!}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2$$

$$= \frac{5\cdot 4\cdot 3!}{3!\cdot 2\cdot 1}\times\frac{1}{64}\times\frac{9}{16} = \frac{45}{512}$$

Hence, the probability of only 3 cards are spades = $\frac{45}{512}$

(iii) P(none of cards is spade) = P[X = 0]

$$= p(0) = {}^5C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{5-0}$$

$$= 1\times 1\times\left(\frac{3}{4}\right)^5 = \frac{243}{1024}$$

Hence, the probability of none of the cards is a spade = $\frac{243}{1024}$

Question 5.
The probability of a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs
(i) none
(ii) not more than one
(iii) more than one
(iv) at least one, will fuse after 150 days of use.
Solution:
Let X = number of fuse bulbs.
p = probability of a bulb produced by a factory will fuse after 150 days of use.
∴ p = 0.05
and q = 1 – p = 1 – 0.05 = 0.95
Given: n = 5
∴ X ~ B(5, 0.05)
The p.m.f. of X is given by
P(X = x) = ${}^nC_x p^x q^{n-x}$
i.e. p(x) = ${}^5C_x (0.05)^x (0.95)^{5-x}$, x = 0, 1, 2, 3, 4, 5
(i) P(none of a bulb produced by a factory will fuse after 150 days of use) = P[X = 0]
= p(0)
= ${}^5C_0 (0.05)^0 (0.95)^{5-0}$
= 1 × 1 × $(0.95)^5$
= $(0.95)^5$
Hence, the probability that none of the bulbs will fuse after 150 days = $(0.95)^5$.

(ii) P(not more than one bulb will fuse after 150 days of j use) = P[X ≤ 1]
= p(0) + p(1)
= ${}^5C_0\cdot (0.05)^0 (0.95)^{5-0} + {}^5C_1 (0.05)^1 (0.95)^4$
= 1 × 1 × $(0.95)^5$ + 5 × (0.05) × $(0.95)^4$
= $(0.95)^4$ [0.95 + 5(0.05)]
= $(0.95)^4$ (0.95 + 0.25)
= $(0.95)^4$ (1.20)
= (1.2) $(0.95)^4$
Hence, the probability that not more than one bulb will fuse after 150 days = $(1.2)(0.95)^4$.

(iii) P(more than one bulb fuse after 150 days)
= P[X > 1]
= 1 – P[X ≤ 1]
= 1 – $(1.2)(0.95)^4$
Hence, the probability that more than one bulb fuse after 150 days = 1 – $(1.2)(0.95)^4$.

(iv) P(at least one bulb fuse after 150 days)
= P[X ≥ 1]
= 1 – P[X = 0]
= 1 – p(0)
= 1 – $^5C_0(0.05)^0(0.95)^{5-0}$
= 1 – 1 × 1 × $(0.95)^5$
= 1 – $(0.95)^5$
Hence, the probability that at least one bulb fuses after 150 days = 1 – $(0.95)^5$.

Question 6.
A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?
Solution:
Let X = number of balls marked with digit 0.
p = probability of drawing a ball from 10 balls marked with the digit 0.
∴ p = $\frac{1}{10}$
and q = 1 – p = 1 – $\frac{1}{10}$ = $\frac{9}{10}$
The p.m.f. of X is given by

$$P(X=x) = {}^nC_x\, p^x q^{n-x}$$

$$\text{i.e. } p(x) = {}^4C_x \left(\frac{1}{10}\right)^x \left(\frac{9}{10}\right)^{4-x},\ x = 0, 1, \ldots, 4$$

P(none of the ball marked with digit 0) = P(X = 0)

$$= p(0) = {}^4C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{4-0}$$

$$= 1 \times 1 \times \left(\frac{9}{10}\right)^4 = \left(\frac{9}{10}\right)^4$$

Hence, the probability that none of the bulb marked with digit 0 is $\left(\frac{9}{10}\right)^4$

Question 7.
On a multiple-choice examination with three possible answers for each of the five questions. What is the probability that a candidate would get four or more correct answers just by guessing?
Solution:
Let X = number of correct answers.
p = probability that a candidate gets a correct answer from three possible answers.
∴ p = $\frac{1}{3}$ and q = 1 – p = 1 – $\frac{1}{3}$ = $\frac{2}{3}$
Given: n = 5
∴ X ~ B(5, $\frac{1}{3}$)
The p.m.f. of X is given by

$$P(X=x) = {}^nC_x\, p^x q^{n-x},\ x = 0, 1, 2, 4, 5$$

$$\text{i.e. } p(x) = {}^5C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x},\ x = 0, 1, 2, 3, 4, 5$$

P(four or more correct answers) = P[X ≥ 4]
= p(4) + p(5)

$$= {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^{5-4} + {}^5C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^{5-5}$$

$$= 5 \times \left(\frac{1}{3}\right)^4 \times \left(\frac{2}{3}\right)^1 + 1 \times \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^0$$

$$= \left(\frac{1}{3}\right)^4 \left[5 \times \frac{2}{3} + \frac{1}{3}\right]$$

$$= \left(\frac{1}{3}\right)^4 \left[\frac{10}{3} + \frac{1}{3}\right] = \frac{1}{81} \times \frac{11}{3} = \frac{11}{243}$$

Hence, the probability of getting four or more correct answers = $\frac{11}{243}$.

Question 8.
A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$, find the probability that he will win a prize
(i) at least once
(ii) exactly once
(iii) at least twice.

Solution:
Let X = number of winning prizes.
p = probability of winning a prize
∴ p = $\frac{1}{100}$
and q = 1 – p = 1 – $\frac{1}{100}$ = $\frac{99}{100}$
Given: n = 50
∴ X ~ B(50, $\frac{1}{100}$)
The p.m.f. of X is given by
$P(X=x) = {}^nC_x p^x q^{n-x}$
i.e., p(x) = ${}^{50}C_x\left(\frac{1}{100}\right)^x\left(\frac{99}{100}\right)^{50-x}$, x = 0, 1, 2,… 50
(i) P(a person wins a prize at least once)

$$= P[X \geqslant 1] = 1 - P[X < 1] = 1 - p(0)$$
$$= 1 - {}^{50}C_0\left(\frac{1}{100}\right)^0\left(\frac{99}{100}\right)^{50-0}$$
$$= 1 - 1 \times 1 \times \left(\frac{99}{100}\right)^{50}$$
$$= 1 - \left(\frac{99}{100}\right)^{50}$$

Hence, probability of winning a prize at least once = 1 – $\left(\frac{99}{100}\right)^{50}$

(ii) P(a person wins exactly one prize) = P[X = 1] = p(1)

$$= {}^{50}C_1\left(\frac{1}{100}\right)^1\left(\frac{99}{100}\right)^{50-1}$$
$$= 50 \times \left(\frac{1}{100}\right) \times \left(\frac{99}{100}\right)^{49}$$
$$= \frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

Hence, probability of winning a prize exactly once = $\frac{1}{2}\left(\frac{99}{100}\right)^{49}$

(iii) P(a persons wins the prize at least twice) = P[X ≥ 2]
= 1 – P[X < 2]
= 1 – [p(0) + p(1)]

$$= 1 - \left[{}^{50}C_0\left(\frac{1}{100}\right)^0\left(\frac{99}{100}\right)^{50-0} + {}^{50}C_1\left(\frac{1}{100}\right)^1\left(\frac{99}{100}\right)^{50-1}\right]$$
$$= 1 - \left[1 \times 1 \times \left(\frac{99}{100}\right)^{50} + 50 \times \frac{1}{100} \times \left(\frac{99}{100}\right)^{49}\right]$$
$$= 1 - \left[\left(\frac{99}{100}\right)^{50} + \frac{1}{2}\left(\frac{99}{100}\right)^{49}\right]$$
$$= 1 - \left(\frac{99}{100}\right)^{49}\left[\frac{99}{100} + \frac{1}{2}\right]$$
$$= 1 - \left(\frac{99}{100}\right)^{49}\left[\frac{149}{100}\right]$$
$$= 1 - 149\left(\frac{99^{49}}{100^{50}}\right)$$

Hence, the probability of winning the prize at least twice = 1 – 149$\left(\frac{99^{49}}{100^{50}}\right)$.

Question 9.
In a box of floppy discs, it is known that 95% will work. A sample of three of the discs is selected at random. Find the probability that (i) none (ii) 1 (iii) 2 (iv) all 3 of the sample will work.
Solution:
Let X = number of working discs.
p = probability that a floppy disc works
∴ p = 95% = $\frac{95}{100} = \frac{19}{20}$
and q = 1 – p = 1 – $\frac{19}{20}$ = $\frac{1}{20}$
Given: n = 3
∴ X ~ B(3, $\frac{19}{20}$)

The p.m.f. of X is given by

$$P(X = x) = {}^nC_x\, p^x q^{n-x}$$

$$\text{i.e. } p(x) = {}^3C_x \left(\frac{19}{20}\right)^x \left(\frac{1}{20}\right)^{3-x}, \ x = 0, 1, 2, 3$$

(i) P(none of the floppy discs work) = P(X = 0)

$$= p(0) = {}^3C_0 \left(\frac{19}{20}\right)^0 \left(\frac{1}{20}\right)^{3-0}$$

$$= 1 \times 1 \times \frac{1}{20^3} = \frac{1}{20^3}$$

Hence, the probability that none of the floppy disc will work = $\frac{1}{20^3}$.

(ii) P(exactly one floppy disc works) = P(X = 1)

$$= p(1) = {}^3C_1 \left(\frac{19}{20}\right)^1 \left(\frac{1}{20}\right)^{3-1}$$

$$= 3 \times \frac{19}{20} \times \left(\frac{1}{20}\right)^2$$

$$= 3\left(\frac{19}{20^3}\right)$$

Hence, the probability that exactly one floppy disc works = $3\left(\frac{19}{20^3}\right)$

(iii) P(exactly two floppy discs work) = P(X = 2)

$$= p(2) = {}^3C_2 \left(\frac{19}{20}\right)^2 \left(\frac{1}{20}\right)^{3-2}$$

$$= \frac{}{2!\cdot 1!} \times \frac{}{(20)^2} \times \frac{}{20} = 3\left(\frac{9^2}{20^3}\right)$$

Hence, the probability that exactly 2 floppy discs work = $3\left(\frac{19^2}{20^3}\right)$

(iv) P(all 3 floppy discs work) = P(X = 3)

$$= p(3) = {}^3C_3 \left(\frac{19}{20}\right)^3 \left(\frac{1}{20}\right)^{3-3}$$

$$= 1 \times \left(\frac{19}{20}\right)^3 \times \left(\frac{1}{20}\right)^0$$

$$= \left(\frac{19}{20}\right)^3$$

Hence, the probability that all 3 floppy discs work = $\left(\frac{19}{20}\right)^3$.

Question 10.
Find the probability of throwing at most 2 sixes in 6 throws of a single die.
Solution:
Let X = number of sixes.
p = probability that a die shows six in a single throw
∴ p = $\frac{1}{6}$
and q = 1 – p = 1 – $\frac{1}{6}$ = $\frac{5}{6}$
Given: n = 6
∴ X ~ B(6, $\frac{1}{6}$)
The p.m.f. of X is given by

$$P(X=x) = {}^{n}C_x p^x q^{n-x}$$

$$\text{i.e. } p(x) = {}^{6}C_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{6-x}, \; x = 0, 1, 2, \ldots, 6$$

$$P(\text{at most 2 sixes}) = P[X \leqslant 2]$$

$$= p(0) + p(1) + p(2)$$

$$= {}^{6}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{6-0} + {}^{6}C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{6-1} + {}^{6}C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{6-2}$$

$$= 1 \times 1 \times \left(\frac{5}{6}\right)^6 + 6 \times \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)^5 + \frac{6!}{2!\ 4!} \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^6 + \left(\frac{5}{6}\right)^5 + \frac{6 \times 5}{2 \times 1} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^6 + \left(\frac{5}{6}\right)^5 + 15 \times \frac{1}{36} \times \left(\frac{5}{6}\right)^4$$

$$= \left[\left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right) + \frac{15}{36}\right] \left(\frac{5}{6}\right)^4$$

$$= \left(\frac{25}{36} + \frac{5}{6} + \frac{15}{36}\right) \cdot \left(\frac{5}{6}\right)^4$$

$$= \left(\frac{25 + 30 + 15}{36}\right) \left(\frac{5}{6}\right)^4$$

$$= \frac{70}{36} \left(\frac{5}{6}\right)^4$$

$$= \frac{7}{3} \times \frac{10}{12} \times \left(\frac{5}{6}\right)^4$$

$$= \frac{7}{3} \times \frac{5}{6} \times \left(\frac{5}{6}\right)^4 = \frac{7}{3} \left(\frac{5}{6}\right)^5$$

Hence, probability of throwing at most 2 sixes = 73(56)5.

Question 11.
It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?
Solution:
Let X = number of defective articles.
p = probability of defective articles.
∴ p = 10% = 10100=110
and q = 1 – p = 1 – 110 = 910
Given: n = 12
∴ X ~ B(12, 110)
The p.m.f. of X is given by

$$P[X=x] = {}^{n}C_x p^x q^{n-x}$$

$$\text{i.e. } p(x) = {}^{12}C_x \left(\frac{1}{10}\right)^x \left(\frac{9}{10}\right)^{12-x}, \; x = 1, 2, 3, \ldots, 12$$

$$P(\text{9 defective articles}) = P[X=9]$$

$$= p(9) = {}^{12}C_9 \left(\frac{1}{10}\right)^9 \left(\frac{9}{10}\right)^{12-9}$$

$$= \frac{12!}{9!3!} \left(\frac{1}{10}\right)^9 \left(\frac{9}{10}\right)^3$$

$$= \frac{12 \times 11 \times 10 \times 9!}{9! \times 3 \times 2 \times 1} \times \frac{1}{10^9} \times \frac{9^3}{10^3}$$

$$= 2 \times 11 \times 10 \cdot \frac{9^3}{10^{12}} = 22 \left(\frac{9^3}{10^{11}}\right)$$

Hence, the probability of getting 9 defective articles = 22(931011)

Question 12.
Given X ~ B(n, P)
(i) If n = 10 and p = 0.4, find E(x) and Var(X).
(ii) If p = 0.6 and E(X) = 6, find n and Var(X).
(iii) If n = 25, E(X) = 10, find p and SD(X).
(iv) If n = 10, E(X) = 8, find Var(X).
Solution:
(i) Given: n = 10 and p = 0.4
∴ q = 1 – p = 1 – 0.4 = 0.6
∴ E(X) = np = 10(0.4) = 4
Var(X) = npq = 10(0.4)(0.6) = 2.4
Hence, E(X) = 4, Var(X) = 2.4.

(ii) Given: p = 0.6, E (X) = 6
E(X) = np
6 = n(0.6)
n = 60.6 = 10
Now, q = 1 – p = 1 – 0.6 = 0.4
∴ Var(X) = npq = 10(0.6)(0.4) = 2.4
Hence, n = 10 and Var(X) = 2.4.

(iii) Given: n = 25, E(X) = 10
E(X) = np
10 = 25p
p = 1025=25
∴ q = 1 – p = 1 – 25 = 35
Var(X) = npq = 25×25×35 = 6
∴ SD(X) = √Var(X) = √6
Hence, p = 25 and S.D.(X) = √6.

(iv) Given: n = 10, E(X) = 8
E(X) = np
8 = 10p
p = 810=45
q = 1 – p = 1 – 45 = 15
Var(X) = npq = 10(45)(15)=85
Hence, Var(X) = 85.

## Maharashtra State Board 12th Maths Solutions Chapter 8 Binomial Distribution Miscellaneous Exercise 8

(I) Choose the correct option from the given alternatives:

Question 1.
The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is
(a) √50
(b) 5
(c) 25
(d) 10

Answer:
(b) 5

Question 2.
The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probablity of 2 successes is
(a) 128256
(b) 219256
(c) 37256
(d) 28256
Answer:
(d) 28256

**Hint :** $np = 4,\ npq = 2 \quad \therefore\ q = \frac{1}{2}$ and $p = \frac{1}{2}$

$$\therefore\ n\left(\frac{1}{2}\right) = 4 \quad \therefore\ n = 8$$

$$\therefore\ P(X=2) = {}^8C_2\left(\frac{1}{2}\right)^8 = \frac{8 \times 7}{1 \times 2} \times \frac{1}{256} = \frac{28}{256}.$$

Question 3.
For a binomial distribution, n = 5. If P(X = 4) = P(X = 3) then p = __________
(a) 13
(b) 34
(c) 1
(d) 23
Answer:
(d) 23

**Hint :** $P(X=4) = P(X=3)$

$$\therefore\ {}^5C_4\ p^4q = {}^5C_3\ p^3q^2$$

$$\therefore\ 5p = 10q = 10(1-p)$$

$$\therefore\ p = 2 - 2p \quad \therefore\ p = \frac{2}{3}.$$

Question 4.
In a binomial distribution, n = 4. If 2 P(X = 3) = 3 P(X = 2) then p = __________
(a) 413
(b) 513
(c) 913
(d) 613
Answer:
(c) 913

Question 5.
If X ~ B (4, p) and P (X = 0) = 1681, then P (X = 4) = __________
(a) 116
(b) 181
(c) 127
(d) 18
Answer:
(b) 181

**Hint :** $P(X=0) = {}^4C_0\ P^0q^4 = \frac{16}{81}$

$$\therefore\ q^4 = \left(\frac{2}{3}\right)^4 \quad \therefore\ q = \frac{2}{3}$$

$$\therefore\ p = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\therefore\ P(X=4) = {}^4C_4\ p^4q^0 = \left(\frac{1}{3}\right)^4 = \frac{1}{81}.$$

Question 6.
The probability of a shooter hitting a target is 34. How many minimum numbers of times must he fie so that the probability of hitting the target at least once is more than 0·99?
(a) 2
(b) 3
(c) 4
(d) 5
Answer:
(c) 4

Hint : $P(X \geqslant 1) > 0.99$

$\therefore\ 1 - P(X = 0) > 0.99$

$\therefore\ P(X = 0) < 0.01 = \frac{1}{100}$

$\therefore\ {}^{n}C_{0}\left(\frac{3}{4}\right)^{0}\left(\frac{1}{4}\right)^{n} < \frac{1}{100}$

$\therefore\ \left(\frac{1}{4}\right)^{n} < \frac{1}{100} \qquad \therefore\ n = 4$

Question 7.
If the mean and variance of a binomial distribution are 18 and 12 respectively, then n = __________
(a) 36
(b) 54
(c) 18
(d) 27
Answer:
(b) 54

Hint : $np = 18$ and $npq = 12$

$\therefore\ \frac{npq}{np} = \frac{12}{18} \qquad \therefore\ q = \frac{2}{3}$

$\therefore\ p = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}$

$\therefore\ n\left(\frac{1}{3}\right) = 18 \qquad \therefore\ n = 54$

(II) Solve the following:

Question 1.
Let X ~ B(10, 0.2). Find
(i) P(X = 1)
(ii) P(X ≥ 1)
(iii) P(X ≤ 8).
Solution:
X ~ B(10, 0.2)
∴ n = 10, p = 0.2
∴ q = 1 – p = 1 – 0.2 = 0.8

The p,m.f. of X is given by

$$P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

$$\therefore\ P(X=x) = {}^{10}C_{x}\,(0.2)^{x}(0.8)^{10-x},\ x = 0, 1, 2, 3, \ldots, 10$$

**(i)** $P(X=1) = {}^{10}C_{1}\ (0.2)^{1}\ (0.8)^{10-1}$

$$= 10\,(0.2)\ (0.8)^{9} = 2\,(0.8)^{9}.$$

**(ii)** $P(X \geqslant 1) = 1 - P(X < 1)$

$$= 1 - P(X=0)$$

$$= 1 - {}^{10}C_{0}\ (0.2)^{0}\ (0.8)^{10-0}$$

$$= 1 - 1 \times 1 \times (0.8)^{10} = 1 - (0.8)^{10}$$

**(iii)** $P(X \leqslant 8) = 1 - P(X > 8)$

$$= 1 - [P(X=9) + P(X=10)]$$

$$= 1 - [{}^{10}C_{9}\,(0.2)^{9}(0.8)^{10-9} + {}^{10}C_{10}(0.2)^{10}(0.8)^{10-10}]$$

$$= 1 - [10(0.2)^{9}(0.8)^{1} + 1(0.2)^{10}(0.8)^{0}]$$

$$= 1 - (0.2)^{9}\,[10(0.8) + (0.2)]$$

$$= 1 - (0.2)^{9}\ [8 + 0.2]$$

$$= 1 - (8.2)\,(0.2)^{9}.$$

Question 2.
Let X ~ B(n, p).
(i) If n = 10, E(X) = 5, find p and Var(X).
(ii) If E(X) = 5 and Var(X) = 2.5, find n and p.
Solution:
X ~ B(n, p)
(i) Given: n = 10 and E(X) = 5
But E(X) = np
∴ np = 5.
∴ 10p = 5
∴ p = 12
∴ q = 1 – p = 1 – 12 = 12
Var(X) = npq = 10(12)(12) = 2.5.
Hence, p = 12 and Var(X) = 2.5

(ii) Given: E(X) = 5 and Var(X) = 2.5
∴ np = 5 and npq = 2.5
∴ npqnp=2.55
∴ q = 0.5 = 510=12
∴ p = 1 – q = 1 – 12 = 12
Substituting p = 12 in np = 5, we get
n(12) = 5
∴ n = 10
Hence, n = 10 and p = 12

Question 3.
If a fair coin is tossed 10 times and the probability that it shows heads (i) 5 times (ii) in the first four tosses and tail in the last six tosses.
Solution:
Let X = number of heads.
p = probability that coin tossed shows a head
∴ p = 12
q = 1 – p = 1 – 12 = 12
Given: n = 10
∴ X ~ B(10, 12)
The p.m.f. of X is given by

P(X = x) = $^nC_x P^x q^{n-x}$

$$\therefore\ p(x) = {}^{10}C_x\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{10-x} = {}^{10}C_x\left(\frac{1}{2}\right)^{10}$$

$$x = 0, 1, 2, \ldots 10$$

(i) P(coin shows heads 5 times) = P[X = 5]

$$= p(5) = {}^{10}C_5\cdot\left(\frac{1}{2}\right)^{10}$$

$$= \frac{10!}{5!\,5!} \times \frac{1}{1024}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5 \times 4 \times 3 \times 2 \times 1} \times \frac{1}{1024} = \frac{63}{256}$$

Hence, the probability that can shows heads exactly 5 times = $\frac{63}{256}$

(ii) P(getting heads in first four tosses and tails in last six tosses) = P(X = 4)

$$= p(4) = {}^{10}C_4\cdot\left(\frac{1}{2}\right)^{10}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6!}{6! \times 4 \times 3 \times 2 \times 1} \times \frac{1}{1024}$$

$$= 210 \times \frac{1}{1024} = \frac{105}{512}$$

Hence, the probability that getting heads in first four tosses and tails in last six tosses = $\frac{105}{512}$.

Question 4.
The probability that a bomb will hit a target is 0.8. Find the probability that out of 10 bombs dropped, exactly 2 will miss the target.
Solution:
Let X = the number of bombs hitting the target.
p = probability that bomb will hit the target
∴ p = 0.8 = $\frac{8}{10} = \frac{4}{5}$
∴ q = 1 – p = 1 – $\frac{4}{5}$ = $\frac{1}{5}$
Given: n = 10
∴ X ~ B(10, $\frac{4}{5}$)
The p.m.f. of X is given as:
P[X = x] = $^nC_x p^x q^{n-x}$
i.e.p(x) = $^{10}C_x\left(\frac{4}{5}\right)^x\left(\frac{1}{5}\right)^{10-x}$
P(exactly 2 bombs will miss the target) = P(exactly 8 bombs will hit the target)
= P[X = 8]
= p(8)

$$= {}^{10}C_8\left(\frac{4}{5}\right)^8\left(\frac{1}{5}\right)^{10-8}$$

$$= {}^{10}C_2\left(\frac{4}{5}\right)^8\left(\frac{1}{5}\right)^2 \qquad \ldots\ [\because\ {}^nC_x = {}^nC_{n-x}]$$

$$= \frac{10 \times 9}{1 \times 2} \times \frac{4^8}{5^{10}} = \frac{45 \times 4^8}{5^{10}} = 45\left(\frac{2^{16}}{5^{10}}\right)$$

Hence, the probability that exactly 2 bombs will miss the target = 45$\left(\frac{2^{16}}{5^{10}}\right)$

Question 5.
The probability that a mountain bike travelling along a certain track will have a tire burst is 0.05. Find the probability that among 17 riders:
(i) exactly one has a burst tyre
(ii) at most three have a burst tyre
(iii) two or more have burst tyres.
Solution:
Let X = number of burst tyres.
p = probability that a mountain bike travelling along a certain track will have a tyre burst.
∴ p = 0.05
∴ q = 1 – p = 1 – 0.05 = 0.95

Given: n = 17
$\therefore X \sim B(17, 0.05)$
The p.m.f. of X is given by
$P(X = x) = {}^nC_x p^x q^{n-x}$
i.e.(x) = ${}^{17}C_x (0.05)^x (0.95)^{17-x}$, x = 0, 1, 2, ……, 17
(i) P(exactly one has a burst tyre)
$P(X = 1) = p(1) = {}^{17}C_1 (0.05)^1 (0.95)^{17-1}$
$= 17(0.05)(0.95)^{16}$
$= 0.85(0.95)^{16}$
Hence, the probability that riders has exactly one burst tyre = $(0.85)(0.95)^{16}$

(ii) P(at most three have a burst tyre) = $P(X \le 3)$
= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)
= p(0) + p(1) + p(2) + p(3)

$$= {}^{17}C_0(0.05)^0(0.95)^{17-0} + {}^{17}C_1(0.05)^1(0.17)^{17-1} + {}^{17}C_2(0.05)^2(0.17)^{17-2} + {}^{17}C_3(0.05)^3(0.17)^{17-3}$$

$$= 1(1)(0.95)^{17} + 17(0.05)(0.95)^{16} + \frac{17 \times 16}{2 \times 1} \times (0.05)^2(0.95)^{15} + \frac{17 \times 16 \times 15}{3 \times 2 \times 1} \times (0.05)^3 \times (0.95)^{14}$$

$$= (0.95)^{17} + 17(0.05) \times (0.95)^{16} + 17(8) \times (0.05)^2 \times (0.95)^{15} + 17(8)(5) \times (0.05)^3 \times (0.95)^{14}$$

$$= (0.95)^{14}[(0.95)^3 + (17)(0.05)(0.95)^2 + 17(8) \times (0.05)^2 \times (0.95)^1 + 17(8)(5)(0.05)^3]$$

$$= (0.95)^{14}[0.8574 + 0.7671 + 0.323 + 0.085]$$

$$= (2.0325)(0.95)^{14}$$

Hence, the probability that at most three riders have burst tyre = $(2.0325)(0.95)^{14}$.

(iii) P(two or more have tyre burst) = $P(X \ge 2)$
$= 1 - P(X < 2)$
= 1 – [P(X = 0) + P(X = 1)]
= 1 – [p(0) + p(1)]
$= 1 - [{}^{17}C_0 (0.05)^0 (0.95)^{17} + {}^{17}C_1 (0.05)(0.95)^{16}]$
$= 1 - [1(1)(0.95)^{17} + 17(0.05)(0.95)^{16}]$
$= 1 - (0.95)^{16}[0.95 + 0.85]$
$= 1 - (1.80)(0.95)^{16}$
$= 1 - (1.8)(0.95)^{16}$
Hence, the probability that two or more riders have tyre burst = $1 - (1.8)(0.95)^{16}$.

Question 6.
The probability that a lamp in a classroom will be burnt out is 0.3. Six such lamps are fitted in the classroom. If it is known that the classroom is unusable if the number of lamps burning in it is less than four, find the probability that the classroom cannot be used on a random occasion.
Solution:
Let X = number of lamps burnt out in the classroom.
p = probability of a lamp in a classroom will be burnt
$\therefore p = 0.3 = \frac{3}{10}$
$\therefore q = 1 - p = 1 - \frac{3}{10} = \frac{7}{10}$
Given: n = 6
$\therefore X \sim B(6, \frac{3}{10})$
The p.m.f. of X is given as:
$P[X = x] = {}^nC_x p^x q^{n-x}$
i.e., $p(x) = {}^6C_x \left(\frac{3}{10}\right)^x \left(\frac{7}{10}\right)^{6-x}$
Since the classroom is unusable if the number of lamps burning in it is less than four, therefore
P(classroom cannot be used) = P[X < 4]
= P[X = 0] + P[X = 1] + P[X = 2] + P[X = 3]
= p(0) + p(1) + p(2) + p(3)

$$= {}^6C_0\left(\frac{3}{10}\right)^0\left(\frac{7}{10}\right)^{6-0} + {}^6C_1\left(\frac{3}{10}\right)^1\left(\frac{7}{10}\right)^{6-1} + {}^6C_2\left(\frac{3}{10}\right)^2\left(\frac{7}{10}\right)^{6-2} + {}^6C_3\left(\frac{3}{10}\right)^3\left(\frac{7}{10}\right)^{6-3}$$

$$= 1 \times 1 \times \left(\frac{7}{10}\right)^6 + 6\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)^5 + \frac{6 \times 5}{1 \times 2}\cdot\left(\frac{3}{10}\right)^2\left(\frac{7}{10}\right)^4 + \frac{6 \times 5 \times 4}{1 \times 2 \times 3}\cdot\left(\frac{3}{10}\right)^3\left(\frac{7}{10}\right)^3$$

$$= [7^6 + 18 \times 7^5 + 15 \times 9 \times 7^4 + 20 \times 27 \times 7^3]\frac{1}{10^6}$$

$$= \frac{117649 + 302526 + 324135 + 185220}{10^6}$$

$$= \frac{929530}{10^6} = 0.92953$$

Hence, the probability that the classroom cannot be used on a random occasion is 0.92953.

Question 7.
A lot of 100 items contain 10 defective items. Five items are selected at random from the lot and sent to the retail store. What is the probability that the store will receive at most one defective item?
Solution:
Let X = number of defective items.
p = probability that item is defective
∴ p = $\frac{10}{100}=\frac{1}{10}$
∴ q = 1 – p = 1 – $\frac{1}{10}$ = $\frac{9}{10}$
Given: n = 5
∴ X ~ B(5, $\frac{1}{10}$)
The p.m.f. of X is given as:
P[X = x] = ${}^nC_x p^x q^{n-x}$
i.e., p(x) = ${}^5C_x\left(\frac{1}{10}\right)^x\left(\frac{9}{10}\right)^{5-x}$
P (store will receive at most one defective item) = P[X ≤ 1]
=P[X = 0] + P[X = 1]
= p(0) + p(1)

$$= {}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{5-0} + {}^5C_1\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^{5-1}$$

$$= 1 \times 1 \times \left(\frac{9}{10}\right)^5 + 5 \times \frac{1}{10} \times \left(\frac{9}{10}\right)^4$$

$$= (0.9)^5 + (0.05)(0.9)^4$$

$$= (0.9 + 0.5)(0.9)^4$$

$$= (1.4)(0.9)^4$$

Hence, the probability that the store will receive at most one defective item is $(1.4)(0.9)^4$.

Question 8.
A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%. The inspector of the retailer picks 20 items from a shipment. What is the probability that the store will receive at most one defective item?
Solution:
Let X = number of defective electronic devices.
p = probability that device is defective
∴ p = 3% = $\frac{3}{100}$
∴ q = 1 – p = 1 – $\frac{3}{100}$ = $\frac{97}{100}$
Given: n = 20
∴ X ~ B(20, $\frac{3}{100}$)
The p.m.f. of X is given as:

$$P[X=x] = {}^nC_x p^x q^{n-x}$$

i.e. $p(x) = {}^{20}C_x \left(\frac{3}{100}\right)^x \left(\frac{97}{100}\right)^{20-x}$

$P$(store will receive at most one defective item)

$= P[X \leqslant 1] = P[X=0] + P[X=1]$

$= p(0) + p(1)$

$= {}^{20}C_0 \left(\frac{3}{100}\right)^0 \left(\frac{97}{100}\right)^{20-0} + {}^{20}C_1 \left(\frac{3}{100}\right)^1 \left(\frac{97}{100}\right)^{20-1}$

$= 1 \times 1 \times (0.97)^{20} + 20 \times (0.03) \times (0.97)^{19}$

$= (0.97 + 0.6)(0.97)^{19}$

$= (1.57)(0.97)^{19}$

Hence, the probability that the store will receive at most one defective item = $(1.57)(0.97)^{19}$.

Question 9.
The probability that a certain kind of component will survive a check test is 0.6. Find the probability that exactly 2 of the next 4 tested components tested survive.
Solution:
Let X = number of tested components survive.
p = probability that the component survives the check test

$\therefore\ p = 0.6 = \frac{6}{10} = \frac{3}{5}$

$\therefore\ q = 1 - p = 1 - \frac{3}{5} = \frac{2}{5}$

Given : $n = 4$

$\therefore\ X \sim B\left(4, \frac{3}{5}\right)$

The p.m.f. of $X$ is given as :

$$P[X=x] = {}^nC_x p^x q^{n-x}$$

i.e. $p(x) = {}^4C_x \left(\frac{3}{5}\right)^x \left(\frac{2}{5}\right)^{4-x}$

$P$(exactly 2 components survive)

$= P[X=2] = p(2)$

$= {}^4C_2 \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^{4-2}$

$= \left(\frac{4 \times 3}{1 \times 2}\right) \times \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^2 = \frac{6 \times 9 \times 4}{625}$

$= \frac{216}{625} = 0.3456$

Hence, the probability that exactly 2 of the 4 tested components survive is 0.3456.

Question 10.
An examination consists of 10 multiple choice questions, in each of which a candidate has to deduce which one of five suggested answers is correct. A completely unprepared student guesses each answer completely randomly. What is the probability that this student gets 8 or more questions correct? Draw the appropriate moral.
Solution:
Let X = number of correct answers.
p = probability that student gets correct answer
$\therefore$ p = $\frac{1}{5}$
$\therefore$ q = 1 – p = 1 – $\frac{1}{5}$ = $\frac{4}{5}$
Given: n = 10 (number of total questions)
$\therefore$ X ~ B(10, $\frac{1}{5}$)
The p.m.f. of X is given by

$$P(X=x) = {}^nC_x\, p^x - q^{n-x}$$

i.e. $p(x) = {}^{10}C_x \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{10-x}$, $x = 0, 1, 2, \ldots, 10$

$P$(student gets 8 or more questions correct)

$$= P(X \geqslant 8) = P(X=8) + P(X=9) + P(X=10)$$

$$= {}^{10}C_8\left(\frac{1}{5}\right)^8\left(\frac{4}{5}\right)^2 + {}^{10}C_9\left(\frac{1}{5}\right)^9\left(\frac{4}{5}\right)^1 + {}^{10}C_{10}\left(\frac{1}{5}\right)^{10}\left(\frac{4}{5}\right)^0$$

$$= \frac{10 \times 9 \times 8!}{8! \times 2 \times 1} \times \left(\frac{1}{5}\right)^8 \times \left(\frac{4}{5}\right)^2 + 10\left(\frac{1}{5}\right)^9\left(\frac{4}{5}\right)^1 + 1 \times \left(\frac{1}{5}\right)^{10}$$

$$= 45 \times \left(\frac{1}{5}\right)^8 \times \left(\frac{4}{5}\right)^2 + 10 \times \left(\frac{1}{5}\right)^9 \times \left(\frac{4}{5}\right) + \left(\frac{1}{5}\right)^{10}$$

$$= \left(\frac{1}{5}\right)^8\left[45 \times \left(\frac{4}{5}\right)^2 + 10 \times \left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right) + \left(\frac{1}{5}\right)^2\right]$$

$$= \left[45 \times \frac{16}{25} + \frac{10}{5} \times \frac{4}{5} + \frac{1}{25}\right]\left(\frac{1}{5}\right)^8$$

$$= \left[\frac{720}{25} + \frac{40}{25} + \frac{1}{25}\right]\left(\frac{1}{5^8}\right)$$

$$= \left(\frac{761}{25}\right) \times \left(\frac{1}{5^8}\right) = \frac{30.44}{5^8}$$

Hence, the probability that student gets 8 or more questions correct = $\frac{30.44}{5^8}$

Question 11.
The probability that a machine will produce all bolts in a production run within specification is 0.998. A sample of 8 machines is taken at random. Calculate the probability that (i) all 8 machines (ii) 7 or 8 machines (iii) at most 6 machines will produce all bolts within specification.
Solution:
Let X = number of machines which produce the bolts within specification.
p = probability that a machine produce bolts within specification
p = 0.998 and q = 1 – p = 1 – 0.998 = 0.002
Given: n = 8
∴ X ~ B(8, 0.998)
The p.m.f. of X is given by
P(X = x) = ${}^nC_x p^x q^{n-x}$
i.e. p(x) = ${}^8C_x (0.998)^x (0.002)^{8-x}$, x = 0, 1, 2, ..., 8
(i) P(all 8 machines will produce all bolts within specification) = P[X = 8]
= p(8)
= ${}^8C_8$ $(0.998)^8$ $(0.002)^{8-8}$
= $1(0.998)^8$ . (1)
= $(0.998)^8$
Hence, the probability that all 8 machines produce all bolts with specification = $(0.998)^8$.

(ii) P(7 or 8 machines will produce all bolts within i specification) = P (X = 7) + P (X = 8)

$$= {}^8C_7(0.998)^7(0.002)^{8-7} + {}^8C_8(0.998)^8(0.002)^{8-8}$$

$$= 8 \times (0.998)^7(0.002)^1 + 1 \times (0.998)^8(0.002)^0$$

$$= (0.998)^7[8(0.002) + 0.998]$$

$$= (0.016 + 0.998)(0.998)^7$$

$$= (1.014) \times (0.998)^7$$

Hence, the probability that 7 or 8 machines produce all bolts within specification = $(1.014)(0.998)^7$.

(iii) P(at most 6 machines will produce all bolts with specification) = P[X ≤ 6]
= 1 – P[x > 6]
= 1 – [P(X = 7) + P(X = 8)]
= 1 – [P(7) + P(8)]
= 1 – $(1.014)(0.998)^7$
Hence, the probability that at most 6 machines will produce all bolts with specification = 1 – $(1.014)(0.998)^7$.

Question 12.
The probability that a machine develops a fault within the first 3 years of use is 0.003. If 40 machines are selected at random, calculate the probability that 38 or more will develop any faults within the first 3 years of use.
Solution:
Let X = the number of machines who develop a fault.
p = probability that a machine develops a fait within the first 3 years of use
∴ p = 0.003 and q = 1 – p = 1 – 0.003 = 0.997
Given: n = 40
∴ X ~ B(40, 0.003)
The p.m.f. of X is given by

$$P(X=x)={}^nC_x p^x q^{n-x},\ x=0,1,2,\dots,n$$

i.e. $p(x)={}^{40}C_x (0.003)^x (0.997)^{40-x},\ x=0,1,2,\dots,40$

$P$(38 or more machines will develop any fault)

$$=P(X\geqslant 38)=P(X=38)+P(X=39)+P(X=40)$$

$$=p(38)+p(39)+p(40)$$

$$={}^{40}C_{38}(0.003)^{38}(0.997)^{40-38}+{}^{40}C_{39}(0.003)^{39}(0.997)^{40-39}+{}^{40}C_{40}(0.003)^{40}(0.997)^{0}$$

$$=\frac{40\times 39}{2\times 1}(0.003)^{38}(0.997)^2+40(0.003)^{39}(0.997)^1+1\cdot(0.003)^{40}(0.997)^0$$

$$=(780)(0.003)^{38}(0.997)^2+(40)(0.003)^{39}(0.997)+1\times(0.003)^{40}\times 1$$

$$=(0.003)^{38}[(780)(0.997)^2+40(0.003)(0.997)+(0.003)^2]$$

$$=(0.003)^{38}[775.327+0.1196+0.000009]$$

$$=(0.003)^{38}(775.446609)$$

$$=(775.446609)(0.003)^{38}$$

$$\approx(775.44)(0.003)^{38}$$

Hence, the probability that 38 or more machines will develop the fault within 3 years of use = $(775.44)(0.003)^{38}$.

Question 13.
A computer installation has 10 terminals. Independently, the probability that anyone terminal will require attention during a week is 0.1. Find the probabilities that (i) 0 (ii) 1 (iii) 2 (iv) 3 or more, terminals will require attention during the next week.
Solution:
Let X = number of terminals which required attention during a week.
p = probability that any terminal will require attention during a week
∴ p = 0.1 and q = 1 – p = 1 – 0.1 = 0.9
Given: n = 10
∴ X ~ B(10, 0.1)
The p.m.f. of X is given by
P(X = x) = ${}^nC_x p^x q^{n-x}$
i.e. p(x) = ${}^{10}C_x(0.1)^x(0.9)^{10-x}$, x = 0, 1, 2, ..., 10
(i) P(no terminal will require attention) = P(X = 0)

$$=p(0)={}^{10}C_0(0.1)^0(0.9)^{10-0}$$

$$=1\times 1\times(0.9)^{10}=(0.9)^{10}$$

Hence, the probability that no terminal requires attention = $(0.9)^{10}$

(ii) P(1 terminal will require attention)

$$P(X=1)=p(1)={}^{10}C_1(0.1)^1(0.9)^{10-1}$$

$$=10(0.1)(0.9)^9$$

$$=(1.0)(0.9)^9$$

$$=(0.9)^9$$

Hence, the probability that 1 terminal requires attention = $(0.9)^9$.

(iii) P(2 terminals will require attention)

$$P(X=2)=p(2)={}^{10}C_2\,(0.1)^2(0.9)^{10-2}$$
$$=\frac{10\times 9}{1\times 2}(0.1)^2(0.9)^8$$
$$=45(0.01)(0.9)^8$$
$$=(0.45)\times(0.9)^8$$

Hence, the probability that 2 terminals require attention = $(0.45)(0.9)^8$.

(iv) P(3 or more terminals will require attention)

$$=P(X\geqslant 3)$$
$$=1-P(x<3)$$
$$=1-[P(X=0)+P(X=1)+P(X=2)]$$
$$=1-[p(0)+p(1)+p(2)]$$
$$=1-[(0.9)^{10}+(0.9)^9+(0.45)(0.9)^8]$$
$$=1-[(0.9)^2+(0.9)^1+0.45](0.9)^8$$
$$=1-[0.81+0.9+0.45](0.9)^8$$
$$=1-(2.16)\times(0.9)^8$$

Hence, the probability that 3 or more terminals require attention = $1-(2.16)\times(0.9)^8$.

Question 14.
In a large school, 80% of the pupil like Mathematics. A visitor to the school asks each of 4 pupils, chosen at random, whether they like Mathematics.
(i) Calculate the probabilities of obtaining an answer yes from 0, 1, 2, 3, 4 of the pupils.
(ii) Find the probability that the visitor obtains answer yes from at least 2 pupils:
(a) when the number of pupils questioned remains at 4.
(b) when the number of pupils questioned is increased to 8.
Solution:
Let X = number of pupils like Mathematics.
p = probability that pupils like Mathematics

$$\therefore\ p=80\%=\frac{80}{100}=\frac{4}{5}$$

and $q=1-p=1-\frac{4}{5}=\frac{1}{5}$

Given : $n=4$

$$\therefore\ X\sim B\left(4,\frac{4}{5}\right)$$

The p.m.f. of $X$ is given by

$$P(X=x)={}^nC_x\,P^x\,q^{n-x}$$

i.e. $p(x)={}^4C_x\left(\frac{4}{5}\right)^x\left(\frac{1}{5}\right)^{4-x}$, $x=0, 1, 2, 3, 4$

(i) The probabilities of obtaining an answer yes from 0, 1, 2, 3, 4 of pupils are P(X = 0), P(X = 1), P(X = 2), P(X = 3) and P(X = 4) respectively

$$\text{i.e. } {}^4C_0\left(\frac{4}{5}\right)^0\left(\frac{1}{5}\right)^{4-0}, {}^4C_1\left(\frac{4}{5}\right)^1\left(\frac{1}{5}\right)^{4-1}, {}^4C_2\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)^{4-2}, {}^4C_3\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)^{4-3} \text{ and } {}^4C_4\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)^{4-4}$$

$$\text{i.e. } 1(1)\left(\frac{1}{5}\right)^4, 4\left(\frac{4}{5}\right)\cdot\left(\frac{1}{5}\right)^3, \frac{4\times 3}{1\times 2}\left(\frac{16}{25}\right)\left(\frac{1}{25}\right), 4\left(\frac{64}{125}\right)\left(\frac{1}{5}\right) \text{ and } 1\times\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)^0$$

$$\text{i.e. } \left(\frac{1}{5}\right)^4, \frac{16}{5}\left(\frac{1}{5}\right)^3, \frac{96}{5^2}\left(\frac{1}{5^2}\right), \frac{256}{5^3}\left(\frac{1}{5}\right) \text{ and } \frac{256}{5^4}$$

$$\text{i.e. } \frac{1}{5^4}, \frac{16}{5^4}, \frac{96}{5^4}, \frac{256}{5^4}, \frac{256}{5^4}.$$

$$OR\ \frac{1}{625}, \frac{16}{625}, \frac{96}{625}, \frac{256}{625} \text{ and } \frac{256}{625}.$$

(ii) (a) P(visitor obtains the answer yes from at least 2 pupils when the number of pupils questioned remains at 4) = P(X ≥ 2)
= P(X = 2) + P(X = 3) + P(X = 4)

$$= {}^4C_2\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)^2 + {}^4C_3\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)^1 + {}^4C_4\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)^0$$

$$= \frac{4\times 3}{1\times 2}\times\frac{16}{5^2}\times\frac{1}{5^2} + 4\times\frac{64}{5^3}\times\frac{1}{5} + 1\times\frac{256}{5^4}$$

$$= \frac{96}{5^4} + \frac{256}{5^4} + \frac{256}{5^4}$$

$$= (96 + 256 + 256)\frac{1}{5^4}$$

$$= \frac{608}{5^4} = \frac{608}{625}.$$

(b) P(the visitor obtains the answer yes from at least 2 pupils when number of pupils questioned is increased to 8)

$$= P(X \geqslant 2)$$

$$= 1 - P(X < 2)$$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - \left[{}^8C_0\left(\frac{4}{5}\right)^0\left(\frac{1}{5}\right)^{8-0} + {}^8C_1\left(\frac{4}{5}\right)^1\left(\frac{1}{5}\right)^{8-1}\right]$$

$$= 1 - \left[1(1)\left(\frac{1}{5}\right)^8 + (8)\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)^7\right]$$

$$= 1 - \left[\frac{1}{5^8} + \frac{32}{5^8}\right]$$

$$= 1 - \frac{33}{5^8}.$$

Question 15.
It is observed that it rains 12 days out of 30 days. Find the probability that
(i) it rains exactly 3 days of the week.
(ii) it will rain at least 2 days of a given week.
Solution:
Let X = the number of days it rains in a week.

p = probability that it rains

$$\therefore\ p = \frac{12}{30} = \frac{2}{5}$$

and $q = 1 - p = 1 - \frac{2}{5} = \frac{3}{5}$

Given : $n = 7$

i.e. $X \sim B\left(7, \frac{2}{5}\right)$

The p.m.f. of $X$ is given by

$P(X = x) = {}^nC_x\, p^x\, q^{n-x}$

i.e. $p(x) = {}^7C_x \left(\frac{2}{5}\right)^x \left(\frac{3}{5}\right)^{7-x}$, $x = 0, 1, 2, \ldots, 7$

(i) P(it rains exactly 3 days of week) = P(X = 3)

$$= p(3) = {}^7C_3 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^{7-3}$$

$$= \frac{7 \times 6 \times 5}{3 \times 2 \times 1}\left(\frac{8}{125}\right)\left(\frac{81}{625}\right)$$

$$= 35\left(\frac{8}{125}\right)\left(\frac{81}{625}\right) = \frac{35 \times 8 \times 81}{5^7}$$

$$= \frac{22680}{78125} = 0.2903$$

Hence, the probability that it rains exactly 3 days of week = 0.2903.

(ii) P(it will rain at least 2 days of the given week)

$$= P(X \geqslant 2) = 1 - P(X < 2)$$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - \left[{}^7C_0 \left(\frac{2}{5}\right)^0 \left(\frac{3}{5}\right)^{7-0} + {}^7C_1 \left(\frac{2}{5}\right)^1 \left(\frac{3}{5}\right)^{7-1}\right]$$

$$= 1 - \left[1(1)\left(\frac{3}{5}\right)^7 + 7\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^6\right]$$

$$= 1 - \left[\frac{3}{5} + \frac{14}{5}\right]\left(\frac{3}{5}\right)^6$$

$$= 1 - \left(\frac{17}{5}\right)\left(\frac{729}{(5)^6}\right) = 1 - \frac{12393}{5^7}$$

$$= 1 - \frac{12393}{78125} = 1 - 0.1586$$

$$= 0.8414$$

Hence, the probability that it rains at least 2 days of a given week = 0.8414

Question 16.
If the probability of success in a single trial is 0.01. How many trials are required in order to have a probability greater than 0.5 of getting at least one success?
Solution:
Let X = number of successes.
p = probability of success in a single trial
∴ p = 0.01
and q = 1 – p = 1 – 0.01 = 0.99
∴ X ~ B(n, 0.01)
The p.m.f. of X is given by
P(X = x) = ${}^nC_x p^x q^{n-x}$

i.e. $p(x) = {}^nC_x (0.01)^x (0.99)^{n-x}$, $x = 1, 2, \ldots, n$

$P$(at least one success)

$= P(X \geqslant 1) = 1 - P(X < 1)$

$= 1 - P(X = 0) = 1 - p(0)$

$= 1 - {}^nC_0 (0.01)^0 (0.99)^{n-0}$

$= 1 - 1\,(1)(0.99)^n$

$= 1 - (0.99)^n$

Given : $P(X \geqslant 1) > 0.5$

i.e. $1 - (0.99)^n > 0.5$

i.e. $1 - 0.5 > (0.99)^n$

i.e. $0.5 > (0.99)^n$

i.e. $(0.99)^n < 0.5$

i.e. $\log (0.99)^n < \log (0.5)$

i.e. $n \log (0.99) < \log 0.5$

i.e. $n < \dfrac{\log\ 0.5}{\log\ 0.99}$

i.e. $n < 68.96$

$\therefore\ n = 68$

Hence, the number of trials required in order to have a probability greater than 0.5 of getting at least one success is $log0.5log0.99$ or 68.

Question 17.
In binomial distribution with five Bernoulli's trials, the probability of one and two success are 0.4096 and 0.2048 respectively. Find the probability of success.
Solution:
Given: X ~ B(n = 5, p)
The probability of X success is

$P(X = x) = {}^nC_x\, p^x\, q^{n-x}$, $x = 0, 1, 2, \ldots, n$

i.e. $P(X = x) = {}^5C_x\, p^x\, q^{5-x}$, $x = 0, 1, 2, 3, 4, 5$

Probabilities of one and two successes are

$P(X = 1) = {}^5C_1\, p^1\, q^{5-1}$

and $P(X = 2) = {}^5C_2\, p^2\, q^{5-2}$ respectively

Given : $P(X = 1) = 0.4096$ and $P(X = 2) = 0.2048$

$\therefore\ \dfrac{P(X = 2)}{P(X = 1)} = \dfrac{0.2048}{0.4096}$

i.e. $\dfrac{{}^5C_2\ p^2\ q^{5-2}}{{}^5C_1\ p^2\ q^{5-1}} = \dfrac{1}{2}$

i.e. $2 \times {}^5C_2\, p^2\, q^3 = 1 \times {}^5C_1\, pq^4$

i.e. $2 \times \dfrac{5 \times 4}{1 \times 2} \times p^2\, q^3 = 1 \times 5 \times pq^4$

i.e. $20p^2q^3 = 5pq^4$

i.e. $4p = q$

i.e. $4p = 1 - p$

i.e. $5v = 1$

$\therefore\ p = \dfrac{1}{5}$

Hence, the probability of success is $15$.