Maharashtra State Board 11th Commerce Maths Solutions Chapter 7 Limits Ex 7.1

I. Evaluate the following limits:

Question 1.
$\lim_{x\to3}\left[\frac{\sqrt{x+6}}{x}\right]$
Solution:

$$\lim_{x\to3}\left[\frac{\sqrt{x+6}}{x}\right]=\frac{\lim_{x\to3}\sqrt{x+6}}{\lim_{x\to3}x}$$
$$=\frac{\sqrt{3+6}}{3}$$
$$=\frac{\sqrt{9}}{3}$$
$$=\frac{3}{3}$$
$$=1$$

Question 2.
$\lim_{x\to2}\left[\frac{x^{-3}-2^{-3}}{x-2}\right]$
Solution:

$$\lim_{x\to2}\frac{x^{-3}-2^{-3}}{x-2}=(-3)\,.\,(2)^{-4}$$
$$\ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$
$$=-3\times\frac{1}{2^4}$$
$$=\frac{-3}{16}$$

Question 3.
$\lim_{x\to5}\left[\frac{x^3-125}{x^5-3125}\right]$
Solution:

$$\lim_{x\to5}\frac{x^3-125}{x^5-3125}$$
$$=\lim_{x\to5}\frac{\left(\frac{x^3-5^3}{x-5}\right)}{\left(\frac{x^5-5^5}{x-5}\right)}\qquad\ldots\left[\begin{array}{l}\because x\to5,\ \therefore x\neq5,\\ \therefore x-5\neq0\end{array}\right]$$
$$=\frac{\lim_{x\to5}\frac{x^3-5^3}{x-5}}{\lim_{x\to5}\frac{x^5-5^5}{x-5}}$$
$$=\frac{3(5)^2}{5(5)^4}\qquad\ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=n.a^{n-1}\right]$$
$$=\frac{3}{(5)^3}$$
$$=\frac{3}{125}$$

Question 4.
If $\lim_{x\to1}\left[\frac{x^4-1}{x-1}\right]=\lim_{x\to a}\left[\frac{x^3-a^3}{x-a}\right]$, find all possible values of a.

Solution:

$$\lim_{x\to 1}\left[\frac{x^4-1}{x-1}\right]=\lim_{x\to a}\left[\frac{x^3-a^3}{x-a}\right]$$

$$\therefore \quad \lim_{x\to 1}\frac{x^4-(1)^4}{x-1}=\lim_{x\to a}\frac{x^3-a^3}{x-a}$$

$$\therefore \quad 4(1)^3=3a^2 \qquad \ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$\therefore \quad 3a^2=4$$

$$\therefore \quad a^2=\frac{4}{3}$$

$$\therefore \quad a=\pm\frac{2}{\sqrt{3}}$$

II. Evaluate the following limits:

Question 1.
$\lim_{x\to 7}\left[\frac{(\sqrt[3]{x}-\sqrt[3]{7})(\sqrt[3]{x}+\sqrt[3]{7})}{x-7}\right]$
Solution:

$$\lim_{x\to 7}\left[\frac{\left(\sqrt[3]{x}-\sqrt[3]{7}\right)\left(\sqrt[3]{x}+\sqrt[3]{7}\right)}{x-7}\right]$$

$$=\lim_{x\to 7}\left[\frac{(x^{\frac{1}{3}}-7^{\frac{1}{3}})(x^{\frac{1}{3}}+7^{\frac{1}{3}})}{x-7}\right]$$

$$=\lim_{x\to 7}\left[\frac{x^{\frac{2}{3}}-7^{\frac{2}{3}}}{x-7}\right] \quad \ldots[\because (a-b)(a+b)=a^2-b^2]$$

$$=\frac{2}{3}(7)^{\frac{-1}{3}} \qquad \ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$=\frac{2}{3}\cdot\frac{1}{7^{\frac{1}{3}}}=\frac{2}{3\sqrt[3]{7}}$$

Question 2.
If $\lim_{x\to 5}\left[\frac{x^k-5^k}{x-5}\right]=500$, find all possible values of k.
Solution:

$$\lim_{x\to 5}\frac{x^k-5^k}{x-5}=500$$

$$\therefore \quad k(5)^{k-1}=500 \qquad \ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$\therefore \quad k(5)^{k-1}=4\times 125$$

$$\therefore \quad k(5)^{k-1}=4\times(5)^3$$

$$\therefore \quad k(5)^{k-1}=4\times(5)^{4-1}$$

Comparing both sides, we get
$k=4$

Question 3.
$\lim_{x\to 0}\left[\frac{(1-x)^8-1}{(1-x)^2-1}\right]$
Solution:

$$\lim_{x \to 0} \frac{(1+x)^8 - 1}{(1-x)^2 - 1}$$

Put $1 - x = y$

As $x \to 0, y \to 1$

$$\therefore \quad \lim_{x \to 0} \frac{(1+x)^8 - 1}{(1-x)^2 - 1} = \lim_{y \to 1} \frac{y^8 - 1^8}{y^2 - 1^2}$$

$$= \lim_{y \to 1} \frac{\frac{y^8 - 1^8}{y-1}}{\frac{y^2 - 1^2}{y-1}} \qquad \ldots \begin{bmatrix} \because y \to 1, \therefore y \neq 1, \\ \therefore y - 1 \neq 0 \end{bmatrix}$$

$$= \frac{\lim\limits_{y \to 1} \frac{y^8 - 1^8}{y-1}}{\lim\limits_{y \to 1} \frac{y^2 - 1^2}{y-1}}$$

$$= \frac{8(1)^7}{2(1)^1} \qquad \ldots \left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$$= 4$$

Alternate method:

$$\lim_{x \to 0} \frac{(1+x)^8 - 1}{(1-x)^2 - 1}$$

Put $1 - x = y$

As $x \to 0, y \to 1$

$$\therefore \quad \lim_{x \to 0} \frac{(1+x)^8 - 1}{(1-x)^2 - 1} = \lim_{y \to 1} \frac{y^8 - 1}{y^2 - 1}$$

$$= \lim_{y \to 1} \frac{(y^4 - 1)(y^4 + 1)}{y^2 - 1}$$

$$= \lim_{y \to 1} \frac{(y^2 - 1)(y^2 + 1)(y^4 + 1)}{y^2 - 1}$$

$$= \lim_{y \to 1} (y^2 + 1)(y^4 + 1) \qquad \ldots \begin{bmatrix} \because y \to 1, \therefore y \neq 1, \\ \therefore y^2 \neq 1 \\ \therefore y^2 - 1 \neq 0 \end{bmatrix}$$

$$= (2)(2)$$

$$= 4$$

III. Evaluate the following limits:

Question 1.

limx→0[1+x√3−1+x√x]

Solution:

$$\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt{1+x}}{x}$$

$$=\lim_{x\to 0}\frac{(1+x)^{\frac{1}{3}}-(1+x)^{\frac{1}{2}}}{x}$$

Put $1+x=y$

As $x\to 0, y\to 1$

$$\lim_{x\to 0}\frac{(1+x)^{\frac{1}{3}}-(1+x)^{\frac{1}{2}}}{x}$$

$$=\lim_{y\to 1}\frac{y^{\frac{1}{3}}-y^{\frac{1}{2}}}{y-1}$$

$$=\lim_{y\to 1}\frac{\left(y^{\frac{1}{3}}-1\right)-\left(y^{\frac{1}{2}}-1\right)}{y-1}$$

$$=\lim_{y\to 1}\left(\frac{y^{\frac{1}{3}}-1}{y-1}-\frac{y^{\frac{1}{2}}-1}{y-1}\right)$$

$$=\lim_{y\to 1}\left(\frac{y^{\frac{1}{3}}-1}{y-1}-\frac{y^{\frac{1}{2}}-1}{y-1}\right)$$

$$=\lim_{y\to 1}\frac{y^{\frac{1}{3}}-1^{\frac{1}{3}}}{y-1}-\lim_{y\to 1}\frac{y^{\frac{1}{2}}-1^{\frac{1}{2}}}{y-1}$$

$$=\frac{1}{3}(1)^{\frac{-2}{3}}-\frac{1}{2}(1)^{\frac{-1}{2}} \qquad \ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$=\frac{1}{3}-\frac{1}{2}$$

$$=\frac{2-3}{6}$$

$$=-\frac{1}{6}$$

Question 2.

$lim_{y\to 1}[2y-27+y\sqrt{3}-2]$

Solution:

$$\lim_{y\to 1}\frac{2y-2}{\sqrt[3]{7+y}-2}$$

$$=\lim_{y\to 1}\frac{2(y-1)}{(7+y)^{\frac{1}{3}}-8^{\frac{1}{3}}} \qquad \ldots\left[\because 2=(2^3)^{\frac{1}{3}}=8^{\frac{1}{3}}\right]$$

$$=\lim_{y\to 1}\frac{2}{\frac{(y+7)^{\frac{1}{3}}-8^{\frac{1}{3}}}{y-1}}$$

$$=\frac{\lim_{y\to 1} 2}{\lim_{y\to 1}\frac{(y+7)^{\frac{1}{3}}-8^{\frac{1}{3}}}{(y+7)-8}}$$

Let $y+7=x$

As $y\to 1, x\to 8$

$$=\frac{2}{\lim_{x\to 8}\frac{x^{\frac{1}{3}}-8^{\frac{1}{3}}}{x-8}}$$

$$=\frac{2}{\frac{1}{3}(8)^{\frac{-2}{3}}} \qquad \ldots\left[\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$=2(3)\,.\,(8)^{\frac{2}{3}}$$

$$=6(2^3)^{\frac{2}{3}}$$

$$=6\times(2)^2=24$$

Question 3.

$lim_{z\to a}[(z+2)32-(a+2)32z-a]$

Solution:

$$\lim_{z\to a}\frac{(z+2)^{\frac{3}{2}}-(a+2)^{\frac{3}{2}}}{z-a}$$

Put $z + 2 = y$ and $a + 2 = b$

As $z \to a$, $z + 2 \to a + 2$

i.e. $y \to b$

$$\lim_{z\to a}\frac{(z+2)^{\frac{3}{2}}-(a+2)^{\frac{3}{2}}}{z-a}$$

$$= \lim_{y\to b}\frac{y^{\frac{3}{2}}-b^{\frac{3}{2}}}{(y-2)-(b-2)}$$

$$= \lim_{y\to b}\frac{y^{\frac{3}{2}}-b^{\frac{3}{2}}}{y-b}$$

$$= \frac{3}{2}.b^{\frac{1}{2}} \quad \ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$= \frac{3}{2}(a+2)^{\frac{1}{2}} \quad \ldots[\because b = a + 2]$$

Question 4.

$lim_{x\to 5}[x3-125x2-25]$

Solution:

$$\lim_{x\to 5}\frac{x^3-125}{x^2-25}$$

$$= \lim_{x\to 5}\frac{\frac{x^3-125}{x-5}}{\frac{x^2-25}{x-5}} \quad \ldots\left[\begin{array}{l}\text{As } x\to 5,\ x\neq 5\\ \therefore x-5\neq 0\\ \text{Divide Numerator and}\\ \text{Denominator by } x-5.\end{array}\right]$$

$$= \lim_{x\to 5}\frac{\left(\frac{x^3-5^3}{x-5}\right)}{\left(\frac{x^2-5^2}{x-5}\right)}$$

$$= \frac{3(5)^2}{2(5)^1} \quad \ldots\left[\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$= \frac{15}{2}$$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 7 Limits Ex 7.2

I. Evaluate the following limits:

Question 1.

$lim_{z\to 2}[z2-5z+6z2-4]$

Solution:

$$\lim_{z \to 2} \frac{z^2 - 5z + 6}{z^2 - 4}$$

$$= \lim_{z \to 2} \frac{(z-3)(z-2)}{(z+2)(z-2)}$$

$$= \lim_{z \to 2} \frac{z-3}{z-2} \quad \cdots \begin{bmatrix} \text{As } z \to 2, z \neq 2 \\ \therefore z-2 \neq 0 \end{bmatrix}$$

$$= \frac{2-3}{2+2}$$

$$= -\frac{1}{4}$$

Question 2.

$\lim_{x \to -3} \left[\frac{x+3}{x^2+4x+3}\right]$

Solution:

$$\lim_{x \to -3} \left[\frac{x+3}{x^2+4x+3}\right]$$

$$= \lim_{x \to -3} \frac{x+3}{(x+3)(x+1)}$$

$$= \lim_{x \to -3} \frac{1}{x+1} \quad \cdots \begin{bmatrix} \text{As } x \to -3, x \neq -3 \\ \therefore x+3 \neq 0 \end{bmatrix}$$

$$= \frac{1}{-3+1}$$

$$= -\frac{1}{2}$$

Question 3.

$\lim_{y \to 0} \left[\frac{5y^3+8y^2}{3y^4-16y^2}\right]$

Solution:

$$\lim_{y \to 0} \left[\frac{5y^3+8y^2}{3y^4-16y^2}\right]$$

$$= \lim_{y \to 0} \frac{y^2(5y+8)}{y^2(3y^2-16)}$$

$$= \lim_{y \to 0} \frac{5y+8}{3y^2-16} \quad \cdots \begin{bmatrix} \text{As } y \to 0, y \neq 0 \\ \therefore y^2 \neq 0 \end{bmatrix}$$

$$= \frac{5(0)+8}{3(0)^2-16}$$

$$= \frac{8}{-16}$$

$$= -\frac{1}{2}$$

Question 4.

$\lim_{x \to -2} \left[\frac{-2x-4}{x^3+2x^2}\right]$

Solution:

$$\lim_{x \to -2} \left[\frac{-2x-4}{x^3+2x^2}\right]$$

$$= \lim_{x \to -2} \frac{-2(x+2)}{x^2(x+2)}$$

$$= \lim_{x \to -2} \frac{-2}{x^2} \quad \cdots \begin{bmatrix} \text{As } x \to -2, x \neq -2 \\ \therefore x+2 \neq 0 \end{bmatrix}$$

$$= \frac{(-2)}{(-2)^2}$$

$$= \frac{-2}{4}$$

$$= \frac{-1}{2}$$

II. Evaluate the following limits:

Question 1.
limu→1[u4−1u3−1]
Solution:

$$\lim_{u \to 1}\left[\frac{u^4-1}{u^3-1}\right]$$

$$= \lim_{u \to 1}\frac{\left[\frac{u^4-1^4}{u-1}\right]}{\left[\frac{u^3-1^3}{u-1}\right]} \quad \ldots\left[\begin{array}{l}\because u \to 1;\ u \neq 1 \\ \therefore u-1 \neq 0\end{array}\right]$$

$$= \frac{4(1)^3}{3(1)^2} \quad \ldots\left|\because \lim_{x \to a}\frac{x^n-a^n}{x-a} = na^{n-1}\right|$$

$$= \frac{4}{3}$$

Question 2.
limx→3[1x−3−9xx3−27]
Solution:

$$\lim_{x \to 3}\left[\frac{1}{x-3}-\frac{9x}{x^3-27}\right]$$

$$= \lim_{x \to 3}\left[\frac{1}{x-3}-\frac{9x}{x^3-3^3}\right]$$

$$= \lim_{x \to 3}\left[\frac{1}{x-3}-\frac{9x}{(x-3)(x^2+3x+9)}\right]$$

$$= \lim_{x \to 3}\left[\frac{x^2+3x+9-9x}{(x-3)(x^2+3x+9)}\right]$$

$$= \lim_{x \to 3}\left[\frac{x^2-6x+9}{(x-3)(x^2+3x+9)}\right]$$

$$= \lim_{x \to 3}\left[\frac{(x-3)^2}{(x-3)(x^2+3x+9)}\right]$$

$$= \lim_{x \to 3}\left[\frac{x-3}{x^2+3x+9}\right] \quad \ldots\left[\begin{array}{l}\because x \to 3,\ x \neq 3 \\ \therefore x-3 \neq 0\end{array}\right]$$

$$= \frac{3-3}{(3)^2+3(3)+9} = \frac{0}{27}$$

$$= 0$$

Question 3.
limx→2[x3−4x2+4xx2−1]
Solution:

$$\lim_{x \to 2}\left[\frac{x^3-4x^2+4x}{x^2-1}\right]$$

$$= \lim_{x \to 2}\frac{x(x^2-4x+4)}{(x^2-1)} = \lim_{x \to 2}\frac{x(x-2)^2}{x^2-1}$$

$$= \frac{2(0)}{(2)^2-1} = \frac{2 \times 0}{3}$$

$$= 0$$

III. Evaluate the following limits:

Question 1.
limx→−2[x7+x5+160x3+8]
Solution:

$$\lim_{x\to -2}\left[\frac{x^7+x^5+160}{x^3+8}\right]$$

$$= \lim_{x\to -2}\frac{\left(x^7+128\right)+\left(x^5+32\right)}{x^3+8}$$

$$= \lim_{x\to -2}\frac{\dfrac{\left(x^7+128\right)+\left(x^5+32\right)}{x+2}}{\dfrac{x^3+8}{x+2}}$$

$$\ldots\left[\begin{array}{l}\text{As } x\to -2, x\neq -2\\ \therefore\ x+2\neq 0\\ \text{Divide Numerator and}\\ \text{Denominator by } x+2\end{array}\right]$$

$$= \frac{\lim_{x\to -2}\left(\dfrac{x^7+2^7}{x+2}+\dfrac{x^5+2^5}{x+2}\right)}{\lim_{x\to -2}\left(\dfrac{x^3+2^3}{x+2}\right)}$$

$$= \frac{\lim_{x\to -2}\dfrac{x^7-(-2)^7}{x-(-2)}+\lim_{x\to -2}\dfrac{x^5-(-2)^5}{x-(-2)}}{\lim_{x\to -2}\dfrac{x^3-(-2)^3}{x-(-2)}}$$

$$= \frac{7(-2)^6+5(-2)^4}{3(-2)^2}\quad\ldots\left[\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$= \frac{7(64)+5(16)}{3(4)}$$

$$= \frac{448+80}{12} = \frac{528}{12}$$

$$= 44$$

Question 2.

$\lim_{y\to\frac{1}{2}}\left[\frac{1-8y^3}{y-4y^3}\right]$

Solution:

$$\lim_{y\to\frac{1}{2}}\left[\frac{1-8y^3}{y-4y^3}\right]$$

$$= \lim_{y\to\frac{1}{2}}\frac{1-8y^3}{y\left(1-4y^2\right)}$$

$$= \lim_{y\to\frac{1}{2}}\frac{(1)^3-(2y)^3}{y\left[(1)^2-(2y)^2\right]}$$

$$= \lim_{y\to\frac{1}{2}}\frac{(1-2y)\left(1+2y+4y^2\right)}{y(1-2y)(1+2y)}$$

$$= \lim_{y\to\frac{1}{2}}\frac{1+2y+4y^2}{y(1+2y)}\quad\ldots\left[\begin{array}{l}\because y\to\frac{1}{2}, \therefore y\neq\frac{1}{2}\\ \therefore 2y\neq 1 \therefore 2y-1\neq 0\\ \therefore 1-2y\neq 0\end{array}\right]$$

$$= \frac{1+2\left(\frac{1}{2}\right)+4\left(\frac{1}{2}\right)^2}{\frac{1}{2}\left[1+2\left(\frac{1}{2}\right)\right]}$$

$$= \frac{1+1+1}{\frac{1}{2}(2)} = 3$$

Question 3.

$\lim_{v\to\sqrt{2}}\left[\frac{v^2+v\sqrt{2}-4}{v^2-3v\sqrt{2}+4}\right]$

Solution:

$$\lim_{v \to \sqrt{2}} \left[\frac{v^2 + v\sqrt{2} - 4}{v^2 - 3v\sqrt{2} + 4}\right]$$

Consider, $v^2 + v\sqrt{2} - 4 = v^2 + \sqrt{2}v - 4$

$$= v^2 + 2\sqrt{2}v - \sqrt{2}v - 4$$

$$= v\left(v + 2\sqrt{2}\right) - \sqrt{2}\left(v + 2\sqrt{2}\right)$$

$$= \left(v + 2\sqrt{2}\right)\left(v - \sqrt{2}\right)$$

$v^2 - 3v\sqrt{2} + 4 = v^2 - 3\sqrt{2}v + 4$

$$= v^2 - 2\sqrt{2}\,v - \sqrt{2}\,v + 4$$

$$= v\left(v - 2\sqrt{2}\right) - \sqrt{2}\left(v - 2\sqrt{2}\right)$$

$$= \left(v - 2\sqrt{2}\right)\left(v - \sqrt{2}\right)$$

$$\lim_{v \to \sqrt{2}} \left[\frac{v^2 + v\sqrt{2} - 4}{v^2 - 3v\sqrt{2} + 4}\right]$$

$$= \lim_{v \to \sqrt{2}} \frac{\left(v + 2\sqrt{2}\right)\left(v - \sqrt{2}\right)}{\left(v - 2\sqrt{2}\right)\left(v - \sqrt{2}\right)}$$

$$= \lim_{v \to \sqrt{2}} \frac{v + 2\sqrt{2}}{v - 2\sqrt{2}} \quad \ldots \begin{bmatrix} \text{As } v \to \sqrt{2},\ v \neq \sqrt{2} \\ \therefore v - \sqrt{2} \neq 0 \end{bmatrix}$$

$$= \frac{\sqrt{2} + 2\sqrt{2}}{\sqrt{2} - 2\sqrt{2}}$$

$$= \frac{3\sqrt{2}}{-\sqrt{2}}$$

$$= -3$$

Question 4.

limx→3[x2+2x−15x2−5x+6]

Solution:

$$\lim_{x \to 3} \left[\frac{x^2 + 2x - 15}{x^3 - 5x + 6}\right]$$

$$= \lim_{x \to 3} \frac{(x + 5)(x - 3)}{(x - 2)(x - 3)}$$

$$= \lim_{x \to 3} \frac{x + 5}{x - 2} \quad \ldots \begin{bmatrix} \text{as } x \to 3,\ x \neq 3 \\ \therefore\ x - 3 \neq 0 \end{bmatrix}$$

$$= \frac{3 + 5}{3 - 2}$$

$$= 8$$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 7 Limits Ex 7.3

I. Evaluate the following limits:

Question 1.

$lim_{x\to0}[6+x+x2\sqrt{}-6\sqrt{}x]$

Solution:

$$\lim_{x\to0}\left[\frac{\sqrt{6+x+x^2}-\sqrt{6}}{x}\right]$$
$$=\lim_{x\to0}\left[\frac{\sqrt{6+x+x^2}-\sqrt{6}}{x}\times\frac{\sqrt{6+x+x^2}+\sqrt{6}}{\sqrt{6+x+x^2}+\sqrt{6}}\right]$$
$$=\lim_{x\to0}\frac{\left(6+x+x^2\right)-6}{x\left(\sqrt{6+x+x^2}+\sqrt{6}\right)}$$
$$=\lim_{x\to0}\frac{x+x^2}{x\left(\sqrt{6+x+x^2}+\sqrt{6}\right)}$$
$$=\lim_{x\to0}\frac{x(1+x)}{x\left(\sqrt{6+x+x^2}+\sqrt{6}\right)}$$
$$=\lim_{x\to0}\frac{1+x}{\sqrt{6+x+x^2}+\sqrt{6}}\quad\cdots[\because x\to0,\therefore x\neq0]$$
$$=\frac{(1+0)}{\sqrt{6}+\sqrt{6}}$$
$$=\frac{1}{2\sqrt{6}}$$

Question 2.

$lim_{y\to0}[1-y2\sqrt{}-1+y2\sqrt{}y2]$

Solution:

$$\lim_{y\to0}\left[\frac{\sqrt{1-y^2}-\sqrt{1+y^2}}{y^2}\right]$$
$$=\lim_{y\to0}\left[\frac{\sqrt{1-y^2}-\sqrt{1+y^2}}{y^2}\times\frac{\sqrt{1-y^2}+\sqrt{1+y^2}}{\sqrt{1-y^2}+\sqrt{1+y^2}}\right]$$
$$=\lim_{y\to0}\frac{\left(1-y^2\right)-\left(1+y^2\right)}{y^2\left(\sqrt{1-y^2}+\sqrt{1+y^2}\right)}$$
$$=\lim_{y\to0}\frac{1-y^2-1-y^2}{y^2\left(\sqrt{1-y^2}+\sqrt{1+y^2}\right)}$$
$$=\lim_{y\to0}\frac{-2y^2}{y^2\left(\sqrt{1-y^2}+\sqrt{1+y^2}\right)}$$
$$=\lim_{y\to0}\frac{-2}{\sqrt{1-y^2}+\sqrt{1+y^2}}\quad\cdots\left[\begin{array}{l}\because y\to0,\therefore y\neq0,\\\therefore y^2\neq0\end{array}\right]$$
$$=\frac{-2}{\sqrt{1-0^2}+\sqrt{1+0^2}}$$
$$=\frac{-2}{1+1}$$
$$=-1$$

Question 3.

$lim_{x\to2}[2+x\sqrt{}-6-x\sqrt{}x\sqrt{}-2\sqrt{}]$

Solution:

$$\lim_{x \to 2}\left(\frac{\sqrt{2+x}-\sqrt{6-x}}{\sqrt{x}-\sqrt{2}}\right)$$

$$= \lim_{x \to 2}\left[\frac{\sqrt{2+x}-\sqrt{6-x}}{\sqrt{x}-\sqrt{2}} \times \frac{\sqrt{2+x}+\sqrt{6-x}}{\sqrt{2+x}+\sqrt{6-x}} \times \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}\right]$$

$$\ldots \left[\begin{array}{l}\text{By taking conjugates} \\ \text{of both, the numerator} \\ \text{as well as the} \\ \text{Denomerator}\end{array}\right]$$

$$= \lim_{x \to 2}\left[\frac{(2+x)-(6-x)}{(x-2)} \times \frac{\sqrt{x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{6-x}}\right]$$

$$= \lim_{x \to 2}\left[\frac{-4+2x}{x-2} \times \frac{\sqrt{x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{6-x}}\right]$$

$$= \lim_{x \to 2}\left[\frac{2(x-2)}{x-2} \times \frac{\sqrt{x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{6-x}}\right]$$

$$= \lim_{x \to 2}\left[\frac{2\left(\sqrt{x}+\sqrt{2}\right)}{\sqrt{2+x}+\sqrt{6-x}}\right] \ldots \left[\begin{array}{l}\because x \to 2;\ x \neq 2 \\ \therefore\ x-2 \neq 0\end{array}\right]$$

$$= \frac{2\left(\sqrt{2}+\sqrt{2}\right)}{\sqrt{2+2}+\sqrt{6-2}}$$

$$= \frac{2\left(2\sqrt{2}\right)}{2+2}$$

$$= \frac{4\sqrt{2}}{4}$$

$$= \sqrt{2}$$

II. Evaluate the following limits:

Question 1.
$\lim_{x \to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$
Solution:

$$\lim_{x \to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$= \lim_{x \to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$$

$$= \lim_{x \to a}\left[\frac{(a+2x)-3x}{(3a+x)-4x} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}\right]$$

$$= \lim_{x \to a}\left[\frac{a-x}{3a-3x} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}\right]$$

$$= \lim_{x \to a}\left[\frac{-(x-a)}{-3(x-a)} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}\right]$$

$$= \lim_{x \to a}\left[\frac{\sqrt{3a+x}+2\sqrt{x}}{3\left(\sqrt{a+2x}+\sqrt{3x}\right)}\right] \ldots \left[\begin{array}{l}\because x \to a,\ x \neq a \\ \therefore x-a \neq 0\end{array}\right]$$

$$= \frac{\sqrt{3a+a}+2\sqrt{a}}{3\left(\sqrt{a+2a}+\sqrt{3a}\right)}$$

$$= \frac{\sqrt{4a}+2\sqrt{a}}{3\left(\sqrt{3a}+\sqrt{3a}\right)}$$

$$= \frac{2\sqrt{a}+2\sqrt{a}}{3\left(2\sqrt{3a}\right)}$$

$$= \frac{4\sqrt{a}}{6\sqrt{3}\,\sqrt{a}} = \frac{2}{3\sqrt{3}}$$

Question 2.
$\lim_{x\to2}\left[\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right]$
Solution:

$$\lim_{x\to2}\left[\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right]$$
$$=\lim_{x\to2}\left[\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\times\frac{\sqrt{x+2}+\sqrt{3x-2}}{\sqrt{x+2}+\sqrt{3x-2}}\right]$$
$$=\lim_{x\to2}\frac{(x^2-4)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{(x+2)-(3x-2)}$$
$$=\lim_{x\to2}\frac{(x^2-4)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2x+4}$$
$$=\lim_{x\to2}\frac{(x+2)(x-2)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2(x-2)}$$
$$=\lim_{x\to2}\frac{(x+2)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2}\quad\ldots\left[\begin{array}{l}\text{As } x\to2,\ x\neq2\\ \therefore x-2\neq0\end{array}\right]$$
$$=\frac{(2+2)\left(\sqrt{2+2}+\sqrt{3(2)-2}\right)}{-2}$$
$$=\frac{4(2+2)}{-2}=\frac{16}{-2}$$
$$=-8$$

III. Evaluate the following limits:

Question 1.
$\lim_{x\to1}\left[\frac{x^2+x\sqrt{x}-2}{x-1}\right]$
Solution:

$$\lim_{x\to1}\left[\frac{x^2+x\sqrt{x}-2}{x-1}\right]$$
$$=\lim_{x\to1}\left[\frac{(x^2-1)+(x\sqrt{x}-1)}{x-1}\right]$$
$$=\lim_{x\to1}\left[\frac{x^2-1}{x-1}+\frac{x^{\frac{3}{2}}-1}{x-1}\right]\ldots\left[\begin{array}{l}\because x\sqrt{x}=x^1\cdot x^{\frac{1}{2}}\\ \qquad=x^{1+\frac{1}{2}}=x^{\frac{3}{2}}\end{array}\right]$$
$$=\lim_{x\to1}\left(\frac{x^2-1^2}{x-1}\right)+\lim_{x\to1}\left(\frac{x^{\frac{3}{2}}-1^{\frac{3}{2}}}{x-1}\right)$$
$$=2(1)^1+\frac{3}{2}(1)^{\frac{1}{2}}\quad\ldots\left[\because\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$
$$=2+\frac{3}{2}=\frac{7}{2}$$

Question 2.
$\lim_{x\to0}\left[\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}\right]$

Solution:

$$\lim_{x \to 0} \frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1+x^2}-\sqrt{1+x}\right)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}{\left(\sqrt{1+x^3}-\sqrt{1+x}\right)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}$$

$$= \lim_{x \to 0} \frac{\left[1+x^2-(1+x)\right]\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left[1+x^3-(1+x)\right]\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}$$

$$= \lim_{x \to 0} \frac{x(x-1)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{x\left(x^2-1\right)\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}$$

$$= \lim_{x \to 0} \frac{x(x-1)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{x(x-1)(x+1)\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}$$

$$= \lim_{x \to 0} \frac{\sqrt{1+x^3}+\sqrt{1+x}}{(x+1)\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}$$

$$\ldots[\because x \to 0,\ \therefore\ x \neq 0,\ \therefore\ x-1 \neq 0]$$

$$= \frac{\sqrt{1+0^3}+\sqrt{1+0}}{(0+1)\left(\sqrt{1+0^2}+\sqrt{1+0}\right)}$$

$$= \frac{1+1}{1(1+1)}$$

$$= 1$$

Question 3.

*lim*x→4[x2+x−203x+4√−4]

Solution:

$$\lim_{x \to 4}\left[\frac{x^2+x-20}{\sqrt{3x+4}-4}\right]$$

$$= \lim_{x \to 4}\left[\frac{(x+5)(x-4)}{\sqrt{3x+4}-4} \times \frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4}\right]$$

$$= \lim_{x \to 4} \frac{(x+5)(x-4)\left(\sqrt{3x+4}+4\right)}{3x+4-16}$$

$$= \lim_{x \to 4} \frac{(x+5)(x-4)\left(\sqrt{3x+4}+4\right)}{3x-12}$$

$$= \lim_{x \to 4} \frac{(x+5)(x-4)\left(\sqrt{3x+4}+4\right)}{3(x-4)}$$

$$= \lim_{x \to 4} \frac{(x+5)\left(\sqrt{3x+4}+4\right)}{3}$$

$$\ldots\left[\begin{array}{l}\because x \to 4, x \neq 4 \\ \therefore x-4 \neq 0\end{array}\right]$$

$$= \frac{(4+5)\left(\sqrt{3(4)+4}+4\right)}{3}$$

$$= \frac{9(4+4)}{3}$$

$$= 3\ (8)$$

$$= 24$$

Question 4.

*lim*x→2[x3−8x+2√−3x−2√]

Solution:

$$\lim_{x\to 2}\left[\frac{x^3-8}{\sqrt{x+2}-\sqrt{3x-2}}\right]$$

$$=\lim_{x\to 2}\left[\frac{x^3-8}{\sqrt{x+2}-\sqrt{3x-2}}\times\frac{\sqrt{x+2}+\sqrt{3x-2}}{\sqrt{x+2}+\sqrt{3x-2}}\right]$$

$$=\lim_{x\to 2}\frac{(x^3-8)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{(x+2)-(3x-2)}$$

$$=\lim_{x\to 2}\frac{(x^3-8)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2x+4}$$

$$=\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2(x-2)}$$

$$=\lim_{x\to 2}\frac{(x^2+2x+4)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2}$$

$$\ldots\left[\begin{array}{l}\text{As } x\to 2,\ x\neq 2\\ \therefore\ x-2\neq 0\end{array}\right]$$

$$=-\frac{1}{2}\lim_{x\to 2}\left(x^2+2x+4\right)\times\lim_{x\to 2}\left(\sqrt{x+2}+\sqrt{3x-2}\right)$$

$$=-\frac{1}{2}\times\left[(2)^2+2(2)+4\right]\times\left(\sqrt{2+2}+\sqrt{3(2)-2}\right)$$

$$=-\frac{1}{2}\times 12\times(2+2)$$

$$=-6\times 4$$

$$=-24$$

IV. Evaluate the following limits:

Question 1.

limy→2[2−y3−y√−1]

Solution:

$$\lim_{y\to 2}\left[\frac{2-y}{\sqrt{3-y}-1}\right]$$

$$=\lim_{y\to 2}\left[\frac{2-y}{\sqrt{3-y}-1}\times\frac{\sqrt{3-y}+1}{\sqrt{3-y}+1}\right]$$

$$=\lim_{y\to 2}\frac{(2-y)\left(\sqrt{3-y}+1\right)}{3-y-1}$$

$$=\lim_{y\to 2}\frac{(2-y)\left(\sqrt{3-y}+1\right)}{2-y}$$

$$=\lim_{y\to 2}\left(\sqrt{3-y}+1\right)\quad\ldots\left[\begin{array}{l}\text{As } y\to 2,\ y\neq 2\\ \therefore\ y-2\neq 0\ \ \therefore\ 2-y\neq 0\end{array}\right]$$

$$=\sqrt{3-2}+1$$

$$=1+1$$

$$=2$$

Question 2.

limz→4[3−5+z√1−5−z√]

Solution:

$$\lim_{z \to 4}\left[\frac{3-\sqrt{5+z}}{1-\sqrt{5-z}}\right]$$

$$= \lim_{z \to 4}\left[\frac{3-\sqrt{5+z}}{1-\sqrt{5-z}} \times \frac{3+\sqrt{5+z}}{1+\sqrt{5-z}} \times \frac{1+\sqrt{5-z}}{3+\sqrt{5+z}}\right]$$

$$= \lim_{z \to 4}\left[\frac{9-(5+z)}{1-(5-z)} \times \frac{1+\sqrt{5-z}}{3+\sqrt{5+z}}\right]$$

$$= \lim_{z \to 4}\left[\frac{4-z}{-4+z} \times \frac{1+\sqrt{5-z}}{3+\sqrt{5+z}}\right]$$

$$= \lim_{z \to 4}\left[\frac{-(z-4)}{z-4} \times \frac{1+\sqrt{5-z}}{3+\sqrt{5+z}}\right]$$

$$= \lim_{z \to 4}\left[\frac{-\left(1+\sqrt{5-z}\right)}{3+\sqrt{5+z}}\right] \quad \ldots\left[\begin{array}{l}\because z \to 4, \therefore z \neq 4, \\ \therefore z-4 \neq 0\end{array}\right]$$

$$= \frac{-\left(1+\sqrt{5-4}\right)}{3+\sqrt{5+4}}$$

$$= \frac{-(1+1)}{3+3} = \frac{-2}{6}$$

$$= -\frac{1}{3}$$

ALLguides

Maharashtra State Board 11th Commerce Maths Solutions Chapter 7 Limits Ex 7.4

I. Evaluate the following:

Question 1.
$\lim_{x \to 0}\left[\frac{9^x-5^x}{4^x-1}\right]$

Solution:

$$\lim_{x\to 0}\frac{9^x-5^x}{4^x-1} = \lim_{x\to 0}\frac{9^x-1+1-5^x}{4^x-1}$$

$$= \lim_{x\to 0}\frac{(9^x-1)-(5^x-1)}{4^x-1}$$

$$= \lim_{x\to 0}\frac{\frac{(9^x-1)-(5^x-1)}{x}}{\frac{(4^x-1)}{x}}$$

$$\ldots[\because x\to 0, \therefore x\neq 0]$$

$$= \lim_{x\to 0}\frac{\left(\frac{9^x-1}{x}\right)-\left(\frac{5^x-1}{x}\right)}{\left(\frac{4^x-1}{x}\right)}$$

$$= \frac{\lim_{x\to 0}\frac{9^x-1}{x}-\lim_{x\to 0}\frac{5^x-1}{x}}{\lim_{x\to 0}\frac{4^x-1}{x}}$$

$$= \frac{\log 9-\log 5}{\log 4} \quad \ldots\left[\because \lim_{x\to 0}\frac{a^x-1}{x}=\log a\right]$$

$$= \frac{1}{\log 4}\log\left(\frac{9}{5}\right)$$

Question 2.

$\lim_{x\to 0}\left[\frac{5^x+3^x-2^x-1}{x}\right]$

Solution:

$$\lim_{x\to 0}\frac{5^x+3^x-2^x-1}{x}$$

$$= \lim_{x\to 0}\frac{(5^x-1)+(3^x-2^x)}{x}$$

$$= \lim_{x\to 0}\frac{(5^x-1)+(3^x-1)-(2^x-1)}{x}$$

$$= \lim_{x\to 0}\left(\frac{5^x-1}{x}+\frac{3^x-1}{x}-\frac{2^x-1}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{5^x-1}{x}\right)+\lim_{x\to 0}\left(\frac{3^x-1}{x}\right)-\lim_{x\to 0}\left(\frac{2^x-1}{x}\right)$$

$$= \log 5+\log 3-\log 2 \quad \ldots\left[\lim_{x\to 0}\frac{a^x-1}{x}=\log a\right]$$

$$= \log\frac{5\times 3}{2}$$

$$= \log\frac{15}{2}$$

Question 3.

$\lim_{x\to 0}\left[\frac{\log(2+x)-\log(2-x)}{x}\right]$

Solution:

$$\lim_{x\to0}\left[\frac{\log(2+x)-\log(2-x)}{x}\right]$$

$$=\lim_{x\to0}\frac{\log\left[2\left(1+\frac{x}{2}\right)\right]-\log\left[2\left(1-\frac{x}{2}\right)\right]}{x}$$

$$=\lim_{x\to0}\frac{\log2+\log\left(1+\frac{x}{2}\right)-\left[\log2+\log\left(1-\frac{x}{2}\right)\right]}{x}$$

$$=\lim_{x\to0}\frac{\log\left(1+\frac{x}{2}\right)-\log\left(1-\frac{x}{2}\right)}{x}$$

$$=\lim_{x\to0}\left[\frac{\log\left(1+\frac{x}{2}\right)}{x}-\frac{\log\left(1-\frac{x}{2}\right)}{x}\right]$$

$$=\lim_{x\to0}\left[\frac{\log\left(1+\frac{x}{2}\right)}{2\left(\frac{x}{2}\right)}-\frac{\log\left(1-\frac{x}{2}\right)}{(-2)\left(-\frac{x}{2}\right)}\right]$$

$$=\frac{1}{2}\lim_{x\to0}\frac{\log\left(1+\frac{x}{2}\right)}{\frac{x}{2}}+\frac{1}{2}\lim_{x\to0}\frac{\log\left(1-\frac{x}{2}\right)}{-\frac{x}{2}}$$

$$=\frac{1}{2}(1)+\frac{1}{2}(1) \quad \ldots\left[\because x\to0,\frac{x}{2}\to0 \text{ and } \lim_{x\to0}\frac{\log(1+x)}{x}=1\right]$$

$$=1$$

II. Evaluate the following:

Question 1.
$\lim_{x\to0}\left[\frac{3^x+3^{-x}-2}{x^2}\right]$
Solution:

$$\lim_{x\to0}\frac{3^x+3^{-x}-2}{x^2}$$

$$=\lim_{x\to0}\frac{3^x+\frac{1}{3^x}-2}{x^2}$$

$$=\lim_{x\to0}\frac{\left(3^x\right)^2+1-2\left(3^x\right)}{3^x\cdot x^2}$$

$$=\lim_{x\to0}\frac{\left(3^x-1\right)^2}{x^2\cdot\left(3^x\right)} \quad \ldots[\because a^2-2ab+b^2=(a-b)^2]$$

$$=\lim_{x\to0}\left(\frac{3^x-1}{x}\right)^2\times\frac{1}{3^x}$$

$$=\lim_{x\to0}\left(\frac{3^x-1}{x}\right)^2\times\frac{1}{\lim_{x\to0}\left(3^x\right)}$$

$$=(\log3)^2\times\frac{1}{3^0}$$

$$=(\log3)^2\times\frac{1}{1} \quad \ldots\left[\lim_{x\to0}\frac{a^x-1}{x}=\log a\right]$$

$$=(\log3)^2$$

Question 2.
$\lim_{x\to0}\left[\frac{3+x}{3-x}\right]^{\frac{1}{x}}$

Solution:

$$\lim_{x\to 0}\left(\frac{3+x}{3-x}\right)^{\frac{1}{x}}$$

$$= \lim_{x\to 0}\left(\frac{1+\frac{x}{3}}{1-\frac{x}{3}}\right)^{\frac{1}{x}} \qquad \ldots\left[\begin{array}{l}\text{Divide numerator and}\\ \text{denominator by } 3\end{array}\right]$$

$$= \lim_{x\to 0}\frac{\left(1+\frac{x}{3}\right)^{\frac{1}{x}}}{\left(1-\frac{x}{3}\right)^{\frac{1}{x}}} = \lim_{x\to 0}\frac{\left(1+\frac{x}{3}\right)^{\frac{3}{x}\times\frac{1}{3}}}{\left(1-\frac{x}{3}\right)^{\frac{-3}{x}\times\frac{1}{-3}}}$$

$$= \frac{\lim_{x\to 0}\left[\left(1+\frac{x}{3}\right)^{\frac{3}{x}}\right]^{\frac{1}{3}}}{\lim_{x\to 0}\left[\left(1-\frac{x}{3}\right)^{\frac{-3}{x}}\right]^{-\frac{1}{3}}}$$

$$= \frac{e^{1/3}}{e^{-1/3}} \qquad \ldots\left[\begin{array}{l}\because x\to 0,\ \frac{x}{3}\to 0, \frac{-x}{3}\to 0 \text{ and}\\ \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e\end{array}\right]$$

$$= e^{\frac{1}{3}+\frac{1}{3}}$$

$$= e^{\frac{2}{3}}$$

Question 3.

$\lim_{x\to 0}\left[\frac{\log(3-x)-\log(3+x)}{x}\right]$

Solution:

ALLgu

$$\lim_{x\to 0}\frac{\log(3-x)-\log(3+x)}{x}$$

$$=\lim_{x\to 0}\frac{1}{x}\log\left(\frac{3-x}{3+x}\right)$$

$$=\lim_{x\to 0}\log\left(\frac{3-x}{3+x}\right)^{\frac{1}{x}}$$

$$=\lim_{x\to 0}\log\left(\frac{1-\frac{x}{3}}{1+\frac{x}{3}}\right)^{\frac{1}{x}}=\log\left[\lim_{x\to 0}\frac{\left(1-\frac{x}{3}\right)^{\frac{1}{x}}}{\left(1+\frac{x}{3}\right)^{\frac{1}{x}}}\right]$$

$$=\log\left[\frac{\left\{\lim_{x\to 0}\left(1-\frac{x}{3}\right)^{\frac{-3}{x}}\right\}^{\frac{-1}{3}}}{\left\{\lim_{x\to 0}\left(1+\frac{x}{3}\right)^{\frac{3}{x}}\right\}^{\frac{1}{3}}}\right]$$

$$=\log\left(\frac{e^{-\frac{1}{3}}}{e^{\frac{1}{3}}}\right)$$

$$\ldots\left[\because\ x\to 0,\ \pm\frac{x}{3}\to 0\ \text{and}\ \lim_{x\to 0}(1+x)^{\frac{1}{x}}=e\right]$$

$$=\log e^{-2/3}$$

$$=-\frac{2}{3}.\log e=-\frac{2}{3}(1)$$

$$=-\frac{2}{3}$$

III. Evaluate the following:

Question 1.
$\lim_{x\to 0}\left[\frac{a^{3x}-b^{2x}}{\log(1+4x)}\right]$
Solution:

ALL

$$\lim_{x\to0}\frac{a^{3x}-b^{2x}}{\log(1+4x)}$$

$$=\lim_{x\to0}\frac{\left(a^{3x}-1\right)-\left(b^{2x}-1\right)}{\log(1+4x)}$$

$$=\lim_{x\to0}\frac{\frac{\left(a^{3x}-1\right)-\left(b^{2x}-1\right)}{x}}{\frac{\log(1+4x)}{x}}$$

$$=\frac{\lim_{x\to0}\left[\frac{\left(a^{3x}-1\right)}{x}-\frac{\left(b^{2x}-1\right)}{x}\right]}{\lim_{x\to0}\frac{\log(1+4x)}{x}}$$

$$=\frac{\lim_{x\to0}\left[\frac{\left(a^{3x}-1\right)}{3x}\right]\times3-\lim_{x\to0}\left[\frac{\left(b^{2x}-1\right)}{2x}\right]\times2}{\lim_{x\to0}\frac{\log(1+4x)}{4x}\times4}$$

$$=\frac{3\log a-2\log b}{1\times4}$$

$$\ldots\left[\begin{array}{l}\because x\to0,\ 2x\to0,\ 3x\to0\\ 4x\to0\text{ and }\lim_{x\to0}\frac{a^x-1}{x}=\log a\\ \text{and }\lim_{x\to0}\frac{\log(1+x)}{x}=1\end{array}\right]$$

$$=\frac{1}{4}\left(\log a^3-\log b^2\right)$$

$$=\frac{1}{4}\log\left(\frac{a^3}{b^2}\right)$$

Question 2.

$\lim_{x\to0}\left[\frac{(2^x-1)^2}{(3^x-1)\cdot\log(1+x)}\right]$

Solution:

$$\lim_{x\to0}\frac{(2^x-1)^2}{(3^x-1)\cdot\log(1+x)}$$

$$=\lim_{x\to0}\frac{\frac{(2^x-1)^2}{x^2}}{\frac{(3^x-1)\cdot\log(1+x)}{x^2}}\cdots\left(\begin{array}{l}\text{Divide Numerator and}\\ \text{Denominator by }x^2\\ \text{As }x\to0,\ x\neq0\\ \therefore\ x^2\neq0\end{array}\right)$$

$$=\frac{\lim_{x\to0}\left(\frac{2^x-1}{x}\right)^2}{\lim_{x\to0}\left[\left(\frac{3^x-1}{x}\right)\times\frac{\log(1+x)}{x}\right]}$$

$$=\frac{\lim_{x\to0}\left(\frac{2^x-1}{x}\right)^2}{\lim_{x\to0}\left(\frac{3^x-1}{x}\right)\times\lim_{x\to0}\frac{\log(1+x)}{x}}$$

$$=\frac{(\log 2)^2}{\log 3\times1}\qquad\cdots\left[\begin{array}{l}\lim_{x\to0}\frac{a^x-1}{x}=\log a,\\ \lim_{x\to0}\frac{\log(1+x)}{x}=1\end{array}\right]$$

$$=\frac{(\log 2)^2}{\log 3}$$

Question 3.

$\lim_{x\to0}\left[\frac{15^x-5^x-3^x+1}{x^2}\right]$

Solution:

$$\lim_{x \to 0} \frac{15^x - 5^x - 3^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{5^x \cdot 3^x - 5^x - 3^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{5^x(3^x - 1) - 1(3^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{(3^x - 1)(5^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \left(\frac{3^x - 1}{x} \times \frac{5^x - 1}{x} \right)$$

$$= \lim_{x \to 0} \frac{3^x - 1}{x} \times \lim_{x \to 0} \frac{5^x - 1}{x}$$

$$= \log 3 \cdot \log 5 \qquad \ldots \left[\lim_{x \to 0} \frac{a^x - 1}{x} = \log a \right]$$

Question 4.

$\lim_{x \to 2} \left[\frac{3^{\frac{x}{2}} - 3}{3^x - 9} \right]$

Solution:

$$\lim_{x \to 2} \left[\frac{3^{\frac{x}{2}} - 3}{3^x - 9} \right]$$

$$= \lim_{x \to 2} \left[\frac{3^{\frac{x}{2}} - 3}{\left(3^{\frac{x}{2}}\right)^2 - (3)^2} \right]$$

$$= \lim_{x \to 2} \frac{3^{\frac{x}{2}} - 3}{\left(3^{\frac{x}{2}} - 3\right)\left(3^{\frac{x}{2}} + 3\right)}$$

$$= \lim_{x \to 2} \frac{1}{3^{\frac{x}{2}} + 3} \qquad \ldots \left[\begin{array}{l} \text{As } x \to 2, \frac{x}{2} \to 1 \\ \therefore\ 3^{\frac{x}{2}} \to 3^1 \quad \therefore\ 3^{\frac{x}{2}} \neq 3 \\ \therefore\ 3^{\frac{x}{2}} - 3 \neq 0 \end{array} \right]$$

$$= \frac{1}{3^{\frac{2}{2}} + 3}$$

$$= \frac{1}{3^1 + 3}$$

$$= \frac{1}{6}$$

IV. Evaluate the following:

Question 1.

$\lim_{x \to 0} \left[\frac{(25)^x - 2(5)^x + 1}{x^2} \right]$

Solution:

$$\lim_{x\to 0}\frac{(25)^x-2(5)^x+1}{x^2}$$
$$=\lim_{x\to 0}\frac{(5^2)^x-2(5^x)+1}{x^2}$$
$$=\lim_{x\to 0}\frac{(5^x)^2-2(5^x)+1}{x^2}$$
$$=\lim_{x\to 0}\frac{(5^x-1)^2}{x^2}$$
$$=\lim_{x\to 0}\left(\frac{5^x-1}{x}\right)^2$$
$$=(\log 5)^2 \qquad \ldots\left[\lim_{x\to 0}\frac{a^x-1}{x}=\log a\right]$$

Question 2.
$lim_{x\to 0}[(49)^x-2(35)^x+(25)^x x^2]$
Solution:

$$\lim_{x\to 0}\frac{(49)^x-2(35)^x+(25)^x}{x^2}$$
$$=\lim_{x\to 0}\frac{(7^2)^x-2(7\times 5)^x+(5^2)^x}{x^2}$$
$$=\lim_{x\to 0}\frac{(7^x)^2-2(7^x)(5^x)+(5^x)^2}{x^2}$$
$$=\lim_{x\to 0}\frac{(7^x-5^x)^2}{x^2}$$
$$=\lim_{x\to 0}\left(\frac{7^x-5^x}{x}\right)^2$$
$$=\lim_{x\to 0}\left[\frac{(7^x-1)-(5^x-1)}{x}\right]^2$$
$$=\lim_{x\to 0}\left(\frac{7^x-1}{x}-\frac{5^x-1}{x}\right)^2$$
$$=\left(\lim_{x\to 0}\frac{7^x-1}{x}-\lim_{x\to 0}\frac{5^x-1}{x}\right)^2$$
$$=(\log 7-\log 5)^2 \qquad \ldots\left[\lim_{x\to 0}\frac{a^x-1}{x}=\log a\right]$$
$$=\left(\log\frac{7}{5}\right)^2$$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 7 Limits Miscellaneous Exercise 7

I.

Question 1.

If $\lim_{x\to2}\frac{x^n-2^n}{x-2}=80$ then find the value of n.

Solution:

$$\lim_{x\to2}\frac{x^n-2^n}{x-2}=80$$

$\therefore \quad n(2)^{n-1}=80 \qquad \ldots\left[\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$

$\therefore \quad n(2)^{n-1}=5\times16$

$\qquad\qquad =5\times(2)^4$

$\therefore \quad n(2)^{n-1}=5\times(2)^{5-1}$

$\therefore \quad n=5$

II. Evaluate the following Limits:

Question 1.

$\lim_{x\to a}\frac{(x+2)^{\frac{5}{3}}-(a+2)^{\frac{5}{3}}}{x-a}$

Solution:

$$\lim_{x\to a}\frac{(x+2)^{\frac{5}{3}}-(a+2)^{\frac{5}{3}}}{x-a}$$

Put $x+2=y$ and $a+2=b$

As $x\to a,\ x+2\to a+2$

i.e. $y\to b$.

$$\lim_{x\to a}\frac{(x+2)^{\frac{5}{3}}-(a+2)^{\frac{5}{3}}}{x-a}$$

$$=\lim_{x\to a}\frac{y^{\frac{5}{3}}-b^{\frac{5}{3}}}{(y-2)-(b-2)}$$

$$=\lim_{y\to b}\frac{y^{\frac{5}{3}}-b^{\frac{5}{3}}}{y-b}$$

$=\frac{5}{3}b^{\frac{2}{3}} \qquad \ldots\left[\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$

$=\frac{5}{3}(a+2)^{\frac{2}{3}} \qquad \ldots[\because b=a+2]$

Question 2.

$\lim_{x\to0}\frac{(1+x)^n-1}{x}$

Solution:

$$\lim_{x\to0}\frac{(1+x)^n-1}{x}$$

Put $1+x=y \quad \therefore \quad x=y-1$

As $x\to0,\ y\to1$

$$\lim_{x\to0}\frac{(1+x)^n-1}{x}$$

$$=\lim_{y\to1}\frac{y^n-1}{y-1}$$

$$=\lim_{y\to1}\frac{y^n-1^n}{y-1}$$

$=n(1)^{n-1} \qquad \ldots\left[\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$

$=n$

Question 3.

$\lim_{x\to2}\left[\frac{(x-2)}{2x^2-7x+6}\right]$

Solution:

$$\lim_{x \to 2}\left(\frac{x-2}{2x^2-7x+6}\right)$$

$$= \lim_{x\to 2}\frac{x-2}{(x-2)(2x-3)}$$

$$= \lim_{x\to 2}\frac{1}{2x-3} \quad \ldots\left[\begin{array}{l}\text{As } x\to 2,\ x\neq 2\\ \therefore\ x-2\neq 0\end{array}\right]$$

$$= \frac{1}{2(2)-3}$$

$$= 1$$

Question 4.

$lim_{x\to1}[x3-1x2+5x-6]$

Solution:

$$\lim_{x\to 1}\frac{x^3-1}{x^2+5x-6}$$

$$= \lim_{x\to 1}\frac{(x-1)\left(x^2+x+1\right)}{(x-1)(x+6)}$$

$$= \lim_{x\to 1}\frac{x^2+x+1}{x+6} \quad \ldots\left[\begin{array}{l}\text{As } x\to 1,\ x\neq 1\\ \therefore\ x-1\neq 0\end{array}\right]$$

$$= \frac{(1)^2+1+1}{1+6}$$

$$= \frac{3}{7}$$

Question 5.

$lim_{x\to3}[x-3x-2\sqrt{}-4-x\sqrt{}]$

Solution:

$$\lim_{x\to 3}\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}$$

$$= \lim_{x\to 3}\left[\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}\times\frac{\sqrt{x-2}+\sqrt{4-x}}{\sqrt{x-2}+\sqrt{4-x}}\right]$$

$$= \lim_{x\to 3}\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{(x-2)-(4-x)}$$

$$= \lim_{x\to 3}\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{2x-6}$$

$$= \lim_{x\to 3}\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{2(x-3)}$$

$$= \lim_{x\to 3}\frac{\sqrt{x-2}+\sqrt{4-x}}{2} \quad \ldots\left[\begin{array}{l}\text{As } x\to 3,\ x\neq 3\\ \therefore\ x-3\neq 0\end{array}\right]$$

$$= \frac{1}{2}\lim_{x\to 3}\left(\sqrt{x-2}+\sqrt{4-x}\right)$$

$$= \frac{1}{2}\left(\sqrt{3-2}+\sqrt{4-3}\right)$$

$$= \frac{1}{2}(1+1)$$

$$= 1$$

Question 6.

$lim_{x\to4}[3-5+x\sqrt{}1-5-x\sqrt{}]$

Solution:

$$\lim_{x\to 4}\left[\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right]$$

$$=\lim_{x\to 4}\left[\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\times\frac{3+\sqrt{5+x}}{1+\sqrt{5-x}}\times\frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right]$$

$$=\lim_{x\to 4}\left[\frac{9-(5+x)}{1-(5-x)}\times\frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right]$$

$$=\lim_{x\to 4}\left[\frac{4-x}{-4+x}\times\frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right]$$

$$=\lim_{x\to 4}\left[\frac{-(x-4)}{x-4}\times\frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right]$$

$$=\lim_{x\to 4}\left[\frac{-\left(1+\sqrt{5-x}\right)}{3+\sqrt{5+x}}\right]\quad\ldots\left[\begin{array}{l}\text{As } x\to 4,\ x\neq 4\\ \therefore x-4\neq 0\end{array}\right]$$

$$=\frac{-\left(1+\sqrt{5-4}\right)}{3+\sqrt{5+4}}$$

$$=\frac{-(1+1)}{3+3}$$

$$=\frac{-2}{6}$$

$$=-\frac{1}{3}$$

Question 7.

$\lim_{x\to 0}\left[\frac{5^x-1}{x}\right]$

Solution:

$$\lim_{x\to 0}\frac{5^x-1}{x}=\log 5\qquad\ldots\left[\lim_{x\to 0}\frac{a^x-1}{x}=\log a\right]$$

Question 8.

$\lim_{x\to 0}\left(1+\frac{x}{5}\right)^{\frac{1}{x}}$

Solution:

$$\lim_{x\to 0}\left(1+\frac{x}{5}\right)^{\frac{1}{x}}$$

$$=\lim_{x\to 0}\left[\left(1+\frac{x}{5}\right)^{\frac{5}{x}}\right]^{\frac{1}{5}}$$

$$=e^{\frac{1}{5}}\qquad\ldots\left[\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e\right]$$

Question 9.

$\lim_{x\to 0}\left[\frac{\log(1+9x)}{x}\right]$

Solution:

$$\lim_{x\to 0}\frac{\log(1+9x)}{x}$$

$$=\lim_{x\to 0}\left[\frac{\log(1+9x)}{9x}\right]\times 9$$

$$=1\times 9\qquad\ldots\left[\lim_{x\to 0}\frac{\log(1+x)}{x}=1\right]$$

$$=9$$

Question 10.

$lim_{x \to 0} (1-x)_5-1(1-x)_3-1$

Solution:

$$\lim_{x \to 0}\left[\frac{(1-x)^5-1}{(1-x)^3-1}\right]$$

Put $1 - x = y$

As $x \to 0, y \to 1$

$$\lim_{x \to 0}\left[\frac{(1-x)^5-1}{(1-x)^3-1}\right]$$

$$= \lim_{y \to 1}\frac{y^5-1}{y^3-1}$$

$$= \lim_{y \to 1}\left(\frac{\frac{y^5-1}{y-1}}{\frac{y^3-1}{y-1}}\right) \quad \ldots \left[\begin{array}{l}\text{As } y \to 1, y \neq 1 \\ \therefore \quad y-1 \neq 0 \\ \text{Divide Numerator and} \\ \text{Denominator by } y-1\end{array}\right]$$

$$= \frac{\lim_{y \to 1}\frac{y^5-1^5}{y-1}}{\lim_{y \to 1}\frac{y^3-1^3}{y-1}}$$

$$= \frac{5(1)^4}{3(1)^2} \quad \ldots \left[\lim_{x \to a}\frac{x^n-a^n}{x-a} = n\,a^{n-1}\right]$$

$$= \frac{5}{3}$$

Question 11.

$lim_{x \to 0}[a_x+b_x+c_x-3x]$

Solution:

$$\lim_{x \to 0}\frac{a^x+b^x+c^x-3}{x}$$

$$= \lim_{x \to 0}\frac{(a^x-1)+(b^x-1)+(c^x-1)}{x}$$

$$= \lim_{x \to 0}\left(\frac{a^x-1}{x}+\frac{b^x-1}{x}+\frac{c^x-1}{x}\right)$$

$$= \lim_{x \to 0}\left(\frac{a^x-1}{x}\right)+\lim_{x \to 0}\left(\frac{b^x-1}{x}\right)+\lim_{x \to 0}\left(\frac{c^x-1}{x}\right)$$

$$= \log a + \log b + \log c$$

$$\ldots \left[\lim_{x \to 0}\frac{a^x-1}{x} = \log a\right]$$

$$= \log(abc).$$

Question 12.

$lim_{x \to 0} e_x+e_{-x}-2x_2$

Solution:

$$\lim_{x\to 0}\frac{e^x+e^{-x}-2}{x^2}$$

$$= \lim_{x\to 0}\frac{e^x+\frac{1}{e^x}-2}{x^2}$$

$$= \lim_{x\to 0}\frac{\left(e^x\right)^2+1-2\,e^x}{x^2\cdot e^x}$$

$$= \lim_{x\to 0}\frac{\left(e^x-1\right)^2}{x^2\cdot e^x}$$

$$= \lim_{x\to 0}\left[\left(\frac{e^x-1}{x}\right)^2\times\frac{1}{e^x}\right]$$

$$= \lim_{x\to 0}\left(\frac{e^x-1}{x}\right)^2\times\frac{1}{\lim\limits_{x\to 0}e^x}$$

$$= (1)^2\times\frac{1}{e^0}\qquad\ldots\left[\lim_{x\to 0}\frac{e^x-1}{x}=1\right]$$

$$= 1\times\frac{1}{1}$$

$$= 1$$

Question 13.

$\lim_{x\to 0}\left[\frac{x(6^x-3^x)}{(2^x-1)\cdot\log(1+x)}\right]$

Solution:

$$\lim_{x\to 0}\frac{x\left(6^x-3^x\right)}{\left(2^x-1\right)\cdot\log(1+x)}$$

$$= \lim_{x\to 0}\frac{x\cdot\left(3^x\cdot 2^x-3^x\right)}{\left(2^x-1\right)\cdot\log(1+x)}$$

$$= \lim_{x\to 0}\frac{x\cdot 3^x\left(2^x-1\right)}{\left(2^x-1\right)\cdot\log(1+x)}$$

$$= \lim_{x\to 0}\frac{x\cdot 3^x}{\log(1+x)}\qquad\ldots\left[\begin{array}{l}\text{As } x\to 0,\ 2^x\to 2^0\\ \text{i.e. } 2^x\to 1\ \therefore\ 2^x\neq 1\\ \therefore 2^x-1\neq 0\end{array}\right]$$

$$= \lim_{x\to 0}\frac{3^x}{\frac{\log(1+x)}{x}}$$

$$= \frac{\lim\limits_{x\to 0}3^x}{\lim\limits_{x\to 0}\frac{\log(1+x)}{x}}$$

$$= \frac{3^0}{1}\qquad\ldots\left[\lim_{x\to 0}\frac{\log(1+x)}{x}=1\right]$$

$$= 1$$

Question 14.

$\lim_{x\to 0}\left[\frac{a^{3x}-a^{2x}-a^x+1}{x^2}\right]$

Solution:

$$\lim_{x \to 0} \frac{a^{3x} - a^{2x} - a^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{a^{2x} \cdot a^x - a^{2x} - a^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{a^{2x}(a^x - 1) - 1(a^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{(a^x - 1) \cdot (a^{2x} - 1)}{x^2}$$

$$= \lim_{x \to 0} \left(\frac{a^x - 1}{x} \times \frac{a^{2x} - 1}{x} \right)$$

$$= \lim_{x \to 0} \left(\frac{a^x - 1}{x} \right) \times \lim_{x \to 0} \left(\frac{a^{2x} - 1}{2x} \right) \times 2$$

$$= \log a \,.\, (2 \log a) \quad \ldots \left[\begin{array}{l} \text{As } x \to 0,\ 2x \to 0 \text{ and} \\ \lim_{x \to 0} \frac{a^x - 1}{x} = \log a \end{array} \right]$$

$$= 2(\log a)^2$$

Question 15.

$lim_{x\to0}[(5^x-1)^2 x \cdot \log(1+x)]$

Solution:

$$\lim_{x \to 0} \frac{(5^x - 1)^2}{x \cdot \log(1+x)}$$

$$= \lim_{x \to 0} \frac{\frac{(5^x - 1)^2}{x^2}}{\frac{x \cdot \log(1+x)}{x^2}} \quad \ldots \left[\begin{array}{l} \text{As } x \to 0, x \neq 0 \therefore x^2 \neq 0 \\ \text{Divide Numerator and} \\ \text{Denominator by } x^2 \end{array} \right]$$

$$= \frac{\lim_{x \to 0} \left(\frac{5^x - 1}{x} \right)^2}{\lim_{x \to 0} \frac{\log(1+x)}{x}}$$

$$= \frac{(\log 5)^2}{1} \quad \ldots \left[\begin{array}{l} \lim_{x \to 0} \frac{a^x - 1}{x} = \log a, \\ \lim_{x \to 0} \frac{\log(1+x)}{x} = 1 \end{array} \right]$$

$$= (\log 5)^2$$

Question 16.

$lim_{x\to0}[a^{4x}-1 b^{2x}-1]$

Solution:

$$\lim_{x \to 0} \frac{a^{4x} - 1}{b^{2x} - 1}$$

$$= \lim_{x \to 0} \frac{\frac{a^{4x} - 1}{x}}{\frac{b^{2x} - 1}{x}}$$

$$= \frac{\lim_{x \to 0} \left(\frac{a^{4x} - 1}{4x} \right) \times 4}{\lim_{x \to 0} \left(\frac{b^{2x} - 1}{2x} \right) \times 2}$$

$$= \frac{4 \log a}{2 \log b} \quad \ldots \left[\begin{array}{l} \text{As } x \to 0,\ 2x \to 0,\ 4x \to 0 \\ \text{and } \lim_{x \to 0} \frac{a^x - 1}{x} = \log a \end{array} \right]$$

$$= \frac{2 \log a}{\log b}$$

Question 17.

$lim_{x\to0}[\log100+\log(0.01+x)x]$

Solution:

$$\lim_{x \to 0}\left[\frac{\log 100 + \log(0.01 + x)}{x}\right]$$

$$= \lim_{x \to 0}\frac{\log[100(0.01 + x)]}{x}$$

$$= \lim_{x \to 0}\frac{\log(1 + 100x)}{x}$$

$$= \lim_{x \to 0}\left[\frac{\log(1 + 100x)}{100x}\right] \times 100$$

$$= 1 \times 100 \quad \ldots\left[\begin{array}{l}\text{As } x \to 0, 100x \to 0 \text{ and} \\ \lim_{x \to 0}\frac{\log(1+x)}{x} = 1\end{array}\right]$$

$$= 100$$

Question 18.

$\lim_{x \to 0}\left[\frac{\log(4-x)-\log(4+x)}{x}\right]$

Solution:

$$\lim_{x \to 0}\frac{\log(4-x)-\log(4+x)}{x}$$

$$= \lim_{x \to 0}\frac{\log\left[4\left(1-\frac{x}{4}\right)\right]-\log\left[4\left(1+\frac{x}{4}\right)\right]}{x}$$

$$= \lim_{x \to 0}\frac{\log 4 + \log\left(1-\frac{x}{4}\right)-\left[\log 4 + \log\left(1+\frac{x}{4}\right)\right]}{x}$$

$$= \lim_{x \to 0}\frac{\log\left(1-\frac{x}{4}\right)-\log\left(1+\frac{x}{4}\right)}{x}$$

$$= \lim_{x \to 0}\left[\frac{\log\left(1-\frac{x}{4}\right)}{x}-\frac{\log\left(1+\frac{x}{4}\right)}{x}\right]$$

$$= \lim_{x \to 0}\frac{\log\left(1-\frac{x}{4}\right)}{(-4)\left(-\frac{x}{4}\right)}-\lim_{x \to 0}\frac{\log\left(1+\frac{x}{4}\right)}{4\left(\frac{x}{4}\right)}$$

$$= -\frac{1}{4}\lim_{x \to 0}\frac{\log\left(1-\frac{x}{4}\right)}{-\frac{x}{4}}-\frac{1}{4}\lim_{x \to 0}\frac{\log\left(1+\frac{x}{4}\right)}{\frac{x}{4}}$$

$$= -\frac{1}{4}(1)-\frac{1}{4}(1) \quad \ldots\left[\begin{array}{l}\text{As } x \to 0, \frac{x}{4} \to 0, \frac{-x}{4} \to 0 \\ \text{and } \lim_{x \to 0}\frac{\log(1+x)}{x} = 1\end{array}\right]$$

$$= -\frac{1}{2}$$

Question 19.

Evaluate the limit of the function if exist at x = 1 where,

$f(x)=\begin{cases}7-4x & x<1 \\ x^2+2 & x \geq 1\end{cases}$

Solution:

$$f(x) = 7 - 4x;\ x < 1$$
$$= x^2 + 2\ ;\ x \geq 1$$
$$\lim_{x\to1^-} f(x) = \lim_{x\to1^-} (7 - 4x)$$
$$= 7 - 4(1)$$
$$= 3$$
$$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} (x^2 + 2)$$
$$= (1)^2 + 2$$
$$= 3$$

$\therefore$ $\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x)$

$\therefore$ $\lim_{x\to1} f(x)$ exists.

$\therefore$ $\lim_{x\to1} f(x) = 3$

ALLguidesite