Maharashtra State Board 12th Physics Solutions Chapter 5 Oscillations

1. Choose the correct option.

i) A particle performs linear S.H.M. starting from the mean position. Its amplitude is A and time period is T. At the instance when its speed is half the maximum speed, its displacement x is
(A) 3√2A
(B) 23√
(C) A/2
(D) 12√
Answer:
(A) 3√2A

ii) A body of mass 1 kg is performing linear S.H.M. Its displacement x (cm) at t (second) is given by x = 6 sin (100t + π/4). Maximum kinetic energy of the body is
(A) 36 J
(B) 9 J
(C) 27 J
(D) 18 J
Answer:
(D) 18 J

iii) The length of second's pendulum on the surface of earth is nearly 1 m. Its length on the surface of moon should be [Given: acceleration due to gravity (g) on moon is 1/6 th of that on the earth's surface]
(A) 16 m
(B) 6 m
(C) 136 m
(D) 16√ m.
Answer:
(A) 16 m

iv) Two identical springs of constant k are connected, first in series and then in parallel. A metal block of mass m is suspended from their combination. The ratio of their frequencies of vertical oscillations will be in a ratio
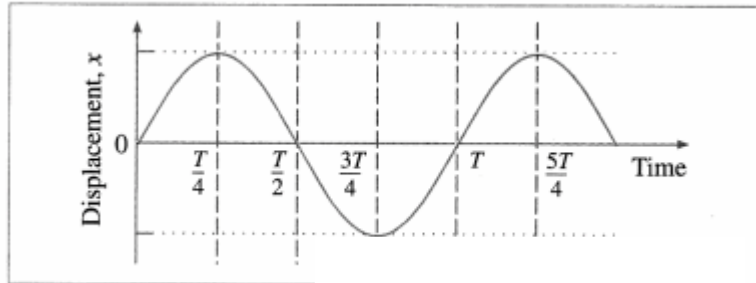(A) 1:4
(B) 1:2
(C) 2:1
(D) 4:1
Answer:
(B) 1:2

v) The graph shows variation of displacement of a particle performing S.H.M. with time t. Which of the following statements is correct from the graph?
(A) The acceleration is maximum at time T.
(B) The force is maximum at time 3T/4.
(C) The velocity is zero at time T/2.
(D) The kinetic energy is equal to total energy at time T/4.

Answer:
(B) The force is maximum at time 3T/4.

2. Answer in brief.

i) Define linear simple harmonic motion.
Answer:
Definition: Linear simple harmonic motion (SHM) is defined as the linear periodic motion of a body, in which the force (or acceleration) is always directed towards the mean position and its magnitude is proportional to the displacement from the mean position.
OR
A particle is said to execute linear SHM if the particle undergoes oscillations about a point of stable equilibrium, subject to a linear restoring force always directed towards that point and whose magnitude is proportional to the magnitude of the displacement of the particle from that point.
Examples : The vibrations of the tines (prongs) of a tuning fork, the oscillations of the needle of a sewing machine.

ii) Using differential equation of linear S.H.M, obtain the expression for
(a) velocity in S.H.M.,
(b) acceleration in S.H.M.
Answer:
The general expression for the displacement of a particle in SHM at time t is $x = A \sin(\omega t + \alpha)$ … (1)
where A is the amplitude, ω is a constant in a particular case and α is the initial phase.
The velocity of the particle is

$$v = \frac{dx}{dt} = \frac{d}{dt}[A \sin(\omega t + \alpha)]$$

$$= \omega A \cos(\omega t + \alpha)$$

$$= \omega A \sqrt{1 - \sin^2(\omega t + \alpha)}$$

From Eq. (1), $\sin(\omega t + \alpha) = x/A$

$$\therefore v = \omega A \sqrt{1 - \frac{x^2}{A^2}}$$

$$\therefore v = \omega \sqrt{A^2 - x^2} \qquad \dots (2)$$

Equation (2) gives the velocity as a function of x.

The acceleration of the particle is

a = dvdt = ddt [Aω cos (ωt + α) J at at

$\therefore$ a = $- \omega^2$ A sin (ωt + α)

But from Eq. (1), A sin (ωt + α) = x

$\therefore$ a = $-\omega^2 x$ ... (3)

Equation (3) gives the acceleration as a function of x. The minus sign shows that the direction of the acceleration is opposite to that of the displacement.

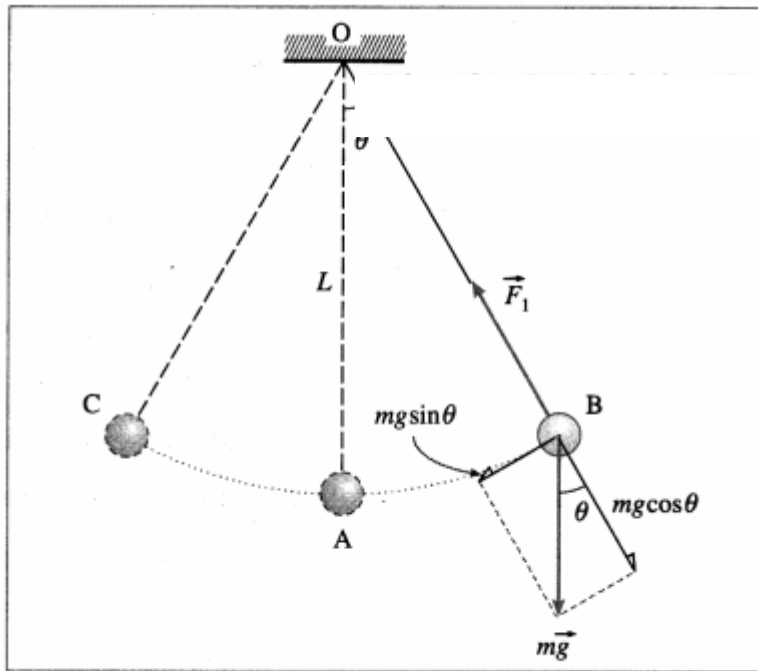iii) Obtain the expression for the period of a simple pendulum performing S.H.M.

Answer:

An ideal simple pendulum is defined as a heavy point mass suspended from a rigid support by a weightless, inextensible and twistless string, and set oscillating under gravity through a small angle in a vertical plane.

In practice, a small but heavy sphere, called the bob, is used. The distance from the point of suspension to the centre of gravity of the bob is called the length of the pendulum.

Consider a simple pendulum of length L_1 – suspended from a rigid support O. When displaced from its initial position of rest through a small angle θ in a vertical plane and released, it performs oscillations between two extremes, B and C, as shown in below figure.

At B, the forces on the bob are its weight $\overrightarrow{mg}$ and the tension $F1-\rightarrow$ in the string.

Resolve $\overrightarrow{mg}$ into two components : mg cos θ in the direction opposite to that of the tension and mg sin θ perpendicular to the string.

Motion of a simple pendulum

mg cos θ balanced by the tension in the string. mg sin θ restores the bob to the equilibrium position.

Restoring force, F = – mg sin θ

If θ is small and expressed in radian,

sin θ ≈ θ = arc radius = ABOB = xL

∴ F = – mgθ = -mgxL …. (1)

Since m, g and L are constant,

F ∝ (-x) …. (2)

Thus, the net force on the bob is in the direction opposite to that of displacement x of the bob from its mean position as indicated by the minus sign, and the magnitude of the force is proportional to the magnitude of the displacement. Hence, it follows that the motion of a simple pendulum is linear SHM.

Acceleration, a = Fm = –gLx … (3)

Therefore, acceleration per unit displacement

= |ax| = gL ….. (4)

Period of SHM,

$$T = \frac{2\pi}{\sqrt{\text{acceleration per unit displacement}}}$$

$$= \frac{2\pi}{\sqrt{g/L}} \qquad \therefore T = 2\pi\sqrt{\frac{L}{g}} \qquad \text{... (5)}$$

This gives the expression for the period of a simple pendulum.

iv) State the laws of simple pendulum.

Answer:

The period of a simple pendulum at a given place is

$T = 2\pi \sqrt{\dfrac{L}{g}}$

where L is the length of the simple pendulum and g is the acceleration due to gravity at that place. From the above expression, the laws of simple pendulum are as follows :

(1) Law of length : The period of a simple pendulum at a given place (g constant) is directly proportional to the square root of its length.

$\therefore T \propto \sqrt{L}$

(2) Law of acceleration due to gravity : The period of a simple pendulum of a given length (L constant) is inversely proportional to the square root of the acceleration due to gravity.

$\therefore T \propto \sqrt{\dfrac{1}{g}}$

(3) Law of mass : The period of a simple pendulum does not depend on the mass or material of the bob of the pendulum.

(4) Law of isochronism : The period of a simple pendulum does not depend on the amplitude of oscillations, provided that the amplitude is small.

v) Prove that under certain conditions a magnet vibrating in uniform magnetic field performs angular S.H.M.

Answer:

Consider a bar magnet of magnetic moment μ, suspended horizontally by a light twistless fibre in a region where the horizontal component of the Earth's magnetic field is B_h. The bar magnet is free to rotate in a horizontal plane. It comes to rest in approximately the North-South direction, along B_h. If it is rotated in the horizontal plane by a small
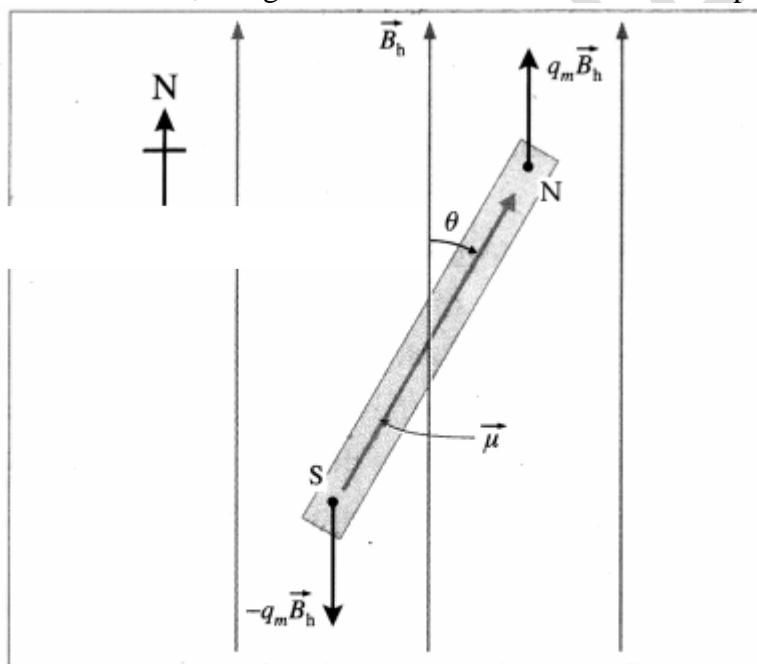
Restoring torque on a bar magnet angular by the horizontal component of the Earth's magnetic field

displacement θ from its rest position ($\theta = 0$), the suspension fibre is twisted. When the magnet is released, it oscillates about the rest position in angular or torsional oscillation.

The bar magnet experiences a torque τ due to the field B_h. Which tends to restore it to its original orientation parallel to B_h. For small θ, this restoring torque is

$\tau = - \mu B_h \sin \theta = - \mu B_h \mu$ …. (1)

where the minus sign indicates that the torque is opposite in direction to the angular displacement θ. Equation (1) shows that the torque (and hence the angular acceleration) is directly proportional in magnitude of the angular displacement but opposite in direction. Hence, for small angular displacement, the oscillations of the bar magnet in a uniform magnetic field is simple harmonic.

Question 3.
Obtain the expression for the period of a magnet vibrating in a uniform magnetic field and performing S.H.M.
Answer:
Definition : Angular SHM is defined as the oscillatory motion of a body in which the restoring torque responsible for angular acceleration is directly proportional to the angular displacement and its direction is opposite to that of angular displacement.
The differential equation of angular SHM is
$I d_2\theta dt_2 + c\theta = 0$ ….. (1)
where I = moment of inertia of the oscillating body, $d_2\theta dt_2$ = angular acceleration of the body when its angular displacement is θ, and c = torsion constant of the suspension wire,
∴ $d_2\theta dt_2 + cI\theta = 0$
Let $cI = \omega^2$, a constant. Therefore, the angular frequency, $\omega = \sqrt{c/I}$ and the angular acceleration,
$a = d_2\theta dt_2 = -\omega^2\theta$ … (2)
The minus sign shows that the α and θ have opposite directions. The period T of angular SHM is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{C/I}} = 2\pi\sqrt{\frac{I}{c}} \qquad \text{... (3)}$$

This is the expression for the period in terms of torque constant. Also, from Eq. (2),

$$\omega = \sqrt{\left|\frac{\alpha}{\theta}\right|}$$

$$= \sqrt{\text{angular acceleration per unit angular displacement}}$$

$$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{|\alpha/\theta|}}$$

$$= \frac{2\pi}{\sqrt{\text{angular acceleration per unit angular displacement}}}$$

Question 4.
Show that a linear S.H.M. is the projection of a U.C.M. along any of its diameter.
Answer:
Consider a particle which moves anticlockwise around a circular path of radius A with a constant angular speed ω. Let the path lie in the x-y plane with the centre at the origin O. The instantaneous position P of the particle is called the reference point and the circle in which the particle moves as the reference circle.
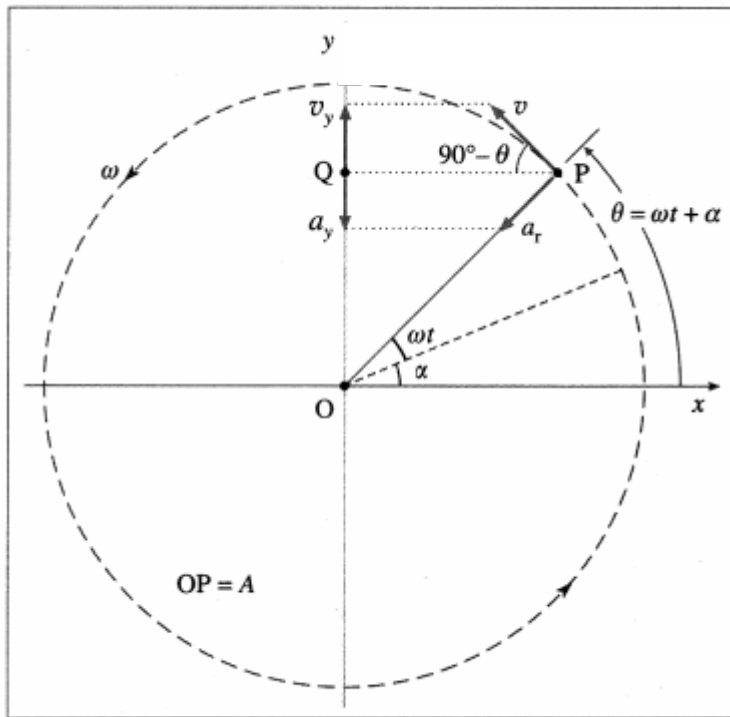
SHM as projection of UCM on a diameter

The perpendicular projection of P onto the y-axis is Q. Then, as the particle travels around the circle, Q moves to-and-fro along the y-axis. Line OP makes an angle α with the x-axis at t = 0. At time t, this angle becomes θ = ωt + α.

The projection Q of the reference point is described by the y-coordinate,

y = OQ = OP sin ∠OPQ, Since ∠OPQ = ωt + α, y = A sin(ωt + α)

which is the equation of a linear SHM of amplitude A. The angular frequency w of a linear SHM can thus be understood as the angular velocity of the reference particle.

The tangential velocity of the reference particle is v = ωA. Its y-component at time t is v_y = ωA sin (90° − θ) = ωA cos θ

∴ v_y = ωA cos (ωt + α)

The centripetal acceleration of the reference particle is a_r = $\omega^2 A$, so that its y-component at time t is a_x = a_r sin ∠OPQ

∴ a_x = − $\omega^2 A$ sin (ωt + α)

Question 5.
Draw graphs of displacement, velocity and acceleration against phase angle, for a particle performing linear S.H.M. from (a) the mean position
(b) the positive extreme position. Deduce your conclusions from the graph.
Answer:
Consider a particle performing SHM, with amplitude A and period T = 2π/ω starting from the mean position towards the positive extreme position where ω is the angular frequency. Its displacement from the mean position (x), velocity (v) and acceleration (a) at any instant are
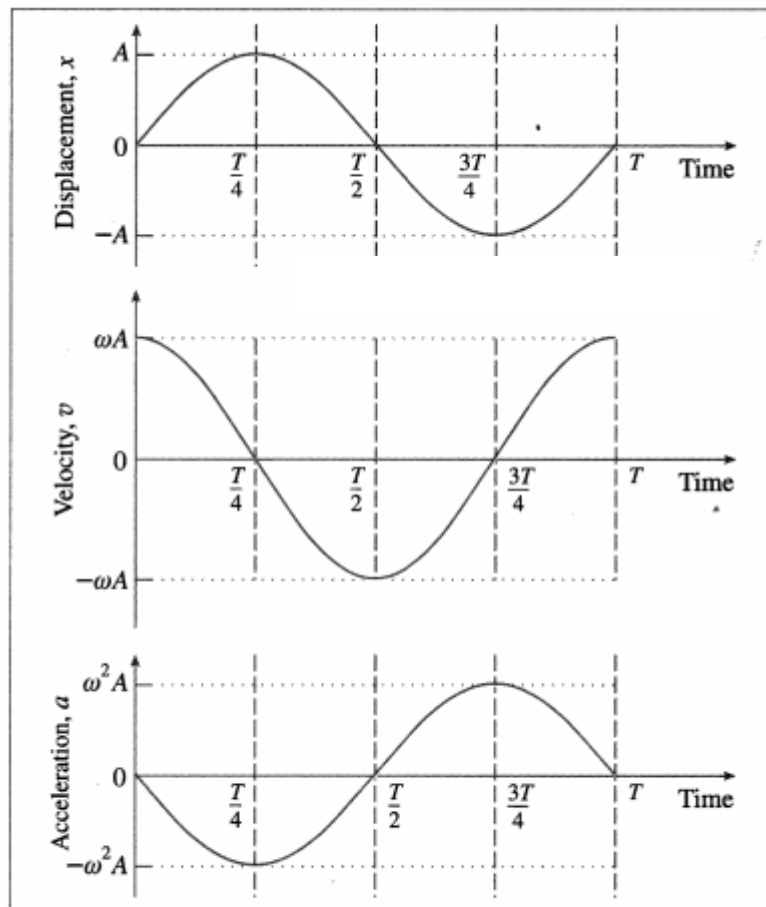
x = A sin ωt = A sin$\left(\frac{2\pi}{T}t\right)$ (∵ω = 2πT)

$v = dxdt = \omega A \cos \omega t = \omega A \cos\left(2\pi Tt\right)$

$a = dvdt = -\omega^2 A \sin \omega t = -\omega^2 A \sin\left(2\pi Tt\right)$ as the initial phase $\alpha = 0$.

Using these expressions, the values of x, v and a at the end of every quarter of a period, starting from t = 0, are tabulated below.

t	0	$T/4$	$T/2$	$3T/4$	T
ωt	0	$\pi/2$	π	$3\pi/2$	2π
x	0	A	0	$-A$	0
v	ωA	0	$-\omega A$	0	ωA
a	0	$-\omega^2 A$	0	$\omega^2 A$	0

Using the values in the table we can plot graphs of displacement, velocity and acceleration with time.

**Graphs of displacement, velocity and
acceleration with time for a particle in SHM
starting from the mean position**

Conclusions :
1. The displacement, velocity and acceleration of a particle performing linear SHM are periodic (harmonic) functions of time. For a particle starting at the mean position, the x-t and a-t graphs are sine curves. The v-t graph is a cosine curve.
2. There is a phase difference of π2 radians between x and v, and between v and a.
3. There is a phase difference of π radians between x and a.

Consider a particle performing linear SHM with amplitude A and period $T = 2\pi/\omega$, starting from the positive extreme position, where ω is the angular frequency. Its displacement from the mean position (x), velocity (y) and acceleration (a) at any instant (t) are

$x = A \cos \omega t = A \cos \left(2\pi T t\right)$ ($\because \omega = 2\pi T$)
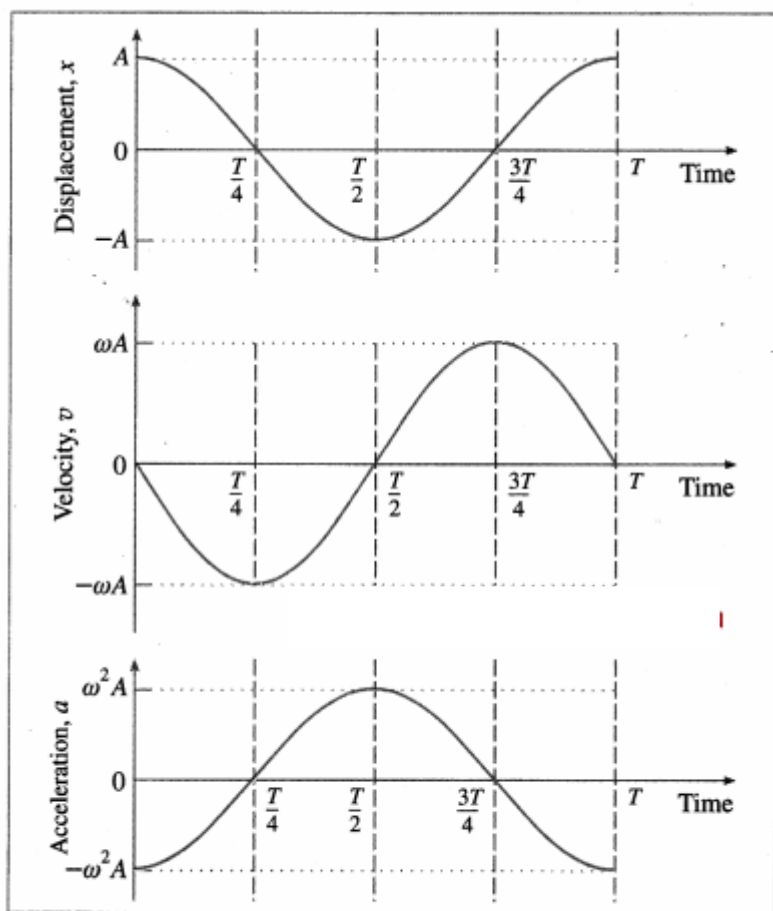
$v = -\omega A \sin \omega t = -\omega A \sin \left(2\pi T t\right)$

$a = -\omega^2 A \cos \omega t = -\omega^2 A \cos \left(2\pi T t\right)$

Using these expressions, the values of x, y and a at the end of every quarter of a period, starting from t = 0, are tabulated below.

t	0	$T/4$	$T/2$	$3T/4$	T
ωt	0	$\pi/2$	π	$3\pi/2$	2π
x	A	0	$-A$	0	A
v	0	$-\omega A$	0	ωA	0
a	$-\omega^2 A$	0	$\omega^2 A$	0	$-\omega^2 A$

Using these values, we can plot graphs showing the variation of displacement, velocity and acceleration with time.

**Variation of displacement, velocity
and acceleration with time for a particle in SHM
starting from the positive extremity**

Conclusions :

1. The displacement, velocity and acceleration of a particle performing linear SHM are periodic (harmonic) functions of time. For a particle starting from an extreme position, the x-t and a-t graphs are cosine curves; the v-t graph is a sine curve.

2. There is a phase difference of π2 radians between x and v, and between v and a.

3. There is a phase difference of n radians between x and a.

Explanations :

(1) v-t graph : It is a sine curve, i.e., the velocity is a periodic (harmonic) function of time which repeats after a phase of 2π rad. There is a phase difference of $\pi/2$ rad between a and v. v is minimum (equal to zero) at the extreme positions (i.e., at $x = \pm A$) and v is maximum (= $\pm \omega A$) at the mean position ($x = 0$).

(2) a-t graph : It is a cosine curve, i.e., the acceleration is a periodic (harmonic) function of time which repeats after a phase of 2π rad. There is a phase difference of π rad between v and a. a is minimum (equal to zero) at the mean position ($x = 0$) and a is maximum (= $\mp\omega^2 A$) at

the extreme positions (x = ±A).
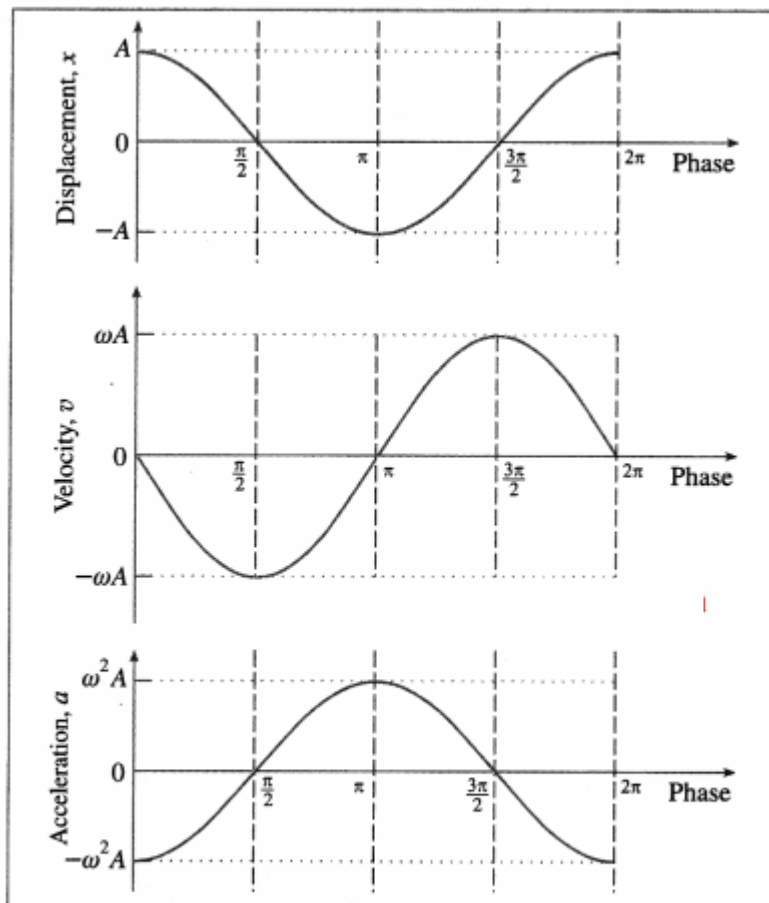
**Graphs of displacement, velocity and
acceleration with phase for a particle in SHM
starting from the positive extremity**

Question 6.
Deduce the expressions for the kinetic energy and potential energy of a particle executing
S.H.M. Hence obtain the expression for total energy of a particle performing S.H.M and
show that the total energy is conserved. State the factors on which total energy depends.
Answer:
Consider a particle of mass m performing linear SHM with amplitude A. The restoring force
acting r on the particle is F = – kx, where k is the force constant and x is the displacement of
the particle from its mean position.
(1) Kinetic energy : At distance x from the mean position, the velocity is

$v = \omega A2-x2-------\sqrt{}$

where $\omega = k/m----\sqrt{}.$ The kinetic energy (KE) of the particle is
KE = 12 mv² = 12 mω² (A² – x²)
= 12k(A² – x²) … (1)
If the phase of the particle at an instant t is θ = ωt + α, where α is initial phase, its velocity at
that instant is
v = ωA cos (ωt + α)
and its KE at that instant is

$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2 \cos^2(\omega t + \alpha)$ ….. (2)

Therefore, the KE varies with time as $\cos^2 \theta$.

(2) Potential energy : The potential energy of a particle in linear SHM is defined as the work done by an external agent, against the restoring force, in taking the particle from its mean position to a given point in the path, keeping the particle in equilibirum.

Suppose the particle in below figure is displaced from P_1 to P_2, through an infinitesimal distance dx against the restoring force F as shown.
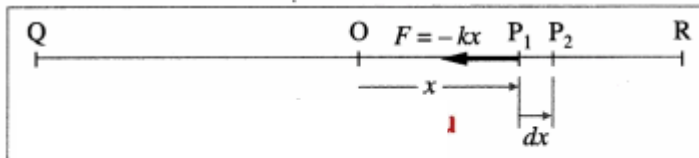
Potential energy of a particle in SHM

The corresponding work done by the external agent will be dW = (– F)dx = kx dx. This work done is stored in the form of potential energy. The potential energy (PE) of the particle when its displacement from the mean position is x can be found by integrating the above expression from 0 to x.

$\therefore PE = \int dW = \int_0^x kx\, dx = \frac{1}{2}kx^2$ … (3)

The displacement of the particle at an instant t being

$x = A \sin(wt + \alpha)$

its PE at that instant is

$PE = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \sin^2(\omega t + \alpha)$ … (4)

Therefore, the PE varies with time as $\sin^2\theta$.

(3) Total energy : The total energy of the particle is equal to the sum of its potential energy and kinetic energy.

From Eqs. (1) and (2), total energy is E = PE + KE

$= \frac{1}{2}kx^2 + \frac{1}{2}k(A^2 - x^2)$

$= \frac{1}{2}kx^2 + \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$\therefore E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2A^2$ … (5)

As m is constant, ω and A are constants of the motion, the total energy of the particle remains constant (or its conserved).

Question 7.
Deduce the expression for period of simple pendulum. Hence state the factors on which its period depends.
Answer:
An ideal simple pendulum is defined as a heavy point mass suspended from a rigid support by a weightless, inextensible and twistless string, and set oscillating under gravity through a small angle in a vertical plane.
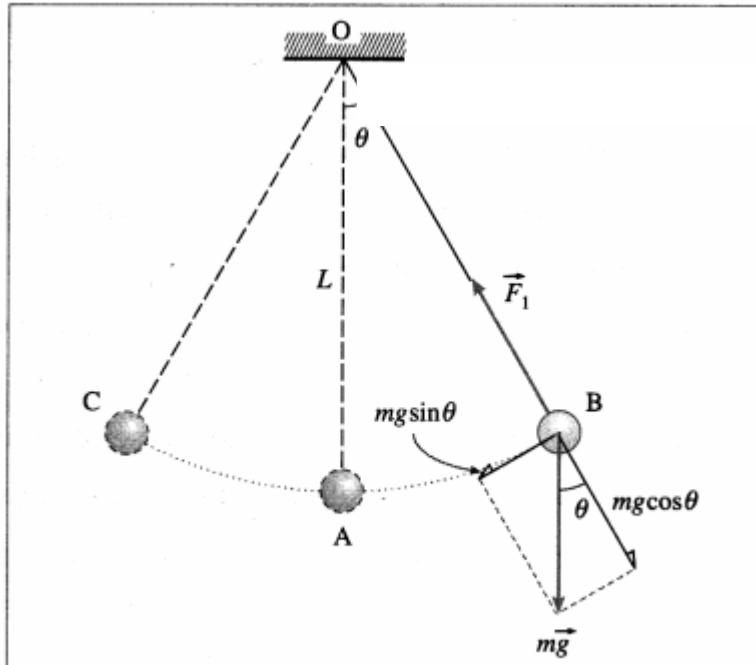
In practice, a small but heavy sphere, called the bob, is used. The distance from the point of suspension to the centre of gravity of the bob is called the length of the pendulum.

Consider a simple pendulum of length L_1 – suspended from a rigid support O. When displaced from its initial position of rest through a small angle θ in a vertical plane and

released, it performs oscillations between two extremes, B and C, as shown in below figure.
At B, the forces on the bob are its weight $\overrightarrow{mg}$ and the tension $F_1{-}{\rightarrow}$ in the string.

Resolve $\overrightarrow{mg}$ into two components : mg cos θ in the direction opposite to that of the tension and mg sin θ perpendicular to the string.

Motion of a simple pendulum

mg cos θ balanced by the tension in the string. mg sin θ restores the bob to the equilibrium position.

Restoring force, F = – mg sin θ

If θ is small and expressed in radian,

sin θ ≈ θ = arc radius = ABOB = xL

∴ F = – mgθ = -mgxL …. (1)

Since m, g and L are constant,

F ∝ (-x) …. (2)

Thus, the net force on the bob is in the direction opposite to that of displacement x of the bob from its mean position as indicated by the minus sign, and the magnitude of the force is proportional to the magnitude of the displacement. Hence, it follows that the motion of a simple pendulum is linear SHM.

Acceleration, a = Fm = –gLx … (3)

Therefore, acceleration per unit displacement

= |ax| = gL ….. (4)

Period of SHM,

$$T = \frac{2\pi}{\sqrt{\text{acceleration per unit displacement}}}$$

$$= \frac{2\pi}{\sqrt{g/L}} \qquad \therefore \ T = 2\pi\sqrt{\frac{L}{g}} \qquad \qquad \dots (5)$$

This gives the expression for the period of a simple pendulum.

The period of a simple pendulum at a given place is

$T = 2\pi\sqrt{\dfrac{L}{g}}$

where L is the length of the simple pendulum and g is the acceleration due to gravity at that place. From the above expression, the laws of simple pendulum are as follows :

(1) Law of length : The period of a simple pendulum at a given place (g constant) is directly proportional to the square root of its length.

$\therefore T \propto \sqrt{L}$

(2) Law of acceleration due to gravity : The period of a simple pendulum of a given length (L constant) is inversely proportional to the square root of the acceleration due to gravity.

$\therefore T \propto \dfrac{1}{\sqrt{g}}$

Question 8.

At what distance from the mean position is the speed of a particle performing S.H.M. half its maximum speed. Given path length of S.H.M. = 10 cm. [Ans: 4.33 cm]

Answer:

Data : $v = \frac{1}{2}v_{max}$, 2A = 10 cm

$\therefore A = 5$ cm

$v = \omega\sqrt{A^2 - x^2}$ and $v_{max} = \omega A$

Since $v = \frac{1}{2}v_{max}$,

$$\omega\sqrt{A^2 - x^2} = \frac{\omega A}{2}$$

$$\therefore A^2 - x^2 = \frac{A^2}{4} \qquad \therefore x^2 = A^2 - \frac{A^2}{4} = \frac{3A^2}{4}$$

$$\therefore x = \pm\frac{\sqrt{3}}{2}A = \pm 0.866 \times 5 = \pm\,\mathbf{4.33\ cm}$$

This gives the required displacement.

Question 9.

In SI units, the differential equation of an S.H.M. is $\frac{d^2x}{dt^2} = -36x$. Find its frequency and period. Find its frequency and period.

[Ans: 0.9548 Hz, 1.047 s]

Answer:

$\frac{d^2x}{dt^2} = -36x$

Comparing this equation with the general equation,

$\frac{d^2x}{dt^2} = -\omega^2 x$

We get, $\omega^2 = 36 \therefore \omega = 6$ rad/s

$\omega = 2\pi f$

$\therefore$ The frequency, $f = \frac{\omega}{2\pi} = \frac{6}{2(3.142)} = \frac{6}{6.284} = 0.9548$ Hz

and the period, $T = \frac{1}{f} = \frac{1}{0.9548} = 1.047$ s

Question 10.

A needle of a sewing machine moves along a path of amplitude 4 cm with frequency 5 Hz. Find its acceleration $\frac{1}{30}$s after it has crossed the mean position. [Ans: 34.2 m/s²]

Answer:

Data : A = 4 cm = 4×10^{-2} m, f = 5Hz, t = $\frac{1}{30}$s

$\omega = 2\pi f = 2\pi (5) = 10\pi$ rad/s

Therefore, the magnitude of the acceleration,

$|a| = \omega^2 x = \omega^2 A \sin \omega t$

$= (10\pi)^2 (4 \times 10^2)$

$= 10\pi^2 \sin \pi 3 = 10 (9.872)(0.866) = 34.20$ m/s²

Question 11.

Potential energy of a particle performing linear S.H.M is $0.1 \pi^2 x^2$ joule. If mass of the particle is 20 g, find the frequency of S.H.M. [Ans: 1.581 Hz]

Answer:

Data : PE $= 0.1 \pi^2 x^2$ J, m $= 20$ g $= 2 \times 10^{-2}$ kg

PE $= \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m (4\pi^2f^2)x^2$

$\therefore \frac{1}{2}m(4\pi^2f^2)x^2 = 0.1 \pi^2 x^2$

$\therefore 2mf^2 = 0.1 \therefore f^2 = 120(2\times10^{-2}) = 2.5$

$\therefore$ The frequency of SHM is f $= \sqrt{2.5} = 1.581$ Hz

Question 12.

The total energy of a body of mass 2 kg performing S.H.M. is 40 J. Find its speed while crossing the centre of the path. [Ans: 6.324 cm/s]

Answer:

Data : m $= 2$ kg, E $= 40$ J

The speed of the body while crossing the centre of the path (mean position) is v_{max} and the total energy is entirely kinetic energy.

$$\therefore \frac{1}{2} mv^2_{max} = E$$

$$\therefore v_{max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 40}{2}} = \textbf{6.324 m/s}$$

Question 13.

A simple pendulum performs S.H.M of period 4 seconds. How much time after crossing the mean position, will the displacement of the bob be one third of its amplitude. [Ans: 0.2163 s]

Answer:

Data : T $= 4$ s, x $= A/3$

The displacement of a particle starting into SHM from the mean position is x $= A \sin \omega t = A \sin \frac{2\pi}{T} t$

$$\therefore A \sin \frac{2\pi}{T} t = \frac{A}{3}$$

$$\therefore \frac{2\pi}{T} t = \sin^{-1} 0.3333 = 19.47° = 19.47 \times \frac{\pi}{180} \text{ rad}$$

$$\therefore \frac{2}{4} t = \frac{19.47}{180} \qquad \therefore t = \frac{19.47}{90} = 0.2163 s$$

$\therefore$ the displacement of the bob will be one-third of its amplitude 0.2163 s after crossing the mean position.

Question 14.

A simple pendulum of length 100 cm performs S.H.M. Find the restoring force acting on its bob of mass 50 g when the displacement from the mean position is 3 cm. [Ans: 1.48×10^{-2} N]

Answer:

Data : L $= 100$ cm, m $= 50$ g $= 5 \times 10^{-2}$ kg, x $= 3$ cm, g $= 9.8$ m/s²

Restoring force, F $= mg \sin \theta = mg\theta$

$= (5 \times 10^{-2})(9.8)(3100)$

$= 1.47 \times 10^{-2}$ N

Question 15.

Find the change in length of a second's pendulum, if the acceleration due to gravity at the place changes from 9.75

m/s^2 to 9.80 m/s^2. [Ans: Decreases by 5.07 mm]

Answer:

Data : $g_1 = 9.75$ m/s^2, $g_2 = 9.8$ m/s^2

Length of a seconds pendulum, $L = \frac{g}{\pi^2}$

$$\therefore L_1 = \frac{g_1}{\pi^2} = \frac{9.75}{9.872} = 0.9876 \text{ m}$$

$$\text{and } L_2 = \frac{g_2}{\pi^2} = \frac{9.8}{9.872} = 0.9927 \text{ m}$$

∴ The length of the seconds pendulum must be increased from 0.9876 m to 0.9927 m, i.e., by 0.0051 m.

Question 16.

At what distance from the mean position is the kinetic energy of a particle performing S.H.M. of amplitude 8 cm, three times its potential energy? [Ans: 4 cm]

Answer:

Data : $A = 8$ cm, KE = 3 PE

KE = $\frac{1}{2}$ ($A^2 - x^2$) and PE = $\frac{1}{2}kx^2$

Given, KE = 3PE.

$\therefore \frac{1}{2}k(A^2 - x^2) = 3(\frac{1}{2}kx^2)$

$\therefore A^2 - x^2 = 3x^2 \therefore 4x^2 = A^2$

∴ the required displacement is

$x = \pm\frac{A}{2} = \pm\frac{8}{2} = \pm 4$ cm

Question 17.

A particle performing linear S.H.M. of period 2π seconds about the mean position O is observed to have a speed of $b\sqrt{3}$ m/s, when at a distance b (metre) from O. If the particle is moving away from O at that instant, find the

time required by the particle, to travel a further distance b. [Ans: $\pi/3$ s]

Answer:

Data : $T = 2\pi$s, $v = b\sqrt{3}$ m/s at $x = b$

$$\therefore \omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1 \text{ rad/s}$$

$$v = \omega\sqrt{A^2 - x^2}$$

$$\therefore \text{At } x = b, \quad b\sqrt{3} = (1)\sqrt{A^2 - b^2}$$

$$\therefore 3b^2 = A^2 - b^2 \qquad \therefore A = 2b$$

∴ Assuming the particle starts from the mean position, its displacement is given by

$x = A \sin \omega t = 2b \sin t$

If the particle is at $x = b$ at $t = t_1$,

$b = 2b \sin t_1$ ∴ $t_1 = \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$ s

Also, with period $T = 2\pi$ s, on travelling a further distance b the particle will reach the positive extremity at time $t_2 = \frac{\pi}{2}$ s.

∴ The time taken to travel a further distance b from $x = b$ is $t_2 - t_1 = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$ s.

Question 18.

The period of oscillation of a body of mass m_1 suspended from a light spring is T. When a body of mass m_2 is tied to the first body and the system is made to oscillate, the period is 2T. Compare the masses m_1 and m_2 [Ans: 1/3]

Answer:

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \therefore \frac{2T}{T} = 2 = \sqrt{\frac{m_1 + m_2}{m_1}}$$

$$\therefore \frac{m_1 + m_2}{m_1} = 4 \qquad \therefore \frac{m_2}{m_1} = \frac{3}{1} \qquad \therefore \frac{m_1}{m_2} = \frac{1}{3}$$

This gives the required ratio of the masses.

Question 19.

The displacement of an oscillating particle is given by $x = a\sin\omega t + b\cos\omega t$ where a, b and ω are constants. Prove that the particle performs a linear S.H.M. with amplitude A

$= \sqrt{a^2 + b^2}$

Answer:

$x = a\sin\omega t + b\cos\omega t$

Let $a = A\cos\varphi$ and $b = A\sin\varphi$, so that

$A^2 = a^2 + b^2$ and $\tan\varphi = \frac{b}{a}$

∴ $x = A\cos\varphi \sin\omega t + A\sin\varphi \cos\omega t$

∴ $x = A\sin(\omega t + \varphi)$

which is the equation of a linear SHM with amplitude $A = \sqrt{a^2 + b^2}$ and phase constant $\varphi = \tan^{-1}\frac{b}{a}$, as required.

Question 20.

Two parallel S.H.M.s represented by $x_1 = 5\sin(4\pi t + \frac{\pi}{3})$ cm and $x_2 = 3\sin(4\pi t + \pi/4)$ cm are superposed on a particle. Determine the amplitude and epoch of the resultant S.H.M. [Ans: 7.936 cm, 54° 23′]

Answer:

Data: $x_1 = 5\sin(4\pi t + \frac{\pi}{3}) = A_1 \sin(\omega t + \alpha)$,

$x_2 = 3\sin(4\pi t + \frac{\pi}{4}) = A_2 \sin(\omega t + \beta)$

∴ $A_1 = 5$ cm, $A_2 = 3$ cm, $\alpha = \frac{\pi}{3}$ rad, $\beta = \frac{\pi}{4}$ rad

(i) Resultant amplitude,

$$R = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\alpha - \beta)}$$

$$= \sqrt{(5)^2 + (3)^2 + 2(5)(3) \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)}$$

$$= \sqrt{25 + 9 + 30 \cos\frac{\pi}{12}} = \sqrt{34 + 30(0.9659)}$$

$$= \sqrt{34 + 28.98} = \sqrt{62.98} = 7.936 \text{ cm}$$

(ii) Epoch of the resultant SHM,

$$\delta = \tan^{-1} \frac{A_1 \sin \alpha + A_2 \sin \beta}{A_1 \cos \alpha + A_2 \cos \beta}$$

$$= \tan^{-1} \frac{5 \sin (\pi/3) + 3 \sin (\pi/4)}{5 \cos (\pi/3) + 3 \cos (\pi/4)}$$

$$= \tan^{-1} \frac{5 (0.866) + 3 (0.7071)}{5 (0.5) + 3 (0.7071)}$$

$$= \tan^{-1} \frac{4.33 + 2.1213}{2.5 + 2.1213} = \tan^{-1} 1.396 = \mathbf{54°23'}$$

Question 21.
A 20 cm wide thin circular disc of mass 200 g is suspended to a rigid support from a thin metallic string. By holding the rim of the disc, the string is twisted through 60° and released. It now performs angular oscillations of period 1 second. Calculate the maximum restoring torque generated in the string under undamped conditions. ($\pi^3 \approx 31$)
[Ans: 0.04133 N m]
Answer:
Data: R = 10cm = 0.1 m, M = 0.2 kg, θ_m = 60° = $\frac{\pi}{3}$ rad, T = 1 s, $\pi^3 \approx 31$
The Ml of the disc about the rotation axis (perperdicular through its centre) is
I = $\frac{1}{2}MR^2$ = $(0.2)(0.1)^2$ = 10^{-3} kg.m^2

The period of torsional oscillation, T = $2\pi\sqrt{\frac{I}{c}}$

∴ The torsion constant, c = $\frac{4\pi^2 I}{T^2}$
The magnitude of the maximum restoring torque,

$$\tau_{max} = c\theta_m = \left(4\pi^2 \frac{I}{T^2}\right)\left(\frac{\pi}{3}\right)$$

$$= \frac{4}{3}\pi^3 \frac{I}{T^2} = \frac{4}{3}(31)\left(\frac{10^{-3}}{1^2}\right)$$

$$= 41.33 \times 10^{-3} = \mathbf{0.04133 \ N \cdot m}$$

Question 22.
Find the number of oscillations performed per minute by a magnet is vibrating in the plane of a uniform field of 1.6×10^{-5} Wb/m^2. The magnet has moment of inertia 3×10^{-6} kgm^2 and magnetic moment 3 A m^2. [Ans:38.19 osc/min.]
Answer:
Data : B = 1.6×10^{-5} T, I = 3×10^{-6} kg/m^2,
μ = 3 A.m^2

The period of oscillation, T = $2\pi\sqrt{\frac{I}{\mu B_h}}$

∴ The frequency of oscillation is

f = $\frac{1}{2\pi}\sqrt{\frac{\mu B}{I}}$

∴ The number of oscillations per minute

$$= 60f = \frac{60}{2\pi} \sqrt{\frac{3(1.6 \times 10^{-5})}{3 \times 10^{-6}}} = \frac{60}{2\pi} \sqrt{16} = \frac{120}{3.142}$$

= 38.19 per minute

Question 23.
A wooden block of mass m is kept on a piston that can perform vertical vibrations of adjustable frequency and amplitude. During vibrations, we don't want the block to leave the contact with the piston. How much maximum frequency is possible if the amplitude of vibrations is restricted to 25 cm? In this case, how much is the energy per unit mass of the block? ($g \approx \pi^2 \approx 10$ m s^{-2})
[Ans: $n_{max} = 1/s$, E/m = 1.25 J/kg]
Answer:
Data : A = 0.25 m, g = π^2 = 10 m/s^2
During vertical oscillations, the acceleration is maximum at the turning points at the top and bottom. The block will just lose contact with the piston when its apparent weight is zero at the top, i. e., when its acceleration is $a_{max} = g$, downwards.

$$|a_{max}| = \omega^2 A = 4\pi^2 f^2_{max} A$$

$$\therefore 4\pi^2 f^2_{max} A = g$$

$$\therefore f_{max} = \sqrt{\frac{g}{\pi^2} \cdot \frac{1}{4A}} = \sqrt{\frac{10}{10} \cdot \frac{1}{4(0.25)}} = 1 \text{ Hz}$$

This gives the required frequency of the piston.

$$E = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} m (4\pi^2 f^2) A^2$$

$$\therefore \frac{E}{m} = 2\pi^2 f^2 A^2 = 2(10)(1)^2 \left(\frac{1}{4}\right)$$

$$= \frac{20}{16} = \frac{5}{4} = 1.25 \text{ J/kg}$$

12th Physics Digest Chapter 5 Oscillations of Waves Intext Questions and Answers

Can you tell? (Textbook Page No. 112)

Question 1.
Why is the term angular frequency (ω) used here for a linear motion ?
Answer:
A linear SHM is the projection of a UCM on a diameter of the circle. The angular speed co of a particle moving along this reference circle is called the angular frequency of the particle executing linear SHM.

Can you tell? (Textbook Page No. 114)

Question 1.
State at which point during an oscillation the oscillator has zero velocity but positive acceleration ?
Answer:
At the left extreme, i.e., $x = - A$, so that $a = - \omega^2 x = - \omega^2(- A) = \omega^2 A = a_{max}$

Question 2.
During which part of the simple harmonic motion velocity is positive but the displacement is negative, and vice versa ?
Answer:
Velocity v is positive (to the right) while displacement x is negative when the particle in SHM is moving from the left extreme towards the mean position. Velocity v is negative (to the left) while displacement x is positive when the particle in SHM is moving from the right extreme towards the mean position.

Can you tell? (Textbook page 76)

Question 1.
To start a pendulum swinging, usually you pull it slightly to one side and release it. What kind of energy is transferred to the mass in doing this?
Answer:
On pulling the bob of a simple pendulum slightly to one side, it is raised to a slightly higher position. Thus, it gains gravitational potential energy.

Question 2.
Describe the energy changes that occur when the mass is released.
Answer:
When released, the bob oscillates in SHM in a vertical plane and the energy oscillates back and forth between kinetic and potential, going completely from one form of energy to the other as the pendulum oscillates. In the case of undamped SHM, the motion starts with all of the energy as gravitational potential energy. As the object starts to move, the gravitational potential energy is converted into kinetic energy, becoming entirely kinetic energy at the equilibrium position. The velocity becomes zero at the other extreme as the kinetic energy is completely converted back into gravitational potential energy, and this cycle then repeats.

Question 3.
Is/are there any other way/ways to start the oscillations of a pendulum? Which energy is supplied in this case/cases?
Answer:
The bob can be given a kinetic energy at its equilibrium position or at any other position of its path. In the first case, the motion starts with all of the energy as kinetic energy. In the second case, the motion starts with partly gravitational potential energy and partly kinetic energy.

Can you tell? (Textbook Page No. 109)

Question 1.
Is the motion of a leaf of a tree blowing in the wind periodic ?
Answer:
The leaf of a tree blowing in the wind oscillates, but the motion is not periodic. Also, its displacement from the equilibrium position is not a regular function of time.