

Maharashtra State Board 12th Commerce Maths Solutions Chapter 3 Linear Regression Ex 3.1

Question 1.

The HRD manager of the company wants to find a measure which he can use to fix the monthly income of persons applying for the job in the production department. As an experimental project. He collected data of 7 persons from that department referring to years of service and their monthly incomes.

Years of service (X)	11	7	9	5	8	6	10
Monthly Income (Rs.1000's) (Y)	10	8	6	5	9	7	11

(i) Find the regression equation of income on years of service.

(ii) What initial start would you recommend for a person applying for the job after having served in a similar capacity in another company for 13 years?

Solution:

x	y	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$
11	10	3	2	6	9
7	8	-1	0	0	1
9	6	1	-2	-2	1
5	5	-3	-3	9	9
8	9	0	1	0	0
6	7	-2	-1	2	4
10	11	2	3	6	4
Total	56	56		21	28

Given, $n = 7$

$$\therefore \bar{x} = \frac{\sum x_i}{n} = \frac{56}{7} = 8$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{56}{7} = 8$$

$$b_{yx} = \frac{\text{Cov}(X, Y)}{\sigma_x^2} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

$$\therefore b_{yx} = \frac{21}{28} = \frac{3}{4} = 0.75$$

(i) Regression equation of Y on X is $(Y - \bar{y}) = b_{yx}(x - \bar{x})$

$$(Y - 8) = 0.75(x - 8)$$

$$Y = 0.75x + 2$$

(ii) When $x = 13$

$$Y = 0.75(13) + 2 = 11.75$$

Recommended income for the person is ₹ 11750.

Question 2.

Calculate the regression equations of X on Y and Y on X from the following data:

X	10	12	13	17	18
Y	5	6	7	9	13

Solution:

x	y	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$
10	5	-4	-3	12	16	9
12	6	-2	-2	4	4	4
13	7	-1	-1	1	1	1
17	9	3	1	3	9	1
18	13	4	5	20	16	25
Total	70	40		40	46	40

Given, $n = 5$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{70}{5} = 14$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{40}{5} = 8$$

$$b_{xy} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (y - \bar{y})^2}$$

$$= \frac{40}{40} = 1$$

$$b_{yx} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

$$= \frac{40}{46} = 0.87$$

Regression equation of X on Y is $(X - \bar{x}) = b_{xy} (Y - \bar{y})$

$$(X - 14) = 1(Y - 8)$$

$$X - 14 = Y - 8$$

$$X = Y + 6$$

Regression equation Y on X is $(Y - \bar{y}) = b_{yx} (X - \bar{x})$

$$(Y - 8) = 0.87(X - 14)$$

$$Y - 8 = 0.87X - 12.18$$

$$Y = 0.87X - 4.18$$

Question 3.

For a certain bivariate data on 5 pairs of observations given

$$\sum x = 20, \sum y = 20, \sum x^2 = 90, \sum y^2 = 90, \sum xy = 76$$

Calculate (i) $\text{cov}(x, y)$, (ii) b_{yx} and b_{xy} , (iii) r

Solution:

$$\text{Cov}(x, y) = \frac{\sum (x \cdot y)}{n} - \bar{x} \cdot \bar{y}$$

$$= \frac{\sum (x \cdot y)}{n} - \left(\frac{\sum x}{n} \right) \left(\frac{\sum y}{n} \right)$$

$$= \frac{76}{5} - \left(\frac{20}{5} \right) \left(\frac{20}{5} \right)$$

$$= 15.2 - 16$$

$$= -0.8$$

$$b_{yx} = \frac{n \sum (x \cdot y) - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$$

$$= \frac{5(76) - (20)(20)}{5(90) - (20)^2} = \frac{380 - 400}{450 - 400} = \frac{-20}{+50} = -0.4$$

$$b_{xy} = \frac{n \sum (x \cdot y) - (\sum x)(\sum y)}{n \sum y^2 - (\sum y)^2}$$

$$= \frac{5(76) - (20)(20)}{5(90) - (20)^2} = \frac{380 - 400}{450 - 400} = \frac{-20}{50} = -0.4$$

$$\text{Now, } r^2 = b_{yx} \cdot b_{xy}$$

$$= (-0.4) (-0.4)$$

$$= 0.16$$

$$\therefore r = \pm 0.4$$

Sine b_{yx} and b_{xy} are negative, $r = -0.4$

Question 4.

From the following data estimate y when x = 125

X	120	115	120	125	126	123
Y	13	15	14	13	12	14

Solution:

Let $u = x - 122$, $v = y - 14$

x	y	u	v	$u \cdot v$	u^2	v^2
120	13	-2	-1	2	4	1
115	15	-7	1	-7	49	1
120	14	-2	0	0	4	0
125	13	3	-1	-3	9	1
126	12	4	-2	-8	16	4
123	14	1	0	0	1	0
Total	729	81	-3	-16	83	7

$$\bar{x} = \frac{\sum x_i}{n} = \frac{729}{6} = 121.5$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{81}{6} = 13.5$$

$$b_{yx} = b_{vu} = \frac{n \sum (u \cdot v) - (\sum u)(\sum v)}{n \sum u^2 - (\sum u)^2}$$

$$= \frac{6(-16) - (-3)(-3)}{6(83) - (-3)^2} = \frac{-96 - 9}{498 - 9} = \frac{-105}{489} = -0.21$$

Regression equation of Y on X is

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$(Y - 13.5) = -0.21(x - 121.5)$$

$$Y - 13.5 = -0.21x + 25.52$$

$$Y = -0.21x + 39.02$$

When $x = 125$

$$Y = -0.21(125) + 39.02$$

$$= -26.25 + 39.02$$

$$= 12.77$$

Question 5.

The following table gives the aptitude test scores and productivity indices of 10 works selected at workers selected randomly.

Obtain the two regression equation and estimate

(i) The productivity index of a worker whose test score is 95.

(ii) The test score when productivity index is 75.

Solution:

x	y	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$
60	68	-5	3	-15	25	9
62	60	-3	-5	15	9	25
65	62	0	-3	0	0	9
70	80	5	15	75	25	225
72	85	7	20	140	49	400
48	40	-17	-25	425	289	625
53	52	-12	-13	156	144	169
73	62	8	-3	-24	64	9
65	60	0	-5	0	0	25
82	81	17	16	272	289	256
Total	650	650		1044	894	1752

$$\bar{x} = \frac{\sum x_i}{n} = \frac{650}{10} = 65$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{650}{10} = 65$$

$$b_{yx} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

$$= \frac{1044}{894} = 1.16$$

$$b_{xy} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (y - \bar{y})^2}$$

$$= \frac{1044}{1752} = 0.59$$

Regression equation of Y on X,

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$(Y - 65) = 1.16 (x - 65)$$

$$Y - 65 = 1.16x - 75.4$$

$$Y = 1.16x - 10.4$$

(i) When $x = 95$

$$Y = 1.16(95) - 10.4$$

$$= 110.2 - 10.4$$

$$= 99.8$$

Regression equation of X on Y,

$$(X - \bar{x}) = b_{xy} (Y - \bar{y})$$

$$(X - 65) = 0.59(y - 65)$$

$$(X - 65) = 0.59y - 38.35$$

$$X = 0.59y + 26.65$$

(ii) When $y = 75$

$$x = 0.59(75) + 26.65$$

$$= 44.25 + 26.65$$

$$= 70.9$$

Question 6.

Compute the appropriate regression equation for the following data.

X [Independent Variable]	2	4	5	6	8	11
Y [Dependent Variable]	18	12	10	8	7	5

Solution:

Since x is the independent variable, and y is the dependent variable, we need to find regression equation of y on x

x	y	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$
2	18	-4	8	-32	16
4	12	-2	2	-4	4
5	10	-1	0	0	1
6	8	0	-2	0	0
8	7	2	-3	-6	4
11	5	5	-5	-25	15
Total	36	60		-67	50

$$\bar{x} = \frac{\sum x_i}{n} = \frac{36}{6} = 6$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{60}{6} = 10$$

$$b_{yx} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} = \frac{-67}{50} = -1.34$$

Regression equation of y on x is $(y - \bar{y}) = b_{yx} (x - \bar{x})$

$$(y - 10) = -13.4(x - 6)$$

$$y - 10 = -1.34x + 8.04$$

$$y = -1.34x + 18.04$$

Question 7.

The following are the marks obtained by the students in Economic (X) and Mathematics (Y)

X	59	60	61	62	63
Y	78	82	82	79	81

Find the regression equation of Y and X.

Solution:

Let $u = x - 61$, $v = y - 80$

	x	y	u	v	$u - v$	u^2
	59	78	-2	-2	4	4
	60	82	-1	2	-2	1
	61	82	0	2	0	0
	62	79	1	-1	-1	1
	63	81	2	1	2	4
Total	305	402	0	2	3	10

$$\bar{x} = \frac{\sum x_i}{n} = \frac{305}{5} = 61$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{402}{5} = 80.4$$

$$b_{yx} = \frac{n \sum (u \cdot v) - (\sum u)(\sum v)}{n \sum u^2 - (\sum u)^2}$$

$$= \frac{5(3) - (0)(2)}{5(10) - (0)^2} = \frac{15}{50} = 0.3$$

Regression equation of Y on X is

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$(Y - 80.4) = 0.3(x - 61)$$

$$Y - 80.4 = 0.3x - 18.3$$

$$Y = 0.3x + 62.1$$

Question 8.

For the following bivariate data obtain the equation of two regressions lines:

X	1	2	3	4	5
Y	5	7	9	11	13

Solution:

x	y	(x - \bar{x})	(y - \bar{y})	(x - \bar{x})(y - \bar{y})	(x - \bar{x})²	(y - \bar{y})²
1	5	-2	-4	8	4	16
2	7	-1	-7	2	1	4
3	9	0	0	0	0	0
4	4	1	2	2	1	4
5	13	2	4	8	4	16
Total	15	45		20	10	40

$$\bar{x} = \frac{\sum x_i}{n} = \frac{15}{5} = 3$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{45}{5} = 9$$

$$b_{yx} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} = \frac{20}{10} = 2$$

$$b_{xy} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (y - \bar{y})^2} = \frac{20}{40} = 0.5$$

Regression equation of Y on X

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$(Y - 9) = 2(x - 3)$$

$$Y - 9 = 2x - 6$$

$$Y = 2x + 3$$

Regression equation of X on Y

$$(X - \bar{x}) = b_{xy} (Y - \bar{y})$$

$$(X - 3) = 0.5(y - 9)$$

$$(X - 3) = 0.5y - 4.5$$

$$X = 0.5y - 1.5$$

Question 9.

Find the following data obtain the equation of two regression lines:

Solution:

$$\bar{x} = \frac{\sum x_i}{n} = \frac{30}{5} = 6$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{40}{5} = 8$$

$$b_{yx} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} = \frac{-26}{40} = -0.65$$

$$b_{xy} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (y - \bar{y})^2} = \frac{-26}{20} = -1.3$$

Regression of Y on X,

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$(Y - 8) = 0.65(x - 6)$$

$$Y - 8 = -0.65x + 3.9$$

$$Y = -0.65x + 11.9$$

Regression of X on Y

$$(X - \bar{x}) = b_{xy} (Y - \bar{y})$$

$$(X - 6) = -1.3(y - 8)$$

$$(X - 6) = -1.3y + 10.4$$

$$X = -1.3y + 16.4$$

Question 10.

For the following data, find the regression line of Y on X

<i>X</i>	1	2	3
<i>Y</i>	2	1	6

Hence find the most likely value of y when x = 4

Solution:

	x	y	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$
	1	2	-1	-1	1	1
	2	1	0	-2	0	0
	3	6	1	3	3	1
Total	6	9			4	2

$$\bar{x} = \frac{\sum x_i}{n} = \frac{6}{3} = 2$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{9}{3} = 3$$

$$b_{yx} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} = \frac{4}{2} = 2$$

Regression equation of Y on X is

$$(Y - \bar{y}) = b_{yx} (X - \bar{x}),$$

$$(Y - 3) = 2(x - 2)$$

$$Y - 3 = 2x - 4$$

$$Y = 2x - 1$$

$$\text{When } x = 4$$

$$Y = 2(4) - 1$$

$$= 8 - 1$$

$$= 7$$

Question 11.

Find the following data, find the regression equation of Y on X, and estimate Y when X = 10.

X	1	2	3	4	5	6
Y	2	4	7	6	5	6

Solution:

	x	y	xy	x^2
	1	2	2	1
	2	4	8	4
	3	7	21	9
	4	6	24	16
	5	5	25	25
	6	6	36	36
Total	21	30	116	91

$$\bar{x} = \frac{\sum x_i}{n} = \frac{21}{6} = 3.5$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{30}{6} = 5$$

$$b_{yx} = \frac{n \sum (x \cdot y) - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$$

$$= \frac{6(116) - (21)(30)}{6(91) - (21)^2}$$

$$= 0.63$$

Regression equation of Y on X is

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$(Y - 5) = (0.63)(x - 3.5)$$

$$Y - 5 = 0.63x - 2.2$$

$$Y = 0.63x + 2.8$$

$$\text{When } x = 10$$

$$Y = 0.63(10) + 2.8$$

$$= 6.3 + 2.8$$

$$= 9.1$$

Question 12.

The following sample gave the number of hours of study (X) per day for an examination and marks (Y) obtained by 12 students.

X	3	3	3	4	4	5	5	5	6	6	7	8
Y	45	60	55	60	75	70	80	75	90	80	75	85

Obtain the line of regression of marks on hours of study.

Solution:

Let $u = x - 5$, $v = y - 70$

	x	y	u	v	$u \cdot v$	u^2
	3	45	-2	-25	50	4
	3	60	-2	-10	20	4
	3	55	-2	-15	30	4
	4	60	-1	-10	10	1
	4	75	-1	5	-5	1
	5	70	0	0	0	0
	5	80	0	10	0	0
	5	75	0	5	0	0
	6	90	1	20	20	1
	6	80	1	10	10	1
	7	75	2	5	10	4
	8	85	3	15	45	9
Total	59	850	-1	10	190	29

$$\bar{x} = \frac{\sum x_i}{n} = \frac{59}{12} = 4.92$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{850}{12} = 70.83$$

$$b_{yx} = b_{vu} = \frac{n \sum (u \cdot v) - (\sum u)(\sum v)}{n \sum u^2 - (\sum u)^2}$$

$$= \frac{12(190) - (-1)(10)}{12(29) - (-1)^2} = 6.6$$

∴ Equation of marks on hours of study is

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$(Y - 70.83) = 6.6(x - 4.92)$$

$$Y - 70.83 = 6.6x - 32.47$$

$$\therefore Y = 6.6x + 38.36$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 3 Linear Regression Ex 3.2

Question 1.

For bivariate data.

$$\bar{x} = 53, \bar{y} = 28, b_{yx} = -1.2, b_{xy} = -0.3$$

Find,

(i) Correlation coefficient between X and Y.

(ii) Estimate Y for X = 50

(iii) Estimate X for Y = 25

Solution:

(i) $r^2 = b_{yx} \cdot b_{xy}$

$$r^2 = (-1.2)(-0.3)$$

$$r^2 = 0.36$$

$$r = \pm 0.6$$

Since, b_{yx} and b_{xy} are negative, $r = -0.6$

(ii) Regression equation of Y on X is

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$Y - 28 = -1.2(50 - 53)$$

$$Y - 28 = -1.2(-3)$$

$$Y - 28 = 3.6$$

$$Y = 31.6$$

(iii) Regression equation of X on Y is

$$(X - \bar{x}) = b_{xy} (Y - \bar{y})$$

$$(X - 53) = -0.3(25 - 28)$$

$$X - 53 = -0.3(-3)$$

$$X - 53 = 0.9$$

$$X = 53.9$$

Question 2.

From the data of 20 pairs of observation on X and Y, following result are obtained $\bar{x} = 199$, $\bar{y} = 94$, $\sum (x_i - \bar{x})^2 = 1200$, $\sum (y_i - \bar{y})^2 = 300$

$$\sum (x_i - \bar{x})(y_i - \bar{y}) = -250$$

Find

(i) The line of regression of Y on X.

(ii) The line of regression of X on Y.

(iii) Correlation coefficient between X on Y.

Solution:

$$24y - 2256 = -5x + 995$$

$$5x + 24y = 3251$$

$$(ii) \quad b_{xy} = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (y - \bar{y})^2} = \frac{-250}{300} = -\frac{5}{6}$$

Line of regression of X on Y is

$$(X - \bar{x}) = b_{xy} (Y - \bar{y})$$

$$(X - 199) = -\frac{5}{6} (Y - 94)$$

$$6x - 1194 = -5y + 470$$

$$6x + 5y = 1664$$

$$(iii) \quad r^2 = b_{yx} \cdot b_{xy}$$

$$r^2 = \frac{-5}{24} \times \frac{-5}{6}$$

$$r^2 = \frac{25}{144}$$

$$\therefore r = \pm \frac{5}{12}$$

$$\text{Since } b_{yx} \text{ and } b_{xy} \text{ are negative, } \therefore r = -\frac{5}{12}$$

Question 3.

From the data of 7 pairs of observations on X and Y following results are obtained.

$$\sum (x_i - 70) = -35, \sum (y_i - 60) = -7, \sum (x_i - 70)^2 = 2989, \sum (y_i - 60)^2 = 476, \sum (x_i - 70)(y_i - 60) = 1064 \text{ [Given } \sqrt{0.7884} = 0.8879]$$

Obtain

(i) The line of regression of Y on X.

(ii) The line of regression of X on Y.

(iii) The correlation coefficient between X and Y.

Solution:

$$\text{Let } u_i = x_i - 70, v_i = y_i - 60$$

$$\text{Given } \Sigma u = -35, \Sigma v = -7$$

$$\Sigma u^2 = 2989, \Sigma v^2 = 476$$

$$\Sigma(u \cdot v) = 1064$$

$$\bar{u} = \bar{x} - 70$$

$$\frac{-35}{7} = \bar{x} - 70$$

$$-5 = \bar{x} - 70$$

$$\therefore \bar{x} = 65$$

$$\bar{v} = \bar{y} - 60$$

$$\frac{-7}{7} = \bar{y} - 60$$

$$-1 = \bar{y} - 60$$

$$\bar{y} = 59$$

$$b_{yx} = b_{vu} = \frac{n \Sigma(u \cdot v) - (\Sigma u)(\Sigma v)}{n(\Sigma u^2) - (\Sigma u)^2}$$

$$= \frac{7(1064) - (-35)(-7)}{7(2989) - (-35)^2} = 0.36$$

$$b_{xy} = b_{uv} = \frac{n \Sigma(u \cdot v) - (\Sigma u)(\Sigma v)}{n(\Sigma v^2) - (\Sigma v)^2}$$

$$= \frac{7(1064) - (-35)(-7)}{7(476) - (-7)^2} = 2.19$$

(i) Line of regression Y on X is

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$(Y - 59) = 0.36(x - 65)$$

$$(Y - 59) = 0.36x - 23.4$$

$$Y = 0.36x + 35.6$$

(ii) Line of regression X on Y is

$$(X - \bar{x}) = b_{xy} (Y - \bar{y})$$

$$(X - 65) = 2.19(y - 59)$$

$$(X - 65) = 2.19y - 129.21$$

$$X = 2.19y - 64.21$$

(iii) $r^2 = b_{yx} \cdot b_{xy}$

$$r^2 = (0.36)(2.19)$$

$$r^2 = 0.7884$$

$$r = \pm\sqrt{0.7884} = \pm 0.8879$$

Since b_{yx} and b_{xy} are positive.

$$\therefore r = 0.8879$$

Question 4.

You are given the following information about advertising expenditure and sales.

	Advertisement expenditure (Rs.in lakh) (X)	Sales (Rs.in lakh) (Y)
Arithmetic mean	10	90
Standard deviation	3	12

Correlation coefficient between X and Y is 0.8

(i) Obtain two regression equations.

(ii) What is the likely sales when the advertising budget is ₹ 15 lakh?

(iii) What should be the advertising budget if the company wants to attain sales target of ₹ 120 lakh?

Solution:

Given, $\bar{x} = 10$, $\bar{y} = 90$, $\sigma_x = 3$, $\sigma_y = 12$, $r = 0.8$

$$b_{yx} = r \frac{\sigma_y}{\sigma_x} = 0.8 \times \frac{12}{3} = 3.2$$

$$b_{xy} = r \frac{\sigma_x}{\sigma_y} = 0.8 \times \frac{3}{12} = 0.2$$

(i) Regression equation of Y on X is

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$(Y - 90) = 3.2(x - 10)$$

$$Y - 90 = 3.2x - 32$$

$$Y = 3.2x + 58$$

Regression equation of X on Y is

$$(X - \bar{x}) = b_{xy} (Y - \bar{y})$$

$$(X - 10) = 0.2(y - 90)$$

$$X - 10 = 0.2y + 18$$

$$X = 0.2y - 8$$

(ii) When $x = 15$,

$$Y = 3.2(15) + 58$$

$$= 48 + 58$$

$$= 106 \text{ lakh}$$

(iii) When $y = 120$

$$X = 0.2(120) - 8$$

$$= 24 - 8$$

$$= 16 \text{ lakh}$$

Question 5.

Bring out inconsistency if any, in the following:

(i) $b_{yx} + b_{xy} = 1.30$ and $r = 0.75$

(ii) $b_{yx} = b_{xy} = 1.50$ and $r = -0.9$

(iii) $b_{yx} = 1.9$ and $b_{xy} = -0.25$

(iv) $b_{yx} = 2.6$ and $b_{xy} = 12.6$

Solution:

(i) Given, $b_{yx} + b_{xy} = 1.30$ and $r = 0.75$

$$b_{yx} + b_{xy} = 1.30 \Rightarrow 0.65$$

But for regression coefficients b_{yx} and b_{xy}

$$|b_{yx} + b_{xy}| \geq r$$

Here, $0.65 < r = 0.75$

∴ The data is inconsistent

(ii) The signs of b_{yx} , b_{xy} and r must be same (all three positive or all three negative)

∴ The data is inconsistent.

(iii) The signs of b_{yx} and b_{xy} should be same (either both positive or both negative)

∴ The data is consistent.

(iv) $b_{yx} \cdot b_{xy} = 2.6 \times 12.6 = 1$

$$\therefore 0 \leq r^2 \leq 1$$

∴ The data is consistent.

Question 6.

Two sample from bivariate populations have 15 observation each. The sample means of X and Y are 25 and 18 respectively. The corresponding sum of square of deviations from respective means are 136 and 150. The sum of product of deviations from respective means is 123. Obtain the equation of line of regression of X on Y.

Solution:

$$\text{Given, } n = 15, \bar{x} = 25, \bar{y} = 18, \Sigma(x - \bar{x})^2 = 136, \Sigma(y - \bar{y})^2 = 150, \Sigma(x - \bar{x})(y - \bar{y}) = 123$$

Regression equation of X on Y is $(X - \bar{x}) = b_{xy} (Y - \bar{y})$

$$(X - 25) = 0.82(y - 18)$$

$$(X - 25) = 0.82y - 14.76$$

$$X = 0.82y + 10.24$$

Question 7.

For a certain bivariate data

	X	Y
Mean	25	20
S.D.	4	3

And $r = 0.5$ estimate y when $x = 10$ and estimate x when $y = 16$

Solution:

Given, $\bar{x} = 25$, $\bar{y} = 20$, $\sigma_x = 4$, $\sigma_y = 3$, $r = 0.5$

$$b_{yx} = r\sigma_y\sigma_x = 0.5 \times 3 \times 4 = 0.375$$

Regression equation of Y on X is

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$(Y - 20) = 0.375(x - 25)$$

$$Y - 20 = 0.375x - 9.375$$

$$Y = 0.375x + 10.625$$

When, $x = 10$

$$Y = 0.375(10) + 10.625$$

$$= 3.75 + 10.625$$

$$= 14.375$$

$$b_{xy} = r\sigma_x\sigma_y = 0.5 \times 4 \times 3 = 0.67$$

Regression equation of X on Y is

$$(X - \bar{x}) = b_{yx} (Y - \bar{y})$$

$$(X - 25) = 0.67(y - 20)$$

$$(X - 25) = 0.67y - 13.4$$

$$X = 0.67y + 11.6$$

When, $Y = 16$

$$x = 0.67(16) + 11.6$$

$$= 10.72 + 11.6$$

$$= 22.32$$

Question 8.

Given the following information about the production and demand of a commodity obtain the two regression lines:

	Production (X)	Demand (Y)
Mean	85	90
S.D.	5	6

Coefficient of correlation between X and Y is 0.6 . Also estimate the problem when demand is 100 .

Solution:

Given $\bar{x} = 85$, $\bar{y} = 90$, $\sigma_x = 5$, $\sigma_y = 6$ and $r = 0.6$

$$b_{xy} = r\sigma_x\sigma_y = 0.6 \times 5 \times 6 = 0.5$$

$$b_{yx} = r\sigma_y\sigma_x = 0.6 \times 6 \times 5 = 0.72$$

Regression equation of X on Y is

$$(X - \bar{x}) = b_{xy} (Y - \bar{y})$$

$$(X - 85) = 0.5(y - 90)$$

$$(X - 85) = 0.5y - 45$$

$$X = 0.5y + 40$$

When $y = 100$,

$$x = 0.5(100) + 40$$

$$= 50 + 40$$

$$= 90$$

Regression equation of Y on X is

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$(Y - 90) = 0.72(x - 85)$$

$$(Y - 90) = 0.72x - 61.2$$

$$Y = 0.72x + 28.8$$

Question 9.

Given the following data, obtain linear regression estimate of X for $Y = 10$

Solution:

$\bar{x} = 7.6$, $\bar{y} = 14.8$, $\sigma_x = 3.2$, $\sigma_y = 16$ and $r = 0.7$

$$b_{xy} = r\sigma_x\sigma_y = 0.7 \times 3.2 \times 16 = 0.14$$

Regression equation of X on Y is

$$(X - \bar{y}) = b_{xy} (Y - \bar{y})$$

$$(X - 7.6) = 0.14(y - 14.8)$$

$$X - 7.6 = 0.14y - 2.072$$

$$X = 0.14y + 5.528$$

When $y = 10$

$$x = 0.14(10) + 5.528$$

$$= 1.4 + 5.528$$

$$= 6.928$$

Question 10.

An inquiry of 50 families to study the relationship between expenditure on accommodation (₹ x) and expenditure on food and entertainment (₹ y) gave the following result:

$$\Sigma x = 8500, \Sigma y = 9600, \sigma_x = 60, \sigma_y = 20, r = 0.6$$

Estimate the expenditure on food and entertainment when expenditure on accommodation is ₹ 200

Solution:

$$n = 50 \text{ (given)}$$

$$b_{yx} = \frac{r\sigma_y}{\sigma_x} = 0.6 \times \frac{20}{60} = 0.2$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{8500}{50} = 170$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{9600}{50} = 192$$

Regression equation of Y on X is

$$Y - \bar{y} = b_{yx} (X - \bar{x})$$

$$(Y - 192) = 0.2(200 - 170)$$

$$Y - 192 = 0.2(30)$$

$$Y = 192 + 6$$

$$Y = 198$$

Question 11.

The following data about the sales and advertisement expenditure of a firms is given below (in ₹ crores)

	Sales	Adv. Exp.
Mean	40	6
S.D.	10	1.5

Also correlation coefficient between X and Y is 0.9

(i) Estimate the likely sales for a proposed advertisement expenditure of ₹ 10 crores.

(ii) What should be the advertisement expenditure if the firm proposes a sales target ₹ 60 crores

Let the sales be X and advertisement expenditure be Y

Solution:

$$\text{Given, } \bar{x} = 40, \bar{y} = 6, \sigma_x = 10, \sigma_y = 1.5, r = 0.9$$

$$b_{yx} = r\sigma_y/\sigma_x = 0.9 \times 1.5/10 = 0.135$$

$$b_{xy} = r\sigma_x/\sigma_y = 0.9 \times 10/1.5 = 6$$

(i) Regression equation of X on Y is

$$(X - \bar{x}) = b_{xy} (Y - \bar{y})$$

$$(X - 40) = 6(y - 6)$$

$$X - 40 = 6y - 36$$

$$X = 6y + 4$$

When $y = 10$

$$x = 6(10) + 4$$

$$= 60 + 4$$

$$= 64 \text{ crores}$$

(ii) Regression equation Y on X is

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$(Y - 6) = 0.135 (x - 40)$$

$$Y - 6 = 0.135x - 5.4$$

$$Y = 0.135x + 0.6$$

When $x = 60$

$$Y = 0.135(60) + 0.6$$

$$= 8.1 + 0.6$$

$$= 8.7 \text{ crores}$$

Question 12.

For certain bivariate data the following information are available

	\bar{X}	\bar{Y}
A.M.	13	17
S.D.	3	2

Correlation coefficient between x and y is 0.6, estimate x when y = 15 and estimate y when x = 10.

Solution:

Given, $\bar{x} = 13$, $\bar{y} = 17$, $\sigma_x = 3$, $\sigma_y = 2$, $r = 0.6$

$$b_{yx} = r\sigma_y\sigma_x = 0.6 \times 2 \times 3 = 0.4$$

$$b_{xy} = r\sigma_x\sigma_y = 0.6 \times 3 \times 2 = 0.9$$

Regression equation of Y on X

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$Y - 17 = 0.4(x - 13)$$

$$Y = 0.4x + 11.8$$

When x = 10

$$Y = 0.4(10) + 11.8$$

$$= 4 + 11.8$$

$$= 15.8$$

Regression equation of X on Y

$$(X - \bar{x}) = b_{xy} (Y - \bar{y})$$

$$(X - 13) = 0.9(y - 17)$$

$$X - 13 = 0.9y - 15.3$$

$$X = 0.9y - 2.3$$

When y = 15

$$X = 0.9(15) - 2.3$$

$$= 13.5 - 2.3$$

$$= 11.2$$

Maharashtra State Board 12th Commerce Maths Solutions Chapter 3 Linear Regression Ex 3.3

Question 1.

From the two regression equations find r, \bar{x} and \bar{y} .

$$4y = 9x + 15 \text{ and } 25x = 4y + 17$$

Solution:

$$\text{Given } 4y = 9x + 15 \text{ and } 25x = 4y + 17$$

$$y = \frac{9x}{4} + \frac{15}{4}$$

$$x = \frac{4y}{25} + \frac{17}{25}$$

$$b_{yx} = \frac{9}{4} \quad b_{xy} = \frac{4}{25}$$

$$b_{yx} \cdot b_{xy} = \frac{9}{4} \times \frac{4}{25}$$

$$= \frac{9}{25} \quad \text{which belongs to } [0, 1]$$

\therefore Our assumption is correct

$$\therefore r^2 = b_{yx} \cdot b_{xy}$$

$$r^2 = \frac{9}{25}$$

$$r = \pm \frac{3}{5}$$

Since b_{yx} and b_{xy} are positive.

$$\therefore r = 35 = 0.6$$

(\bar{x}, \bar{y}) is the point of intersection of the regression lines

$$9x - 4y = -15 \quad \dots\dots(i)$$

$$25x - 4y = 17 \quad \dots\dots(ii)$$

$$-16x = -32$$

$$x = 2$$

$$\therefore \bar{x} = 2$$

Substituting $x = 2$ in equation (i)

$$9(2) - 4y = -15$$

$$18 + 15 = 4y$$

$$33 = 4y$$

$$y = 33/4 = 8.25$$

$$\therefore \bar{y} = 8.25$$

Question 2.

In a partially destroyed laboratory record of an analysis of regression data, the following data are legible:

Variance of $X = 9$

Regression equations:

$$8x - 10y + 66 = 0 \text{ And } 40x - 18y = 214.$$

Find on the basis of the above information

(i) The mean values of X and Y .

(ii) Correlation coefficient between X and Y .

(iii) Standard deviation of Y .

Solution:

$$\text{Given, } \sigma_x^2 = 9, \sigma_x = 3$$

(i) (\bar{x}, \bar{y}) is the point of intersection of the regression lines

$$40x - 50y = -330 \quad \dots\dots(i)$$

$$40x - 50y = +214 \quad \dots\dots(ii)$$

$$-32y = -544$$

$$y = 17$$

$$\therefore \bar{y} = 17$$

$$8x - 10(17) + 66 = 0$$

$$8x = 104$$

$$x = 13$$

$$\therefore \bar{x} = 13$$

$$(ii) \quad 8x - 10y + 66 = 0, \quad 40x - 18y = 214$$

$$10y = 8x + 66, \quad 40x = 18y + 214$$

$$y = \frac{8x}{10} + \frac{66}{10} \quad x = \frac{18}{40} + \frac{214}{40}$$

$$\therefore b_{yx} = \frac{8}{10} = \frac{4}{5} \quad b_{xy} = \frac{18}{40} = \frac{9}{20}$$

$$b_{yx} \cdot b_{xy} = \frac{4}{5} \times \frac{9}{20} = \frac{9}{25} \text{ which belongs to } [0, 1]$$

\therefore Our assumption is correct

$$r^2 = b_{yx} \cdot b_{xy}$$

$$r^2 = \frac{9}{25}$$

$$r = \pm \frac{3}{5}$$

Since b_{yx} and b_{xy} are positive $\therefore r = \frac{3}{5}$

$$(iii) \quad b_{yx} = r \frac{\sigma_y}{\sigma_x}$$

$$\frac{4}{5} = \frac{3}{5} \times \frac{\sigma_y}{3}$$

$$\sigma_y = 4$$

Question 3.

For 50 students of a class, the regression equation of marks in statistics (X) on the marks in Accountancy (Y) is $3y - 5x + 180 = 0$. The mean marks in accountancy is 44 and the variance of marks in statistics is (916) th of the variance of marks in accountancy. Find the mean in statistics and the correlation coefficient between marks in two subjects.

Solution:

$$\text{Given, } n = 50, \bar{y} = 44$$

$$\sigma_{2x} = 916 \sigma_{2y}$$

$$\therefore \sigma_x \sigma_x = 34$$

Since (\bar{x}, \bar{y}) is the point intersection of the regression line.

$\therefore (\bar{x}, \bar{y})$ satisfies the regression equation.

$$3\bar{y} - 5\bar{x} + 180 = 0$$

$$3(44) - 5\bar{x} + 180 = 0$$

$$\therefore 5\bar{x} = 132 + 180$$

$$\bar{x} = \frac{312}{5} = 62.4$$

\therefore Mean marks in statistics is 62.4

Regression equation of X on Y is $3y - 5x + 180 = 0$

$$\therefore 5x = 3y + 180$$

Question 4.

For bivariate data, the regression coefficient of Y on X is 0.4 and the regression coefficient of X on Y is 0.9. Find the value of the variance of Y if the variance of X is 9.

Solution:

$$\text{Given, } b_{yx} = 0.4, b_{xy} = 0.9, \sigma_x^2 = 9, \sigma_x = 3$$

$$r^2 = b_{yx} \cdot b_{xy}$$

$$r^2 = 0.4 \times 0.9$$

$$r^2 = 0.36$$

$$r = \pm 0.6$$

Since b_{yx} and b_{xy} are positive $\therefore r = 0.6$

$$b_{yx} = \frac{r \cdot \sigma_y}{\sigma_x}$$

$$0.4 = 0.6 \times \frac{\sigma_y}{3}$$

$$\frac{4}{10} = \frac{6}{10} \times \frac{\sigma_y}{3}$$

$$\sigma_y = 2$$

$$\therefore \sigma_y^2 = 4$$

\therefore Variance of y is 4.

Question 5.

The equation of two regression lines are $2x + 3y - 6 = 0$ and $3x + 2y - 12 = 0$

Find (i) Correlation coefficient (ii) $\sigma_x \sigma_y$

Solution:

$$(i) \quad 2x + 3y = 6, \quad 3x + 2y = 12$$

$$3y = -2x + 6 \quad 3x = -2y + 12$$

$$y = \frac{-2}{3}x + 2 \quad x = \frac{-2}{3}y + 4$$

$$b_{yx} = \frac{-2}{3} \quad b_{xy} = \frac{-2}{3}$$

$$b_{yx} \cdot b_{xy} = \frac{-2}{3} \times \frac{-2}{3} = \frac{4}{9} \in [0, 1]$$

\therefore Our assumption is correct.

$$\therefore r^2 = b_{yx} \cdot b_{xy}$$

$$r^2 = \frac{4}{9}$$

$$r = \pm \frac{2}{3}$$

Since b_{yx} and b_{xy} are negative $\therefore r = \frac{-2}{3}$

$$(ii) \quad b_{xy} = \frac{r \cdot \sigma_y}{\sigma_x}$$

$$\frac{-2}{3} = \frac{-2}{3} \cdot \frac{\sigma_x}{\sigma_y}$$

$$\therefore \frac{\sigma_x}{\sigma_y} = 1$$

Question 6.

For a bivariate data $\bar{x} = 53, \bar{y} = 28, b_{yx} = -1.5$ and $b_{xy} = -0.2$. Estimate Y when X = 50.

Solution:

Regression equation of Y on X is

$$(Y - \bar{y}) = b_{yx} (X - \bar{x})$$

$$(Y - 28) = -1.5(50 - 53)$$

$$Y - 28 = -1.5(-3)$$

$$Y - 28 = 4.5$$

$$Y = 32.5$$

Question 7.

The equation of two regression lines are $x - 4y = 5$ and $16y - x = 64$. Find means of X and Y. Also, find the correlation coefficient between X and Y.

Solution:

Since (\bar{x}, \bar{y}) is the point of intersection of the regression lines.

$$x - 4y = 5 \dots (i)$$

$$-x + 16y = 64 \dots (ii)$$

$$12y = 69$$

$$y = 5.75$$

Substituting $y = 5.75$ in equation (i)

$$x - 4(5.75) = 5$$

$$x - 23 = 5$$

$$x = 28$$

$$\therefore \bar{x} = 28, \bar{y} = 5.75$$

$$x - 4y = 5$$

$$x = 4y + 5$$

$$\therefore b_{xy} = 4$$

$$16y - x = 64$$

$$16y = x + 64$$

$$y = \frac{1}{16}x + 4$$

$$b_{yx} = \frac{1}{16}$$

$$b_{yx} \cdot b_{xy} = \frac{1}{16} \times 4 = \frac{1}{4} \in [0, 1]$$

\therefore Our assumption is correct

$$\therefore r^2 = b_{yx} \cdot b_{xy}$$

$$r^2 = \frac{1}{4}$$

$$r = \pm \frac{1}{2}$$

Since b_{yx} and b_{xy} are positive,

$$\therefore r = \frac{1}{2} = 0.5$$

Question 8.

In partially destroyed record, the following data are available variance of $X = 25$. Regression equation of Y on X is $5y - x = 22$ and

Regression equation of X on Y is $64x - 45y = 22$ Find

(i) Mean values of X and Y .

(ii) Standard deviation of Y .

(iii) Coefficient of correlation between X and Y .

Solution:

$$\text{Given } \sigma_x^2 = 25, \therefore \sigma_x = 5$$

(i) Since (\bar{x}, \bar{y}) is the point of intersection of regression lines

$$-x + 5y = 22 \dots (i)$$

$$64x - 45y = 22 \dots (ii)$$

equation (i) becomes

$$-9x + 45y = 198$$

$$64y - 45y = 22$$

$$55x = 220$$

$$x = 4$$

Substituting $x = 4$ in equation (i)

$$-4 + 5y = 22$$

$$5y = 26$$

$$\therefore y = 5.2$$

$$\therefore \bar{x} = 4, \bar{y} = 5.2$$

Regression equation of X on Y is

$$64x - 45y = 22$$

$$64x = 45y + 22$$

$$x = \frac{45}{64}y + \frac{22}{64}$$

$$b_{xy} = \frac{45}{64}$$

(ii) Regression equation of Y on X is

$$5y - x = 22$$

$$5y = x + 22$$

$$y = \frac{1}{5}x + \frac{22}{5}$$

$$b_{yx} = \frac{1}{5}$$

$$r^2 = b_{yx} \cdot b_{xy}$$

$$= \frac{1}{5} \times \frac{45}{64}$$

$$= \frac{9}{64}$$

$$r = \pm \frac{3}{8}$$

Since b_{yx} and b_{xy} are positive

$$\therefore r = \frac{3}{8} = 0.375$$

$$(iii) b_{yx} = \frac{r \cdot \sigma_y}{\sigma_x}$$

$$\frac{1}{5} = \frac{3}{8} \times \frac{\sigma_y}{5}$$

$$\sigma_y = \frac{8}{3}$$

Question 9.

If the two regression lines for a bivariate data are $2x = y + 15$ (x on y) and $4y - 3x + 25$ (y on x) find

(i) \bar{x}

(ii) \bar{y}

(iii) b_{yx}

(iv) b_{xy}

(v) r [Given $\sqrt{0.375} = 0.61$]

Solution:

Since (\bar{x}, \bar{y}) is the point of intersection of the regression line

$$2x = y + 15$$

$$4y = 3x + 25$$

$$2x - y = 15 \dots\dots(i)$$

$$3x - 4y = -25 \dots\dots(ii)$$

Multiplying equation (i) by 4

$$8x - 4y = 60$$

$$3x - 4y = -25$$

on Subtracting,

$$5x = 85$$

$$\therefore x = 17$$

Substituting x in equation (i)

$$2(17) - y = 15$$

$$34 - 15 = y$$

$$\therefore y = 19$$

$$\bar{x} = 17, \bar{y} = 19$$

$$2x = y + 15$$

$$4y = 3x + 25$$

$$x = \frac{1}{2}y + 7.5$$

$$y = \frac{3}{4}x + 6.25$$

$$b_{xy} = \frac{1}{2}$$

$$b_{yx} = \frac{3}{4}$$

$$b_{xy} \cdot b_{yx} = \frac{1}{2} \times \frac{3}{4} = \frac{3}{8} \in [0, 1]$$

\therefore Our assumption is correct

$$r^2 = b_{xy} \cdot b_{yx}$$

$$r^2 = \frac{3}{8}$$

$$= 0.375$$

$$r = \pm \sqrt{0.375} = \pm 0.61$$

Since b_{yx} and b_{xy} are positive, $\therefore r = 0.61$

Question 10.

The two regression equation are $5x - 6y + 90 = 0$ and $15x - 8y - 130 = 0$. Find \bar{x} , \bar{y} , r .

Solution:

Since (\bar{x}, \bar{y}) is the point of intersection of the regression lines

$$5x - 6y + 90 = 0 \dots\dots(i)$$

$$15x - 8y - 130 = 0$$

$$15x - 18y + 270 = 0$$

$$15x - 8y - 130 = 0$$

on subtracting,

$$-10y + 400 = 0$$

$$y = 40$$

Substituting $y = 40$ in equation (i)

$$5x - 6(40) + 90 = 0$$

$$5x = 150$$

$$x = 30$$

$$\therefore \bar{x} = 30, \bar{y} = 40$$

$$5x - 6y + 90 = 0$$

$$15x - 8y - 130 = 0$$

$$6y = 5x + 90$$

$$15x = 8y + 130$$

$$y = \frac{5}{6}x + 15$$

$$x = \frac{8}{15}y + \frac{130}{15}$$

$$b_{yx} = \frac{5}{6}$$

$$b_{xy} = \frac{8}{15}$$

$$b_{yx} \cdot b_{xy} = \frac{5}{6} \times \frac{8}{15} = \frac{4}{9} \in [0, 1]$$

\therefore Our assumption is correct

$$\therefore r^2 = b_{yx} \cdot b_{xy}$$

$$= \frac{4}{9}$$

$$\therefore r = \pm \frac{2}{3}$$

Since b_{yx} and b_{xy} are positive

$$\therefore r = \frac{2}{3}$$

Question 11.

Two lines of regression are $10x + 3y - 62 = 0$ and $6x + 5y - 50 = 0$ Identify the regression equation equation of x on y . Hence find \bar{x} , \bar{y} , and r .

Solution:

$$10x + 3y = 62$$

$$6x + 5y = 50$$

$$10x = -3y + 62$$

$$5y = -6x + 50$$

$$x = \frac{-3}{10}y + \frac{62}{10}$$

$$y = \frac{-6}{5}x + 10$$

$$b_{xy} = \frac{-3}{10}$$

$$b_{yx} = \frac{-6}{5}$$

$$b_{xy} \cdot b_{yx} = \frac{-3}{10} \times \frac{-6}{5} = \frac{18}{50} = \frac{9}{25} \in [0, 1]$$

\therefore Our assumption is correct.

\therefore Regression equation of X on Y is $10x + 3y - 62 = 0$

$$r^2 = b_{yx} \cdot b_{xy}$$

$$r^2 = \frac{9}{25}$$

$$r = \pm \frac{3}{5}$$

Since, b_{yx} and b_{xy} are negative, $r = -\frac{3}{5} = -0.6$

Also (\bar{x}, \bar{y}) is the point of intersection of the regression lines

$$50x + 15y = 310$$

$$18x + 15y = 150$$

on subtracting

$$32x = 160$$

$$x = 5$$

Substituting $x = 5$ in $10x + 3y = 62$

$$10(5) + 3y = 62$$

$$3y = 12$$

$$\therefore y = 4$$

$$\therefore \bar{x} = 5, \bar{y} = 4$$

Question 12.

For certain X and Y series, which are correlated the two lines of regression are $10y = 3x + 170$ and $5x + 70 = 6y$. Find the correlation coefficient between them. Find the mean values of X and Y.

Solution:

$$10y = 3x + 170$$

$$5x + 70 = 6y$$

$$y = \frac{3}{10}x + 17$$

$$x = \frac{6}{5}y - 14$$

$$b_{yx} = \frac{3}{10}$$

$$b_{yx} = \frac{6}{5}$$

$$b_{yx} \cdot b_{xy} = \frac{3}{10} \times \frac{6}{5} = \frac{18}{50} = \frac{9}{25} \in [0, 1]$$

\therefore Our assumption is correct

$$r^2 = b_{yx} \cdot b_{xy}$$

$$= \frac{9}{25}$$

$$r = \pm \frac{3}{5}$$

Since b_{yx} and b_{xy} are positive,

$$r = \frac{3}{5} = 0.6$$

Since, (\bar{x}, \bar{y}) is the point of intersection of the regression lines

$$3x - 10y = -170 \dots\dots(i)$$

$$5x - 6y = -70 \dots\dots(ii)$$

$$9x - 30y = -510$$

$$25x - 30y = -350$$

on subtracting

$$-16x = -160$$

$$x = 10$$

Substituting $x = 10$ in equation (i)

$$3(10) - 10y = -170$$

$$30 + 170 = 10y$$

$$200 = 10y$$

$$y = 20$$

$$\therefore \bar{x} = 10, \bar{y} = 20$$

Question 13.

Regression equation of two series are $2x - y - 15 = 0$ and $4y + 25 = 0$ and $3x - 4y + 25 = 0$. Find \bar{x}, \bar{y} and regression coefficients, Also find coefficients of correlation. [Given $\sqrt{0.375} = 0.61$]

Solution:

Since (\bar{x}, \bar{y}) is the point of intersection of the regression line

$$2x = y + 15$$

$$4y = 3x + 25$$

$$2x - y = 15 \dots\dots(i)$$

$$3x - 4y = -15 \dots\dots(ii)$$

Multiply equation (i) by 4

$$8x - 4y = 60$$

$$3x - 4y = -25$$

on subtracting,

$$5x = 85$$

$$x = 17$$

Substituting x in equation (i)

$$2(17) - y = 15$$

$$34 - 15 = y$$

$$y = 19$$

$$\therefore \bar{x} = 17, \bar{y} = 19$$

$$2x = y + 15$$

$$4y = 3x + 25$$

$$x = \frac{1}{2}y + 7.5$$

$$y = \frac{3}{4}x + 6.25$$

$$b_{xy} = \frac{1}{2}$$

$$b_{yx} = \frac{3}{4}$$

$$b_{xy} \cdot b_{yx} = \frac{1}{2} \times \frac{3}{4} = \frac{3}{8} \in [0, 1]$$

∴ Our assumption is correct

$$r^2 = b_{xy} \cdot b_{yx}$$

$$r^2 = \frac{3}{8} = 0.375$$

$$r = \pm\sqrt{0.375} = \pm 0.61$$

Since, b_{yx} and b_{xy} are positive, ∴ $r = 0.61$

Question 14.

The two regression lines between height (X) in inches and weight (Y) in kgs of girls are $4y - 15x + 500 = 0$ and $20x - 3y - 900 = 0$. Find the mean height and weight of the group. Also, estimate the weight of a girl whose height is 70 inches.

Solution:

Since (\bar{x}, \bar{y}) is the point intersection of the regression lines

$$15x - 4y = 500 \dots\dots(i)$$

$$20x - 3y = 900 \dots\dots(ii)$$

$$60x - 16y = 2000$$

$$60x - 9y = 2700$$

on subtracting,

$$-7y = -700$$

$$y = 100$$

Substituting $y = 100$ in equation (i)

$$15x - 4(100) = 500$$

$$15x = 900$$

$$x = 60$$

∴ Our assumption is correct

∴ Regression equation of Y on X is

$$Y = 1.54x - 125$$

When $x = 70$

$$Y = 1.54 \times 70 - 125$$

$$= 262.5 - 125$$

$$= 137.5 \text{ kg}$$