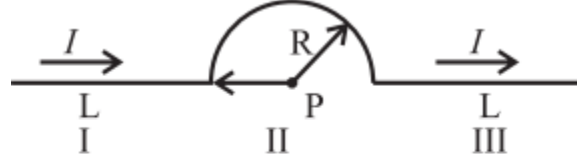


Maharashtra State Board 12th Physics Solutions Chapter 10 Magnetic Fields due to Electric Current

1. Choose the correct option.

i) A conductor has 3 segments; two straight and of length L each and a semicircular with radius R . It carries a current I . What is the magnetic field B at point P ?

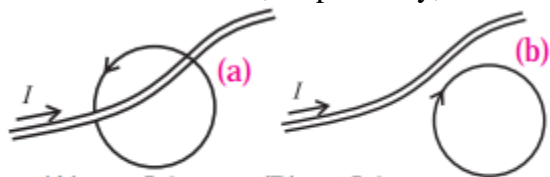


- (A) $\frac{\mu_0 I}{4\pi R}$ (B) $\frac{\mu_0 I}{4\pi R^2}$
(C) $\frac{\mu_0 I}{4 R}$ (D) $\frac{\mu_0 I}{4\pi}$

Answer:

(C) $\mu_0 4IR$

ii) Figure a, b show two Amperian loops associated with the conductors carrying current I in the sense shown. The $\oint \vec{B} \cdot d\vec{l}$ in the cases a and b will be, respectively,

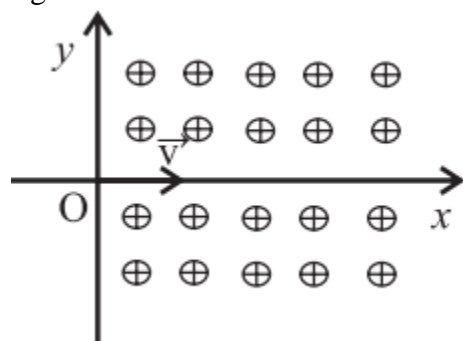


- (A) $-\mu_0 I, 0$ (B) $\mu_0 I, 0$
(C) $0, \mu_0 I$ (D) $0, -\mu_0 I$

Answer:

(A) $-\mu_0 I, 0$

iii) A proton enters a perpendicular uniform magnetic field B at origin along the positive x axis with a velocity v as shown in the figure. Then it will follow the following path. [The magnetic field is directed into the paper].



- (A) It will continue to move along positive x axis.
(B) It will move along a curved path, bending towards positive x axis.
(C) It will move along a curved path, bending towards negative y axis.
(D) It will move along a sinusoidal path along the positive x axis.

Answer:

(C) It will move along a curved path, bending towards negative y axis.

(iv) A conducting thick copper rod of length 1 m carries a current of 15 A and is located on the Earth's equator. There the magnetic flux lines of the Earth's magnetic field are horizontal, with the field of 1.3×10^{-4} T, south to north. The magnitude and direction of the force on the rod, when it is oriented so that current flows from west to east, are

- (A) 14×10^{-4} N, downward.
(B) 20×10^{-4} N, downward.
(C) 14×10^{-4} N, upward.
(D) 20×10^{-4} N, upward.

Answer:

(D) 20×10^{-4} N, upward.

v) A charged particle is in motion having initial velocity \vec{V} when it enters into a region of uniform magnetic field perpendicular to \vec{V} . Because of the magnetic force the kinetic energy of the particle will

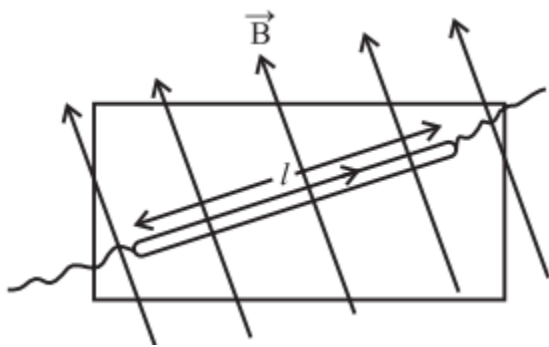
- (A) remain unchanged.
(B) get reduced.
(C) increase.
(D) be reduced to zero.

Answer:

(A) remain unchanged.

Question 2.

A piece of straight wire has mass 20 g and length 1 m. It is to be levitated using a current of 1 A flowing through it and a perpendicular magnetic field B in a horizontal direction. What must be the magnetic of B ?



Answer:

Data: $m = 20 \text{ g} = 2 \times 10^{-2} \text{ kg}$, $l = 1 \text{ m}$, $I = 1 \text{ A}$,
 $g = 9.8 \text{ m/s}^2$

To balance the wire, the upward magnetic force must be equal in magnitude to the downward force due to gravity.

$$\therefore F_m = IlB = mg$$

Therefore, the magnitude of the magnetic field,

$$B = \frac{mgl}{Il} = \frac{(2 \times 10^{-2})(9.8)(1)}{(1)(1)} = 0.196 \text{ T}$$

Question 3.

Calculate the value of magnetic field at a distance of 2 cm from a very long straight wire carrying a current of 5 A (Given: $\mu_0 = 4\pi \times 10^{-7} \text{ Wb/Am}$).

Answer:

Data : $I = 5 \text{ A}$, $a = 0.02 \text{ m}$, $\mu_0 4\pi = 10^{-7} \text{ T}\cdot\text{m/A}$

The magnetic induction,

$$B = \frac{\mu_0 I 2\pi a}{l} = \frac{\mu_0 4\pi I a}{2l} = 10^{-7} \times 2(5)2 \times 10^{-2} = 5 \times 10^{-5} \text{ T}$$

Question 4.

An electron is moving with a speed of $3.2 \times 10^6 \text{ m/s}$ in a magnetic field of $6.00 \times 10^{-4} \text{ T}$ perpendicular to its path. What will be the radius of the path? What will be frequency and the kinetic energy in keV ? [Given: mass of electron = $9.1 \times 10^{-31} \text{ kg}$, charge $e = 1.6 \times 10^{-19} \text{ C}$, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$]

Answer:

Data : $v = 3 \times 10^7 \text{ m/s}$, $B = 6 \times 10^{-4} \text{ T}$,

$m_e = 9 \times 10^{-31} \text{ kg}$, $e = 1.6 \times 10^{-19} \text{ C}$,

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

The radius of the circular path,

$$r = \frac{m_e v}{|e|B} = \frac{(9 \times 10^{-31})(3 \times 10^7)}{(1.6 \times 10^{-19})(6 \times 10^{-4})} = \frac{2.7}{9.6} = 0.2812 \text{ m}$$

The frequency of revolution,

$$f = \frac{|e|B}{2\pi m_e} = \frac{(1.6 \times 10^{-19})(6 \times 10^{-4})}{2 \times 3.142 \times (9 \times 10^{-31})} = \frac{9.6}{18 \times 3.142} \times 10^8$$

$$= 16.97 \text{ MHz}$$

Since the magnetic force does not change the kinetic energy of the charge,

$$\text{KE} = \frac{1}{2} m_e v^2 = \frac{1}{2} (9 \times 10^{-31})(3 \times 10^7)^2 = \frac{81}{2} \times 10^{-17} \text{ J}$$

$$= \frac{81}{2(1.6 \times 10^{-19})} \times 10^{-17} \text{ eV}$$

$$= \frac{8.1}{3.2} \times 10^3 = 2.531 \text{ keV}$$

Question 5.

An alpha particle (the nucleus of helium atom) (with charge $+2e$) is accelerated and moves in a vacuum tube with kinetic energy = 10.00 MeV. On applying a transverse uniform magnetic field of 1.851 T, it follows a circular trajectory of radius 24.60 cm. Obtain the mass of the alpha particle. [charge of electron = $1.62 \times 10^{-19} \text{ C}$]

Answer:

Data: $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$,

$E = 10 \text{ MeV} = 10^7 \times 1.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-12} \text{ J}$

$B = 1.88 \text{ T}$, $r = 0.242 \text{ m}$, $e = 1.6 \times 10^{-19} \text{ C}$

Charge of an α -particle,

$$q = 2e = 2(1.6 \times 10^{-19}) = 3.2 \times 10^{-19} \text{ C}$$

$$r = \frac{m_\alpha v}{qB} \quad \text{and} \quad E = \frac{1}{2} m_\alpha v^2$$

$$\therefore r^2 = \frac{(m_\alpha v)^2}{q^2 B^2} \quad \text{and} \quad 2Em_\alpha = (m_\alpha v)^2$$

$$\therefore r^2 = \frac{2Em_\alpha}{q^2 B^2}$$

$$\therefore m_\alpha = \frac{(qBr)^2}{2E} = \frac{[(3.2 \times 10^{-19})(1.88)(0.242)]^2}{2(1.6 \times 10^{-12})}$$

$$= (3.2 \times 10^{-26})(1.88 \times 0.242)^2$$

$$= (3.2 \times 10^{-26})(0.455)^2$$

$$= (3.2 \times 10^{-26})(0.2070) = \mathbf{6.624 \times 10^{-27} \text{ kg}}$$

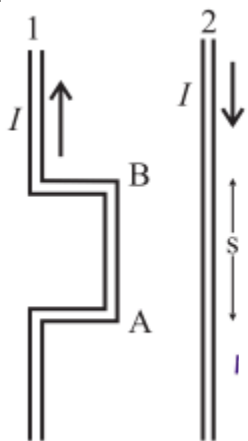
This gives the mass of the α -particle.

[Note : The value of r has been adjusted to match with the answer. The CODATA (Committee on Data for Science and Technology) accepted value of m_α is approximately $6.6446 \times 10^{-27} \text{ kg}$.]

Question 6.

Two wires shown in the figure are connected in a series circuit and the same amount of current of 10 A passes through both, but in apposite directions. Separation between the two wires is 8 mm. The length AB is $S = 22 \text{ cm}$. Obtain the direction and magnitude of the magnetic field due to current in wire 2 on the section AB of wire 1. Also obtain the magnitude and direction of the force on wire 1.

$[\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}]$



Answer:

Data: $I_1 = I_2 = 10 \text{ A}$, $s = 8 \text{ mm} = 8 \times 10^{-3} \text{ m}$, $l = 0.22 \text{ m}$

By right hand grip rule, the direction of the magnetic field $\mathbf{B}_2 \rightarrow$ due to the current in wire 2 at AB is into the page and its magnitude is

$$B_2 = \mu_0 4\pi 2 I s = 10^{-7} \times 2(10)8 \times 10^{-3} = 14 \times 10^{-3} \text{ T}$$

The current in segment AB is upwards. Then, by Fleming's left hand rule, the force on it due to $\mathbf{B}_2 \rightarrow$ is to the left of the diagram, i.e., away from wire 1, or repulsive. The magnitude of the force is

$$F_{\text{on 1 by 2}} = I_1 B_2 = (10)(0.22) \times 14 \times 10^{-3}$$

$$= 5.5 \times 10^{-4} \text{ N}$$

Question 7.

A very long straight wire carries a current 5.2 A. What is the magnitude of the magnetic field at a distance 3.1 cm from the wire? [

$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}]$

Answer:

Data : $I = 5.2 \text{ A}$, $a = 0.031 \text{ m}$, $\mu_0 4\pi = 10^{-7} \text{ T}\cdot\text{m/A}$

The magnetic induction,

$$B = \mu_0 I 2\pi a = \mu_0 4\pi 2 I a = 10^{-7} \times 2(5.2)3.1 \times 10^{-2}$$

$$= 3.35 \times 10^{-5} \text{ T}$$

Question 8.

Current of equal magnitude flows through two long parallel wires having separation of 1.35 cm. If the force per unit length on each of the wires is $4.76 \times 10^{-2} \text{ N}$, what must be I ?

Answer:

$$\text{Data : } I_1 = I_2 = I, s = 1.35 \times 10^{-2} \text{ m}, \frac{F}{l} = 4.76 \times 10^{-2} \text{ N}$$

$$F = \left(\frac{\mu_0}{4\pi}\right) \frac{2I_1 I_2 l}{s} = \left(\frac{\mu_0}{4\pi}\right) \frac{2I^2 l}{s}$$

$$\therefore I^2 = \frac{F}{l} \frac{s}{2(\mu_0/4\pi)}$$

$$= (4.76 \times 10^{-2}) \frac{1.35 \times 10^{-2}}{2 \times 10^{-7}} = 3.213 \times 10^3$$

$$\therefore I = \sqrt{32.13 \times 10^2} = \mathbf{56.68 \text{ A}}$$

Question 9.

Magnetic field at a distance 2.4 cm from a long straight wire is $16 \mu\text{T}$. What must be current through the wire?

Answer:

Data: $a = 2.4 \times 10^{-2} \text{ m}$, $B = 1.6 \times 10^{-5} \text{ T}$,

$\mu_0 4\pi = 10^{-7} \text{ T}\cdot\text{m/A}$

$B = \mu_0 I 2\pi a = \mu_0 4\pi 2Ia$

The current through the wire,

$$I = \frac{1}{\mu_0 4\pi a B^2} = \frac{1}{10^{-7} (2.4 \times 10^{-2}) (1.6 \times 10^{-5})^2} = 1.92 \text{ A}$$

Question 10.

The magnetic field at the centre of a circular current carrying loop of radius 12.3 cm is $6.4 \times 10^{-6} \text{ T}$. What will be the magnetic moment of the loop?

Answer:

Data: $R = 12.3 \text{ cm} = 12.3 \times 10^{-2} \text{ m}$,

$B = 6.4 \times 10^{-6} \text{ T}$, $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2R} \times \frac{2\pi R^2}{2\pi R^2} = \frac{\mu_0}{4\pi} \frac{2I(A)}{R^3} \quad (\because A = \pi R^2)$$

$$= \frac{\mu_0}{4\pi} \frac{2\mu}{R^3}$$

where $\mu = IA$ is the magnetic dipole moment of the coil.

$$\therefore \mu = \frac{BR^3}{2(\mu_0/4\pi)} = \frac{(6.4 \times 10^{-6})(0.123)^3}{2 \times 10^{-7}}$$

$$= 5.955 \times 10^{-2} \text{ J/T (or A}\cdot\text{m}^2)$$

Question 11.

A circular loop of radius 9.7 cm carries a current 2.3 A. Obtain the magnitude of the magnetic field (a) at the centre of the loop and (b) at a distance of 9.7 cm from the centre of the loop but on the axis.

Answer:

Data: $R = z = 9.7 \text{ cm} = 9.7 \times 10^{-2} \text{ m}$, $I = 2.3 \text{ A}$, $N = 1$

(a) At the centre of the coil :

The magnitude of the magnetic induction,

$$B = \mu_0 NI 2R$$

Question 12.

A circular coil of wire is made up of 100 turns, each of radius 8.0 cm. If a current of 0.40 A passes through it, what be the magnetic field at the centre of the coil?

Answer:

Data: $N = 100$, $R = 8 \times 10^{-2} \text{ m}$, $I = 0.4 \text{ A}$,

$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$

$B = \mu_0 NI 2R$

$$= (4\pi \times 10^{-7})(100)(0.4)^2(8 \times 10^{-2}) = 3.142 \times 10^{-4} \text{ T}$$

Question 13.

For proton acceleration, a cyclotron is used in which a magnetic field of 1.4 Wb/m^2 is applied. Find the time period for reversing the electric field between the two Ds.

Answer:

Data: $B = 1.4 \text{ Wb/m}^2$, $m = 1.67 \times 10^{-27} \text{ kg}$,

$q = 1.6 \times 10^{-19} \text{ C}$

$$T = 2\pi m q B$$

$$t = \frac{T}{2} = \frac{\pi m q B}{1} = \frac{(3.142)(1.67 \times 10^{-27})(1.6 \times 10^{-19})(1.4)}{1}$$

$$= 2.342 \times 10^{-8} \text{ s}$$

This is the required time interval.

Question 14.

A moving coil galvanometer has been fitted with a rectangular coil having 50 turns and dimensions $5 \text{ cm} \times 3 \text{ cm}$. The radial magnetic field in which the coil is suspended is of 0.05 Wb/m^2 . The torsional constant of the spring is $1.5 \times 10^{-9} \text{ Nm/degree}$. Obtain the current required to be passed through the galvanometer so as to produce a deflection of 30° .

Answer:

Data : $N = 50$, $C = 1.5 \times 10^{-9} \text{ Nm/degree}$,

$A = lb = 5 \text{ cm} \times 3 \text{ cm} = 15 \text{ cm}^2 = 15 \times 10^{-4} \text{ m}^2$,

$B = 0.05 \text{ Wb/m}^2$, $\theta = 30^\circ$

$$NIAB = C\theta$$

$$\therefore \text{The current through the coil, } I = \frac{C\theta}{NAB}$$

$$= \frac{1.5 \times 10^{-9} \times 30}{50 \times 15 \times 10^{-4} \times 0.05} = 3 \times 10^{-5} \times 0.5$$

$$= 1.2 \times 10^{-5} \text{ A}$$

Question 15.

A solenoid of length $\pi \text{ m}$ and 5 cm in diameter has winding of 1000 turns and carries a current of 5 A. Calculate the magnetic field at its centre along the axis.

Answer:

Data: $L = 3.142 \text{ m}$, $N = 1000$, $I = 5 \text{ A}$,

$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$

The magnetic induction,

$$B = \mu_0 nI = \mu_0(NL)I$$

$$= (4\pi \times 10^{-7})(1000)(3.142)(5) = 20 \times 3.142 \times 10^{-4} \times 3.142$$

$$= 2 \times 10^{-3} \text{ T}$$

Question 16.

A toroid of narrow radius of 10 cm has 1000 turns of wire. For a magnetic field of $5 \times 10^{-2} \text{ T}$ along its axis, how much current is required to be passed through the wire?

Answer:

Data : Central radius, $r = 10 \text{ cm} = 0.1 \text{ m}$, $N = 1000$,

$$B = 5 \times 10^{-2} \text{ T}, = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$$

The magnetic induction,

$$B = \mu_0 NI 2\pi r = \mu_0 4\pi N I r$$

$$2\pi r 4\pi r$$

$$\therefore 5 \times 10^{-2} = 10^{-7} \times 2(1000)I 0.1$$

$$\therefore I = 502 = 25 \text{ A}$$

This is the required current.

Question 17.

In a cyclotron protons are to be accelerated. Radius of its D is 60 cm. and its oscillator frequency is 10 MHz. What will be the kinetic energy of the proton thus accelerated?

(Proton mass = $1.67 \times 10^{-27} \text{ kg}$, $e = 1.60 \times 10^{-19} \text{ C}$, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$)

Answer:

Data : $R = 0.6 \text{ m}$, $f = 10^7 \text{ Hz}$, $m_p = 1.67 \times 10^{-27} \text{ kg}$,

$e = 1.6 \times 10^{-19} \text{ C}$, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

$$f = \frac{qB}{2\pi m_p} \quad \text{and} \quad KE = \frac{q^2 B^2 R^2}{2m_p}$$

$$\therefore f^2 = \frac{q^2 B^2}{4\pi^2 m_p^2} \quad \therefore \frac{q^2 B^2}{2m_p} = 2\pi^2 f^2 m_p$$

$$\therefore KE = 2\pi^2 f^2 m_p R^2$$

$$= 2 \times 9.872 \times (10^7)^2 (1.67 \times 10^{-27}) (0.6)^2$$

$$= 11.87 \times 10^{-13} \text{ J} = \frac{11.87 \times 10^{-13}}{1.6 \times 10^{-19}}$$

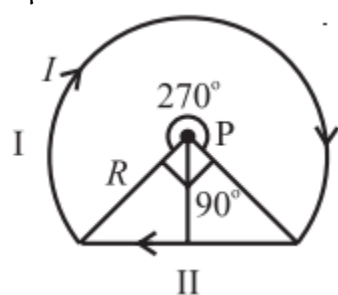
$$= 7.419 \times 10^6 \text{ eV}$$

$$= 7.419 \text{ MeV}$$

Question 18.

A wire loop of the form shown in the figure carries a current I . Obtain the magnitude and direction of the magnetic field at P . Given

$$: \mu_0 I 4\pi R 2 - \sqrt{}$$



Answer:

The wire loop is in the form of a circular arc AB of radius R and a straight conductor BCA. The arc AB subtends an angle of $\Phi = 270^\circ$

$= 3\pi/2 \text{ rad}$ at the centre of the loop P . Since $PA = PB = R$ and C is the midpoint of AB , $AB = 2\sqrt{R^2 - R^2/2} = 2\sqrt{R^2/2} = R\sqrt{2}$.

Therefore, $a = PC = R/\sqrt{2}$.

The magnetic inductions at P due to the arc AB and the straight conductor BCA are respectively,

$$B_1 = \mu_0 I \Phi / 4\pi R \text{ and } B_2 = \mu_0 I 2a / 4\pi R$$

Therefore, the net magnetic induction at P is

$$B_{\text{net}} = B_1 + B_2 = \frac{\mu_0 I}{4\pi R} \left[\frac{\Phi}{R} + \frac{2}{R/\sqrt{2}} \right]$$

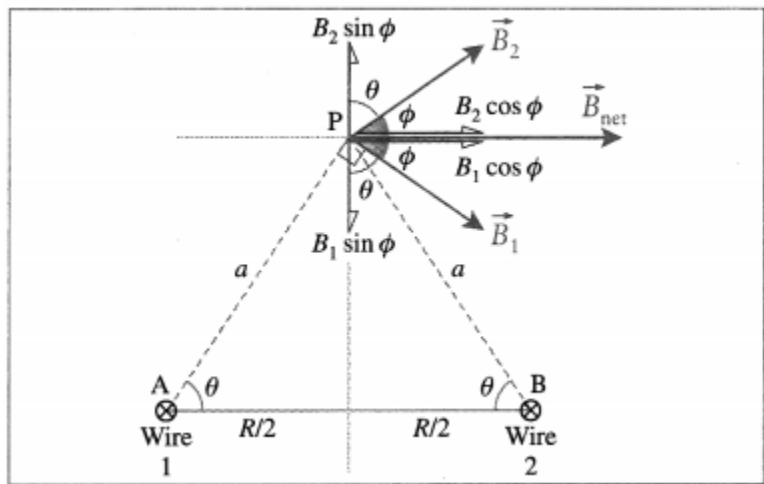
$$= \frac{\mu_0 I}{4\pi R} \left[\frac{3\pi}{2} + 2\sqrt{2} \right] = \frac{\mu_0 I}{4\pi R} \left[\frac{3\pi}{2} + 2\sqrt{2} \right]$$

This is the required expression.

Question 19.

Two long parallel wires going into the plane of the paper are separated by a distance R , and carry a current I each in the same direction. Show that the magnitude of the magnetic field at a point P equidistant from the wires and subtending angle θ from the plane containing the wires, is $B = \mu_0 \pi I R \sin 2\theta$ What is the direction of the magnetic field?

Answer:



In above figure, \vec{B}_1 and \vec{B}_2 are the magnetic fields in the plane of the page due to the currents in wires 1 and 2, respectively. Their directions are given by the right hand grip rule: \vec{B}_1 is perpendicular to AP and makes an angle Φ with the horizontal. \vec{B}_2 is perpendicular to BP and also makes an angle Φ with the horizontal. $AP = BP = a = R/2\cos\theta$ and $B_1 = B_2 = \mu_0 4\pi 2Ia = \mu_0 4\pi 2I(2\cos\theta)R = \mu_0 \pi IR \cos\theta$ Since the vertical components cancel out, the magnitude of the net magnetic induction at P is $B_{net} = 2B_1 \cos\Phi = 2B_1 \cos(90^\circ - \theta) = 2B_1 \sin\theta = 2(\mu_0 \pi IR \cos\theta) \sin\theta = \mu_0 \pi IR \sin 2\theta$ as required. is in the plane parallel to that of the wires and to the right as shown in the figure.

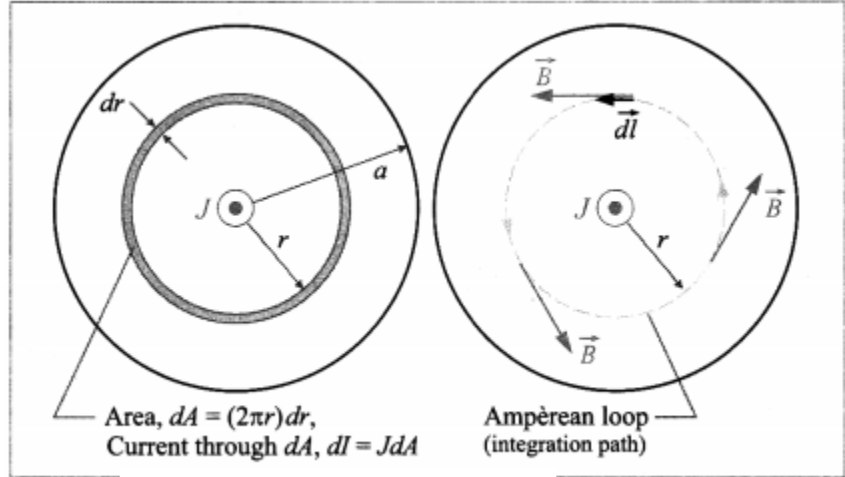
Question 20.

Figure shows a section of a very long cylindrical wire of diameter a, carrying a current I. The current density which is in the direction of the central axis of the wire varies linearly with radial distance r from the axis according to the relation $J = J_0 r/a$. Obtain the magnetic field B inside the wire at a distance r from its centre. [Answer: $B J r$]



Answer:

Consider an annular differential element of radius r and width dr. The current through the area dA of this element is $dI = JdA = (J_0 r/a)2\pi r dr = 2\pi J_0 r^2 da \dots\dots\dots (1)$



To apply the Ampere's circuital law to the circular path of integration, we note that the wire has perfect cylindrical symmetry with all the charges moving parallel to the wire. So, the magnetic field must be tangent to circles that are concentric with the wire. The

enclosed current is the current within radius r . Thus,

$$\oint B dl = \mu_0 I_{\text{encl}}$$
$$\therefore \oint B dl = \mu_0 \int_0^r dl = \mu_0 \int_0^r \frac{2\pi J_0}{a} r^2 dr \quad \dots (2)$$
$$\therefore B(2\pi r) = \frac{\mu_0 2\pi J_0}{a} \left(\frac{r^3}{3} \right)$$
$$\therefore B = \frac{\mu_0 J_0}{3a} r^2 \quad \dots (3)$$

which is the required expression.

[Note : Integrating Eq. (1) from $r=0$ to $r=a$,

$$I = \int dI = \int_0^a \frac{2\pi J_0}{a} r^2 dr = \frac{2\pi J_0}{a} \left[\frac{r^3}{3} \right]_0^a = \frac{2\pi J_0 a^2}{3}. \quad \text{Therefore,}$$

$$J_0 = \frac{3I}{2\pi a^2}, \text{ so that in terms of the total current } I \text{ carried}$$

$$\text{by the wire, } B = \frac{\mu_0}{3} \frac{r^2}{a} \left(\frac{3I}{2\pi a^2} \right) = \frac{\mu_0 I}{2\pi a^3} r^2.]$$

Question 21.

In the above problem, what will be the magnetic field B inside the wire at a distance r from its axis, if the current density J is uniform across the cross section of the wire?

Answer:

Figure shows the cross section of a long straight wire of radius a that carries a current I out of the page. Because the current is uniformly distributed over the cross section of the wire, the magnetic field \vec{B} due to the current must be cylindrically symmetrical.

Thus, along the Amperian loop of radius r ($r < a$), symmetry suggests that \vec{B} is tangent to the loop, as shown in the figure.

$$\oint \vec{B} \cdot d\vec{l} \Rightarrow B \oint dl = B(2\pi r) \quad \dots (1)$$

Because the current is uniformly distributed, the current I_{encl} enclosed by the loop is proportional to the area encircled by the loop; that is,

$$I_{\text{encl}} = J\pi r^2$$

By right-hand rule, the sign of d is positive. Then, by Ampere's law,

$$B(2\pi r) = \mu_0 I_{\text{encl}} = \mu_0 J\pi r^2 \quad \dots (2)$$

$$\therefore B = \mu_0 J 2r \quad \dots (3)$$

OR

$$I_{\text{encl}} = I \pi r^2 / \pi a^2$$

By right-hand rule, the sign of $\frac{\pi r^2}{\pi a^2}$ is positive. Then, by Ampere's law,

$$\oint B dl = \mu_0 I_{\text{encl}}$$

$$\therefore B(2\pi r) = \mu_0 I \frac{r^2}{a^2} \quad \dots (4)$$

$$\therefore B = (\mu_0 I / 2\pi a^2) r \quad \dots (5)$$

[Note: Thus, inside the wire, the magnitude B of the magnetic field is proportional to distance r from the centre. At a distance r outside a straight wire, $B = (\mu_0 I / 2\pi r)$ i.e., $B \propto 1/r$.]

Theory Exercise

Question 1.

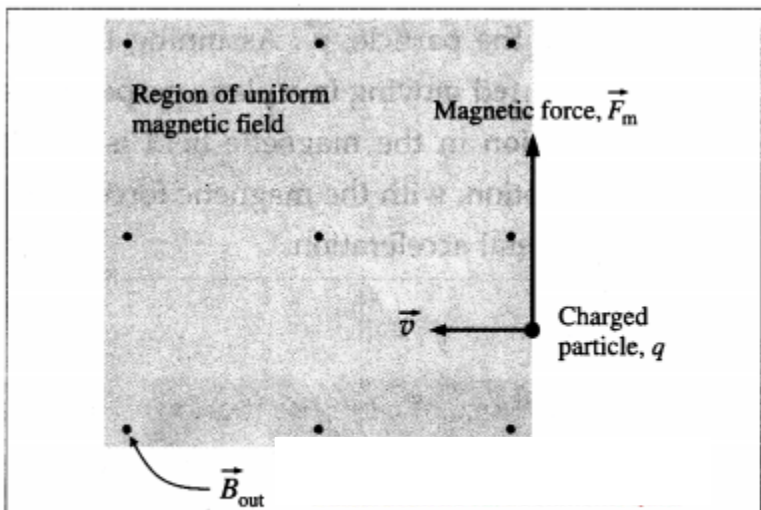
Distinguish between the forces experienced by a moving charge in a uniform electric field and in a uniform magnetic field.

Answer:

A charge q moving with a velocity \vec{v} through a magnetic field of induction \vec{B} experiences a magnetic force perpendicular both to \vec{B} and \vec{v} . Experimental observations show that the magnitude of the force is proportional to the magnitude of \vec{B} , the speed of the particle, the charge q and the sine of the angle θ between \vec{v} and \vec{B} . That is, the magnetic force, $F_m = qv B \sin \theta$

$$\therefore \vec{F}_m = q(\vec{v} \times \vec{B})$$

Therefore, at every instant \vec{F}_m acts in a direction perpendicular to the plane of \vec{v} and \vec{B} .



Force on a moving charge in a uniform magnetic field pointing out of the page

If the moving charge is negative, the direction of the force \vec{F}_m acting on it is opposite to that given by the right-handed screw rule for the cross-product $\vec{v} \times \vec{B}$.

If the charged particle moves through a region of space where both electric and magnetic fields are present, both fields exert forces on the particle.

The force due to the electric field \vec{E} is $\vec{F}_e = q\vec{E}$.

The total force on a moving charge in electric and magnetic fields is called the Lorentz force :

$$\vec{F} = \vec{F}_e + \vec{F}_m = q(\vec{E} + \vec{v} \times \vec{B})$$

Special cases :

(i) \vec{v} is parallel or antiparallel to \vec{B} : In this case, $F_m = qvB \sin 0^\circ = 0$. That is, the magnetic force on the charge is zero.

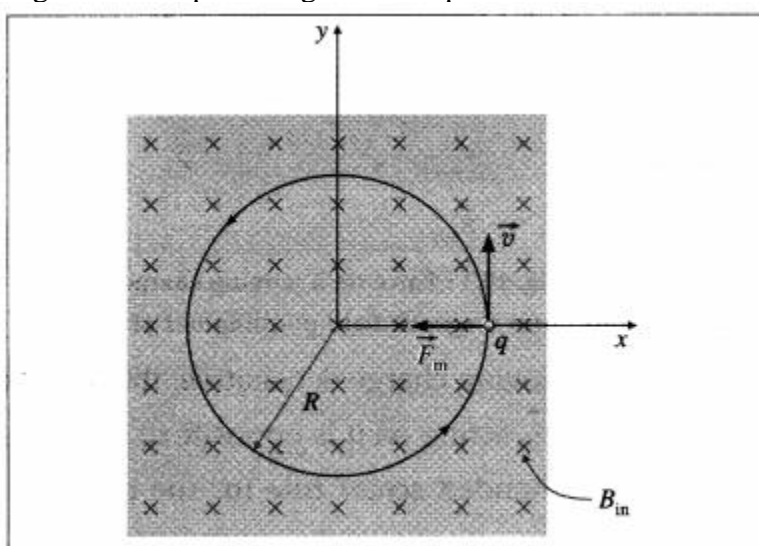
(ii) The charge is stationary ($v = 0$) : In this case, even if $q \neq 0$ and $B \neq 0$, $F_m = q(0)B \sin \theta = 0$. That is, the magnetic force on a stationary charge is zero.

Question 2.

Under what condition a charge undergoes uniform circular motion in a magnetic field? Describe, with a neat diagram, cyclotron as an application of this principle. Obtain an expression for the frequency of revolution in terms of the specific charge and magnetic field.

Answer:

Suppose a particle of mass m and charge q enters a region of uniform magnetic field of induction \vec{B} . In Fig., \vec{B} points into the page. The magnetic force \vec{F}_m on the particle is always perpendicular to the velocity of the particle, \vec{v} . Assuming the charged particle started moving in a plane perpendicular to \vec{B} , its motion in the magnetic field is a uniform circular motion, with the magnetic force providing the centripetal acceleration.



Charge q moving anticlockwise in a plane perpendicular to \vec{B} into the page

If the charge moves in a circle of radius R ,

$$F_m = |q|vB = mv^2/R$$

$$\therefore mv = p = |q|BR \dots\dots\dots (1)$$

where $p = mv$ is the linear momentum of the particle. Equation (1) is known as the cyclotron formula because it describes the motion of a particle in a cyclotron-the first of the modern particle accelerators.

Question 3.

What is special about a radial magnetic field ? Why is it useful in a moving coil galvanometer ?

Answer:

1. Advantage of radial magnetic field in a moving- coil galvanometer:
 - As the coil rotates, its plane is always parallel to the field. That way, the deflecting torque is always a ximum depending only on the current in the coil, but not on the position of the coil.
 - The restoring torque is proportional to the deflection so that a radial field makes the deflection proportional to the current. The instrument then has a linear scale, i.e., the divisions of the scale are evenly spaced. This makes it particularly straight forward to calibrate and to read.
2. Producing radial magnetic field :
 - The pole pieces of the permanent magnet are made cylindrically concave, concentric with the axis of the coil.
 - A soft iron cylinder is centred between the pole pieces so that it forms a narrow cylindrical gap in which the sides of the coil can move. Together, they produce a radial magnetic, field; that is, the magnetic lines of force in the gap are along radii to the central axis.

Question 4.

State Biot-Savart law. Apply it to

(i) infinitely long current carrying conductor and (ii) a point on the axis of a current carrying circular loop.

Answer:

Consider a very short segment of length dl of a wire carrying a current I . The product $I d\vec{l}$ is called a current element; the direction of the vector $d\vec{l}$ is along the wire in the direction of the current.

Biot-Savart law (Laplace law) : The magnitude of the incremental magnetic induction $d\vec{B}$ produced by a current element

$I d\vec{l}$ at a distance r from it is directly proportional to the magnitude $I d\vec{l}$ of the current element, the sine of the angle between the current element $I d\vec{l}$ and the unit vector \hat{r} directed from the current element toward the point in question, and inversely proportional to the square of the distance of the point from the current element; the magnetic induction is directed perpendicular to both

$I d\vec{l}$ and \hat{r} as per the cross product rule.

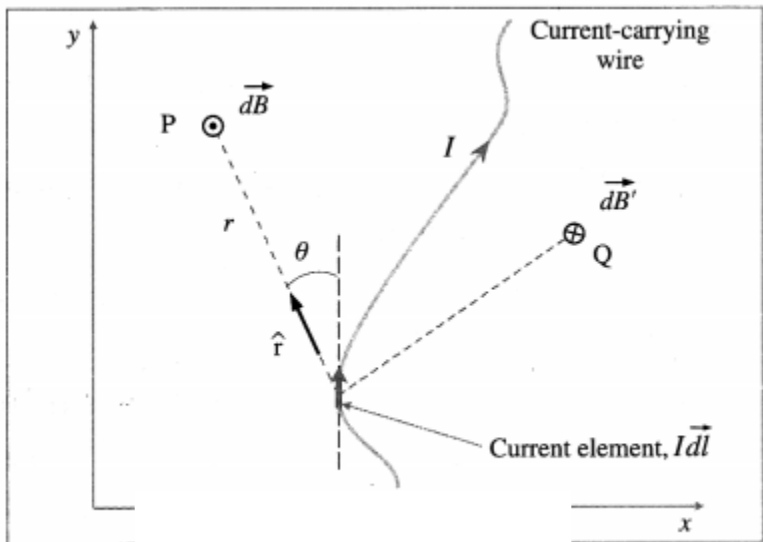
$$dB \propto \frac{Idl \sin \theta}{r^2}$$

$$\therefore dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2} \quad \dots (1)$$

$$\text{In vector form, } \vec{dB} = \left(\frac{\mu_0}{4\pi} \right) \frac{I \vec{dl} \times \hat{r}}{r^2} \quad \dots (2)$$

where $\hat{r} = \frac{\vec{r}}{r}$ and the constant μ_0 is the permeability of free space. Equations (1) and (2) are called Biot-Savart law.

The incremental magnetic induction \vec{dB} is given by the right-handed screw rule of vector crossproduct $I \vec{dl} \times \hat{r}$. In Fig, the current element $I \vec{dl}$ and \hat{r} are in the plane of the page, so that \vec{dB} points out of the page at point P shown by \odot ; at the point Q, \vec{dB} points into the page shown by \otimes .



Magnetic induction due to a current element

The magnetic induction \vec{B} at the point due to the entire wire is, by the principle of superposition, the vector sum of the contributions \vec{dB} from all the current elements making up the wire.

From Eq. (2),

$$\vec{B} = \int \vec{dB} = \frac{\mu_0}{4\pi} \int I \vec{dl} \times \hat{r} / r^2$$

[Notes : (1) The above law is based on experiments by Jean Baptiste Biot (1774-1862) and Felix Savart (1791-1841), French physicists. From their observations Laplace deduced the law mathematically. (2) The Biot- Savart law plays a similar role in magnetostatics as Coulomb's law does in electrostatics.]

Question 5.

State Ampere's law. Explain how is it useful in different situations.

Answer:

Ampere's circuital law : In free space, the line integral of magnetic induction around a closed path in a magnetic field is equal to μ_0 times the net steady current enclosed by the path.

In mathematical form,

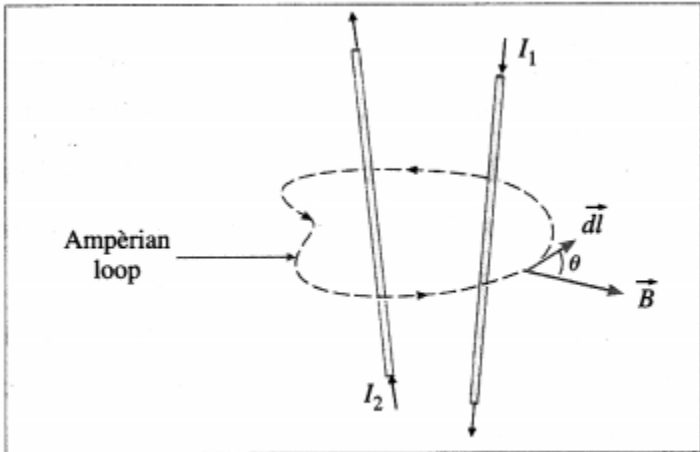
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \quad \dots (1)$$

where \vec{B} is the magnetic induction at any point on the path in vacuum, $d\vec{l}$ is the length element of the path, I is the net steady current enclosed and μ_0 is the permeability of free space.

Explanation : Figure shows two wires carrying currents I_1 and I_2 in vacuum. The magnetic induction \vec{B} at any point is the net effect of these currents.

To find the magnitude B of the magnetic induction :

We construct an imaginary closed curve around the conductors, called an Amperian loop, and imagine it divided into small elements of length $d\vec{l}$. The direction of $d\vec{l}$ is the direction along which the loop is traced.



Ampère's law

(ii) We assign signs to the currents using the right hand rule : If the fingers of the right hand are curled in the direction in which the loop is traced, then a current in the direction of the outstretched thumb is taken to be positive while a current in the opposite direction is taken to be negative.

For each length element of the Amperian loop, $\vec{B} \cdot d\vec{l} \rightarrow$ gives the product of the length dl of the element and the component of \vec{B} parallel to $d\vec{l} \rightarrow$. If θ is the angle between $d\vec{l} \rightarrow$ and \vec{B} ,

$$\vec{B} \cdot d\vec{l} \rightarrow = (B \cos \theta) dl$$

Then, the line integral,

$$\oint \vec{B} \cdot d\vec{l} \rightarrow = \oint B \cos \theta dl \dots\dots\dots (2)$$

For the case shown in Fig., the net current I through the surface bounded by the loop is

$$I = I_2 - I_1$$

$$\therefore \oint B \cos \theta dl = \mu_0 I$$

$$= \mu_0 (I_2 - I_1) \dots\dots\dots (3)$$

Equation (3) can be solved only when B is uniform and hence can be taken out of the integral.

[Note : Ampere's law in magnetostatics plays the part of Gauss's law of electrostatics. In particular, for currents with appropriate symmetry, Ampere's law in integral form offers an efficient way of calculating the magnetic field. Like Gauss's law, Ampere's law is always true (for steady currents), but it is useful only when the symmetry of the problem enables B to be taken out of the integral $\oint \vec{B} \cdot d\vec{l} \rightarrow$. The current configurations that can be handled by Ampere's law are infinite straight conductor, infinite plane, infinite solenoid and toroid.]

12th Physics Digest Chapter 10 Magnetic Fields due to Electric Current Intext Questions and Answers

[Do you know \(Textbook Page No. 230\)](#)

Question 1.

You must have noticed high tension power transmission lines, the power lines on the big tall steel towers. Strong magnetic fields are created by these lines. Care has to be taken to reduce the exposure levels to less than 0.5 milligauss (mG).

Answer:

With increasing population, many houses are constructed near high voltage overhead power transmission lines, if not right below them. Large transmission lines configurations with high voltage and current levels generate electric and magnetic fields and raises concerns about their effects on humans located at ground surfaces. With conductors typically 20 m above the ground, the electric field 2 m above the ground is about 0.2 kV / m to 1 kV / m. In comparison, that due to thunderstorms can reach 20 kV/m. For the same conductors, magnetic field 2 m above the ground is less than 6 μ T. In comparison, that due to the Earth is about 40 μ T.

[Do you know \(Textbook Page No. 232\)](#)

Question 1.

Magnetic Resonance Imaging (MRI) technique used for medical imaging requires a magnetic field with a strength of 1.5 T and even upto 7 T. Nuclear Magnetic Resonance experiments require a magnetic field upto 14 T. Such high magnetic fields can be produced using superconducting coil electromagnet. On the other hand, Earth's magnetic field on the surface of the Earth is about 3.6×10^{-5} T = 0.36 gauss.

Answer:

Magnetic Resonance Imaging (MRI) is a non-invasive imaging technology that produces three dimensional detailed anatomical images. Although MRI does not emit the ionizing radiation that is found in X-ray imaging, it does employ a strong magnetic field, e.g., medical MRIs usually have strengths between 1.5 T and 3 T.

The 21.1 T superconducting magnet at Maglab (Florida, US) is the world's strongest MRI scanner used for Nuclear Magnetic Resonance (NMR) research. Since its inception in 2004, it has been continually conducting electric current of 284 A by itself. Because it is superconducting, the current runs through some 152 km of wire without resistance, so no outside energy source is needed. However, 2400 litres of liquid helium is cycled to keep the magnet at a superconducting temperature of 1.7 K. Even when not in use this magnet is kept cold; if it warms up to room temperature, it takes at least six weeks to cool it back down to operating temperature. The 45 T Hybrid Magnet of the Lab (which combines a superconducting magnet of 11.5 T with a resistive magnet of 33.5 T) is kept at 1.8 K using 2800 L of liquid helium and 15142 L of cold water.

Question 2.

Let us look at a charged particle which is moving in a circle with a constant speed. This is uniform circular motion that you have studied earlier. Thus, there must be a net force acting on the particle, directed towards the centre of the circle. As the speed is constant, the force also must be constant, always perpendicular to the velocity of the particle at any given instant of time. Such a force is provided by the uniform magnetic field \vec{B} perpendicular to the plane of the circle along which the charged particle moves.

Answer:

When a charged particle moves in uniform circular motion inside a uniform magnetic field \vec{B} in a plane perpendicular to \vec{B} , the centripetal force is the magnetic force on the particle. As in any UCM, this magnetic force is constant in magnitude and perpendicular to the velocity of the particle.

[Remember this \(Textbook Page No. 233\)](#)

Question 1.

Field penetrating into the paper is represented as \otimes , while that coming out of the paper is shown by \odot .

Answer:

In a two-dimensional diagram, a vector pointing perpendicularly into the plane of the diagram is shown by a cross \otimes while that pointing out of the plane is shown by a dot \odot .

[Do you know \(Textbook Page No. 234\)](#)

Question 1.

Particle accelerators are important for a variety of research purposes. Large accelerators are used in particle research. There have been several accelerators in India since 1953. The Department of Atomic Energy (DAE), Govt. of India, had taken initiative in setting up accelerators for research. Apart from ion accelerators, the DAE has developed and commissioned a 2 GeV electron accelerator which is a radiation source for research in science. This accelerator, 'Synchrotron', is fully functional at Raja Ramanna Centre for Advanced Technology, Indore. An electron accelerator, Microtron with electron energy 8-10 MeV is functioning at Physics Department, Savitribai Phule Pune University, Pune.

Answer:

Particle accelerators are machines that accelerate charged subatomic particles to high energy for research and applications. They play a major role in the field of basic and applied sciences, in our understanding of nature and the universe. The size and cost of particle accelerators increase with the energy of the particles they produce. Medical Cyclotrons across the country are dedicated for medical isotope productions and for medical sciences. There are many existing and upcoming particle accelerators in India in different parts of country.

(https://www.researchgate.net/publication/320948083_Existing_and_upcoming_particle_accelerators_in_India). For cutting-edge high energy particle physics, Indian particle physicists collaborate with those at Large Hadron Collider CERN, Geneva.

[Can you recall \(Textbook Page No. 238\)](#)

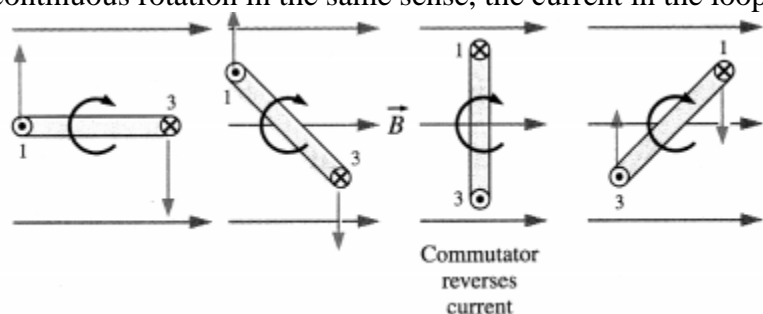
Question 1.

How does the coil in a motor rotate by a full rotation? In a motor, we require continuous rotation of the current carrying coil. As the plane of the coil tends to become parallel to the magnetic field \vec{B} , the current in the coil is reversed externally. Referring to Fig. the segment ab occupies the position cd. At this position of rotation, the current is reversed. Instead of from b to a, it flows from a to b, force \vec{F} continues to act in the same direction so that the torque continues to rotate the coil. The reversal of the current is achieved by using a commutator which connects the wires of the power supply to the coil via carbon brush contacts.

Answer:

Electric Motor

From Fig. we see that the torque on a current loop rotates the loop to smaller values of θ until the torque becomes zero, when the plane of the loop is perpendicular to the magnetic field and $\theta = 0$. If the current in the loop remains in the same direction when the loop turns past this position, the torque will reverse direction and turn the loop in the opposite direction, i.e., anticlockwise. To provide continuous rotation in the same sense, the current in the loop must periodically reverse direction, as shown in Fig.



In an electric motor, the current reversal is achieved externally by brushes and a split-ring commutator.

[Use your brain power \(Textbook Page No. 242\)](#)

Question 1.

Currents in two infinitely long, parallel wires exert forces on each other. Is this consistent with Newton's third law?

Answer:

Yes, they are equal in magnitude and opposite in direction and act on the contrary parts : $\vec{F}_{\text{on 2 by 1}} = \vec{F}_{\text{on 1 by 2}}$. Thus, they form action-reaction pair.

[Do you know \(Textbook Page No. 244\)](#)

Question 1.

So far we have used the constant μ_0 everywhere. This means in each such case, we have carried out the evaluation in free space (vacuum). μ_0 is the permeability of free space.

Answer:

Permeability of free space or vacuum, $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$.

Earlier SI (2006) had fixed this value of μ_0 as exact but revised SI fixes the value of e , requiring μ_0 (and ϵ_0) to be determined experimentally.

[Use your brain power \(Textbook Page No. 244\)](#)

Question 1.

Using electrostatic analogue, obtain the magnetic field \vec{B}_{equator} at a distance d on the perpendicular bisector of a magnetic dipole of magnetic length $2l$ and moment \vec{M} . For far field, verify that

$$\vec{B}_{\text{equator}} = \frac{\mu_0 4\pi}{r^3} \vec{M}$$

Answer:

The magnitude of the electric intensity at a point at a distance r from an electric charge q in vacuum is given by

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

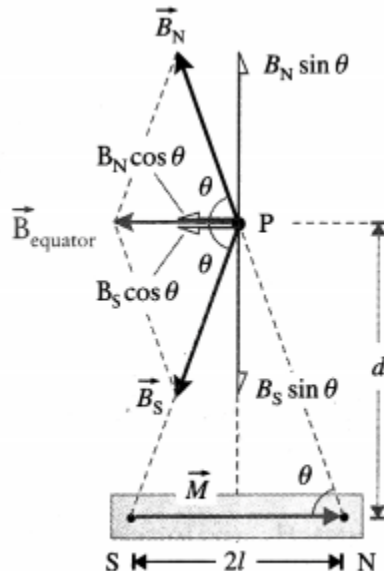
where ϵ_0 is the permittivity of free space. This intensity is directed away from the charge, if the charge is positive and towards the charge, if the charge is negative.

A magnetic pole is similar to an electric charge. The N-pole is similar to a positive charge and the S-pole is similar to a negative charge. Like an electric charge, a magnetic pole is assumed to produce a magnetic field in the surrounding region. The magnetic field at any point is denoted by a vector quantity called magnetic induction. Thus, by analogy, the magnitude of the magnetic induction at a point at a distance r from a magnetic pole of strength q_m is given by

$$B = \mu_0 4\pi q_m r^2$$

This induction is directed away from the pole if it is an N-pole (strength $+q_m$) and towards the pole if it is an S-pole (strength $-q_m$).

Consider a point P on the equator of a magnetic dipole with pole strengths $+q_m$ and $-q_m$ and of magnetic length $2l$. Let P be at a distance d from the centre of the dipole,



The magnetic induction of a bar magnet at an equatorial point

The magnetic induction at P due to the N-pole is directed along NP (away from the N-pole) while that due to the S-pole is along PS (towards the S-pole), each having a magnitude

$$B_N = B_S = (\mu_0 4\pi) q_m (d^2 + l^2)^{-3/2}$$

$$(\because NP = SP = \sqrt{d^2 + l^2})$$

The inductions due to the two poles are equal in magnitude so that the two, oppositely directed equatorial components, $B_N \sin \theta$ and $B_S \sin \theta$, cancel each other.

Therefore, the resultant induction is in a direction parallel to the axis of the magnetic dipole and has direction opposite to that of the magnetic moment of the magnetic dipole. The component of the induction due to the two poles along the axis is

$$(\mu_0 4\pi) q_m (d^2 + l^2)^{-3/2} \cos \theta$$

where θ is the angle shown in the diagram. From the diagram,

$$\cos \theta = \frac{l}{(d^2 + l^2)^{1/2}}$$

The total induction at P is then

$$B = (B_N + B_S) \cos \theta$$

$$= \left(\frac{\mu_0}{4\pi} \right) \frac{2q_m}{(d^2 + l^2)} \cos \theta$$

$$= \left(\frac{\mu_0}{4\pi} \right) \frac{2q_m}{(d^2 + l^2)} \cdot \frac{l}{(d^2 + l^2)^{1/2}}$$

$$\therefore B = \left(\frac{\mu_0}{4\pi} \right) \frac{M}{(d^2 + l^2)^{3/2}}$$

where $M = 2q_m l$ is the magnetic moment of the dipole.

$$\vec{B}_{\text{equator}} = \left(\frac{\mu_0}{4\pi} \right) \frac{-\vec{M}}{(d^2 + l^2)^{3/2}}$$

The minus sign shows that the direction of \vec{B}_{equator} is opposite to that of \vec{M} .

If the dipole is very short, i.e., $l \ll d$, we can ignore l^2 in comparison with d^2 . We then get

$$\vec{B}_{\text{equator}} = \left(\frac{\mu_0}{4\pi} \right) \frac{(-\vec{M})}{d^3}$$

Thus, for a short dipole the induction varies in-versely as the cube of the distance from it.

Question 2.

What is the fundamental difference between an electric dipole and a magnetic dipole?

Answer:

If a magnet is carefully and repeatedly cut, it would expose two new faces with opposite poles such that each piece would still be a

magnet. This suggests that magnetic fields are essentially dipolar in character. The most elementary magnetic structure always behaves as a pair of two magnetic poles of opposite types and of equal strengths. Hence, analogous to an electric dipole, we hypothesize that there are positive and negative magnetic charges (or north and south poles) of equal strengths a finite distance apart within a magnet. Also, they are assumed to act as the source of the magnetic field in exactly the same way that electric charges act as the source of electric field. The magnitude of each 'magnetic charge' is referred to as its 'pole strength' and is equal to $q_m = M/2l$, where \vec{M} is the magnetic dipole moment, pointing from the negative (or south, S) pole to the positive (or north, N) pole. However, while two types of electric charges exist in nature and have separate existence, isolated magnetic charges, or magnetic monopoles, are not observed. A magnetic pole is not an experimental fact: there are no real poles. To put it in another way, there are no point sources for \vec{B} , as there are for \vec{E} ; there exists no magnetic analog to electric charge. Every experimental effort to demonstrate the existence of magnetic charges has failed. Hence, magnetic poles are called fictitious. The electric field diverges away from a (positive) charge; the magnetic field line curls around a current. Electric field lines originate on positive charges and terminate on negative ones; magnetic field lines do not begin or end anywhere, they typically form closed loops or extend out to infinity.

[Do you know \(Textbook Page No. 247\)](#)

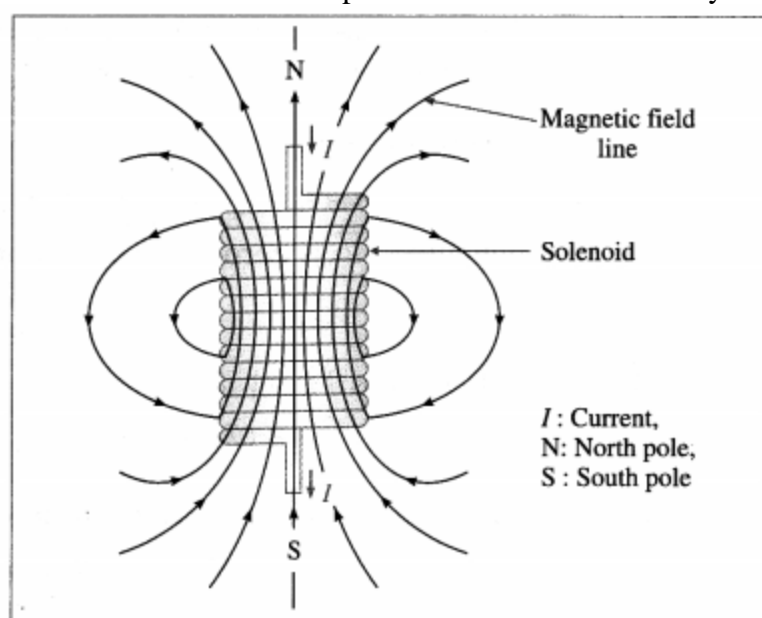
Question 1.

What is an ideal solenoid?

Answer:

A solenoid is a long wire wound in the form of a helix. An ideal solenoid is tightly wound and infinitely long, i.e., its turns are closely spaced and the solenoid is very long compared to its crosssectional radius.

Each turn of a solenoid acts approximately as a circular loop. Suppose the solenoid carries a steady current I . The net magnetic field due to the current in the solenoid is the vector sum of the fields due to the current in all the turns. In the case of a tightly-wound solenoid of finite length, Fig. 10.39, the magnetic field lines are approximately parallel only near the centre of the solenoid, indicating a nearly uniform field there. However, close to the ends, the field lines diverge from one end and converge at the other end. This field distribution is similar to that of a bar magnet. Thus, one end of the solenoid behaves like the north pole of a magnet and the opposite end behaves like the south pole. The field outside is very weak near the midpoint.



Magnetic field lines produced by a steady current in a tightly-wound solenoid of finite length

For an ideal solenoid, the magnetic field inside is reasonably uniform over the cross section and parallel to the axis throughout the volume enclosed by the solenoid. The field outside is negligible in this case.

[Use your brain power \(Textbook Page No. 248\)](#)

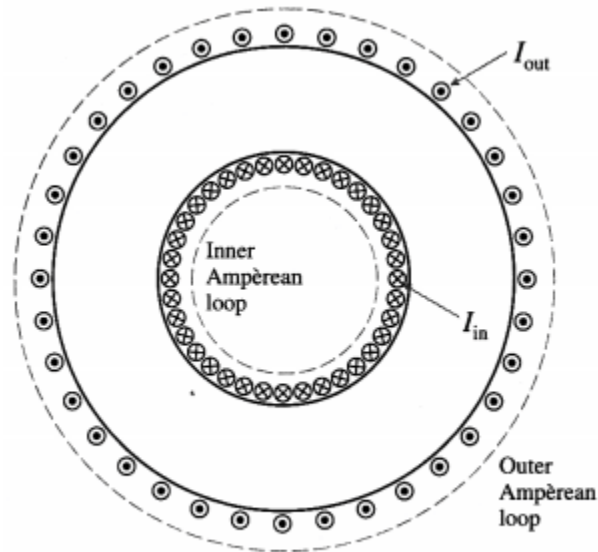
Question 1.

Choosing different Amperian loops, show that out-side an ideal toroid $B = 0$.

Answer:

From below figure, the inner Amperian loop does not enclose any current while the outer Amperian loop encloses an equal number of

I_{in} and I_{out} . Hence, by Ampere's law, $B = 0$ outside an ideal toroid.

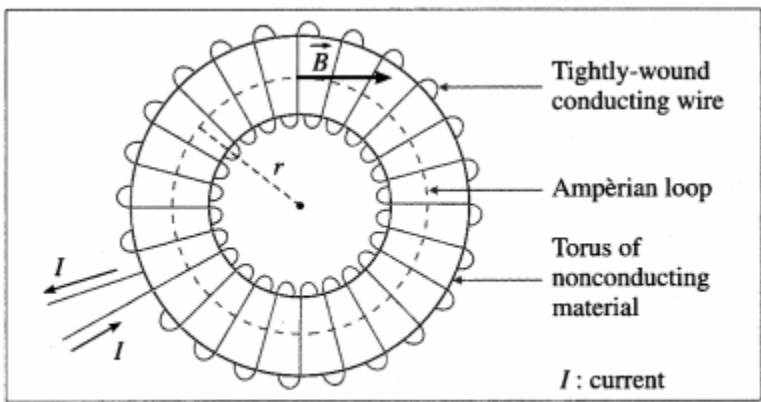


Question 2.

What is an ideal toroid?

Answer:

A toroid is a toroidal solenoid. An ideal toroid consists of a long conducting wire wound tightly around a torus, a doughnut-shaped ring, made of a nonconducting material.



A toroid carrying a steady current

In an ideal toroid carrying a steady current, the magnetic field in the interior of the toroid is tangential to any circle concentric with the axis of the toroid and has the same value on this circle (the dashed line in figure). Also, the magnitude of the magnetic induction external to the toroid is negligible.

Categories