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Maharashtra State Board 11th Maths Solutions Chapter 3 Trigonometry – II Ex 3.1

Question 1.
Find the values of:
i. sin 150°
ü. cos 75°
iii. tan 105°
iv. cot 225°
Solution:
i. sin 15° = sin (45° – 30°)
= sin 45° cos 30° – cos 45° sin 30°
$\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)=\frac{\sqrt{3}-1}{2\sqrt{2}}$
[Note: Answer given in the textbook is $\frac{\sqrt{3}+1}{2\sqrt{2}}$ However, as per our calculation it is $\frac{\sqrt{3}-1}{2\sqrt{2}}$

ii. cos 75° = cos (45° + 30°)
= cos 45° cos 30° – sin 45° sin 30°

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

iii. tan 105° = tan (60° +45°)

$$= \frac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$$

$$= \frac{\sqrt{3}+1}{1-\left(\sqrt{3}\right)(1)}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}}$$

iv. cot 225°

$$\cot 225° = \frac{1}{\tan 225°} = \frac{1}{\tan\left(180° + 45°\right)}$$

$$= \frac{1}{\left(\frac{\tan 180° + \tan 45°}{1 - \tan 180° \tan 45°}\right)}$$

$$= \frac{1}{\left(\frac{0+1}{1-0(1)}\right)}$$

$$= \frac{1}{\left(\frac{1}{1}\right)}$$

$$= 1$$

Question 2.
Perove the following:
i. $\cos\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-y\right)-\sin\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-y\right)=-\cos(x+y)$
Solution:
L.H.S

$$= \cos\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - y\right) - \sin\left(\frac{\pi}{2} - x\right) \sin\left(\frac{\pi}{2} - y\right)$$

$$= \sin x \sin y - \cos x \cos y \quad \ldots \left[\because \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta, \ \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta\right]$$

= -(cos x cos y – sin x sin y)
= – cos (x+y)
= R.H.S

ii. $tan(\pi4+\theta)=1+tan\theta1-tan\theta$

L.H.S $=tan(\pi4+\theta)$

$$= \frac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\tan\theta}$$

$$= \frac{1 + \tan\theta}{1-(1)\tan\theta}$$

$$= \frac{1 + \tan\theta}{1-\tan\theta}$$

R.H.S.
[Note : The question has been modified.]

iii. $(1+tanx1-tanx)2=tan(\pi4+x)tan(\pi4-x)$
Solution:

$$\text{R.H.S.} = \frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$$

$$= \frac{\left(\dfrac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}\right)} = \frac{\left(\dfrac{1 + \tan x}{1-(1)\tan x}\right)}{\left(\dfrac{1 - \tan x}{1+(1)\tan x}\right)}$$

$$= \left(\frac{1 + \tan x}{1 - \tan x}\right) \times \left(\frac{1 + \tan x}{1 - \tan x}\right)$$

$$= \left(\frac{1 + \tan x}{1 - \tan x}\right)^2 = \text{L.H.S.}$$

iv. sin [(n+1)A] . sin [(n+2)A] + cos [(n+1)A] . cos [(n+2)A] = cos A
Solution:
L.H.S. = sin [(n + 1)A] . sin [(n + 2)A] + cos [(n + 1)A] . cos [(n + 2)A]
= cos [(n + 2)A] . cos [(n + 1)A] + sin [(n + 2)A] . sin [(n + 1)A]
Let(n+2)Aaand(n+l)Ab ...(i)
∴ L.H.S. = cos a. cos b + sin a. sin b
= cos (a — b)
= cos [(n + 2)A — (n + I)A]
...[From (i)]
cos[(n+2 – n – 1)A]
= cos A
= R.H.S.

v. 2−√cos(π4−A)=cosA+sinA

Solution:

$$\text{L.H.S.} = \sqrt{2}\cos\left(\frac{\pi}{4} - \text{A}\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4}\cos\text{A} + \sin\frac{\pi}{4}\sin\text{A}\right)$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos\text{A} + \frac{1}{\sqrt{2}}\sin\text{A}\right)$$

$$= \frac{\sqrt{2}}{\sqrt{2}}(\cos\text{A} + \sin\text{A})$$

$$= \cos\text{A} + \sin\text{A}$$

$$= \text{R.H.S.}$$

vi. cos(x−y)cos(x+y)=cotxcoty+1cotxcoty−1

Solution:

$$\text{L.H.S.} = \frac{\cos(x-y)}{\cos(x+y)}$$

$$= \frac{\cos x\cos y + \sin x\sin y}{\cos x\cos y - \sin x\sin y}$$

Dividing numerator and denominator by $\sin x \sin y$, we get

$$\text{L.H.S.} = \frac{\left(\dfrac{\cos x\cos y}{\sin x\sin y} + 1\right)}{\left(\dfrac{\cos x\cos y}{\sin x\sin y} - 1\right)}$$

$$= \frac{\cot x\cot y + 1}{\cot x\cot y - 1}$$

$$= \text{R.H.S.}$$

vii. cos (x + y). cos (x – y) = $\cos^2 y - \sin^2 x$

Solution:

L.H.S. = cos(x + y). cos(x – y)

= (cos x cos y – sin x sin y). (cos x cos y + sin x sin y)

= $\cos^2 x \cos^2 y - \sin^2 x \sin^2 y$

...[∵ (a – b) (a + b) = $a^2 - b^2$]

= $(1 - \sin^2 x)\cos^2 y - \sin^2 x(1 - \cos^2 y)$

...[∵ $\sin^2 e + \cos^2 0 = 1$]

= $\cos^2 y - \cos^2 y \sin^2 x - \sin^2 x + \sin^2 x \cos^2 y$

= $\cos^2 y - \sin^2 x$

=R.H.S.

viii.tan5A–tan3Atan5A+tan3A=sin2Asin8A

Solution:

$$\text{L.H.S.} = \frac{\tan 5A - \tan 3A}{\tan 5A + \tan 3A}$$

$$= \frac{\frac{\sin 5A}{\cos 5A} - \frac{\sin 3A}{\cos 3A}}{\frac{\sin 5A}{\cos 5A} + \frac{\sin 3A}{\cos 3A}}$$

$$= \frac{\left(\frac{\sin 5A \cos 3A - \sin 3A \cos 5A}{\cos 5A \cos 3A}\right)}{\left(\frac{\sin 5A \cos 3A + \sin 3A \cos 5A}{\cos 5A \cos 3A}\right)}$$

$$= \frac{\sin 5A \cos 3A - \cos 5A \sin 3A}{\sin 5A \cos 3A + \cos 5A \sin 3A}$$

$$= \frac{\sin(5A - 3A)}{\sin(5A + 3A)}$$

$$= \frac{\sin 2A}{\sin 8A}$$

$$= \text{R.H.S.}$$

ix. tan 8θ – tan 5θ – tan 3θ = tan 8θ tan 5θ tan 3θ

Solution:

Since, 8θ = 5θ + 3θ

∴ tan 8θ = tan (5θ + 3θ)

∴ tan 8θ = tan5θ+tan3θ1–tan5θtan3θ

∴ tan 8θ (1 – tan 5θ.tan 3θ) = tan 5θ + tan 3θ

∴ tan 8θ – tan8θ.tan5θ.tan3θ = tan5θ + tan 3θ

∴ tan 8θ – tan 5θ – tan 3θ = tan 8θ.tan 5θ.tan 3θ

x. tan 50° = tan 40° + 2tan 10°

Solution:

Since, 50° = 10° +40°

∴ tan 50° = tan (10° + 40°)

∴ tan10∘ +tan40∘ 1–tan10∘ tan40∘

∴ tan 50° (1 – tan 10° tan 40°) = tan 10° + tan 40°

∴ tan 50° – tan 10° tan 40° tan 50° = tan 10° + tan 40°

∴ tan 50° – tan 10° tan 40° tan (90° – 40°) = tan 10° + tan 40°

∴ tan 50° – tan 10° tan 40° cot 40°

= tan 10° + tan 40° …[∵ tan (90° – θ) = cot θ]

∴ tan 50° – tan 10° tan 40°. 1tan40∘ = tan 10° + tan 40°

∴ tan 50° – tan 10°. 1 = tan 10° + tan 40°

∴ tan 50° = tan 40° + 2 tan 10°

xi. cos27∘ +sin27∘ cos27∘ –sin27∘ = tan 72°

Solution:

cos27∘ +sin27∘ cos27∘ –sin27∘

Dividing numerator and cos 27°, we get denominator by cos 27°, we get

$$\text{L.H.S.} = \frac{1 + \frac{\sin 27^\circ}{\cos 27^\circ}}{1 - \frac{\sin 27^\circ}{\cos 27^\circ}}$$

$$= \frac{1 + \tan 27^\circ}{1 - \tan 27^\circ}$$

$$= \frac{\tan 45^\circ + \tan 27^\circ}{1 - \tan 45^\circ \tan 27^\circ} \quad \ldots[\because \tan 45^\circ = 1]$$

$$= \tan(45^\circ + 27^\circ)$$

$$= \tan 72^\circ = \text{R.H.S.}$$

= tan (45° + 27°)

= tan 72° = R.H.S

xii. $\frac{\cos 27^\circ + \sin 27^\circ}{\cos 27^\circ - \sin 27^\circ} = \tan 72^\circ$

Solution:
Since 45° = 10° + 35°,
tan 45° = tan (10° +35°)
∴ $\frac{\tan 10^\circ + \tan 35^\circ}{1 - \tan 10^\circ \tan 35^\circ}$
∴ 1 – tan 10° tan 35o = tan 10° + tan 35°
∴ tan 10° + tan 35° + tan 10° tan 35° = 1

xiii. tan 10° + tan 35° + tan 10°. tan 35° = 1
Solution:

$$\text{L.H.S.} = \frac{\cot A \cot 4A + 1}{\cot A \cot 4A - 1}$$

$$= \frac{\frac{\cos A}{\sin A} \cdot \frac{\cos 4A}{\sin 4A} + 1}{\frac{\cos A}{\sin A} \cdot \frac{\cos 4A}{\sin 4A} - 1}$$

$$= \frac{\frac{\cos A \cos 4A + \sin A \sin 4A}{\sin A \sin 4A}}{\frac{\cos A \cos 4A - \sin A \sin 4A}{\sin A \sin 4A}}$$

$$= \frac{\cos 4A \cos A + \sin 4A \sin A}{\cos 4A \cos A - \sin 4A \sin A}$$

$$= \frac{\cos(4A - A)}{\cos(4A + A)}$$

$$= \frac{\cos 3A}{\cos 5A} = \text{R.H.S.}$$

xiv. $\frac{\cos 15^\circ - \sin 15^\circ}{\cos 15^\circ + \sin 15^\circ} = \frac{1}{\sqrt{3}}$

Solution:
Dividing numerator and cos 15°, we get

$$\text{L.H.S.} = \frac{1 - \frac{\sin 15^\circ}{\cos 15^\circ}}{1 + \frac{\sin 15^\circ}{\cos 15^\circ}}$$

$$= \frac{1 - \tan 15^\circ}{1 + \tan 15^\circ}$$

$$= \frac{\tan 45^\circ - \tan 15^\circ}{1 + (\tan 45^\circ)(\tan 15^\circ)} \ldots [\because \tan 45^\circ = 1]$$

$$= \tan(45^\circ - 15^\circ)$$

$$= \tan 30^\circ = \frac{1}{\sqrt{3}} = \text{R.H.S.}$$

= tan (45° + 15°)
= tan 30° = $\frac{1}{\sqrt{3}}$ = R.H.S

Question 3.
If sin A = $-\frac{5}{13}$, π < A < $\frac{3\pi}{2}$ and cos B = $\frac{3}{5}$, $\frac{3\pi}{2}$ < B < 2π, find
i. sin (A+B)
ii. cos (A-B)
iii. tan (A + B)
Solution:
Given, sin A = $-\frac{5}{13}$
We know that,
$\cos^2 A = 1 - \sin^2 A = 1 - \left(-\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}$
∴ cos A = $\pm\frac{12}{13}$
Since, π < A < $\frac{3\pi}{2}$
∴ 'A' lies in the 3rd quadrant.

$\therefore$ cos A<0
cos A = $-\frac{12}{13}$
Also,cos B = $\frac{3}{5}$

$\therefore$ $\sin^2 B = 1 - \cos^2 B = 1-\left(\frac{3}{5}\right)^2 = 1-\frac{9}{25} = \frac{16}{25}$

$\therefore$ sin B = $\pm\frac{4}{5}$
Since, $\frac{3\pi}{2} < B < 2\pi$
$\therefore$ 'B' lies in the 4th quadrant.
$\therefore$ sin B<0
Sin B = $-\frac{4}{5}$

i. sin (A + B) = sin A cos B+cos A sin B

$$= \left(-\frac{5}{13}\right)\left(\frac{3}{5}\right)+\left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right)$$

$$= -\frac{15}{65}+\frac{48}{65}=\frac{33}{65}$$

ii. cos (A -B) = cos A cos B + sin A sin B

$$= \left(-\frac{12}{13}\right)\left(\frac{3}{5}\right)+\left(-\frac{5}{13}\right)\left(-\frac{4}{5}\right)$$

$$= -\frac{36}{65}+\frac{20}{65}=-\frac{16}{65}$$

iii.

$$\tan A = \frac{\sin A}{\cos A} = \frac{\left(-\frac{5}{13}\right)}{\left(-\frac{12}{13}\right)} = \frac{5}{12}$$

$$\tan B = \frac{\sin B}{\cos B} = \frac{\left(-\frac{4}{5}\right)}{\left(\frac{3}{5}\right)} = -\frac{4}{3}$$

$$\tan (A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B} = \frac{\frac{5}{12}-\frac{4}{3}}{1-\left(\frac{5}{12}\right)\left(-\frac{4}{3}\right)}$$

$$= \frac{\left(-\frac{33}{36}\right)}{\left(\frac{56}{36}\right)} = -\frac{33}{56}$$

Question 4.
If tan A = $\frac{5}{6}$, tan B = $\frac{1}{11}$ prove that A + B = $\frac{\pi}{4}$
Solution:
Given tan A = $\frac{5}{6}$, tan B = $\frac{1}{11}$

$$\tan (A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B} = \frac{\frac{5}{6}+\frac{1}{11}}{1-\left(\frac{5}{6}\right)\left(\frac{1}{11}\right)}$$

$$= \frac{\left(\frac{61}{66}\right)}{\left(\frac{61}{66}\right)} = 1$$

$\therefore$ tan (A + B) = tan $\frac{\pi}{4}$
$\therefore$ A + B = $\frac{\pi}{4}$

Maharashtra State Board 11th Maths Solutions Chapter 3 Trigonometry – II Ex 3.2

Question 1.
Find the values of:
i. sin 690°
ii. sin 495°
iii. cos 315°
iv. cos 600°
v. tan 225°
vi. tan (- 690°)
vii. sec 240°
viii. sec (- 855°)
ix. cosec 780°
x. cot (-1110°)
Solution:
i. sin 690° = sin (720° -30°)
Solution:
i. sin 690° = sin (720° -30°)
= sin (2 x 360° – 30°)
= – sin 30°
= -12

ii. sin 495° = sin (360° + 135°)
= sin (135°)
= sin (90° + 45°)
= cos 45°
= 12√

iii. cos 315° = cos (270° + 45°)
sin 45° = 12√

iv. cos 600° = cos (360° + 240°)
= cos 240°
= cos (180° + 60°)
= – cos 60°
= −12

v. tan 225° = tan (180° + 45°)
= tan 45°
= 1 .

vi. tan (- 690°) = – tan 690°
= – tan (720° – 30°)
= – tan (2 x 360° – 30°)
= – (- tan 30°)
= tan 30°
= 13√

vii. sec 240° = sec (180° + 60°)
= – sec 60°
= – 2

viii. sec (-855°) = sec (855°)
= sec (720°+135°)
= sec (2 x360°+ 135°) = sec 135°
= sec (90° + 45°)
= – cosec 45°
= −2–√

ix. cosec 780° = cosec (720° + 60°)
= cosec (2 x 360° + 60°)

= cosec 60°
= 23√

x. cot (-1110°) =-cot (1110°)
= -cot (1080°+ 30°)
= – cot (3 x 360° + 30°)
= – cot 30°
= – 3–√

Question 2.
Prove the following:
i. cos(π+x)cos(−x)sin(π−x)cos(π2+x)=cot2x
ii. cos(3π2+x)cos(2π+x)[cot(3π2−x)+cot(2π+x)]
iii. sec 840° cot (- 945°) + sin 600° tan (- 690°) = 3/2
iv. cosec(90∘−x)sin(180∘−x)cot(360∘−x)sec(180∘+x)tan(90∘+x)sin(−x)=1
v. sin3(π+x)sec2(π−x)tan(2π−x)cos2(π2+x)sin(π−x)cosec2(−x)=tan3x
vi. cos θ + sin (270° + θ) – sin (270° – θ) + cos (180° + θ) = 0
Solution:
i.

$$\text{L.H.S.} = \frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)}$$

$$= \frac{(-\cos x)(\cos x)}{(\sin x)(-\sin x)}$$

$$= \frac{\cos^2 x}{\sin^2 x}$$

$$= \cot^2 x$$

$$= \text{R.H.S.}$$

ii. L.H.S.
= cos (3π2 + x) cos (2π + x) . [cot (– x) + (2π + x)]
= (sin x)(cos x) (tan x + cot x)
= sin x cos x ((sinxcosx+cosxsinx))
= sin x cos x (sin2x+cos2xsinxcosx)
= sin x cos x (1sinxcosx)
= 1 = R.H.S

iii. sec 840° = sec (720° + 120°)
= sec (2 x 360° + 120°)
= sec (120°)
= sec (90° + 30°)
= – cosec 30°
= -2

cot(-945°) = -cot 945°
= -cot (720° + 225°)
= -cot (2 x 360° +225°)
= -cot (225°)
= -cot (180° + 459)
= -cot 45°
= -1

sin 600° = sin (360° + 240°)
= sin (240°)
= sin (180° +60°)
= – sin 60° = −3√2

tan (-690°) = – tan 690°
= – tan (360° +330°)
= -tan (330°)
=- tan (360° – 30°)

=-(-tan 30°)
= tan 30°0 = 13√

L.H.S. = sec 840° cot (-945°) + sin 600° tan (-690°)
= (-2)(-1) + (−3√2)(13√)

= 2 – 12=32

= R. H. S.

iv.

$$\text{L.H.S.} = \frac{\operatorname{cosec}(90^\circ - x).\sin(180^\circ - x).\cot(360^\circ - x)}{\sec(180^\circ + x).\tan(90^\circ + x).\sin(-x)}$$

$$= \frac{\sec x.\sin x.(-\cot x)}{(-\sec x).(-\cot x).(-\sin x)}$$

= 1
= R.H.S

v.

$$\text{L.H.S.} = \frac{\sin^3(\pi + x)\sec^2(\pi - x)\tan(2\pi - x)}{\cos^2\left(\frac{\pi}{2} + x\right)\sin(\pi - x)\operatorname{cosec}^2(-x)}$$

$$= \frac{[\sin(\pi + x)]^3[\sec(\pi - x)]^2\tan(2\pi - x)}{\left[\cos\left(\frac{\pi}{2} + x\right)\right]^2\sin(\pi - x).(-\operatorname{cosec} x)^2}$$

$$= \frac{(-\sin x)^3(-\sec x)^2(-\tan x)}{(-\sin x)^2.\sin x.\operatorname{cosec}^2 x}$$

$$= \frac{(-\sin^3 x).\sec^2 x.(-\tan x)}{\sin^2 x.\sin x.\frac{1}{\sin^2 x}}$$

$$= \frac{\sin^3 x.\sec^2 x.\tan x}{\sin x}$$

$$= \sin^2 x \cdot \frac{1}{\cos^2 x} \cdot \tan x$$

$$= \tan^2 x.\tan x$$

$$= \tan^3 x$$

= R.H.S.

vi. L.H.S. = cos θ + sin (270° + θ) – sin (270° – θ) + cos (180° + θ)
= cos θ + (- cos θ)-(- cos θ) – cos θ
= cos θ – cos θ + cos θ – cos θ
= 0
= R.H.S.

Maharashtra State Board 11th Maths Solutions Chapter 3 Trigonometry – II Ex 3.3

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Question 1.
Find the values of:
i. sin $\frac{\pi}{8}$
ii. $\frac{\pi}{8}$
Solution:
We know that $\sin^2 \theta$ = 1−cos2θ2[/atex]Substitutingθ=[latex]π8, we get

$$\sin^2\frac{\pi}{8} = \frac{1-\cos\frac{\pi}{4}}{2} = \frac{1-\frac{1}{\sqrt{2}}}{2} = \frac{\sqrt{2}-1}{2\sqrt{2}}$$

$$\therefore \quad \sin\frac{\pi}{8} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}} \quad \ldots\left[\because \sin\frac{\pi}{8} \text{ is positive}\right]$$

$$\therefore \quad \sin\frac{\pi}{8} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}}$$

$$= \sqrt{\frac{2-\sqrt{2}}{4}}$$

$$\therefore \quad \sin\frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

ii. We know that, $\cos^2 \theta = \frac{1+\cos 2\theta}{2}$
Substituting θ = $\frac{\pi}{8}$, we get

$$\cos^2\frac{\pi}{8} = \frac{1+\cos\frac{\pi}{4}}{2}$$

$$= \frac{1+\frac{1}{\sqrt{2}}}{2} = \frac{\sqrt{2}+1}{2\sqrt{2}}$$

$$\therefore \quad \cos\frac{\pi}{8} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}} \quad \ldots\left[\because \cos\frac{\pi}{8} \text{ is positive}\right]$$

$$\therefore \quad \cos\frac{\pi}{8} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}}$$

$$= \sqrt{\frac{2+\sqrt{2}}{4}}$$

$$\therefore \quad \cos\frac{\pi}{8} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

Question 2.
Find sin 2x, cos 2x, tan 2x if sec x = $\frac{-13}{5}$, $\frac{\pi}{2}$ < x < π
Solution:
sec x = $\frac{-13}{5}$, $\frac{\pi}{2}$ < x < π
We know that
$\sec^2 x = 1 + \tan^2 x$
$\tan^2 x = \frac{169}{25} - 1 = \frac{144}{25}$
tan x = $\pm\frac{12}{5}$
Since $\frac{\pi}{2}$ < x < π
x lies in the 2nd quadrant.
tan x < 0

$$\therefore \quad \tan x = -\frac{12}{5}$$

$$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$$

$$= \frac{2\left(-\frac{12}{5}\right)}{1+\left(-\frac{12}{5}\right)^2}$$

$$= \frac{-\frac{24}{5}}{1+\frac{144}{25}} = \frac{\frac{-24}{5}}{\frac{169}{25}}$$

$$= \frac{-24}{5} \times \frac{25}{169} = \frac{-120}{169}$$

$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$$

$$= \frac{1-\left(-\frac{12}{5}\right)^2}{1+\left(-\frac{12}{5}\right)^2}$$

$$= \frac{1-\frac{144}{25}}{1+\frac{144}{25}} = \frac{25-144}{25+144}$$

$$= -\frac{119}{169}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$= \frac{2\left(-\frac{12}{5}\right)}{1-\left(-\frac{12}{5}\right)^2}$$

$$= \frac{-\frac{24}{5}}{1-\frac{144}{25}}$$

$$= \frac{-\frac{24}{5}}{-\frac{119}{25}} = \frac{24}{5} \times \frac{25}{119}$$

$$= \frac{120}{119}$$

Question 3.
i. = $\tan^2 \theta$
Solution:
L. H. S. = $\frac{1-\cos 2\theta}{1+\cos 2\theta}$
= $\frac{2\sin^2\theta}{2\cos^2\theta}$
= $2\tan^2 \theta$
= R.H.S.

ii. (sin 3x + sin x) sin x + (cos 3x – cos x) cos x = 0
Solution:
L.H.S. = (sin 3x + sin x) sin x + (cos 3x – cosx)cosx
= sin 3x sin x + $\sin^2 x$ + cos 3x cos x – $\cos^2 x$
= (cos 3x cos x + sin 3x sin x)
— ($\cos^2 x$ — $\sin^2 x$)
= cos (3x – x) – cos 2x
= cos 2x – cos 2x
= 0
= R.H.S.

iii. $(\cos x + \cos y)^2 + (\sin x + \sin y)^2 = 4\cos^2\left(\frac{x-y}{2}\right)$

Solution:

L.H.S. = $(\cos x + \cos y)^2 + (\sin x + \sin y)^2$
$= \cos^2 x + \cos^2 y + 2\cos x.\cos y + \sin^2 x + \sin^2 y + 2\sin x.\sin y$
$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2(\cos x.\cos y + \sin x.\sin y)$
$= 1 + 1 + 2\cos(x - y)$
$= 2 + 2\cos(x - y)$
$= 2[1 + \cos(x - y)]$
$= 2[2\cos^2[(\left(\frac{x-y}{2}\right))]$ … [$\because 1 + \cos\theta = 2\cos^2\frac{\theta}{2}$]
$= 4\cos^2\left(\frac{x-y}{2}\right)$
= R.H.S.

[Note: The question has been modified]

iv. $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\left(\frac{x-y}{2}\right)$

Solution:

L.H.S. = $(\cos x - \cos y)^2 + (\sin x - \sin y)^2$
$= \cos^2 x + \cos^2 y + 2\cos x.\cos y + \sin^2 x + \sin^2 y + 2\sin x.\sin y$
$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2(\cos x.\cos y + \sin x.\sin y)$
$= 1 + 1 - 2\cos(x - y)$
$= 2 - 2\cos(x - y)$
$= 2[1 - \cos(x - y)]$
$= 2[2\sin^2[(\left(\frac{x-y}{2}\right))]$ … [$\because 1 - \cos\theta = 2\sin^2\frac{\theta}{2}$]
$= 4\sin^2\left(\frac{x-y}{2}\right)$
= R.H.S.

v. $\tan x + \cot x = 2\operatorname{cosec} 2x$

Solution:

L.H.S. = $\tan x + \cot x$

$$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$= \frac{1}{\sin x \cos x} = \frac{2}{2\sin x \cos x}$$

$$= \frac{2}{\sin 2x}$$

$$= 2\operatorname{cosec} 2x = \text{R.H.S.}$$

vi. $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$

Solution:

$$\text{L.H.S.} = \frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x}$$

$$= \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$$

$$= \frac{(\cos^2 x + \sin^2 x + 2\sin x \cos x) - (\cos^2 x + \sin^2 x - 2\sin x \cos x)}{\cos^2 x - \sin^2 x}$$

$$= \frac{1 + 2\sin x \cos x - 1 + 2\sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$= \frac{2(2\sin x \cos x)}{\cos^2 x - \sin^2 x}$$

$$= \frac{2\sin 2x}{\cos 2x}$$

$$= 2\tan 2x$$

= R.H.S.

vii. 2+2+2+2cos8x−−−−−−−−−√−−−−−−−−−−−−−−√−−−−−−−−−−−−−−−−−−−√ = 2 cos x
Solution:

$$\text{L.H.S.} = \sqrt{2+\sqrt{2+\sqrt{2+2\cos 8x}}}$$
$$= \sqrt{2+\sqrt{2+\sqrt{2[1+\cos 2(4x)]}}}$$
$$= \sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2 4x}}}$$
$$\ldots[\because 1+\cos 2\theta = 2\cos^2\theta]$$
$$= \sqrt{2+\sqrt{2+2\cos 4x}}$$
$$= \sqrt{2+\sqrt{2[1+\cos 2(2x)]}}$$
$$= \sqrt{2+\sqrt{2\times 2\cos^2 2x}}$$
$$= \sqrt{2+2\cos 2x} = \sqrt{2(1+\cos 2x)}$$
$$= \sqrt{2\times 2\cos^2 x}$$

= 2 cos x
= R.H.S.
[Note : The question has been modified.]

viii. 16 sin θ cos θ cos 2θ cos 4θ cos 8θ = sin 16θ
Solution:
L.H.S. = 16 sin θ cos θ cos 2θ cos 4θ cos 8θ
= 8(2sinθ cosθ) cos2θ cos 4θ cos 8θ
= 8sin 2θ cos 2θ cos 4θ cos 8θ
= 4(2sin 2θ cos 2θ) cos 4θ cos 8θ
= 4sin 4θ cos 4θ cos 8θ
= 2(2sin 4θ cos 4θ) cos 8θ
= 2sin 8θ cos 8θ
= sin 16θ
= R.H.S.

ix. \(= 2 cot 2x
Solution:

$$\text{L.H.S.} = \frac{\sin 3x}{\cos x} + \frac{\cos 3x}{\sin x}$$
$$= \frac{\sin 3x \sin x + \cos 3x \cos x}{\sin x \cos x}$$
$$= \frac{\cos(3x-x)}{\sin x \cos x}$$
$$= \frac{2\cos 2x}{2\sin x \cos x}$$
$$= \frac{2\cos 2x}{\sin 2x}$$
$$= 2\cot 2x$$
$$= \text{R.H.S.}$$

x. [latex]\frac{\cos x}{1+\sin x}=\frac{\cot \left(\frac{x}{2}\right)-1}{\cot \left(\frac{x}{2}\right)+1}\)
Solution:

$$\text{L.H.S.} = \frac{\cos x}{1+\sin x}$$

$$= \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos^2\frac{x}{2} + \sin^2\frac{x}{2}\right) + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}$$

$$= \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$$

Dividing numerator and denominator by $\sin\frac{x}{2}$,

we get

$$\text{L.H.S.} = \frac{\dfrac{\cos\frac{x}{2}}{\sin\frac{x}{2}} - 1}{\dfrac{\cos\frac{x}{2}}{\sin\frac{x}{2}} + 1}$$

$$= \frac{\cot\frac{x}{2} - 1}{\cot\frac{x}{2} + 1}$$

$$= \text{R.H.S.}$$

xi. tan(θ2)+cot(θ2)cot(θ2)−tan(θ2)=secθ

Solution:

$$\text{L.H.S.} = \frac{\tan\frac{\theta}{2} + \cot\frac{\theta}{2}}{\cot\frac{\theta}{2} - \tan\frac{\theta}{2}}$$

$$= \frac{\dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} + \dfrac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}}{\dfrac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} - \dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}$$

$$= \frac{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta = \text{R.H.S.}$$

xii. 1tan3A−tanA−1cot3A−cotA = cot 2A

Solution:

$$\text{L.H.S.} = \frac{1}{\tan 3A - \tan A} - \frac{1}{\cot 3A - \cot A}$$
$$= \frac{1}{\tan 3A - \tan A} - \frac{1}{\frac{1}{\tan 3A} - \frac{1}{\tan A}}$$
$$= \frac{1}{\tan 3A - \tan A} - \frac{\tan 3A . \tan A}{\tan A - \tan 3A}$$
$$= \frac{1}{\tan 3A - \tan A} + \frac{\tan 3A . \tan A}{\tan 3A - \tan A}$$
$$= \frac{1 + \tan 3A . \tan A}{\tan 3A - \tan A}$$
$$= \frac{1}{\frac{\tan 3A - \tan A}{1 + \tan 3A . \tan A}}$$
$$= \frac{1}{\tan(3A - A)}$$
$$= \frac{1}{\tan 2A}$$
$$= \cot 2A = \text{R.H.S.}$$

xiii. cos 7° cos 14° cos 28° cos 56° sin68∘ 16cos83∘
Solution:
L.H.S. = cos 7° cos 14° cos 28° cos 56°
= 12sin7∘ (2sin 7°cos 7°)cos 14°cos 28°cos 56°
= 12sin7∘ (sin 14° cos 14° cos 28° cos 56°)
…[∵ 2sinθ cosθ = sin 2θ]
= [\frac{1}{2\left(2 \sin 7^{\circ}\right)}latex][/latex] (2sin 14° cos 14°) cos 28° cos 56°
= 14sin7∘ (sin 28° cos 28° cos 56°)
= 12(4sin7∘)(2 sin 28° cos 28°) cos 56°
= 18sin7∘ (sin 56° cos 56°)
= 18sin7∘ (2 sin 56° cos 56°)
= 116sin7∘ (sin 112°)
= sin(180∘ −68∘)16sin(90∘ −83∘)
= sin68∘ 16cos83∘
= R.H.S.

xiv. = sin2(−160∘)sin270∘ +sin(180∘ −θ)sinθ = sec2 20°
Solution:

$$\text{L.H.S.} = \frac{\sin^2(-160^\circ)}{\sin^2 70^\circ} + \frac{\sin(180^\circ - \theta)}{\sin\theta}$$
$$= \frac{(-\sin 160^\circ)^2}{\sin^2 70^\circ} + \frac{\sin\theta}{\sin\theta}$$
$$= \frac{\sin^2 160^\circ}{\sin^2 70^\circ} + 1$$
$$= 1 + \frac{(\sin 160^\circ)^2}{(\sin 70^\circ)^2}$$
$$= 1 + \frac{[\sin(180^\circ - 20^\circ)]^2}{[\sin(90^\circ - 20^\circ)]^2}$$
$$= 1 + \frac{\sin^2 20^\circ}{\cos^2 20^\circ}$$
$$= 1 + \tan^2 20^\circ$$
$$= \sec^2 20^\circ = \text{R.H.S.}$$

xv. 2cos4x+12cosx+1 = (2 cos x – 1)(2 cos 2x – 1)

Solution:

$$\text{L.H.S.} = \frac{2\cos 4x+1}{2\cos x+1}$$

$$= \frac{2\left[2\cos^2(2x)-1\right]+1}{2\cos x+1}$$

$$\ldots[\because \cos 2\theta = 2\cos^2\theta - 1]$$

$$= \frac{4\cos^2 2x-2+1}{2\cos x+1}$$

$$= \frac{(2\cos 2x)^2-(1)^2}{2\cos x+1}$$

$$= \frac{(2\cos 2x+1)(2\cos 2x-1)}{2\cos x+1}$$

$$= \frac{\left[2\left(2\cos^2 x-1\right)+1\right](2\cos 2x-1)}{2\cos x+1}$$

$$= \frac{\left(4\cos^2 x-2+1\right)(2\cos 2x-1)}{2\cos x+1}$$

$$= \frac{\left[(2\cos x)^2-(1)^2\right](2\cos 2x-1)}{2\cos x+1}$$

$$= \frac{(2\cos x+1)(2\cos x-1)(2\cos 2x-1)}{2\cos x+1}$$

$$= (2\cos x-1)(2\cos 2x-1)$$

= R.H.S.

xvi. = cos2 x + cos2 (x + 120°) + cos2(x – 120°) = 32

Solution:

L.H.S = cos2 x + cos2 (x + 120°) + cos2(x – 120°) =

$$= \frac{1+\cos 2x}{2} + \frac{1+\cos 2(x+120^\circ)}{2} + \frac{1+\cos 2(x-120^\circ)}{2}$$

$$\ldots\left[\because \cos^2\theta = \frac{1+\cos 2\theta}{2}\right]$$

32+12 [cos 2x + cos(2x + 240°) + cos(2x 240°)]

= 32+12(cos 2x + cos 2x cos 240°— sin 2x sin 240° + cos 2x cos 240° + sin 2x sin 240°)

= 32+12(cos 2x + 2 cos 2x cos 240°)

= 32+12 [cos 2x + 2 cos 2x cos(180° + 60°)]

= 32+12 [cos 2x + 2cos 2x(-cos 600)]

= 32+12 [cos 2x —2 cos 2x(12)]

= 32+12 (cos 2x – cos 2x)

= 32+12 (0)

= 32 = R.H.S.

xvii. 2 cosec 2x + cosec x = sec cot $\frac{x}{2}$

Solution:

$$\begin{aligned}\text{L.H.S.} &= 2\operatorname{cosec} 2x + \operatorname{cosec} x \\ &= \frac{2}{\sin 2x} + \frac{1}{\sin x} \\ &= \frac{2}{2\sin x \cos x} + \frac{1}{\sin x} \\ &= \frac{1}{\sin x \cos x} + \frac{1}{\sin x} \\ &= \frac{1+\cos x}{\sin x \cos x} \\ &= \frac{2\cos^2 \frac{x}{2}}{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)\cos x} \\ &= \frac{1}{\cos x} \cdot \frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)} \\ &= \sec x \cot\left(\frac{x}{2}\right) \\ &= \text{R.H.S.}\end{aligned}$$

xviii. 4 cos x cos ($\frac{\pi}{3}$ + x) cos ($\frac{\pi}{3}$ – x) = cos 3x

Solution:

$$\begin{aligned}\text{L.H.S.} &= 4\cos x \cdot \cos\left(\frac{\pi}{3}+x\right) \cdot \cos\left(\frac{\pi}{3}-x\right) \\ &= 4\cos x\left(\cos\frac{\pi}{3}\cos x - \sin\frac{\pi}{3}\sin x\right)\left(\cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x\right) \\ &= 4\cos x\left(\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x\right)\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right) \\ &= 4\cos x\left[\left(\frac{1}{2}\cos x\right)^2 - \left(\frac{\sqrt{3}}{2}\sin x\right)^2\right] \\ &= 4\cos x\left(\frac{1}{4}\cos^2 x - \frac{3}{4}\sin^2 x\right)\end{aligned}$$

= $\cos^3 x$ — 3cos x.$\sin^2 x$

= $\cos^3$ x — 3cos x (1— $\cos^2$ x)

= $\cos^3 x$ — 3cos x + 3 $\cos^3 x$

=4 $\cos^3 x$ — 3cos x

= cos 3x = R.H.S.

INote: The question has been modijìed.I

xix. sin x tan $\frac{x}{2}$ + 2cos x = $\frac{2}{1+\tan^2(\frac{x}{2})}$

Solution:

L.H.S. = sin x tan (x/2)+ 2cos x

= $\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)$ + 2cos x

= $\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)$ + 2 cos x

= $2\sin^2$ x/2 + 2cosx

= 1 – cosx + 2cosx

= 1 + cos x

=$2\cos^2$ x/2

= $\frac{2}{\sec^2\frac{x}{2}} = \frac{2}{1+\tan^2\frac{x}{2}}$ =R.H.S.

Maharashtra State Board 11th Maths Solutions Chapter 3 Trigonometry – II Ex 3.4

Question 1.
Express the following as a sum or difference of two trigonometric functions.
i. 2sin 4x cos 2x
ii. 2sin $\frac{2\pi}{3}$ cos $\frac{\pi}{2}$
iii. 2cos 4θ cos 2θ
iv. 2cos 35° cos 75°
Solution:
i. 2sin 4x cos 2x = sin(4x + 2x) + sin (4x – 2x)
= sin 6x + sin 2x

ii.

$$2\sin\frac{2\pi}{3}\cos\frac{\pi}{2} = \sin\left(\frac{2\pi}{3}+\frac{\pi}{2}\right)+\sin\left(\frac{2\pi}{3}-\frac{\pi}{2}\right)$$

$$= \sin\frac{7\pi}{6}+\sin\frac{\pi}{6}$$

[Note: Answer given in the textbook is sin $\frac{7\pi}{12}$ + sin $\frac{\pi}{12}$ However, as per our calculation it is sin $\frac{7\pi}{6}$ + sin $\frac{\pi}{6}$

iii. 2cos 4θ cos 2θ = cos(4θ + 2θ)+cos (4θ – 2θ)
= cos 6θ + cos 2θ

iv. 2cos 35° cos75°
= cos(35° + 75°) + cos (35° – 75°)
= cos 110° + cos (-40)°
= cos 110° + cos 40° … [∵ cos(-θ) = cos θ]

Question 2.
Prove the following:
i. $\frac{\sin 2x+\sin 2y}{\sin 2x-\sin 2y}=\frac{\tan(x+y)}{\tan(x-y)}$
Solution:

$$\text{L.H.S.} = \frac{\sin 2x+\sin 2y}{\sin 2x-\sin 2y}$$

$$= \frac{2\sin\left(\frac{2x+2y}{2}\right)\cos\left(\frac{2x-2y}{2}\right)}{2\cos\left(\frac{2x+2y}{2}\right)\sin\left(\frac{2x-2y}{2}\right)}$$

$$= \frac{\sin(x+y)\cos(x-y)}{\cos(x+y)\sin(x-y)}$$

$$= \tan(x+y)\,.\,\cot(x-y)$$

$$= \tan(x+y)\cdot\frac{1}{\tan(x-y)}$$

$$= \frac{\tan(x+y)}{\tan(x-y)}$$

$$= \text{R.H.S.}$$

ii. sin 6x + sin 4x – sin 2x = 4 cos x sin 2x cos 3x
Solution:
L.H.S. = sin 6x + sin 4x — sin 2x
= 2sin $\left(\frac{6x+4x}{2}\right)$ cos $\left(\frac{6x-4x}{2}\right)$ – 2 sin x cos x

= 2 sin 5x cos x — 2 sin x cos x
= 2 cos x (sin 5x — sin x)
= 2 cos [2cos(5x+x2)sin(5x−x2)]
= 2 cos x (2 cos 3x sin 2x)
= 4 cos x sin 2x cos 3x
= R.H.S.
[Note: The question has been modified.]

iii. sinx−sin3x+sin5x−sin7xcosx−cos3x−cos5x+cos7x = cot 2x
Solution:

$$\text{L.H.S.} = \frac{\sin x - \sin 3x + \sin 5x - \sin 7x}{\cos x - \cos 3x - \cos 5x + \cos 7x}$$

$$= \frac{(\sin 5x + \sin x) - (\sin 7x + \sin 3x)}{(\cos x - \cos 5x) - (\cos 3x - \cos 7x)}$$

$$= \frac{2\sin\left(\frac{5x+x}{2}\right)\cdot\cos\left(\frac{5x-x}{2}\right) - 2\sin\left(\frac{7x+3x}{2}\right)\cdot\cos\left(\frac{7x-3x}{2}\right)}{2\sin\left(\frac{x+5x}{2}\right)\cdot\sin\left(\frac{5x-x}{2}\right) - 2\sin\left(\frac{3x+7x}{2}\right)\cdot\sin\left(\frac{7x-3x}{2}\right)}$$

$$= \frac{2\sin 3x\cdot\cos 2x - 2\sin 5x\cdot\cos 2x}{2\sin 3x\cdot\sin 2x - 2\sin 5x\cdot\sin 2x}$$

$$= \frac{2\cos 2x\,(\sin 3x - \sin 5x)}{2\sin 2x\,(\sin 3x - \sin 5x)}$$

$$= \frac{\cos 2x}{\sin 2x} = \cot 2x = \text{R.H.S.}$$

iv. sin 18° cos 39° + sin 6° cos 15° = sin 24° cos 33°
Solution:
L.H.S. = sin 18°.cos 39° + sin 6°.cos 15°
= 12 (2 cos 39°sin 18° + 2.cos 15°.sin 6°)
= 12[sin(39° + 18°) — sin(39° — 18°) + sin (15° + 6°) — sin (15° — 6°)]
= 12(sin57° – sin21° + sin 21°- sin9°)
= 12(sin57° – sin9°)
= 12 x 2. cos (57∘ +9∘ 2)⋅ sin(57∘ −9∘ 2)
= cos 33° .sin 24°
= sin 24°. cos 33°
= R.H.S.

v. cos 20° cos 40° cos 60°cos 80° = 1/16
Solution:
L.H.S. = cos 20°.cos 40°.cos 60°.cos 80°
= cos 20°.cos 40°.12 .cos 80°
= 12×2(2 cos 40°.cos 20°).cos 80°
= 14[cos(40° + 20°) + cos(40°- 20°)].cos80°
= 14(cos 60° + cos 20°) cos 80°
=14cos 60°. cos 80° + 14 cos 20°. cos 80°
= 14(12)cos80∘ +12×4(2cos80∘ cos20∘)
= 18 cos 80° + 18[cos (80° + 20°) + cos (80° — 20°)]
= 18cos 80° + 18(cos 100° + cos 60°)
= 18 cos 80° + 18cos 100° + 18cos 60°
= 18 cos 80° = 18 cos (180° – 80°) + 18×12
= 18 cos 80° – 18 cos 80° + 116 ... [∵ cos (180 – θ) = – cos θ]
= 116 = R.H.S

vi. sin 20° sin 40° sin 60° sin 80° = 3/16
Solution:

$$\text{L.H.S.} = \sin 20^\circ . \sin 40^\circ . \sin 60^\circ . \sin 80^\circ$$

$$= \sin 20^\circ . \sin 40^\circ . \left(\frac{\sqrt{3}}{2}\right) . \sin 80^\circ$$

$$= \frac{\sqrt{3}}{2} . \frac{1}{2} (2.\sin 40^\circ.\sin 20^\circ).\sin 80^\circ$$

$$= \frac{\sqrt{3}}{4} [\cos(40^\circ - 20^\circ) - \cos(40^\circ + 20^\circ)].\sin 80^\circ$$

$$= \frac{\sqrt{3}}{4} (\cos 20^\circ - \cos 60^\circ)\sin 80^\circ$$

$$= \frac{\sqrt{3}}{4} .\cos 20^\circ.\sin 80^\circ - \frac{\sqrt{3}}{4} \cos 60^\circ.\sin 80^\circ$$

$$= \frac{\sqrt{3}}{2 \times 4} (2 \sin 80^\circ.\cos 20^\circ) - \frac{\sqrt{3}}{4} \left(\frac{1}{2}\right).\sin 80^\circ$$

$$= \frac{\sqrt{3}}{8} .[\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ)] - \frac{\sqrt{3}}{8} .\sin 80^\circ$$

$$= \frac{\sqrt{3}}{8} .(\sin 100^\circ + \sin 60^\circ) - \frac{\sqrt{3}}{8} .\sin 80^\circ$$

$$= \frac{\sqrt{3}}{8} \sin 100^\circ + \frac{\sqrt{3}}{8} \sin 60^\circ - \frac{\sqrt{3}}{8} \sin 80^\circ$$

$$= \frac{\sqrt{3}}{8} .\sin(180^\circ - 80^\circ) + \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{8} .\sin 80^\circ$$

$$= \frac{\sqrt{3}}{8} \sin 80^\circ + \frac{3}{16} - \frac{\sqrt{3}}{8} \sin 80^\circ \quad \ldots[\because \sin(180^\circ - \theta) = \sin\theta]$$

$$= \frac{3}{16} = \text{R.H.S.}$$

Maharashtra State Board 11th Maths Solutions Chapter 3 Trigonometry – II Ex 3.5

Question 1.
In Δ ABC, A + B + C = π, show that
cos2A + cos2B + cos2C = – 1 – 4 cosA cosB cosC
Solution:
L.H.S. = cos 2A + cos 2B + cos 2C
$= 2 \cdot \cos\left(\frac{2A+2B}{2}\right) \cdot \cos\left(\frac{2A-2B}{2}\right) + \cos 2C$
= 2.cos(A + B).cos (A – B) + 2$\cos^2$C – 1
In ΔABC, A + B + C = π
∴ A + B = π – C
∴ cos(A + B) = cos(π – C)
∴ cos(A + B) = – cosC ....(i)

∴ L.H.S. = – 2.cos C.cos (A – B) + 2.$\cos^2$C – 1 ...[From(i)]
= – 1 – 2.cosC.[cos(A – B) – cosC]
= – 1 – 2.cos C.[cos(A – B) + cos(A + B)]
... [From (i)]
= – 1 – 2.cos C.(2.cos A.cos B)
= – 1 – 4.cos A.cos B.cos C = R.H.S.

Question 2.
sin A + sin B + sin C = 4 cos A/2 cos B/2 cos C/2
Solution:

$$\text{L.H.S.} = \sin A + \sin B + \sin C$$
$$= 2.\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right) + 2.\sin\frac{C}{2}.\cos\frac{C}{2}$$

In Δ ABC, A + B + C = π

$$\therefore \quad A + B = \pi - C$$
$$\therefore \quad \sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{\pi - C}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cos\frac{C}{2} \quad \text{...(i)}$$
$$\text{and } \cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{\pi - C}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{C}{2}\right) = \sin\frac{C}{2} \quad \text{...(ii)}$$
$$\therefore \quad \text{L.H.S.} = 2.\cos\frac{C}{2}.\cos\left(\frac{A-B}{2}\right) + 2.\cos\left(\frac{A+B}{2}\right).\cos\frac{C}{2} \quad \text{...[From (i) and (ii)]}$$
$$= 2.\cos\frac{C}{2}.\left[\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)\right]$$
$$= 2.\cos\frac{C}{2}.2\cos\left[\frac{\frac{A+B}{2} + \frac{A-B}{2}}{2}\right].\cos\left[\frac{\frac{A+B}{2} - \frac{A-B}{2}}{2}\right]$$
$$= 2.\cos\frac{C}{2}.\left(2.\cos\frac{A}{2}.\cos\frac{B}{2}\right)$$
$$= 4.\cos\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2} = \text{R.H.S.}$$

Question 3.
cos A + Cos B + Cos C = 4 cos A/2 cos B/2 cos C/2
= 2 · cos(A+B2) · cos(A−B2)−(1−2sin2C2)
Solution:
L.H.S. = sin A + sin B + sin C
= 2 · cos(A+B2) · cos(A−B2)−(1−2sin2C2)
In Δ ABC, A + B + C = π ,

$\therefore$ A + B = π – C

$$\therefore \quad \cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{\pi-C}{2}\right) = \cos\left(\frac{\pi}{2}-\frac{C}{2}\right)$$

$$= \sin\frac{C}{2} \qquad \ldots(i)$$

$$\therefore \quad \text{L.H.S.} = 2.\sin\frac{C}{2}.\cos\left(\frac{A-B}{2}\right) - 1 + 2.\sin^2\frac{C}{2}$$

...[From (i)]

$$= 2.\sin\frac{C}{2}.\left[\cos\left(\frac{A-B}{2}\right)+\sin\frac{C}{2}\right] - 1$$

$$= 2.\sin\frac{C}{2}.\left[\cos\left(\frac{A-B}{2}\right)+\cos\left(\frac{A+B}{2}\right)\right] - 1$$

...[From (i)]

$$= 2\sin\frac{C}{2}.2\cos\left[\frac{\frac{A+B}{2}+\frac{A-B}{2}}{2}\right] .\cos\left[\frac{\frac{A+B}{2}-\frac{A-B}{2}}{2}\right] - 1$$

$$= 2.\sin\frac{C}{2}.\left(2.\cos\frac{A}{2}.\cos\frac{B}{2}\right) - 1$$

$$= 4.\cos\frac{A}{2}.\cos\frac{B}{2}.\sin\frac{C}{2} - 1$$

= R.H.S.

Question 4.
$\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin A \sin B \cos C$
Solution:
We know that, $\sin^2 = 1-cos2\theta 2$
L.H.S.
$= \sin^2 + \sin^2 B + \sin^2 C$

$$= \frac{1-\cos 2A}{2} + \frac{1-\cos 2B}{2} - \sin^2 C$$

$$= \frac{1}{2}[2 - (\cos 2A + \cos 2B)] - \sin^2 C$$

$$= \frac{1}{2}\left[2 - 2.\cos\left(\frac{2A+2B}{2}\right).\cos\left(\frac{2A-2B}{2}\right)\right] - \sin^2 C$$

= 1 – cos(A + B). cos(A – B) – $\sin^2 C$
= (1 – $\sin^2$ C) – cos (A + B). cos (A – B)
= $\cos^2$ C – cos(A + B). cos(A – B)
$\therefore$ cos(A + B) = cos(it — C)
$\therefore$ cos(A + B) = — cos C ...(i)
$\therefore$ L.H.S. = $\cos^2 C$ + cos C.cos(A – B)
... [From (i)]
= cos C[cos C + cos(A – B)]
= cos C[- cos(A + B) + cos(A – B)]
... [From (i)]
= cos C[cos (A-B) – cos(A + B)]
= cos C(2 sin A.sin B)
= 2 sin A.sin B. cos C
= R.H.S.
[Note: The question has been modified.]

Question 5.
$sin2A2+sin2B2-sin2C2 = 1-2cosA2cosB2sinC2$
Solution:

We know that, $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

L.H.S.

$= \sin^2\frac{A}{2} + \sin^2\frac{B}{2} - \sin^2\frac{C}{2}$

$= \frac{1-\cos A}{2} + \frac{1-\cos B}{2} - \sin^2\frac{C}{2}$

$= \frac{1}{2}[2-(\cos A + \cos B)] - \sin^2\frac{C}{2}$

$= \frac{1}{2}\left[2-2.\cos\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)\right] - \sin^2\frac{C}{2}$

$= 1 - \cos\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right) - \sin^2\frac{C}{2}$

In Δ ABC, $A + B + C = \pi$

$\therefore$ $A + B = \pi - C$

$\therefore$ $\cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{\pi-C}{2}\right) = \cos\left(\frac{\pi}{2}-\frac{C}{2}\right)$

$= \sin\frac{C}{2}$ ...(i)

$\therefore$ L.H.S. $= 1 - \sin\frac{C}{2}.\cos\left(\frac{A-B}{2}\right) - \sin^2\frac{C}{2}$

...[From (i)]

$= 1 - \sin\frac{C}{2}.\left[\cos\left(\frac{A-B}{2}\right) + \sin\frac{C}{2}\right]$

$= 1 - \sin\frac{C}{2}.\left[\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)\right]$

...[From (i)]

$= 1 - \sin\frac{C}{2}.2\cos\left[\frac{\frac{A+B}{2}+\frac{A-B}{2}}{2}\right]$

$.\cos\left[\frac{\frac{A+B}{2}-\frac{A-B}{2}}{2}\right]$

$= 1 - \sin\frac{C}{2}.\left(2.\cos\frac{A}{2}.\cos\frac{B}{2}\right)$

$= 1 - 2.\cos\frac{A}{2}.\cos\frac{B}{2}.\sin\frac{C}{2}$

= R.H.S.

Question 6.

tan A2 tan B2 tan B2 tan C2 tan C2 tan A2 = 1

Solution:

In Δ ABC,

A + B + C = π

∴ A + B = π – C

$$\therefore \quad \tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi - C}{2}\right)$$

$$\therefore \quad \tan\left(\frac{A}{2}+\frac{B}{2}\right) = \tan\left(\frac{\pi}{2}-\frac{C}{2}\right)$$

$$\therefore \quad \frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}.\tan\frac{B}{2}} = \cot\frac{C}{2}$$

$$\therefore \quad \frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}.\tan\frac{B}{2}} = \frac{1}{\tan\frac{C}{2}}$$

$$\therefore \quad \tan\frac{C}{2}.\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right) = 1 - \tan\frac{A}{2}.\tan\frac{B}{2}$$

$$\therefore \quad \tan\frac{A}{2}.\tan\frac{B}{2} + \tan\frac{B}{2}.\tan\frac{C}{2} + \tan\frac{C}{2}.\tan\frac{A}{2} = 1$$

Question 7.
$\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$
Solution:
In Δ ABC,
A + B + C = π
∴ A + B = π – C

$$\therefore \quad \tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi - C}{2}\right)$$

$$\therefore \quad \tan\left(\frac{A}{2}+\frac{B}{2}\right) = \tan\left(\frac{\pi}{2}-\frac{C}{2}\right)$$

$$\therefore \quad \frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}.\tan\frac{B}{2}} = \cot\frac{C}{2}$$

$$\therefore \quad \frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}.\tan\frac{B}{2}} = \frac{1}{\tan\frac{C}{2}}$$

$$\therefore \quad \tan\frac{C}{2}.\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right) = 1 - \tan\frac{A}{2}.\tan\frac{B}{2}$$

$$\therefore \quad \tan\frac{B}{2}.\tan\frac{C}{2} + \tan\frac{A}{2}.\tan\frac{C}{2} + \tan\frac{A}{2}.\tan\frac{B}{2} = 1$$

Dividing throughout by $\tan\frac{A}{2}.\tan\frac{B}{2}.\tan\frac{C}{2}$,

we get

$$\frac{1}{\tan\frac{A}{2}} + \frac{1}{\tan\frac{B}{2}} + \frac{1}{\tan\frac{C}{2}} = \frac{1}{\tan\frac{A}{2}.\tan\frac{B}{2}.\tan\frac{C}{2}}$$

$$\therefore \quad \cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}.\cot\frac{B}{2}.\cot\frac{C}{2}$$

Question 8.
tan 2A + tan 2B + tan 2C = tan 2A tan 2B + tan 2C
Solution:
In Δ ABC,
A + B + C = π
∴ 2A + 2B + 2C = 2π
∴ 2A + 2B = 2π – 2C
tan(2A + 2B) = tan(2n — 2C)
$\frac{\tan 2A+\tan 2B}{1-\tan 2A\cdot\tan 2B}$ = -tan 2C
∴ tan2A+tan2B=—tan2C.(1-tan2A.tan2B)

$\therefore$ tan 2A + tan 2B = – tan2C+ tan2A.tan2B.tan2C
$\therefore$ tan 2A + tan 2B + tan 2C = tan2A.tan2B.tan2C

Question 9.
$\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2 \sin A \sin B \sin C$
Solution:
we know that $\cos^2\theta = \frac{1+\cos 2\theta}{2}$
L.H.S.
$= \cos^2 A + \cos^2 B + \cos^2 C$

$$= \frac{1+\cos 2A}{2} + \frac{1+\cos 2B}{2} - \cos^2 C$$

$$= \frac{1}{2}[2 + (\cos 2A + \cos 2B)] - \cos^2 C$$

$$= \frac{1}{2}\left[2 + 2.\cos\left(\frac{2A+2B}{2}\right).\cos\left(\frac{2A-2B}{2}\right)\right] - \cos^2 C$$

= 1 + cos (A + B).cos(A — B) – cos2 C
In ΔABC,
A + B + C = π
A + B = π — C
cos(A + B) = cos(π — C)
cos(A + B) = -cosC .... (i)
L.H.S. = 1 — cos C.cos(A — B) — $\cos^2$ C
...[From(i)]
= 1 — cos C.[cos(A — B) + cos C]
= 1 — cos C.[cos(A — B) — cos(A + B)]
.. .[From (i)]
= 1 — cos C.(2.sin A.sin B)
= 1 — 2.sinA.sin B.cos C
= R.H.S.

Maharashtra State Board 11th Maths Solutions Chapter 3 Trigonometry – II Miscellaneous Exercise 3

I. Select the correct option from the given alternatives.

Question 1.
The value of sin(n + 1) A sin(n + 2) A + cos(n + 1) A cos(n + 2) A is equal to
(a) sin A
(b) cos A
(c) -cos A
(d) sin 2A
Answer:
(b) cos A
Hint:
L.H.S. = sin [(n + 1)A] . sin [(n + 2)A] + cos [(n + 1)A] . cos [(n + 2)A]
= cos [(n + 2)A] . cos [(n + 1)A] + sin [(n + 2)A] . sin [(n + 1)A]
Let (n + 2)A = a and (n + 1)A = b ... (i)
$\therefore$ L.H.S. = cos a . cos b + sin a . sin b
= cos (a – b)
= cos [(n + 2)A – (n + 1)A] ....[From (i)]
= cos [(n + 2 – n – 1)A]
= cos A
= R.H.S.

Question 2.
If tan A – tan B = x and cot B – cot A = y, then cot (A – B) = _______

(a) $\frac{1}{y}-\frac{1}{x}$

(b) $\frac{1}{x}-\frac{1}{y}$

(c) $\frac{1}{x}+\frac{1}{y}$

(d) $\frac{xy}{x-y}$

Answer:

(c) $\frac{1}{x}+\frac{1}{y}$

Hint:

$$x = \frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}$$

$$= \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B} = \frac{\sin(A-B)}{\cos A \cos B}$$

$$y = \frac{\cos B}{\sin B} - \frac{\cos A}{\sin A}$$

$$= \frac{\sin A \cos B - \cos A \sin B}{\sin A \sin B}$$

$$= \frac{\sin(A-B)}{\sin A \sin B}$$

$$\frac{y}{x} = \frac{\cos A \cos B}{\sin A \sin B} = \cot A \cot B$$

$$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$= \frac{\frac{y}{x}+1}{y} = \frac{x+y}{xy}$$

$$= \frac{1}{x}+\frac{1}{y}$$

Question 3.
If sin θ = n sin(θ + 2α), then tan(θ + α) is equal to
(a) $\frac{1+n}{2-n}$ tan α
(b) $\frac{1-n}{1+n}$ tan α
(c) tan α
(d) $\frac{1+n}{1-n}$ tan α
Answer:
(d) $\frac{1+n}{1-n}$ tan α
Hint:

$$\sin\theta = n \sin(\theta + 2\alpha)$$

$$\frac{n}{1} = \frac{\sin\theta}{\sin(\theta+2\alpha)}$$

By componendo-dividendo, we get

$$\frac{n+1}{n-1} = \frac{\sin\theta + \sin(\theta+2\alpha)}{\sin\theta - \sin(\theta+2\alpha)}$$

$$= \frac{2\sin(\theta+\alpha)\cos\alpha}{-2\cos(\theta+\alpha)\sin\alpha}$$

$$\tan(\theta+\alpha) = -\left(\frac{n+1}{n-1}\right)\tan\alpha = \frac{1+n}{1-n}\tan\alpha$$

Question 4.
The value of $\frac{\cos\theta}{1+\sin\theta}$ is equal to _______

(a) $\tan\left(\frac{\theta}{2}-\frac{\pi}{4}\right)$

(b) $\tan\left(-\frac{\pi}{4}-\frac{\theta}{2}\right)$

(c) $\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

(d) $\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

Answer:

(c) $\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

Hint:

$$\frac{\cos\theta}{1+\sin\theta} = \frac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2}$$

$$= \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}$$

Dividing numerator and denominator by $\cos\frac{\theta}{2}$, we get

$$\frac{\cos\theta}{1+\sin\theta} = \frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}} = \tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

Question 5.
The value of cos A cos (60° – A) cos (60° + A) is equal to _______
(a) 12 cos 3A
(b) cos 3A
(c) 14 cos 3A
(d) 4cos 3A
Answer:
(c) 14 cos 3A
Hint:

$$\cos A \cos(60° - A)\cos(60° + A)$$
$$= (\cos A)(\cos 60° \cos A + \sin 60° \sin A) \cdot (\cos 60° \cos A - \sin 60° \sin A)$$
$$= (\cos A)\left(\frac{1}{2}\cos A + \frac{\sqrt{3}}{2}\sin A\right)\left(\frac{1}{2}\cos A - \frac{\sqrt{3}}{2}\sin A\right)$$
$$= \frac{1}{4}\cos A(\cos^2 A - 3\sin^2 A)$$
$$= \frac{1}{4}[\cos^3 A - 3\cos A(1-\cos^2 A)]$$
$$= \frac{1}{4}(4\cos^3 A - 3\cos A) = \frac{1}{4}\cos 3A$$

Question 6.
The value of sinπ14sin3π14sin5π14sin7π14sin9π14sin11π14sin13π14 is _______
(a) 116
(b) 164
(c) 1128
(d) 1256
Answer:
(b) 164
Hint:

$$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$$

$$= \sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14} \times 1$$

$$\times \sin\left(\pi-\frac{5\pi}{14}\right)\sin\left(\pi-\frac{3\pi}{14}\right)\sin\left(\pi-\frac{\pi}{14}\right)$$

$$\ldots\left[\because \sin\frac{7\pi}{14} = \sin\frac{\pi}{2} = 1\right]$$

$$=\left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)^2 \ldots[\because \sin(\pi-\theta) = \sin\theta]$$

$$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$$

$$= \sin\left(\frac{\pi}{2}-\frac{3\pi}{7}\right)\sin\left(\frac{\pi}{2}-\frac{2\pi}{7}\right)\sin\left(\frac{\pi}{2}-\frac{\pi}{7}\right)$$

$$= \cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}$$

$$= \frac{1}{2\sin\left(\frac{\pi}{7}\right)}\left[\sin\left(\frac{2\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\right]\cos\frac{3\pi}{7}$$

$$= \frac{1}{4\sin\left(\frac{\pi}{7}\right)}\left(\sin\frac{4\pi}{7}\right)\cos\left(\pi-\frac{4\pi}{7}\right)$$

$$= -\frac{1}{4\sin\left(\frac{\pi}{7}\right)}\left(\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}\right)$$

$$= -\frac{1}{8\sin\left(\frac{\pi}{7}\right)}\sin\left(\frac{8\pi}{7}\right)$$

$$= -\frac{1}{8\sin\left(\frac{\pi}{7}\right)}\left(-\sin\left(\frac{\pi}{7}\right)\right)$$

$$\ldots\left[\sin\left(\frac{8\pi}{7}\right)=\sin\left(\pi+\frac{\pi}{7}\right)=-\sin\left(\frac{\pi}{7}\right)\right]$$

$$= \frac{1}{8}$$

$\therefore$ Required expression $= \left(\frac{1}{8}\right)^2 = \frac{1}{64}$

Question 7.
If α + β + γ = π, then the value of $\sin^2$ α + $\sin^2$ β – $\sin^2$ γ is equal to _______
(a) 2 sin α
(b) 2 sin α cos β sin γ
(c) 2 sin α sin β cos γ
(d) 2 sin α sin β sin γ
Answer:
(c) 2 sin α sin β cos γ
Hint:
$\sin^2$ α + $\sin^2$ β – $\sin^2$ γ
= $\frac{1-\cos 2\alpha}{2}+\frac{1-\cos 2\beta}{2}-\sin^2\gamma$
= 1 – $\frac{1}{2}$ (cos 2α + cos 2β) – 1 + $\cos^2$ γ
= $-\frac{1}{2}$ × 2 cos(α + β) cos(α – β) + $\cos^2$ γ
= cos γ cos (α – β) + $\cos^2$ γ …..[∵ α + β + γ = π]
= cos γ [cos (α – β) + cos γ]
= cos γ [cos (α – β) – cos (α + β)]
= 2 sin α sin β cos γ

Question 8.
Let 0 < A, B < $\frac{\pi}{2}$ satisfying the equation 3$\sin^2$ A + 2$\sin^2$ B = 1 and 3sin 2A – 2sin 2B = 0, then A + 2B is equal to _______
(a) π
(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$
(d) 2π
Answer:
(b) $\frac{\pi}{2}$
Hint:
$3 \sin 2A - 2\sin 2B = 0$
$\sin 2B = \frac{3}{2} \sin 2A$ …….(i)
$3 \sin^2 A + 2 \sin^2 B = 1$
$3 \sin^2 A = 1 - 2 \sin^2 B$
$3 \sin^2 A = \cos 2B$ ……(ii)
$\cos(A + 2B) = \cos A \cos 2B - \sin A \sin 2B$
$= \cos A (3 \sin^2 A) - \sin A (\frac{3}{2} \sin 2A)$ …..[From (i) and (ii)]
$= 3 \cos A \sin^2 A - \frac{3}{2} (\sin A) (2 \sin A \cos A)$
$= 3 \cos A \sin^2 A - 3 \sin^2 A \cos A$
$= 0$
$= \cos \frac{\pi}{2}$
$\therefore A + 2B = \frac{\pi}{2}$ ……..[$\because 0 < A + 2B < \frac{3\pi}{2}$]

Question 9.
In ΔABC if cot A cot B cot C > 0, then the triangle is ________
(a) acute-angled
(b) right-angled
(c) obtuse-angled
(d) isosceles right-angled
Answer:
(a) acute angled
Hint:
cot A cot B cot C > 0
Case I:
cot A, cot B, cot C > 0
∴ cot A > 0, cot B > 0, cot C > 0
$\therefore 0 < A < \frac{\pi}{2}, 0 < B < \frac{\pi}{2}, 0 < C < \frac{\pi}{2}$
∴ ΔABC is an acute angled triangle.
Case II:
Two of cot A, cot B, cot C < 0
0 < A, B, C < π and two of cot A, cot B, cot C < 0
∴ Two angles A, B, C are in the 2nd quadrant which is not possible.

Question 10.
The numerical value of tan 20° tan 80° cot 50° is equal to ________
(a) √3
(b) $\frac{1}{\sqrt{3}}$
(c) 2√3
(d) $\frac{1}{2\sqrt{3}}$
Answer:
(a) √3
Hint:
L.H.S. = tan 20° tan 80° cot 50°
= tan 20° tan 80° cot (90° – 40°)
= tan 20° tan 80° tan 40°
= tan 20° tan (60° + 20°) tan (60° – 20°)

$$= \tan 20^\circ \left(\frac{\tan 60^\circ + \tan 20^\circ}{1 - \tan 60^\circ \tan 20^\circ}\right)\left(\frac{\tan 60^\circ - \tan 20^\circ}{1 + \tan 60^\circ \tan 20^\circ}\right)$$

$$= \tan 20^\circ \left(\frac{\sqrt{3} + \tan 20^\circ}{1 - \sqrt{3}\tan 20^\circ}\right)\left(\frac{\sqrt{3} - \tan 20^\circ}{1 + \sqrt{3}\tan 20^\circ}\right)$$

$$= \tan 20^\circ \left[\frac{\left(\sqrt{3}\right)^2 - \tan^2 20^\circ}{1^2 - \left(\sqrt{3}\tan 20^\circ\right)^2}\right]$$

$$= \tan 20^\circ \left(\frac{3 - \tan^2 20^\circ}{1 - 3\tan^2 20^\circ}\right)$$

$$= \frac{3\tan 20^\circ - \tan^3 20^\circ}{1 - 3\tan^2 20^\circ}$$

= tan 3(20°)
= tan 60°
= √3
= R.H.S.

II. Prove the following.

Question 1.
tan 20° tan 80° cot 50° = √3
Solution:
L.H.S. = tan 20° tan 80° cot 50°
= tan 20° tan 80° cot (90° – 40°)
= tan 20° tan 80° tan 40°
= tan 20° tan (60° + 20°) tan (60° – 20°)

$$= \tan 20^\circ \left(\frac{\tan 60^\circ + \tan 20^\circ}{1 - \tan 60^\circ \tan 20^\circ}\right)\left(\frac{\tan 60^\circ - \tan 20^\circ}{1 + \tan 60^\circ \tan 20^\circ}\right)$$

$$= \tan 20^\circ \left(\frac{\sqrt{3} + \tan 20^\circ}{1 - \sqrt{3}\tan 20^\circ}\right)\left(\frac{\sqrt{3} - \tan 20^\circ}{1 + \sqrt{3}\tan 20^\circ}\right)$$

$$= \tan 20^\circ \left[\frac{\left(\sqrt{3}\right)^2 - \tan^2 20^\circ}{1^2 - \left(\sqrt{3}\tan 20^\circ\right)^2}\right]$$

$$= \tan 20^\circ \left(\frac{3 - \tan^2 20^\circ}{1 - 3\tan^2 20^\circ}\right)$$

$$= \frac{3\tan 20^\circ - \tan^3 20^\circ}{1 - 3\tan^2 20^\circ}$$

= tan 3(20°)
= tan 60°
= √3
= R.H.S.

Question 2.
If sin α sin β – cos α cos β + 1 = 0, then prove that cot α tan β = -1.
Solution:
sin α sin β – cos α cos β + 1 = 0
∴ cos α cos β – sin α sin β = 1
∴ cos (α + β) = 1
∴ α + β = 0 ......[∵ cos 0 = 1]
∴ β = -α
L.H.S. = cot α tan β
= cot α tan(-α)
= -cot α tan α
= -1
= R.H.S.

Question 3.
cos2π15cos4π15cos8π15cos16π15=116

Solution:

$$\text{L.H.S.} = \cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$$

$$= \frac{1}{2\sin\frac{2\pi}{15}}\left(2\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\right)$$

$$.\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$$

$$= \frac{1}{2\sin\frac{2\pi}{1}}\left(\sin\frac{4\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}\right)$$

$$\ldots[\because 2\sin\theta\cos\theta = \sin 2\theta]$$

$$= \frac{1}{2\times 2\sin\frac{2\pi}{15}}\left(2\sin\frac{4\pi}{15}\cos\frac{4\pi}{15}\right)\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$$

$$= \frac{1}{4\sin\frac{2\pi}{15}}\left(\sin\frac{8\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}\right)$$

$$= \frac{1}{2\times 4\sin\frac{2\pi}{15}}\left(2\sin\frac{8\pi}{15}\cos\frac{8\pi}{15}\right)\cos\frac{16\pi}{15}$$

$$= \frac{1}{8\sin\frac{2\pi}{15}}\left(\sin\frac{16\pi}{15}\cos\frac{16\pi}{15}\right)$$

$$= \frac{1}{2\times 8\sin\frac{2\pi}{15}}\left(2\sin\frac{16\pi}{15}\cos\frac{16\pi}{15}\right)$$

$$= \frac{1}{16\sin\frac{2\pi}{15}}\sin\left(\frac{32\pi}{15}\right)$$

$$= \frac{1}{16\sin\frac{2\pi}{15}}\sin\left(2\pi+\frac{2\pi}{15}\right)$$

$$= \frac{1}{16\sin\frac{2\pi}{15}}\left(\sin\frac{2\pi}{15}\right)\ldots[\because \sin(2\pi+\theta) = \sin\theta]$$

$$= \frac{1}{16}$$

$$= \text{R. H. S.}$$

Question 4.
$(1+\cos\frac{\pi}{8})(1+\cos\frac{3\pi}{8})(1+\cos\frac{5\pi}{8})(1+\cos\frac{7\pi}{8})=\frac{1}{8}$
Solution:

L.H.S.

$= \left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

Since, $\cos(\pi-\theta) = -\cos\theta$

$\therefore \quad \cos\frac{7\pi}{8} = \cos\left(\pi-\frac{\pi}{8}\right) = -\cos\frac{\pi}{8}$ ...(i)

and $\cos\frac{5\pi}{8} = \cos\left(\pi-\frac{3\pi}{8}\right) = -\cos\frac{3\pi}{8}$ ...(ii)

$\therefore \quad$ L.H.S. $= \left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$

...[From (i) and (ii)]

$= \left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$

$= \sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$

$= \frac{1}{4}\left(2\sin\frac{\pi}{8}\sin\frac{3\pi}{8}\right)^2$

$= \frac{1}{4}\left[\cos\left(\frac{\pi}{8}-\frac{3\pi}{8}\right)-\cos\left(\frac{\pi}{8}+\frac{3\pi}{8}\right)\right]^2$

$= \frac{1}{4}\left[\cos\left(-\frac{\pi}{4}\right)-\cos\left(\frac{\pi}{2}\right)\right]^2$

$= \frac{1}{4}\left(\cos\left(\frac{\pi}{4}\right)-0\right)^2 = \frac{1}{4}\left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{4}\left(\frac{1}{2}\right)$

$= \frac{1}{8}$

= R. H. S.

Question 5.

cos 12° + cos 84° + cos 156° + cos 132° = $-\frac{1}{2}$

Solution:

L.H.S.

$= \cos 12° + \cos 84° + \cos 156° + \cos 132°$

$= (\cos 132° + \cos 12°) + (\cos 156° + \cos 84°)$

$= 2\cos\left(\frac{132°+12°}{2}\right)\cos\left(\frac{132°-12°}{2}\right) + 2\cos\left(\frac{156°+84°}{2}\right)\cos\left(\frac{156°-84°}{2}\right)$

$= 2\cos 72° \cos 60° + 2\cos 120° \cos 36°$

$= 2\cos 72° \cos 60° + 2\cos(180° - 60°)\cos 36°$

$= 2\cos 72° \cos 60° + 2(-\cos 60°)\cos 36°$

$= 2\cos 72°\left(\frac{1}{2}\right) - 2\left(\frac{1}{2}\right)\cos 36°$

$= \cos 72° - \cos 36°$

$= 2\sin\left(\frac{72°+36°}{2}\right)\sin\left(\frac{36°-72°}{2}\right)$

$= 2\sin 54° \sin(-18°)$

$= -2\sin 54°.\sin 18°$

$= -2\left(\frac{\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}-1}{4}\right)$

$= -\frac{1}{8}(5-1)$

$= -\frac{1}{2}$ = R.H.S.

Question 6.

$\cos(\frac{\pi}{4}+x)+\cos(\frac{\pi}{4}-x)=\sqrt{2}\cos x$

Solution:

L.H.S.

$$= \cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right)$$

$$= 2\cos\left(\frac{\frac{\pi}{4}+x+\frac{\pi}{4}-x}{2}\right)\cos\left(\frac{\frac{\pi}{4}+x-\left(\frac{\pi}{4}-x\right)}{2}\right)$$

$$= 2\cos\frac{\pi}{4}\cos x$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)\cos x$$

$$= \sqrt{2}\cos x$$

= R.H.S.

Alternate Method:

$$\text{L.H.S.} = \cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right)$$

$$= \cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x$$

$$= 2\cos\frac{\pi}{4}\cos x$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)\cos x$$

$$= \sqrt{2}\cos x = \text{R.H.S.}$$

Question 7.

$\frac{\sin 5x-2\sin 3x+\sin x}{\cos 5x-\cos x}=\tan x$

Solution:

$$\text{L.H.S.} = \frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x}$$

$$= \frac{(\sin 5x + \sin x) - 2\sin 3x}{\cos 5x - \cos x}$$

$$= \frac{2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right) - 2\sin 3x}{-2\sin\left(\frac{5x+x}{2}\right)\sin\left(\frac{5x-x}{2}\right)}$$

$$= \frac{2\sin 3x\cos 2x - 2\sin 3x}{-2\sin 3x\sin 2x}$$

$$= \frac{2\sin 3x(\cos 2x - 1)}{-2\sin 3x\sin 2x}$$

$$= \frac{-(1-\cos 2x)}{-\sin 2x}$$

$$= \frac{1-\cos 2x}{\sin 2x}$$

$$= \frac{2\sin^2 x}{2\sin x\cos x}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x = \text{R.H.S.}$$

Question 8.

$\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Solution:

$$\begin{aligned}\text{L.H.S.} &= \sin^2 6x - \sin^2 4x\\ &= (\sin 6x)^2 - (\sin 4x)^2\\ &= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)\\ &= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\\ &\qquad \left[2\cos\left(\frac{6x+4x}{2}\right)\sin\left(\frac{6x-4x}{2}\right)\right]\\ &= (2\sin 5x\cos x)(2\cos 5x\sin x)\\ &= (2\sin x\cos x)(2\sin 5x\cos 5x)\\ &= \sin 2x\sin 10x\\ &= \text{R.H.S.}\end{aligned}$$

Question 9.

$\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Solution:

$$\begin{aligned}\text{L.H.S.} &= \cos^2 2x - \cos^2 6x\\ &= (\cos 2x)^2 - (\cos 6x)^2\\ &= (\cos 2x + \cos 6x)(\cos 2x - \cos 6x)\\ &= \left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\\ &\qquad \cdot\left[2\sin\left(\frac{2x+6x}{2}\right)\sin\left(\frac{6x-2x}{2}\right)\right]\\ &= [2\cos 4x\cos(-2x)][2\sin 4x\sin 2x]\\ &= (2\cos 4x\cos 2x)(2\sin 4x\sin 2x)\\ &= (2\sin 2x\cos 2x)(2\sin 4x\cos 4x)\\ &= \sin 4x\sin 8x\\ &= \text{R.H.S.}\end{aligned}$$

Question 10.

$\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

Solution:

$$\begin{aligned}\text{L.H.S.} &= \cot 4x(\sin 5x + \sin 3x)\\ &= \cot 4x\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]\\ &= \frac{\cos 4x}{\sin 4x}(2\sin 4x\cos x)\\ &= 2\cos 4x\cos x \qquad \ldots\text{(i)}\\ \text{R.H.S.} &= \cot x(\sin 5x - \sin 3x)\\ &= \cot x\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]\\ &= \frac{\cos x}{\sin x}(2\cos 4x\sin x)\\ &= 2\cos 4x\cos x \qquad \ldots\text{(ii)}\end{aligned}$$

From (i) and (ii), we get

L.H.S. = R.H.S.

Question 11.

$\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

Solution:

$$\begin{aligned}\text{L.H.S.} &= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}\\ &= \frac{2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{5x-9x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)}\\ &= \frac{2\sin 7x\sin(-2x)}{2\cos 10x\sin 7x}\\ &= \frac{-\sin 2x}{\cos 10x}\\ &= \text{R.H.S.}\end{aligned}$$

Question 12.
If sin 2A = λ sin 2B, then prove that $\frac{\tan(A+B)}{\tan(A-B)}=\frac{\lambda+1}{\lambda-1}$
Solution:

$$\sin 2A = \lambda \sin 2B$$

$$\therefore \quad \frac{\sin 2A}{\sin 2B} = \frac{\lambda}{1}$$

By componendo-dividendo, we get

$$\frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B} = \frac{\lambda+1}{\lambda-1}$$

$$\therefore \quad \frac{2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right)}{2\cos\left(\frac{2A+2B}{2}\right)\sin\left(\frac{2A-2B}{2}\right)} = \frac{\lambda+1}{\lambda-1}$$

$$\therefore \quad \frac{2\sin(A+B)\cos(A-B)}{2\cos(A+B)\sin(A-B)} = \frac{\lambda+1}{\lambda-1}$$

$$\therefore \quad \tan(A+B) \,.\, \cot(A-B) = \frac{\lambda+1}{\lambda-1}$$

$$\therefore \quad \frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}$$

Question 13.
$\frac{2\cos 2A+1}{2\cos 2A-1}$ = tan (60° + A) tan (60° – A)
Solution:

R.H.S. = tan (60° + A) tan (60° – A)

$$= \frac{\sin(60^\circ+A)\sin(60^\circ-A)}{\cos(60^\circ+A)\cos(60^\circ-A)}$$

$$= \frac{2\sin(60^\circ+A)\sin(60^\circ-A)}{2\cos(60^\circ+A)\cos(60^\circ-A)}$$

$$= \frac{\cos\left[60^\circ+A-(60^\circ-A)\right]-\cos(60^\circ+A+60^\circ-A)}{\cos(60^\circ+A+60^\circ-A)+\cos\left[60^\circ+A-(60^\circ-A)\right]}$$

$$= \frac{\cos 2A-\cos 120^\circ}{\cos 120^\circ-\cos 2A}$$

$$= \frac{\cos 2A-\cos(180^\circ-60^\circ)}{\cos(180^\circ-60^\circ)+\cos 2A}$$

$$= \frac{\cos 2A-(-\cos 60^\circ)}{-\cos 60^\circ+\cos 2A}$$

$$= \frac{\cos 2A+\frac{1}{2}}{-\frac{1}{2}+\cos 2A}$$

$$= \frac{2\cos 2A+1}{2\cos 2A-1}$$

= L.H.S.

Question 14.
tan A + tan (60° + A) + tan (120° + A) = 3 tan 3A
Solution:

L.H.S.

$= \tan A + \tan(60° + A) + \tan(120° + A)$

$$= \tan A + \frac{\tan 60° + \tan A}{1 - \tan 60° \tan A} + \frac{\tan 120° + \tan A}{1 - \tan 120° \tan A}$$

$$= \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3} \tan A} + \frac{-\tan 60° + \tan A}{1 - \left(-\tan 60°\right)\tan A}$$

$$\ldots \begin{bmatrix} \because \tan 120° = \tan\left(180° - 60°\right) \\ = -\tan 60° \end{bmatrix}$$

$$= \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3} \tan A} - \frac{\sqrt{3} - \tan A}{1 + \sqrt{3} \tan A}$$

$= \tan A$

$$+ \frac{\sqrt{3} + 3\tan A + \tan A + \sqrt{3}\tan^2 A - \sqrt{3} + \tan A + 3\tan A - \sqrt{3}\tan^2 A}{\left(1 - \sqrt{3}\tan A\right)\left(1 + \sqrt{3}\tan A\right)}$$

$$= \tan A + \frac{8\tan A}{1 - 3\tan^2 A}$$

$$= \frac{\tan A - 3\tan^3 A + 8\tan A}{1 - 3\tan^2 A}$$

$$= \frac{9\tan A - 3\tan^3 A}{1 - 3\tan^2 A}$$

$$= 3\left(\frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}\right)$$

$= 3 \tan 3A$

$=$ R.H.S.

Question 15.

3 tan6 10° – 27 tan4 10° + 33 tan2 10° = 1

Solution:

Since, $\tan 30° = \frac{1}{\sqrt{3}}$

$\therefore \quad \tan 3(10°) = \frac{1}{\sqrt{3}}$

$$\therefore \quad \frac{3\tan 10° - \tan^3 10°}{1 - 3\tan^2 10°} = \frac{1}{\sqrt{3}}$$

Squaring both the sides, we get

$$\frac{\left(3\tan 10° - \tan^3 10°\right)^2}{\left(1 - 3\tan^2 10°\right)^2} = \frac{1}{3}$$

$$\therefore \quad \frac{9\tan^2 10° - 6\tan^4 10° + \tan^6 10°}{1 - 6\tan^2 10° + 9\tan^4 10°} = \frac{1}{3}$$

$$\therefore \quad 3(9\tan^2 10° - 6\tan^4 10° + \tan^6 10°) = 1 - 6\tan^2 10° + 9\tan^4 10°$$

$$\therefore \quad 27\tan^2 10° - 18\tan^4 10° + 3\tan^6 10° = 1 - 6\tan^2 10° + 9\tan^4 10°$$

$$\therefore \quad 3\tan^6 10° - 27\tan^4 10° + 33\tan^2 10° = 1$$

Question 16.

cosec 48° + cosec 96° + cosec 192° + cosec 384° = 0

Solution:

L.H.S. = cosec 48° + cosec 96° + cosec 192° + cosec 384°

= cosec 48° + cosec (180° – 84°) + cosec (180° + 12°) + cosec (360° + 24°)

= cosec 48° + cosec 84° + cosec (-12°) + cosec 24°

$$= \frac{1}{\sin 48^\circ} + \frac{1}{\sin 84^\circ} + \frac{1}{-\sin 12^\circ} + \frac{1}{\sin 24^\circ}$$

$$= \left(\frac{1}{\sin 48^\circ} - \frac{1}{\sin 12^\circ}\right) + \left(\frac{1}{\sin 84^\circ} + \frac{1}{\sin 24^\circ}\right)$$

$$= -\frac{(\sin 48^\circ - \sin 12^\circ)}{\sin 48^\circ \sin 12^\circ} + \frac{(\sin 84^\circ + \sin 24^\circ)}{\sin 84^\circ \sin 24^\circ}$$

$$= -\frac{2\cos\left(\frac{48^\circ + 12^\circ}{2}\right)\sin\left(\frac{48^\circ - 12^\circ}{2}\right)}{\frac{1}{2}\left[\cos\left(48^\circ - 12^\circ\right) - \cos\left(48^\circ + 12^\circ\right)\right]}$$

$$+ \frac{2\sin\left(\frac{84^\circ + 24^\circ}{2}\right)\cos\left(\frac{84^\circ - 24^\circ}{2}\right)}{\frac{1}{2}\left[\cos\left(84^\circ - 24^\circ\right) - \cos\left(84^\circ + 24^\circ\right)\right]}$$

$$= -\frac{2\cos 30^\circ \sin 18^\circ}{\frac{1}{2}(\cos 36^\circ - \cos 60^\circ)}$$

$$+ \frac{2\sin 54^\circ \cos 30^\circ}{\frac{1}{2}(\cos 60^\circ - \cos 108^\circ)}$$

$$= \frac{4\cos 30^\circ \sin 18^\circ}{\cos 60^\circ - \cos 36^\circ} + \frac{4\sin 54^\circ \cos 30^\circ}{\cos 60^\circ + \sin 18^\circ}$$

$$= 4\cos 30^\circ\left[\frac{\sin 18^\circ}{\cos 60^\circ - \cos 36^\circ} + \frac{\sin 54^\circ}{\cos 60^\circ + \sin 18^\circ}\right]$$

$$= 4\cos 30^\circ\left[\frac{\sin 18^\circ}{\cos 60^\circ - \cos 36^\circ} + \frac{\cos 36^\circ}{\cos 60^\circ + \sin 18^\circ}\right]$$

$$\ldots\left[\because \sin 54^\circ = \sin\left(90^\circ - 36^\circ\right) = \cos 36^\circ\right]$$

$$= 4\cos 30^\circ\left[\frac{\frac{\sqrt{5}-1}{4}}{\frac{1}{2} - \frac{\sqrt{5}+1}{4}} + \frac{\frac{\sqrt{5}+1}{4}}{\frac{1}{2} + \frac{\sqrt{5}-1}{4}}\right]$$

$$= 4\cos 30^\circ(-1 + 1) = 0 = \text{R.H.S.}$$

Question 17.
$3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = 13$
Solution:
$(\sin x - \cos x)^4$
$= [(\sin x - \cos x)^2]^2$
$= (\sin^2 x + \cos^2 x - 2 \sin x \cos x)^2$
$= (1 - 2 \sin x \cos x)^2$
$= 1 - 4 \sin x \cos x + 4 \sin^2 x \cos^2 x$
$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2 \sin x \cos x = 1 + 2 \sin x \cos x$
$\sin^6 x + \cos^6 x$
$= (\sin^2 x)^3 + (\cos^2 x)^3$
$= (\sin^2 x + \cos^2 x)^3 - 3 \sin^2 x \cos^2 x (\sin^2 x + \cos^2 x)$ .....$[\because a^3 + b^3 = (a + b)^3 - 3ab(a + b)]$
$= 1^3 - 3 \sin^2 x \cos^2 x (1)$
$= 1 - 3 \sin^2 x \cos^2 x$
L.H.S. $= 3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$
$= 3(1 - 4 \sin x \cos x + 4 \sin^2 x \cos^2 x) + 6(1 + 2 \sin x \cos x) + 4(1 - 3 \sin^2 x \cos^2 x)$
$= 3 - 12 \sin x \cos x + 12 \sin^2 x \cos^2 x + 6 + 12 \sin x \cos x + 4 - 12 \sin^2 x \cos^2 x$
$= 13$
= R.H.S.

Question 18.
$\tan A + 2 \tan 2A + 4 \tan 4A + 8 \cot 8A = \cot A$
Solution:
We have to prove that,
$\tan A + 2 \tan 2A + 4 \tan 4A + 8 \cot 8A = \cot A$
i.e., to prove,
$\cot A - \tan A - 2 \tan 2A - 4 \tan 4A - 8 \cot 8A = 0$

$$\cot\theta - \tan\theta = \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$
$$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta\ \sin\theta}$$
$$= \frac{2\cos 2\theta}{2\sin\theta\cos\theta}$$
$$= \frac{2\cos 2\theta}{\sin 2\theta}$$

∴ $\cot\theta - \tan\theta = 2\cot 2\theta$ .....(i)

L.H.S. = cot A – tan A – 2 tan 2A – 4 tan 4A – 8 cot 8A
= 2 cot 2A – 2 tan 2A – 4 tan 4A – 8 cot 8A .....[From (i)]
= 2(cot 2A – tan 2A) – 4 tan 4A – 8 cot 8A
= 2 × 2 cot 2(2A) – 4 tan 4A – 8 cot 8A ......[From (i)]
= 4(cot 4A – tan 4A) – 8 cot 8A
= 4 × 2 cot 2(4A) – 8 cot 8A ......[From (i)]
= 8 cot 8A – 8 cot 8A = 0
= R.H.S.

Alternate Method:

$$\text{L.H.S.} = \tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A$$
$$= \tan A + 2\tan 2A + 4\left(\frac{\sin 4A}{\cos 4A} + \frac{2\cos 8A}{\sin 8A}\right)$$
$$= \tan A + 2\tan 2A + 4\left(\frac{\sin 4A\sin 8A + 2\cos 8A\cos 4A}{\sin 8A\cos 4A}\right)$$
$$= \tan A + 2\tan 2A + 4\left[\frac{(\cos 8A\cos 4A + \sin 8A\sin 4A) + \cos 8A\cos 4A}{\sin 8A\cos 4A}\right]$$

$$= \tan A + 2\tan 2A + 4\left[\frac{\cos(8A-4A)+\cos 8A\cos 4A}{\sin 8A\cos 4A}\right]$$

$$= \tan A + 2\tan 2A + 4\left(\frac{\cos 4A + \cos 8A\cos 4A}{\sin 8A\cos 4A}\right)$$

$$= \tan A + 2\tan 2A + 4\left[\frac{\cos 4A(1+\cos 8A)}{\sin 8A\cos 4A}\right]$$

$$= \tan A + 2\tan 2A + 4\left[\frac{1+\cos 2(4A)}{\sin 2(4A)}\right]$$

$$= \tan A + 2\tan 2A + 4\left(\frac{2\cos^2 4A}{2\sin 4A\cos 4A}\right)$$

$$= \tan A + 2\frac{\sin 2A}{\cos 2A} + 4\frac{\cos 4A}{\sin 4A}$$

$$= \tan A + 2\left(\frac{\sin 2A}{\cos 2A} + \frac{2\cos 4A}{\sin 4A}\right)$$

$$= \tan A + 2\left(\frac{\sin 4A\sin 2A + 2\cos 4A\cos 2A}{\sin 4A\cos 2A}\right)$$

$$= \tan A + 2\left[\frac{(\cos 4A\cos 2A + \sin 4A\sin 2A) + \cos 4A\cos 2A}{\sin 4A\cos 2A}\right]$$

$$= \tan A + 2\left[\frac{\cos(4A-2A) + \cos 4A\cos 2A}{\sin 4A\cos 2A}\right]$$

$$= \tan A + 2\left(\frac{\cos 2A + \cos 4A\cos 2A}{\sin 4A\cos 2A}\right)$$

$$= \tan A + 2\left[\frac{\cos 2A(1+\cos 4A)}{\sin 4A\cos 2A}\right]$$

$$= \tan A + 2\left(\frac{2\cos^2 2A}{2\sin 2A\cos 2A}\right)$$

$$= \frac{\sin A}{\cos A} + \frac{2\cos 2A}{\sin 2A}$$

$$= \frac{\sin 2A\sin A + 2\cos 2A\cos A}{\sin 2A\cos A}$$

$$= \frac{\cos 2A\cos A + \sin 2A\sin A + \cos 2A\cos A}{\sin 2A\cos A}$$

$$= \frac{\cos(2A-A) + \cos 2A\cos A}{\sin 2A\cos A}$$

$$= \frac{\cos A(1+\cos 2A)}{\sin 2A\cos A}$$

$$= \frac{2\cos^2 A}{2\sin A\cos A}$$

$$= \cot A$$

$$= \text{R.H.S.}$$

Question 19.
If A + B + C = $\frac{3\pi}{2}$, then cos 2A + cos 2B + cos 2C = 1 – 4 sin A sin B sin C

Solution:

In Δ ABC,

$$A + B + C = \frac{3\pi}{2}$$

$$\therefore \quad A + B = \frac{3\pi}{2} - C$$

$$\therefore \quad \cos(A + B) = \cos\left(\frac{3\pi}{2} - C\right)$$

$$= -\sin C \quad \ldots(i)$$

L.H.S. $= \cos 2A + \cos 2B + \cos 2C$

$$= 2\cos\left(\frac{2A + 2B}{2}\right)\cos\left(\frac{2A - 2B}{2}\right) + \cos 2C$$

$$= 2\cos(A + B)\cos(A - B) + \cos 2C$$

$$= 2(-\sin C)\cos(A - B) + 1 - 2\sin^2 C \quad \ldots[\text{From (i)}]$$

$$= 1 - 2\sin C[\cos(A - B) + \sin C]$$

$$= 1 - 2\sin C\{\cos(A - B) - \cos(A + B)\} \quad \ldots[\text{From (i)}]$$

$$= 1 - 2\sin C \times 2\sin\left(\frac{A - B + A + B}{2}\right)\sin\left(\frac{A + B - A + B}{2}\right)$$

$$= 1 - 2\sin C(2\sin A\sin B)$$

$$= 1 - 4\sin A\sin B\sin C$$

= R.H.S.

Question 20.
In any triangle ABC, sin A – cos B = cos C. Show that $\angle B = \frac{\pi}{2}$.
Solution:
sin A – cos B = cos C
∴ sin A = cos B + cos C

$$\therefore \quad 2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cos\left(\frac{B + C}{2}\right)\cos\left(\frac{B - C}{2}\right)$$

$$\therefore \quad 2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cos\left(\frac{\pi}{2} - \frac{A}{2}\right)\cos\left(\frac{B - C}{2}\right) \quad \ldots\left[\begin{array}{l}\because A + B + C = \pi, \\ \therefore \frac{B + C}{2} = \frac{\pi}{2} - \frac{A}{2}\end{array}\right]$$

$$\therefore \quad 2\sin\frac{A}{2}\cos\frac{A}{2} = 2\sin\frac{A}{2}\cos\left(\frac{B - C}{2}\right)$$

$$\therefore \quad \cos\frac{A}{2} = \cos\left(\frac{B - C}{2}\right) \quad \ldots\left[\because \sin\frac{A}{2} \neq 0\right]$$

$$\therefore \quad \frac{A}{2} = \frac{B - C}{2}$$

A = B – C ....(i)
In ΔABC,
A + B + C = π
∴ B – C + B + C = π
∴ 2B = π
∴ B = $\frac{\pi}{2}$

Question 21.
$\frac{\tan^3 x}{1 + \tan^2 x} + \frac{\cot^3 x}{1 + \cot^2 x}$ = sec x cosec x – 2 sin x cos x

Solution:

$$\text{L.H.S.} = \frac{\tan^3 x}{1+\tan^2 x} + \frac{\cot^3 x}{1+\cot^2 x}$$

$$= \frac{\left(\frac{\sin^3 x}{\cos^3 x}\right)}{\sec^2 x} + \frac{\frac{\cos^3 x}{\sin^3 x}}{\operatorname{cosec}^2 x}$$

$$= \frac{\sin^3 x}{\cos^3 x} \times \cos^2 x + \frac{\cos^3 x}{\sin^3 x} \times \sin^2 x$$

$$= \frac{\sin^3 x}{\cos x} + \frac{\cos^3 x}{\sin x}$$

$$= \frac{\sin^4 x + \cos^4 x}{\cos x \sin x}$$

$$= \frac{\left(\sin^2 x\right)^2 + \left(\cos^2 x\right)^2}{\cos x \sin x}$$

$$= \frac{\left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x \cos^2 x}{\cos x \sin x}$$

$$\ldots\left[\because a^2+b^2=(a+b)^2-2ab\right]$$

$$= \frac{1^2 - 2\sin^2 x \cos^2 x}{\cos x \sin x}$$

$$= \frac{1}{\cos x \sin x} - \frac{2\sin^2 x \cos^2 x}{\cos x \sin x}$$

$$= \sec x \cos x - 2\sin x \cos x$$

$$= \text{R.H.S.}$$

Question 22.
sin 20° sin 40° sin 80° = 3√8
Solution:
L.H.S. = sin 20°. sin 40°. sin 80°
= sin 20°. sin 40°. sin 80°
= 12 (2 . sin 40°. sin 20°) . sin 80°
= 12 [cos(40° – 20°) – cos (40° + 20°)] . sin 80°
= 12 (cos 20° – cos 60°) sin 80°
= 12 . cos 20° . sin 80° – 12 . cos 60° . sin 80°
= 12×2 (2 sin 80° . cos 20°) – 12×2 . sin 80°
= 14 [sin(80° + 20°) + sin (80° – 20°)] – 12 . sin 80°

$$= \frac{1}{4}.(\sin 100° + \sin 60°) - \frac{1}{4}.\sin 80°$$

$$= \frac{1}{4}\sin 100° + \frac{1}{4}\sin 60° - \frac{1}{4}\sin 80°$$

$$= \frac{1}{4}.\sin(180° - 80°) + \frac{1}{4} \times \frac{\sqrt{3}}{2} - \frac{1}{4}.\sin 80°$$

$$= \frac{1}{4}\sin 80° + \frac{\sqrt{3}}{8} - \frac{1}{4}\sin 80°$$

$$\ldots[\because \sin(180° - \theta) = \sin\theta]$$

$$= \frac{\sqrt{3}}{8} = \text{R.H.S.}$$

Question 23.
sin 18° = 5√-14
Solution:
Let θ = 18°
∴ 5θ = 90°
∴ 2θ + 3θ = 90°
∴ 2θ = 90° – 3θ
∴ sin 2θ = sin (90° – 3θ)
∴ sin 2θ = cos 3θ
∴ 2 sin θ cos θ = 4 $\cos^3$ θ – 3 cos θ
∴ 2 sin θ = 4 $\cos^2$ θ – 3 …..[∵ cos θ ≠ 0]
∴ 2 sin θ = 4 (1 – $\sin^2$ θ) – 3
∴ 2 sin θ = 1 – 4 $\sin^2$ θ

∴ $4 \sin^2 \theta + 2 \sin \theta - 1 = 0$
∴ sin θ = -2±4+16√8
= -2±25√8
= -1±5√4
Since, sin 18° > 0
∴ sin 18°= 5√-14

Question 24.
cos 36° = 5√+14
Solution:
We know that,
$\cos 2\theta = 1 - 2 \sin^2 \theta$
cos 36° = cos 2(18°)
$= 1 - 2 \sin^2 18°$

$$= 1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2$$

$$= \frac{8-\left(5+1-2\sqrt{5}\right)}{8}$$

$$= \frac{8-\left(6-2\sqrt{5}\right)}{8}$$

$$= \frac{2+2\sqrt{5}}{8}$$

∴ cos 36° = 5√+14

Question 25.
sin 36° = 10-25√√4
Solution:
We know that, $\sin^2 \theta = 1 - \cos^2 \theta$
$\sin^2 36° = 1 - \cos^2 36°$
= 1 – (5√+14)2
= 16-(5+1+25√)16
= 10-25√16
∴ sin 36° = 10-25√√4 ......[∵ sin 36° is positive]

Question 26.
sinπc8=122-2-√------√
Solution:

We know that cos $\frac{\pi}{4} = \cos 45^\circ = \frac{1}{\sqrt{2}}$

Also $\frac{\pi}{8}$ lies in the first quadrant, hence sin $\frac{\pi}{8}$ is positive.

Now, cos 2θ = 1 – 2sin²θ

By putting θ = $\frac{\pi}{8}$, we get,

$\cos \frac{\pi}{4} = 1 - 2\sin^2 \frac{\pi}{8}$

$\therefore 2\sin^2 \frac{\pi}{8} = 1 - \cos \frac{\pi}{4}$

$= 1 - \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{2}-1}{\sqrt{2}}$

$\therefore \sin^2 \frac{\pi}{8} = \frac{\sqrt{2}-1}{2\sqrt{2}}$

$= \frac{\sqrt{2}\left(\sqrt{2}-1\right)}{4}$

$= \frac{2-\sqrt{2}}{4}$

$\therefore \sin\frac{\pi}{8} = \frac{1}{\sqrt{2-\sqrt{2}}}$ ...... $\left[\because \sin \frac{\pi}{8} \text{ is positive}\right]$

Question 27.
tan $\frac{\pi}{8}$ = √2 – 1
Solution:

We know that,

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\tan \frac{\pi}{4} = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

Let $\tan \frac{\pi}{8} = t$

$\therefore \frac{2t}{1-t^2} = 1$

$\therefore 2t = 1 - t^2$

$\therefore t^2 + 2t - 1 = 0$

$\therefore t = \frac{-2 \pm \sqrt{4+4}}{2}$

$= \frac{-2 \pm 2\sqrt{2}}{2}$

$= -1 \pm \sqrt{2}$

$t = \tan \frac{\pi}{8} > 0$

$\therefore \tan \frac{\pi}{8} = \sqrt{2} - 1$

Question 28.
tan 6° tan 42° tan 66° tan 78° = 1
Solution:

L.H.S. = tan 6° tan 42° tan 66° tan 78°

$$= \frac{\sin 6°}{\cos 6°} \cdot \frac{\sin 42°}{\cos 42°} \cdot \frac{\sin 66°}{\cos 66°} \cdot \frac{\sin 78°}{\cos 78°}$$

$$= \frac{(2\sin 66° \sin 6°)(2\sin 78° \sin 42°)}{(2\cos 66° \cos 6°)(2\cos 78° \cos 42°)}$$

$$= \frac{\cos(66° - 6°) - \cos(66° + 6°)}{\cos(66° + 6°) + \cos(66° - 6°)} \cdot \frac{\cos(78° - 42°) - \cos(78° + 42°)}{\cos(78° + 42°) + \cos(78° - 42°)}$$

$$= \frac{(\cos 60° - \cos 72°)(\cos 36° - \cos 120°)}{(\cos 60° + \cos 72°)(\cos 36° + \cos 120°)}$$

$$= \frac{(\cos 60° - \sin 18°)(\cos 36° + \sin 30°)}{(\cos 60° + \sin 18°)(\cos 36° - \sin 30°)}$$

$$\ldots[\because \cos(90° + \theta) = -\sin\theta]$$

$$= \frac{\left(\frac{1}{2} - \frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4} + \frac{1}{2}\right)}{\left(\frac{1}{2} + \frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right)}$$

$$= \frac{9-5}{5-1}$$

$= 1$

= R.H.S.

Question 29.
sin 47° + sin 61° – sin 11° – sin 25° = cos 7°
Solution:

L.H.S. = sin 47° + sin 61° – sin 11° – sin 25°

$= (\sin 47° - \sin 25°) + (\sin 61° - \sin 11°)$

$$= 2\cos\left(\frac{47° + 25°}{2}\right)\sin\left(\frac{47° - 25°}{2}\right) + 2\cos\left(\frac{61° + 11°}{2}\right)\sin\left(\frac{61° - 11°}{2}\right)$$

$= 2\cos 36° \sin 11° + 2\cos 36° \sin 25°$

$= 2\cos 36° (\sin 11° + \sin 25°)$

$$= 2\cos 36°\left[2\sin\left(\frac{25° + 11°}{2}\right)\cos\left(\frac{25° - 11°}{2}\right)\right]$$

$= 2\cos 36° (2\sin 18° \cos 7°)$

$$= 4\left(\frac{\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}-1}{4}\right)\cos 7°$$

$$= \frac{4(5-1)}{16}\cos 7°$$

$= \cos 7°$

= R.H.S.

Question 30.
√3 cosec 20° – sec 20° = 4
Solution:

$$\text{L. H. S.} = \sqrt{3}\operatorname{cosec} 20^\circ - \sec 20^\circ$$
$$= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$$
$$= 2\left(\frac{\frac{\sqrt{3}}{2}}{\sin 20^\circ} - \frac{\frac{1}{2}}{\cos 20^\circ}\right)$$
$$= 2\left(\frac{\sin 60^\circ}{\sin 20^\circ} - \frac{\cos 60^\circ}{\cos 20^\circ}\right)$$
$$= 2\left(\frac{\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ}{\sin 20^\circ \cos 20^\circ}\right)$$
$$= \frac{2\sin(60^\circ - 20^\circ)}{\frac{1}{2}(2\sin 20^\circ \cos 20^\circ)}$$
$$= \frac{4\sin 40^\circ}{\sin 40^\circ}$$
$$= 4$$
$$= \text{R.H.S.}$$

Question 31.
In ΔABC, ∠C = $\frac{2\pi}{3}$, then prove that $\cos^2 A + \cos^2 B - \cos A \cos B = \frac{3}{4}$.
Solution:

$$\angle C = \frac{2\pi}{3}$$
In ΔABC,
$$A + B + C = \pi$$
$$\therefore \quad A + B + \frac{2\pi}{3} = \pi$$
$$\therefore \quad A + B = \frac{\pi}{3}$$
$$\therefore \quad \cos(A + B) = \cos\frac{\pi}{3} = \frac{1}{2} \qquad \ldots(i)$$
$$\text{L.H.S.} = \cos^2 A + \cos^2 B - \cos A \cos B$$
$$= \frac{1}{2}(2\cos^2 A + 2\cos^2 B) - \frac{1}{2}(2\cos A \cos B)$$
$$= \frac{1}{2}(1 + \cos 2A + 1 + \cos 2B) - \frac{1}{2}(2\cos A \cos B)$$
$$= \frac{1}{2}\left[2 + 2\cos\left(\frac{2A + 2B}{2}\right)\cos\left(\frac{2A - 2B}{2}\right)\right] - \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$
$$= \frac{1}{2}[2 + 2\cos(A + B)\cos(A - B)] - \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$
$$= \frac{1}{2}\left[2 + 2\left(\frac{1}{2}\right)\cos(A - B)\right] - \frac{1}{2}\left[\frac{1}{2} + \cos(A - B)\right] \qquad \ldots[\text{From (i)}]$$
$$= \frac{1}{2}\left[2 + \cos(A - B) - \frac{1}{2} - \cos(A - B)\right]$$
$$= \frac{1}{2}\left(\frac{3}{2}\right) = \frac{3}{4}$$
$$= \text{R.H.S.}$$