Maharashtra State Board 11th Commerce Maths Solutions Chapter 7 Probability Ex 7.1

Question 1.
State the sample space and n(S) for the following random experiments.
(i) A coin is tossed twice. If a second throw results in a tail, a die is thrown.
(ii) A coin is tossed twice. If a second throw results in a head, a die is thrown, otherwise, a coin is tossed.
Solution:
(i) When a coin is tossed twice, the outcomes are HH, HT, TH, TT.
A coin is tossed twice and if the second throw results in a tail, a die is thrown.
Out of the above 4 possibilities, on second throw tail occurs in two cases only i.e., HT, TT.
∴ S = {HH, TH, HT1, HT2, HT3, HT4, HT5, HT6, TT1, TT2, TT3, TT4, TT5, TT6}
∴ n(S) = 14

(ii) When a coin is tossed twice, the outcomes are HH, HT, TH, TT.
Let A be the event that the second throw results in the head when a coin is tossed twice followed by a die is thrown.
∴ A = {HH1, HH2, HH3, HH4, HH5, HH6, TH1, TH2, TH3, TH4, TH5, TH6}
The remaining outcomes i.e., HT, TT are followed by the tossing of a coin.
Let us consider this as event B.
∴ B = {HTT, HTH, TTT, TTH}
The sample space S of the experiment is A ∪ B.
∴ S = {HH1, HH2, HH3, HH4, HH5, HH6, TH1, TH2, TH3, TH4, TH5, TH6, HTT, HTH, TTT, TTH}
∴ n(S) = 16

Question 2.
In a bag, there are three balls; one black, one red, and one green. Two balls are drawn one after another with replacement. State sample space and n(S).
Solution:
The bag contains 3 balls out of which one is black (B), one is red (R) and the other one is green (G).

Two balls are drawn one after the other, with replacement, from the bag.
∴ the sample space S is given by
S = {BB, BR, BG, RB, RR, RG, GB, GR, GG}
∴ n(S) = 9

Question 3.
A coin and die are tossed. State sample space of following events.
(i) A: Getting a head and an even number.
(ii) B: Getting a prime number.
(iii) C: Getting a tail and perfect square.
Solution:
When a coin and a die are tossed the sample space S is given by
S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}
(i) A: getting a head and an even number
∴ A = {H2, H4, H6}

(ii) B: getting a prime number
∴ B = {H2, H3, H5, T2, T3, T5}

(iii) C: getting a tail and a perfect square.
∴ C = {T1, T4}

Question 4.
Find the total number of distinct possible outcomes n(S) for each of the following random experiments.
(i) From a box containing 25 lottery tickets and 3 tickets are drawn at random.
(ii) From a group of 4 boys and 3 girls, any two students are selected at random.
(iii) 5 balls are randomly placed into five cells, such that each cell will be occupied.
(iv) 6 students are arranged in a row for photographs.
Solution:
(i) Let S be the event that 3 tickets are drawn at random from 25 tickets
∴ 3 tickets can be selected in ${}^{25}C_3$ ways
∴ n(S) = ${}^{25}C_3$
= $25\times24\times233\times2\times1$
= 2300

(ii) There are 4 boys and 3 girls i.e., 7 students.
2 students can be selected from these 7 students in ${}^7C_2$ ways.
∴ n(S) = ${}^7C_2$
= 7×62×1
= 21

(iii) 5 balls have to be placed in 5 cells in such a way that each cell is occupied.
∴ The first ball can be placed into one of the 5 cells in 5 ways, the second ball placed into one of the remaining 4 cells in 4 ways.
Similarly, the third, fourth, and fifth balls can be placed in 3 ways, 2 ways, and 1 way, respectively.
∴ a total number of ways of filling 5 cells such that each cell is occupied = 5!
= 5 × 4 × 3 × 2 × 1
= 120
∴ n(S) = 120

(iv) Six students can be arranged in a row for a photograph in ${}^6P_6$ = 6! ways.
∴ n(S) = 6!
= 6 × 5 × 4 × 3 × 2 × 1
= 720

Question 5.
Two dice are thrown. Write favourable outcomes for the following events.
(i) P: The sum of the numbers on two dice is divisible by 3 or 4.
(ii) Q: sum of the numbers on two dice is 7.
(iii) R: sum of the numbers on two dice is a prime number.
Also, check whether
(a) events P and Q are mutually exclusive and exhaustive.
(b) events Q and R are mutually exclusive and exhaustive.
Solution:
When two dice are thrown, all possible outcomes are
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

(i) P: sum of the numbers on two dice is divisible by 3 or 4.
∴ P = {(1, 2), (1, 3), (1, 5), (2, 1), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (3, 6), (4, 2), (4, 4), (4, 5), (5, 1), (5, 3), (5, 4), (6, 2), (6, 3), (6, 6)}

(ii) Q: sum of the numbers on two dice is 7.
∴ Q = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

(iii) R: sum of the numbers on two dice is a prime number.
∴ R = {(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)}
(a) P and Q are mutually exclusive events as $P \cap Q = \varphi$ and
$P \cup Q$ = {(1, 2), (1, 3), (1, 5), (1, 6), (2, 1), (2, 2), (2, 4), (2, 5), (2, 6), (3, 1), (3, 3), (3, 4), (3, 5), (3, 6), (4, 2), (4, 3), (4, 4), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 6)} ≠ S
∴ P and Q are not exhaustive events as $P \cup Q \neq S$.

(b) $Q \cap R$ = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}
∴ $Q \cap R \neq \varphi$
Also, $Q \cup R$ = {(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)} ≠ S
∴ Q and R are neither mutually exclusive nor exhaustive events.

Question 6.
A card is drawn at random from an ordinary pack of 52 playing cards. State the number of elements in the sample space, if
consideration of suits
(i) is not taken into account.
(ii) is taken into account.
Solution:
(i) If consideration of suits is not taken into account, then one card can be drawn from the pack of 52 playing cards in $^{52}C_1 = 52$ ways.
∴ n(S) = 52

(ii) If consideration of suits is taken into account, then one card can be drawn from each suit in $^{13}C_1 \times {}^{4}C_1$
= 13 × 4
= 52 ways.
∴ n(S) = 52

Question 7.
Box-I contains 3 red (R11, R12, R13) and 2 blue (B11, B12) marbles while Box-II contains 2 red (R21, R22) and 4 blue (B21, B22, B23, B24) marbles. A fair coin is tossed. If the coin turns up heads, a marble is chosen from Box-I; if it turns up tails, a marble is chosen from Box-II. Describe the sample space.
Solution:
Box I contains 3 red and 2 blue marbles i.e., (R11, R12, R13, B11, B12)
Box II contains 2 red and 4 blue marbles i.e., (R21, R22, B21, B22, B23, B24)
It is given that a fair coin is tossed and if a head comes then marble is chosen from box I otherwise it is chosen from box II
∴ the sample space is
S = {(H, R11), (H, R12), (H, R13), (H, B11), (H, B12), (T, R21), (T, R22), (T, B21), (T, B22), (T, B23), (T, B24)}

Question 8.
Consider an experiment of drawing two cards at random from a bag containing 4 cards marked 5, 6, 7, and 8. Find the sample space, if cards are drawn
(i) with replacement
(ii) without replacement.
Solution:
The bag contains 4 cards marked 5, 6, 7, and 8.
Two cards are to be drawn from this bag.
(i) If the two cards are drawn with replacement, then the sample space is
S = {(5, 5), (5, 6), (5, 7), (5, 8), (6, 5), (6, 6), (6, 7), (6,8), (7, 5), (7, 6), (7, 7), (7, 8), (8, 5), (8, 6), (8, 7), (8, 8)}

(ii) If the two cards are drawn without replacement, then the sample space is
S = {(5, 6), (5, 7), (5, 8), (6, 5), (6, 7), (6, 8), (7, 5), (7, 6), (7, 8), (8, 5), (8, 6), (8, 7)}

Maharashtra State Board 11th Commerce Maths Solutions Chapter 7 Probability Ex 7.2

Question 1.
A fair die is thrown two times. Find the chance that
(i) product of the numbers on the upper face is 12.
(ii) sum of the numbers on the upper face is 10.
(iii) sum of the numbers on the upper face is at least 10.
(iv) sum of the numbers on the upper face is 4.
(v) the first throw gives an odd number and the second throw gives a multiple of 3.
(vi) both the times die to show the same number (doublet).
Solution:
If a fair die is thrown twice, the sample space is
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(S) = 36
(i) Let A be the event that the product of the numbers on uppermost face is 12.
∴ A = {(2, 6), (3, 4), (4, 3), (6, 2)}
∴ n(A) = 4
∴ P(A) = $n(A)n(S)=436=19$

(ii) Let B be the event that sum of the numbers on uppermost face is 10.
∴ B = {(4, 6), (5, 5), (6, 4)}
∴ n(B) = 3
∴ P(B) = $n(B)n(S)=336=112$

(iii) Let C be the event that sum of the numbers on uppermost face is at least 10 (i.e., 10 or more than 10 which are 10 or 11 or 12)
∴ C = {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}
∴ n(C) = 6
∴ P(C) = $n(C)n(S)=636=16$

(iv) Let D be the event that sum of the numbers on uppermost face is 4.
∴ D = {(1, 3), (2, 2), (3, 1)}

∴ n(D) = 3
∴ P(D) = $\frac{n(D)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

(v) Let E be the event that 1st throw gives an odd number and 2nd throw gives multiple of 3.
∴ E = {(1, 3), (1, 6), (3, 3), (3, 6), (5, 3), (5, 6)}
∴ n(E) = 6
∴ P(E) = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(vi) Let F be the event that both times die shows same number.
∴ F = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}
∴ n(F) = 6
∴ P(F) = $\frac{n(F)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

Question 2.
Two cards are drawn from a pack of 52 cards. Find the probability that
(i) both are black.
(ii) both are diamonds.
(iii) both are ace cards.
(iv) both are face cards.
(v) one is a spade and the other is a non-spade.
(vi) both are from the same suit.
(vii) both are from the same denomination.
Solution:
Two cards can be drawn from a pack of 52 cards in $^{52}C_2$ ways.
∴ n(S) = $^{52}C_2$
(i) Let A be the event that both the cards drawn are black.
The pack of 52 cards contains 26 black cards.
∴ 2 cards can be drawn from them in $^{26}C_2$ ways
∴ n(A) = $^{26}C_2$
∴ P(A) = $\frac{n(A)}{n(S)} = \frac{^{26}C_2}{^{52}C_2}$

(ii) Let B be the event that both the cards drawn are diamond.
There are 13 diamond cards in a pack of 52 cards.
∴ 2 diamond cards can be drawn from 13 diamond cards in $^{13}C_2$ ways

∴ $n(B) = {}^{13}C_2$
∴ $P(B) = \frac{n(B)}{n(S)} = \frac{{}^{13}C_2}{{}^{52}C_2}$

(iii) Let C be the event that both the cards drawn are aces.
In a pack of 52 cards, there are 4 ace cards.
∴ 2 ace cards can be drawn from 4 ace cards in ${}^{4}C_2$ ways
∴ $n(C) = {}^{4}C_2$
∴ $P(C) = \frac{n(C)}{n(S)} = \frac{{}^{4}C_2}{{}^{52}C_2}$

(iv) Let D be the event that both the cards drawn are face cards.
There are 12 face cards in a pack of 52 cards.
∴ 2 face cards can be drawn from 12 face cards in ${}^{12}C_2$ ways.
∴ $n(D) = {}^{12}C_2$
∴ $P(D) = \frac{n(D)}{n(S)} = \frac{{}^{12}C_2}{{}^{52}C_2}$

(v) Let E be the event that out of the two cards drawn one is a spade and other is non-spade.
There are 13 spade cards and 39 cards are non-spade cards in a pack of 52 cards.
∴ One spade card can be drawn from 13 spade cards in ${}^{13}C_1$ ways and one non-spade card can be drawn from 39 non-spade cards in 39C1 ways.
∴ $n(E) = {}^{13}C_1 \cdot {}^{39}C_1$
∴ $P(E) = \frac{n(E)}{n(S)} = \frac{{}^{13}C_1 \cdot {}^{39}C_1}{{}^{52}C_2}$

(vi) Let F be the event that both the cards drawn are of the same suit.
A pack of 52 cards consists of 4 suits each containing 13 cards.
2 cards can be drawn from a suit in ${}^{13}C_2$ ways.
A suit can be selected in 4 ways.
∴ $n(F) = {}^{13}C_2 \times 4$
∴ $P(F) = \frac{n(F)}{n(S)} = \frac{4 \times {}^{13}C_2}{{}^{52}C_2}$

(vii) Let G be the event that both the cards drawn are of same denominations.
A pack of cards has 13 denominations and 4 different cards for each denomination
∴ $n(G) = 13 \times {}^{4}C_2$
∴ $P(G) = \frac{n(G)}{n(S)} = \frac{13 \times {}^{4}C_2}{{}^{52}C_2}$

Question 3.
Four cards are drawn from a pack of 52 cards. Find the probability that
(i) 3 are Kings and 1 is Jack.
(ii) all the cards are from different suits.
(iii) at least one heart.
(iv) all cards are club and one of them is a jack.
Solution:
4 cards can be drawn out of 52 cards in ${}^{52}C_4$ ways.
∴ $n(S) = {}^{52}C_4$
(i) Let A be the event that out of the four cards drawn, 3 are kings and 1 is a jack.
There are 4 kings and 4 jacks in a pack of 52 cards.
∴ 3 kings can be drawn from 4 kings in ${}^4C_3$ ways.
Similarly, 1 jack can be drawn out of 4 jacks in ${}^4C_1$ ways.
∴ Total number of ways in which 3 kings and 1 jack can be drawn is ${}^4C_3 \times {}^4C_1$
∴ $n(A) = {}^4C_3 \times {}^4C_1$
∴ $P(A) = \frac{n(A)}{n(S)} = \frac{{}^4C_3 \times {}^4C_1}{{}^{52}C_4}$

(ii) Let B be the event that all the cards drawn are of different suits.
A pack of 52 cards consists of 4 suits each containing 13 cards.
∴ A card can be drawn from each suit in ${}^{13}C_1$ ways.
∴ 4 cards can be drawn from 4 different suits in ${}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$ ways.
∴ $n(B) = {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$
∴ $P(B) = \frac{n(B)}{n(S)} = \frac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_4}$

(iii) Let C be the event that out of the four cards drawn at least one is a heart.
∴ C′ is the event that all 4 cards drawn are non-heart cards.
In a pack of 52 cards, there are 39 non-heart cards.
∴ 4 non-heart cards can be drawn in ${}^{39}C_4$ ways.
∴ $n(C') = {}^{39}C_4$
∴ $P(C') = \frac{n(C')}{n(S)} = \frac{{}^{39}C_4}{{}^{52}C_4}$
∴ $P(C) = 1 - P(C') = 1 - \frac{{}^{39}C_4}{{}^{52}C_4}$

(iv) Let D be the event that all the 4 cards drawn are clubs and one of them is a jack.
In a pack of 52 cards, there are 13 club cards having 1 jack card.

∴ 1 jack can be drawn in ${}^{1}C_{1}$ way and the other 3 cards can be drawn from remaining 12 club cards in ${}^{12}C_{3}$ ways.
∴ n(D) = ${}^{12}C_{3} \times {}^{1}C_{1}$
∴ P(D) = n(D)n(S)=12C3×1C152C4

Question 4.
A bag contains 15 balls of three different colours: Green, Black, and Yellow. A ball is drawn at random from the bag. The probability of a green ball is 1/3. The probability of yellow is 1/5.
(i) What is the probability of blackball?
(ii) How many balls are green, black, and yellow?
Solution:
(i) The bag contains 15 balls of three different colours i.e., green (G), black (B) and yellow (Y)
∴ P(G) = 13 and P(Y) = 15
If a ball is drawn from the bag, then it can be any one of the green, black and yellow.
∴ P(G) + P(B) + P(Y) = 1
∴ 13 + P(B) + 15 = 1
∴ P(B) + 815 = 1
∴ P(B) = 1 – 815 = 715
∴ Probability of black ball is 715

(ii) Total number of balls = 15 and
P(G) = 13, P(B) = 715, P(Y) = 15
∴ number of green balls = 13 × 15 = 5
number of black balls = 715 × 15 = 7
and number of yellow balls = 15 × 15 = 3.

Question 5.
A box contains 75 tickets numbered 1 to 75. A ticket is drawn at random from the box. What is the probability that the
(i) number on the ticket is divisible by 6?
(ii) number on the ticket is a perfect square?
(iii) number on the ticket is prime?
(iv) number on the ticket is divisible by 3 and 5?

Solution:
The box contains 75 tickets numbered 1 to 75.
∴ 1 ticket can be drawn from the box in ${}^{75}C_1$ = 75 ways.
∴ n(S) = 75
(i) Let A be the event that number on ticket is divisible by 6.
∴ A = {6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72}
∴ n(A) = 12
∴ P(A) = n(A)n(S)=1275=425

(ii) Let B be the event that number on ticket is a perfect square.
∴ B = {1, 4, 9, 16, 25, 36, 49, 64}
∴ n(B) = 8
∴ P(B) = n(B)n(S)=875

(iii) Let C be the event that the number on the ticket is a prime number.
∴ C = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73}
∴ n(C) = 21
∴ P(C) = n(C)n(s)=2175=725

(iv) Let D be the event that number on ticket is divisible by 3 and 5 i.e., divisible by L.C.M. of 3 and 5 i.e., 15
∴ D = {15, 30, 45, 60, 75}
∴ n(D) = 5
∴ P(D) = n(D)n(S)=575=115

Question 6.
From a group of 8 boys and 5 girls, a committee of five is to be formed. Find the probability that the committee contains
(i) 3 boys and 2 girls
(ii) at least 3 boys.
Solution:
The group consists of 8 boys and 5 girls i.e., 8 + 5 = 13 persons.
A committee of 5 is to be formed from this group.
∴ 5 persons from 13 persons can be selected in ${}^{13}C_5$ ways
∴ n(S) = ${}^{13}C_5$
(i) Let A be the event that the committee contains 3 boys and 2 girls.
3 boys from 8 boys can be selected in ${}^{8}C_3$ ways and 2 girls from 5 girls can

be selected in $^5C_2$ ways
∴ n(A) = $^8C_3 \cdot {}^5C_2$
∴ P(A) = n(A)n(S)=8C3·5C213C5

(ii) Let B be the event that the committee contains at least 3 boys (i.e., 3 boys and 2 girls or 4 boys and 1 girl or 5 boys and no girl)
∴ n(B) = $^8C_3 \cdot {}^5C_2 + {}^8C_4 \cdot {}^5C_1 + {}^8C_5 \cdot {}^5C_0$
∴ P(B) = n(B)n(S)=8C3·5C2+8C4·5C1+8C5·5C013C5

Question 7.
A room has three sockets for lamps. From a collection of 10 light bulbs of which 6 are defective, a person selects 3 bulbs at random and puts them in a socket. What is the probability that the room is lit?
Solution:
Total number of bulbs = 10
Number of defective bulbs = 6
∴ Number of non-defective bulbs = 4
3 bulbs can be selected out of 10 light bulbs in $^{10}C_3$ ways.
∴ n(S) = $^{10}C_3$
Let A be the event that room is lit.
∴ A′ is the event that the room is not lit.
For A′ the bulbs should be selected from the 6 defective bulbs.
This can be done in $^6C_3$ ways.
∴ n(A′) = $^6C_3$
∴ P(A′) = n(A′)n(S)=6C310C3
∴ P(Room is lit) = 1 – P(Room is not lit)

$$\begin{aligned} \therefore \quad P(A) &= 1 - P(A') \\ &= 1 - \frac{^6C_3}{^{10}C_3} \\ &= 1 - \frac{6\times5\times4}{10\times9\times8} \\ &= 1 - \frac{1}{6} = \frac{5}{6} \end{aligned}$$

Question 8.
The letters of the word LOGARITHM are arranged at random. Find the

probability that
(i) Vowels are always together.
(ii) Vowels are never together.
(iii) Exactly 4 letters between G and H
(iv) begins with O and ends with T
(v) Start with a vowel and ends with a consonant.
Solution:
There are 9 letters in the word LOGARITHM.
These letters can be arranged among themselves in ${}^9P_9 = 9!$ ways.
$\therefore$ n(S) = 9!
(i) Let A be the event that vowels are always together.
The word LOGARITHM consists of 3 vowels (O, A, I) and 6 consonants (L, G, R, T, H, M).
3 vowels can be arranged among themselves in = ${}^3P_3 = 3!$ ways.
Considering 3 vowels as one group, 6 consonants and this group (i.e., altogether 7) can be arranged in ${}^7P_7 = 7!$ ways.
$\therefore$ n(A) = 3! × 7!
$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3! \times 7!}{9!}$

(ii) Let B be the event that vowels are never together.
Consider the following arrangement
_C_C_C_C_C_C_
6 consonants create 7 gaps.
$\therefore$ 3 vowels can be arranged in 7 gaps in ${}^7P_3$ ways.
Also 6 consonants can be arranged among themselves in ${}^6P_6 = 6!$ ways.
$\therefore n(B) = 6! \times {}^7P_3$
$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{6! \times {}^7P_3}{9!}$

(iii) Let C be the event that exactly 4 letters are arranged between G and H.
Consider the following arrangement
1 2 3 4 5 6 7 8 9
$\therefore$ Out of 9 places, G and H can occupy any one of following 4 positions in 4 ways.
1st and 6th, 2nd and 7th, 3rd and 8th, 4th and 9th
Now, G and H can be arranged among themselves in ${}^2P_2 = 2! = 2$ ways.
Also, the remaining 7 letters can be arranged in remaining 7 places in ${}^7P_7 = 7!$ ways.

∴ n(C) = 4 × 2 × 7! = 8 × 7! = 8!
∴ P(C) = n(C)n(S)=8!9!=8!9×8!=19

(iv) Let D be the event that word begins with O and ends with T.
Thus first and last letter can be arranged in one way each and the remaining 7 letters can be arranged in remaining 7 places in ${}^7P_7$ = 7! ways
∴ n(D) = 7! × 1 × 1 = 7!
∴ P(D) = n(D)n(S)=7!9!

(v) Let E be the event that word starts with vowel and ends with consonant.
There are 3 vowels and 6 consonants in the word LOGARITHM.
∴ The first place can be filled in 3 different ways and the last place can be filled in 6 ways.
Now, remaining 7 letters can be arranged in 7 places in ${}^7P_7$ = 7! ways
∴ n(E) = 3 × 6 × 7!
∴ P(E) = n(E)n(S)=3×6×7!9!

Question 9.
The letters of the word SAVITA are arranged at random. Find the probability that vowels are always together.
Solution:
The word SAVITA contains 6 letters. Out of 6 letters, 3 are vowels (A, A, I) and 3 are consonants (S, V, T).
6 letters in which A repeats twice can be arranged among themselves in 6!2! ways.
∴ n(S) = 6!2!
Let A be the event that vowels are always together.
3 vowels (A, A, I) can be arranged among themselves in 3!2! ways.
Considering 3 vowels as one group, 3 consonants and this group (i.e.

altogether 4) can be arranged in ${}^4P_4 = 4!$ ways.

$$\therefore \quad n(A) = 4! \times \frac{3!}{2!}$$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{4! \times \frac{3!}{2!}}{\frac{6!}{2!}} = \frac{4!\,3!}{6!} = \frac{1}{5}$$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 7 Probability Ex 7.3

Question 1.
Two dice are thrown together. What is the probability that sum of the numbers on two dice is 5 or the number on the second die is greater than or equal to the number on the first die?
Solution:
When two dice are thrown, the sample space is
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
$\therefore$ n(S) = 36
Let A be the event that sum of numbers on two dice is 5.
$\therefore$ A = {(1, 4), (2, 3), (3, 2), (4, 1)}
$\therefore$ n(A) = 4

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{4}{36}$
Let B be the event that number on second die is greater than or equal to number on first die.
B = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 5), (5, 6), (6, 6)}
∴ n(B) = 21
∴ P(B) = $\frac{n(B)}{n(S)} = \frac{21}{36}$
Now, A ∩ B = {(1, 4), (2, 3)}
∴ n(A ∩ B) = 2
∴ P(A ∩ B) = $\frac{n(A \cap B)}{n(S)} = \frac{2}{36}$
∴ Required probability = P(A ∪ B)
P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
= $\frac{4}{36} + \frac{21}{36} - \frac{2}{36}$
= $\frac{23}{36}$

Question 2.
A card is drawn from a pack of 52 cards. What is the probability that,
(i) card is either red or black?
(ii) card is either red or face card?
Solution:
One card can be drawn from the pack of 52 cards in $^{52}C_1$ = 52 ways
∴ n(S) = 52
Also, the pack of 52 cards consists of 26 red and 26 black cards.
(i) Let A be the event that a red card is drawn Red card can be drawn in $^{26}C_1$ = 26 ways
∴ n(A) = 26
∴ P(A) = $\frac{26}{52}$
Let B be the event that a black card is drawn
∴ Black card can be drawn in $^{26}C_1$ = 26 ways.
∴ n(B) = 26
∴ P(B) = $\frac{n(B)}{n(S)} = \frac{26}{52}$
Since A and B are mutually exclusive and exhaustive events
∴ P(A ∩ B) = 0
∴ required probability = P(A ∪ B)
∴ P(A ∪ B) = P(A) + P(B)
= $\frac{26}{52} + \frac{26}{52}$

= 5252
= 1

(ii) Let A be the event that a red card is drawn
∴ red card can be drawn in $^{26}C_1$ = 26 ways
∴ n(A) = 26
∴ P(A) = n(A)n(S)=2652
Let B be the event that a face card is drawn There are 12 face cards in the pack of 52 cards
∴ 1 face card can be drawn in $^{12}C_1$ = 12 ways
∴ n(B) = 12
∴ P(B) = n(B)n(S)=1252
There are 6 red face cards.
∴ n(A ∩ B) = 6
∴ P(A ∩ B) = n(A∩B)n(S)=652
∴ Required probability = P(A ∪ B)
P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
= 2652+1252−652
= 3252
= 813

Question 3.
Two cards are drawn from a pack of 52 cards. What is the probability that,
(i) both the cards are of the same colour?
(ii) both the cards are either black or queens?
Solution:
Two cards can be drawn from 52 cards in $^{52}C_2$ ways.
∴ n(S) = $^{52}C_2$
Also, the pack of 52 cards consists of 26 red and 26 black cards.
(i) Let A be the event that both cards are red.
∴ 2 red cards can be drawn in $^{26}C_2$ ways.
∴ n(A) = $^{26}C_2$
∴ P(A) = n(A)n(S)=26C252C2=26×2552×51=25102
Let B be the event that both cards are black.
∴ 2 black cards can be drawn in $^{26}C_2$ ways
∴ n(B) = $^{26}C_2$
∴ P(B) = n(B)n(S)=26C252C2=26×2552×51=25102
Since A and B are mutually exclusive and exhaustive events

∴ P(A ∩ B) = 0
∴ Required probability = P(A ∪ B)
∴ P(A ∪ B) = P(A) + P(B)
= 25102+25102
= 2551

(ii) Let A be the event that both cards are black.
∴ 2 black cards can be drawn in $^{26}C_2$ ways.
∴ n(A) = $^{26}C_2$
∴ P(A) = n(A)n(S)=26C252C2=26×2552×51=25102
Let B be the event that both cards are queens.
There are 4 queens in a pack of 52 cards
∴ 2 queen cards can be drawn in $^{4}C_2$ ways.
∴ n(B) = $^{4}C_2$
∴ P(B) = n(B)n(S)=4C252C2=4×352×51=1221
There are two black queen cards.
∴ n(A ∩ B) = $^{2}C_2$ = 1

$$\therefore \quad P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{^{52}C_2} = \frac{1 \times 2 \times 1}{52 \times 51}$$
$$= \frac{1}{1326}$$

$$\therefore \quad \text{Required probability} = P(A \cup B)$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{25}{102} + \frac{1}{221} - \frac{1}{1326}$$
$$= \frac{325 + 6 - 1}{1326} = \frac{330}{1326}$$
$$= \frac{55}{221}$$

Question 4.
A bag contains 50 tickets, numbered from 1 to 50. One ticket is drawn at random. What is the probability that
(i) number on the ticket is a perfect square or divisible by 4?
(ii) number on the ticket is a prime number or greater than 30?
Solution:
Out of the 50 tickets, a ticket can be drawn in $^{50}C_1$ = 50 ways.

∴ n(S) = 50

(i) Let A be the event that the number on the ticket is a perfect square.

∴ A = {1, 4, 9, 16, 25, 36, 49}

∴ n(A) = 7

∴ P(A) = n(A)n(S)=750

Let B be the event that the number on the ticket is divisible by 4.

∴ B = {4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48}

∴ n(B) = 12

∴ P(B) = n(B)n(S)=1250

Now, A ∩ B = {4, 16, 36}

∴ n(A ∩ B) = 3

∴ P(A ∩ B) = n(A∩B)n(S)=350

Required probability = P (A u B)

P (A ∪ B) = P(A) + P(B) – P(A ∩ B)

= 750+1250−350

= 825

(ii) Let A be the event that the number on the ticket is a prime number.

∴ A = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47}

∴ n(A) = 15

∴ P(A) = n(A)n(S)=1550

Let B be the event that the number is greater than 30.

∴ B = {31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50}

∴ n(B) = 20

∴ P(B) = n(B)n(S)=2050

Now, A ∩ B = {31, 37, 41, 43, 47}

∴ n(A ∩ B) = 5

∴ P(A ∩ B) = n(A∩B)n(S)=550

∴ Required probability = P(A ∪ B)

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

= 1550+2050−550

= 15+20−550

= 35

Question 5.
A hundred students appeared for two examinations. 60 passed the first, 50 passed the second, and 30 passed in both. Find the probability that students selected at random
(i) passed at least one examination.
(ii) passed in exactly one examination.
(iii) failed in both examinations.
Solution:
Out of hundred students 1 student can be selected in ${}^{100}C_1$ = 100 ways.
∴ n(S) = 100
Let A be the event that the student passed in the first examination.
Let B be the event that student passed in second examination.
∴ n(A) = 60, n(B) = 50 and n(A ∩ B) = 30

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{60}{100} = \frac{6}{10}$$

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{50}{100} = \frac{5}{10}$$

$$\therefore \quad P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{30}{100} = \frac{3}{10}$$

(i) P(student passed in at least one examination) = P(A ∪ B)
= P(A) + P(B) – P (A ∩ B)
= 610+510−310
= 45

(ii) P(student passed in exactly one examination) = P(A) + P(B) – 2.P(A ∩ B)
= 610+510−2(310)
= 12

(iii) P(student failed in both examinations) = P(A′ ∩ B′)
= P(A ∪ B)′ …..[De Morgan’s law]
= 1 – P(A ∪ B)
= 1 – 45
= 15

Question 6.
If P(A) = 14, P(B) = 25 and P(A ∪ B) = 12. Find the values of the following probabilities.
(i) P(A ∩ B)

(ii) P(A ∩ B′)
(iii) P(A′ ∩ B)
(iv) P(A′ ∪ B′)
(v) P(A′ ∩ B′)
Solution:
Here, P(A) = 14, P(B) = 25 and P(A ∪ B) = 12
(i) P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
∴ P(A ∩ B) = P(A) + P(B) – P(A ∪ B)
= 14+25−12
= 320

(ii) P(A′ ∩ B′) = P(A) – P(A ∩ B)
= 14−320
= 110

(iii) P(A′ ∩ B) = P(B) – P(A ∩ B)
= 25−320
= 14

(iv) P(A′ ∪ B′) = P(A ∩ B)′ …..[De Morgan’s law]
= 1 – P(A ∩ B)
= 1 – 320
= 1720

(v) P(A′ ∩ B′) = P(A ∪ B)′ …..[De Morgan’s law]
= 1 – P(A ∪ B)
= 1 – 12
= 12

Question 7.
A computer software company is bidding for computer programs A and B. The probability that the company will get software A is 35, the probability that the company will get software B is 13 and the probability that company will get both A and B is 18. What is the probability that the company will get at least one software?
Solution:
Let A be the event that the company will get software A.
∴ P(A) = 35

Let B be the event that company will get software B.
∴ P(B) = 13
Also, P(A ∩ B) = 18
∴ P(the company will get at least one software) = P(A ∪ B)
= P(A) + P(B) – P(A ∩ B)
= 35+13−18
= 72+40−15120
= 97120

Question 8.
A card is drawn from a well-shuffled pack of 52 cards. Find the probability of it being a heart or a queen.
Solution:
One card can be drawn from the pack of 52 cards in $^{52}C_1$ = 52 ways
∴ n(S) = 52
Also, the pack of 52 cards consists of 13 heart cards and 4 queen cards
Let A be the event that a card drawn is the heart.
A heart card can be drawn from 13 heart cards in $^{13}C_1$ ways
∴ n(A) = $^{13}C_1$
∴ P(A) = n(A)n(S)=13C152=1352
Let B be the event that a card drawn is queen.
A queen card can be drawn from 4 queen cards in $^{4}C_1$ ways
∴ n(B) = $^{4}C_1$
∴ P(B) = n(B)n(S)=4C152=452
There is one queen card out of 4 which is also a heart card
∴ n(A ∩ B) = $^{1}C_1$
∴ P(A ∩ B) = n(A∩B)n(S)=1C152=152
∴ P(card is a heart or a queen) = P(A ∪ B)
= P(A) + P(B) – P(A ∩ B)
= 1352+452−152
= 13+4−152
= 1652
∴ P(A ∪ B) = 413

Question 9.
In a group of students, there are 3 boys and 4 girls. Four students are to be

selected at random from the group. Find the probability that either 3 boys and 1 girl or 3 girls and 1 boy are selected.

Solution:

The group consists of 3 boys and 4 girls i.e., 7 students.

4 students can be selected from this group in $^7C_4$

$= 7×6×5×44×3×2×1$

= 35 ways.

∴ n(S) = 35

Let A be the event that 3 boys and 1 girl are selected.

3 boys can be selected in $^3C_3$ ways while a girl can be selected in $^4C_1$ ways.

∴ n(A) = $^3C_3 \times {}^4C_1 = 4$

∴ P(A) = $n(A)n(S)=435$

Let B be the event that 3 girls and 1 boy are selected.

3 girls can be selected in $^4C3$ ways and a boy can be selected in $^3C_1$ ways.

∴ n(B) = $^4C_3 \times {}^3C_1 = 12$

∴ P(B) = $n(B)n(S)=1235$

Since A and B are mutually exclusive and exhaustive events

∴ P(A ∩ B) = 0

∴ Required probability = P(A ∪ B)

= P(A) + P(B)

$= 435+1235$

$= 1635$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 7 Probability Ex 7.4

Question 1.
Two dice are thrown simultaneously, if at least one of the dice shows a number 5, what is the probability that sum of the numbers on two dice is 9?
Solution:
When two dice are thrown simultaneously, the sample space is
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
∴ n(S) = 36
Let A be the event that at least one die shows number 5.
∴ A = {(1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 5)}
∴ n(A) = 11
∴ P(A) = $\frac{n(A)}{n(S)} = \frac{11}{36}$
Let B be the event that sum of the numbers on two dice is 9.
∴ B = {(3, 6), (4, 5), (5, 4), (6, 3)}
Also, A ∩ B = {(4, 5), (5, 4)}
∴ n(A ∩ B) = 2
∴ P(A ∩ B) = $\frac{n(A \cap B)}{n(S)} = \frac{2}{36}$
∴ Probability of sum of numbers on two dice is 9, given that one dice shows number 5, is given by
P(B/A) = $\frac{P(A \cap B)}{P(A)} = \frac{\frac{2}{36}}{\frac{11}{36}} = \frac{2}{11}$

Question 2.
A pair of dice is thrown. If sum of the numbers is an even number, what is the probability that it is a perfect square?
Solution:
When two dice are thrown simultaneously, the sample space is
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

∴ n(S) = 36
Let A be the event that sum of the numbers is an even number.
∴ A = {(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)}
∴ n(A) = 18
∴ P(A) = $\frac{n(A)}{n(S)}=\frac{18}{36}$
Let B be the event that sum of outcomes is a perfect square.
∴ B = {(1, 3), (2, 2), (3, 1), (3, 6), (4, 5), (5, 4), (6, 3)}
Also, A n B= {(1, 3), (2, 2), (3, 1)}
∴ n(A ∩ B) = 3
∴ P(A ∩ B) = $\frac{n(A\cap B)}{n(S)}=\frac{3}{36}$
∴ Probability of sum of the numbers is a perfect square, given that sum of numbers is an even number, is given by
P(B/A) = $\frac{P(A\cap B)}{P(A)}=\frac{\frac{3}{36}}{\frac{18}{36}}=\frac{3}{18}=\frac{1}{6}$

Question 3.
A box contains 11 tickets numbered from 1 to 11. Two tickets are drawn at random with replacement. If the sum is even, find the probability that both the numbers are odd.
Solution:
Two tickets can be drawn from 11 tickets with replacement in 11 × 11 = 121 ways.
∴ n(S) = 121
Let A be the event that the sum of two numbers is even.
The event A occurs, if either both the tickets with odd numbers or both the tickets with even numbers are drawn.
There are 6 odd numbers (1, 3, 5, 7, 9, 11) and 5 even numbers (2, 4, 6, 8, 10) from 1 to 11.
∴ n(A) = 6 × 6 + 5 × 5
= 36 + 25
= 61
∴ P(A) = $\frac{n(A)}{n(S)}=\frac{61}{121}$
Let B be the event that the numbers tickets drawn are odd
∴ n(B) = 6 × 6 = 36
∴ P(B) = $\frac{n(B)}{n(S)}=\frac{36}{121}$
Since 6 odd numbers are common between A and B.
∴ n(A ∩ B) = 6 × 6 = 36
∴ P(A ∩ B) = $\frac{n(A\cap B)}{n(S)}=\frac{36}{121}$

∴ Probability of both the numbers are odd, given that sum is even, is given by

P(B/A) = P(A∩B)P(A)=3612161121=3661

Question 4.

A card is drawn from a well-shuffled pack of 52 cards. Consider two events A and B as

A: a club card is drawn.

B: an ace card is drawn.

Determine whether events A and B are independent or not.

Solution:

One card can be drawn out of 52 cards in $^{52}C_1$ ways.

∴ $n(S) = {}^{52}C_1$

Let A be the event that a club card is drawn.

1 club card out of 13 club cards can be drawn in $^{13}C_1$ ways.

∴ $n(A) = {}^{13}C_1$

∴ $P(A) = \frac{n(A)}{n(S)} = \frac{^{13}C_1}{^{52}C_1}$

Let B be the event that an ace card is drawn.

An ace card out of 4 aces can be drawn in $^{4}C_1$ ways.

∴ $n(B) = {}^{4}C_1$

∴ $P(B) = \frac{n(B)}{n(S)} = \frac{^{4}C_1}{^{52}C_1}$

Since 1 card is common between A and B

∴ $n(A \cap B) = {}^{1}C_1$

∴ $P(A \cap B) = \frac{n(A\cap B)}{n(S)} = \frac{^{1}C_1}{^{52}C_1} = \frac{1}{52}$ …….(i)

∴ $P(A) \times P(B) = \frac{^{13}C_1}{^{52}C_1} \times \frac{^{4}C_1}{^{52}C_1} = \frac{13\times4}{52\times52} = \frac{1}{52}$ …….(ii)

From (i) and (ii), we get

$P(A \cap B) = P(A) \times P(B)$

∴ A and B are independent events.

Question 5.

A problem in statistics is given to three students A, B, and C. Their chances of solving the problem are 1/3, 1/4, and 1/5 respectively. If all of them try independently, what is the probability that,

(i) problem is solved?

(ii) problem is not solved?

(iii) exactly two students solve the problem?

Solution:
Let A be the event that student A can solve the problem.
B be the event that student B can solve the problem.
C be the event that student C can solve problem.
∴ P(A) = 13, P(B) = 14, P(C) = 15
∴ P(A′) = 1 – P(A) = 1 – 13 = 23
P(B′) = 1 – P(B) = 1 – 14 = 54
P(C′) = 1 – P(C) = 1 – 15 = 45
Since A, B, C are independent events
∴ A′, B′, C′ are also independent events
(i) Let X be the event that problem is solved.
Problem can be solved if at least one of the three students solves the problem.
P(X) = P(at least one student solves the problem)
= 1 – P(no student solved problem)
= 1 – P(A′ ∩ B′ ∩ C′)
= 1 – P(A′) P(B′) P(C′)
= 1 – 23×34×45
= 1 – 25
= 35

(ii) Let Y be the event that problem is not solved
∴ P(Y) = P(A′ ∩ B′ ∩ C′)
= P(A′) P(B′) P(C′)
= 23×34×45
= 25

(iii) Let Z be the event that exactly two students solve the problem.
∴ P(Z) = P(A ∩ B ∩ C′) ∪ P(A ∩ B′ ∩ C) ∪ P(A′ ∩ B ∩ C)
= P(A) . P(B) . P(C′) + P(A) . P(B′) . P(C) + P(A′) . P(B) . P(C)
= (13×14×45)+(13×34×15)+(23×14×15)
= 460+360+260
= 320

Question 6.
The probability that a 50-year old man will be alive till age 60 is 0.83 and

the probability that a 45-year old woman will be alive till age 55 is 0.97. What is the probability that a man whose age is 50 and his wife whose age is 45 will both be alive for the next 10 years?
Solution:
Let A be the event that man will be alive at 60.
∴ P(A) = 0.83
Let B be the event that a woman will be alive at 55.
∴ P(B) = 0.97
A ∩ B = Event that both will be alive.
Also, A and B are independent events
∴ P(both man and his wife will be alive) = P(A ∩ B)
= P(A) . P(B)
= 0.83 × 0.97
= 0.8051

Question 7.
In an examination, 30% of the students have failed in subject I, 20% of the students have failed in subject II and 10% have failed in both subjects I and subject II. A student is selected at random, what is the probability that the student
(i) has failed in the subject I, if it is known that he is failed in subject II?
(ii) has failed in at least one subject?
(iii) has failed in exactly one subject?
Solution:
Let A be the event that the student failed in Subject I
B be the event that the student failed in Subject II
Then P(A) = 30% = 30100
P(B) = 20% = 20100
and P(A ∩ B) = 10% = 10100
(i) P (student failed in Subject I, given that he has failed in Subject II) = P(A/B)
P(A∩B)P(B)=(10100)(20100)=1020=12

(ii) P(student failed in at least one subject) = P(A ∪ B)
= P(A) + P(B) – P(A ∩ B)
= 30100+20100−10100
= 0.40

(iii) P(student failed in exactly one subject) = P(A) + P(B) – 2P(A ∩ B)
= $30100 + 20100 - 2(10100)$
= 0.30

Question 8.
One-shot is fired from each of the three guns. Let A, B, and C denote the events that the target is hit by the first, second and third gun respectively. Assuming that A, B, and C are independent events and that P(A) = 0.5, P(B) = 0.6, and P(C) = 0.8, then find the probability that at least one hit is registered.
Solution:
A be the event that first gun hits the target
B be the event that second gun hits the target
C be the event that third gun hits the target
P(A) = 0.5, P(B) = 0.6, P(C) = 0.8
∴ P(A′) = 1 – P(A) = 1 – 0.5 = 0.5
∴ P(B′) = 1 – P(B) = 1 – 0.6 = 0.4
∴ P(C′) = 1 – P(C) = 1 – 0.8 = 0.2
Now A, B, C are independent events
∴ A′, B′, C are also independent events.
∴ P (at least one hit is registered)
= 1 – P(no hit is registered)
= 1 – P(A′ ∩ B′ ∩ C′)
= 1 – P(A′) P(B′) P(C′)
= 1 – (0.5) (0.4) (0.2)
= 1 – 0.04
= 0.96

Question 9.
A bag contains 10 white balls and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that
(i) first is white and second is black?
(ii) one is white and the other is black?
Solution:
Total number of balls = 10 + 15 = 25
Let S be an event that two balls are drawn at random without replacement

in succession

$\therefore n(S) = {}^{25}C_1 \times {}^{24}C_1 = 25 \times 24$

(i) Let A be the event that the first ball is white and the second is black.

First white ball can be drawn from 10 white balls in ${}^{10}C_1$ ways

and second black ball can be drawn from 15 black balls in ${}^{15}C_1$ ways.

$\therefore n(A) = {}^{10}C_1 \times {}^{15}C_1$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{{}^{10}C_1 \times {}^{15}C_1}{25 \times 24} = \frac{10 \times 15}{25 \times 24} = \frac{1}{4}$

$$\therefore \quad n(B) = {}^{10}C_1 . {}^{15}C_1 + {}^{15}C_1 . {}^{10}C_1$$

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{10 \times 15}{25 \times 24} + \frac{15 \times 10}{25 \times 24}$$

$$= \frac{150}{25 \times 24} + \frac{150}{25 \times 24}$$

$$= \frac{300}{25 \times 24} = \frac{1}{2}$$

Question 10.

An urn contains 4 black, 5 white, and 6 red balls. Two balls are drawn one after the other without replacement, what is the probability that at least one ball is black?

Solution:

Total number of balls in the urn = 4 + 5 + 6 = 15

Two balls can be drawn without replacement in ${}^{15}C_2 = \frac{15 \times 14}{1 \times 2} = 105$ ways

$\therefore n(S) = 105$

Let A be the event that at least one ball is black

i.e., 1 black and 1 non-black or 2 black and 0 non-black.

1 black ball can be drawn out of 4 black balls in ${}^{4}C_1 = 4$ ways

and 1 non-black ball can be drawn out of remaining 11 non-black balls in ${}^{11}C_1 = 11$ ways

$\therefore$ 1 black and 1 non black ball can be drawn in 4 × 11 = 44 ways

Also, 2 black balls can be drawn from 4 black balls in ${}^{4}C_2 = \frac{4 \times 3}{1 \times 2} = 6$ ways

$\therefore n(A) = 44 + 6 = 50$

$\therefore$ Required probability = $P(A) = \frac{n(A)}{n(S)} = \frac{50}{105} = \frac{10}{21}$

Alternate Solution:

Total number of balls = 15

Required probability = 1 – P(neither of two balls is black)

Balls are drawn without replacement

Probability of first non-black ball drawn = 1115
Probability of second non-black ball drawn = 1014
Probability of neither of two balls is black = 1115×1014=1121
Required probability = 1 – 1121 = 1021

Question 11.
Two balls are drawn from an urn containing 5 green, 3 blue, 7 yellow balls one by one without replacement. What is the probability that at least one ball is blue?
Solution:
Total number of balls in the urn = 5 + 3 + 7 = 15
Out of these 12 are non-blue balls.
Two balls can be drawn from 15 balls without replacement in $^{15}C_2$
= 15×141×2
= 105 ways.
∴ n(S) = 105
Let A be the event that at least one ball is blue,
i.e., 1 blue and other non-blue or both are blue.
∴ n(A) = $^{3}C_1 \times {}^{12}C_1 + {}^{3}C_2$
= 3 × 12 + 3
= 36 + 3
= 39
∴ P(A) = n(A)n(S)=39105=1335

Alternate solution:
Total number of balls in the urn = 15
Required probability = 1 – P(neither of two balls is blue)
Balls are drawn one by one without replacement.
Probability of first non-blue ball drawn = 1215
Probability of second non-blue ball drawn = 1114
Probability of neither of two ball is blue = 1215×1114=2235
∴ Required probability = 1 – 2235 = 1335

Question 12.
A bag contains 4 blue and 5 green balls. Another bag contains 3 blue and 7 green balls. If one ball ¡s drawn from each bag, what is the Probability that

two balls are of the same colour?

Solution:

Let A be the event that a blue ball is drawn from each bag.

Probability of drawing one blue ball out of 4 blue balls where there are a total of 9 balls in the first bag and that of drawing one blue ball out of 3 blue balls where there are a total of 10 balls in the second bag is

P(A) = $\frac{4}{9}\times\frac{3}{10}$

Let B be the event that a green ball is drawn from each bag.

Probability of drawing one green ball out of 5 green balls where there are a total of 9 balls in the first bag and that of drawing one green ball out of 7 green balls where there are a total of 10 balls in the second bag is

P(B) = $\frac{5}{9}\times\frac{7}{10}$

Since both, the events are mutually exclusive and exhaustive events

∴ P(that both the balls are of the same colour) = P(both are of blue colour) or P(both are of green colour)

= P(A) + P(B)

= $\frac{4}{9}\times\frac{3}{10}+\frac{5}{9}\times\frac{7}{10}$

= $\frac{12}{90}+\frac{35}{90}$

= $\frac{47}{90}$

Question 13.

Two cards are drawn one after the other from a pack of 52 cards with replacement. What is the probability that both the cards are drawn are face cards?

Solution:

Two cards are drawn from a pack of 52 cards with replacement.

∴ n(S) = 52 × 52

Let A be the event that two cards drawn are face cards.

First card from 12 face cards is drawn with replacement in $^{12}C_1$ = 12 ways and second face card is drawn from 12 face card in $^{12}C_1$ = 12 ways after replacement.

∴ n(A) = 12 × 12

∴ P(that both the cards drawn are face cards) = P(A)

= $\frac{n(A)}{n(S)}=\frac{12\times12}{52\times52}=\frac{9}{169}$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 7 Probability Miscellaneous Exercise 7

Question 1.
From a group of 2 men (M_1, M_2) and three women (W_1, W_2, W_3), two persons are selected. Describe the sample space of the experiment. If E is the event in which one man and one woman are selected, then which are the cases favourable to E?
Solution:
Let S be the sample space of the given event.
∴ S = {(M_1, M_2), (M_1, W_1), (M_1, W_2), (M_1, W_3), (M_2, W_1), (M_2, W_2), (M_2, W_3), (W_1, W_2) (W_1, W_3), (W_2, W_3)}
Let E be the event that one man and one woman are selected.
∴ E = {(M_1, W_1), (M_1, W_2), (M_1, W_3), (M_2, W_1), (M_2, W_2), (M_2, W_3)}
Here, the order is not important in which 2 persons are selected e.g. (M_1, M_2) is the same as (M_2, M_1)

Question 2.
Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys and 1 girl and 3 boys. One child is selected at random from each group. What is the chance that the three selected consist of 1 girl and 2 boys?
Solution:

Group - I		Group - II		Group – III	
Girls	Boys	Girls	Boys	Girls	Boys
3	1	2	2	1	3

Let G_1, G_2, G_3 denote events for selecting a girl,
and B_1, B_2, B_3 denote events for selecting a boy from 1st, 2nd and 3rd groups respectively.
Then P(G_1) = 34, P(G_2) = 24, P(G_3) = 14

P(B_1) = 14, P(B_2) = 24, P(B_3) = 34
Where G_1, G_2, G_3, B_1, B_2 and B_3 are mutually exclusive events.
Let E be the event that 1 girl and 2 boys are selected
∴ E = ($G_1 \cap B_2 \cap B_3$) ∪ ($B_1 \cap G_2 \cap B_3$) ∪ ($B_1 \cap B_2 \cap G_3$)
∴ P(E) = P($G_1 \cap B_2 \cap B_3$) + P($B_1 \cap G_2 \cap B_3$) + P($B_1 \cap B_2 \cap G_3$)

$$= \left(\frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}\right) + \left(\frac{1}{4}\times\frac{2}{4}\times\frac{3}{4}\right) + \left(\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}\right)$$

$$= \frac{18+6+2}{64} = \frac{26}{64}$$

$$= \frac{13}{32}$$

Question 3.
A room has 3 sockets for lamps. From a collection of 10 light bulbs, 6 are defective. A person selects 3 at random and puts them in every socket. What is the probability that the room, will be lit?
Solution:
Total number of bulbs = 10
Number of defective bulbs = 6
∴ Number of non-defective bulbs = 4
3 bulbs can be selected out of 10 light bulbs in ${}^{10}C_3$ ways.
∴ n(S) = ${}^{10}C_3$
Let A be the event that room is lit.
∴ A′ is the event that the room is not lit.
For A′ the bulbs should be selected from the 6 defective bulbs.
This can be done in ${}^{6}C_3$ ways.
∴ n(A′) = ${}^{6}C_3$
∴ P(A′) = n(A′)n(S)=6C310C3
∴ P(Room is lit) = 1 – P(Room is not lit)

$$\therefore \quad P(A) = 1 - P(A')$$

$$= 1 - \frac{{}^{6}C_3}{{}^{10}C_3}$$

$$= 1 - \frac{6\times5\times4}{10\times9\times8}$$

$$= 1 - \frac{1}{6} = \frac{5}{6}$$

Question 4.
There are 2 red and 3 black balls in a bag. 3 balls are taken out at random from the bag. Find the probability of getting 2 red and 1 black ball or 1 red and 2 black balls.
Solution:
There are 2 + 3 = 5 balls in the bag and 3 balls can be drawn out of these in $^5C_3 = \frac{5\times4\times3}{1\times2\times3}$ = 10 ways.
$\therefore$ n(S) = 10
Let A be the event that 2 balls are red and 1 ball is black
2 red balls can be drawn out of 2 red balls in $^2C_2$ = 1 way
and 1 black ball can be drawn out of 3 black balls in $^3C_1$ = 3 ways.
$\therefore n(A) = {^2C_2} \times {^3C_1} = 1 \times 3 = 3$
$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{10}$
Let B be the event that 1 ball is red and 2 balls are black
1 red ball out of 2 red balls can be drawn in $^2C_1$ = 2 ways
and 2 black balls out of 3 black balls can be drawn in $^3C_2 = \frac{3\times2}{1\times2}$ = 3 ways.
$\therefore n(B) = {^2C_1} \times {^3C_2} = 2 \times 3 = 6$
$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{6}{10}$
Since A and B are mutually exclusive and exhaustive events
$\therefore P(A \cap B) = 0$
$\therefore$ Required probability = $P(A \cup B) = P(A) + P(B)$
$= \frac{3}{10} + \frac{6}{10}$
$= \frac{9}{10}$

Question 5.
A box contains 25 tickets numbered 1 to 25. Two tickets are drawn at random. What is the probability that the product of the numbers is even?
Solution:
Two tickets can be drawn out of 25 tickets in $^{25}C_2 = \frac{25\times24}{1\times2}$ = 300 ways.
$\therefore$ n(S) = 300
Let A be the event that product of two numbers is even.
This is possible if both numbers are even, or one number is even and other is odd.
As there are 13 odd numbers and 12 even numbers from 1 to 25.
$\therefore n(A) = {^{12}C_2} + {^{12}C_1} \times {^{13}C_1}$
$= \frac{12\times11}{1\times2} + 12 \times 13$

= 66 + 156
= 222
∴ Required probability = P(A)
= n(A)n(S)
= 222300
= 3750

Question 6.
A, B and C are mutually exclusive and exhaustive events associated with the random experiment. Find P(A), given that
P(B) = 32 P(A) and P(C) = 12 P(B)
Solution:
P(B) = 32 P(A) and P(C) = 12 P(B)
Since A, B, C are mutually exclusive and exhaustive events,

$$\therefore \quad P(A \cup B \cup C) = P(A) + P(B) + P(C) = 1$$

$$\therefore \quad P(A) + \frac{3}{2}P(A) + \frac{1}{2}P(B) = 1$$

$$\therefore \quad P(A) + \frac{3}{2}P(A) + \frac{1}{2} \times \frac{3}{2}P(A) = 1$$

$$\therefore \quad P(A) + \frac{3}{2}P(A) + \frac{3}{4}P(A) = 1$$

$$\therefore \quad P(A) \times \left(1 + \frac{3}{2} + \frac{3}{4}\right) = 1$$

$$\therefore \quad P(A) \times \left(\frac{13}{4}\right) = 1$$

$$\therefore \quad P(A) = \frac{4}{13}$$

Question 7.
An urn contains four tickets marked with numbers 112, 121, 122, 222, and one ticket is drawn at random. Let A_i (i = 1, 2, 3) be the event that ith digit of the number of the ticket drawn is 1. Discuss the independence of the events A_1, A_2, and A_3.
Solution:
One ticket can be drawn out of 4 tickets in ${}^4C_1$ = 4 ways.
∴ n(S) = 4
According to the given information,
Let A_1 be the event that 1st digit of the number of tickets is 1

A_2 be the event that the 2nd digit of the number of tickets is 1
A_3 be the event that the 3rd digit of the number of tickets is 1
$\therefore A_1 = \{112, 121, 122\}, A_2 = \{112\}, A_3 = \{121\}$

$$\therefore \quad P(A_1) = \frac{n(A_1)}{n(S)} = \frac{3}{4},$$

$$P(A_2) = \frac{n(A_2)}{n(S)} = \frac{1}{4},$$

$$P(A_3) = \frac{n(A_3)}{n(S)} = \frac{1}{4}$$

$$\left.\begin{aligned} P(A_1)\,P(A_2) &= \frac{3}{16}, \\ P(A_2)\,P(A_3) &= \frac{1}{16} \\ P(A_1)\,P(A_3) &= \frac{3}{16} \end{aligned}\right\} \quad \ldots\text{(i)}$$

$$A_1 \cap A_2 = \{112\}, A_2 \cap A_3 = \phi, A_1 \cap A_3 = \{121\}$$

$$\left.\begin{aligned} P(A_1 \cap A_2) &= \frac{n(A_1 \cap A_2)}{n(S)} = \frac{1}{4}, \\ P(A_2 \cap A_3) &= 0, \\ P(A_1 \cap A_3) &= \frac{1}{4} \end{aligned}\right\} \quad \ldots\text{(ii)}$$

$\therefore$ From (i) and (ii),

$$\left.\begin{aligned} P(A_1).P(A_2) &\neq P(A_1 \cap A_2), \\ P(A_2).P(A_3) &\neq P(A_2 \cap A_3), \\ P(A_1).P(A_3) &\neq P(A_1 \cap A_3) \end{aligned}\right\} \quad \ldots\text{(iii)}$$

$\therefore A_1, A_2, A_3$ are not pairwise independent
For mutual independence of events A_1, A_2, A_3
We require to have
$P(A_1 \cap A_2 \cap A_3) = P(A_1)\,P(A_2)\,P(A_3)$
and $P(A_1)\,P(A_2) = P(A_1 \cap A_2)$,
$P(A_2)\,P(A_3) = P(A_2 \cap A_3)$,
$P(A_1)\,P(A_3) = P(A_1 \cap A_3)$
$\therefore$ From (iii),
A_1, A_2, A_3 are not mutually independent.

Question 8.
The odds against a certain event are 5 : 2 and the odds in favour of another independent event are 6 : 5. Find the chance that at least one of the events

will happen.
Solution:
Let A and B be two independent events.
Odds against A are 5 : 2
∴ the probability of occurrence of event A is given by
P(A) = $25+2=27$
Odds in favour of B are 6 : 5
∴ the probability of occurrence of event B is given by
P(B) = $66+5=611$
∴ P(at least one event will happen) = P(A ∪ B)
= P(A) + P(B) – P(A ∩ B)
= P(A) + P(B) – P(A) P(B) ……[∵ A and B are independent events]

$$= \frac{2}{7} + \frac{6}{11} - \frac{2}{7} \times \frac{6}{11}$$
$$= \frac{2}{7} + \frac{6}{11} - \frac{12}{77} = \frac{22+42-12}{77} = \frac{52}{77}$$

Question 9.
The odds against a husband who is 55 years old living till he is 75 is 8 : 5 and it is 4 : 3 against his wife who is now 48, living till she is 68. Find the probability that
(i) the couple will be alive 20 years hence
(ii) at least one of them will be alive 20 years hence.
Solution:
Let A be the event that husband would be alive after 20 years.
Odds against A are 8 : 5
∴ the probability of occurrence of event A is given by
P(A) = $58+5=513$
∴ P(A′) = 1 – P(A)
= 1 – 513
= 813
Let B be the event that wife would be alive after 20 years.
Odds against B are 4 : 3
∴ the probability of occurrence of event B is given by
P(B) = $34+3=37$
∴ P(B′) = 1 – P(B)
= 1 – 37
= 47

Since A and B are independent events
∴ A′ and B′ are also independent events
(i) Let X be the event that both will be alive after 20 years.
∴ P(X) = (A ∩ B)
∴ P(X) = P(A) . P(B)
= 513×37
= 1591

(ii) Let Y be the event that at least one will be alive after 20 years.
∴ P(Y) = P(at least one would be alive)
= 1 – P(both would not be alive)
= 1 – P(A′ ∩ B′)
= 1 – P(A′). P(B′)
= 1 – 813×47
= 1 – 3291
= 5991

Question 10.
Two throws are made, the first with 3 dice and the second with 2 dice. The faces of each die are marked with the number 1 to 6. What is the probability that the total in the first throw is not less than 15 and at the same time the total in the second throw is not less than 8?
Solution:
When 3 dice are thrown, then the sample space S_1 has 6 × 6 × 6 = 216 sample points.
∴ $n(S_1)$ = 216
Let A be the event that the sum of the numbers is not less than 15.
∴ A = {(3, 6, 6), (4, 5, 6), (4, 6, 5), (4, 6, 6), (5, 4, 6), (5, 5, 5), (5, 5, 6), (5, 6, 4), (5, 6, 5), (5, 6, 6), (6, 3, 6), (6, 4, 5), (6, 4, 6), (6, 5, 4), (6, 5, 5), (6, 5, 6), (6, 6, 3), (6, 6, 4), (6, 6, 5), (6, 6, 6)}
∴ n(A) = 20
∴ P(A) = n(A)n(S1)=20216=554
When 2 dice are thrown, the sample space S_2 has 6 × 6 = 36 sample points.
∴ $n(S_2)$ = 36
Let B be the event that sum of numbers is not less than 8.
∴ B = {(2, 6), (3, 5), (3,6), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6,

3), (6, 4), (6, 5), (6, 6)}
∴ n(B) = 15
∴ P(B) = $\frac{n(B)}{n(S_2)} = \frac{15}{36} = \frac{5}{12}$
A ∩ B = Event that the total in the first throw is not less than 15 and at the same time the total in the second throw is not less than 8.
∴ A and B are independent events
∴ P(A ∩ B) = P(A) . P(B)
= $\frac{5}{54} \times \frac{5}{12}$
= $\frac{25}{648}$

Question 11.
Two-thirds of the students in a class are boys and the rest are girls. It is known that the probability of a girl getting first class is 0.25 and that of a boy getting is 0.28. Find the probability that a student chosen at random will get first class.
Solution:
Let A be the event that student chosen is a boy
B be the event that student chosen is a girl
C be the event that student gets first class
∴ P(A) = $\frac{2}{3}$, P(B) = $\frac{1}{3}$
Probability of student getting first class, given that student is boy
Probability of student getting first class given that student is a girl, is
P(C/A) = 0.28 = $\frac{28}{100}$
and P(C/B) = 0.25 = $\frac{25}{100}$
∴ Required probability = P((A ∩ C) ∪ (B ∩ C))
Since A ∩ C and B ∩ C are mutually exclusive events
∴ Required probability = P(A ∩ C) + P(B ∩ C)
= P(A) . P(C/A) + P(B) . P(C/B)

$$= \left(\frac{2}{3} \times \frac{28}{100}\right) + \left(\frac{1}{3} \times \frac{25}{100}\right)$$

$$= \frac{56 + 25}{300} = \frac{81}{300} = \frac{27}{100} = 0.27$$

Question 12.
A number of two digits is formed using the digits 1, 2, 3,……, 9. What is the probability that the number so chosen is even and less than 60?
Solution:

The number of two digits can be formed from the given 9 digits in 9 × 9 = 81 different ways.
∴ n(S) = 81
Let A be the event that the number is even and less than 60.
Since the number is even, the unit place of two digits can be filled in $^4P_1$ = 4 different ways by any one of the digits 2, 4, 6, 8.
Also the number is less than 60, so tenth place can be filled in $^5P_1$ = 5 different ways by any one of the digits 1, 2, 3, 4, 5.
∴ n(A) = 4 × 5 = 20
∴ Required probability = P(A) = n(A)n(S)=2081

Question 13.
A bag contains 8 red balls and 5 white balls. Two successive draws of 3 balls each are made without replacement. Find the probability that the first drawing will give 3 white balls and the second drawing will give 3 red balls.
Solution:
Total number of balls = 8 + 5 = 13.
3 balls can be drawn out of 13 balls in $^{13}C_3$ ways.
∴ n(S) = $^{13}C_3$
Let A be the event that all 3 balls drawn are white.
3 white balls can be drawn out of 5 white balls in $^5C_3$ ways.
∴ n(A) = $^5C_3$
∴ P(A) = n(A)n(S)=5C313C3=5×4×313×12×11=5143
After drawing 3 white balls which are not replaced in the bag, there are 10 balls left in the bag out of which 8 are red balls.
Let B be the event that the second draw of 3 balls are red.
∴ Probability of drawing 3 red balls, given that 3 white balls have been already drawn, is given by
P(B/A) = 8C310C3=8×7×610×9×8=715
∴ Required probability = P(A ∩ B)
= P(A) . P(B/A)
= 5143×715
= 7429

Question 14.
The odds against student X solving a business statistics problem are 8 : 6

and the odds in favour of student Y solving the same problem are 14 : 16
(i) What is the chance that the problem will be solved, if they try independently?
(ii) What is the probability that neither solves the problem?
Solution:
(i) Let A be the event that X solves the problem B be the event that Y solves the problem.
Since the odds against student X solving the problem are 8 : 6
∴ Probability of occurrence of event A is given by
P(A) = 68+6=614
and P(A′) = 1 – P(A)
= 1 – 614
= 814
Also, the odds in favour of student Y solving the problem are 14 : 16
∴ Probability of occurrence of event B is given by
P(B) = 1414+16=1430 and
P(B′) = 1 – P(B)
= 1 – 1430
= 1630
Now A and B are independent events.
∴ A′ and B′ are independent events.
∴ A′ ∩ B′ = Event that neither solves the problem
= P(A′ ∩ B′)
= P(A′) . P(B′)
= 814×1630
= 32105
A ∪ B = the event that the problem is solved
∴ P(problem will be solved) = P(A ∪ B)
= 1 – P(A ∪ B)′
= 1 – P(A′ ∩ B′)
= 1 – 32105
= 1 – 73105

(ii) P (neither solves the problem) = P(A′ ∩ B′)
= P(A′) P(B′)

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$= 814 \times 1630$

$= 32105$