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Maharashtra State Board 11th Maths Solutions Chapter 8 Measures of Dispersion Ex 8.1

Question 1.
Find a range of the following data:
19, 27, 15, 21, 33, 45, 7, 12, 20, 26
Solution:
Here, largest value (L) = 45, smallest value (S) = 7
∴ Range = L – S = 45 – 7 = 38

Question 2.
Find range of the following data:
575, 609, 335, 280, 729, 544, 852, 427, 967, 250
Solution:
Here, largest value (L) = 967, smallest value (S) = 250
∴ Range = L – S = 967 – 250 = 717

Question 3.
The following data gives a number of typing mistakes done by Radha during a week. Find the range of the data.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
No. of mistakes	15	20	21	12	17	10

Solution:
Here, largest value (L) = 21, smallest value (S) = 10
∴ Range = L – S = 21 – 10 = 11

Question 4.
The following results were obtained by rolling a die 25 times. Find the range of the data.

Score	1	2	3	4	5	6
Frequency	4	6	2	7	3	3

Solution:
Here, largest value (L) = 6, smallest value (S) = 1
∴ Range = L – S = 6 – 1 = 5

Question 5.
Find range for the following data:

Classes	62-64	64-66	66-68	68-70	70-72
Frequency	5	3	4	5	3

Solution:
Here, upper limit of the highest class (L) = 72
lower limit of the lowest class (S) = 62
∴ Range = L – S = 72 – 62 = 10

Maharashtra State Board 11th Maths Solutions Chapter 8 Measures of Dispersion Ex 8.2

Question 1.
Find variance and S.D. for the following set of numbers.
7, 11, 2, 4, 9, 6, 3, 7, 11, 2, 5, 8, 3, 6, 8, 8, 2, 6
Solution:

Given data:
7, 11, 2, 4, 9, 6, 3, 7, 11, 2, 5, 8, 3, 6, 8, 8, 2, 6
The tabulated form of the above data is

x_i	f_i
2	3
3	2
4	1
5	1
6	3
7	2
8	3
9	1
11	2

Calculation of variance and S.D.

x_i	f_i	f_ix_i	x_i^2	$f_ix_i^2$
2	3	6	4	12
3	2	6	9	18
4	1	4	16	16
5	1	5	25	25
6	3	18	36	108
7	2	14	49	98
8	3	24	64	192
9	1	9	81	81
11	2	22	121	242
Total	N = 18	$\sum f_ix_i = 108$		$\sum f_ix_i^2 = 792$

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{108}{18} = 6$$

$$\text{Var (X)} = \sigma_x^2 = \frac{\sum f_i x_i^2}{N} - (\bar{x})^2$$

$$= \frac{792}{18} - (6)^2 = 44 - 36 = 8$$

$$\text{S. D.} = \sigma_x = \sqrt{\text{Var(X)}} = \sqrt{8} = 2.82$$

Question 2.
Find variance and S.D. for the following set of numbers.
65, 77, 81, 98, 100, 80, 129
Solution:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
65	–25	625
77	–13	169
81	–9	81
98	8	64
100	10	100
80	–10	100
129	39	1521
$\sum x_i = 630$		$\sum(x_i - \bar{x})^2 = 2660$

Here, n = 7

$$\bar{x} = \frac{\sum x_i}{n} = \frac{630}{7} = 90$$

$$\text{Var (X)} = \sigma_x^2 = \frac{1}{n}\sum(x_i - \bar{x})^2 = \frac{2660}{7} = 380$$

$$\text{S. D.} = \sigma_x = \sqrt{\text{Var(X)}} = \sqrt{380} = 19.49$$

Question 3.
Compute variance and standard deviation for the following data:

X	2	4	6	8	10	12	14	16	18	20
F	8	10	10	7	6	4	3	4	2	6

Solution:

x_i	f_i	$f_i x_i$	$f_i x_i^2$
2	8	16	32
4	10	40	160
6	10	60	360
8	7	56	448
10	6	60	600
12	4	48	576
14	3	42	588
16	4	64	1024
18	2	36	648
20	6	120	2400
	N = 60	$\Sigma f_i x_i = 542$	$\Sigma f_i x_i^2 = 6836$

$$\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{542}{60} = 9.03$$

$$\text{Var}(X) = \sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2$$

$$= \frac{6836}{60} - (9.03)^2$$

$$= 113.93 - 81.54$$

$$= 32.39$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{32.39} = 5.69$$

Question 4.
Compute the variance and S.D.

X	31	32	33	34	35	36	37
Frequency	15	12	10	8	9	10	6

Solution:

Let u = x-Ah=x-341

Calculation of variance of u:

x_i	f_i	$u_i = \frac{x_i - 34}{1}$	$f_i u_i$	$f_i u_i^2$
31	15	−3	−45	135
32	12	−2	−24	48
33	10	−1	−10	10
34	8	0	0	0
35	9	1	9	9
36	10	2	20	40
37	6	3	18	54
	N = 70		$\Sigma f_i u_i = -32$	$\Sigma f_i u_i^2 = 296$

$$\bar{u} = \frac{\Sigma f_i u_i}{N} = \frac{-32}{70} = -0.4571$$

$$\text{Var}(u) = \sigma_u^2 = \frac{\Sigma f_i u_i^2}{N} - (\bar{u})^2 = \frac{296}{70} - (-0.4571)^2$$

$$= 4.2286 - 0.2089$$

$$= 4.0197$$

$$\text{Var}(X) = h^2\,\text{Var}(u) = (1)^2 \times 4.0197 = 4.0197$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{4.0197} = 2.005$$

Question 5.
Following data gives ages of 100 students in a college. Calculate variance and S.D.

Age (In years)	16	17	18	19	20	21
No. of Students	20	7	11	17	30	15

Solution:

Let u = x-Ah=x-191

Age (in years) (x_i)	No. of Students (f_i)	$u_i = \frac{x_i - 19}{1}$	f_iu_i	$f_iu_i^2$
16	20	–3	–60	180
17	7	–2	–14	28
18	11	–1	–11	11
19	17	0	0	0
20	30	1	30	30
21	15	2	30	60
	N = 100		$\Sigma f_iu_i = -25$	$\Sigma f_iu_i^2 = 309$

$$\bar{u} = \frac{\Sigma f_i u_i}{N} = \frac{-25}{100} = -0.25$$

$$\text{Var (u)} = \sigma_u^2 = \frac{\Sigma f_i u_i^2}{N} - (\bar{u})^2 = \frac{309}{100} - (-0.25)^2$$

$$= 3.09 - 0.0625$$

$$= 3.0275$$

$$\text{Var (X)} = h^2 \text{ Var (u)} = (1)^2 (3.0275) = 3.0275$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var(X)}} = \sqrt{3.0275} = 1.74$$

Question 6.
Find mean, variance and S.D. of the following data.

Classes	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Freq.	7	14	6	13	9	15	11	10	15

Solution:

Classes	Mid value (x_i)	f_i	f_ix_i	$f_ix_i^2$
10 – 20	15	7	105	1575
20 – 30	25	14	350	8750
30 – 40	35	6	210	7350
40 – 50	45	13	585	26325
50 – 60	55	9	495	27225
60 – 70	65	15	975	63375
70 – 80	75	11	825	61875
80 – 90	85	10	850	72250
90 – 100	95	15	1425	135375
		N = 100	$\Sigma f_ix_i = 5820$	$\Sigma f_ix_i^2 = 404100$

$$\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{5820}{100} = 58.2$$

$$\text{Var (X)} = \sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2$$

$$= \frac{404100}{100} - (58.2)^2$$

$$= 4041 - 3387.24$$

$$= 653.76$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var(X)}} = \sqrt{653.76} = 25.56$$

Alternate Method:

Let u = $x-Ah=x-5510$

Calculation of variance of u:

Classes	Mid value (x_i)	f_i	$u_i = \frac{x_i - 55}{10}$	f_iu_i	$f_iu_i^2$
10 – 20	15	7	–4	–28	112
20 – 30	25	14	–3	–42	126
30 – 40	35	6	–2	–12	24
40 – 50	45	13	–1	–13	13
50 – 60	55	9	0	0	0
60 – 70	65	15	1	15	15
70 – 80	75	11	2	22	44
80 – 90	85	10	3	30	90
90 – 100	95	15	4	60	240
		N = 100		$\Sigma f_iu_i = 32$	$\Sigma f_iu_i^2 = 664$

$$\bar{u} = \frac{\Sigma f_iu_i}{N} = \frac{32}{100} = 0.32$$

$$\begin{aligned}\bar{x} &= \bar{u} \times h + A \\ &= 0.32 \times 10 + 55 \\ &= 58.2\end{aligned}$$

$$\begin{aligned}\text{Var}(u) = \sigma_u^2 &= \frac{\Sigma f_iu_i^2}{N} - (\bar{u})^2 \\ &= \frac{664}{100} - (0.32)^2 \\ &= 6.64 - 0.1024 \\ &= 6.5376\end{aligned}$$

$$\begin{aligned}\text{Var}(X) &= h^2\,\text{Var}(u) \\ &= (10)^2 \times 6.5376 \\ &= 100 \times 6.5376 \\ &= 653.76\end{aligned}$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{653.76} = 25.56$$

Question 7.
Find the variance and S.D. of the following frequency distribution which gives the distribution of 200 plants according to their height.

Height (in cm)	14-18	19-23	24-28	29-33	34-38	39-43	44-48
No. of plants	5	18	44	70	36	22	5

Solution:
Since data is not continuous, we have to make it continuous.

Let u = $\frac{x-A}{h} = \frac{x-31}{5}$

Calculation of variance of u:

Height (in cm) C. I.	Mid value (x_i)	No. of plants (f_i)	$u_i = \frac{x_i - 31}{5}$	f_iu_i	$f_iu_i^2$
13.5 – 18.5	16	5	–3	–15	45
18.5 – 23.5	21	18	–2	–36	72
23.5 – 28.5	26	44	–1	–44	44
28.5 – 33.5	31	70	0	0	0
33.5 – 38.5	36	36	1	36	36
38.5 – 43.5	41	22	2	44	88
43.5 – 48.5	46	5	3	15	45
		N = 200		$\Sigma f_iu_i = 0$	$\Sigma f_iu_i^2 = 330$

$$\bar{u} = \frac{\Sigma f_iu_i}{N} = \frac{0}{200} = 0$$

$$\begin{aligned}\text{Var}(u) = \sigma_u^2 &= \frac{\Sigma f_iu_i^2}{N} - (\bar{u})^2 \\ &= \frac{330}{200} - 0^2 \\ &= 1.65\end{aligned}$$

$$\text{Var}(X) = h^2\,\text{Var}(u) = (5)^2 \times 1.65 = 25 \times 1.65 = 41.25$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{41.25} = 6.42$$

Question 8.
The mean of 5 observations is 4.8 and the variance is 6.56. If three of the five observations are 1, 3, and 8, find the other two observations.
Solution:
$\bar{x}$ = 4.8, Var (X) = 6.56, n = 5, x_1 = 1, x_2 = 3, x_3 = 8 ......(given)
Let the remaining two observations be x_4 and x_5.

$$\bar{x} = \frac{\sum x_i}{n}$$

$$\therefore \quad \sum x_i = n\bar{x} = 5 \times 4.8 = 24$$

$$\text{Var (X)} = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$\therefore \quad 6.56 = \frac{\sum x_i^2}{5} - (4.8)^2$$

$$\therefore \quad \Sigma x_i^2 = 5[6.56 + (4.8)^2]$$

$$= 5 \times 29.6 = 148$$

$$\text{Now, } \sum x_i = 1 + 3 + 8 + x_4 + x_5$$

$$\therefore \quad 24 = 12 + x_4 + x_5$$

$$\therefore \quad x_4 + x_5 = 12$$

$$\therefore \quad x_5 = 12 - x_4 \quad \text{...... (i)}$$

$$\sum x_i^2 = 1^2 + 3^2 + 8^2 + x_4^2 + x_5^2$$

$$\therefore \quad 148 = 1 + 9 + 64 + x_4^2 + (12 - x_4)^2 \quad \text{...... [From (i)]}$$

$$\therefore \quad x_4^2 + x_4^2 - 24x_4 + 144 + 74 = 148$$

$$\therefore \quad 2x_4^2 - 24x_4 + 70 = 0$$

$$\therefore \quad x_4^2 - 12x_4 + 35 = 0$$

$$\therefore \quad x_4^2 - 7x_4 - 5x_4 + 35 = 0$$

$$\therefore \quad x_4(x_4 - 7) - 5(x_4 - 7) = 0$$

$$\therefore \quad (x_4 - 7)(x_4 - 5) = 0$$

$$\therefore \quad x_4 = 7 \text{ or } x_4 = 5$$

From (i), we get
x_5 = 5 or x_5 = 7
∴ The two numbers are 5 and 7.

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Question 1.
The means of two samples of sizes 60 and 120 respectively are 35.4 and 30.9 and the standard deviations are 4 and 5. Obtain the standard deviation of the sample of size 180 obtained by combining the two samples.
Solution:

Let $n_1 = 60, n_2 = 120, \bar{x}_1 = 35.4, \bar{x}_2 = 30.9, \sigma_1 = 4, \sigma_2 = 5$

Combined mean $(\bar{x}_c) = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{60 \times 35.4 + 120 \times 30.9}{60 + 120} = \frac{2124 + 3708}{180} = \frac{5832}{180} = 32.4$

Now, $d_1 = \bar{x}_1 - \bar{x}_c = 35.4 - 32.4 = 3$

$d_2 = \bar{x}_2 - \bar{x}_c = 30.9 - 32.4 = -1.5$

$\therefore$ Combined standard deviation $(\sigma_c) = \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}} = \sqrt{\frac{60(4^2 + 3^2) + 120[5^2 + (-1.5)^2]}{60 + 120}}$

$$= \sqrt{\frac{60(16 + 9) + 120(25 + 2.25)}{180}}$$

$$= \sqrt{\frac{60(25) + 120(27.25)}{180}}$$

$$= \sqrt{\frac{1500 + 3270}{180}}$$

$$= \sqrt{\frac{4770}{180}} = \sqrt{26.5} = 5.15$$

Question 2.
For certain data, the following information is available.

	X	Y
Mean	13	17
S. D.	3	2
Size	20	30

Obtain the combined standard deviation.
Solution:

Here, $\bar{x} = 13, \bar{y} = 17, \sigma_x = 3, \sigma_y = 2, n_x = 20, n_y = 30$

Combined mean $(\bar{x}_c) = \frac{n_x\bar{x} + n_y\bar{y}}{n_x + n_y} = \frac{20(13) + 30(17)}{20+30}$

$$= \frac{260+510}{50}$$

$$= \frac{770}{50} = 15.4$$

Now, $d_x = \bar{x} - \bar{x}_c = 13 - 15.4 = -2.4$

$d_y = \bar{y} - \bar{x}_c = 17 - 15.4 = 1.6$

$\therefore$ Combined standard deviation $(\sigma_c) = \sqrt{\frac{n_x(\sigma_x^2 + d_x^2) + n_y(\sigma_y^2 + d_y^2)}{n_x + n_y}}$

$$= \sqrt{\frac{20[3^2 + (-2.4)^2] + 30(2^2 + 1.6^2)}{20 + 30}}$$

$$= \sqrt{\frac{20(9+5.76)+30(4+2.56)}{50}}$$

$$= \sqrt{\frac{20(14.76)+30(6.56)}{50}}$$

$$= \sqrt{\frac{295.2+196.8}{50}}$$

$$= \sqrt{\frac{492}{50}} = \sqrt{9.84} = 3.14$$

Question 3.
Calculate the coefficient of variation of marks secured by a student in the exam, where the marks are:
85, 91, 96, 88, 98, 82

Solution:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
85	–5	25
91	1	1
96	6	36
88	–2	4
98	8	64
82	–8	64
$\Sigma x_i = 540$		$\Sigma(x_i - \bar{x})^2 = 194$

Here, n = 6, $\bar{x} = \frac{\Sigma x_i}{n} = \frac{540}{6} = 90$

$\text{Var}(X) = \sigma_x^2 = \frac{1}{n}\Sigma(x_i - \bar{x})^2 = \frac{194}{6} = 32.33$

$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)}$

$= \sqrt{32.33}$

$= 5.69$

Now, $\text{C.V.} = 100 \times \frac{\sigma_x}{\bar{x}}$

$= 100 \times \frac{5.69}{90} = 6.32\%$

Question 4.
Find the coefficient of variation of a sample that has a mean equal to 25 and a standard deviation of 5.
Solution:

Given, $\bar{x} = 25$, $\sigma_x = 5$

$\text{C.V.} = 100 \times \frac{\sigma_x}{\bar{x}}$

$= 100 \times \frac{5}{25} = 20\%$

Question 5.
A group of 65 students of class XI has their average height as 150.4 cm with a coefficient of variance of 2.5%. What is the standard deviation of their heights?
Solution:

Given, n = 65, $\bar{x}$ = 150.4, C.V. = 2.5%

$\text{C.V.} = 100 \times \frac{\sigma_x}{\bar{x}}$

$\therefore \quad 2.5 = 100 \times \frac{\sigma_x}{150.4}$

$\therefore \quad \frac{2.5 \times 150.4}{100} = \sigma_x$

$\therefore \quad \sigma_x = 3.76$

∴ The standard deviation of students' height is 3.76 cm.

Question 6.
Two workers on the same job show the following results:

	Worker P	Worker Q
Mean time for completing the job (hours)	33	21
Standard Deviation (hours)	9	7

(i) Regarding the time required to complete the job, which worker is more consistent?
(ii) Which worker seems to be faster in completing the job?
Solution:

Here, $\bar{p} = 33$, $\bar{q} = 21$, $\sigma_p = 9$, $\sigma_q = 7$

$$\text{C.V. (P)} = 100 \times \frac{\sigma_p}{\bar{p}} = 100 \times \frac{9}{33} = 27.27\%$$

$$\text{C.V. (Q)} = 100 \times \frac{\sigma_q}{\bar{q}} = 100 \times \frac{7}{21} = 33.33\%$$

(i) Since C.V. (P) < C.V.(Q), Worker P is more consistent regarding the time required to complete the job. (ii) Since $\bar{p} > \bar{q}$,
i.e., the expected time for completing the job is less for worker Q.
∴ Worker Q seems to be faster in completing the job.

Question 7.
A company has two departments with 42 and 60 employees respectively. Their average weekly wages are ₹ 750 and ₹ 400. The standard deviations are 8 and 10 respectively.
(i) Which department has a larger bill?
(ii) Which department has larger variability in wages?
Solution:

Let $n_1 = 42$, $n_2 = 60$, $\bar{x}_1 = 750$, $\bar{x}_2 = 400$, $\sigma_1 = 8$, $\sigma_2 = 10$

$$\text{C.V. (1)} = 100 \times \frac{\sigma_1}{\bar{x}_1} = 100 \times \frac{8}{750} = 1.07\%$$

$$\text{C.V. (2)} = 100 \times \frac{\sigma_2}{\bar{x}_2} = 100 \times \frac{10}{400} = 2.5\%$$

(i) Since $\bar{x}_1 > \bar{x}_2$,
i.e., average weekly wages are more for the first department.
∴ The first department has a larger bill.

(ii) Since C.V. (1) < C.V. (2),
second department is less consistent.
∴ The second department has larger variability in wages.

Question 8.
The following table gives the weights of the students of two classes. Calculate the coefficient of variation of the two distributions. Which series is more variable?

Weight (in kg)	Class A	Class B
30-40	22	13
40-50	16	10
50-60	12	17

Solution:
Let x denote the data of class A and y denote the data of class B.
Calculation of S.D. for class A:

Weight (in kg) C.I.	Mid value (x_i)	f_i	f_ix_i	$f_ix_i^2$
30 – 40	35	22	770	26950
40 – 50	45	16	720	32400
50 – 60	55	12	660	36300
		N = 50	$\Sigma f_ix_i = 2150$	$\Sigma f_ix_i^2 = 95650$

$$\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{2150}{50} = 43$$

$$\sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2$$

$$\sigma_x^2 = \frac{95650}{50} - (43)^2 = 1913 - 1849 = 64$$

$\sigma_x = \sqrt{64} = 8$

Calculation of S.D. for class B:

Weight (in kg) C.I.	Mid value (y_i)	f_i	$f_i y_i$	$f_i y_i^2$
30 – 40	35	13	455	15925
40 – 50	45	10	450	20250
50 – 60	55	17	935	51425
		N = 40	$\Sigma f_i y_i = 1840$	$\Sigma f_i y_i^2 = 87600$

$$\bar{y} = \frac{\Sigma f_i y_i}{N} = \frac{1840}{40} = 46$$

$$\sigma_y^2 = \frac{\Sigma f_i y_i^2}{N} - (\bar{y})^2$$

$$\therefore \quad \sigma_y^2 = \frac{87600}{40} - (46)^2 = 2190 - 2116 = 74$$

$$\therefore \quad \sigma_y = \sqrt{74} = 8.60$$

Now, C.V. (X) $= 100 \times \frac{\sigma_x}{\bar{x}} = 100 \times \frac{8}{43} = 18.6\%$

C.V. (Y) $= 100 \times \frac{\sigma_y}{\bar{y}} = 100 \times \frac{8.60}{46} = 18.7\%$

Since C.V. (Y) > C.V.(X),
C.V. (B) > C.V. (A)
∴ Series B is more variable.

Question 9.
Compute the coefficient of variation for team A and team B.

No. of goals	0	1	2	3	4
No. of matches played by team A	19	6	5	16	14
No. of matches played by team B	16	14	10	14	16

Which team is more consistent?
Solution:
Let f_1 denote no. of goals of team A and f_2 denote no. of goals of team B.

No. of goals (x_i)	f_{1i}	$f_{1i}x_i$	$f_{1i}x_i^2$
0	19	0	0
1	6	6	6
2	5	10	20
3	16	48	144
4	14	56	224
	$N_1 = 60$	$\Sigma f_{1i}x_i = 120$	$\Sigma f_{1i}x_i^2 = 394$

No. of goals (x_i)	f_{2i}	$f_{2i}x_i$	$f_{2i}x_i^2$
0	16	0	0
1	14	14	14
2	10	20	40
3	14	42	126
4	16	64	256
	$N_2 = 70$	$\Sigma f_{2i}x_i = 140$	$\Sigma f_{2i}x_i^2 = 436$

$$\bar{x}_1 = \frac{\Sigma f_{1i} x_i}{N_1} = \frac{120}{60} = 2$$

$$\sigma_{x_1}{}^2 = \frac{1}{N_1}\Sigma f_{1i} x_i^2 - (\bar{x}_1)^2 = \frac{394}{60} - 2^2 = 6.5666 - 4 = 2.5666$$

$$\sigma_{x_1} = \sqrt{2.5666} = 1.60$$

$$\text{C.V. of team A} = 100 \times \frac{\sigma_{x_1}}{\bar{x}_1} = 100 \times \frac{1.60}{2} = 80\%$$

$$\bar{x}_2 = \frac{\Sigma f_{2i} x_i}{N_2} = \frac{140}{70} = 2$$

$$\sigma_{x_2}{}^2 = \frac{1}{N_2}\Sigma f_{2i} x_i^2 - (\bar{x}_2)^2 = \frac{436}{70} - 2^2 = 6.2285 - 4 = 2.2285$$

$$\sigma_{x_2} = \sqrt{2.2285} = 1.49$$

$$\text{C.V. of team B} = 100 \times \frac{\sigma_{x_2}}{\bar{x}_2} = 100 \times \frac{1.49}{2} = 74.5\%$$

Since C.V. of team A > C.V. of team B,
Team B is more consistent.

Question 10.
Given below is the information about marks obtained in Mathematics and Statistics by 100 students in a class. Which subject shows the highest variability in marks?

	Mathematics	Statistics
Mean	20	25
S.D.	2	3

Solution:

Here, $\bar{x}_m = 20$, $\bar{x}_s = 25$, $\sigma_m = 2$, $\sigma_s = 3$

$$\text{C.V. (M)} = 100 \times \frac{\sigma_m}{\bar{x}_m}$$

$$= 100 \times \frac{2}{20} = 10\%$$

$$\text{C.V. (S)} = 100 \times \frac{\sigma_s}{\bar{x}_s} = 100 \times \frac{3}{25} = 12\%$$

Since C.V. (S) > C.V. (M),
The subject statistics show higher variability in marks.

Maharashtra State Board 11th Maths Solutions Chapter 8 Measures of Dispersion Miscellaneous Exercise 8

(I) Select the correct option from the given alternatives:

Question 1.
If there are 10 values each equal to 10, then S.D. of these values is ________
(A) 100
(B) 20
(C) 0
(D) 6
Answer:
(C) 0

Hint:
Var (X) = $\sigma^2_x = \frac{\Sigma x_i^2}{n} - (\bar{X})^2$
= $\frac{1000}{10}$ – 100
= 0
∴ S.D. = 0

Question 2.
The number of patients who visited cardiologists are 13, 17, 11, 15 in four days, then variance (approximately) is
(A) 5 patients
(B) 4 patients
(C) 10 patients
(D) 15 patients
Answer:
(A) 5 patients

Question 3.
If the observations of a variable X are, -4, -20, -30, -44 and -36, then the value of the range will be:
(A) -48
(B) 40
(C) -40
(D) 48
Answer:
(B) 40

Question 4.
The standard deviation of a distribution divided by the mean of the distribution and expressed in percentage is called:
(A) Coefficient of Standard deviation
(B) Coefficient of skewness
(C) Coefficient of quartile deviation
(D) Coefficient of variation
Answer:
(D) Coefficient of variation

Question 5.
If the S.D. of first n natural numbers is √2, then the value of n must be
(A) 5
(B) 4
(C) 7
(D) 6
Answer:
(A) 5

Question 6.
The positive square root of the mean of the squares of the deviations of observations from their mean is called:
(A) Variance
(B) Range
(C) S.D.
(D) C.V.
Answer:
(C) S.D.

Question 7.
The variance of 19, 21, 23, 25 and 27 is 8. The variance of 14, 16, 18, 20 and 22 is:
(A) Greater than 8
(B) 8
(C) Less than 8
(D) 8 – 5 = 3
Answer:
(B) 8

Question 8.
For any two numbers, SD is always
(A) Twice the range
(B) Half of the range
(C) Square of the range
(D) None of these

Answer:
(B) Half of the range

Question 9.
Given the heights (in cm) of two groups of students:
Group A: 131 cm, 150 cm, 147 cm, 138 cm, 144 cm
Group B: 139 cm, 148 cm, 132 cm, 151 cm, 140 cm
Which of the following is/are true?
I. The ranges of the heights of the two groups of students are the same.
II. The means of the heights of the two groups of students are the same.
(A) I only
(B) II only
(C) Both I and II
(D) None
Answer:
(C) Both I and II

Question 10.
The standard deviation of data is 12 and mean is 72, then the coefficient of variation is
(A) 13.67%
(B) 16.67%
(C) 14.67%
(D) 15.67%
Answer:
(B) 16.67%

(II) Answer the following:

Question 1.
Find the range for the following data.
76, 57, 80, 103, 61, 63, 89, 96, 105, 72
Solution:
Here, largest value (L) = 105, smallest value (S) = 57
∴ Range = L – S
= 105 – 57
= 48

Question 2.
Find the range for the following data.
116, 124, 164, 150, 149, 114, 195, 128, 138, 203, 144
Solution:
Here, largest value (L) = 203, smallest value (S) = 114
∴ Range = L – S
= 203 – 114
= 89

Question 3.
Given below is the frequency distribution of weekly wages of 400 workers. Find the range.

Weekly wages (in '00 Rs.)	10	15	20	25	30	35	40
No. of workers	45	63	102	55	74	36	25

Solution:
Here, largest value (L) = 40, smallest value (S) = 10
∴ Range = L – S
= 40 – 10
= 30

Question 4.
Find the range of the following data.

Classes	115-125	125-135	135-145	145-155	155-165	165-175
Frequency	1	4	6	1	3	5

Solution:

Here, upper limit of the highest class (L) = 175
lower limit of the lowest class (S) = 115
∴ Range = L – S
= 175 – 115
= 60

Question 5.
Find variance and S.D. for the following set of numbers.
25, 21, 23, 29, 27, 22, 28, 23, 21, 25
Solution:

x_i	x_i^2
25	625
21	441
23	529
29	841
27	729
22	484
28	784
23	529
21	441
25	625
$\Sigma x_i = 244$	$\Sigma x_i^2 = 6028$

Here, n = 10

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{244}{10} = 24.4$$

$$\text{Var (X)} = \sigma_x^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2 = \frac{6028}{10} - (24.4)^2$$

$$= 602.8 - 595.36 = 7.44$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var(X)}} = \sqrt{7.44} = 2.72$$

Question 6.
Find variance and S.D. for the following set of numbers.
125, 130, 150, 165, 190, 195, 210, 230, 245, 260
Solution:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
125	–65	4225
130	–60	3600
150	–40	1600
165	–25	625
190	0	0
195	5	25
210	20	400
230	40	1600
245	55	3025
260	70	4900
$\Sigma x_i = 1900$		$\Sigma (x_i - \bar{x})^2 = 20000$

Here, n = 10

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{1900}{10} = 190$$

$$\text{Var (X)} = \sigma_x^2 = \frac{1}{n}\ \Sigma(x_i - \bar{x})^2 = \frac{20000}{10} = 2000$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var(X)}} = \sqrt{2000} = 44.72$$

Question 7.
Following data gives no. of goals scored by a team in 90 matches. Compute the standard deviation.

No. of Goals Scored	0	1	2	3	4	5
No. of matches	5	20	25	15	20	5

Solution:

No. of goals scored (x_i)	No. of matches (f_i)	$f_i x_i$	x_i^2	$f_i x_i^2$
0	5	0	0	0
1	20	20	1	20
2	25	50	4	100
3	15	45	9	135
4	20	80	16	320
5	5	25	25	125
	N = 90	$\Sigma f_i x_i = 220$		$\Sigma f_i x_i^2 = 700$

$$\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{220}{90} = 2.44$$

$$\text{Var}(X) = \sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2 = \frac{700}{90} - (2.44)^2$$

$$= 7.78 - 5.9536 = 1.83$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{1.83} = 1.35$$

Question 8.
Compute the variance and S.D. for the following data:

X	62	30	64	47	63	46	35	28	60
F	5	8	3	4	5	7	8	3	7

Solution:

x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
62	5	310	3844	19220
30	8	240	900	7200
64	3	192	4096	12288
47	4	188	2209	8836
63	5	315	3969	19845
46	7	322	2116	14812
35	8	280	1225	9800
28	3	84	784	2352
60	7	420	3600	25200
	N = 50	$\Sigma f_i x_i = 2351$		$\Sigma f_i x_i^2 = 119553$

$$\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{2351}{50} = 47.02$$

$$\text{Var}(X) = \sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2$$

$$= \frac{119553}{50} - (47.02)^2 = 2391.06 - 2210.88 = 180.18$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{180.18} = 13.42$$

Question 9.
Calculate S.D. from the following data.

Age (In yrs)	20-29	30-39	40-49	50-59	60-69	70-79	80-89
Freq.	65	100	55	87	42	38	13

Solution:
Since data is not continuous, we have to make it continuous.

let u = $\frac{x-A}{h} = \frac{x-54.5}{10}$

Calculation of variance of u:

Age (in years) C. I.	Mid value (x_i)	f_i	$u_i = \frac{x_i - 54.5}{10}$	f_iu_i	$f_iu_i^2$
19.5 – 29.5	24.5	65	–3	–195	585
29.5 – 39.5	34.5	100	–2	–200	400
39.5 – 49.5	44.5	55	–1	–55	55
49.5 – 59.5	54.5	87	0	0	0
59.5 – 69.5	64.5	42	1	42	42
69.5 – 79.5	74.5	38	2	76	152
79.5 – 89.5	84.5	13	3	39	117
		N = 400		$\Sigma f_iu_i = -293$	$\Sigma f_iu_i^2 = 1351$

$$\bar{u} = \frac{\Sigma f_i u_i}{N} = \frac{-293}{400} = -0.7325$$

$$\text{Var}(u) = \sigma_u^2 = \frac{\Sigma f_i u_i^2}{N} - (\bar{u})^2$$

$$= \frac{1351}{400} - (-0.7325)^2$$

$$= 3.3775 - 0.5366 = 2.8409$$

$$\therefore \quad \text{Var}(X) = h^2 \text{ var}(u) = (10)^2 \times 2.8409$$

$$= 100 \times 2.8409$$

$$= 284.09$$

$$\therefore \quad \text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{284.09} = 16.85$$

Question 10.
Given below is the frequency distribution of marks obtained by 100 students. Compute arithmetic mean and S.D.

Marks	40-49	50-59	60-69	70-79	80-89	90-99
No. of students	4	12	25	28	26	5

Solution:
Since data is not continuous, we have to make it continuous.

Let u = $x-Ah=x-74.510$

Calculation of variance of u:

Marks C. I.	Mid value (x_i)	No. of students (f_i)	$u_i = \frac{x_i - 74.5}{10}$	f_iu_i	$f_iu_i^2$
39.5 – 49.5	44.5	4	–3	–12	36
49.5 – 59.5	54.5	12	–2	–24	48
59.5 – 69.5	64.5	25	–1	–25	25
69.5 – 79.5	74.5	28	0	0	0
79.5 – 89.5	84.5	26	1	26	26
89.5 – 99.5	94.5	5	2	10	20
		N = 100		$\Sigma f_iu_i = -25$	$\Sigma f_iu_i^2 = 155$

$$\bar{u} = \frac{\Sigma f_i u_i}{N} = \frac{-25}{100} = -0.25$$

$$\bar{x} = \bar{u} \times h + A$$

$$= -0.25 \times 10 + 74.5$$

$$= 72$$

$$\text{Var}(u) = \sigma_u^2 = \frac{\Sigma f_i u_i^2}{N} - (\bar{u})^2$$

$$= \frac{155}{100} - (-0.25)^2$$

$$= 1.55 - 0.0625$$

$$= 1.4875$$

∴ Var (X) = $h2$ var (u)
= $(10)2$ × 1.4875
= 100 × 1.4875
= 148.75
∴ S.D. = σx = √Var(X)
= √148.75
= 12.2

Question 11.
The arithmetic mean and standard deviation of a series of 20 items were calculated by a student as 20 cm and 5 cm respectively. But while calculating them, item 13 was misread as 30. Find the corrected mean and standard deviation.
Solution:
n = 20, $\bar{x}$ = 20, σ_x = 5 .....(given)

$\bar{x} = \frac{1}{n}\sum x_i$

$\therefore \sum x_i = n\bar{x}$ = 20 × 20 = 400

Corrected Σx_i = Σx_i – (incorrect observation) + (correct observation)
= 400 – 30 + 13
= 383

$$\text{Corrected mean} = \frac{\text{corrected} \sum x_i}{n}$$
$$= \frac{383}{20} = 19.15 \text{ cm}$$

Now, $\sigma_x = 5$

$\therefore \quad \sigma_x^2 = (5)^2$

$\sigma_x^2 = \frac{1}{n}(\sum x_i^2) - (\bar{x})^2$

$\therefore \quad (5)^2 = \frac{1}{20}(\sum x_i^2) - (20)^2$

$\therefore \quad 25 = \frac{(\sum x_i^2)}{20} - (400)$

$\therefore \quad \frac{(\sum x_i^2)}{20} = 425$

$\therefore \quad \sum x_i^2 = 8500$

Corrected $\sum x_i^2$ = $\sum x_i^2$ – (incorrect observation)^2 + (correct observation)^2
= 8500 – $(30)^2$ + $(13)^2$
= 8500 – 900 + 169
= 7769

$$\text{Corrected S.D.} = \sqrt{\frac{\text{Corrected} \sum x_i^2}{n} - (\text{Corrected mean})^2}$$
$$= \sqrt{\frac{7769}{20} - (19.15)^2}$$
$$= \sqrt{388.45 - 366.72}$$
$$= \sqrt{21.73}$$
$$= 4.66 \text{ cm}$$

Question 12.
The mean and S.D. of a group of 50 observations are 40 and 5 respectively. If two more observations 60 and 72 are added to the set, find the mean and S.D. of 52 items.
Solution:

$n = 50, \bar{x} = 40, \sigma_x = 5$ ....(given)

$\bar{x} = \frac{1}{n}\sum x_i$

$\therefore \sum x_i = n\bar{x} = 50 \times 40 = 2000$

New $\sum x_i = \sum x_i + 60 + 72$
$= 2000 + 60 + 72$
$= 2132$

$\therefore$ New Mean $= \frac{2132}{52} = 41$

Now, $\sigma_x = 5$

$\therefore \sigma_x^2 = 25$

$\sigma_x^2 = \frac{1}{n}(\sum x_i^2) - (\bar{x})^2$

$\therefore 25 = \frac{1}{50}(\sum x_i^2) - (40)^2$

$\therefore 25 = \frac{1}{50}(\sum x_i^2) - 1600$

$\therefore \frac{\sum x_i^2}{50} = 25 + 1600 = 1625$

$\therefore \sum x_i^2 = 50 \times 1625 = 81250$

New $\sum x_i^2 = \sum x_i^2 + (60)^2 + (72)^2$
$= 81250 + 3600 + 5184 = 90034$

$\therefore$ New S.D. $= \sqrt{\frac{\text{New} \sum x_i^2}{n} - (\text{New mean})^2}$

$= \sqrt{\frac{90034}{52} - (41)^2}$

$= \sqrt{\frac{90034}{52} - 1681}$

$= \sqrt{\frac{90034 - 87412}{52}}$

$= \sqrt{\frac{2622}{52}} = \sqrt{50.42} = 7.1$

Question 13.
The mean height of 200 students is 65 inches. The mean heights of boys and girls are 70 inches and 62 inches respectively and the standard deviations are 8 and 10 respectively. Find the number of boys and combined S.D.
Solution:
Let n_1 and n_2 be the number of boys and girls respectively.
Let $n = 200$, $\bar{X}_c = 65$, $\bar{X}_1 = 70$, $\bar{X}_2 = 62$, $\sigma_1 = 8$, $\sigma_2 = 10$
Here, $n_1 + n_2 = n$
$n_1 + n_2 = 200$ ......(i)
Combined mean ($\bar{X}_c$) = $\frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$
$65 = \frac{n_1(70) + n_2(62)}{200}$ ...... [From (i)]
$70n_1 + 62n_2 = 13000$
$35n_1 + 31n_2 = 6500$ .....(ii)
Solving (i) and (ii), we get
$n_1 = 75$, $n_2 = 125$
Number of boys = 75

$d_1 = \bar{x}_1 - \bar{x}_c = 70 - 65 = 5$

$d_2 = \bar{x}_2 - \bar{x}_c = 62 - 65 = -3$

Combined S.D. $(\sigma_c) = \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$

$= \sqrt{\frac{75(64+25) + 125(100+9)}{200}}$

$= \sqrt{\frac{6675 + 13625}{200}}$

$= \sqrt{\frac{20300}{200}} = \sqrt{101.5} = 10.07$

Question 14.
From the following data available for 5 pairs of observations of two variables x and y, obtain combined S.D. for all 10 observations.

Where, $\sum_{i=1}^{n} x_i = 30$, $\sum_{i=1}^{n} y_i = 40$, $\sum_{i=1}^{n} x_i^2 = 220$,
$\sum_{i=1}^{n} y_i^2 = 340$

Solution:

Here, $\sum_{i=1}^{n} x_i = 30$, $\sum_{i=1}^{n} y_i = 40$, $\sum_{i=1}^{n} x_i^2 = 220$, $\sum_{i=1}^{n} y_i^2 = 340$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{30}{5} = 6,$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{40}{5} = 8$$

$$\text{Combined mean}(\bar{x}_c) = \frac{n_x\bar{x} + n_y\bar{y}}{n_x + n_y} = \frac{5(6)+5(8)}{5+5} = \frac{30+40}{10} = \frac{70}{10} = 7$$

Now, $d_x = \bar{x} - \bar{x}_c = 6 - 7 = -1$

$d_y = \bar{y} - \bar{x}_c = 8 - 7 = 1$

$$\sigma_x^2 = \frac{1}{n}\sum x_i^2 - (\bar{x})^2 = \frac{1}{5}(220) - (6)^2 = 44 - 36 = 8$$

$$\sigma_y^2 = \frac{1}{n}\sum y_i^2 - (\bar{y})^2 = \frac{1}{5}(340) - (8)^2 = 68 - 64 = 4$$

$$\text{Combined standard deviation } (\sigma_c) = \sqrt{\frac{n_x(\sigma_x^2 + d_x^2) + n_y(\sigma_y^2 + d_y^2)}{n_x + n_y}}$$

$$= \sqrt{\frac{5\left[8+(-1)^2\right]+5\left[4+(1)^2\right]}{5+5}}$$

$$= \sqrt{\frac{5(8+1)+5(4+1)}{10}}$$

$$= \sqrt{\frac{5(9)+5(5)}{10}}$$

$$= \sqrt{\frac{45+25}{10}}$$

$$= \sqrt{\frac{70}{10}}$$

$$= \sqrt{7}$$

$$= 2.65$$

Question 15.
Calculate the coefficient of variation of the following data.
23, 27, 25, 28, 21, 14, 16, 12, 18, 16
Solution:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
23	3	9
27	7	49
25	5	25
28	8	64
21	1	1
14	–6	36
16	–4	16
12	–8	64
18	–2	4
16	–4	16
$\Sigma x_i = 200$		$\Sigma(x_i - \bar{x})^2 = 284$

Here, n = 10, $\bar{x} = \frac{\Sigma x_i}{n} = \frac{200}{10} = 20$

$\text{Var}(X) = \sigma_x^2 = \frac{1}{n}\Sigma(x_i - \bar{x})^2 = \frac{284}{10} = 28.4$

$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{28.4} = 5.33$

Now, $\text{C.V.} = 100 \times \frac{\sigma_x}{\bar{x}} = 100 \times \frac{5.33}{20} = 26.65\%$

Question 16.
The following data relates to the distribution of weights of 100 boys and 80 girls in a school.

	Boys	Girls
Mean	60	47
Variance	16	9

Which of the two is more variable?
Solution:

Let $n_1 = 100$, $n_2 = 80$, $\bar{x}_1 = 60$, $\bar{x}_2 = 47$, $\sigma_1 = \sqrt{16} = 4$, $\sigma_2 = \sqrt{9} = 3$

$\text{C.V. of boys} = 100 \times \frac{\sigma_1}{\bar{x}_1} = 100 \times \frac{4}{60} = \frac{400}{600} = 6.67\%$

$\text{C.V. of girls} = 100 \times \frac{\sigma_2}{\bar{x}_2} = 100 \times \frac{3}{47} = \frac{300}{47} = 6.38\%$

Here, C.V. of boys > C.V. of girls
∴ The Series of boys is more variable.

Question 17.
The mean and standard deviations of the two brands of watches are given below:

	Brand-I	Brand-II
Mean	36 months	48 months
S.D.	8 months	10 months

Calculate the coefficient of variation of the two brands and interpret the results.
Solution:

Here, $\bar{x}_I = 36$, $\bar{x}_{II} = 48$, $\sigma_I = 8$, $\sigma_{II} = 10$

$\text{C.V. (I)} = 100 \times \frac{\sigma_I}{\bar{x}_I} = 100 \times \frac{8}{36} = 22.22\%$

$\text{C.V. (II)} = 100 \times \frac{\sigma_{II}}{\bar{x}_{II}} = 100 \times \frac{10}{48} = 20.83\%$

Here, C.V. (I) > C.V. (II)
∴ The brand I is more variable.

Question 18.
Calculate the coefficient of variation for the data given below:

Size (cm)	5-8	8-11	11-14	14-17	17-20	20-23	23-26
No of items	3	14	13	16	19	24	11

Solution:

Size (cm)	Mid value (x_i)	No. of students (f_i)	f_ix_i	$f_ix_i^2$
5 – 8	6.5	3	19.5	126.75
8 – 11	9.5	14	133	1263.5
11 – 14	12.5	13	162.5	2031.25
14 – 17	15.5	16	248	3844
17 – 20	18.5	19	351.5	6502.75
20 – 23	21.5	24	516	11094
23 – 26	24.5	11	269.5	6602.75
		N = 100	$\Sigma f_ix_i = 1700$	$\Sigma f_ix_i^2 = 31465$

$$\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{1700}{100} = 17$$

$$\text{Var (X)} = \sigma_x^2 = \frac{1}{N}\Sigma f_i x_i^2 - (\bar{x})^2$$

$$= \frac{1}{100}(31465) - (17)^2$$

$$= 314.65 - 289$$

$$= 25.65$$

$$\sigma_x = \sqrt{25.65} = 5.06$$

$$\text{C.V.} = 100 \times \frac{\sigma_x}{\bar{x}} = 100 \times \frac{5.06}{17} = 29.76\ \%$$

Question 19.
Calculate the coefficient of variation for the data given below:

Income (Rs.)	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000
No. of families	24	13	15	28	12	8	10

Solution:

Let u = x-Ah=x-65001000

Calculation of variance of u:

Income (₹)	Mid value (x_i)	No. of families (f_i)	$u_i = \frac{x_i - 6500}{1000}$	f_iu_i	$f_iu_i^2$
3000 – 4000	3500	24	–3	–72	216
4000 – 5000	4500	13	–2	–26	52
5000 – 6000	5500	15	–1	–15	15
6000 – 7000	6500	28	0	0	0
7000 – 8000	7500	12	1	12	12
8000 – 9000	8500	8	2	16	32
9000 – 10000	9500	10	3	30	90
		N = 110		$\Sigma f_iu_i = -55$	$\Sigma f_iu_i^2 = 417$

$$\bar{u} = \frac{\Sigma f_i u_i}{N} = \frac{-55}{110} = -0.5$$

$$\bar{x} = \bar{u} \times h + A = (-0.5) \times 1000 + 6500 = -500 + 6500 = 6000$$

$$\text{Var (u)} = \sigma_u^2 = \frac{\Sigma f_i u_i^2}{N} - (\bar{u})^2 = \frac{417}{110} - (-0.5)^2$$

$$= 3.79 - 0.25$$

$$= 3.54$$

$$\text{Var (X)} = h^2\ \text{Var (u)} = (1000)^2 \times 3.54 = 3540000$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var (X)}} = \sqrt{3540000} = 1881.48$$

$$\text{C.V.} = 100 \times \frac{\sigma_x}{\bar{x}} = 100 \times \frac{1881.48}{6000} = 31.36\%$$

Question 20.
Compute the coefficient of variations for the following data to show whether the variation is greater in the field or in the area of the field.

Year	Area (in acres)	Yield (in lakhs)
2011-12	156	62
2012-13	135	70
2013-14	128	68
2014-15	117	76
2015-16	141	65
2016-17	154	69
2017-18	142	71

Solution:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
156	17	289
135	– 4	16
128	– 11	121
117	– 22	484
141	2	4
154	15	225
142	3	9
$\Sigma x_i = 973$		$\Sigma(x_i - \bar{x})^2 = 1148$

Here, n = 7, $\bar{x} = \frac{\sum x_i}{n} = \frac{973}{7} = 139$

Var (X) $= \sigma_x^2 = \frac{1}{n}\sum(x_i - \bar{x})^2 = \frac{1148}{7} = 164$

S.D. $= \sigma_x = \sqrt{\text{Var (X)}} = \sqrt{164} = 12.8062$

Now, C.V. $= 100 \times \frac{\sigma_x}{\bar{x}} = 100 \times \frac{12.8062}{139} = 9.21\%$

y_i	$y_i - \bar{y}$	$(y_i - \bar{y})^2$
62	– 6.71	45.02
70	1.29	1.66
68	– 0.71	0.50
76	7.29	53.14
65	– 3.71	13.76
69	0.29	0.0841
71	2.29	5.24
$\Sigma y_i = 481$		$\Sigma(y_i - \bar{y})^2 = 119.4$

Here, n = 7, $\bar{y} = \frac{\sum y_i}{n} = \frac{481}{7} = 68.71$

Var (Y) $= \sigma_y^2 = \frac{1}{n}\sum(y_i - \bar{y})^2 = \frac{119.4}{7} = 17.0571$

S.D. $= \sigma_y = \sqrt{\text{Var (Y)}} = \sqrt{17.0571} = 4.13$

Now, C.V. $= 100 \times \frac{\sigma_y}{\bar{y}} = 100 \times \frac{4.13}{68.71} = 100 \times 0.0601 = 6.01\%$

∴ the variation is greater in the area of the field.

Question 21.
There are two companies U and V which manufacture cars. A sample of 40 cars each from these companies is taken and the average running life (in years) is recorded.

Life (in years)	No of Cars	
	Company U	Company V
0-5	5	14
5-10	18	8
10-15	17	18

Which company shows greater consistency?
Solution:
Let f_1 denote no. of cars of company U and f_2 denote no. of cars of company V.

Life (in years) C.I.	Mid value (x_i)	f_{1i}	$f_{1i}x_i$	$f_{1i}x_i^2$
0 – 5	2.5	5	12.5	31.25
5 – 10	7.5	18	135	1012.5
10 – 15	12.5	17	212.5	2656.25
		$N_1 = 40$	$\Sigma f_{1i}x_i = 360$	$\Sigma f_{1i}x_i^2 = 3700$

Life (in years) C.I.	Mid value (x_i)	f_{2i}	$f_{2i}x_i$	$f_{2i}x_i^2$
0 – 5	2.5	14	35	87.5
5 – 10	7.5	8	60	450
10 – 15	12.5	18	225	2812.5
		$N_2 = 40$	$\Sigma f_{2i}x_i = 320$	$\Sigma f_{2i}x_i^2 = 3350$

$$\bar{x}_1 = \frac{\Sigma f_{1i}x_i}{N_1} = \frac{360}{40} = 9$$

$$\text{Var}(X_1) = \sigma_{x_1}^2 = \frac{\Sigma f_{1i}x_i^2}{N_1} - (\bar{x}_1)^2 = \frac{3700}{40} - (9)^2$$

$$= 92.5 - 81 = 11.5$$

$$\text{S.D.} = \sigma_{x_1} = \sqrt{\text{Var}(X_1)} = \sqrt{11.5} = 3.39$$

$$\text{C.V. (U)} = 100 \times \frac{\sigma_{x_1}}{\bar{x}_1} = 100 \times \frac{3.39}{9} = 37.67\%$$

$$\bar{x}_2 = \frac{\Sigma f_{2i}x_i}{N_2} = \frac{320}{40} = 8$$

$$\text{Var}(X_2) = \sigma_{x_2}^2 = \frac{\Sigma f_{2i}x_i^2}{N_2} - (\bar{x}_2)^2 = \frac{3350}{40} - (8)^2 = 83.75 - 64 = 19.75$$

$$\text{S.D.} = \sigma_{x_2} = \sqrt{\text{Var}(X_2)} = \sqrt{19.75} = 4.44$$

$$\text{C.V.(V)} = 100 \times \frac{\sigma_{x_2}}{\bar{x}_2} = 100 \times \frac{4.44}{8} = 55.5\%$$

Here, C.V. (U) < C.V. (V)
∴ Company U shows greater consistency in performance.

Question 22.
The means and S.D. of weights and heights of 100 students of a school are as follows.

	Weights	Heights
Mean	56.5 kg	61 inches
S.D.	8.76 kg	12.18 inches

Which shows more variability, weights, or heights?
Solution:

Here, $\bar{x}_w = 56.5$, $\bar{x}_h = 61$, $\sigma_w = 8.76$, $\sigma_h = 12.18$

$$\text{C.V. for weight} = 100 \times \frac{\sigma_w}{\bar{x}_w} = 100 \times \frac{8.76}{56.5} = 15.50\%$$

$$\text{C.V. for height} = 100 \times \frac{\sigma_h}{\bar{x}_h} = 100 \times \frac{12.18}{61} = 19.96\%$$

Here, C.V. for weight < C.V. for height
∴ Height shows more variability.