Maharashtra State Board 12th Commerce Maths Solutions Chapter 7 Assignment Problem and Sequencing Ex 7.1

Question 1.
A job production unit has four jobs A, B, C, D which can be manufactured on each of the four machines P, Q, R, and S. The processing cost of each job for each machine is given in the following table:

Job	Machines (Processing Cost in Rs.)			
	I	II	III	IV
P	31	25	33	29
Q	25	24	23	21
R	19	21	23	24
S	38	36	34	40

Find the optimal assignment to minimize the total processing cost.
Solution:
The cost matrix is given by

Job	Machines (Processing Cost in ₹)			
	I	II	III	IV
P	31	25	33	29
Q	25	24	23	21
R	19	21	23	24
S	38	36	34	40

Subtracting row minimum from all the elements in that row we get

Job	Machines (Processing Cost in ₹)			
	I	II	III	IV
P	6	0	8	4
Q	4	3	2	0
R	0	2	4	5
S	4	2	0	6

Subtracting column minimum from all the elements in that column we get the same matrix.
As all the rows and columns have single zeros the allotment can be done as follows.

Job	Machines (Processing Cost in ₹)			
	I	II	III	IV
P	6	$\boxed{0}$	8	4
Q	4	3	2	$\boxed{0}$
R	$\boxed{0}$	2	4	5
S	4	2	$\boxed{0}$	6

As per the table, the job allotments are
P → II, Q → IV, R → I, S → III
The total minimum cost = 25 + 21 + 19 + 34 = ₹ 99

Question 2.
Five wagons are available at stations 1, 2, 3, 4, and 5. These are required at 5 stations I, II, III, IV, and V. The mileage between various stations are given in the table below. How should the wagons be transported so as to minimize the mileage covered?

	I	II	III	IV	V
1	10	5	9	18	11
2	13	9	6	12	14
3	3	2	4	4	5
4	18	9	12	17	15
5	11	6	14	19	10

Solution:
The mileage matrix is given by

	I	II	III	IV	V
1	10	5	9	18	11
2	13	9	6	12	14
3	3	2	4	4	5
4	18	9	12	17	15
5	11	6	14	19	10

Subtracting row minimum from all elements in that row we get

	I	II	III	IV	V
1	5	0	4	13	6
2	7	3	0	6	8
3	1	0	2	2	3
4	9	0	3	8	6
5	5	0	8	13	4

Subtracting column minimum from all elements in that column we get

	I	II	III	IV	V
1	4	0	4	11	3
2	6	3	0	4	5
3	0	0	2	0	0
4	8	0	3	6	3
5	4	0	8	11	1

Draw minimum lines covering all the zeros

	I	II	III	IV	V
1	4	0	4	11	3
2	6	3	0	4	5
3	0	0	2	0	0
4	8	0	3	6	3
5	4	0	8	11	1

The number of lines covering all the zeros (3) is less than the order of the matrix (5). Hence an assignment is not possible. The modification is required. The minimum uncovered value 1 is subtracted from uncovered values and added to the values at the intersection. The numbers on the lines remain the same we get

	I	II	III	IV	V
1	3	0	3	10	2
2	6	4	0	4	5
3	0	1	2	0	0
4	7	0	2	5	2
5	3	0	7	10	0

Drawing a minimum number of lines covering all the zeros.
No. of lines covering all the zeros (4) is less than the order of the matrix (5).
Hence assignment is not possible.
Again modification is required. The minimum uncovered value 3 is subtracted from the uncovered values and added to the values at the intersection.
The numbers on the lines remain the same we get

	I	II	III	IV	V
1	0	0	3	7	2
2	3	4	0	1	5
3	0	4	5	0	3
4	4	0	2	2	2
5	0	0	7	7	0

No. of lines covering all the zeros (5) are equal to the order of the matrix so the assignment is possible.

	I	II	III	IV	V
1	$\boxed{0}$	$\cancel{0}$	3	7	2
2	3	4	$\boxed{0}$	1	5
3	$\cancel{0}$	4	5	$\boxed{0}$	3
4	4	$\boxed{0}$	2	2	2
5	$\cancel{0}$	$\cancel{0}$	7	7	$\boxed{0}$

According to the table the assignment is
1 → I, 2 → II, 3 → IV, 4 → II, 5 → V
Total minimum mileage = 10 + 6 + 4 + 9 + 10 = 39 units

Question 3.
Five different machines can do any of the five required jobs, with different profits five required jobs, with different profits resulting from each assignment as shown below:

Job	Machines (Profit in Rs.)				
	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

Find the optimal assignment schedule.
Solution:
This profit matrix has to be reduced to cost matrix by subtracting all the values of the matrix from the largest value (62) we get

Cost matrix					
Job	A	B	C	D	E
1	32	25	22	34	22
2	22	38	35	41	26
3	22	30	29	32	27
4	37	24	22	26	26
5	33	0	21	28	23

Subtracting row minimum value from all the elements in that column we get

Cost matrix					
Job	A	B	C	D	E
1	10	3	0	12	0
2	0	16	13	19	4
3	0	8	7	10	5
4	15	2	0	4	4
5	33	0	21	28	23

Subtracting column minimum from all the elements in that column we get

	A	B	C	D	E
1	10	3	0	8	0
2	0	16	13	15	4
3	0	8	7	6	5
4	15	2	0	0	4
5	33	0	21	24	23

Drawing minimum lines covering all zeros we get

	A	B	C	D	E
1	10	3	0	8	0
2	0	16	13	15	4
3	0	8	7	6	5
4	15	2	0	0	4
5	33	0	21	24	23

No. of lines (4) is less than the order of the matrix (5). Hence assignment is not possible. The modification is required. The minimum uncovered value (4) is subtracted from the uncovered value and added to the values at the intersection. The values on the lines remain the same, we get

	A	B	C	D	E
1	14	7	$\boxed{0}$	8	0
2	0	16	9	11	$\boxed{0}$
3	$\boxed{0}$	8	3	2	1
4	19	6	0	$\boxed{0}$	4
5	33	$\boxed{0}$	17	20	19

No. of lines (5) are equal to the order of the matrix (5). So assignments are possible
1 → C, 2 → E, 3 → A, 4 → D, 5 → B
For the minimum profit look at the corresponding in the profit matrix given.
Maximum profit = 40 + 36 + 40 + 36 + 62 = 214 units

Question 4.
Four new machines M_1, M_2, M_3, and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D, and E available. Because of limited space, machine M_2 cannot be placed at C and M_1 cannot be placed at A. The cost matrix is given below.

Machines	Places				
	A	B	C	D	E
M1	4	6	10	5	6
M2	7	4	-	5	4
M3	-	6	9	6	2
M4	9	3	7	2	3

Find the optimal assignment schedule.
Solution:
This is a restricted assignment so we assign a very high cost '∞' to the prohibited all.
Also as it is an unbalanced problem we add a dummy row M_5 with all values as '0', we get

	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	∞	5	4
M_3	∞	6	9	6	2
M_4	9	3	7	2	3
M_5	0	0	0	0	0

Subtracting row minimum from all the elements in that row, we get

	A	B	C	D	E
M_1	0	2	6	1	2
M_2	3	0	∞	1	0
M_3	∞	4	7	4	0
M_4	7	1	5	0	1
M_5	0	0	0	0	0

Subtracting column minimum from all the elements in that column we get the same matrix.

	A	B	C	D	E
M_1	0	2	6	1	2
M_2	3	0	∞	1	0
M_3	∞	4	7	4	0
M_4	7	1	5	0	1
M_5	0	0	0	0	0

As minimum no. of lines covering all zeros (5) is equal to the order of the matrix, Assignment is possible

	A	B	C	D	E
M_1	$\boxed{0}$	2	6	1	2
M_2	3	$\boxed{0}$	∞	1	$\cancel{0}$
M_3	∞	4	7	4	$\boxed{0}$
M_4	7	1	5	$\boxed{0}$	1
M_5	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

The assignments are given by
$M_1 \to A$, $M_2 \to B$, $M_3 \to E$, $M_4 \to D$, $M_5 \to C$
As M_5 is dummy no machine is installed at C
For minimum cost taking the corresponding values in the cost matrix we get
Minimum cost = 4 + 4 + 2 + 2 = 12 units

Question 5.
A company has a team of four salesmen and there is four districts where the company wants to start its business. After taking into account the capabilities of salesmen and the nature of districts, the company estimates that the profit per day in rupees for each salesman in each district is as below:

Salesman	District			
	1	2	3	4
A	16	10	12	11
B	12	13	15	15
C	15	15	11	14
D	13	14	14	15

Find the assignments of a salesman to various districts which will yield maximum profit.
Solution:
The profit matrix has to be reduced to the cost matrix. Subtracting all the values from the maximum value (16) we get

Cost District				
	1	2	3	4
A	0	6	4	5
B	4	3	1	1
C	1	1	5	2
D	3	2	2	1

Subtracting row minimum from all values in that row we get

	1	2	3	4
A	0	6	4	5
B	3	2	0	0
C	0	0	4	1
D	2	1	1	0

Subtracting column minimum from each column we get the same matrix

	1	2	3	4
A	0	6	4	5
B	3	2	0	0
C	0	0	4	1
D	2	1	1	0

As minimum no. of lines covering all zeros (4) is equal to the order of the matrix (4) Assignment is possible

	1	2	3	4
A	$\boxed{0}$	6	4	5
B	3	2	$\boxed{0}$	$\cancel{0}$
C	$\cancel{0}$	$\boxed{0}$	4	1
D	2	1	1	$\boxed{0}$

∴ A → 1, B → 3, C → 2, D → 4
For maximum profit, we take the corresponding values in the profit matrix. We get
Maximum profit = 16 + 15 + 15 + 15 = ₹ 61

Question 6.
In the modification of a plant layout of a factory four new machines M_1, M_2, M_3, and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D, and E available. Because of limited space, machine M_2 can not be placed at C and M_3 can not be placed at A the cost of locating a machine at a place (in hundred rupees) is as follows.

Machines	Location				
	A	B	C	D	E
M1	9	11	15	10	11
M2	12	9	-	10	9
M3	-	11	14	11	7
M4	14	8	12	7	8

Find the optimal assignment schedule.
Solution:
This is an unbalanced problem so we add a dummy row M5 with all values as '0'.
Also, this is on restricted assignment problem. So we assign a very high-cost W to the prohibited cells we have

	A	B	C	D	E
M_1	9	11	15	10	11
M_2	12	9	∞	10	9
M_3	∞	11	14	11	7
M_4	14	8	12	7	8
M_5	0	0	0	0	0

Subtracting row minimum from all values in that row we get

	A	B	C	D	E
M_1	0	2	6	1	2
M_2	3	0	∞	1	0
M_3	∞	4	7	4	0
M_4	7	1	5	0	1
M_5	0	0	0	0	0

Subtracting column minimum from all values in that column we get the same matrix

	A	B	C	D	E
M_1	0	2	6	1	2
M_2	3	0	∞	1	0
M_3	∞	4	7	4	0
M_4	7	1	5	0	1
M_5	0	0	0	0	0

As minimum no. of lines covering all zeros (5) is equal to the order of the matrix (5) assignment is possible.

	A	B	C	D	E
M_1	$\boxed{0}$	2	6	1	2
M_2	3	$\boxed{0}$	∞	1	$\cancel{0}$
M_3	∞	4	7	4	$\boxed{0}$
M_4	7	1	5	$\boxed{0}$	1
M_5	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

The assignment is
M_1 → A, M_2 → B, M_3 → E, M_4 → D, M_5 → C

As M5 is dummy, no machine is installed at C.
The minimum cost is found by taking the corresponding values in the cost matrix
Minimum cost = 9 + 9 + 7 + 7 + 0 = 32 (in hundred ₹)

Maharashtra State Board 12th Commerce Maths Solutions Chapter 7 Assignment Problem and Sequencing Ex 7.2

Question 1.
A machine operator has to perform two operations, turning and threading on 6 different jobs. The time required to perform these operations (in minutes) for each job is known. Determine the order in which the jobs should be processed in order to minimize the total time required to complete all the jobs. Also, find the total processing time and idle times for turning and threading operations.

Job	1	2	3	4	5	6
Time for turning	3	12	5	2	9	11
Time for threading	8	10	9	6	3	1

Solution:
Let turning to be A and threading be B.

Job	1	2	3	4	5	6
A	3	12	5	2	9	11
B	8	10	9	6	3	1

∴ Observe Min{A, B} = 1 for job 6 on B.

					6

Then the problem reduces to

Job	1	2	3	4	5
A	3	12	5	2	9
B	8	10	9	6	3

∴ Now Min {A, B} = 2 for job 4 on A

4					6

Then the problem reduce to

Job	1	2	3	5
A	3	12	5	9
B	8	10	9	3

Now Min {A, B} = 3 for job 1 on A and job 5 on B

4	1			5	6

Then the problem reduces to

Job	2	3
A	12	5
B	10	9

Now Min {A, B) = 5 for job 3 on A

4	1	3		5	6

Only job 2 is left so the optimal sequence is

4	1	3	2	5	6

Worktable is given by

Jobs	A		B		Idle time for B
	In	Out	In	Out	
4	0	2	2	8	2
1	2	5	8	16	
3	5	10	16	25	
2	10	22	25	35	
5	22	31	35	38	
6	31	42	42	43	4
					6

Total elapsed time = 43 minutes
Idle time for A (turning) = 43 – 42 = 1 min
Idle time for B (threshing) = 2 + 4 = 6 min

Question 2.
A company has three jobs on hand, Each of these must be processed through two departments, in the AB where
Department A: Press shop and
Department B: Finishing
The table below gives the number of days required by each job each department

Job	I	II	III
Department A	8	6	5
Department B	8	3	4

Find the sequence in which the three jobs should be processed so as to take minimum time to finish all the three jobs. Also find idle time for both the departments.
Solution:
Observe Min {A, B} = 3 for job II on B.

		II

Then the problem is reduced to

Job	I	III
A	8	5
B	8	4

Now Min {A, B} = 4 for job III at B

III	II

Now only job I in left
∴ the optimal sequence is given by

I	III	II

The work table is

Jobs	A		B		Idle time for B
	In	Out	In	Out	
I	0	8	8	16	8
II	8	13	16	20	
III	13	19	20	23	
					8

Total elapsed time = 23 days
Idle time for A = 23 – 19 = 4 days
Idle time for B = 8 days

Question 3.
An insurance company receives three types of policy application bundles daily from its head office for data entry and filing. The time (in minutes) required for each type for these two operations is given in the following table:

Policy	1	2	3
Data Entry	90	120	180
Filing	140	110	100

Find the sequence that minimizes the total time required to complete the entire task. Also, find the total elapsed time and idle times for each operation.
Solution:
Let Data entry be A and filing be B. So

Policy	1	2	3
A	90	120	180
B	140	110	100

Observe min {A, B} = 90 for policy 1 at A

1		

Then the problem reduces to

Policy	2	3
A	120	180
B	110	100

Observe min {A, B} = 100 for policy 3 at B

1		3

Now only policy 2 is left
∴ The optimal sequence is

1	2	3

Worktable

Policy	A		B		Idle time for B
	In	Out	In	Out	
1	0	90	$\boxed{90}$	230	90
2	90	120	230	340	
3	210	$\boxed{390}$	390	$\boxed{490}$	50
					140

So Total elapsed time = 490 min
Idle time for A (data entry) = 490 – 390 = 100 min
Idle time for B (filing) = 140 min.

Question 4.
There are five jobs, each of which must go through two machines in the order XY. Processing times (in hours) are given below. Determine the sequence for the jobs that will minimize the total elapsed time. Also, find the total elapse time and idle time for each machine.

Job	A	B	C	D	E
Machine X	10	2	18	6	20
Machine Y	4	12	14	16	8

Solution:
Observe min {x, y} = 2 for job B on x

B				

The problem reduces to

Job	A	C	D	E
X	10	18	6	20
Y	4	14	16	8

Now min [x, y] = 4 for job A on x

B				A

The problem reduces to

Job	C	D	E
X	18	6	20
Y	14	16	8

Now min [x, y] = 6 for job D on x

B	D			A

The problem reduces to

B	D		E	A

Now min [x, y] = 8 for job E on y

Job	C	E
X	18	20
Y	14	8

Now only job C in left
∴ The optimal sequence is

B	D	C	E	A

Worktable

Job	X		Y		Idle time for y
	In	Out	In	Out	
B	0	2	$\boxed{2}$	14	2
D	2	8	14	30	
C	8	26	30	44	
E	26	46	46	54	2
A	46	$\boxed{56}$	56	$\boxed{60}$	2
					6

Total elapsed time = 60 hrs
Idle time for X = 60 – 56 = 4 hrs
Idle time for Y = 6 hrs

Question 5.
Find the sequence that minimizes the total elapsed time to complete the following jobs in the order AB. Find the total elapsed time and idle times for both machines.

Job	I	II	III	IV	V	VI	VII
Machine A	7	16	19	10	14	15	5
Machine B	12	14	14	10	16	5	7

Solution:
Observe min {A, B} = 5 for job VI for B and job VII for A

VII						VI

The problem reduces to

Job	I	II	III	IV	V
A	7	16	19	10	14
B	12	14	14	10	16

Now min {A, B] = 7 for job I on A

VII	I					VI

The problem reduces to

Job	II	III	IV	V
A	16	19	10	14
B	14	14	10	16

Now min {A, B] = 10 for job IV on A and B so we have two options.

VII	I	IV				VI

Or

VII	I				IV	VI

we take the 1st one.
The problem reduces to

Job	II	III	V
A	16	19	14
B	14	14	16

Now min {A, B} = 14 for job V on A and job II and III for job B.
∴ We have

VII	I	IV	V	III	II	VI

Or

VII	I	IV	V	II	III	VI

We take the optimal sequence as.
VII – I – IV – V – III – II – VI
Worktable

Job	A		B		Idle time for B
	In	Out	In	Out	
VII	0	5	$\boxed{5}$	12	5
I	5	12	12	24	
IV	12	22	24	34	2
V	22	36	36	52	3
III	36	55	55	69	2
II	55	71	71	85	1
VI	71	$\boxed{86}$	86	$\boxed{91}$	
					13

Total elapsed time = 91 units
Idle time for A = 91 – 86 = 5 units
Idle time for B = 13 units

Question 6.
Find the optimal sequence that minimizes the total time required to complete the following jobs in the order ABC. The processing times are given in hrs.
(i)

Job	I	II	III	IV	V	VI	VII
Machine A	6	7	5	11	6	7	12
Machine B	4	3	2	5	1	5	3
Machine C	3	8	7	4	9	8	7

(ii)

Job	1	2	3	4	5
Machine A	5	7	6	9	5
Machine B	2	1	4	5	3
Machine C	3	7	5	6	7

Solution:
(i) Min A = 5, Max B = 5
As min A ≥ max B.
The problem can be converted into two machine problems.
Let G and H be two fictitious machines such that G = A + B and H = B + C we get

Job	I	II	III	IV	V	VI	VII
G	10	10	7	16	7	12	15
H	7	11	9	9	10	13	10

Now min {G, H} = 7 for job III & V for G and job I for H
∴ We have two options

V	III					I

Or

III	V					I

We take the first one
The problem reduces to

Job	II	IV	VI	VII
G	10	16	12	15
H	11	9	13	10

Min {G, H} = 9 for job IV on H

V	III				IV	I

The problem reduces to

Job	II	VI	VII
G	10	12	15
H	11	13	10

Now min {G, H} = 10 for job II for G and job VII for H

V	III	II		VII	IV	I

Now job VI is left
∴ The optimal sequence is

V	III	II	VI	VII	IV	I

The work table is

Job	A		B		Idle time for B	C		Idle time for B
	In	Out	In	Out		In	Out	
V	0	6	6	7	6	7	16	7
III	6	11	11	13	4	16	23	
II	11	18	18	21	5	23	31	
VI	18	25	25	30	4	31	39	
VII	25	37	37	40	7	40	47	1
IV	37	48	48	53	8	53	57	6
I	48	54	54	58	1	58	61	1
					35			15

Total elapsed time = 61 hrs
Idle time for A = 61 – 54 = 7 hrs
Idle time for B = 35 + [61 – 58] = 38 hrs
Idle time for C = 15 hrs

(ii) Min A = 5, Max B = 5
Min A ≥ Max B.
The problem can be converted into two machine problems.
Let G and H be two fictitious machines such that G = A + B and H = B + C we get

Job	1	2	3	4	5
G	7	8	10	14	8
H	5	8	9	11	10

Now min {G, H} = 5 for job 1 for H.

				1

The problem reduces to

Job	2	3	4	5
G	8	10	14	8
H	8	9	11	10

Now min {G, H} = 8 for job 2 for G and job H also job 5 for G
∴ We have two options

2	5			1

Or

5	2			1

We take the first one
The problem reduces to

Job	3	4
G	10	14
H	9	11

Now min {G, H} = 9 for job 3 for H

2	5		3	1

Now only job 4 is left
∴ The optimal sequence is

2	5	4	3	1

Worktable

Job	A		B		Idle time for B	C		Idle time for C
	In	Out	In	Out		In	Out	8
2	0	7	$\boxed{7}$	8	7	$\boxed{8}$	15	
5	7	12	12	15	4	15	22	
4	12	21	21	26	6	26	32	4
3	21	27	27	31	1	32	37	
1	27	$\boxed{32}$	32	$\boxed{34}$	1	37	$\boxed{40}$	
					19			12

Total elapsed time = 40 hrs
Idle time for A = 40 – 32 = 8 hrs
Idle time for B = 19 + [40 – 34] = 25 hrs
Idle time for C = 12 hrs

Question 7.
A publisher produces 5 books on Mathematics. The books have to go through composing, printing, and binding was done by 3 machines P, Q, E. The time schedule for the entire task in the proper unit is as follows.

Book	A	B	C	D	E
Machine P	4	9	8	6	5
Machine Q	5	6	2	3	4
Machine R	8	10	6	7	11

Determine the optimum time required to finish the entire task.
Solution:
Min R = 6, Max Q = 6
As min R ≥ max Q.
The problem can be converted into a two-machine problem.
Let G and H be two fictitious machines such that G = P + Q and H = Q + R we get

Book	A	B	C	D	E
G	9	15	10	9	9
H	13	16	8	10	15

Min {G, H} = 9 for books A, D, E for G.
∴ We have more than one option we take

D	A	E		

The problem reduces to

Book	B	C
G	15	10
H	16	8

Min {G, H} = 8 for book C on H

D	A	E		C

Now only B is left. So the optimal sequence is

D	A	E	B	C

Worktable

Books	P		Q		Idle time for Q	R		Idle time for R
	In	Out	In	Out		In	out	9
D	0	6	$\boxed{6}$	9	6	$\boxed{9}$	16	
A	6	10	10	15	1	16	24	
E	10	15	15	19		24	35	
B	15	24	24	30	5	35	45	
C	24	$\boxed{32}$	32	$\boxed{34}$	2	45	$\boxed{51}$	
					14			9

Total elapsed time = 51 units
Idle time for P = 51 – 32 = 19 units
Idle time for Q = 14 + [51 – 34] = 31 units
Idle time for R = 9 units

Maharashtra State Board 12th Commerce Maths Solutions Chapter 7 Assignment Problem and Sequencing Miscellaneous Exercise 7

(I) Choose the correct alternative.

Question 1.
In sequencing, an optimal path is one that minimizes __________
(a) Elapsed time
(b) Idle time
(c) Both (a) and (b)
(d) Ready time
Answer:
(c) Both (a) and (b)

Question 2.
If job A to D have processing times as 5, 6, 8, 4 on first machine and 4, 7, 9, 10 on the second machine then the optimal sequence is:
(a) CDAB
(b) DBCA
(c) BCDA
(d) ABCD
Answer:
(b) DBCA

Question 3.
The objective of sequence problem is
(a) to find the order in which jobs are to be made
(b) to find the time required for the completing all the job on hand
(c) to find the sequence in which jobs on hand are to be processed to minimize the total time required for processing the jobs
(d) to maximize the cost
Answer:
(c) to find the sequence in which jobs on hand are to be processed to minimize the total time required for processing the jobs

Question 4.
If there are n jobs and m machines, then there will be __________ sequences of doing the jobs.

(a) mn
(b) m(n!)
(c) n^m
(d) $(n!)^m$
Answer:
(d) $(n!)^m$

Question 5.
The Assignment Problem is solved by
(a) Simple method
(b) Hungarian method
(c) Vector method
(d) Graphical method
Answer:
(b) Hungarian method

Question 6.
In solving 2 machine and n jobs sequencing problem, the following assumption is wrong
(a) No passing is allowed
(b) Processing times are known
(c) Handling times is negligible
(d) The time of passing depends on the order of machining
Answer:
(d) The time of passing depends on the order of machining

Question 7.
To use the Hungarian method, a profit maximization assignments problem requires
(a) Converting all profit to opportunity losses
(b) A dummy person or job
(c) Matrix expansion
(d) Finding the maximum number of lines to cover all the zeros in the reduced matrix
Answer:
(a) Converting all profits to opportunity losses

Question 8.
Using the Hungarian method the optimal assignment obtained for the following assignment problem to minimize the total cost is:

Agent	Job			
	A	B	C	D
1	10	12	15	25
2	14	11	19	32
3	18	21	23	29
4	15	20	26	28

(a) 1 – C, 2 – B, 3 – D, 4 – A
(b) 1 – B, 2 – C, 3 – A, 4 – D
(c) 1 – A, 2 – B, 3 – C, 4 – D
(d) 1 – D, 2 – A, 3 – B, 4 – C
Answer:
(a) 1 – C, 2 – B, 3 – D, 4 – A

Question 9.
The assignment problem is said to be unbalanced if
(a) Number of rows is greater than the number of columns
(b) Number of rows is lesser than number of columns
(c) Number of rows is equal to the number of columns
(d) Both (a) and (b)
Answer:
(d) Both (a) and (b)

Question 10.
The assignment problem is said to be balanced if
(a) Number of rows is greater than the number of columns
(b) Number of rows is lesser than number of columns
(c) Number of rows is equal to the number of columns
(d) If the entry of rows is zero
Answer:
(c) Number of rows is equal to number of columns

Question 11.
The assignment problem is said to be balanced if it is a
(a) Square matrix
(b) Rectangular matrix
(c) Unit matrix
(d) Triangular matrix
Answer:
(a) Square matrix

Question 12.
In an assignment problem if the number of rows is greater than the number of columns then
(a) Dummy column is added
(b) Dummy row is added
(c) Row with cost 1 is added
(d) Column with cost 1 is added
Answer:
(a) Dummy column is added

Question 13.
In a 3 machine and 5 jobs problem, the least of processing times on machines A, B, and C are 5, 1 and 3 hours and the highest processing times are 9, 5 and 7 respectively, then it can be converted to a 2 machine problem if the order of the machines is:
(a) B – A – C
(b) A – B – C
(c) C – B – A
(d) Any order
Answer:
(b) A – B – C

Question 14.
The objective of an assignment problem is to assign
(a) Number of jobs to equal number of persons at maximum cost
(b) Number of jobs to equal number of persons at minimum cost
(c) Only the maximize cost
(d) Only to minimize cost
Answer:
(b) Number of jobs to equal number of persons at minimum cost

(II) Fill in the blanks.

Question 1.
An assignment problem is said to be unbalanced when __________
Answer:
the number of rows is not equal to the number of columns

Question 2.
When the number of rows is equal to the Number of columns then the problem is said to be __________ assignment problem.
Answer:
balanced

Question 3.
For solving assignment problem the matrix should be a __________
Answer:
square matrix

Question 4.
If the given matrix is not a __________ matrix, the assignment problem is called an unbalanced problem.
Answer:
square

Question 5.
A dummy row(s) or column(s) with the cost elements as __________ the matrix of an unbalanced assignment problem as a square matrix.
Answer:
zero

Question 6.
The time interval between starting the first job and completing the last, job including the idle time (if any) in a particular order by the given set of machines is called __________

Answer:
Total elapsed time

Question 7.
The time for which a machine j does not have a job to process to the start of job i is called __________
Answer:
Idle time

Question 8.
The maximization assignment problem is transformed to minimization problem by subtracting each entry in the table from the __________ value in the table.
Answer:
maximum

Question 9.
When the assignment problem has more than one solution, then it is __________ optimal solution.
Answer:
multiple

Question 10.
The time required for printing four books A, B, C, and D is 5, 8, 10, and 7 hours. While its data entry requires 7, 4, 3, and 6 hrs respectively. The sequence that minimizes total elapsed time is __________
Answer:
A – D – B – C

(III) State whether each of the following is True or False.

Question 1.
One machine – one job is not an assumption in solving sequencing problems.
Answer:
False

Question 2.
If there are two least processing times for machine A and machine B, priority is given for the processing time which has the lowest time of the adjacent machine.
Answer:
True

Question 3.
To convert the assignment problem into a maximization problem, the smallest element in the matrix is deducted from all other elements.
Answer:
False

Question 4.
The Hungarian method operates on the principle of matrix reduction, whereby the cost table is reduced to a set of opportunity costs.
Answer:
True

Question 5.
In a sequencing problem, the processing times are dependent on the order of processing the jobs on machines.
Answer:
False

Question 6.
The optimal assignment is made in the Hungarian method to cells in the reduced matrix that contain a Zero.
Answer:
True

Question 7.
Using the Hungarian method, the optimal solution to an assignment problem is fund when the minimum number of lines required to cover the zero cells in the reduced matrix equals the number of people.
Answer:
True

Question 8.
In an assignment problem, if a number of columns are greater than the number of rows, then a dummy column is added.

Answer:
False

Question 9.
The purpose of a dummy row or column in an assignment problem is to obtain a balance between a total number of activities and a total number of resources.
Answer:
True

Question 10.
One of the assumptions made while sequencing n jobs on 2 machines is: two jobs must be loaded at a time on any machine.
Answer:
False

(IV) Solve the following problems.

Part – I

Question 1.
A plant manager has four subordinates, and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. This estimate of the times each man would take to perform each task is given in the effectiveness matrix below.

	I	II	III	IV
A	7	25	26	10
B	12	27	3	25
C	37	18	17	14
D	18	25	23	9

How should the tasks be allocated, one to a man, as to minimize the total man-hours?
Solution:
The hr matrix is given by

	I	II	III	IV
A	7	25	26	10
B	12	27	3	25
C	37	18	17	14
D	18	25	23	9

Subtracting row minimum from all values in that row we get

	I	II	III	IV
A	0	18	19	3
B	9	24	0	22
C	23	4	3	0
D	9	16	14	0

Subtracting column minimum from all values in that column we get

	I	II	III	IV
A	0	14	19	3
B	9	20	0	22
C	23	0	3	0
D	9	12	14	0

The minimum no. of lines covering ail the zeros (4) is equal to the order of the matrix (4)
∴ The assignment is possible.

	I	II	III	IV
A	$\boxed{0}$	14	19	3
B	9	20	$\boxed{0}$	22
C	23	$\boxed{0}$	3	$\cancel{0}$
D	9	12	14	$\boxed{0}$

The assignment is
A → I, B → III, C → II, D → IV
For the minimum hrs. take the corresponding value from the hr matrix.
Minimum hrs = 7 + 3 + 18 + 9 = 37 hrs

Question 2.
A dairy plant has five milk tankers, I, II, III, IV & V. These milk tankers are to be used on five delivery routes A, B, C, D & E. The distances (in kms) between the dairy plant and the delivery routes are given in the following distance matrix.

	I	II	II	IV	V
A	150	120	175	180	200
B	125	110	120	150	165
C	130	100	145	160	175
D	40	40	70	70	100
E	45	25	60	70	95

How should the milk tankers be assigned to the chilling centre so as to minimize the distance travelled?
Solution:
The distance matrix is given by

	I	II	III	IV	V
A	150	120	175	180	200
B	125	110	120	150	165
C	130	100	145	160	175
D	40	40	70	70	100
E	45	25	60	70	95

Subtracting row minimum from all values in that row we get

	I	II	III	IV	V
A	30	0	55	60	80
B	15	0	10	40	55
C	30	0	45	60	75
D	0	0	30	30	60
E	20	0	35	45	70

Subtracting column minimum from each value in that column we get

	I	II	III	IV	V
A	30	0	45	30	25
B	15	0	0	10	0
C	30	0	35	30	20
D	0	0	20	0	5
E	20	0	25	15	15

The number of lines covering all the zeros (3) is less than the order of the matrix (5) so the assignment is not possible. The modification is required.
The minimum uncovered value (15) is subtracted from uncovered values and added to the values at the intersection. The numbers on the lines remain the same. We get

	I	II	III	IV	V
A	15	0	30	15	10
B	15	15	0	10	0
C	15	0	20	15	5
D	0	15	20	0	5
E	5	0	10	0	0

The minimum lines covering all the zeros (4) are less than the order of the matrix (5) so the assignment is not possible. The modification is required the minimum uncovered value (5) is subtracted from uncovered values and added to the values at the intersection. The numbers on the lines remain the same we get

	I	II	III	IV	V
A	10	$\boxed{0}$	25	10	5
B	15	25	$\boxed{0}$	10	0
C	10	0	15	10	$\boxed{0}$
D	$\boxed{0}$	20	20	0	5
E	5	5	10	$\boxed{0}$	0

The minimum number of lines covering all the zeros (5) is equal to the order of the matrix (5) So assignment is possible.
The assignment is
A → II, B → III, C → V, D → I, E → IV
Total minimum distance is = 120 + 120 + 175 + 40 + 70 = 525 kms.

Question 3.
Solve the following assignment problem to maximize sales:

Salesmen	Territories				
	I	II	III	IV	V
A	11	16	18	15	15
B	7	19	11	13	17
C	9	6	14	14	7
D	13	12	17	11	13

Solution:
As it is a maximization problem so we need to convert it into a minimization problem.
Subtracting all the values from the maximum value (19) we get

	I	II	II	IV	V
A	8	3	1	4	4
B	12	0	8	6	2
C	10	13	5	5	12
D	6	7	2	8	6

Also, it is an unbalanced problem so we need to add a dummy row (E) with all values zero, we get

	I	II	III	IV	V
A	8	3	1	4	4
B	12	0	8	6	2
C	10	13	5	5	12
D	6	7	2	8	6
E	0	0	0	0	0

Subtracting row minimum from all values in that row we get

	I	II	II	IV	V
A	7	2	0	3	3
B	12	0	8	6	2
C	5	8	0	0	7
D	6	5	0	6	4
E	0	0	0	0	0

Subtracting column minimum from all values in that column we get the same matrix

	I	II	III	IV	V
A	7	2	0	3	3
B	12	0	8	6	2
C	5	8	0	0	7
D	4	5	0	6	4
E	0	0	0	0	0

The minimum number of lines covering all the zero (4) is less than the order of the matrix (5) So assignment is not possible. The modification is required. The minimum uncovered value (2) is subtracted from the uncovered values and added to the values at the intersection. The values on the lines remain the same. We get

	I	II	III	IV	V
A	5	2	0	1	1
B	10	0	8	4	0
C	5	10	2	0	7
D	2	5	0	4	2
E	0	2	2	0	0

The minimum number of lines covering all the zeros (4) is less than the order of the matrix (5) so the assignment is not possible. The modification is required. The minimum uncovered value (1) is subtracted from the uncovered value and added to the values at the intersection. The values on the lines remain the same. We get

	I	II	III	IV	V
A	4	1	0	0	0
B	10	0	9	4	0
C	5	10	3	0	7
D	1	4	0	3	1
E	0	2	3	0	0

The minimum number of lines covering all the zeros (5) is equal to the order of the matrix (5) so the assignment is possible.

	I	II	III	IV	V
A	4	1	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$
B	10	$\boxed{0}$	9	4	$\cancel{0}$
C	5	10	3	$\boxed{0}$	7
D	1	4	$\boxed{0}$	3	1
E	$\boxed{0}$	2	3	$\cancel{0}$	$\cancel{0}$

The assignment is

The assignment is

A → V, B → II, C → IV, D → III, E → I
No salesman goes to I as E is a dummy row.
For the maximum value take the corresponding values from the original matrix.
We get Maximum value = 15 + 19 + 14 + 17 + 0 = 65 units

Question 4.
The estimated sales (tons) per month in four different cities by five different managers are given below:

Manager	Cities			
	P	Q	R	S
I	34	36	33	35
II	33	35	31	33
III	37	39	35	35
IV	36	36	34	34
V	35	36	35	33

Find out the assignment of managers to cities in order to maximize sales.
Solution:
This is a maximizing problem. To convert it into minimizing problem subtract all the values of the matrix from the maximum (largest) value (39) we get

	P	Q	R	S
I	5	3	6	4
II	6	4	8	6
III	2	0	4	4
IV	3	3	5	5
V	4	3	4	6

Also as it is an unbalanced problem so we have to add a dummy column (T) with all the values as zero. We get

	P	Q	R	S	T
I	5	3	6	4	0
II	6	4	8	6	0
III	2	0	4	4	0
IV	3	3	5	5	0
V	4	3	4	6	0

Subtracting row minimum from all values in that row we get the same matrix
Subtracting column minimum from all values in that column we get

	P	Q	R	S	T
I	3	3	2	0	0
II	4	4	4	6	0
III	0	0	0	0	0
IV	1	3	5	1	0
V	2	3	0	2	0

The minimum number of lines covering all the zeros (4) is less than the order of the matrix (5) so assignments are not possible. The modification is required. The minimum uncovered value (1) is subtracted from the uncovered values and added to the values at the intersection. The values on the lines remain the same. We get

	P	Q	R	S	T
I	2	2	2	0	0
II	3	3	4	6	0
III	0	0	1	1	1
IV	0	2	5	1	0
V	1	2	0	2	0

The minimum number of lines covering all the zeros (5) is equal to the order of the matrix (5) so the assignment is possible.

	P	Q	R	S	T
I	2	2	2	$\boxed{0}$	$\cancel{0}$
II	3	3	4	6	$\boxed{0}$
III	$\cancel{0}$	$\boxed{0}$	1	1	1
IV	$\boxed{0}$	2	5	1	$\cancel{0}$
V	1	2	$\boxed{0}$	2	$\cancel{0}$

So I → S, II → T, III → Q, IV → P, V → R.
As T is dummy manager II is not given any city.
To find the maximum sales we take the corresponding value from the original matrix
Total maximum sales = 35 + 39 + 36 + 35 = 145 tons

Question 5.
Consider the problem of assigning five operators to five machines. The assignment costs are given in the following table.

Operator	Machine				
	1	2	3	4	5
A	6	6	-	3	7
B	8	5	3	4	5
C	10	4	6	-	4
D	8	3	7	8	3
E	7	6	8	10	2

Operator A cannot be assigned to machine 3 and operator C cannot be assigned to machine 4. Find the optimal assignment schedule.
Solution:
This is a restricted assignment problem, so we assign a very high cost (oo) to the prohibited cells we get

	1	2	3	4	5
A	6	6	∞	3	7
B	8	5	3	4	5
C	10	4	6	∞	4
D	8	3	7	8	3
E	7	6	8	10	2

Subtracting row minimum from all values in that row we get.

	1	2	3	4	5
A	3	3	∞	0	4
B	5	2	0	1	2
C	6	0	2	∞	0
D	5	0	4	5	0
E	5	4	6	8	0

Subtracting column minimum from all values in that column we get

	1	2	3	4	5
A	0	3	∞	0	4
B	2	2	0	1	2
C	3	0	2	∞	0
D	2	0	4	5	0
E	2	4	6	8	0

As the minimum number of lines covering all the zeros (4) is equal to the order of the matrix (5) so the assignment is not possible. The modification is required. The minimum uncovered value (2) is subtracted from all the uncovered values and added to the values at the intersection. The values on the lines remain the same. We get

	1	2	3	4	5
A	0	5	∞	0	6
B	2	4	0	1	4
C	1	0	0	∞	0
D	0	0	2	3	0
E	0	4	4	6	0

As the minimum number of lines covering all the zeros (5) is equal to the order of the matrix, assignment is the possible

	1	2	3	4	5
A	$\cancel{0}$	5	∞	$\boxed{0}$	6
B	2	4	$\boxed{0}$	1	4
C	1	$\boxed{0}$	$\cancel{0}$	∞	$\cancel{0}$
D	$\boxed{0}$	$\cancel{0}$	2	3	$\cancel{0}$
E	$\cancel{0}$	4	4	0	$\boxed{0}$

So A → 4, B → 3, C → 2, D → 1, E → 5
For the minimum cost take the corresponding values from the cost matrix we get
Total minimum cost = 3 + 3 + 4 + 3 + 7 = 20 units

Question 6.
A chartered accountant's firm has accepted five new cases. The estimated number of days required by each of their five employees for each case are given below, where-means that the particular employee can not be assigned the particular case. Determine the optimal assignment of cases of the employees so that the total number of days required to complete these five cases will be minimum. Also, find the minimum number of days.

Employee	Cases				
	I	II	III	IV	V
E_1	6	4	5	7	8
E_2	7	-	8	6	9
E_3	8	6	7	9	10
E_4	5	7	-	4	6
E_5	9	5	3	10	-

Solution:
This is a restricted assignment problem so we assign a very high cost (∞) to all the prohibited cells. The day matrix becomes

	I	II	III	IV	V
E_1	6	4	5	7	8
E_2	7	∞	8	6	9
E_3	8	6	7	9	10
E_4	5	7	∞	4	6
E_5	9	5	3	10	∞

Subtracting row minimum from all values in that row we get

	I	II	III	IV	V
E_1	2	0	1	3	4
E_2	1	∞	2	0	3
E_3	2	0	1	3	4
E_4	1	3	∞	0	2
E_5	6	2	0	7	∞

Subtracting column minimum from all values in that column we get

	I	II	III	IV	V
E_1	1	0	1	3	2
E_2	0	∞	2	0	1
E_3	1	0	1	3	2
E_4	0	3	∞	0	0
E_5	5	2	0	7	∞

The minimum number of lines covering all the zeros (4) is less than the order of the matrix (5) so the assignment is not possible, The modification is required. The minimum uncovered value (1) is subtracted from all the uncovered values and added to the values at the intersection. The values on the lines remain the same, we get

	I	II	III	IV	V
E_1	0	0	1	2	1
E_2	0	∞	3	0	1
E_3	0	0	1	2	1
E_4	0	4	∞	0	0
E_5	4	2	0	6	∞

The minimum number of lines covering all the zeros (5) is equal to the order of the matrix (5) so the assignment is possible.

	I	II	III	IV	V
E_1	$\boxed{0}$	$\cancel{0}$	1	2	1
E_2	$\cancel{0}$	∞	3	$\boxed{0}$	1
E_3	$\cancel{0}$	$\boxed{0}$	1	2	1
E_4	$\cancel{0}$	4	∞	$\cancel{0}$	$\boxed{0}$
E_5	4	2	$\boxed{0}$	6	∞

So $E_1 \rightarrow$ I, $E_2 \rightarrow$ IV, $E_3 \rightarrow$ II, $E_4 \rightarrow$ V, $E_5 \rightarrow$ III
To find the minimum number of days we take the corresponding values from the day matrix.
Total minimum number of days = 6 + 6 + 6 + 6 + 3 = 27 days

Part – II

Question 1.
A readymade garments manufacture has to process 7 items through two stages of production, namely cutting and sewing. The time taken in hours for each of these items in different stages are given below:

Items	1	2	3	4	5	6	7
Time for Cutting	5	7	3	4	6	7	12
Time for Sewing	2	6	7	5	9	5	8

Find the sequence in which these items are to be processed through these stages so as to minimize the total processing time. Also, find the idle time of each machine.
Solution:
Let A = cutting and B = sewing. So we have

Items	1	2	3	4	5	6	7
A	5	7	3	4	6	7	12
B	2	6	7	5	9	5	8

Observe min {A, B} = 2 for item 1 for B.

					1

The problem reduces to

Items	2	3	4	5	6	7
A	7	3	4	6	7	12
B	6	7	5	9	5	8

Now min {A, B} = 3 for item 3 for A

3						1

The problem reduces to

Items	2	4	5	6	7
A	7	4	6	7	12
B	6	5	9	5	8

New min {A , B} = 4 for item 4 for A.

3	4					1

The problem reduces to

Items	2	5	6	7
A	7	6	7	12
B	6	9	5	8

Now min(A, B} = 5 for item 6 for B

3	4				6	1

The problem reduces to

Items	2	5	7
A	7	6	12
B	6	9	8

Now min {A, B} = 6 for item 5 for A and item 2 for B

3	4	5		2	6	1

Now only 7 is left

∴ The optimal sequence is

3	4	5	7	2	6	1

Worktable

Items	A		B		Idle time for B
	In	out	In	out	
3	0	3	$\boxed{3}$	10	3
4	3	7	10	15	
5	7	13	15	24	
7	13	25	25	33	1
2	25	32	33	39	
6	32	39	39	44	
1	39	$\boxed{44}$	44	$\boxed{46}$	
					4

Total elapsed time = 46 hrs

Idle time for A (cutting) = 46 – 44 = 2 hrs

Idle time for B (Sewing) = 4 hrs

Question 2.

Five jobs must pass through a lathe and a surface grinder, in that order. The processing times in hours are shown below. Determine the optimal sequence of the jobs. Also, find the idle time of each machine.

Job	I	II	III	IV	V
Lathe	4	1	5	2	5
Surface grinder	3	2	4	3	6

Solution:

Let A = lathe and B = surface grinder. We have

Job	I	II	III	IV	V
A	4	1	5	2	5
B	3	2	4	3	6

Observe min {A, B} = 1 for job II for A

II				

The problem reduces to

Job	I	III	IV	V
A	4	5	2	5
B	3	4	3	6

Now min {A, B} = 2 for job IV for A

II	IV			

The problem reduces to

Job	I	III	V
A	4	5	5
B	3	4	6

Now min {A, B} = 3 for job I for B

II	IV			I

The problem reduces to

Job	III	V
A	5	5
B	4	6

Now min {A, B} = 5 for jobs III and V for A
∴ We have two options

II	IV	V	III	I

or

II	IV	III	V	I

We take the first one.
Worktable

Job	A		B		Idle time for B
	In	out	In	out	
II	0	1	1	3	1
IV	1	3	3	6	
V	3	8	8	14	2
III	8	13	14	18	
I	13	$\boxed{17}$	18	$\boxed{21}$	
					3

Total elapsed time = 21 hrs
Idle time for A (lathe) = 21 – 17 = 4 hrs
Idle time for B (surface grinder) = 3 hrs

Question 3.
Find the sequence that minimizes the total elapsed time to complete the following jobs. Each job is processed in order AB.

	Jobs (Processing times in minutes						
	I	II	III	IV	V	VI	VII
Machine A	12	6	5	11	5	7	6
Machine B	7	8	9	4	7	8	3

Determine the sequence for the jobs so as to minimize the processing time. Find the total elapsed time and the idle time for both machines.
Solution:
Observe min {A, B} = 3 for job VII on B.

						VII

The problem reduces to

Job	I	II	III	IV	V	VI
A	12	6	5	11	5	7
B	7	8	9	4	7	8

Now min {A, B} = 4 for job IV on B.

					IV	VII

The problem reduces to

Job	I	II	III	V	VI
A	12	6	5	5	7
B	7	8	9	7	8

Now min {A, B} = 5 for job III & V on A. we have two options

III	V				IV	VII

or

V	III				IV	VII

We take the first one
The problem reduces to

Job	I	II	VI
A	12	6	7
B	7	8	8

Now min {A, B} = 5 for job II on A

III	V	II			IV	VII

The problem reduces to

Job	I	VI
A	12	7
B	7	8

Now min {A, B} = 7 for a job I on B and for job VI on A
∴ The optional sequence is

III	V	II	VI	I	IV	VII

Worktable

Job	A		B		Idle time for B
	In	Out	In	Out	
III	0	5	$\boxed{5}$	14	5
V	5	10	14	21	
II	10	16	21	29	
VI	16	23	29	37	
I	23	35	37	44	
IV	35	46	46	50	2
VII	46	52	52	$\boxed{55}$	2
					9

Total elapsed time = 55 units
Idle time for A = 55 – 52 = 3 units
Idle time for B = 9 units.

Question 4.
A toy manufacturing company has five types of toys. Each toy has to go through three machines A, B, C in the order ABC. The time required in hours for each process is given in the following table.

Type	1	2	3	4	5
Machine A	16	20	12	14	22
Machine B	10	12	4	6	8
Machine C	8	18	16	12	10

Solve the problem for minimizing the total elapsed time.
Solution:
Min A = 12, Max B = 12
As min A ≥ max B.
The problem can be converted into two machine problems.
Let G and H be two fictitious machines such that G = A + B and H = B + C, We get

Type	1	2	3	4	5
G	26	32	16	20	30
H	18	30	20	18	18

Now min {G, H} = 16 for type 3 on G

3				

The problem reduces to

Type	1	2	4	5
G	26	32	20	30
H	18	30	18	18

Min (G, H} = 18 for type 1, 4 & 5 on H
We have more than one option, we take

3		4	5	1

Now only type 2 is left.
∴ The optional sequence is

3	2	4	5	1

Worktable

Type	A		B		Idle time For B	C		Idle time For C
	In	Out	In	Out		In	Out	
3	0	12	$\boxed{12}$	16	12	$\boxed{16}$	32	16
2	12	32	32	44	16	44	62	12
4	32	54	54	62	10	62	72	
5	54	68	68	74	6	74	86	2
1	68	$\boxed{84}$	84	$\boxed{94}$	10	94	$\boxed{102}$	8
					54			38

Total elapsed time = 102 hours
Idle time for A = 102 – 84 = 18 hours
Idle time for B = 54 + (102 – 94) = 62 hours
Idle time for C = 38 hours

Question 5.
A foreman wants to process 4 different jobs on three machines: a shaping machine, a drilling machine, and a tapping, the sequence of operations being shaping-drilling-tapping. Decide the optimal sequence for the four jobs to minimize the total elapsed time. Also, find the total elapsed time and the idle time for every machine.

Job	Shaping (minutes)	Drilling (minutes)	Trapping (Minutes)
1	13	3	18
2	18	8	4
3	8	6	13
4	23	6	8

Solution:
The time matrix is

Job	1	2	3	4
Shaping (A)	13	18	8	23
Drilling (B)	3	8	6	6
Trapping(C)	18	4	13	8

Min A = 8, Max B = 8, as min A ≥ max B.
The problem can be converted into a two-machine problem.
Let G and H be two fictitious machines such that
G = A + B and H = B + C we get

Job	1	2	3	4
G	16	26	14	29
H	21	12	19	14

Observe min (G, H} = 12 for job 2 on H

			2

The problem reduces to

Job	1	3	4
G	16	14	29
H	21	19	14

Now min {G, H} = 14 for job 3 on G and job 4 on H

3		4	2

Now only job 1 is left.

∴ The optimal sequence is

3	1	4	2

Worktable

Job	A		B		Idle time	C		Idle time
	In	Out	In	Out	For B	In	Out	For C
3	0	8	$\boxed{8}$	14	8	$\boxed{14}$	27	14
1	8	21	21	24	7	27	45	
4	21	44	44	50	20	50	58	5
2	44	$\boxed{62}$	62	$\boxed{70}$	12	70	$\boxed{74}$	12
					47			31

Total elapsed time = 74 min

Idle time for A (shapping) = 74 – 62 = 12 min

Idle time for B (Drilling) = 47 + (74 – 70) = 51 min

Idle time for C (trapping) = 31 min