

Practice Set 2.1 Algebra 10th Std Maths Part 1 Answers Chapter 2 Quadratic Equations

Question 1.

Write any two quadratic equations.

Solution:

i. $y^2 - 7y + 12 = 0$

ii. $x^2 - 8 = 0$

Question 2.

Decide which of the following are quadratic

i. $x^2 - 7y + 2 = 0$

ii. $y^2 = 5y - 10$

iii. $y^2 + 1y = 2$

iv. $x + 1x = -2$

v. $(m + 2)(m - 5) = 0$

vi. $m^3 + 3m^2 - 2 = 3m^3$

Solution:

i. The given equation is $x^2 + 5x - 2 = 0$

Here, x is the only variable and maximum index of the variable is 2.

a = 1, b = 5, c = -2 are real numbers and $a \neq 0$.

∴ The given equation is a quadratic equation.

ii. The given equation is

$y^2 = 5y - 10$

∴ $y^2 - 5y + 10 = 0$

Here, y is the only variable and maximum index of the variable is 2.

a = 1, b = -5, c = 10 are real numbers and $a \neq 0$.

∴ The given equation is a quadratic equation.

iii. The given equation is

$y^2 + 1y = 2$

∴ $y^3 + 1 = 2y$...[Multiplying both sides by y]

∴ $y^3 - 2y + 1 = 0$

Here, y is the only variable and maximum index of the variable is not 2.

∴ The given equation is not a quadratic equation.

iv. The given equation is

$x + 1x = -2$

∴ $x^2 + 1 = -2x$...[Multiplying both sides by x]

∴ $x^2 + 2x + 1 = 0$

Here, x is the only variable and maximum index of the variable is 2.

a = 1, b = 2, c = 1 are real numbers and $a \neq 0$.

∴ The given equation is a quadratic equation.

v. The given equation is

$(m + 2)(m - 5) = 0$

∴ $m(m - 5) + 2(m - 5) = 0$

∴ $m^2 - 5m + 2m - 10 = 0$

∴ $m^2 - 3m - 10 = 0$

Here, m is the only variable and maximum index of the variable is 2.

a = 1, b = -3, c = -10 are real numbers and $a \neq 0$.

∴ The given equation is a quadratic equation.

vi. The given equation is

$m^3 + 3m^2 - 2 = 3m^3$

∴ $3m^3 - m^3 - 3m^2 + 2 = 0$

∴ $2m^3 - 3m^2 + 2 = 0$

Here, m is the only variable and maximum

index of the variable is not 2.

∴ The given equation is not a quadratic equation.

Question 3.

Write the following equations in the form $ax^2 + bx + c = 0$, then write the values of a, b, c for each equation.

i. $2y = 10 - y^2$

ii. $(x - 1)^2 = 2x + 3$

iii. $x^2 + 5x = -(3 - x)$

iv. $3m^2 = 2m^2 - 9$

v. $P(3 + 6p) = -5$

vi. $x^2 - 9 = 13$

Solution:

i. $2y - 10 - y^2$

$$\therefore y^2 + 2y - 10 = 0$$

Comparing the above equation with

$$ay^2 + by + c = 0, \text{ we get}$$

$$a = 1, b = 2, c = -10$$

$$\text{ii. } (x - 1)^2 = 2x + 3$$

$$\therefore x^2 - 2x + 1 = 2x + 3$$

$$x^2 - 2x + 1 - 2x - 3 = 0$$

$$\therefore x^2 - 4x - 2 = 0$$

Comparing the above equation with

$$ax^2 + bx + c = 0, \text{ we get}$$

$$a = 1, b = -4, c = -2$$

$$\text{iii. } x^2 + 5x = -(3 - x)$$

$$\therefore x^2 + 5x = -3 + x$$

$$\therefore x^2 + 5x - x + 3 = 0$$

$$\therefore x^2 + 4x + 3 = 0$$

Comparing the above equation with

$$ax^2 + bx + c = 0, \text{ we get}$$

$$a = 1, b = 4, c = 3$$

$$\text{iv. } 3m^2 = 2m^2 - 9$$

$$\therefore 3m^2 - 2m^2 + 9 = 0$$

$$\therefore m^2 + 9 = 0$$

$$\therefore m^2 + 0m + 9 = 0$$

Comparing the above equation with

$$am^2 + bm + c = 0, \text{ we get}$$

$$a = 1, b = 0, c = 9$$

$$\text{v. } p(3 + 6p) = -5$$

$$\therefore 3p + 6p^2 = -5$$

$$\therefore 6p^2 + 3p + 5 = 0$$

Comparing the above equation with

$$ap^2 + bp + c = 0, \text{ we get}$$

$$a = 6, b = 3, c = 5$$

$$\text{vi. } x^2 - 9 = 13$$

$$\therefore x^2 - 9 - 13 = 0$$

$$\therefore x^2 - 22 = 0$$

$$\therefore x^2 + 0x - 22 = 0$$

Comparing the above equation with

$$ax^2 + bx + c = 0, \text{ we get}$$

$$a = 1, b = 0, c = -22$$

Question 4.

Determine whether the values given against each of the quadratic equation are the roots of the equation.

$$\text{i. } x^2 + 4x - 5 = 0; x = 1, -1$$

$$\text{ii. } 2m^2 - 5m = 0; m = 2, 5/2$$

Solution:

i. The given equation is

$$x^2 + 4x - 5 = 0 \dots(i)$$

Putting $x = 1$ in L.H.S. of equation (i), we get

$$\text{L.H.S.} = (1)^2 + 4(1) - 5 = 1 + 4 - 5 = 0$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$\therefore x = 1$ is the root of the given quadratic equation.

Putting $x = -1$ in L.H.S. of equation (i), we get

$$\text{L.H.S.} = (-1)^2 + 4(-1) - 5 = 1 - 4 - 5 = -8$$

$$\therefore \text{L.H.S.} \neq \text{R.H.S.}$$

$\therefore x = -1$ is not the root of the given quadratic equation.

ii. The given equation is

$$2m^2 - 5m = 0 \dots(i)$$

Putting $m = 2$ in L.H.S. of equation (i), we get

$$\text{L.H.S.} = 2(2)^2 - 5(2) = 2(4) - 10 = 8 - 10 = -2$$

$$\therefore \text{L.H.S.} \neq \text{R.H.S.}$$

$\therefore m = 2$ is not the root of the given quadratic equation.

Putting $m = \frac{5}{2}$ in L.H.S. of equation (i), we get

$$\begin{aligned} \text{L.H.S.} &= 2\left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) = 2\left(\frac{25}{4}\right) - \frac{25}{2} \\ &= \frac{25}{2} - \frac{25}{2} \\ &= 0 \end{aligned}$$

\therefore L.H.S. = R.H.S.

\therefore $m = \frac{5}{2}$ is the root of the given quadratic equation.

Question 5.

Find k if $x = 3$ is a root of equation $kx^2 - 10x + 3 = 0$.

Solution:

$x = 3$ is the root of the equation $kx^2 - 10x + 3 = 0$.

Putting $x = 3$ in the given equation, we get

$$k(3)^2 - 10(3) + 3 = 0$$

$$\therefore 9k - 30 + 3 = 0$$

$$\therefore 9k - 27 = 0$$

$$\therefore 9k = 27$$

$$\therefore k = \frac{27}{9}$$

$$\therefore k = 3$$

Question 6.

One of the roots of equation $5m^2 + 2m + k = 0$ is $-\frac{7}{5}$. Complete the following activity to find the value of ' k '.

Solution:

$-\frac{7}{5}$ is a root of quadratic equation

$$5m^2 + 2m + k = 0.$$

\therefore Put $m = -\frac{7}{5}$ in the equation.

$$\therefore 5 \times \left(-\frac{7}{5}\right)^2 + 2 \times \left(-\frac{7}{5}\right) + k = 0$$

$$\therefore \frac{49}{5} + \frac{-14}{5} + k = 0$$

$$\therefore 7 + k = 0$$

$$\therefore k = -7$$

Question 1.

$x^2 + 3x - 5$, $3x^2 - 5x$, $5x^2$; Write the polynomials in the index form. Observe the coefficients and fill in the boxes. (Textbook p. no. 31)

Answer:

Index form of the given polynomials:

$$x^2 + 3x - 5, 3x^2 - 5x + 0, 5x^2 + 0x + 0$$

i. Coefficients of x^2 are [1], [3] and [5] respectively, and these coefficients are non zero.

ii. Coefficients of x are 3, [-5] and [0] respectively.

iii. Constant terms are [-5], [0] and [0] respectively.

Here, constant terms of second and third polynomial is zero.

Question 2.

Complete the following table (Textbook p. no. 31)

Quadratic Equation	General form	a	b	c
$x^2 - 4 = 0$	$x^2 + 0x - 4 = 0$	1	0	-4
$y^2 = 2y - 7$
$x^2 + 2x = 0$

Answer:

Quadratic Equation	General form	a	b	c
$x^2 - 4 = 0$	$x^2 + 0x - 4 = 0$	1	0	-4
$y^2 = 2y - 7$	$y^2 - 2y + 7 = 0$	1	-2	7
$x^2 + 2x = 0$	$x^2 + 2x + 0 = 0$	1	2	0

Question 3.

Decide which of the following are quadratic equations? (Textbook pg. no. 31)

i. $9y^2 + 5 = 0$

ii. $m^3 - 5m^2 + 4 = 0$

iii. $(l + 2)(l - 5) = 0$

Solution:

i. In the equation $9y^2 + 5 = 0$, $[y]$ is the only variable and maximum index of the variable is $[2]$.

∴ It [is] a quadratic equation.

ii. In the equation $m^3 - 5m^2 + 4 = 0$, $[m]$ is the only variable and maximum index of the variable is not 2.

∴ It [is not] a quadratic equation.

iii. $(l + 2)(l - 5) = 0$

∴ $l(l - 5) + 2(l - 5) = 0$

∴ $l^2 - 5l + 2l - 10 = 0$

∴ $l^2 - 3l - 10 = 0$.

In this equation $[l]$ is the only variable and maximum index of the variable is $[2]$

∴ it [is] a quadratic equation.

Question 4.

If $x = 5$ is a root of equation $kx^2 - 14x - 5 = 0$, then find the value of k by completing the following activity. (Textbook pg, no. 33)

Solution:

One of the roots of equation $kx^2 - 14x - 5 = 0$

is $\boxed{5}$.

∴ Put $x = \boxed{5}$ in the given equation.

∴ $k\boxed{5}^2 - 14\boxed{(5)} - 5 = 0$

∴ $25k - 70 - 5 = 0$

∴ $25k - \boxed{75} = 0$

∴ $25k = \boxed{75}$

∴ $k = \frac{\boxed{75}}{\boxed{25}} = 3$

Practice Set 2.2 Algebra 10th Std Maths Part 1 Answers Chapter 2 Quadratic Equations

Question 1.

Solve the following quadratic equations by factorisation.

i. $x^2 - 15x + 54 = 0$

ii. $x^2 + x - 20 = 0$

iii. $2y^2 + 27y + 13 = 0$

iv. $5m^2 = 22m + 15$

v. $2x^2 - 2x + 12 = 0$

vi. $6x - 2x = 1$

vii. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ to solve this quadratic equation by factorisation complete the following activity

viii. $3x^2 - 2\sqrt{6}x + 2 = 0$

ix. $2m(m - 24) = 50$

x. $252 = 9$

xi. $7m^2 = 21m$

xii. $m^2 - 11 = 0$

Solution:

i.	$x^2 - 15x + 54 = 0$	
∴	$x^2 - 9x - 6x + 54 = 0$	$\begin{array}{c} 54 \\ \swarrow \quad \searrow \\ -9 \quad -6 \end{array}$
∴	$x(x - 9) - 6(x - 9) = 0$	$-9 \times -6 = +54$
∴	$(x - 9)(x - 6) = 0$	$-9 - 6 = -15$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

∴ $x - 9 = 0$ or $x - 6 = 0$

∴ $x = 9$ or $x = 6$

∴ The roots of the given quadratic equation are 9 and 6.

ii.	$x^2 + x - 20 = 0$	
∴	$x^2 + 5x - 4x - 20 = 0$	$\begin{array}{c} -20 \\ \swarrow \quad \searrow \\ 5 \quad -4 \end{array}$
∴	$x(x + 5) - 4(x + 5) = 0$	$5 \times -4 = -20$
∴	$(x + 5)(x - 4) = 0$	$5 - 4 = 1$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

∴ $x + 5 = 0$ or $x - 4 = 0$

∴ $x = -5$ or $x = 4$

∴ The roots of the given quadratic equation are -5 and 4.

iii.	$2y^2 + 27y + 13 = 0$	
∴	$2y^2 + 26y + y + 13 = 0$	$2 \times 13 = 26$
∴	$2y(y + 13) + 1(y + 13) = 0$	$\begin{array}{c} 26 \quad 1 \\ \swarrow \quad \searrow \end{array}$
∴	$(y + 13)(2y + 1) = 0$	$26 \times 1 = 26$
		$26 + 1 = 27$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

∴ $y + 13 = 0$ or $2y + 1 = 0$

∴ $y = -13$ or $2y = -1$

∴ $y = -13$ or $y = -\frac{1}{2}$

∴ The roots of the given quadratic equation are -13 and $-\frac{1}{2}$

iv.	$5m^2 = 22m + 15$	
∴	$5m^2 - 22m - 15 = 0$	$5 \times -15 = -75$
∴	$5m^2 - 25m + 3m - 15 = 0$	$\begin{array}{c} -25 \quad +3 \\ \swarrow \quad \searrow \end{array}$
∴	$5m(m - 5) + 3(m - 5) = 0$	$-25 + 3 = -22$
∴	$(m - 5)(5m + 3) = 0$	$-25 \times 3 = -75$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

∴ $m - 5 = 0$ or $5m + 3 = 0$

∴ $m = 5$ or $5m = -3$

∴ $m = 5$ or $m = -\frac{3}{5}$

∴ The roots of the given quadratic equation are 5 and $-\frac{3}{5}$

$$v. \quad 2x^2 - 2x + \frac{1}{2} = 0$$

$$\therefore 4x^2 - 4x + 1 = 0$$

...[Multiplying both sides by 2]

$$\therefore 4x^2 - 2x - 2x + 1 = 0$$

$$\therefore 2x(2x - 1) - 1(2x - 1) = 0$$

$$\therefore (2x - 1)(2x - 1) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore 2x - 1 = 0 \text{ or } 2x - 1 = 0$$

$$\therefore 2x = 1 \text{ or } 2x = 1$$

$$\therefore x = \frac{1}{2} \text{ or } x = \frac{1}{2}$$

The roots of the given quadratic equation are $\frac{1}{2}$ and $\frac{1}{2}$.

$$\begin{array}{c} 4 \\ \swarrow \quad \searrow \\ -2, -2 \\ -2 - 2 = -4 \\ -2 \times -2 = 4 \end{array}$$

$$vi. \quad 6x - \frac{2}{x} = 1$$

$$\therefore 6x^2 - 2 = x \quad \dots[\text{Multiplying both sides by } x]$$

$$\therefore 6x^2 - x - 2 = 0$$

$$\therefore 6x^2 - 4x + 3x - 2 = 0$$

$$\therefore 2x(3x - 2) + 1(3x - 2) = 0$$

$$\therefore (3x - 2)(2x + 1) = 0$$

$$\begin{array}{c} 6 \times -2 = -12 \\ \swarrow \quad \searrow \\ -4, +3 \\ -4 + 3 = -1 \\ -4 \times 3 = -12 \end{array}$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore 3x - 2 = 0 \text{ or } 2x + 1 = 0$$

$$\therefore 3x = 2 \text{ or } 2x = -1$$

$$\therefore x = \frac{2}{3} \text{ or } 2x = -1$$

\therefore The roots of the given quadratic equation are $\frac{2}{3}$ and $-\frac{1}{2}$.

$$vii. \quad \sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

$$\therefore \sqrt{2}x^2 + \boxed{5x} + \boxed{2x} + 5\sqrt{2} = 0$$

$$\therefore x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$$

$$\therefore (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$$

$$\begin{array}{c} \sqrt{2} \times 5\sqrt{2} = 10 \\ \swarrow \quad \searrow \\ 5, 2 \\ 5 + 2 = 7 \\ 5 \times 2 = 10 \end{array}$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore (\sqrt{2}x + 5) = 0 \text{ or } (x + \sqrt{2}) = 0$$

$$\therefore x = \boxed{\frac{-5}{\sqrt{2}}} \text{ or } x = -\sqrt{2}$$

$$\therefore \boxed{\frac{-5}{\sqrt{2}}} \text{ and } -\sqrt{2} \text{ are roots of the equation.}$$

$$viii. \quad 3x^2 - 2\sqrt{6}x + 2 = 0$$

$$\therefore 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$$

$$\therefore \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$$

$$\therefore (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$$

$$\begin{array}{c} 3 \times 2 = 6 \\ \swarrow \quad \searrow \\ -\sqrt{6} \quad -\sqrt{6} \\ -\sqrt{6} - \sqrt{6} = -2\sqrt{6} \\ -\sqrt{6} \times -\sqrt{6} = 6 \end{array}$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore \sqrt{3}x - \sqrt{2} = 0 \text{ or } \sqrt{3}x - \sqrt{2} = 0$$

$$\therefore \sqrt{3}x = \sqrt{2} \text{ or } \sqrt{3}x = \sqrt{2}$$

$$\therefore x = \frac{\sqrt{2}}{\sqrt{3}} \text{ or } x = \frac{\sqrt{2}}{\sqrt{3}}$$

The roots of the given quadratic equation are $\frac{\sqrt{2}}{\sqrt{3}}$ and $\frac{\sqrt{2}}{\sqrt{3}}$.

$$ix. \quad 2m(m - 24) = 50$$

$$\therefore 2m^2 - 48m = 50$$

$$\therefore 2m^2 - 48m - 50 = 0$$

$$\therefore m^2 - 24m - 25 = 0 \quad \dots[\text{Dividing both sides by 2}]$$

$$\begin{aligned} \therefore m^2 - 25m + m - 25 &= 0 \\ \therefore m(m - 25) + 1(m - 25) &= 0 \\ \therefore (m - 25)(m + 1) &= 0 \end{aligned}$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\begin{array}{c} -25 \\ \wedge \\ -25 \quad +1 \\ -25 \times 1 = -25 \\ -25 + 1 = -24 \end{array}$$

$$\therefore m - 25 = 0 \text{ or } m + 1 = 0$$

$$\therefore m = 25 \text{ or } m = -1$$

\therefore The roots of the given quadratic equation are 25 and -1.

x. $25m^2 = 9$

$$\therefore 25m^2 - 9 = 0$$

$$\therefore (5m)^2 - (3)^2 = 0$$

$$\therefore (5m + 3)(5m - 3) = 0$$

$$\dots [\because a^2 - b^2 = (a + b)(a - b)]$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore 5m + 3 = 0 \text{ or } 5m - 3 = 0$$

$$\therefore 5m = -3 \text{ or } 5m = 3$$

$$\therefore m = -\frac{3}{5} \text{ or } m = \frac{3}{5}$$

\therefore The roots of the given quadratic equation are $-\frac{3}{5}$ and $\frac{3}{5}$.

xi. $7m^2 = 21m$

$$\therefore 7m^2 - 21m = 0$$

$$\therefore m^2 - 3m = 0 \dots [\text{Dividing both sides by 7}]$$

$$\therefore m(m - 3) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore m = 0 \text{ or } m - 3 = 0$$

$$\therefore m = 0 \text{ or } m = 3$$

\therefore The roots of the given quadratic equation are 0 and 3.

xii. $m^2 - 11 = 0$

$$\therefore m^2 - (\sqrt{11})^2 = 0$$

$$\therefore (m + \sqrt{11})(m - \sqrt{11}) = 0$$

$$\dots [\because (a)^2 - (b)^2 = (a + b)(a - b)]$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore m + \sqrt{11} = 0 \text{ or } m - \sqrt{11} = 0$$

$$\therefore m = -\sqrt{11} \text{ or } m = \sqrt{11}$$

\therefore The roots of the given quadratic equation are $-\sqrt{11}$ and $\sqrt{11}$

Practice Set 2.3 Algebra 10th Std Maths Part 1 Answers Chapter 2 Quadratic Equations

Question 1.

Solve the following quadratic equations by completing the square method.

1. $x^2 + x - 20 = 0$

2. $x^2 + 2x - 5 = 0$

3. $m^2 - 5m = -3$

4. $9y^2 - 12y + 2 = 0$

5. $2y^2 + 9y + 10 = 0$

6. $5x^2 = 4x + 7$

Solution:

1. $x^2 + x - 20 = 0$

If $x^2 + x + k = (x + a)^2$, then

$$x^2 + x + k = x^2 + 2ax + a^2$$

Comparing the coefficients, we get

$$1 = 2a \text{ and } k = a^2$$

$$\therefore a = \frac{1}{2} \text{ and } k = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\text{Now, } x^2 + x - 20 = 0$$

$$\therefore x^2 + x + \frac{1}{4} - \frac{1}{4} - 20 = 0$$

$$\therefore \left(x + \frac{1}{2}\right)^2 - \left(\frac{1+80}{4}\right) = 0$$

$$\therefore \left(x + \frac{1}{2}\right)^2 - \left(\frac{81}{4}\right) = 0$$

$$\therefore \left(x + \frac{1}{2}\right)^2 = \frac{81}{4}$$

Taking square root of both sides, we get

$$x + \frac{1}{2} = \pm \frac{9}{2}$$

$$\therefore x + \frac{1}{2} = \frac{9}{2} \quad \text{or} \quad x + \frac{1}{2} = -\frac{9}{2}$$

$$\therefore x = \frac{9}{2} - \frac{1}{2} \quad \text{or} \quad x = -\frac{9}{2} - \frac{1}{2}$$

$$\therefore x = \frac{8}{2} = 4 \quad \text{or} \quad x = -\frac{10}{2} = -5$$

\therefore The roots of the given quadratic equation are 4 and -5.

$$2. x^2 + 2x - 5 = 0$$

If $x^2 + 2x + k = (x + a)^2$, then

$$x^2 + 2x + k = x^2 + 2ax + a^2$$

Comparing the coefficients, we get

$$2 = 2a \text{ and } k = a^2$$

$$\therefore a = 1 \text{ and } k = (1)^2 = 1$$

$$\text{Now, } x^2 + 2x - 5 = 0$$

$$\therefore x^2 + 2x + 1 - 1 - 5 = 0$$

$$\therefore (x + 1)^2 - 6 = 0$$

$$\therefore (x + 1)^2 = 6$$

Taking square root of both sides, we get

$$x + 1 = \pm \sqrt{6}$$

$$\therefore x + 1 = \sqrt{6} \text{ or } x + 1 = -\sqrt{6}$$

$$\therefore x = \sqrt{6} - 1 \text{ or } x = -\sqrt{6} - 1$$

\therefore The roots of the given quadratic equation are $\sqrt{6} - 1$ and $-\sqrt{6} - 1$.

$$3. m^2 - 5m = -3$$

$$\therefore m^2 - 5m + 3 = 0$$

If $m^2 - 5m + k = (m + a)^2$, then

$$m^2 - 5m + k = m^2 + 2am + a^2$$

Comparing the coefficients, we get

$$-5 = 2a \text{ and } k = a^2$$

$$\therefore a = \frac{-5}{2} \text{ and } k = \left(\frac{-5}{2}\right)^2 = \frac{25}{4}$$

$$\text{Now, } m^2 - 5m + 3 = 0$$

$$\therefore m^2 - 5m + \frac{25}{4} - \frac{25}{4} + 3 = 0$$

$$\therefore \left(m - \frac{5}{2}\right)^2 + \left(\frac{-25 + 12}{4}\right) = 0$$

$$\therefore \left(m - \frac{5}{2}\right)^2 - \frac{13}{4} = 0$$

$$\therefore \left(m - \frac{5}{2}\right)^2 = \frac{13}{4}$$

Taking square root of both sides, we get

$$m - \frac{5}{2} = \pm \frac{\sqrt{13}}{2}$$

$$\therefore m - \frac{5}{2} = \frac{\sqrt{13}}{2} \quad \text{or} \quad m - \frac{5}{2} = -\frac{\sqrt{13}}{2}$$

$$\therefore m = \frac{\sqrt{13} + 5}{2} \quad \text{or} \quad m = \frac{-\sqrt{13} + 5}{2}$$

The roots of the given quadratic equation are $\frac{\sqrt{13} + 5}{2}$ and $\frac{-\sqrt{13} + 5}{2}$.

$$4. 9y^2 - 12y + 2 = 0$$

$$\therefore y^2 - \frac{4}{3}y + \frac{2}{9} = 0 \quad \dots [\text{Dividing both sides by 9}]$$

$$\text{If } y^2 - \frac{4}{3}y + k = (y + a)^2, \text{ then}$$

$$y^2 - \frac{4}{3}y + k = y^2 + 2ay + a^2$$

Comparing the coefficients, we get

$$-\frac{4}{3} = 2a \text{ and } k = a^2$$

$$\therefore a = \frac{-2}{3} \text{ and } k = \left(\frac{-2}{3}\right)^2 = \frac{4}{9}$$

$$\text{Now, } y^2 - \frac{4}{3}y + \frac{2}{9} = 0$$

$$\therefore y^2 - \frac{4}{3}y + \frac{4}{9} - \frac{4}{9} + \frac{2}{9} = 0$$

$$\therefore \left(y - \frac{2}{3}\right)^2 - \frac{2}{9} = 0$$

$$\therefore \left(y - \frac{2}{3}\right)^2 = \frac{2}{9}$$

Taking square root of both sides, we get

$$y - \frac{2}{3} = \pm \frac{\sqrt{2}}{3}$$

$$\therefore y - \frac{2}{3} = \frac{\sqrt{2}}{3} \quad \text{or} \quad y - \frac{2}{3} = -\frac{\sqrt{2}}{3}$$

$$\therefore y = \frac{\sqrt{2} + 2}{3} \quad \text{or} \quad y = \frac{-\sqrt{2} + 2}{3}$$

The roots of the given quadratic equation are $\frac{\sqrt{2} + 2}{3}$ and $\frac{-\sqrt{2} + 2}{3}$.

5. $2y^2 + 9y + 10 = 0$

$$\therefore y^2 + \frac{9}{2}y + 5 = 0 \text{ ...[Dividing both sides by 2]}$$

If $y^2 + \frac{9}{2}y + k = (y + a)^2$, then

$$y^2 + \frac{9}{2}y + k = y^2 + 2ay + a^2$$

Comparing the coefficients, we get

$$\frac{9}{2} = 2a \text{ and } k = a^2$$

$$\therefore a = \frac{9}{4} \text{ and } k = \left(\frac{9}{4}\right)^2 = \frac{81}{16}$$

Now, $y^2 + \frac{9}{2}y + 5 = 0$

$$\therefore y^2 + \frac{9}{2}y + \frac{81}{16} - \frac{81}{16} + 5 = 0$$

$$\therefore \left(y + \frac{9}{4}\right)^2 + \left(\frac{-81 + 80}{16}\right) = 0$$

$$\therefore \left(y + \frac{9}{4}\right)^2 - \frac{1}{16} = 0$$

$$\therefore \left(y + \frac{9}{4}\right)^2 = \frac{1}{16}$$

Taking square root of both sides, we get

$$y + \frac{9}{4} = \pm \frac{1}{4}$$

$$\therefore y + \frac{9}{4} = \frac{1}{4} \quad \text{or} \quad y + \frac{9}{4} = -\frac{1}{4}$$

$$\therefore y = \frac{1}{4} - \frac{9}{4} \quad \text{or} \quad y = -\frac{1}{4} - \frac{9}{4}$$

$$\therefore y = \frac{-8}{4} = -2 \quad \text{or} \quad y = -\frac{10}{4} = \frac{-5}{2}$$

\therefore The roots of the given quadratic equation are -2 and $-\frac{5}{2}$.

6. $5x^2 = 4x + 7$

$$\therefore 5x^2 - 4x - 7 = 0$$

$$\therefore x^2 - \frac{4}{5}x - \frac{7}{5} = 0 \text{ ...[Dividing both sides by 5]}$$

If $x^2 - \frac{4}{5}x + k = (x + a)^2$, then

$$x^2 - \frac{4}{5}x + k = x^2 + 2ax + a^2$$

Comparing the coefficients, we get

$$-\frac{4}{5} = 2a \text{ and } k = a^2$$

$$\therefore a = -\frac{2}{5} \text{ and } k = \left(-\frac{2}{5}\right)^2 = \frac{4}{25}$$

$$\text{Now, } x^2 - \frac{4}{5}x - \frac{7}{5} = 0$$

$$\therefore x^2 - \frac{4}{5}x + \frac{4}{25} - \frac{4}{25} - \frac{7}{5} = 0$$

$$\therefore \left(x - \frac{2}{5}\right)^2 - \left(\frac{4+35}{25}\right) = 0$$

$$\therefore \left(x - \frac{2}{5}\right)^2 - \frac{39}{25} = 0$$

$$\therefore \left(x - \frac{2}{5}\right)^2 = \frac{39}{25}$$

Taking square root of both sides, we get

$$x - \frac{2}{5} = \pm \frac{\sqrt{39}}{5}$$

$$\therefore x - \frac{2}{5} = \frac{\sqrt{39}}{5} \quad \text{or} \quad x - \frac{2}{5} = -\frac{\sqrt{39}}{5}$$

$$\therefore x = \frac{2 + \sqrt{39}}{5} \quad \text{or} \quad x = \frac{2 - \sqrt{39}}{5}$$

$$\therefore \text{The roots of the given quadratic equation are } \frac{2 + \sqrt{39}}{5} \text{ and } \frac{2 - \sqrt{39}}{5}.$$

Practice Set 2.4 Algebra 10th Std Maths Part 1 Answers Chapter 2 Quadratic Equations

Question 1.

Compare the given quadratic equations to the general form and write values of a, b, c.

i. $x^2 - 7x + 5 = 0$

ii. $2m^2 = 5m - 5$

iii. $y^2 = 7y$

Solution:

i. $x^2 - 7x + 5 = 0$

Comparing the above equation with

$$ax^2 + bx + c = 0, \text{ we get}$$

$$a = 1, b = -7, c = 5$$

ii. $2m^2 = 5m - 5$

$$\therefore 2m^2 - 5m + 5 = 0$$

Comparing the above equation with

$$am^2 + bm + c = 0, \text{ we get}$$

$$a = 2, b = -5, c = 5$$

iii. $y^2 = 7y$

$$\therefore y^2 - 7y + 0 = 0$$

Comparing the above equation with

$$ay^2 + by + c = 0, \text{ we get}$$

$$a = 1, b = -7, c = 0$$

Question 2.

Solve using formula.

i. $x^2 + 6x + 5 = 0$

ii. $x^2 - 3x - 2 = 0$

iii. $3m^2 + 2m - 7 = 0$

iv. $5m^2 - 4m - 2 = 0$

v. $y^2 + 13y = 2$

vi. $5x^2 + 13x + 8 = 0$

Solution:

i. $x^2 + 6x + 5 = 0$

Comparing the above equation with

$ax^2 + bx + c = 0$, we get

$a = 1, b = 6, c = 5$

$\therefore b^2 - 4ac = (6)^2 - 4 \times 1 \times 5$

$= 36 - 20 = 16$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{16}}{2(1)}$$

$$= \frac{-6 \pm 4}{2} = \frac{2(-3 \pm 2)}{2}$$

$$\therefore x = -3 \pm 2$$

$\therefore x = -3 + 2$ or $x = -3 - 2$

$\therefore x = -1$ or $x = -5$

\therefore The roots of the given quadratic equation are -1 and -5.

ii. $x^2 - 3x - 2 = 0$

Comparing the above equation with

$ax^2 + bx + c = 0$, we get

$a = 1, b = -3, c = -2$

$\therefore b^2 - 4ac = (-3)^2 - 4 \times 1 \times (-2)$

$= 9 + 8 = 17$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{17}}{2(1)}$$

$$\therefore x = \frac{3 \pm \sqrt{17}}{2}$$

$$\therefore x = \frac{3 + \sqrt{17}}{2} \text{ or } x = \frac{3 - \sqrt{17}}{2}$$

\therefore **The roots of the given quadratic equation are $\frac{3 + \sqrt{17}}{2}$ and $\frac{3 - \sqrt{17}}{2}$.**

iii. $3m^2 + 2m - 7 = 0$

Comparing the above equation with

$am^2 + bm + c = 0$, we get

$a = 3, b = 2, c = -7$

$\therefore b^2 - 4ac = (2)^2 - 4 \times 3 \times (-7)$

$= 4 + 84 = 88$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{88}}{2(3)}$$

$$= \frac{-2 \pm \sqrt{4 \times 22}}{6}$$

$$= \frac{-2 \pm 2\sqrt{22}}{6}$$

$$= \frac{2(-1 \pm \sqrt{22})}{6}$$

$$\therefore m = \frac{-1 \pm \sqrt{22}}{3}$$

$$\therefore m = \frac{-1 + \sqrt{22}}{3} \text{ or } m = \frac{-1 - \sqrt{22}}{3}$$

\therefore **The roots of the given quadratic equation are $\frac{-1 + \sqrt{22}}{3}$ and $\frac{-1 - \sqrt{22}}{3}$.**

iv. $5m^2 - 4m - 2 = 0$

Comparing the above equation with

$am^2 + bm + c = 0$, we get

$a = 5, b = -4, c = -2$

$$\therefore b^2 - 4ac = (-4)^2 - 4 \times 5 \times (-2)$$

$$= 16 + 40 = 56$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{56}}{2(5)}$$

$$= \frac{4 \pm \sqrt{4 \times 14}}{10}$$

$$= \frac{4 \pm 2\sqrt{14}}{10}$$

$$= \frac{2(2 \pm \sqrt{14})}{10}$$

$$\therefore m = \frac{2 \pm \sqrt{14}}{5}$$

$$\therefore m = \frac{2 + \sqrt{14}}{5} \quad \text{or} \quad m = \frac{2 - \sqrt{14}}{5}$$

\therefore **The roots of the given quadratic equation are $\frac{2 + \sqrt{14}}{5}$ and $\frac{2 - \sqrt{14}}{5}$.**

$$v. y^2 + 13y = 2$$

$$\therefore 3y^2 + y = 6 \text{ ... (Multiplying both sides by 3)}$$

$$\therefore 3y^2 + y - 6 = 0$$

Comparing the above equation with

$ay^2 + by + c = 0$, we get

$a = 3, b = 1, c = -6$

$$\therefore b^2 - 4ac = (1)^2 - 4 \times 3 \times (-6)$$

$$= 1 + 72 = 73$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{73}}{2(3)}$$

$$\therefore y = \frac{-1 \pm \sqrt{73}}{6}$$

$$\therefore y = \frac{-1 + \sqrt{73}}{6} \quad \text{or} \quad y = \frac{-1 - \sqrt{73}}{6}$$

\therefore **The roots of the given quadratic equation are $\frac{-1 + \sqrt{73}}{6}$ and $\frac{-1 - \sqrt{73}}{6}$.**

$$vi. 5x^2 + 13x + 8 = 0$$

Comparing the above equation with

$ax^2 + bx + c = 0$, we get

$a = 5, b = 13, c = 8$

$$\therefore b^2 - 4ac = (13)^2 - 4 \times 5 \times 8$$

$$= 169 - 160 = 9$$

$$\therefore b^2 - 4ac = (13)^2 - 4 \times 5 \times 8$$

$$= 169 - 160 = 9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-13 \pm \sqrt{9}}{2(5)}$$

$$= \frac{-13 \pm 3}{10}$$

$$\therefore x = \frac{-13 + 3}{10} \quad \text{or} \quad x = \frac{-13 - 3}{10}$$

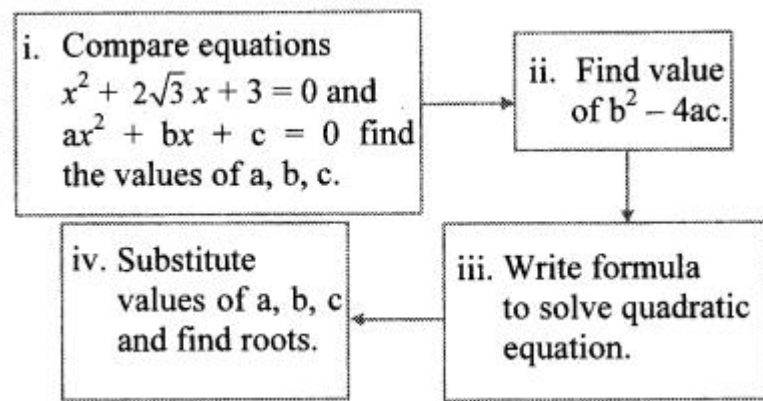
$$\therefore x = \frac{-10}{10} \quad \text{or} \quad x = \frac{-16}{10}$$

$$\therefore x = -1 \quad \text{or} \quad x = \frac{-8}{5}$$

The roots of the given quadratic equation are -1 and $-\frac{8}{5}$.

Question 3.

With the help of the flow chart given below solve the equation $x^2 + 2\sqrt{3}x + 3 = 0$ using the formula.



Solution:

i. $x^2 + 2\sqrt{3}x + 3 = 0$

Comparing the above equation with

$ax^2 + bx + c = 0$, we get

$a = 1, b = 2\sqrt{3}, c = 3$

ii. $b^2 - 4ac = (2\sqrt{3})^2 - 4 \times 1 \times 3$
 $= 12 - 12$
 $= 0$

iii. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

iv. $x = \frac{-2\sqrt{3} \pm 0}{2(1)}$

$\therefore x = \frac{-2\sqrt{3} + 0}{2}$ or $x = \frac{-2\sqrt{3} - 0}{2}$

$\therefore x = -\sqrt{3}$ or $x = -\sqrt{3}$

\therefore The roots of the given quadratic equation are $-\sqrt{3}$ and $-\sqrt{3}$.

Question 1.

Solve the equation $2x^2 + 13x + 15 = 0$ by factorisation method, by completing the square method and by using the formula. Verify that you will get the same roots every time. (Textbook pg. no. 43)

Solution:

i. Factorisation method:

$2x^2 + 13x + 15 = 0$

$\therefore 2x^2 + 10x + 3x + 15 = 0$

$\therefore 2x(x + 5) + 3(x + 5) = 0$

$\therefore (x + 5)(2x + 3) = 0$

30
10 3
10 × 3 = 10
10 + 3 = 13

By using the property, if the product of two numbers is zero, then at least zero, we get

$\therefore x + 5 = 0$ or $2x + 3 = 0$

$\therefore x + -5 = 0$ or $2x = -3 = 0$

$\therefore x + -5 =$ or $x = -\frac{3}{2}$

\therefore The roots of the given quadratic equation are $-\frac{3}{2}$ and -5 .

ii. Completing the square method:

$2x^2 + 13x + 15 = 0$

$\therefore x^2 + \frac{13}{2}x + \frac{15}{2} = 0$...[Dividing both sides by 2]

If $x^2 + \frac{13}{2}x + k = (x + a)^2$, then

$x^2 + \frac{13}{2}x + k = x^2 + 2ax + a^2$

Comparing the coefficients, we get

$\frac{13}{2} = 2a$ and $k = a^2$

$\therefore a = \frac{13}{4}$ and $k = \left(\frac{13}{4}\right)^2 = \frac{169}{16}$

Now, $x^2 + \frac{13}{2}x + \frac{15}{2} = 0$

$$\therefore x^2 + \frac{13}{2}x + \frac{169}{16} - \frac{169}{16} + \frac{15}{2} = 0$$

$$\therefore \left(x + \frac{13}{4}\right)^2 + \left(\frac{-169 + 120}{16}\right) = 0$$

$$\therefore \left(x + \frac{13}{4}\right)^2 - \frac{49}{16} = 0$$

$$\therefore \left(x + \frac{13}{4}\right)^2 = \frac{49}{16}$$

Taking square root of both sides, we get

$$x + \frac{13}{4} = \pm \frac{7}{4}$$

$$\therefore x + \frac{13}{4} = \frac{7}{4} \quad \text{or} \quad x + \frac{13}{4} = -\frac{7}{4}$$

$$\therefore x = \frac{7}{4} - \frac{13}{4} \quad \text{or} \quad x = -\frac{7}{4} - \frac{13}{4}$$

$$\therefore x = \frac{7-13}{4} \quad \text{or} \quad x = \frac{-7-13}{4}$$

$$\therefore x = \frac{-6}{4} \quad \text{or} \quad x = \frac{-20}{4}$$

$$\therefore x = \frac{-3}{2} \quad \text{or} \quad x = -5$$

\therefore The roots of the given quadratic equation are $-\frac{3}{2}$ and -5 .

iii. Formula method:

$$2x^2 + 13x + 15 = 0$$

Comparing the above equation with

$$ax^2 + bx + c = 0, \text{ we get}$$

$$a = 2, b = 13, c = 15$$

$$\therefore b^2 - 4ac = (13)^2 - 4 \times 2 \times 15$$

$$= 169 - 120 = 49$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-13 \pm \sqrt{49}}{2(2)} \\ &= \frac{-13 \pm 7}{4} \end{aligned}$$

$$\therefore x = \frac{-13+7}{4} \quad \text{or} \quad x = \frac{-13-7}{4}$$

$$\therefore x = \frac{-6}{4} \quad \text{or} \quad x = \frac{-20}{4}$$

$$\therefore x = \frac{-3}{2} \quad \text{or} \quad x = -5$$

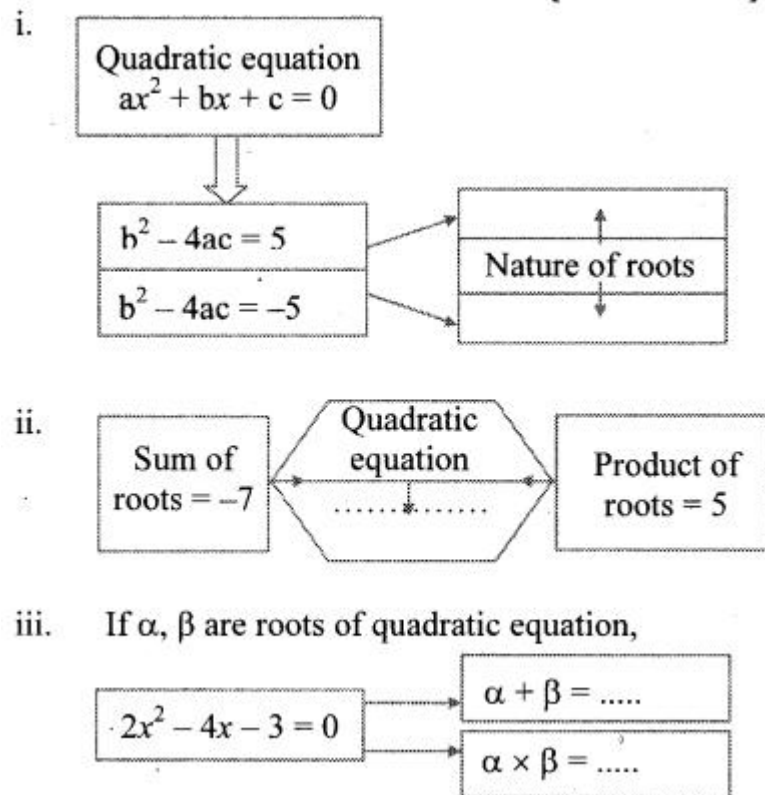
\therefore The roots of the given quadratic equation are $-\frac{3}{2}$ and -5 .

\therefore By all the above three methods, we get the same roots of the given quadratic equation.

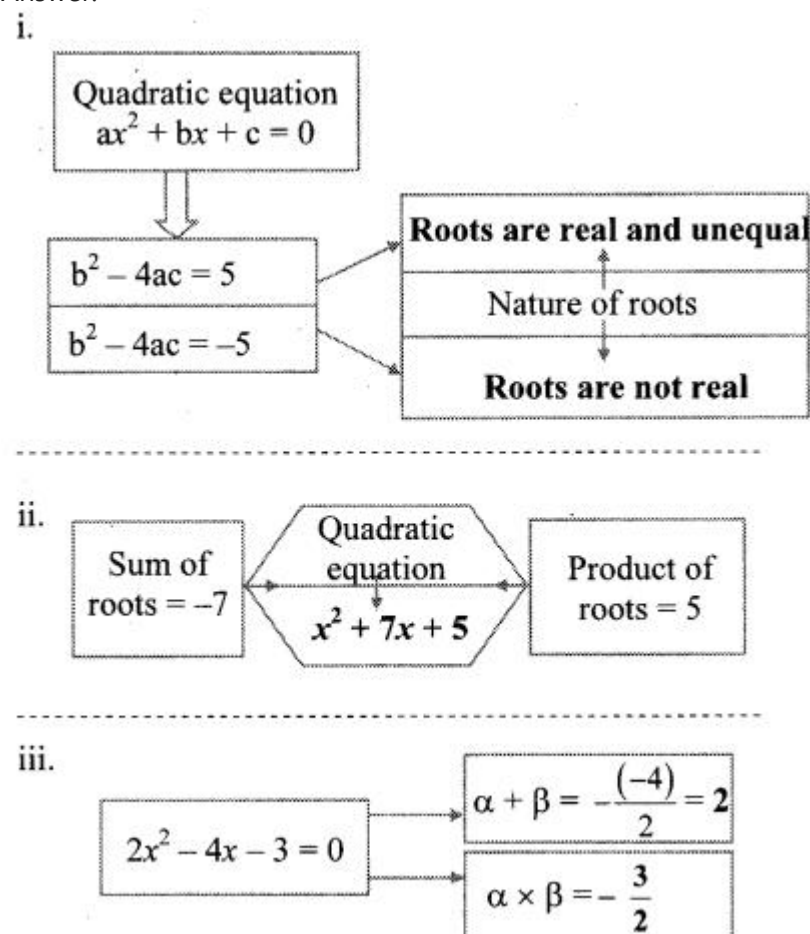
Practice Set 2.5 Algebra 10th Std Maths Part 1 Answers Chapter 2 Quadratic Equations

Question 1.

Fill in the gaps and complete.



Answer:



Question 2.

Find the value of discriminant.

- i. $x^2 + 7x - 1 = 0$
ii. $2y^2 - 5y + 10 = 0$
iii. $\sqrt{2}x^2 + 4x + 2\sqrt{2} = 0$

Solution:

i. $x^2 + 7x - 1 = 0$

Comparing the above equation with
 $ax^2 + bx + c = 0$, we get

$a = 1, b = 7, c = -1$

$\therefore b^2 - 4ac = (7)^2 - 4 \times 1 \times (-1)$
 $= 49 + 4$

$\therefore b^2 - 4ac = 53$

ii. $2y^2 - 5y + 10 = 0$

Comparing the above equation with
 $ay^2 + by + c = 0$, we get

$a = 2, b = -5, c = 10$

$\therefore b^2 - 4ac = (-5)^2 - 4 \times 2 \times 10$
 $= 25 - 80$

$\therefore b^2 - 4ac = -55$

iii. $\sqrt{2}x^2 + 4x + 2\sqrt{2} = 0$

Comparing the above equation with
 $ax + bx + c = 0$, we get

$$a = \sqrt{2}, b = 4, c = 2\sqrt{2}$$

$$\therefore b^2 - 4ac = (4)^2 - 4 \times \sqrt{2} \times 2\sqrt{2}$$

$$= 16 - 16$$

$$\therefore b^2 - 4ac = 0$$

Question 3.

Determine the nature of roots of the following quadratic equations.

i. $x^2 - 4x + 4 = 0$

ii. $2y^2 - 7y + 2 = 0$

iii. $m^2 + 2m + 9 = 0$

Solution:

i. $x^2 - 4x + 4 = 0$

Comparing the above equation with

$$ax^2 + bx + c = 0, \text{ we get}$$

$$a = 1, b = -4, c = 4$$

$$\therefore \Delta = b^2 - 4ac$$

$$= (-4)^2 - 4 \times 1 \times 4$$

$$= 16 - 16$$

$$\therefore \Delta = 0$$

\therefore Roots of the given quadratic equation are real and equal.

ii. $2y^2 - 7y + 2 = 0$

Comparing the above equation with

$$ay^2 + by + c = 0, \text{ we get}$$

$$a = 2, b = -7, c = 2$$

$$\therefore \Delta = b^2 - 4ac$$

$$= (-7)^2 - 4 \times 2 \times 2$$

$$= 49 - 16$$

$$\therefore \Delta = 33$$

$$\therefore \Delta > 0$$

\therefore Roots of the given quadratic equation are real and unequal.

iii. $m^2 + 2m + 9 = 0$

Comparing the above equation with

$$am^2 + bm + c = 0, \text{ we get}$$

$$a = 1, b = 2, c = 9$$

$$\therefore \Delta = b^2 - 4ac$$

$$= (2)^2 - 4 \times 1 \times 9$$

$$= 4 - 36$$

$$\therefore \Delta = -32$$

$$\therefore \Delta < 0$$

\therefore Roots of the given quadratic equation are not real.

Question 4.

Form the quadratic equation from the roots given below.

i. 0 and 4

ii. 3 and -10

iii. $\frac{1}{2}, \frac{1}{2}$

iv. $2 - \sqrt{5}, 2 + \sqrt{5}$

Solution:

i. Let $\alpha = 0$ and $\beta = 4$

$$\therefore \alpha + \beta = 0 + 4 = 4$$

$$\text{and } \alpha \times \beta = 0 \times 4 = 0$$

\therefore The required quadratic equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\therefore x^2 - 4x + 0 = 0$$

$$\therefore x^2 - 4x = 0$$

ii. Let $\alpha = 3$ and $\beta = -10$

$$\therefore \alpha + \beta = 3 - 10 = -7$$

$$\text{and } \alpha \times \beta = 3 \times -10 = -30$$

\therefore The required quadratic equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\therefore x^2 - (-7)x + (-30) = 0$$

iii. Let $\alpha = \frac{1}{2}$ and $\beta = -\frac{1}{2}$

$\therefore \alpha + \beta = \frac{1}{2} - \frac{1}{2} = 0$

and $\alpha \times \beta = \frac{1}{2} \times -\frac{1}{2} = -\frac{1}{4}$

\therefore The required quadratic equation is
 $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore x^2 - 0x + \left(-\frac{1}{4}\right) = 0$

$\therefore x^2 - \frac{1}{4} = 0$

$\therefore 4x^2 - 1 = 0$

iv. Let $\alpha = 2 - \sqrt{5}$ and $\beta = 2 + \sqrt{5}$

$\therefore \alpha + \beta = 2 - \sqrt{5} + 2 + \sqrt{5} = 4$

and $\alpha \times \beta = (2 - \sqrt{5})(2 + \sqrt{5})$

$= (2)^2 - (\sqrt{5})^2$

$\dots[\because (a + b)(a - b) = a^2 - b^2]$

$= 4 - 5 = -1$

\therefore The required quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore x^2 - 4x - 1 = 0$

Question 5.

Sum of the roots of a quadratic equation is double their product. Find k if equation is $x^2 - 4kx + k + 3 = 0$.

Solution:

$x^2 - 4kx + k + 3 = 0$

Comparing the above equation with

$ax^2 + bx + c = 0$, we get

$a = 1, b = -4k, c = k + 3$

Let α and β be the roots of the given quadratic equation.

Then, $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

According to the given condition,

$\alpha + \beta = 2\alpha\beta$

$\therefore \frac{-b}{a} = \frac{2c}{a}$

$\therefore -b = 2c$

$\therefore -(-4k) = 2(k + 3)$

$\therefore 4k = 2k + 6$

$\therefore 4k - 2k = 6$

$\therefore 2k = 6$

$\therefore k = \frac{6}{2}$

$\therefore k = 3$

Question 6.

α, β are roots of $y^2 - 2y - 7 = 0$ find,

i. $\alpha^2 + \beta^2$

ii. $\alpha^3 + \beta^3$

Solution:

$y^2 - 2y - 7 = 0$

Comparing the above equation with

$ay^2 + by + c = 0$, we get

$a = 1, b = -2, c = -7$

$$\therefore \alpha + \beta = -\frac{b}{a} = -\frac{(-2)}{1} = 2$$

$$\alpha\beta = \frac{c}{a} = -\frac{7}{1} = -7$$

i. $(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$$\therefore \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (2)^2 - 2(-7) = 4 + 14$$

$$\therefore \alpha^2 + \beta^2 = 18$$

ii. $(\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta)$

$$\therefore \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$= (2)^3 - 3(-7)(2) = 8 + 42$$

$$\therefore \alpha^3 + \beta^3 = 50$$

Question 7.

The roots of each of the following quadratic equations are real and equal, find k.

i. $3y^2 + ky + 12 = 0$

ii. $kx(x-2) + 6 = 0$

Solution:

i. $3y^2 + ky + 12 = 0$

Comparing the above equation with

$$ay^2 + by + c = 0, \text{ we get}$$

$$a = 3, b = k, c = 12$$

$$\therefore \Delta = b^2 - 4ac$$

$$= (k)^2 - 4 \times 3 \times 12$$

$$= k^2 - 144 = k^2 - (12)^2$$

$$\therefore \Delta = (k + 12)(k - 12) \dots [\because a^2 - b^2 = (a + b)(a - b)]$$

Since, the roots are real and equal.

$$\therefore \Delta = 0$$

$$\therefore (k + 12)(k - 12) = 0$$

$$\therefore k + 12 = 0 \text{ or } k - 12 = 0$$

$$\therefore k = -12 \text{ or } k = 12$$

ii. $kx(x-2) + 6 = 0$

$$\therefore kx^2 - 2kx + 6 = 0$$

Comparing the above equation with

$$ax^2 + bx + c = 0, \text{ we get}$$

$$a = k, b = -2k, c = 6$$

$$\therefore \Delta = b^2 - 4ac$$

$$= (-2k)^2 - 4 \times k \times 6$$

$$= 4k^2 - 24k$$

$$\therefore \Delta = 4k(k - 6)$$

Since, the roots are real and equal.

$$\therefore \Delta = 0$$

$$\therefore 4k(k - 6) = 0$$

$$\therefore k(k - 6) = 0$$

$$\therefore k = 0 \text{ or } k - 6 = 0$$

But, if $k = 0$ then quadratic coefficient becomes zero.

$$\therefore k \neq 0$$

$$\therefore k = 6$$

Question 1.

Fill in the blanks. (Textbook pg. no. 44)

Solution:

Value of discriminant	Nature of roots
50	Real and unequal
-30	Not real
0	Real and equal

Question 2.

Determine nature of roots of the quadratic equation: $x^2 + 2x - 9 = 0$ (Textbook pg. no. 45)

Solution:

Comparing $x^2 + 2x - 9 = 0$ with
 $ax^2 + bx + c = 0$, we get

$$a = \boxed{1}, b = 2, c = \boxed{-9}$$

$$\therefore b^2 - 4ac = 2^2 - 4 \times \boxed{1} \times \boxed{-9}$$

$$\therefore \Delta = 4 - \boxed{-36}$$

$$\therefore \Delta = 4 + 36$$

$$\therefore \Delta = 40$$

$$\therefore b^2 - 4ac > 0$$

\therefore The roots of the given equation are real and unequal.

Question 3.

Fill in the empty boxes properly. (Textbook pg. no. 46)

Solution:

$$10x^2 + 10x + 1 = 0$$

Comparing the above equation with

$$ax^2 + bx + c = 0, \text{ we get}$$

$$a = 10, b = 10, c = 1$$

$$\therefore \alpha + \beta = \frac{-b}{a} = \frac{-10}{10} = -1$$

$$\text{and } \alpha \times \beta = \frac{c}{a} = \frac{1}{10}$$

Question 4.

Write the quadratic equation if addition of the roots is 10 and product of the roots is 9. (Textbook pg, no. 48)

Answer:

$$\text{Quadratic equation: } x^2 - \boxed{10}x + \boxed{9} = \boxed{0}$$

Question 5.

What will be the quadratic equation if $\alpha = 2, \beta = 5$. (Textbook pg. no, 48)

Solution:

$$x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$$

$$\therefore x^2 - (\boxed{2} + \boxed{5})x + \boxed{2} \times \boxed{5} = 0$$

$$\therefore \boxed{1}x^2 - \boxed{7}x + \boxed{10} = 0$$

Practice Set 2.6 Algebra 10th Std Maths Part 1 Answers Chapter 2 Quadratic Equations

Question 1.

Product of Pragati's age 2 years ago and years hence is 84. Find her present age.

Solution:

Let the present age of Pragati be x years.

\therefore 2 years ago,

Age of Pragati = $(x - 2)$ years

After 3 years,

Age of Pragati = $(x + 3)$ years

According to the given condition,

$$(x - 2)(x + 3) = 84$$

$$\therefore x(x + 3) - 2(x + 3) = 84$$

$$\therefore x^2 + 3x - 2x - 6 = 84$$

$$\therefore x^2 + x - 6 - 84 = 0$$

$$\therefore x^2 + x - 90 = 0$$

$$x^2 + 10x - 9x - 90 = 0$$

$$\therefore x(x + 10) - 9(x + 10) = 0$$

$$\therefore (x + 10)(x - 9) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore x + 10 = 0 \text{ or } x - 9 = 0$$

$$\therefore x = -10 \text{ or } x = 9$$

But, age cannot be negative.

$$\therefore x = 9$$

\therefore Present age of Pragati is 9 years.

Question 2.

The sum of squares of two consecutive even natural numbers is 244; find the numbers.

Solution:

Let the first even natural number be x .

\therefore the next consecutive even natural number will be $(x + 2)$.

According to the given condition,

$$x^2 + (x + 2)^2 = 244$$

$$\therefore x^2 + x^2 + 4x + 4 = 244$$

$$\therefore 2x^2 + 4x + 4 - 244 = 0$$

$$\therefore 2x^2 + 4x - 240 = 0$$

$$\therefore x^2 + 2x - 120 = 0 \text{ ... [Dividing both sides by 2]}$$

$$\therefore x^2 + 12x - 10x - 120 = 0$$

$$\therefore x(x + 12) - 10(x + 12) = 0$$

$$\therefore (x + 12)(x - 10) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore x + 12 = 0 \text{ or } x - 10 = 0$$

$$\therefore x = -12 \text{ or } x = 10$$

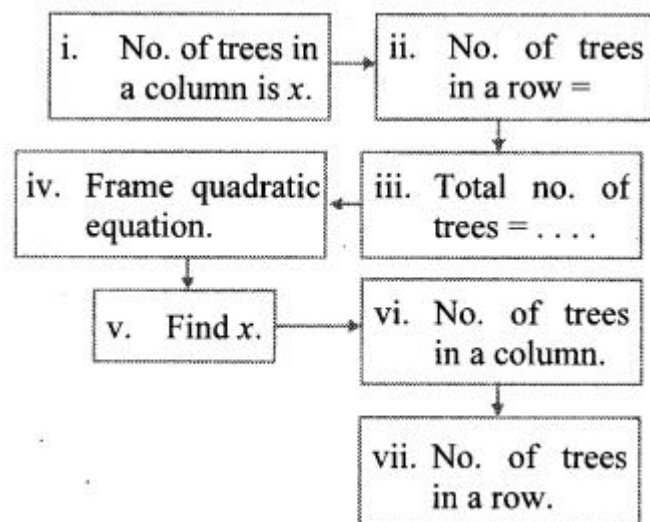
But, natural number cannot be negative.

$$\therefore x = 10 \text{ and } x + 2 = 10 + 2 = 12$$

\therefore The two consecutive even natural numbers are 10 and 12.

Question 3.

In the orange garden of Mr. Madhusudan there are 150 orange trees. The number of trees in each row is 5 more than that in each column. Find the number of trees in each row and each column with the help of following flow chart.



Solution:

i. Number of trees in a column is x .

ii. Number of trees in a row = $x + 5$

iii. Total number of trees = $x \times (x + 5)$

iv. According to the given condition,

$$x(x + 5) = 150$$

$$\therefore x^2 + 5x = 150$$

$$\therefore x^2 + 5x - 150 = 0$$

$$\therefore x^2 + 15x - 10x - 150 = 0$$

$$\therefore x(x + 15) - 10(x + 15) = 0$$

$$\therefore (x + 15)(x - 10) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore x + 15 = 0 \text{ or } x - 10 = 0$$

$$\therefore x = -15 \text{ or } x = 10$$

But, number of trees cannot be negative.

$$\therefore x = 10$$

vi. Number of trees in a column is 10.

vii. Number of trees in a row = $x + 5 = 10 + 5 = 15$

\therefore Number of trees in a row is 15.

Question 4.

Vivek is older than Kishor by 5 years. The Find their present ages is 16 Find their Present ages

Solution:

Let the present age of Kishor be x .

\therefore Present age of Vivek = $(x + 5)$ years

According to the given condition,

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$$

$$\therefore \frac{x+5+x}{x(x+5)} = \frac{1}{6}$$

$$\therefore \frac{2x+5}{x(x+5)} = \frac{1}{6}$$

$$\therefore 6(2x + 5) = x(x + 5)$$

$$\therefore 12x + 30 = x^2 + 5x$$

$$\therefore x^2 + 5x - 12x - 30 = 0$$

$$\therefore x^2 - 7x - 30 = 0$$

$$\therefore x^2 - 10x + 3x - 30 = 0$$

$$\therefore x(x - 10) + 3(x - 10) = 0$$

$$\therefore (x - 10)(x + 3) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore x - 10 = 0 \text{ or } x + 3 = 0$$

$$\therefore x = 10 \text{ or } x = -3$$

But, age cannot be negative.

$$\therefore x = 10 \text{ and } x + 5 = 10 + 5 = 15$$

\therefore Present ages of Kishor and Vivek are 10 years and 15 years respectively.

Question 5.

Suyash scored 10 marks more in second test than that in the first. 5 times the score of the second test is the same as square of the score in the first test. Find his score in the first test.

Solution:

Let the score of Suyash in the first test be x .

\therefore Score in the second test = $x + 10$ According to the given condition,

$$5(x + 10) = x^2$$

$$\therefore 5x + 50 = x^2$$

$$\therefore x^2 - 5x - 50 = 0$$

$$\therefore x^2 - 10x + 5x - 50 = 0$$

$$\therefore x(x - 10) + 5(x - 10) = 0$$

$$\therefore (x - 10)(x + 5) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore x - 10 = 0 \text{ or } x + 5 = 0$$

$$\therefore x = 10 \text{ or } x = -5$$

But, score cannot be negative.

$$\therefore x = 10$$

\therefore The score of Suyash in the first test is 10.

Question 6.

'Mr. Kasam runs a small business of making earthen pots. He makes certain number of pots on daily basis. Production cost of each pot is ₹ 40 more than 10 times total number of pots, he makes in one day. If production cost of all pots per day is ₹ 600, find production cost of one pot and number of pots he makes per day.

Solution:

Let Mr. Kasam make x number of pots on daily basis.

Production cost of each pot = ₹ $(10x + 40)$

According to the given condition,

$$x(10x + 40) = 600$$

$$\therefore 10x^2 + 40x = 600$$

$$\therefore 10x^2 + 40x - 600 = 0$$

$$\therefore x^2 + 4x - 60 = 0 \text{ ...[Dividing both sides by 10]}$$

$$\therefore x^2 + 10x - 6x - 60 = 0$$

$$\therefore x(x + 10) - 6(x + 10) = 0$$

$$\therefore (x + 10)(x - 6) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore x + 10 = 0 \text{ or } x - 6 = 0$$

$$\therefore x = -10 \text{ or } x = 6$$

But, number of pots cannot be negative.

$$\therefore x = 6$$

$$\therefore \text{Production cost of each pot} = 7(10x + 40)$$

$$= ₹ [(10 \times 6) + 40]$$

$$= ₹(60 + 40) = ₹ 100$$

Production cost of one pot is ₹ 100 and the number of pots Mr. Kasam makes per day is 6.

Question 7.

Pratik takes 8 hours to travel 36 km downstream and return to the same spot. The speed of boat in still water is 12 km. per hour. Find the speed of water current.

Solution:

Let the speed of water current be x km/hr. Speed of boat is 12 km/hr. ($x < 12$)

In upstream, speed of the water current decreases the speed of the boat and it is the opposite in downstream.

\therefore speed of the boat in upstream = $(12 - x)$ km/hr and speed of the boat in downstream = $(12 + x)$ km/hr.

$$\text{Now, Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\begin{aligned} \text{Time required to cover 36 km upstream} \\ = \frac{36}{12-x} \text{ hrs} \end{aligned}$$

$$\begin{aligned} \text{Time required to cover 36 km downstream} \\ = \frac{36}{12+x} \text{ hrs} \end{aligned}$$

According to the given condition,

$$\frac{36}{12-x} + \frac{36}{12+x} = 8$$

$$\therefore 36 \left(\frac{1}{12-x} + \frac{1}{12+x} \right) = 8$$

$$\therefore \frac{1}{12-x} + \frac{1}{12+x} = \frac{8}{36}$$

$$\therefore \frac{(12+x) + (12-x)}{(12-x)(12+x)} = \frac{2}{9}$$

$$\therefore \frac{24}{144-x^2} = \frac{2}{9} \quad \dots [\because (a+b)(a-b) = a^2 - b^2]$$

$$\therefore 24 \times 9 = 2(144 - x^2)$$

$$\therefore 216 = 288 - 2x^2$$

$$\therefore 2x^2 = 288 - 216$$

$$\therefore 2x^2 = 72$$

$$\therefore x^2 = 36$$

$$\therefore x = \pm 6 \quad \dots [\text{Taking square root of both sides}]$$

But, speed cannot be negative.

$$\therefore x = 6$$

\therefore The speed of water current is 6 km/hr.

Question 8.

Pintu takes 6 days more than those of Nishu to complete certain work. If they work together they finish it in 4 days. How many days would it take to complete the work if they work alone.

Solution:

Let Nishu take x days to complete the work alone.

$$\therefore \text{Total work done by Nishu in 1 day} = \frac{1}{x}$$

Also, Pintu takes $(x + 6)$ days to complete the work alone.

$$\therefore \text{Total work done by Pintu in 1 day} = \frac{1}{x+6}$$

$$\therefore \text{Total work done by both in 1 day} = \left(\frac{1}{x} + \frac{1}{x+6} \right)$$

But, both take 4 days to complete the work together.

$$\therefore \text{Total work done by both in 1 day} = \frac{1}{4}$$

According to the given condition,

$$\frac{1}{x} + \frac{1}{x+6} = \frac{1}{4}$$

$$\therefore \frac{x+6+x}{x(x+6)} = \frac{1}{4}$$

$$\therefore \frac{2x+6}{x(x+6)} = \frac{1}{4}$$

$$\therefore 4(2x + 6) = x(x + 6)$$

$$\therefore 8x + 24 = x^2 + 6x$$

$$\therefore x^2 + 6x - 8x - 24 = 0$$

$$\therefore x^2 - 2x - 24 = 0$$

$$\therefore x^2 - 6x + 4x - 24 = 0$$

$$\therefore x(x - 6) + 4(x - 6) = 0$$

$$\therefore (x - 6)(x + 4) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore x - 6 = 0 \text{ or } x + 4 = 0$$

$$\therefore x = 6 \text{ or } x = -4$$

But, number of days cannot be negative,

$$\therefore x = 6 \text{ and } x + 6 = 6 + 6 = 12$$

\therefore Number of days taken by Nishu and Pintu to complete the work alone is 6 days and 12 days respectively.

Question 9.

If 460 is divided by a natural number, quotient is 6 more than five times the divisor and remainder is 1. Find quotient and divisor.

Solution:

Let the natural number be x .

$$\therefore \text{Divisor} = x, \text{ Quotient} = 5x + 6$$

Also, Dividend = 460 and Remainder = 1

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$\begin{aligned}
 \therefore 460 &= x \times (5x + 6) + 1 \\
 \therefore 460 &= 5x^2 + 6x + 1 \\
 \therefore 5x^2 + 6x + 1 - 460 &= 0 \\
 \therefore 5x^2 + 6x - 459 &= 0 \\
 \therefore 5x^2 - 45x + 51x - 459 &= 0 \\
 \therefore 5x(x - 9) + 51(x - 9) &= 0 \\
 \therefore (x - 9)(5x + 51) &= 0
 \end{aligned}$$

$$\begin{aligned}
 &5 \times -459 \\
 &= 5 \times -9 \times 51 \\
 &= -45, +51
 \end{aligned}$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore x - 9 = 0 \text{ or } 5x + 51 = 0$$

$$\therefore x = 9 \text{ or } x = -51.5$$

But, natural number cannot be negative,

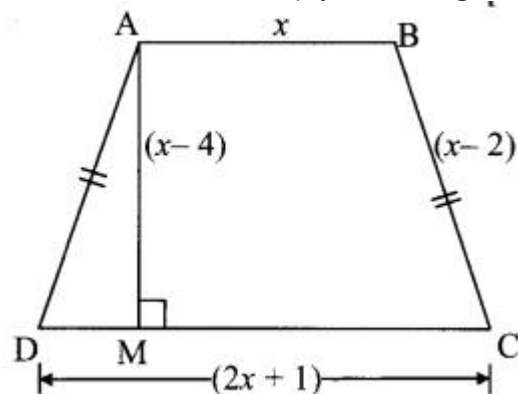
$$\therefore x = 9$$

$$\therefore \text{Quotient} = 5x + 6 = 5(9) + 6 = 45 + 6 = 51$$

$$\therefore \text{Quotient is 51 and Divisor is 9.}$$

Question 10.

In the given fig. □ABCD is a trapezium, AB || CD and its area is 33 cm². From the information given in the figure find the lengths of all sides of the □ABCD. Fill in the empty boxes to get the solution.



Solution:

□ABCD is a trapezium. AB || CD

Area of trapezium

$$= \frac{1}{2} \times (\text{Sum of parallel sides}) \times \text{Height}$$

$$\therefore A(\square ABCD) = \frac{1}{2} \times (AB + CD) \times \boxed{AM}$$

$$\therefore 33 = \frac{1}{2} (x + 2x + 1) \times \boxed{(x - 4)}$$

$$\therefore \boxed{66} = (3x + 1) \times \boxed{(x - 4)}$$

$$\therefore 66 = 3x^2 - 12x + x - 4$$

$$\therefore 3x^2 - \boxed{11x} - \boxed{70} = 0$$

$$\therefore 3x^2 - 21x + 10x - 70 = 0$$

$$\therefore 3x(x - 7) + 10(x - 7) = 0$$

$$\therefore (3x + 10)(x - 7) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore (3x + 10) = 0 \text{ or } \boxed{(x - 7)} = 0$$

$$\therefore x = \frac{-10}{3} \text{ or } x = \boxed{7}$$

But, length is never negative.

$$\therefore x \neq \frac{-10}{3}$$

$$\therefore x = \boxed{7}$$

$$AB = \boxed{x = 7 \text{ cm}}$$

$$CD = \boxed{2x + 1 = 2(7) + 1 = 15 \text{ cm}}$$

$$AD = BC = \boxed{x - 2 = 7 - 2 = 5 \text{ cm}}$$

Problem Set 2 Algebra 10th Std Maths Part 1 Answers Chapter 2 Quadratic Equations

Question 1.

Choose the correct answers for the following questions.

i. Which one is the quadratic equation?

- (A) $\frac{5}{x} - 3 = x^2$
 (B) $x(x + 5) = 2$
 (C) $n - 1 = 2n$
 (D) $\frac{1}{x^2}(x + 2) = x$

Answer:

(B)

ii. Out of the following equations which one is not a quadratic equation?

- (A) $x^2 + 4x = 11 + x^2$
 (B) $x = 4x$
 (C) $5x^2 = 90$
 (D) $2x - x^2 = x^2 + 5$

Answer:

(A)

iii. The roots of $x^2 + kx + k = 0$ are real and equal, find k.

- (A) 0
 (B) 4
 (C) 0 or 4
 (D) 2

Answer:

(C)

iv. For $\sqrt{2}x^2 - 5x + \sqrt{2} = 0$, find the value of the discriminant.

- (A) -5
 (B) 17
 (C) $\sqrt{2}$
 (D) $2\sqrt{2} - 5$

Answer:

(B)

v. Which of the following quadratic equations has roots 3,5?

- (A) $x^2 - 15x + 8 = 0$
 (B) $x^2 - 8x + 15 = 0$
 (C) $x^2 + 3x + 5 = 0$
 (D) $x^2 + 8x - 15 = 0$

Answer:

(B)

vi. Out of the following equations, find the equation having the sum of its roots -5.

- (A) $3x^2 - 15x + 3 = 0$
 (B) $x^2 - 5x + 3 = 0$
 (C) $x^2 + 3x - 5 = 0$
 (D) $3x^2 + 15x + 3 = 0$

Answer:

(D)

vii. $\sqrt{5}m^2 - \sqrt{5}m + \sqrt{5} = 0$ which of the following statement is true for this given equation?

- (A) Real and unequal roots
 (B) Real and equal roots
 (C) Roots are not real
 (D) Three roots

Answer:

(C)

viii. One of the roots of equation $x^2 + mx - 5 = 0$ is 2; find m.

- (A) -2
 (B) -12
 (C) 12
 (D) 2

Answer:

(C)

Question 2.

Which of the following equations is quadratic

i. $x^2 + 2x + 11 = 0$

ii. $x^2 - 2x + 5 = x^2$

iii. $(x + 2)^2 = 2x^2$

Solution:

i. The given equation is

$$x^2 + 2x + 11 = 0$$

Here, x is the only variable and maximum index of the variable is 2.

$a = 1$, $b = 2$, $c = 11$ are real numbers and

$a \neq 0$.

The given equation is a quadratic equation.

ii. The given equation is

$$x^2 - 2x + 5 = x^2$$

$$\therefore x^2 - x^2 + 2x - 5 = 0$$

$$\therefore 2x - 5 = 0$$

Here, x is the only variable and maximum index of the variable is not 2.

\therefore The given equation is not a quadratic equation.

iii. The given equation is

$$(x + 2)^2 = 2x^2$$

$$\therefore x^2 + 4x + 4 = 2x^2$$

$$\therefore 2x^2 - x^2 - 4x - 4 = 0$$

$$\therefore x^2 - 4x - 4 = 0$$

Here, x is the only variable and maximum index of the variable is 2.

$a = 1$, $b = -4$, $c = -4$ are real numbers and

$a \neq 0$.

\therefore The given equation is a quadratic equation.

Question 3.

Find the value of discriminant for each of the following equations.

i. $2y^2 - y + 2 = 0$

ii. $5m^2 - m = 0$

iii. $\sqrt{5}x^2 - x - \sqrt{5} = 0$

Solution:

i. $2y^2 - y + 2 = 0$

Comparing the above equation with

$ay^2 + by + c = 0$, we get

$$a = 2, b = -1, c = 2$$

$$\therefore b^2 - 4ac = (-1)^2 - 4 \times 2 \times 2$$

$$= 1 - 16$$

$$\therefore b^2 - 4ac = -15$$

ii. $5m^2 - m = 0$

$$\therefore 5m^2 - m + 0 = 0$$

Comparing the above equation with

$am^2 + bm + c = 0$, we get

$$a = 5, b = -1, c = 0$$

$$\therefore b^2 - 4ac = (-1)^2 - 4 \times 5 \times 0$$

$$= 1 - 0$$

$$\therefore b^2 - 4ac = 1$$

iii. $\sqrt{5}x^2 - x - \sqrt{5} = 0$

Comparing the above equation with

$ax^2 + bx + c = 0$, we get

$$a = \sqrt{5}, b = -1, c = -\sqrt{5}$$

$$\therefore b^2 - 4ac = (-1)^2 - 4 \times \sqrt{5} \times \sqrt{5}$$

$$= 1 - 20$$

$$\therefore b^2 - 4ac = -19$$

Question 4.

One of the roots of quadratic equation $2x^2 + kx - 2 = 0$ is -2 , find k .

Solution:

-2 is one of the roots of the equation

$$2x^2 + kx - 2 = 0.$$

\therefore Putting $x = -2$ in the given equation, we get

$$2(-2)^2 + k(-2) - 2 = 0$$

$$\therefore 8 - 2k - 2 = 0$$

$$\therefore 6 - 2k = 0$$

$$\therefore 2k = 6$$

$$\therefore k = 62$$

$$\therefore k = 3$$

Question 5.

Two roots of quadratic equations are given; frame the equation.

i. 10 and -10

ii. $1 - 3\sqrt{5}$ and $1 + 3\sqrt{5}$

iii. 0 and 7

Solution:

i. Let $\alpha = 10$ and $\beta = -10$

$$\therefore \alpha + \beta = 10 - 10 = 0$$

$$\text{and } \alpha \times \beta = 10 \times -10 = -100$$

\therefore The required quadratic equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\therefore x^2 - 0x + (-100) = 0$$

$$\therefore x^2 - 100 = 0$$

ii. Let $\alpha = 1 - 3\sqrt{5}$ and $\beta = 1 + 3\sqrt{5}$

$$\alpha + \beta = 1 - 3\sqrt{5} + 1 + 3\sqrt{5} = 2$$

$$\text{and } \alpha \times \beta = (1 - 3\sqrt{5})(1 + 3\sqrt{5})$$

$$= (1)^2 - (3\sqrt{5})^2$$

$$= 1 - 45$$

$$= -44$$

\therefore The required quadratic equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\therefore x^2 - 2x - 44 = 0$$

iii. Let $\alpha = 0$ and $\beta = 7$

$$\therefore \alpha + \beta = 0 + 7 = 7$$

$$\text{and } \alpha \times \beta = 0 \times 7 = 0$$

\therefore The required quadratic equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\therefore x^2 - 7x + 0 = 0$$

$$\therefore x^2 - 7x = 0$$

Question 6.

Determine the nature of roots for each of the quadratic equation.

i. $3x^2 - 5x + 7 = 0$

ii. $\sqrt{3}x^2 + \sqrt{2}x - 2\sqrt{3} = 0$

iii. $m^2 - 2m + 1 = 0$

Solution:

i. $3x^2 - 5x + 7 = 0$

Comparing the above equation with

$ax^2 + bx + c = 0$, we get

$$a = 3, b = -5, c = 7$$

$$\therefore \Delta = b^2 - 4ac$$

$$= (-5)^2 - 4 \times 3 \times 7$$

$$= 25 - 84$$

$$\therefore \Delta = -59$$

$$\therefore \Delta < 0$$

\therefore Roots of the given quadratic equation are not real.

ii. $\sqrt{3}x^2 + \sqrt{2}x - 2\sqrt{3} = 0$

Comparing the above equation with

$ax^2 + bx + c = 0$, we get

$$a = \sqrt{3}, b = \sqrt{2}, c = -2\sqrt{3}$$

$$\therefore \Delta = b^2 - 4ac$$

$$= (\sqrt{2})^2 - 4 \times \sqrt{3} \times (-2\sqrt{3})$$

$$= 2 + 24$$

$$\therefore \Delta = 26$$

$$\therefore \Delta > 0$$

\therefore Roots of the given quadratic equation are real and unequal.

iii. $m^2 - 2m + 1 = 0$

Comparing the above equation with

$am^2 + bm + c = 0$, we get

$$a = 1, b = -2, c = 1$$

$$\therefore \Delta = b^2 - 4ac$$

$$= (-2)^2 - 4 \times 1 \times 1$$

$$= 4 - 4$$

$$\therefore \Delta = 0$$

\therefore Roots of the given quadratic equation are real and equal

Question 7.

Solve the following quadratic equations.

i. $\frac{1}{x+5} = \frac{1}{x^2}$

ii. $x^2 - \frac{3x}{10} - \frac{1}{10} = 0$

iii. $(2x+3)^2 = 25$

iv. $m^2 + 5m + 5 = 0$

v. $5m^2 + 2m + 1 = 0$

vi. $x^2 - 4x - 3 = 0$

Solution:

i. $\frac{1}{x+5} = \frac{1}{x^2}$

$\therefore x^2 = x + 5$

$\therefore x^2 - x - 5 = 0$

Comparing the above equation with

$ax^2 + bx + c = 0$, we get

$a = 1, b = -1, c = -5$

$\therefore b^2 - 4ac = (-1)^2 - 4 \times 1 \times -5$
 $= 1 + 20 = 21$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-1) \pm \sqrt{21}}{2(1)}$

$\therefore x = \frac{1 \pm \sqrt{21}}{2}$

$\therefore x = \frac{1 + \sqrt{21}}{2}$ or $x = \frac{1 - \sqrt{21}}{2}$

\therefore **The roots of the given quadratic equation are $\frac{1 + \sqrt{21}}{2}$ and $\frac{1 - \sqrt{21}}{2}$.**

ii. $x^2 - 3x - 1 = 0$

$\therefore 10x^2 - 3x - 1 = 0$

...[Multiplying both sides by 10]

$\therefore 10x^2 - 5x + 2x - 1 = 0$

$\therefore 5x(2x - 1) + 1(2x - 1) = 0$

$\therefore (2x - 1)(5x + 1) = 0$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$\therefore 2x - 1 = 0$ or $5x + 1 = 0$

$\therefore 2x = 1$ or $5x = -1$

$\therefore x = \frac{1}{2}$ or $x = -\frac{1}{5}$

\therefore The roots of the given quadratic equation are $\frac{1}{2}$ and $-\frac{1}{5}$

iii. $(2x + 3)^2 = 25$

$\therefore (2x + 3)^2 - 25 = 0$

$\therefore (2x + 3)^2 - (5)^2 = 0$

$\therefore (2x + 3 - 5)(2x + 3 + 5) = 0$ [$\because a^2 - b^2 = (a - b)(a + b)$]

$\therefore (2x - 2)(2x + 8) = 0$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$\therefore 2x - 2 = 0$ or $2x + 8 = 0$

$\therefore 2x = 2$ or $2x = -8$

$\therefore x = 1$ or $x = -4$

\therefore The roots of the given quadratic equation are 1 and -4.

iv. $m^2 + 5m + 5 = 0$

Comparing the above equation with

$am^2 + bm + c = 0$, we get

$a = 1, b = 5, c = 5$

$\therefore b^2 - 4ac = (5)^2 - 4 \times 1 \times 5$

$$= 25 - 20 = 5$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5}}{2(1)}$$

$$\therefore m = \frac{-5 \pm \sqrt{5}}{2}$$

$$\therefore m = \frac{-5 + \sqrt{5}}{2} \text{ or } m = \frac{-5 - \sqrt{5}}{2}$$

\therefore The roots of the given quadratic equation are $\frac{-5 + \sqrt{5}}{2}$ and $\frac{-5 - \sqrt{5}}{2}$.

v. $5m^2 + 2m + 1 = 0$

Comparing the above equation with

$am^2 + bm + c = 0$, we get

$$a = 5, b = 2, c = 1$$

$$\therefore b^2 - 4ac = (2)^2 - 4 \times 5 \times 1$$

$$= 4 - 20$$

$$= -16$$

$$\therefore b^2 - 4ac < 0$$

\therefore Roots of the given quadratic equation are not real.

vi. $x^2 - 4x - 3 = 0$

Comparing the above equation with

$ax^2 + bx + c = 0$, we get

$$a = 1, b = -4, c = -3$$

$$\therefore b^2 - 4ac = (-4)^2 - 4 \times 1 \times -3$$

$$= 16 + 12$$

$$= 28$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{28}}{2(1)}$$

$$= \frac{4 \pm \sqrt{4 \times 7}}{2}$$

$$= \frac{4 \pm 2\sqrt{7}}{2}$$

$$= \frac{2(2 \pm \sqrt{7})}{2}$$

$$\therefore x = 2 \pm \sqrt{7}$$

$$\therefore x = 2 + \sqrt{7} \text{ or } x = 2 - \sqrt{7}$$

\therefore The roots of the given quadratic equation are $2 + \sqrt{7}$ and $2 - \sqrt{7}$.

Question 8.

Find m, if $(m - 12)x^2 + 2(m - 12)x + 2 = 0$ has real and equal roots.

Solution:

$$(m - 12)x^2 + 2(m - 12)x + 2 = 0$$

Comparing the above equation with

$ax^2 + bx + c = 0$, we get

$$a = m - 12, b = 2(m - 12), c = 2$$

$$\therefore \Delta = b^2 - 4ac$$

$$= [2(m - 12)]^2 - 4 \times (m - 12) \times 2$$

$$= 4(m - 12)^2 - 8(m - 12)$$

$$= 4(m - 12)(m - 12 - 2)$$

$$\therefore \Delta = 4(m - 12)(m - 14)$$

Since, the roots are real and equal.

$$\therefore \Delta = 0$$

$$\therefore 4(m - 12)(m - 14) = 0 \quad (m - 12)(m - 14) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore m - 12 = 0 \text{ or } m - 14 = 0$$

$$\therefore m = 12 \text{ or } m = 14$$

But, if $m = 12$, then quadratic coefficient becomes zero.

$$\therefore m \neq 12$$

$$\therefore m = 14$$

Question 9.

The sum of two roots of a quadratic equation is 5 and sum of their cubes is 35, find the equation.

Solution:

Let α and β be the roots of the quadratic equation.

According to the given conditions,

$$\alpha + \beta = 5 \text{ and } \alpha^3 + \beta^3 = 35$$

$$\text{Now, } (\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$$

$$\therefore (\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta)$$

$$\therefore (5)^3 = 35 + 3\alpha\beta(5)$$

$$\therefore 125 = 35 + 15\alpha\beta$$

$$\therefore 125 - 35 = 15\alpha\beta$$

$$\therefore 15\alpha\beta = 90$$

$$\therefore \alpha\beta = 90 \div 15$$

$$\therefore \alpha\beta = 6$$

\therefore The required quadratic equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\therefore x^2 - 5x + 6 = 0$$

Question 10.

Find quadratic equation such that its roots are square of sum of the roots and square of difference of the roots of equation

$$2x^2 + 2(p + q)x + p^2 + q^2 = 0.$$

Solution:

The given quadratic equation is

$$2x^2 + 2(p + q)x + p^2 + q^2 = 0$$

$$\text{Here, } a = 2, b = 2(p + q), c = p^2 + q^2$$

$$\therefore \alpha + \beta = \frac{-b}{a} = \frac{-2(p + q)}{2} = -(p + q)$$

$$\text{and } \alpha\beta = \frac{c}{a} = \frac{p^2 + q^2}{2}$$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$(\alpha - \beta)^2 = [-(p + q)]^2 - 4 \times \frac{p^2 + q^2}{2}$$

$$= (p + q)^2 - 2(p^2 + q^2)$$

$$= p^2 + 2pq + q^2 - 2p^2 - 2q^2$$

$$= -p^2 + 2pq - q^2$$

$$= -(p^2 - 2pq + q^2)$$

$$= -(p - q)^2$$

$$\dots [\because (a^2 - 2ab + b^2) = (a - b)^2]$$

According to the given condition,

Roots of the required quadratic equation are

$$(\alpha + \beta)^2 \text{ and } (\alpha - \beta)^2$$

Now, Sum of the roots

$$= (\alpha + \beta)^2 + (\alpha - \beta)^2$$

$$= [-(p + q)]^2 - (p - q)^2$$

$$= (p + q)^2 - (p - q)^2$$

$$= p^2 + 2pq + q^2 - (p^2 - 2pq + q^2)$$

$$\dots [\because (a + b)^2 = a^2 + 2ab + b^2 \text{ and}$$

$$(a - b)^2 = a^2 - 2ab + b^2]$$

$$= p^2 + 2pq + q^2 - p^2 + 2pq - q^2$$

$$= 4pq$$

Product of the roots

$$= (\alpha + \beta)^2 (\alpha - \beta)^2$$

$$= [-(p + q)]^2 [-(p - q)]^2$$

$$= -(p + q)^2 (p - q)^2$$

$$= -[(p + q)(p - q)]^2 = -(p^2 - q^2)^2$$

\therefore The required quadratic equation is

$$x^2 - [(\alpha + \beta)^2 + (\alpha - \beta)^2]x + [(\alpha + \beta)^2 (\alpha - \beta)^2] = 0$$

$$\therefore x^2 - (4pq)x - (p^2 - q^2)^2 = 0$$

Question 11.

Mukund possesses ₹ 50 more than what Sagar possesses. The product of the amount they have is 15,000. Find the amount each one has.

Solution:

Let the amount Sagar possesses be ₹ x.

$$\therefore \text{the amount Mukund possesses} = ₹ (x + 50)$$

According to the given condition,

$$x(x + 50) = 15000$$

$$\therefore x^2 + 50x - 15000 = 0$$

$$\therefore x^2 + 150x - 100x - 15000 = 0$$

$$\therefore x(x + 150) - 100(x + 150) = 0$$

$$\therefore (x + 150)(x - 100) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore x + 150 = 0 \text{ or } x - 100 = 0$$

$$\therefore x = -150 \text{ or } x = 100$$

But, amount cannot be negative.

$$\therefore x = 100 \text{ and } x + 50 = 100 + 50 = 150$$

\therefore The amount possessed by Sagar and Mukund are ₹ 100 and ₹150 respectively.

Question 12.

The difference between squares of two numbers is 120. The square of smaller number is twice the greater number. Find the numbers.

Solution:

Let the numbers be x and y ($x > y$).

According to the given condition,

$$x^2 - y^2 = 120 \dots(i)$$

$$y^2 = 2x \dots(ii)$$

Substituting $y^2 = 2x$ in equation (i), we get

$$x^2 - 2x = 120$$

$$\therefore x^2 - 2x - 120 = 0$$

$$\therefore x^2 - 12x + 10x - 120 = 0$$

$$\therefore x(x - 12) + 10(x - 12) = 0$$

$$\therefore (x - 12)(x + 10) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore x - 12 = 0 \text{ or } x + 10 = 0$$

$$\therefore x = 12 \text{ or } x = -10$$

But $x \neq -10$

$$\text{as, } y^2 = 2x = 2(-10) = -20 \dots[\text{Since, the square of number cannot be negative}]$$

$$\therefore x = 12$$

$$\text{Smaller number} = y^2 = 2x$$

$$\therefore y^2 = 2 \times 12$$

$$\therefore y^2 = 24$$

$$\therefore y = \pm \sqrt{24} \dots[\text{Taking square root of both sides}]$$

\therefore The smaller number is $\sqrt{24}$ and greater number is 12 or the smaller number is $-\sqrt{24}$ and greater number is 12.

Question 13.

Ranjana wants to distribute 540 oranges among some students. If 30 students were more each would get 3 oranges less. Find the number of students.

Solution:

Let the number of students be x .

Total number of oranges = 540

$$\therefore \text{the number of oranges each student gets} = \frac{540}{x}$$

$$\text{If there were 30 more students, the total number of students} = (x + 30) \text{ and the total number of oranges each student gets} = \frac{540}{x+30}$$

According to the given condition,

$$\begin{aligned} \frac{540}{x} - \frac{540}{x+30} &= 3 & \therefore 2x^2 + 10x &= 20 \times \left(\frac{x^2}{9}\right) \\ \therefore 540 \left(\frac{1}{x} - \frac{1}{x+30}\right) &= 3 & \therefore x^2 + 5x &= \frac{10x^2}{9} \dots[\text{Dividing both sides by 2}] \\ \therefore \frac{x+30-x}{x(x+30)} &= \frac{3}{540} & \therefore 9x^2 + 45x &= 10x^2 \dots[\text{Multiplying both sides by 9}] \\ \therefore \frac{30}{x^2 + 30x} &= \frac{3}{540} & \therefore 10x^2 - 9x^2 - 45x &= 0 \\ & & \therefore x^2 - 45x &= 0 \\ & & \therefore x(x - 45) &= 0 \end{aligned}$$

$$\therefore 30 \times 540 = 3x^2 + 90x$$

$$\therefore 3x^2 + 90x = 16200$$

$$\therefore x^2 + 30x - 5400 = 0$$

...[Dividing both sides by 3]

$$\therefore x^2 + 90x - 60x - 5400 = 0$$

$$\therefore x(x + 90) - 60(x + 90) = 0$$

$$\therefore (x + 90)(x - 60) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore x + 90 = 0 \text{ or } x - 60 = 0$$

$$\therefore x = -90 \text{ or } x = 60$$

But, number of students cannot be negative,

$$x = 60$$

\therefore The total number of students is 60.

Question 14.

Mr. Dinesh owns an rectangular agricultural farm at village Talvel. The length of the farm is 10 metre more than twice the breadth. In order to harvest rain water, he dug a square shaped pond inside the farm. The side of pond is $\frac{1}{3}$ of the breadth of the farm. The area of the farm is 20 times the area of the pond. Find the length and breadth of the farm and side of the pond.

Solution:

Let the breadth of the rectangular farm be x m.

∴ Length of rectangular farm = $(2x + 10)$ m

Area of rectangular farm = Length × Breadth

$$= (2x + 10) \times x$$

$$= (2x^2 + 10x) \text{ sq. m}$$

Now, side of square shaped pond = x^3 m

∴ Area of square shaped pond = $(\text{side})^2$

$$= (x^3)^2 \text{ m}$$

$$= x^6 \text{ m}$$

According to the given condition,

Area of rectangular farm = 20 × Area of pond

$$\therefore 2x^2 + 10x = 20 \times \left(\frac{x^2}{9} \right)$$

$$\therefore x^2 + 5x = \frac{10x^2}{9} \quad \dots [\text{Dividing both sides by 2}]$$

$$\therefore 9x^2 + 45x = 10x^2 \quad \dots [\text{Multiplying both sides by 9}]$$

$$\therefore 10x^2 - 9x^2 - 45x = 0$$

$$\therefore x^2 - 45x = 0$$

$$\therefore x(x - 45) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore x = 0 \text{ or } x - 45 = 0$$

$$x = 0 \text{ or } x = 45$$

But, breadth of the rectangular farm cannot be zero,

$$\therefore x = 45$$

Length of rectangular farm

$$= 2x + 10 = 2(45) + 10 = 100 \text{ m}$$

$$\text{Side of the pond} = x^3 = 45^3 = 15 \text{ m}$$

∴ Length and breadth of the farm and the side of the pond are 100 m, 45 m and 15 m respectively.

Question 15.

A tank fills completely in 2 hours if both the taps are open. If only one of the taps is open at the given time, the smaller tap takes 3 hours more than the larger one to fill the tank. How much time does each tap take to fill the tank completely?

Solution:

Let the larger tap take x hours to fill the tank completely.

∴ Part of tank filled by the larger tap in 1 hour = $\frac{1}{x}$

Also, the smaller tap takes $(x + 3)$ hours to fill the tank completely.

∴ Part of tank filled by the smaller tap in 1 hour = $\frac{1}{x+3}$

∴ Part of tank filled by both the taps in 1 hour

$$= \left(\frac{1}{x} + \frac{1}{x+3} \right)$$

But, the tank gets filled in 2 hours by both the taps.

∴ Part of tank filled by both the taps in 1 hour = $\frac{1}{2}$

According to the given condition,

$$\frac{1}{x} + \frac{1}{x+3} = \frac{1}{2}$$

$$\therefore \frac{x+3+x}{x(x+3)} = \frac{1}{2}$$

$$\therefore \frac{2x+3}{x(x+3)} = \frac{1}{2}$$

$$\therefore 2(2x + 3) = x(x + 3)$$

$$\therefore 4x + 6 = x^2 + 3x$$

$$\therefore x^2 + 3x - 4x - 6 = 0$$

$$\therefore x^2 - x - 6 = 0$$

$$\therefore x^2 - 3x + 2x - 6 = 0$$

$$\therefore x(x - 3) + 2(x - 3) = 0$$

$$\therefore (x - 3)(x + 2) = 0$$

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$$\therefore x - 3 = 0 \text{ or } x + 2 = 0$$

$$\therefore x = 3 \text{ or } x = -2$$

But, time cannot be negative.

$$\therefore x = 3 \text{ and } x + 3 = 3 + 3 = 6$$

∴ The larger tap takes 3 hours and the smaller tap takes 6 hours to fill the tank completely.