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# Maharashtra State Board 12th Commerce Maths Solutions Chapter 1 Mathematical Logic Ex 1.1

State which of the following sentences are statements. Justify your answer. In case of statements, write down the truth value:

Question (i).
A triangle has ' n' sides.
Solution:
It is a statement that is false, hence its truth value is 'F'.

Question (ii).
The sum of interior angles of a triangle is 180°.
Solution:
It is a statement that is true, hence its truth value is 'T'.

Question (iii).
You are amazing!
Solution:
It is an exclamatory sentence, hence it is not a statement.

Question (iv).
Please grant me a loan.
Solution:
It is an imperative sentence, hence it is not a statement.

Question (v).
$\sqrt{-4}$ is an irrational number.
Solution:
It is a statement that is false, hence its truth value is 'F'.

Question (vi).
$x^2 - 6x + 8 = 0$ implies $x = -4$ or $x = -2$.
Solution:
It is a statement that is false, hence its truth value is 'F'.

Question (vii).
He is an actor.
Solution:
It is an open sentence, hence it is not a statement.

Question (viii).
Did you eat lunch yet?
Solution:
It is an interrogative sentence, hence it is not a statement.

Question (ix).
Have a cup of cappuccino.
Solution:
It is an imperative sentence, hence it is not a statement.

Question (x).
$(x + y)^2 = x^2 + 2xy + y^2$ for all $x, y \in R$.
Solution:
It is a mathematical identity that is true, hence its truth value is 'T'.

Question (xi).
Every real number is a complex number.
Solution:
It is a statement that is true, hence its truth value is 'T.

Question (xii).
1 is a prime number.
Solution:
It is a statement that is false, hence its truth value is 'F'.

Question (xiii).
With the sunset, the day ends.

Solution:
It is a statement that is true, hence its truth value is ‘T’.

Question (xiv).
1! = 0.
Solution:
It is a statement that is false, hence its truth value is

Question (xv).
3 + 5 > 11.
Solution:
It is a statement that is false, hence its truth value is ‘F’.

Question (xvi).
The number π is an irrational number.
Solution:
It is a statement that is true, hence its truth value is ‘T’.

Question (xvii).
$x^2 – y^2 = (x + y)(x – y)$ for all $x, y \in R$.
Solution:
It is a mathematical identity that is true, hence its truth value is ‘T’.

Question (xviii).
The number 2 is only even a prime number.
Solution:
It is a statement that is true, hence its truth value is ‘T’.

Question (xix).
Two coplanar lines are either parallel or intersecting.
Solution:
It is a statement that is true, hence its truth value is ‘T’.

Question (xx).
The number of arrangements of 7 girls in a row for a photograph is 7!
Solution:
It is a statement that is true, hence its truth value is ‘T’.

Question (xxi).
Give me a compass box.
Solution:
It is an imperative sentence, hence it is not a statement.

Question (xxii).
Bring the motor car here.
Solution:
It is an imperative sentence, hence it is not a statement.

Question (xxiii).
It may rain today.
Solution:
It is an open sentence, hence it is not a statement.

Question (xxiv).
If $a + b < 7$, where $a \geq 0$ and $b \geq 0$, then $a < 7$ and $b < 7$.
Solution:
It is a statement that is true, hence its truth value is ‘T’.

Question (xxv).
Can you speak English?
Solution:
It is an interrogative sentence, hence it is not a statement.

# Maharashtra State Board 12th Commerce Maths Solutions Chapter 1 Mathematical Logic Ex 1.2

Question 1.
Express the following statements in symbolic form:
(i) e is a vowel or 2 + 3 = 5.
Solution:
Let p : e is a vowel.
q: 2 + 3 = 5.
Then the symbolic form of the given statement is $p \lor q$.

(ii) Mango is a fruit but potato is a vegetable.
Solution:
Let p : Mango is a fruit.
q : Potato is a vegetable.
Then the symbolic form of the given statement is $p \land q$.

(iii) Milk is white or grass is green.
Solution:
Let p : Milk is white.
q : Grass is green.
Then the symbolic form of the given statement is $p \lor q$.

(iv) I like playing but not singing.
Solution:
Let p : I like playing.
q : I am not singing.
Then the symbolic form of the given statement is $p \land q$.

(v) Even though it is cloudy, it is still raining.
Solution:
The given statement is equivalent to:
It is cloudy and it is still raining.
Let p : It is cloudy.
q : It is still raining.
Then the symbolic form of the given statement is $p \land q$.

Question 2.
Write the truth values of the following statements:
(I) Earth is a planet and Moon is a star.
Solution:
Let p : Earth is a planet.
q : Moon is a star.
Then the symbolic form of the given statement is $p \land q$.
The truth values of p and q are T and F respectively.
$\therefore$ the truth value of $p \land q$ is F. …[$T \land F \equiv F$]

(ii) 16 is an even number and 8 is a perfect square.
Solution:
Let p : 16 is an even number.
q : 8 is a perfect square.
Then the symbolic form of the given statement is $p \land q$.
The truth values of p and q are T and F respectively.
$\therefore$ the truth value of $p \land q$ is F. …..[$T \land F \equiv F$]

(iii) A quadratic equation has two distinct roots or 6 has three prime factors.
Solution:
Let p : A quadratic equation has two distinct roots.
q : 6 has three prime factors.
Then the symbolic form of the given statement is $p \lor q$.
The truth values of both p and q are F.
$\therefore$ the truth value of $p \lor q$ is F. …..[$F \lor F \equiv F$]

(iv) The Himalayas are the highest mountains but they are part of India in the northeast.
Solution:
Let p : the Himalayas are the highest mountains.
q : They are part of India in the northeast.

Then the symbolic form of the given statement is $p \land q$.
The truth values of both p and q are T.
∴ the truth value of $p \land q$ is T. .....[$T \land T \equiv T$]

## Maharashtra State Board 12th Commerce Maths Solutions Chapter 1 Mathematical Logic Ex 1.3

Question 1.
Write the negation of each of the following statements:
(i) All men are animals.
Solution:
Some men are not animals.

(ii) 3 is a natural number.
Solution:
-3 is not a natural number.

(iii) It is false that Nagpur is the capital of Maharashtra.
Solution:
Nagpur is the capital of Maharashtra.

(iv) $2 + 3 \neq 5$.
Solution:
$2 + 3 = 5$.

Question 2.
Write the truth value of the negation of each of the following statements:
(i) √5 is an irrational number.
Solution:
Let p : √5 is an irrational number.
The truth value of p is T.
Therefore, the truth value of ~p is F.

(ii) London is in England.
Solution:
Let p : London is in England.
The truth value of p is T.
Therefore, the truth value of ~p is F.

(iii) For every $x \in N$, $x + 3 < 8$.
Solution:
Let p : For every $x \in N$, $x + 3 < 8$.
The truth value of p is F.
Therefore, the truth value of ~p is T.

## Maharashtra State Board 12th Commerce Maths Solutions Chapter 1 Mathematical Logic Ex 1.4

Question 1.
Write the following statements in symbolic form:
(i) If the triangle is equilateral, then it is equiangular.
Solution:
Let p : Triangle is equilateral.
q : It is equiangular.
Then the symbolic form of the given statement is $p \to q$.

(ii) It is not true that 'i' is a real number.
Solution:
Let p : 'i' is a real number.
Then the symbolic form of the given statement is ~p.

(iii) Even though it is not cloudy, it is still raining.
Solution:
Let p : It is cloudy.
q : It is still raining.
Then the symbolic form of the given statement is ~p ∧ q.

(iv) Milk is white if and only if the sky is not blue.
Solution:
Let p : Milk is white.
q : Sky is blue.
Then the symbolic form of the given statement is p ↔ (~q).

(v) Stock prices are high if and only if stocks are rising.
Solution:
Let p : Stock prices are high.
q : stocks are rising.
Then the symbolic form of the given statement is p ↔ q

(vi) If Kutub-Minar is in Delhi, then Taj Mahal is in Agra.
Solution:
Let p : Kutub-Minar is in Delhi.
q : Taj Mahal is in Agra.
Then the symbolic form of the given statement is p → q

Question 2.
Find the truth value of each of the following statements:
(i) It is not true that 3 – 7i is a real number.
Solution:
Let p : 3 – 7i be a real number.
Then the symbolic form of the given statement is ~p.
The truth value of p is F.
∴ the truth value of ~p is T. ….[~F ≡ T]

(ii) If a joint venture is a temporary partnership, then a discount on purchase is credited to the supplier.
Solution:
Let p : Joint venture is a temporary partnership.
q : Discount on purchases is credited to the supplier.
Then the symbolic form of the given statement is p → q.
The truth values of p and q are T and F respectively.
∴ the truth value of p → q is F. …..[T → F ≡ F]

(iii) Every accountant is free to apply his own accounting rules if and only if machinery is an asset.
Solution:
Let p : Every accountant is free to apply his own accounting rules.
q : Machinery is an asset.
Then the symbolic form of the given statement is p ↔ q.
The truth values of p and q are F and T respectively.
∴ the truth value of p ↔ q is F. ….[F ↔ T ≡ F]

(iv) Neither 27 is a prime number nor divisible by 4.
Solution:
Let p : 27 is a prime number.
q : 27 is divisible by 4.
Then the symbolic form of the given statement is ~p ∧ ~q.
The truth values of both p and q are F.
∴ the truth value of ~p ∧ ~q is T. …..[~F ∧ ~F ≡ T ∧ T ≡ T]

(v) 3 is a prime number and an odd number.
Solution:
Let p : 3 be a prime number.
q : 3 is an odd number.
Then the symbolic form of the given statement is p ∧ q

The truth values of both p and q are T.
∴ the truth value of $p \wedge q$ is T. …..[$T \wedge T \equiv T$]

Question 3.
If p and q are true and r and s are false, find the true value of each of the following statements:
(i) $p \wedge (q \wedge r)$
Solution:
Truth values of p and q are T and truth values of r and s are F.
$p \wedge (q \wedge r) \equiv T \wedge (T \wedge F)$
$\equiv T \wedge F$
$\equiv F$
Hence, the truth value of the given statement is false.

(ii) $(p \rightarrow q) \vee (r \wedge s)$
Solution:
$(p \rightarrow q) \vee (r \wedge s) \equiv (T \rightarrow T) \vee (F \wedge F)$
$\equiv T \vee F$
$\equiv T$
Hence, the truth value of the given statement is true.

(iii) $\sim[(\sim p \vee s) \wedge (\sim q \wedge r)]$
Solution:
$\sim[(\sim p \vee s) \wedge (\sim q \wedge r)] \equiv \sim[(\sim T \vee F) \wedge (\sim T \wedge F)]$
$\equiv \sim[(F \vee F) \wedge (F \wedge F)]$
$\equiv \sim(F \wedge F)$
$\equiv \sim F$
$\equiv T$
Hence, the truth value of the given statement is true.

(iv) $(p \rightarrow q) \leftrightarrow \sim(p \vee q)$
Solution:
$(p \rightarrow q) \leftrightarrow \sim(p \vee q) = (T \rightarrow T) \leftrightarrow \sim(T \vee T)$
$\equiv T \leftrightarrow \sim (T)$
$\equiv T \leftrightarrow F$
$\equiv F$
Hence, the truth value of the given statement is false.

(v) $[(p \vee s) \rightarrow r] \vee [\sim(p \rightarrow q) \vee s]$
Solution:
$[(p \vee s) \rightarrow r] \vee \sim[\sim(p \rightarrow q) \vee s]$
$\equiv [(T \vee F) \rightarrow F] \vee \sim[\sim(T \rightarrow T) \vee F]$
$\equiv (T \rightarrow F) \vee \sim(\sim T \vee F)$
$\equiv F \vee \sim (F \vee F)$
$\equiv F \vee \sim F$
$\equiv F \vee T$
$\equiv T$
Hence, the truth value of the given statement is true.

(vi) $\sim[p \vee (r \wedge s)] \wedge \sim[(r \wedge \sim s) \wedge q]$
Solution:
$\sim[p \vee (r \wedge s)] \wedge \sim[(r \wedge \sim s) \wedge q]$
$\equiv \sim[T \vee (F \wedge F)] \wedge \sim[(F \wedge \sim F) \wedge T]$
$\equiv \sim[T \vee F] \wedge \sim[(F \wedge T) \wedge T]$
$\equiv \sim T \wedge \sim(F \wedge T)$
$\equiv F \wedge \sim F$
$\equiv F \wedge T$
$\equiv F$
Hence, the truth value of the given statement is false.

Question 4.
Assuming that the following statements are true:
p : Sunday is a holiday.
q : Ram does not study on holiday.
Find the truth values of the following statements:
(i) Sunday is not holiday or Ram studies on holiday.
Solution:
The symbolic form of the statement is $\sim p \vee \sim q$.

| $p$ | $q$ | $\sim p$ | $\sim q$ | $\sim p \vee \sim q$ |
|---|---|---|---|---|
| T | T | F | F | F |

∴ the truth value of the given statement is F.

(ii) If Sunday is not a holiday, then Ram studies on holiday.
Solution:
The symbolic form of the given statement is $\sim p \rightarrow \sim q$.

| $p$ | $q$ | $\sim p$ | $\sim q$ | $\sim p \rightarrow \sim q$ |
|---|---|---|---|---|
| T | T | F | F | T |

∴ the truth value of the given statement is T.

(iii) Sunday is a holiday and Ram studies on holiday.
Solution:
The symbolic form of the given statement is $p \wedge q$.

| $p$ | $q$ | $\sim q$ | $p \wedge \sim q$ |
|---|---|---|---|
| T | T | F | F |

∴ the truth value of the given statement is F.

Question 5.
If p : He swims.
q : Water is warm.
Give the verbal statements for the following symbolic statements:
(i) $p \leftrightarrow \sim q$
Solution:
$p \leftrightarrow \sim q$
He swims if and only if the water is not warm.

(ii) $\sim(p \vee q)$
Solution:
$\sim(p \vee q)$
It is not true that he swims or water is warm.

(iii) $q \rightarrow p$
Solution:
$q \rightarrow p$
If water is warm, then he swims.

(iv) $q \wedge \sim p$
Solution:
$q \wedge \sim p$
The water is warm and he does not swim.

## Maharashtra State Board 12th Commerce Maths Solutions Chapter 1 Mathematical Logic Ex 1.5

Question 1.
Use qualifiers to convert each of the following open sentences defined on N, into a true statement:
(i) $x^2 + 3x - 10 = 0$
Solution:
$\exists$ x $\in$ N, such that $x^2 + 3x - 10 = 0$ is a true statement
($x = 2 \in N$ satisfy $x^2 + 3x - 10 = 0$)

(ii) $3x - 4 < 9$
Solution:
$\exists$ x $\in$ N, such that $3x - 4 < 9$ is a true statement.
($x = 1, 2, 3, 4 \in N$ satisfy $3x - 4 < 9$)

(iii) $n^2 \geq 1$
Solution:
$\forall$ n $\in$ N, $n^2 \geq 1$ is a true statement.
(All n $\in$ N satisfy $n^2 \geq 1$)

(iv) 2n – 1 = 5
Solution:
$\exists$ x $\in$ N, such that 2n – 1 = 5 is a true statement.
(n = 3 $\in$ N satisfy 2n – 1 = 5)

(v) y + 4 > 6
Solution:
$\exists$ y $\in$ N, such that y + 4 > 6 is a true statement.
(y = 3, 4, 5, … $\in$ N satisfy y + 4 > 6

(vi) 3y – 2 ≤ 9
Solution:
$\exists$ y $\in$ N, such that 2y ≤ 9 is a true statement.
(y = 1, 2, 3 $\in$ N satisfy 3y – 2 ≤ 9).

Question 2.
If B = {2, 3, 5, 6, 7}, determine the truth value of each of the following:
(i) $\forall$ x $\in$ B, x is a prime number.
Solution:
(i) x = 6 $\in$ B does not satisfy x is a prime number.
So, the given statement is false, hence its truth value is F.

(ii) $\exists$ n $\in$ B, such that n + 6 > 12.
Solution:
Clearly n = 7 $\in$ B satisfies n + 6 > 12.
So, the given statement is true, hence its truth value is T.

(iii) $\exists$ n $\in$ B, such that 2n + 2 < 4.
Solution:
No element n $\in$ B satisfy 2n + 2 < 4.
So, the given statement is false, hence its truth value is F.

(iv) $\forall$ y $\in$ B, $y^2$ is negative.
Solution:
No element y $\in$ B satisfy $y^2$ is negative.
So, the given statement is false, hence its truth value is F.

(v) $\forall$ y $\in$ B, (y – 5) $\in$ N.
Solution:
y = 2 $\in$ B, y = 3 $\in$ B and y = 5 $\in$ B do not satisfy (y – 5) $\in$ N.
So, the given statement is false, hence its truth value is F.

# Maharashtra State Board 12th Commerce Maths Solutions Chapter 1 Mathematical Logic Ex 1.6

Question 1.
Prepare the truth tables for the following statement patterns:
(i) p → (~p ∨ q)
Solution:
Here are two statements and three connectives.

∴ there are 2 × 2 = 4 rows and 2 + 3 = 5 columns in the truth table.

| $p$ | $q$ | $\sim p$ | $\sim p \vee q$ | $p \rightarrow (\sim p \vee q)$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

(ii) (~p ∨ q) ∧ (~p ∨ ~q)
Solution:

| $p$ | $q$ | $\sim p$ | $\sim q$ | $\sim p \vee q$ | $\sim p \vee \sim q$ | $(\sim p \vee q) \wedge (\sim p \vee \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F |
| T | F | F | T | F | T | F |
| F | T | T | F | T | T | T |
| F | F | T | T | T | T | T |

(iii) (p ∧ r) → (p ∨ ~q)
Solution:
Here are three statements and 4 connectives.
∴ there are 2 × 2 × 2 = 8 rows and 3 + 4 = 7 columns in the truth table.

| $p$ | $q$ | $r$ | $\sim q$ | $p \wedge r$ | $p \vee \sim q$ | $(p \wedge r) \rightarrow (p \vee \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | T | F | T | T | T |
| T | T | F | F | F | T | T |
| T | F | T | T | T | T | T |
| T | F | F | T | F | T | T |
| F | T | T | F | F | F | T |
| F | T | F | F | F | F | T |
| F | F | T | T | F | T | T |
| F | F | F | T | F | T | T |

(iv) (p ∧ q) ∨ ~r
Solution:

| $p$ | $q$ | $r$ | $\sim r$ | $p \wedge q$ | $(p \wedge q) \vee \sim r$ |
|---|---|---|---|---|---|
| T | T | T | F | T | T |
| T | T | F | T | T | T |
| T | F | T | F | F | F |
| T | F | F | T | F | T |
| F | T | T | F | F | F |
| F | T | F | T | F | T |
| F | F | T | F | F | F |
| F | F | F | T | F | T |

Question 2.
Examine, whether each of the following statement patterns is a tautology or a contradiction or a contingency:
(i) q ∨ [~(p ∧ q)]
Solution:

| $p$ | $q$ | $p \wedge q$ | $\sim(p \wedge q)$ | $q \vee [\sim(p \wedge q)]$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | F | T | T |

All the entries in the last column of the above truth table are T.
∴ q ∨ [~(p ∧ q)] is a tautology.

(ii) (~q ∧ p) ∧ (p ∧ ~p)
Solution:

| $p$ | $q$ | $\sim p$ | $\sim q$ | $\sim q \wedge p$ | $p \wedge \sim p$ | $(\sim q \wedge p) \wedge (p \wedge \sim p)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | F | F | F |
| T | F | F | T | T | F | F |
| F | T | T | F | F | F | F |
| F | F | T | T | F | F | F |

All the entries in the last column of the above truth table are F.
∴ (~q ∧ p) ∧ (p ∧ ~p) is a contradiction.

(iii) (p ∧ ~q) → (~p ∧ ~q)
Solution:

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \wedge \sim q$ | $\sim p \wedge \sim q$ | $(p \wedge \sim q) \to (\sim p \wedge \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | F | F | T |
| T | F | F | T | T | F | F |
| F | T | T | F | F | F | T |
| F | F | T | T | F | T | T |

The entries in the last column are neither all T nor all F.
∴ (p ∧ ~q) → (~p ∧ ~q) is a contingency.

(iv) ~p → (p → ~q)
Solution:

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \to \sim q$ | $\sim p \to (p \to \sim q)$ |
|---|---|---|---|---|---|
| T | T | F | F | F | T |
| T | F | F | T | T | T |
| F | T | T | F | T | T |
| F | F | T | T | T | T |

All the entries in the last column of the truth table are T.
∴ p → (p → ~q) is a tautology.

Question 3.
Prove that each of the following statement pattern is a tautology:
(i) (p ∧ q) → q
Solution:

| $p$ | $q$ | $p \wedge q$ | $(p \wedge q) \to q$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | T |
| F | T | F | T |
| F | F | F | T |

All the entries in the last column of the above truth table are T.
∴ (p ∧ q) → q is a tautology.

(ii) (p → q) ↔ (~q → ~p)
Solution:

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \rightarrow q$ | $\sim q \rightarrow \sim p$ | $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | T | T |
| T | F | F | T | F | F | T |
| F | T | T | F | T | T | T |
| F | F | T | T | T | T | T |

All the entries in the last column of the above truth table are T.
∴ (p → q) ↔ (~q → ~p) is a tautology.

(iii) (~p ∧ ~q) → (p → q)
Solution:

| $p$ | $q$ | $\sim p$ | $\sim q$ | $\sim p \wedge \sim q$ | $p \rightarrow q$ | $(\sim p \wedge \sim q) \rightarrow (p \wedge q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | F | T | T |
| T | F | F | T | F | F | T |
| F | T | T | F | F | T | T |
| F | F | T | T | T | T | T |

All the entries in the last column of the above truth table are T.
∴ (~p ∧ ~q) → (p → q) is a tautology.

(iv) (~p ∨ ~q) ↔ ~(p ∧ q)
Solution:

| $p$ | $q$ | $\sim p$ | $\sim q$ | $\sim p \vee \sim q$ | $p \wedge q$ | $\sim (p \wedge q)$ | $(\sim p \vee \sim q) \leftrightarrow \sim (p \wedge q)$ |
|---|---|---|---|---|---|---|---|
| T | T | F | F | F | T | F | T |
| T | F | F | T | T | F | T | T |
| F | T | T | F | T | F | T | T |
| F | F | T | T | T | F | T | T |

All the entries in the last column of the above truth table are T.
∴ (~p ∨ ~q) ↔ ~(p ∧ q) is a tautology.

Question 4.
Prove that each of the following statement pattern is a contradiction:
(i) (p ∨ q) ∧ (~p ∧ ~q)
Solution:

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \vee q$ | $\sim p \wedge \sim q$ | $(p \vee q) \wedge (\sim p \wedge \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F |
| T | F | F | T | T | F | F |
| F | T | T | F | T | F | F |
| F | F | T | T | F | T | F |

All the entries in the last column of the above truth table are F.
∴ (p ∨ q) ∧ (~p ∧ ~q) is a contradiction.

(ii) (p ∧ q) ∧ ~p
Solution:

| $p$ | $q$ | $\sim p$ | $p \wedge q$ | $(p \wedge q) \wedge \sim p$ |
|---|---|---|---|---|
| T | T | F | T | F |
| T | F | F | F | F |
| F | T | T | F | F |
| F | F | T | F | F |

All the entries in the last column of the above truth table are T.
∴ (p ∧ q) ∧ ~p is a contradiction.

(iii) (p ∧ q) ∧ (~p ∨ ~q)
Solution:

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \wedge q$ | $\sim p \vee \sim q$ | $(p \wedge q) \wedge (\sim p \vee \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F |
| T | F | F | T | F | T | F |
| F | T | T | F | F | T | F |
| F | F | T | T | F | T | F |

All the entries in the last column of the above truth table are F.
∴ (p ∧ q) ∧ (~p ∨ ~q) is a contradiction.

(iv) (p → q) ∧ (p ∧ ~q)
Solution:

| $p$ | $q$ | $\sim q$ | $p \to q$ | $p \wedge \sim q$ | $(p \to q) \wedge (p \wedge \sim q)$ |
|---|---|---|---|---|---|
| T | T | F | T | F | F |
| T | F | T | F | T | F |
| F | T | F | T | F | F |
| F | F | T | T | F | F |

All the entries in the last column of the above truth table are F.
∴ (p → q) ∧ (p ∧ ~q) is a contradiction.

Question 5.
Show that each of the following statement pattern is a contingency:
(i) (p ∧ ~q) → (~p ∧ ~q)
Solution:

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \wedge \sim q$ | $\sim p \wedge \sim q$ | $(p \wedge \sim q) \to (\sim p \wedge \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | F | F | T |
| T | F | F | T | T | F | F |
| F | T | T | F | F | F | T |
| F | F | T | T | F | T | T |

The entries in the last column of the above truth table are neither all T nor all F.
∴ (p ∧ ~q) → (~p ∧ ~q) is a contingency.

(ii) (p → q) ↔ (~p ∧ q)
Solution:

| $p$ | $q$ | $\sim p$ | $p \to q$ | $\sim p \wedge q$ | $(p \to q) \leftrightarrow (\sim p \wedge q)$ |
|---|---|---|---|---|---|
| T | T | F | T | F | F |
| T | F | F | F | F | T |
| F | T | T | T | T | T |
| F | F | T | T | F | F |

The entries in the last column of the above truth table are neither all T nor all F.
∴ (p → q) ↔ (~p ∧ q) is a contingency.

(iii) p ∧ [(p → ~q) → q]
Solution:

| $p$ | $q$ | $\sim q$ | $p \to \sim q$ | $(p \to \sim q) \to q$ | $p \wedge [(p \to \sim q) \to q]$ |
|---|---|---|---|---|---|
| T | T | F | F | T | T |
| T | F | T | T | F | F |
| F | T | F | T | T | F |
| F | F | T | T | F | F |

The entries in the last column of the above truth table are neither all T nor all F.
∴ p ∧ [(p → ~q) → q] is a contingency.

(iv) (p → q) ∧ (p → r)
Solution:

| $p$ | $q$ | $r$ | $p \to q$ | $p \to r$ | $(p \to q) \wedge (p \to r)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | T | F | T | F | F |
| T | F | T | F | T | F |
| T | F | F | F | F | F |
| F | T | T | T | T | T |
| F | T | F | T | T | T |
| F | F | T | T | T | T |
| F | F | F | T | T | T |

The entries in the last column of the above truth table are neither all T nor all F.
∴ (p → q) ∧ (p → r) is a contingency.

Question 6.
Using the truth table, verify:
(i) p ∨ (q ∧ r) = (p ∨ q) ∧ (p ∨ r)
Solution:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| $p$ | $q$ | $r$ | $q \wedge r$ | $p \vee (q \wedge r)$ | $p \vee q$ | $p \vee r$ | $(p \vee q) \wedge (p \vee r)$ |
| T | T | T | T | T | T | T | T |
| T | T | F | F | T | T | T | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | T | T | T |
| F | T | T | T | T | T | T | T |
| F | T | F | F | F | T | F | F |
| F | F | T | F | F | F | T | F |
| F | F | F | F | F | F | F | F |

The entries in columns 5 and 8 are identical.
∴ p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r).

(ii) p → (p → q) ≡ ~q → (p → q)
Solution:

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $p$ | $q$ | $\sim q$ | $p \to q$ | $p \to (p \to q)$ | $\sim q \to (p \to q)$ |
| T | T | F | T | T | T |
| T | F | T | F | F | F |
| F | T | F | T | T | T |
| F | F | T | T | T | T |

The entries in columns 5 and 6 are identical.
∴ p → (p → q) ≡ ~q → (p → q)

(iii) ~(p → ~q) ≡ p ∧ ~(~q) ≡ p ∧ q
Solution:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| $p$ | $q$ | $\sim q$ | $p \to \sim q$ | $\sim (p \to \sim q)$ | $\sim (\sim q)$ | $p \wedge \sim (\sim q)$ | $p \wedge q$ |
| T | T | F | F | T | T | T | T |
| T | F | T | T | F | F | F | F |
| F | T | F | T | F | T | F | F |
| F | F | T | T | F | F | F | F |

The entries in columns 5, 7 and 8 are identical.
∴ ~(p → ~q) ≡ p ∧ ~(~q) ≡ p ∧ q.

(iv) ~(p ∨ q) ∨ (~p ∧ q) ≡ ~p
Solution:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| $p$ | $q$ | $\sim p$ | $p \vee q$ | $\sim (p \vee q)$ | $\sim p \wedge q$ | $\sim (p \vee q) \vee (\sim p \wedge q)$ |
| T | T | F | T | F | F | F |
| T | F | F | T | F | F | F |
| F | T | T | T | F | T | T |
| F | F | T | F | T | F | T |

The entries in columns 3 and 7 are identical.
∴ ~(p ∨ q) ∨ (~p ∧ q) ≡ ~p.

Question 7.
Prove that the following pairs of statement patterns are equivalent:
(i) p ∨ (q ∧ r) and (p ∨ q) ∧ (p ∨ r)
Solution:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| $p$ | $q$ | $r$ | $q \wedge r$ | $p \vee (q \wedge r)$ | $p \vee q$ | $p \vee r$ | $(p \vee q) \wedge (p \vee r)$ |
| T | T | T | T | T | T | T | T |
| T | T | F | F | T | T | T | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | T | T | T |
| F | T | T | T | T | T | T | T |
| F | T | F | F | F | T | F | F |
| F | F | T | F | F | F | T | F |
| F | F | F | F | F | F | F | F |

The entries in columns 5 and 8 are identical.
∴ p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)

(ii) p ↔ q and (p → q) ∧ (q → p)
Solution:

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $p$ | $q$ | $p \leftrightarrow q$ | $p \to q$ | $q \to p$ | $(p \to q) \wedge (q \to p)$ |
| T | T | T | T | T | T |
| T | F | F | F | T | F |
| F | T | F | T | F | F |
| F | F | T | T | T | T |

The entries in columns 3 and 6 are identical.
∴ p ↔ q ≡ (p → q) ∧ (q → p)

(iii) p → q and ~q → ~p and ~p ∨ q
Solution:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \to q$ | $\sim q \to \sim p$ | $\sim p \vee q$ |
| T | T | F | F | T | T | T |
| T | F | F | T | F | F | F |
| F | T | T | F | T | T | T |
| F | F | T | T | T | T | T |

The entries in columns 5, 6 and 7 are identical.
∴ p → q ≡ ~q → ~p ≡ ~p ∨ q.

(iv) ~(p ∧ q) and ~p ∨ ~q
Solution:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \wedge q$ | $\sim (p \wedge q)$ | $\sim p \vee \sim q$ |
| T | T | F | F | T | F | F |
| T | F | F | T | F | T | T |
| F | T | T | F | F | T | T |
| F | F | T | T | F | T | T |

The entries in columns 6 and 7 are identical.
∴ ~(p ∧ q) ≡ ~p ∨ ~q.

## Maharashtra State Board 12th Commerce Maths Solutions Chapter 1 Mathematical Logic Ex 1.7

Question 1.
Write the dual of each of the following:
(i) (p ∨ q) ∨ r
Solution:
(p ∧ q) ∧ r

(ii) ~(p ∨ q) ∧ [p ∨ ~(q ∧ ~r)]
Solution:
~(p ∧ q) ∨ [p ∧ ~(q ∨ ~r)]

(iii) $p \lor (q \lor r) \equiv (p \lor q) \lor r$
Solution:
$p \land (q \land r) \equiv (p \land q) \land r$

(iv) $\sim(p \land q) \equiv \sim p \lor \sim q$
Solution:
$\sim(p \lor q) \equiv \sim p \land \sim q$

Question 2.
Write the dual statement of each of the following compound statements:
(i) 13 is prime number and India is a democratic country.
Solution:
13 is prime number or India is a democratic country.

(ii) Karina is very good or everybody likes her.
Solution:
Karina is very good and everybody likes her.

(iii) Radha and Sushmita can not read Urdu.
Solution:
Radha or Sushmita can not read Urdu.

(iv) A number is real number and the square of the number is non-negative.
Solution:
A number is real number or the square of the number is non-negative.



## Maharashtra State Board 12th Commerce Maths Solutions Chapter 1 Mathematical Logic Ex 1.8

Question 1.
Write the negation of each of the following statements:
(i) All the stars are shining if it is night.
Solution:
The given statement can be written as:
If it is night, then all the stars are shining.
Let p : It is night.
q : All the stars are shining.
Then the symbolic form of the given statement is $p \to q$
Since, $\sim(p \to q) \equiv p \land \sim q$,
the negation of the given statement is ‘It is night and all the stars are not shining.’

(ii) $\forall\ n \in N, n + 1 > 0$.
Solution:
The negation of the given statement is
‘$\exists\ n \in N$, such that $n + 1 \leq 0$.’

(iii) $\exists\ n \in N$, such that $(n^2 + 2)$ is odd number.
Solution:
The negation of the given statement is
‘$\forall\ n \in N$, $n^2 + 2$ is not an odd number.’

(iv) Some continuous functions are differentiable.
Solution:
The negation of a given statement is ‘All continuous functions are not differentiable.’

Question 2.
Using the rules of negation, write the negations of the following:
(i) $(p \to r) \land q$

Solution:
The negation of $(p \rightarrow r) \wedge q$ is
$\sim[(p \rightarrow r) \wedge q] \equiv \sim(p \rightarrow r) \vee (\sim q)$ .....[Negation of conjunction]
$\equiv (p \wedge \sim r) \vee (\sim q)$ ......[Negation of implication]

(ii) $\sim(p \vee q) \rightarrow r$
Solution:
The negation of $\sim(p \vee q) \rightarrow r$ is
$\sim[\sim(p \vee q) \rightarrow r] \equiv \sim(p \vee q) \wedge (\sim r)$ .....[Negation of implication]
$\equiv (\sim p \wedge \sim q) \wedge (\sim r)$ ......[Negation of disjunction]

(iii) $(\sim p \wedge q) \wedge (\sim q \vee \sim r)$
Solution:
The negation of $(\sim p \wedge q) \wedge (\sim q \vee \sim r)$ is
$\sim[(\sim p \wedge q) \wedge (\sim q \vee \sim r)] \equiv \sim(\sim p \wedge q) \vee \sim(\sim q \vee \sim r)$ ......[Negation of conjunction]
$\equiv [\sim(\sim p) \vee \sim q] \vee [\sim(\sim q) \wedge \sim(\sim r)]$ ... [Negation of conjunction and disjunction]
$\equiv (p \vee \sim q) \vee (q \wedge r)$ .....[Negation of negation]

Question 3.
Write the converse, inverse, and contrapositive of the following statements:
(i) If it snows, then they do not drive the car.
Solution:
Let p : It snows.
q : They do not drive the car.
Then the symbolic form of the given statement is $p \rightarrow q$.
Converse: $q \rightarrow p$ is the converse of $p \rightarrow q$.
i.e. If they do not drive the car, then it snows.
Inverse: $\sim p \rightarrow \sim q$ is the inverse of $p \rightarrow q$.
i.e. If it does not snow, then they drive the car.
Contrapositive: $\sim q \rightarrow \sim p$ is the contrapositive of $p \rightarrow q$.
i.e. If they drive the car, then it does not snow.

(ii) If he studies, then he will go to college.
Solution:
Let p : He studies.
q : He will go to college.
Then two symbolic form of the given statement is $p \rightarrow q$.
Converse: $q \rightarrow p$ is the converse of $p \rightarrow q$.
i.e. If he will go to college, then he studies.
Inverse: $\sim p \rightarrow \sim q$ is the inverse of $p \rightarrow q$.
i.e. If he does not study, then he will not go to college.
Contrapositive: $\sim q \rightarrow \sim p$ is the contrapositive of $p \rightarrow q$.
i.e. If he will not go to college, then he does not study.

Question 4.
With proper justification, state the negation of each of the following:
(i) $(p \rightarrow q) \vee (p \rightarrow r)$
Solution:
The negation of $(p \rightarrow q) \vee (p \rightarrow r)$ is
$\sim[(p \rightarrow q) \vee (p \rightarrow r)] \equiv \sim(p \rightarrow q) \wedge \sim(p \rightarrow r)$ .....[Negation of disjunction]
$\equiv (p \wedge \sim q) \wedge (p \wedge \sim r)$ ...[Negation of implication]

(ii) $(p \leftrightarrow q) \vee (\sim q \rightarrow \sim r)$
Solution:
The negation of $(p \leftrightarrow q) \vee (\sim q \rightarrow \sim r)$ is
$\sim[(p \leftrightarrow q) \vee (\sim q \rightarrow \sim r)] \equiv \sim(p \leftrightarrow q) \wedge \sim(\sim q \rightarrow \sim r)$ .....[Negation of disjunction]
$\equiv [(p \wedge \sim q) \vee (q \wedge \sim p)] \wedge [\sim q \wedge \sim(\sim r)]$ ......[Negation of biconditional and implication]
$\equiv [(p \wedge \sim q) \vee (q \wedge \sim p)] \wedge (\sim q \wedge r)$ ......[Negation of negation]

(iii) $(p \rightarrow q) \wedge r$
Solution:
The negation of $(p \rightarrow q) \wedge r$ is
$\sim[(p \rightarrow q) \wedge r] \equiv \sim(p \rightarrow q) \vee (\sim r)$ .....[Negation of conjunction]
$\equiv (p \wedge \sim q) \vee (\sim r)$ .....[Negation of implication]

# Maharashtra State Board 12th Commerce Maths Solutions Chapter 1 Mathematical Logic Ex 1.9

Question 1.
Without using truth table, show that
(i) $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$
Solution:
LHS = $p \leftrightarrow q$
$\equiv (p \rightarrow q) \wedge (q \rightarrow p)$
$\equiv (\sim p \vee q) \wedge (\sim(q \vee p)$ .....(Conditional Law)
$\equiv [\sim p \wedge (\sim(q \vee p)] \vee [q \wedge (\sim q \vee p]$ .....(Distributive Law)
$\equiv [(\sim p \wedge \sim q) \vee (\sim p \wedge p)] \vee [(q \wedge \sim q) \vee (q \wedge p)]$ ......(Distributive Law)
$\equiv [(\sim p \wedge \sim q) \vee c] \vee [c \vee (q \wedge p)]$ .....(Complement Law)
$\equiv (\sim p \wedge \sim q) \vee (q \wedge p)$ ......(Identity Law)
$\equiv (\sim p \wedge \sim q) \vee (p \wedge q)$ ......(Commutative Law)
$\equiv (p \wedge q) \vee (\sim p \wedge q)$ ......(Commutative Law)
≡ RHS.

(ii) $p \wedge [\sim p \vee q) \vee (\sim q)] \equiv p$
Solution:
LHS = $p \wedge [(\sim p \vee q) \vee (\sim q)]$
$\equiv p \wedge [\sim p \vee (q \vee \sim q)]$ ......(Associative Law)
$\equiv p \wedge [\sim p \vee t]$ .......(Complement Law)
$\equiv p \wedge t$ ......(Identity Law)
$\equiv p$ ......(Identity Law)
= RHS.

(iii) $\sim[(p \wedge q) \rightarrow \sim(q)] \equiv p \wedge q$
Solution:
LHS = $\sim[(p \wedge q) \rightarrow \sim(\sim q)]$
$\equiv (p \wedge q) \wedge \sim(\sim q)$ ......(Negation of implication)
$\equiv (p \wedge q) \wedge q$ .....(Negation of negation)
$\equiv p \wedge (q \wedge q)$ .....(Associative Law)
$\equiv P \wedge q$ ......(Idempotent Law)
= RHS

(iv) $\sim r \rightarrow \sim(p \wedge q) \equiv [\sim(q \rightarrow r)] \rightarrow (\sim p)$
Solution:
LHS = $\sim r \rightarrow \sim(p \wedge q)$
$\equiv \sim q \rightarrow (\sim p \vee \sim q)$ ......(De Morgan's Law)
$\equiv \sim(\sim r) \vee (\sim p \vee \sim q)$ .....(Conditional Law)
$\equiv r \vee (\sim p \vee \sim q)$ .....(Involution Law)
$\equiv r \vee \sim q \vee \sim p$ .....(Commutative Law)
$\equiv (\sim q \vee r) \vee (\sim p)$ ......(Commutative Law)
$\equiv \sim(q \rightarrow r) \vee (\sim p)$ ......(Conditional Law)
$\equiv \sim(q \rightarrow r) \rightarrow (\sim p)$ ......(Conditional Law)
= RHS.

(v) $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$
Solution:
LHS = $(p \vee q) \rightarrow r$
$\equiv \sim(p \rightarrow q) \vee r$ ........(Conditional Law)
$\equiv (\sim p \wedge \sim q) \vee r$ ..........(De Morgan's Law)
$\equiv (\sim p \vee r) \wedge (\sim q \vee r)$ ...........(Distributive Law)
$\equiv (p \rightarrow r) \wedge (q \rightarrow r)$ .......(Conditional Law)
= RHS.

Question 2.
Using the algebra of statement, prove that:
(i) $[p \wedge (q \vee r)] \vee [\sim r \wedge \sim q \wedge p] \equiv p$
Solution:
LHS = $[p \wedge (q \vee r)] \vee [\sim r \wedge \sim q \wedge p]$
$\equiv [p \wedge (q \vee r)] \vee [(\sim r \wedge \sim q) \wedge p]$ ......(Associative Law)
$\equiv [p \wedge (q \vee r)] \vee [(\sim q \wedge \sim r) \wedge p]$ ......(Commutative Law)
$\equiv [p \wedge (q \vee r)] \vee [\sim (q \vee r) \wedge p]$ ......(De Morgan's Law)

≡ [p ∧ (q ∨ r)] ∨ [p ∧ ~(q ∨ r)] ……(Commutative Law)
≡ p ∧ [(q ∨ r) ∨ ~ (q ∨ r) ] …..(Distributive Law)
≡ p ∧ t …….(Complement Law)
≡ p ……(Identity Law)
= RHS.

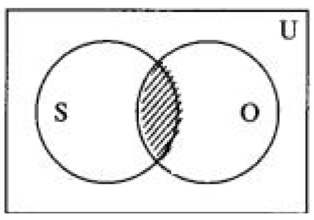
(ii) (p ∧ q) ∨ (p ∧ ~q) ∨ (~p ∧ ~q) ≡ p ∨ ~q
Solution:
LHS = (p ∧ q) ∨ (p ∧ ~q) ∨ (~p ∧ ~ q)
≡ (p ∧ q) ∨ [(p ∧ ~q) ∨ (~p ∧ ~q)] ……(Associative Law)
≡ (p ∧ q) ∨ [(~q ∧ p) ∨ (~q ∧ ~p)] …..(CommutativeLaw)
≡ (p ∧ q) ∨ [ ~q ∧ (p ∨ ~ p)] …..(Distributive Law)
≡ (p ∧ q) ∨ (~q ∧ t) ……(Complement Law)
≡ (p ∧ q) ∨ (~q) …….(Identity Law)
≡ (p ∨ ~q) ∧ (q ∨ ~q) ……(Distributive Law)
≡ (p ∨ ~q) ∧ t …….(Complement Law)
≡ p ∨ ~q …..(Identity Law)
= RHS.

(iii) (p ∨ q) ∧ (~p ∨ ~q) ≡ (p ∧ ~q) ∨ (~p ∧ q)
Solution:
LHS = (p ∨ q) ∧ (~p ∨ ~q)
≡ [p ∧ (~p ∨ ~q)] ∨ [q ∧ (~p ∨ ~q)] ……(Distributive Law)
≡ [(p ∧ ~p) ∨ (p ∧ ~q)] ∨ [q ∧ ~p) ∨ (q ∧ ~q)] ……(Distributive Law)
≡ [c ∨ (p ∧ ~q)] ∨ [(q ∧ ~p) ∨ c] ……(Complement Law)
≡ (p ∧ ~q) ∨ (q ∧ ~p) ……..(Identity Law)
≡ (p ∧ ~q) ∨ (~p ∧ q) ………(Commutative Law)
= RHS.



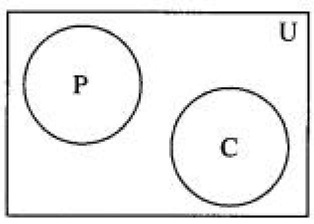
## Maharashtra State Board 12th Commerce Maths Solutions Chapter 1 Mathematical Logic Ex 1.10

Question 1.
Express the truth of each of the following statements by Venn diagrams:
(i) Some hardworking students are obedient.
Solution:
Let U : set of all students
S : set of all hardworking students
O : set of all obedient students.
Then the Venn diagram represents the truth of the given statement is as below:



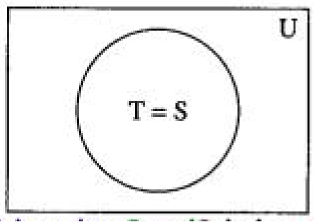

S ∩ O ≠ φ

(ii) No circles are polygons.
Solution:
Let U : set of closed geometrical figures in the plane
P : set of all polygons
C : set of all circles.
Then the Venn diagram represents the truth of the given statement is as follows:

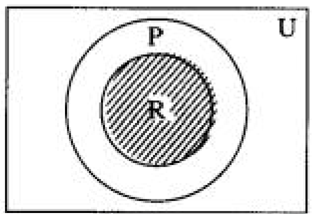



P ∩ C = φ

(iii) All teachers are scholars and scholars are teachers.
Solution:
Let U : set of all human beings
T : set of all teachers
S : set of all scholars.
Then the Venn diagram represents the truth of the given statement is as below:

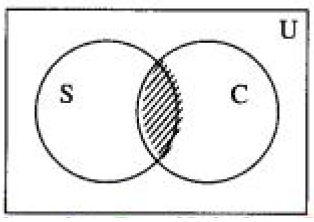



(iv) If a quadrilateral is a rhombus, then it is a parallelogram.
Solution:
Let U : set of all quadrilaterals
R : set of all rhombus
P : set of all parallelograms.
Then the Venn diagram represents the truth of the given statement is as below:



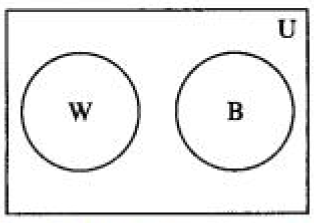

$R \subset P$

Question 2.
Draw the Venn diagrams for the truth of the following statements:
(i) Some share brokers are chartered accountants.
Solution:
Let U : set of all human beings
S : set of all share brokers
C : set of all chartered accountants.
Then the Venn diagram represents the truth of the given statement is as below:



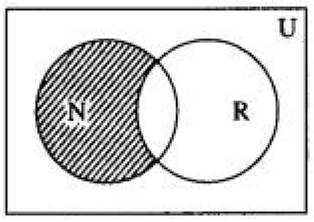

$S \cap C \neq \varphi$

(ii) No wicket-keeper is a bowler in a cricket team.
Solution:
Let U : set of all human beings
W : set of all wicket keepers
B : set of all bowlers.
Then the Venn diagram represents the truth of the given statement is as follows:



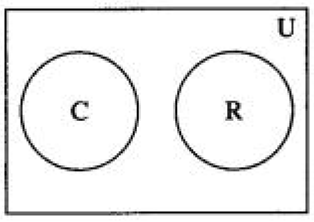

$W \cap B = \varphi$

Question 3.
Represent the following statements by Venn diagrams:
(i) Some non-resident Indians are not rich.
Solution:
Let U : set of all human beings
N : set of all non-resident Indians
R : set of all rich people.
Then the Venn diagram represents the truth of the given statement is as below:



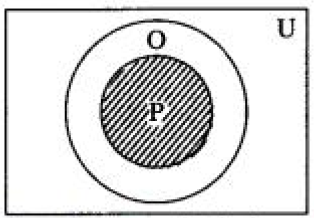

N – R ≠ φ

(ii) No circle is a rectangle.
Solution:
Let U : set of all geometrical figures
C : set of all circles
R : set of all rectangles
Then the Venn diagram represents the truth of the given statement is as below:




C ∩ R = φ

(iii) If n is a prime number and n ≠ 2, then it is odd.
Solution:
Let U : set of all real numbers
P : set of all prime numbers n, where n ≠ 2
O : set of all odd numbers.
Then the Venn diagram represents the truth of the given statement is as below:




P ⊂ O

## Maharashtra State Board 12th Commerce Maths Solutions Chapter 1 Mathematical Logic Miscellaneous Exercise 1

(I) Choose the correct alternative:

Question 1.
Which of the following is not a statement?
(a) Smoking is injurious to health
(b) 2 + 2 = 4
(c) 2 is only even prime number
(d) Come here
Answer:
(d) Come here

Question 2.
Which of the following is an open statement?
(a) x is a natural number
(b) Give me a glass of water
(c) Wish you best of luck
(d) Good morning to all

Answer:
(a) x is a natural number

Question 3.
Let $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$. Then this law is known as
(a) Commutative law
(b) Associative law.
(c) De Morgan's law
(d) Distributive law
Answer:
(d) Distributive law

Question 4.
The false statement in the following is:
(a) $p \wedge (\sim p)$ is a contradiction
(b) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a contradiction
(c) $\sim(\sim p) \leftrightarrow p$ is a tautology
(d) $p \vee (\sim p) \leftrightarrow p$ is a tautology.
Answer:
(b) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a contradiction

Question 5.
Consider the following three statements
p : 2 is an even number.
q : 2 is a prime number.
r : Sum of two prime numbers is always even.
Then, the symbolic statement $(p \wedge q) \rightarrow \sim r$ means:
(a) 2 is an even and prime number and the sum of two prime numbers is always even.
(b) 2 is an even and prime number and the sum of two prime numbers is not always even.
(c) If 2 is an even and prime number, then the sum of two prime numbers is not always even.
(d) If 2 is an even and prime number, then the sum of two prime numbers is also even.
Answer:
(c) If 2 is an even and prime number, then the sum of two prime numbers is not always even.

Question 6.
If p : He is intelligent.
q : He is strong.
Then, symbolic form of statement: 'It is wrong that, he is intelligent or strong' is
(a) $\sim p \vee \sim p$
(b) $\sim(p \wedge q)$
(c) $\sim(p \vee q)$
(d) $p \vee \sim q$
Answer:
(c) $\sim(p \vee q)$

Question 7.
The negation of the proposition 'If 2 is prime, then 3 is odd', is
(a) If 2 is not prime, then 3 is not odd
(b) 2 is prime and 3 is not odd
(c) 2 is not prime and 3 is odd
(d) If 2 is not prime, then 3 is odd
Answer:
(b) 2 is prime and 3 is not odd

Question 8.
The statement $(\sim p \wedge q) \vee \sim q$ is
(a) $p \vee q$
(b) $p \wedge q$
(c) $\sim(p \vee q)$
(d) $\sim(p \wedge q)$
Answer:
(d) $\sim(p \wedge q)$
Hint:
$(\sim p \wedge q) \vee \sim q = (\sim p \vee \sim q) \wedge (q \vee \sim q)$
$= (\sim p \vee \sim q) \wedge t$
$= \sim p \vee \sim q$
$= \sim(p \wedge q)$

Question 9.
Which of the following is always true?
(a) $\sim(p \rightarrow q) \equiv \sim q \rightarrow \sim p$
(b) $\sim(p \vee q) \equiv \sim p \vee \sim q$
(c) $\sim(p \rightarrow q) \equiv p \wedge \sim q$
(d) $\sim(p \wedge q) \equiv \sim p \wedge \sim q$
Answer:
(c) $\sim(p \rightarrow q) \equiv p \wedge \sim q$

Question 10.
$\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to
(a) $\sim p$
(b) $p$
(c) $q$
(d) $\sim q$
Answer:
(a) $\sim p$
Hint:
$\sim(p \vee q) \vee (\sim p \wedge q) \equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$
$\equiv \sim p \wedge (\sim q \vee q)$
$\equiv \sim p \wedge t$
$\equiv \sim p$

Question 11.
If p and q are two statements, then $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is
(a) contradiction
(b) tautology
(c) neither (a) nor (b)
(d) none of these
Answer:
(b) tautology

Question 12.
If p is the sentence 'This statement is false', then
(a) truth value of p is T
(b) truth value of p is F
(c) p is both true and false
(d) p is neither true nor false
Answer:
(d) p is neither true nor false

Question 13.
Conditional $p \rightarrow q$ is equivalent to
(a) $p \rightarrow \sim q$
(b) $\sim p \vee q$
(c) $\sim p \rightarrow \sim q$
(d) $p \vee \sim q$
Answer:
(b) $\sim p \vee q$

Question 14.
Negation of the statement 'This is false or That is true' is
(a) That is true or This is false
(b) That is true and This is false
(c) This is true and That is false
(d) That is false and That is true
Answer:
(c) This is true and That is false

Question 15.
If p is any statement, then $(p \vee \sim p)$ is a
(a) contingency
(b) contradiction
(c) tautology
(d) none of them
Answer:
(c) tautology

(II) Fill in the blanks:

Question 1.
The statement q → p is called as the __________ of the statement p → q.
Answer:
Converse

Question 2.
Conjunction of two statements p and q is symbolically written as
Answer:
$p \wedge q$

Question 3.
If $p \vee q$ is true, then truth value of $\sim p \vee \sim q$ is __________
Answer:
False

Question 4.
Negation of 'some men are animal' is __________
Answer:
All men are not animal.
OR
No men are animals.

Question 5.
Truth value of if x = 2, then $x^2 = -4$ is __________
Answer:
False

Question 6.
Inverse of statement pattern p → q is given by __________
Answer:
~p → ~q

Question 7.
p ↔ q is false when p and q have __________ truth values.
Answer:
Different

Question 8.
Let p : The problem is easy. r : It is not challenging. Then verbal form of ~p → r is __________
Answer:
If the problem is not easy, then it is not challenging.

Question 9.
Truth value of 2 + 3 = 5 if and only if -3 > -9 is __________
Answer:
T [Hint: T ↔ T = T]

(III) State whether each of the following is True or False:

Question 1.
Truth value of 2 + 3 < 6 is F.
Answer:
False

Question 2.
There are 24 months in a year is a statement.
Answer:
True

Question 3.
$p \wedge q$ has truth value F if both p and q have truth value F.
Answer:
False

Question 4.
The negation of 10 + 20 = 30 is, it is false that 10 + 20 ≠ 30.
Answer:
False

Question 5.
Dual of $(p \wedge \sim q) \vee t$ is $(p \vee \sim q) \vee c$.
Answer:
False

Question 6.
Dual of 'John and Ayub went to the forest' is 'John or Ayub went to the forest.'
Answer:
True

Question 7.
'His birthday is on 29th February' is not a statement.
Answer:
True

Question 8.
$x^2 = 25$ is true statement.
Answer:
False

Question 9.
The truth value of '$\sqrt{5}$ is not an irrational number' is T.
Answer:
False

Question 10.
$p \wedge t = p$.
Answer:
True

(IV) Solve the following:

Question 1.
State which of the following sentences are statements in logic:
(i) Ice cream Sundaes are my favourite.
Solution:
It is a statement.

(ii) $x + 3 = 8$, x is variable.
Solution:
It is a statement.

(iii) Read a lot to improve your writing skill.
Solution:
It is an imperative sentence, hence it is not a statement.

(iv) z is a positive number.
Solution:
It is an open sentence, hence it is not a statement.

(v) $(a + b)^2 = a^2 + 2ab + b^2$ for all $a, b \in R$.
Solution:
It is a statement.

(vi) $(2 + 1)^2 = 9$.
Solution:
It is a statement.

(vii) Why are you sad?
Solution:
It is an interrogative sentence, hence it is not a statement.

(viii) How beautiful the flower is!
Solution:
It is an exclamatory sentence, hence it is not a statement.

(ix) The square of any odd number is even.
Solution:
It is a statement.

(x) All integers are natural numbers.
Solution:
It is a statement.

(xi) If x is a real number, then $x^2 \geq 0$.
Solution:
It is a statement.

(xii) Do not come inside the room.
Solution:
It is an imperative sentence, hence it is not a statement.

(xiii) What a horrible sight it was!
Solution:
It is an exclamatory sentence, hence it is not a statement.

Question 2.
Which of the following sentences are statements? In case of a statement, write down the truth value:
(i) What is a happy ending?
Solution:
It is an interrogative sentence, hence it is not a statement.

(ii) The square of every real number is positive.
Solution:
It is a statement that is false, hence its truth value is F.

(iii) Every parallelogram is a rhombus.
Solution:
It is a statement that is true, hence its truth value is T.

(iv) $a^2 - b^2 = (a + b)(a - b)$ for all $a, b \in R$.
Solution:
It is a mathematical identity that is true, hence its truth value is T.

(v) Please carry out my instruction.
Solution:
It is an imperative sentence, hence it is not a statement.

(vi) The Himalayas is the highest mountain range.
Solution:
It is a statement that is true, hence its truth value is T.

(vii) $(x - 2)(x - 3) = x^2 - 5x + 6$ for all $x \in R$.
Solution:
It is a mathematical identity that is true, hence its truth value is T.

(viii) What are the causes of rural unemployment?
Solution:
It is an interrogative sentence, hence it is not a statement.

(ix) $0! = 1$.
Solution:
It is a statement that is true, hence its truth value is T.

(x) The quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ always has two real roots.
Solution:
It is a statement that is false, hence its truth value is F.

Question 3.
Assuming the first statement as p and second as q, write the following statements in symbolic form:
(i) The Sun has set and Moon has risen.
Solution:
Let p : The Sun has set.
q : Moon has risen.
Then the symbolic form of the given statement is $p \wedge q$.

(ii) Mona likes Mathematics and Physics.
Solution:
Let p : Mona likes Mathematics.
q : Mona likes Physics.
Then the symbolic form of the given statement is $p \wedge q$.

(iii) 3 is a prime number if 3 is a perfect square number.
Solution:
Let p : 3 be a prime number.
q : 3 is a perfect square number.
Then the symbolic form of the given statement is $p \leftrightarrow q$.

(iv) Kavita is brilliant and brave.
Solution:
Let p : Kavita is brilliant.
q : Kavita is brave.
Then the symbolic form of the given statement is $p \wedge q$.

(v) If Kiran drives a car, then Sameer will walk.
Solution:
Let p : Kiran drives a car.
q : Sameet will walk.
Then the symbolic form of the given statement is $p \rightarrow q$.

(vi) The necessary condition for the existence of a tangent to the curve of the function is continuity.
Solution:
The given statement can be written as:
'If the function is continuous, then the tangent to the curve exists.'
Let p : The function is continuous.
q : Tangent to the curve exists.
Then the symbolic form of the given statement is $p \rightarrow q$.

(vii) To be brave is necessary and sufficient condition to climb Mount Everest.
Solution:
Let p : To be brave.
q : Climb Mount Everest.
Then the symbolic form of the given statement is $p \leftrightarrow q$.

(viii) $x^3 + y^3 = (x + y)^3$, iff $xy = 0$.
Solution:
Let p : $x^3 + y^3 = (x + y)^3$.
q : $xy = 0$.
Then the symbolic form of the given statement is $p \leftrightarrow q$.

(ix) The drug is effective though it has side effects.
Solution:
Let p : The drug is effective.
q : It has side effects.
Then the symbolic form of the given statement is $p \wedge q$.

(x) If a real number is not rational, then it must be irrational.
Solution:
Let p : A real number is not rational.
q : It must be irrational.
Then the symbolic form of the given statement is $p \rightarrow q$.

(xi) It is not true that Ram is tall and handsome.
Solution:
Let p : Ram is tall.
q : Ram is handsome.
Then the symbolic form of the given statement is $\sim(p \wedge q)$.

(xii) Even though it is not cloudy, it is still raining.
Solution:
The given statement is equivalent to:
It is not cloudy and it is still raining,
Let p : It is not cloudy.
q : It is still raining.
Then the symbolic form of the given statement is $p \wedge q$.

(xiii) It is not true that intelligent persons are neither polite nor helpful.
Solution:
Let p : Intelligent persons are neither polite nor helpful.
Then the symbolic form of the given statement is ~p.

(xiv) If the question paper is not easy, then we shall not pass.
Solution:
Let p : The question paper is not easy.
q : We shall not pass.
Then the symbolic form of the given statement is p → q.

Question 4.
If p : Proof is lengthy.
q : It is interesting.
Express the following statements in symbolic form:
(i) Proof is lengthy and it is not interesting.
(ii) If the proof is lengthy, then it is interesting.
(iii) It is not true that the proof is lengthy but it is interesting.
(iv) It is interesting iff the proof is lengthy.
Solution:
The symbolic form of the given statements are:
(i) p ∧ ~q
(ii) p → q
(iii) ~(p ∧ q)
(iv) q ↔ p

Question 5.
Let p : Sachin win the match.
q : Sachin is a member of the Rajya Sabha.
r : Sachin is happy.
Write the verbal statement for each of the following:
(i) (p ∧ q) ∨ r
Solution:
Sachin wins the match and he is a member of the Rajya Sabha or Sachin is happy.

(ii) p → r
Solution:
If Sachin wins the match, then he is happy.

(iii) ~p ∨ q
Solution:
Sachin does not win the match or he is a member of the Rajya Sabha.

(iv) p → (q ∨ r)
Solution:
If Sachin wins the match, then he is a member of the Rajya Sabha or he is happy.

(v) p → q
Solution:
If Sachin wins the match, then he is a member of the Rajya Sabha.

(vi) (p ∧ q) ∧ ~r
Solution:
Sachin wins the match and he is a member of the Rajya Sabha but he is not happy.

(vii) ~(p ∨ q) ∧ r
Solution:
It is false that Sachin wins the match or he is a member of the Rajya Sabha but he is happy.

Question 6.
Determine the truth values of the following statements:
(i) 4 + 5 = 7 or 9 – 2 = 5.
Solution:
Let p : 4 + 5 = 7.
q : 9 – 2 = 5.
Then the symbolic form of the given statement is p ∨ q.
The truth values of both p and q are F.
∴ the truth value of p ∨ q is F. …….[F ∨ F ≡ F]

(ii) If 9 > 1, then $x^2 – 2x + 1 = 0$ for x = 1.
Solution:

Let p : 9 > 1.
q : $x^2 - 2x + 1 = 0$ for x = 1.
Then the symbolic form of the given statement is $p \to q$.
The truth values of both p and q are T.
∴ the truth value of $p \to q$ is T. …..[$T \to T \equiv T$]

(iii) x + y = 0 is the equation of a straight line if and only if $y^2 = 4x$ is the equation of the parabola.
Solution:
Let p : x + y = 0 is the equation of a straight line.
q : $y^2 = 4x$ is the equation of the parabola.
Then the symbolic form of the given statement is $p \leftrightarrow q$.
The truth values of both p and q are T.
∴ the truth value of $p \leftrightarrow q$ is T. …..[$T \leftrightarrow T \equiv T$]

(iv) It is not true that 2 + 3 = 6 or 12 + 3 = 5.
Solution:
Let p : 2 + 3 = 6.
q : 12 + 3 = 5.
Then the symbolic form of the given statement is $\sim(p \vee q)$.
The truth values of both p and q are F.
∴ the truth value of $\sim(p \vee q)$ is T. …..[$\sim(F \vee F) \equiv \sim F \equiv T$]

Question 7.
Assuming the following statements
p : Stock prices are high.
q : Stocks are rising.
to be true, find the truth values of the following:

(i) Stock prices are not high or stocks are rising.
Solution:
p and q are true, i.e. T.
∴ ~p and ~q are false, i.e. F.
The given statement in symbolic form is $\sim p \vee q$.
Since, $\sim T \vee T \equiv F \vee T \equiv T$, the given statement is true.
Hence, its truth value is 'T'.

(ii) Stock prices are high and stocks are rising if and only if stock prices are high.
Solution:
The given statement in symbolic form is $(p \wedge q) \leftrightarrow p$.
Since $(T \wedge T) \leftrightarrow T \equiv T \leftrightarrow T \equiv T$, the given statement is true.
Hence, its truth value is 'T'.

(iii) If stock prices are high, then stocks are not rising.
Solution:
The given statement in symbolic form is $p \to \sim q$.
Since, $T \to \sim T \equiv T \to F \equiv F$, the given statement is false.
Hence, its truth value is 'F'.

(iv) It is false that stocks are rising and stock prices are high.
Solution:
The given statement in symbolic form is $\sim(q \wedge p)$.
Since, $\sim(T \wedge T) \equiv \sim T \equiv F$, the given statement is false.
Hence, its truth value is 'F'.

(v) Stock prices are high or stocks are not rising iff stocks are rising.
Solution:
The given statement in symbolic form is $(p \vee \sim q) \leftrightarrow q$.
Since $(T \vee \sim T) \leftrightarrow T \equiv (T \vee F) \leftrightarrow T$
$\equiv T \leftrightarrow T$
$\equiv T$, the given statement is true.
Hence, its truth value is 'T'.

Question 8.
Rewrite the following statements without using conditional:
[Hint: $P \to q \equiv \sim p \vee q$]
(i) If price increases, then demand falls.
(ii) If demand falls, then the price does not increase.
Solution:
Since, $p \to q \equiv \sim p \vee q$, the given statements can be written as:
(i) Price does not increase or demand falls.
(ii) Demand does not fall or price does not increase.

Question 9.
If p, q, r are statements with truth values T, T, F respectively, determine the truth values of the following:
(i) $(p \wedge q) \rightarrow \sim p$
Solution:
Truth values of p, q, r are T, T, F respectively.
$(p \wedge q) \rightarrow \sim p \equiv (T \wedge T) \rightarrow \sim T$
$\equiv T \rightarrow F$
$\equiv F$
Hence, the truth value of the given statement is false, i.e. F.

(ii) $p \leftrightarrow (q \rightarrow \sim p)$
Solution:
$p \leftrightarrow (q \rightarrow \sim p) \equiv T \leftrightarrow (T \rightarrow \sim T)$
$\equiv T \leftrightarrow (T \rightarrow F)$
$\equiv T \leftrightarrow F$
$\equiv F$
Hence, the truth value of the given statement is false, i.e. F.

(iii) $(p \wedge \sim q) \vee (\sim p \wedge q)$
Solution:
$(p \wedge \sim q) \vee (\sim p \wedge q) \equiv (T \wedge \sim T) \vee (\sim T \wedge T)$
$\equiv (T \wedge F) \vee (F \wedge T)$
$\equiv F \vee F$
$\equiv F$
Hence, the truth value of the given statement is false, i.e. F.

(iv) $\sim(p \wedge q) \rightarrow \sim(q \wedge p)$
Solution:
$\sim(p \wedge q) \rightarrow \sim(q \wedge p) \equiv \sim(T \wedge T) \rightarrow \sim(T \wedge T)$
$\equiv \sim T \rightarrow \sim T$
$\equiv F \rightarrow F$
$\equiv T$
Hence, the truth value of the given statement is true, i.e. T.

(v) $\sim[(p \rightarrow q) \leftrightarrow (p \wedge \sim q)]$
Solution:
$\sim[(p \rightarrow q) \leftrightarrow (p \wedge \sim q)]$
$\equiv \sim[(T \rightarrow T) \leftrightarrow (T \wedge \sim T)]$
$\equiv \sim[T \leftrightarrow (T \wedge F)]$
$\equiv \sim[T \leftrightarrow F]$
$\equiv \sim F$
$\equiv T$.
Hence, the truth value of the given statement is true, i.e. T.

Question 10.
Write the negations of the following:
(i) If ΔABC is not equilateral, then it is not equiangular.
Solution:
Let p : ΔABC is not equilateral.
q : It is not equiangular.
Then the symbolic form of the given statement is $p \rightarrow q$.
Since, $\sim(p \rightarrow q) \equiv p \wedge \sim q$, the negation of the given statement is:
'ΔABC is not equilateral and it is equiangular.'

(ii) Ramesh is intelligent and he is hard working.
Solution:
Let p : Ramesh is intelligent.
q : He is hard working.
Then the symbolic form of the given statement is $p \wedge q$.
Since, $\sim(p \wedge q) \equiv \sim p \vee \sim q$, the negation of the given statement is:
'Ramesh is not intelligent or he is not hard-working.'

(iii) A angle is a right angle if and only if it is of measure 90°.
Solution:
Let p : An angle is a right angle.
q : It is of measure 90°.
Then the symbolic form of the given statement is $p \leftrightarrow q$.
Since, $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$, the negation of the given statement is:
'An angle is a right angle and it is not of measure 90° or an angle is of measure 90° and it is not a right angle.'

(iv) Kanchanjunga is in India and Everest is in Nepal.
Solution:
Let p : Kanchenjunga is in India.
q : Everest is in Nepal.
Then the symbolic form of the given statement is $p \wedge q$.
Since, $\sim(p \wedge q) \equiv \sim p \vee \sim q$, the negation of the given statement is:
'Kanchenjunga is not in India or Everest is not in Nepal.'

(v) If $x \in A \cap B$, then $x \in A$ and $x \in B$.
Solution:
Let $p : x \in A \cap B$, $q : x \in A$, $r : x \in B$.
Then the symbolic form of the given statement is $P \rightarrow (q \wedge r)$
Since, $\sim(p \rightarrow q) \equiv p \wedge \sim q$ and $\sim(p \wedge q) = \sim p \vee \sim q$,
the negation of the given statement is:
'$x \in A \cap B$ and $x \notin A$ or $x \notin B$.

Question 11.
Construct the truth table for each of the following statement patterns:
(i) $(p \wedge \sim q) \leftrightarrow (q \rightarrow p)$
Solution:
$(p \wedge \sim q) \leftrightarrow (q \rightarrow p)$

| $p$ | $q$ | $\sim q$ | $p \wedge \sim q$ | $q \rightarrow p$ | $(p \wedge \sim q) \leftrightarrow (q \rightarrow p)$ |
|---|---|---|---|---|---|
| T | T | F | F | T | F |
| T | F | T | T | T | T |
| F | T | F | F | F | T |
| F | F | T | F | T | F |

(ii) $(\sim p \vee q) \wedge (\sim p \wedge \sim q)$
Solution:
$(\sim p \vee q) \wedge (\sim p \wedge \sim q)$

| $p$ | $q$ | $\sim p$ | $\sim q$ | $\sim p \vee q$ | $\sim p \wedge \sim q$ | $(\sim p \vee q) \wedge (\sim p \wedge \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F |
| T | F | F | T | F | F | F |
| F | T | T | F | T | F | F |
| F | F | T | T | T | T | T |

(iii) $(p \wedge r) \rightarrow (p \vee \sim q)$
Solution:
$(p \wedge r) \rightarrow (p \vee \sim q)$

| $p$ | $q$ | $r$ | $\sim q$ | $p \wedge r$ | $p \vee \sim q$ | $(p \wedge r) \rightarrow (p \vee \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | T | F | T | T | T |
| T | T | F | F | F | T | T |
| T | F | T | T | T | T | T |
| T | F | F | T | F | T | T |
| F | T | T | F | F | F | T |
| F | T | F | F | F | F | T |
| F | F | T | T | F | T | T |
| F | F | F | T | F | T | T |

(iv) $(p \vee r) \rightarrow \sim(q \wedge r)$
Solution:

(p ∨ r) → ~(q ∧ r)

| $p$ | $q$ | $r$ | $p \vee r$ | $q \wedge r$ | $\sim(q \wedge r)$ | $(p \vee r) \to \sim(q \wedge r)$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | F | F |
| T | T | F | T | F | T | T |
| T | F | T | T | F | T | T |
| T | F | F | T | F | T | T |
| F | T | T | T | T | F | F |
| F | T | F | F | F | T | T |
| F | F | T | T | F | T | T |
| F | F | F | F | F | T | T |

(v) (p ∨ ~q) → (r ∧ p)
Solution:
(p ∨ ~q) → (r ∧ p)

| $p$ | $q$ | $r$ | $\sim q$ | $p \vee \sim q$ | $r \wedge p$ | $(p \vee \sim q) \to (r \wedge p)$ |
|---|---|---|---|---|---|---|
| T | T | T | F | T | T | T |
| T | T | F | F | T | F | F |
| T | F | T | T | T | T | T |
| T | F | F | T | T | F | F |
| F | T | T | F | F | F | T |
| F | T | F | F | F | F | T |
| F | F | T | T | T | F | F |
| F | F | F | T | T | F | F |

Question 12.
What is a tautology? What is a contradiction? Show that the negation of a tautology is a contradiction and the negation of a contradiction is a tautology.
Solution:
Tautology: A statement pattern that has all the entries in the last column of its truth table as T is called a tautology.
For example:

| $p$ | $\sim p$ | $p \vee \sim p$ |
|---|---|---|
| T | F | T |
| F | T | T |

In the above truth table for the statement p ∨ ~p,
we observe that all the entries in the last column are T.
Hence, the statement p ∨ ~p is a tautology.

Contradiction: A statement pattern that has all the entries in the last column of its truth table as F is called a contradiction.
For example:

| $p$ | $\sim p$ | $p \wedge \sim p$ |
|---|---|---|
| T | F | F |
| F | T | F |

In the above truth table for the statement p ∧ ~p,
we observe that all the entries in the last column are F.
Hence, the statement p ∧ ~p is a contradiction.

To show that the negation of a tautology is a contradiction and vice versa:
A tautology is true on every row of its truth table.
Since, ~T = F and ~F = T, when we negate a tautology, the resulting statement is false on every row of its table.

i.e. the negation of tautology is a contradiction.
Similarly, the negation of a contradiction is a tautology.

Question 13.
Determine whether the following statement patterns is a tautology or a contradiction or a contingency:
(i) [(p ∧ q) ∨ (~p)] ∨ [p ∧ (~q)]
Solution:
[(p ∧ q) ∨ (~p)] ∨ [p ∧ (~q)]

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \wedge q$ | $(p \wedge q) \vee (\sim p)$ | $p \wedge (\sim q)$ | $[(p \wedge q) \vee (\sim p)] \vee [(p \wedge \sim q)]$ |
|---|---|---|---|---|---|---|---|
| T | T | F | F | T | T | F | T |
| T | F | F | T | F | F | T | T |
| F | T | T | F | F | T | F | T |
| F | F | T | T | F | T | F | T |

All the entries in the last column of the above truth table are T.
∴ [(p ∧ q) ∨ (~p)] ∨ [p ∧ (~q)] is a tautology.

(ii) [(~p ∧ q) ∧ (q ∧ r)] ∨ (~q)
Solution:
[(~p ∧ q) ∧ (q ∧ r)] ∨ (~q)

| $p$ | $q$ | $r$ | $\sim p$ | $\sim q$ | $\sim p \wedge q$ | $q \wedge r$ | $(\sim p \wedge q) \wedge (q \wedge r)$ | $[(\sim p \wedge q) \wedge (q \wedge r)] \vee (\sim q)$ |
|---|---|---|---|---|---|---|---|---|
| T | T | T | F | F | F | T | F | F |
| T | T | F | F | F | F | F | F | F |
| T | F | T | F | T | F | F | F | T |
| T | F | F | F | T | F | F | F | T |
| F | T | T | T | F | T | T | T | T |
| F | T | F | T | F | T | F | F | F |
| F | F | T | T | T | F | F | F | T |
| F | F | F | T | T | F | F | F | T |

The entries in the last column of the above truth table are neither all T nor all F.
∴ [(~p ∧ q) ∧ (q ∧ r)] ∨ (~q) is a contingency.

(iii) [~(p ∨ q) → p] ↔ [(~p) ∧ (~q)]
Solution:
[~(p ∨ q) → p] ↔ [(~p) ∧ (~q)]

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \vee q$ | $\sim (p \vee q)$ | $\sim (p \vee q) \to p$ | $\sim p \wedge \sim q$ | $[\sim (p \vee q) \to p] \leftrightarrow [(\sim p) \wedge (\sim q)]$ |
|---|---|---|---|---|---|---|---|---|
| T | T | F | F | T | F | T | F | F |
| T | F | F | T | T | F | T | F | F |
| F | T | T | F | T | F | T | F | F |
| F | F | T | T | F | T | F | T | F |

All the entries in the last column of the above truth table are F.
∴ [~(p ∨ q) → p] ↔ [(~p) ∧ (~q)] is a contradiction.

(iv) [~(p ∧ q) → p] ↔ [(~p) ∧ (~q)]
Solution:
[~(p ∧ q) → p] ↔ [(~p) ∧ (~q)]

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \wedge q$ | $\sim (p \wedge q)$ | $\sim (p \wedge q) \to p$ | $\sim p \wedge \sim q$ | $[\sim (p \wedge q) \to p] \leftrightarrow [(\sim p) \wedge (\sim q)]$ |
|---|---|---|---|---|---|---|---|---|
| T | T | F | F | T | F | T | F | F |
| T | F | F | T | F | T | T | F | F |
| F | T | T | F | F | T | F | F | T |
| F | F | T | T | F | T | F | T | F |

The entries in the last column of the above truth table are neither all T nor all F.
∴ [~(p ∧ q) → p] ↔ [(~p) ∧ (~q)] is a contingency.

(v) [p → (~q ∨ r)] ↔ ~[p → (q → r)]
Solution:
[p → (~q ∨ r)] ↔ ~[p → (q → r)]

| $p$ | $q$ | $r$ | $\sim q$ | $\sim q \vee r$ | $p \to (\sim q \vee r)$ (I) | $q \to r$ | $p \to (q \to r)$ | $\sim[p \to (q \to r)]$ (II) | (I) ↔ (II) |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | F | T | T | T | T | F | F |
| T | T | F | F | F | F | F | F | T | F |
| T | F | T | T | T | T | T | T | F | F |
| T | F | F | T | T | T | T | T | F | F |
| F | T | T | F | T | T | T | T | F | F |
| F | T | F | F | F | T | F | T | F | F |
| F | F | T | T | T | T | T | T | F | F |
| F | F | F | T | T | T | T | T | F | F |

All the entries in the last column of the above truth table are F.
∴ [p → (~q ∨ r)] ↔ ~[p → (q → r)] is a contradiction.

Question 14.
Using the truth table, prove the following logical equivalences:
(i) p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r)
Solution:
p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r)

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| $p$ | $q$ | $r$ | $q \vee r$ | $p \wedge (q \vee r)$ | $p \wedge q$ | $p \wedge r$ | $(p \wedge q) \vee (p \wedge r)$ |
| T | T | T | T | T | T | T | T |
| T | T | F | T | T | T | F | T |
| T | F | T | T | T | F | T | T |
| T | F | F | F | F | F | F | F |
| F | T | T | T | F | F | F | F |
| F | T | F | T | F | F | F | F |
| F | F | T | T | F | F | F | F |
| F | F | F | F | F | F | F | F |

The entries in columns 5 and 8 are identical.
∴ p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r)

(ii) [~(p ∨ q) ∨ (p ∨ q)] ∧ r ≡ r
Solution:
[~(p ∨ q) ∨ (p ∨ q)] ∧ r ≡ r

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| $p$ | $q$ | $r$ | $p \vee q$ | $\sim(p \vee q)$ | $\sim(p \vee q) \vee (p \vee q)$ | $[\sim(p \vee q) \vee (p \vee q)] \wedge r$ |
| T | T | T | T | F | T | T |
| T | T | F | T | F | T | F |
| T | F | T | T | F | T | T |
| T | F | F | T | F | T | F |
| F | T | T | T | F | T | T |
| F | T | F | T | F | T | F |
| F | F | T | F | T | T | T |
| F | F | F | F | T | T | F |

The entries in columns 3 and 7 are identical.
∴ [~(p ∨ q) ∨ (p ∨ q)] ∧ r ≡ r

(iii) p ∧ (~p ∨ q) ≡ p ∧ q
Solution:
p ∧ (~p ∨ q) ≡ p ∧ q

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $p$ | $q$ | $\sim p$ | $\sim p \vee q$ | $p \wedge (\sim p \vee q)$ | $p \wedge q$ |
| T | T | F | T | T | T |
| T | F | F | F | F | F |
| F | T | T | T | F | F |
| F | F | T | T | F | F |

The entries in columns 5 and 6 are identical.
∴ $p \wedge (\sim p \vee q) \equiv p \wedge q$

(iv) $p \leftrightarrow q \equiv \sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)$
Solution:
$p \leftrightarrow q \equiv \sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)$

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| $p$ | $q$ | $p \leftrightarrow q$ | $\sim p$ | $\sim q$ | $p \wedge \sim q$ | $\sim(p \wedge \sim q)$ | $q \wedge \sim p$ | $\sim(q \wedge \sim p)$ | $\sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)$ |
| T | T | T | F | F | F | T | F | T | T |
| T | F | F | F | T | T | F | F | T | F |
| F | T | F | T | F | F | T | T | F | F |
| F | F | T | T | T | F | T | F | T | T |

The entries in columns 3 and 10 are identical.
∴ $p \leftrightarrow q \equiv \sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)$

(v) $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$
Solution:
$\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $p$ | $q$ | $\sim p$ | $\sim p \wedge q$ | $p \vee q$ | $(p \vee q) \wedge \sim p$ |
| T | T | F | F | T | F |
| T | F | F | F | T | F |
| F | T | T | T | T | T |
| F | F | T | F | F | F |

The entries in columns 4 and 6 are identical.
∴ $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

Question 15.
Write the converse, inverse, contrapositive of the following statements:
(i) If 2 + 5 = 10, then 4 + 10 = 20.
Solution:
Let p : 2 + 5 = 10.
q : 4 + 10 = 20.
Then the symbolic form of the given statement is $p \rightarrow q$.
Converse: $q \rightarrow p$ is the converse of $p \rightarrow q$
i.e. If 4 + 10 = 20, then 2 + 5 = 10.
Inverse: $\sim p \rightarrow \sim q$ is the inverse of $p \rightarrow q$
i.e. If 2 + 5 ≠ 10, then 4 + 10 ≠ 20.
Cotrapositive: $\sim q \rightarrow \sim p$ is the contrapositive of $p \rightarrow q$,
i.e. If 4 +10 ≠ 20, then 2 + 5 ≠ 10.

(ii) If a man is a bachelor, then he is happy.
Solution:
Let p : A man is a bachelor.
q : He is happy.
Then the symbolic form of the given statement is $p \rightarrow q$.
Converse: $q \rightarrow p$ is the converse of $p \rightarrow q$
i.e. If a man is happy, then he is a bachelor.
Inverse: $\sim p \rightarrow \sim q$ is the inverse of $p \rightarrow q$
i.e. If a man is not a bachelor, then he is not happy.
Contrapositive: $\sim q \rightarrow \sim p$ is the contrapositive of $p \rightarrow q$
i.e., If a man is not happy, then he is not a bachelor.

(iii) If I do not work hard, then I do not prosper.
Solution:
Let p : I do not work hard.
q : I do not prosper.
Then the symbolic form of the given statement is $p \rightarrow q$.
Converse: $q \rightarrow p$ is the converse of $p \rightarrow q$
i.e. If I do not prosper, then I do not work hard.
Inverse: $\sim p \rightarrow \sim q$ is the inverse of $p \rightarrow q$
i.e. If I work hard, then I prosper.
Contrapositive: $\sim q \rightarrow \sim p$ is the contrapositive of $p \rightarrow q$
i.e. If I prosper, then I work hard.

Question 16.
State the dual of each of the following statements by applying the principle of duality:
(i) $(p \wedge \sim q) \vee (\sim p \wedge q) \equiv (p \vee q) \wedge \sim(p \wedge q)$
(ii) $p \vee (q \vee r) \equiv \sim[(p \wedge q) \vee (r \vee s)]$
(iii) 2 is an even number or 9 is a perfect square.
Solution:
The duals are given by:
(i) $(p \vee \sim q) \wedge (\sim p \vee q) \equiv (p \wedge q) \vee \sim(p \vee q)$
(ii) $p \wedge (q \wedge r) \equiv \sim[(p \vee q) \wedge (r \wedge s)]$
(iii) 2 is an even number and 9 is a perfect square.

Question 17.
Rewrite the following statements without using the connective 'If … then':
(i) If a quadrilateral is a rhombus, then it is not a square.
(ii) If $10 - 3 = 7$, then $10 \times 3 \neq 30$.
(iii) If it rains, then the principal declares a holiday.
Solution:
Since, $p \rightarrow q \equiv \sim p \vee q$ the given statements can be written as:
(i) A quadrilateral is not a rhombus or it is not a square.
(ii) $10 - 3 \neq 7$ or $10 \times 3 \neq 30$.
(iii) It does not rain or the principal declares a holiday.

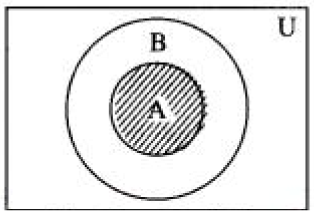
Question 18.
Write the dual of each of the following:
(i) $(\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$
(ii) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
(iii) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
(iv) $\sim(p \vee q) \equiv \sim p \wedge \sim q$.
Solution:
The duals are given by:
(i) $(\sim p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee \sim q)$
(ii) $(p \vee q) \vee r \equiv p \vee (q \vee r)$
(iii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
(iv) $\sim(p \wedge q) \equiv \sim p \wedge \sim q$

Question 19.
Consider the following statements:
(i) If D is a dog, then D is very good.
(ii) If D is very good, then D is a dog.
(iii) If D is not very good, then D is not a dog.
(iv) If D is not a dog, then D is not very good.
Identify the pairs of statements having the same meaning. Justify.
Solution:
Let p : D is a dog. and q : D is very good.
Then the given statements in the symbolic form are:
(i) $p \rightarrow q$
(ii) $q \rightarrow p$
(iii) $\sim q \rightarrow \sim p$
(iv) $\sim p \rightarrow \sim q$

| | | | | (i) | (ii) | (iii) | (iv) |
|---|---|---|---|---|---|---|---|
| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \to q$ | $q \to p$ | $\sim q \to \sim p$ | $\sim p \to \sim q$ |
| T | T | F | F | T | T | T | T |
| T | F | F | T | F | T | F | T |
| F | T | T | F | T | F | T | F |
| F | F | T | T | T | T | T | T |

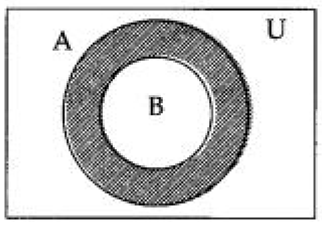
The entries in columns (i) and (iii) are identical. Hence, these statements are equivalent.
∴ the statements (i) and (iii) have the same meaning.
Similarly, the entries in columns (ii) and (iv) are identical. Hence, these statements are equivalent.
∴ the statements (ii) and (iv) have the same meaning.

Question 20.
Express the truth of each of the following statements by Venn diagrams:
(i) All men are mortal.
Solution:
Let U : a set of all human being
A : set of all men
B : set of all mortals.
Then the Venn diagram represents the truth of the given statement is as below:



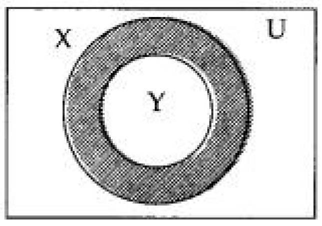

$A \subset B$

(ii) Some persons are not politicians.
Solution:
Let U : set of all human being
A : set of all persons
B : set of all politicians.
Then the Venn diagram represents the truth of the given statement is as follows:




$A - B \neq \phi$

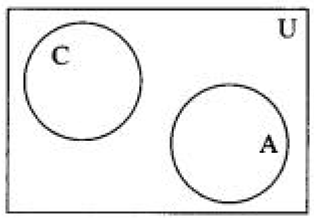
(iii) Some members of the present Indian cricket are not committed.
Solution:
Let U : set of all human being
X : set of all members of present Indian cricket
Y : set of all committed members of the present Indian cricket.
Then the Venn diagram represents the truth of the given statement is as below:




$X - Y \neq \phi$

(iv) No child is an adult.
Solution:
Let U : set of all human beings
C : set of all children
A : set of all adults.

Then the Venn diagram represents the truth of the given statement is as below:




$C \cap A = \phi$

Question 21.
If A = {2, 3, 4, 5, 6, 7, 8}, determine the truth value of each of the following statements:
(i) $\exists\ x \in A$, such that $3x + 2 > 9$.
Solution:
Clearly x = 3, 4, 5, 6, 7, 8 $\in$ A satisfy $3x + 2 > 9$.
So, the given statement is true, hence its truth value is T.

(ii) $\forall x \in A$, $x^2 < 18$.
Solution:
x = 5, 6, 7, 8 $\in$ A do not satisfy $x^2 < 18$.
So the given statement is false, hence its truth value is F.

(iii) $\exists x \in A$, such that $x + 3 < 11$.
Solution:
Clearly x = 2, 3, 4, 5, 6, 7 $\in$ A which satisfy $x + 3 < 11$.
So, the given statement is True, hence its truth value is T.

(iv) $\forall x \in A$, $x^2 + 2 \geq 5$.
Solution:
$x^2 + 2 \geq 5$ for all $x \in A$.
So, the given statement is true, hence its truth value is T.

Question 22.
Write the negations of the following statements:
(i) 7 is a prime number and the Taj Mahal is in Agra.
Solution:
Let p : 7 be a prime number.
q : Taj Mahal is in Agra.
Then the symbolic form of the given statement is $p \wedge q$.
Since, $(p \wedge q) \equiv \sim p \vee \sim q$,
the negation of the given statement is:
'7 is not a prime number or Taj Mahal is not in Agra.'

(ii) 10 > 5 and 3 < 8.
Solution:
Let p : 10 > 5.
q : 3 < 8.
Then the symbolic form of the given statement is $P \wedge q$.
Since, $\sim(p \wedge q) = \sim p \vee \sim q$, the negation of the given statement is:
'$10 \leq 5$ or $3 \geq 8$'
OR
'$10 \ngtr 5$ or $3 \nless 8$'

(iii) I will have tea or coffee.
Solution:
The negation of the given statement is:
'I will not have tea and coffee.'

(iv) $\forall n \in N$, $n + 3 > 9$.
Solution:
The negation of the given statement is:
'$\exists n \in N$, such that $n + 3 \ngtr 9$.'
OR
'$\exists n \in N$, such that $n + 3 \leq 9$.'

(v) $\exists x \in A$, such that $x + 5 < 11$.
Solution:
The negation of the given statement is:
'$\forall x \in A$, $x + 5 \nless 1$.'
OR
'$\forall x \in A$, $x + 5 \geq 11$.'