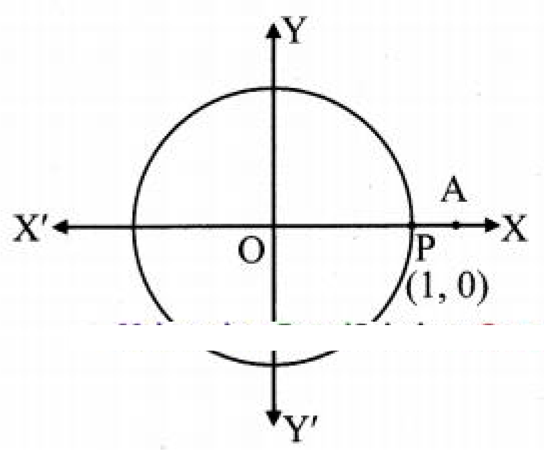
Maharashtra State Board 11th Maths Solutions Chapter 2 Trigonometry – I Ex 2.1

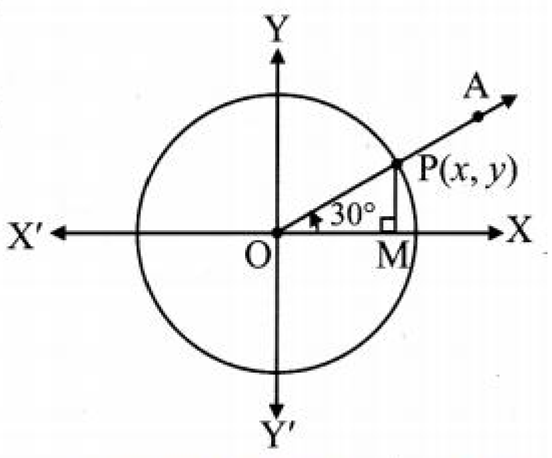
Question 1.
Find the trigonometric functions of 0°, 30°, 45°, 60°, 150°, 180°, 210°, 300°, 330°, – 30°, – 45°, – 60°, – 90°, – 120°, – 225°, – 240°, – 270°, – 315°
Solution:
Angle of measure 0°:



Let m∠XOA = 0° = 0^c
Its terminal arm (ray OA) intersects the standard
unit circle in P(1, 0).
Hence,x = 1 and y = 0
sin 0° = y = 0,
cos 0° = x = 1,
tan 0° = $\frac{y}{x}=\frac{0}{1}$ = 0
cot 0° = $\frac{x}{y}=\frac{1}{0}$ which is not defined
sec 0° = $\frac{1}{x}=\frac{1}{1}$ = 1
cot 0° = $\frac{1}{y}=\frac{1}{0}$ which is not defined,

Angle of measure 30°:
Let m∠XOA = 30°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y)
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 30° – 60° – 90° triangle.
OP= 1

$$OM = \frac{\sqrt{3}}{2} OP = \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$$



$$PM = \frac{1}{2} OP = \frac{1}{2}(1) = \frac{1}{2}$$

Since point P lies in 1st quadrant, x > 0, y > 0

∴ x = OM = 3√2 and y = PM = 12

$$\therefore \quad P \equiv \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\sin 30° = y = \frac{1}{2}$$

$$\cos 30° = x = \frac{\sqrt{3}}{2}$$

$$\tan 30° = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

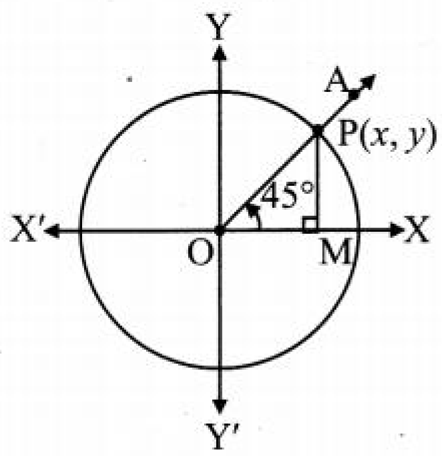
$$\operatorname{cosec} 30° = \frac{1}{y} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$\sec 30° = \frac{1}{x} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$$

$$\cot 30° = \frac{x}{y} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Angle of measure 45°:
Let m∠XOA = 45°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 45° – 45° – 90° triangle.
OP = 1,

$$OM = \frac{1}{\sqrt{2}} OP$$
$$= \frac{1}{\sqrt{2}} (1)$$
$$= \frac{1}{\sqrt{2}}$$
$$PM = \frac{1}{\sqrt{2}} OP$$
$$= \frac{1}{\sqrt{2}} (1)$$
$$= \frac{1}{\sqrt{2}}$$



Since point P lies in the 1st quadrant, x > 0, y > 0
∴ x = OM = 12√ and
y = PM = 12√

∴ P = (12√, 12√)

$$\sin 45^\circ = y = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = x = \frac{1}{\sqrt{2}}$$

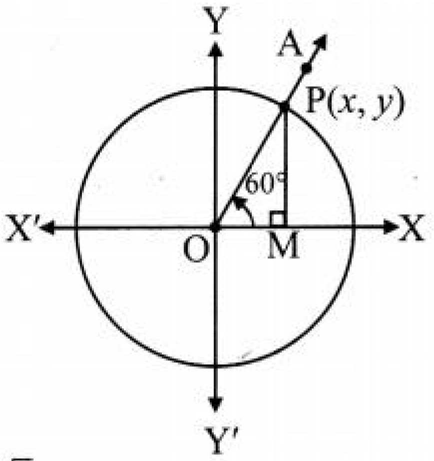
$$\tan 45^\circ = \frac{y}{x} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

$$\operatorname{cosec} 45^\circ = \frac{1}{y} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}$$

$$\sec 45^\circ = \frac{1}{x} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}$$

$$\cot 45^\circ = \frac{x}{y} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

Angle of measure 60°:
Let m∠XOA = 60°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 30° – 60° – 90° triangle.
OP= 1,



$$OM = \frac{1}{2} OP$$

$$= \frac{1}{2} (1)$$

$$= \frac{1}{2}$$

$$PM = \frac{\sqrt{3}}{2} OP$$

$$= \frac{\sqrt{3}}{2} (1) = \frac{\sqrt{3}}{2}$$

Since point P lies in the 1st quadrant,

$x > 0, y > 0$

$\therefore \quad x = OM = \frac{1}{2}$ and $y = PM = \frac{\sqrt{3}}{2}$

$\therefore \quad P \equiv \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$$\sin 60° = y = \frac{\sqrt{3}}{2}$$

$$\cos 60° = x = \frac{1}{2}$$

$$\tan 60° = \frac{y}{x} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \sqrt{3}$$

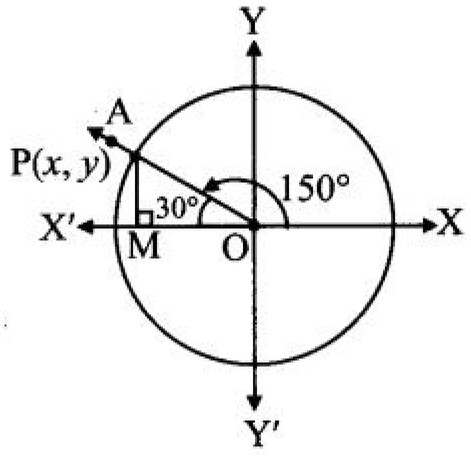
$$\text{cosec } 60° = \frac{1}{y} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$$

$$\sec 60° = \frac{1}{x} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$\cot 60° = \frac{x}{y} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}}$$

Angle of measure 150°:
Let m∠XOA = 150°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 30° – 60° – 90° triangle.
OP= 1,

$$OM = \frac{\sqrt{3}}{2} OP$$
$$= \frac{\sqrt{3}}{2}(1)$$
$$= \frac{\sqrt{3}}{2}$$
$$PM = \frac{1}{2} OP$$
$$= \frac{1}{2}(1)$$
$$= \frac{1}{2}$$



Since point P lies in the 2nd quadrant, x < 0, y > 0

$$\therefore \quad x = -OM = \frac{-\sqrt{3}}{2} \text{ and } y = PM = \frac{1}{2}$$

$$\therefore \quad P \equiv \left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\sin 150^\circ = y = \frac{1}{2}$$

$$\cos 150^\circ = x = -\frac{\sqrt{3}}{2}$$

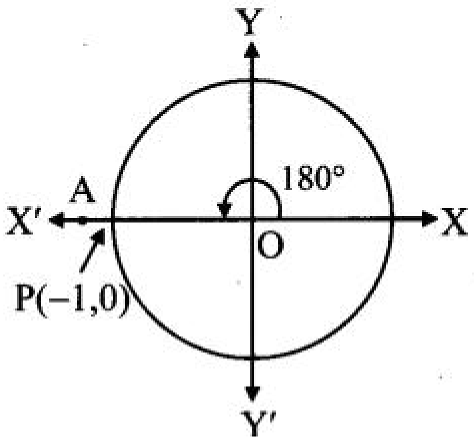
$$\tan 150^\circ = \frac{y}{x} = \frac{\frac{1}{2}}{-\left(\frac{\sqrt{3}}{2}\right)} = -\frac{1}{\sqrt{3}}$$

$$\operatorname{cosec} 150^\circ = \frac{1}{y} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$\sec 150^\circ = \frac{1}{x} = \frac{1}{-\left(\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\cot 150^\circ = \frac{x}{y} = \frac{-\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}} = -\sqrt{3}$$

Angle of measure 180°:
Let m∠XOA = 180°
Its terminal arm (ray OA) intersects the standard unit circle at P(-1, 0).
∴ x = – 1 and y = 0
sin 180° =y = 0
cos 180° = x = -1



tan 180° = yx
= 0-1 = 0
Cosec 180° = 1y
= 10

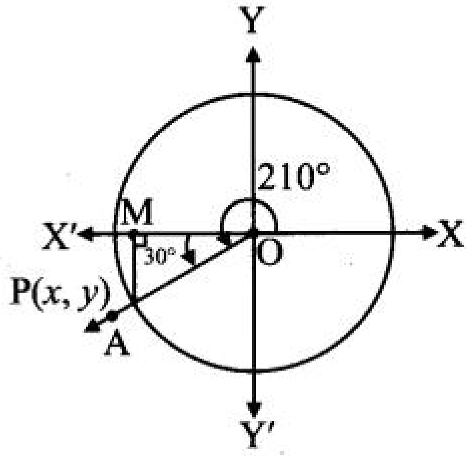
which is not defined.

sec 180°= $\frac{1}{x}=\frac{1}{-1}$ = -1

cot 180° = $\frac{x}{y}=\frac{-1}{0}$, which is not defined.

Angle of measure 210°:
Let m∠XOA = 210°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 30° – 60° – 90° triangle.
OP= 1,

$$\begin{aligned} OM &= \frac{\sqrt{3}}{2} OP \\ &= \frac{\sqrt{3}}{2}(1) \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$



$$\begin{aligned} PM &= \frac{1}{2} OP \\ &= \frac{1}{2}(1) \\ &= \frac{1}{2} \end{aligned}$$

Since point P lies in the 3rd quadrant, x < 0,y < 0
∴ x = -OM = $-\frac{\sqrt{3}}{2}$ and y = -PM = $-\frac{1}{2}$

∴ P ≡($-\frac{\sqrt{3}}{2},-\frac{1}{2}$)

$$\sin 210° = y = \frac{-1}{2}$$

$$\cos 210° = x = \frac{-\sqrt{3}}{2}$$

$$\tan 210° = \frac{y}{x} = \frac{\left(-\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}}$$

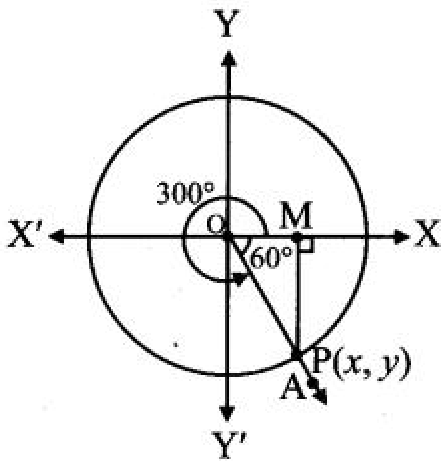
$$\operatorname{cosec} 210° = \frac{1}{y} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\sec 210° = \frac{1}{x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\cot 210° = \frac{x}{y} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

Angle of measure 300°:
Let m∠XOA = 300°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 30° – 60° – 90° triangle.
OP = 1,

$$OM = \frac{1}{2} OP$$
$$= \frac{1}{2} (1)$$
$$= \frac{1}{2}$$
$$PM = \frac{\sqrt{3}}{2} OP$$
$$= \frac{\sqrt{3}}{2} (1)$$
$$= \frac{\sqrt{3}}{2}$$



Since point P lies in the 1st quadrant, x > 0,y > 0
x = OM = 12 = and y = -PM = -3√2
sin 300° = y = -3√2
cos 300° = x = 12
tan 300° = yx=-3√212=-3-√

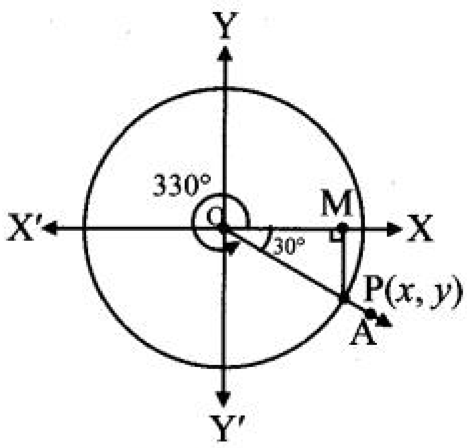
$$\operatorname{cosec} 300° = \frac{1}{y} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\sec 300° = \frac{1}{x} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$\cot 300° = \frac{x}{y} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

Angle of measure 330°:
Let m∠XOA = 330°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 30° – 60° – 90° triangle.
OP= 1,

$$OM = \frac{\sqrt{3}}{2} OP$$
$$= \frac{\sqrt{3}}{2} (1)$$
$$= \frac{\sqrt{3}}{2}$$
$$PM = \frac{1}{2} OP$$
$$= \frac{1}{2} (1) = \frac{1}{2}$$



Since point P lies in the 4th quadrant, x > 0, y < 0

$\therefore \quad x = OM = \frac{\sqrt{3}}{2}$ and $y = -PM = -\frac{1}{2}$

$\therefore \quad P \equiv \left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$

$$\sin 330° = y = -\frac{1}{2}$$

$$\cos 330° = x = \frac{\sqrt{3}}{2}$$

$$\tan 330° = \frac{y}{x} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

$$\operatorname{cosec} 330° = \frac{1}{y} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

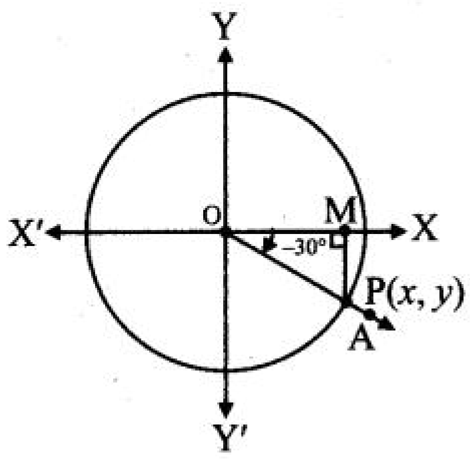
$$\sec 330° = \frac{1}{x} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$$

$$\cot 330° = \frac{x}{y} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$

Angle of measure 30°
Let m∠XOA = -30°
Its terminal arm (ray OA) intersects the standard unit circle at P(x,y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 30° – 60 — 90° triangle.
op = 1,

$OP = 1,$

$$OM = \frac{\sqrt{3}}{2} OP = \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$$



$$PM = \frac{1}{2} OP = \frac{1}{2}(1) = \frac{1}{2}$$

Since point P lies in the 4th quadrant x > 0, y < 0

$\therefore \quad x = \text{OM} = \frac{\sqrt{3}}{2}$ and $y = -\text{PM} = \frac{-1}{2}$

$\therefore \quad \text{P} \equiv \left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$

$$\sin(-30°) = y = -\frac{1}{2}$$

$$\cos(-30°) = x = \frac{\sqrt{3}}{2}$$

$$\tan(-30°) = \frac{y}{x} = \frac{\left(-\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = -\frac{1}{\sqrt{3}}$$

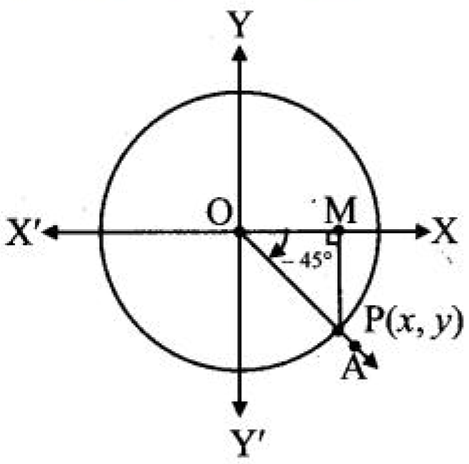
$$\text{cosec}(-30°) = \frac{1}{y} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\sec(-30°) = \frac{1}{x} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$$

$$\cot(-30°) = \frac{x}{y} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = -\sqrt{3}$$

Angle of measure 45°:
Let m∠XOA = 45°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 45° – 45° – 90° triangle.
OP = 1,

$$\text{OM} = \frac{1}{\sqrt{2}}\text{OP} = \frac{1}{\sqrt{2}}(1) = \frac{1}{\sqrt{2}}$$



$$\text{PM} = \frac{1}{\sqrt{2}}\text{OP} = \frac{1}{\sqrt{2}}(1) = \frac{1}{\sqrt{2}}$$

Since point P lies in the 4th quadrant x > 0, y < 0

$\therefore \quad x = OM = \frac{1}{\sqrt{2}}$ and $y = -PM = \frac{-1}{\sqrt{2}}$

$\therefore \quad P \equiv \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

$\sin(-45°) = y = -\frac{1}{\sqrt{2}}$

$\cos(-45°) = x = \frac{1}{\sqrt{2}}$

$\tan(-45°) = \frac{y}{x} = \frac{\left(-\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}\right)} = -1$

$\operatorname{cosec}(-45°) = \frac{1}{y} = \frac{1}{\left(-\frac{1}{\sqrt{2}}\right)} = -\sqrt{2}$

$\sec(-45°) = \frac{1}{x} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}$

$\cot(-45°) = \frac{x}{y} = \frac{\left(\frac{1}{\sqrt{2}}\right)}{\left(-\frac{1}{\sqrt{2}}\right)} = -1$

[Note : Answer given in the textbook of sin (45°) = – 1/2. However, as per our calculation it is −12√]

Angle of measure (-60°):
Let m∠XOA = -60°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
ΔOMP is a 30° – 60° – 90° triangle.
OP = 1,

$OM = \frac{1}{2} OP$

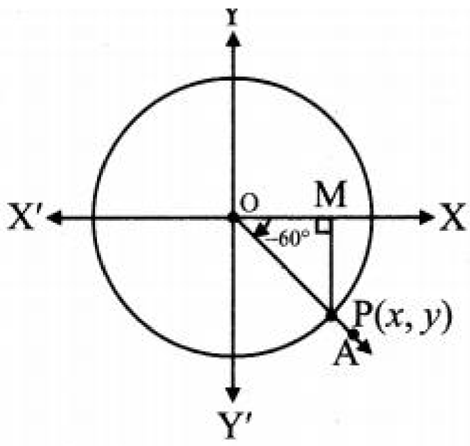
$= \frac{1}{2}(1)$

$= \frac{1}{2}$

$PM = \frac{\sqrt{3}}{2} OP$

$= \frac{\sqrt{3}}{2}(1)$

$= \frac{\sqrt{3}}{2}$



Since point P lies in the 4′ quadrant,
x > 0, y < 0

x = OM =12 and y = -PM = −3√2

$$\therefore \quad P \equiv \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$$\sin(-60^\circ) = y = -\frac{\sqrt{3}}{2}$$

$$\cos(-60^\circ) = x = \frac{1}{2}$$

$$\tan(-60^\circ) = \frac{y}{x} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= -\sqrt{3}$$

$$\operatorname{cosec}(-60^\circ) = \frac{1}{y} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)}$$

$$= -\frac{2}{\sqrt{3}}$$

$$\sec(-60^\circ) = \frac{1}{x} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$\cot(-60^\circ) = \frac{x}{y} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$$

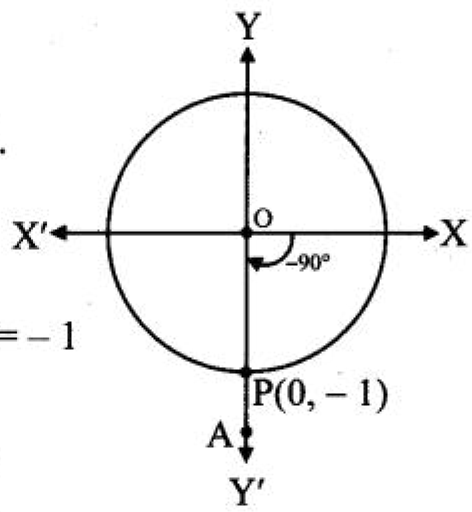
$$= -\frac{1}{\sqrt{3}}$$

Angle of measure (-90°):
Let m∠XOA = -90°
It terminal arm (ray OA) intersects the standard unit circle at P(0, -1)
∴ x = 0 and y = -1
sin (-90°) = y = -1
cos (-90°) = s = 0

$$\tan(-90^\circ) = \frac{y}{x}$$

$$= \frac{-1}{0},$$

which is not defined.



$$\operatorname{cosec}(-90^\circ) = \frac{1}{y}$$

$$= \frac{1}{-1} = -1$$

$$\sec(-90^\circ) = \frac{1}{x} = \frac{1}{0},$$

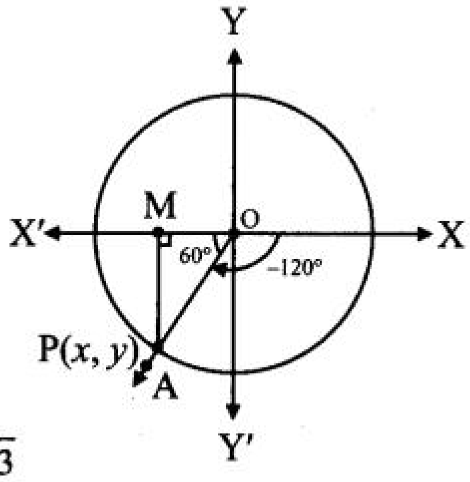
which is not defined.

$$\cot(-90^\circ) = \frac{x}{y} = \frac{0}{-1} = 0$$

Angle of measure (-120°):
Let m∠XOA = – 120°

Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 30° – 60° – 90° triangle.
OP = 1,

$$OM = \frac{1}{2}OP = \frac{1}{2}(1) = \frac{1}{2}$$



$$PM = \frac{\sqrt{3}}{2}OP = \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$$

Since point P lies in the 3rd quadrant, x < 0, y < 0

$$\therefore \quad x = -OM = \frac{-1}{2} \text{ and } y = -PM = \frac{-\sqrt{3}}{2}$$

$$\therefore \quad P \equiv \left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$$

$$\sin(-120°) = y = -\frac{\sqrt{3}}{2}$$

$$\cos(-120°) = x = -\frac{1}{2}$$

$$\tan(-120°) = \frac{y}{x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

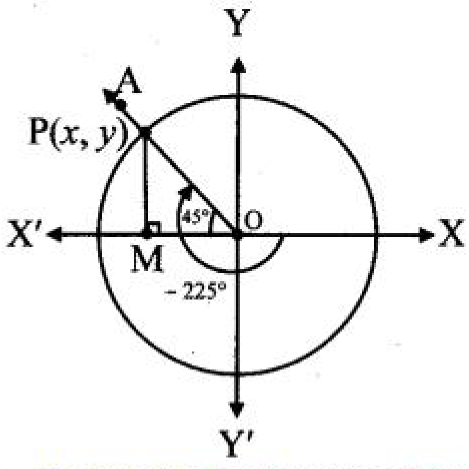
$$\operatorname{cosec}(-120°) = \frac{1}{y} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\sec(-120°) = \frac{1}{x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\cot(-120°) = \frac{x}{y} = \frac{\left(-\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}}$$

Angle of measure (- 225°):
Let m∠XOA = – 225°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
ΔOMP is a 45° – 45° – 90° triangle.
OP = 1,

$$OM = \frac{1}{\sqrt{2}} OP$$
$$= \frac{1}{\sqrt{2}} (1)$$
$$= \frac{1}{\sqrt{2}}$$



$$PM = \frac{1}{\sqrt{2}} OP$$
$$= \frac{1}{\sqrt{2}} (1) = \frac{1}{\sqrt{2}}$$

Since point P lies in the 2nd quadrant, x < 0, y > 0

$$\therefore \quad x = -OM = \frac{-1}{\sqrt{2}} \text{ and } y = PM = \frac{1}{\sqrt{2}}$$

$$\therefore \quad P \equiv \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\sin(-225°) = y = \frac{1}{\sqrt{2}}$$

$$\cos(-225°) = x = -\frac{1}{\sqrt{2}}$$

$$\tan(-225°) = \frac{y}{x} = \frac{\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = -1$$

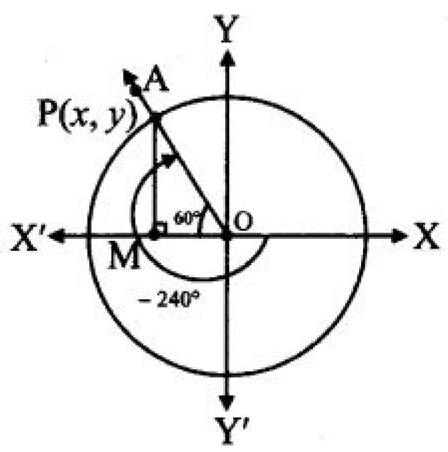
$$\operatorname{cosec}(-225°) = \frac{1}{y} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}$$

$$\sec(-225°) = \frac{1}{x} = \frac{1}{\left(-\frac{1}{\sqrt{2}}\right)} = -\sqrt{2}$$

$$\cot(-225°) = \frac{x}{y} = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -1$$

Angle of measure 2400):
Let m∠XOA = 240°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 30° – 60° – 900 triangle.

$$OP = 1,$$
$$PM = \frac{\sqrt{3}}{2} OP$$
$$= \frac{\sqrt{3}}{2} (1)$$
$$= \frac{\sqrt{3}}{2}$$



$$OM = \frac{1}{2} OP$$
$$= \frac{1}{2} (1) = \frac{1}{2}$$

Since point P lies in the 2nd quadrant, x<0, y>0

$\therefore \quad x = -\text{OM} = -\frac{1}{2}$ and $y = \text{PM} = \frac{\sqrt{3}}{2}$

$\therefore \quad \text{P} \equiv \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$$\sin(-240°) = y = \frac{\sqrt{3}}{2}$$

$$\cos(-240°) = x = -\frac{1}{2}$$

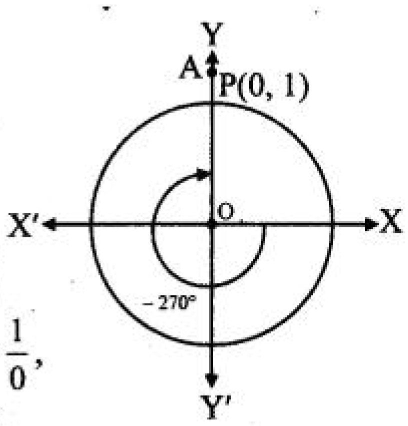
$$\tan(-240°) = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$

$$\text{cosec}(-240°) = \frac{1}{y} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$$

$$\sec(-240°) = \frac{1}{x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\cot(-240°) = \frac{x}{y} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

Angle of measure (- 270°):
Let m∠XOA = – 270°
Its terminal arm (ray OA)
intersects the standard unit,
circle at P(0, 1).
∴ x = 0 and y = 1
sin (- 270°) = y = 1
cos (- 270°) = x = 0
tan(-270°)= $\frac{y}{x}=\frac{1}{0}$
which is not defined.



$$\text{cosec}(-270°) = \frac{1}{y} = \frac{1}{1} = 1$$

$$\sec(-270°) = \frac{1}{x} = \frac{1}{0},$$

which is not defined.

$$\cot(-270°) = \frac{x}{y} = \frac{0}{1} = 0$$

Angle of measure (315°):
Let m∠XOA 315°
Its terminal arm (ray OA) intersects the standard unit circle at P(x,y).
Draw seg PM perpendicular to the X-axis.
ΔOMP is a 45° – 45° – 90° triangle.

OP = 1,

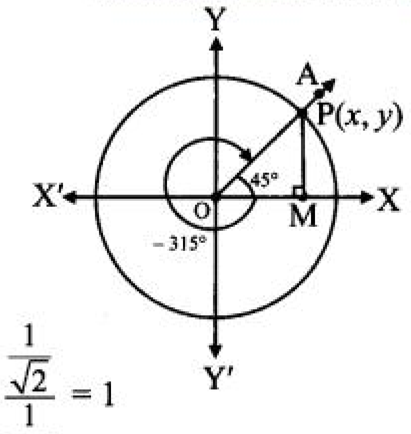
$OM = \frac{1}{\sqrt{2}}\ OP = \frac{1}{\sqrt{2}}\ (1) = \frac{1}{\sqrt{2}}$

$PM = \frac{1}{\sqrt{2}}\ OP = \frac{1}{\sqrt{2}}\ (1) = \frac{1}{\sqrt{2}}$

Since point P lies in the 1st quadrant,
$x > 0, y > 0$

$\therefore\quad x = OM = \frac{1}{\sqrt{2}}$ and $y = PM = \frac{1}{\sqrt{2}}$

$\therefore\quad P \equiv \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$



$\sin(-315°) = y$
$= \frac{1}{\sqrt{2}}$

$\cos(-315°) = x$
$= \frac{1}{\sqrt{2}}$

$$\tan(-315°) = \frac{y}{x} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

$$\text{cosec}(-315°) = \frac{1}{y} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}$$

$$\sec(-315°) = \frac{1}{x} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}$$

$$\cot(-315°) = \frac{x}{y} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

Question 2.
State the signs of:
i. tan 380°
ii. cot 230°
iii 468°
Solution:
1. 380° = 360° + 20°
∴ 380° and 20° are co-terminal angles.
Since 0° < 20° <90°0,
20° lies in the I quadrant.
∴ 380° lies in the 1st quadrant,
∴ tan 380° is positive.

ii. Since, 180° <230° <270°
∴ 230° lies in the 3rd quadrant.
∴ cot 230° is positive.

iii. 468° = 360°+108°
∴ 468° and 108° are co-terminal angles.
Since 90° < 108° < 180°,
108° lies in the 2nd quadrant.
∴ 468° lies in the 2nd quadrant.
∴ sec 468° is negative.

Question 3.
State the signs of cos 4^c and cos 4°. Which of these two functions is greater?
Solution:
Since 0° < 4° < 90°, 4° lies in the first quadrant. ∴ cos4° >0 ...(i)
Since 1^c = 57° nearly,
180° < 4^c < 270°
∴ 4^c lies in the third quadrant.
∴ cos 4^c < 0 ....(ii)
From (i) and (ii),
cos 4° is greater.

Question 4.
State the quadrant in which 6 lies if
i. sin θ < 0 and tan θ > 0
ii. cos θ < 0 and tan θ > 0
Solution:
i. sin θ < 0 sin θ is negative in 3rd and 4th quadrants, tan 0 > 0
tan θ is positive in 1st and 3rd quadrants.
∴ θ lies in the 3rd quadrant.

ii. cos θ < 0 cos θ is negative in 2nd and 3rd quadrants, tan 0 > 0
tan θ is positive in 1st and 3rd quadrants.
∴ θ lies in the 3rd quadrant.

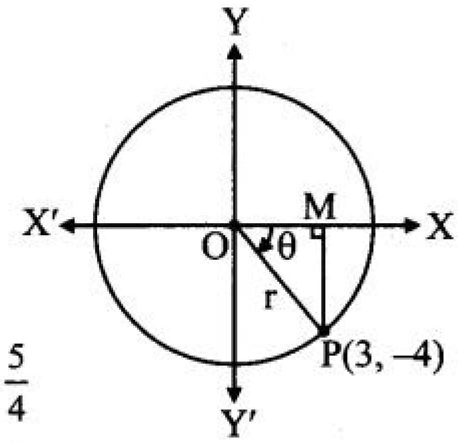
Question 5.
Evaluate each of the following:
i. sin 30° + cos 45° + tan 180°
ii. cosec 45° + cot 45° + tan 0°
iii. sin 30° x cos 45° x lies tan 360°
Solution:
i. We know that,
sin30° = 1/2, cos 45° = 12√ =, tan 180° = 0
sin30° + cos 45° +tan 180°
= 12+12√+0=2√+12

ii. We know that,
cosec 45° = 2–√ , cot 45° = 1, tan 0° = 0
cosec 45° + cot 45° + tan 0°
= 2–√ + 1 + 0 = 2–√ + 1

iii. We know that,
sin 30° = 12, cos 45° = 12√ =, tan 360° = 0
sin 30° x cos 45° x tan 360°
= (12)(12√) = 0

Question 6.
Find all trigonometric functions of angle in standard position whose terminal arm passes through point (3, – 4).
Solution:
Let θ be the measure of the angle in standard position whose terminal arm passes through P(3, -4).
∴ x = 3 and y = -4
r = OP

$$= \sqrt{3^2 + (-4)^2}$$
$$= \sqrt{9+16}$$
$$= 5$$



$$\sin\theta = \frac{y}{r} = -\frac{4}{5}$$
$$\cos\theta = \frac{x}{r} = \frac{3}{5}$$
$$\tan\theta = \frac{y}{x} = -\frac{4}{3}$$
$$\text{cosec}\,\theta = \frac{r}{y} = -\frac{5}{4}$$
$$\sec\theta = \frac{r}{x} = \frac{5}{3}$$
$$\cot\theta = \frac{x}{y} = -\frac{3}{4}$$

Question 7.
If cos θ = 1213,0<θ<π2 find the value of sin2θ−cos2θ2sinθcosθ,1tan2θ
Solution:
cos θ = 1213

We know that,
$\sin^2 \theta = 1 - \cos^2\theta$

$$= 1 - \left(\frac{12}{13}\right)^2$$

$$= 1 - \frac{144}{169}$$

$$= \frac{25}{169}$$

$\therefore \sin \theta = \pm \frac{5}{13}$

Since $0 < \theta < \frac{\pi}{2}$, θ lies in the 1st quadrant, $\therefore \sin \theta > 0$

$$\therefore \quad \sin \theta = \frac{5}{13}$$

$$\therefore \quad \frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} = \frac{\left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2}{2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)}$$

$$= \frac{\frac{25}{169} - \frac{144}{169}}{\frac{120}{169}}$$

$$= \frac{-\frac{119}{169}}{\frac{120}{169}}$$

$$= -\frac{119}{120}$$

$$\tan \theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$$

$$\therefore \quad \frac{1}{\tan^2\theta} = \frac{1}{\left(\frac{5}{12}\right)^2}$$

$$= \frac{1}{\frac{25}{144}} = \frac{144}{25}$$

Question 8.
Using tables evaluate the following:
i. 4 cot 45° – $\sec^2$ 60° + sin 30°
ii. $\cos^2 0 + \cos^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} + \cos^2 \frac{\pi}{2}$

Solution:
i. We know that,
cot 45° = 1, sec 60° = 2, sin 30° = 1/2
4 cot 45° – $\sec^2$ 60° + sin 30°
$= 4(1) - (2)^2 + \frac{1}{2}$
$= 4 - 4 + \frac{1}{2} = \frac{1}{2}$

ii. We know that,

$$\cos 0 = 1, \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, \cos\frac{\pi}{3} = \frac{1}{2},$$

$$\cos\frac{\pi}{2} = 0$$

$$\cos^2 0 + \cos^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{2}$$

$$= 1 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 0$$

$$= 1 + \frac{3}{4} + \frac{1}{4} = 2$$

Question 9.
Find the other trigonometric functions if

i. cot θ = −35, and 180 < θ < 270

ii. Sec A = −257 and A lies in the second quadrant.

iii cot x = 34, x lies in the third quadrant.

iv. tan x = -512 x lies in the fourth quadrant.

Solution:

i. cot θ = −35

we know that,
$\sin^2\theta = 1 - \cos^2\theta$

= 1 – (−35)2

= 1 – 925=1625

∴ sin θ = ± 45

Since 180° < 0 < 270°,
θ lies in the 3rd quadrant.
∴ sin θ < 0

$$\therefore \quad \sin\theta = -\frac{4}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \frac{4}{3}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{3}{5}\right)} = -\frac{5}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(\frac{4}{3}\right)} = \frac{3}{4}$$

Since A lies in the 2nd quadrant,

tan A < 0

$$\therefore \quad \tan A = -\frac{24}{7}$$

$$\therefore \quad \cot A = \frac{1}{\tan A} = \frac{1}{\left(-\frac{24}{7}\right)} = -\frac{7}{24}$$

$$\cos A = \frac{1}{\sec A} = \frac{1}{\left(-\frac{25}{7}\right)} = -\frac{7}{25}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\therefore \quad \sin A = \tan A \cos A = -\frac{24}{7} \times -\frac{7}{25} = \frac{24}{25}$$

$$\therefore \quad \operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{\frac{24}{25}} = \frac{25}{24}$$

iii. Given, cot x = 34

We know that,

cosec2 x = 1 + cot2 x

= 1 + (34)2=1+916=2516

∴ cosec x = ± 54

Since x lies in the 3rd quadrant, cosec x < 0

$$\therefore \quad \operatorname{cosec} x = -\frac{5}{4}$$

$$\therefore \quad \sin x = \frac{1}{\operatorname{cosec} x} = \frac{1}{\left(-\frac{5}{4}\right)} = -\frac{4}{5}$$

$$\tan x = \frac{1}{\cot x} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\therefore \quad \cos x = \cot x \sin x = \frac{3}{4}\left(-\frac{4}{5}\right) = -\frac{3}{5}$$

$$\therefore \quad \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{3}{5}\right)} = -\frac{5}{3}$$

iv. Given, tan x = −512

sec2 x = 1 + tan2

= 1 + (−512)2

= 1 + 25144=169144

∴ sec x = ± 1312

Since x lies in the 4th quadrant,

sec x > 0

$$\therefore \quad \sec x = \frac{13}{12}$$

$$\therefore \quad \cos x = \frac{1}{\sec x} = \frac{1}{\frac{13}{12}} = \frac{12}{13}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\therefore \quad \sin x = \tan x \cos x = -\frac{5}{12} \times \frac{12}{13} = -\frac{5}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{5}{13}\right)} = -\frac{13}{5}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

Maharashtra State Board 11th Maths Solutions Chapter 2 Trigonometry – I Ex 2.2

Question 1.
If 2sin A = 1 = 2–√ cos B and π2 < A < π, 3π2
Solution:
Given, 2sin A = 1
∴ sin A = 1/2
we know that,
cos2 A = 1 – sin2 A = 1 – (12)2=1−14=34
∴ cos A = ±3√2
Since π2 < A < π
A lies in the 2nd quadrant.

$$\therefore \quad \cos A = \pm\frac{\sqrt{3}}{2}$$

Since $\frac{\pi}{2} < A < \pi$,
A lies in the 2^{nd} quadrant.

$$\therefore \quad \cos A = -\frac{\sqrt{3}}{2}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

Also, $\sqrt{2} \cos B = 1$

$$\therefore \quad \cos B = \frac{1}{\sqrt{2}}$$

We know that,
Sin2 B = 1 – cos2 B = 1 – (12√)212=12
∴ sin B = ±12√
Since 3π2 < B < 2π

B lies in the 4th quadrant,

$$\therefore \quad \sin B = -\frac{1}{\sqrt{2}}$$

$$\tan B = \frac{\sin B}{\cos B} = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -1$$

$$\therefore \quad \frac{\tan A + \tan B}{\cos A - \cos B} = \frac{-\frac{1}{\sqrt{3}} - 1}{-\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}} = \frac{\frac{-1-\sqrt{3}}{\sqrt{3}}}{\frac{-\sqrt{3}-\sqrt{2}}{2}}$$

$$= \frac{-\left(1+\sqrt{3}\right)}{\sqrt{3}} \times \frac{2}{-\left(\sqrt{3}+\sqrt{2}\right)}$$

$$= \frac{2\left(1+\sqrt{3}\right)}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}\right)}$$

Question 2.
If and A, B are angles in the second quadran, then prove that 4cosA + 3 cos B = -5
Solution:
Given, sinA3=sinB4=15
∴ sin A = 35 and sin B = 45
We know that,
cos2 A = 1 – sin2 = 1 – (35)2 = 1 – 925=1625
∴ Cos A = ± [45
Since A lies in the second quadrant,
cos A < 0
∴ Cos A = −45
Sin B = 4/5
We know that,
cos2B = 1 – sin2B = 1 – (45)2=1−1625=925
∴ Cos B = ±45
Since B lies in the second quadrant, cos B < 0

$$\therefore \quad \cos B = -\frac{3}{5}$$

$$\therefore \quad 4\cos A + 3\cos B = 4\left(-\frac{4}{5}\right) + 3\left(-\frac{3}{5}\right)$$

$$= -\frac{16}{5} - \frac{9}{5} = -\frac{25}{5} = -5$$

Question 3.
If tan θ = 12, evaluate 2sinθ+3cosθ4cosθ+3sinθ
Solution:

$$\text{Given, } \tan\theta = \frac{1}{2}$$

$$\frac{2\sin\theta + 3\cos\theta}{4\cos\theta + 3\sin\theta} = \frac{\frac{2\sin\theta}{\cos\theta} + 3}{4 + \frac{3\sin\theta}{\cos\theta}}$$

...[Dividing numerator and denominator by cos θ]

$$= \frac{2\tan\theta + 3}{4 + 3\tan\theta}$$

$$= \frac{2\left(\frac{1}{2}\right) + 3}{4 + 3\left(\frac{1}{2}\right)} = \frac{4}{\left(\frac{11}{2}\right)} = \frac{8}{11}$$

Question 4.
Eliminate 0 from the following:
i. x = 3sec θ, y = 4tan θ
ii. x = 6cosec θ,y = 8cot θ
iii. x = 4cos θ – 5sin θ, y = 4sin θ + 5cos θ
iv. x = 5 + 6 cosec θ,y = 3 + 8 cot θ
v. x = 3 – 4tan θ,3y = 5 + 3sec θ
Solution:
i. x = 3sec θ, y = 4tan θ
∴ sec θ = x3 and tan θ= y4
We know that,
$\sec^2\theta - \tan^2\theta = 1$

$$\therefore \quad \left(\frac{x}{3}\right)^2 - \left(\frac{y}{4}\right)^2 = 1$$

$$\therefore \quad \frac{x^2}{9} - \frac{y^2}{16} = 1$$

∴ $16x^2 - 9y^2 = 144$

ii. x = 6cosec θ and y = 8cot θ
∴ cosec θ = and cot θ =
We know that,
$\operatorname{cosec}^2\theta - \cot^2\theta =$

$$\therefore \quad \left(\frac{x}{6}\right)^2 - \left(\frac{y}{8}\right)^2 = 1$$

$$\therefore \quad \frac{x^2}{36} - \frac{y^2}{64} = 1$$

$16x^2 - 9y^2 = 576$

iii. x = 4cos θ – 5 sin θ … (i)
y = 4sin θ + 5cos θ .. .(ii)
Squaring (i) and (ii) and adding, we get
$x^2 + y^2 = (4\cos\theta - 5\sin\theta)^2 + (4\sin\theta + 5\cos\theta)^2$
$= 16\cos^2\theta - 40\sin\theta\cos\theta + 25\sin^2\theta + 16\sin^2\theta + 40\sin\theta\cos\theta + 25\cos^2\theta$
$= 16(\sin^2\theta + \cos^2\theta) + 25(\sin^2\theta + \cos^2\theta)$
= 16(1) + 25(1)
= 41

iv. x = 5 + 6cosec θ andy = 3 + 8cot θ
∴ x – 5 = 6cosec θ and y – 3 = 8cot θ
∴ cosec θ = x−56 and cot θ = y−38
We know that,
$\operatorname{cosec}^2\theta - \cot^2\theta = 1$
∴ (x−56)2−(y−38)2 = 1

v. 2x = 3 – 4tan θ and 3y = 5 + 3sec θ
∴ 2x – 3 = -4tan θ and 3y – 5 = 3sec θ
∴ tan θ = 3−2x4 and sec θ = 3y−53θ
We know that, $\sec^2\theta - \tan^2\theta = 1$
∴ (3y−53)2−(3−2x4)2 = 1
∴ (3y−53)2−(2x−34)2 = 1

Question 5.
If $2\sin^2\theta + 3\sin\theta = 0$, find the permissible values of cosθ.
Solution:
$2\sin^2\theta + 3\sin\theta = 0$
∴ sin θ (2sin θ + 3) = 0
∴ sin θ = 0 or sin θ = −32
Since – 1 ≤ sin θ ≤ 1,
sin θ = 0
1−cos2θ−−−−−−−−√ = 0 …[∵ $\sin^2\theta = 1 - \cos^2\theta$]
∴ $1 - \cos^2\theta = 0$
∴ $\cos^2\theta = 1$
∴ cos θ = ±1 …[∵ – 1 ≤ cos θ ≤ 1]

Question 6.
If $2\cos^2 \theta - 11 \cos \theta + 5 = 0$, then find the possible values of $\cos \theta$.
Solution:
$2\cos^2\theta - 11 \cos \theta + 5 = 0$
$\therefore 2\cos^2 \theta - 10 \cos \theta - \cos \theta + 5 = 0$
$\therefore 2\cos \theta(\cos \theta - 5) - 1 (\cos \theta - 5) = 0$
$\therefore (\cos \theta - 5) (2\cos \theta - 1) = 0$
$\cos \theta - 5 = 0$ or $2\cos \theta - 1 = 0$
$\therefore \cos \theta = 5$ or $\cos \theta = 1/2$
Since, $-1 \le \cos \theta \le 1$
$\therefore \cos \theta = 1/2$

Question 7.
Find the acute angle θ such $2\cos^2 \theta = 3\sin \theta$.
Solution:
$2\cos^2 0 = 3\sin \theta$
$\therefore 2(1 - \sin^2 \theta) = 3\sin \theta$
$\therefore 2 - 2\sin^2 \theta = 3\sin \theta$
$\therefore 2\sin^2 \theta + 3\sin 9 - 2 = \theta$
$\therefore 2\sin^2 \theta + 4\sin \theta - \sin \theta - 2 = \theta$
$\therefore 2\sin \theta(\sin \theta + 2) -1 (\sin \theta + 2) = \theta$
$\therefore (\sin \theta + 2) (2\sin \theta - 1) = 0$
$\therefore \sin \theta + 2 = 0$ or $2\sin \theta - 1 = 0$
$\therefore \sin \theta = -2$ or $\sin \theta = 1/2$
Since, $-1 \le \sin \theta \le 1$
$\therefore \text{Sin } \theta = 1/2$
$\therefore \theta = 30^\circ$ ...[$\because \sin 30 = 1/2$]

Question 8.
Find the acute angle 0 such that 5tan2 0 + 3 = 9sec 0.
Solution:
$5\tan^2 \theta + 3 = 9\sec \theta$
$\therefore 5(\sec^2 \theta - 1) + 3 = 9\sec \theta$
$\therefore 5\sec^2 \theta - 5 + 3 = 9\sec \theta$
$\therefore 5\sec^2 \theta - 9\sec \theta - 2 = 0$
$\therefore 5\sec^2 \theta - 10 \sec \theta + \sec \theta - 2 = 0$
$\therefore 5\sec \theta(\sec \theta - 2) + 1(\sec \theta - 2) = 0$
$\therefore (\sec \theta - 2) (5\sec \theta + 1) = 0$
$\therefore \sec \theta - 2 = 0$ or $5\sec \theta + 1 = 0$
$\therefore \sec \theta = 2$ or $\sec \theta = -1/5$
Since $\sec \theta \ge 1$ or $\sec \theta \le -1$,
$\sec \theta = 2$
$\therefore \theta = 60^\circ$ ... [$\because \sec 60^\circ = 2$]

Question 9.
Find $\sin \theta$ such that $3\cos \theta + 4\sin \theta = 4$.
Solution:
$3\cos \theta + 4\sin \theta = 4$
$\therefore 3\cos \theta = 4(1 - \sin \theta)$
Squaring both the sides, we get .
$9\cos^2\theta = 16(1 - \sin \theta)^2$
$\therefore 9(1 - \sin^2 \theta) = 16(1 + \sin^2 \theta - 2\sin \theta)$
$\therefore 9 - 9\sin^2 \theta = 16 + 16\sin^2 \theta - 32\sin \theta$
$\therefore 25\sin^2 \theta - 32\sin \theta + 7 = 0$
$\therefore 25\sin^2 \theta - 25\sin \theta - 7\sin \theta + 7 = 0$
$25\sin \theta (\sin \theta - 1) - 7 (\sin \theta - 1) = 0$
$\therefore (\sin \theta - 1) (25\sin \theta - 7) = 0$
$\therefore \sin \theta - 1 = 0$ or $25 \sin \theta - 7 = 0$
$\therefore \sin \theta = 1$ or $\sin \theta = 725$
Since, $-1 \le \sin \theta \le 1$
$\therefore \sin \theta = 1$ or 725
[Note: Answer given in the textbook is 1. However, as per our calculation it is 1 or 725.]

Question 10.
If $\operatorname{cosec} \theta + \cot \theta = 5$, then evaluate $\sec \theta$.
Solution:
$\operatorname{cosec} \theta + \cot \theta = 5$
$\therefore 1\sin\theta+\cos\theta\sin\theta=5$

$\therefore \frac{1+\cos\theta}{\sin\theta}=5$
$\therefore$ 1 + cos θ = 5.sin θ
Squaring both the sides, we get
1 + 2 cos θ + $\cos^2$ θ = 25 $\sin^2$ θ
$\therefore$ $\cos^2$ θ + 2 cos θ + 1 = 25 (1 – $\cos^2$ θ)
$\therefore$ $\cos^2$ θ + 2 cos θ + 1 = 25 – 25 $\cos^2$ θ
$\therefore$ 26 $\cos^2$ θ + 2 cos θ – 24 = 0
$\therefore$ 26 $\cos^2$ θ + 26 cos θ – 24 cos θ – 24 = 0
$\therefore$ 26 cos θ (cos θ + 1) – 24 (cos θ + 1) = 0
$\therefore$ (cos θ + 1) (26 cos θ – 24) = 0
$\therefore$ cos θ + 1 = θ or 26 cos θ – 24 = 0
$\therefore$ cos θ = -1 or cos θ = $\frac{24}{26}=\frac{12}{13}$
When cos θ = -1, sin θ = 0
$\therefore$ cot θ and cosec x are not defined,
$\therefore$ cos θ ≠ -1
$\therefore$ cos θ = $\frac{12}{13}$
$\therefore$ sec θ = $\frac{1}{\cos\theta}=\frac{13}{12}$
[Note: Answer given in the textbook is -1 or $\frac{13}{12}$.
However, as per our calculation it is only $\frac{13}{12}$.]

Question 11.
If cot θ = $\frac{3}{4}$ and π < θ < $\frac{3\pi}{2}$, then find the value of 4 cosec θ + 5 cos θ.
Solution:
We know that,
cosec^2θ = 1 + $\cot^2$ θ = $\left(\frac{3}{4}\right)^2$ = 1 + $\frac{9}{16}$
$\therefore$ cosec^2 θ = $\frac{25}{16}$
$\therefore$ cosec θ = $\pm\frac{5}{4}$
Since π < θ < $\frac{3\pi}{2}$
θ lies in the third quadrant.
$\therefore$ cosec θ < 0
$\therefore$ cosec θ = $-\frac{5}{4}$
cot θ = $\frac{3}{4}$
tan θ = $\frac{1}{\cot\theta}=\frac{4}{3}$
We know that,
$\sec^2$ θ = 1 + $\tan^2$ θ = 1 + $\left(\frac{4}{3}\right)^2$
= 1 + $\frac{16}{9}=\frac{25}{9}$
$\therefore$ sec θ = $\pm\frac{5}{3}$
Since θ lies in the third quadrant,
sec θ < 0
$\therefore$ sec θ = $-\frac{5}{3}$
cos θ = $\frac{1}{\sec\theta}=-\frac{3}{5}$
$\therefore$ 4cosec θ + 5cos θ
= $4\left(-\frac{5}{4}\right)+5\left(-\frac{3}{5}\right)$
= -5 – 3 = -8
[Note: The question has been modified.]

Question 12.
Find the Cartesian co-ordinates of points whose polar co-ordinates are:
i. (3, 90°) ii. (1, 180°)
Solution:
i. (r, θ) = (3, 90°)
Using x = r cos θ and y = r sin θ, where (x, y) are the required cartesian co-ordinates, we get
x = 3cos 90° and y = 3sin 90°
$\therefore$ x = 3(0) = 0 and y = 3(1) = 3
$\therefore$ the required cartesian co-ordinates are (0, 3).

ii. (r, θ) = (1, 180°)
Using x = r cos θ and y = r sin θ, where (x, y) are the required cartesian co-ordinates, we get
x = 1(cos 180°) and y = 1(sin 180°)
$\therefore$ x = -1 and y = 0
$\therefore$ the required cartesian co-ordinates are (-1, 0).

Question 13.
Find the polar co-ordinates of points whose Cartesian co-ordinates are:
1. (5, 5) ii. (1, $\sqrt{3}$)

ii. (-1, -1) iv. (-$\sqrt{3}$, 1)

Solution:
i. (x, y) = (5, 5)

∴ r = $\sqrt{x^2+y^2}$ = $\sqrt{25+25}$

$=\sqrt{50}=5\sqrt{2}$

tan θ = $\frac{y}{x}=\frac{5}{5}$ = 1

Since the given point lies in the 1st quadrant,
θ = 45° ...[∵ tan 45° = 1]

∴ the required polar co-ordinates are ($5\sqrt{2}$, 45°).

ii. (x, y) = (1, $\sqrt{3}$)

∴ r = $\sqrt{x^2+y^2}=\sqrt{1+3}=\sqrt{4}=2$

tan θ = $\frac{y}{x}=\frac{\sqrt{3}}{1}=\sqrt{3}$

Since the given point lies in the 1st quadrant,

θ = 60° ...[∵ tan 60° = $\sqrt{3}$]

∴ the required polar co-ordinates are (2, 60°).

iii. (x, y) = (-1, -1)

∴ r = $\sqrt{x^2+y^2}=\sqrt{1+1}=\sqrt{2}$

tan θ = $\frac{y}{x}=\frac{-1}{-1}=1$

∴ tan θ = tan $\frac{\pi}{4}$

Since the given point lies in the 3rd quadrant,

tan θ = tan $\left(\pi+\frac{\pi}{4}\right)$ ...[∵ tan (n + x) = tanx]

∴ tan θ = tan $\left(\frac{5\pi}{4}\right)$

∴ θ = $\frac{5\pi}{4}$ = 225°

∴ the required polar co-ordinates are ($\sqrt{2}$, 225°).

iv. (x, y) = (-$\sqrt{3}$, 1)

∴ r = $\sqrt{x^2+y^2}=\sqrt{3+1}=\sqrt{4}=2$

tan θ = $\frac{y}{x}=\frac{1}{-\sqrt{3}}$ = -tan $\frac{\pi}{6}$

Since the given point lies in the 2nd quadrant,

tan θ = tan $\left(\pi-\frac{\pi}{6}\right)$ ...[∵ tan (π – x) = – tanx]

∴ tan θ = tan $\left(\frac{5\pi}{6}\right)$

∴ θ = $\frac{5\pi}{6}$ = 150°

∴ the required polar co-ordinates are (2, 150°)

Question 14.
Find the values of:
i. sin$\frac{19\pi}{3}$
ii. cos 1140°
iii. cot $\frac{25\pi}{3}$
Solution:
i. We know that sine function is periodic with period 2π.

sin $\frac{19\pi}{3}$ = sin $\left(6\pi+\frac{\pi}{3}\right)$ = sin $\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

ii. We know that cosine function is periodic with period 2π.
cos 1140° = cos (3 × 360° + 60°)
= cos 60° = $\frac{1}{2}$

iii. We know that cotangent function is periodic with period π.

cot $\frac{25\pi}{3}$ = cot $\left(8\pi+\frac{\pi}{3}\right)$ = cot $\frac{\pi}{3}$ = $\frac{1}{\sqrt{3}}$

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Maharashtra State Board 11th Maths Solutions Chapter 2 Trigonometry – I Miscellaneous Exercise 2

I. Select the correct option from the given alternatives.

Question 1.
The value of the expression
cos1°. cos2°. cos3° … cos 179° =
(A) -1
(B) 0
(C) $\frac{1}{\sqrt{2}}$
(D) 1
Answer:
(B) 0

Explanation:
cos 1° cos 2° cos 3° … cos 179°
= cos 1° cos 2° cos 3° … cos 90°… cos 179°
= 0 …[∵ cos 90° = 0]

Question 2.
$\frac{\tan A}{1+\sec A}+\frac{1+\sec A}{\tan A}$ is equal to
(A) 2cosec A
(B) 2 sec A
(C) 2 sin A
(D) 2 cos A
Answer:
(A) 2cosec A

Explanation:

$$\frac{\tan A}{1+\sec A}+\frac{1+\sec A}{\tan A}$$

$$=\frac{\tan^2 A+1+\sec^2 A+2\sec A}{(1+\sec A)\tan A}$$

$$=\frac{\sec^2 A+\sec^2 A+2\sec A}{(1+\sec A)\tan A} \quad \ldots[\because 1+\tan^2 A=\sec^2 A]$$

$$=\frac{2\sec A(\sec A+1)}{(1+\sec A)\tan A}=\frac{2\sec A}{\tan A}$$

$$=\frac{2}{\sin A}=2\operatorname{cosec} A$$

Question 3.
If α is a root of $25\cos^2 θ + 5\cos θ – 12 = 0$, $\frac{\pi}{2} < α < π$, then sin 2α is equal to
(A) $-\frac{24}{25}$
(B) $-\frac{13}{18}$
(C) $\frac{13}{18}$
(D) $\frac{24}{25}$
Answer:
(A) $-\frac{24}{25}$

Explanation:

$25 \cos^2 θ + 5 \cos θ – 12 = 0$
∴ (5cos θ + 4) (5 cos θ – 3) = 0
∴ cos θ = $-\frac{4}{5}$ or cos θ = $\frac{3}{5}$
Since $\frac{\pi}{2}$ < α < π,
cos α < 0

∴ cos α = −45

$\sin^2 \alpha = 1 - \cos^2 \alpha$ = 1 – 1625=925

∴ sin α = ±35

Since π2 < α < π sin α > 0

∴ sin α = 3/5

sin 2 α = 2 sin α cos α

= 2(35)(-45)=−2425

Question 4.

If θ = 60°, then 1+tan2θ2tanθ is equal to

(A) 3√2

(B) 23√

(C) 13√

(D) 3–√

Answer:

(B) 23√

Explanation:

$$\frac{1+\tan^2\theta}{2\tan\theta} = \frac{1+\tan^2 60^\circ}{2\tan 60^\circ}$$

$$= \frac{1+\left(\sqrt{3}\right)^2}{2\sqrt{3}}$$

$$= \frac{1+3}{2\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}$$

Question 5.

If sec θ = m and tan θ = n, then 1m{(m+n)+1(m+n)} is equal to

(A) 2

(B) mn

(C) 2m

(D) 2n

Answer:

(A) 2

Explanation:

$$\frac{1}{m}\left[(m+n)+\frac{1}{(m+n)}\right]$$

$$= \frac{1}{\sec\theta}\left[(\sec\theta+\tan\theta)+\frac{1}{(\sec\theta+\tan\theta)}\right]$$

$$= \frac{1}{\sec\theta}\left(\sec\theta+\tan\theta+\frac{\sec\theta-\tan\theta}{\sec^2\theta-\tan^2\theta}\right)$$

$$= \frac{1}{\sec\theta}\left(\sec\theta+\tan\theta+\frac{\sec\theta-\tan\theta}{1}\right)$$

$$= \frac{1}{\sec\theta}\ (2\sec\theta) = 2$$

Question 6.

If cosec θ + cot θ = 52, then the value of tan θ is

(A) 1425

(B) 2021

(C) 2120

(D) 1516

Answer:

(B) 2021

Explanation:

cosec θ + cot θ = 52 ....(i)

$\text{cosec}^2\theta - \cot^2\theta = 1$
∴ (cosec θ + cot θ) (cosec θ – cot θ) = 1
∴ $\frac{5}{2}$ (cosec θ – cot θ) = 1
∴ cosec θ – cot θ = $\frac{2}{5}$ ...(ii)
Subtracting (ii) from (i), we get
2 cot θ = $\frac{5}{2}-\frac{2}{5}=\frac{21}{10}$
∴ cot θ = $\frac{21}{20}$
∴ tan θ = $\frac{20}{21}$

Question 7.
$1-\frac{\sin^2\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}-\frac{\sin\theta}{1-\cos\theta}$ equals
(A) 0
(B) 1
(C) sin θ
(D) cos θ
Answer:
(D) cos θ

Explanation:

$$1-\frac{\sin^2\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}-\frac{\sin\theta}{1-\cos\theta}$$
$$=1-\frac{1-\cos^2\theta}{1+\cos\theta}+\frac{1-\cos^2\theta}{\sin\theta(1-\cos\theta)}-\frac{\sin\theta}{1-\cos\theta}$$
$$=1-(1-\cos\theta)+\frac{\sin\theta}{1-\cos\theta}-\frac{\sin\theta}{1-\cos\theta}$$
$$=\cos\theta$$

Question 8.
If cosec θ – cot θ = q, then the value of cot θ is
(A) $\frac{2q}{1+q^2}$
(B) $\frac{2q}{1-q^2}$
(C) $\frac{1-q^2}{2q}$
(D) $\frac{1+q^2}{2q}$
Answer:
(C) $\frac{1-q^2}{2q}$

Explanation:

cosec θ – cot θ = q ......(i)
$\text{cosec}^2\theta - \cot^2\theta = 1$
∴ (cosec θ + cot θ) (cosec θ – cot θ) = 1
∴ (cosec θ + cot θ)q = 1
∴ cosec θ + cot θ = 1/q ....(ii)
Subtracting (i) from (ii), we get
2cot θ = $\frac{1}{q}-q$
∴ cot θ = $\frac{1-q^2}{2q}$

Question 9.
The cotangent of the angles $\frac{\pi}{3},\frac{\pi}{4}$ and $\frac{\pi}{6}$ are in
(A) A.P.
(B) G.P.
(C) H.P.
(D) Not in progression
Answer:
(B) G.P.

Explanation:

$$\cot\frac{\pi}{3}=\frac{1}{\sqrt{3}},\ \cot\frac{\pi}{4}=1,\ \cot\frac{\pi}{6}=\sqrt{3}$$

$$\therefore\quad \cot\frac{\pi}{3}\cot\frac{\pi}{6}=1=\left(\cot\frac{\pi}{4}\right)^2$$

$$\therefore\quad \cot\frac{\pi}{3},\ \cot\frac{\pi}{4},\ \cot\frac{\pi}{6}\text{ are in G.P.}$$

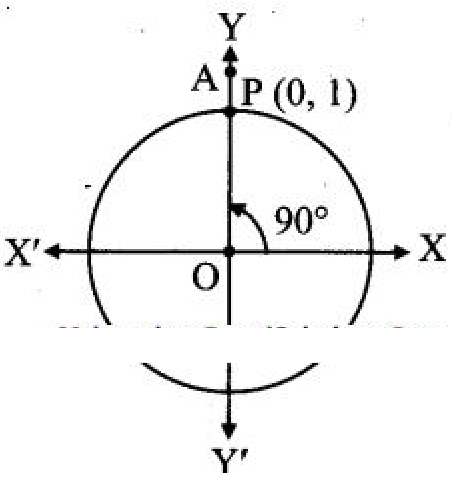
Question 10.
The value of tan 1°.tan 2° tan 3° equal to
(A) -1
(B) 1
(C) $\frac{\pi}{2}$
(D) 2
Answer:
(B) 1

Explanation:

tan1° tan2° tan3° … tan89°
= (tan 1° tan 89°) (tan 2° tan 88°)
…(tan 44° tan 46°) tan 45°
= (tan 1 ° cot 1 °) (tan 2° cot 2°)
…(tan 44° cot 44°) . tan 45°
…tan(∵ 90° – θ) = cot θ]
= 1 x 1 x 1 x … x 1 x tan 45° =1

II. Answer the following:

Question 1.
Find the trigonometric functions of:
90°, 120°, 225°, 240°, 270°, 315°, -120°, -150°, -180°, -210°, -300°, -330°
Solution:
Angle of measure 90° :
Let m∠XOA = 90°
Its terminal arm (ray OA)
intersects the standard, unit circle at P(0, 1).



∴ x = 0 and y = 1
sin 90° = y = 1
cos 90° = x = 0

tan 90° = $\frac{y}{x}=\frac{1}{0}$, which is not defined

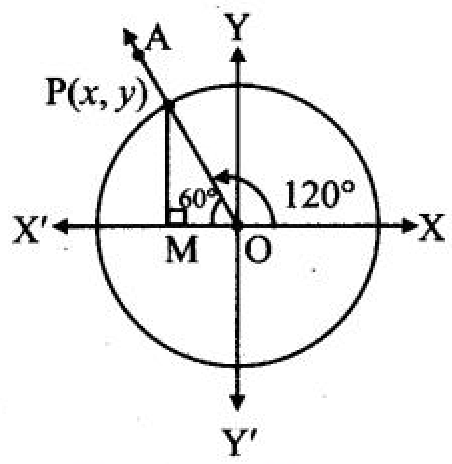
cosec 90° = $\frac{1}{y}=\frac{1}{1}$ = 1

sec 90° = $\frac{1}{x}=\frac{1}{0}$, which is not defined

cot 90° = $\frac{x}{y}=\frac{0}{1}$ = 0

Angle of measure 120° :
Let m∠XOA =120°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 30° – 60° – 90° triangle.
OP = 1

$$PM = \frac{\sqrt{3}}{2}\ OP$$



$$= \frac{\sqrt{3}}{2}(1)$$

$$= \frac{\sqrt{3}}{2}$$

$$OM = \frac{1}{2}\ OP$$

$$= \frac{1}{2}\ (1)$$

$$= \frac{1}{2}$$

Since point P lies in the 2nd quadrant, x < 0, y > 0

$$\therefore \quad x = -\,OM = -\,\frac{1}{2} \text{ and } y = PM = \frac{\sqrt{3}}{2}$$

$$\therefore \quad P \equiv \left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\sin 120° = y = \frac{\sqrt{3}}{2}$$

$$\cos 120° = x = -\frac{1}{2}$$

$$\tan 120° = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$

$$\operatorname{cosec} 120° = \frac{1}{y} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$$

$$\sec 120° = \frac{1}{x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\cot 120° = \frac{x}{y} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

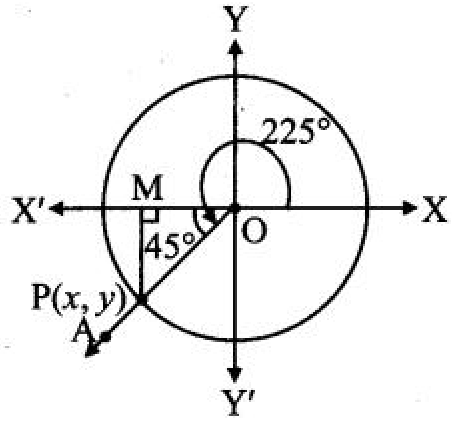
[Note: Answer given in the textbook of tan 120° is $-\frac{1}{\sqrt{3}}$ and cot 120° is $-\sqrt{3}$. However, as per our $-\sqrt{3}$ calculation the answer of tan 120° is $-\sqrt{3}$ and cot 120° is $-\frac{1}{\sqrt{3}}$

Angle of measure 225° :
Let m∠XOA = 225°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
ΔOMP is a 45° – 45° – 90° triangle.
OP = 1

$$OM = \frac{1}{\sqrt{2}} OP$$

$$= \frac{1}{\sqrt{2}} (1)$$

$$= \frac{1}{\sqrt{2}}$$



$$PM = \frac{1}{\sqrt{2}} OP$$

$$= \frac{1}{\sqrt{2}} (1)$$

$$= \frac{1}{\sqrt{2}}$$

Since point P lies in the 3rd quadrant, $x < 0$, $y < 0$

$$\therefore \quad x = -OM = -\frac{1}{\sqrt{2}} \text{ and } y = -PM = -\frac{1}{\sqrt{2}}$$

$$\therefore \quad P \equiv \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

$$\sin 225° = y = -\frac{1}{\sqrt{2}}$$

$$\cos 225° = x = -\frac{1}{\sqrt{2}}$$

$$\tan 225° = \frac{y}{x} = \frac{-\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = 1$$

$$\operatorname{cosec} 225° = \frac{1}{y} = \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}$$

$$\sec 225° = \frac{1}{x} = \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}$$

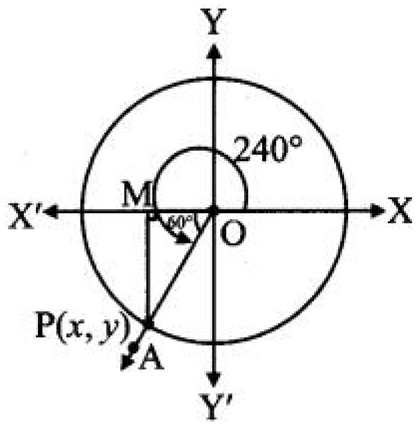
$$\cot 225° = \frac{x}{y} = \frac{-\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = 1$$

Angle of measure 240° :
Let m∠XOA = 240°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
ΔOMP is a 30° – 60° – 90° triangle.
OP = 1

$$OM = \frac{1}{2} OP$$

$$= \frac{1}{2} (1)$$

$$= \frac{1}{2}$$



$$PM = \frac{\sqrt{3}}{2} OP$$

$$= \frac{\sqrt{3}}{2} (1)$$

$$= \frac{\sqrt{3}}{2}$$

Since point P lies in the 3rd quadrant, x < 0, y < 0

$$\therefore \quad x = -\text{OM} = -\frac{1}{2} \text{ and } y = -\text{PM} = -\frac{\sqrt{3}}{2}$$

$$\therefore \quad \text{P} \equiv \left(\frac{-1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$$\sin 240° = y = -\frac{\sqrt{3}}{2}$$

$$\cos 240° = x = -\frac{1}{2}$$

$$\tan 240° = \frac{y}{x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

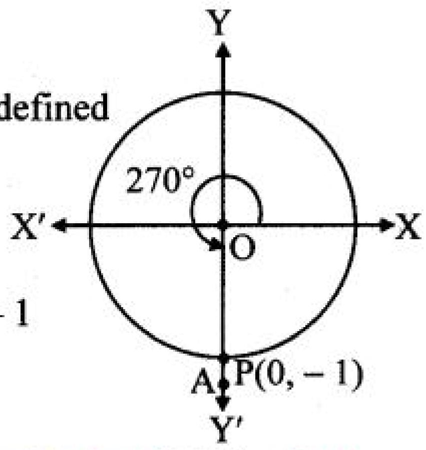
$$\operatorname{cosec} 240° = \frac{1}{y} = \frac{1}{\left(\frac{-\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\sec 240° = \frac{1}{x} = \frac{1}{-\frac{1}{2}} = -2$$

$$\cot 240° = \frac{x}{y} = \frac{\left(\frac{-1}{2}\right)}{\left(\frac{-\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}}$$

Angle of measure 270° :
Let m∠XOA = 270°
Its terminal arm (ray OA) intersects the standard unit circle at P(0, – 1).
x = 0 andy = – 1
sin 270° = y = -1
cos 270° = x = 0
tan 270° = $\frac{y}{x}$

$$= -\frac{1}{0}, \text{ which is not defined}$$



$$\operatorname{cosec} 270° = \frac{1}{y} = \frac{1}{-1} = -1$$

$$\sec 270° = \frac{1}{x} = \frac{1}{0}, \text{ which is not defined}$$

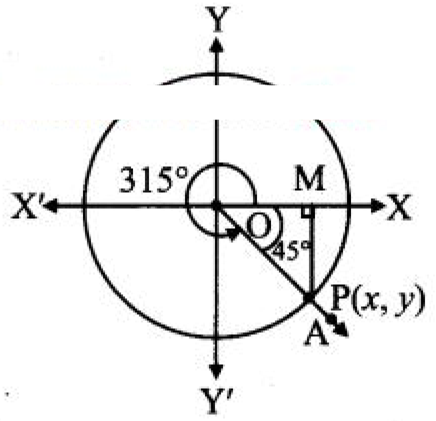
$$\cot 270° = \frac{x}{y} = \frac{0}{-1} = 0$$

Angle of measure 315° :
Let m∠XOA = 315°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 45° – 45° – 90° triangle.
OP = 1

$$OM = \frac{1}{\sqrt{2}} OP$$

$$= \frac{1}{\sqrt{2}} (1)$$

$$= \frac{1}{\sqrt{2}}$$



$$PM = \frac{1}{\sqrt{2}} OP$$

$$= \frac{1}{\sqrt{2}} (1) = \frac{1}{\sqrt{2}}$$

Since point P lies in the 4^{th} quadrant,
$x > 0, y < 0$

$\therefore \quad x = OM = \frac{1}{\sqrt{2}}$ and $y = -PM = -\frac{1}{\sqrt{2}}$

$\therefore \quad P \equiv \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$$\sin 315° = y = -\frac{1}{\sqrt{2}}$$

$$\cos 315° = x = \frac{1}{\sqrt{2}}$$

$$\tan 315° = \frac{y}{x} = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -1$$

$$\operatorname{cosec} 315° = \frac{1}{y} = \frac{1}{\left(-\frac{1}{\sqrt{2}}\right)} = -\sqrt{2}$$

$$\sec 315° = \frac{1}{x} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}$$

$$\cot 315° = \frac{x}{y} = \frac{\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = -1$$

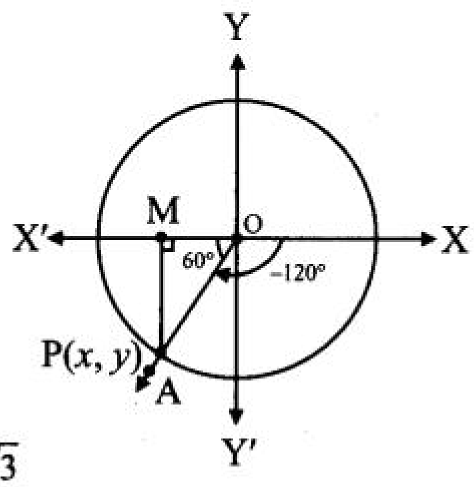
[Note: Answer given in the textbook of cot 315° is 1. However, as per our calculation it is -1.]

Angle of measure (-120°):
Let $m\angle XOA = -120°$
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
$\therefore$ ΔOMP is a 30° – 60° – 90° triangle.
OP = 1,

$$OM = \frac{1}{2} OP$$

$$= \frac{1}{2} (1)$$

$$= \frac{1}{2}$$



$$PM = \frac{\sqrt{3}}{2} OP$$

$$= \frac{\sqrt{3}}{2} (1) = \frac{\sqrt{3}}{2}$$

Since point P lies in the 3rd quadrant, x < 0, y < 0

$\therefore \quad x = -\text{OM} = \dfrac{-1}{2}$ and $y = -\text{PM} = \dfrac{-\sqrt{3}}{2}$

$\therefore \quad \text{P} \equiv \left(\dfrac{-1}{2}, \dfrac{-\sqrt{3}}{2}\right)$

$$\sin(-120^\circ) = y = -\frac{\sqrt{3}}{2}$$

$$\cos(-120^\circ) = x = -\frac{1}{2}$$

$$\tan(-120^\circ) = \frac{y}{x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

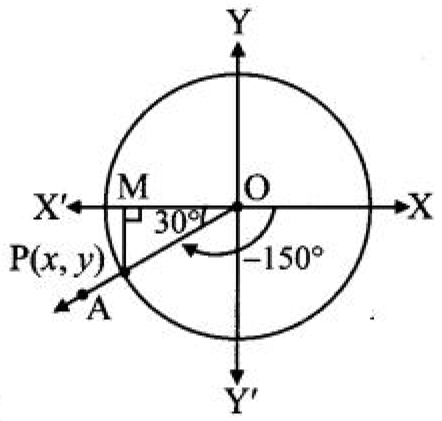
$$\text{cosec}(-120^\circ) = \frac{1}{y} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\sec(-120^\circ) = \frac{1}{x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\cot(-120^\circ) = \frac{x}{y} = \frac{\left(-\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}}$$

Angle of measure (-150°) :
Let m∠XOA = – 150°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 30° – 60° – 90° triangle.
OP = 1

$$OM = \frac{\sqrt{3}}{2} OP$$
$$= \frac{\sqrt{3}}{2}(1)$$
$$= \frac{\sqrt{3}}{2}$$



$$PM = \frac{1}{2} OP$$
$$= \frac{1}{2}(1) = \frac{1}{2}$$

Since point P lies in the 3[rd] quadrant,
$x < 0, y < 0$

$\therefore$ $x = -OM = -\frac{\sqrt{3}}{2}$ and $y = -PM = -\frac{1}{2}$

$\therefore$ $P \equiv \left(-\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$

$$\sin(-150°) = y = -\frac{1}{2}$$

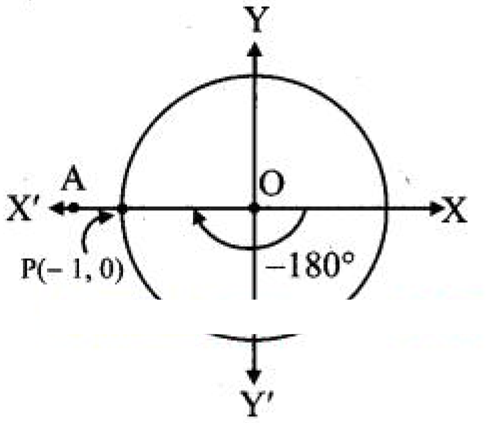
$$\cos(-150°) = x = \frac{-\sqrt{3}}{2}$$

$$\tan(-150°) = \frac{y}{x} = \frac{-\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\sec(-150°) = \frac{1}{x} = \frac{1}{\left(\frac{-\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\cot(-150°) = \frac{x}{y} = \frac{\frac{-\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$$

Angle of measure (-180°):
Let m∠XOA = – 180°
Its terminal arm (ray OA) intersects the standard unit circle at P(- 1, 0).
∴ x = – 1 andy = 0
sin (-180°) = y = 0
cos (-180°) = x
= -1



$$\tan(-180°) = \frac{y}{x} = \frac{0}{-1} = 0$$

$$\operatorname{cosec}(-180°) = \frac{1}{y} = \frac{1}{0}, \text{ which is not defined}$$

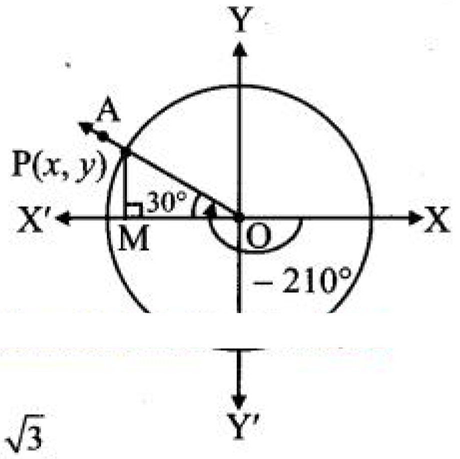
$$\sec(-180°) = \frac{1}{x} = \frac{1}{-1} = -1$$

$$\cot(-180°) = \frac{x}{y} = \frac{-1}{0}, \text{ which is not defined}$$

Angle of measure (- 210°):
Let m∠XOA = -210°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 30° – 60° – 90° triangle.
OP = 1

OP = 1

$$PM = \frac{1}{2} OP = \frac{1}{2}(1) = \frac{1}{2}$$



$$OM = \frac{\sqrt{3}}{2} OP = \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$$

Since point P lies in the 2[nd] quadrant,
$x < 0, y > 0$

$\therefore \quad x = -OM = -\frac{\sqrt{3}}{2}$ and $y = PM = \frac{1}{2}$

$\therefore \quad P \equiv \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$$\sin(-210°) = y = \frac{1}{2}$$

$$\cos(-210°) = x = \frac{-\sqrt{3}}{2}$$

$$\tan(-210°) = \frac{y}{x} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{-\sqrt{3}}{2}\right)} = -\frac{1}{\sqrt{3}}$$

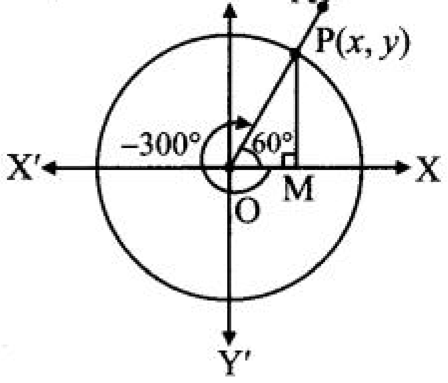
$$\operatorname{cosec}(-210°) = \frac{1}{y} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$\sec(-210°) = \frac{1}{x} = \frac{1}{\left(\frac{-\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\cot(-210°) = \frac{x}{y} = \frac{\left(\frac{-\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = -\sqrt{3}$$

Angle of measure (- 300°):
Let m∠XOA = – 300° Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
ΔOMP is a 30° – 60° – 90° triangle.
OP = 1

$$OM = \frac{1}{2} OP = \frac{1}{2}(1) = \frac{1}{2}$$



$$PM = \frac{\sqrt{3}}{2} OP = \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$$

Since point P lies in the 1st quadrant, x>0,y>0

x = OM = 12 and
y = PM = 3√2

$$\therefore \quad P \equiv \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\sin(-300^\circ) = y = \frac{\sqrt{3}}{2}$$

$$\cos(-300^\circ) = x = \frac{1}{2}$$

$$\tan(-300^\circ) = \frac{y}{x} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \sqrt{3}$$

$$\operatorname{cosec}(-300^\circ) = \frac{1}{y} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$$

$$\sec(-300^\circ) = \frac{1}{x} = \left(\frac{1}{\frac{1}{2}}\right) = 2$$

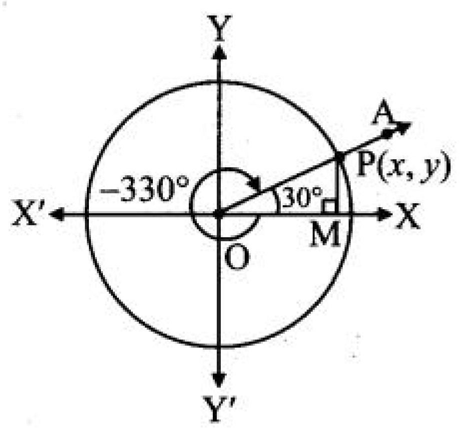
$$\cot(-300^\circ) = \frac{x}{y} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Angle of measure (- 330°):
Let m∠XOA = – 330°
Its terminal arm (ray OA) intersects the standard unit circle at P(x, y).
Draw seg PM perpendicular to the X-axis.
∴ ΔOMP is a 30° – 60° – 90° triangle.
OP= 1

$$OM = \frac{\sqrt{3}}{2}\ OP$$

$$= \frac{\sqrt{3}}{2}(1)$$

$$= \frac{\sqrt{3}}{2}$$



$$PM = \frac{1}{2}\ OP$$

$$= \frac{1}{2}(1)$$

$$= \frac{1}{2}$$

Since point P lies in the 1st quadrant, x > 0, y > 0

∴ x = OM = 3√2 and y = PM = 12

$$\therefore \quad P \equiv \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\sin(-330°) = y = \frac{1}{2}$$

$$\cos(-330°) = x = \frac{\sqrt{3}}{2}$$

$$\tan(-330°) = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\operatorname{cosec}(-330°) = \frac{1}{y} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$\sec(-330°) = \frac{1}{x} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$$

$$\cot(-330°) = \frac{x}{y} = \frac{\frac{\sqrt{3}}{2}}{\left(\frac{1}{2}\right)} = \sqrt{3}$$

Question 2.
State the signs of:
i. cosec 520°
ii. cot 1899°
iii. sin 986°
Solution:
i. 520° =360° + 160°
∴ 520° and 160° are co-terminal angles.
Since 90° < 160° < 180°,
160° lies in the 2nd quadrant.
∴ 520° lies in the 2nd quadrant,
∴ cosec 520° is positive.

ii. 1899° = 5 x 360° + 99°
∴ 1899° and 99° are co-terminal angles.
Since 90° < 99° < 180°,
99° lies in the 2nd quadrant.
∴ 1899° lies in the 2nd quadrant.
∴ cot 1899° is negative.

iii. 986° = 2x 360° + 266°
∴ 986° and 266° are co-terminal angles.
Since 180° < 266° < 270°,
266° lies in the 3rd quadrant.
∴ 986° lies in the 3rd quadrant.
∴ sin 986° is negative.

Question 3.
State the quadrant in which 6 lies if
i. tan θ < 0 and sec θ > 0
ii. sin θ < 0 and cos θ < 0
iii. sin θ > 0 and tan θ < 0
Solution:
i. tan θ < 0 tan θ is negative in 2nd and 4th quadrants, sec θ > 0
sec θ is positive in 1st and 4th quadrants.
∴ θ lies in the 4th quadrant.

ii. sin θ < 0
sin θ is negative in 3rd and 4th quadrants, cos θ < 0
cos θ is negative in 2nd and 3rd quadrants.
∴ θ lies in the 3rd quadrant.

iii. $\sin \theta > 0$
$\sin \theta$ is positive in 1st and 2nd quadrants, $\tan \theta < 0$
$\tan \theta$ is negative in 2nd and 4th quadrants.
∴ θ lies in the 2nd quadrant.

Question 4.
Which is greater?
sin (1856°) or sin (2006°)
Solution:
1856° = 5 x 360° + 56°
∴ 1856° and 56° are co-terminal angles.
Since 0° < 56° < 90°, 56° lies in the 1st quadrant.
∴ 1856° lies in the 1st quadrant,
∴ sin 1856° >0 …(i)
2006° = 5 x 360° + 206°
∴ 2006° and 206° are co-terminal angles.
Since 180° < 206° < 270°,
206° lies in the 3rd quadrant.
∴ 2006° lies in the 3rd quadrant,
∴ sin 2006° <0 …(ii)
From (i) and (ii),
sin 1856° is greater.

Question 5.
Which of the following is positive?
sin(-310°) or sin(310°)
Solution:
Since 270° <310° <360°,
310° lies in the 4th quadrant.
∴ sin (310°) < 0
-310° = -360°+ 50°
∴ 50° and – 310° are co-terminal angles.
Since 0° < 50° < 90°, 50° lies in the 1st quadrant.
∴ – 310° lies in the 1st quadrant.
∴ sin (- 310°) > 0
∴ sin (- 310°) is positive.

Question 6.
Show that $1 - 2\sin \theta \cos \theta \geq 0$ for all $\theta \in R$.
Solution:
$1 - 2 \sin \theta \cos \theta$
$= \sin^2 \theta + \cos^2 \theta - 2\sin \theta \cos \theta$
$= (\sin \theta - \cos \theta)^2 \geq 0$ for all $\theta \in R$

Question 7.
Show that $\tan^2 \theta + \cot^2 \theta \geq 2$ for all $\theta \in R$.
Solution:

$$\tan^2 \theta + \cot^2 \theta = \tan^2 \theta + \frac{1}{\tan^2 \theta}$$

$$= (\tan \theta)^2 + \left(\frac{1}{\tan \theta}\right)^2$$

$$= \left(\tan \theta - \frac{1}{\tan \theta}\right)^2 + 2\tan \theta \cdot \frac{1}{\tan \theta}$$

$$\ldots[\because a^2 + b^2 = (a-b)^2 + 2ab]$$

$$= \left(\tan \theta - \frac{1}{\tan \theta}\right)^2 + 2 \geq 2 \text{ for all } \theta \in R.$$

Question 8.
If $\sin \theta = \frac{x^2-y^2}{x^2+y^2}$, then find the values of $\cos \theta$, $\tan \theta$ in terms of x and y.
Solution:
Given, $\sin \theta = \frac{x^2-y^2}{x^2+y^2}$
we know that

$\cos^2\theta = 1 - \sin^2\theta$

$$= 1 - \frac{\left(x^2 - y^2\right)^2}{\left(x^2 + y^2\right)^2}$$

$$= \frac{\left(x^2 + y^2\right)^2 - \left(x^2 - y^2\right)^2}{\left(x^2 + y^2\right)^2}$$

$$= \frac{x^4 + 2x^2y^2 + y^4 - \left(x^4 - 2x^2y^2 + y^4\right)}{\left(x^2 + y^2\right)^2}$$

$$\therefore \quad \cos^2\theta = \frac{4x^2y^2}{\left(x^2 + y^2\right)^2}$$

$$\therefore \quad \cos\theta = \pm\frac{2xy}{\left(x^2 + y^2\right)}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\frac{x^2 - y^2}{x^2 + y^2}}{\pm\frac{2xy}{x^2 + y^2}}$$

$$= \pm\frac{x^2 - y^2}{2xy}$$

[Note: Answer given in the textbook of cos θ = 2xyx2+y2 and tan θ
= .However,asperourcalculationtheanswerofcosθ=±[latex]2xyx2+y2 and tan θ = ± x2−y22xy.]

Question 9.
If sec θ = 2–√ and 3π2 < θ < 2π, then evaluate 1+tanθ+cosecθ1+cotθ−cosecθ
Solution:
Given sec θ = 2–√
We know that,
$\tan^2\theta = \sec^2\theta - 1$
= (2–√) – 1
= 2 – 1 = 1
∴ tan θ = ±1
Since 3π2 < θ < 2π
θ lies in the 4th quadrant.
∴ tan θ < 0

∴ tan θ = -1

$$\cot \theta = \frac{1}{\tan \theta} = -1$$

$$\cos \theta = \frac{1}{\sec \theta} = \frac{1}{\sqrt{2}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\therefore \quad \sin \theta = \tan \theta \cos \theta$$

$$= (-1)\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$$

$$\therefore \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta} = -\sqrt{2}$$

$$\therefore \quad \frac{1+\tan \theta+\operatorname{cosec} \theta}{1+\cot \theta-\operatorname{cosec} \theta}$$

$$= \frac{1-1-\sqrt{2}}{1-1+\sqrt{2}} = \frac{-\sqrt{2}}{\sqrt{2}} = -1$$

Question 10.
Prove the following:

i. $\sin^2 A \cos^2 B + \cos^2 A \sin^2 B + \cos^2 A \cos^2 B + \sin^2 A \sin^2 B = 1$
Solution:
L.H.S. = $\sin^2 A \cos^2 B + \cos^2 A \sin^2 B + \cos^2 A \cos^2 B + \sin^2 A \sin^2 B$
= $\sin^2 A (\cos^2 B + \sin^2 B) + \cos^2 A (\sin^2 B + \cos^2 B)$
= $\sin^2 A(1) + \cos^2 A(1)$
= 1 = R.H.S.

ii. $\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta-\operatorname{cosec}^3\theta}=\sin^2\theta\cos^2\theta$
Solution:

$$\text{L.H.S.} = \frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta-\operatorname{cosec}^3\theta}$$

$$= \frac{\left(1+\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\right)(\sin\theta-\cos\theta)}{\frac{1}{\cos^3\theta}-\frac{1}{\sin^3\theta}}$$

$$= \frac{\left(\frac{\sin\theta\cos\theta+\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}\right)(\sin\theta-\cos\theta)}{\frac{\sin^3\theta-\cos^3\theta}{\sin^3\theta\cos^3\theta}}$$

$$= \frac{(\sin\theta\cos\theta+\cos^2\theta+\sin^2\theta)(\sin\theta-\cos\theta)}{\sin\theta\cos\theta} \times \frac{\sin^3\theta\cos^3\theta}{\sin^3\theta-\cos^3\theta}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)\sin^2\theta\cos^2\theta}{\sin^3\theta-\cos^3\theta}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)\sin^2\theta\cos^2\theta}{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)}$$

$$\ldots[\because a^3-b^3=(a-b)(a^2+ab+b^2)]$$

$$= \sin^2\theta\cos^2\theta$$
$$= \text{R.H.S.}$$

iii. L.H.S. = $\left(\tan\theta+\frac{1}{\cos\theta}\right)^2+\left(\tan\theta-\frac{1}{\cos\theta}\right)^2=2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$
Solution:
L.H.S. = $\left(\tan\theta+\frac{1}{\cos\theta}\right)^2+\left(\tan\theta-\frac{1}{\cos\theta}\right)^2$

$= (\tan\theta + \sec\theta)^2 + (\tan\theta - \sec\theta)^2$
$= \tan^2\theta + 2\tan\theta\sec\theta + \sec^2\theta$
$+ \tan^2\theta - 2\tan\theta\sec\theta + .\sec^2\theta$
$= 2(\tan^2\theta + \sec^2\theta)$

iv. $2.\sec^2\theta - \sec^4\theta - 2.\operatorname{cosec}^2\theta + \operatorname{cosec}^4\theta = \cot^4\theta - \tan^4\theta$
Solution:
LHS.
$= 2.\sec^2\theta - \sec^4\theta - 2.\operatorname{cosec}^2\theta + \operatorname{cosec}^4\theta = = 2\sec^2\theta - (\sec^2\theta)^2 - 2\operatorname{cosec}^2\theta + (\operatorname{cosec}^2\theta)^2$
$= 2(1+\tan^2\theta) - (1+\tan^2\theta)^2 - 2(1+\cot^2\theta)$
$+ (1+\cot^2\theta)^2$
$= 2 + 2\tan^2\theta - (1 + 2\tan^2\theta + \tan^4\theta)$
$- 2 - 2\cot^2\theta + 1 + 2\cot^2\theta + \cot^4\theta$
$= 2 + 2.\tan^2\theta - 1 - 2\tan^2\theta - \tan^4\theta - 2$
$- 2\cot^2\theta + 1 + 2\cot^2\theta + \cot^4\theta$
$= \cot^4\theta - \tan^4\theta$ = R.H.S.

v. $\sin^4\theta + \cos^4\theta = \sin^4\theta + \cos^4\theta$
Solution:
L.H.S. $= \sin^4\theta + \cos^4\theta$
$= (\sin^2\theta)^2 + (\cos^2\theta)^2 = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$
… [v $a^2 + b^2 = (a + b)^2 - 2ab$]
$= 1 - 2\sin^2\theta\cos^2\theta$
= R.H.S.

vi. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$
L.H.S =
$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1=0$
$= \sin^6\theta + \cos^6\theta$
$= (\sin^2\theta)^3 + (\cos^2\theta)^3 = (\sin^2\theta + \cos^2\theta)^3$
$- 3\sin^2\theta\cos^2\theta$ (sin2 0 + cos2 0)
…[••• $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$]
$= (1)^3 - 3\sin^2\theta\cos^2\theta(1)$
$= 1-3\sin^2\theta\cos^2\theta\ \sin^4\theta + \cos^4\theta$
$= (\sin^2\theta)^2 + (\cos^2\theta)^2 = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$
…[Y $a^2 + b^2 = (a + b)^2 - 2ab$]
$= 1-2\sin^2\theta\cos^2\theta$
L.H.S.$= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$
$= 2(1-3\sin^2\theta\cos^2\theta) -3(1 - 2\sin^2\theta\cos^2\theta) + 1$
$= 2-6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta + 1 = c$
= R.H.S.

vii. $\cos^4\theta - \sin^4\theta + 1 = 2\cos^2\theta$
L.H.S. $= \cos^4\theta - \sin^4\theta + 1$
$= (\cos^2\theta)^2 - (\sin^2\theta)^2 + 1 = (\cos^2\theta + \sin^2\theta)\ c(os^2\theta - \sin^2\theta) +1$
$= (1)(\cos^2\theta - \sin^2\theta) + 1 = \cos^2\theta + (1 - \sin^2\theta)$
$= \cos^2\theta + \cos^2\theta = 2\cos^2\theta$ = R.H.S.

viii. $\sin^4\theta + 2\sin^2\theta\cos^2\theta = 1 - \cos^4\theta$
L.H.S. $= \sin^4\theta + 2\sin^2\theta\cos^2\theta = \sin^2\theta(\sin^2\theta + 2\cos^2\theta)$
$= (\sin^2\theta)(\sin^2\theta + \cos^2\theta + \cos^2\theta) = (1 - \cos^2\theta)(1 + \cos^2\theta)$
$= 1 - \cos^4\theta$ = R.H.S.

ix. $\sin^3\theta+\cos^3\theta\sin\theta+\cos\theta+\sin^3\theta-\cos^3\theta\sin\theta-\cos\theta=2$
Solution:

$$= 2\left(\frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{\cos^2\theta}\right)$$

$$= 2\left(\frac{\sin^2\theta+1}{\cos^2\theta}\right)$$

$$= 2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$$

= R.H.S.

$= (\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta) + (\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)$
$= 2(\sin^2\theta + \cos^2\theta)$
$= 2(1)$
$= 2$ = R.H.S.

x. $\tan^2 \theta - \sin^2 \theta = \sin^4 \theta \sec^2 \theta$
Solution:
L.H.S. = $\tan^2 \theta - \sin^2 \theta$
= $\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$
= $\sin^2 \theta \left(\frac{1}{\cos^2\theta} - 1\right)$
= $\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$
= $(\sin^2 \theta)(\sin^2 \theta)\sec^2 \theta$
= $\sin^4 \theta \sec^2 \theta$
= R.H.S

xi. $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos \theta + \text{see } \theta)^2 = \tan^2 \theta + \cot^2 \theta + 7$
Solution:
L.H.S. = $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos \theta + \text{see } \theta)^2$
= $\sin^2 \theta + \operatorname{cosec}^2 \theta + 2\sin\theta \operatorname{cosec} \theta$
+ $\cos^2 \theta + \sec^2 \theta + 2\sec 0 \cos 0$
= $(\sin^2 \theta + \cos^2 \theta) + \operatorname{cosec}^2 \theta + 2 + \sec^2 \theta + 2$
= $1 + (1 + \cot^2 \theta) + 2 + (1 + \tan^2 \theta) + 2 = \tan^2 \theta + \cot^2 \theta + 7$
= R.H.S.

xii. $\sin^8\theta - \cos^8\theta = (\sin^2 \theta - \cos^2 \theta)(1 - 2\sin^2 \theta \cos^2 \theta)$
Solution:
L.H.S. = $\sin^8\theta - \cos^8\theta$
= $(\sin^4\theta)^2 - (\cos^4\theta)^2$
= $(\sin^4\theta - \cos^4\theta)(\sin^4\theta + \cos^4\theta)$
= $[(\sin^2 \theta)^2 - (\cos^2 \theta)^2]$
. $[(\sin^2 \theta)^2 + (\cos^2 \theta)^2]$
= $(\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta)$. $[(\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta.\cos^2 \theta]$ ...[Y $a^2 + b^2 = (a + b)^2 - 2ab$]
= $(1)(\sin^2 \theta - \cos^2 \theta)(1^2 - 2\sin^2 \theta \cos^2 \theta)$
= $(\sin^2 \theta - \cos^2 \theta)(1 - 2\sin^2 \theta \cos^2 \theta)$
= R.H.S.

xiii. $\sin^6 A + \cos^6 A = 1 - 3 \sin^2 A + 3\sin^4 A$
Soluiton:
L.H.S. = $\sin^6 A + \cos^6 A$
= $(\sin^2 A)^3 + (\cos^2 A)^3$
= $(\sin^2 A + \cos^2 A)^3$
$- 3\sin^2 A \cos^2 A(\sin^2 A + \cos^2 A)$
...[$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$]
= $1^3 - 3\sin^2 A \cos^2 A (1)$
= $1 - 3\sin^2 A \cos^2 A$
= $1 - 3 \sin^2 A (1 - \sin^2 A)$
= $1 - 3 \sin^2 A + 3\sin^4 A$
= R.H.S.

xiv. $(1 + \tan A \tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A \sec^2 B$
Solution:
L.H.S. = $(1 + \tan A \tan B)^2 + (\tan A - \tan B)^2$
= $1 + 2\tan A \tan B + \tan^2 A \tan^2 + \tan^2 A - 2\tan A \tan B + \tan^2 B$
= $1 + \tan^2 A + \tan^2 B + \tan^2 A \tan^2 B$
= $1(1 + \tan^2 A) + \tan^2 B(1 + \tan^2 A)$
= $(1 + \tan^2 A)(1 + \tan^2 B)$
= $\sec^2 A \sec^2 B$ = R.H.S.

xv. $\frac{1+\cot\theta+\operatorname{cosec}\theta}{1-\cot\theta+\operatorname{cosec}\theta} = \frac{\operatorname{cosec}\theta+\cot\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}$
Solution:
We know that $\operatorname{cosec}^2\theta - \cot^2 \theta = 1$

$\therefore$ (cosec θ – cot θ) (cosec θ + cot θ) = 1

$$\text{L.H.S.} = \frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\frac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}$$

$$=\frac{(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)}{\sin\theta+\cos\theta}$$

$$+\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta-\cos\theta}$$

$$\ldots\left[\begin{array}{l}a^3+b^3=(a+b)(a^2-ab+b^2)\\ a^3-b^3=(a-b)(a^2+ab+b^2)\end{array}\right]$$

xvi. *tanθ+secθ−1tanθ+secθ+1=tanθsecθ+1*

Solution:
We know that
$\tan^2\theta = \sec^2\theta - 1$
$\therefore$ tan θ. tanθ = (sec θ + 1)(sec θ – 1)

$$\therefore \quad \frac{\operatorname{cosec}\theta+\cot\theta}{1}=\frac{1}{\operatorname{cosec}\theta-\cot\theta}$$

By componendo-dividendo, we get

$$\frac{\operatorname{cosec}\theta+\cot\theta+1}{\operatorname{cosec}\theta+\cot\theta-1}=\frac{1+\operatorname{cosec}\theta-\cot\theta}{1-(\operatorname{cosec}\theta-\cot\theta)}$$

$$\therefore \quad \frac{\operatorname{cosec}\theta+\cot\theta+1}{\operatorname{cosec}\theta+\cot\theta-1}=\frac{1+\operatorname{cosec}\theta-\cot\theta}{1-\operatorname{cosec}\theta+\cot\theta}$$

$$\therefore \quad \frac{\operatorname{cosec}\theta+\cot\theta+1}{1+\operatorname{cosec}\theta-\cot\theta}=\frac{\operatorname{cosec}\theta+\cot\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}$$

xvii. *cosecθ+cotθ−1cosecθ+cotθ+1=1−sinθcosθ*

Solution:
We know that,
$\cot^2\theta = \operatorname{cosec}^2\theta - 1$
$\therefore$ cot θ . cot θ = (cosec θ + 1)(cosec θ – 1)

$$\therefore \quad \frac{\tan\theta}{\sec\theta+1}=\frac{\sec\theta-1}{\tan\theta}$$

By the theorem on equal ratios, we get

$$\frac{\tan\theta}{\sec\theta+1}=\frac{\sec\theta-1}{\tan\theta}=\frac{\tan\theta+\sec\theta-1}{\sec\theta+1+\tan\theta}$$

$$\therefore \quad \frac{\tan\theta+\sec\theta-1}{\tan\theta+\sec\theta+1}=\frac{\tan\theta}{\sec\theta+1}$$

Alternate Method:

$$\therefore \quad \frac{\cot\theta}{\operatorname{cosec}\theta+1}=\frac{\operatorname{cosec}\theta-1}{\cot\theta}$$

By the theorem on equal ratios, we get

$$\frac{\cot\theta}{\operatorname{cosec}\theta+1}=\frac{\operatorname{cosec}\theta-1}{\cot\theta}=\frac{\cot\theta+\operatorname{cosec}\theta-1}{\operatorname{cosec}\theta+1+\cot\theta}$$

$$\therefore \quad \frac{\operatorname{cosec}\theta-1}{\cot\theta}=\frac{\cot\theta+\operatorname{cose}\theta-1}{\operatorname{cosec}\theta+1+\cot\theta}$$

$$\therefore \quad \frac{\frac{1}{\sin\theta}-1}{\frac{\cos\theta}{\sin\theta}}=\frac{\operatorname{cosec}\theta+\cot\theta-1}{\operatorname{cosec}\theta+\cot\theta+1}$$

$$\therefore \quad \frac{\operatorname{cosec}\theta+\cot\theta-1}{\operatorname{cosec}\theta+\cot\theta+1}=\frac{1-\sin\theta}{\cos\theta}$$

xviii. *cosecθ+cotθ+1cotθ+cosecθ−1=cotθcosecθ−1*

solution:

We know that,
$\cot^2 \theta = \operatorname{cosec}^2 \theta - 1$
$\therefore \cot \theta . \cot \theta = (\operatorname{cosec} \theta + 1)(\operatorname{cosec} \theta - 1)$

$$\therefore \quad \frac{\cot\theta}{\operatorname{cosec}\theta - 1} = \frac{\operatorname{cosec}\theta + 1}{\cot\theta}$$

By the theorem on equal ratios, we get

$$\frac{\cot\theta}{\operatorname{cosec}\theta - 1} = \frac{\operatorname{cosec}\theta + 1}{\cot\theta} = \frac{\cot\theta + \operatorname{cosec}\theta + 1}{\operatorname{cosec}\theta - 1 + \cot\theta}$$

$$\therefore \quad \frac{\operatorname{cosec}\theta + \cot\theta + 1}{\cot\theta + \operatorname{cosec}\theta - 1} = \frac{\cot\theta}{\operatorname{cosec}\theta - 1}$$