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Maharashtra State Board 11th Commerce Maths Solutions Chapter 2 Measures of Dispersion Ex 2.1

Question 1.
Find range of the following data:
575, 609, 335, 280, 729, 544, 852, 427, 967, 250
Solution:
Here, largest value (L) = 967, smallest value (S) = 250
∴ Range = L – S
= 967 – 250
= 717

Question 2.
The following data gives the number of typing mistakes done by Radha during a week. Find the range of the data.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
No. of mistakes	15	20	21	12	17	10

Solution:
Here, largest value (L) = 21, smallest value (S) = 10
∴ Range = L – S
= 21 – 10
= 11

Question 3.
Find range for the following data:

Classes	62-64	64-66	66-68	68-70	70-72
Frequency	5	3	4	5	3

Solution:
Here, upper limit of the highest class (L) = 72, lower limit of the lowest class (S) = 62

∴ Range = L – S
= 72 – 62
= 10

Question 4.
Find the Q. D. for the following data.
3, 16, 8, 15, 19, 11, 5, 17, 9, 5, 3.
Solution:
The given data can be arranged in ascending order as follows:
3, 3, 5, 5, 8, 9, 11, 15, 16, 17, 19
Here, n = 11
Q_1 = value of $\left(\frac{n+1}{4}\right)^{th}$ observation
= value of $\left(\frac{11+1}{4}\right)^{th}$ observation
= value of 3rd observation
∴ Q_1 = 5
Q_3 = value of $3\left(\frac{n+1}{4}\right)^{th}$ observation
= value of $3\left(\frac{11+1}{4}\right)^{th}$ observation
= value of (3 × 3)th observation
= value of 9th observation
= 16
∴ Q.D.= $\frac{Q_3-Q_1}{2}$
= $\frac{16-5}{2}$
= $\frac{11}{2}$
= 5.5

Question 5.
Given below are the prices of shares of a company for the last 10 days. Find Q.D.:
172, 164, 188, 214, 190, 237, 200, 195, 208, 230.
Solution:
The given data can be arranged in ascending order as follows:
164, 172, 188, 190, 195, 200, 208, 214, 230, 237
Here, n = 10
Q_1 = value of $\left(\frac{n+1}{4}\right)^{th}$ observation
= value of $\left(\frac{10+1}{4}\right)^{th}$ observation
= value of (2.75)th observation
= value of 2nd observation + 0.75(value of 3rd observation – value of 2nd observation)

= 172 + 0.75(188 – 172)
= 172 + 0.75(16)
= 172 + 12
= 184
∴ Q_3 = value of 3(n+14)th observation
= value of 3(10+14)th observation
= value of (3 × 2.75)th observation
= value of (8.25)th observation
= value of 8th observation + 0.25(value of 9th observation – value of 8th observation)
= 214 + 0.25(230 – 214)
= 214 + 0.25(16)
= 214 + 4
= 218
∴ Q.D. = Q3−Q12
= 218−1842
= 342
= 17

Question 6.
Calculate Q.D. for the following data.

X	24	25	26	27	28	29	30
F	6	5	3	2	4	7	3

Solution:
Since the given data is arranged in ascending order, we construct less than cumulative frequency table as follows:

x	f	Less than cumulative frequency (c.f.)
24	6	6
25	5	11 ← Q_1
26	3	14
27	2	16
28	4	20
29	7	27 ← Q_3
30	3	30
Total	N = 30	

Here, n = 30
Q_1 = value of $(n+14)$th observation
= value of $(30+14)$th observation
= value of (7.75)th observation
Cumulative frequency which is just greater than (or equal to) 7.75 is 11.
∴ Q_1 = 25
Q_3 = value of $[3(n+14)]$th observation
= value of $[3(30+14)]$th observation
= value of (3 × 7.75)th observation
= value of (23.25)th observation
Cumulative frequency which is just greater than (or equal to) 23.25 is 27.
∴ Q_3 = 29
∴ Q.D. = $Q3-Q12$
= $29-252$
∴ Q.D. = 2

Question 7.
Following data gives the age distribution of 240 employees of a firm.
Calculate Q.D. of the distribution.

Age (In years)	20-25	25-30	30-35	35-40	40-45	45-50
No. of employees	30	40	60	50	46	14

Solution:
We construct the less than cumulative frequency table as follows:

Age (In years)	No. of employees	Less than cumulative frequency (c.f.)
20-25	30	30
25-30	40	70 ← Q_1
30-35	60	130
35-40	50	180 ← Q_3
40-45	46	226
45-50	14	240
Total	N = 240	

Here, N = 240
Q_1 class = class containing $(N4)$th observation
∴ $N4=2404$ = 60

Cumulative frequency which is just greater than (or equal to) 60 is 70.

∴ Q_1 lies in the class 25 – 30.

∴ L = 25, c.f. = 30, f = 40, h = 5

$$\therefore \quad Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f.\right)$$

$$= 25 + \frac{5}{40}(60 - 30)$$

$$= 25 + \frac{1}{8} \times 30$$

$$= 25 + 3.75$$

$$\therefore \quad Q_1 = 28.75$$

Q_3 class = class containing $\left(\frac{3N}{4}\right)^{th}$ observation

$$\therefore \quad \frac{3N}{4} = \frac{3 \times 240}{4} = 180$$

Cumulative frequency which is just greater than (or equal to) 180 is 180.

∴ Q_3 lies in the class 35-40.

∴ L = 35, c.f. = 130, f = 50, h = 5

$$\therefore \quad Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$$

$$= 35 + \frac{5}{50}(180 - 130)$$

$$= 35 + \frac{1}{10} \times 50$$

$$= 35 + 5$$

$$\therefore \quad Q_3 = 40$$

$$\therefore \quad Q.D. = \frac{Q_3 - Q_1}{2} = \frac{40 - 28.75}{2} = \frac{11.25}{2} = 5.625$$

Question 8.

Following data gives the weight of boxes. Calculate Q.D. for the data.

Weights (kg.)	10-12	12-14	14-16	16-18	18-20	20-22
No. of boxes	3	7	16	14	18	2
c.f.	3	10	26	40	58	60

Solution:

Weights (kg.)	No. of boxes (f)	c.f.
10-12	3	3
12-14	7	10
14-16	16	26 ← Q_1
16-18	14	40
18-20	18	58 ← Q_3
20-22	2	60
Total	N = 60	

Here, N = 60

Q_1 class = class containing $\left(\frac{N}{4}\right)^{th}$ observation

∴ $\frac{N}{4} = \frac{60}{4}$ = 15

Cumulative frequency which is just greater than (or equal to) 15 is 26.

∴ Q_1 lies in the class 14 – 16.

∴ L = 14, c.f. = 10, f = 16, h = 2

$$\therefore \quad Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - \text{c.f.}\right)$$

$$= 14 + \frac{2}{16}(15 - 10)$$

$$= 14 + \frac{1}{8} \times 5$$

$$= 14 + 0.625$$

$$\therefore \quad Q_1 = 14.625$$

Q_3 class = class containing $\left(\frac{3N}{4}\right)^{th}$ observation

$$\therefore \quad \frac{3N}{4} = \frac{3 \times 60}{4} = 45$$

Cumulative frequency which is just greater than (or equal to) 45 is 58.

∴ Q_3 lies in the class 18 – 20.

$\therefore$ L = 18, c.f. = 40, f = 18, h = 2

$$\therefore \quad Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$$

$$= 18 + \frac{2}{18}(45 - 40)$$

$$= 18 + \frac{1}{9} \times 5$$

$$= 18 + 0.5556$$

$$\therefore \quad Q_3 = 18.555$$

$$\therefore \quad Q.D. = \frac{Q_3 - Q_1}{2}$$

$$= \frac{18.5556 - 14.625}{2}$$

$$= \frac{3.9306}{2} = 1.9653$$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 2 Measures of Dispersion Ex 2.2

Question 1.
Find the variance and S.D. for the following set of numbers.
7, 11, 2, 4, 9, 6, 3, 7, 11, 2, 5, 8, 3, 6, 8, 8, 2, 6
Solution:
Given data: 7, 11, 2, 4, 9, 6, 3, 7, 11, 2, 5, 8, 3, 6, 8, 8, 2, 6
The tabulated form of the above data is as given below:

x_i	2	3	4	5	6	7	8	9	11
f_i	3	2	1	1	3	2	3	1	2

We prepare the following table for the calculation of variance and S. D.

x_i	f_i	$f_i x_i$	$f_i x_i^2$
2	3	6	12
3	2	6	18
4	1	4	16
5	1	5	25
6	3	18	108
7	2	14	98
8	3	24	192
9	1	9	81
11	2	22	242
Total	N = 18	$\sum f_i x_i = 108$	$\sum f_i x_i^2 = 792$

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{108}{18} = 6$$

$$\text{Var (X)} = \sigma_x^2 = \frac{\sum f_i x_i^2}{N} - (\bar{x})^2 = \frac{792}{18} - (6)^2 = 44 - 36 = 8$$

$\therefore$ $\text{S. D.} = \sigma_x = \sqrt{\text{Var(X)}} = \sqrt{8} = 2\sqrt{2}$

Question 2.
Find the variance and S.D. for the following set of numbers.
65, 77, 81, 98, 100, 80, 129
Solution:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
65	–25	625
77	–13	169
81	–9	81
98	8	64
100	10	100
80	–10	100
129	39	1521
$\sum x_i = 630$		$\sum (x_i - \bar{x})^2 = 2660$

Here, n = 7

$$\bar{x} = \frac{\sum x_i}{n} = \frac{630}{7} = 90$$

$$\text{Var (X)} = \sigma_x^2 = \frac{1}{n}\sum (x_i - \bar{x})^2 = \frac{2660}{7} = 380$$

$\therefore$ $\text{S. D.} = \sigma_x = \sqrt{\text{Var(X)}} = \sqrt{380} = 2\sqrt{95}$

Question 3.
Compute the variance and standard deviation for the following data:

x	2	4	6	8	10
f	5	4	3	2	1

Solution:
We prepare the following table for the calculation of variance and S.D.:

x_i	f_i	f_ix_i	$f_ix_i^2$
2	5	10	20
4	4	16	64
6	3	18	108
8	2	16	128
10	1	10	100
Total	N = 15	$\sum f_ix_i = 70$	$\sum f_ix_i^2 = 420$

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{70}{15} = 4.6667$$

$$\text{Var}(X) = \sigma_x^2 = \frac{\sum f_i x_i^2}{N} - (\bar{x})^2 = \frac{420}{15} - (4.6667)^2 = 28 - 21.7781 = 6.2219$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{6.2219}$$

Question 4.
Compute the variance and S.D.

x	1	3	5	7	9
Frequency	5	10	20	10	5

Solution:

We prepare the following table for the calculation of variance and S.D.:

x_i	f_i	$f_i x_i$	$f_i x_i^2$
1	5	5	5
3	10	30	90
5	20	100	500
7	10	70	490
9	5	45	405
Total	N = 50	$\Sigma f_i x_i = 250$	$\Sigma f_i x_i^2 = 1490$

$$\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{250}{50} = 5$$

$$\text{Var}(X) = \sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2 = \frac{1490}{50} - (5)^2 = 29.8 - 25 = 4.8$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{4.8}$$

Question 5.

The following data gives the age of 100 students in a school. Calculate variance and S.D.

Age (In years)	10	11	12	13	14
No. of Students	10	20	40	20	10

Solution:

We prepare the following table for the calculation of variance and S.D:

Age (In years) x_i	No. of students f_i	$f_i x_i$	$f_i x_i^2$
10	10	100	1000
11	20	220	2420
12	40	480	5760
13	20	260	3380
14	10	140	1960
Total	N = 100	$\Sigma f_i x_i = 1200$	$\Sigma f_i x_i^2 = 14520$

$$\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{1200}{100} = 12$$

$$\text{Var}(X) = \sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2 = \frac{14520}{100} - (12)^2 = 145.2 - 144 = 1.2$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{1.2}$$

Question 6.
The mean and variance of 5 observations are 3 and 2 respectively. If three of the five observations are 1, 3, and 5, find the values of the other two observations.
Solution:
$\bar{x} = 3$, Var (X) = 2, n = 5, $x_1 = 1$, $x_2 = 3$, $x_3 = 5$ …[Given]
Let the remaining two observations be x_4 and x_5.

$$\bar{x} = \frac{\sum x_i}{n}$$

$$\therefore \quad \sum x_i = n\bar{x} = 5 \times 3 = 15$$

$$\text{Var (X)} = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$\therefore \quad 2 = \frac{\sum x_i^2}{5} - (3)^2$$

$$\therefore \quad \frac{\sum x_i^2}{5} = 2 + 9$$

$$\therefore \quad \sum x_i^2 = 5 \times 11$$

$$\therefore \quad \sum x_i^2 = 55$$

Now,

$$\sum x_i = 1 + 3 + 5 + x_4 + x_5$$

$$\therefore \quad 15 = 9 + x_4 + x_5$$

$$\therefore \quad x_4 + x_5 = 15 - 9$$

$$\therefore \quad x_4 + x_5 = 6$$

$$\therefore \quad x_5 = 6 - x_4 \quad \text{…(i)}$$

$$\sum x_i^2 = 1^2 + 3^2 + 5^2 + x_4^2 + x_5^2$$

$$\therefore \quad 55 = 1 + 9 + 25 + x_4^2 + (6 - x_4)^2 \quad \text{…[From (i)]}$$

$$\therefore \quad 55 = 35 + x_4^2 + 36 - 12x_4 + x_4^2$$

$$\therefore \quad 2x_4^2 - 12x_4 + 16 = 0$$

$$\therefore \quad x_4^2 - 6x_4 + 8 = 0$$

$$\therefore \quad x_4^2 - 4x_4 - 2x_4 + 8 = 0$$

$$\therefore \quad x_4(x_4 - 4) - 2(x_4 - 4) = 0$$

$\therefore$ $(x_4 – 4)(x_4 – 2) = 0$
$\therefore$ $x_4 = 4$ or $x_4 = 2$
From (i), we get
$x_5 = 2$ or $x_5 = 4$
$\therefore$ The two numbers are 2 and 4.

Question 7.
Obtain standard deviation for the following data:

Height (in inches)	60-62	62-64	64-66	66-68	68-70
Number of students	4	30	45	15	6

Solution:
We prepare the following table for the calculation of standard deviation.

Height (in inches)	Mid value x_i	Number of students f_i	$f_i x_i$	$f_i x_i^2$
60-62	61	4	244	14884
62-64	63	30	1890	119070
64-66	65	45	2925	190125
66-68	67	15	1005	67335
68-70	69	6	414	28566
Total		N = 100	$\sum f_i x_i = 6478$	$\sum f_i x_i^2 = 419980$

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{6478}{100} = 64.78$$

$$\text{Var}(X) = \sigma_x^2 = \frac{\sum f_i x_i^2}{N} - (\bar{x})^2 = \frac{419980}{100} - (64.78)^2 = 4199.80 - 4196.4484 = 3.3516$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{3.3516}$$

Question 8.
The following distribution was obtained by change of origin and scale of variable X.

d_i	−4	−3	−2	−1	0	1	2	3	4
f_i	4	8	14	18	20	14	10	6	6

If it is given tnat mean ana variance are 59.5 and 413 respectively, determine actual class intervals.
Solution:
Here, Mean = $\bar{x}$ = 59.5, and
Var(X) = σ^2 = 413
Let x_i be a mid value of class and
d = x−ah, where a is assumed mean and h is class width.
We prepare the following table for calculation of mean and variance of d_i.

d_i	f_i	f_id_i	$f_id_i^2$
–4	4	–16	64
–3	8	–24	72
–2	14	–28	56
–1	18	–18	18
0	20	0	0
1	14	14	14
2	10	20	40
3	6	18	54
4	6	24	96
Total	N = 100	$\sum f_id_i = -10$	$\sum f_id_i^2 = 414$

$$\bar{d} = \frac{1}{N}\sum f_id_i = \frac{1}{100} \times (-10) = -0.1$$

Here, $\bar{d} = \frac{\bar{x} - a}{h}$

$\therefore \quad -0.1 = \frac{59.5 - a}{h}$

$\therefore \quad -0.1h = 59.5 - a$

$\therefore \quad -0.1h + a = 59.5 \quad \ldots(i)$

$$\text{Var}(D) = \sigma_d^2 = \frac{1}{N}\sum f_id_i^2 - (\bar{d})^2 = \frac{1}{100} \times 414 - (-0.1)^2 = 4.14 - 0.01 = 4.13$$

Now, Var(X) = h^2. Var(D)

$\therefore$ 413 = $h^2 \times 4.13$

$\therefore$ h^2 = 100

$\therefore$ h = 10

Substituting h = 10 in (i), we get

-0.1 × 10 + a = 59.5

$\therefore$ -1 + a = 59.5

$\therefore$ a = 59.5 + 1

$\therefore$ a = 60.5

We prepare the following table to determine actual class intervals for corresponding values of d_i.

d_i	Mid value $x_i = d_i \times h + a$	Class interval
–4	20.5	15.5–25.5
–3	30.5	25.5–35.5
–2	40.5	35.5–45.5
–1	50.5	45.5–55.5
0	60.5	55.5–65.5
1	70.5	65.5–75.5
2	80.5	75.5–85.5
3	90.5	85.5–95.5
4	100.5	95.5–105.5

∴ The actual class intervals are 15.5 – 25.5, 25.5 – 35.5, …….., 95.5 – 105.5

Maharashtra State Board 11th Commerce Maths Solutions Chapter 2 Measures of Dispersion Ex 2.3

Question 1.
The mean and standard deviation of two distributions of 100 and 150 items are 50, 5, and 40, 6 respectively. Find the mean and standard deviation of all the 250 items taken together.
Solution:

Here, $n_1 = 100$, $n_2 = 150$, $\bar{x}_1 = 50$, $\bar{x}_2 = 40$, $\sigma_1 = 5$, $\sigma_2 = 6$

Combined mean is given by,

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{(100 \times 50) + (150 \times 40)}{100 + 150} = \frac{5000 + 6000}{250} = \frac{11000}{250} = 44$$

Combined standard deviation is given by,

$$\sigma_c = \sqrt{\frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2}}$$

where $d_1 = \bar{x}_1 - \bar{x}_c$, $d_2 = \bar{x}_2 - \bar{x}_c$

$d_1 = 50 - 44 = 6$ and $d_2 = 40 - 44 = -4$

$$\sigma_c = \sqrt{\frac{100\left(5^2 + 6^2\right) + 150\left[6^2 + (-4)^2\right]}{100 + 150}}$$

$$= \sqrt{\frac{100(25 + 36) + 150(36 + 16)}{250}}$$

$$= \sqrt{\frac{(100 \times 61) + (150 \times 52)}{250}}$$

$$= \sqrt{\frac{6100 + 7800}{250}}$$

$$= \sqrt{\frac{13900}{250}}$$

$$= \sqrt{55.6}$$

∴ The mean and standard deviation of all 250 items taken together are 44 and √55.6 respectively.

Question 2.

For certain bivariate data, the following information is available.

	X	Y
Mean	13	17
S. D.	3	2
Size	10	10

Obtain the combined standard deviation.

Solution:

Here, $\bar{x} = 13$, $\bar{y} = 17$, $\sigma_x = 3$, $\sigma_y = 2$, $n_x = 10$, $n_y = 10$

Combined mean is given by $\bar{x}_c = \dfrac{n_x\bar{x} + n_y\bar{y}}{n_x + n_y} = \dfrac{10(13)+10(17)}{10+10} = \dfrac{130+170}{20} = \dfrac{300}{20} = 15$

Combined standard deviation is given by,

$$\sigma_c = \sqrt{\frac{n_x\left(\sigma_x^2 + d_x^2\right) + n_y\left(\sigma_y^2 + d_y^2\right)}{n_x + n_y}}$$

where $d_1 = \bar{x} - \bar{x}_c$, $d_2 = \bar{y} - \bar{x}_c$

$d_1 = 13 - 15 = -2$ and $d_2 = 17 - 15 = 2$

$$\sigma_c = \sqrt{\frac{10\left[3^2 + (-2)^2\right] + 10\left(2^2 + 2^2\right)}{10+10}}$$

$$= \sqrt{\frac{10(9+4) + 10(4+4)}{20}}$$

$$= \sqrt{\frac{10(13) + 10(8)}{20}}$$

$$= \sqrt{\frac{130+80}{20}}$$

$$= \sqrt{\frac{210}{20}} = \sqrt{10.5}$$

Question 3.
Calculate the coefficient of variation of marks secured by a student in the exam, where the marks are: 2, 4, 6, 8, 10. (Given: √8 = 2.8284)
Solution:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
2	–4	16
4	–2	4
6	0	0
8	2	4
10	4	16
$\sum x_i = 30$		$\Sigma(x_i - \bar{x})^2 = 40$

Here, n = 5,

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{30}{5} = 6$$

$$\text{Var}(X) = \sigma_x^2 = \frac{1}{n}\Sigma(x_i - \bar{x})^2 = \frac{40}{5} = 8$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{8} = 2.8284$$

$$\text{Now, C.V.} = 100 \times \frac{\sigma}{\bar{x}} = 100 \times \frac{2.8284}{6} = 47.14\%$$

Question 4.
Find the coefficient of variation of a sample that has a mean equal to 25 and a standard deviation of 5.
Solution:
Given, $\bar{x}$ = 25, σ = 5
C.V. = 100 × $\frac{\sigma}{\bar{x}}$
= 100 × $\frac{5}{25}$
= 20%

Question 5.
A group of 65 students of class XI has their average height as 150.4 cm with a coefficient of variation of 2.5%. What is the standard deviation of their height?
Solution:
Given, n = 65, $\bar{x}$ = 150.4, C.V. = 2.5%
C.V. = 100 × $\frac{\sigma}{\bar{x}}$
∴ 2.5 = 100 × $\frac{\sigma}{150.4}$
∴ $\frac{2.5 \times 150.4}{100}$ = σ
∴ σ = 3.76
∴ the standard deviation of students’ height is 3.76.

Question 6.
Two workers on the same job show1 the following results:

	Worker P	Worker Q
Mean time for completing the job (hours)	33	21
Standard Deviation (hours)	9	7

(i) Regarding the time required to complete the job, which worker is more consistent?
(ii) Which worker seems to be faster in completing the job?
Solution:

Here, $\bar{p} = 33$, $\bar{q} = 21$, $\sigma_p = 9$, $\sigma_q = 7$

$$\text{C.V. (P)} = 100 \times \frac{\sigma_p}{\bar{p}} = 100 \times \frac{9}{33} = 27.27\%$$

$$\text{C.V. (Q)} = 100 \times \frac{\sigma_q}{\bar{q}} = 100 \times \frac{7}{21} = 33.33\%$$

(i) Since, C.V. (P) < C.V.(Q)
∴ Worker P is more consistent regarding the time required to complete the job.

(ii) Since, $\bar{p} > \bar{q}$
i.e., the expected time for completing the job is less for worker Q.
∴ Worker Q seems to be faster in completing the job.

Question 7.
A company has two departments with 42 and 60 employees respectively. Their average weekly wages are ₹ 750 and ₹ 400. The standard deviations are 8 and 10 respectively.
(i) Which department has a larger bill?
(ii) Which department has larger variability in wages?
Solution:

Let $n_1 = 42$, $n_2 = 60$, $\bar{x}_1 = 750$, $\bar{x}_2 = 400$, $\sigma_1 = 8$, $\sigma_2 = 10$

$$\text{C.V. (1)} = 100 \times \frac{\sigma_1}{\bar{x}_1} = 100 \times \frac{8}{750} = 1.07\%$$

$$\text{C.V. (2)} = 100 \times \frac{\sigma_2}{\bar{x}_2} = 100 \times \frac{10}{400} = 2.5\%$$

(i) Since, $\bar{x}_1 > \bar{x}_2$
i.e., average weekly wages are more for the first department.
∴ the first department has a larger bill.
(ii) Since, C.V. (1) < C.V. (2)
∴ the second department is less consistent.
∴ the second department has larger variability in wages.

Question 8.
The following table gives the weights of the students of class A. Calculate the coefficient of variation (Given √8 = 0.8944)

Weight (in kg)	Class A
25-35	8
35-45	4
45-55	8

Solution:

Weight (in kg)	Mid value (x_i)	f_i	$f_i x_i$	$f_i x_i^2$
25-35	30	8	240	7200
35-45	40	4	160	6400
45-55	50	8	400	20000
Total		N = 20	$\sum f_i x_i = 800$	$\sum f_i x_i^2 = 33600$

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{800}{20} = 40$$

$$\text{Var}(X) = \sigma_x^2 = \frac{\sum f_i x_i^2}{N} - (\bar{x})^2$$

$$= \frac{33600}{20} - (40)^2$$

$$= 1680 - 1600$$

$$= 80$$

$$\text{S. D.} = \sigma_x = \sqrt{80} = \sqrt{\frac{100 \times 8}{10}} = 10\sqrt{0.8} = 10(0.8944) = 8.944$$

$$\text{C.V.}(X) = 100 \times \frac{\sigma_x}{\bar{x}} = 100 \times \frac{8.944}{40} = 22.36\%$$

Question 9.
Compute coefficient of variation for team A and team B. (Given: √2.5162 = 1.5863, √2.244 = 1.4980)

No. of goals	0	1	2	3	4
No. of matches played by team A	19	6	5	16	14
No. of matches played by team B	16	16	5	18	15

Which team is more consistent?
Solution:
Let f_1 denote no. of goals of team A and f_2 denote no. of goals of team B.

No. of goals (x_i)	f_{1i}	f_{2i}	$f_{1i}x_i$	$f_{2i}x_i$	$f_{1i}x_i^2$	$f_{2i}x_i^2$
0	18	14	0	0	0	0
1	7	16	7	16	7	16
2	5	5	10	10	20	20
3	16	18	48	54	144	162
4	14	17	56	68	224	272
	$N_1 = 60$	$N_2 = 70$	$\Sigma f_{1i}x_i = 121$	$\Sigma f_{2i}x_i = 148$	$\Sigma f_{1i}x_i^2 = 395$	$\Sigma f_{2i}x_i^2 = 470$

$$\bar{x}_1 = \frac{\Sigma f_{1i}x_i}{N_1} = \frac{121}{60} = 2.0167$$

$$\sigma_{x_1}^2 = \frac{1}{N_1}\Sigma f_{1i}x_i^2 - (\bar{x}_1)^2$$

$$= \frac{395}{60} - (2.0167)^2$$

$$= 6.5833 - 4.0671 = 2.5162$$

$$\sigma_{x_1} = \sqrt{2.5162} = 1.5863$$

$$\text{C.V. of team A} = \text{C.V. }(x_1) = 100 \times \frac{\sigma_{x_1}}{\bar{x}_1} = 100 \times \frac{1.5863}{2.0167} = 78.66\ \%$$

$$\bar{x}_2 = \frac{\Sigma f_{2i}x_i}{N_2} = \frac{148}{70} = 2.1143$$

$$\sigma_{x_2}^2 = \frac{1}{N_2}\Sigma f_{2i}x_i^2 - (\bar{x}_2)^2$$

$$= \frac{470}{70} - (2.1143)^2 = 6.7143 - 4.4703 = 2.244$$

$$\sigma_{x_2} = \sqrt{2.244} = 1.4980$$

$$\text{C.V. of team B} = \text{C.V. }(x_2) = 100 \times \frac{\sigma_{x_2}}{\bar{x}_2} = 100 \times \frac{1.4980}{2.1143} = 70.85\ \%$$

Since C.V. of team A > C.V. of team B.
∴ team B is more consistent.

Question 10.
Given below is the information about marks obtained in Mathematics and Statistics by 100 students in a class. Which subject shows the highest variability in marks?

	Mathematics	Statistics
Mean	20	25
S.D.	2	3

Solution:

Here, $\bar{x}_m = 20$, $\bar{x}_s = 25$, $\sigma_m = 2$, $\sigma_s = 3$

$$\text{C.V. (M)} = 100 \times \frac{\sigma_m}{\bar{x}_m}$$

$$= 100 \times \frac{2}{20} = 10\%$$

$$\text{C.V. (S)} = 100 \times \frac{\sigma_s}{\bar{x}_s} = 100 \times \frac{3}{25} = 12\%$$

Since C.V. (S) > C.V. (M)

∴ The subject statistics show higher variability in marks.

Maharashtra State Board 11th Commerce Maths Solutions Chapter 2 Measures of Dispersion Miscellaneous Exercise 2

Question 1.
Find the range for the following data.
116, 124, 164, 150, 149, 114, 195, 128, 138, 203, 144
Solution:
Here, largest value (L) = 203, smallest value (S) = 114
∴ Range = L – S
= 203 – 114
= 89

Question 2.
Given below the frequency distribution of weekly w ages of 400 workers.
Find the range.

Weekly wages (in ’00 Rs.)	10	15	20	25	30	35	40
No. of workers	45	63	102	55	74	36	25

Solution:
Here, largest value (L) = 40, smallest value (S) = 10
∴ Range = L – S
= 40 – 10
= 30

Question 3.
Find the range of the following data.

Classes	115-125	125-135	135-145	145-155	155-165	165-175
Fre-quency	1	4	6	1	3	5

Solution:
Here, upper limit of the highest class (L) = 175, lower limit of the lowest class (S) = 115
∴ Range = L – S
= 175 – 115
= 60

Question 4.
The city traffic police issued challans for not observing the traffic rules:

Day of the Week	Mon	Tue	Wed	Thus	Fri	Sat
No. of Challans	40	24	36	58	62	80

Find Q.D.
Solution:
The given data can be arranged in ascending order as follows:
24, 36, 40, 58, 62, 80
Here, n = 6
Q_1 = value of $(n+14)\mathrm{th}$ observation
= value of $(6+14)\mathrm{th}$ observation
= value of (1.75)th observation
= value of 1st observation + 0.75(value of 2nd observation – value of 1st observation)
= 24 + 0.75(36 – 24)
= 24 + 0.75(12)
= 24 + 9
∴ Q_1 = 33
Q_3 = value of $3(n+14)\mathrm{th}$ observation
= value of $3(6+14)\mathrm{th}$ observation

= value of (3 × 1.75)th observation
= value of (5.25)th observation
= value of 5th observation + 0.25(value of 6th observation – value of 5th observation)
= 62 + 0.25(80 – 62)
= 62 + 0.25(18)
= 62 + 4.5
= 66.5
∴ Q.D. = $\frac{Q_3-Q_1}{2}=\frac{66.5-33}{2}=\frac{33.5}{2}$ = 16.75

Question 5.
Calculate Q.D. from the following data.

X (less than)	10	20	30	40	50	60	70
Frequency	5	8	15	20	30	33	35

Solution:
We construct the less than cumulative frequency table as follows:

Class	f	Less than cumulative frequency (c.f.)
0-10	5	5
10-20	3	8
20-30	7	15 ← Q_1
30-40	5	20
40-50	10	30 ← Q_3
50-60	3	33
60-70	2	35
Total	N = 35	

Here, N = 35
Q_1 class = class containing $\left(\frac{N}{4}\right)^{th}$ observation
∴ $\frac{N}{4}=\frac{35}{4}$ = 8.75
Cumulative frequency which is just greater than (or equal to) 8.75 is 15.
∴ Q_1 lies in the class 20-30.

$\therefore$ L = 20, c.f. = 8, f = 7, h = 10

$$\therefore \quad Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - \text{c.f.}\right)$$

$$= 20 + \frac{10}{7}(8.75 - 8)$$

$$= 20 + \frac{10}{7} \times 0.75 = 20 + 1.07$$

$$\therefore \quad Q_1 = 21.07$$

Q_3 class = class containing $\left(\frac{3N}{4}\right)^{th}$ observation

$$\therefore \quad \frac{3N}{4} = \frac{3 \times 35}{4} = 26.25$$

Cumulative frequency which is just greater than (or equal to) 26.25 is 30.

$\therefore$ Q_3 lies in the class 40-50.

$\therefore$ L = 40, c.f. = 20, f = 10, h = 10

$$\therefore \quad Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - \text{c.f.}\right) = 40 + \frac{10}{10}(26.25 - 20) = 40 + 6.25$$

$$\therefore \quad Q_3 = 46.25$$

$$\text{Q.D.} = \frac{Q_3 - Q_1}{2} = \frac{46.25 - 21.07}{2} = \frac{25.18}{2} = 12.59$$

Question 6.
Calculate the appropriate measure of dispersion for the following data.

Wages (In Rs.)	Less than 35	35-40	40-45	45-50	50-55	55-60
No. of Workers	15	50	85	40	27	33

Solution:
Since open-ended classes are given, the appropriate measure of dispersion that we can compute is the quartile deviation.
We construct the less than cumulative frequency table as follows:

Wages(in ₹)	No. of workers (f)	Less than cumulative frequency (c.f.)
Less than 35	15	15
35-40	50	65 ← Q_1
40-45	85	150
45-50	40	190 ← Q_3
50-55	27	217
55-60	33	250
Total	N = 250	

Here N = 250

Q_1 class class containing (N4)th observation

∴ N4=2504 = 62.5

Cumulative frequency which is just greater than (or equal to) 62.5 is 65.

∴ Q_1 lies in the class 35-40.

∴ L = 35, c.f. = 15, f = 50, h = 5

$$\therefore \quad Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - \text{c.f.}\right)$$

$$= 35 + \frac{5}{50}(62.5 - 15)$$

$$= 35 + \frac{1}{10} \times 47.5$$

$$= 35 + 4.75$$

$$\therefore \quad Q_1 = 39.75$$

Q_3 class = class containing $\left(\frac{3N}{4}\right)$ observation

$$\therefore \quad \frac{3N}{4} = \frac{3 \times 250}{4} = 187.5$$

The cumulative frequency which is just greater than (or equal to) 187.5 is 190.

∴ Q_3 lies in the class 45-50.

∴ L = 45, c.f. = 150, f = 40, h = 5

$$\therefore \quad Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$$

$$= 45 + \frac{5}{40}(187.5 - 150)$$

$$= 45 + \frac{1}{8} \times 37.5$$

$$= 45 + 4.6$$

$$\therefore \quad Q_3 = 49.6875$$

$$\therefore \quad Q.D. = \frac{Q_3 - Q_1}{2} = \frac{49.6875 - 39.75}{2} = \frac{9.9375}{2}$$

$$\therefore \quad Q.D. = 4.9688$$

Question 7.
Calculate Q.D. of the following data.

height of plants (in feet)	2-4	4-6	6-8	8-10	10-12	12-14	14-16
No. of plants	15	20	25	12	18	13	17

Solution:
We construct the less than cumulative frequency table as follows:

Height of plants (in feet)	No. of plants (f)	Less than cumulative frequency (c.f.)
2-4	15	15
4-6	20	35 ← Q_1
6-8	25	60
8-10	12	72
10-12	18	90← Q_3
12-14	13	103
14-16	17	120
Total	N = 120	

Here, N = 120
Q_1 class = class containing (N4)th observation
∴ N4=1204 = 30
Cumulative frequency which is just greater than (or equal to) 30 is 35.
∴ Q_1 lies in the class 4-6.
∴ L = 4, c.f. = 15, f = 20, h = 2

$$\therefore \quad Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f.\right)$$

$$= 4 + \frac{2}{20}\ (30 - 15)$$

$$= 4 + \frac{1}{10} \times 15$$

$$= 4 + 1.5$$

$$= 5.5$$

Q_3 class = class containing $\left(\frac{3N}{4}\right)^{th}$ observation

$$\therefore \quad \frac{3N}{4} = \frac{3 \times 120}{4} = 90$$

Cumulative frequency which is just greater than (or equal to) 90 is 90.
∴ Q_3 lies in the class 10-12.
∴ L = 10, c.f. = 72, f = 18, h = 2

$$\therefore \quad Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$$

$$= 10 + \frac{2}{18}\ (90 - 72)$$

$$= 10 + \frac{2}{18} \times 18$$

$$= 10 + 2$$

$$\therefore \quad Q_3 = 12$$

$$\therefore \quad Q.D. = \frac{Q_3 - Q_1}{2}$$

$$= \frac{12 - 5.5}{2} = \frac{6.5}{2} = 3.25$$

Question 8.
Find variance and S.D. for the following set of numbers.
25, 21, 23, 29, 27, 22, 28, 23, 27, 25 (Given $\sqrt{6.6} = 2.57$)
Solution:

We prepare the following table for the calculation of variance and S.D.:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
25	0	0
21	–4	16
23	–2	4
29	4	16
27	2	4
22	–3	9
28	3	9
23	–2	4
27	2	4
25	0	0
$\sum x_i = 250$		$\Sigma(x_i - \bar{x})^2 = 66$

Here, n = 10

$$\bar{x} = \frac{\sum x_i}{n} = \frac{250}{10} = 25$$

$$\text{Var (X)} = \sigma_x^2 = \frac{1}{n}\Sigma(x_i - \bar{x})^2 = \frac{1}{10} \times 66 = 6.6$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var (X)}} = \sqrt{6.6} = 2.57$$

Question 9.
Following data gives no. of goals scored by a team in 90 matches.

No. of Goals Scored	0	1	2	3	4	5
No. of matches	15	20	25	15	20	5

Compute the variance and standard deviation for the above data.
Solution:

We prepare the following table for the calculation of variance and S.D:

No. of goals scored (x_i)	No. of matches (f_i)	$f_i x_i$	$f_i x_i^2$
0	5	0	0
1	20	20	20
2	25	50	100
3	15	45	135
4	20	80	320
5	5	25	125
Total	N = 90	$\Sigma f_i x_i = 220$	$\Sigma f_i x_i^2 = 700$

$$\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{220}{90} = 2.44$$

$$\text{Var}(X) = \sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2 = \frac{700}{90} - (2.44)^2 = 7.78 - 5.9536 = 1.83$$

$$\text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{1.83}$$

Question 10.
Compute the arithmetic mean and S.D. and C.V. (Given √296 = 17.20)

C.I.	45-55	55-65	65-75	75-85	85-95	95-105
f	4	2	5	3	6	5

Solution:

We prepare the following table for calculation of arithmetic mean and S.D.:

C.I.	Mid value (x_i)	f_i	$f_i x_i$	$f_i x_i^2$
45-55	50	4	200	10000
55-65	60	2	120	7200
65-75	70	5	350	24500
75-85	80	3	240	19200
85-95	90	6	540	48600
95-105	100	5	500	50000
Total		N = 25	$\Sigma f_i x_i = 1950$	$\Sigma f_i x_i^2 = 159500$

Arithmetic mean $= \bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{1950}{25} = 78$

Var (X) $= \sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2 = \frac{159500}{25} - (78)^2 = 6380 - 6084 = 296$

S.D. $= \sigma_x = \sqrt{\text{Var (X)}} = \sqrt{296} = 17.20$

C.V. $= 100 \times \frac{\sigma_x}{\bar{x}} = 100 \times \frac{17.20}{78} = 22.05\%$

Question 11.
The mean and S.D. of 200 items are found to be 60 and 20 respectively. At the time of calculation, two items were wrongly taken as 3 and 67 instead of 13 and 17. Find the correct mean and variance.
Solution:
Here, n = 200, $\bar{x}$ = Mean = 60, S.D. = 20
Wrongly taken items are 3 and 67.
Correct items are 13 and 17.
Now, $\bar{x}$ = 60

$$\therefore \quad \frac{1}{n}\sum_{i=1}^{n} x_i = 60$$

$$\therefore \quad \frac{1}{200}\sum_{i=1}^{n} x_i = 60$$

$$\therefore \quad \sum_{i=1}^{n} x_i = 200 \times 60$$

$$\therefore \quad \sum_{i=1}^{n} x_i = 12000$$

Correct value of $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i$ (sum of wrongly taken items) + (sum of correct items)

= 12000 – (3 + 67) + (13 + 17)
= 12000 – 70 + 30
= 11960
Correct value of mean = $\frac{1}{n}$ × correct value of $\sum_{i=1}^{n} x_i$
= $\frac{1}{200}$ × 11960
= 59.8
Now, S.D. = 20
Variance = $(S.D.)^2 = 20^2$
∴ Variance = 400
∴ $\frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2 = 400$

$$\therefore \quad \frac{1}{200}\sum_{i=1}^{n} x_i^2 - (60)^2 = 400$$

$$\therefore \quad \frac{1}{200}\sum_{i}^{n} x_i^2 = 400 + 3600$$

$$\therefore \quad \sum_{i=1}^{n} x_i^2 = 4000 \times 200$$

$$\therefore \quad \sum_{i=1}^{n} x_i^2 = 800000$$

Correct value of $\sum_{i=1}^{n} x_i^2$
= $\sum_{i=1}^{n} x_i^2$ – (Sum of squares of wrongly taken items) + (Sum of squares of correct items)
= 800000 – $(3^2 + 67^2)$ + $(13^2 + 17^2)$
= 800000 – (9 + 4489) + (169 + 289)
= 800000 – 4498 + 458
= 795960
∴ Correct value of Variance = $\left(\frac{1}{n} \times \sum_{i=1}^{n} x_i^2\right)$ – (correct value of $\bar{x}$)^2
= $\frac{1}{200}$ × 795960 – $(59.8)^2$
= 3979.8 – 3576.04
= 403.76
∴ The correct mean is 59.8 and correct variance is 403.76.

Question 12.
The mean and S.D. of a group of 48 observations are 40 and 8 respectively. If two more observations 60 and 65 are added to the set, find the mean and S.D. of 50 items.

Solution:

$n = 48,\ \bar{x} = 40,\ \sigma_x = 8$ ...[Given]

$\bar{x} = \frac{1}{n}\sum x_i$

$\therefore \sum x_i = n\bar{x} = 48 \times 40 = 1920$

New $\sum x_i = \sum x_i + 60 + 65 = 1920 + 60 + 65 = 2045$

$\therefore$ New Mean $= \frac{2045}{50} = 40.9$

Now, $\sigma_x = 8$

$\therefore \sigma_x^2 = 64$

Since, $\sigma_x^2 = \frac{1}{n}(\sum x_i^2) - (\bar{x})^2$

$\therefore 64 = \frac{1}{48}(\sum x_i^2) - (40)^2$

$\therefore 64 = \frac{1}{48}(\sum x_i^2) - 1600$

$\therefore \frac{\sum x_i^2}{48} = 64 + 1600 = 1664$

$\therefore \sum x_i^2 = 48 \times 1664 = 79872$

New $\sum x_i^2 = \sum x_i^2 + (60)^2 + (65)^2 = 79872 + 3600 + 4225 = 87697$

$\therefore$ New S.D. $= \sqrt{\frac{\text{New}\sum x_i^2}{n} - (\text{New mean})^2}$

$= \sqrt{\frac{87697}{50} - (40.9)^2} = \sqrt{1753.94 - 1672.81} = \sqrt{81.13}$

Question 13.

The mean height of 200 students is 65 inches. The mean heights of boys and girls are 70 inches and 62 inches respectively and the standard deviations are 8 and 10 respectively. Find the number of boys and combined S.D.

Solution:

Let n_1 and n_2 be the number of boys and girls respectively.

Let $n = 200$, $\bar{x}_c = 65$, $\bar{x}_1 = 70$, $\bar{x}_1 = 62$, $\sigma_1 = 8$, $\sigma_2 = 10$

Here, $n_1 + n_2 = n$

$\therefore n_1 + n_2 = 200$ ....(i)

Combined mean is given by

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$65 = \frac{n_1(70) + n_2(62)}{200}$$

$\therefore$ $70n_1 + 62n_2 = 13000$

$\therefore$ $35n_1 + 31n_2 = 6500$ ……..(ii)

Solving (i) and (ii), we get

$n_1 = 75$, $n_2 = 125$

Combined standard deviation is given by,

$$\sigma_c = \sqrt{\frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2}}$$

where $d_1 = \bar{x}_1 - \bar{x}_c$, $d_2 = \bar{x}_2 - \bar{x}_c$

$\therefore$ $d_1 = 70 - 65 = 5$ and $d_2 = 62 - 65 = -3$

$$\therefore \quad \sigma_c = \sqrt{\frac{75(64+25) + 125(100+9)}{2}}$$

$$= \sqrt{\frac{6675 + 13625}{200}} = \sqrt{\frac{20300}{200}} = \sqrt{101.5} = 10.07$$

Question 14.

From the following data available for 5 pairs of observations of two variables x and y, obtain the combined S.D. for all 10 observations, where $\sum_{i=1}^{n} x_i = 30$, $\sum_{i=1}^{n} y_i = 40$, $\sum_{i=1}^{n} x_i^2 = 225$, $\sum_{i=1}^{n} y_i^2 = 340$

Solution:

Here, $\sum_{i=1}^{n} x_i = 30$, $\sum_{i=1}^{n} y_i = 40$, $\sum_{i=1}^{n} x_i^2 = 225$, $\sum_{i=1}^{n} y_i^2 = 340$, $n_x = 5$, $n_y = 5$

$$\bar{x} = \frac{\sum x_i}{n_x} = \frac{30}{5} = 6,$$

$$\bar{y} = \frac{\sum y_i}{n_y} = \frac{40}{5} = 8$$

Combined mean is given by $\bar{x}_c = \frac{n_x \bar{x} + n_y \bar{y}}{n_x + n_y} = \frac{5(6)+5(8)}{5+5} = \frac{30+40}{10} = \frac{70}{10} = 7$

Combined standard deviation is given by,

$$\sigma_c = \sqrt{\frac{n_x(\sigma_x^2 + d_x^2) + n_y(\sigma_y^2 + d_y^2)}{n_x + n_y}}$$

where $d_x = \bar{x} - \bar{x}_c$, $d_y = \bar{y} - \bar{x}_c$

$$\sigma_x^2 = \frac{1}{n_x}\sum x_i^2 - (\bar{x})^2 = \frac{1}{5}(225) - (6)^2 = 45 - 36 = 9$$

$$\sigma_y^2 = \frac{1}{n_y}\sum y_i^2 - (\bar{y})^2 = \frac{1}{5}(340) - (8)^2 = 68 - 64 = 4$$

$d_x = 6 - 7 = -1$ and $d_y = 8 - 7 = 1$

$$\therefore \quad \sigma_c = \sqrt{\frac{5\left[9+(-1)^2\right]+5\left[4+(1)^2\right]}{5+5}}$$

$$= \sqrt{\frac{5(9+1)+5(4+1)}{10}} = \sqrt{\frac{5(10)+5(5)}{10}} = \sqrt{\frac{50+25}{10}} = \sqrt{\frac{75}{10}} = \sqrt{7.5}$$

Question 15.
The mean and standard deviations of two brands of watches are given below:

	Brand-I	Brand-II
Mean	36 months	48 months
S.D.	8 months	10 months

Calculate the coefficient of variation of the two brands and interpret the results.
Solution:

Here, $\bar{x}_I = 36$, $\bar{x}_{II} = 48$, $\sigma_I = 8$, $\sigma_{II} = 10$

$$\text{C.V. (I)} = 100 \times \frac{\sigma_I}{\bar{x}_I} = 100 \times \frac{8}{36} = 22.22\%$$

$$\text{C.V. (II)} = 100 \times \frac{\sigma_{II}}{\bar{x}_{II}} = 100 \times \frac{10}{48} = 20.83\%$$

Since C.V. (I) > C.V. (II)

∴ the brand I is more variable.

Question 16.

Calculate the coefficient of variation for the data given below. [Given √3.3 = 1.8166]

C.I.	5-15	15-25	25-35	35-45	45-55	55-65	65-75
f	6	7	15	25	8	18	21

Solution:

C.I.	Mid value (x_i)	Frequency (f_i)	$f_i x_i$	$f_i x_i^2$
5-15	10	6	60	600
15-25	20	7	140	2800
25-35	30	15	450	13500
35-45	40	25	1000	40000
45-55	50	8	400	20000
55-65	60	18	1080	64800
65-75	70	21	1470	102900
Total		N = 100	$\sum f_i x_i = 4600$	$\sum f_i x_i^2 = 244600$

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{4600}{100} = 46$$

$$\text{Var (X)} = \sigma_x^2 = \frac{1}{N}\sum f_i x_i^2 - (\bar{x})^2 = \frac{1}{100} \times 244600 - (46)^2 = 2446 - 2116 = 330$$

$$\text{S.D.} = \sigma_x = \sqrt{330} = \sqrt{3.3 \times 100} = 10\sqrt{3.3} = 10 \times 1.8166 = 18.166$$

$$\text{C.V.} = 100 \times \frac{\sigma_x}{\bar{x}} = 100 \times \frac{18.166}{46} = 39.49\%$$