Maharashtra State Board 11th Commerce Maths Solutions Chapter 6 Determinants Ex 6.1

Question 1.
Evaluate the following determinants:

i) $\begin{vmatrix}4 & 7\\ -7 & 0\end{vmatrix}$ ii) $\begin{vmatrix}3 & -5 & 2\\ 1 & 8 & 9\\ 3 & 7 & 0\end{vmatrix}$ iii) $\begin{vmatrix}1 & i & 3\\ i^3 & 2 & 5\\ 3 & 2 & i^4\end{vmatrix}$

iv) $\begin{vmatrix}5 & 5 & 5\\ 5 & 4 & 4\\ 5 & 4 & 8\end{vmatrix}$ v) $\begin{vmatrix}2i & 3\\ 4 & -i\end{vmatrix}$ vi) $\begin{vmatrix}3 & -4 & 5\\ 1 & 1 & -2\\ 2 & 3 & 1\end{vmatrix}$

vii) $\begin{vmatrix}a & h & g\\ h & b & f\\ g & f & c\end{vmatrix}$ viii) $\begin{vmatrix}0 & a & -b\\ -a & 0 & -c\\ b & c & 0\end{vmatrix}$

Solution:
(i) $\begin{vmatrix}4 & 7\\ -7 & 0\end{vmatrix}$
= 4(0) – (-7)(7)
= 0 + 49
= 49

ii. $\begin{vmatrix}3 & -5 & 2\\ 1 & 8 & 9\\ 3 & 7 & 0\end{vmatrix} = 3\begin{vmatrix}8 & 9\\ 7 & 0\end{vmatrix} - (-5)\begin{vmatrix}1 & 9\\ 3 & 0\end{vmatrix} + 2\begin{vmatrix}1 & 8\\ 3 & 7\end{vmatrix}$
= 3(0 – 63) – 5(0 – 27) + 2(7 – 24)
= 3(-63) + 5 (-27) + 2(-17)
= – 189 – 135 – 34
= -358

iii. $\begin{vmatrix}1 & i & 3\\ i^3 & 2 & 5\\ 3 & 2 & i^4\end{vmatrix} = \begin{vmatrix}1 & i & 3\\ -i & 2 & 5\\ 3 & 2 & 1\end{vmatrix}$ $\ldots[\because i^2 = -1, i^4 = 1]$

$= 1\begin{vmatrix}2 & 5\\ 2 & 1\end{vmatrix} - i\begin{vmatrix}-i & 5\\ 3 & 1\end{vmatrix} + 3\begin{vmatrix}-i & 2\\ 3 & 2\end{vmatrix}$
= 1(2 – 10) – i(-i – 15) + 3(-2i – 6)
= -8 + i^2 + 15i – 6i – 18
= i^2 – 26 + 9i
= -1 – 26 + 9i …[∵ i^2 = -1]
= -27 + 9i

iv. $\begin{vmatrix}5 & 5 & 5\\ 5 & 4 & 4\\ 5 & 4 & 8\end{vmatrix} = 5\begin{vmatrix}4 & 4\\ 4 & 8\end{vmatrix} - 5\begin{vmatrix}5 & 4\\ 5 & 8\end{vmatrix} + 5\begin{vmatrix}5 & 4\\ 5 & 4\end{vmatrix}$
= 5(32 – 16) – 5(40 – 20) + 5(20 – 20)
= 5(16) – 5(20) + 5(0)
= 80 – 100
= -20

(v) $\begin{vmatrix}2i & 3\\ 4 & -i\end{vmatrix}$
= 2i(-i) – 3(4)
= -2i^2 – 12
= -2(-1) – 12 …[∵ i^2 = -1]
= 2 – 12
= -10

vi. $\begin{vmatrix}3 & -4 & 5\\ 1 & 1 & -2\\ 2 & 3 & 1\end{vmatrix} = 3\begin{vmatrix}1 & -2\\ 3 & 1\end{vmatrix} - (-4)\begin{vmatrix}1 & -2\\ 2 & 1\end{vmatrix} + 5\begin{vmatrix}1 & 1\\ 2 & 3\end{vmatrix}$
= 3(1 + 6) + 4(1 + 4) + 5(3 – 2)

= 3(7)+ 4(5) + 5(1)
= 21 + 20 + 5
= 46

vii. $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = a\begin{vmatrix} b & f \\ f & c \end{vmatrix} - h\begin{vmatrix} h & f \\ g & c \end{vmatrix} + g\begin{vmatrix} h & b \\ g & f \end{vmatrix}$

= $a(bc - f^2) - h(hc - gf) + g(hf - gb)$
= $abc - af^2 - h^2c + fgh + fgh - g^2b$
= $abc + 2fgh - af^2 - bg^2 - ch^2$

$\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0\begin{vmatrix} 0 & -c \\ c & 0 \end{vmatrix} - a\begin{vmatrix} -a & -c \\ b & 0 \end{vmatrix} - b\begin{vmatrix} -a & 0 \\ b & c \end{vmatrix}$

= 0 – a(0 + bc) – b(-ac – 0)
= -a(bc) – b(-ac)
= -abc + abc
= 0

Question 2.
Find the value(s) of x, if

i) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

ii) $\begin{vmatrix} 2 & 1 & x+1 \\ -1 & 3 & -4 \\ 0 & -5 & 3 \end{vmatrix} = 0$ iii) $\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$

Solution:
(i) ∣ ∣ ∣ 2435 ∣ ∣ ∣ = ∣ ∣ ∣ x2x35 ∣ ∣ ∣
∴ 10 – 12 = 5x – 6x
∴ -2 = -x
∴ x = 2

Check:
We can check if our answer is right or wrong.
In order to do so, substitute x = 2 in the given determinant.
For x = 2,
L.H.S. = ∣ ∣ ∣ 2435 ∣ ∣ ∣
= 10 – 12
= -2
R.H.S. = ∣ ∣ ∣ x2x35 ∣ ∣ ∣
= ∣ ∣ ∣ 2435 ∣ ∣ ∣
= 10 – 12
= -2
Thus, our answer is correct.

ii. $\begin{vmatrix} 2 & 1 & x+1 \\ -1 & 3 & -4 \\ 0 & -5 & 3 \end{vmatrix} = 0$

∴ $2\begin{vmatrix} 3 & -4 \\ -5 & 3 \end{vmatrix} - 1\begin{vmatrix} -1 & -4 \\ 0 & 3 \end{vmatrix} + (x+1)\begin{vmatrix} -1 & 3 \\ 0 & -5 \end{vmatrix} = 0$

∴ 2(9 – 20) – 1 (-3 – 0) + (x + 1) (5 – 0) = 0
∴ 2(-11) – 1(-3) + (x + 1)(5) = 0
∴ -22 + 3 + 5x + 5 = 0
∴ 5x = 14
∴ x = 145

iii. $\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$

$\therefore \quad (x-1)\begin{vmatrix} x-2 & x-3 \\ 0 & x-3 \end{vmatrix} - x\begin{vmatrix} 0 & x-3 \\ 0 & x-3 \end{vmatrix} + (x-2)\begin{vmatrix} 0 & x-2 \\ 0 & 0 \end{vmatrix} = 0$

$\therefore$ (x – 1)[(x – 2)(x – 3) – 0] – x(0 – 0) + (x – 2)(0 – 0) = 0
$\therefore$ (x – 1)(x – 2)(x – 3) = 0
$\therefore$ x – 1 = 0 or x – 2 = 0 or x – 3 = 0
$\therefore$ x = 1 or x = 2 or x = 3

Question 3.
Solve the following equations.

i) $\begin{vmatrix} x & 2 & 2 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix} = 0$ ii) $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

Solution:

i. $\begin{vmatrix} x & 2 & 2 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix} = 0$

$\therefore \quad x\begin{vmatrix} x & 2 \\ 2 & x \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 2 & x \end{vmatrix} + 2\begin{vmatrix} 2 & x \\ 2 & 2 \end{vmatrix} = 0$

$\therefore$ x(x^2 – 4) – 2(2x – 4) + 2(4 – 2x) = 0
$\therefore$ x(x^2 – 4) – 2(2x – 4) – 2(2x – 4) = 0
$\therefore$ x(x + 2)(x – 2) – 4(2x – 4) = 0
$\therefore$ x(x + 2)(x – 2) – 8(x – 2) = 0
$\therefore$ (x – 2)[x(x + 2) – 8] = 0
$\therefore$ (x – 2)(x^2 + 2x – 8) = 0
$\therefore$ (x – 2)(x^2 + 4x – 2x – 8) = 0
$\therefore$ (x – 2)(x + 4)(x – 2) = 0
$\therefore$ $(x – 2)^2$ (x + 4) = 0
$\therefore$ $(x – 2)^2$ = 0 or x + 4 = 0
$\therefore$ x – 2 = 0 or x = -4
$\therefore$ x = 2 or x = -4

ii. $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

$\therefore \quad 1\begin{vmatrix} -2 & 5 \\ 2x & 5x^2 \end{vmatrix} - 4\begin{vmatrix} 1 & 5 \\ 1 & 5x^2 \end{vmatrix} + 20\begin{vmatrix} 1 & -2 \\ 1 & 2x \end{vmatrix} = 0$

$\therefore$ 1(-10x^2 – 10x) – 4(5x^2 – 5) + 20(2x + 2) = 0
$\therefore$ -10x^2 – 10x – 20x^2 + 20 + 40x + 40 = 0
$\therefore$ -30x^2 + 30x + 60 = 0
$\therefore$ x^2 – x – 2 = 0 .....[Dividing throughout by (-30)]
$\therefore$ x^2 – 2x + x – 2 = 0
$\therefore$ (x – 2)(x + 1) = 0
$\therefore$ x – 2 = 0 or x + 1 = 0 x = 2 or x = -1

Question 4.
Find the value of x, if
∣ ∣ ∣ ∣ x2x3−11−42−35 ∣ ∣ ∣ ∣ = 29
Solution:

$$\begin{vmatrix} x & -1 & 2 \\ 2x & 1 & -3 \\ 3 & -4 & 5 \end{vmatrix} = 29$$

$$\therefore \quad x\begin{vmatrix} 1 & -3 \\ -4 & 5 \end{vmatrix} - (-1)\begin{vmatrix} 2x & -3 \\ 3 & 5 \end{vmatrix} + 2\begin{vmatrix} 2x & 1 \\ 3 & -4 \end{vmatrix} = 29$$

∴ x(5 – 12) + 1(10x + 9) + 2(-8x – 3) = 29
∴ -7x + 10x + 9 – 16x – 6 = 29
∴ -13x + 3 = 29
∴ -13x = 26
∴ x = -2

Question 5.
Find x and y if ∣∣∣∣ 4i15i33i2−32i4i ∣∣∣∣ = x + iy, where i = √-1.
Solution:

$$\begin{vmatrix} 4i & i^3 & 2i \\ 1 & 3i^2 & 4 \\ 5 & -3 & i \end{vmatrix}$$

$$= \begin{vmatrix} 4i & -i & 2i \\ 1 & -3 & 4 \\ 5 & -3 & i \end{vmatrix} \qquad \ldots[\because i^2 = -1]$$

$$= 4i\begin{vmatrix} -3 & 4 \\ -3 & i \end{vmatrix} - (-i)\begin{vmatrix} 1 & 4 \\ 5 & i \end{vmatrix} + 2i\begin{vmatrix} 1 & -3 \\ 5 & -3 \end{vmatrix}$$

= 4i(-3i + 12) + i(i – 20) + 2i(-3 + 15)
= $-12i^2$ + 48i + i^2 – 20i + 24i
= $-11i^2$ + 52i
= -11(-1) + 52i …..[∵ i^2 = -1]
= 11 + 52i
Comparing with x + iy, we get
x = 11, y = 52

Maharashtra State Board 11th Commerce Maths Solutions Chapter 6 Determinants Ex 6.2

Question 1.
Without expanding, evaluate the following determinants.
(i) ∣∣∣∣ 111abcb+cc+aa+b ∣∣∣∣

Solution:

Let $D = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

Applying $C_3 \to C_3 + C_2$, we get

$D = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$

Taking $(a + b + c)$ common from C_3, we get

$D = (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$

$\therefore \quad D = (a + b + c)(0)$

…[$\because$ C_1 and C_3 are identical]

$\therefore \quad D = 0$

(ii) | | | | 256x369x4812x | | | |

Solution:

Let $D = \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

Taking $(3x)$ common from R_3, we get

$D = 3x \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}$

$= (3x)(0)$ …[$\because$ R_1 and R_3 are identical]

$= 0$

(iii) | | | | 235789657586 | | | |

Solution:

Let $D = \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$

Applying $C_3 \to C_3 - 9C_2$, we get

$D = \begin{vmatrix} 2 & 7 & 2 \\ 3 & 8 & 3 \\ 5 & 9 & 5 \end{vmatrix}$

$= 0$ …[$\because$ C_1 and C_3 are identical]

Question 2.

Using properties of determinants, show that | | | | a+babaa+ccbcb+c | | | | = 4abc

Solution:

$$\text{L.H.S.} = \begin{vmatrix} a+b & a & b \\ a & a+c & c \\ b & c & b+c \end{vmatrix}$$

Applying $C_1 \to C_1 - (C_2 + C_3)$, we get

$$\text{L.H.S.} = \begin{vmatrix} 0 & a & b \\ -2c & a+c & c \\ -2c & c & b+c \end{vmatrix}$$

Taking (–2) common from C_1, we get

$$\text{L.H.S.} = -2\begin{vmatrix} 0 & a & b \\ c & a+c & c \\ c & c & b+c \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get

$$\text{L.H.S.} = -2\begin{vmatrix} 0 & a & b \\ c & a & 0 \\ c & 0 & b \end{vmatrix}$$

$$\begin{aligned} &= -2[0(ab-0) - a(bc-0) + b(0-ac)] \\ &= -2(0 - abc - abc) \\ &= -2(-2abc) \\ &= 4abc \\ &= \text{R.H.S.} \end{aligned}$$

Question 3.
Solve the following equation.

$$\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$$

Solution:

$$\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} x+2 & x+6 & x-1 \\ 4 & -7 & 3 \\ -3 & -4 & 7 \end{vmatrix} = 0$$

∴ $(x + 2)(-49 + 12) - (x + 6)(28 + 9) + (x - 1)(-16 - 21) = 0$
∴ $(x + 2)(-37) - (x + 6)(37) + (x - 1)(-37) = 0$
∴ $-37(x + 2 + x + 6 + x - 1) = 0$
∴ $3x + 7 = 0$
∴ $x = -\frac{7}{3}$

Question 4.
If $\begin{vmatrix} 4+x & 4-x & 4-x \\ 4-x & 4+x & 4-x \\ 4-x & 4-x & 4+x \end{vmatrix} = 0$, then find the values of x.

Solution:

$$\begin{vmatrix} 4+x & 4-x & 4-x \\ 4-x & 4+x & 4-x \\ 4-x & 4-x & 4+x \end{vmatrix} = 0$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 12-x & 4-x & 4-x \\ 12-x & 4+x & 4-x \\ 12-x & 4-x & 4+x \end{vmatrix} = 0$$

Taking $(12 - x)$ common from C_1, we get

$$(12-x)\begin{vmatrix} 1 & 4-x & 4-x \\ 1 & 4+x & 4-x \\ 1 & 4-x & 4+x \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$(12-x)\begin{vmatrix} 1 & 4-x & 4-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$$

∴ $(12 - x)[1(4x^2 - 0) - (4 - x)(0 - 0) + (4 - x)(0 - 0)] = 0$
∴ $(12 - x)(4x^2) = 0$
∴ $x^2(12 - x) = 0$

∴ x = 0 or 12 – x = 0
∴ x = 0 or x = 12

Question 5.
Without expanding determinants, show that
∣∣∣∣1633176412∣∣∣∣+4∣∣∣∣221317326∣∣∣∣=10∣∣∣∣133212176∣∣∣∣
Solution:

$$\text{L.H.S.} = \begin{vmatrix} 1 & 3 & 6 \\ 6 & 1 & 4 \\ 3 & 7 & 12 \end{vmatrix} + 4\begin{vmatrix} 2 & 3 & 3 \\ 2 & 1 & 2 \\ 1 & 7 & 6 \end{vmatrix}$$

In 1[st] determinant, taking 2 common from C_3,
we get

$$\text{L.H.S.} = 2\begin{vmatrix} 1 & 3 & 3 \\ 6 & 1 & 2 \\ 3 & 7 & 6 \end{vmatrix} + 4\begin{vmatrix} 2 & 3 & 3 \\ 2 & 1 & 2 \\ 1 & 7 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 & 3 \\ 12 & 1 & 2 \\ 6 & 7 & 6 \end{vmatrix} + \begin{vmatrix} 8 & 3 & 3 \\ 8 & 1 & 2 \\ 4 & 7 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 2+8 & 3 & 3 \\ 12+8 & 1 & 2 \\ 6+4 & 7 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 10 & 3 & 3 \\ 20 & 1 & 2 \\ 10 & 7 & 6 \end{vmatrix}$$

Interchanging rows and columns, we get

$$\text{L.H.S.} = \begin{vmatrix} 10 & 20 & 10 \\ 3 & 1 & 7 \\ 3 & 2 & 6 \end{vmatrix}$$

Taking 10 common from R_1, we get

$$\text{L.H.S.} = 10\begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 7 \\ 3 & 2 & 6 \end{vmatrix}$$

$$= \text{R.H.S.}$$

Question 6.
Without expanding determinants, find the value of
(i) ∣∣∣∣101215576478107124153∣∣∣∣
Solution:

$$\text{Let } D = \begin{vmatrix} 10 & 57 & 107 \\ 12 & 64 & 124 \\ 15 & 78 & 153 \end{vmatrix}$$

Applying $C_3 \to C_3 - C_2$, we get

$$D = \begin{vmatrix} 10 & 57 & 50 \\ 12 & 64 & 60 \\ 15 & 78 & 75 \end{vmatrix}$$

Taking (5) common from C_3, we get

$$D = 5\begin{vmatrix} 10 & 57 & 10 \\ 12 & 64 & 12 \\ 15 & 78 & 15 \end{vmatrix}$$

$$= 5(0) \qquad \ldots[\because C_1 \text{ and } C_3 \text{ are identical}]$$

$$= 0$$

(ii) ∣∣∣∣201420202023201720232026111∣∣∣∣

Solution:

Let $D = \begin{vmatrix} 2014 & 2017 & 1 \\ 2020 & 2023 & 1 \\ 2023 & 2026 & 1 \end{vmatrix}$

Applying $C_2 \to C_2 - C_1$, we get

$D = \begin{vmatrix} 2014 & 3 & 1 \\ 2020 & 3 & 1 \\ 2023 & 3 & 1 \end{vmatrix}$

Taking (3) common from C_2, we get

$D = 3\begin{vmatrix} 2014 & 1 & 1 \\ 2020 & 1 & 1 \\ 2023 & 1 & 1 \end{vmatrix}$

$= 3(0)$ …[∵ C_2 and C_3 are identical]

$= 3$

Question 7.

Without expanding determinants, prove that

(i) ∣∣∣∣a1a2a3b1b2b3c1c2c3∣∣∣∣=∣∣∣∣b1b2b3c1c2c3a1a2a3∣∣∣∣=∣∣∣∣c1c2c3a1a2a3b1b2b3∣∣∣∣

Solution:

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ …(i)

Let $E = \begin{vmatrix} b_1 & c_1 & a_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix}$

Applying $C_1 \leftrightarrow C_2$, we get

$$E = -\begin{vmatrix} c_1 & b_1 & a_1 \\ c_2 & b_2 & a_2 \\ c_3 & b_3 & a_3 \end{vmatrix}$$

Applying $C_1 \leftrightarrow C_3$, we get

$$E = -(-1)\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$E = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \ldots\text{(ii)}$$

Let $F = \begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix}$

Applying $C_1 \leftrightarrow C_2$, we get

$$F = -\begin{vmatrix} a_1 & c_1 & b_1 \\ a_2 & c_2 & b_2 \\ a_3 & c_3 & b_3 \end{vmatrix}$$

Applying $C_2 \leftrightarrow C_3$, we get

$$F = -(-1)\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$F = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \ldots\text{(iii)}$$

From (i), (ii) and (iii), we get

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} b_1 & c_1 & a_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix} = \begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix}$$

(ii) $\begin{vmatrix} 1 & yz & y+z \\ 1 & zx & z+x \\ 1 & xy & x+y \end{vmatrix} = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$

Solution:

$$\text{L.H.S.} = \begin{vmatrix} 1 & yz & y+z \\ 1 & zx & z+x \\ 1 & xy & x+y \end{vmatrix}$$

$$= \begin{vmatrix} 1 & yz & x+y+z-x \\ 1 & zx & y+z+x-y \\ 1 & xy & z+x+y-z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & yz & x+y+z \\ 1 & zx & x+y+z \\ 1 & xy & x+y+z \end{vmatrix} + \begin{vmatrix} 1 & yz & -x \\ 1 & zx & -y \\ 1 & xy & -z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & yz & x+y+z \\ 1 & zx & x+y+z \\ 1 & xy & x+y+z \end{vmatrix} - \begin{vmatrix} 1 & yz & x \\ 1 & zx & y \\ 1 & xy & z \end{vmatrix}$$

In 1st determinant, taking (x + y + z) common from C_3 and in 2nd determinant, taking $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ common from R_1, R_2, R_3 respectively, we get

$$\text{L.H.S.} = (x+y+z)\begin{vmatrix} 1 & yz & 1 \\ 1 & zx & 1 \\ 1 & xy & 1 \end{vmatrix} - \frac{1}{xyz}\begin{vmatrix} x & xyz & x^2 \\ y & xyz & y^2 \\ z & xyz & z^2 \end{vmatrix}$$

In 2^{nd} determinant, taking xyz common from C_2, we get

$$\text{L.H.S.} = (x+y+z)(0) - \frac{xyz}{xyz}\begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix}$$

....[$\because$ C_1 and C_3 are identical in 1^{st} determinant]

$$= -\begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix}$$

Applying $C_1 \leftrightarrow C_2$, we get

$$\text{L.H.S.} = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = \text{R.H.S.}$$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 6 Determinants Ex 6.3

Question 1.
Solve the following equations using Cramer's Rule.
(i) x + 2y – z = 5, 2x – y + z = 1, 3x + 3y = 8
Solution:
Given equations are
x + 2y – z = 5
2x – y + z = 1
3x + 3y = 8 i.e. 3x + 3y + 0z = 8
∴ D = ∣∣∣∣ 1232−13−110 ∣∣∣∣
= 1(0 – 3) – 2(0 – 3) – 1(6 + 3)
= -3 + 6 – 9
= -6
D_x = ∣∣∣∣ 5182−13−110 ∣∣∣∣
= 5(0 – 3) – 2(0 – 8) + (-1)(3 + 8)
= -15 + 16 – 11
= -10
D_y = ∣∣∣∣ 123518−110 ∣∣∣∣
= 1(0 – 8) – 5(0 – 3) + 1(16 – 3)
= -8 + 15 – 13
= -6
D_z = ∣∣∣∣ 1232−13518 ∣∣∣∣
= 1(-8 – 3) – 2(16 – 3) + 5(6 + 3)

= -11 – 26 + 45
= 8
By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{-10}{-6} = \frac{5}{3}$$

$$y = \frac{D_y}{D} = \frac{-6}{-6} = 1$$

$$z = \frac{D_z}{D} = \frac{8}{-6} = \frac{-4}{3}$$

x = 53, y = 1 and z = −43 are the solutions of the given equations.

Check:
We can check if our answer is right or wrong.
In order to do so, substitute the values of x, y and z in the given equations.
x = 53, y = 1 and z = −43 satisfy the given equations.
If either one of the equations is not satisfied, then our answer is wrong.
If x = 53, y = 1 and z = −43 are the solutions of the given equations.
L.H.S. = x + 2y – z
= 53+2−43
= 73
≠ R.H.S.
L.H.S. = 2x – y + z
= 103−1+43
= 113
≠ R.H.S.
L.H.S. = 3x + 3y
= 5 + 3
= 8
= R.H.S.

(ii) 2x – y + 6z = 10, 3x + 4y – 5z = 11, 8x – 7y – 9z = 12
Solution:
Given equations are
2x – y + 6z = 10
3x + 4y – 5z = 11
8x – 7y – 9z = 12
∴ D = ∣∣∣∣238−14−76−5−9∣∣∣∣
= 2(-36 – 35) – (-1)(-27 + 40) + 6(-21 – 32)
= -142 + 13 – 318
= -447
D_x = ∣∣∣∣101112−14−76−5−9∣∣∣∣
= 10(-36 – 35) – (-1)(-99 + 60) + 6(-77 – 48)
= -710 – 39 – 750
= -1499
D_y = ∣∣∣∣2381011126−5−9∣∣∣∣
= 2(-99 + 60) – 10(-27 + 40) + 6(36 – 88)
= -78 – 130 – 312
= -520
D_z = ∣∣∣∣238−14−7101112∣∣∣∣
= 2(48 + 77) – (-1)(36 – 88) + 10(-21 – 32)
= 250 – 52 – 530
= -332
By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{-1499}{-447} = \frac{1499}{447}$$

$$y = \frac{D_y}{D} = \frac{-520}{-447} = \frac{520}{447}$$

$$z = \frac{D_z}{D} = \frac{-332}{-447} = \frac{332}{447}$$

∴ x = 1499447, y = 520447 and z = 332447 are the solutions of the given equations.

(iii) 11x – y – z = 31, x – 6y + 2z = -26, x + 2y – 7z = -24
Solution:
Given equations are
11x – y – z = 31
x – 6y + 2z = -26
x + 2y – 7z = -24
D = ∣∣∣∣1111-1-62-12-7∣∣∣∣
= 11(42 – 4) – (-1)(-7 – 2) + (-1)(2 + 6)
= 418 – 9 – 8
= 401
D_x = ∣∣∣∣31-26-24-1-62-12-7∣∣∣∣
= 31(42 – 4) – (-1)(182 + 48) + (-1)(-52 – 144)
= 1178 + 230 + 196
= 1604
D_y = ∣∣∣∣111131-26-24-12-7∣∣∣∣
= 11(182 + 48) – 31(-7 – 2) + (-1)(-24 + 26)
= 2530 + 279 – 2
= 2807
D_z = ∣∣∣∣1111-1-6231-26-24∣∣∣∣
= 11(144 + 52) – (-1)(-24 + 26) + 31(2 + 6)
= 2156 + 2 + 248
= 2406
By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{1604}{401} = 4$$

$$y = \frac{D_y}{D} = \frac{2807}{401} = 7$$

$$z = \frac{D_z}{D} = \frac{2406}{401} = 6$$

∴ x = 4, y = 7 and z = 6 are the solutions of the given equations.

(iv) 1x+1y+1z=−2, 1x−2y+1z=3, 2x−1y+3z=−1
Solution:
Let 1x = p, 1y = q, 1z = r
The given equations become
p + q + r = -2
p – 2q + r = 3
2p – q + 3r = -1
D = ∣∣∣∣1121-2-1113∣∣∣∣
= 1(-6 + 1) – 1(3 – 2) + 1(-1 + 4)
= -5 – 1 + 3
= -3
D_p = ∣∣∣∣-23-11-2-1113∣∣∣∣
= -2(-6 + 1) – 1(9 + 1) + 1(-3 – 2)
= 10 – 10 – 5
= -5
D_q = ∣∣∣∣112-23-1113∣∣∣∣
= 1(9 + 1) + 2(3 – 2) + 1(-1 – 6)
= 10 + 2 – 7
= 5
D_r = ∣∣∣∣1121-2-1-23-1∣∣∣∣
= 1(2 + 3) – 1(-1 – 6) – 2(-1 + 4)
= 5 + 7 – 6
= 6
By Cramer's Rule,

$$p = \frac{D_p}{D} = \frac{-5}{-3} = \frac{5}{3}$$

$$q = \frac{D_q}{q} = \frac{-5}{3},$$

$$r = \frac{D_r}{D} = \frac{6}{-3} = -2$$

$$\frac{1}{x} = \frac{5}{3},\ \frac{1}{y} = \frac{-5}{3},\ \frac{1}{z} = -2$$

∴ x = 35, y = -35, z = -12 are the solutions of the given equations.

(v) 2x−1y+3z=4,1x−1y+1z=2,3x+1y−1z=2
Solution:
Let 1x = p, 1y = q, 1z = r
The given equations become
2p – q – 3r = 4
p – q + r = 2
3p + q – r = 2
D = ∣∣∣∣213-1-1131-1∣∣∣∣
= 2(1 – 1) – (-1)(-1 – 3) + 3(1 + 3)
= 0 – 4 + 12
= 8
D_p = ∣∣∣∣422-1-1131-1∣∣∣∣
= 4(1 – 1) – (-1)(-2 – 2) + 3(2 + 2)
= 0 – 4 + 12
= 8
D_q = ∣∣∣∣21342231-1∣∣∣∣
= 2(-2 – 2) – 4(-1 – 3) + 3(2 – 6)
= -8 + 16 – 12
= -4
D_r = ∣∣∣∣213-1-11422∣∣∣∣
= 2(-2 – 2) – (-1)(2 – 6) + 4(1 + 3)
= -8 – 4 + 16
= 4
By Cramer's Rule,

$$p = \frac{D_p}{D} = \frac{8}{8} = 1$$

$$q = \frac{D_q}{D} = \frac{-4}{8} = \frac{-1}{2}$$

$$r = \frac{D_r}{D} = \frac{4}{8} = \frac{1}{2}$$

$$\frac{1}{x} = 1,\ \frac{1}{y} = \frac{-1}{2},\ \frac{1}{z} = \frac{1}{2}$$

∴ x = 1, y = -2 and z = 2 are the solutions of the given equations.

Question 2.
An amount of ₹ 5,000 is invested in three plans at rates 6%, 7% and 8% per annum respectively. The total annual income from these investments is ₹ 350. If the total annual income from first two investments is ₹ 70 more than the income from the third, find the amount invested in each plan by using Cramer's Rule.
Solution:
Let the amount of each investment be ₹ x, ₹ y and ₹ z.
According to the given conditions,
x + y + z = 5000
6%x + 7%y + 8%z = 350
∴ 6100x+7100y−8100z=350
∴ 6x + 7y + 8z = 35000
6%x + 7%y = 8%z + 70
∴ 6100x+7100y=8100z+70
∴ 6x + 7y = 8z + 7000
∴ 6x + 7y – 8z = 7000

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{vmatrix}$$

$= 1(-56-56) - 1(-48-48) + 1(42-42)$
$= -112 + 96 + 0$
$= -16$

$$D_x = \begin{vmatrix} 5000 & 1 & 1 \\ 35000 & 7 & 8 \\ 7000 & 7 & -8 \end{vmatrix}$$

Taking 100(

$$D_x = \begin{vmatrix} 5 & 1 & 1 \\ 35 & 7 & 8 \\ 7 & 7 & -8 \end{vmatrix}$$

Applying $C_1 \to C_1 - 5C_3$ and $C_2 \to C_2 - C_3$,
we get

$$D_x = 1000 \begin{vmatrix} 0 & 0 & 1 \\ -5 & -1 & 8 \\ 47 & 15 & -8 \end{vmatrix}$$

$= 1000[0 - 0 + 1(-75 + 47)]$
$= 1000 \times (-28) = -28000$

$$D_y = \begin{vmatrix} 1 & 5000 & 1 \\ 6 & 35000 & 8 \\ 6 & 7000 & -8 \end{vmatrix}$$

Taking 1000 common from C_2, we get

$$D_y = 1000 \begin{vmatrix} 1 & 5 & 1 \\ 6 & 35 & 8 \\ 6 & 7 & -8 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - 5C_3$,
We get

$$D_y = 1000 \begin{vmatrix} 0 & 0 & 1 \\ -2 & -5 & 8 \\ 14 & 47 & -8 \end{vmatrix}$$

$= 1000[0 - 0 + 1(-94 + 70)]$
$= 1000(-24) = -24000$

$$D_z = 1000 \begin{vmatrix} 0 & 1 & 5 \\ 6 & 7 & 35 \\ 6 & 7 & 7 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$ and $C_3 \to C_3 - 5C_2$,
we get

$$Dz = 1000 \begin{vmatrix} 0 & 1 & 0 \\ -1 & 7 & 0 \\ -1 & 7 & -28 \end{vmatrix}$$

$= 1000[0 - 1(28 - 0) + 0]$
$= 1000 \times (-28) = -28000$

By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{-28000}{-16} = 1750$$

$$y = \frac{D_y}{D} = \frac{-24000}{-16} = 1500$$

$$z = \frac{D_z}{D} = \frac{-28000}{-16} = 1750$$

∴ Amounts of investments are ₹ 1750, ₹ 1500, and ₹ 1750.

Check:
First condition:
1750 + 1500 + 1750 = 5000

Second condition:
6% of 1750 + 7% of 1500 + 8% of 1750
= 105 + 105 + 140
= 350
Third condition:
Combined income = 105 + 105
= 210
= 140 + 70
Thus, all the conditions are satisfied.

Question 3.
Show that the following equations are consistent.
2x + 3y + 4 = 0, x + 2y + 3 = 0, 3x + 4y + 5 = 0
Solution:
Given equations are
2x + 3y + 4 = 0
x + 2y + 3 = 0
3x + 4y + 5 = 0
$\therefore \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix}$
= 2(10 – 12) – 3(5 – 9) + 4(4 – 6)
= 2(-2) – 3(-4) + 4(-2)
= -4 + 12 – 8
= 0
∴ The given equations are consistent.

Question 4.
Find k, if the following equations are consistent.
(i) x + 3y + 2 = 0, 2x + 4y – k = 0, x – 2y – 3k = 0
Solution:
Given equations are
x + 3y + 2 = 0
2x + 4y – k = 0
x – 2y – 3k = 0
Since, these equations are consistent.
$\therefore \begin{vmatrix} 1 & 3 & 2 \\ 2 & 4 & -k \\ 1 & -2 & -3k \end{vmatrix} = 0$
∴ 1(-12k – 2k) – 3(-6k + k) + 2(-4 – 4) = 0
∴ -14k + 15k – 16 = 0
∴ k – 16 = 0
∴ k = 16
Check:
If the value of k satisfies the condition for the given equations to be consistent, then our answer is correct.
Substitute k = 16 in the given equation.
$\begin{vmatrix} 1 & 3 & 2 \\ 2 & 4 & -16 \\ 1 & -2 & -48 \end{vmatrix}$
= 1(-192 – 32) – 3(-96 + 16) + 2(-4 – 4)
= 0
Thus, our answer is correct.

(ii) (k – 2)x + (k – 1)y = 17, (k – 1)x + (k – 2)y = 18, x + y = 5
Solution:
Given equations are
(k – 2)x + (k – 1)y = 17
(k – 1)x + (k – 2)y = 18
x + y = 5
Since, these equations are consistent.
$\therefore \begin{vmatrix} k-2 & k-1 & -17 \\ k-1 & k-2 & -18 \\ 1 & 1 & -5 \end{vmatrix} = 0$
Applying $R_1 \to R_1 - R_2$, we get
$\begin{vmatrix} -1 & 1 & 1 \\ k-1 & k-2 & -18 \\ 1 & 1 & -5 \end{vmatrix} = 0$
∴ -1(-5k + 10 + 18) – 1(-5k + 5 + 18) + 1(k – 1 – k + 2) = 0
∴ -1(-5k – 28) – 1(- 5k + 23) + 1(1) = 0
∴ 5k – 28 + 5k – 23 – 1 = 0
∴ 10k – 50 = 0
∴ k = 5

Question 5.
Find the area of the triangle whose vertices are:
(i) (4, 5), (0, 7), (-1, 1)

Solution:
Here, A(x_1, y_1) ≡ A(4, 5), B(x_2, y_2) ≡ B(0, 7), C(x_3, y_3) ≡ C(-1, 1)

Area of a triangle = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

∴ A(ΔABC) = $\frac{1}{2}\begin{vmatrix} 4 & 5 & 1 \\ 0 & 7 & 1 \\ -1 & 1 & 1 \end{vmatrix}$

= $\frac{1}{2}$ [4(7 – 1) – 5(0 + 1) + 1(0 + 7)]

= $\frac{1}{2}$ (24 – 5 + 7)

= 13 sq.units.

(ii) (3, 2), (-1, 5), (-2, -3)
Solution:
Here, A(x_1, y_1) ≡ A(3, 2), B(x_2, y_2) = B(-1, 5), C(x_3, y_3) ≡ C(-2, -3)

Area of a triangle = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

∴ A(ΔABC) = $\frac{1}{2}\begin{vmatrix} 3 & 2 & 1 \\ -1 & 5 & 1 \\ -2 & -3 & 1 \end{vmatrix}$

= $\frac{1}{2}$ [3(5 + 3) – 2(-1 + 2) + 1(3 + 10)]

= $\frac{1}{2}$ (24 – 2 + 13)

= $\frac{35}{2}$ sq. units

(iii) (0, 5), (0, -5), (5, 0)
Solution:
Here, A(x_1, y_1) ≡ A(0, 5), B(x_2, y_2) ≡ B(0, -5), C(x_3, y_3) ≡ C(5,0)

Area of a triangle = $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

∴ A(ΔABC) = $\frac{1}{2}\begin{vmatrix} 0 & 5 & 1 \\ 0 & -5 & 1 \\ 5 & 0 & 1 \end{vmatrix}$

= $\frac{1}{2}$ [0(-5 – 0) – 5(0 – 5) + 1(0 + 25)]

= $\frac{1}{2}$ (0 + 25 + 25)

= $\frac{50}{2}$

= 25 sq.units

Question 6.
Find the value of k, if the area of the triangle with vertices at A(k, 3), B(-5, 7), C(-1, 4) is 4 square units.
Solution:
Here, A(x_1, y_1) ≡ A(k, 3), B(x_2, y_2) ≡ B(-5, 7), C(x_3, y_3) ≡ C(-1, 4)
A(ΔABC) = 4 sq.units

Area of a triangle = $\frac{1}{2}\begin{vmatrix} x_b & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

∴ $\frac{1}{2}\begin{vmatrix} k & 3 & 1 \\ -5 & 7 & 1 \\ -1 & 4 & 1 \end{vmatrix}$ = ±4

∴ k(7 – 4) – 3(-5 + 1) + 1(-20 + 7) = ±8
∴ 3k + 12 – 13 = ±8
∴ 3k – 1 = ±8
∴ 3k – 1 = 8 or 3k – 1 = -8
∴ 3k = 9 or 3k = -7
∴ k = 3 or k = $\frac{-7}{3}$

Check:
For k = 3,

$$A(\Delta ABC) = \frac{1}{2}\begin{vmatrix} 3 & 3 & 1 \\ -5 & 7 & 1 \\ -1 & 4 & 1 \end{vmatrix} = 4$$

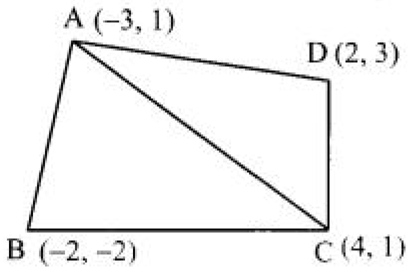
For k = $\frac{-7}{3}$

$$A(\Delta ABC) = \frac{1}{2}\begin{vmatrix} \frac{-7}{3} & 3 & 1 \\ -5 & 7 & 1 \\ -1 & 4 & 1 \end{vmatrix} = -4 = 4$$

…[∵ area cannot be negative]

Thus, our answer is correct.

Question 7.
Find the area of the quadrilateral whose vertices are A(-3, 1), B(-2, -2), C(4, 1), D(2, 3).
Solution:
A(-3, 1), B(-2, -2), C(4, 1), D(2, 3)



A(ABCD) = A(ΔABC) + A(ΔACD)

Area of a triangle = $\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$

∴ A(ΔABC) $= \frac{1}{2}\begin{vmatrix}-3 & 1 & 1\\ -2 & -2 & 1\\ 4 & 1 & 1\end{vmatrix}$

$= \frac{1}{2}\ [-3(-2-1) - 1(-2-4) + 1(-2+8)]$

$= \frac{1}{2}\ (9 + 6 + 6)$

∴ A(Δ ABC) $= \frac{21}{2}$ sq.units.

∴ A(ΔACD) $= \frac{1}{2}\begin{vmatrix}-3 & 1 & 1\\ 4 & 1 & 1\\ 2 & 3 & 1\end{vmatrix}$

$= \frac{1}{2}[-3(1-3) - 1(4-2) + 1(12-2)]$

$= \frac{1}{2}\ (6 - 2 + 10)$

∴ A(Δ ACD) = 7 sq.units.

A(ABCD) = A(ΔABC) + A(ΔACD)
$= \frac{21}{2} + 7$
$= \frac{35}{2}$ sq.units.

Question 8.
By using determinant, show that the following points are collinear.
P(5, 0), Q(10, -3), R(-5, 6)
Solution:
Here, $P(x_1, y_1)$ ≡ P(5, 0), $Q(x_2, y_2)$ ≡ Q(10, -3), $R(x_3, y_3)$ ≡ R(-5, 6)
If A(ΔPQR) = 0, then the points P, Q, R are collinear.
∴ A(ΔPQR) $= \frac{1}{2}\begin{vmatrix}5 & 0 & 1\\ 10 & -3 & 1\\ -5 & 6 & 1\end{vmatrix}$
$= \frac{1}{2}$ [5(-3 – 6) – 0(10 + 5) + 1(60 – 15)]
$= \frac{1}{2}$ (-45 + 0 + 45)
= 0
∴ A(ΔPQR) = 0
∴ Points P, Q and R are collinear.

Question 9.
The sum of three numbers is 15. If the second number is subtracted from the sum of first and third numbers, then we get 5. When the third number is subtracted from the sum of twice the first number and the second number, we get 4. Find the three numbers.
Solution:
Let the three numbers be x, y and z.
According to the given conditions,
x + y + z = 15
x + z – y = 5 i.e. x – y + z = 5
2x + y – z = 4
D $= \begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 1\\ 2 & 1 & -1\end{vmatrix}$
= 1(1 – 1) – 1 (-1 – 2) + 1(1 + 2)
= 1(0) – 1(-3) + 1(3)
= 0 + 3 + 3
= 6 ≠ 0
$D_x = \begin{vmatrix}15 & 1 & 1\\ 5 & -1 & 1\\ 4 & 1 & -1\end{vmatrix}$
= 15(1 – 1) – 1(-5 – 4) + 1(5 + 4)
= 15(0) – 1(-9) + 1(9)

= 0 + 9 + 9
= 18
D_y = | | | | 112155411-1 | | | |
= 1(-5 – 4) – 15(-1 – 2) + 1(4 – 10)
= 1(-9) – 15(-3) + 1(-6)
= -9 + 45 – 6
= 30
D_z = | | | | 1121-111554 | | | |
= 1(-4 – 5) – 1(4 – 10) + 15(1 + 2)
= 1(-9) – 1(-6) + 15(3)
= -9 + 6 + 45
= 42
By Cramer’s Rule,
x = D_xD=186 = 3
y = D_yD=306 = 5
z = D_zD=426 = 7
∴ The three numbers are 3, 5 and 7.

Maharashtra State Board 11th Commerce Maths Solutions Chapter 6 Determinants Miscellaneous Exercise 6

Question 1.
Evaluate:
(i) | | | | 259-520712 | | | |
Solution:

$$\begin{vmatrix} 2 & -5 & 7 \\ 5 & 2 & 1 \\ 9 & 0 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} - (-5)\begin{vmatrix} 5 & 1 \\ 9 & 2 \end{vmatrix} + 7\begin{vmatrix} 5 & 2 \\ 9 & 0 \end{vmatrix}$$

= 2(4 – 0) + 5(10 – 9) + 7(0 – 18)
= 2(4) + 5(1) + 7(-18)
= 8 + 5 – 126
= -113

(ii) | | | | 109-32712-42 | | | |
Solution:

$$\begin{vmatrix} 1 & -3 & 12 \\ 0 & 2 & -4 \\ 9 & 7 & 2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & -4 \\ 7 & 2 \end{vmatrix} - (-3)\begin{vmatrix} 0 & -4 \\ 9 & 2 \end{vmatrix} + 12\begin{vmatrix} 0 & 2 \\ 9 & 7 \end{vmatrix}$$

= 1(4 + 28) + 3(0 + 36) + 12(0 – 18)
= 1(32) + 3(36) + 12(-18)
= 32 + 108 – 216
= -76

Question 2.
Find the value(s) of x, if
(i) ∣∣∣∣1114−22x20−55x2∣∣∣∣=0
Solution:
∣∣∣∣1114−22x20−55x2∣∣∣∣=0
∴ $1(-10x^2 + 10x) – 4(5x^2 + 5) + 20(2x + 2) = 0$
∴ $-10x^2 + 10x – 20x^2 – 20 + 40x + 40 = 0$
∴ $-30x^2 + 50x + 20 = 0$
∴ $3x^2 – 5x – 2 = 0$ ……[Dividing throughout by (-10)]
∴ $3x^2 – 6x + x – 2 = 0$
∴ 3x(x – 2) + 1(x – 2) = 0
∴ (x – 2) (3x + 1) = 0
∴ x – 2 = 0 or 3x + 1 = 0
∴ x = 2 or x = −13

(ii) ∣∣∣∣1112x414x161∣∣∣∣=0
Solution:
∣∣∣∣1112x414x161∣∣∣∣=0
∴ 1(4 – 16) – 2x(1 – 16) + 4x(1 – 4) = 0
∴ 1(-12) – 2x(-15) + 4x(-3) = 0
∴ -12 + 30x – 12x = 0
∴ 18x = 12
∴ x = 23

Question 3.
By using properties of determinants, prove that ∣∣∣∣x+yz1y+zx1z+xy1∣∣∣∣=0.
Solution:

$$\text{L.H.S.} = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2$, we get

$$\text{L.H.S.} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

Taking $(x + y + z)$ common from R_1, we get

$$\text{L.H.S.} = (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$= (x + y + z)(0)$
…[∵ R_1 and R_3 are identical]
$= 0$
= R.H.S.

Question 4.
Without expanding the determinants, show that

i) $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix} = 0$

ii) $\begin{vmatrix} xa & yb & zc \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} x & y & z \\ a & b & c \\ bc & ca & ab \end{vmatrix}$

iii) $\begin{vmatrix} l & m & n \\ e & d & f \\ u & v & w \end{vmatrix} = \begin{vmatrix} n & f & w \\ l & e & u \\ m & d & v \end{vmatrix}$

iv) $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0$

Solution:

$$\text{L.H.S.} = \begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix}$$

Taking bc, ca, ab common from R_1, R_2, R_3 respectively, we get

$$\text{L.H.S.} = (bc)(ca)(ab) \begin{vmatrix} \frac{b+c}{bc} & 1 & bc \\ \frac{c+a}{ca} & 1 & ca \\ \frac{a+b}{ab} & 1 & ab \end{vmatrix}$$

Taking abc common from C_3, we get

$$\text{L.H.S.} = (a^2b^2c^2)(abc) \begin{vmatrix} \frac{1}{c}+\frac{1}{b} & 1 & \frac{1}{a} \\ \frac{1}{a}+\frac{1}{c} & 1 & \frac{1}{b} \\ \frac{1}{b}+\frac{1}{a} & 1 & \frac{1}{c} \end{vmatrix}$$

Applying $C_1 \to C_1 + C_3$, we get

$$\text{L.H.S.} = a^3b^3c^3 \begin{vmatrix} \frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1 & \frac{1}{a} \\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1 & \frac{1}{b} \\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1 & \frac{1}{c} \end{vmatrix}$$

Taking $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ common from C_1, we get

$$\text{L.H.S.} = a^3b^3c^3 \left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 1 & 1 & \frac{1}{a} \\ 1 & 1 & \frac{1}{b} \\ 1 & 1 & \frac{1}{c} \end{vmatrix}$$

$= a^3b^3c^3 \left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)(0)$

...[$\because$ C_1 and C_2 are identical]

$= 0 =$ R.H.S.

ii. L.H.S. $= \begin{vmatrix} xa & yb & zc \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix}$

Taking a, b, c common from C_1, C_2, C_3 respectively, we get

L.H.S. $= abc \begin{vmatrix} x & y & z \\ a & b & c \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \end{vmatrix}$

$= \begin{vmatrix} x & y & z \\ a & b & c \\ \frac{abc}{a} & \frac{abc}{b} & \frac{abc}{c} \end{vmatrix}$

$= \begin{vmatrix} x & y & z \\ a & b & c \\ bc & ca & ab \end{vmatrix}$

$=$ R.H.S.

iii. L.H.S. $= \begin{vmatrix} l & m & n \\ e & d & f \\ u & v & w \end{vmatrix}$

Interchanging rows and columns, we get

L.H.S. $= \begin{vmatrix} l & e & u \\ m & d & v \\ n & f & w \end{vmatrix}$

Applying $R_2 \leftrightarrow R_3$, we get

L.H.S. $= -\begin{vmatrix} l & e & u \\ n & f & w \\ m & d & v \end{vmatrix}$

Applying $R_1 \leftrightarrow R_2$, we get

L.H.S. $= \begin{vmatrix} n & f & w \\ l & e & u \\ m & d & v \end{vmatrix}$

$=$ R.H.S.

iv. Let D $= \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$

Taking (-1) common from R_1, R_2, R_3, we get

$D = (-1)^3 \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$

Interchanging rows and columns, we get

$D = -1 \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$

$\therefore$ $D = -1(D)$

$\therefore$ $2D = 0$

$\therefore$ $D = 0$

$\therefore$ $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0$

Question 5.
Solve the following linear equations by Cramer's Rule.
(i) 2x – y + z = 1, x + 2y + 3z = 8, 3x + y – 4z = 1
Solution:
Given equations are
2x – y + z = 1
x + 2y + 3z = 8
3x + y – 4z = 1
D = ∣∣∣∣213-12113-4∣∣∣∣
= 2(-8 – 3) – (-1)(-4 – 9) + 1(1 – 6)
= 2(-11) + 1(-13) + 1(-5)
= -22 – 13 – 5
= -40 ≠ 0
D_x = ∣∣∣∣181-12113-4∣∣∣∣
= 1(-8 – 3) – (-1)(-32 – 3) + 1(8 – 2)
= 1(-11) + 1(-35) + 1(6)
= -11 – 35 + 6
= -40
D_y = ∣∣∣∣21318113-4∣∣∣∣
= 2(-32 – 3) – 1(-4 – 9) + 1(1 – 24)
= 2(-35) – 1(-13) + 1(-23)
= -70 + 13 – 23
= -80
D_z = ∣∣∣∣213-121181∣∣∣∣
= 2(2 – 8) – (-1)(1 – 24) + 1(1 – 6)
= 2(-6) + 1(-23) + 1(-5)
= -12 – 23 – 5
= -40
By Cramer's Rule,
x = DxD=-40-40 = 1
y = DyD=-80-40 = 2
z = DzD=-40-40 = 1
∴ x = 1, y = 2 and z = 1 are the solutions of the given equations.

(ii) 1x+1y+1z=-2, 1x-2y+1z=3, 2x-1y+3z=-1
Solution:
Let 1x = p, 1y = q, 1z = r
The given equations become
p + q + r = -2
p – 2q + r = 3
2p – q + 3r = -1
D = ∣∣∣∣1121-2-1113∣∣∣∣
= 1(-6 + 1) – 1(3 – 2) + 1(-1 + 4)
= -5 – 1 + 3
= -3
D_p = ∣∣∣∣-23-11-2-1113∣∣∣∣
= -2(-6 + 1) – 1(9 + 1) + 1(-3 – 2)
= 10 – 10 – 5
= -5
D_q = ∣∣∣∣112-23-1113∣∣∣∣
= 1(9 + 1) + 2(3 – 2) + 1(-1 – 6)
= 10 + 2 – 7
= 5
D_r = ∣∣∣∣1121-2-1-23-1∣∣∣∣
= 1(2 + 3) – 1(-1 – 6) – 2(-1 + 4)
= 5 + 7 – 6
= 6
By Cramer's Rule,
p = DpD=-5-3=53
q = DyD=-80-40 = 2

r = D_2D=-40-40 = 1
∴ x = 35, y = -35, z = -12 are the solutions of the given equations.

(iii) x – y + 2z = 7, 3x + 4y – 5z = 5, 2x – y + 3z = 12
Solution:
Given equations are
x – y + 2z = 1
3x + 4y – 5z = 5
2x – y + 3z = 12
D = ∣∣∣∣132-14-12-53∣∣∣∣
= 1(12 – 5) – (-1)(9 + 10) + 2(-3 – 8)
= 1(7) + 1(19) + 2(-11)
= 7 + 19 – 22
= 4 ≠ 0
D_x = ∣∣∣∣7512-14-12-53∣∣∣∣
= 7(12 – 5) – (-1)(15 + 60) + 2(-5 – 48)
= 7(7)+ 1(75) +2(-53)
= 49 + 75 – 106
= 18
D_y = ∣∣∣∣13275122-53∣∣∣∣
= 1(15 + 60) – 7(9 + 10) + 2(36 – 10)
= 1(75) – 7(19) + 2(26)
= 75 – 133 + 52
= -6
D_z = ∣∣∣∣132-14-17512∣∣∣∣
= 1(48 + 5) – (-1)(36 – 10) + 7(-3 – 8)
= 1(53)+ 1(26) + 7(-11)
= 53 + 26 – 77
= 2
By Cramer's Rule,
x = D_xD=184=92
y = D_yD=-64=-32
z = D_zD=24=12
∴ x = 92, y = -32 and z = 12 are the solutions of the given equations.

Question 6.
Find the value(s) of k, if the following equations are consistent.
(i) 3x + y – 2 = 0, kx + 2y – 3 = 0 and 2x – y = 3
Solution:
Given equations are
3x + y – 2 = 0
kx + 2y – 3 = 0
2x – y = 3 i.e. 2x – y – 3 = 0
Since, these equations are consistent.
∴ ∣∣∣∣3k212-1-2-3-3∣∣∣∣=0
∴ 3(-6 – 3) – 1(-3k + 6) – 2(-k – 4) = 0
∴ 3(-9) – 1 (-3k + 6) – 2(-k – 4) = 0
∴ -27 + 3k – 6 + 2k + 8 = 0
∴ 5k – 25 = 0
∴ k = 5

(ii) kx + 3y + 4 = 0, x + ky + 3 = 0, 3x + 4y + 5 = 0
Solution:
Given equations are
kx + 3y + 4 = 0
x + ky + 3 = 0
3x + 4y + 5 = 0
Since, these equations are consistent.
∴ ∣∣∣∣k133k4435∣∣∣∣=0
∴ k(5k – 12) – 3(5 – 9) + 4(4 – 3k) = 0
∴ $5k^2$ – 12k + 12 + 16 – 12k = 0
∴ $5k^2$ – 24k + 28 = 0

∴ $5k^2 – 10k – 14k + 28 = 0$
∴ 5k(k – 2) – 14(k – 2) = 0
∴ (k – 2) (5k – 14) = 0
∴ k – 2 = 0 or 5k – 14 = 0
∴ k = 2 or k = 145

Question 7.
Find the area of triangles whose vertices are
(i) A(-1, 2), B(2, 4), C(0, 0)
Solution:
Here, A(x_1, y_1) ≡ A(-1, 2), B(x_2, y_2) ≡ B(2, 4), C(x_3, y_3) ≡ C(0, 0)
Area of a triangle = 12 ∣∣∣∣ x1x2x3y1y2y3111 ∣∣∣∣
∴ A(ΔABC) = 12 ∣∣∣∣ -120240111 ∣∣∣∣
= 12 [-1(4 – 0) – 2(2 – 0) + 1(0 – 0)]
= 12 (-4 – 4)
= 12 (-8)
= -4
Since, area cannot be negative.
∴ A(ΔABC) = 4 sq.units

(ii) P(3, 6), Q(-1, 3), R(2, -1)
Solution:
Here, P(x_1, y_1) ≡ P(3, 6), Q(x_2, y_2) ≡ Q(-1, 3), R(x_3, y_3) ≡ R(2, -1)
Area of a triangle = 12 ∣∣∣∣ x1x2x3y1y2y3111 ∣∣∣∣
A(ΔPQR) = 12 ∣∣∣∣ 3-1263-1111 ∣∣∣∣
= 12 [3(3 + 1) – 6(-1 – 2) + 1(1 – 6)]
= 12 [3(4) – 6(-3) + 1(-5)]
= 12 (12 + 18 – 5)
∴ A(ΔPQR) = 252 sq.units

(iii) L(1, 1), M(-2, 2), N(5, 4)
Solution:
Here, L(x_1, y_1) ≡ L(1, 1), M(x_2, y_2) ≡ M(-2, 2), N(x_3, y_3) ≡ N(5, 4)
Area of a triangle = 12 ∣∣∣∣ x1x2x3y1y2y3111 ∣∣∣∣
A(ΔLMN) = 12 ∣∣∣∣ 1-25124111 ∣∣∣∣
= 12 [1(2 – 4) -1(-2 – 5) + 1(-8 – 10)]
= 12 [1(-2) – 1(-7) + 1(-18)]
= 12 (-2 + 7 – 18)
= -132
Since, area cannot be negative.
∴ A(ΔLMN) = 132 sq.units

Question 8.
Find the value of k,
(i) if the area of ΔPQR is 4 square units and vertices are P(k, 0), Q(4, 0), R(0, 2).
Solution:
Here, P(x_1, y_1) ≡ P(k, 0), Q(x_2, y_2) ≡ Q(4, 0), R(x_3, y_3) ≡ R(0, 2)
A(ΔPQR) = 4 sq.units
Area of a triangle = 12 ∣∣∣∣ x1x2x3y1y2y3111 ∣∣∣∣
∴ ±4 = 12 ∣∣∣∣ k40002111 ∣∣∣∣
∴ ±4 = 12 [k(0 – 2) – 0 + 1(8 – 0)]
∴ ±8 = -2k + 8
∴ 8 = -2k + 8 or -8 = -2k + 8
∴ -2k = 0 or 2k = 16
∴ k = 0 or k = 8

(ii) if area of ΔLMN is 332 square units and vertices are L(3, -5), M(-2, k), N(1, 4).
Solution:

Here, $L(x_1, y_1) \equiv L(3, -5)$, $M(x_2, y_2) \equiv M(-2, k)$, $N(x_3, y_3) \equiv N(1, 4)$
$A(\Delta LMN) = \frac{33}{2}$ sq.units

Area of a triangle $= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$\therefore \pm\frac{33}{2} = \frac{1}{2} \begin{vmatrix} 3 & -5 & 1 \\ -2 & k & 1 \\ 1 & 4 & 1 \end{vmatrix}$

$\therefore \pm\frac{33}{2} = \frac{1}{2} [3(k - 4) - (-5)(-2 - 1) + 1(-8 - k)]$

$\therefore \pm 33 = 3k - 12 - 15 - 8 - k$

$\therefore 33 = 2k - 35$

$\therefore 2k - 35 = 33$ or $2k - 35 = -33$

$\therefore 2k = 68$ or $2k = 2$

$\therefore k = 34$ or $k = 1$