Maharashtra State Board 11th Maths Solutions Chapter 1 Angle and its Measurement Ex 1.1

Question 1.
(A) Determine which of the following pairs of angles are co-terminal.
i. 210°, 150°
ii. 360°, -30°
iii. -180°, 540°
iv. -405°, 675°
v. 860°, 580°
vi. 900°, -900°
Solution:
210°,- 150°
210°-(- 150°) = 210°+ 150°
= 360°
= 1 (360°),
which is a multiple of 360°.
∴ The given pair of angles is co-terminal.

ii. 360°, – 30°
360° – (- 30°) = 360° + 30°
= 390°,
which is not a multiple of 360°.
∴ The given pair of angles is not co-terminal.

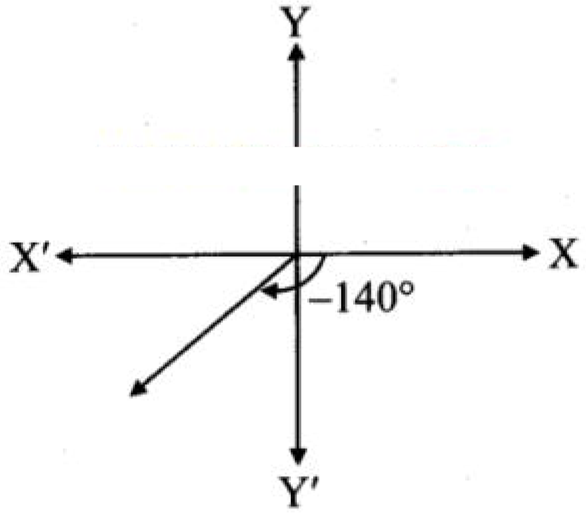
iii. -180°, 540°
540° -(-180°) = 540°+ 180°
= 720°
= 2(360°),
which is a multiple of 360°.
∴ The given pair of angles is co-terminal.

iv. – 405°, 675°
675° – (- 405°) = 675° + 405°
= 1080°
= 3(360°),
which is a multiple of 360°.
∴ The given pair of angles is co-terminal.

v. 860°, 580°
860° – 580° = 280°
which is not a multiple of 360, °.
∴ The given pair of angles is not co-terminal.

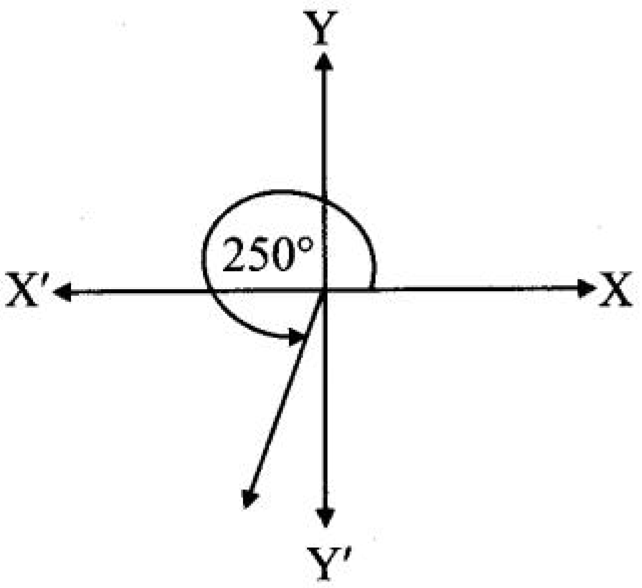
vi. 900°, 900°
900° – (-900°) = 900° + 900°
= 1800°
= 5(360°)
which is a multiple of 360°
∴ The given pair of angles is co-terminal.

Question 1.
(B) Draw the angles of the following measures and determine their quadrants.
i. -140°
ii. 250°
iii. 420°
iv. 750°
v. 945°
vi. 1120°
vii. – 80°
viii. – 330°
ix. – 500°
x. – 820°
Solution:



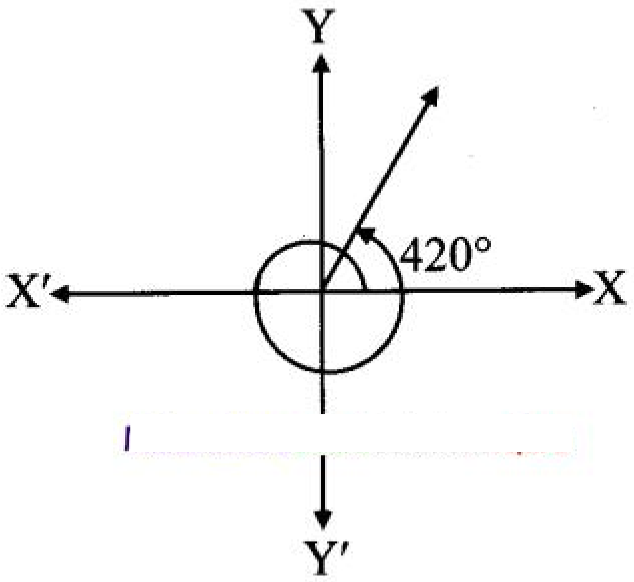
From the figure, the given angle terminates in quadrant III.

ii.



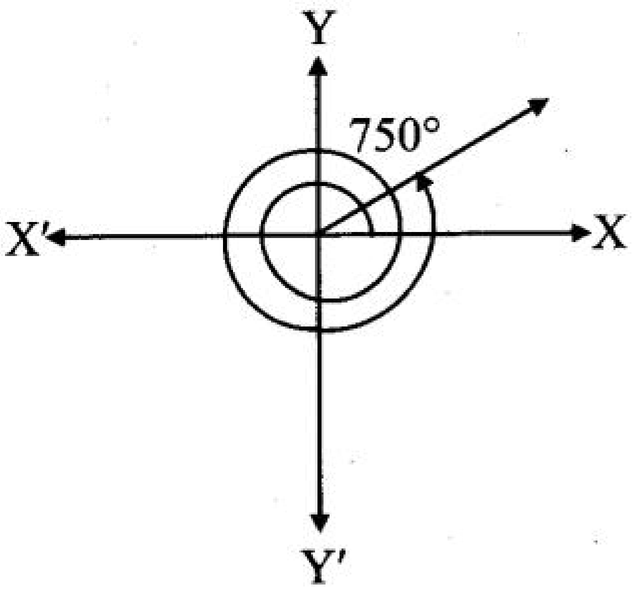
From the figure, the given angle terminates in quadrant III.

iii.



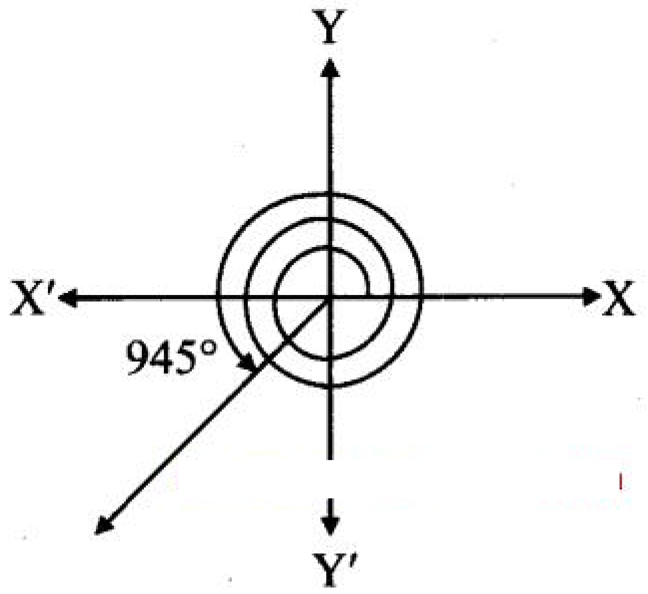
From the figure, the given angle terminates in quadrant I.

iv.



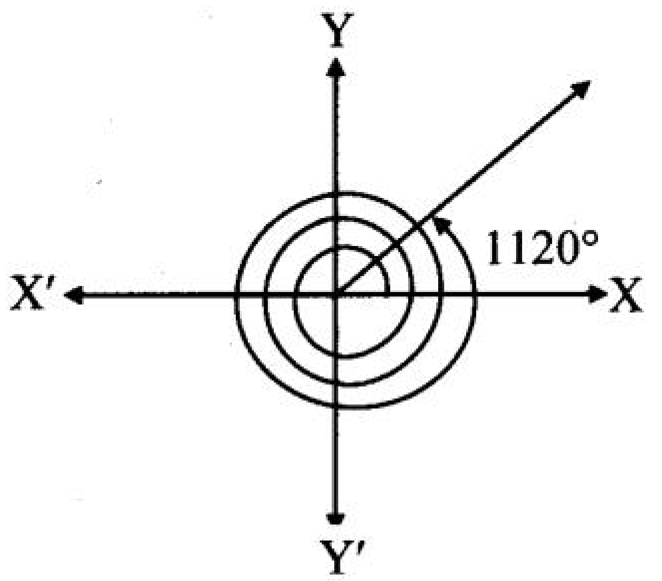
From the figure, the given angle terminates in quadrant I.

v.



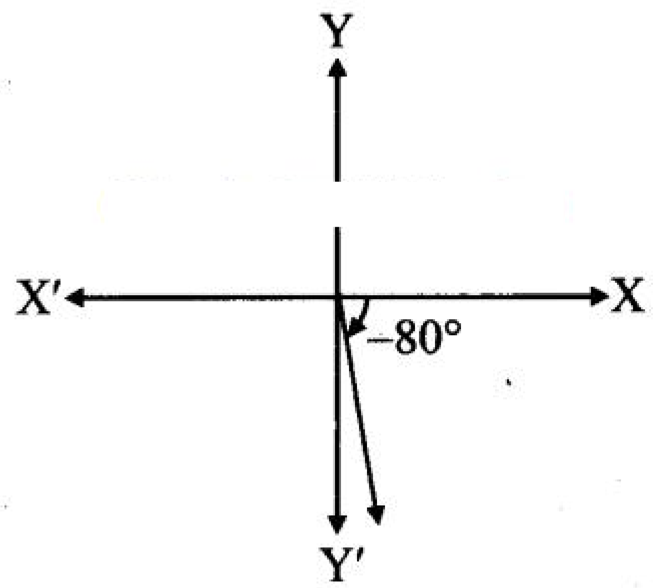
From the figure, the given angle terminates in quadrant III.

vi.



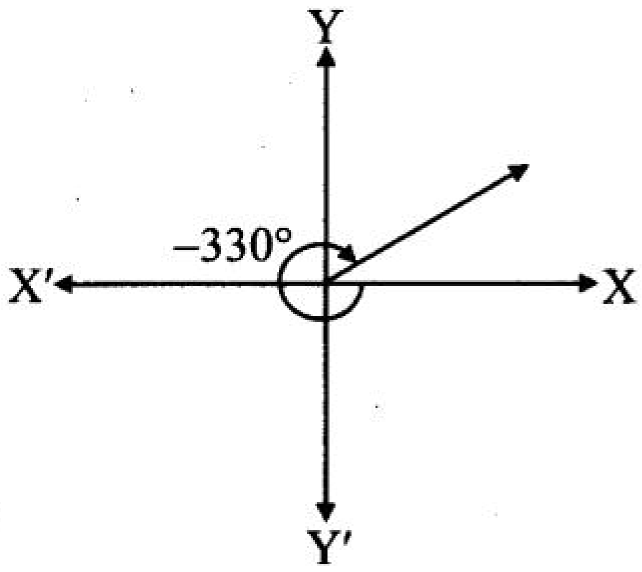
From the figure, the given angle terminates in quadrant I.

vii.



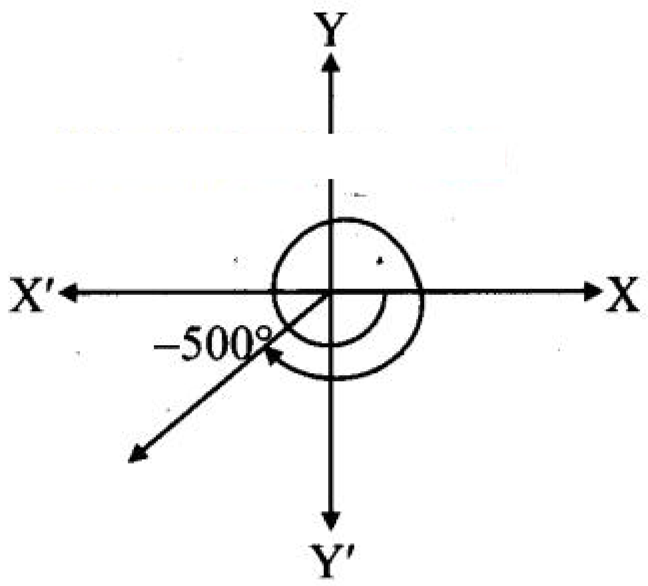
From the figure, the given angle terminates in quadrant IV.

viii.



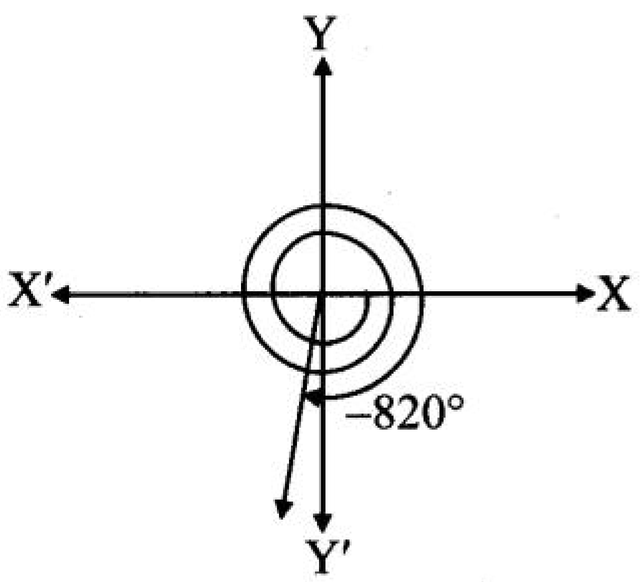
From the figure, the given angle terminates in quadrant I.

ix.



From the figure, the given angle terminates in quadrant III.
[Note: Answer given in the textbook is 'Angle lies in quadrant II'. However, we found that it lies in quadrant III.]

x.



From the figure, the given angle terminates in quadrant III.

Question 2.
Convert the following angles into radians,
i. 85°
ii. 250°
iii. -132°
iv. 65°30′
v. 75°30′
vi. 40°48′
Solution:
we know that = $\theta^\circ = \left(\theta \times \frac{\pi}{180}\right)^c$

i. 85° = $\left(85 \times \frac{\pi}{180}\right)^c = \left(\frac{17\pi}{36}\right)^c$

ii. 250° = $\left(250 \times \frac{\pi}{180}\right)^c = \left(\frac{25\pi}{18}\right)^c$

iii. 132° = $\left(-132 \times \frac{\pi}{180}\right)^c = \left(-\frac{11\pi}{15}\right)^c$

[Note : Answer given in the textbook is $\frac{11\pi}{15}$ However, as per our calculation it is $\left(-\frac{11\pi}{15}\right)^c$]

iv. 65° 30′ = 65° + 30′

= 65° + $\left(\frac{30}{60}\right)^\circ$ ... [1′ = (1/60)°]

= 65° + (1/2)°

$$= \left(65 + \frac{1}{2}\right)^\circ$$

$$= \left(\frac{131}{2}\right)^\circ$$

$$= \left(\frac{131}{2} \times \frac{\pi}{180}\right)^c$$

$$= \left(\frac{131\pi}{360}\right)^c$$

v. 75° 30′ = 75° + 30′

$$= 75^\circ + \left(\frac{30}{60}\right)^\circ \quad \ldots \left[\because 1' = \left(\frac{1}{60}\right)^\circ\right]$$

$$= 75^\circ + \left(\frac{1}{2}\right)^\circ$$

$$= \left(75 + \frac{1}{2}\right)^\circ$$

$$= \left(\frac{151}{2}\right)^\circ$$

$$= \left(\frac{151}{2} \times \frac{\pi}{180}\right)^c$$

$$= \left(\frac{151\pi}{360}\right)^c$$

vi. 40°48′ = 40° + 48′

$$= 40^\circ + \left(\frac{48}{60}\right)^\circ \quad \ldots \left[\because 1' = \left(\frac{1}{60}\right)^\circ\right]$$

$$= 40^\circ + \left(\frac{4}{5}\right)^\circ$$

$$= \left(40 + \frac{4}{5}\right)^\circ$$

$$= \left(\frac{204}{5}\right)^\circ$$

$$= \left(\frac{204}{5} \times \frac{\pi}{180}\right)^c$$

$$= \left(\frac{17\pi}{75}\right)^c$$

Question 3.
Convert the following angles in degrees.
i. $\frac{7\pi^c}{12}$
ii. $-\frac{5\pi^c}{3}$
iii. 5^c
iv. $\frac{11\pi^c}{18}$
v. $\left(-\frac{1}{4}\right)^c$

Solution:

$$\text{We know that } \theta^c = \left(\theta \times \frac{180}{\pi}\right)^\circ$$

i. $$\frac{7\pi^c}{12} = \left(\frac{7\pi}{12} \times \frac{180}{\pi}\right)^\circ = 105^\circ$$

ii. $$-\frac{5\pi^c}{3} = \left(-\frac{5\pi}{3} \times \frac{180}{\pi}\right)^\circ = -300^\circ$$

iii. $$5^c = \left(5 \times \frac{180}{\pi}\right)^\circ = \left(\frac{900}{\pi}\right)^\circ$$

iv. $$\frac{11\pi^c}{18} = \left(\frac{11\pi}{18} \times \frac{180}{\pi}\right)^\circ = 110^\circ$$

v. $$\left(-\frac{1}{4}\right)^c = \left(-\frac{1}{4} \times \frac{180}{\pi}\right)^\circ = \left(-\frac{45}{\pi}\right)^\circ$$

Question 4.
Express the following angles in degrees, minutes and seconds.
i. $(183.7)^\circ$
ii. $(245.33)^\circ$
iii. $\left(\frac{1}{5}\right)^c$

Solution:
We know that $1^\circ = 60'$ and $1' = 60''$
i. $(183.7)^\circ = 183^\circ + (0.7)^\circ$
$= 183^\circ + (0.7 \times 60)'$
$= 183^\circ + 42'$
$= 183^\circ\ 42'$

ii. $(245.33)^\circ = 245^\circ + (0.33)^\circ$
$= 245^\circ + (0.33 \times 60)'$
$= 245^\circ + (19.8)'$
$= 245^\circ + 19' + (0.8)'$
$= 245^\circ\ 19' + (0.8 \times 60)''$
$= 245^\circ\ 19' + 48''$
$= 245^\circ\ 19'\ 48''$

iii. We know that $\theta^c = \left(\theta \times \frac{180}{\pi}\right)^\circ$

$$\therefore \quad \left(\frac{1}{5}\right)^c = \left(\frac{1}{5} \times \frac{180}{\pi}\right)^\circ$$

$$= \left(\frac{36}{\pi}\right)^\circ$$

$$= \left(\frac{36}{3.14}\right)^\circ \quad \ldots[\because \pi = 3.14]$$

$= (11.46)^\circ$
$= 11^\circ + (0.46)^\circ$
$= 11^\circ + (0.46 \times 60)'$
$= 11^\circ + (27.6)'$
$= 11^\circ + 27' + (0.6)'$

= 11° + 27′ + (0.6×60)″
= 11°27′ + 36″
= 11°27′36″ (approx.)

Question 5.
In △ABC, if m∠A = $\frac{7\pi^c}{36}$, m∠B = 120°, find m∠C in degree and radian.
Solution:
We know that $\theta^c = \left(\theta \times \frac{180}{\pi}\right)^\circ$
In △ABC,
m∠A = $\frac{7\pi^c}{36} = \left(\frac{7\pi}{36} \times \frac{180}{\pi}\right)^\circ$ = 35°
m∠B = 120°
∴ m∠A + m∠B + m∠C = 180°
… [Sum of the angles of a triangle is 180°]
∴ 35° + 120° + m∠C = 180° m∠C = 180° – 35° – 120°
∴ m∠C = 25°

$$= \left(25 \times \frac{\pi}{180}\right)^c \qquad \ldots\left[\because \theta^\circ = \left(\theta \times \frac{\pi}{180}\right)^c\right]$$

$$= \left(\frac{5\pi}{36}\right)^c$$

$$\therefore \quad m\angle C = 25^\circ = \left(\frac{5\pi}{36}\right)^c$$

$$\text{i.e., } \left(\frac{5\pi}{9} \times \frac{180}{\pi}\right)^\circ, \left(\frac{5\pi}{18} \times \frac{180}{\pi}\right)^\circ,$$

Question 6.
Two angles of a triangle are $\frac{5\pi}{9}^c$ and $\frac{5\pi}{18}^c$ Find the degree and radian measures of third angle.
Solution:
We know that $\theta^c = \left[\theta \times \right]^\circ$ Themeasuresoftwoanglesofatriangleare[latex] $\frac{5\pi^c}{9}, \frac{5\pi^c}{18}$,
i.e., $\left(\frac{5\pi}{9} \times \frac{180}{\pi}\right)^\circ, \left(\frac{5\pi}{18} \times \frac{180}{\pi}\right)^0$
i.e., 100°, 50°
Let the measure of third angle of the triangle be x°.
∴ 100°+50°+x° = 180°
…[Sum of the angles of a triangle is 180°]
∴ x° = 180°- 100° – 50°
∴ x° = 30°

$$= \left(30 \times \frac{\pi}{180}\right)^c \qquad \ldots\left[\because \theta^\circ = \left(\theta \times \frac{\pi}{180}\right)^c\right]$$

$$= \left(\frac{\pi}{6}\right)^c$$

∴ The degree and radian measures of the third angle are 30° and $\left(\frac{\pi}{6}\right)^c$ respectively.

Question 7.
In a right angled triangle, the acute angles are in the ratio 4:5. Find the angles of the triangle in degrees and radians.
Solution:
Since the triangle is aright angled triangle, one of the angles is 90°.
In the right angled triangle, the acute angles are in the ratio 4:5.
Let the measures of the acute angles of the triangle in degrees be 4k and 5k, where k is a constant.
∴ 4k + 5k+ 90°= 180°
… [Sum of the angles of a triangle is 180°]
∴ 9k = 180° – 90°
∴ 9k = 90°
∴ k = 10°
∴ The measures of the angles in degrees are
4k = 4 x 10° = 40°,
5k = 5 x 10° = 50°
and 90°.

we known that $\theta° = \left(\theta \times \frac{\pi}{180}\right)^c$
∴ The measure of the angles in radius are

$$40° = \left(40 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{9}\right)^c$$

$$50° = \left(50 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{18}\right)^c$$

$$90° = \left(90 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{2}\right)^c$$

Question 8.
The sum of two angles is $5\pi^c$ and their difference is 60°. Find their measures in degrees.
Solution:
Let the measures of the two angles in degrees be x and y.
Sum of two angles is $5\pi^c$
$x + y = 5\pi^c$
$x + y = \left(5\pi \times \frac{180}{\pi}\right)$ ...[$\theta^c = \left(\theta \times \frac{180}{\pi}\right)^\circ$]
∴ x + y = 900° ………..(i)
∴ Difference of two angles is 60°.
x – y = 60° ….(ii)
Adding (i) and (ii), we get
2x = 960°
∴ x = 480°
Substituting the value of x in (i), we get
480° + y = 900°
∴ y = 900° — 480° = 420°
∴ The measures of the two angles in degrees are 480° and 420°.

Question 9.
The measures of the angles of a triangle are in the ratio 3:7:8. Find their measures in degrees and radians.
Solution:
The measures of the angles of the triangle are in the ratio 3:7:8.
Let the measures of the angles of the triangle in degrees be 3k, 7k and 8k, where k is a constant.
∴ 3k + 7k + 8k = 180°
… [Sum of the angles of a triangle is 180°]
∴ 18k =180°
∴ k = 10°
∴ The measures of the angles in degrees are
3k = 3 x 10° = 30°,
7k = 7 x 10° = 70° and
8k = 8 x 10° = 80°.

$$30° = \left(30 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{6}\right)^c$$

$$70° = \left(70 \times \frac{\pi}{180}\right)^c = \left(\frac{7\pi}{18}\right)^c$$

$$80° = \left(80 \times \frac{\pi}{180}\right)^c = \left(\frac{4\pi}{9}\right)^c$$

Question 10.
The measures of the angles of a triangle are in A.P. and the greatest is 5 times the smallest (least). Find the angles in degrees and radians.
Solution:
Let the measures of the angles of the triangle in degrees be a – d, a, a + d, where a > d> 0.
∴ a – d + a + a + d = 180°
…[Sum of the angles of a triangle is 180°]
∴ 3a = 180°
∴ a = 60° …(i)
According to the given condition, greatest angle is 5 times the smallest angle.
∴ a + d = 5 (a – d)

∴ a + d = 5a – 5d
∴ 6d = 4a
∴ 3d = 2a
∴ 3d = 2(60°) …[From (i)]
∴ d = $\frac{120^\circ}{3}$ = 40°
∴ The measures of the angles in degrees are
a – d = 60° – 40° = 20°
a = 60° and
a + d = 60° + 40° = 100°

$$\text{We know that } \theta^\circ = \left(\theta \times \frac{\pi}{180}\right)^c$$

∴ The measures of the angles in radians are

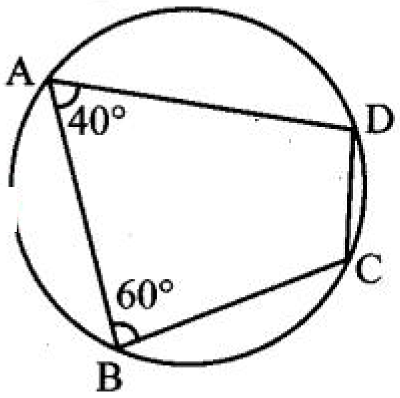
$$20^\circ = \left(20 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{9}\right)^c$$

$$60^\circ = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$$

$$100^\circ = \left(100 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{9}\right)^c$$

Question 11.
In a cyclic quadrilateral two adjacent angles are 40 and $\frac{\pi^c}{3}$. Find the angles of the quadralateral in degrees.
Solution:
Let ABCD be the cyclic quadrilateral such that
∠A = 40° and

$$\angle B = \frac{\pi^c}{3} = \left(\frac{\pi}{3} \times \frac{180}{\pi}\right)^\circ \quad \ldots\left[\because \theta^c = \left(\theta \times \frac{180}{\pi}\right)^\circ\right]$$
$$= 60^\circ$$



∴ ∠A + ∠C = 180°
∴ 40° + ∠C = 180°
∴ ∠C= 180°- 40°= 140°
Also, ∠B + ∠D = 180°
… [Opposite angles of a cyclic quadrilateral are supplementary]
∴ 60° + ∠D =180°
∴ ∠D = 180°- 60° = 120°
∴ The angles of the quadrilateral in degrees are 40°, 60°, 140° and 120°.

Question 12.
One angle of a quadrilateral has measure $\frac{2\pi^c}{5}$ and the measures of other three angles are in the ratio 2:3:4. Find their measures in degrees and radians.
Solution:
We know that $\theta^c = \left(\theta \times \frac{180}{\pi}\right)^\circ$)
One angle of the quadrilateral has measure $\frac{2\pi^c}{5} = \left(\frac{2\pi}{5} \times \frac{180}{\pi}\right)^\circ = 72^\circ$
Measures of other three angles are in the ratio 2:3:4.
Let the measures of the other three angles of the quadrilateral in degrees be 2k, 3k, 4k, where k is a constant.
∴ 72° + 2k + 3k + 4k = 360°
…[Sum of the angles of a quadrilateral is 360°]
∴ 9k = 288°
k = 32°
∴ The measures of the angles in degrees are

2k = 2 x 32° = 64°
3k = 3 x 32° = 96°
4k = 4 x 32°= 128°
We know that θ° = (θ x $\frac{\pi}{180}$)^c
∴ The measures of the angles in radians are

$$64^\circ = \left(64 \times \frac{\pi}{180}\right)^c = \left(\frac{16\pi}{45}\right)^c$$

$$96^\circ = \left(96 \times \frac{\pi}{180}\right)^c = \left(\frac{8\pi}{15}\right)^c$$

$$128^\circ = \left(128 \times \frac{\pi}{180}\right)^c = \left(\frac{32\pi}{45}\right)^c$$

Question 13.
Find the degree and radian measures of exterior and interior angles of a regular
i. pentagon
ii. hexagon
iii. septagon
iv. octagon
Solution:
i. Pentagon:
Number of sides = 5
Number of exterior angles = 5
Sum of exterior angles = 360°

$$\therefore \quad \text{Each exterior angle} = \frac{360^\circ}{\text{no. of sides}} = \frac{360^\circ}{5} = 72^\circ$$

$$= \left(72 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{5}\right)^c$$

Interior angle + Exterior angle = 180°
∴ Each interior angle = 180° — 72° = 108°
= \(

ii. Hexagon:
Number of sides = 6
Number of exterior angles = 6
Sum of exterior angles = 360°

$$\therefore \quad \text{Each exterior angle} = \frac{360^\circ}{\text{no. of sides}} = \frac{360^\circ}{6} = 60^\circ$$

$$= \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$$

Interior angle + Exterior angle = 180°
∴ Each interior angle = 180° – 60° = 120°
= (120 x [latex]\frac{\pi}{180}\))$_c$ = (\(\frac{2 \pi}{3}[latex])$_c$

iii. Septagon:
Number of sides = 7
Number of exterior angles = 7
Sum of exterior angles = 360°
∴ Each exterior angle = [latex]\frac{360^{\circ}}{\text { no. of sides }}=\frac{360^{\circ}}{7}\)
= (51.43)°
= $(\frac{360}{7} \times \frac{\pi}{180})^c = (\frac{2\pi}{7})^c$
Interior angle + Exterior angle = 180°

$\therefore$ Each interior angle = 180° – $\left(\frac{360}{7}\right)^\circ$

$$\therefore \quad \text{Each interior angle} = 180^\circ - \left(\frac{360}{7}\right)^\circ$$

$$= \left(\frac{1260-360}{7}\right)^\circ$$

$$= \left(\frac{900}{7}\right)^\circ = (128.57)^\circ$$

$$= \left(\frac{900}{7} \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{7}\right)^c$$

iv. Octagon:
Number of sides = 8
Number of exterior angles = 8
Sum of exterior angles = 360°

$\therefore$ Each exterior angle = $\frac{360^\circ}{\text{no. of sides}} = \frac{360^\circ}{8}$

= 45°

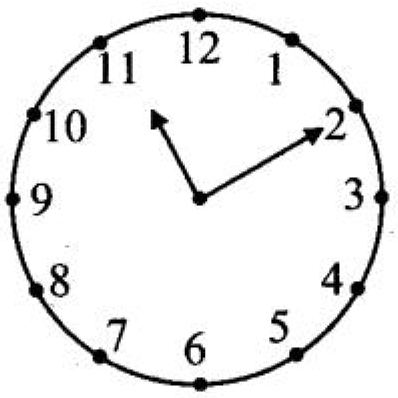
= $\left(45 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{4}\right)^c$

Interior angle + Exterior angle = 180°
Each interior angle = 180° – 45° = 135°

= $\left(135 \times \frac{\pi}{180}\right)^c = \left(\frac{3\pi}{4}\right)^c$

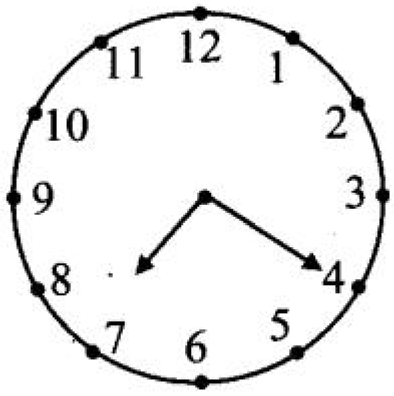
Question 14.
Find the angle between hour-hand and minute-hand in a clock at
i. ten past eleven
ii. twenty past seven
iii. thirty five past one
iv. quarter to six
v. 2:20
vi. 10:10
Solution:

i. At 11:10, the minute-hand is at mark 2 and hour-hand has crossed $\left(\frac{1}{6}\right)^{th}$ of the angle between 11 and 12.



Angle between two consecutive marks = $\frac{360^\circ}{12}$ = 30°
Angle traced by hour-hand in 10 minutes
= $\frac{1}{6}$ (30°) = 5°
Angle between marks 11 and 2 = 3 x 30° = 90°
$\therefore$ Angle between two hands of the clock at ten past eleven = 90° – 5° = 85°

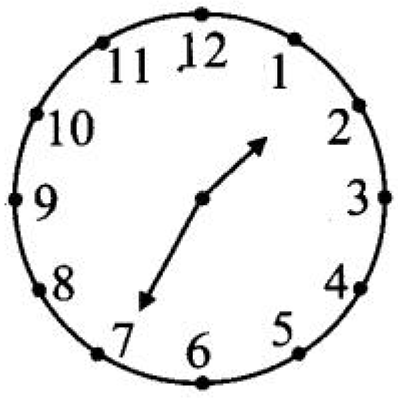
ii. At 7 : 20 the minute -hand is at mark 4 and hour -hand has crossed $\left(\frac{1}{3}\right)$rd of angle between 7 and 8.



Angle between two consecutive marks

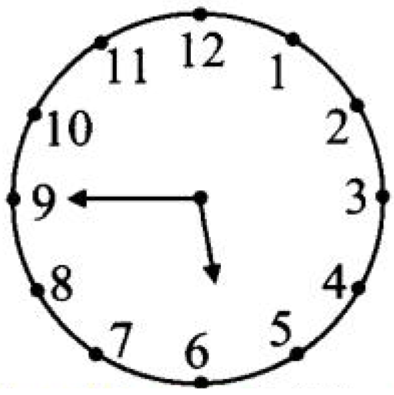
= 360°/12 = 30°
Angle traced by hour-hand in 20 minutes
= 13(30°)= 10°
Angle between marks 4 and 7 = 3 x 30° = 90°
Angle between two hands of the clock at twenty past seven = 90° – 10° = 100°

iii. At 1 : 35 the minute -hand is at mark 7 and hour -hand has crossed (712)th of angle between 1 and 2.



Angle between two consecutive marks
= 360°/12 = 30°
Angle traced by hour-hand in 35 minutes
= 712(30◦)=(352)◦ =(1712)◦ 13
Angle between marks 1 and 7 = 6 x 30° = 180°
Angle between two hands of the clock at thirty five past one = 180° – (1712)◦ =(16212)◦
= 162° + 12 = 162°30′

iv. At 5:45, the minute-hand is at mark 9 and hour- hand has crossed (frac34)th of the angle between 5 and 6.



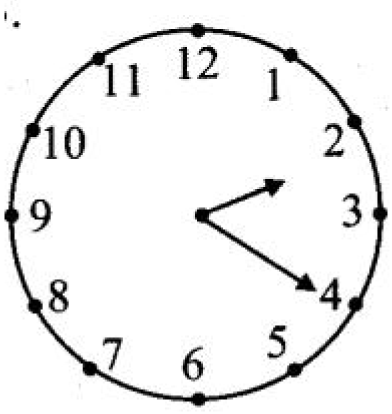
Angle between two consecutive marks
= 360°/12 = 30°
Angle traced by hour-hand in 45 minutes
34(30◦)=(22.5)◦ =(2212)◦
Angle between marks 5 and 9
= 4 x 30° = 120°
∴ Angle between two hands of the clock at quarter to six = 120◦ −(2212)o

$$= \left(97\frac{1}{2}\right)^{\circ}$$

$$= 97^{\circ} + \frac{1}{2}^{\circ}$$

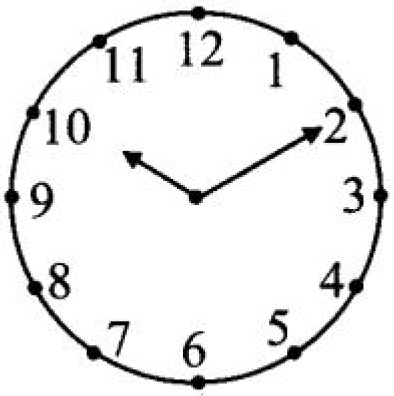
$$= 97^{\circ}30'$$

v. At 2 : 20, the minute-hand is at mark 4 hour hand has crossed $\frac{1}{3}$rd of the angle between 2 and 3.



Angle between two consecutive marks = 360°/12 = 30°
Angle traced by hour-hand in 20 minutes
= frac13(30°)= 10°
Angle between marks 2 and 4 = 2 x 30° = 60°
∴ Angle between two hands of the clock at 2 :20 = 60° – 10° = 50°

vi. At 10:10, the minute-hand is at mark 2 and hour-hand has crossed\frac{1}{6}[/latex] th between 10 and 11.



Angle between two consecutive marks
360°/12 = 30°
Angle traced by hour-hand in 10 minutes
= $\frac{1}{6}$ (30°) = 5°
Angle between marks 10 and 2= 4 x 30° = 120°
... Angle between two hands of the clock at 10:10
= 120° – 5°= 115°

Maharashtra State Board 11th Maths Solutions Chapter 1 Angle and its Measurement Ex 1.2

Question 1.
Find the length of an arc of a circle which subtends an angle of 108° at the centre, if the radius of the circle is 15 cm.
Solution:
Here, r = 15cm and
θ = 108° = $\left(108 \times \frac{\pi}{180}\right)^c = \left(\frac{3\pi}{5}\right)^c$
Since S = r.θ
S = 15 x $\frac{3\pi}{5}$
= 9π cm.

Question 2.
The radius of a circle is 9 cm. Find the length of an arc of this circle which cuts off a chord of length equal to length of radius.
Solution:
Here, r = 9cm
Let the arc AB cut off a chord equal to the radius of the circle.
Since OA = OB = AB,
ΔOAB is an equilateral triangle.
m∠AOB = 60°
θ = 60°

$= \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$

Since S = r.θ,

S = 9 x $\frac{\pi}{3}$ = 3π cm.

Question 3.

Find the angle in degree subtended at the centre of a circle by an arc whose length is 15 cm, if the radius of the circle is 25 cm.

Solution:

Here, r = 25 cm and S = 15 cm

Since S = r.θ,

15 = 25 x θ

$$\therefore \quad \theta = \left(\frac{15}{25}\right)^c$$

$$\therefore \quad \theta = \left(\frac{3}{5}\right)^c = \left(\frac{3}{5} \times \frac{180}{\pi}\right)^\circ$$

$$= \left(\frac{108}{\pi}\right)^\circ = \left(\frac{108}{3.14}\right)^\circ \quad ...[\because \pi = 3.14]$$

$$= (34.40)^\circ \text{ (approx.)}$$

∴ The required angle in degree is $\left(\frac{108}{\pi}\right)^\circ$ or (34.40)°(approx.).

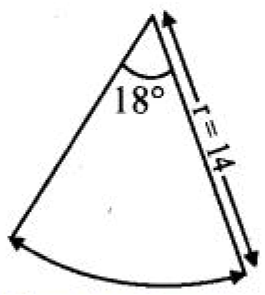
Question 4.

A pendulum of length 14 cm oscillates through an angle of 18°. Find the length of its path.

Solution:

Here, r = 14 cm and

$$\theta = 18^\circ = \left(18 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{10}\right)^c$$



Since S = r.θ,

$$S = 14 \times \frac{\pi}{10}$$

$$\therefore \quad S = \frac{7\pi}{5} = \frac{7(3.14)}{5} \quad ...[\because \pi = 3.14]$$

$$= \frac{21.98}{5} = 4.4 \text{ cm. (approx.)}$$

Question 5.

Two arcs of the same length subtend angles of 60° and 75° at the centres of the two circles. What is the ratio of radii of two circles?

Solution:

Let r_1, and r_2 be the radii of the two circles and let their arcs of same length S subtend angles of 60° and 75° at their centres.

Angle subtended at the centre of the first circle,

$\theta_1 = 60^\circ = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$

∴ $S = r_1\theta_1 = r_1\left(\left(\frac{\pi}{3}\right)\right)$

Angle subtended at the centre of the second circle,

$$\theta_2 = 75^\circ = \left(75 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{12}\right)^c$$

$$\therefore \quad S = r_2\theta_2 = r_2\left(\frac{5\pi}{12}\right) \quad ...(ii)$$

From (i) and (ii), we get

$$r_1\left(\frac{\pi}{3}\right) = r_2\left(\frac{5\pi}{12}\right)$$

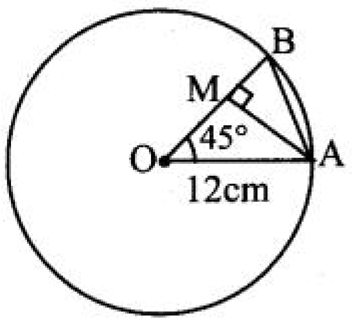
$$\therefore \quad \frac{r_1}{r_2} = \frac{15}{12}$$

$$\therefore \quad \frac{r_1}{r_2} = \frac{5}{4}$$

$$\therefore \quad r_1 : r_2 = 5 : 4.$$

Question 6.
The area of the circle is 2571 sq.cm. Find the length of its arc subtending an angle of 144° at the centre. Also find the area of the corresponding sector.
Solution:
Area of circle = πr^2
But area is given to be 25 π sq.cm
∴ $25\pi = \pi r^2$
∴ $r^2 = 25$
∴ r = 5 cm
θ = 144° = =(144×π180)c=(4π5)c
Since s = rθ
S = 5(4π5) = 4π
Also, A(sector) = 12 x r x S = 12 x 5 x 4π
= 10π sq. cm

Question 7.
OAB is a sector of the circle having centre at O and radius 12 cm. If m∠AOB = 45°, find the difference between the area of sector OAB and ΔAOB.
Solution:
Here, r = 12 cm
θ=45∘ =(45×π180)c=(π4)c



Draw AM ⊥ OB
In ΔOAM,

$$\therefore \quad \frac{1}{\sqrt{2}} = \frac{AM}{12}$$

$$\therefore \quad AM = \frac{12}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 6\sqrt{2} \text{ cm}$$

$$\therefore \quad A(\text{sector OAB}) - A(\Delta AOB)$$

$$= \frac{1}{2}r^2\theta - \frac{1}{2} \times OB \times AM$$

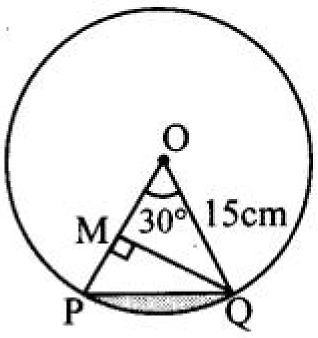
$$= \frac{1}{2} \times (12)^2 \times \frac{\pi}{4} - \frac{1}{2} \times 12 \times 6\sqrt{2}$$

$$= \frac{1}{2} \times 144 \times \frac{\pi}{4} - 36\sqrt{2} = 18\pi - 36\sqrt{2}$$

$$= 18(\pi - 2\sqrt{2}) \text{ sq.cm.}$$

[Note: The question has been modified.]

Question 8.
OPQ is the sector of a circle having centre at O and radius 15 cm. If m∠POQ = 30°, find the area enclosed by arc PQ and chord PQ.
Solution:
Here, r = 15 cm
m∠POQ = 30°
\(\left(30 \times \frac{\pi}{180}\right)^{c}[/larex]



∴ θ = [latex]\left(\frac{\pi}{6}\right)^{c}\)
Draw QM ⊥ OP
In ΔOQM,

sin 30° = QM 15

QM= 15 x 12=152

Shaded portion indicates the area enclosed by arc PQ and chord PQ.
∴ A(shaded portion)
= A(sector OPQ) – A(ΔOPQ)

$$= \frac{1}{2}r^2\theta - \frac{1}{2} \times OP \times QM$$

$$= \frac{1}{2} \times (15)^2 \times \frac{\pi}{6} - \frac{1}{2} \times 15 \times \frac{15}{2}$$

$$= \frac{225\pi}{12} - \frac{225}{4}$$

$$= \frac{225}{4}\left(\frac{\pi}{3} - 1\right) \text{ sq.cm.}$$

Question 9.
The perimeter of a sector of the circle of area 25π sq.cm is 20 cm. Find the area of sector.
Solution:
Area of circle = πr2
But area is given to be 25π sq.cm.
∴ 25π = πr2
∴ r2 = 25
∴ r = 5 cm
Perimeter of sector = 2r + S
But perimeter is given to be 20 cm.
∴ 20 = 2(5) + S
∴ 20 = 10 + S
∴ S = 10 cm
Area of sector = 12 x r x S
= 12 x 5 x 10
= 25sq.cm.

Question 10.
The perimeter of a sector of the circle of area 64 7i sq.cm is 56 cm. Find the area of the sector.
Solution:
Area of circle = πr2
But area is given to be 25π sq.cm.
∴ 64π = πr2
∴ r2 = 64
∴ r = 8 cm
Perimeter of sector = 2r + S
But perimeter is given to be 20 cm.
∴ 56 = 2(5) + S
∴ 56 = 16 + S
∴ S = 40 cm
Area of sector = 12 x r x S
= 12 x 8 x 40
= 160sq.cm.

Maharashtra State Board 11th Maths Solutions Chapter 1 Angle and its Measurement Miscellaneous Exercise 1

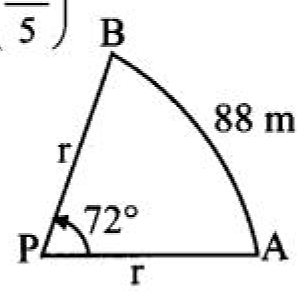
I. Select the correct option from the given alternatives.

Question 1.
$(22\pi15)cX$ is equal to
(A) 246°
(B) 264°
(C) 224°
(D) 426°
Answer:
(B) 264°

Question 2.
156° is equal to
(A) $\left(\frac{17\pi}{15}\right)^c$ (B) $\left(\frac{13\pi}{15}\right)^c$
(C) $\left(\frac{11\pi}{15}\right)^c$ (D) $\left(\frac{7\pi}{15}\right)^c$
Answer:
(B)

Question 3.
A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 metres when it traces the angle of 12° at the centre, then the length of the rope is
(A) 70 m
(B) 55 m
(C) 40 m
(D) 35 m
Answer:
(A) 70 m

$$\theta = 72° = \left(72 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{5}\right)^c$$



$$S = 88 \text{ m}$$
$$S = r\theta$$
$$\therefore \quad 88 = r\left(\frac{2\pi}{5}\right)$$
$$\therefore \quad r = 88 \times \frac{5}{2\pi}$$
$$= 88 \times \frac{5}{2\left(\frac{22}{7}\right)} = 70 \text{ m}$$

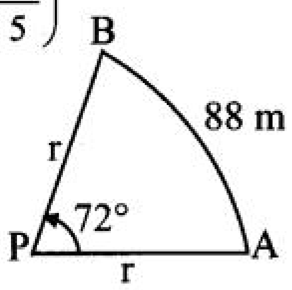
Question 4.
A pendulum 14 cm long oscillates through an angle of 12°, then the angle of the path described by its extremities is
(A) $\frac{13\pi}{14}$ (B) $\frac{14\pi}{13}$
(C) $\frac{15\pi}{14}$ (D) $\frac{14\pi}{15}$
Answer:
(D)

Question 5.
Angle between hands of a clock when it shows the time 9 :45 is
(A) (7.5)°
(B) (12.5)°
(C) (17.5)°
(D) (22.5)°
Answer:
(D) (22.5)°

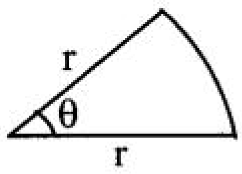
Question 6.
20 metres of wire is available for fencing off a flower-bed in the form of a circular sector of radius 5 metres, then .the maximum area (in

sq. m.) of the flower-bed is
(A) 15
(B) 20
(C) 25
(D) 30
Answer:
(C) 25
r + r + rθ = 20m
2r + rθ = 20

$$\theta = 72^\circ = \left(72 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{5}\right)^c$$



$$S = 88 \text{ m}$$
$$S = r\theta$$
$$\therefore \quad 88 = r\left(\frac{2\pi}{5}\right)$$
$$\therefore \quad r = 88 \times \frac{5}{2\pi}$$
$$= 88 \times \frac{5}{2\left(\frac{22}{7}\right)} = 70 \text{ m}$$
$$\therefore \quad \theta = \frac{20 - 2r}{r}$$



$$r = 5\text{m}$$
$$\text{Area} = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2}r^2\left(\frac{20 - 2r}{r}\right)$$
$$= \frac{1}{2}(5)^2\left(\frac{20 - 10}{5}\right)$$
$$= 25 \text{ sq. m}$$

Question 7.
If the angles of a triangle are in the ratio 1:2:3, then the smallest angle in radian is
(A) π3
(B) π6
(C) π2
(D) π9
Answer:
(B) π6

Question 8.
A semicircle is divided into two sectors whose angles are in the ratio 4:5. Find the ratio of their areas?
(A) 5:1
(B) 4:5
(C) 5:4
(D) 3:4
Answer:
(B) 4:5

Question 9.
Find the measure of the angle between hour- hand and the minute hand of a clock at twenty minutes past two.
(A) 50°
(B) 60°
(C) 54°
(D) 65°
Answer:
(A) 50°

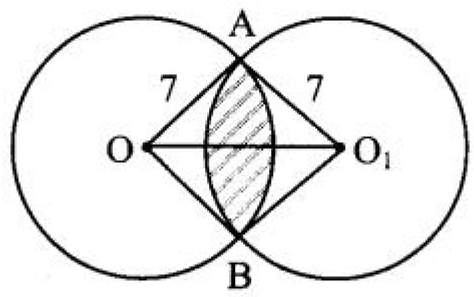
Question 10.
The central angle of a sector of circle of area 9π sq.cm is 60°, the perimeter of the sector is
(A) π
(B) 3 + π
(C) 6 + π
(D) 6
Answer:
(C) 6 + π

II. Answer the following.

Question 1.
Find the number of sides of a regular polygon, if each of its interior angles is $\frac{3\pi^c}{4}$.
Solution:
Each interior angle of a regular polygon
= $\frac{3\pi}{4} = \left(\frac{3\pi}{4} \times \frac{180}{\pi}\right)^\circ$ = 135°
Interior angle + Exterior angle = 180°
∴ Exterior angle = 180° – 135° = 45°
Let the number of sides of the regular polygon be n.
But in a regular polygon, exterior angle = $\frac{360^\circ}{\text{no.of sides}}$
∴ 45° = $\frac{360^\circ}{n}$
∴ n = $\frac{360^\circ}{45^\circ}$ = 8
∴ Number of sides of a regular polygon = 8.

Question 2.
Two circles each of radius 7 cm, intersect each other. The distance between their centres is 7√2 cm. Find the area common to both the circles.
Solution:
Let O and O_1 be the centres of two circles intersecting each other at A and B.
Then OA = OB = O_1A = O_1B = 7 cm
and OO_1 = 7√2 cm
OO_1^2 = 98 ………………(i)
Since $OA^2 + O_1A^2 = 7^2$
= 98
= OO_1^2 …..[from (i)]
m∠OAO_1 = 90°
□ OAO_1B is a square.
m∠AOB = m∠AO_1B = 90°

$$= \left(90 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{2}\right)^c$$



$$\text{Now, A(sector OAB)} = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 7^2 \times \frac{\pi}{2}$$
$$= \frac{49\pi}{4} \text{ sq.cm}$$

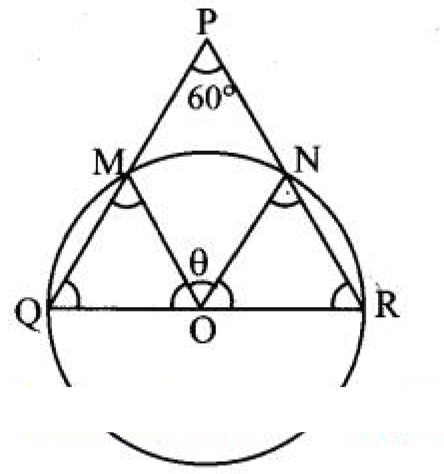
$$\text{and A(sector } O_1AB) = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 7^2 \times \frac{\pi}{2}$$
$$= \frac{49\pi}{4} \text{ sq.cm}$$

A(□ OAO_1B) = $(\text{side})^2 = (7)^2$ = 49 sq.cm

∴ Required area = area of shaded portion = A(sector OAB) + A(sector O_1AB)) – A(□ OAO_1B)

$$= \frac{49\pi}{4} + \frac{49\pi}{4} - 49$$
$$= \frac{49\pi}{2} - 49$$
$$= 49\left(\frac{\pi}{2} - 1\right) \text{ sq.cm}$$

Question 3.
ΔPQR is an equilateral triangle with side 18 cm. A circle is drawn on segment QR as diameter. Find the length of the arc of this circle within the triangle.
Solution:
Let 'O' be the centre of the circle drawn on QR as a diameter.
Let the circle intersect seg PQ and seg PR at points M and N respectively.
Since l(OQ) = l(OM),
m∠OM Q = m∠OQM = 60°
m∠MOQ = 60°
Similarly, m∠NOR = 60°
Given, QR =18 cm.
r = 9 cm



$\theta = 60° = \left(60 \times \frac{\pi}{180}\right)^c$
$= \left(\frac{\pi}{3}\right)^c$
∴ l(arc MN) = S = rθ = $9 \times \frac{\pi}{3}$ = 3π cm.

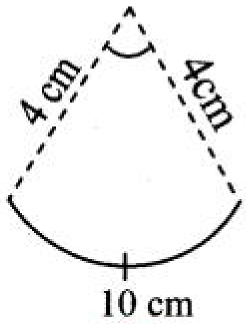
Question 4.
Find the radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm.
Solution:
Let S be the length of the arc and r be the radius of the circle.
$\theta = 60° = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$
S = 37.4 cm
Since S = rθ,

$$37.4 = r \times \frac{\pi}{3}$$
$$\therefore \quad 3 \times 37.4 = r \times \frac{22}{7} \qquad \ldots\left[\because \pi = \frac{22}{7}\right]$$
$$\therefore \quad r = \frac{3 \times 37.4 \times 7}{22}$$
$$\therefore \quad r = 35.7 \text{ cm}$$

Question 5.
A wire of length 10 cm is bent so as to form an arc of a circle of radius 4 cm. What is the angle subtended at the centre in degrees?
Solution:
S = 10 cm and r = 4 cm
Since S = rθ,

$10 = 4 \times \theta$

$$\therefore \quad \theta = \left(\frac{5}{2}\right)^c$$
$$= \left(\frac{5}{2} \times \frac{180}{\pi}\right)^\circ$$
$$= \left(\frac{450}{\pi}\right)^\circ$$



Question 6.
If two arcs of the same length in two circles subtend angles 65° and 110° at the centre. Find the ratio of their radii.
Solution:
Let r_1 and r_2 be the radii of the two circles and let their arcs of same length S subtend angles of 65° and 110° at their centres.
Angle subtended at the centre of the first circle,

$$\theta_1 = 65^\circ = \left(65 \times \frac{\pi}{180}\right)^c = \left(\frac{13\pi}{36}\right)^c$$

$$\therefore \quad S = r_1\theta_1 = r_1\left(\frac{13\pi}{36}\right) \quad \ldots(i)$$

Angle subtended at the centre of the second circle,

$$\theta_2 = 110^\circ = \left(110 \times \frac{\pi}{180}\right)^c = \left(\frac{11\pi}{18}\right)^c$$

$$\therefore \quad S = r_2\theta_2 = r_2\left(\frac{11\pi}{18}\right) \quad \ldots(ii)$$

From (i) and (ii), we get

$$r_1\left(\frac{13\pi}{36}\right) = r_2\left(\frac{11\pi}{18}\right)$$

$$\therefore \quad \frac{r_1}{r_2} = \frac{22}{13}$$

$$\therefore \quad r_1 : r_2 = 22 : 13$$

Question 7.
The area of a circle is 81TH sq.cm. Find the length of the arc subtending an angle of 300° at the centre and also the area of corresponding sector.
Solution:
Area of circle = πr^2
But area is given to be 81 n sq.cm
$\therefore \pi r^2 = 81\pi$
$\therefore r^2 = 81$
$\therefore r = 9$ cm
$\theta = 300^\circ = \left(300 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{3}\right)^c$
Since $S = r\theta$
$S = 9 \times \frac{5\pi}{3} = 15\pi$ cm
Area of sector = $\frac{1}{2} \times r \times S$
$= \frac{1}{2} \times 9 \times 15\pi = \frac{135\pi}{2}$ sq.cm

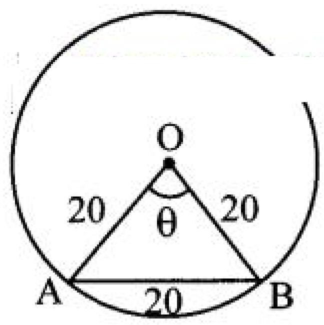
Question 8.
Show that minute-hand of a clock gains 5° 30′ on the hour-hand in one minute.
Solution:
Angle made by hour-hand in one minute
$= \frac{360^\circ}{12 \times 60} = \left(\frac{1}{2}\right)^\circ$
Angle made by minute-hand in one minute = $\frac{360^\circ}{60} = 6^\circ$
$\therefore$ Gain by minute-hand on the hour-hand in one minute
$= 6^\circ - \left(\frac{1}{2}\right)^\circ = \left(5\frac{1}{2}\right)^\circ = 5^\circ 30'$
[Note: The question has been modified.]

Question 9.
A train is running on a circular track of radius 1 km at the rate of 36 km per hour. Find the angle to the nearest minute, through which it will turn in 30 seconds.
Solution:
r = 1km = 1000m
l(Arc covered by train in 30 seconds)
= 30 x $\frac{36000}{60 \times 60}$m
∴ S = 300 m
Since S = rθ,
300 = 1000 x θ

$$\therefore \quad \theta = \left(\frac{3}{10}\right)^c = \left(\frac{3}{10} \times \frac{180}{\pi}\right)^\circ$$
$$= \left(\frac{54}{\pi}\right)^\circ$$
$$= \left(\frac{54 \times 7}{22}\right)^\circ \quad \ldots\left[\because \pi = \frac{22}{7}\right]$$

= (17.18)°
= 17° +(0.18)°
= 17° + (0.18 x 60)′ = 17° + (10.8)′
∴ θ = 17°11′(approx.)

Question 10.
In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.
Solution:



Let 'O' be the centre of the circle and AB be the chord of the circle.
Here, d = 40 cm
∴ r = $\frac{40}{2}$ = 20 cm
Since OA = OB = AB,
ΔOAB is an equilateral triangle.
The angle subtended at the centre by the minor
arc AOB is θ = 60° = $\left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$
= l(minor arc of chord AB) = rθ = 20 x $\frac{\pi}{3}$
= $\frac{20\pi}{3}$ cm

Question 11.
The angles of a quadrilateral are in A.P. and the greatest angle is double the least. Find angles of the quadrilateral in radians.
Solution:
Let the measures of the angles of the quadrilateral in degrees be a – 3d, a – d, a + d, a + 3d, where a > d > 0
∴ (a – 3d) + (a – d) + (a + d) + (a + 3d) = 360°
… [Sum of the angles of a quadrilateral is 360°]
∴ 4a = 360°
∴ a = 90°
According to the given condition, the greatest angle is double the least,
∴ a + 3d = 2.(a – 3d)
∴ 90° + 3d = 2.(90° – 3d)
∴ 90° + 3d = 180° – 6d 9d = 90°
∴ d = 10°
∴ The measures of the angles in degrees are
a – 3d = 90° – 3(10°) = 90° – 30° = 60°,
a – d = 90° – 10° = 80°,
a + d = 90°+ 10°= 100°,

$a + 3d = 90° + 3(10°) = 90° + 30° = 120°$

We know that $\theta° = \left(\theta \times \frac{\pi}{180}\right)^c$

$\therefore$ The measures of the angles in radians are

$60° = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$

$80° = \left(80 \times \frac{\pi}{180}\right)^c = \left(\frac{4\pi}{9}\right)^c$

$100° = \left(100 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{9}\right)^c$

$120° = \left(120 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{3}\right)^c$