Maharashtra State Board 11th Maths Solutions Chapter 5 Sets and Relations Ex 5.1

Question 1.
Describe the following sets in Roster form:
(i) A = {x/x is a letter of the word 'MOVEMENT'}
(ii) B = {x/x is an integer, −32 < x < 92>
(iii) C = {x/x = 2n + 1, n ∈ N}
Solution:
(i) A = {M, O, V, E, N, T}
(ii) B = {-1, 0, 1, 2, 3, 4}
(iii) C = {3, 5, 7, 9, ... }

Question 2.
Describe the following sets in Set-Builder form:
(i) {0}
(ii) {0, ±1, ±2, ±3}
(iii) {12,25,310,417,526,637,750}
(iv) {0, -1, 2, -3, 4, -5, 6,...}
Solution:
(i) Let A = {0}
0 is a whole number but it is not a natural number.
∴ A = {x / x ∈ W, x ∉ N}

(ii) Let B = {0, ±1, ±2, ±3}
B is the set of elements which belongs to Z from -3 to 3.
∴ B = {x /x ∈ Z, -3 ≤ x ≤ 3}

(iii) Let C = {12,25,310,417,526,637,750}
∴ C = {x / x = nn^2+1, n ∈ N, n ≤ 7}

(iv) Let D = {0, -1, 2, -3, 4, -5, 6, ...}
∴ D = {x/x = (-1)n-1 × (n – 1), n ∈ N}

Question 3.
If A = {x / $6x^2$ + x – 15 = 0}, B = {x / $2x^2$ – 5x – 3 = 0}, C = {x / $2x^2$ – x – 3 = 0}, then find
(i) (A ∪ B ∪ C)
(ii) (A ∩ B ∩ C)
Solution:
A = [x/$6x^2$ + x – 15 = 0)
$6x^2$ + x – 15 = 0
$6x^2$ + 10x – 9x – 15 = 0
2x(3x + 5) – 3(3x + 5) = 0
(3x + 5) (2x – 3) = 0
3x + 5 = 0 or 2x – 3 = 0
x = −53 or x = 32
A = {−53, 32}

B = {x/$2x^2$ – 5x – 3 = 0}
$2x^2$ – 5x – 3 = 0
$2x^2$ – 6x + x – 3 = 0
2x(x – 3) + 1(x – 3) = 0
(x – 3)(2x + 1) = 0
x – 3 = 0 or 2x + 1 = 0
x = 3 or x = −12
B = (−12, 3)

C = {x/$2x^2$ – x – 3 = 0}
$2x^2$ – x – 3 = 0
$2x^2$ – 3x + 2x – 3 = 0
x(2x – 3) + 1(2x – 3) = 0
(2x – 3) (x + 1) = 0
2x – 3 = 0 or x + 1 = 0
x = 32 or x = -1
C = {-1, 32}

(i) $A \cup B \cup C = \{-53, 32\} \cup \{-12, 3\} \cup \{-1, 32\} = \{-53, -1, -12, 32, 3\}$

(ii) $A \cap B \cap C = \{\ \}$

Question 4.
If A, B, C are the sets for the letters in the words ‘college’, ‘marriage’ and ‘luggage’ respectively, then verify that $[A - (B \cup C)] = [(A - B) \cap (A - C)]$.
Solution:
$A = \{c, o, l, g, e\}$
$B = \{m, a, r, i, g, e\}$
$C = \{l, u, g, a, e\}$
$B \cup C = \{m, a, r, i, g, e, l, u\}$
$A - (B \cup C) = \{c, o\}$
$A - B = \{c, o, l\}$
$A - C = \{c, o\}$
$\therefore [(A - B) \cap (A - C)] = \{c, o\} = A - (B \cup C)$
$\therefore [A - (B \cup C)] = [(A - B) \cap (A - C)]$

Question 5.
If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{4, 5, 6, 7, 8\}$ and universal set $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then verify the following:
(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
(iii) $(A \cup B)' = A' \cap B'$
(iv) $(A \cap B)' = A' \cup B'$
(v) $A = (A \cap B) \cup (A \cap B')$
(vi) $B = (A \cap B) \cup (A' \cap B)$
(vii) $(A \cup B) = (A - B) \cup (A \cap B) \cup (B - A)$
(viii) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$
(ix) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
(x) $n(B) = n(A' \cap B) + n(A \cap B)$
Solution:
$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{4, 5, 6, 7, 8\}$,
$X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
(i) $B \cap C = \{4, 5, 6\}$
$\therefore A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\}$ .....(i)
$A \cup B = \{1, 2, 3, 4, 5, 6\}$
$A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$\therefore (A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\}$ ....(ii)
From (i) and (ii), we get
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $B \cup C = \{3, 4, 5, 6, 7, 8\}$
$\therefore A \cap (B \cup C) = \{3, 4\}$ ....(i)
$A \cap B = \{3, 4\}$
$A \cap C = \{4\}$
$\therefore (A \cap B) \cup (A \cap C) = \{3, 4\}$ ....(ii)
From (i) and (ii), we get
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iii) $A \cup B = \{1, 2, 3, 4, 5, 6\}$
$\therefore (A \cup B)' = \{7, 8, 9, 10\}$ ....(i)
$A' = \{5, 6, 7, 8, 9, 10\}$,
$B' = \{1, 2, 7, 8, 9, 10\}$
$\therefore A' \cap B' = \{7, 8, 9, 10\}$ ....(ii)
From (i) and (ii), we get
$(A \cup B)' = A' \cap B'$

(iv) $A \cap B = \{3, 4\}$
$(A \cap B)' = \{1, 2, 5, 6, 7, 8, 9, 10\}$ ....(i)
$A' = \{5, 6, 7, 8, 9, 10\}$
$B' = \{1, 2, 7, 8, 9, 10\}$
$\therefore A' \cup B' = \{1, 2, 5, 6, 7, 8, 9, 10\}$ ....(ii)
From (i) and (ii), we get
$(A \cap B)' = A' \cup B'$

(v) $A = \{1, 2, 3, 4\}$ ......(i)
$A \cap B = \{3, 4\}$

B′ = {1, 2, 7, 8, 9, 10}
A ∩ B′ = {1, 2}
∴ (A ∩ B) ∪ (A ∩ B′) = {1, 2, 3, 4} …..(ii)
From (i) and (ii), we get
A = (A ∩ B) ∪ (A ∩ B′)

(vi) B = {3, 4, 5, 6} …..(i)
A ∩ B = {3, 4}
A′ = {5, 6, 7, 8, 9, 10}
A′ ∩ B = {5, 6}
∴ (A ∩ B) ∪ (A′ ∩ B) = {3, 4, 5, 6} …..(ii)
From (i) and (ii), we get
B = (A ∩ B) ∪ (A′ ∩ B)

(vii) A ∪ B = {1, 2, 3, 4, 5, 6} …….(i)
A – B = {1, 2}
A ∩ B = {3, 4}
B – A = {5, 6}
∴ (A – B) ∪ (A ∩ B) ∪ (B – A) = {1, 2, 3, 4, 5, 6} ……(ii)
From (i) and (ii), we get
A ∪ B = (A – B) ∪ (A ∩ B) ∪ (B – A)

(viii) B – C = {3}
C – B = {7, 8}
B Δ C = (B – C) ∪ (C – B) = {3, 7, 8}
∴ A ∩ (B Δ C) = {3} ……(i)
A ∩ B = {3, 4}
A ∩ C = {4}
∴ (A ∩ B) Δ (A ∩ C) = [(A ∩ B) – (A ∩ C)] ∪ [(A ∩ C) – (A ∩ B)] = {3} …..(ii)
From (i) and (ii), we get
A ∩ (B Δ C) = (A ∩ B) Δ (A ∩ C)

(ix) A = {1, 2, 3, 4}, B = {3, 4, 5, 6}
A ∩ B = {3, 4}, A ∪ B = {1, 2, 3, 4, 5, 6}
∴ n(A) = 4, n(B) = 4,
n(A ∩ B) = 2, n(A ∪ B) = 6 ……(i)
∴ n(A) + n(B) – n(A ∩ B) = 4 + 4 – 2
∴ n(A) + n(B) – n(A ∩ B) = 6 …..(ii)
From (i) and (ii), we get
n(A ∪ B) = n(A) + n(B) – n(A ∩ B)

(x) B = {3, 4, 5, 6}
∴ n(B) = 4 …..(i)
A′ = {5, 6, 7, 8, 9, 10}
A′ ∩ B = {5, 6}
∴ n(A′ ∩ B) = 2
A ∩ B = {3, 4}
∴ n(A ∩ B) = 2
∴ n(A′ ∩ B) + n(A ∩ B) = 2 + 2 = 4 …..(ii)
From (i) and (ii), we get
n(B) = n(A′ ∩ B) + n (A ∩ B)

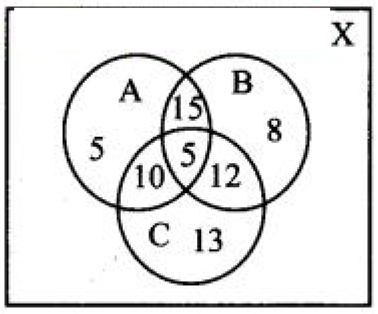
Question 6.
If A and B are subsets of the universal set X and n(X) = 50, n(A) = 35, n(B) = 20, n(A′ ∩ B′) = 5, find
(i) n(A ∪ B)
(ii) n(A ∩ B)
(iii) n(A′ ∩ B)
(iv) n(A ∩ B′)
Solution:
n(X) = 50, n(A) = 35, n(B) = 20, n(A′ ∩ B′) = 5
(i) n(A ∪ B) = n(X) – [n(A ∪ B)′]
= n(X) – n(A′ ∩ B′)
= 50 – 5
= 45

(ii) n(A ∩ B) = n(A) + n(B) – n(A ∪ B)
= 35 + 20 – 45
= 10

(iii) $n(A' \cap B) = n(B) - n(A \cap B)$
$= 20 - 10$
$= 10$

(iv) $n(A \cap B') = n(A) - n(A \cap B)$
$= 35 - 10$
$= 25$

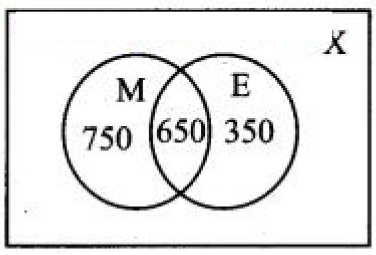
Question 7.
In a class of 200 students who appeared in certain examinations, 35 students faded in CET, 40 in NEET and 40 in JEE, 20 faded in CET and NEET, 17 in NEET and JEE, 15 in CET and JEE and 5 faded in ad three examinations. Find how many students
(i) did not fail in any examination.
(ii) faded in NEET or JEE entrance.
Solution:
Let A = set of students who failed in CET
B = set of students who failed in NEET
C = set of students who failed in JEE
X = set of all students
$\therefore n(X) = 200, n(A) = 35, n(B) = 40, n(C) = 40, n(A \cap B) = 20, n(B \cap C) = 17, n(A \cap C) = 15, n(A \cap B \cap C) = 5$



(i) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
$= 35 + 40 + 40 - 20 - 17 - 15 + 5$
$= 68$
$\therefore$ No. of students who did not fail in any exam = $n(X) - n(A \cup B \cup C)$
$= 200 - 68$
$= 132$

(ii) No. of students who failed in NEET or JEE entrance = $n(B \cup C)$
$= n(B) + n(C) - n(B \cap C)$
$= 40 + 40 - 17$
$= 63$

Question 8.
From amongst 2000 Uterate individuals of a town, 70% read Marathi newspapers, 50% read English newspapers and 32.5% read both Marathi and English newspapers. Find the number of individuals who read
(i) at least one of the newspapers.
(ii) neither Marathi nor English newspaper.
(iii) only one of the newspapers.
Solution:
Let M = set of individuals who read Marathi newspapers
E = set of individuals who read English newspapers
X = set of all literate individuals
$\therefore n(X) = 2000$,
$n(M) = 70100 \times 2000 = 1400$
$n(E) = 50100 \times 2000 = 1000$
$n(M \cap E) = 32.5100 \times 2000 = 650$
(i) $n(M \cup E) = n(M) + n(E) - n(M \cap E)$
$= 1400 + 1000 - 650$
$= 1750$



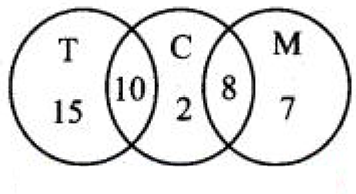
No. of individuals who read at least one of the newspapers = $n(M \cup E) = 1750$.

(ii) No. of individuals who read neither Marathi nor English newspaper = $n(M' \cap E')$
$= n(M \cup E)'$

= n(X) – n(M ∪ E)
= 2000 – 1750
= 250

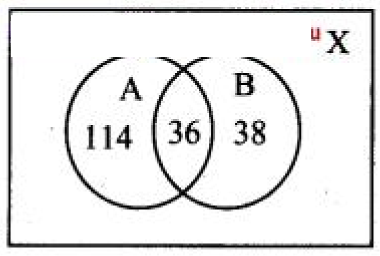
(iii) No. of individuals who read only one of the newspapers = n(M ∩ E′) + n(M′ ∩ E)
= n(M ∪ E) – n(M ∩ E)
= 1750 – 650
= 1100

Question 9.
In a hostel, 25 students take tea, 20 students take coffee, 15 students take milk, 10 students take both tea and coffee, 8 students take both milk and coffee. None of them take tea and milk both and everyone takes atleast one beverage, find the total number of students in the hostel.
Solution:
Let T = set of students who take tea
C = set of students who take coffee
M = set of students who take milk
∴ n(T) = 25, n(C) = 20, n(M) = 15, n(T ∩ C) = 10, n(M ∩ C) = 8, n(T ∩ M) = 0, n(T ∩ M ∩ C) = 0



∴ Total number of students in the hostel = n(T ∪ C ∪ M)
= n(T) + n(C) + n(M) – n(T ∩ C) – n(M ∩ C) – n(T ∩ M) + n(T ∩ M ∩ C)
= 25 + 20 + 15 – 10 – 8 – 0 + 0
= 42

Question 10.
There are 260 persons with skin disorders. If 150 had been exposed to the chemical A, 74 to the chemical B, and 36 to both chemicals A and B, find the number of persons exposed to
(i) Chemical A but not Chemical B
(ii) Chemical B but not Chemical A
(iii) Chemical A or Chemical B
Solution:
Let A = set of persons exposed to chemical A
B = set of persons exposed to chemical B
X = set of all persons
∴ n(X) = 260, n(A) = 150, n(B) = 74, n(A ∩ B) = 36



(i) No. of persons exposed to chemical A but not to chemical B = n(A ∩ B′)
= n(A) – n(A ∩ B)
= 150 – 36
= 114

(ii) No. of persons exposed to chemical B but not to chemical A = n(A′ ∩ B)
= n(B) – n(A ∩ B)
= 74 – 36
= 38

(iii) No. of persons exposed to chemical A or chemical B = n(A ∪ B)
= n(A) + n(B) – n(A ∩ B)
= 150 + 74 – 36
= 188

Question 11.
Write down the power set of A = {1, 2, 3}.
Solution:
A = {1, 2, 3}

The power set of A is given by
P(A) = {{Φ}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}}

Question 12.
Write the following intervals in Set-Builder form:
(i) (-3, 0)
(ii) [6, 12]
(iii) (6, ∞)
(iv) (-∞, 5]
(v) (2, 5]
(vi) [-3, 4)
Solution:
(i) (-3, 0) = $\{x / x \in R, -3 < x < 0\}$

(ii) [6, 12] = $\{x / x \in R, 6 \leq x \leq 12\}$

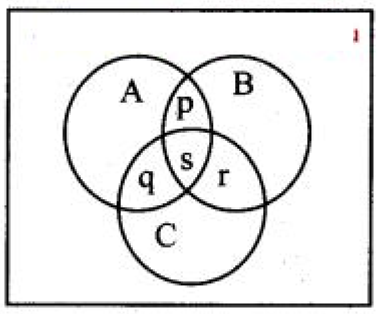
(iii) (6, ∞) = $\{x / x \in R, x > 6\}$

(iv) (-∞, 5] = $\{x / x \in R, x \leq 5\}$

(v) (2, 5] = $\{x / x \in R, 2 < x \leq 5\}$

(vi) [-3, 4) = $\{x / x \in R, -3 \leq x < 4\}$

Question 13.
A college awarded 38 medals in volleyball, 15 in football, and 20 in basketball. The medals were awarded to a total of 58 players and only 3 players got medals in all three sports. How many received medals in exactly two of the three sports?
Solution:
Let A = Set of students who received medals in volleyball
B = Set of students who received medals in football
C = Set of students who received medals in basketball
$n(A) = 38, n(B) = 15, n(C) = 20, n(A \cup B \cup C) = 58, n(A \cap B \cap C) = 3$
$n(A \cup B \cup C) = n(A) + n(B) + n(C) – n(A \cap B) – n(B \cap C) – n(A \cap C) + n(A \cap B \cap C)$
$58 = 38 + 15 + 20 – n(A \cap B) – n(B \cap C) – n(A \cap C) + 3$
$\therefore n(A \cap B) + n(B \cap C) + n(A \cap C) = 18$ ……(i)
Number of players who got exactly two medals = p + q + r
Here, $s = n(A \cap B \cap C) = 3$



$n(A \cap B) + n(B \cap C) + n(A \cap C) = 18$ …..[From (i)]
$\therefore p + s + s + r + q + s = 18$
$\therefore p + q + r + 3s = 18$
$\therefore p + q + r + 3(3) = 18$
$\therefore p + q + r = 18 – 9 = 9$
∴ Number of players who received exactly two medals = 9.

Question 14.
Solve the following inequalities and write the solution set using interval notation.
(i) $-9 < 2x + 7 \leq 19$
(ii) $x^2 – x > 20$
(iii) $\frac{2x}{x-4} \leq 5$
(iv) $6x^2 + 1 \leq 5x$
Solution:
(i) $-9 < 2x + 7 \leq 19$
$\therefore -16 < 2x \leq 12$
$\therefore -8 < x \leq 6$
$\therefore x \in (-8, 6]$

(ii) $x^2 – x > 20$
$\therefore x^2 – x – 20 > 0$
$\therefore x^2 – 5x + 4x – 20 > 0$

∴ (x – 5) (x + 4) > 0
∴ either x – 5 > 0 and x + 4 > 0 or x – 5 < 0 and x + 4 < 0

Case I: x – 5 > 0 and x + 4 > 0
∴ x > 5 and x > -4
∴ x > 5 ….(i)

Case II:
x – 5 < 0 and x + 4 < 0
∴ x < 5 and x < -4
∴ x < -4 …..(ii)
From (i) and (ii), we get
x ∈ (-∞, – 4) ∪ (5, ∞)

(iii) 2xx−4 ≤ 5
∴ 2xx−4 – 5 ≤ 0
∴ 2x−5x+20x−4 ≤ 0
∴ 20−3xx−4 ≤ 0
When ab ≤ 0,
a ≥ 0 and b < 0 or a ≤ 0 and b > 0
∴ either 20 – 3x ≥ 0 and x – 4 < 0 or 20 – 3x ≤ 0 and x – 4 > 0
Case I:
20 – 3x ≥ 0 and x – 4 < 0
∴ x ≤ 203 and x < 4
∴ x < 4 ……(I)

Case II: 20 – 3x ≤ 0 and x – 4 > 0
∴ x ≥ 203 and x > 4
∴ x ≥ 203 ……(ii)
From (i) and (ii), we get
x ∈ (-∞, 4) ∪ [203, ∞)

(iv) 6x2 + 1 ≤ 5x
6x2 – 5x + 1 ≤ 0
6x2 – 3x – 2x + 1 ≤ 0
(3x – 1) (2x – 1) ≤ 0
either 3x – 1 ≤ 0 and 2x – 1 ≥ 0 or 3x – 1 ≥ 0 and 2x – 1 ≤ 0
Case I:
3x – 1 ≤ 0 and 2x – 1 ≥ 0
∴ x ≤ 13 and x ≥ 12, which is not possible.

Case II:
3x – 1 ≥ 0 and 2x – 1 ≤ 0
∴ x ≥ 13 and x ≤ 12
∴ x ∈ [13, 12]

Question 15.
If A = (-7, 3], B = [2, 6] and C = [4, 9], then find
(i) A ∪ B
(ii) B ∪ C
(iii) A ∪ C
(iv) A ∩ B
(v) B ∩ C
(vi) A ∩ C
(vii) A′ ∩ B
(viii) B′ ∩ C′
(ix) B – C
(x) A – B
Solution:
A = (-7, 3], B = [2, 6], C = [4, 9]
(i) A ∪ B = (-7, 6]

(ii) B ∪ C = [2, 9]

(iii) A ∪ C = (-7, 3] ∪ [4, 9]

(iv) A ∩ B = [2, 3]

(v) B ∩ C = [4, 6]

(vi) A ∩ C = { }

(vii) A′ = (-∞, – 7] ∪ (3, ∞)
∴ A′ ∩ B = (3, 6]

(viii) B′ = (-∞, 2) ∪ (6, ∞)
C′ = (-∞, 4) ∪ (9, ∞)
∴ B′ ∩ C′ = (-∞, 2) ∪ (9, ∞)

(ix) B – C = [2, 4)

(x) A – B = (-7, 2)

Maharashtra State Board 11th Maths Solutions Chapter 5 Sets and Relations Ex 5.2

Question 1.
If (x – 1, y + 4) = (1, 2), find the values of x and y.
Solution:
(x – 1, y + 4) = (1, 2)
By the definition of equality of ordered pairs, we have
x – 1 = 1 and y + 4 = 2
∴ x = 2 and y = -2

Question 2.
If $(x+13, y3-1)=(12, 32)$, find x and y.
Solution:
$(x+13, y3-1)=(12, 32)$
By the definition of equality of ordered pairs, we have
x + 13 = 12 and y3 – 1 = 32
∴ x = 12 – 13 and y3 = 32 + 1
∴ x = 16 and y = 152

Question 3.
If A = {a, b, c}, B = {x, y}, find A × B, B × A, A × A, B × B.
Solution:
A = (a, b, c}, B = {x, y}
A × B = {(a, x), (a, y), (b, x), (b, y), (c, x), (c, y)}
B × A = {(x, a), (x, b), (x, c), (y, a), (y, b), (y, c)}
A × A = {(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)}
B × B = {(x, x), (x, y), (y, x), (y, y)}

Question 4.
If P = {1, 2, 3} and Q = {1, 4}, find sets P × Q and Q × P.
Solution:
P = {1, 2, 3}, Q = {1, 4}
∴ P × Q = {(1, 1), (1, 4), (2, 1), (2, 4), (3, 1), (3, 4)}
and Q × P = {(1, 1), (1, 2), (1, 3), (4, 1), (4, 2), (4, 3)}

Question 5.
Let A = {1, 2, 3, 4}, B = {4, 5, 6}, C = {5, 6}. Verify,
(i) A × (B ∩ C) = (A × B) ∩ (A × C)
(ii) A × (B ∪ C) = (A × B) ∪ (A × C)
Solution:
A = {1, 2, 3, 4}, B = {4, 5, 6}, C = {5, 6}
(i) B ∩ C = {5, 6}
A × (B ∩ C) = = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)}
A × B = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}
A × C = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)}
∴ (A × B) ∩ (A × C) = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)}
∴ A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) B ∪ C = {4, 5, 6}
A × (B ∪ C) = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3,4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}
A × B = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}
A × C = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)}
∴ (A × B) ∪ (A × C) = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}
∴ A × (B ∪ C) = (A × B) ∪ (A × C)

Question 6.
Express {(x, y) / $x^2 + y^2 = 100$, where x, y ∈ W} as a set of ordered pairs.
Solution:
{(x, y) / $x^2 + y^2 = 100$, where x, y ∈ W}
We have, $x^2 + y^2 = 100$
When x = 0 and y = 10,
$x^2 + y^2 = 0^2 + 10^2 = 100$
When x = 6 and y = 8,
$x^2 + y^2 = 6^2 + 8^2 = 100$
When x = 8 and y = 6,
$x^2 + y^2 = 8^2 + 6^2 = 100$
When x = 10 and y = 0,
$x^2 + y^2 = 10^2 + 0^2 = 100$
∴ Set of ordered pairs = {(0, 10), (6, 8), (8, 6), (10, 0)}

Question 7.
Let A = {6, 8} and B = {1, 3, 5}. Show that R_1 = {(a, b) / a ∈ A, b ∈ B, a – b is an even number} is a null relation, R_2 = {(a, b) / a ∈ A, b ∈ B, a + b is an odd number} is a universal relation.
Solution:
A = {6, 8}, B = {1, 3, 5}
R_1 = {(a, b)/ a ∈ A, b ∈ B, a – b is an even number}
a ∈ A
∴ a = 6, 8
b ∈ B
∴ b = 1, 3, 5
When a = 6 and b = 1, a – b = 5, which is odd
When a = 6 and b = 3, a – b = 3, which is odd
When a = 6 and b = 5, a – b = 1, which is odd
When a = 8 and b = 1, a – b = 7, which is odd
When a = 8 and b = 3, a – b = 5, which is odd
When a = 8 and b = 5, a – b = 3, which is odd
Thus, no set of values of a and b gives a – b as even.
∴ R_1 has a null relation from A to B.
A × B = {(6, 1), (6, 3), (6, 5), (8, 1), (8, 3), (8, 5)}
When a = 6 and b = 1, a + b = 7, which is odd
When a = 6 and b = 3, a + b = 9, which is odd
When a = 6 and b = 5, a + b = 11, which is odd
When a = 8 and b = 1, a + b = 9, which is odd
When a = 8 and b = 3, a + b = 11, which is odd
When a = 8 and b = 5, a + b = 13, which is odd
∴ R_2 = {(6, 1), (6, 3), (6, 5), (8, 1), (8, 3), (8, 5)}
Here, R_2 = A × B
∴ R_2 has a universal relation from A to B.

Question 8.
Write the relation in the Roster form. State its domain and range.

(i) $R_1 = \{(a, a^2) / a$ is a prime number less than $15\}$
(ii) $R_2 = \{(a, \frac{1}{a}) / 0 < a \le 5, a \in N\}$
(iii) $R_3 = \{(x, y / y = 3x, y \in \{3, 6, 9, 12\}, x \in \{1, 2, 3\}\}$
(iv) $R_4 = \{(x, y) / y > x + 1, x = 1, 2$ and $y = 2, 4, 6\}$
(v) $R_5 = \{(x, y) / x + y = 3, x, y \in \{0, 1, 2, 3\}\}$
(vi) $R_6 = \{(a, b) / a \in N, a < 6$ and $b = 4\}$
(vii) $R_7 = \{(a, b) / a, b \in N, a + b = 6\}$
(viii) $R_8 = \{(a, b)/ b = a + 2, a \in Z, 0 < a < 5\}$
Solution:
(i) $R_1 = \{(a, a^2) / a$ is a prime number less than $15\}$
$\therefore a = 2, 3, 5, 7, 11, 13$
$\therefore a^2 = 4, 9, 25, 49, 121, 169$
$\therefore R_1 = \{(2, 4), (3, 9), (5, 25), (7, 49), (11, 121), (13, 169)\}$
$\therefore$ Domain $(R_1) = \{a/a$ is a prime number less than $15\}$
$= \{2, 3, 5, 7, 11, 13\}$
Range $(R_1) = \{a^2/a$ is a prime number less than $15\}$
$= \{4, 9, 25, 49, 121, 169\}$

ii. $R_2 = \left\{\left(a, \frac{1}{a}\right) / 0 < a \le 5, a \in N\right\}$

$\therefore$ $a = 1, 2, 3, 4, 5$

$\therefore$ $\frac{1}{a} = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

$\therefore$ $R_2 = \left\{(1, 1), \left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right), \left(4, \frac{1}{4}\right), \left(5, \frac{1}{5}\right)\right\}$

$\therefore$ Domain $(R_2) = \{a / 0 < a \le 5, a \in N\}$
$= \{1, 2, 3, 4, 5\}$

Range $(R_2) = \left\{\frac{1}{a} / 0 < a \le 5, a \in N\right\}$
$= \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

(iii) $R_3 = \{(x, y) / y = 3x, x \in \{1, 2, 3\}, y \in \{3, 6, 9, 12\}\}$
Here $y = 3x$
When $x = 1, y = 3(1) = 3$
When $x = 2, y = 3(2) = 6$
When $x = 3, y = 3(3) = 9$
$\therefore R_3 = \{(1, 3), (2, 6), (3, 9)\}$
$\therefore$ Domain $(R_3) = \{1, 2, 3\}$
$\therefore$ Range $(R_3) = \{3, 6, 9\}$

(iv) $R_4 = \{(x, y) / y > x + 1, x = 1, 2$ and $y = 2, 4, 6\}$
Here, $y > x + 1$
When $x = 1$ and $y = 2, 2 \ngtr 1 + 1$
When $x = 1$ and $y = 4, 4 > 1 + 1$
When $x = 1$ and $y = 6, 6 > 1 + 1$
When $x = 2$ and $y = 2, 2 \ngtr 2 + 1$
When $x = 2$ and $y = 4, 4 > 2 + 1$
When $x = 2$ and $y = 6, 6 > 2 + 1$
$\therefore R_4 = \{(1, 4), (1, 6), (2, 4), (2, 6)\}$
Domain $(R_4) = \{1, 2\}$
Range $(R_4) = \{4, 6\}$

(v) $R_5 = \{(x, y) / x + y = 3, x, y \in (0, 1, 2, 3)\}$
Here, $x + y = 3$
When $x = 0, y = 3$
When $x = 1, y = 2$
When $x = 2, y = 1$
When $x = 3, y = 0$
$\therefore R_5 = \{(0, 3), (1, 2), (2, 1), (3, 0)\}$
Domain $(R_5) = \{0, 1, 2, 3\}$
Range $(R_5) = \{3, 2, 1, 0\}$

(vi) $R_6 = \{(a, b)/ a \in N, a < 6$ and $b = 4\}$
$a \in N$ and $a < 6$
$\therefore a = 1, 2, 3, 4, 5$ and $b = 4$

R_6 = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4)}
Domain (R_6) = {1, 2, 3, 4, 5}
Range (R_6) = {4}

(vii) R_7 = {(a, b) / a, b ∈ N, a + b = 6}
Here, a + b = 6
When a = 1, b = 5
When a = 2, b = 4
When a = 3, b = 3
When a = 4, b = 2
When a = 5, b = 1
∴ R_7 = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)}
Domain (R_7) = {1, 2, 3, 4, 5}
Range (R_7) = {5, 4, 3, 2, 1}

(viii) R_8 = {(a, b) / b = a + 2, a ∈ Z, 0 < a < 5}
Here, b = a + 2
When a = 1, b = 3
When a = 2, b = 4
When a = 3, b = 5
When a = 4, b = 6
∴ R_8 = {(1, 3), (2, 4), (3, 5), (4, 6)}
Domain (R_8) = {1, 2, 3, 4}
Range (R_8) = {3, 4, 5, 6}

Question 9.
Identify which of the following relations are reflexive, symmetric, and transitive.

Relation	Reflexive	Symmetrice	Transitive
$R = \{(a,b) : a,b \in Z,\ a-b$ is an integer$\}$			
$R = \{(a,b) : a,b \in N,\ a+b$ is even$\}$	✓	✓	×
$R = \{(a,b) : a,b \in N,\ a$ divides $b\}$			
$R = \{(a,b) : a,b \in N,\ a^2 - 4ab + 3b^2 = 0\}$			
$R = \{(a,b) : a$ is sister of b and $a,b \in G =$ Set of girls$\}$			
$R = \{(a,b) :$ Line a is perpendicular to line b in a place$\}$			
$R = \{(a,b) : a,b \in R,\ a < b\}$			
$R = \{(a,b) : a,b \in R,\ a \le b^3\}$			

Solution:

	Relation	Reflexive	Symmetric	Transitive
i.	R = {(a, b) : a, b ∈ Z, a – b is an integer}	✓	✓	✓
ii.	R = {(a, b) : a, b ∈ N, a + b is even}	✓	✓	✓
iii.	R = {(a, b) : a, b ∈ N, a divides b}	✓	×	✓
iv.	R = {(a, b) : a, b ∈ N, $a^2 - 4ab + 3b^2 = 0$}	✓	×	×
v.	R = {(a, b) : a is sister of b and a, b ∈ G = Set of girls}	×	✓	✓
vi.	R = {(a, b) : Line a is perpendicular to line b in a plane}	×	✓	×
vii.	R = {(a, b) : a, b ∈ R, a < b}	×	×	✓
viii.	R = {(a, b) : a, b ∈ R, a ≤ b^3}	×	×	×

(i) Given, R = {(a, b): a, b ∈ Z, a – b is an integer}
Let a ∈ Z, then a – a ∈ Z
∴ (a, a) ∈ R
∴ R is reflexive.
Let (a, b) ∈ R
∴ a – b ∈ Z
∴ -(a – b) ∈ Z, i.e., b – a ∈ Z
∴ (b, a) ∈ R
∴ R is symmetric.
Let (a, b) and (b, c) ∈ R
∴ a – b ∈ Z and b – c ∈ Z
∴ (a – b) + (b – c) ∈ Z
∴ a – c ∈ Z
∴ (a, c) ∈ R
∴ R is transitive.

(ii) Given, R = {(a, b) : a, b ∈ N, a + b is even}
Let a ∈ N, then a + a = 2a, which is even.
∴ (a, a) ∈ R
∴ R is reflexive.
Let (a, b) ∈ R

∴ a + b is even
∴ b + a is even
∴ (b, a) ∈ R
∴ R is symmetric.
Let (a, b) and (b, c) ∈ R
∴ a + b and b + c is even
Let a + b = 2x and b + c = 2y for x, y ∈ N
∴ (a + b) + (b + c) = 2x + 2y
∴ a + 2b + c = 2(x + y)
∴ a + c = 2(x + y) – 2b = 2(x + y – b)
∴ a + c is even ……..[∵ x, y, b ∈ N, x + y – b ∈ N]
∴ (a, c) ∈ R
∴ R is transitive.

(iii) Given, R = {(a, b) : a, b ∈ N, a divides b}
Let a ∈ N, then a divides a.
∴ (a, a) ∈ R
∴ R is reflexive.
Let a = 2 and b = 8, then 2 divides 8
∴ (a, b) ∈ R
But 8 does not divide 2.
∴ (b, a) ∉ R
∴ R is not symmetric.
Let (a, b) and (b, c) ∈ R
∴ a divides b and b divides c.
Let b = ax and c = by for x, y ∈ N.
∴ c = (ax) y = a(xy)
i.e., a divides c.
∴ (a, c) ∈ R
∴ R is transitive.

(iv) Given, R = {(a, b) : a, b ∈ N, $a^2 - 4ab + 3b^2 = 0$}
Let a ∈ N, then $a^2 - 4aa + 3a^2 = a^2 - 4a^2 + 3a^2 = 0$
∴ (a, a) ∈ R
∴ R is reflexive.
Let a = 3 and b = 1,
then $a^2 - 4ab + 3b^2 = 9 - 12 + 3 = 0$
∴ (a, b) ∈ R
Consider, $b^2 - 4ba + 3a^2 = 1 - 12 + 9 = -2 \neq 0$
∴ (b, a) ∉ R
∴ R is not symmetric.
Let a = 3, b = 1 and c = 13,
then $a^2 - 4ab + 3b^2 = 9 - 12 + 3 = 0$ and
$b^2 - 4bc + 3c^2 = 1 - 43 + 13 = 1 - 1 = 0$
∴ we get (a, b) and (b, c) ∈ R.
Consider, $a^2 - 4ac + 3c^2 = 9 - 4 + 13 = 163 \neq 0$
∴ (a, c) ∉ R
∴ R is not transitive.

(v) Given, R = {(a, b) : a is sister of b and a, b ∈ G = Set of girls}
Let a ∈ G, then 'a' cannot be a sister of herself.
∴ (a, a) ∉ R
∴ R is not reflexive.
Let (a, b) ∈ R
∴ 'a' is a sister of 'b'.
∴ 'b' is a sister of 'a'.
∴ (b, c) ∈ R
∴ R is symmetric.
Let (a, b) and (b, c) ∈ R
∴ 'a' is a sister of 'b' and 'b' is a sister of 'c'
∴ 'a' is a sister of 'c'.
∴ (a, c) ∈ R
∴ R is transitive.

(vi) Given, R = {(a, b) : Line a is perpendicular to line b in a plane}
Let a be any line in the plane, then a cannot be perpendicular to itself.
∴ (a, a) ∉ R
∴ R is not reflexive.

Let $(a, b) \in R$
$\therefore$ a is perpendicular to b.
$\therefore$ b is perpendicular to a.
$\therefore (b, a) \in R$.
$\therefore$ R is symmetric.
Let (a, b) and $(b, c) \in R$.
$\therefore$ a is perpendicular to b and b is perpendicular to c.
$\therefore$ a is parallel to c.
$\therefore (a, c) \notin R$
$\therefore$ R is not transitive.

(vii) Given, $R = \{(a, b) : a, b \in R, a < b\}$
Let $a \in R$, then $a \nless a$.
$\therefore (a, a) \notin R$
$\therefore$ R is not reflexive.
Let a = 1 and b = 2, then 1 < 2
$\therefore (a, b) \in R$
But $2 \nless 1$
$\therefore (b, a) \notin R$
$\therefore$ R is not symmetric.
Let (a, b) and $(b, c) \in R$
$\therefore a < b$ and $b < c$
$\therefore a < c$
$\therefore (a, c) \in R$
$\therefore$ R is transitive.

(viii) Given, $R = \{(a, b) : a, b \in R, a \leq b^3\}$
Let a = -3, then $a^3 = -27$.
Here, $a \nless a$
$\therefore (a, a) \notin R$
$\therefore$ R is not reflexive.
Let a = 2 and b = 9, then $b^3 = 729$
Here, $a < b^3$
$\therefore (a, b) \in R$
Consider, a3 = 8
Here, $b \nless a^3$
$\therefore (b, a) \notin R$
$\therefore$ R is not symmetric.
Let a = 10, b = 3, c = 2,
then $b^3 = 27$ and $c^3 = 8$
Here, $a < b^3$ and $b < c^3$.
$\therefore$ (a, b) and $(b, c) \in R$
But $a \nless c^3$
$\therefore (a, c) \notin R$.
$\therefore$ R is not transitive.

Maharashtra State Board 11th Maths Solutions Chapter 5 Sets and Relations Miscellaneous Exercise 5

(I) Select the correct answer from the given alternative.

Question 1.
For the set A = {a, b, c, d, e} the correct statement is

(A) {a, b} ∈ A
(B) {a} ∈ A
(C) a ∈ A
(D) a ∉ A
Answer:
(C) a ∈ A

Question 2.
If aN = {ax : x ∈ N}, then set 6N ∩ 8N =
(A) 8N
(B) 48N
(C) 12N
(D) 24N
Answer:
(D) 24N
Hint:
6N = {6x : x ∈ N} = {6, 12, 18, 24, 30, ……}
8N = {8x : x ∈ N} = {8, 16, 24, 32, ……}
∴ 6N ∩ 8N = {24, 48, 72, …..}
= {24x : x ∈ N}
= 24N

Question 3.
If set A is empty set then n[P[P[P(A)]]] is
(A) 6
(B) 16
(C) 2
(D) 4
Answer:
(D) 4
Hint:
A = Φ
∴ n(A) = 0
∴ n[P(A)] = $2^{n(A)} = 2^0 = 1$
∴ n[P[P(A)]] = $2^{n[P(A)]} = 2^1 = 2$
∴ n[P[P[P(A)]]] = $2^{n[P[P(A)]]} = 2^2 = 4$

Question 4.
In a city 20% of the population travels by car, 50% travels by bus and 10% travels by both car and bus. Then, persons travelling by car or bus are
(A) 80%
(B) 40%
(C) 60%
(D) 70%
Answer:
(C) 60%
Hint:
Let C = Population travels by car
B = Population travels by bus
n(C) = 20%, n(B) = 50%, n(C ∩ B) = 10%
n(C ∪ B) = n(C) + n(B) – n(C ∩ B)
= 20% + 50% – 10%
= 60%

Question 5.
If the two sets A and B are having 43 elements in common, then the number of elements common to each of the sets A × B and B × A is
(A) 43^2
(B) 2^{43}
(C) 43^{43}
(D) 2^{86}
Answer:
(A) 43^2

Question 6.
Let R be a relation on the set N be defied by {(x, y) / x, y ∈ N, 2x + y = 41} Then R is
(A) Reflexive
(B) Symmetric
(C) Transitive

(D) None of these
Answer:
(D) None of these

Question 7.
The relation ">" in the set of N (Natural number) is
(A) Symmetric
(B) Reflexive
(C) Transitive
(D) Equivalent relation
Answer:
(C) Transitive
Hint:
For any a ∈ N, a ≯ a
∴ (a, a) ∉ R
∴ > is not reflexive.
For any a, b ∈ N, if a > b, then b ≯ a.
∴ > is not symmetric.
For any a, b, c ∈ N,
if a > b and b > c, then a > c
∴ > is transitive.

Question 8.
A relation between A and B is
(A) only A × B
(B) An Universal set of A × B
(C) An equivalent set of A × B
(D) A subset of A × B
Answer:
(D) A subset of A × B

Question 9.
If (x, y) ∈ N × N, then xy = x^2 is a relation that is
(A) Symmetric
(B) Reflexive
(C) Transitive
(D) Equivalence
Answer:
(D) Equivalence
Hint:
Let x ∈ R, then xx = x^2
∴ x is related to x.
∴ Given relation is reflexive.
Letx = 0 and y = 2,
then xy = 0 × 2 = 0 = x^2
∴ x is related to y.
Consider, yx = 2 × 0 = 0 ≠ y^2
∴ y is not related to x.
∴ Given relation is not symmetric.
Let x be related to y and y be related to z.
∴ xy = x^2 and yz = y^2
∴ x = $\frac{x^2}{y}$ and z = $\frac{y^2}{y}$ = y .....[if y ≠ 0]
Consider, xz = $\frac{x^2}{y}$ × y = x^2
∴ x is related to z.
∴ Given relation is transitive.

Question 10.
If A = {a, b, c}, The total no. of distinct relations in A × A is
(A) 3
(B) 9
(C) 8
(D) 29
Answer:
(D) 29

(II) Answer the following.

Question 1.
Write down the following sets in set builder form:
(i) {10, 20, 30, 40, 50}
(ii) {a, e, i, o, u}
(iii) {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}
Solution:
(i) Let A = {10, 20, 30, 40, 50}
$\therefore$ A = {x/x = 10n, n $\in$ N and n $\le$ 5}

(ii) Let B = {a, e, i, o, u}
$\therefore$ B = {x/x is a vowel of English alphabets}

(iii) Let C = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}
$\therefore$ C = {x/x is a day of a week}

Question 2.
If U = {x/x $\in$ N, 1 $\le$ x $\le$ 12}, A = {1,4, 7,10}, B = {2, 4, 6, 7, 11}, C = {3, 5, 8, 9, 12}. Write down the sets.
(i) A $\cup$ B
(ii) B $\cap$ C
(iii) A – B
(iv) B $\cap$ C′
(v) A $\cup$ B $\cup$ C
(vi) A $\cap$ (B $\cup$ C)
Solution:
U = {x/x $\in$ N, 1 $\le$ x $\le$ 12} = {1, 2, 3, …., 12}
A = {1, 4, 7, 10}, B = {2, 4, 6, 7, 11}, C = {3, 5, 8, 9, 12}
(i) A $\cup$ B = {1, 2, 4, 6, 7, 10, 11}

(ii) B $\cap$ C = {}

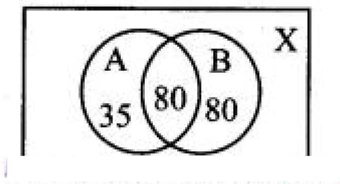
(iii) A – B = {1, 10}

(iv) C′ = {1, 2, 4, 6, 7, 10, 11}
$\therefore$ B $\cap$ C′ = {2, 4, 6, 7, 11}

(v) A $\cup$ B $\cup$ C = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

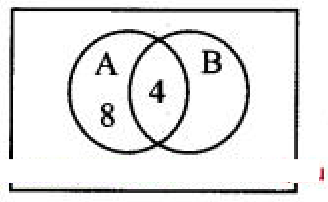
(vi) B $\cup$ C = {2, 3, 4, 5, 6, 7, 8, 9, 11, 12}
$\therefore$ A $\cap$ (B $\cup$ C) = {4, 7}

Question 3.
In a survey of 425 students in a school, it was found that 115 drink apple juice, 160 drink orange juice, and 80 drink both apple as well as orange juice. How many drinks neither apple juice nor orange juice?
Solution:
Let A = set of students who drink apple juice
B = set of students who drink orange juice
X = set of all students
$\therefore$ n(X) = 425, n(A) = 115, n(B) = 160, n(A $\cap$ B) = 80



No. of students who neither drink apple juice nor orange juice = n(A′ $\cap$ B′) = n(A $\cup$ B)′
= n(X) – n(A $\cup$ B)
= 425 – [n(A) + n(B) – n(A $\cap$ B)]
= 425 – (115 + 160 – 80)
= 230

Question 4.
In a school, there are 20 teachers who teach Mathematics or Physics. Of these, 12 teach Mathematics and 4 teach both Physics and Mathematics. How many teachers teach Physics?
Solution:
Let A = set of teachers who teach Mathematics
B = set of teachers who teach Physics
$\therefore$ n(A $\cup$ B) = 20, n(A) = 12, n(A $\cap$ B) = 4



Since n(A ∪ B) = n(A) + n(B) – n(A ∩ B),
20 = 12 + n(B) – 4
∴ n(B) = 12
∴ Number of teachers who teach physics = 12

Question 5.
(i) If A = {1, 2, 3} and B = {2, 4}, state the elements of A × A, A × B, B × A, B × B, (A × B) ∩ (B × A).
(ii) If A = {-1, 1}, find A × A × A.
Solution:
(i) A = {1, 2, 3} and B = {2, 4}
A × A = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}
A × B = {(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)}
B × A = {(2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)}
B × B = {(2, 2), (2, 4), (4, 2), (4, 4)}
∴ (A × B) ∩ (B × A) = {(2, 2)}

(ii) A = {-1, 1}
∴ A × A × A = {(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)}

Question 6.
If A = {1, 2, 3}, B = {4, 5, 6}, check if the following are relations from A to B. Also, write its domain and range.
(i) R_1 = {(1, 4), (1, 5), (1, 6)}
(ii) R_2 = {(1, 5), (2, 4), (3, 6)}
(iii) R_3 = {(1, 4), (1, 5), (3, 6), (2, 6), (3, 4)}
(iv) R_4 = {(4, 2), (2, 6), (5, 1), (2, 4)}
Solution:
A = {1, 2, 3}, B = {4, 5, 6}
∴ A × B = {(1, 4), (1, 5), (1, 6), (2,4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}
(i) R_1 = {(1, 4), (1, 5), (1, 6)}
Since R_1 ⊆ A × B,
R_1 is a relation from A to B.
Domain (R_1) = Set of first components of R_1 = {1}
Range (R_1) = Set of second components of R_1 = {4, 5, 6}

(ii) R_2 = {(1, 5),(2, 4),(3, 6)}
Since R_2 ⊆ A × B,
R_2 is a relation from A to B.
Domain (R_2) = Set of first components of R_2 = {1, 2, 3}
Range (R_2) = Set of second components of R_2 = {4, 5, 6}

(iii) R_3 = {(1, 4), (1, 5), (3, 6), (2, 6), (3, 4)}
Since R_3 ⊆ A × B,
R_3 is a relation from A to B.
Domain (R_3) = Set of first components of R_3 = {1, 2, 3}
Range (R_3) = Set of second components of R_3 = {4, 5, 6}

(iv) R_4 = {(4, 2), (2, 6), (5, 1), (2, 4)}
Since (4, 2) ∈ R_4, but (4, 2) ∉ A × B,
R_4 ⊄ A × B
∴ R_4 is not a relation from A to B.

Question 7.
Determine the domain and range of the following relations.
(i) R = {(a, b) / a ∈ N, a < 5, b = 4}
(ii) R = {(a, b) / b = |a – 1|, a ∈ Z, |a| < 3}
Solution:
(i) R = {(a, b) / a ∈ N, a < 5, b = 4}
∴ Domain (R) = {a / a ∈ N, a < 5} = {1, 2, 3, 4}
Range (R) = {b / b = 4} = {4}

(ii) R = {(a, b) / b = |a – 1|, a ∈ Z, |a| < 3}
Since a ∈ Z and |a| < 3,
a < 3 and a > -3

∴ $-3 < a < 3$
∴ a = -2, -1, 0, 1, 2
b = |a – 1|
When a = -2, b = 3
When a = -1, b = 2
When a = 0, b = 1
When a = 1, b = 0
When a = 2, b = 1
Domain (R) = {-2, -1, 0, 1, 2}
Range (R) = {0, 1, 2, 3}

Question 8.
Find R : A → A when A = {1, 2, 3, 4} such that
(i) R = {(a, b) / a – b = 10}
(ii) R = {(a, b) / |a – b| ≥ 0}
Solution:
R : A → A, A = {1, 2, 3,4}
(i) R = {(a, b)/a – b = 10} = { }

(ii) R = {(a, b) / |a – b| ≥ 0}
= {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)}
A × A = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)}
∴ R = A × A

Question 9.
R : {1, 2, 3} → {1, 2, 3} given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)}. Check if R is
(i) reflexive
(ii) symmetric
(iii) transitive
Solution:
R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)}
(i) Here, (x, x) ∈ R, for x ∈ {1, 2, 3}
∴ R is reflexive.

(ii) Here, (1, 2) ∈ R, but (2, 1) ∉ R.
∴ R is not symmetric.

(iii) Here, (1, 2), (2, 3) ∈ R,
But (1, 3) ∉ R.
∴ R is not transitive.

Question 10.
Check if R : Z → Z, R = {(a, b) | 2 divides a – b} is an equivalence relation.
Solution:
(i) Since 2 divides a – a,
(a, a) ∈ R
∴ R is reflexive. .

(ii) Let (a, b) ∈ R
Then 2 divides a – b
∴ 2 divides b – a
∴ (b, a) ∈ R
∴ R is symmetric.

(iii) Let (a, b) ∈ R, (b, c) ∈ R
Then a – b = 2m, b – c = 2n,
∴ a – c = 2(m + n), where m, n are integers.
∴ 2 divides a – c
∴ (a, c) ∈ R
∴ R is transitive.
Thus, R is an equivalence relation.

Question 11.
Show that the relation R in the set A = {1, 2, 3, 4, 5} Given by R = {(a, b) / |a – b| is even} is an equivalence relation.
Solution:
(i) Since |a – a| is even,
∴ (a, a) ∈ R
∴ R is reflexive.

(ii) Let (a, b) ∈ R
Then |a – b| is even
∴ |b – a| is even
∴ (b, a) ∈ R
∴ R is symmetric.

(iii) Let (a, b), (b, c) ∈ R
Then a – b = ±2m, b – c = ±2n
∴ a – c = ±2(m + n), where m, n are integers.
∴ (a, c) ∈ R
∴ R is transitive
Thus, R is an equivalence relation.

Question 12.
Show that the following are equivalence relations:
(i) R in A is set of all books given by R = {(x, y) / x and y have same number of pages}
(ii) R in A = {x ∈ Z | 0 ≤ x ≤ 12} given by R = {(a, b) / |a – b| is a multiple of 4}
(iii) R in A = (x ∈ N/x ≤ 10} given by R = {(a, b) | a = b}
Solution:
(i) a. Clearly (x, x) ∈ R
∴ R is reflexive.

b. If (x, y) ∈ R then (y, x) ∈ R.
∴ R is symmetric.

c. Let (x, y) ∈ R, (y, x) ∈ R.
Then x, y, and z are 3 books having the same number of pages.
∴ (x, z) ∈ R as x, z has the same number of pages.
∴ R is transitive.
Thus, R is an equivalence relation.

(ii) a. Since |a – a| is a multiple of 4,
(a, a) ∈ R
∴ R is reflexive.

b. Let (a, b) ∈ R
Then a – b = ±4m,
∴ b – a = ±4m, where m is an integer
∴ (b, a) ∈ R
∴ R is symmetric.

c. Let (a, b), (b, c) ∈ R
a – b = ± 4m, b – c = ± 4n,
∴ a – c = ±4(m + n), where m, n are integers
∴ (a, c) ∈ R
∴ R is transitive
Thus, R is an equivalence relation.

(iii) a. Since a = a
∴ (a, a) ∈ R
∴ R is reflexive.

b. Let (a, b) ∈ R Then a = b
∴ b = a
∴ (b, a) ∈ R
∴ R is symmetric.

c. Let (a, b), (b, c) ∈ R
Then, a = b, b = c
∴ a = c
∴ (a, c) ∈ R
∴ R is transitive.
Thus, R is an equivalence relation.