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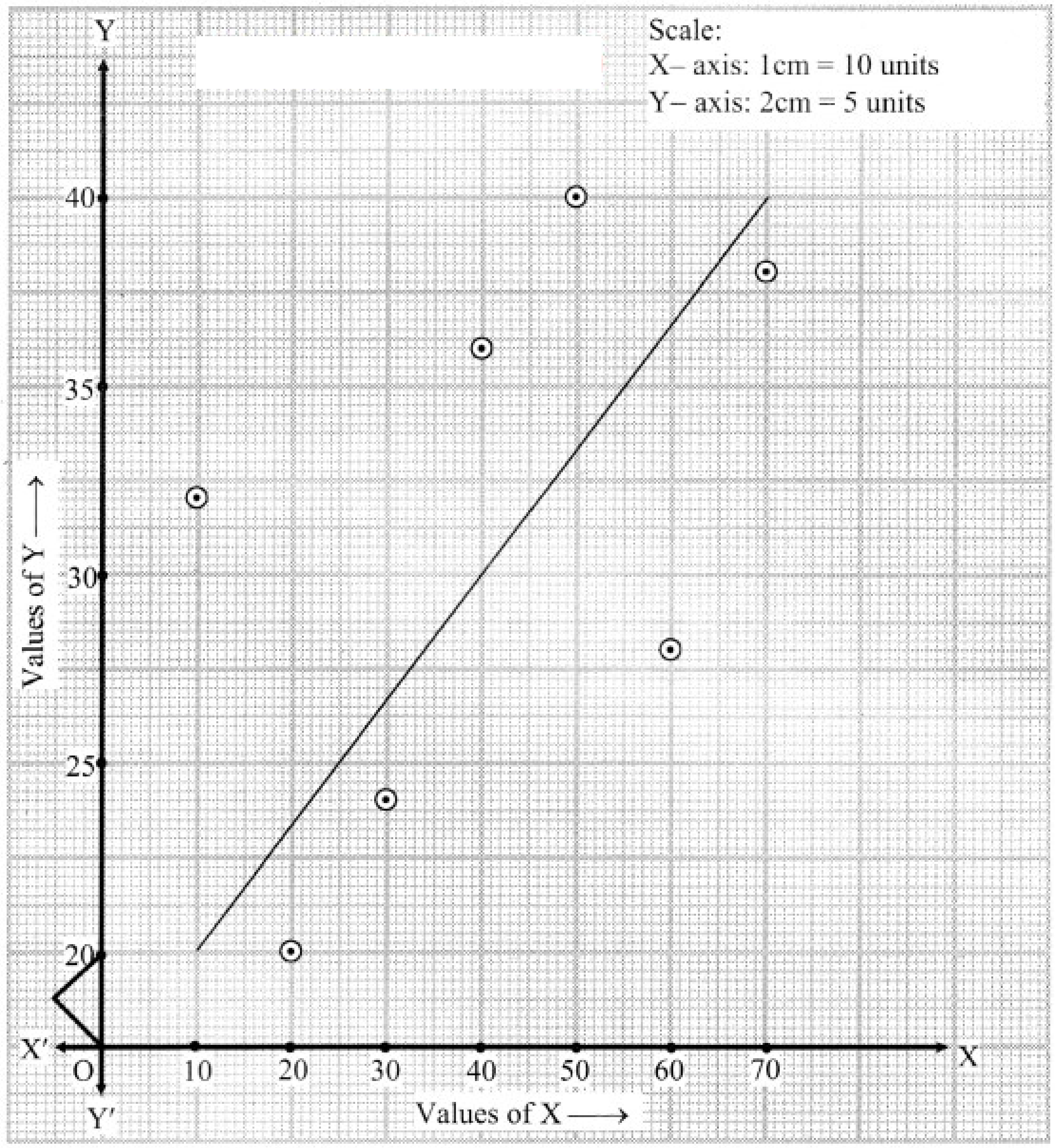
Maharashtra State Board 11th Commerce Maths Solutions Chapter 5 Correlation Ex 5.1

Question 1.
Draw a scatter diagram for the data given below and interpret it.

x	10	20	30	40	50	60	70
y	32	20	24	36	40	28	38

Solution:



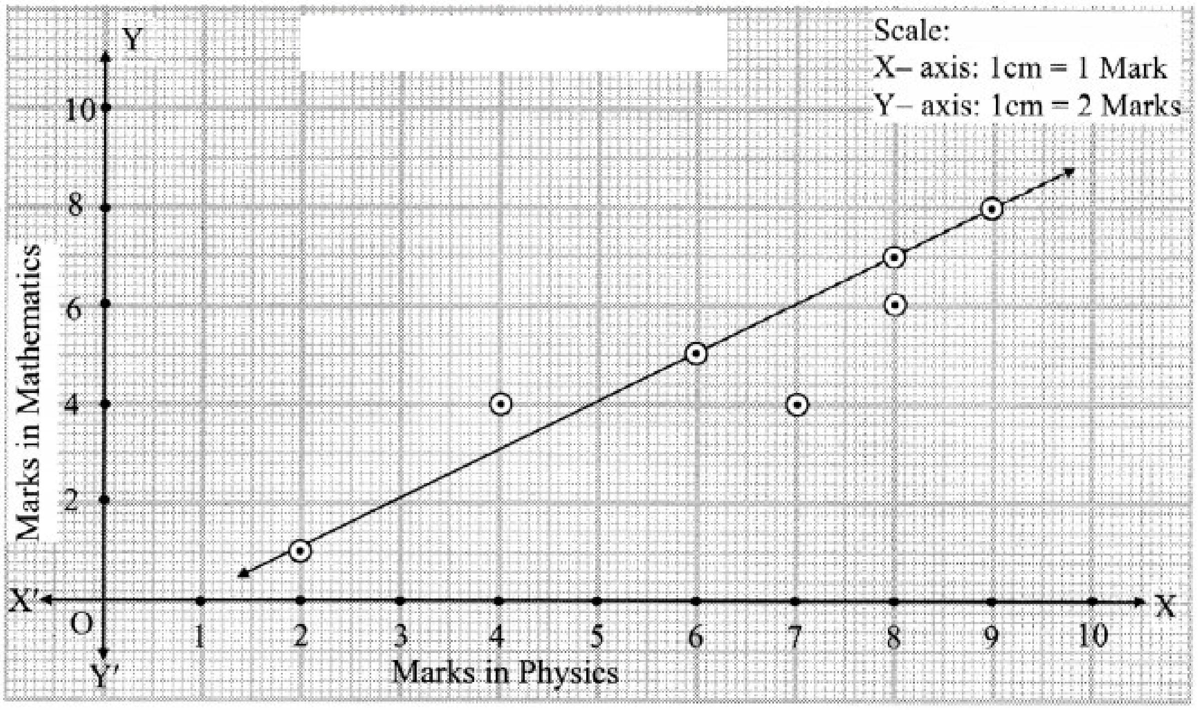
Since all the points lie in a band rising from left to right.

Therefore, there is a positive correlation between the values of X and Y respectively.

Question 2.
For the following data of marks of 7 students in Physics (x) and Mathematics (y), draw scatter diagram and state the type of correlation.

x	8	6	2	4	7	8	9
y	6	5	1	4	4	7	8

Solution:
We take marks in Physics on X-axis and marks in Mathematics on Y-axis and plot the points as below.

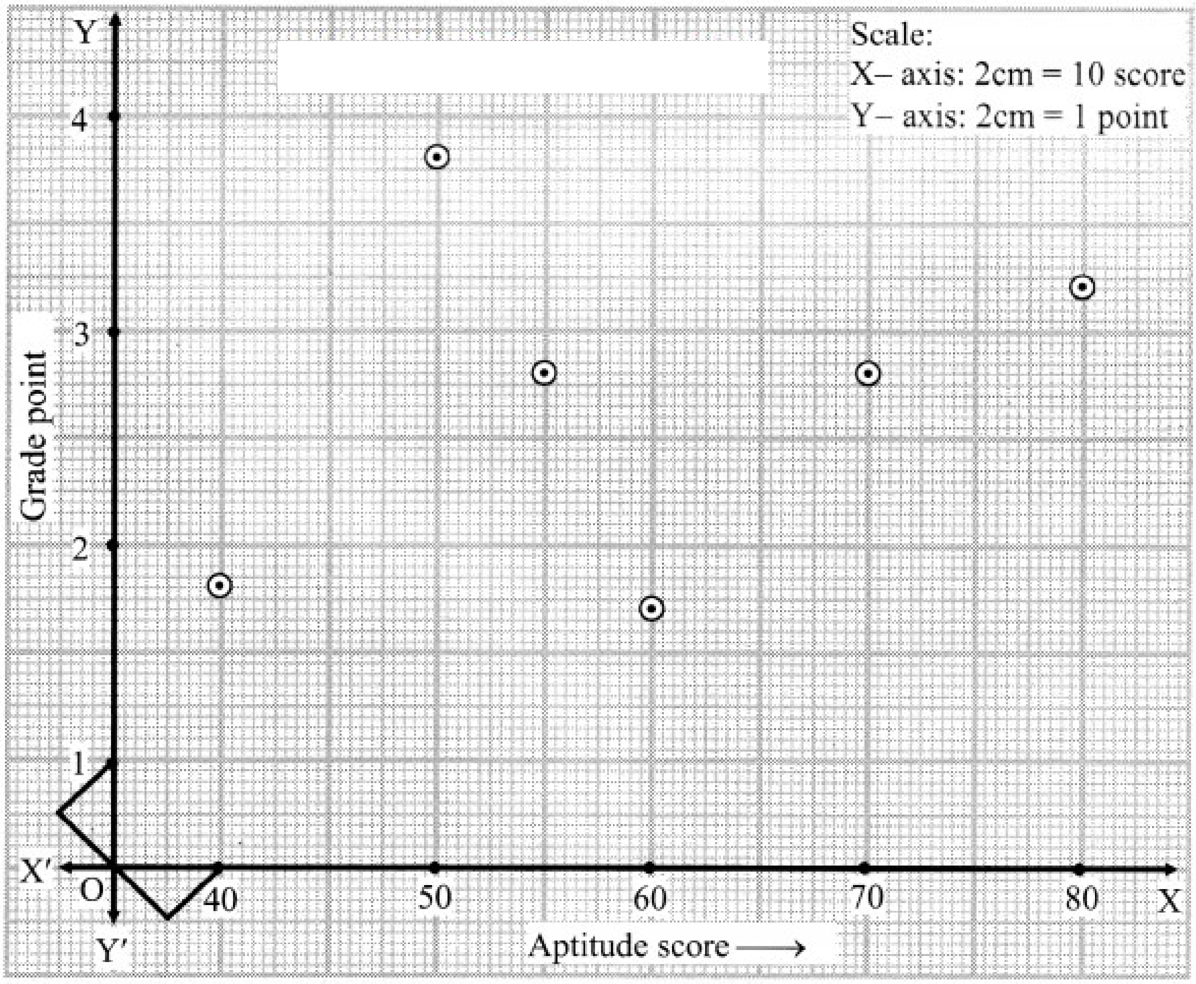


We get a band of points rising from left to right. This indicates the positive correlation between marks in Physics and marks in Mathematics.

Question 3.
Draw a scatter diagram for the data given below. Is there any correlation between Aptitude score and Grade points?

Aptitude score	40	50	55	60	70	80
Grade points	1.8	3.8	2.8	1.7	2.8	3.2

Solution:



The points are completely scattered i.e., no trend is observed.
∴ there is no correlation between Aptitude score (X) and Grade point (Y).

Question 4.
Find correlation coefficient between x andy series for the following data:
n = 15, $\bar{x}$ = 25, $\bar{y}$ = 18, σ_x = 3.01, σ_y = 3.03, $\sum(x_i-\bar{x})(y_i-\bar{y})$ = 122
Solution:

Here, n = 15, $\bar{x}$ = 25, $\bar{y}$ = 18, σ_x = 3.01, σ_y = 3.03, $\sum(x_i-\bar{x})(y_i-\bar{y})$ = 122

$$\text{Since, Cov (X, Y)} = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})$$

$$\therefore \quad \text{Cov (X, Y)} = \frac{1}{15} \times 122$$

$$= 8.13$$

$$\text{Since, } r = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}$$

$$\therefore \quad r = \frac{8.13}{3.01 \times 3.03}$$

$$= \frac{8.13}{9.1203}$$

$$\therefore \quad r = 0.89$$

Question 5.
The correlation coefficient between two variables x and y are 0.48. The covariance is 36 and the variance of x is 16. Find the standard deviation of y.
Solution:
Given, r = 0.48, Cov(X, Y) = 36
Since σ^2_X = 16
∴ σ_x = 4

$$\text{Since, } r = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}$$

$$\therefore \quad 0.48 = \frac{36}{4 \times \sigma_Y}$$

$$\therefore \quad \sigma_Y = \frac{36}{0.48 \times 4} = \frac{9}{0.48}$$

$$= \frac{900}{48} = 18.75$$

∴ the standard deviation of y is 18.75.

Question 6.
In the following data, one of the values of y is missing. Arithmetic means of x and y series are 6 and 8 respectively. (√2 = 1.4142)

x	6	2	10	4	8
y	9	11	?	8	7

(i) Estimate missing observation.

(ii) Calculate correlation coefficient.

Solution:

(i) Let X = x_i, Y = y_i and missing observation be 'a'.

Given, $\bar{x}$ = 6, $\bar{y}$ = 8, n = 5

∴ 8 = $\frac{35+a}{5}$

∴ 40 = 35 + a

∴ a = 5

(ii) We construct the following table:

	x_i	y_i	x_i^2	y_i^2	$x_i y_i$
	6	9	36	81	54
	2	11	4	121	22
	10	a = 5	100	25	50
	4	8	16	64	32
	8	7	64	49	56
Total	**30**	**40**	**220**	**340**	**214**

From the table, we have

$\sum x_i = 30,\ \sum y_i = 40,\ \sum x_i^2 = 220,\ \sum y_i^2 = 340,\ \sum x_i y_i = 214$

Since, $\text{Cov}(X, Y) = \frac{1}{n}\sum x_i y_i - \bar{x}\,\bar{y}$

$\therefore \quad \text{Cov}(X, Y) = \frac{1}{5} \times 214 - 6 \times 8$

$= 42.8 - 48$

$= -5.2$

$\sigma_X^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$

$= \frac{220}{5} - (6)^2 = 44 - 36$

$\therefore \quad \sigma_X^2 = 8$

$\therefore \quad \sigma_X = \sqrt{8} = 2\sqrt{2} = 2(1.4142) = 2.83$

$\sigma_Y^2 = \frac{\sum y_i^2}{n} - (\bar{y})^2$

$= \frac{340}{5} - (8)^2 = 68 - 64$

$\therefore \quad \sigma_Y^2 = 4$

$\therefore \quad \sigma_Y = \sqrt{4} = 2$

Thus, the correlation coefficient between X and Y is

$r = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}$

$= \frac{-5.2}{2.83 \times 2}$

$= \frac{-2.6}{2.83}$

$= -0.92$

Question 7.
Find correlation coefficient from the following data. [Given: √3 = 1.732]

X	3	6	2	9	5
Y	4	5	8	6	7

Solution:

	x_i	y_i	x_i^2	y_i^2	x_iy_i
	3	4	9	16	12
	6	5	36	25	30
	2	8	4	64	16
	9	6	81	36	54
	5	7	25	49	35
Total	**25**	**30**	**155**	**190**	**147**

From the table, we have

$n = 5,\ \sum x_i = 25,\ \sum y_i = 30,\ \sum x_i^2 = 155,\ \sum y_i^2 = 190,\ \sum x_i y_i = 147$

$$\bar{x} = \frac{\sum x_i}{n}$$

$$= \frac{25}{5}$$

$$= 5$$

$$\bar{y} = \frac{\sum y_i}{n}$$

$$= \frac{30}{5}$$

$$= 6$$

Since, $\text{Cov}(X, Y) = \frac{1}{n}\sum x_i y_i - \bar{x}\bar{y}$

$$\text{Cov}(X, Y) = \frac{1}{5} \times 147 - (5 \times 6)$$

$$= 29.4 - 30$$

$$= -0.6$$

$$\sigma_X^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$= \frac{155}{5} - (5)^2$$

$$= 31 - 25$$

$\therefore \quad \sigma_X^2 = 6$

$\therefore \quad \sigma_X = \sqrt{6}$

$$\sigma_Y^2 = \frac{\sum y_i^2}{n} - (\bar{y})^2$$

$$= \frac{190}{5} - (6)^2$$

$$= 38 - 36$$

$\therefore \quad \sigma_Y^2 = 2$

$\therefore \quad \sigma_Y = \sqrt{2}$

$\therefore \quad \sigma_X \sigma_Y = \sqrt{6}\sqrt{2} = \sqrt{12}$

$$= 2\sqrt{3}$$

$$= 2(1.732) = 3.464$$

Thus, the correlation coefficient between X and Y is

$$r = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

$$= \frac{-0.6}{3.464}$$

$$= -0.1732$$

Question 8.
The correlation coefficient between x and y is 0.3 and their covariance is 12. The variance of x is 9, find the standard deviation of y.
Solution:
Given, r = 0.3, Cov(X, Y) = 12,
$\sigma_X^2 = 9$
$\therefore \sigma_X = 3$

$$\text{Since, } r = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

$\therefore \quad 0.3 = \frac{12}{(3)(\sigma_Y)}$

$\therefore \quad \sigma_Y = \frac{12}{(3)(0.3)}$

$\therefore \quad \sigma_Y = \frac{4}{0.3}$

$\therefore \quad \sigma_Y = 13.33$

$\therefore$ the standard deviation of y is 13.33.

Maharashtra State Board 11th Commerce Maths Solutions Chapter 5 Correlation Miscellaneous Exercise 5

Question 1.
Two series of x and y with 50 items each have standard deviations of 4.8 and 3.5 respectively. If the sum of products of deviations of x and y series from respective arithmetic means is 420, then find the correlation coefficient between x and y.
Solution:

Given, n = 50, $\sigma_X = 4.8$, $\sigma_Y = 3.5$, $\sum(x_i - \bar{x})(y_i - \bar{y}) = 420$

$$\text{Cov}(X, Y) = \frac{1}{n} \sum(x_i - \bar{x})(y_i - \bar{y})$$

$$= \frac{1}{50} \times 420$$

$\therefore$ Cov (X, Y) = 8.4

$$r = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{8.4}{(4.8)(3.5)} = \frac{84 \times 10}{48 \times 35} = \frac{1}{2} = 0.5$$

Question 2.
Find the number of pairs of observations from the following data,
r = 0.15, $\sigma_y = 4$, $\sum(x_i - \bar{x})(y_i - \bar{y}) = 12$, $\Sigma(x_i - \bar{x})^2 = 40$.
Solution:
Given, r = 0.15, $\sigma_y = 4$, $\sum(x_i - \bar{x})(y_i - \bar{y}) = 12$, $\Sigma(x_i - \bar{x})^2 = 40$

Since, $\sigma_X = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2} = \sqrt{\frac{40}{n}}$

$\text{Cov}(X, Y) = \frac{1}{n}\sum(x_i - \bar{x})(y_i - \bar{y})$

$= \frac{1}{n} \times 12$

$\therefore \quad \text{Cov}(X, Y) = \frac{12}{n}$

Since, $r = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$

$\therefore \quad 0.15 = \frac{\frac{12}{n}}{\sqrt{\frac{40}{n}} \times 4}$

$\therefore \quad 0.15 = \frac{3}{n \times \sqrt{\frac{40}{n}}}$

$\therefore \quad 0.05 = \frac{1}{\sqrt{n} \times \sqrt{40}}$

Squaring on both the sides, we get

$0.0025 = \frac{1}{n \times 40}$

$\therefore \quad n = \frac{1}{0.0025 \times 40}$

$= \frac{10000}{25 \times 40}$

$= \frac{10000}{1000}$

$\therefore \quad n = 10$

Question 3.

Given that r = 0.4, $\sigma_y = 3$, $\sum(x_i-\bar{x})(y_i-\bar{y}) = 108$, $\sum(x_i-\bar{x})^2 = 900$. Find the number of pairs of observations.

Solution:

Given, r = 0.4, σ_y = 3, $\sum(x_i-\bar{x})(y_i-\bar{y})$ = 108, $\sum(x_i-\bar{x})^2$ = 900

$$\text{Cov}(X, Y) = \frac{1}{n}\sum\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)$$

$$= \frac{1}{n} \times 108$$

$$\therefore \quad \text{Cov}(X, Y) = \frac{108}{n}$$

$$\sigma_X = \sqrt{\frac{1}{n} \times \sum\left(x_i - \bar{x}\right)^2}$$

$$= \sqrt{\frac{1}{n} \times 900}$$

$$= \sqrt{\frac{900}{n}} = \frac{30}{\sqrt{n}}$$

$$\text{Since, } r = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

$$\therefore \quad 0.4 = \frac{\frac{108}{n}}{\frac{30}{\sqrt{n}} \times 3}$$

$$\therefore \quad 0.4 = \frac{108}{n} \times \frac{\sqrt{n}}{30 \times 3}$$

$$\therefore \quad 0.4 = \frac{12}{10\sqrt{n}}$$

$$\therefore \quad \sqrt{n} = \frac{12}{4} = 3$$

$$\therefore \quad n = 9$$

Question 4.
Given the following information, Σx_i^2 = 90, $\Sigma x_i y_i$ = 60, r = 0.8, σ_y = 2.5, where x_i and y_i are the deviations from their respective means, find the number of items.
Solution:
Here, Σx_i^2 = 90, $\Sigma x_i y_i$ = 60, r = 0.8, σ_y = 2.5
Here, x_i and y_i are the deviations from their respective means.
∴ If X_i, Y_i are elements of x and y series respectively, then
$X_i - \bar{x} = x_i$ and $Y_i - \bar{y} = y_i$

$\therefore \Sigma x_i y_i = \Sigma(X_i - \bar{x})(Y_i - \bar{y}) = 60$, $\sum x_i^2 = \sum(X_i - \bar{x})^2 = 90$

Now, $\sigma_X^2 = \frac{\Sigma(X_i - \bar{x})^2}{n}$

$\therefore \quad \sigma_X^2 = \frac{90}{n}$

$\therefore \quad \sigma_X = \sqrt{\frac{90}{n}}$

Also, $\text{Cov}(X, Y) = \frac{1}{n}\Sigma(X_i - \bar{x})(Y_i - \bar{y})$

$\therefore \quad \text{Cov}(X, Y) = \frac{60}{n}$

$r = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}$

$\therefore \quad 0.8 = \frac{\frac{60}{n}}{\sqrt{\frac{90}{n}} \times 2.5}$

$\therefore \quad 0.8 \times 2.5 \times \sqrt{\frac{90}{n}} = \frac{60}{n}$

$\therefore \quad 2 \times \frac{\sqrt{90}}{\sqrt{n}} = \frac{60}{n}$

$\therefore \quad \frac{n}{\sqrt{n}} = \frac{60}{2 \times \sqrt{90}}$

$\therefore \quad \frac{\sqrt{n} \times \sqrt{n}}{\sqrt{n}} = \frac{30}{\sqrt{90}} = \frac{\sqrt{30} \times \sqrt{30}}{\sqrt{3}\sqrt{30}}$

$\therefore \quad \sqrt{n} = \sqrt{10}$

$\therefore \quad n = 10$

Question 5.

A sample of 5 items is taken from the production of a firm. The length and weight of 5 items are given below. [Given: √0.8823 = 0.9393]

Length(cm)	3	4	6	7	10
Weight(gm.)	9	11	14	15	16

Calculate the correlation coefficient between length and weight and interpret the result.

Solution:

Let length = x_i (in cm), Weight = y_i (in gm)

	x_i	y_i	x_i^2	y_i^2	$x_i y_i$
	3	9	9	81	27
	4	11	16	121	44
	6	14	36	196	84
	7	15	49	225	105
	10	16	100	256	160
Total	**30**	**65**	**210**	**879**	**420**

From the table, we have

$n = 5,\ \sum x_i = 30,\ \sum y_i = 65,\ \sum x_i^2 = 210,\ \sum y_i^2 = 879,\ \sum x_i y_i = 420$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{30}{5} = 6,\quad \bar{y} = \frac{\sum y_i}{n} = \frac{65}{5} = 13$$

$$\text{Cov}(X, Y) = \frac{1}{n}\sum x_i y_i - \bar{x}\,\bar{y}$$

$$= \frac{1}{5} \times 420 - 6 \times 13$$

$$= 84 - 78$$

$$= 6$$

$$\sigma_X^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2 = \frac{210}{5} - (6)^2 = 42 - 36$$

$\therefore\quad \sigma_X^2 = 6$

$\therefore\quad \sigma_X = \sqrt{6}$

$$\sigma_Y^2 = \frac{\sum y_i^2}{n} - (\bar{y})^2$$

$$= \frac{879}{5} - (13)^2$$

$$= 175.8 - 169$$

$\sigma_Y^2 = 6.8$

$\therefore\quad \sigma_Y = \sqrt{6.8}$

Thus, the coefficient of correlation between X and Y is

$$r = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{6}{\sqrt{6}\,\sqrt{6.8}} = \sqrt{\frac{6}{6.8}} = \sqrt{\frac{60}{68}} = \sqrt{\frac{15}{17}} = \sqrt{0.8823}$$

$\therefore\quad r = 0.9393 \approx 0.94$

∴ the value of r indicates a high degree of positive correlation between length and weight of items.

Question 6.
Calculate the correlation coefficient from the following data and interpret it.

X	1	3	5	7	9	11	13
Y	12	10	8	6	4	2	0

Solution:

	x_i	y_i	x_i^2	y_i^2	$x_i y_i$
	1	12	1	144	12
	3	10	9	100	30
	5	8	25	64	40
	7	6	49	36	42
	9	4	81	16	36
	11	2	121	4	22
	13	0	169	0	0
Total	**49**	**42**	**455**	**364**	**182**

From the table, we have

$n = 7,\ \sum x_i = 49,\ \sum y_i = 42,\ \sum x_i^2 = 455,\ \sum y_i^2 = 364,\ \sum x_i y_i = 182.$

$$\therefore \quad \bar{x} = \frac{\sum x_i}{n} = \frac{49}{7} = 7,$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{42}{7} = 6$$

$$\text{Cov}(X, Y) = \frac{1}{n}\sum x_i y_i - \bar{x}\,\bar{y}$$

$$= \frac{1}{7} \times 182 - (7 \times 6)$$

$$= 26 - 42$$

$$\therefore \quad \text{Cov}(X, Y) = -16$$

$$\sigma_X^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$= \frac{455}{7} - (7)^2 = 65 - 49$$

$\therefore \quad \sigma_X^2 = 16$

$\therefore \quad \sigma_X = 4$

$$\sigma_Y^2 = \frac{\sum y_i^2}{n} - (\bar{y})^2$$

$$= \frac{364}{7} - (6)^2 = 52 - 36$$

$\sigma_Y^2 = 16$

$\therefore \quad \sigma_Y = 4$

Thus, the coefficient of correlation between X and Y is

$$r = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{-16}{4 \times 4} = -1$$

∴ the value of r indicates a perfect negative correlation between x and y.

Question 7.
Calculate the correlation coefficient from the following data and interpret it.

x	9	7	6	8	9	6	7
y	19	17	16	18	19	16	17

Solution:

	x_i	y_i	x_i^2	y_i^2	$x_i y_i$
	9	19	81	361	171
	7	17	49	289	119
	6	16	36	256	96
	8	18	64	324	144
	9	19	81	361	171
	6	16	36	256	96
	7	17	49	289	119
Total	**52**	**122**	**396**	**2136**	**916**

From the table, we have

$n = 7, \sum x_i = 52, \sum y_i = 122, \sum x_i^2 = 396, \sum y_i^2 = 2136, \sum x_i y_i = 916$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{52}{7}$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{122}{7}$$

$$\therefore \quad \bar{x}\,\bar{y} = \frac{52 \times 122}{49} = \frac{6344}{49}$$

$$\text{Cov}(X, Y) = \frac{1}{n}\sum x_i y_i - \bar{x}\,\bar{y}$$

$$= \frac{916}{7} - \frac{6344}{49}$$

$$= \frac{6412 - 6344}{49} = \frac{68}{49}$$

$$\sigma_X^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$= \frac{396}{7} - \left(\frac{52}{7}\right)^2 = \frac{2772 - 2704}{49} = \frac{68}{49}$$

Question 8.
If the correlation coefficient between X and Y is 0.8, what is the correlation coefficient between
(i) 2X and Y
(ii) X2 and Y
(iii) X and 3Y
(iv) X – 5 and Y – 3
(v) X + 7 and Y + 9
(vi) X−57 and Y−38?
Solution:
The correlation coefficient remains unaffected by the change of origin and scale.
i.e., if u_i = xi−ah and v_i = yi−bk, then Corr(U, V) = ±Corr(X, Y).
according to the same or opposite signs of h and k.
(i) u_i = 2(xi−0)1, v_i = yi−01
Here, h = 1 and k = 1 are of the same signs.
∴ Corr (U, V) = Corr (X, Y) = 0.8

(ii) $u_i = \frac{x_i - 0}{2}$, $v_i = \frac{y_i - 0}{1}$
Here, h = 2 and k = 1 are of the same signs.
∴ Corr (U, V) = Corr (X, Y) = 0.8

(iii) Corr (X, 3Y) = Corr (X, Y) = 0.8

(iv) Corr (X – 5, Y – 3) = Corr(X, Y) = 0.8

(v) Corr (X + 7, Y + 9) = Corr(X, Y) = 0.8

(vi) Corr$\left(\frac{X-5}{7}, \frac{Y-3}{8}\right)$ = Corr(X, Y) = 0.8

Question 9.
In the calculation of the correlation coefficient between the height and weight of a group of students of a college, one investigator took the measurements in inches and pounds while the other investigator took the measurements in cm. and kg. Will they get the same value of the correlation coefficient or different values? Justify your answer.
Solution:
The coefficient of correlation is a ratio of covariance and standard deviations.
Since covariance and standard deviations are independent of units of measurement.
∴ coefficient of correlation is also independent of units of measurement.
∴ values of coefficient of correlation obtained by first and second investigators are the same.