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Practice Set 3.1 Algebra 9th Std Maths Part 1 Answers Chapter 3 Polynomials

Question 1.
State whether the given algebraic expressions are polynomials? Justify.
i. $y + 1y$
ii. $2 – 5\sqrt{x}$
iii. $x^2 + 7x + 9$
iv. $2m^{-2} + 7m – 5$
v. 10
Answer:
i. No, because power of v in the term $5\sqrt{x}$ is -1 (negative number).
ii. No, because the power of x in the term $5\sqrt{x}$ is
i. e. 0.5 (decimal number).
iii. Yes. All the coefficients are real numbers. Also, the power of each term is a whole number.
iv. No, because the power of m in the term $2m^{-2}$ is -2 (negative number).
v. Yes, because 10 is a constant polynomial.
Question 2.
Write the coefficient of m^3 in each of the given polynomial.
i. m^3
ii. $2-\sqrt{}_{-3} + m – \sqrt{3}m^3$
iii. $3-\sqrt{}_{-2}m^3 + 5m^2 – 7m -1$
Answer:
i. 1
ii. $-\sqrt{3}$
iii. – 23
Question 3.
Write the polynomial in x using the given information. [1 Mark each]
i. Monomial with degree 7
ii. Binomial with degree 35
iii. Trinomial with degree 8
Answer:
i. $5x^7$
ii. $x^{35} – 1$
iii. $3x^8 + 2x^6 + x^5$

Question 4.
Write the degree of the given polynomials.
i. $\sqrt{5}$
ii. $x°$
iii. x^2
iv. $\sqrt{2}m^{10} – 7$
v. $2p – \sqrt{7}$
vi. $7y – y^3 + y^5$
vii. xyz +xy-z

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viii. $m^3n^7 – 3m^5n + mn$
Answer:
i. $\sqrt{5} = \sqrt{5}\ x^\circ$
∴ Degree of the polynomial = 0

ii. x°
∴Degree of the polynomial = 0

iii. x^2
∴Degree of the polynomial = 2

iv. $\sqrt{2}m^{10} – 7$
Here, the highest power of m is 10.
∴Degree of the polynomial = 10

v. $2p – \sqrt{7}$
Here, the highest power of p is 1.
∴ Degree of the polynomial = 1

vi. $7y – y^3 + y^5$
Here, the highest power of y is 5.
∴Degree of the polynomial = 5

vii. xyz + xy – z
Here, the sum of the powers of x, y and z in the term xyz is 1 + 1 + 1= 3,
which is the highest sum of powers in the given polynomial.
∴Degree of the polynomial = 3

viii. $m^3n^7 – 3m^5n + mn$
Here, the sum of the powers of m and n in the term m^3n^7 is 3 + 7 = 10,
which is the highest sum of powers in the given polynomial.
∴ Degree of the polynomial = 10

Question 5.
Classify the following polynomials as linear, quadratic and cubic polynomial. [2 Marks]
i. $2x^2 + 3x + 1$
ii. 5p
iii. $\sqrt{2} – 12$
iv. $m^3 + 7m^2 + 2 - \sqrt{5}m – \sqrt{7}$
v. a^2
vi. $3r^3$
Answer:
Linear polynomials: ii, iii
Quadratic polynomials: i, v
Cubic polynomials: iv, vi

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Question 6.
Write the following polynomials in standard form.
i. $m^3 + 3 + 5m$
ii. $– 7y + y^5 + 3y^3 – 12+ 2y^4 – y^2$
Answer:
i. $m^3 + 5m + 3$
ii. $y^5 + 2y^4 + 3y^3 – y^2 – 7y – 12$
Question 7.
Write the following polynomials in coefficient form.
i. $x^3 – 2$
ii. $5y$
iii. $2m^4 – 3m^2 + 7$
iv. $– 23$
Answer:
i. $x^3 – 2 = x^3 + 0x^2 + 0x – 2$
∴ Coefficient form of the given polynomial = (1, 0, 0, -2)
ii. $5y = 5y + 0$
∴Coefficient form of the given polynomial = (5,0)

iii. $2m^4 – 3m^2 + 7$
$= 2m^4 + Om^3 – 3m^2 + 0m + 7$
∴ Coefficient form of the given polynomial = (2, 0, -3, 0, 7)

iv. – 23
∴Coefficient form of the given polynomial = (- 23)
Question 8.
Write the polynomials in index form.
i. (1, 2, 3)
ii. (5, 0, 0, 0 ,-1)
iii. (-2, 2, -2, 2)
Answer:
i. Number of coefficients = 3
∴ Degree = 3 – 1 = 2
∴ Taking x as variable, the index form is $x^2 + 2x + 3$

ii. Number of coefficients = 5
∴ Degree = 5 – 1=4
∴ Taking x as variable, the index form is $5x^4 + 0x^3 + 0x^2 + 0x – 1$

iii. Number of coefficients = 4
∴Degree = 4 – 1 = 3
∴Taking x as variable, the index form is $-2x^3 + 2x^2 – 2x + 2$

Question 9.
Write the appropriate polynomials in the boxes.

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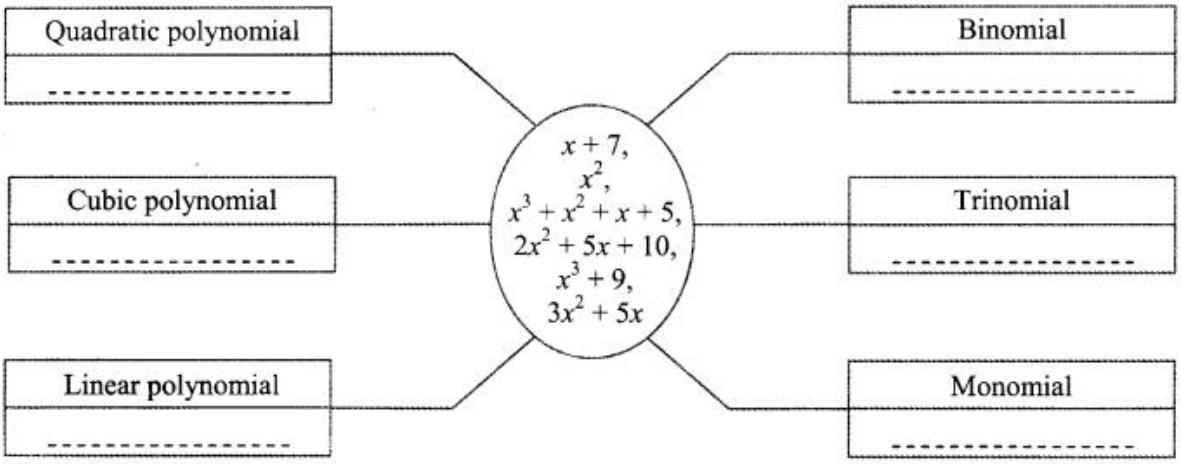

Answer:
i. Quadratic polynomial: x^2; $2x^2 + 5x + 10$; $3x^2 + 5x$
ii. Cubic polynomial: $x^3 + x^2 + x + 5$; $x^3 + 9$
iii. Linear polynomial: $x + 7$
iv. Binomial: $x + 7$; $x^3 + 9$; $3x^2 + 5x$
v. Trinomial: $2x^2 + 5x + 10$
vi. Monomial: x^2

Question 1.
Write an example of a monomial, a binomial and a trinomial having variable x and degree 5. (Textbook pg. no. 3)
Answer:
Monomial: x^5
Binomial: $x^5 + x$
Trinomial: $2x^5 – x^2 + 5$

Question 2.
Give example of a binomial in two variables having degree 5. (Textbook pg. no. 38)
Answer:
$x^3y^2 + xy$

Practice Set 3.2 Algebra 9th Std Maths Part 1 Answers Chapter 3 Polynomials

Question 1.
Use the given letters to write the answer.
i. There are ‘a’ trees in the village Lat. If the number of trees increases every year by ‘b’. then how many trees will there be after ‘x’ years?
ii. For the parade there are y students in each row and x such row are

formed. Then, how many students are there for the parade in all ?
iii. The tens and units place of a two digit number is m and n respectively. Write the polynomial which represents the two digit number.
Solution:
i. Number of trees in the village Lat = a
Number of trees increasing each year = b
∴ Number of trees after x years = a + bx
∴ There will be a + bx trees in the village Lat after x years.

ii. Total rows = x
Number of students in each row = y
∴ Total students = Total rows × Number of students in each row
= x × y
= xy
∴ There are in all xy students for the parade.

iii. Digit in units place = n
Digit in tens place = m
∴ The two digit number = 10 x digit in tens place + digit in units place
= 10m + n
∴ The polynomial representing the two digit number is 10m + n.

Question 2.
Add the given polynomials.
i. $x^3 – 2x^2 – 9$; $5x^3 + 2x + 9$
ii. $-7m^4 + 5m^3 + \sqrt{2}$; $5m^4 – 3m^3 + 2m^2 + 3m – 6$
iii. $2y^2 + 7y + 5$; $3y + 9$; $3y^2 – 4y – 3$
Solution:
i. $(x^3 – 2x^2 – 9) + (5x^3 + 2x + 9)$
$= x^3 – 2x^2 – 9 + 5x^3 + 2x + 9$
$= \underline{x^3 + 5x^3} – 2x^2 + 2x – \underline{9 + 9}$
$= 6x^3 – 2x^2 + 2x$

ii. $(-7m^4 + 5m^3 + \sqrt{2}) + (5m^4 – 3m^3 + 2m^2 + 3m – 6)$
$= -7m^4 + 5m^3 + \sqrt{2} + 5m^4 – 3m^3 + 2m^2 + 3m – 6$
$= \underline{-7m^4 + 5m^4} + \underline{5m^3 – 3m^3} + 2m^2 + 3m + \underline{\sqrt{2} – 6}$
$= -2m^4 + 2m^3 + 2m^2 + 3m + \sqrt{2} – 6$

iii. $(2y^2 + 7y + 5) + (3y + 9) + (3y^2 – 4y – 3)$
$= 2y^2 + 7y + 5 + 3y + 9 + 3y^2 – 4y – 3$
$= \underline{2y^2 + 3y^2} + \underline{7y + 3y – 4y} + \underline{5 + 9 – 3}$
$= 5y^2 + 6y + 11$

Question 3.
Subtract the second polynomial from the first.
i. $x^2 – 9x + \sqrt{3}$; $– 19x + \sqrt{3} + 7x^2$
ii. $2ab^2 + 3a^2b – 4ab$; $3ab – 8ab^2 + 2a^2b$
Solution:
i. $x^2 – 9x + \sqrt{3}$ -$(- 19x + \sqrt{3} + 7x^2)$
$= x^2 – 9x + \sqrt{3} + 19x – \sqrt{3} – 7x^2$
$= \underline{x^2 – 7x^2} – \underline{9x + 19x} + \underline{\sqrt{3} – \sqrt{3}}$
$= – 6x^2 + 10x$

ii. $(2ab^2 + 3a^2b – 4ab) – (3ab – 8ab^2 + 2a^2b)$
$= 2ab^2 + 3a^2b – 4ab – 3ab + 8ab^2 – 2a^2b$
$= \underline{2ab^2 + 8ab^2} + \underline{3a^2b – 2a^2b} – \underline{4ab – 3ab}$
$= 10ab^2 + a^2b – 7ab$

Question 4.
Multiply the given polynomials.
i. $2x$; $x^2 – 2x – 1$
ii. $x^5 – 1$; $x^3 + 2x^2 + 2$
iii. $2y + 1$; $y^2 – 2y + 3y$
Solution:
i. $(2x) \times (x^2 – 2x – 1) = 2x^3 – 4x^2 – 2x$

ii. $(x^5 – 1) \times (x^3 + 2x^2 + 2)$
$= x^5 (x^3 + 2x^2 + 2)$ -$1(x^3 + 2x^2 + 2)$
$= x^8 + 2x^7 + 2x^5 – x^3 – 2x^2 – 2$

iii. $(2y + 1) \times (y^2 – 2y^3 + 3y)$
$= 2y(y^2 – 2y^3 + 3y) + 1(y^2 – 2y^3 + 3y)$
$= 2y^3 – 4y^4 + 6y^2 + y^2 – 2y^3 + 3y$
$= -4y^4 + \underline{2y^3 – 2y^3} + \underline{6y^2 + y^2} + 3y$
$= -4y^4 + 7y^2 + 3y$

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Question 5.

Divide first polynomial by second polynomial and write the answer in the form 'Dividend = Divisor x Quotient + Remainder'.

i. $x^3 – 64; x – 4$

ii. $5x^5 + 4x^4 – 3x^3 + 2x^2 + 2 ; x^2 – x$

Solution:

i. $x^3 – 64 = x3 + 0x^2 + 0x – 64$

$$
\begin{array}{r|l}
x - 4 & x^2 + 4x + 16 \\
\hline
 & x^3 + 0x^2 + 0x - 64 \\
 & x^3 - 4x^2 \\
 & - \quad + \\
\hline
 & 4x^2 + 0x \\
 & 4x^2 - 16x \\
 & - \quad + \\
\hline
 & 16x - 64 \\
 & 16x - 64 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

∴ Quotient = $x^2 + 4x + 16$, Remainder = 0

Now, Dividend = Divisor x Quotient + Remainder

∴ $x^3 – 64 = (x – 4)(x^2 + 4x + 16) + 0$

ii. $5x^5 + 4x^4 – 3x^3 + 2x^2 + 2 = 5x^5 + 4x^4 – 3x^3 + 2x + 0x + 2$

$$
\begin{array}{r|l}
x^2 - x & 5x^5 + 4x^4 - 3x^3 + 2x^2 + 0x + 2 \\
\hline
 & 5x^5 - 5x^4 \\
 & - \quad + \\
\hline
 & 9x^4 - 3x^3 \\
 & 9x^4 - 9x^3 \\
 & - \quad + \\
\hline
 & 6x^3 + 2x^2 \\
 & 6x^3 - 6x^2 \\
 & - \quad + \\
\hline
 & 8x^2 + 0x \\
 & 8x^2 - 8x \\
 & - \quad + \\
\hline
 & 8x + 2
\end{array}
$$

∴ Quotient = $5x^3 + 9x^2 + 6x + 8$,

Remainder = $8x + 2$

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Now, Dividend = Divisor x Quotient + Remainder

$\therefore 5x^5 + 4x^4 – 3x^3 + 2x^2 + 2 = (x^2 – x)(5x^3 + 9x^2 + 6x + 8) + (8x + 2)$

Question 6.

Write down the information in the form of algebraic expression and simplify.

There is a rectangular farm with length $(2a^2 + 3b^2)$ metre and breadth $(a^2 + b^2)$ metre. The farmer used a square shaped plot of the farm to build a house. The side of the plot was (a2 – b2) metre. What is the area of the remaining part of the farm? [4 Marks]

Solution:

Length of the rectangular farm = $(2a^2 + 3b^2)$ m

Breadth of the rectangular farm = $(a^2 + b^2)$ m

Area of the farm = length x breadth = $(2a^2 + 3b^2) \times (a^2 + b^2)$

$= 2a^2(a^2 + b^2) + 3b^2(a^2 + b^2)$

$= 2a^2 + \underline{2a^2b^2 + 3a^2b^2} + 3b^4$

$= (2a^4 + 5a^2b^2 + 3b^4)$ sq. m … (i)

The farmer used a square shaped plot of the farm to build a house.

Side of the square shaped plot = $(a^2 – b^2)$ m

$\therefore$ Area of the plot = $(side)^2$

$= (a^2 – b^2)^2$

$= (a^4 – 2a^2b^2 + b^4)$ sq m… .(ii)

$\therefore$ Area of the remaining farm = Area of the farm – Area of the plot

$= (2a^4 + 5a^2b^2 + 3b^4) – (a^4 – 2a^2b^2 + b^4)$ … [From (i) and (ii)]

$= 2a^4 + 5a^2b^2 + 3b^4 – a^4 + 2a^2b^2 – b^4$

$= \underline{2a^4 – a^4} + \underline{5a^2b^2 + 2a^2b^2} + \underline{3b^4 – b^4}$

$= a^4 + 7a^2b^2 + 2b^4$

$\therefore$ The area of the remaining farm is $(a^4 + 7a^2b^2 + 2b^4)$ sq. m.

Practice Set 3.3 Algebra 9th Std Maths Part 1 Answers Chapter 3 Polynomials

Question 1.

Divide each of the following polynomials by synthetic division method and

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also by linear division method. Write the quotient and the remainder.

i. $(2m^2 – 3m + 10) ÷ (m – 5)$

ii. $(x^4 + 2x^3 + 3x^2 + 4x + 5) ÷ (x + 2)$

iii. $(y^3 – 216) ÷ (y – 6)$

iv. $(2x^4 + 3x^3 + 4x – 2x^2) ÷ (x + 3)$

v. $(x^4 – 3x^2 – 8) ÷ (x + 4)$

vi. $(y^3 – 3y^2 + 5y – 1) ÷ (y – 1)$

Solution:

i. Synthetic division:

$(2m^2 – 3m + 10) ÷ (m – 5)$

Dividend = $2m^2 – 3m + 10$

∴ Coefficient form of dividend = (2, -3, 10)

Divisor = m – 5

∴ Opposite of -5 is 5.

5	2	– 3	10
		10	35
	2	7	45

Coefficient form of quotient = (2, 7)

∴ Quotient = 2m + 7,

Remainder = 45

Linear division method:

$2m^2 – 3m + 10$

To get the term $2m^2$, multiply (m – 5) by 2m and add 10m,

$= 2m(m – 5) + 10m - 3m + 10$

$= 2m(m – 5) + 7m + 10$

To get the term 7m, multiply (m – 5) by 7 and add 35

$= 2m(m – 5) + 7(m - 5) + 35 + 10$

$= (m – 5) (2m + 7) + 45$

∴ Quotient = 2m + 7,

Remainder = 45

ii. Synthetic division:

$(x^4 + 2x^3 + 3x^2 + 4x + 5) ÷ (x + 2)$

Dividend = $x^4 + 2x^3 + 3x^2 + 4x + 5$

∴ Coefficient form of dividend = (1, 2, 3, 4, 5)

Divisor = x + 2

∴ Opposite of + 2 is -2.

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−2	1	2	3	4	5
		−2	0	−6	4
	1	0	3	−2	9

Coefficient form of quotient = (1, 0, 3, -2)
∴ Quotient = $x^3 + 3x - 2$,
Remainder = 9

Linear division method:
$x^4 + 2x^3 + 3x^2 + 4x + 5$
To get the term x^4, multiply (x + 2) by x^3 and subtract $2x^3$,
$= x^3(x + 2) - 2x^3 + 2x^3 + 3x^2 + 4x + 5$
$= x^3(x + 2) + 3x^2 + 4x + 5$
To get the term $3x^2$, multiply (x + 2) by 3x and subtract 6x,
$= x^3(x + 2) + 3x(x + 2) - 6x + 4x + 5$
$= x^3(x + 2) + 3x(x + 2) - 2x + 5$
To get the term -2x, multiply (x + 2) by -2 and add 4,
$= x^3(x + 2) + 3x(x + 2) - 2(x + 2) + 4 + 5$
$= (x + 2)(x3 + 3x - 2) + 9$
∴ Quotient = $x^3 + 3x - 2$,
Remainder – 9

iii. Synthetic division:
$(y^3 - 216) \div (y - 6)$
Dividend = $y^3 - 216$
∴ Index form = $y^3 + 0y^3 + 0y - 216$
∴ Coefficient form of dividend = (1, 0, 0, -216)
Divisor = y – 6
∴ Opposite of – 6 is 6.

6	1	0	0	−216
		6	36	216
	1	6	36	0

Coefficient form of quotient = (1, 6, 36)
∴ Quotient = $y^2 + 6y + 36$,
Remainder = 0

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Linear division method:

$y^3 – 216$

To get the term y^3, multiply $(y – 6)$ by y^2 and add $6y^2$,

$= y^2(y – 6) + 6y^2 – 216$

$= y^2(y – 6) + 6ysup>2 – 216$

To get the, term $6 y^2$ multiply $(y – 6)$ by $6y$ and add $36y$,

$= y^2(y – 6) + 6y(y – 6) + 36y – 216$

$= y^2(y – 6) + 6y(y – 6) + 36y – 216$

To get the term $36y$, multiply $(y- 6)$ by 36 and add 216,

$= y^2 (y – 6) + 6y(y – 6) + 36(y – 6) + 216 – 216$

$= (y – 6) (y^2 + 6y + 36) + 0$

Quotient $= y^2 + 6y + 36$

Remainder = 0

iv. Synthetic division:

$(2x^4 + 3x^3 + 4x – 2x^2) \div (x + 3)$

Dividend $= 2x^4 + 3x^3 + 4x – 2x^2$

$\therefore$ Index form $= 2x^4 + 3x^3 – 2x^2 + 4x + 0$

$\therefore$ Coefficient form of the dividend = (2,3, -2,4,0)

Divisor $= x + 3$

$\therefore$ Opposite of + 3 is -3

-3	2	3	-2	4	0
		-6	9	-21	51
	2	-3	7	-17	51

Coefficient form of quotient = (2, -3, 7, -17)

$\therefore$ Quotient $= 2x^3 – 3x^2 + 7x – 17$,

Remainder = 51

Linear division method:

$2x^4 + 3x^3 + 4x – 2x^2 = 2x^2 + 3x^3 – 2x^2 + 4x$

To get the term $2x^4$, multiply $(x + 3)$ by $2x^3$ and subtract $6x^3$,

$= 2x^3(x + 31 – 6x^3 + 3x^3 – 2x^2 + 4x$

$= 2x^3(x + 3) – 3x^3 – 2x^2 + 4x$

To get the term $– 3x^3$, multiply $(x + 3)$ by $-3x^2$ and add $9x^2$,

$= 2x^3(x + 3) – 3x^2(x + 3) + 9x^2 – 2x^2 + 4x$

$= 2x^3(x + 3) – 3x^2(x + 3) + 7x^2 + 4x$

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To get the term $7x^2$, multiply (x + 3) by 7x and subtract 21x,
$= 2x^3(x + 3) – 3x^2(x + 3) + 7x(x + 3) – 21x + 4x$
$= 2x^3(x + 3) – 3x^2(x + 3) + 7x(x + 3) – 17x$

To get the term -17x, multiply (x + 3) by -17 and add 51,
$= 2x^3(x + 3) – 3x^2(x + 3) + 7x(x+3) – 17(x + 3) + 51$
$= (x + 3) (2x^3 – 3x^2 + 7x- 17) + 51$
∴ Quotient $= 2x^3 – 3x^2 + 7x – 17$,
Remainder = 51

v. Synthetic division:
$(x^4 – 3x^2 – 8) + (x + 4)$
Dividend $= x^4 – 3x^2 – 8$
∴ Index form $= x^4 + 0x^3 – 3x^2 + 0x – 8$
∴ Coefficient form of the dividend = (1,0, -3,0, -8)
Divisor = x + 4
∴ Opposite of + 4 is -4

–4	1	0	–3	0	–8
		–4	16	–52	208
	1	–4	13	–52	200

∴ Coefficient form of quotient = (1, -4, 13, -52)
∴ Quotient $= x^3 – 4x^2 + 13x – 52$,
Remainder = 200

Linear division method:
$x^4 – 3x^2 – 8$
To get the term x^4, multiply (x + 4) by x^3 and subtract $4x^3$,
$= x^3(x + 4) – 4x^3 – 3x^2 – 8$
$= x^3(x + 4) – 4x^3 – 3x^2 – 8$
To get the term $– 4x^3$, multiply (x + 4) by $-4x^2$ and add $16x^2$,
$= x^3(x + 4) – 4x^2 (x + 4) + 16x^2 – 3x^2 – 8$
$= x^3(x + 4) – 4x^2 (x + 4) + 13x^2 – 8$
To get the term $13x^2$, multiply (x + 4) by 13x and subtract 52x,
$= x^3(x + 4) – 4x^2(x + 4) + 13x(x + 4) – 52x – 8$
$= x^3(x + 4) – 4x^2(x + 4) + 13x(x + 4) – 52x – 8$
To get the term -52x, multiply (x + 4) by – 52 and add 208,
$= x^3(x + 4) – 4x^2(x + 4) + 13x(x + 4) – 52(x + 4) + 208 – 8$

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$= (x + 4)(x^3 - 4x^2 + 13x - 52) + 200$
$\therefore$ Quotient $= x^3 - 4x^2 + 13x - 52$,
Remainder 200

vi. Synthetic division:
$(y^3 - 3y^2 + 5y - 1) \div (y - 1)$
Dividend $= y^3 - 3y^2 + 5y - 1$
Coefficient form of the dividend = (1, -3, 5, -1)
Divisor $= y - 1$
$\therefore$Opposite of -1 is 1.

1	1	–3	5	–1
		1	–2	3
	1	–2	3	2

$\therefore$ Coefficient form of quotient = (1, -2, 3)
$\therefore$ Quotient $= y^2 - 2y + 3$,
Remainder = 2

Linear division method:
$y^3 - 3y^2 + 5y - 1$
To get the term y^3 , multiply $(y - 1)$ by y^2 and add y^2
$= y^2(y - 1) + y^2 - 3y^2 + 5y - 1$
$= y^2(y - 1) - 2y^2 + 5y - 1$
To get the term $-2y^2$, multiply $(y - 1)$ by $-2y$ and subtract $2y$,
$= y^2(y - 1) - 2y(y - 1) - 2y + 5y - 1$
$= y^2(y - 1) - 2y(y - 1) + 3y - 1$
To get the term $3y$, multiply $(y - 1)$ by 3 and add 3,
$= y^2(y - 1) - 2y(y - 1) + 3(y - 1) + 3 - 1$
$= (y - 1)(y^2 - 2y + 3) + 2$
$\therefore$ Quotient $= y^2 - 2y + 3$,
Remainder = 2.

Practice Set 3.4 Algebra 9th Std Maths Part 1 Answers Chapter 3 Polynomials

Question 1.
For $x = 0$, find the value of the polynomial $x^2 - 5x + 5$.

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Solution:
$p(x) = x^2 – 5x + 5$
Put x = 0 in the given polynomial.
$\therefore P(0) = (0)^2 – 5(0) + 5$
$= 0 – 0 + 5$
$\therefore p(0) = 5$

Question 2.
If $p(y) = y^2 – 3\sqrt{2} + 1$, then find $p(3\sqrt{2})$.
Solution:
$p(y) = y^2 – 3\sqrt{2}\, y + 1$
Putp= $3\sqrt{2}$ in the given polynomial.
$\therefore p(3\sqrt{2}) = (3\sqrt{2})^2 – 3\sqrt{2} (3\sqrt{2}) + 1$
$= 9 \times 2 – 9 \times 2 + 1$
$= 18 – 18 + 1$
$\therefore p(3\sqrt{2}) = 1$

Question 3.
If $p(m) = m^3 + 2m^2 – m + 10$, then P(a) + p(-a) = ?
Solution:
$p(m) = m^3 + 2m^2 – m + 10$
Put m = a in the given polynomial.
$\therefore p(a) = a^3 + 2a^2 – a + 10$ ...(i)
Put m = -a in the given polynomial.
$p(-a) = (-a)^3 + 2(-a)^2 – (-a) +10$
$\therefore p (-a) = -a^3 + 2a^2 + a + 10$ ...(ii)
Adding (i) and (ii),
$p(a) + p(-a) = (a^3 + 2a^2 – a + 10) + (-a^3 + 2a^2 + a + 10)$
$= \underline{a^3 – a^3} + \underline{2a^2 + 2a^2} – \underline{a + a} + \underline{10 + 10}$
$\therefore p(a) + p(-a) = 4a^2 + 20$

Question 4.
If $p(y) = 2y^3 – 6y^2 – 5y + 7$, then find p(2).
Solution:
$p(y) = 2y^3 – 6y^2 – 5y + 7$
Put y = 2 in the given polynomial.
$\therefore p(2) = 2(2)^3 – 6(2)^2 – 5(2) + 7$

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= 2 x 8 – 6 x 4 – 10 + 7
= 16 – 24 – 10 + 7
∴ P(2) = -11

Practice Set 3.5 Algebra 9th Std Maths Part 1 Answers Chapter 3 Polynomials

Question 1.
Find the value of the polynomial $2x – 2x^3 + 7$ using given values for x.
i. x = 3
ii. x = -1
iii. x = 0
Solution:
i. $p(x) = 2x – 2x^3 + 7$
Put x = 3 in the given polynomial.
∴ $p(3) = 2(3) – 2(3)^3 + 7$
= 6 – 2 x 27 + 7
= 6 – 54 + 7
∴ P(3) = – 41

ii. $p(x) = 2x – 2x^3 + 7$
Put x = -1 in the given polynomial.
∴ $p(- 1) = 2(- 1) – 2(-1)^3 + 7$
= – 2 – 2(-1) + 7
= -2 + 2 + 7
∴ p(-1) = 7

iii. $p(x) = 2x – 2x^3 + 7$
Put x = 0 in the given polynomial.
∴ $p(0) = 2(0) – 2(0)^3 + 7$
= 0 – 0 + 7
∴ P(0) = 7

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- *Arjun*

Question 2.
For each of the following polynomial, find p(1), p(0) and p(- 2).
i. $p(x) = x^3$
ii. $p(y) = y^2 – 2y + 5$
ii. $p(y) = x^4 – 2x^2 + x$
Solution:
i. $p(x) = x^3$
$\therefore p(1) = 1^3 = 1$
$p(x) = x^3$
$\therefore p(0) = 0^3 = 0$
$p(x) = x^3$
$\therefore p(-2) = (-2)^3 = -8$

ii. $p(y) = y^2 – 2y + 5$
$\therefore p(1) = 1^2 – 2(1) + 5$
$= 1 – 2 + 5$
$\therefore P(1) = 4$
$p(y) = y^2 – 2y + 5$
$\therefore p(0) = 0^2 – 2(0) + 5$
$= 0 – 0 + 5$
$\therefore p(0) = 5$
$p(y) = y^2 – 2y + 5$
$\therefore p(- 2) = (- 2)^2 – 2(- 2) + 5$
$= 4 + 4 + 5$
$\therefore p(-2) = 13$

iii. $p(x) = x^4 – 2x^2 – x$
$\therefore p(1) = (1)^4 – 2(1)^2 – 1$
$= 1 – 2 – 1$
$\therefore p(1) = -2$
$\therefore p(x) = x^4 – 2x^2 – x$
$\therefore p(0) = (0)^4 – 2(0)^2 – 0$
$= 0 – 0 – 0$
$\therefore p(0) = 0$
$p(x) = x^4 – 2x^2 – x$
$\therefore p(-2) = (-2)^4 – 2(-2)^2 – (-2)$
$= 16 – 2(4) + 2$

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$= 16 – 8 + 2$
$\therefore p(-2) = 10$

Question 3.
If the value of the polynomial $m^3 + 2m + a$ is 12 for m = 2, then find the value of a.
Solution:
$p(m) = m^3 + 2m + a$
$\therefore p(2) = (2)^3 + 2(2) + a$
$\therefore 12 = 8 + 4 + a$ ... [$\because p(2) = 12$]
$\therefore 12 = 12 + a$
$\therefore a = 12 – 12$
$\therefore a = 0$

Question 4.
For the polynomial $mx^2 – 2x + 3$ if $p(-1) = 7$, then find m.
Solution:
$p(x) = mx^2 – 2x + 3$
$\therefore p(-1) = m(-1)^2 – 2(-1) + 3$
$\therefore 7 = m(1) + 2 + 3$ ...[$\because p(-1) = 7$]
$\therefore 7 = m + 5$
$\therefore m = 7 – 5$
$\therefore m = 2$

Question 5.
Divide the first polynomial by the second polynomial and find the remainder using remainder theorem.
i. $(x^2 – 1x + 9); (x + 1)$
ii. $(2x^3 – 2x^2 + ax – a); (x – a)$
iii. $(54m^3 + 18m^2 – 27m + 5); (m – 3)$
Solution:
i. $p(x) = x^2 – 7x + 9$
Divisor = x + 1
$\therefore$ take $x = – 1$
$\therefore$ By remainder theorem,
$\therefore$ Remainder $= p(-1)$
$p(x) = x^2 – 7x + 9$

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$\therefore p(-1) = (-1)^2 - 7(-1) + 9$
$= 1 + 7 + 9$
$\therefore$ Remainder $=17$

ii. $p(x) = 2x^3 - 2x^2 + ax - a$
Divisor $= x - a$
$\therefore$ take $x = a$
By remainder theorem,
Remainder $= p(a)$
$p(x) = 2x^3 - 2x^2 + ax - a$
$\therefore p(a) = 2a^3 - 2a^2 + a(a) - a$
$= 2a^3 - 2a^2 + a^2 - a$
$\therefore$ Remainder $= 2a^3 - a^2 - a$

iii. $p(m) = 54m^3 + 18m^2 - 27m + 5$
Divisor $= m - 3$
$\therefore$ take $m = 3$
$\therefore$ By remainder theorem,
Remainder $= p(3)$
$p(m) = 54m^3 + 18m^2 - 27m + 5$
$\therefore p(3) = 54(3)^3 + 18(3)^2 - 27(3) + 5$
$= 54(27) + 18(9) - 81 + 5$
$= 1458 + 162 - 81 + 5$
$\therefore$ Remainder $= 1544$

Question 6.
If the polynomial $y^3 - 5y^2 + 7y + m$ is divided by $y + 2$ and the remainder is 50, then find the value of m.
Solution:
$p(y) = y^3 - 5y^2 + 7y + m$
Divisor $= y + 2$
$\therefore$ take $y = -2$
$\therefore$ By remainder theorem,
Remainder $= p(-2) = 50$
$P(y) = y^3 - 5y^2 + 7y + m$
$\therefore P(-2) = (-2)^3 - 5(-2)^2 + 7(-2) + m$
$\therefore 50 = -8 - 5(4) - 14 + m$

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$\therefore$ 50 = -8 – 20 – 14 + m
$\therefore$ 50 = – 42 + m
$\therefore$ m = 50 + 42
$\therefore$ m = 92

Question 7.
Use factor theorem to determine whether x + 3 is a factor of x^2 + 2x – 3 or not.
Solution:
p(x) = x^2 + 2x – 3
Divisor = x + 3
$\therefore$ take x = – 3
$\therefore$ Remainder = p(-3)
p(x) = x^2 + 2x – 3
$\therefore$ p(-3) = $(-3)^2$ + 2(- 3) – 3
= 9 – 6 – 3
$\therefore$ p(-3) = 0
$\therefore$ By factor theorem, x + 3 is a factor of x^2 + 2x – 3.

Question 8.
If (x – 2) is a factor of x^3 – mx^2 + 10x – 20, then find the value of m.
Solution:
p(x) = x^3 – mx^2 + 10x – 20 x – 2 is a factor of x3 – mx2 + lOx – 20.
$\therefore$By factor theorem,
Remainder = p(2) = 0
p(x) = x^3 – mx^2 + 10x – 20
$\therefore$ p(2) = $(2)^3$ – m$(2)^2$ + 10(2) – 20
$\therefore$ 0 = 8 – 4m + 20 – 20
$\therefore$ 0 = 8 – 4m
$\therefore$ 4m = 8
$\therefore$ m = 2

Question 9.
By using factor theorem in the following examples, determine whether q(x) is a factor of p(x) or not.
i. p(x) = x^3 – x^2 – x -1 ; q(x) = x – 1
ii. p(x) = 2x^3 – x^2 – 45 ; q(x) = x – 3

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Solution:
i. $p(x) = x^3 – x^2 – x – 1$
Divisor = $q(x) = x – 1$
∴ take $x = 1$
Remainder = $p(1)$
$p(x) = x^3 – x^2 – x – 1$
∴ $P(1) = (1)^3 – (1)^2 – 1 – 1$
$= 1 – 1 – 1 – 1$
$= -2 \neq 0$
∴ By factor theorem, $x – 1$ is not a factor of $x^3 – x^2 – x – 1$.

ii. $p(x) = 2x^3 – x – 45$
Divisor = $q(x) = x – 3$
take $x = 3$
Remainder = $p(3)$
$p(x) = 2x^3 – x^2 – 45$
$P(3) = 2(3)^3 – (3)^2 – 45$
$= 2(27) – 9 – 45$
$= 54 – 9 – 45$
$= 0$
∴ By factor theorem, $x – 3$ is a factor of $2x^3 – x^2 – 45$.

Question 10.
If $(x^{31} + 31)$ is divided by $(x + 1)$, then find the remainder.
Solution:
$p(x) = x^{31} + 31$
Divisor = $x + 1$
∴ take $x = – 1$
∴ By remainder theorem,
Remainder = $p(-1)$
$p(x) = x^{31} + 31$ ...
∴ $p(-1) = (-1)^{31} + 31$
$= -1 + 31 = 30$
∴ Remainder = 30

Question 11.
Show that $m – 1$ is a factor of $m^{21} – 1$ and $m^{22} – 1$. [3 Marks]

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Solution:
i. $p(m) = m^{21} – 1$
Divisor = m – 1
∴ take m = 1
Remainder = p(1)
$p(m) = m^{21} – 1$
∴ $P(1) = 1^{21} – 1 = 1 – 1 = 0$
∴ By factor theorem, m -1 is a factor of m^{21} -1.

ii. $p(m) = m^{22} – 1$
Divisor = m – 1
∴ take m = 1
Remainder = p(1)
$p(m) = m^{22} – 1$
∴ $P(1) = 1^{22} – 1 = 1 – 1 = 0$
∴ By factor theorem, m -1 is a factor of $m^{22} – 1$.

Question 12.
If x – 2 and $x – 12$ both are the factors of the polynomial $nx^2 – 5x + m$, then show that m = n = 2.
Solution:
$p(x) = nx^2 – 5x + m$
(x – 2) is a factor of $nx^2 – 5x + m$.
∴ By factor theorem,
P(2) = 0
∴ $p(x) = nx^2 – 5x + m$
∴ $p(2) = n(2)^2 – 5(2) + m$
∴ 0 = n(4) – 10 + m
∴ 4n – 10 + m = 0 …(i)
Also, ($x = 12$) is a factor of $nx^2 – 5x + m$.
∴ By factor theorem,
$p(12) = 0$
$p(x) = nx^2 – 5x + m$
∴ $p(12) = n(12)^2 – 512 + m$
$0 = n4 – 52 + m$
∴ 0 = n- 10 +4m … [Multiplying both sides by 4]
∴ n = 10 – 4m ……(ii)

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Substituting n = 10 – 4m in equation (i),
4(10 – 4m) – 10 + m = 0
∴ 40 – 16m – 10 + m = 0
∴ -15m+ 30 = 0
∴ -15m = -30
∴ m = 2
Substituting m = 2 in equation (ii),
n = 10 – 4(2)
= 10 – 8
∴ n = 2
∴ m = n = 2
Question 13.
i. If p(x) = 2 + 5x, then find the value of p(2) + p(- 2) – p(1).
Solution:
p(x) = 2 + 5x
∴ P(2) = 2 + 5(2)
= 2 + 10
= 12
p(x) = 2 + 5x
P(- 2) = 2 + 5(- 2)
= 2 – 10 = – 8
p(x) = 2 + 5x
P(1) = 2 + 5(1)
= 2 + 5 = 7
∴ P(2) + P(- 2) – p(1) = 12 + (- 8) – 7
∴ P(2) + p(- 2) – p(1) = – 3

ii. If $p(x) = 2x^2 – 5\sqrt{3}\, x + 5$, then find the value of $p(5\sqrt{3})$.
Solution:
$p(x) = 2x^2 – 5\sqrt{3}\, x + 5$
$\therefore p(5\sqrt{3}) = 2(5\sqrt{3})^2 – 5\sqrt{3}\,(5\sqrt{3}) + 5$
= 2 (25 x 3) – 25 x 3 + 5
= 150-75 + 5
$\therefore p(5\sqrt{3}) = 80$

Question 1.
1. Divide $p(x) = 3x^2 + x + 7$ by x + 2. Find the remainder.

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2. Find the value of $p(x) = 3x^2 + x + 7$ when $x = -2$.
3. See whether remainder obtained by division is same as the value of p(-2). Take one more example and verify. (Textbook pg. no. 50)

Solution:

1.
$$
\begin{array}{r}
3x-5 \\
x+2\overline{)\,3x^2+x+7} \\
3x^2+6x \\
\hline
-5x+7 \\
-5x-10 \\
\hline
17
\end{array}
$$

∴ Remainder = 17

2. $p(x) = 3x^2 + x + 7$
Substituting $x = -2$, we get
$p(-2) = 3(2)^2 + (-2) + 7$
$= 12 - 2 + 7$
∴ $p(-2) = 17$

3. Yes, remainder = p(-2)

Another Example:
If the polynomial $t^3 - 3t^2 + kt + 50$ is divided by $(t - 3)$, the remainder is 62. Find the value of k.
Solution:
When given polynomial is divided by $(t - 3)$ the remainder is 62. It means the value of the polynomial when $t = 3$ is 62.
$p(t) = t^3 - 3t^3 + kt + 50$
By remainder theorem,
Remainder = $p(3) = 33 - 3^2 + k \times 3 + 50$
$= 27 - 3 \times 9 + 3k + 50$
$= 27 - 27 + 3k + 50$
$= 3k + 50$
But remainder is 62.
∴ $3k + 50 = 62$
∴ $3k = 62 - 50$

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$\therefore 3k = 12$
$\therefore k = 4$

Question 2.
Verify that (x – 1) is a factor of the polynomial $x^3 + 4x – 5$. (Textbook pg. no. 51)
Solution:
Here, $p(x) = x^3 + 4x – 5$
Substituting x = 1 in p(x), we get
$p(1) = (1)^3 + 4(1) – 5$
$= 1 + 4 – 5$
$P(1) = 0$
$\therefore$ By remainder theorem,
Remainder = 0
$\therefore$ (x -1) is the factor of $x^3 + 4x – 5$.

Practice Set 3.6 Algebra 9th Std Maths Part 1 Answers Chapter 3 Polynomials

Question 1.
Find the factors of the polynomials given below:
i. $2x^2 + x – 1$
ii. $2m^2 + 5m – 3$
iii. $12x^2 + 61x + 77$
iv. $3y^2 – 2y – 1$
v. $\sqrt{3}x^2 + 4x + \sqrt{3}$
vi. $12x^2 – 3x + 4$
Solution:
i. $2x^2 + x – 1$
$= 2x^2 + 2x – x – 1$
$= 2x(x + 1)- 1(x + 1)$

= (x + 1)(2x – 1)

$2 \times -1 = -2$

$2 \quad -1$

$2 \times -1 = -2$

$2 - 1 = 1$

ii. $2m^2 + 5m - 3$

$= 2m^2 + 6m - m - 3$

$= 2m(m + 3) - 1(m + 3)$

$= (m + 3)(2m - 1)$

$2 \times -3 = -6$

$6 \quad -1$

$6 \times -1 = -6$

$6 - 1 = 5$

iii. $12x^2 + 61x + 77$

$= 12x^2 + 28x + 33x + 77$

$= 4x(3x + 7)\ 4 + 11(3x + 7)$

$= (3x + 7)(4x + 11)$

$12 \times 77 = 4 \times 3 \times 11 \times 7$

$= 28 \times 33$

$28 \quad 33$

$28 + 33 = 61$

$28 \times 33 = 924$

iv. $3y^2 - 2y - 1$

$= 3y^2 - 3y + y - 1$

$= 3y(y - 1) + 1\ (y - 1)$

$= (y - 1)(3y + 1)$

$3 \times -1 = -3$

$-3 \quad 1$

$-3 \times 1 = -3$

$-3 + 1 = -2$

v. $\sqrt{3} \times 2 + 4x + \sqrt{3}$

$= \sqrt{3} \times 2 + 3x + x + \sqrt{3}$

$= \sqrt{3} \times 2 + \sqrt{3}\ x\ \sqrt{3}x + x + \sqrt{3}$

$= \sqrt{3}x(x + \sqrt{3}) + 1\ (\ x + \sqrt{3}\ )$

= (x + √3)(√3x + 1)

$\sqrt{3} \times \sqrt{3} = 3$

3 1

$3 \times 1 = 3$

$3 + 1 = 4$

vi. $12x^2 - 3x + 4$

$= 12x^2 - 2x - x + 4$

$= 12x^2 - 2 \times 2 2x - x + 4$

$= 12\ x(x - 4) - 1\ (x - 4)$

$= (x - 4)\ (12\ x - 1)$

$\frac{1}{2} \times 4 = 2$

−2 −1

$-2 \times -1 = 2$

$-2 - 1 = -3$

Alternate method

$12\ x^2 - 3x + 4 = 12\ (x^2 - 6x + 8)$

$= 12\ (x^2 - 4x - 2x + 8)$

$= 12\ [x(x - 4) - 2(x - 4)]$

$= 12\ (x - 2)(x - 4)$

8

−4 −2

$-4 \times -2 = 8$

$-4 - 2 = -6$

Question 2.

Factorize the following polynomials.

i. $(x^2 - x)^2 - 8(x^2 - x) + 12$

iii. $(x^2 - 6x)^2 - 8(x^2 - 6x + 8) - 64$

v. $(y + 2)\ (y - 3)\ (y + 8)\ (y + 3) + 56$

vii. $(x - 3)\ (x - 4)^2\ (x - 5) - 6$

Solution:

i. $(x^2 - x)^2 - 8(x^2 - x) + 12$

$= m^2 - 8m + 12$ ...[Putting $x^2 - x = m$]

$= m^2 - 6m - 2m + 12$

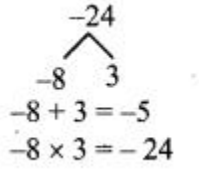
$= m(m - 6) - 2(m - 6)$

$= (m - 6)(m - 2)$

$= (x^2 - x - 6)\ (x^2 - x - 2)$ ...[Replacing $m = x^2 - x$]

$= (x^2 – 3x + 2x – 6) (x^2 – 2x + x – 2)$
$= [x(x – 3) + 2(x – 3)] [x(x – 2) + 1 (x-2)]$
$= (x – 3) (x + 2) (x – 2) (x + 1)$

ii. $(x – 5)^2 – (5x – 25) – 24$
$= (x – 5)^2 – (5x – 25) – 24$
$= (x – 5)^2 – 5(x – 5) – 24$
$= m^2 – 5m – 24$ … [Putting $x – 5 = m$]
$= m^2 – 8m + 3m – 24$
$= m(m – 8) + 3(m – 8)$
$= (m – 8) (m + 3)$
$= (x – 5 – 8) (x – 5 + 3)$ … [Replacing $m = x – 5$]
$= (x – 13) (x – 2)$



iii. $(x^2 – 6x)^2 – 8(x^2 – 6x + 8) – 64$
$= m^2 – 8(m + 8)-64$ …[Putting $x^2 – 6x = m$]
$= m^2 – 8m – 64 – 64$
$= m^2 – 8m – 128$
$= m^2 – 16m + 8m- 128$
$= m(m – 16) + 8(m – 16)$
$= (m – 16)(m + 8)$
$= (x^2 – 6x – 16) (x^2 – 6x + 8)$ … [Replacing $m = x^2 – 6x$]
$= (x^2 – 8x + 2x – 16) (x^2 – 4x – 2x + 8)$
$= [x(x – 8) + 2(x – 8)] [x(x – 4) – 2(x – 4)]$
$= (x – 8) (x + 2) (x – 4) (x – 2)$

iv. $(x^2– 2x + 3) (x^2 – 2x + 5) – 35$
$= (m + 3) (m + 5) – 35$ … [Putting $x^2 – 2x = m$]
$= m (m + 5) + 3(m + 5) – 35$
$= m^2 + 5m + 3m + 15 – 35$
$= m^2 + 8m – 20$
$= m^2 + 10m – 2m – 20$
$= m(m + 10) – 2(m + 10)$

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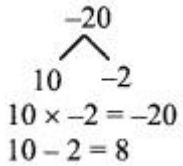
- *Arjun*

= (m + 10) (m – 2)
= (x^2 – 2x + 10) (x^2 – 2x – 2) … [Replacing m = x^2 – 2x]



v. (y + 2) (y – 3) (y + 8) (y + 3) + 56
= (y + 2)(y + 3)(y – 3)(y + 8) + 56
= (y^2 + 3y + 2y + 6) (y^2 + 8y – 3y – 24) + 56
= (y^2 + 5y + 6) (y^2 + 5y – 24) + 56
= (m + 6) (m – 24) + 56 … [Putting y2 + 5y = m]
= m (m – 24) + 6 (m – 24) + 56
= m^2 – 24m + 6m – 144 + 56
= m^2 – 18m – 88
= m^2 – 22m + 4m – 88
= m(m – 22) + 4(m – 22)
= (m – 22) (m + 4)
= (y^2 + 5y – 22)(y^2 + 5y + 4) … [Replacing m = y^2 + 5y]
= (y^2 + 5y – 22) (y^2 + 4y + y + 4)
= (y^2 + 5y – 22) [y(y + 4) + 1(y + 4)]
= (y^2 + 5y – 22) (y + 4) (y + 1)

vi. (y^2 + 5y) (y^2 + 5y – 2) – 24
= (m)(m – 2) – 24 … [Putting y^2 + 5y = m]
= m^2 – 2m – 24
= m^2 – 6m + 4m – 24
= m(m – 6) + 4(m – 6)
= (m – 6) (m + 4)
= (y^2 + 5y – 6) (y^2 + 5y + 4) … [Replacing m = y^2 + 5y]
= (y^2 + 6y – y – 6) (y^2 + 4y + y + 4)
= [y(y + 6) – 1(y + 6)] [y(y + 4) + 1(y + 4)]
= (y + 6) (y – 1) (y + 4) (y + 1)

vii. (x – 3) (x – 4)2 (x – 5) – 6
= (x – 3) (x – 5) $(x – 4)^2$ – 6
= (x^2 – 5x – 3x + 15) (x^2 – 8x + 16) – 6
= (x^2 – 8x + 15) (x^2 – 8x + 16) – 6

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- *Arjun*

= (m + 15) (m+ 16) – 6 … [Putting $x^2 – 8x = m$]
= m (m + 16) + 15 (m + 16) – 6
= m^2 + 16m + 15m + 240 – 6
= m^2 + 31m + 234
= m^2 + 18m + 13m + 234
= m(m + 18) + 13(m + 18)
= (m + 18) (m + 13)
= ($x^2 – 8x + 18$) ($x^2 – 8x + 13$) … [Replacing $m = x^2 – 8x$]

Problem Set 3 Algebra 9th Std Maths Part 1 Answers Chapter 3 Polynomials

Question 1.
Write the correct alternative answer for each of the following questions.

i. Which of the following is a polynomial?

(A) $\frac{x}{y}$ (B) $x^{\sqrt{2}} - 3x$

(C) $x^{-2} + 7$ (D) $\sqrt{2}\,x^2 + \frac{1}{2}$

Answer:
(D) $\sqrt{2}x^2 + 12$
ii. What is the degree of the polynomial $\sqrt{7}$?
(A) 12
(B) 5
(C) 2
(D) 0
Answer:
(D) 0
iii. What is the degree of the polynomial ?
(A) 0
(B) 1
(C) undefined

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(D) any real number
Answer:
(C) undefined

iv. What is the degree of the polynomial $2x^2$ + 5xsup>3 + 7?
(A) 3
(B) 2
(C) 5
(D) 7
Answer:
(A) 3

v. What is the coefficient form of x^3 – 1 ?
(A) (1, -1)
(B) (3, -1)
(C) (1, 0, 0, -1)
(D) (1, 3, -1)
Answer:
(C) (1, 0, 0, -1)

vi. p(x) = x^2 – x + 3, then p (7√7) = ?
(A) 3
(B) 7√7
(C) 42√7+3
(D) 49√7
Answer:
(D) 49√7

vii. When x = – 1, what is the value of the polynomial $2x^3$ + 2x ?
(A) 4
(B) 2
(C) -2
(D) -4
Answer:
(A) 4

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viii. If x – 1 is a factor of the polynomial $3x^2 + mx$, then find the value of m.
(A) 2
(B) -2
(C) -3
(D) 3
Answer:
(C) -3

ix. Multiply $(x^2 – 3) (2x – 7x^3 + 4)$ and write the degree of the product.
(A) 5
(B) 3
(C) 2
(D) 0
Answer:
(A) 5

x. Which is the following is a linear polynomials?
(A) $x + 5$
(B) $x^2 + 5$
(C) $x^3 + 5$
(D) $x^4 + 5$
Answer:
(A) $x + 5$

Hints:
v. $x^3 – 1 = x^3 + 0x^2 + 0x – 1$

vi. $p(7\sqrt{7}) = (7\sqrt{7})^2 (7\sqrt{7}) (7\sqrt{7}) + 3$
$= 3$

vii. $p(-1) = 2(-1)^3 + 2(-1)$
$= -2 – 2 = -4$

vii. $p(1) = 0$
$\therefore 3(1)^2 + m(1) = 0$
$\therefore 3 + m = 0$
$\therefore m = -3$

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ix. Here, degree of first polynomial = 2 and
degree of second polynomial 3
∴ Degree of polynomial obtained by multiplication = 2 + 3 = 5

Question 2.
Write the degree of the polynomial for each of the following.
i. $5 + 3x^4$
ii. 7
iii. $ax^7 + bx^9$ (a, b are constants)
Answer:
i. $5 + 3x^4$
Here, the highest power of x is 4.
∴Degree of the polynomial = 4

ii. $7 = 7x^\circ$
∴ Degree of the polynomial = 0

iii. $ax^7 + bx^9$
Here, the highest power ofx is 9.
∴Degree of the polynomial = 9

Question 3.
Write the following polynomials in standard form. [1 Mark each]
i. $4x^2 + 7x^4 – x^3 – x + 9$
ii. $p + 2p^3 + 10p^2 + 5p^4 – 8$
Answer:
i. $7x^4 – x^3 + 4x^2 – x + 9$
ii. $5p^4 + 2p^3 + 10p^2 + p – 8$

Question 4.
Write the following polynomial in coefficient form.
i. $x^4 + 16$
ii. $m^5 + 2m^2 + 3m+15$
Answer:
i. $x^4 + 16$
Index form = $x^4 + 0x^3 + 0x^2 + 0x + 16$
∴ Coefficient form of the polynomial = (1,0,0,0,16)

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ii. $m^5 + 2m^2 + 3m + 15$
Index form = $m^5 + 0m^4 + 0m^3 + 2m^2 + 3m + 15$
∴ Coefficient form of the polynomial = (1, 0, 0, 2, 3, 15)

Question 5.
Write the index form of the polynomial using variable x from its coefficient form.
i. (3, -2, 0, 7, 18)
ii. (6, 1, 0, 7)
iii. (4, 5, -3, 0)
Answer:
i. Number of coefficients = 5
∴ Degree = 5 – 1 = 4
∴Index form = $3x^4 – 2x^3 + 0x^2 + 7x + 18$

ii. Number of coefficients = 4
∴Degree = 4 – 1 = 3
∴ Index form = $6x^3 + x^2 + 0x + 7$

iii. Number of coefficients = 4
∴ Degree = 4 – 1 = 3
∴ Index form = $4x^3 + 5x^2 – 3x + 0$

Question 6.
Add the following polynomials.
i. $7x^4 – 2x^3 + x + 10$;
$3x^4 + 15x^3 + 9x^2 – 8x + 2$
ii. $3p^3q + 2p^2q + 7$;
$2p^2q + 4pq – 2p^3q$
Solution:
i. $(7x^4 – 2x^3 + x + 10) + (3x^4 + 15x^3 + 9x^2 – 8x + 2)$
$= 7x^4 – 2x^3 + x + 10 + 3x^4 + 15x^3 + 9x^2 – 8x + 2$
$= 7x^4 + 3x^4 – 2x^3 + 1\ 5x^3 + 9x^2 + x – 8x + 10 + 2$
$= 10x^4 + 13x^3 + 9x^2 – 7x + 12$

ii. $(3p^3q + 2p^2q + 7) + (2p^2q + 4pq – 2p^3q)$
$= 3p^3q + 2p^2q + 7 + 2p^2q + 4pq – 2p^3q$

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$= 3p^3q – 2p^3q + 2p^2q + 2p^2q + 4pq + 7$
$= p^3q + 4p^2q + 4pq + 7$

Question 7.
Subtract the second polynomial from the first.
i. $5x^2 – 2y + 9 ; 3x^2 + 5y – 7$
ii. $2x^2 + 3x + 5 ; x^2 – 2x + 3$
Solution:
i. $(5x^2 – 2y + 9) – (3x^2 + 5y – 7)$
$= 5x^2 – 2y+ 9 – 3x^2 – 5y + 1$
$= 5x^2 – 3x^2 – 2y – 5y + 9 + 7$
$= 2x^2 – 1y + 16$

ii. $(2x^2+ 3x + 5) – (x^2 – 2x + 3)$
$= 2x^2 + 3x + 5 – x^2 + 2x – 3$
$= 2x^2 – x^2 + 3x + 2x + 5 – 3$
$= x^2 + 5x + 2$

Question 8.
Multiply the following polynomials.
i. $(m^3 – 2m + 3) (m^4 – 2m^2 + 3m + 2)$
ii. $(5m^3 – 2) (m^2 – m + 3)$
Solution:
i. $(m^3 – 2m + 3) (m^4 – 2m^2 + 3m + 2)$
$= m^3(m^4 – 2m^2 + 3m + 2) – 2m(m^4 – 2m^2 + 3m + 2) + 3(m^4 – 2m^2 + 3m + 2)$
$= m^7 – 2m^5 + 3m^4 + 2m^3 – 2m^5 + 4m^3 – 6m^2 – 4m + 3m^4 – 6m^2 + 9m + 6$
$= m^7 – 2m^5 – 2m^5 + 3m^4 + 3m^4 + 2m^3 + 4m^3 – 6m^2 – 6m^2 – 4m + 9m + 6$
$= m^7 – 4m^5 + 6m^4 + 6m^3 – 12m^2 + 5m + 6$

ii. $(5m^3 – 2) (m^2 – m + 3)$
$= 5m^3(m^2 – m + 3) – 2(m^2 – m + 3)$
$= 5m^5 – 5m^4 + 15m^3 – 2m^2 + 2m – 6$

Question 9.
Divide polynomial $3x^3 – 8x^2 + x + 7$ by $x – 3$ using synthetic method and write the quotient and remainder.
Solution:

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Dividend = $3x^3 – 8x^2 + x + 7$
∴ Coefficient form of dividend = (3, – 8, 1,7)
Divisor = x – 3
∴ Opposite of – 3 is 3

3	3	–8	1	7
		9	3	12
	3	1	4	19

Coefficient form of quotient = (3, 1,4)
∴ Quotient = $3x^2 + x + 4$ and
Remainder =19

Question 10.
For which value of m, x + 3 is the factor of the polynomial $x^3 – 2mx + 21$?
Solution:
Here, $p(x) = x^3 – 2mx + 21$
(x + 3) is a factor of $x^3 – 2mx + 21$.
∴ By factor theorem,
Remainder = 0
∴ P(- 3) = 0
$p(x) = x^3 – 2mx + 21$
∴ $p(-3) = (-3)^3 – 2(m)(-3) + 21$
∴ $0 = – 27 + 6m + 21$
∴ $6 + 6m = 0$
∴ $6m = 6$
∴ $m = 1$
∴ x + 3 is the factor of $x^3 – 2mx + 21$ for m = 1.

Question 11.
At the end of the year 2016, the population of villages Kovad, Varud, Chikhali is $5x^2 – 3y^2$, $7y^2 + 2xy$ and $9x^2 + 4xy$ respectively. At the beginning of the year 2017, $x^2 + xy – y^2$, 5xy and $3x^2 + xy$ persons from each of the three villages respectively went to another village for education, then what is the remaining total population of these three villages ?
Solution:
Total population of villages at the end of 2016 = $(5x^2 – 3y^2) + (7y^2 + 2xy) + (9x^2 + 4xy)$

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$= 5x^2 + 9x^2 – 3y^2 + 7y^2 + 2xy + 4xy$
$= 14x^2 + 4y^2 + 6xy$ .....(i)
Total number of persons who went to other village at the beginning of 2017 $= (x^2 + xy – y^2) + (5xy) + (3x^2 + xy)$
$= x^2 + 3x^2 – y^2 + xy + 5xy + xy$
$= 4x^2 – y^2 + 7xy$ ... (ii)
Remaining total population of villages = Total population at the end of 2016 – total number of persons who went to other village at the beginning of 2017
$= 14x^2 + 4y^2 + 6xy – (4x^2 – y^2 + 7xy)$ ... [From (i) and (ii)]
$= 14x^2 + 4y^2 + 6xy – 4x^2 + y^2 – 7xy$
$= 14x^2 – 4x^2 + 4y^2 + y^2 + 6xy – 7xy = 1$
$= 10x^2 + 5y^2 – xy$
∴ The remaining total population of the three villages is $10x^2 + 5y^2 – xy$.

Question 12.
Polynomials $bx^2 + x + 5$ and $bx^3 – 2x + 5$ are divided by polynomial $x – 3$ and the remainders are m and n respectively. If $m – n = 0$, then find the value of b.
Solution:
When polynomial $bx^2 + x + 5$ is divided by $(x – 3)$, the remainder is m.
∴ By remainder theorem,
Remainder $= p(3) = m$
$p(x) = bx^2 + x + 5$
$\therefore p(3) = b(3)^2 + 3 + 5$
$\therefore m = b(9) + 8$
$m = 9b + 8$ ...(i)
When polynomial $bx^3 – 2x + 5$ is divided by $x – 3$ the remainder is n
∴ remainder $= p(3) = n$
$p(x) = bx^3 – 2x + 5$
$\therefore P(3) = b(3)^3 – 2(3) + 5$
$\therefore n = b(27) – 6 + 5$
$\therefore n = 27b – 1$ ...(ii)
Now, $m – n = 0$ ...[Given]
$\therefore m = n$
$\therefore 9b + 8 = 27b – 1$ ...[From (i) and (ii)]
$\therefore 8 + 1 = 27b – 9b$

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∴ 9 = 18b
∴ b = 12
Question 13.
Simplify.
$(8m^2 + 3m – 6) – (9m – 7) + (3m^2 – 2m + 4)$
Solution:
$(8m^2 + 3m – 6) – (9m – 7) + (3m^2 – 2m + 4)$
$= 8m^2 + 3m – 6 – 9m + 7 + 3m^2 – 2m + 4$
$= 8m^2 + 3m^2 + 3m – 9m – 2m – 6 + 7 + 4$
$= 11m^2 – 8m + 5$

Question 14.
Which polynomial is to be subtracted from $x^2 + 13x + 7$ to get the polynomial $3x^2 + 5x – 4$?
Solution:
Let the required polynomial be A.
$\therefore (x^2 + 13x + 7) – A = 3x^2 + 5x – 4$
$\therefore A = (x^2 + 13x + 7) – (3x^2 + 5x – 4)$
$= x^2 + 13x + 7 – 3x^2 – 5x + 4$
$= x^2 – 3x^2 + 13x – 5x + 7+4$
$= -2x^2 + 8x + 11$
$\therefore – 2x^2 + 8x + 11$ must be subtracted from $x^2 + 13x + 7$ to get $3x^2 + 5x – 4$.

Question 15.
Which polynomial is to be added to 4m + 2n + 3 to get the polynomial 6m + 3n + 10?
Solution:
Let the required polynomial be A.
∴ (4m + 2n + 3) + A = 6m + 3n + 10
∴ A = 6m + 3n + 10 – (4m + 2n + 3)
= 6m + 3n + 10 – 4m – 2n – 3
= 6m – 4m + 3n – 2n + 10 – 3
= 2m + n + 7
∴ 2m + n + 7 must be added to 4m + 2n + 3 to get 6m + 3n + 10.

Question 1.
Read the following passage, write the appropriate amount in the boxes and

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discuss.

Govind, who is a dry land farmer from Shiralas has a 5 acre field. His family includes his wife, two children and his old mother. He borrowed one lakh twenty five thousand rupees from the bank for one year as agricultural loan at 10 p.c.p.a. He cultivated soyabean in x acres and cotton and tur in y acres. The expenditure he incurred was as follows :

He spent ₹10,000 on seeds. The expenses for fertilizers and pesticides for the soyabean crop was ₹ 2000x and ₹ $4000x^2$ were spent on wages and cultivation of land. He spent ₹ 8000y on fertilizers and pesticides and ₹ 9000y2 for wages and cultivation of land for the cotton and tur crops.

Let us write the total expenditure on all the crops by using variables x and y.

₹ $10000 + 2000x + 4000 \times 2 + 8000y + 9000y^2$

He harvested $5x^2$ quintals soyabean and sold it at ₹ 2800 per quintal. The cotton crop yield was $53\ y^2$ quintals which fetched ₹ 5000 per quintal.

The tur crop yield was 4y quintals and was sold at ₹ 4000 per quintal. Write the total income in rupees that was obtained by selling the entire farm produce, with the help of an expression using variables x and y. (Textbook pg. no. 44)

Answer:

Total income = income on soyabean crop + income on cotton crop + income on tur crop

= ₹ $(5x^2 \times 2800)$ + ₹$(53\ y^2 \times 5000)$ + ₹ $(4y \times 4000)$

= ₹ $(14000x^2 + 250003y^2 + 16000y)$

Question 2.

We have seen the example of expenditure and income (in terms of polynomials) of Govind who is a dry land farmer. He has borrowed rupees one lakh twenty-five thousand from the bank as an agriculture loan and repaid the said loan at 10 p.c.p.a. He had spent ₹ 10,000 on seeds. The expenses on soyabean crop was ₹ 2000x for fertilizers and pesticides and ₹ $4000x^2$ was spent on wages and cultivation. He spent ₹ 8000y on fertilizers and pesticides and ₹$9000y^2$ on cultivation and wages for cotton and tur crop.

His total income was

₹ $(14000x^2 + 250003y^2 + 16000y)$

By taking x = 2, y = 3 write the income expenditure account of Govind's

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farming. (Textbook pg. no. 52)

Solution:

–	Credit (Income)
₹ 1,25,000	Bank loan
₹ 56000	Income from soyabean
₹ 75000	Income from cotton
₹ 48000	Income from tur
₹ 304000	Total income
–	Debit (Expenses)
₹ 1,37,000	loan paid with interest for seeds
₹ 10000	For seeds
₹ 4000	Fertilizers and pesticides for soyabean
₹ 16000	Wages and cultivation charges for soyabean
₹ 24000	Fertilizers and pesticides for cotton & tur
₹ 81000	Wages and cultivation charges for cotton & tur
₹ 272000	Total expenditure