Maharashtra State Board 11th Maths Solutions Chapter 3 Permutations and Combination Ex 3.1

Question 1.
A teacher wants to select the class monitor in a class of 30 boys and 20 girls. In how many ways can the monitor be selected if the monitor must be a girl?
Solution:
There are 30 boys and 20 girls.
A teacher can select any boy as a class monitor from 30 boys in 30 different ways and he can select any girl as a class monitor from 20 girls in 20 different ways.
∴ by the fundamental principle of addition, the total number of ways a teacher can select a class monitor = 30 + 20 = 50
Hence, there are 50 different ways to select a class monitor.

Question 2.
A Signal is generated from 2 flags by putting one flag above the other. If 4 flags of different colours are available, how many different signals can be generated?
Solution:
A signal is generated from 2 flags and there are 4 flags of different colours available.
∴ 1st flag can be any one of the available 4 flags.
∴ It can be selected in 4 ways.
Now, 2nd flag is to be selected for which 3 flags are available for a different signal.
∴ 2nd flag can be anyone from these 3 flags.
∴ It can be selected in 3 ways.
∴ By using the fundamental principle of multiplication, total no. of ways a signal can be generated = 4 × 3 = 12
∴ 12 different signals can be generated.

Question 3.
How many two-letter words can be formed using letters from the word SPACE, when repetition of letters (i) is allowed (ii) is not allowed
Solution:
A two-letter word is to be formed out of the letters of the word SPACE.
(i) When repetition of the letters is allowed 1st letter can be selected in 5 ways.
2nd letter can be selected in 5 ways.
∴ By using the fundamental principle of multiplication, the total number of 2-letter words = 5 × 5 = 25

(ii) When repetition of the letters is not allowed 1st letter can be selected in 5 ways.
2nd letter can be selected in 4 ways.
∴ By using the fundamental principle of multiplication, the total number of 2-letter words = 5 × 4 = 20

Question 4.
How many three-digit numbers can be formed from the digits 0, 1, 3, 5, 6 if repetitions of digits (i) are allowed (ii) are not allowed?
Solution:
A three-digit number is to be formed from the digits 0, 1, 3, 5, 6.
(i) When repetition of digits is allowed,
100's place digit should be a non-zero number.
Hence, it can be anyone from digits 1, 3, 5,6.
∴ 100's place digit can be selected in 4 ways.
10's and unit's place digit can be zero and digits can be repeated.
∴ 10's place digit can be selected in 5 ways and the unit's place digit can be selected in 5 ways.
∴ By using the fundamental principle of multiplication, the total number of three-digit numbers = 4 × 5 × 5 = 100

(ii) When repetition of digits is not allowed,
100's place digit should be a non-zero number.
Hence, it can be anyone from digits 1, 3, 5, 6.
∴ 100's place digit can be selected in 4 ways.
10's and unit's place digit can be zero and digits can't be repeated.
∴ 10's place digit can be selected in 4 ways and the unit's place digit can be selected in 3 ways.
∴ By using the fundamental principle of multiplication, the total number of two-digit numbers = 4 × 4 × 3 = 48

Question 5.
How many three-digit numbers can be formed using the digits 2, 3, 4, 5, 6 if digits can be repeated?
Solution:
A 3-digit number is to be formed from the digits 2, 3, 4, 5, 6 where digits can be repeated.
∴ Unit's place digit can be selected in 5 ways.
10's place digit can be selected in 5 ways.
100's place digit can be selected in 5 ways.
∴ By using fundamental principle of multiplication, total number of 3-digit numbers = 5 × 5 × 5 = 125

Question 6.
A letter lock contains 3 rings and each ring contains 5 letters. Determine the maximum number of false trails that can be made before the lock is opened.
Solution:
A letter lock has 3 rings, each ring containing 5 different letters.
∴ A letter from each ring can be selected in 5 ways.
∴ By using the fundamental principle of multiplication, a total number of trials that can be made = 5 × 5 × 5 = 125.
Out of these 124 wrong attempts are made and in the 125th attempt, the lock gets opened.
∴ A maximum number of false trials = 124.

Question 7.
In a test, 5 questions are of the form 'state, true or false. No student has got all answers correct. Also, the answer of every student is different. Find the number of students who appeared for the test.
Solution:
Every question can be answered in 2 ways. (True or False)
∴ By using the fundamental principle of multiplication, the total number of set of answers possible = 2 × 2 × 2 × 2 × 2 = 32.
Since One of them is the case where all questions are answered correctly,
The number of wrong answers = 32 – 1 = 31.
Since no student has answered all the questions correctly, the number of students who appeared for the test are 31.

Question 8.
How many numbers between 100 and 1000 have 4 in the unit's place?
Solution:
Numbers between 100 and 1000 are 3-digit numbers.
A 3-digit number is to be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, where the unit place digit is 4.
Unit's place digit is 4.
∴ it can be selected in 1 way only.
10's place digit can be selected in 10 ways.
For a 3-digit number, 100's place digit should be a non-zero number.
∴ 100's place digit can be selected in 9 ways.
∴ By using the fundamental principle of multiplication,
total numbers between 100 and 1000 which have 4 in the units place = 1 × 10 × 9 = 90.

Question 9.
How many numbers between 100 and 1000 have the digit 7 exactly once?
Solution:
Numbers between 100 and 1000 are 3-digit numbers.
A 3-digit number is to be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, where exactly one of the digits is 7.
When 7 is in unit's place:
Unit's place digit is 7.
∴ it can be selected in 1 way only.
10's place digit can be selected in 9 ways.
100's place digit can be selected in 8 ways.
Total numbers which have 7 in unit's place = 1 × 9 × 8 = 72.

When 7 is in 10's place:
Unit's place digit can be selected in 9 ways.
10's place digit is 7.
∴ it can be selected in 1 way only.
100's place digit can be selected in 8 ways.
∴ A total number of numbers which have 7 in 10's place = 9 × 1 × 8 = 72.

When 7 is in 100's place:
Unit's place digit can be selected in 9 ways.
10's place digit can be selected in 9 ways.
100's place digit is 7.
∴ it can be selected in 1 way only.
∴ Total numbers which have 7 in 100's place = 9 × 9 × 1 = 81.
∴ Total numbers between 100 and 1000 having digit 7 exactly once = 72 + 72 + 81 = 225

Question 10.
How many four-digit numbers will not exceed 7432 if they are formed using the digits 2, 3, 4, 7 without repetition?
Solution:
Between any set of digits, the greatest number is possible when digits are arranged in descending order.
∴ 7432 is the greatest number, formed from the digits 2, 3, 4, 7.
Since a 4-digit number is to be formed from the digits 2, 3, 4, 7, where repetition of the digit is not allowed,

1000’s place digit can be selected in 4 ways,
100’s place digit can be selected in 3 ways,
10’s place digit can be selected in 2 ways,
Unit’s place digit can be selected in 1 way.
∴ Total number of numbers not exceeding 7432 that can be formed with the digits 2, 3, 4, 7 = Total number of four-digit numbers possible from the digits 2, 3, 4, 7
= 4 × 3 × 2 × 1
= 24

Question 11.
If numbers are formed using digits 2, 3, 4, 5, 6 without repetition, how many of them will exceed 400?
Solution:
Case I: Number of three-digit numbers formed from 2, 3, 4, 5, 6, greater than 400.
100’s place can be filled by any one of the numbers 4, 5, 6.
100’s place digit can be selected in 3 ways.
Since repetition is not allowed, 10’s place can be filled by any one of the remaining four numbers.
∴ 10’s place digit can be selected in 4 ways.
Unit’s place digit can be selected in 3 ways.
∴ Total number of three-digit numbers formed = 3 × 4 × 3 = 36

Case II: Number of four-digit numbers formed from 2, 3, 4, 5, 6.
Since repetition of digits is not allowed,
1000’s place digit can be selected in 5 ways.
100’s place digit can be selected in 4 ways.
10’s place digit can be selected in 3 ways.
Unit’s place digit can be selected in 2 ways.
∴ Total number of four-digit numbers formed = 5 × 4 × 3 × 2 = 120

Case III: Number of five-digit numbers formed from 2, 3, 4, 5, 6.
Similarly, since repetition of digits is not allowed,
Total number of five digit numbers formed = 5 × 4 × 3 × 2 × 1 = 120.
∴ Total number of numbers that exceed 400 = 36 + 120 + 120 = 276

Question 12.
How many numbers formed with the digits 0, 1, 2, 5, 7, 8 will fall between 13 and 1000 if digits can be repeated?
Solution:
Case I: 2-digit numbers more than 13, less than 20, formed from the digits 0, 1, 2, 5, 7, 8.
10’s place digit is 1.
∴ it can be selected in 1 way only.
Unit’s place can be filled by any one of the numbers 5, 7, 8.
∴ Unit’s place digit can be selected in 3 ways.
∴ Total number of such numbers = 1 × 3 = 3.

Case II: 2-digit numbers more than 20 formed from 0, 1, 2, 5, 7, 8.
10’s place can be filled by any one of the numbers 2, 5, 7, 8.
∴ 10’s place digit can be selected in 4 ways.
Since repetition is allowed, the unit’s place can be filled by one of the remaining 6 digits.
∴ Unit’s place digit can be selected in 6 ways.
∴ Total number of such numbers = 4 × 6 = 24.

Case III: 3-digit numbers formed from 0, 1, 2, 5, 7, 8.
Similarly, since repetition of digits is allowed, the total number of such numbers = 5 × 6 × 6 = 180.
All cases are mutually exclusive.
∴ Total number of required numbers = 3 + 24 + 180 = 207

Question 13.
A school has three gates and four staircases from the first floor to the second floor. How many ways does a student have to go from outside the school to his classroom on the second floor?
Solution:
A student can go inside the school from outside in 3 ways and from the first floor to the second floor in 4 ways.
∴ A number of ways to choose gates = 3.
The number of ways to choose a staircase = 4.
By using the fundamental principle of multiplication,
number of ways in which a student has to go from outside the school to his classroom = 4 × 3 = 12

Question 14.
How many five-digit numbers formed using the digit 0, 1, 2, 3, 4, 5 are divisible by 5 if digits are not repeated?
Solution:

Here, repetition of digits is not allowed.
For a number to be divisible by 5,
unit's place digit should be 0 or 5.
Case I: when unit's place is 0
Unit's place digit can be selected in 1 way.
10's place digit can be selected in 5 ways.
100's place digit can be selected in 4 ways.
1000's place digit can be selected in 3 ways.
10000's place digit can be selected in 2 ways.
∴ Total number of numbers = 1 × 5 × 4 × 3 × 2 = 120.

Case II: when the unit's place is 5
Unit's place digit can be selected in 1 way.
10000's place should be a non-zero number.
∴ It can be selected in 4 ways.
1000's place digit can be selected in 4 ways.
100's place digit can be selected in 3 ways.
10's place digit can be selected in 2 ways.
∴ Total number of numbers = 1 × 4 × 4 × 3 × 2 = 96
∴ Total number of required numbers = 120 + 96 = 216

Maharashtra State Board 11th Maths Solutions Chapter 3 Permutations and Combination Ex 3.2

Question 1.
Evaluate:
(i) 8!
Solution:
8!
= 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1
= 40320

(ii) 10!
Solution:
10!
= 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1
= 3628800

(iii) 10! – 6!
Solution:
10! – 6!
= 10 × 9 × 8 × 7 × 6! – 6!
= 6! (10 × 9 × 8 × 7 – 1)
= 6! (5040 – 1)
= 6 × 5 × 4 × 3 × 2 × 1 × 5039
= 3628080

(iv) (10 – 6)!
Solution:
(10 – 6)!
= 4!
= 4 × 3 × 2 × 1
= 24

Question 2.
Compute:
(i) $12!6!$
Solution:
$12!6!=12\times11\times10\times9\times8\times7\times6!6!$
= 12 × 11 × 10 × 9 × 8 × 7
= 665280

(ii) $(126)!$
Solution:
$(126)!$
= 2!
= 2 × 1
= 2

(iii) (3 × 2)!
Solution:
(3 × 2)!
= 6!
= 6 × 5 × 4 × 3 × 2 × 1
= 720

(iv) 3! × 2!
Solution:
3! × 2!
= 3 × 2 × 1 × 2 × 1
= 12

(v) $9!3!6!$
Solution:
$9!3!6!=9\times8\times7\times6!(3\times2\times1)\times6!=84$

(vi) $6!-4!4!$
Solution:
$6!-4!4!=6\times5\times4!-4!4!=4!(6\times5-1)4!=29$

(vii) $8!6!-4!$
Solution:
$8!6!-4!=8\times7\times6\times5\times4!6\times5\times4!-4!$
$= 8\times7\times6\times5\times4!4!(6\times5-1)$
$= 168029$
= 57.93

(viii) $8!(6-4)!$
Solution:
$8!(6-4)!=8!2!$
$= 8\times7\times6\times5\times4\times3\times2!2!$
= 20160

Question 3.
Write in terms of factorials
(i) 5 × 6 × 7 × 8 × 9 × 10
Solution:
5 × 6 × 7 × 8 × 9 × 10 = 10 × 9 × 8 × 7 × 6 × 5
Multiplying and dividing by 4!, we get
$= 10\times9\times8\times7\times6\times5\times4!4!$
$= 10\times9\times8\times7\times6\times5\times4\times3\times2\times14!$
$= 10!4!$

(ii) 3 × 6 × 9 × 12 × 15
Solution:
3 × 6 × 9 × 12 × 15
= 3 × (3 × 2) × (3 × 3) × (3 × 4) × (3 × 5)

= (3^5) (5 × 4 × 3 × 2 × 1)
= 3^5 (5!)

(iii) 6 × 7 × 8 × 9
Solution:
6 × 7 × 8 × 9 = 9 × 8 × 7 × 6
Multiplying and dividing by 5!, we get
$= \frac{9\times8\times7\times6\times5!}{5!}$
$= \frac{9\times8\times7\times6\times5\times4\times3\times2\times1}{5!}$
$= \frac{9!}{5!}$

(iv) 5 × 10 × 15 × 20
Solution:
5 × 10 × 15 × 20
= (5 × 1) × (5 × 2) × (5 × 3) × (5 × 4)
= (5^4) (4 × 3 × 2 × 1)
= (5^4) (4!)

Question 4.
Evaluate: $\frac{n!}{r!(n-r)!}$ for
(i) n = 8, r = 6
(ii) n = 12, r = 12
(iii) n = 15, r = 10
(iv) n = 15, r = 8
Solution:
i. n = 8, r = 6

$$\therefore \frac{n!}{r!(n-r)!} = \frac{8!}{6!(8-6)!} = \frac{8\times7\times6!}{6!\,2!}$$
$$= \frac{8\times7}{2!} = \frac{8\times7}{1\times2} = 28$$

ii. n = 12, r = 12

$$\therefore \frac{n!}{r!(n-r)!} = \frac{12!}{12!(12-12)!} = \frac{12!}{12!\,0!}$$
$$= 1 \qquad \ldots[\because 0! = 1]$$

iii. n = 15, r = 10

$$\therefore \frac{n!}{r!(n-r)!} = \frac{15!}{10!(15-10)!} = \frac{15!}{10!\times5!}$$
$$= \frac{15\times14\times13\times12\times11\times10!}{10!\times(5\times4\times3\times2\times1)}$$
$$= \frac{15\times14\times13\times12\times11}{5\times4\times3\times2\times1}$$
$$= 7\times13\times3\times11$$
$$= 3003$$

iv. n = 15, r = 8

$$\therefore \frac{n!}{r!(n-r)!} = \frac{15!}{8!(15-8)!}$$
$$= \frac{15\times14\times13\times12\times11\times10\times9\times8!}{8!\times7!}$$
$$= \frac{15\times14\times13\times12\times11\times10\times9}{7\times6\times5\times4\times3\times2\times1}$$
$$= 5\times13\times11\times9$$
$$= 6435$$

Question 5.
Find n, if
(i) $\frac{n}{8!}=\frac{3}{6!}+\frac{1}{4!}$

Solution:

$$\frac{n}{8!}=\frac{3}{6!}+\frac{1}{4!}$$

$$\therefore \quad \frac{n}{8!}=\frac{3}{6!}+\frac{6\times5}{6\times5\times4!}$$

$$\therefore \quad \frac{n}{8!}=\frac{3}{6!}+\frac{30}{6!}$$

$$\therefore \quad \frac{n}{8\times7\times6!}=\frac{33}{6!}$$

$$\therefore \quad \frac{n}{56}=33$$

$$\therefore \quad n=56\times33=1848$$

(ii) $\frac{n}{6!}=\frac{4}{8!}+\frac{3}{6!}$

Solution:

$$\frac{n}{6!}=\frac{4}{8!}+\frac{3}{6!}$$

$$\therefore \quad \frac{n}{6!}-\frac{3}{6!}=\frac{4}{8!}$$

$$\therefore \quad \frac{n-3}{6!}=\frac{4}{8\times7\times6!}$$

$$\therefore \quad n-3=\frac{4}{8\times7}$$

$$\therefore \quad n-3=\frac{1}{14}$$

$$\therefore \quad n=\frac{1}{14}+3=\frac{43}{14}$$

(iii) $\frac{1}{n!}=\frac{1}{4!}-\frac{4}{5!}$

Solution:

$$\frac{1!}{n!}=\frac{1!}{4!}-\frac{4}{5!}$$

$$\therefore \quad \frac{1}{n!}=\frac{1}{4!}-\frac{4}{5!}$$

$$\therefore \quad \frac{1}{n!}=\frac{5}{5\times4!}-\frac{4}{5!}$$

$$\therefore \quad \frac{1}{n!}=\frac{5}{5!}-\frac{4}{5!}$$

$$\therefore \quad \frac{1}{n!}=\frac{1}{5!}$$

$$\therefore \quad n!=5!$$

$$\therefore \quad n=5$$

(iv) $(n + 1)! = 42 \times (n - 1)!$

Solution:

$(n + 1)! = 42(n – 1)!$

$\therefore (n + 1)\, n\, (n – 1)! = 42(n – 1)!$

$\therefore n^2 + n = 42$

$\therefore n^2 + n – 42 = 0$

$\therefore (n + 7)(n – 6) = 0$

$\therefore n = -7$ or $n = 6$

But $n \neq -7$ as $n \in N$

$\therefore n = 6$

(v) $(n + 3)! = 110 \times (n + 1)!$

Solution:

$(n + 3)! = (110)(n + 1)!$

$\therefore (n + 3)(n + 2)(n + 1)! = 110(n + 1)!$

$\therefore (n + 3)(n + 2) = (11)(10)$

Comparing on both sides, we get

n + 3 = 11
∴ n = 8

Question 6.
Find n, if:

(i) $\frac{(17-n)!}{(14-n)!}=5!$

Solution:

$$\frac{(17-n)!}{(14-n)!}=5!$$

$$\frac{(17-n)(16-n)(15-n)(14-n)!}{(14-n)!}=5\times4\times3\times2\times1$$

∴ (17 – n) (16 – n) (15 – n) = 6 × 5 × 4
Comparing on both sides, we get
17 – n = 6
∴ n = 11

(ii) $\frac{(15-n)!}{(13-n)!}=12$

Solution:

$\frac{(15-n)!}{(13-n)!}=12$

∴ $\frac{(15-n)(14-n)(13-n)!}{(13-n)!}=12$

∴ (15 – n) (14 – n) = 4 × 3
Comparing on both sides, we get
∴ 15 – n = 4
∴ n = 11

(iii) $\frac{n!}{3!(n-3)!}:\frac{n!}{5!(n-5)!}=5:3$

Solution:

$$\frac{n!}{3!(n-3)!}:\frac{n!}{5!(n-5)!}=5:3$$

$$\therefore \quad \frac{n!}{3!(n-3)!}\times\frac{5!(n-5)!}{n!}=\frac{5}{3}$$

$$\therefore \quad \frac{n!}{3!(n-3)(n-4)(n-5)!}\times\frac{5\times4\times3!(n-5)!}{n!}=\frac{5}{3}$$

$$\therefore \quad \frac{5\times4}{(n-3)(n-4)}=\frac{5}{3}$$

∴ 12 = (n – 3)(n – 4)
(n – 3)(n – 4) = 4 × 3
Comparing on both sides, we get
n – 3 = 4
∴ n = 7

(iv) $\frac{n!}{3!(n-3)!}:\frac{n!}{5!(n-7)!}=1:6$

Solution:

$$\frac{n!}{3!(n-3)!}:\frac{n!}{5!(n-7)!}=1:6$$

$$\therefore \quad \frac{n!}{3!(n-3)!}\times\frac{5!(n-7)!}{n!}=\frac{1}{6}$$

$$\therefore \quad \frac{n!}{3!(n-3)(n-4)(n-5)(n-6)(n-7)!}\times\frac{5\times4\times3!(n-7)!}{n!}=\frac{1}{6}$$

$$\therefore \quad \frac{5\times4}{(n-3)(n-4)(n-5)(n-6)}=\frac{1}{6}$$

∴ 120 = (n – 3)(n – 4) (n – 5)(n – 6)
∴ (n – 3)(n – 4) (n – 5)(n – 6) = 5 × 4 × 3 × 2
Comparing on both sides, we get
n – 3 = 5
∴ n = 8

(v) $\frac{(2n)!}{7!(2n-7)!} : \frac{n!}{4!(n-4)!} = 24:1$

Solution:

$$\frac{(2n)!}{7!(2n-7)!} : \frac{n!}{4!(n-4)!} = 24:1$$

$$\therefore \quad \frac{(2n)!}{7!(2n-7)!} \times \frac{4!(n-4)!}{n!} = 24$$

$$\therefore \quad \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)(2n-5)(2n-6)(2n-7)!}{7\times6\times5\times4!(2n-7)!} \times \frac{4!(n-4)!}{n(n-1)(n-2)(n-3)(n-4)!} = 24$$

$$\therefore \quad \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)(2n-5)(2n-6)}{7\times6\times5} \times \frac{1}{n(n-1)(n-2)(n-3)} = 24$$

$$\therefore \quad \frac{(2n)(2n-1)2(n-1)(2n-3)2(n-2)(2n-5)2(n-3)}{7\times6\times5} \times \frac{1}{n(n-1)(n-2)(n-3)} = 24$$

$$\therefore \quad \frac{16(2n-1)(2n-3)(2n-5)}{7\times6\times5} = 24$$

$(2n-1)(2n-3)(2n-5) = \frac{24\times7\times6\times5}{16}$

$\therefore (2n-1)(2n-3)(2n-5) = 9 \times 7 \times 5$

Comparing on both sides. We get

$\therefore 2n-1 = 9$

$\therefore n = 5$

Question 7.

Show that $\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} = \frac{(n+1)!}{r!(n-r+1)!}$

Solution:

$$\text{L.H.S.} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

$$= \frac{n!}{r(r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$

$$= \frac{(n+1).n!}{r.(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{(n+1)!}{r!(n-r+1)!}$$

$$= \text{R.H.S}$$

Question 8.

Show that $\frac{9!}{3!6!} + \frac{9!}{4!5!} = \frac{10!}{4!6!}$

Solution:

$$\text{L.H.S.} = \frac{9!}{3!\,6!} + \frac{9!}{4!\,5!} = \frac{9!}{3!\times6\times5!} + \frac{9!}{4\times3!\times5!}$$

$$= \frac{9!}{3!\,5!}\left[\frac{1}{6} + \frac{1}{4}\right]$$

$$= \frac{9!}{3!\,5!}\left[\frac{4+6}{6\times4}\right]$$

$$= \frac{10\times9!}{6\times5!\times4\times3!}$$

$$= \frac{10!}{6!\,4!} = \frac{10!}{4!\,6!}$$

$$= \text{R.H.S.}$$

Question 9.
Show that $\frac{(2n)!}{n!}$ = 2n (2n – 1)(2n – 3)…5.3.1
Solution:

$$\text{L.H.S.} = \frac{(2n)!}{n!}$$
$$= \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)...6\times5\times4\times3\times2\times1}{n!}$$
$$= \frac{(2n)(2n-1)[2(n-1)](2n-3)[2(n-2)]...(2\times3)\times5\times(2\times2)\times3\times(2\times1)\times1}{n!}$$
$$= \frac{2^n[n(n-1)(n-2)...3.2.1][(2n-1)(2n-3)...5.3.1]}{n!}$$
$$= \frac{2^n(n!)(2n-1)(2n-3)...5.3.1}{n!}$$
$$= 2^n(2n-1)(2n-3)\ldots5.3.1 = \text{R.H.S.}$$

Question 10.
Simplify
(i) $\frac{(2n+2)!}{(2n)!}$
Solution:

$$\frac{(2n+2)!}{(2n)!} = \frac{(2n+2)(2n+1)(2n)!}{(2n)!}$$
$$= (2n+2)(2n+1)$$

(ii) $\frac{(n+3)!}{(n^2-4)(n+1)!}$
Solution:

$$\frac{(n+3)!}{(n^2-4)(n+1)!} = \frac{(n+3)(n+2)(n+1)!}{(n-2)(n+2)(n+1)!}$$
$$= \frac{n+3}{n-2}$$

(iii) $\frac{1}{n!}-\frac{1}{(n-1)!}-\frac{1}{(n-2)!}$
Solution:

$$\frac{1}{n!}-\frac{1}{(n-1)!}-\frac{1}{(n-2)!}$$
$$= \frac{1}{n(n-1)(n-2)!}-\frac{1}{(n-1)(n-2)!}-\frac{1}{(n-2)!}$$
$$= \frac{1}{(n-2)!}\left[\frac{1}{n(n-1)}-\frac{1}{(n-1)}-1\right]$$
$$= \frac{1}{(n-2)!}\left[\frac{1-n-n(n-1)}{n(n-1)}\right]$$
$$= \frac{1-n^2}{n(n-1)(n-2)!}$$
$$= \frac{-(n-1)(n+1)}{n(n-1)(n-2)!}$$
$$= \frac{-(n+1)}{n(n-2)!}$$

(iv) n[n! + (n – 1)!] + n^2(n – 1)! + (n + 1)!

Solution:

$$\begin{aligned}
&n[n! + (n-1)!] + n^2(n-1)! + (n+1)! \\
&= n[n(n-1)! + (n-1)!] + n^2(n-1)! + (n+1)! \\
&= n(n-1)!\,[n+1] + n^2(n-1)! + (n+1)! \\
&= n(n-1)!\,(n+1) + n^2(n-1)! \\
&\qquad\qquad\qquad + (n+1)\,n(n-1)! \\
&= n(n-1)!\,[n+1+n+n+1] \\
&= n!(3n+2) \\
&= (3n+2)\,n!
\end{aligned}$$

(v) n+2n!−3n+1(n+1)!

Solution:

$$\begin{aligned}
\frac{n+2}{n!} - \frac{3n+1}{(n+1)!} &= \frac{n+2}{n!} - \frac{3n+1}{(n+1).n!} \\
&= \frac{(n+1)(n+2)-(3n+1)}{(n+1).n!} \\
&= \frac{n^2+3n+2-3n-1}{(n+1).n!} \\
&= \frac{n^2+1}{(n+1)!}
\end{aligned}$$

(vi) 1(n−1)!+1−n(n+1)!

Solution:

$$\begin{aligned}
\frac{1}{(n-1)!} + \frac{1-n}{(n+1)!} &= \frac{1}{(n-1)!} + \frac{1-n}{(n+1)n(n-1)!} \\
&= \frac{n(n+1)+1-n}{(n+1)n(n-1)!} \\
&= \frac{n^2+n+1-n}{(n+1)!} \\
&= \frac{n^2+1}{(n+1)!}
\end{aligned}$$

(vii) 1n!−3(n+1)!−n2−4(n+2)!

Solution:

$$\begin{aligned}
&\frac{1}{n!} - \frac{3}{(n+1)!} - \frac{n^2-4}{(n+2)!} \\
&= \frac{1}{n!} - \frac{3}{(n+1)n!} - \frac{n^2-4}{(n+2)(n+1)n!} \\
&= \frac{(n+1)(n+2)-3(n+2)-n^2+4}{(n+2)(n+1)n!} \\
&= \frac{n^2+3n+2-3n-6-n^2+4}{(n+2)!} \\
&= 0
\end{aligned}$$

(viii) $\frac{n^2-9}{(n+3)!}+\frac{6}{(n+2)!}-\frac{1}{(n+1)!}$

Solution:

$$\frac{n^2-9}{(n+3)!}+\frac{6}{(n+2)!}-\frac{1}{(n+1)!}$$

$$=\frac{n^2-9}{(n+3)(n+2)(n+1)!}+\frac{6}{(n+2)(n+1)!}-\frac{1}{(n+1)!}$$

$$=\frac{n^2-9+6(n+3)-(n+3)(n+2)}{(n+3)(n+2)(n+1)!}$$

$$=\frac{n^2-9+6n+18-n^2-5n-6}{(n+3)(n+2)(n+1)!}$$

$$=\frac{n+3}{(n+3)(n+2)(n+1)!}$$

$$=\frac{1}{(n+2)(n+1)!}$$

$$=\frac{1}{(n+2)!}$$

Maharashtra State Board 11th Maths Solutions Chapter 3 Permutations and Combination Ex 3.3

Question 1.
Find n, if ${}^nP_6 : {}^nP_3 = 120 : 1$.
Solution:
${}^nP_6 : {}^nP_3 = 120 : 1$

$$\therefore \quad \frac{n!}{(n-6)!} \div \frac{n!}{(n-3)!} = \frac{120}{1}$$

$$\therefore \quad \frac{n!}{(n-6)!} \times \frac{(n-3)!}{n!} = 120$$

$$\therefore \quad \frac{n!}{(n-6)!} \times \frac{(n-3)(n-4)(n-5)(n-6)!}{n!} = 120$$

∴ (n – 3) (n – 4) (n – 5) = 120
∴ (n – 3) (n – 4) (n – 5) = 6 × 5 × 4
Comparing on both sides, we get
n – 3 = 6
∴ n = 9

Question 2.
Find m and n, if ${}^{(m+n)}P_2 = 56$ and ${}^{(m-n)}P_2 = 12$.
Solution:
${}^{(m+n)}P_2 = 56$

$$\therefore \quad \frac{(m+n)!}{(m+n-2)!} = 56$$

$$\therefore \quad \frac{(m+n)(m+n-1)(m+n-2)!}{(m+n-2)!} = 56$$

(m + n) (m + n – 1) = 56
Let m + n = t
t(t – 1) = 56

$t^2 - t - 56 = 0$
$(t - 8)(t + 7) = 0$
$t = 8$ or $t = -7$
$m + n = 8$ or $m + n = -7$
But $m + n \neq -7$
$\therefore m + n = 8$ ......(i)
Also, ${}^{(m-n)}P_2 = 12$

$$\therefore \quad \frac{(m-n)!}{(m-n-2)!} = 12$$

$$\therefore \quad \frac{(m-n)(m-n-1)(m-n-2)!}{(m-n-2)!} = 12$$

$(m - n)(m - n - 1) = 12$
Let $m - n = a$
$a(a - 1) = 12$
$a^2 - a - 12 = 0$
$(a - 4)(a + 3) = 0$
$a = 4$ or $a = -3$
$m - n = 4$ or $m - n = -3$
But $m - n \neq -3$
$\therefore m - n = 4$ ......(ii)
Adding (i) and (ii), we get
$2m = 12$
$\therefore m = 6$
Substituting $m = 6$ in (ii), we get
$6 - n = 4$
$\therefore n = 2$

Question 3.
Find r, if ${}^{12}P_{r-2} : {}^{11}P_{r-1} = 3 : 14$.
Solution:

$${}^{12}P_{r-2} : {}^{11}P_{r-1} = 3 : 14$$

$$\therefore \quad \frac{12!}{(12-r+2)!} \div \frac{11!}{(11-r+1)!} = \frac{3}{14}$$

$$\therefore \quad \frac{12!}{(14-r)!} \times \frac{(12-r)!}{11!} = \frac{3}{14}$$

$$\therefore \quad \frac{12 \times 11!}{(14-r)(13-r)(12-r)!} \times \frac{(12-r)!}{11!} = \frac{3}{14}$$

$$\therefore \quad \frac{12}{(14-r)(13-r)} = \frac{3}{14}$$

$(14 - r)(13 - r) = 8 \times 7$
Comparing on both sides, we get
$14 - r = 8$
$\therefore r = 6$

Question 4.
Show that $(n + 1)({}^{n}P_r) = (n - r + 1)[{}^{(n+1)}P_r]$
Solution:

$$\text{L.H.S.} = (n+1)\,{}^{n}P_r = (n+1)\frac{n!}{(n-r)!} = \frac{(n+1)!}{(n-r)!}$$

$$\text{R.H.S.} = (n-r+1)\,{}^{(n+1)}P_r$$

$$= (n-r+1)\,\frac{(n+1)!}{(n-r+1)!}$$

$$= \frac{(n-r+1)(n+1)!}{(n-r+1)(n-r)!}$$

$$= \frac{(n+1)!}{(n-r)!}$$

L.H.S. = R.H.S.

Question 5.
How many 4 letter words can be formed using letters in the word MADHURI, if (a) letters can be repeated (b) letters cannot be repeated.
Solution:
There are 7 letters in the word MADHURI.
(a) A 4 letter word is to be formed from the letters of the word MADHURI and repetition of letters is allowed.

∴ 1st letter can be filled in 7 ways.
2nd letter can be filled in 7 ways.
3rd letter can be filled in 7 ways.
4th letter can be filled in 7 ways.
∴ Total no. of ways a 4-letter word can be formed = 7 × 7 × 7 × 7 = 2401

(b) When repetition of letters is not allowed, the number of 4-letter words formed from the letters of the word MADHURI
is ${}_7P_4 = \frac{7!}{(7-4)!} = \frac{7 \times 6 \times 5 \times 4 \times 3!}{3!}$ = 840

Question 6.
Determine the number of arrangements of letters of the word ALGORITHM if
(a) vowels are always together.
(b) no two vowels are together.
(c) consonants are at even positions.
(d) O is the first and T is the last letter.
Solution:
There are 9 letters in the word ALGORITHM.
(a) When vowels are always together.
There are 3 vowels in the word ALGORITHM (i.e., A, I, O).
Let us consider these 3 vowels as one unit.
This unit with 6 other letters is to be arranged.
∴ The number of arrangement = ${}_7P_7$ = 7! = 5040
3 vowels can be arranged among themselves in ${}_3P_3$ = 3! = 6 ways.
∴ Required number of arrangements = 7! × 3!
= 5040 × 6
= 30240

(b) When no two vowels are together.
There are 6 consonants in the word ALGORITHM,
they can be arranged among themselves in ${}_6P_6$ = 6! = 720 ways.
Let consonants be denoted by C.
_C _C_ C _C_C_C
There are 7 places marked by '_' in which 3 vowels can be arranged.
∴ Vowels can be arranged in ${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7 \times 6 \times 5 \times 4!}{4!}$ = 210 ways.
∴ Required number of arrangements = 720 × 210 = 151200

(c) When consonants are at even positions.
There are 4 even places and 6 consonants in the word ALGORITHM.
∴ 6 consonants can be arranged at 4 even positions in 6P4 = $\frac{6!}{(6-4)!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$ = 360 ways.
Remaining 5 letters (3 vowels and 2 consonants) can be arranged in odd position in ${}_5P_5$ = 5! = 120 ways.
∴ Required number of arrangements = 360 × 120 = 43200

(d) When O is the first and T is the last letter.
All the letters of the word ALGORITHM are to be arranged among themselves such that arrangement begins with O and ends with T.
∴ Position of O and T are fixed.
∴ Other 7 letters can be arranged between O and T among themselves in ${}_7P_7$ = 7! = 5040 ways.
∴ Required number of arrangements = 5040

Question 7.
In a group photograph, 6 teachers and principal are in the first row and 18 students are in the second row. There are 12 boys and 6 girls among the students. If the middle position is reserved for the principal and if no two girls are together, find the number of arrangements.
Solution:
In 1st row, 6 teachers can be arranged among themselves in ${}_6P_6$ = 6! ways.
In the 2nd row, 12 boys can be arranged among themselves in ${}_{12}P_{12}$ = 12! ways.
No two girls are together.
So, there are 13 places formed by 12 boys in which 6 girls occupy any 6 places in ${}_{13}P_6$ ways.

∴ Required number of arrangements
$= 6! \times 12! \times {}^{13}P_6$
$= 6! \times 12! \times \frac{13!}{(13-6)!}$
$= 6! \times 12! \times \frac{13!}{7!}$
$= \frac{6! \times 12! \times 13!}{7 \times 6!}$
$= \frac{12!13!}{7}$

Question 8.
Find the number of ways so that letters of the word HISTORY can be arranged as
(a) Y and T are together
(b) Y is next to T
(c) there is no restriction
(d) begin and end with a vowel
(e) end in ST
(f) begin with S and end with T
Solution:
There are 7 letters in the word HISTORY
(a) When 'Y' and 'T' are together.
Let us consider 'Y' and 'T' as one unit.
This unit with other 5 letters are to be arranged.
∴ The number of arrangement of one unit and 5 letters = ${}^6P_6$ = 6! = 720
Also, 'Y' and 'T' can be arranged among themselves in ${}^2P_2$ = 2! = 2 ways.
∴ A total number of arrangements when Y and T are always together = 6! × 2!
= 120 × 2
= 1440

(b) When 'Y' is next to 'T'.
Let us take this ('Y' next to 'T') as one unit.
This unit with 5 other letters is to be arranged.
∴ The number of arrangements of 5 letters and one unit = ${}^6P_6$ = 6! = 720
Also, 'Y' has to be always next to 'T'.
∴ They can be arranged among themselves in 1 way only.
∴ Total number of arrangements possible when Y is next to T = 720 × 1 = 720

(c) When there is no restriction.
7 letters can be arranged among themselves in ${}^7P_7$ = 7! ways.
∴ The total number of arrangements possible if there is no restriction = 7!

(d) When begin and end with a vowel.
There are 2 vowels in the word HISTORY.
All other letters of the word HISTORY are to be arranged between 2 vowels such that the arrangement begins and ends with a vowel.
The other 5 letters can be filled between the two vowels in ${}^5P_5$ = 5! = 120 ways.
Also, 2 vowels can be arranged among themselves at first and last places in ${}^2P_2$ = 2! = 2 ways.
∴ Total number of arrangements when the word begins and ends with vowel = 120 × 2 = 240

(e) When a word ends in ST.
As the arrangement ends with ST,
the remaining 5 letters can be arranged among themselves in ${}^5P_5$ = 5! = 120 ways.
∴ Total number of arrangements when the word ends with ST = 120

(f) When a word begins with S and ends with T.
As arrangement begins with S and ends with T,
the remaining 5 letters can be arranged between S and T among themselves in ${}^5P_5$ = 5! = 120 ways.
Total number of arrangements when the word begins with S and ends with T = 120

Question 9.
Find the number of arrangements of the letters in the word SOLAPUR so that consonants and vowels are placed alternately.
Solution:
There are 4 consonants S, L, P, R, and 3 vowels A, O, U in the word SOLAPUR.
Consonants and vowels are to be alternated.

∴ Vowels must occur in even places and consonants in odd places.
∴ 3 vowels can be arranged at 3 even places in ${}^3P_3$ = 3! = 6 ways.
Also, 4 consonants can be arranged at 4 odd places in ${}^4P_4$ = 4! = 24 ways.
Required number of arrangements = 6 × 24 = 144

Question 10.
Find the number of 4-digit numbers that can be formed using the digits 1, 2, 4, 5, 6, 8 if
(a) digits can be repeated.
(b) digits cannot be repeated.
Solution:
(a) A 4 digit number is to be made from the digits 1, 2, 4, 5, 6, 8 such that digits can be repeated.
∴ Unit's place digit can be filled in 6 ways.
10's place digit can be filled in 6 ways.
100's place digit can be filled in 6 ways.
1000's place digit can be filled in 6 ways.
∴ Total number of numbers that can be formed = 6 × 6 × 6 × 6 = 1296

(b) A 4 different digit number is to be made from the digits 1, 2, 4, 5, 6, 8 without repetition of digits.
∴ 4 different digits are to be arranged from 6 given digits which can be done in ${}^6P_4$ ways.
∴ Total number of numbers that can be formed
$= \frac{6!}{(6-4)!}$
$= \frac{6\times5\times4\times3\times2!}{2!}$
= 360

Question 11.
How many numbers can be formed using the digits 0, 1, 2, 3, 4, 5 without repetition so that resulting numbers are between 100 and 1000?
Solution:
A number between 100 and 1000 that can be formed from the digits 0, 1, 2, 3, 4, 5 is of 3 digits, and repetition of digits is not allowed.
∴ 100's place can be filled in 5 ways as it is a non-zero number.
10's place digits can be filled in 5 ways.
Unit's place digit can be filled in 4 ways.
∴ Total number of ways the number can be formed = 5 × 5 × 4 = 100

Question 12.
Find the number of 6-digit numbers using the digits 3, 4, 5, 6, 7, 8 without repetition. How many of these numbers are (a) divisible by 5 (b) not divisible by 5
Solution:
A number of 6 different digits is to be formed from the digits 3, 4, 5, 6, 7, 8 which can be done in ${}^6P_6$ = 6! = 720 ways.
(a) If the number is to be divisible by 5,
the unit's place digit can be 5 only.
∴ it can be arranged in 1 way only.
The other 5 digits can be arranged among themselves in ${}^5P_5$ = 5! = 120 ways.
∴ Required number of numbers divisible by 5 = 1 × 120 = 120

(b) If the number is not divisible by 5,
unit's place can be any digit from 3, 4, 6, 7, 8.
∴ it can be arranged in 5 ways.
Other 5 digits can be arranged in ${}^5P_5$ = 5! = 120 ways.
∴ Required number of numbers not divisible by 5 = 5 × 120 = 600

Question 13.
A code word is formed by two different English letters followed by two non-zero distinct digits. Find the number of such code words. Also, find the number of such code words that end with an even digit.
Solution:
There is a total of 26 alphabets.
A code word contains 2 English alphabets.
∴ 2 alphabets can be filled in ${}^{26}P_2$
$= \frac{26!}{(26-2)!}$
$= \frac{26\times25\times24!}{24!}$
= 650 ways.
Also, alphabets to be followed by two distinct non-zero digits from 1 to 9 which can be filled in
${}^9P_2 = \frac{9!}{(9-2)!} = \frac{9\times8\times7!}{7!}$ = 72 ways.
∴ Total number of a code words = 650 × 72 = 46800.
To find the number of codewords end with an even integer.
2 alphabets can be filled in 650 ways.
The digit in the unit's place should be an even number between 1 to 9, which can be filled in 4 ways.

Also, 10’s place can be filled in 8 ways.
∴ Total number of codewords = 650 × 4 × 8 = 20800

Question 14.
Find the number of ways in which 5 letters can be posted in 3 post boxes if any number of letters can be posted in a post box.
Solution:
There are 5 letters and 3 post boxes and any number of letters can be posted in all three post boxes.
∴ Each letter can be posted in 3 ways.
∴ Total number of ways 5 letters can be posted = 3 × 3 × 3 × 3 × 3 = 243

Question 15.
Find the number of arranging 11 distinct objects taken 4 at a time so that a specified object (a) always occurs (b) never occurs
Solution:
There are 11 distinct objects and 4 are to be taken at a time.
(a) The number of permutations of n distinct objects, taken r at a time, when one particular object will always occur is $r \times {}^{(n-1)}P_{(r-1)}$

Here, r = 4, n = 11

$$\therefore \quad r \times {}^{(n-1)}P_{(r-1)} = 4 \times {}^{10}P_3$$
$$= 4 \times \frac{10!}{(10-3)!}$$
$$= 4 \times \frac{10!}{7!}$$
$$= 4 \times \frac{10 \times 9 \times 8 \times 7!}{7!}$$
$$= 2880$$

∴ In 2880 permutations of 11 distinct objects, taken 4 at a time, one particular object will always occur.

(b) When one particular object will not occur, then 4 objects are to be arranged from 10 objects which can be done in ${}^{10}P_4$ = 10 × 9 × 8 × 7 = 5040 ways.
∴ In 5040 permutations of 11 distinct objects, taken 4 at a time, one particular object will never occur.

Question 16.
In how many ways can 5 different books be arranged on a shelf if
(i) there are no restrictions
(ii) 2 books are always together
(iii) 2 books are never together
Solution:
(i) 5 books arranged in ${}^5P_5$ = 5! = 120 ways.

(ii) 2 books are together.
Let us consider two books as one unit. This unit with the other 3 books can be arranged in ${}^4P_4$ = 4! = 24 ways.
Also, two books can be arranged among themselves in ${}^2P_2$ = 2 ways.
∴ Required number of arrangements = 24 × 2 = 48

(iii) Say books are B_1, B_2, B_3, B_4, B_5 are to be arranged with B_1, B_2 never together.
B_3, B_4, B_5 can be arranged among themselves in ${}^3P_3$ = 3! = 6 ways.
B_3, B_4, B_5 create 4 gaps in which B_1, B_2 are arranged in ${}^4P_2$ = 4 × 3 = 12 ways.
∴ Required number of arrangements = 6 × 12 = 72

Question 17.
3 boys and 3 girls are to sit in a row. How many ways can this be done if
(i) there are no restrictions.
(ii) there is a girl at each end.
(iii) boys and girls are at alternate places.
(iv) all-boys sit together.
Solution:
3 boys and 3 girls are to be arranged in a row.
(i) When there are no restrictions.
∴ Required number of arrangements = 6! = 720

(ii) When there is a girl at each end.
3 girls can be arranged at two ends in
${}^3P_2 = \frac{3!}{1!}$ = 3 × 2 = 6 ways.
And remaining 1 girl and 3 boys can be arranged between the two girls in ${}^4P_4$ = 4! = 24 ways.
∴ Required number of arrangements = 6 × 24 = 144

(iii) Boys and girls are at alternate places.
We can first arrange 3 girls among themselves in ${}_3P_3$ = 3! = 6 ways.
Let girls be denoted by G.
G – G – G –
There are 3 places marked by '-' where 3 boys can be arranged in 3! = 6 ways.
∴ Total number of such arrangements = 6 × 6 = 36
OR
Similarly, we can first arrange 3 boys in 3! = 6 ways
and then arrange 3 girls alternately in 3! = 6 ways.
∴ Total number of such arrangements = 6 × 6 = 36
∴ Required number of arrangements = 36 + 36 = 72

(iv) All boys sit together.
Let us consider all boys as one group.
This one group with the other 3 girls can be arranged ${}_4P_4$ = 4! = 24 ways.
Also, 3 boys can be arranged among themselves in ${}_3P_3$ = 3! = 6 ways.
∴ Required number of arrangements = 24 × 6 = 144

Maharashtra State Board 11th Maths Solutions Chapter 3 Permutations and Combination Ex 3.4

Question 1.
Find the number of permutations of letters in each of the following words.
(i) DIVYA
(ii) SHANTARAM
(iii) REPRESENT
(iv) COMBINE
(v) BAL BHARATI
Solution:
(i) There are 5 distinct letters in the word DIVYA.
∴ Number of permutations of the letters of the word DIVYA = 5! = 120

(ii) There are 9 letters in the word SHANTARAM in which 'A' is repeated 3 times.
∴ Number of permutations of the letters of the word SHANTARAM = $\frac{9!}{3!}$
= 9 × 8 × 7 × 6 × 5 × 4
= 60480

(iii) There are 9 letters in the word REPRESENT in which 'E' is repeated 3 times and 'R' is repeated 2 times.
∴ Number of permutations of the letters of the word REPRESENT = $\frac{9!}{3!2!}$
= $\frac{9\times8\times7\times6\times5\times4}{2}$
= 30240

(iv) There are 7 distinct letters in the word COMBINE.
∴ Number of permutations of the letters of the word COMBINE = 7! = 5040

(v) There are 10 letters in the word BALBHARATI in which 'B' is repeated 2 times and 'A' is repeated 3 times.
∴ Number of permutations of the letters of the word BALBHARATI = $\frac{10!}{2!3!}$
= $\frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{2\times3\times2}$
= 302400

Question 2.
You have 2 identical books on English, 3 identical books on Hindi, and 4 identical books on Mathematics. Find the number of distinct ways of arranging them on a shelf.
Solution:
There are total 9 books to be arranged on a shelf.
Out of these 9 books, 2 books on English, 3 books on Hindi and 4 books on mathematics are identical.
∴ Total number of arrangements possible = $\frac{9!}{2!3!4!}$
= $\frac{9 \times 8 \times 7 \times 6 \times 5 \times 4!}{2 \times 3 \times 2 \times 4!}$
= 9 × 4 × 7 × 5
= 1260

Question 3.
A coin is tossed 8 times. In how many ways can we obtain (a) 4 heads and 4 tails? (b) at least 6 heads?
Solution:
A coin is tossed 8 times. All heads are identical and all tails are identical.
(a) 4 heads and 4 tails are to be obtained.
∴ Number of ways it can be obtained = $\frac{8!}{4!4!}$
= $\frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2}$
= 70

(b) At least 6 heads are to be obtained.
∴ Outcome can be (6 heads and 2 tails) or (7 heads and 1 tail) or (8 heads)
∴ Number of ways it can be obtained = $\frac{8!}{6!2!} + \frac{8!}{7!1!} + \frac{8!}{8!}$
= $\frac{8 \times 7}{2}$ + 8 + 1
= 28 + 8 + 1
= 37

Question 4.
A bag has 5 red, 4 blue, and 4 green marbles. If all are drawn one by one and their colours are recorded, how many different arrangements can be found?
Solution:
There is a total of 13 marbles in a bag.
Out of these 5 are Red, 4 Blue, and 4 are Green marbles.
All balls of the same colour are taken to be identical.
∴ Required number of arrangements = $\frac{13!}{5!4!4!}$

Question 5.
Find the number of ways of arranging letters of the word MATHEMATICAL. How many of these arrangements have all vowels together?
Solution:
There are 12 letters in the word MATHEMATICAL in which 'M' is repeated 2 times, 'A' repeated 3 times and 'T' repeated 2 times.
∴ Required number of arrangements = $\frac{12!}{2!3!2!}$
When all the vowels,
i.e., 'A', 'A', 'A', 'E', 'I' are to be kept together.
Let us consider them as one unit.
Number of arrangements of these vowels among themselves = $\frac{5!}{3!}$ ways.
This unit is to be arranged with 7 other letters in which 'M' and 'T' repeated 2 times each.
∴ Number of such arrangements = $\frac{8!}{2!2!}$
∴ Required number of arrangements = $\frac{8! \times 5!}{2!2!3!}$

Question 6.
Find the number of different arrangements of letters in the word MAHARASHTRA. How many of these arrangements have (a) letters R and H never together? (b) all vowels together?
Solution:
There are 11 letters in the word MAHARASHTRA in which 'A' is repeated 4 times, 'H' repeated 2 times, and 'R' repeated 2 times.
∴ Total number of words can be formed = $\frac{11!}{4!2!2!}$

(a) When letters R and H are never together.
Other than 2R, 2H there are 4A, 1S, 1T, 1M.
These letters can be arranged in $\frac{7!}{4!}$ ways = 210.
These seven letters create 8 gaps in which 2R, 2H are to be arranged.
Number of ways to do = $\frac{{}^8P_4}{2!2!}$ = 420
Required number of arrangements = 210 × 420 = 88200.

(b) When all vowels are together.
There are 4 vowels in the word MAHARASHTRA, i.e., A, A, A, A.
Let us consider these 4 vowels as one unit, which can be arranged among themselves in $\frac{4!}{4!}$ = 1 way.

This unit is to be arranged with 7 other letters in which ‘H’ is repeated 2 times, ‘R’ is repeated 2 times.
∴ Total number of arrangements = 8!2!2!

Question 7.
How many different words are formed if the letter R is used thrice and letters S and T are used twice each?
Solution:
To find the number of different words when ‘R’ is taken thrice, ‘S’ is taken twice and ‘T’ is taken twice.
∴ Total number of letters available = 7, of which ‘S’ and ‘T’ repeat 2 times each, ‘R’ repeats 3 times.
∴ Required number of words = 7!2!2!3!
= 7×6×5×4×3!2×1×2×1×3!
= 7 × 6 × 5
= 210

Question 8.
Find the number of arrangements of letters in the word MUMBAI so that the letter B is always next to A.
Solution:
There are 6 letters in the word MUMBAI.
These letters are to be arranged in such a way that ‘B’ is always next to ‘A’.
Let us consider AB as one unit.
This unit with the other 4 letters in which ‘M’ repeats twice is to be arranged.
∴ Required number of arrangements = 5!2!
= 5×4×3×2!2!
= 60

Question 9.
Find the number of arrangements of letters in the word CONSTITUTION that begin and end with N.
Solution:
There are 12 letters in the word CONSTITUTION, in which ‘O’, ‘N’, ‘I’ repeat two times each, ‘T’ repeats 3 times.
When the arrangement starts and ends with ‘N’,
other 10 letters can be arranged between two N,
in which ‘O’ and ‘I’ repeat twice each and ‘T’ repeats 3 times.
∴ Required number of arrangements = 10!2!2!3!

Question 10.
Find the number of different ways of arranging letters in the word ARRANGE. How many of these arrangements do not have the two R’s and two A’s together?
Solution:
There are 7 letters in the word ARRANGE in which ‘A’ and ‘R’ repeat 2 times each.
∴ Number of ways to arrange the letters of word ARRANGE = 7!2!2! = 1260
Consider the words in which 2A are together and 2R are together.
Let us consider 2A as one unit and 2R as one unit.
These two units with remaining 3 letters can be arranged in = 5!2!2! = 30 ways.
Number of arrangements in which neither 2A together nor 2R are together = 1260 – 30 = 1230

Question 11.
How many distinct 5 digit numbers can be formed using the digits 3, 2, 3, 2, 4, 5.
Solution:
5 digit numbers are to be formed from 2, 3, 2, 3, 4, 5.
Case I: Numbers formed from 2, 2, 3, 4, 5 OR 2, 3, 3, 4, 5
Number of such numbers = 5!2!+5!2! = 5! = 120
Case II: Numbers are formed from 2, 2, 3, 3 and any one of 4 or 5
Number of such numbers = 5!2!2!+5!2!2! = 60
Required number of numbers = 120 + 60 = 180

Question 12.
Find the number of distinct numbers formed using the digits 3, 4, 5, 6, 7, 8, 9, so that odd positions are occupied by odd digits.
Solution:
A number is to be formed with digits 3, 4, 5, 6, 7, 8, 9 such that odd digits always occupy the odd places.
There are 4 odd digits, i.e. 3, 5, 7, 9.
∴ They can be arranged at 4 odd places among themselves in 4! = 24 ways.
There are 3 even digits, i.e. 4, 6, 8.
∴ They can be arranged at 3 even places among themselves in 3! = 6 ways.
∴ Required number of numbers formed = 24 × 6 = 144

Question 13.
How many different 6-digit numbers can be formed using digits in the number 659942? How many of them are divisible by 4?
Solution:
A 6-digit number is to be formed using digits of 659942, in which 9 repeats twice.
∴ Required number of numbers formed = $\frac{6!}{2!}$
= $\frac{6\times5\times4\times3\times2!}{2!}$
= 360
A 6-digit number is to be formed using the same digits that are divisible by 4.
For a number to be divisible by 4, the last two digits should be divisible by 4,
i.e. 24, 52, 56, 64, 92 or 96.
Case I: When the last two digits are 24, 52, 56 or 64.
As the digit 9 repeats twice in the remaining four numbers, the number of arrangements = $\frac{4!}{2!}$ = 12
∴ 6-digit numbers that are divisible by 4 so formed are 12 + 12 + 12 + 12 = 48.
Case II: When the last two digits are 92 or 96.
As each of the remaining four numbers are distinct, the number of arrangements = 4! = 24
∴ 6-digit numbers that are divisible by 4 so formed are 24 + 24 = 48.
∴ Required number of numbers framed = 48 + 48 = 96

Question 14.
Find the number of distinct words formed from letters in the word INDIAN. How many of them have the two N's together?
Solution:
There are 6 letters in the word INDIAN in which I and N are repeated twice.
Number of different words that can be formed using the letters of the word INDIAN = $\frac{6!}{2!2!}$
= $\frac{6\times5\times4\times3\times2!}{2\times2!}$
= 180
When two N's are together.
Let us consider the two N's as one unit.
They can be arranged with 4 other letters in $\frac{5!}{2!}$
= $\frac{5\times4\times3\times2!}{2!}$
= 60 ways.
∴ 2 N can be arranged in $\frac{2!}{2!}$ = 1 way.
∴ Required number of words = 60 × 1 = 60

Question 15.
Find the number of different ways of arranging letters in the word PLATOON if (a) the two O's are never together. (b) consonants and vowels occupy alternate positions.
Solution:
There are 7 letters in the words PLATOON in which 'O' repeat 2 times.
(a) When the two O's are never together.
Let us arrange the other 5 letters first, which can be done in 5! = 120 ways.
The letters P, L, A, T, N create 6 gaps, in which O's are arranged.
Two O's can take their places in ${}^6P_2$ ways.
But 'O' repeats 2 times.
∴ Two O's can be arranged in $\frac{{}^6P_2}{2!}$
= $\frac{\frac{6!}{(6-2)!}}{2!}$
= $\frac{6\times5\times4!}{4!\times2\times1}$
= 3 × 5
= 15 ways
∴ Required number of arrangements = 120 × 15 = 1800

(b) When consonants and vowels occupy alternate positions.
There are 4 consonants and 3 vowels in the word PLATOON.
∴ At odd places, consonants occur and at even places, vowels occur.
4 consonants can be arranged among themselves in 4! ways.
3 vowels in which O occurs twice and A occurs once.
∴ They can be arranged in $\frac{3!}{2!}$ ways.
Now, vowels and consonants should occupy alternate positions.
∴ Required number of arrangements = 4! × $\frac{3!}{2!}$
= 4 × 3 × 2 × $\frac{3\times2!}{2!}$
= 72

Maharashtra State Board 11th Maths Solutions Chapter 3 Permutations and Combination Ex 3.4

Question 1.
Find the number of permutations of letters in each of the following words.
(i) DIVYA
(ii) SHANTARAM
(iii) REPRESENT
(iv) COMBINE
(v) BAL BHARATI
Solution:
(i) There are 5 distinct letters in the word DIVYA.
∴ Number of permutations of the letters of the word DIVYA = 5! = 120

(ii) There are 9 letters in the word SHANTARAM in which 'A' is repeated 3 times.
∴ Number of permutations of the letters of the word SHANTARAM = $\frac{9!}{3!}$
= 9 × 8 × 7 × 6 × 5 × 4
= 60480

(iii) There are 9 letters in the word REPRESENT in which 'E' is repeated 3 times and 'R' is repeated 2 times.
∴ Number of permutations of the letters of the word REPRESENT = $\frac{9!}{3!2!}$
= $\frac{9\times8\times7\times6\times5\times4}{2}$
= 30240

(iv) There are 7 distinct letters in the word COMBINE.
∴ Number of permutations of the letters of the word COMBINE = 7! = 5040

(v) There are 10 letters in the word BALBHARATI in which 'B' is repeated 2 times and 'A' is repeated 3 times.
∴ Number of permutations of the letters of the word BALBHARATI = $\frac{10!}{2!3!}$
= $\frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{2\times3\times2}$
= 302400

Question 2.
You have 2 identical books on English, 3 identical books on Hindi, and 4 identical books on Mathematics. Find the number of distinct ways of arranging them on a shelf.
Solution:
There are total 9 books to be arranged on a shelf.
Out of these 9 books, 2 books on English, 3 books on Hindi and 4 books on mathematics are identical.
∴ Total number of arrangements possible = $\frac{9!}{2!3!4!}$
= $\frac{9\times8\times7\times6\times5\times4!}{2\times3\times2\times4!}$
= 9 × 4 × 7 × 5
= 1260

Question 3.
A coin is tossed 8 times. In how many ways can we obtain (a) 4 heads and 4 tails? (b) at least 6 heads?
Solution:
A coin is tossed 8 times. All heads are identical and all tails are identical.
(a) 4 heads and 4 tails are to be obtained.
∴ Number of ways it can be obtained = $\frac{8!}{4!4!}$
= $\frac{8\times7\times6\times5}{4\times3\times2}$
= 70

(b) At least 6 heads are to be obtained.
∴ Outcome can be (6 heads and 2 tails) or (7 heads and 1 tail) or (8 heads)
∴ Number of ways it can be obtained = $\frac{8!}{6!2!}+\frac{8!}{7!1!}+\frac{8!}{8!}$
= $\frac{8\times7}{2}$ + 8 + 1
= 28 + 8 + 1
= 37

Question 4.
A bag has 5 red, 4 blue, and 4 green marbles. If all are drawn one by one and their colours are recorded, how many different arrangements can be found?
Solution:
There is a total of 13 marbles in a bag.

Out of these 5 are Red, 4 Blue, and 4 are Green marbles.
All balls of the same colour are taken to be identical.
∴ Required number of arrangements = $\frac{13!}{5!4!4!}$

Question 5.
Find the number of ways of arranging letters of the word MATHEMATICAL. How many of these arrangements have all vowels together?
Solution:
There are 12 letters in the word MATHEMATICAL in which ‘M’ is repeated 2 times, ‘A’ repeated 3 times and ‘T’ repeated 2 times.
∴ Required number of arrangements = $\frac{12!}{2!3!2!}$
When all the vowels,
i.e., ‘A’, ‘A’, ‘A’, ‘E’, ‘I’ are to be kept together.
Let us consider them as one unit.
Number of arrangements of these vowels among themselves = $\frac{5!}{3!}$ ways.
This unit is to be arranged with 7 other letters in which ‘M’ and ‘T’ repeated 2 times each.
∴ Number of such arrangements = $\frac{8!}{2!2!}$
∴ Required number of arrangements = $\frac{8!\times 5!}{2!2!3!}$

Question 6.
Find the number of different arrangements of letters in the word MAHARASHTRA. How many of these arrangements have (a) letters R and H never together? (b) all vowels together?
Solution:
There are 11 letters in the word MAHARASHTRA in which ‘A’ is repeated 4 times, ‘H’ repeated 2 times, and ‘R’ repeated 2 times.
∴ Total number of words can be formed = $\frac{11!}{4!2!2!}$

(a) When letters R and H are never together.
Other than 2R, 2H there are 4A, 1S, 1T, 1M.
These letters can be arranged in $\frac{7!}{4!}$ ways = 210.
These seven letters create 8 gaps in which 2R, 2H are to be arranged.
Number of ways to do = $\frac{{}^8P_4}{2!2!}$ = 420
Required number of arrangements = 210 × 420 = 88200.

(b) When all vowels are together.
There are 4 vowels in the word MAHARASHTRA, i.e., A, A, A, A.
Let us consider these 4 vowels as one unit, which can be arranged among themselves in $\frac{4!}{4!}$ = 1 way.
This unit is to be arranged with 7 other letters in which ‘H’ is repeated 2 times, ‘R’ is repeated 2 times.
∴ Total number of arrangements = $\frac{8!}{2!2!}$

Question 7.
How many different words are formed if the letter R is used thrice and letters S and T are used twice each?
Solution:
To find the number of different words when ‘R’ is taken thrice, ‘S’ is taken twice and ‘T’ is taken twice.
∴ Total number of letters available = 7, of which ‘S’ and ‘T’ repeat 2 times each, ‘R’ repeats 3 times.
∴ Required number of words = $\frac{7!}{2!2!3!}$
= $\frac{7\times 6\times 5\times 4\times 3!}{2\times 1\times 2\times 1\times 3!}$
= 7 × 6 × 5
= 210

Question 8.
Find the number of arrangements of letters in the word MUMBAI so that the letter B is always next to A.
Solution:
There are 6 letters in the word MUMBAI.
These letters are to be arranged in such a way that ‘B’ is always next to ‘A’.
Let us consider AB as one unit.
This unit with the other 4 letters in which ‘M’ repeats twice is to be arranged.
∴ Required number of arrangements = $\frac{5!}{2!}$
= $\frac{5\times 4\times 3\times 2!}{2!}$
= 60

Question 9.
Find the number of arrangements of letters in the word CONSTITUTION that begin and end with N.
Solution:
There are 12 letters in the word CONSTITUTION, in which ‘O’, ‘N’, ‘I’ repeat two times each, ‘T’ repeats 3 times.
When the arrangement starts and ends with ‘N’,
other 10 letters can be arranged between two N,

in which ‘O’ and ‘I’ repeat twice each and ‘T’ repeats 3 times.
∴ Required number of arrangements = $10!2!2!3!$

Question 10.
Find the number of different ways of arranging letters in the word ARRANGE. How many of these arrangements do not have the two R’s and two A’s together?
Solution:
There are 7 letters in the word ARRANGE in which ‘A’ and ‘R’ repeat 2 times each.
∴ Number of ways to arrange the letters of word ARRANGE = $7!2!2!$ = 1260
Consider the words in which 2A are together and 2R are together.
Let us consider 2A as one unit and 2R as one unit.
These two units with remaining 3 letters can be arranged in = $5!2!2!$ = 30 ways.
Number of arrangements in which neither 2A together nor 2R are together = 1260 – 30 = 1230

Question 11.
How many distinct 5 digit numbers can be formed using the digits 3, 2, 3, 2, 4, 5.
Solution:
5 digit numbers are to be formed from 2, 3, 2, 3, 4, 5.
Case I: Numbers formed from 2, 2, 3, 4, 5 OR 2, 3, 3, 4, 5
Number of such numbers = $5!2!+5!2!$ = 5! = 120
Case II: Numbers are formed from 2, 2, 3, 3 and any one of 4 or 5
Number of such numbers = $5!2!2!+5!2!2!$ = 60
Required number of numbers = 120 + 60 = 180

Question 12.
Find the number of distinct numbers formed using the digits 3, 4, 5, 6, 7, 8, 9, so that odd positions are occupied by odd digits.
Solution:
A number is to be formed with digits 3, 4, 5, 6, 7, 8, 9 such that odd digits always occupy the odd places.
There are 4 odd digits, i.e. 3, 5, 7, 9.
∴ They can be arranged at 4 odd places among themselves in 4! = 24 ways.
There are 3 even digits, i.e. 4, 6, 8.
∴ They can be arranged at 3 even places among themselves in 3! = 6 ways.
∴ Required number of numbers formed = 24 × 6 = 144

Question 13.
How many different 6-digit numbers can be formed using digits in the number 659942? How many of them are divisible by 4?
Solution:
A 6-digit number is to be formed using digits of 659942, in which 9 repeats twice.
∴ Required number of numbers formed = $6!2!$
= $6\times5\times4\times3\times2!2!$
= 360
A 6-digit number is to be formed using the same digits that are divisible by 4.
For a number to be divisible by 4, the last two digits should be divisible by 4,
i.e. 24, 52, 56, 64, 92 or 96.
Case I: When the last two digits are 24, 52, 56 or 64.
As the digit 9 repeats twice in the remaining four numbers, the number of arrangements = $4!2!$ = 12
∴ 6-digit numbers that are divisible by 4 so formed are 12 + 12 + 12 + 12 = 48.
Case II: When the last two digits are 92 or 96.
As each of the remaining four numbers are distinct, the number of arrangements = 4! = 24
∴ 6-digit numbers that are divisible by 4 so formed are 24 + 24 = 48.
∴ Required number of numbers framed = 48 + 48 = 96

Question 14.
Find the number of distinct words formed from letters in the word INDIAN. How many of them have the two N’s together?
Solution:
There are 6 letters in the word INDIAN in which I and N are repeated twice.
Number of different words that can be formed using the letters of the word INDIAN = $6!2!2!$
= $6\times5\times4\times3\times2!2!\times2!$
= 180
When two N’s are together.
Let us consider the two N’s as one unit.
They can be arranged with 4 other letters in $5!2!$
= $5\times4\times3\times2!2!$
= 60 ways.
∴ 2 N can be arranged in $2!2!$ = 1 way.
∴ Required number of words = 60 × 1 = 60

Question 15.
Find the number of different ways of arranging letters in the word PLATOON if (a) the two O's are never together. (b) consonants and vowels occupy alternate positions.
Solution:
There are 7 letters in the words PLATOON in which 'O' repeat 2 times.
(a) When the two O's are never together.
Let us arrange the other 5 letters first, which can be done in 5! = 120 ways.
The letters P, L, A, T, N create 6 gaps, in which O's are arranged.
Two O's can take their places in $^6P_2$ ways.
But 'O' repeats 2 times.
∴ Two O's can be arranged in $\frac{^6P_2}{2!}$
= $\frac{\frac{6!}{(6-2)!}}{2!}$
= $\frac{6\times5\times4!}{4!\times2\times1}$
= 3 × 5
= 15 ways
∴ Required number of arrangements = 120 × 15 = 1800

(b) When consonants and vowels occupy alternate positions.
There are 4 consonants and 3 vowels in the word PLATOON.
∴ At odd places, consonants occur and at even places, vowels occur.
4 consonants can be arranged among themselves in 4! ways.
3 vowels in which O occurs twice and A occurs once.
∴ They can be arranged in $\frac{3!}{2!}$ ways.
Now, vowels and consonants should occupy alternate positions.
∴ Required number of arrangements = 4! × $\frac{3!}{2!}$
= 4 × 3 × 2 × $\frac{3\times2!}{2!}$
= 72

Maharashtra State Board 11th Maths Solutions Chapter 3 Permutations and Combination Ex 3.5

Question 1.
In how many different ways can 8 friends sit around a table?
Solution:
We know that 'n' persons can sit around a table in (n – 1)! ways.
∴ 8 friends can sit around a table in 7!
= 7 × 6 × 5 × 4 × 3 × 2 × 1
= 5040 ways.

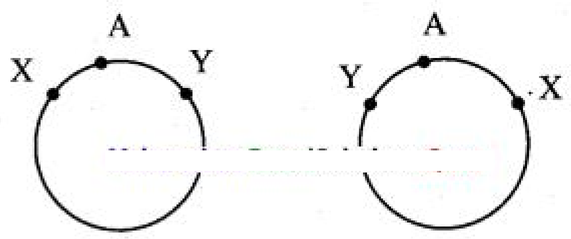
Question 2.
A party has 20 participants. Find the number of distinct ways for the host to sit with them around a circular table. How many of these ways have two specified persons on either side of the host?
Solution:
A party has 20 participants.
All of them and the host (i.e., 21 persons) can be seated at a circular table in (21 – 1)! = 20! ways.
When two particular participants are seated on either side of the host.
The host takes the chair in 1 way.
These 2 persons can sit on either side of the host in 2! ways.
Once the host occupies his chair, it is not circular permutation more.
The remaining 18 people occupy their chairs in 18! ways.
∴ A total number of arrangements possible if two particular participants are seated on either side of the host = 2! × 18! = 2 × 18!

Question 3.
Delegates from 24 countries participate in a round table discussion. Find the number of seating arrangements where two specified delegates are (a) always together. (b) never together.

Solution:
(a) Delegates of 24 countries are to participate in a round table discussion such that two specified delegates are always together.
Let us consider these 2 delegates as one unit. They can be arranged among themselves in 2! ways.
Also, these two delegates are to be seated with 22 other delegates (i.e. total of 23) which can be done in (23 – 1)! = 22! ways.
∴ Required number of arrangements = 2! × 22!

(b) When 2 specified delegates are never together then, other 22 delegates can be participate in a round table discussion in (22 – 1)! = 21! ways.
∴ There are 22 places of which any 2 places can be filled by those 2 delegates so that they are never together.
∴ Two specified delegates can be arranged in ${}^{22}P_2$ ways.
∴ Required number of arrangements = ${}^{22}P_2$ × 21!
= $\frac{22!}{(22-2)!} \times 21!$
= $\frac{22!}{20!}$ × 21!
= 22 × 21 × 21!
= 21 × 22 × 21!
= 21 × 22!

Question 4.
Find the number of ways for 15 people to sit around the table so that no two arrangements have the same neighbours.
Solution:
There are 15 people to sit around a table.
∴ They can be arranged in(15 – 1)! = 14! ways.
But, they should not have the same neighbour in any two arrangements.
Around the table, arrangements (i.e., clockwise and anticlockwise) coincide.



∴ Required number of arrangements = $\frac{14!}{2}$

Question 5.
A committee of 10 members sits around a table. Find the number of arrangements that have the President and the Vice-president together.
Solution:
A committee of 10 members sits around a table.
But, President and Vice-president sit together.
Let us consider President and Vice-president as one unit.
They can be arranged among themselves in 2! ways.
Now, this unit with the other 8 members of the committee is to be arranged around a table, which can be done in (9 – 1)! = 8! ways.
∴ Required number of arrangements = 8! × 2! = 2 × 8!

Question 6.
Five men, two women, and a child sit around a table. Find the number of arrangements where the child is seated (a) between the two women. (b) between two men.
Solution:
5 men, 2 women, and a child sit around a table.
(a) When the child is seated between two women.
5 men, 2 women, and a child are to be seated around a round table such that the child is seated between two women.
∴ the two women can be seated on either side of the child in 2! ways.
Let us consider these 3 (two women and a child) as one unit.
Now, this one unit is to be arranged with the remaining 5 men,
i.e., a total of 6 units are to be arranged around a round table, which can be done in (6 – 1)! = 5! ways.
∴ Required number of arrangements = 5! × 2!
= 120 × 2
= 240

(b) Two men can be selected from 5 men in
${}^{5}C_2 = \frac{5!}{2!(5-2)!} = \frac{5\times4\times3!}{2\times3!}$ = 10 ways.
Also, these two men can sit on either side of the child in 2! ways.
Let us take two men and a child as one unit.
Now, this one unit is to be arranged with the remaining 3 men and 2 women,
i.e., a total of 6 units (3 + 2 + 1) are to be arranged around a round table, which can be done in (6 – 1)! = 5! ways.
∴ Required number of arrangements = 10 × 2! × 5!
= 10 × 2 × 120
= 2400

Question 7.
Eight men and six women sit around a table. How many sitting arrangements will have no two women together?
Solution:
8 men can be seated around a table in (8 – 1)! = 7! ways.
No two women should sit together.
There are 8 gaps created by 8 men's seats.
∴ Women can be seated in 8 gaps in $^8P_6$ ways.
∴ Required number of arrangements = 7! × $^8P_6$

Question 8.
Find the number of seating arrangements for 3 men and 3 women to sit around a table so that exactly two women are together.
Solution:
2 women (who wish to sit together) can be selected from 3 in
$^3C_2 = \frac{3!}{2!(3-2)!} = \frac{3\times2!}{2!\times1!}$ = 3 ways.
Also, these two women can sit together in 2! ways.
Let us take two women as one unit.
Now, this one unit is to be arranged with the remaining 3 men and 1 woman,
i.e., a total of 5 units are to be arranged around a round table, which can be done in (5 – 1)! = 4! ways.
∴ Required number of arrangements = 3 × 2! × 4!
= 3 × 2 × 24
= 144

Question 9.
Four objects in a set of ten objects are alike. Find the number of ways of arranging them in a circular order.
Solution:
Ten things can be arranged in a circular order of which 4 are alike in $\frac{9!}{4!}$ ways.
∴ Required number of arrangements = $\frac{9!}{4!}$

Question 10.
Fifteen persons sit around a table. Find the number of arrangements that have two specified persons not sitting side by side.
Solution:
Since 2 particular persons can't be sitting side by side,
the other 13 persons can be arranged around the table in (13 – 1)! = 12! ways.
The two persons who are not sitting side by side may take 13 positions created by 3 persons in $^{13}P_2$ ways.
∴ Required number of arrangements = 12! × $^{13}P_2$
= 12! × 13 × 12
= 13 × 12! × 12
= 12 × 13!

Maharashtra State Board 11th Maths Solutions Chapter 3 Permutations and Combination Ex 3.6

Question 1.
Find the value of
(a) $^{15}C_4$
Solution:

$$^{15}C_4 = \frac{15!}{4!(15-4)!} = \frac{15!}{4!11!}$$
$$= \frac{15\times14\times13\times12\times11!}{4\times3\times2\times1\times11!}$$
$$= \frac{15\times14\times13\times12}{4\times3\times2\times1} = 1365$$

(b) $^{80}C_2$
Solution:

$$^{80}C_2 = \frac{80!}{2!(80-2)!} = \frac{80!}{2!\ 78!}$$
$$= \frac{80 \times 79 \times 78!}{2 \times 78!}$$
$$= 40 \times 79 = 3160$$

(c) $^{15}C_4 + {}^{15}C_5$
Solution:

$$^{15}C_4 + {}^{15}C_5 = {}^{15}C_5 + {}^{15}C_4 = {}^{15}C_5 + {}^{15}C_{5-1}$$
$$= {}^{16}C_5 \quad \ldots[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$$

(d) $^{20}C_{16} - {}^{19}C_{16}$
Solution:

$$^{20}C_{16} - {}^{19}C_{16} = \frac{20!}{16!\,4!} - \frac{19!}{16!\,3!}$$
$$= \frac{20 \times 19!}{16! \times 4 \times 3!} - \frac{19!}{16!\,3!}$$
$$= \frac{19!}{3!\,16!}\left[\frac{20}{4} - 1\right]$$
$$= \frac{19!}{3!\,16!}(4)$$
$$= \frac{19!}{3!(16)(15!)}\ 4$$
$$= \frac{19!}{4(3!)(15!)}$$
$$= \frac{19!}{4!\,15!} = {}^{19}C_{15} \text{ or } {}^{19}C_4$$

Question 2.
Find n if
(a) $^6P_2 = n(^6C_2)$
Solution:

$$^6P_2 = n\left({}^6C_2\right)$$
$$\therefore \quad \frac{6!}{(6-2)!} = n\ \frac{6!}{2!(6-2)!}$$
$$\therefore \quad \frac{6!}{4!} = n\ \frac{6!}{2!\,4!}$$
$$\therefore \quad n = 2! = 2 \times 1 = 2$$

(b) ${}^{2n}C_3 : {}^{n}C_2 = 52 : 3$
Solution:

$${}^{2n}C_3 : {}^{n}C_2 = 52 : 3$$

$$\therefore \quad \frac{(2n)!}{3!(2n-3)!} \div \frac{n!}{2!(n-2)!} = \frac{52}{3}$$

$$\therefore \quad \frac{(2n)!}{3!(2n-3)!} \times \frac{2!(n-2)!}{n!} = \frac{52}{3}$$

$$\therefore \quad \frac{(2n)(2n-1)(2n-2)(2n-3)!}{3\times 2!(2n-3)!} \times \frac{2!(n-2)!}{n(n-1)(n-2)!} = \frac{52}{3}$$

$$\therefore \quad \frac{2n(2n-1).2(n-1)}{3} \times \frac{1}{n(n-1)} = \frac{52}{3}$$

$$\therefore \quad \frac{4(2n-1)}{3} = \frac{52}{3}$$

$$\therefore \quad 2n - 1 = 13$$

$$\therefore \quad 2n = 14$$

$$\therefore \quad n = 7$$

(c) ${}^{n}C_{n-3} = 84$
Solution:

$${}^{n}C_{n-3} = 84$$

$$\therefore \quad \frac{n!}{(n-3)![n-(n-3)]!} = 84$$

$$\therefore \quad \frac{n(n-1)(n-2)(n-3)!}{(n-3)!\times 3!} = 84$$

$$\therefore \quad n(n-1)(n-2) = 84 \times 6$$

$$\therefore \quad n(n-1)(n-2) = 9 \times 8 \times 7$$

Comparing on both sides, we get
$n = 9$

Question 3.
Find r if ${}^{14}C_{2r} : {}^{10}C_{2r-4} = 143 : 10$.
Solution:

$${}^{14}C_{2r} : {}^{10}C_{2r-4} = 143 : 10$$

$$\therefore \quad \frac{14!}{2r!(14-2r)!} \div \frac{10!}{(2r-4)!(14-2r)!} = \frac{143}{10}$$

$$\therefore \quad \frac{14!}{2r!(14-2r)!} \times \frac{(2r-4)!(14-2r)!}{10!} = \frac{143}{10}$$

$$\therefore \quad \frac{14\times 13\times 12\times 11\times 10!}{2r(2r-1)(2r-2)(2r-3)(2r-4)!(14-2r)!} \times \frac{(2r-4)!(14-2r)!}{10!} = \frac{143}{10}$$

$$\therefore \quad \frac{14\times 13\times 12\times 11}{2r(2r-1)\times(2r-2)(2r-3)} = \frac{143}{10}$$

$\therefore$ $2r(2r-1)(2r-2)(2r-3) = 14 \times 12 \times 10$
$\therefore$ $2r(2r-1)(2r-2)(2r-3) = 8 \times 7 \times 6 \times 5$
Comparing on both sides, we get
$\therefore$ $r = 4$

Question 4.
Find n and r if,
(a) ${}^{n}P_{r} = 720$ and ${}^{n}C_{n-r} = 120$

Solution:

$^nP_r = 720$

$\therefore \quad \frac{n!}{(n-r)!} = 720 \quad \ldots(i)$

Also, $^nC_{n-r} = 120$

$\therefore \quad \frac{n!}{(n-r)!(n-n+r)!} = 120$

$\therefore \quad \frac{n!}{r!(n-r)!} = 120 \quad \ldots(ii)$

Dividing (i) by (ii), we get

$$\frac{\frac{n!}{(n-r)!}}{\frac{n!}{r!(n-r)!}} = \frac{720}{120}$$

$\therefore \quad r! = 6$

$\therefore \quad r = 3$

Substituting r = 3 in (i), we get

$\frac{n!}{(n-3)!} = 720$

$\therefore \quad \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 720$

$\therefore \quad n(n-1)(n-2) = 10 \times 9 \times 8$

$\therefore \quad n = 10$

(b) $^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 20 : 35 : 42$

Solution:

$^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 20 : 35 : 42$

$\therefore \quad {}^nC_{r-1} : {}^nC_r = \frac{20}{35}$

$\therefore \quad \frac{n!}{(r-1)![n-(r-1)]!} \div \frac{n!}{r!(n-r)!} = \frac{4}{7}$

$\therefore \quad \frac{n!}{(r-1)!(n+1-r)!} \times \frac{r!(n-r)!}{n!} = \frac{4}{7}$

$\therefore \quad \frac{n!}{(r-1)!(n+1-r)!} \times \frac{r(r-1)!(n-r)!}{n!} = \frac{4}{7}$

$\therefore \quad \frac{r(n-r)!}{(n+1-r)(n-r)!} = \frac{4}{7}$

$\therefore \quad 7r = 4(n+1-r)$

$\therefore \quad 7r = 4n + 4 - 4r$

$\therefore \quad 11r = 4n + 4 \quad \ldots(i)$

Also, $^nC_r : {}^nC_{r+1} = \frac{35}{42}$

$\therefore \quad \frac{n!}{r!(n-r)!} \div \frac{n!}{(r+1)!(n-r-1)!} = \frac{5}{6}$

$\therefore \quad \frac{n!}{r!(n-r)!} \times \frac{(r+1)!(n-r-1)!}{n!} = \frac{5}{6}$

$\therefore \quad \frac{(r+1)r!(n-r-1)!}{r!(n-r)(n-r-1)!} = \frac{5}{6}$

$\therefore \quad \frac{r+1}{n-r} = \frac{5}{6}$

$\therefore \quad 6r + 6 = 5n - 5r$

$\therefore \quad 11r = 5n - 6$

$\therefore \quad 4n + 4 = 5n - 6 \quad \ldots$[From (i)]

$\therefore \quad n = 10$

$\therefore \quad 11r = 4(10) + 4 \quad \ldots$[From (i)]

$\quad = 44$

$\therefore \quad r = 4$

Question 5.
If ${}^nP_r$ = 1814400 and ${}^nC_r$ = 45, find ${}^{n+4}C_{r+3}$.
Solution:

$${}^nP_r = 1814400,\ {}^nC_r = 45$$

$$\therefore \quad \frac{{}^nP_r}{{}^nC_r} = \frac{1814400}{45}$$

$$\therefore \quad \frac{n!}{(n-r)!} \times \frac{r!(n-r)!}{n!} = \frac{1814400}{45}$$

$$\therefore \quad r! = 40320$$

$$\therefore \quad r! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$\therefore \quad r! = 8!$$

$$\therefore \quad r = 8$$

Also, ${}^nC_r = 45$

$$\therefore \quad {}^nC_8 = 45$$

$$\therefore \quad \frac{n!}{8!(n-8)!} = 45$$

$$\therefore \quad \frac{n!}{8!(n-8)!} = 45$$

$$\therefore \quad n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)(n-7) = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3$$

Comparing on both sides, we get

$n = 10$

$$\therefore \quad {}^{n+4}C_{r+3} = {}^{14}C_{11} = \frac{14!}{11!(14-11)!} = \frac{14!}{11!\,3!}$$

$$= \frac{14 \times 13 \times 12 \times 11!}{11! \times 3 \times 2 \times 1}$$

$$= 364$$

Question 6.
If ${}^nC_{r-1}$ = 6435, ${}^nC_r$ = 5005, ${}^nC_{r+1}$ = 3003, find ${}^rC_5$.
Solution:

$${}^nC_{r-1} = 6435,\ {}^nC_r = 5005,\ {}^nC_{r+1} = 3003$$

$$\therefore \quad \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{6435}{5005}$$

$$\therefore \quad \frac{\dfrac{n!}{(r-1)!(n-r+1)!}}{\dfrac{n!}{r!(n-r)!}} = \frac{9}{7}$$

$\therefore \quad \frac{n!}{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{9}{7}$

$\therefore \quad \frac{n!}{(r-1)!(n-r+1)(n-r)!} \times \frac{r(r-1)!(n-r)!}{n!} = \frac{9}{7}$

$\therefore \quad \frac{r}{n-r+1} = \frac{9}{7}$

$\therefore \quad 7r = 9n - 9r + 9$

$\therefore \quad 16r - 9n = 9 \qquad \ldots(i)$

Now, $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{5005}{3003}$

$\therefore \quad \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{5}{3}$

$\therefore \quad \frac{n!}{r!(n-r)!} \times \frac{(r+1)!(n-r-1)!}{n!} = \frac{5}{3}$

$\therefore \quad \frac{n!}{r!(n-r)(n-r-1)!} \times \frac{(r+1)r!(n-r-1)!}{n!} = \frac{5}{3}$

$\therefore \quad \frac{r+1}{n-r} = \frac{5}{3}$

$\therefore \quad 3r + 3 = 5n - 5r$

$\therefore \quad 8r - 5n = -3 \qquad \ldots(ii)$

Solving (i) and (ii), we get

$n = 15, r = 9$

$${}^rC_5 = {}^9C_5 = \frac{9!}{4!5!}$$
$$= \frac{9\times8\times7\times6\times5!}{4\times3\times2\times1\times5!}$$
$$= 126$$

Question 7.
Find the number of ways of drawing 9 balls from a bag that has 6 red balls, 8 green balls, and 7 blue balls so that 3 balls of every colour are drawn.
Solution:
9 balls are to be selected from 6 red, 8 green, 7 blue balls such that the selection consists of 3 balls of each colour.
∴ 3 red balls can be selected from 6 red balls in ${}^6C_3$ ways.
3 reen balls can be selected from 8 green balls in ${}^8C_3$ ways.
3 blue balls can be selected from 7 blue balls in ${}^7C_3$ ways.
∴ Number of ways selection can be done if the selection consists of 3 balls of each colour

$$= {}^6C_3.\ {}^8C_3.\ {}^7C_3$$
$$= \frac{6!}{3!3!} \times \frac{8!}{3!5!} \times \frac{7!}{3!4!}$$
$$= \frac{6\times5\times4\times3!}{3\times2\times1\times3!} \times \frac{8\times7\times6\times5!}{3\times2\times1\times5!} \times \frac{7\times6\times5\times4!}{3\times2\times1\times4!}$$
$$= 20 \times 56 \times 35$$
$$= 39200$$

Question 8.
Find the number of ways of selecting a team of 3 boys and 2 girls from 6 boys and 4 girls.
Solution:
There are 6 boys and 4 girls.
A team of 3 boys and 2 girls is to be selected.
∴ 3 boys can be selected from 6 boys in ${}^6C_3$ ways.
2 girls can be selected from 4 girls in ${}^4C_2$ ways.
∴ Number of ways the team can be selected

$$= {}^6C_3 \times {}^4C_2$$
$$= \frac{6!}{3!3!} \times \frac{4!}{2!2!}$$
$$= \frac{6\times5\times4\times3!}{3\times2\times1\times3!} \times \frac{4\times3\times2!}{2\times2!}$$
$$= 20 \times 6$$
$$= 120$$

Question 9.
After a meeting, every participant shakes hands with every other participants. If the number of handshakes is 66, find the number of participants in the meeting.
Solution:
Let there be n participants present in the meeting.
A handshake occurs between 2 persons.
∴ Number of handshakes = ${}^nC_2$
Given 66 handshakes were exchanged.
66 = ${}^nC_2$
66 = $\frac{n!}{2!(n-2)!}$
66 × 2 = $\frac{n(n-1)(n-2)!}{(n-2)!}$
132 = n (n – 1)
n(n – 1) = 12 × 11
Comparing on both sides, we get n = 12
∴ 12 participants were present at the meeting.

Question 10.
If 20 points are marked on a circle, how many chords can be drawn?
Solution:
To draw a chord we need to join two points on the circle.
There are 20 points on a circle.
∴ Total number of chords possible from these points

$$= {}^{20}C_2$$
$$= \frac{20!}{2!18!}$$
$$= \frac{20 \times 19 \times 18!}{2 \times 1 \times 18!}$$
$$= 190$$

Question 11.
Find the number of diagonals of an n-sided polygon. In particular, find the number of diagonals when
(i) n = 10
(ii) n = 15
(iii) n = 12
(iv) n = 8
Solution:
In n-sided polygon, there are 'n' points and 'n' sides.
∴ Through 'n' points we can draw ${}^nC_2$ lines including sides.
∴ Number of diagonals in n sided polygon = ${}^nC_2$ – n (n = number of sides)

i. n = 10,
$${}^nC_2 - n = {}^{10}C_2 - 10$$
$$= \frac{10 \times 9}{1 \times 2} - 10$$
$$= 45 - 10$$
$$= 35$$

ii. n = 15, ${}^nC_2 - n = {}^{15}C_2 - 15$
$$= \frac{15 \times 14}{1 \times 2} - 15$$
$$= 105 - 15$$
$$= 90$$

iii. n = 12, ${}^nC_2 - n = {}^{12}C_2 - 12$
$$= \frac{12 \times 11}{1 \times 2} - 12$$
$$= 66 - 12$$
$$= 54$$

iv. n = 8, ${}^nC_2 - n = {}^8C_2 - 8$
$$= \frac{8 \times 7}{1 \times 2} - 8$$
$$= 28 - 8$$
$$= 20$$

Question 12.
There are 20 straight lines in a plane so that no two lines are parallel and no three lines are concurrent. Determine the number of points of intersection.
Solution:
There are 20 lines such that no two of them are parallel and no three of them are concurrent.
Since no two lines are parallel, they intersect at a point.
∴ Number of points of intersection if no two lines are parallel and no three lines are concurrent = ${}^{20}C_2$
$= \frac{20!}{2!18!}$
$= \frac{20 \times 19 \times 18!}{2 \times 1 \times 18!}$
= 190

Question 13.
Ten points are plotted on a plane. Find the number of straight lines obtained by joining these points if (a) no three points are collinear (b) four points are collinear
Solution:
There are 10 points on a plane.
(a) When no three of them are collinear.
A line is obtained by joining 2 points.
∴ Number of lines passing through these points = ${}^{10}C_2$
$= \frac{10!}{2!8!}$
$= \frac{10 \times 9 \times 8!}{2 \times 1 \times 8!}$
= 5 × 9
= 45

(b) When 4 of them are collinear.
If no three points are collinear, we get a total of ${}^{10}C_2$ = 45 lines by joining them. .....[From (i)]
Since 4 points are collinear, only one line passes through these points instead of ${}^{4}C_2$ lines.
∴ ${}^{4}C_2$ – 1 extra lines are included in 45 lines.
Number of lines passing through these points
= 45 – (${}^{4}C_2$ – 1)
$= 45 - \frac{4!}{2!2!} + 1$
$= 45 - \frac{4 \times 3 \times 2!}{2 \times 2!} + 1$
= 45 – 6 + 1
= 40

Question 14.
Find the number of triangles formed by joining 12 points if
(a) no three points are collinear
(b) four points are collinear
Solution:
There are 12 points on the plane.
(a) When no three of them are collinear.
A triangle can be drawn by joining any three non-collinear points.
∴ Number of triangles that can be obtained from these points = ${}^{12}C_3$
$= \frac{12!}{3!9!}$
$= \frac{12 \times 11 \times 10 \times 9!}{3 \times 2 \times 1 \times 9!}$
= 220

(b) When 4 of these points are collinear.
If no three points are collinear, total we get ${}^{12}C_3$ = 220 triangles by joining them. ......[From (i)]
Since 4 points are collinear, no triangle can be formed by joining these four points.
∴ ${}^{4}C_3$ extra triangles are included in 220 triangles.
∴ Number of triangles that can be obtained from these points = ${}^{12}C_3 - {}^{4}C_3$
$= 220 - \frac{4!}{3! \times 1!}$
$= 220 - \frac{4 \times 3!}{3!}$
= 220 – 4
= 216

Question 15.
A word has 8 consonants and 3 vowels. How many distinct words can be formed if 4 consonants and 2 vowels are chosen?
Solution:
There are 8 consonants and 3 vowels.
From 8 consonants, 4 can be selected in ${}^{8}C_4$
$= \frac{8!}{4!4!}$
$= \frac{8 \times 7 \times 6 \times 5 \times 4!}{4 \times 3 \times 2 \times 1 \times 4!}$
= 70 ways.
From 3 vowels, 2 can be selected in ${}^{3}C_2$
$= \frac{3!}{2!1!}$
$= \frac{3 \times 2!}{2!}$

= 3 ways.
Now, to form a word, these 6 ietters (i.e., 4 consonants and 2 vowels) can be arranged in ${}^6P_6$ = 6! ways.
∴ Total number of words that can be formed = 70 × 3 × 6!
= 70 × 3 × 720
= 151200
∴ 151200 words of 4 consonants and 2 vowels can be formed.

Question 16.
Find n if,
(i) ${}^nC_8 = {}^nC_{12}$
Solution:
${}^nC_8 = {}^nC_{12}$
If ${}^nC_x = {}^nC_y$, then either x = y or x = n – y
∴ 8 = 12 or 8 = n – 12
But 8 = 12 is not possible
∴ 8 = n – 12
∴ n = 20

(ii) ${}^{23}C_{3n} = {}^{23}C_{2n+3}$
Solution:
${}^{23}C_{3n} = {}^{23}C_{2n+3}$
If ${}^nC_x = {}^nC_y$, then either x = y or x = n – y
∴ 3n = 2n + 3 or 3n = 23 – 2n – 3
∴ n = 3 or n = 4

(iii) ${}^{21}C_{6n} = {}^{21}C_{(n^2+5)}$
Solution:
${}^{21}C_{6n} = {}^{21}C_{(n^2+5)}$
If ${}^nC_x = {}^nC_y$, then either x = y or x = n – y
∴ 6n = n^2 + 5 or 6n = 21 – (n^2 + 5)
∴ n^2 – 6n + 5 = 0 or 6n = 21 – n^2 – 5
∴ n^2 – 6n + 5 = 0 or n^2 + 6n – 16 = 0
If n^2 – 6n + 5 = 0, then (n – 1)(n – 5) = 0
∴ n = 1 or n = 5
If n = 5 then
n^2 + 5 = 30 > 21
∴ n ≠ 5
∴ n = 1
If n^2 + 6n – 16 = 0, then (n + 8)(n – 2) = 0
n = -8 or n = 2
n ≠ -8
∴ n = 2
∴ n = 1 or n = 2

Check:
n = 2
∴ n^2 + 5 = 2^2 + 5 = 9
${}^{21}C_{6n} = {}^{21}C_{12}$
and ${}^{21}C_{(n^2+5)} = {}^{21}C_9$
∴ ${}^nC_r = {}^nC_{n-r}$
∴ ${}^{21}C_{12} = {}^{21}C_9$
∴ n = 2 is a right answer.

(iv) ${}^{2n}C_{r-1} = {}^{2n}C_{r+1}$
Solution:
${}^{2n}C_{r-1} = {}^{2n}C_{r+1}$
If ${}^nC_x = {}^nC_y$, then either x = y or x = n – y
∴ r – 1 = r + 1 or r – 1 = 2n – (r + 1)
But r – 1 = r + 1 is not possible
∴ r – 1 = 2n – (r + 1)
∴ r + r = 2n
∴ r = n

Check:
${}^{2n}C_{r-1} = {}^{2n}C_{n-1}$
and ${}^{2n}C_{r+1} = {}^{2n}C_{n+1}$
using ${}^nC_r = {}^nC_{n-r}$, we have
${}^{2n}C_{n+1} = {}^{2n}C_{2n-(n+1)} = {}^{2n}C_{n-1}$
∴ ${}^{2n}C_{r-1} = {}^{2n}C_{r+1}$

(v) ${}^nC_{n-2}$ = 15
Solution:
${}^nC_{n-2}$ = 15
∴ ${}^nC_2$ = 15 …..[∵ ${}^nC_r = {}^nC_{n-r}$]
∴ $\frac{n!}{(n-2)!2!}=15$
∴ $\frac{n(n-1)(n-2)!}{(n-2)!\times2\times1}=15$
∴ n(n – 1) = 30
∴ n(n – 1) = 6 × 5
Equating both sides, we get
∴ n = 6

Question 17.
Find x if ${}^nP_r = x\,{}^nC_r$.
Solution:

$${}^nP_r = x({}^nC_r)$$
$$x = \frac{{}^nP_r}{{}^nC_r}$$
$$= \frac{\frac{n!}{(n-r)!}}{\frac{n!}{(n-r!)r!}}$$
$$= r!$$

Question 18.
Find r if ${}^{11}C_4 + {}^{11}C_5 + {}^{12}C_6 + {}^{13}C_7 = {}^{14}C_r$.
Solution:

$${}^{11}C_4 + {}^{11}C_5 + {}^{12}C_6 + {}^{13}C_7 = {}^{14}C_r$$
$$\therefore \left({}^{11}C_4 + {}^{11}C_5\right) + {}^{12}C_6 + {}^{13}C_7 = {}^{14}C_r$$
$$\therefore ({}^{12}C_5 + {}^{12}C_6) + {}^{13}C_7 = {}^{14}C_r$$
$$\therefore ({}^{13}C_6 + {}^{13}C_7) = {}^{14}C_r \quad \ldots\left[{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\right]$$
$$\therefore {}^{14}C_7 = {}^{14}C_r$$
If ${}^nC_x = {}^nC_y$, then either $x = y$ or $x = n - y$
$$\therefore r = 7 \text{ or } r = 14 - 7 = 7$$
$$\therefore r = 7$$

Question 19.
Find the value of $\sum_{r=1}^{4} {}^{(21-r)}C_4$.
Solution:

$$\sum_{r=1}^{4} {}^{(21-r)}C_4$$
$$= {}^{20}C_4 + {}^{19}C_4 + {}^{18}C_4 + {}^{17}C_4$$
$$= {}^{20}C_4 + {}^{19}C_4 + {}^{18}C_4 + {}^{18}C_5 - {}^{17}C_5$$
$$= {}^{20}C_4 + {}^{19}C_4 + ({}^{18}C_4 + {}^{18}C_5) - {}^{17}C_5$$
$$= {}^{20}C_4 + {}^{19}C_4 + {}^{19}C_5 - {}^{17}C_5$$
$$\ldots\left[{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\right]$$
$$= {}^{20}C_4 + {}^{20}C_5 - {}^{17}C_5$$
$$= {}^{21}C_5 - {}^{17}C_5$$
$$= \frac{21\times20\times19\times18\times17}{1\times2\times3\times4\times5} - \frac{17\times16\times15\times14\times13}{1\times2\times3\times4\times5}$$
$$= 21 \times 19 \times 3 \times 17 - 17 \times 2 \times 14 \times 13$$
$$= 20349 - 6188 = 14161$$

Question 20.
Find the differences between the greatest values in the following:
(a) ${}^{14}C_r$ and ${}^{12}C_r$
Solution:
Greatest value of ${}^{14}C_r$.
Here, n = 14, which is even.
Greatest value of nCr occurs at r = $\frac{n}{2}$ if n is even.
∴ r = $\frac{n}{2}$
∴ r = $\frac{14}{2}$ = 7

$\therefore$ Greatest value of ${}^{14}C_r = {}^{14}C_7 = \frac{14!}{7!7!}$

$= \frac{14\times13\times12\times11\times10\times9\times8\times7!}{7\times6\times5\times4\times3\times2\times1\times7!}$

$= 3432$

Also, for greatest value of ${}^{12}C_r$

n = 12, which is even.

$\therefore$ $r = \frac{12}{2} = 6$

$\therefore$ ${}^{12}C_6 = \frac{12!}{6!6!}$

$= \frac{12\times11\times10\times9\times8\times7\times6!}{6\times5\times4\times3\times2\times1\times6!}$

$= 924$

$\therefore$ Difference between the greatest values of ${}^{14}C_r$ and ${}^{12}C_r$ = ${}^{14}C_r$ – ${}^{12}C_r$
= 3432 – 924
= 2508

(b) ${}^{13}C_r$ and ${}^{8}C_r$
Solution:
Greatest value of ${}^{13}C_r$.
Here n = 13, which is odd.
Greatest value of nCr occurs at r = n−12 if n is odd.
$\therefore$ r = n−12
$\therefore$ r = 13−12 = 6

$\therefore$ Greatest value of ${}^{13}C_r = {}^{13}C_6 = \frac{13!}{6!7!}$

$= \frac{13\times12\times11\times10\times9\times8\times7!}{6\times5\times4\times3\times2\times1\times7!}$

$= 1716$

Also, for greatest value of ${}^{8}C_r$

n = 8, which is even.

$\therefore$ $r = \frac{8}{2} = 4$

$\therefore$ ${}^{8}C_4 = \frac{8!}{4!4!}$

$= \frac{8\times7\times6\times5\times4!}{4\times3\times2\times1\times4!}$

$= 70$

$\therefore$ Difference between the greatest values of ${}^{13}C_r$ and ${}^{8}C_r$ = ${}^{13}C_r$ – ${}^{8}C_r$
= 1716 – 70
= 1646

(c) ${}^{15}C_r$ and ${}^{11}C_r$
Solution:
Greatest value of ${}^{15}C_r$.
Here n = 15, which is odd.
Greatest value of nCr occurs at r = n−12 if n is odd.
$\therefore$ r = n−12
$\therefore$ r = 15−12 = 7

$\therefore$ Greatest value of ${}^{15}C_r = {}^{15}C_7 = \frac{15!}{7!8!}$

$= \frac{15\times14\times13\times12\times11\times10\times9\times8!}{7\times6\times5\times4\times3\times2\times1\times8!}$

$= 6435$

Also, for greatest value of ${}^{11}C_r$

n = 11, which is odd.

$\therefore$ $r = \frac{11-1}{2} = 5$

$\therefore$ ${}^{11}C_5 = \frac{11!}{5!6!}$

$= \frac{11\times10\times9\times8\times7\times6!}{5\times4\times3\times2\times1\times6!}$

$= 462$

$\therefore$ Difference between the greatest values of ${}^{15}C_r$ and ${}^{11}C_r$ = ${}^{15}C_r$ – ${}^{11}C_r$

= 6435 – 462
= 5973

Question 21.
In how many ways can a boy invite his 5 friends to a party so that at least three join the party?
Solution:
Boy can invite = (3 or 4 or 5 friends)
Consider the following table:

Case I	Case II	Case III
3 friends are invited	4 friends are invited	All 5 friends are invited
Number of ways = ${}^5C_3$ = 10	Number of ways = ${}^5C_4$ = 5	Number of ways = ${}^5C_5$ = 1

∴ Number of ways a boy can invite his friends to a party so that three or more of join the party = 10 + 5 + 1 = 16

Question 22.
A group consists of 9 men and 6 women. A team of 6 is to be selected. How many of possible selections will have at least 3 women?
Solution:
There are 9 men and 6 women.
A team of 6 persons is to be formed such that it consist of at least 3 women.
Consider the following table:

	Case I	Case II	Case III	Case IV
	3W 3M	4W 2M	5W 1M	6W –
Number of ways	${}^6C_3 \times {}^9C_3$ = 20 × 84 = 1680	${}^6C_4 \times {}^9C_2$ = 15 × 36 = 540	${}^6C_5 \times {}^9C_1$ = 6 × 9 = 54	1

∴ No. of ways this can be done = 1680 + 540 + 54 + 1 = 2275
∴ 2275 teams can be formed if team consists of at least 3 women.

Question 23.
A committee of 10 persons is to be formed from a group of 10 women and 8 men. How many possible committees will have at least 5 women? How many possible committees will have men in majority?
Solution:
(i) A committee of 10 persons is to be formed from 10 women and 8 men such that the committee contains at least 5 women.
Consider the following table:

		Number of ways
Case I	5W, 5M	${}^{10}C_5 \times {}^8C_5 = 252 \times 26 = 14112$
Case II	6W, 4M	${}^{10}C_6 \times {}^8C_4 = 210 \times 70 = 14700$
Case III	7W, 3M	${}^{10}C_7 \times {}^8C_3 = 120 \times 56 = 6720$
Case IV	8W, 2M	${}^{10}C_8 \times {}^8C_2 = 45 \times 28 = 1260$
Case V	9W, 1M	${}^{10}C_9 \times {}^8C_1 = 10 \times 8 = 80$
Case VI	10 W	1

∴ Number of committees with at least 5 women
= 14112 + 14700 + 6720 + 1260 + 81
= 36873

(ii) Number of committees with men in majority = Total number of committees – (Number of committees with women in majority + women and men equal in number)
= ${}^{18}C_{10}$ – 36873
= ${}^{18}C_8$ – 36873
= 43758 – 36873
= 6885

Question 24.
A question paper has two sections. Section I has 5 questions and section II has 6 questions. A student must answer at least two questions from each section among 6 questions he answers. How many different choices does the student have in choosing questions?
Solution:
There are 11 questions, out of which 5 questions are from section I and 6 questions are from section II.

The student has to select 6 questions taking at least 2 questions from each section.
Consider the following table:

	Case I	Case II	Case III
No. of questions	Sec I (2Q) Sec II (4Q)	Sec I (3Q) Sec II (3Q)	Sec I (4Q) Sec II (2Q)
Number of ways	${}^5C_2 \times {}^6C_4$ $= 10 \times 15$ $= 150$	${}^5C_3 \times {}^6C_3$ $= 10 \times 20$ $= 200$	${}^5C_4 \times {}^6C_2$ $= 5 \times 15$ $= 75$

∴ Number of choices = 150 + 200 + 75 = 425
∴ In 425 ways students can select 6 questions, taking at least 2 questions from each section.

Question 25.
There are 3 wicketkeepers and 5 bowlers among 22 cricket players. A team of 11 player is to be selected so that there is exactly one wicketkeeper and at least 4 bowlers in the team. How many different teams can be formed?
Solution:
There are 22 cricket players, of which 3 are wicketkeepers and 5 are bowlers.
A team of 11 players is to be chosen such that exactly one wicketkeeper and at least 4 bowlers are to be included in the team.
Consider the following table:

	Case I	Case II
	1 wicketkeeper + 4 bowlers + 6 players	1 wicketkeeper + 5 bowlers + 5 players
Number of ways	${}^3C_1 \times {}^5C_4 \times {}^{14}C_6$ $= 3 \times 5 \times 3003$ $= 45045$	${}^3C_1 \times {}^5C_5 \times {}^{14}C_5$ $= 3 \times 1 \times 2002$ $= 6006$

∴ Number of ways a team of 11 players can be selected
= 45045 + 6006
= 51051

Question 26.
Five students are selected from 11. How many ways can these students be selected if
(a) two specified students are selected?
(b) two specified students are not selected?
Solution:
5 students are to be selected from 11 students.
(a) When 2 specified students are included,
then remaining 3 students can be selected from (11 – 2) = 9 students.
∴ Number of ways of selecting 3 students from 9 students = ${}^9C_3$
$= 9!3!\times6!$
$= 9\times8\times7\times6!3\times2\times1\times6!$
= 84
∴ Selection of students is done in 84 ways when 2 specified students are included.

(b) When 2 specified students are not included, then 5 students can be selected from the remaining (11 – 2) = 9 students.
∴ Number of ways of selecting 5 students from 9 students = ${}^9C_5$
$= 9!5!4!$
$= 9\times8\times7\times6\times5!5!\times4\times3\times2\times1$
= 126
∴ Selection of students is done in 126 ways when 2 specified students are not included.