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Maharashtra State Board 11th Commerce Maths Solutions Chapter 4 Sequences and Series Ex 4.1

Question 1.
Verify whether the following sequences are G.P. If so, write tn.
(i) 2, 6, 18, 54, ……
(ii) 1, -5, 25, -125, …….
(iii) $5-\surd,15\surd,155\surd,1255\surd,\ldots$
(iv) 3, 4, 5, 6, ……
(v) 7, 14, 21, 28, …..
Solution:
(i) 2, 6, 18, 54, …….
$t_1 = 2$, $t_2 = 6$, $t_3 = 18$, $t_4 = 54$, …..
Here, $t_2t_1=t_3t_2=t_4t_3=3$
Since, the ratio of any two consecutive terms is a constant, the given sequence is a geometric progression.
Here, a = 2, r = 3
$t_n= ar_{n-1}$
∴ $t_n = 2(3_{n-1})$

(ii) 1, -5, 25, -125, ……
$t_1 = 1$, $t_2 = -5$, $t_3 = 25$, $t_4 = -125$, …..
Here, $t_2t_1=t_3t_2=t_4t_3=-5$
Since, the ratio of any two consecutive terms is a constant, the given sequence is a geometric progression.
Here, a = 1, r = -5
$t_n = ar_{n-1}$
∴ $t_n = (-5)_{n-1}$

(iii) $5-\surd,15\surd,155\surd,1255\surd,\ldots$

$$t_1 = \sqrt{5},\ t_2 = \frac{1}{\sqrt{5}},\ t_3 = \frac{1}{5\sqrt{5}},\ t_4 = \frac{1}{25\sqrt{5}}, \ldots$$

$$\text{Here, } \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \frac{1}{5}$$

Since, the ratio of any two consecutive terms is a constant, the given sequence is a geometric progression.

$$\text{Here, } a = \sqrt{5},\ r = \frac{1}{5}$$

$$t_n = ar^{n-1}$$

$$t_n = \sqrt{5}\left(\frac{1}{5}\right)^{n-1} = (5)^{\frac{1}{2}}\,(5)^{1-n} = (5)^{\frac{3}{2}-n}$$

(iv) 3, 4, 5, 6,……
$t_1 = 3$, $t_2 = 4$, $t_3 = 5$, $t_4 = 6$, …..
Here, $t_2t_1=43, t_3t_2=54, t_4t_3=65$
Since, $t_2t_1 \neq t_3t_2 \neq t_4t_3$
∴ the given sequence is not a geometric progression.

(v) 7, 14, 21, 28, …..
$t_1 = 7$, $t_2 = 14$, $t_3 = 21$, $t_4 = 28$, …..
Here, $t_2t_1=2, t_3t_2=32, t_4t_3=43$
Since, $t_2t_1 \neq t_3t_2 \neq t_4t_3$
∴ the given sequence is not a geometric progression.

Question 2.
For the G.P.,
(i) if r = 13, a = 9, find t_7.
(ii) if a = 7243, r = 13, find t_3.
(iii) if a = 7, r = -3, find t_6.
(iv) if a = 23, t_6 = 162, find r.
Solution:

i. Given, $r = \frac{1}{3}$, $a = 9$

$t_n = ar^{n-1}$

$\therefore \quad t_7 = 9 \times \left(\frac{1}{3}\right)^{7-1} = \frac{9}{3^6} = \frac{1}{81}$

ii. Given, $a = \frac{7}{243}$, $r = \frac{1}{3}$

$t_n = ar^{n-1}$

$\therefore \quad t_3 = \frac{7}{243} \times \left(\frac{1}{3}\right)^{3-1} = \frac{7}{243} \times \left(\frac{1}{3}\right)^2$

$= \frac{7}{243} \times \frac{1}{9} = \frac{7}{2187}$

iii. Given, $a = 7$, $r = -3$

$t_n = ar^{n-1}$

$\therefore \quad t_6 = 7 \times (-3)^{6-1}$

$= 7 \times (-3)^5$

$= 7 \times (-243) = -1701$

iv. Given, $a = \frac{2}{3}$, $t_6 = 162$

$t_n = ar^{n-1}$

$\therefore \quad t_6 = \left(\frac{2}{3}\right)(r^{6-1})$

$\therefore \quad 162 = \frac{2}{3} r^5$

$\therefore \quad r^5 = 162 \times \frac{3}{2}$

$\therefore \quad r^5 = 3^5$

$\therefore \quad r = 3$

Question 3.
Which term of the G. P. 5, 25, 125, 625, ….. is 510?
Solution:

Here, $t_1 = a = 5$, $r = \frac{t_2}{t_1} = \frac{25}{5} = 5$, $t_n = 5^{10}$

$t_n = ar^{n-1}$

$\therefore \quad 5^{10} = 5 \times 5^{(n-1)}$

$\therefore \quad 5^{10} = 5^{(1+n-1)}$

$\therefore \quad 5^{10} = 5^n$

$\therefore \quad n = 10$

$\therefore \quad 5^{10}$ is the 10^{th} term of the G.P.

Question 4.
For what values of x, 43, x, 427 are in G. P.?
Solution:

$\frac{4}{3}, x, \frac{4}{27}$ are in geometric progression.

$\therefore \quad \frac{t_2}{t_1} = \frac{t_3}{t_2}$

$\therefore \quad \frac{x}{\frac{4}{3}} = \frac{\frac{4}{27}}{x}$

$\therefore \quad x^2 = \frac{4}{3} \times \frac{4}{27}$

$\therefore \quad x^2 = \frac{16}{81}$

$\therefore \quad x = \pm \frac{4}{9}$

Question 5.

If for a sequence, $t_n = \frac{5^{n-3}}{2^{n-3}}$, show that the sequence is a G. P. Find its first term and the common ratio.

Solution:

The sequence (tn) is a G.P., if $\frac{t_n}{t_{n-1}}$ = constant, for all n ∈ N

$$\text{Now, } t_n = \frac{5^{n-3}}{2^{n-3}}$$

$$t_{n-1} = \frac{5^{(n-1)-3}}{2^{(n-1)-3}} = \frac{5^{n-4}}{2^{n-4}}$$

$$\therefore \quad \frac{t_n}{t_{n-1}} = \frac{5^{n-3}}{2^{n-3}} \times \frac{2^{n-4}}{5^{n-4}}$$

$$= \frac{5^{(n-3)} \cdot 5^{-(n-4)}}{2^{(n-3)} \cdot 2^{-(n-4)}}$$

$$= \frac{5^{-3+4}}{2^{-3+4}}$$

$$= \frac{5}{2}, \text{ which is a constant, for all } n \in N.$$

$$\therefore \quad r = \frac{5}{2}$$

∴ the sequence is a G. P. with common ratio $\frac{5}{2}$

First term, $t_1 = \frac{5^{1-3}}{2^{1-3}} = \frac{2^2}{5^2} = \frac{4}{25}$

Question 6.

Find three numbers in G. P. such that their sum is 21 and sum of their squares is 189.

Solution:

Let the three numbers in G. P. be $\frac{a}{r}$, a, ar.

According to the first condition,

$$\frac{a}{r} + a + ar = 21$$

$$\therefore \quad \frac{1}{r} + 1 + r = \frac{21}{a}$$

$$\therefore \quad \frac{1}{r} + r = \frac{21}{a} - 1 \qquad \ldots(i)$$

According to the second condition,

$$\frac{a^2}{r^2} + a^2 + a^2r^2 = 189$$

$$\therefore \quad \frac{1}{r^2} + 1 + r^2 = \frac{189}{a^2}$$

$$\therefore \quad \frac{1}{r^2} + r^2 = \frac{189}{a^2} - 1 \qquad \ldots(ii)$$

On squaring equation (i), we get

$$\frac{1}{r^2} + r^2 + 2 = \frac{441}{a^2} - \frac{42}{a} + 1$$

$$\therefore \quad \left(\frac{189}{a^2} - 1\right) + 2 = \frac{441}{a^2} - \frac{42}{a} + 1 \qquad \ldots[\text{From (ii)}]$$

$$\therefore \quad \frac{189}{a^2} + 1 = \frac{441}{a^2} - \frac{42}{a} + 1$$

$$\therefore \quad \frac{441}{a^2} - \frac{189}{a^2} - \frac{42}{a} = 0$$

$$\therefore \quad \frac{252}{a^2} = \frac{42}{a}$$

$$\therefore \quad 252 = 42a$$

$$\therefore \quad a = 6$$

Substituting the value of a in (i), we get

$$\frac{1}{r} + r = \frac{21}{6} - 1$$

$$\therefore \quad \frac{1+r^2}{r} = \frac{15}{6}$$

$\therefore \quad \frac{1+r^2}{r} = \frac{5}{2}$

$\therefore \quad 2r^2 - 5r + 2 = 0$

$\therefore \quad 2r^2 - 4r - r + 2 = 0$

$\therefore \quad (2r - 1)(r - 2) = 0$

$\therefore \quad r = \frac{1}{2}$ or 2

When $a = 6$, $r = \frac{1}{2}$,

$\frac{a}{r} = 12$, $a = 6$, $ar = 3$

When $a = 6$, $r = 2$

$\frac{a}{r} = 3$, $a = 6$, $ar = 12$

$\therefore$ the three numbers are 12, 6, 3 or 3, 6, 12.
Check:
First condition:
12, 6, 3 are in G.P. with $r = \frac{1}{2}$
12 + 6 + 3 = 21
Second condition:
$12^2 + 6^2 + 3^2 = 144 + 36 + 9 = 189$
Thus, both the conditions are satisfied.

Question 7.
Find four numbers in G. P. such that sum of the middle two numbers is $\frac{10}{3}$ and their product is 1.
Solution:
Let the four numbers in G.P. be $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.
According to the second condition,
$\frac{a}{r^3}\left(\frac{a}{r}\right)(ar)(ar^3) = 1$
$\therefore a^4 = 1$
$\therefore a = 1$
According to the first condition,

$\frac{a}{r} + ar = \frac{10}{3}$

$\therefore \quad \frac{1}{r} + (1)r = \frac{10}{3}$

$\therefore \quad \frac{1+r^2}{r} = \frac{10}{3}$

$\therefore \quad 3 + 3r^2 = 10r$

$\therefore \quad 3r^2 - 10r + 3 = 0$

$\therefore \quad (r - 3)(3r - 1) = 0$

$\therefore \quad r = 3$ or $r = \frac{1}{3}$

When $r = 3$, $a = 1$

$\frac{a}{r^3} = \frac{1}{(3)^3} = \frac{1}{27}$, $\frac{a}{r} = \frac{1}{3}$, $ar = 1(3) = 3$ and

$ar^3 = 1(3)^3 = 27$

When $r = \frac{1}{3}$, $a = 1$

$\frac{a}{r^3} = \frac{1}{\left(\frac{1}{3}\right)^3} = 27$, $\frac{a}{r} = \frac{1}{\left(\frac{1}{3}\right)} = 3$,

$ar = 1\left(\frac{1}{3}\right) = \frac{1}{3}$ and $ar^3 = 1\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

$\therefore$ the four numbers in G.P.are

$\frac{1}{27}, \frac{1}{3}, 3, 27$ or $27, 3, \frac{1}{3}, \frac{1}{27}$.

Question 8.
Find five numbers in G. P. such that their product is 1024 and the fifth term is square of the third term.
Solution:

Let the five numbers in G. P. be

ar2,ar,a,ar,ar2

According to the given conditions,

$$\frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 = 1024$$

$\therefore \quad a^5 = 4^5$

$\therefore \quad a = 4 \quad \ldots(i)$

Also, $ar^2 = a^2$

$\therefore \quad r^2 = a$

$\therefore \quad r^2 = 4 \quad \ldots$[From (i)]

$\therefore \quad r = \pm 2$

When $a = 4$, $r = 2$

$$\frac{a}{r^2} = 1, \ \frac{a}{r} = 2, \ a = 4, \ ar = 8, \ ar^2 = 16$$

When a = 4, r = -2

ar2 = 1, ar = -2, a = 4, ar = -8, ar2 = 16

$\therefore$ the five numbers in G.P. are 1, 2, 4, 8, 16 or 1, -2, 4, -8, 16.

Question 9.

The fifth term of a G. P. is x, eighth term of the G. P. is y and eleventh term of the G. P. is z. Verify whether $y^2 = xz$.

Solution:

Given, $t_5 = x$, $t_8 = y$, $t_{11} = z$

Since, $t_n = ar^{n-1}$

$\therefore \quad t_5 = ar^4$, $t_8 = ar^7$, $t_{11} = ar^{10}$

Consider,

L.H.S. $= y^2 = (t_8)^2 = (ar^7)^2 = a^2r^{14}$

R.H.S. $= xz = t_5.t_{11} = ar^4.ar^{10} = a^2r^{14}$

$\therefore$ L.H.S. = R.H.S.

$\therefore \quad y^2 = xz$

Question 10.

If p, q, r, s are in G. P., show that p + q, q + r, r + s are also in G.P.

Solution:

p, q, r, s are in G.P.

$$\therefore \quad \frac{q}{p} = \frac{r}{q} = \frac{s}{r}$$

Let $\frac{q}{p} = \frac{r}{q} = \frac{s}{r} = k$

$\therefore \quad q = pk$, $r = qk$, $s = rk$

We have to prove that p + q, q + r, r + s are in G.P.

i.e., to prove that $\frac{q+r}{p+q} = \frac{r+s}{q+r}$

$$\text{L.H.S.} = \frac{q+r}{p+q} = \frac{q+qk}{p+pk} = \frac{q(1+k)}{p(1+k)} = \frac{q}{p} = k$$

$$\text{R.H.S.} = \frac{r+s}{q+r} = \frac{r+rk}{q+qk} = \frac{r(1+k)}{q(1+k)} = \frac{r}{q} = k$$

$$\therefore \quad \frac{q+r}{p+q} = \frac{r+s}{q+r}$$

$\therefore$ p + q, q + r, r + s are in G.P.

Maharashtra State Board 11th Commerce Maths Solutions Chapter 4 Sequences and Series Ex 4.2

Question 1.
For the following G.P.'s, find S_n.
(i) 3, 6, 12, 24, ...
(ii) $p, q, \frac{q^2}{p}, \frac{q^3}{p^2}, \ldots$
Solution:

i. 3, 6, 12, 24, ...

Here, $a = 3$, $r = \frac{6}{3} = 2 > 1$

$S_n = \frac{a(r^n - 1)}{r - 1}$, for $r > 1$

$\therefore \quad S_n = \frac{3(2^n - 1)}{2 - 1}$

$\therefore \quad S_n = 3(2^n - 1)$

ii. $p, q, \frac{q^2}{p}, \frac{q^3}{p^2}, \ldots$

Here, $a = p$, $r = \frac{q}{p}$

Let $\frac{q}{p} < 1$

$S_n = \frac{a(1 - r^n)}{1 - r}$, for $r < 1$

$$\therefore \quad S_n = \frac{p\left[1 - \left(\frac{q}{p}\right)^n\right]}{1 - \frac{q}{p}}$$

$$\therefore \quad S_n = \frac{p^2}{p - q}\left[1 - \left(\frac{q}{p}\right)^n\right]$$

Let $\frac{q}{p} > 1$

$S_n = \frac{a(r^n - 1)}{r - 1}$, for $r > 1$

$$\therefore \quad S_n = \frac{p\left[\left(\frac{q}{p}\right)^n - 1\right]}{\frac{q}{p} - 1} = \frac{p^2}{q - p}\left[\left(\frac{q}{p}\right)^n - 1\right]$$

Question 2.
For a G.P., if
(i) $a = 2$, $r = -\frac{2}{3}$, find S_6.
(ii) $S_5 = 1023$, $r = 4$, find a.

Solution:

i. $a = 2, r = -\frac{2}{3}$

$S_n = \frac{a(1-r^n)}{1-r}$, for $r < 1$

$\therefore \quad S_6 = \frac{2\left[1-\left(-\frac{2}{3}\right)^6\right]}{1-\left(-\frac{2}{3}\right)}$

$= \frac{2\left[1-\left(\frac{2}{3}\right)^6\right]}{\frac{5}{3}}$

$= \frac{6}{5}\left[\frac{729-64}{3^6}\right] = \frac{6}{5}\left[\frac{665}{729}\right]$

$\therefore \quad S_6 = \frac{266}{243}$

ii. $r = 4, S_5 = 1023$

$S_n = a\left(\frac{r^n-1}{r-1}\right)$, for $r > 1$

$\therefore \quad S_5 = a\left(\frac{4^5-1}{4-1}\right)$

$\therefore \quad 1023 = a\left(\frac{1024-1}{3}\right)$

$\therefore \quad 1023 = \frac{a}{3}(1023)$

$\therefore \quad a = 3$

Question 3.
For a G. P., if
(i) a = 2, r = 3, S_n = 242, find n.
(ii) sum of the first 3 terms is 125 and the sum of the next 3 terms is 27, find the value of r.
Solution:
(i) a = 2, r = 3, S_n = 242
Sn = $a\left(\frac{r^n-1}{r-1}\right)$, for r > 1

$\therefore \quad 242 = 2\left(\frac{3^n - 1}{3 - 1}\right)$

$\therefore \quad 242 = 3^n - 1$

$\therefore \quad 3^n = 243$

$\therefore \quad 3^n = 3^5$

$\therefore \quad n = 5$

$S_5 = 2\left(\frac{3^5 - 1}{3 - 1}\right) = 2\left(\frac{243 - 1}{2}\right) = 242$

Thus, our answer is correct.

ii. $S_3 = 125$, $S_6 = 125 + 27 = 152$

$S_n = a\left(\frac{1 - r^n}{1 - r}\right)$

$\therefore \quad S_3 = a\left(\frac{1 - r^3}{1 - r}\right)$

$\therefore \quad 125 = a\left(\frac{1 - r^3}{1 - r}\right) \quad \ldots(i)$

Also, $S_6 = a\left(\frac{1 - r^6}{1 - r}\right)$

$\therefore \quad 152 = a\left(\frac{1 - r^6}{1 - r}\right) \quad \ldots(ii)$

Dividing (ii) by (i), we get

$\frac{152}{125} = \frac{1-r^6}{1-r^3}$

$\therefore \quad \frac{152}{125} = \frac{(1+r^3)(1-r^3)}{(1-r^3)}$

$\therefore \quad 1 + r^3 = \frac{152}{125}$

$\therefore \quad r^3 = \frac{152}{125} - 1$

$\therefore \quad r^3 = \frac{27}{125}$

$\therefore \quad r^3 = \left(\frac{3}{5}\right)^3$

$\therefore \quad r = \frac{3}{5}$

Question 4.
For a G. P.,
(i) if $t_3 = 20$, $t_6 = 160$, find S_7.
(ii) if $t_4 = 16$, $t_9 = 512$, find S_{10}.
Solution:
(i) $t_3 = 20$, $t_6 = 160$
$t_n = ar^{n-1}$
$\therefore t_3 = ar^{3-1} = ar^2$
$\therefore ar^2 = 20$
$\therefore a = \frac{20}{r^2}$ ......(i)

Also, $t_6 = ar^5$

$\therefore \quad ar^5 = 160$

$\therefore \quad \left(\frac{20}{r^2}\right)r^5 = 160 \qquad \ldots$[From (i)]

$\therefore \quad r^3 = \frac{160}{20} = 8$

$\therefore \quad r = 2$

Substituting the value of r in (i), we get

$a = \frac{20}{2^2} = 5$

Now, $S_n = \frac{a(r^n - 1)}{r - 1}$, for $r > 1$

$\therefore \quad S_7 = \frac{5(2^7 - 1)}{2 - 1} = 5(128 - 1) = 635$

ii. $t_4 = 16$, $t_9 = 512$

$t_n = ar^{n-1}$

$\therefore \quad t_4 = ar^{4-1} = ar^3$

$\therefore \quad ar^3 = 16$

$\therefore \quad a = \frac{16}{r^3} \qquad \ldots$(i)

Also, $t_9 = ar^8$

$\therefore \quad ar^8 = 512$

$\therefore \quad \frac{16}{r^3} \times r^8 = 512$

$\therefore \quad r^5 = 32$

$\therefore \quad r = 2$

Substituting r = 2 in (i), we get

$a = \frac{16}{2^3} = \frac{16}{8} = 2$

Now, $S_n = \frac{a(r^n - 1)}{r - 1}$ for $r > 1$

$$\therefore \quad S_{10} = \frac{2(2^{10} - 1)}{2 - 1}$$
$$= 2(1024 - 1)$$
$$= 2046$$

Question 5.
Find the sum to n terms:
(i) 3 + 33 + 333 + 3333 + ……
(ii) 8 + 88 + 888 + 8888 + ……..
Solution:
(i) S_n = 3 + 33 + 333 +….. upto n terms
= 3(1 + 11 + 111 +….. upto n terms)
= $\frac{3}{9}$(9 + 99 + 999 + … upto n terms)
= $\frac{3}{9}$[(10 – 1) + (100 – 1) + (1000 – 1) +… upto n terms]
= $\frac{3}{9}$[(10 + 100 + 1000 + … upto n terms) – (1 + 1 + 1 + … n times)]
But 10, 100, 1000, … n terms are in G.P.
with a = 10, r = $\frac{100}{10}$ = 10

$$\therefore \quad S_n = \frac{3}{9}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right]$$
$$= \frac{3}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$$
$$\therefore \quad S_n = \frac{1}{27}\,[10(10^n - 1) - 9n]$$

(ii) S_n = 8 + 88 + 888 + … upto n terms
= 8(1 + 11 + 111 + … upto n terms)
= $\frac{8}{9}$ (9 + 99 + 999 + … upto n terms)
= $\frac{8}{9}$ [(10 – 1) + (100 – 1) + (1000 – 1) +… upto n terms]
= $\frac{8}{9}$ [(10 + 100 + 1000 + … upto n terms) – (1 + 1 + 1 + … n times)]
But 10, 100, 1000, … n terms are in G.P. with

a = 10, r = 10010 = 10

$$\therefore \quad S_n = \frac{8}{9}\left[10\left(\frac{10^n-1}{10-1}\right)-n\right]$$

$$= \frac{8}{9}\left[\frac{10}{9}\left(10^n-1\right)-n\right]$$

$$\therefore \quad S_n = \frac{8}{81}\left[10(10^n-1)-9n\right]$$

Question 6.
Find the sum to n terms:
(i) 0.4 + 0.44 + 0.444 + ……
(ii) 0.7 + 0.77 + 0.777 + …..
Solution:
(i) S_n = 0.4 + 0.44 + 0.444 + ….. upto n terms
= 4(0.1 + 0.11 + 0.111 + …. upto n terms)
= 49 (0.9 + 0.99 + 0.999 + … upto n terms)
= 49 [(i – 0.1) + (1 – 0.01) + (1 – 0.001) … upto n terms]
= 49 [(1 + 1 + 1 + …n times) – (0.1 + 0.01 + 0.001 +… upto n terms)]
But 0.1, 0.01, 0.001, … n terms are in G.P.
with a = 0.1, r = 0.010.1 = 0.1
∴ S_n = 49{n–0.1[1-(0.1)n1-0.1]}

$$\therefore \quad S_n = \frac{4}{9}\left\{n-\frac{0.1}{0.9}\left[1-(0.1)^n\right]\right\}$$

$$\therefore \quad S_n = \frac{4}{9}\left[n-\frac{1}{9}\left(1-(0.1)^n\right)\right]$$

$$\therefore \quad S_n = \frac{4}{81}\left\{9n-\left(1-\frac{1}{10^n}\right)\right\}$$

(ii) S_n = 0.7 + 0.77 + 0.777 + … upto n terms
= 7(0.1 + 0.11 + 0.111 + … upto n terms)
= 79 (0.9 + 0.99 + 0.999 + … upto n terms)
= 79 [(1 – 0.1) + (1 – 0.01) + (1 – 0.001) +… upto n terms]
= 79 [(1 + 1 + 1 +… n times) – (0.1 + 0.01 + 0.001 +… upto n terms)]
But 0.1, 0.01, 0.001, … n terms are in G.P.
with a = 0.1, r = 0.010.1 = 0.1

$$S_n = \frac{7}{9}\left\{n-0.1\left[\frac{1-(0.1)^n}{1-0.1}\right]\right\}$$

$$= \frac{7}{9}\left\{n-\frac{0.1}{0.9}\left[1-(0.1)^n\right]\right\}$$

$$= \frac{7}{9}\left\{n-\frac{1}{9}\left[1-(0.1)^n\right]\right\}$$

$$S_n = \frac{7}{81}\left\{9n-\left(1-\frac{1}{10^n}\right)\right\}$$

Question 7.
Find the nth terms of the sequences:
(i) 0.5, 0.55, 0.555,…..
(ii) 0.2, 0.22, 0.222,…..
Solution:
(i) Let t_1 = 0.5, t_2 = 0.55, t_3 = 0.555 and so on.
t_1 = 0.5
t_2 = 0.55 = 0.5 + 0.05
t_3 = 0.555 = 0.5 + 0.05 + 0.005
∴ t_n = 0.5 + 0.05 + 0.005 + … upto n terms
But 0.5, 0.05, 0.005, … upto n terms are in G.P. with a = 0.5 and r = 0.1

$\therefore$ t_n = the sum of first n terms of the G.P.

$$\therefore \quad t_n = 0.5 \left\{\frac{1-(0.1)^n}{1-0.1}\right\}$$

$$\therefore \quad t_n = \frac{0.5}{0.9} \{1-(0.1)^n\}$$

$$\therefore \quad t_n = \frac{5}{9} \{1-(0.1)^n\}$$

(ii) Let $t_1 = 0.2$, $t_2 = 0.22$, $t_3 = 0.222$ and so on
$t_1 = 0.2$
$t_2 = 0.22 = 0.2 + 0.02$
$t_3 = 0.222 = 0.2 + 0.02 + 0.002$
$\therefore t_n = 0.2 + 0.02 + 0.002 + \dots$ upto n terms
But 0.2, 0.02, 0.002, … upto n terms are in G.P. with a = 0.2 and r = 0.1
$\therefore$ t_n = the sum of first n terms of the G.P.

$$\therefore \quad t_n = 0.2 \left\{\frac{1-(0.1)^n}{1-0.1}\right\}$$

$$\therefore \quad t_n = \frac{0.2}{0.9} \{1-(0.1)^n\}$$

$$\therefore \quad t_n = \frac{2}{9} \{1-(0.1)^n\}$$

Question 8.
For a sequence, if $S_n = 2(3^n - 1)$, find the nth term, hence showing that the sequence is a G.P.
Solution:

$$S_n = 2(3^n - 1)$$

$$\therefore \quad S_{n-1} = 2(3^{n-1} - 1)$$

But $t_n = S_n - S_{n-1}$

$$= 2(3^n - 1) - 2(3^{n-1} - 1)$$

$$= 2(3^n - 1 - 3^{n-1} + 1)$$

$$= 2(3^n - 3^{n-1}) = 2(3^{n-1+1} - 3^{n-1})$$

$$\therefore \quad t_n = 2.3^{n-1}(3-1) = 4.3^{n-1}$$

$$\therefore \quad t_{n-1} = 4.3^{(n-1)-1} = 4.3^{n-2}$$

The sequence (t_n) is a G. P.,

if $\frac{t_n}{t_{n-1}}$ = constant

for all $n \in N$

$$\therefore \quad \frac{t_n}{t_{n-1}} = \frac{4.3^{n-1}}{4.3^{n-2}} = \frac{3^{n-1}}{3^{n-1}.3^{(-1)}}$$

$$= 3 = \text{ constant for all } n \in N$$

$$\therefore \quad r = 3$$

$\therefore$ the sequence is a G.P. with $t_n = 4.3^{n-1}$.

Question 9.
If S, P, R are the sum, product and sum of the reciprocals of n terms of a G.P. respectively, then verify that $\left(\frac{S}{R}\right)^n = P^2$.
Solution:
Let a be the 1st term and r be the common ratio of the G.P.
$\therefore$ the G.P. is a, ar, ar^2, ar^3, …, ar^{n-1}

$S = a + ar + ar^2 + \dots + ar^{n-1} = a\left(\frac{r^n - 1}{r - 1}\right)$

$P = a(ar)(ar^2)\dots(ar^{n-1})$

$= a^n . r^{1+2+3+\dots+(n-1)}$

$= a^n . r^{\frac{n(n-1)}{2}}$

$\therefore \quad P^2 = a^{2n} . r^{n(n-1)} \quad \dots(i)$

$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \dots + \frac{1}{ar^{n-1}}$

$= \frac{r^{n-1} + r^{n-2} + r^{n-3} + \dots + r^2 + r + 1}{a . r^{n-1}}$

$= \frac{1 + r + r^2 + \dots + r^{n-2} + r^{n-1}}{a . r^{n-1}}$

$1, r, r^2, \dots, r^{n-1}$ are in G.P., with $a = 1$, $r = r$

$\therefore \quad 1 + r + r^2 + \dots + r^{n-1} = 1.\left(\frac{r^n - 1}{r - 1}\right)$

$\therefore \quad R = \frac{1}{ar^{n-1}}\left(\frac{r^n - 1}{r - 1}\right) = \frac{1}{a^2 . r^{n-1}} \times a \times \left(\frac{r^n - 1}{r - 1}\right)$

$\therefore \quad R = \frac{1}{a^2 . r^{n-1}} S$

$\therefore \quad a^2 . r^{n-1} = \frac{S}{R}$

$\therefore \quad (a^2 . r^{n-1})^n = \left(\frac{S}{R}\right)^n$

$\therefore \quad a^{2n} . r^{n(n-1)} = \left(\frac{S}{R}\right)^n$

$\therefore \quad P^2 = \left(\frac{S}{R}\right)^n \quad \dots$[From (i)]

Question 10.
If S_n, S_{2n}, S_{3n} are the sum of n, 2n, 3n terms of a G.P. respectively, then verify that $S_n (S_{3n} - S_{2n}) = (S_{2n} - S_n)^2$.
Solution:
Let a and r be the 1st term and common ratio of the G.P. respectively.

$\therefore \quad S_n = a\left(\frac{r^n - 1}{r-1}\right), S_{2n} = a\left(\frac{r^{2n} - 1}{r-1}\right), S_{3n} = a\left(\frac{r^{3n} - 1}{r-1}\right)$

$\therefore \quad S_{2n} - S_n = a\left(\frac{r^{2n} - 1}{r-1}\right) - a\left(\frac{r^n - 1}{r-1}\right)$

$= \frac{a}{r-1}(r^{2n} - 1 - r^n + 1)$

$= \frac{a}{r-1}(r^{2n} - r^n)$

$= \frac{a\,r^n}{r-1}(r^n - 1)$

$\therefore \quad S_{2n} - S_n = r^n.\frac{a(r^n - 1)}{r-1}$ ....(i)

$S_{3n} - S_{2n} = a\left(\frac{r^{3n} - 1}{r-1}\right) - a\left(\frac{r^{2n} - 1}{r-1}\right)$

$= \frac{a}{r-1}(r^{3n} - 1 - r^{2n} + 1)$

$= \frac{a}{r-1}(r^{3n} - r^{2n})$

$= \frac{a}{r-1}.\,r^{2n}(r^n - 1) = a.\left(\frac{r^n - 1}{r-1}\right).r^{2n}$

$\therefore \quad S_n(S_{3n} - S_{2n}) = \left[a.\left(\frac{r^n - 1}{r-1}\right)\right]\left[a.\left(\frac{r^n - 1}{r-1}\right)r^{2n}\right]$

$= \left[r^n.\frac{a(r^n - 1)}{r-1}\right]^2$

$\therefore \quad S_n(S_{3n} - S_{2n}) = (S_{2n} - S_n)^2$ ...[From (i)]

Maharashtra State Board 11th Commerce Maths Solutions Chapter 4 Sequences and Series Ex 4.3

Question 1.
Determine whether the sum to infinity of the following G.P'.s exist. If exists, find it.

(i) 12,14,18,116,…

(ii) 2,43,89,1627,…

(iii) −3,1,−13,19,…

(iv) 15,−25,45,−85,165,…

Solution:

i. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

Here, $a = \frac{1}{2}$, $r = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

Since, $|r| = \left|\frac{1}{2}\right| < 1$

∴ Sum to infinity exists.

Sum to infinity $= \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$

ii. $2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \ldots$

$a = 2$, $r = \frac{\frac{4}{3}}{2} = \frac{2}{3}$

Since, $|r| = \left|\frac{2}{3}\right| < 1$

∴ Sum to infinity exists.

Sum to infinity $= \frac{a}{1-r} = \frac{2}{1-\frac{2}{3}} = 6$

iii. $-3, 1, \frac{-1}{3}, \frac{1}{9}, \ldots$

$a = -3$, $r = -\frac{1}{3}$

Since, $|r| = \left|-\frac{1}{3}\right| < 1$

∴ Sum to infinity exists.

Sum to infinity $= \frac{a}{1-r} = \frac{-3}{1-\left(-\frac{1}{3}\right)} = -\frac{9}{4}$

iv. $\frac{1}{5}, \frac{-2}{5}, \frac{4}{5}, \frac{-8}{5}, \frac{16}{5}, \ldots$

$a = \frac{1}{5}$, $r = \frac{\frac{-2}{5}}{\frac{1}{5}}$

Since, $|r| = |-2| > 1$

∴ Sum to infinity does not exist.

Question 2.
Express the following recurring decimals as a rational number.

(i) $0.\overline{32}$

(ii) 3.5

(iii) $4.\overline{18}$

(iv) $0.3\overline{45}$

(v) $3.\overline{456}$

Solution:

(i) $0.\overline{32}$ = 0.323232…..

= 0.32 + 0.0032 + 0.000032 + …..

Here, 0.32, 0.0032, 0.000032, … are in G.P. with a = 0.32 and r = 0.01

Since, |r| = |0.01| < 1

∴ Sum to infinity exists.

∴ Sum to infinity = $\frac{a}{1-r}$

$$\therefore \quad 0.\overline{32} = \frac{0.32}{1-(0.01)} = \frac{0.32}{0.99}$$

$$\therefore \quad 0.\overline{32} = \frac{32}{99}$$

(ii) 3.5 = 3.555… = 3 + 0.5 + 0.05 + 0.005 + …

Here, 0.5, 0.05, 0.005, … are in G.P. with a = 0.5 and r = 0.1

Since, |r| = |0.1| < 1

∴ Sum to infinity exists.

$$\therefore \quad \text{Sum to infinity} = \frac{a}{1-r} = \frac{0.5}{1-(0.1)} = \frac{0.5}{0.9} = \frac{5}{9}$$

$$\therefore \quad 3.\dot{5} = 3 + \frac{5}{9} = \frac{32}{9}$$

(iii) $4.\overline{18}$ = 4.181818…..

= 4 + 0.18 + 0.0018 + 0.000018 + …..

Here, 0.18, 0.0018, 0.000018, … are in G.P. with a = 0.18 and r = 0.01

Since, |r| = |0.01| < 1

∴ Sum to infinity exists.

$$\text{Sum of infinity} = \frac{a}{1-r} = \frac{0.18}{1-(0.01)} = \frac{0.18}{0.99}$$

$$= \frac{18}{99} = \frac{2}{11}$$

$$4.\overline{18} = 4 + \frac{2}{11} = \frac{46}{11}$$

(iv) 0.345 = 0.3454545…..

= 0.3 + 0.045 + 0.00045 + 0.0000045 + …..

Here, 0.045, 0.00045, 0.0000045, … are in G.P. with a = 0.045, r = 0.01

Since, |r| = |0.01| < 1

∴ Sum to infinity exists.

$\therefore$ Sum to infinity $= \frac{a}{1-r}$

$= \frac{0.045}{1-0.01}$

$= \frac{0.045}{0.99} = \frac{45}{990}$

$\therefore \quad 0.3\overline{45} = 0.3 + \frac{45}{990}$

$= \frac{3}{10} + \frac{1}{22}$

$= \frac{33+5}{110}$

$= \frac{38}{110} = \frac{19}{55}$

Alternate method:

$0.3\overline{45} = \frac{3.\overline{45}}{10}$

$= \frac{3 + 0.45 + 0.0045 + 0.000045 + \ldots}{10}$

Here, 0.45, 0.0045, 0.000045… are in G.P.
with a = 0.45 and r = 0.01
Since, | r | = |0.01| < 1

$\therefore$ Sum to infinity exists.

$\therefore$ Sum to infinity $= \frac{a}{1-r} = \frac{0.45}{1-0.01} = \frac{0.45}{0.99}$

$= \frac{45}{99} = \frac{5}{11}$

$\therefore \quad 0.3\overline{45} = \frac{3+\frac{5}{11}}{10} = \frac{\frac{38}{11}}{10} = \frac{19}{55}$

(v) $3.4\overline{56}$ = 3.4565656 .....
= 3.4 + 0.056 + 0.00056 + 0.0000056 + ...
Here, 0.056, 0.00056, 0.0000056, … are in G.P. with a = 0.056 and r = 0.01
Since, |r| = |0.01| < 1
∴ Sum to infinity exists.

Sum to infinity $= \frac{a}{1-r}$

$= \frac{0.056}{1-0.01}$

$= \frac{0.056}{0.99} = \frac{56}{990}$

$3.4\overline{56} = 3.4 + \frac{56}{990}$

$= \frac{34}{10} + \frac{56}{990}$

$= \frac{3366+56}{990} = \frac{3422}{990} = \frac{1711}{495}$

Question 3.
If the common ratio of a G.P. is $\frac{2}{3}$ and sum of its terms to infinity is 12. Find the first term.
Solution:
r = $\frac{2}{3}$, sum to infinity = 12 … [Given]
Sum to infinity = $\frac{a}{1-r}$
∴ 12 = $\frac{a}{1-\frac{2}{3}}$
∴ a = 12 × $\frac{1}{3}$
∴ a = 4

Question 4.
If the first term of a G.P. is 16 and sum of its terms to infinity is 1765, find the common ratio.
Solution:
a = 16, sum to infinity = 1765 … [Given]
Sum to infinity = a1−r

∴ 1765=161−r

∴ 115=11−r

∴ 11 – 11r = 5
∴ 11r = 6
∴ r = 611

Question 5.
The sum of the terms of an infinite G.P. is 5 and the sum of the squares of those terms is 15. Find the G.P.
Solution:
Let the required G.P. be a, ar, ar2, ar3, …..
Sum to infinity of this G.P. = 5
∴ 5 = a1−r
∴ a = 5(1 – r) ……(i)
Also, the sum of the squares of the terms is 15.

$\therefore \quad (a^2 + a^2r^2 + a^2r^4 + \ldots) = 15$

$\therefore \quad 15 = \dfrac{a^2}{1-r^2}$

$\therefore \quad 15(1 - r^2) = a^2$

$\therefore \quad 15(1 - r)(1 + r) = 25(1 - r)^2 \qquad \ldots$[From (i)]

$\therefore \quad 3(1 + r) = 5(1 - r)$

$\therefore \quad 3 + 3r = 5 - 5r$

$\therefore \quad 8r = 2$

$\therefore \quad r = \dfrac{1}{4}$

$\therefore \quad a = 5\left(1 - \dfrac{1}{4}\right) = 5\left(\dfrac{3}{4}\right) = \dfrac{15}{4}$

$\therefore$ Required G.P. is a, ar, ar^2, ar^3, …
i.e., $\dfrac{15}{4}, \dfrac{15}{16}, \dfrac{15}{64}, \ldots$

Maharashtra State Board 11th Commerce Maths Solutions Chapter 4 Sequences and Series Ex 4.4

Question 1.
Verify whether the following sequences are H.P.
(i) 13,15,17,19,…

(ii) 13,16,19,112,……….

(iii) 17,19,111,113,115,…
Solution:
(i) 13,15,17,19,…
Here, the reciprocal sequence is 3, 5, 7, 9, …
∴ t_1 = 3, t_2 = 5, t_3 = 7, …..
∵ $t_2 - t_1 = t_3 - t_2 = t_4 - t_3$ = 2, constant
∴ The reciprocal sequence is an A.P.
∴ the given sequence is H.P.

(ii) 13,16,19,112,…………

Here, the reciprocal sequence is 3, 6, 9, 12 …
∴ $t_1 = 3$, $t_2 = 6$, $t_3 = 9$, $t_4 = 12$, …
∵ $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = 3$, constant
∴ The reciprocal sequence is an A.P.
∴ The given sequence is H.P.

(iii) 17,19,111,113,115,…

Here, the reciprocal sequence is 7, 9, 11, 13, 15, ……
∴ $t_1 = 7$, $t_2 = 9$, $t_3 = 11$, $t_4 = 13$, …..
∵ $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = 2$, constant
∴ The reciprocal sequence is an A.P.
∴ The given sequence is H.P.

Question 2.
Find the nth term and hence find the 8th term of the following H.P.s:

(i) 12,15,18,111,………

(ii) 14,16,18,110,…………

(iii) 15,110,115,120,… … …

Solution:

i. $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \ldots$ are in H.P.

∴ 2, 5, 8, 11, … are in A.P.

∴ $a = 2, d = 3$

$t_n = a + (n - 1)d$
$= 2 + (n - 1)(3)$
$= 3n - 1$

∴ n^{th} term of H.P. is $\frac{1}{3n-1}$

∴ 8^{th} term of H.P. $= \frac{1}{3(8)-1} = \frac{1}{23}$

ii. $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots$ are in H.P.

∴ 4, 6, 8, 10, … are in A.P.

∴ $a = 4, d = 2$

$t_n = a + (n - 1)d$
$= 4 + (n - 1)(2)$
$= 2n + 2$

∴ n^{th} term of H.P. $= \frac{1}{2n+2}$

∴ 8^{th} term of H.P. $= \frac{1}{2(8)+2} = \frac{1}{18}$

iii. $\frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \ldots$ are in H.P.

5, 10, 15, 20, … are in H.P.

∴ $a = 5, d = 5$

$t_n = a + (n - 1)d$
$= 5 + (n - 1)(5)$
$= 5n$

∴ n^{th} term of H.P. $= \frac{1}{5n}$

∴ 8^{th} term of H.P. $= \frac{1}{5(8)} = \frac{1}{40}$

Question 3.
Find A.M. of two positive numbers whose G.M. and H.M. are 4 and 165.
Solution:
G.M. = 4, H.M. = 165

∵ $(G.M.)^2$ = (A.M.) (H.M.)
∴ 16 = A.M. × 165
∴ A.M. = 5

Question 4.
Find H.M. of two positive numbers whose A.M. and G.M. are 152 and 6.
Solution:
A.M. = 152, G.M. = 6
Now, $(G.M.)^2$ = (A.M.) (H.M.)
∴ 6^2 = 152 × H.M.
∴ H.M. = 36 × 215
∴ H.M. = 245

Question 5.
Find G.M. of two positive numbers whose A.M. and H.M. are 75 and 48.
Solution:
A.M. = 75, H.M. = 48
$(G.M.)^2$ = (A.M.) (H.M.)
∵ $(G.M.)^2$ = 75 × 48
∵ $(G.M.)^2$ = 25 × 3 × 16 × 3
∵ $(G.M.)^2 = 5^2 \times 4^2 \times 3^2$
∴ G.M. = 5 × 4 × 3
∴ G.M. = 60

Question 6.
Insert two numbers between 17 and 113 so that the resulting sequence is a H.P.
Solution:
Let the required numbers be $1H_1$ and $1H_2$.
∴ $17,1H_1,1H_2,113$ are in H.P.
∴ 7, H_1, H_2 and 13 are in A.P.
∴ t_1 = a = 7 and t_4 = a + 3d = 13
∴ 7 + 3d = 13
∴ 3d = 6
∴ d = 2
∴ $H_1 = t_2$ = a + d = 7 + 2 = 9
and $H_2 = t_3$ = a + 2d = 7 + 2(2) = 11
∴ 19 and 111 are the required numbers to be inserted between 17 and 113 so that the resulting sequence is a H.P.

Question 7.
Insert two numbers between 1 and -27 so that the resulting sequence is a G.P.
Solution:
Let the required numbers be G_1 and G_2.
∴ 1, G_1, G_2, -27 are in G.P.
∴ $t_1 = 1$, $t_2 = G_1$, $t_3 = G_2$, $t_4 = -27$
∴ t_1 = a = 1
$t_n = ar^{n-1}$
∴ $t_4 = (1)\ r^{4-1}$
∴ $-27 = r^3$
∴ $r^3 = (-3)^3$
∴ r = -3
∴ $G_1 = t_2$ = ar = 1(-3) = -3
∴ $G_2 = t_3$ = ar = $1(-3)^2$ = 9
∴ -3 and 9 are the required numbers to be inserted between 1 and -27 so that the resulting sequence is a G.P.

Question 8.
Find two numbers whose A.M. exceeds their G.M. by 12 and their H.M. by 2526.
Solution:
Let a, b be the two numbers.

$$A = \frac{a+b}{2},\ G = \sqrt{ab},\ H = \frac{2ab}{a+b}$$

According to the given conditions,

$$A = G + \frac{1}{2},\ A = H + \frac{25}{26}$$

$$\therefore \quad G = A - \frac{1}{2},\ H = A - \frac{25}{26} \quad \ldots(i)$$

Now, $G^2 = AH$

$$\left(A - \frac{1}{2}\right)^2 = A\left(A - \frac{25}{26}\right)$$

$$\therefore \quad A^2 - A + \frac{1}{4} = A^2 - \frac{25}{26}A$$

$$\therefore \quad A - \frac{25}{26}A = \frac{1}{4}$$

$$\therefore \quad \frac{1}{26}A = \frac{1}{4}$$

$$\therefore \quad A = \frac{13}{2} \quad \ldots(ii)$$

$$\therefore \quad G = 6 \quad \ldots[\text{From (i) and (ii)}]$$

$$\therefore \quad \frac{a+b}{2} = \frac{13}{2} \text{ and } \sqrt{ab} = 6$$

∴ a + b = 13
∴ b = 13 – a …….(iii)
and ab = 36
∴ a(13 – a) = 36 …… [From (iii)]
∴ a^2 – 13a + 36 = 0
∴ (a – 4)(a – 9) = 0
∴ a = 4 or a = 9
When a = 4, b = 13 – 4 = 9
When a = 9, b = 13 – 9 = 4
∴ the two numbers are 4 and 9.

Question 9.
Find two numbers whose A.M. exceeds G.M. bv 7 and their H.M. by 635.
Solution:
Let a, b be the two numbers.

$$A = \frac{a+b}{2},\ G = \sqrt{ab},\ H = \frac{2ab}{a+b}$$

According to the given conditions,

$$A = G + 7,\ A = H + \frac{63}{5}$$

$$\therefore \quad G = A - 7, \quad \ldots(i)$$

$$H = A - \frac{63}{5}$$

Now, $G^2 = AH$

$$\therefore \quad (A - 7)^2 = A\left(A - \frac{63}{5}\right)$$

$$\therefore \quad A^2 - 14A + 49 = A^2 - \frac{63A}{5}$$

$$\therefore \quad 14A - \frac{63A}{5} = 49$$

$$\therefore \quad \frac{7A}{5} = 49$$

$$\therefore \quad A = 35$$

$$\therefore \quad \frac{a+b}{2} = 35$$

∴ a + b = 70
∴ b = 70 – a …..(ii)
∴ G = A – 7 = 35 – 7 = 28 …….[From (i)]
∴ √ab = 28

∴ ab = 28^2 = 784
∴ a(70 – a) = 784 ……[From (ii)]
∴ 70a – a^2 = 784
∴ a^2 – 70a + 784 = 0
∴ a^2 – 56a – 14a + 784 = 0
∴ (a – 56) (a – 14) = 0
∴ a = 14 or a = 56
When a = 14, b = 70 – 14 = 56
When a = 56, b = 70 – 56 = 14
∴ the two numbers are 14 and 56.

Maharashtra State Board 11th Commerce Maths Solutions Chapter 4 Sequences and Series Ex 4.5

Question 1.
Find the sum $\sum_{r=1}^{n}(r+1)(2r-1)$.
Solution:

$$\sum_{r=1}^{n}(r+1)(2r-1)$$
$$=\sum_{r=1}^{n}\left(2r^2+r-1\right)$$
$$=2\sum_{r=1}^{n}r^2+\sum_{r=1}^{n}r-\sum_{r=1}^{n}1$$
$$=2.\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}-n$$
$$=\frac{n}{6}[2(2n^2+3n+1)+3(n+1)-6]$$
$$=\frac{n}{6}(4n^2+6n+2+3n+3-6)$$
$$=\frac{n}{6}(4n^2+9n-1)$$

Question 2.
Find $\sum_{r=1}^{n}(3r^2-2r+1)$.
Solution:

$$\sum_{r=1}^{n}\left(3r^2-2r+1\right)$$
$$=3\sum_{r=1}^{n}r^2-2\sum_{r=1}^{n}r+\sum_{r=1}^{n}1$$
$$=3.\frac{n(n+1)(2n+1)}{6}-2\frac{n(n+1)}{2}+n$$
$$=\frac{n}{2}[(2n^2+3n+1)-2(n+1)+2]$$
$$=\frac{n}{2}(2n^2+3n+1-2n-2+2)$$
$$=\frac{n}{2}(2n^2+n+1)$$

Question 3.

Find $\sum_{r=1}^{n} \frac{1+2+3+\ldots+r}{r}$.

Solution:

$$\sum_{r=1}^{n}\left(\frac{1+2+3+\ldots+r}{r}\right)$$

$$= \sum_{r=1}^{n}\frac{r(r+1)}{2r}$$

$$= \frac{1}{2}\sum_{r=1}^{n}(r+1)$$

$$= \frac{1}{2}\left[\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1\right]$$

$$= \frac{1}{2}\left[\frac{n(n+1)}{2} + n\right]$$

$$= \frac{n}{4}\,[(n+1)+2]$$

$$= \frac{n}{4}\,(n+3)$$

Question 4.

Find $\sum_{r=1}^{n} \frac{1^3+2^3+\ldots+r^3}{r(r+1)}$.

Solution:

We know that,

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$\therefore \quad 1^3 + 2^3 + 3^3 + \ldots + r^3 = \frac{r^2(r+1)^2}{4}$$

$$\therefore \quad \frac{1^3 + 2^3 + 3^3 + \ldots + r^3}{r(r+1)} = \frac{r(r+1)}{4}$$

$$\therefore \quad \sum_{r=1}^{n}\left[\frac{1^3+2^3+3^3+\ldots+r^3}{r(r+1)}\right]$$

$$= \sum_{r=1}^{n}\frac{r(r+1)}{4}$$

$$= \frac{1}{4}\sum_{r=1}^{n}(r^2+r) = \frac{1}{4}\left(\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r\right)$$

$$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{1}{4}\cdot\frac{n(n+1)}{2}\left(\frac{2n+1}{3}+1\right)$$

$$= \frac{n(n+1)}{8}\left(\frac{2n+1+3}{3}\right)$$

$$= \frac{n(n+1)(2n+4)}{24}$$

$$= \frac{2n(n+1)(n+2)}{24}$$

$$= \frac{n(n+1)(n+2)}{12}$$

Question 5.

Find the sum 5 × 7 + 7 × 9 + 9 × 11 + 11 × 13 + …… upto n terms.

Solution:

5 × 7 + 7 × 9 + 9 × 11 + 11 × 13 + ….. upto n terms

Now, 5, 7, 9, 11, … are in A.P.

rth term = 5 + (r – 1) (2) = 2r + 3

7, 9, 11,. … are in A.P.

rth term = 7 + (r – 1) (2) = 2r + 5

∴ 5 × 7 + 7 × 9 + 9 × 11 + 11 × 13 + …… upto n terms

$$= \sum_{r=1}^{n}(2r+3)(2r+5)$$

$$= \sum_{r=1}^{n}\left(4r^2+16r+15\right)$$

$$= 4\sum_{r=1}^{n} r^2 + 16\sum_{r=1}^{n} r + 15\sum_{r=1}^{n} 1$$

$$= 4\frac{n(n+1)(2n+1)}{6} + 16\frac{n(n+1)}{2} + 15n$$

$$= \frac{n}{3}[2(2n^2+3n+1)+24(n+1)+45]$$

$$= \frac{n}{3}(4n^2+6n+2+24n+24+45)$$

$$= \frac{n}{3}(4n^2+30n+71)$$

Question 6.
Find the sum $2^2 + 4^2 + 6^2 + 8^2 +$ …… upto n terms.
Solution:
$2^2 + 4^2 + 6^2 + 8^2 +$ …… upto n terms
$= (2 \times 1)^2 + (2 \times 2)^2 + (2 \times 3)^2 + (2 \times 4)^2 +$ ……

Maharashtra State Board 11th Commerce Maths Solutions Chapter 4 Sequences and Series Ex 4.5

Question 1.
Find the sum $\sum_{r=1}^{n}(r+1)(2r-1)$.
Solution:

$$\sum_{r=1}^{n}(r+1)(2r-1)$$

$$= \sum_{r=1}^{n}\left(2r^2+r-1\right)$$

$$= 2\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r - \sum_{r=1}^{n} 1$$

$$= 2.\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} - n$$

$$= \frac{n}{6}[2(2n^2+3n+1)+3(n+1)-6]$$

$$= \frac{n}{6}(4n^2+6n+2+3n+3-6)$$

$$= \frac{n}{6}(4n^2+9n-1)$$

Question 2.

Find $\sum_{r=1}^{n}(3r^2-2r+1)$.

Solution:

$$\sum_{r=1}^{n}\left(3r^2-2r+1\right)$$

$$=3\sum_{r=1}^{n}r^2-2\sum_{r=1}^{n}r+\sum_{r=1}^{n}1$$

$$=3.\frac{n(n+1)(2n+1)}{6}-2\frac{n(n+1)}{2}+n$$

$$=\frac{n}{2}\left[(2n^2+3n+1)-2(n+1)+2\right]$$

$$=\frac{n}{2}(2n^2+3n+1-2n-2+2)$$

$$=\frac{n}{2}(2n^2+n+1)$$

Question 3.

Find $\sum_{r=1}^{n}\frac{1+2+3+\ldots+r}{r}$.

Solution:

$$\sum_{r=1}^{n}\left(\frac{1+2+3+\ldots+r}{r}\right)$$

$$=\sum_{r=1}^{n}\frac{r(r+1)}{2r}$$

$$=\frac{1}{2}\sum_{r=1}^{n}(r+1)$$

$$=\frac{1}{2}\left[\sum_{r=1}^{n}r+\sum_{r=1}^{n}1\right]$$

$$=\frac{1}{2}\left[\frac{n(n+1)}{2}+n\right]$$

$$=\frac{n}{4}\left[(n+1)+2\right]$$

$$=\frac{n}{4}(n+3)$$

Question 4.

Find $\sum_{r=1}^{n}\frac{1^3+2^3+\ldots+r^3}{r(r+1)}$.

Solution:

We know that,

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$\therefore \quad 1^3 + 2^3 + 3^3 + \dots + r^3 = \frac{r^2(r+1)^2}{4}$$

$$\therefore \quad \frac{1^3 + 2^3 + 3^3 + \dots + r^3}{r(r+1)} = \frac{r(r+1)}{4}$$

$$\therefore \quad \sum_{r=1}^{n}\left[\frac{1^3 + 2^3 + 3^3 + \dots + r^3}{r(r+1)}\right]$$

$$= \sum_{r=1}^{n}\frac{r(r+1)}{4}$$

$$= \frac{1}{4}\sum_{r=1}^{n}(r^2 + r) = \frac{1}{4}\left(\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r\right)$$

$$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{1}{4}.\frac{n(n+1)}{2}\left(\frac{2n+1}{3} + 1\right)$$

$$= \frac{n(n+1)}{8}\left(\frac{2n+1+3}{3}\right)$$

$$= \frac{n(n+1)(2n+4)}{24}$$

$$= \frac{2n(n+1)(n+2)}{24}$$

$$= \frac{n(n+1)(n+2)}{12}$$

Question 5.
Find the sum 5 × 7 + 7 × 9 + 9 × 11 + 11 × 13 + …… upto n terms.
Solution:
5 × 7 + 7 × 9 + 9 × 11 + 11 × 13 + ….. upto n terms
Now, 5, 7, 9, 11, … are in A.P.
rth term = 5 + (r – 1) (2) = 2r + 3
7, 9, 11,. … are in A.P.
rth term = 7 + (r – 1) (2) = 2r + 5
∴ 5 × 7 + 7 × 9 + 9 × 11 + 11 × 13 + …… upto n terms

$$= \sum_{r=1}^{n}(2r+3)(2r+5)$$

$$= \sum_{r=1}^{n}(4r^2 + 16r + 15)$$

$$= 4\sum_{r=1}^{n} r^2 + 16\sum_{r=1}^{n} r + 15\sum_{r=1}^{n} 1$$

$$= 4\frac{n(n+1)(2n+1)}{6} + 16\frac{n(n+1)}{2} + 15n$$

$$= \frac{n}{3}\left[2(2n^2 + 3n + 1) + 24(n+1) + 45\right]$$

$$= \frac{n}{3}(4n^2 + 6n + 2 + 24n + 24 + 45)$$

$$= \frac{n}{3}(4n^2 + 30n + 71)$$

Question 6.
Find the sum $2^2 + 4^2 + 6^2 + 8^2$ + …… upto n terms.
Solution:
$2^2 + 4^2 + 6^2 + 8^2$ + …… upto n terms

$= (2 \times 1)^2 + (2 \times 2)^2 + (2 \times 3)^2 + (2 \times 4)^2 + \ldots\ldots$

$$= \sum_{r=1}^{n} (2r)^2$$

$$= 4\sum_{r=1}^{n} r^2$$

$$= \frac{4.n(n+1)(2n+1)}{6}$$

$$= \frac{2n(n+1)(2n+1)}{3}$$

Question 7.
Find $(70^2 - 69^2) + (68^2 - 67^2) + (66^2 - 65^2) + \ldots\ldots. + (2^2 - 1^2)$
Solution:
Let $S = (70^2 - 69^2) + (68^2 - 67^2) + \ldots\ldots + (2^2 - 1^2)$
$\therefore S = (2^2 - 1^2) + (4^2 - 3^2) + \ldots\ldots + (70^2 - 69^2)$
Here, 2, 4, 6,…, 70 is an A.P. with rth term = 2r
and 1, 3, 5,….., 69 in A.P. with rth term = 2r – 1

$$S = \sum_{r=1}^{35}\left[(2r)^2 - (2r-1)^2\right]$$

$$= \sum_{r=1}^{35}\left[4r^2 - \left(4r^2 - 4r + 1\right)\right]$$

$$= \sum_{r=1}^{35}(4r-1)$$

$$= 4\sum_{r=1}^{35} r - \sum_{r=1}^{35} 1$$

$$= 4.\frac{35\times36}{2} - 35$$

$$= (72 - 1)(35)$$

$$= 71 \times 35$$

$$= 2485$$

Question 8.
Find the sum $1 \times 3 \times 5 + 3 \times 5 \times 7 + 5 \times 7 \times 9 + \ldots\ldots + (2n - 1)(2n + 1)(2n + 3)$
Solution:
$1 \times 3 \times 5 + 3 \times 5 \times 7 + 5 \times 7 \times 9 + \ldots\ldots + (2n - 1)(2n + 1)(2n + 3)$
Now, 1, 3, 5, 7, … are in A.P. with a = 1 and d = 2.
$\therefore$ rth term = 1 + (r – 1)2 = 2r – 1
3, 5, 7, 9, … are in A.P. with a = 3 and d = 2
$\therefore$ rth term = 3 + (r – 1)2 = 2r + 1
and 5, 7, 9, 11, … are in A.P. with a = 5 and d = 2
$\therefore$ rth term = 5 + (r – 1)2 = 2r + 3
$\therefore 1 \times 3 \times 5 + 3 \times 5 \times 7 + 5 \times 7 \times 9 + \ldots\ldots$ upto n terms

$$= \sum_{r=1}^{n}(2r-1)(2r+1)(2r+3)$$

$$= \sum_{r=1}^{n}\left(4r^2 - 1\right)(2r+3)$$

$$= \sum_{r=1}^{n}\left(8r^3 + 12r^2 - 2r - 3\right)$$

$$= 8\sum_{r=1}^{n} r^3 + 12\sum_{r=1}^{n} r^2 - 2\sum_{r=1}^{n} r - 3\sum_{r=1}^{n} 1$$

$$= 8\left\{\frac{n(n+1)}{2}\right\}^2 + 12\left\{\frac{n(n+1)(2n+1)}{6}\right\} - 2\left\{\frac{n(n+1)}{2}\right\} - 3n$$

$$= 2n^2(n+1)^2 + 2n(n+1)(2n+1) - n(n+1) - 3n$$

$= n(n + 1)[2n(n + 1) + 4n + 2 - 1] - 3n$
$= n(n + l)(2n^2 + 6n + 1) - 3n$
$= n(2n^3 + 8n^2 + 7n + 1 - 3)$
$= n(2n^3 + 8n^2 + 7n - 2)$

Question 9.
Find n, if 1×2+2×3+3×4+4×5+…+ upto n terms 1+2+3+4+…+ upto n terms = 1003

Solution:

$$\frac{1\times 2+2\times 3+3\times 4+\ldots \text{ upto n terms}}{1+2+3+4+\ldots \text{ upto n terms}}=\frac{100}{3}$$

$$\therefore \quad \frac{\sum_{r=1}^{n} r(r+1)}{\sum_{r=1}^{n} r}=\frac{100}{3}$$

$$\therefore \quad \frac{\sum_{r=1}^{n} r^2+\sum_{r=1}^{n} r}{\sum_{r=1}^{n} r}=\frac{100}{3}$$

$$\therefore \quad \frac{\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}}{\frac{n(n+1)}{2}}=\frac{100}{3}$$

$$\therefore \quad \frac{\frac{n(n+1)}{6}\left[(2n+1)+3\right]}{\frac{n(n+1)}{2}}=\frac{100}{3}$$

$$\therefore \quad \frac{2(n+2)}{3}=\frac{100}{3}$$

$$\therefore \quad n+2=50$$

$$\therefore \quad n=48$$

Question 10.
If S_1, S_2, and S_3 are the sums of first n natural numbers, their squares, and their cubes respectively, then show that:
$9S_2^2 = S_3(1 + 8S_1)$.

Solution:

$$S_1 = 1 + 2 + 3 + \ldots + n = \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

$$S_2 = 1^2 + 2^2 + 3^2 + \ldots + n^2$$

$$= \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

$$S_3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

$$\text{R.H.S.} = S_3(1 + 8S_1)$$

$$= \frac{n^2(n+1)^2}{4}\left[1+8.\frac{n(n+1)}{2}\right]$$

$$= \frac{n^2(n+1)^2}{4}(1 + 4n^2 + 4n)$$

$$= \frac{n^2(n+1)^2}{4}(2n+1)^2$$

$$= \frac{9.n^2(n+1)^2(2n+1)^2}{36}$$

$$= 9\left[\frac{n(n+1)(2n+1)}{6}\right]^2$$

$$= 9S_2^2$$

$$= \text{L.H.S.}$$

Question 7.
Find $(70^2 – 69^2) + (68^2 – 67^2) + (66^2 – 65^2) + \ldots\ldots + (2^2 – 1^2)$

Solution:
Let $S = (70^2 – 69^2) + (68^2 – 67^2) + \ldots\ldots + (2^2 – 1^2)$
$\therefore S = (2^2 – 1^2) + (4^2 – 3^2) + \ldots\ldots + (70^2 – 69^2)$
Here, 2, 4, 6,…, 70 is an A.P. with rth term = 2r

and 1, 3, 5,....., 69 in A.P. with rth term = 2r – 1

$$S = \sum_{r=1}^{35}\left[(2r)^2 - (2r-1)^2\right]$$
$$= \sum_{r=1}^{35}\left[4r^2 - \left(4r^2 - 4r + 1\right)\right]$$
$$= \sum_{r=1}^{35}(4r-1)$$
$$= 4\sum_{r=1}^{35} r - \sum_{r=1}^{35} 1$$
$$= 4.\frac{35\times36}{2} - 35$$
$$= (72-1)(35)$$
$$= 71 \times 35$$
$$= 2485$$

Question 8.
Find the sum 1 × 3 × 5 + 3 × 5 × 7 + 5 × 7 × 9 + ... + (2n – 1) (2n + 1) (2n + 3)
Solution:
1 × 3 × 5 + 3 × 5 × 7 + 5 × 7 × 9 + ... + (2n – 1) (2n + 1) (2n + 3)
Now, 1, 3, 5, 7, ... are in A.P. with a = 1 and d = 2.
∴ rth term = 1 + (r – 1)2 = 2r – 1
3, 5, 7, 9, ... are in A.P. with a = 3 and d = 2
∴ rth term = 3 + (r – 1)2 = 2r + 1
and 5, 7, 9, 11, ... are in A.P. with a = 5 and d = 2
∴ rth term = 5 + (r – 1)2 = 2r + 3
∴ 1 × 3 × 5 + 3 × 5 × 7 + 5 × 7 × 9 + ... upto n terms

$$= \sum_{r=1}^{n}(2r-1)(2r+1)(2r+3)$$
$$= \sum_{r=1}^{n}\left(4r^2-1\right)(2r+3)$$
$$= \sum_{r=1}^{n}\left(8r^3+12r^2-2r-3\right)$$
$$= 8\sum_{r=1}^{n} r^3 + 12\sum_{r=1}^{n} r^2 - 2\sum_{r=1}^{n} r - 3\sum_{r=1}^{n} 1$$
$$= 8\left\{\frac{n(n+1)}{2}\right\}^2 + 12\left\{\frac{n(n+1)(2n+1)}{6}\right\} - 2\left\{\frac{n(n+1)}{2}\right\} - 3n$$
$$= 2n^2(n+1)^2 + 2n(n+1)(2n+1) - n(n+1) - 3n$$

= n(n + 1)[2n(n + 1) + 4n + 2 – 1] – 3n
= n(n + l)($2n^2$ + 6n + 1) – 3n
= n($2n^3$ + $8n^2$ + 7n + 1 – 3)
= n($2n^3$ + $8n^2$ + 7n – 2)

Question 9.
Find n, if $\frac{1\times2+2\times3+3\times4+4\times5+\ldots+\text{ upto n terms}}{1+2+3+4+\ldots+\text{ upto n terms}} = 1003$

Solution:

$$\frac{1\times2+2\times3+3\times4+\ldots \text{ upto n terms}}{1+2+3+4+\ldots \text{ upto n terms}}=\frac{100}{3}$$

$$\therefore \quad \frac{\sum_{r=1}^{n} r(r+1)}{\sum_{r=1}^{n} r}=\frac{100}{3}$$

$$\therefore \quad \frac{\sum_{r=1}^{n} r^2+\sum_{r=1}^{n} r}{\sum_{r=1}^{n} r}=\frac{100}{3}$$

$$\therefore \quad \frac{\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}}{\frac{n(n+1)}{2}}=\frac{100}{3}$$

$$\therefore \quad \frac{\frac{n(n+1)}{6}\left[(2n+1)+3\right]}{\frac{n(n+1)}{2}}=\frac{100}{3}$$

$$\therefore \quad \frac{2(n+2)}{3}=\frac{100}{3}$$

$\therefore \quad n + 2 = 50$

$\therefore \quad n = 48$

Question 10.
If S_1, S_2, and S_3 are the sums of first n natural numbers, their squares, and their cubes respectively, then show that:
$9S_2^2 = S_3(1 + 8S_1)$.
Solution:

$$S_1 = 1 + 2 + 3 + \ldots + n = \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

$$S_2 = 1^2 + 2^2 + 3^2 + \ldots + n^2$$

$$= \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

$$S_3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

R.H.S. $= S_3(1 + 8S_1)$

$$= \frac{n^2(n+1)^2}{4}\left[1+8.\frac{n(n+1)}{2}\right]$$

$$= \frac{n^2(n+1)^2}{4}(1 + 4n^2 + 4n)$$

$$= \frac{n^2(n+1)^2}{4}(2n + 1)^2$$

$$= \frac{9.n^2(n+1)^2(2n+1)^2}{36}$$

$$= 9\left[\frac{n(n+1)(2n+1)}{6}\right]^2$$

$$= 9S_2^{\,2}$$

= L.H.S.

Maharashtra State Board 11th Commerce Maths Solutions Chapter 4 Sequences and Series Miscellaneous Exercise 4

Question 1.
In a G.P., the fourth term is 48 and the eighth term is 768. Find the tenth term.
Solution:

Given, $t_4 = 48$, $t_8 = 768$

$t_n = ar^{n-1}$

$\therefore \quad t_4 = ar^3$

$\therefore \quad ar^3 = 48 \quad \ldots(i)$

and $ar^7 = 768 \quad \ldots(ii)$

Equation (ii) ÷ equation (i), we get

$$\frac{ar^7}{ar^3} = \frac{768}{48}$$

$\therefore \quad r^4 = 16$

$\therefore \quad r = 2$

Substituting r = 2 in (i), we get

$a.(2^3) = 48$

$\therefore \quad a = 6$

$\because \quad t_{10} = ar^9$

$\therefore \quad t_{10} = ar^9 = 6\,(2^9) = 3072$

Question 2.
For a G.P. a = 43 and t_7 = 2431024, find the value of r.
Solution:

Given, $a = \frac{4}{3}$, $t_7 = \frac{243}{1024}$

$t_n = ar^{n-1}$

$\therefore \quad t_7 = ar^6$

$\therefore \quad \frac{243}{1024} = ar^6$

$\therefore \quad \frac{243}{1024} = \frac{4}{3}r^6$

$\therefore \quad r^6 = \frac{3^6}{4^6}$

$\therefore \quad r = \frac{3}{4}$

Question 3.
For a sequence, if t_n = $5_{n-2}7_{n-3}$, verify whether the sequence is a G.P. If it is a G.P., find its first term and the common ratio.
Solution:
The sequence (t_n) is a G.P., if $5_{n-2}7_{n-3}$ = constant, for all n ∈ N.

$$\text{Now, } t_n = \frac{5^{n-2}}{7^{n-3}}$$

$$\therefore \quad t_{n-1} = \frac{5^{(n-1)-2}}{7^{(n-1)-3}} = \frac{5^{n-3}}{7^{n-4}}$$

$$\therefore \quad \frac{t_n}{t_{n-1}} = \frac{5^{n-2}}{7^{n-3}} \times \frac{7^{n-4}}{5^{n-3}}$$

$$= \frac{5^{n-2}}{7^{n-3}} \times \frac{7^{n-3} \cdot 7^{(-1)}}{5^{n-2} \cdot 5^{(-1)}}$$

$$= \frac{7^{(-1)}}{5^{(-1)}}$$

$$= \frac{5}{7} = \text{constant, for all } n \in N$$

$$\therefore \quad r = \frac{5}{7}$$

∴ the sequence is a G.P. with common ratio = $\frac{5}{7}$

∴ first term = t_1 = $\frac{5^{1-2}}{7^{1-3}} = \frac{5^{-1}}{7^{-2}} = \frac{7^2}{5} = \frac{49}{5}$

Question 4.
Find three numbers in G.P., such that their sum is 35 and their product is 1000.
Solution:
Let the three numbers in G.P. be $\frac{a}{r}$, a, ar.
According to the first condition,

$$\frac{a}{r} + a + ar = 35$$

$$\therefore \quad a\left(\frac{1}{r} + 1 + r\right) = 35 \quad \ldots(i)$$

According to the second condition,

$$\left(\frac{a}{r}\right)(a)(ar) = 1000$$

$$\therefore \quad a^3 = 1000$$

$$\therefore \quad a = 10$$

Substituting the value of a in (i), we get

$$10\left(\frac{1}{r} + 1 + r\right) = 35$$

$$\therefore \quad \frac{1}{r} + r + 1 = \frac{35}{10}$$

$$\therefore \quad \frac{1}{r} + r = \frac{35}{10} - 1$$

$$\therefore \quad \frac{1}{r} + r = \frac{25}{10}$$

$$\therefore \quad \frac{1}{r} + r = \frac{5}{2}$$

$$\therefore \quad 2r^2 - 5r + 2 = 0$$

$$\therefore \quad (2r - 1)(r - 2) = 0$$

$$\therefore \quad r = \frac{1}{2} \text{ or } r = 2$$

$$\text{When } r = \frac{1}{2},\ a = 10$$

$$\frac{a}{r} = \frac{10}{\left(\frac{1}{2}\right)} = 20,\ a = 10 \text{ and } ar = 10\left(\frac{1}{2}\right) = 5$$

$$\text{When } r = 2,\ a = 10$$

$$\frac{a}{r} = \frac{10}{2} = 5,\ a = 10 \text{ and } \quad ar = 10(2) = 20$$

∴ the three numbers in G.P. are 20, 10, 5 or 5, 10, 20.

Question 5.
Find 4 numbers in G. P. such that the sum of the middle 2 numbers is $\frac{10}{3}$ and their product is 1.
Solution:

Let the four numbers in G.P. be $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.

According to the second condition,

$\frac{a}{r^3}\left(\frac{a}{r}\right)(ar)(ar^3)=1$

$\therefore a^4 = 1$

$\therefore a = 1$

According to the first condition,

$$\frac{a}{r} + ar = \frac{10}{3}$$

$$\therefore \quad \frac{1}{r} + (1)r = \frac{10}{3}$$

$$\therefore \quad \frac{1+r^2}{r} = \frac{10}{3}$$

$$\therefore \quad 3 + 3r^2 = 10r$$

$$\therefore \quad 3r^2 - 10r + 3 = 0$$

$$\therefore \quad (r-3)(3r-1) = 0$$

$$\therefore \quad r = 3 \text{ or } r = \frac{1}{3}$$

When $r = 3$, $a = 1$

$$\frac{a}{r^3} = \frac{1}{(3)^3} = \frac{1}{27}, \ \frac{a}{r} = \frac{1}{3}, \ ar = 1(3) = 3 \text{ and}$$

$$ar^3 = 1(3)^3 = 27$$

When $r = \frac{1}{3}$, $a = 1$

$$\frac{a}{r^3} = \frac{1}{\left(\frac{1}{3}\right)^3} = 27, \ \frac{a}{r} = \frac{1}{\left(\frac{1}{3}\right)} = 3,$$

$$ar = 1\left(\frac{1}{3}\right) = \frac{1}{3} \text{ and } ar^3 = 1\left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$\therefore$ the four numbers in G.P.are

$$\frac{1}{27}, \frac{1}{3}, 3, 27 \text{ or } 27, 3, \frac{1}{3}, \frac{1}{27}.$$

Question 6.

Find five numbers in G.P. such that their product is 243 and the sum of the second and fourth numbers is 10.

Solution:

Let the five numbers in G.P. be

$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

According to the first condition,

$\frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 = 243$

$\therefore \quad a^5 = 243$

$\therefore \quad a = 3$

According to the second condition,

$\frac{a}{r} + ar = 10$

$\therefore \quad \frac{1}{r} + r = \frac{10}{a}$

$\therefore \quad \frac{1+r^2}{r} = \frac{10}{3}$

$\therefore \quad 3r^2 - 10r + 3 = 0$

$\therefore \quad 3r^2 - 9r - r + 3 = 0$

$\therefore \quad (3r - 1)(r - 3) = 0$

$\therefore \quad r = \frac{1}{3}, 3$

When $a = 3$, $r = \frac{1}{3}$

$\frac{a}{r^2} = 27, \frac{a}{r} = 9, a = 3, ar = 1, ar^2 = \frac{1}{3}$

When $a = 3$, $r = 3$

$\frac{a}{r^2} = \frac{1}{3}, \frac{a}{r} = 1, a = 3, ar = 9, ar^2 = 27$

$\therefore$ the five numbers in G.P. are

$27, 9, 3, 1, \frac{1}{3}$ or $\frac{1}{3}, 1, 3, 9, 27$

Question 7.
For a sequence, $S_n = 4(7^n - 1)$, verify whether the sequence is a G.P.
Solution:

$S_n = 4(7^n - 1)$

$\therefore \quad S_{n-1} = 4(7^{n-1} - 1)$

But, $t_n = S_n - S_{n-1}$

$= 4(7^n - 1) - 4(7^{n-1} - 1)$

$= 4(7^n - 1 - 7^{n-1} + 1)$

$= 4(7^n - 7^{n-1})$

$= 4(7^{n-1+1} - 7^{n-1})$

$= 4.7^{n-1}(7 - 1)$

$\therefore \quad t_n = 24.7^{n-1}$

$\therefore \quad t_{n-1} = 24.7^{(n-1)-1} = 24.7^{n-2}$

The sequence is a G.P., if $\frac{t_n}{t_{n-1}}$ = constant

for all $n \in N$

$\therefore \quad \frac{t_n}{t_{n-1}} = \frac{24.7^{n-1}}{24.7^{n-2}} = \frac{7^{n-1}}{7^{n-1}.7^{(-1)}}$

$= 7$ = constant, for all $n \in N$

$\therefore$ the sequence is a G.P.

Question 8.
Find 2 + 22 + 222 + 2222 + …… upto n terms.
Solution:
S_n = 2 + 22 + 222 +….. upto n terms
= 2(1 + 11 + 111 +…… upto n terms)
= $\frac{2}{9}$ (9 + 99 + 999 + … upto n terms)
= $\frac{2}{9}$ [(10 – 1) + (100 – 1) + (1000 – 1) +…… upto n terms]
= $\frac{2}{9}$ [(10 + 100 + 1000 + … upto n terms) – (1 + 1 + 1 + ….. n times)]
Since, 10, 100, 1000, …… n terms are in G.P.
with a = 10, r = $\frac{100}{10}$ = 10

$\therefore \quad S_n = \frac{2}{9}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right] = \frac{2}{9}\left[\frac{10}{9}\left(10^n - 1\right) - n\right]$

$\therefore \quad S_n = \frac{2}{81}[10(10^n - 1) - 9n]$

Question 9.
Find the nth term of the sequence 0.6, 0.66, 0.666, 0.6666,.....
Solution:
0.6, 0.66, 0.666, 0.6666, ...
$\therefore t_1 = 0.6$
$t_2 = 0.66 = 0.6 + 0.06$
$t_3 = 0.666 = 0.6 + 0.06 + 0.006$
Hence, in general
$t_n = 0.6 + 0.06 + 0.006 + \ldots$ upto n terms.
The terms are in G.P.with
a = 0.6, r = $\frac{0.06}{0.6}$ = 0.1
$\therefore t_n$ = the sum of first n terms of the G.P.

$$\therefore \quad t_n = 0.6\left[\frac{1-(0.1)^n}{1-0.1}\right] = \frac{0.6}{0.9}[1-(0.1)^n]$$

$$\therefore \quad t_n = \frac{6}{9}[1-(0.1)^n] = \frac{2}{3}[1-(0.1)^n]$$

Question 10.
Find $\sum_{r=1}^{n}(5r^2+4r-3)$.
Solution:

$$\sum_{r=1}^{n}\left(5r^2+4r-3\right)$$
$$= 5\sum_{r=1}^{n}r^2 + 4\sum_{r=1}^{n}r - 3\sum_{r=1}^{n}1$$
$$= 5.\ \frac{n(n+1)(2n+1)}{6} + 4.\frac{n(n+1)}{2} - 3n$$
$$= \frac{n}{6}\ [5(2n^2+3n+1)+12(n+1)-18]$$
$$= \frac{n}{6}(10n^2+15n+5+12n+12-18)$$
$$= \frac{n}{6}(10n^2+27n-1)$$

Question 11.
Find $\sum_{r=1}^{n} r(r-3)(r-2)$.
Solution:

$$\sum_{r=1}^{n} r(r-3)(r-2)$$
$$= \sum_{r=1}^{n}\left(r^3-5r^2+6r\right)$$
$$= \sum_{r=1}^{n}r^3 - 5\sum_{r=1}^{n}r^2 + 6\sum_{r=1}^{n}r$$
$$= \frac{n^2(n+1)^2}{4} - 5\frac{n(n+1)(2n+1)}{6} + 6.\frac{n(n+1)}{2}$$
$$= \frac{n(n+1)}{12}\left[3n(n+1)-10(2n+1)+36\right]$$
$$= \frac{n(n+1)}{12}\ (3n^2+3n-20n-10+36)$$
$$= \frac{n(n+1)}{12}\left(3n^2-17n+26\right)$$

Question 12.
Find $\sum_{r=1}^{n}\frac{1^2+2^2+3^2+\ldots+r^2}{2r+1}$
Solution:

We know that,

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\therefore \quad 1^2 + 2^2 + 3^2 + \ldots + r^2 = \frac{r(r+1)(2r+1)}{6}$$

$$\therefore \quad \frac{1^2 + 2^2 + 3^2 + \ldots + r^2}{2r+1} = \frac{r(r+1)}{6}$$

$$\therefore \quad \sum_{r=1}^{n}\left(\frac{1^2 + 2^2 + 3^2 + \ldots + r^2}{2r+1}\right)$$

$$= \sum_{r=1}^{n}\frac{r(r+1)}{6} = \frac{1}{6}\sum_{r=1}^{n}\left(r^2 + r\right)$$

$$= \frac{1}{6}\left(\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r\right)$$

$$= \frac{1}{6}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{1}{6} \times \frac{n(n+1)}{2}\left(\frac{2n+1}{3} + 1\right)$$

$$= \frac{n(n+1)}{12}\left(\frac{2n+1+3}{3}\right)$$

$$= \frac{n(n+1)(2n+4)}{36}$$

$$= \frac{2n(n+1)(n+2)}{36}$$

$$= \frac{n(n+1)(n+2)}{18}$$

Question 13.
Find $\sum_{r=1}^{n} \frac{1^3+2^3+3^3+\ldots+r^3}{(r+1)^2}$
Solution:

$$\sum_{r=1}^{n} \frac{1^3 + 2^3 + 3^3 + \ldots + r^3}{(r+1)^2}$$

$$= \sum_{r=1}^{n} \frac{r^2(r+1)^2}{4} \times \frac{1}{(r+1)^2}$$

$$= \frac{1}{4}\sum_{r=1}^{n} r^2$$

$$= \frac{1}{4} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)(2n+1)}{24}$$

Question 14.
Find 2 × 6 + 4 × 9 + 6 × 12 + …… upto n terms.
Solution:
2, 4, 6, … are in A.P.
∴ rth term = 2 + (r – 1)2 = 2r
6, 9, 12, … are in A.P.
∴ rth term = 6 + (r – 1) (3) = (3r + 3)
∴ 2 × 6 + 4 × 9 + 6 × 12 +…… upto n terms

$$= \sum_{r=1}^{n} 2r \times (3r+3)$$

$$= 6\sum_{r=1}^{n} r^2 + 6\sum_{r=1}^{n} r$$

$$= 6.\frac{n(n+1)(2n+1)}{6} + 6\frac{n(n+1)}{2}$$

= n(n + 1) (2n + 1 + 3)
= 2n(n + 1)(n + 2)

Question 15.
Find $12^2 + 13^2 + 14^2 + 15^2 + \ldots\ldots + 20^2$.
Solution:
$12^2 + 13^2 + 14^2 + 15^2 + \ldots\ldots + 20^2$
$= (1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + 20^2) - (1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + 11^2)$

$$= \sum_{r=1}^{20} r^2 - \sum_{r=1}^{11} r^2$$

$$= \frac{20(20+1)(2\times 20+1)}{6} - \frac{11(11+1)(2\times 11+1)}{6}$$

$$= \frac{20\times 21\times 41}{6} - \frac{11\times 12\times 23}{6}$$

= 2870 – 506
= 2364

Question 16.
Find $(50^2 - 49^2) + (48^2 - 47^2) + (46^2 - 45^2) + \ldots\ldots + (2^2 - 1^2)$.
Solution:
$(50^2 - 49^2) + (48^2 - 47^2) + (46^2 - 45^2) + \ldots\ldots + (2^2 - 1^2)$
$= (50^2 + 48^2 + 46^2 + \ldots\ldots + 2^2) - (49^2 + 47^2 + 45^2 + \ldots\ldots + 1^2)$
$= \sum_{r=1}^{25}(2r)^2 - \sum_{r=1}^{25}(2r-1)^2$

$$= \sum_{r=1}^{25} 4r^2 - \sum_{r=1}^{25}(4r^2 - 4r + 1)$$

$$= \sum_{r=1}^{25}\left[4r^2 - (4r^2 - 4r + 1)\right]$$

$$= \sum_{r=1}^{25}(4r - 1)$$

$$= 4\sum_{r=1}^{25} r - \sum_{r=1}^{25} 1$$

$$= 4 \times \frac{25(25+1)}{2} - 25$$

$$= \frac{4(25)(26)}{2} - 25$$

= 1300 – 25
= 1275

Question 17.
In a G.P., if $t_2 = 7$, $t_4 = 1575$, find r.

Solution:

Given $t_2 = 7$, $t_4 = 1575$

$t_n = ar^{n-1}$

$\therefore \quad t_2 = ar$

$\therefore \quad 7 = ar$

$\therefore \quad a = \frac{7}{r} \quad \ldots(i)$

$t_4 = ar^3$

$\therefore \quad ar^3 = 1575$

$\therefore \quad r^3 \times \left(\frac{7}{r}\right) = 1575 \quad \ldots[\text{From (i)}]$

$\therefore \quad r^2 \times 7 = 1575$

$\therefore \quad r^2 = \frac{1575}{7}$

$\therefore \quad r^2 = 225$

$\therefore \quad r = \pm 15$

Question 18.
Find k so that k – 1, k, k + 2 are consecutive terms of a G.P.
Solution:
Since k – 1, k, k + 2 are consecutive terms of a G.P.

$\therefore \frac{k}{k-1} = \frac{k+2}{k}$

$\therefore k^2 = k^2 + k - 2$

$\therefore k - 2 = 0$

$\therefore k = 2$

Question 19.
If pth, qth and rth terms of a G.P. are x, y, z respectively, find the value of $x^{q-r} \cdot y^{r-p} \cdot z^{p-q}$.
Solution:
Let a be the first term and R be the common ratio of the G.P.

$\therefore t_n = a.R^{n-1}$

$\therefore x = a.R^{p-1}$, $y = a.R^{q-1}$, $z = a.R^{r-1}$

$\therefore \quad x^{q-r}.y^{r-p}.z^{p-q}$

$= (a.R^{p-1})^{q-r}.(a.R^{q-1})^{r-p}.(a.R^{r-1})^{p-q}$

$= a^{q-r}R^{(p-1)(q-r)}.a^{r-p}R^{(q-1)(r-p)}.a^{p-q}R^{(r-1)(p-q)}$

$= a^{(q-r+r-p+p-q)}.R^{[(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)]}$

$= a^0.R^{(pq-pr-q+r+qr-pq-r+p+pr-qr-p+q)}$

$= (1).R^0 = 1$