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Peer-to-Peer Energy Sharing among Smart Energy Buildings by Distributed Transaction

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***Abstract*—Efficient building energy management is very essential for energy saving and green society. This paper investigates a sustainable energy management for an energy building cluster with distributed transaction. The building cluster consists of several types of energy buildings, e.g., office, industrial, and commercial buildings. We firstly formulate utility functions for the buildings of consuming energy based on the characteristics of their controllable loads. Then a two-stage energy sharing strategy is presented. In the first stage the total social energy cost is minimized through finding the optimal energy sharing profiles in a distributed way. In the second stage the clearing for mutual energy sharing is modeled as a non-cooperative game, and the existence of the equilibrium of the game is illustrated and a relaxation-based algorithm is introduced to search for the equilibrium. Moreover, a real-time model for each building to overcome real-time uncertainties such as renewable energy generation and base loads is provided. The simulation results show that the proposed energy sharing strategy is economically beneficial for the energy buildings, computationally efficient, and is promising to facilitate a sustainable regional building cluster.**

***Index Terms*—Energy building, peer-to-peer, energy sharing, distributed optimization, non-cooperative game.**

I. Introduction

About 30% of total energy consumption and 60% of electricity consumption are spent on buildings in recent years, thus the energy efficiency in building sector has been a major concern today [1]. In some countries, the energy building has been presented and policies are made to promote the development of nearly zero energy buildings (ZEB) achieved by enhancing the energy efficiency performance of building equipments, such as the heating, ventilation and air conditioning (HVAC) units, shiftable electrical appliances (SEA), and flexible commercial services (FCS), using the renewable energy and energy storage on site [2]. Such a nearly ZEB requires a very high energy performance. The very low amount of energy demand should be covered almost by renewable energy resources on site or nearby and energy from storage system of high capital cost due to uncertainties [3]. Therefore, achieving a nearly ZEB without grid will be very difficult nowadays. This paper, thus, proposes another promising way by extending a single building to a cluster of buildings, that is achieving a sustainable regional building cluster, e.g., minimizing the total social energy cost as a whole, by promoting mutual energy sharing among the buildings with new and secure technologies.

Efforts have been made to achieve the energy sharing among individual energy agents, such as prosumers, microgrids and buildings. The works can be roughly classified into three groups: *leader-follower structure*, *aggregation structure*, and *peer-to-peer structure*. For *leader-follower structure*, a leader, e.g., retailer or operator, is always required to make proper prices for internal energy trading, and agents decide their energy sharing profiles in response to the prices [4–7]. Specifically, [4, 5] formulate the structure as bi-level problems solved in a centralized way where the leader should gather the global information of the agents. [6, 7] present the Stackelberg game for energy sharing within a prosumer microgrid including a microgrid operator and PV prosumers. In general, the leader coordinates the energy sharing just to minimize its own profit, thus the structure may not be reliable for reducing social cost.

As for *aggregation structure*, a proper price mechanism [8, 9] must be firstly designed and accepted by the agents. [8] introduces a price mechanism based on the relationship of energy supply and demand, and a virtual energy sharing provider is assumed to coordinate the sharing activities for a prosumer community. [9] presents a Cournot-based price function and formulates an aggregation game for multi-energy microgrids. Limited by the price mechanism with inside non-cooperative characteristics, the works in this group can not minimize the total social energy cost, either.

Moreover, the aforementioned results always require a center not only to coordinate energy sharing activities but also to control the cash flow of energy trading. In other words, each agent only knows his energy demand and supply and the cash flow with the center, but has no knowledge about who uses his surplus energy or who provides energy for him, and even the center may not be entirely trustworthy. That is to say, the transparent energy trading profiles are required. Fortunately, another class of works focusing on the *peer-to-peer* energy trading for agents is promising. [10] evaluates the feasibility of peer-to-peer energy trading among household prosumer in low-voltage electricity networks. [11] presents a middlemen to facilitate customer-to-customer energy trading in a localized event-driven market, but the middlemen acts a center to facilitate the energy trading. [12] proposes a new scalable market design of bilateral contract networks for peer-to-peer energy trading among suppliers and consumers.

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However, [12] is not suitable for energy agents such as prosumers which can behave as energy suppliers or consumers in favorable conditions in practice. [13] investigates peer-to-peer energy sharing of nanogrid cluster with a central controller. Meanwhile, the transparent peer-to-peer energy trading and the total social energy costs haven't ever been covered in [10–13]. [14–16] propose models with transparent mutual energy and monetary exchange for a prosumer community to minimize total community energy costs, but the models are still solved by a centralized hub or management system with computation and privacy concerns. Nevertheless, efficient distributed methods are essential for reliable application. Therefore, as a summary, a new energy sharing framework is required to cover the advantages of transparent energy sharing, minimum social energy cost, and secure distributed implementation for the energy sharing problem of a cluster of energy buildings.

Motivated by these discussion, in this paper, we present a two-stage peer-to-peer energy sharing strategy for a building cluster. In the first stage, we derive optimal energy sharing profiles of the buildings to minimize the total social energy cost. The social energy cost includes the trading cost with the retail energy provider (REP), the operation cost of energy storage system (ESS), and the negative utilities of controllable loads of consuming energy. The profiles include the energy supplies/demands of each building to/from other buildings. In the second stage, we determined the clearing prices of mutual energy sharing via modeling a non-cooperative game. Since each building tends to charge more for the energy supplies to other buildings and pay less for the energy demands from other buildings, the clearing prices should be equilibrium prices such that each building will pay/charge for the energy demands/supplies. In both stage, we introduce distributed algorithms to solve their problems. In addition, we introduce the distributed transaction technology on which transactions can succeed without the need of having a trusted third party to enable secure decentralized market platforms [17] as the application tool. Thus, this work may be the first to present an energy sharing framework for an energy building cluster with the minimum total social energy cost, transparent energy sharing profiles, and totally distributed implementation as far as the authors know. More specifically, the contributions of the paper are summarized as follows:

1) Focusing on a cluster of energy buildings with peer-to-peer energy sharing, a sustainable regional building cluster is achieved in the sense that the cluster has a low and smooth net demand profile and thus has a low dependence on the main grid.
2) A new peer-to-peer energy sharing strategy is proposed, where the total social cost of the building cluster is minimized and the clearing for peer-to-peer energy sharing is reasonable and economical. The strategy is achieved in a totally distributed fashion with transparent energy and cash flow with privacy information security guaranteed.
3) A real-time optimization model is presented for each building to overcome the real-time mismatch between the prediction and actual values such as renewable energy outputs and base loads.
4) The simulation results show the energy efficiency and economical benefits of the proposed framework as well as the satisfying algorithm performance.

The rest of the paper is organized as follows. Section II models the energy system of the energy buildings. Section III proposes the peer-to-peer energy sharing strategy with related algorithms. Section IV provides a real-time optimization model for each building to handle uncertainties. Section V presents the case results. Section VI concludes the paper and points out the future research.

II. System Modeling for Each Building

We consider a cluster of smart energy buildings close to each other and mutually interconnected with DC lines. The building cluster is denoted by $\mathcal{N} = \{1, 2, ..., n, ..., N\}$, and a commercial retail electricity providers (REP) is on behalf of the main grid for energy retailing. Each building $n \in \mathcal{N}$ contains base loads, controllable loads $\mathcal{A}_n = \{1, 2, ..., A_n\}$, local renewable energy generation system, and possibly energy storage system (ESS). Then the energy buildings are classified into three sets $\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3$ according to the types of their controllable loads. $\mathcal{N}_1$ includes N_1 office buildings with controllable HVAC units maintaining a comfortable indoor temperature. $\mathcal{N}_2$ includes N_2 industrial buildings with shiftable electrical appliances (SEA). $\mathcal{N}_3$ includes N_3 commercial buildings with flexible commercial services (FCS). Thus $\mathcal{N} = \mathcal{N}_1 + \mathcal{N}_2 + \mathcal{N}_3$ and $N = N_1 + N_2 + N_3$. To facilitate the interconnected energy sharing with distributed transaction such as physical implementation and information communication, the buildings are also deployed with local energy management systems with smart meters, and advanced metering infrastructure [18, 19]. We consider an operation horizon of one working day $\mathcal{H} = \{1, ..., H\}$ divided into $H = 10$ equal time slots, e.g., hours, since the buildings mainly carry out the activities in daytime generally. Assume the buildings can make their energy schedules based on the prediction values such as the daily renewable energy generation profile and base loads. Thus we research on the day-ahead energy sharing among the buildings with distributed transaction. Note that the practice of day-ahead schedule is conductive to the main grid, since the main grid can determine energy supplies for the next day and only needs to cope with a relatively small fluctuation of demands in the real time. The energy mismatch between the prediction and the actual values in real time such as the renewable energy generation will also be further discussed in the next section.

A. Controllable loads

The controllable loads are essential parts for smart economical energy management for the buildings for which we model utility functions to quantify the benefits by consuming energy.

1) Adjustable HVAC Units: For office buildings, the HVAC units are the main controllable loads for energy management. Based on the physical law of HVAC units studied in [20–22], the indoor temperature of the thermal dynamics for the HVAC unit $\forall k \in \mathcal{A}_n, n \in \mathcal{N}_1$, is shown below. From the formula it can be seen that the indoor temperature varies with the

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last indoor temperature $T_{\text{in},k}^{h-1}$, the current outdoor temperature $T_{\text{out},k}^{h}$, and the power consumption q_k^h for heating or cooling:

$$T_{\text{in},k}^{h} = T_{\text{in},k}^{h-1} - \frac{\Delta h}{C_k R_k}(T_{\text{in},k}^{h-1} - T_{\text{out},k}^{h} + \eta R_k q_k^h \Delta h), \quad (1)$$

where Δh is the time period, C_k and R_k are physical parameters of the HVAC unit k, η denotes the behavior of the HVAC unit k (η is positive for cooling and negative for heating), and q_k^h is the power of the energy consumption of the HVAC unit k at time slot h. Generally, we can set a reference temperature T_{ref} which will be the most comfortable temperature for most people [23]. Accordingly, we define the following comfort model as the utility function of the HVAC unit k:

$$U_k(\boldsymbol{q}_k) = M_k - \sum_{h\in\mathcal{H}} \alpha_k (T_{\text{in},k}^{h} - T_{\text{ref}})^2, \quad (2)$$

where $\boldsymbol{q}_k = \{q_k^h, \forall h \in \mathcal{H}\}$ is the power profiles of the energy consumption of the HVAC unit k, M_k is a given positive constant to make the utility positive in actual applications. Since it is a constant, it has no influence on the optimization framework of this paper. α_k is the coefficient for the quantification the comfort model indicating the energy consumption preference. Besides, the indoor temperature must be limited in a suitable range to ensure the indoor temperature acceptable as follows:

$$T_{\text{in},k}^{\min} \le T_{in,k}^{h} \le T_{\text{in},k}^{\max}, \forall h \in \mathcal{H}, \quad (3)$$

where $T_{\text{in},k}^{\min}$ and $T_{\text{in},k}^{\max}$ are the minimum and maximum acceptable indoor temperature for the HVAC unit k.

2) Shiftable Electrical Appliances: Usually, there are preferred energy consumption profiles for the SEA $\forall k \in \mathcal{A}_n, n \in \mathcal{N}_2$, in production scheduling. Thus the deviation between the actual consumption profiles and the preset consumption profiles will bring inconvenience for the owner of the buildings. Similar with the inconvenience measure in [8, 24], we define the quantified inconvenience function as the utility function of the SEA k:

$$U_k(\boldsymbol{q}_k) = M_k - \sum_{h\in\mathcal{H}} \beta_k (q_k^h \Delta h - q_{\text{ref},k}^h \Delta h)^2, \quad (4)$$

where $\boldsymbol{q}_k = \{q_k^h, \forall h \in \mathcal{H}\}$ is the power profiles of energy consumption of SEA k, β_k is to indicate the sensitivity towards the load shifting, and $q_{\text{ref},k}^h$ is the preferred consumption level of SEA k at time slot h. M_k is a given positive constant. The energy consumption power must satisfy the following constraints:

$$\sum_{h\in H} q_k^h \Delta h = D_k, \quad (5a)$$

$$q_k^{h,\min} \le q_k^h \le q_k^{h,\max}, \forall h \in \mathcal{H}, \quad (5b)$$

where D_k is the prescribed total energy requirement which must be satisfied in the whole day, and $q_k^{h,\min}$ and $q_k^{h,\max}$ are the lower and upper bound of the energy consumption power at time slot h for the SEA k.

3) Flexible Commercial Services: The FCS can achieve utilities by consuming electrical power. Similar with [25], we introduce the following utility function for the flexible commercial service $\forall k \in \mathcal{A}_n, n \in \mathcal{N}_3$, as follows:

$$U_k(\boldsymbol{q}_k) = \sum_{h\in\mathcal{H}} \gamma_k \ln(q_k^h \Delta h + 1), \quad (6)$$

where $\boldsymbol{q}_k = \{q_k^h, \forall h \in \mathcal{H}\}$ is the energy consumption power vector of FCS k, and γ_k is the preference parameter of FCS k [25]. The natural logarithm $\ln(\cdot)$ has been widely adopted to design utility models [26], and has also been shown suitable for designing the utilities achieved by commercial services [6, 27]. The energy consumption of FCS k must meet the following constraints:

$$q_k^{\min} \le q_k^h \le q_k^{\max}, \forall h \in \mathcal{H}, \quad (7)$$

where $q_k^{\min}$ and $q_k^{\max}$ are the minimum and maximum energy consumption limits to ensure the safe operation for FCS k.

Note that the constraints for the three types of demands are all convex, we can define a general denotation for the constraint set for the above controllable loads:

$$\Phi_k(\boldsymbol{q}_k) := \begin{cases} \{(1),(3)\}, k \in \mathcal{A}_n, n \in \mathcal{N}_1, \\ \{(5a),(5b)\}, k \in \mathcal{A}_n, n \in \mathcal{N}_2, \\ \{(7)\}, k \in \mathcal{A}_n, n \in \mathcal{N}_3. \end{cases}$$

B. *Energy Storage System*

We suppose the building $n \in \mathcal{N}$ may be equipped with the ESS with a capacity $S_n^{\max}$. We model the operation cost of the degradation for the ESS caused by charging and discharging [28] as follows:

$$C_n^{\text{ESS}}(\boldsymbol{c}_n, \boldsymbol{d}_n) = \sum_{h\in H} \left(\varepsilon_n^c c_n^h \Delta h + \varepsilon_n^d d_n^h \Delta h\right), \quad (8)$$

where $\boldsymbol{c}_n = \{c_n^h, \forall h \in \mathcal{H}\}$ and $\boldsymbol{d}_n = \{d_n^h, \forall h \in \mathcal{H}\}$ are the charging and discharging power vectors of the ESS in the building n, and ε_n^c and ε_n^d are the coefficients for the amortized cost of charging and discharging over the lifetime [29]. The charging and discharging power for ESS must meet the following constraints to ensure a normal operation:

$$0 \le c_n^h \le c_n^{\max}, 0 \le d_n^h \le d_n^{\max}, \ \forall h \in \mathcal{H}, \quad (9a)$$

$$S_n^h = S_n^{h-1} + \eta_n^c c_n^h \Delta h - \eta_n^d d_n^h \Delta h. \quad (9b)$$

$$(1 - SoC_n) S_n^{\max} \le S_n^h \le SoC_n S_n^{\max} \ \forall h \in \mathcal{H}. \quad (9c)$$

$$S_n^H \ge S_n^0. \quad (9d)$$

where $c_n^{\max}$ and $d_n^{\max}$ are the charging and discharging power limits for the ESS n. η_n^c and η_n^d denote the charging and discharging efficiency for the ESS. Note that the operation cost function C_n^{ESS} and the charging/discharging efficiency η_n^c, η_n^d will lead to extra energy loss and energy cost in the case of simultaneous charge/discharge, the case of simultaneous charge/discharge will never happen in an optimal energy schedule for ESS. The energy level S_n^h at each time slot h should be limited in a rated SoC range (9c) to sustain the life time [30]. Moreover, the energy level at the terminal time slot H cannot be lower than the initial level S_n^0 for emergent requirement and decoupling across different time horizons (9d). Let $\Upsilon_n := \{(9a),(9b),(9c),(9d)\}$ denote the constraints of the ESS owned by the building n.

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C. Energy Trading with REP

The buildings can buy energy from and feed-in energy to REP with the market prices set by REP. Let p_b^h and p_s^h be the power buying price and selling price at time slot h for the buildings. Generally speaking, the buying price p_b^h is supposed to be greater than the selling p_s^h such that the setting is to avoid the arbitrage from purchasing and selling energy and to promote the on-site consumption of local renewable energy. Let $\boldsymbol{b}_n = \{b_n^h, \forall h \in \mathcal{H}\}$ and $\boldsymbol{s}_n = \{s_n^h, \forall h \in \mathcal{H}\}$ denote the energy demand and supply profiles from and to REP over the time horizon. Thus the energy trading cost of the building with REP over the time horizon is formulated as follows:

$$C_n^{\mathrm{REP}}(\boldsymbol{b}_n, \boldsymbol{s}_n) = \sum_{h \in H} \left((p_b^h + \epsilon^h) b_n^h \Delta h - p_s^h s_n^h \Delta h\right), \quad (10)$$

where ϵ^h is the grid marginal emissions for additional taxes of carbon emissions [31].

D. Energy Sharing with Other Buildings

The energy sharing profiles among the buildings are totally peer-to-peer cleared through the totally distributed transaction, which means each building can purchase/sell energy from/to any other buildings with their privately compromised prices. Through sharing energy with each other, interconnected buildings can make full use of local renewable energy, improve the energy efficiency, reduce the reliance on the main grid, and decrease the carbon emissions for the environmental issues so as to achieve a sustainable regional building cluster. Let $\boldsymbol{e}_n = \{e_{n,m}^h, \forall h \in \mathcal{H}, \forall m \in \mathcal{N} \backslash n\}$ denote the energy sharing profile of building n with other buildings $m \in \mathcal{N} \backslash n$ over the time horizon. Specifically, $e_{n,m}^t$ denotes the energy the building n shares with the building m at time slot h. If the building n purchases energy from the building m, $e_{n,m}^t$ is positive; If the building n sells energy to the building n, $e_{n,m}^t$ is negative. Obviously, the energy sharing among the buildings must satisfy the following coupled constraints:

$$e_{n,m}^h + e_{m,n}^h = 0, \forall h \in \mathcal{H}, \forall n \in \mathcal{N}, \forall m \in \mathcal{N} \backslash n. \quad (11)$$

The buildings are located close to each other, thus the communication network and the power network are assumed to work well without congestion, and the loss of energy sharing is negligible. Let $\mathcal{Y} = \{(11)\}$ be the constraint set for $\{\boldsymbol{e}_n, \forall n \in \mathcal{N}\}$. The problem is how to clear the peer-to-peer energy sharing profiles. Since the energy sharing is distributed and encrypted, the distributed transaction mechanism is adopted for payment clearing of mutual energy sharing, where the energy sharing among each pair of buildings can be totally decided and negotiated by themselves. Let $\boldsymbol{\lambda}_n := \{\lambda_{n,m}, m \in \mathcal{N} \backslash n\}$ denote the prices of the building $n \in \mathcal{N}$ sets for the energy sharing with the buildings $m \in \mathcal{N} \backslash n$. Considering the balance of payments in the clearing of energy sharing between each pair of buildings, the following constraints must be satisfied:

$$\lambda_{n,m} = \lambda_{m,n}, \forall n \in \mathcal{N}, \forall m \in \mathcal{N} \backslash n. \quad (12)$$

Let $\mathcal{W} := \{(12)\}$ denote the set of above coupled constraints. In this case, the clearing of energy sharing of the building n can be formulated as follows:

$$C_n^{\mathrm{ES}} = \sum_{m \in \mathcal{N} \backslash n} \sum_{h \in \mathcal{H}} \lambda_{n,m} e_{n,m}^h, \quad (13)$$

III. Problem Formulation and Algorithms

In this section, we first formulate the problem without considering the interconnected energy sharing as the traditional practice and energy scheduling problem. Then we formulate the coupled energy sharing problem with distributed transaction. The algorithms will also be introduced.

A. The Problem Without Energy Sharing

Since there exits no coupled relationship among the buildings without considering energy sharing, each building can schedule their energy demand and supply individually. Thus the problem of energy scheduling for building $n \in \mathcal{N}$ is formulated below:

$$\begin{aligned} \min\ & C_n^{\mathrm{P0}}(\boldsymbol{q}_k, \boldsymbol{c}_n, \boldsymbol{d}_n, \boldsymbol{b}_n, \boldsymbol{s}_n) = \\ & C_n^{\mathrm{ESS}}(\boldsymbol{c}_n, \boldsymbol{d}_n) + C_n^{\mathrm{REP}}(\boldsymbol{b}_n, \boldsymbol{s}_n) - \sum_{k \in \mathcal{A}_n} U_k(\boldsymbol{q}_k), \end{aligned} \quad (14a)$$

subject to:

$$\sum_{k \in \mathcal{A}_n} q_k^h + c_n^h - d_n^h - b_n^h + s_n^h = g_n^h - l_n^h, \ \forall h \in \mathcal{H}, \quad (14b)$$

$$\boldsymbol{q}_k \in \Phi_k, \forall k \in \mathcal{A}_n; \ \boldsymbol{c}_n, \boldsymbol{d}_n \in \Upsilon_n, \quad (14c)$$

where g_n^h and l_n^h are the predicted renewable energy generation and the basic demand for the building n at time slot h, and (14b) is the energy balance constraint for the building n at any time slot. Since the objective function $C_n^{\mathrm{P0}}(\cdot)$ and the constraints are all convex and well-defined, the above problem is convex, which means that there exits optimal solutions for the minimum individual social cost C_n^{P0} [32] (denoted as C_n^0).

B. The Problem with Energy Sharing

In this part, we present a two-stage optimization strategy. In the first stage we derive the optimal energy sharing profiles for the building cluster to minimize the total social energy cost, where each building decides the energy supplies/demands to/from other buildings. In the second stage we propose a game-based clearing for mutual energy sharing, where each building will charge/pay for the energy supplies/demands to/from other buildings.

1) Minimizing total social cost: The total social cost C^{SC} is to evaluate the sustainability of the system including the energy trading costs with REP, operation cost of ESS, and utilities of controllable loads for all the energy buildings as a whole. Thus C^{SC} is formulated as follows:

$$C^{\mathrm{SC}} = \sum_{n \in \mathcal{N}} C_n^{\mathrm{P1}}(\boldsymbol{x}_n), \quad (15)$$

where $\boldsymbol{x}_n := \{\boldsymbol{q}_k, \boldsymbol{c}_n, \boldsymbol{d}_n, \boldsymbol{b}_n, \boldsymbol{s}_n, \boldsymbol{e}_n\}$ denote the decision variables of the building n and $C_n^{\mathrm{P1}}(\boldsymbol{x}_n) = C_n^{\mathrm{REP}}(\cdot) + C_n^{\mathrm{ESS}}(\cdot) -$

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$\sum_{k\in\mathcal{A}_n} U_k(\cdot)$. Note that the clearing for mutual energy sharing is balanced, clearing for energy sharing has no influence on the total social cost. Therefore, the clearing $C_n^{\mathrm{ES}}(\boldsymbol{e}_n)$ for the buildings is neglected in this stage. The problem of minimizing C^{SC} must meet the coupled constraint set $\mathcal{Y}$ for $\{\boldsymbol{e}_n, \forall n\}$ and the individual constraint set $\mathcal{X}_n$:

$$\begin{aligned}\mathcal{X}_n = \{&\boldsymbol{q}_k, \boldsymbol{c}_n, \boldsymbol{d}_n, \boldsymbol{b}_n, \boldsymbol{s}_n, \boldsymbol{e}_n | \\ &\sum_{k\in\mathcal{A}_n} q_k^h + c_n^h - d_n^h - b_n^h + s_n^h - \sum_{m\in\mathcal{N}/n} e_{n,m}^h \\ &= g_n^h - l_n^h, \ \forall h \in \mathcal{H}, \\ &\boldsymbol{q}_k \in \Phi_k, \forall k \in \mathcal{A}_n; \ \boldsymbol{c}_n, \boldsymbol{d}_n \in \Upsilon_n.\}\end{aligned} \tag{16}$$

Note that the objective function can be decomposed into the objective of each building $n \in \mathcal{N}$, the problem minimizing social cost C^{SC} with constraints $\mathcal{Y}$ & $\{\mathcal{X}_n, \forall n\}$ can be solved by distributed optimization algorithm such as ADMM. Since the problem (15) with directly coupled constraints (11) is a N-block structure with N blocks of subproblems, we can turn the N-block problem into a two-block problem as a standard ADMM form by introducing auxiliary variables $\boldsymbol{\varepsilon}$ to reformulate the coupled constraints (11) as follows:

$$e_{n,m}^h - \varepsilon_{n,m}^h = 0, \forall n \in \mathcal{N}, \forall m \in \mathcal{N}\backslash n, \forall h \in \mathcal{H}, \tag{17a}$$
$$\varepsilon_{n,m}^h + \varepsilon_{m,n}^h = 0, \forall n \in \mathcal{N}, \forall m \in \mathcal{N}\backslash n, \forall h \in \mathcal{H}. \tag{17b}$$

In such case, the augmented Lagrangian for the problem is shown as follows:

$$\begin{aligned}&L(\boldsymbol{q}_k, \boldsymbol{c}_n, \boldsymbol{d}_n, \boldsymbol{b}_n, \boldsymbol{s}_n, \boldsymbol{e}_n, \boldsymbol{\varepsilon}_n) = \\ &\sum_{n\in\mathcal{N}} \left[\begin{array}{l} C_n^{\mathrm{P1}}(\boldsymbol{q}_k, \boldsymbol{c}_n, \boldsymbol{d}_n, \boldsymbol{b}_n, \boldsymbol{s}_n, \boldsymbol{e}_n) + \\ \frac{\tau}{2} \sum_{m\in\mathcal{N}\backslash n} \sum_{h\in\mathcal{H}} \left(e_{n,m}^h - \varepsilon_{n,m}^h + \frac{\sigma_{n,m}^h}{\tau} \right)^2 \end{array} \right],\end{aligned} \tag{18}$$

where $\boldsymbol{\sigma} = \{\sigma_{n,m}^h, \forall n \in \mathcal{N}, \forall m \in \mathcal{N}/n, \forall h \in \mathcal{H}\}$ is the dual variables related to the coupled constraints (17a), and τ is a well-defined given positive parameter. The ADMM for solving the problem consists of three steps in each iteration. The first step S1 involves the building deciding their own decision variables $\boldsymbol{x}_n, \forall n$, by (18) with the current auxiliary and dual variables:

$$\min \left[\begin{array}{l} C_n^{\mathrm{P1}}(\boldsymbol{q}_k, \boldsymbol{c}_n, \boldsymbol{d}_n, \boldsymbol{b}_n, \boldsymbol{s}_n, \boldsymbol{e}_n) + \\ \frac{\tau}{2} \sum_{m\in\mathcal{N}\backslash n} \sum_{h\in\mathcal{H}} \left(e_{n,m}^h - \varepsilon_{n,m}^h(t) + \frac{\sigma_{n,m}^h(t)}{\tau} \right)^2 \end{array} \right],$$
$$\text{Variables}: \boldsymbol{x}_n,$$
$$\text{Given}: \boldsymbol{\varepsilon}_{n,m}(t), \boldsymbol{\sigma}_{n,m}(t), \forall m \in \mathcal{N}\backslash n,$$
$$\text{Subject to}: \mathcal{X}_n.$$

The second step S2 involves updating the auxiliary variable vectors $\boldsymbol{\varepsilon}_n, \forall n$, according to the optimization results $\boldsymbol{e}_n(t+1)$:

$$\min \frac{\tau}{2} \sum_{n\in\mathcal{N}} \sum_{m\in\mathcal{N}\backslash n} \sum_{h\in\mathcal{H}} \left(e_{n,m}^h(t+1) - \varepsilon_{n,m}^h + \frac{\sigma_{n,m}^h(t)}{\tau} \right)^2,$$
$$\text{Auxiliary variables}: \boldsymbol{\varepsilon}_n, \forall n \in \mathcal{N},$$
$$\text{Given}: \boldsymbol{e}_{n,m}(t+1), \boldsymbol{\sigma}_{n,m}(t), \forall m \in \mathcal{N}\backslash n,$$
$$\text{Subject to}: \text{(17b)}.$$

The third step S3 involves updating the dual variables $\boldsymbol{\sigma}$ by evaluating the constraints (17a) with $\boldsymbol{e}_n(t+1)$ and $\boldsymbol{\varepsilon}_n(t+1)$:

$$\sigma_{n,m}^h(t+1) = \sigma_{n,m}^h(t) + \tau \left(e_{n,m}^h(t+1) - \varepsilon_{n,m}^h(t+1) \right),$$
$$\text{Dual variables}: \boldsymbol{\varepsilon}_n, \forall n \in \mathcal{N},$$
$$\text{Given}: \boldsymbol{e}_{n,m}(t+1), \boldsymbol{\varepsilon}_{n,m}(t+1), \forall n, \forall m \in \mathcal{N}\backslash n.$$

The fast ADMM [33] is the ADMM with a predictor-corrector-type acceleration step which is stable for the cases when the objective functions are strongly convex. The algorithm of the fast ADMM for solving the problem of minimizing the social energy cost is presented in Algorithm 1. Compared to the ADMM with the convergence rate of order $\mathcal{O}(1/t)$, the faster ADMM attains a convergence rate of order $\mathcal{O}(1/t^2)$.

Algorithm 1 Minimizing the social costs by fast ADMM with relaxation.

1: Initialize $\{\boldsymbol{\varepsilon}_n(1) = \mathbf{0}, \forall n \in \mathcal{N}\}$;
2: Initialize $\boldsymbol{\sigma}(0) = \hat{\boldsymbol{\sigma}}(1) = \mathbf{0}$, $t = 1$, the parameter $\tau = 0.03$, the accuracy ξ, $\theta(1) = 1$;
3: **repeat**
4: **for** Each building $n = 1, 2, ..., N$ **do**
5: Updates $\boldsymbol{x}_n(t+1)$ by S1 with $\boldsymbol{\varepsilon}_n(t), \hat{\boldsymbol{\sigma}}_n(t)$;
6: **end for**
7: Update $\boldsymbol{\varepsilon}_n(t+1), \forall n$, by S2 with $\boldsymbol{e}_n(t), \forall n, \hat{\boldsymbol{\sigma}}(t)$;
8: Update the dual variables $\boldsymbol{\sigma}(t+1)$ by modified S3:
$$\sigma_{n,m}^h(t) = \hat{\sigma}_{n,m}^h(t) + \tau \left(e_{n,m}^h(t+1) - \varepsilon_{n,m}^h(t+1) \right);$$
9: Update acceleration operator: $\theta(t+1) = \frac{1+\sqrt{1+\theta(t)^2}}{2}$;
10: Accelerate dual variables:
$$\hat{\boldsymbol{\sigma}}(t+1) = \boldsymbol{\sigma}(t) + \frac{\theta(t) - 1}{\theta(t+1)} (\boldsymbol{\sigma}(t) - \boldsymbol{\sigma}(t-1));$$
11: Iteration time: $t = t + 1$;
12: **until** $\|\sigma(t+1) - \sigma(t)\| < \xi$

Let $\boldsymbol{x}_n^*, \forall n \in \mathcal{N}$, denote the optimal solution of the above problem, and $C_n^1 = C_n^{\mathrm{P1}}(\boldsymbol{x}_n^*)$ denote the individual social cost.

2) Non-cooperative game for clearing : Though the balance of payments is required, each building always tends to pay less for energy purchasing from other buildings or charge more for energy selling to other buildings. Therefore, the clearing where each building clears for the energy sharing with other buildings is essentially a non-cooperative game with coupled constraints. Define $f_n(\boldsymbol{\lambda}_n, \boldsymbol{\lambda}_{-n})$ as the cost function for the clearing game of energy sharing shown below:

$$f_n(\boldsymbol{\lambda}_n, \boldsymbol{\lambda}_{-n}) = \kappa \sum_{m\in\mathcal{N}\backslash n} \lambda_{n,m} \sum_{h\in H} e_{n,m}^h, \forall n, \tag{19}$$

where κ is a given constant parameter, $\boldsymbol{\lambda}_{-n} := \{\boldsymbol{\lambda}_m, \forall m \in \mathcal{N}/n\}$ since the coupled constraints (12) must be satisfied. The clearing of energy sharing must bring economical benefits for the buildings compared with the social cost C_n^0 without energy sharing, thus we have:

$$C_n^1 + C_n^{\mathrm{ES}}(\boldsymbol{\lambda}_n) < C_n^0, \ \boldsymbol{\lambda}_n \succ \mathbf{0}, \tag{20}$$

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Let $\mathcal{Z}_n := \{(20)\}$ denote the individual constraint set for each building. Define $\Gamma = \{\mathcal{N}, \boldsymbol{\lambda} \in \prod_n \mathcal{Z}_n \cup \mathcal{W}, \{f_n(\boldsymbol{\lambda}_n, \boldsymbol{\lambda}_{-n}), \forall n\}\}$ as the clearing game of energy sharing, which contains:

1) Players: all the buildings in the set $\mathcal{N}$;
2) Strategy sets: $\{\prod_n \mathcal{Z}_n \cup \mathcal{W}\}$ as the strategy sets;
3) Cost function: $f_n(\boldsymbol{\lambda}_n, \boldsymbol{\lambda}_{-n})$ of each building n with the decision variable vector $\boldsymbol{\lambda}_n$.

A vector $\boldsymbol{\lambda}^* = (\boldsymbol{\lambda}_n^*, \boldsymbol{\lambda}_{-n}^*)$ is called a GNE of the game if $f_n(\boldsymbol{\lambda}_n^*, \lambda_{-n}^*) \leq f_n(\boldsymbol{\lambda}_n, \lambda_{-n}^*)$ with constraints (12)(20) satisfied for all $n \in \mathcal{N}$. In such a GNE, each building cannot decrease its cost function by changing its strategy $\boldsymbol{\lambda}_n^*$ to any other feasible strategy. Before introducing ways to seek the GNE, it is essential to guarantee the existence of the GNE, which can be investigated by convex analysis.

Theorem 1. *There always exists a GNE of the clearing game* Γ *for energy sharing.*

Proof. Note that the cost functions $f_n(\boldsymbol{\lambda}_n, \boldsymbol{\lambda}_{-n})$ are convex. The constraint set $\prod_{n\in\mathcal{N}} \mathcal{Z}_n \cup \mathcal{W}$ are well-defined, thus the feasible region is nonempty, compact convex. Therefore, the existence of GNE is guaranteed. □

We introduce a dynamic response based algorithm [34] to search for GNE through a regularized Nikaido-Isoda-function (NI-function) $\Psi_\rho(\boldsymbol{\lambda}, \boldsymbol{\omega}) : \{\prod_{n\in\mathcal{N}} \mathcal{Z}_n \cup \mathcal{W}\} \times \{\prod_{n\in\mathcal{N}} \mathcal{Z}_n \cup \mathcal{W}\} \to R^{N(N-1)\times N(N-1)}$ formulated below:

$$\Psi_\rho(\boldsymbol{\lambda}, \boldsymbol{\omega}) = \sum_{n\in\mathcal{N}} [f_n(\boldsymbol{\lambda}_n, \boldsymbol{\lambda}_{-n}) - f_n(\boldsymbol{\omega}_n, \boldsymbol{\lambda}_{-n})] - \frac{\rho}{2} \|\boldsymbol{\lambda} - \boldsymbol{\omega}\|_2^2, \tag{21}$$

where $\boldsymbol{\omega} = \{\boldsymbol{\omega}_n, \forall n \in \mathcal{N}\} \in \{\prod_{n\in\mathcal{N}} \mathcal{Z}_n \cup \mathcal{W}\}$ and ρ is a given parameter. The regularized NI-function is a measure which indicates how much a player gains if the player changes its strategy $\boldsymbol{\lambda}_n$ to a new strategy $\boldsymbol{\omega}_n$ as a best response while all other players continue to hold their strategies $\boldsymbol{\lambda}_{-n}$. Let $\boldsymbol{\omega}_{(\rho)} \in \{\prod_{n\in\mathcal{N}} \mathcal{Z}_n \cup \mathcal{W}\}$ denote the maximizer of function (21):

$$\boldsymbol{\omega}_{(\rho)}(\boldsymbol{\lambda}) = \arg\min \sum_{n\in\mathcal{N}} \Big[f_n(\boldsymbol{\omega}_n, \boldsymbol{\lambda}_{-n}) + \frac{\rho}{2} \|\boldsymbol{\lambda}_n - \boldsymbol{\omega}_n\|_2^2\Big]. \tag{22}$$

Note that the decomposability of the objective function and the coupled constraints are very similar with the problem in the first stage, therefore the above problem can also be solved in the fast ADMM framework of Algorithm 1. Let $V_\rho(\boldsymbol{\lambda}) := \Psi_\rho(\boldsymbol{\lambda}, \boldsymbol{\omega}_{(\rho)}(\boldsymbol{\lambda}))$ and a point $\boldsymbol{\lambda}^* \in \{\prod_{n\in\mathcal{N}} \mathcal{Z}_n \cup \mathcal{W}\}$ is defined as the equilibrium point which is the GNE of the game Γ if $V_\rho(\boldsymbol{\lambda}^*) = 0$ holds. Based on the definitions, we introduce the relaxation-based method [35, 36] as an efficient algorithm to search for the GNE in a distributed way in Algorithm 2.

Algorithm 2 Search for the GNE by the relaxation-based method.

1: Initialize iteration index $i = 1$, parameter $\rho = 0.3$, error ζ, $\kappa = 0.01$;
2: **for** Each building $n = 1, 2, ..., N$ **do**
3: Initialize individual strategy $\boldsymbol{\lambda}_n(i)$;
4: **end for**
5: **repeat**
6: The buildings acquire their best responses $\boldsymbol{\omega}_n(i), \forall n \in \mathcal{N}$, via solving the problem (22) with constraint (20) by the fast ADMM framework.
7: **for** Each building $n = 1, 2, ..., N$ **do**
8: Update $\boldsymbol{\lambda}_n(i+1) = (1 - \frac{1}{i+1})\boldsymbol{\lambda}_n(i) + \frac{1}{i+1}\boldsymbol{\omega}_n(i)$;
9: **end for**
10: Set $i = i + 1$;
11: **until** $\|\boldsymbol{\omega}(i+1) - \boldsymbol{\lambda}(i)\| < \zeta$

IV. Real-time Balancing

Due to the prediction errors of uncertain parameters such as the renewable energy generation and base loads, there will be mismatch between the day-ahead schedule and real-time energy demands. A widely used way is to balance the mismatch directly by real-time trading with REP, which is not economical due to the unfavorable prices. Generally, the buildings know exact values of the uncertain parameters in the current time slot, and will have new prediction for the parameters of the residual time slots. At each time slot the buildings are supposed to adjust their energy schedules for residual time slots according to the current state and their latest renewable energy generation prediction [37]. The energy sharing profiles among the buildings are the optimization results in the first stage, and it will be inconvenient to carry out the optimization in each time slot in real-time stage considering the communication and computation burden. Thus we can assume each building can adjust the energy schedule and trade with REP in the real time while keeping the preset energy sharing profiles with other buildings. Based on the assumption, a real-time optimization model is presented for each building at each time slot in real time to adjust its energy schedule for the following finite horizon.

At the time slot h in real-time stage, let $\tilde{\boldsymbol{q}}_k = [\tilde{q}_k^h, ..., \tilde{q}_k^H]$ denote the power consumption profile of the controllable load k for the residual time slots, and $\tilde{\boldsymbol{c}}_n$ and $\tilde{\boldsymbol{d}}_n$ can be defined similarly. Let $\tilde{\boldsymbol{b}}_n$ and $\tilde{\boldsymbol{s}}_n$ denote the real-time energy purchasing from and selling to REP for the residual time slots at the real-time prices $\tilde{\boldsymbol{p}}_b$ and $\tilde{\boldsymbol{s}}_b$. Let $\tilde{U}_k(\tilde{\boldsymbol{q}}_k)$ denote the utilities achieved through the energy consumption profile $\tilde{\boldsymbol{q}}_k$, and so is $\tilde{C}_n^{\mathrm{ESS}}(\tilde{\boldsymbol{c}}_n, \tilde{\boldsymbol{d}}_n)$. Let $\tilde{C}_n^{\mathrm{REP}}(\tilde{\boldsymbol{b}}_n, \tilde{\boldsymbol{s}}_n) = \sum_{t=h}^{H} \left(\tilde{b}_n^t \tilde{p}_b^t \Delta h - \tilde{s}_n^t \tilde{p}_s^t \Delta h\right)$. We formulate the following real-time optimization model for building $n \in \mathcal{N}$:

$$\min\ \tilde{C}_n^{\mathrm{ESS}}(\tilde{\boldsymbol{c}}_n, \tilde{\boldsymbol{d}}_n) - \sum_{k\in\mathcal{A}_n} \tilde{U}_k(\tilde{\boldsymbol{q}}_k) + \tilde{C}_n^{\mathrm{REP}}(\tilde{\boldsymbol{b}}_n, \tilde{\boldsymbol{s}}_n), \tag{23a}$$

subject to:

$$\begin{aligned} &\sum_{k\in\mathcal{A}_n} \tilde{q}_k^t + \tilde{c}_n^t - \tilde{d}_n^t - \tilde{b}_n^t + \tilde{s}_n^t = b_n^t - s_n^t + \\ &\tilde{g}_n^t - \tilde{l}_n^t + \sum_{m\in\mathcal{N}\backslash n} \tilde{e}_{n,m}^t,\ \forall t = h, ..., H, \end{aligned} \tag{23b}$$

$$\tilde{\boldsymbol{q}}_k \in \tilde{\Phi}_k, \forall k \in \mathcal{A}_n;\ \tilde{\boldsymbol{c}}_n, \tilde{\boldsymbol{d}}_n \in \tilde{\Upsilon}_n;\ \tilde{\boldsymbol{b}}_n, \tilde{\boldsymbol{s}}_n \geq \boldsymbol{0}, \tag{23c}$$

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where $\tilde{\boldsymbol{g}}_n$ and $\tilde{\boldsymbol{l}}_n$ are the latest renewable energy generation and uncontrollable load, and $\tilde{e}^t_{n,m}$ denotes the practical energy sharing quantity which may deviate from the scheduled energy sharing quantity $e^t_{n,m}$ due to some possible practical faults such as buildings being unavailable. (23b) is the real-time energy balance for the residual time slots. In fact, only the first option of the decision vectors $(\tilde{\boldsymbol{q}}_k, \tilde{\boldsymbol{c}}_n, \tilde{\boldsymbol{d}}_n, \tilde{\boldsymbol{b}}_n, \tilde{\boldsymbol{s}}_n)$ will be applied for real-time energy schedule, and the optimization will be carried out again based on the latest state at the next time slot $h+1$. The aim of presenting such a real-time optimization problem for each building is to provide an economical way to overcome the uncertainties of the uncertain parameters.

Overall, the framework of the proposed energy management scheme for the energy building cluster in the paper is shown in Fig. 1. Ahead of the day, the buildings will predict their uncontrollable load profiles and renewable energy generation profiles over the day. Then the buildings in the cluster carry out the peer-to-peer energy sharing strategy with coordination in a distributed fashion to obtain the optimal energy sharing profiles and the corresponding clearing for the profiles. The clearing will be executed immediately, while the energy sharing profiles with energy exchanges will be operated in real time of the day. All the mismatches for each building are going to be handled by itself through the real-time optimization model at each time slot of the day.

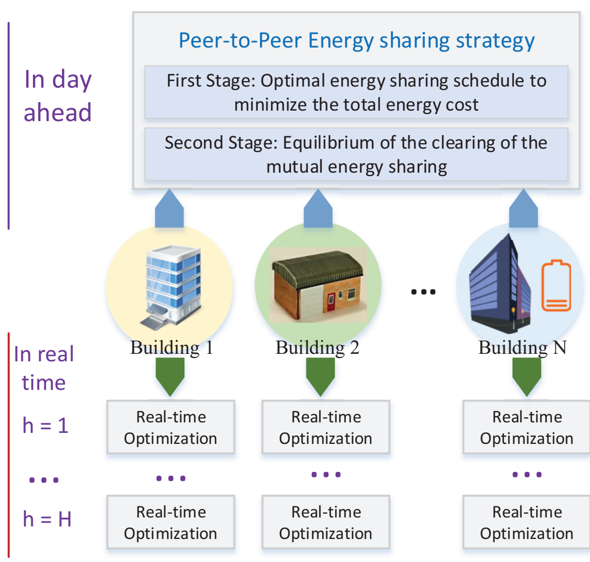

Fig. 1. Framework of the energy management of the building cluster.

V. Simulation Cases

In this section, we carry out simulation cases to illustrate the energy efficiency and algorithm performance of the proposed energy sharing framework.

A. Basic Data

We assume the building cluster includes one office building owning HVAC units [22], two industrial buildings equipped with SEA [8], and one commercial building providing FCS [25]. The main parameters of these buildings for their utility functions are shown in Table. I. We set $M_k = 50, k \in \mathcal{A}_n, \forall n$, for the HVAC units and SEAs. Due to the high capital cost for ESS, we only assume the commercial building $n = 4$ has an ESS with $S_4^{\max} = 300kWh$, $S_4^{\min} = 40kWh$, $S_4^0 = 60kWh$, $c_4^{\max} = 80kW$, $d_4^{\max} = 80kW$, $\eta_4^c = 0.94$ and $\eta_4^d = 1.06$.

TABLE I
Parameters for the controllable loads.

1 : HVAC 1,2	C	R	η	α
	3.3,3,3	1.35,1,35	$0.185, 0.148$	$1, 1$
2 : SEA 1,2	$\beta_1, \beta_2 = 0.1, 0.1$		$D_1, D_2 = 200, 300$	
3 : SEA 1,2	$\beta_1, \beta_2 = 0.1, 0.1$		$D_1, D_2 = 240, 280$	
4 : FCS 1,2	$\gamma_1, \gamma_2 = 3, 3.9$			

We select a typical day on July 10th, 2018 in Wuhan, China to simulate the energy performance of the buildings. We show the predicted solar power generation [38] and outdoor temperature of the day in Fig. 2. The power prices provided by the REP are flat prices set as $p_b^h = 0.23\$/kWh$, $p_s^h = 0.10\$/kWh$, $\epsilon^h = 0.02\$/kWh$. Other parameters for the algorithms are set as $\tau = 0.03$, $\xi = 0.02$ for Algorithm 1, and $\rho = 0.2$, $\zeta = 0.02$ for Algorithm 2.

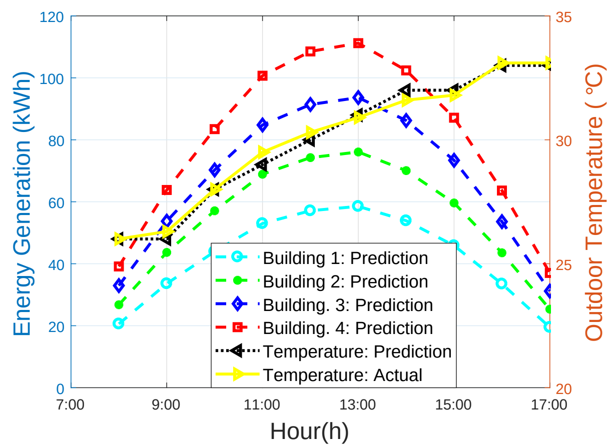

Fig. 2. Solar power generation of the buildings and Temperature curves.

B. Optimal Energy Trading

We compare the optimal REP energy trading profiles of the buildings for the problem without energy sharing in III-A with those for the problem with energy sharing in III-B. Without energy sharing, the buildings will purchase the lack of energy from REP for its energy demand, and sell the surplus energy directly to REP. As shown in Fig. 3(a), due to their similar energy characteristics, the total net demand profile of the cluster shows a strong fluctuation in the range $[-40, 60]$kWh over the day, such as demanding a large amount of energy in periods of low PV generation at hours 8:00, 9:00, 17:00 and selling much in periods of high PV generation at hours 10:00-15:00. However, under the energy sharing framework, the net energy demand is relatively small, and nearly limited in $[-20, 20]$kWh according to Fig. 3(b). The comparison shows that the proposed energy sharing strategy has the capability of

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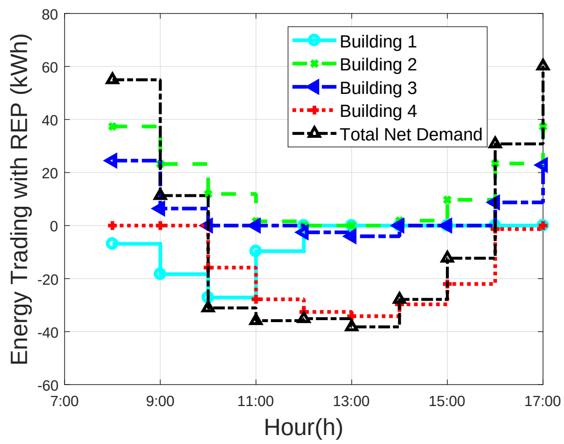
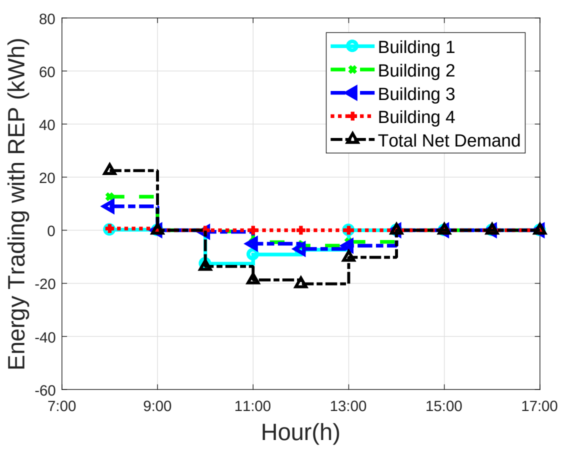
reducing the energy reliance on the main grid of the building cluster so as to facilitate a sustainable local microgrid.



(a) Optimal Energy Trading with REP without Energy Sharing .



(b) Optimal Energy Trading with REP with Energy Sharing.

Fig. 3. Energy Trading with REP and total net demand of the building cluster.

C. Energy Sharing Profiles and Clearing

There are six pairs of buildings in the cluster to facilitate the energy sharing and carry out distributed transaction, and their energy sharing profiles are shown in Fig. 4, where the positive values mean the energy purchasing of the former building from the latter one, and the negatives mean the energy selling of the former one to the latter one. The clearing prices which are also the equilibrium of the clearing game Γ for energy sharing as well as the total energy exchange over the horizon are shown in Fig. 5. More specifically, we show their social costs, energy trading cost (REP Cost), and clearing cost (ES Cost) of the buildings without and with the energy sharing strategy (denoted as No and Yes, respectively) in Table. II. Under the energy sharing framework with interconnected energy exchange, the total social cost has reduced a lot, and the total cost for each building is also smaller. The results show that the proposed energy sharing framework is beneficial for a sustainable society and economical for each energy building.

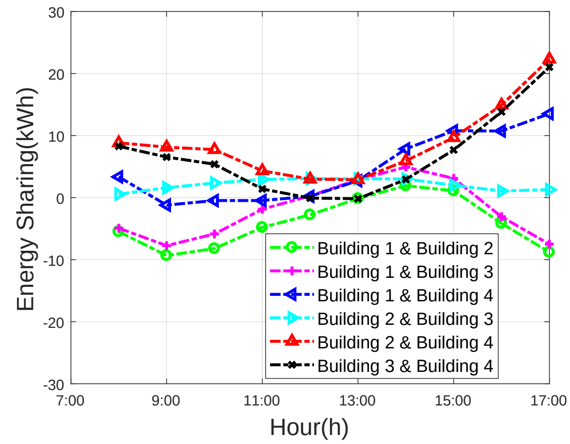


Fig. 4. Energy Sharing Profiles among the Buildings.

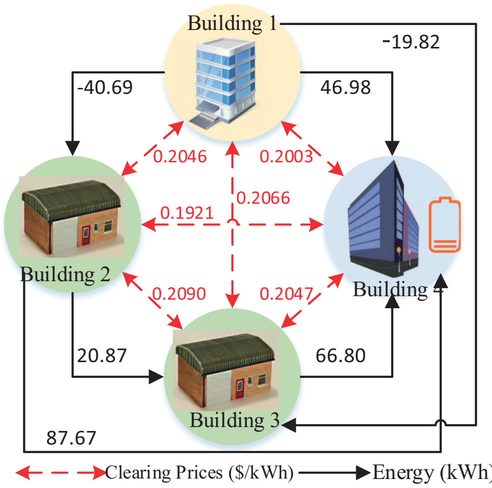


Fig. 5. Energy Sharing Clearing among the Buildings.

TABLE II
ENERGY COSTS OF THE BUILDINGS.

	Cost ($)	B. 1	B. 2	B. 3	B. 4	Sum
No	Social	-94.79	-63.32	-82.43	-223.50	-464.04
	REP	-6.20	36.62	14.96	-16.35	29.03
ES	Social	-95.18	-97.55	-98.58	-199.16	-490.47
	REP	-2.86	1.65	0.40	0.16	-0.98
	ES	-3.01	29.53	13.40	-39.93	0
	Total	-98.19	-68.02	-85.18	-239.08	-490.47

D. Energy Schedule in Day Ahead and Real Time

We show the energy schedules of the buildings dispatching controllable loads in Fig. 6. Specifically, Fig. 6(a) shows the scheduled indoor temperature verus the most comfortable indoor reference temperature and outdoor temperature. In most of hours, the scheduled temperature is closed to the reference temperature with enough PV generation while the building has to sacrifice the comfortness for energy saving in hours 16:00-17:00 with low PV generation. With the real-time optimization (23) to handle the real-time mismatch of prediction values and actual values, the real-time indoor temperature profile verus that without real-time optimization is also obtained and shown

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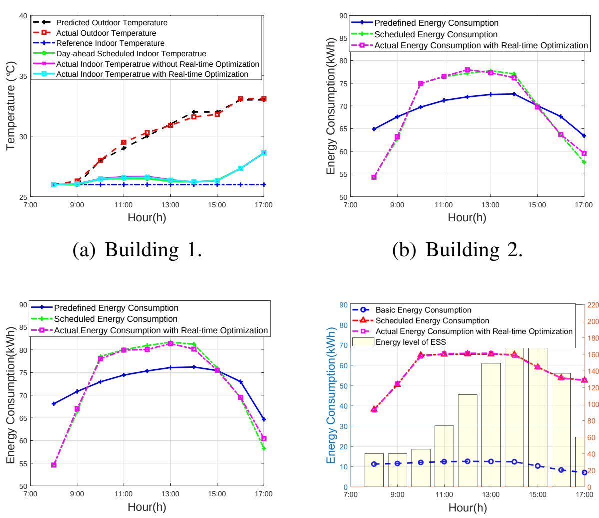
in Fig. 6. The buildings 2,3 transfer part of their flexible loads from hours 8:00-9:00 and 16:00-17:00 with low PV generation to hours 10:00-14:00 with high PV generation shown by Fig. 6(b) and Fig. 6(c). The real-time energy consumption profiles of building 2 and building 3 with real-time optimization are also shown in Fig. 6(b) and Fig. 6(c). The commercial building 4 consumes the maximum power to provide service for economical interests in hours 10:00-14:00 with enough renewable energy according to Fig. 6(d). The energy level is also shown by the yellow bar in Fig. 6(d). Building 4 purchases surplus energy from other buildings and stores the energy in ESS and then discharges the ESS to provide energy for other buildings shown, which shows the importance of ESS in energy sharing. In other words, the ESS of a building is also shared in the building cluster. The simulation results show that efficient scheduling for controllable loads is very essential and important for economical and sustainable energy system.

Numerical comparisons between the simulation results with and without real-time optimization in real time are shown in Table. III. With the mismatch of prediction values and actual values such as PV generations, base loads and outdoor temperatures, the actual utilities of the buildings consuming the controllable loads have all increased compared to the general practice of directly balancing the mismatch by real-time trading with REP without real-time optimization. The reason is that the real-time optimization balances the mismatch by both adjusting the controllable loads and real-time trading with REP. Meanwhile, the real-time trading costs (T.Costs) are also lower than those without real-time optimization. The simulation results verifie the efficiency of the proposed real-time optimization model for each building.



(a) Building 1. (b) Building 2.

(c) Building 3. (d) Building 4.

Fig. 6. Optimal Energy Schedule of the buildings.

E. Algorithm Evaluation

We carry out the simulation in Matlab 2017a environment running on Intel-i5 personal computer with 8GB RAM. We show the iteration times and calculation time to reach the setting accuracy for the proposed Algorithm 1 and Algorithm 2 for the aforementioned 4-building case and another 10-building case in Table. IV. The energy sharing problem as aforementioned is formulated and solved by a two-stage optimization framework. The goal in the first stage is to minimize the total social costs in a distributed way shown in Algorithm 1. The goal in the second stage is to search for the equilibrium for the clearing game of energy sharing in a distributed way shown in Algorithm 2. For the 4-building case, when we set the accuracy $\xi = 0.02$, the Algorithm 1 just requires 17 times of iterations within the running time of 60.67s. When we set the accuracy $\zeta = 0.02$, the Algorithm 2 requires 6 times of iterations within 8.20s. For the 10-building case, Algorithm 1 requires 17 times of iteration within 433.47s to reach the accuracy of $\xi = 0.02$, and Algorithm 2 requires 10 times of iteration within 57.84s to reach the accuracy of $\xi = 0.025$. Moreover, in each iteration of Algorithm 2, the inner fast ADMM only requires 14 and 15 iteration times in 4-building and 10-building cases, respectively, to reach a high accuracy. The relatively small iteration times of the inner fast ADMM shows the efficiency of the algorithms. Therefore, the proposed framework is computationally efficient for limited iteration times to reach a high accuracy. Average Time in Table. IV is the quotient of total calculation time and building number, and the results show that the computation time will be reduced a lot if it can be implemented in a distributed and parallel computation environment. In addition, the calculation performances of 4-building case and 10-building case show that the framework is also scalable.

TABLE III
NEGATIVE UTILITIES AND TRADING COST WITH REP IN REAL TIME.

Cost ($)		B. 1	B. 2	B. 3	B. 4
Utilities	No Real-time	-108.13	-103.85	-102.02	200.61
	Real-time	-107.96	-103.76	-101.90	200.54
T.Costs	No Real-time	0.399	0.572	0.351	0.341
	Real-time	-1.434	0.460	0.318	0.214

TABLE IV
CALCULATION PERFORMANCE FOR 4-BUILDING AND 10-BUILDING CASES

		4 Buildings	10 Buildings
A.1	Accuracy Setting	0.02	0.02
	Iteration Times	17	17
	Calculation Time (s)	60.67	433.47
	Average Time	15.17	43.35
A.2	Accuracy Setting	0.02	0.025
	Iteration Times	6	10
	Total Calculation Time	8.20	57.84
	Average Time	2.05	5.78
	Accuracy Setting of Inner Loop	0.02	0.02
	Iteration Times of Inner Loop	14	15

VI. CONCLUSION

This paper investigates the energy sharing among smart energy buildings to facilitate a sustainable regional building cluster with the distributed transaction technology. Specifically, we present a two-stage strategy for the energy sharing problem, where in the first stage the energy sharing is to minimize the

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total social cost of the buildings, and in the second stage a clearing game for energy sharing is modeled for buildings negotiating their payments for mutual energy sharing. The fast ADMM algorithm and NI-function based method are used to solve the energy sharing problem in a distributed way. In addition, a real-time optimization model is presented to handle real-time uncertainties. The simulation results show the proposed energy sharing framework is economically beneficial for the energy buildings and computationally efficient. In our future work, we intend to broaden the range of local building cluster to a wider network such as a city, where the physical network constraints would be taken into account.

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