UWE Bristol | University of the West of England

# Faculty of Environment and Technology
# Academic Year: 2022/2023
# End of Teaching Block or Date: 2

**Module Leader:** Mokhtar Nibouche
**Module Code** UFMFMT-30-2
**Module Title:** Signals and Systems
**Examination Duration:** 120 minutes

## ONLINE EXAM

### Instructions to Students:

- There are FOUR (04) questions in the exam paper. **Answer all questions**.
- Each question carries 25 marks.
- As is usual for an exam, for this assessment you are not expected to include full referencing, but are encouraged to cite the sources of key theories, models, case studies, statutes etc.
- This is an individual assessment: do not copy and paste work from any other source or work with any other person during this exam. Text-matching software will be used on all submissions.

### Formatting

- Please use **.pdf** file format when submitting. We cannot ensure that other formats are compatible with markers' software and cannot guarantee to mark incorrect formats.
- **Please include the module name and number and your student number (not your name).** Please indicate clearly which questions you are answering.

### Instructions for submission

You must submit your assignment before the stated deadline by electronic submission through Blackboard.

- Multiple submissions can be made to the portal, but only the final one will be accepted. Please save your work frequently.
- **It is your responsibility to submit exam in a format stipulated above** Your marks may be affected if your tutor cannot open or properly view your submission.
- **Do not leave submission to the very last minute.** Always allow time in case of technical issues.
- The date and time of your submission is taken from the Blackboard server and is recorded when your submission is complete, not when you click Submit.
- **It is essential that you check that you have submitted the correct file(s),** and that each complete file was received. Submission receipts are accessed from the Coursework tab.

**There is no late submission permitted on this timed assessment.**

### QUESTION 1 [25 marks]

**a.** Using the phasor graphical method, determine $Y(t) = Y_1(t) + Y_2(t)$, where

$$Y_1(t) = 5\sin(5t + 50^o) \text{ and } Y_2(t) = 3\cos(5t)i$$

**[8 marks]**

**b.** Determine the transfer function $H(s) = \frac{V_2(s)}{V_1(s)}$ of the electrical circuit of Figure Q1 and then put it under the normalised form:

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\,\omega_n + \omega_n^2}$$

**[8 marks]**

Calculate the numerical values of the static gain, **K**, the natural frequency, $\boldsymbol{\omega_n}$, and the damping factor, $\boldsymbol{\zeta}$.

**[3 marks]**

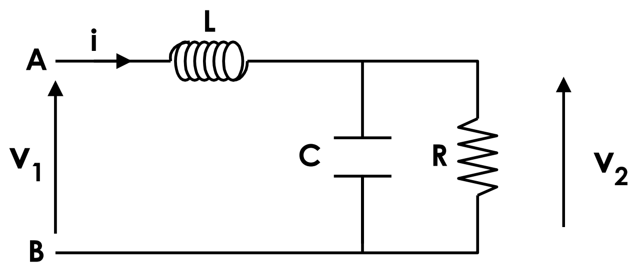


**Figure Q1**

$R = 2.5\Omega$, $L = 1.25$ H and $C = 0.1$ F

**c.** Determine the gain and the phase shift of the circuit. Then, calculate the output voltage $v_2(t)$ in response to an input $v_1(t) = 3\sin(2t + 35^o)$

**[6 marks]**

**QUESTION 2 [25 marks]**

Consider the Laplace transfer function:

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{30(s+10)}{(s+1)(s+20)}$$

**a)** Put H(s) under a normalised form. **[2 marks]**

**b)** Determine the expressions of both the gain (in decibels) and the phase of H(s). **[5 marks]**

**c)** Sketch the Bode plots (gain and phase) of H(s). **[8 marks]**

**d.** Consider the transfer function of a low pass digital filter, where X(z) and Y(z), represent the z-transforms of the input and output, respectively:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z+1}{z-0.8}$$

i. Draw the pole-zero diagram of H(z). **[2 marks]**

ii. Specify the filter's difference equation. **[3 marks]**

iii. Sketch its magnitude frequency response over the range $0 < \Omega < \pi$. **[5 marks]**

**QUESTION 3 [25 marks]**

**a)** Determine the Fourier series $a_0, a_n$ and $b_n$ of the periodic signal, x(t), illustrated in Figure Q3(a). Then, calculate the first 3 non-zero harmonics of the series. **[10 marks]**

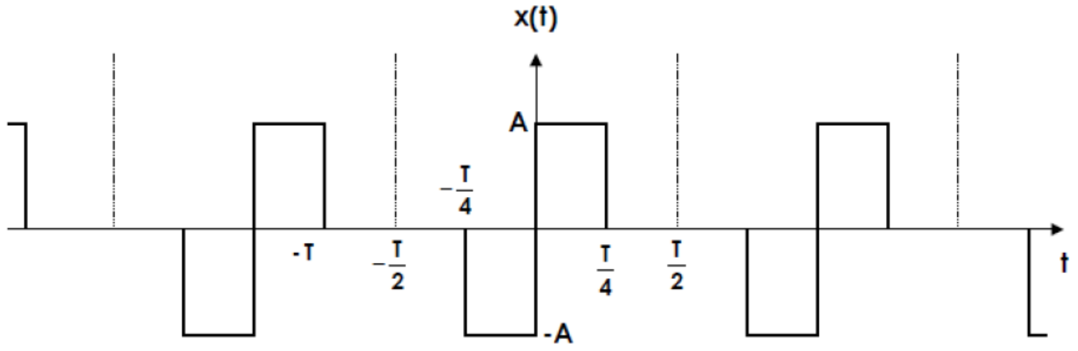


**Figure Q3(a):** Periodic signal

**b)** Knowing that the values of A = 1 and T = 4, put x(t) under the form:

$$x(t) = a_0 + \sum_{n=1}^{n=4} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

where $\omega_0 = \frac{2\pi}{T}$

**[2 marks]**

**c)** The signal x(t) as described in b) is applied to a circuit whose transfer function is:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{0.25s}{1 + 0.25s}$$

where X(s) and Y(s) are the Laplace transfer functions of the x(t) and the output y(t) of the circuit, respectively.

i. Determine the gain and the phase of the circuit.

**[3 marks]**

ii. Find then the output of the circuit y(t). Explain and reflect on the results.

**[10 marks]**

**QUESTION 4 [25 marks]**

Consider the circuit describing a battery model and consisting of resistors and capacitors as shown in Figure Q4. The aim is to find the state-space model of the circuit, where the state space variables are the voltage $v_1(t)$ across the capacitor $C_1$, and the voltage $v_2(t)$ across the capacitor $C_2$, respectively. The output is represented by $v_0(t)$. The numerical values are: $C_1 = 200\mu F$, $C_2 = 100\mu F$, $R_1 = 2.5k\Omega$ and $R_2 = 2k\Omega$.

Note that the general relationship that links the current to the voltage for a capacitor is given by: $C\frac{dv(t)}{dt} = i(t)$, where v(t) and i(t) are respectively, the voltage across the capacitor and the current flowing through it.

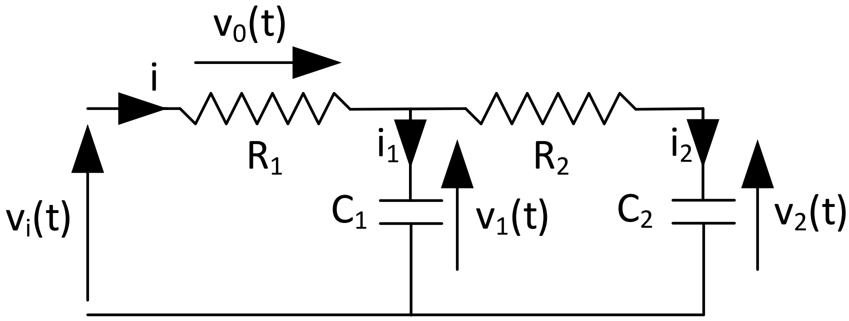


**Figure Q4**

**a)** Determine the differential equations for both the capacitor voltage, $\mathbf{v_1(t)}$**,** and the capacitor voltage, $\mathbf{v_2(t)}$. Give the equation of the output voltage, $\mathbf{v_0(t)}$, in terms of the capacitor voltage, $\mathbf{v_1(t)}$. **[10 marks]**

**b)** Let's rewrite the state variables $v_1(t)$ and $v_2(t)$ as $x_1(t)$ and $x_2(t)$, respectively. Let's also rewrite the output voltage $v_0(t)$ and the input voltage as y(t) and u(t), respectively.

i. Using the state variables $x_1(t)$, $x_2(t)$ and the variables y(t) and u(t), write down the state equations and the output equation describing the circuit. **[3 marks]**

j. Express the equations in a matrix form:

**[5 marks]**

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

Where $x(t)$ is state vector and $\dot{x}(t)$ is the derivative of x(t) with respect to time.

**c)** Determine the Laplace transfer function from the state space representation. **[7 marks]**

## End of Exam Paper