ShipX Vessel Responses (VERES)

Theory Manual

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February 13, 2004

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1 HYDRODYNAMIC COEFFICIENTS AND EXCITING FORCES

In this chapter an outline of the main aspects of the strip theory[1] of Salvesen, Tuck & Faltinsen [9], and the high speed theory of Faltinsen & Zhao [1] will be given. Further details may be found in the references.

1.1 General

Consider a ship advancing at constant mean forward speed with arbitrary heading in regular sinusoidal waves. It is assumed that the resulting oscillatory motions of the ship are linear and harmonic. A right-handed coordinate system (x, y, z) fixed with respect to the mean oscillatory position of the ship is used. The z-axis is positive vertically upwards through the center of gravity of the ship. The origin is located in the plane of the undisturbed free surface. The ship is assumed to have the x-z plane as a plane of symmetry in its mean oscillatory position. The x-axis is pointing towards the stern. The translatory displacements in the x, y and z directions are η_1, η_2 and η_3, respectively. η_1 is the surge, η_2 is the sway and η_3 is the heave displacement. Furthermore, the angular displacements of the rotational motions about the x, y and z axes are η_4, η_5 and η_6, respectively. η_4 is the roll, η_5 is the pitch and η_6 is the yaw angle.

If viscous effects are neglected, the fluid motion can be assumed to be irrotational, so that the problem can be formulated in terms of potential flow theory. The total velocity potential $\Phi(x, y, z)$ must then satisfy the Laplace equation and the following exact boundary conditions:

$$\frac{DF}{Dt} = 0, \tag{1.1}$$

on the hull surface where the hull is defined by $F(x', y', z') = 0$ with (x', y', z') a coordinate system fixed in the ship, and

$$\frac{Dp}{Dt} = -\rho \frac{D}{Dt}\left(\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left|\nabla\Phi\right|^2 + gz\right) = 0, \tag{1.2}$$

on the unknown free surface given by $z = Z(x, y; t)$. In addition, suitable radiation conditions at infinity must be satisfied. Here g is the acceleration of gravity and ρ is the mass density of the fluid.

Separating the velocity potential $\Phi(x, y, z; t)$ into two parts, one the time-independent steady contribution due to the forward motion of the ship and the other the time-dependent part

[1] Please notice the difference in the definition of the coordinate systems. The theory in this manual has been re-formulated to fit the coordinate definitions and unit normal definitions used in VERES.

associated with the incident wave system and the unsteady body motions, we get

$$\Phi(x,y,z;t) = [Ux + \phi_S(x,y,z)] + \phi_T(x,y,z)e^{i\omega t}. \tag{1.3}$$

Here $Ux + \phi_S$ is the steady contribution with U the forward speed of the ship, ϕ_T is the complex amplitude of the unsteady potential and ω is the frequency of encounter in the moving reference frame. It is understood that real part is to be taken in expressions involving $e^{i\omega t}$.

In order to linearize the boundary conditions (1.1) and (1.2), it will be assumed that the geometry of the hull is such that the steady perturbation potential ϕ_S and its derivatives are small, and further that by considering only small oscillatory motions, the potential ϕ_T and its derivatives can also be assumed to be small. Under these assumptions the problem can be linearized by disregarding higher-order terms in both ϕ_S and ϕ_T as well as terms involving cross products between ϕ_S and ϕ_T.

Furthermore, in linearizing the problem it will be convenient to linearly decompose the amplitude of the time-dependent part of the potential

$$\phi_T = \phi_I + \phi_D + \sum_{j=1}^{6} \phi_j \eta_j, \tag{1.4}$$

where ϕ_I is the incident wave potential, ϕ_D is the diffraction potential, and ϕ_j is the contribution to the velocity potential from the jth mode of motion.

The incident wave potential is written as

$$\phi_I = \frac{g\zeta_a}{\omega_0} e^{kz} e^{-ik(x\cos\beta + y\sin\beta)}, \tag{1.5}$$

where ζ_a is the wave amplitude, k is the wave number, β is the heading angle and $\omega_0 = \sqrt{gk}$ is the wave frequency which is related to the frequency of encounter by

$$\omega = \omega_0 + kU\cos\beta. \tag{1.6}$$

Including only linear terms and applying Taylor expansions about the mean hull position in the hull boundary condition (1.1) and about the undisturbed free surface, $z = 0$ in the free surface condition (1.2), it can be shown that the individual potentials must satisfy the following linear boundary conditions:

1. The steady perturbation potential, ϕ_S, must satisfy the body condition

   $$\frac{\partial}{\partial n}[Ux + \phi_S] = 0, \tag{1.7}$$

   on the mean position of the hull and the free surface condition

   $$U^2 \frac{\partial^2 \phi_S}{\partial x^2} + g\frac{\partial \phi_S}{\partial z} = 0, \tag{1.8}$$

   on $z = 0$.

2. The incident wave potential, ϕ_I, and the diffraction potential, ϕ_D, must satisfy

$$\frac{\partial \phi_I}{\partial n} + \frac{\partial \phi_D}{\partial n} = 0, \tag{1.9}$$

on the mean position of the hull and the free surface condition

$$\left[\left(i\omega + U\frac{\partial}{\partial x}\right)^2 + g\frac{\partial}{\partial z}\right](\phi_I, \phi_D) = 0, \tag{1.10}$$

on $z = 0$.

3. The oscillatory potential components ϕ_j, $j = 1, 2, \ldots, 6)$, must satisfy

$$\frac{\partial \phi_j}{\partial n} = i\omega n_j - U m_j, \tag{1.11}$$

on the mean position of the hull and the free surface condition

$$\left(i\omega + U\frac{\partial}{\partial x}\right)^2 \phi_j + g\frac{\partial}{\partial z}\phi_j = 0, \tag{1.12}$$

on $z = 0$. Here the generalized normal n_j, is defined by

$$(n_1, n_2, n_3) \quad = \quad \vec{n}, \tag{1.13}$$

$$(n_4, n_5, n_6) \quad = \quad \vec{r} \times \vec{n}, \tag{1.14}$$

with $\vec{r} = (x, y, z)$ is the position vector with respect to the origin of the coordinate system and $\vec{n}$ is the outward unit normal vector pointing *into* the fluid. Further

$$(m_1, m_2, m_3) \quad = \quad \vec{m} = (\vec{n} \cdot \nabla)\nabla(x + \frac{1}{U}\phi_S) \tag{1.15}$$

$$(m_4, m_5, m_6) \quad = \quad \vec{r} \times \vec{m} - \nabla(x + \frac{1}{U}\phi_S) \tag{1.16}$$

$$m_j \quad = \quad 0 \qquad j = 1, 2, 3, 4 \tag{1.17}$$

$$m_5 \quad = \quad n_3 \tag{1.18}$$

$$m_6 \quad = \quad -n_2. \tag{1.19}$$

In addition to these linear boundary conditions the potentials ϕ_S, ϕ_I, ϕ_D and ϕ_j must each satisfy the Laplace equation in the fluid domain and the appropriate radiation conditions at infinity.

The pressure in the fluid is obtained from the Bernoulli equation

$$p = -\rho\left(\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left|\nabla\Phi\right|^2 + gz\right). \tag{1.20}$$

If the pressure is expanded in a Taylor series about the undisturbed position of the hull and the pressure expression is then linearized by including only term of first order in ϕ_S and ϕ_T, it follows that the linearized time-dependent pressure on the hull is

$$p = -\rho \left(i\omega + U \frac{\partial}{\partial x} \right) \phi_T e^{i\omega t}. \tag{1.21}$$

Integration of the pressure over the hull surface yields the hydrodynamic force and moment amplitudes

$$H_j = \rho \int \int_S n_j \left(i\omega + U \frac{\partial}{\partial x} \right) \phi_T \, ds, j = 1, 2, \ldots, 6. \tag{1.22}$$

Here the integration is over the mean position of the hull surface S. H_1, H_2 and H_3 are the force components in the x, y and z directions, while H_4, H_5 and H_6 are the moments about the x, y and z axes. The force and moments can be divided into two parts as

$$H_j = F_j + G_j, \tag{1.23}$$

where F_j is the exciting force and moment:

$$F_j = \rho \int \int_S n_j \left(i\omega + U \frac{\partial}{\partial x} \right) (\phi_I + \phi_D) \, ds, \tag{1.24}$$

and G_j is the force and moment due to the body motions $\eta_1, \ldots, \eta_6$:

$$G_j = \rho \int \int_S n_j \left(i\omega + U \frac{\partial}{\partial x} \right) \sum_{k=1}^{6} \phi_k \eta_k \, ds \tag{1.25}$$

$$= \sum_{k=1}^{6} T_{jk} \eta_k. \tag{1.26}$$

Here T_{jk} denotes the hydrodynamic force and moment in the jth direction per unit oscillatory displacement in the kth mode:

$$T_{jk} = \rho \int \int_S n_j \left(i\omega + U \frac{\partial}{\partial x} \right) \phi_k \, ds. \tag{1.27}$$

T_{jk} may be separated into real and imaginary parts as

$$T_{jk} = \omega^2 A_{jk} - i\omega B_{jk}, \tag{1.28}$$

where A_{jk} and B_{jk} are the added mass and damping coefficients, respectively.

1.2 Local analysis at each strip

The diffraction and radiation problems are solved by matching of a near-field solution and a far-field solution. The near-field solution is obtained by a numerical boundary-element formulation, while the far-field solution is obtained by an asymptotic analysis.

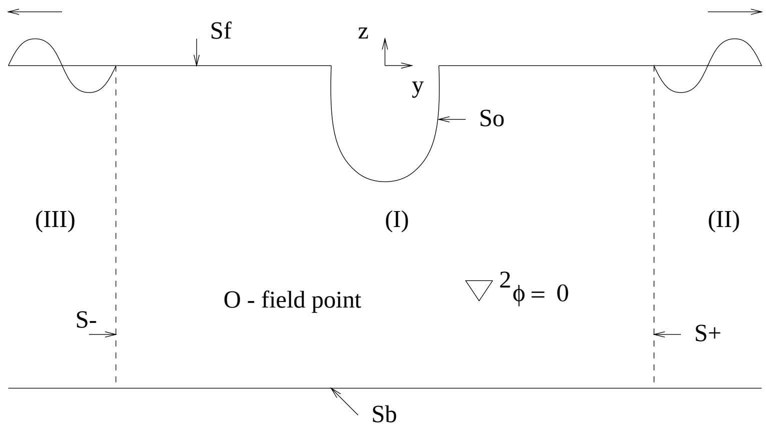

Figure 1.1: The fluid domain and control surfaces

The local analysis in the cross-sectional plane is carried out using Green's second identity to represent the velocity potential in terms of a distribution of fundamental two-dimensional sources and dipoles over a closed surface containing the body surface, the free surface and a control surface far away from the body. The fluid domain and the control surfaces are sketched in Figure 1.1.
Straight line segments are used to approximate the control surfaces, and constant values of the velocity potential and its normal derivatives are assumed at each segment. For each cross section, the velocity potential at a point (y,z) can be represented using Green's second identity:

$$-2\pi\phi = \int_s \left(\phi \frac{\partial \log(r)}{\partial n} - \log(r) \frac{\partial \phi}{\partial n} \right) ds \tag{1.29}$$

To obtain our equation system we let the field point approach the center of each element. At the hull surface, S_o, the velocity potential ϕ is the unknown. At the free surface, S_f, the normal velocity $\frac{\partial \phi}{\partial n}$ is the unknown, since ϕ is known through the free surface condition.

This method is applied both in the low speed and high speed theory. The differences are in the boundary conditions on the free surface, and the integration over the body. The different theories are described in the following sections.

1.3 Low Speed Strip Theory Formulation

The following section describes the part of the theory which is particular for the low speed strip theory formulation by Salvesen, Tuck & Faltinsen [9]. Due to different definitions of coordinate systems, the evaluation of the coefficients is included here, with the definitions which are applied in VERES.

In the low speed strip theory, the hull condition (1.11) is further simplified by dividing the oscillatory potential into one speed dependent, and one speed independent part

$$\phi_j = \phi_j^0 - \frac{U}{i\omega}\phi_j^U, \tag{1.30}$$

where ϕ_j^0 is speed independent. This results in the two hull conditions

$$\frac{\partial \phi_j^0}{\partial n} = i\omega n_j, \tag{1.31}$$

$$\frac{\partial \phi_j^U}{\partial n} = i\omega m_j. \tag{1.32}$$

Now, since both ϕ_j^0 and ϕ_j^U must satisfy the Laplace equation, the same free-surface condition and the same radiation conditions, it follows from the hull conditions above that

$$\phi_j^U = 0, \quad j = 1, 2, 3, 4 \tag{1.33}$$

$$\phi_5^U = \phi_3^0, \tag{1.34}$$

$$\phi_6^U = -\phi_2^0. \tag{1.35}$$

Thus, we see that the oscillatory potential components can be expressed in terms of the speed-independent part of the potential, ϕ_j^0, as

$$\phi_j = \phi_j^0, \quad j = 1, 2, 3, 4, \tag{1.36}$$

$$\phi_5 = \phi_5^0 - \frac{U}{i\omega}\phi_3^0, \tag{1.37}$$

$$\phi_6 = \phi_6^0 + \frac{U}{i\omega}\phi_2^0, \tag{1.38}$$

where $\phi_j^0, (j = 1, 2, \ldots, 6)$ must satisfy the conditions

$$\frac{\partial \phi_j^0}{\partial n} = i\omega n_j, \tag{1.39}$$

on the mean position of the hull and

$$\left(i\omega + U\frac{\partial}{\partial x}\right)^2 \phi_j^0 + g\frac{\partial \phi_j^0}{\partial z} = 0, \tag{1.40}$$

on $z = 0$.

Stoke's theorem may be expressed in the following form [Milne-Thomson [7], 2.51]

$$\int\int_S (\vec{n} \times \nabla) \times \vec{q}\, ds = -\int_C \vec{dl} \times \vec{q}, \tag{1.41}$$

where S is a surface situated in the fluid with the closed curve C as boundary. Here $\vec{q}$ is any vector function and $\vec{dl}$ is the direction element of arc C.

Letting $q = U\phi\vec{i}$ for the case $j = 1, 2, 3$ gives

$$\int\int_S (\vec{n} \times \nabla) \times (U\phi\vec{i})\, ds = -\int_C \vec{dl} \times U\phi\vec{i}. \tag{1.42}$$

By use of various vector theorems, [Ogilvie and Tuck [8]], the integrand of the surface integral can be rewritten

$$\begin{aligned}(\vec{n}\times\nabla)\times U\phi\vec{i} &= \phi\vec{n}\times(\nabla\times U\vec{i})+\phi(\vec{n}\cdot\nabla)U\vec{i}-\vec{n}(U\vec{i}\cdot\nabla\phi)\\ &\quad +(\vec{n}\cdot U\vec{i})\nabla\phi-\phi\vec{n}(\nabla\cdot U\vec{i})\\ &= \phi(\vec{n}\cdot\nabla)U\vec{i}-\vec{n}(U\vec{i}\cdot\nabla\phi).\end{aligned} \tag{1.43}$$

Thus the surface integral is

$$\int\int_S\left[\phi(\vec{n}\cdot\nabla)U\vec{i}-\vec{n}(U\vec{i}\cdot\nabla\phi)\right]\,ds=-\int\int_S\left[\phi m+\vec{n}(U\vec{i}\cdot\nabla\phi)\right]\,ds. \tag{1.44}$$

The surface integral may now be written as

$$\int\int_S\left[\phi(\vec{n}\cdot\nabla)U\vec{i}-\vec{n}(U\vec{i}\cdot\nabla\phi)\right]\,ds=\int\int_S\left[U\phi\vec{m}-\vec{n}(U\vec{i}\cdot\nabla\phi)\right]\,ds. \tag{1.45}$$

Neglecting the contribution from the line integral along the free surface one may write

$$\vec{dl}=dl(\vec{i}\times\vec{n}), \tag{1.46}$$

and then use some standard theorems to show that

$$\begin{aligned}(\vec{i}\times\vec{n})\times\phi U\vec{i} &= -\left[\vec{n}(\vec{i}\cdot\phi U\vec{i})\right]\\ &= -\vec{n}\phi U.\end{aligned} \tag{1.47}$$

This gives the following theorem

$$\int\int_S n_j U\frac{\partial}{\partial x}\phi\,ds=U\int\int_S m_j\phi\,ds+U\int_C n_j\phi\,dl. \tag{1.48}$$

It may be shown [Ogilvie and Tuck (1969)], that this form also applies to the case $j=4,5,6$.

Applying the above theorem in the relationships for the added mass and damping coefficients will give

$$T_{jk}=\rho\int\int_S n_j i\omega\phi_k\,ds+U\rho\int\int_S m_j\phi_k\,ds+U\rho\int_{C_A} n_j\phi_k\,dl. \tag{1.49}$$

Here C_A refers to the aftermost cross section of the ship.

The speed-independent part of T_{jk} may now be defined as

$$T^0_{jk}=\rho i\omega\int\int_S n_j\phi^0_k\,ds, \tag{1.50}$$

and the speed-independent part of the line integral at any cross section C_x as

$$t_{jk}=\rho i\omega\int_{C_x} n_j\phi^0_k\,dl. \tag{1.51}$$

The added mass and damping coefficients can now be expressed in terms of the speed-independent terms as follows

$$
\begin{aligned}
T_{jk} &= T^0_{jk} + \frac{U}{i\omega} t^A_{jk}, \quad j,k = 1,2,3,4, && (1.52)\\
T_{5k} &= T^0_{5k} + \frac{U}{i\omega} T^0_{3k} + \frac{U}{i\omega} t^A_{5k}, \quad k = 1,2,3,4, && (1.53)\\
T_{6k} &= T^0_{6k} - \frac{U}{i\omega} T^0_{2k} + \frac{U}{i\omega} t^A_{6k}, \quad k = 1,2,3,4, && (1.54)\\
T_{j5} &= T^0_{j5} - \frac{U}{i\omega} T^0_{j3} + \frac{U}{i\omega} t^A_{j5} + \frac{U^2}{\omega^2} t^A_{j3}, \quad j = 1,2,3,4, && (1.55)\\
T_{j6} &= T^0_{j6} + \frac{U}{i\omega} T^0_{j2} + \frac{U}{i\omega} t^A_{j6} - \frac{U^2}{\omega^2} t^A_{j2}, \quad j = 1,2,3,4, && (1.56)\\
T_{55} &= T^0_{55} + \frac{U^2}{\omega^2} T^0_{33} + \frac{U}{i\omega} t^A_{55} + \frac{U^2}{\omega^2} t^A_{53}, && (1.57)\\
T_{66} &= T^0_{66} + \frac{U^2}{\omega^2} T^0_{22} + \frac{U}{i\omega} t^A_{66} - \frac{U^2}{\omega^2} t^A_{62}, && (1.58)
\end{aligned}
$$

where t^A_{jk} refers to the line integral evaluated at the aftermost section. In obtaining the expressions for T_{55} and T_{66} the symmetry relationship for the zero speed coefficients has been applied

$$T^0_{jk} = T^0_{kj}. \tag{1.59}$$

Above, the speed-dependent coefficients have been expressed in terms of the speed-independent surface integral and line integral. The zero-speed terms will now be reduced to a form suitable for numerical evaluation. This is done by introducing the following strip theory approximations. If it is assumed that the beam and draft of the ship are much smaller than the length, then it is consistent with the previous assumptions to set $ds = dld\xi$ in the surface integral

$$T^0_{jk} = \rho i\omega \int_L \int_{C_x} n_j \phi^0_k dl\, d\xi = \int_L t_{jk} d\xi, \tag{1.60}$$

where L implies that the integration is over the length of the ship and ξ is the variable of integration in the x-direction. Since the hull is assumed to be long and slender it follows that in the neighborhood of the hull $\partial/\partial x \ll \partial/\partial y$ or $\partial/\partial z$. It also follows that the component of the hull normal in the x-direction is much smaller than the normal components in the y- and z-directions

$$n_1 \ll n_2 \text{ or } n_3, \tag{1.61}$$

so that the components of the three-dimensional generalized normal $n_j, j = 2,3,4$, with the two-dimensional generalized normal in the yz-plane, $N_j, j = 2,3,4$ and then set

$$
\begin{aligned}
n_5 &= -xN_3, && (1.62)\\
n_6 &= xN_2. && (1.63)
\end{aligned}
$$

In order to reduce the free surface condition it will be necessary to assume that the frequency of encounter is high, $\omega \gg U(\partial/\partial x)$, which requires that the wavelength is approximately of the same order as the ship beam.

Under these assumptions the three-dimensional Laplace equation and the boundary conditions to be satisfied by ϕ_k^0 for $k = 2, 3, 4$ reduce to the two-dimensional Laplace equation and the conditions for the two-dimensional problem of a cylinder with cross section C_x oscillating in the free surface, so that at a given cross section

$$\phi_k^0 = \psi_k \text{ for } k = 2, 3, 4, \tag{1.64}$$

where ψ_k is the potential for the sectional two-dimensional problem. It also follows that at a given section

$$\phi_5^0 = -x\psi_3, \tag{1.65}$$

$$\phi_6^0 = x\psi_2, \tag{1.66}$$

while $\phi_1^0 \ll \phi_k^0, k = 2, 3, \ldots, 6$. Hence, for $j = 2, 3, 4$

$$t_{jj} = \rho i\omega \int_{C_x} N_j \psi_j dl = \omega^2 a_{jj} - i\omega b_{jj}, \tag{1.67}$$

where a_{jj} and b_{jj} are the sectional two-dimensional added mass and damping coefficients for sway, heave and roll. Similarly, the sectional sway-roll cross-coupling coefficients is

$$t_{24} = \rho i\omega \int_{C_x} N_2 \psi_4 dl = \omega^2 a_{24} - i\omega b_{24}. \tag{1.68}$$

The zero-speed added mass and damping coefficients, T_{jk}^0 can now be expressed in terms of the sectional two-dimensional added mass and damping coefficients, t_{22}, t_{33}, t_{44} and t_{24}. For ships with lateral symmetry, the only nonzero coefficients are

$$T_{22}^0 = \int t_{22} d\xi, \tag{1.69}$$

$$T_{26}^0 = T_{62}^0 = \int \xi t_{22} d\xi, \tag{1.70}$$

$$T_{66}^0 = \int \xi^2 t_{22} d\xi, \tag{1.71}$$

$$T_{33}^0 = \int t_{33} d\xi, \tag{1.72}$$

$$T_{35}^0 = T_{53}^0 = -\int \xi t_{33} d\xi, \tag{1.73}$$

$$T_{55}^0 = \int \xi^2 t_{33} d\xi, \tag{1.74}$$

$$T_{44}^0 = \int t_{44} d\xi, \tag{1.75}$$

$$T_{24}^0 = T_{42}^0 = \int t_{24} d\xi, \tag{1.76}$$

$$T_{46}^0 = T_{64}^0 = \int \xi t_{24} d\xi, \tag{1.77}$$

where the integration is over the length of the ship.

This implies that the following expressions are obtained for the added mass and damping coefficients

$$A_{22} = \int_L a_{22} d\xi - \frac{U}{\omega^2} b_{22}^A, \tag{1.78}$$

$$B_{22} = \int_L b_{22} d\xi + U a_{22}^A, \tag{1.79}$$

$$A_{24} = A_{42} = \int_L a_{24} d\xi - \frac{U}{\omega^2} b_{24}^A, \tag{1.80}$$

$$B_{24} = B_{42} = \int_L b_{24} d\xi + U a_{24}^A, \tag{1.81}$$

$$A_{33} = \int_L a_{33} d\xi - \frac{U}{\omega^2} b_{33}^A, \tag{1.82}$$

$$B_{33} = \int_L b_{33} d\xi + U a_{33}^A, \tag{1.83}$$

$$A_{44} = \int_L a_{44} d\xi - \frac{U}{\omega^2} b_{44}^A, \tag{1.84}$$

$$B_{44} = \int_L b_{44} d\xi + U a_{44}^A, \tag{1.85}$$

$$A_{53} = -\int_L x a_{33} d\xi - \frac{U}{\omega^2} \int_L b_{33} d\xi + \frac{U}{\omega^2} x_A b_{33}^A, \tag{1.86}$$

$$B_{53} = -\int_L x b_{33} d\xi + U \int_L a_{33} d\xi - U x_A a_{33}^A, \tag{1.87}$$

$$A_{62} = \int_L x a_{22} d\xi + \frac{U}{\omega^2} \int_L b_{22} d\xi - \frac{U}{\omega^2} x_A b_{22}^A, \tag{1.88}$$

$$B_{62} = \int_L x b_{22} d\xi - U \int_L a_{22} d\xi + U x_A a_{22}^A, \tag{1.89}$$

$$A_{64} = \int_L x a_{24} d\xi + \frac{U}{\omega^2} \int_L b_{24} d\xi - \frac{U}{\omega^2} x_A b_{24}^A, \tag{1.90}$$

$$B_{64} = \int_L x a_{24} d\xi - U \int_L a_{24} d\xi + U x_A a_{24}^A, \tag{1.91}$$

$$A_{35} = -\int_L x a_{33} d\xi + \frac{U}{\omega^2} \int_L b_{33} d\xi + \frac{U}{\omega^2} x_A b_{33}^A + \frac{U^2}{\omega^2} a_{33}^A, \tag{1.92}$$

$$B_{35} = -\int_L x b_{33} d\xi - U \int_L a_{33} d\xi - U x_A a_{33}^A + \frac{U^2}{\omega^2} b_{33}^A, \tag{1.93}$$

$$A_{26} = \int_L x a_{22} d\xi - \frac{U}{\omega^2} \int_L b_{22} d\xi - \frac{U}{\omega^2} x_A b_{22}^A - \frac{U^2}{\omega^2} a_{22}^A, \tag{1.94}$$

$$B_{26} = \int_L x b_{22} d\ell + U \int_L a_{22} d\ell + U x_A a_{22}^A - \frac{U^2}{\omega^2} b_{22}^A, \tag{1.95}$$

$$A_{46} = \int_L x a_{24} d\xi - \frac{U}{\omega^2} \int_L b_{24} d\xi - \frac{U}{\omega^2} x_A b_{24}^A - \frac{U^2}{\omega^2} a_{24}^A, \tag{1.96}$$

$$B_{46} = \int_L x b_{24} d\xi + U \int_L a_{24} d\xi + U x_A a_{24}^A - \frac{U^2}{\omega^2} b_{24}^A, \tag{1.97}$$

$$A_{55} = \int_L x^2 a_{33} d\xi + \frac{U^2}{\omega^2} \int_L a_{33} d\xi - \frac{U}{\omega^2} x_A^2 b_{33}^A - \frac{U^2}{\omega^2} x_A a_{33}^A, \tag{1.98}$$

$$B_{55} = \int_L x^2 b_{33} d\xi + \frac{U^2}{\omega^2} \int_L b_{33} d\xi + U x_A^2 a_{33}^A - \frac{U^2}{\omega^2} x_A b_{33}^A, \tag{1.99}$$

$$A_{66} = \int_L x^2 a_{22} d\xi + \frac{U^2}{\omega^2} \int_L a_{22} d\xi - \frac{U}{\omega^2} x_A^2 b_{22}^A - \frac{U^2}{\omega^2} x_A a_{22}^A, \tag{1.100}$$

$$B_{66} = \int_L x^2 b_{22} d\xi + \frac{U^2}{\omega^2} \int_L b_{22} d\xi + U x_A^2 a_{22}^A - \frac{U^2}{\omega^2} x_A b_{22}^A. \tag{1.101}$$

The exiting force and moment is expressed as

$$F_j = \rho \int \int_S n_j \left(i\omega + U \frac{\partial}{\partial x} \right) (\phi_I + \phi_D) ds, \quad j = 1, 2, \ldots, 6. \tag{1.102}$$

It is convenient to separate the exciting force into two parts, the incident wave part F_j^I, and the diffraction part F_j^D, so that

$$F_j = F_j^I + F_j^D, \tag{1.103}$$

with

$$F_j^I = \rho \int \int_S n_j \left(i\omega + U \frac{\partial}{\partial x} \right) \phi_I ds, \tag{1.104}$$

and

$$F_j^D = \rho \int \int_S n_j \left(i\omega + U \frac{\partial}{\partial x} \right) \phi_D ds. \tag{1.105}$$

The incident wave part of the exciting force, denoted the Froude-Krylov force, may now be written as

$$F_j^I = \rho i \int\int_S n_j \left(\omega - kU\cos\beta\right)\phi_I ds, \tag{1.106}$$

which may be reduced to

$$F_j^I = \rho i \omega_0 \int\int_S n_j \phi_I ds. \tag{1.107}$$

Applying the Stokes theorem to the diffraction part of the exciting force gives

$$F_j^D = \rho \int\int_S \left(i\omega n_j + U m_j\right)\phi_D ds + \rho U \int_{C_A} n_j \phi_D dl. \tag{1.108}$$

The sectional Froude-Krylov force is defined by

$$f_j(x) = ige^{-ikx\cos\beta}\int_{C_x} N_j e^{kz} e^{-iky\sin\beta} dl, \quad j = 2, 3, 4. \tag{1.109}$$

The sectional diffraction force is obtained by writing the diffraction potential as

$$\phi_D = \psi_D \zeta_a e^{-ikx\cos\beta}, \tag{1.110}$$

$$h_j(x) = i\omega e^{-ikx\cos\beta}\int_{C_x} N_j \psi_D dl, \quad j = 2, 3, 4. \tag{1.111}$$

This enables the exciting force and moment to be written in the final form

$$F_j = \rho\zeta_a \int_L (f_j + h_j) d\xi + \rho\zeta_a \frac{U}{i\omega} h_j^A, \quad j = 2, 3, 4, \tag{1.112}$$

$$F_5 = -\rho\zeta_a \int_L \left[x(f_3 + h_3) - \frac{U}{i\omega} h_3\right] d\xi - \rho\zeta_a \frac{U}{i\omega} x_A h_3^A, \tag{1.113}$$

$$F_6 = \rho\zeta_a \int_L \left[x(f_2 + h_2) - \frac{U}{i\omega} h_2\right] d\xi + \rho\zeta_a \frac{U}{i\omega} x_A h_2^A. \tag{1.114}$$

$F_1 \ll F_k, k = 2, 3, \ldots, 6$, and is neglected. h_j^A refers to $h_j(x)$ evaluated at the aftermost section.

1.4 High Speed Formulation

The high-speed formulation [1] is based on a strip theory approach, where the free-surface condition is used to step the solution in the downstream direction. The solution is started assuming that both the velocity potential and its x-derivative are zero at the first strip, counted from the bow.

In the solution procedure, the radiation and diffraction potentials are re-written as

$$\phi = e^{-i(\omega/U)x}\Psi(y, z) \tag{1.115}$$

where $\Psi(y,z)$ is independent of x and the time dependency is in the first term.

The following boundary conditions have to be satisfied at each cross-section for Ψ, where the conditions holds for both radiation and diffraction:

- Laplace's equation in the fluid:

$$\frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2} = 0. \tag{1.116}$$

- Free-surface condition:

$$U^2\frac{\partial^2\Psi}{\partial x^2} + g\frac{\partial\Psi}{\partial z} = 0 \text{ on } z = 0. \tag{1.117}$$

  This free-surface condition differs from the ordinary strip theory formulation in that the terms proportional to U^2 are retained, while they are neglected in the ordinary strip theory.

- Body boundary condition:

$$\frac{\partial\Psi_j}{\partial n} = i\omega n_j - Um_j, \quad j = 2,\ldots,6 \tag{1.118}$$

$$\frac{\partial\Psi_D}{\partial n} = (in_x - n_z)\omega_0\zeta_a e^{kz+i(\omega_0/U)x-iky\sin\beta} \tag{1.119}$$

- Furthermore, the potentials and their x-derivatives are set to zero at the foremost part of the vessel, i.e:

$$\Psi = 0 \tag{1.120}$$

$$\text{on} \quad x = x_B$$

$$\frac{\partial\Psi}{\partial x} = 0$$

  where x_B denotes the first strip. This solution procedure requires that there are no upstream waves. This condition is satisfied when the waveform parameter $\tau = \omega U/g > 1/4$.

When the radiation potentials Ψ_j and the diffraction potential Ψ_D has been solved for each strip, the total solution can be found by applying Eqn. 1.115 and integrating over the hull surface:

- **Hydrodynamic coefficients**

  The added mass and damping coefficients are calculated by the equations

$$A_{jk} = \frac{Re(T_{jk})}{\omega^2} \quad , \quad B_{jk} = \frac{-Im(T_{jk})}{\omega} \tag{1.121}$$

  where

$$T_{jk} = \rho\int\int_S n_j\left(i\omega + U\frac{\partial}{\partial x}\right)\phi_k ds \tag{1.122}$$

- **Exciting forces**

  The wave excitation forces are calculated by integrating the Froude-Krylov and diffraction forces over the hull surface, i.e:

$$F_j = \rho \int \int_S n_j \left(i\omega_0 \phi_I + i\omega \phi_D + U \frac{\partial \phi_D}{\partial x} \right) ds. \tag{1.123}$$

2 RESTORING COEFFICIENTS

The hydrostatic restoring coefficients which are independent of frequency and forward speed, follow directly from hydrostatic considerations.
For a vessel in the free surface symmetric about the $x-z$ plane, the only coefficients different from zero are

$$C_{33} = \rho g \int_L b dx = \rho g A_{wp}, \tag{2.1}$$

$$C_{35} = C_{53} = -\rho g \int_L bx dx, \tag{2.2}$$

$$C_{44} = \rho g \nabla (z_B - z_G) + \rho g \int_L y^2 b dx = \rho g \nabla \overline{GM}_T, \tag{2.3}$$

$$C_{55} = \rho g \nabla (z_B - z_G) + \rho g \int_L x^2 b dx = \rho g \nabla \overline{GM}_L, \tag{2.4}$$

Here b is the sectional breadth, A_{wp} is the waterplane area, ∇ is the displaced volume of water and z_B and z_G are the z-coordinates of the center of buoyancy and center of gravity, respectively. $\overline{GM}_L$ and $\overline{GM}_T$ is the longitudinal and transverse metacentric heights, respectively.

3 VISCOUS ROLL DAMPING COEFFICIENTS

3.1 Frictional roll damping

For turbulent flow, Kato [6] applies Hughes formula for the frictional coefficient, giving the following contribution to the roll damping coefficients:

$$B_{44}^{V1} = \frac{8}{3\pi} \cdot 0.9275 \cdot \rho \cdot S \cdot r_s^2 \cdot \omega^{0.5} \cdot \nu^{0.5} \tag{3.1}$$

$$B_{44}^{V2} = 0.00755 \cdot \rho \cdot S \cdot r_s^{2.772} \cdot \omega^{-0.114} \cdot \nu^{0.114} \cdot \eta_{4a}^{-0.228}. \tag{3.2}$$

Note that the nonlinear damping coefficient B_{44}^{V2} is dependent on the roll amplitude. This requires that the wave amplitude must be given as input to VERES .

Correction for forward speed effects

The coefficients are corrected for forward speed effects, as explained in Himeno [2], giving the corrected damping coefficient as

$$B = B_0 \left(1 + 4.1 \cdot \frac{U}{\omega L}\right), \tag{3.3}$$

where the coefficient B_0 represents the friction damping coefficient at zero forward speed.

3.2 Eddy damping

This damping component is caused by flow separation at the bilge of the cross section. Based on results from forced roll tests for a number of two dimensional cylinders without bilge keels, Ikeda et.al. [3] has proposed a prediction method giving the following contribution to the roll damping coefficients:

$$B_{44}^{V1} = 0.0 \tag{3.4}$$

$$B_{44}^{V2} = 0.5 \cdot \rho \cdot r_{max}^2 \int_S c_p(s)\ell(s)ds \tag{3.5}$$

where r_{max} is the maximum distance from the roll axis to the hull surface, $c_p(s)$ is the pressure coefficient and $\ell(s)$ is the roll moment lever. The integration is taken over the wetted surface.

Correction for forward speed effects

The coefficients are corrected for forward speed effects as explained in Himeno [2] giving the corrected eddy damping coefficient as

$$B = B_0 \cdot \frac{0.04 \cdot \omega^2 \cdot L^2}{U^2 + 0.04 \cdot \omega^2 \cdot L^2} \tag{3.6}$$

3.3 Bilge keel damping

Damping due to normal forces on bilge keels

The contribution to the roll damping coefficients is based on Ikeda et.al. [4] giving:

$$B_{44}^{V1} = \frac{8}{3\pi^2} \cdot 22.5 \cdot b_{bk}^2 \cdot r_{bk}^2 \cdot f \cdot \omega \tag{3.7}$$

$$B_{44}^{V2} = 2.4 \cdot b_{bk} \cdot f^2 \cdot r_{bk}^3, \tag{3.8}$$

where b_{bk} is the breadth of the bilge keel, r_{bk} is the distance from the roll axis to the bilge keel and f is a correction factor for velocity increment at the bilge given as

$$f = 1.0 + 0.3 \cdot \exp\{-160 \cdot (1.0 - \sigma)\}. \tag{3.9}$$

Here σ is the area coefficient of the cross-section.

Damping due to hull pressure created by bilge keels

The contribution to the roll damping coefficients is based on Ikeda et.al. [5] giving:

- **Damping due to pressure increase on the hull:**

$$B_{1H}^{+} = 0.0 \tag{3.10}$$

$$B_{2H}^{+} = 0.5 \cdot f^2 \cdot r_{bk}^2 \cdot 1.2 \cdot (I_2 + I_4), \tag{3.11}$$

where

$$I_2 = \int_{s2}^{s3} \ell(s)ds \tag{3.12}$$

and

$$I_4 = \int_{s5}^{s6} \ell(s)ds. \tag{3.13}$$

- **Damping due to pressure decrease on the hull:**

$$B_{1H}^{-} = \frac{4}{3\pi^2} \cdot f \cdot r_{bk} \cdot b_{bk} \cdot \omega \cdot 22.5(I_1 + I_3) \tag{3.14}$$

$$B_{2H}^{-} = 0.5 \cdot f^2 \cdot r_{bk}^2 \cdot 1.2 \cdot (I_1 + I_3), \tag{3.15}$$

where

$$I_1 = \int_{s1}^{s2} \ell(s)ds \tag{3.16}$$

and

$$I_3 = \int_{s4}^{s5} \ell(s)ds. \tag{3.17}$$

- **Total contribution:**

  The total contributions from the hull pressure damping due to bilge keels can then be expressed as:

$$B_{44}^{V1} = B_{1H}^{+} + B_{1H}^{-} \tag{3.18}$$

$$B_{44}^{V2} = B_{2H}^{+} + B_{2H}^{-} \tag{3.19}$$

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