2

Modelling the ROV

This chapter will describe how our ROV is modelled using Fossen [2011] 6 DOF model for ROV's.

According to Fossen [2011] a underwater vehicle with 6 DOF, like our ROV, can be modelled the following way

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}(\boldsymbol{\eta}) = \boldsymbol{\tau} \tag{2.1}$$

where

$$\boldsymbol{\eta} = [x, y, z, \phi, \theta, \psi]^T$$

$$\boldsymbol{\nu} = [u, v, w, p, q, r]^T$$

are the generalised vectors for describing motion in 6 DOF. The matrices $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{D}$ and $\boldsymbol{g}$ respectively describes how, inertia, Coriolis forces, damping forces, gravity and buoyancy effect the ROV. The vector $\boldsymbol{\tau}$ describes the forces and torques produced by the vehicle's actuators.

TODO: Write somethin about the differer coordinate systems

In this chapter the waiting cross product $\boldsymbol{S}(\cdot)$ is defined as $\boldsymbol{S}(a\boldsymbol{A})\boldsymbol{B} = a\boldsymbol{A} \times \boldsymbol{B}$. The notation used for the parameters in this chapter and chapter 3 can be seen in table 2.1. The notation for forces, moments, linear and angular velocities, positions and Euler angles used in the model is summarised in table 2.2.

2.1 The body-fixed and global coordinate systems

During modelling it is important to choose proper coordinate systems in which to describe the systems behaviour. For modelling the ROV used in this thesis,

Table 2.1: *The notation and description of the parameters used in the ROV model.*

Notation	Description
$\boldsymbol{I}_b$	Inertia matrix for rotation around CO.
$\boldsymbol{I}_g$	Inertia matrix for rotation around CG.
K_p, M_q, N_r	Linear damping coefficients for rotation in water.
$K_{p\|p\|}, M_{q\|q\|}, N_{r\|r\|}$	Quadratic damping coefficients for rotation in water.
$K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}$	Increased inertia about x, y, z-axis due to rotation in water.
X_u, Y_v, Z_w	Linear damping coefficients for translation in water.
$X_{u\|u\|}, Y_{v\|v\|}, Z_{w\|w\|}$	Quadratic damping coefficients for translation in water.
$X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}}$	Added mass in x, y, z-direction due to translation in water.
$l_{x_i}, l_{y_i}, l_{z_i}$.	Moment arms from CG to each thruster i.
m	The ROV's mass
z_B	Distance between CB and CG along the z-axis.
V	Displaced volume.
ρ	Water density.
g	Gravity.
r_b^g	The distance between CO and CG.

Table 2.2: *The notation of SNAME [1950] for marine vessels.*

DOF	Description	Forces and moments	Linear and angular velocities	Positions and Euler angles
1	Motions in the x direction (surge).	X	u	x
2	Motions in the y direction (sway).	Y	v	y
3	Motions in the z direction (heave).	Z	w	z
4	Rotation about the x axis (roll, heel).	K	p	ϕ
5	Rotation about the y axis (pitch, trim).	M	q	θ
6	Rotation about the z axis (yaw).	N	r	ψ

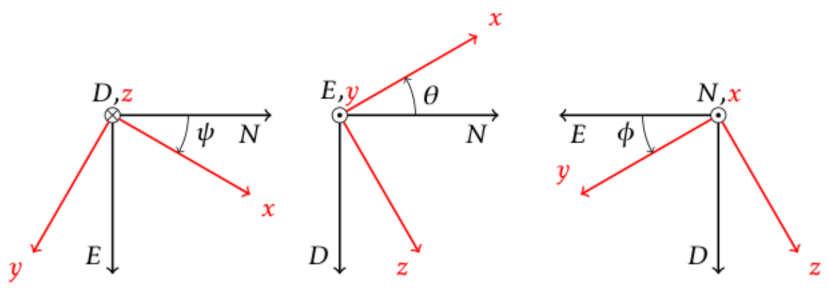

Figure 2.1: The local and global coordinate systems relate to each other by the rotations ψ, $\theta$and ϕ.

two separate coordinate systems were chosen. The first system, the body-fixed coordinate system, is like its name implies fixed to the ROV and rotates with with the ROV. The body-fix coordinate system is a right-hand system, the x-axis is placed along the length of the ROV, the y-axis points to the right, and the z-axis points downwards. The coordinate system is centred in the ROV's CG. The body-fixed coordinate system makes it easier to describe sensor readings, since the sensors rotate with the ROV. It also easier to express the effect of each thruster in forces and moments described in the body-fixed coordinate system.

The global coordinate system is earth fix, with axes N, E and D. N points in the direction of our decided North, E points in the direction of our decided East and D points down towards the CG of the Earth. This coordinate system is used to express buoyancy and gravitational forces on the ROV, their effects are transformed to the local coordinate system by a rotation matrix. How the local and global coordinate systems relate to each other can be seen in Figure 2.1.

TODO: Picture of the ro with the coordinat system.

2.2 Inertia

The inertia matrix, $\boldsymbol{M}$, describes the resistance of moving and rotating the ROV in its 6 DOF. It also describes the added mass that comes from rotating and translating a body in a liquid media. The inertia matrix is defined according to Fossen [2011] as

$$\boldsymbol{M} = \boldsymbol{M}_{RB} + \boldsymbol{M}_A \tag{2.2}$$

where $\boldsymbol{M}_{RB}$ is the inertia for a rigid-body and $\boldsymbol{M}_A$ is the added mass.

The inertia matrix, $\boldsymbol{M}_{RB}$, for a rigid-body is defined in Fossen [2011, p.52] as

$$\boldsymbol{M}_{RB} = \begin{pmatrix} m\boldsymbol{I}_{3x3} & -m\boldsymbol{S}(r_g^b) \\ m\boldsymbol{S}(r_g^b) & \boldsymbol{I}_b \end{pmatrix} \tag{2.3}$$

where r_g^b is the distance between the ROV's CO and CG. It has been assumed that the ROV's CO and CG coincide, thus simplifying (2.3) to

$$\boldsymbol{M}_{RB} = \begin{pmatrix} m\boldsymbol{I}_{3x3} & \boldsymbol{0}_{3x3} \\ \boldsymbol{0}_{3x3} & \boldsymbol{I}_g \end{pmatrix} \tag{2.4}$$

According to Fossen [2011, p.121] the added mass of the water acting upon the ROV is defined as

$$\boldsymbol{M}_A = -\begin{pmatrix} X_{\dot{u}} & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{\dot{v}} & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{\dot{w}} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{\dot{p}} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{\dot{q}} & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{\dot{r}} \end{pmatrix} \tag{2.5}$$

under the assumption that the ROV moves at low speeds relative to the water.

2.3 Coriolis forces

Since the ROV travels in a rotating reference frame, the Earth, the ROV is subjected to inertial forces called Coriolis forces. The Coriolis forces acting on the ROV are described in a similar manner to the inertia matrix, $\boldsymbol{M}$, with

$$\boldsymbol{C} = \boldsymbol{C}_{RB} + \boldsymbol{C}_A \tag{2.6}$$

according to Fossen [2011, p.110]. $\boldsymbol{C}_{RB}$ describes the Coriolis and centripetal forces caused by the rigid body's mass, while $\boldsymbol{C}_A$ describe the same effects but caused by the added mass.

The rigid-body Coriolis matrix is, according to Fossen [2011, p.55], given as

$$\boldsymbol{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} = \begin{pmatrix} m\boldsymbol{S}(\boldsymbol{\nu}_2) & -m\boldsymbol{S}(\boldsymbol{\nu}_2)\boldsymbol{S}(r_g^b) \\ m\boldsymbol{S}(r_g^b)\boldsymbol{S}(\boldsymbol{\nu}_2) & -\boldsymbol{S}(\boldsymbol{I}_b\boldsymbol{\nu}_2) \end{pmatrix}\boldsymbol{\nu} = \begin{pmatrix} m(qw - rv) \\ m(ru - pw) \\ m(pv - qu) \\ qr(I_y - I_z) \\ rp(I_z - I_x) \\ qp(I_x - I_y) \end{pmatrix} \tag{2.7}$$

where

$$\boldsymbol{\nu} = [u, v, w, p, q, r]^T$$

The assumption that the ROV is symmetric about xyz-plane has been made in order to eliminate cross-terms in $\boldsymbol{C}$.

The Coriolis and centripetal effects from the added mass are described as

$$
\begin{aligned}
\boldsymbol{C}_A\boldsymbol{\nu} &= \begin{pmatrix}
0 & 0 & 0 & 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v \\
0 & 0 & 0 & Z_{\dot{w}}w & 0 & -X_{\dot{u}}u \\
0 & 0 & 0 & -Y_{\dot{v}}v & X_{\dot{u}}u & 0 \\
0 & -Z_{\dot{w}}w & Y_{\dot{v}}v & 0 & -N_{\dot{r}}r & M_{\dot{q}}q \\
Z_{\dot{w}}w & 0 & -X_{\dot{u}}u & N_{\dot{r}}r & 0 & -K_{\dot{p}}p \\
-Y_{\dot{v}}v & X_{\dot{u}}u & 0 & -M_{\dot{q}}q & K_{\dot{p}}p & 0
\end{pmatrix}\boldsymbol{\nu} = \\
&= \begin{pmatrix}
Y_{\dot{v}}vr - Z_{\dot{w}}wq \\
Z_{\dot{w}}wp - X_{\dot{u}}ur \\
X_{\dot{u}}uq - Y_{\dot{v}}vp \\
(Y_{\dot{v}} - Z_{\dot{w}})vw + (M_{\dot{q}} - N_{\dot{r}})qr \\
(Z_{\dot{w}} - X_{\dot{u}})uw + (N_{\dot{r}} - K_{\dot{p}})pr \\
(X_{\dot{u}} - Y_{\dot{v}})uv + (K_{\dot{p}} - M_{\dot{q}})pq
\end{pmatrix}
\end{aligned} \tag{2.8}
$$

according to Fossen [2011, p.121]. Under the assumption that the ROV is moving slowly and has three planes of symmetry.

2.4 Viscous damping

According to Fossen [2011, p.122] there are four main sources of hydrodynamic damping acting upon a submersed vehicle. Potential damping, skin friction, wave drift damping and damping from vortex shedding. The effects of these four sources on a 6 DOF vehicle can, according to Fossen [2011] be described as two 6-by-6 matrices. The first matrix, $\boldsymbol{D}$, contains the linear damping terms, while the second matrix, $\boldsymbol{D}_n(\boldsymbol{\nu})$, contains the quadratic, or non-linear, damping terms. Together these two matrices form the Viscous damping matrix $\boldsymbol{D}(\boldsymbol{\nu})$ which in turn can be simplified to

$$
\begin{aligned}
&\boldsymbol{D}(\boldsymbol{\nu}) = \boldsymbol{D} + \boldsymbol{D}_n(\boldsymbol{\nu}) = \\
&-\begin{pmatrix}
X_u + X_{u|u|}|u| & 0 & 0 & 0 & 0 & 0 \\
0 & Y_v + Y_{v|v|}|v| & 0 & 0 & 0 & 0 \\
0 & 0 & Z_w + Z_{w|w|}|w| & 0 & 0 & 0 \\
0 & 0 & 0 & K_p + K_{p|p|}|p| & 0 & 0 \\
0 & 0 & 0 & 0 & M_q + M_{q|q|}|q| & 0 \\
0 & 0 & 0 & 0 & 0 & N_r + N_{r|r|}|r|
\end{pmatrix}
\end{aligned} \tag{2.9}
$$

When $\boldsymbol{D}(\boldsymbol{\nu})$ is multiplied with $\boldsymbol{\nu}$, the following row-vector is obtained

$$
\boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} = -\begin{pmatrix}
(X_u + X_{u|u|}|u|)u \\
(Y_v + Y_{v|v|}|v|)v \\
(Z_w + Z_{w|w|}|w|)w \\
(K_p + K_{p|p|}|p|)p \\
(M_q + M_{q|q|}|q|)q \\
(N_r + N_{r|r|}|r|)r
\end{pmatrix} \tag{2.10}
$$

where

$$\boldsymbol{\nu} = [u, v, w, p, q, r]^T$$

According to Fossen [2011, p.129-130] if the ROV is symmetric about the xz-plane and the damping is assumed to be decoupled $\boldsymbol{D}(\boldsymbol{\nu})$ is a diagonal matrix.

2.5 Restoring forces

A submerged vehicle will according to Fossen [2011] be acted upon by forces and moments caused by the earths gravitational pull and the buoyancy force. Fossen [2011] further states that these forces and moments are, in hydrostatic terms, called restoring forces and that they act as spring forces on the ROV. The restoring forces and moments are calculated using four main parameters; the weight of the vehicle, W, its buoyancy B and lastly the coordinates for the CG and the ROV's center of buoyancy (CB). The weight of the ROV is

$$W = mg \tag{2.11}$$

and its buoyancy is

$$B = \rho g V \tag{2.12}$$

where m is the ROV's mass, g the gravitational constant, ρ the density of water, and V the volume of displaced water. In other words the magnitude of the buoyancy forces is equal to the weight of the displaced water. For a fully submerged vehicle V will naturally be equal the the volume of the vehicle.

The restoring forces matrix, $\boldsymbol{g}(\boldsymbol{\eta})$, according to Fossen [2011, p.60] defined as

$$g(\boldsymbol{\eta}) = \begin{pmatrix} (W - B)\sin\theta \\ -(W - B)\cos\theta\sin\phi \\ -(W - B)\cos\theta\sin\phi \\ -z_B B\cos\theta\sin\phi \\ -z_B B\sin\theta \\ 0 \end{pmatrix} \tag{2.13}$$

and describes how the forces and moments caused by the buoyancy force and gravitational pull of the Earth act on a 6 DOF model. In these calculations it is assumed that the CG coincides with the CO and that the CB lies at coordinate $[0, 0, z_B]$, directly above or below the CO. Take note that the positions of the three centers are described using the coordinate system described in Section 2.1, a roll and pitch stable ROV should thus have a $z_B < 0$.

TODO: Picture of the moments and forces including the name of the thrusters.

2.6 Thrust Matrix

According to Fossen [2011, p.401] the ROV's actuators can be modelled as

$$\boldsymbol{\tau} = \boldsymbol{T}\boldsymbol{f}(\boldsymbol{u}) \tag{2.14}$$

where $\boldsymbol{T}$ is the geometry of the actuators, see Figure **??**.

$$
\begin{aligned}
\boldsymbol{\tau} = \boldsymbol{T}f(\boldsymbol{u}) &= \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 & -1 & 0 \\ l_{y_1} & -l_{y_2} & 0 & 0 & 0 & l_{z_6} \\ l_{x_1} & l_{x_2} & 0 & 0 & -l_{x_5} & 0 \\ 0 & 0 & l_{y_3} & -l_{y_4} & 0 & 0 \end{pmatrix} \begin{pmatrix} gf(u_1) \\ gf(u_2) \\ gf(u_3) \\ gf(u_4) \\ gf(u_5) \\ gf(u_6) \end{pmatrix} = \\
&= \begin{pmatrix} f(u_3)g + f(u_5)g \\ -f(u_6)g \\ -f(u_1)g - f(u_2)g - f(u_4)g \\ f(u_2)gl_{y_2} - f(u_1)gl_{y_1} + f(u_6)gl_{z_6} \\ f(u_2)gl_{x_2} - f(u_1)gl_{x_1} - f(u_4)gl_{x_4} \\ f(u_3)gl_{y_3} - f(u_5)gl_{y_5} \end{pmatrix}
\end{aligned}
\tag{2.15}
$$

where $l_{x_1}, l_{x_2}, l_{y_1}, l_{y_2}, l_{y_3}, l_{y_4}$ and l_{z_6} are the offsets in the x, y or z direction of the n:th thruster. $f(u_i)$ is a lookup table from control signal to thrust, see appendix B for details.

TODO: Is the thruster func tion correct. Shoul g be included c not?

TODO: Write appendix B.

2.7 Equations in Component Form

If (2.1) is solved for $\dot{\boldsymbol{\nu}}$ the following equations can be derived

$$
\begin{aligned}
\dot{u} = \frac{f(u_3) + f(u_4)}{m - X_{\dot{u}}} + \frac{u(X_u + X_{u|u|}|u|)}{m - X_{\dot{u}}} + \frac{\sin(\theta)(B - W)}{m - X_{\dot{u}}} + \\
\frac{m(rv - qw)}{m - X_{\dot{u}}} + \frac{-Y_{\dot{v}} rv}{m - X_{\dot{u}}} + \frac{Z_{\dot{w}} qw}{m - X_{\dot{u}}}
\end{aligned}
\tag{2.16}
$$

$$
\begin{aligned}
\dot{v} = \frac{-f(u_6)}{m - Y_{\dot{v}}} + \frac{v(Y_v + Y_{v|v|}|v|)}{m - Y_{\dot{v}}} + \frac{-\cos\theta\sin\phi(B - W)}{m - Y_{\dot{v}}} + \\
\frac{m(pw - ru)}{m - Y_{\dot{v}}} + \frac{X_{\dot{u}} ru}{m - Y_{\dot{v}}} + \frac{-Z_{\dot{w}} pw}{m - Y_{\dot{v}}}
\end{aligned}
\tag{2.17}
$$

$$
\begin{aligned}
\dot{w} = \frac{-f(u_1) - f(u_2) - f(u_5)}{m - Z_{\dot{w}}} + \frac{w(Z_w + Z_{w|w|}|w|)}{m - Z_{\dot{w}}} + \frac{-\cos\phi\cos\theta(B - W)}{m - Z_{\dot{w}}} + \\
\frac{m(qu - pv)}{m - Z_{\dot{w}}} + \frac{-X_{\dot{u}} qu}{m - Z_{\dot{w}}} + \frac{Y_{\dot{v}} pv}{m - Z_{\dot{w}}}
\end{aligned}
\tag{2.18}
$$

$$\dot{p} = \frac{f(u_1)l_{y_1} - f(u_2)l_{y_2} + f(u_6)l_{z_6}}{I_x - K_{\dot{p}}} + \frac{p(Kp + K_{p|p|}|p|)}{I_x - K_{\dot{p}}} + \frac{-M_{\dot{q}}qr}{I_x - K_{\dot{p}}} + \frac{N_{\dot{r}}qr}{I_x - K_{\dot{p}}} + \frac{qr(I_y - I_z)}{I_x - K_{\dot{p}}} + \frac{-Y_{\dot{v}}vw}{I_x - K_{\dot{p}}} + \frac{Z_{\dot{w}}vw}{I_x - K_{\dot{p}}} + \frac{B\cos\theta\sin\phi z_B}{I_x - K_{\dot{p}}} \tag{2.19}$$

$$\dot{q} = \frac{f(u_1)l_{x_1} + f(u_2)l_{x_2} - f(u_5)l_{x_5}}{I_y - M_{\dot{q}}} + \frac{q(M_q + M_{q|q|}|q|)}{I_y - M_{\dot{q}}} + \frac{K_{\dot{p}}pr}{I_y - M_{\dot{q}}} + \frac{-N_{\dot{r}}pr}{I_y - M_{\dot{q}}} + \frac{pr(I_z - I_x)}{I_y - M_{\dot{q}}} + \frac{-Z_{\dot{w}}uw}{I_y - M_{\dot{q}}} + \frac{X_{\dot{u}}uw}{I_y - M_{\dot{q}}} + \frac{B\sin\theta z_B}{I_y - M_{\dot{q}}} \tag{2.20}$$

$$\dot{r} = \frac{f(u_3)l_{y_3} - f(u_4)l_{y_4}}{I_z - N_{\dot{r}}} + \frac{r(N_r + N_{r|r|}|r|)}{I_z - N_{\dot{r}}} + \frac{-K_{\dot{p}}pq}{I_z - N_{\dot{r}}} + \frac{M_{\dot{q}}pq}{I_z - N_{\dot{r}}} + \frac{pq(I_x - I_y)}{I_z - N_{\dot{r}}} + \frac{-X_{\dot{u}}uv}{I_z - N_{\dot{r}}} + \frac{Y_{\dot{v}}uv}{I_z - N_{\dot{r}}} \tag{2.21}$$