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k-color ASS

$\sum_{i=1}^{k} x_{vi} \geq 1 \quad \forall v \in V$

$x_{ui} + x_{vi} \leq 1 \quad \forall \{u,v\} \in E \quad i = 1..k$

k-color PoP

k SAT

$\bigvee_{i=1} x_{vi} \quad \forall v \in V$

$\neg x_{ui} \vee \neg x_{vi} \quad \forall u,v \in E \quad i = 1 \dots k$

SAT

$y_{k,v} = 0$ $\quad \neg y_{k,v} \quad v \in V$

$y_{i-1,v} - y_{i,v} \geq 0$ $\quad \neg y_{i-1,v} \vee \neg y_{i,v} \quad v \in V \; i = 2..k$

$y_{i,u} + y_{i,v} \geq 1$ $\quad y_{i,u} \vee y_{i,v} \quad \{u,v\} \in E$

$(y_{i-1,u} - y_{i,u}) + (y_{i-1,v} - y_{i,v}) \leq 1$ $\quad \neg y_{i-1,u} \vee y_{i,u} \vee \neg y_{i-1,v} \vee y_{i,v} \quad \{u,v\} \in E \quad i = 2..k$

$y_{i,q} - y_{i,v} \geq 0$ $\quad y_{i,q} \vee \neg y_{i,v} \quad v \in V \quad i = 1..k$

At most k constraints (binary encoding):

$(\neg X_i \vee T_{1,i} \vee \dots \vee T_{k,i}) \quad i = 1..n$

$\neg T_{y,i} \vee \phi(i, y, j) \quad y = 1..k \quad j = 1..\lceil \log_2 n \rceil$

At least k constraints $\Rightarrow$
At most n-k negative

$(X_i \vee T_{1,i} \vee \dots \vee T_{n-k,i}) \quad i = 1..n$

$\neg T_{y,i} \vee \phi(i, y, j) \quad y = 1..n-k \quad j = 1..\lceil \log_2 n \rceil$

$\Rightarrow$ Equitability-Constraints (For exact k-coloring)

LP:

$\sum_{v \in V} x_{vj} \geq \lfloor \frac{n}{k} \rfloor \quad j = 1..k \longrightarrow$ at most $\lfloor \frac{n}{k} \rfloor$ constraint

$\sum_{v \in V} x_{vj} \leq \lceil \frac{n}{k} \rceil \longrightarrow$ at least $\lceil \frac{n}{k} \rceil$ constraint

Problem: No binary search possible

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Pre coloring variables:

$x_i = 1 \Rightarrow$ remove all literals $\neg x_i$ From clauses
remove all clauses containing x_i

For $x_i = 0$ analogue

Todo: Model $\sum x_{vj} \geq \sum (w_j - w_{j+1}) \cdot k_j$ as SAT,
Alternative at-most-k-encodings, strengthened Formulations

LP: $x_{vj} \leq w_j \quad \forall v \in V, j = 1 \dots k$

$w_{j+1} \leq w_j$

SAT:

$\neg x_{vj} \vee w_j$

$\neg w_{j+1} \vee w_j$

k-color POP

$y_{k,v} = 0$

$y_{i-1,v} - y_{i,v} \geq 0$

$y_{i,u} + y_{i,v} \geq 1$

$x_{i,v} = y_{i-1,v} - y_{i,v}$

$x_{1,v} = 1 - y_{1,v}$

$x_{ui} + x_{vi} \leq 1 \quad \forall uv \in E, i = 1 \dots k$

$y_{i,u} - y_{i,v} \geq 0$

SAT

$\neg y_{k,v} \quad v \in V$

$y_{i-1,v} \vee \neg y_{i,v} \quad v \in V, i = 2 \dots k$

$y_{i,u} \vee y_{i,v} \quad uv \in E$

$\neg x_{i,v} \vee y_{i-1,v} \wedge \neg x_{i,v} \vee \neg y_{i,v} \wedge x_{i,v} \vee \neg y_{i-1,v} \vee y_{i,v}$

$x_{1,v} \vee y_{1,v} \wedge \neg x_{1,v} \vee \neg y_{1,v}$

$\neg x_{ui} \vee \neg x_{vi} \quad \forall uv \in E, i = 1 \dots k$

$y_{i,u} \vee \neg y_{i,v} \quad v \in V, i = 2 \dots k$

$y_{i,q} - y_{i,v} \geq 0$ $y_{i,q} \vee \neg y_{i,v}$ $v \in V$