UNIT 10: Integration (part 1)

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SPECIFICATION REFERENCES

8.2 Integrate x^n, (including $\frac{1}{x}$) and integrate e^{kx}, $\sin kx$, $\cos kx$ and related sums, differences and constant multiples

To include integration of standard functions such as $\sin 3x$, $\sec^2 2x$, $\tan x$, e^{5x}, $\frac{1}{2x}$.
Students are expected to be able to use trigonometric identities to integrate, for example, $\sin^2 x$, $\tan^2 x$, $\cos^2 3x$.

8.5 Students should recognise integrals of the form $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$.

PRIOR KNOWLEDGE

Covered so far

- Knowledge of e^x and $\ln x$
- Laws of logarithms
- Trigonometry
- Differentiation

AS Mathematics – Pure Mathematics content

6.1, 6.3 Knowledge of e^x and $\ln x$ (See Unit 8 of SoW)
6.4 Laws of logarithms (See Unit 8 of SoW)
5.1 Trigonometry (See Unit 4 of SoW)
7, 8 Differentiation and integration (See Units 6 &7 of SoW)

KEYWORDS

Integral, inverse, differential, coefficient, index, power, negative, reciprocal, natural logarithm, $\ln|x|$, coefficient, exponential, identity, sin, cos, tan, sec, cosec, cot, e^x.

NOTES

This first part of Integration is about using the reverse process of differentiation and applying previously leant skills. The next part will use further techniques for the integration of combined functions as well as looking at applications of integration.

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10a. Integrating x^n (including when $n = -1$), exponentials and trigonometric functions (8.2)

Teaching time
4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to integrate expressions by inspection using the reverse of differentiation;
- be able to integrate x^n for all values of n and understand that the integral of $\frac{1}{x}$ is $\ln|x|$;
- be able to integrate expressions by inspection using the reverse of the chain rule (or function of a function);
- be able to integrate trigonometric expressions;
- be able to integrate expressions involving e^x.

TEACHING POINTS

Recap all the methods of differentiation covered earlier in the course. This can also be used as a starting point for introducing the different rules for integration.

Consider the integral of $x^{-1} = \frac{1}{x}$. Using the rule from AS Mathematics – Pure Mathematics gives. $\frac{1}{0}$. However, if we recall that the differential of $\ln|x|$ is $\frac{1}{x}$, then the reverse operation tells us that the integral of $\frac{1}{x}$ is $\ln|x| + c$. Similarly, the differential of e^x is e^x, so the integral will also give the same result.

Finally, the differential of trig expressions should be recapped as this also leads to some standard results for trigonometric integrals.

Take care to show how the integral of $\sin x$ is $-\cos x + c$ (as the differential of $\cos x$ leads to $-\sin x$).

The integral of $\sec^2 x$ looks difficult but is only the reverse of the differential of $\tan x$.

Students must end all indefinite integrations with $+ c$ and use correct notation when integrating and must include dx.

Encourage students to develop their own technique for integrating problems which require the reverse chain rule. If good examples are used, most students will be able to work out their own method and soon be able to write down the answers directly for integrals like $3e^{2x}$ and $4\sin(3x)$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

It is always a good idea to advise students to differentiate their answer to see if it goes back to the original expression (pre-integration). This is a good way to check for sign errors, particularly with the trigonometric questions.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

In exam situations, many students incorrectly integrate functions involving e^x by dividing by the x.

Algebraic errors are also fairly common; clear working and good notation can help here.

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NOTES

Make sure students are fluent in these basic integrals, as this will increase the likelihood of success with the remainder of this unit.

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10b. Using the reverse of differentiation and using trigonometric identities to manipulate integrals (8.2)	Teaching time 5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- recognise integrals of the form $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$;
- be able to use trigonometric identities to manipulate and simplify expressions to a form which can be integrated directly.

TEACHING POINTS

Consider the rule for differentiating $\ln|f(x)|$. This was $\frac{f'(x)}{f(x)}$. A special case of this is the integral of $\frac{1}{x}$, which is $\ln|x|$ $(+ c)$.

So, if we have to integrate an expression in which the top of the fraction is the exact differential of the denominator (or a multiple of it), then the answer is the natural log of the denominator $(+ c)$.

Make sure students can adjust questions like the integral of $\frac{4x^2}{x^3}$.

Consider examples like the integral of $\tan x$ by rewriting it as $\frac{\sin x}{\cos x}$, leading to a natural log answer (be careful with the minus!)

One of the most common integrals is $\cos^2 x$. The standard method for integrating this is to rearrange the appropriate double angle formula to create an integral involving not x^2 but $2x$ which is much easier to directly integrate (as shown in the previous section).

Students will need lots of practice in selecting the correct version of $\cos 2x$, which involves only $\cos^2 x$ terms and then rearranging it.

The specification states: 'students are expected to be able to use trigonometric identities to integrate, for example, $\sin^2 x$, $\tan^2 x$, $\cos^2 3x$'.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students must have lots of practice at working with logarithms and exponentials when integrating, and should leave their answers in exact form. They also need to be fluent in knowing the key trig identities and how to manipulate them from the ones in the formula book.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

The most common errors seen include: mistakes when arranging and substituting identities into integrals; and incorrectly applying laws of logarithms.

NOTES

Log integrals are vital when working with the partial fractions and many of the differential equations in the next unit.

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