UNIT 6: Applications of kinematics Projectiles (7.5)	Teaching Time 6 Hours

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SPECIFICATION REFERENCES

7.5 Model motion under gravity in a vertical plane using vectors; projectiles.

PRIOR KNOWLEDGE

GCSE (9-1) in Mathematics at Higher Tier

G20 Trigonometry

AS Mathematics – Mechanics content

7.3 *suvat* formulae (See Unit 7b of the SoW)

8.3 Vertical motion under gravity (See Unit 7b of the SoW)

8.2 **i**, **j** (2D) vectors (See Unit 8a of the SoW)

AS Mathematics – Pure Mathematics content

10.1 **i**, **j** (2D) vectors (See Unit 5 of the SoW)

10.2 Magnitude and direction of a vector

5 Trigonometry (See Unit 4 of the SoW)

A level Mathematics – Pure Mathematics content

5.5 $\sec^2 x = 1 + \tan^2 x$ identity and solving trigonometric equations (See Unit 6 of the SoW)

KEYWORDS

Projectile, range, vertical, horizontal, component, acceleration, gravity, initial velocity, vector, angle of projection, position, trajectory, parabola.

NOTES

The guidance on the specification document states: 'Derivation of formulae for time of flight, range and greatest height and the derivation of the equation of the path of a projectile may be required.'

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OBJECTIVES

By the end of the sub-unit, students should:

- be able to find the time of flight of a projectile;
- be able to find the range and maximum height of a projectile;
- be able to derive formulae to find the greatest height, the time of flight and the horizontal range (for a full trajectory);
- know how to modify projectile equations to take account of the height of release;
- be able to derive and use the equation of the path.

TEACHING POINTS

Define a projectile as an object dropped or thrown in the air. Show a video from the net of a golf chip or shot-putter. Explain that the path is called a parabola which is the old Greek word for throw.

Discuss the modelling assumptions: the object is treated as a particle so it does not spin and has no air resistance. Therefore the only force on the object is gravity. (Link this back to vertical motion under gravity.)

Discuss the fact that displacement, velocity and acceleration are vectors with components in the horizontal and vertical directions. These components obey the *suvat* formulae, and the horizontal and vertical directions can be treated separately.

Begin with *horizontal* projection examples and encourage student to make two lists for the motion in the horizontal and vertical directions. It is easier to start this way as the initial *vertical* velocity is zero for this type of question, hence $u = 0$ for the vertical equation of motion.

For **all** Projectile questions:

$a = 0$ (in the horizontal direction), so the horizontal velocity is constant.

$a = 9.8\ \text{m s}^{-2}$ or $-9.8\ \text{m s}^{-2}$ (in the vertical direction) depending on whether downwards is taken as positive or upwards is taken as positive.

The two equations of motion often will have time, t, as a common term.

Move on to projection with speed $U\ \text{m s}^{-1}$ at *any* angle α (above the horizontal ground) and introduce the concept of the initial velocity having horizontal and vertical components. (It may be advisable to revise magnitude and direction, Pythagoras and basic trigonometry.)

Horizontally, $u = U\cos\alpha$ and if upwards is positive, vertically, $u = U\sin\alpha$.

Derive the formulae for the time of flight, greatest height (when the vertical velocity is zero) and horizontal range (for the maximum range you will need to use the identity $\sin 2\alpha = 2\sin\alpha\cos\alpha$ which is covered in A level Mathematics - Pure Mathematics content, see SoW Unit 6d)

Emphasise the fact that s is displacement. So, for example, for the vertical equation of motion, we use $s = 0$ if the projectile returns to the ground, and if it is projected from a height and lands lower than its starting point, then, if upwards is positive s will be negative in the vertical direction.

Show examples with the initial velocity as an **i - j** vector (the **i** coefficient is u for the horizontal equation of motion). This actually makes it easier as the components are done for you.

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OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

The general equation of the path, if we know the projection speed and the projection angle α, is a useful equation. It reduces to a quadratic (which you can show is a parabola with a negative x^2 term), but requires the identity $1 + \tan^2 \alpha = \sec^2 \alpha$ (A level Mathematics – Pure Mathematics content – see SoW Unit 6c). It can be used to find the possible angle(s) of projection to reach any point on the trajectory. There is also a symmetry of path in the absence of air resistance.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students often find projectile questions challenging, sometimes confusing the horizontal and vertical aspects of the motion, for example by including the horizontal component of velocity in an equation for the vertical motion.

Other common mistakes include considering only one component of velocity when finding speeds and making sign errors when producing quadratic equations (to find t).

NOTES

General formulae can be derived to obtain the maximum height, time of flight and range in terms of initial velocity U, acceleration g and angle α, but these only work for a full trajectory. This means learning them to use in exams should be done with caution; it is probably better for students to work from first principles, rather than learn and substitute.

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