UNIT 4: Momentum and impulse (part 2)

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SPECIFICATION REFERENCES

1.2 Momentum as a vector. The impulse-momentum principle in vector form.

PRIOR KNOWLEDGE

AS Mathematics – Pure content

10.1 – 10.5 Work with **i**, **j** vectors; magnitude and direction (See SoW Year 1 Unit 5)

AS Mathematics – Mechanics content

Intuitive idea of momentum and S.I. units

7.3 Kinematics – constant acceleration formulae (See SoW Year 1 Unit 8b)

A level Mathematics – Mechanics content

7.3 Further kinematics – constant acceleration formulae in 2D (See SoW Year 2 Unit 5)

KEYWORDS

Mass, velocity, speed, Ns, momentum, impulse, force, time, collisions, direct, smooth, body, sphere, coalesce, conservation, vector, **i**, **j**, **unit vector**, magnitude, string, light, inextensible, jerk, impulsive tension.

NOTES

The specification states that the spheres may be modelled as particles. This topic was in legacy Mechanics A level in units M1 (1 dimension) and M2 (2dimensions).

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4a. Momentum as a vector (i, j problems); Impulse-momentum principle in vector form (1.2)

Teaching time
7 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to extend the definition of linear momentum and impulse to 2-D using vectors.
- be able to use the impulse-momentum principle in vector form i.e. $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$.

TEACHING POINTS

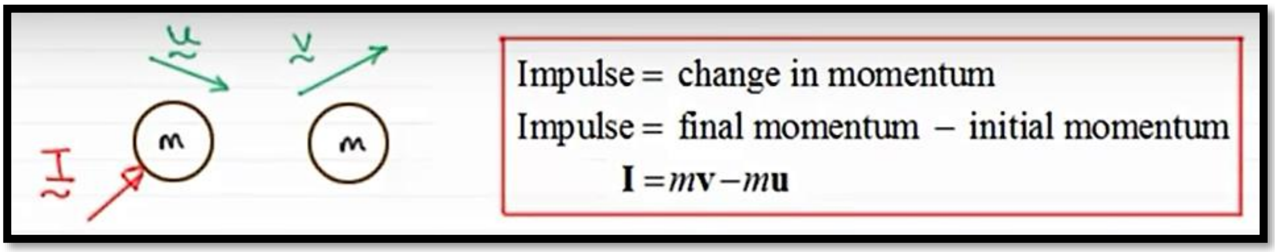
Oblique impacts are considered later in this course, but here the questions in 2-D are limited to the impulse – momentum principle.

Linear momentum is a vector quantity (since ‘ momentum = mass × velocity’ where mass is a scalar and velocity is a vector). Impulse is defined as ‘change in momentum’ so that is also a vector quantity. We apply the formula for impulse in vector form; we consider an impulse applied to a particle moving in a straight line where the impulse acts at an angle to that straight line (see diagram below).

Time is a scalar quantity, but force and velocity are vectors, therefore the formula can be written:

$\mathbf{F}t = m\mathbf{v} - m\mathbf{u}$ or

$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$



It is good if the students can visualise the problems and draw realistic diagrams, rather than just apply the formula.

The following numerical example illustrates the use of the formula; students should be encouraged to show the relevant directions in a diagram.

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If an impulse of $6\mathbf{i}+3\mathbf{j}$ Ns is applied to a particle of mass 3kg. Given that the initial velocity is $4\mathbf{i}-2\mathbf{j}\ ms^{-1}$. Find the final speed and the angle made with the vector $\mathbf{i}$.

$$6\mathbf{i}+3\mathbf{j} = 3\mathbf{v} - 3(4\mathbf{i}-2\mathbf{j})$$
$$\therefore 6\mathbf{i}+3\mathbf{j} = 3\mathbf{v} - 12\mathbf{i} + 6\mathbf{j}$$
$$\therefore 18\mathbf{i} - 3\mathbf{j} = 3\mathbf{v}$$
$$\therefore \mathbf{v} = 6\mathbf{i} - \mathbf{j}$$

$$\therefore \text{speed} = \sqrt{6^2+1^2} = \sqrt{37}\ \text{ms}^{-1}$$
$$\tan\theta = \frac{1}{6} \Rightarrow \theta = \tan^{-1}\frac{1}{6}$$
$$\therefore \theta = 9.462...^\circ = 9.5^\circ\ (1\text{dp})$$

Students could work in column or **i**, **j** vectors.

Working separately for the coefficients of the components **i** and **j** may lead to a simpler solution in some problems.

At other times, is useful to use $m(\mathbf{v} - \mathbf{u}) = \mathbf{I}$ as the velocities can be simplified in terms of **i** and **j**.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

This example involves use of the above formula.

A particle of mass 0.2 kg is moving with velocity $(3a\mathbf{i} - a\mathbf{j})\ \text{ms}^{-1}$, where a is a positive constant, when it receives an impulse of magnitude $2\sqrt{5}$ N s. Immediately after receiving the impulse the particle has velocity $(a\mathbf{i} - 2a\mathbf{j})\ \text{ms}^{-1}$.

(a) Find the value of a.

You should cover questions where the velocity of one of the particles is given in speed and direction form, rather than directly in components.

For example, suppose the velocity **u** is in the direction 3**i** + 4**j** and the speed is 10 m s^{-1}.

To find **u** the concept of a unit vector needs to be covered. The magnitude of the direction part gives a magnitude (speed) of 5.

The unit vector $\hat{\mathbf{u}}$ has a magnitude of 1, but preserves the angle properties i.e. $\hat{\mathbf{u}} = \frac{1}{5}(3\mathbf{i} + 4\mathbf{j})$. Multiplying by the speed required gives $\mathbf{u} = 10 \times \frac{1}{5}(3\mathbf{i} + 4\mathbf{j}) = (6\mathbf{i} + 8\mathbf{j})$ m s^{-1}.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

In these sorts of exam questions, the majority of errors are due to slips in the arithmetic though vectors can cause confusion for some students who combine **i** and **j** terms inappropriately.

Students must always read the question carefully so they know how to give their final answers. For example, they may unnecessarily find the magnitude of **v**, either because they do not read the question properly or

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they do not know the difference between speed and velocity. Similarly, they sometimes only state the velocity when the speed is actually required thereby losing marks unnecessarily.

NOTES

Conservation of momentum is further extended in Unit 6 (elastic collisions in two dimensions) to include oblique impacts.

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