UNIT 2: Work, energy and power

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SPECIFICATION REFERENCES

2.1 Kinetic and potential energy, work and power. The work-energy principle. The principle of conservation of mechanical energy.

PRIOR KNOWLEDGE

GCSE Physics

- Intuitive idea of work and energy and S.I. units

AS Mathematics – Mechanics content

7.3	Kinematics – motion of a particle in a straight line with constant acceleration (See SoW Unit 8b)
8.2, 8.4	Types of forces and Newton's laws (See SoW Unit 9)

A level Mathematics – Mechanics content

8.4	Resolving forces (See SoW Unit 4)
8.6	Frictional forces (See SoW Unit 7)
8.2, 8.4, 8.5, 8.6	Dynamics (See SoW Unit 8b)

KEYWORDS

Work, energy, power, joules, gravitational potential energy (GPE), kinetic energy (KE), energy change, resistance, force, distance, displacement, speed, velocity, conservation of mechanical energy, external force, work-energy principle, reaction, power, watts, KW, tractive (driving) force , acceleration, inclined plane, resistance, rate of working, rough/smooth surface, friction.

NOTES

Kinetic energy will also be required for Unit 3: Elastic collisions in one dimension.

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2a. Work and kinetic energy; derivation of units and formulae (2.1)

Teaching time
5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- **understand the derivation, units and definitions of work and energy;**
- **be able to define kinetic energy (KE);**
- **understand that work done on a body moving in a horizontal plane is the change in kinetic energy.**

TEACHING POINTS

[The introduction to this topic is very similar to that of Unit 1: Momentum and impulse.]

Begin with two equations familiar from AS Mathematics:

$v^2 = u^2 + 2as$ and $F = ma$

Rearrange $F = ma$ to give $a = \frac{F}{m}$, substitute into the kinematics equation and rearrange to give:

$Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Note that we are assuming that F is constant.

What is the quantity mass × speed^2?

Its unit could be based on the Fs part of the formula above which gives newtons × metres (N m), but this is easily confused with moments, which also have the units N m

One of the pioneers in formalising the concepts of work and energy was James Joule (1818-1889).

He defined work done as force × distance and the unit was named the joule (J)

So, $\frac{1}{2}$ × mass × speed^2 is defined as the kinetic energy (KE) of a body (i.e. energy of movement). It is a measure of a body's capacity to do work by moving and is measured in joules (J).

In practice one joule is a tiny amount of energy as it is defined as the work done in moving 1 N a distance of 1 m. Most functional applications use kJ (or 1000 J)

From the definition of work done and the equation above we have the work–energy principle for a particle moving in a horizontal plane under a constant force:

work done (force × distance) = change in kinetic energy

(Note that KE is always positive as the speed is squared.)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Discuss situations where there are changes in kinetic energy and what types of forces cause these changes; for example, friction causing a sliding object to slow down or the driving force of an engine making a car go faster. Problems involving these scenarios will be considered in the next two sub-units.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Sign errors and confusing the different equations involving forces and energy are the main mistakes students make in these questions.

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NOTES

In the next part we will consider gravitational potential energy and include this in the work-energy principle.

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2b. Potential energy, work-energy principle, conservation of mechanical energy, problem solving (2.1)	Teaching time 8 hours

OBJECTIVES

By the end of the sub-unit, students should:

- **understand the concept of gravitational potential energy (GPE);**
- **be able to include GPE when applying the work-energy principle;**
- **know the conditions for conservation of mechanical energy;**
- **be able to solve problems involving work and energy.**

TEACHING POINTS

Consider a package of mass m kg at rest on a shelf which is h metres above the ground. The package has the potential to fall off the shelf, so it is possible to think of the energy being stored. If the package did fall (vertically), then this energy would be released and its speed would begin to increase (and so its kinetic energy would increase).

Can we measure this gravitation potential energy as the package rests on the shelf? It is the work done against gravity, which is force × distance. The force on the package is its weight, mg newtons, and the distance is the height it could potentially fall, h metres. Therefore, GPE = mgh joules.

(Note that GPE can be treated as a negative term – see Problem Solving below)

We can now extend the idea of the work-energy principle from the previous section to a particle not restricted to moving in a horizontal plane:

work done = total change in sum of KE and GPE

We generally relate a change in height of a particle to a change in its GPE rather than as work done by gravity i.e. it is included on just the right hand side of the equation.

What conditions would mean that there is no total energy change during a system's motion? This happens in the absence of any *external* forces (excluding gravity), such as a driving force, air resistance or friction.

In these situations we have conservation of (mechanical) energy:

KE + GPE (before) = KE + GPE (after)

We say mechanical energy as we are not considering, for example, heat energy or sound energy.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

When solving problems involving work and energy, it is important to understand whether mechanical energy is conserved or not. Are there any external forces like friction? It is always a good idea to read the whole question first to be able to picture fully the situation.

When a body is moving up or down an inclined plane, the GPE will be calculated using the change in *vertical* height (despite the path not being vertical). Remember that GPE can also be considered as work done by gravity; since the force of gravity acts vertically, the vertical height is used in the force × distance (mgh) formula. A particle could be moving up or down a curved path; the principle of using the change in vertical height to calculate the change in GPE still applies.

However, the work done (by friction) will be based on the *actual* distance along the plane using the force × distance formula.

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Try to encourage students to avoid applying "gain in KE = loss in GPE" unless it is absolutely clear that there are no external forces (excluding gravity) acting. In most problems, a better approach is to look at the particle at the start and at the end of the motion being considered. (Put in a dotted line to indicate the GPE level at the start of the motion – if the particle is higher at the start, still put GPE = 0 J, which will give a negative GPE at the lower point indicating loss of GPE.)

Now, calculate GPE + KE (start) and GPE + KE (end).

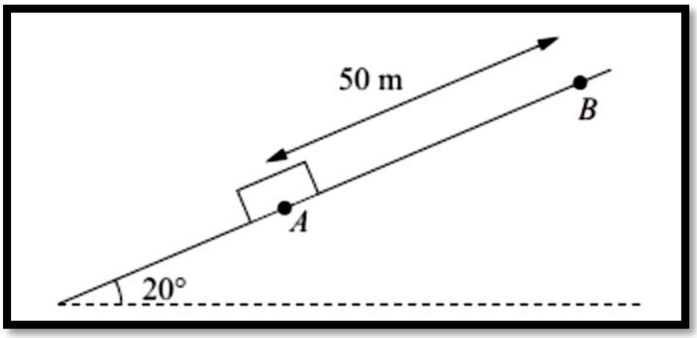
If there is no external force other than gravity, then energy is conserved so we can equate the two energy totals (often solving for a final velocity).

If the total energies cannot be equated (due to the presence of an external force), then work out the overall energy change by subtraction (loss in energy = energy 'before' – energy 'after').

If there is a total loss in energy we have work done by a resisting force. Conversely, if there is more energy at the end, then an external force has put energy into the system (e.g. cyclist pedalling up a hill).

In either case, the overall energy change must equal force × distance, where the force is the *external* force (assumed to be constant) that caused the energy change and the distance is measured in the same direction as the force.

The past exam question below illustrates when this approach would be used.



The box is released from rest at the point B and slides down the slope which is assumed to be smooth. Using conservation of mechanical energy, or otherwise, find the speed of the box as it reaches A.

Discuss what the 'otherwise' method could refer to (use of $F=ma$ and *suvat*). Also discuss the situation if the slope were not assumed to be smooth.

It may be required to apply the work-energy principle to a whole system rather than a single particle, for example two particles connected by a string passing over a pulley as shown in the diagram below.

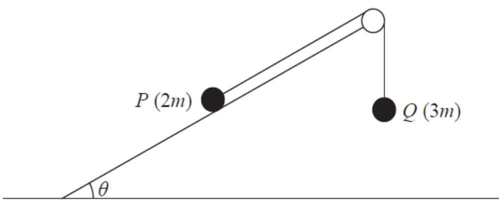


Figure 1

If the system is released from rest and the plane is smooth then both particles will gain KE, particle Q will lose GPE and particle P will gain GPE, but overall **total** mechanical energy will be conserved. If, however, the plane is not smooth then there will an overall loss in energy which, by the work energy principle, will be equal to the work done by friction as P moves up the plane.

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COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Common errors in these sorts of questions include: sign errors, double counting, mistakenly thinking work is done against gravity when moving in a downwards direction, forgetting to include both the kinetic and gravitation potential energy terms.

In the case of two particles connected by a string over a pulley students generally realise that they need to include KE, GPE and (if appropriate) work done against friction but a common error is to omit the KE (or GPE) of one of the particles or just to write ½ mv^2 without indicating which mass is being used.

Students should also be aware that if a question states a particular method (such as using the work-energy principle) then marks will not be awarded if an alternative method (such as $F = ma$ and *suvat*) is used.

NOTES

Frictional forces and knowledge of the coefficient of friction, μ, may be required for these questions.

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2c. Power; derivation of units and formula (2.1)

Teaching time
5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- **understand that power in watts is the rate of doing work;**
- **be able to calculate the power (P) of a vehicle with a tractive (driving) force F, moving with velocity v;**
- **be able to use the formula $P = Fv$ in problem solving.**

TEACHING POINTS

What is power? It is the rate at which we do work:

$$\text{Power} = \frac{d}{dt}(\text{work done}) = \frac{d(Fs)}{dt}$$

When the force is constant,

$$\text{Power} = F\frac{ds}{dt} = F\,v$$

So, for a moving vehicle with a constant driving or tractive force, F newtons, moving at speed v ms^{-1} we have:

Power = tractive force × speed or $\boxed{P = Fv}$

The dimensional unit is J s^{-1}, but electrical power in SI units is measured in watts (W) (after James Watt). Therefore mechanical power is also measured in watts. (Again kW is a more common unit for heavy vehicles moving at high speed.)

Emphasise that the tractive force is *not* the resultant force, but the external force exerted by the engine of the vehicle trying to drive it forwards. None of these forces need be constant; the formula holds at any particular instant during the motion.

What is interesting is that if we have a fixed power, say 500 W for an electric car, and the tractive force is 100 N, then the speed will be 5 m s^{-1}. But, if we increase the speed to 10 m s^{-1}, then the tractive force decreases to 50 N (i.e. half the force for double the speed!) This seems counter-intuitive, but if we think about the momentum of the vehicle, this is greater at higher speed and helps to 'keep the car moving' and so less force is needed.

Questions may involve a vehicle travelling on a horizontal plane or on an inclined plane. It is important to note whether it is moving up or down the plane. The resistance to motion is usually taken to be constant although it could depend on v.

Some problems require, or state, that the vehicle is travelling at *maximum* or *constant* speed. This indicates that the acceleration = 0 m s^{-2} and when we resolve along the direction of motion, the resultant force is zero. It therefore follows that the tractive force of the engine and the external resistances, including components of weights if the motion is not horizontal, are then equal. This implies equilibrium, despite the fact that the car is moving. This is dynamic equilibrium, which can be thought of as the experience when flying at a constant 500 mph in a plane and you cannot experience any forces other than hearing the sound of the engine.

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OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

To model everyday life, the speed of the vehicle could be given in km h^{-1} and the power in kW. This means that before starting to answer the problem, the units need changing to m s^{-1} and watts respectively.

It is also possible that the angle of an inclined plane could be given in arcsin, arctan or arccos form. The most common form is arcsin since the component of weight parallel to the plane is $mg\sin\theta$ (which means it is not necessary to evaluate the angle itself).A diagram is essential in order to identify all the forces, velocities and acceleration(where relevant).

This past exam question illustrates a typical example.

A truck of mass of 300 kg moves along a straight horizontal road with a constant speed of 10 m s^{-1}. The resistance to motion of the truck has magnitude 120 N.

(a) Find the rate at which the engine of the truck is working.

On another occasion the truck moves at a constant speed up a hill inclined at θ to the horizontal, where $\sin\theta = \frac{1}{14}$.

The resistance to motion of the truck from non-gravitational forces remains at magnitude 120 N and the rate at which the engine works is the same as in part (*a*).

(b) Find the speed of the truck.

Note that 'non-gravitational' forces are referred to in this question. When the truck is moving up the hill the component of weight acting parallel to the plane could be considered as acting as a resistance. To ensure that this is not included the resistance term here the expression 'non-gravitational' forces is used.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

A common error is to miss one of the forces when writing equations. This can often be avoided if students draw a diagram of forces before they start.

Sign errors may also creep in when decelerations are involved.

There is sometimes confusion between 'tractive (driving) force' and 'resultant force'.

Students need to be aware of what is an appropriate level of accuracy to use for their final answer. For example, only 2 significant figures are appropriate following the use of g = 9.8 (but 3 are accepted).

NOTES

The work-energy principle can also be used to extend these power questions.

The resistance to the motion of a car when its speed is v m s^{-1} could be modelled as a variable force of magnitude, say $(200 + 2v)$ N. This is a more realistic model as the resistance to motion increases with the speed of the vehicle. Although this topic was in legacy Mechanics A level, unit M2, questions were restricted to considering only constant resistance; however, this is not the case here.

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