UNIT 3: Elastic collisions in one dimension

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SPECIFICATION REFERENCES

3.1 **Direct impact of elastic spheres. Newton's law of restitution. Loss of mechanical energy due to impact.**

3.2 **Successive impacts of spheres and/or a spheres with a smooth plane surface**

PRIOR KNOWLEDGE

- Intuitive idea about collisions and S.I. units

Covered so far

- Momentum and impulse (See SoW Unit 1)
- Mechanical energy (See SoW Unit 2)

A level Mathematics – Pure content

4.5 Geometric Sequences & Series (including the sum to infinity) (See SoW Unit 6a)

KEYWORDS

Mass, velocity, N s, momentum, impulse, force, collisions, direct, impact, smooth, sphere, elastic, conservation, coefficient of restitution (*e*), Newton's (experimental) law of restitution, approach speed, separation speed, opposite direction, perfectly elastic, inelastic, plane, energy, kinetic energy, joules, 'loss' of mechanical energy.

NOTES

This topic builds on, and extends, Unit 1: Momentum and impulse, by considering collisions of spheres taking into account elasticity. It also considers how kinetic energy is changed by a collision. This topic was in legacy Mechanics A level in unit M2. The specification states that the spheres may be modelled as particles.
The collisions in this unit will still be direct and not involve oblique impacts.

3a. Direct impact of elastic spheres, Newton's Law of restitution and loss of kinetic energy due to impact (3.1)

Teaching time
8 hours

OBJECTIVES

By the end of the sub-unit, students should:

- **be able to express the 'compressibility', 'bounciness' or 'elasticity' of an object by a value called the coefficient of restitution (*e*);**
- **know that $0 \le e \le 1$ [and that $e = 0$ means inelastic and $e = 1$ means perfectly elastic];**
- **know and be able to use Newton's (experimental) law of restitution for direct impacts of elastic spheres;**
- **be able to calculate the change in kinetic energy due to an impact.**

TEACHING POINTS

When we considered collisions in Unit 1: Momentum and impulse we had no information about the nature of the materials from which the particles were made. But what if we now consider collisions where we have this information (such as spheres made of, for example, wood and steel) and what if we could somehow include this information in our equations?

Newton performed some experiments, mainly by bouncing balls off a solid wall, and found that the ratio of the approach speed to separation speed varied according to the type of materials making up the two surfaces in contact.

If you throw a ball made up of damp mud (or a rotten tomato!) it may not even bounce off the wall, but 'splat' and stick to the wall. In this case the separation speed would be zero.

He decided to call the ratio of separation speed to approach speed the coefficient of restitution (*e*). It is a measure the 'elasticity' of the objects hitting each other. Its value will be a number between 0 and 1. (For the damp mud example, the coefficient of restitution would be zero, whereas if the collision is perfectly elastic then $e = 1$.)

This definition leads to a simple formula which states that the coefficient of restitution (*e*) for two colliding particles is such that:

$$e = \frac{\text{speed of separation of particles}}{\text{speed of approach of particles}}$$

This is known as Newton's (experimental) law of restitution.

Notice that we use *speed* rather than velocity (so the ratio is always positive).

For a ball bouncing off a fixed plane (either a wall or a floor) we can further simplify the formula, giving:

$$e = \frac{v}{u} \quad \text{or} \quad v = eu$$

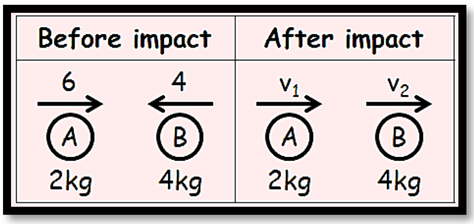
(where u is the speed with which it hits the plane and v is the rebound speed).

This shows that if $e = 1$, the ball is so 'hard' that it rebounds at the same speed as it approaches.

However, the most useful application of Newton's law of restitution is for collisions between two moving objects. Here the approach and separation speeds are relative speeds, rather than just u and v (as in situations where the wall is fixed).

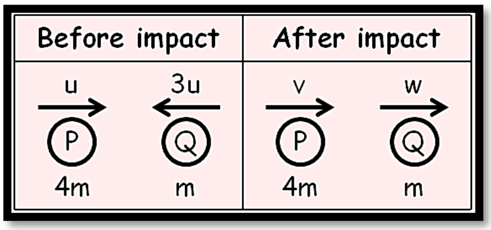
Encourage students to draw carefully labelled diagrams and to be careful when finding the approach and separation speeds. For example, for two objects moving directly towards each other (a head on collision) the approach speed is the sum of the two speeds. It is possible to think intuitively, rather than apply a formulaic approach when solving problems (see Examiner's Report below). However, students may prefer to use the formula: $v_1 - v_2 = -e\,(u_1 - u_2)$ where the velocities are all measured in the same direction so, as in the conservation of momentum equation, consideration of the signs of terms is crucial.

For questions like this one, where the final velocities of both spheres need to be found, the conservation of linear momentum still applies (using velocities rather than speeds – so directions, and consequently signs, count). This can be used along with Newton's law of restitution to form simultaneous equations which can be solved to find, for example, missing velocities or masses.



OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Include examples with algebraic velocities and masses, like the example below:-



When two objects collide, there will be a bang and heat is generated. These are forms of sound energy and heat energy which indicates that some of the mechanical energy has been transformed into a different form of energy.

This means that if we consider the kinetic energy of the system before and after the impact, there will be a 'loss of KE'. This is how a past exam question worded this concept:

Given that $e = \frac{1}{2}$, find the total kinetic energy lost in the first collision between A and B.

The terms may be algebraic (often in the form mu^2) for this comparison. Note that if $e = 1$, the collision is said to be perfectly elastic and no energy is lost. Remember also that although overall energy is lost in a collision ($e < 1$) it is possible for an individual particle to gain KE.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

This quote from a recent examiner's report highlights the main sources of errors:

Most candidates understood the principles of conservation of momentum and Newton's experimental law. However, many lost a mark here because they did not pay sufficient attention to the direction of motion of the particles after the collision, leading to inconsistent signs between their two equations. Even if they had indicated directions on a diagram, this was not always consistent with their equations. It was also common to see substitution into "template equations" rather than understanding of the equations.

The final 2 marks were very often lost because many failed to realise that the final direction of motion of P was the key to a solution. There were a few sign errors and arithmetic errors which resulted in attempts to 'fudge' the answer rather than find the source of the error.

NOTES

The next section will look at applications and more complex problems involving elastic impacts.

3b. Problem solving (including 'successive' impacts) (3.2)	**Teaching time** 9 hours

OBJECTIVES

By the end of the sub-unit, students should:

- **be able to solve problems of the following types involving elastic impacts:**
  - **a) successive collisions between pairs of spheres (horizontal motion);**
  - **b) bouncing ball (off a horizontal elastic plane);**
  - **c) successive collisions including two spheres and sphere against a wall;**
  - **d) determination of number of collisions or deriving the possible range of *e*.**

TEACHING POINTS

Although this topic is essentially made up of the application of two main principles (conservation of momentum and Newton's Law of restitution), there are many types of problems which could be set.

Consider three spheres A, B, C (masses $2m$, $3m$, $4m$ respectively) lying in a straight line, with B and C stationary, and A is projected towards B with speed u m s^{-1}. (The coefficient of restitution for all impacts is e.) Pose the problem: A hits B, then B hits C. But will A hit B again?

The condition for the third impact is often dependent on the range of values of e, which is set up by means of an inequality originating from $v_A > v_B$ or $v_A - v_B > 0$. This inequality may require some algebraic manipulation.

Sometimes, one of the spheres will hit and rebound from a fixed vertical wall. Two such examples are below.

> Two small spheres A and B move on a smooth horizontal table. The mass of A is m and the mass of B is $4m$. Initially A is moving with speed u when it collides directly with B, which is at rest on the table. As a result of the collision, the direction of motion of A is reversed. The coefficient of restitution between the particles is e.
>
> (a) Find expressions for the speed of A and the speed of B immediately after the collision.
>
> In the subsequent motion, B strikes a fixed vertical wall and rebounds.
>
> The wall is perpendicular to the direction of motion of B.
>
> The coefficient of restitution between B and the wall is $\frac{4}{5}$.
>
> (b) Given that there is a second collision between A and B show that $\frac{1}{4} < e < \frac{9}{16}$.

> Two small spheres A and B have mass $3m$ and $2m$ respectively. They are moving towards each other in opposite directions on a smooth horizontal plane, both with speed $2u$, when they collide directly. As a result of the collision, the direction of motion of B is reversed and its speed is unchanged.
>
> (a) Find the coefficient of restitution between the spheres.
>
> Subsequently, B collides directly with another small sphere C of mass $5m$ which is at rest. The coefficient of restitution between B and C is $\frac{3}{5}$.
>
> (b) Show that, after B collides with C, there will be no further collisions between the spheres.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Consider examples in which a small ball (mass m) is dropped from a height, h on to an elastic horizontal plane, in which the coefficient of restitution for all bounces is e.

Either kinematics or the conservation of mechanical energy can be used to find the speed of the ball immediately before the first impact, namely $\sqrt{2gh}$. However the rebound speed will be $e\sqrt{2gh}$, which is less than the speed of impact. What will be the new greatest height after this first bounce?

You will find that the height is e^2h. By repeating the process, you will find the greatest heights form the terms of a geometric sequence with common ratio e^2 and, since $0 \leq e \leq 1$, the sum of these terms will converge (see work on Geometric Series in AL Pure).

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

When considering multiple collisions and trying to work out whether subsequent collisions will occur, a common error is to only consider collisions between two of the three spheres. Students may also come unstuck if they do not produce a correct inequality, for example by simply considering the speed of one spheres rather than its speed relative to the other spheres.

Sign errors in equations and inequalities can be avoided by use of a clear diagram.

Students should be advised that substituting values into equations at early stages may make things easier and help avoid algebraic slips.

NOTES

Collisions involving oblique impacts are not included in this unit.