Unit 4: Linear Programming (part 1)

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SPECIFICATION REFERENCES

5.1 Formulation of problems as linear programs.

5.2 Graphical solution of two variable problems using objective line and vertex methods including cases where integer solutions are required.

PRIOR KNOWLEDGE

GCSE (9-1) in Mathematics at Higher Tier

A22 Solve linear inequalities; represent the solution set using set notation and on a graph

KEYWORDS

Decision variables, constraints, objective function, feasible solution, feasible region, optimal solution.

4a. Formulation of problems (5.1)

Teaching time
2 hours

OBJECTIVES

By the end of the sub-unit, students should:

- **know how to formulate a linear programming problem from a real-life problem (write inequalities from worded questions);**
- **be able to form an appropriate objective function to maximise or minimise.**

TEACHING POINTS

First determine and define the decision variables (the quantities you need to know to solve the problem). Second, decide what the constraints are, including the non-negativity constraints. Third, find the objective function that you need to maximise or minimise.

The decision variables must be carefully defined. Students should get into the habit of starting a LP problem with "Let *x* be the *number* of …" etc. Examiners often comment about poorly defined variables.

Students struggle to formulate the constraints as inequalities from worded questions. A useful method is to draw a two-way table showing all the information before attempting to write the inequalities. Some comparative constraints are tricky; first get the algebra correct and then the inequality correct.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Use questions where students have to change units before writing their constraints. Many students struggle to write and simplify the constraint inequalities. Use as varied a selection of problems as possible, for example include constraints given as percentages.

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Students can be easily put off when things look different, for example when inequalities are written using percentages.

Weak algebra skills can cause difficulties when dealing with inequalities.

Common mistakes include omitting the instruction to maximise the objective, omitting non-negativity constraints, trying to combine several conditions into one inequality, wasting time by starting to solve the LP problem.

4b. Graphical solutions (5.2)

Teaching time
6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- **know how to represent a linear programming problem graphically and identify the feasible region;**
- **be able to solve linear programming problems to find a maximum or minimum;**
- **be able to interpret solutions in the context of the original real life problem.**

TEACHING POINTS

All students, even those who are more able, will need to practise plotting inequalities. It is usual practice to shade out the region that is not required, leaving the feasible region unshaded.

Students should understand that any point in the feasible region is a feasible solution. LP problems are therefore solved by finding which member of the set of feasible solutions gives the optimal (maximum or minimum) value of the objective function.

Students should be able to use the objective line method (ruler method), and the vertex method. They should also know how to proceed when the decision variables *must* be integers.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Consider scaling more difficult axes. Practice scenarios where the equations are in percentage and fraction form.

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Marks are easily lost in exams for inaccurate lines or inequalities so that the feasible region is not clear. Using different scales on the axes can also add a level of complication which can derail some students.

Students should be careful when it comes to the constraints – neither omitting any nor adding others. For example, thinking that integer values are needed despite this constraint not having been specified.

When point testing, not all student test all vertices. Whilst it may be obvious to students that other vertices would not give optimal values for the objective function, the algorithm requires that all vertices of the feasible region are tested and this is required to gain full marks.

NOTES

Students should plot the objective function and label it if using the objective line method.

Shade out the region you cannot use.