UNIT 5: Forces at any angle

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SPECIFICATION REFERENCES

8.4 Resolving forces in 2 dimensions. Problems may be set where forces need to be resolved.

8.6 Understand and use the $F \leq \mu R$ model for friction; coefficient of friction; motion of a body on a rough surface; limiting friction and limiting equilibrium.

PRIOR KNOWLEDGE

- 2D trigonometry
- Cosine and sine rules
- $\frac{\sin x}{\cos x} = \tan x$ (to find the angle of the resultant)
- Basic vectors, magnitude and direction (kinematics)
- **i**, **j** vectors
- Force diagrams and assumptions

KEYWORDS

Force, weight, tension, thrust, friction, coefficient of friction, μ, limiting, reaction, resultant, magnitude, direction, bearing, force diagram, equilibrium, inextensible, light, negligible, particle, smooth, rough, uniform, perpendicular.

NOTES

This unit is designed to help students develop the tools to enable modelling of the statics and dynamics problems in Unit 7.

The specification guidance for 8.4 states: 'Restricted to forces in two perpendicular directions or simple cases of forces given as 2D vectors.'

5a. Resolving forces (8.4)	**Teaching time** 3 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand the language relating to forces;
- be able to identify the forces acting on a particle and represent them in a force diagram;
- understand how to find the resultant force (magnitude and direction);
- be able to find the resultant of several concurrent forces by vector addition;
- be able to resolve a force into components and be able to select suitable directions for resolution.

TEACHING POINTS

Begin by considering two forces acting at right angles to one another (horizontal and vertical), use Pythagoras and trigonometry to find the hypotenuse (resultant R) and angle (direction θ above the horizontal) respectively. [You could also link to velocity from speed and vector addition rule.]

It is easy going from component form to magnitude/direction; but can we go backwards?

Guide students to consider the right-angled triangle and use trigonometry to show that the horizontal component is $R\cos\theta$ and the vertical component is $R\sin\theta$ of the Resultant, R (hypotenuse).

Show that forces given in the form **i**, **j** can be simply drawn as a right-angled triangle and the resultant and direction can be found the same way. Extend to finding the resultant of a system of forces given in **i** – **j** form by adding **i** and **j** components.

Look at two forces acting at *any* angle and show that the triangle can be solved using the cosine rule (to find the resultant) and sine rule (to find the direction).

Extend to more than two forces and resolve the system using $R(\rightarrow)$ and $R(\uparrow)$ to create two perpendicular forces, then use Pythagoras and trigonometry to calculate the resultant and direction.

Show that the weight component of a particle on an inclined plane acts in two directions: along and perpendicular to the plane. This will be a critical skill for solving the statics/dynamics questions in the next unit.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

This topic can be linked with Unit 7 and these techniques used to solve statics and dynamics problems.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

When resolving common errors are: to omit g; sign errors; reversal or confusion* of when to use cos and/or sin; to omit one force (usually weight).

Students may also easily get confused by the vocabulary and mix up ‘resultant’ and ‘reaction’.

NOTES

For students who find these concepts difficult it is possible to simplify most questions by restricting the resolving of a force to using *just* cos θ. This can be done by using the method of '**cos** a**cr**o**ss** the number of degrees the force has to be turned to reach the direction we want to resolve in'.

(*This eliminates the choice of cos or sin for weaker students and can avoid the confusion mentioned in the paragraph above.)

The next section looks at the concept of friction forces, which will lead to a refined mathematical model involving rough planes.

5b. Friction forces (including coefficient of friction μ) (8.6)	**Teaching time** 3 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand that a rough plane will have an associated frictional force, which opposes relative motion (i.e. the direction of the frictional force is always opposite to how the object is moving or 'wants' to move);
- understand that the 'roughness' of two surfaces is represented by a value called the coefficient of friction represented by μ;
- know that $0 \le \mu$ but that there is no theoretical upper limit for μ although for most surfaces it tends to be less than 1 and that a 'smooth' surface has a value of $\mu = 0$;
- be able to draw force diagrams involving rough surfaces which include the frictional force
- understand and be able to use the formula $F \le \mu R$.

TEACHING POINTS

Start by asking students to rub their hands together vigorously. The warmth is caused by microscopic peaks and troughs on the surface of the skin interlocking. The rougher the surface, the 'sharper' these peaks and troughs. Explain to students that this principle applies even to the smoothest looking surfaces and the force which opposes motion is called the frictional force. The value which represents the roughness is called the coefficient of friction (μ) and is zero for a smooth surface.

If we consider a book on a rough horizontal table, it will be *harder* to move the book if:-

- we put a 'paperweight' on it (increasing the reaction force)

or

- we put it on a rougher surface (increasing the value of μ).

Therefore the expression to model frictional forces uses these two factors (in direct proportion) and is given by μR. This is the maximum resistance any surface can provide before the book begins to move, so the inequality $F \le \mu R$ applies until the force wanting to cause motion reaches the limiting value μR, called limiting friction.

Consider a 10 kg book on a rough horizontal plane. If $\mu = 0.5$, investigate the value of the frictional force if the pushing force, P is **a** 10 N, **b** 98 N, **c** 100 N

[Link to *resultant force* = *ma* from AS Mathematics – Mechanics content, see SoW Unit 8.]

Now place the book on an inclined plane and analyse the limiting friction being careful to stress that the reaction force is NOT the weight in this case. Will the book begin to slide for different angles of plane? What is the maximum angle achievable before the book slides?

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

This topic can be linked with Unit 7 and these techniques used to solve statics and dynamics problems.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students are often good at drawing force diagrams, but common errors are omitting arrowheads, incorrectly labelling (e.g. 4 kg rather than $4g$) and missing off the normal reaction or friction forces. Students can sometimes struggle to work out the direction of the frictional force.

Some students may mistakenly think that the coefficient of friction changes if the mass of an object or the angle of the slope changes.

NOTES

This topic is designed to help students develop the tools to enable modelling of the statics and dynamics problems in Unit 7.