UNIT 7: Kinematics 1 (constant acceleration)

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SPECIFICATION REFERENCES

7.1 Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration

7.2 Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of gradient; velocity against time and interpretation of gradient and area under the graph

7.3 Understand, use and derive the formulae for constant acceleration for motion in a straight line

8.3 Understand and use weight and motion in a straight line under gravity; gravitational acceleration, *g*, and its value in S.I. units to varying degrees of accuracy

PRIOR KNOWLEDGE

GCSE (9-1) in Mathematics at Higher Tier

R1 Change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts

R11 Use compound units such as speed, rates of pay, unit pricing, density and pressure

A2 Substitute numerical values into formulae and expressions, including scientific formulae

A5 Understand and use standard mathematical formulae; rearrange formulae to change the subject

A14 Plot and interpret graphs (including reciprocal graphs and exponential graphs) and graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

A15 Calculate or estimate gradients of graphs and area under graphs (including quadratic and non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts

A17 Solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation)

A18 Solve quadratic equations (including those that require rearrangement) algebraically by factorising, by completing the square and by using the quadratic formula

AS Mathematics – Pure Mathematics content

3.1 Gradient (See Unit 2a of the SoW)

KEYWORDS

Distance (m), displacement (m), speed (m s^{-1}), velocity (m s^{-1}), acceleration (m s^{-2}), retardation (m s^{-2}), deceleration (m s^{-2}), scalar, vector, 2D, linear, area, trapezium, gradient, equations of motion, gravity, constant, 9.8 m s^{-2}, vertical.

NOTES

The guidance on the specification document states that graphical solutions to problems may be required. This section assumes *constant* acceleration; hence the graphical approach involves linear line segments and the familiar equations of linear motion *suvat*, formulae for constant acceleration. (N.B. ‘equation of motion’ refers to $F = ma$, and is nothing to do with these formulae).

The guidance also states that derivation of constant acceleration formulae may use knowledge of sections 7.2 and/or 7.4 (Unit 9).

Kinematics 2 (Unit 9) analyses particles’ motion under a <u>variable</u> force, hence a variable acceleration. The mathematical model for this requires calculus which is covered in AS Mathematics – Pure Mathematics content, see SoW Units 6 and 7.

The usual value for g in this course is 9.8 m s^{-2}, but some questions may specify a different value. Students may assume that g is constant, but should be aware that it is not a universal constant but depends on location.

7a. Graphical representation of velocity, acceleration and displacement (7.1) (7.2)

Teaching time
4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to draw and interpret kinematics graphs, knowing the significance (where appropriate) of their gradients and the areas underneath them.

TEACHING POINTS

Introduce this topic by making links to the GCSE (9-1) in Mathematics prior knowledge for distance–time (travel) and speed–time graphs. Kinematics is the analysis of a particle's motion without reference to the resultant force that caused that motion.

Stress that forces causing the motion of the body in this section are *constant*, therefore acceleration is constant and this results in a *straight line* travel speed-time or velocity-time graph.

Extend the ideas to displacement by considering a particle which moves in reverse direction back beyond the starting point.

For a velocity–time graph, consider the units for the area of a unit square 1 m s^{-1} by 1 s. The 's' cancels, leaving 'm', therefore the area represents the displacement.

Discuss and interpret graphs that model real situations. For example, the distance–time graph for a particle moving with constant speed, the velocity–time graph for a particle with constant acceleration.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Throw an object straight (vertically) up in the air. Time the flight and estimate the greatest height, to scale the graphs correctly, and keep for possible later use. Draw the displacement–time and velocity–time graphs (upward direction positive and initial velocity non-zero). If the object is caught at the same height at which it was thrown, what is the average velocity for the motion?

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Many students can draw a velocity time graph with the correct shape, but do not always label the required speeds and times clearly on the axes. Students often tend to add a scale (for example 4, 8, 12, 16, …) unnecessarily, rather than just indicating the initial and final speeds.

Candidates are able to find distance travelled and the acceleration from velocity–time graphs and can find an average speed, but some struggle with the vocabulary of velocity and displacement.

NOTES

This unit can be linked with Unit 7b by drawing a general velocity–time graph for a particle with initial velocity (u), final velocity (v), taking time (t) moving under constant acceleration (a). The gradient of the line is $\frac{v-u}{t} = a$, which rearranges to $v = u + at$. Finding the area under the graph in 3 different ways will lead to 3 of the other *suvat* formulae but $v^2 = u^2 + 2as$ will have to be derived by eliminating t between two of them.

7b. Motion in a straight line under constant acceleration; *suvat* formulae for constant acceleration; Vertical motion under gravity (7.3) (8.3)

Teaching time
6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- recognise when it is appropriate to use the *suvat* formulae for constant acceleration;
- be able to solve kinematics problems using constant acceleration formulae;
- be able to solve problems involving vertical motion under gravity.

TEACHING POINTS

Make links back to Unit 7a and contrast the previous graphical approach with this algebraic approach. Note that there are five quantities, s, u, v, a and t (four vectors and one scalar) and each formula relates four of them hence there are five formulae. The formulae that must be derived and learnt are:

- $v = u + at$
- $s = \frac{(u+v)t}{2}$
- $s = ut + \frac{1}{2}at^2$
- $v^2 = u^2 + 2as$
- $s = vt - \frac{1}{2}at^2$

These formulae are only valid for *constant* acceleration in a straight line (and are referred to as the *suvat* formulae).

When solving problems, write down known variables and the variable(s) to be found – this should help to identify which one (or more, as some problems will involve simultaneous equations) of the *suvat* formulae to select. Emphasise to students the need to make sure units are compatible.

Model the good practice of drawing a diagram to illustrate the situation whenever possible, especially when considering vertical motion under gravity. This will encourage students to draw their own diagrams.

Mark the positive direction on the diagram and take acceleration due to gravity (g) to be 9.8 m s^{-2} unless directed otherwise. Students may assume that g is constant, but they should be aware that g is not a universal constant but depends on location.

If an object is thrown upwards and upwards is taken as being positive then $a = -9.8$ m s^{-2}. Explain that the velocity is zero at the greatest height and there is symmetry in the path (up and down to the same point) due to the fact that we model air resistance as being negligible.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

One of the more demanding problems is when two objects are released (or dropped) at different times, say 2 seconds apart, and students are asked to find the common position when one catches-up or passes the other. Students may find it difficult to select the times (values of t) to assign in the equations; they may need guiding towards t and $(t - 2)$ or t and $(t + 2)$.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students are generally able to use *suvat* formulae in 2D to find unknown heights, velocities etc. However, students sometimes ignore the significance of a negative value for velocity, acceleration or displacement and don't refer their answer back to the original problem. They need to recognise that $s = -3$ m means the object is 3 m *below* its starting point in the negative direction i.e. s is effectively a coordinate. This is where a diagram helps students understand the physics of the situation.

NOTES

End the section by looking forwards to Kinematics 2 (Unit 9) with a problem illustrating a variable acceleration e.g. $v = 2t^2 + 3t$. Explain that this will give a curved velocity–time graph so the *suvat* formulae will not work and instead we may need to find the gradient of the tangent and the area under the curve (Link back to GCSE (9-1) in Mathematics at Higher Tier or to calculus in AS Mathematics – Pure Mathematics content, see SoW Unit 7) You could also consider how to analyse motion in 2D (or 3D); this will be addressed during Kinematics 2 (Unit 9).