UNIT 2: Coordinate geometry in the (x, y) plane

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SPECIFICATION REFERENCES

2.7 Understand and use proportional relationships and their graphs

3.1 Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$
Gradient conditions for two straight lines to be parallel or perpendicular
Be able to use straight line models in a variety of contexts

3.2 Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$
Completing the square to find the centre and radius of a circle
Use of the following properties:

- the angle in a semicircle is a right angle
- the perpendicular from the centre to a chord bisects the chord
- the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point

PRIOR KNOWLEDGE

Algebraic manipulation covered so far

- Simultaneous equations
- Completing the square

<u>GCSE (9-1) in Mathematics at Higher Tier</u>

A9 Equation of a line
Parallel and perpendicular lines
G20 Pythagoras
A14 Conversion graphs
R10 Calculating the proportionality constant *k*
G10 Circle theorems

KEYWORDS

Equation, bisect, centre, chord, circle, circumcircle, coefficient, constant, diameter, gradient, hypotenuse, intercept, isosceles, linear, midpoint, parallel, perpendicular, proportion, Pythagoras, radius, right angle, segment, semicircle, simultaneous, tangent.

Pearson

2a. Straight-line graphs, parallel/perpendicular, length and area problems (3.1) (2.7)	Teaching time 6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and use the equation of a straight line;
- know and be able to apply the gradient conditions for two straight lines to be parallel or perpendicular;
- be able to find lengths and areas using equations of straight lines;
- be able to use straight-line graphs in modelling.

TEACHING POINTS

Students should be encouraged to draw sketches when answering questions or, if a diagram is given, annotate the diagram.

Equations can be given or asked for in the forms $y = mx + c$ and $ax + by + c = 0$ where a, b and c are integers. Students will need to be familiar with both forms, so questions should be asked where different forms are given or required in the answer. Given either form, students should be able to find the intercepts with the axes and the gradient. The x-intercept often causes students more difficulty, so will need more practice, but is useful for sketches and questions involving area or perimeter.

Students should be able to find the equation of a line given the gradient and a point, either the formula $y - y_1 = m(x - x_1)$ can be used or the values substituted into $y = mx + c$. To find the equation of a line from two points the gradient can be found and then one of the previous two methods used or the formula $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$ can be used. If this formula is used, care needs to be taken to ensure that the y-values are substituted into the correct places and that negative signs are taken into account. It should be emphasised that in the majority of cases, the form $y - y_1 = m(x - x_1)$ is far more efficient and less prone to errors than other methods.

The gradient conditions for parallel and perpendicular lines may be remembered from GCSE (9-1), but are still worth revising. They need to be well understood as they are used further when dealing with circles and in differentiation. Students should be able to identify whether lines are parallel, perpendicular or neither and find the equation of a parallel or perpendicular line when given a point on the line.

The length of a line segment is found by using Pythagoras' theorem, which can be written as the formula $d = \sqrt{(x - x_1)^2 + (y_2 - y_1)^2}$. This can be linked to proof with students being encouraged to show how to go from Pythagoras to the formula. Answers to length and distance questions are likely to be given in surd form, giving further practice in simplifying surds. Students should be encouraged to give answers in exact form unless specified otherwise.

Make shapes using lines and the axes; students can then be asked to find the area or perimeter of composite shapes. Answers should be given in exact form to practise combining and simplifying surds.

Real-life situations such as conversions can be modelled using straight-line graphs, this is likely to be familiar from GCSE (9-1) Mathematics.

Students should also be familiar with finding the relationship between two variables and expressing this using the proportion symbol $\propto$ or using an equation involving a constant (k). This can be extended to straight-line graphs through the origin with a gradient of k. Students should be able to calculate and interpret the gradient.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

To help students see how much information is given in the equation of a line, a good activity is to give an equation and ask students to find everything they know about that line, e.g. the intercepts, a point on the line, the gradient, a sketch, a parallel line, etc.

Students can be given sketches and asked to suggest equations that would/would not work.

Modelling with straight-line graphs gives the opportunity to collect data that can then be plotted and a line of best fit used to find an equation. It might also be possible to compare the data to a theoretical model. Students should be encouraged to consider strengths and limitations of modelling.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

In exams, students should be encouraged to quote formulae before using them. This allows method marks to be awarded even if arithmetical slips are made or incorrect values substituted.

Questions may specify a particular form for an answer (for example integer coefficients). Emphasise to students the importance of following these instructions carefully so as not to lose marks.

Students should be encouraged to draw diagrams while working on solutions as this often results in fewer mistakes and can act as a sense check for answers. At the same time, where diagrams are given in questions, students should be aware that these are not to be relied upon and 'spotting' answers by looking at a diagram without providing evidence to support this will not gain full marks. However, candidates should be encouraged to use any diagrams provided to help them answer the question.

The usual sorts of algebraic and numerical slips cause marks to be lost and students should be encouraged to carefully check their working. A common error is to incorrectly calculate the gradient of a straight line when it is given in the form $ax + by + c = 0$, so students should be encouraged to practice this technique.

NOTES

Dynamic geometry programs can be used to make changes and observe the effect, helping students to discover and visualise the effect of changing equations.

2b. Circles: equation of a circle, geometric problems on a grid (3.2)	**Teaching time** 7 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to find the midpoint of a line segment;
- understand and use the equation of a circle;
- be able to find points of intersection between a circle and a line;
- know and be able to use the properties of chords and tangents.

TEACHING POINTS

Drawing sketches or annotating given diagrams will help students to understand the question in many cases and so should be encouraged.

Students should be able to find the midpoint given two points from GCSE (9-1) Mathematics. This can be built upon to find the coordinate of a point given the midpoint and one of the end points. The midpoint can be used to find the perpendicular bisector, recapping the work from straight-line graphs.

The equation of the circle $(x-a)^2+(y-b)^2=r^2$ can be derived from Pythagoras' theorem, giving students the opportunity to look at proof.

Students should be able to find the radius and the coordinates of the centre of the circle given the equation of the circle, and vice versa. Students should be familiar with the equations $x^2+y^2+2fx+2gy+c=0$ and $(x-a)^2+(y-b)^2=r^2$. 'Complete the square' method should be used to factorise the equation into the more useful form. Students will need practice within this context to ensure that they are confident with the algebraic manipulation needed, in particular mistakes are often made with the signs and forgetting the constant term.

Circle theorems from GCSE (9-1) Mathematics can be used in questions so a quick recap could be useful and then they should be incorporated into questions. Examples of this include: finding the equation of the circumcircle of a triangle with given vertices; or finding the equation of a tangent using the perpendicular property of tangent and radius.

Simultaneous equations can be used to find the points of intersection between a circle and a straight line. Students can also be asked to show that a line and circle do not intersect, for which the discriminant can be used. Finding intersections with the axes should also be covered.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

The conditions in which a circle and a line intersect can be investigated, with students justifying which will and will not intersect.

Investigate finding the equation of a circle given 3 points on its circumference.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Most errors when completing the square to find the equation of a circle involve the constant term. Students may forget to subtract it or perhaps add it instead. Having found the equation, when giving the coordinates of the centre students must take care to get the signs the right way round as marks are easily lost by getting this wrong.

When substituting into equations to find the intersections with axes, students sometimes substitute for the wrong variable, for example substituting $y = 0$ when trying to find the intersection with the y-axis. Another error is substituting the entire bracket $(x - a)$ for 0 rather than just x.

When finding the equation of a tangent to a point on the circle, typical errors are: finding the gradient of the radius; finding a line parallel to the radius; and finding a line through the centre of the circle.