Unit 3: Algorithms on Graphs II (part 1)

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SPECIFICATION REFERENCES

3.1 Algorithm for finding the shortest route around a network, travelling along every edge at least once and ending at the start vertex (The Route Inspection Algorithm)

PRIOR KNOWLEDGE

Covered so far

- Introduction to graph theory (See Unit 1c)
- Dijkstra's algorithm (See Unit 2b)
- Eulerian and semi-Eulerian graphs (for the Route Inspection Problem) (See Unit 1c)

KEYWORDS

Traversable, odd valency, Eulerian, Semi-Eulerian, minimum weight.

3a. Route inspection problem (3.1)	**Teaching time** 5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- **be able to determine whether a graph is traversable;**
- **be able to apply an algorithm to solve the route inspection problem;**
- **find a route by inspection;**
- **understand the importance of the order of vertices of the graph in finding a route.**

TEACHING POINTS

This is a relatively straightforward algorithm to complete. Recap earlier teaching on Eulerian graphs and order of vertices. Use a selection of graphs to demonstrate if a graph is traversable and list the condition that this requires. Revisit the Bridges of Königsberg problem.

In an exam, students are usually given the sum of the values of the edges in the network. (Though apparently some students find the need to recalculate this leading to incorrect answers!)

Students must clearly state the odd vertices and the minimum route between each pair. They must be aware that they need to check *all possible routes* between the two vertices to find the minimum route not just the one which passes through the least number of vertices. The direct route may not be the shortest route.

They are not usually asked to find a particular route but should be aware of the number of times a given vertex will appear in the optimal route. It should be emphasized that this is the degree of the vertex ÷ 2, unless it is also part of a repeated route.

The shortest route can be found by inspection in the examination – if Dijkstra's algorithm is required it will be explicitly asked for.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Euler's hand-shaking lemma.

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

While most students show the correct three distinct pairings of the correct four odd nodes many candidates do not show the total for each pairing. These totals need to be given as evidence that the correct arcs, which need to be traversed twice, have been chosen.

Some students in an exam did not realise that 'the number of visits' = 'degree of vertex' ÷ 2. They resorted to finding a detailed route, with varying degrees of success, and then counting each occurrence of a vertex.

Modified situations in the Chinese postman problems are often not handled well so students should spend time studying semi-Eulerian situations.

NOTES

This is also known as the ‘Chinese postman’ problem. It was first studied by the Chinese mathematician Mei-Ko Kwan in 1962.

Students will be expected to use inspection to consider all possible pairings of odd nodes. The network will contain at most four odd nodes.