UNIT 1: Momentum and impulse (part 1)

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SPECIFICATION REFERENCES

1.1 **Momentum and impulse. The impulse-momentum principle. The principle of conservation of momentum applied to two spheres colliding directly**

PRIOR KNOWLEDGE

AS Mathematics – Mechanics content

Intuitive idea of momentum and S.I. units

7.3 Kinematics – constant acceleration formulae (See SoW Unit 8b)

KEYWORDS

Mass, velocity, speed, Ns, momentum, impulse, force, time, collisions, direct, smooth, body, sphere, particle, coalesce, conservation, vector, magnitude, string, light, inextensible, jerk, impulsive tension.

NOTES

This topic was in legacy Mechanics A level in unit M1.

1a. Momentum and impulse; derivation of units and formulae. Momentum-impulse principle. Conservation of momentum applied to collisions and 'jerking' string problems (1.1)

Teaching time
10 hours

OBJECTIVES

By the end of the unit, students should:

- **understand the definitions, derivation, and units of momentum and impulse;**
- **understand what happens to the momentum of a sphere as a result of a collision;**
- **be able to use the principle of conservation of momentum applied to direct collisions in 1-dimension.**

TEACHING POINTS

Begin with two equations familiar from AS Mechanics:

$v = u + at$ and $F = ma$

Rearrange the latter to make $a = \frac{F}{m}$, substitute into $v = u + at$ and then rearrange to give

$Ft = mv - mu$.

Note that we are assuming F is constant in this equation.

What is the quantity mass × velocity? Its unit is based on the Ft part of the formula above which gives newtons × seconds (N s).

Momentum is defined as mass × velocity and it is basically a measure of a body's capacity to keep moving (or stop moving).

The fact that the formula has initial and final velocities suggests a change of velocity, which means that an acceleration exists, therefore a force must cause that change. However, this could be a very sudden change, due to a collision or a strike. The quantity Ft is defined as the impulse of the force.

This gives us the impulse–momentum principle which states:

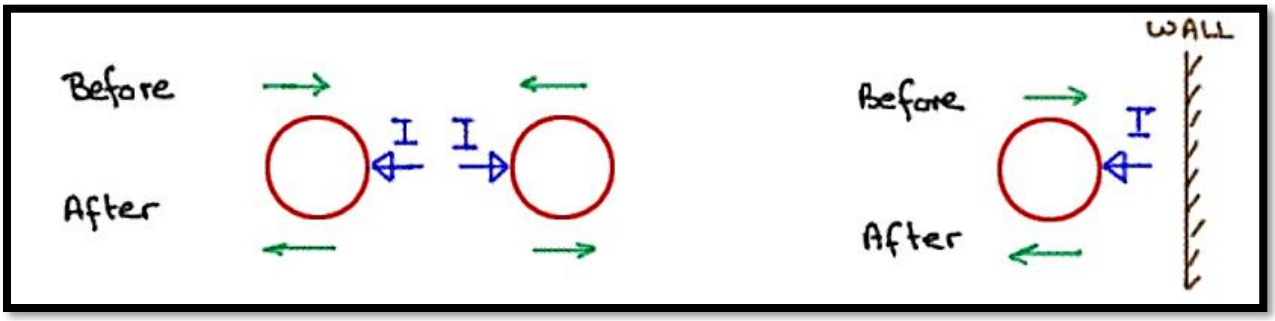
Impulse = change in momentum for a body ($I = mv - mu$)

Note that velocities are vectors which therefore, in one dimension, could be positive or negative depending on their directions; this means that the corresponding impulse could also be positive or negative.

The idea of an impulse can be illustrated by considering a hammer hitting a nail. Each strike on the nail is for a very short time, so the quantity Ft will have a large value of F (as the t is very small). This is why the hammer is effective. F is called an 'impulsive' force.

Think of flicking someone with a tea towel; it is in contact with the victim for only a very short time and although the tea towel is not very heavy, the flick hurts because the impulsive force is large.

Consider a single body reversing direction after being hit (e.g. a cricket ball hit by a bat) or bouncing off a vertical wall. The impulse can be calculated using the impulse–momentum principle (i.e. impulse = change in momentum for the ball).



The formula for impulse ($Ft = mv - mu$) is a consequence of Newton's Second Law. During a collision a force acts on each particle for a short time when the particles are in contact. By Newton's Third Law these forces are of equal magnitude and act in opposite directions. So the impulses acting on the particles are also of equal magnitude and acting in opposite directions.

Hence: $m_1 (v_1 - u_1) = - m_2 (v_2 - u_2)$

This can be rearranged to give the law of conservation of (linear) momentum for a collision which is of fundamental importance in mechanics, namely: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

In order to illustrate this you could show some clips of collisions between two vehicles from the internet (e.g. car racing crash) or balls in a game of snooker.

Students could experiment by flicking coins (one large and one small) to see how they move after collisions. (Sometimes the direction of the velocity is reversed if a small coin strikes a much heavier coin.)

This is also a good opportunity to mention the other assumptions we need to make for this topic. We model the colliding bodies as particles; this ensures that collisions are direct and there is no spin. Generally the surface on which they are moving is modelled as smooth and the particles will be travelling along the same straight line. However, remember that the smooth surface only affects the motion before and after the collision and not the impact itself. For collisions of two particles, it is important that students draw diagrams which cover before and after the collision; the velocities, masses and direction of impulse should be included as appropriate for each particle. The law of conservation of momentum will apply to the colliding particles.

Momentum before = Momentum after

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Students must decide which direction to take as positive and then be consistent throughout their solution. Emphasise that it is velocities not speeds (i.e. vector quantities) in the equation and so signs of terms are important.

Remind the students that coalesce means that bodies move off together as one body after the collision.

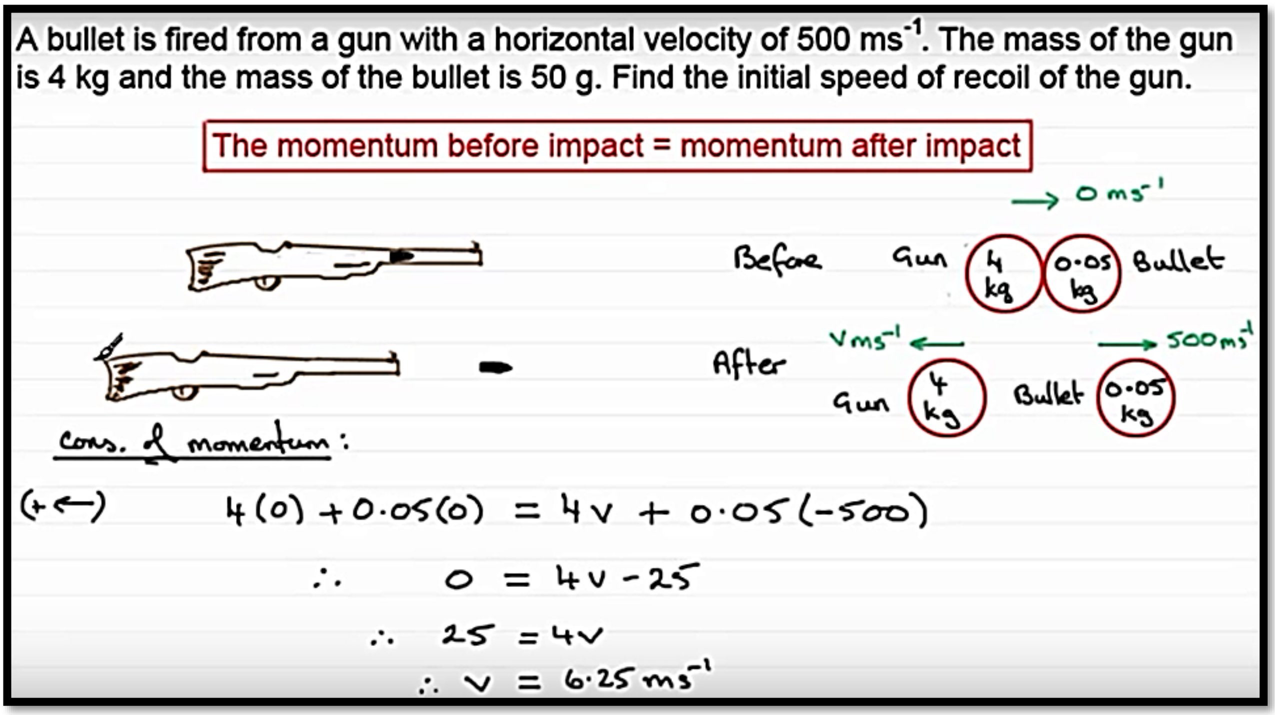
Cover lots of examples of collisions and also include examples of two particles joined by a slack light inextensible string. When the one in front moves off (or is travelling faster than the one behind), the string will get tight and 'jerk' the other particle. This 'jerk' acts as an impulse (equal and opposite on each particle). The particles will then move off together in the subsequent motion, effectively as one body and total momentum is conserved. It is a good idea to set a variety of questions with algebraic rather than numerical values for the quantities.

During a collision each particle receives an impulse. As mentioned previously these impulses are equal in magnitude but opposite in direction. When asked for the magnitude of the impulse of A on B, make sure the students are clear that only the momentum of one particle is to be considered (and not the system as a whole). If the question refers to '*A* on *B*' it is probably advisable to find the impulse on *B*; however, the impulse on *A* has the same magnitude and so it would be equally correct to find the impulse on *A*. Emphasise that the magnitude denotes a positive answer and so, even if the impulse turns out to be negative, the final answer must be given as positive. Note that it is also worth calculating the impulse of *B* on *A* to confirm that it is equal in magnitude but of opposite sign to the impulse of *A* on *B*.

The result will be a positive impulse if the body's velocity has been increased as a result of the collision and negative if the velocity has decreased. For example, if *A* were moving at -4 m s^{-1} before the collision and 3 m s^{-1} after being hit, there is a positive impulse as $3 > -4$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Most exam questions have an element of problem solving but an example worth considering is a gun and recoil. (Worked example below; note that units must be consistent)



You should also cover successive collisions between pairs of spheres (e.g. *A* hits *B* then *B* hits *C*). Note that when a sphere hits a wall it is possible to find the impulse on the sphere (using change in momentum) but remember to take account of the change in direction of motion due to the impact.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Questions on momentum in 1- dimension are generally completed well. The main reason for loss of marks is arithmetic errors or sign errors. It should be remembered that both speed and magnitude of impulse are positive (scalar) quantities.

Occasionally students forget to take account of the directions, particularly when considering impulses, and so run into difficulties. Remind students that any direction of motion should be described in the context of the problem (such as 'direction reversed' or 'opposite direction') and not relative to their diagram (such as 'to the right' or by drawing an arrow).

NOTES

Conservation of momentum will be used again in Unit 3 (elastic collisions in one dimension); this will contain further analysis of the motions of colliding particles where extra information is available about the elasticity of the materials involved.