## UNIT 6: Proof

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### SPECIFICATION REFERENCES

**1.1 Construct proofs using mathematical induction**
**Contexts include sums of series, divisibility and powers of matrices**

### PRIOR KNOWLEDGE

Covered so far

- Series notation
- Index laws
- Matrix multiplication

### KEYWORDS

Mathematical induction, general statement, basis, assumption, inductive, conclusion, integer, summation, divisible, matrix.


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| 6a. Proof by mathematical induction (1.1) | **Teaching Time** 6 Hours |
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## OBJECTIVES

By the end of the sub-unit, students should:

- be able to obtain a proof for the summation of a series, using induction;
- be able to use proof by induction to prove that an expression is divisible by a certain integer;
- be able to use mathematical induction to prove general statements involving matrix multiplication.

## TEACHING POINTS

Need to revisit and highlight the nature of proof in mathematics and that this is one tool that will help but does have limitations. If the proof in P1 and 2 is not covered yet, this should could be discussed.

All the situations basically follow the same procedure:

1 Prove the statement is true for $n = 1$, ie, show that when $n = 1$, LHS = RHS.

2 Assume the statement to prove IS true for $n = k$ (where $k$ is a positive integer). This is just a matter of rewriting the original statement with $n$ replaced by $k$.

3 Write an expression for the next term i.e. $n = k + 1$.

4 Manipulate this expression and simplify it to look like the original expression with a $(k + 1)$ replacing all the $k$'s.

5 This has proved that the situation works for $k$ +1 if it works for $k$.

6 Therefore, it can be induced that it would work for $k + 2$, $k + 3$, etc. in a similar way. Hence it works for all positive integers $k$.

Students need to demonstrate their understanding of the concept of proof by induction, and not just learn the appropriate statements.

Give examples of the three different types of proof covered in this specification, with particular care given to proving that an expression is divisible by a certain integer, as this is the type students will typically find the most challenging.

## OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Use mathematical induction to produce a proof for a general term of a recurrence relation.

Proof by Strong Induction.

## COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Setting out is crucial at each stage. 'Almost all candidates started by attempting to check for $n = 1$, but often neglected to state that they had shown it to be true next to their working at this stage'.

Candidates may lose marks for not including all parts of their proof. An example of a minimum acceptable conclusion, following on from completely correct work, would be 'if the result is true for $n = k$ then it has been shown to be true for $n = k + 1$ and as it was shown true for $n = 1$ then the result is true for all positive integers'.

It is important to point out that for some proofs, students must show that the formula is true for both $n = 1$ and $n = 2$.

Simply trying to use standard results will result in a loss of a number of marks.


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For proofs of the summation of a series, it is usually best to expand brackets as little as possible. Common factors of bracketed expressions should be factorised out, as well as fractions. 'Candidates who took (2$k$+1) out as a factor at the beginning were generally successful'.

For proofs involving matrix multiplication, students must show sufficient work to justify being awarded full marks for their solution.

The proofs for divisibility tend to be the most problematic, with many students getting stuck after forming $f(k+1) - f(k)$ and this should be practised.

'Statements were often ones that had been learned, rather than being used in the appropriate context. The conclusions were often ill-conceived, particularly when defining the values for which the proof was valid'.

## NOTES

Proof by induction is not a method to derive formulae from first principles. It is simply used to check whether or not a general statement is true.


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