UNIT 3: Further algebra

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SPECIFICATION REFERENCES

2.6 Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem

1.1 Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including: proof by deduction, proof by exhaustion, disproof by counter-example

4.1 Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n; the notations $n!$ and ${}_nC_r$; link to binomial probabilities

PRIOR KNOWLEDGE

Algebraic manipulation covered so far

- Factorising quadratics
- notation

GCSE (9-1) in Mathematics at Higher Tier

A4 Expanding brackets

A2 Substitution

A6 Proof

KEYWORDS

Binomial, coefficient, probability, proof, assumptions, deduction, exhaustion, disproof, counter-example, polynomials, factorisation, quadratic, cubic, quartic, conjecture, prediction, rational number, implies, necessary, sufficient, converse, fully factorise, factor, expand, therefore, conclusion.

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3a. Algebraic division, factor theorem and proof (2.6) (1.1)

Teaching time
8 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to use algebraic division;
- know and be able to apply the factor theorem;
- be able to fully factorise a cubic expression;
- understand and be able to use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion;
- be able to use methods of proof, including proof by deduction, proof by exhaustion and disproof by counter-example.

TEACHING POINTS

When using algebraic division, only division by $(ax + b)$ or $(ax - b)$ will be required.

Different methods for algebraic division should be considered depending on students' prior experience and preferred ways of working. Whichever method is used, clear working out should be shown.

Equations in which the coefficient of x or x^2 is 0 for example $x^3 + 3x^2 - 4$ or $2x^3 + 5x - 20$ will need additional explanation and practice.

Students should know that if $f(x) = 0$ when $x = a$, then $(x - a)$ is a factor of $f(x)$. Questions in the form $(ax + b)$ should be covered.

Where a negative is being substituted into the equation the distinction between $(-2)^2$ and -2^2 will be important especially when students are using a calculator as examiners often comment on the fact that students will sometimes evaluate $(-2)^2$ as -4.

Factor theorem can be used to find an unknown constant. For example: Find a given that $(x - 2)$ is a factor of $x^3 + ax^2 - 4x + 6$. Two conditions can also be given in order to form simultaneous equations to solve.

When fully factorising a cubic, emphasis should be placed on choosing appropriate values. The final answer may need to be written as a factorised cubic or, alternatively, as the solutions to an equation which can then be used to sketch the curve. Students sometimes use the roots of a polynomial equation to help them factorise but this method must be used with care. Questions sometimes use the word 'hence' and so students must be careful which method they chose in these cases.

This is an excellent opportunity to review curve sketching by asking students to give a sketch following factorisation.

Students should be familiar with basic proofs from GCSE (9-1) Mathematics this knowledge can be built upon to look at the different types of proof. Students will need to understand how to set out each type of proof; the correct conventions in language and layout should be encouraged.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

The factor theorem can be introduced through investigation by substituting different values and checking against division to look for patterns.

Proof gives the opportunity to review previous concepts in a different way for example coordinate geometry. Proof will also be included in later topics.

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COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

The majority of errors seen in exam questions are not due to misunderstanding the method, but instead arithmetic and algebraic mistakes. For example, incorrect simplification of terms – especially those involving fractions; mistakes with negative numbers; and writing expressions rather than equations.

Students should be aware that long division is not always the best or quickest method to use and sometimes results in some complicated algebra.

When using the factor theoremf, stress the importance of checking the value that is substituted; a common error is to use, for example, f(1) rather than f(–1).

You should also emphasise the importance of fully factorising expressions, as a fairly significant number of students stop when they have reached one linear factor and a quadratic factor.

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3b. The binomial expansion (4.1)	**Teaching time** 7 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use the binomial expansion of $(a + bx)^n$ for positive integer n;
- be able to find an unknown coefficient of a binomial expansion.

TEACHING POINTS

Students should initially be introduced to Pascal's triangle, which can be used to expand simple brackets.

Students will need to be familiar with factorials and the ${}_nC_r$ notation.

Introduce the formal binomial expansion in the same way as the formula booklet and discuss the various terms to ensure all students understand.

Setting out work clearly and logically will be invaluable in helping students to achieve the final answer and also to spot mistakes if necessary.

Where there is a coefficient of x (other than 1) students will need to be reminded that the power applies to the whole term, not just the x, and that answers must be simplified appropriately. Negative and fractional coefficients will also need practice.

The limitations of the binomial expansion should be discussed.

Students should practice finding the coefficient of a single term, they should also be able to deal with setting up simple algebraic equations to find unknown constants.

Use of the binomial expansion can be linked to basic probability and approximations.

[Links can also be made with the statistics work in A level Mathematics.]

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students can be encouraged to discover the link between Pascal's triangle and the expansion of simple brackets.

Students could look at find the general term of a particular expansion.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Marks are most commonly lost in exam questions because of errors in expanding terms. For example not including the coefficient when calculating, say, $(ax)^2$; not simplifying terms fully; sign errors; and omitting brackets. Good notation will help to avoid many of these mistakes.

When writing expansions which involve unknown constants, some students fail to also include the x's in their expansion.

When using their expansions to work out the value of a constant, a significant number of students do not understand that the coefficient does not include the x or x^2 part and so are often unable to form an equation in the unknown alone.

Questions often go on to ask students to use their binomial expansion to evaluate a number raised to a power. For example, evaluating $(1.025)^8$ by substituting $x = 0.025$ into an expansion for $(1 + x)^8$. Students should be advised that simply using their calculator to evaluate $(1.025)^8$ will gain no marks as it is not answering the question.

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NOTES

Be aware of an alternative notation such as $\binom{n}{r}$ and ${}^{n}C_{r}$.

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