UNIT 7: Applications of forces

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SPECIFICATION REFERENCES

8.2 Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2D vectors); extend to situations where forces need to be resolved (restricted to 2 dimensions).

8.4 Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line; application to problems involving smooth pulleys and connected particles; resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces.

8.5 Understand and use addition of forces; resultant forces; dynamics for motion of a particle in a plane.

8.6 An understanding of $F \le \mu R$ in a situation of equilibrium.

9.1 Moments: problems involving parallel and non-parallel coplanar forces e.g. ladder problems.

PRIOR KNOWLEDGE

- Types of forces and force diagrams
- Assumptions made throughout this course (e.g. particle, rigid, light, etc.)
- S.I. units
- Moments and frictional forces
- Resolving forces

AS Mathematics – Mechanics content

7	Kinematics (constant acceleration) (See Unit 7 of the SoW)
8.1, 8.2, 8.4	Newton's laws of motion (See Unit 8 of the SoW)
8.4	Basic equilibrium (See Unit 8 of the SoW)

KEYWORDS

Force, resultant, component, resolving, plane, parallel, perpendicular, weight, tension, thrust, friction, air resistance, reaction, driving force, braking force, force diagram, equilibrium, inextensible, light, negligible, particle, rough, smooth, incline, uniform, friction, coefficient of friction, concurrent, coplanar.

NOTES

The guidance on the specification document specifies: 'Problems may be set where forces need to be resolved, e.g. At least one of the particles is moving on an inclined plane.'

7a. Equilibrium and statics (including ladder problems) (8.4) (8.5) (9.1)	Teaching time 4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand that a body is in equilibrium under a set of concurrent (acting through the same point) forces is if their resultant is zero;
- know that vectors representing forces in equilibrium form a closed polygon;
- understand how to solve problems involving equilibrium of a particle under coplanar forces, including particles on inclined planes and 2D vectors;
- be able to solve statics problems for a system of forces which are not concurrent (e.g. ladder problems), thus applying the principle of moments for forces at any angle.

TEACHING POINTS

This topic is a natural extension of AS Mathematics – Mechanics content (see SoW Unit 8a), which considers statics for systems whose forces are perpendicular (and do not need resolving at any angle) and **i**, **j** vector examples.

Recall the previous definition of equilibrium: the vector sum of the forces is zero, so the sum of their resolved parts in any direction is zero.

The book on an inclined plane provides the most common example of a weight on a slope. Stress the importance of key phrases like 'rough plane', which will introduce a frictional force. Also highlight the part of the sentence that says 'the book is *on the point* of moving *down* the plane' and emphasise that this indicates that the frictional force is in the up direction and is at its limiting value.

Cover examples

- Where the angle of incline is given in arctan or arcsin form, so students have to construct and read off sin and cos of the angle.
- Where weights are held in equilibrium by two strings at any angle (this is the same as a weight being tied onto a particular point of a single string – the knot makes it effectively two pieces of string with two different tensions).You could show an alternative graphical solution. For example, combining. the three forces to form a closed triangle (equilibrium means no resultant). Applying the sine rule to this triangle gives a useful result called Lami's theorem, but it can only be used for three forces in equilibrium.
- Where a ring is free to slide on a string (hence one tension).
- Where the forces are given in terms of **i** and **j**.

Finally, move on to ladder-type problems which will revise moments and then extend to any angle, as the forces will not be concurrent. Extend the moments formula to '*perpendicular force* × *distance*' and resolve the force to find its component at right angles to the full distance from the moments point.

Show students how to use the alternative formula '*force* × *perpendicular distance*', by measuring the perpendicular distance from the moments point to the line of action of the force.

Also make sure that students are clear about the directions of the frictional force (for examples involving rough surfaces) and the reactions at the wall and ground being labelled differently.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Extension: consider a uniform rod which has one end *freely hinged* to a wall and the other end tied to a point above the wall, making the bar horizontal. Discuss the fact that the reaction at the hinge is *not* perpendicular to the wall and that the lines of actions of all the forces in the system will all meet at one point for equilibrium. Representing the reaction at the hinge as two perpendicular forces, the 'resolving and taking moments' solution would be fairly straightforward.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students are often good at drawing force diagrams, but common errors are omitting arrowheads, incorrectly labelling (e.g. 4 kg rather than $4g$) and missing off the normal reaction or friction forces. Students can sometimes struggle to work out the direction of the frictional force.

Common errors in questions involving moments are ignored the weight of the ladder, sine/cosine confusion and missing a distance in one or more terms.

NOTES

The guidance on the specification document states 'Problems may be set where forces need to be resolved. (Restricted to forces in two perpendicular directions or simple cases of forces given as 2D vectors.)'

7b. Dynamics of a particle (8.2) (8.4) (8.5) (8.6)	**Teaching time** 4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- know and understand the meaning of Newton's second law;
- be able to formulate the equation of motion for a particle in 1-dimensional motion where the resultant force is mass × acceleration;
- be able to formulate the equation of motion for a particle in 2-dimensional motion where the resultant force is mass × acceleration;
- be able to formulate and solve separate equations of motion for connected particles, where one of the particles could be on an inclined and/or rough plane.

TEACHING POINTS

This topic is a natural extension of AS Mathematics – Mechanics content (see SoW Unit 8a), which considers dynamics for systems whose forces are perpendicular (and do not need resolving at any angle) and **i**, **j** vector examples.

Recall the previous definition of dynamics: the *vector sum of the forces* = *mass* × *acceleration*, so the sum of their resolved parts in any direction can now be represented as a *single* force. This force is called the resultant and is equal to *mass* × *acceleration* (Newton's second law).

We can use the equations of motion for constant acceleration to describe the motion in more detail e.g. time taken to come to rest etc.

The basic mathematical modelling is identical to that of setting up a statics problem, except when you resolve in the direction of motion; there will be a 'winning' resultant force.

For inclined plane problems stress, that it is often easier, to resolve along and perpendicular to the plane. Some students find it hard to understand that even though the particle is moving up/down, the forces are 'balanced' if we resolve perpendicular to the plane.

Make sure you cover examples in which a force 'pushing' up the plane is removed at a certain point. This means the frictional force and component of weight now influence the subsequent motion and act as 'braking forces' causing a retardation, bringing the particle to instantaneous rest (and then the friction changes direction, as the particle wants to slide back down the plane).

Provide some examples where the forces are given in terms of **i** and **j**. These are solved by applying Newton's Second Law in vector form, hence $\boldsymbol{F} = m\boldsymbol{a}$.

Connected particle problems (previously covered in AS Mathematics – Mechanics content, see SoW Unit 8b) can now be extended so at least one of the particles is placed on a rough or smooth inclined plane and/or a rough horizontal plane. This introduces the resolving and frictional concepts from the previous unit.

For 'car and caravan' type questions, the tow rope or tow-bar can now be modelled at an angle rather than horizontally.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

To make this dynamics topic more real, you could set up experiments, involving connected particles for example, and video the motions. You can then see that when one particle hits the ground, the second particle continuing to move up and the string become slack.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Common errors candidates make include: confusing the terms 'resultant' and 'reaction'; incorrectly treated the scenario as a statics problem and assuming the forces are in equilibrium; omitting g from the weight term; and, more rarely, including g in the 'ma' term.

NOTES

The guidance on the specification document states 'Connected particle problems could include problems with particles in contact e.g. lift problems.'

Students may be required to resolve a vector into two components or use a vector diagram, e.g. problems involving two or more forces, given in magnitude–direction form.