UNIT 12: Vectors (3D)
Use of vectors in three dimensions; knowledge of column vectors and i, j and k unit vectors (10.1)

Teaching time
5 hours

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SPECIFICATION REFERENCES

10.1 Use vectors in three dimensions
Knowledge of column vectors and **i**, **j** and **k** unit vectors in three dimensions

PRIOR KNOWLEDGE

GCSE (9-1) in Mathematics at Higher Tier

G24, G25 Vectors

AS Mathematics – Pure Mathematics content

10 Vectors (See Unit 5 of SoW)

KEYWORDS

Vector, scalar, column, 3D coordinates, vertices, Cartesian, **i**, **j**, **k**, magnitude, origin, distance, direction, angle, position vector, unit vector, orthogonal, vector addition/subtraction.

NOTES

This topic is a natural extension of the vector work in AS Pure Mathematics. It extends to 3 dimensions via an additional vector **k**, or a third entry in the column vector.

OBJECTIVES

By the end of the sub-unit, students should:

- be able to extend the work on vectors from AS Pure Mathematics to 3D with column vectors and with the use of **i**, **j** and **k** unit vectors;
- be able to calculate the magnitude of a 3D vector;
- know the definition of a unit vector in 3D;
- be able to add 3D vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations;
- understand and use position vectors, and calculate the distance between two 3D points represented by position vectors;
- be able to use vectors to solve problems in pure mathematics and in contexts (e.g. mechanics).

TEACHING POINTS

Begin by showing some 3D coordinates on x, y, z axes. (Graph drawing packages are very useful here, especially if you can turn the grid to view from different positions.)

Consider a cuboid (2 by 3 by 4), with one corner at the origin. Ask the class to write down the coordinates of all the vertices.

Remind students of 2D work and extend to 3D column vectors, orthogonal unit vectors **i**, **j**, **k** and position vectors.

Write all the vectors from the 'origin' corner of the cuboid as position vectors (e.g. OA = $\underline{\mathbf{a}}$, etc.)

Calculate the magnitude of these vectors as $\sqrt{2^2 + 3^2 + 4^2}$ for example.

Extend this idea to calculating the distance d between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) using $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$

Extend the ideas of vector addition and subtraction to 3D: $\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

Cover the triangle and parallelogram laws of addition, as well as demonstrating parallel vectors.

Show how to find a unit vector in the direction of $\underline{\mathbf{a}}$, and make sure students are familiar with the notation $|\boldsymbol{a}|$ (extended to 3D).

Use vectors to solve problems in pure mathematics and discuss the 3D geometrical interpretations of solutions.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Link examples to mechanics (kinematics and forces). For example, consider questions such as:

The velocity of an object is given by vector $\mathbf{v} = 3t\mathbf{i} + t^2\mathbf{j} + 4\mathbf{k}$

What is its speed after 5 seconds?

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Encourage students to draw diagrams to help their geometrical thinking when answering vector questions.

Stress the importance of reading the question carefully and giving answers in the correct way, for example coordinates or column vectors may be requested.

Emphasise the importance of good notation. Students do not always understand that AP^2 represents the square of the length AP.

NOTES

Please note that vector equations of straight lines and the scalar product are *not* on this specifications.