UNIT 4: Further algebra and functions (series)

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SPECIFICATION REFERENCES

4.4 Understand and use the method of differences for summation of series including use of partial fractions.

4.5 Find the Maclaurin series of a function including the general term.

4.6 Recognise and use the Maclaurin series for e^x, $\ln(1 + x)$, $\sin x$, $\cos x$ and $(1 + x)^n$, and be aware of the range of values of x for which they are valid (proof not required).

PRIOR KNOWLEDGE

AS Further Mathematics – Core Pure Mathematics

4.3	Sums of Series (See SoW Unit 4)
4.1, 4.2	Algebra and functions (See SoW Unit 5)

A level Mathematics – Pure content

2.6	Algebraic fraction manipulation (See SoW Year 2 Unit 1a)
2.10	Partial fractions (See SoW Year 2 Unit 1b)
7.2, 7.4, 7.5	Implicit differentiation (See SoW Year 2 Unit 2c)
7.4	Product rule (See SoW Year 2 Unit 2c)

KEYWORDS

Series, sigma, differences, sum, term, general term, partial fraction, function, Maclaurin, range, power.

4a. Method of differences (4.4)	**Teaching time** 6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to use the method of differences to sum simple finite series.

TEACHING POINTS

Start this topic with differences that do not involve fractions to introduce the idea e.g.
$\sum_{r=1}^{n}\frac{1}{2}\left(r(r+1)-r(r-1)\right) = \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$. Carefully listing the beginning and the end of the series in pairs, showing a sufficient number of terms to show how the cancelling process works will help understanding.

Move on to expressing the general term as $\mathrm{f}(r+1) - \mathrm{f}(r)$.

With the move to problems with partial fractions, the revisiting of how to develop the necessary fractions.

Stress the need to set out working carefully, especially with differences that involve fractions.

Show some examples without the summation sign and that this is just as easy and can they find the Summation function that can used to prove the function.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Consider problems with multiples of *n* on both the lower and upper limits leading to the difference between series found by the method of differences.

Look at proving the standard formulae from FP1 using this method.

It can be shown that there are some standard expressions that they can develop

e.g. $(k+1)^2 - k^2 = 2k + 1$ (Natural Numbers)

$(k+1)^3 - k^3 = 3k^2+3k+1$ (sum of square positive numbers)

$(k+1)^4 - k^4 = 4k^3+6k^2+4k+1$ (sum of cube positive numbers)

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Emphasise the importance of presentation as poor layout can lead to a student miscopying their own work or making other errors and so achieving a lower score.

Students should take care to include only the terms required and not extend beyond the limits given on the summation sign.

Where an answer is given, it is essential to show all the steps required to produce this answer; making leaps in working can result in loss of marks.

4b. Maclaurin series (4.5)(4.6)	**Teaching time** 6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to find and use higher derivatives of functions;
- know how to express functions as an infinite series in ascending powers using Maclaurin's expansion;
- be able to find the series expansion of composite functions.

TEACHING POINTS

Start with Taylor's series to show its usefulness in solving problems. Why do we use Maclaurin's and not Taylor's. Highlight the nature of these series and that they will converge for a specific domain. Include as many different types of function as possible to practise techniques including implicit differentiation and differentiation of a product. Finding values of these derivatives at $x = 0$ will facilitate the introduction of the Maclaurin series. A good time to prove the small angle formula and the point when x is small. How small?

Look explicitly at some binomial expansions to show how they can be expressed as infinite series, then suggest other familiar functions can also be expressed as series by considering a simple function, say e^x. Develop the formula for the Maclaurin expansion from here and stress the factorial denominators of the coefficients as these can be forgotten.

Emphasise the need for clear layout and proper use of brackets when moving on to composite functions.

For example, the expansion of $\cos(2x^2)$ has a second term of $-\frac{(2x^2)^2}{2!}$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Consider functions such as $\ln x$ and discuss if they have a Maclaurin expansion. Include the range of validity and consider how many terms are required to give a good degree of accuracy to the actual value of the function.

Use of technology to show the accuracy that you can go to with the series expansion. modelling the function and the expansion on a graph and if you truncate the series expansion. Sliders can be used to develop this.

How accurate does this need to be for a summation of a converging series. Can develop the understanding of radius of convergence. Valid domains can be linked here for the functions.

Good problem to link here with differentiation of inverse trig functions is Leibniz proof of finding an approximation for π using infinite series and arctan. Great discussion point of how and why he did this and how sophisticated it was for the 18th century.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

The general form of the series should be quoted before an attempt is made to substitute derivatives. Without it, method marks may not be awarded.