UNIT 7: Integration

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SPECIFICATION REFERENCES

8.1 Know and use the Fundamental Theorem of Calculus

8.2 Integrate x^n (excluding $n = -1$), and related sums, differences and constant multiples

8.3 Evaluate definite integrals; use a definite integral to find the area under a curve

PRIOR KNOWLEDGE

Covered so far

- Algebraic manipulation
- Differentiation

KEYWORDS

Calculus, differentiate, integrate, reverse, indefinite, definite, constant, evaluate, intersection.

7a. Definition as opposite of differentiation, indefinite integrals of x^n (8.1) (8.2)

Teaching time
6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- know and be able to use the Fundamental Theorem of Calculus;
- be able to integrate x^n (excluding $n = -1$), and related sums, differences and constant multiples.

TEACHING POINTS

Integration can be introduced as the reverse process of differentiation. Students need to know that for indefinite integrals a constant of integration is required.

Similarly to differentiation, students should be confident with algebraic manipulation. For example, the ability to integrate expressions such as $\frac{1}{2}x^2 - 3x^{-\frac{1}{2}}$ and $\frac{(x+2)^2}{x^{\frac{1}{2}}}$ is expected. Introduce students to the integral sign; this can be useful in setting work out clearly on these sorts of questions and will be used later in definite integration.

Given f′(x) and a point on the curve, students should be able to find an equation of the curve in the form y = f(x).

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to explain the need for the + c in indefinite integration.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students sometimes have difficulty when integrating expressions involving negative indices. Forgetting to add + c when working out indefinite integrals is also a very common mistake.

7b. Definite integrals and areas under curves (8.3)	Teaching time 5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to evaluate definite integrals;
- be able to use a definite integral to find the area under a curve.

TEACHING POINTS

It is important that students show their working out clearly as mistakes are easily made when putting values into a calculator. Students should also be encouraged to check their answers. Calculators that perform numerical integration can be used as a check, but a full method will be needed.

Students will be expected to understand the implication of a negative answer from indefinite integration.

Links can be made with curve sketching in questions where students need to find the points of intersection with the x-axis for a curve in order to find the limits of integration.

Areas can be made up of a combination of a curve and a line so further links can be made to coordinate geometry.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Discuss the implication of a negative answer to encourage students reasoning skills.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Lack of algebraic fluency can cause problems for some students, particularly when negative/fractional indices are involved or when a negative number is raised to a power. Arithmetic slips are also a common cause of lost marks, often when negative numbers are substituted and subtracted after integration.

Students are generally more successful if they expand any brackets before attempting to integrate the function.