UNIT 8: Further kinematics

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SPECIFICATION REFERENCES

7.3 Extend the constant acceleration formulae of motion to 2 dimensions using vectors.

7.4 Use calculus in kinematics for (variable acceleration) motion in a straight line. Extend to 2 dimensions using vectors.

PRIOR KNOWLEDGE

- Basic trigonometry, Pythagoras and vectors
- Find the magnitude and direction of vectors

<u>AS Mathematics – Mechanics content</u>

7 Kinematics 1 and equations of motion (See Unit 7b of the SoW)

7 Kinematics 2 (variable force) (See Unit 9 of the SoW)

<u>AS Mathematics – Pure Mathematics content</u>

10.1 2D vectors – **i**, **j** system (See Unit 5 of the SoW)

KEYWORDS

Distance, displacement, speed, velocity, constant acceleration, constant force, variable force, variable acceleration, retardation, deceleration, initial (t = 0), stationary (speed = 0), at rest (speed = 0), instantaneously, differentiate, integrate, turning point.

NOTES

This topic builds on the kinematics covered in AS Mathematics – Mechanics content, see SoW Units 7 and 9.

8a. Constant acceleration (equations of motion in 2D; the i, j system) (7.3)	Teaching time 3 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to recognise when the use of constant acceleration formulae is appropriate;
- be able to write positions, velocities and accelerations in vector form;
- understand the language of kinematics appropriate to motion in 2 dimensions
- be able to find the magnitude and direction of vectors;
- be able to extend techniques for motion in 1 dimension to 2 dimensions by using vectors;
- know how to use velocity triangles to solve simple problems;
- understand and use *suvat* formulae for constant acceleration in 2D;
- know how to apply the equations of motion to **i**, **j** vector problems;
- be able to use $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$, $\boldsymbol{r} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$ etc. with vectors given in **i** , **j** or column vector form.

TEACHING POINTS

This topic enables us to use the familiar *suvat* formulae for constant acceleration for more complex motions in two dimensions. It is important to stress that the acceleration may have different values for the **i** and **j** components, but is fixed in value for that direction and is therefore constant. Illustrate this by reviewing projectile motion (covered in Unit 6), which showed that the acceleration was zero in the horizontal direction and ± 9.8 m s^{-2} in the vertical direction, hence for a full trajectory $\boldsymbol{a} = (0\mathbf{i} - 9.8\mathbf{j})$ m s^{-2}. This gives a curved (parabolic) path even though the accelerations are constant.

Cover examples which ask for the speed, distance and direction of motion. Make sure that students can pick out the keywords, and realise when the answer can be left in **i**, **j** form and when to form a triangle and use Pythagoras and tan to calculate the magnitude and direction (e.g. when asked for the speed and direction of motion of a particle).

Also stress that the *angle* of the velocity vector gives the true direction of motion and that the acceleration's magnitude does not have a special keyword, but will just be asked for as magnitude of the acceleration.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Projectile questions could also be tackled using the vector equations of motion rather than separating out the horizontal and vertical motions (Unit 6).

Some graphical packages will draw the graphs using **i - j** vectors; these can be used to help students visualise the problems.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Candidates are generally able to use *suvat* equations in 2D to find unknown heights, velocities etc. However, some common errors are: finding a solution in vector form and not extracting one component e.g. to find the height; incorrectly finding velocity rather than speed and vice versa; and equating scalars and vectors and forgetting to split e.g. velocities into **i** and **j** components.

NOTES

If there is change in motion, we have a dynamics problem. These are solved by applying Newton's second law in vector form: $\boldsymbol{F} = m\boldsymbol{a}$. This naturally leads onto the next section in which the force is variable.

8b. Variable acceleration (use of calculus and finding vectors $\dot{\boldsymbol{r}}$ and $\ddot{\boldsymbol{r}}$ at a given time) (7.4)	Teaching time 3 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to extend techniques for motion in 1 dimension to 2 dimensions by using calculus and vector versions of equations for variable force/acceleration problems;
- understand the language and notation of kinematics appropriate to variable motion in 2 dimensions, i.e. knowing the notation $\dot{\boldsymbol{r}}$ and $\ddot{\boldsymbol{r}}$ for variable acceleration in terms of time.

TEACHING POINTS

This topic links directly to, and is an extension of AS Mathematics – Mechanics content (see SoW Unit 9), which used:

$$v = \frac{\mathrm{d}s}{\mathrm{d}t},\ a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} \text{ and } s = \int v\,\mathrm{d}t,\ v = \int a\,\mathrm{d}t$$

to model the rates of change for motion of a particle subject to a variable force.

Motions can now be more complicated as the forces in the **i** and **j** directions can differ and be variable (i.e. $\boldsymbol{F} = m\boldsymbol{a}$). Also the notation for 2D motion replaces the displacement, s, with position vector, $\boldsymbol{r}$. Velocity, $\boldsymbol{v}$, can be defined as $\dot{\boldsymbol{r}}$ and the acceleration vector can be called $\ddot{\boldsymbol{r}}$ (rather than $\boldsymbol{a}$).

Introduce this notation to students, explaining how the dot above the $\boldsymbol{r}$ denotes how many times the $\boldsymbol{r}$ has been differentiated with respect to time. Hence $\ddot{\boldsymbol{r}}$ (representing the acceleration) effectively means $\boldsymbol{r}$ differentiated *twice* with respect to time or $\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2}$.

The other vital point to stress is when we integrate $\dot{\boldsymbol{r}}$ (or $\boldsymbol{v}$) to obtain the displacement $\boldsymbol{r}$, we have to introduce a vector constant of integration in the form $c\mathbf{i} + k\mathbf{j}$ (rather than just $+ c$). Any conditions provided in the question (e.g. the particle is initially at the point with position vector $(3\mathbf{i} + 2\mathbf{j})$ m) allow us to substitute into the expression for $\boldsymbol{r}$ and calculate the constants.

Ask questions along the lines of:

Consider an aeroplane taking off. Its position is given by $\boldsymbol{r} = (80t\,\mathbf{i} + 0.5t^3\,\mathbf{j})$ m. What is its velocity and acceleration at time t? Now criticise the model. (Hint: consider motion in the x-direction)

Reverse the process: a particle has acceleration $\boldsymbol{a} = (4t\mathbf{i} + 2\mathbf{j})$ m s^{-2} and is initially at the origin moving with velocity $2\mathbf{i}$ m s^{-1}. Find $\dot{\boldsymbol{r}}$ and $\mathbf{r}$ using integration. (Be careful with the constants of integration!)

Just as in the 1-dimensional case, we do not need to use calculus every time; if the acceleration vector is constant, we can use vector forms of the *suvat* formulae as in Unit 8a.

Questions on this topic often ask about the direction of motion: stress that this is given by the direction of the velocity vector. To find when an object is moving due North, the East component of the velocity vector is zero and the North component positive.

Finally, a question may ask for the force acting on the particle of mass m kg. In this situation students will need to find the acceleration ($\ddot{\boldsymbol{r}}$) at time t and then state the force $\boldsymbol{F}$ as $\boldsymbol{F} = m\ddot{\boldsymbol{r}}$ or $\boldsymbol{F} = m\boldsymbol{a}$ (in terms of **i** and **j**).

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

The variable is always t for this unit. (See Further Mathematics – Further Mechanics 2 content, for when the force is dependent on other factors and variables such on the displacement or velocity.)

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Some common errors students make include: forgetting the constant of integration; giving the final answer as a vector when the question asked for the speed; and not being careful about changes of direction and so, for example, finding the displacement rather than the distance travelled.

NOTES

The following diagram may help students decide whether to differentiate or integrate to solve a problem. '**d**' for the **d**own arrow means '**d**ifferentiate'. Hence, down from $\boldsymbol{r}$ gives $\boldsymbol{v}$ or $\dot{\boldsymbol{r}}$ or $\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = v$. Integration is the opposite of differentiation so up is integrate. Up from $\boldsymbol{a}(\ddot{\boldsymbol{r}})$ gives $v(\dot{\boldsymbol{r}})$ or integral of $\boldsymbol{a}(\ddot{\boldsymbol{r}})$ with respect to t gives $\boldsymbol{v}(\dot{\boldsymbol{r}})$.

↓ $\boldsymbol{s}$

diff ↓ $\boldsymbol{v}$ $(\dot{\boldsymbol{r}})$ ↑ int

$\boldsymbol{a}$ $(\ddot{\boldsymbol{r}})$ ↑