UNIT 8: Exponentials and logarithms
Exponential functions and natural logarithms
(6.1) (6.2) (6.3) (6.4) (6.5) (6.6) (6.7)

Teaching time
12 hours

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SPECIFICATION REFERENCES

6.1 Know and use the function a^x and its graph, where a is positive

Know and use the function e^x and its graph

6.2 Know that the gradient of e^{kx} is equal to $k\mathrm{e}^{kx}$ and hence understand why the exponential model is suitable in many applications

6.3 Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$

Know and use the function $\ln x$ and its graph

Know and use $\ln x$ as the inverse function of e^x

6.4 Understand and use the laws of logarithms:

$$\log_a x + \log_a y = \log_a (xy)$$

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$$

$$k\log_a x = \log_a x^k \text{(including, for example, } k = -1 \text{ and} = -\tfrac{1}{2}\text{)}$$

6.5 Solve equations of the form $a^x = b$

6.6 Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y

6.7 Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models

PRIOR KNOWLEDGE

Covered so far

- Indices

<u>GCSE (9-1) in Mathematics at Higher Tier</u>

R16 Compound interest

KEYWORDS

Exponential, exponent, power, logarithm, base, initial, rate of change, compound interest

OBJECTIVES

By the end of the sub-unit, students should:

- know and be able to use the function $a^x$and its graph, where a is positive;
- know and be able to use the function e^x and its graph;
- know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications;
- know and be able to use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$;
- know and be able to use the function $\ln x$ and its graph;
- know and be able to use $\ln x$ as the inverse function of e^x ;
- understand and use the laws of logarithms:

$$\log_a x + \log_a y = \log_a (xy)$$
$$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$$
$$k\log_a x = \log_a x^k \text{(including, for example, } k = -1 \text{ and } k = -\tfrac{1}{2})$$

- be able to solve equations of the form $a^x = b$;
- be able to use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y;
- understand and be able to use exponential growth and decay in modelling, giving consideration to limitations and refinements of exponential models.

TEACHING POINTS

When sketching the graph of a^xstudents should understand the difference in shape between $a < 1$ and $a > 1$.

Explain to students that e^x is a special case of a^x. Graphs of the function e^x should include those in the form $y = e^{ax+b} + c$.

Students should realise that when the rate of change is proportional to the y-value, an exponential model should be used.

An ability to solve equations of the form $e^{ax+b} = p$ and $\ln(ax + b) = q$ is expected.

Students can use the laws of indices to prove the laws of logarithms and show that $\log_a a = 1$.

In solving equations students may use the change of base formula. Solving equations questions may be in the form $2^{3x-1} = 3$.

Students should be able to plot $\log y$ against $\log x$ and obtain a straight line where the intercept is $\log_a$ and the gradient is n and plot $\log y$ against x to obtain a straight line where the intercept is $\log k$ and the gradient is $\log b$. There should be discussion about why this is an appropriate model and why it is only an estimate.

Contexts for modelling should could include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth. Students should be familiar with terms such as initial, meaning when $t = 0$. They may need to explore the behaviour for large values of t or to consider whether the range of values predicted is appropriate. Consideration of a second improved model may be required.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students can look at different models for population growth using the exponential function.

Use graphing software to investigate varying the parameters of a population model.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Errors seen in exam questions where students have to sketch exponential curves include: stopping the curve at $x = 0$; getting the wrong y-intercept; and believing the curve levels off to $y = 1$ for $x < 0$.

When using laws of logs to answer proof or ‘show that’ questions, students must show all the steps clearly and not have jumps in their working out.