UNIT 7: Parametric equations

Return to overview

SPECIFICATION REFERENCES

3.3 Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms

3.4 Use parametric equations in modelling in a variety of contexts

PRIOR KNOWLEDGE

Covered so far

- Trigonometric identities
- Knowledge of a variety of functions involving powers, roots, trigonometric functions, exponentials and logarithms

GCSE (9-1) in Mathematics at Higher Tier

G11 Coordinate geometry

A2, A5 Changing the subject of the formula, and substitution

A12 Graphs of linear, quadratic and trigonometric functions

AS Mathematics – Pure Mathematics content

2.7, 3.1, 3.2 Coordinate geometry (See Unit 2 of SoW)

5.5, 5.7 Trigonometric identities (See Unit 4b of SoW)

KEYWORDS

Parametric, Cartesian, convert, parameter t, identity, eliminate, substitute, circle, hyperbola, parabola, ellipse, domain, modelling.

NOTES

Later in the course, students will need to be able to differentiate (using the chain rule) parametric equations to find tangents, normals, turning points etc.

7a. Definition and converting between parametric and Cartesian forms (3.3)	Teaching time 3 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand the difference between the Cartesian and parametric system of expressing coordinates;
- be able to convert between parametric and Cartesian forms.

TEACHING POINTS

Begin by explaining the difference between the Cartesian system, when a graph is described using $y = \mathrm{f}(x)$, and the parametric system, which uses $x = \mathrm{f}(t)$ and $y = \mathrm{g}(t)$ for some parameter t.

Illustrate this by asking the class to consider $x = 5t$ and $y = 3t^2$ and to try to eliminate t from the two equations. This will give $y = \frac{3}{25}x^2$ or $25y = 3x^2$. (This is a quadratic equation – parabola.)

Repeat for $x = 5t$ and $y = \frac{5}{t}$. This becomes $y = \frac{25}{x}$ (a hyperbola).

Sometimes we need to eliminate the parameter, t, by using identities rather than substitution.

Consider $x = 3\cos t$ and $y = 3\sin t$. Squaring both equations and adding means we can use $\cos^2 t + \sin^2 t = 1$ to give $x^2 + y^2 = 9$. (This is a circle, centre (0, 0) of radius 3.)

Ask students to use similar methods to show that $x = 2 + 5\cos t,\ \ y = -4 + 5\sin t$ describes a circle centre (2, −4) with radius 5.

How do we convert from Cartesian to parametric? (We need to be in radians) For example, what are the pair of parametric equations for a circle, centre (3, 5) radius 10?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

What shape is given by $x = 4\cos t$, $y = 2\sin t$?

Name and properties of curve? (See sub-unit 7b for plotting.)

The trigonometric identities in Unit 6 (such as $\sec^2 x = 1 + \tan^2 x$) can be used to convert from parametric to Cartesian form.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students may have difficulties making any progress with these sorts of questions if they cannot work out which trigonometric identity to apply when eliminating the parameter t.

NOTES

The next section will look at how to plot parametric equations and modelling examples.

7b. Curve sketching and modelling (3.3) (3.4)	**Teaching time** 2 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to plot and sketch curves given in parametric form;
- recognise some standard curves in parametric form and how they can be used for modelling.

TEACHING POINTS

It is often easier to match the properties of a curve in parametric form than it is in its Cartesian form.

In order to establish the shapes of some well-known curves such as circles, ellipses etc., ask the students to plot the pair of parametric equations in the form of a table of values.

When plotting $x = 4 \cos t$, $y = 4 \sin t$ what will the range of t be? (Remember to use radians.)

Now plot $x = 4 \cos t$, $y = 2 \sin t$. (This is the shape mentioned in the reasoning/problem solving section of sub-unit 7a.)

What values of t will we need for $x = 5t$, $y = \frac{5}{t}$?

Investigate parametric equations which give closed loops. These will be integrated later in course to find the area of a loop, so we need to establish how values of t link plotting (direction vital).

The specification states 'Students should pay particular attention to the domain of the parameter t, as a specific section of a curve may be described.'

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

A shape may be modelled using parametric equations (e.g. an object moves with constant velocity from (1, 8) at $t = 0$ to (6, 20) at $t = 5$), or students may be asked to find parametric equations for a motion.

Make links to Unit 10 (Kinematics) of Applied Paper 3.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

The examiner comments for these questions illustrate how difficult students find this topic:

The final part proved very demanding and only a minority of students were able to use one of the trigonometric forms of Pythagoras to eliminate t and manipulate the resulting equation to obtain an answer in the required form.

Few even attempted the domain and the fully correct answer $0° \le t \le 2\pi$, was very rarely seen.

NOTES

Parametric equations is assumed knowledge for the calculus work in Further Mathematics – Further Pure Mathematics where students must find the volume of revolution for a solid formed by a pair of parametric equations