Pearson

UNIT 1: Complex numbers

Return to Overview

SPECIFICATION REFERENCES

2.1 **Solve any quadratic equation with real coefficients.**
Solve cubic or quartic equations with real coefficients.

2.2 **Add, subtract, multiply and divide complex numbers in the form $x + \mathrm{i}y$ with x and y real.**
Understand and use the terms 'real part' and 'imaginary part'.

2.4 **Use and interpret Argand diagrams.**

2.5 **Convert between the Cartesian form and the modulus-argument form of a complex number.**

2.6 **Multiply and divide complex numbers in modulus argument form.**

2.7 **Construct and interpret simple loci in the Argand diagram such as $|z - a| > r$ and $\arg(z - a) = \theta$.**

PRIOR KNOWLEDGE

GCSE (9-1) in Mathematics at Higher Tier

- Like terms, expanding brackets and factorising (including difference of two squares)
- Surds
- Solving quadratic equations by factorising, completing the square and using the quadratic formula
- Trigonometric ratios, such as $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
- Use of the discriminant
- Factor theorem
- Dividing polynomials
- Binomial expansion
- Vectors (**i** and **j** components)
- Radians
- Compound angle formulae
- Equation of a straight line
- Equation of a circle

AS Mathematics – Pure content

2.3 Quadratic functions (See SoW Year 1 Unit 1b)
2.6 Algebraic division (See SoW Year 1 Unit 2b)
3.2 Equation of a circle (See SoW Year 1 Unit 6b)
9.1 Vectors in two dimensions (See SoW Year 1 Unit 5a)

A level Mathematics – Pure content

5.2 Radians (See SoW Year 2 Unit 8a)

5.6 Compound angle formulae (See SoW Year 2 Unit 8d)

KEYWORDS

Conjugate, real part, imaginary part, complex conjugate, root, discriminant, Argand diagram, Cartesian coordinates, vector, magnitude, modulus, argument, principal argument, radians, modulus-argument form, polynomial, coefficient, quadratic, quartic, cubic, locus, loci.

1a. Introduction of complex numbers, basic manipulation (2.1, 2.2)	Teaching Time 3 Hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to solve any quadratic equation with real coefficients;
- be able to add, subtract and multiply complex numbers in the form $x + \mathrm{i}y$ with x and y real;
- understand and use the terms 'real part' and 'imaginary part'.

TEACHING POINTS

Begin by looking at previously 'unsolvable' quadratics, (relate to discriminant: $b^2 - 4ac < 0$). Sketch the graph and show that it does not cross the x-axis (no **real** roots). Put the idea across that we need an **imaginary** axis to cross for some sort of root to exist.

Define i as $\sqrt{-1}$. Hence $\mathrm{i}^2 = -1$.

Emphasise that in a complex number, the real part and the imaginary part cannot be combined to form a single term. Introduce addition and subtraction of complex numbers as adding (or subtracting) the real parts and adding (or subtracting) the imaginary parts. Similarly, multiplying complex numbers uses the same techniques as for multiplying brackets in algebra, and then simplifying powers of i and using $\mathrm{i}^2 = -1$.

Show students both methods of using the quadratic formula or completing the square to find complex solutions of a quadratic equation. Particular care must be taken with signs when using the quadratic formula.

It is worth noting that if the quadratic formula is quoted incorrectly, no marks will be gained.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Expand brackets such as $(2 + 3\mathrm{i})^8$ using the binomial theorem.

Can you multiply two different complex numbers to get a real number?

How would you deal with a complex number fraction? Links to the next section and that complex numbers follow the same rules as surds.

What happens when you square or cube complex numbers?

Can some complex numbers be squared to become imaginary numbers?

Equating Complex numbers to use the real to real and imaginary to imaginary e.g. find the values of a and b if the following complex numbers are equal. $z_1 = a(2 + \mathrm{i}) + (b - \mathrm{i})$, $z_2 = a(3 + 2\mathrm{i}) + 4\mathrm{i} + b\mathrm{i}$

Use of complex numbers in Geometric series, e.g. $u_{n+1} = (2 + \mathrm{i})u_n$, $u_1 = 3$, Find the first 5 terms. The Sum to n terms, etc. Can link to another series and look at common ratios.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

The examiners commented on how handwriting was difficult to read at times. In particular, the number "2" and the letter "x" were often written badly by many candidates. It is important that candidates write numbers, letters and symbols clearly so that marks are not lost unnecessarily.

Students should be encouraged to write down the quadratic formula before they use it.

Care must be taken when using the quadratic formula; a common error is the loss of the square root sign so that $\frac{\sqrt{1-4\times1\times2}}{2}$ becomes $\frac{1\pm7\mathrm{i}}{2}$.

NOTES

Practice simplifying terms such as $\sqrt{-24}$

1b. Argand diagrams (2.4)

Teaching Time
2 Hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to use and interpret Argand diagrams.

TEACHING POINTS

Link Argand diagrams with **i**, **j** vectors, if previously covered in AS Mathematics.

The position of the complex number on the Argand diagram (the quadrant in which it appears) will determine whether its argument is positive or negative and whether its argument is acute or obtuse. Students should be encouraged to take care when drawing Argand diagrams, and to use an accurate scale.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Begin thinking about loci diagrams, e.g. what would $|z| = 3$ look like?

What would the square or cube of a complex number look like on an argand diagram? Can you explain what happens? Can you explore this graphically with variables.

What would the Argand diagram look like when you add complex numbers together? (Geogebra)

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Students will lose marks due to lack of required labelling. 'Many Argand diagrams were messy with little regard for scale, but labelling enabled the candidates to earn the marks'.

'Errors made included plotting the pure imaginary roots on the real axis or not plotting the complex roots as a conjugate pair'.

1c. Modulus and argument (2.5, 2.6)

Teaching Time
6 Hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to convert between the Cartesian form and the modulus-argument form of a complex number;
- be able to multiply and divide complex numbers in modulus-argument form.

TEACHING POINTS

Link modulus and argument with magnitude and direction of vectors as introduced in AS Mathematics.

Encourage students to always draw a diagram when calculating $\arg(z)$. $\tan^{-1}\left(\frac{y}{x}\right)$ only works for the 1st and 4th quadrants. Remember that principal argument is $-\pi < \arg(z) \leq \pi$.

Make the mod-argument form clear and note that it can also be written as $z = r\cos(\theta)$.

Show that when complex numbers are multiplied the modulus is multiplied and the arguments are added.

$z = r_1\cos(\theta_1)$, $w = r_2\cos(\theta_2)$

$zw = r_1r_2\cos(\theta_1 + \theta_2)$

This can be shown using the cartesian form to prove the solution.

For division the moduli are divided and the arguments subtracted.

Emphasise that using the mod-arg form for dividing is much easier and quicker that dividing the Cartesian from. Students should be very confident that they can easily work from one to the other quickly.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Find the square roots of a complex number.

Using compound angle formulae to prove the arguments are added in multiplication.

How to deal with a complex number in the form $z = \cos\theta - \mathrm{i}\sin\theta$

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Students are advised to draw a diagram so they can visualise the argument. Some students will use $\tan\left(\frac{2}{3}\right)$ rather than $\arctan\left(\frac{2}{3}\right)$.

i should not be included in the calculation of the modulus.

1d. Loci (2.7)	**Teaching Time** 4 Hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to construct and interpret simple loci in the Argand diagram such as $|z-a|>r$ and $\arg(z-a)=\theta$.

TEACHING POINTS

Consider sums and differences and how they look in 'vector' diagrams. Illustrate how modulus diagrams look. Stress the vital fact that the angle can vary for a fixed distance – hence loci.

The locus of z (the path which represents all possible values of z) will form a curve or a straight line.

To work out which region is required for the inequalities, choose a point (for circles, the centre of the circle is usually a good choice) to find whether it is in the region required.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Extend to equations such as $|z-3|=|z+\mathrm{i}|$.

Sketch the locus of $arg\left(\frac{z-6}{z-2}\right)=\frac{\pi}{4}$ and find the Cartesian equation of this locus.

For the complex number $z=x+\mathrm{i}y$, find the minimum and maximum values of $|z|$ given that z satisfies an equation such as $|z-12-5\mathrm{i}|=3$.

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

'Many lost a mark by failing to conclude explicitly that their equation represented a circle'.

'A surprising number of candidates shaded the outside of the circle rather than the inside and few gave any reason for their decision. Few thought to use the coordinates of the centre of the circle to find whether it was in the region required'.