UNIT 4: Series

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SPECIFICATION REFERENCES

4.3 Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series

PRIOR KNOWLEDGE

GCSE (9-1) in Mathematics at Higher Tier

- Expanding and factorising quadratic and cubic expressions

A level Mathematics - Pure content

4.3 Sigma notation (See SoW Year 2 Unit 6b)

KEYWORDS

Sigma notation, series, sum, arithmetic series, geometric series, binomial series, integer, natural numbers,

4a. Sums of series (4.3)	**Teaching Time** 4 Hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to use sigma notation;
- understand and use formulae for the sums of integers, squares and cubes;
- be able to use known formulae to sum more complex series.

TEACHING POINTS

If arithmetic and geometric series in A level Mathematics has been covered, this could be reviewed, showing that $\sum r$ is an arithmetic series with $a = 1$ and $d = 1$.

Knowledge that $\sum_1^n 1 = n$ is expected.

Always multiply brackets before attempting to evaluate summations of series.

Any number of terms can be 'split up' using the addition rule and the multiple rule. For example, $\sum_{r=1}^{n}(r^2 + r - 2)$should first be 'split up' to $\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r - 2\sum_{r=1}^{n} 1$. Then the standard results for $\sum_{r=1}^{n} r^2$, $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} 1$ can be used.

Show how the proofs of the standard results are generated. The student should have an understanding of how they are generated.

Look carefully at the limits for the summation. In general: $\sum_{r=k}^{n} \mathrm{f}(r) = \sum_{r=1}^{n} \mathrm{f}(r) + \sum_{r=1}^{k-1} \mathrm{f}(r)$; however a common mistake is to use $\sum_{r=k}^{n} \mathrm{f}(r) = \sum_{r=1}^{n} \mathrm{f}(r) + \sum_{r=1}^{k} \mathrm{f}(r)$.

When asked to find a general result for a sum it is good practice to test it for small values of n. This does not prove it is correct, but if one value of n does not work, then it is obvious that the result is incorrect.

To find $\sum_{r=1}^{2n} r^2$, use the standard result $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n + 1)(2n + 1)$ and replace n by $2n$ to give $\sum_{r=1}^{2n} r^2 = \frac{2n}{6}(2n + 1)(4n + 1)$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Using partial fractions and the method of differences to sum series involving fractions.

Prove $\sum_{r=1}^{n} r^2$. Can you find a general term for $\sum_{r=1}^{n} r^3$?

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

It is a common mistake to write $5\sum_{r=1}^{n} 1$ as 5, instead of $5n$.

As shown above, a common mistake is to use $\sum_{r=k}^{n} \mathrm{f}(r) = \sum_{r=1}^{n} \mathrm{f}(r) + \sum_{r=1}^{k} \mathrm{f}(r)$ instead of $\sum_{r=k}^{n} \mathrm{f}(r) = \sum_{r=1}^{n} \mathrm{f}(r) + \sum_{r=1}^{k-1} \mathrm{f}(r)$.

In expanding $(3r-2)^2$, it is not unusual to see $(3r)^2 = 3r^2$. Students almost always use the correct formulae for $\sum r^2$ and $\sum r$ but evaluating $\sum 4$ as 4 rather than $4n$ is common. Taking out a factor of $\frac{n}{3}$ or $\frac{n}{6}$ in order to achieve the required result usually results in less errors than expanding completely to obtain a cubic before attempting a factor. The most common errors result from taking out the factor of $\frac{n}{3}$ or $\frac{n}{6}$ and not compensating for the $\frac{1}{3}$ or the $\frac{1}{6}$ on all the terms inside the bracket.

The approach to take out common factors as early as possible should be encouraged if it is appropriate to the question.

NOTES

$$\sum_{r=1}^{n} r^3 = \left(\sum_{r=1}^{n} r\right)^2$$

The formulae for $\sum_{r=1}^{n} 1 = n$ and $\sum_{r=1}^{n} r = \frac{n}{2}(n+1)$ should be learnt.