UNIT 7: Vectors

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SPECIFICATION REFERENCES

6.1 **Understand and use the vector and Cartesian forms of an equation of a straight line in 3D**

6.2 **Understand and use the vector and Cartesian forms of the equation of a plane**

6.3 **Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane**

6.4 **Check whether vectors are perpendicular by using the scalar product**

6.5 **Find the intersection between a line and a plane**

Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane

PRIOR KNOWLEDGE

AS Mathematics - Pure content

10.1 Vectors in 2D (See SoW Year 1 Units 5a and 5b)

A level Mathematics - Pure content

10.1 Vectors in 3D (See SoW Year 2 Unit 11)

KEYWORDS

Vector, scalar, magnitude, modulus, direction, vector quantity, scalar quantity, displacement vector, zero vector, unit vector, base vector, component, equal vectors, localised vector, free vector, resultant, triangle law, parallelogram law, position vector, vector equation, Cartesian equation, scalar product, dot product, collinear, skew, concurrent, direction ratio, perpendicular, Pythagoras' Theorem.

7a. Vector and Cartesian equations of a line and a plane (6.1, 6.2)	**Teaching Time** 9 Hours

OBJECTIVES

By the end of the sub-unit, students should:

- know how to find the vector equation of a line in both two and three dimensions;
- understand and use the Cartesian forms of an equation of a straight line in three dimensions;
- understand and use the vector and Cartesian forms of the equation of a plane.

TEACHING POINTS

Students should be familiar with both the vector form (**r** = **a** + λ**b**) and the Cartesian form ($\frac{x-a_1}{b_1} = \frac{y-a_2}{b_2} = \frac{z-a_3}{b_3}$) of the equation of a straight line. Show similarities with $y = mx + c$ and **r** = **a** + λ**b**.

Similarly, the forms **r** = **a** + λ**b** + μ**c** (vector form) and $ax + by + cz = d$ (Cartesian form) of the equation of a plane should be familiar.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Can link to mechanics and the intersection of vectors, both in 2D and 3D. Can touch on Euclidean geometry as use of these in the real world context.

Show problems that link vector to Cartesian forms of a plane and how the Cartesian form is used.

Look at the link of vectors and complex numbers in Quaternions.

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Students must give the equation of a line or plane as an equation and not as an expression, i.e. they will lose marks for not writing **r** =…

They need to know the difference between giving an answer in vector form or Cartesian form.

7b. Scalar product (6.3, 6.4)	**Teaching Time** 5 Hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to find the scalar product of two vectors;
- be able to check whether vectors are perpendicular by using the scalar product;
- be able to use the scalar product to express the equation of a plane;
- be able to use the scalar product to calculate the angle between two lines;
- be able to use the scalar product to calculate the angle between two planes;
- be able to use the scalar product to calculate the angle between a line and a plane.

TEACHING POINTS

The scalar form of the equation of a plane (**r•c** = p) should be familiar.

The formulae for angle between two lines: $\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|a||\mathbf{b}|}$, the angle between two planes: $\cos\theta = \left|\frac{\mathbf{n}_1\cdot\mathbf{n}_2}{|n_1||\mathbf{n}_2|}\right|$ and the angle between a line and a plane: $\sin\theta = \left|\frac{\mathbf{b}\cdot\mathbf{n}}{|\mathbf{b}||\mathbf{n}|}\right|$, are all similar, but they need to be learnt.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Proof of the Cosine rule using vectors and that it is uses the scalar product.

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Students must check whether the answers for angles should be given in degrees or radians, and also whether the angle should be acute or obtuse.

When applying the scalar product formula, the direction of the vectors is important. The vectors should be coming 'out' of the angle.

7c. Problems involving points, lines and planes (6.5)

Teaching Time
7 Hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to find the points of intersection of lines and planes which meet;
- be able to calculate the perpendicular distance between two lines;
- be able to calculate the perpendicular distance from a point to a line or to a plane.

TEACHING POINTS

The intersection of line and planes

The intersection of planes and the system of 3 solutions and the use of matrices and technology to help solve these. The emphasis of unique solution, an infinite solution and no solutions.

The perpendicular distance between two lines or between a point and a line or a plane gives the shortest distance between them. The perpendicular distance from (α, β, γ) to $n_1x + n_2y + n_3z + d = 0$ is $\frac{|n_1\alpha+n_2\beta+n_3\gamma+d|}{\sqrt{n_1{}^2+n_2{}^2+n_3{}^2}}$. This is given in the formula booklet.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Nrich problem - Walls. Perpendicular distance between 2 planes with 2 common points.

Extend the perpendicular distance to look at distance between 2 planes. Could bring in here the cross product.

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

When finding points of intersection, students will sometimes successfully find μ or λ, but then proceed no further and not attempt to find the actual point of intersection.

Take care when using the formula for perpendicular distance from a point to a plane not to miss out the '+*d*' in the numerator.