## Unit 5: Critical Path Analysis (part 1)

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### SPECIFICATION REFERENCES

**4.1 Modelling of a project by an activity network, from a precedence table.**

**4.2 Completion of the precedence table for a given activity network.**

**4.3 Algorithm for finding the critical path. Earliest and latest event times. Earliest and latest start and finish times for activities. Identification of critical activities and critical path(s).**

**4.4 Calculation of the total float of an activity. Construction of Gantt (cascade) charts.**

### PRIOR KNOWLEDGE

Covered so far

- Introduction to graph theory (See Unit 1c)

### KEYWORDS

Activities, events, precedence table, activity networks, source node, sink node, dummies, earliest event times, latest event times, critical path, critical activities, total float, Gantt (cascade) chart, lower bound.

## 5a. Activity networks; precedence tables (4.1) (4.2)

**Teaching time**
5 hours

### OBJECTIVES

By the end of the sub-unit, students should:

- **be able to model a project by an activity network from a precedence table;**
- **be able to complete a precedence table from a given network;**
- **understand the use of dummies.**

### TEACHING POINTS

There are lots of relevant examples of projects that will be familiar to students. This section is about dividing the amount of work up into tasks and then ordering these tasks in a deliberate way. Students need to follow a systematic approach and use clear labelling.

Introduce a project such as a referendum or general election – what tasks have to be completed to ensure that the results can be published within 24 hours of the ballot closing? Ask students to come up with as many tasks as they can and use their responses to introduce the concepts of dependencies (voting can't start until the ballot boxes are in place).

Lead students to drawing a table to identify dependencies, and then discuss how complex the table would become in large projects.

The duration of each task is not yet required, but it will be in the next section so mention it here.

When formalising precedence tables, show that only immediate dependence is shown in the table, and that we only include activities that are not already written down.

It is often tricky to get the layout of activity networks correct first time; students should draw in pencil and be prepared to rub out! Key points for activity networks are:

- they are a visual representation of the project
- only *activity on arc* networks will be used – the activities are represented by the arcs, and the completion of the activities, called events, are shown by nodes
- each arc is labelled with letters and arrows, and is a straight line
- start with the source node (labelled 0), finish with the sink node
- there should only be one end point.

Students often find dummies difficult; they should understand that a dummy carries no weight (no time or cost) and that its direction is important. Exam questions frequently ask what the purpose of a specific dummy is, so this should be carefully explored.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should formulate their own project e.g. organising the prom, applying for university etc. and list the activities in a precedence table and draw up an activity network.

Students should be able to answer questions about activity networks, such as 'what activities that must have finished before X can start?' or 'what is the only activity that Y does not depend on?'.

## COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Common errors in exams include: using activity on vertex; lack of arrows (particularly serious on dummy arcs); not having a single start and a single finish; and incorrect dummies.

Students are encouraged to produce a rough, and then a neat version of their network in an exam, as it is very difficult to draw a clear diagram on the first attempt. When constructing a clear diagram, students should check carefully that they have one start and one finish, the required number of dummies (if stated), and all activities present.

## NOTES

Activity on arc will be used. The use of dummies is included.

In a precedence network, precedence tables will only show immediate predecessors.

## 5b. Critical path algorithm; earliest and latest event times (4.3)

**Teaching time**
4 hours

### OBJECTIVES

By the end of the sub-unit, students should:

- **know how to carry out a forward pass and backward pass using early and late event times;**
- **be able to interpret and use dummies;**
- **be able to identify critical activities and critical paths.**

### TEACHING POINTS

Adapt the activity networks to include the duration of each activity. Explain early and late event times, and forward and backward passes using a fairly simple project, like planning and delivering an assembly. Students find these times difficult if the projects are abstract or if they are presented with only a part of an activity network.

Dummies need careful attention here, as students sometimes forget that they carry no weight.

Students should be able to identify critical activities and critical paths.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Choose examples where there is more than one critical path, and some where an activity connecting two critical events isn't a critical activity.

Explore early and late event times and list each activity's earliest and latest start and finish times. Examine the effect of increasing the duration of a critical activity.

### COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Dummy activities can cause problems for students when calculating early and late times.

### NOTES

Some students may easily confuse the working methods used here with those used in Dijkstra's algorithm.

## 5c. Total float; Gantt charts (4.4)

**Teaching time**
3 hours

### OBJECTIVES

By the end of the sub-unit, students should:

- **know how to determine the total float of activities;**
- **be able to construct and interpret Gantt (cascade) charts.**

### TEACHING POINTS

Define the total float and then use the formula

Total float = latest finish time – duration – earliest start time

Explain how a Gantt (cascade) chart provides a graphical way of representing the range of possible start and finish times for all activities on a single diagram. It is often easiest to construct with the critical activities running in a single line along the top of the chart.

Ensure that students can read the scale showing elapsed time.

Students can then use Gantt charts to identify which activities *must* be happening or *could* be happening at any stage during the project.

### OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Begin investigating the minimum number of people (workers) needed to complete the project in the minimum time. As well as asking which activities must be happening at any given time, look at how many workers that would entail. What if an activity required more than one worker?

### COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Floats can cause problems – as indicated by the following comments:

"Most students gained full marks, showing the three numbers used in the calculation of each of the floats. Some just stated the value of each float, losing two of the three marks."

"Many calculated the floats correctly and showed the three numbers they used to calculate each float as requested, others wasted time creating a table of all the early and late start and finish times and the durations, for all activities, but then did not indicate which numbers they had used to find the floats and so lost marks."

"A number thought that a zero float was a consequence of the early and late times being the same. Many used 'flow' instead of 'float', many confused event and activity and arc with node."

"Many incorrect total float calculations were seen, candidates MUST show their working here if they are to gain full credit. As always a few candidates found the sum of their total floats."

Students should be reminded of the need to present their diagrams clearly; many Gantt charts can be extremely difficult to read – e.g. the line between the activity and its float is not always clear, or floats are frequently very faint. It would be advisable for candidates to check to see if their diagrams include all the activities and to re-check the precedences of their activities when they have finished their diagrams.

When using a cascade chart to determine a lower bound, students need to state both the time they are looking at and the activities that must be happening then.

Many students struggle to interpret the "time" on the scale for a Gantt chart in the context of the question. For example, if the time referred to is days, note that "time 0-1" is the same as "day 1", and that "time 0.5" would be midday on day 1. It is therefore often safer to refer to time unless context is specifically requested.

**NOTES**

Gantt charts show the degree of flexibility in the timing of each activity however they do not show the dependencies – students should read Gantt charts alongside either a dependency table or the activity network. Alternatively, vertical lines can be introduced to represent dependencies.