UNIT 3: Complex numbers (part 2)

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SPECIFICATION REFERENCES

2.1 **Solve any quadratic equation with real coefficients.**
Solve cubic or quartic equations with real coefficients.

2.3 **Understand and use the complex conjugate.**
Know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs.

PRIOR KNOWLEDGE

GCSE (9-1) in Mathematics at Higher Tier

- Like terms, expanding brackets and factorising (including difference of two squares)
- Surds
- Solving quadratic equations by factorising, completing the square and using the quadratic formula
- Trigonometric ratios, such as $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
- Use of the discriminant
- Factor theorem
- Dividing polynomials
- Binomial expansion
- Vectors (**i** and **j** components)
- Radians
- Compound angle formulae
- Equation of a straight line
- Equation of a circle

AS Mathematics – Pure content

2.3 Quadratic functions (See SoW Year 1 Unit 1b)
2.6 Algebraic division (See SoW Year 1 Unit 2b)
3.2 Equation of a circle (See SoW Year 1 Unit 6b)
9.1 Vectors in two dimensions (See SoW Year 1 Unit 5a)

A level Mathematics – Pure content

5.2 Radians (See SoW Year 2 Unit 8a)
5.6 Compound angle formulae (See SoW Year 2 Unit 8d)

KEYWORDS

Conjugate, real part, imaginary part, complex conjugate, root, discriminant, Argand diagram, Cartesian coordinates, vector, magnitude, modulus, argument, principal argument, radians, modulus-argument form, polynomial, coefficient, quadratic, quartic, cubic, complex conjugate pair, locus, loci.

3a. Complex conjugate, division and solving polynomial equations (2.3, 2.1)	**Teaching Time** 6 Hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and use the complex conjugate of a complex number;
- be able to divide two complex numbers by using the complex conjugate of the denominator;
- know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs;
- be able to solve cubic or quartic equations with real coefficients.

TEACHING POINTS

Use the difference of two squares and surds (rationalisation) to illustrate the method of manipulation of complex conjugates. The process for dividing complex numbers is similar to the process used to divide surds. For surds the denominator is rationalised. For complex numbers the denominator is made real.

Emphasise that zz* is real.

Revisit the factor theorem.

Highlight the fundamental theorem of Algebra and the nature of polynomials and roots.

Take care when finding the complex conjugate of a number given in the form '(imaginary part)i + real part', e.g. 8i - 3. The complex conjugate is – 8i – 3, and not 8i + 3.

For cubic equations, either:

i. all three roots are real; or
ii. one root is real and the other two roots from a complex conjugate pair

For quartic equations, either:

i. all four roots are real; or
ii. two roots are real and the other two roots form a complex conjugate pair; or
iii. two roots form a complex conjugate pair and the other two roots also form a complex conjugate pair

Show students both methods of using the quadratic formula or completing the square to find complex solutions of a quadratic.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Questions involving area of shapes such as rectangles and triangles.

Relate the roots of cubic and quartic equations to the graphs.

Find the equation of the polynomial given the roots.

Extend the nature of the roots to higher powers of polynomials.

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Students should be encouraged to write down the quadratic formula before they use it.

Care must be taken when using the quadratic formula; a common error is the loss of the square root sign so that $\frac{\sqrt{1-4\times1\times2}}{2}$ becomes $\frac{1\pm7\text{i}}{2}$.

'It is worth noting that a significant number of candidates did not know what was meant by z^*'.

Remind students to be careful not to negate the real part instead of the imaginary part.

'Almost all students knew that they were required to multiply through by the conjugate of the denominator, but some lost accuracy because of sign errors or not collecting like terms correctly. There were a number of students who did not present the solution in the required form'.

Common algebraic errors include $(2\text{i})^2 = 2\text{i}^2$ and equivalent.

'Candidates need to be reminded that correct answers may not get full marks if insufficient working is shown'.