UNIT 4: Trigonometry

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SPECIFICATION REFERENCES

5.1 Understand and use the definitions of sine, cosine and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form $\frac{1}{2}ab\sin C$

5.3 Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity

5.5 Understand and use $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Understand and use $\sin^2\theta + \cos^2\theta = 1$

5.7 Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle

PRIOR KNOWLEDGE

Algebra covered so far

- Basic algebraic manipulation
- Quadratics
- Graph transformations

<u>GCSE (9-1) in Mathematics at Higher Tier</u>

G20 Pythagoras' Theorem

Trigonometry in right-angled triangles

G22 The sine rule

The cosine rule

G23 The area of a triangle

G15 Bearings

KEYWORDS

Sine, cosine, tangent, interval, period, amplitude, function, inverse, angle of elevation, angle of depression, bearing, degree, identity, special angles, unit circle, symmetry, hypotenuse, opposite, adjacent, intercept.

4a. Trigonometric ratios and graphs (5.1) (5.3)	**Teaching time** 6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use the definitions of sine, cosine and tangent for all arguments;
- understand and be able to use the sine and cosine rules;
- understand and be able to use the area of a triangle in the form $\frac{1}{2}ab\sin C$;
- understand and be able to use the sine, cosine and tangent functions; their graphs, symmetries and periodicity.

TEACHING POINTS

Students should be shown the x and y coordinates of points on the unit circle can be used to give cosine and sine respectively.

Use of trigonometric ratios will have been covered at GCSE (9-1) Mathematics; questions should now be focused more on multi-step problems and questions set in context.

When using the sine rule the ambiguous case should be covered.

Links to proof can be made, for example proving the area of a triangle.

Students should be encouraged to write down any formulae they will be using before substituting in the numbers.

Students should be able to solve questions in various contexts; these could include coordinate geometry or real-life situations. Questions may involve bearings, which may not be well remembered from GCSE so should be reviewed. Students should be encouraged to check that their answers are realistic as this check can show up errors.

When completing multi-step questions emphasise to students that they should show all working out and use the answer function on their calculators to avoid rounding errors. It can be a useful teaching point to divide the class asking one side to round all answers and the other to keep values stored in their calculator to show how this affects the final answer.

The unit circle can again be used to show how the trigonometric graphs are formed. Characteristics such as the period and amplitude should be discussed. Knowledge of graphs of curves with equations such as $y = \sin x$, $y = \cos (x + 30)$, $y = \tan 2x$ is expected so this is a good opportunity to recap transformations.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Use of the graphs can be linked to modelling situations such as yearly temperatures, wave lengths and tidal patterns.

Proof of the sine and cosine rules.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students occasionally assume that triangles given in exam questions are right-angled and so use right-angled trigonometric ratios rather than the sine and cosine rules.

A frequently seen error in these questions is students using the cosine rule to calculate an incorrect angle, sometimes despite having drawn a correctly labelled diagram. This indicates a lack of understanding of how the labelling of edges and angles on a diagram relates to the application of the cosine rule formula.

4b. Trigonometric identities and equations (5.5) (5.7)

Teaching time
10 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to solve trigonometric equations within a given interval
- understand and be able to use $\tan\theta = \frac{\cos\theta}{\sin\theta}$
- Understand and use $\sin^2\theta + \cos^2\theta = 1$

TEACHING POINTS

When solving trigonometric equations, finding multiple values within a range can initially be illustrated using the graphs of the functions. The decision can then be made whether to move on to using CAST diagrams or continue using graphs. Whichever method is used students will need plenty of practice in identifying all values within the limits correctly.

Intervals with negative solutions as well as positive solutions should be used.

Students should be able to solve equations such as $\sin(x + 70°) = 0.5$ for $0 < x < 360°$; $3 + 5\cos 2x = 1$ for $-180° < x < 180°$; and $6\cos 2x + \sin x - 5 = 0$ for $0 < x < 360°$, giving their answers in degrees.

Students should be comfortable factorising quadratic trigonometric equations and finding all possible solutions. It should be noted that in some cases only one of the factorisations will give solutions but in most case there will be two sets of solutions. Situations where one answer is equal to zero can cause some confusion with students then not looking for further solutions. This sort of example should be covered in class. For example, the equation, $\sin\theta(3\sin\theta + 1) = 0$ will often be simplified to just $3\sin\theta + 1 = 0$, resulting in the loss of solutions to the original equation.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Following from the previous section, if graphs are used to model situations then the equations can be used to find values at given points.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Common errors include: not finding values in the given range; finding extra, incorrect, solutions; not going on to find the values of x and instead leaving the values for, say $2x$ or $x + 30$; algebraic slips when rearranging the equation; and not giving answers to the correct degree of accuracy. The loss of accuracy in the final answers to trigonometric equations is common and often results in the unnecessary loss of marks.

Sketches of the trigonometric functions are often helpful to check all solutions have been found.