UNIT 3: Polar coordinates

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SPECIFICATION REFERENCES

7.1 Understand and use polar coordinates and be able to convert between polar and Cartesian coordinates.

7.2 Sketch curves with r given as a function of θ, including use of trigonometric functions.

7.3 Find the area enclosed by a polar curve.

PRIOR KNOWLEDGE

A level Mathematics – Pure content

3.3	Cartesian coordinates (See SoW Year 2 Unit 9a)
3.3, 3.4	Curve sketching (See SoW Year 2 Unit 9b)
8.2 – 8.8	Integration (See SoW Year 2 Unit 3)

KEYWORDS

Polar, Cartesian, coordinates, convert, parallel, point, enclosed, area

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3a. Convert between Cartesian and polar (7.1) and sketch $r(\theta)$ (7.2)	**Teaching time** 5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use polar coordinates and be able to convert between polar and Cartesian coordinates;
- know how to sketch standard polar curves.

TEACHING POINTS

Introduce polar coordinates using a simple curve e.g. a circle centered at the origin. Relate to work already covered with Complex Numbers. Support the development of ideas by sketching points whenever possible. Reminding students that $x = r\cos\theta$ and $y = r\sin\theta$ can also be a very useful prompt. Online calculators can be used to check the answers of more demanding questions.

Online polar plotting tools give allow you to experiment quickly with the range of values chosen for θ. Using sliders to vary the values $r = p\cos(q\theta)$ *or* $a + b\cos(q\theta)$.

Good opportunity to develop some enquiry based learning with how the constant coefficients affects the function and how it looks like on a polar plot. This will develop the student understanding for the shape that they are looking for when hand plotting and the necessary turning points. For example, plot $r = 3\cos(2\theta)$ for 0 to π and then 0 to 2π. Look at the symmetries of the functions as they are developed. This will help with the sketching and their understanding of what the plots are for various different types.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

How can you have a straight line on a polar plot, this is a good to link Cartesian and polar.

Set problems such as finding all the sets of polar coordinates that represent the same point in space. This will emphasise the characteristics of polar coordinates.

Derive $r^2 = x^2 + y^2$ and make links to the (unit) circle.

How can you develop a cardioid for Valentine’s Day using their understanding of the shapes?

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

A sketch is an essential part of any solution strategy and should be used whenever appropriate.

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3b. Area enclosed by a polar curve (7.3)

Teaching time
5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to find tangents parallel and perpendicular to the initial line;
- be able to find (compound) areas under polar graphs using the formula $\frac{1}{2}\int r^2 \mathrm{d}\theta$

TEACHING POINTS

Use and understand $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}}$ to derive gradients of tangents for each problem rather than using a general formula. Be careful to stress that the product rule typically results in more than one term on the numerator and denominator.

Emphasise that typically areas to be found are ‘enclosed by’ rather than ‘under’. Choose examples that have been sketched in the earlier sub-unit and encourage sketching whenever possible.

Online graphing/calculating tools can be used to confirm the areas being found, especially when problems are more complex e.g. finding the area enclosed by inner loops.

Integration of trigonometric functions is required here, so it would be good to begin with a reminder of standard integrals and the formulae for double angles to make the resulting integrals more accessible.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Identify areas to be found from sketches e.g., the area between inner and outer loops of the curve $r = 2 + 4\cos\theta$

Find tangents not parallel to the initial line e.g. $r = 3 + 8\sin\theta$ at $\theta = \frac{\pi}{6}$

Find the equation of a normal to a polar curve.

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Students should take care to provide their answers in the required form and to the required accuracy. For example, marks will be lost for a decimal approximation if the question asks for an exact area.

Errors in integration are often caused by incorrect limits or by substituting limits incorrectly. Marks are also lost due to basic errors in expanding expressions and algebraic manipulation.

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