**Year 13 Further Maths Half Term 2 Assessment**

**1.**

**a** Using the definitions for cosh *x* and sinh *x* in terms of exponentials, show that

cosh2 *x* – sinh2 *x* = 1

**(3 marks)**

**b** Find the exact value of *x* for which

2 tanh *x* – cosh *x* = 0

giving your answer as a natural logarithm.

**(5 marks)**

**2**

**a** Find the polar equation for the Cartesian equation 

giving your answer in the form 

**(3 marks)**

**b** Show that the curve with Cartesian equation  has polar equation 

where *A* is a constant to be found.

**(3 marks)**

**3** Sketch the following curves, where *a* is a positive constant.

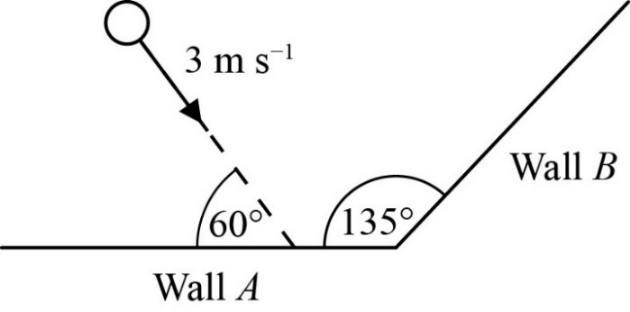
**a** 

giving the coordinates of the point where the curve intersects the half line 

**(2 marks)**

**b** ,

giving the coordinates of the points where the curve intersects the initial line and the half line  **(3 marks)**

**4** Two infinitely long, smooth vertical walls, *A* and *B*, are fixed on a smooth horizontal surface and intersect at an angle of 135o. A smooth sphere of mass 0.25 kg is projected across the surface towards wall A with a speed of 3 m s-1 and at an angle of 60o to wall *A,* as shown in the diagram below.

The coefficient of restitution between the sphere and each of the walls is denoted by *e*.

After its initial collision with wall *A* the ball is known to rebound and collide, at some point, with wall *B*

**a** Show that the range of possible values for *e* is given by 

**(3 marks)**

The exact value of *e* is found to be 

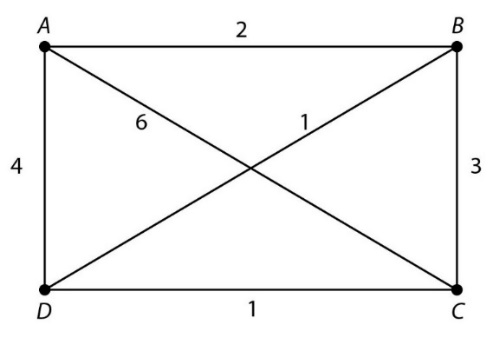
**b** After colliding with wall *B* the direction of motion of the sphere makes an angle *θ* ° with wall *B*.

Find the value of *θ*

**(3 marks)**

**c** Find the overall kinetic energy lost due to the two collisions.

**(6 marks)**

**5** The figure shows a network with the weights on the arcs representing distances.

Two iterations of Floyd’s algorithm have been completed. The resulting distance table and route table are shown below.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** | **D** |  |  | **A** | **B** | **C** | **D** |
| **A** | - | 2 | **5** | **3** |  | **A** | A | B | B | B |
| **B** | 2 | - | 3 | 1 |  | **B** | A | B | C | D |
| **C** | **5** | 3 | - | 1 |  | **C** | B | B | C | D |
| **D** | **3** | 1 | 1 | - |  | **D** | B | B | C | D |

**a** Apply Floyd’s algorithm for two more iterations to obtain the final distance and route tables.

You should show both the distance table and the route table after each iteration. Use the blank tables in the insert to help you.

**(4 marks)**

**b** Explain how to use your final matrix to find the shortest route from vertex *A* to vertex *C*.

State this distance. **(3 marks)**

**6** The cubic equation  has roots  and 

**a** Write down the values of ,  and 

**(1 mark)**

**b** Given that  and find the value of 

**(3 marks)**

**c** Find the value of *k*.

**(2 marks)**

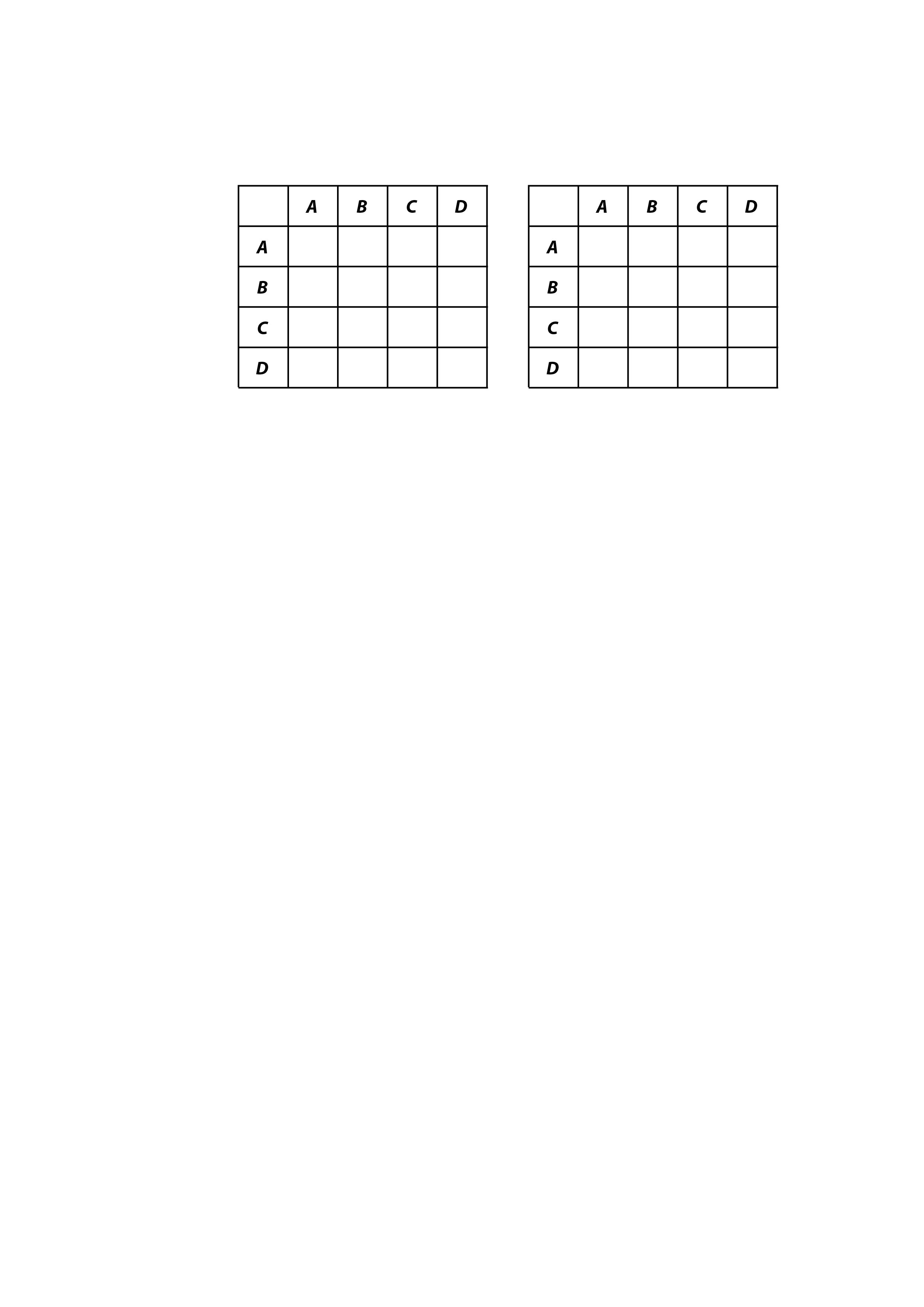
**7** Prove by induction that for all positive integer *n*, 

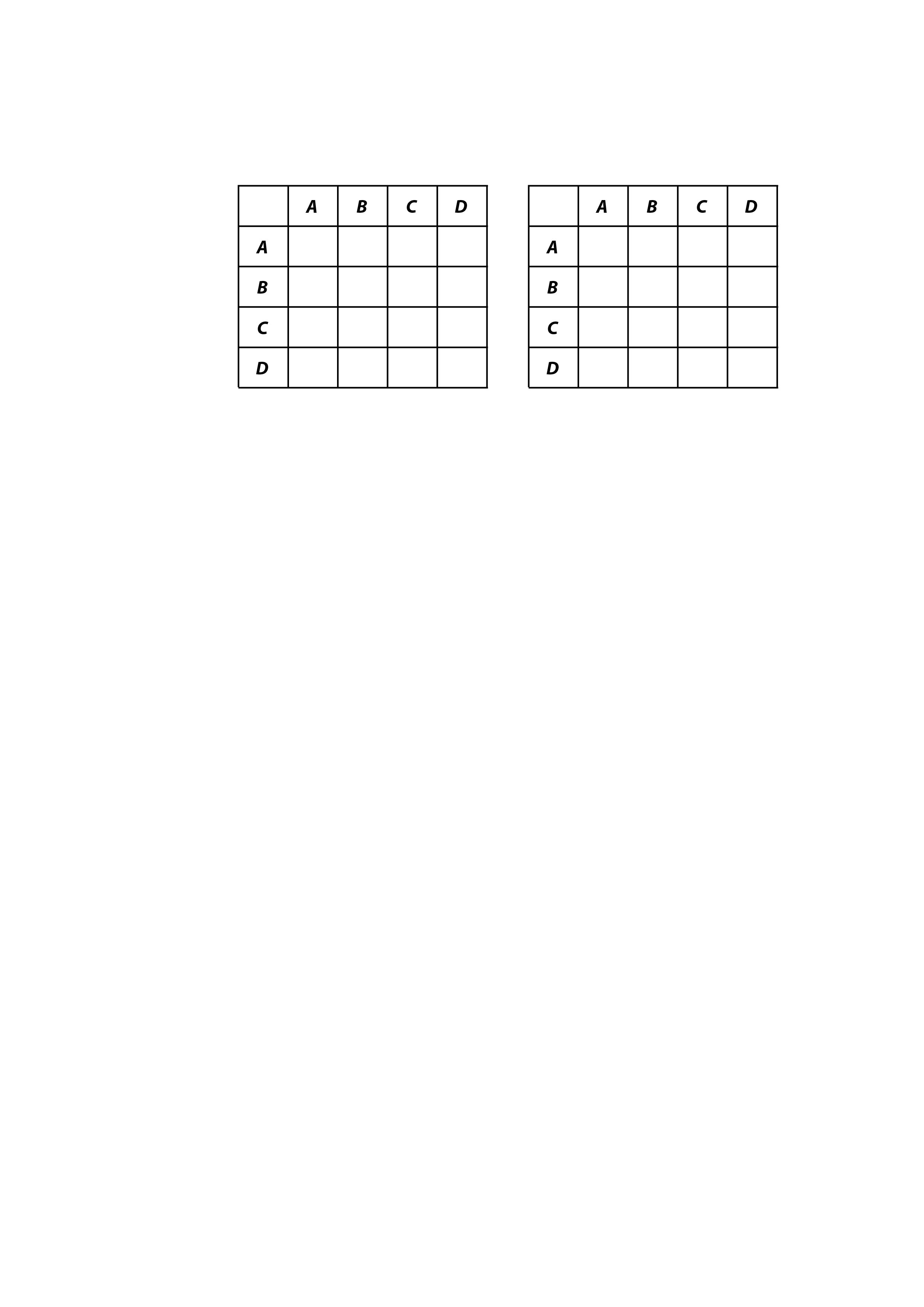
**(6 marks)**

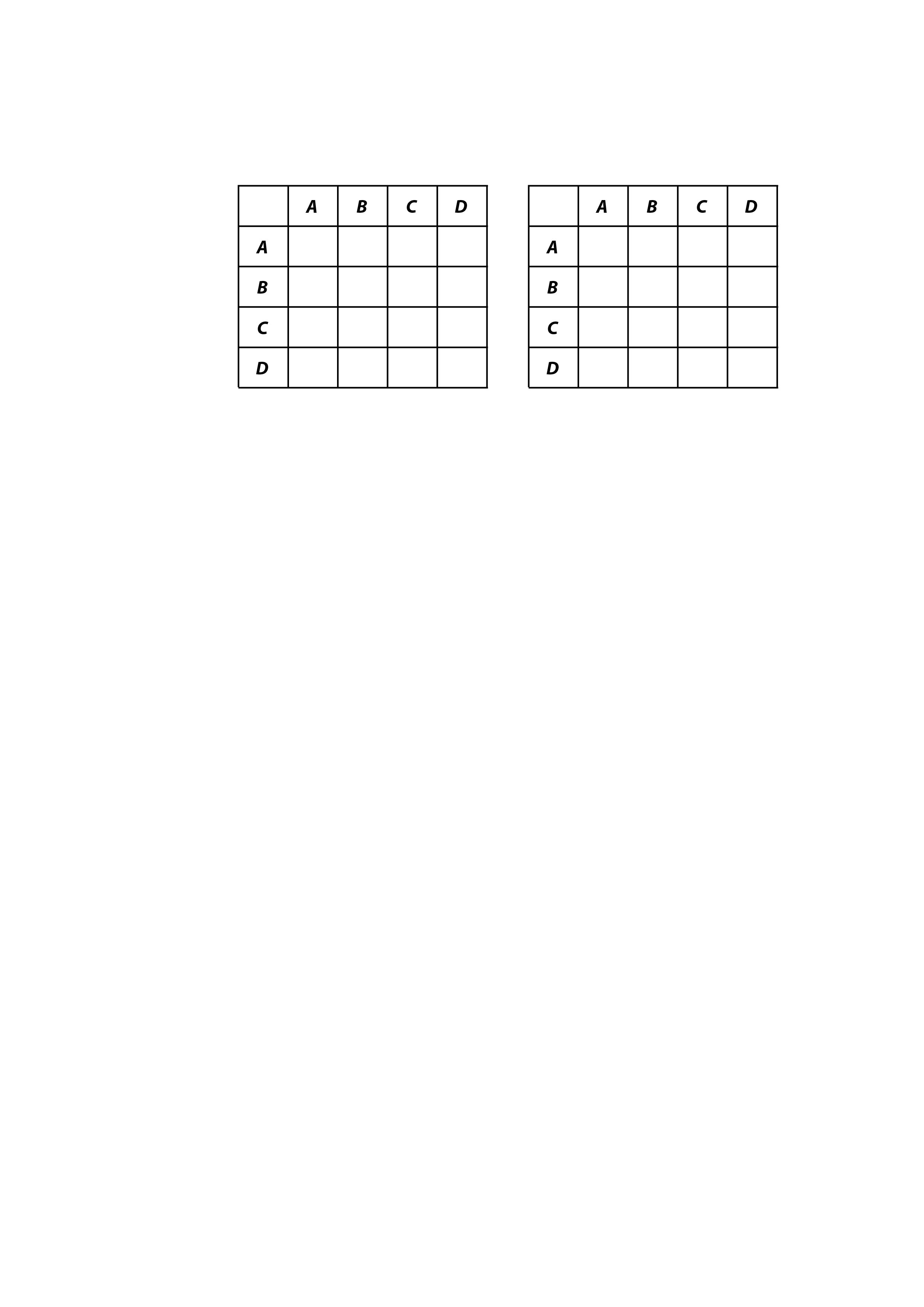
**END OF TEST**

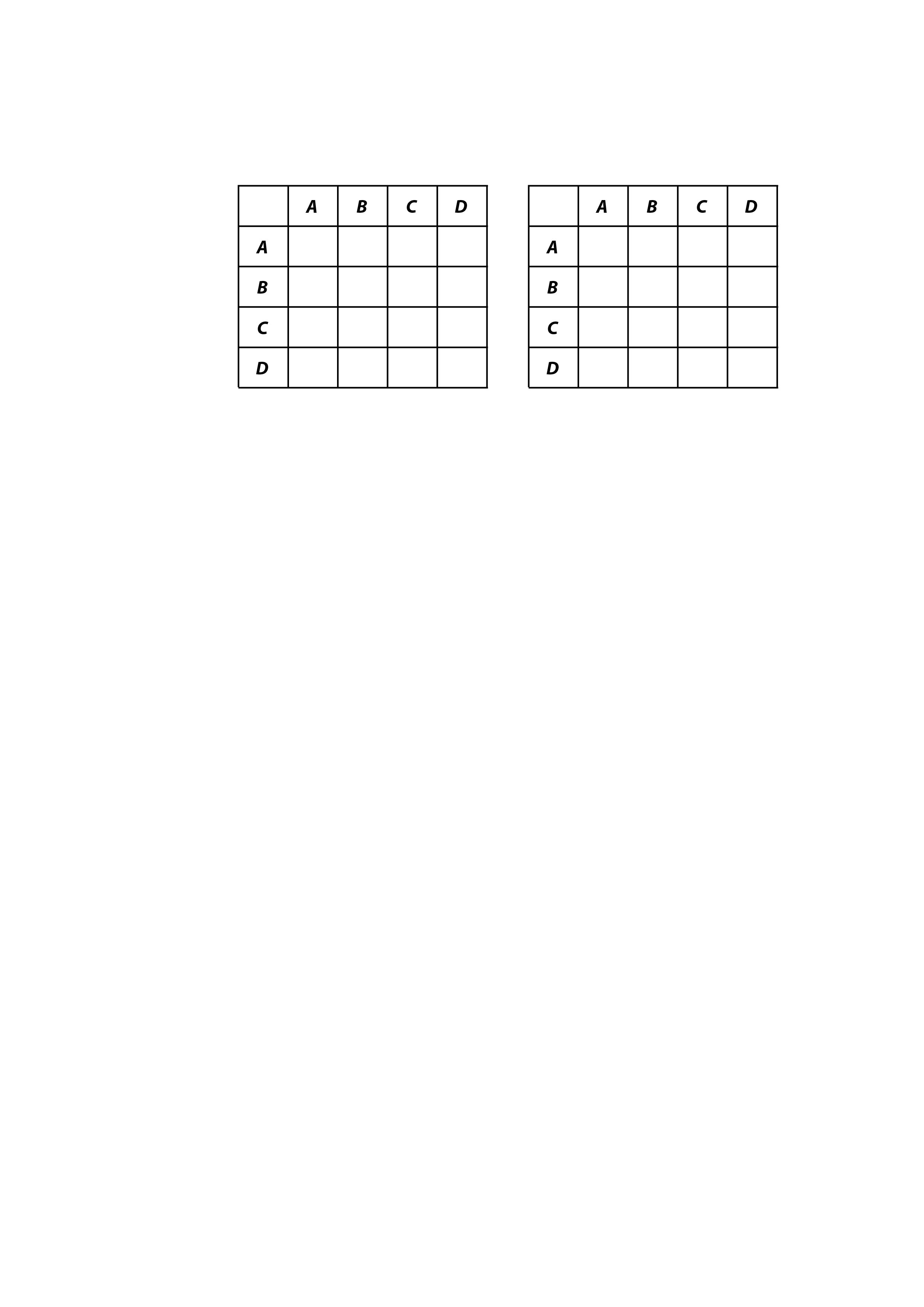
**TOTAL MARKS 50**

**INSERT FOR QUESTION 5**









EXTRA QUESTION for Mechanics 2 strand

To replace Question 5

**5M2**

A town planner has created a new horizontal road, which includes a curved corner modelled by a quarter circle of radius *r*. She knows that the coefficient of friction between any vehicle and the new road will be 0.75 in wet conditions.

**a** Show that the maximum safe speed,  for any vehicle to travel around this curved corner, without slipping, is given by , where *g* is the constant of acceleration due to gravity. **(4 marks)**

The planner ensures that a speed limit sign, equal to 10% below  is visible. A driver sees the speed limit sign of 72 km/h and takes the curve at this constant speed.

**b** Find the value of in ms–1 **(2 marks)**

**c** Find the time, in seconds to 3 significant figures, for the driver to travel along this curved corner. **(4 marks)**

**MARK SCHEME**

|  |  |  |
| --- | --- | --- |
| **1a** |  | **M1** |
|  | **M1** |
|  | **A1\*** |
| **Alt** | cosh2 *x* – sinh2 *x* = (cosh *x* + sinh *x*)(cosh *x* – sinh *x*) | **M1** |
|  | **M1** |
|  | **A1\*** |
|  | **(3)** |
| **1b** |  | **M1** |
| 2 sinh *x* = cosh2 *x*  2 sinh *x* = 1 + sinh2 *x*  sinh2 *x* – 2 sinh *x* + 1 = 0 | **M1** |
| (sinh *x* – 1)2 = 0  sinh *x* = 1 | **M1**  **A1** |
|  | **A1** |
|  | **(5)** |
| (8 marks) | | | |

|  |
| --- |
| Notes  **1a M1:** Uses correct exponential forms of sinh *x* and cosh *x*  **M1:** Expands at least one bracket correctly  **A1\*:** Obtains 1 with no errors. Both LHS and RHS of given answer to have been seen  **Alt M1:** Correct factorisation of cosh2 *x* – sinh2 *x*  **M1:** Uses correct exponential forms of sinh *x* and cosh *x*  **A1\*:** Obtains 1 with no errors. Both LHS and RHS of given answer to have been seen  **1b M1:** Uses  **M1:** Uses cosh2 *x* = 1 + sinh2 *x* and obtains 3TQ in sinh *x*  **M1:** Solves their 3TQ in sinh *x* (usual rules)  **A1:** Correct value for sinh *x*  **A1:**  only |

|  |  |  |
| --- | --- | --- |
| **2a** |  | **M1**  **M1**  **A1** |
|  |  | **(3)** |
| **2b** |  | **M1**  **M1**  **A1\*** |
|  |  | **(3)** |

**2a M1** Attempts to substitute

**M1** Makes *r* the subject

**A1** Any correct form

**2b M1** Attempts to substitute

**M1** Use of double angle formula

**A1\*** cao

|  |  |  |
| --- | --- | --- |
| **3a** |  | **B1**  **B1** |
|  |  | **(2)** |
| **3b** |  | **B1**  **B1**  **B1** |
|  |  | **(3)** |

|  |  |  |
| --- | --- | --- |
| **4a** | Derives (or simply states) relationship between angles before (60) and after  colliding with Wall *A*: | **M1** |
| Deduces for sphere to hit wall *B*: | **M1** |
| Thus: | **A1\*** |
|  | **(3)** |
| **4b** | Uses  with  to find: | **M1** |
| Thus deduces sphere meets wall *B* at an angle: | **M1** |
| Hence:  (5.1 or better) | **A1** |
|  | **(3)** |

|  |  |  |
| --- | --- | --- |
| **4c** | Uses parallel and perpendicular directions, and NEL formula, to find, for speed of sphere, *v*, after colliding with wall *A*:  and | **M1** |
| Thus uses Pythagoras to find: | **M1** |
| Similarly establishes parallel and perpendicular relationships for before and after the sphere collides with wall *B*, where the sphere leaves with speed *w*, at an angle , such that:  and/or | **M1** |
| Uses Pythagoras or  value from part **4bi** to find *w* or : | **M1** |
| Uses correct formula to find KE lost over both collisions, *L*: | **M1** |
| Thus Overall KE lost,  joules (accept 0.77 or better) | **A1** |
|  | **(6)** |

|  |  |  |
| --- | --- | --- |
| **5a**  **5b** | Unchanged d and r tables shown: | **M1** |
| AC,BC,CA,CB distances changed correctly  Routes updated with Ds | **M1,A1**  **A1** |
| *A* to *C* via *D* (row 1, col 3) so *ADC* | **M1** |
| *A* to *D* is via *B* (row1 col 4) so *ABDC* | **A1** |
| (row 1, col 3) from distance matrix |  |
| Distance = 4 | **A1** |

|  |  |  |
| --- | --- | --- |
| **6a** | States   and | **B1** |
|  | **(1)** |
| **6b** | Deduces that  is a root. | **M1** |
| Finds | **M1** |
| Uses  to state | **A1** |
|  | **(3)** |
| **6c** | Uses  to write | **M1** |
| Solves to find | **A1** |
|  | **(2)** |

|  |  |  |
| --- | --- | --- |
| **7** | ,  and | **B1** |
| Assume the general statement is time for *n* = *k*  So assume  is true. | **M1** |
| Begins to build an expression for *n* = *k* + 1  So,  Therefore | **M1** |
| Multiplies  by  to obtain a common denominator, | **M1** |
| Attempts to simplify: | **M1** |
| Demonstrates an understanding of the process of mathematical induction,  Then the general statement is true for .  As the general result has been shown to be true for   then the general result is true for all | **A1** |

**5M2**

|  |  |  |
| --- | --- | --- |
| Q | Scheme | Marks |
| **2a** | States Max Friction = *µmg*, where *m* is the mass of a vehicle. | **M1** |
| Using *F* = *ma,* states radial force in terms of *v*: *F* = | **M1** |
| Equates to find maximum safe speed: *µm*g = | **M1** |
| Substitutes  and shows correct working: | **A1\*** |
|  | **(4)** |
| **2b** | Finds value of  km/h oe | **M1** |
| Changes units using 1000/3600 and obtains  ms–1 | **A1\*** |
|  | **(2)** |
| **2c** | Uses their  and equates: | **M1** |
| Solves correctly for *r*:  oe | **A1** |
| Uses *r* value and converts km/h to m/s with correct sdt form: | **M1** |
| Finds time = 5.28 *s* | **A1** |
|  | **(4)** |