Powers and roots

- Cubing positive and negative numbers
- Finding the cube root of a number
- Using power notation

Keywords

You should know

explanation 1a explanation 1b online

1 Fiona thinks 20^2 is 40 and Ben thinks 20^2 is 400. Explain who is right.

2 **a** Copy and complete the list of square numbers.

$1^2 = 1$	$11^2 =$	$21^2 =$
$2^2 = 4$	$12^2 =$	$22^2 =$
$3^2 = 9$	$13^2 =$	$23^2 =$
$4^2 = 16$	$14^2 =$	$24^2 =$
$5^2 = 25$	$15^2 =$	$25^2 =$
$6^2 = 36$	$16^2 =$	$26^2 =$
$7^2 = 49$	$17^2 =$	$27^2 =$
$8^2 = 64$	$18^2 =$	$28^2 =$
$9^2 = 81$	$19^2 =$	$29^2 =$
$10^2 = 100$	$20^2 =$	$30^2 =$

b The last digit of a number is 0.
What is the last digit of the square of this number?

c If the last digit of a number is 1 or 9, the last digit of the number squared is 1.
Write down four more similar facts by looking at the last digits of the numbers in the list.

d A palindromic number reads the same in reverse, for example 2002.
Write three palindromic square numbers.

e Brian has found two square numbers that add up to another square number.

$3^2 + 4^2 = 5^2$ since $9 + 16 = 25$

Find two more sets of numbers like Brian's.

3 Lagrange (1736–1813) was a famous mathematician who proved that any whole number can be expressed as the sum of four square numbers.

$6 = 2^2 + 1^2 + 1^2 + 0^2$ $\quad$ $25 = 4^2 + 2^2 + 2^2 + 1^2$

a Write each number as the sum of four square numbers.

i 7 **ii** 20 **iii** 100 **iv** 18 **v** 35

b The teacher asks the class to write 124 as the sum of four square numbers. Catherine was the first to get an answer. This is her answer.

$124 = 10^2 + 4^2 + 2^2 + 2^2$

i Is Catherine's answer right?

ii Other pupils in the class claim that they have different answers that are also correct. Find four other possible answers.

c Write down five different possible ways of expressing 50 as the sum of four square numbers. Here is one to get you started.

$5^2 + 5^2 + 0^2 + 0^2 = 25 + 25 + 0 + 0 = 50$

d Repeat part **c** using your own choice of number. Challenge the person sitting next to you to find all your ways.

4 Work these out.

a $\sqrt{16}$ **b** $\sqrt{49}$ **c** $\sqrt{25}$ **d** $\sqrt{100}$

e $\sqrt{144}$ **f** $\sqrt{100} + \sqrt{49}$ **g** $\sqrt{196} - \sqrt{64}$ **h** $\sqrt{81} + \sqrt{25}$

i $3^2 \times \sqrt{121}$ **j** $\sqrt{169} \times \sqrt{36}$ **k** $\sqrt{16} \times \sqrt{100}$ **l** $\sqrt{1600}$

5 Copy and complete these statements.

a $\sqrt{400} = \sqrt{(\square \times 100)} = \sqrt{\square} \times \sqrt{\square} = \square \times 10 = \square$

b $\sqrt{2500} = \sqrt{(\square \times \square)} = \sqrt{\square} \times \sqrt{\square} = \square \times \square = \square$

c $\sqrt{6400} = \sqrt{(\square \times \square)} = \sqrt{\square} \times \sqrt{\square} = \square \times \square = \square$

d $\sqrt{900} = \sqrt{(\square \times \square)} = \sqrt{\square} \times \sqrt{\square} = \square \times \square = \square$

6 $2^2 = 4$. This can be written as the sum of two prime numbers: $2 + 2 = 4$.

$3^2 = 9$. This can also be written as the sum of two prime numbers: $2 + 7 = 9$.

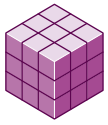
Is it possible to write every square number up to 12^2 as the sum of two prime numbers?

explanation 2a explanation 2b

7 Here is a sequence of diagrams showing the **cube numbers** 1, 8, 27.

$1^3 = 1 \times 1 \times 1 = 1$

$2^3 = 2 \times 2 \times 2 = 8$



$3^3 = 3 \times 3 \times 3 = 27$

a Copy and complete this table showing the first eighteen cube numbers. You will need a calculator.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
n^3	1	8	27															

b The number 212 is a palindromic number because it reads the same when the digits are reversed.

Write the palindromic cube numbers from the table.

c $27^3 = 19\,683$ and $1 + 9 + 6 + 8 + 3 = 27$.
Find two other cube numbers from the table with digits that add up to the number that is being cubed.

d Every whole number can be written as the sum of nine or fewer cube numbers.

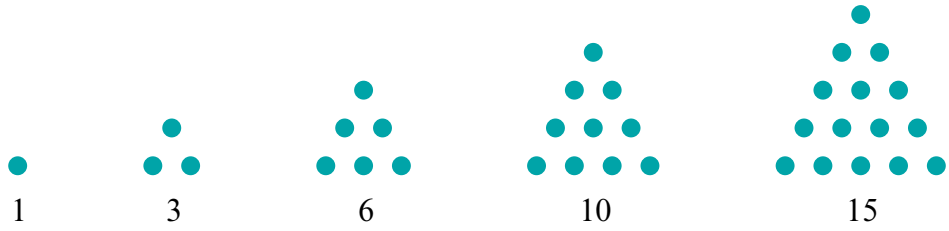
For example $40 = 3^3 + 2^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3$
because $27 + 8 + 1 + 1 + 1 + 1 + 1 = 40$

Write each of these numbers as the sum of cube numbers. Try to use the smallest amount of cube numbers you can.

i 20 **ii** 100 **iii** 19 **iv** 31 **v** 65

e Find a number between 1 and 30 that must be written in this way as the sum of exactly nine cube numbers.

8 **a** The diagram shows the first five triangular numbers.
Write the next two triangular numbers.



b Work these out.

i 1^3 **ii** $1^3 + 2^3$ **iii** $1^3 + 2^3 + 3^3$ **iv** $1^3 + 2^3 + 3^3 + 4^3$

c Is there a link between the triangular numbers and the sums of cube numbers?

d Use what you have discovered to explain how you could find the answer to $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3$.

9 Copy the table and put one number in each space until the table is filled.

3, 4, 8, 9, 12, 16, 25, 27, 64

Use each number once.

	Even	Cube number	Odd
Factors of 24			
Square number			
Multiples of 3			

10 Work these out.

a $\sqrt[3]{1}$ **b** $\sqrt[3]{64}$ **c** $\sqrt[3]{216}$ **d** $3 \times \sqrt[3]{8}$ **e** $\sqrt[3]{27} + \sqrt[3]{64}$

f $2 \times \sqrt[3]{512}$ **g** $\sqrt[3]{1} + \sqrt[3]{343} - \sqrt[3]{125}$ **h** $\sqrt[3]{1000} \times \sqrt[3]{8}$

11 Write each expression using powers. The first one has been completed for you.

a $3 \times 3 \times 3 \times 3 = 3^4$

b $7 \times 7 \times 7 \times 7 \times 7$

c $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

d $6 \times 6 \times 6$

e $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$

f $13 \times 13 \times 13 \times 13$

g $10 \times 10 \times 10 \times 10 \times 10$

h $10 \times 10 \times 10$

12 The pyramid numbers are 1, 5, 14, 30, …

Notice that

$1 = 1^2$

$5 = 1^2 + 2^2$

$14 = 1^2 + 2^2 + 3^2$

$30 = 1^2 + 2^2 + 3^2 + 4^2$

a Write down the next two numbers in the sequence.

b What will the 10th pyramid number be?

c Why do you think these numbers are called pyramid numbers?

explanation 3

13 Work these out.

a $(-2)^3$ **b** $(-3)^3$ **c** $(-5)^3$ **d** $(-6)^3$ **e** $(-9)^3$

14 The teacher says that the equation

$x^3 - 7x + 6 = 0$

has three integer solutions between −4 and +4.
He asks the class to work out which of
the numbers −4, −3, −2, −1, 0, 1, 2 , 3, 4
are solutions.

a Chris tries −3. Is this a solution?

b Tony tries 3. Is this a solution?

c Find the other two solutions.

d Use the same numbers to solve $x^3 - 13x + 12 = 0$.

$$\begin{aligned}&(-3)^3 - 7 \times -3 + 6\\ &= -27 - -21 + 6\\ &= -27 + 21 + 6\\ &= 0\end{aligned}$$

$$\begin{aligned}&3^3 - 7 \times 3 + 6\\ &= 27 - 21 + 6\\ &= 12\end{aligned}$$

explanation 4 online

15 Use a calculator to work these out.

a 18^2 **b** 27^2 **c** 65^2 **d** 81^2

e 120^2 **f** $109^2 - 96^2$ **g** $74^2 + 33^2$ **h** $55^2 - 16^2 - 39^2$

16 Use a calculator to work these out.

a 2.5^2 **b** 5.8^2 **c** 6.1^2 **d** 8.9^2 **e** 7.3^2 **f** 10.2^2

g 13^3 **h** 20^3 **i** 16^3 **j** 25^3 **k** 1.8^3 **l** 10.1^3

explanation 5 online

17 Complete these statements by finding the two consecutive whole numbers that are on either side of these square roots.

a $\square < \sqrt{6} < \square$ **b** $\square < \sqrt{24} < \square$ **c** $\square < \sqrt{45} < \square$

d $\square < \sqrt{88} < \square$ **e** $\square < \sqrt{152} < \square$ **f** $\square < \sqrt{200} < \square$

18 Chi is trying to find x if $x^3 = 5$.

a Look at Chi's work, explain his strategy.

b What number should Chi try next?

c Continue with Chi's table and find the closest numbers with 2 decimal places to complete this statement.

$\square < x < \square$

x	Working out x^3	Comment
1	$1^3 = 1$	Too small
2	$2^3 = 8$	Too big
1.5	$1.5^3 = 3.375$	Too small
1.7	$1.7^3 = 4.913$	Too small
1.8	$1.8^3 = 5.832$	Too big

19 Phil has a harder equation to solve.

Find x if $x^3 + x = 3$.

a How does Phil know the answer lies between 1 and 2?

b What number should he try next?

c Find the closest numbers with 2 decimal places to complete the statement $\square < x < \square$.

x	Working out $x^3 + x$	Comment
1	$1^3 + 1 = 2$	Too small
2	$2^3 + 2 = 10$	Too big

20 Use a similar method to find numbers to complete $\square < x < \square$ for the equation $x^3 - x = 50$.

Your numbers should have 2 decimal places.