Expressions

- Expanding single brackets in more complicated algebraic expressions
- Factorising expressions by taking out common factors
- Expanding pairs of brackets
- Factorising quadratic expressions
- Solving quadratic equations by factorising

Keywords

You should know

explanation 1a explanation 1b

1 Copy and complete these grids to expand the brackets.

a $2(x + 5)$

×	x	+5
2		

b $5(2x + 7)$

×	$2x$	+7
5		

c $3(4x - 5)$

×	$4x$	−5
3		

d $6(5 - x)$

×	5	$-x$
6		

e $-2(3x + 9)$

×	$3x$	+9
−2		

f $-3(4 - 2x)$

×	4	$-2x$
−3		

2 Expand the brackets. You may use any method.

a $3(x + 5)$ **b** $6(a - 4)$ **c** $2(3x + 7)$ **d** $5(3a + 2b)$

e $-2(3x + 7)$ **f** $-4(6 + x)$ **g** $-5(3 - x)$ **h** $-6(2x - 3)$

3 Expand the brackets.

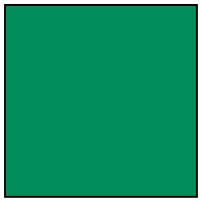
a $a(a + 2)$ **b** $2x(3x - 5)$ **c** $-3x(x + 7)$ **d** $-6(5a - 2b)$

e $3x(2x + 5y)$ **f** $4xy(3y + 5x^2)$ **g** $-6a(3a - 2)$ **h** $3xyz(z + 2xy)$

explanation 2a | explanation 2b | online

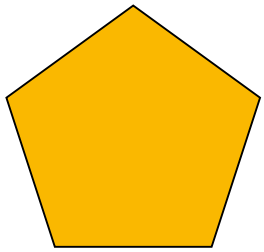
4 Write equivalent expressions, with brackets and without brackets, for the perimeters of these regular polygons.

a



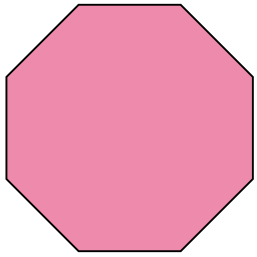
$(2x - 3)$

b



$(8 - 2x)$

c



$(5x + y)$

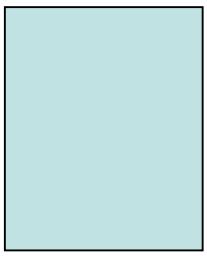
5 Write equivalent expressions, with brackets and without brackets, for the perimeters of these rectangles.

a

$(2x - 5)$

$(3x + 1)$

b



$(18 - 3x)$

$(2x - 1)$

6 Expand the brackets and simplify each expression.

a $3(2x + 1) + 2(x - 3)$

b $4(a + 3) + 5(3a - 1)$

c $5(d - 4) + 4(3 + d)$

d $6(3p - 3) + 3(-2p - 2)$

e $3(y - 2) - 2(y + 3)$

f $5(2m - 3) + 3(2 - m)$

7 Expand the brackets and simplify each expression.

a $5m(m + 2n) - m(3m - n)$

b $4a(6b - 3a) - 2a(4a - 2b)$

c $2x(5 - x) + 3x(4x + 1)$

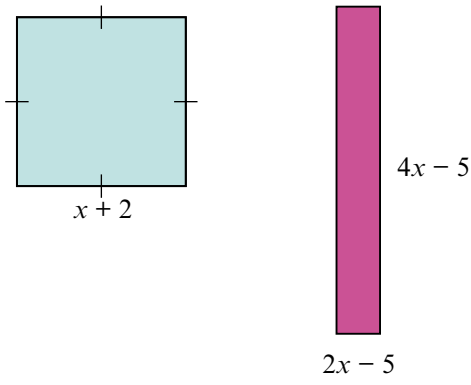
d $7y(2y - 5) - 4y(5 - 2y)$

e $-5m(2m - 3n) - 5n(3m - 2n)$

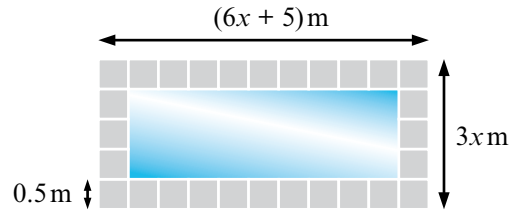
f $-4y(7x + y) - 5x(3y - 2x)$

8 The square and the rectangle have equal perimeters.

Write an equation using brackets and solve it to find the value of x.



9 Tom has a garden pond with a path around it. The length and width of the path and pond are shown on the diagram. The path has width 0.5 m. Find a simplified expression for the perimeter of the pond.



explanation 3a | explanation 3b | online

10 Copy and complete these partly factorised expressions.

a $8x + 12 = 4(\square + \square)$

b $9a + 15b = \square(3a + \square)$

c $4x + 6y + 8z = 2(\square + \square + \square)$

d $3x^2 + 9xy = \square\,(\square + 3y)$

11 Factorise fully these expressions.

a $5x + 10$

b $14a + 21$

c $10 - 2x$

d $21s + 28t$

e $6x + 9y + 12z$

f $18s - 15t$

g $20x + 25st$

h $56r - 63s$

i $24f + 16g$

j $26x + 28y$

k $3x + 6y + 9z$

12 Factorise fully these expressions.

a $x^2 + 2x$

b $5y + y^2$

c $15x - 7x^2$

d $pqr + 2r$

e $7st - 8s$

f $4gy + 5py$

g $4x^2 + x^3$

h $2p^3 - 3p$

i $x + xy - x^2$

j $3x^2 + x^3 - x^2z$

k $wxy - wd + 5w$

13 Factorise fully these expressions.

a $3y^2 + 3y$ **b** $2m^2 - 4m$ **c** $8y^2 - 2y$ **d** $4b^2 + 8b^3$

e $10m^2 - 5m^3$ **f** $12n^2 + 14n$ **g** $8f - 24f^2$ **h** $3x^2 + 6xy$

i $12ab - 48ab^2$ **j** $2p^2 - 2p + 4pq$ **k** $4a^2b - 4ab + 8ab^2$

l $15p^2q + 25pq - 5q^2p$

14 The area of a rectangle is $12x^2 + 16x$.

One possible expression for the length × the width of the rectangle is $x(12x + 16)$.

Write two more possible expressions for the length × width of the rectangle.

explanation 4a explanation 4b

For questions **15**, **16**, **18** and **19**, use either of the methods given in Explanation 4.

15 Multiply out these brackets and give each answer in its simplest form.

a $(x + 3)(x + 5)$ **b** $(x + 5)(x + 7)$ **c** $(x + 8)(x + 1)$

d $(x + 1)(x - 6)$ **e** $(x - 4)(x + 5)$ **f** $(x - 2)(x - 9)$

g $(x - 3)(x - 4)$ **h** $(x - 4)(x + 3)$ **i** $(x - 7)(2x - 3)$

16 Expand the brackets and simplify the result.

$(x + 1)^2 = (x + 1)(x + 1)$

a $(x + 3)^2$ **b** $(x + 1)^2$ **c** $(x + 2)^2$

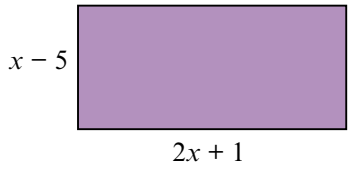
d $(x + 6)^2$ **e** $(x + 9)^2$ **f** $(x - 7)^2$

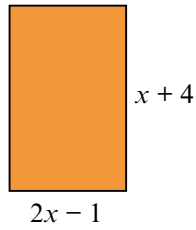
g $(a - 2)^2$ **h** $(p - 10)^2$ **i** $(n - 12)^2$

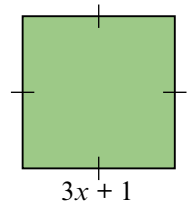
17 What do you notice about the results for question **16**?

Write down the identities for $(a + b)^2$ and $(a - b)^2$.

18 Find the area of each rectangle and give each answer in its simplest form.







19 Expand and simplify these.

a $(x + 3)(x - 3)$ **b** $(x - 5)(x + 5)$ **c** $(x + 10)(x - 10)$

20 What do you notice about the results for question **19**?

Write the expanded form of $(a + b)(a - b)$.

explanation 5a explanation 5b

21 Use the difference of two squares to factorise these.

a $x^2 - 64$ **b** $x^2 - 144$ **c** $a^2 - 100$ **d** $p^2 - 81$

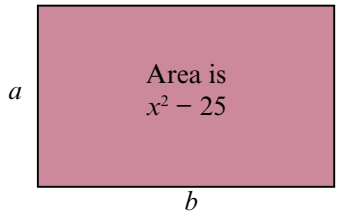
22 Which of these expressions can be factorised using the difference of two squares?

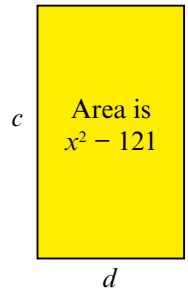
A $x^2 + 4x$ **B** $x^2 - 16x$ **C** $x^2 - 36$ **D** $x^2 + 36$

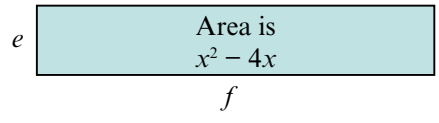
E $36x^2 - 49$ **F** $x^2 - 49x$ **G** $x^2 + 49$

23 Which of the expressions in question **22** cannot be factorised by either using the difference of two squares or by taking out a common factor?

24 Factorise the expression for the area of each rectangle, using one of the methods from question **23**. Use your answer to find expressions for the lengths of the sides of these rectangles.







explanation 6a | explanation 6b | explanation 6c | online

25 Copy and complete these factorisations.

a $x^2 + 8x + 7 = (x + 7)(\square + \square)$ **b** $x^2 + 5x + 6 = (x + \square)(x + \square)$

c $x^2 + 6x + 9 = (\square + \square)(\square + \square)$ **d** $x^2 + 10x + 16 = (\square + \square)(\square + \square)$

e $x^2 + 7x + 10 = (\square + \square)(\square + \square)$ **f** $x^2 + 16x + 28 = (\square + \square)(\square + \square)$

26 Factorise these.

a $x^2 + 4x + 3$ **b** $x^2 + 6x + 5$ **c** $x^2 + 8x + 12$

d $x^2 + 10x + 21$ **e** $x^2 + 9x + 20$ **f** $x^2 - 4x + 3$

g $x^2 - 13x + 42$ **h** $x^2 - 8x + 12$ **i** $x^2 - 20x + 100$

j $x^2 - 17x + 70$ **k** $x^2 + 15x + 56$ **l** $x^2 - 7x + 12$

explanation 7a | explanation 7b

27 Factorise these.

a $x^2 - 2x - 3$ **b** $x^2 + 4x - 5$ **c** $x^2 - 5x - 6$

d $x^2 - 2x - 8$ **e** $x^2 + 3x - 10$ **f** $x^2 + 2x - 15$

g $x^2 - 5x - 24$ **h** $x^2 + 5x - 36$ **i** $x^2 - 2x - 99$

j $x^2 + 5x - 14$ **k** $x^2 - 20x - 300$ **l** $x^2 + 28x - 60$

28 Factorise these.

a $3x^2 - 9x$ **b** $x^2 + 5x - 14$ **c** $x^2 - 15x - 34$

d $x^2 - 121$ **e** $x^2 + 6x - 40$ **f** $x^2 - 12x + 27$

g $16x^2 + 4x$ **h** $36 - x^2$ **i** $x^2 - 9x - 220$

j $x^2 + 12x + 36$ **k** $x^2 - 15x$ **l** $400 - x^2$

explanation 8a | explanation 8b | explanation 8c | online

29 Solve these quadratic equations by factorising.

a $x^2 - 15x + 56 = 0$ **b** $a^2 + 5a = 0$ **c** $b^2 - 1 = 0$

d $d^2 - 5d - 150 = 0$ **e** $15x^2 - 20x = 0$ **f** $f^2 - 400 = 0$

g $x^2 + 20x + 96 = 0$ **h** $x^2 + 9x - 36 = 0$ **i** $x^2 + 6x + 9 = 0$

30 Solve these quadratic equations by factorising.
You will need to rearrange them first.

a $x^2 + 3x = 40$ **b** $x^2 = 18 - 7x$ **c** $14x - 24 = x^2$

d $x^2 = 5x$ **e** $x^2 - 6x = -9$ **f** $-2x = -x^2 + 8$

31 The area of this rectangle is $24\,\text{cm}^2$.

a Show that $x^2 + 12x + 11 = 0$.

b Solve the equation $x^2 + 12x + 11 = 0$.

c Which of your solutions is not sensible for this problem?

d Write the length and width of the rectangle.

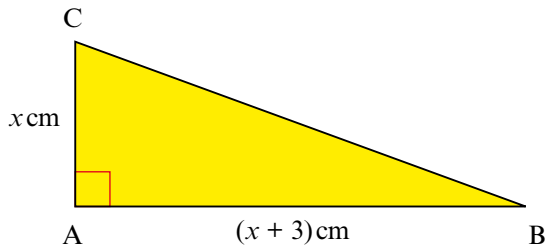
32 Stephen is x years old and Anna is two years older than Stephen.
The product of their ages is 80.

a Show that $x^2 + 2x - 80 = 0$.

b Solve the quadratic equation $x^2 + 2x - 80 = 0$.

c How old is Stephen?

33 The area of the right-angled triangle is $27\,\text{cm}^2$.



a Show that $x^2 + 3x - 54 = 0$.

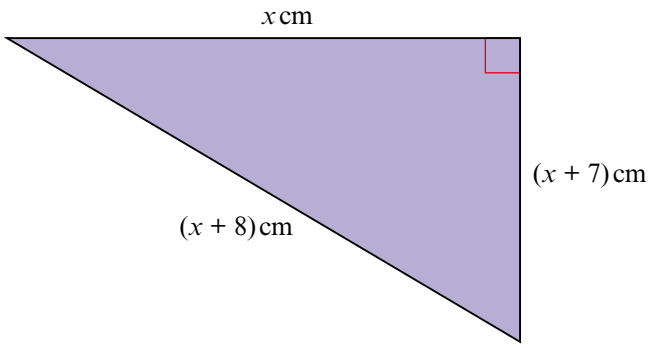
b Solve the quadratic equation $x^2 + 3x - 54 = 0$ and find the length of AB.

34 Two positive numbers differ by 3 and their product is 180.
The smaller of the two numbers is x.

a Show that $x^2 + 3x - 180 = 0$.

b Solve this quadratic equation to find the two numbers.

35



a Pythagoras' theorem states that for any right-angled triangle the square of the length of the longest side is equal to the sum of the squares of the other two sides.

Use Pythagoras' theorem to show that $x^2 - 2x - 15 = 0$ for this right-angled triangle.

b Solve the quadratic equation to find the side lengths of the triangle.