Factors and powers

- Writing a number as a product of primes
- Writing fractions and square and cube roots using index notation
- Calculating approximate values for square roots and cube roots
- Simplifying expressions that involve surds

Keywords

You should know

explanation 1a explanation 1b

1 Which of the numbers in the box is *not* a factor of the given number?

a 120

24	12	
	4	48

b 252

9	53	
	21	126

2 Which of the numbers in the box is *not* a multiple of the given number?

a 12

24	84	
	158	228

b 15

45	90	
	180	365

3 Find the first number greater than 25 that is a multiple of 6 *and* a factor of 96.

4 For each list, write down the numbers that are *not* prime numbers.

a 1, 3, 7, 17, 27, 31, 33, 47, 51, 67 **b** 11, 28, 43, 53, 77, 83, 93, 101, 153, 179

5 Why is 2 the only even number that is a prime number?

6 Find values for these without using a calculator.

a 4^3 **b** -6^2 **c** $(-6)^2$ **d** 5^3

e $(-3)^4$ **f** 5×2^2 **g** $-2^3 \times 3^2$ **h** $(-7)^2 \times 2$

explanation 2a | explanation 2b | explanation 2c

7 Colin started to find 72 as the product of prime factors using a table.

2	72
2	36
2	18

a Copy and complete the table.

b Write 72 as a product of prime factors using index notation.

8 Use a factor tree or a table to express each number as a product of primes.

Write the product of prime factors using index notation.

a 84 **b** 252 **c** 450 **d** 3168

9 Find the HCF of each pair of numbers.

a 32 and 144 **b** 45 and 210 **c** 28 and 350 **d** 84 and 252

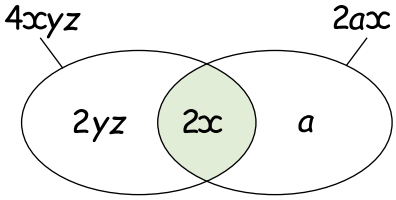
10 Find the LCM of each pair of numbers.

a 12 and 18 **b** 30 and 75 **c** 15 and 24 **d** 84 and 252

11 Two numbers have a HCF of 8 and a LCM of 480.

What might they be? Find all possible answers.

12 Suzie used a Venn diagram to find the HCF of two algebraic expressions, $4xyz$ and $2ax$.



HCF = $2x$

Use Venn diagrams to find the HCFs of these expressions.

a $2xy$ and $6y$ **b** $4ab$ and $8bc$

c $5abc$ and $10b^2$ **d** $12xy^2z$ and $4x^2y$

explanation 3 | online

13 Use index laws to multiply these expressions. Leave your answers in index form.

a $4^4 \times 4^5$ **b** $6^2 \times 6^4$ **c** $3^5 \times 3^7$ **d** $5^5 \times 5^6$

e $8^3 \times 8^6$ **f** $3^3 \times 3^0$ **g** $3^5 \times 5^2 \times 3^2$ **h** $3^2 \times 4^3 \times 3^2 \times 4^4$

14 Use index laws to divide these expressions. Leave your answers in index form.

a $4^8 \div 4^5$ **b** $7^7 \div 7^4$ **c** $9^5 \div 9^2$ **d** $5^5 \div 5^3$

e $\frac{3^3}{3^0}$ **f** $\frac{3^9}{3^2}$ **g** $2^9 \div 2^2 \div 2^5$ **h** $3^{12} \div 3^2 \div 3^4$

15 Use index laws to simplify these.

Leave your answers in index form.

a $2^5 \times 2^4 \div 2^3$ **b** $5^4 \div 5^3 \times 5^6$ **c** $10^7 \times 10^8 \div 10^6$

d $5^6 \times 3^4 \div 5^4$ **e** $7^5 \times 6^5 \times 7^2 \div 6^4$ **f** $\frac{3^7 \div 4^5 \times 4^6}{3^3}$

16 To simplify $(3^2)^3$ Jaydeep wrote $3^2 \times 3^2 \times 3^2 = 3^6$.

a Simplify these powers in a similar way.

i $(4^3)^2$ **ii** $(5^3)^3$ **iii** $(4^3)^4$ **iv** $(3^4)^2$

b What relationship do you notice between the powers in the question and the answer?

c Copy and complete this law. $(a^n)^m = a^{m \square n}$

explanation 4a explanation 4b explanation 4c

17 Copy and complete the bottom row of this table with fractions or whole numbers.

5^{-3}	5^{-2}	5^{-1}	5^0	5^1	5^2	5^3
						125

18 Copy and complete these.

a $\frac{1}{6^2} = 6^{\square}$ **b** $4^{-5} = \frac{1}{4^{\square}}$ **c** $\frac{1}{25} = \frac{1}{5^{\square}} = 5^{\square}$ **d** $\frac{1}{81} = 9^{\square}$

19 Without using a calculator write these without powers.

a 2^{-3} **b** 3^{-2} **c** 5^{-1} **d** 4^{-2} **e** 6^{-2} **f** 5^{-3}

20 Write these fractions using negative powers.

a $\frac{1}{2^2}$ **b** $\frac{1}{4^5}$ **c** $\frac{1}{6^3}$ **d** $\frac{1}{4^4}$

e $\frac{1}{4}$ **f** $\frac{1}{36}$ **g** $\frac{1}{27}$ **h** $\frac{1}{16}$

For parts **e** to **h** first write the denominator as a power of another number.

21 Copy and complete these.

a $\sqrt{36} = 36^{\square}$ **b** $\sqrt{28} = 28^{\square}$ **c** $\sqrt{100} = 100^{\square}$

d $\sqrt[3]{64} = 64^{\square}$ **e** $\sqrt[3]{125} = 125^{\square}$

22 Without using a calculator find these.

a $4^{\frac{1}{2}}$ **b** $16^{\frac{1}{2}}$ **c** $9^{\frac{1}{2}}$ **d** $100^{\frac{1}{2}}$ **e** $49^{\frac{1}{2}}$

f $81^{\frac{1}{2}}$ **g** $144^{\frac{1}{2}}$ **h** $8^{\frac{1}{3}}$ **i** $27^{\frac{1}{3}}$ **j** $125^{\frac{1}{3}}$

explanation 5a explanation 5b online

23 Find these positive and negative roots.

a $\pm\sqrt{25}$ **b** $\pm\sqrt{121}$ **c** $\pm\sqrt{144}$ **d** $\pm\sqrt{400}$ **e** $\pm\sqrt{10\,000}$

24 Find the answers to these. Use a calculator if necessary.

a $\sqrt[3]{27}$ **b** $\sqrt[3]{-27}$ **c** $\sqrt[3]{-125}$ **d** $\sqrt[3]{64}$ **e** $\sqrt[3]{-216}$

25 Which of these are true?

A $\sqrt{9} + \sqrt{25} = \sqrt{9 + 25}$ **B** $\sqrt{25} - \sqrt{9} = \sqrt{25 - 9}$

C $\sqrt{9} \times \sqrt{25} = \sqrt{9 \times 25}$ **D** $\sqrt{9} \div \sqrt{4} = \sqrt{9 \div 4}$

26 The square root of some numbers can be can be found by factorising.

Find these square roots by factorising.

a $\sqrt{400}$ **b** $\sqrt{256}$ **c** $\sqrt{324}$ **d** $\sqrt{441}$ **e** $\sqrt{484}$

$$\begin{aligned}\sqrt{225} &= \sqrt{9 \times 25}\\ &= \sqrt{9} \times \sqrt{25}\\ &= 3 \times 5\\ &= 15\end{aligned}$$

27 **a** Copy and complete these statements about $\sqrt{42}$.

i 42 lies between the consecutive square numbers $\square$ and $\square$.

ii This can be written as $\square < 42 < \square$.

iii So $6 < \sqrt{42} < \square$.

b Find an approximate value for $\sqrt{42}$ to two decimal places.

Write your answer in the form $a < \sqrt{42} < b$.

Do not use the $\sqrt{\ }$ key on your calculator.

28 Find approximations for these to two decimal places.

a $\sqrt{8}$ **b** $\sqrt{32}$ **c** $\sqrt{90}$ **d** $\sqrt{24}$

explanation 6a explanation 6b

29 Which of these are surds? Use a calculator if necessary.

a $\sqrt{4}$ **b** $\sqrt{7}$ **c** $\sqrt{17}$ **d** $\sqrt{25}$

e $\sqrt{58}$ **f** $\sqrt{144}$ **g** $\sqrt{20.25}$ **h** $\sqrt{5.68}$

i $\sqrt{4.84}$ **j** $\sqrt{70.56}$ **k** $\sqrt{110.25}$ **l** $\sqrt{218.12}$

30 Find an exact answer or simplified surd form for each of these.

a $\sqrt{7} \times \sqrt{7}$ **b** $\sqrt{3} \times \sqrt{3} \times \sqrt{3}$ **c** $\sqrt{9} \times \sqrt{9}$

d $\sqrt{13} \times \sqrt{13} \times \sqrt{13}$ **e** $\sqrt{8} \times \sqrt{8}$ **f** $\sqrt{20} \times \sqrt{20}$

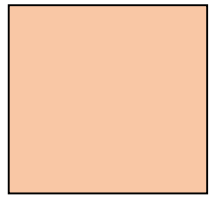
g $\sqrt{4} \times \sqrt{4} \times \sqrt{4} \times \sqrt{4}$ **h** $\sqrt{4} \times \sqrt{4} \times \sqrt{4} \times \sqrt{4} \times \sqrt{4}$ **i** $\sqrt{3} \times \sqrt{3} \times \sqrt{3}$

31 What is the exact area of each square?

a

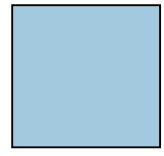
$\sqrt{1}$ cm

b



$\sqrt{27}$ cm

c



$\sqrt{2}$ cm

32 What is the length of one side of a square with these areas?

Where appropriate leave answers in surd form.

a 81 cm^2 **b** 28 cm^2 **c** 50 m^2 **d** 56.25 mm^2

33 Find an exact answer for these. Do not use a calculator.

a $\sqrt{4} \times \sqrt{9}$ **b** $\sqrt{32} \times \sqrt{2}$ **c** $\sqrt{2} \times \sqrt{50}$

d $\sqrt{3} \times \sqrt{27}$ **e** $\sqrt{8} \times \sqrt{18}$ **f** $\sqrt{10} \times \sqrt{2} \times \sqrt{5}$

g $\sqrt{5} \times \sqrt{10} \times \sqrt{8}$ **h** $\sqrt{5} \times \sqrt{45}$ **i** $\sqrt{5} \times \sqrt{10} \times \sqrt{18}$

j $\sqrt{5} \times \sqrt{10} \times \sqrt{50}$ **k** $\sqrt{8} \times \sqrt{10} \times \sqrt{20}$ **l** $\sqrt{5} \times \sqrt{125}$