Factors, multiples, primes and powers

- Writing numbers as the product of prime factors
- Using index notation
- Calculating approximate ranges for square roots and cube roots
- Using the index laws
- Using prime factors to find the highest common factor and lowest common multiple

Keywords

(explanation 1a) (explanation 1b) (explanation 1c)

1 a Write down five multiples of 9 that are greater than 100.

   b What is the smallest multiple of 9 that has no odd digits?

   c Write down three multiples of 6 that are between 50 and 70.

   d Write down all the multiples of 8 that are between 80 and 120.

2 Write down all the factors of each of these numbers.

   a 24   **b** 30   **c** 45   **d** 64

   e 60   **f** 81   **g** 144   **h** 154

3 Write which of the following are *not* prime numbers, and why.

   17, 93, 71, 59, 87, 1, 91, 32

4 Write the prime factors of these numbers by using a factor tree.

   a 24   **b** 72   **c** 144   **d** 256

5 Write each of these numbers as the product of prime factors by using a factor tree.

   a 42   **b** 33   **c** 24   **d** 45   **e** 53

6 Use a factor tree to write these numbers as the product of prime factors.

   a 360   **b** 1020   **c** 576

7 This fraction sum consists of four different digits and adds up to 1.

$$\frac{4}{8} + \frac{1}{2} = 1$$

Find five other possible fraction sums which add up to 1 and are each made from four different digits.

8 Think of a two-digit number where the tens digit is equal to the units digit. Show that this number is divisible by 11. Is this true for any two-digit number where the digits are the same? Try to prove it.

> explanation 2

9 Write these without indices. The first one has been done for you.

   a $16^5 = 16 \times 16 \times 16 \times 16 \times 16$ **b** 23^3 **c** 100^4

   d 36^5 **e** 3^6 **f** 7^4 **g** 16^3 **h** 25^5

10 Copy and complete these.

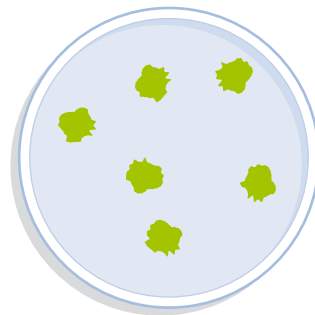   **a** $4 = 2^{\square}$ **b** $27 = 3^{\square}$ **c** $16 = 2^{\square}$ **d** $625 = 5^{\square}$ **e** $400 = \square^{\square}$

11 At the start of an experiment, a colony of bacteria contains 6 bacteria.
Every hour, the colony doubles in size.

   a Write down how many bacteria there are after 1 hour, 2 hours, 3 hours, 4 hours and 5 hours.

   b Try to find a formula to work out how many bacteria there would be after any number of hours.

   c How many would there be after 100 hours?

12 We can write 3 as $2^2 - 1$. Is $2^3 - 1$ a prime number?

Find two more prime numbers which can be written in the form $2^n - 1$ (a power of 2 minus 1). These are known as Mersenne primes.

explanation 3

13 Work these out using a calculator.

a $15^3 + 27^2$ b $26^3 - 17^3$ c $72^4 - 8^3$ d $101^3 + 92^3$

e $34^5 - 98^2$ f $67^3 + 33^2 - 24^3$ g $3(57^4 - 38^3)$ h $\dfrac{(8^6 - 4^5)}{2}$

14 a Work out the difference of these two squares: $83^2 - 71^2$.

b Now work out $(83 - 71)(83 + 71)$.

c Comment on your answers to parts **a** and **b**.

d Is this true for the difference of any two squares?
Investigate by choosing your own numbers.

15 Work these out without a calculator.

a $\sqrt{25} + \sqrt{64}$ b $\sqrt{121} - \sqrt{49}$ c $\sqrt{144} + \sqrt{81}$ d $\sqrt{100} - \sqrt{16}$

explanation 4a   explanation 4b

16 Work out these cube roots using the information
given in the box.

Use your calculator to check the answers.

a $\sqrt[3]{64}$ b $\sqrt[3]{125}$

c $\sqrt[3]{4913}$ d $\sqrt[3]{1\,000\,000}$

$4^3 = 64$
$5^3 = 125$
$17^3 = 4913$
$4913 \div 3 = 1637.67$
$64 \div 3 = 21.33$
$100^3 = 1\,000\,000$

17 Imagine that the $\sqrt{}$ button on your calculator is broken.
Use another method to estimate the value of these square roots.
Give each answer as an inequality involving numbers with 2 decimal places.

a $\sqrt{29}$ b $\sqrt{57}$ c $\sqrt{93}$ d $\sqrt{101}$

18 Using a similar method, estimate the value of $\sqrt[3]{20}$.

explanation 5a   explanation 5b

19 Write each expression as a single power of a number.

a $5^2 \times 5^9$

b $17^6 \div 17^3$

c $24^8 \div 24^7$

d $37^4 \times 37^6$

e $54^3 \times 54^6$

f $21^{12} \div 21^4$

g $26^{15} \times 26$

h $23^5 \div 23$

i $14^7 \div 14^5$

20 Simplify each expression, then use your calculator to work out the answer.

a $6^2 \times 6^3$

b $12^{13} \div 12^9$

c $5^3 \times 5^4$

d $4^{30} \div 4^{20}$

e $14^5 \div 14^2$

f $18^4 \times 18^1$

g $11^7 \div 11^3$

h $23^2 \times 23$

i $32^{16} \div 32^{14}$

21 Work these out without using a calculator.

a $11^8 \div 11^6$

b $10^5 \times 10^2$

c $7^9 \div 7^7$

d $18^{23} \div 18^{22}$

e $8^6 \div 8^4$

f $2^3 \times 2^1$

g $3^2 \times 3^2$

h $9^{11} \div 9^9$

i $1^2 \times 1^3$

22 Work these out without using a calculator.

a $16^8 \div 16^8$

b $2^{16} \div 2^{11}$

c $6^2 \div 6^0$

d $2^{13} \div 2^{15}$

e $1^4 \div 1^3$

f $3^7 \div 3^4$

23 Work these out without using a calculator. Show your working.

Remember to use BIDMAS.

a $(7 - 4)^2$

b $(2 + 3)^3$

c $(9 \div 3)^2$

d $(4 \times 3)^2$

e $(3 \times 5 - 11)^3$

f $(24 - 19 + 2)^2$

g $(48 - 92 \div 2)^4$

h $(108 \div 12 - 21 \div 7)^2$

24 Simplify these. (Remember to work from left to right when there are multiplications and divisions in a question.)

   a $3^4 \times 3^7 \times 3^6$ **b** $6^5 \div 6^4 \times 6^2$ **c** $7^8 \times 7^8 \div 7^{15}$

   d $9^{14} \div 9^7 \div 9^8$ **e** $4^7 \times 4^3 \div 4^8$ **f** $12^8 \div 12^5 \div 12^2$

> explanation 6a   explanation 6b

25 Find the highest common factor (HCF) of each pair.

   a 16 and 36   **b** 30 and 48   **c** 30 and 625

   d 44 and 132   **e** 35 and 50   **f** 126 and 28

26 Find the lowest common multiple (LCM) of each pair.

   a 8 and 28   **b** 15 and 48   **c** 7 and 54

   d 8 and 63   **e** 12 and 9   **f** 16 and 6

27 Find the HCF and LCM of these pairs of numbers.

   a 15 and 45   **b** 48 and 60   **c** 24 and 56

   d 20 and 8   **e** 35 and 56   **f** 35 and 125

28 **a** Find the LCM of 12 and 16.   **b** Find the LCM of 20 and 40.

   c Find the LCM of 30 and 42.   **d** Find the HCF of 22 and 42.

   e Find the HCF of 132 and 156.   **f** Find the HCF of 144 and 30.

29 Identify the odd one out in each list of numbers and give a reason for your choice.

   a 3, 15, 20, 30, 45   **b** 65, 90, 104, 135, 195

   c 17, 29, 51, 79, 97   **d** 24, 108, 140, 96, 132