



## Prisms and cylinders

- Recognising a prism
- Finding the volume of a prism
- Finding the volume of a cylinder
- Finding the surface area of a prism

Keywords

You should know

explanation 1a

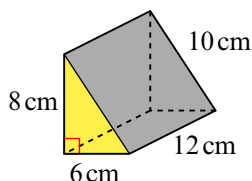
explanation 1b

explanation 1c

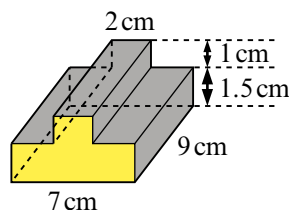
explanation 1d

- 1 This question is about what prisms are.
  - a What is a prism?
  - b Name some everyday objects that are prisms.
  - c Design and draw your own prism.
- 2 Write a set of easy-to-follow instructions for how to find the volume of a prism.
- 3 Find the volume of each prism.

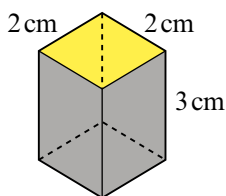
a



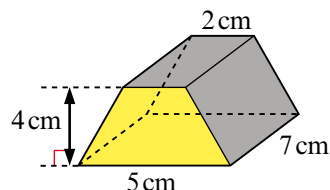
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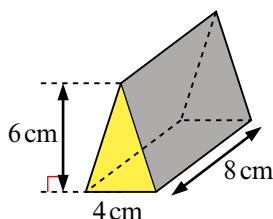
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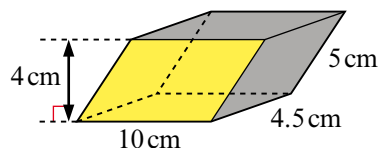
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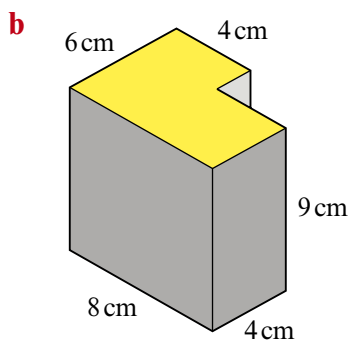
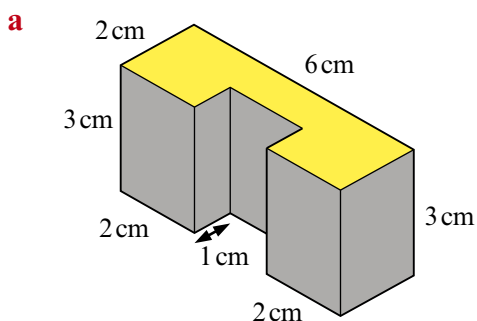
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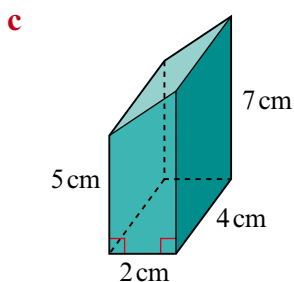
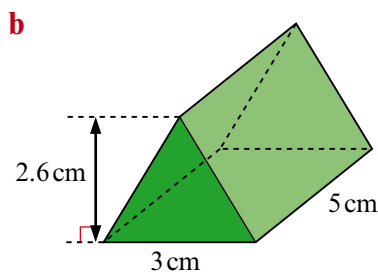
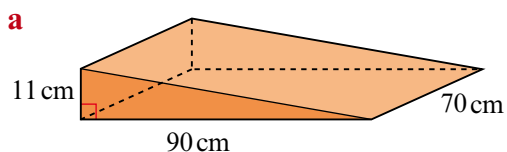
f



**4** Find the area of the cross-section first and then find the volume of each prism.



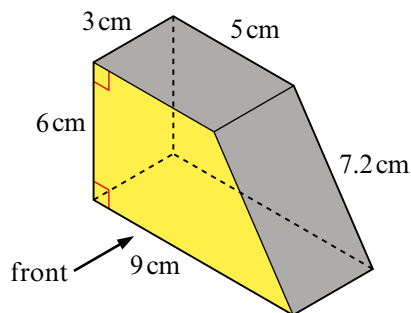
**5** Find the volume of each prism.



**6** The diagram shows a prism.

- Draw the front view (the yellow face). Label all the side lengths.
- Find the area of the front face.
- Find the volume of the prism.
- The density of silver is  $10.5 \text{ g/cm}^3$ .

How many grams would this prism weigh, if it were made from silver?



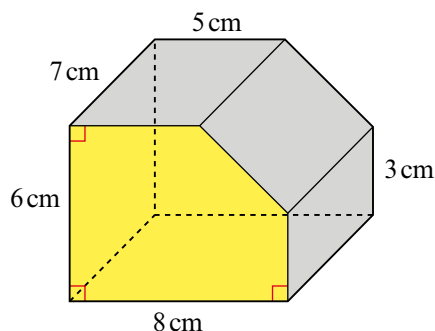
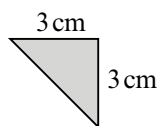
Remember:  $\text{density} = \frac{\text{mass}}{\text{volume}}$

**7** The diagram shows a triangle and a prism.

**a** Find the area of the triangle.

**b** Find the area of the front face of the prism.

**c** Find the volume of the prism.



**d** The density of glass is  $2.4 \text{ g/cm}^3$ .

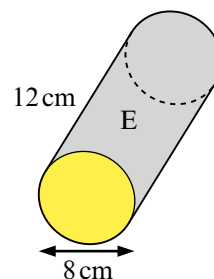
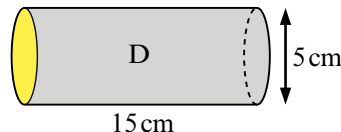
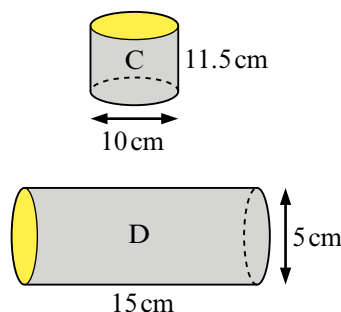
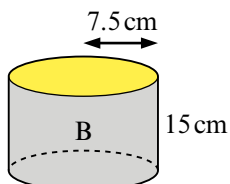
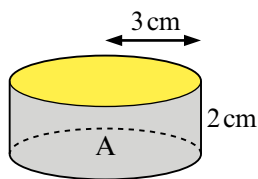
If this prism were made of glass, how many grams would it weigh?

explanation 2a

explanation 2b

explanation 2c

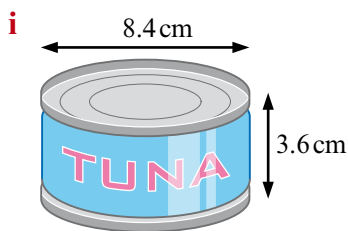
**8** Here are five cylindrical objects.



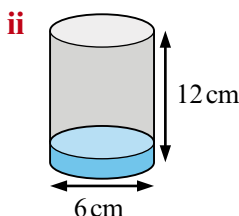
**a** Find the volume of each cylinder to 2 decimal places.

**b** Which cylinder represents a 750 ml can of paint?

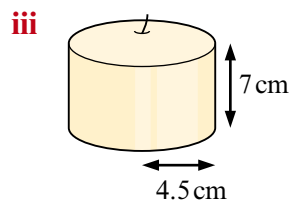
**9** These objects are all cylinders.



Tin of tuna



Drinking glass



Candle

**a** Find the volume of each object correct to 2 decimal places.

**b** How many millilitres will the glass hold?

**c** The density of candle wax is  $0.92 \text{ g/cm}^3$ . What is the mass of the candle?

**d** There is 130 g of tuna in the tin.

Work out the density of tuna to 1 decimal place.

**10** A can of cola has a diameter of 6.4 cm and a height of 10.4 cm.

**a** Sketch the can of cola and find the volume of this can in cubic centimetres.

**b**  $1 \text{ cm}^3$  is equivalent to 1 ml.

What is the maximum number of millilitres of cola in the tin?

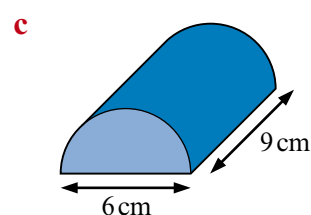
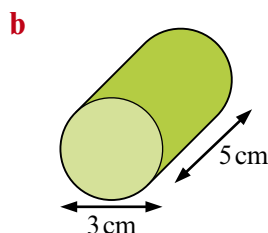
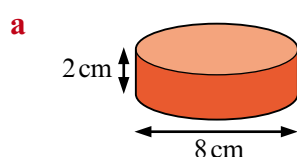
**c** The diameter of a can of energy drink is 4.8 cm.

Copy and complete the trial and improvement table to find the height of the can that holds just over 330 ml.

$\pi \times 2.4^2$	Height	Volume $\text{cm}^3$
	10	

**11** Find the volume of each wooden brick.

Give each answer correct to 2 decimal places.

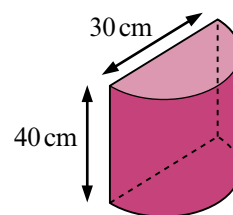


**12** A company makes semicircular bins that can be fixed to a wall.

**a** Find the area of the semicircular top opening to 1 decimal place.

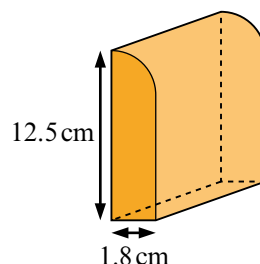
**b** Find the volume of the bin in cubic centimetres to 1 decimal place.

**c** The capacity of the bin is measured in litres.  $1 \text{ cm}^3$  is equivalent to 1 ml. Find the capacity of the bin in litres, correct to 1 decimal place.



**13** The diagram shows the cross-section of a length of skirting board.

- a** Find the area of a circle with a radius 1.8 cm.
- b** Find the area of the cross-section of the skirting board.

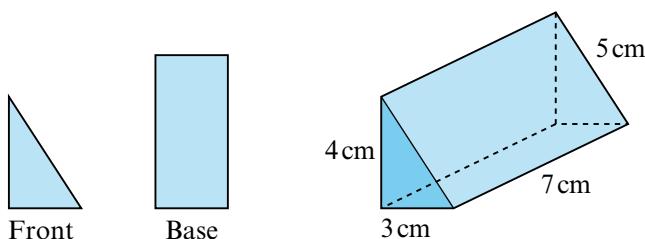


- c** This skirting board is sold in 2 m lengths. Find the volume in cubic centimetres of one of these lengths.
- d** The skirting board is made from MDF, which has a density of  $0.73 \text{ g/cm}^3$ . What is the weight of a 2 m length of the skirting board?

explanation 3a

explanation 3b

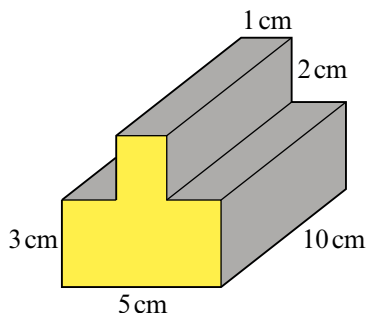
**14** Brian is finding the surface area of this prism. He starts by making a list of the faces of the prism.



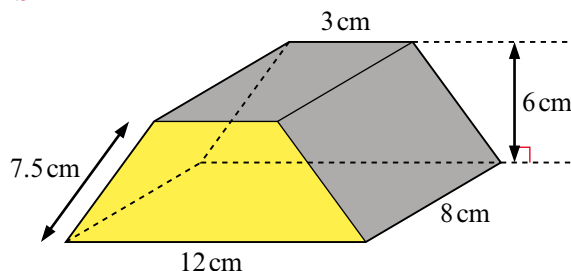
- a** Copy and complete Brian's list of faces.
- b** Find the area of each face.
- c** Find the total surface area of the prism.

**15** For each prism, make a list of the faces and use this to find the surface area.

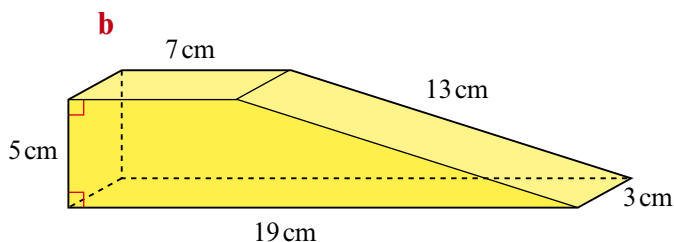
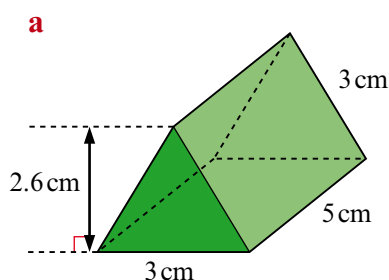
**a**



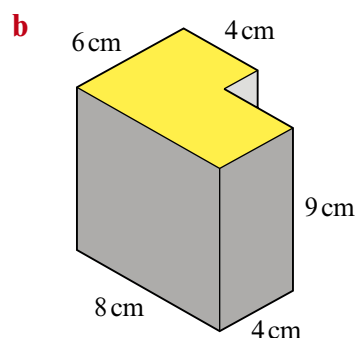
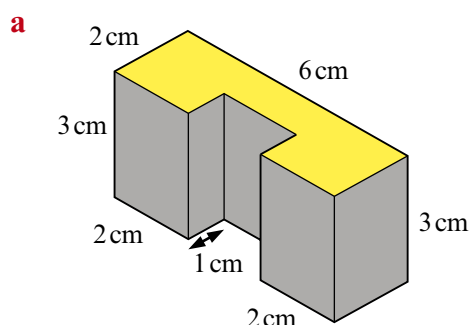
**b**



**16** For each prism, make a list of the faces and use this to find the surface area.



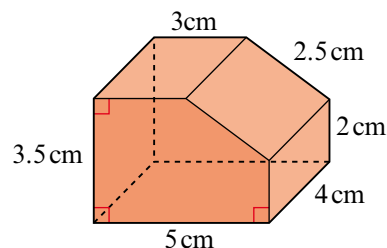
**17** Find the surface area of each prism.



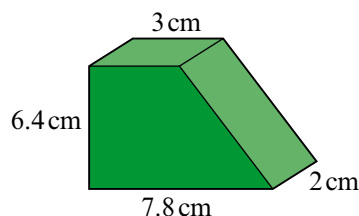
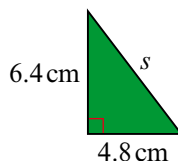
**18 a** Make a list of all the faces for this solid and find the area of each face.

**b** Find the total surface area.

**c** Find the volume of the solid.



**19** Look at the diagrams.



**a** Explain why the base of the right-angled triangle is 4.8 cm.

**b** Use Pythagoras' Theorem to find  $s$ .

**c** Find the surface area of the prism.

**d** Find the volume of the prism.

#### Keywords

- Prism
- Constant cross-section
- Cylinder

#### You should know

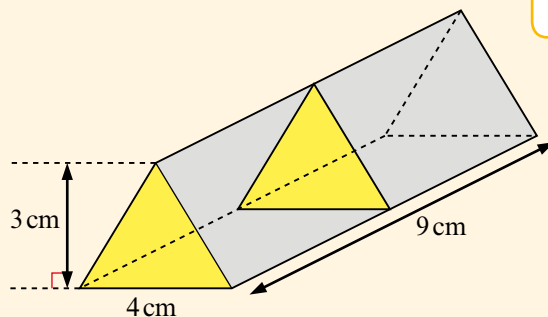
- How to find the volume of a cuboid
- The units used to measure volume
- How to find the area of a rectangle, triangle, parallelogram and trapezium
- How to find the area and circumference of a circle
- How to use Pythagoras' Theorem

#### explanation 1a

This is a triangular **prism**.

It is a **prism** because it has a **constant cross-section**.

The vertical cross-section is always a triangle.



Level 7

$$\begin{aligned}\text{The area of the triangular cross-section} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 4 \times 3 = 6 \text{ cm}^2\end{aligned}$$

If the length of the prism were 1 cm, the volume would be  $6 \text{ cm}^3$ .

The length of this prism is 9 cm.

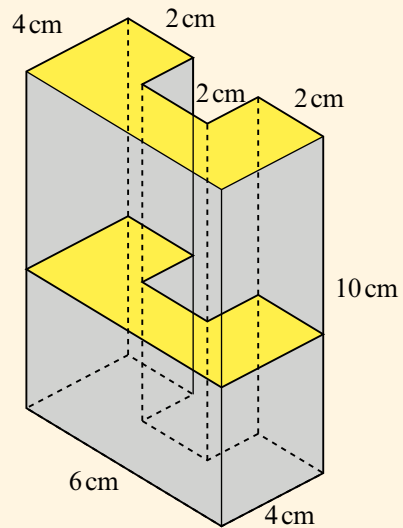
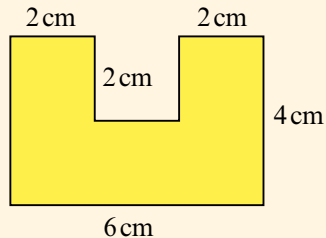
So, the volume is  $6 \text{ cm}^2 \times 9 \text{ cm} = 54 \text{ cm}^3$ .

In general,

$$\text{volume of a prism} = (\text{area of cross-section}) \times \text{length}$$

This is a U-shaped prism.

The horizontal cross-section is constant.



$$\begin{aligned}\text{Area of big rectangle} &= 4\text{ cm} \times 6\text{ cm} \\ &= 24\text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of square cut-out} &= 2\text{ cm} \times 2\text{ cm} \\ &= 4\text{ cm}^2\end{aligned}$$

$$\text{Area of U-shape} = 24\text{ cm}^2 - 4\text{ cm}^2 = 20\text{ cm}^2$$

The length of the prism is 10 cm.

$$\begin{aligned}\text{Volume of prism} &= (\text{area of cross-section}) \times \text{length} \\ &= 20\text{ cm}^2 \times 10\text{ cm} \\ &= 200\text{ cm}^3\end{aligned}$$

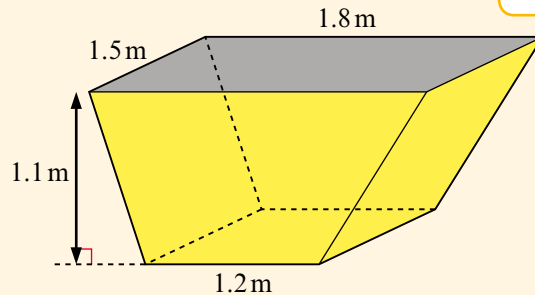


explanation 1c

Level 7

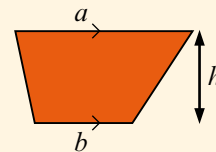
Look at this example.

Find the volume of the skip  
in cubic metres.



The vertical cross-section of the skip is a trapezium.

$$\begin{aligned}\text{Area of cross-section} &= \frac{1}{2}(a + b)h \\ &= \frac{1}{2} \times (1.8 + 1.2) \times 1.1 \\ &= 1.65 \text{ m}^2\end{aligned}$$

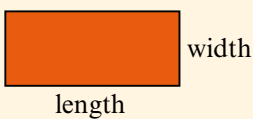


$$\begin{aligned}\text{Volume of a prism} &= (\text{area of cross-section}) \times \text{length} \\ &= 1.65 \text{ m}^2 \times 1.5 \text{ m} \\ &= 2.475 \text{ m}^3\end{aligned}$$

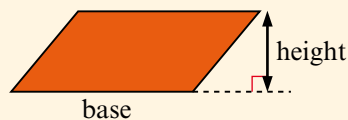
explanation 1d

Level 7

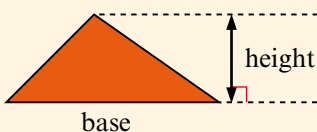
Common cross-sections



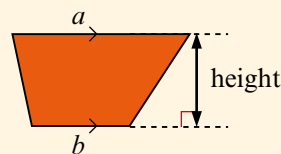
$$\text{Area} = \text{length} \times \text{width}$$



$$\text{Area} = \text{base} \times \text{height}$$



$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$



$$\text{Area} = \frac{1}{2} \times (a + b) \times \text{height}$$

explanation 2a

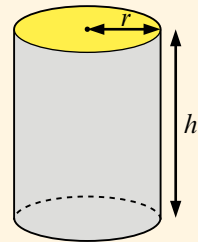
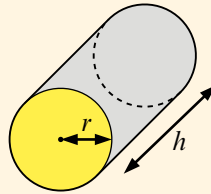
Level 7

When the cross-section of a prism is a circle, then the prism is called a **cylinder**.

$$\text{Area of cross-section} = \pi r^2$$

The length of the prism is  $h$ .

$$\begin{aligned}\text{Volume of a cylinder} \\ &= (\text{area of cross-section}) \times \text{length} \\ &= \pi r^2 \times h\end{aligned}$$



explanation 2b

Level 7

A tin of baked beans has the diameter and height shown.

$$\begin{aligned}\text{Area of cross-section} &= \pi r^2 \\ &= \pi \times 3.75^2 \\ &= 44.178\,646\,69\text{ cm}^2\end{aligned}$$

Length of prism = 10.2 cm

$$\begin{aligned}\text{Volume of a cylinder} \\ &= (\text{area of cross-section}) \times \text{length} \\ &= 44.178\,646\,69\text{ cm}^2 \times 10.2\text{ cm} \\ &= 450.622\,196\,2\text{ cm}^3 \\ &= 450.6\text{ cm}^3 \text{ to 1 d.p.}\end{aligned}$$



Diameter	= 7.5 cm
Radius	= 7.5 ÷ 2
	= 3.75 cm

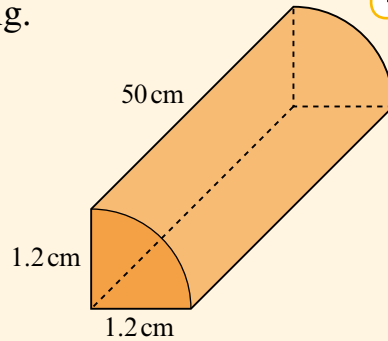
Find the volume of the wooden beading.

Area of a circle with radius 1.2 cm

$$= \pi r^2$$

$$= \pi \times 1.2^2$$

$$= 4.523893421 \text{ cm}^2$$



$$\text{Area of cross-section} = 4.523893421 \text{ cm}^2 \div 4 = 1.130973355 \text{ cm}^2$$

$$\text{Volume of beading} = (\text{area of cross-section}) \times \text{length}$$

$$= 1.130973355 \text{ cm}^2 \times 50 \text{ cm}$$

$$= 56.54866776 \text{ cm}^3$$

$$= 56.5 \text{ cm}^3 \text{ to 1 d.p.}$$

**explanation 3a****Level 7**

Find the surface area of the prism.

The front is a trapezium.

$$\begin{aligned}\text{Area of front} &= \frac{1}{2}(a + b)h \\ &= \frac{1}{2}(10\text{ cm} + 7\text{ cm}) \times 4\text{ cm} \\ &= 34\text{ cm}^2\end{aligned}$$

$$\text{Area of back} = 34\text{ cm}^2$$

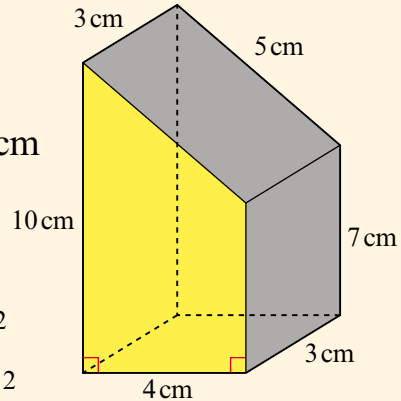
$$\text{Area of right side} = 3\text{ cm} \times 7\text{ cm} = 21\text{ cm}^2$$

$$\text{Area of left side} = 10\text{ cm} \times 3\text{ cm} = 30\text{ cm}^2$$

$$\text{Area of base} = 4\text{ cm} \times 3\text{ cm} = 12\text{ cm}^2$$

$$\text{Area of top} = 5\text{ cm} \times 3\text{ cm} = 15\text{ cm}^2$$

$$\begin{aligned}\text{Total surface area} &= (34 + 34 + 21 + 30 + 12 + 15)\text{ cm}^2 \\ &= 146\text{ cm}^2\end{aligned}$$



To find the surface area of this prism, you must find the length of the sloping side.

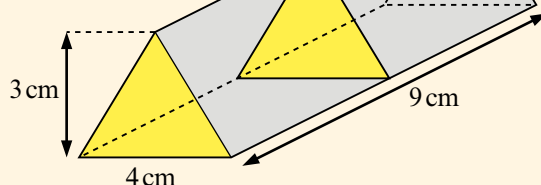
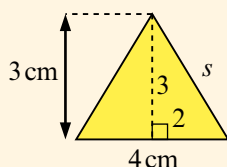
Using Pythagoras' Theorem:

$$3^2 + 2^2 = s^2$$

$$9 + 4 = s^2$$

$$13 = s^2$$

$$s = \sqrt{13} = 3.61 \text{ to 2 d.p.}$$



$$\text{Area of front face} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 3 = 6 \text{ cm}^2$$

$$\text{Area of back face} = 6 \text{ cm}^2$$

$$\text{Area of base} = 4 \times 9 = 36 \text{ cm}^2$$

$$\begin{aligned} \text{Area of sloping side} &= \sqrt{13} \times 9 \\ &= 32.45 \text{ cm}^2 \text{ to 2 d.p.} \end{aligned}$$

$$\begin{aligned} \text{Total surface area} &= 6 + 6 + 36 + 32.45 + 32.45 \\ &= 112.90 \text{ cm}^2 \text{ to 2 d.p.} \end{aligned}$$

Using  $\sqrt{13}$  instead of the rounded answer 3.61 gives a more accurate result.