Planner

Sequences

- Generating sequences from the term-to-term rule using ICT
- Generating quadratic sequences from the position-to-term rule
- Generating sequences from practical problems
- Finding the *n*th term of an arithmetic sequence
- Finding the *n*th term of a quadratic sequence

Keywords

You should know

explanation 1a | explanation 1b | online

1 Find the next three terms and write the term-to-term rule for each sequence.

a 12, 19, 26, …

b 81, 27, 9, …

c $\frac{1}{4}, \frac{1}{2}, 1, \ldots$

d 9, 5, 1, …

e 6, 12, 24, …

f −16, −12, −8, …

g 0.25, 0.5, 0.75, …

h −5, 10, −20, …

i 64, −16, 4, …

2 Copy and complete this table. Work out the terms without a calculator and leave your answers as fractions where necessary.

	First term(s)	Term-to-term rule	First five terms
a	2	+ 8	2, □, □, □, □
b	25	÷ 5	25, □, □, □, □
c	0	− 3	0, □, □, □, □
d	$\frac{1}{3}$	+ $\frac{1}{2}$	$\frac{1}{3}$, □, □, □, □
e	1	× $\frac{1}{3}$	1, □, □, □, □
f	−16	− 4	−16, □, □, □, □
g	7	÷ 4	7, □, □, □, □
h	1, 1	Add the two previous terms	1, 1, □, □, □

3 Write the next three terms of each sequence.

a 0.3, 0.6, 0.9, 1.2, … **b** 3, 6, 12, 24, 48, … **c** 10 000, 1000, 100, 10, …

d 320, 160, 80, 40, … **e** 0, 1, 1, 2, 3, 5, 8, … **f** 1, 2, 4, 7, 11, 16, …

4 What is the term-to-term rule for each sequence in question **3**?

5 Look at these sequences. Some terms in each sequence are missing.

i What is the term-to-term rule for each sequence?

ii Write the missing terms of each sequence.

a $\square$, 1.6, $\square$, 2.0, $\square$, 2.4 **b** 3, $\square$, 15, $\square$, 27

c 32, $\square$, 8, $\square$, 2 **d** 1, $\square$, 9, $\square$, 81, $\square$

explanation 2a explanation 2b online

6 Write down the first five terms in each arithmetic sequence from these position-to-term (nth term) rules. Show your working.

a $n + 8$ **b** $7n$ **c** $\frac{n}{2}$ **d** $-2n$ **e** $n - 0.4$

f $2n - 1$ **g** $n + \frac{1}{2}$ **h** $3n - 5$ **i** $4 + 5n$ **j** $88 - 12n$

7 For each of these position-to-term (nth term) rules, write the constant difference between terms of the sequence.
You should not need to work out any terms in the sequence.

a $6n + 3$ **b** $7 - 2n$ **c** $\frac{3n}{2} + 7$ **d** $4(2n - 1)$

8 Match each sequence with its correct nth term.

a 3, 7, 11, 15, 19, …	$11 - 4n$
b 6, 10, 14, 18, 22, …	$5n + 4$
c 30, 27, 24, 21, 18, …	$2n - 11$
d −9, −7, −5, −3, −1, ….	$4n - 1$
e 9, 14, 19, 24, 29, …	$11 - 5n$
f 7, 3, −1, −5, −9, …	$33 - 3n$
g 6, 1, −4, −9, −14, …	$4n + 2$

9 The nth term of a sequence is $5n - 3$. Which of these numbers appears in this sequence? Give the position of the number in the sequence where appropriate and explain your reasoning.

a 67 **b** 143 **c** 682 **d** 5347

10 These are the nth terms of four sequences.

$$3n - 1 \qquad 300 - 2n \qquad \frac{5n}{2} + 7 \qquad 13n - 4$$

Which of these sequences include the number 139? Show your working.

explanation 3a explanation 3b

11 These are the first five terms of some arithmetic sequences.

For each sequence, write the term-to-term rule, the position-to-term (nth term) rule and the 20th term.

a 5, 10, 15, 20, 25 **b** 3, 7, 11, 15, 19 **c** $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$

d 20, 23, 26, 29, 32 **e** 7, 19, 31, 43, 55 **f** 30, 40, 50, 60, 70

g 6, 9, 12, 15, 18 **h** 35, 42, 49, 56, 63 **i** 120, 124, 128, 132, 136

12 These are the first five terms of some arithmetic sequences.

For each sequence, write the term-to-term rule, the position-to-term (nth term) rule and the 20th term.

a −3, −6, −9, −12, −15 **b** −1, −3, −5, −7, −9 **c** 3, −1, −5, −9, −13

d 18, 15, 12, 9, 6 **e** 90, 83, 76, 69, 62 **f** 45, 30, 15, 0, −15

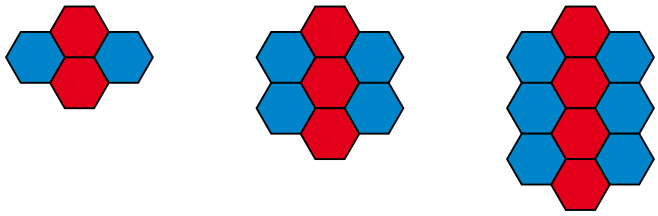
g 7, 15, 23, 31, 39 **h** 21, 23, 25, 27, 29 **i** 15, 16, 17, 18, 19

13 Calculate the number of terms in these sequences.

a 3, 7, 11, 15, …, 419 **b** 11, 16, 21, 26, …, 701

c 50, 43, 36, 29, …, −118 **d** 2, 11, 18, 27, …, 2315

14 Remla made tile designs with red and blue tiles.



Design 1 **Design 2** **Design 3**

a Copy and complete this table.

Design number	1	2	3	4	5	…	10
Number of red tiles							
Number of blue tiles							
Total number of tiles							

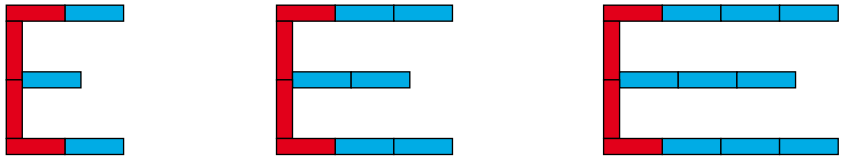
b Find an expression for the number of these tiles in the *n*th design.

i red tiles

ii blue tiles

iii the total number of tiles

15 E-mail Expert made a sequence of neon signs of the letter E to put on their shop window.



Size 1 **Size 2** **Size 3**

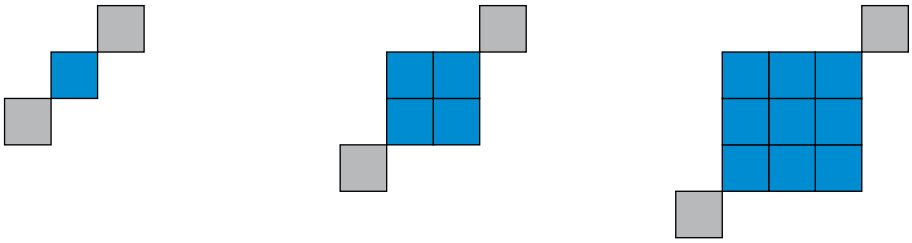
a Copy and complete this table.

Size	1	2	3	4	5	…	15
Number of lights	7	10	13			…	

b What is the *n*th term of the sequence? Justify your expression by referring to the diagrams. How can the rule be linked to the design structure?

explanation 4a | explanation 4b | explanation 4c | explanation 4d

16 Fred, a landscape gardener, is designing a new range of fish ponds with paving slabs at the opposite corners.



Pattern 1 **Pattern 2** **Pattern 3**

a How many grey squares would be in the 10th pattern?

b How many blue squares would be in the nth pattern?

c What is the nth term rule for the total number of grey squares and blue squares in this sequence?

17 Find the next two terms in each of these sequences.

a 1, 2, 4, 7, 11 **b** −4, 0, 6, 14, 24 **c** 10, 22, 38, 58, 82

d 30, 26, 20, 12, 2 **e** 2, 2.5, 3.5, 5, 7 **f** 10, 30, 60, 100, 150

18 Find the nth term of each of these quadratic sequences.
Use the inspection method.

a 0, 3, 8, 15, 24 **b** 3, 6, 11, 18, 27 **c** 3, 12, 27, 48, 75

d −5, −2, 3, 10, 19 **e** 0.5, 2, 4.5, 8, 12.5 **f** 4, 9, 16, 25, 36

19 Work out the first five terms in each sequence from the position-to-term (nth term) rule. Show your working.

a $n^2 + 7$ **b** $n^2 + n$ **c** $5 - n^2$ **d** $n^2 + 2n - 7$

e n^3 **f** $2n^3 - 5$ **g** $\frac{n}{n+2}$ **h** $\frac{10}{n}$

20 Match each sequence with its correct nth term.

a	10, 13, 18, 25, 34, …	$2n^2 + n$
b	−3, −12, −27, −48, −75, …	$n^2 + 9$
c	3, 10, 21, 36, 55, …	$10 - n^2$
d	2, 10, 24, 44, 70, …	$-3n^2$
e	3, 7, 13, 21, 31, …	$6n^2$
f	6, 24, 54, 96, 150, …	$3n^2 - n$
g	9, 6, 1, −6, −15, …	$n^2 + n + 1$

21 Calculate the number of terms in each sequence.

a 1, 4, 9, 16, …, 400 **b** −1, 2, 7, 14, …, 888 **c** 2, 6, 12, 20, …, 272

explanation 5a explanation 5b

22 Use a difference tree to find the nth term of each quadratic sequence.

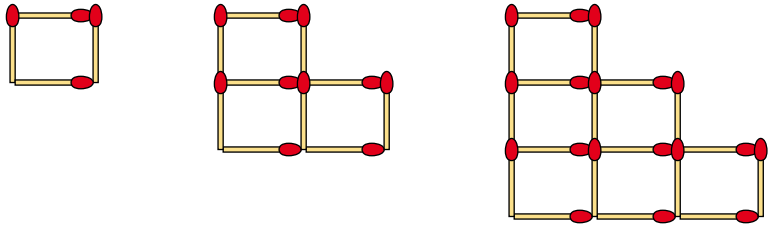
a 2, 7, 14, 23, 34, … **b** 2, 11, 26, 47, 74, …

c 1, 13, 27, 43, 61, … **d** 8, 14, 24, 38, 56, …

e 0, 5, 12, 21, 32, … **f** 2, 6, 12, 20, 30, …

g 6, 12, 22, 36, 54, … **h** 1, 6, 15, 28, 45, …

23 Sam is making staircase patterns from matchsticks.
Here are his first three patterns.

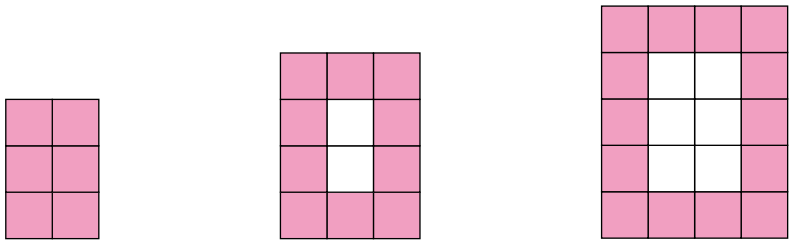


a Draw the fourth staircase pattern in the sequence and write the number of matchsticks used for each of the first four patterns.

b Find the rule for the number of matchsticks used to make the nth pattern in the sequence.

24 Aimee constructs a rectangular sequence pattern.

Each of the squares on the perimeter of the rectangle has been shaded pink.



Pattern 1 **Pattern 2** **Pattern 3**

a Draw the next two patterns in this sequence.

b Use a difference method to find the *n*th terms of these sequences.

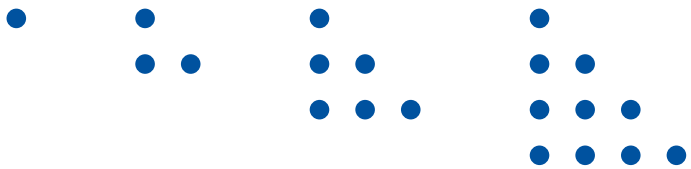
i squares shaded pink

ii unshaded squares

iii all the squares

c Show that the sum of your two *n*th terms from parts **b i** and **b ii** is equivalent to the *n*th term for part **b iii**.

25



1 3 6 10

These dot patterns form the first four terms of the sequence known as triangular numbers.

a Write the first 10 triangular numbers.

b Find the *n*th term for the triangular number sequence.

c Is 2600 a triangular number? Explain your reasoning.

d Use dot pattern diagrams to explain why the sum of any two consecutive triangular numbers is always a square number.

e Use algebra to prove that the sum of any two consecutive triangular numbers is a square number.

> Use x for one term and $x + 1$ for the next term in your rule from part **b**.