Formulae

- Finding the value of a formula
- **Obtaining a formula**
- Checking that a formula works

Keywords

You should know

explanation 1

- **1** Fatima thinks 7^2 is 14. What mistake has she made?
- **2** Sam has made mistakes. Copy his work out correctly. Explain how each calculation should be done.

a
$$2 \times 5^2 = 100 \times$$
 b $2^3 = 6 \times$ c $1^2 = 2 \times$ d $3^2 + 4^2 = 49 \times$ e $(-3)^2 = -9 \times$ f $(-2)^3 = 8 \times$

b
$$2^{3} = 6 \times$$

$$c 1^2 = 2 x$$

d
$$3^2 + 4^2 = 49 \times$$

$$e (-3)^2 = -9$$

$$f (-2)^3 = 8 \times$$

3 Which cards match each other? (Some of the cards will be left over.)

$$5x^2 + 3x^2$$

$$15x^{4}$$

$$x^2$$

$$x \times x^2$$

$$x^2 \qquad x \times x^2 \qquad 64 \qquad x^7 \times x^3 \qquad x^m \times x^n$$

16

$$x + x$$

$$x + x^2$$

$$x + x^2 \qquad 2x \qquad 5x^2 \times 3x^2$$

explanation 2a

explanation 2b

4 R = 3, S = 2 and T = -4. Find the value of each expression.

a
$$R^2$$

b
$$10R^2$$

a
$$R^2$$
 b $10R^2$ **c** $1+R^3$ **d** $5+3S^3$

d
$$5 + 3S^3$$

e
$$R^3 - 4S^3$$
 f T^3 **g** $2T^2$ **h** S^2T^3

$$\mathbf{f} = T^3$$

$$\mathbf{g} \quad 2T^2$$

h
$$S^2T^3$$

5 a = 3, b = 4 and c = 10. Which envelope contains the value of each expression?

A 43	B -6	<u>C</u> 18
34	E 1	F -7
G 54	36	28
405	K 2	8.5

- **a** 2(a+b-c) **b** a^2b **c** $10-a^2$ **d** $5(c-1)^2$ **e** 2ab+c **f** $a(b^2-c)$

- **g** $a^2 b^2$ **h** $\frac{b+c}{a+b}$
 - i (a-1)(b-1)(c-1)
- $\frac{a+b+c}{2}$ **k** 3+bc
- 6 One envelope in question 5 was not used. Make up three different expressions that could go inside this envelope.
- 7 u = 4, v = 2 and w = -3. Find the value of each expression.

- $u^2 + 6v^2$

- **a** u + w **b** u w **c** $u^2 + 6v^2$ **d** $u^3 + v^3$ **e** uv^3 **f** $2(v w)^2$ **g** $5wv^2$ **h** $2w^3$ **i** 7 2vw

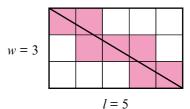
- j $(u+2w)^3$ k $(v^3-w^2)(2u+v)$ l $(v+w)(u+v)^2$
- **8** p = 5, q = 6 and r = -2. Find the value of each expression.
 - **a** 5p 2q
- **b** 2p + 3r
- c $2q^2 + 5r^2$

- **d** $8p^2 + 1$ **e** $\frac{26 + r^3}{p + q + r}$ **f** $4r^3 + 7$
- **g** $2p^2 3r$ **h** $10q^2 7$ **i** $2p^3 6p$

explanation 3a

explanation 3b

9 A rectangle has length *l* and width *w*. This is the formula for c, the number of squares that are cut by a diagonal of the rectangle.



c = l + w - (HCF of l and w)

- a Paul checks the formula on a 5 by 3 rectangle. He shades the squares that have been cut by the diagonal and counts them. The answer is c = 7. Does the formula give the same answer?
- **b** Draw rectangles for these values of *l* and *w*. Count the number of squares that are cut by the diagonal. Then check whether the formula gives the same answer.

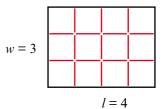
$$l = 7, w = 2$$

i
$$l = 7, w = 2$$
 ii $l = 10, w = 4$

iii
$$l = 8, w = 4$$

10 This rectangle is 3 units wide and 4 units long. It has 17 red dividers.

Arjun says that this formula gives the number of dividers, d, needed by a rectangle of length l and width w.



$$d = (w - 1)l + (l - 1)w$$

Divider -

- a Does the formula work for this rectangle?
- **b** Check whether the formula works for the following rectangles. Draw diagrams to show your working.

$$i l = 5, w = 3$$

i
$$l = 5, w = 3$$
 ii $l = 6, w = 2$ iii $l = 7, w = 1$

iii
$$l = 7, w = 1$$

The sum of the first *n* square numbers is *S*. Copy and complete the table.

n	1	2	3	4	5
S	1	1 + 4 = 5	1 + 4 + 9 =	1 + 4 + 9 + 16 =	1+4+9+16+25

b Here are three possible formulae for S. Only one is correct.

Which is the correct formula? Show how you decided.

i
$$S = 4n - 3$$

ii
$$S = \frac{5n^2 - 7n + 4}{2}$$

i
$$S = 4n - 3$$
 ii $S = \frac{5n^2 - 7n + 4}{2}$ iii $S = \frac{n(n+1)(2n+1)}{6}$

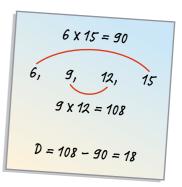
12 Brian writes four consecutive multiples of 3.

He multiplies the largest and smallest numbers together.

He subtracts this from the product of the middle two numbers.

The difference is D = 108 - 90 = 18

a Write another four consecutive multiples of 3 and find the difference *D*. Is the difference always 18?



- **b** What is the difference, D, for four consecutive multiples of 2?
- **c** Find the difference, *D*, for any four consecutive multiples of 4.
- **d** What is *D* for any four consecutive multiples of 1?
- e Copy and complete the table.

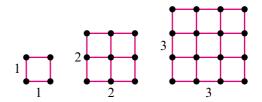
 m represents the multiple and D the difference.

Multiple (m)	1	2	3	4	5	6
Difference (D)			18			

- **f** Complete the formula. $D = 2 \times m^{\square}$
- **g** Find D for these values of m.
 - i 7

ii 9

- **iii** 10
- **13** Sally uses 12 red connectors to make a square of side length 2 units.



a Draw the next two squares. Copy and complete the table.

Side length of square (s)	1 unit	2 units	3 units	4 units	5 units
Number of connectors (c)	4	12			

- **b** Copy and complete this formula. $c = \Box s(s+1)$
- **c** Use your formula to find the number of connectors when s = 8.
- d Sally has 200 connectors. What is the largest square she can make?

14 The diagram shows squares of side length 2 units and of side length 3 units. To find T, multiply the numbers in opposite corners of the square, then add the products.

1	2
3	4

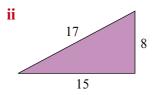
1	2	3
4	5	6
7	8	9

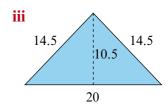
- **a** Draw a similar square that has side length 4 units.
- b For the square of side length 2 units, $T = (3 \times 2) + (1 \times 4) = 6 + 4 = 10$ For the square of side length 3 units, $T = (7 \times 3) + (1 \times 9) = 21 + 9 = 30$ Find T for the square of side length 4 units.
- **c** *n* is the side length of a square. Check whether $T = n^3 + n$ is the correct formula for *T* by finding the value of *T* when n = 2, 3, and 4.
- **15** This is the formula for the area of a triangle.

Area = $\frac{1}{2}$ × base × height

a Use the formula to find the areas of these triangles.

i 5 13 12





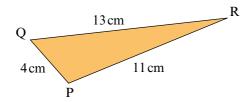
b The letters a, b, and c stand for the length of each side of a triangle.

$$s = \frac{a+b+c}{2}$$

$$T = \sqrt{s(s-a)(s-b)(s-c)}$$

For each triangle in part \mathbf{a} , find the values of s and T.

- **c** What does the formula for *T* give?
- **16** Explain how to find the area of the triangle PQR. Find the area of the triangle to the nearest square centimetre.



Use your answer to question **15c**.