Planner

Describing sequences

- Generating a sequence from a position-to-term rule
- Describing a sequence using a position-to-term rule
- Writing a position-to-term rule using algebra
- Using the relationship between a term-to-term rule and a rule for the nth term

Keywords

You should know

explanation 1a explanation 1b

1 Copy and complete the table below for each position-to-term rule.

Position	1	2	3	4	5
Term					

a Position → + 2 → Term

b Position → × 2 → Term

c Position → × 4 → − 3 → Term

d Position → × 1.5 → − 2 → Term

e Position → × −2 → + 2 → Term

f Position → − 1 → × −2 → Term

2 What do you notice about your answers to questions **1e** and **1f**? Why is this?

3 Find the position-to-term rules for these arithmetic sequences.
Write your position-to-term rules as function machines.

a

Position	1	2	3	4	5
Term	6	7	8	9	10

b

Position	1	2	3	4	5
Term	3	6	9	12	15

c

Position	1	2	3	4	5
Term	−4	−8	−12	−16	−20

4 Find the position-to-term rules for these arithmetic sequences.

a

Position	1	2	3	4	5
Term	2	5	8	11	14

b

Position	1	2	3	4	5
Term	3	5	7	9	11

Write your position-to-term rules as function machines.

Position → × d → + □ → Term

c

Position	1	2	3	4	5
Term	−3	−1	1	3	5

d

Position	1	2	3	4	5
Term	−3	−7	−11	−15	−19

e

Position	1	2	3	4	5
Term	2.5	3	3.5	4	4.5

explanation 2 online

5 **a** Convert each position-to-term rule below into an expression for the nth term.

b Calculate the 10th term of each sequence in part **a**.

i Position → + 3 → Term

ii Position → × 2 → − 3 → Term

iii Position → × 4 → − 1 → Term

iv Position → × $\frac{1}{2}$ → + 3 → Term

v Position → ÷ 2 → + 3 → Term

6 These are the rules for the nth terms of some arithmetic sequences. Write the first five terms of each sequence.

a $2n$ **b** $2n - 1$ **c** $3n + 4$ **d** $4n - 4$

e $\frac{1}{2}n + 1$ **f** $-2n$ **g** $-3n + 6$ **h** $-n + 4$

i $-\frac{1}{4}n + 2$ **j** $1 - 3n$ **k** $2(n - 3)$ **l** $\frac{n + 1}{2}$

7 Each table shows an arithmetic sequence.

i Write the term-to-term rule or difference, d, for each sequence.

ii Write the rule for the nth term of each sequence.

iii Write the 100th term of each sequence.

a

Position	1	2	3	4	5
Term	5	9	13	17	21

b

Position	1	2	3	4	5
Term	1	4	7	10	13

c

Position	1	2	3	4	5
Term	7	9	11	13	15

d

Position	1	2	3	4	5
Term	−3	−1	1	3	5

e

Position	1	2	3	4	5
Term	−2	−4	−6	−8	−10

f

Position	1	2	3	4	5
Term	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$

g

Position	1	2	3	4	5
Term	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$

8 What do you notice about the common differences, d, and the rules for the nth terms for the arithmetic sequences in question **7**?

9 Copy and complete these sentences.

a The expression for the nth term is $3n + 1$. The term-to-term rule is …

b The expression for the nth term is $-3n + 1$. The term-to-term rule is …

e The expression for the nth term is $-5n - 2$. The term-to-term rule is …

d The difference is + 2. A possible expression for the nth term is …

e The difference is − 7. A possible expression for the nth term is …

f The difference is $-\frac{1}{4}$. A possible expression for the nth term is …

10 Write an expression for the nth term of these arithmetic sequences.

a 4, 5, 6, 7, 8 **b** −2, 1, 4, 7, 10 **c** 12, 22, 32, 42, 52

d −3, 2, 7, 12, 17 **e** 7, 13, 19, 25, 31 **f** 0, −1, −2, −3, −4

g 6, 4, 2, 0, −2 **h** 23, 18, 13, 8, 3 **i** −5.5, −5, −4.5, −4, −3.5

11 Angus has spilt ink over his homework.

The ink covers parts of four arithmetic sequences.

Make a copy for him, completing all the boxes correctly.

Position	1	2	3	4	5	6	n
a		5	7	9		13	
b	14			5		−1	
c		−2				0	$\frac{n}{2} - 3$
d	5.7				6.5		

12 The oval contains sequences. The rectangle contains expressions for nth terms. Four of the sequences match four of the expressions.

a Which sequence matches which nth term?

b Write the first five terms of the sequence that matches the remaining nth term.

c Write a possible expression for the nth term of the remaining sequence.

d Which of the nth terms give sequences that are *not* arithmetic?

i …, 71, 78, 85, 92, 99, …

ii …, $3\frac{1}{2}$, $5\frac{1}{2}$, $7\frac{1}{2}$, $9\frac{1}{2}$, …

iii …, 34.5, 34.8, 35.1, 35.4, …

iv …, −68, −75, −82, −89, …

v …, $\frac{1}{23}$, $\frac{1}{27}$, $\frac{1}{31}$, $\frac{1}{35}$, …

A $\frac{3n}{10} + 30$

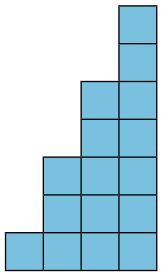
B $\frac{1}{4n + 3}$

C $2 - 7n$

D $7n - 13$

E $\frac{2}{n + 3}$

13 Jules is making staircase patterns using square tiles.



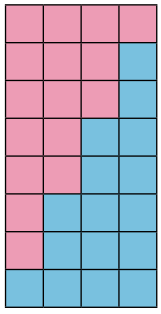
The first step is 1 tile high.
After that, each step is 2 tiles higher than the one before.
The second step is 3 tiles high, the third is 5 tiles high, and so on. The diagram shows the pattern with 4 steps.

a **i** Jules continues the pattern, and writes down the heights of the first six steps. Copy and complete his list.
1, 3, 5, ☐, ☐, ☐

ii Write an expression for the height of the nth step.

iii Write down the height of the 20th step.

b Jules makes two copies of his 4-step pattern and fits them together to form a rectangle, as shown.



i How many tiles are there in the rectangle?

ii Use your answer to write down the value of 1 + 3 + 5 + 7. Explain how you know.

c Jules makes two identical 20-step patterns.
He fits them together to make a rectangle as described in part **b**.

i What is the width of the rectangle? What is its length?

ii Use your answers to work out the sum of the first 20 odd numbers.

iii Describe an easy way to add the first 20 terms of any arithmetic sequence.

14 The school theatre has rows with different numbers of seats.
Row A has 13 seats, row B has 15 seats, row C has 17 seats and so on.
All the rows are lettered in alphabetical order up to and including T.

a How many seats are there in row H?

b How many seats in row Q?

c How many seats are there in total in the theatre?

15 One of the machines in a screw factory has developed a fault.

Each screw *should* be 25.4 mm long. In fact, the first screw was 25.39 mm long, the second was 25.38 mm, the third was 25.37 mm and so on.

a What was the length of the 50th screw?

b The machine was stopped after a screw of length 16.07 mm was produced.
How many incorrect screws were made?