## **Trial and improvement**

Using a calculator to find approximate solutions to an equation

Using a spreadsheet to find approximate solutions to an equation

**Keywords** 

You should know

explanation 1a

explanation 1b

explanation 1c

1 Solve these equations.

**a**  $x^2 + 7 = 32$  **b**  $x^2 - 1 = 35$  **c**  $x^2 + 1 = 10$ 

**d**  $x^3 = 8$  **e**  $x^3 - 1 = 26$  **f**  $2 - x^2 = -14$ 

**2** Find two solutions to each of these equations.

**a**  $x^2 + 8 = 72$  **b**  $y^2 - 10 = 134$  **c**  $5x^2 = 125$  **d**  $3x^2 - 4 = 44$ 

**3** Copy the table and use trial and improvement to find an approximate value of x which satisfies the equation  $x^2 + 2x = 58$ .

Give your answer to one decimal place.

The first trial has been done for you.

Value of x	$x^2 + 2x$ working and result of trial	Comment
7	$7^2 + 2 \times 7 = 63$	Too big

- 4 a Show that one solution to  $x^2 3x = 38$  lies between 7 and 8.
  - **b** Use trial and improvement to find this solution to one decimal place. Use a table to show your working.
- **5** a Show that a solution to  $x^3 + 3x 40 = 0$  lies between 3 and 4.
  - **b** Use trial and improvement to find this solution to one decimal place. Use a table to show your working.

**6** Use a calculator and trial and improvement to solve these equations to 1 d.p.

**a** 
$$t^3 = 30$$

**b** 
$$x^3 + x = 11$$

$$y^3 - y = 40$$

**a** 
$$t^3 = 30$$
 **b**  $x^3 + x = 11$  **c**  $y^3 - y = 40$  **d**  $3.25g^3 = 7.1$ 

**e** 
$$m^3 - m = 80$$
 **f**  $x^3 + x = 97$  **g**  $2y^3 + y = 6$  **h**  $2y^3 + 3y = 4$ 

$$x^3 + x = 97$$

$$y^3 + y = 6$$

**h** 
$$2y^3 + 3y = 4$$

**7** Use a calculator to find an approximate solution to 2 d.p. for each equation.

$$e^3 + 2e = 65$$

**b** 
$$z^3 - z = 4$$

**a** 
$$e^3 + 2e = 65$$
 **b**  $z^3 - z = 4$  **c**  $p^3 + p = 124$ 

**d** 
$$q(q^2 + 5) = 35$$

$$e^{-}$$
  $w^4 - 3w^2 = 300$ 

**d** 
$$q(q^2 + 5) = 35$$
 **e**  $w^4 - 3w^2 = 300$  **f**  $x(5 - x^2) = -43$ 

explanation 2a

explanation 2b explanation 2c

8 Peter is using a spreadsheet to solve the equation  $x^3 + 4x = 700$  by trial and improvement.

He wants to give his answer to one decimal place.

B2 <b>▼</b> fx							
	А	В	С	D	Е		
1	Value of x	Result of trial					
2	1						
3							
4							
5							
6							

- a What formula should he type in cell A3 to increase his x values by 1 each time?
- **b** What formula should he type in cell B2 to work out the value of each trial?
- Set up this spreadsheet and solve the equation, giving your solution to one decimal place.
- **9** Use a spreadsheet to solve each equation to two decimal places.

**a** 
$$z^3 + 3z = 420$$

**a** 
$$z^3 + 3z = 420$$
 **b**  $(x-6)(x+2)(2x-5) = 310$  **c**  $x^4 - 2x^3 + x^2 = 25$ 

$$\mathbf{c} \quad x^4 - 2x^3 + x^2 = 25$$

explanation 3

**10** Use a calculator to solve this problem.

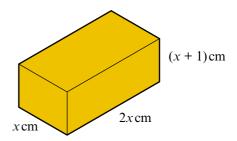
This cube has volume 50 cm<sup>3</sup>.



Let the length of the side be x, then find a formula for the volume in terms of x.

Work out the length of the side of the cube to two decimal places.

- 11 This cuboid has a volume of 400 cm<sup>3</sup>.
  - a Write an expression for the volume of this cuboid.
  - Form an equation and solve it using trial and improvement.Give your answer to two decimal places.



**12** Ilana is trying to solve this problem.

A number to the power 5 is equal to the twice the number cubed add 75.

Use a suitable method to find an approximation for this number to one decimal place.

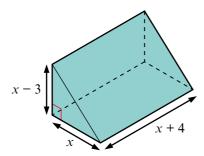
**13** The volume of a sphere is given by the formula  $V = \frac{4}{3}\pi r^3$  where V is the volume and r is the radius of the sphere.



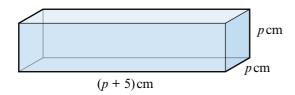
The Earth has an approximate volume of  $1.0868 \times 10^{12} \, \text{km}^3$ .

Use trial and improvement to find the approximate diameter of the Earth. Give your answer to the nearest kilometre.

14 The diagram shows a triangular prism. The prism has a volume of 300 cm<sup>3</sup>.



- a Show that the equation x(x-3)(x+4) = 600 can be used in this problem.
- **b** Use trial and improvement to find the value of x correct to one decimal place.
- **15** A storage box in the shape of a cuboid is manufactured from Perspex. Its dimensions are shown below. The manufacturer has allowed 1000 cm<sup>2</sup> for each box.
  - a Find the surface area of the box in terms of p.
  - **b** Find, by a numerical method, the value of p (to 1 d.p.) for which the surface area is 1000 cm<sup>2</sup>.



16 For each of the equations given below, decide whether an algebraic or numerical method is most efficient for finding solutions, then solve the equation (to 1 d.p. where appropriate).

**a** 
$$4x = 10 - x$$
 **b**  $x^2 = 121$ 

**b** 
$$x^2 = 121$$

**c** 
$$3x^3 + 2x = 64$$

**d** 
$$x(x^2-2)=25$$

**d** 
$$x(x^2-2)=25$$
 **e**  $x^2+3x-54=0$  **f**  $x^3+3x+2=0$ 

$$\mathbf{f} \quad x^3 + 3x + 2 = 0$$

$$\mathbf{g} \quad x^2 - 49 = 0$$

h 
$$x^3 - 27 = 0$$

**g** 
$$x^2 - 49 = 0$$
 **h**  $x^3 - 27 = 0$  **i**  $4x^4 + 16 = 8x^2$