



Formulae and expressions

- Solving problems involving more complicated formulae
- Changing the subject of a formula
- Writing a formula from a sentence

Keywords

You should know

explanation 1

- 1** Find the value of each expression by substituting in the value of the variable. Do not use a calculator.

a $3s + 2$ when $s = 1.5$

c $5(y^2 - 1)$ when $y = 3$

e $\frac{g^2}{4}$ when $g = 5$

g $20 - k$ when $k = -5$

i $\sqrt{9s^2}$ when $s = -3$

b $x^2 - 3x$ when $x = 4$

d $2z(z^2 - 4z)$ when $z = 4$

f $2m + 12$ when $m = -3$

h $\frac{2y + 6}{y}$ when $y = -6$

j $\frac{2p^2(p - 3)}{2}$ when $p = 10$

- 2** Find the value of each expression when $x = 7.4$ and $y = 2.1$. Give your answers to one decimal place.

a $3x - 4y$

c $3(x^2 - y^2)$

e $\frac{(y - 4)^2}{3y}$

g $\sqrt{(x^2 - y^2)}$

b $2y^2 + 3x$

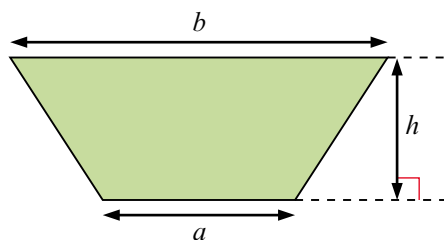
d $4xy(y - x)^2$

f $\frac{(xy)^2}{(y - 2)}$

h $\frac{4\sqrt{x}}{y}$

- 3** Work out the area of the trapezium with $a = 4.4$ cm, $b = 8.2$ cm and $h = 1.7$ cm.

$$\text{Area} = \frac{h}{2}(a + b)$$



- 4** The formula for the n th triangular number is $\frac{n(n+1)}{2}$.

For the first triangular number, $n = 1$. Substitute this into the formula: $\frac{1 \times 2}{2} = 1$.

So the first triangular number is 1.

Find these triangular numbers using the formula.

- a** 3rd **b** 5th **c** 10th **d** 25th **e** 100th

- 5** Copy and complete this table to generate terms in each sequence.

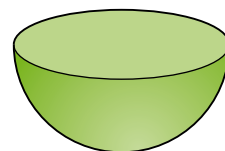
	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 10$
$n^2 + 3n - 2$	2	8	16	26	
$2n - 1$					
$n^3 - 2n^2$					
$n(n-3)(4-n)$					

- 6 a** The volume of a sphere is $\frac{4}{3}\pi r^3$, where r is the radius of the sphere. Find the volumes of spheres with these radii. Use the value of π from your calculator. Give each answer correct to 2 decimal places with the correct units.

- i** 5 cm **ii** 7.5 cm **iii** 1 m **iv** 25 mm

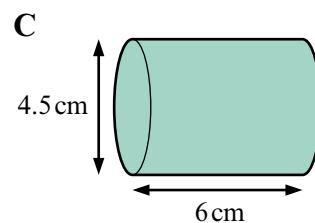
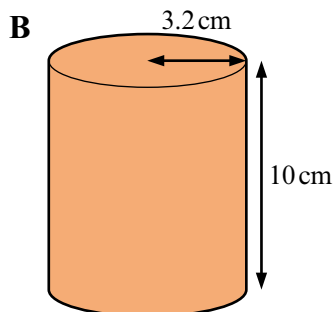
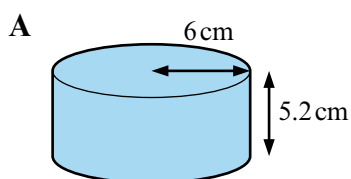
- b** If the radius of the Earth is 6378 km, find the volume of the Earth. Give your answer in standard form to three significant figures. (You may assume that the Earth is spherical and use the formula from part **a**.)

- c** What is the volume of a hemisphere of radius 20 m?
(A hemisphere is a sphere cut in half through the centre.)



- 7** To find the total surface area of a cylinder, use the formula $A = 2\pi r(r + h)$, where r is the base radius and h is the height of the cylinder.

Which cylinder has the greatest surface area? Show how you decided.



explanation 2a

explanation 2b

explanation 2c

8 Make r the subject of these formulae.

a $s = r + 6$

b $4y = 3r$

c $p = r - 3$

d $f = \frac{r}{5}$

e $4r = dg$

f $4 - r = t$

g $mnr = 7t$

h $\frac{1}{3}r = g$

9 Make x the subject of these formulae.

a $y = 3x + 1$

b $2x + y = 7$

c $y = \frac{x}{5} - 7$

d $y = 4t - x$

e $s = 5x + 3$

f $g = \frac{4}{x}$

g $b = \frac{3x}{2} + 8$

h $hx + 5 = t$

10 Make p the subject of the following formulae.

a $4p = s$

b $p - 3 = 2d$

c $2p + 7 = l$

d $\frac{p}{2} - 5 = q$

e $7p - r = 4z$

f $5q = 3 + p^2$

g $\frac{3pq}{5} + 2 = g$

11 The cost of hiring a minibus is given by the formula $C = 25 + 10d + 0.2m$ where C is the cost in pounds, d is the number of days the minibus is hired for and m is the number of miles the minibus travels.**a** The minibus is hired for 7 days and travels 400 miles. Calculate the total cost of hiring the minibus.**b** Ahmed hires the minibus for 5 days and his total bill comes to £460. How many miles did he travel?**c** Rearrange the formula to make d the subject.**12** The general equation of a straight line is $y = mx + c$.Rearrange this formula to make m the subject.**13** The formula for the area of a trapezium is $A = \frac{h}{2}(a + b)$.**a** Rearrange the formula to make b the subject.**b** Find the value of b when $A = 45$, $h = 2.5$ and $a = 8$

14 Make a the subject of these formulae.

a $y = a^2$

b $r = 2a^3$

c $a^2 + 4 = h$

d $t = s + 3a^2$

e $\frac{5a}{4} = u + v$

f $\frac{a^2}{4} + 5 = d$

g $\sqrt{a+5} = h$

h $3a^4 - 8 = k$

15 Make c the subject of these formulae.

a $6(4 + c) = n$

b $4(2d - c) = t$

c $fg = 5(3c - j)$

d $\frac{3c^2 - 5}{3} = y$

e $\sqrt{\frac{cd}{4}} = t$

f $\sqrt[3]{c} - 5 = h$

g $4\sqrt{c} - 7 = r$

h $fc^3 - 5 = 4p$

16 Make c the subject of these formulae.

a $y = \frac{t}{c+p}$

b $\sqrt{\frac{5r}{2c}} = 7$

c $y = \sqrt{\frac{5}{2+c}}$

d $z = 3\sqrt{\frac{t}{3-4c}}$

17 The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$, where V is the volume and r is the radius of the sphere.

Rearrange the formula to make r the subject.

18 The time taken for a pendulum to complete its swing is given by this formula.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

a Find T when $l = 1.2$ and $g = 9.8$.

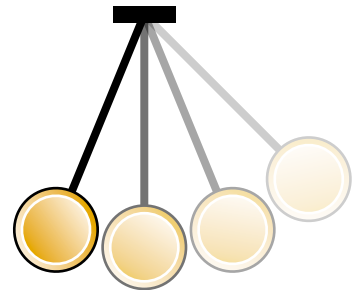
Give your answer to one decimal place.

b Rearrange the formula to make l the subject.

c Find l when $T = 1.7$ and $g = 9.8$.

Give your answer to one decimal place.

d Rearrange the formula to make g the subject.

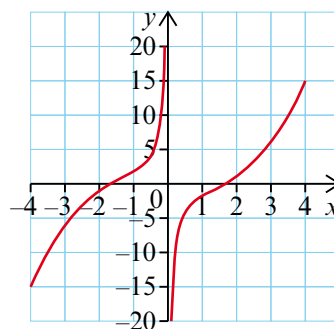


- 19** The graph shown has equation $y = \frac{x^3}{4} - \frac{4}{x^2}$.

Find the value of y for these values of x .

- a** $x = 2$ **b** $x = 0.5$
c $x = -2.5$ **d** $x = -0.5$
e Use trial and improvement to find the value of x when $y = 10$.

Give your answer to one decimal place.



explanation 3

- 20** The number of children at a party is three times the number of adults.

- a** Write a formula for the total number of people at the party.
b Two teenagers are helping with the party.
 How could you modify your formula to include this fact?

- 21** On each long side of a conference table, the number of seats is equal to the length of the table (in metres) multiplied by 2.



Three more people can sit at each end of the table.

- a** Write a formula that relates the number of people at a table to the length of the table.
b Find the number of people who can sit at a table 12m long.
c What length of table would have 30 seats?
d Make a chart showing the length of table (in metres) and the number of seats for table lengths 1, 2, 3, 4, 5, 6. What do you notice?

- 22** A taxi driver charges a fare of £2.50 added to 50p per mile added to £1.20 per passenger.

Write a formula that relates the fare to the number of miles and passengers.

- 23** Nicola invests £ P in a savings account for n years. The amount of compound interest, £ I , that she earns is calculated using this rule.

Add 100 to the annual percentage interest rate R , then divide by 100, and then raise this to the power n . Now multiply this value by the amount £ P invested. Finally subtract the original amount £ P .

- a** Write a formula for I in terms of R , n and P .
- b** Calculate the compound interest received on £18 000 invested for 5 years at a rate of 6% per year.

- 24** Planets that orbit the sun all have this relationship.

The cube of the distance of the planet in miles from the Sun is equal to the square of the time in years taken to orbit the Sun multiplied by a constant, K .

- a** Write the relationship as a formula.
- b** The Earth takes one year to make an orbit and is 9.3×10^7 miles from the Sun.

Find the value of the constant K . Give your answer in standard form.

- c** Rearrange your formula from part **a** to make T the subject.
- d** The planet Mars is 1.42×10^8 miles from the Sun.
Calculate the time it takes to make one orbit of the Sun.

- 25** Weather experts use the following formula to calculate the wind chill factor in degrees Fahrenheit.

$$W = (0.3\sqrt{V} + 0.474 - 0.02V)(T - 91.4) + 91.4$$

W is the wind chill factor, V is the wind speed in miles per hour and T is the temperature in degrees Fahrenheit.

- a** Calculate the wind chill factor when $V = 20$ km/h and $T = -2^\circ\text{F}$.
- b** Find the temperature when the wind chill is -20°F and the wind speed is 10 mph.