Mental methods

- Using mental strategies to solve word problems
- Using factors, powers and roots in solving problems
- Using known facts to derive unknown facts

Keywords

You should know

You should answer the questions in this topic without using a calculator.

explanation 1a

explanation 1b

explanation 1c

1 Clare started these using some number facts she already knew.

Finish her calculations or find a way of your own to find the answers.

a
$$34 \times 26 = (34 \times 25) + 34 = (34 \times 100 \div 4) + 34 =$$

b
$$82 \times 49 = (82 \times 50) - 82 =$$

$$c$$
 102 × 16 = (100 × 16) + (2 × 16) =

d
$$73 \div 25 = 73 \div 100 \times 4 =$$

e
$$91 \div 50 = 91 \div 100 \times 2 =$$

f
$$66 \div 8 = 66 \div 2 \div 2 \div 2 =$$

2 Find the answers to these using mental strategies for multiplying and dividing.

a
$$67 \times 5$$

c
$$36 \div 50$$

d
$$48 \times 25$$

f
$$32 \times 12.5$$

h
$$925 \div 25$$
 i 47×69

$$\mathbf{i}$$
 47 × 69

3 Work these out.

$$\mathbf{a} \quad 500 \times 8000$$

c
$$56000 \div 700$$

d
$$300 \div 6000$$

f
$$2500 \times 4000$$

4 Use your knowledge of place value to find the missing numbers.

a
$$0.34 \times 0.1 = \Box$$

b
$$8.6 \times 0.01 = \Box$$

c
$$0.0675 \times 0.01 = \Box$$

d
$$0.01 \times \square = 9.2$$

e
$$0.6 \div 0.1 = \square$$

d
$$0.01 \times \square = 9.2$$
 e $0.6 \div 0.1 = \square$ **f** $0.017 \div 0.01 = \square$

g
$$4.28 \div 0.01 = \square$$

h
$$\Box \div 0.1 = 20.09$$
 i $64 \div \Box = 6400$

i
$$64 \div \square = 6400$$

- **5** Work these out.
 - **a** 0.36×0.03

b 600 × 0.9

c $4.2 \div 6$

d $42 \div 0.07$

e 0.056×0.02

- \mathbf{f} 8400 ÷ 0.03
- **6** Given that $3.2 \times 48 = 153.6$, find
 - **a** 320×480

- **b** 0.32×4.8
- **c** 0.032×480

d 1536 ÷ 3.2

- e 1.536 ÷ 48
- **f** $153.6 \div 4.8$
- **7** Find three calculations that give each answer.
 - $\mathbf{a} \quad \square \times \square = 13.2$
- **b** $\Box \div \Box = 0.036$
- $\mathbf{c} \quad \square \times \square \times \square = 1.44$

explanation 2

- **8** Write whether each number is divisible by the digit shown.
 - **a** 735 (by 3)

- **b** 958 (by 4)
- c 1038 (by 6)

- **d** 4120 (by 8)
- e 3762 (by 9)
- **f** 80314 (by 6)

- **g** 73 053 (by 9)
- **h** 824 (by 3)
- i 92896 (by 4)

- j 101 836 (by 8)
- **k** 473 615 (by 3)
- 1 632 026 (by 6)
- **9** Which of these numbers are divisible by both 2 and 6?
 - **a** 84

b 326

c 432

d 573

e 812

- **f** 948
- **10** Which of these numbers are divisible by both 5 and 8?
 - **a** 96

b 520

c 645

d 355

e 1040

- **f** 3908
- 11 Which of the numbers 216, 504, 732, 756 and 1722 are divisible by each of these?
 - **a** 8

b 3

c 6

d 4

e 5

f 9

explanation 3

12 Work these out.

a
$$12 \times -5$$

a
$$12 \times -5$$
 b -12×-5 **c** -12×5

$$c -12 \times 5$$

d
$$60 \div -12$$

$$e -60 \div 12$$

d
$$60 \div -12$$
 e $-60 \div 12$ **f** $-60 \div -12$

13 Work out the answers to these.

a
$$3 \times -7$$

a
$$3 \times -7$$
 b -6×-4 **c** -4×8 **d** 5×-11

$$c -4 \times 8$$

d
$$5 \times -11$$

$$e -6 \times -2$$

$$f -12 \times 4$$

$$\mathbf{g} \quad 8 \div -2$$

e
$$-6 \times -2$$
 f -12×4 **g** $8 \div -2$ **h** $-12 \div 3$

$$i -20 \div -5$$

i
$$-20 \div -5$$
 j $-36 \div -3$ k $24 \div -3$ l $-48 \div -4$

$$k 24 \div -3$$

$$-48 \div -4$$

$$\mathbf{m} \quad 4 \times -20$$

$$\mathbf{n} - 8 \times -12$$

o
$$45 \div -3$$

m
$$4 \times -20$$
 n -8×-12 **o** $45 \div -3$ **p** $-120 \div -20$

q
$$-12 \times -12$$

$$r -500 \div 50$$

q
$$-12 \times -12$$
 r $-500 \div 50$ **s** $200 \div -20$ **t** 50×-8

t
$$50 \times -8$$

14 Copy and complete these multiplication tables.

a

×	-5		8
		-24	-32
2			
		42	
-3			

×		-9	-12
7			
	-45		
-11			
	150		-120

15 Copy and complete these division tables. Divide the numbers in the top row by the numbers in the first column.

For example, $108 \div -12 = -9$ has been done for you.

a

÷	108	-36	-72
3			
-12	-9		
18	6	-4	-4
-9			

b

		200	0.0
÷		300	-90
-10			
	12	20	-6
2			
	-3	-5	

- **16** Write down the next three numbers in each of these sequences.
 - **a** 5, -10, 20, ...

b -8, 24, -72, ...

c 7, -14, 28, ...

d -10, 10, -10, ...

- **17** True or false?
 - **a** $-48 \times -3 = 144$

b $-72 \div -18 = -4$

c $35 \times -5 = -175$

d $175 \div -5 = -35$

- **18** Work these out.

 - **a** $3 \times -6 \times -5$ **b** $-4 \times -7 \times -2$ **c** $-8 \times 5 \times 9$
 - d $-11 \times -3 \times 2$ e $-2 \times 4 \times -5$ f $3 \times 3 \times -3$

- **g** $-2 \times 4 \times 5$ **h** $12 \div -4 \times 2$ **i** $18 \times -2 \div -6$

- j $-20 \div -4 \times 2$ k $-9 \times 3 \div -3$ l $36 \div -4 \div -3$
- **19** Work these out.

a
$$\frac{-8 \times -4}{-2}$$
 b $\frac{-24 \times 3}{-8}$ **c** $\frac{56 \times -2}{14}$ **d** $\frac{63 \times -4}{-9}$

b
$$\frac{-24 \times 3}{-8}$$

$$c \frac{56 \times -2}{14}$$

d
$$\frac{63 \times -4}{-9}$$

e
$$\frac{-9 \times -4}{3}$$
 f $\frac{18 \times -2}{-9}$ g $\frac{-45 \times -4}{5}$ h $\frac{81 \times -3}{9}$

$$\frac{18 \times -2}{-9}$$

$$\frac{-45 \times -4}{5}$$

h
$$\frac{81 \times -3}{9}$$

$$i \quad \frac{-6 \times -8}{-6}$$

$$\frac{26+-2}{-8}$$

i
$$\frac{-6 \times -8}{-6}$$
 j $\frac{26 + -2}{-8}$ k $\frac{-65 - -5}{5}$ l $\frac{59 + -3}{8}$

$$\frac{59 + -3}{8}$$

20 Work these out.

a
$$(-6+11) \times -20$$

b
$$(7 + -19) \times (3 - 6)$$

c
$$(10-24) \div (7-9)$$

d
$$(9 \times -3) + (18 \div -2)$$

- **21** a Find two negative numbers whose sum is -9 and product is 14.
 - **b** Find two negative numbers whose sum is -12 and product is 35.
 - c Find two numbers whose sum is -32 and product is 60.
 - **d** Find two numbers whose sum is -32 and product is 240.

explanation 4

22 Work these out.

$$(-3)^2$$

b
$$(2)^4$$

$$(5)^3$$

d
$$(-6)^2$$

$$(10)^3$$

$$f (4)^3$$

23 Find the value of each of these expressions.

$$(-4)^2 + 2^3$$

b
$$3^3 - (2)^4$$

$$(-5)^2 + 5^2$$

d
$$10^6 - (10)^3$$

a
$$(-4)^2 + 2^3$$
 b $3^3 - (2)^4$ **c** $(-5)^2 + 5^2$ **d** $10^6 - (10)^3$ **e** $20^2 + (-10)^2$ **f** $5^2 - (-5)^2$

$$\mathbf{f} = 5^2 - (-5)^2$$

24 Find these square roots.

a
$$\sqrt{25}$$

b
$$\sqrt{81}$$

c
$$\sqrt{121}$$
 d $\sqrt{144}$ **e** $\sqrt{169}$

$$d \sqrt{144}$$

25 a Find two different numbers which, when squared, give the answer 16.

b Find a number which, when cubed, gives the answer 64.

c Find two square numbers with a difference of 13.

26 In an experiment it was found that the number of bacteria cells doubled each hour as shown by this table.

Hour	Number of cells	
0	$1 = 2^0$	
1	$2 = 2^1$	
2	$4 = 2^2$	
3	$8 = 2^3$	

How many cells will there be after these times?

- **a** 6 hours
- **b** 12 hours

explanation 5

27 Work these out using the difference of two squares.

$$a 6^2 - 5^2$$

b
$$12^2 - 8^2$$

a
$$6^2 - 5^2$$
 b $12^2 - 8^2$ **c** $20^2 - 11^2$ **d** $9^2 - 7^2$

d
$$9^2 - 7^2$$

$$e 12^2 - 9^2$$

$$f 20^2 - 14^2$$

$$16^2 - 15^2$$

e
$$12^2 - 9^2$$
 f $20^2 - 14^2$ g $16^2 - 15^2$ h $100^2 - 50^2$

28 The sign shows the cost of visiting a castle. How much would a family of 2 adults and 2 children save by buying a family pass?

Adults £8.50 Children £4 Family pass (2 adults and 2 children) £22.50

- **29** Johnny's camper van was stuck in second gear. He could only travel at about 25 miles per hour. How long would it take him to travel 225 miles at this speed?
- **30** A bird flies at an average speed of 30 kilometres per hour. How far will it fly in one minute at this speed?
- **31** Imogen measured her kitten's tail. It was 5.6 cm long. When the cat was fully grown its tail was 3 times longer. How long was the cat's tail then?



- **32** Adesh bought 24 lollipops at £0.36 each for his son's birthday party. How much did he pay?
- **33** A charity sold tickets for dinner with a celebrity at £70 each. The total amount received was £4900. How many tickets were sold?
- **34** Bella bought scary masks for all her friends who came to a horror movie night. They cost £0.90 each and she spent £27 altogether. How many masks did she buy?
- **35** A toy train travels around a 50 m track. In one circuit of the track the wheels turn 200 times. What distance is travelled when the wheels turn just once?

36 Ramesh and Amber were given large identical bags of counters.

Ramesh put his counters into piles of 3 and there were none left over.

Amber put hers into piles of 4 and there were none left over.

Which of these could have been the number of counters they had in the large bag?

78 128 192 270 288

- 37 There is a 0.6 chance that Emily will beat Tom in a tennis game. Out of 40 games, how many would you expect her to win?
- 38 Kalem wanted to make orange drink that was 10% concentrated juice. How many litres of concentrate would he need to make 0.26 litres of juice?
- 39 Forty years ago, Britain used pounds, shillings and pence. There were 20 shillings in a pound.A gallon of petrol cost about 7 shillings at that time.About how many pounds did a gallon of petrol cost?
- **40** a If 625 out of 1000 of people in a particular town travel to work alone, what percentage of the people travel to work with someone else?



- **b** If about 1% of the population has no television, what fraction has at least one television?
- c If 6.25% of households have no central heating, find the percentage of households that do have central heating.
- **41** a 15 miles per hour is approximately 24 kilometres per hour. How many kilometres per hour faster is 40 miles per hour?
 - b A group of students has £82 to share between them. After spending £10, each of them receives £4.50. How many students were there?
 - c The mean number of CDs owned by a group of friends is 15. After buying 8 more CDs between them, they own 98 in total. How many friends are there in the group?