Mental methods (1)

- Using facts you know to answer unfamiliar questions
- Working with multiples, factors, powers and roots

Keywords

You should know

Answer the questions in this section without using a calculator, unless told otherwise.

explanation 1a   explanation 1b

1 Write these fractions as decimals.

a $\frac{1}{5}$ b $\frac{3}{4}$ c $\frac{7}{20}$

d $\frac{6}{25}$ e $\frac{3}{8}$ f $\frac{4}{5}$

g $\frac{5}{8}$ h $\frac{9}{10}$ i $\frac{7}{50}$

2 a   i Use the fact that $\frac{1}{8} = 0.125$ to write $\frac{3}{8}$ as a decimal.

ii Check your answer to part **i** by calculating $3 \div 8$.

iii What is $\frac{7}{8}$ as a decimal?

How did you work it out?

Explain another method you could have used.

b Use the fact that $\frac{1}{8} = 0.125$ to work out these fractions.

i $\frac{1}{80}$ ii $\frac{1}{16}$ iii $\frac{1}{40}$ iv $\frac{3}{16}$

3 Write these decimals as fractions.
Give each answer in its simplest form.

a 0.4   b 0.85   c 0.84

d 0.625   e 1.9   f 2.375

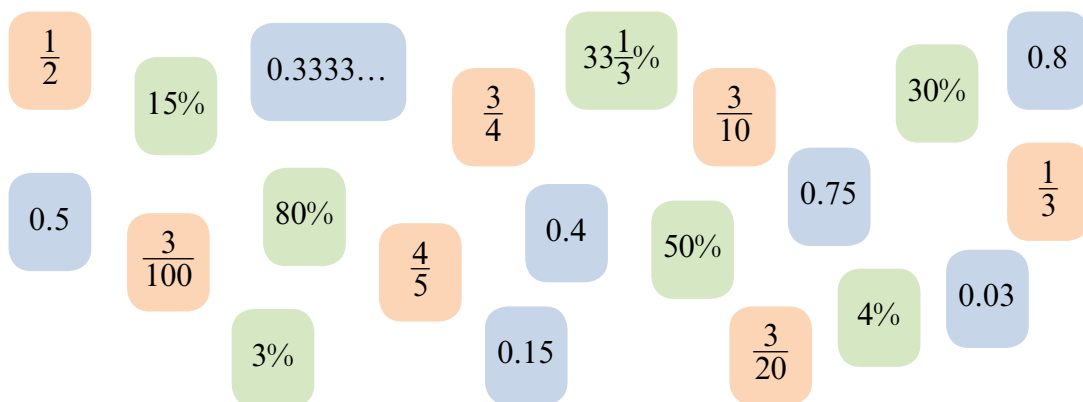g 3.72   h 4.44   i 10.16

4 Group these cards into sets so that each set contains an equivalent fraction, decimal and percentage.

Some cards are missing.

Work out what should be on the missing cards to complete the sets.

$\frac{1}{2}$ 0.3333…   15% $33\frac{1}{3}\%$ $\frac{3}{4}$ $\frac{3}{10}$ 30%   0.8

0.5 $\frac{3}{100}$ 80%   0.4   50%   0.75 $\frac{1}{3}$ 0.03   4%

$\frac{4}{5}$ 3%   0.15 $\frac{3}{20}$

5 Rewrite each article using percentages where appropriate.

a The chocolate digestive is Britain's favourite biscuit. The chocolate hobnob came second in a poll of 5000 people. About a third of people in the UK eat biscuits as a mid-morning snack but the largest proportion, four out of ten, eat them as they watch TV.

b Nine out of ten people say that parents should be told if their children are obese. More than $\frac{7}{10}$ said the government should abandon its plan to allow parents to opt out of being told. The poll used a research panel of a hundred families, each of two adults and up to four children.

explanation 2a   explanation 2b

6 Work out these quantities.

a 15% of £38   **b** 35% of 72 kg   **c** 21% of £62   **d** 30% of £56

e 64% of 39 km   **f** $33\frac{1}{3}\%$ of 69 m   **g** 5% of £20   **h** 3% of 15 km

7 This table gives details of the first week's viewing figures for a television programme over a 3-year period.

Work out estimates for the audience in each year.

Year	2008	2007	2006
% of total TV audience	19.9%	24.8%	33.4%
Estimated total TV audience in millions	20.5	20.0	21.6
Estimated programme audience in millions			

8 The prices shown do not include VAT.

£200   £550   £120

a Work out the VAT, at a rate of 17.5%, for each item.

b Find the total cost of each item, including VAT.

9 These are the normal prices of some items.
In a sale all items are reduced by 15%. Find the sale prices.

a trainers costing £45

b tennis racquet costing £38

c football costing £16

d hockey stick costing £55

e mountain bike costing £160

f table-tennis ball costing 60p

10 **a**   **i** Find 25% of 10% of £800

　　　　ii Now work out 10% of 25% of £800

　　b   **i** Find 50% of 5% of £800

　　　　ii Now work out 5% of 50% of £800

　　c   **i** Find 10% of 50% of £400

　　　　ii Now work out 50% of 10% of £400

　　d Write a sentence explaining what you notice about your answers to **a**, **b** and **c**.

explanation 3a   explanation 3b

11 Copy and complete this table.

3	×	4	=	
2	×	4	=	
1	×	4	=	
0.1	×	4	=	
0.2	×	4	=	
0.3	×	4	=	

12 Use the fact that $8 \times 4 = 32$ to work out these products.

　　a 8×0.4　　　　**b** 0.8×0.4　　　　**c** 80×0.4

　　d 8×0.04　　　　**e** 80×40　　　　**f** 0.8×400

13 Use the fact that $9 \times 7 = 63$ to work out these products.

　　a 0.9×7　　　　**b** 9×0.7　　　　**c** 90×7

　　d 90×0.7　　　　**e** 0.09×0.7　　　　**f** 900×0.7

14 Use your answer to part **a** to work out the answers to parts **b**, **c** and **d**.

　　a $20 \div 4$　　　　**b** $2 \div 4$　　　　**c** $0.2 \div 4$　　　　**d** $0.02 \div 4$

15 Use your answer to part **a** to work out the answers to parts **b**, **c** and **d**.

　　a $18 \div 2$　　　　**b** $1.8 \div 2$　　　　**c** $0.18 \div 2$　　　　**d** $0.018 \div 2$

16 Copy and complete these statements. Do not use the number 1!

a $\square \times \square = 6$ b $\square \times \square = 0.8$

c $\square \times \square = 0.4$ d $\square \div \square = 3$

e $\square \div \square = 0.2$ f $\square \div \square = 0.5$

17 Make 36 using the digits 1, 3 , 3 and 5 once each, together with any combination of the symbols +, −, ×, ÷ and brackets.

18 Andy was born in 1982. Using the digits of that year, in any order, together with any combination of the symbols +, −, ×, ÷ and brackets, how many numbers between 1 and 30 can you make?

19 To multiply a number by 25 Ann multiplies by 5 then by 5 again.

Bushra multiplies by 100 then divides by 4.

Connie halves the number twice then moves the digits two places to the left.

Explain why each pupil has a correct method.

20 Copy and complete this table of squares and cubes.

Number (x)	1	2	3	4	5	6	7	8	9	10
Square (x^2)	1		9							
Cube (x^3)	1			64						

21 In 1770 Joseph Louis Lagrange, a French mathematician, proved that every whole number can be expressed as the sum of no more than four squares. For example

$31 = 5^2 + 2^2 + 1^2 + 1^2$

$21 = 4^2 + 2^2 + 1^2$

a Using your table from question **20** to help you, show that this is true for all the whole numbers from 80 to 90 inclusive.

b Which of the numbers in part **a** can you make in more than one way?

22 Find those multiples of 5 less than 100 that can be expressed as the sum or difference of two squares. Use your table from question **20** to help you.

explanation 4

23 Split the numbers into factors to help work out these products.

    a 16×25 **b** 75×28 **c** 35×8

    d $64 \times 125 \times 15$ **e** $425 \div 25$ **f** $462 \div 14$

24 Square roots of large numbers can sometimes be found by splitting the number into factors.

$$\sqrt{225} = \sqrt{9 \times 25} = \sqrt{9} \times \sqrt{25} = 3 \times 5 = 15$$

Work these out by splitting each number into factors that are square numbers.

    a $\sqrt{324}$ **b** $\sqrt{900}$ **c** $\sqrt{729}$

    d $\sqrt{196}$ **e** $\sqrt{2500}$ **f** $\sqrt{1225}$

explanation 5

25 Change just one digit in each number to make it a multiple of 9.

    a   367   **b**   12 345   **c**   62 628   **d**   22 222

26 Change just one digit in each number to make it a multiple of 9 and 5.

    a   365   **b**   27 280   **c**   62 025   **d**   48 102

27

    35 875   4 283 604

    24 255   5 377 779

       160 600

    a   Which of these numbers are multiples of 5?

    b   Which of these numbers are multiples of 9?