# CS 331: Artificial Intelligence
# Naïve Bayes

Thanks to Andrew Moore for some course material

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## Naïve Bayes

- A special type of Bayesian network
- Makes a conditional independence assumption
- Typically used for classification

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# Classification

Suppose you are trying to classify situations that determine whether or not Canvas will be down. You've come up with the following list of variables (which are all Boolean):

| | |
|---|---|
| Monday | Is a Monday |
| Assn | CS331 assignment due |
| Grades | CS331 instructor needs to enter grades |
| Win | The Beavers won the football game |

We also have a Boolean variable called CD which stands for "Canvas down"



# Classification

These are called features or attributes

This is called the "class" variable (because we're trying to classify it)

| Monday | Assn | Grades | Win | CD |
|---|---|---|---|---|
| true | true | true | false | true |
| false | true | true | true | false |
| true | false | false | false | false |
| false | true | true | false | true |
| true | true | true | false | true |
| false | false | true | false | true |
| true | true | false | true | false |

These entries in the CD column are called "class labels"

# Classification

| Monday | Assn | Grades | Win | CD |
|---|---|---|---|---|
| true | true | true | false | true |
| false | true | true | true | false |
| true | false | false | false | false |
| false | true | false | false | true |
| true | true | true | false | true |
| false | false | true | false | true |
| true | true | false | true | false |

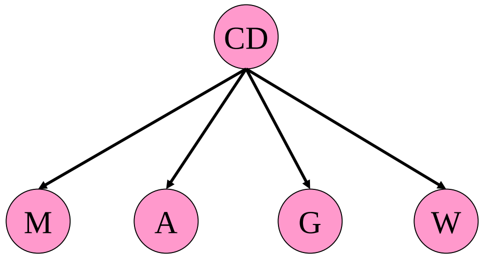
You create a dataset out of your past experience. This is called “training data”.

| Monday | Assn | Grades | Win |
|---|---|---|---|
| true | true | true | true |
| false | true | true | false |

You now have 2 new situations and you would like to predict if Canvas will go down. This is called “test data”.



# Naïve Bayes Structure

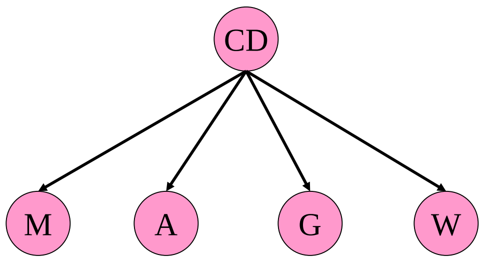



Notice the conditional independence assumption:

The features are conditionally independent given the class variable.

# Naïve Bayes Parameters

***P***$(CD)$ = ?




***P***$(M \mid CD)$ = ? ***P***$(A \mid CD)$ = ? ***P***$(G \mid CD)$ = ? ***P***$(W \mid CD)$ = ?

How do you get these parameters from the training data?

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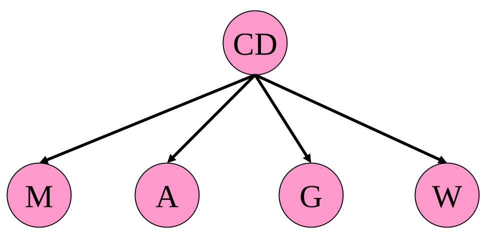
# Naïve Bayes Parameters




| CD | P( CD ) |
|---|---|
| false | (# of records in training data with CD = false) / (# of records in training data) |
| true | (# of records in training data with CD = true) / (# of records in training data) |

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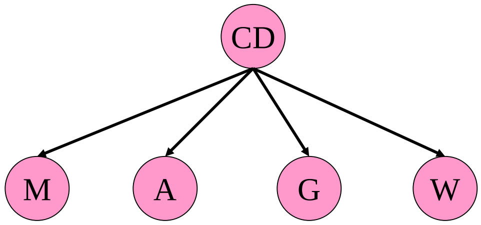
# Naïve Bayes Parameters




| M | CD | P( M \| CD ) |
|---|---|---|
| false | false | (# of records with M = false and CD = false) / (# of records with CD = false) |
| false | true | (# of records with M = false and CD = true) / (# of records with CD = true) |
| true | false | (# of records with M = true and CD = false) / (# of records with CD = false) |
| true | true | (# of records with M = true and CD = true) / (# of records with CD = true) |

# Inference in Naïve Bayes

$$\boldsymbol{P}(CD \mid M, A, G, W)$$

$$= \frac{\boldsymbol{P}(M, A, G, W \mid CD)P(CD)}{\boldsymbol{P}(M, A, G, W)}$$

By Bayes Rule

$$= \alpha \boldsymbol{P}(M, A, G, W \mid CD)\boldsymbol{P}(CD)$$

Treat denominator as constant

$$= \alpha \boldsymbol{P}(CD)\boldsymbol{P}(M \mid CD)\boldsymbol{P}(A \mid CD)\boldsymbol{P}(G \mid CD)\boldsymbol{P}(W \mid CD)$$

From conditional independence

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# Prediction

- Suppose you are now in a day when M=true, A=true, G=true, W=true.
- You need to predict if CD=true or CD=false.
- We will use the notation that CD=true is equivalent to cd and CD=false is equivalent to $\neg$cd.

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# Prediction

- You need to compare:
  - $P( cd \mid m, a, g, w ) = \alpha P( cd ) P( m \mid cd ) P( a \mid cd ) P( g \mid cd ) P( w \mid cd )$
  - $P(\neg cd \mid m, a, g, w) = \alpha P(\neg cd ) P( m \mid \neg cd ) P( a \mid \neg cd ) P( g \mid \neg cd ) P( w \mid \neg cd )$
- Whichever probability is the bigger of the two above, that is your prediction for CD
- Because you take the max of the two probabilities above, you can ignore $\alpha$ (since it is the same in both)

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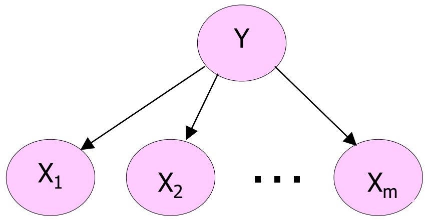
# The General Case




1. Estimate P(Y=v) as fraction of records with Y=v
2. Estimate P($X_i$=u | Y=v) as fraction of "Y=v" records that also have X=u.
3. To predict the Y value given observations of all the $X_i$ values, compute

$$Y^{\text{predict}} = \underset{v}{\operatorname{argmax}}\; P(Y=v \mid X_1=u_1 \cdots X_m=u_m)$$

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# Naïve Bayes Classifier

$$Y^{\text{predict}} = \underset{v}{\operatorname{argmax}}\; P(Y=v \mid X_1=u_1 \cdots X_m=u_m)$$

$$Y^{\text{predict}} = \underset{v}{\operatorname{argmax}} \frac{P(Y=v, X_1=u_1 \cdots X_m=u_m)}{P(X_1=u_1 \cdots X_m=u_m)}$$

$$Y^{\text{predict}} = \underset{v}{\operatorname{argmax}} \frac{P(X_1=u_1 \cdots X_m=u_m \mid Y=v)P(Y=v)}{P(X_1=u_1 \cdots X_m=u_m)}$$

$$Y^{\text{predict}} = \underset{v}{\operatorname{argmax}}\; P(X_1=u_1 \cdots X_m=u_m \mid Y=v)P(Y=v)$$

Because of the structure of the Bayes Net

$$Y^{\text{predict}} = \underset{v}{\operatorname{argmax}}\; P(Y=v)\prod_{j=1}^{m} P(X_j=u_j \mid Y=v)$$

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## Technical Point #1

- The probabilities P($X_j = u_j \mid Y = v$) can sometimes be really small
- This can result in numerical instability since floating point numbers are not represented exactly on any computer architecture
- To get around this, use the log of the last line in the previous slide i.e.

$$Y^{\text{predict}} = \underset{v}{\operatorname{argmax}} \left[ \log(P(Y = v)) + \sum_{j=1}^{m} \log(P(X_j = u_j \mid Y = v)) \right]$$

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## Technical Point #2

- When estimating parameters, what happens if you don't have any records that match a certain combination of features?
- For example, in our training data, we didn't have M=false, A=false, G=false, W=false
- This means that P($X_j = u_j \mid Y = v$) in the formula below will be 0 and the entire expression will be 0.

$$P(Y = v) \prod_{j=1}^{m} P(X_j = u_j \mid Y = v)$$

Even more horrible things happen if you had this expression in log space

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## Uniform Dirichlet Priors

Let $N_j$ be the number of values that $X_j$ can take on.

$$P(X_j = u_j \mid V = v) = \frac{(\#\text{records with } X_j = u_j \text{ and } V = v) + 1}{(\#\text{records with } V = v) + N_j}$$

What happens when you have no records with $V = v$ ?

$$P(X_j = u_j \mid V = v) = \frac{1}{N_j}$$

This means that each value of $X_j$ is equally likely in the absence of data. If you have a lot of data, it dominates the $1/N_j$ value. We call this trick a “uniform Dirichlet prior”.

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## What You Should Know

- How to learn the parameters for a Naïve Bayes model
- How to make predictions with a Naïve Bayes model
- How to implement a Naïve Bayes Model

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# Programming Assignment #3

You will classify text into two classes.

There are two files:

1. Training data: trainingSet.txt
2. Testing data: testSet.txt

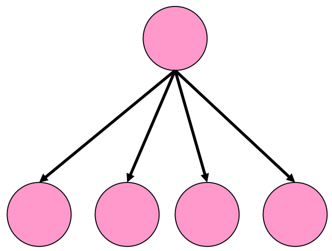


# Programming Assignment #3

Two parts to this assignment:

1. Pre-processing step
2. Classification step

# 1. Preprocessing Step



- Recall that naïve Bayes has the structure shown to the right
- The nodes correspond to random variables, which are the features or attributes in the data
- What are the features in the documents?
- Note: a "document" in our assignment is a Yelp review



# The Vocabulary

- The features of the documents will be the presence/absence of words in the vocabulary
- The vocabulary is the list of words that are known to the classifier
- Ideally, the vocabulary would be all the words in the English language
- For this assignment, you will form the vocabulary using all the words in the training data

# Bag of Words

Suppose you have the following documents:

| Training Data | Class Label |
|---|---|
| This is an excellent laptop | Class 0 |
| No, this is not sarcasm! | Class 1 |

| Test Data | |
|---|---|
| Excellent Laptop =P | Class 1 |

You will ignore punctuation for this assignment

The vocabulary will be:
this, is, an, excellent, laptop, no, not, sarcasm



# Bag of Words

Vocab: this, is, an, excellent, laptop, no, not, sarcasm

⇩ Keep this in alphabetical order to help with debugging

Vocab: an, excellent, is, laptop, no, not, sarcasm, this

# Training data

Next, convert your training and test data into features

Training Data

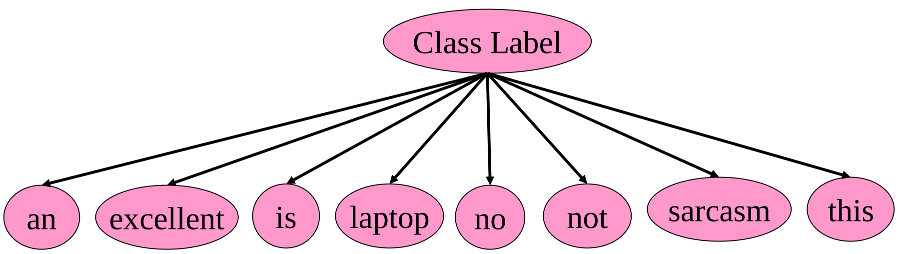
| an | excellent | is | laptop | no | not | sarcasm | this | Class Label |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |

Test Data

| an | excellent | is | laptop | no | not | sarcasm | this | Class Label |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |

You will output the training data in feature form, with the features alphabetized (we will grade you on this output).

# 2. Classification Step (Training Phase)




- Your naïve Bayes classifier now looks something like the above
- You still need to fill in the conditional probability tables in each node
- This is done in the training phase (as described on slides 9 and 10)
- Remember to use the uniform Dirichlet prior trick (see slide 17)

# 2. Classification Step (Testing Phase)

Testing phase

- Load the featurized test data
- For each document in the test data, predict its class label
- This requires computing:

  P(Class label | Words in document)

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# 2. Classification Step (Testing Phase)

Suppose you have the following test instance:

| an | excellent | is | laptop | no | not | sarcasm | this | Class Label |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | (to be predicted) |

$$P(Class = 1 \mid an = 0, excellent = 1, is = 0, laptop = 1, no = 0, not = 0, sarcasm = 0, this = 0)$$
$$= \alpha P(Class = 1) * P(an = 0|Class = 1) * P(excellent = 1|Class = 1) * P(is = 0|Class = 1) * P(laptop = 1|Class = 1) * P(no=0/Class =1) * P(not = 0|Class = 1) * P(sarcasm = 0|Class = 1) * P(this = 0|Class = 1)$$

Note: Use P(Word = 1 | Class) if you have a 1 for the word. Otherwise use P(Word = 0 | Class)

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## 2. Classification Step (Testing Phase)

| an | excellent | is | laptop | no | not | sarcasm | this | Class Label |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | (to be predicted) |

Then compute the following:

$$P(Class = 0 \mid an = 0, excellent = 1, is = 0, laptop = 1, no = 0, not = 0, sarcasm = 0, this = 0)$$
$$= \alpha P(Class = 0) * P(an = 0|Class = 0) * P(excellent = 1|Class = 0) * P(is = 0|Class = 0) * P(laptop = 1|Class = 0) * P(no=0/Class=0) * P(not = 0|Class = 0) * P(sarcasm = 0|Class = 0) * P(this = 0|Class = 0)$$



## 2. Classification Step (Testing Phase)

| an | excellent | is | laptop | no | not | sarcasm | this | Class Label |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | (to be predicted) |

If

$$P(Class = 1 \mid an = 0, excellent = 1, is = 0, laptop = 1, no = 0, not = 0, sarcasm = 0, this = 0)$$
$$>$$
$$P(Class = 0 \mid an = 0, excellent = 1, is = 0, laptop = 1, no = 0, not = 0, sarcasm = 0, this = 0)$$

Predict Class = 1 otherwise predict Class = 0

## 2. Classification Step (Testing Phase)

- For each document in the testing data set, predict its class label
- Compare the predicted class label to the actual class label
- Output the accuracy for each class:

$$\frac{\#\text{ correctly predicted class labels}}{\text{total }\#\text{ of predictions}}$$

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## Results

There are two sets of results we require:

1. Results #1:
   – Use trainingSet.txt for the training phase
   – Use trainingSet.txt for the testing phase
   – Report accuracy
2. Results #2:
   – Use trainingSet.txt for the training phase
   – Use testSet.txt for the testing phase
   – Report accuracy

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