CS 331: Artificial Intelligence
Local Search II

3. Beam Search

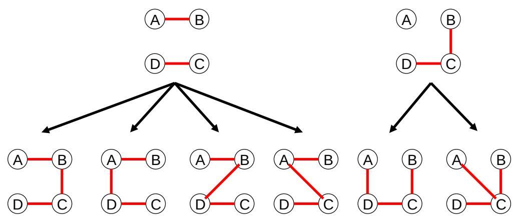
Local Beam Search

Travelling Salesman Problem

Keeps track of k states rather than just 1. k=2 in this example. Start with k randomly generated states.

Local Beam Search Example

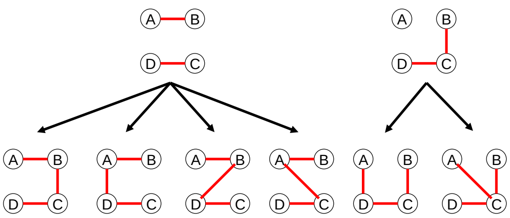
Travelling Salesman Problem (k=2)



Generate all successors of all the k states

Local Beam Search Example

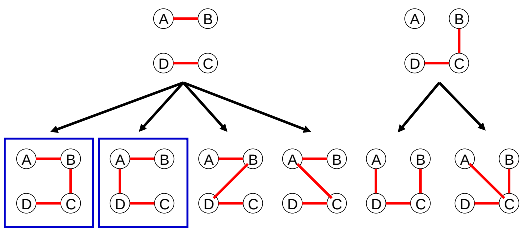
Travelling Salesman Problem (k=2)



None of these is a goal state so we continue

Local Beam Search Example

Travelling Salesman Problem (k=2)



Select the best k successors from the complete list

Local Beam Search Example

Travelling Salesman Problem (k=2)

Repeat the process until goal found

Local Beam Search

- How is this different from k random restarts in parallel?
- Random-restart search: each search runs independently of the others
- Local beam search: useful information is passed among the k parallel search threads
- Eg. One state generates good successors while the other k-1 states all generate bad successors, then the more promising states are expanded

Local Beam Search

- Disadvantage: all k states can become stuck in a small region of the state space
- To fix this, use stochastic beam search
- Stochastic beam search:
  - Doesn't pick best k successors
  - Chooses k successors at random, with probability of choosing a given successor being an increasing function of its value

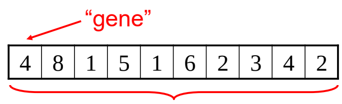
4. Genetic Algorithms

Genetic Algorithms

- Like natural selection in which an organism creates offspring according to its fitness for the environment
- Essentially a variant of stochastic beam search that combines two parent states (just like sexual reproduction)
- Over time, population contains individuals with high fitness

Definitions

- Fitness function: Evaluation function in GA terminology
- Population: k randomly generated states (called individuals)
- Individual: String over a finite alphabet



Definitions

- Selection: Pick two random individuals for reproduction
- Crossover: Mix the two parent strings at the crossover point

Parents

4	8	1	5	1	6	2	3	4	2
5	4	1	7	3	7	3	6	1	7

Offspring

4	8	1	5	3	7	3	6	1	7
5	4	1	7	1	6	2	3	4	2

Crossover point randomly chosen

Definitions

- Mutation: randomly change a location in an individual's string with a small independent probability

4	8	1	5	1	6	2	3	4	2

4	8	1	5	1	6	0	3	4	2

Randomness aids in avoiding small local extrema

GA Overview

Population = Initial population
Iterate until some individual is fit enough or enough time has elapsed:
  NewPopulation = Empty
  For 1 to size(Population)
    Select pair of parents (P_1,P_2) using Selection(P,Fitness Function)
    Child C = Crossover(P_1, P_2)
    With small random probability, Mutate(C)
    Add C to NewPopulation
  Population = NewPopulation
Return individual in Population with best Fitness Function

GA Overview

Population = Initial population
Iterate until some individual is fit enough or enough time has elapsed:
  NewPopulation = Empty
  For 1 to size(Population)
    Select pair of parents (P_1,P_2) using Selec
    Child C = Crossover(P_1, P_2)
    With small random probability, Mutate(C)
    Add C to NewPopulation
  Population = NewPopulation
Return individual in Population with best Fitness Function

This pseudocode only produces one child. Could also do a variant like before where we produce 2 children

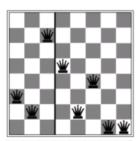
Lots of variants

- Variant 1: Culling - individuals below a certain threshold are removed
- Variant 2: Selection based on:

$$P(X\ selected) = \frac{Eval(X)}{\sum_{Y \in Population} Eval(Y)}$$

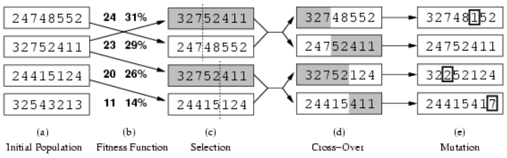
Example: 8-queens

- Fitness Function: number of nonattacking pairs of queens (28 is the value for the solution)
- Represent 8-queens state as an 8 digit string in which each digit represents position of queen

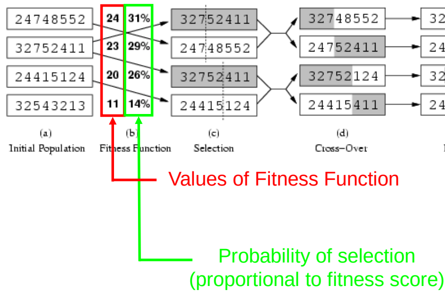


3	2	7	5	2	4	1	1

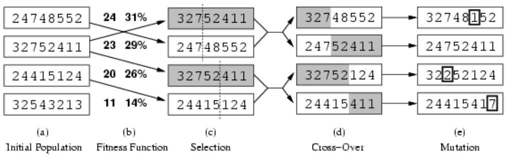
Example: 8-queens



Example: 8-queens (Fitness Function)

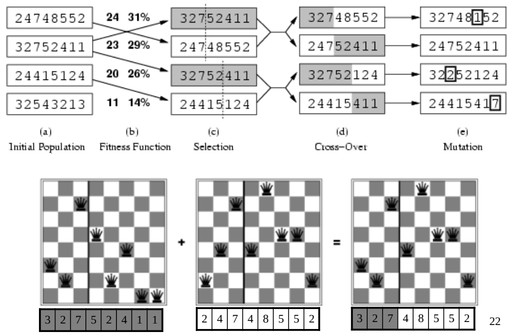


Example: 8-queens (Selection)

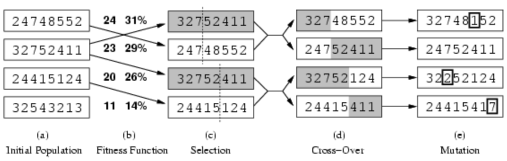


Notice 3 2 7 5 2 4 1 1 is selected twice while 3 2 5 4 3 2 1 3 is not selected at all

Example: 8-queens (Crossover)



Example: 8-queens (Mutation)



Mutation corresponds to randomly selecting a queen and randomly moving it in its column

Implementation details on Genetic Algorithms

- Initially, population is diverse and crossover produces big changes from parents
- Over time, individuals become quite similar and crossover doesn't produce such a big change
- Crossover is the big advantage:
  - Preserves a big block of "genes" that have evolved independently to perform useful functions
  - E.g. Putting first 3 queens in positions 2, 4, and 6 is a useful block

Schemas

- A substring in which some of the positions can be left unspecified eg. 246*****
- Instances: strings that match the schema
- If the average fitness of the instances of a schema is above the mean, then the number of instances of the schema within the population will grow over time

25

Schemas

- Schemas are important if contiguous blocks provide a consistent benefit
- Genetic algorithms work best when schemas correspond to meaningful components of a solution

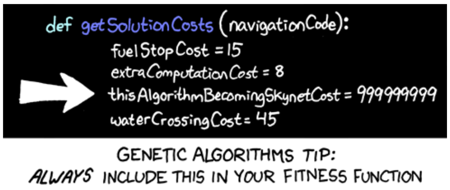
26

The fine print…

- The representation of each state is critical to the performance of the GA
- Lots of parameters to tweak but if you get them right, GAs can work well
- Limited theoretical results (skeptics say it's just a big hack)

27

And remember….



(From http://www.xkcd.com/534/)

28

Gradient Descent

29

Discrete Environments

Hillclimbing pseudocode

X ← Initial configuration
Iterate:
  E ← Eval(X)
  N ← Neighbors(X)
  For each X_i in N
    E_i ← Eval(X_i)
  E* ← Highest E_i
  X* ← X_i with highest E_i
  If E* > E
    X ← X*
  Else
    Return X

- In discrete state spaces, the # of neighbors is finite.
- What if there is a continuum of possible moves leading to an infinite # of neighbors?

30

Local Search in Continuous State Spaces

- Almost all real world problems involve continuous state spaces
- To perform local search in continuous state spaces, you need techniques from calculus
- The main technique to find a minimum is called gradient descent (or gradient ascent if you want to find the maximum)

Gradient Descent

- What is the gradient of a function f(x)?
  - Usually written as

$$\nabla f(x) = \frac{\partial}{\partial x} f(x)$$

  - $\nabla f(x)$ (the gradient itself) represents the direction of the steepest slope
  - $|\nabla f(x)|$ (the magnitude of the gradient) tells you how big the steepest slope is

Gradient Descent

Suppose we want to find a local minimum of a function f(x). We use the gradient descent rule:

$$x \leftarrow x - \alpha \nabla f(x)$$

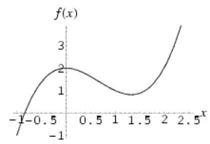
α is the learning rate, which is usually a small number like 0.05

Suppose we want to find a local maximum of a function f(x). We use the gradient ascent rule:

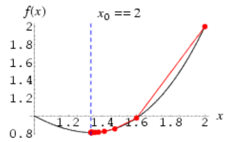
$$x \leftarrow x + \alpha \nabla f(x)$$

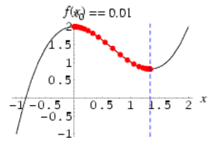
Gradient Descent Examples

$f(x) = x^3 - 2x^2 + 2$



These pictures were taken from Wolfram Mathworld





Question of the Day

- Why not just calculate the global optimum using $\nabla f(x) = 0$?
  - May not be able to solve this equation in closed form
  - If you can't solve it globally, you can still compute the gradient locally (like we are doing in gradient descent)

Multivariate Gradient Descent

- What happens if your function is multivariate eg. $f(x_1,x_2,x_3)$?
- Then

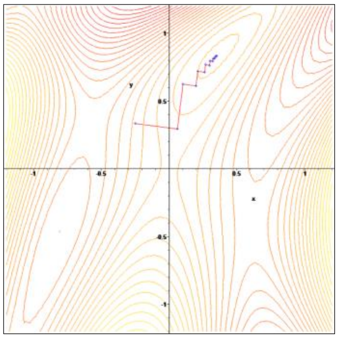
$$\nabla f(x_1, x_2, x_3) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_3} \right)$$

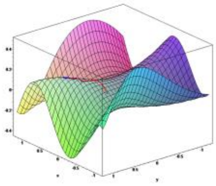
- The gradient descent rule becomes:

$$x_1 \leftarrow x_1 - \alpha \frac{\partial f}{\partial x_1} \qquad x_3 \leftarrow x_3 - \alpha \frac{\partial f}{\partial x_3}$$

$$x_2 \leftarrow x_2 - \alpha \frac{\partial f}{\partial x_2}$$

Multivariate Gradient Ascent





Pictures taken from Wikipedia entry on gradient descent

More About the Learning Rate

- If α is too large
  - Gradient descent overshoots the optimum point
- If α is too small
  - Gradient descent requires too many steps and will take a very long time to converge

Weaknesses of Gradient Descent

1. Can be very slow to converge to a local optimum, especially if the curvature in different directions is very different
2. Good results depend on the value of the learning rate α
3. What if the function f(x) isn't differentiable at x?

What you should know

- Be able to formulate a problem as a Genetic Algorithm
- Understand what crossover and mutation do and why they are important
- Differences between hillclimbing, simulated annealing, local beam search, and genetic algorithms
- Understand how gradient descent works, including its strengths and weaknesses
- Understand how to derive the gradient descent rule