

CS 331: Artificial Intelligence

Local Search 1

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Tough real-world problems



Suppose you had to solve VLSI layout problems (minimize distance between components, unused space, etc.)...



Or schedule airlines...



Or schedule workers with specific skill sets to do tasks that have resource and ordering constraints

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What do these problems have in common?

- These problems are unlike the search problems from last class:
 - The path to the goal is irrelevant -- all you care about is the final configuration
 - These are often optimization problems in which you find the best state according to an objective function
- These problems are examples of **local search problems**

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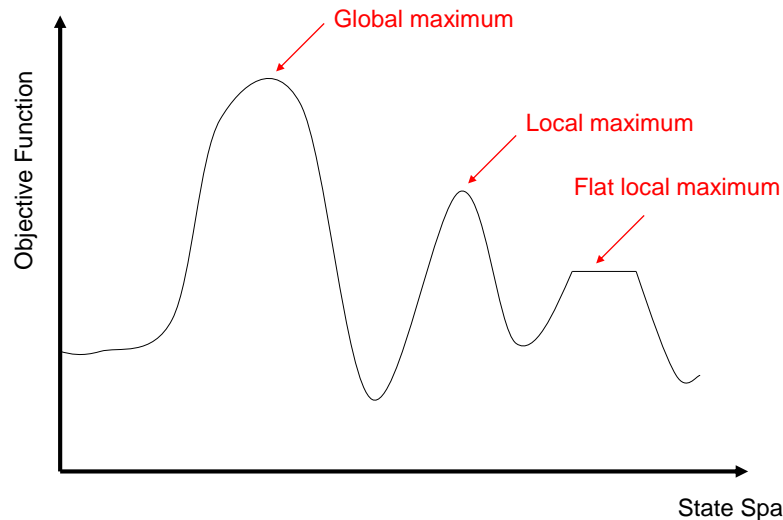
Local Search Problems

- Given a set of states $S = \{X_1, \dots, X_m\}$
- And an objective function $\text{Eval}(X_i)$ that returns the “goodness” of a state
- Find the state X^* that maximizes the objective function

Note: Sometimes $\text{Eval}(X_i)$ is a cost function instead of an objective function. In this case, we want to find the state X^* that minimizes the cost function. We will deal with objective functions in these slides but it's easy to just flip the signs and think of cost functions.

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1D State-Space Landscape



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Why is this hard?

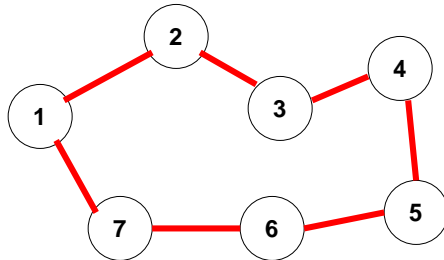
- Lots of states (sometimes an infinite number)
- Most of these problems are NP-complete
- Objective function might be expensive

But:

- Use very little memory (usually constant)
- Find reasonable (but usually not optimal) solutions in large or infinite state spaces

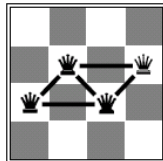
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More examples Traveling Salesman Problem

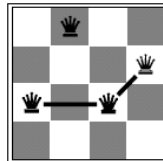


- State X = order of cities visited such that all cities are visited exactly once
- $\text{Eval}(X)$ = length of the tour defined by state X
- Want to minimize the length of the tour

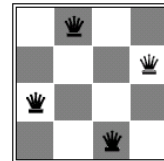
More Examples n-queens



$\text{Eval}(X) = 5$




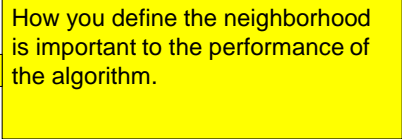

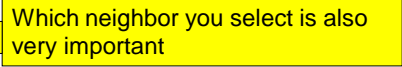

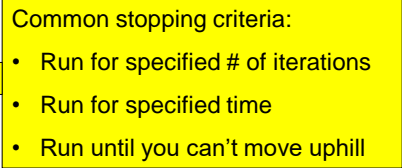
$\text{Eval}(X) = 2$



$\text{Eval}(X) = 0$

- State X = placement of the queens on the board
- $\text{Eval}(X)$ = # of pairs of queens attacking each other
- Want to minimize $\text{Eval}(X)$

Local Search Algorithm Recipe

1. Start with initial configuration X
2. Evaluate its neighbors ie. the set of all states reachable in one move from X

3. Select one of its neighbors X^*

4. Move to X^* and repeat until the current configuration is satisfactory


Outline

1. Hill-climbing
2. Simulated Annealing
3. Beam Search
4. Genetic Algorithms

1. Hill-climbing

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Hill-climbing (Intuitively)

- “...resembles trying to find the top of Mount Everest in a thick fog while suffering from amnesia.”
- Starting at initial state X , keep moving to the neighbor with the highest objective function value greater than X 's.



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Hillclimbing Pseudocode

$X \leftarrow$ Initial configuration

Iterate:

$E \leftarrow \text{Eval}(X)$

$N \leftarrow \text{Neighbors}(X)$

For each X_i in N

$E_i \leftarrow \text{Eval}(X_i)$

$E^* \leftarrow$ Highest E_i

$X^* \leftarrow X_i$ with highest E_i

If $E^* > E$

$X \leftarrow X^*$

Else

Return X

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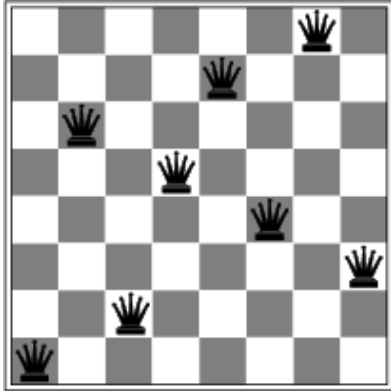
Example: 8-queens

18	12	14	13	13	12	14	14
14	16	13	15	12	14	12	16
14	12	18	13	15	12	14	14
15	14	14	♙	13	16	13	16
♙	14	17	15	♙	14	16	16
17	♙	16	18	15	♙	15	♙
18	14	♙	15	15	14	♙	16
14	14	13	17	12	14	12	18

These numbers are the value of the heuristic cost function if you move the queen in that column to that square

- State: a board with 8 queens
- Successor function: move a single queen to any other square in same column
- Heuristic cost function (h): # of pairs of queens attacking each other

Example: 8-queens



- Local minimum with $h=1$
- Note: at global minimum, $h=0$

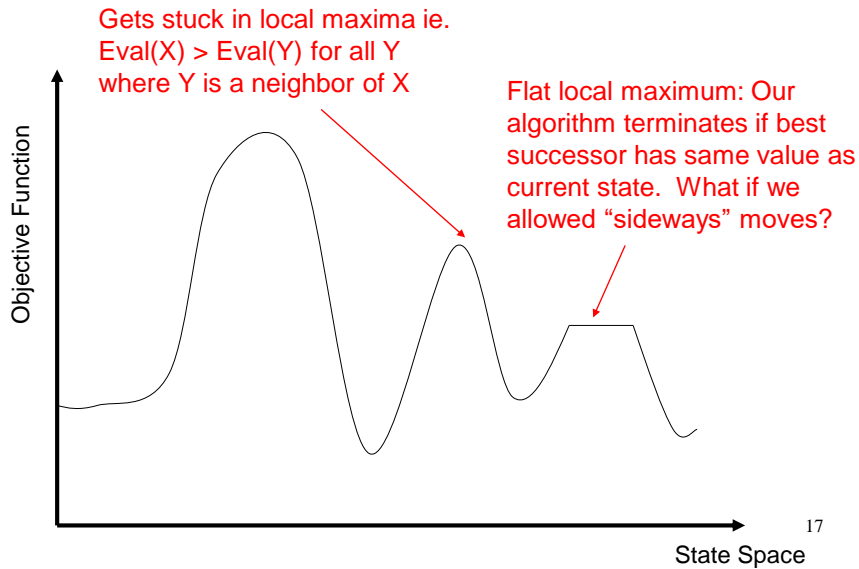
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More on hill-climbing

- Hill-climbing also called **greedy** local search
- Greedy because it takes the best immediate move
- Greedy algorithms often perform quite well

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Problems with Hill-climbing



Problems with Hill-climbing (Ridges)

Sequence of local maxima not directly connected to each other

All available actions from each local maxima go downhill

Neighborhoods

- Recall that we said that defining the neighborhood correctly is critical to the performance of the algorithm
- Large neighborhood: Lots of evaluations to do at each state but better chance of finding a good maximum
- Small neighborhood: Fewer evaluations at each state but can potentially get stuck in more local maxima

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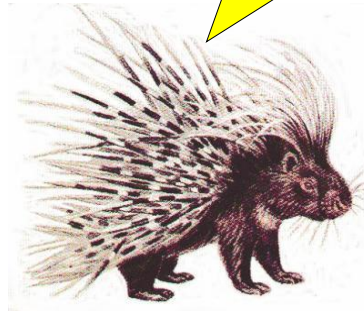
Variants of Hill-climbing

- Stochastic hill climbing:
 - Chooses at random among the uphill moves
 - Probability of selection varies with steepness
- First-choice hill climbing:
 - Generates successors randomly until one is generated that is better than the current state
 - Good when state has many successors
- Random-restart hill-climbing
 - Good for dealing with local maxima
 - Conduct a series of hill-climbing searches from randomly generated initial states
 - Stop when a goal state is found (or until time runs out, in which case return the best state found so far)

Which variant?

- Depends on the state-space landscape (which is difficult to know a priori)
- Typical real-world problems have an exponential number of local maxima
- But hill-climbing still works reasonably well

What if a state-space landscape looked like this guy's back?



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Simulated Annealing

- Hill-climbing never makes a downhill move
- What if we added some random moves to hill-climbing to help it get out of local maxima?
- This is the motivation for **simulated annealing**

If you're curious, **annealing** refers to the process used to harden metals by heating them to a high temperature and then gradually cooling them

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2. Simulated Annealing

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Simulated Annealing Pseudocode

$X \leftarrow$ Initial configuration

Iterate:

$E \leftarrow \text{Eval}(X)$

$X' \leftarrow$ Randomly selected neighbor of X

$E' \leftarrow \text{Eval}(X')$

If $E' \geq E$

$X \leftarrow X'$

$E \leftarrow E'$

Else with probability p

$X \leftarrow X'$

$E \leftarrow E'$

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Simulated Annealing Pseudocode

$X \leftarrow$ Initial configuration

Iterate:

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If $E' \geq E$

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Else with probability p

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This part is different
from hillclimbing

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Simulated Annealing Pseudocode

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How do you set p ?

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Setting p

- What if p is too low?
 - We don't make many downhill moves and we might not get out of many local maxima
- What if p is too high?
 - We may be making too many suboptimal moves
- Should p be constant?
 - We might be making too many random moves when we are near the global maximum

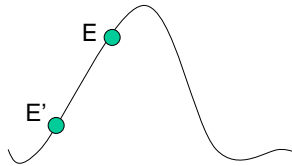
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Setting p

- **Decrease p as iterations progress**
 - Accept more downhill moves early, accept fewer as search goes on
 - Intuition: as search progresses, we are moving towards more promising areas and quite likely toward a global maximum
- **Decrease p as $E-E'$ increases**
 - Accept fewer downhill moves if slope is high
 - See next slide for intuition

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Decreasing p as $E-E'$ increases



$E-E'$ is large: we are likely moving towards a sharp (interesting) maximum so don't move downhill too much



$E-E'$ is small: we are likely moving towards a smooth (uninteresting) maximum so we want to escape this local maximum

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Setting p

- Needs a temperature parameter T
- If $E' \leq E$, accept the downhill move with probability $p = e^{-(E-E')/T}$
- Start with high temperature T , (more downhill moves allowed at the start)
- Decrease T gradually as iterations increase (less downhill moves allowed)
- **Annealing schedule** describes how T is decreased at each step

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Complete Simulated Annealing Pseudocode

```
X ← Initial configuration
Iterate:
  Do K times:
    E ← Eval(X)
    X' ← Randomly selected neighbor of X
    E' ← Eval(X')
    If E' ≥ E
      X ← X'
      E ← E'
    Else with probability  $p = e^{-(E-E')/T}$ 
      X ← X'
      E ← E'
  T =  $\alpha T$ 
```

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Complete Simulated Annealing Pseudocode

```
X ← Initial configuration
Iterate:
  Do K times:
    E ← Eval(X)
    X' ← Randomly selected neighbor of X
    E' ← Eval(X')
    If E' ≥ E
      X ← X'
      E ← E'
    Else with probability  $p = e^{-(E-E')/T}$ 
      X ← X'
      E ← E'
  T =  $\alpha T$ 
```

We keep the temperature fixed and we iterate K times

Exponential cooling schedule
 $T(n) = \alpha T(n-1)$ with $0 < \alpha < 1$.

Convergence

- If the schedule lowers T slowly enough, the algorithm will find a global optimum with probability approaching 1
- In practice, reaching the global optimum could take an enormous number of iterations

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The fine print...

- Design of neighborhood is critical
- Lots of parameters to tweak eg. α , K , initial temperature
- Simulated annealing is usually better than hillclimbing if you can find the right parameters

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What You Should Know

- Be able to formulate a problem as a local search problem
- Know the difference between local search and uninformed and informed search
- Know how hillclimbing works
- Know how simulated annealing works
- Know the pros and cons of both methods

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Exercise

You have to move from your old apartment to your new one.

You have the following:

- A list $L = \{a_1, a_2, \dots, a_n\}$ of n items, each with a size $s(a_i) > 0$.
- There are M moving boxes available, each with a box capacity C (assume MC far exceeds the sum of the sizes of your items). You can put as many items into a box as long as the sum of their sizes does not exceed the box capacity C .

Your job is to pack your stuff into as few boxes as possible.
Formulate this as a local search problem.

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Exercise (continued)

- States?
- Neighborhood?
- Evaluation function?
- How to avoid local maxima?

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