Different Types of Factoring

Warm Up:

a.) Given $p(x) = 2x^3 - 7x^2 + 3x - 1$, state the remainder if $p(x)$ is divided by $x+2$

$$p(-2) = 2(-2)^3 - 7(-2)^2 + 3(-2) - 1$$
$$= -16 - 28 - 6 \quad -1$$
$$= -51 \leftarrow \text{remainder}$$

Try it:

b.) Factor $p(x) = x^4 - 2x^3 - 3x^2 + 4x + 4$

$p(-1) = 0 \nearrow p(x) = (x+1)\underbrace{(x^3 - 3x^2 + 4)}_{Q(x)}$

$Q(-1) = 0 \nearrow p(x) = (x+1)^2 \underbrace{(x^2 - 4x + 4)}_{\text{go again}} \leftarrow \text{Factors (Perf Sq)}$

$\boxed{\therefore p(x) = (x+1)^2 (x-2)^2}$
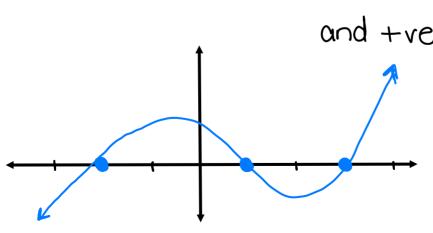
Factoring Polynomials

Why do we need to factor?

Sketch the graph of $\boxed{f(x) = x^3 - 2x^2 - 5x + 6}$.

not so easy without the zeros

$\curvearrowright$ SAME EQUATION

Now sketch the graph of $\boxed{f(x) = (x-3)(x-1)(x+2)}$

Much easier to work with! use zeros, degree 3, and +ve L.C.

opposite end behav.

as $x \to \infty$, $f(x) \to \infty$

Factoring Polynomials

- <u>Always</u> common factor 1st $(!!!)^{1000}$ (Gr 10 and Gr 11)

- Binomial to factor? $\overset{\text{look for}}{\text{difference}}$, sum/diff
    of squares   of cubes
    Gr. 10   Gr. 11

- Trinomial to factor? use tricky trinomial chart
    or decomposition. (Gr 10)

- More than 3 terms? → try factor by grouping (Gr 11)
    → Use factor theorem (Gr 12)

FACTORING Polynomials

Ex: What are the zeros of $p(x) = (x+2)(x-5)(2x-7)$?

zeros are $-2, +5, +\frac{7}{2}$

Since they are zeros we know that

$p(-2) = 0$ since $(x+2)$ is a factor

$p(5) = 0$ since $(x-5)$ is a factor

$p(\frac{7}{2}) = 0$ since $(2x-7)$ is a factor

In general, if $p(k) = 0$, then $(x-k)$ is a factor

FACTOR $x^4 - 4x^3 + 4x^2 - 4x + 3$

Decompose

$x^4 - 4x^3 + 3x^2 + 1x^2 - 4x + 3$

GROUP   GROUP

$(x^4 - 4x^3 + 3x^2) + (x^2 - 4x + 3)$

Common factor brackets 1st

$x^2(x^2 - 4x + 3) + 1(x^2 - 4x + 3)$

COMMON FACTOR OUT

looking at both terms, what is common?

$(x^2 - 4x + 3)(x^2 + 1)$

Doesn't factor

$(x - 3)(x - 1)(x^2 + 1)$

✷ FACTORING worksheet only truly needs factor theorem x2

↖ twice

if you choose the most efficient method!

Factor the following:

1.) $x^4 - 8x^3 - 3x^2 + 24x$

2.) $6x^3 - 17x^2 + 15x - 4$

3.) $2x^4 + 5x^3 - 14x^2 - 5x + 12$

4.) $4x^4 - 7x^3 - 18x^2 + 28x + 8$

5.) $x^4 - x^3 + 8x - 8$

6.) $x^4 - 4x^3 + 6x^2 - 4x + 1$

7.) $2x^3 + x^2 + x - 1$

8.) $x^4 + 6x^2 + 9$

FACTOR the following:

1.) $x^4 - 8x^3 - 3x^2 + 24x$

2.) $6x^3 - 17x^2 + 15x - 4$

FACTOR the following:

3.) $2x^4 + 5x^3 - 14x^2 - 5x + 12$

4.) $4x^4 - 7x^3 - 18x^2 + 28x + 8$

FACTOR the following:

5.) $x^4 - x^3 + 8x - 8$

6.) $x^4 - 4x^3 + 6x^2 - 4x + 1$

FACTOR the following:

7.) $2x^3 + x^2 + x - 1$

8.) $x^4 + 6x^2 + 9$

PRACTICE: complete factoring examples from class.

(SECT 3.7) pg 182 #2cdh, 3bc, 5bd, 7

CHAPTER REVIEW pg 183 #1, 7,
and #10-18 ←select appropriate questions to review.

GETTING STARTED pg 194 #4

SOLUTIONS

FACTOR the following:

1.) $x^4 - 8x^3 - 3x^2 + 24x$

C.F. 1ST

$= (x)(\underline{x^3 - 8x^2} + \underline{-3x + 24})$

by group

$= (x)(x^2(x-8) + -3(x-8))$

$= (x)(x-8)(x^2-3)$

$= (x-1)(6x^2 - 8x - 3x + 4)$
$= (x-1)(2x(3x-4) - 1(3x-4))$
$= (x-1)(3x-4)(2x-1)$

$= (x-1)(3x-4)(2x-1)$

2.) $6x^3 - 17x^2 + 15x - 4$

Try $x=1 \rightarrow 6 - 17 + 15 - 4$
$ = 0$

$\therefore (x-1)$ is a factor

$$\begin{array}{r} 6x^2 - 11x + 4 \\ x-1 \overline{)\ 6x^3 - 17x^2 + 15x - 4} \\ \underline{-\ 6x^3 + 6x^2} \\ -11x^2 + 15x \\ \underline{-\ +11x^2 \pm 11x} \\ 4x - 4 \\ \underline{-\ 4x + 4} \\ 0 \end{array}$$

$\leftarrow = (x-1)(6x^2 - 11x + 4)$

use decomp: Multiply to 24
ADD to −11

#'s: −8, −3

FACTOR the following:

3.) $2x^4 + 5x^3 - 14x^2 - 5x + 12$

Try $x = 1 \rightarrow 2 + 5 - 14 - 5 + 12 = 0$

∴ $(x-1)$ is a factor!

	$2x^3$	$+7x^2$	$-7x$	-12
x	$2x^4$	$+7x^3$	$-7x^2$	$-12x$
-1	$-2x^3$	$-7x^2$	$+7x$	$+12$

OR

DON'T FORGET!

$= (x-1)(2x^3 + 7x^2 - 7x - 12)$

Continue factoring

try $x = -1$ into $\uparrow$ $-2 + 7 + 7 - 12 = 0$

∴ $(x+1)$ is a factor

$(2x^3 + 7x^2 - 7x - 12) \div (x+1)$

	$2x^2$	$5x$	-12
x	$2x^3$	$5x^2$	$-12x$
$+1$	$2x^2$	$5x$	-12

OR

$= (x-1)(x+1)(2x^2 + 5x - 12)$

FACTOR

$= (x-1)(x+1)(2x-3)(x+4)$

Factor the following:

3.) $2x^4 + 5x^3 - 14x^2 - 5x + 12$

$2x^4 + 5x^3 - 12x^2 - 2x^2 - 5x + 12$

$x^2(2x^2 + 5x - 12) - 1(2x^2 + 5x - 12)$

$(x^2 - 1)(2x^2 + 5x - 12)$

$= (x+1)(x-1)(2x-3)(x+4)$

$= (x-2)(x^2 - 4)(4x+1)$

$= (x-2)(x-2)(x+2)(4x+1)$

$= (x-2)^2(x+2)(4x+1)$

4.) $4x^4 - 7x^3 - 18x^2 + 28x + 8$

try $x=1 \rightarrow 4 - 7 - 18 + 28 + 8 = 15$

try $x=-1 \rightarrow 4 + 7 - 18 - 28 + 8 = -27$

try $x=2 \rightarrow 64 - 56 - 72 + 56 + 8 = 0$

$\therefore (x-2)$ is a factor!

	$4x^3$	x^2	$-16x$	-4	
x	$4x^4$	x^3	$-16x^2$	$-4x$	OR
-2	$-8x^3$	$-2x^2$	$+32x$	$+8$	$=8$

$= (x-2)(4x^3 + x^2 - 16x - 4)$
↙ Factor By Grouping

$= (x-2)(x^2(4x+1) - 4(4x+1))$

FACTOR the following:

5.) $x^4 - x^3 + 8x - 8$
    by grouping

$= (x^4 - x^3) + (8x - 8)$

$= x^3(x-1) + 8(x-1)$

$= (x-1)(x^3 + 8)$
            SUM OF CUBES!

$= (x-1)(x+2)(x^2 - 2x + 4)$

NOTE: FACTOR THEOREM CAN

FACTOR $x^3 + 8$
use factor $x+2$ AND $\div$

6.) $1x^4 - 4x^3 + 6x^2 - 4x + 1$

Remember Pascal's $\triangle$?

$= (x-1)^4$

```
              1
             1 1
            1 2 1
           1 3 3 1
row 4 →   1 4 6 4 1
```

FACTOR the following:

7.) $2x^3 + x^2 + x - 1$

try $x = 1 \rightarrow 2 + 1 + 1 - 1 = 3$

try $x = -1 \rightarrow -2 + 1 - 1 - 1 = -3$

hmm... try K/J or $\frac{1}{2} \rightarrow = 0$ ✓

∴ $(x - \frac{1}{2})$ or $(2x - 1)$ is a factor!

$$\begin{array}{r} x^2 + x + 1 \\ 2x - 1 \overline{) 2x^3 + x^2 + x - 1} \\ \underline{-2x^3 + x^2} \\ 2x^2 + x \\ \underline{-2x^2 + x} \\ 2x - 1 \\ \underline{-2x + 1} \\ \text{OR} \end{array}$$

$(2x - 1)(x^2 + x + 1)$
↑ Doesn't factor

8.) $x^4 + 6x^2 + 9$ x: 9
    +: 6

Simple Trinomial

OR PERFECT SQUARE TRI

$= (x^2 + 3)^2$