Homo irrationalis: On Solving a *non-trivial* Numerical Game by Human Players

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Abstract

One of the questions in the application form for the *non-trivial* fellowship is to choose a number that maximizes the player's score and present the reasoning that led to its selection. Those who try to apply Nash equilibrium to this problem forget one fact: humans are not rational beings, and it is precisely the assumption of rational players that is key here. Other deductive approaches, on the other hand, require certain unverifiable initial assumptions to be made, thus leading to unpredictable results. In our micro-paper [1], we describe an inductive approach to solving the given problem, which consists of approximating, using samples collected from people, the distribution from which future answers will come. The code used for analysis is available at: github.com/MrFishPL/NonTrivialApplication.git

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1. Introduction and the problem definition

The application form for the *non-trivial* fellowship contains three brain-teaser questions. One of them reads as follows:

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[1] This future fellow achieved 5th place in Poland at National *Panda* Physics Olympiad and earned a top score in the first stage of the Poland's Regional Physics Olympiad (still ongoing). He's 14yo btw.

[2] This future fellow won a silver medal on the IOAI (Beijing, 2025), took 1st place in the world in the Team Competition (Beijing, 2025) and co-authored 2 research papers [2, 3], now he want to ship the most impactful research in his life so far.

> You are the CEO of a biotech start-up, competing with others to search caves for bacteria that could produce novel antibiotics. However, exploration risks damage to the delicate ecosystem. To simulate this, you will play a game with four random applicants.
>
> How to play:
>
> You and four others submit a number. We subtract the sum of all five numbers from 50. This represents the possibility of harm from exploration. Then, we multiply the resulting number by your submission – that's your score. This represents your incentive to find the bacteria as fast as you can. Your goal is to maximize your score.

So we are dealing with a game in which players make decisions without knowing the choices of other players. This problem can be understood in two different ways: (1) maximizing the expected value of the score or (2) maximizing the probability of winning. Furthermore, it is not clear what exactly is meant by the phrase *We subtract the sum of all five numbers from 50. This represents the possibility of harm from exploration.* We therefore assume that there is simply a monotonic function $f : \mathbb{R} \to (0, 1)$, and $f(S)$ can be used to interpret the result.

In our work, we assume that the task is to maximize the probability of winning. Additionally, let us make an observation: if S falls below 0, the player who gave the lowest number wins. If S remains above 0, the player who gave the highest number wins. If S is 0, all players achieve the same result (a tie).

Assuming that all players are rational, it would be reasonable to give a number that results in Nash equilibrium. However, this assumption is false because players are human beings and human beings are not rational [4, 5].

Another approach to solving this problem is to assume a certain distribution of opponents' responses, e.g., a uniform distribution or a normal distribution, and use the Monte Carlo method [6] to find the optimal result. However, this again requires making certain assumptions that are likely to prove inaccurate in a real game. The same applies to behavioral assumptions—for example, the statement that due to the greed of the players, the sum is likely to fall below 0 is unverifiable at the response stage.

All deductive approaches seem to have unacceptable weaknesses, so we decided to reverse our thinking and switch to induction. Using an online survey, we asked people what number they would choose in this numerical game. At the time of compiling the results, we had 238 responses with explanations, of which 134 showed consistent reasoning. Based on these 134, we developed models that allow us to answer questions (1) and (2). Our final answer includes the answer to question (2). In the next section, we discuss the process of collecting and processing data.

2. Data pipeline

In order to approximate the distribution of people's responses to this numerical game, we decided to conduct an online survey. The survey was conducted in Polish among communities close to the authors, including the schools they attend, the MD Fellowship[3] community, and others. Our study revealed three types of responses: (a) given without in-depth thought, humorous, (b) completely wrong, e.g., ignoring the fact that S can be negative, (c) showing the presence of in-depth thought. Since we assume that in the target research group (people applying to *non-trivial*), all participants want to give the best answer and present a coherent thought process, we reject all answers from group (a). On the other hand, due to the specific nature of the group (international Olympiad medalists and other extraordinary people), we assume that, apart from outliers, there will be no completely incorrect answers in the responses, so we also reject group (b). We construct the model based on group (c), which contains 134 samples.

Since we had a very tight deadline for conducting the study, we decided to simplify the content of the game so that its length and form would not discourage participation in the survey. The modified, translated content is as follows:

> Rules of the game:
>
> You and four other participants each choose a number. From 50, we subtract the sum of all five numbers. The resulting value is then multiplied by the number you chose - this gives your score (see the formula below). Your goal is to maximize your score.
>
> Auxiliary formula:
>
> x_1: your number

[3] https://mdfellows.com/

x_2, ..., x_5: numbers chosen by the other participants

S = $50 - x_1 - x_2 - ... - x_5$

Your score = $x_1 \cdot S$

(The score of the second participant is $x_2 \cdot S$, etc.)

What number will you choose?

We are aware that modifying content may lead to changes in distribution, but we believe that this is a good compromise between fidelity to the original and data collection efficiency.

3. The model

See also: Ethical AI usage - section 5

3.1. Game formalization

Each of the five players submits a number $x_i \in \mathbb{R}$. Define

$$S = 50 - \sum_{i=1}^{5} x_i, \qquad u_i(x_1, \ldots, x_5) = x_i \cdot S.$$

All players' scores are multiplied by the same factor S, hence the ranking is determined by the sign of S: if $S > 0$, the highest submitted number wins; if $S < 0$, the lowest submitted number wins; if $S = 0$, all players tie.

3.2. Empirical opponent model and data split

Let $\mathcal{D}$ be the filtered set of coherent (non-humorous, non-incorrect) survey responses. We split the data into training and validation sets in a 7:3 ratio:

$$\mathcal{D} = \mathcal{D}_{\text{tr}} \cup \mathcal{D}_{\text{val}}, \qquad |\mathcal{D}_{\text{tr}}| : |\mathcal{D}_{\text{val}}| = 7 : 3.$$

We model an opponent's number as a draw from the empirical distribution induced by $\mathcal{D}_{\text{tr}}$:

$$\hat{F}_{\text{tr}}(t) = \frac{1}{|\mathcal{D}_{\text{tr}}|} \sum_{z \in \mathcal{D}_{\text{tr}}} \mathbf{1}(z \leq t), \qquad X_2, X_3, X_4, X_5 \overset{iid}{\sim} \hat{F}_{\text{tr}}.$$

Analogously, for evaluation on the validation set we use $\hat{F}_{\text{val}}$ induced by $\mathcal{D}_{\text{val}}$.

3.3. Maximizing the probability of winning

For a chosen value x (player 1), define the strict win indicator

$$\mathrm{win}(x; X_2, \ldots, X_5) = \mathbf{1}(u_1(x, X_2, \ldots, X_5) > u_j(x, X_2, \ldots, X_5) \quad \forall j \in \{2, 3, 4, 5\}) .$$

Let

$$Y = \sum_{j=2}^{5} X_j, \qquad T = 50 - Y, \qquad S = T - x,$$

and denote the opponents' extrema

$$M = \max(X_2, \ldots, X_5), \qquad m = \min(X_2, \ldots, X_5).$$

A strict win occurs exactly when x lies in the (open) winning interval

$$I(X_2, \ldots, X_5) = \begin{cases} (M,\, T) & \text{if } M < T, \\ (T,\, m) & \text{if } T < m, \\ \varnothing & \text{otherwise,} \end{cases} \qquad \mathrm{win}(x; X_2, \ldots, X_5) = \mathbf{1}(x \in I).$$

Our objective is to maximize

$$p(x) = \mathbb{P}(\mathrm{win}(x; X_2, \ldots, X_5) = 1) .$$

We approximate $p(x)$ via Monte Carlo sampling: for $k = 1, \ldots, K$, draw $(X_2^{(k)}, \ldots, X_5^{(k)})$ i.i.d. from the empirical distribution and compute $I^{(k)}$. Then

$$\widehat{p}_K(x) = \frac{1}{K} \sum_{k=1}^{K} \mathbf{1}\big(x \in I^{(k)}\big) , \qquad \hat{x}_{\mathrm{win}} \in \arg\max_{x \in \mathcal{X}} \widehat{p}_K(x),$$

where $\mathcal{X}$ is the allowed set of submissions (in our experiments, real-valued x). In practice, maximizing $\widehat{p}_K(x)$ is equivalent to finding a point covered by the maximum number of winning intervals $\{I^{(k)}\}_{k=1}^{K}$.

3.4. Train/validation results (win probability)

Optimizing $\widehat{p}_K(x)$ on $\hat{F}_{\mathrm{tr}}$ yields

$$\hat{x}_{\mathrm{win}} = 15.000005, \qquad \widehat{p}_K(\hat{x}_{\mathrm{win}}; \mathcal{D}_{\mathrm{tr}}) = 0.8, \qquad \widehat{p}_K(\hat{x}_{\mathrm{win}}; \mathcal{D}_{\mathrm{val}}) = 0.856.$$

We refer to $\widehat{p}_K(\cdot)$ as the *win accuracy* (estimated probability of a strict win against four opponents drawn from the corresponding empirical distribution).

3.5. Baseline: maximizing expected value

As a baseline, consider maximizing the expected score

$$\mathrm{EV}(x) = \mathbb{E}[u_1(x, X_2, \dots, X_5)] = \mathbb{E}\left[x\left(50 - x - \sum_{j=2}^{5} X_j\right)\right].$$

Let $\mu = \mathbb{E}[X]$ for $X \sim \hat{F}_{\mathrm{tr}}$. By linearity of expectation,

$$\mathrm{EV}(x) = -x^2 + (50 - 4\mu)\,x, \qquad x_{\mathrm{EV}} = \arg\max_x \mathrm{EV}(x) = \frac{50 - 4\mu}{2} = 25 - 2\mu.$$

On the training set this gives

$$x_{\mathrm{EV,tr}} = 14.42, \qquad \mathrm{EV}_{\mathrm{tr}}(x_{\mathrm{EV,tr}}) = 207.98,$$

and when evaluated on the validation distribution, it attains an average score of 212.03.

4. Limitations

We are aware of the imperfections of this approach. First of all, due to the very limited time frame for collecting responses, we were only able to obtain 134 qualitative samples, so our distribution approximation may be inaccurate. Furthermore, it is mainly extraordinary people who apply to *non-trivial*, so their responses may differ from those we have collected. Nevertheless, we believe that this approach is better than making top-down, unverifiable assumptions.

5. Ethical AI usage

AI was used to translate and edit text, and write code. Also, AI was used to formalize our intuitions in the section 3. Although we fully understand these formulas and endorse them, we would not have been able to write them down ourselves. We hope that our research mentor will help us acquire this skill. ChatGPT[4] and DeepL[5] were used for the described purpose.

[4] https://chatgpt.com/
[5] https://www.deepl.com/

6. References

References

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