# Secure Byzantine-Robust Machine Learning

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### **Abstract**

Increasingly machine learning systems are being deployed to edge servers and devices (e.g. mobile phones) and trained in a collaborative manner. Such distributed/federated/decentralized training raises a number of concerns about the robustness, privacy, and security of the procedure. While extensive work has been done in tackling with robustness, privacy, or security individually, their combination has rarely been studied. In this paper, we propose a secure two-server protocol that offers both input privacy and Byzantine-robustness. In addition, this protocol is communication-efficient, fault-tolerant and enjoys local differential privacy.

# 1 Introduction

Recent years have witnessed fast growth of successful machine learning applications based on data collected from decentralized user devices. Unfortunately, however, currently most of the important machine learning models on a societal level do not have their utility, control, and privacy aligned with the data ownership of the participants. This issue can be partially attributed to a fundamental conflict between the two leading paradigms of traditional centralized training of models on one hand, and decentralized/collaborative training schemes on the other hand: While centralized training violates the privacy rights of participating users, existing alternative training schemes are typically not robust. Malicious participants can sabotage the training system by feeding it wrong data intentionally, known as *data poisoning*. In this paper, we tackle this problem and propose a novel distributed training algorithm which offers both *privacy* and *robustness*.

When applied to datasets containing personal data, the use of privacy-preserving techniques is currently required under regulations such as the *General Data Protection Regulation* (GDPR) or *Health Insurance Portability and Accountability Act* (HIPAA). The idea of training models on decentralized datasets and incrementally aggregating model updates via a central server motivates the federated learning paradigm [24].

However, an honest-but-curious server may still be able to infer a user's input from its corresponding model update [41]. To solve this problem, *secure* aggregation rules as proposed in [7] achieve guaranteed *input privacy*. The additive aggregation rule is a simple and efficient case of secure multi-party computation (MPC), which refers to schemes where multiple parties jointly compute the output of a function, without revealing their individual inputs. Input privacy here means the server provably learns nothing more than the function output, which in our case is the sum of all contributed model updates. Such secure aggregation rules have found wider industry adoption recently e.g. by Google on Android phones [6, 31] where input privacy guarantees can offer e.g. efficiency and exactness benefits compared to differential privacy, but both can also be combined.

The concept of Byzantine robustness has received considerable attention in the past few years for practical applications, as a way to make the training process robust to malicious actors. A Byzantine participant or worker can behave arbitrarily malicious, e.g. sending arbitrary updates to the server. This poses great challenge to the most widely used aggregation rules, e.g. simple average, since a single Byzantine worker can compromise the results of aggregation. A number of Byzantine-robust

aggregation rules have been proposed recently [4, 27, 2, 25, 40, 28] and can be used as a building block for our proposed technique.

Combining input privacy and Byzantine robustness, however, has rarely been studied. The work closest to our approach is [30] which tolerates data poisoning but does not offer Byzantine robustness, as it still requires all workers to correctly compute aggregation weights according to the protocol. This combination is not realistic to enforce in practice because it is hard and inefficient for servers to verify the client-side correctness of such non-affine weights. Prio [11] is a private and robust aggregation system relying on a different technique, called secret-shared non-interactive proofs (SNIP). While their setting is similar to ours, the robustness they offer is limited to check the range of the input. Besides, the encoding for SNIP has to be affine-aggregable and is expensive for clients to compute.

In this paper, we propose a secure aggregation rule with the help of two non-colluding honest-butcurious servers. This rule also tolerates server-worker collusion. In addition, we combine robustness and privacy to at the cost of leaking only worker similarity information which is marginal for high-dimensional neural networks.

**Main contributions.** In this paper, we propose a novel distributed training protocol which is

- **Privacy-preserving:** our method keeps the input data of each user secure against any other user, and against our honest-but-curious servers.
- **Byzantine robust:** our method offers Byzantine robustness and allows to incorporate existing robust aggregation rules, e.g. [4, 2]. The results are exact, i.e. identical to the non-private robust methods.
- Fault tolerant and easy to use: our method natively supports workers dropping out or newly joining the training process. The protocol is easy to implement and to understand for users.
- Efficient and scalable: the computation and communication overhead of our method is negligible (less than a factor of 2) compared to non-private methods. Scalability in terms of cost including setup and communication is linear in the number of workers.

# 2 Problem setup, privacy, and robustness

We consider the distributed setup of n user devices, which we call workers, with the help of two additional servers. Each worker i has its own private part of the training dataset. The workers want to collaboratively train a public machine learning model benefitting from the joint training data of all participants.

In every training step, each worker computes its own private model update (for example a gradient based on its own data) denoted by the vector  $\boldsymbol{x}_i$ . The aggregation protocol aims to compute the sum  $\boldsymbol{z} = \sum_{i=1}^n \boldsymbol{x}_i$  (or a robust version of this aggregation), which is then used to update a public model. While the result  $\boldsymbol{z}$  is public in all cases, the protocol must keep each  $\boldsymbol{x}_i$  private from any adversary or other workers.

**Security model.** We consider honest-but-curious servers which do not collude with each other but may collude with malicious workers. An honest-but-curious server follows the protocol but may try to inspect all messages. We also assume that all communication channels are secure. We guarantee the strong notion of  $input\ privacy$ , which means the servers and workers know nothing more about each other than what can be inferred from the public output of the aggregation z.

**Byzantine robustness model.** We allow the standard Byzantine worker model which assumes that workers can send arbitrary adversarial messages trying to compromise the process. We assume that a fraction of up to  $\alpha$  (< 0.5) of the workers is Byzantine, i.e. are *malicious* and not follow the protocol.

Additive secret sharing. Secret sharing is a way to split any secret into multiple parts such that no part leaks the secret. Formally, suppose a scalar a is a secret and the secret holder shares it with k parties through secret-shared values  $\langle a \rangle$ . In this paper, we only consider additive secret-sharing where  $\langle a \rangle$  is a notation for the set  $\{a_i\}_{i=1}^k$  which satisfy  $a = \sum_{p=1}^k a_p$ , with  $a_p$  held by party p. Crucially, it must not be possible to reconstruct a from any  $a_p$ . For vectors like x, their secret-shared values  $\langle x \rangle$  are simply component-wise scalar secret-shared values.

**Two-server setting.** We assume there are two non-colluding servers: model server (S1) and worker server (S2). S1 holds the output of each aggregation and thus also the machine learning model which is public to all workers. S2 holds intermediate values to perform Byzantine aggregation. Another

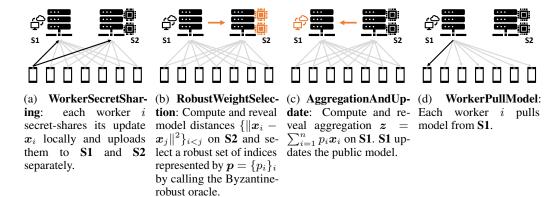


Figure 1: Illustration of Algorithm 2. The orange components on servers represent the computation-intensive operations at low communication cost between servers.

key assumption is that the servers have no incentive to collude with workers, perhaps enforced via a potential huge penalty if exposed. It is realistic to assume that the communication link between the two servers S1 and S2 is faster than the individual links to the workers. To perform robust aggregation, the servers will need access to a sufficient number of *Beaver's triples*. These are data-independent values required to implement secure multiplication in MPC on both servers, and can be precomputed beforehand. For completeness, the classic algorithm for multiplication is given in Appendix B.1.

**Byzantine-robust aggregation oracles.** Most of existing robust aggregation algorithms rely on distance measures to identity potential adversarial behavior [4, 40, 25, 21, 13]. All such distance-based aggregation rules can be directly incorporated into our proposed scheme, making them secure. While many aforementioned papers assume that the workers have i.i.d datasets, our protocol is oblivious to the distribution of the data across the workers. In particular, our protocol also works with schemes such as [21, 13] designed for non-iid data.

### 3 Secure aggregation protocol: two-server model

Each worker first splits its private vector  $x_i$  into two additive secret shares, and transmits those to each corresponding server, ensuring that neither server can reconstruct the original vector on its own. The two servers then execute our secure aggregation protocol. On the level of servers, the protocol is a two-party computation (2PC). In the case of non-robust aggregation, servers simply add all shares (we present this case in detail in Algorithm 1). In the robust case which is of our main interest here, the two servers exactly emulate an existing Byzantine robust aggregation rule, at the cost of revealing only distances of worker gradients on the server (the robust algorithm is presented in Algorithm 2). Finally, the resulting aggregated output vector z is sent back to all workers and applied as the update to the public machine learning model.

## 3.1 Non-robust aggregation

In each round, Algorithm 1 consists of two stages:

- WorkerSecretSharing (Figure 1a): each worker i randomly splits its private input  $x_i$  into two additive secret shares  $x_i = x_i^{(1)} + x_i^{(2)}$ . This can be done e.g. by sampling a large noise  $\xi_i$  and then using  $x_i \pm \xi_i$  as the shares. Worker i sends  $x_i^{(1)}$  to S1 and  $x_i^{(2)}$  to S2. We write  $\langle x_i \rangle$  for the secret-shared values distributed over the two servers.
- AggregationAndUpdate (Figure 1c): Given some weights  $\{p_i\}_{i=1}^n$ , each server locally computes  $\langle \sum_{i=1}^n p_i x_i \rangle$ . Then S2 sends its share  $\langle \sum_{i=1}^n p_i x_i \rangle^{(2)}$  to S1 so that S1 can then compute  $z = \sum_{i=1}^n p_i x_i$ . S1 updates the public model with z.

Our secure aggregation protocol is extremely simple, and as we will discuss later, has very low communication overhead, does not require cryptographic primitives, gives strong input privacy and is compatible with differential privacy, and is robust to worker dropouts and failures. We believe this makes our protocol especially attractive for federated learning applications.

We now argue about correctness and privacy. It is clear that the output z of the above protocol satisfies  $z = \sum_{i=1}^{n} p_i x_i$ , ensuring that all workers compute the right update. Now we argue about the privacy guarantees. We track the values stored by each of the servers and workers:

- S1: The secret share  $\{\boldsymbol{x}_i^{(1)}\}_{i=1}^n$  and the sum of other share  $\sum_{i=1}^n \boldsymbol{x}_i^{(2)}$ . S2: The secret share  $\{\boldsymbol{x}_i^{(2)}\}_{i=1}^n$ . Worker i:  $\boldsymbol{x}_i$  and  $\boldsymbol{z} = \sum_{i=1}^n p_i \boldsymbol{x}_i$ .

Clearly, the workers have no information other than the aggregate z and their own data. S2 only has the secret share which on their own leak no information about any data. Hence surprisingly, S2 learns no information in this process. S1 has its own secret share and also the sum of the other share. If n=1, then  $z=x_i$  and hence S1 is allowed to learn everything. If n>1, then S1 cannot recover information about any individual secret share  $x_i^{(2)}$  from the sum. Thus, S1 learns z and nothing else.

#### Robust aggregation

We now describe how Algorithm 2 replaces the simple aggregation with any distance-based robust aggregation rule such as e.g. Multi-Krum [4] which relies on computing  $\|x_i - x_j\|^2$  for all i, j pairs. The key idea is to use two-party MPC to do these computations in a secure way.

- WorkerSecretSharing (Figure 1a): As before, each worker i secret shares  $\langle x_i \rangle$  distributed over the two servers S1 and S2.
- **RobustWeightSelection** (Figure 1b): After collecting all secret-shared values  $\{\langle x_i \rangle\}_i$ , the servers compute pairwise difference  $\{\langle x_i - x_j \rangle\}_{i < j}$  locally. S2 then reveals—to itself exclusively—in plain text all of the pairwise Euclidean distances between workers  $\{\|x_i - x_j\|^2\}_{i < j}$  with the help of precomputed Beaver's triples and Algorithm 3. The distances are kept private from S1 and workers. S2 then feeds these distances to the distance-based robust aggregation rule (e.g. Multi-Krum), returning (on S2) a weight vector  $p = \{p_i\}_{i=1}^n$  (a selected subset indices can be converted to a vector of binary values), and secret-sharing them with **S1** for aggregation.
- AggregationAndUpdate (Figure 1c): Given weight vector p from previous step, we would like S1 to compute  $\sum_{i=1}^{n} p_i x_i$ . S2 secret shares with S1 the values of  $\{\langle p_i \rangle\}$  instead of sending in plain-text since they may be private. Then, S1 reveals to itself, but not to S2, in plain text the value of  $z = \sum_{i=1}^{n} p_i x_i$  using secret-shared multiplication and updates the public model.
- WorkerPullModel (Figure 1d): Workers pull the latest public model on S1 and update it locally.

The key difference between the robust and the non-robust aggregation scheme is the weight selection phase where S2 computes all pairwise distances and uses this to run a robust-aggregation rule in a black-box manner. S2 computes these distances i) without leaking any information to S1, and ii) without itself learning anything other than the pair-wise distances (and in particular none of the actual values of  $x_i$ ). To perform such a computation, S1 and S2 use precomputed Beaver's triplets (Algorithm 3 in the Appendix), which can be made available in a scalable way [36].

#### 3.3 Salient features

Overall, our protocols are very resource-light and straightforward from the perspective of the workers. Further, since we use Byzantine-robust aggregation, our protocols are provably fault-tolerant even if a large fraction of workers misbehave. This further lowers the requirements of a worker. In particular,

**Communication overhead.** We denote by  $t_{\uparrow}$  the time to upload a model (or gradient) to a server, and accordingly  $t_{\downarrow}$  for the time to download from a server, and  $t_{\leftrightarrow}$  for the time to transmit data between servers. In applications, individual uplink speed from worker and servers is typically the main bottleneck, as it is typically much slower than downlink, and the bandwidth between servers can be very large. For our protocols, the time spent on the uplink is within a factor of 2 of the non-secure variants. Besides, our protocol only requires one round of communication, which is an advantage over interactive proofs.

Fault tolerance. The workers in Algorithm 1 and Algorithm 2 are completely stateless across multiple rounds and there is no *offline* phase required. This means that workers can start participating in the protocols simply by pulling the latest public model. Further, our protocols are unaffected if

# **Algorithm 1** Two-Server Secure Aggregation (Non-robust variant)

**Setup**: n workers (non-Byzantine) with private vectors  $x_i$ . Two non-colluding servers S1 and S2. Workers: (WorkerSecretSharing)

- 1. split private  $x_i$  into additive secret shares  $\langle x_i \rangle = \{x_i^{(1)}, x_i^{(2)}\}$  (such that  $x_i = x_i^{(1)} + x_i^{(2)}$ )
- 2. send  $x_i^{(1)}$  to **S1** and  $x_i^{(2)}$  to **S2**

# Servers:

- 1.  $\forall i$ , **S1** collects  $\boldsymbol{x}_i^{(1)}$  and **S2** collects  $\boldsymbol{x}_i^{(2)}$
- 2. (AggregationAndUpdate):
  - (a) On S1 and S2, compute  $\langle \sum_{i=1}^{n} x_i \rangle$  locally (b) S2 sends its share of  $\langle \sum_{i=1}^{n} x_i \rangle$  to S1 (c) S1 reveals  $z = \sum_{i=1}^{n} x_i$  to everyone

# Algorithm 2 Two-Server Secure Robust Aggregation (Distance-Based)

**Setup**: n workers,  $\alpha n$  of which are Byzantine. Two non-colluding servers **S1** and **S2**.

## Workers: (WorkerSecretSharing)

- 1. split private  $x_i$  into additive secret shares  $\langle x_i \rangle = \{x_i^{(1)}, x_i^{(2)}\}$  (such that  $x_i = x_i^{(1)} + x_i^{(2)}$ )
  2. send  $x_i^{(1)}$  to  $\mathbf{S1}$  and  $x_i^{(2)}$  to  $\mathbf{S2}$

### Servers

- 1.  $\forall i$ , **S1** collects gradient  $x_i^{(1)}$  and **S2** collects  $x_i^{(2)}$
- 2. (RobustWeightSelection):
  - (a) For each pair  $(x_i, x_j)$  compute their Euclidean distance (i < j):
    - ullet On S1 and S2, compute  $\langle x_i x_j \rangle = \langle x_i \rangle \langle x_j \rangle$  locally
    - Use precomputed Beaver's triples (see Algorithm 3) to compute the distance  $\|\boldsymbol{x}_i - \boldsymbol{x}_i\|^2$
  - (b) **S2** perform robust aggregation rule  $p = \text{MultiKrum}(\{\|x_i x_j\|^2\}_{i < j})$
  - (c) **S2** secret-shares  $\langle p \rangle$  with **S1**
- 3. (AggregationAndUpdate):
  - (a) On S1 and S2, use MPC multiplication to compute  $\langle \sum_{i=1}^{n} p_i x_i \rangle$  locally
  - (b) **S2** sends its share of  $\langle \sum_{i=1}^{n} p_i \boldsymbol{x}_i \rangle^{(2)}$  to **S1** (c) **S1** reveals  $\boldsymbol{z} = \sum_{i=1}^{n} p_i \boldsymbol{x}_i$  to all workers.

# Workers:

1. (WorkerPullModel): Collect z and update model locally

some workers drop out in the middle of a round. Unlike in [7], there is no entanglement between workers and we don't face unbounded recovery issues.

Compatibility with local differential privacy. One byproduct of our protocol can be used to convert differentially private mechanisms, such as [1] which only of the aggregate model which guarantees privacy, into the stronger locally differentially private mechanisms which guarantee user-level privacy.

Other Byzantine-robust oracles. We can also use some robust-aggregation rules which are not based on pair-wise distances such as Byzantine SGD [2]. Since the basic structures are very similar to Algorithm 2, we put Algorithm 5 in the appendix.

**Security.** The security of Algorithm 1 is straightforward as we previously discussed. The security of Algorithm 2 again relies on the separation of information between S1 and S2 with neither the workers nor S1 learning anything other than the aggregate z. We will next formally prove that this is true even in the presence of malicious workers.

# Theoretical guarantees

# Exactness

In the following lemma we show that Algorithm 2 gives the exact same result as non-privacypreserving version.

**Lemma 1** (Exactness of Algorithm 2). The resulting z in Algorithm 2 is identical to the output of the non-privacy-preserving version of the used robust aggregation rule.

*Proof.* After secret-sharing  $x_i$  to  $\langle x_i \rangle$  to two servers, Algorithm 2 performs local differences  $\{\langle x_i - x_j \rangle\}_{i < j}$ . Using shared-values multiplication via Beaver's triple, **S2** obtains the list of true Euclidean distances  $\{\|x_i - x_j\|^2\}_{i < j}$ . The result is fed to a *distance-based* robust aggregation rule oracle, all solely on **S2**. Therefore, the resulting indices  $\{p_i\}_i$  as used in  $z := \sum_{i=1}^n p_i x_i$  are identical to the aggregation of non-privacy-preserving robust aggregation.

With the exactness of the protocol established, we next focus on the privacy guarantee.

### 4.2 Privacy

We prove probabilistic (information-theoretic) notions of privacy which gives the strongest guarantee possible. Formally, we will aim to show that the distribution of the secret does not change even after being conditioned on all observations made by all participants, i.e. each worker i, S1 and S2. This implies that the observations carry absolutely no information about the secret. Our results rely on the existence of simple additive secret-sharing protocols as discussed in the Appendix.

Each worker i only receives the final aggregated z at the end of the protocol and is not involved in any other manner. Hence no information can be leaked to them. We will now examine S1. The proofs below rely on Beaver's triples which we summarize in the following lemma.

**Lemma 2** (Beaver's triples). Suppose we secret share  $\langle x \rangle$  and  $\langle y \rangle$  between **S1** and **S2** and want to compute xy on **S2**. There exists a protocol which enables such computation which uses precomputed shares  $BV = (\langle a \rangle, \langle b \rangle, \langle c \rangle)$  such that **S1** does not learn anything and **S2** only learns xy.

Due to the page limit, we put the details about Beaver's triples, multiplying secret shares, as well as the proofs for the next two theorems to the Appendix.

**Theorem I** (Privacy for S1). Let  $z = \sum_{i=1}^n p_i x_i$  where  $\{p_i\}_{i=1}^n$  is the output of byzantine oracle or a vetor of 1s (non-private). Let  $BV_{ij} = \langle a_{ij}, b_{ij}, c_{ij} \rangle$  and  $BVp_i = \langle a_i^p, b_i^p, c_i^p \rangle$  be the Beaver's triple used in the multiplications. Let  $\langle \cdot \rangle^{(1)}$  be the share of the secret-shared values  $\langle \cdot \rangle$  on S1. Then for all workers i

$$\mathbb{P}(\boldsymbol{x}_{i} = x_{i} \mid \{\langle \boldsymbol{x}_{i} \rangle^{(1)}, \langle p_{i} \rangle^{(1)}\}_{i=1}^{n}, \{BV_{i,j}^{(1)}, \boldsymbol{x}_{i} - \boldsymbol{x}_{j} - \boldsymbol{a}_{ij}, \boldsymbol{x}_{i} - \boldsymbol{x}_{j} - \boldsymbol{b}_{ij}\}_{i < j}, \\
\{\langle \|\boldsymbol{x}_{i} - \boldsymbol{x}_{j}\|^{2} \rangle^{(1)}\}_{i < j}, \{BVp_{i}^{(1)}, p_{i} - \boldsymbol{a}_{i}^{p}, p_{i} - \boldsymbol{b}_{i}^{p}\}_{i=1}^{n}, \boldsymbol{z}) = \mathbb{P}(\boldsymbol{x}_{i} = x_{i} \mid \boldsymbol{z})$$

Note that the conditioned values are what **S1** observes throughout the algorithm.  $\{BV_{ij}^{(1)}, \boldsymbol{x}_i - \boldsymbol{x}_j - \boldsymbol{a}_{ij}, \boldsymbol{x}_i - \boldsymbol{x}_j - \boldsymbol{b}_{ij}\}_{i < j}$  and  $\{BVp_i^{(1)}, p_i - \boldsymbol{a}_i^p, p_i - \boldsymbol{b}_i^p\}_{i=1}^n$  are intermediate values during shared values multiplication.

For S2, the theorem to prove is a bit different because in this case S2 doesn't know the output of aggregation z. In fact, this is more similar to an independent system which knows little about the underlying tasks, model weights, etc. We show that while S2 has observed many intermediate values, it can only learn no more than what can be inferred from model distances.

**Theorem II** (Privacy for S2). Let  $\{p_i\}_{i=1}^n$  is the output of byzantine oracle or a vetor of 1s (non-private). Let  $BV_{ij} = \langle \mathbf{a}_{ij}, \mathbf{b}_{ij}, \mathbf{c}_{ij} \rangle$  and  $BVp_i = \langle \mathbf{a}_i^p, \mathbf{b}_i^p, \mathbf{c}_i^p \rangle$  be the Beaver's triple used in the multiplications. Let  $\langle \cdot \rangle^{(1)}$  be the share of the secret-shared values  $\langle \cdot \rangle$  on S2. Then for all workers i

$$\mathbb{P}(\boldsymbol{x}_{i} = x_{i} \mid \{\langle \boldsymbol{x}_{i} \rangle^{(2)}, \langle p_{i} \rangle^{(2)}, p_{i} \}_{i=1}^{n}, \{BV_{i,j}^{(2)}, \boldsymbol{x}_{i} - \boldsymbol{x}_{j} - \boldsymbol{a}_{ij}, \boldsymbol{x}_{i} - \boldsymbol{x}_{j} - \boldsymbol{b}_{ij} \}_{i < j}, \\
\{\langle \|\boldsymbol{x}_{i} - \boldsymbol{x}_{j}\|^{2} \rangle^{(2)}, \|\boldsymbol{x}_{i} - \boldsymbol{x}_{j}\|^{2} \}_{i < j}, \{BVp_{i}^{(2)}, p_{i} - \boldsymbol{a}_{i}^{p}, p_{i} - \boldsymbol{b}_{i}^{p} \}_{i=1}^{n}) \\
= \mathbb{P}(\boldsymbol{x}_{i} = x_{i} \mid \{\|\boldsymbol{x}_{i} - \boldsymbol{x}_{j}\|^{2} \}_{i < j}) \tag{1}$$

Note that the conditioned values are what **S1** observed throughout the algorithm.  $\{BV_{ij}^{(2)}, \boldsymbol{x}_i - \boldsymbol{x}_j - \boldsymbol{a}_{ij}, \boldsymbol{x}_i - \boldsymbol{x}_j - \boldsymbol{b}_{ij}\}_{i < j}$  and  $\{BVp_i^{(2)}, p_i - \boldsymbol{a}_i^p, p_i - \boldsymbol{b}_i^p\}_{i = 1}^n$  are intermediate values during shared values multiplication.

The model distances indeed only leaks similarity among the workers. Such similarity, however, does not tell **S2** information about the parameters; in [25] the *leeway attack* attacks distance based-rules because they don't distinguish two gradients with evenly distributed noise and two different gradients very different in one parameter. This means the leaked information has low impact to the privacy.

It is also worth noting that curious workers can only inspect others' values by learning from the public model/update. This is because in our scheme, workers don't interact directly and there is only one round of communication between servers and workers. So the only message a worker receives is the public model update.

#### 4.3 Combining with differential privacy

While input privacy is our main goal, our approach is naturally compatible with other orthogonal notions of privacy. Global differential privacy (DP) [35, 1, 8] is mainly concerned about the privacy of the *aggregated* model, and whether it leaks information about the training data. On the other hand, local differential privacy (LDP) [12, 18] is stronger notions which is also concerned with the training process itself. It requires that every communication transmitted by the worker does not leak information about their data. In general, it is hard to learn deep learning models satisfying LDP using iterate perturbation (which is the standard mechanism for DP) [7].

Our non-robust protocol is naturally compatible with local differential privacy. Consider the usual iterative optimization algorithm which in each round t performs

$$p_t \leftarrow p_{t-1} - \eta(x_t + \nu_t)$$
, where  $x_t = \frac{1}{n} \sum_{i=1}^n x_{t,i}$ . (2)

Here  $x_t$  is the aggregate update,  $p_t$  is the model parameters, and  $\nu_t$  is the noise added for DP [1].

**Theorem III** (from DP to LDP). Suppose that the noise  $\nu_t$  in (2) is sufficient to ensure that the set of model parameters  $\{p_t\}_{t\in[T]}$  satisfy  $(\varepsilon,\delta)$ -DP for  $\varepsilon\geq 1$ . Then, running (2) with using Alg. 1 to compute  $(x_t+\eta_t)$  by securely aggregating  $\{x_{1,t}+n\eta_t,x_{2,t},\ldots,x_{n,t}\}$  satisfies  $(\varepsilon,\delta)$ -LDP.

Unlike existing approaches, we do not face a tension between differential privacy which relies on real-valued vectors and cryptographic tools which operate solely on discrete/quantized objects. This is because our protocols do not rely on any cryptographic primitives, in contrast to e.g. [7]. In particular, the vectors  $x_i$  can be full-precision (real-valued), and do not need to be quantized. Thus, our secure aggregation protocol can be integrated with a mechanism which has global DP properties e.g. [1], and prove *local* DP guarantees for the resulting mechanism.

# 5 Empirical analysis of overhead

We present an illustrative simulation on a local machine (i7-8565U) to demonstrate the overhead of our scheme. We use PyTorch with MPI to train a neural network of 1.2 million parameters on the MNIST dataset. We compare the following three settings: simple aggregation with 1 server, secure aggregation with 2 servers, robust secure aggregation with 2 servers (with Krum [4]). The number of workers is always 5.

Figure 2 shows the time spent on all parts of training for one aggregation step.  $T_{grad}$  is the time spent on batch gradient computation;  $T_{w2s}$  refers to the time spend on uploading and downloading gradients;  $T_{s2s}$  is the time spend on communication between servers. Note that the server-to-server communication could be further

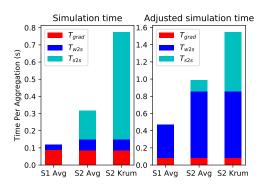


Figure 2: Left: Actual time spent; Right: Time adjusted for network bandwidth.

reduced by employing more efficient aggregation rules. Since the simulation is run on a local machine, time spent on communication is underestimated. In the right hand side figure, we adjusts time by assuming the worker-to-server link has 100Mbps bandwidth and 1Gbps respectively for the server-to-server link. Even in this scenario, we can see that the overhead from private aggregation is small. Furthermore, the additional overhead by the robustness module is moderate comparing to the standard training, even for realistic deep-learning settings. For comparison, a zero-knowledge-proof-based approach need to spend 0.03 seconds to encode a submission of 100 integers [11].

# 6 Literature review

Secure Aggregation. In the standard distributed setting with 1 server, [7] proposes a secure aggregation rule which is also fault tolerant. They generate a shared secret key for each pair of users. The secret keys are used to construct masks to the input gradients so that masks cancel each other after aggregation. To achieve fault tolerance, they employ Shamir's secret sharing. To deal with active adversaries, they use a public key infrastructure (PKI) as well as a second mask applied to the input. A followup work [23] minimizes the pairwise communication by outsourcing the key generation to two non-colluding cryptographic secret providers. However, both protocols are still not scalable because each worker needs to compute a shared-secret key and a noise mask for every other client. When recovering from failures, all live clients need to be notified and send their corresponding masks to the server, which introduces significant extra communications. In contrast, workers in our scheme are freed from coordinating with other workers, which leads to a more scalable system.

**Byzantine-Robust Aggregation/SGD.** [4] first proposes Krum and Multi-Krum for training machine learning models in the presence of Byzantine workers. [25] proposes a general enhancement recipe for Byzantine-resilient rules, termed Bulyan, to defend poisoning attacks. [2] proves a robust SGD training scheme with optimal sample complexity and the number of SGD computations. [28] uses HMM to detect and exclude Byzantine workers for federated learning. [40] proposes median and trimmed-mean -based Byzantine robust algorithms which achieve optimal statistical performance. Many of the aforementioned papers require i.i.d. datasets. Byzantine robust rules for non-i.i.d dataset have appeared only recently [21, 13]. Further, [37] extends the Byzantine setting to attackers manipulating data transfer between workers and server and [38] extends it to tolerate an arbitrary number of Byzantine workers.

[30] proposes a robust aggregation rule RFA which is also privacy-preserving. However, it is only robust to the noise in the dataset but not Byzantine-tolerant because it relies on workers to compute aggregation weights according to the protocol. [11] proposes a private and robust aggregation system (Prio) based on secret-shared non-interactive proof (SNIP). Similar to our settings, the clients' secret share their input and send them the servers for aggregation. In fact, each client sends input shares along with a SNIP proof which will be used by servers to validate the submission. However, the generation of a SNIP proof on client is expansive and the cost increases with the dimension of vectors submitted. Besides, the robustness in this paper is limited to validating the range of the data, e.g. validate that input for a binary value should be 0 or 1. But it does not work if malicious clients misreport their private data.

**Inference** As A Service. An orthogonal line of work is inference as a service or oblivious inference. A user encrypts its own data and uploads it to the server for inference. [14, 33, 15, 22, 26, 33, 10, 16, 32] falls into a general category of 2-party computation (2PC). A number of issues have to be taken into account: the non-linear activations should be replaced with MPC-friendly activations, represent the floating number as integers. [34] uses functional encryption on polynomial networks. [14] also have to adapt activations to polynomial activations and max pooling to scaled mean pooling.

**Server-Aided MPC.** One common setting for training machine learning model with MPC is the server-aided case [26, 9]. In previous works, both the model weights and the data are stored in shared values, which in turn makes the inference process computationally very costly. Another issue is that only a limited number of operations (function evaluations) are supported by shared values. Therefore, approximating non-linear activation functions again introduces significant overhead. In our paper, the computation of gradients are local to the workers, only output gradients are sent to the servers. Thus no adaptations of the worker's neural network architectures for MPC are required.

#### 7 Conclusion

In this paper, we propose a novel secure and Byzantine-robust aggregation rule. To our knowledge, this is the first work to address these two key properties jointly. Our algorithm is simple and fault tolerant and scales well with the number of workers. The protocol is based on two non-colluding honest-but-curious auxiliary servers. In addition, the Byzantine-robust aggregation rule used internally can be replaced by any existing distance-based robust rule. The communication overhead of our algorithm is roughly bounded by a factor of 2. The computation overhead, as shown in Algorithm 3, is simply the cost of a few linear operations which is marginal. No computational overhead is incurred on the workers.

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# **Appendix**

#### A Proofs

**Theorem I** (Privacy for S1). Let  $z = \sum_{i=1}^n p_i x_i$  where  $\{p_i\}_{i=1}^n$  is the output of byzantine oracle or a vetor of 1s (non-private). Let  $BV_{ij} = \langle \mathbf{a}_{ij}, \mathbf{b}_{ij}, \mathbf{c}_{ij} \rangle$  and  $BVp_i = \langle \mathbf{a}_i^p, \mathbf{b}_i^p, \mathbf{c}_i^p \rangle$  be the Beaver's triple used in the multiplications. Let  $\langle \cdot \rangle^{(1)}$  be the share of the secret-shared values  $\langle \cdot \rangle$  on S1. Then for all workers i

$$\mathbb{P}(\boldsymbol{x}_{i} = x_{i} \mid \{\langle \boldsymbol{x}_{i} \rangle^{(1)}, \langle p_{i} \rangle^{(1)}\}_{i=1}^{n}, \{BV_{i,j}^{(1)}, \boldsymbol{x}_{i} - \boldsymbol{x}_{j} - \boldsymbol{a}_{ij}, \boldsymbol{x}_{i} - \boldsymbol{x}_{j} - \boldsymbol{b}_{ij}\}_{i < j}, \\
\{\langle \|\boldsymbol{x}_{i} - \boldsymbol{x}_{j}\|^{2} \rangle^{(1)}\}_{i < j}, \{BVp_{i}^{(1)}, p_{i} - \boldsymbol{a}_{i}^{p}, p_{i} - \boldsymbol{b}_{i}^{p}\}_{i=1}^{n}, \boldsymbol{z}) = \mathbb{P}(\boldsymbol{x}_{i} = x_{i} \mid \boldsymbol{z})$$

Note that the conditioned values are what **S1** observes throughout the algorithm.  $\{BV_{ij}^{(1)}, \boldsymbol{x}_i - \boldsymbol{x}_j - \boldsymbol{a}_{ij}, \boldsymbol{x}_i - \boldsymbol{x}_j - \boldsymbol{b}_{ij}\}_{i < j}$  and  $\{BVp_i^{(1)}, p_i - \boldsymbol{a}_i^p, p_i - \boldsymbol{b}_i^p\}_{i=1}^n$  are intermediate values during shared values multiplication.

*Proof.* First, we use the independence of Beaver's triple to simplify the conditioned term.

- The Beaver's triples are data-independent. Since  $\langle \boldsymbol{a}_i^p \rangle^{(2)}$  and  $\langle \boldsymbol{b}_i^p \rangle^{(2)}$  only exist in  $\{p_i \boldsymbol{a}_i^p, p_i \boldsymbol{b}_i^p\}_i$  and they are independent of all other variables, we can remove  $\{p_i \boldsymbol{a}_i^p, p_i \boldsymbol{b}_i^p\}_i$  from conditioned terms.
- $\bullet$  For the same reason  $\{BVp_i^{(1)}\}_{i=1}^n$  are independent of all other variables and can be removed.
- The secret shares of aggregation weights  $\langle p_i \rangle^{(1)} := (p_i + \eta_i)/2$  and  $\langle p_i \rangle^{(2)} := (p_i \eta_i)/2$  where  $\eta_i$  is random noise. Then  $\{\langle p_i \rangle^{(1)}\}_i$  are independent of all other variables. Thus it can be removed.

Now the left hand side (LHS) can be simplified as

$$LHS = \mathbb{P}(\boldsymbol{x}_{i} = \boldsymbol{x}_{i} | \{\langle \boldsymbol{x}_{i} \rangle^{(1)} \}_{i=1}^{n},$$

$$\{BV_{i,j}^{(1)}, \boldsymbol{x}_{i} - \boldsymbol{x}_{j} - \boldsymbol{a}_{ij}, \boldsymbol{x}_{i} - \boldsymbol{x}_{j} - \boldsymbol{b}_{ij},$$

$$\langle \|\boldsymbol{x}_{i} - \boldsymbol{x}_{j}\|^{2} \rangle^{(1)} \}_{i < j}, \boldsymbol{z})$$
(3)

There are other independence properties:

- The secret shares of the input  $\langle \boldsymbol{x}_i \rangle$  can be seen as generated by random noise  $\xi_i$ . Thus  $\langle \boldsymbol{x}_i \rangle^{(1)} := (\xi_i + \boldsymbol{x}_i)/2$  and  $\langle \boldsymbol{x}_i \rangle^{(2)} := (-\xi_i + \boldsymbol{x}_i)/2$  are independent of others like  $\boldsymbol{x}_i$ . Besides, for all  $j \neq i$ ,  $\langle \boldsymbol{x}_i \rangle^{(\cdot)}$  and  $\langle \boldsymbol{x}_j \rangle^{(\cdot)}$  are independent.
- Beaver's triple  $\{BV_{i,j}^{(1)}\}_{i < j}$  and  $\{BV_{i,j}^{(2)}\}_{i < j}$  are clearly independent. Since they are generated before the existance of data, they are always independent of  $\{\boldsymbol{x}_i^{(\cdot)}\}_j$ .

Next, according to Beaver's multiplication Algorithm 3,

$$\langle \|m{x}_i - m{x}_j\|^2 
angle^{(1)} = m{c}_{ij}^{(1)} + (m{x}_i - m{x}_j - m{a}_{ij}) m{b}_{ij}^{(1)} + (m{x}_i - m{x}_j - m{b}_{ij}) m{a}_{ij}^{(1)}$$

we can remove this term from condition:

$$LHS = \mathbb{P}(\mathbf{x}_i = x_i | \{ \langle \mathbf{x}_i \rangle^{(1)} \}_{i=1}^n, \mathbf{z},$$

$$\{ BV_{i,j}^{(1)}, \mathbf{x}_i - \mathbf{x}_j - \mathbf{a}_{ij}, \mathbf{x}_i - \mathbf{x}_j - \mathbf{b}_{ij} \}_{i < j} )$$
(4)

By the independence between  $\langle x_i \rangle^{(\cdot)}$  and  $BV_{ij}^{(\cdot)}$ , we can further simplify the conditioned term

$$LHS = \mathbb{P}(\boldsymbol{x}_{i} = x_{i} | \{\langle \boldsymbol{x}_{i} \rangle^{(1)}\}_{i=1}^{n}, \boldsymbol{z},$$

$$\{BV_{i,j}^{(1)}, \langle \boldsymbol{x}_{i} - \boldsymbol{x}_{j} - \boldsymbol{a}_{ij} \rangle^{(2)}, \langle \boldsymbol{x}_{i} - \boldsymbol{x}_{j} - \boldsymbol{b}_{ij} \rangle^{(2)}\}_{i < j})$$

$$(5)$$

Since  $BV_{ij}^{(1)}$  and  $BV_{ij}^{(2)}$  are always independent of all other variables, we know that

$$LHS = \mathbb{P}(\boldsymbol{x}_i = x_i | \{ \langle \boldsymbol{x}_i \rangle^{(1)} \}_{i=1}^n, \boldsymbol{z})$$
(6)

For worker  $i, \forall j \neq i, \langle x_i \rangle^{(\cdot)}$  and  $\langle x_j \rangle^{(1)}$  are independent

$$LHS = \mathbb{P}(\boldsymbol{x}_i = x_i | \boldsymbol{z}).$$

**Theorem II** (Privacy for **S2**). Let  $\{p_i\}_{i=1}^n$  is the output of byzantine oracle or a vetor of 1s (non-private). Let  $BV_{ij} = \langle \boldsymbol{a}_{ij}, \boldsymbol{b}_{ij}, \boldsymbol{c}_{ij} \rangle$  and  $BVp_i = \langle \boldsymbol{a}_i^p, \boldsymbol{b}_i^p, \boldsymbol{c}_i^p \rangle$  be the Beaver's triple used in the multiplications. Let  $\langle \cdot \rangle^{(1)}$  be the share of the secret-shared values  $\langle \cdot \rangle$  on S2. Then for all workers i

$$\mathbb{P}(\boldsymbol{x}_{i} = x_{i} \mid \{\langle \boldsymbol{x}_{i} \rangle^{(2)}, \langle p_{i} \rangle^{(2)}, p_{i} \}_{i=1}^{n}, \{BV_{i,j}^{(2)}, \boldsymbol{x}_{i} - \boldsymbol{x}_{j} - \boldsymbol{a}_{ij}, \boldsymbol{x}_{i} - \boldsymbol{x}_{j} - \boldsymbol{b}_{ij} \}_{i < j}, \\
\{\langle \|\boldsymbol{x}_{i} - \boldsymbol{x}_{j}\|^{2} \rangle^{(2)}, \|\boldsymbol{x}_{i} - \boldsymbol{x}_{j}\|^{2} \}_{i < j}, \{BVp_{i}^{(2)}, p_{i} - \boldsymbol{a}_{i}^{p}, p_{i} - \boldsymbol{b}_{i}^{p} \}_{i=1}^{n}) \\
= \mathbb{P}(\boldsymbol{x}_{i} = x_{i} \mid \{\|\boldsymbol{x}_{i} - \boldsymbol{x}_{j}\|^{2} \}_{i < j}) \tag{1}$$

Note that the conditioned values are what **S1** observed throughout the algorithm.  $\{BV_{ij}^{(2)}, x_i - x_j - x_$  $\{a_{ij}, x_i - x_j - b_{ij}\}_{i < j}$  and  $\{BVp_i^{(2)}, p_i - a_i^p, p_i - b_i^p\}_{i=1}^n$  are intermediate values during shared values multiplication.

*Proof.* Similar to the proof of Theorem I, we can first conclude

- $\{p_i a_i^p, p_i b_i^p\}_i$  and  $\{BVp_i^{(2)}\}_{i=1}^n$  could be dropped because these they are data independent and no other terms depend on them.
- $\{\langle p_i \rangle^{(2)}\}_{i=1}^n$  is independent of the others so it can be dropped.  $\{p_i\}_{i=1}^n$  can be inferred from  $\{\|\boldsymbol{x}_i \boldsymbol{x}_j\|^2\}_{ij}$  so it can also be dropped.
- By the definition of  $\{\langle \|x_i x_j\|^2 \rangle^{(2)}\}_{ij}$ , it can be represented by  $\{x_i\}^{(2)}$  and  $\{BV_{ij}^{(2)}, x_i C_{ij}\}_{ij}$  $\{x_i - a_{ij}, x_i - x_j - b_{ij}\}_{i < j}$ .

Now the left hand side (LHS) can be simplified as

$$LHS = \mathbb{P}(\mathbf{x}_{i} = x_{i} | \{\langle \mathbf{x}_{i} \rangle^{(2)} \}_{i=1}^{n},$$

$$\{BV_{ij}^{(2)}, \mathbf{x}_{i} - \mathbf{x}_{j} - \mathbf{a}_{ij}, \mathbf{x}_{i} - \mathbf{x}_{j} - \mathbf{b}_{ij},$$

$$\|\mathbf{x}_{i} - \mathbf{x}_{j}\|^{2} \}_{i < j})$$
(7)

Because  $x_i$  is independent of  $\{\langle x_i \rangle^{(2)}\}_{i=1}^n$  as well as data independent terms like  $\{BV_{ij}^{(2)}, \boldsymbol{a}_{ij}^{(1)}, \boldsymbol{b}_{ij}^{(1)}\}_{i < j}$ , we have

$$LHS = \mathbb{P}(\boldsymbol{x}_i = x_i \mid ||\boldsymbol{x}_i - \boldsymbol{x}_j||^2)_{i < j})$$

**Theorem III** (from DP to LDP). Suppose that the noise  $\nu_t$  in (2) is sufficient to ensure that the set of model parameters  $\{p_t\}_{t\in[T]}$  satisfy  $(\varepsilon,\delta)$ -DP for  $\varepsilon\geq 1$ . Then, running (2) with using Alg. 1 to compute  $(x_t + \eta_t)$  by securely aggregating  $\{x_{1,t} + n\eta_t, x_{2,t}, \dots, x_{n,t}\}$  satisfies  $(\varepsilon, \delta)$ -LDP.

*Proof.* Suppose that worker  $i \in [n]$  copmutes it gradient  $x_i$  based on data  $d_i \in \mathcal{D}$ . For the sake of simplicity, let us assume that the arregate model satisfies  $\varepsilon$ -DP. The proof is identical for the more relaxed notion of  $(\varepsilon, \delta)$ -DP for  $\varepsilon \geq 1$ . This implies that for any  $j \in [n]$  and  $d_j, d_j \in \mathcal{D}$ ,

$$\frac{\Pr\left[\frac{1}{n}\left(\sum_{i=1}^{n} \boldsymbol{x}_{i}(d_{i})\right) + \nu = \boldsymbol{y}\right]}{\Pr\left[\frac{1}{n}\left(\sum_{i\neq j} \boldsymbol{x}_{i}(d_{i})\right) + \frac{1}{n}\boldsymbol{x}_{j}(\tilde{d}_{j}) + \nu = \boldsymbol{y}\right]} \leq \varepsilon, \forall \boldsymbol{y}.$$
(8)

Now, we examine the communication received by each server and measure how much information is revealed about any given worker  $j \in [n]$ . The values stored and seen are:

- S1: The secret share  $(x_1 + n\nu)^{(1)}$ ,  $\{x_i(d_i)^{(1)}\}_{i=2}^n$  and the sum of other shares  $(x_1 + n\nu)^{(2)} + \sum_{i=2}^n x_i(d_i)^{(2)} = ((\sum_{i=1}^n x_i(d_i)) + n\nu)^{(2)}$ . S2: The secret share  $(x_1 + n\nu)^{(2)}$ ,  $\{x_i(d_i)^{(2)}\}_{i=2}^n$ . Worker i:  $z = (\sum_{i=1}^n x_i(d_i)) + n\nu$ .

The equality above is because our secret shares are *linear*. Now, the values seen by any worker satisfy  $\varepsilon$ -LDP directly by (8). For the server, note that by the definition of our secret shares, we have for any worker j,

$$\begin{aligned} & \boldsymbol{x}_j(d_j)^{(1)} \text{ is independent of } \boldsymbol{x}_j(d_j) \\ & \Rightarrow \Pr[\boldsymbol{x}_j(d_j)^{(1)} = y] = \Pr[\boldsymbol{x}_j(d_j)^{(1)} = \tilde{y}] \,, \forall \boldsymbol{y}, \tilde{\boldsymbol{y}} \\ & \Rightarrow \Pr[\boldsymbol{x}_j(d_j)^{(1)} = y] = \Pr[\boldsymbol{x}_j(\tilde{d}_j)^{(1)} = y] \,, \forall d_j, \tilde{d}_j \in \mathcal{D} \,. \end{aligned}$$

A similar statement holds for the second share. This proves that the values computed/seen by the workers or servers satisfy  $\varepsilon$ -LDP.

# B Notes on security

#### **B.1** Beaver's MPC Protocol

# Algorithm 3 [3]'s MPC Protocol

```
\begin{array}{l} \underline{\textbf{input}}\colon \langle x\rangle; \langle y\rangle; \text{Beaver's triple } (\langle a\rangle, \langle b\rangle, \langle c\rangle) \text{ s.t. } c = ab \\ \underline{\textbf{output}}\colon \langle z\rangle \text{ s.t. } z = xy \\ \hline{\textbf{for all party } i \ \textbf{do}} \\ \text{locally compute } x_i - a_i \text{ and } y_i - b_i \text{ and then broadcast them to all parties } \\ \text{collect all shares and reveal } x - a = \Sigma_i(x_i - a_i), y - b = \Sigma_i(y_i - b_i) \\ \text{compute } z_i := c_i + (x - a)b_i + (y - b)a_i \\ \textbf{end for} \\ \hline{\text{The first party 1 updates }} z_1 := z_1 + (x - a)(y - b) \\ \end{array}
```

In this section, we briefly introduce [3]'s classic implementations of addition  $\langle x+y \rangle$  and multplication  $\langle xy \rangle$  given additive secret-shared values  $\langle x \rangle$  and  $\langle y \rangle$  where each party i holding  $x_i$  and  $y_i$ . The algorithm for multiplication is given in Algorithm 3.

Addition. The secret-shared values form of sum,  $\langle x + y \rangle$ , is obtained by simply each party i locally compute  $x_i + y_i$ .

*Multiplication.* Assume we already have three secret-shared values called a triple,  $\langle a \rangle$ ,  $\langle b \rangle$ , and  $\langle c \rangle$  such that c=ab.

Then note that if each party broadcasts  $x_i - a_i$  and  $y_i - b_i$ , then each party i can compute x - a and y - b (so these values are publicly known), and hence compute

$$z_i := c_i + (x - a)b_i + (y - b)a_i$$

Additionally, one party (chosen arbitrarily) adds on the public value (x - a)(y - b) to their share so that summing all the shares up, the parties get

$$\Sigma_i z_i = c + (x - a)b + (y - b)a + (x - a)(y - b) = xy$$

and so they have a secret sharing  $\langle z \rangle$  of xy.

The generation of Beaver's triples. There are many different implementations of the offline phase of the MPC multiplication. For example, semi-homomorphic encryption based implementations [20] or oblivious transfer-based implementations [19]. Since their security and performance have been demonstrated, we may assume the Beaver's triples are ready for use at the initial step of our protocol.

### **B.2** Notes on obtaining a secret share

Suppose that we want to secret share a bounded real vector  $\boldsymbol{x} \in (-B,B]^d$  for some  $B \geq 0$ . Then, we sample a random vector  $\boldsymbol{\xi}$  uniformly from  $(-B,B]^d$ . This is easily done by sampling each coordinate independently from (-B,B]. Then the secret shares become  $(\boldsymbol{\xi},\boldsymbol{x}-\boldsymbol{\xi})$ . Since  $\boldsymbol{\xi}$  is drawn from a uniform distribution from  $[-B,B]^d$ , the distribution of  $\boldsymbol{x}-\boldsymbol{\xi}$  conditioned on  $\boldsymbol{x}$  is still uniform over  $(-B,B]^d$  and (importantly) independent of  $\boldsymbol{x}$ . All arithmetic operations are then carried out modulo [-B,B] i.e.  $B+1\equiv -B+1$  and  $-B-1\equiv B-1$ . This simple scheme ensures information theoretic input-privacy for continuous vectors.

The scheme described above requires access to true randomness i.e. the ability to sample uniformly from (-B, B]. We make this assumption to simplify the proofs and the presentation. We note that differential privacy techniques such as [1] also assume access to a similar source of true randomness. In practice, however, this would be replaced with a pseudo-random-generator (PRG) [5, 39].

## **B.3** Computational indistinguishability

Let  $\{X_n\}$ ,  $\{Y_n\}$  be sequences of distributions indexed by a security parameter n (like the length of the input).  $\{X_n\}$  and  $\{Y_n\}$  are *computationally indistinguishable* if for every polynomial-time A and polynomially-bounded  $\varepsilon$ , and sufficiently large n

$$\left|\Pr[A(X_n) = 1] - \Pr[A(Y_n) = 1]\right| \le \varepsilon(n) \tag{9}$$

If a pseudorandom generator, instead of true randomness, is used in Appendix B.2, then the shares are indistinguishable from a uniform distribution over a field of same length. Thus in Theorem I and Theorem II, the secret shares can be replaced by an independent random variable of uniform distribution with negligible change in probability.

# **B.4** Notes on the security of S2

Theorem II proves that **S2** does not learn anything besides the pairwise distances between the various models. While this does leak some information about the models, **S2** cannot use this information to reconstruct any  $x_i$ . This is because the pair-wise distances are invariant to translations, rotations, and shuffling of the coordinates of  $\{x_i\}$ .

This remains true even if S2 additionally learns the global model too.

# **Example: Two-server protocol with ByzantineSGD oracle**

We can replace MultiKrum with ByzantineSGD in [2]. To fit into our protocol, we make some minor modifications but still guarantee that output is same. The core part of [2] is listed in Algorithm 4.

# Algorithm 4 ByzantineSGD [2]

```
\underline{\textbf{input}} \colon \mathcal{I} \text{ is the set of good workers, } \{A_i\}_{i \in [m]}, \{\|B_i - B_j\|\}_{i < j} \ \{\|\nabla_{k,i} - \nabla_{k,j}\|\}_{i < j} \ (i,j \in [m]),
thresholds \mathfrak{T}_A, \mathfrak{T}_B > 0
output: Subset good workers \mathcal{S}
 A_{\text{med}} := \text{median}\{A_1, \dots, A_m\};
B_{\text{med}} \leftarrow B_i \text{ where } i \in [m] \text{ is any machine s.t. } |\{j \in [m] : \|B_j - B_i\| \leq \mathfrak{T}_B\}| > m/2;
\nabla_{\text{med}} \leftarrow \nabla_{k,i} \text{ where } i \in [m] \text{ is any machine s.t. } |\{j \in [m] : \|\nabla_{k,j} - \nabla_{k,i}\| \leq 2\nu\}| > m/2;
\mathcal{S} \leftarrow \{i \in \mathcal{I} : |A_i - A_{\text{med}}| \leq \mathfrak{T}_A \wedge \|B_i - B_{\text{med}}\| \leq \mathfrak{T}_B \wedge \|\nabla_{k,j} - \nabla_{k,i}\| \leq 4\nu\};
```

The main algorithm can be summarized in Algorithm 5, the red lines highlights the changes. Different from Multi-KRUM [4], [2] uses states in their algorithm. As a result, the servers need to keep track of such states.

## **Algorithm 5** Two-Server Secure ByzantineSGD

#### Setup:

- n workers, at most  $\alpha$  percent of which are Byzantine.
- Two non-colluding servers S1 and S2
- ByzantineSGD Oracle: returns an indices set S.

  - With thresholds  $\mathfrak{T}_A$  and  $\mathfrak{T}_B$  Oracle state  $A_i^{\mathrm{old}},\,\langle B_i^{\mathrm{old}}\rangle$  for each worker i

#### Workers:

- 1. (WorkerSecretSharing):
  - (a) randomly split private  $x_i$  into additive secret shares  $\langle x_i \rangle = \{x_i^{(1)}, x_i^{(2)}\}$  (such that  $oldsymbol{x}_i = oldsymbol{x}_i^{(1)} + oldsymbol{x}_i^{(2)}$  (b) sends  $oldsymbol{x}_i^{(1)}$  to  $oldsymbol{\mathbf{S1}}$  and  $oldsymbol{x}_i^{(2)}$  to  $oldsymbol{\mathbf{S2}}$

#### Servers:

- 1.  $\forall i$ , **S1** collects gradient  $\boldsymbol{x}_i^{(1)}$  and **S2** collects  $\boldsymbol{x}_i^{(2)}$ .

  (a) Use Beaver's triple to compute  $A_i := \langle \langle \boldsymbol{x}_i \rangle, \langle \boldsymbol{w} \boldsymbol{w}_0 \rangle \rangle_{\text{inner}} + A_i^{\text{old}}$ 
  - (b)  $\langle B_i \rangle := \langle \boldsymbol{x}_i \rangle + \langle B_i^{\text{old}} \rangle$
- 2. (RobustSubsetSelection):
  - (a) For each pair (i, j) of gradients computes their distance (i < j):
    - On S1 and S2, compute  $\langle B_i B_j \rangle = \langle B_i \rangle \langle B_j \rangle$  locally
    - Use precomputed Beaver's triple and Algorithm 3 to compute the distance  $||B_i - B_i||^2$
    - On S1 and S2, compute  $\langle x_i x_j \rangle = \langle x_i \rangle \langle x_j \rangle$  locally
    - Use precomputed Beaver's triple and Algorithm 3 to compute the distance  $\|\boldsymbol{x}_i - \boldsymbol{x}_j\|_2^2$
  - (b) S2 perform Byzantine SGD  $\ddot{S}$ =ByzantineSGD( $\{A_i\}_i, \{\|B_i B_j\|\}_{i < j}, \{\|x_i B_j\|\}_{i < j}, \{\|x_i B_j\|\}_{i < j}$  $|x_j||_{i< j}, \mathfrak{T}_A, \mathfrak{T}_B$ ); if  $|\mathcal{S}| < 2$ , exit; Convert  $\mathcal{S}$  to a weight vector p of length n
  - (c) S2 secret-shares  $\langle p \rangle$  with S1
- 3. (AggregationAndUpdate):
  - (a) On **S1** and **S2**, use MPC multiplication to compute  $\langle \sum_{i=1}^{n} p_i x_i \rangle$  locally
  - (b) **S2** sends its share of  $\langle \sum_{i=1}^n p_i \boldsymbol{x}_i \rangle^{(2)}$  to **S1** (c) **S1** reveals  $\boldsymbol{z} = \sum_{i=1}^n p_i \boldsymbol{x}_i$  to all workers. (d) **S2** updates  $A_i^{\text{old}} \leftarrow A_i, \langle B_i^{\text{old}} \rangle \leftarrow \langle B_i \rangle$

#### Workers:

1. (WorkerPullModel): Collect z and update model  $w \leftarrow w + z$  locally

# D Data ownership diagram

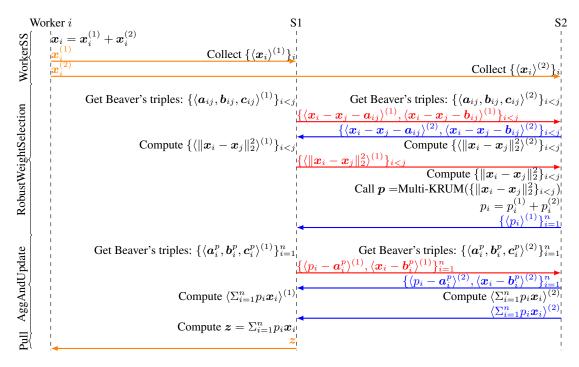


Figure 3: Overview of data ownership and Algorithm 1. The underlying Byzantine-robust oracle is Multi-KRUM.

In Figure 3, we show a diagram of data ownership to demonstrate of the data transmitted among workers and servers. Note that the Beaver's triples are already local to each server so that no extra communication is needed.