

EP30/H/02

Mathematics Paper 2

Marking Instructions

These Marking Instructions have been provided to show how SQA would mark this Exemplar Question Paper.

The information in this publication may be reproduced to support SQA qualifications only on a non-commercial basis. If it is to be used for any other purpose, written permission must be obtained from SQA's Marketing team on permissions@sqa.org.uk.

Where the publication includes materials from sources other than SQA (ie secondary copyright), this material should only be reproduced for the purposes of examination or assessment. If it needs to be reproduced for any other purpose it is the user's responsibility to obtain the necessary copyright clearance.



General Marking Principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the Detailed Marking Instructions, which identify the key features required in candidate responses.

- (a) Marks for each candidate response must <u>always</u> be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.
- **(b)** Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.
- (c) Credit must be assigned in accordance with the specific assessment guidelines.
- (d) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.
- (e) Working subsequent to an error must be followed through, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is easier, candidates lose the opportunity to gain credit.
- (f) Where transcription errors occur, candidates would normally lose the opportunity to gain a processing mark.
- (g) Scored-out or erased working which has not been replaced should be marked where still legible. However, if the scored-out or erased working has been replaced, only the work which has not been scored out should be judged.
- (h) Unless specifically mentioned in the specific assessment guidelines, do not penalise:
 - working subsequent to a correct answer
 - correct working in the wrong part of a question
 - legitimate variations in solutions
 - a repeated error within a question

Definitions of Mathematics-specific command words used in this Paper are:

Determine: obtain an answer from given facts, figures or information;

Expand: multiply out an algebraic expression by making use of the distributive law or a compound trigonometric expression by making use of one of the addition formulae for $\sin(A\pm B)$ or $\cos(A\pm B)$;

Express: use given information to rewrite an expression in a specified form;

Find: obtain an answer showing relevant stages of working;

Hence: use the previous answer to proceed;

Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used;

Identify: provide an answer from a number of possibilities;

Justify: show good reason(s) for the conclusion(s) reached;

Show that: use mathematics to prove something, eg that a statement or given value is correct - all steps, including the required conclusion, must be shown;

Sketch: give a general idea of the required shape or relationship and annotate with all relevant points and features;

Solve: obtain the answer(s) using algebraic and/or numerical and/or graphical methods.

Detailed Marking Instructions for each question

Question			•	Max nark	Additional Guidance (Illustration of evidence for awarding a mark at each •)		
1	(a)		$u_1 = 8$ and $u_2 = -4$	1			
			•¹ find terms of sequence		$\bullet^1 u_1 = 8 \text{ and } u_2 = -4$		
1	(l)	p=2 or $q=-3$	3			
			•² interpret sequence		• 2 eg $4p+q=5$ and $5p+q=7$		
			•³ solve for one variable		• 3 $p = 2$ or $q = -3$		
			• 4 state second variable		• $^{4} q = -3 \text{ or } p = 2$		
Note	S		1	-	11 as one of their equations at ●².		
			Treat equations like $p4+q=5$ or $p(4)+q=5$ as bad form. Candidates should not be penalised for using $u_{n+1}=pu_n+q$.				
			Garrandates smooth first b	oc pc			
1	(c)	(i)	$l = 0, -1$	3			
			• s know how to find a valid limit		•5 $l = -\frac{1}{2}l$ or $l = \frac{0}{1 - \left(-\frac{1}{2}\right)}$		
			• 6 calculate a valid limit only		$ullet^6 l = 0$		
		(ii)	• ⁷ state reason		\bullet ⁷ outside interval -1		
Note	Notes		4 Just stating that $l = al + b$ or $l = \frac{b}{1-a}$ is not sufficient for \bullet^5 .				
			5 Any calculations based on formulae masquerading as a limit rule cannot gain ● □				
			and \bullet^6 . For candidates who use " $b=0$ ", \bullet^6 is only available to those who simplify $\frac{0}{2}$ to 0 .				
			 Accept 2>1 or p>1 for •⁷. This may be expressed in words. Candidates who use a without reference to p or 2 cannot gain •⁷. 				

2	(a)	P (-3, -1) Q (1, 7)	6	
				Substituting for <i>y</i>
		●¹ rearrange linear equation		• $y = 2x + 5$ stated or implied by • 2
		•² substitute into circle		$\bullet^2 (2x+5)^22(2x+5)$
		•³ express in standard form		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
		• ⁴ start to solve		
		● ⁵ state roots		\bullet^5 $x = -3$ and $x = 1$
		• determine corresponding <i>y</i> coordinates		•6 $y = -1$ and $y = 7$
				Substituting for x
				$\bullet^1 \ x = \frac{y-5}{2}$ stated or implied by \bullet^2
				$\bullet^2 \left(\frac{y-5}{2}\right)^2 \dots -6 \left(\frac{y-5}{2}\right) \dots$
				$ \begin{vmatrix} \bullet^3 & 5y^2 - 30y - 35 = 0 \\ \bullet^4 \text{ eg} & 5(y+1)(y-7) \end{vmatrix} = 0 \text{ must appear at the } \bullet^3 \text{ or } \bullet^4 \text{ stage to gain } \bullet^3 $
				• 5 $y = -1$ and $y = 7$
				$\bullet^6 \ x = -3 \ \text{and} \ x = 1$
Note	es	available. 2 Cross marking is avai	ilable	here for \bullet^5 and \bullet^6 . o distinguish between points P and Q.
		5 Canadaces do Not No		and Q

2	(b)	$(x+5)^2 + (y-5)^2 = 40$	6	
		• ⁷ centre of original circle		• ⁷ (3, 1)
		•8 radius of original circle		$\bullet^8 \sqrt{40}$ accept $r^2 = 40$
		Method 1: Using midpoint		Method 1: Using midpoint
		• 9 midpoint of chord		• ⁹ (-1, 3)
		• 10 evidence for finding new centre		● ¹⁰ eg stepping out or midpoint formula
		• ¹¹ centre of new circle		• ¹¹ (-5, 5)
		• ¹² equation of new circle		$ \bullet^{12} (x+5)^2 + (y-5)^2 = 40 $
		Method 2: Stepping out using P and Q		Method 2: Stepping out using P and Q
		• 9 evidence of C ₁ to P or C ₁ to Q		• ⁹ eg stepping out or vector approach
		$ullet^{10}$ evidence of Q to C_2 or P to C_2		• 10 eg stepping out or vector approach
		• ¹¹ centre of new circle		• ¹¹ (-5, 5)
		•12 equation of new circle		$ \bullet^{12} (x+5)^2 + (y-5)^2 = 40 $
Note	es	The evidence for • ⁷ a Centre (-5, 5) withou in method 2 may still working in method 1 • ¹⁰ , • ¹¹ or • ¹² .	ıt wor	may appear in (a). king in method 1 may still gain \bullet^{12} but not \bullet^{10} or \bullet^{11} , \bullet^{12} but not \bullet^{9} , \bullet^{10} or \bullet^{11} . Any other centre without not gain \bullet^{10} , \bullet^{11} or \bullet^{12} , in method 2 does not gain \bullet^{9} ,
		6 The centre must have	_	n clearly indicated before it is used at the $ullet^{12}$ stage.
			may no	39.69, or any other approximations for ●12. ot appear until the candidate states the radius or the cle.
3		-7 < <i>p</i> < 5	4	
		•¹ substitute into discriminant		$\bullet^1 (p+1)^2 - 4 \times 1 \times 9$
		•² know condition for no real roots		$\bullet^2 b^2 - 4ac < 0$
		•³ factorise		$ \bullet^3 (p-5)(p+7) < 0 $
		\bullet^4 solve for p		$\bullet^3 (p-5)(p+7) < 0$ $\bullet^4 -7$

			ı	
4		$\frac{27}{4}$	5	
		4		
		• 1 know to integrate and interpret limits		•¹ ∫ ₋₃
		•² use "upper—lower"		
		• ³ integrate		$\bullet^3 \frac{1}{4}x^4 + x^3$
		• 4 substitute limits		$ \bullet^4 \ 0 - \left(\frac{1}{4}(-3)^4 + (-3)^3\right) $
		● ⁵ evaluate area		$\bullet^5 \frac{27}{4} \text{ units}^2$
Note	es		ifferer	ntiates one or more terms at \bullet^3 then \bullet^4 and \bullet^5 are not
				without integrating at \bullet^2 do not gain \bullet^3 , \bullet^4 and \bullet^5 . lence that they have considered the upper limit
		4 Where candidates sh	ow no	evidence for both \bullet^3 and \bullet^4 , but arrive at the correct
		area, then \bullet^3 , \bullet^4 and 5 The omission of dx a		e not available. nould not be penalised.
	l			
5	(a)	$\overrightarrow{OB} = 4\mathbf{i} + 4\mathbf{j}$	1	
		●¹ state \overrightarrow{OB} in unit vector form		$\bullet^1 4\mathbf{i} + 4\mathbf{j}$
5	(b)	(2)	3	
		$\overrightarrow{DB} = \begin{bmatrix} -1\\2\\-6 \end{bmatrix}$		
		` ,		
		$\overrightarrow{DM} = \begin{pmatrix} 0 \\ -2 \\ \zeta \end{pmatrix}$		
		[-6]		
		\bullet^2 state components of \overrightarrow{DB}		2)
				$\begin{vmatrix} \bullet^2 & 2 \\ -6 \end{pmatrix}$
		• 3 state coordinates of M		$ullet^3$ (2,0,0) stated, or implied by $ullet^4$
				$\left \begin{array}{c} 0 \\ -2 \end{array} \right $
				(-6)

5	(c)	40 · 3° or 0 · 703 rads	5	
	\			
		● ⁵ know to use scalar product		• $\cos B \hat{D} M = \frac{\overline{DB}. \overline{DM}}{ \overline{DB} . \overline{DM} }$ stated or implied by • 9
		● ⁶ find scalar product		● ⁶ DB.DM = 32
		• ⁷ find magnitude of a vector		
		• 8 find magnitude of a vector		$ \bullet^8 \left \overline{DM} \right = \sqrt{40} $
		• 9 evaluate angle BDM		● ⁹ 40 · 3° or 0 · 703 rads
Notes		2 If candidates do not relates to the labelli 3 •9 should be awarded	attem ng in t d to ar :h mag	ny answer which rounds to 40° or 0.7 radians.
6		27	4	
		$\frac{2}{2}$		
		●¹ use distributive law		•¹ p.p+p.q+p.r
		•² calculate scalar product		$\bullet^2 \mathbf{p.p} = 9$
		•³ calculate scalar product		$\bullet^3 \mathbf{p.q} = \frac{9}{2}$
		• process scalar product = 0 and complete		
7	(a)	k ≈ 0·028	4	
		•¹ interpret half-life		$\bullet^1 \frac{1}{2} P_0 = P_0 e^{-25k} \text{ stated or implied by } \bullet^2$
		•² process equation		$\bullet^2 e^{-25k} = \frac{1}{2}$
		•³ write in logarithmic form		$\bullet^3 \log_e \frac{1}{2} = -25k$
		\bullet^4 process for k		$\bullet^4 k \approx 0.028$
Note	es es	1 Do not penalise cand	idates	who substitute a numerical value for $P_{ m 0}$ in part (a).

-,	/L \	No with		
7	(b)	No, with reason	4	
		• ⁵ interpret equation		$\bullet^5 P_t = P_0 e^{-80 \times 0.028}$
		● ⁶ process		$\bullet^6 P_t \approx 0.1065 P_0$
		• ⁷ state percentage decrease		● ⁷ 89%
		• ⁸ justify answer		• 8 No, the concentration will not have decreased by over 90%. 89% decrease.
Note	es	available unless already available unless already available of k example of k example of k is only available for k and k are available for k an	ady pe nihilo or can	value of k which does not round to 0.028 , \bullet^5 is not enalised in part (a). then \bullet^5 , \bullet^6 and \bullet^7 are not available. Indidates who express P_t as a multiple of P_0 . In group proportion. This is not a valid strategy.
8		$\frac{3\pi}{8}$	6	
		•¹ start to integrate		$\bullet^1 - \frac{5}{4} \cos \dots$
		•² complete integration		$\bullet^2 - \frac{5}{4} \cos \left(4x - \frac{\pi}{2} \right)$
		•³ process limits		$\bullet^3 - \frac{5}{4}\cos\left(4a - \frac{\pi}{2}\right) + \frac{5}{4}\cos\left(\frac{4\pi}{8} - \frac{\pi}{2}\right)$
		• 4 simplify numeric term and equate to $\frac{10}{4}$		
		• 5 start to solve equation		$\bullet^5 \cos\left(4a - \frac{\pi}{2}\right) = -1$ $\bullet^6 a = \frac{3\pi}{8}$
		$ullet^6$ solve for a		$\bullet^6 \ a = \frac{3\pi}{8}$
2 The inclusion of + c 3 • 6 is only available for 4 Where the candidate		at ● ¹ or a va e diffe e integ	lutions outwith the range cannot gain \bullet^6 . or \bullet^2 should be treated as bad form. alid numerical answer. rentiates, \bullet^1 , \bullet^2 and \bullet^3 are not available. trates incorrectly, \bullet^3 , \bullet^4 , \bullet^5 and \bullet^6 are still wen in radians.	

9	(a)	4 cm	5				
		•¹ prepare to differentiate		●¹ 48 <i>x</i> ⁻¹			
		•² differentiate		$\bullet^2 3 - 48x^{-2}$			
		•³ equate derivative to 0		$\bullet^3 3-48x^{-2}=0$			
		\bullet^4 process for x		$\bullet^4 x = 4$			
		• ⁵ verify nature		● ⁵ nature table or 2 nd derivative			
Note	es	1 Do not penalise the r	1 Do not penalise the non-appearance of −4 at •⁴.				
9	(b)	No, (£198 > £195)	2				
		$ullet^6$ evaluate L		$\bullet^6 L = 24$			
		• ⁷ calculate cost and justify answer		• 7 24×£8·25 = £198. No and reason (£198 > £195)			
Note	es	2 Candidates who process $x = -4$ to obtain $L = -24$ do not gain \bullet^6 .					
		y = 24 is not awarde	d ● ⁶ .				
10	(a)	$a(t) = -16\sin\left(2t - \frac{\pi}{2}\right)$	3				
		•¹ know to differentiate		$\bullet^1 \ a = v'(t)$			
		•² differentiate trig function		$\bullet^2 -8\sin\left(2t-\frac{\pi}{2}\right)$			
		•³ applies chain rule		$ullet^3$ × 2 and complete			
				$a(t) = -16\sin\left(2t - \frac{\pi}{2}\right)$			
Note	es	1 Alternatively, $8\cos\left(2t - \frac{\pi}{2}\right) = 8\sin 2t$					
		$\bullet^1 \ v'(t) \ \bullet^2 = 8 \ \mathrm{co}$	os 2 <i>t</i>	•³ =×2			

10	(b)	a(10) > 0 therefore increasing	2	
		• 4 know to and evaluate $a(10)$		$\bullet^4 a(10) = 6.53$
		• 5 interpret result		• 5 $a(10) > 0$ therefore increasing
Note	es	2 \bullet^4 and \bullet^5 are not as \bullet^2 and \bullet^3 may be as	vailabl warde	onsequence of substituting into a derivative. le to candidates who work in degrees. d if they appear in the working for $10(b)$. ear link between acceleration and $v'(t)$.
10	(c)	$s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$	3	
		• 6 know to integrate		$\bullet^6 \ s(t) = \int v(t)dt$
		• ⁷ integrate correctly		$\bullet^7 \ s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + c$
		• 8 determine constant and complete		•8 $c = 8$ so $s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$
Notes		4 • 7 and •8 are not a accept $\int 8\cos(2t-9)$		le to candidates who work in degrees. However, for ● ⁶ .

[END OF EXEMPLAR MARKING INSTRUCTIONS]