X100/302

MATHEMATICS
HIGHER
Paper 2

Read Carefully

1   **Calculators may be used in this paper.**

2   Full credit will be given only where the solution contains appropriate working.

3   Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a.b = |a|\,|b| \cos \theta$, where θ is the angle between a and b

or $a.b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\dfrac{1}{a} \cos ax + C$
$\cos ax$	$\dfrac{1}{a} \sin ax + C$

Marks

1. The vertices of triangle ABC are A(7, 9), B(−3, −1) and C(5, −5) as shown in the diagram.

   The broken line represents the perpendicular bisector of BC.

   (*a*) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$.

   4

   (*b*) Find the equation of the median from C.

   3

   (*c*) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C.
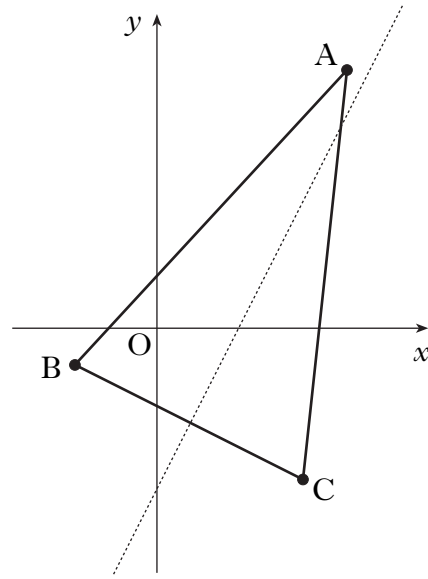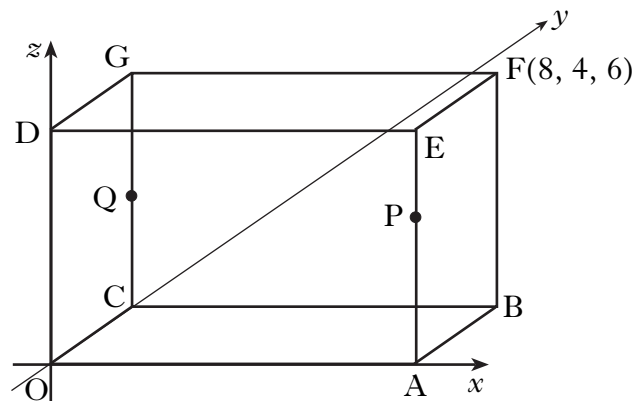
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2. The diagram shows a cuboid OABC, DEFG.

   F is the point (8, 4, 6).

   P divides AE in the ratio 2:1.

   Q is the midpoint of CG.

   (*a*) State the coordinates of P and Q.

   2

   (*b*) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$.

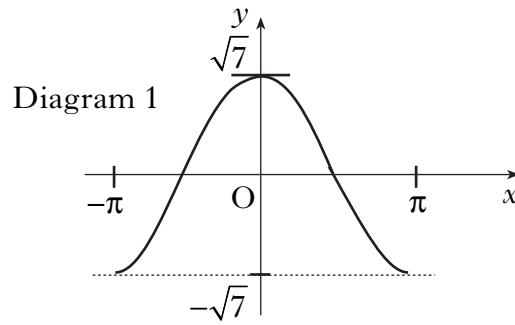   **2**

   (*c*) Find the size of angle QPA.
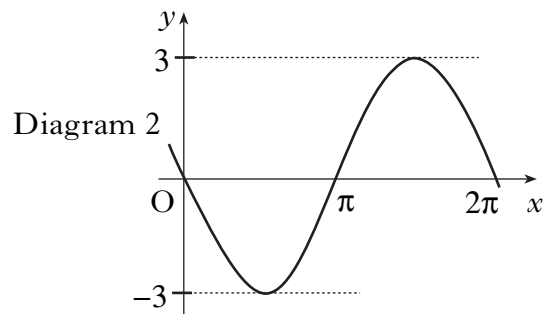
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3. (*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of p.   Diagram 1

(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

Write down the value of q.   Diagram 2   **2**

(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \dfrac{\pi}{2}$.   **4**

(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression.   **2**

4. (*a*) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$.   **2**

(*b*) A second circle has equation $(x - 4)^2 + (y - 6)^2 = 26$.

Find the distance between the centres of these two circles and hence show that the circles intersect.   **4**

(*c*) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles.
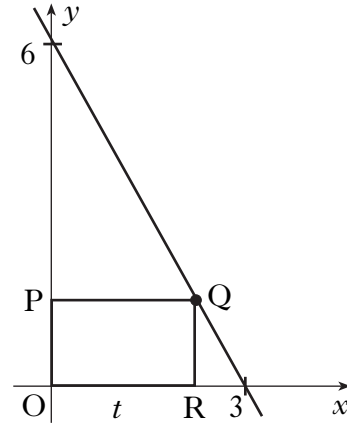
Find the coordinates of the points of intersection of the two circles.   **5**

5. Solve the equation $\cos 2x^\circ + 2\sin x^\circ = \sin^2 x^\circ$ in the interval $0 \leq x < 360$.   **5**

6. In the diagram, Q lies on the line joining (0, 6) and (3, 0).

OPQR is a rectangle, where P and R lie on the axes and OR = t.

(*a*) Show that QR = $6 - 2t$.   **3**

(*b*) Find the coordinates of Q for which the rectangle has a maximum area.   **6**
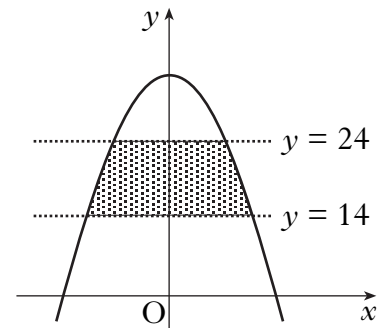

7. The parabola shown in the diagram has equation

$$y = 32 - 2x^2.$$

The shaded area lies between the lines $y = 14$ and $y = 24$.

Calculate the shaded area.   **8**

[*END OF QUESTION PAPER*]

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