H

National
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2016 Mathematics

Higher Paper 1

Finalised Marking Instructions

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General Marking Principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

For each question the marking instructions are generally in two sections, namely Illustrative Scheme and Generic Scheme. The Illustrative Scheme covers methods which are commonly seen throughout the marking. The Generic Scheme indicates the rationale for which each mark is awarded. In general, markers should use the Illustrative Scheme and only use the Generic Scheme where a candidate has used a method not covered in the Illustrative Scheme.

(a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader.

(d) Credit must be assigned in accordance with the specific assessment guidelines.

(e) One mark is available for each •. There are no half marks.

(f) Working subsequent to an error must be **followed through**, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

(g) As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

(h) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(i) As a consequence of an error perceived to be trivial, casual or insignificant, eg $6\times6=12$ candidates lose the opportunity of gaining a mark. However, note the second example in comment **(j)**.

(j) Where a transcription error (paper to script or within script) occurs, the candidate should normally lose the opportunity to be awarded the next process mark, eg

This is a transcription error and so the mark is not awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$
Eased as no longer a solution of a quadratic equation so mark is not awarded.	$x=1$
Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$ $(x-3)(x-1)=0$ $x=1$ or 3

(k) **Horizontal/vertical marking**

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example:

	•5	•6
•5	$x = 2$	$x = -4$
•6	$y = 5$	$y = -7$

Horizontal: •5 $x = 2$ and $x = -4$ Vertical: •5 $x = 2$ and $y = 5$
•6 $y = 5$ and $y = -7$ •6 $x = -4$ and $y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

(l) In final answers, unless specifically mentioned in the detailed marking instructions, numerical values should be simplified as far as possible, eg:

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(m) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(n) Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a correct answer
- Correct working in the wrong part of a question
- Legitimate variations in numerical answers/algebraic expressions, eg angles in degrees rounded to nearest degree
- Omission of units
- Bad form (bad form only becomes bad form if subsequent working is correct), eg $(x^3+2x^2+3x+2)(2x+1)$ written as $(x^3+2x^2+3x+2)\times 2x+1$

  $2x^4+4x^3+6x^2+4x+x^3+2x^2+3x+2$ written as $2x^4+5x^3+8x^2+7x+2$ gains full credit

- Repeated error within a question, but not between questions or papers

(o) In any 'Show that…' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow-through from a previous error unless specified in the detailed marking instructions.

(p) All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

(q) Scored-out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.

(r) Where a candidate has made multiple attempts using the same strategy and not identified their final answer, mark all attempts and award the lowest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Specific Marking Instructions for each question

Question			Generic Scheme	Illustrative Scheme	Max Mark
1.			$\bullet^1$ find the gradient $\bullet^2$ state equation	$\bullet^1$ -4 $\bullet^2$ $y+4x=-5$	**2**

Notes:

1. Accept any rearrangement of $y=-4x-5$ for $\bullet^2$.
2. On this occasion accept $y-3=-4\left(x-(-2)\right)$; however, in future candidates should expect that the final equation will only be accepted when it involves a single constant term.
3. For any acceptable answer without working, award 2/2.
4. $\bullet^2$ is not available as a consequence of using a perpendicular gradient.
5. For candidates who **explicitly state** $m=4$ leading to $y-3=4\left(x-(-2)\right)$, award 1/2.

   For candidates who state $y-3=4\left(x-(-2)\right)$ with no other working, award 0/2.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
2.			$\bullet^1$ write in differentiable form $\bullet^2$ differentiate first term $\bullet^3$ differentiate second term	$\bullet^1$ $\cdots+8x^{\frac{1}{2}}$ stated or implied by $\bullet^3$ $\bullet^2$ $36x^2$ $\bullet^3$ $4x^{-\frac{1}{2}}$	**3**

Notes:

1. $\bullet^3$ is only available for differentiating a term with a fractional index.
2. Where candidates attempt to integrate throughout, only $\bullet^1$ is available.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
3.	(a)		•1 interpret recurrence relation and calculate u_4	•1 $u_4 = 12$	**1**

Notes:

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•2 communicate condition for limit to exist	•2 A limit exists as the recurrence relation is linear and $-1 < \frac{1}{3} < 1$	**1**

Notes:

1. On this occasion for •2 accept:

   any of $-1 < \frac{1}{3} < 1$ or $\left|\frac{1}{3}\right| < 1$ or $0 < \frac{1}{3} < 1$ with no further comment;

   or statements such as:

   "$\frac{1}{3}$ lies between -1 and 1" or "$\frac{1}{3}$ is a proper fraction"

2. •2 is not available for: $-1 \le \frac{1}{3} \le 1$ or $\frac{1}{3} < 1$

   or statements such as:

   "It is between -1 and 1" or "$\frac{1}{3}$ is a fraction"

3. Candidates who state $-1 < a < 1$ can only gain •2 if it is **explicitly stated** that $a = \frac{1}{3}$.

Commonly Observed Responses:

Candidate A	Candidate B
$a = \frac{1}{3}$ $-1 < a < 1$ so a limit exists. •2 ✓	$u_{n+1} = au_n + b$ $u_{n+1} = \frac{1}{3}u_n + 10$ $-1 < a < 1$ so a limit exists. •2 ^

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(c)		•3 Know how to calculate limit	•3 $\dfrac{10}{1-\frac{1}{3}}$ or $L = \frac{1}{3}L + 10$	**2**
			•4 calculate limit	•4 15	

Notes:

4. Do not accept $L = \dfrac{b}{1-a}$ with no further working for •3.
5. •3 and •4 are not available to candidates who conjecture that $L = 15$ following the calculation of further terms in the sequence.
6. For $L = 15$ with no working, award 0/2.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
4.			$\bullet^1$ find the centre $\bullet^2$ calculate the radius $\bullet^3$ state equation of circle	$\bullet^1$ $(-3,4)$ stated or implied by $\bullet^3$ $\bullet^2$ $\sqrt{17}$ $\bullet^3$ $(x+3)^2+(y-4)^2=17$ or equivalent	**3**

Notes:

1. Accept $\dfrac{\sqrt{68}}{2}$ for $\bullet^2$.
2. $\bullet^3$ is not available to candidates who do not simplify $\left(\sqrt{17}\right)^2$ or $\left(\dfrac{\sqrt{68}}{2}\right)^2$.
3. $\bullet^3$ is not available to candidates who do not attempt to half the diameter.
4. $\bullet^3$ is not available to candidates who use either A or B for the centre.
5. $\bullet^3$ is not available to candidates who substitute a negative value for the radius.
6. $\bullet^2$&$\bullet^3$ are not available to candidates if the diameter or radius appears ex nihilo.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
5.			$\bullet^1$ start to integrate $\bullet^2$ complete integration	$\bullet^1$ $\ldots\times\sin(4x+1)$ $\bullet^2$ $2\sin(4x+1)+c$	**2**

Notes:

1. An answer which has not been fully simplified, eg $\dfrac{8\sin(4x+1)}{4}+c$ or $\dfrac{4\sin(4x+1)}{2}+c$, does not gain $\bullet^2$.
2. Where candidates have differentiated throughout, or in part (indicated by the appearance of a negative sign or $\times4$), see candidates A to F.
3. No marks are available for a line of working containing $\sin(4x+1)^2$ or for any working thereafter.

Commonly Observed Responses:

Candidate A Differentiated throughout: $-32\sin(4x+1)+c$ award 0/2	**Candidate C** Differentiated in part: $32\sin(4x+1)+c$ award 1/2	**Candidate E** Differentiated in part: $-2\sin(4x+1)+c$ award 1/2
Candidate B Differentiated throughout: $-32\sin(4x+1)$ award 0/2	**Candidate D** Differentiated in part: $32\sin(4x+1)$ award 0/2	**Candidate F** Differentiated in part: $-2\sin(4x+1)$ award 0/2

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.	(a)			**Method 1:**	**3**
			•¹ equate composite function to x	•¹ $f\left(f^{-1}(x)\right)=x$	
			•² write $f\left(f^{-1}(x)\right)$ in terms of $f^{-1}(x)$	•² $3f^{-1}(x)+5=x$	
			•³ state inverse function	•³ $f^{-1}(x)=\dfrac{x-5}{3}$	
				Method 2:	**3**
			•¹ write as $y=3x+5$ and start to rearrange	•¹ $y-5=3x$	
			•² complete rearrangement	•² $x=\dfrac{y-5}{3}$	
			•³ state inverse function	•³ $f^{-1}(x)=\dfrac{x-5}{3}$	
				Method 3	**3**
			•¹ interchange variables	•¹ $x=3y+5$	
			•² complete rearrangement	•² $\dfrac{x-5}{3}=y$	
			•³ state inverse function	•³ $f^{-1}(x)=\dfrac{x-5}{3}$	

Notes:

1. $y=\dfrac{x-5}{3}$ does not gain •³.
2. At •³ stage, accept f^{-1} expressed in terms of any dummy variable eg $f^{-1}(y)=\dfrac{y-5}{3}$.
3. $f^{-1}(x)=\dfrac{x-5}{3}$ with no working gains 3/3.

Commonly Observed Responses:

Candidate A

$$\times 3 \quad +5$$
$$x \rightarrow 3x \rightarrow 3x+5=f(x)$$
$$\div 3 \quad -5$$

•¹ ✓

$\dfrac{x-5}{3}$ •² ✓

$f^{-1}(x)=\dfrac{x-5}{3}$ •³ ✓

•¹ awarded for knowing to perform inverse operations in reverse order.

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		$\bullet^1$ correct value	$\bullet^1$ 2	**1**

Notes:

Commonly Observed Responses:

Candidate B

$g(x) = 3x + 1$

$g(2) = 3 \times 2 + 1 = 7$

$g^{-1}(x) = \dfrac{x-1}{3}$

$g^{-1}(7) = \dfrac{7-1}{3} = 2$ $\bullet^4$ ✘

If the candidate had followed this by stating that this would be true for all functions g for which $g(2) = 7$ and g^{-1} exists then $\bullet^4$ would be awarded.

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)		$\bullet^1$ identify pathway $\bullet^2$ state $\overrightarrow{FH}$	$\bullet^1$ $\overrightarrow{FG}+\overrightarrow{GH}$ $\bullet^2$ $\mathbf{i}+3\mathbf{j}-4\mathbf{k}$	**2**

Notes:

1. Award $\bullet^1$ for $(-2\mathbf{i}-6\mathbf{j}+3\mathbf{k})+(3\mathbf{i}+9\mathbf{j}-7\mathbf{k})$.
2. For $\mathbf{i}+3\mathbf{j}-4\mathbf{k}$ without working, award both $\bullet^1$ and $\bullet^2$.
3. Accept, throughout the question, solutions written as column vectors.
4. $\bullet^2$ is not available for adding or subtracting vectors within an invalid strategy.
5. Where candidates choose specific points consistent with the given vectors only $\bullet^1$ and $\bullet^4$ are available. However, should the statement 'without loss of generality' precede the selected points then all 4 marks are available.

Commonly Observed Responses:

Candidate A

$\overrightarrow{FH}=\overrightarrow{FG}+\overrightarrow{EH}$ $\quad\bullet^1$ ✘

$$\begin{pmatrix}-2\\-6\\3\end{pmatrix}+\begin{pmatrix}2\\3\\1\end{pmatrix}$$

$$\begin{pmatrix}0\\-3\\4\end{pmatrix} \quad \bullet^2 \ \boxed{\checkmark 2}$$

	(b)		$\bullet^1$ identify pathway $\bullet^2$ $\overrightarrow{FE}$	$\bullet^1$ $\overrightarrow{FH}+\overrightarrow{HE}$ or equivalent $\bullet^2$ $-\mathbf{i}-5\mathbf{k}$	**2**

Notes:

6. Award $\bullet^3$ for $(\mathbf{i}+3\mathbf{j}-4\mathbf{k})-(2\mathbf{i}+3\mathbf{j}+\mathbf{k})$
   or $(\mathbf{i}+3\mathbf{j}-4\mathbf{k})+(-2\mathbf{i}-3\mathbf{j}-\mathbf{k})$
   or $(-2\mathbf{i}-6\mathbf{j}+3\mathbf{k})+(3\mathbf{i}+9\mathbf{j}-7\mathbf{k})-(2\mathbf{i}+3\mathbf{j}+\mathbf{k})$
   or $(-2\mathbf{i}-6\mathbf{j}+3\mathbf{k})+(3\mathbf{i}+9\mathbf{j}-7\mathbf{k})+(-2\mathbf{i}-3\mathbf{j}-\mathbf{k})$.
7. For $-\mathbf{i}-5\mathbf{k}$ without working, award 0/2.
8. $\bullet^4$ is not available for simply adding or subtracting vectors. There must be evidence of a valid strategy at $\bullet^3$.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.			●[1] substitute for y	●[1] $x^2+(3x-5)^2+2x-4(3x-5)-5...$	5
			Method 1 & 2	**Method 1**	
			●[2] express in standard quadratic form	●[2] $10x^2-40x+40$ $\left.\right\}=0$	
			●[3] factorise or use discriminant	●[3] $10(x-2)^2$	
			●[4] interpret result to demonstrate tangency	●[4] only one solution implies tangency (or repeated factor implies tangency)	
			●[5] find coordinates	●[5] $x=2,\ y=1$	
				Method 2	
				●[2] $10x^2-40x+40=0$ **stated explicitly**	
				●[3] $(-40)^2-4\times10\times40$ or $(-4)^2-4\times1\times4$	
				●[4] $b^2-4ac=0$ so line is a tangent	
				●[5] $x=2,\ y=1$	
			Method 3	**Method 3**	
			●[1] make inference and state m_{rad}	●[1] If $y=3x-5$ is a tangent, $m_{rad}=\frac{-1}{3}$	
			●[2] find the centre and the equation of the radius	●[2] $(-1,2)$ and $3y=-x+5$	
			●[3] solve simultaneous equations	●[3] $\begin{matrix}3y=-x+5\\y=3x-5\end{matrix}\rightarrow(2,1)$	
			●[4] verify location of point of intersection	●[4] check $(2,1)$ lies on the circle.	
			●[5] communicates result	●[5] $\therefore$ the line is a tangent to the circle	

Notes:

1. In Method 1 $"=0"$ must appear at ●[2] or ●[3] stage for ●[2] to be awarded.
2. Award ●[3] and ●[4] for correct use of quadratic formula to get equal (repeated) roots $\Rightarrow$ line is a tangent.

<table>
<tr><th colspan="3">Question</th><th>Generic Scheme</th><th>Illustrative Scheme</th><th>Max Mark</th></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td colspan="4">Candidate A
$x^2+(3x-5)^2+2x-4(3x-5)-5=0$ •1 ✓
$10x^2-40x+40=0$ •2 ✓
$b^2-4ac=(-40)^2-4\times10\times40=0\Rightarrow$ tgt •3 ✓</td><td colspan="2">Candidate B
$x^2+(3x-5)^2+2x-4(3x-5)-5=0$ •1 ✓
$10x^2-40x+40$ •2 ^
$b^2-4ac=(-40)^2-4\times10\times40=0\Rightarrow$ tgt •3 $\boxed{✓1}$</td></tr>
<tr><td colspan="4">Candidate C
$x^2+(3x-5)^2+2x-4(3x-5)-5=0$ •1 ✓
$x^2+\underline{9x^2+25}+2x-12x+20-5=0$
$10x^2-10x+40=0$ •2 ✗
$b^2-4ac=(-10)^2-4\times10\times40=-1500\Rightarrow$
no real roots so line is not a tangent •3 $\boxed{✓1}$
•4 and •5 are unavailable.</td><td colspan="2">Candidate D
$x^2+(3x-5)^2+2x-4(3x-5)-5=0$ •1 ✓
$10x^2-40x+40=0$ •2 ✓
$10(x-2)^2$ •3 ✓
Repeated root $\Rightarrow$ Only one point of contact.
•4 ✓</td></tr>
<tr><td>9</td><td>(a)</td><td></td><td>•1 know to and differentiate one term

•2 complete differentiation and equate to zero

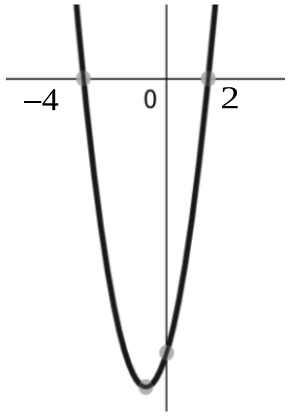
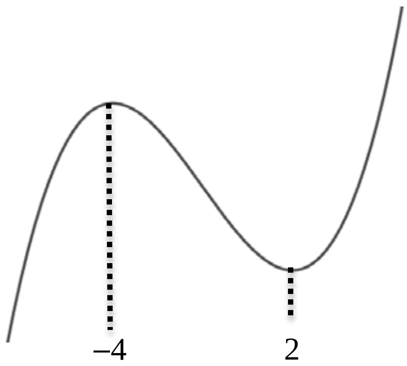
•3 factorise derivative

•4 process for x</td><td>•1 eg $f'(x)=3x^2...$

•2 $3x^2+6x-24=0$

•3 $3(x+4)(x-2)$

•4 -4 and 2</td><td>4</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">1. •2 is only available if $"=0"$ appears at •2 or •3 stage.
2. •3 is available for substituting correctly in the quadratic formula.
3. At •3 do not penalise candidates who fail to extract the common factor or who have divided the quadratic equation by 3.
4. •3 and •4 are not available to candidates who arrive at a linear expression at •2.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td colspan="6"></td></tr>
</table>

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		●[5] know how to identify where curve is increasing	●[5] **Method 1**  **Method 2** $3x^2+6x-24>0$ **Method 3** Table of signs for a derivative – see the additional page for acceptable responses. **Method 4** 	2
			●[6] state range	●[6] $x<-4$ and $x>2$	

Notes:

5. For $x<-4$ and $x>2$ without working award 0/2.
6. $2<x<-4$ does not gain ●[6].

Commonly Observed Responses:

Table of signs for a derivative - acceptable responses.

x	-4^-	-4	-4^+
$\frac{dy}{dx}$ *or* $f'(x)$	$+$	0	$-$
Shape or Slope	/	—	\

x	2^-	2	2^+
$\frac{dy}{dx}$ *or* $f'(x)$	$-$	0	$+$
Shape or Slope	\	—	/

x	$\rightarrow$	-4	$\rightarrow$
$\frac{dy}{dx}$ *or* $f'(x)$	$+$	0	$-$
Shape or Slope	/	—	\

x	$\rightarrow$	2	$\rightarrow$
$\frac{dy}{dx}$ *or* $f'(x)$	$-$	0	$+$
Shape or Slope	\	—	/

Arrows are taken to mean "in the neighbourhood of"

x	a	-4	b	2	c
$\frac{dy}{dx}$ *or* $f'(x)$	$+$	0	$-$	0	$+$
Shape or Slope	/	—	\	—	/

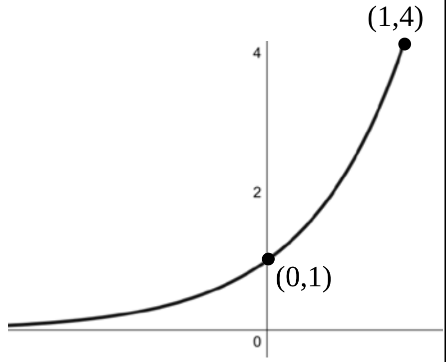
Where: $a < -4$, $-4 < b < 2$, $c > 2$
Since the function is continuous '$-4 < b < 2$' is acceptable.

x	$\rightarrow$	-4	$\rightarrow$	2	$\rightarrow$
$\frac{dy}{dx}$ *or* $f'(x)$	$+$	0	$-$	0	$+$
Shape or Slope	/	—	\	—	/

Since the function is continuous '$-4 \rightarrow 2$' is acceptable.

General Comments

- Since this question refers to both y and $f(x)$, $\frac{dy}{dx}$ and $f'(x)$ are accepted.
- The row labelled 'shape' or 'slope' is not required in this question since the sign of the derivative is sufficient to indicate where the function is increasing.
- For this question, do not penalise the omission of 'x' on the top row of the table.

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.			$\bullet^1$ graph reflected in $y = x$ $\bullet^2$ correct annotation	$\bullet^1$  $\bullet^2$ $(0,1)$ and $(1,4)$	2

Notes:

1. For $\bullet^1$ accept any graph of the correct shape and orientation which crosses the $y-\text{axis}$. The graph must not cross the $x-\text{axis}$.
2. Both $(0,1)$ and $(1,4)$ must be marked and labelled on the graph for $\bullet^2$ to be awarded.
3. $\bullet^2$ is only available where the candidate has attempted to reflect the given curve in the line $y = x$.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
11.	(a)		●[1] interpret ratio ●[2] determine coordinates	●[1] $\frac{1}{3}$ ●[2] $(2,1,0)$	**2**

Notes:

1. ●[1] may be implied by ●[2] or be evidenced by their working.
2. For $(3,-1,2)$ award 1/2.
3. For $(2,1,0)$ without working award 2/2.
4. $\begin{pmatrix}2\\1\\0\end{pmatrix}$ gains 1/2.
5. $\begin{pmatrix}3\\-1\\2\end{pmatrix}$ gains 0/2.

Commonly Observed Responses:

Candidate A		Candidate B	
$\overrightarrow{BC}=\frac{1}{3}\overrightarrow{AC}$	●[1] ✗	$\frac{\overrightarrow{AB}}{\overrightarrow{BC}}=\frac{1}{2}$	
$(3,-1,2)$	●[2] $\boxed{✓1}$	$2\overrightarrow{AB}=\overrightarrow{BC}$	
		$2(\mathbf{b}-\mathbf{a})=\mathbf{c}-\mathbf{b}$	
		$3\mathbf{b}=\mathbf{c}+2\mathbf{a}$	●[1] ✓
		$3\mathbf{b}=\begin{pmatrix}6\\3\\0\end{pmatrix}$	
		$\mathbf{b}=\begin{pmatrix}2\\1\\0\end{pmatrix}$ B(2,1,0)	●[2] ✓

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•¹ find $\overrightarrow{AC}$	•¹ $\overrightarrow{AC} = \begin{pmatrix} 3 \\ -6 \\ 6 \end{pmatrix}$	**3**
			•² find $\lvert\overrightarrow{AC}\rvert$	•² 9	
			•³ determine k	•³ $\frac{1}{9}$	

Notes:

6. Evidence for •³ may appear in part (a).
7. •³ may be implied at •⁴ stage by :
   - $\sqrt{3^2 + (-6)^2 + 6^2}$
   - $\sqrt{3^2 - 6^2 + 6^2} = 9$
   - $\sqrt{3^2 + -6^2 + 6^2} = 9$.
8. $\sqrt{81}$ must be simplified at the •⁴ or •⁵ stage for •⁴ to be awarded.
9. •⁵ can only be awarded as a consequence of a valid strategy at •⁴. k must be > 0 .

Commonly Observed Responses:

Candidate A		**Candidate B**		**Candidate C**	
$\lvert\overrightarrow{AC}\rvert = \sqrt{81}$	•⁴ ✓	$\lvert\overrightarrow{AC}\rvert = \sqrt{81}$	•⁴ $\boxed{✓2}$	$\lvert\overrightarrow{AC}\rvert = \sqrt{81}$	•⁴ $\boxed{✓2}$
$\frac{1}{9}$	•⁵ ✓	$\frac{1}{\sqrt{81}}$	•⁵ $\boxed{✓1}$		•⁵ ^

ALTERNATIVE STRATEGY

Where candidates use the distance formulae to determine the distance from A to C, award •³ for $AC = \sqrt{3^2 + 6^2 + 6^2}$.

Question			Generic Scheme	Illustrative Scheme	Max Mark
12.	(a)		●¹ interpret notation ●² demonstrate result	●¹ $2(3-x)^2-4(3-x)+5$ ●² $18-12x+2x^2-12+4x+5$ leading to $2x^2-8x+11$	**2**

Notes:

1. At ●² there must be a relevant intermediate step between ●¹ and the final answer for ●² to be awarded.
2. $f(3-x)$ alone is not sufficient to gain ●¹.
3. Beware of candidates who fudge their working between ●¹ and ●².

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)			**Method 1**	**3**
			●¹ identify common factor	●¹ $2\left[x^2-4x\ldots\right.$ stated or implied by ●²	
			●² start to complete the square	●² $2(x-2)^2\ldots$	
			●³ write in required form	●³ $2(x-2)^2+3$	
				Method 2	
			●¹ expand completed square form	●¹ $px^2+2pqx+pq^2+r$	
			●² equate coefficients	●² $p=2,\ 2pq=-8,\ pq^2+r=11$	
			●³ process for q and r and write in required form	●³ $2(x-2)^2+3$	

Notes:

4. At ●⁵ $2(x+(-2))^2+3$ must be simplified to $2(x-2)^2+3$.
5. $2(x-2)^2+3$ with no working gains ●⁵ only; however, see Candidate G.
6. Where a candidate has used the function they arrived at in part (a) as $h(x)$, ●³ is not available. However, ●⁴ and ●⁵ can still be gained for dealing with an expression of equivalent difficulty.
7. ●⁵ is only available for a calculation involving both the multiplication and addition of integers.

Question	Generic Scheme	Illustrative Scheme	Max Mark
Commonly Observed Responses:			

Candidate A

$2\left(x^2-4x+\frac{11}{2}\right)$ $\bullet^3$ ✓

$2\left(x^2-4x+4-4+\frac{11}{2}\right)$ $\bullet^4$ not awarded at this line.

$2(x-2)^2+\frac{3}{2}$ $\bullet^4$ ✓ $\bullet^5$ ✗

Candidate B

$2x^2-8x+11=2(x-4)^2-16+11$ $\bullet^3$ ✗ $\bullet^4$ ✗

$=2(x-4)^2-5$ $\bullet^5$ $\boxed{\checkmark 2}$

Candidate C

$px^2+2pqx+pq^2+r$ $\bullet^3$ ✓

$p=2,\ 2pq=-8,\ q^2+r=11$ $\bullet^4$ ✗

$p=2,\ q=-2,\ r=7$

$2(x-2)^2+7$ $\bullet^5$ $\boxed{\checkmark 1}$

Candidate D

$2\left[\left(x^2-8x\right)+11\right]$ $\bullet^3$ ✗

$2\left[(x-4)^2-16\right]+11$ $\bullet^4$ $\boxed{\checkmark 1}$

$2(x-4)^2-21$ $\bullet^5$ $\boxed{\checkmark 1}$

Candidate E

$p(x+q)^2+r=px^2+2pqx+pq^2+r$ $\bullet^3$ ✓

$p=2,\ 2pq=-8,\ pq^2+r=11$ $\bullet^4$ ✓

$q=-2,\ r=3$ $\bullet^5$ ✓

$\bullet^5$ is awarded as all working relates to completed square form

Candidate F

$px^2+2pqx+pq^2+r$ $\bullet^3$ ✓

$p=2,\ 2pq=-8,\ pq^2+r=11$ $\bullet^4$ ✓

$q=-2,\ r=3$ $\bullet^5$ ✗

$\bullet^5$ is lost as no reference is made to completed square form

Candidate G

$2(x-2)^2+3$

Check: $2\left(x^2-4x+4\right)+3$

$=2x^2-8x+8+3$

$2x^2-8x+11$

Award 3/3

Candidate H

$2x^2-8x+11$

$=2(x-2)^2-4+11$ $\bullet^3$ ✓ $\bullet^4$ ✓

$=2(x-2)^2+7$ $\bullet^5$ ✗

Question			Generic Scheme	Illustrative Scheme	Max Mark
13.			●[1] calculate lengths AC and AD ●[2] select appropriate formula and express in terms of p and q ●[3] calculate two of $\cos p,\ \cos q,\ \sin p,\ \sin q$ ●[4] calculate other two and substitute into formula ●[5] arrange into required form	●[1] $AC=\sqrt{17}$ and $AD=5$ stated or implied by ●[3] ●[2] $\cos q\cos p+\sin q\sin p$ stated or implied by ●[4] ●[3] $\cos p=\frac{4}{\sqrt{17}}$, $\cos q=\frac{4}{5}$ $\sin p=\frac{1}{\sqrt{17}}$, $\sin q=\frac{3}{5}$ ●[4] $\frac{4}{5}\times\frac{4}{\sqrt{17}}+\frac{3}{5}\times\frac{1}{\sqrt{17}}$ ●[5] $\frac{19}{5\sqrt{17}}\times\frac{\sqrt{17}}{\sqrt{17}}=\frac{19\sqrt{17}}{85}$ or $\frac{19}{5\sqrt{17}}=\frac{19\sqrt{17}}{5\times17}=\frac{19\sqrt{17}}{85}$	5

Notes:

1. For any attempt to use $\cos(q-p)=\cos q-\cos p$, only ●[1] and ●[3] are available.
2. At the ●[3] and ●[4] stages, do not penalise the use of fractions greater than 1 resulting from an error at ●[1]. ●[5] will be lost.
3. Candidates who write $\cos\left(\frac{4}{5}\right)\times\cos\left(\frac{4}{\sqrt{17}}\right)+\sin\left(\frac{3}{5}\right)\times\sin\left(\frac{1}{\sqrt{17}}\right)$ gain ●[1], ●[2] and ●[3].

   ●[4] and ●[5] are unavailable.
4. Clear evidence of multiplying by $\frac{\sqrt{17}}{\sqrt{17}}$ must be seen between ●[4] and ●[5] for ●[5] to be awarded.
5. ●[4] implies ●[1], ●[2] and ●[3].

Commonly Observed Responses:

Candidate A		Candidate B	
$\frac{4}{5}\times\frac{4}{\sqrt{17}}+\frac{3}{5}\times\frac{1}{\sqrt{17}}$	●[4] ✓	$AC=\sqrt{17}$ and $AD=\sqrt{21}$	●[1] ✗
$\frac{19}{5\sqrt{17}}\times\sqrt{17}$		$\cos q\cos p+\sin q\sin p$	●[2] ✓
$\frac{19\sqrt{17}}{85}$	●[5] ✗	$\cos p=\frac{4}{\sqrt{17}}$ $\sin p=\frac{1}{\sqrt{17}}$	●[3] ✓
		$\frac{\sqrt{17}}{\sqrt{21}}\times\frac{4}{\sqrt{17}}+\frac{2}{\sqrt{21}}\times\frac{1}{\sqrt{17}}$	●[4] ✗
		=…●[5] not available	

Question			Generic Scheme	Illustrative Scheme	Max Mark
14.	(a)		$\bullet^1$ state value	$\bullet^1$ 2	**1**

Notes:

1. Evidence for $\bullet^1$ may not appear until part (b).

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		$\bullet^2$ use result of part (a)	$\bullet^2$ $\log_4 x + \log_4 (x-6) = 2$	**5**
			$\bullet^3$ use laws of logarithms	$\bullet^3$ $\log_4 x(x-6) = 2$	
			$\bullet^4$ use laws of logarithms	$\bullet^4$ $x(x-6) = 4^2$	
			$\bullet^5$ write in standard quadratic form	$\bullet^5$ $x^2 - 6x - 16 = 0$	
			$\bullet^6$ solve for x and identify appropriate solution	$\bullet^6$ 8	

Notes:

2. $\bullet^3$ & $\bullet^4$ can only be awarded for use of laws of logarithms applied to algebraic expressions of equivalent difficulty.
3. $\bullet^4$ is not available for $x(x-6) = 2^4$; however candidates may still gain $\bullet^5$ & $\bullet^6$.
4. $\bullet^6$ is only available for solving a polynomial of degree 2 or higher.
5. $\bullet^6$ is not available for responses which retain invalid solutions.

Commonly Observed Responses:

Candidate A		Candidate B		Candidate C	
$\log_5 25 = 5$	$\bullet^1$ ✗	$\log_5 25 = 2$	$\bullet^1$ ✓	$\log_5 25 = 2$	$\bullet^1$ ✓
$\log_4 x(x-6) = 5$	$\bullet^2$ $\boxed{✓1}$	$\log_4 x(x-6) = 2$	$\bullet^2$ ✓	$\log_4 x(x-6) = 2$	$\bullet^2$ ✓
	$\bullet^3$ $\boxed{✓1}$		$\bullet^3$ ✓		$\bullet^3$ ✓
$x(x-6) = 4^5$	$\bullet^4$ $\boxed{✓1}$	$x(x-6) = 8$	$\bullet^4$ ✗	$x(x-6) = 8$	$\bullet^4$ ✗
$x^2 - 6x - 1024 = 0$	$\bullet^5$ $\boxed{✓1}$	$x^2 - 6x - 8 = 0$	$\bullet^5$ $\boxed{✓1}$	$x^2 - 6x + 8 = 0$	$\bullet^5$ ✗
$35.14...$	$\bullet^6$ $\boxed{✓1}$	$7.12...$	$\bullet^6$ $\boxed{✓1}$	$x = 2, 4$	$\bullet^6$ ✗
				or	
				$x = \cancel{2}, \cancel{4}$	$\bullet^6$ ✗

Question			Generic Scheme	Illustrative Scheme	Max Mark
15.	(a)		$\bullet^1$ value of a $\bullet^2$ value of b $\bullet^3$ calculate k	$\bullet^1$ $a = 4$ $\bullet^2$ $b = -5$ $\bullet^3$ $k = -\frac{1}{12}$	**3**

Notes:

1. Evidence for the values of a and b may first appear in an expression for $f(x)$. Where marks have been awarded for a and b in an expression for $f(x)$ ignore any values attributed to a and b in subsequent working.

Commonly Observed Responses:

Candidate A	**Candidate B**	**Candidate C**
Both roots interchanged $a = -5$ $\bullet^1$ ✗ $b = 4$ $\bullet^2$ $\boxed{\checkmark 1}$ $k = \frac{1}{6}$ $\bullet^3$ $\boxed{\checkmark 1}$	$a = 4$ $\bullet^1$ ✓ $b = 5$ $\bullet^2$ ✗ $k = -\frac{3}{16}$ $\bullet^3$ $\boxed{\checkmark 1}$	**Using** $(1,9)$ $a = -4$ $\bullet^1$ ✗ $b = 5$ $\bullet^2$ $\boxed{\checkmark 1}$ $k = \frac{9}{80}$ $\bullet^3$ $\boxed{\checkmark 1}$

Candidate D - BEWARE	**Summary for expressions of $f(x)$ for $\bullet^1$ and $\bullet^2$:**
Using $(0,9)$ $a = -5$ $\bullet^1$ ✗ $b = 4$ $\bullet^2$ $\boxed{\checkmark 1}$ $k = \frac{9}{80}$ $\bullet^3$ ✗	**signs correct, brackets correct** $f(x) = (x-4)(x+5)^2$ $\bullet^1$ ✓ $\bullet^2$ ✓ **signs incorrect, brackets correct** $f(x) = (x+4)(x-5)^2$ $\bullet^1$ ✗ $\bullet^2$ $\boxed{\checkmark 1}$ **signs correct, brackets incorrect** $f(x) = (x+5)(x-4)^2$ $\bullet^1$ ✗ $\bullet^2$ $\boxed{\checkmark 1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		$\bullet^1$ state range of values	$\bullet^1$ $d > 9$	**1**

Notes:

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]