H

National
Qualifications
SPECIMEN ONLY

S847/76/11

Mathematics
Paper 1 (Non-calculator)

Date — Not applicable

Duration — 1 hour 30 minutes

Total marks — 70

Attempt ALL questions.

You may NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

SQA ©

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FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product:

$\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\,\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(\mathrm{A} \pm \mathrm{B}) = \sin \mathrm{A} \cos \mathrm{B} \pm \cos \mathrm{A} \sin \mathrm{B}$$

$$\cos(\mathrm{A} \pm \mathrm{B}) = \cos \mathrm{A} \cos \mathrm{B} \mp \sin \mathrm{A} \sin \mathrm{B}$$

$$\sin 2\mathrm{A} = 2 \sin \mathrm{A} \cos \mathrm{A}$$

$$\begin{aligned} \cos 2\mathrm{A} &= \cos^2 \mathrm{A} - \sin^2 \mathrm{A} \\ &= 2\cos^2 \mathrm{A} - 1 \\ &= 1 - 2\sin^2 \mathrm{A} \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

MARKS

Attempt ALL questions

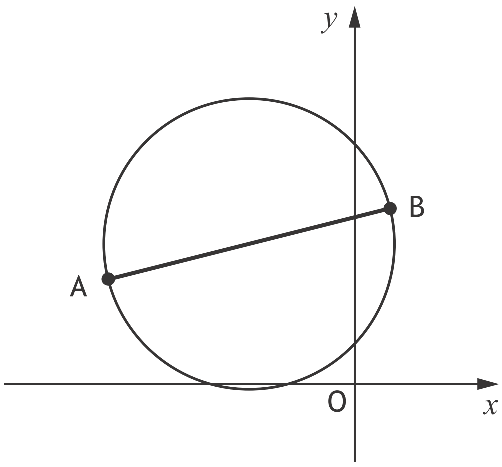
Total marks — 70

1. A curve has equation $y = x^2 - 4x + 7$.

Find the equation of the tangent to this curve at the point where $x = 5$. **4**

2. A and B are the points $(-7, 3)$ and $(1, 5)$.

AB is a diameter of a circle.



Find the equation of this circle. **3**

3. Line l_1 has equation $\sqrt{3}y - x = 0$.

(a) Line l_2 is perpendicular to l_1. Find the gradient of l_2. **2**

(b) Calculate the angle l_2 makes with the positive direction of the x-axis. **2**

MARKS

4. Evaluate $\int_{1}^{2} \frac{1}{6}x^{-2}\, dx$. **3**

5. The points A(0, 9, 7), B(5, −1, 2), C(4, 1, 3) and D(x, −2, 2) are such that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CD}$.

Determine the value of x. **4**

6. Determine the range of values of p such that the equation $x^2 + (p+1)x + 9 = 0$ has no real roots. **4**

7. Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact. **5**

MARKS

8. For the polynomial, $x^3 - 4x^2 + ax + b$

- $x - 1$ is a factor
- -12 is the remainder when it is divided by $x - 2$

(a) Determine the values of a and b. **5**

(b) Hence solve $x^3 - 4x^2 + ax + b = 0$. **3**

9. A sequence is generated by the recurrence relation $u_{n+1} = mu_n + 6$ where m is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. **2**

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$. **1**

(ii) Calculate this limit. **2**

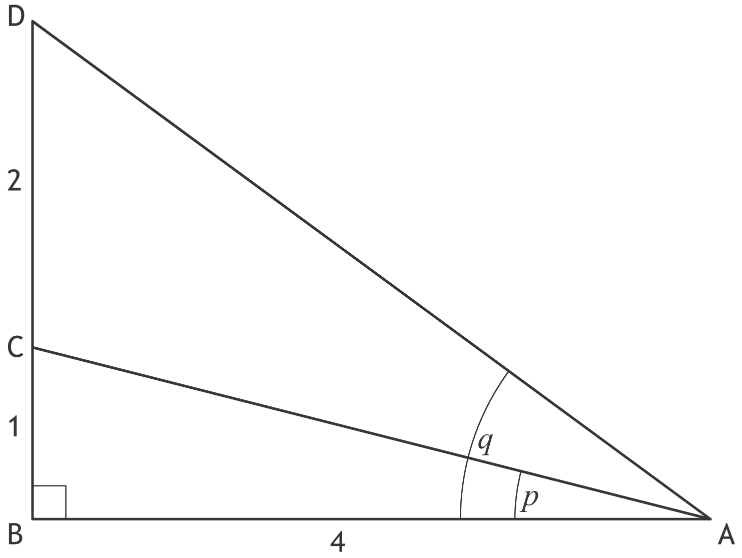
10. (a) Evaluate $\log_5 25$. **1**

(b) Hence solve $\log_4 x + \log_4 (x - 6) = \log_5 25$, where $x > 6$. **5**

11. Find the rate of change of the function $f(x) = 4\sin^3 x$ when $x = \frac{5\pi}{6}$. **3**

MARKS

12. Triangle ABD is right-angled at B with angles BAC $= p$ and BAD $= q$ and lengths as shown in the diagram below.



Show that the exact value of $\cos(q-p)$ is $\frac{19\sqrt{17}}{85}$. **5**

13. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x. **4**

14. (a) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \leq x < 360$. **5**

(b) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \leq x < 360$. **2**

MARKS

15. Functions f and g are defined on suitable domains by $f(x) = x^3 - 1$ and $g(x) = 3x + 1$.

(a) Find an expression for $k(x)$, where $k(x) = g(f(x))$. **2**

(b) If $h(k(x)) = x$, find an expression for $h(x)$. **3**

[END OF SPECIMEN QUESTION PAPER]