2008 Mathematics

Higher – Paper 1 and Paper 2

Finalised Marking Instructions

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (√). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗ or **X**√). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✗✗).

5. • The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, **not** a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked. Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   - working subsequent to a correct answer
   - legitimate variations in numerical answers
   - correct working in the "wrong" part of a question
   - omission of units
   - bad form

9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referal to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13 Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pr mark.

14 Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pr mark.

15 **Do not write any comments on the scripts**. A **revised** summary of acceptable notation is given on page 4.

16 Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1 **Tick** correct working.
2 Put a mark in the **outer right-hand margin to match the marks allocations on the question paper.**
3 Do **no**t write marks as fractions.
4 Put each mark **at the end** of the candidate's response to the question.
5 **Follow through** errors to see if candidates can score marks subsequent to the error.
6 Do **not** write any comments on the scripts.

Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs

✓ The tick. You are not expected to tick every line but of course you must check through the whole of a response.

___ ✗ The cross and underline. Underline an error and place a cross at the end of the line.

The tick-cross. Use this to show correct work where you are **following through** subsequent to an error.

∧ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

~ The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

Bullets showing where marks are being allotted may be shown on scripts

margins

$\frac{dy}{dx} = 4x - 7$ ✓ •

$4x - 7 = \underline{0}$ ✗

$x = \frac{7}{4}$

$y = 3\frac{7}{8}$ • 2

$C = \underline{(1,-1)}$ ✗

$m = \dfrac{3-(-1)}{4-1}$

$m_{rad} = \frac{4}{3}$ •

$m_{tgt} = \dfrac{-1}{\frac{4}{3}}$

$m_{tgt} = -\frac{3}{4}$ •

$y - 3 = -\frac{3}{4}(x-2)$ • 3

$x^2 - 3x = 28$ ✓ •

$x = 7$ ∧ 1

$\sin(x) = 0.75 = inv\sin(0.75) = 48.6°$ ✓ • 1

Remember - No comments on the scripts. No abreviations. No new signs.
Please use the above and nothing else.

All of these are to help us be more consistent and **accurate**.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).

1	2		UNIT 1
		A1	*determine range/domain*
		A2	*recognise general features of graphs:poly,exp,log*
		A3	*sketch and annotate related functions*
		A4	*obtain a formula for composite function*
		A5	*complete the square*
		A6	*interpret equations and expressions*
		A7	*determine function(poly,exp,log) from graph & vv*
		A8	*sketch/annotate graph given critical features*
		A9	*interpret loci such as st.lines,para,poly,circle*
		A10	*use the notation* u_n *for the nth term*
		A11	*evaluate successive terms of a RR*
		A12	*decide when RR has limit/interpret limit*
		A13	*evaluate limit*
		A14	*apply A10-A14 to problems*
		G1	*use the distance formula*
		G2	*find gradient from 2 pts,/angle/equ. of line*
		G3	*find equation of a line*
		G4	*interpret all equations of a line*
		G5	*use property of perpendicular lines*
		G6	*calculate mid-point*
		G7	*find equation of median, altitude,perp. bisector*
		G8	*apply G1-G7 to problems eg intersect.,concur.,collin.*
		C1	*differentiate sums, differences*
		C2	*differentiate negative & fractional powers*
		C3	*express in differentiable form and differentiate*
		C4	*find gradient at point on curve & vv*
		C5	*find equation of tangent to a polynomial/trig curve*
		C6	*find rate of change*
		C7	*find when curve strictly increasing etc*
		C8	*find stationary points/values*
		C9	*determinenature of stationary points*
		C10	*sketch curvegiven the equation*
		C11	*apply C1-C10 to problems eg optimise, greatest/least*
		T1	*use gen. features of graphs of* $f(x)=k\sin(ax+b)$,
			$f(x)=k\cos(ax+b)$; *identify period/amplitude*
		T2	*use radians inc conversion from degrees & vv*
		T3	*know and use exact values*
		T4	*recognise form of trig. function from graph*
		T5	*interpret trig. equations and expressions*
		T6	*apply T1-T5 to problems*

1	2		UNIT 2
		A15	*use the general equation of a parabola*
		A16	*solve a quadratic inequality*
		A17	*find nature of roots of a quadratic*
		A18	*given nature of roots, find a condition on coeffs*
		A19	*form an equation with given roots*
		A20	*apply A15-A19 to solve problems*
		A21	*use Rem Th. For values, factors, roots*
		A22	*solve cubic and quartic equations*
		A23	*find intersection of line and polynomial*
		A24	*find if line is tangent to polynomial*
		A25	*find intersection of two polynomials*
		A26	*confiirm and improve on approx roots*
		A27	*apply A21-A26 to problems*
		G9	*find C/R of a circle from its equation/other data*
		G10	*find the equation of a circle*
		G11	*find equation of a tangent to a circle*
		G12	*find intersection of line & circle*
		G13	*find if/when line is tangent to circle*
		G14	*find if two circles touch*
		G15	*apply G9-G14 to problems*
		C12	*find integrals of* px^n *and sums/diffs*
		C13	*integrate with negative & fractional powers*
		C14	*express in integrable form and integrate*
		C15	*evaluate definite integrals*
		C16	*find area between curve and x-axis*
		C17	*find area between two curves*
		C18	*solve differential equations(variables separable)*
		C19	*apply C12-C18 to problems*
		T7	*solve linear & quadratic equations in radians*
		T8	*apply compound and double angle (c & da) formulae*
			in numerical & literal cases
		T9	*apply c & da formulae in geometrical cases*
		T10	*use c & da formulaewhen solving equations*
		T11	*apply T7-T10 to problems*

1	2		UNIT 3 **Year**
		A28	*use the laws of logs to simplify/find equiv. expression*
		A29	*sketch associated graphs*
		A30	*solve equs of the form* $A = Be^{kt}$ *for A,B,k or t*
		A31	*solve equs of the form* $\log_b(a) = c$ *for a,b or c*
		A32	*solve equations involving logarithms*
		A33	*use relationships of the form* $y = ax^n$ *or* $y = ab^x$
		A34	*apply A28-A33 to problems*
		G16	*calculate the length of a vector*
		G17	*calculate the 3rd given two from A,B and vector AB*
		G18	*use unit vectors*
		G19	*use: if* $\mathbf{u}$, $\mathbf{v}$ *are parallel then* $\mathbf{v} = k\mathbf{u}$
		G20	*add, subtract, find scalar mult. of vectors*
		G21	*simplify vector pathways*
		G22	*interpret 2D sketches of 3D situations*
		G23	*find if 3 points in space are collinear*
		G24	*find ratio which one point divides two others*
		G25	*given a ratio, find/interpret 3rd point/vector*
		G26	*calculate the scalar product*
		G27	*use: if* $\mathbf{u}$, $\mathbf{v}$ *are perpendicular then* $\mathbf{v.u=0}$
		G28	*calculate the angle between two vectors*
		G29	*use the distributive law*
		G30	*apply G16-G29 to problems eg geometry probs.*
		C20	*differentiate* $p\sin(ax+b)$, $p\cos(ax+b)$
		C21	*differentiate using the chain rule*
		C22	*integrate* $(ax + b)^n$
		C23	*integrate* $p\sin(ax+b)$, $p\cos(ax+b)$
		C24	*apply C20-C23 to problems*
		T12	*solve sim.equs of form* $k\cos(a)=p$, $k\sin(a)=q$
		T13	*express* $p\cos(x)+q\sin(x)$ *in form* $k\cos(x\pm a)$ *etc*
		T14	*find max/min/zeros of* $p\cos(x)+q\sin(x)$
		T15	*sketch graph of* $y=p\cos(x)+q\sin(x)$
		T16	*solve equ of the form* $y=p\cos(rx)+q\sin(rx)$
		T17	*apply T12-T16 to problems*

2008 Higher Mathematics
Paper 1 Section A

1	C
2	D
3	C
4	B
5	A
6	B
7	C
8	D
9	B
10	A
11	B
12	C
13	A
14	B
15	C
16	A
17	C
18	C
19	B
20	D

1.21

QU	part	mk	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
1.21	a	6	C8,C9	NC		1	3	2	6				6		
	b	5	A21,A22			1	3	1	5					5	
	c	4	C10					4	2	2			4		

A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

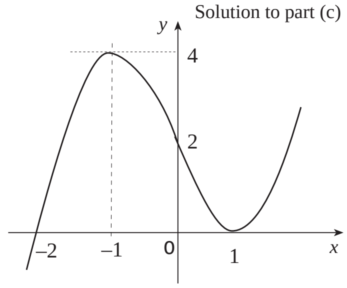
(a) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. 6

(b) (i) Show that $(x-1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. 5

(c) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. 4

Solution to part (c)



The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ss	set derivative to zero
$\bullet^2$	pd	differentiate
$\bullet^3$	pd	solve
$\bullet^4$	pd	evaluate y-coordinates
$\bullet^5$	ic	justification
$\bullet^6$	ic	state conclusions
$\bullet^7$	ss	know to use $x = 1$
$\bullet^8$	pd	complete eval. & conclusion
$\bullet^9$	ic	start to find quadratic factor
$\bullet^{10}$	pd	complete quadratic factor
$\bullet^{11}$	pd	factorise completely
$\bullet^{12}$	ic	interpret y-intercept
$\bullet^{13}$	ic	interpret x-intercepts
$\bullet^{14}$	ic	sketch : showing turning points
$\bullet^{15}$	ic	sketch : showing intercepts

Primary Method : Give 1 mark for each •

$\bullet^1$ $f'(x) = 0$

$\bullet^2$ $3x^2 - 3$

$\bullet^3$ $\bullet^4$

$$\begin{array}{c|c|c|}
 & \bullet^3 & \bullet^4 \\
x & -1 & 1 \\
y & 4 & 0
\end{array}$$

$\bullet^5$ $\bullet^6$

$$\begin{array}{c|ccc|ccc|}
 & & \bullet^5 & & & \bullet^6 & \\
 & \dots & -1 & \dots & \dots & 1 & \dots \\
f' & + & 0 & - & - & 0 & + \\
 & \text{max} & at\,x = & -1 & \text{min} & at\,x = & 1
\end{array}$$

$\bullet^7$ know to use $x = 1$

$\bullet^8$ $1 - 3 + 2 = 0 \Rightarrow x - 1$ is a factor

$\bullet^9$ $(x-1)(x^2 ...)$

$\bullet^{10}$ $(x-1)(x^2 + x - 2)$

$\bullet^{11}$ $(x-1)(x-1)(x+2)$ ***stated explicitly***

$\bullet^{12}$ $(0,2)$

$\bullet^{13}$ $(-2,0),\ (1,0)$

$\bullet^{14}$ Sketch with turning pts marked

$\bullet^{15}$ Sketch with $(0,2)$ or $(-2,0)$

Notes

1 The "=0" shown at $\bullet^1$ must appear at least once before the $\bullet^3$ stage.

2 An unsimplified $\sqrt{1}$ should be penalised at the first occurrence.

3 $\bullet^3$ is only available as a consequence of solving $f'(x) = 0$.

4 The nature table must reflect previous working from $\bullet^3$.

5 Candidates who introduce an extra solution at the $\bullet^3$ stage cannot earn $\bullet^3$.

6 The use of the 2nd derivative is an acceptable strategy for $\bullet^5$.

7 As shown in the Primary Method, ($\bullet^3$ and $\bullet^4$) and ($\bullet^5$ and $\bullet^6$) can be marked in series or in parallel.

8 The working for (b) may appear in (a) or vice versa. Full marks are available wherever the working occurs.

Notes

9 In Primary method $\bullet^8$ and alternative $\bullet^9$, candidates must show some acknowledgement of the resulting "0". Although a statement wrt the zero is preferable, accept something as simple as "underlining the zero".

Alternative Method : $\bullet^7$ to $\bullet^{10}$

$\bullet^7$

$$\begin{array}{c|cccc}
1 & 1 & 0 & -3 & 2 \\
\hline
 & & & &
\end{array}$$

$\bullet^8$

$$\begin{array}{c|cccc}
1 & 1 & 0 & -3 & 2 \\
 & & 1 & 1 & -2 \\
\hline
 & 1 & 1 & -2 & 0
\end{array}$$

$\bullet^9$ $f(1) = 0$ *so* $(x-1)$ *is a factor*

$\bullet^{10}$ $x^2 + x - 2$

Notes

10 Evidence for $\bullet^{12}$ and $\bullet^{13}$ may not appear until the sketch.

11 $\bullet^{14}$ and $\bullet^{15}$ are only available for the graph of a cubic.

Nota Bene

For candidates who omit the x^2 coeff. leading to

$\bullet^7$ X

$\bullet^8$ $\surd$

$$\begin{array}{c|ccc}
1 & 1 & -3 & 2 \\
 & & 1 & -2 \\
\hline
 & 1 & -2 & 0
\end{array}$$

$\bullet^9$ $\surd$ $f(1) = 0$ *so* $(x-1)$.........

$\bullet^{10}$ X $x^2 - 2x$

$\bullet^{11}$ $\surd$ $x(x-1)(x-2)$

but

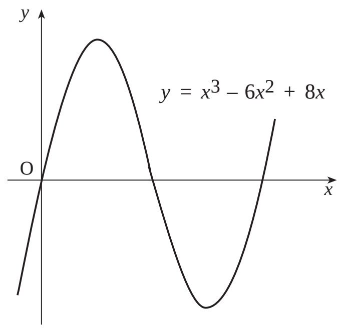
$\bullet^{10}$ X $x - 2$

$\bullet^{11}$ X $(x-1)(x-2)$

1.22

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
1.22	a	5	C4	NC		2	3		5				5		
	b	2	C11			1		1	2					2	

The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.



(a) Find the coordinates of the points on the curve where the gradient of the tangent is -1. 5

(b) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. 2

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ss	know to differentiate
$\bullet^2$	pd	differentiate
$\bullet^3$	ss	set derivative to -1
$\bullet^4$	pd	factorise and solve
$\bullet^5$	pd	solve for y
$\bullet^6$	ss	use gradient
$\bullet^7$	ic	interpret result

Primary Method : Give 1 mark for each •

$\bullet^1$ $\frac{dy}{dx} = ..(1\ term\ correct)$ $s\ /\ i\ by\ \bullet^2$

$\bullet^2$ $3x^2 - 12x + 8$ $s\ /\ i\ by\ \bullet^3$

$\bullet^3$ $3x^2 - 12x + 8 = -1$

		$\bullet^4$	$\bullet^5$
$\bullet^4$	x	1	3
$\bullet^5$	y	3	-3

$\bullet^6$ $y = 4 - x$ has gradient $= -1$

$\bullet^7$ *check* $(3,-3)$ and reject
check $(1,3)$ and accept

Notes

1 *in (a)*

$\bullet^1$ $\surd$ $\frac{dy}{dx} = ..(1\ term\ correct)$

$\bullet^2$ $\surd$ $3x^2 - 12x + 8$

For candidates who now guess $x = 1$ and check that $\frac{dy}{dx} = -1$, only one further mark ($\bullet^3$) can be awarded. Guessing and checking further answers gains no more credit.

2 An "=0" must appear at least once in the two lines shown in the alternative for $\bullet^6$ and $\bullet^7$.

Common Error

$\bullet^1$ $\surd$ $\frac{dy}{dx} = ..(1\ term\ correct)$

$\bullet^2$ $\surd$ $3x^2 - 12x + 8$

$\bullet^3$ X $3x^2 - 12x + 8 = 0$

$\bullet^4$ X irrespective of what is written.

$\bullet^5$ X

Alternative for $\bullet^6$ and $\bullet^7$

$\bullet^6$ $\begin{cases} x^3 - 6x^2 + 8x = 4 - x \\ x^3 - 6x^2 + 9x - 4 = 0 \\ (x-1)(x^2 - 5x + 4) \\ \quad (x-4)(x-1) \end{cases}$

$\bullet^7$ $\begin{cases} \text{repeated root implies} \\ \text{tangent at } (1,3). \end{cases}$

1.23

qu	part	mk	A3	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
1.23	a	3	A4	NC				3	3				3		
	b	5	A31			2	2	1		1	4				5

Functions f, g and h are defined on suitable domains by $f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$.

(a) Find expressions for $h\big(f(x)\big)$ and $h\big(g(x)\big)$. 3

(b) Hence solve $h\big(f(x)\big) - h\big(g(x)\big) = 3$ 5

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ic	interpretation composition
$\bullet^2$	ic	interpretation composition
$\bullet^3$	ic	interpretation composition
$\bullet^4$	ss	use log laws
$\bullet^5$	ss	convert to exponential form
$\bullet^6$	pd	process conversion
$\bullet^7$	pd	express in standard form
$\bullet^8$	ic	find valid solutions

Primary Method : Give 1 mark for each •

$\bullet^1$	$h\big(f(x)\big) = h\big(x^2 - x + 10\big)$ $s\ /\ i\ by\ \bullet^2$
$\bullet^2$	$\log_2\left(x^2 - x + 10\right)$
$\bullet^3$	$\log_2(5 - x)$
$\bullet^4$	$\log_2\left(\dfrac{x^2 - x + 10}{5 - x}\right)$
$\bullet^5$	$\dfrac{x^2 - x + 10}{5 - x} = 2^3$
$\bullet^6$	$x^2 - x + 10 = 8(5 - x)$
$\bullet^7$	$x^2 + 7x - 30 = 0$
$\bullet^8$	$x = 3,\ -10$

Notes

1 In (a) 2 marks are available for finding one of $h(f(x))$ or $h(g(x))$ and the third mark is for the other.

2 Treat $\log_2 x^2 - x + 10$ *and* $\log_2 5 - x$ as bad form.

3 The omission of the base should not be penalised in $\bullet^2$ to $\bullet^4$.

4 $\bullet^7$ is only available for a quadratic equation and $\bullet^8$ must be the follow-through solutions.

Common Error 1

$\bullet^4$	X	$\log_2(x^2 + 5) = 3$
$\bullet^5$	$\surd$	$x^2 + 5 = 2^3$
$\bullet^6$	X	$x^2 = 3$
$\bullet^7$	X	$x = \pm\sqrt{3}$
$\bullet^8$	X	not available

Common Error 2

$\bullet^4$	$\surd$	$\log_2\left(\dfrac{x^2 - x + 10}{5 - x}\right)$
		$\log_2\left(\dfrac{x^2 \cancel{-} \cancel{x} + \cancel{10}}{\cancel{5} \cancel{-} \cancel{x}}\right)$
		$\log_2\left(x^2 + 2\right) = 3$
$\bullet^5$	$X\surd$	$x^2 + 2 = 2^3$
$\bullet^6$	X	$x = \pm\sqrt{6}$
$\bullet^7$	X	*not available*
$\bullet^8$	X	*not available*

Common Error 3

$\bullet^4$	X	not available
$\bullet^5$	$\surd$	$\log_2(x^2 - x + 10) - \log_2(5 - x) = \log_2 8$
$\bullet^6$	X	$x^2 - x + 10 - (5 - x) = 8$
$\bullet^7$	X	not available
$\bullet^8$	X	not available