National
Qualifications
SPECIMEN ONLY

S847/76/12

Mathematics
Paper 2

Date — Not applicable

Duration — 1 hour 30 minutes

Total marks — 65

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

S8477612

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

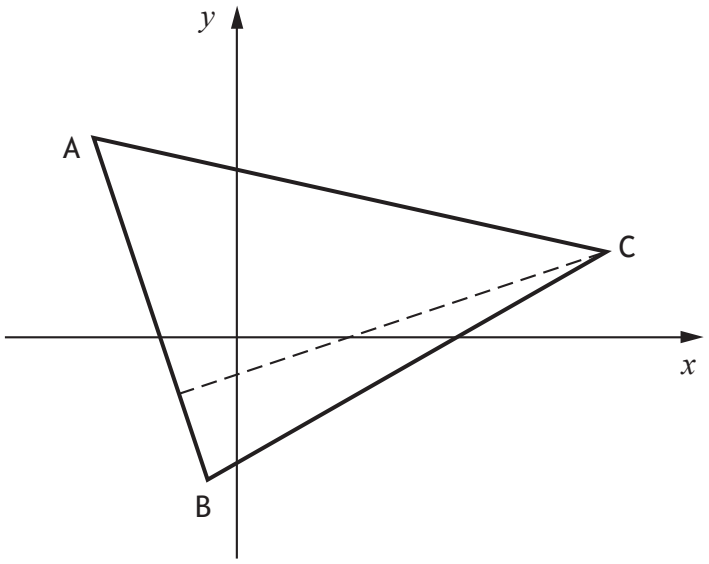
MARKS

Total marks — 65

Attempt ALL questions

1. The vertices of triangle ABC are A(−5, 7), B(−1, −5) and C(13, 3) as shown in the diagram.

The broken line represents the altitude from C.



(a) Find the equation of the altitude from C. **3**

(b) Find the equation of the median from B. **3**

(c) Find the coordinates of the point of intersection of the altitude from C and the median from B. **2**

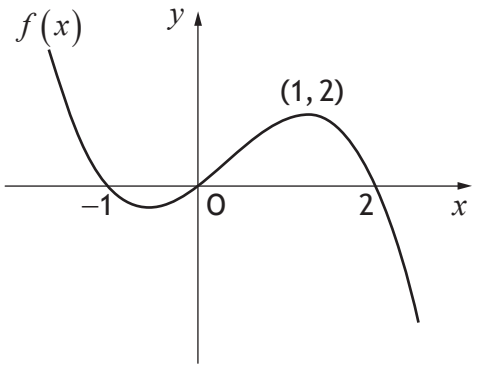
2. Find $\int \frac{4x^3 + 1}{x^2} dx$, $x \neq 0$. **4**

[Turn over

MARKS

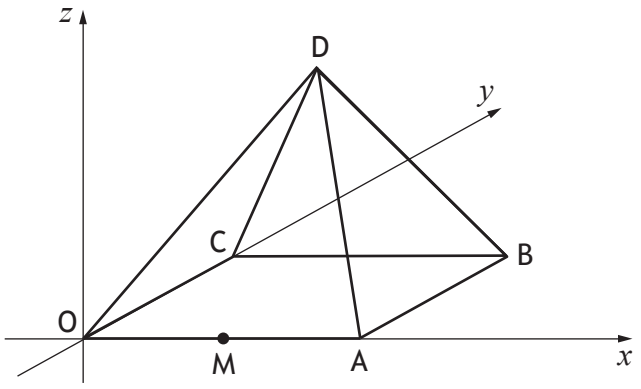
3. The diagram shows the curve with equation $y = f(x)$, where $f(x) = kx(x+a)(x+b)$.

The curve passes through $(-1, 0)$, $(0, 0)$, $(1, 2)$ and $(2, 0)$.



Find the values of a, b and k. **3**

4. D,OABC is a square-based pyramid as shown.



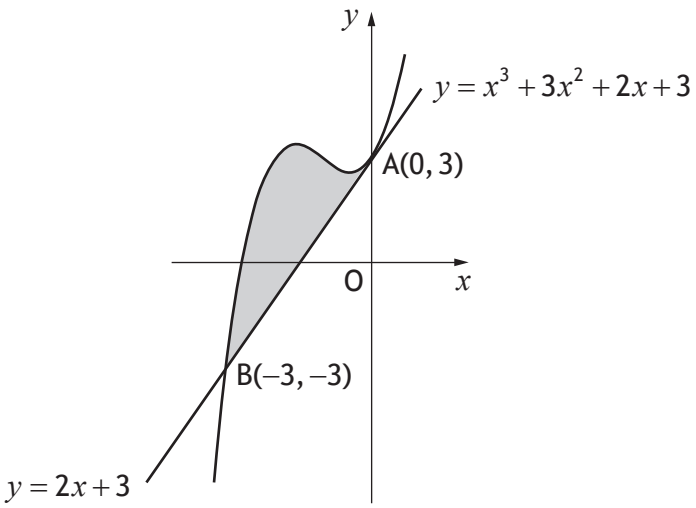
- O is the origin and OA = 4 units.
- M is the mid-point of OA.
- $\overrightarrow{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

(a) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3**

(b) Find the size of angle BDM. **5**

MARKS

5. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown.



The line meets the curve again at B(−3, −3).

Find the area enclosed by the line and the curve. **5**

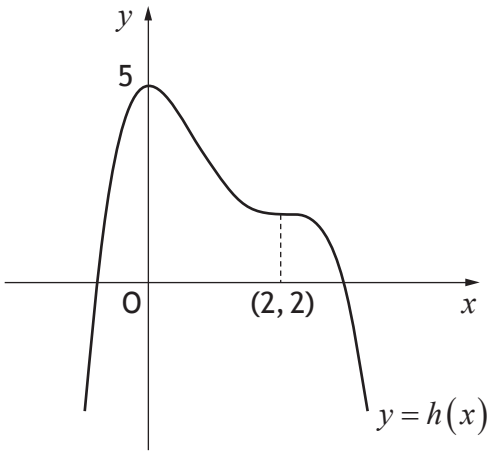
6. (a) Express $3x^2 + 24x + 50$ in the form $a(x + b)^2 + c$. **3**

(b) Given that $f(x) = x^3 + 12x^2 + 50x - 11$, find $f'(x)$. **2**

(c) Hence, or otherwise, explain why the curve with equation $y = f(x)$ is strictly increasing for all values of x. **2**

[Turn over

7. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at (0, 5) and (2, 2).



On separate diagrams sketch the graphs of:

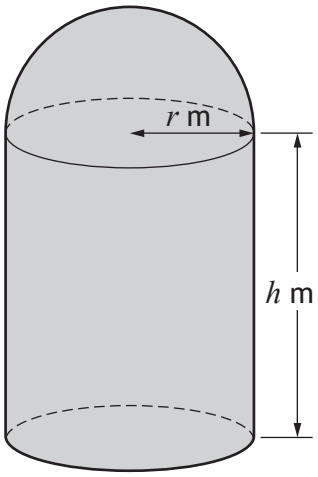
(a) $y = 2 - h(x)$. **2**

(b) $y = h'(x)$. **3**

MARKS

8. A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres.

The volume of the **cylindrical** part of the container needs to be 100 cubic metres.



(a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by:

$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$ **3**

(b) Determine the value of r which minimises the amount of metal needed to build the container. **6**

9. Given that

$$\int_{\frac{\pi}{8}}^{a} \sin\left(4x - \frac{\pi}{2}\right) dx = \frac{1}{2}, \quad 0 \leq a < \frac{\pi}{2},$$

calculate the value of a. **6**

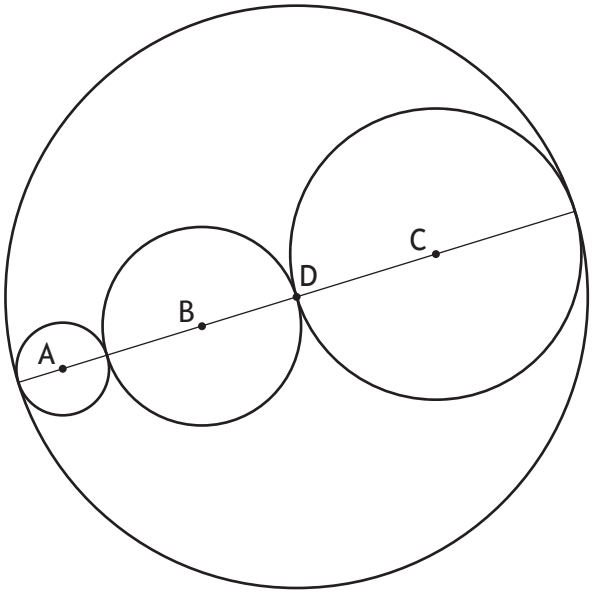
[Turn over

MARKS

10. Show that $\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \frac{\pi}{2}$. **3**

11. (a) Show that the points A(−7, −2), B(2, 1) and C(17, 6) are collinear. **3**

Three circles with centres A, B and C are drawn inside a circle with centre D as shown.



The circles with centres A, B and C have radii r_A, r_B and r_C respectively.

- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

(b) Determine the equation of the circle with centre D. **4**

[END OF SPECIMEN QUESTION PAPER]