

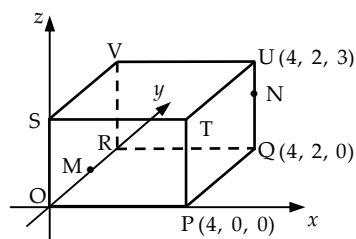
1 The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point  $(4, 0, 0)$ ,

Q is  $(4, 2, 0)$  and U is  $(4, 2, 3)$ .

M is the midpoint of OR.

N is the point on UQ such that  $UN = \frac{1}{3}UQ$ .



(a) State the coordinates of M and N.

2

(b) Express the vectors  $\overrightarrow{VM}$  and  $\overrightarrow{VN}$  in component form.

2

**Treat as bad form, coordinates written as components and vice versa, throughout this question.**

### Generic Scheme

### Illustrative Scheme

1 (a)

•<sup>1</sup> ic interpret midpoint for M

•<sup>2</sup> ic interpret ratio for N

•<sup>1</sup>  $(0, 1, 0)$

•<sup>2</sup>  $(4, 2, 2)$

1 (b)

•<sup>3</sup> ic interpret diagram

•<sup>4</sup> pd process vectors

$$\begin{aligned} \bullet^3 \quad \overrightarrow{VM} &= \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix} \\ \bullet^4 \quad \overrightarrow{VN} &= \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix} \end{aligned}$$

Using evidence from (a) or may have been taken directly from diagram.

### Notes

1. V is the point  $(0, 2, 3)$ , which may or may not appear in the working to (b).

### Regularly occurring responses

#### Response 1

(a)  $M(2, 0, 0) \times$  •<sup>1</sup>  $N(4, 2, -1) \times$  •<sup>2</sup>

0 marks out of 2

(b)  $\overrightarrow{VM} = \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} \times$  •<sup>3</sup>

Consistent with (a)

$\overrightarrow{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix} \checkmark$  •<sup>4</sup>

From diagram

2 marks out of 2

#### Response 2

(b)  $V(0, 3, 2)$

$$\begin{aligned} \overrightarrow{VM} &= \begin{pmatrix} 0 \\ -2 \\ -2 \end{pmatrix} \times \bullet^3 \\ \overrightarrow{VN} &= \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} \times \bullet^4 \end{aligned}$$

1 mark out of 2

Incorrect V stated

#### Response 3

(a)  $M(0, 2, 0) \times$  •<sup>1</sup>  $N(4, 2, 2) \checkmark$  •<sup>2</sup>

1 mark out of 2

(b)  $\overrightarrow{VM} = \begin{pmatrix} -4 \\ 0 \\ -3 \end{pmatrix} \times \bullet^3$   
 $\overrightarrow{VN} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \times \bullet^4$

$V(4, 2, 3)$  used in both but not stated

0 marks out of 2

- 1 The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

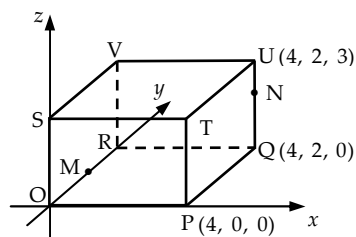
P is the point  $(4, 0, 0)$ ,

Q is  $(4, 2, 0)$  and U is  $(4, 2, 3)$ .

M is the midpoint of OR.

N is the point on UQ such that  $UN = \frac{1}{3} UQ$ .

- (c) Calculate the size of angle MVN.



5

**Treat as bad form, coordinates written as components and vice versa, throughout this question.**

### Generic Scheme

### Illustrative Scheme

1 (c)

#### Method 1 : Vector Approach

- <sup>5</sup> ss know to use scalar product
- <sup>6</sup> pd find scalar product
- <sup>7</sup> pd find magnitude of a vector
- <sup>8</sup> pd find magnitude of a vector
- <sup>9</sup> pd evaluate angle

#### Method 2 : Cosine Rule Approach

- <sup>5</sup> ss know to use cosine rule
- <sup>6</sup> pd find magnitude of a side
- <sup>7</sup> pd find magnitude of a side
- <sup>8</sup> pd find magnitude of a side
- <sup>9</sup> pd evaluate angle

#### Method 1 : Vector Approach

- <sup>5</sup>  $\cos \hat{M}\hat{V}\hat{N} = \frac{\overrightarrow{VM} \cdot \overrightarrow{VN}}{|\overrightarrow{VM}| |\overrightarrow{VN}|}$  *stated, or implied by* •<sup>9</sup>
- <sup>6</sup>  $\overrightarrow{VM} \cdot \overrightarrow{VN} = 3$
- <sup>7</sup>  $|\overrightarrow{VM}| = \sqrt{10}$
- <sup>8</sup>  $|\overrightarrow{VN}| = \sqrt{17}$
- <sup>9</sup>  $76.7^\circ$  or  $1.339$  rads or  $85.2$  grads

#### Method 2 : Cosine Rule Approach

- <sup>5</sup>  $\cos \hat{M}\hat{V}\hat{N} = \frac{VM^2 + VN^2 - MN^2}{2 \times VM \times VN}$  *stated, or implied by* •<sup>9</sup>
- <sup>6</sup>  $VM = \sqrt{10}$
- <sup>7</sup>  $VN = \sqrt{17}$
- <sup>8</sup>  $MN = \sqrt{21}$
- <sup>9</sup>  $76.7^\circ$  or  $1.339$  rads or  $85.2$  grads

### Notes

2. •<sup>5</sup> is not available to candidates who choose to evaluate an incorrect angle.
3. For candidates who do not attempt •<sup>9</sup>, then •<sup>5</sup> is only available if the formula quoted relates to the labelling in the question.
4. •<sup>9</sup> should be awarded for any answer that rounds to  $77^\circ$  or  $1.3$  rads or  $85$  grads (i.e. correct to two significant figures.)

### Regularly occurring responses

#### Response 1

$$\cos \hat{M}\hat{O}\hat{N} = \frac{\overrightarrow{OM} \cdot \overrightarrow{ON}}{|\overrightarrow{OM}| |\overrightarrow{ON}|} \quad \text{✗} \quad \bullet^5 \quad \text{Wrong angle}$$

$$\overrightarrow{OM} \cdot \overrightarrow{ON} = 2 \quad \text{✗} \quad \bullet^6$$

$$|\overrightarrow{OM}| = 1 \quad \text{✗} \quad \bullet^7 \quad \text{Eased because only one non-zero component.}$$

$$|\overrightarrow{ON}| = \sqrt{24} \quad \text{✗} \quad \bullet^8$$

$$65.9^\circ \text{ or } 1.150 \text{ rads or } 73.2 \text{ grads} \quad \text{✗} \quad \bullet^9 \quad \text{3 marks out of 5}$$

#### Response 2

$$\cos \hat{M}\hat{V}\hat{N} = \frac{\overrightarrow{VM} \cdot \overrightarrow{VN}}{|\overrightarrow{VM}| |\overrightarrow{VN}|} \quad \text{✓} \quad \bullet^5 \quad \text{Going directly to } 90^\circ \text{ from } \bullet^6 \text{ would lose } \bullet^7 \text{ and } \bullet^8.$$

$$\overrightarrow{VM} \cdot \overrightarrow{VN} = 0 \quad \text{✗} \quad \bullet^6$$

$$|\overrightarrow{VM}| = \sqrt{17} \quad \text{✗} \quad \bullet^7$$

$$|\overrightarrow{VN}| = 2 \quad \text{✗} \quad \bullet^8$$

$$90^\circ \text{ or equivalent} \quad \text{✗} \quad \bullet^9 \quad \text{4 marks out of 5}$$

- 2 (a)  $12 \cos x^\circ - 5 \sin x^\circ$  can be expressed in the form  $k \cos(x+a)^\circ$ , where  $k > 0$  and  $0 \leq a < 360$ .  
Calculate the values of  $k$  and  $a$ .

4

### Generic Scheme

### Illustrative Scheme

(a)

- <sup>1</sup> ss use addition formula
- <sup>2</sup> ic compare coefficients
- <sup>3</sup> pd process  $k$
- <sup>4</sup> pd process  $a$

- <sup>1</sup>  $k \cos x^\circ \cos a^\circ - k \sin x^\circ \sin a^\circ$  or  $k(\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ)$   
*stated explicitly*
- <sup>2</sup>  $k \cos a^\circ = 12$  and  $k \sin a^\circ = 5$  or  $-k \sin a^\circ = -5$   
*stated explicitly*
- <sup>3</sup> 13 *no justification required, but do not accept  $\sqrt{169}$*
- <sup>4</sup>  $22 \cdot 6$  *accept any answer which rounds to 23*

### Notes

- Do not penalise the omission of the degree symbol.
- Treat  $k \cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ$  as bad form only if the equations at the •<sup>2</sup> stage both contain  $k$ .
- $13 \cos x^\circ \cos a^\circ - 13 \sin x^\circ \sin a^\circ$  or  $13(\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ)$  is acceptable for •<sup>1</sup> and •<sup>3</sup>.
- <sup>2</sup> is not available for  $k \cos x^\circ = 12$  and  $k \sin x^\circ = 5$  or  $-k \sin x^\circ = -5$ , however, •<sup>4</sup> is still available.
- <sup>4</sup> is lost to candidates who give  $a$  in radians only.
- <sup>4</sup> may be gained only as a consequence of using evidence at •<sup>2</sup> stage.
- Candidates may use any form of the wave equation for •<sup>1</sup>, •<sup>2</sup> and •<sup>3</sup>, however •<sup>4</sup> is only available if the value of  $a$  is interpreted for the form  $k \cos(x+a)^\circ$ .

### Regularly occurring responses

#### Response 1A

$$k(\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ) \checkmark \bullet^1$$

$$\sin a = 5 \quad \times \bullet^2$$

$$\cos a = 12$$

$$\tan a^\circ = \frac{5}{12}$$

$$a = 22 \cdot 6 \quad \times \bullet^4$$

$$13 \cos(x + 22 \cdot 6) \quad \checkmark \bullet^3$$

2 marks out of 4

#### Response 1B

$$k \cos x \cos a - k \sin x \sin a \quad \checkmark \bullet^1$$

$$k = 13 \quad \checkmark \bullet^3 \quad \wedge \bullet^2$$

$$\tan a^\circ = \frac{5}{12}$$

$$a = 22 \cdot 6 \quad \times \bullet^4$$

2 marks out of 4

#### Response 2

$$k \cos(x-a)$$

$$= k \cos x \cos a + k \sin x \sin a \quad \times \bullet^1$$

$$= 13 \cos x \cos a + 13 \sin x \sin a \quad \checkmark \bullet^3$$

$$13 \cos a = 12 \quad 13 \sin a = -5 \quad \times \bullet^2$$

$$\text{then } a = 22 \cdot 6 \quad \times \bullet^4 \quad \text{See note 6}$$

$$\text{or } a = 337 \cdot 4 \quad \times \bullet^4 \quad \text{See note 7}$$

3 marks out of 4

#### Response 3A

$$k \cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ$$

$$k \sin a = 5 \quad \checkmark \bullet^1 \quad \checkmark \bullet^2$$

$$k \cos a = 12$$

$$k = 13 \quad \checkmark \bullet^3 \quad \tan a^\circ = \frac{12}{5} \quad \times \bullet^4$$

$$a = 67 \cdot 4$$

3 marks out of 4

#### Response 3B

$$k \cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ$$

$$k \sin a = 12 \quad \checkmark \bullet^1 \quad \times \bullet^2$$

$$k \cos a = 5$$

$$k = 13 \quad \checkmark \bullet^3 \quad \tan a^\circ = \frac{12}{5}$$

$$a = 67 \cdot 4 \quad \times \bullet^4$$

3 marks out of 4

#### Response 4

$$k \cos x^\circ \cos a^\circ - k \sin x^\circ \sin a^\circ \quad \checkmark \bullet^1$$

$$k \cos a = 12 \quad \times \bullet^2$$

$$k \sin a = -5$$

$$k = 13 \quad \checkmark \bullet^3 \quad \tan a^\circ = -\frac{5}{12}$$

$$a = 337 \cdot 4 \quad \times \bullet^4 \quad \text{See note 6}$$

3 marks out of 4

- 2 (b) (i) Hence state the maximum and minimum values of  $12 \cos x^\circ - 5 \sin x^\circ$ .  
 (ii) Determine the values of  $x$ , in the interval  $0 \leq x < 360$ , at which these maximum and minimum values occur.

3

## Generic Scheme

## Illustrative Scheme

(b)

- <sup>5</sup> ss state maximum and minimum
- <sup>6</sup> ic find  $x$  corresponding to max. value
- <sup>7</sup> pd find  $x$  corresponding to min. value

- <sup>5</sup> 13, -13
- <sup>6</sup> maximum at  $337.4$  and no others
- <sup>7</sup> minimum at  $157.4$  and no others
- or
- <sup>6</sup>  $337.4$  and  $157.4$  and no others
- <sup>7</sup> maximum at  $337.4$  and minimum at  $157.4$

## Notes

8. •<sup>5</sup> is available for  $\sqrt{169}$  and  $-\sqrt{169}$  only if  $\sqrt{169}$  has been penalised at •<sup>3</sup>.
9. Accept answers which round to 337 and 157 for •<sup>6</sup> and •<sup>7</sup>.
10. Candidates who continue to work in radian measure should not be penalised further.
11. Extra solutions, correct or incorrect, should be penalised at •<sup>6</sup> or •<sup>7</sup> but not both.
12. •<sup>6</sup> and •<sup>7</sup> are not available to candidates who work with  $13 \cos(x + 22.6)^\circ = 0$  or  $13 \cos(x + 22.6)^\circ = 1$ .
13. Candidates who use  $13 \cos(x - 22.6)^\circ$  from a correct (a) lose •<sup>6</sup> but •<sup>7</sup> is still available.

## Regularly occurring responses

## Response 1

From (a)  $a = 67.4$ 

max/ min =  $\pm 13$  ✓ •<sup>5</sup>  
 max at  $292.6$  ✗ •<sup>6</sup>  
 min at  $112.6$  ✗ •<sup>7</sup>

3 marks out of 3

## Response 2

From (a)  $\sqrt{169} \cos(x + 22.6)^\circ$ 

max =  $\sqrt{169}$  min =  $-\sqrt{169}$  ✗ •<sup>5</sup>  
 max at  $22.6$  ✗ •<sup>6</sup>  
 min at  $202.6$  ✗ •<sup>7</sup>

2 marks out of 3

 $\sqrt{169}$  already  
penalised at (a)

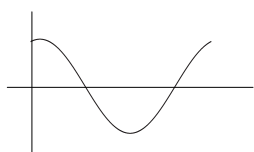
## Response 3A

$13 \cos(x + 22.6)^\circ$  max at  $22.6$  ✗ •<sup>6</sup>  
 $\times 13$  min at  $202.6$  ✗ •<sup>7</sup>

1 mark out of 3

Insufficient evidence for •<sup>5</sup>

## Response 3B



$13 \cos(x + 22.6)^\circ$  max at  $22.6$  ✗ •<sup>6</sup>  
 min at  $202.6$  ✗ •<sup>7</sup>

1 mark out of 3

**N.B.** Candidates who use differentiation in (b) can gain •<sup>5</sup> only, as a direct result of their response in (a). This question is in degrees and so calculus is not appropriate for •<sup>6</sup> and •<sup>7</sup>.

- 3 (a) (i) Show that the line with equation  $y = 3 - x$  is a tangent to the circle with equation  $x^2 + y^2 + 14x + 4y - 19 = 0$ .
- (ii) Find the coordinates of the point of contact, P.

5

### Generic Scheme

### Illustrative Scheme

(a)

•<sup>1</sup> ss substitute

•<sup>2</sup> pd express in standard form

•<sup>3</sup> ic start proof

•<sup>4</sup> ic complete proof

•<sup>5</sup> pd coordinates of P

$$\bullet^1 \quad x^2 + (3 - x)^2 + 14x + 4(3 - x) - 19 = 0$$

**Method 1 : Factorising**

$$\bullet^2 \quad 2x^2 + 4x + 2 \quad \bullet^3 \quad 2(x+1)(x+1) \quad \left. \vphantom{\begin{matrix} \bullet^2 \\ \bullet^3 \end{matrix}} \right\} = 0 \quad \text{see note 1}$$

•<sup>4</sup> equal roots so line is a tangent

**Method 2 : Discriminant**

$$\bullet^2 \quad 2x^2 + 4x + 2 = 0 \quad \text{stated explicitly}$$

$$\bullet^3 \quad 4^2 - 4 \times 2 \times 2$$

$$\bullet^4 \quad b^2 - 4ac = 0 \text{ so line is a tangent}$$

$$\bullet^5 \quad x = -1, y = 4$$

### Notes

**For method 1 :**

- <sup>2</sup> is only available if “= 0” appears at either •<sup>2</sup> or •<sup>3</sup> stage.
- Alternative wording for •<sup>4</sup> could be e.g. ‘repeated roots’, ‘repeated factor’, ‘only one solution’, ‘only one point of contact’ **along with** ‘line is a tangent’.

**For both methods :**

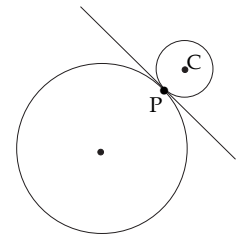
- Candidates must work with a quadratic equation at the •<sup>3</sup> and •<sup>4</sup> stages.
- Simply stating the tangency condition without supporting working cannot gain •<sup>4</sup>.
- For candidates who obtain two distinct roots, •<sup>4</sup> is still available for ‘not equal roots so not a tangent’ or ‘ $b^2 - 4ac \neq 0$  so line is not a tangent’, but •<sup>5</sup> is not available.

- 3 (b) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (a) and a second smaller circle with centre C.

The line  $y = 3 - x$  is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle.



$$x^2 + y^2 + 14x + 4y - 19 = 0$$

6

## Generic Scheme

## Illustrative Scheme

(b)

**Method 1 :** via centre and radius

- <sup>6</sup> ic state centre of larger circle
- <sup>7</sup> ss find radius of larger circle
- <sup>8</sup> pd find radius of smaller circle
- <sup>9</sup> ss strategy for finding centre
- <sup>10</sup> ic interpret centre of smaller circle
- <sup>11</sup> ic state equation

**Method 2 :** via ratios

- <sup>6</sup> ic state centre of larger circle
- <sup>7</sup> ss strategy for finding centre
- <sup>8</sup> ic state centre of smaller circle
- <sup>9</sup> ss strategy for finding radius
- <sup>10</sup> pd find radius of smaller circle
- <sup>11</sup> ic state equation

**Method 1 :** via centre and radius

- <sup>6</sup>  $(-7, -2)$  see note 11
- <sup>7</sup>  $\sqrt{72}$  see note 6 *stated, or implied by* •<sup>8</sup>
- <sup>8</sup>  $\sqrt{8}$  see note 7
- <sup>9</sup> e.g. "Stepping out"
- <sup>10</sup>  $(1, 6)$
- <sup>11</sup>  $(x-1)^2 + (y-6)^2 = 8$  or  $x^2 + y^2 - 2x - 12y + 29 = 0$

**Method 2 :** via ratios

- <sup>6</sup>  $(-7, -2)$  see note 11
- <sup>7</sup> e.g. "Stepping out"
- <sup>8</sup>  $(1, 6)$
- <sup>9</sup>  $\sqrt{2^2 + 2^2}$
- <sup>10</sup>  $\sqrt{8}$  see note 10
- <sup>11</sup>  $(x-1)^2 + (y-6)^2 = 8$  or  $x^2 + y^2 - 2x - 12y + 29 = 0$

## Notes

## For method 1:

6. Acceptable alternatives for •<sup>7</sup> are  $6\sqrt{2}$  or decimal equivalent which rounds to  $8.5$  i.e. to two significant figures.
7. Acceptable alternatives for •<sup>8</sup> are  $\frac{\sqrt{72}}{3}$  or  $2\sqrt{2}$  or decimal equivalent which rounds to  $2.8$ .
8.  $(1, 6)$  without working gains •<sup>10</sup> but loses •<sup>9</sup>.

## For method 2:

9.  $(1, 6)$  without working gains •<sup>8</sup> but loses •<sup>7</sup>.
10. Acceptable alternatives for •<sup>10</sup> are  $2\sqrt{2}$  or decimal equivalent which rounds to  $2.8$ .

## In both methods:

11. If  $m = 1$  is used in a 'stepping out' method the centre of the larger circle need not be stated explicitly for •<sup>6</sup>.
12. For the smaller circle, candidates who 'guess' values for either the centre or radius cannot be awarded •<sup>11</sup>.
13. At •<sup>11</sup> e.g.  $\sqrt{8^2}$ ,  $2.8^2$  are unacceptable, but any decimal which rounds to  $7.8$  is acceptable.
14. •<sup>11</sup> is not available to candidates who divide the coordinates of the centre of the larger circle by 3.

## Generic Scheme

## Illustrative Scheme

4

- <sup>1</sup> ss know to use double angle formula
- <sup>2</sup> ic express as quadratic in  $\cos x$
- <sup>3</sup> ss start to solve

- <sup>4</sup> pd reduce to equations in  $\cos x$  only
- <sup>5</sup> pd complete solutions to include only one where  $\cos x = k$  with  $|k| > 1$

**Method 1** : Using factorisation

- <sup>1</sup>  $2 \times (2\cos^2 x - 1) \dots$
  - <sup>2</sup>  $4\cos^2 x - 5\cos x - 6$
  - <sup>3</sup>  $(4\cos x + 3)(\cos x - 2)$
- } = 0 must appear at either of these lines to gain •<sup>2</sup>.

**Method 2** : Using quadratic formula

- <sup>1</sup>  $2 \times (2\cos^2 x - 1) \dots$
- <sup>2</sup>  $4\cos^2 x - 5\cos x - 6 = 0$
- <sup>3</sup>  $\cos x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 4 \times (-6)}}{2 \times 4}$

**In both methods :**

- <sup>4</sup>  $\cos x = -\frac{3}{4}$  and  $\cos x = 2$
  - <sup>5</sup>  $2 \cdot 419, 3 \cdot 864$  and no solution
- or**
- <sup>4</sup>  $\cos x = 2$  and no solution
  - <sup>5</sup>  $\cos x = -\frac{3}{4}$  and  $2 \cdot 419, 3 \cdot 864$

## Notes

1. •<sup>1</sup> is not available for simply stating that  $\cos 2A = 2\cos^2 A - 1$  with no further working.
2. Substituting  $\cos 2A = 2\cos^2 A - 1$  or  $\cos 2a = 2\cos^2 a - 1$  etc. should be treated as bad form throughout.
3. In the event of  $\cos^2 x - \sin^2 x$  or  $1 - 2\sin^2 x$  being substituted for  $\cos 2x$ , •<sup>1</sup> cannot be given until the equation reduces to a quadratic in  $\cos x$ .
4. Candidates may express the quadratic equation obtained at the •<sup>2</sup> stage in the form  $4c^2 - 5c + 6 = 0$ ,  $4x^2 - 5x + 6 = 0$  etc. For candidates who do not solve a trig. equation at •<sup>5</sup>,  $\cos x$  must appear explicitly to gain •<sup>4</sup>.
5. •<sup>4</sup> and •<sup>5</sup> are only available as a consequence of solving a quadratic equation subsequent to a substitution.
6. Any attempt to solve  $a\cos^2 x + b\cos x = c$  loses •<sup>3</sup>, •<sup>4</sup> and •<sup>5</sup>.
7. Accept answers given as decimals which round to  $2 \cdot 4$  and  $3 \cdot 9$ .
8. There must be an indication after  $\cos x = 2$  that there are no solutions to this equation.  
Acceptable evidence : e.g. " ~~$\cos x = 2$~~ ", "NA", "out of range", "invalid" and " $\cos x = 2$  no", " $\cos x = 2 \times$ ".  
Unacceptable evidence : e.g. " $\cos x = 2$ ", " $\cos x = 2$  ???", "Maths Error".
9. •<sup>5</sup> is not available to candidates who work throughout in degrees and do not convert their answer into radian measure.
10. Do not accept e.g.  $221 \cdot 4$ ,  $138 \cdot 6$ ,  $\frac{221 \cdot 4\pi}{180}$ ,  $\frac{221\pi}{180}$ ,  $1 \cdot 23\pi$ .
11. Ignore correct solution outside the interval  $0 \leq x < 2\pi$ .

4

## Regularly occurring responses

## Response 1

$$2 \times 2 \cos^2 x - 1 \dots \checkmark \bullet^1$$

$$4 \cos^2 x - 5 \cos x - 5 = 0 \quad \times \bullet^2$$

$$\cos x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 4 \times (-5)}}{2 \times 4} \quad \times \bullet^3$$

$$\cos x = \frac{5 - \sqrt{105}}{8} \quad \text{and} \quad \cos x = \frac{5 + \sqrt{105}}{8} \quad \times \bullet^4$$

$$2 \cdot 286, 3 \cdot 997 \quad \text{and} \quad \text{no solution} \quad \times \bullet^5$$

$\bullet^5$  is only available to candidates where one, but not both, of their equations has no solution for  $\cos x$ .

4 marks out of 5

## Response 2

$$4 \cos^2 x - 1 \dots \checkmark \bullet^1$$

$$4 \cos^2 x - 5 \cos x - 5 = 0 \quad \times \bullet^2$$

$$(2 \cos x + 1)(2 \cos x - 5) = 0 \quad \times \bullet^3$$

$$\cos x = -\frac{1}{2} \quad \text{and} \quad \cancel{\cos x = \frac{5}{2}} \quad \times \bullet^4$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3} \quad \times \bullet^5$$

$4 \cos^2 x - 1$  with no further working cannot gain  $\bullet^1$ ; however if a quadratic in  $\cos x$  subsequently appears then  $\bullet^1$  is awarded but  $\bullet^2$  is not available.

3 marks out of 5

$\bullet^1$  is lost here as it is not clear whether the candidate has used  $2 \cos^2 x - 1$  or  $\cos^2 x - 1$  as their substitution.

## Response 3A

$$2 \cos^2 x - 1 - 5 \cos x - 4 = 0 \quad \times \bullet^1$$

$$2 \cos^2 x - 5 \cos x - 5 = 0 \quad \times \bullet^2$$

$$\cos x = \frac{5 \pm \sqrt{25 + 40}}{4} \quad \times \bullet^3$$

$$\cos x = -0.766 \quad \text{and} \quad \cos x = 3.267 \quad \times \bullet^4$$

$$x = 2.44, 3.84 \quad \text{and} \quad \text{undefined} \quad \times \bullet^5$$

4 marks out of 5

## Response 3B

$$\cos 2x = 2 \cos^2 x - 1$$

$$2 \cos^2 x - 1 - 5 \cos x - 4 = 0 \quad \checkmark \bullet^1$$

$$2 \cos^2 x - 5 \cos x - 5 = 0 \quad \times \bullet^2$$

$$\cos x = \frac{5 \pm \sqrt{25 + 40}}{4} \quad \times \bullet^3$$

$$\cos x = -0.766 \quad \text{and} \quad \cos x = 3.267 \quad \times \bullet^4$$

$$x = 2.44, 3.84 \quad \text{and} \quad \text{undefined} \quad \times \bullet^5$$

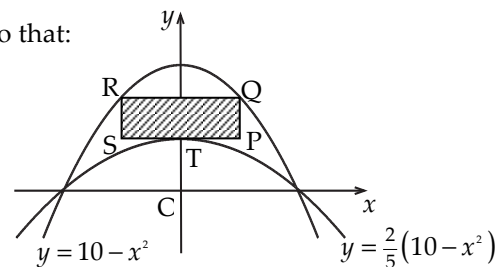
4 marks out of 5



- 5 The parabolas with equations  $y = 10 - x^2$  and  $y = \frac{2}{5}(10 - x^2)$  are shown in the diagram below.

A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola.
- RQ and SP are parallel to the  $x$ -axis.
- T, the turning point of the lower parabola, lies on SP.



- (a) (i) If  $TP = x$  units, find an expression for the length of PQ.  
(ii) Hence show that the area,  $A$ , of rectangle PQRS is given by

$$A(x) = 12x - 2x^3$$

3

- (b) Find the maximum area of this rectangle.

6

### Generic Scheme

### Illustrative Scheme

5 (a)

- <sup>1</sup> ss know to and find OT
- <sup>2</sup> ic obtain an expression for PQ
- <sup>3</sup> ic complete area evaluation

- <sup>1</sup> 4 or (0,4) *stated, or implied by* •<sup>2</sup>
- <sup>2</sup>  $10 - x^2 - 4$
- <sup>3</sup>  $2x \times (6 - x^2) = 12x - 2x^3$

### Notes

1. The evidence for •<sup>1</sup> and •<sup>2</sup> may appear on a sketch.
2. No marks are available to candidates who work backwards from the area formula.
3. •<sup>3</sup> is only available if •<sup>2</sup> has been awarded.

5 (b)

- <sup>4</sup> ss know to and start to differentiate
- <sup>5</sup> pd complete differentiation
- <sup>6</sup> ic set derivative to zero
- <sup>7</sup> pd obtain  $x$
- <sup>8</sup> ss justify nature of stationary point
- <sup>9</sup> ic interpret result and evaluate area

- <sup>4</sup>  $A'(x) = 12 \dots$  *stated, or implied by* •<sup>5</sup>
- <sup>5</sup>  $12 - 6x^2$
- <sup>6</sup>  $12 - 6x^2 = 0$
- <sup>7</sup>  $\sqrt{2}$  or decimal equivalent (ignore inclusion of  $-\sqrt{2}$ )
- <sup>8</sup>

$x$	$\dots$	$\sqrt{2}$	$\dots$
$A'(x)$	$+$	$0$	$-$

(Note : accept  $12 - 6x^2$  in lieu of  $A'(x)$  in the nature table.)
- <sup>9</sup> Max **and**  $8\sqrt{2}$  or decimal equivalent  
N.B. To conclude a maximum the evidence must come from •<sup>8</sup>.

### Notes

4. At •<sup>7</sup> accept any answer which rounds to 1.4.
5. Throughout this question treat the use of  $f'(x)$  or  $\frac{dy}{dx}$  as bad form.
6. At •<sup>8</sup> the nature can be determined using the second derivative.
7. At •<sup>9</sup> accept any answer which rounds to 11.3 or 11.4.

5

## Regularly occurring responses

## Response 1

$$A(x) = 12x - 2x^3$$

$$A'(x) = 24x^2 - 6x^3 \quad \times \bullet^4 \quad \times \bullet^5$$

$$24x^2 - 6x^3 = 0 \quad \times \bullet^6$$

$A'(x) = 0$  on its own would not be sufficient for  $\bullet^6$ .

## Response 2

At stationary points,  $A'(x) = 0$

$$12 - 6x^2 \quad \checkmark \bullet^4 \quad \checkmark \bullet^5 \quad \checkmark \bullet^6$$

$$x = \sqrt{2} \quad \checkmark \bullet^7$$

## Response 3

$$A(x) = 12x - 2x^3$$

$$= 12 - 6x^2 \quad \checkmark \bullet^4 \quad \checkmark \bullet^5$$

Bad form

## Response 4A

	...	$\sqrt{2}$	...
$A'(x)$	+	0	-

x missing

$\times \bullet^8$

## Response 4B

x	1	$\sqrt{2}$	2
$A'(x)$	6	0	-12

$\checkmark \bullet^8$

## Response 4C

x	...	$\sqrt{2}$	...
$A'(x)$	/	-	/

or 'slope' etc.

signs or values are necessary

$\times \bullet^8$

## Response 5

Maximum at  $x = \sqrt{2}$

$$y = 12\sqrt{2} - 2\sqrt{2}^3 = 8\sqrt{2} \quad \checkmark \bullet^9$$

$$\text{Area} = 2\sqrt{2} \times 8\sqrt{2} = 32$$

Treat this as an error subsequent to a correct answer.

- 6 (a) A curve has equation  $y = (2x - 9)^{\frac{1}{2}}$ .

Show that the equation of the tangent to this curve at the point where  $x = 9$  is  $y = \frac{1}{3}x$ .

5

- (b) Diagram 1 shows part of the curve and the tangent.

The curve cuts the  $x$ -axis at the point A.

Find the coordinates of point A.

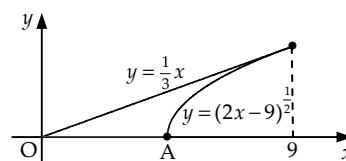


Diagram 1

1

### Generic Scheme

### Illustrative Scheme

6 (a)

- <sup>1</sup> ss know to and start to differentiate
- <sup>2</sup> pd complete chain rule derivative
- <sup>3</sup> pd gradient via differentiation
- <sup>4</sup> pd obtain  $y_{\text{CURVE}}$  at  $x = 9$
- <sup>5</sup> ic state equation and complete

- <sup>1</sup>  $\frac{1}{2}(2x - 9)^{-\frac{1}{2}}$
- <sup>2</sup>  $\dots \times 2$
- <sup>3</sup>  $\frac{1}{3}$
- <sup>4</sup> 3
- <sup>5</sup>  $y - 3 = \frac{1}{3}(x - 9)$  and complete to  $y = \frac{1}{3}x$

### Notes

- <sup>3</sup> is only available as a consequence of differentiating equation of the curve.
- Candidates must arrive at the equation of the tangent via the point (9, 3) and not the origin.
- For •<sup>3</sup> accept  $9^{-\frac{1}{2}}$ .

### Regularly occurring responses

#### Response 1

Candidates who equate derivatives:

$$\frac{1}{2}(2x - 9)^{-\frac{1}{2}} \times 2 = \frac{1}{3} \quad \checkmark \bullet^3$$

$\checkmark \bullet^1 \quad \checkmark \bullet^2$

leading to  $x = 9$  and  $y = 3$  from curve  $\checkmark \bullet^4$

Also obtaining  $y = 3$  from line and so line is a tangent  $\checkmark \bullet^5$

5 marks out of 5

#### Response 2

Candidates who intersect curve and line:

$$(2x - 9)^{\frac{1}{2}} = \frac{1}{3}x \quad \checkmark \bullet^1$$

$$2x - 9 = \left(\frac{1}{3}x\right)^2 \quad \checkmark \bullet^2$$

$$\frac{1}{9}x^2 - 2x + 9 = 0 \quad \checkmark \bullet^3$$

Factorising or using discriminant  $\checkmark \bullet^4$

Equal roots or  $b^2 - 4ac = 0$  so line is a tangent  $\checkmark \bullet^5$

5 marks out of 5

(b)

- <sup>6</sup> ic obtain coordinates of A

$$\bullet^6 \left(\frac{9}{2}, 0\right)$$

### Notes

- Accept  $x = \frac{9}{2}$ ,  $y = 0$  where  $y = 0$  may appear from  $(2x - 9)^{\frac{1}{2}} = 0$ .
- For  $\left(\frac{9}{2}, 0\right)$  without working •<sup>6</sup> is awarded, but from erroneous working •<sup>6</sup> is lost (see response below).

### Regularly occurring responses

#### Response 1

$$\sqrt{2x - 9} = 0 \Rightarrow \sqrt{2x} - 3 = 0 \Rightarrow 2x - 9 = 0 \Rightarrow x = 4.5$$

Here •<sup>6</sup> cannot be awarded due to the erroneous working.

6 (c) Calculate the shaded area shown in diagram 2.

7

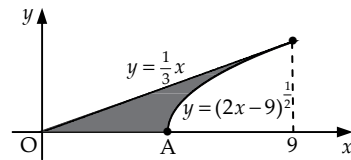


Diagram 2

## Generic Scheme

## Illustrative Scheme

6 (c)

**Method 1** : Area of triangle – area under curve

- <sup>7</sup> ss strategy for finding shaded area
- <sup>8</sup> ss know to integrate  $(2x-9)^{\frac{1}{2}}$
- <sup>9</sup> pd start integration
- <sup>10</sup> pd complete integration
- <sup>11</sup> ic limits  $x_A$  and 9
- <sup>12</sup> pd substitute limits
- <sup>13</sup> pd evaluate area and complete strategy

**Method 2** : Area between line and curve

- <sup>7</sup> ss strategy for finding shaded area
- <sup>8</sup> ss know to integrate  $(2x-9)^{\frac{1}{2}}$
- <sup>9</sup> pd start integration
- <sup>10</sup> pd complete integration
- <sup>11</sup> ic limits  $x_A$  and 9
- <sup>12</sup> pd 'upper – lower' and substitute limits
- <sup>13</sup> pd evaluate area and complete strategy

**Method 1** : Area of triangle – area under curve

- <sup>7</sup> Shaded area = Area of large  $\Delta$  – Area under curve
- <sup>8</sup>  $\int (2x-9)^{\frac{1}{2}} dx$
- <sup>9</sup>  $\frac{(2x-9)^{\frac{3}{2}}}{\frac{3}{2}}$
- <sup>10</sup>  $\dots \times \frac{1}{2}$
- <sup>11</sup>  $\frac{9}{2}$  and 9
- <sup>12</sup>  $\frac{1}{3}(18-9)^{\frac{3}{2}} - 0$
- <sup>13</sup>  $\frac{27}{2} - 9 = \frac{9}{2}$  or  $4\frac{1}{2}$  or  $4.5$

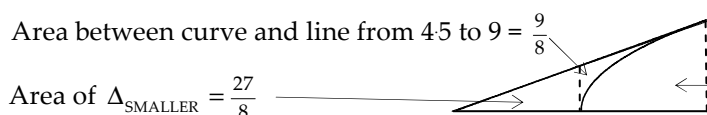
**Method 2** : Area between line and curve

- <sup>7</sup> Area of small  $\Delta$  + area between line and curve
- <sup>8</sup>  $\int \dots (2x-9)^{\frac{1}{2}} dx$
- <sup>9</sup>  $\dots \frac{(2x-9)^{\frac{3}{2}}}{\frac{3}{2}}$
- <sup>10</sup>  $\dots \times \frac{1}{2}$
- <sup>11</sup>  $\frac{9}{2}$  and 9
- <sup>12</sup>  $\left( \frac{1}{6} \times 9^2 - \frac{1}{3} (18-9)^{\frac{3}{2}} \right) - \left( \frac{1}{6} \times \left( \frac{9}{2} \right)^2 - \frac{1}{3} (9-9)^{\frac{3}{2}} \right)$
- <sup>13</sup>  $\frac{27}{8} + \frac{9}{8} = \frac{9}{2}$  or  $4\frac{1}{2}$  or  $4.5$

## Notes

6. •<sup>7</sup> may not be obvious until the final line of working and may be implied by final answer or a diagram.
7. At •<sup>11</sup> the value of  $x_A$  must lie between 0 and 9 exclusively, however, •<sup>12</sup> and •<sup>13</sup> are only available if  $4.5 \leq x_A < 9$ .
8. Full marks are available to candidates who integrate with respect to  $y$ .

You may find the following helpful in marking this question:

Area of  $\Delta_{\text{LARGER}} = 13.5$  or  $\frac{27}{2}$ 

Area under curve from 4.5 to 9 = 9

## Generic Scheme

## Illustrative Scheme

(a)

- <sup>1</sup> ss convert from log to exponential form
- <sup>2</sup> ss know to and convert back to log form
- <sup>3</sup> pd process and complete

- <sup>1</sup>  $x = 4^P$
- <sup>2</sup>  $\log_{16} x = \log_{16} 4^P$
- <sup>3</sup>  $\log_{16} x = P \times \log_{16} 4$  and complete

## Notes

1. No marks are available to candidates who simply substitute in values and verify the result.

e.g.  $\log_4 4 = 1$  and  $\log_{16} 4 = \frac{1}{2}$  ✗  
 $\log_4 x = P$  and  $\log_{16} x = \frac{1}{2}P$

## Regularly occurring responses

## Response 1

$$\begin{aligned}\log_4 x &= P \\ x &= 4^P \quad \checkmark \bullet^1 \\ \log_{16} x &= \frac{1}{2}P \quad \text{✗} \bullet^2 \\ x &= 16^{\frac{1}{2}P} \\ &= 4^P \quad \wedge \bullet^3\end{aligned}$$

1 mark out of 3

## Response 2

$$\begin{aligned}\log_4 x &= P \\ x &= 4^P \quad \checkmark \bullet^1 \\ x^2 &= 4^{2P} \\ &= 16^P \quad \text{✗} \bullet^2 \\ \log_{16} x^2 &= P \\ 2\log_{16} x &= P \\ \log_{16} x &= \frac{1}{2}P \quad \checkmark \bullet^3\end{aligned}$$

2 marks out of 3

## Response 3

$$\begin{aligned}x &= 4^P \quad \checkmark \bullet^1 \\ x &= \left(16^{\frac{1}{2}}\right)^P \quad \text{or} \quad 16^{\frac{1}{2} \times P} \quad \checkmark \bullet^2 \\ x &= 16^{\frac{1}{2}P} \\ \log_{16} x &= \frac{1}{2}P \quad \checkmark \bullet^3\end{aligned}$$

Without this step  $\bullet^2$  would be lost but  $\bullet^3$  is still available as follow through.

3 marks out of 3

## Response 4

$$\begin{aligned}x &= 4^P \quad \checkmark \bullet^1 & \log_{16} x &= kP \\ \log_4 x &= P & 16^{\log_{16} x} &= 16^{kP} \\ 4^{\log_4 x} &= 4^P = x & x &= 16^{kP} \\ 4^{\log_4 x} &= x \\ 4^P &= 16^{kP} \\ 4 &= 16^k \quad \text{✗} \bullet^2 \\ k &= \frac{1}{2} \\ \therefore \log_{16} x &= \frac{1}{2}P \text{ as } 16^{\frac{1}{2}} = 4 \quad \checkmark \bullet^3\end{aligned}$$

2 marks out of 3

## Response 5

**Beware** that some candidates give a circular argument.

This is only worth  $\bullet^1$ .

$$\begin{aligned}\log_4 x &= P & \text{then} & & \log_{16} x &= \frac{1}{2}P \\ x &= 4^P \quad \checkmark \bullet^1 & & & x &= 16^{\frac{1}{2}P} \\ \log_4 x &= \log_4 4^P & & & \log_{16} x &= \log_{16} 16^{\frac{1}{2}P} \\ \log_4 x &= P \log_4 4 & & & \log_{16} x &= \frac{1}{2}P \log_{16} 16 \\ \log_4 x &= P & & & \log_{16} x &= \frac{1}{2}P \quad \text{✗}\end{aligned}$$

1 mark out of 3

## Response 6

$$\log_{16} x = \frac{\log_4 x}{\log_4 16} = \frac{\log_4 x}{2} = \frac{1}{2}P$$

$\checkmark \bullet^1 \quad \checkmark \bullet^2 \quad \checkmark \bullet^3$

3 mark out of 3

Using change of base result.

7 (b) Solve  $\log_3 x + \log_9 x = 12$ .

3

## Generic Scheme

## Illustrative Scheme

(b)

- <sup>4</sup> ss use appropriate strategy
- <sup>5</sup> pd start solving process
- <sup>6</sup> pd complete process via log to expo form

- <sup>4</sup>  $\log_3 x + \frac{1}{2} \log_3 x = 12$
- <sup>5</sup>  $\log_3 x = 8$
- <sup>6</sup>  $x = 3^8 (= 6561)$

or

- <sup>4</sup>  $Q + \frac{1}{2}Q = 12$   
 $Q = 8$
- <sup>5</sup>  $\log_3 x = 8$
- <sup>6</sup>  $x = 3^8 (= 6561)$

- <sup>4</sup>  $2\log_9 x + \log_9 x = 12$
- <sup>5</sup>  $\log_9 x = 4$
- <sup>6</sup>  $x = 9^4 (= 6561)$

or

- <sup>4</sup>  $2Q + Q = 12$   
 $Q = 4$
- <sup>5</sup>  $\log_9 x = 4$
- <sup>6</sup>  $x = 9^4 (= 6561)$

## Notes

2. At •<sup>4</sup> any letter except  $x$  may be used in lieu of  $Q$ .
3. Candidates who use a trial and improvement technique by substituting values for  $x$  gain no marks.
4. The answer with no working gains no marks.

## Regularly occurring responses

## Response 1

$$\begin{aligned} Q + 2Q &= 12 & \checkmark \bullet^4 & \text{ or } \times \bullet^4 \\ 3Q &= 12 & \times \bullet^5 & \\ Q &= 4 & \times \bullet^5 & \\ \log_3 x &= 4 & \times \bullet^5 & \\ x &= 3^4 & \checkmark \bullet^6 & \\ &= 81 & & \end{aligned}$$

2 marks out of 3

## Response 2

$$\begin{aligned} \log_3 x + 2\log_3 x &= 12 & \times \bullet^4 & \\ 3\log_3 x &= 12 & & \\ \log_3 x &= 4 & \checkmark \bullet^5 & \\ x &= 3^4 & \times \bullet^6 & \\ &= 81 & & \end{aligned}$$

2 marks out of 3

The marks allocated are dependent on what substitution is used for  $Q$ .

## Response 3

$$\begin{aligned} 2\log_9 x + \log_9 x &= 12 & \checkmark \bullet^4 & \\ \log_9 x^2 + \log_9 x &= 12 & & \\ \log_9 x^3 &= 12 & \checkmark \bullet^5 & \\ x^3 &= 9^{12} & & \\ x &= \sqrt[3]{9^{12}} & & \\ x &= 9^4 & \checkmark \bullet^6 & \\ x &= 3^8 & & \\ &= 6561 & & \end{aligned}$$

3 marks out of 3

## Response 4

$$\begin{aligned} \log_3 x &= 8 & \times \bullet^4 & \times \bullet^5 \\ x &= 3^8 & \checkmark \bullet^6 & \\ &= 6561 & & \end{aligned}$$

Without justification, •<sup>4</sup> and •<sup>5</sup> are not available.