# 2007 Mathematics

# Higher – Paper 2

# Finalised Marking Instructions

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓ ). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick ( ✗ or **X**✓ ). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick ( ✗✗ ).

5. • The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, **not** a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked. Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   - working subsequent to a correct answer
   - legitimate variations in numerical answers
   - correct working in the "wrong" part of a question
   - omission of units
   - bad form

9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referal to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13 Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.

14 Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pd mark.

15 **Do not write any comments on the scripts**. A **revised** summary of acceptable notation is given on page 4.

16 Working that has been crossed out by the candidate cannot receive any credit. If you feel that a candidate has been disadvantaged by this action, make a P.A. Referral.

17 Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

**Summary**

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1 **Tick** correct working.
2 Put a mark in the **outer right-hand margin to match the marks allocations on the question paper.**
3 Do **no**t write marks as fractions.
4 Put each mark **at the end** of the candidate's response to the question.
5 **Follow through** errors to see if candidates can score marks subsequent to the error.
6 Do **not** write any comments on the scripts.

## Higher Mathematics : A Guide to Standard Signs and Abbreviations

**Remember - No comments on the scripts. Please use the following and nothing else.**

### Signs

✓ The tick. You are not expected to tick every line but of course you must check through the whole of a response.

✗ The cross and underline. Underline an error and place a cross at the end of the line.

The tick-cross. Use this to show correct work where you are **following through** subsequent to an error.

$\wedge$ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

$\sim$ The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

Bullets showing where marks are being allotted may be shown on scripts

margins

$\frac{dy}{dx} = 4x - 7$ ✓ •

$4x - 7 = \underline{0}$ ✗

$x = \frac{7}{4}$

$y = 3\frac{7}{8}$ • 2

$C = \underline{(1,-1)}$ ✗

$m = \dfrac{3-(-1)}{4-1}$

$m_{rad} = \frac{4}{3}$ •

$m_{tgt} = \dfrac{-1}{\frac{4}{3}}$

$m_{tgt} = -\frac{3}{4}$ •

$y - 3 = -\frac{3}{4}\left(x - 2\right)$ • 3

$x^2 - 3x = 28$ ✓ •

$x = 7$ $\wedge$ 1

$\sin(x) = 0.75 = inv\sin(0.75) = 48.6°$ 1

✓ •

**Remember - No comments on the scripts. No abreviations. No new signs.**
**Please use the above and nothing else.**

All of these are to help us be more consistent and **accurate**.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).

| 1 | 2 | | UNIT 1 |
|---|---|---|---|
| | | A1 | *determine range/domain* |
| | | A2 | *recognise general features of graphs:poly,exp,log* |
| | | A3 | *sketch and annotate related functions* |
| | | A4 | *obtain a formula for composite function* |
| | | A5 | *complete the square* |
| | | A6 | *interpret equations and expressions* |
| | | A7 | *determine function(poly,exp,log) from graph & vv* |
| | | A8 | *sketch/annotate graph given critical features* |
| | | A9 | *interpret loci such as st.lines,para,poly,circle* |
| | | A10 | *use the notation* $u_n$ *for the nth term* |
| | | A11 | *evaluate successive terms of a RR* |
| | | A12 | *decide when RR has limit/interpret limit* |
| | | A13 | *evaluate limit* |
| | | A14 | *apply A10-A14 to problems* |
| | | G1 | *use the distance formula* |
| | | G2 | *find gradient from 2 pts,/angle/equ. of line* |
| | | G3 | *find equation of a line* |
| | | G4 | *interpret all equations of a line* |
| | | G5 | *use property of perpendicular lines* |
| | | G6 | *calculate mid-point* |
| | | G7 | *find equation of median, altitude,perp. bisector* |
| | | G8 | *apply G1-G7 to problems eg intersect.,concur.,collin.* |
| | | C1 | *differentiate sums, differences* |
| | | C2 | *differentiate negative & fractional powers* |
| | | C3 | *express in differentiable form and differentiate* |
| | | C4 | *find gradient at point on curve & vv* |
| | | C5 | *find equation of tangent to a polynomial/trig curve* |
| | | C6 | *find rate of change* |
| | | C7 | *find when curve strictly increasing etc* |
| | | C8 | *find stationary points/values* |
| | | C9 | *determinenature of stationary points* |
| | | C10 | *sketch curvegiven the equation* |
| | | C11 | *apply C1-C10 to problems eg optimise, greatest/least* |
| | | T1 | *use gen. features of graphs of* $f(x)=k\sin(ax+b)$, |
| | | | $f(x)=k\cos(ax+b)$; *identify period/amplitude* |
| | | T2 | *use radians inc conversion from degrees & vv* |
| | | T3 | *know and use exact values* |
| | | T4 | *recognise form of trig. function from graph* |
| | | T5 | *interpret trig. equations and expressions* |
| | | T6 | *apply T1-T5 to problems* |

| 1 | 2 | | UNIT 2 |
|---|---|---|---|
| | | A15 | *use the general equation of a parabola* |
| | | A16 | *solve a quadratic inequality* |
| | | A17 | *find nature of roots of a quadratic* |
| | | A18 | *given nature of roots, find a condition on coeffs* |
| | | A19 | *form an equation with given roots* |
| | | A20 | *apply A15-A19 to solve problems* |
| | | A21 | *use Rem Th. For values, factors, roots* |
| | | A22 | *solve cubic and quartic equations* |
| | | A23 | *find intersection of line and polynomial* |
| | | A24 | *find if line is tangent to polynomial* |
| | | A25 | *find intersection of two polynomials* |
| | | A26 | *confiirm and improve on approx roots* |
| | | A27 | *apply A21-A26 to problems* |
| | | G9 | *find C/R of a circle from its equation/other data* |
| | | G10 | *find the equation of a circle* |
| | | G11 | *find equation of a tangent to a circle* |
| | | G12 | *find intersection of line & circle* |
| | | G13 | *find if/when line is tangent to circle* |
| | | G14 | *find if two circles touch* |
| | | G15 | *apply G9-G14 to problems* |
| | | C12 | *find integrals of* $px^n$ *and sums/diffs* |
| | | C13 | *integrate with negative & fractional powers* |
| | | C14 | *express in integrable form and integrate* |
| | | C15 | *evaluate definite integrals* |
| | | C16 | *find area between curve and x-axis* |
| | | C17 | *find area between two curves* |
| | | C18 | *solve differential equations(variables separable)* |
| | | C19 | *apply C12-C18 to problems* |
| | | T7 | *solve linear & quadratic equations in radians* |
| | | T8 | *apply compound and double angle (c & da) formulae* |
| | | | *in numerical & literal cases* |
| | | T9 | *apply c & da formulae in geometrical cases* |
| | | T10 | *use c & da formulaewhen solving equations* |
| | | T11 | *apply T7-T10 to problems* |

| 1 | 2 | | UNIT 3 |
|---|---|---|---|
| | | A28 | *use the laws of logs to simplify/find equiv. expression* |
| | | A29 | *sketch associated graphs* |
| | | A30 | *solve equs of the form* $A = Be^{kt}$ *for A,B,k or t* |
| | | A31 | *solve equs of the form* $\log_b(a) = c$ *for a,b or c* |
| | | A32 | *solve equations involving logarithms* |
| | | A33 | *use relationships of the form* $y = ax^n$ *or* $y = ab^x$ |
| | | A34 | *apply A28-A33 to problems* |
| | | G16 | *calculate the length of a vector* |
| | | G17 | *calculate the 3rd given two from A,B and vector AB* |
| | | G18 | *use unit vectors* |
| | | G19 | *use: if* $\boldsymbol{u}$, $\boldsymbol{v}$ *are parallel then* $\boldsymbol{v} = k\boldsymbol{u}$ |
| | | G20 | *add, subtract, find scalar mult. of vectors* |
| | | G21 | *simplify vector pathways* |
| | | G22 | *interpret 2D sketches of 3D situations* |
| | | G23 | *find if 3 points in space are collinear* |
| | | G24 | *find ratio which one point divides two others* |
| | | G25 | *given a ratio, find/interpret 3rd point/vector* |
| | | G26 | *calculate the scalar product* |
| | | G27 | *use: if* $\boldsymbol{u}$, $\boldsymbol{v}$ *are perpendicular then* $\boldsymbol{v.u=0}$ |
| | | G28 | *calculate the angle between two vectors* |
| | | G29 | *use the distributive law* |
| | | G30 | *apply G16-G29 to problems eg geometry probs.* |
| | | C20 | *differentiate* $p\sin(ax+b)$, $p\cos(ax+b)$ |
| | | C21 | *differentiate using the chain rule* |
| | | C22 | *integrate* $(ax + b)^n$ |
| | | C23 | *integrate psin(ax+b), pcos(ax+b)* |
| | | C24 | *apply C20-C23 to problems* |
| | | T12 | *solve sim.equs of form* $k\cos(a)=p$, $k\sin(a)=q$ |
| | | T13 | *express* $p\cos(x)+q\sin(x)$ *in form* $k\cos(x\pm a)$ *etc* |
| | | T14 | *find max/min/zeros of* $p\cos(x)+q\sin(x)$ |
| | | T15 | *sketch graph of* $y=p\cos(x)+q\sin(x)$ |
| | | T16 | *solve equ of the form* $y=p\cos(rx)+q\sin(rx)$ |
| | | T17 | *apply T12-T16 to problems* |

# 2.01

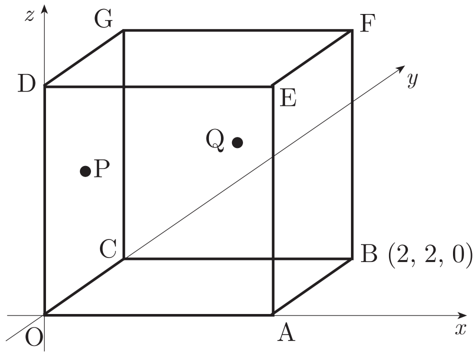
| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.01 | a | 1 | G21, G28 | CN | 7044 | | | 1 | 1 | | |
| | b | 2 | | CN | | | | 2 | 2 | | |
| | c | 5 | | CN | | 1 | 4 | | 5 | | |

OABCDEFG is a cube with side 2 units, as shown in the diagram.
B has coordinates (2, 2, 0).
P is the centre of face OCGD and Q is the centre of face CBFG.

(*a*) Write down the coordinates of G. **1**

(*b*) Find $\boldsymbol{p}$ and $\boldsymbol{q}$, the position vectors of points P and Q. **2**

(*c*) Find the size of angle POQ. **5**




The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- $\bullet^1$ ic interpret 2-D sketch of 3-D situation
- $\bullet^2$ ic interpret coordinates to vector
- $\bullet^3$ ic interpret coordinates to vector
- $\bullet^4$ ss knows to use scalar product
- $\bullet^5$ pd process length
- $\bullet^6$ pd process length
- $\bullet^7$ pd process process scalar product
- $\bullet^8$ pd process angle

**Primary Method : Give 1 mark for each •**

$\bullet^1$ $G = (0,2,2)$

$\bullet^2$ $\boldsymbol{p} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$\bullet^3$ $\boldsymbol{q} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$\boldsymbol{p}$ and $\boldsymbol{q}$ must be stated explicitly as a column (or row) vector

$\bullet^4$ $\cos \text{P}\hat{\text{O}}\text{Q} = \dfrac{\boldsymbol{p}.\boldsymbol{q}}{|\boldsymbol{p}||\boldsymbol{q}|}$ stated or implied (s/i) by $\bullet^8$

$\bullet^5$ $|\boldsymbol{p}| = \sqrt{2}$

$\bullet^6$ $|\boldsymbol{q}| = \sqrt{6}$

$\bullet^7$ $\boldsymbol{p}.\boldsymbol{q} = 3$

$\bullet^8$ $\text{P}\hat{\text{O}}\text{Q} = 30°$

[*radians* : $\frac{\pi}{6}$ (0.524); *gradians* : 33.3 ]

***Notes 1***

1. Treat coordinates written as column vectors as bad form
2. In (b), if $\boldsymbol{p}$ is wrong, this ***may*** be a follow through from (a) which has wrong coordinates for G.
3. For candidates who do not attempt $\bullet^8$, the formula quoted at $\bullet^4$ must relate to the labelling in the question for $\bullet^4$ to be awarded.
4. In (c) for $\bullet^8$ accept answers which round to 30°(2 s.f.)
5. In (c) $\bullet^4$ is not available for candidates who choose to calculate an incorrect angle (e.g. angle OPQ).

***Alternative Method for*** $\bullet^4$ ***to*** $\bullet^8$

$\bullet^4$ $\cos \text{P}\hat{\text{O}}\text{Q} = \dfrac{\text{OP}^2 + \text{OQ}^2 - \text{PQ}^2}{2 \times \text{OP} \times \text{OQ}}$ stated or implied (s/i) by $\bullet^8$

$\bullet^5$ $\text{OP} = \sqrt{2}$

$\bullet^6$ $\text{OQ} = \sqrt{6}$

$\bullet^7$ $\text{PQ} = \sqrt{2}$

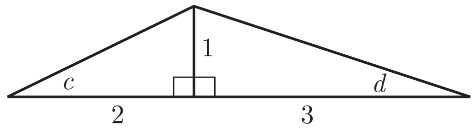
$\bullet^8$ $\text{P}\hat{\text{O}}\text{Q} = 30°$

[*radians* : $\frac{\pi}{6}$ (0.524); *gradians* : 33.3 ]

# 2.02

| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.02 | a | 4 | T9 | CN | 7098 | 1 | 1 | 2 | 4 | | |
| | b | 4 | | | | 2 | 1 | 1 | 4 | | |

The diagram shows two right-angled triangles with angles $c$ and $d$ marked as shown.




$(a)$ Find the exact value of $\sin(c+d)$. **4**

$(b)$ $(i)$ Find the exact value of $\sin 2c$

$(ii)$ Show that $\cos 2d$ has the same exact value. **4**

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

| | | |
|---|---|---|
| $\bullet^1$ | ic | interpret the diagram |
| $\bullet^2$ | ss | expand |
| $\bullet^3$ | ic | substitute |
| $\bullet^4$ | pd | simplify |
| $\bullet^5$ | ss | use double angle formula |
| $\bullet^6$ | pd | process |
| $\bullet^7$ | ss | use double angle formula |
| $\bullet^8$ | ic | complete proof of equality |

**Primary Method : Give 1 mark for each •**

| | | |
|---|---|---|
| $\bullet^1$ | $\sqrt{5}$ *and* $\sqrt{10}$ | s/i by $\bullet^3$ |
| $\bullet^2$ | $\sin(c)\cos(d)+\cos(c)\sin(d)$ | s/i by $\bullet^3$ |
| $\bullet^3$ | $\frac{1}{\sqrt{5}}\times\frac{3}{\sqrt{10}}+\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{10}}$ | |
| $\bullet^4$ | $\frac{1}{\sqrt{2}}$ (accept any equivalent single fraction) | |
| $\bullet^5$ | $2\sin(c)\cos(c)$ | |
| $\bullet^6$ | $2\times\frac{1}{\sqrt{5}}\times\frac{2}{\sqrt{5}}=\frac{4}{5}$ | *or equivalent* |
| $\bullet^7$ | *e.g.* $\cos^2(d)-\sin^2(d)$ | |
| $\bullet^8$ | $\frac{9}{10}-\frac{1}{10}=\frac{8}{10}=\frac{4}{5}$ | |

***Notes 1***

1 Any attempt to use $\sin(c+d)=\sin c+\sin d$ loses $\bullet^2$, $\bullet^3$ and $\bullet^4$

2 At $\bullet^3$ treat $\sin\left(\frac{1}{\sqrt{5}}\right)\cos\left(\frac{3}{\sqrt{10}}\right)+\cos\left(\frac{2}{\sqrt{5}}\right)\sin\left(\frac{1}{\sqrt{10}}\right)$ as bad form if the trig functions disappear to give the answer

3 At the $\bullet^3$ stage do not penalise the use of fractions which are greater than 1

4 Neither $\bullet^4$ nor $\bullet^6$ are available for answers $>1$

5 Any work based on $\sin 2c = 2\sin c$ loses $\bullet^5$ and $\bullet^6$

6 Any work based on $\cos 2d = 2\cos d$ loses $\bullet^7$ and $\bullet^8$

7 In (b) candidates may calculate $\sin 2c$ and $\cos 2d$ in any order. If either $\sin 2c$ or $\cos 2d$ is correct that may be awarded 2 of the 4 marks available

8 Any working based on numerical values for c and d (eg 27° and 18°) earns no credit but $\bullet^1$, $\bullet^2$, $\bullet^5$ and $\bullet^7$ are still available.

9 $\bullet^8$ is only available if the answer to (b)(ii) is shown to be equivalent to the answer to (b)(i)

10 If $\sqrt{5}$ and $\sqrt{10}$ are approximated to decimal values then $\bullet^4$,$\bullet^6$ and $\bullet^8$ are not available.

***Common Errors***

1 $\sin 2c = 2\sin d\cos d$

$\sin 2c = 2\frac{1}{\sqrt{10}}\frac{3}{\sqrt{10}}$ award 1 mark from $\bullet^5$ and $\bullet^6$

2 $\cos 2d = \cos^2 c - \sin^2 c$

$\cos 2d = \frac{2}{\sqrt{5}}\frac{2}{\sqrt{5}} - \frac{1}{\sqrt{5}}\frac{1}{\sqrt{5}}$ award 1 mark from $\bullet^7$ and $\bullet^8$

# 2.03

| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.03 | | 6 | G13 | CN | | 1 | 1 | 4 | 6 | | |

Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle. **6**

The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- $\bullet^1$ ss substitute
- $\bullet^2$ pd expand brackets
- $\bullet^3$ ic express in standard form
- $\bullet^4$ ic factorise
- $\bullet^5$ ic complete proof
- $\bullet^6$ ic state coordinates

**Primary Method : Give 1 mark for each •**

- $\bullet^1$ $x^2 + (6-2x)^2 + 6x - 4(6-2x) - 7 = 0$
- $\bullet^2$ $.....36 - 24x + 4x^2 .... - 24 + 8x....$
- $\bullet^3$ $5x^2 - 10x + 5 = 0$
- $\bullet^4$ $(x-1)^2 = 0$
- $\bullet^5$ equal roots $\Rightarrow$ line is tangent
- $\bullet^6$ $x = 1, \;\; y = 4$

*alternatives for* $\bullet^4$ *and* $\bullet^5$

- $\bullet^4$ $b^2 - 4ac = 0 \Rightarrow$ tangent
- $\bullet^5$ $(-10)^2 - 4 \times 5 \times 5 = 0$

- $\bullet^4$ use quad. formula to get roots
- $\bullet^5$ equal roots $\Rightarrow$ line is tangent

**Alternative Method : Give 1 mark for each •**

- $\bullet^1$ $m_{line} = -2$
- $\bullet^2$ $(-3,2)$ *and* $\frac{1}{2}$
- $\bullet^3$ *equ. of radius* $: y - 2 = \frac{1}{2}(x+3)$
- $\bullet^4$ $x = 1$
- $\bullet^5$ $y = 4$
- $\bullet^6$ check that (1,4) lies on the circle

***Notes 1***

1. An " $= 0$ " must appear somewhere in the working between $\bullet^1$ and $\bullet^4$ stage. Failure to appear will lose one of these marks
2. For candidates who obtain 2 roots: $\bullet^5$ is still available for "not equal roots so NO tangent" but $\bullet^6$ is not available

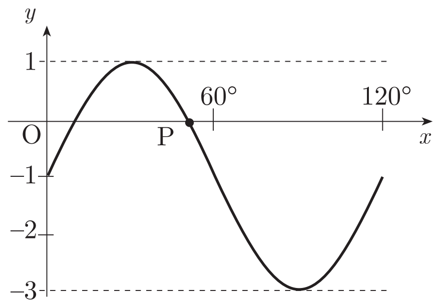
# 2.04

| qu | part | mk | code | calc | source | ss | | ic | C | B | A |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.04 | a | 3 | T4, T7 | CN | 7102 | | | 3 | 3 | | |
| | b | 3 | | CN | | 1 | 2 | | 3 | | |

The diagram shows part of the graph of a function whose equation is of the form $y = a\sin(bx°) + c$.

(*a*) Write down the values of $a$, $b$ and $c$. **3**

(*b*) Determine the exact value of the $x$-coordinate of P, the point where the graph intersects the $x$-axis as shown in the diagram. **3**




The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$ ic interpret vertical scaling
$\bullet^2$ ic interpret period
$\bullet^3$ ic interpret vertical translation
$\bullet^4$ ss set to zero
$\bullet^5$ pd process exact value
$\bullet^6$ ic interpret diagram

solution via a graphics calculator

$\bullet^4$ ss sketch and annotate
$\bullet^5$ ic interpret scale
$\bullet^6$ ic check exact value

**Primary Method : Give 1 mark for each •**

$\bullet^1$ $a = 2$
$\bullet^2$ $b = 3$
$\bullet^3$ $c = -1$
$\bullet^4$ $2\sin(3x°) - 1 = 0$
$\bullet^5$ one answer from 10° or 50°
$\bullet^6$ $x_P = 50°$

*alternative for* $\bullet^4, \bullet^5$ *and* $\bullet^6$

$\bullet^4$ sketch of graph with pointer to sol.point
$\bullet^5$ extraction of 50°
$\bullet^6$ confirmation of $2\sin(3\times 50°) - 1$ does $= 0$

***Notes 1***

1 $\bullet^4$ may be awarded for $a\sin(bx) + c = 0$
2 For $\bullet^2$ accept " $b = 3x$ " as bad form
3 $\bullet^6$ may only be awarded for a value of $x$ such that $30 < x < 60$
4 $\bullet^6$ may be awarded for (50°,0) but NOT for (0,50°)

# 2.05

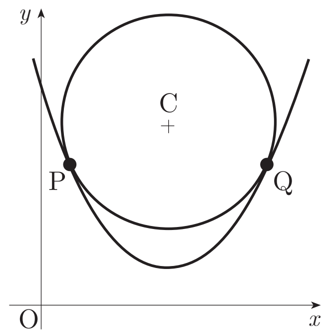
| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.05 | a | 5 | C5,G10,G11 | CN | 7017 | 2 | 2 | 1 | 5 | | |
| | b | 2 | | | | 1 | | 1 | | 2 | |
| | c | 2 | | | | | | 2 | | 2 | |

A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q.

(*a*) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q. **5**

(*b*) Find the coordinates of P. **2**

(*c*) Find the coordinates of C, the centre of the circle. **2**




The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

| | | |
|---|---|---|
| $\bullet^1$ | ss | know to differentiate |
| $\bullet^2$ | pd | process |
| $\bullet^3$ | ss | equate gradients |
| $\bullet^4$ | pd | process |
| $\bullet^5$ | ic | interpret $y$-coordinate |
| $\bullet^6$ | ss | use symmetry of diagram |
| $\bullet^7$ | ic | interpret coordinates |
| $\bullet^8$ | ic | interpret centre |
| $\bullet^9$ | ic | interpret centre |

**Primary Method : Give 1 mark for each •**

$\bullet^1$ $\frac{dy}{dx} = ...$(1 term correct)
$\bullet^2$ $x - 8$
$\bullet^3$ $x - 8 = 4$
$\bullet^4$ $x = 12$
$\bullet^5$ $y = 10$
$\bullet^6$ $m_P = -4$
$\bullet^7$ $\text{P} = (4{,}10)$
$\bullet^8$ $x_C = 8$
$\bullet^9$ $y_C = 11$

***Notes 1***

1 Treat $y = x - 8$ as bad form provided $y$ is replaced by 4 at $\bullet^3$

2 *Cave*
Look out for the following:
$\bullet^5$ is not available to candidates who substitute the gradient of 4 into the equation in order to find the value of $y_Q$

3 Alt. strategies for $\bullet^6$
(a) substitute $y = 10$ into the parabola
(b) use the t.p. as a step to P

4 *Cave*
There are other legitimate methods for finding the coordinates of Q

5 Candidates who solve the tangents at P and Q AND then state that $x_C = 8$ may be awarded $\bullet^8$.

***Alternative Method for (c)***

Solving the normals

i.e. $y - 10 = -\frac{1}{4}(x - 12)$
$y - 10 = \frac{1}{4}(x - 4)$

may be used. Marks are awarded as normal:
$x = 8$ ($\bullet^8$) and $y = 11$ ($\bullet^9$)

***Common Errors***

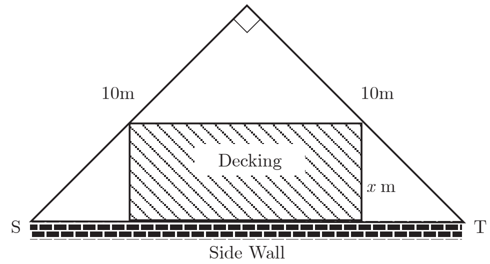
1 $\frac{dy}{dx} = x - 8$ $\quad\surd\,\bullet^1, \surd\,\bullet^2$
$x - 8 = 0 \Rightarrow x = 8, y = 2$ $\quad\surd\,\bullet^5$

2 For the occasional candidate who starts with $x - 8 = 4$
award $\bullet^1, \bullet^2$ and $\bullet^3$

# 2.06

| qu | part | mk | code | calc | source | ss | | ic | C | B | A |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.06 | a | 3 | C11 | CN | 7062 | | 1 | 2 | | | 3 |
| | b | 5 | | CN | | 1 | 3 | 1 | 1 | 4 | |

A householder has a garden in the shape of a right-angled isosceles triangle.
It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.




(a) (i) Find the exact value of ST.

(ii) Given that the breadth of the decking is $x$ metres, show that the area of the decking, $A$ square metres, is given by

$$A = \left(10\sqrt{2}\right)x - 2x^2$$ **3**

(b) Find the dimensions of the decking which maximises its area. **5**

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- $\bullet^1$ pd calculate ST
- $\bullet^2$ ic interpret the triangle
- $\bullet^3$ ic complete proof
- $\bullet^4$ ss set derivative zero
- $\bullet^5$ pd differentiate
- $\bullet^6$ pd solve for breadth
- $\bullet^7$ ic justify s.p.s with e.g. nature table
- $\bullet^8$ pd find corresponding length

**Primary Method : Give 1 mark for each •**

- $\bullet^1$ $ST = \sqrt{200}$
- $\bullet^2$ $length = \sqrt{200} - 2x$ s/i by their method
- $\bullet^3$ $\left(\sqrt{200} - 2x\right) \times x$ *and complete proof*
- $\bullet^4$ $\frac{dA}{dx} = 0$
- $\bullet^5$ $\frac{dA}{dx} = 10\sqrt{2} - 4x$
- $\bullet^6$ $x = \frac{10\sqrt{2}}{4}$ or equivalent (3.5)
- $\bullet^7$ *justification* : e.g. nature table
- $\bullet^8$ $length = 5\sqrt{2}$ (7.1)

***Notes 1***

In (b)

1 An " = 0 " must appear somewhere in the working between $\bullet^4$ and $\bullet^6$

2 For $\bullet^7$ accept $\frac{d^2A}{dx^2} = -4 < 0$ at $x = \frac{10\sqrt{2}}{4} \Rightarrow$ maximum

Minimum requirement of a nature table

| | ... | 3.5 | ... |
|---|---|---|---|
| $f'(x)$ | $+$ | 0 | $-$ |

hence maximum

***better*** would be

| $x$ | $\rightarrow$ | $\frac{5\sqrt{2}}{2}$ | $\rightarrow$ |
|---|---|---|---|
| $f'(x)$ | $+$ | 0 | $-$ |
| $f(x)$ | $\nearrow$ | $\cdots$ | $\searrow$ |

hence maximum at $x = \frac{5\sqrt{2}}{2}$

# 2.07

| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.07 | | 4 | C23, T3 | CR | 7046 | | 3 | 1 | | 3 | 1 |

Find the value of $\int_0^2 \sin(4x+1)\,dx$. **4**

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide
but only where a candidate does not use the primary method or any
alternative method shown in detail in the marking scheme.

- $\bullet^1$ pd integrate the trig function
- $\bullet^2$ pd deal with the "4"
- $\bullet^3$ ic substitute the limits
- $\bullet^4$ pd evaluate

**Primary Method : Give 1 mark for each •**

- $\bullet^1$ $-\cos(4x+1)$
- $\bullet^2$ $\times\dfrac{1}{4}$
- $\bullet^3$ $-\dfrac{1}{4}\cos(4\times 2+1)-\left(-\dfrac{1}{4}\cos(4\times 0+1)\right)$
- $\bullet^4$ 0.36

***Notes 1***

1. $\bullet^2$ is only available if it follows on from $\pm\sin(4x+1)$ *or* $\pm\cos(4x+1)$
2. $\bullet^3$ is available for substituting the limits correctly into any trig. function except the original one
3. $\bullet^4$ is available for using any trig. function except the original one
4. If candidates leave the calculator in degree mode obtaining 0.000304 then $\bullet^4$ is NOT awarded

***Alternative Method***

$\sin 4x\cos 1+\cos 4x\sin 1$

- $\bullet^1$ $-\frac{1}{4}\cos 4x\cos 1$
- $\bullet^2$ $\frac{1}{4}\sin 4x\sin 1$
- $\bullet^3$ $\left(-\frac{1}{4}\cos 8\cos 1+\frac{1}{4}\sin 8\sin 1\right)-\left(-\frac{1}{4}\cos 0\cos 1+\frac{1}{4}\sin 0\sin 1\right)$
- $\bullet^4$ 0.36

# 2.08

| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.08 | | 4 | A31 | CR | 7049 | 2 | 1 | 1 | | 4 | |

The curve with equation $y = \log_3(x-1) - 2.2$, where $x > 1$, cuts the $x$-axis at the point $(a, 0)$.
Find the value of $a$. **4**

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

| | | |
|---|---|---|
| $\bullet^1$ | ic | substitute |
| $\bullet^2$ | ss | isolate the log term |
| $\bullet^3$ | ss | convert to exponential form |
| $\bullet^4$ | pd | process |

**Primary Method : Give 1 mark for each •**

$\bullet^1$ $\log_3(a-1) - 2.2 = 0$ s/i by $\bullet^2$
$\bullet^2$ $\log_3(a-1) = 2.2$
$\bullet^3$ $a - 1 = 3^{2.2}$
$\bullet^4$ $a = 12.2$

***Alt.method 1***

$\bullet^1$ $\log_3(a-1) - 2.2 = 0$ s/i by $\bullet^2$
$\bullet^2$ $\log_3(a-1) = 2.2$
$\bullet^3$ $\log_3(a-1) = \log_3(11.21)$
$\bullet^4$ $a = 12.2$

***Alt.method 2***

$\bullet^1$ $\log_3(a-1) - 2.2 = 0$ s/i by $\bullet^2$
$\log_3(a-1) - 2.2\log_3 3 = 0$
$\bullet^2$ $\log_3(a-1) - \log_3(11.21) = 0$
$\bullet^3$ $\log_3 \frac{(a-1)}{11.21} = 0$
$\bullet^4$ $a = 12.2$

***Notes 1***

1 Solutions given in terms of $x$ rather than $a$ should be treated as bad form.

***Common Error 1***

$\bullet^1$ $\surd$ $\log_3(a-1) - 2.2 = 0$
$\bullet^2$ $\surd$ $\log_3(a-1) = 2.2$
$\bullet^3$ $X$ $\log_3(a-1) = \log_3 2.2$
$\bullet^4$ $X$ $a - 1 = 2.2 \Rightarrow a = 3.2$ $[eased]$

***Common Error 2***

$\bullet^1$ $\surd$ $\log_3(a-1) - 2.2 = 0$
$\bullet^2$ $\surd$ $\log_3(a-1) = 2.2$
$\bullet^3$ $X$ $\log_3 a - \log_3 1 = 2.2$
$\log_3 a = 2.2$
$\bullet^4$ $X\surd$ $a = 3^{2.2} = 11.2$

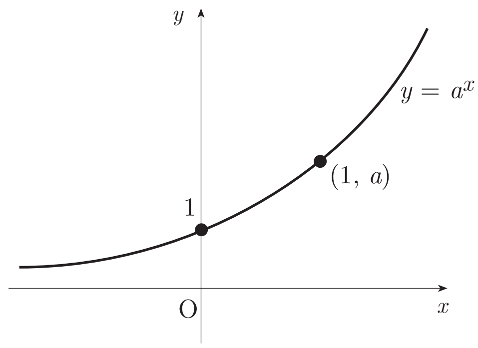
# 2.09

| qu | part | mk | code | calc | source | ss | | ic | C | B | A | | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.09 | a | 2 | A3 | CN | 7071 | | | 2 | | 2 | | | 2 | | |
| | b | 2 | | CN | | | | 2 | | | 2 | | 2 | | |

The diagram shows the graph of $y = a^x$, $a > 1$.
On separate diagrams sketch the graphs of:

(*a*) $y = a^{-x}$ **2**

(*b*) $y = a^{1-x}$ **2**

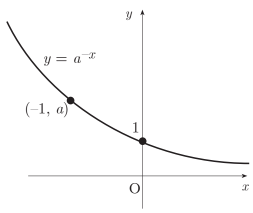


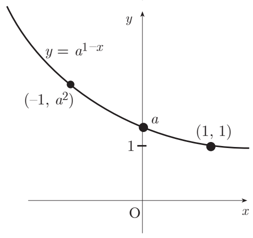

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- $\bullet^1$ ic determine the requ. transformation
- $\bullet^2$ ic state coordinates of pt. on graph
- $\bullet^3$ ic determine the requ. transformation
- $\bullet^4$ ic state coordinates of pt. on graph

**Primary Method : Give 1 mark for each •**

- $\bullet^1$ reflecting in *y-axis* and passing thr' e.g. (0,1)
- $\bullet^2$ passing thr' 1 more point e.g. $(-1, a)$ *or* $\left(1, \dfrac{1}{a}\right)$
- $\bullet^3$ vertical scaling of "$a$" and passing thr' e.g. $(0, a)$
- $\bullet^4$ passing thr' 1 more point e.g. $(-1, a^2)$ *or* (1,1)



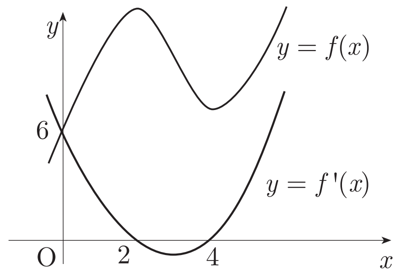

***Notes 1***

1. For $\bullet^1$ and $\bullet^3$ the shape must be an exponential decay graph lying above the $x$-axis
2. There are no follow-through marks available to candidates who use an incorrect graph from (a) as a basis for their answer to (b).

# 2.10

| qu | part | mk | code | calc | source | ss | | ic | C | B | A |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.10 | a | 3 | C18, C19 | CN | 7028 | 1 | 1 | 1 | 1 | 2 | |
| | b | 4 | | CN | | 1 | 1 | 2 | | | 4 |

The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.
Both graphs pass through the point (0,6).
The graph of $y = f'(x)$ also passes through the points (2,0) and (4,0).




$(a)$ Given that $f'(x)$ is of the form $k(x-a)(x-b)$

$(i)$ Write down the values of $a$ and $b$.

$(ii)$ Find the value of $k$. **3**

$(b)$ Find the equation of the graph of the cubic function $y = f(x)$. **4**

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- $\bullet^1$ ic interpret roots on diagram
- $\bullet^2$ ss know to use $y$-intercept
- $\bullet^3$ pd process
- $\bullet^4$ ss know to integrate
- $\bullet^5$ pd integrate
- $\bullet^6$ ic express as an equation
- $\bullet^7$ ic interpret constant of integration

**Primary Method : Give 1 mark for each •**

- $\bullet^1$ $a = 2$ *and* $b = 4$ ***or*** $k(x-2)(x-4)$
- $\bullet^2$ $6 = k(0-2)(0-4)$
- $\bullet^3$ $k = \dfrac{3}{4}$
- $\bullet^4$ $\int \left(\frac{3}{4}(x-2)(x-4)\right) dx$ s/i by $\bullet^5$
- $\bullet^5$ *any two terms integrated correctly* ( $\frac{3}{12}x^3$ *etc*)
- $\bullet^6$ $y = \frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x + c$
- $\bullet^7$ $c = 6$

***Notes 1***

1 For candidates who fail to complete (a) but produce values for $k, a$ and $b$ *ex nihilo*, all 4 marks are available in (b).
A deduction of 1 mark may be made if their choice eases the working.

2 In (b)
For candidates who use $k = 1$, a "fully correct" follow-through solution may be awarded 3 out of the last 4 marks

3 For candidates who retain "$k$", "$a$" and "$b$", $\bullet^4, \bullet^5, \bullet^6$ and $\bullet^7$ are still available.

# 2.11

| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.11 | a | 1 | A33 | CR | 7014 | | | 1 | | 1 | | | | 1 |
| | b | 1 | | | | | | 1 | 1 | | | | | 1 |
| | c | 4 | | | | 1 | | 3 | | | 4 | | | 4 |

Two variables $x$ and $y$ satisfy the equation $y = 3 \times 4^x$.

$(a)$ Find the value of $a$ if $(a, 6)$ lies on the graph with equation $y = 3 \times 4^x$ . **1**

$(b)$ If $(-\frac{1}{2}, b)$ also lies on the graph, find $b$. **1**

$(c)$ A graph is drawn of $\log_{10} y$ against $x$. Show that its equation will be of the form $\log_{10} y = Px + Q$ and state the gradient of this line. **4**

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$ ic interprets equation
$\bullet^2$ ic interprets equation
$\bullet^3$ ss introduces logs
$\bullet^4$ ic uses log law
$\bullet^5$ ic uses log law and completes
$\bullet^6$ ic interprets equation

**Primary Method : Give 1 mark for each •**

$\bullet^1$ $a = \frac{1}{2}$

$\bullet^2$ $b = \frac{3}{2}$

$\bullet^3$ $\log_{10}(y) = \log_{10}\left(3 \times 4^x\right)$

$\bullet^4$ $\log_{10}(y) = \log_{10}(3) + \log_{10}\left(4^x\right)$

$\bullet^5$ $\log_{10}(y) = x\log_{10}(4) + \log_{10}(3)$

$\bullet^6$ $gradient = \log_{10}(4)$ or equivalent

***Notes***

1 Do not penalise $x = \frac{1}{2}$, $y = \frac{3}{2}$

2 Candidates who start their "proof" with the wrong form (e.g. $y = Px^Q$) earn no credit in part (c).

***Alternative Method***

$\bullet^1$ $y = 10^{Px+Q}$

$\bullet^2$ $y = 10^Q \times (10^P)^x$

$\bullet^3$ $10^Q = 3$ *and* $10^P = 4$

$\bullet^4$ $P = \log_{10} 4$

***Cave***

*In (a) look out for the following :*

$$6 = 3 \times 4^a$$
$$2 = 4^a$$
$$\frac{2}{4} = a$$
$$a = \frac{1}{2}$$

This is not awarded $\bullet^1$