

# **2010 Mathematics**

# Higher

# **Finalised Marking Instructions**

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2010 Mathematics Higher Marking Instructions

Exam date: 21 May 2010

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# **Strictly Confidential**

These instructions are **strictly confidential** and, in common with the scripts you will view and mark, they must never form the subject of remark of any kind, except to Scottish Qualifications Authority staff.

# Marking

The utmost care must be taken when entering Item level marks into Appointees Online.

It is of particular importance that you enter a zero (0) when the candidate has attempted a question but has not gained a mark and a dash (-) when the candidate has not attempted a question.

#### Part One: General Marking Principles for Mathematics Higher

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the specific Marking Instructions for each question.

For each question the marking instructions are split into two sections, namely Generic Scheme and Illustrative Scheme. The Generic Scheme indicates what each mark is being awarded for. The Illustrative Scheme cover methods which you will commonly see throughout your marking. In general you should use the Illustrative Scheme for marking and revert to the Generic Scheme only where a candidate has used a method not covered in the Illustrative Scheme or you are unsure of whether you should award a mark or not.

All markers should apply the following general marking principles throughout their marking:

- 1. Marks for each candidate response must **always** be assigned in line with these general marking principles and the specific Marking Instructions for the relevant question. If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader. You can do this by e-mailing/phoning your team leader. Alternatively, you can refer the issue directly to your Team Leader by checking the 'Referral' box on the marking screen.
- **2.** Marking should always be positive, i.e. marks should be awarded for what is correct and not deducted for errors or omissions.
- **3.** Award one mark for each •. Each error should be underlined in **red** at the point where it first occurs, and not at any subsequent stage of the working.
- 4 The total mark for each question should be entered in **red** in the **outer** right hand margin, opposite the end of the working concerned. Only the mark, as a whole number, should be written; do not use fractions. The marks should correspond to those on the question paper and these marking instructions.
- 5 Where a candidate has scored zero for any question, or part of a question, 0 should be written in the right hand margin against their answer.
- 6 Working subsequent to an error must be followed through; with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction of mark(s) should be made.
- 7 There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. In general, as a consequence of one of these errors, candidates lose the opportunity of gaining the appropriate *ic* mark or *pd* mark.
- 8 As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking scheme, a correct answer with no working receives no credit.

- 9 Normally, do not penalise:
  - Working subsequent to a correct answer;
- Omission of units;
- Legitimate variations in numerical answers;
- Bad form;
- Correct working in the wrong part of a question;

unless specifically mentioned in the marking scheme.

- No piece of working should be ignored without careful checking even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme. Answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two; provided it satisfies the criteria for the marks.
- 11 If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.
- 12 No marks at this stage should be deducted for careless or badly arranged work.
- 13 It is of great importance that the utmost care should be exercised in adding up the marks. Using the Electronic Marks Capture (EMC) screen to tally marks for you is not recommended. A manual check of the total, using the grid issued with this marking scheme, can be confirmed by the EMC system.
- 14 In cases of difficulty, covered neither in detail nor in principle in these instructions, attention may be directed to the assessment of particular answers by making a referral to the Principal Assessor (P.A.) Please see the general Instructions for P.A. referrals.
- **15** If a candidate presents multiple solutions for a question <u>and</u> it is not clear which is intended as their final attempt, mark each one and award the lowest mark.

# **Marking Scripts**

No comments, words or acronyms should be written on scripts.

Please use the following and **nothing else**.

✓ Tick when a piece of working is correct and gains a mark. You are not expected to tick every line of working but you must check through the whole of a response.

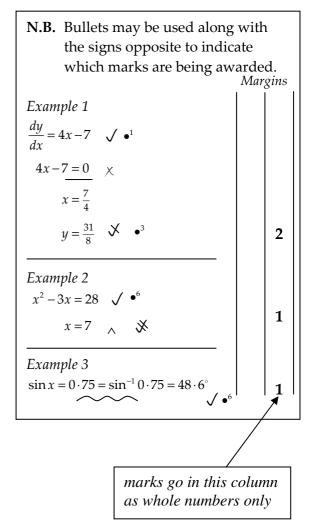
\_\_\_\_ X Underline each error and place a cross at the end of the line where it occurs.

A cross-tick should be used to indicate 'correct working' where a mark is awarded as a result of follow through from an error.

A double cross-tick should be used to indicate correct working which is inadequate to score any marks e.g. incorrect method which is mathematically correct or eased working.

A tilde should be used to indicate a minor transgression which is not being penalised, e.g. bad form.

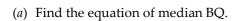
These are essential for later stages of the SQA procedures.



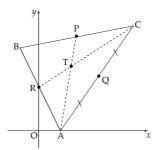
	<b>Question</b>	<u>Answer</u>
	1	A
	2	C
	3	D
	4	A
	5	В
	6	D
	7	C
	8	В
	9	C
	10	В
	11	D
	12	A
	13	В
	14	C
	15	C
	16	Α
	17	В
	18	В
	19	C
	20	A
<b>Summary</b>	A	5
<u>o ammun y</u>	В	6
	C	6
	D	3
	D	3

Triangle ABC has vertices A(4, 0), B(-4, 16) and C(18, 20), as shown in the diagram opposite.

> Medians AP and CR intersect at the point T(6, 12).



(b) Verify that T lies on BQ.



3

1

### **Generic Scheme**

#### **Illustrative Scheme**

# 21 (a)

- ss know and find midpoint of AC
- pd calculate gradient of BQ
- ic state equation

- $-\frac{6}{15}$  or equivalent  $y-16=-\frac{2}{5}(x-(-4))$  or  $y-10=-\frac{2}{5}(x-11)$

#### **Notes**

- 1. Candidates who do not use a midpoint lose  $\bullet^2$  and  $\bullet^3$ .
- 2. There is no need to simplify  $m_{BQ}$  for  $\bullet^2$ . It must, however, be simplified before  $\bullet^3$  can be awarded. Do not award  $\bullet^3$  for 6x+15y-216=0, although  $\bullet^3$  would be awarded for 6x+15y-216=0then 2x + 5y - 72 = 0.
- 3. If  $m_{\rm BO}$  cannot be simplified, due to an error, then  $\bullet^3$  is still available.
- is available for using y = mx + c where  $m = -\frac{2}{5}$  and  $c = \frac{72}{5}$ .
- 5. Accept y = -0.4x + 14.4.
- Candidates who find the equations of AP or CR can only gain 1 mark.

AP: 
$$y-0=6(x-4)$$
 or  $y-12=6(x-6)$ 

CR: 
$$y-20 = \frac{2}{3}(x-18)$$
 or  $y-12 = \frac{2}{3}(x-6)$ 

#### 21 (b)

- ic substitute in for T and complete
- e.g. Substitution: 2(6) + 5(12) = 12 + 60 = 72

Gradients:  $m_{BT} = -\frac{4}{10} = -\frac{2}{5} = m_{BQ}$ 

Vectors:  $\overrightarrow{BT} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$ ,  $\overrightarrow{TQ} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$  and  $\overrightarrow{BT} = 2\overrightarrow{TQ}$ 

#### Notes

- is available as follow through with an appropriate communication statement, e.g. 'T does not lie on BQ'.
- Statements such as 'PA, RC and BQ are all medians and therefore all share the same point T' do not gain 4.
- Since only 1 mark is available here, do not penalise the omission of any reference to a "common point" or "parallel".

#### Regularly occurring responses

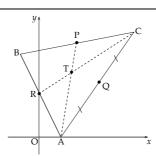
 $m_{\rm BT} = -\frac{4}{10} = -\frac{2}{5} = m_{\rm BQ}$  leading to 2:1 in (c), without further working, gains  $\bullet^4$  and Gradient approach: (b)

 $m_{\rm BQ} = -\frac{6}{15}$  and  $m_{\rm TQ} = -\frac{2}{5}$  leading to  $m_{\rm BQ} = 3m_{\rm TQ}$  so T lies on BQ leading to 2:1 in (c), but without further working, loses  $ullet^4$  and  $ullet^5$  but gains  $ullet^6$  .

21 Triangle ABC has vertices A (4, 0), B(-4, 16) and C(18, 20), as shown in the diagram opposite.

> Medians AP and CR intersect at the point T(6, 12).

(c) Find the ratio in which T divides BQ.



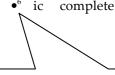
2

#### Generic Scheme

#### Illustrative Scheme

21 (c)

- ss valid method for finding the ratio
- ic complete to simplified ratio



For 2:1 **without** working, only  $\bullet$ <sup>6</sup> is awarded.

Be aware that the working may appear in (b). Some candidates obtain 2:1 from erroneous working thus losing  $\bullet^6$ .

Method 1: Vector approach

• 6 e.g. 
$$\overrightarrow{BT} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$$
 and  $\overrightarrow{TQ} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ 

Method 2: "Stepping out" approach

•5 e.g. 
$$\downarrow 10$$
  $\downarrow 5$  or  $-4$   $\downarrow 6$   $\downarrow 11$   $\downarrow 10$   $\downarrow 5$ 

 $\bullet^6$  2:1

Method 3: Distance Formula approach

• e.g. 
$$d_{BT} = \sqrt{116}$$
 and  $d_{TQ} = \sqrt{29}$ 

Notes

10. Any evidence of appropriate steps, e.g. 10 and 5 or 4 and 2 but not 2 and 1, can be awarded ● leading to  $\bullet^6$ 

e.g.  $\begin{array}{c|c} 2 & 1 \\ \hline B & T & O \end{array}$  is not sufficient on its own and so loses  $\bullet^5$  but gains  $\bullet^6$ .

- 11.  $\sqrt{116}$ :  $\sqrt{29}$  with no further simplification may be awarded  $\bullet^5$  but not  $\bullet^6$ .
- 12. In this question working for (c) may appear in (b), where the working appears for •<sup>4</sup>

#### Regularly occurring responses

Response 1

(b) 
$$\overrightarrow{BT} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$$
  $\overrightarrow{TQ} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$   $\overrightarrow{BT} = 2\overrightarrow{TQ}$ 

Response 2

 $\overrightarrow{BQ} = \begin{pmatrix} 15 \\ -6 \end{pmatrix} = -3\overrightarrow{QT} \checkmark \bullet^{4} \qquad (b) \qquad \overrightarrow{QT} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} \qquad \overrightarrow{TB} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ 

(b) 
$$\overrightarrow{QT} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$$
  $\overrightarrow{TB} = \begin{pmatrix} -10 \\ 4 \end{pmatrix}$ 

(c)  $2:1\sqrt{\bullet^6}$ 

3 marks out of 3

3 marks out of 3

3 marks out of 3

Response 4

(b) 
$$\overrightarrow{BT} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$$
  $\overrightarrow{TQ} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$  so  $2\overrightarrow{BT} = \overrightarrow{TQ} \times \bullet^4$  (c)  $2:1 \times \bullet^6$  but  $1:2$  would have gained  $\bullet^6$ 

(a) (i) Show that (x-1) is a factor of  $f(x) = 2x^3 + x^2 - 8x + 5$ . 22

(ii) Hence factorise f(x) fully.

5

1

(b) Solve  $2x^3 + x^2 - 8x + 5 = 0$ .

Illustrative Scheme

#### Generic Scheme

know to use x = 1

complete evaluation ic

state conclusion

pd find quadratic factor

factorise completely

Method 1: Using synthetic division

 $\bullet^3$  (x-1) is a factor see note 2

stated, or implied by •5  $e^4 2x^2 + 3x - 5$ 

(x-1)(x-1)(2x+5)stated explicitly

Method 2: Using substitution and inspection

• know to use x = 1

 $\bullet^2$  2+1-8+5=0

 $\bullet^3$  (x-1) is a factor see note 2

 $(x-1)(2x^2+3x-5)$ 

stated explicitly (x-1)(x-1)(2x+5)

#### Notes

22 (a)

- 1. Communication at  $\bullet^3$  must be consistent with working at  $\bullet^2$ . i.e. candidate's working must arrive legitimately at zero before  $ullet^3$  is awarded. If the remainder is not 0 then an appropriate statement would be (x-1) is not a factor.
- For  $\bullet^3$ , minimum acceptable statement is 'factor'. Unacceptable statements: x = 1 is a factor, (x + 1) is a factor, x = 1 is a root, (x - 1) is a root etc.
- 3. At  $\bullet^5$  the expression may be written as  $(x-1)^2(2x+5)$ .

22 (b)

 $\bullet^6$  ic state solutions •6 x = 1 and  $x = -\frac{5}{2}$  or -2.5 or  $-2\frac{1}{2}$ 

These may appear in the working at (a).

#### **Notes**

- 4. From (a) (x-1)(x-1)(2x+5) leading to x=1,  $x=-\frac{5}{2}$  then (1,0) and  $(-\frac{5}{2},0)$  gains  $\bullet^6$ . However, (x-1)(x-1)(2x+5) leading to (1, 0) and  $(-\frac{5}{2}, 0)$  only does not gain  $\bullet^6$ .
- 5. From (a) (x-1)(2x+5) only leading to x=1,  $x=-\frac{5}{2}$  does not gain  $\bullet$ <sup>6</sup> as equation solved is not a cubic, but (x-1)(x+1)(2x-5) leading to x=1, x=-1 and  $x=\frac{5}{2}$  gains  $\bullet^6$  as follow through from a cubic equation.

**Illustrative Scheme** 

22 (c) The line with equation y = 2x - 3 is a tangent to the curve with equation  $y = 2x^3 + x^2 - 6x + 2$  at the point G.

Find the coordinates of G.

(*d*) This tangent meets the curve again at the point H. Write down the coordinates of H.

5

1

#### Generic Scheme

# Method 1 : Equating curve and line

- ss set  $y_{\text{CURVE}} = y_{\text{LINE}}$
- •<sup>8</sup> ic express in standard form
- ss compare with (a) or factorise
- ic identify  $x_c$
- 11 pd evaluate  $y_G$

#### Method 2: Differentiation

- ss know to and differentiate curve
- set derivative to gradient of line
- 9 pd solve quadratic equation
- 10 ss process to identify  $x_G$
- ic complete to  $y_{\text{CURVE}} = y_{\text{LINE}}$

#### Method 1: Equating curve and line

- $7 2x^3 + x^2 6x + 2 = 2x 3$  stated explicitly
- $\bullet^8 \quad 2x^3 + x^2 8x + 5$
- $\bullet^9$  (x-1)(x-1)(2x+5) = 0 see note 6
- $\bullet^{10} \quad \mathbf{r} = 1$
- $\bullet^{11}$  y = -1

#### Method 2: Differentiation

- $6x^2 + 2x 6$
- $6x^2 + 2x 6 = 2$
- $x = -\frac{4}{3}$  and 1
- 10 at x = 1 evaluate  $y_{CURVE}$  and  $y_{LINE}$
- y = -1 from both curve and line

#### **Notes**

22 (c)

#### In method 1:

- 6. \* is only available if '= 0' appears at either the \* or \* stage.
- 7.  $\bullet^9$ ,  $\bullet^{10}$  and  $\bullet^{11}$  are only available via the working from  $\bullet^7$  and  $\bullet^8$ .
- 8. If (x-1)(x-1)(2x+5) does not appear at  $\bullet^9$  stage, it can be implied by  $\bullet^5$  and  $\bullet^{10}$ .
- 9. At  $\bullet^9$  a quadratic used from (a) may gain  $\bullet^9$ ,  $\bullet^{11}$  and  $\bullet^{12}$  but a quadratic from  $\bullet^8$  may gain  $\bullet^{11}$  and  $\bullet^{12}$  only.
- 10. If G and H are interchanged then  $\bullet^{10}$  is lost but  $\bullet^{11}$  and  $\bullet^{12}$  are still available.
- 11. Candidates who obtain three distinct factors at  $\bullet^9$  can gain  $\bullet^{11}$  for evaluating all y values, but lose  $\bullet^{10}$  and  $\bullet^{12}$
- 12. A repeated factor at  $\bullet^5$  or  $\bullet^9$  stage is required for  $\bullet^{10}$  to be awarded without justification.

#### In both methods:

13. All marks in (c) are available as a result of differentiating  $2x^3 + x^2 - 6x + 2$  and solving this equal to 2 (from method 2).

Only marks  $\bullet^7$  and  $\bullet^8$  (from method 1) are available to those candidates who choose to differentiate  $2x^3 + x^2 - 8x + 5$  and solve this equal to 0.

14. Candidates may choose a combination of making equations equal and differentiation.

# 22 (d)

•12 pd state solution

- $\bullet^{12} \left(-\frac{5}{2},-8\right)$
- may appear in (c)

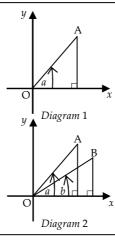
#### Notes

15. Method 2 from (c) would not yield a value for H and so  $\bullet^{12}$  is not available.

- 23 (a) Diagram 1 shows a right angled triangle, where the line OA has equation 3x - 2y = 0.
  - (i) Show that  $\tan a = \frac{3}{2}$ .
  - (ii) Find the value of sin a.
  - (b) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation 3x - 4y = 0.

Find the values of  $\sin b$  and  $\cos b$ .



4

4

# **Generic Scheme**

#### **Illustrative Scheme**

23 (a)

- •¹ ss write in slope/intercept form
- ic connect gradient and tan a
- pd calculate hypotenuse
- ic state value of sine ratio
- •¹  $y = \frac{3}{2}x$  or y = 1.5x stated explicitly •²  $m = \frac{3}{2}$  and  $\tan a = \frac{3}{2}$  or  $m = \tan a$  and  $\tan a = \frac{3}{2}$ •³  $\sqrt{13}$  stated, or implied by •⁴ •⁴  $\frac{3}{\sqrt{13}}$  or  $\frac{3\sqrt{13}}{\sqrt{13}}$  may not appear until (c)

- may not appear until (c)

#### Notes

- 1. 4 is only available if  $-1 \le \sin a \le 1$ .
- 2. Only numerical answers are acceptable for  $\bullet^3$  and  $\bullet^4$ .

# Regularly occurring responses

Response 1



$$3\times(2) - 2\times(3) = 0 \quad \wedge \bullet^{1}$$
$$\tan a = \frac{3}{2} \qquad \qquad \checkmark \bullet^{2}$$

2 marks out of 4

23 (b)

- ss determine tan b
- ss know to complete triangle
- pd determine hypotenuse
- ic state values of sine and cosine ratios
- stated, or implied by  $ullet^6$
- •6 right angled triangle with 3 and 4 correctly shown
- •<sup>7</sup> 5 stated, or implied by •<sup>8</sup> •<sup>8</sup>  $\sin b = \frac{3}{5}$  and  $\cos b = \frac{4}{5}$  may not appear until (c)

#### **Notes**

- sin  $b \le 1$  and  $-1 \le \cos b \le 1$ .
- $\sin b = \frac{3}{5}$  and  $\cos b = \frac{4}{5}$  without working is awarded 3 marks only.
- 5. Only numerical answers are acceptable for  $\bullet^7$  and  $\bullet^8$ .

**23** (*c*) (i) Find the value of  $\sin(a-b)$ .

(ii) State the value of  $\sin(b-a)$ .

4

# **Generic Scheme**

#### Illustrative Scheme

23 (c)

know to use addition formula

substitute into expansion

evaluate sine of compound angle

use  $\sin(-x) = -\sin x$ 

 $\sin a \cos b - \cos a \sin b$ 

•10  $\frac{3}{\sqrt{13}} \times \frac{4}{5} - \frac{2}{\sqrt{13}} \times \frac{3}{5}$ •11  $\frac{6}{5\sqrt{13}}$ •12  $-\frac{6}{5\sqrt{13}}$ 

#### Notes

6.  $\sin(A - B) = \sin A \cos B - \cos A \sin B$ , or just  $\sin A \cos B - \cos A \sin B$ , with no further working does not gain  $\bullet$ <sup>9</sup>.

7. Candidates should not be penalised further at  $\bullet^{10}$ ,  $\bullet^{11}$  and  $\bullet^{12}$  for values of sine and cosine outside the range

8. Candidates who use  $\sin(a-b) = \sin a - \sin b$  lose  $\bullet^9$ ,  $\bullet^{10}$  and  $\bullet^{11}$  but can gain  $\bullet^{12}$ , as follow through, only for a non-zero answer which is obtained from the result  $\sin(-x) = -\sin x$ .

9. Treat  $\sin \frac{3}{\sqrt{13}} \cos \frac{4}{5} - \cos \frac{2}{\sqrt{3}} \sin \frac{3}{5}$  as bad form only if 'sin' and 'cos' subsequently disappear.

10. It is acceptable to work through the whole expansion again for  $\bullet^{12}$ .

#### Regularly occurring responses

Response 1

$$\sin(a-b) = \sin a - \sin b \quad \times \bullet^{9}$$
$$= 6 - 6 \quad \times \bullet^{10}$$
$$= 0 \quad \times \bullet^{11}$$

 $\sin(b-a)=0 \times \bullet^{12}$ 

0 marks out of 4

Response 2

$$\sin a = 3$$
  $\cos a = 2$  Marks lost in (a) or (b)  
 $\sin b = 3$   $\cos b = 4$   
 $\sin(a-b) = \sin a \cos b - \cos a \sin b$   $\checkmark \bullet^9$ 

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$= 3 \times 4 - 2 \times 3 \quad \checkmark \bullet^{10}$$

$$= 6 \quad \checkmark \bullet^{11}$$
Faces

Eased - not dealing with fraction containing a surd.

 $\sin(b-a) = -6 \quad \checkmark \bullet^{12}$ 

3 marks out of 4

# Response 3

From (a) and (b)  $\sin a = \frac{2}{3}$   $\cos a = \frac{1}{3}$  $\sin b = \frac{2}{3} \qquad \cos b = \frac{3}{5}$ 

> (c) (i)  $\sin(a-b) = \sin a \cos b - \cos a \sin b \checkmark \bullet^9$  $=\frac{2}{3}\times\frac{3}{5}-\frac{1}{3}\times\frac{2}{5}$   $\checkmark$  •<sup>10</sup>  $=\frac{4}{15}$   $\times$  •<sup>11</sup>

(ii)  $\sin(b-a) = \sin b \cos a - \cos b \sin a$  $=\frac{2}{5}\times\frac{1}{3}-\frac{3}{5}\times\frac{2}{3}$  $=-\frac{4}{15} \times \bullet^{12}$ 

3 marks out of 4

Response 4

(i)  $\sin(a-b) = \sin a \sin b - \cos a \cos b \times \bullet^9$  $=\frac{3}{\sqrt{13}}\times\frac{3}{5}-\frac{2}{\sqrt{13}}\times\frac{4}{5}$   $\checkmark$  •<sup>10</sup>

$$=\frac{1}{5\sqrt{13}}$$
  $\chi_{\bullet^{11}}$ 

(ii)  $\sin(b-a) = -\frac{1}{5\sqrt{13}}$   $(5)^{-12}$ 

3 marks out of 4

Here the working was not necessary; the answer would gain  $\bullet^{12}$ , provided it is non zero.