H

National
Qualifications
2017

2017 Mathematics Paper 1 (Non-calculator)

Higher

Finalised Marking Instructions

General marking principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

For each question the marking instructions are generally in two sections, namely Illustrative Scheme and Generic Scheme. The illustrative scheme covers methods which are commonly seen throughout the marking. The generic scheme indicates the rationale for which each mark is awarded. In general, markers should use the illustrative scheme and only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Marks for each candidate response must always be assigned in line with these general marking principles and the detailed marking instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) If a specific candidate response does not seem to be covered by either the principles or detailed marking instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader.

(d) Credit must be assigned in accordance with the specific assessment guidelines.

(e) One mark is available for each •. There are no half marks.

(f) Working subsequent to an error must be **followed through**, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

(g) As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

(h) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(i) As a consequence of an error perceived to be trivial, casual or insignificant, eg $6\times6=12$ candidates lose the opportunity of gaining a mark. However, note the second example in comment **(j)**.

(j) Where a transcription error (paper to script or within script) occurs, the candidate should normally lose the opportunity to be awarded the next process mark, eg

This is a transcription error and so the mark is not awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$
Eased as no longer a solution of a quadratic equation so mark is not awarded.	$x=1$
Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$ $(x-3)(x-1)=0$ $x=1$ or 3

(k) **Horizontal/vertical marking**
Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

(l) In final answers, unless specifically mentioned in the detailed marking instructions, numerical values should be simplified as far as possible, eg:

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50 $\qquad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(m) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(n) Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a correct answer
- Correct working in the wrong part of a question
- Legitimate variations in numerical answers/algebraic expressions, eg angles in degrees rounded to nearest degree
- Omission of units
- Bad form (bad form only becomes bad form if subsequent working is correct), eg $(x^3+2x^2+3x+2)(2x+1)$ written as $(x^3+2x^2+3x+2)\times 2x+1$

  $2x^4+4x^3+6x^2+4x+x^3+2x^2+3x+2$ written as $2x^4+5x^3+8x^2+7x+2$ gains full credit

- Repeated error within a question, but not between questions or papers

(o) In any 'Show that…' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow-through from a previous error unless specified in the detailed marking instructions.

(p) All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

(q) Scored-out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.

(r) Where a candidate has made multiple attempts using the same strategy and not identified their final answer, mark all attempts and award the lowest mark.

Where a candidate has tried different valid strategies, apply the above ruling to attempts within each strategy and then award the highest resultant mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Specific marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		$\bullet^1$ evaluate expression	$\bullet^1$ 10	1

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
1.	(b)		$\bullet^2$ interpret notation $\bullet^3$ state expression for $g\left(f\left(x\right)\right)$	$\bullet^2$ $g(5x)$ $\bullet^3$ $2\cos 5x$	2

Notes:

1. For $2\cos 5x$ without working, award both $\bullet^2$ and $\bullet^3$.
2. Candidates who interpret the composite function as either $g\left(x\right)\times f\left(x\right)$ or $g\left(x\right)+f\left(x\right)$ do not gain any marks.
3. $g\left(f\left(x\right)\right)=10\cos x$ award $\bullet^2$. However, $10\cos x$ with no working does not gain any marks.
4. $g\left(f\left(x\right)\right)$ leading to $2\cos\left(5x\right)$ followed by incorrect 'simplification' of the function award $\bullet^2$ and $\bullet^3$.

Commonly Observed Responses:

Candidate A

$g\left(f\left(x\right)\right)=2\cos\left(5x\right)$ $\bullet^2$✓ $\bullet^3$✓

$=10\cos\left(x\right)$

Question			Generic scheme	Illustrative scheme	Max mark
2.			$\bullet^1$ state coordinates of centre $\bullet^2$ find gradient of radius $\bullet^3$ state perpendicular gradient $\bullet^4$ determine equation of tangent	$\bullet^1$ (4, 3) $\bullet^2$ $\frac{1}{3}$ $\bullet^3$ -3 $\bullet^4$ $y=-3x-5$	4

Notes:

1. Accept $\frac{2}{6}$ for $\bullet^2$.
2. The perpendicular gradient must be simplified at $\bullet^3$ or $\bullet^4$ stage for $\bullet^3$ to be available.
3. $\bullet^4$ is only available as a consequence of trying to find and use a perpendicular gradient.
4. At $\bullet^4$, accept $y+3x+5=0$, $y+3x=-5$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.			$\bullet^1$ start to differentiate $\bullet^2$ complete differentiation	$\bullet^1$ $12(4x-1)^{11}$..... $\bullet^2$ ....$\times 4$	2

Notes:

1. $\bullet^2$ is awarded for correct application of the chain rule.

Commonly Observed Responses:

Candidate A	Candidate B
$\frac{dy}{dx}=12(4x-1)^{11}\times 4$ $\bullet^1$✓ $\bullet^2$✓ $\frac{dy}{dx}=36(4x-1)^{11}$ **Working subsequent to a correct answer: General Marking Principle (n)**	$\frac{dy}{dx}=36(4x-1)^{11}$ $\bullet^1$✗ $\bullet^2$✗ **Incorrect answer with no working**

Question			Generic scheme	Illustrative scheme	Max mark
4.			Method 1 •¹ use the discriminant •² apply condition and simplify •³ determine the value of k	Method 1 •¹ $4^2-4\times1\times(k-5)$ •² $36-4k=0$ or $36=4k$ •³ $k=9$	3
			Method 2 •¹ communicate and express in factorised form •² expand and compare •³ determine the value of k	Method 2 •¹ equal roots $\Rightarrow x^2+4x+(k-5)=(x+2)^2$ •² x^2+4x+4 leading to $k-5=4$ •³ $k=9$	

Notes:

1. At the •¹ stage, treat $4^2-4\times1\times k-5$ as bad form only if the candidate treats '$k-5$' as if it is bracketed in their next line of working. See Candidates A and B.
2. In Method 1 if candidates use any condition other than 'discriminant $=0$' then •² is lost and •³ is unavailable.

Commonly Observed Responses:

Candidate A		Candidate B		
$4^2-4\times1\times k-5$	•¹ ✓	$4^2-4\times1\times \underline{k-5}$	•¹ ✗	
$36-4k=0$	•² ✓	$11-4k=0$	•² $\boxed{✓1}$	
$k=9$	•³ ✓	$k=\dfrac{11}{4}$	•³ $\boxed{✓1}$	

Question		Generic scheme	Illustrative scheme	Max mark
5.	(a)	•[1] evaluate scalar product	•[1] 1	1

Notes:

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
5.	(b)	•[2] calculate $\lvert\mathbf{u}\rvert$	•[2] $\sqrt{27}$	
		•[3] use scalar product	•[3] $\sqrt{27}\times\sqrt{3}\times\cos\frac{\pi}{3}$	
		•[4] evaluate $\mathbf{u.w}$	•[4] $\frac{9}{2}$ or $4\cdot5$	3

Notes:

1. Candidates who treat negative signs with a lack of rigour and arrive at $\sqrt{27}$ gain •[2].
2. Surds must be fully simplified for •[4] to be awarded.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.			**Method 1**	**Method 1**	
			•¹ equate composite function to x	•¹ $h\left(h^{-1}(x)\right)=x$	
			•² write $h\left(h^{-1}(x)\right)$ in terms of $h^{-1}(x)$	•² $\left(h^{-1}(x)\right)^3+7=x$	
			•³ state inverse function	•³ $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$	**3**
			Method 2	**Method 2**	
			•¹ write as $y=x^3+7$ and start to rearrange	•¹ $y-7=x^3$	
			•² complete rearrangement	•² $x=\sqrt[3]{y-7}$	
			•³ state inverse function	•³ $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$	**3**
			Method 3	**Method 3**	
			•¹ interchange variables	•¹ $x=y^3+7$	
			•² complete rearrangement	•² $y=\sqrt[3]{x-7}$	
			•³ state inverse function	•³ $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$	**3**

Notes:

1. $y=\sqrt[3]{x-7}\left(\text{or } y=(x-7)^{\frac{1}{3}}\right)$ does not gain •³.

2. At •³ stage, accept h^{-1} expressed in terms of any dummy variable eg $h^{-1}(y)=\sqrt[3]{y-7}$.

3. $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$ with no working gains 3/3.

Question	Generic scheme	Illustrative scheme	Max mark
Commonly Observed Responses:			

Candidate A

$x \to x^3 \to x^3 + 7 = h(x)$ $\wedge 3 \to +7$ $\therefore -7 \to \sqrt[3]{}$	$\bullet^1$ ✓ awarded for knowing to perform the inverse operations in reverse order
$\sqrt[3]{x-7}$	$\bullet^2$ ✓
$h^{-1}(x) = \sqrt[3]{x-7}$	$\bullet^3$ ✓

Candidate B - BEWARE	**Candidate C**
$h'(x) = \ldots \bullet^3$ ✗	$h^{-1}(x) = \sqrt[3]{x} - 7 \quad \bullet^3$ ✗ **With no working 0/3**

Question			Generic scheme	Illustrative scheme	Max mark
7.			$\bullet^1$ **find midpoint of AB** $\bullet^2$ demonstrate the line is vertical $\bullet^3$ state equation	$\bullet^1$ $(2,7)$ $\bullet^2$ m_{median} undefined $\bullet^3$ $x=2$	3

Notes:

1. $m_{median}=\frac{\pm 4}{0}$ alone is not sufficient to gain $\bullet^2$. Candidates must use either 'vertical' or 'undefined'. However $\bullet^3$ is still available.
2. '$m_{median}=\frac{4}{0}$ ✗' '$m_{median}=\frac{4}{0}$ impossible' '$m_{median}=\frac{4}{0}$ infinite' are **not** acceptable for $\bullet^2$. However, if these are followed by either 'vertical' or 'undefined' then award $\bullet^2$, and $\bullet^3$ is still available.
3. '$m_{median}=\frac{4}{0}=0$ undefined' '$m_{median}=\frac{}{0}$ undefined' are **not** acceptable for $\bullet^2$.
4. $\bullet^3$ is not available as a consequence of using a numeric gradient; however, see notes 5 and 6.
5. For candidates who find an incorrect midpoint (a,b), using the coordinates of A and B and find the 'median' through C without any further errors award 1/3. However, if $a=2$, then both $\bullet^2$ and $\bullet^3$ are available.
6. For candidates who find $15y=2x+121$ (median through B) or $3y=2x+21$ (median through A) award 1/3.

Commonly Observed Responses:

Candidate A $(2,7)$ $\bullet^1$ ✓ $m=\frac{4}{0}$ $=0$ **undefined** $\bullet^2$ ✗ $x=2$ $\bullet^3$ $\boxed{✓1}$	**Candidate B** $(2,7)$ $\bullet^1$ ✓ $m=\frac{4}{0}$ $=0$ $\bullet^2$ ✗ $y=7$ $\bullet^3$ $\boxed{✓2}$	**Candidate C** $(2,7)$ $\bullet^1$ ✓ $m=\frac{4}{0}$ $\bullet^2$ ^ $y-7=\frac{4}{0}(x-2)$ $0=4x-8$ $x=2$ $\bullet^3$ ✗
Candidate D $(2,7)$ $\bullet^1$ ✓ Median passes through $(2,7)$ and $(2,11)$ $\bullet^2$ ✗ $x=2$ $\bullet^3$ $\boxed{✓1}$	**Candidate E** $(2,7)$ $\bullet^1$ ✓ Both coordinates have an x value $2\Rightarrow$ vertical line $\bullet^2$ ✓ $x=2$ $\bullet^3$ ✓	

Question			Generic scheme	Illustrative scheme	Max mark
8.			•1 write in differentiable form	•1 $\frac{1}{2}t^{-1}$	
			•2 differentiate	•2 $-\frac{1}{2}t^{-2}$	
			•3 evaluate derivative	•3 $-\frac{1}{50}$	3

Notes:

1. Candidates who arrive at an expression containing more than one term at •1 award 0/3.
2. •2 is only available for differentiating a term containing a negative power of t.

Commonly Observed Responses:

Candidate A		Candidate B		Candidate C	
$2t^{-1}$	•1 ✗	$2t^{-1}$	•1 ✗	$-\frac{1}{2}t^{-2}$	•1 ✓ implied by •2 ✓
$-2t^{-2}$	•2 $\boxed{✓1}$	$-2t^{-2}$	•2 $\boxed{✓1}$		
$-\frac{2}{25}$	•3 $\boxed{✓1}$	$-\frac{1}{50}$	•3 ✗	$-\frac{1}{50}$	•3 ✓

Candidate D		Candidate E		Candidate F Bad form of chain rule		Candidate G	
$(2t)^{-1}$	•1 ✓	$(2t)^{-1}$	•1 ✓	$2t^{-1}$	•1 ✓	$2t^{-1}$	•1 ✗
$-(2t)^{-2}$	•2 ✗	$-(2t)^{-2}$	•2 ✗	$-2t^{-2}\times 2$	•2 ✓	$-2t^{-2}\times 2$	•2 ✗
$-\frac{1}{100}$	•3 $\boxed{✓1}$	$-\frac{2}{25}$	•3 ✗	$-\frac{1}{50}$	•3 ✓	$-\frac{4}{25}$	•3 $\boxed{✓1}$

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•[1] interpret information •[2] state the value of m	•[1] $13 = 28m + 6$ **stated explicitly or in a rearranged form** •[2] $m = \frac{1}{4}$ or $m = 0\cdot25$	2

Notes:

1. Stating '$m = \frac{1}{4}$' or simply writing '$\frac{1}{4}$' with no other working gains only •[2].

Commonly Observed Responses:

Candidate A		Candidate B	
$13 = 28\underline{u_n} + 6$	•[1] ✘	$28 = 13m + 6$	•[1] ✘
$u_n = \frac{1}{4}$	•[2] $\boxed{✓1}$	$m = \frac{22}{13}$	•[2] $\boxed{✓1}$

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)	(i)	$\bullet^3$ communicate condition for limit to exist	$\bullet^3$ a limit exists as the recurrence relation is linear and $-1 < \frac{1}{4} < 1$	**1**

Notes:

2. For $\bullet^3$ accept:

   any of $-1 < \frac{1}{4} < 1$ or $\left|\frac{1}{4}\right| < 1$ or $0 < \frac{1}{4} < 1$ with no further comment;

   or statements such as:

   "$\frac{1}{4}$ lies between -1 and 1" or "$\frac{1}{4}$ is a proper fraction"

3. $\bullet^3$ is not available for:

   $-1 \le \frac{1}{4} \le 1$ or $\frac{1}{4} < 1$

   or statements such as:

   "It is between -1 and 1." or "$\frac{1}{4}$ is a fraction."

4. Candidates who state $-1 < m < 1$ can only gain $\bullet^3$ if it is **explicitly stated** that $m = \frac{1}{4}$ in part (a).

5. Do not accept '$-1 < a < 1$' for $\bullet^3$.

Commonly Observed Responses:

Candidate C			Candidate D		
(a)	$m = \frac{1}{4}$	$\bullet^1$ ✓ $\bullet^2$ ✓	(a)	$\frac{1}{4}$	$\bullet^1$ ✓ $\bullet^2$ ✓
(b)	$-1 < m < 1$	$\bullet^3$ ✓	(b)	$-1 < m < 1$	$\bullet^3$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)	(ii)	•[4] know how to calculate limit •[5] calculate limit	•[4] $\frac{6}{1-\frac{1}{4}}$ or $L=\frac{1}{4}L+6$ •[5] 8	**2**

Notes:

6. Do not accept $L=\frac{b}{1-a}$ with no further working for •[4].
7. •[4] and •[5] are not available to candidates who conjecture that $L=8$ following the calculation of further terms in the sequence.
8. For $L=8$ with no working, award 0/2.
9. For candidates who use a value of m appearing ex nihilo or which is inconsistent with their answer in part (a) •[4] and •[5] are not available.

Commonly Observed Responses:

Candidate E - no valid limit	
(a) $m=4$ •[1] ✗ **(b)** $L=\frac{6}{1-4}$ •[4] $\boxed{✓1}$ $L=-2$ •[5] ✗	

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)			**Method 1**	
			•¹ know to integrate between appropriate limits	•¹ $\int_0^2 \ldots\, dx$	
			•² use "*upper* – *lower*"	•² $\int_0^2 \left(\left(x^3-4x^2+3x+1\right)-\left(x^2-3x+1\right)\right)$	
			•³ integrate	•³ $\frac{x^4}{4}-\frac{5x^3}{3}+3x^2$	
			•⁴ substitute limits	•⁴ $\left(\frac{2^4}{4}-\frac{5\times 2^3}{3}+3\times 2^2\right)-(0)$	
			•⁵ evaluate area	•⁵ $\frac{8}{3}$	
				Method 2	
			•¹ know to integrate between appropriate limits for both integrals	•¹ $\int_0^2 \ldots\, dx$ and $\int_0^2 \ldots\, dx$	
			•² integrate both functions	•² $\frac{x^4}{4}-\frac{4x^3}{3}+\frac{3x^2}{2}+x$ and $\frac{x^3}{3}-\frac{3x^2}{2}+x$	
			•³ substitute limits into both functions	•³ $\left(\frac{2^4}{4}-\frac{4\left(2^3\right)}{3}+\frac{3\left(2^2\right)}{2}+2\right)-0$ and $\left(\frac{2^3}{3}-\frac{3\left(2^2\right)}{2}+2\right)-0$	
			•⁴ evaluation of both functions	•⁴ $\frac{4}{3}$ and $\frac{-4}{3}$	
			•⁵ evidence of subtracting areas	•⁵ $\frac{4}{3}-\frac{-4}{3}=\frac{8}{3}$	**5**

Question	Generic scheme	Illustrative scheme	Max mark

Notes:

1. $\bullet^1$ is not available to candidates who omit 'dx'.
2. Treat the absence of brackets at $\bullet^2$ stage as bad form only if the correct integral is obtained at $\bullet^3$. See Candidates A and B.
3. Where a candidate differentiates one or more terms at $\bullet^3$, then $\bullet^3$, $\bullet^4$ and $\bullet^5$ are unavailable.
4. Accept unsimplified expressions at $\bullet^3$ e.g. $\frac{x^4}{4}-\frac{4x^3}{3}+\frac{3x^2}{2}+x-\frac{x^3}{3}+\frac{3x^2}{2}-x$.
5. Do not penalise the inclusion of '$+c$'.
6. Candidates who substitute limits without integrating do not gain $\bullet^3$, $\bullet^4$ or $\bullet^5$.
7. $\bullet^4$ is only available if there is evidence that the lower limit '0' has been considered.
8. Do not penalise errors in substitution of $x=0$ at $\bullet^3$.

Commonly Observed Responses:

Candidate A	Candidate B
$\bullet^1$ ✓	$\bullet^1$ ✓
$\int_0^2 x^3-4x^2+3x+1-x^2-3x+1\ dx$	$\int_0^2 x^3-4x^2+3x+1-x^2-3x+1\ dx$ $\bullet^2$ ✗
$\frac{x^4}{4}-\frac{5x^3}{3}+3x^2$ $\bullet^3$ ✓ $\Rightarrow \bullet^2$ ✓	$\frac{x^4}{4}-\frac{5x^3}{3}+2x$ $\bullet^3$ $\boxed{✓1}$
	$\int \ldots = -\frac{16}{3}$ cannot be negative so $=\frac{16}{3}$ $\bullet^5$ ✗
	However, $\int \ldots = -\frac{16}{3}$ so Area $=\frac{16}{3}$ $\bullet^5$ ✓

	Treating individual integrals as areas	
Candidate C - Method 2	**Candidate D** - Method 2	**Candidate E** - Method 2
$\bullet^1$ ✓	$\bullet^1$ ✓	$\bullet^1$ ✓
$\bullet^2$ ✓	$\bullet^2$ ✓	$\bullet^2$ ✓
$\bullet^3$ ✓	$\bullet^3$ ✓	$\bullet^3$ ✓
$\frac{4}{3}$ and $\frac{-4}{3}$ $\bullet^4$✓	$\frac{4}{3}$ and $\frac{-4}{3}$ $\bullet^4$✓	$\frac{4}{3}$ and $\frac{-4}{3}$ $\bullet^4$✓
	$=\frac{4}{3}$	Area cannot be negative
$\therefore$ Area is $\frac{4}{3}-\left(-\frac{4}{3}\right)=\frac{8}{3}$ $\bullet^5$ ✓	$\therefore$ Area is $\frac{4}{3}+\frac{4}{3}=\frac{8}{3}$ $\bullet^5$ ✗	$\therefore$ Area is $\frac{4}{3}+\frac{4}{3}=\frac{8}{3}$ $\bullet^5$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
10.	(b)			**Method 1**	
			•[6] use "*line* – *quadratic*"	•[6] $\int\left(\left(1-x\right)-\left(x^2-3x+1\right)\right)dx$	
			•[7] integrate	•[7] $-\frac{x^3}{3}+x^2$	
			•[8] substitute limits and evaluate integral	•[8] $\left(-\frac{2^3}{3}+2^2\right)-\left(0\right)=\frac{4}{3}$	
			•[9] state fraction	•[9] $\frac{1}{2}$	
				Method 2	
			•[6] use "cubic - *line*"	•[6] $\int\left(\left(x^3-4x^2+3x+1\right)-\left(1-x\right)\right)dx$	
			•[7] integrate	•[7] $\frac{x^4}{4}-\frac{4x^3}{3}+2x^2$	
			•[8] substitute limits and evaluate integral	•[8] $\left(\frac{2^4}{4}-4\times\frac{2^3}{3}+2\times2^2\right)-\left(0\right)=\frac{4}{3}$	
			•[9] state fraction	•[9] $\frac{1}{2}$	
				Method 3	
			•[6] integrate line	•[6] $\int\left(1-x\right)dx=\left[x-\frac{x^2}{2}\right]_0^2$	
			•[7] substitute limits and evaluate integral	•[7] $\left(2-\frac{2^2}{2}\right)-\left(0\right)=0$	
			•[8] evidence of subtracting integrals	•[8] $0-\left(-\frac{4}{3}\right)=\frac{4}{3}$ or $\frac{4}{3}-0$	
			•[9] state fraction	•[9] $\frac{1}{2}$	4

Question	Generic scheme	Illustrative scheme	Max mark

Notes:

IMPORTANT: Notes prefixed by * may be subject to General Marking Principle (n). If a candidate has been penalised for the error in (a) then they must not be penalised a second time for the same error in (b).**

9. *** $\bullet^6$ is not available to candidates who omit 'dx'.
10. In Methods 1 and 2 only, treat the absence of brackets at $\bullet^6$ stage as bad form only if the correct integral is obtained at $\bullet^7$.
11. Candidates who have an incorrect expression to integrate at the $\bullet^3$ and $\bullet^7$ stage due solely to the absence of brackets lose $\bullet^2$, but are awarded $\bullet^6$.
12. Where a candidate differentiates one or more terms at $\bullet^7$, then $\bullet^7$, $\bullet^8$ and $\bullet^9$ are unavailable.
    *** In cases where Note 3 has applied in part (a), $\bullet^7$ is lost but $\bullet^8$ and $\bullet^9$ are available.
13. In Methods 1 and 2 only, accept unsimplified expressions at $\bullet^7$ e.g. $x-\frac{x^2}{2}-\frac{x^3}{3}+\frac{3x^2}{2}-x$
14. Do not penalise the inclusion of '$+c$'.
15. *** $\bullet^8$ in Methods 1 and 2 and $\bullet^7$ in method 3 is only available if there is evidence that the lower limit '0' has been considered.
16. At the $\bullet^9$ stage, the fraction must be consistent with the answers at $\bullet^5$ and $\bullet^8$ for $\bullet^9$ to be awarded.
17. Do not penalise errors in substitution of $x=0$ at $\bullet^8$ in Method 1 & 2 or $\bullet^7$ in Method 3.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
11.				**Method 1**	
			•1 determine the gradient of given line or of AB	•1 $\frac{2}{3}$ or $\frac{a-2}{12}$	
			•2 determine the other gradient	•2 $\frac{a-2}{12}$ or $\frac{2}{3}$	
			•3 find a	•3 10	
				Method 2	
			•1 determine the gradient of given line	•1 $\frac{2}{3}$ stated or implied by •2	
			•2 equation of line and substitute	•2 $y-2=\frac{2}{3}(x+7)$ $a-2=\frac{2}{3}(5+7)$	
			•3 solve for a	•3 10	3

Notes:

Commonly Observed Responses:

Candidate A - using simultaneous equations	Candidate B	Candidate C - Method 2
$m_{\text{line}}=\frac{2}{3}$ •1 ✓ $3y=2x+20$ $3y=2x-10+3a$ •2 ✓ $0=0+30-3a$ $3a=30$ $a=10$ •3 ✓	$m_{\text{AB}}=\frac{a-2}{12}$ •1 ✓ $\frac{a-2}{12}=\underline{-2}$ •2 ✗ $a=-22$ •3 $\boxed{✓1}$	•1 ✓ $y-2=\frac{2}{3}(x+7)$ $3y=2x+20$ $3y=2\times5+20$ •2 ✓ $3y=30$ $y=10$ No mention of a •3 ^

Question			Generic scheme	Illustrative scheme	Max mark
12.			•¹ use laws of logs •² write in exponential form •³ solve for a	•¹ $\log_a 9$ •² $a^{\frac{1}{2}} = 9$ •³ 81	3

Notes:

1. $\frac{36}{4}$ must be simplified at •¹ or •² stage for •¹ to be awarded.
2. Accept $\log 9$ at •¹.
3. •² may be implied by •³.

Commonly Observed Responses:

Candidate A	Candidate B	Candidate C
$\log_a 144$ •¹ ✗ $a^{\frac{1}{2}} = 144$ •² ✓1 $a = 12$ •³ ✗	$\log_a 32$ •¹ ✗ $a^{\frac{1}{2}} = 32$ •² ✓1 •³ ^	$\log_a 9$ •¹ ✓ $a = 9^{\frac{1}{2}}$ •² ✗ $a = 3$ •³ ✓2
Candidate D $2\log_a 36 - 2\log_a 4 = 1$ $\log_a 36^2 - \log_a 4^2 = 1$ •¹ ✓ $\log_a \frac{36^2}{4^2} = 1$ $\log_a 81 = 1$ •² ✓ $a = 81$ •³ ✓		

Question	Generic scheme	Illustrative scheme	Max mark
13.	•[1] write in integrable form	•[1] $(5-4x)^{-\frac{1}{2}}$	
	•[2] start to integrate	•[2] $\dfrac{(5-4x)^{\frac{1}{2}}}{\frac{1}{2}}\ldots$	
	•[3] process coefficient of x	•[3] $\ldots\times\dfrac{1}{(-4)}$	
	•[4] complete integration and simplify	•[4] $-\frac{1}{2}(5-4x)^{\frac{1}{2}}+c$	4

Notes:

1. For candidates who differentiate throughout, only •[1] is available.
2. For candidates who 'integrate the denominator' without attempting to write in integrable form award 0/4.
3. If candidates start to integrate individual terms within the bracket or attempt to expand a bracket no further marks are available.
4. $'+c'$ is required for•[4].

Commonly Observed Responses:

Candidate A		Candidate B	
$(5-4x)^{-\frac{1}{2}}$	•[1] ✓	$(5-4x)^{\frac{1}{2}}$	•[1] ×
$\dfrac{(5-4x)^{\frac{1}{2}}}{\frac{1}{2}}$	•[2] ✓ •[3] ^	$\dfrac{(5-4x)^{\frac{3}{2}}}{\frac{3}{2}}\times\dfrac{1}{(-4)}$	•[2] $\boxed{✓1}$ •[3] ✓
$2(5-4x)^{\frac{1}{2}}+c$	•[4] $\boxed{✓2}$	$-\dfrac{(5-4x)^{\frac{3}{2}}}{6}+c$	•[4] $\boxed{✓1}$

Candidate C		Candidate D	
Differentiate in part:		Differentiate in part:	
$(5-4x)^{-\frac{1}{2}}$	•[1] ✓	$(5-4x)^{-\frac{1}{2}}$	•[1] ✓
$-\frac{1}{2}(5-4x)^{-\frac{3}{2}}\times\dfrac{1}{(-4)}$	•[2] × •[3] ✓	$\dfrac{(5-4x)^{\frac{1}{2}}}{\frac{1}{2}}\times(-4)$	•[2] ✓ •[3] ×
$\frac{1}{8}(5-4x)^{-\frac{3}{2}}+c$	•[4] $\boxed{✓1}$	$-8(5-4x)^{\frac{1}{2}}+c$	•[4] $\boxed{✓1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
14.	(a)		$\bullet^1$ use compound angle formula	$\bullet^1$ $k\sin x°\cos a° - k\cos x°\sin a°$ **stated explicitly**	
			$\bullet^2$ compare coefficients	$\bullet^2$ $k\cos a° = \sqrt{3},\ k\sin a° = 1$ **stated explicitly**	
			$\bullet^3$ process for k	$\bullet^3$ $k = 2$	
			$\bullet^4$ process for a and express in required form	$\bullet^4$ $2\sin(x-30)°$	4

Notes:

1. Accept $k(\sin x°\cos a° - \cos x°\sin a°)$ for $\bullet^1$. Treat $k\sin x°\cos a° - \cos x°\sin a°$ as bad form only if the equations at the $\bullet^2$ stage both contain k.
2. Do not penalise the omission of degree signs.
3. $2\sin x°\cos a° - 2\cos x°\sin a°$ or $2(\sin x°\cos a° - \cos x°\sin a°)$ is acceptable for $\bullet^1$ and $\bullet^3$.
4. In the calculation of $k = 2$, do not penalise the appearance of -1.
5. Accept $k\cos a° = \sqrt{3},\ -k\sin a° = -1$ for $\bullet^2$.
6. $\bullet^2$ is not available for $k\cos x° = \sqrt{3},\ k\sin x° = 1$, however, $\bullet^4$ is still available.
7. $\bullet^3$ is only available for a single value of $k,\ k > 0$.
8. $\bullet^3$ is not available to candidates who work with $\sqrt{4}$ throughout parts (a) and (b) without simplifying at any stage.
9. $\bullet^4$ is not available for a value of a given in radians.
10. Candidates may use any form of the wave equation for $\bullet^1$, $\bullet^2$ and $\bullet^3$, however, $\bullet^4$ is only available if the value of a is interpreted in the form $k\sin(x-a)°$
11. Evidence for $\bullet^4$ may only appear as a label on the graph in part (b).

Commonly Observed Responses:

Responses with missing information in working:

Candidate A	Candidate B	
$\bullet^1$ ^	$k\sin x\cos a - k\cos x\sin a$ $\bullet^1$ ✓	
$2\cos a = \sqrt{3}$ $2\sin a = 1$ $\bullet^2$ ✓ $\bullet^3$ ✓	$\cos a = \sqrt{3}$ $\sin a = 1$ $\bullet^2$ ✗	
$\tan a = \frac{1}{\sqrt{3}},\ a = 30$	$\tan a = \frac{1}{\sqrt{3}}$ $a = 30$ Not consistent with equations at $\bullet^2$.	
$2\sin(x-30)°$ $\bullet^4$ ✓	$2\sin(x-30)°$ $\bullet^3$ ✓ $\bullet^4$ ✗	

<table>
<tr><th>Question</th><th>Generic Scheme</th><th>Illustrative Scheme</th><th>Max Mark</th></tr>
<tr><td colspan="4">Responses with the correct expansion of $k\sin(x-a)°$ but errors for either $\bullet^2$ or $\bullet^4$.</td></tr>
<tr><td colspan="4">
<table>
<tr>
<td>Candidate C

$k\cos a=\sqrt{3},\ k\sin a=1$ $\bullet^2$ ✓

$\tan a=\sqrt{3}$ $\bullet^4$ ✗
$a=60$</td>
<td>Candidate D

$k\cos a=1,\ k\sin a=\sqrt{3}$ $\bullet^2$ ✗

$\tan a=\sqrt{3}$
$a=60$
$2\sin(x-60)°$ $\bullet^4$ $\boxed{\checkmark 1}$</td>
<td>Candidate E

$k\cos a=\sqrt{3},\ k\sin a=-1$ $\bullet^2$ ✗

$\tan a=-\frac{1}{\sqrt{3}},\ a=330$

$2\sin(x-330)°$ $\bullet^4$ $\boxed{\checkmark 1}$</td>
</tr>
</table>
</td></tr>
<tr><td colspan="4">Responses with the incorrect labelling; $k\sin A\cos B-k\cos A\sin B$ from formula list.</td></tr>
<tr><td colspan="4">
<table>
<tr>
<td>Candidate F
$k\sin A\cos B-k\cos A\sin B$ $\bullet^1$ ✗
$k\cos a=\sqrt{3}$
$k\sin a=1$ $\bullet^2$ ✓
$\tan a=\frac{1}{\sqrt{3}},\ a=30$
$2\sin(x-30)°$ $\bullet^3$ ✓ $\bullet^4$ ✓</td>
<td>Candidate G
$k\sin A\cos B-k\cos A\sin B$ $\bullet^1$ ✗
$k\cos x=\sqrt{3}$
$k\sin x=1$ $\bullet^2$ ✗
$\tan x=\frac{1}{\sqrt{3}},\ x=30$
$2\sin(x-30)°$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{\checkmark 1}$</td>
<td>Candidate H
$k\sin A\cos B-k\cos A\sin B$ $\bullet^1$ ✗
$k\cos B=\sqrt{3}$
$k\sin B=1$ $\bullet^2$ ✗
$\tan B=\frac{1}{\sqrt{3}},\ B=30$
$2\sin(x-30)°$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{\checkmark 1}$</td>
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Question			Generic scheme	Illustrative scheme	Max mark
14.	(b)		$\bullet^5$ roots identifiable from graph $\bullet^6$ coordinates of both turning points identifiable from graph $\bullet^7$ y-intercept and value of y at $x = 360$ identifiable from graph	$\bullet^5$ 30 and 210 $\bullet^6$ $(120, 2)$ and $(300, -2)$ $\bullet^7$ -1	3

Notes:

12. $\bullet^5$, $\bullet^6$and $\bullet^7$ are only available for attempting to draw a "sine" graph with a period of 360°.
13. Ignore any part of a graph drawn outwith $0 \le x \le 360$.
14. Vertical marking is not applicable to $\bullet^5$ and $\bullet^6$.
15. Candidates sketch arrived at in (b) must be consistent with the equation obtained in (a), see also candidates I and J.
16. For any incorrect horizontal translation of the graph of the wave function arrived at in part(a) only $\bullet^6$ is available.

Commonly Observed Responses:

Candidate I	**Candidate J**
(a) $2\sin(x-30)$ correct equation (b) Incorrect translation: Sketch of $2\sin(x+30)$ Only $\bullet^6$ is available	(a) $2\sin(x+30)$ incorrect equation (b) Sketch of $2\sin(x+30)$ All 3 marks are available

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		$\bullet^1$ state value of a $\bullet^2$ state value of b	$\bullet^1$ -5 $\bullet^2$ 3	2

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative Scheme	Max Mark
15.	(b)		$\bullet^3$ state value of integral	$\bullet^3$ 10	1

Notes:

1. Candidates answer at (b) must be consistent with the value of b obtained in (a).
2. In parts (b) and (c), candidates who have 10 and -6 accompanied by working, the working must be checked to ensure that no errors have occurred prior to the correct answer appearing.

Commonly Observed Responses:

Candidate A

From (a)

$a = -3$ $\bullet^1$ ✗

$b = 5$ $\bullet^2$ ✗

$\int h(x)\,dx = 14$ $\bullet^3$ $\boxed{✓1}$

Question			Generic scheme	Illustrative scheme	Max mark
15.	(c)		$\bullet^4$ state value of derivative	$\bullet^4$ -6	1

Notes:

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]