X100/12/02

NATIONAL QUALIFICATIONS 2014

TUESDAY, 6 MAY
1.00 PM – 2.30 PM

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on Page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and **Centre Name** printed on it.

Do not change any of these details.

4 If any of this information is wrong, tell the Invigilator immediately.

5 If this information is correct, **print** your name and seat number in the boxes provided.

6 The answer to each question is A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below).

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on your answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where x = 2?

A 8

B 1

C 0

D -4

The correct answer is **A—**8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).

Changing an answer

If you decide to change your answer, carefully erase your first answer and, using your pencil, fill in the answer you want. The answer below has been changed to **D**.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a}.\mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos\theta$, where θ is the angle between **a** and **b**

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

[Turn over

SECTION A

ALL questions should be attempted.

1. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 1$, with $u_2 = 15$.
What is the value of u_4?

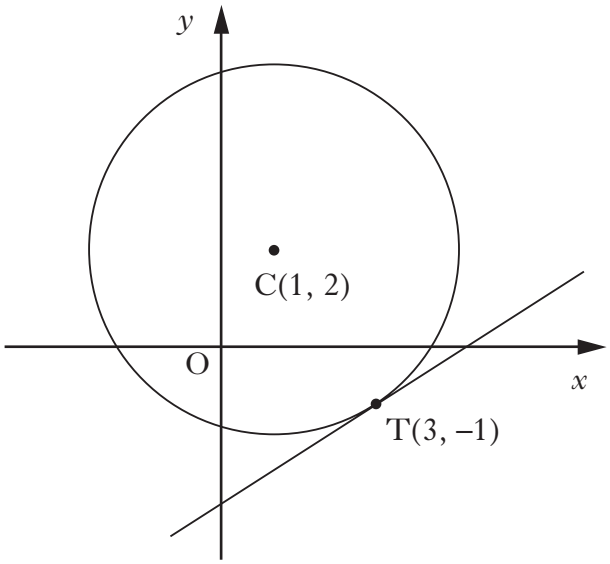
A $2\frac{1}{9}$

B $2\frac{1}{3}$

C 3

D 30

2. The diagram shows a circle with centre C(1, 2) and the tangent at T(3, –1).



What is the gradient of this tangent?

A $\frac{1}{4}$

B $\frac{2}{3}$

C $\frac{3}{2}$

D 4

3. If $\log_4 12 - \log_4 x = \log_4 6$, what is the value of x?

A 2

B 6

C 18

D 72

4. If $3\sin x - 4\cos x$ is written in the form $k\cos(x - a)$, what are the values of $k\cos a$ and $k\sin a$?

	$k\cos a$	$k\sin a$
A	–3	4
B	3	–4
C	4	–3
D	–4	3

5. Find $\int (2x+9)^5 \, dx$.

A $10\,(2x + 9)^4 + c$

B $\frac{1}{4}\,(2x + 9)^4 + c$

C $10\,(2x + 9)^6 + c$

D $\frac{1}{12}\,(2x + 9)^6 + c$

[Turn over

6. Given that $\boldsymbol{u} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, find $2\boldsymbol{u} - 3\boldsymbol{v}$ in component form.

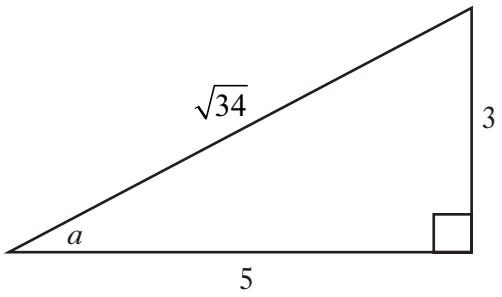
A $\begin{pmatrix} -9 \\ 5 \\ -6 \end{pmatrix}$

B $\begin{pmatrix} -9 \\ -1 \\ -4 \end{pmatrix}$

C $\begin{pmatrix} -3 \\ -1 \\ 6 \end{pmatrix}$

D $\begin{pmatrix} 11 \\ -5 \\ 4 \end{pmatrix}$

7. A right-angled triangle has sides and angles as shown in the diagram.



What is the value of $\sin 2a$?

A $\frac{8}{17}$

B $\frac{3}{\sqrt{34}}$

C $\frac{15}{17}$

D $\frac{6}{\sqrt{34}}$

8. What is the derivative of $\left(4-9x^4\right)^{\frac{1}{2}}$?

A $-\frac{9}{2}\left(4-9x^4\right)^{-\frac{1}{2}}$

B $\frac{1}{2}\left(4-9x^{-4}\right)^{-\frac{1}{2}}$

C $2\left(4-9x^4\right)^{-\frac{1}{2}}$

D $-18x^3\left(4-9x^4\right)^{-\frac{1}{2}}$

9. $\sin x+\sqrt{3}\cos x$ can be written as $2\cos\left(x-\frac{\pi}{6}\right)$.

The maximum value of $\sin x+\sqrt{3}\cos x$ is 2.

What is the maximum value of $5\sin 2x+5\sqrt{3}\cos 2x$?

A 20

B 10

C 5

D 2

10. A sequence is defined by the recurrence relation

$u_{n+1}=(k-2)u_n+5$ with $u_0=3$.

For what values of k does this sequence have a limit as $n\rightarrow\infty$?

A $-3<k<-1$

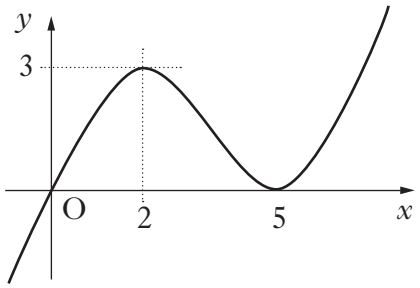
B $-1<k<1$

C $1<k<3$

D $k<3$

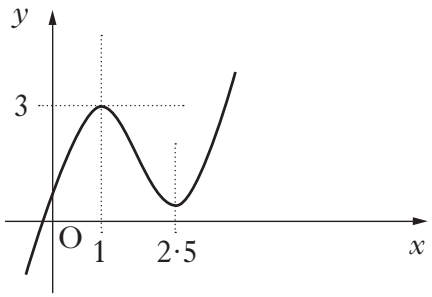
[Turn over

11. The diagram shows part of the graph of $y = f(x)$.

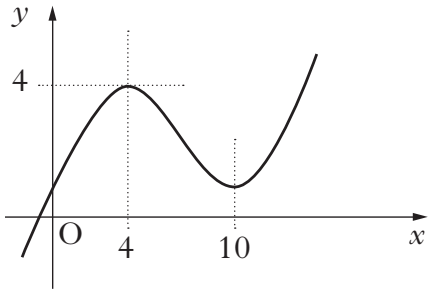


Which of the following diagrams could be the graph of $y = 2f(x) + 1$?

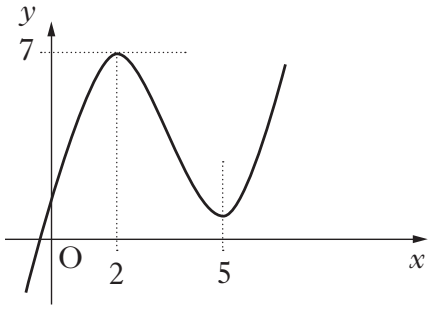
A



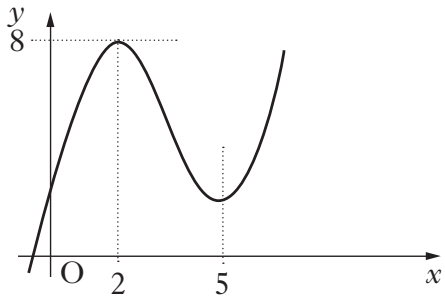
B



C



D



12. A function f, defined on a suitable domain, is given by $f(x) = \dfrac{6x}{x^2 + 6x - 16}$.

What restrictions are there on the domain of f?

A $x \neq -8$ or $x \neq 2$

B $x \neq -4$ or $x \neq 4$

C $x \neq 0$

D $x \neq 10$ or $x \neq 16$

13. What is the value of $\sin\left(\frac{\pi}{3}\right) - \cos\left(\frac{5\pi}{4}\right)$?

A $\dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}}$

B $\dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}}$

C $\dfrac{1}{2} - \dfrac{1}{\sqrt{2}}$

D $\dfrac{1}{2} + \dfrac{1}{\sqrt{2}}$

14. The vectors $\boldsymbol{u} = \begin{pmatrix} 1 \\ k \\ k \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -6 \\ 2 \\ 5 \end{pmatrix}$ are perpendicular.

What is the value of k?

A $\dfrac{-6}{7}$

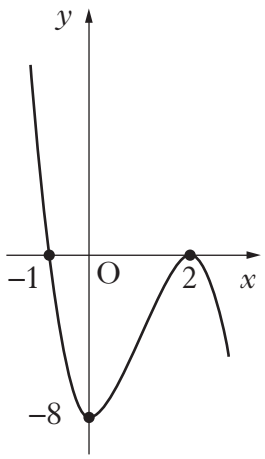
B -1

C 1

D $\dfrac{6}{7}$

[Turn over

15. The diagram shows a cubic curve passing through (–1, 0), (2, 0) and (0, –8).



What is the equation of the curve?

A $y = -2(x + 1)^2(x + 2)$

B $y = -2(x + 1)(x - 2)^2$

C $y = 4(x + 1)(x - 2)$

D $y = -8(x + 1)(x - 2)^2$

16. The unit vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are such that $\boldsymbol{a}.\boldsymbol{b} = \frac{2}{3}$. Determine the value of $\boldsymbol{a}.(\boldsymbol{a} + 2\boldsymbol{b})$.

A $\frac{2}{3}$

B $\frac{4}{3}$

C $\frac{7}{3}$

D 3

17. $3x^2 + 12x + 17$ is expressed in the form $3(x + p)^2 + q$.
What is the value of q?

A 1

B 5

C 17

D –19

18. What is the value of $1 - 2\sin^2 15°$?

A $\frac{1}{2}$

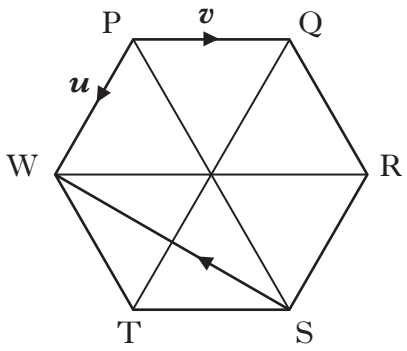
B $\frac{3}{4}$

C $\frac{\sqrt{3}}{2}$

D $\frac{7}{8}$

19. The diagram shows a regular hexagon PQRSTW.

$\overrightarrow{PW}$ and $\overrightarrow{PQ}$ represent vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ respectively.



What is $\overrightarrow{SW}$ in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$?

A $-\boldsymbol{u} - 2\boldsymbol{v}$

B $-\boldsymbol{u} - \boldsymbol{v}$

C $\boldsymbol{u} - \boldsymbol{v}$

D $\boldsymbol{u} + 2\boldsymbol{v}$

20. Evaluate $2 - \log_5 \frac{1}{25}$.

A -3

B 0

C $\frac{3}{2}$

D 4

[*END OF SECTION A*]

SECTION B

Marks

ALL questions should be attempted.

21. A curve has equation $y = 3x^2 - x^3$.

(*a*) Find the coordinates of the stationary points on this curve and determine their nature. **6**

(*b*) State the coordinates of the points where the curve meets the coordinate axes and sketch the curve. **2**

22. For the polynomial $6x^3 + 7x^2 + ax + b$,

- $x + 1$ is a factor
- 72 is the remainder when it is divided by $x - 2$.

(*a*) Determine the values of a and b. **4**

(*b*) Hence factorise the polynomial completely. **3**

23. (*a*) Find P and Q, the points of intersection of the line $y = 3x - 5$ and the circle C_1 with equation $x^2 + y^2 + 2x - 4y - 15 = 0$. **4**

(*b*) T is the centre of C_1.

Show that PT and QT are perpendicular. **3**

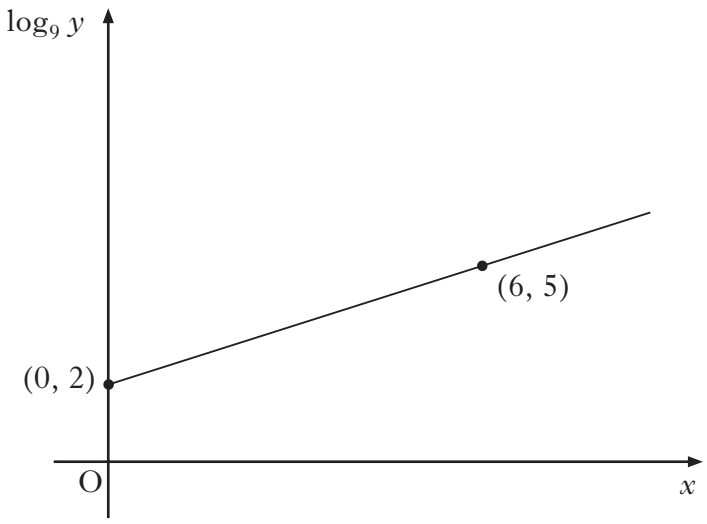
(*c*) A second circle C_2 passes through P, Q and T.

Find the equation of C_2. **3**

24. Two variables, x and y, are related by the equation *Marks*

$$y = ka^x.$$

When $\log_9 y$ is plotted against x, a straight line passing through the points (0, 2) and (6, 5) is obtained, as shown in the diagram.



Find the values of k and a. **5**

[*END OF SECTION B*]

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