H

National
Qualifications
SPECIMEN ONLY

S847/76/12

# Mathematics
# Paper 2

Date — Not applicable

Duration — 1 hour 45 minutes

**Total marks — 80**

Attempt ALL questions.

**You may use a calculator.**

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.



*S8477612*

## FORMULAE LIST

### Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

### Scalar product:

$\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where $\theta$ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

### Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

### Table of standard derivatives:

| $f(x)$ | $f'(x)$ |
|---|---|
| $\sin ax$ | $a\cos ax$ |
| $\cos ax$ | $-a\sin ax$ |

### Table of standard integrals:

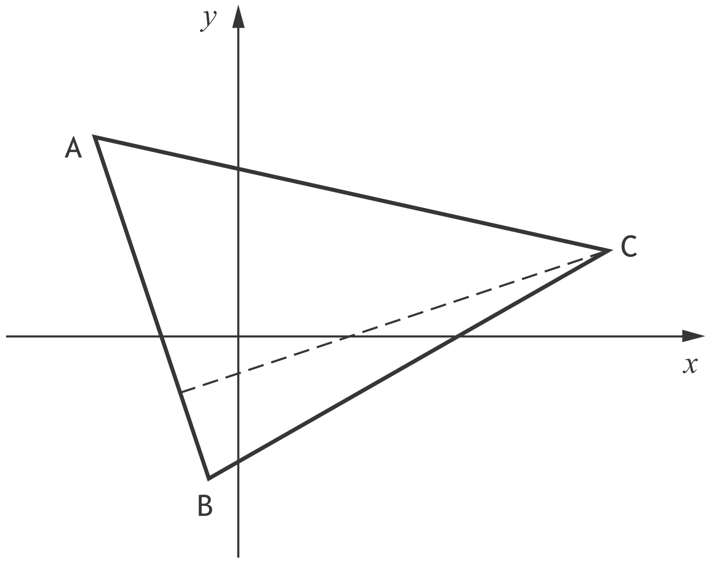
| $f(x)$ | $\int f(x)dx$ |
|---|---|
| $\sin ax$ | $-\frac{1}{a}\cos ax + c$ |
| $\cos ax$ | $\frac{1}{a}\sin ax + c$ |

**Attempt ALL questions**

**Total marks — 80**

**1.** The vertices of triangle ABC are A(−5, 7), B(−1, −5) and C(13, 3) as shown in the diagram.

The broken line represents the altitude from C.




(a) Find the equation of the altitude from C. **3**

(b) Find the equation of the median from B. **3**

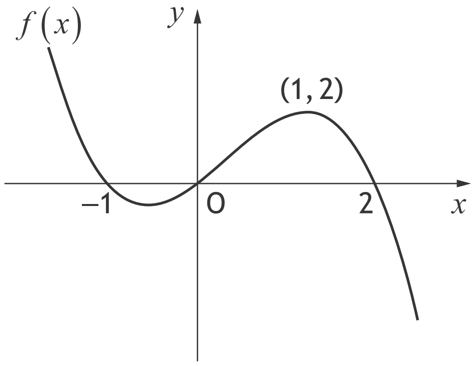
(c) Find the coordinates of the point of intersection of the altitude from C and the median from B. **2**

**2.** Find $\int \frac{4x^3+1}{x^2}\,dx,\ x \neq 0$. **4**

MARKS

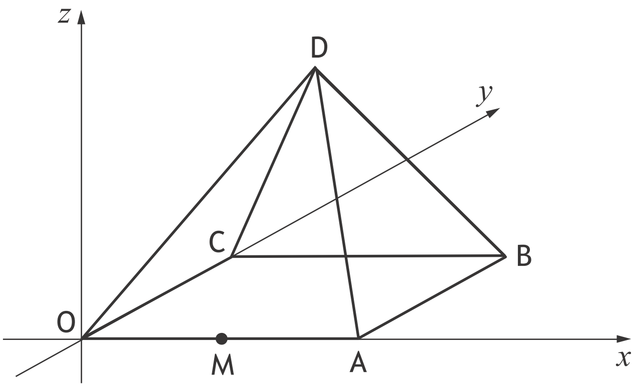
**3.** The diagram shows the curve with equation $y = f(x)$, where $f(x) = kx(x+a)(x+b)$.

The curve passes through $(-1, 0)$, $(0, 0)$, $(1, 2)$ and $(2, 0)$.




Find the values of $a$, $b$ and $k$. **3**

**4.** D,OABC is a square-based pyramid as shown.



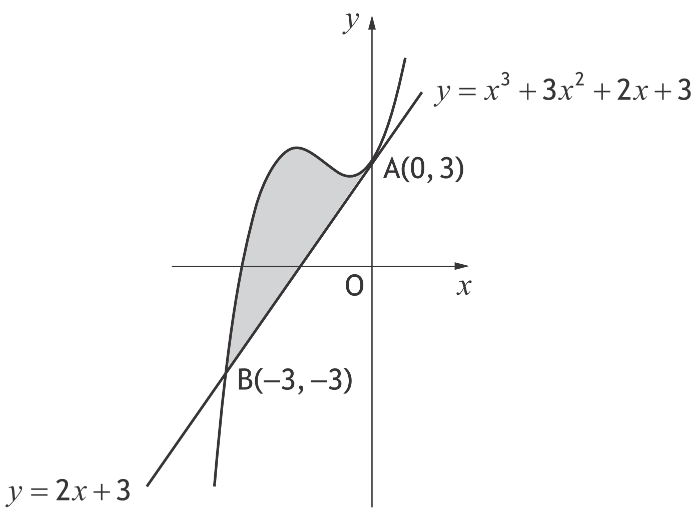

- O is the origin and OA = 4 units.
- M is the mid-point of OA.
- $\overrightarrow{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

(a) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3**

(b) Find the size of angle BDM. **5**

MARKS

**5.** The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown.




The line meets the curve again at B(−3, −3).

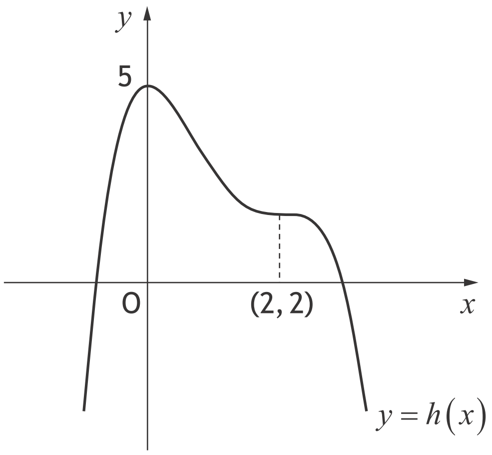
Find the area enclosed by the line and the curve. **5**

**6.** (a) Express $3x^2 + 24x + 50$ in the form $a(x+b)^2 + c$. **3**

(b) Given that $f(x) = x^3 + 12x^2 + 50x - 11$, find $f'(x)$. **2**

(c) Hence, or otherwise, explain why the curve with equation $y = f(x)$ is strictly increasing for all values of $x$. **2**

**7.** The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $(0, 5)$ and $(2, 2)$.




On separate diagrams sketch the graphs of:

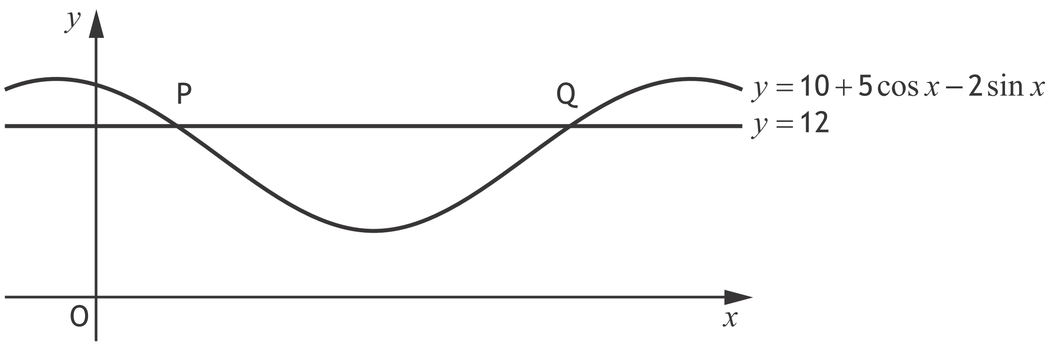
(a) $y = 2 - h(x)$. **2**

(b) $y = h'(x)$. **3**

**8.** (a) Express $5\cos x - 2\sin x$ in the form $k\cos(x + a)$, where $k > 0$ and $0 < a < 2\pi$. **4**

(b) The diagram shows a sketch of part of the graph of $y = 10 + 5\cos x - 2\sin x$ and the line with equation $y = 12$.

The line cuts the curve at the points P and Q.

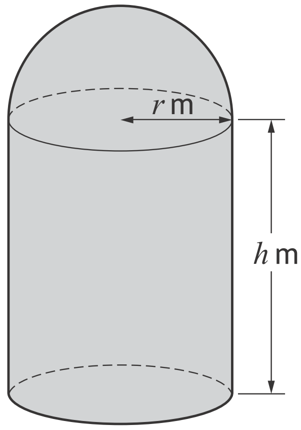



Find the $x$-coordinates of P and Q. **4**

MARKS

**9.** A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is $r$ metres, and the height is $h$ metres.

The volume of the **cylindrical** part of the container needs to be 100 cubic metres.




(a) Given that the curved surface area of a hemisphere of radius $r$ is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by:

$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$ **3**

(b) Determine the value of $r$ which minimises the amount of metal needed to build the container. **6**

**10.** Given that

$$\int_{\frac{\pi}{8}}^{a} \sin\left(4x - \frac{\pi}{2}\right) dx = \frac{1}{2}, \quad 0 \le a < \frac{\pi}{2},$$

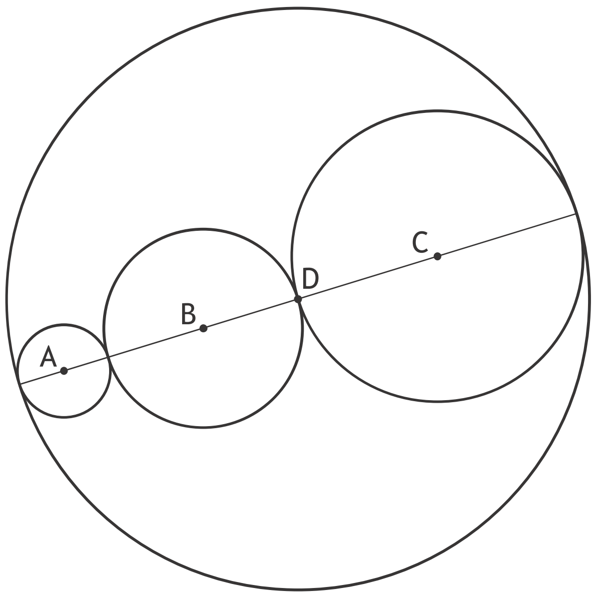
calculate the value of $a$. **6**

**MARKS**

**11.** Show that $\dfrac{\sin 2x}{2\cos x} - \sin x\cos^2 x = \sin^3 x$ , where $0 < x < \dfrac{\pi}{2}$. **3**

**12.** (a) Show that the points A$(-7,-2)$, B$(2,1)$ and C$(17,6)$ are collinear. **3**

Three circles with centres A, B and C are drawn inside a circle with centre D as shown.




The circles with centres A, B and C have radii $r_A$, $r_B$ and $r_C$ respectively.

- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

(b) Determine the equation of the circle with centre D. **4**

MARKS

**13.** The concentration of a pesticide in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- $P_0$ is the initial concentration;
- $P_t$ is the concentration at time $t$;
- $t$ is the time, in days, after the application of the pesticide.

(a) It takes 25 days for the concentration of the pesticide to be reduced to one half of its initial concentration.

Calculate the value of $k$. **4**

(b) Eighty days after the initial application, what is the percentage decrease in concentration of the pesticide? **3**

**[END OF SPECIMEN QUESTION PAPER]**