X100/302

NATIONAL QUALIFICATIONS 2009

THURSDAY, 21 MAY
10.50 AM – 12.00 NOON

MATHEMATICS
HIGHER
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

1. Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature. **8**

2. Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

(a) (i) Find $p(x)$ where $p(x) = f(g(x))$.

(ii) Find $q(x)$ where $q(x) = g(f(x))$. **3**

(b) Solve $p'(x) = q'(x)$. **3**

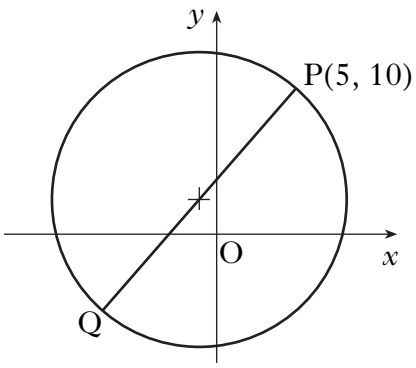
3. (a) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.

(ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. **4**

(b) Solve $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$. **5**

4. (a) Show that the point P(5, 10) lies on circle C_1 with equation $(x + 1)^2 + (y - 2)^2 = 100$. **1**

(b) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. **5**



(c) Two circles, C_2 and C_3, touch circle C_1 at Q.

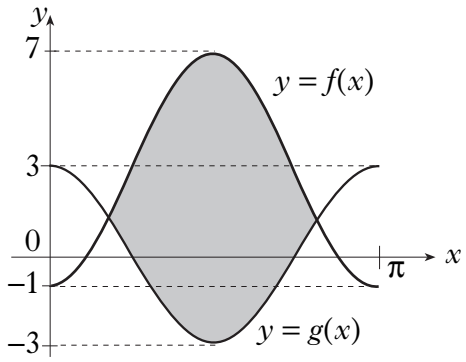
The radius of each of these circles is twice the radius of circle C_1.

Find the equations of circles C_2 and C_3. **4**

[Turn over

Marks

5. The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.



$f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$.

(*a*) Write down the values of m and n. **1**

(*b*) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \leq x \leq \pi$. **5**

(*c*) Calculate the shaded area. **6**

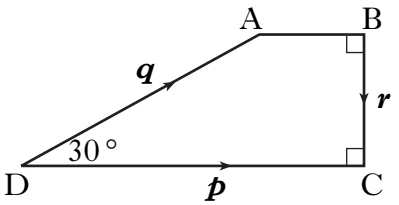
6. The size of the human population, N, can be modelled using the equation $N = N_0e^{rt}$ where N_0 is the population in 2006, t is the time in years since 2006, and r is the annual rate of increase in the population.

(*a*) In 2006 the population of the United Kingdom was approximately 61 million, with an annual rate of increase of 1·6%. Assuming this growth rate remains constant, what would be the population in 2020? **2**

(*b*) In 2006 the population of Scotland was approximately 5·1 million, with an annual rate of increase of 0·43%.

Assuming this growth rate remains constant, how long would it take for Scotland's population to double in size? **3**

7. Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are represented on the diagram shown where angle ADC = 30°.



It is also given that $|\boldsymbol{p}| = 4$ and $|\boldsymbol{q}| = 3$.

(*a*) Evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$ and $\boldsymbol{r}.(\boldsymbol{p} - \boldsymbol{q})$. **6**

(*b*) Find $|\boldsymbol{q} + \boldsymbol{r}|$ and $|\boldsymbol{p} - \boldsymbol{q}|$. **4**

[*END OF QUESTION PAPER*]