

| qu   | Mk | Code  | cal | Source | ss | pd | ic | C | B | A | U1 | U2 | U3 |
|------|----|-------|-----|--------|----|----|----|---|---|---|----|----|----|
| 2.01 | 8  | C8,C9 | cn  | 08507  | 3  | 4  | 1  | 8 |   |   | 8  |    |    |

**2.01**

Find the coordinates of the turning points of the curve with equation  $y = x^3 - 3x^2 - 9x + 12$  and determine their nature.

8

The primary method m.s. is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide

but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- <sup>1</sup> ss know to differentiate
- <sup>2</sup> pd differentiate
- <sup>3</sup> ss set derivative to zero
- <sup>4</sup> pd factorise
- <sup>5</sup> pd solve for  $x$
- <sup>6</sup> pd evaluate  $y$ -coordinates
- <sup>7</sup> ss know to, and justify turning points
- <sup>8</sup> ic interpret result

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $\frac{dy}{dx} = \dots$  (1 term correct)
- <sup>2</sup>  $3x^2 - 6x - 9$
- <sup>3</sup>  $\frac{dy}{dx} = 0$
- <sup>4</sup>  $3(x+1)(x-3)$

|                |                |                |
|----------------|----------------|----------------|
|                | • <sup>5</sup> | • <sup>6</sup> |
| • <sup>5</sup> | $x = -1$       | $x = 3$        |
| • <sup>6</sup> | $y = 17$       | $y = -15$      |

|                |                                       |                         |
|----------------|---------------------------------------|-------------------------|
|                | • <sup>7</sup>                        | • <sup>8</sup>          |
| • <sup>7</sup> | $x \dots -1 \dots$                    | $\dots 3 \dots$         |
|                | $\frac{dy}{dx} \dots + \dots - \dots$ | $\dots - \dots + \dots$ |
| • <sup>8</sup> | max                                   | min                     |

## Notes

- The "=0" (shown at •<sup>3</sup>) **must** occur at least once before •<sup>5</sup>.
- <sup>4</sup> is only available as a consequence of solving  $\frac{dy}{dx} = 0$ .
- The nature table must reflect previous working from •<sup>4</sup>.
- For •<sup>4</sup>, accept  $(x+1)(x-3)$ .
- The use of the 2nd derivative is an acceptable strategy.
- As shown in the Primary Method, (•<sup>5</sup> and •<sup>6</sup>) and (•<sup>7</sup> and •<sup>8</sup>) can be marked horizontally or vertically.
- <sup>1</sup>, •<sup>2</sup> and •<sup>3</sup> are the only marks available to candidates who solve  $3x^2 - 6x = 9$ .

## Notes cont

- If •<sup>7</sup> is not awarded, •<sup>8</sup> is only available as follow-through if there is clear evidence of where the signs at the •<sup>7</sup> stage have been obtained.
- For •<sup>7</sup> and •<sup>8</sup>  
The completed nature table is worth 2 marks if correct.  
If the labels "x" and/or " $\frac{dy}{dx}$ " are missing from an otherwise correct table then **award 1 mark**.  
If the labels "x" and/or " $\frac{dy}{dx}$ " are missing from a table where either •<sup>7</sup> or •<sup>8</sup> (vertically) would otherwise have been awarded, then **award 0 marks**.

## Alternatives

This would be fairly common:

- <sup>1</sup>  $\sqrt{\frac{dy}{dx} = \dots}$  (1 term correct)
- <sup>2</sup>  $\sqrt{3x^2 - 6x - 9}$
- <sup>3</sup>, •<sup>4</sup>  $\sqrt{\sqrt{(3x-9)(x+1)} = 0}$   
or  $(3x+3)(x-3) = 0$

## Min. requirements of a nature table

|                 |         |      |         |
|-----------------|---------|------|---------|
| $x$             | $\dots$ | $-1$ | $\dots$ |
| $\frac{dy}{dx}$ | $+$     | $0$  | $-$     |
|                 |         | max  |         |

## Preferred nature table

|                 |         |      |              |
|-----------------|---------|------|--------------|
| $x$             | $\dots$ | $-1$ | $\dots$      |
| $\frac{dy}{dx}$ | $+$     | $0$  | $-$          |
|                 | $/$     | $-$  | $\backslash$ |
|                 |         | max  |              |

# Higher Mathematics 2009 v10

| qu   |   | Mk | Code | cal | Source | ss | pd | ic | C | B | A |  | U1 | U2 | U3 |  |
|------|---|----|------|-----|--------|----|----|----|---|---|---|--|----|----|----|--|
| 2.02 | a | 3  | A4   | cn  | 09011  | 1  |    | 2  | 3 |   |   |  | 3  |    |    |  |
|      | b | 3  | C1   | cn  |        | 2  | 1  |    | 3 |   |   |  | 3  |    |    |  |

2.02

Functions  $f$  and  $g$  are given by  $f(x) = 3x + 1$  and  $g(x) = x^2 - 2$ .

- (a) (i) Find  $p(x)$  where  $p(x) = f(g(x))$   
(ii) Find  $q(x)$  where  $q(x) = g(f(x))$ . 3  
(b) Solve  $p'(x) = q'(x)$ . 3

The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide

but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- <sup>1</sup> ss substitute for  $g(x)$  in  $f(x)$
- <sup>2</sup> ic complete
- <sup>3</sup> ic sub. and complete for  $q(x)$
- <sup>4</sup> ss simplify
- <sup>5</sup> pd differentiate
- <sup>6</sup> pd solve

**Primary Method: Give 1 mark for each •**

- <sup>1</sup>  $f(x^2 - 2)$  s / i by •<sup>2</sup>
- <sup>2</sup>  $3(x^2 - 2) + 1$
- <sup>3</sup>  $(3x + 1)^2 - 2$
- <sup>4</sup>  $3x^2 - 5$  s / i by •<sup>5</sup>
- <sup>5</sup>  $6x$   $9x^2 + 6x - 1$   $18x + 6$  or equiv.
- <sup>6</sup>  $x = -\frac{1}{2}$

## Notes

- In (a)  
2 marks are available for finding either  $f(g(x))$  or  $g(f(x))$  and 1 mark for finding the other.
- In (b)  
candidates who start by equating  $p(x)$  and  $q(x)$  and then differentiate may earn •<sup>4</sup> and •<sup>6</sup> only.

## Common Errors

1

$p(x)$  and  $q(x)$  switched round:

- X •<sup>1</sup>  $p(x) = g(3x + 1)$
- X ✓ •<sup>2</sup>  $p(x) = (3x + 1)^2 - 2$
- X ✓ •<sup>3</sup>  $q(x) = \dots = 3(x^2 - 2) + 1$

2

Candidates who find  $f(f(x))$  and  $g(g(x))$  can earn no marks in (a) but

- X ✓ •<sup>4</sup>  $9x + 4$  and  $x^4 - 4x^2 + 2$
- X ✓ •<sup>5</sup>  $9 = 4x^3 - 8x$
- XX •<sup>6</sup> not available

3

- X •<sup>4</sup>  $3x^2 - 1$  and  $9x^2 + 6x - 1$
- X ✓ •<sup>5</sup>  $6x$  and  $18x + 6$
- X ✓ •<sup>6</sup>  $x = -\frac{1}{2}$

**Alternative for •<sup>1</sup> to •<sup>3</sup>:**

- <sup>1</sup>  $f(g(x)) = 3 \times g(x) + 1$
- <sup>2</sup>  $f(g(x)) = 3(x^2 - 2) + 1$   
 $g(f(x)) = (f(x))^2 - 2$
- <sup>3</sup>  $g(f(x)) = (3x + 1)^2 - 2$

# Higher Mathematics 2009 v10

| qu   |   | Mk | Code | cal | Source | ss | pd | ic | C | B | A |  | U1 | U2 | U3 |  |
|------|---|----|------|-----|--------|----|----|----|---|---|---|--|----|----|----|--|
| 2.03 | a | 4  | A21  | cn  | 09008  | 1  | 1  | 2  | 4 |   |   |  |    | 4  |    |  |
|      | b | 5  | A32  | cn  |        | 2  | 1  | 2  |   | 5 |   |  |    |    | 5  |  |

2.03

- (a) (i) Show that  $x = 1$  is a root of  $x^3 + 8x^2 + 11x - 20 = 0$ .  
(ii) Hence factorise  $x^3 + 8x^2 + 11x - 20$  fully. 4  
(b) Solve  $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$ . 5

The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide

but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- <sup>1</sup> ss know and use  $f(a) = 0 \Leftrightarrow a$  is a root
- <sup>2</sup> ic start to find quadratic factor
- <sup>3</sup> ic complete quadratic factor
- <sup>4</sup> pd factorise fully
- <sup>5</sup> ss use log laws
- <sup>6</sup> ss know to & convert to exponential form
- <sup>7</sup> ic write cubic in standard form
- <sup>8</sup> pd solve cubic
- <sup>9</sup> ic interpret valid solution

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $f(1) = 1 + 8 + 11 - 20 = 0$  so  $x = 1$  is a root **See Note 1**
- <sup>2</sup>  $(x - 1)(x^2 + \dots)$
- <sup>3</sup>  $(x^2 + 9x + 20)$
- <sup>4</sup>  $(x - 1)(x + 4)(x + 5)$  **Stated explicitly**
- <sup>5</sup>  $\log_2((x + 3)(x^2 + 5x - 4))$  **s / i by •<sup>6</sup>**
- <sup>6</sup>  $(x + 3)(x^2 + 5x - 4) = 2^3$
- <sup>7</sup>  $x^3 + 8x^2 + 11x - 20 = 0$
- <sup>8</sup>  $x = 1$  or  $x = -4$  or  $x = -5$  **Stated explicitly here**
- <sup>9</sup>  $x = 1$  only

## Notes

- For candidates evaluating the function, some acknowledgement of the resulting zero must be shown in order to gain •<sup>1</sup>.
- For candidates using synthetic division (shown in Alt. box), some acknowledgement of the resulting zero must be shown in order to gain •<sup>2</sup>.
- In option 2 the "zero" has been highlighted by underlining. This can also appear in colour, bold or boxed. Some acknowledgement of the resulting zero must be shown in order to gain •<sup>1</sup> as indicated in each option.

## Common Errors

- 1
- <sup>5</sup> X  $\log_2 \frac{x^2 + 5x - 4}{x + 3} = 3$
  - <sup>6</sup> X ✓  $\frac{x^2 + 5x - 4}{x + 3} = 2^3$
  - <sup>7</sup> X  $x^2 - 3x - 28 = 0$
  - <sup>8</sup> X  $x = 7$  or  $-4$
  - <sup>9</sup> X ✓  $x = 7$  **ONLY**

## Options

Alternative for •<sup>1</sup> to •<sup>2</sup>.

**1**

- <sup>1</sup>

|   |   |    |     |
|---|---|----|-----|
| 1 | 8 | 11 | -20 |
| 1 | 1 |    |     |
| 1 | 9 |    |     |
| 1 | 8 | 11 | -20 |
| 1 | 1 | 9  | 20  |
| 1 | 9 | 20 | 0   |

**rem. = 0**  
so  $x = 1$  is root  
**see note 2**

**2**

- <sup>1</sup>

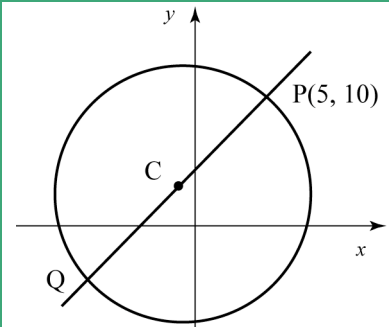
|   |   |    |     |
|---|---|----|-----|
| 1 | 8 | 11 | -20 |
| 1 | 1 |    |     |
| 1 | 9 |    |     |
| 1 | 8 | 11 | -20 |
| 1 | 1 | 9  | 20  |
| 1 | 9 | 20 | 0   |

**so  $x = 1$  is root**  
**see note 3**

| qu   |   | Mk | Code | Cal | Source | ss | pd | ic | C | B | A |  | U1 | U2 | U3 |  |
|------|---|----|------|-----|--------|----|----|----|---|---|---|--|----|----|----|--|
| 2.04 | a | 1  | A6   | cn  | 08026  |    | 1  |    | 1 |   |   |  | 1  |    |    |  |
|      | b | 5  | G11  | cn  |        | 2  |    | 3  | 5 |   |   |  |    | 5  |    |  |
|      | c | 4  | G15  | nc  |        | 1  | 1  | 2  |   |   | 4 |  |    | 4  |    |  |

**2.04**

- (a) Show that the point P(5, 10) lies on circle  $C_1$  with equation  $(x + 1)^2 + (y - 2)^2 = 100$ . 1
- (b) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. 5
- (c) Two circles,  $C_2$  and  $C_3$ , touch circle  $C_1$  at Q. The radius of each of these circles is twice the radius of circle  $C_1$ . Find the equations of circles  $C_2$  and  $C_3$ . 4



The primary method m.s. is based on the following generic m.s.  
This generic marking scheme may be used as an equivalence g  
but only where a candidate does not use the primary method or  
alternative method shown in detail in the marking scheme.

- <sup>1</sup> pd substitute
- <sup>2</sup> ic find centre
- <sup>3</sup> ss use mid-point result for Q
- <sup>4</sup> ss know to, and find gradient of radi
- <sup>5</sup> ic find gradient of tangent
- <sup>6</sup> ic state equation of tangent
- <sup>7</sup> ic state radius
- <sup>8</sup> ss know how to find centre
- <sup>9</sup> ic state equation of one circle
- <sup>10</sup> ic state equation of the other circle

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $(5 + 1)^2 + (10 - 2)^2 = 100$
- <sup>2</sup>  $centre = (-1, 2)$
- <sup>3</sup>  $Q = (-7, -6)$  (no evidence requ.)
- <sup>4</sup>  $m_{rad} = \frac{8}{6}$
- <sup>5</sup>  $m_{tgt} = -\frac{3}{4}$  s / i by •<sup>6</sup>
- <sup>6</sup>  $y - (-6) = -\frac{3}{4}(x - (-7))$
- <sup>7</sup>  $radius = 20$  s / i by •<sup>9</sup> or •<sup>10</sup>
- <sup>8</sup>  $centre = (5, 10)$  s / i by •<sup>9</sup>
- <sup>9</sup>  $(x - 5)^2 + (y - 10)^2 = 400$
- <sup>10</sup>  $(x + 19)^2 + (y + 22)^2 = 400$

## Notes

- In (a), candidates may choose to show that distance CP = the radius. Markers should note that evidence for •<sup>2</sup>, which is in (b), may appear in (a).
- The minimum requirement for •<sup>1</sup> is as shown in the Primary Method.
- <sup>6</sup> is only available as a consequence of attempting to find a perp. gradient.
- For candidates who choose a Q *ex nihilo*, •<sup>6</sup> is only available if the chosen Q lies in the 3rd quadrant.

## Notes cont

- <sup>9</sup> and/or •<sup>10</sup> are only available as follow-through if a centre with numerical coordinates has been stated explicitly.
- <sup>10</sup> is not available as a follow-through; it must be correct.

## Alternative for •<sup>8</sup>, •<sup>9</sup> and •<sup>10</sup>

- <sup>8</sup>  $centre = (-19, -22)$  s / i by •<sup>9</sup>
- <sup>9</sup>  $(x + 19)^2 + (y + 22)^2 = 400$
- <sup>10</sup>  $(x - 5)^2 + (y - 10)^2 = 400$

| qu   |   | Mk | Code    | Cal | Source | ss | pd | ic | C | B | A |  | U1 | U2 | U3 |  |
|------|---|----|---------|-----|--------|----|----|----|---|---|---|--|----|----|----|--|
| 2.05 | a | 1  | T4      | cn  | 09026  |    |    | 1  | 1 |   |   |  | 1  |    |    |  |
|      | b | 5  | T6      | cr  |        | 1  | 3  | 1  | 5 |   |   |  |    | 5  |    |  |
|      | c | 6  | C17, 23 | cr  |        | 1  | 3  | 2  |   | 6 |   |  |    | 6  |    |  |

2.05

The graphs of  $y = f(x)$  and  $y = g(x)$  are shown in the diagram.

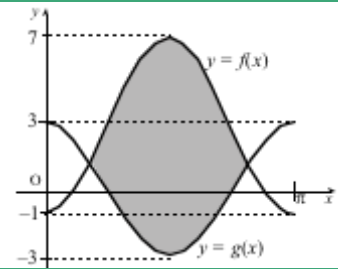
$f(x) = -4 \cos(2x) + 3$  and  $g(x)$  is of the form  $g(x) = m \cos(nx)$ .

- (a) Write down the values of  $m$  and  $n$ .  
 (b) Find, correct to 1 decimal place, the coordinates of the points of intersection of the two graphs in the interval shown.  
 (c) Calculate the shaded area.

1

5

6



The primary method m.s. is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- <sup>1</sup> ic interprets graph
- <sup>2</sup> ss knows how to find intersection
- <sup>3</sup> pd starts to solve
- <sup>4</sup> pd finds  $x$ -coordinate in the 1st quadrant
- <sup>5</sup> pd finds  $x$ -coordinate in the 2nd quadrant
- <sup>6</sup> pd finds  $y$ -coordinates
- <sup>7</sup> ss knows how to find area
- <sup>8</sup> ic states limits
- <sup>9</sup> pd integrate
- <sup>10</sup> pd integrate
- <sup>11</sup> ic substitute limits
- <sup>12</sup> pd evaluate area

**Continued on next page**

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $m = 3$  and  $n = 2$
- <sup>2</sup>  $3 \cos 2x = -4 \cos 2x + 3$
- <sup>3</sup>  $\cos 2x = \frac{3}{7}$
- <sup>4</sup>  $x = 0.6$
- <sup>5</sup>  $x = 2.6$
- <sup>6</sup>  $y = 1.3, 1.3$
- <sup>7</sup>  $\int (-4 \cos 2x + 3 - 3 \cos 2x) dx$
- <sup>8</sup>  $\int_{0.6}^{2.6}$
- <sup>9</sup>  $-7 \sin 2x$
- <sup>10</sup>  $3x - \frac{7}{2} \sin 2x$
- <sup>11</sup>  $(3 \times 2.6 - \frac{7}{2} \sin 5.2) - (3 \times 0.6 - \frac{7}{2} \sin 1.2)$
- <sup>12</sup> 12.4

**Continued on next page**

Question 2.05 cont.

Notes 1

- Answers which are not rounded should be treated as "bad form" and not penalised.
- If  $n = 1$  from (a), then in (b) the follow-through solution is 0.697 and 5.586.  
•<sup>5</sup> is not available in (b)  
and •<sup>8</sup> is not available in (c).
- If  $n = 3$  from (a), then in (b) only •<sup>2</sup> is available.
- At •<sup>5</sup> :  
 $x = 2.5$  can only come from calculating  $\pi - 0.6$ . For this to be accepted, candidates must state that it comes from symmetry of the graph.
- For •<sup>6</sup>  
Acceptable values of  $y$  will lie in the range 1.1 to 1.6  
(due to early rounding !!)
- Values of  $x$  used for the limits must lie between 0 and  $\pi$ ,  
i.e.  $0 < \text{limits} < \pi$ , else •<sup>8</sup> is lost.
- <sup>8</sup>, •<sup>11</sup> and •<sup>12</sup> are not available to candidates who use  $-3$  and  $7$  as the limits.
- Candidates must deal appropriately with any extraneous negative signs which may appear before •<sup>12</sup> can be awarded.

**It is considered inappropriate to write ..... = -12.4 = 12.4**

Common Errors

- For candidates who work in degrees throughout this question, the following marks are available:  

| In (b)                         | In (c)                          |
|--------------------------------|---------------------------------|
| • <sup>2</sup> $\sqrt{\quad}$  | • <sup>7</sup> $\sqrt{\quad}$   |
| • <sup>3</sup> $\sqrt{\quad}$  | • <sup>8</sup> $X$              |
| • <sup>4</sup> $X$             | • <sup>9</sup> $X$              |
| • <sup>5</sup> $X\sqrt{\quad}$ | • <sup>10</sup> $X\sqrt{\quad}$ |
| • <sup>6</sup> $\sqrt{\quad}$  | • <sup>11</sup> $X$             |
|                                | • <sup>12</sup> $X$             |
- In (c) candidates who deal with  $f(x)$  and  $g(x)$  separately and **add** can only earn at most
  - <sup>8</sup> correct limits
  - <sup>9</sup> for correct integral of  $f(x)$
  - <sup>10</sup> for correct integral of  $g(x)$
  - <sup>11</sup> for correct substitution.

Alternative for •<sup>3</sup>, •<sup>4</sup>, •<sup>5</sup>

Option 1

$$\begin{aligned} \bullet^3 \quad \cos^2 x &= \frac{10}{14} \\ \bullet^4 \quad \cos x &= \sqrt{\frac{10}{14}}, \quad \cos x = -\sqrt{\frac{10}{14}} \\ \bullet^5 \quad x &= 0.6 \quad x = 2.6 \end{aligned}$$

Option 2

$$\begin{aligned} \bullet^3 \quad \cos^2 x &= \frac{10}{14} \\ \bullet^4 \quad \cos x &= \sqrt{\frac{10}{14}} \quad \text{and} \quad x = 0.6 \\ \bullet^5 \quad \cos x &= -\sqrt{\frac{10}{14}} \quad \text{and} \quad x = 2.6 \end{aligned}$$

Option 3

$$\begin{aligned} \bullet^3 \quad \sin^2 x &= \frac{4}{14} \\ \bullet^4 \quad \sin x &= \sqrt{\frac{4}{14}} \\ \bullet^5 \quad x &= 0.6, \quad x = 2.6 \end{aligned}$$

Alternative for •<sup>9</sup>, •<sup>10</sup>

$$\begin{aligned} \bullet^9 \quad &-4 \sin 2x - 3 \sin 2x \\ \bullet^{10} \quad &3x - \frac{4}{2} \sin 2x - \frac{3}{2} \sin 2x \end{aligned}$$

# Higher Mathematics 2009 v10

| qu   |   | Mk | Code    | cal | Source | ss | pd | ic | C | B | A |  | U1 | U2 | U3 |  |
|------|---|----|---------|-----|--------|----|----|----|---|---|---|--|----|----|----|--|
| 2.06 | a | 2  | A30, 34 | cr  | 08532  |    | 1  | 1  |   | 2 |   |  |    |    | 2  |  |
|      | b | 3  | A30, 34 | cr  |        | 1  | 1  | 1  |   | 3 |   |  |    |    | 3  |  |

2.06

The size of the human population,  $N$ , can be modelled using the equation  $N = N_0 e^{rt}$  where  $N_0$  is the population in 2006,  $t$  is the time in years since 2006, and  $r$  is the annual rate of increase in the population.

- (a) In 2006 the population of the United Kingdom was approximately 61 million, with an annual rate of increase of 1.6%. Assuming this growth rate remains constant, what would be the population in 2020 ? 2
- (b) In 2006 the population of Scotland was approximately 5.1 million, with an annual rate of increase of 0.43%. Assuming this growth rate remains constant, how long would it take for Scotland's population to double in size ? 3

The primary method m.s. is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide

but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- <sup>1</sup> ic substitute into equation
- <sup>2</sup> pd evaluate exponential expression
- <sup>3</sup> ic interpret info and substitute
- <sup>4</sup> ss convert expo. equ. to log. equ.
- <sup>5</sup> pd process

**Primary Method : Give 1 mark for each •**

- <sup>1</sup>  $61e^{0.016 \times 14}$
- <sup>2</sup> 76 million *or equiv.*
- <sup>3</sup>  $10.2 = 5.1e^{0.0043t}$
- <sup>4</sup>  $0.0043t = \ln 2$
- <sup>5</sup>  $t = 161.2$  years

## Notes

- For •<sup>2</sup>, do not accept 76.  
Accept any answer which rounds to 76 million and was obtained from legitimate sources.
- <sup>5</sup> is for a rounded up answer or implying a rounded-up answer.  
Acceptable answers would include 162 and 161.2 but not 161.
- Cave**  
Beware of poor imitations which yield results similar/same to that given in the paradigm, e.g.  
compound percentage  
*or* recurrence relations.  
These can receive no credit but see Common Error 2 for exception.

## Common Errors

- Candidates who misread the rate of increase:

- <sup>1</sup>  $X$   $61e^{1.6 \times 14}$
- <sup>2</sup>  $X \checkmark$   $3.26 \times 10^{11}$  million
- <sup>3</sup>  $X \checkmark$   $10.2 = 5.1e^{0.43t}$
- <sup>4</sup>  $X \checkmark$   $0.43t = \ln 2$
- <sup>5</sup>  $X \checkmark$   $t = 1.612$

## 2

- <sup>1</sup>  $X$   $61 \times 1.016^{14}$
- <sup>2</sup>  $X$  76 million
- <sup>3</sup>  $X$   $10.2 = 5.1 \times 1.0043^t$
- <sup>4</sup>  $X \checkmark$   $t \ln 1.0043 = \ln 2$
- <sup>5</sup>  $X \checkmark$   $t = 162$

**i.e. award 2 marks**

## Options

- <sup>1</sup>  $61000000e^{0.016 \times 14}$
  - <sup>2</sup> 76000000
- <sup>1</sup>  $(61 \text{ million}) \times e^{0.016 \times 14}$
  - <sup>2</sup> 76 million
- <sup>1</sup>  $61000000e^{0.224}$
  - <sup>2</sup> 76 million
- <sup>1</sup>  $(61 \text{ million}) \times e^{0.224}$
  - <sup>2</sup> 76000000

| qu   |   | Mk | Code    | cal | Source | ss | pd | ic | C | B | A |  | U1 | U2 | U3 |  |
|------|---|----|---------|-----|--------|----|----|----|---|---|---|--|----|----|----|--|
| 2.07 | a | 6  | G29, 26 | cn  | 09031  | 1  | 2  | 3  |   | 6 |   |  |    |    | 6  |  |
|      | b | 4  | G21, 30 | cr  |        | 1  | 1  | 2  |   | 2 | 2 |  |    |    | 4  |  |

2.07

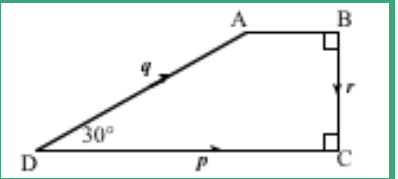
Vectors  $p$ ,  $q$  and  $r$  are represented on the diagram shown where angle  $ADC = 30^\circ$ . It is also given that  $|p| = 4$  and  $|q| = 3$ .

(a) Evaluate  $p \cdot (q + r)$  and  $r \cdot (p - q)$ .

6

(b) Find  $|q + r|$  and  $|p - q|$ .

4



The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide

but only where a candidate does not use the primary method or any

alternative method shown in detail in the marking scheme.

- <sup>1</sup> ss use distributive law
- <sup>2</sup> ic interpret scalar product
- <sup>3</sup> pd processing scalar product
- <sup>4</sup> ic interpret perpendicularity
- <sup>5</sup> ic interpret scalar product
- <sup>6</sup> pd complete processing
- <sup>7</sup> ic interpret vectors on a 2-D diagram
- <sup>8</sup> pd evaluate magnitude of vector sum
- <sup>9</sup> ic interpret vectors on a 2-D diagram
- <sup>10</sup> pd evaluate magnitude of vector difference

Primary Method : Give 1 mark for each •

- <sup>1</sup>  $p \cdot q + p \cdot r$  s / i by (•<sup>2</sup> and •<sup>4</sup>)
- <sup>2</sup>  $4 \times 3 \cos 30^\circ$  s / i by •<sup>3</sup>
- <sup>3</sup>  $6\sqrt{3}$  (10.4)
- <sup>4</sup>  $p \cdot r = 0$  explicitly stated
- <sup>5</sup>  $-|r| \times 3 \cos 120^\circ$
- <sup>6</sup>  $r = \frac{3}{2}$  and ...  $\frac{9}{4}$
- <sup>7</sup>  $q + r \equiv$  from D to the projection of A onto DC
- <sup>8</sup>  $|q + r| = \frac{3\sqrt{3}}{2}$
- <sup>9</sup>  $p - q \equiv \overline{AC}$
- <sup>10</sup>  $|p - q| = \sqrt{\left(4 - \frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$  (2.05)

### Notes

- $p \cdot (q + r) = pq + pr$  gains no marks unless the "vectors" are treated correctly further on. In this case treat this as bad form.
- The evidence for •<sup>7</sup> and •<sup>9</sup> will likely appear in a diagram with the vectors  $q + r$  and  $p - q$  clearly marked.

### Common Errors

- For •<sup>1</sup> to •<sup>4</sup>  
 $p \cdot (q + r) = p \cdot q + p \cdot r$   
 $= 4 \times 3 + 4 \times \frac{3}{2}$   
 $= 18$   
 can only be awarded •<sup>1</sup>.

### Alternatives 1

- For •<sup>7</sup> and •<sup>8</sup> :  
 $\sqrt{p \cdot (q + r)} = |p| |q + r| \cos 0$   
 $6\sqrt{3} = 4 |q + r| \times 1$   
 $\sqrt{p \cdot (q + r)} = \frac{6\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}$
- For •<sup>9</sup>, •<sup>10</sup> :  
 Using right-angled  $\Delta ABC$   
 $\overline{AC} = p - q$ ,  
 and  $|\overline{AB}| = 4 - \frac{3\sqrt{3}}{2}$ ,  $|\overline{BC}| = \frac{3}{2}$   
 and  $\hat{ACB} = 43.06^\circ$   
 use  $r \cdot (p - q) = \frac{9}{4}$   
 to get  $|p - q| = 2.05$

### Alternatives 2

- For •<sup>7</sup>, •<sup>8</sup>, •<sup>9</sup>, •<sup>10</sup> :  
 Set up a coord system with origin at D  
 $C = (4, 0)$ ,  $A = \left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ ,  $B = \left(4, \frac{3}{2}\right)$
- $p = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ ,  $q = \begin{pmatrix} \frac{3\sqrt{3}}{2} \\ \frac{3}{2} \end{pmatrix}$ ,  $r = \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix}$
- $q + r = \begin{pmatrix} \frac{3\sqrt{3}}{2} \\ 0 \end{pmatrix}$  and  $|q + r| = 2.60$
- $p - q = \begin{pmatrix} 4 - \frac{3\sqrt{3}}{2} \\ -\frac{3}{2} \end{pmatrix}$  and  $|p - q| = 2.05$



# Higher Mathematics 2009 v10

Marks : May 2009

| Centre/group |   |  |  |  |  |        |   |  |  |  |  | totals |   |
|--------------|---|--|--|--|--|--------|---|--|--|--|--|--------|---|
| cand no.     |   |  |  |  |  |        |   |  |  |  |  |        |   |
| 21a          | 1 |  |  |  |  | 21a    | 1 |  |  |  |  | 21a    | 1 |
| 21b          | 3 |  |  |  |  | 21b    | 3 |  |  |  |  | 21b    | 3 |
| 21c          | 4 |  |  |  |  | 21c    | 4 |  |  |  |  | 21c    | 4 |
| 22a          | 4 |  |  |  |  | 22a    | 4 |  |  |  |  | 22a    | 4 |
| 22b          | 4 |  |  |  |  | 22b    | 4 |  |  |  |  | 22b    | 4 |
| 23a          | 2 |  |  |  |  | 23a    | 2 |  |  |  |  | 23a    | 2 |
| 23b          | 3 |  |  |  |  | 23b    | 3 |  |  |  |  | 23b    | 3 |
| 24a          | 3 |  |  |  |  | 24a    | 3 |  |  |  |  | 24a    | 3 |
| 24b          | 2 |  |  |  |  | 24b    | 2 |  |  |  |  | 24b    | 2 |
| 24c          | 4 |  |  |  |  | 24c    | 4 |  |  |  |  | 24c    | 4 |
| 1            | 8 |  |  |  |  | 1      | 8 |  |  |  |  | 1      | 8 |
| 2a           | 3 |  |  |  |  | 2a     | 3 |  |  |  |  | 2a     | 3 |
| 2b           | 3 |  |  |  |  | 2b     | 3 |  |  |  |  | 2b     | 3 |
| 3a           | 4 |  |  |  |  | 3a     | 4 |  |  |  |  | 3a     | 4 |
| 3b           | 5 |  |  |  |  | 3b     | 5 |  |  |  |  | 3b     | 5 |
| 4a           | 1 |  |  |  |  | 4a     | 1 |  |  |  |  | 4a     | 1 |
| 4b           | 5 |  |  |  |  | 4b     | 5 |  |  |  |  | 4b     | 5 |
| 4c           | 4 |  |  |  |  | 4c     | 4 |  |  |  |  | 4c     | 4 |
| 5a           | 1 |  |  |  |  | 5a     | 1 |  |  |  |  | 5a     | 1 |
| 5b           | 5 |  |  |  |  | 5b     | 5 |  |  |  |  | 5b     | 5 |
| 5c           | 6 |  |  |  |  | 5c     | 6 |  |  |  |  | 5c     | 6 |
| 6a           | 2 |  |  |  |  | 6a     | 2 |  |  |  |  | 6a     | 2 |
| 6b           | 3 |  |  |  |  | 6b     | 3 |  |  |  |  | 6b     | 3 |
| 7a           | 6 |  |  |  |  | 7a     | 6 |  |  |  |  | 7a     | 6 |
| 7b           | 4 |  |  |  |  | 7b     | 4 |  |  |  |  | 7b     | 4 |
| totals       |   |  |  |  |  | totals |   |  |  |  |  |        |   |

| Centre/group |   |  |  |  |  |        |   |  |  |  |  | totals |   |
|--------------|---|--|--|--|--|--------|---|--|--|--|--|--------|---|
| cand.no      |   |  |  |  |  |        |   |  |  |  |  |        |   |
| 21a          | 1 |  |  |  |  | 21a    | 1 |  |  |  |  | 21a    | 1 |
| 21b          | 3 |  |  |  |  | 21b    | 3 |  |  |  |  | 21b    | 3 |
| 21c          | 4 |  |  |  |  | 21c    | 4 |  |  |  |  | 21c    | 4 |
| 22a          | 4 |  |  |  |  | 22a    | 4 |  |  |  |  | 22a    | 4 |
| 22b          | 4 |  |  |  |  | 22b    | 4 |  |  |  |  | 22b    | 4 |
| 23a          | 2 |  |  |  |  | 23a    | 2 |  |  |  |  | 23a    | 2 |
| 23b          | 3 |  |  |  |  | 23b    | 3 |  |  |  |  | 23b    | 3 |
| 24a          | 3 |  |  |  |  | 24a    | 3 |  |  |  |  | 24a    | 3 |
| 24b          | 2 |  |  |  |  | 24b    | 2 |  |  |  |  | 24b    | 2 |
| 24c          | 4 |  |  |  |  | 24c    | 4 |  |  |  |  | 24c    | 4 |
| 1            | 8 |  |  |  |  | 1      | 8 |  |  |  |  | 1      | 8 |
| 2a           | 3 |  |  |  |  | 2a     | 3 |  |  |  |  | 2a     | 3 |
| 2b           | 3 |  |  |  |  | 2b     | 3 |  |  |  |  | 2b     | 3 |
| 3a           | 4 |  |  |  |  | 3a     | 4 |  |  |  |  | 3a     | 4 |
| 3b           | 5 |  |  |  |  | 3b     | 5 |  |  |  |  | 3b     | 5 |
| 4a           | 1 |  |  |  |  | 4a     | 1 |  |  |  |  | 4a     | 1 |
| 4b           | 5 |  |  |  |  | 4b     | 5 |  |  |  |  | 4b     | 5 |
| 4c           | 4 |  |  |  |  | 4c     | 4 |  |  |  |  | 4c     | 4 |
| 5a           | 1 |  |  |  |  | 5a     | 1 |  |  |  |  | 5a     | 1 |
| 5b           | 5 |  |  |  |  | 5b     | 5 |  |  |  |  | 5b     | 5 |
| 5c           | 6 |  |  |  |  | 5c     | 6 |  |  |  |  | 5c     | 6 |
| 6a           | 2 |  |  |  |  | 6a     | 2 |  |  |  |  | 6a     | 2 |
| 6b           | 3 |  |  |  |  | 6b     | 3 |  |  |  |  | 6b     | 3 |
| 7a           | 6 |  |  |  |  | 7a     | 6 |  |  |  |  | 7a     | 6 |
| 7b           | 4 |  |  |  |  | 7b     | 4 |  |  |  |  | 7b     | 4 |
| totals       |   |  |  |  |  | totals |   |  |  |  |  |        |   |

**the end**