H

National
Qualifications
EXEMPLAR PAPER ONLY

EP30/H/02

Mathematics
Paper 2

Marking Instructions

These Marking Instructions have been provided to show how SQA would mark this Exemplar Question Paper.

SQA

General Marking Principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the Detailed Marking Instructions, which identify the key features required in candidate responses.

(a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) Credit must be assigned in accordance with the specific assessment guidelines.

(d) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(e) Working subsequent to an error must be followed through, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is easier, candidates lose the opportunity to gain credit.

(f) Where transcription errors occur, candidates would normally lose the opportunity to gain a processing mark.

(g) Scored-out or erased working which has not been replaced should be marked where still legible. However, if the scored-out or erased working has been replaced, only the work which has not been scored out should be judged.

(h) Unless specifically mentioned in the specific assessment guidelines, do not penalise:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in solutions
- a repeated error within a question

Definitions of Mathematics-specific command words used in this Paper are:

Determine: obtain an answer from given facts, figures or information;

Expand: multiply out an algebraic expression by making use of the distributive law or a compound trigonometric expression by making use of one of the addition formulae for $\sin(A \pm B)$ or $\cos(A \pm B)$;

Express: use given information to rewrite an expression in a specified form;

Find: obtain an answer showing relevant stages of working;

Hence: use the previous answer to proceed;

Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used;

Identify: provide an answer from a number of possibilities;

Justify: show good reason(s) for the conclusion(s) reached;

Show that: use mathematics to prove something, eg that a statement or given value is correct — all steps, including the required conclusion, must be shown;

Sketch: give a general idea of the required shape or relationship and annotate with all relevant points and features;

Solve: obtain the answer(s) using algebraic and/or numerical and/or graphical methods.

Detailed Marking Instructions for each question

<table>
<tr><th colspan="3">Question</th><th>Expected Response
(Give one mark for each •)</th><th>Max mark</th><th>Additional Guidance
(Illustration of evidence for awarding a mark at each •)</th></tr>
<tr><td rowspan="2">1</td><td colspan="2" rowspan="2">(a)</td><td>$u_1 = 8$ and $u_2 = -4$</td><td rowspan="2">1</td><td></td></tr>
<tr><td>●¹ find terms of sequence</td><td>●¹ $u_1 = 8$ and $u_2 = -4$</td></tr>
<tr><td rowspan="2">1</td><td colspan="2" rowspan="2">(b)</td><td>$p = 2$ or $q = -3$</td><td rowspan="2">3</td><td></td></tr>
<tr><td>●² interpret sequence
●³ solve for one variable
●⁴ state second variable</td><td>●² eg $4p + q = 5$ and $5p + q = 7$
●³ $p = 2$ or $q = -3$
●⁴ $q = -3$ or $p = 2$</td></tr>
<tr><td colspan="3">Notes</td><td colspan="3">1 Candidates may use $7p + q = 11$ as one of their equations at ●².
2 Treat equations like $p4 + q = 5$ or $p(4) + q = 5$ as bad form.
3 Candidates should not be penalised for using $u_{n+1} = pu_n + q$.</td></tr>
<tr><td rowspan="3">1</td><td rowspan="3">(c)</td><td rowspan="2">(i)</td><td>$l = 0$, $-1 < p < 1$</td><td rowspan="3">3</td><td></td></tr>
<tr><td>●⁵ know how to find a valid limit
●⁶ calculate a valid limit only</td><td>●⁵ $l = -\frac{1}{2}l$ or $l = \frac{0}{1-\left(-\frac{1}{2}\right)}$
●⁶ $l = 0$</td></tr>
<tr><td>(ii)</td><td>●⁷ state reason</td><td>●⁷ outside interval $-1 < p < 1$</td></tr>
<tr><td colspan="3">Notes</td><td colspan="3">4 Just stating that $l = al + b$ or $l = \frac{b}{1-a}$ is not sufficient for ●⁵.
5 Any calculations based on formulae masquerading as a limit rule cannot gain ●⁵ and ●⁶.
6 For candidates who use "$b = 0$", ●⁶ is only available to those who simplify $\frac{0}{\ldots}$ to 0.
7 Accept $2 > 1$ or $p > 1$ for ●⁷. This may be expressed in words.
8 Candidates who use a without reference to p or 2 cannot gain ●⁷.</td></tr>
</table>

2	**(a)**	P (-3, -1) Q (1, 7)	6	
		$\bullet^1$ rearrange linear equation		Substituting for y $\bullet^1$ $y=2x+5$ **stated or implied by** $\bullet^2$
		$\bullet^2$ substitute into circle		$\bullet^2$ $\ldots(2x+5)^2\ldots-2(2x+5)\ldots$
		$\bullet^3$ express in standard form		$\bullet^3$ $5x^2+10x-15=0$ = 0 must appear at the $\bullet^3$ or $\bullet^4$ stage to gain $\bullet^3$
		$\bullet^4$ start to solve		$\bullet^4$eg $5(x+3)(x-1)$
		$\bullet^5$ state roots		$\bullet^5$ $x=-3$ and $x=1$
		$\bullet^6$ determine corresponding y coordinates		$\bullet^6$ $y=-1$ and $y=7$
				Substituting for x $\bullet^1$ $x=\dfrac{y-5}{2}$ **stated or implied by** $\bullet^2$
				$\bullet^2$ $\left(\dfrac{y-5}{2}\right)^2\ldots-6\left(\dfrac{y-5}{2}\right)\ldots$
				$\bullet^3$ $5y^2-30y-35=0$ = 0 must appear at the $\bullet^3$ or $\bullet^4$ stage to gain $\bullet^3$
				$\bullet^4$eg $5(y+1)(y-7)$
				$\bullet^5$ $y=-1$ and $y=7$
				$\bullet^6$ $x=-3$ and $x=1$
Notes		1 At $\bullet^4$ the quadratic must lead to two real distinct roots for $\bullet^5$ and $\bullet^6$ to be available. 2 Cross marking is available here for $\bullet^5$ and $\bullet^6$. 3 Candidates do not need to distinguish between points P and Q.		

2	(b)	$(x+5)^2+(y-5)^2=40$	6	
		$\bullet^7$ centre of original circle		$\bullet^7$ (3, 1)
		$\bullet^8$ radius of original circle		$\bullet^8$ $\sqrt{40}$ accept $r^2=40$
		Method 1: Using midpoint		**Method 1: Using midpoint**
		$\bullet^9$ midpoint of chord		$\bullet^9$ (-1, 3)
		$\bullet^{10}$ evidence for finding new centre		$\bullet^{10}$ eg stepping out or midpoint formula
		$\bullet^{11}$ centre of new circle		$\bullet^{11}$ (-5, 5)
		$\bullet^{12}$ equation of new circle		$\bullet^{12}$ $(x+5)^2+(y-5)^2=40$
		Method 2: Stepping out using P and Q		**Method 2: Stepping out using P and Q**
		$\bullet^9$ evidence of C_1 to P **or** C_1 to Q		$\bullet^9$ eg stepping out or vector approach
		$\bullet^{10}$ evidence of Q to C_2 **or** P to C_2		$\bullet^{10}$ eg stepping out or vector approach
		$\bullet^{11}$ centre of new circle		$\bullet^{11}$ (-5, 5)
		$\bullet^{12}$ equation of new circle		$\bullet^{12}$ $(x+5)^2+(y-5)^2=40$

Notes

4 The evidence for $\bullet^7$ and $\bullet^8$ may appear in (a).
5 Centre (-5, 5) **without working** in method 1 may still gain $\bullet^{12}$ but not $\bullet^{10}$ or $\bullet^{11}$, in method 2 may still gain $\bullet^{12}$ but not $\bullet^9$, $\bullet^{10}$ or $\bullet^{11}$. Any other centre **without working** in method 1 does not gain $\bullet^{10}$, $\bullet^{11}$ or $\bullet^{12}$, in method 2 does not gain $\bullet^9$, $\bullet^{10}$, $\bullet^{11}$ or $\bullet^{12}$.
6 The centre must have been clearly indicated before it is used at the $\bullet^{12}$ stage.
7 Do not accept, eg $\sqrt{40}^2$ or $39\cdot69$, or any other approximations for $\bullet^{12}$.
8 The evidence for $\bullet^8$ may not appear until the candidate states the radius or equation of the second circle.

3		$-7<p<5$	4	
		$\bullet^1$ substitute into discriminant		$\bullet^1$ $(p+1)^2-4\times1\times9$
		$\bullet^2$ know condition for no real roots		$\bullet^2$ $b^2-4ac<0$
		$\bullet^3$ factorise		$\bullet^3$ $(p-5)(p+7)<0$
		$\bullet^4$ solve for p		$\bullet^4$ $-7<p<5$

4		$\frac{27}{4}$	5	
		$\bullet^1$ know to integrate and interpret limits $\bullet^2$ use "upper − lower" $\bullet^3$ integrate $\bullet^4$ substitute limits $\bullet^5$ evaluate area		$\bullet^1$ $\int_{-3}^{0} \ldots\ldots\ldots$ $\bullet^2$ $\int_{-3}^{0}\left(x^3+3x^2+2x+3\right)-\left(2x+3\right)dx$ $\bullet^3$ $\frac{1}{4}x^4+x^3$ $\bullet^4$ $0-\left(\frac{1}{4}(-3)^4+(-3)^3\right)$ $\bullet^5$ $\frac{27}{4}$ units^2
Notes		1 Where a candidate differentiates one or more terms at $\bullet^3$ then $\bullet^4$ and $\bullet^5$ are not available. 2 Candidates who substitute without integrating at $\bullet^2$ do not gain $\bullet^3$, $\bullet^4$ and $\bullet^5$. 3 Candidates must show evidence that they have considered the upper limit 0 at $\bullet^4$. 4 Where candidates show no evidence for both $\bullet^3$ and $\bullet^4$, but arrive at the correct area, then $\bullet^3$, $\bullet^4$ and $\bullet^5$ are not available. 5 The omission of dx at $\bullet^2$ should not be penalised.		
5	(a)	$\overrightarrow{OB} = 4\mathbf{i} + 4\mathbf{j}$	1	
		$\bullet^1$ state $\overrightarrow{OB}$ in unit vector form		$\bullet^1$ $4\mathbf{i}+4\mathbf{j}$
5	(b)	$\overrightarrow{DB}=\begin{pmatrix}2\\2\\-6\end{pmatrix}$ $\overrightarrow{DM}=\begin{pmatrix}0\\-2\\-6\end{pmatrix}$	3	
		$\bullet^2$ state components of $\overrightarrow{DB}$ $\bullet^3$ state coordinates of M $\bullet^4$ state components of $\overrightarrow{DM}$		$\bullet^2$ $\begin{pmatrix}2\\2\\-6\end{pmatrix}$ $\bullet^3$ $(2,0,0)$ stated, or implied by $\bullet^4$ $\bullet^4$ $\begin{pmatrix}0\\-2\\-6\end{pmatrix}$

5	**(c)**	$40 \cdot 3°$ or $0 \cdot 703$ rads	5	
		$\bullet^5$ know to use scalar product		$\bullet^5$ $\cos B\hat{D}M = \dfrac{\overrightarrow{DB}.\ \overrightarrow{DM}}{\lvert\overrightarrow{DB}\rvert.\lvert\overrightarrow{DM}\rvert}$ **stated or implied by** $\bullet^9$
		$\bullet^6$ find scalar product		$\bullet^6$ $\overrightarrow{DB}.\overrightarrow{DM} = 32$
		$\bullet^7$ find magnitude of a vector		$\bullet^7$ $\lvert\overrightarrow{DB}\rvert = \sqrt{44}$
		$\bullet^8$ find magnitude of a vector		$\bullet^8$ $\lvert\overrightarrow{DM}\rvert = \sqrt{40}$
		$\bullet^9$ evaluate angle BDM		$\bullet^9$ $40 \cdot 3°$ or $0 \cdot 703$ rads
Notes		1 $\bullet^5$ is not available to candidates who evaluate the wrong angle. 2 If candidates do not attempt $\bullet^9$, then $\bullet^5$ is only available if the formula quoted relates to the labelling in the question. 3 $\bullet^9$ should be awarded to any answer which rounds to $40°$ or $0 \cdot 7$ radians. 4 In the event that both magnitudes are equal **or** there is only one non-zero component, $\bullet^8$ is not available.		
6		$\dfrac{27}{2}$	4	
		$\bullet^1$ use distributive law		$\bullet^1$ $\mathbf{p.p} + \mathbf{p.q} + \mathbf{p.r}$
		$\bullet^2$ calculate scalar product		$\bullet^2$ $\mathbf{p.p} = 9$
		$\bullet^3$ calculate scalar product		$\bullet^3$ $\mathbf{p.q} = \dfrac{9}{2}$
		$\bullet^4$ process scalar product $=0$ and complete		$\bullet^4$ $\mathbf{p.r} = 0$ and $\dfrac{27}{2}$
7	**(a)**	$k \approx 0 \cdot 028$	4	
		$\bullet^1$ interpret half-life		$\bullet^1$ $\dfrac{1}{2}P_0 = P_0 e^{-25k}$ **stated or implied by** $\bullet^2$
		$\bullet^2$ process equation		$\bullet^2$ $e^{-25k} = \dfrac{1}{2}$
		$\bullet^3$ write in logarithmic form		$\bullet^3$ $\log_e \dfrac{1}{2} = -25k$
		$\bullet^4$ process for k		$\bullet^4$ $k \approx 0 \cdot 028$
Notes		1 Do not penalise candidates who substitute a numerical value for P_0 in part (a).		

7	**(b)**	No, with reason	4	
		●[5] interpret equation ●[6] process ●[7] state percentage decrease ●[8] justify answer		●[5] $P_t = P_0 e^{-80\times 0\cdot028}$ ●[6] $P_t \approx 0\cdot1065 P_0$ ●[7] 89% ●[8] No, the concentration will not have decreased by over 90%. 89% decrease.
Notes		2 For candidates who use a value of k which does not round to $0\cdot028$, ●[5] is not available unless already penalised in part (a). 3 For a value of k ex-nihilo then ●[5], ●[6] and ●[7] are not available. 4 ●[6] is only available for candidates who express P_t as a multiple of P_0. 5 Beware of candidates using proportion. This is not a valid strategy.		
8		$\frac{3\pi}{8}$	6	
		●[1] start to integrate ●[2] complete integration ●[3] process limits ●[4] simplify numeric term and equate to $\frac{10}{4}$ ●[5] start to solve equation ●[6] solve for a		●[1] $-\frac{5}{4}\cos\ldots$ ●[2] $-\frac{5}{4}\cos\left(4x-\frac{\pi}{2}\right)$ ●[3] $-\frac{5}{4}\cos\left(4a-\frac{\pi}{2}\right)+\frac{5}{4}\cos\left(\frac{4\pi}{8}-\frac{\pi}{2}\right)$ ●[4] $-\frac{5}{4}\cos\left(4a-\frac{\pi}{2}\right)+\frac{5}{4}=\frac{10}{4}$ ●[5] $\cos\left(4a-\frac{\pi}{2}\right)=-1$ ●[6] $a=\frac{3\pi}{8}$
Notes		1 Candidates who include solutions outwith the range cannot gain ●[6]. 2 The inclusion of $+c$ at ●[1] or ●[2] should be treated as bad form. 3 ●[6] is only available for a valid numerical answer. 4 Where the candidate differentiates, ●[1], ●[2] and ●[3] are not available. 5 Where the candidate integrates incorrectly, ●[3], ●[4], ●[5] and ●[6] are still available. 6 The value of a **must** be given in radians.		

9	**(a)**	4 cm	5	
		●¹ prepare to differentiate ●² differentiate ●³ equate derivative to 0 ●⁴ process for x ●⁵ verify nature		●¹ $\ldots 48x^{-1}$ ●² $3-48x^{-2}$ ●³ $3-48x^{-2}=0$ ●⁴ $x=4$ ●⁵ nature table or 2nd derivative
Notes		1 Do not penalise the non-appearance of -4 at ●⁴.		
9	**(b)**	No, (£198 > £195)	2	
		●⁶ evaluate L ●⁷ calculate cost and justify answer		●⁶ $L=24$ ●⁷ $24\times £8\cdot 25$ = £198. No and reason (£198 > £195)
Notes		2 Candidates who process $x=-4$ to obtain $L=-24$ do not gain ●⁶. 3 $y=24$ is not awarded ●⁶.		
10	**(a)**	$a(t)=-16\sin\left(2t-\frac{\pi}{2}\right)$	3	
		●¹ know to differentiate ●² differentiate trig function ●³ applies chain rule		●¹ $a=v'(t)$ ●² $-8\sin\left(2t-\frac{\pi}{2}\right)$..... ●³ ....$\times 2$ and complete $a(t)=-16\sin\left(2t-\frac{\pi}{2}\right)$
Notes		1 Alternatively, $8\cos\left(2t-\frac{\pi}{2}\right)=8\sin 2t$ ●¹ $v'(t)$...... ●² $=8\cos 2t\ldots$ ●³ $=\ldots\times 2$		

10	**(b)**	$a(10) > 0$ therefore increasing	2	
		●[4] know to and evaluate $a(10)$ ●[5] interpret result		●[4] $a(10) = 6 \cdot 53$ ●[5] $a(10) > 0$ therefore increasing
Notes		1 ●[5] is available only as a consequence of substituting into a derivative. 2 ●[4] and ●[5] are not available to candidates who work in degrees. 3 ●[2] and ●[3] may be awarded if they appear in the working for 10(b). However, ●[1] requires a clear link between acceleration and $v'(t)$.		
10	**(c)**	$s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$	3	
		●[6] know to integrate ●[7] integrate correctly ●[8] determine constant and complete		●[6] $s(t) = \int v(t)dt$ ●[7] $s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + c$ ●[8] $c = 8$ so $s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$
Notes		4 ●[7] and ●8 are not available to candidates who work in degrees. However, accept $\int 8\cos(2t - 90)dt$ for ●[6].		

[END OF EXEMPLAR MARKING INSTRUCTIONS]