H

National
Qualifications
SPECIMEN ONLY

S847/76/12

Mathematics
Paper 2

Marking Instructions

These marking instructions have been provided to show how SQA would mark this specimen question paper.

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If a candidate response does not seem to be covered by either the principles or detailed marking instructions, and you are uncertain how to assess it, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.	$x^2 + 5x + 7 = 9x + 4$ $x - 4x + 3 = 0$
This is no longer a solution of a quadratic equation, so the mark is not awarded.	$x = 1$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2 + 5x + 7 = 9x + 4$ $x - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $x = 1$ or 3

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	•5	•6
•5	$x = 2$	$x = -4$
•6	$y = 5$	$y = -7$

Horizontal: •5 $x = 2$ and $x = -4$ Vertical: •5 $x = 2$ and $y = 5$

•6 $y = 5$ and $y = -7$ •6 $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3+2x^2+3x+2)(2x+1)$ written as

  $(x^3+2x^2+3x+2)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$

  gains full credit

- repeated error within a question, but not between questions or papers

(l) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(m) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(n) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(o) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

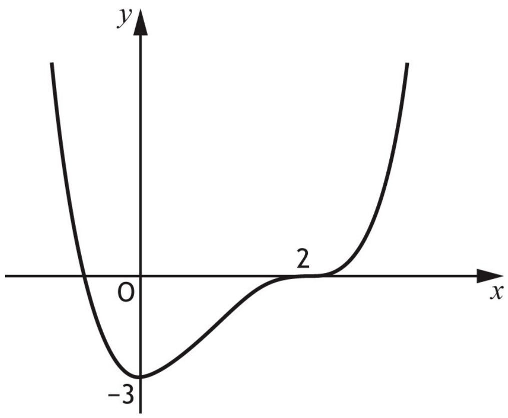
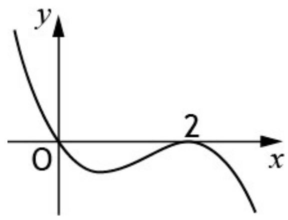
Marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1. (a)	•¹ calculate gradient of AB •² use property of perpendicular lines •³ determine equation of altitude	•¹ $m_{AB} = -3$ •² $m_{alt} = \frac{1}{3}$ •³ $x - 3y = 4$	**3**
1. (b)	•⁴ calculate midpoint of AC •⁵ calculate gradient of median •⁶ determine equation of median	•⁴ $(4,5)$ •⁵ $m_{BM} = 2$ •⁶ $y = 2x - 3$	**3**
1. (c)	•⁷ find x or y coordinate •⁸ find remaining coordinate	•⁷ $x = 1$ or $y = -1$ •⁸ $y = -1$ or $x = 1$	**2**
2.	•¹ write in integrable form •² integrate one term •³ integrate other term •⁴ complete integration and simplify	•¹ $4x + x^{-2}$ •² eg $\frac{4}{2}x^2 + \ldots$ •³ $\ldots\frac{x^{-1}}{-1}$ •⁴ $2x^2 - x^{-1} + c$	**4**
3.	•¹ value of a •² value of b •³ calculate k	•¹ 1 •² -2 •³ -1	**3**

Question	Generic scheme	Illustrative scheme	Max mark
4. (a)			**3**
	●[1] state components of $\overrightarrow{DB}$	●[1] $\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$	
	●[2] state coordinates of M	●[2] $(2,0,0)$ stated or implied by ●[3]	
	●[3] state components of $\overrightarrow{DM}$	●[3] $\begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$	
4. (b)			**5**
	●[4] evaluate $\overrightarrow{DB}.\overrightarrow{DM}$	●[4] 32	
	●[5] evaluate $\lvert\overrightarrow{DB}\rvert$	●[5] $\sqrt{44}$	
	●[6] evaluate $\lvert\overrightarrow{DM}\rvert$	●[6] $\sqrt{40}$	
	●[7] use scalar product	●[7] $\cos BDM = \frac{32}{\sqrt{44}\sqrt{40}}$	
	●[8] calculate angle	●[8] $40 \cdot 3^\circ$ or 0.703 rads	

Question	Generic scheme	Illustrative scheme	Max mark
5.	•¹ know to integrate and interpret limits •² use 'upper — lower' •³ integrate •⁴ substitute limits •⁵ evaluate area	•¹ $\int_{-3}^{0} \ldots dx$ •² $\int_{-3}^{0} \left(x^3+3x^2+2x+3\right)-\left(2x+3\right)dx$ •³ $\frac{1}{4}x^4+x^3$ •⁴ $0-\left(\frac{1}{4}(-3)^4+(-3)^3\right)$ •⁵ $\frac{27}{4}$ units^2	5

Question	Generic scheme	Illustrative scheme	Max mark
6. (a)	**Method 1** •1 identify common factor •2 complete the square •3 process for c and write in required form	**Method 1** •1 $3(x^2+8x\ldots\ldots$ stated or implied by •2 •2 $3(x+4)^2\ldots\ldots$ •3 $3(x+4)^2+2$	3
	Method 2 •1 expand completed square form •2 equate coefficients •3 process for b and c and write in required form	**Method 2** •1 $ax^2+2abx+ab^2+c$ •2 $a=3,\ 2ab=24,\ ab^2+c=50$ •3 $3(x+4)^2+2$	3
6. (b)	•4 differentiate two terms •5 complete differentiation	•4 $3x^2+24x\ldots.$ •5 $\ldots.+50$	2
6. (c)	**Method 1** •6 link with (a) and identify sign of $(x+4)^2$ •7 communicate reason	**Method 1** •6 $f'(x)=3(x+4)^2+2$ and $(x+4)^2\geq 0\ \forall\ x$ •7 $\therefore 3(x+4)^2+2>0 \Rightarrow$ always strictly increasing	2
	Method 2 •6 identify minimum value of $f'(x)$ •7 communicate reason	**Method 2** •6 eg minimum value $=2$ or annotated sketch •7 $2>0 \therefore (f'(x)>0) \Rightarrow$ always strictly increasing	2

Question	Generic scheme	Illustrative scheme	Max mark
7. (a)	●¹ evidence of reflecting in x-axis ●² vertical translation of 2 units identifiable from graph	●¹ reflection of graph in x-axis ●² graph moves parallel to y-axis by 2 units upwards  	**2**
7. (b)	●³ identify roots ●⁴ interpret point of inflexion ●⁵ complete cubic curve	●³ 0 **and** 2 only ●⁴ turning point at $(2,0)$ ●⁵ cubic passing through origin with negative gradient  	**3**

Question	Generic scheme	Illustrative scheme	Max mark
8. (a)	•[1] use compound angle formula •[2] compare coefficients •[3] process for k •[4] process for a and express in required form	•[1] $k\cos x\cos a - k\sin x\sin a$ **stated explicitly** •[2] $k\cos a = 5, k\sin a = 2$ **stated explicitly** •[3] $k = \sqrt{29}$ •[4] $\sqrt{29}\cos(x + 0\cdot38)$	**4**
8. (b)	•[5] equate to 12 and simplify constant terms •[6] use result of part (a) and rearrange •[7] solve for $x + a$ •[8] solve for x	•[5] $5\cos x - 2\sin x = 2$ or $5\cos x - 2\sin x - 2 = 0$ •[6] $\cos(x + 0\cdot3805\ldots) = \frac{2}{\sqrt{29}}$ •[7] $1\cdot1902\ldots,$ | $5\cdot0928\ldots$ •[8] $0\cdot8097\ldots,$ | $4\cdot712\ldots$	**4**

Question		Generic scheme	Illustrative scheme	Max mark
9.	(a)	•[1] equate volume to 100 •[2] obtain an expression for h •[3] demonstrate result	•[1] $V = \pi r^2 h = 100$ •[2] $h = \dfrac{100}{\pi r^2}$ •[3] $A = \pi r^2 + 2\pi r^2 + 2\pi r \times \dfrac{100}{\pi r^2}$ leading to $A = \dfrac{200}{r} + 3\pi r^2$	3
9.	(b)	•[4] start to differentiate •[5] complete differentiation •[6] set derivative to zero •[7] obtain r •[8] verify nature of stationary point •[9] interpret and communicate result	•[4] $A'(r) = 6\pi r \ldots$ •[5] $A'(r) = 6\pi r - \dfrac{200}{r^2}$ •[6] $6\pi r - \dfrac{200}{r^2} = 0$ •[7] $r = \sqrt[3]{\dfrac{100}{3\pi}}$ $(\approx 2\cdot 20)$ metres •[8] table of signs for a derivative when $r = 2\cdot 1974\ldots$ •[9] minimum when $r \approx 2\cdot 20$ (m) or •[8] $A''(r) = 6\pi + \dfrac{400}{r^3}$ •[9] $A''(2\cdot 1974\ldots) > 0 \therefore$ minimum when $r \approx 2\cdot 20$ (m)	6

Question	Generic scheme	Illustrative scheme	Max mark
10.	●¹ start to integrate	●¹ $-\frac{1}{4}\cos\ldots$	6
	●² complete integration	●² $-\frac{1}{4}\cos\left(4x-\frac{\pi}{2}\right)$	
	●³ process limits	●³ $-\frac{1}{4}\cos\left(4a-\frac{\pi}{2}\right)+\frac{1}{4}\cos\left(\frac{4\pi}{8}-\frac{\pi}{2}\right)$	
	●⁴ simplify numeric term and equate to $\frac{1}{2}$	●⁴ $-\frac{1}{4}\cos\left(4a-\frac{\pi}{2}\right)+\frac{1}{4}=\frac{1}{2}$	
	●⁵ start to solve equation	●⁵ $\cos\left(4a-\frac{\pi}{2}\right)=-1$	
	●⁶ solve for a	●⁶ $a=\frac{3\pi}{8}$	
11.	**Method 1**	**Method 1**	3
	●¹ substitute for $\sin 2x$	●¹ $\frac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above	
	●² simplify and factorise	●² $\sin x\left(1-\cos^2 x\right)$	
	●³ substitute for $1-\cos^2 x$ and simplify	●³ $\sin x\times\sin^2 x$ leading to $\sin^3 x$	
	Method 2	**Method 2**	3
	●¹ substitute for $\sin 2x$	●¹ $\frac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above	
	●² simplify and substitute for $\cos^2 x$	●² $\sin x-\sin x\left(1-\sin^2 x\right)$	
	●³ expand and simplify	●³ $\sin x-\sin x+\sin^3 x$ leading to $\sin^3 x$	

Question	Generic scheme	Illustrative scheme	Max mark
12. (a)	**Method 1**	**Method 1**	**3**
	•1 calculate m_{AB}	•1 eg $m_{AB}=\frac{3}{9}=\frac{1}{3}$	
	•2 calculate m_{BC}	•2 eg $m_{BC}=\frac{5}{15}=\frac{1}{3}$	
	•3 interpret result and state conclusion	•3 ... $\Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	
	Method 2	**Method 2**	**3**
	•1 calculate an appropriate vector, eg $\overrightarrow{AB}$	•1 eg $\overrightarrow{AB}=\begin{pmatrix}9\\3\end{pmatrix}$	
	•2 calculate a second vector, eg $\overrightarrow{BC}$ and compare	•2 eg $\overrightarrow{BC}=\begin{pmatrix}15\\5\end{pmatrix}$ $\therefore$ $\overrightarrow{AB}=\frac{3}{5}\overrightarrow{BC}$	
	•3 interpret result and state conclusion	•3 ... $\Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	
	Method 3	**Method 3**	**3**
	•1 calculate m_{AB}	•1 $m_{AB}=\frac{3}{9}=\frac{1}{3}$	
	•2 find equation of line and substitute point	•2 eg, $y-1=\frac{1}{3}(x-2)$ leading to $6-1=\frac{1}{3}(17-2)$	
	•3 communication	•3 since C lies on line A, B and C are collinear	
12. (b)	•4 find radius	•4 $6\sqrt{10}$	**4**
	•5 determine an appropriate ratio	•5 eg $2:3$ or $\frac{2}{5}$ (using B and C) or $3:5$ or $\frac{8}{5}$ (using A and C)	
	•6 find centre	•6 $(8,3)$	
	•7 state equation of circle	•7 $(x-8)^2+(y-3)^2=360$	

Question	Generic scheme	Illustrative scheme	Max mark
13. (a)	•1 interpret half-life	•1 $\frac{1}{2}P_0 = P_0e^{-25k}$ **stated or implied by •2**	**4**
	•2 process equation	•2 $e^{-25k} = \frac{1}{2}$	
	•3 write in logarithmic form	•3 $\log_e \frac{1}{2} = -25k$	
	•4 process for k	•4 $k \approx 0\cdot028$	
13. (b)	•5 interpret equation	•5 $P_t = P_0e^{-80\times0\cdot028}$	**3**
	•6 process	•6 $P_t \approx 0\cdot1065P_0$	
	•7 state percentage decrease	•7 89%	

[END OF SPECIMEN MARKING INSTRUCTIONS]