H

National Qualifications
2023 MODIFIED

X847/76/12

Mathematics
Paper 2

THURSDAY, 4 MAY

10:45 AM – 12:15 PM

Total marks — 65

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product
$\mathbf{a} . \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a} . \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$
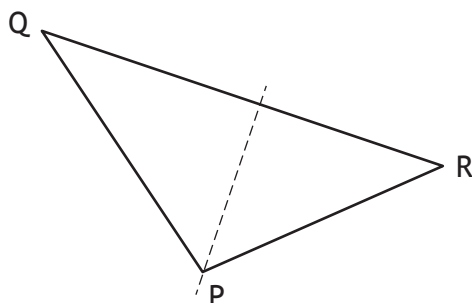
Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\dfrac{1}{a} \cos ax + c$
$\cos ax$	$\dfrac{1}{a} \sin ax + c$

**Total marks — 65**

Attempt ALL questions

1. Triangle PQR has vertices P(5, −1), Q(−2, 8) and R(13, 3).

   (a) Find the equation of the altitude from P.   **3**

   (b) Calculate the angle that the side PR makes with the positive direction of the *x*-axis.   **2**

2. Find the equation of the tangent to the curve with equation $y = 2x^5 - 3x$ at the point where $x = 1$.   **4**

3. Find $\displaystyle\int 7\cos\left(4x + \frac{\pi}{3}\right)dx$.   **2**

[Turn over

4. The diagram shows the cubic graph of $y = f(x)$, with stationary points at $(2, 0)$ and $(0, -2)$.
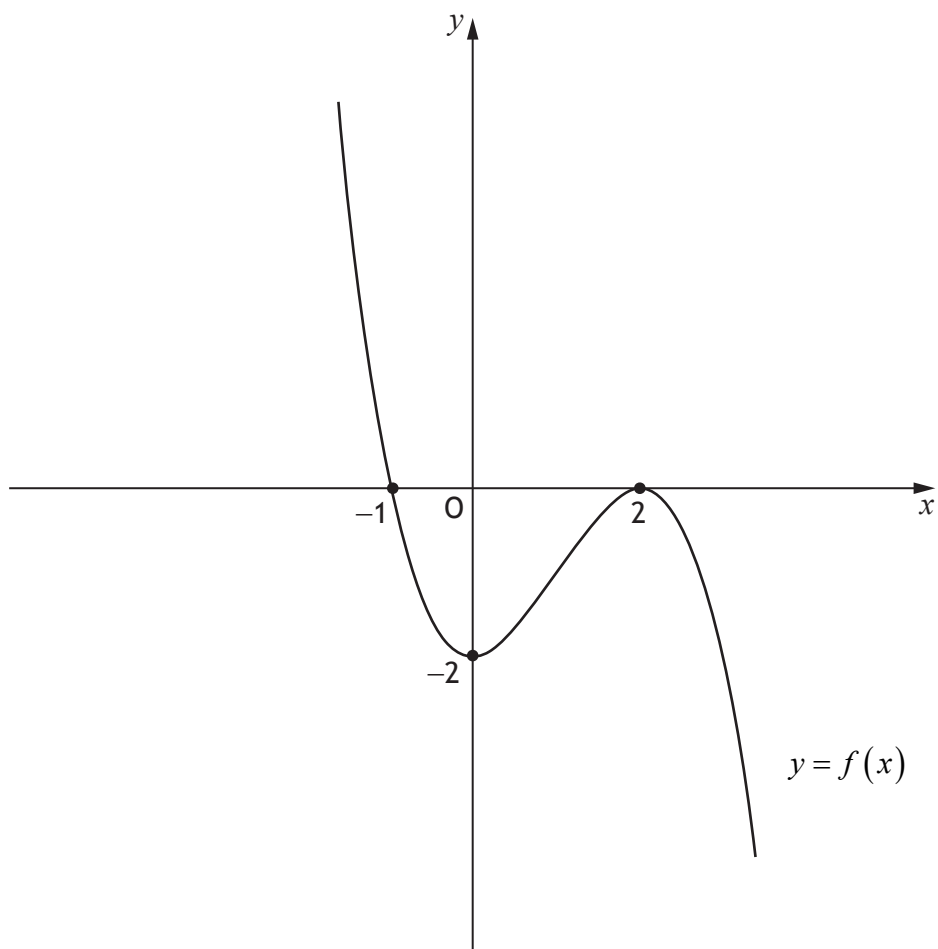
On the diagram in your answer booklet, sketch the graph of $y = 2f(-x)$.　　**2**

5. A function, f, is defined by $f(x) = (3 - 2x)^4$, where $x \in \mathbb{R}$.

Calculate the rate of change of f when $x = 4$.　　**3**

6. A function $f(x)$ is defined by $f(x) = \dfrac{2}{x} + 3$, $x > 0$.
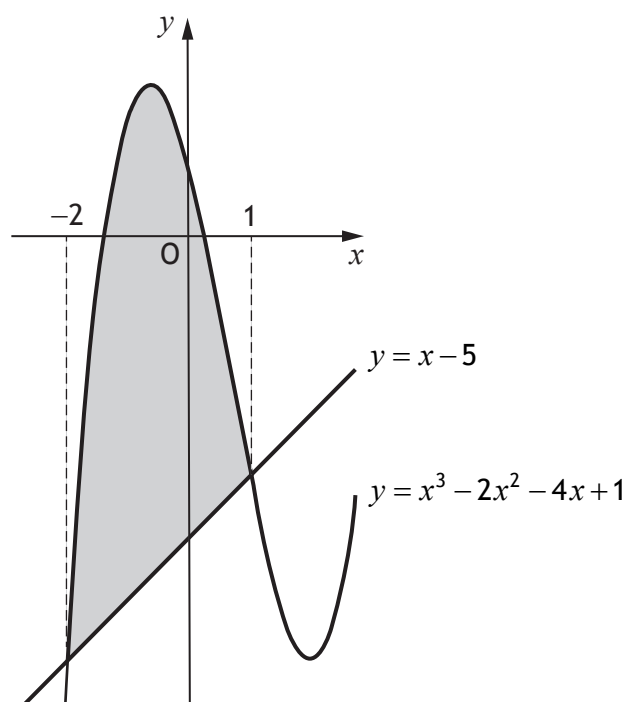
Find the inverse function, $f^{-1}(x)$.

3

7. Solve the equation $\sin x° + 2 = 3\cos 2x°$ for $0 \le x < 360$.

5

8. The diagram shows part of the curve with equation $y = x^3 - 2x^2 - 4x + 1$ and the line with equation $y = x - 5$.

The curve and the line intersect at the points where $x = -2$ and $x = 1$.

Calculate the shaded area.

5

9. (a) Express $7\cos x° - 3\sin x°$ in the form $k\sin(x+a)°$ where $k > 0$, $0 < a < 360$. **4**

    (b) Hence, or otherwise, find:

       (i) the maximum value of $14\cos x° - 6\sin x°$ **1**

       (ii) the value of x for which it occurs where $0 \le x < 360$. **2**

10. Determine the range of values of x for which the function $f(x) = 2x^3 + 9x^2 - 24x + 6$ is strictly decreasing. **4**

11. Circle C_1 has equation $(x-4)^2 + (y+2)^2 = 37$.
Circle C_2 has equation $x^2 + y^2 + 2x - 6y - 7 = 0$.

    (a) Calculate the distance between the centres of C_1 and C_2. **3**

    (b) Hence, show that C_1 and C_2 intersect at two distinct points. **3**

12. A curve, for which $\dfrac{dy}{dx} = 8x^3 + 3$, passes through the point $(-1, 3)$.

Express y in terms of x. **4**

13. A patient is given a dose of medicine.

The concentration of the medicine in the patient's blood is modelled by

$$C_t = 11e^{-0.0053\,t}$$

where:

- t is the time, in minutes, since the dose of medicine was given
- C_t is the concentration of the medicine, in mg/l, at time t.

(a) Calculate the concentration of the medicine 30 minutes after the dose was given. **1**

The dose of medicine becomes ineffective when its concentration falls to 0.66 mg/l.

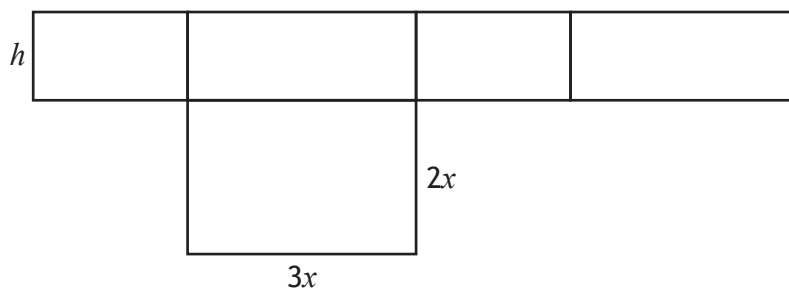(b) Calculate the time taken for this dose of the medicine to become ineffective. **3**

[Turn over

**14.** A net of an open box is shown.

The box is a cuboid with height h centimetres.

The base is a rectangle measuring $3x$ centimetres by $2x$ centimetres.

(a)　(i)　Express the area of the net, A cm^2, in terms of h and x.　　**1**

　　　(ii)　Given that $A = 7200$ cm^2, show that the volume of the box, V cm^3, is given by $V = 4320x - \dfrac{18}{5}x^3$.　　**2**

(b)　Determine the value of x that maximises the volume of the box.　　**4**

15. The line $x + 3y = 17$ is a tangent to a circle at the point $(2, 5)$.
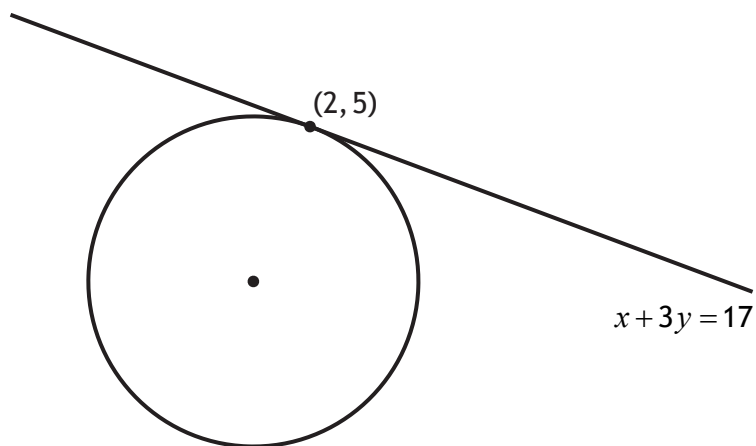
The centre of the circle lies on the y-axis.

Find the coordinates of the centre of the circle.　　**4**

[END OF QUESTION PAPER]