

X100/301

NATIONAL
QUALIFICATIONS
2007

TUESDAY, 15 MAY
9.00 AM – 10.10 AM

**MATHEMATICS
HIGHER**

Units 1, 2 and 3

Paper 1

(Non-calculator)

Read Carefully

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

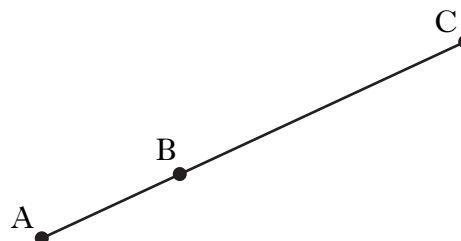
1. Find the equation of the line through the point $(-1, 4)$ which is parallel to the line with equation $3x - y + 2 = 0$.

3

2. Relative to a suitable coordinate system A and B are the points $(-2, 1, -1)$ and $(1, 3, 2)$ respectively.

A, B and C are collinear points and C is positioned such that $BC = 2AB$.

Find the coordinates of C.



4

3. Functions f and g , defined on suitable domains, are given by $f(x) = x^2 + 1$ and $g(x) = 1 - 2x$.

Find:

(a) $g(f(x))$;

2

(b) $g(g(x))$.

2

4. Find the range of values of k such that the equation $kx^2 - x - 1 = 0$ has no real roots.

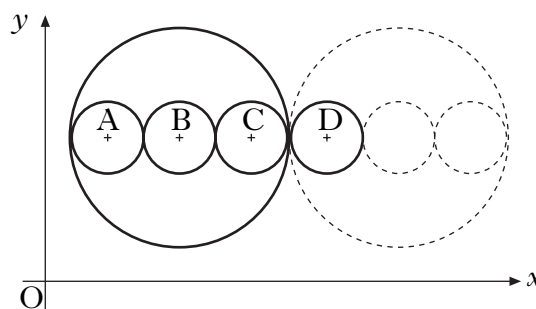
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5. The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x -axis.

This pattern is continued, as shown in the diagram.

Find the equation of the circle with centre D.



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[Turn over

6. Solve the equation $\sin 2x^\circ = 6\cos x^\circ$ for $0 \leq x \leq 360$.

4

7. A sequence is defined by the recurrence relation

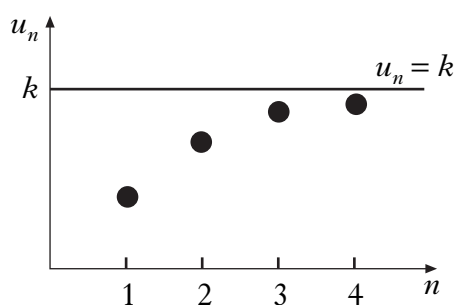
$$u_{n+1} = \frac{1}{4}u_n + 16, \quad u_0 = 0.$$

- (a) Calculate the values of u_1 , u_2 and u_3 .

3

Four terms of this sequence, u_1 , u_2 , u_3 and u_4 are plotted as shown in the graph.

As $n \rightarrow \infty$, the points on the graph approach the line $u_n = k$, where k is the limit of this sequence.



- (b) (i) Give a reason why this sequence has a limit.

- (ii) Find the exact value of k .

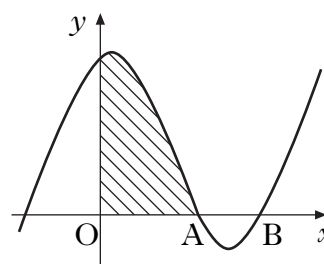
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8. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.

- (a) Show that the graph cuts the x -axis at $(3, 0)$.

- (b) Hence or otherwise find the coordinates of A.

- (c) Find the shaded area.



1

3

5

9. A function f is defined by the formula $f(x) = 3x - x^3$.

- (a) Find the exact values where the graph of $y = f(x)$ meets the x - and y -axes.

2

- (b) Find the coordinates of the stationary points of the function and determine their nature.

7

- (c) Sketch the graph of $y = f(x)$.

1

10. Given that $y = \sqrt{3x^2 + 2}$, find $\frac{dy}{dx}$. 3

11. (a) Express $f(x) = \sqrt{3} \cos x + \sin x$ in the form $k \cos(x - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$. 4

(b) Hence or otherwise sketch the graph of $y = f(x)$ in the interval $0 \leq x \leq 2\pi$. 4

[END OF QUESTION PAPER]