H

National
Qualifications
2019

2019 Mathematics

Higher Paper 2

Finalised Marking Instructions

SQA
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General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$
This is no longer a solution of a quadratic equation, so the mark is not awarded.	$x=1$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$ $(x-3)(x-1)=0$ $x=1$ or 3

(i) Horizontal/vertical marking
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to .. or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3 + 2x^2 + 3x + 2\right)\left(2x + 1\right)$ written as
  $\left(x^3 + 2x^2 + 3x + 2\right) \times 2x + 1$
  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$
  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•¹ calculate the midpoint of AC •² calculate the gradient of BD •³ determine equation of BD	•¹ $(-4, -3)$ •² $-\frac{1}{3}$ •³ $3y = -x - 13$	3

Notes:

1. •² is only available to candidates who use a midpoint to find a gradient.
2. •³ is only available as a consequence of using the midpoint of AC and the point B.
3. At •³ accept any arrangement of a candidate's equation where constant terms have been simplified.
4. •³ is not available as a consequence of using a perpendicular gradient.

Commonly Observed Responses:

Candidate A - Perpendicular Bisector of AC $\text{Midpoint}_{AC}\ (-4,-3)$ •¹ ✓ $m_{AC} = 9 \Rightarrow m_{\perp} = -\frac{1}{9}$ •² ✗ $9y + x + 31 = 0$ •³ $\boxed{✓ 2}$ For other perpendicular bisectors award 0/3	**Candidate B** - Altitude through B $m_{AC} = 9$ •¹ ^ $m_{\perp} = -\frac{1}{9}$ •² ✗ $9y + x = -61$ •³ $\boxed{✓ 2}$
Candidate C - Median through A $\text{Midpoint}_{BC}\ (4,-1)$ •¹ ✗ $m_{AM} = \frac{11}{9}$ •² $\boxed{✓ 1}$ $9y - 11x + 53 = 0$ •³ $\boxed{✓ 2}$	**Candidate D** - Median through C $\text{Midpoint}_{AB}\ (3,-10)$ •¹ ✗ $m_{CM} = -\frac{8}{3}$ •² $\boxed{✓ 1}$ $3y + 8x + 6 = 0$ •³ $\boxed{✓ 2}$

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[4] calculate gradient of BC •[5] use property of perpendicular lines •[6] determine equation of AE	•[4] -1 •[5] 1 •[6] $y = x - 7$	3

Notes:

5. •[6] is only available to candidates who find and use a perpendicular gradient.
6. At •[6] accept any arrangement of a candidate's equation where constant terms have been simplified.

Commonly Observed Responses:

Candidate E

Correct gradient from incorrect substitution

$m_{BC} = \dfrac{-3-11}{6+8} = -1$	•[4] ✘
$m_{AE} = 1$	•[5] $\boxed{✓ 1}$
$y = x - 7$	•[6] $\boxed{✓ 1}$

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[7] find x or y coordinate •[8] find remaining coordinate of the point of intersection	•[7] $x = 2$ or $y = -5$ •[8] $y = -5$ or $x = 2$	2

Notes:

7. For $(2, -5)$ with no working, award 0/2.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] express $6\sqrt{x}$ in integrable form •[2] integrate first term •[3] integrate second term •[4] complete integration	•[1] $6x^{\frac{1}{2}}$ •[2] $\dfrac{6x^{\frac{3}{2}}}{\frac{3}{2}}\ldots$ •[3] $\ldots-\dfrac{4x^{-2}}{-2}\ldots$ •[4] $4x^{\frac{3}{2}}+2x^{-2}+5x+c$	**4**

Notes:

1. •[2] is only available for integrating a term with a fractional index.
2. All coefficients must be simplified at •[4] stage for •[4] to be awarded.
3. Do not penalise the appearance of an integral sign throughout.
4. Do not penalise the omission of '$+c$' at •[2] and •[3].

Commonly Observed Responses:

Candidate A

$\int\left(6x^{\frac{1}{2}}-4x^{-3}+5\right)dx$ •[1] ✓

$=\dfrac{6x^{\frac{3}{2}}}{\frac{3}{2}}-\dfrac{4x^{-2}}{-2}+5x+c$ •[2] ✓ •[3] ✓

$=\boxed{\dfrac{12}{3}}x^{\frac{3}{2}}+2x^{-2}+5x+c$

$=4\sqrt{x^3}+\boxed{\dfrac{2}{\sqrt{x}}}+5x+c$ •[4] ✗

•[4] cannot be awarded over two lines of working

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		$\bullet^1$ identify pathway	$\bullet^1$ $-\mathbf{p}+\mathbf{r}$	**1**

Notes:

1. Accept $-\mathbf{P}+\mathbf{R}$ for $\bullet^1$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^2$ state an appropriate pathway	$\bullet^2$ eg $\overrightarrow{EB}+\overrightarrow{BF}$ stated or implied by $\bullet^3$	**2**
			$\bullet^3$ express pathway in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$	$\bullet^3$ $\mathbf{p}-\mathbf{r}+\frac{3}{4}\mathbf{q}$ or equivalent	

Notes:

2. $\bullet^3$ can only be awarded for a vector expressed in terms of all three of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$.

Commonly Observed Responses:

Candidate A - incorrect expression in $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ and no pathway stated $\mathbf{p}-\mathbf{r}\ldots$ Award 1/2	**Candidate B -** incorrect expression in $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ and no pathway stated $\ldots+\frac{3}{4}\mathbf{q}$ or $\ldots+\mathbf{q}-\frac{1}{4}\mathbf{q}$ Award 1/2

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•1 state values of a and b	•1 $a = 0\cdot973,\ b = 30$	1

Notes:

1. Accept $u_{n+1} = 0\cdot973u_n + 30$ for •1.

Commonly Observed Responses:

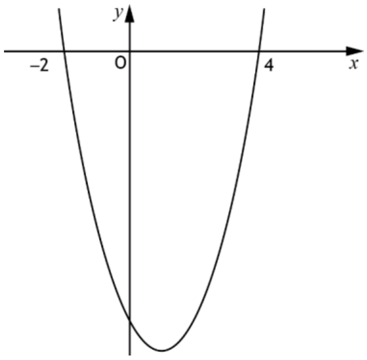
Question			Generic scheme	Illustrative scheme	Max mark
	(b)	(i)	•2 communicate condition for limit to exist	•2 a limit exists as the recurrence relation is linear and $-1 < 0\cdot973 < 1$	1
		(ii)	•3 know how to find limit •4 process limit and state estimated population	•3 $L = 0\cdot973L + 30$ or $L = \dfrac{30}{1-0\cdot973}$ •4 1100	2

Notes:

2. For •2 accept:
   $-1 < 0\cdot973 < 1$ or $\lvert 0\cdot973 \rvert < 1$ or $0 < 0\cdot973 < 1$ with no further comment;
   or statements such as "$0\cdot973$ lies between -1 and 1";
   or $-1 < a < 1$ (as a is previously defined).
3. •2 is not available for:
   $-1 \le 0\cdot973 \le 1$ or $0\cdot973 < 1$;
   or statements such as "it is between -1 and 1"
4. Do not accept $L = \dfrac{b}{1-a}$ with no further working for •3.
5. For $L = 1100$ with no working award •3 and •4.

Commonly Observed Responses:

Candidate A - no rounding required $u_{n+1} = 0\cdot97u_n + 30$ •1 ✗ ⋮ $L = \dfrac{30}{1-0\cdot97}$ •3 ✓1 $L = 1000$ •4 ✓2	**Candidate B** - correct rounding $u_{n+1} = 0\cdot027u_n + 30$ •1 ✗ ⋮ $L = \dfrac{30}{1-0\cdot027}$ •3 ✓1 $L = 0$ •4 ✓1
Candidate C - no valid limit $u_{n+1} = 2\cdot7u_n + 30$ •1 ✗ A limit does not exist as $2\cdot7 > 1$ •2 ✗ $L = \dfrac{30}{1-2\cdot7}$ •3 ✓1 $L = 0$ •4 ✗	

Question			Generic scheme	Illustrative scheme	Max mark
5.			•¹ identify shape and roots •² interpret shape	•¹ parabola with roots at −2 and 4 •² parabola with a minimum turning point at $x=1$ 	**2**

Notes:

1. •¹ and •² are only available for attempting to draw a 'parabola'.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•[1] use compound angle formula •[2] compare coefficients •[3] process for k •[4] process for a and express in required form	•[1] $k\cos x^\circ\cos a^\circ - k\sin x^\circ\sin a^\circ$ **stated explicitly** •[2] $k\cos a^\circ = 2,\ k\sin a^\circ = 3$ **stated explicitly** •[3] $\sqrt{13}$ •[4] $\sqrt{13}\cos(x + 56\cdot 3\ldots)^\circ$	4

Notes:

1. Accept $k(\cos x^\circ\cos a^\circ - \sin x^\circ\sin a^\circ)$ for •[1].
   Treat $k\cos x^\circ\cos a^\circ - \sin x^\circ\sin a^\circ$ as bad form only if the equations at the •[2] stage both contain k.
2. Do not penalise the omission of degree signs.
3. $\sqrt{13}\cos x^\circ\cos a^\circ - \sqrt{13}\sin x^\circ\sin a^\circ$ or $\sqrt{13}(\cos x^\circ\cos a^\circ - \sin x^\circ\sin a^\circ)$ is acceptable for •[1] and •[3].
4. •[2] is not available for $k\cos x^\circ = 2$, $k\sin x^\circ = 3$, however •[4] may still be gained. See Candidate F.
5. Accept $k\cos a^\circ = 2,\ -k\sin a^\circ = -3$ for •[2].
6. •[3] is only available for a single value of k, $k > 0$.
7. •[4] is not available for a value of a given in radians.
8. Accept values of a which round to 56.
9. Candidates may use any form of the wave function for •[1], •[2] and •[3].
   However, •[4] is only available if the wave is interpreted in the form $k\cos(x + a)^\circ$.
10. Evidence for •[4] may not appear until part (b).

Commonly Observed Responses:

Candidate A	Candidate B	Candidate C
•[1] ^	$k\cos x^\circ\cos a^\circ - k\sin x^\circ\sin a^\circ$ •[1] ✓	$\cos x^\circ\cos a^\circ - \sin x^\circ\sin a^\circ$ •[1] ✗
$\sqrt{13}\cos a^\circ = 2$ $\sqrt{13}\sin a^\circ = 3$ •[2] ✓ •[3] ✓	$\cos a^\circ = 2$ $\sin a^\circ = 3$ •[2] ✗	$\cos a^\circ = 2$ $\sin a^\circ = 3$ •[2] $\boxed{✓ 2}$
		$k = \sqrt{13}$ •[3] ✓
$\tan a^\circ = \frac{3}{2}$ $a = 56\cdot 3$	$\tan a^\circ = \frac{3}{2}$ $a = 56\cdot 3$ Not consistent with equations at •[2].	$\tan a^\circ = \frac{3}{2}$ $a = 56\cdot 3$
$\sqrt{13}\cos(x + 56\cdot 3)^\circ$ •[4] ✓	$\sqrt{13}\cos(x + 56\cdot 3)^\circ$ •[3] ✓ •[4] ✗	$\sqrt{13}\cos(x + 56\cdot 3)^\circ$ •[4] ✗

Question	Generic scheme	Illustrative scheme	Max mark
Candidate D - errors at $\bullet^2$ $k\cos x^\circ \cos a^\circ - k\sin x^\circ \sin a^\circ$ $\bullet^1$ ✓ $k\cos a^\circ = 3$ $k\sin a^\circ = 2$ $\bullet^2$ ✗ $\tan a^\circ = \frac{2}{3}$ $a = 33\cdot 7$ $\sqrt{13}\cos(x+33\cdot 7)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓\ 1}$	**Candidate E** - errors at $\bullet^2$ $k\cos x^\circ \cos a^\circ - k\sin x^\circ \sin a^\circ$ $\bullet^1$ ✓ $k\cos a^\circ = 2$ $k\sin a^\circ = -3$ $\bullet^2$ ✗ $\tan a^\circ = -\frac{3}{2}$ $a = 303\cdot 7$ $\sqrt{13}\cos(x+303\cdot 7)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓\ 1}$	**Candidate F** - use of x $k\cos x^\circ \cos a^\circ - k\sin x^\circ \sin a^\circ$ $\bullet^1$ ✓ $k\cos x^\circ = 2$ $k\sin x^\circ = 3$ $\bullet^2$ ✗ $\tan a^\circ = \frac{3}{2}$ $x = 56\cdot 3$ $\sqrt{13}\cos(x+56\cdot 3)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓\ 1}$	
Candidate G $k\cos \mathrm{A} \cos \mathrm{B} - k\sin \mathrm{A} \sin \mathrm{B}$ $\bullet^1$ ✗ $k\cos \mathrm{A}^\circ = 2$ $k\sin \mathrm{A}^\circ = 3$ $\bullet^2$ ✗ Unclear at this stage whether A relates to a or to x. $\tan \mathrm{A}^\circ = \frac{3}{2}$ $a = 56\cdot 3$ $\sqrt{13}\cos(x+56\cdot 3)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓\ 1}$			
(b)	$\bullet^5$ link to (a) $\bullet^6$ solve for $x+a$ $\bullet^7$ solve for x	$\bullet^5$ $\sqrt{13}\cos(x+56\cdot 3\ldots)^\circ = 3$ $\bullet^6$ $33\cdot 69\ldots(393{\cdot}69\ldots)$ $\bullet^6$ $326\cdot 31\ldots$ $\bullet^7$ $337\cdot 38\ldots$ $\bullet^7$ 270	3

Notes:

11. Do not penalise working which rounds to 34, 326, 394 leading to 270 and 337.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		**Method 1** •¹ identify common factor •² complete the square •³ process for r and write in required form	**Method 1** •¹ $-6\left(x^2-4x\ldots\right.$ stated or implied by •² •² $-6\left(x-2\right)^2\ldots$ •³ $-6\left(x-2\right)^2-1$	3
			Method 2 •¹ expand completed square form •² equate coefficients •³ process for q and r and write in required form	**Method 2** •¹ $px^2+2pqx+pq^2+r$ •² $p=-6$, $2pq=24$ $pq^2+r=-25$ •³ $-6\left(x-2\right)^2-1$	

Notes:

1. $-6\left(x-2\right)^2-1$ **with no working** gains •¹ and •² only. However, see Candidate E.
2. •³ is not available in cases where $p>0$.

Commonly Observed Responses:

Candidate A

$-6\left(x^2-4\right)-25$

$-6\left(\left(x-2\right)^2-4\right)-25$ •¹ ✓ •² ✓

$-6\left(x-2\right)^2-1$ •³ ✓

See the exception to general marking principle (h)

Candidate B

$px^2+2pqx+pq^2+r$ •¹ ✓

$p=-6,\ 2pq=24,\ pq^2+r=-25$ •² ✓

$q=-2,\ r=-1$ •³ ^

•³ is lost as answer is not in completed square form

Candidate C

$-6\left(x^2+24x\right)-25$ •¹ ✗

$-6\left(\left(x+12\right)^2-144\right)-25$ •² $\boxed{✓ 1}$

$-6\left(x+12\right)^2+839$ •³ $\boxed{✓ 1}$

Candidate D

$-6\left(\left(x+12\right)^2-144\right)-25$ •¹ ^ •² ✗

$-6\left(x+12\right)^2+839$ •³ $\boxed{✓ 1}$

Candidate E

$-6\left(x-2\right)^2-1$

Check: $=-6\left(x^2-4x+4\right)-1$

$=-6x^2+24x-24-1$

$=-6x^2+24x-25$

Award 3/3

Candidate F

$-6x^2+24x-25$

$=6x^2-24x+25$ •¹ ✗

$=6\left(x^2-4x\ldots\right.$

$=6\left(x-2\right)^2\ldots$ •² $\boxed{✓ 1}$

$=-6\left(x-2\right)^2\ldots$ •³ ✗

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		**Method 1** •4 differentiate •5 link with (a) and identify sign of $(x-2)^2$ •6 communicate reason	**Method 1** •4 $-6x^2+24x-25$ •5 $f'(x)=-6(x-2)^2-1$ and $(x-2)^2\geq 0\ \forall x$ •6 eg $\therefore -6(x-2)^2-1<0\ \forall x$ $\Rightarrow$ always strictly decreasing	3
			Method 2 •4 differentiate •5 identify maximum value of $f'(x)$ •6 communicate reason	**Method 2** •4 $-6x^2+24x-25$ •5 'maximum value is -1' or annotated sketch including x-axis •6 $-1<0$ or 'graph lies below x-axis' $\therefore f'(x)<0\ \forall x$ $\Rightarrow$ always strictly decreasing	

Notes:

3. In Method 1, do not penalise $(x-2)^2>0$ or the omission of $f'(x)$ at •5.
4. In Method 1, accept $-6(x-2)^2\leq 0$ or $-6(x-2)^2<0$ at •5.
5. At •5 communication must be explicitly in terms of the derivative of the given function. Do not accept statements such as '$(\text{something})^2\geq 0$', '$\text{something squared}\geq 0$'.
   However, •6 is still available.

Commonly Observed Responses:

Candidate G $f'(x)=-6x^2+24x-25$ •4 ✓ $f'(x)=-6(x-2)^2-1$ •5 ^ $-6(x-2)^2-1<0$ $\Rightarrow$ strictly decreasing •6 ^	

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		**Method 1** $\bullet^1$ equate composite function to x $\bullet^2$ write $f\left(f^{-1}(x)\right)$ in terms of $f^{-1}(x)$ $\bullet^3$ state inverse function	**Method 1** $\bullet^1$ $f\left(f^{-1}(x)\right)=x$ $\bullet^2$ $\sqrt[3]{f^{-1}(x)}+8=x$ $\bullet^3$ $f^{-1}(x)=(x-8)^3$	**3**
			Method 2 $\bullet^1$ write as $y=f(x)$ and start to rearrange $\bullet^2$ express x in terms of y $\bullet^3$ state inverse function	**Method 2** $\bullet^1$ $y=f(x)\Rightarrow x=f^{-1}(y)$ $y-8=\sqrt[3]{x}$ $\bullet^2$ $x=(y-8)^3$ $\bullet^3$ $f^{-1}(y)=(y-8)^3$ $\Rightarrow f^{-1}(x)=(x-8)^3$	

Notes:

1. In Method 2, accept '$y-8=\sqrt[3]{x}$' without reference to $y=f(x)\Rightarrow x=f^{-1}(y)$ at $\bullet^1$.
2. In Method 2, accept $f^{-1}(x)=(x-8)^3$ without reference to $f^{-1}(y)$ at $\bullet^3$.
3. At $\bullet^3$ stage, accept f^{-1} written in terms of any dummy variable eg $f^{-1}(y)=(y-8)^3$.
4. $y=(x-8)^3$ does not gain $\bullet^3$.
5. $f^{-1}(x)=(x-8)^3$ with no working gains 3/3.

Question			Generic scheme	Illustrative scheme	Max mark
Commonly Observed Responses:					

Candidate A - multiple expressions for $y=f(x)$ $f(x)=\sqrt[3]{x}+8$ $y=\sqrt[3]{x}+8$ $y-8=\sqrt[3]{x}$ $x=(y-8)^3$ $y=(x-8)^3$ $f^{-1}(x)=(x-8)^3$ Award 2/3	**Candidate B** - multiple expressions for $y=f(x)$ $f(x)=\sqrt[3]{x}+8$ $y=\sqrt[3]{x}+8$ $x=\sqrt[3]{y}+8$ $y=(x-8)^3$ $f^{-1}(x)=(x-8)^3$ Award 2/3
Candidate C - **BEWARE** $f'(x)=\ldots$ $\bullet^3$ ✘	**Candidate D** $f^{-1}(x)=x-8^3$ with no working Award 0/3

Candidate E

$x \to \sqrt[3]{x} \to \sqrt[3]{x}+8 = f(x)$

$\sqrt[3]{\ } \to +8$

$\therefore -8 \to (\ )^3$ $\bullet^1$ ✓

$(x-8)^3$ $\bullet^2$ ✓

$f^{-1}(x)=(x-8)^3$ $\bullet^3$ ✓

awarded for knowing to perform inverse operations in reverse

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^4$ state domain	$\bullet^4$ $9 \le x \le 18,\ x \in \mathbb{R}$	1
Notes:					
1. Do not penalise the omission of $x \in \mathbb{R}$.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		$\bullet^1$ identify initial power	$\bullet^1$ 120	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^2$ interpret information	$\bullet^2$ $102 = 120e^{-0\cdot0079t}$ stated or implied by $\bullet^3$	**4**
			$\bullet^3$ process equation	$\bullet^3$ $e^{-0\cdot0079t} = 0\cdot85$	
			$\bullet^4$ write in logarithmic form	$\bullet^4$ $\log_e 0\cdot85 = -0\cdot0079t$	
			$\bullet^5$ process for t	$\bullet^5$ 20·572...	

Notes:

1. Candidates who interpret 15% incorrectly do not gain $\bullet^2$, but $\bullet^3$, $\bullet^4$ and $\bullet^5$ are still available. See Candidate E.
2. $\bullet^3$ may be implied by $\bullet^4$.
3. Any base may be used at $\bullet^4$ stage. See Candidate A.
4. Accept $\ln 0\cdot85 = -0\cdot0079t \ln e$ for $\bullet^4$.
5. Accept 20·57 or 20·6 at $\bullet^5$.
6. The calculation at $\bullet^5$ must follow from the valid use of exponentials and logarithms at $\bullet^3$ and $\bullet^4$.
7. For candidates who take an iterative approach to arrive at $t = 20\cdot6$ award 1/4. However, if, in the iterations P_t is evaluated for $t = 20\cdot55$ and $t = 20\cdot65$ then award 4/4.

Commonly Observed Responses:

Candidate A

$102 = 120e^{-0\cdot0079t}$ $\bullet^2$ ✓

$e^{-0\cdot0079t} = 0\cdot85$ $\bullet^3$ ✓

$\log_{10} 0\cdot85 = -0\cdot0079t \log_{10} e$ $\bullet^4$ ✓

$20\cdot6$ $\bullet^5$ ✓

Candidate B

$102 = 120e^{-0\cdot0079t}$ $\bullet^2$ ✓

$e^{-0\cdot0079t} = 0\cdot85$ $\bullet^3$ ✓

$t = 20\cdot6$ $\bullet^4$ ^ $\bullet^5$ $\boxed{\checkmark\ 1}$

Candidate C

$\log_e 0\cdot85 = -0\cdot0079t$ $\bullet^4$ ✓

$t = 20\cdot6$ years $\bullet^5$ ✓

$t = 20$ years 6 months

Incorrect conversion subsequent to answer is not penalised

Candidate D

$\log_e 0\cdot85 = -0\cdot0079t$ $\bullet^4$ ✓

$t = 20$ years 6 months $\bullet^5$ ✗

Candidate E

$15 = 100e^{-0\cdot0079t}$

$e^{-0\cdot0079t} = 0\cdot15$ $\bullet^3$ $\boxed{\checkmark\ 1}$

$\log_e 0\cdot15 = -0\cdot0079t$ $\bullet^4$ $\boxed{\checkmark\ 1}$

$240\cdot1\ldots$ $\bullet^5$ $\boxed{\checkmark\ 1}$

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		$\bullet^1$ use -3 in synthetic division or in evaluation of quartic $\bullet^2$ complete division/evaluation and interpret result	$\bullet^1$ $\begin{array}{r\|rrrrr} -3 & 3 & 10 & 1 & -8 & -6 \\ \hline & 3 & & & & \end{array}$ or $3\times(-3)^4+10\times(-3)^3+(-3)^2-8\times(-3)-6$ $\bullet^2$ $\begin{array}{r\|rrrr\|r} -3 & 3 & 10 & 1 & -8 & -6 \\ & & -9 & -3 & 6 & 6 \\ \hline & 3 & 1 & -2 & -2 & 0 \end{array}$ Remainder $=0\therefore(x+3)$ is a factor or $f(-3)=0\therefore(x+3)$ is a factor	**2**

Notes:

1. Communication at $\bullet^2$ must be consistent with working at that stage ie a candidate's working must arrive legitimately at 0 before $\bullet^2$ can be awarded.
2. Accept any of the following for $\bullet^2$:
   - '$f(-3)=0$ so $(x+3)$ is a factor'
   - 'since remainder = 0, it is a factor'
   - the '0' from any method linked to the word 'factor' by 'so', 'hence', $\therefore$, $\rightarrow$, $\Rightarrow$ etc.
3. Do not accept any of the following for $\bullet^2$:
   - double underlining the '0' or boxing the '0' without comment
   - '$x=-3$ is a factor', '… is a root'
   - the word 'factor' only, with no link.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•³ identify cubic and attempt to factorise	•³ eg $\begin{array}{c\|cccc} \ldots & 3 & 1 & -2 & -2 \\ & \ldots & \ldots & \ldots & \\ \hline & \ldots & \ldots & \ldots & \ldots \end{array}$	5
			•⁴ find second factor	•⁴ eg $\begin{array}{c\|cccc} 1 & 3 & 1 & -2 & -2 \\ & & 3 & 4 & 2 \\ \hline & 3 & 4 & 2 & 0 \end{array}$ leading to $(x-1)$	
			•⁵ identify quadratic	•⁵ $3x^2+4x+2$	
			•⁶ evaluate discriminant	•⁶ -8	
			•⁷ interpret discriminant and factorise fully	•⁷ since $-8<0$, quadratic has no (real) factors leading to $(x+3)(x-1)(3x^2+4x+2)$	

Notes:

4. Candidates who arrive at $(x+3)(x-1)(3x^2+4x+2)$ by using algebraic long division or by inspection gain •³, •⁴ and •⁵.
5. Evidence for •⁶ may appear in the quadratic formula.
6. Accept '$-8<0$ so no real roots' **with the fully factorised quartic** for •⁷:
7. Do not accept any of the following for •⁷:
   - $(x+3)(x-1)(3x^2+4x+2)$ does not factorise
   - $(x+3)(x-1)(\ldots\ \ldots)(\ldots\ \ldots)$ cannot factorise further.
8. Accept $(x+3)(x-1)3x^2+4x+2$, with a valid reason for •⁷.
9. Where the quadratic factor obtained at •⁵ can be factorised, •⁶ and •⁷ are not available.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$(x+3)(x-1)(3x^2+4x+2)$	•⁵ ✓	$(x+3)(x-1)(3x^2+4x+2)$	•⁵ ✓
$b^2-4ac=16-24<0$	•⁶ ^	$b^2-4ac<0$	•⁶ ^
so does not factorise	•⁷ $\boxed{✓\ 1}$	so does not factorise	•⁷ ^

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		$\bullet^1$ express A in terms of x and h	$\bullet^1$ $(A=)16x^2+16xh$	**3**
			$\bullet^2$ express height in terms of x	$\bullet^2$ $h=\frac{2000}{8x^2}$	
			$\bullet^3$ substitute for h and complete proof	$\bullet^3$ $A=16x^2+16x\times\frac{2000}{8x^2}$ leading to $A=16x^2+\frac{4000}{x}$	

Notes:

1. At $\bullet^1$ accept any unsimplified form of $16x^2+16xh$.
2. The substitution for h at $\bullet^3$ must be clearly shown for $\bullet^3$ to be available.
3. For candidates who omit some of the surfaces of the box, only $\bullet^2$ is available.

Commonly Observed Responses:

	(b)		$\bullet^4$ express A in differentiable form	$\bullet^4$ $16x^2+4000x^{-1}$	**6**
			$\bullet^5$ differentiate	$\bullet^5$ $32x-4000x^{-2}$	
			$\bullet^6$ equate expression for derivative to 0	$\bullet^6$ $32x-4000x^{-2}=0$	
			$\bullet^7$ process for x	$\bullet^7$ 5	
			$\bullet^8$ verify nature	$\bullet^8$ table of signs for a derivative (see below) $\therefore$ minimum or $A''(x)=96>0\ \Rightarrow$ minimum	
			$\bullet^9$ evaluate A	$\bullet^9$ $A=1200$ or min value $=1200$	

Notes:
4. For a numerical approach award 0/6. 5. $\bullet^6$ can be awarded for $32x = 4000x^{-2}$. 6. For candidates who integrate any term at the $\bullet^5$ stage, only $\bullet^6$ is available on follow through for setting their 'derivative' to 0. 7. $\bullet^7$, $\bullet^8$ and $\bullet^9$ are only available for working with a derivative which contains an index ≤ -2. 8. $\sqrt[3]{\frac{4000}{32}}$ must be simplified at $\bullet^7$ or $\bullet^8$ for $\bullet^7$ to be awarded. 9. $\bullet^8$ is not available to candidates who consider a value of $x \leq 0$ in the neighbourhood of 5. 10. $\bullet^9$ is still available in cases where a candidate's table of signs does not lead legitimately to a minimum at $\bullet^8$. 11. $\bullet^8$ and $\bullet^9$ are not available to candidates who state that the minimum exists at a negative value of x. See Candidates C and D.

For the table of signs for a derivative, accept:

x	5^-	5	5^+
$A'(x)$	$-$	0	$+$
Shape or slope	$\backslash$	$\_$	$/$

x	$\rightarrow$	5	$\rightarrow$
$A'(x)$	$-$	0	$+$
Shape or slope	$\backslash$	$\_$	$/$

Arrows are taken to mean 'in the neighbourhood of'

x	a	5	b
$A'(x)$	$-$	0	$+$
Shape or slope	$\backslash$	$\_$	$/$

Where $0 < a < 5$ and $b > 5$

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $A'(x)$ in the table is an acceptable alternative to writing '+' or '–' signs. Values must be checked for accuracy.
- The only acceptable variations of $A'(x)$ are: A', $\frac{dA}{dx}$ and $32x - 4000x^{-2}$.

Commonly Observed Responses:

Candidate A - differentiating over multiple lines

	$\bullet^4$ ^
$A'(x) = 32x + 4000x^{-1}$	
$A'(x) = 32x - 4000x^{-2}$	$\bullet^5$ ✗
$32x - 4000x^{-2} = 0$	$\bullet^6$ $\boxed{✓\ 1}$

Candidate B - differentiating over multiple lines

$A = 16x^2 + 4000x^{-1}$	$\bullet^4$ ✓
$A'(x) = 32x + 4000x^{-1}$	
$A'(x) = 32x - 4000x^{-2}$	$\bullet^5$ ✗
$32x - 4000x^{-2} = 0$	$\bullet^6$ $\boxed{✓\ 1}$

Candidate C - only considers 5

$A = 16x^2 + 4000x^{-1}$	$\bullet^4$ ✓
$A' = 32x - 4000x^{-2} = 0$	$\bullet^5$ ✓ $\bullet^6$ ✓
$x = \pm 5$	$\bullet^7$ ✗

x	$\rightarrow$	5	$\rightarrow$
A'	$-$	0	$+$
	$\backslash$	$\_$	$/$

$\therefore$ minimum	$\bullet^8$ $\boxed{✓\ 1}$
$A = 1200$ or min value $= 1200$	$\bullet^9$ $\boxed{✓\ 1}$

Candidate D - considers 5 and negative 5 in **separate** tables

$A = 16x^2 + 4000x^{-1}$	$\bullet^4$ ✓
$A' = 32x - 4000x^{-2} = 0$	$\bullet^5$ ✓ $\bullet^6$ ✓
$x = \pm 5$	$\bullet^7$ ✗

x	$\rightarrow$	5	$\rightarrow$
A'	$-$	0	$+$
	$\backslash$	$\_$	$/$

x	$\rightarrow$	-5	$\rightarrow$
A'	$-$	0	$+$
	$/$	$-$	$\backslash$

$\therefore$ minimum when $x = 5$	$\bullet^8$ $\boxed{✓\ 1}$
$A = 1200$ or min value $= 1200$	$\bullet^9$ $\boxed{✓\ 1}$

Ignore incorrect working in second table

Question			Generic scheme	Illustrative scheme	Max mark
12.			**Method 1** •[1] state linear equation •[2] introduce logs •[3] use laws of logs •[4] use laws of logs •[5] state a and b	**Method 1** •[1] $\log_4 y = 3x - 1$ •[2] $\log_4 y = 3x\log_4 4 - \log_4 4$ •[3] $\log_4 y = \log_4 4^{3x} - \log_4 4$ •[4] $\log_4 y = \log_4\left(\frac{4^{3x}}{4}\right)$ or $\log_4 y = \log_4 4^{-1}4^{3x}$ •[5] $a = \frac{1}{4}$, $b = 64$	5
			Method 2 •[1] state linear equation •[2] convert to exponential form •[3] use laws of indices •[4] state a •[5] state b	**Method 2** •[1] $\log_4 y = 3x - 1$ •[2] $y = 4^{3x-1}$ •[3] $y = 4^{-1}4^{3x}$ •[4] $a = \frac{1}{4}$ •[5] $b = 64$	5
			Method 3 •[1] introduce logs to $y = ab^x$ •[2] use laws of logs •[3] interpret intercept •[4] interpret gradient •[5] state a and b	**Method 3** **The equations at •[1], •[2], •[3] and •[4] must be stated explicitly.** •[1] $\log_4 y = \log_4 ab^x$ •[2] $\log_4 y = \log_4 a + x\log_4 b$ •[3] $-1 = \log_4 a$ •[4] $3 = \log_4 b$ •[5] $a = \frac{1}{4}$, $b = 64$	5

Question			Generic scheme	Illustrative scheme	Max mark
			Method 4 ●¹ interpret point on log graph ●² convert from log to exponential form ●³ interpret point and convert ●⁴ substitute into $y = ab^x$ and evaluate a ●⁵ substitute other point into $y = ab^x$ and evaluate b	**Method 4** ●¹ $x = 3$ **and** $\log_4 y = 8$ ●² $x = 3$ **and** $y = 4^8$ ●³ $x = 0$ **and** $\log_4 y = -1$ $x = 0$ **and** $y = 4^{-1}$ ●⁴ $4^{-1} = ab^0 \Rightarrow a = \frac{1}{4}$ ●⁵ $4^8 = \frac{1}{4}b^3 \Rightarrow b = 64$	**5**

Notes:

1. In any method, marks may only be awarded within a valid strategy using $y = ab^x$.
2. Accept $y = \frac{1}{4}\cdot 64^x$ for ●⁵.
3. Markers must identify the method which best matches the candidates approach; they must not mix and match between methods.
4. Penalise the omission of base 4 at most once in any method.
5. Do not accept $a = 4^{-1}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.			•1 interpret information given •2 integrate any two terms •3 complete integration •4 interpret information given and substitute •5 process for c and state expression for $f(x)$	•1 $f'(x)=3x^2-16x+11$ or $f(x)=\int(3x^2-16x+11)\,dx$ •2 eg $\frac{3x^3}{3}-\frac{16x^2}{2}\ldots$ •3 $\ldots+11x+c$ •4 $0=7^3-8\times7^2+11\times7+c$ •5 $f(x)=x^3-8x^2+11x-28$	5

Notes:

1. For candidates who make no attempt to integrate to find $f(x)$ award 0/5.
2. Do not penalise the omission of $f(x)$ or dx or the appearance of $+c$ at •1.
3. If any two terms have been integrated correctly •1 may be implied by •2.
4. For candidates who omit $+c$, only •1 and •2 are available.
5. For candidates who differentiate **any** term, •3 •4 and •5 are not available.
6. Candidates must attempt to integrate both terms containing x for •4 and •5 to be available. See Candidate B.
7. Accept $y=x^3-8x^2+11x-28$ at •5 since $y=f(x)$ is defined in the question.
8. Candidates must simplify coefficients in <u>**their**</u> final line of working for the last mark available in that line of working to be awarded.

Commonly Observed Responses:

Candidate A - incomplete substitution		Candidate B - partial integration	
$f(x)=x^3-8x^2+11x+c$	•1 ✓ •2 ✓ •3 ✓	$f(x)=x^3-8x^2+11+c$	•1 ✓ •2 ✓ •3 ✗
$f(x)=7^3-8\times7^2+11\times7+c$	•4 ^	$0=7^3-8\times7^2+11+c$	•4 $\boxed{✓ 1}$
$c=-28$		$c=38$	
$f(x)=x^3-8x^2+11x-28$	•5 $\boxed{✓ 1}$	$f(x)=x^3-8x^2+49$	•5 $\boxed{✓ 1}$

Question			Generic scheme	Illustrative scheme	Max mark
14.			•[1] expand •[2] evaluate $\mathbf{u.u}$ •[3] determine equation in $\cos\theta$ •[4] evaluate angle	•[1] $\mathbf{u.u}+\mathbf{u.v}$ •[2] 16 •[3] $20\cos\theta = 5$ or $\cos\theta = \frac{5}{20}$ •[4] $75\cdot5...^\circ$ or $1\cdot31...$ radians	4

Notes:

1. Do not accept $\mathbf{u}^2$ for •[1], however •[2], •[3] and •[4] are still available.
2. Where there is no evidence for •[1], then •[2], •[3] and •[4] are not available, however see Candidates C and D.
3. Where candidates use $|\mathbf{u}| \neq 4$, then •[3] and •[4] are not available.
4. Where there is no evidence of using $|\mathbf{u}|^2$, •[3] is not available. See Candidate A.
5. Do not penalise omission of units in final answer.
6. Ignore the appearance of $284\cdot5^\circ$.
7. Accept answers which round to 76° or 1·3 radians.

Commonly Observed Responses:

Candidate A $\mathbf{u}.(\mathbf{u}+\mathbf{v}) = \mathbf{u.u}+\mathbf{u.v}$ •[1] ✓ $4+20\cos\theta = 21$ •[2] ✗ $\cos\theta = \frac{17}{20}$ •[3] $\boxed{✓\ 2}$ $\theta = 31\cdot7...^\circ$ •[4] $\boxed{✓\ 1}$	**Candidate B** $16+\mathbf{u.v} = 21$ •[1] ✓ •[2] ✓ $\mathbf{u.v} = 5$ $\cos\theta = \frac{5}{20}$ •[3] ✓ $\theta = 75\cdot5^\circ$ •[4] ✓
Candidate C - missing working $\mathbf{u.u} = 16$ •[2] ✓ $\mathbf{u.v} = 21-16$ $\cos\theta = \frac{5}{20}$ •[1] ✓ •[3] ✓ $\theta = 75\cdot5^\circ$ •[4] ✓	**Candidate D** - missing working $21-16 = 5$ •[1] ^ $\cos\theta = \frac{5}{20}$ •[2] ✓ •[3] ✓ $\theta = 75\cdot5^\circ$ •[4] ✓

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		•¹ find gradient of radius •² state gradient of tangent •³ state equation of tangent	•¹ $-\frac{1}{3}$ •² 3 •³ $y=3x-2$	**3**

Notes:

1. Do not accept $y=\frac{3}{1}x-2$ for •³.
2. •³ is only available as a consequence of trying to find and use a perpendicular gradient.
3. At •³ accept, $y-3x+2=0$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)	(i)	•⁴ find coordinates of T	•⁴ (0,–2)	**1**
		(ii)	•⁵ find midpoint CT •⁶ find radius of circle with diameter CT •⁷ state equation of circle	•⁵ (4,5) •⁶ $\sqrt{65}$ stated or implied by •⁷ •⁷ $(x-4)^2+(y-5)^2=65$	**3**

Notes:

4. Answers in part (b)(i) must be consistent with answers from part (a).
5. Accept $x=0$, $y=-2$ for •⁴.
6. $(x-4)^2+(y-5)^2=\left(\sqrt{65}\right)^2$ does not gain •⁷.
7. •⁷ is not available to candidates who use a line other than CT as the diameter of the circle.

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]