Common (PDF) Distributions with plotting and Brief Explanation

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(◕‿◕✿)

Purpose & Outcome:

- Show Common Distributions and how to plot them using R
- Show their relative PDF formulas and brief explanations

Suggested Book: `Statistics: An Introduction using R, by Michael J. Crawley`

If you want to see a specific topic let me know!

In [124]:
```
library(dplyr)
library(ggplot2)
library(tidyverse)
library(reshape2)
library(tidyr)
```

In [8]:
```
set.seed(65432)
```

Binomial:

discrete

PDF

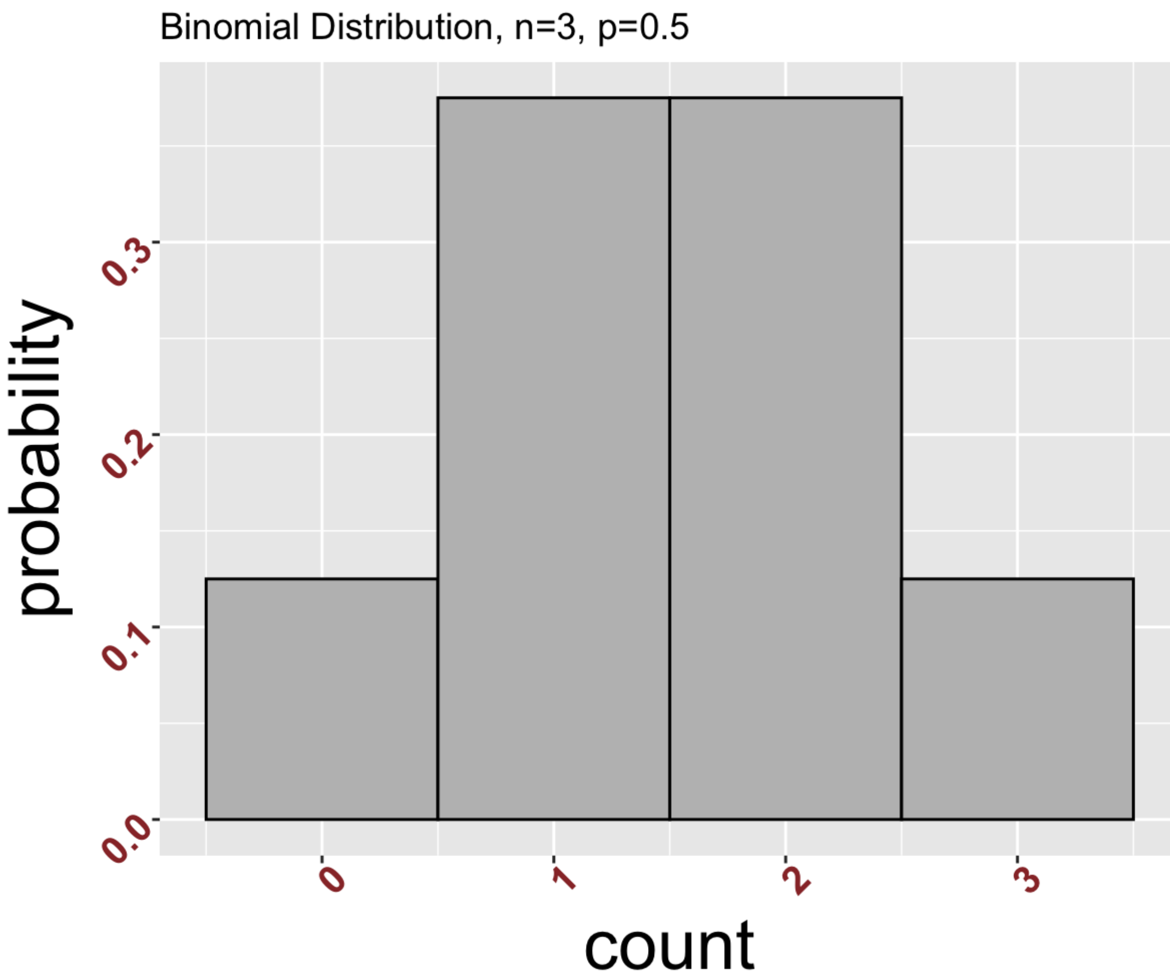
$f(x) = \binom{n}{k} p^x (1 - x)^{(n-x)}$, where x=1,2,3,... n

- `Discrete Probability Function` : describes (n) independent trials.
  - Each `trial` has an outcome of (T/F), (success/failure)
- `If you have (n) trials, with (p) as the successful trials then (x) will be probability of successful trials.`

`Mean` np

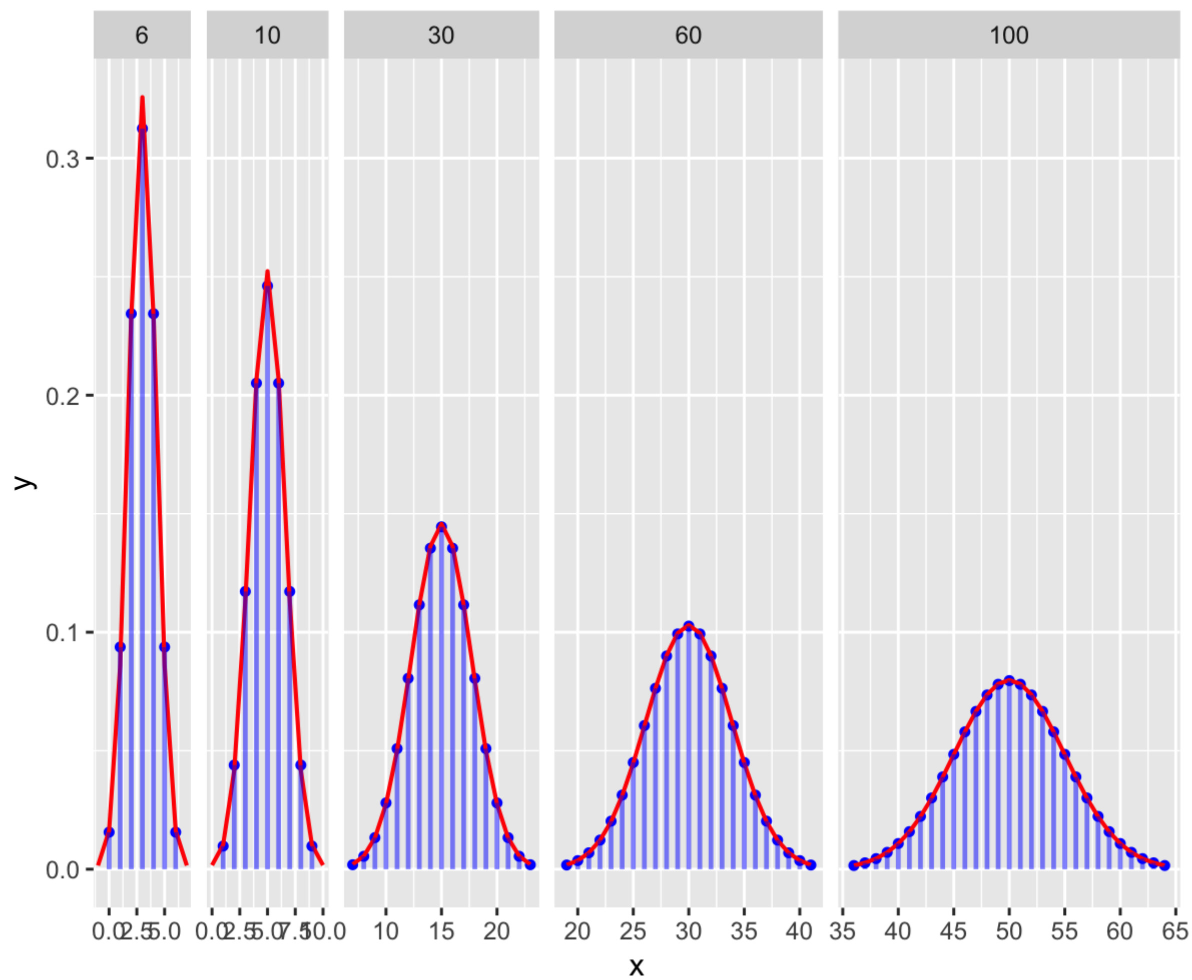
`Standard Deviation` $\sqrt{np(1 - p)}$

In [45]:
```
dat<-data.frame(count=0:3,probability=dbinom(0:3,3,.5))
g<-ggplot(dat,aes(x=count,y=probability))+geom_col(width=1,fill="gray",c
olor="black")
g<-g+labs(title="Binomial Distribution, n=3, p=0.5")

g<-g + theme(axis.title.y = element_text(size = rel(2.3), angle = 90))
g<-g + theme(axis.title.x = element_text(size = rel(2.3), angle = 00))

g<-g + theme(axis.text.x = element_text(face="bold", color="#993333",
                           size=14, angle=45),
          axis.text.y = element_text(face="bold", color="#993333",
                           size=14, angle=45))
g
```



Show the progression as samples increase with p=0.5

In [52]:
```
x <- -5:250
n = c(6,10,30,60,100)
p = 0.5
ymin = 1e-3
normal=data.frame(x=rep(x, length(n)),
                  y=dnorm(x, rep(n,each=length(x))*p,
                (rep(n, each=length(x))*p*(1-p))^0.5),
                  n=rep(n, each=length(x)))
binom = data.frame(x=rep(x, length(n)),
                   y=dbinom(x, rep(n, each=length(x)), p),
                   n=rep(n, each=length(x)))

ggplot(binom %>% filter(y > ymin), aes(x, y)) +
  geom_point(size=1.2, colour="blue") +
  geom_line(data=normal %>% filter(y > ymin), lwd=0.7, colour="red") +
  geom_segment(aes(x=x, xend=x, y=0, yend=y), lwd=0.8, alpha=0.5, colour
="blue") +
  facet_grid(. ~ n, scales="free", space="free")

# citation for link
```



Poisson:

`discrete distribution`

`--------------`

`PDF`

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

- `Probability Distribution` : of independent events **(x)**, for some expected number of events for a time interval **(λ)**.

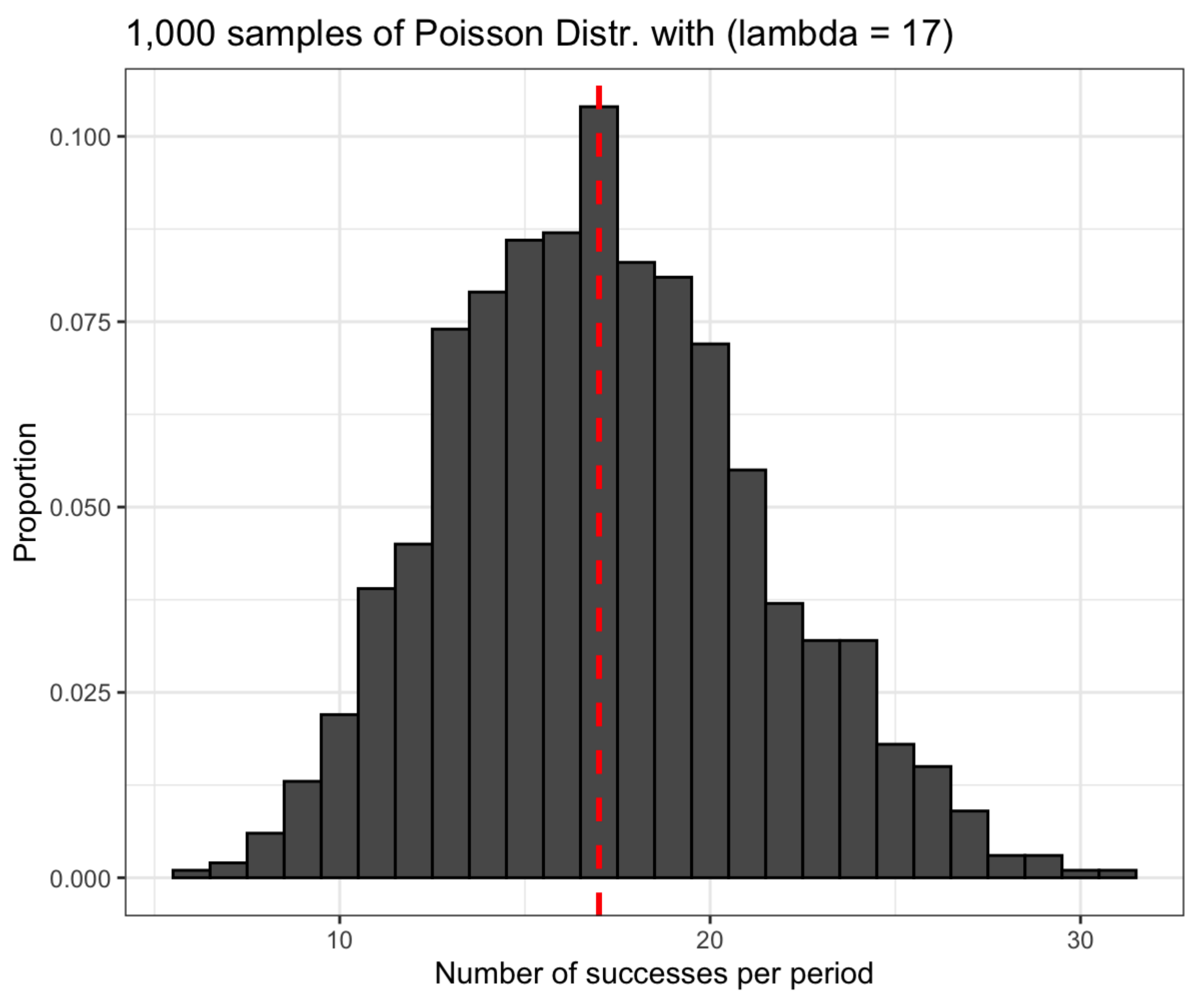
`--------------`

Examples of use cases: `stock market changes, radioactive decay, transportation related to arrivals for example.`

`Mean` λ

`Standard Deviation` $\sqrt{\lambda}$

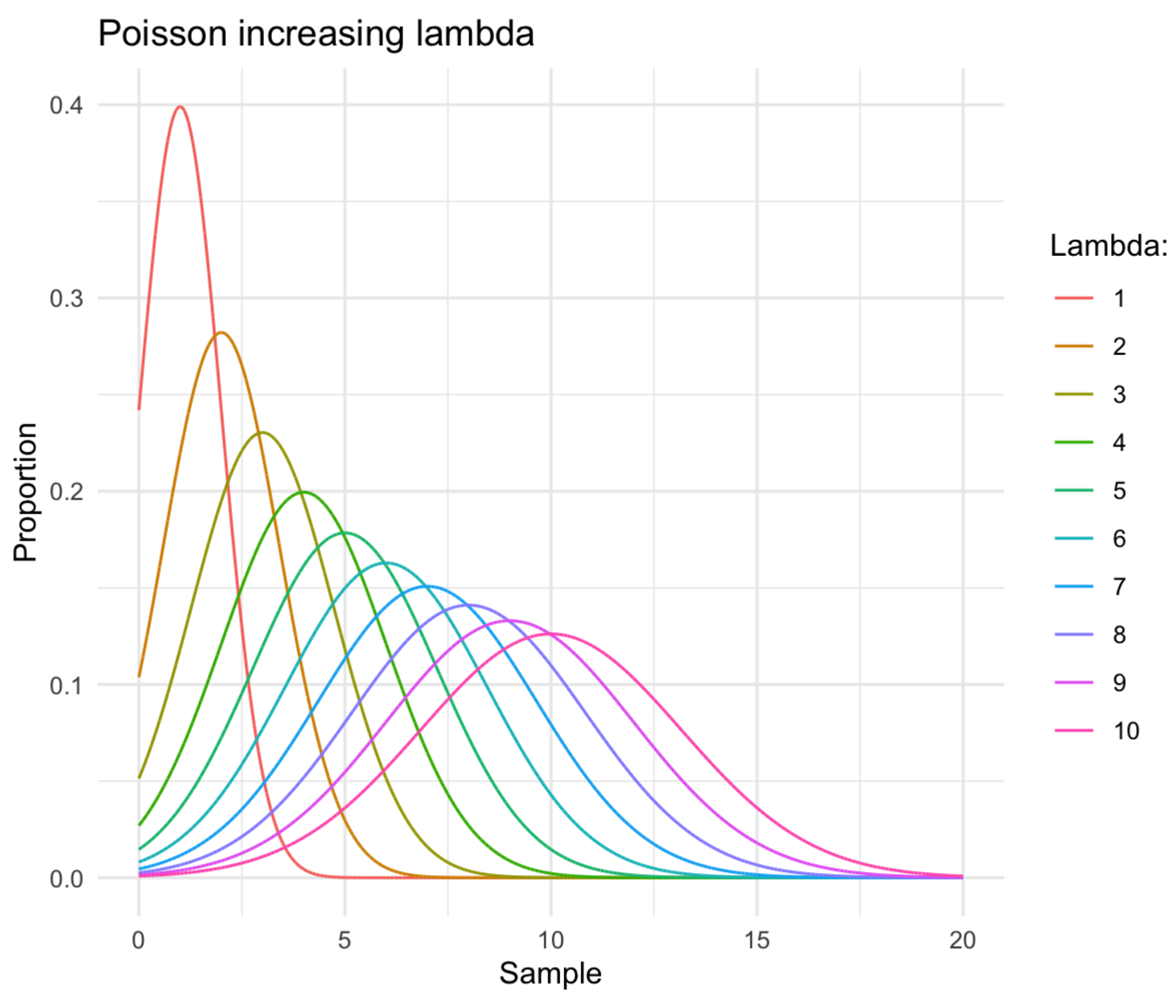
In [88]:
```
poisson_data <- data.frame('data' = rpois(1000, 17))

poisson_data %>% ggplot() +
  geom_histogram(aes(x = data,
                     y = stat(count / sum(count))),
                     color = 'black',
                 binwidth = 1) +
  geom_vline(xintercept = 17,
             size = 1,
             linetype = 'dashed',
             color = 'red') +
  theme_bw() +
  labs(x = 'Number of successes per period',
       y = 'Proportion',
       title = '1,000 samples of Poisson Distr. with (lambda = 17)')

#citation below
```



Notice the trend with λ:

this will be your μ and as you increase the number it will start going from a right skewed toward a normal distribution

In [183]:
```
# Build Poisson distributions

p_dat <- map_df(1:10, ~ tibble(
  l = paste(.),
  x = 0:20,
  y = dpois(0:20, .)
))

# Build Normal distributions

n_dat <- map_df(1:10, ~ tibble(
  l = paste(.),
  x = seq(0, 20, by = 0.001),
  y = dnorm(seq(0, 20, by = 0.001), ., sqrt(.))
))

# Use ggplot2 to plot

ggplot(n_dat, aes(x, y, color = factor(l, levels = 1:10))) +
  geom_line() +
  labs(color = "Lambda:",x = 'Sample',
       y = 'Proportion',
       title = 'Poisson increasing lambda') +
  theme_minimal()

#citation below
```



Weibull:

continuous

PDF

$$f(T) = \frac{\alpha}{\beta}(\frac{x}{\beta})^{\alpha-1}e^{(-\frac{x}{\beta})^{\alpha}}$$

- Think of using this when you want to find `how likely something is going to fail, given that it has survied so far` if used as the `hazard function`
  - You have a few distinct behaviors:

    1.) If you are expecting failure it will most likely occur at the start

    2.) Failure rates are fairly constant

    3.) As time goes on there is a bigger chance of failure

You have a `shape` parameter α, `scaling` parameter β, if T is some quantity of time for failure.

- If $\alpha > 1$: Then your failure rate will gain over time.
- $\alpha = 1$: Constant rate of failure:
  - suggesting that you may have external events causing failure.
- $\alpha < 1$: decrease over time known as (Lindy Effect), think of defective items for instance

----

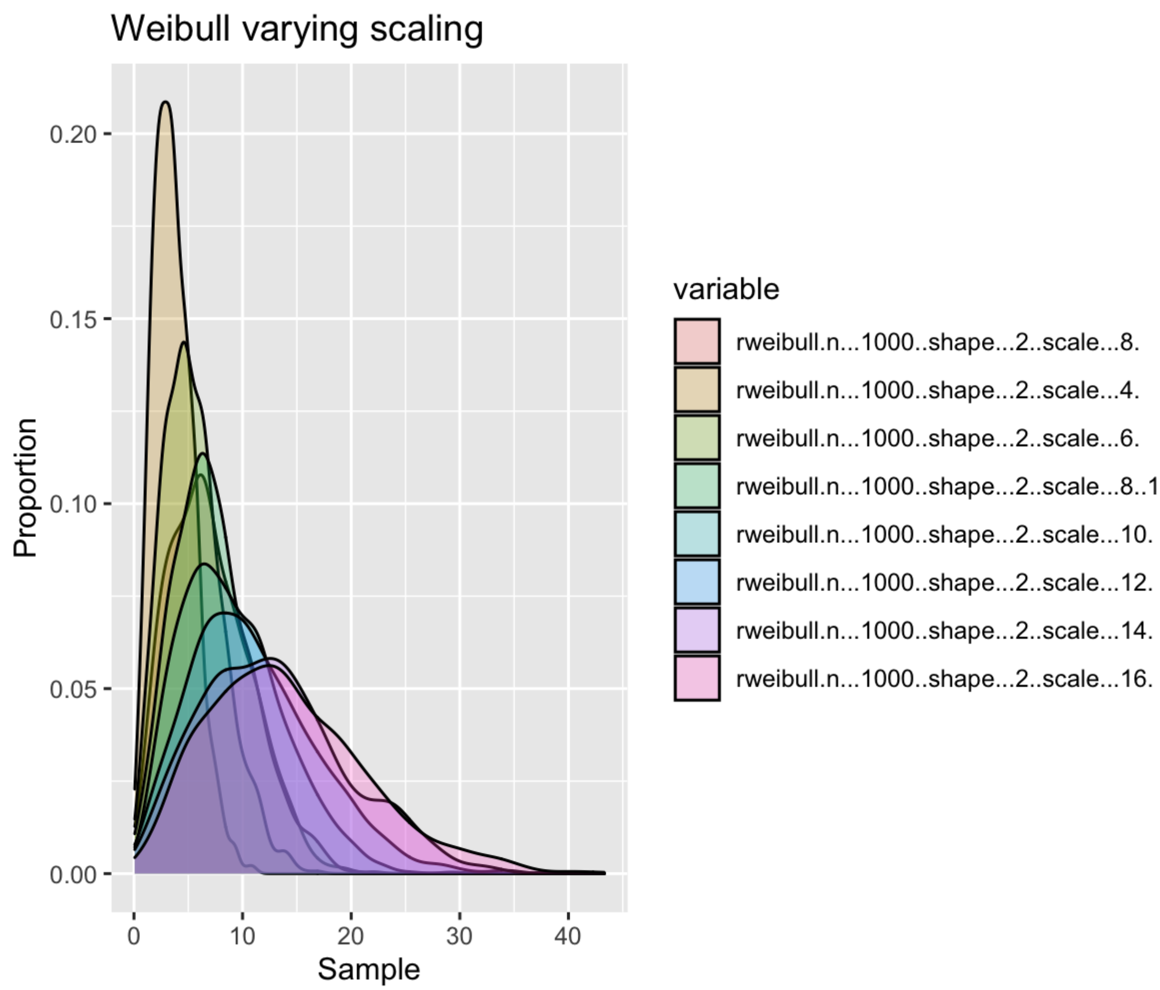
- **Some real use cases would include**: `Cancer, Stocks Crashes, Machine Failure, Divorce, Earthquakes, Medical, Engineering, Social Sciences`

\`Mean\` $\beta\Gamma(1 + \alpha^{-1})$

\`Standard Deviation\`$\beta^2[\Gamma(1 + 2\alpha^{-1}) - \Gamma^2(1 + \alpha^{-1})]$

In [312]:
```
# curve(dweibull(x, shape=2, scale = 1), from=0, to=4)
# curve(dweibull(x, shape=2, scale = 2), from=0, to=4)
# curve(dweibull(x, shape=2, scale = 3), from=0, to=4)

x <- data.frame(std.nromal=rnorm(1000, m=0, sd=1),
                     wide.normal=rnorm(1000, m=0, sd=2),
                     exponent=rexp(1000, rate=1),
                     uniform=runif(1000, min=-3, max=3)
                     )
nx<-data.frame(rweibull(n = 1000,shape=2,scale=8),
  rweibull(n=1000, shape=2, scale = 4),
rweibull(n=1000, shape=2, scale = 6),
rweibull(n=1000, shape=2, scale = 8),rweibull(n=1000, shape=2, scale = 1
0),
rweibull(n=1000, shape=2, scale =12),rweibull(n=1000, shape=2, scale = 1
4),
rweibull(n=1000, shape=2, scale = 16))

library(reshape2)
data<- melt(nx)
g_<-ggplot(data,aes(x=value, fill=variable)) + geom_density(alpha=0.25)
g_+labs(x = 'Sample',
       y = 'Proportion',
       title = 'Weibull varying scaling')
# ggplot(data,aes(x=value, fill=variable)) + geom_histogram(alpha=0.25)
# ggplot(data,aes(x=variable, y=value, fill=variable)) + geom_boxplot()

#citations below
```

```
No id variables; using all as measure variables
```

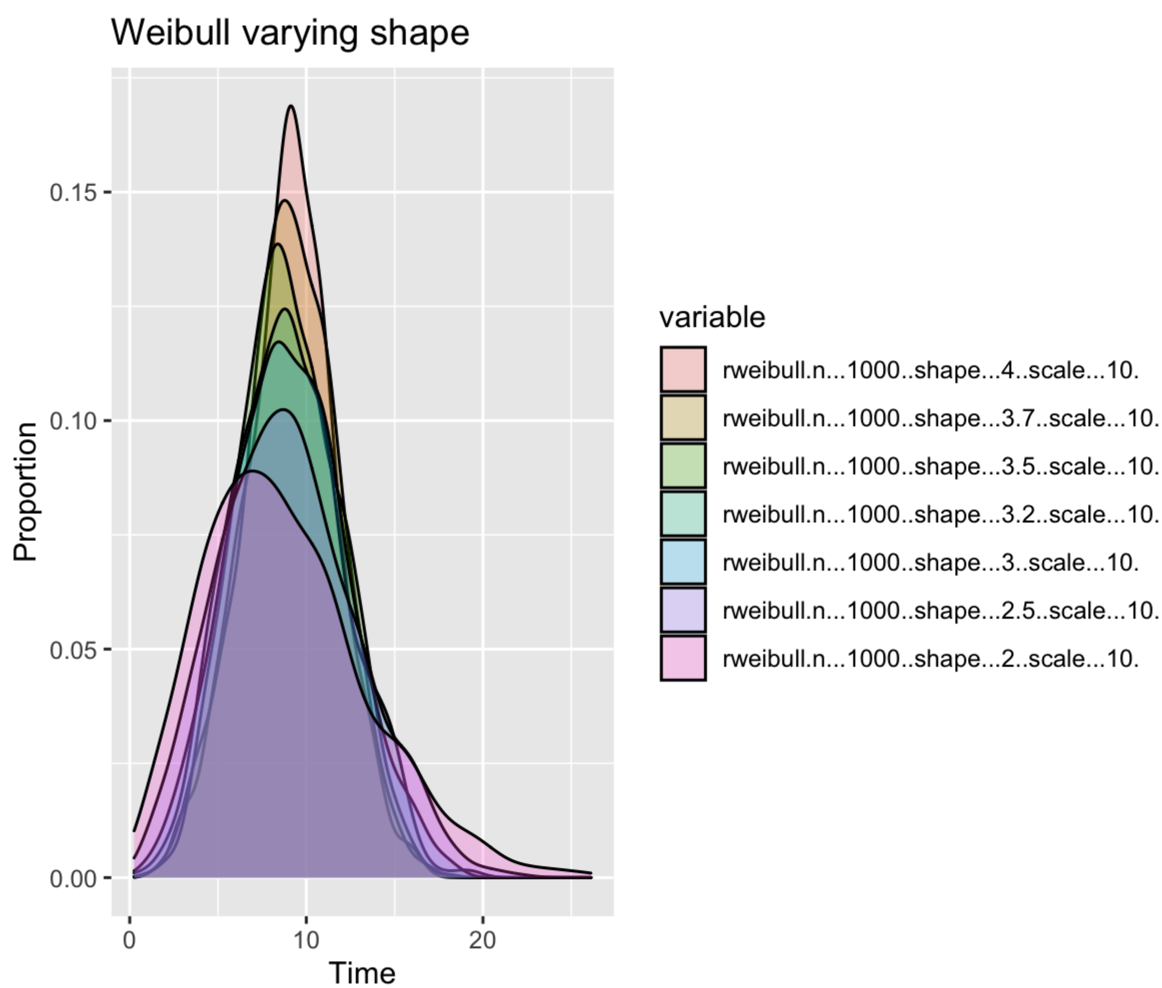


In [ ]:
```
#
```

In [202]:

```
x <- data.frame(std.nromal=rnorm(1000, m=0, sd=1),
                     wide.normal=rnorm(1000, m=0, sd=2),
                     exponent=rexp(1000, rate=1),
                     uniform=runif(1000, min=-3, max=3)
                     )
nx<-data.frame(rweibull(n = 1000,shape=4,scale=10),
  rweibull(n=1000, shape=3.7, scale = 10),
rweibull(n=1000, shape=3.5, scale = 10),
rweibull(n=1000, shape=3.2, scale = 10),rweibull(n=1000, shape=3, scale
= 10),
rweibull(n=1000, shape=2.5, scale =10),rweibull(n=1000, shape=2, scale =
10))

library(reshape2)
data<- melt(nx)
ggplot(data,aes(x=value, fill=variable)) + geom_density(alpha=0.25)+
labs(x = 'Time',
       y = 'Proportion',
       title = 'Weibull varying shape')
```

```
No id variables; using all as measure variables
```



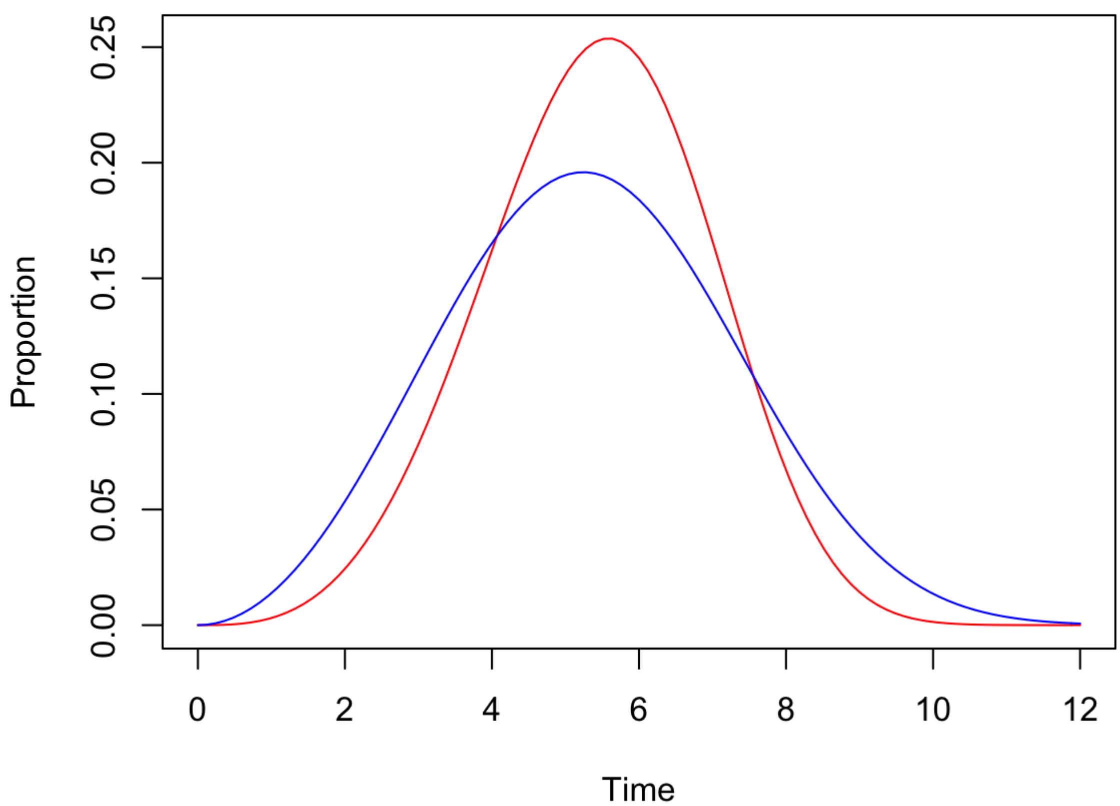
Shape parameter between values of (3-4):

`we notice something`

- We have a bell curve, which is always comforting but it tells us something useful: we will notice that failures will become prone at the end of life for a product such as the life of parts just failing.

citation at bottom: from minitab

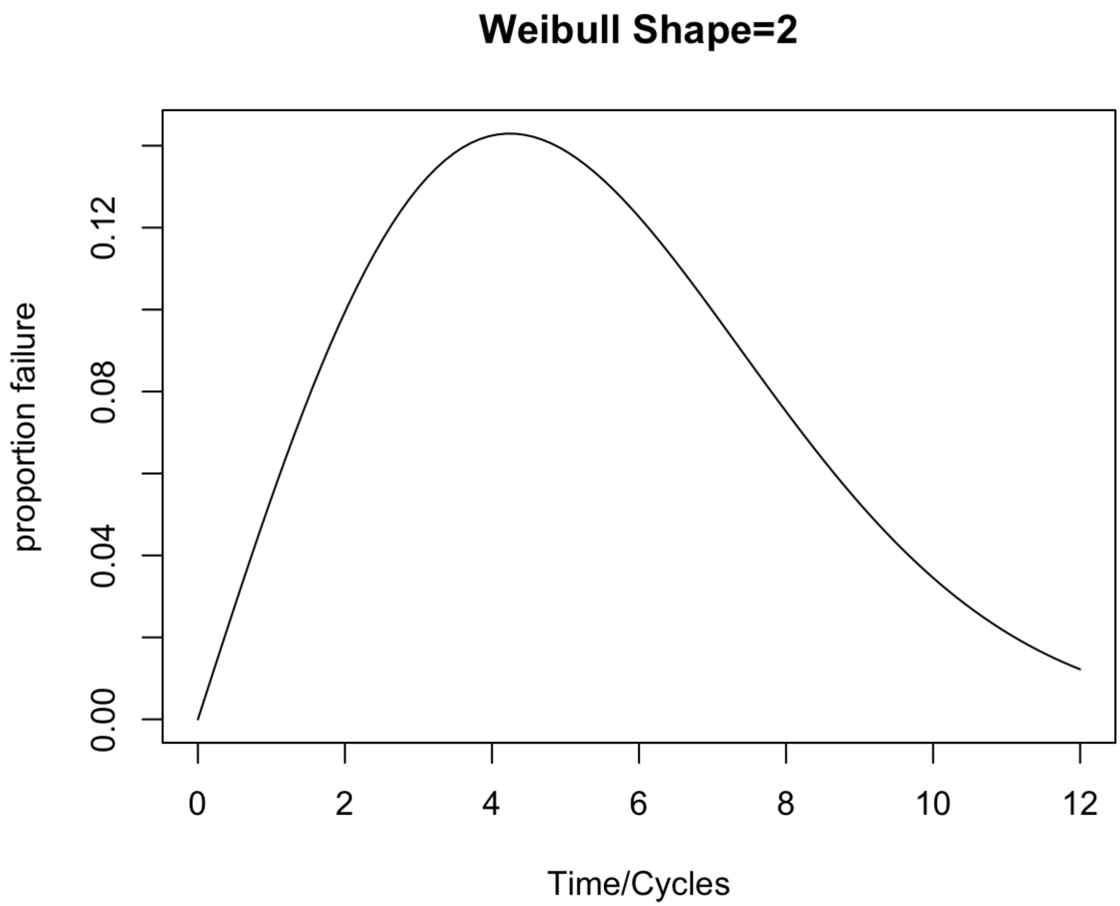
In [213]:
```
curve(dweibull(x, shape=4, scale = 6), from=0, to=12, col='red',xlab = "Time",
      ylab = "Proportion")
curve(dweibull(x, shape=3, scale = 6), from=0, to=12, col='blue', add=TRUE)
```



Shape parameter of 2:

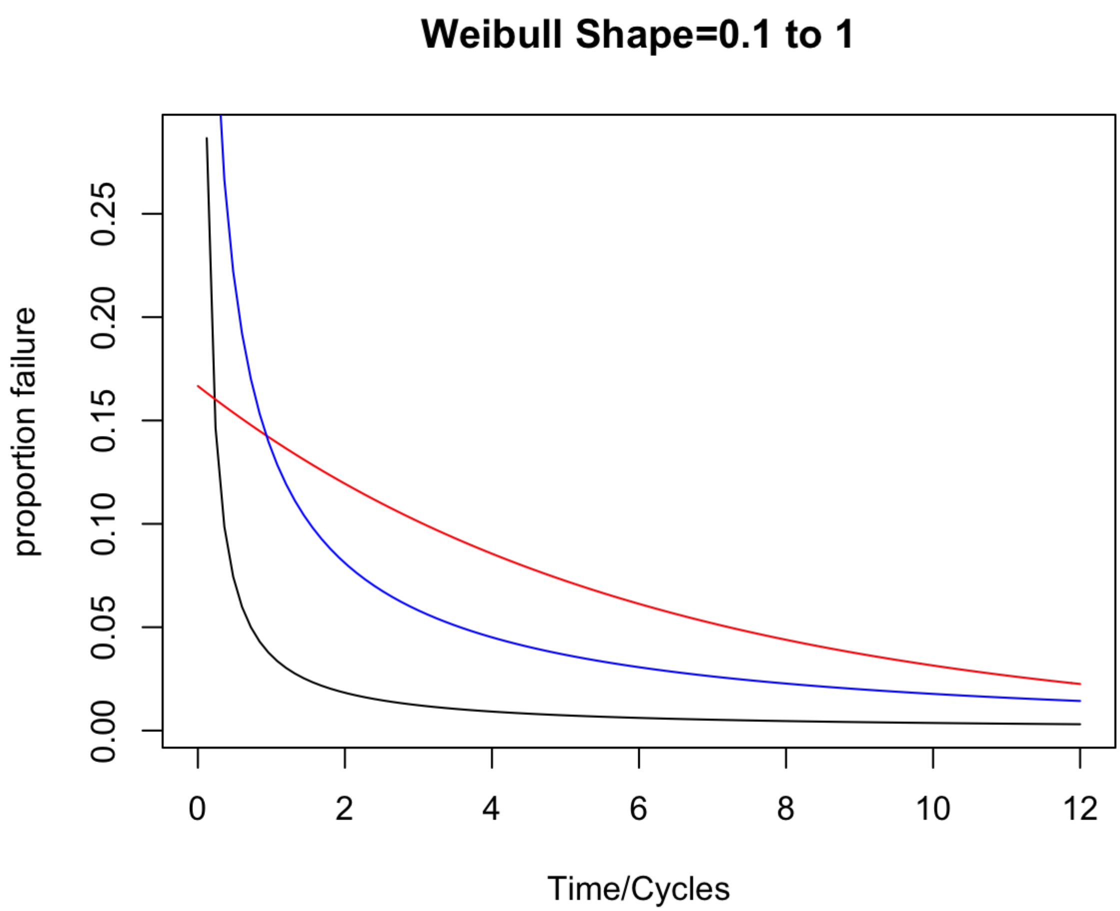
- notice: that you have a sharp increase and then the number of failures decrease over time

In [220]:
```
curve(dweibull(x, shape=2, scale = 6), from=0, to=12, col='black',
      xlab="Time/Cycles",ylab="proportion failure",main="Weibull Shape=
2")
```



You are seeing a **high number of failures initially** with **Shape < 1**

In [229]:
```
curve(dweibull(x, shape=.1, scale = 6), from=0, to=12, col='black',
      xlab="Time/Cycles",ylab="proportion failure",main="Weibull Shape=0.
1 to 1")
curve(dweibull(x, shape=1, scale = 6), from=0, to=12, col='red',
      xlab="Time/Cycles",add=TRUE)
curve(dweibull(x, shape=.5, scale = 6), from=0, to=12, col='blue',
      xlab="Time/Cycles",add=TRUE)
```



`Gamma:`

`continuous`

- *good if you have pretty skewed data*
- *measuring time between occurences of an event*

`PDF`

$$f(x) = \frac{(\frac{x-\mu}{\beta})^{\gamma-1} e^{(-\frac{x-\mu}{\beta})}}{\beta\Gamma\gamma}$$

- γ: shape parameter
  - μ: locations
    - β: scaling
- Γ: gamma function $\int_0^\infty t^{a-1} e^{-t} dt$

`Alternate More Common Version:`

$$f(x) = \frac{x^{\alpha-1} e^{(-\frac{x}{\beta})}}{\beta^{\alpha}\Gamma(\alpha)}$$

- α: shape parameter
  - β: rate parameter

`---------------------`

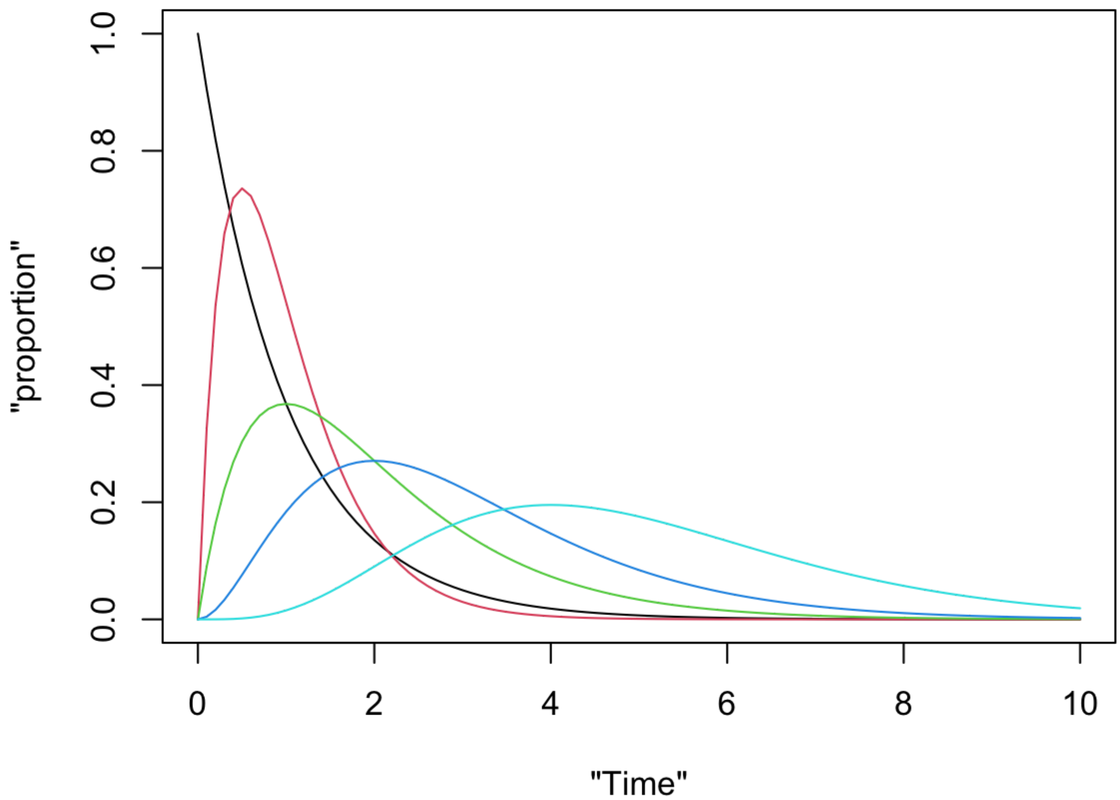
Some use cases: `insurance risk, rainfall data, economics, inventory control, particulate concentrations`

`Mean :`$E[x] = \alpha\beta$

`Variance :` $\alpha\beta^2$

In [327]:
```
# # library(lmtest)
# sample<- rweibull(5000, shape=1, scale = 2) + 10
# ussample<-sample-10
# m.shape10 <-  rgamma(5000, shape=10, scale = 1)
# # hist(ussample,freq=F,breaks=100,main="Take a look to data...")
# m=mean(ussample);std=sd(ussample);m;std
# # hist( m.shape10, freq=F )

# x<-seq(0,max(m.shape10),100)
# # curve(dgamma(x,shape=10,scale=1),col=2,add=T)

# hist( m.shape10, freq=F )
# x<-seq(0,max(m.shape10),100)
# curve(dgamma(x,shape=10,scale=1),col=2,add=T)


k <- c(1, 2, 2,3,5)
mu <- c(1, 1, 2,3,5)
theta <- mu/k
plot('Time', 'proportion', xlim = c(0, 10), ylim = c(0, 1), type = "n")
for(i in seq_along(k))
    curve(dgamma(x, shape = k[i], scale = theta[i]), from = 0,
          to = 10, col = i, add = TRUE)
```

```
Warning message in xy.coords(x, y, xlabel, ylabel, log):
“NAs introduced by coercion”
Warning message in xy.coords(x, y, xlabel, ylabel, log):
“NAs introduced by coercion”
```



Notice that the `Gamma` plot:

- As you increase: shape (k) in plot you will have a spike and a flattening if increasing scale (mu) here.
- When k=mu=1 then you have an exponential plot in (black)

`F Distribution:`

`continuous`

$$f(x) = \frac{\Gamma(\frac{m+n}{2})m^{\frac{m}{2}}(\frac{n}{m})^{\frac{n}{2}}}{\Gamma\frac{n}{2}\Gamma\frac{m}{2}[x\frac{n}{m}+1]^{\frac{n+m}{2}}}$$

The two degrees of freedom are (*m=numerator*) and (*n=denominator*) and the end points dictate the curve formed by the probabilities

Use cases: `used to test difference between two variances or two way ANOVA and used for inference`

- defined for positive values
- not symmetric about the mean (μ)

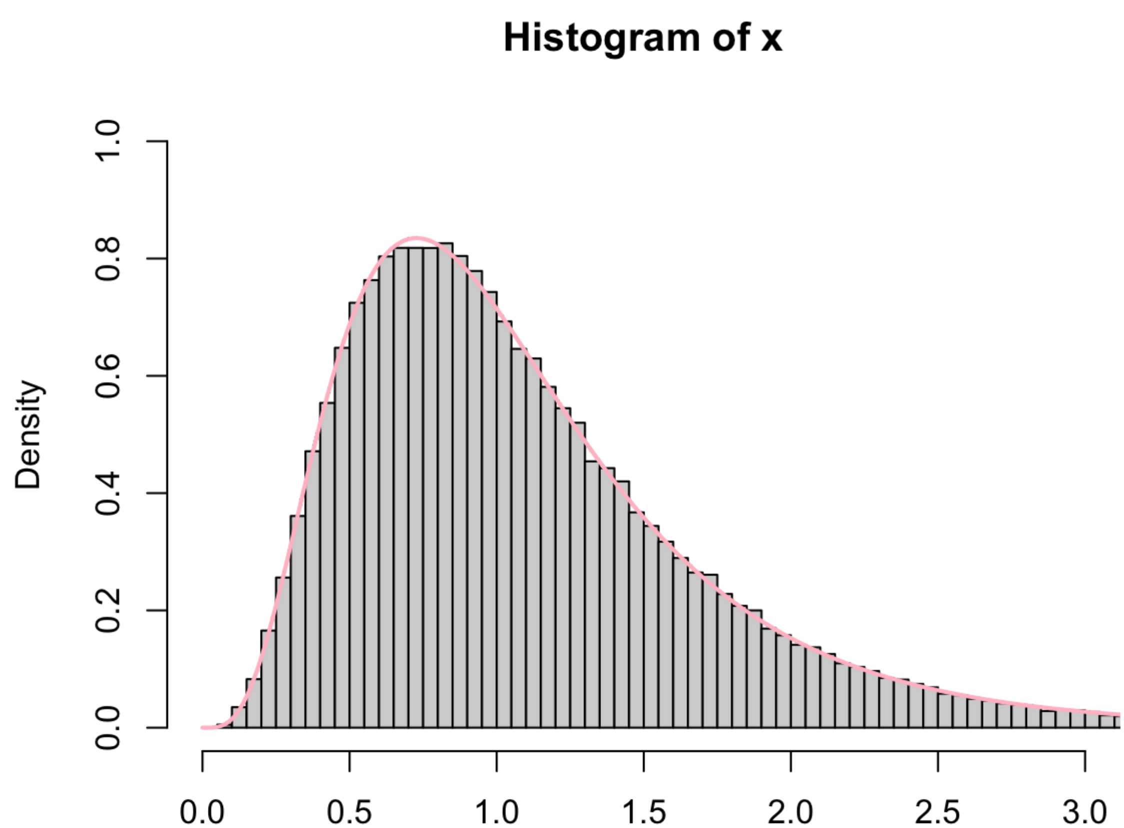
Tips:

- If you have data with large `variance` put that in the numerator to keep a right tailed plot
- If using a 2 tailed test: use $\frac{\alpha}{2}$
- If given the standard deviation you will need to square them to get the variance
- If the degrees of freedom aren't given, use the critical value from a table to avoid type-I error.

`Mean:` $\frac{V_2}{V_2-2}$, for $V_2 > 2$

`Variance:` $\sqrt{\frac{[2*V_2^2(V_1+V_2-2)]}{[V_1(V_2-2)^2(V_2-4)]}}$,for $V_2 > 4$

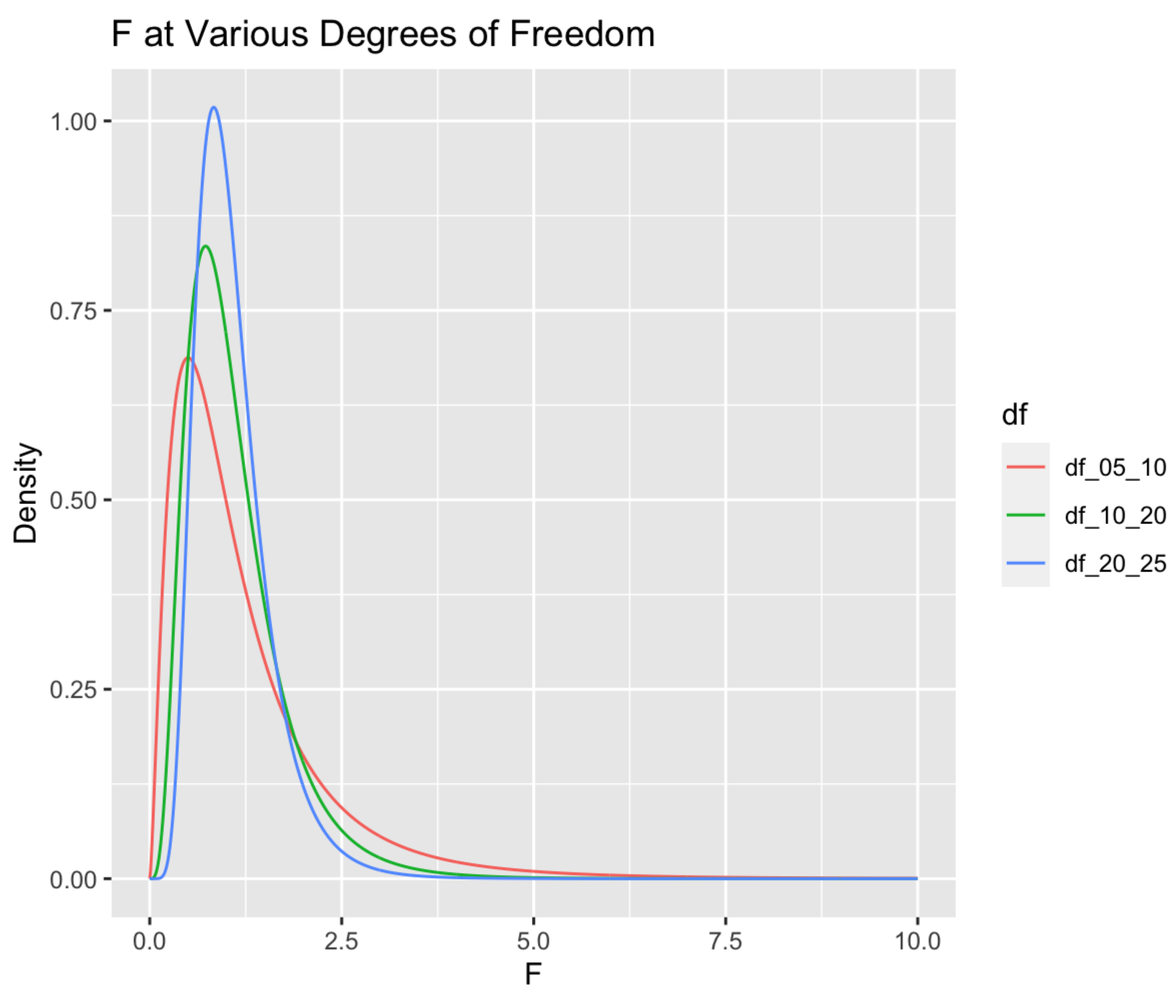
In [248]:
```
x <- rf(100000, df1 = 10, df2 = 20)
hist(x,
     breaks = 'Scott',
     freq = FALSE,
     xlim = c(0,3),
     ylim = c(0,1),
     xlab = '')
#      main = (TeX('Histogram for a $\\F$-distribution with $\\v_1 = 10$
and $\\v_2 = 20$ degrees of freedom (df)')), cex.main=0.9)

curve(df(x, df1 = 10, df2 = 20), from = 0, to = 4, n = 5000, col= 'pink'
, lwd=2, add = T)


# df_1 <- c(10, 10, 10)
# df_2 <- c(20, 30, 60)
# # theta <- mu/k
# plot(0, 0, xlim = c(0, 70), ylim = c(0, 1), type = "n")
# for(i in seq_along(df_1))
#   curve(dgamma(x=x[i], df1 = df_1[i], df2 = df_2[i]), from = 0, to = 7
0, col = i, add = TRUE)
```



As you increase degrees of freedom for both:

you will start getting closer to a (approx.) normal distribution

In [246]:
```
# library(tidyr)

data.frame(f = 0:1000 / 100) %>%
           mutate(df_10_20 = df(x = f, df1 = 10, df2 = 20),
                  df_05_10 = df(x = f, df1 = 5, df2 = 10),
                 df_20_25 = df(x = f, df1 = 20, df2 = 25)
                 ) %>%
  gather(key = "df", value = "density", -f) %>%
ggplot() +
  geom_line(aes(x = f, y = density, color = df)) +
  labs(title = "F at Various Degrees of Freedom",
       x = "F",
       y = "Density")
```



Chi Square Distribution:

skewed to the right, continuous

$V = X_1^2 + X_2^2 + \ldots X_n^2 \sim X_n^2$, with (n) random variables with a standard normal distribution

PDF

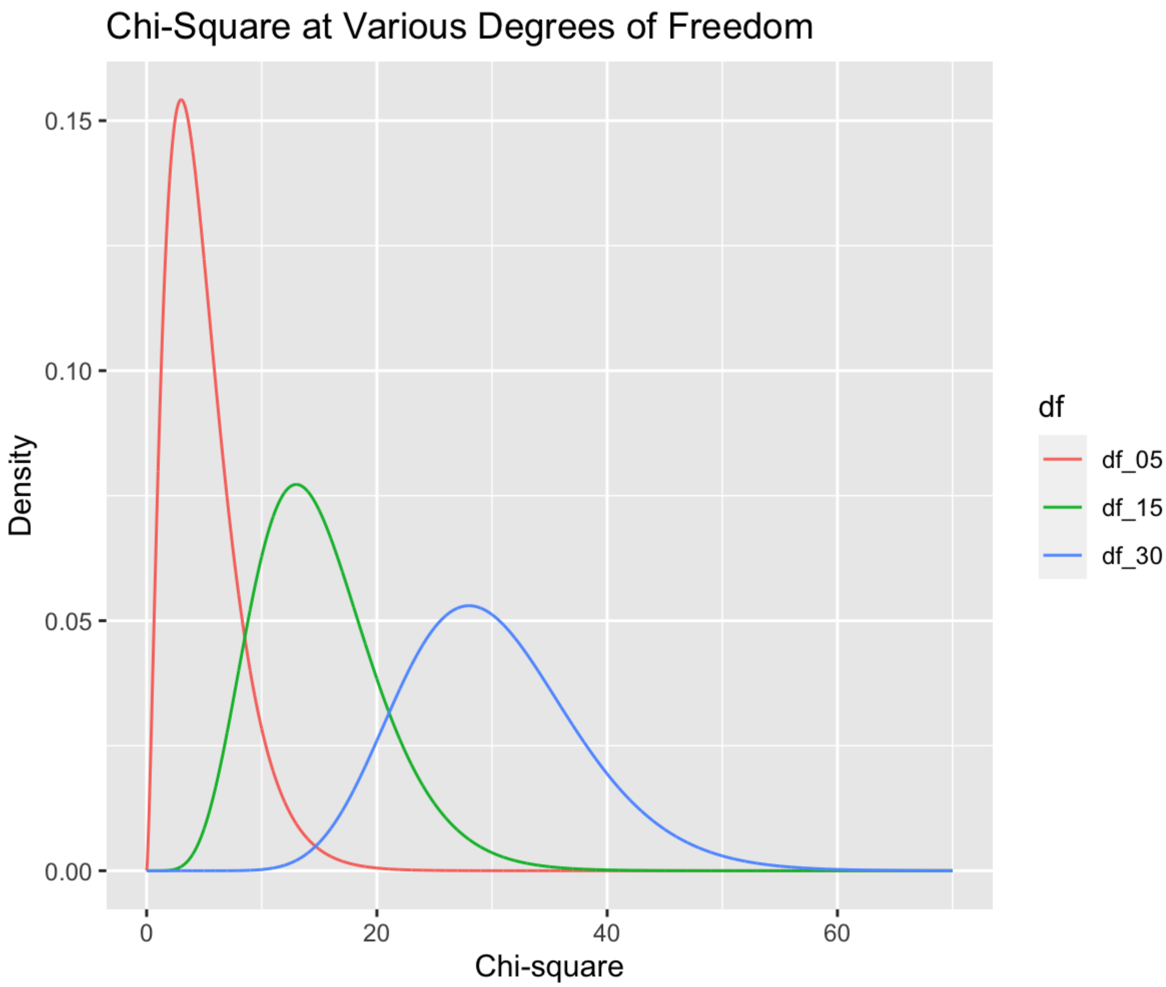
$$f(x;k) = \frac{x^{\frac{k}{2}-1} e^{-\frac{x}{2}}}{2^{\frac{k}{2}} \Gamma \frac{k}{2}}$$

Use Cases:

- `Estimate Confidence Intervals` from a population (std dev.) of a normal distribution for a given sample
- `Check for independence`
- `Find differences between observed and expected`
- Can use for `goodness of fit` using Chi-Sq test

The distribution is dependent upon the number of `degrees of freedom` which will be `df = (# rows-1)(# columns-1)` or `number of samples - 1`

In [249]:
```
data.frame(chisq = 0:7000 / 100) %>%
           mutate(df_05 = dchisq(x = chisq, df = 5),
                  df_15 = dchisq(x = chisq, df = 15),
                  df_30 = dchisq(x = chisq, df = 30)) %>%
  gather(key = "df", value = "density", -chisq) %>%
ggplot() +
  geom_line(aes(x = chisq, y = density, color = df)) +
  labs(title = "Chi-Square at Various Degrees of Freedom",
       x = "Chi-square",
       y = "Density")

# plot from R-documentation and in citations
```



Student T-test:

`Continuous`

`-------------`

- Great for hypothesis testing, used when you want to `estimate the mean of a normal distribution with a small sample size` and used to check the `statistical significance between two sample means or confidence intervals`

$$t = \frac{Z}{\sqrt(\frac{V}{m})} \sim t_m$$

- Assuming that `Z` has a (*standard*) `normal distribution` , where `V` has a `Chi Sq. distribution` with `m degrees of freedom`

Simplify this (more common version):

`PDF`

$$t = \frac{\bar{X} - \mu}{\frac{\sigma_{sample}}{\sqrt{n}}}$$

Also, the `t distribution` has `df=n-1`

`---------------`

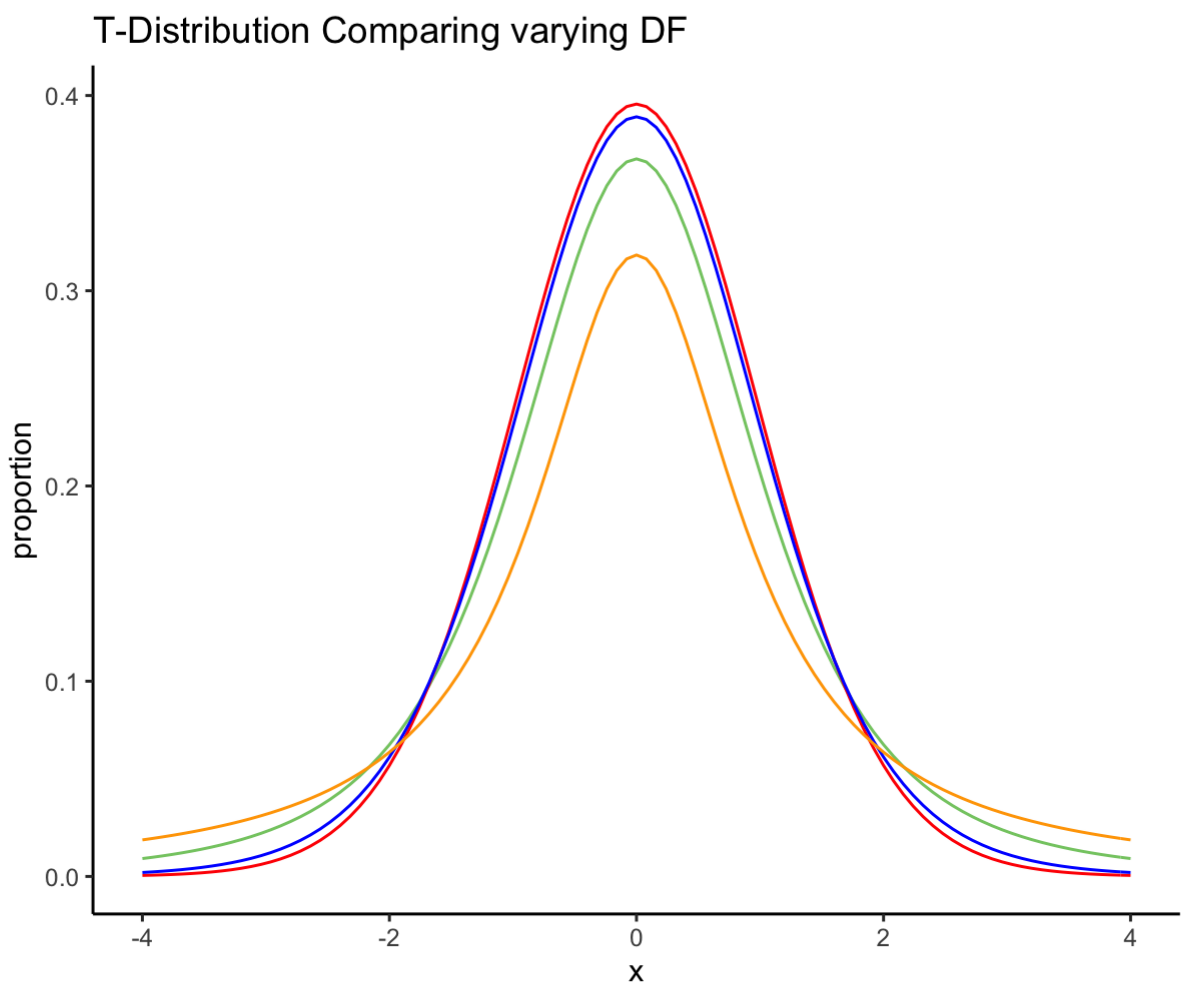
Simplify above statement: `think of it like this, you are looking to see if two populations are the same with respect to the variable you are testing and returning a probability`

- Consider using this when: you have small sample sizes and have a somewhat normal distribution without outliers and not a lot of skewness.
  - The higher number of degrees of freedom you have; this will approach a normal distribution

`Since you don't usually know the population you are instead doing an estimatation`

- As you increase the number of degrees of freedom you will approach the standard normal distribution

In [280]:
```
# As You ICREASE THE DEGREES OF FREEDOM APPROACHES NORMAL DISTRIBUTION

p <- data_frame(x = c(-4,4)) %>%
    ggplot(aes(x = x))

p + stat_function(fun = dt, args = list(df = 3), color = "#84CA72", size
= .5) +
    stat_function(fun = dt, args = list(df = 30), size = .5,color='red')
+
    stat_function(fun = dt, args = list(df = 10), size = .5,color='blue'
)+
    stat_function(fun = dt, args = list(df = 1), size = .5,color='orang
e')+
    ggtitle("T-Distribution Comparing varying DF") +
    xlab("x") +
    ylab("proportion") +
    theme_classic()
```



`Logistic:`

`continuous`

Resembles the Normal distribution but has longer (heavier) tails

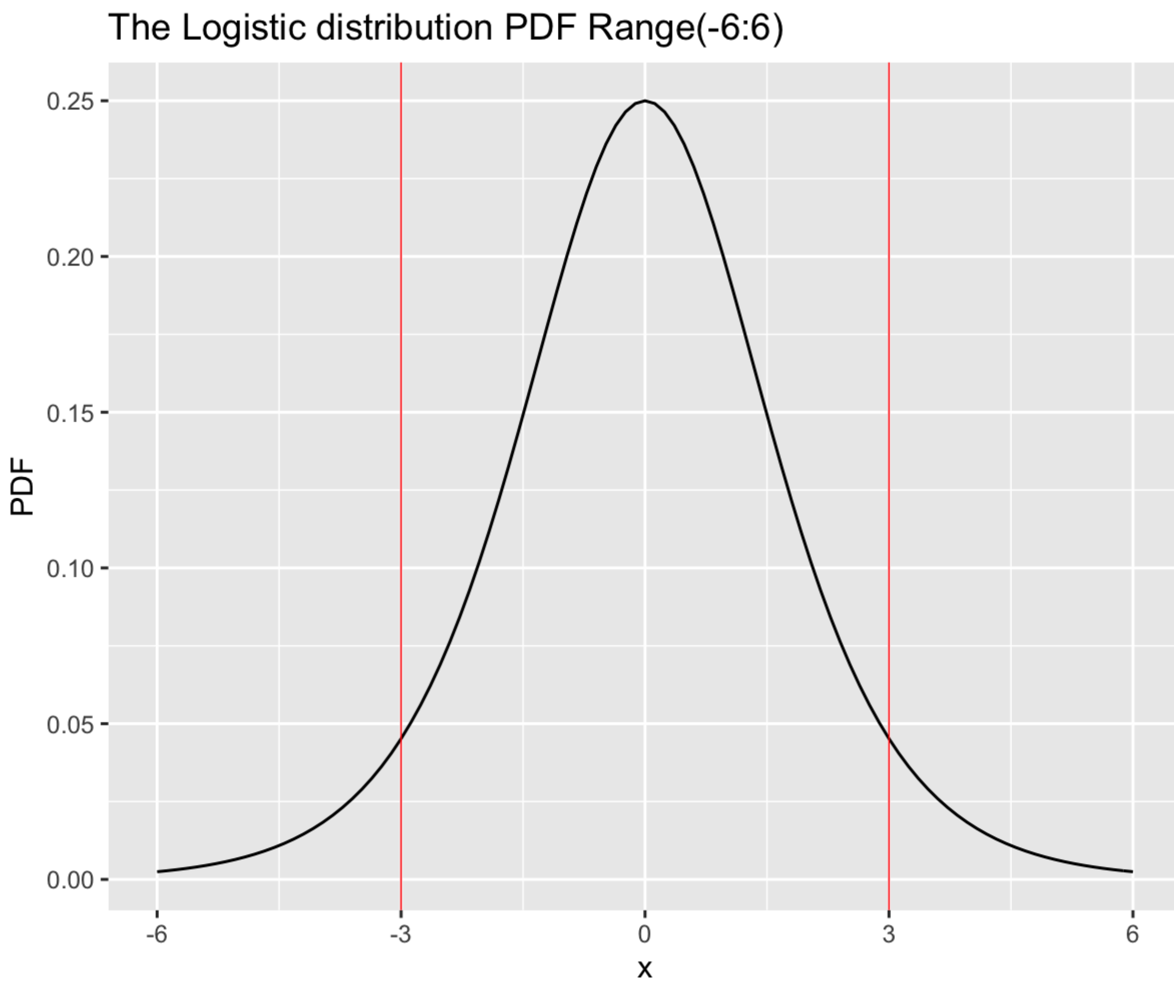
`PDF`

$$f(x) = \frac{e^{\frac{x-\mu}{\sigma}}}{\sigma(1+e^{\frac{x-\mu}{\sigma}})^2}, x \in \mathbb{R}$$

Common uses `used with logistic regression (very common for data analysis), used with feedforward neural networks`

- Uses with `categorical data` with dependent variables such as (*binary classification*)
  - good for population growth, chemical reactions

In [291]:
```
PDF_LD <- function(x) { y <- exp(x)/((1+exp(x))^2)}

# Plot PDF

ggplot(data.frame(x=c(-6, 6)), aes(x)) + stat_function(fun=PDF_LD) + geo
m_vline(xintercept = 3,
                color = "red", size=.25) + geom_vline(xintercept = -3,
                color = "red", size=.25) + labs(title = "The Logistic di
stribution PDF Range(-6:6)", y = "PDF")
```



Lets look at what our data look like when you vary parameters

[Logistic Distribution Wiki Ex. (https://en.wikipedia.org/wiki/Logistic_distribution)](https://en.wikipedia.org/wiki/Logistic_distribution)

Pareto:

power law probability distribution,continuous

PDF

$$f(x) = \frac{\alpha\beta^{\alpha}}{x^{1+\alpha}}$$

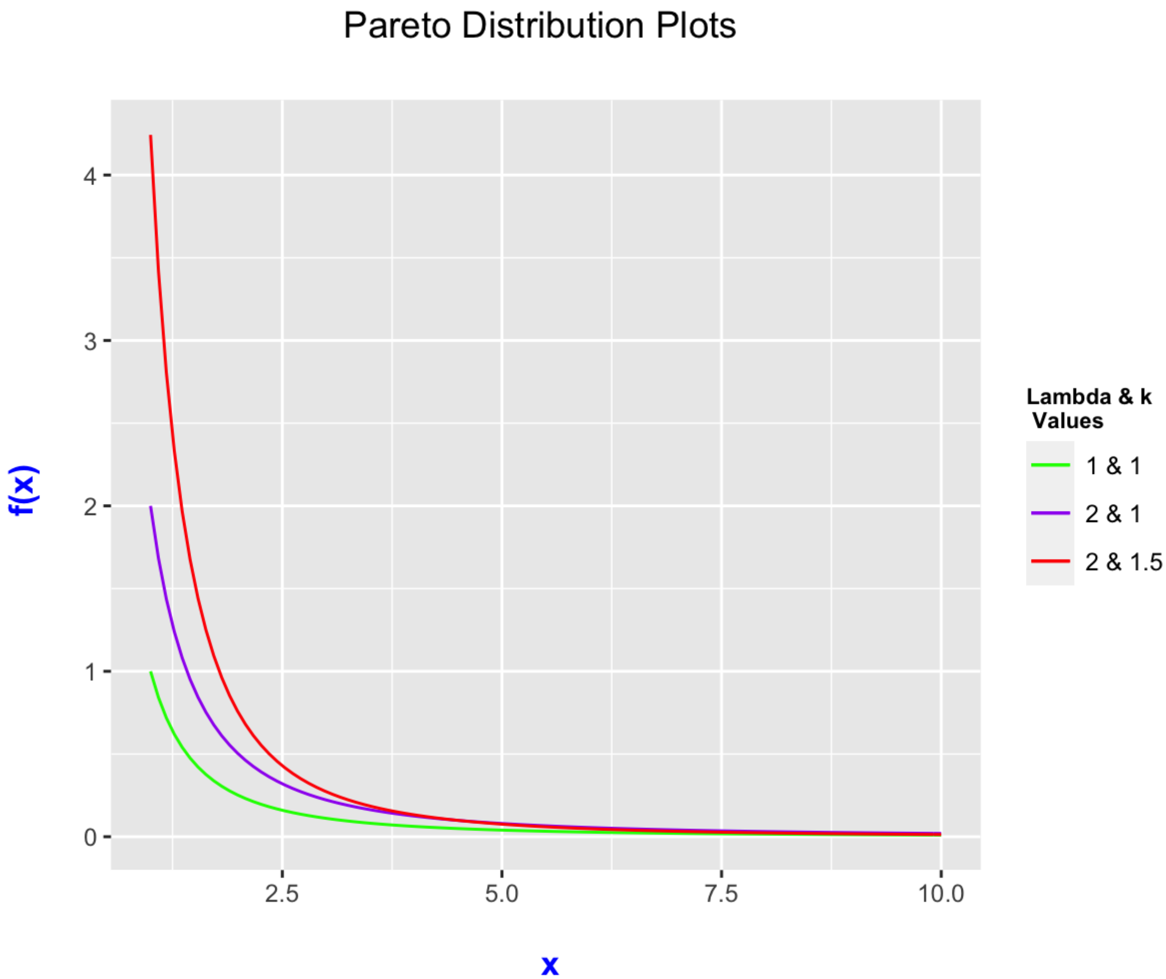
x_m: minimum value of x

α: shape parameter (slope), used for finding concentration near mode

β: scale parameter

Uses: good for looking at wealth distribution, changes in stock prices, natural resource occurences, geophysical, quality control, scientific, actuarial applications, natural phenomena

In [285]:
```
# Multiple Pareto Distributions:
# Pareto Density Function:
pareto_pdf <- function(x, lambda = 1, k = 1){
    density <- (k*(lambda^k)) / (x^(k + 1))
    return(density)
}

x_lower_pareto <- 1
x_upper_pareto <- 10
ggplot(data.frame(x = c(x_lower_pareto, x_upper_pareto)), aes(x = x)) +
  xlim(c(x_lower_pareto, x_upper_pareto)) +
  stat_function(fun = pareto_pdf, args = list(lambda = 1, k = 1), aes(co
lour = "1 & 1")) +
  stat_function(fun = pareto_pdf, args = list(lambda = 2, k = 1), aes(co
lour = "2 & 1")) +
  stat_function(fun = pareto_pdf, args = list(lambda = 2, k = 1.5), aes(
colour = "2 & 1.5")) +
  scale_color_manual("Lambda & k \n Values", values = c("green", "purpl
e", "red")) +
  labs(x = "\n x", y = "f(x) \n",
       title = "Pareto Distribution Plots \n") +
  theme(plot.title = element_text(hjust = 0.5),
        axis.title.x = element_text(face="bold", colour="blue", size = 1
2),
        axis.title.y = element_text(face="bold", colour="blue", size = 1
2),
        legend.title = element_text(face="bold", size = 8),
        legend.position = "right")

# citaiton below for this
```



Take away from `Pareto`:

```
 as you increase the slope (alpha or here 'k') you get a steep drop-off and scaling
just moves to the right
```

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SUBscrbe

Citations & Help:

https://rafalab.github.io/dsbook/distributions.html (https://rafalab.github.io/dsbook/distributions.html)

https://www.r-tutor.com/elementary-statistics/probability-distributions (https://www.r-tutor.com/elementary-statistics/probability-distributions)

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`Latex Help`

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`Poisson`

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`Weibull`

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`Gamma`

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```
General Distribution Stuff
```

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```
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```

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```
Chi Square
```

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https://www.dummies.com/education/math/business-statistics/how-to-measure-the-moments-of-the-f-distribution/ (https://www.dummies.com/education/math/business-statistics/how-to-measure-the-moments-of-the-f-distribution/)

Pareto

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plotting help

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