Explaining Logistic Regression

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Purpose & Outcome:

- Cover the theory of Logistic Regression and get a feel of the underworkings
- Some plotting and examples to help

stats book (https://mregresion.files.wordpress.com/2012/08/agresti-introduction-to-categorical-data.pdf) | great lecture notes: UCLA (http://web.cs.ucla.edu/~yzsun/classes/2018Fall_CS145/Discussion/Discussion_Week2.pdf)

Let me know if there are any videos you would like to see.

Check me out on Buy Me A Coffee: @mrfugudatasci

In [14]:
```
import numpy as np
import math
import matplotlib.pyplot as plt
import statsmodels.api as sm
from sklearn import linear_model
%matplotlib inline
```

Background:

- `Logistic (Logit) regression` is used for `Classification`. We showed an example of this in the previous notebook (*`skLearn_01`*) where we looked for Ham/Spam for sms messages.
- You are trying to estimate the probability that some instance, belongs to a particular class.
  - Using logistic regression, you are preserving the marginal probabilities of training data.
  - You are also using the coefficients to aide in detecting what input variables are important.

But, why not use `Linear Regression` ?

- In order to use `Linear Regression` we would have to create a threshold for the classification.
  - This is a problem if we are only trying to do a (Pass/Fail).
  - Unbounded results; unlike when using logistic which are [0:1]
    - Hmm, what does that mean?
      - Well: Linear regression will have an instance where the values are > 1 or <0.
  - Outliers, are a problem because they will change the best fit line and increase error.

Differences:

- `Linear Regression:`
  - Fits a line, to the points (data) which can be used to estimate/predicate a new value

- `Logistic Regression:`
  - fits a line to best separate between classes

We can connect Linear Regression to Logistic Regression:

- We are taking the labels and finding a predicted label and output a probability

$y = \hat{y}$, meaning (y) predicted, y= class label

- Using the sigma function we can take the linear regression and comfort to the range of [0,1]

$$\hat{y} = sigmafunction(\beta_o + \beta_1 x_1 + \beta_2 x_2 + \ldots) \rightarrow \hat{y} \in [0, 1]$$

- We can equate the $(\beta_o + \beta_1 x_1 + \beta_2 x_2 + \ldots)$ as the distribution perpendicular to the line for our decision boundary. This means that the points residing in our plot are separated by a descision boundary.
  - The points will be separated by a distance perpendicular to that line.
  - That distance is therefore, giving a probability to a class or not.
    - The farther they are from that line (decision boundary), the higher the probability toward that class label.

What are we trying to find?

- The parameters "coefficients" β are estimated using one of two methods:
  - `Least Squares Optimization:` using iterative approach IRWLS (https://en.wikipedia.org/wiki/Iteratively_reweighted_least_squares)
  - `Max Likelihood Estimation`

What Are We Exactly Calculating with Logistic Regression?

- `You are computing a weighted sum of input features, plus a bias 'intercept' term`
  - There is one coefficient per input.

`Starting from Linear Regression` : we have our hypothesis f (x_i) representing the predicted response for the i^{th} oberservation of x_i.

$$P(X) = f(x_i) = \hat{Y} = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_m x_{im}$$

`At this point the output would be a (real) value, not a class label.`

Here comes the `Logistic Function` (*sigmoid function*)

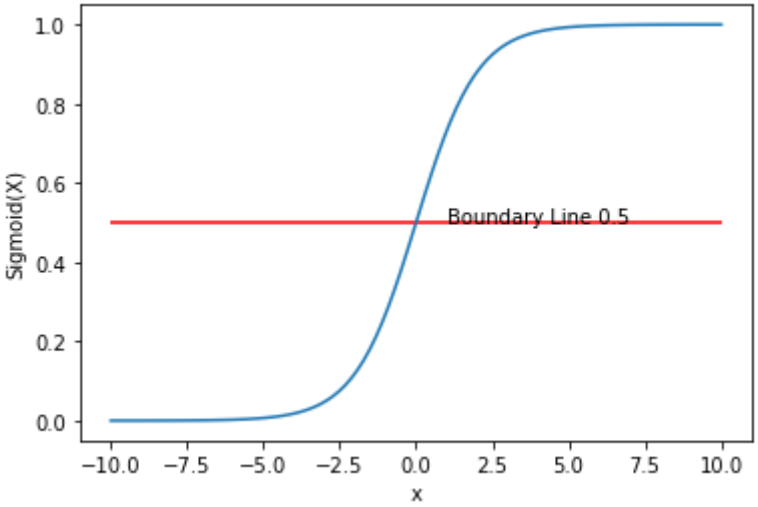
`Logistic Function:` $f(x) = \frac{1}{(1+e^{-x})}$

For input (x) we will replace with the weighted sum

$$f(x) = \frac{1}{(1+e^{-(\beta_o+\beta x)})}$$

In [2]:
```
# Plot sigmoid
x = np.linspace(-10, 10, 100)
z = 1/(1 + np.exp(-x))

plt.plot(x, z)
plt.xlabel("x")
plt.ylabel("Sigmoid(X)")
plt.hlines(y=.5, xmin=-10, xmax=10, colors='red', linestyles='solid')
plt.text(1,0.5,'Boundary Line 0.5')
plt.show()
```

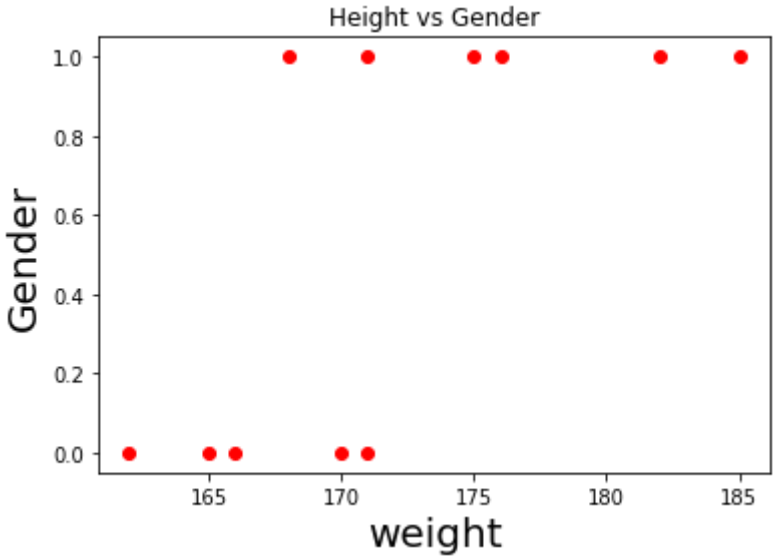


We are trying to find the best parameters for our example: from the family of logistics

source (https://www.sjsu.edu/faculty/guangliang.chen/Math251F18/lec5logistic.pdf)

What we would like is to fit a line to these data:

In [3]:
```
height = np.transpose(np.array([[162, 165, 166, 170, 171, 168, 171, 175,
176,
182, 185]]))
y = np.transpose(np.array([[0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1]]))
plt.xlabel("weight",fontsize=20)
plt.ylabel("Gender",fontsize=20)
plt.plot(height,y,'ro')
plt.title('Height vs Gender')
plt.show()
```



Now, we have to get the `Log-Odds:`

- Think of when you did an introduction to probability and were introduced to gambling: (wins:loses) ratio.

We can convert that to a probability of success 'your odds': $\frac{P}{1-P}$

`And to get the log-odds for success:` $log(\frac{P}{1-P})$

Now we will plug this into our logistic function from above:

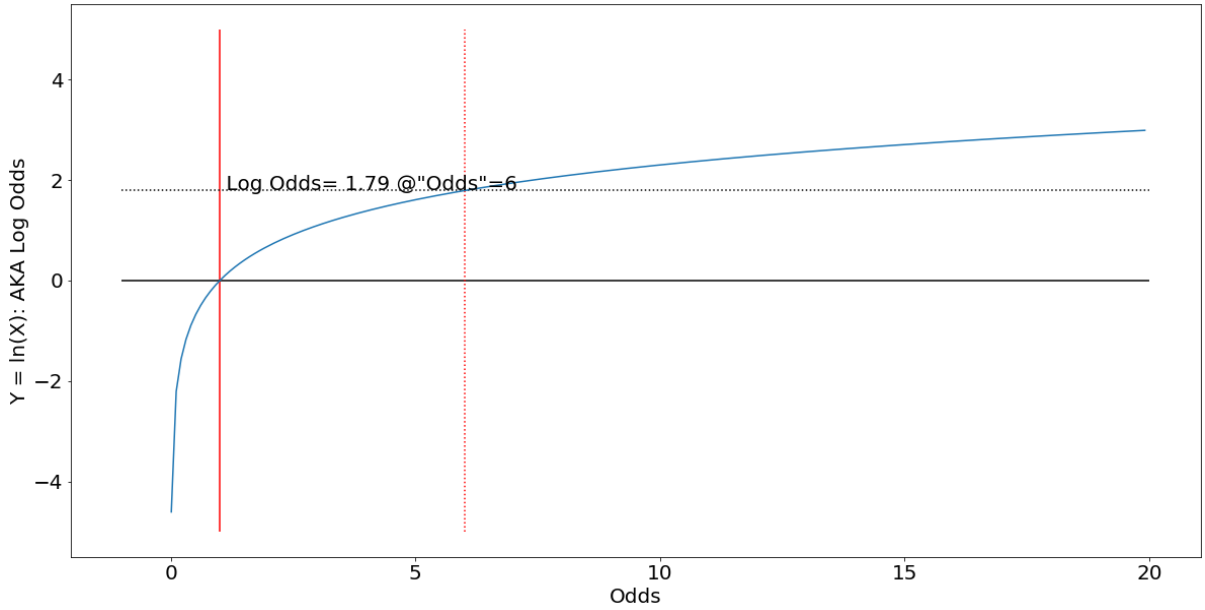
$$log(\frac{P}{1-P}) = \beta_o + \beta X$$

$$\frac{P}{1-P} = e^{\beta_o + \beta X}$$

In [4]:
```
# Log Odds:


x = np.arange(0.01,20,0.1)
y = np.log(x)
plt.figure(figsize=(20,10))
plt.rc('xtick', labelsize=20)
plt.rc('ytick', labelsize=20)
plt.vlines(x=1,ymin=-5,ymax=5, colors='red', linestyles='solid')
plt.hlines(y=0,xmin=-1,xmax=20, colors='black', linestyles='solid')
plt.xlabel("Odds",size=20)
plt.ylabel("Y = ln(X): AKA Log Odds",size=20)

plt.vlines(x=6,ymin=-5,ymax=5, colors='red', linestyles='dotted')
plt.hlines(y=np.log(6),xmin=-1,xmax=20, colors='black', linestyles='dott
ed')
plt.text(1,np.log(6),' Log Odds= 1.79 @"Odds"=6',size=20)
plt.plot(x,y)
plt.show()
```



Let's Have fun and do some interpreting:

In [5]:
```
log_odds_=np.log(6)
print('Log-Odds ln(x); when Odds = 6: returns %.2f' % log_odds_)
print('--------------------')
print('')
print('--------------------')
prob_=1 / (1 + np.exp(-np.log(6)))
print('Probability when you convert Log-Odds value 1 / (1 + np.exp(-np.l
og(6)))= %.2f'
      % prob_ )
print('--------------------')
print('')
odds_ = 0.7/.3
print("If Prob. = 0.7, let's find the Odds-ratio (P/(1-P)) = (%.2f : 1)"
% odds_)
print('you will have a 2.33 to 1 chance')
```

```
Log-Odds ln(x); when Odds = 6: returns 1.79
--------------------

--------------------
Probability when you convert Log-Odds value 1 / (1 + np.exp(-np.log
(6)))= 0.86
--------------------

If Prob. = 0.7, let's find the Odds-ratio (P/(1-P)) = (2.33 : 1)
you will have a 2.33 to 1 chance
```

In `Logistic Regression` the dependent variable is the Logit:

$$Log(Odds) = logit(P) = ln(\frac{P}{1-P})$$

Therefore the odds is a function of probability of getting a 1.

How about we use an example to clarify

Ex.) Among men 40 years old, the odds of acting weird before the age of 50 are 2X's greater for never married men.

$$\frac{'oddsOfBeingWeirdGivenNeverMarried'}{oddsOfWeirdBeingMarried} = 2$$

$$\frac{'oddsOfBeingWeirdGivenNeverMarried'}{oddsOfWeirdBeingMarried} = \frac{e^{\beta_0}e^{\beta_1}}{e^{\beta_0}} = e^{\beta_1}$$

Group	X	Odds Weird
Never Married	1	$e^{\beta_0}e^{\beta_1}$
Married	0	e^{β_0}

Therefore, when we increase x_k by 1 unit then the odds variable e^{β_k} is multiplied by Y=1. with all other independent variables held constant.

Before we go further let's get some insight:

if we take the right hand side of the equation and set to $\sum_{j=0}^{K} e^{b_j x_j}$ noticing that we have $\prod_{j=0}^{K} e^{b_j x_j}$

- **Notice** that you have a `multiplicative effect instead of additive like linear regression`

Ex.) Now, let's give $b_j = 0.789$, and set $x_j = 1$ then we end up with 2.201 Therefore, we would end up with each value odds for our response (e^{b_j}) being True will (Increase/Decrease) as x_j increase 1 unit.

Here, we noticed a doubling effect.

`If you were using` x_j `as some value such as months, or years then for each time interval of (x) you will double the odds of the response being true.` (All other things being held constant!)

`ex.) if the categories were: Rich/Poor and encoded (Rich=1,Poor=0) then we would have if the response was Rich; twice as likely or being true (all other things being constant)`

$$\frac{P}{1-P} = e^{\beta_o+\beta X}$$

$$P = e^{\beta_o+\beta X}(1-P)$$

Substitute

$$P = e^{\beta_o+\beta X} - P(e^{\beta_o+\beta X})$$

move to other side

$$P + P(e^{\beta_o+\beta X}) = e^{\beta_o+\beta X}$$

combine

$$P(1 + e^{\beta_o+\beta X}) = e^{\beta_o+\beta X}$$

move to other side

$$P = \frac{e^{\beta_o+\beta X}}{(1+e^{\beta_o+\beta X})}$$

equate 1 to exp. form

$$P = \frac{e^{\beta_o+\beta X}}{\frac{e^{\beta_o+\beta X}}{e^{\beta_o+\beta X}}+e^{\beta_o+\beta X}}$$

pull out common term and cancel

$$P = \frac{e^{\beta_o+\beta X}}{e^{\beta_o+\beta X}[\frac{1}{e^{\beta_o+\beta X}}+1]}$$

$$P(X) = \frac{1}{(1+e^{-(\beta_o+\beta X)})}$$

Note: if the log-odds are linearly related to capital (X), then the relation between X and P are nonlinear. Then you will have the S-curve we had above.

- The **decision boundary:**
  - $\beta_o + \beta x = 0$ will be your solution to the separating line for the predicted classes.
  - Distance between the decision line is: $\frac{\beta_o}{||\beta||} + x\frac{\beta}{||\beta||}$

Logistic Regression WILL tell you where the boundary is between classes as well as the dependence of class probability boundaries

ex. and help (http://faculty.cas.usf.edu/mbrannick/regression/Logistic.html)

We are trying to use the separation boundary to classify. The likelihood estimation will be maximized so we can get the best separation between class

source (https://github.com/DrIanGregory/MachineLearning-LogisticRegressionWithGradientDescentOrNewton)

Decision Boundary:

Understand that the properties of the decision boundary are `generated from the hypothesis` , not the dataset. This means that for instance:

$h_o(x) = g(\beta_0 + x\beta_1 + x_2\beta_2)$

we would be able to take this had some $\beta = \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$

and we were predicting that y=1, then $-5 + x_1 + x_2 >= 0$ becomes $x_1 + x_2 >= 5$

- **`Decission Boundary Consideration:`** Ideally, you aim for `Recall & Precision = 1`
  - `Low Recall: High Precision:` situation where you may want to `reduce number of false positives without adjusting false negaitives` you will choose a value with either a `High Recall or Low Precision` .
    - This can occur in situations where you have adevertising and you want to make a clear, positive impression.
  - `Low Precision: High Recall:` opposite of above.
    - Think of falsly labeling someone with cancer as not having cancer.

source (https://www.geeksforgeeks.org/understanding-logistic-regression/)

Important:

Understand, that we are trying to find our parameters in order to create that decision boundary. That is what the next steps are for!

- we want the best predicted weights $\beta_o, \beta_1, \ldots$ such that the p(x) will be as close as possible to the response y:label.
  - To get the best weights, we can do log-likelihood estimation (*maximize*)

source (https://realpython.com/logistic-regression-python/)

Likelihood:

- Since, `logistic regression` predicts probabilities and not just classes; we can use the likelihood.

Before we get started we will need the `Bernoulli Trials` $p(x_i)^{y_i}(1-p(x_i))^{1-y_i}$

where:

- x_i = vector of features
- y_i = class labels

$$Y_i = \begin{cases} p=1 \\ 1-p=0 \end{cases}$$

`Likelihood, with Bernoulli trials:` $L(\beta_o, \beta) = \prod_{i=1}^{n} P(x_i)^{y_i}(P(x_i)^{1-y_i})$

- The `Likelihood:` is the conditional probability of getting the values we observed, as a function of $\beta's$
  - your given some data (y_i) which is a function of the parameters β here

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Bernoulli/Binomial Review (http://galton.uchicago.edu/~eichler/stat22000/Handouts/l12.pdf)

In [6]:
```
# ex.)
print('Given a class (0,1) labeled (y) and some probability yhat:')
print('-------------')


def likelihood(y, yhat):
    return yhat * y + (1 - yhat) * (1 - y)

# test for y=1
y, yhat = 1, 0.8
print('y=%.1f, yhat=%.1f, likelihood: %.2f' % (y, yhat, likelihood(y, yh
at)))
y, yhat = 1, 0.1
print('y=%.1f, yhat=%.1f, likelihood: %.2f' % (y, yhat, likelihood(y, yh
at)))
# test for y=0
y, yhat = 0, 0.1
print('y=%.1f, yhat=%.1f, likelihood: %.2f' % (y, yhat, likelihood(y, yh
at)))
y, yhat = 0, 0.8
print('y=%.1f, yhat=%.1f, likelihood: %.2f' % (y, yhat, likelihood(y, yh
at)))

# adapted from:
# https://machinelearningmastery.com/logistic-regression-with-maximum-li
kelihood-estimation/
```

```
Given a class (0,1) labeled (y) and some probability yhat:
-------------
y=1.0, yhat=0.8, likelihood: 0.80
y=1.0, yhat=0.1, likelihood: 0.10
y=0.0, yhat=0.1, likelihood: 0.90
y=0.0, yhat=0.8, likelihood: 0.20
```

Log-Likelihood:

- We want to maximize the `log-likelihood` with respect to the $\beta's$
  - `side note` : Using Sums vs Multiplications are easier to calculate/compute
  - There are benefits with plotting and concavity as well when you are reaching the max

`Log-Likelihood:`

$$l(\beta_o, \beta) = \sum_{i=1}^{n} y_i log(P(x_i)) + \sum_{i=1}^{n} (y_i - 1) log(1 - P(x_i))$$

$$l(\beta_o, \beta) = \sum_{i=1}^{n} y_i log(P(x_i)) + \sum_{i=1}^{n} log(P(1 - x_i)) - y_i log(P(1 -$$

`Manipulate logs:`

$$l(\beta_o, \beta) = \sum_{i=1}^{n} y_i log(\frac{P(x_i)}{1-P(x_i)}) + log(1 - P(x_i)))$$

`Substitute:` $log(\frac{P(x_i)}{1-P(x_i)}) = \beta_o + x\beta$

$$l(\beta_o, \beta) = \sum_{i=1}^{n} y_i(\beta_o + x\beta) + log(1 - P(x_i)))$$

`Substitute:` $P(x_i)$, also pay attention to the signs!

$$l(\beta_o, \beta) = \sum_{i=1}^{n} y_i(\beta_o + x\beta) - log(1 + e^{\beta_o + x\beta}))$$

Training our data:

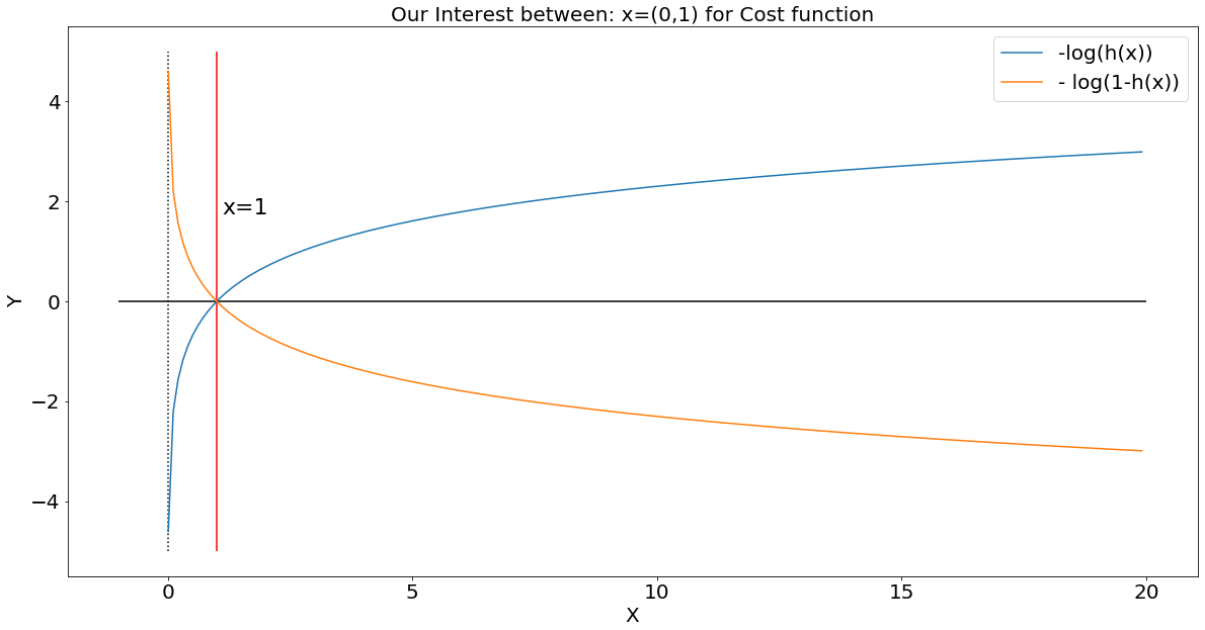
That is where the `Cost Function` comes into play: aims to estimate β

- **`Since the log-likelihood estimates the MAX and we want the MIN we need to put a (-) in front of it`**
- Since logistic regression estimates probabilities and makes predictions; we will use the β so that the model will estimate high probabilities for $y = 1$ and low probalilities of $y = 0$.

$$Y_i = \begin{cases} -log(P(x)), y=1 \\ (-log(1-P(x)), y=0 \end{cases}$$

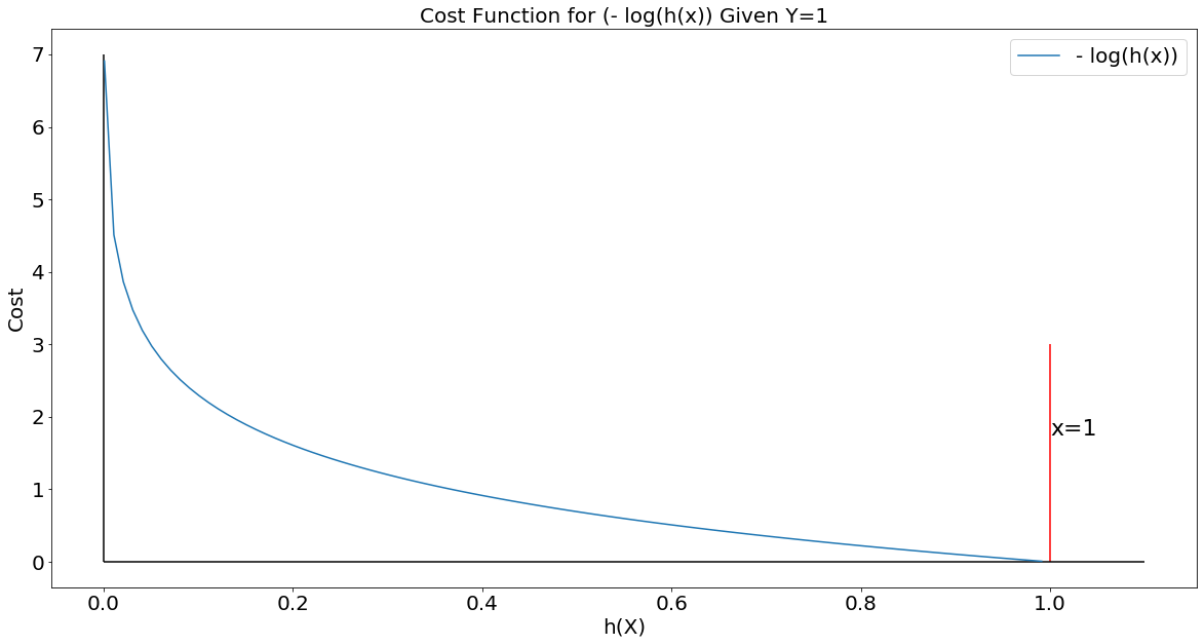
source (https://www.oreilly.com/library/view/hands-on-machine-learning/9781491962282/ch04.html#logisticregression_model_estimated_probability_vectorized_form) | source2 (https://towardsdatascience.com/logistic-regression-detailed-overview-46c4da4303bc)

In [7]:
```
x = np.arange(0.01,20,0.1)
y = np.log(x)
y_ = -np.log(x)
plt.figure(figsize=(20,10))
plt.rc('xtick', labelsize=20)
plt.rc('ytick', labelsize=20)
plt.vlines(x=1,ymin=-5,ymax=5, colors='red', linestyles='solid')
plt.text(1.1,1.75,'x=1',size=22)
plt.hlines(y=0,xmin=-1,xmax=20, colors='black', linestyles='solid')
plt.xlabel("X",size=20)
plt.ylabel("Y",size=20)
plt.plot(x,y,label='-log(h(x))')
plt.plot(x,-y,label='- log(1-h(x))')
plt.vlines(x=0,ymin=-5,ymax=5, colors='black', linestyles='dotted')
plt.title('Our Interest between: x=(0,1) for Cost function',fontsize=20)
plt.legend(fontsize=20)
plt.show()
```



Y=1 which is $-log(h(x))$

In [8]:
```
x = np.arange(0.001,1,0.01)
y = -np.log(x)
plt.figure(figsize=(20,10))
plt.rc('xtick', labelsize=20)
plt.rc('ytick', labelsize=20)
plt.vlines(x=1,ymin=0,ymax=3, colors='red', linestyles='solid')
plt.text(1,1.75,'x=1',size=22)
plt.hlines(y=0,xmin=0,xmax=1.1, colors='black', linestyles='solid')
plt.vlines(x=0,ymin=0,ymax=7, colors='black', linestyles='solid')
plt.xlabel("h(X)",size=20)
plt.ylabel("Cost",size=20)
plt.plot(x,y,label='- log(h(x))')
plt.title("Cost Function for (- log(h(x)) Given Y=1",fontsize=20)
plt.legend(fontsize=20)
plt.show()
```

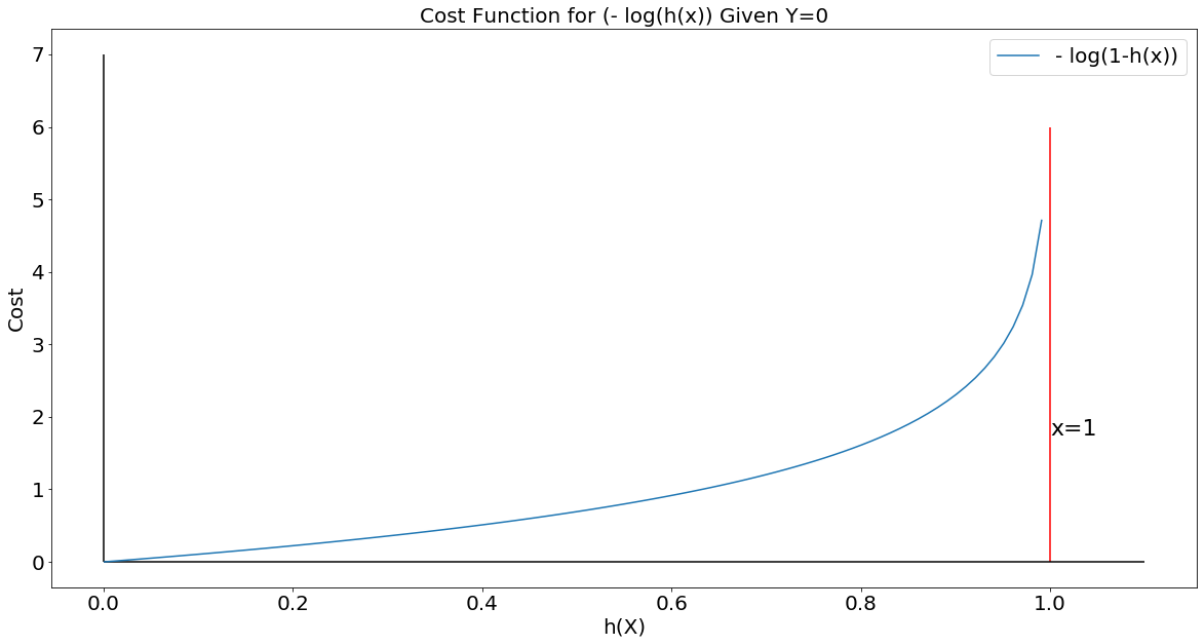


- If y=1 then the $Cost(h(x), y) = -log(h(x))$, then P(x)=1 and Cost =0
- If y=0 then the cost will be quite high and explode, P(x)=0, y=0, and high cost.

where X is our $\bar{Y}$

This is saying that if we predict 1, given that y=1 then the cost is zero.

In [9]:
```
x = np.arange(0.001,1,0.01)
y = -np.log(1-x)
plt.figure(figsize=(20,10))
plt.rc('xtick', labelsize=20)
plt.rc('ytick', labelsize=20)
plt.vlines(x=1,ymin=0,ymax=6, colors='red', linestyles='solid')
plt.text(1,1.75,'x=1',size=22)
plt.hlines(y=0,xmin=0,xmax=1.1, colors='black', linestyles='solid')
plt.vlines(x=0,ymin=0,ymax=7, colors='black', linestyles='solid')
plt.xlabel("h(X)",size=20)
plt.ylabel("Cost",size=20)
plt.plot(x,y,label='- log(1-h(x))')
plt.title("Cost Function for (- log(h(x)) Given Y=0",fontsize=20)
plt.legend(fontsize=20)
plt.show()
```



This is telling us: if y=0,which is $-log(h(1-x))$

- IF y=0 and we predicted 0, then the cost is 0.
- But, if y=0 and we predicted 1, then the cost is quite high and explodes.

Now what?

- **Well, we have the log-likelihood but, we would like to maximize this. But, unfortunately we are unable to go directly to finding derivatives w.r.t parameters.**

This is not a closed form solution, and we need to use approximation methods:

Choose one:

such as

- **Gradient Descent**
- **Newton-Raphson**
- Other Iterative approaches
- Cross Entropy

We want to use optimization (cost function) to do this

Gradient Descent: can have some issues that need to be considered; for instance you can get stuck in a local minima with nonlinear functions.

Newton-Raphson: is a root finding algorithm

- Since, we can't use the dirivative and set to zero let's get on with this!

Take the log-likelihood from above:

$$l(\beta_o, \beta) = \sum_{i=1}^{n} y_i(\beta_o + x\beta) - log(1 + e^{\beta_o + x\beta})$$

The **Newton-Raphson Equation (1 iteration!):** $\beta_{i+1} = \beta_i - \frac{f'(\beta_i)}{f''(\beta_i)}$

Take first partial derivatives w.r.t parameters:

$$\frac{\partial l}{\partial \beta_1} = -\sum_{i=1}^{n} \frac{1}{1+e^{\beta_o + x_i\beta_1}} x_i e^{\beta_o + \beta_1 x_i} + \sum_{i=1}^{n} y_i x_i$$

Let's do some substitution here:

$$\frac{\partial l}{\partial \beta_1} = -\sum_{i=1}^{n} P(x_i)x_i + \sum_{i=1}^{n} y_i x_i$$

Combine terms and bring out sum:

$$\frac{\partial l}{\partial \beta_1} = \sum_{i=1}^{n} x_i(y_i - P(x_i))$$

Now We need the Second Partial Derivative:

$$\frac{\partial^2 l}{\partial \beta_1 \beta_o} = -\sum_{i=1}^{n} x_{i,o}\left(\frac{e^{\beta_o + \beta_{1,x_i}}}{1+e^{\beta_o + \beta_{1,x_i}}}\right)\left(\frac{1}{1+e^{\beta_o + \beta_{1,x_i}}}\right)x_{i,1}$$

$$\frac{\partial^2 l}{\partial \beta_1 \beta_o} = \sum_{i=1}^{n} x_{i,o}x_{i,1}P(x_i)(1 - P(x_i))$$

This is 1 iteration only!

Let us generalize each of the partials:

$$\frac{\partial l}{\partial \beta_j} = \sum_{i=1}^{n} x_j(y_i - P(x_i j))$$

$$\frac{\partial^2 l}{\partial \beta_j \beta_k} = \sum_{i=1}^{n} x_{i,j} x_{i,k} P(x_i)(1 - P(x_i))$$

Then you would have to take these and combine:

$$\beta* = \beta_j - \frac{\frac{\partial l}{\partial \beta_j}}{\frac{\partial^2 l}{\partial \beta_j \beta_k}}$$

computational considerations (https://sites.stat.washington.edu/mmp/courses/stat535/fall15/Handouts/l3slides-training.pdf) | derivation (https://whyml.wordpress.com/tag/logistic-regression/) | derivation and thesis: explained logist regression (https://mds.marshall.edu/cgi/viewcontent.cgi?article=2169&context=etd)

Now we have went in 1D, and 1 iteration: But, ...What if we want to go in multiple dimensions/variables?

CHECK THESE OUT FOR HELP:

more than 1 variable derivation (http://www.cs.columbia.edu/~amoretti/smac_04_tutorial.html) | ex more than 1 variable (http://web.cs.ucla.edu/~yzsun/classes/2018Fall_CS145/Discussion/Discussion_Week2.pdf) | Suppl. text as well (https://www.statlect.com/fundamentals-of-statistics/logistic-model-maximum-likelihood) | more help (http://www.cs.columbia.edu/~amoretti/smac_04_tutorial.html) | alt help (http://fourier.eng.hmc.edu/e176/lectures/NM/node21.html)

Here is Multiple Dimensions of Newton-Raphson: (Optimization)

Since we are starting from the knowledge of:

This is for 1D:

The **Newton-Raphson Equation (1 iteration!):** $\beta_{i+1} = \beta_i - \frac{f'(\beta_i)}{f''(\beta_i)}$

We want `Multiple Dimensions` : therefore we will use the Taylor expansion second order, with Hessian and Gradient.

`Let me show you something to let this resonate and make sense:`

If we simplify the Taylor Expansion to: $f(\beta_{x+1}) \simeq f(\beta_o) + (\beta_{i+1} - \beta_o) f'(x_o)$

with some shuffling:

$$\beta_{i+1} = \beta_o - \frac{f(\beta_o)}{f'(\beta_o)}$$

or

$$\beta_{i+1} = \beta_o - \frac{f'(\beta_o)}{f''(\beta_o)}$$

The distinction is that the first one will use a tangent line crossing the x-axis and that will be your next point to start. The seond one will use a function 'parabola' curve to approximate and it will instead use the minimum for its point to move to next location.

`Here is what we want in the end from all the math:`

Answer: $\beta_{i+1} = \beta_o - H(\beta_o)^{-1} \nabla f(\beta_o)$

Lets visualize what is happending with Newton-Raphson for a few iterations if you were using a cost function

Source of plots (http://www.cs.columbia.edu/~amoretti/smac_04_tutorial.html)

Alternate way is plotting:

- using a quadratic which will do the same thing; except for each iteration the minimum of the function will be used for next moving point instead of where the point crosses the axis.

The parameters that are found here:

are able to be placed back into your equation and used to find your probabilities

What is the take away?

- Each iteration will approach the min. for the cost function
- You are trying to find the best fit, the parameters that are found will be those!
- Take those parameters and put them into your function and then you will be able to best estimate your given data.
- You can then make predictions based on these parameters using your data and find out how well your actual data, measuare against your predictions by accuracy, recall, precision etc.

In [27]:
```
# import statsmodels.api as sm
# from sklearn import linear_model


x_ = np.transpose(np.array([[162, 165, 166, 170, 171, 168, 171, 175, 176
,
182, 185]]))
x = sm.add_constant(x_)
y=np.array([0,0,0,0,0,1,1,1,1,1,1])
result=sm.Logit(y,x)
mod=result.fit(method='newton')
print('params',mod.params)
print('predicted_prob',mod.predict(x))
```

```
Optimization terminated successfully.
         Current function value: 0.345900
         Iterations 8
params [-84.83310945   0.49853544]
predicted_prob [0.01678456 0.07078045 0.11142901 0.47949025 0.60263354
0.25366578
 0.60263354 0.91763145 0.9482958  0.99726897 0.99938664]
```

In [28]:
```
x_ = np.transpose(np.array([[162, 165, 166, 170, 171, 168, 171, 175, 176
,
182, 185]]))

predicted_probs=[]
log_odds_=[]
odds_scale=[]
for i in x_:
    log_odds_.append(-84.833+.4985*i)
    odds_scale.append(np.exp(-84.833+.4985*i))
    predicted_probs.append(np.exp(-84.833+.4985*i)/(1+np.exp(-84.833+.49
85*i)))

# http://www.pmean.com/13/predicted.html (math motivation)
# https://aaronschlegel.me/newtons-method-equation-roots.html (good exam
ple worked)
```

In [29]:
```
print('log_odds:',log_odds_)
print('---------------------------------')
print('odds:',odds_scale)
print('---------------------------------')
print('predicted_prob:',predicted_probs)
print('---------------------------------')
```

```
log_odds: [array([-4.076]), array([-2.5805]), array([-2.082]), array([-
0.088]), array([0.4105]), array([-1.085]), array([0.4105]), array([2.40
45]), array([2.903]), array([5.894]), array([7.3895])]
---------------------------------
odds: [array([0.01697523]), array([0.07573613]), array([0.1246806]), ar
ray([0.91576088]), array([1.50757138]), array([0.33790179]), array([1.5
0757138]), array([11.07289245]), array([18.22874967]), array([362.85380
061]), array([1618.89646231])]
---------------------------------
predicted_prob: [array([0.01669188]), array([0.070404]), array([0.11085
868]), array([0.47801419]), array([0.60120776]), array([0.25256098]), a
rray([0.60120776]), array([0.91716981]), array([0.94799454]), array([0.
99725164]), array([0.99938268])]
---------------------------------
```

In [56]:
```
#confusion matrix:
mod.pred_table()

# now you can take the 4+5=9 therefore 90% accuracy btw
```

Out[56]:
```
array([[4., 1.],
       [1., 5.]])
```

In [85]:
```
# Now with SkLearn:
from sklearn.linear_model import LogisticRegression
from sklearn.metrics import classification_report, confusion_matrix

model = LogisticRegression(solver='liblinear', random_state=0).fit(x_, y
)
print('coeff:beta',model.coef_)
print('intercept',model.intercept_)
print('predicted_labels',model.predict(x_))
print('actual_labels',y)
print('accuracy',model.score(x_,y))
print('confusion_mat',confusion_matrix(y, model.predict(x_)))
```

```
coeff:beta [[0.00223001]]
intercept [-0.14698449]
predicted_labels [1 1 1 1 1 1 1 1 1 1 1]
actual_labels [0 0 0 0 0 1 1 1 1 1 1]
accuracy 0.5454545454545454
confusion_mat [[0 5]
 [0 6]]
```

Gradient Descent vs Newton-Raphson

Newton-Raphson:

- Finds roots
- converges quickly
- when the second derivative is known, it computes quickly such as `logistic regression`
- need to provide a starting point, this can be critical!
- `Runtime:` $O(np(n+p)+P^3)$
- `Memory used:` $O(p^2)$ for each iteration
- Computationally intensive, but since you have low iterations it will converge (if it converges) faster than gradient descent
  - The second derivative is what helps convergence, because it is used for each step
- Very large datasets can be a problem with computing the Hessian matrix.
  - To get around this you may decrease complexity/dimensions but result in more iterations
  - Paying attention to the Hessian matrix is important as well: make sure it is positive definite.
- `When using sklearn` : consider using different solvers such as

```
sklearn.linear_model.LogisticRegression and use solver: 'lbfgs'
```

Gradient Descent:

- finds the min/max using first derivative
- Can be troublesome with finding local min/max
  - this can be combated using a random approach
- parametric: we need to adjust parameters for hyper-tuning
- `Runtime:` $O(np)$
- `Memory:` $O(p)$

Newton-Raphson vs Gradient Descent (https://medium.com/@papillonbee/logistic-regression-from-scratch-with-gradient-descent-and-newtons-method-ff4307e3cb30) | Good background Training predictors (https://sites.stat.washington.edu/mmp/courses/stat535/fall15/Handouts/l3slides-training.pdf) | worked ex. gradient descent (https://kseow.com/logisticregression.html)

`SOLVER Choices over Gradient Descent:`

- `LBFGS,BFGS,Conjugate Gradient`
  - `Pros` :
    - Dont need to pick a parameter for learning rate
    - usually runs faster due to less iterations for exampl
    - can work out gradient, but there are some hiccups
  - `Cons`
    - Complex
    - Somewhat of a blackbox unless you know what your doing

Pros & Cons: Logistic Regression

Pros

- good because it classifies and returns probabilities as well
- you are able to determine the relevance of a predictor by its sign and size which is good for interpreting and training.

Cons

- Potential overfitting with large datasets, use of regularization terms should be considered
- perfect separation is a problem if two classes could be completely separated. Machine learning isn't needed in this scenario. But, adding penalized terms to the weights or prior probabilities weights.

source (https://www.knowledgehut.com/blog/data-science/logistic-regression-for-machine-learning)

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```
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```

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