PEER REVIEW PROBLEMS We are using this assignment to give you an opportunity to test the mechanics of the peer review system that will be used to evaluate the final exam. You have to take the final exam and participate in the calibrated peer review process to be eligible for distinction in the course. The use of the mechanics of the review system for this assignment does not involve grading calibration, nor will your work be numerically graded by other students, though it will be viewed by three other students, selected randomly from the class. (This is purely to let you become familiar with the peer review interface, the peer review process, and to test legibility of uploaded work, all in advance of the final exam.) This entire process is carried out in full, mutual anonymity.

See the peer review module on the website for an interactive version of this assignment that gives full details of the submission procedure. The deadline for completing this assignment is noon PDT on Friday October 19. The deadline for submitting your "reviews" of others' solutions is noon PDT on Tuesday October 23.

There will be no tutorial session on this assignment. Model solutions to the questions on this assignment will be accessible on the course website after the (simplified) peer review process is completed.

1. Express as concisely and accurately as you can the relationship between $b|a$ and a/b.

2. Determine whether each of the following is true or false and prove your answer. (You saw these questions in the in-lecture quiz, so th first part is a repeat, except that now you should know the right answers.) The focus of this assignment is to *prove* each of your answers.

   (a) $0|7$ (b) $9|0$ (c) $0|0$ (d) $1|1$

   (e) $7|44$ (f) $7|(-42)$ (g) $(-7)|49$ (h) $(-7)|(-56)$

   (i) $(\forall n \in \mathcal{Z})[1|n]$ (j) $(\forall n \in \mathcal{N})[n|0]$ (k) $(\forall n \in \mathcal{Z})[n|0]$

3. Prove all the parts of the theorem in the lecture, giving the basic properties of divisibility. Namely, show that for any integers a, b, c, d, with $a \neq 0$:

   (a) $a|0,\ \ a|a$;
   (b) $a|1$ if and only if $a = \pm 1$;
   (c) if $a|b$ and $c|d$, then $ac|bd$ (for $c \neq 0$) ;
   (d) if $a|b$ and $b|c$, then $a|c$ (for $b \neq 0$) ;
   (e) $[a|b$ and $b|a]$ if and only if $a = \pm b$;
   (f) if $a|b$ and $b \neq 0$, then $|a| \leq |b|$;
   (g) if $a|b$ and $a|c$, then $a|(bx + cy)$ for any integers x, y.

4. Prove that if p is prime, then $\sqrt{p}$ is irrational. (You can assume that if p is prime, then whenever p divides a product ab, p divides at least one of a, b.)

OPTIONAL PROBLEMS (NOT PART OF THE REVIEW PROCESS)

1. It is a standard result about primes (known as Euclid's Lemma) that if p is prime, then whenever p divides a product ab, p divides at least one of a, b. Prove the converse, that any natural number having this property (for any pair a, b) must be prime.

2. Try to prove Euclid's Lemma. If you do not succeed, you can find proofs in most textbooks on elementary number theory, and on the Web. Find a proof and make sure you understand it. If you find a proof on the Web, you will need to check that it is correct. There are false mathematical proofs all over the Internet. Though false proofs on Wikipedia usually get corrected fairly quickly, they also occasionally become corrupted when a well-intentioned contributor makes an attempted simplification that renders the proof incorrect. Learning how to make good use of Web resources is an important part of being a good mathematical thinker.