1. Let $A = \{r \in \mathcal{Q} \mid r > 0 \wedge r^2 > 3\}$. Show that A has a lower bound in $\mathcal{Q}$ but no greatest lower bound in $\mathcal{Q}$. Give all details of the proof along the lines of the proof given in the lecture that the rationals are not complete.

2. In addition to the completeness property, the *Archimedean property* is an important fundamental property of $\mathcal{R}$. It says is that if $x, y \in \mathcal{R}$ and $x, y > 0$, there is an $n \in \mathcal{N}$ such that $nx > y$.

   Use the Archimedean property to show that if $r, s \in \mathcal{R}$ and $r < s$, there is a $q \in \mathcal{Q}$ such that $r < q < s$. (Hint: pick $n \in \mathcal{N},\ n > 1/(s-r)$, and find an $m \in \mathcal{N}$ such that $r < (m/n) < s$.)

3. Formulate both in symbols and in words what it means to say that $a_n \not\to a$ as $n \to \infty$.

4. Prove that $(n/(n+1))^2 \to 1$ as $n \to \infty$.

5. Prove that $1/n^2 \to 0$ as $n \to \infty$.

6. Prove that $1/2^n \to 0$ as $n \to \infty$.

7. We say a sequence $\{a_n\}_{n=1}^{\infty}$ *tends to infinity* if, as n increases, a_n increases without bound. For instance, the sequence $\{n\}_{n=1}^{\infty}$ tends to infinity, as does the sequence $\{2^n\}_{n=1}^{\infty}$. Formulate a precise definition of this notion, and prove that both of these examples fulfil the definition.

8. Let $\{a_n\}_{n=1}^{\infty}$ be an increasing sequence (i.e. $a_n < a_{n+1}$ for each n). Suppose that $a_n \to a$ as $n \to \infty$. Prove that $a = \text{lub}\{a_n | n \in \mathcal{N}\}$.

9. Prove that if $\{a_n\}_{n=1}^{\infty}$ is increasing and bounded above, then it tends to a limit.