Motivation & introduction

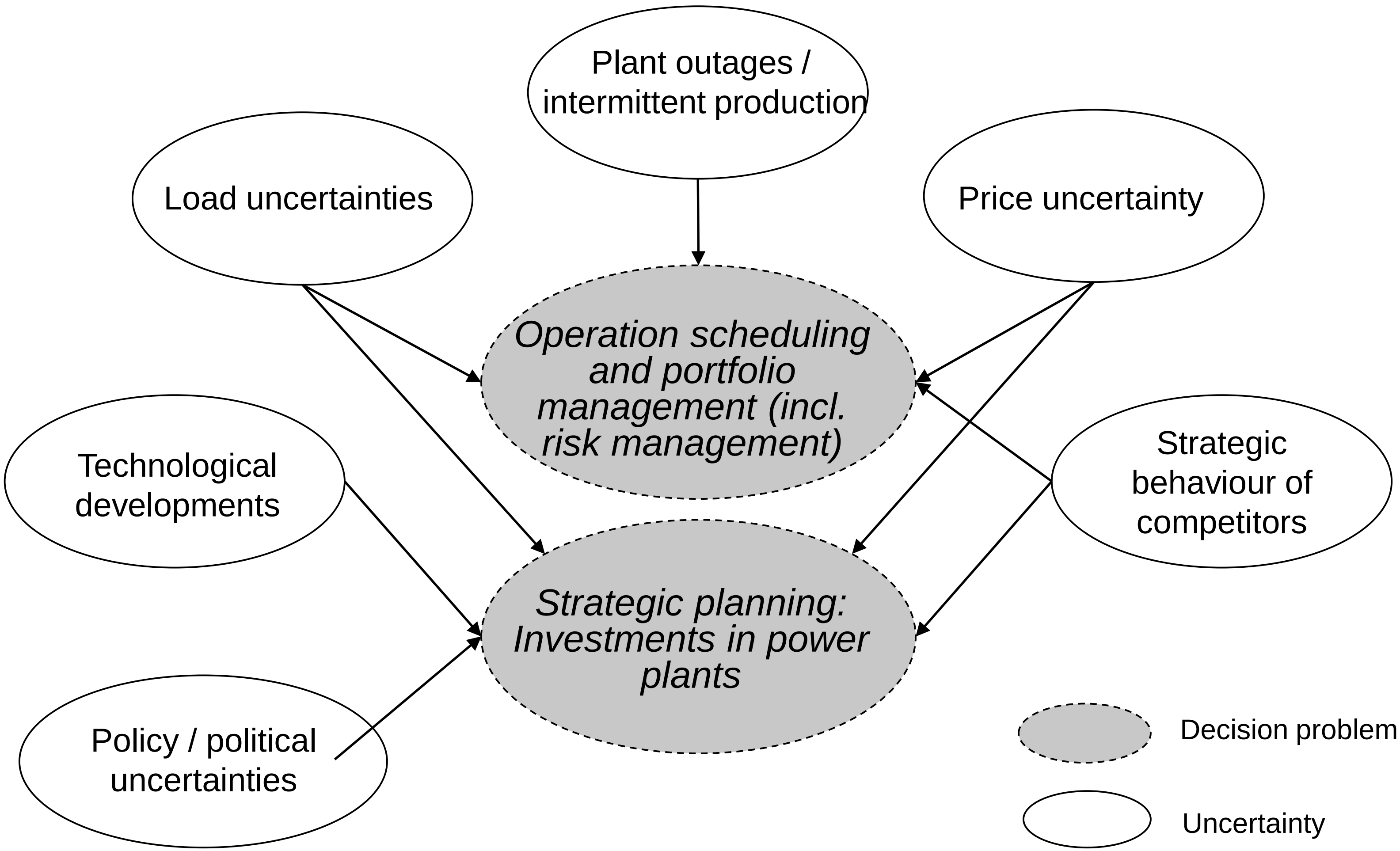
Decision-making problems in (electricity) markets are subject to uncertainties

- … *under risk*: probabilities p_j of possible future situations s_j are known!
  => stochastic decision model
- … *under insecurity*: possible future situations s_j are known, but not their probabilities p_j.
  => often "Laplace assumption" => all situations have same probability p
  => or instead: "playing" with p_j and its impact on the result => robustness?

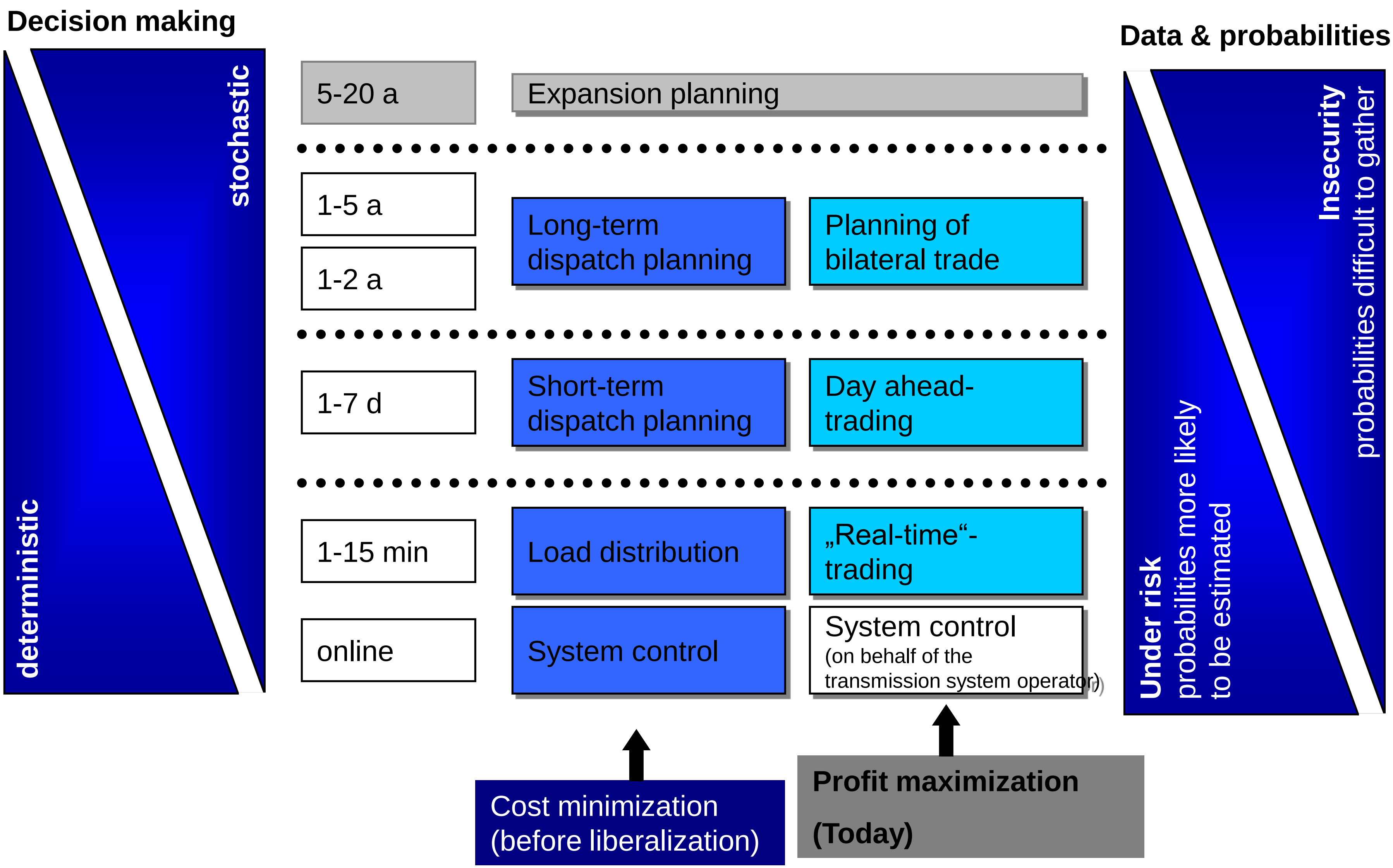
Uncertainties in energy markets

- prices, demand, intermittent production, plant availability, …

Uncertainties and key decisions in electricity markets



Planning task, different lead times and uncertainty



Introduction (2)

Uncertain parameters have to be modelled first!

- in general this is done with stochastic processes (first day!)
- in this course: probabilities p_j and future situations s_j are given / known!

Each uncertain parameter is modelled

- by a set of finite outcomes or scenarios
- where each outcome represents a realization of the uncertain parameter(s) with an associated probability.

Usually / often the number of outcomes is very high

- not “manageable” in stochastic models (due to limitations in calculation time or calculation power)

=> Scenario reduction: Reducing amount of outcomes while retaining the statistical properties of the uncertain parameters

Methods considering uncertainty

What is the general objective in decision making, when a decision has to be made under uncertainty?

- Identify a „optimal“ decision, which is „robust“ under uncertain parameters!

=> *Different methods can be applied!*

Methods dealing with uncertainty

- *Correction Method*: mark-up for risk or mark-down for uncertain parameter

  + Can be used within standard methods (e.g. higher rate of return in DCF)

  + Easy to apply

  - In general, only one parameter can be considered

  - No information about different developments

Methods considering uncertainty (cont.)

- *Scenario analysis*: define different possible/expected developments („worlds/futures“)

  + Easy to apply, especially in combination with deterministic methods

  - Challenge: selection of adequate scenarios (without calculating „thousands“ of scenarios“, e.g. three uncertain parameters with three developments – worst case, reference, best case - results in $3^3 = 27$ scenarios)

  - Robust decisions are not implicitly identified => difficult to identify

- *Sensitivity analysis*: identify impact of different parameters on „robustness“

  + Can be used within standard methods
  => *can be used as preselection method for more sophisticated method*

  + Easy to apply

  - Difficult to develop conclusions, when several parameters are varied

  - Robust decisions are not implicitly identified => difficult to identify

Methods considering uncertainty (cont.)

- *one-stage stochastic programming*: nearly analogue to deterministic problem

  + still quite easy to set-up (but depends on type of model):
    introduction of risk coefficient (risk aversion) in objective function and/or restriction!
    if „coefficient“ on restriction: probabilistically constrained problems / Chanced-constraint programming (can also get difficult)!

  + often also feasible for „real world“ problems (as similar to deterministic model)
  - but: comparable to correction method, which has several shortfalls

- *two-stage / multi-stage stochastic model*: implicit consideration of uncertainty

  + „strongest“ method to consider uncertainty in Operations Research

  + robust decision can implicitly be determined
  - large effort, data availability and calculation time

Stochastic programming fundamentals

Random variable

- … a finite set of realizations or scenarios

For instance, random variable λ can be represented by $\lambda(\omega),\ \omega = 1, \ldots, N_\Omega$ where ω is the scenario index, N_Ω is the number of scenarios considered and Ω is the set of scenarios.

Each realization $\lambda(\omega)$ is associated with a probability $\pi(\omega)$ defined as

$$\pi(\omega) = P(\omega | \lambda = \lambda(\omega)), \quad \text{where} \quad \sum_{\omega \in \Omega} \pi(\omega) = 1.$$

Example: Random variable: electricity price

- Characterisation of electricity price at 12 o'clock
  - 45 €/MWh with probability 0.2
  - 35 €/MWh with probability 0.6
  - 30 €/MWh with probability 0.2

Stochastic programming fundamentals

Stochastic process

- Evolution of a random variable over time
- A stochastic process is constituted by a set of dependent random variables sequentially arranged in time
  - For each time period, the corresponding random variable (e.g. price at 12) depends on the random variables (e.g. prices in other hours of the day).

Example continued:

- Electricity price at 12 o'clock characterised as before
- Pool price at 1 pm represented by the random variable:
  - If 45 €/MWh at noon, probabilities at 1 pm of the price being 50, 40 or 35 €/MWh are 0.8, 0.1 or 0.1.
  - If 35 €/MWh at noon, probabilities at 1pm being 50, 40 or 35 €/MWh are 0.2, 0.7 or 0.1.
  - If 30 €/MWh at noon, probabilities at 1pm being 50, 40 or 35 €/MWh are 0.1, 0.4 or 0.5.

=> Dependent variables *price at noon* and *price at 1pm* constitute a **stochastic process.**

- Time series analysis and modelling of stochastic processes can be (!), but must not, a basis for stochastic programming => in this course, we apply both!

Stochastic programming fundamentals

Scenarios

- Can be based on a stochastic process, but must not be based on it!
- Definition (now): A scenario is a single realization of a stochastic process!
- To adequately describe a stochastic process, it is crucial to generate a sufficient number of scenarios, so that most plausible realizations of the stochastic process are covered!
- Generally, it is required to calculate a very large amount of scenarios resulting in associated stochastic programs, which are difficult to solve due to the size!
- Number of initially generated scenarios must be reduced, without loosing information!
  - Scenario generation and scenario-reduction procedures are necessary

Stochastic programming fundamentals

Example Stochastic process: scenarios

Scenario #	Price at 12 (€/MWh)	Price at 1pm (€/MWh)	Probability (%)
1	45	50	0.2 x 0.8 = 0.16
2	45	40	0.2 x 0.1 = 0.02
3	45	35	0.2 x 0.1 = 0.02
4	35	50	0.6 x 0.2 = 0.12
5	35	40	0.6 x 0.7 = 0.42
6	35	35	0.6 x 0.1 = 0.06
7	30	50	0.2 x 0.1 = 0.02
8	30	40	0.2 x 0.4 = 0.08
9	30	35	0.2 x 0.5 = 0.10

- Number of scenarios is small (9), but this is not necessarily the case.

Stochastic programming fundamentals

Stochastic programming problems

- Decision making under uncertainty: decision maker has to make a optimal decision throughout a decision horizon with incomplete information.
- A *stage* represents a point in time where decisions are made or where uncertainty partially or totally vanishes.
  - Information available is usually different from stage to stage
  - According to number of stages: two-stage and multi-stage stochastic problems can be distinguished
  - Be aware: each stage requires a decision based on different / new information (else, the problem would be the same)

Stochastic programming fundamentals

Two stage problem:

- Decision x is made before knowing the actual value of the stochastic process λ, while y is determined after the realisation of λ.
- Consequently, decision y depends on the decision x previously made and on the realisation $\lambda(\omega)$ of the stochastic process λ. y can be expressed as $y(x, \omega)$.
- Decision-making process:
  1. Decision x is made.
  2. Stochastic process λ is realized as $\lambda(\omega)$.
  3. Decision $y(x, \omega)$ is made.

Stochastic programming fundamentals

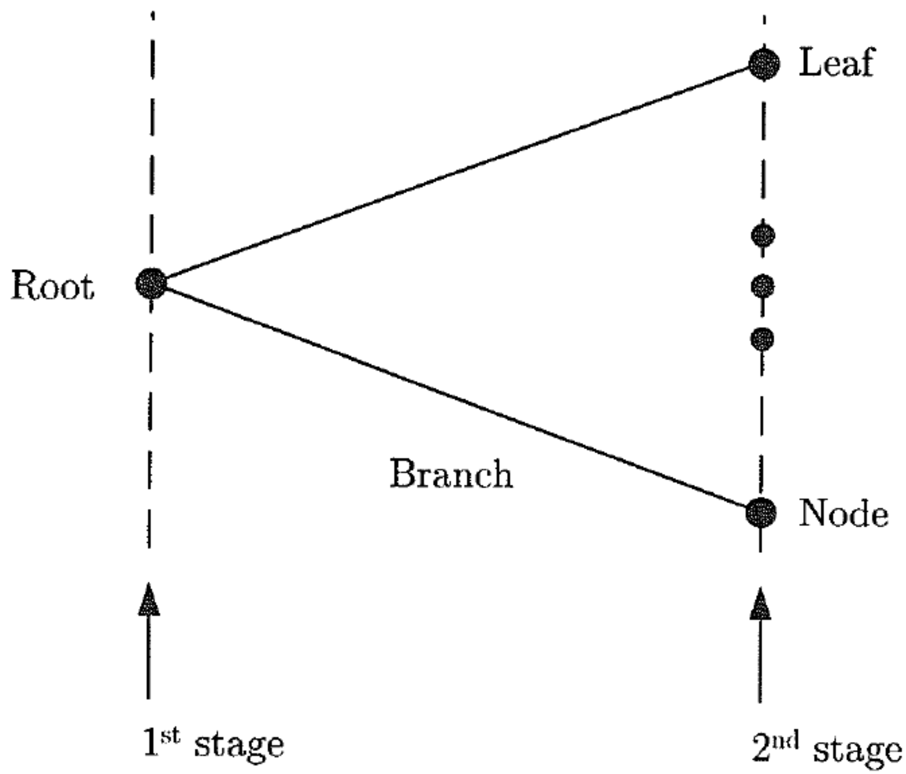
Two kind of decisions in a two stage problem:

- *First-stage* or *here-and-now* decisions:
  - Decisions are made before the realization of the stochastic process.
  - Variables representing here-and-now decisions do not depend on each realization of the stochastic process.

- *Second-stage* or *wait-and-see* decisions:
  - Decisions are made after knowing the actual realization of the stochastic process.
  - Consequently, decisions depend on each realization vector of the stochastic process.
  - If the stochastic process is represented by a set of scenarios, a second-stage decision variable is defined for each single scenario.

Stochastic programming fundamentals

Visualization of two stage problem with a scenario tree:



Here-and-now decisions
(e.g. plant investments)

Wait-and-see decisions
(e.g. plant dispatch under given investments)