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A multi-stage stochastic programming model for dynamic pricing and lead time decisions in multi-class make-to-order firm

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Abstract Make-to-order firms use different strategies, such as dynamic pricing and due date management, to influence their performance. In these strategies, orders are segmented into classes based on their sensitivity to lead time and price. Quoting different prices and lead times to different classes of customer can increase a firm's profit and its capacity utilization. Most research in this area does not consider the effects of production constraints on price and lead time decisions. In this paper, we consider the role of flexibility in dynamically choosing the price, lead time and segmentation of customers in make-to-order environments with limited production capacity and multi-period horizon under a stochastic demand function. To reflect the dynamic variations of a system's conditions, we propose a Multi-stage Stochastic Programming (MSP) method to jointly determine prices, lead time and production. Furthermore, we assume that demand is a linear function of price, lead time and time. Through numerical analyses, we indicate the benefits of dynamic pricing and lead time decisions, based on different customer classes in various environments.

1. Introduction

Today's world is more competitive than ever before. For this reason, revenue management and pricing policies are the most effective tools that managers can use to influence market demand, and balance supply and demand. Revenue management is the art of maximizing profit from a limited capacity of products over a finite horizon. This is done by selling each product to the right customer at the right time for the right price [1]. The application of revenue management has been most effective when demand can be segmented, and price sensitivity varies across these segments [2].

While revenue management techniques and dynamic pricing policies have been successfully applied by airlines, hotels and retail chains, it has great potential also for manufacturing environments [2–4]. Some benefits of revenue management and pricing include potential increases in profit and improvements, such as a reduction in demand or production variability, which results in more efficient supply chains.

In revenue management, segmentation of orders is based on their sensitivity to price. Moreover, the segmentation and quotation lead times, based on an order's time sensitivity, is a managerial challenge in make-to-order environments.

In Make-To-Order (MTO) environments, various attributes of the product, such as price and lead time, are evaluated by the buyer. Thus, for each customer, the firm should determine the due date and price based on customer preference, available capacity and other potential orders that could demand resources [5]. In fact, by using both lead time differentiation and price differentiation in a revenue management system, manufacturers can coordinate pricing and production decisions to improve the firm's performance, and increase the overall capacity utilization by dynamically differentiating products. Dell Computers and Amazon.com are examples of companies that separate their customers into different classes and change prices based on parameters, such as demand variation, inventory levels or production schedules [6].

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Based on pricing and scheduling decisions, as proposed by Charnsirisakskul et al. [7], this paper proposes an extended model that incorporates joint pricing and lead time control problems in a stochastic production environment with multiple customer segments, which are classified based on the parameters of willingness to pay and sensitivity to short delivery time.

We model an MTO production system where demand is a function of price, lead time and time. We take into account that orders arrive within pre-determined intervals. Price and due-date decisions over a planning horizon are assumed to be made dynamically to upcoming orders in these time intervals, based upon available capacity, operating costs associated with the production of the order, tardiness and holding costs. The orders in each time interval can be classified differently.

The firm bears holding costs and tardiness penalties incurred for orders that are completed in advance of their due dates and orders shipped after the preferred due date, respectively.

To reflect the dynamic variations of system conditions, we propose Multi-stage Stochastic Programming (MSP) methods.

This paper is organized as follows: Section 2 discusses the relevant literature. Section 3 describes the pricing and lead time quotation problem. In Section 4, we describe the algorithm used to approximate the stochastic demand process, and to reformulate our problem in the scenario representation. Section 5 presents some numerical examples for our model. Conclusions are summarized in Section 6.

2. Literature review

Joint pricing and inventory control strategies in a single period (news vendor) manufacturing environment, assuming static prices and demand, were first analyzed by Whitin [8]. Whitin's work was extended by Mills [9] who studied the effect of uncertainty on a pricing policy under a linear-additive functional form for demand. Static pricing with a multiplicative form of demand was formulated by Zabel [10] and Karlin and Carr [11]. Additionally, Petruzzi and Dada [12] examined an extension of the news vendor problem. In this problem, demand depends on both price and inventory levels. Other researchers considered multi-period, finite-horizon pricing models [13–15]. The retail industry also uses dynamic pricing techniques to match demand with capacity, maximize revenue or achieve other strategic goals, as shown by Gallego and Vanryzin [16,17], Federgruen and Heching [18], Chen and Simchi-Levi [19], You and Wu [20] and others. Chan et al. [21] and Elmaghraby and Keskinocak [22] presented a thorough review of both single and multi-period models combining pricing and inventory strategies. Most research on integrating pricing strategies with inventory control policies has ignored production capacity limitations.

The second body of research related to our paper focuses on due-date management in which the key decisions are due-date setting and scheduling (e.g. [23–31]). Keskinocak and Tayur [5] presented a review on due date management and dynamic pricing. They categorized all due-date assignment methods into two categories: exogenous (determined by the sales department, without knowing the actual production schedule) and endogenous (assigned internally by the scheduling model). Some researchers used mathematical programming to solve the due-date setting problem. For example, Chen et al. [27] developed mixed-integer programming models for quantity and due-date quoting to customer orders that arrive within a pre-determined time interval. Sawik [31] proposed a dual-objective problem of due-date setting over a rolling planning horizon in make-to-order manufacturing. They considered the total number of delayed products as a primary optimality criterion, and the total or maximum delay of orders as a secondary criterion. The results in the papers cited above indicate that proper due-date management offers a much larger improvement in performance than priority sequencing.

This literature also assumes that the demand process is independent of any pricing and/or due-date decisions. Some researchers considered that quoted lead times (or due dates) and price affect the customers' decisions on placing an order. Duenyas and Hopp [32] were the first to analyze a queuing model, where lead time quotation affects the demand rate. They developed queuing models that allow customers to leave if the due date offered by the firm is too late. The objective is to maximize profit; the decisions involve sequencing and due date setting. Duenyas [33] also developed an effective heuristic for quoting due dates and sequencing orders.

Palaka et al. [34], So and Song [35] and Webster [36] studied the optimal selection of price and delivery time, assuming a fixed scheduling rule. They formulated the problem as a steady state queuing model.

Keskinocak et al. [37] proposed several online and offline algorithms for quoting lead times to different customer classes for maximizing revenue, subject to the constraint that quoted lead times are 100% reliable, when the processing time per customer is known.

Easton and Moodie [38] developed a probabilistic model that determined the optimal pricing and due date by setting decisions with contingent orders. In their model, the probability that the customer will accept a quoted price/due-date pair follows an S-shaped Logit model. Watanapa and Techanitisawad [39] extended Easton and Moodie's model by proposing a model that incorporates the market segmentability by applying different policies to different demand classes. In their model, the multi-class policies are managed by the interrelationship of the price, due date, competition level and quality requirement in determining winning probability. Charnsirisakskul et al. [7] expanded on previous research by formulating a deterministic mixed-integer programming model for the problem of order selection, due date setting and production scheduling. We modified this model as follows.

1. In Charnsirisakskul's model, the demand function is deterministic. Thus the production lot size is determined over the production planning time. The significance of uncertainties has prompted us to consider the multiple classes of orders with stochastic demand. We assume that demand is a function of price, lead time and time as a linear-additive form (defined as $d(p, l, t, \xi) = D(p, l, t) + \xi$, where $d(p, l, t, \xi)$ is demand and ξ is a zero-mean random variable that does not depend on the price and lead time. In this case, the mean demand is $D(p, l, t)$, and the noise term, ξ, shifts the demand randomly about this mean [1]). Whitin [8] was the first to examine the linear-additive functional demand form. This form of demand was also studied by Mills [9], Thowsen [14], Federgruen and Heching [18], Dana and Petruzzi [40], Chen and Simchi-Levi [19] and Yin and Rajaram [41].
2. Under the time-dependent and stochastic demand assumed above, we extend Charnsirisakskul's model to dynamic pricing and lead time decisions to match demand with capacity. The actual orders arrive gradually over a planning horizon. Therefore, a better solution method would track actual demand and production and then adjust the next production control policy, accordingly. This would be a dynamic solution method.

In this paper, we use multi-stage stochastic programming to explicitly deal with uncertain demand, dynamic pricing and lead time decision making. A standard approach for solving multi-stage stochastic models uses scenario trees to model the uncertainty of the relevant data. We developed a scenario generation method for pricing and due date management.

To the best of our knowledge, there is no pricing and lead time model based upon stochastic programming and scenario generation methods.

The scenarios and their probabilities form a discrete approximation of the probability distribution of the data process. There are several approaches to generate scenario trees for stochastic programs. Dupacova et al. [42] categorized them as:

(i) Bound-based constructions;
(ii) Monte Carlo-based schemes, or Quasi Monte Carlo-based methods;
(iii) EVPI-based sampling and reduction within decomposition schemes;
(iv) Moment-matching principles;
(v) Probability metric-based approximations.

In this paper, a probability metric-based approximation for a scenario tree construction approach, developed by Dupacova et al. [43] and Heitsch and Romisch [44–46], is used.

With an initial set of discrete probability distributions, they determined a scenario subset of prescribed cardinality and a probability measure based on this set that is the closest to the initial distribution in terms of a natural (or canonical) probability metric. They also used their algorithm to approximate passenger demand and cancellation processes in multi-stage stochastic programming for a revenue management model in [47,48].

3. Multi-stage stochastic pricing and lead time decision model

3.1. Problem description

We model the MTO manufacturing facility as a single machine. This might be the bottleneck in the system with negligible setup times (and costs), where order preemption is allowed. The planning horizon is assumed to be finite and divided into periods of equal length. The capacity in each period may differ. We assume that the demand dynamically arrives in R stages and in each arrival (equivalently, stage) there are different classes of orders, classified based on their sensitivity to quoted price and due date. Demand for each order is stochastic with a continuous function of price and lead time. Within stage r, the price and lead time are defined for order class i, based on stochastic demand and remaining capacities. A firm can accept or reject any order. For accepted orders arriving at stage r, and after realization of stochastic demand, the manufacturer decides on a production schedule within a finite horizon. This, in turn, affects the due-dates. Production scheduling must occur within any time period between the arrival time of the order and the end of the planning horizon. Scheduled orders should be delivered in periods between the quoted due date and the end of the planning horizon.

The production capacity in each period is finite and perishable if left unused. The capacity for periods before the current time period is lost.

If an order is scheduled within any period prior to its negotiated due date, it is stored in a third party warehousing facility and incurs holding costs. An order shipped after the commitment due date is considered late and incurs a tardiness penalty proportional to the number of periods and the quantity delay. Shortages are allowed, and unmet demands are lost.

The manufacturer's objective is to maximize net profit, which is the sum of revenues from accepted orders minus the production, holding, tardiness and shortage costs, subject to capacity, delivery time, and demand constraints. The notations of the multi-stage stochastic pricing and lead time decision model are defined as follows:

$\Psi = \{1 \cdots I\}$	set of order classes, classified based on sensitivity to price and lead time;
$\mathbf{T} = \{1 \cdots T\}$	set of planning periods;
$\mathbf{R} = \{1 \cdots R\}$	set of stages;
$\Pr(i, r)$	set of prices $\{p^i_{1,r}, \ldots, p^i_{j,r}, \ldots, p^i_{n^r_i,r}\}$ (per unit), the manufacturer can charge for order $i \in \Psi$ in stage $r \in \mathbf{R}$, where n^r_i is the number of prices offered to order class i in stage $r \in \mathbf{R}$;
$e(r)$	arrival time for customers in stage $r \in \mathbf{R}$;
$Du(i, r)$	set of due dates $\{l^i_{1,r}, \ldots, l^i_{k,r}, \ldots, l^i_{L^r_i,r}\}$, the manufacturer can charge for order $i \in \Psi$ in stage $r \in \mathbf{R}$, where $e(r) \le l^i_{1,r} < l^i_{2,r}, \ldots, < l^i_{L^r_i,r} \le T$ and L^r_i is the number of due dates offered to order class i in stage $r \in \mathbf{R}$;
Ch_i	third party holding cost per time period per unit of order $i \in \Psi$;
Ca_i	tardiness penalty per period per unit of order $i \in \Psi$;
Cs_i	shortage penalty per unit of order $i \in \Psi$;
Cp^i_t	production cost per unit of order $i \in \Psi$ in period $t \in \mathbf{T}$;
K^r_t	units of production capacity available within the production period, t, at beginning of stage $r \in \mathbf{R}$, where $t = e(r), \ldots, T$;
$d^r_{i,j,k}$	demand of order $i \in \Psi$ in stage $r \in \mathbf{R}$, corresponding to price, $p^i_{j,r} \in \Pr(i, r)$, and due date, $l^i_{k,r} \in Du(i, r)$, expressed in units of production capacity required.

Note: Demands at different stages are independent of each other. We consider a linear additive demand function of the following form:

$$\begin{aligned} d^r_{i,j,k} &= D(p^i_{j,r}, l^i_{k,r}, r) + \zeta^i_r = a^i_r - b^i p^i_{j,r} \\ &\quad - c^i((l^i_{k,r} - e(r) + 1) - l^i_0)^2 + \zeta^i_r, a^i_t, b^i, \\ &\quad c^i > 0,\ i \in \Psi,\ r \in \mathbf{R}, \end{aligned}$$

where a^i_r represents the market size in stage $r \in \mathbf{R}$ for order class $i \in \Psi$, b^i and c^i represent the customer price and lead time sensitivity, l^i_0 is the preferred lead time, ζ^i_r is a random variable of the demand with PDF, $f(.)$, and CDF, $F(.)$. $l^i_{k,r} - e(r) + 1$ is a time interval between the arrival time of an order and quoted due date, referred to as *lead time*. The decision variables are as follows:

$Z^{r-1}_{i,j,k}$	1 if price $p^i_{j,r} \in \Pr(i, r)$, and due date $l^i_{k,r} \in Du(i, r)$ are selected (quoted) for order $i \in \Psi$ at beginning of stage r ($r = 1, \ldots, R$); 0, otherwise;

$x^r_{i,j,t,t'}$ quantity produced (in units of capacity) for order $i \in \Psi$ in production period t and delivered in period t' within stage r if price $p^i_{j,r}$ is selected: $\{t = e(r), \ldots, t',\ t' = l^i_{1,r}, \ldots, T,\ r = 1, \ldots, R\}$;

U^r_i the total quantity produced for order $i \in \Psi$ within stage $r(r = 1, \ldots, R)$;

H^r_i the total quantity-period inventory of order $i \in \Psi$ within stage $r(r = 1, \ldots, R)$;

$A^r_{i,k}$ the total quantity-period tardiness of order $i \in \Psi$ with quoted due date, $l^i_{k,r} \in Du(i, r)$, within stage $r(r = 1, \ldots, R)$;

$\hat{\mathbf{Y}}^r$ the vector of the above decision variables at stage r $\hat{\mathbf{Y}} = (Z^r, x^r, U^r, H^r, A^r)$.

The stochastic input parameters are the unconstrained demand components represented by a discrete time stochastic process, $\xi = (\xi_0, \ldots, \xi_R)$, on some probability space, (Ω, F, P), where $\xi_r := (\zeta^1_r, \ldots, \zeta^I_r)$. The first stage data, i.e. vector ξ_0, are deterministic.

Consider $\xi_{[r]} := (\xi_0, \ldots, \xi_r)$ to denote the history of the process up to stage r. We assume that the process is stagewise independent, i.e. ξ_r is stochastically independent of $\xi_{[r-1]}$.

We model our problem with multi-stage stochastic programming to explicitly deal with uncertain demand, dynamic pricing and lead time decision making.

3.2. Stochastic model

The multi-stage stochastic programming problem for multi-period pricing and lead time decision making is formulated as:

$$\max f_0(\hat{\mathbf{Y}}_0) + \mathbf{E}[\max f_1(\hat{\mathbf{Y}}_1, \xi_1) + \mathbf{E}[\cdots + \mathbf{E}[\max f_R(\hat{\mathbf{Y}}_R, \xi_R)]]],$$

where f_r is the profit function at stage r, and defined as:

$$f_0 = 0, \tag{3.1}$$

S.T.

$$\sum_{j=1}^{n^1_i} \sum_{k=1}^{L^1_i} Z^0_{i,j,k} \le 1, \quad \forall i \in \Psi, \tag{3.2}$$

and:

$$\begin{aligned} f_r(\hat{\mathbf{Y}}_r, \xi_r) = {} & \sum_{i=1}^{I} \sum_{j=1}^{n^r_i} \sum_{k=1}^{L^r_i} p^i_{j,r} \times \sum_{t'=l^i_{1,r}}^{T} \sum_{t=e(r)}^{t'} x^r_{i,j,t,t'} \\ & - \sum_{i=1}^{I} \sum_{j=1}^{n^r_i} \sum_{t'=l^i_1}^{T} \sum_{t=e(r)}^{t'} x^r_{i,j,t,t'} \times Cp^i_t \\ & - \sum_{i=1}^{I} H^r_i \times Ch_i - \sum_{i=1}^{I} \sum_{k=1}^{L^r_i} A^r_{i,k} \times Ca_i - \sum_{i=1}^{I} Cs_i \\ & \times \left(\left(\sum_{j=1}^{n^r_i} \sum_{k=1}^{L^r_i} d^r_{i,j,k} \times Z^{r-1}_{i,j,k} \right) - U^r_i \right), \end{aligned} \tag{3.3}$$

S.T.

$$\sum_{j=1}^{n^r_i} \sum_{k=1}^{L^r_i} Z^{r-1}_{i,j,k} \le 1, \quad \forall i \in \Psi,\ \mathbf{P}\text{-a.s.}, \tag{3.4}$$

$$\sum_{t'=l^i_{1,r}}^{T} \sum_{t=e(r)}^{t'} x^r_{i,j,t,t'} \le \sum_{j=1}^{n^r_i} \sum_{k=1}^{L^r_i} Z^{r-1}_{i,j,k} d^r_{i,j,k}, \quad \forall i \in \Psi,\ j = 1, \ldots, n^r_i,\ \mathbf{P}\text{-a.s.}, \tag{3.5}$$

$$\sum_{i\in\Psi} \sum_{j=1}^{n^r_i} \sum_{t'=\max\{t, l^i_{1,r}\}}^{T} x^r_{i,j,t,t'} \le K^{r-1}_t, \quad t = e(r), \ldots, T,\ \mathbf{P}\text{-a.s.}, \tag{3.6}$$

$$K^r_t = K^{r-1}_t - \sum_{i\in\Psi} \sum_{j=1}^{n^r_i} \sum_{t'=\max\{t, l^i_{1,r}\}}^{T} x^r_{i,j,t,t'}, \quad t = e(r), \ldots, T,\ \mathbf{P}\text{-a.s.}, \tag{3.7}$$

$$\sum_{j=1}^{n^r_i} \sum_{t'=1}^{l^i_{k,r}-1} \sum_{t=e(r)}^{t'} x^r_{i,j,t,t'} \le M_1 \left(1 - \sum_{j=1}^{n^r_i} Z^{r-1}_{i,j,k} \right), \quad \forall i \in \Psi, \quad k = 1, \ldots, L^r_i,\ \mathbf{P}\text{-a.s.}, \tag{3.8}$$

$$H^r_i \ge \sum_{j=1}^{n^r_i} \sum_{t'=l^i_{1,r}}^{T} \sum_{t=e(r)}^{t'} (t' - t) x^r_{i,j,t,t'}, \quad \forall i \in \Psi,\ \mathbf{P}\text{-a.s.}, \tag{3.9}$$

$$\begin{aligned} A^r_{i,k} \ge {} & \sum_{j=1}^{n^r_i} \sum_{t'=l^i_{k,r}}^{T} \sum_{t=e(r)}^{t'} (t' - l^i_{k,r}) x^r_{i,j,t,t'} \\ & + M_2 \left(\sum_{j=1}^{n^r_i} Z^{r-1}_{i,j,k} - 1 \right), \\ & \forall i \in \Psi,\ k = 1, \ldots, L^r_i,\ \text{P-a.s.}, \end{aligned} \tag{3.10}$$

$$U^r_i = \sum_{j=1}^{n^r_i} \sum_{t'=l^i_{1,r}}^{T} \sum_{t=e(r)}^{t'} x^r_{i,j,t,t'}, \quad \forall i \in \Psi,\ \mathbf{P}\text{-a.s.}, \tag{3.11}$$

$$\begin{aligned} & x^r_{i,j,t,t'} \ge 0, \quad \forall i \in \Psi,\ t' = l^i_{1,t}, \ldots, T, \\ & t = e(r), \ldots, T,\ \mathbf{P}\text{-a.s.}, \\ & H^r_i \ge 0, \quad \forall i \in \Psi,\ t = 1, \ldots, T,\ \mathbf{P}\text{-a.s.}, \\ & A^r_{i,k} \ge 0, \quad \forall i \in \Psi,\ k = 1, \ldots, L^r_i,\ \mathbf{P}\text{-a.s.}, \\ & Z^{r-1}_{i,j,k} \in \{0, 1\}, \quad \forall i \in \Psi,\ j = 1, \ldots, n^r_i, \\ & k = 1, \ldots, L^r_i,\ \mathbf{P}\text{-a.s.} \end{aligned} \tag{3.12}$$

The values of decision vector $\hat{\mathbf{Y}}^r$, chosen at stage r, may depend on the information (data), $\xi_0, \ldots, \xi_r$, available up to stage r, but not on the results of future observations. This is the basic requirement of nonanticipativity.

At the beginning of stage 1, the only event is the price and lead time definition for customers at stage 1, and there are not any terms in function f_0. Constraint 3–2 ensures that each order is either rejected or accepted; only one price and one lead time are chosen for each accepted order at the beginning of stage 1.

The five terms in Constraint 3–3 correspond to total revenue, production cost, inventory holding cost, tardiness penalty and shortage penalty, respectively. Constraint 3–4 is like Constraint 3–2 and ensures that each order is either rejected or accepted; only one price and one lead time are chosen for each accepted order at the beginning of stage r. Constraint 3–5 ensures that if the price, $p^i_{j,r}$, and due date, $l^i_{k,r}$, are selected for order i at stage r, at most, $d^r_{i,j,k}$ units must be produced and delivered for order i.

Constraint 3–6 are capacity constraints that ensure the production capacity in each period is not exceeded. The remaining capacity for each production period is determined dynamically by Constraint 3–7. An order can be delivered between the quoted due date and the end of the planning horizon only if the order is accepted. Constraint 3–8 indicates that order i cannot be delivered before $l^i_{k,r}$ if due date $l^i_{k,r}$ is selected.

The total quantity-period inventory, total quantity-period tardiness and total quantity produced by each order are defined by Constraints 3–9, 3–10 and 3–11, respectively.

4. Stochastic programming model in node representation

We assume random vector $\xi = (\xi_0, \ldots, \xi_R)$ has a finite discrete distribution defined by S realizations $\xi^s = (\xi^s_0, \ldots, \xi^s_R)$ called scenarios, and corresponding probabilities as $\pi^s, s = 1 \cdots S$. It is assumed the scenarios have a common root, i.e. $(\xi^1_0 = \xi^2_0 = \cdots = \xi^S_0)$. This information structure can be represented in the form of a scenario tree where the nodes at stage (or level) r of the tree indicate the information available up to stage r.

There are a finite set, $N = \{0, \ldots, N\}$, of nodes and a total of R stages in the scenario tree. We denote the stage (or level) belonging to node n by $r(n)$. The root node, $n = 0$, is the only node at stage $r = 0$. Each node, n, of the scenario tree, except for the root node, has a unique parent node denoted by $a(n)$ and a set $C(n)$ of successor nodes. We assume that $D(t) \subset N$ is the set of nodes at stage r. The path from the root node to node n will be denoted by *Path(n)*. If n is a terminal (leaf) node, i.e. $n \in D(T)$, then *Path(n)* corresponds to a scenario. For each node, $n \in N$, we obtain a probability, π^n, where $\pi^n = \sum_{m \in C(n)} \pi^m$, and the probability of leaf nodes, $n \in D(T)$, are the same probability of realized scenarios, $\{\pi^n\}_{n \in D(T)} = \{\pi^s\}^S_{s=1}$.

To solve our multi-stage stochastic model, we rewrite it in node representation, according to the description of the scenario tree. Thus we get:

$$\max \sum_{n=1}^{N} \sum_{i=1}^{I} \sum_{j=1}^{n_i^{r(n)}} \pi^n \times p^i_{j,r(n)} \times \sum_{t'=l^i_{1,r(n)}}^{T} \sum_{t=e(r(n))}^{t'} x_{i,j,t,t',n} - \sum_{n=1}^{N} \sum_{i=1}^{I} \sum_{j=1}^{n_i^{r(n)}} \sum_{t'=l^i_{1,r(n)}}^{T} \sum_{t=e(r(n))}^{t'} \pi^n \times x_{i,j,t,t',n} \times Cp^i_t - \sum_{n=1}^{N} \sum_{i=1}^{I} \pi^n \times H_{i,n} \times Ch_i - \sum_{n=1}^{N} \sum_{i=1}^{I} \sum_{k=1}^{L_i^{r(n)}} \pi^n \times A_{i,k,n} \times Ca_i - \pi^n \times Cs_i \times \sum_{n=1}^{N} \sum_{i=1}^{I} \left(\left(\sum_{j=1}^{n_i^{r(n)}} \sum_{k=1}^{L_i^{r(n)}} d^n_{i,j,k} \times Z_{i,j,k,a(n)} \right) - U_{i,n} \right), \tag{4.1}$$

S.T.

$$\sum_{j=1}^{n_i^{r(n)+1}} \sum_{k=1}^{L_i^{r(n)+1}} Z_{i,j,k,n} \leq 1, \quad \forall i \in \Psi,\ n \in N/D(T), \tag{4.2}$$

$$\sum_{t'=l^i_{1,r(n)}}^{T} \sum_{t=e(r(n))}^{t'} x_{i,j,t,t',n} \leq \sum_{k=1}^{L_i^{r(n)}} Z_{i,j,k,a(n)} d^n_{i,j,k}, \quad \forall i \in \Psi,\ j = 1, \ldots, n_i^{r(n)},\ n \in N/0, \tag{4.3}$$

$$\sum_{i \in \Psi} \sum_{j=1}^{n_i^{r(n)}} \sum_{t'=\max\{t, l^i_{1,r(n)}\}}^{T} x_{i,j,t,t',n} \leq K_t^{a(n)}, \quad t = e(r(n)), \ldots, T, \quad \in N/0, \tag{4.4}$$

$$K^n_t = K_t^{a(n)} - \sum_{i \in \Psi} \sum_{j=1}^{n_i^{r(n)}} \sum_{t'=\max\{t, l^i_{1,r(n)}\}}^{T} x_{i,j,t,t',n}, \quad t = e(r(n)), \ldots, T,\ n \in N/0, \tag{4.5}$$

$$\sum_{j=1}^{n_i^{r(n)}} \sum_{t'=1}^{l^i_{k,r(n)}-1} \sum_{t=e(r(n))}^{t'} x_{i,j,t,t',n} \leq M_1 \left(1 - \sum_{j=1}^{n_i^{r(n)}} Z_{i,j,k,a(n)} \right), \quad \forall i \in \Psi, \quad k = 1, \ldots, L_i^{r(n)}, \quad n \in N/0, \tag{4.6}$$

$$H_{i,n} \geq \sum_{j=1}^{n_i^{r(n)}} \sum_{t'=l^i_{1,r(n)}}^{T} \sum_{t=e(r(n))}^{t'} (t' - t) x_{i,j,t,t',n}, \quad \forall i \in \Psi, \quad n \in N/0, \tag{4.7}$$

$$A_{i,k,n} \geq \sum_{j=1}^{n_i^{r(n)}} \sum_{t'=l^i_{k,r(n)}}^{T} \sum_{t=e(r(n))}^{t'} (t' - l^i_{k,r(n)}) x_{i,j,t,t',n} + M_2 \left(\sum_{j=1}^{n_i^{r(n)}} Z_{i,j,k,a(n)} - 1 \right), \quad \forall i \in \Psi,\ k = 1, \ldots, L_i^{r(n)},\ n \in N/0, \tag{4.8}$$

$$U_{i,n} = \sum_{j=1}^{n_i^{r(n)}} \sum_{t'=l^i_{1,r(n)}}^{T} \sum_{t=e(r(n))}^{t'} x_{i,j,t,t',n}, \quad \forall i \in \Psi,\ n \in N/0, \tag{4.9}$$

$$\begin{aligned} & x_{i,j,t,t',n} \geq 0, \quad \forall i \in \Psi, \\ & \quad t = e(r(n)), \ldots, t', \qquad t' = l^i_{1,r(n)}, \ldots, T, \\ & \quad j = 1, \ldots, n_i^{r(n)}, \qquad n \in N/0, \\ & H_{i,n} \geq 0, \qquad A_{i,k,n} \geq 0, \\ & \quad \forall i \in \Psi,\ k = 1, \ldots, L_i^{r(n)},\ n \in N/0, \\ & Z_{i,j,k,n} \in \{0, 1\}, \quad \forall i \in \Psi,\ j = 1, \ldots, n_i^{r(n)+1}, \\ & \quad k = 1, \ldots, L_i^{r(n)+1},\ n \in N/D(T). \end{aligned} \tag{4.10}$$

The decision variables $x_{i,j,t,t',n}$, $H_{i,n}$, $A_{i,k,n}$ and $Z_{i,j,k,n}$ in Constraints 4-1 to 4-10 are variables defined in node n after realization of random data, ξ, at stage $r(n)$. Demand for order class

i in node n is equal to:

$$d^n_{i,j,k} = D(p^i_{j,r(n)}, l^i_{k,r(n)}, r(n)) + \zeta^i_n,$$

with probability π^n, where $p^i_{j,r(n)}$ and $l^i_{k,r(n)}$ are the price and lead time quoted to customer class i in node $a(n)$. This means price and lead time are determined before the realization of random data at each stage. ζ^i_n is realization of the random parameter in node n for customer class i. Depending on the number of realizations of ξ, the mixed-integer linear programming problem denoted by Constraints 4-1 to 4-10 becomes (very) large in scale.

One way to overcome this difficulty is to use decomposition methods that exploit the special structures of the model [49]. Reducing the originally designed tree can also reduce the model dimensions [43–46]. This approach makes use of probability metrics, i.e. metric distances on spaces of probability measures, where the metrics are selected such that the optimal values of the original and approximate stochastic programs are similar if the distance between the original probability distribution and its approximation is small. We will briefly describe the approach of Heitsch and Römisch [46] that is used for scenario reduction in our model. They assume a stochastic process, $\hat{\xi}$, with finite and large sample sizes of individual scenarios, $\{\xi^s_t\}_{t=0}^{t=T}$, with probabilities $p_s > 0, s = 1, \dots, S$, and common root nodes, i.e. $\xi^1_0 = \cdots = \xi^S_0$, are given. They then determine a stochastic process, ξ_{tr}, on (Ω, F, P) by bundling and deletion processes, which have tree form scenarios and satisfy the condition:

$$\|\hat{\xi} - \xi_{tr}\|_1 \le \varepsilon. \tag{4.11}$$

$\varepsilon > 0$ is a prescribed tolerance. The bundling and deletion processes rely on computing and bounding the Kantorovich distance, $\hat{\mu}_r(P, Q)$, between the original probability distribution given by the individual scenarios, $P = \sum_{i=1}^{S} p_i \delta_{\xi^i}$, and their weights, and the reduced probability distribution, $Q = \sum_{j=1, j\notin J}^{S} q_j \delta_{\xi^j}$. J denotes some index set of deleted scenarios. The Kantorovich distance is given by Relation 4–12, where P and Q are the fixed Borel probability measures on a closed subset of $\mathbf{R}^s$, Ω, i.e. $P, Q \in \mathbf{P}(\Omega)$, and function $c : \Omega \times \Omega \to \mathbf{R}$ is given by Relation 4–13.

$$\hat{\mu}_r(P, Q) = \min\left\{ \sum_{\substack{i,j=1\\ j\notin J}}^{S} c_r(\xi^i, \xi^j)\eta_{ij} : \eta_{ij} \ge 0, \right. \\ \left. \sum_{i=1}^{S} \eta_{ij} = q_j, \sum_{\substack{j=1\\ j\notin J}}^{S} \eta_{ij} = p_i \right\}, \tag{4.12}$$

$$c_r(\xi^i, \xi^j) := \max\{1, |\xi^i - \xi_0|^{r-1}, \\ |\xi^j - \xi_0|^{r-1}\}|\xi^i - \xi^j|, \quad \forall \xi^i, \xi^j \in \Omega. \tag{4.13}$$

Also $J \subset \{1, \dots, N\}$ and $\delta_\xi \in \mathbf{P}(\Omega)$ denote the Dirac measures that place the unit mass at ξ.

A set-covering problem represents the optimal choice of an index set, J, for scenario reduction. It can be formulated as a 0-1 integer program that is NP-hard. In [46], a heuristic algorithm is approximated to determine a reduced probability distribution, Q of ξ, in multi-stage stochastic programming. Using the algorithm in [46], we determined a reduced stochastic process of ξ_{tr} in scenario tree form, and employed it in a stochastic programming model denoted by Constraints 4-1 to 4-10. The details for the backward reduction algorithm are given in the Appendix.

Table 1: Characteristics of demand.

Customer class	Price sensitivity b^i	Lead time sensitivity c^i	Preferred lead time l^i_0
1	0.2	2	1
2	0.5	1	2
3	0.7	0.6	3

5. Numerical study

In this section, we perform a numerical study of our basic model to investigate the effect of dynamic pricing and segmentation of orders on the profitability of the firm. Also, we illustrate the advantage of our modeling approach compared to the expected value solution approach.

5.1. Flexibility in time, price and lead time

In our analysis, we consider three different policies:

1. Price flexibility,
2. Lead time flexibility,
3. Time flexibility.

With price (lead time) flexibility, we charge different prices (lead times) to different customer classes and classify customers based on their sensitivity to changes in dominating factors, such as price and lead time. When there is no price (lead time) flexibility, a single (fixed) price (lead time) is charged to all customer classes [7]. With time flexibility, we change the price and lead time dynamically to manage demand. We also investigated the effect of demand variation over time in our model. To this purpose, we consider combinations of two (low and high) levels for demand variation over time (L–D, H–D). We illustrate demand variation by varying market size, a^i_t, with time, t. We consider an example with three classes of customer and a planning horizon, $T = 4$. Customer classes arrive at the beginning of each period, so there are $R = T = 4$ stages. The market size for each class at each stage is as follows:

$$\text{L--D} = \begin{array}{c} \begin{array}{cccc} t1 & t2 & t3 & t4 \end{array} \\ \begin{pmatrix} 9 & 10 & 11 & 12 \\ 13 & 12 & 11 & 0 \\ 14 & 13 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{l} \\ \text{class1} \\ \text{class2} \\ \text{class3} \end{array}$$

$$\text{H--D} = \begin{array}{c} \begin{array}{cccc} t1 & t2 & t3 & t4 \end{array} \\ \begin{pmatrix} 3 & 8 & 13 & 18 \\ 19 & 12 & 5 & 0 \\ 18 & 9 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{l} \\ \text{class1} \\ \text{class2} \\ \text{class3} \end{array}$$

Table 1 shows other demand function components for each customer class in each time period.

The random perturbation, ζ^i_r, follows a discrete distribution with probability mass function:

$$\mathbf{P}(\zeta^i_r = \zeta^i_{r,z}) = pr^i_{r,z}, \qquad i \in \Psi, \qquad r = 1 \cdots R.$$

We assume probability functions are the same at all stages and for all classes; therefore, we can remove two indices, i and r. The ζ_z and pr_z are presented in Table 2.

The holding and tardiness costs per period per unit and the shortage costs per unit for each class of customer are as follows:

$$Ch_1 = Ch_2 = Ch_3 = 1, \qquad Ca_1 = Ca_2 = Ca_3 = 4,$$
$$Cs_1 = Cs_2 = Cs_3 = 15.$$

The production costs in all periods are 3.

Table 2: Random perturbation for demand function.

Z	1	2	3	4	5	6	7	8	9	10
ζ_z	−3	−2	−1.5	−0.75	−0.35	0.35	0.75	1.5	2	3
pr_z	0.02	0.04	0.09	0.15	0.2	0.2	0.15	0.09	0.04	0.02

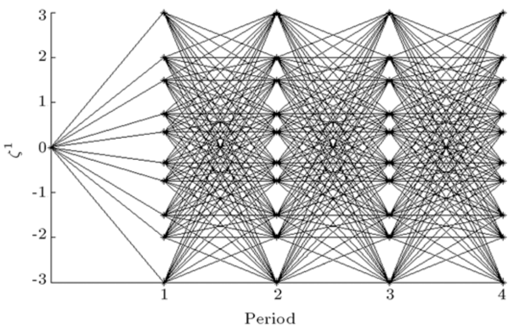

Figure 1a: Original scenario tree for customer class 1.

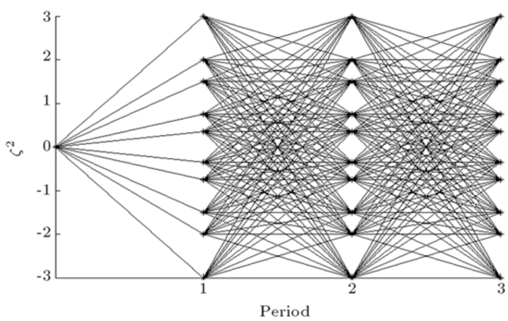

Figure 1b: Reduced scenario tree for customer class 2.

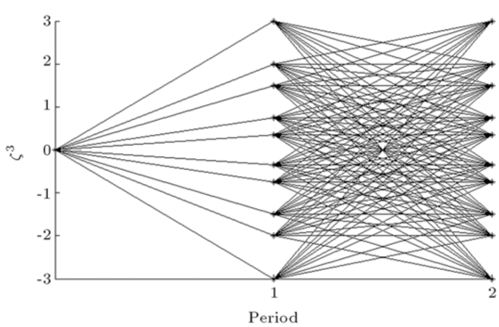

Figure 1c: Original scenario tree for customer class 2.

The production capacity in each period at the beginning of the planning horizon is $\hat{K}_0 = (10, 10, 10, 10)$. By using discrete distributions with finite realizations for each customer class in each period from the data in Table 2, we get $S = 10^9$ realizations or scenarios for the joint distribution over the total time horizon, for all customers. The resulting original scenario tree is illustrated in Figures 1a–1c for each respective customer class.

Using the initial sets of scenarios and *algorithms* described in [46], a scenario set consisting of 144 scenarios and 212 nodes is generated using a procedure implemented in MATLAB software for the stochastic process, ξ, where $\|\hat{\xi} - \xi_{tr}\|_1 \leq 4.2026$ in Relation 4–11. The reduced trees for each customer class are shown in Figures 2a–2c.

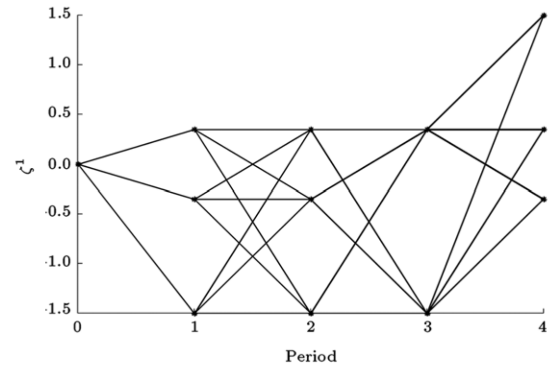

Figure 2a: Reduced scenario tree for customer class 1.

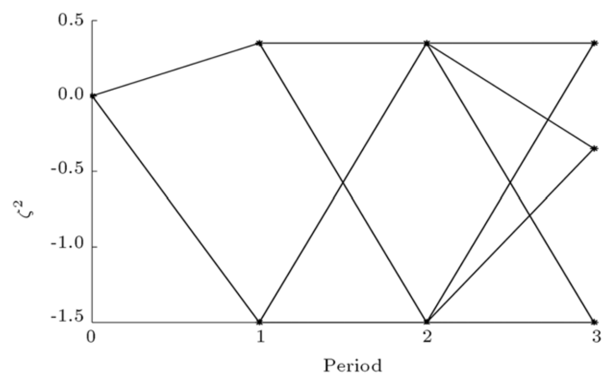

Figure 2b: Reduced scenario tree for customer class 2.

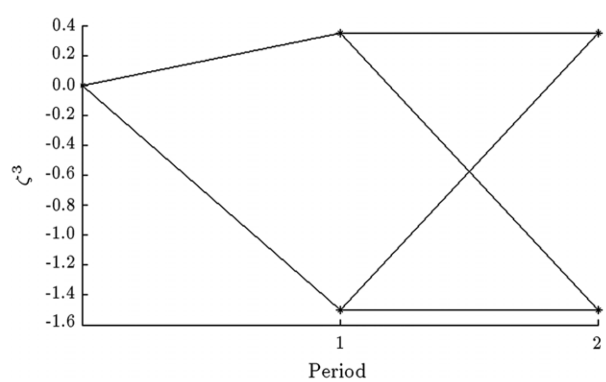

Figure 2c: Reduced scenario tree for customer class 3.

In each figure, there are 144 scenarios, some of which overlap. We are interested in simultaneously maximizing the expected total profit, while determining the following decisions dynamically:

1. Optimal price and lead time quoted to each class of order at the beginning of each period;
2. Production amount for each class in each period according to each realized scenario.

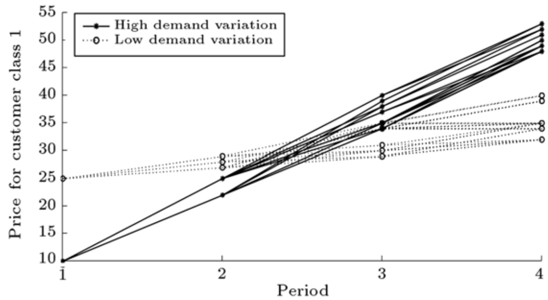

Figure 3a: Scenario tree for customer class 1 price.

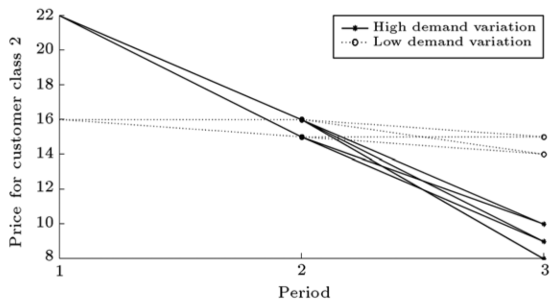

Figure 3b: Scenario tree for customer class 2 price.

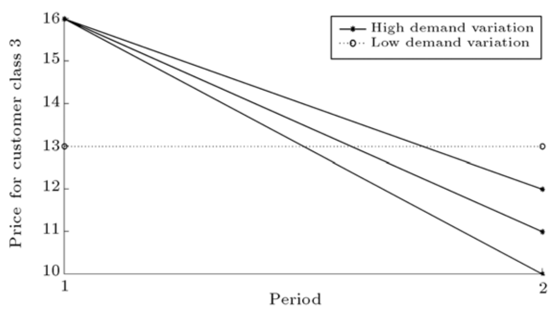

Figure 3c: Scenario tree for customer class 3 price.

Table 3: The optimal prices and lead times of each customer class at the beginning of the time horizon in the numerical example.

		Optimal price	Optimal lead time
High demand variability	Class 1	10	1
	Class 2	22	2
	Class 3	16	2
Low demand variability	Class 1	25	1
	Class 2	16	2
	Class 3	13	2

We used the model expressed in Section 4 to solve this pricing model.

Table 3 illustrates the price and lead time selected at the beginning of period 1 for each customer class under two considered demand variations. Figures 3a–3c illustrate the scenario trees for the price selected at the beginning of each period for each customer class.

The demand for class 1 increases with time; hence the price is also increasing with time. For customer classes 2 and 3, the price decreases with time because of the decreasing demand. In fact, by increasing the price during periods of high demand and decreasing the price during periods of low demand, the firms can smooth the demand and improve their capacity utilization.

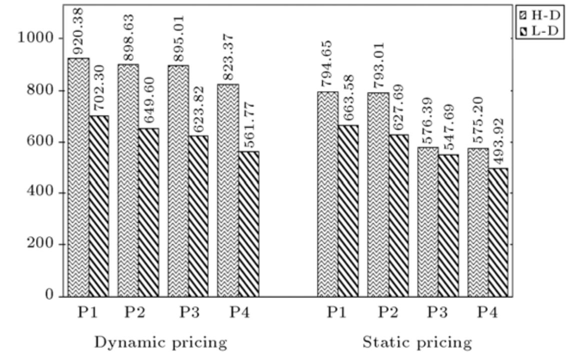

Figure 4: Expected profit under different combinations of lead time and price flexibility for two dynamic and static cases under high demand variability (H–D) and low demand variability (L–D).

Table 4: Sensitivity analysis with respect to various combinations of demand in the model on increased percentage profit of the policies over the base case policy.

levels of demand variability	Policies	Expected profit	Increased percentage profit over the base case
H–D	P1-Dynamic	920.3819	60.01076
	P2-Dynamic	898.6272	56.22865
	P3-Dynamic	895.008	55.59944
	P4-Dynamic	823.3712	43.1452
	P1-Static	794.6499	38.15193
	P2-Static	793.0111	37.86702
	P3-Static	576.3918	0.207197
	P4-Static	575.2	0
L–D	P1-Dynamic	702.2992	42.18995
	P2-Dynamic	649.6049	31.52128
	P3-Dynamic	623.8175	26.30027
	P4-Dynamic	561.77	13.73792
	P1-Static	663.5824	34.35121
	P2-Static	627.6874	27.08378
	P3-Static	547.6881	10.88685
	P4-Static	493.9162	0

We consider different policies for price and lead time flexibility and dynamic pricing as follows:

P1: price flexibility, lead time flexibility,
P2: price flexibility, no lead time flexibility,
P3: no price flexibility, lead time flexibility,
P4: no price flexibility, no lead time flexibility.

These combinations are considered for two cases: Dynamic pricing (D) and Static pricing (S).

The maximum expected profit of each policy for two demand variation scenarios is represented in Figure 4. We also consider the base case in which there is no flexibility of price or lead time with static pricing. The percentage profit increases over the base case for all policies under different combinations of demand is summarized in Table 4.

The following inferences may be drawn from Table 4 and Figure 4:

1. With high demand variability, dynamic pricing leads to higher expected profit under all policies.
2. Under low demand variability, price and lead time flexibility are more useful than dynamic pricing.
3. The percentage profit increases over the base case for P2-Dynamic and P3-Dynamic are close to each other under both demand environments, and are considered interchangeable.

4. Under a no price flexibility-static pricing policy, lead time flexibility can increase profit in a low demand variability environment.
5. In the high demand variation, the percentage profit increases over the base case for dynamic pricing and price, and lead time flexibility, are 43.14%, 37.86%, and 0.20%. Thus the manufacturer chooses dynamic pricing and price flexibility in descending order. But in an environment with low demand variability, the ranking of policies, according to their percentage profit increases over the base case, is:
   1. Price flexibility (27.083%),
   2. Dynamic pricing (13.73%),
   3. Lead time flexibility (10.88%).
6. If the manufacturer can choose two types of flexibilities for high demand variability, we rank the policies according to their percentage profit increases over the base case as follows:
   1. Price flexibility-dynamic pricing (56.23%),
   2. Lead time flexibility-dynamic pricing (55.60%),
   3. Lead time flexibility- price flexibility (38.15%),

   and for low demand variability, we rank the policies as follows:
   1. Lead time flexibility- price flexibility (34.35%),
   2. Price flexibility-dynamic pricing (31.52%),
   3. Lead time flexibility-dynamic pricing (26.30%).

5.2. Value of the stochastic program

Stochastic programs are computationally difficult to solve. Therefore, for real-world problems, people have a tendency to solve much simpler versions. For example, researchers may solve the deterministic program by replacing all random variables with their expected values, or they may solve deterministic programs, each corresponding to one particular scenario, and then combine these different solutions by some heuristic rule. The accuracy of such approaches is to be evaluated by introducing two concepts, the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS) [50].

Expected Value of Perfect Information (EVPI): The EVPI concept measures the maximum amount a decision maker would be ready to pay in return for complete information about the future. Let ξ be the random variable whose realizations correspond to the various scenarios. Let Q_0^* be the optimal value of the stochastic programming, and $\bar{Q}(\xi)$ the optimal value for the deterministic problem corresponding to one particular scenario, ξ.

The wait-and-see value (WS), which corresponds to the expected value of the optimal objective for each scenario, is $\mathrm{WS} = E_\xi(\bar{Q}(\xi))$. The expected value of perfect information (EVPI) is then defined by $\mathrm{EVPI} = \mathrm{WS} - Q_0^*$.

Value of the Stochastic Solution (VSS): The stochastic programming approach considers the entire range of uncertain scenarios. On this score, it may be better than its deterministic correspondents. However, it also increases computational complexity dramatically. Therefore, the majority of people would solve the deterministic problem by replacing the random variables by their corresponding expected values. The concept of value of the stochastic programming solution (VSS) can be used to justify whether the extra effort on modeling and solving stochastic programming is worthwhile. Let $\bar{Z}(\bar{\xi})$ be the optimal decision of the first stage in the deterministic problem where all random variables are replaced by their expected values. The Value of the Stochastic Solution (VSS) is then defined as $\mathrm{VSS} = Q_0^* - \mathrm{EEV}$, with $\mathrm{EEV} = E_\xi(Q(\bar{Z}(\bar{\xi}), \xi))$. In general, a bigger VSS indicates higher benefit from using the stochastic programming approach.

In a maximization problem, the relation between the defined values has been established by Madansky [51] as follows:

$$\mathrm{EEV} \le Q_0^* \le \mathrm{WS}.$$

In this section, we compute these measures for five problem instances. For each problem, we randomly choose the parameters, a_t^i, b^i, c^i, in customers demand functions and cost parameters from a uniform distribution. To see how the lower and upper bounds, and the Value of the Stochastic Solution (VSS) are represented under different probability distributions, the sensitivity analysis, with respect to random data, is studied. To this purpose, we solve generated examples considering four probability distribution functions for random variable ξ in the demand function as follows:

1. Exponential distribution with a mean of 3: $\xi \sim \exp(3)$,
2. Exponential distribution with a mean of 10: $\xi \sim \exp(10)$,
3. Uniform distribution on interval $[-3, 3] : \xi \sim U(-3, 3)$,
4. Uniform distribution on interval $[-10, 10] : \xi \sim U(-10, 10)$.

With the initial set consisting of $S = 10\,500$ scenarios and the backward reduction algorithm described in [46], a scenario set consisting of 300 scenarios is generated for the stochastic process, ξ, by a procedure implemented in MATLAB software

Tables 5 and 6 show the WS, EVPI and VSS of five problem instances, and the best objective value for the multi-stage stochastic programming problem, which optimized with 300 scenarios. The column "LB" represents the lower bound for true problems with 10 500 scenarios, and is defined as:

$$\mathrm{LB} = E_{\xi_0}(Q(Z^*(\xi), \xi_0)),$$

where $Z^*(\xi)$ is the optimal decision in the reduced stochastic problem with 300 scenarios, and ξ_0 is the random variable consisting of $S = 10\,500$ scenarios. We can obtain the relative optimality gap, $\left(\frac{Q^*-\mathrm{LB}}{\mathrm{LB}} \times 100\right)$, of the solution, $Z^*(\xi)$, using the lower bound estimate, (LB), and the objective function value estimate from the reduced stochastic program (Q^*). The optimal objective value of the reduced stochastic program is an upper bound for the true stochastic program, consisting of initial scenarios. Therefore, a smaller gap indicates smaller error from using the reduced stochastic programming approach.

As we can see from Tables 5 and 6, the values of stochastic programming increase with increasing the exponential distribution's mean or increasing the uniform distribution's interval length. In fact, the solutions given by the deterministic models would not be able to define best price and lead time for problem instances that had enormous variance in demand function. This relatively large value for VSS justifies the use of more sophisticated modeling techniques and the extra computational efforts.

As we can see from Table 5 and 6, all instances, except instances with a $U(-3, 3)$ distribution function, have high EVPI, meaning that perfect information would be helpful to substantially improve the objective function.

The small relative optimality gap, in almost all instances, indicates that scenario reduction can be a good approximation of the main problem.

Table 5: Computational results for wait-and-see, EVPI, and lower bound of multi-stage stochastic programming under exponential distribution.

	Exp(3)							Exp(10)						
	Q_0^*	WS	EVPI	EEV	VSS	LB	Gap (%)	Q_0^*	WS	EVPI	EEV	VSS	LB	Gap (%)
$N1$	1029.717	1079.370	49.653	1010.780	18.937	1025.045	0.456	1587.375	2131.380	544.005	1542.830	44.545	1580.133	0.458
$N2$	1184.674	1244.123	59.449	1171.930	12.744	1181.487	0.270	1765.889	2257.620	491.731	1586.670	179.219	1752.174	0.783
$N3$	493.581	524.627	31.046	490.510	3.071	491.964	0.329	826.122	1024.770	198.648	783.750	42.372	818.285	0.958
$N4$	1215.623	1274.340	58.717	1198.700	16.923	1212.243	0.279	1594.970	1827.780	232.810	1501.670	93.300	1583.723	0.710
$N5$	361.905	387.670	25.765	351.870	10.035	358.825	0.858	691.550	805.700	114.150	601.980	89.570	680.632	1.604

Table 6: Computational results for wait-and-see, EVPI, and lower bound of multi-stage stochastic programming under uniform distribution.

	$U[-3, 3]$							$U[-10, 10]$						
	Q_0^*	WS	EVPI	EEV	VSS	LB	Gap (%)	Q_0^*	WS	EVPI	EEV	VSS	LB	Gap (%)
$N1$	715.550	763.006	47.456	711.040	4.510	712.545	0.422	674.290	775.050	100.760	531.240	143.050	658.763	2.357
$N2$	926.339	970.880	44.541	900.870	25.469	917.560	0.957	906.869	992.360	85.491	695.990	210.879	889.278	1.978
$N3$	324.916	328.870	3.954	322.760	2.156	323.051	0.577	301.095	358.070	56.975	215.400	85.695	289.367	4.053
$N4$	1002.104	1006.870	4.766	993.040	9.064	999.249	0.286	969.605	1026.130	56.525	900.500	69.105	960.031	0.997
$N5$	238.861	241.550	2.689	236.700	2.161	238.748	0.047	201.780	250.670	48.890	190.580	11.200	198.456	1.675

6. Conclusion

We have presented a multi-stage stochastic programming approach to dynamically determine price and lead time for MTO firms with multiple customer classes. We used an additive form for the demand function, in which the stochastic parameter is approximated by a scenario tree. The scenario tree is generated by the forward reduction algorithm obtained in [46]. Through numerical examples, we compared benefits of price and lead time flexibility and dynamic pricing under two demand environments.

We can summarize the following findings from the results of numerical examples:

- Firstly, under both demand environments, price and lead time flexibility leads to higher profits than no flexibility in price and lead time.
- Secondly, the rankings for price, lead time flexibility and dynamic pricing are dependent on the demand environment. Under high demand variability, the dynamic pricing is more beneficial than price and lead time flexibility; under low demand variability, price and lead time flexibility are more beneficial than dynamic pricing.
- Thirdly, under a no price flexibility-static pricing policy, lead time flexibility can increase profit in low demand variability, but having lead time flexibility in a high demand variability environment does not have a significant benefit.

Scenario representation of pricing and lead time quotation problems correspond to large scale mixed integer programming. Future research may focus on using other methods, like decomposition or Monte Carlo, to solve the resulting large scale linear program, and compare the solutions with our results in this paper.

Appendix. Simultaneous backward reduction [46]

let P be a fixed Borel probability measure on Ω, i.e. $P \in P(\Omega)$, with scenarios $\{\omega_1, \omega_2, \ldots, \omega_N\}$ and probability weights $\{p_1, p_2, \ldots, p_N\}$. Thus the simultaneous backward reduction algorithms, according to [46], are as in the following steps: *Step* 1.

$c_{kj} := c(\omega_k, \omega_j), \quad j = 1, \ldots, N.$

Sorting of:

$\{c_{kj} : j = 1, \ldots, N\}, \quad k = 1, \ldots, N,$

$c_{ll}^{[1]} := \min_{j \neq l} c_{lj}, \quad l = 1, \ldots, N,$

$z_l^{[1]} := p_l c_{ll}^{[1]}, \qquad l = 1, \ldots, N,$

$l_1 \in \arg_{l \in \{1,\ldots,N\}} \min z_l^{[1]}, \quad J^{[1]} := \{l_1\}.$

Step i:

$c_{kl}^{[i]} := \min_{j \neq J^{[i-1]} \cup \{l\}} c_{kj},$

$l \notin J^{[i-1]}, \quad k \in J^{[i-1]} \cup \{l\},$

$z_l^{[i]} := \sum_{j \neq J^{[i-1]} \cup \{l\}} p_k c_{kl}^{[i]}, \quad l \notin J^{[i-1]},$

$l_i \in \arg_{j \neq J^{[i-1]}} \min z_l^{[i]}, \quad J^{[i]} := J^{[i-1]} \cup \{l_i\}.$

Step $N - n + 1$. *Redistribution by* Eq. (A.1) where:

$$\bar{q}_j = p_j + \sum_{i \in J_j} p_i, \quad \text{for each} \quad j \notin J, \tag{A.1}$$

$J_j := \{i \in J : j = j(i)\},$

and:

$$j(i) \in \arg\min_{j \notin J} c(\omega_i, \omega_j), \quad \text{for each} \quad i \in J, \tag{A.2}$$

and function $c : \Omega \times \Omega \to R$ is given by:

$$c(\omega, \tilde{\omega}) = \max\{1, \|\omega - \omega_0\|, \|\tilde{\omega} - \omega_0\|\} \|\omega - \tilde{\omega}\|, \quad \forall \omega, \tilde{\omega} \in \Omega. \tag{A.3}$$

References

[1] Talluri, K.T. and van Ryzin, G.J., *The Theory and Practice of Revenue Management*, Kluwer Academic Publishers, New York (2004).
[2] Harris, F.H.d. and Pinder, J.P. "A revenue management approach to demand management and order booking in assemble-to-order manufacturing", *Journal of Operations Management*, 13(4), pp. 299–309 (1995).
[3] Defregger, F. and Kuhn, H. "Revenue management for a make-to-order company with limited inventory capacity", *OR Spectrum*, 29(1), pp. 137–156 (2007).
[4] Spengler, T., Rehkopf, S. and Volling, T. "Revenue management in make-to-order manufacturing: an application to the iron and steel industry", *OR Spectrum*, 29(1), pp. 157–171 (2007).

[5] Keskinocak, P. and Tayur, S. "Due-date management policies", In *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era*, D. Simchi-Levi, S.D. Wu and Z.J.M. Shen, Eds., In *International Series on Operations Research and Management Science*, pp. 485–553, Kluwer Academic Publishers, Norwell, MA (2004).
[6] Biller, S., et al. "Dynamic pricing and the direct to customer model in the automotive industry", *Electronic Commerce Research*, 5(2), pp. 309–334 (2005).
[7] Charnsirisakskul, K., Griffin, P.M. and Keskinocak, P. "Pricing and scheduling decisions with lead-time flexibility", *European Journal of Operational Research*, 171(1), pp. 153–169 (2006).
[8] Whitin, T.M. "Inventory control and price theory", *Management Science*, 2(1), pp. 61–68 (1955).
[9] Mills, E.S. "Uncertainty and price theory", *The Quarterly Journal of Economics*, 73(1), pp. 116–130 (1959).
[10] Zabel, E. "Monopoly and uncertainty", *Review of Economic Studies*, 37(2), pp. 205–219 (1970).
[11] Karlin, S. and Carr, C.R. "Prices and optimal inventory policy", In *Studies in Applied Probability and Management Science*, K. Arrow, S. Karlin and H. Scarf, Eds., pp. 159–172, Stanford University Press, Stanford, CA (1962).
[12] Petruzzi, N.C. and Dada, M. "Pricing and the newsvendor model: a review with extensions", *Operations Research*, 47(2), pp. 183–194 (1999).
[13] Thomas, L. "Price production decision with deterministic demand", *Management Science*, 16(11), pp. 747–750 (1970).
[14] Thowsen, G.T. "A dynamic, non-stationary inventory problem for a price/quantity setting firm", *Naval Research Logistics Quarterly*, 22(3), pp. 461–476 (1975).
[15] Zabel, E. "Multi-period monopoly under uncertainty", *Journal of Economic Theory*, 5(3), pp. 524–536 (1972).
[16] Gallego, G. and Vanryzin, G. "Optimal dynamic pricing of inventories with stochastic demand over finite horizons", *Management Science*, 40(8), pp. 999–1020 (1994).
[17] Gallego, G. and Vanryzin, G. "A multi-product dynamic pricing problem and its applications to network yield management", *Operations Research*, 45(1), pp. 24–41 (1997).
[18] Federgruen, A. and Heching, A. "Combined pricing and inventory control under uncertainty", *Operations Research*, 47(3), pp. 454–475 (1999).
[19] Chen, X. and Simchi-Levi, D. "Coordinating inventory control and pricing strategies with random demand and fixed ordering cost: the finite horizon case", *Operations Research*, 52(6), pp. 887–896 (2004).
[20] You, P.S. and Wu, M.T. "Optimal ordering and pricing policy for an inventory system with order cancellations", *OR Spectrum*, 29(4), pp. 661–679 (2007).
[21] Chan, L.M.A., et al. "Coordination of pricing and inventory decisions: a survey and classification", In *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era*, D. Simchi-Levi, S.D. Wu and Z.J.M. Shen, Eds., In *International Series on Operations Research and Management Science*, pp. 335–392, Kluwer Academic Publishers, Norwell, MA (2004).
[22] Elmaghraby, W. and Keskinocak, P. "Dynamic pricing in the presence of inventory considerations: research overview, current practices, and future directions", *Management Science*, 49(10), pp. 1287–1309 (2003).
[23] Seidmann, A. and Smith, M. "Due date assignment for production systems", *Management Science*, 27(5), pp. 101–413 (1981).
[24] Bertrand, J. "The effect of workload dependent due-dates on job shop performance", *Management Science*, 29(7), pp. 799–816 (1983).
[25] Wein, L.M. "Due-date setting and priority sequencing in a multiclass M/G/1 queue", *Management Science*, 37(7), pp. 834–850 (1991).
[26] Wein, L.M. and Chevalier, P. "A broader view of the job-shop scheduling problem", *Management Science*, 38(7), pp. 1018–1033 (1992).
[27] Chen, C.Y., Zhao, Z.Y. and Ball, M.O. "Quantity and due date quoting available to promise", *Information Systems Frontiers*, 3(4), pp. 477–488 (2001).
[28] Hegedus, M.G. and Hopp, W.J. "Due date setting with supply constraints in systems using MRP", *Computers & Industrial Engineering*, 39(3–4), pp. 293–305 (2001).
[29] Moses, S., Grant, H., Gruenwald, L. and Pulat, S. "Real-time due-date promising by build-to-order environments", *International Journal of Production Research*, 42(20), pp. 4353–4375 (2004).
[30] Zorzini, M., Corti, D. and Pozzetti, A. "Due date (DD) quotation and capacity planning in make-to-order companies: results from an empirical analysis", *International Journal of Production Economics*, 112(2), pp. 919–933 (2008).
[31] Sawik, T. "Multi-objective due-date setting in a make-to-order environment", *International Journal of Production Research*, 47(22), pp. 6205–6231 (2009).
[32] Duenyas, I. and Hopp, W.J. "Quoting customer lead times", *Management Science*, 41(1), pp. 43–57 (1995).
[33] Duenyas, I. "Single facility due date setting with multiple customer classes", *Management Science*, 41(4), pp. 608–619 (1995).
[34] Palaka, K., Erlebacher, S. and Kropp, D.H. "Lead-time setting, capacity utilization, and pricing decisions under lead-time dependent demand", *IIE Transactions*, 30(2), pp. 151–163 (1998).
[35] So, K.C. and Song, J.S. "Price, delivery time guarantees and capacity selection", *European Journal of Operational Research*, 111(1), pp. 28–49 (1998).
[36] Webster, S. "Dynamic pricing and lead time policies for make to order systems", *Decision Sciences*, 33(4), pp. 579–599 (2002).
[37] Keskinocak, P., Ravi, R. and Tayur, S. "Scheduling and reliable lead time quotation for orders with availability intervals and lead time sensitive revenues", *Management Science*, 47(2), pp. 264–279 (2001).
[38] Easton, F.F. and Moodie, D.R. "Pricing and lead time decisions for make-to-order firms with contingent orders", *European Journal of Operational Research*, 116(2), pp. 305–318 (1999).
[39] Watanapa, B. and Techanitisawad, A. "Simultaneous price and due date settings for multiple customer classes", *European Journal of Operational Research*, 166(2), pp. 351–368 (2005).
[40] Dana, J.D. and Petruzzi, N.C. "Note: the newsvendor model with endogenous demand", *Management Science*, 47(11), pp. 1488–1497 (2001).
[41] Yin, R. and Rajaram, K. "Joint pricing and inventory control with a Markovian demand model", *European Journal of Operational Research*, 182(1), pp. 113–126 (2007).
[42] Dupacova, J., Consigli, G. and Wallace, S.W. "Scenarios for multistage stochastic programs", *Annals of Operations Research*, 100(1–4), pp. 25–53 (2000).
[43] Dupacova, J., Growekuska, N. and Romisch, W. "Scenario reduction in stochastic programming", *Mathematical Programming, Series A*, 95(3), pp. 493–511 (2003).
[44] Heitsch, H. and Romisch, W. "Scenario reduction algorithms in stochastic programming", *Computational Optimization and Applications*, 24(2–3), pp. 187–206 (2003).
[45] Heitsch, H. and Römisch, W. "Generation of multivariate scenario trees to model stochasticity in power management", in: *IEEE Power Tech. Conference*, St. Petersburg, Russia, 2005, pp. 1–7.
[46] Heitsch, H. and Römisch, W. "Scenario tree modeling for multistage stochastic programs", *Mathematical Programming, Series A and B*, 118(2), pp. 371–406 (2009).
[47] Möller, A., Romisch, W. and Weber, K. "A new approach to O&D revenue management based on scenario trees", *Journal of Revenue and Pricing Management*, 3, pp. 265–276 (2004).
[48] Möller, A., Romisch, W. and Wever, K. "Airline network revenue management by multistage stochastic programming", *Computational Management Science*, 5(4), pp. 355–377 (2008).
[49] Ruszczynski, A. and Shapiro, A. "Stochastic programming models", In *Handbooks in Operations Research and Management Science*, A. Ruszczynski and A. Shapiro, Eds., pp. 1–63, Elsevier Science, Amsterdam (2003).
[50] Birge, J.R. and Louveaux, F., *Introduction to Stochastic Programming*, Springer, New York (1997).
[51] Madansky, A. "Inequalities for stochastic linear programming problems", *Management Science*, 6(2), pp. 197–204 (1960).

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