European Journal of Operational Research 308 (2023) 400–421

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Stochastic optimization of trading strategies in sequential electricity markets

Emil Kraft[a,*], Marianna Russo[b,c], Dogan Keles[d], Valentin Bertsch[e]

[a] Karlsruhe Institute of Technology, Chair of Energy Economics, Hertzstraße 16, Karlsruhe 76187, Germany
[b] NEOMA Business School, Department of Finance, Mont-Saint-Aignan, France
[c] NTNU Business School, Trondheim, Norway
[d] Technical University of Denmark, Department of Technology, Management and Economics, Lyngby, Denmark
[e] Ruhr-University Bochum, Chair of Energy Systems and Energy Economics, Bochum, Germany

ARTICLE INFO

Article history:
Received 23 June 2021
Accepted 30 October 2022
Available online 4 November 2022

Keywords:
OR in energy
Electricity markets
Multi-stage stochastic programming
Uncertainty modeling
Risk modeling

ABSTRACT

Quantity and price risks determine key uncertainties market participants face in electricity markets with increased volatility, for instance due to high shares of renewables. In the time from day-ahead until real-time, there lies a large variation in best available information, such as between forecasts and realizations of uncertain parameters like renewable feed-in and electricity prices. This uncertainty reflects on both the market outcomes and the quantity of renewable generation, making the determination of sound trading strategies across different market segments a complex task. The scope of the paper is to optimize day-ahead and intraday trading decisions jointly for a portfolio with controllable and intermittent renewable generation under consideration of risk. We include a reserve market, a day-ahead market and an intraday market in stochastic modeling and develop a multi-stage stochastic Mixed Integer Linear Program. We assess the profitability as well as the risk exposure, quantified by the conditional value at risk metric, of trading strategies following different risk preferences. We conclude that a risk-neutral trader mainly relies on the opportunity of higher expected profits in intraday trading, whereas risk can be hedged effectively by trading on the day-ahead. Finally, we show that reserve market participation implies various rationales, including the relation of expected reserve prices among each other, the relation of expected reserve prices to spot market prices, as well as the relation of the spot market prices among each other.

1. Introduction

With increasing uncertainties in the energy system in recent years, reserve and spot electricity markets have been moving towards higher granularity and trading decision times appear to move more and more to the short-term and even close to real-time. The increasing shares of weather-dependent intermittent renewable generation and the introduction of intraday markets imply many changes in the design of short-term markets, but also in the trading rationales in sequential market structures and the risk exposure of individual market participants. The complexity of trading decisions leads to myriads of possible strategies to bring the flexibility and energy of a power plant portfolio profitably to the electricity markets. Hereby, not only the market segments themselves are subject to uncertainty, but also the relationship and interplay of the market segments need to be considered when deriving trading decisions. Another aspect that is relevant in the course of the energy transition is the actor structure in the energy sector. As more and more small market participants enter the market, that are sensitive to risk but unable to develop sophisticated methods, the demand for insights and approaches to determine sound trading strategies for all market segments increases.

As of today, trading decisions are typically determined with the help of deterministic programming approaches, basic stochastic considerations as well as the gut feeling of traders, and do not consider all market segments as a whole. In the literature, there are several studies that highlight the need to consider different market segments and associated uncertainties in thorough (e.g., Boomsma, Juul, & Fleten, 2014; Möst & Keles, 2010). Most of the existing studies focus on modeling the relationship between market prices and uncertain renewable power generation on the day-ahead stage, but neglect the relationship with the parameters on the intraday stage. We provide an approach for consistent modeling of the different uncertain parameters, especially between both spot market stages, day-ahead and intraday. These uncertainties are considered jointly with the uncertainty of reserve power market prices in a coor-

* Corresponding author.
E-mail address: emil.kraft@kit.edu (E. Kraft).

https://doi.org/10.1016/j.ejor.2022.10.040
0377-2217/

Sets and indices

H	hours of day
I	scenarios for stage i (reserve market)
J	scenarios for stage j (day-ahead market)
K	scenarios for stage k (intraday market)
L, M, N	cardinality of sets I, J, K
LDA	bidding price levels for day-ahead market
LID	bidding price levels for intraday market
LN	bidding price levels for negative reserve
LP	bidding price levels for positive reserve
QH	quarter hours of day
$QH(H)$	mapping of quarter hours to respective hours
$QH(TS)$	mapping of quarter hours to respective 4 hours time slices
RES	intermittent renewable units in portfolio
TS	4 hours time slices of day
U	controllable units in portfolio

Parameters

α	probability level for VaR and CVaR
β^{m}	binary acceptance parameter of bids in market m
κ^{var}	variable costs
λ	risk aversion weight parameter
$\nu^{\mathrm{min/max}}$	minimum/maximum daily generation of controllable unit as share of baseload operation at P^{U}
$\phi^{\mathrm{DA/ID}}$	day-ahead/intraday renewable generation forecast
$\triangle P^{\mathrm{U/RES}}$	maximum load change of controllable/renewable unit within reserve activation time as share of $P^{\mathrm{U/RES}}$
$\triangle t$	correction factor between quarter hours and hours
$BIGM$	sufficiently large number, e.g. 100,000
P^{min}	minimum load of controllable unit
$P^{\mathrm{U/RES}}$	nominal capacity of controllable/renewable unit
pr	scenario probability
$q^{\mathrm{short/long}}$	maximum short/long position as proportion of maximum volume on spot market
y^{m}	market price of market m

Variables

η	Value-at-Risk (VaR)
κ	cost
π	contribution margin
ρ	revenue
$x^{\mathrm{m,trade}}$	trade volume in market m, negative value represents buy
x^{imb}	imbalance volume

Positive variables

$\triangle x^{\mathrm{m,U,+/-}}$	upward/downward (+/−) load change of controllable unit in market m
s	auxiliary variable for CVaR modeling
$x^{\mathrm{m,bid}}$	bid volume in market $m \in$ {aFRRpos, aFRRneg, DA, ID}
$x^{\mathrm{rm,U/RES}}$	volume of $rm \in$ {aFRRpos, aFRRneg} provided by controllable/renewable unit U/RES
$x^{\mathrm{dispatch,U/RES}}$	dispatch volume of controllable/renewable unit
$x^{\mathrm{imb,+/-}}$	absolute value of positive/negative (+/−) imbalance volume
$x^{\mathrm{m,dispatch,U/RES}}$	dispatch volume of controllable/renewable unit in market m
$x^{\mathrm{m,gen,bid}}$	sell bid volume for dispatch generation in market m
$x^{\mathrm{m,gen,trade}}$	sell volume of dispatch generation in market m
$x^{\mathrm{m,long,bid}}$	ask bid volume in market m to get a long position
$x^{\mathrm{m,long,trade}}$	buy volume on market m to get a long position
$x^{\mathrm{m,short,bid}}$	sell bid volume in market m to get a short position
$x^{\mathrm{m,short,trade}}$	sell volume on market m to get a short position

Binary variables

$\delta^{\mathrm{m,ask/sell}}$	auxiliary variable to ensure either an ask or a sell bid for one price level
δ^{imb}	auxiliary variable for absolute value of imbalance
$\delta^{\mathrm{m,U}}$	auxiliary variable for (potential) load change in commitment on market m
$\delta^{\mathrm{rm,U}}$	auxiliary variable for minimum load requirement of controllable unit provision
θ	auxiliary variable for VaR modeling

dinated bidding problem. This requires to extend existing modeling approaches, that consider one or a few sources of uncertainty independently from each other, by including the conditional relations of uncertainties of the relevant parameters, too (Russo, Kraft, Bertsch, & Keles, 2022). In doing so, we provide a major contribution, as existing studies do not combine bidding on the reserve market and both segments of the spot market (day-ahead and intraday) under uncertainty, while considering risk preferences and their implications on bidding strategies.

With the proposed approach, we model the relevant quantity and price risks from the morning of the day ahead until the gate closure of intraday trading and include all key characteristics of the reserve market, the day-ahead spot market as well as the intraday spot market. We intend to focus on day-ahead decisions, i.e. bidding in the reserve and day-ahead market, that lead to an optimal pre-positioning for recourse actions, when new information becomes available on the intraday market. The proposed approach could be used on a day-to-day basis by energy companies in supporting the decision-making, but also provides manifold insights for risk management. To estimate and apply the developed models, we provide a case study for the German electricity market design and a renewable generation portfolio consisting of intermittent and controllable units.

The results consist on the one hand in a transparent assessment of the expected profits and risks under different trading strategies. In this study, besides the expected value we include risk metrics such as the conditional value at risk into decision-making, as introduced in Conejo, Carrión, & Morales (2010). We present efficient frontiers and profit distributions associated with optimal trading decisions. On the other hand, we derive and discuss valuable insights on trading rationales both within and across the market segments. Like that, we provide not only an innovative application of stochastic programming to a complex real-world problem, but also interesting insights for scholars, traders, and ultimately policy makers designing markets for the energy transition.

The remainder of the paper is organized as follows: In Section 2, we discuss approaches in the literature to face the trading problem with stochastic optimization and further specify the research gap. Section 3 presents the considered sequential market setting and key characteristics of the single market segments. In Section 4, we describe the trading problem and develop a methodology to derive optimal trading strategies based on a multi-stage stochastic mixed-integer linear problem. Further, Section 4 includes the stochastic modeling of uncertainties that serves as input for the trading problem. Section 5 applies the developed approach to the case of a power plant portfolio in the German market and discusses results and underlying trading rationales. Finally, Section 6 summarizes the main conclusions for different stakeholder groups and provides an outlook to future developments and applications.

2. Literature review and research gap

Decisions in energy economics are often categorized into strategic (i.e., mostly investment) and operational or short-term decisions. This paper sets a clear focus on short-term decisions. Optimization approaches to provide decision support for short-term decisions of actors in energy economics can be further distinguished into the optimal use of the technical units and the optimal interaction with revenue streams, i.e. the markets for flexibility and energy. As this paper focuses on the European setting of a self-dispatch system with balancing responsible parties (BRP), we will not address ISO optimization approaches, which are deployed mainly in the US (e.g., CAISO in California). We are focusing on approaches from the perspective of individuals.

In the literature, the two short-term optimization problems that are relevant for this paper are, the optimal dispatch problem or unit commitment problem for a power plant portfolio, and the optimal trading problem. However, in most cases these two cannot be separated strictly from each other. Whereas the first describes the problem of delivering a defined schedule of energy or providing a defined flexibility on activation request at minimal cost, the second enhances the scope by taking into account the (expected) market outcomes and optimizing the bids, which lead to the profit-maximizing operation. As the objective function is defined to maximize the contribution margins and as the market commitments are not known ex ante, the unit commitment problem is not modeled explicitly but implicitly. Whereas unit commitment rather focuses on technical constraints of the plant or the plant portfolio, the trading problem rather addresses the market operations in more detail.

Obviously, there are many previous works considering the deterministic unit commitment and trading problem. Typically, although technical constraints are non-linear in reality (e.g. efficiency for partial load), the problem is formulated as a Mixed-Integer Linear Program (MILP) to keep the problem mathematically tractable with standard solvers. However, deterministic approaches fail to address for increasing uncertainty and to depict the risk, even more so with rising shares of weather-dependent renewable generation and uncertain market prices. We therefore focus on approaches of stochastic programming. For handling uncertainty, the main stochastic optimization approaches include exact solution methods and approximation techniques (see, e.g., Birge & Louveaux, 2011, for an overview). Zheng, Wang, & Liu (2015) provide a review of stochastic optimization approaches for the unit commitment problem and distinguish between stochastic programming, robust programming and (approximate) stochastic dynamic programming. For the literature overview to remain concise, at this point we focus on works that apply stochastic programming approaches to electricity market bidding in sequential market settings and refer to Klaboe & Fosso (2013); Möst & Keles (2010), and Zheng et al. (2015) for more thorough reviews of stochastic modeling in energy economics.

Fleten & Kristoffersen (2007) deploy stochastic programming to determine optimal bidding strategies for hydropower plants with a cascade structure. Boomsma et al. (2014) model coordinated bidding in electricity spot and balancing power markets in the Nordic market design with the help of a multi-stage stochastic program and compare the risk exposure of different bidding strategies. Ottesen, Tomasgard, & Fleten (2018) deploy a multi-stage stochastic program to derive an optimal trading strategy for a portfolio of demand side management units in three market segments: Starting with an option market that is cleared for an entire week, followed by a daily spot market and an hourly flexibility market, the trader faces three possible revenue streams with uncertain prices as in the Nordic market design. Klæboe, Braathen, Eriksrud, & Fleten (2019) continue the investigation of coordinated bidding strategies for hydropower plants in the Nordic market with a similar multi-stage stochastic approach.

Plazas, Conejo, & Prieto (2005) deploy the case of the Spanish market design to investigate bidding strategies in three market segments of the electricity, aiming at maximizing the expected profit. Morales, Conejo, & Perez-Ruiz (2010) and Pandžić, Morales, Conejo, & Kuzle (2013) formulate a multi-stage stochastic problem for offering and operating a virtual power plant in a market setting with spot and adjustment market as two decision stages. Wozabal & Rameseder (2020) formulate a similar multi-stage bidding problem for a virtual power plant on the Spanish day-ahead and intraday market as a Markov decision process and solve it with stochastic dual dynamic programming. For the perspective of the operator of a local energy market, Laur, Nieto-Martin, Bunn, & Vicente-Pastor (2018) present a multi-stage stochastic approach to procure flexibility services in distribution network and discuss risk implications.

This paper stands in line and pursuits similar ideas with the presented papers, although we face a significantly different problem structure. The existing studies do not match the structure of short-term electricity markets and all their relevant design elements that we observe in a real-world market setting as presented in Section 3. For this reason, the study fills this gap by presenting a comprehensive problem description for the trading decisions of a portfolio manager with intermittent and controllable renewables and applies it to the case of the German market design. Furthermore, we cover the entire market risk and uncertainties such a portfolio faces in the operations on the short-term markets.

We address these by considering a stochastic programming approach with three decision stages depicting the trading in sequential markets including different temporal resolutions, pay-as-bid and uniform pricing, and high uncertainty of prices and volumes stemming from various sources. This raises the need to consider a stochastic approach for trading in sequential markets. Forecast errors on the day-ahead are unavoidable and lead to significant price and quantity risks for any market participant. Addressing these adequately in the decision process requires a thorough analysis and modeling of the stochastics and consequently an approach that considers all aspects, the uncertainty and the technical and market constraints. With the approach presented in this paper, we aim at filling this gap in the existing literature.

3. Market description

In European power markets, the products relevant for a power plant portfolio trader can be distinguished into the mere delivery of energy and the provision of reserve power for the transmission system operator (TSO) to balance the system. Whereas the former is organized in large exchanges and the market design has been harmonized internationally to a large extent, the latter is still organized in distinct national designs. Rising shares of generation by renewable energy sources (RES) led to an increasing relevance of close-to-realtime decisions. This applies not only for reserve products that are procured with shorter lead times, but particularly for the interplay of day-ahead and intraday spot market operations.

The setting that is studied in this paper includes a balancing reserve power market with separate products for positive and negative direction, a day-ahead electricity market, and an intraday elec-

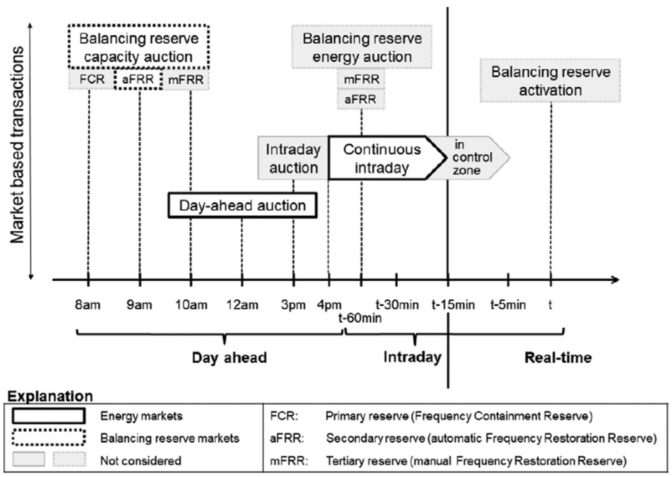

Fig. 1. Sequence of markets for electricity and flexibility with gate-closure times in the German market design from November 2020 on. The considered market segments reserve ("aFRR capacity auction"), day-ahead spot market ("Day-ahead auction") and intraday spot market ("Continuous intraday") are marked with black boxes.

tricity market. This threefold organization is typical for European power markets and can be found with small variations in many countries. With regard to the balancing reserve, we focus on the secondary reserve market (i.e., automatic Frequency Restoration Reserve, aFRR) with its lead times and product requirements. For better readability, the term "reserve" is used synonymously with secondary reserve and aFRR in the following.

Fig. 1 provides an overview over the market sequence until real-time and highlights the market segments relevant for this study. To substantiate the products and lead times in a concrete case, we choose the market design setting of Germany. The German market is selected to be representative, because the structure with three sequential markets is common in many electricity market designs: A reserve segment, a day-ahead market and an intraday market. Therefore, the developed approach addresses not only the German market. With slight adaptations, it can be used to model uncertainty and trading decisions in most other market settings, even if slightly different bidding rules and market setups apply. Further, the German setting with auctions to procure flexible capacity as options to be activated by the TSOs serves as the blueprint in the ongoing reserve market harmonization process within Europe (cf. European Commission, 2017). The procurement of reserve products by the TSOs is organized in a two-stage procedure. The first stage, the so-called capacity market auction for the next day, takes place prior to the day-ahead spot market at 9 am and determines the reserve providers for the following day. In this auction, the prequalified reserve providers can place bids consisting of the capacity price (in EUR/MW) and a volume (in MW). Providers are allowed to submit several distinct bids. The 24 hours of the day are split into six time slices consisting of four hours each (0–4, 4–8, ..., 20–24). Further, reserves for the negative (downward regulation) and the positive (upward regulation) direction are auctioned as separate products. This leads to twelve distinct reserve auctions each day, which are remunerated according to pay-as-bid pricing. In the second step of reserve procurement, the so-called reserve energy market auction takes place before the gate closure time of the intraday market, and determines the prices and merit order of activation. The successful bid in the capacity market obliges the trader to offer in the reserve energy market, however also free energy bids are allowed. For the scope of this paper, the reserve energy market is left out for two reasons. First, the energy price bid can only lead to positive contribution margins if above the variable costs of provision and therefore poses no risk of losses to the portfolio profit. Second and most important, as bids into the reserve energy market do not constrain the action space for bids in other market segments or interfere with their profitability, they can be considered independently from the other trading decisions. Therefore, it is rather a complementing element than an opportunity and can be neglected.

After the reserve capacity market, at 12 am the day-ahead spot market auction takes place, in which energy delivery for the next day is traded in hourly resolution with uniform pricing. Subsequently, at 3 pm the intraday auction takes place, in which energy can be traded in quarter-hourly resolution. The intraday market is then open for continuous trading of energy in quarter-hourly products until 15 minutes before delivery.[1] Intuitively, the continuous intraday trading could be represented as up to 96 individual stages. However, on the day ahead, the unveiling of information can hardly be anticipated or modeled in such detail that a trader could exploit it. To address the challenge of deriving optimal bids in the sequence of markets, it is more important to properly represent the opportunity that the intraday market poses to the trader than to represent all potential price processes and patterns in the intraday market.[2] A typical way to do so is to abstract the intraday market to one decision stage for all quarter hourly delivery periods instead of modeling each gate-closure time as a single stage. In accordance with Ottesen et al. (2018) and Laur et al. (2018) the decisions are made by approximating the intraday trading with one hypothetical auction for each quarter hour, with the index price ID3 of the trades completed in the last three hours as representative price for each quarter hour. As the index averages completed trades, we handle the hypothetical auction as a uniform pricing auction. In that way, we are able to capture the profit opportunity as well as the involved risk in both upward and downward direction appropriately.We acknowledge that this simplification neglects profits from potential re-positioning in reaction to the volatility of the continuous price process during intraday trading. Further, we assume that the trader acts as a price-taker and is always able to find a counterparty, which is well reasonable considering the liquidity and the trading volumes on the intraday market.

In conclusion, the trader faces three markets to be considered: The reserve capacity market with six four-hour products, the day ahead market with 24 hourly products and the intraday market with 96 quarter-hourly products. Considering both the price uncertainty of the three markets and the volume uncertainty of the renewable generation, the trader faces a complex decision problem with numerous decision variables.

4. Methodology

4.1. The trading problem

For the bidder, the determination of the optimal bids implies both price risks and a quantity risks. The information available to the trader in the optimization problem is summarized in Fig. 2. Information like the residual load forecast and its updates are not explicitly provided to the problem, but are contained in the price processes as described above. Note further, that logically the

[1] Note, that half-hour and hour products are also traded in the intraday market. However, as the 15 minute product is the best approximation for the value of the 15 minutes period if contained within a product with larger resolution, including further product resolutions for intraday trading would not essentially change the trading strategies but result in substitution trades. We do not aspire to optimize for trades across intraday products, but for trading strategies across the stages and therefore focus on the 15 minute products alone.

[2] To consider different price processes and patterns is particularly important in the presence of time-coupling constraints, e.g., when optimizing trading decisions for storages. However, this exceeds the scope of this paper.

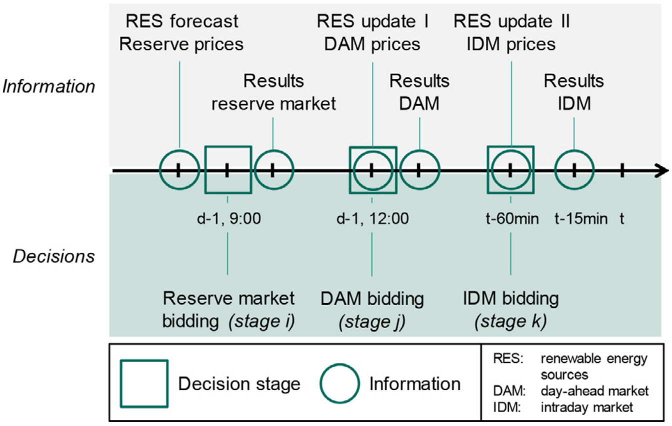

Fig. 2. Overview over available information and decisions in the modeled trading problem (illustration adapted from Wozabal & Rameseder, 2020). Based on the renewable forecast and reserve price information for the type day, the reserve market bidding decision takes place on the day ahead at 9am, followed by the reserve market results, the updated renewable forecast and day-ahead market prices. This is the information, which is known before the day-ahead market bidding decision at noon, which is followed by unveiling the market results as well as a second renewable generation forecast update and intraday prices 60 minutes before real-time. Based on this information follows the intraday market bidding decision.

stochastic information on later stages is implicitly considered in the decisions of the early stages. The structure of the decisions on the three stages will be explained in more detail in Section 4.2.

4.2. Target function

We formulate the target for the risk-neutral problem straight forward as maximization of the sum of expected contribution margins π throughout the scenarios on the three stages i, j and k in all market segments and time steps as the revenues (ρ) minus the costs (κ). The scenarios on each stage can be denoted as $il \in I = \{i1, \ldots, iL\}$ for stage i, $jm \in J = \{j1, \ldots, jM\}$ for stage j, and $kn \in K = \{k1, \ldots, kN\}$ for stage k, with l, m and n counting from 1 to the cardinality of the respective set on each stage. This finite probability space is denoted by Ω. For better readability, in the following the counting indices are dropped where no confusion can be expected, and the scenarios on the considered stages are denoted simplified as $(i, j, k) \in \Omega$.

$$\max \mathbb{E}_{(i,j,k)\in\Omega}(\pi_{i,j,k}) = \max \left(\mathbb{E}_{(i,j,k)\in\Omega}(\rho_{i,j,k}) - \mathbb{E}_{(i,j,k)\in\Omega}(\kappa_{i,j,k})\right) \tag{1}$$

with the expected revenues ρ being the sum of reserve (aFRRpos and aFRRneg) market, day-ahead (DA) market and intraday (ID) market revenues.

$$\mathbb{E}_{(i,j,k)\in\Omega}\left(\rho_{i,j,k}\right) = \mathbb{E}_{(i)\in\Omega}\left(\rho_i^{\text{aFRRpos}}\right) + \mathbb{E}_{(i)\in\Omega}\left(\rho_i^{\text{aFRRneg}}\right) + \mathbb{E}_{(i,j)\in\Omega}\left(\rho_{i,j}^{\text{DA}}\right) + \mathbb{E}_{(i,j,k)\in\Omega}\left(\rho_{i,j,k}^{\text{ID}}\right) \tag{2}$$

One key challenge in the formulation of the trading problem consists in addressing the reserve market design with its particularities. As pay-as-bid pricing intuitively comes with both the price and the volume as decision variables, an alternative formulation must be deployed for the problem to remain a mixed-integer linear problem (MILP). However, the modeling of uncertainty yields discrete values for reserve prices for positive and negative direction (LP and LN, respectively) for each reserve price scenario i. We therefore define these price levels p_{lp}^{aFRRpos} (p_{ln}^{aFRRneg}) as fixed bidding levels and define only the bid volume $x_{lp,i,ts}^{\text{aFRRpos,bid}}$ ($x_{ln,i,ts}^{\text{aFRRneg,bid}}$) on price level lp (ln) as decision variable for positive (negative) reserve market bidding. In this way, we define a bidding curve with volumes on several price levels to be submitted to each segment of the reserve market.

As we model the uncertainty with a discrete probability space, the trader has no incentive to bid on price levels distinct from the given scenario prices. The eventual acceptance of a bid on price level $lp \in LP$ for the positive reserve product (or on price level $ln \in LN$ for negative reserve) in time slice ts and scenario i is modeled with the help of the binary acceptance parameters $\beta_{lp,i,ts}^{\text{aFRRpos}}$ and $\beta_{ln,i,ts}^{\text{aFRRneg}}$, that translate the marginal prices into acceptance or decline of a bid as described in Eq. (49). The resulting expected revenues from the positive reserve market over all time steps and scenarios are then defined as (negative follows analogously):

$$\mathbb{E}_{(i)\in\Omega}\left(\rho_i^{\text{aFRRpos}}\right) = \sum_{i\in I} pr_i \sum_{ts\in TS} \sum_{lp\in LP} \left(\beta_{lp,i,ts}^{\text{aFRRpos}} \cdot y_{lp}^{\text{aFRRpos}} \cdot x_{lp,i,ts}^{\text{aFRRpos}}\right) \tag{3}$$

pr_i, pr_j, pr_k denote scenario probabilities for $i \in I$, $j \in J$, and $k \in K$, as described in 4.6. Note, that this formulation takes into account the reserve market to be cleared according to pay-as-bid pricing and the spot market segments according to uniform pricing. The price y_{lp}^{aFRRpos} for an accepted bid is therefore indexed with the respective price level and reflects the pay-as-bid pricing.

In contrast, the day-ahead and intraday market are cleared with uniform pricing.[3] Hereby, $y_{j,h}^{\text{DA}}$, $y_{i,j,k,qh}^{\text{ID}}$ denote the prices on the day-ahead and intraday market for the different time steps and scenarios, respectively. The expected revenues on the day-ahead market are defined as the trading volume $x_{i,j,h}^{\text{DA,trade}}$ multiplied by the uniform price $y_{j,h}^{\text{DA}}$ in scenario j, summed up over all hours.

$$\mathbb{E}_{(i,j)\in\Omega}\left(\rho_{i,j}^{\text{DA}}\right) = \sum_{i\in I} pr_i \sum_{j\in J} pr_j \sum_{h\in H} y_{j,h}^{\text{DA}} \cdot x_{i,j,h}^{\text{DA,trade}} \tag{4}$$

The day-ahead market is modeled such that the trader submits a bid curve to the market, consisting of volume bids on defined fixed price levels lda. For the evaluation of bidding strategies, we distinguish between bids to sell generation, $x_{i,j,h}^{\text{DA,gen,bid}}$, bids to take a short position (i.e., selling more than is expected to be generated), $x_{i,j,h}^{\text{DA,short,bid}}$, and bids to take a long position (i.e., buying electricity on the day-ahead market), $x_{i,j,h}^{\text{DA,long,bid}}$.

Again, there is no incentive to deviate from the market prices contained in the scenarios J for the day-ahead market decision stage. With the help of the binary parameter $\beta_{lda,j,h}^{\text{DA}}$, denoting the accepted price levels lda of day-ahead market bids for selling electricity in hour h and scenario j, the traded volume is defined as follows. We stress at this point that the trader does not only have the option of purely selling the generation, but also to prepare a good position for potential intraday trading. Thus, both building a short position that exceeds the expected generation and going long (i.e., buying electricity to sell it later and profit from rising prices) is within the trader's action space. The bids to take a long position in the day-ahead market (i.e., buying electricity) have the opposite acceptance structure of selling bids $(1 - \beta_{lda,j,h}^{\text{DA}})$.[4]

$$x_{i,j,h}^{\text{DA,trade}} = x_{i,j,h}^{\text{DA,gen,trade}} + x_{i,j,h}^{\text{DA,short,trade}} - x_{i,j,h}^{\text{DA,long,trade}} \tag{5}$$

[3] As described in Section 3, we model the intraday auction as an uniform pricing auction with the ID3 price as clearing price.

[4] This formulation implies that in case $y_{j,h}^{\text{DA}} = y_{lda,j,h}^{\text{DA}}$ a selling bid gets accepted at price level lda, whereas an ask bid is declined. A successful ask bid must be at least one price level above, despite being valued with price level lda. We thereby reflect a certain bid-ask spread and avoid opposite bids that cancel each other out and only inflate trading volumes.

with

$$x^{\mathrm{DA,gen,trade}}_{i,j,h} = \sum_{lda\in LDA} \beta^{\mathrm{DA}}_{lda,j,h} \cdot x^{\mathrm{DA,gen,bid}}_{lda,i,j,h} \tag{6}$$

$$x^{\mathrm{DA,short,trade}}_{i,j,h} = \sum_{lda\in LDA} \beta^{\mathrm{DA}}_{lda,j,h} \cdot x^{\mathrm{DA,short,bid}}_{lda,i,j,h} \tag{7}$$

$$x^{\mathrm{DA,long,trade}}_{i,j,h} = \sum_{lda\in LDA} \left(1-\beta^{\mathrm{DA}}_{lda,j,h}\right) \cdot x^{\mathrm{DA,long,bid}}_{lda,i,j,h} \tag{8}$$

However, the long and the short position of the portfolio are constrained to the extent allowed in BRP contracts as will be presented in Eqs. (21) and (22). The revenues from the intraday market are defined analogously. The factor $\triangle t$ captures the difference in temporal resolution between hour h and quarter hour qh (i.e. $\triangle t = 0.25$), so that the energy amount equals the integral of the power output.

$$\mathbb{E}_{(i,j,k)\in\Omega}\left(\rho^{\mathrm{ID}}_{i,j,k}\right) = \sum_{i\in I} pr_i \sum_{j\in J} pr_j \sum_{k\in K} pr_k \sum_{qh\in QH} y^{\mathrm{ID}}_{j,k,qh} \cdot x^{\mathrm{ID,trade}}_{i,j,k,qh} \cdot \triangle t \tag{9}$$

The realized intraday trades, $x^{\mathrm{ID,trade}}_{i,j,k,qh}$, and the breakdown into intraday positions are defined analogously as for the day-ahead market in (6)–(8). The expected costs occurring in each market segment sum up to the total costs. To account for potential active schedule violations in a future application, the term $\kappa^{\mathrm{Imb}}_{i,j,k}$ completes the formulation of the trading problem.

$$\begin{aligned}\mathbb{E}_{(i,j,k)\in\Omega}(\kappa_{i,j,k}) = {}& \mathbb{E}_{(i)\in\Omega}\left(\kappa^{\mathrm{aFRRpos}}_{i}\right) + \mathbb{E}_{(i)\in\Omega}\left(\kappa^{\mathrm{aFRRneg}}_{i}\right)\\ &+\mathbb{E}_{(i,j)\in\Omega}\left(\kappa^{\mathrm{DA}}_{i,j}\right)\\ &+\mathbb{E}_{(i,j,k)\in\Omega}\left(\kappa^{\mathrm{ID}}_{i,j,k}\right) + \mathbb{E}_{(i,j,k)\in\Omega}\left(\kappa^{\mathrm{Imb}}_{i,j,k}\right)\end{aligned} \tag{10}$$

The pure provision of capacity is valued at no costs. For positive reserve (aFRRpos), the reserve activation may lead to additional fuel consumption and thus additional variable costs. On the other hand, an activation of negative reserve (aFRRneg) may lead to fuel savings and thus a reduction of the costs arising from the spot market operation. However, the cost effects of potential reserve activation can be easily addressed by appropriate energy bids. As the costs of a potential positive reserve activation are independent from the bidding decision on the reserve power market, we consider it reasonable to value them at zero costs. The same applies for the negative reserve.

The activation of reserves is not further considered in this paper. The only assumption that needs to be made is that the energy to meet the activation is available for the controllable plants u. The costs for the day-ahead market operation consist of variable costs for the controllable plant u and the renewable source res. The costs for a potential long position are already accounted for in the revenues in Eqs. (4) and (9).

$$\begin{aligned}\mathbb{E}_{(i,j)\in\Omega}\left(\kappa^{\mathrm{DA}}_{i,j}\right) = \sum_{i\in I} pr_i \sum_{j\in J} pr_j \sum_{h\in H} \Bigg(&\sum_{u\in U} \kappa^{\mathrm{var}}_{u} \cdot x^{\mathrm{DA,dispatch,U}}_{i,j,u,h}\\ &+ \sum_{res\in RES} \kappa^{\mathrm{var}}_{res} \cdot x^{\mathrm{DA,dispatch,RES}}_{i,j,res,h}\Bigg)\end{aligned} \tag{11}$$

$\kappa^{\mathrm{var}}_{u}$ denotes the variable cost of unit u, $x^{\mathrm{DA,dispatch,U}}_{i,j,u,h}$ denotes the energy dispatched (i.e., sold with $x^{\mathrm{DA,gen,bid}}_{lda,i,j,h}$) from unit u on the day-ahead market. The dispatch for renewable source res is defined analogously.

$$\begin{aligned}\mathbb{E}_{(i,j,k)\in\Omega}(\kappa^{\mathrm{ID}}_{i,j,k}) = \sum_{i\in I} pr_i \sum_{j\in J} pr_j \sum_{k\in K} pr_k \sum_{qh\in QH} \Bigg(&\sum_{u\in U} c^{\mathrm{var}}_{u} \cdot x^{\mathrm{ID,dispatch,U}}_{i,j,k,u,qh}\\ &+ \sum_{res\in RES} c^{\mathrm{var}}_{res} \cdot x^{\mathrm{ID,dispatch,RES}}_{i,j,k,res,qh}\Bigg) \cdot \triangle t\end{aligned} \tag{12}$$

$$\begin{aligned}\mathbb{E}_{(i,j,k)\in\Omega}\left(\kappa^{\mathrm{Imb}}_{i,j,k}\right) = {}&\sum_{i\in I} pr_i \sum_{j\in J} pr_j \sum_{k\in K} pr_k\\ &\sum_{qh\in QH} \left(x^{\mathrm{imb},+}_{i,j,k,qh} + x^{\mathrm{imb},-}_{i,j,k,qh}\right) \cdot y^{\mathrm{imb}}_{qh} \cdot \triangle t\end{aligned} \tag{13}$$

with the absolute value of the energy imbalance $x^{\mathrm{imb}}_{i,j,k,qh}$ denoted by the sum of the positive and negative share ($x^{\mathrm{imb},+}_{i,j,k,qh} + x^{\mathrm{imb},-}_{i,j,k,qh}$) valued with the imbalance price y^{imb}_{qh}.[5] Logically, the imbalance can only either be positive or negative, which is reflected by the constraints (19) and (20) in Section 4.5.

4.3. Consideration of risk

A major advantage of the stochastic over the deterministic problem formulation consists in the ability of the presented approach to quantify and to take into account risks when determining the trading strategy. Based on theory provided in textbooks such as Conejo et al. (2010) and Birge & Louveaux (2011), we distinguish between risk-neutral decision making and decision making under consideration of risk. Whereas the risk-neutral decision is based solely on the expected value of the profits over all scenarios as presented in (1), a real-world trader most likely will also want to consider the risk exposure related with the trading decision. In order to determine trading decisions with less risk exposure, we therefore introduce risk to our approach. This enables us to make use of the characterization of uncertainty, which contains more information than a single figure, such as the expected value, can capture. Risk exposure is quantified with the help of risk metrics. Commonly used metrics include the variance, the shortfall probability, the expected shortage and as well as value at risk (VaR) and conditional value at risk (CVaR) (Conejo et al., 2010). However, for the trading problem to remain scalable and flexible, the use of a coherent risk metric,[6] particularly one satisfying sub-additivity, is of practical use. As the CVaR brings these desirable properties and including the CVaR is also computationally more efficient than including the VaR,[7] we modify the problem formulation in order to include the CVaR into the target function (adaptation of Conejo et al., 2010). Further details and the definition of the CVaR and the VaR are provided in Annex A.2.

4.4. Modeling the conditional value at risk

In adaption of Conejo et al. (2010) the target function is augmented by the variable η, that corresponds to the VaR, the parameter α representing the probability level of the VaR and the non-negative continuous variable $s_{i,j,k}$ defined by Eq. (15) to the maximum of the VaR η minus the contribution margin $\pi_{i,j,k}$ in a

[5] As intentional imbalances are prohibited by the up-to-date German BRP contract, $x^{\mathrm{imb}}_{i,j,k,qh}$ is forced to equal zero with a sufficiently large number $BIGM$ as y^{imb}_{qh}. In the presented case study $BIGM$ equals 100,000 EUR/MWh. In the general formulation, y^{imb}_{qh} may be equipped with a close-to-real-time forecast to reflect an expected imbalance price. However, this exceeds the scope of this paper.

[6] Coherent risk metrics satisfy the conditions of monotonicity, sub-additivity, homogeneity, and translational invariance.

[7] The VaR formulation, as presented in Annex A.2, requires the introduction of additional binary variables, whereas the CVaR representation requires only continuous variables.

scenario and zero. The optimization objective is now the weighted sum of expected value and the CVaR of the contribution margins throughout the scenarios, with $\lambda \in (0, 1)$ as weight in the target function. λ can be referred to as parameter of risk aversion.

$$\max \quad (1-\lambda) \cdot \mathbb{E}_{(i,j,k) \in \Omega}(\pi_{i,j,k}) + \lambda \cdot \left(\eta - \frac{1}{1-\alpha} \sum_{i \in I} pr_i \sum_{j \in J} pr_j \sum_{k \in K} pr_k \cdot s_{i,j,k} \right) \tag{14}$$

$$\eta - \pi_{i,j,k} \leq s_{i,j,k} \quad \forall (i, j, k) \tag{15}$$

$$s_{i,j,k} \geq 0 \quad \forall (i, j, k) \tag{16}$$

In the remainder of this paper, sets will be dropped from the notation to remain concise, $\forall (i, j, k) \in \Omega$ is equivalent to $\forall (i, j, k)$, $\forall qh \in QH$ equivalent to $\forall qh$, and so on. The chosen multi-criteria formulation as weighted sum allows us to consider both the expected value of contribution margins and the CVaR at level α. The parameters λ and α will be used in the case study to distinguish between and evaluate different risk strategies. Further, for the interested reader we provide the problem formulation using the VaR as risk metric in Annex A.2.

4.5. Constraints

Besides the aforementioned constraints for modeling the risk, we include constraints from three categories in the problem formulation that will be presented in the subsequent paragraphs. First, several constraints regarding the trading logic, the market design and the market rules need to be considered. Further, the operational constraints of the technical units in the portfolio to fulfill energy delivery and provide the reserve products need to be considered in modeling the trading decision. Hereby, we distinguish between the intermittent renewable energy sources and controllable (renewable) units (e.g., a biogas power plant). The third category of constraints comprises the stochastic programming constraints, in which we summarize the constraints required for the formulation of the multi-stage stochastic problem and further auxiliary constraints.

4.5.1. Market constraints

As introduced in Section 4.2, the formulation shall provide the trader with the option to apply bidding strategies between the spot market stages (day-ahead and intraday market). To do so, we include variables to capture short- and long-selling positions on the respective stage. However, as the market rules prohibit intentional imbalances, the traded energy volumes on the day-ahead and intraday market must equal the generation or consumption volumes of the portfolio. The slack variable $x^{\text{imb}}_{i,j,k,qh}$ ensures equality, and enters the target function as formulated in Eq. (13). Eventually, the energy schedule needs to be balanced for each qh. In the following, $qh(h)$ denotes the mapping of the quarter hours contained in an hour to the respective hour h (e.g., $qh(1) = \{1, 2, 3, 4\}$). As mentioned earlier, the imbalances enter the target function as positive variables, penalized with BIGM, and are thus forced to equal zero.

$$x^{\text{DA,trade}}_{i,j,h} + x^{\text{ID,trade}}_{i,j,k,qh} + x^{\text{imb}}_{i,j,k,qh} = x^{\text{DA,gen,trade}}_{i,j,h} + x^{\text{ID,gen,trade}}_{i,j,k,qh} \quad \forall (i, j, k), h, qh(h) \tag{17}$$

with

$$x^{\text{imb}}_{i,j,k,qh} = x^{\text{imb},+}_{i,j,k,qh} - x^{\text{imb},-}_{i,j,k,qh} \quad \forall (i, j, k), h, qh(h) \tag{18}$$

For the absolute value consideration of the imbalance volume in (13), a $BIGM$ formulation with the auxiliary binary $\delta^{\text{imb}}_{i,j,k,qh}$ leads to the following equations to ensure that $x^{\text{imb},+}_{i,j,k,qh}$ and $x^{\text{imb},-}_{i,j,k,qh}$ are not greater than zero at the same time.

$$x^{\text{imb},+}_{i,j,k,qh} \leq BIGM \cdot \delta^{\text{imb}}_{i,j,k,qh} \quad \forall (i, j, k), qh \tag{19}$$

$$x^{\text{imb},-}_{i,j,k,qh} \leq BIGM \cdot \left(1 - \delta^{\text{imb}}_{i,j,k,qh}\right) \quad \forall (i, j, k), qh \tag{20}$$

Traders have incentives, if one spot market is dominating the other (e.g. price expectations for the intraday are favorable compared to the day-ahead market), to realize unlimited profit opportunities[8] of short and long trades between the markets. To account for the trades to be related to the portfolio and not to a purely speculative arbitrage strategy, we introduce volume limits for the short and long position related to the portfolio generation.[9] Eqs. (21) and (22) limit the short and the long trade volume in the day-ahead market. In consequence, the respective trade volumes are implicitly limited for the intraday market, too.

$$x^{\text{DA,short,trade}}_{i,j,h} \leq \left(\max_{qh} \left(\sum_{res \in RES} P^{\text{RES}}_{res} \cdot \phi^{\text{DA}}_{qh,res} \right) + \sum_{u \in U} P^{\text{U}}_{u} \right) \cdot q^{\text{short}} \quad \forall (i, j), h \tag{21}$$

$$x^{\text{DA,long,trade}}_{i,j,h} \leq \left(\max_{qh} \left(\sum_{res \in RES} P^{\text{RES}}_{res} \cdot \phi^{\text{DA}}_{qh,res} \right) + \sum_{u \in U} P^{\text{U}}_{u} \right) \cdot q^{\text{long}} \quad \forall (i, j), h \tag{22}$$

Further, to avoid bids on the same price level that cancel each other out and inflate bidding volumes, Eqs. (23) and (24) ensure for the day-ahead market that the trader either submits a sell bid or an ask bid on level lda. The analog formulation applies for the intraday market.

$$x^{\text{DA,gen,bid}}_{lda,i,j,h} + x^{\text{DA,short,bid}}_{lda,i,j,h} \leq BIGM \cdot \delta^{\text{DA,ask/sell}}_{lda,i,j,h} \quad \forall lda, (i, j), h \tag{23}$$

$$x^{\text{DA,long,bid}}_{lda,i,j,h} \leq BIGM \cdot \left(1 - \delta^{\text{DA,ask/sell}}_{lda,i,j,h}\right) \quad \forall lda, (i, j), h \tag{24}$$

Finally, the bids the trader submits to the day-ahead and the intraday market are summed up for evaluation purposes in the variables $x^{\text{DA,bid}}_{lda,i,j,h}$ and $x^{\text{ID,bid}}_{lid,i,j,k,qh}$ as defined in Eqs. (25) and (26). Aggregated over the considered price levels, these can be interpreted as bid curves submitted to the markets (cf. Figs. 7, 8, and 11).

$$x^{\text{DA,bid}}_{lda,i,j,h} = x^{\text{DA,gen,bid}}_{lda,i,j,h} + x^{\text{DA,short,bid}}_{lda,i,j,h} - x^{\text{DA,long,bid}}_{lda,i,j,h} \tag{25}$$

$$x^{\text{ID,bid}}_{lid,i,j,k,qh} = x^{\text{ID,gen,bid}}_{lid,i,j,k,qh} + x^{\text{ID,short,bid}}_{lid,i,j,k,qh} - x^{\text{ID,long,bid}}_{lid,i,j,k,qh} \tag{26}$$

4.5.2. Technical constraints

Several technical constraints need to be respected in the formulation of the bids in order to guarantee the feasibility of the market results for the operation of the plant portfolio. Firstly, the portfolio must be able to provide the reserve commitments. As the reserve commitment can be covered by a pool of technical units with any spatial distribution within the market area, each single unit can contribute with its flexibility to reach the market commitment. Thereby, we consider the flexibility contributions of the units to the portfolio to be constant for quarter hours. We assume

[8] An alternative term sometimes used in this context is arbitrage opportunity. However, we define an arbitrage trade to lead to risk-free profit. As this is not necessarily given, yet the trader might still favor one market expectation over another, we refer to expected profit under risk as profit opportunity.

[9] The up-to-date contracts for balancing responsible parties in Germany provide the regulation that the short or long volume must not exceed a proportion $q^{\text{short/long}}$ of 10 percent of the maximum schedule value of the day. To avoid the maximum operator with decision variables, we consider the sum of the maximum value in the renewable generation forecast and the installed capacity of the controllable plants as approximation for the maximum schedule value of the day instead of $\max_{i,j,k,h,qh(h)}(x^{\text{DA,gen,trade}}_{i,j,h} + x^{\text{ID,gen,trade}}_{i,j,k,qh})$.

reserve provision from both, the units u and the intermittent renewable energy sources res, to be technically feasible.[10] The condition is formulated in (27) for the positive direction, the condition for the negative direction is derived analogously. The elements of the set of intermittent renewable units RES and the set of controllable units U are denoted as $\{res1, \ldots\}$ and $\{u1, \ldots\}$. The index notation is simplified to res and u, where no confusion is expected.

$$\sum_{lp\in LP} \beta^{pos}_{lp,i,ts} \cdot x^{\mathrm{aFRRpos}}_{lp,i,ts} \le \sum_{res\in RES} x^{\mathrm{aFRRpos,RES}}_{i,qh,res} + \sum_{u\in U} x^{\mathrm{aFRRpos,U}}_{i,qh,u} \quad \forall ts, qh(ts), i \tag{27}$$

Secondly, the portfolio must fulfill the schedule defined by spot market commitments, which are derived from the accepted bids for each hour in the day-ahead market (and analogously for each quarter hour in the intraday market).

$$x^{\mathrm{DA,gen,trade}}_{i,j,h} = \sum_{u\in U} x^{\mathrm{DA,dispatch,U}}_{i,j,h,u} + \sum_{res\in RES} x^{\mathrm{DA,dispatch,RES}}_{i,j,h,res} \quad \forall (i,j), h \tag{28}$$

To obtain a feasible dispatch schedule, the minimum load requirement of controllable unit u is modeled with the help of a semi-continuous variable $x^{\mathrm{dispatch,U}}_{i,j,k,qh,u}$ which can only take values that are either 0 or in the range between minimum load $P^{\min}_u$ and the nominal capacity P^{U}_u. As the market constraints distinguish between day-ahead and intraday market dispatch, the overall scheduled dispatch for u is described in (29).

$$x^{\mathrm{dispatch,U}}_{i,j,k,qh,u} = x^{\mathrm{DA,dispatch,U}}_{i,j,h,u} + x^{\mathrm{ID,dispatch,U}}_{i,j,k,qh,u} \quad \forall (i,j,k), h, qh(h), u \tag{29}$$

Thirdly, for a unit to provide negative reserve, it must at least run on that level to be able to decrease the generation. The potential activation of negative reserve capacity must also not violate the minimum load requirement. Likewise, to provide positive reserve, it must at least run on minimum load and a potential activation must not violate the capacity constraint. Eqs. (30)–(34) formulate these minimum load constraints.

$$x^{\mathrm{aFRRneg,RES}}_{i,qh,res} \le x^{\mathrm{dispatch,RES}}_{i,j,k,qh,res} \quad \forall (i,j,k), qh, res \tag{30}$$

$$x^{\mathrm{dispatch,U}}_{i,j,k,qh,u} - x^{\mathrm{aFRRneg,U}}_{i,qh,u} \ge P^{\min}_u \cdot \delta^{\mathrm{aFRRneg,U}}_{i,j,k,qh,u} \quad \forall (i,j,k), qh, u \tag{31}$$

$$x^{\mathrm{aFRRneg,U}}_{i,qh,u} \le BIGM \cdot \delta^{\mathrm{aFRRneg,U}}_{i,j,k,qh,u} \quad \forall (i,j,k), qh, u \tag{32}$$

$$x^{\mathrm{dispatch,U}}_{i,j,k,qh,u} \ge P^{\min}_u \cdot \delta^{\mathrm{aFRRpos,U}}_{i,j,k,qh,u} \quad \forall (i,j,k), qh, u \tag{33}$$

$$x^{\mathrm{aFRRpos,U}}_{i,qh,u} \le BIGM \cdot \delta^{\mathrm{aFRRpos,U}}_{i,j,k,qh,u} \quad \forall (i,j,k), qh, u \tag{34}$$

Fourthly, the provision of positive reserve and the dispatch of a unit u is limited by its nominal capacity P^{U}_u, leading to capacity constraint (35). Likewise, a unit res is naturally limited by its nominal capacity P^{RES}_{res} derated with the generation forecast $\phi^{\mathrm{DA/ID}} \in [0,1]$. As described in Section 4.6 we distinguish between the deterministic day-ahead forecast and the scenario-based intraday update. Hereby, to respect the potential downward or upward correction of an intraday forecast update $\phi^{\mathrm{ID}}_{k,qh,res}$ compared to $\phi^{\mathrm{DA}}_{qh,res}$, the possible reserve provision of a renewable unit $x^{\mathrm{aFRRpos,RES}}_{i,qh,res}$ and dispatch must satisfy two capacity constraints (36) and (37). With this formulation, the positive reserve provision is implicitly limited to the minimum out of the day-ahead and the intraday updates of the generation forecast contained in Ω.[11]

$$x^{\mathrm{aFRRpos,U}}_{i,qh,u} + x^{\mathrm{DA,dispatch,U}}_{i,j,h,u} + x^{\mathrm{ID,dispatch,U}}_{i,j,k,qh,u} \le P^{\mathrm{U}}_u \quad \forall i,j,k,h,qh(h),u \tag{35}$$

$$x^{\mathrm{aFRRpos,RES}}_{i,qh,res} + x^{\mathrm{DA,dispatch,RES}}_{i,j,res,h} \le P^{\mathrm{RES}}_{res} \cdot \phi^{\mathrm{DA}}_{qh,res} \quad \forall (i,j), h, qh(h), res \tag{36}$$

$$x^{\mathrm{aFRRpos,RES}}_{i,qh,res} + x^{\mathrm{DA,dispatch,RES}}_{i,j,h,res} + x^{\mathrm{ID,dispatch,RES}}_{i,j,k,qh,res} \le P^{\mathrm{RES}}_{res} \cdot \phi^{\mathrm{ID}}_{k,qh,res} \quad \forall (i,j,k), h, qh(h), res \tag{37}$$

Further, the fuel storage capability for the dispatchable RES plants onsite (e.g., for a biogas plant) is limited, which leads to a minimum and maximum daily generation $\nu^{\min/\max}$ of unit u as proportion of a baseload operation of the installed capacity.

$$|QH| \cdot P^U_u \cdot \nu^{\min}_u \le \sum_{qh\in QH} x^{\mathrm{dispatch,U}}_{i,j,k,qh,u} \le |QH| \cdot P^U_u \cdot \nu^{\max}_u \quad \forall (i,j,k), u \tag{38}$$

with $|.|$ as the cardinality of a set. Finally, each unit u has a limited load change gradient $\triangle P_u$, which needs to be respected to obtain technically feasible results. We consider it to be the same for upward and downward load changes and define it as proportion of the installed capacity P^{U}_u. At the transition between two quarter hours, load changes can originate from all considered market segments. Therefore, some additional constraints are required. Logically, the maximum possible load changes from all market segments, including potential reserve activation gradients, must comply with the load change gradient of u. To depict the potential reserve activation gradient between two consecutive quarter hours qh and $qh+1$, the change in the flexibility contribution of u is split into the negative and positive part with positive variables. To model the negative reserve direction, the summands for qh and $qh+1$ are swapped, leading to (39) and (40).

$$x^{\mathrm{aFRRpos,U}}_{i,qh,u} - x^{\mathrm{aFRRpos,U}}_{i,qh+1,u} = \triangle x^{\mathrm{aFRRpos,U,+}}_{i,qh,u} - \triangle x^{\mathrm{aFRRpos,U,-}}_{i,qh,u} \quad \forall i, qh, u \tag{39}$$

$$x^{\mathrm{aFRRneg,U}}_{i,qh+1,u} - x^{\mathrm{aFRRneg,U}}_{i,qh,u} = \triangle x^{\mathrm{aFRRneg,U,+}}_{i,qh,u} - \triangle x^{\mathrm{aFRRneg,U,-}}_{i,qh,u} \quad \forall i, qh, u \tag{40}$$

Similarly, the spot market schedule changes for u are split with positive variables in upward and downward direction. As for the overall scheduled dispatch in (29), the day-ahead and the intraday market are considered together.

$$x^{\mathrm{dispatch,U}}_{i,j,k,qh,u} - x^{\mathrm{dispatch,U}}_{i,j,k,qh+1,u} = \triangle x^{\mathrm{spot,U,+}}_{i,j,k,qh,u} - \triangle x^{\mathrm{spot,U,-}}_{i,j,k,qh,u} \quad \forall (i,j,k), qh, u \tag{41}$$

To ensure that the changes in market commitments are not simultaneously non-zero, following $BIGM$ formulations must hold for all the considered market segments. For conciseness, only the constraints for the changes in spot market commitments are presented. Formulations for the reserve segments follow analogously.

$$\triangle x^{\mathrm{spot,U+}}_{i,j,k,qh,u} \le BIGM \cdot \delta^{\mathrm{spot,U}}_{i,j,k,qh,u} \quad \forall (i,j,k), qh, u \tag{42}$$

[10] Although relatively few capacity of renewable sources is prequalified in today's reserve market, this is caused by rather economical than by technical consideration (see e.g., Brauns, Jansen, Jost, Siefert, Speckmann, & Widdel, 2014, for a feasibility study). The main barriers are necessary investments in communication infrastructure and the sheer economics of renewables in providing reserve.

[11] This goes perfectly in line with the feed-in potential based approach to quantify reserve provision potential of intermittent renewable energy sources (Brauns et al., 2014).

$$\triangle x^{\text{spot,U}-}_{i,j,k,qh,u} \leq BIGM \cdot \left(1 - \delta^{\text{spot,U}}_{i,j,k,qh,u}\right) \quad \forall (i,j,k), qh, u \tag{43}$$

The binary variable $\delta^{\text{spot,U}}_{i,j,k,qh,u}$ indicates whether the dispatch changes in upward or downward direction at the transition from qh to $qh+1$. With (39)–(41), the load change constraint in upward and in downward direction for a technical unit u formulates as denoted by (44) and (45). Note, that the chosen formulation with quarter hour resolution of spot market commitments and flexibility contributions to the portfolio's reserve provision implicitly respects the different temporal resolutions of reserve (4 hours), day-ahead (1 hour) and intraday (15 minutes) markets. If these were modeled according to the respective product duration, separate cases for (a) the transition to the first quarter hour of a time slice, (b) the transition to the first quarter hour of an hour, and (c) the transition to an intra-hour quarter hour should be distinguished.

$$\begin{aligned} x^{\text{aFRRpos,U}}_{i,qh,u} &+ x^{\text{aFRRneg,U}}_{i,qh,u} + \triangle x^{\text{aFRRpos,U},+}_{i,qh,u} + \triangle x^{\text{aFRRneg,U},+}_{i,qh,u} \\ &+ \triangle x^{\text{spot,U},+}_{i,j,k,qh,u} - \triangle x^{\text{spot,U},-}_{i,j,k,qh,u} \\ &\leq P_u \cdot \triangle P_u \quad \forall (i,j,k), qh \end{aligned} \tag{44}$$

$$\begin{aligned} x^{\text{aFRRpos,U}}_{i,qh,u} &+ x^{\text{aFRRneg,U}}_{i,qh,u} + \triangle x^{\text{aFRRpos,U},-}_{i,qh,u} + \triangle x^{\text{aFRRneg,U},-}_{i,qh,u} \\ &+ \triangle x^{\text{spot,U},-}_{i,j,k,qh,u} - \triangle x^{\text{spot,U},+}_{i,j,k,qh,u} \\ &\leq P_u \cdot \triangle P_u \quad \forall (i,j,k), qh \end{aligned} \tag{45}$$

Whereas (35) only considers the sheer capacity of u, (44) and (45) emphasize the value of the flexibility, both in upward and downward direction, and make clear that all market segments are competing for the flexibility of the portfolio. For example, during a scarcity of upward flexibility, a reduction of the spot market dispatch releases upward flexibility to be used for other commitments. It enhances the reserve potential (both positive and negative) and creates the option to increase the positive (decrease the negative) reserve contribution of u to the portfolio's reserve provision. Obviously, it is upon the trader to determine a strategy in which market segments and at which price to allocate the available resources.

4.5.3. Stochastic programming constraints

The problem is formulated as a multi-stage stochastic program. However, for one trading decision with the same information (at the same node of the tree), the decisions have to be consistent for the consecutive stages. Therefore, we introduce non-anticipativity constraints for all trading decisions. With $Ord(.)$ defined as the ordinal number of an element in its set and $|.|$ as the cardinality of the set, we formulate the constraints for the positive reserve (46) and the day-ahead market (47) bids.[12]

$$x^{\text{aFRRpos,bid}}_{lp,i,ts} = x^{\text{aFRRpos,bid}}_{lp,i+1,ts} \quad \forall lp, \{i \mid Ord(i) < |I|\}, ts \tag{46}$$

$$x^{\text{DA,bid}}_{lda,i,j,h} = x^{\text{DA,bid}}_{lda,i,j+1,h} \quad \forall lda, i, \{j \mid Ord(j) < |J|\}, h \tag{47}$$

4.5.4. Recourse properties

To get a better understanding of its complexity, it is worth noting some aspects of the mathematical properties of the formulated problem. The problem involves binary recourse, e.g., for the third-stage semi-continuous variables for the controllable unit dispatch, as well as for the third-stage binary variables in the BIGM formulations. Further, building upon the definition in Wets (1974), we can characterize the problem as with fixed recourse. Finally, the formulation with slack variables for the imbalances makes the problem solvable for all feasible first and second stage solutions, so that we are able to characterize it as with relatively complete recourse (Wets, 1974).

[12] Note, that considering only the aggregated bids enables the switch between short volume and dispatch of generation units as scenarios unfold. Formulations for the negative reserve and intraday market bids follow respectively.

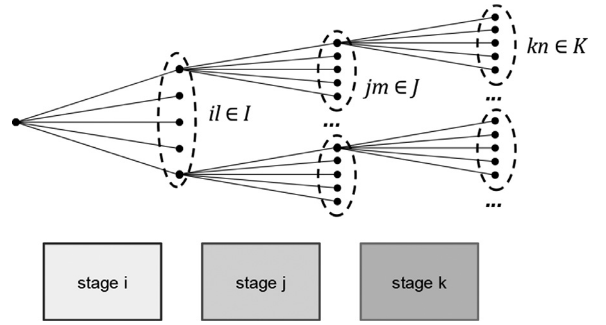

Fig. 3. Illustration of the scenario tree with decision stages i, j and k and scenario sets on each stage. The stages correspond to the market segments balancing reserve, day-ahead spot market and intraday spot market.

4.6. Characterization and modeling of uncertainties

For the methodology to deliver meaningful results, it is essential to describe the uncertainty of the real-world problem as accurate as possible. For the sake of computational tractability, we describe the uncertainty as a finite probability space Ω defined by scenarios ω spanning a scenario tree in three stages with scenarios $il \in I = \{i1, \dots, iL\}$ for stage i, $jm \in J = \{j1, \dots, jM\}$ for stage j, and $kn \in K = \{k1, \dots, kN\}$ for stage k, with counting indices l, m and n from 1 to the cardinality of the respective set on each stage (cf. Fig. 3). As mentioned in Section 4.2, for simplicity the counting indices are dropped, where no confusion can be expected. The first stage scenario branching, denoted by i, represents the uncertainty of the marginal auction price of the reserve market for the positive and negative product. For our models we use information that is available at 8 am on the day before delivery, one hour before the reserve auction takes place.

As mentioned in Section 2, the reserve market prices cannot be completely explained by fundamental drivers. This creates a situation where the reserve market prices are subject to an inherent uncertainty, that especially small market participants are exposed to. The marginal reserve prices[13] are modeled with the help of an additive model and a simulation of the stochastic components applying mean-reverting processes with jump regimes, as proposed by Keles, Genoese, Möst, & Fichtner (2012) for modeling uncertain prices in electricity markets. The additive model explains the marginal prices $y^{\text{reserve}}_{s,w,t}$ and includes the mean seasonal price for the time slice $\bar{y}^{\text{reserve}}_{s,t}$, the day-ahead PV generation forecast x^{PV}_t, the day-ahead residual load forecast x^{RL}_t, and the price of the previous day's auction for the respective time slice y_{t-6}. Further, weekend days and working days are distinguished with a dummy variable δ_w. Combined with seasonal distinction by winter, summer, and transitional season (i.e., spring and fall),[14] six logarithmic models are estimated by the following model equation:

$$\begin{aligned} \log y^{\text{reserve}}_{s,w,t} = {} & c_s + \beta_1 \cdot \log \bar{y}^{\text{reserve}}_{s,t} + \beta_2 \cdot \log x^{\text{PV}}_t + \beta_3 \cdot \log x^{\text{RL}}_t \\ & + \beta_4 \cdot \log y_{t-6} + \beta_5 \cdot \delta_w \\ & + \beta_6 \cdot \delta_w \cdot \log \bar{y}^{\text{reserve}}_{s,t} + \beta_7 \cdot \delta_w \cdot \log x^{\text{PV}}_t \\ & + \beta_8 \cdot \delta_w \cdot \log x^{\text{RL}}_t + \epsilon_t \end{aligned} \tag{48}$$

[13] Precisely, to obtain a smoother time series, the 90 percent quantiles of the accepted bids in each auction are considered as marginal reserve prices.

[14] Winter: December, January, February; Summer: June, July, August; Transition: rest.

Alternative potential fundamental drivers like carbon emission and fuel prices or derivatives like the clean dark spread[15] have not been found significantly improving the goodness-of-fit of the additive model or suffered from strong multicollinearity and were therefore not further considered. We provide model results for model alternatives including the omitted variables in Table 3 in Annex A.1. Please note, that the seasonal mean $\bar{y}^{\text{reserve}}_{s,t}$ to some extent implicitly captures the price influence of the omitted fundamentals.

The series of stochastic residuals ϵ_t is modeled with the help of a mean-reversion process (Ornstein–Uhlenbeck process, see Uhlenbeck & Ornstein, 1930) and jump regimes. Regime switching probabilities and calibration of the mean-reversion process are modeled adapting the procedure introduced by Keles et al. (2012). To determine scenarios for the optimization, the obtained stochastic models are simulated 1000 times and reduced by means of k-means clustering. In Annex A.1, more details for the modeling of the reserve prices as well as a validation are provided. To describe the uncertainty of reserve prices in the case study, the scenario set I for stage i consists of ten scenarios, with probabilities according to the clustering.

The marginal reserve prices $y^{\text{aFRRpos}}_{i,ts}$ (and $y^{\text{aFRRneg}}_{i,ts}$) determine the acceptance for potential reserve bids on price level lp (and ln) for the positive (and the negative) product in scenario set I and time slice ts. For the formulation of the optimization problem, the acceptance of a bid is translated to the binary parameter $\beta^{\text{aFRRpos}}_{lp,i,ts}$ as follows (analogously $\beta^{\text{aFRRneg}}_{ln,i,ts}$ for the negative product):

$$\beta^{\text{aFRRpos}}_{lp,i,ts} = \begin{cases} 1, & \text{if } lp \le y^{\text{aFRRpos}}_{i,ts} \\ 0, & \text{otherwise.} \end{cases} \tag{49}$$

The second stage scenario branching depicts the price uncertainty of day-ahead market prices. This uncertainty is on the one hand driven by potential changes in renewable generation and load forecasts between the gate closure of the reserve market (9 am, d-1) and that of the day-ahead spot market (12 am, d-1). On the other hand, a stochastic component for the day-ahead market itself is modeled. Potential changes in residual load forecasts (including the modeling of renewable generation and consecutively residual load) until gate closure of the day-ahead market, influence of forecast changes and stochastic nature of day-ahead market prices, are modeled with a mean-reversion process (Ornstein–Uhlenbeck process, see Uhlenbeck & Ornstein, 1930), provided with the information at 12 am on the day-ahead (d-1). As in this paper a price-taking trader is modeled, the scenarios consist of price levels and respective probabilities. Analogously to the reserve market, to use the price scenarios in the problem formulation, the price levels are translated into a binary parameter $\beta^{\text{DA}}_{lda,j,h}$ indicating whether a bid on a certain price level lda in hour h is accepted or declined in scenario j. Note, that the second stage is assumed to be independent from the first stage, meaning that the realization of reserve prices (i) has no influence on day-ahead spot market prices.

Based on the second stage the third stage scenario branching captures an updated renewable generation and load forecast under conditional expectation, resulting in an updated residual load forecast. These are sources of quantity uncertainty and result in a price uncertainty with regard to intraday market prices in the quarter hours qh in the scenarios contained in set K, reflecting the information uncertainty 60 minutes from real-time.

To thoroughly model the effects as well as the stochastics of changing residual load forecasts on spot market prices, three different sources of uncertainty are distinguished and modeled: the uncertainty of solar generation, the stochastic component of the residual load (to be interpreted as changes in either wind generation or the system load) and the stochastic component of the day-ahead and intraday market themselves. Note, that wind generation as a source of uncertainty is not modeled explicitly. In comparison to the solar generation, that follows a usual daily pattern and can be categorized into levels relatively straight forward, the wind generation does not follow a usual daily pattern but rather day-specific patterns and is therefore difficult to model with the use of a limited number of type days. We therefore include the forecasting uncertainty of wind generation in the model for the residual load uncertainty. Referring to the idea of type days, in this way we consider for the uncertainty of wind generation but relate these to seasonal-average wind days. For deriving trading decisions in real-world applications, we consider it reasonable to include forecasts for the renewable generation and consequently for the residual load on a daily basis into the approach. In particular, this allows to account for day-specific wind patterns and their risk implications.

The modeling of the solar infeed, the residual load, as well as the spot market prices follows the basic ideas of Keles et al. (2012) and Lingohr & Müller (2019). However, the integrated modeling of mutual dependencies requires an enhanced approach. To capture the character and relations adequately, multivariate mean-reverting processes and stochastic differential equations are estimated based on empirical data and simulated with Monte Carlo.

Based on stochastic simulations and k-means clustering, we derive scenarios for the average working day and the average weekend day across different seasons, as detailed in Russo et al. (2022). Specifically, we identify three seasons (summer, winter, transition) and distinguish between working and weekend days across them. We therefore identify three levels of residual load (low, medium, high) and generate distinct scenario trees for the corresponding 18 day types to be used in the optimization problem. The transition from stage j to stage k is not assumed to be stage-wise independent, but the presented approach deploys conditional expectations to obtain consistent and arbitrage-free scenarios across the stages.

The simulation of the individual parameters is done as accurately as possible. Still, there are small errors left in the simulations. For example, in the case of the average day-ahead price during the transition season, we observe a standard deviation of 7.04 against standard deviations in the simulated series ranging from a minimum of 6.38 to a max of 7.62 across the three scenarios. Furthermore, for scenarios in stages j and k, different probabilities are assumed: the scenarios in the range up to one standard deviation (e.g., {j1, j2, j3}) from the mean are assumed to be equally probable, with a probability of 25%. Equal probability of 12.5% is also assumed for the scenarios above and below one standard deviation from the mean (e.g., {j4, j5}), thus resembling probabilities drawn from a normal distribution. Nonetheless, this does not imply that possible errors cumulate: For instance, the error of price simulation implicitly includes the error of the solar PV simulation since this simulation is one input to the price simulation. Since the individual errors are in line with those reported in the literature (e.g., Keles et al., 2012), we follow similar approaches proposed in the stochastic modeling literature and use these simulations as input to the stochastic optimization model (e.g., Fleten & Kristoffersen, 2007; Green, Staffell, & Vasilakos, 2014; Osório, Lujano-Rojas, Matias, & Catalão, 2015; Zhang, Hu, Cheng, Zhang, Hong, & Gu, 2021, and references therein). For the case study, the scenario sets on stage j and k contain five scenarios each. These are derived with probabilities according to the assumed normal distribution of the σ-ranges[16] in the scenario definition. Analogously to the previous

[15] The clean dark spread is defined as the difference between the spot market price and the costs for fuel and emission certificates of a hard coal power plant with typical efficiency.

[16] The scenarios are derived from the expected mediod value of the stochastic simulation by deviating defined multiples of the standard deviation for each time

market segments, to use the price scenarios in the problem formulation, the price levels are translated into a binary parameter $\beta^{\mathrm{ID}}_{lid,j,k,qh}$ denoting whether a bid on a certain price level *lid* in quarter hour *qh* is accepted or declined. Combined with the 10 reserve price scenarios in set *I*, each of the type days' scenario tree considers 250 possible scenarios to characterize the uncertainty from the day-ahead towards real-time.

An illustrative scenario tree to demonstrate the extent of the price uncertainty in the considered segments and the solar generation uncertainty estimated with the stochastic modeling based on empirical data is provided for the typeday *tra*2[17] in the Annex A.1 and A.3 in Figs. 15–17.

5. Case study

In this section, we conduct a case study in which we apply the proposed methodology to an exemplary power plant portfolio in the Germany market design setting. Germany with its high renewable shares, high data availability, and liquid day-ahead and intraday trading is particularly suitable to demonstrate our approach. We present characteristics of optimal bidding strategies under different considerations of risk. To define risk preferences, we enumerate combinations of the parameter α at levels 85%, 90%, and 95% (i.e., consideration of the 15%, 10%, and 5% worst cases) and λ at levels 10%, 25%, and 50% (i.e., the weighting of the CVaR on α-level compared to the expected value of contribution margins). Where necessary, for the results to remain concise, we present mainly the results for the risk strategy with $\alpha = 95\%$ and $\lambda = 50\%$, corresponding to a strong risk aversion. As a benchmark, we present results for the risk-neutral case (i.e., $\lambda = 0\%$). The complete results for the other strategies are provided in the supplementary material.

5.1. Data and practical implications

The German reserve market results are published by regelleistung.net (2021), the spot market results are published by (EPEX Spot, 2021). As no historical feed-in forecast data other than the ones from ENTSO-E (ENTSO-E, 2021) are available publicly, the generation forecast for the PV plants in the trader's portfolio are assumed to be perfectly correlated with the system-wide generation forecast, in line with the assumption in Russo & Bertsch (2020). This implies a spatial dispersion of the PV plants that is representative for the overall German PV portfolio.[18] Hereby, the stochastic process for the PV intraday forecast update is fed by two inputs to model the information available to the trader as accurate as possible: firstly, the intraday forecast $\phi^{\mathrm{ID}}_{qh,res}$ update for the respective renewable plant *res* and quarter hour *qh*, provided by ENTSO-E in the morning of the trading day. Secondly, to account for the latest available information at the time of the intraday decision on stage three, the forecast error between the intraday forecast for $qh-2$ and the (meanwhile available) realisation of $qh-2$, denoted by $x^{\mathrm{real}}_{qh-2,res}$, is considered. Including the latest realised quarter hour $qh-2$ enables to capture the latest information on PV generation in the decision. We stress at this point, that the difference $(\phi^{\mathrm{ID}}_{qh,res} - x^{\mathrm{real}}_{qh,res})$ is strongly auto-correlated at a lag of two.

With regard to the price risk, the stochastic price process captures effects of intraday renewable generation and load forecast updates. As mentioned in Section 3, a simplification is applied by taking the price index p^{ID3} (published by EPEX Spot, 2021), the weighted average price of all trades closed in the last three hours before the delivery period. The selection of an appropriate recent time horizon of data to estimate the models turns out to be challenging, as both the spot markets and the reserve markets in Germany went through several adjustments in recent years. We therefore use data from the time horizon between July 2019 and the first COVID19-caused lockdown in March 2020 to estimate the stochastic price processes. In this period, we observe stable market circumstances and no structural changes on either the demand or the supply side.

5.2. Composition of portfolio

The portfolio, for which the presented methodology is particularly relevant, should meet certain characteristics regarding the cost-structure. In order to compete with the reserve market prices, the variable costs of the portfolio should be close to average spot market price levels. Just & Weber (2015) refer to the relation between variable costs and spot market price levels as marginality and distinguish between infra-marginal (i.e., $\kappa^{\mathrm{var}} < p^{\mathrm{spot}}$), marginal (i.e., $\kappa^{\mathrm{var}} = p^{\mathrm{spot}}$) and extra-marginal (i.e., $\kappa^{\mathrm{var}} > p^{\mathrm{spot}}$) power plants. In this terminology, the portfolio should change between being infra-marginal, marginal, and extra-marginal for different time steps and scenarios.

For the case study, we consider a portfolio consisting of a set of electricity-led and thus controllable biogas power plants, and a set of PV plants as intermittent renewable source to be suitable to reflect the strengths of the presented approach. Firstly, it depends upon both, the uncertainty of generation quantities (relevant for *res* units) and the uncertainty of price levels (relevant for both *u* and *res* units). Secondly, the variable costs $\kappa^{\mathrm{var}}_{u1}$ are roughly in the range of price variations of the spot markets. Finally, the portfolio is able to provide flexibility in both upward and downward direction or to use the flexibility to profit from spot market prices.

The portfolio for the case study is therefore defined as presented in Table 1. For conciseness, the plants are aggregated and handled as single unit *u*1 and *res*1, respectively.[19]

5.3. Evaluation of trading strategies for the portfolio

As the results for all investigated 18 type days are very extensive, we present the results for an exemplary type day in detail, and based on that we discuss the findings more generally. We present the results for a weekday with a medium level of PV generation and a medium residual load level in the transition season, in the remainder referred to as typeday *tra*2.

Before investigating the decision variables as well as the rationales and characteristics behind the different trading strategies, we first look at the values of the target function and the overall results. To visualize the terms contained in the target function with risk consideration (i.e., expected contribution margins and CVaR),

step and type day respectively. To derive five scenarios, the expected mediod (denoted with j1) is complemented by the values $+/-\sigma$ (denoted with j2/j3) and the values $+/-2\cdot\sigma$ (denoted with j4/j5) with probabilities according to an assumed Normal distribution. This leads to $pr_j = 0.25$ for *j*1, *j*2, and *j*3, and $pr_j = 0.125$ for *j*4 and *j*5. Same applies for the scenarios on stage *k*.

[17] A working day with medium PV feed-in and medium residual load in the transition season.

[18] However, in the real-world (commercial) application, the availability of the required portfolio-specific forecast data would be no limiting factor.

[19] The aggregation of several units to one unit also makes the problem better tractable, as the number of decision variables and constraints grows proportionally with the number of units in set *U*. The main limitation associated with this is that if multiple smaller units are considered, the minimum load requirement (P^{min}_u) could be handled more flexibly. Further, in case the investigation concentrates on part-load efficiency of single units, a split into single units and a non-linear or piece-wise linear consideration is necessary. For the scope of this paper, the aggregation is considered acceptable.

Table 1
Composition of plant portfolio and techno-economic parameters for case study.

Parameter	Symbol	Unit	Value
Installed capacity PV generation *res*1	P^{RES}_{res1}	[MW]	100
Maximum load change of *res*1 within 5 minutes, as share of P^{res}_{res1}	$\triangle P^{RES}_{res1}$	[−]	1.00
Installed capacity controllable generation *u*1	P^{U}_{u1}	[MW]	100
Minimum load requirement of *u*1, as share of P^{U}_{u1}	P^{min}_{u1}	[−]	0.20
Minimum daily generation of *u*1, as share of baseload operation at P^{U}_{u1}	ν^{min}_{u1}	[−]	0.50
Maximum daliy generation of *u*1	ν^{max}_{u1}	[−]	0.95
Variable costs of *u*1	κ^{var}_{u1}	[EUR/MWh]	40
Maximum load change of *u*1 within 5 minutes, as share of P^{U}_{u1}	$\triangle P^{U}_{u1}$	[−]	0.50

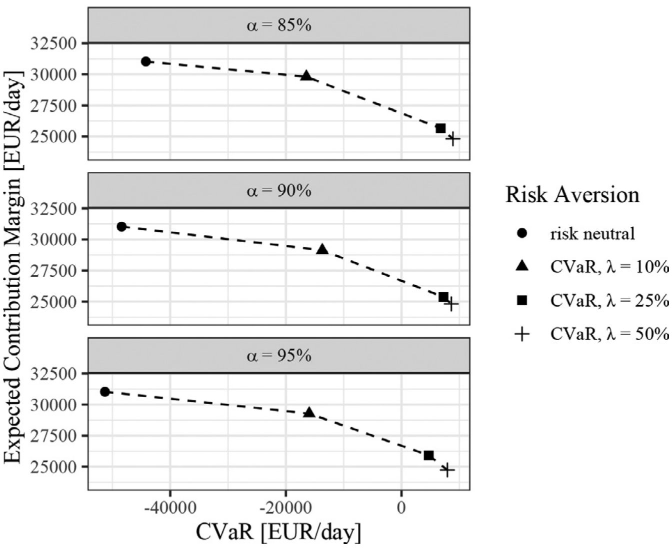

Fig. 4. Efficient frontier of trading decisions for different levels of CVaR interval (α) and risk aversion (λ), medium weekday transition season (tra2).

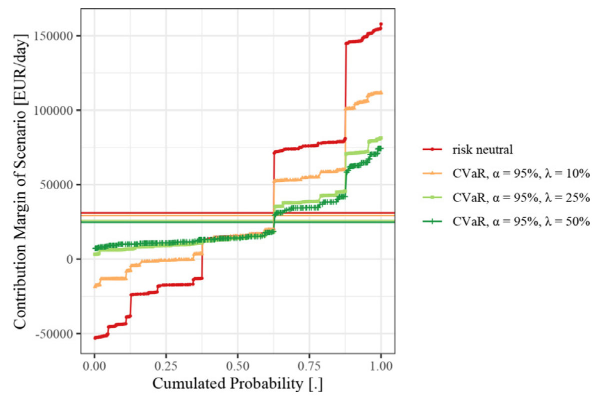

Fig. 5. Empirical cumulative distributions of contribution margins throughout scenarios for trading strategies considering the $\alpha = 95\%$ level for the CVaR, medium weekday transition season (tra2).

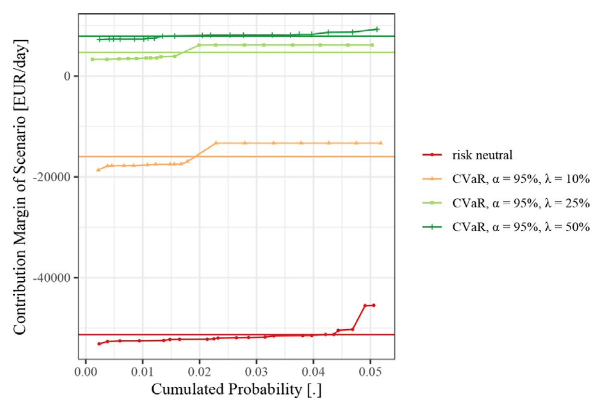

Fig. 6. Empirical cumulative distributions of contribution margins in the 5% of worst case scenarios (CVaR range), medium weekday transition season (tra2).

we define the efficient frontier as all combinations of expected contribution margins and CVaR for trading strategies, that were found not to be dominated by another set of decision variables.

Fig. 4 presents the efficient frontier plot for all considered combinations of α and λ, as well as for the risk-neutral case. Overall, a concave shape of the efficient frontier can be observed for all α levels. Obviously, the consideration of risk decreases the expected value of the trading decision, and at the same time with an increasing weight of the risk metric the risk exposure is reduced. Noteworthy, the increase in λ from risk-neutral (0%) to 10% comes with a large impact, likewise the increase from 10% to 25%. In the former, the expected contribution margins decrease considerably (decrease of 1228–1903 EUR/day), whereas the CVaR increases strongly (27,763–35,330 EUR/day), indicating a lower risk exposure. In the increase from 10% to 25%, the expected contribution margins decrease stronger (decrease of 5110–5659 EUR/day compared to risk-neutral), but yields further benefits with regard to the risk exposure (CVaR increase of 51,000–56,003 EUR/day compared to risk-neutral). However, the increase in λ from 25% to 50% has a comparably small impact. The reduction of the expected contribution margins outweighs the small changes in risk exposure. Table 4 in Annex A provides an overview over the numbers determining the efficient frontier. We stress at this point, that none of the defined and evaluated risk strategies strictly dominates another. In a real-world application, a trader would still be required to choose the risk preference, yet based on information about the opportunities and risks related with the trading decision. In order to gain insights about the distributional patterns of the contribution margins in the different scenarios, we arrange the contribution margins in increasing order for each trading strategy. This visualization can be considered as an empirical cumulative distribution function (ECDF) of a distribution and reads as follows: The contribution margin is lower or equal to the value on the vertical axis with the probability level indicated by the horizontal axis. To be able to identify fat tails in the distribution, a second plot zooms into the interval considered for the risk metric CVaR. The expected value of the contribution margin and the CVaR are respectively included into the plots as horizontal lines. Figs. 5 and 6 show the EDCF for the strategies with $\alpha = 95\%$. One intuitive result derived from the visualization of the risk distribution is that the consideration of risk drastically lowers the spread between the worst and the best cases and thus narrows the distribution, at the expense of a moderate loss in expected contribution margins. Evaluations for the other α-values show analogous patterns.

Investigating the contribution of each market segment to the portfolio's contribution margins in the different scenarios for different trading strategies, we find the main source of risk and the

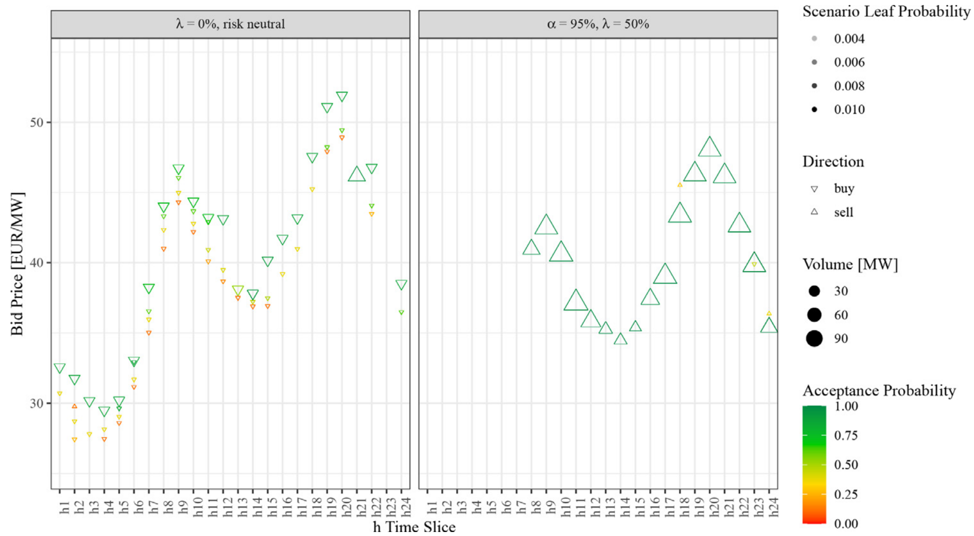

Fig. 7. Day-ahead market bids for risk-neutral strategy, medium weekday transition season (tra2). day-ahead market bids for risk-averse strategy ($\alpha = 95\%, \lambda = 50\%$), medium weekday transition season (tra2).

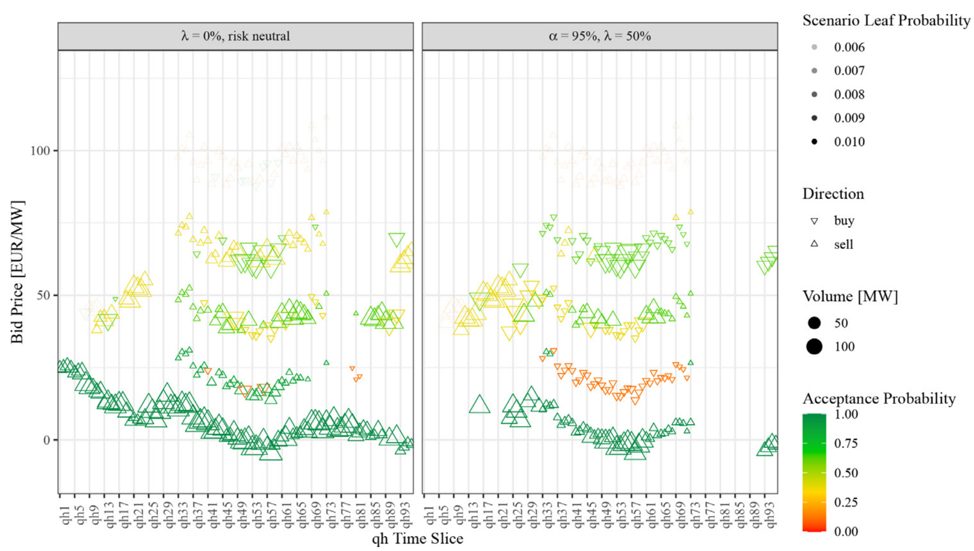

Fig. 8. Intraday market bids for risk-neutral strategy, medium weekday transition season (tra2). Intraday market bids for risk-averse strategy ($\alpha = 95\%, \lambda = 50\%$), medium weekday transition season (tra2).

major lever of risk hedging strategies to lie in the spot market decisions. To present key differences of risk-neutrality and risk aversion in trading strategies, Figs. 7 and 8 show the trader's optimal bids on the day-ahead market and intraday market for the risk-neutral and the strongly risk-averse strategy ($\alpha = 95\%, \lambda = 50\%$). As these bids are submitted on the second (day-ahead) and third (intraday) stage of the problem, we summarize the bids for the representative realization $i1$ of the first stage. Note that the actual bids submitted on a stage do not anticipate realized information from later stages and are thus consistent for all j and k for the day-ahead and the intraday market, respectively (see Eqs. (46)–(47)).

Fig. 7 shows the day-ahead market bids for the risk-neutral strategy. The expectation of potentially higher prices in the intraday market yields buying bids, indicating the trader is willing to take a long position. These bids are in the range of the trader's allowed long/short position range, and are not linked to the portfolio dispatch. The risk-neutral trader seeks to sell the electricity generation on the intraday market, that offers (in expectation) a slightly higher, yet more volatile price level, and even increases a high selling position to sell more volume than the generation capability. On the other hand, the risk-averse strategy focuses on reducing the risk exposure early on by selling most of the generation on the day-ahead market at secure yet (in expectation) slightly lower prices. Notably, the day-ahead trading volumes of the risk-averse trading strategy are considerably higher. Further, the risk-averse strategy contains ask bids on price levels below the variable costs of the dispatchable unit u, allowing to re-buy generation that was sold in the day-ahead market at a higher price. The risk-neutral intraday bids contain less bids to re-position the portfolio, but rely on the (in expectation) higher prices on the intraday market.

The submitted bids on the day-ahead market and the eventual market result determine the position of the trader when facing the intraday stage. Fig. 8 shows the submitted intraday market bids, that follow the respective day-ahead market bids for the risk-neutral and the risk-averse trading strategy. It can be observed in the lower string of bids, that driven by the pre-positioning (long position) from the day-ahead market, the risk-neutral strategy mainly consists in selling the electricity at any price on the in-

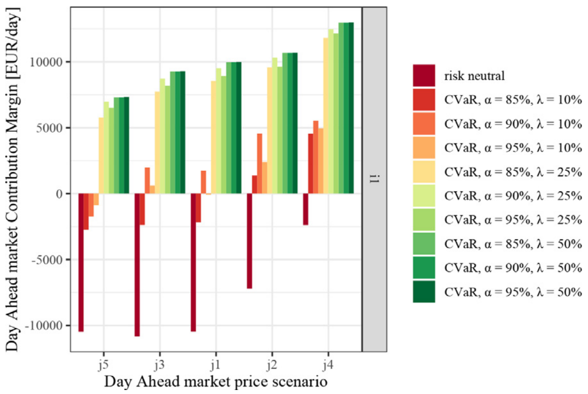

Fig. 9. Diagram of contribution margins on the day-ahead market for realizations of scenario j as successor of $i1$ under different risk attitudes.

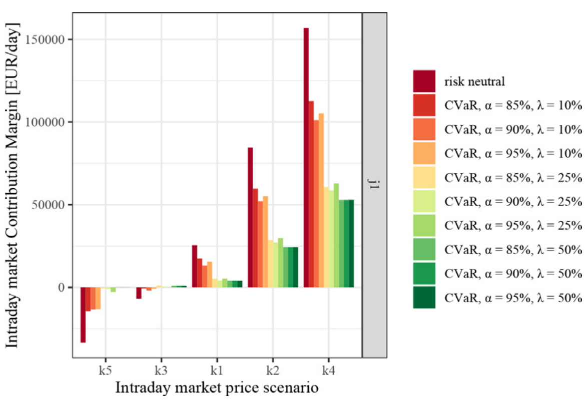

Fig. 10. Diagram of contribution margins on the intraday market for realizations of scenario k as successor of $i1$ and $j1$ under different risk attitudes.

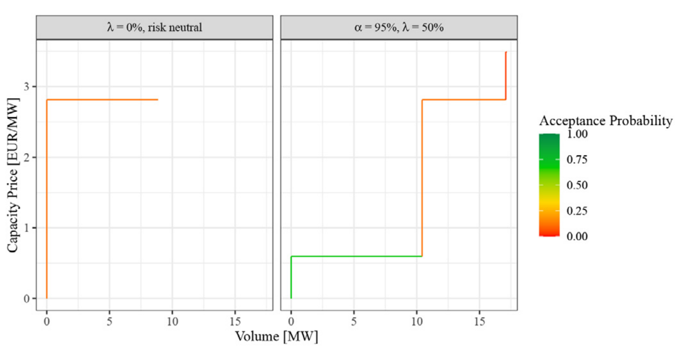

Fig. 11. Reserve market (aFRRneg) bids for the time slice 12–16 hours for the risk-neutral strategy and for a risk-averse strategy ($\alpha = 95\%$, $\lambda = 50\%$), medium weekday transition season (tra2).

traday market. In case the intraday market settles at a higher price level, these bids will be remunerated with the higher price, too, which stands in opposition to bids on the reserve market. It can further be observed, that a considerable amount of bids is placed on price levels, that are linked to a low acceptance probability.

For the representative scenario realizations $i1$ and $j1$, Figs. 9 and 10 present the resulting contribution margins from the day-ahead and the intraday market for the considered trading strategies. As discussed before, the risk-averse bids prevent the trader from taking a long position on the day-ahead market and result in profitable sales, which to some extent depends on the realization of j. On the other hand, the risk-neutral ask bids lead to considerable costs (i.e., negative contribution margins), and no positive contribution margins on the day-ahead market. The value of the long position then strongly swings with the realized intraday market scenario, leading to further negative contribution margins if prices are low ($k5$ and $k3$) and to very profitable sales if prices are high ($k2$ and $k4$).[20] Inspecting the successors of the lower ($j3$ and $j5$) and higher ($j2$ and $j4$) day-ahead market price scenarios (see Fig. 9) yields the same relations yet differently pronounced. The remaining strategies with a moderate risk-aversion follow a compromise.

The day-ahead bids do not contain ask bids to profit from potentially higher prices in the intraday market, yet a share of the (expected) portfolio generation is offered only at high price levels with low acceptance probabilities on the day-ahead market. Such a mixed bidding behavior allows for both, profiting from secure day-ahead market revenues as well as profiting from higher or lower prices in the intraday market. It avoids excessive ask or sell pressure in the intraday market and provides a balance between the advantages of the day-ahead market (low uncertainty, in expectation lower prices) and the ones of the intraday market (high uncertainty, in expectation higher prices). Whether and which bids are submitted to the reserve market is determined by the opportunities the reserve market and the following spot markets offer and at which costs in terms of operational constraints, such as scheduling restrictions and inflexibility, the reserve can be provided by the portfolio. The opportunities are three-fold: First, the pay-as-bid remuneration opens up a strategy space to decide among different reserve price levels and volumes to bid on them. Hereby, the rationale is to avoid the winner's curse introduced to economic theory by Thaler (1988). Second, the expected level of spot market prices and thus the expected profits require to consider which market is more profitable. Third, the opportunity to profit from price variations between the day-ahead and the intraday spot market by flexibly adjusting to new information.

It therefore occurs, as in the case of the considered typeday $tra2$, that the spot market opportunities dominate the reserve market prices for the positive direction and no bids are submitted to the positive reserve segment. Investigating the economics of the reserve market segments (see Just & Weber, 2008; 2015, for formal descriptions of the interplay between spot and reserve markets), it becomes clear that providing positive reserve competes with spot market operations, whereas providing negative reserve can be considered a complementary element to spot market operations,[21] and brings only little operational restrictions. A main finding is therefore, that the reserve bids determined with the stochastic optimization differ fundamentally for negative and positive reserve provision.

The bids submitted for the positive reserve, if any are submitted, are placed only on high price levels. In case of a high reserve price scenario, the trader profits from the accepted bid(s). In case the submitted bids are rejected, the following spot markets offer similar profit opportunities. This rationale is further confirmed by the pay-as-bid remuneration, as it incentivizes riskier reserve market bids. On the other hand, the bids submitted for the negative reserve are not balancing potential reserve market profits with potential spot market profits. The rationale aims at balancing the opportunities among the reserve price levels. Therefore, more diversification in bids and patterns for different risk strategies can be observed (see also Fig. 20 in A for an illustration). Additional re-

[20] Note that the scenarios are not equally probable. As mentioned in Section 4.6, the probability of the moderate scenarios (three scenarios in the center of the plot) is 25% each, and the probability of the outer scenarios is 12.5% each.

[21] This holds for the case that spot prices are higher than the variable costs. For the case that spot prices are below the variable costs, depending on operational restrictions such as minimum fuel consumption, the revenues for providing negative reserve must compensate for the unprofitable spot market operation of the minimum run capacity and the offered reserve volume. However, the revenues for providing of positive reserve must only compensate for the minimum run capacity.

sults, such as detailed bidding curves for typeday *tra*2, are provided in Annex A.4. To present results for another typeday, we also provide the bids for the negative reserve for a summer weekend day with high residual load (*sum*6, see (Figs. 22 and 23). On this typeday, the diversification of bids is particularly pronounced and underlines the presented findings. Regarding to the problem size, solver parameters and computational performance, we found the following configuration to be suitable. In its reduced form, the problem for a single type day and risk strategy has 720,000 restrictions (inequality/equality), 560,000 variables (230,000 binary), and 2,000,000 coefficients. The problem was implemented in GAMS and solved with the CLPEX solver, applying parallel mode with 36 threads and a MIP gap of 0.01. Using a computer equipped with an Intel Xeon Gold 6248R (3.0 gigahertz, 24 Cores, 48 Threads) and 64 gigabytes RAM, the solution time with cold start amounts to roughly three hours for a single bidding strategy. However, providing the solver with the solution of a similar strategy (warm start), the solution time can be reduced to below 30 minutes for a single bidding strategy. Further, for days with strictly dominant markets (i.e., the spot market offers higher profit opportunities than the reserve markets), the solver determines within a matter of minutes. Sensitivity runs with only five instead of ten reserve price scenarios reduced computational more than proportionally. Further sensitivities splitting the controllable unit up into five and ten controllable units increased the computational time considerably, yet within tractable ranges. Considering the structure of the problem and the curse of dimensionality, this finding is intuitive.

5.4. Discussion

The results presented for the case study and the analysis of underlying bidding rationales allows to derive general conclusions with regard to decisions under uncertainty in the sequential electricity market context. The first conclusion we draw is that trading more and thereby taking a position at an earlier stage can reduce the risk significantly. Intuitively, placing reserve bids at low prices with high acceptance probability and selling generation on the day-ahead market sacrifices profit opportunities but secures revenues early on.

Measures to increase the expected profit and the associated risk exposure include (a) betting on high reserve prices, (b) betting on intraday prices higher than the day-ahead prices (no/few reserve bids, selling offers only at high price levels or even buying to go long on the day-ahead market), and (c) betting on intraday prices lower than the day-ahead prices (no/few reserve bids, selling offers at low price levels or even selling to go short on the day-ahead market). It can be concluded that the intraday market offers the highest risk, but also the highest reward, and that the main task of a trader is to balance these out by participating in all markets.

Therefore, a reasonable strategy appears to determine the operation decision based on the reserve and the day-ahead market results, but to not sell all generation capacity and flexibility on the day-ahead. In that way, one can profit from opportunities of higher or lower prices in the intraday market and the portfolio risk is reasonably hedged. However, the discussion remains without one strictly dominant strategy as this is no one-size-fits-it-all case. The faced uncertainties and the myriads of potential decisions in the markets require a sound decision support.

At this point, we critically reflect that further efforts can improve the developed approach in the future. On the one hand, the neglected reserve market segments as well as activation of reserve energy could be included to the approach. Introducing more decision stages might also necessitate investigating time consistency for the consideration of risk (cf. Homem de Mello & Pagnoncelli, 2016). On the other hand, the representation of the intraday market as a single uniform pricing auction neglects arrival processes of prices in continuous trading and therefore potential repositioning profits. Further, trading strategies for larger portfolios with the potential to change market prices should adequately account for the price effects of the submitted bids in each market segment. Moreover, the degree of technical representation of the controllable units could be improved by considering part-load efficiencies and by not aggregating the units in the portfolio as it is done in this paper. However, for the purpose of modeling the interrelated uncertainties and trading decisions in a complex setting as presented in this study, these simplifications are considered necessary to remain able to identify the central implications and to keep the model mathematically tractable.

Finally, we acknowledge that the simulations are based on empirical data and therefore do not contain unprecedented levels or dynamics of and across the parameters. While we agree that in energy markets in general and the electricity market in particular there can be turbulent and unprecedented price movements over the run of several weeks or months, with a day-to-day perspective the forecasting and modeling of uncertainties have strongly advanced by the use of empirical data and is commonly applied for accurate short-term trading decisions. In this regard, the selected approach is considered the best available for the scope of this research.

6. Conclusions and outlook

The methodological framework proposed in this study aims to cope with the uncertainty of trading in electricity markets, and considers the sequential clearing of the reserve, day-ahead and intraday markets. The main source of uncertainty for a trader in the short-term is induced by high shares of intermittent renewable generation. The German market setting is used as a representative case, as it is already characterized by a high renewable share and a sequence of markets that is typical for many electricity market designs in Europe and other parts of the world. The developed methodology models the uncertainty related to the trading decisions of a trader on the day-ahead towards real-time on three decision stages (balancing reserve market, day-ahead spot market, intraday spot market) with the help of scenario trees and under conditional expectation. We present a multi-stage stochastic optimization approach that allows to determine optimal bids for the sequence of the reserve market, the day-ahead spot market, and the intraday spot market. Thereby, we consider all market segments that might appear as opportunities to each other.

The approach provides valuable insights with regard to profit distributions under uncertainty and allows for an extension of the target function to include risk. As the characterization of uncertainty contains much more information than a risk-neutral optimization (maximizing the expected value) can capture, we evaluate trading strategies with different risk preferences. An efficient frontier is derived as the set of optimal and non-dominated tuples of expected profit and risk exposure of the considered trading strategies. We discuss trading implications for the individual market segments, but most importantly for their interplay. Amongst others, the results lead to the conclusion that risk-hedging trading strategies prefer securing revenues on earlier stages and thereby being independent of the more volatile prices on the intraday stage. Risk can be effectively reduced by placing reserve bids on lower price levels and selling generation mostly on the day-ahead spot market. However, in that way potentially higher revenues in the reserve market and on the intraday stage are disregarded as they inevitably increase the risk exposure of the revenues. We provide new insights to short-term market decisions under uncertainty, that are interesting for several stakeholder groups, such as traders, policy makers, and research.

Taking the developed approach, next steps could go in the following directions. Firstly, traders may seek the commercial application to a real-world portfolio. This implies using portfolio-specific forecasting information distinct from the overall system renewable generation, and integrating fuel and carbon prices as well as scenario trees on a daily basis instead of using typedays.

Secondly, policy makers may be interested in the individual behavior of participants in short-term markets in extreme events, such as scarcity scenarios that push the system to the limits, or future electricity systems based on renewable generation. Such scenarios are not covered by our data and scope. However, coupling our approach with other electricity market models as well as insights from recent scarcity events (e.g., in France or Texas) offer a solid basis to model these uncertainties and to assess policy implications. Thirdly, electricity storage and sector-coupling with other energy carriers such as hydrogen may be included in the approach, interesting for both traders and scholars. This extension implies a temporal coupling and complicates the determination of optimal bids, particularly if considering reserve energy activation for storages.

Finally, an extension of the presented approach towards investment appraisal based on uncertain revenue streams from multiple markets appears to become more and more relevant. Especially considering the increasing necessity of flexibility and its optimal use for the energy system, all stakeholder groups are interested in methods to assess investment options more sound than based on established valuation approaches.

Acknowledgments

We thank Christian Füllner for the insightful discussions. The authors acknowledge the financial support of the German Federal Ministry of Education and Research under grant number 03SFK1F0-2 and of the German Federal Ministry for Economic Affairs and Energy under grant number 03SIN120. This work has emanated from Marianna Russo's research time at the Economic and Social Research Institute, Dublin. Marianna Russo acknowledges funding from the ESRI's Energy Policy Research Center and the SFI Energy Systems Integration Partnership Programme (ESIPP) number SFI/15/SPP/E3125. The opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the Energy Policy Research Center and Science Foundation Ireland. All errors remain our own.

Appendix A. Annex

A1. Modeling of reserve prices

For each season and reserve direction, a separate model is estimated from empirical data. As an example, Table 2 presents the coefficients, standard error and *t*-value of the robust estimation for the negative product in the transition season. The standard errors

Table 2
Coefficients for robust estimation of negative reserve product in transition season. All coefficients are significant.

Coeff.	Value	Std. error	*t*-value
c_s	2.250	0.316	7.130
β_1	0.469	0.085	5.550
β_2	−0.066	0.020	−3.355
β_3	−0.628	0.077	−8.195
β_4	0.679	0.016	42.725
β_5	4.508	0.478	9.436
β_6	−0.428	0.146	−2.944
β_7	0.076	0.037	2.056
β_8	−1.169	0.117	−10.001

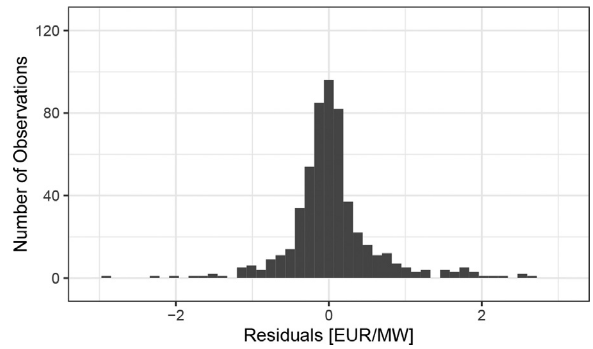

Fig. 12. Residuals of the additive model estimation for the negative reserve product and transition season. The center, the outliers in upward direction and the outliers in downward direction are covered by the base regime, the upward jump regime and the downward jump regime, respectively.

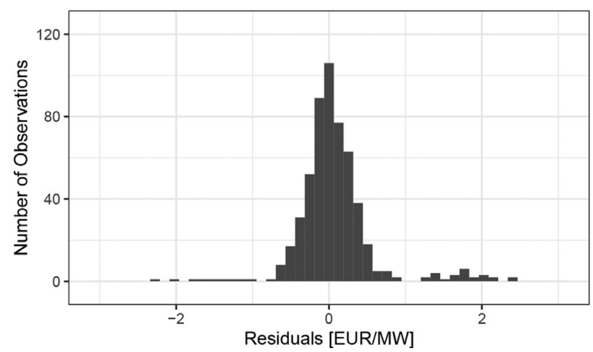

Fig. 13. Residuals simulated with the stochastic process for the negative reserve product and transition season. The modeling yields a good fit to the distribution of the residuals in Fig. 12. The strict definition of the regimes leads to a slight under-representation of values around two standard deviations away from the mean.

Table 3
Goodness-of-fit of the additive reserve price models with residual standard error of the robust estimation as a measure for model fit for the used model, alternative 1 (carbon price as additional explanatory variable), alternative 2 (coal price as additional explanatory variable), alternative 3 (clean dark spread as additional explanatory variable), and alternative 4 (carbon price and coal price as additional explanatory variables). No significant improvement of model fit by including additional explanatory variables is observed. Required data for currency exchange rates, carbon and coal prices taken from EPEX Spot (EPEX Spot, 2021) and investing.com (investing.com, 2021).

Season	Direction	Used model	Alt. 1	Alt. 2	Alt. 3	Alt. 4
Winter	negative	0.372	0.356	0.372	0.358	0.383
Transition	negative	0.298	0.301	0.302	0.297	0.297
Summer	negative	0.383	0.390	0.373	0.408	0.372
Winter	positive	0.360	0.356	0.358	0.353	0.353
Transition	positive	0.576	0.563	0.546	0.550	0.560
Summer	positive	0.534	0.536	0.545	0.538	0.520

and *t*-values of all coefficients suggest high significance. The residuals of the estimation ϵ_t are considered as a stochastic process, which consists of three regimes: A base regime as well as a downward and an upward jump regime, with jump regime observations defined as observations more than two standard deviations away from the mean. Fig. 12 presents the residuals of the conducted robust estimation for the negative product in the transition season.

Table 3 presents the residual standard errors to compare the used model configuration to less parsimonious ones. It can be observed that the overall model fits support the literature and the hypothesis that reserve prices are not completely explainable by the use of fundamental drivers as the residual standard errors are considerable. Yet, the model fit does not suffer significantly from only considering five explanatory variables (seasonal average, solar generation, residual load, lag 6, and a dummy for the distinction of weekdays and weekends). This observation remains valid

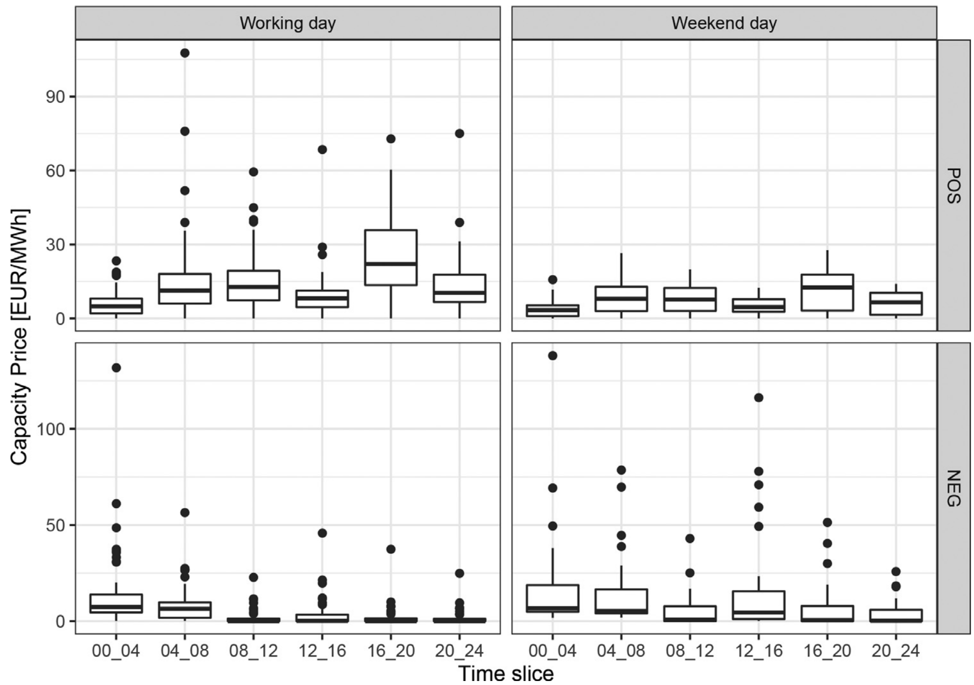

Fig. 14. Boxplots of empirical reserve prices in the transition months of 2019, distinguished by reserve product and type of weekday ($n = 1092$).

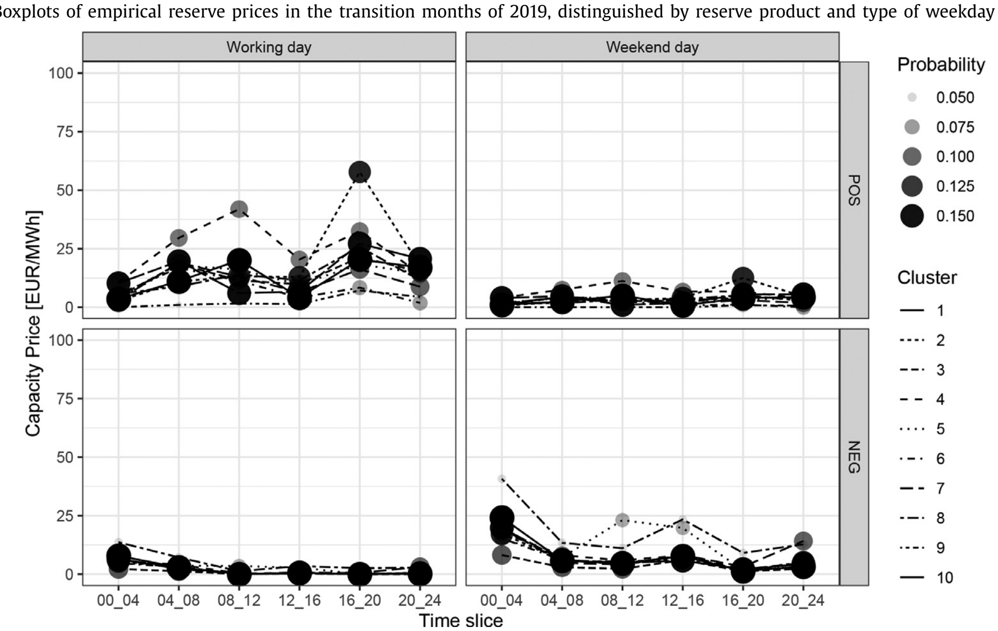

Fig. 15. Scenarios derived from the stochastic modeling for the transition season on medium levels of PV generation and residual load, distinguished by reserve product and type of weekday. Reserve price scenarios for typeday *tra*2 correspond to the left column.

for all seasons and both directions. Therefore, the stochastic residuals are estimated based on the parsimonious model configuration presented in Section 4.6.

Besides the distributional also the auto-regressive characteristics need to be modeled for the process to yield sound simulation results. Therefore, an Ornstein-Uhlenbeck process is estimated for the base and the jump regimes. Further, regime switching probabilities are derived from the historical data and used in a simulation of the stochastic residuals. Fig. 13 presents the results of one simulation of the residuals.

In total, 1000 stochastic residual time series are simulated. These time series are used in the simulation of the reserve price scenarios for the respective type days. In accordance with Russo et al. (2022), three PV generation and coherent residual load levels are distinguished for each season and type of weekday, resulting in 18 type days in total. The levels of the exogenous variables and the stochastic residuals for the respective type day are fed into the additive model. Note, that the lag y_{t-6} enters the model as exogenous variable. To obtain a steady process with the stochastic components respected for accordingly, therefore the last step consists in simulating 15 days of each day type with the 1000 stochastic residual time series, respectively. Finally, the 1000 observations of the 15th day are clustered with k-means clustering ($k = 10$), to obtain the reserve price scenarios used in the stochastic optimization.

Figs. 14 and 15 present the empirically observed values for the transition months and the scenarios derived from the stochastic modeling for the days with medium forecasts for PV generation and residual load. Note, that on the one hand, with variation to low and high levels the reserve price levels become more pronounced. On the other hand, especially days with steep ramps of wind generation are not modeled in the stochastic process but are well contained in the empirical data. The main purpose of the scenario generation is to derive typical days and consistent reserve price patterns. In a real-world application, one would use day-ahead forecasts for the exogenous variables instead of seasonal averages and their variations in upward and downward direction.

A2. Definition and modeling of risk

The VaR η is defined as the (1-α)-quantile of the contribution margin distribution, leading to the following definition for a discrete probability distribution:

$$\mathrm{VaR}(\alpha, x) = \max\left\{\eta : P(\omega | f(x,\omega) < \eta)\right\}, \quad \forall \alpha \in (0,1) \tag{50}$$

with f the distribution of contribution margins, the deterministic parameters x and the stochastic parameters ω in probability space Ω. Including the VaR into the developed approach requires an extension of the model described in the previous sections. However, it is mainly a modification of the target function, two additional sets of constraints and auxiliary variables added to consider for the calculation of the VaR. The target function is augmented by the variable η, that corresponds to the VaR. The target of optimization is now the weighted sum of expected value and the VaR, with $\lambda \in (0,1)$ as parameter for risk aversion (e.g. $\lambda = 0.2$).

$$\max \quad (1-\lambda) \cdot \mathbb{E}_{(i,j,k)\in\Omega}(\pi_{i,j,k}) + \lambda \cdot \eta \tag{51}$$

All constraints from above remain unchanged. In addition, the following two constraints are included in the model. Parameter α represents the probability level of the VaR measure (e.g. $\alpha = 0.95$), $\theta_{i,j,k}$ is a binary variable equal to 1 if the contribution margin $\pi_{i,j,k}$ in scenario (i,j,k) is lower than η and equal to 0 otherwise. With the means of (52) and (53), we ensure that with a probability of $1-\alpha$ percent the contribution margin is lower or equal η.

$$\sum_{i\in I} pr_i \sum_{j\in J} pr_j \sum_{k\in K} pr_k \cdot \theta_{i,j,k} \leq 1-\alpha \tag{52}$$

$$\eta - \pi_{i,j,k} \leq BIGM \cdot \theta_{i,j,k} \quad \forall (i,j,k) \in \Omega \tag{53}$$

The CVaR is defined as the expected value of the contribution margin in the $(1-\alpha)$ worst cases of the distribution, or the expected value if the contribution margins fall below η, leading to following definition for a discrete probability distribution:

$$\mathrm{CVaR}(\alpha, x) = \max\left\{\eta - \frac{1}{1-\alpha} \cdot \mathbb{E}_{\omega\in\Omega}\left\{\max\{\eta - f(x,\omega), 0\}\right\}\right\} \tag{54}$$

with the Value-at-Risk η at α-level ($\alpha \in (0,1)$), the deterministic parameters x and the stochastic parameters ω in probability space Ω. The main advantage of using the CVaR instead of the VaR, besides the coherence, consists in the consideration of so-called fat-tails in the distribution of contribution margins.

A3. Exemplary scenario tree

Figs. 16, 18 and 19.

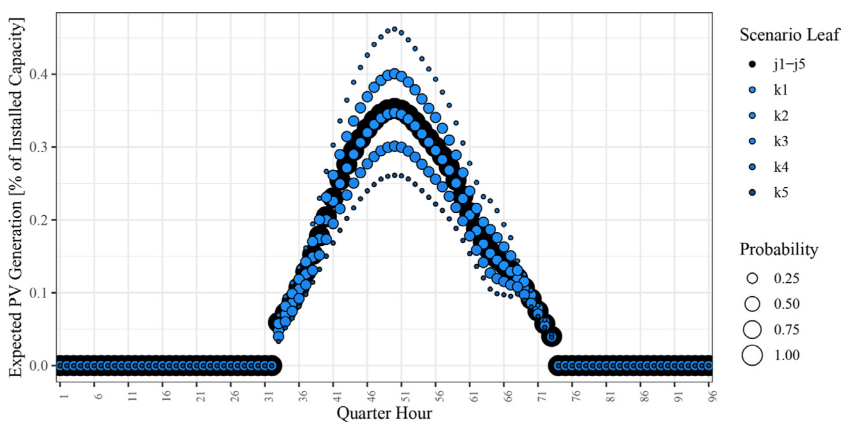

Fig. 16. PV generation scenarios derived from the stochastic modeling for the transition season on medium level of PV generation (e.g., *tra*2). The scenario tree captures the daily pattern as well as forecast updates in the intraday stage.

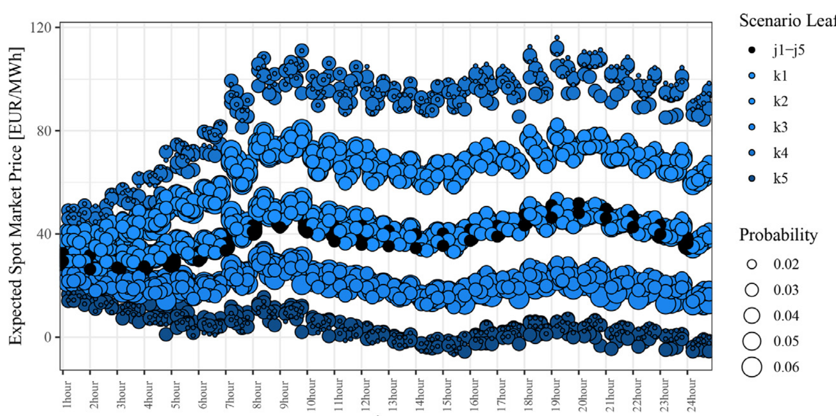

Fig. 17. Spot price scenarios derived from the stochastic modeling for the transition season on medium levels of PV generation and residual load. The scenario tree is arbitrage-free and captures the daily pattern as well as the intra-hourly patterns of the quarter-hourly intraday prices.

A4. Additional results

Fig. 21.

Table 4
Quantitative results for efficient frontier under different risk aversion levels (λ) and different intervals considered for the risk metric CVaR (α). The columns $\triangle\mathbb{E}(\Pi)$ and $\triangle$CVaR present the differences of the strategies with risk aversion in comparison to the according risk neutral optimization.

Risk metric Unit	α [%]	λ [%]	$\mathbb{E}(\Pi)$ [EUR/day]	$\triangle\mathbb{E}(\Pi)$ [EUR/day]	CVaR [EUR/day]	$\triangle$CVaR [EUR/day]
risk neutral	85	0	31,033	–	−44,220	–
CVaR	85	10	29,805	−1228	−16,457	27,763
CVaR	85	25	25,658	−5375	6781	51,000
CVaR	85	50	24,822	−6210	8897	53,116
risk neutral	85	0	31,033	–	−48,410	–
CVaR	90	10	29,130	−1903	−13,701	34,709
CVaR	90	25	25,374	−5659	7259	55,668
CVaR	90	50	24,821	−6212	8611	57,021
risk neutral	85	0	31,033	–	−51,295	–
CVaR	95	10	29,274	−1758	−15,965	35,330
CVaR	95	25	25,922	−5110	4708	56,003
CVaR	95	50	24,730	−6302	7936	59,231

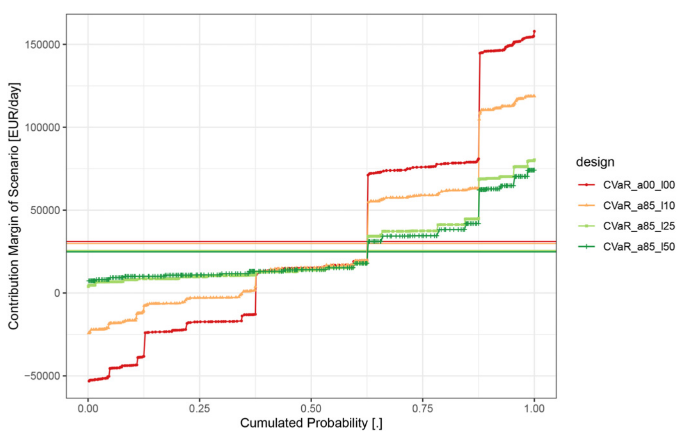

Fig. 18. Empirical Cumulative Distributions of Contribution Margins throughout scenarios for trading strategies considering the $\alpha = 85$ % level for the CVaR, medium weekday transition season (tra2).

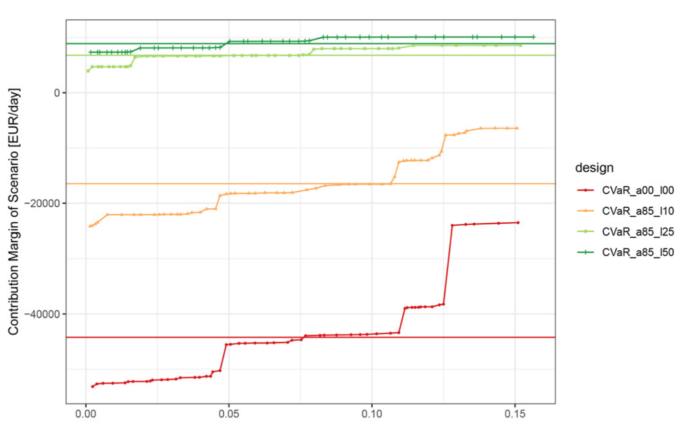

Fig. 19. Empirical Cumulative Distributions of Contribution Margins in the 15 % of worst case scenarios (CVaR range), medium weekday transition season (tra2).

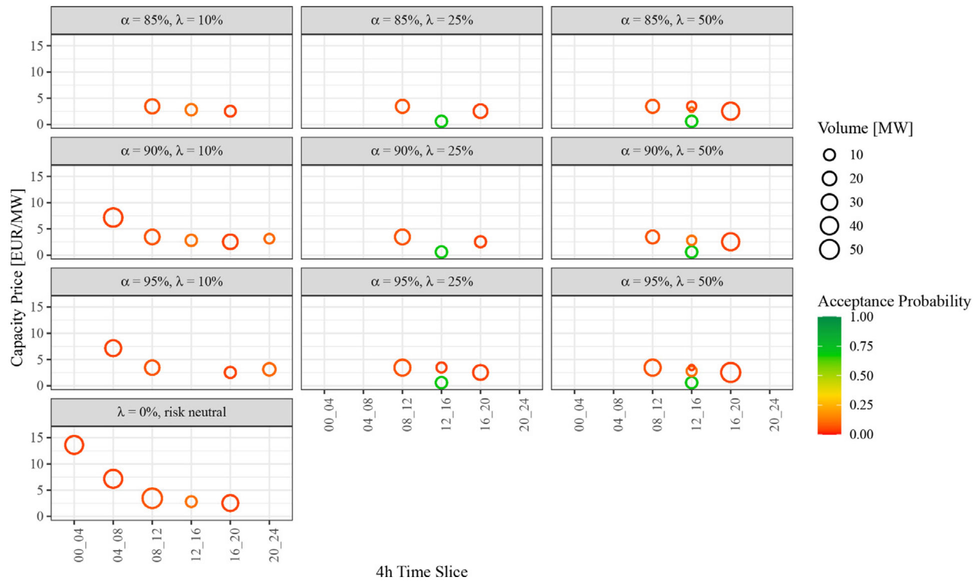

Fig. 21. Reserve market (aFRRneg) bids for different levels of CVaR range (α) and risk aversion (λ), working day with medium residual load, transition season (tra2).

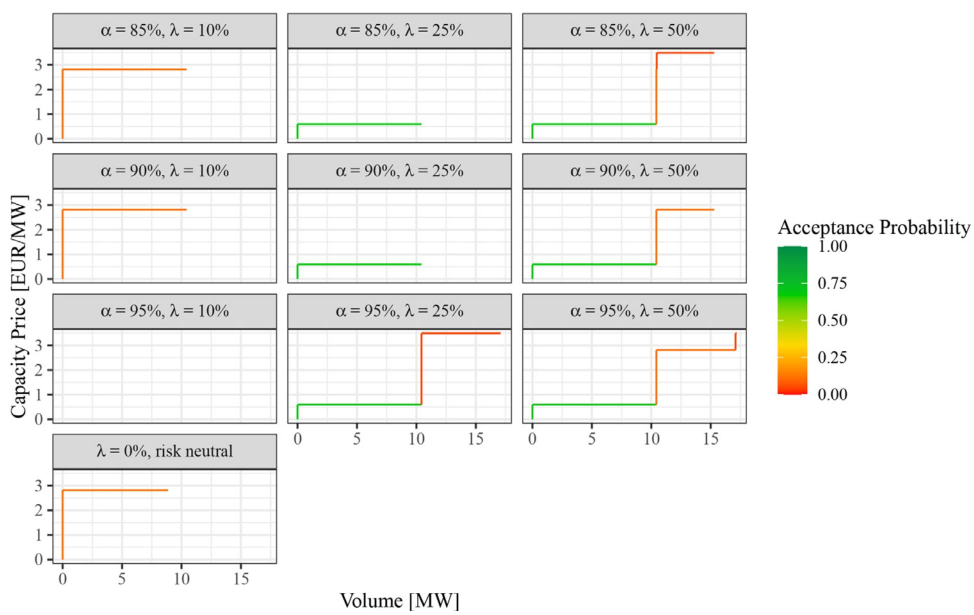

Fig. 20. Reserve market (aFRRneg) bid curves for slice 12-16 for different levels of CVaR range (α) and risk aversion (λ), working day with medium residual load, transition season (tra2). The first observation to note is that the risk aversion leads to a diversification in terms of price levels and aims at securing revenues, whereas the risk-neutral strategy places the reserve bids on a high price level. This can be interpreted as betting on a high price scenario. Secondly, the volume offered in the risk-averse strategy exceeds the volume offered in the risk-neutral strategy, which corresponds to accepting more operational constraints for the spot market decisions.

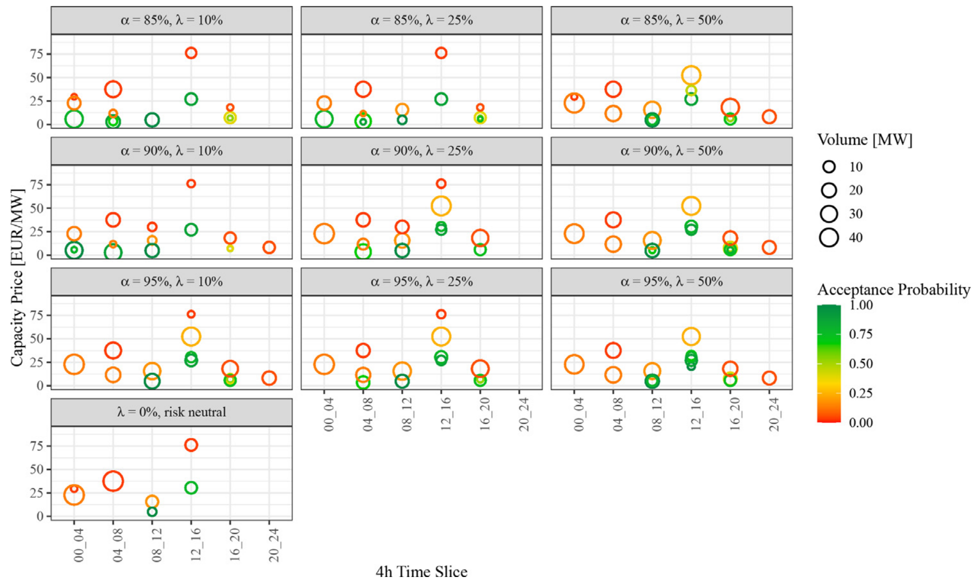

Fig. 22. Reserve market (aFRRneg) bids for different levels of CVaR range (α) and risk aversion (λ), weekend day with high residual load, summer season (sum6).

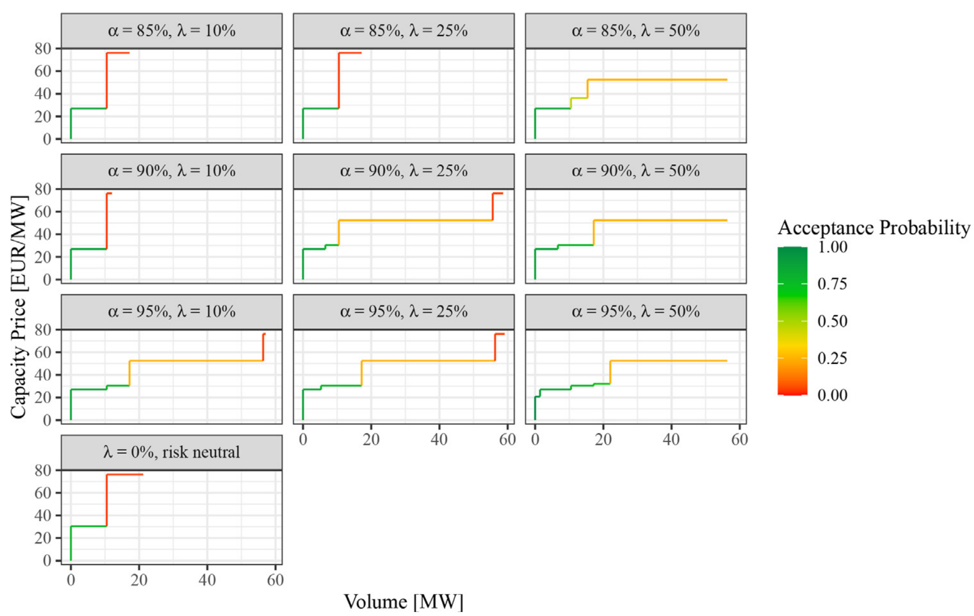

Fig. 23. Reserve market (aFRRneg) bid curves for slice 12-16 for different levels of CVaR range (α) and risk aversion (λ), weekend day with high residual load, summer season (sum6).

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2022.10.040.

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