ResEco - formulas

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1 base formulas

general supplier function for maximizing profit

for one supplier

$$max \sum_{region} Q_{sell}(region) * \left(price(region) - C_{trans}(region)\right)$$

$$-Q_{prod} * C_{prod}$$

for all supplier:

$$\pi(supplier) = \sum_{region} Q_{sell}(supplier, region) * \Big(price(region) - C_{trans}(supplier, region)\Big)$$

$$-Q_{prod}(supplier)*C_{prod}(supplier) \ \forall \ supplier, region$$

Constraints

There are several supplier constraints that must be taken into account.

They have the following logic:

 $"Q_{sell} \leq transCap(supplier, region) \leq Q_{prod} \leq productionCap(supplier) \forall supplier, regions"$

We can sepperate this into 3 constraints:

Transport constraint:

 $Q_{sell}(supplier, region) \leq transCap(supplier, region) \ \forall \ supplier, regions$

Selling cap constraint:

$$\sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \, \forall \, supplier$$

Production cap constraint:

$$Q_{prod}(supplier) \leq ProdCap(supplier) \ \forall \ supplier$$

For gams:

We need to apply the following steps for usability in gams:

- 1. introduce lower bound
- 2. split into 2 sepperate equations
- 3. get the 0 on one side
- 4. change equations to ≤ 0
- 5. introduce dual variables (μ 's)

for transport constraint:

- 1. introduce lower bound
 - $0 \le Q_{sell}(supplier, region) \le transCap(supplier, region) \ \forall \ supplier, regions$
- 2. split into 2 sepperate equations
 - $0 \leq Q_{sell}(supplier, region) \ \forall \ supplier, regions$
 - $Q_{sell}(supplier, region) \leq transCap(supplier, region) \ \forall \ supplier, regions$
- 3. get the 0 on one side
 - $0 \leq Q_{sell}(supplier, region) \ \forall \ supplier, regions$
 - $Q_{sell}(supplier, region) \leq transCap(supplier, region) \ \forall \ supplier, regions$
- 4. change equations to ≤ 0

 $-Q_{sell}(supplier, region) \le 0 \ \forall \ supplier, regions$

 $Q_{sell}(supplier, region) - transCap(supplier, region) \leq 0 \ \forall \ supplier, regions$

- 5. introduce Lagrange variable (μ 's)
 - $-Q_{sell}(supplier, region) \leq 0 \perp \mu_{transCapLow} \forall supplier, regions$

 $Q_{sell}(supplier, region) - transCap(supplier, region) \le 0 \perp \mu_{transCapUp} \forall supplier, regions$

for selling cap constraint:

- 1. introduce lower bound
 - $0 \leq \sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \, \forall \, supplier$
- 2. split into 2 sepperate equations
 - $0 \leq \sum_{region} Q_{sell}(supplier, region) \, \forall \, supplier$
 - $\sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \, \forall \, supplier$
- 3. get the 0 on one side
 - $0 \leq \sum_{region} Q_{sell}(supplier, region) \, \forall \, supplier$
 - $\sum_{region} Q_{sell}(supplier, region) Q_{prod}(supplier) \le 0 \ \forall \ supplier$
- 4. change equations to ≤ 0
 - $-\sum_{region} Q_{sell}(supplier, region) \leq 0 \ \forall \ supplier$

$$\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \leq 0 \; \forall \; supplier$$

- 5. introduce dual variables (μ 's)
 - $-\sum_{region} Q_{sell}(supplier, region) \leq 0 \perp \mu_{sellCapLow} \, \forall \, supplier$
 - $\sum_{region} Q_{sell}(supplier, region) Q_{prod}(supplier) \leq 0 \perp \mu_{sellCapUp} \, \forall \, supplier$

for production cap constraint:

- 1. introduce lower bound
 - $0 \le Q_{prod}(supplier) \le ProdCap(supplier) \ \forall \ supplier$

2. split into 2 sepperate equations

$$0 \le Q_{prod}(supplier)$$

$$Q_{prod}(supplier) \leq ProdCap(supplier)$$

3. get the 0 on one side

$$0 \le Q_{prod}(supplier)$$

$$Q_{prod}(supplier) - ProdCap(supplier) \le 0$$

4. change equations to ≤ 0

$$-Q_{prod}(supplier) \leq 0$$

$$Q_{prod}(supplier) - ProdCap(supplier) \le 0$$

5. introduce dual variables (μ 's)

$$-Q_{prod}(supplier) \leq 0 \perp \mu_{prodCapLow} \, \forall \, supplier$$

$$Q_{prod}(supplier) - ProdCap(supplier) \leq 0 \perp \ \mu_{prodCapUp} \ \forall \ supplier$$

Object function for gams:

$$\begin{split} \max \sum_{region} Q_{sell}(supplier, region) * \Big(price(region) - C_{trans}(supplier, region) \Big) \\ - Q_{prod}(supplier) * C_{prod}(supplier) \forall supplier \end{split}$$

 \rightarrow chang max to min:

$$\begin{split} & \min \sum_{region} Q_{sell}(supplier, region) * \Big(C_{trans}(supplier, region) - price(region) \Big) \\ & + Q_{prod}(supplier) * C_{prod}(supplier) \ \forall \ supplier \end{split}$$

 \rightarrow add constraints:

$$-\mu_{transCapLow} * Q_{sell}(supplier, region)$$

$$\mu_{transCapUp} * \left(Q_{sell}(supplier, region) - transCap(supplier, region)\right)$$

$$-\mu_{sellCapLow} * \sum_{region} Q_{sell}(supplier, region)$$

$$\mu_{sellCapUp} * \left(\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \right)$$

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-\mu_{prodCapLow}*Q_{prod}(supplier)
  \mu_{prodCapUp} * \left(Q_{prod}(supplier) - ProdCap(supplier)\right)
gams obj for supplier (Q_{sell})
   \frac{\partial f}{\partial Q_{sell}} \sum_{region} Q_{sell}(supplier, region) * \left(C_{trans}(supplier, region) - price(region)\right)
  +Q_{prod}(supplier)*C_{prod}(supplier) \ \forall \ supplier, region
   -\mu_{transCapLow}*Q_{sell}(supplier, region)
  +\mu_{transCapUp} * (Q_{sell}(supplier, region) - transCap(supplier, region))
  -\mu_{sellCapLow} * \sum_{region} Q_{sell}(supplier, region)
  +\mu_{sellCapUp}*\left(\sum_{region}Q_{sell}(supplier, region) - Q_{prod}(supplier)\right)
   -\mu_{prodCapLow}*Q_{prod}(supplier)
  +\mu_{prodCapUp} * \left(Q_{prod}(supplier) - ProdCap(supplier)\right)
  \forall supplier, region
  \sum_{region} C_{trans}(supplier, region) - price(region)
   -\mu_{transCapLow}
  +\mu_{transCapUp}
   -\sum_{region} \mu_{sellCapLow}
  +\sum_{region}\mu_{sellCapUp}
  \forall supplier, region
gams obj for supplier (Q_{prod}):
   \frac{\partial f}{\partial Q_{prod}} \sum_{region} Q_{sell}(supplier, region) * \Big(C_{trans}(supplier, region) - price(region)\Big)
  +Q_{prod}(supplier) * C_{prod}(supplier)
   -\mu_{transCapLow}*Q_{sell}(supplier, region)
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 + \mu_{transCapUp} * \left( Q_{sell}(supplier, region) - transCap(supplier, region) \right) 
 - \mu_{sellCapLow} * \sum_{region} Q_{sell}(supplier, region) 
 + \mu_{sellCapUp} * \left( \sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \right) 
 - \mu_{prodCapLow} * Q_{prod}(supplier) 
 + \mu_{prodCapUp} * \left( Q_{prod}(supplier) - ProdCap(supplier) \right) 
 \forall supplier, region 
 = 
 + C_{prod}(supplier) 
 - \mu_{sellCapUp} 
 - \mu_{prodCapLow} 
 + \mu_{prodCapLow} 
 \forall supplier, region 
 \forall supplier, region
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