ResEco - formulas

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June 2024

1 base formulas

general supplier function for maximizing profit

for one supplier

$max \sum_{region} Q_{sell}(region) * \Big(price(region) - C_{trans}(region)\Big)$

$-Q_{prod} * C_{prod}$

for all supplier:

$\pi(supplier) = \sum_{region} Q_{sell}(supplier, region) * \Big(price(region) - C_{trans}(supplier, region)\Big)$

$-Q_{prod}(supplier) * C_{prod}(supplier) \; \forall \; supplier, region$

Constraints

There are several supplier constraints that must be taken into account.

They have the following logic:

"$Q_{sell} \leq transCap(supplier, region) \leq Q_{prod} \leq productionCap(supplier) \forall supplier, regions$"

We can sepperate this into 3 constraints:

Transport constraint:

$$Q_{sell}(supplier, region) \leq transCap(supplier, region) \; \forall \; supplier, regions$$

Selling cap constraint:

$$\sum_{region} Q_{sell}(supplier, region) \leq \; Q_{prod}(supplier) \; \forall \; supplier$$

Production cap constraint:

$$Q_{prod}(supplier) \leq ProdCap(supplier) \; \forall \; supplier$$

For gams:

We need to apply the following steps for usability in gams:

1. introduce lower bound
2. split into 2 sepperate equations
3. get the 0 on one side
4. change equations to ≤ 0
5. introduce dual variables (μ's)

for transport constraint:

1. introduce lower bound

   $$0 \leq Q_{sell}(supplier, region) \leq transCap(supplier, region) \; \forall \; supplier, regions$$

2. split into 2 sepperate equations

   $$0 \leq Q_{sell}(supplier, region) \; \forall \; supplier, regions$$

   $$Q_{sell}(supplier, region) \leq transCap(supplier, region) \; \forall \; supplier, regions$$

3. get the 0 on one side

   $$0 \leq Q_{sell}(supplier, region) \; \forall \; supplier, regions$$

   $$Q_{sell}(supplier, region) \leq transCap(supplier, region) \; \forall \; supplier, regions$$

4. change equations to ≤ 0

$-Q_{sell}(supplier, region) \leq 0 \, \forall \, supplier, regions$

$Q_{sell}(supplier, region) - transCap(supplier, region) \leq 0 \, \forall \, supplier, regions$

5. introduce Lagrange variable (μ's)

$-Q_{sell}(supplier, region) \leq 0 \perp \mu_{transCapLow} \, \forall \, supplier, regions$

$Q_{sell}(supplier, region) - transCap(supplier, region) \leq 0 \perp \mu_{transCapUp} \forall supplier, regions$

for selling cap constraint:

1. introduce lower bound

$0 \leq \sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \, \forall \, supplier$

2. split into 2 sepperate equations

$0 \leq \sum_{region} Q_{sell}(supplier, region) \, \forall \, supplier$

$\sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \, \forall \, supplier$

3. get the 0 on one side

$0 \leq \sum_{region} Q_{sell}(supplier, region) \, \forall \, supplier$

$\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \leq 0 \, \forall \, supplier$

4. change equations to ≤ 0

$-\sum_{region} Q_{sell}(supplier, region) \leq 0 \, \forall \, supplier$

$\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \leq 0 \, \forall \, supplier$

5. introduce dual variables (μ's)

$-\sum_{region} Q_{sell}(supplier, region) \leq 0 \perp \mu_{sellCapLow} \, \forall \, supplier$

$\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \leq 0 \perp \mu_{sellCapUp} \, \forall \, supplier$

for production cap constraint:

1. introduce lower bound

$0 \leq Q_{prod}(supplier) \leq ProdCap(supplier) \, \forall \, supplier$

2. split into 2 sepperate equations

$0 \leq Q_{prod}(supplier)$

$Q_{prod}(supplier) \leq ProdCap(supplier)$

3. get the 0 on one side

$0 \leq Q_{prod}(supplier)$

$Q_{prod}(supplier) - ProdCap(supplier) \leq 0$

4. change equations to ≤ 0

$-Q_{prod}(supplier) \leq 0$

$Q_{prod}(supplier) - ProdCap(supplier) \leq 0$

5. introduce dual variables (μ's)

$-Q_{prod}(supplier) \leq 0 \perp \ \mu_{prodCapLow} \, \forall \, supplier$

$Q_{prod}(supplier) - ProdCap(supplier) \leq 0 \perp \ \mu_{prodCapUp} \, \forall \, supplier$

Object function for gams:

$max \sum_{region} Q_{sell}(supplier, region) * \Big(price(region) - C_{trans}(supplier, region) \Big)$

$-Q_{prod}(supplier) * C_{prod}(supplier) \, \forall \, supplier$

$\rightarrow$ **chang max to min:**

$min \sum_{region} Q_{sell}(supplier, region) * \Big(C_{trans}(supplier, region) - price(region) \Big)$

$+Q_{prod}(supplier) * C_{prod}(supplier) \, \forall \, supplier$

$\rightarrow$ **add constraints:**

$-Q_{sell}(supplier, region) \leq 0 \perp \ \mu_{transCapLow}$

$\rightarrow -\mu_{transCapLow} * Q_{sell}(supplier, region)$

$Q_{sell}(supplier, region) - transCap(supplier, region) \leq 0 \perp \ \mu_{transCapUp}$

$\rightarrow \mu_{transCapUp} * \Big(Q_{sell}(supplier, region) - transCap(supplier, region) \Big)$

$$-\sum_{region} Q_{sell}(supplier, region) \leq 0 \perp \mu_{sellCapLow}$$

$$\rightarrow -\mu_{sellCapLow} * \sum_{region} Q_{sell}(supplier, region)$$

$$\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \leq 0 \perp \mu_{sellCapUp}$$

$$\rightarrow \mu_{sellCapUp} * \Big(\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \Big)$$

$$-Q_{prod}(supplier) \leq 0 \perp \mu_{prodCapLow}$$

$$\rightarrow -\mu_{prodCapLow} * Q_{prod}(supplier)$$

$$Q_{prod}(supplier) - ProdCap(supplier) \leq 0 \perp \mu_{prodCapUp}$$

$$\rightarrow \mu_{prodCapUp} * \Big((Q_{prod}(supplier) - ProdCap(supplier)) \Big)$$

gams obj for supplier (Q_{sell})

$$\frac{\partial f}{\partial Q_{sell}} \sum_{region} Q_{sell}(supplier, region) * \Big(C_{trans}(supplier, region) - price(region) \Big)$$

$$+Q_{prod}(supplier) * C_{prod}(supplier)$$

$$-\mu_{transCapLow} * Q_{sell}(supplier, region)$$

$$+\mu_{transCapUp} * \Big(Q_{sell}(supplier, region) - transCap(supplier, region) \Big)$$

$$-\mu_{sellCapLow} * \sum_{region} Q_{sell}(supplier, region)$$

$$+\mu_{sellCapUp} * \Big(\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \Big)$$

$$-\mu_{prodCapLow} * Q_{prod}(supplier)$$

$$+\mu_{prodCapUp} * \Big(Q_{prod}(supplier) - ProdCap(supplier) \Big)$$

$$=$$

$$\sum_{region} C_{trans}(supplier, region) - price(region)$$

$$-\mu_{transCapLow}$$

$$+\mu_{transCapUp}$$

$$-\sum_{region} \mu_{sellCapLow}$$

$$+\sum_{region} \mu_{sellCapUp}$$

gams obj for supplier (Q_{prod}):

$$\frac{\partial f}{\partial Q_{prod}} \sum_{region} Q_{sell}(supplier, region) * \Big(C_{trans}(supplier, region) - price(region) \Big)$$

$$+ Q_{prod}(supplier) * C_{prod}(supplier)$$

$$- \mu_{transCapLow} * Q_{sell}(supplier, region)$$

$$+ \mu_{transCapUp} * \Big(Q_{sell}(supplier, region) - transCap(supplier, region) \Big)$$

$$- \mu_{sellCapLow} * \sum_{region} Q_{sell}(supplier, region)$$

$$+ \mu_{sellCapUp} * \Big(\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \Big)$$

$$- \mu_{prodCapLow} * Q_{prod}(supplier)$$

$$+ \mu_{prodCapUp} * \Big(Q_{prod}(supplier) - ProdCap(supplier) \Big)$$

$$=$$

$$+ C_{prod}(region)$$

$$- \mu_{sellCapUp}$$

$$- \mu_{prodCapLow}$$

$$+ \mu_{prodCapUp}$$