Resource Economics - formulas

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1 Base formulas

General supplier function for maximizing profit

for one supplier

$$max \sum_{region} \Big(q_{sell}(region) * (price(region) - C_{trans}(region)) \Big)$$
$$- q_{prod} * C_{prod}$$

$\rightarrow$

$$min \sum_{region} \Big(q_{sell}(region) * (C_{trans}(region) - price(region)) \Big)$$
$$+ q_{prod} * C_{prod}$$

with:

$$price(region) = IntersectionPoint(region) - Slope * \sum_{supplier} q_{sell}(region)$$

$\rightarrow$

$$min \sum_{region} \Big(q_{sell}(region)$$
$$*(C_{trans}(region) - IntersectionPoint(region) + Slope * \sum_{supplier} q_{sell}(region)) \Big)$$
$$+ q_{prod} * C_{prod}$$

For transport constraint:

1. Define constraint

   $q_{sell}(region) \leq TransCap(region) \, \forall \, region$

3. For Laplace: bring constraints into standard form

   $q_{sell}(region) - TransCap(region) \leq 0 : \mu_{TransCap} \, \forall \, regions$

4. For GAMS: change equations to ≥ 0

   $TransCap(region) - q_{sell}(region) \geq 0 \, \forall \, regions$

For selling cap constraint:

1. Define constraint

   $\sum_{region} q_{sell}(region) \leq q_{prod}$

3. For Laplace: bring constraints into standard form

   $\sum_{region} q_{sell}(region) - q_{prod} \leq 0 : \mu_{massBal}$

4. For GAMS: change equations to ≥ 0

   $q_{prod} - \sum_{region} q_{sell}(region) \geq 0$

For production cap constraint:

1. Define constraint

   $q_{prod} \leq ProdCap$

3. For Laplace: bring constraints into standard form

   $q_{prod} - ProdCap \leq 0 : \mu_{prodCap}$

4. For GAMS: change equations to ≥ 0

   $ProdCap - q_{prod} \geq 0$

Lagrange function

$$min \sum_{region} \Big(q_{sell}(region) * (C_{trans}(region) - price(region)) \Big) + q_{prod} * C_{prod}$$

add constraints:

$$+\mu_{TransCap} * (q_{sell}(region) - TransCap(region))$$
$$+\mu_{massBal} * (\sum_{region} q_{sell}(region) - q_{prod})$$
$$+\mu_{prodCap} * (q_{prod} - ProdCap)$$

Derive KKT's:

for q_{sell} :

$$\frac{\partial f}{\partial Q_{sell}} =$$
$$C_{trans}(region)$$
$$-q_{sell}(region) * price'(region)$$
$$-1 * price(region)$$
$$+\mu_{TransCap}$$

$+\mu_{massBal}$

$=$

$C_{trans}(region)$

$-q_{sell}(region) * (Slope(region))$

$-(IntersectionPoint(region) + Slope * \sum_{supplier} q_{sell}(region)))$

$+\mu_{TransCap}$

$+\mu_{massBal}$

for q_{prod} :

$\frac{\partial f}{\partial Q_{prod}} = C_{prod} - \mu_{massBal} + \mu_{prodCap}$