

Resource Economics - formulas

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1 Base formulas

General supplier function for maximizing profit

for one supplier

$$\begin{aligned} & \max \sum_{region} \left(q_{sell}(region) * (price(region) - C_{trans}(region)) \right) \\ & - q_{prod} * C_{prod} \end{aligned}$$

→

$$\begin{aligned} & \min \sum_{region} \left(q_{sell}(region) * (C_{trans}(region) - price(region)) \right) \\ & + q_{prod} * C_{prod} \end{aligned}$$

with:

$$price(region) = IntersectionPoint(region) - Slope * \sum_{supplier} q_{sell}(region)$$

→

$$\begin{aligned} & \min \sum_{region} \left(q_{sell}(region) \right. \\ & \left. * (C_{trans}(region) - IntersectionPoint(region) + Slope * \sum_{supplier} q_{sell}(region)) \right) \\ & + q_{prod} * C_{prod} \end{aligned}$$

For transport constraint:

1. Define constraint

$$q_{sell}(region) \leq TransCap(region) \forall region$$

3. For Laplace: bring constraints into standard form

$$q_{sell}(region) - TransCap(region) \leq 0 : \mu_{TransCap} \forall regions$$

4. For GAMS: change equations to ≥ 0

$$TransCap(region) - q_{sell}(region) \geq 0 \forall regions$$

For selling cap constraint:

1. Define constraint

$$\sum_{region} q_{sell}(region) \leq q_{prod}$$

3. For Laplace: bring constraints into standard form

$$\sum_{region} q_{sell}(region) - q_{prod} \leq 0 : \mu_{massBal}$$

4. For GAMS: change equations to ≥ 0

$$q_{prod} - \sum_{region} q_{sell}(region) \geq 0$$

For production cap constraint:

1. Define constraint

$$q_{prod} \leq ProdCap$$

3. For Laplace: bring constraints into standard form

$$q_{prod} - ProdCap \leq 0 : \mu_{prodCap}$$

4. For GAMS: change equations to ≥ 0

$$ProdCap - q_{prod} \geq 0$$

Lagrange function

$$\begin{aligned} &min \sum_{region} \left(q_{sell}(region) \right. \\ &\quad \left. * (C_{trans}(region) - price(region)) \right) \\ &\quad + q_{prod} * C_{prod} \end{aligned}$$

add constraints:

$$\begin{aligned} &+ \mu_{TransCap} * (q_{sell}(region) - TransCap(region)) \\ &+ \mu_{massBal} * (\sum_{region} q_{sell}(region) - q_{prod}) \\ &+ \mu_{prodCap} * (q_{prod} - ProdCap) \end{aligned}$$

Derive KKT's:

for q_{sell} :

$$\begin{aligned} &\frac{\partial f}{\partial Q_{sell}} = \\ &\quad C_{trans}(region) \\ &\quad - q_{sell}(region) * price'(region) \\ &\quad - 1 * price(region) \\ &\quad + \mu_{TransCap} \end{aligned}$$

$$\begin{aligned}
& +\mu_{massBal} \\
& = \\
& C_{trans}(region) \\
& -q_{sell}(region) * (Slope(region)) \\
& -IntersectionPoint(region) - Slope * \sum_{supplier} q_{sell}(region)) \\
& +\mu_{TransCap} \\
& +\mu_{massBal} \\
& \text{for } q_{prod} : \\
& \frac{\partial f}{\partial Q_{prod}} = C_{prod} - \mu_{massBal} + \mu_{prodCap}
\end{aligned}$$