ResEco - formulas

Mr Green Pepper

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1 base formulas

general supplier function for maximizing profit

for one supplier

$max \sum_{region} Q_{sell}(region) * \Big(price(region) - C_{trans}(region)\Big)$

$-Q_{prod} * C_{prod}$

for all supplier:

$\pi(supplier) = \sum_{region} Q_{sell}(supplier, region) * \Big(price(region) - C_{trans}(supplier, region)\Big)$

$-Q_{prod}(supplier) * C_{prod}(supplier) \; \forall \; supplier, region$

Constraints

There are several supplier constraints that must be taken into account.

They have the following logic:

"$Q_{sell} \leq transCap(supplier, region) \leq Q_{prod} \leq productionCap(supplier) \forall supplier, regions$"

We can sepperate this into 3 constraints:

Transport constraint:

$Q_{sell}(supplier, region) \leq transCap(supplier, region) \, \forall \, supplier, regions$

Selling cap constraint:

$\sum_{region} Q_{sell}(supplier, region) \leq \; Q_{prod}(supplier) \, \forall \, supplier$

Production cap constraint:

$Q_{prod}(supplier) \leq ProdCap(supplier) \, \forall \, supplier$

For gams:

We need to apply the following steps for usability in gams:

1. introduce lower bound

2. split into 2 sepperate equations

3. get the 0 on one side

4. change equations to ≥ 0

5. introduce dual variables (μ's)

for transport constraint:

1. introduce lower bound

   $0 \leq Q_{sell}(supplier, region) \leq transCap(supplier, region) \, \forall \, supplier, regions$

2. split into 2 sepperate equations

   $0 \leq Q_{sell}(supplier, region) \, \forall \, supplier, regions$

   $Q_{sell}(supplier, region) \leq transCap(supplier, region) \, \forall \, supplier, regions$

3. get the 0 on one side

   $0 \leq Q_{sell}(supplier, region) \, \forall \, supplier, regions$

   $Q_{sell}(supplier, region) - transCap(supplier, region) \leq 0 \, \forall \, supplier, regions$

4. change equations to ≥ 0

$Q_{sell}(supplier, region) \geq 0 \, \forall \, supplier, regions$

$transCap(supplier, region) - Q_{sell}(supplier, region) \geq 0 \, \forall \, supplier, regions$

5. introduce Lagrange variable (μ's)

$Q_{sell}(supplier, region) \geq 0 \perp \ \mu_{transCapLow} \, \forall \, supplier, regions$

$transCap(supplier, region) - Q_{sell}(supplier, region) \geq 0 \perp \mu_{transCapUp} \forall supplier, regions$

for selling cap constraint:

1. introduce lower bound

$0 \leq \sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \, \forall \, supplier$

2. split into 2 sepperate equations

$0 \leq \sum_{region} Q_{sell}(supplier, region) \, \forall \, supplier$

$\sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \, \forall \, supplier$

3. get the 0 on one side

$0 \leq \sum_{region} Q_{sell}(supplier, region) \, \forall \, supplier$

$Q_{prod}(supplier) - \sum_{region} Q_{sell}(supplier, region) \geq 0 \, \forall \, supplier$

4. change equations to ≥ 0

$\sum_{region} Q_{sell}(supplier, region) \geq 0 \, \forall \, supplier$

$Q_{prod}(supplier) - \sum_{region} Q_{sell}(supplier, region) \geq 0 \, \forall \, supplier$

5. introduce dual variables (μ's)

$\sum_{region} Q_{sell}(supplier, region) \geq 0 \perp \mu_{sellCapLow} \, \forall \, supplier$

$Q_{prod}(supplier) - \sum_{region} Q_{sell}(supplier, region) \geq 0 \perp \mu_{sellCapUp} \, \forall \, supplier$

for production cap constraint:

1. introduce lower bound

$0 \leq Q_{prod}(supplier) \leq ProdCap(supplier) \, \forall \, supplier$

2. split into 2 sepperate equations

   $0 \leq Q_{prod}(supplier)$

   $Q_{prod}(supplier) \leq ProdCap(supplier)$

3. get the 0 on one side

   $0 \leq Q_{prod}(supplier)$

   $Q_{prod}(supplier) - ProdCap(supplier) \leq 0$

4. change equations to ≥ 0

   $Q_{prod}(supplier) \geq 0$

   $ProdCap(supplier) - Q_{prod}(supplier) \geq 0$

5. introduce dual variables (μ's)

   $Q_{prod}(supplier) \geq 0 \perp \ \mu_{prodCapLow} \ \forall \ supplier$

   $ProdCap(supplier) - Q_{prod}(supplier) \geq 0 \perp \ \mu_{prodCapUp} \ \forall \ supplier$

Object function for gams:

$max \sum_{region} Q_{sell}(supplier, region) * \Big(price(region) - C_{trans}(supplier, region)\Big)$

$-Q_{prod}(supplier) * C_{prod}(supplier) \ \forall \ supplier$

$\rightarrow$ **chang max to min:**

$min \sum_{region} Q_{sell}(supplier, region) * \Big(C_{trans}(supplier, region) - price(region)\Big)$

$+Q_{prod}(supplier) * C_{prod}(supplier) \ \forall \ supplier$

$\rightarrow$ **add constraints:**

$-\mu_{transCapLow} * Q_{sell}(supplier, region)$

$+\mu_{transCapUp} * \Big(transCap(supplier, region) - Q_{sell}(supplier, region)\Big)$

$-\mu_{sellCapLow} * \sum_{region} Q_{sell}(supplier, region)$

$+\mu_{sellCapUp} * \Big(Q_{prod}(supplier) - \sum_{region} Q_{sell}(supplier, region)\Big)$

$-\mu_{prodCapLow} * Q_{prod}(supplier)$

$+\mu_{prodCapUp} * \Big(ProdCap(supplier) - Q_{prod}(supplier)\Big)$

derivation obj for supplier (Q_{sell})

$\frac{\partial f}{\partial Q_{sell}} = \sum_{region}(C_{trans}(supplier, region) - price(region))$

$-\mu_{transCapLow}$

$-\mu_{transCapUp}$

$-\sum_{region} \mu_{sellCapLow}$

$-\sum_{region} \mu_{sellCapUp}$

$\forall\, supplier$

derivation obj for supplier (Q_{prod}):

$\frac{\partial f}{\partial Q_{prod}} = C_{prod}(supplier)$

$+\mu_{sellCapUp}$

$-\mu_{prodCapLow}$

$-\mu_{prodCapUp}$

$\forall\, supplier$

demand

max demand

$maxQ_d(region) * (Utility(region) - price(region))$

constraints:

$0 \leq Q_d(region) \leq MaxConsumption(region)$

We need to apply the following steps for usability in gams:

1. introduce lower bound
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4. change equations to ≥ 0
5. introduce dual variables (μ's)

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   $0 \leq Q_d(region) \leq MaxConsumption(region)$

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3. get the 0 on one side

   $0 \leq Q_d(region)$

   $Q_d(region) - MaxConsumption(region) \leq 0$

4. change equations to ≥ 0

   $Q_d(region) \geq 0$

   $MaxConsumption(region) - Q_d(region) \geq 0$

5. introduce dual variables (μ's)