ResEco - formulas

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1 base formulas

general supplier function for maximizing profit

for one supplier

$$max \sum_{region} Q_{sell}(region) * \Big(price(region) - C_{trans}(region)\Big)$$

$$-Q_{prod} * C_{prod}$$

for all supplier:

$$\pi(supplier) = \sum_{region} Q_{sell}(supplier, region) * \Big(price(region) - C_{trans}(supplier, region)\Big)$$

$$-Q_{prod}(supplier) * C_{prod}(supplier) \; \forall \; supplier, region$$

Constraints

There are several supplier constraints that must be taken into account.

They have the following logic:

"$Q_{sell} \leq transCap(supplier, region) \leq Q_{prod} \leq productionCap(supplier) \forall supplier, regions$"

We can sepperate this into 3 constraints:

Transport constraint:

$$Q_{sell}(supplier, region) \leq transCap(supplier, region) \, \forall \, supplier, regions$$

Selling cap constraint:

$$\sum_{region} Q_{sell}(supplier, region) \leq \; Q_{prod}(supplier) \, \forall \, supplier$$

Production cap constraint:

$$Q_{prod}(supplier) \leq ProdCap(supplier) \, \forall \, supplier$$

For gams:

We need to apply the following steps for usability in gams:

1. introduce lower bound

2. split into 2 sepperate equations

3. get the 0 on one side

4. change equations to ≤ 0

5. introduce dual variables (μ's)

for transport constraint:

1. introduce lower bound

   $$0 \leq Q_{sell}(supplier, region) \leq transCap(supplier, region) \, \forall \, supplier, regions$$

2. split into 2 sepperate equations

   $$0 \leq Q_{sell}(supplier, region) \, \forall \, supplier, regions$$

   $$Q_{sell}(supplier, region) \leq transCap(supplier, region) \, \forall \, supplier, regions$$

3. get the 0 on one side

   $$0 \leq Q_{sell}(supplier, region) \, \forall \, supplier, regions$$

   $$Q_{sell}(supplier, region) \leq transCap(supplier, region) \, \forall \, supplier, regions$$

4. change equations to ≤ 0

$$-Q_{sell}(supplier, region) \leq 0 \; \forall \; supplier, regions$$

$$Q_{sell}(supplier, region) - transCap(supplier, region) \leq 0 \; \forall \; supplier, regions$$

5. introduce Lagrange variable (μ's)

$$-Q_{sell}(supplier, region) \leq 0 \perp \mu_{transCapLow} \; \forall \; supplier, regions$$

$$Q_{sell}(supplier, region) - transCap(supplier, region) \leq 0 \perp \mu_{transCapUp} \forall supplier, regions$$

for selling cap constraint:

1. introduce lower bound

$$0 \leq \sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \; \forall \; supplier$$

2. split into 2 sepperate equations

$$0 \leq \sum_{region} Q_{sell}(supplier, region) \; \forall \; supplier$$

$$\sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \; \forall \; supplier$$

3. get the 0 on one side

$$0 \leq \sum_{region} Q_{sell}(supplier, region) \; \forall \; supplier$$

$$\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \leq 0 \; \forall \; supplier$$

4. change equations to ≤ 0

$$- \sum_{region} Q_{sell}(supplier, region) \leq 0 \; \forall \; supplier$$

$$\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \leq 0 \; \forall \; supplier$$

5. introduce dual variables (μ's)

$$- \sum_{region} Q_{sell}(supplier, region) \leq 0 \perp \mu_{sellCapLow} \; \forall \; supplier$$

$$\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \leq 0 \perp \mu_{sellCapUp} \; \forall \; supplier$$

for production cap constraint:

1. introduce lower bound

$$0 \leq Q_{prod}(supplier) \leq ProdCap(supplier) \; \forall \; supplier$$

2. split into 2 sepperate equations

$$0 \leq Q_{prod}(supplier)$$

$$Q_{prod}(supplier) \leq ProdCap(supplier)$$

3. get the 0 on one side

$$0 \leq Q_{prod}(supplier)$$

$$Q_{prod}(supplier) - ProdCap(supplier) \leq 0$$

4. change equations to ≤ 0

$$-Q_{prod}(supplier) \leq 0$$

$$Q_{prod}(supplier) - ProdCap(supplier) \leq 0$$

5. introduce dual variables (μ's)

$$-Q_{prod}(supplier) \leq 0 \perp \mu_{prodCapLow} \; \forall \, supplier$$

$$Q_{prod}(supplier) - ProdCap(supplier) \leq 0 \perp \mu_{prodCapUp} \; \forall \, supplier$$

Object function for gams:

$$max \sum_{region} Q_{sell}(supplier, region) * \Big(price(region) - C_{trans}(supplier, region) \Big)$$

$$-Q_{prod}(supplier) * C_{prod}(supplier) \; \forall \, supplier$$

$\rightarrow$ **chang max to min:**

$$min \sum_{region} Q_{sell}(supplier, region) * \Big(C_{trans}(supplier, region) - price(region) \Big)$$

$$+Q_{prod}(supplier) * C_{prod}(supplier) \; \forall \, supplier$$

$\rightarrow$ **add constraints:**

$$-\mu_{transCapLow} * Q_{sell}(supplier, region)$$

$$\mu_{transCapUp} * \Big(Q_{sell}(supplier, region) - transCap(supplier, region) \Big)$$

$$-\mu_{sellCapLow} * \sum_{region} Q_{sell}(supplier, region)$$

$$\mu_{sellCapUp} * \Big(\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \Big)$$

$$-\mu_{prodCapLow} * Q_{prod}(supplier)$$

$$\mu_{prodCapUp} * \Big(Q_{prod}(supplier) - ProdCap(supplier)\Big)$$

gams obj for supplier (Q_{sell})

$$\frac{\partial f}{\partial Q_{sell}} \sum_{region} Q_{sell}(supplier, region) * \Big(C_{trans}(supplier, region) - price(region)\Big)$$

$$+ Q_{prod}(supplier) * C_{prod}(supplier) \ \forall \ supplier, region$$

$$- \mu_{transCapLow} * Q_{sell}(supplier, region)$$

$$+ \mu_{transCapUp} * \Big(Q_{sell}(supplier, region) - transCap(supplier, region)\Big)$$

$$- \mu_{sellCapLow} * \sum_{region} Q_{sell}(supplier, region)$$

$$+ \mu_{sellCapUp} * \Big(\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier)\Big)$$

$$- \mu_{prodCapLow} * Q_{prod}(supplier)$$

$$+ \mu_{prodCapUp} * \Big(Q_{prod}(supplier) - ProdCap(supplier)\Big)$$

$$\forall \ supplier, region$$

$$=$$

$$\sum_{region} C_{trans}(supplier, region) - price(region)$$

$$- \mu_{transCapLow}$$

$$+ \mu_{transCapUp}$$

$$- \sum_{region} \mu_{sellCapLow}$$

$$+ \sum_{region} \mu_{sellCapUp}$$

$$\forall \ supplier, region$$

gams obj for supplier (Q_{prod}):

$$\frac{\partial f}{\partial Q_{prod}} \sum_{region} Q_{sell}(supplier, region) * \Big(C_{trans}(supplier, region) - price(region)\Big)$$

$$+ Q_{prod}(supplier) * C_{prod}(supplier)$$

$$- \mu_{transCapLow} * Q_{sell}(supplier, region)$$

$$+\mu_{transCapUp} * \Big(Q_{sell}(supplier, region) - transCap(supplier, region)\Big)$$

$$-\mu_{sellCapLow} * \sum_{region} Q_{sell}(supplier, region)$$

$$+\mu_{sellCapUp} * \Big(\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier)\Big)$$

$$-\mu_{prodCapLow} * Q_{prod}(supplier)$$

$$+\mu_{prodCapUp} * \Big(Q_{prod}(supplier) - ProdCap(supplier)\Big)$$

$$\forall \, supplier, region$$

$$=$$

$$+C_{prod}(supplier)$$

$$-\mu_{sellCapUp}$$

$$-\mu_{prodCapLow}$$

$$+\mu_{prodCapUp}$$

$$\forall \, supplier, region$$