

# ResEco - formulas

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## 1 base formulas

### general supplier function for maximizing profit

for one supplier

$$\max \sum_{region} Q_{sell}(region) * (price(region) - C_{trans}(region)) \\ - Q_{prod} * C_{prod}$$

for all supplier:

$$\pi(supplier) = \sum_{region} Q_{sell}(supplier, region) * (price(region) - C_{trans}(supplier, region)) \\ - Q_{prod}(supplier) * C_{prod}(supplier) \forall supplier, region$$

### Constraints

There are several supplier constraints that must be taken into account.

They have the following logic:

$$Q_{sell} \leq transCap(supplier, region) \leq Q_{prod} \leq productionCap(supplier) \forall supplier, regions$$

We can sepperate this into 3 constraints:

Transport constraint:

$$Q_{sell}(supplier, region) \leq transCap(supplier, region) \forall supplier, regions$$

Selling cap constraint:

$$\sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \forall supplier$$

Production cap constraint:

$$Q_{prod}(supplier) \leq ProdCap(supplier) \forall supplier$$

**For gams:**

We need to apply the following steps for usability in gams:

1. introduce lower bound
2. split into 2 sepperate equations
3. get the 0 on one side
4. change equations to  $\geq 0$
5. introduce dual variables ( $\mu$ 's)

**for transport constraint:**

1. introduce lower bound

$$0 \leq Q_{sell}(supplier, region) \leq transCap(supplier, region) \forall supplier, regions$$

2. split into 2 sepperate equations

$$0 \leq Q_{sell}(supplier, region) \forall supplier, regions$$

$$Q_{sell}(supplier, region) \leq transCap(supplier, region) \forall supplier, regions$$

3. get the 0 on one side

$$0 \leq Q_{sell}(supplier, region) \forall supplier, regions$$

$$Q_{sell}(supplier, region) - transCap(supplier, region) \leq 0 \forall supplier, regions$$

4. change equations to  $\geq 0$

$$Q_{sell}(supplier, region) \geq 0 \forall supplier, regions$$

$$transCap(supplier, region) - Q_{sell}(supplier, region) \geq 0 \forall supplier, regions$$

5. introduce Lagrange variable ( $\mu$ 's)

$$Q_{sell}(supplier, region) \geq 0 \perp \mu_{transCapLow} \forall supplier, regions$$

$$transCap(supplier, region) - Q_{sell}(supplier, region) \geq 0 \perp \mu_{transCapUp} \forall supplier, regions$$

**for selling cap constraint:**

1. introduce lower bound

$$0 \leq \sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \forall supplier$$

2. split into 2 sepperate equations

$$0 \leq \sum_{region} Q_{sell}(supplier, region) \forall supplier$$

$$\sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \forall supplier$$

3. get the 0 on one side

$$0 \leq \sum_{region} Q_{sell}(supplier, region) \forall supplier$$

$$Q_{prod}(supplier) - \sum_{region} Q_{sell}(supplier, region) \geq 0 \forall supplier$$

4. change equations to  $\geq 0$

$$\sum_{region} Q_{sell}(supplier, region) \geq 0 \forall supplier$$

$$Q_{prod}(supplier) - \sum_{region} Q_{sell}(supplier, region) \geq 0 \forall supplier$$

5. introduce dual variables ( $\mu$ 's)

$$\sum_{region} Q_{sell}(supplier, region) \geq 0 \perp \mu_{sellCapLow} \forall supplier$$

$$Q_{prod}(supplier) - \sum_{region} Q_{sell}(supplier, region) \geq 0 \perp \mu_{sellCapUp} \forall supplier$$

**for production cap constraint:**

1. introduce lower bound

$$0 \leq Q_{prod}(supplier) \leq ProdCap(supplier) \forall supplier$$

2. split into 2 sepperate equations

$$0 \leq Q_{prod}(supplier)$$

$$Q_{prod}(supplier) \leq ProdCap(supplier)$$

3. get the 0 on one side

$$0 \leq Q_{prod}(supplier)$$

$$Q_{prod}(supplier) - ProdCap(supplier) \leq 0$$

4. change equations to  $\geq 0$

$$Q_{prod}(supplier) \geq 0$$

$$ProdCap(supplier) - Q_{prod}(supplier) \geq 0$$

5. introduce dual variables ( $\mu$ 's)

$$Q_{prod}(supplier) \geq 0 \perp \mu_{prodCapLow} \forall supplier$$

$$ProdCap(supplier) - Q_{prod}(supplier) \geq 0 \perp \mu_{prodCapUp} \forall supplier$$

**Object function for gams:**

$$max \sum_{region} Q_{sell}(supplier, region) * (price(region) - C_{trans}(supplier, region))$$

$$- Q_{prod}(supplier) * C_{prod}(supplier) \forall supplier$$

→ **chang max to min:**

$$min \sum_{region} Q_{sell}(supplier, region) * (C_{trans}(supplier, region) - price(region))$$

$$+ Q_{prod}(supplier) * C_{prod}(supplier) \forall supplier$$

→ **add constraints:**

$$\mu_{transCapLow} * Q_{sell}(supplier, region)$$

$$\mu_{transCapUp} * (transCap(supplier, region) - Q_{sell}(supplier, region))$$

$$\mu_{sellCapLow} * \sum_{region} Q_{sell}(supplier, region)$$

$$\mu_{sellCapUp} * (Q_{prod}(supplier) - \sum_{region} Q_{sell}(supplier, region))$$

$$\mu_{prodCapLow} * Q_{prod}(supplier)$$

$$\mu_{prodCapUp} * (ProdCap(supplier) - Q_{prod}(supplier))$$

**derivation obj for supplier ( $Q_{sell}$ )**

$$\frac{\partial f}{\partial Q_{sell}} = \sum_{region} (C_{trans}(supplier, region) - price(region))$$

$$+ \mu_{transCapLow}$$

$$- \mu_{transCapUp}$$

$$+ \sum_{region} \mu_{sellCapLow}$$

$$- \sum_{region} \mu_{sellCapUp}$$

$$\forall supplier, region$$

**derivation obj for supplier ( $Q_{prod}$ ):**

$$\frac{\partial f}{\partial Q_{prod}} = C_{prod}(supplier)$$

$$+ \mu_{sellCapUp}$$

$$+ \mu_{prodCapLow}$$

$$- \mu_{prodCapUp}$$

$$\forall supplier$$

## demand

max demand

$$max Q_d(region) * (Utility(region) - price(region))$$

constraints:

$$0 \leq Q_d(region) \leq MaxConsumption(region)$$

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$$0 \leq Q_d(region)$$

$$Q_d(region) - MaxConsumption(region) \leq 0$$

4. change equations to  $\geq 0$

$$Q_d(region) \geq 0$$

$$MaxConsumption(region) - Q_d(region) \geq 0$$

5. introduce dual variables ( $\mu$ 's)