

Resource Economics - formulas

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1 Base formulas

General supplier function for maximizing profit

for one supplier

$$\max \sum_{region} Q_{sell}(region) * (price(region) - C_{trans}(region)) \\ - Q_{prod} * C_{prod}$$

for all supplier:

$$\pi(supplier) = \sum_{region} Q_{sell}(supplier, region) * (price(region) - C_{trans}(supplier, region)) \\ - Q_{prod}(supplier) * C_{prod}(supplier) \forall supplier$$

Constraints

There are several supplier constraints that must be taken into account.

They have the following logic:

$$Q_{sell} \leq transCap(supplier, region) \leq Q_{prod} \leq productionCap(supplier) \forall supplier, regions$$

We can sepperate this into 3 constraints:

Transport constraint:

$$Q_{sell}(supplier, region) \leq transCap(supplier, region) \forall supplier, regions$$

Selling cap constraint:

$$\sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \forall supplier$$

Production cap constraint:

$$Q_{prod}(supplier) \leq ProdCap(supplier) \forall supplier$$

Make the constraints useable:

We need to apply the following steps for usability in gams:

1. introduce lower bound
2. split into 2 sepperate equations
3. For Laplace: bring constraints into standard form (constraint $\leq 0 \perp$ dual-Variable)
4. For GAMS: change equations to ≥ 0 (for gams)

For transport constraint:

1. introduce lower bound

$$0 \leq Q_{sell}(supplier, region) \leq transCap(supplier, region) \forall supplier, regions$$
2. split into 2 sepperate equations

$$0 \leq Q_{sell}(supplier, region) \forall supplier, regions$$

$$Q_{sell}(supplier, region) \leq transCap(supplier, region) \forall supplier, regions$$
3. For Laplace: bring constraints into standard form

$$-Q_{sell}(supplier, region) \leq 0 : \mu_{transCapLow} \forall supplier, regions$$

$$Q_{sell}(supplier, region) - transCap(supplier, region) \leq 0 : \mu_{transCapUp} \forall supplier, regions$$
4. For GAMS: change equations to ≥ 0

$$Q_{sell}(supplier, region) \geq 0 \forall supplier, regions$$

$$transCap(supplier, region) - Q_{sell}(supplier, region) \geq 0 \forall supplier, regions$$

For selling cap constraint:

1. introduce lower bound

$$0 \leq \sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \forall supplier$$
2. split into 2 sepperate equations

$$0 \leq \sum_{region} Q_{sell}(supplier, region) \forall supplier$$

$$\sum_{region} Q_{sell}(supplier, region) \leq Q_{prod}(supplier) \forall supplier$$
3. For Laplace: bring constraints into standard form

$$-\sum_{region} Q_{sell}(supplier, region) \leq 0 : \mu_{sellCapLow} \forall supplier$$

$$\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier) \leq 0 : \mu_{sellCapUp} \forall supplier$$
4. For GAMS: change equations to ≥ 0

$$\sum_{region} Q_{sell}(supplier, region) \geq 0 \forall supplier$$

$$Q_{prod}(supplier) - \sum_{region} Q_{sell}(supplier, region) \geq 0 \forall supplier$$

For production cap constraint:

1. introduce lower bound

$$0 \leq Q_{prod}(supplier) \leq ProdCap(supplier) \forall supplier$$

2. split into 2 sepperate equations

$$0 \leq Q_{prod}(supplier)$$

$$Q_{prod}(supplier) \leq ProdCap(supplier)$$

3. For Laplace: bring constraints into standard form

$$-Q_{prod}(supplier) \leq 0 : \mu_{prodCapLow}$$

$$Q_{prod}(supplier) - ProdCap(supplier) \leq 0 : \mu_{prodCapUp}$$

4. For GAMS: change equations to ≥ 0

$$Q_{prod}(supplier) \geq 0$$

$$ProdCap(supplier) - Q_{prod}(supplier) \geq 0$$

Object function

$$\begin{aligned} &max \sum_{region} Q_{sell}(supplier, region) * (price(region) - C_{trans}(supplier, region)) \\ &- Q_{prod}(supplier) * C_{prod}(supplier) \forall supplier \end{aligned}$$

→ **chang max to min:**

$$\begin{aligned} &min \sum_{region} Q_{sell}(supplier, region) * (C_{trans}(supplier, region) - price(region)) \\ &+ Q_{prod}(supplier) * C_{prod}(supplier) \forall supplier \end{aligned}$$

→ **add constraints:**

$$\begin{aligned} &-\mu_{transCapLow} * Q_{sell}(supplier, region) \\ &+\mu_{transCapUp} * (Q_{sell}(supplier, region) - transCap(supplier, region)) \\ &-\mu_{sellCapLow} * \sum_{region} Q_{sell}(supplier, region) \\ &+\mu_{sellCapUp} * (\sum_{region} Q_{sell}(supplier, region) - Q_{prod}(supplier)) \\ &-\mu_{prodCapLow} * Q_{prod}(supplier) \\ &+\mu_{prodCapUp} * (Q_{prod}(supplier) - ProdCap(supplier)) \end{aligned}$$

derivation obj for supplier (Q_{sell})

$$\begin{aligned} \frac{\partial f}{\partial Q_{sell}} &= \sum_{region} (C_{trans}(supplier, region) - price(region)) \\ &-\mu_{transCapLow} \\ &+\mu_{transCapUp} \end{aligned}$$

$$-\sum_{region} \mu_{sellCapLow}$$

$$+\sum_{region} \mu_{sellCapUp}$$

$$\forall supplier$$

derivation obj for supplier (Q_{prod}):

$$\frac{\partial f}{\partial Q_{prod}} = C_{prod}(supplier)$$

$$-\mu_{sellCapUp}$$

$$-\mu_{prodCapLow}$$

$$+\mu_{prodCapUp}$$

$$\forall supplier$$