Bilevel - GAMS Model Explanation

Sets

- region: {NorthAmerica, EU, Russia, OPEC, FarEast}
- leader $\subseteq$ region: {OPEC}
- follower $\subseteq$ region: {NorthAmerica, EU, Russia, FarEast}

Parameters

model horizon is 1 day

Base Utility for Each Consumer Dollar/M Barrel $U^c_{\mathbf{base}}$

- $U^{\text{NorthAmerica}}_{\text{base}} = 10$
- $U^{\text{EU}}_{\text{base}} = 10$
- $U^{\text{Russia}}_{\text{base}} = 10$
- $U^{\text{OPEC}}_{\text{base}} = 10$
- $U^{\text{FarEast}}_{\text{base}} = 10$

Maximum Consumption for Each Consumer M BarrelC^c_{max}

- $C^{\text{NorthAmerica}}_{\text{max}} = 20$
- $C^{\text{EU}}_{\text{max}} = 20$
- $C^{\text{Russia}}_{\text{max}} = 20$
- $C^{\text{OPEC}}_{\text{max}} = 20$
- $C^{\text{FarEast}}_{\text{max}} = 20$

Production Capacity for Each Supplier M Barrel P^s_{max}

- $P^{\text{NorthAmerica}}_{\text{max}} = 19.25$
- $P^{\text{EU}}_{\text{max}} = 3.75$
- $P^{\text{Russia}}_{\text{max}} = 11.7$
- $P^{\text{OPEC}}_{\text{max}} = 33.87$
- $P^{\text{FarEast}}_{\text{max}} = 3.9691$

Interception Point of the Inverse Demand Function for Each Consumer Dollar per M Barrel I_c

- $I_{\text{NorthAmerica}} = 0.190556$
- $I_{\text{EU}} = 2.21038$
- $I_{\text{Russia}} = 0.24444$
- $I_{\text{OPEC}} = 0.23766$
- $I_{\text{FarEast}} = 2.77584$

Slope of the Inverse Demand Function for Each Consumer Dollar Per M Barrel^2 S_c

- $S_{\text{NorthAmerica}} = -0.00008$
- $S_{\text{EU}} = -0.00016$
- $S_{\text{Russia}} = -0.00005$
- $S_{\text{OPEC}} = -0.00002$
- $S_{\text{FarEast}} = -0.00014$

Linear Cost Factor in the Cost Function of Each Supplier Dollar per M Barrel Cost$_s$

- $\text{Cost}_{\text{NorthAmerica}} = 22.85$
- $\text{Cost}_{\text{EU}} = 29.09$
- $\text{Cost}_{\text{Russia}} = 19.21$
- $\text{Cost}_{\text{OPEC}} = 9.39$
- $\text{Cost}_{\text{FarEast}} = 29.9$

Transportation Cost from Supplier s to Consumer c $T^{s,c}_{\text{cost}}$ Dollar M Barrel

	NorthAmerica	EU	Russia	OPEC	FarEast
NorthAmerica	0	91.63	91.63	126.48	126.48
EU	91.63	0	7.33	57.17	152.46
Russia	91.63	7.33	0	57.17	152.46
OPEC	126.48	57.17	57.17	0	95.29
FarEast	126.48	152.46	152.46	95.29	0

Transportation Capacity from Supplier s to Consumer c $T^{s,c}_{\max}$ M Barrel

	NorthAmerica	EU	Russia	OPEC	FarEast
NorthAmerica	35	35	35	35	35
EU	35	35	35	35	35
Russia	35	35	35	35	35
OPEC	35	35	35	35	35
FarEast	35	35	35	35	35

Total Cost per Unit of Oil from Supplier s to Consumer c $\text{cost}^{s,c}$

Dollar per M Barrel

$$\text{cost}^{s,c} = \text{Cost}_s + T^{s,c}_{\text{cost}}$$

Variables

- Π_{leader}: Negative Profit of the leader
- p_c: Regional marginal price for each consumer c
- Π^f_{follower}: Profit of each follower f
- d_c: Demand for each consumer c
- $q_{f,c}$: Quantity sold by follower f to consumer c
- $q_{l,c}$: Quantity sold by leader to consumer c
- $q_{\text{sold},c}$: Sum of quantities sold to consumer c

Positive Variables (Duals denoted by μ)

- $\mu^c_{d,\min}$: Lower limit dual of d_c
- $\mu^c_{d,\max}$: Upper limit dual of d_c
- $\mu^{f,c}_{qf,\min}$: Lower limit dual of q_{fc}
- $\mu^{f,c}_{qf,\max}$: Upper limit dual of q_{fc}
- $\mu^{l,c}_{ql,\min}$: Lower limit dual of q_{lc}
- $\mu^{l,c}_{ql,\max}$: Upper limit dual of q_{lc}

Binary Variables (denoted by ψ)

- $\psi^{c}_{d,\min}$
- $\psi^{c}_{d,\max}$
- $\psi^{f,c}_{qf,\min}$
- $\psi^{f,c}_{qf,\max}$
- $\psi^{l,c}_{ql,\min}$
- $\psi^{l,c}_{ql,\max}$

Scalars

- Big Ms: $M^{d}_{1,\min}$, $M^{d}_{1,\max}$, $M^{qf}_{1,\min}$, $M^{qf}_{1,\max}$, $M^{ql}_{1,\min}$, $M^{ql}_{1,\max}$ - Big Ms: $M^{d}_{2,\min}$, $M^{d}_{2,\max}$, $M^{qf}_{2,\min}$, $M^{qf}_{2,\max}$, $M^{ql}_{2,\min}$, $M^{ql}_{2,\max}$

Equations

Objective Functions

Non-Linear Original Objective Function

$$\text{of}: \quad \Pi_{\text{leader}} = \sum_{(l,c)} \left(\text{cost}^{l,c} \cdot q_{l,c} - p_c \cdot q_{l,c} \right)$$

Linear Objective Function

$$\text{of}: \quad \Pi_{\text{leader}} = \sum_{c} \left(- \sum_{l} q_{l,c} \cdot \text{cost}^{l,c} - \sum_{f} q_{f,c} \cdot \text{cost}^{f,c} - \sum_{f} T^{f,c}_{\max} \cdot qf^{f,c}_{\max} \right) + \sum_{c} U^{c}_{\text{base}} \cdot d_c - \sum_{c} C^{c}_{\max} \cdot d^{c}_{\max}$$

Upper Level Constraints

Production Capacity for Leader

$$\text{production\_cap\_l}(l): \quad \sum_{c} q_{l,c} \leq P^{l}_{\max}$$

Production Capacity for Follower

$$\text{production\_cap\_f}(f): \quad \sum_{c} q_{f,c} \leq P^{f}_{\max}$$

Lagrangian Equilibrium Equations for Followers and Consumers

Leader's Equilibrium

$$\text{leaderEqu}(l,c): \quad \text{cost}^{l,c} - p_c - \mu^{l,c}_{ql,\min} + \mu^{l,c}_{ql,\max} = 0$$

Follower's Equilibrium

$$\text{followerEqu}(f,c): \quad \text{cost}^{f,c} - p_c - \mu^{f,c}_{qf,\min} + \mu^{f,c}_{qf,\max} = 0$$

Consumer's Equilibrium

$$\text{consumerEqu}(c): \quad p_c - U^c_{\text{base}} + \mu^c_{d,\max} - \mu^c_{d,\min} = 0$$

Overall Balance

$$\text{balanceEqu}(c): \quad d_c - \sum_l q_{l,c} - \sum_f q_{f,c} = 0$$

Price Equation

$$\text{price}(c): \quad p_c = I_c + S_c \cdot d_c$$

Constraints

Primals

Minimum Demand

$$\text{eq1\_d\_min}(c): \quad d_c \leq \psi^c_{d,\min} \cdot M^d_{1,\min}$$

Maximum Demand

$$\text{eq1\_d\_max}(c): \quad C^c_{\max} - d_c \leq \psi^c_{d,\max} \cdot M^d_{1,\max}$$

Minimum Quantity Sold by Follower

$$\text{eq1\_qf\_min}(f,c): \quad q_{f,c} \leq \psi^{f,c}_{qf,\min} \cdot M^{qf}_{1,\min}$$

Maximum Quantity Sold by Follower

$$\text{eq1\_qf\_max}(f,c): \quad T^{f,c}_{\max} - q_{f,c} \leq \psi^{f,c}_{qf,\max} \cdot M^{qf}_{1,\max}$$

Minimum Quantity Sold by Leader

$$\text{eq1\_ql\_min}(l,c): \quad q_{l,c} \leq \psi^{l,c}_{ql,\min} \cdot M^{ql}_{1,\min}$$

Maximum Quantity Sold by Leader

$$\text{eq1\_ql\_max}(l,c): \quad T^{l,c}_{\max} - q_{l,c} \leq \psi^{l,c}_{ql,\max} \cdot M^{ql}_{1,\max}$$

Duals

Dual Minimum Demand

$$\text{eq2\_d\_min}(c): \quad \mu^c_{d,\min} \leq (1 - \psi^c_{d,\min}) \cdot M^d_{2,\min}$$

Dual Maximum Demand

$$\text{eq2\_d\_max}(c): \quad \mu^{c}_{d,\max} \leq (1 - \psi^{c}_{d,\max}) \cdot M^{d}_{2,\max}$$

Dual Minimum Quantity Sold by Follower

$$\text{eq2\_qf\_min}(f,c): \quad \mu^{f,c}_{qf,\min} \leq (1 - \psi^{f,c}_{qf,\min}) \cdot M^{qf}_{2,\min}$$

Dual Maximum Quantity Sold by Follower

$$\text{eq2\_qf\_max}(f,c): \quad \mu^{f,c}_{qf,\max} \leq (1 - \psi^{f,c}_{qf,\max}) \cdot M^{qf}_{2,\max}$$

Dual Minimum Quantity Sold by Leader

$$\text{eq2\_ql\_min}(l,c): \quad \mu^{l,c}_{ql,\min} \leq (1 - \psi^{l,c}_{ql,\min}) \cdot M^{ql}_{2,\min}$$

Dual Maximum Quantity Sold by Leader

$$\text{eq2\_ql\_max}(l,c): \quad \mu^{l,c}_{ql,\max} \leq (1 - \psi^{l,c}_{ql,\max}) \cdot M^{ql}_{2,\max}$$

Model and Solution

Model stckl /all/;
solve stckl using minlp (or mip) minimizing Π_{leader};
display p.l, ql.l, qf.l, d.l, ;