FORM 3

PROPOSAL MODERATION

BAMS3216 PROJECT

Student's Name: Yee Jun Hoong **Programme: RMM2G3**

Supervisor's Name: Chong Kam Yoon **Date: 27 March 2021**

Project Title: Recent Development and Analysis of Taylor Series

Proposal Moderation (by Moderator) (√)

Project Requirements	**Comply**	**Does not Comply**
Math content. The project has substantial amount of Mathematics content.		
Concepts or Techniques. The project requires the students to apply various concepts or techniques learnt.		
Knowledge Expansion. The project allows the students the opportunity to expand their existing knowledge.		
Scope. The project should be of scope acceptable within the limits of resources and capability of students.		

Project Scope Moderation (by Moderator)

Changes Recommended (write 'None' if no changes are required)	**Actions Taken** (by Supervisor)

Moderator's Signature: ___________________________

Moderator Name: ________________________________

Proposal

BAMS3216 Project

Student: Yee Jun Hoong	**ID: 20WMR09194**
Supervisor: Chong Kam Yoon	**Date: 27 March 2021**
Title: Recent Development and Analysis of Taylor Series	

Introduction and Definitions:

In mathematics, a function can be represented by the Taylor series as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point. The concept of a Taylor series and its name was formally introduced by the English mathematician Brook Taylor. If the Taylor series is centered at zero, then the series can also be called as Maclaurin series, named after the Scottish mathematician Colin Maclaurin. It is common practice to use a finite number of terms of Taylor series to approximate a function. The Taylor series may be regarded as the limit of the Taylor polynomials.

The definition of the Taylor series is a series expansion of a function about a point. A one-dimensional Taylor series is an expansion of a real function $f(x)$ about a point $x=a$ is given by;

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^3(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots \tag{1.1}$$

The expansion is known as a Maclaurin Series if $a=0$. The equation (1.1) can be written in the more compact sigma notation as follows:

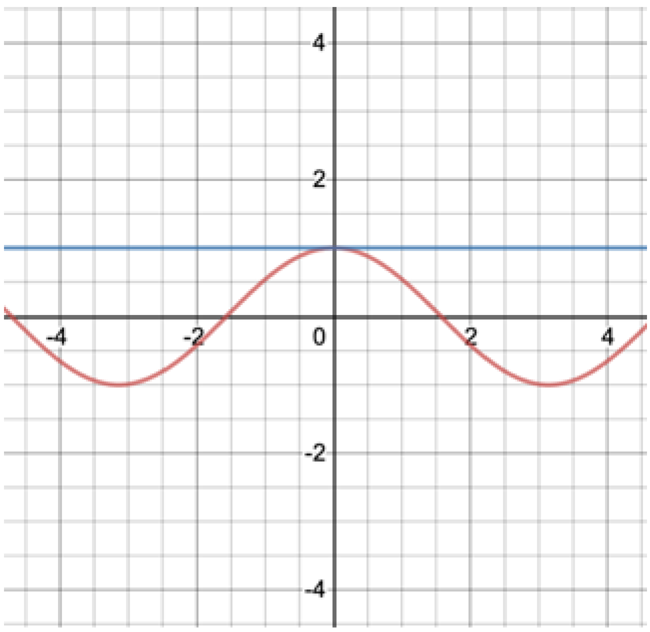
$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \tag{1.2}$$

The $n!$ in the equation (1.2) is mathematical notation for factorial n and $f^{(n)}(a)$ denotes the nth derivation of function f evaluated at the point a. The $0th$ derivation of f is defined to be f itself and both $(x-a)^0$ and $0!$ by their mathematical definitions are set equal to 1.

Literature Review:

Linear functions are one of the simplest types of functions that we can have, which is in the form of $y = mx + b$. Linear functions are easy to graph and to evaluate because the graph of a linear function is a straight line and there are only 1 independent variable (x) and 1 dependent variable (y) in the linear function. For example, we have a complicated function like $f(x) = \cos(x)$ that is hard to evaluate precisely without the help of a calculator or other mathematics tools. If we wanted to estimate the value of $\cos(x)$ near $x = 0$, we could do the value estimation with a linear approximation. Then, We will construct a linear function $g(x) = mx + b$ that is close to $f(x) = \cos(x)$ at $x = 0$. Next, we should think about what values of b and m give us the best approximation of $f(x) = \cos(x)$.

As we are looking for an approximation near $x = 0$, a suitable starting point is to have $g(x) = f(x)$ at $x = 0$. Since $\cos(0) = 1$, we want $g(0) = m(0) + b = 1$, and $b = 1$. To figure out what value of m works best, we want $g(x)$ to be tangent to $f(x)$ at $x = 0$, so the slope m should be equal to $f'(0) = -\sin(0) = 0$. This gives us $g(x) = 1$ as the linear approximation. We can see $f(x) = \cos(x)$ and $g(x) = 1$ as the red curve and the blue curve respectively as follows:



(2.1)

From the graph above, we can see that for x close to zero, a reasonable approximation of $\cos(x)$ will be $g(x) = 1$. We can also conclude that for very small values of x, the $\cos(x)$ is close to 1. However, the approximation is not good enough to approximate the larger values of x.

In my humble two cents, I think that the author explains what Taylor series can do in a very simple and easy-to-understand way. From the example above, I also know that whenever I want to approximate a complicated function numerically, I can take the first few terms of the Taylor series expansion of the function to get a very accurate polynomial approximation.

Objectives:

Taylor series is fundamental to the understanding of mathematical analysis in both theoretical and practical terms in the fields of mathematics. Therefore, I carried out research on recent development and analysis of Taylor series to further expand my understanding of mathematical analysis. I will find reviews of Taylor series from others and to learn from it by analyzing it. Other than that, I will also recreate the missing steps in journals, articles or books that are related to the Taylor series.

Methodology:

During the research on recent development and analysis of Taylor series, I will use mathematics software such as Matlab to calculate, expand and verify Taylor series.

```
syms x
f = 1/(5 + 4*cos(x));
T = taylor(f, 'Order', 8)
```

(3.1)

The picture (3.1) shown in above is a simple example of using the Matlab to calculate the Taylor series. The code will output the result $T = \frac{(49*x\hat{}6)}{131220} + \frac{(5*x\hat{}4)}{1458} + \frac{(2*x\hat{}2)}{81} + \frac{1}{9}$ which is all the terms up to, but not including, order eight in the Taylor series for $f(x)$, in which is the same as the equation (1.2). Technically, T is a Maclaurin series because its expansion point is $a = 0$.

Time Framework:

Plans and goals schedule	**Expected date of completion (2021)**									
	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Obtain a project title and to understand it										
Literature review										
Preparation of progress report										
Finding Journals										
Reading Journals and understand it										
Learn to use tools such as Matlab										
Result analysis										
Report writing										
Preparation of oral presentation slides										
Documentation of the whole report										

References:

[1] Correa, F. J. (2012, March 23). TAYLOR series - numerical Methods Projects. Retrieved March 23, 2021, from https://sites.google.com/site/numericalmethodsprojects/theory-of-error/taylor-series

[2] Jane. (2005). Where do Taylor series come from and Why do we learn about them? Retrieved March 27, 2021, from http://blog.cambridgecoaching.com/where-do-taylor-series-come-from-and-why-do-we-learn-about-them

[3] Select a web site. (1994). Retrieved March 27, 2021, from https://www.mathworks.com/help/symbolic/taylor-series.html