Progress Report

BAMS3216 Project

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Title: Recent Development and Analysis of Taylor Series	

1. Introduction

1.1 Objective

Taylor series is fundamental to the understanding of mathematical analysis in both theoretical and practical terms in the fields of mathematics. The objective of my project is to enhance my current knowledge of Taylor series, as well as to further expand my understanding of mathematical analysis.

In order to have a better understanding and a clearer vision about the development of Taylor series in recent years, I carried out an in-depth research to analyse, evaluate and use information gathered from journals, books, websites, articles and other formal sources related to Taylor series.

Throughout the progress of the project report, I have browsed some useful and knowledgeable information from journals, books, websites and so on that were related to the Taylor series, and carried out a detailed review about them after I have identified and reviewed the sources. Besides that, finding more journals that are related to the Taylor series to expand my works in the literature review is also being listed in my to-do-list as a part of my plans for my next progress.

The missing steps in the journals will be recreated using mathematical software such as Matlab. Furthermore, my humble opinions such as my thoughts about the journals, what are the positive impacts of the results shown in the journal to the Taylor series and so on will be shared after I have reviewed them.

One of my goals of this project is to make sure that I will be able to build a strong foundation and strengthen my knowledge of Taylor series as I progress step by step in this project. Besides that, one of the goals of this project and will also be the expected outcome is I want to learn about the trend and real-world application of Taylor series because it will be very useful to me when I step into the working field in the near future.

1.2 Project Scope

A description of all the progress I have made so far in this progress report will be provided. Throughout the duration of this project, I have done some reviews on journals, made calculations and expansion of Taylor series using mathematical software such as Matlab, and done other research to enhance my knowledge for this project.

I have reviewed 5 different topics of journals that are related to the Taylor series. After a discussion with my supervisor, Mr. Chong Kam Yoon, I have chosen the topic of "History of Taylor series" as my first journal to review. The following topic is able to lead me to learn about who invented Taylor series and what is Taylor series, which has helped me to build a stronger foundation in Taylor series.

Next, the second topic I reviewed was "A new type of Taylor series expansion". This topic has shown the value of Taylor series being worthy to do an in-depth discussion, and its potential to be improved. The rest of the 3 topics I have reviewed showed the Taylor series method is being very useful and played an important role in different areas.

To calculate, expand, and to recreate the missing parts of the Taylor series equation, I used Matlab to obtain the results I wanted. In the methodology part, the examples were given to show how to use the Matlab to do the calculation, plotting and to obtain results.

Last but not least, all the journals, books, articles and so on that I have reviewed, the reference is provided in the last section of this progress report to acknowledge the works of the authors.

1.3 Planning

I have made a table of contents including the schedule of my plans and goals and the contents of the to-do-list I want to accomplish in my next progress.

Plans and goals schedule	Expected date of completion (2021)					
	July	Aug	Sept	Oct	Nov	Dec
Modify and enhance the current journals	██					
Find more Journals	██	██				
Use Matlab to add in more calculations and expansion of Taylor series equations.	██	██	██			
Analyse and obtain result	██	██	██			
Writing the final report	██	██	██	██	██	
Preparation of oral presentation slides				██	██	
Documentation of the whole report					██	██

2. Literature Review

2.1 History of Taylor series

The articles below showed us how the Taylor series was invented, and what the Taylor series is. Besides that, it also showed the history of the Maclaurin series and how it is related to the Taylor series.

2.1.1 Taylor series

A Scottish mathematician named James Gregory has formulated the concept of the Taylor series. The Taylor series are named after a English mathematician, Brook Taylor, who formally introduced the Taylor series in 1715.

The Taylor series represents a function as an infinite sum of terms calculated from the values of its derivatives at a single point. The Taylor series can also be regarded as the limit of the Taylor polynomials.

2.1.2 History of Maclaurin series

The Maclaurin series is the expansion of the Taylor series about 0, and it can be recognized as a special case of Taylor series.

If the series is centered at zero, the series will be called the Maclaurin series, which was named after the Scottish mathematician Colin Maclaurin. In the 18th century, Colin Maclaurin made extensive use of this Maclaurin series.

2.2 A new type of Taylor series expansion

A new type of Taylor series expansion will be introduced by presenting a variant of the classical integration by parts.

This journal presented some closed forms for integrals involving Jacobi and Laguerre polynomials, and an error analysis was given in the sequel for the introduced expansion.

The main goal of this journal is to present a new type of Taylor series expansion through a variant of the classical integration by parts.

In conclusion, this journal provided good examples and very detailed error analysis, which has shown the development of Taylor series and its great potential to be improved.

2.3 Taylor series solution for Lane-Emden equation

The Lane-Emden equation plays a significant role in physics, chemistry and astronomy. In this journal, it showed the suggestion of the simplest method, which is the Taylor series technology to solve the Lane-Emden equation.

This journal gave a suggestion of a simple approach to the Lane-Emden equation, with only a total of 18 steps. The idea in this journal can be extended to all differential equations with initial conditions and fractional calculus.

In this journal, it has concluded that the Taylor series method is straightforward compared with other analytical methods because it has a simple solutions process and accurate results. The redundant terms will not be produced, and the series converges to the exact solution is the most advantageous method of the Taylor series method.

2.4 Taylor series solution for a third order boundary value problem arising in Architectural Engineering

Recently, I found a paper that is about how the Taylor series is used in solving a third order boundary value problem arising in Architectural Engineering.

By clarifying the problem, giving an example and a summary discussion, this paper has successfully applied the technology to the third-order boundary value problems.

The simple solution process and accurate results shown in this paper, have also proved that the Taylor series method is much more attractive for practical applications.

In conclusion, I think that this paper has shown that the Taylor series method is not only extremely simple and straightforward, but also reliable and effective. Furthermore, this paper has shown that the Taylor series is very useful in different areas.

2.5 An improved C4.5 model classification algorithm based on Taylor series

C4.5 is one of the most well known algorithms for rule base classification. This journal proposed the modification of the splitting information of C4.5 by using the Taylor series method which brought the result of a modified model, EC4.5.

The proposed modification offers solutions to overcome the limitations of C4.5. The result obtained by using the Taylor series method suggested a far better result than when the C4.5 was introduced.

Taylor series has been proved to be advantageous and beneficial in a vast range of different areas such as bringing improvement and efficiency to the problems, and delivering a better and effective result.

3. Preliminary Results

3.1 Methodology/Model

Throughout the progress of the progress report, I used the mathematical tool such as Matlab to verify, calculate, expand and recreate missing parts of Taylor series equations.

3.1.1 Taylor series

```
syms x
f = 1/(5 + 4*cos(x));
T = taylor(f, 'Order', 8)
```

(3.1.1a)

The picture (3.1.1a) shown above is a simple example of using Matlab to calculate the Taylor series. By declaring the x by using syms x, it allowed us to use the x variable to calculate the following equations. Let $f = \dfrac{1}{5 + 4\cos x}$, and calculate the T with $T = taylor(f, 'Order', 8)$ will produce the output result of $T = \dfrac{(49 * x^\wedge 6)}{131220} + \dfrac{(5 * x^\wedge 4)}{1458} + \dfrac{(2 * x^\wedge 2)}{81} + \dfrac{1}{9}$, which is all the terms up to, but not more than order eight in the Taylor series for $f(x)$ based on the code typed in for the result T.

```
syms x
g = exp(x*sin(x));
t = taylor(g, 'ExpansionPoint', 2, 'Order', 12);
```

(3.1.1b)

The picture (3.1.1b) showed a similar result compared to (3.1.1a), but with different equations, which generated the expanded result of the first 12 nonzero terms of the Taylor series for $g = \exp(x\sin(x))$ about x=2.

```
t = simplify(t);
size(char(t))
```

$$(3.1.1c)$$

The input $t = simplify(t);$ shown in (3.1.1c) was used to simplify its presentation. Due to the t being a large expression, the input $size(char(t))$ was entered to discover that t has about one hundred thousand characters in its printed form.

```
xd = 1:0.05:3;
yd = subs(g,x,xd);
fplot(t, [1, 3])
hold on
plot(xd, yd, 'r-.')
title('Taylor approximation vs. actual function')
legend('Taylor','Function')
```

(3.1.1d)
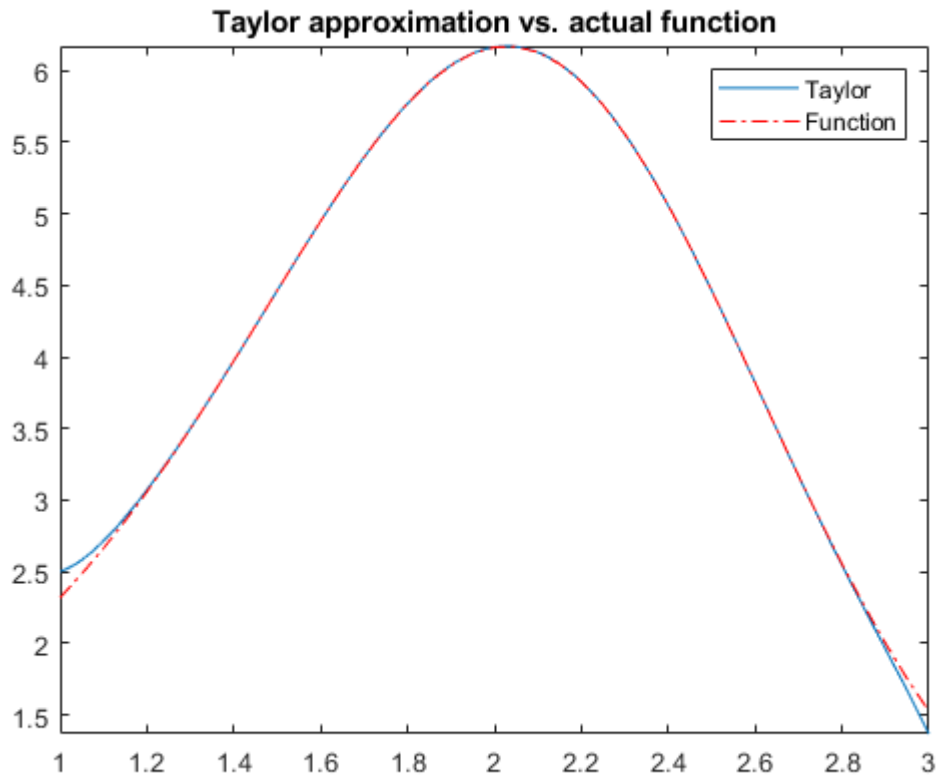
(3.1.1e)

These functions will be plotted together to know how well this Taylor approximation compares to the actual function g as shown in (3.1.1d). The graph in the picture (3.1.1e) was the result we got for the example.

In conclusion, this example has shown the ease of using Matlab to calculate, expand and plot a graph for the Taylor series and produce a good result.

3.2 Classification/Discussion

Throughout the work I have done for this project, I have faced some problems. When I first started with the progress report, one of the problems that I have faced was I cannot find journals that are related to the Taylor series, and are in recent years.

After searching multiple times by using online browsers such as Google and Youtube, I have come to a solution of using Google Scholar. For example, I needed to find recent journals about Taylor series, therefore, the Google Scholar came in handy to me because it has a really convenient setting for the range of years. I can set the range of years and find the journals I want with ease. Furthermore, Google Scholar brought the convenience and efficiency of being able to cite the selected website easily, which saved me a lot of time without needing to cite on the other specific website.

Although Google Scholar has a lot of advantages, I have met a problem when using it. Google Scholar is very keyword sensitive. For instance, if I type in a long sentence, such as "Recent development and analysis of Taylor series", it may show me the recent development of other things that are not related to the Taylor series.

To solve this problem, my supervisor has given me a solution which is a better online browser, the TARUC libraries online browser. The TARUC browser has all the advantages that Google Scholar has, but overall it is better than it. With all the solutions to my problems, I have accomplished my progress report smoothly.

4. References

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