RECENT DEVELOPMENT AND ANALYSIS OF TAYLOR SERIES

By

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By

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DECLARATION

The project report submitted herewith is a result of my own efforts in totality and in every aspects of the works. All information that has been obtained from other sources had been fully acknowledged. I understand that any plagiarism, cheating or collusion of any sorts constitutes a breach of Tunku Abdul Rahman University College rules and regulations and would be subjected to disciplinary actions.

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APPROVAL FOR SUBMISSION

I certify that this project report entitled **"RECENT DEVELOPMENT AND ANALYSIS OF TAYLOR SERIES"** was prepared by **YEE JUN HOONG** and has met the required standard for submission in partial fulfillment of the requirements for the award of Bachelor of Science (Hons.) Management Mathematics with Computing at Tunku Abdul Rahman University College.

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Date: ___________________________ **Date:** ___________________________

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ABSTRACT

In mathematics, a Taylor series function can be known as an infinite sum of terms that are expressed in terms of derivatives of the function at a single point. The function and the sum of its Taylor series are equal near this point for most common functions. In this project, I present research about the development and analysis of Taylor series in recent years, in order to produce results by calculating, simplifying, and recreating the missing parts of Taylor series equation in the formal sources.

Keywords: Taylor series, Maclaurin series, Matlab, Integration by parts, Recent development of Taylor series, Taylor series expansion.

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CHAPTER 1

INTRODUCTION

1.1 Background

Taylor series is fundamental to the understanding of mathematical analysis in both theoretical and practical terms in the fields of mathematics.

In order to have a better understanding and a clearer vision about the development of Taylor series in recent years, I carried out an in-depth research to analyse, evaluate and use information gathered from formal sources related to Taylor series such as journals, research papers, websites, and so on.

Next, a mathematical tool such as Matlab will be used to give some simple examples related to Taylor series, such as calculate, simplify, and plot graphs for Taylor series functions.

After that, I will expand, recreate, and simplify by manually calculating, using online mathematics calculators such as Wolfram Alpha and Symbolab, or using Matlab based on the Taylor series equations from the formal sources. All the missing and extra steps obtained will be shown in Chapter 4 as the results.

1.2 Aims and Objectives

This research aims to enhance my current knowledge of Taylor series, as well as to further expand and enhance my understanding of mathematical analysis.

As the title for my research is “Recent Development and Analysis of Taylor Series”, the first part of my work will be finding relative information from recent journals, papers, websites, and so on that range from 2018 to 2021. After that, I will read, summarise and classify them by sections, and include them into the literature review section.

Moreover, after I have included others’ works into the literature review, I will review some of the most important works of others, and then I will comment critically on the literature in the area of my research. I will make sure the literature review indicates what diversity of view exists among the authors in the specific area of study.

Other than that, I will use Matlab as a mathematical tool for my research. I will use Matlab to give some simple examples in the methodology section. In addition, any missing steps found in the journals will be recreated using mathematical software such as Matlab. Online mathematics calculators such as Wolfram Alpha will also be used to do calculations if needed.

After I have reviewed others’ works in the literature review, I will pay my effort to produce my own results, by calculating, simplifying, and recreating the missing steps of Taylor series equations. Last but not least, I will conclude what I have learned throughout the development of this project, and the new knowledge I have obtained upon the completion of this project.

The objective of this research is to make sure that I am able to build a strong foundation and strengthen my knowledge of Taylor series as I progress step by step in this research. Besides that, I want to learn about the trend and real-world application of Taylor series, such as how to use Taylor series in real life situations, and how to solve real life problems by using Taylor series. This is due to the fact that I believe it will be very useful to me when I step into the working field in the near future.

CHAPTER 2

LITERATURE REVIEW

2.1 Taylor series

This article will briefly explain how the Taylor series was invented, and what the Taylor series is, as well as the Maclaurin series and how it is related to the Taylor series will also be explained.

2.1.1 History of Taylor series

In mathematics, Taylor series is an expression of a function f for which the derivatives of orders exist at a point a in the domain of f in the form of the power series

$$\Sigma^{\infty}{}_{n=0} f^{(n)}(a)\,(z-a)^n/n!$$

in which Σ denotes the addition of each element in the series as n ranges from zero to infinity, $f^{(n)}$ denotes the nth derivative of f, and $n!$ is the standard factorial function (William L. Hosch, 2021).

The concept of Taylor series is formulated by a Scottish mathematician named James Gregory. The Taylor series is named after an English mathematician, Brook Taylor, who formally introduced the Taylor series in 1715.

The Taylor series represents a function as an infinite sum of terms calculated from the values of its derivatives at a single point. The Taylor series can also be regarded as the limit of the Taylor polynomials.

2.1.2 History of Maclaurin series

The Maclaurin series is the expansion of the Taylor series about 0, which can be recognized as a special case of Taylor series.

$f'(0)$ will be the first derivative evaluated at $x = 0$, $f''(0)$ is the second derivative evaluated at $x = 0$, and so on and so forth.

If the series is centered at zero, the series will be called a Maclaurin series, which was named after the Scottish mathematician Colin Maclaurin. In the 18th century, Colin Maclaurin made extensive use of this Maclaurin series (Ltd, 2018).

2.2 A new type of Taylor series expansion

The main aim of this research paper is to introduce a new type of Taylor series expansion through a variant of the classical integration by parts.

Before discussing the new type of Taylor series expansion, this paper starts off with some Taylor series equations and explanations as an introduction. After that, this paper will present the general form of this expansion and consider some interesting cases of it leading to new closed forms for integrals involving Jacobi and Laguerre polynomials. An error analysis is given for the introduced expansion (Masjed-Jamei et al., 2018).

This paper is remarkable in introducing the new type of Taylor series expansion. This is due to the fact that this paper provides a good amount of examples that come with proof, maintain a comfortable flow while introducing and explaining every formula, equations, and so on. Moreover, a clear and easy-to-understand explanation is provided for the error analysis.

In conclusion, this research paper came out with good examples and detailed error analysis, which has shown the development of Taylor series and its great potential to be improved.

2.3 Taylor series solution for Lane-Emden equation

The Lane-Emden equation plays a significant role in physics, chemistry and astronomy. It was reported that a reaction diffusion process can be modelled by Lane-Emden equation, and the process can be optimized by its analytical solution (Ji-Huan & Fei-Yu, 2019).

Although there are many analytical methods for singular boundary value problems, it is relatively difficult to be solved analytically due to its singular property. In this research paper, it will show the suggestion of the simplest method to the problem, which is the Taylor series technology to solve the Lane-Emden equation (Ji-Huan & Fei-Yu, 2019).

This paper is prominent and outstanding in explaining their topics. This is because this paper provides a suggestion of a simple approach to the Lane-Emden equation, with only a total of 18 steps. The idea in this paper can be extended to all differential equations with initial conditions and fractional calculus (Ji-Huan & Fei-Yu, 2019).

This paper has concluded that the Taylor series method is straightforward compared with other analytical methods because it has a simple solutions process and accurate results. The best advantage of the Taylor series method is that the redundant terms will not be produced, and the series converges to the exact solution (Ji-Huan & Fei-Yu, 2019).

2.4 Taylor series solution for a third order boundary value problem arising in Architectural Engineering

The boundary value problems can be used to model many architectural systems. Although there are many numerical and analytical methods to solve such problems, this research is able to suggest a simple yet effective way to the third-order ordinary differential equations by the Taylor series technology (He, 2020).

By clarifying the problem, giving an example and a summary discussion, this paper has successfully applied the technology to the third-order boundary value problems. A comparison is made between the approximate solution obtained in this paper with the variational iteration method, which shows that the result of the present method is simpler and more straightforward (He, 2020).

This paper shows the simple solution process and accurate results, which will prove that the Taylor series method is much more attractive for practical applications (He, 2020).

There are three obvious advantages stated in this paper which are that the solution procedure is simple and straightforward, making it valid for various boundary value problems and initial value problems. Besides that, the series solution converges to the exact solution, and any accuracy can be obtained for a practical problem. Furthermore, the method can be extended to complex boundary conditions (He, 2020).

As a conclusion, this paper has shown a favourable development of the Taylor series through the method in this paper. It also showed that the Taylor series is reliable and effective with the three advantages. Moreover, it indicated that the Taylor series is very useful in different areas.

2.5 An improved C4.5 model classification algorithm based on Taylor series

C4.5 is one of the most well known algorithms for rule base classification. The C4.5 algorithm is an improvement of the ID3 algorithm. This research paper proposes a technique that will handle the setback reported in C4.5. The performance of the proposed technique is measured based on better accuracy (Idriss & Lawan, 2019).

To overcome the limitations of C4.5, the researchers used Taylor series to modify the splitting information of C4.5, which brought the result of a modified model that can be called EC4.5. The researchers apply exponential splitting information, EC4.5, in utilizing the central attribute of the same dataset, which brought the result obtained on introducing Taylor series suggesting a far better result than when the C4.5 was introduced (Idriss & Lawan, 2019).

The proposed modification offers solutions to the limitations associated with C4.5 in terms of presenting an equivalent result with ID3 when the same number of attributes is used. Idriss & Lawan (2019) states that the result of the experiment shows that EC4.5 outperformed, with an accuracy of 99.40%, whereas C4.5 has an accuracy of 51.27%.

Based on the result obtained in this research paper, we can summarize that the Taylor series has been proved to be advantageous and beneficial in a vast range of different areas such as bringing improvement and efficiency to the problems, and delivering a better and effective result. This is because the result obtained in this paper by using the Taylor series method (EC4.5) suggested a far better result than when the C4.5 was introduced.

Based on the result of this research, EC4.5 is selected as the optimal algorithm. Thanks to this research, future work can and is suggested to consider a hybrid approach to handle multi-dimensional data with large intervals using EC4.5 algorithm (Idriss & Lawan, 2019).

2.6 A numerical method based on Taylor series for bifurcation analyses within Föppl-von Karman plate theory

A combination of the asymptotic numerical method (ANW) and the Taylor meshless method (TMM) presents a new numerical technique for post-buckling analysis. These two methods are based on Taylor series, with respect to a scalar load parameter for ANM, and with respect to the space variables for TMM (Tian et al., 2018).

Tian et al. (2018) states that the advantage of ANM is an adaptive step length, which is very efficient near bifurcation points. The specificity of TMM is a quasi-exact solution of the partial differential equations inside the domain, which leads to a strong reduction of the number of degrees of freedom.

In short, the new method shown in this paper is very efficient to solve a quasi-perfect bifurcation response and this does not require a strong numerical expertise. It is possible to combine Taylor series in space and in loading parameters, which can be easily extended to other hyperelastic models or to Newtonian fluids. This double Taylor series leads to an efficient path following technique (Tian et al., 2018).

This paper has done a brilliant job on creating a new method by analyzing the Taylor series with detailed descriptions. This paper provided very thorough explanations and examples while explaining their topic. The method described in this paper is assessed with only a single example. Last but not least, this paper definitely showed a good sign of the non-stoppable development of the Taylor series.

2.7 FPGA implementation of adaptive digital pre-distorter with improving accuracy of lookup table by Taylor series method

In this research paper, it shows the FPGA implementation with improving accuracy of the lookup table by using the Taylor series method.

Power amplifier (PA) is a critical component and inherently non-linear device of modern wireless base stations. Due to the non-linear behaviour of the PA, the amplification of signals with fluctuating envelopes will inevitably lead to loss of adjacent spectrum, adjacent interference and the crossing intermodulation, in-band distortion as well as out-of-band spectral growth in the transmitted signal. The digital pre-distortion (DPD) with the adaptation and updating of the lookup table (LUT) can counteract these non-linear effects (Ren, 2018).

In this Letter, a low-complexity LUT implemented by FPGA of pre-distortion PA lineariser is proposed in order to obtain more accurate linearisation. The algorithm utilises interpolation of the LUT with the method of Taylor series. This experiment showed that the method can be used to obtain the more accurate indexed value of LUT to estimate the PA behaviour for effective DPD (Ren, 2018).

This Letter proposed a method for reducing the LUT indexed quantisation of a DPD system caused by the input value not always being located on the LUT index. Three sections are assigned in this Letter to discuss the detailed issues about the approximation of the LUT with the method of Taylor series implemented by FPGA to increase the performance of the adaptive digital predistorter (Ren, 2018).

This Letter has shown that the Taylor series is a superior method that can be widely used in many other areas such as the FPGA implementation in this Letter.

2.8 Taylor’s series method for solving the nonlinear reaction-diffusion equation in the electroactive polymer film

This research paper analytically solves the nonlinear reaction-diffusion equation in the electroactive polymer film (Usha Rani & Rajendran, 2020).

This mathematical model describes a substrate to form a complex with the immobilized catalyst. The Taylor series method will be applied for an analytical approximation of the substrate in the electroactive polymer film (Usha Rani & Rajendran, 2020).

The obtained analytical solution is used to compare with the numerical results and it is found to be in satisfactory agreement. Present analytical expression is compared with the previous result. Usha Rani & Rajendran (2020) states that the Taylor series method yields a rapidly convergent, easily computable, and rapidly verifiable sequence of analytic approximations that are convenient for parametric simulations.

This paper shows fascinating yet plentiful information when solving its title by using the Taylor series method. Taylor series is undoubtedly a simple, effective, and straightforward method when compared to most of the other methods.

2.9 Full-Wave Computation of the Electric Field in the Partial Element Equivalent Circuit Method Using Taylor Series Expansion of the Retarded Green's Function

This article presents new analytical formulas for the efficient computation of the full-wave electric field generated by conductive, dielectric, and magnetic media in the framework of the partial element equivalent circuit (PEEC) method (Kovacevic-Badstuebner et al., 2020).

To this aim, the full-wave Green's function is handled by the Taylor series expansion leading to three types of integrals for which new analytical formulas are provided in order to avoid slower numerical integration (Kovacevic-Badstuebner et al., 2020).

This article is indeed a very detailed article that explains its own title enormously well. It covers almost every aspect of the title by applying the Taylor series expansion wisely.

This article presents a fast computation of the electric field radiated by electrical currents, magnetization, and charges, in the framework of the PEEC full-wave modeling, assuming an orthogonal tessellation of volumes and surfaces (Kovacevic-Badstuebner et al., 2020).

In short, this article has shown a brilliant development of the Taylor series and how beneficial and effective it is when applied in many different areas.

2.10 Analytical Evaluation of Partial Elements Using a Retarded Taylor Series Expansion of the Green's Function

The computation of the fundamental magnetic and electric field coupling terms for the full-wave partial element equivalent circuit method is time consuming for large problems (Lombardi et al., 2018).

Therefore, this paper presents new full-wave analytical formulas for their computation, which are based on several different Taylor series expansion approaches for the case of rectangular elementary volumes and surfaces (Lombardi et al., 2018).

This paper provides a lot of examples, steps, graphs, plots, and so on, in order to give us the idea of how to apply the Taylor series expansion to its title. The algorithms in this paper represent a new approach for the computation of partial elements for PEEC with retardation (Lombardi et al., 2018).

The development of Taylor series has been successfully shown in this paper as the new formulation using the Taylor series expansion removes the frequency dependence from the integral part. Moreover, the errors in the evaluation of the formulas are sufficiently small for the analytical formulas (Lombardi et al., 2018).

2.11 Taylor series and twisting-index invariants of coupled spin-oscillators

In this paper, the list of invariants for the coupled spin-oscillator is completed by calculating higher order terms of the Taylor series invariant and by computing the twisting index (Alonso et al., 2019).

This paper successfully proves that the Taylor series invariant has certain symmetry properties that make the even powers in one of the variables vanish and allow to show superintegrability of the coupled spin-oscillator on the zero energy level (Alonso et al., 2019).

A lot of inspiring ideas are shown to us in this paper in order to achieve the results they want. A lot of theorems made in this paper come with very thorough and detailed explanations and examples.

This paper carries out a detailed analysis on the Taylor series, and successfully applied it on the title of this paper, and shows us a qualified and proficient development of Taylor series.

CHAPTER 3

METHODOLOGY

3.1 Matlab

In this Methodology section, the mathematical tool Matlab will be used to calculate, expand, plot, and simplify the Taylor series equations. Examples are provided in the section below.

3.1.1 Taylor Series Expansion

(3.1.1a)

```
syms x
f = exp(-x)
fprintf('==============================================')

%-----------------------------
taylor(f)
fprintf('==============================================')
% The default truncation order is 6.

pretty(taylor(f))
fprintf('==============================================')
% Prints the answer in a plain-text format that resembles typeset mathematics.

taylor(f, 'ExpansionPoint', 1)
taylor(f, x, 1)
% Find the Taylor series expansions at x=1 for f.
```

The diagram (3.1.1a) indicates the code used in the Matlab and the result of it. First, I show the truncation order for f $= e^{-x}$ by using taylor(f). It shows a default truncation order for f which is 6.

Next, the "pretty" symbolic expression is used to print the answer in a plain-text format. The difference in between is shown in the diagram (3.1.1b).

After that, to find the Taylor series expansion at x = 1 for f, I use "ExpansionPoint" to specify a specific expansion point. Alternatively, specify the expansion point as the third argument of taylor will also work as shown in the diagram (3.1.1b).

(3.1.1b)

f = e^{-x}

```
==============================================
ans =
```

$$1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{120}$$

```
==============================================
        2    3    4     5
       x    x    x     x
1 - x + -- - -- + -- - ---
        2    6   24   120
==============================================
ans =
```

$$e^{-1} - e^{-1}\,(-1+x) + \frac{e^{-1}\,(-1+x)^2}{2} - \frac{e^{-1}\,(-1+x)^3}{6} + \frac{e^{-1}\,(-1+x)^4}{24} - \frac{e^{-1}\,(-1+x)^5}{120}$$

```
ans =
```

$$e^{-1} - e^{-1}\,(-1+x) + \frac{e^{-1}\,(-1+x)^2}{2} - \frac{e^{-1}\,(-1+x)^3}{6} + \frac{e^{-1}\,(-1+x)^4}{24} - \frac{e^{-1}\,(-1+x)^5}{120}$$

3.1.2 Maclaurin Series of Univariate Expressions

(3.1.2a)

```
syms x

T1 = taylor(exp(x))
T2 = taylor(sin(x))
T3 = taylor(cos(x))
% Find the Maclaurin series expansions

sympref('PolynomialDisplayStyle', 'ascend');
fprintf('==============================================')
T1
T2
T3
% Use the sympref function to modify the output order of symbolic
% polynomials, and redisplay it in ascending order.

sympref('default');
```

The diagram (3.1.2a) shows the Maclaurin series expansions of e^x, $\sin x$, and $\cos x$. The "sympref" function is used to modify the output order of symbolic polynomials, which redisplay the polynomials in ascending order, such as $T1 = \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1$ will be redisplayed as $T1 = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}$ as shown in the diagram (3.1.2b).

(3.1.2b)

T1 =

$$\frac{x^5}{120}+\frac{x^4}{24}+\frac{x^3}{6}+\frac{x^2}{2}+x+1$$

T2 =

$$\frac{x^5}{120}-\frac{x^3}{6}+x$$

T3 =

$$\frac{x^4}{24}-\frac{x^2}{2}+1$$

==

T1 =

$$1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}$$

T2 =

$$x-\frac{x^3}{6}+\frac{x^5}{120}$$

T3 =

$$1-\frac{x^2}{2}+\frac{x^4}{24}$$

3.1.3 Specify Truncation Order for Maclaurin series expansion

(3.1.3a)

```
syms x

g = sin(x)/x
fprintf('==============================================')
T6 = taylor(g)
% The default truncation order is 6.

T10 = taylor(g, x, 'Order', 10)
T14 = taylor(g, x, 'Order', 14)
% Use Order to control the truncation order.

%------------------------------
fprintf('==============================================')
fplot([T6 T10 T14 g])
xlim([-4 4])
grid on

legend('approximation of sin(x)/x up to O(x^6)', ...
    'approximation of sin(x)/x up to O(x^{10})', ...
    'approximation of sin(x)/x up to O(x^{14})', ...
    'sin(x)/x', 'Location', 'Best')
title('Taylor Series Expansion')
% Plot the original expression g and its approximations T6, T10, and T14.
```

The diagram (3.1.3a) shows the way to find the Maclaurin series expansion for $g = \sin(x)/x$. As the Taylor series approximation of this expression does not have a fifth-degree term, therefore, the taylor approximates this expression with the fourth-degree polynomial as shown in the diagram (3.1.3b).

The original expression g and its approximations T6, T10, and T14 are plotted to show the accuracy of the approximation depends on the truncation order.

(3.1.3b)

```
g =
```

$$\frac{\sin(x)}{x}$$

```
==============================================
T6 =
```

$$\frac{x^4}{120}-\frac{x^2}{6}+1$$

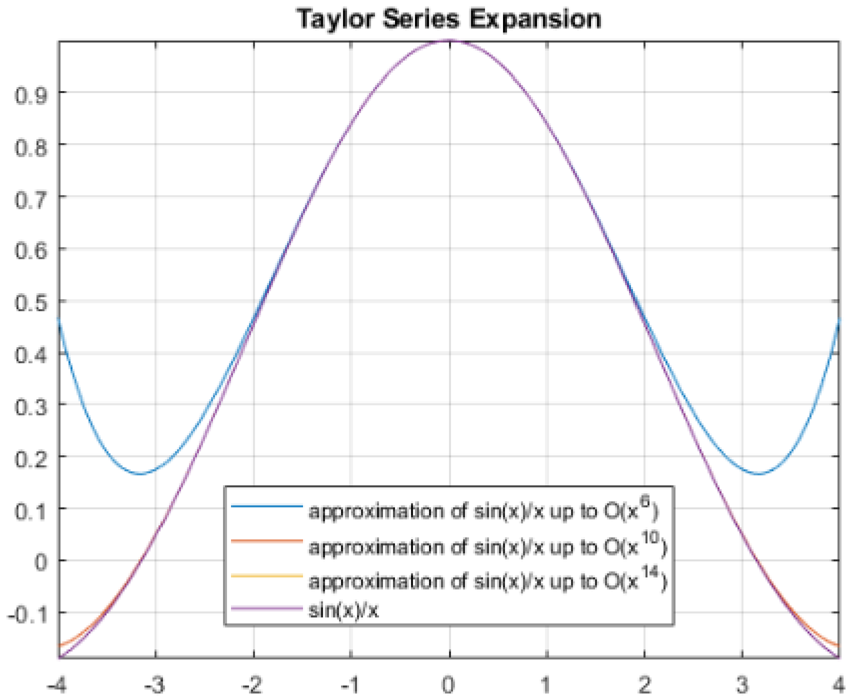
```
T10 =
```

$$\frac{x^8}{362880}-\frac{x^6}{5040}+\frac{x^4}{120}-\frac{x^2}{6}+1$$

```
T14 =
```

$$\frac{x^{12}}{6227020800}-\frac{x^{10}}{39916800}+\frac{x^8}{362880}-\frac{x^6}{5040}+\frac{x^4}{120}-\frac{x^2}{6}+1$$

```
==============================================
```

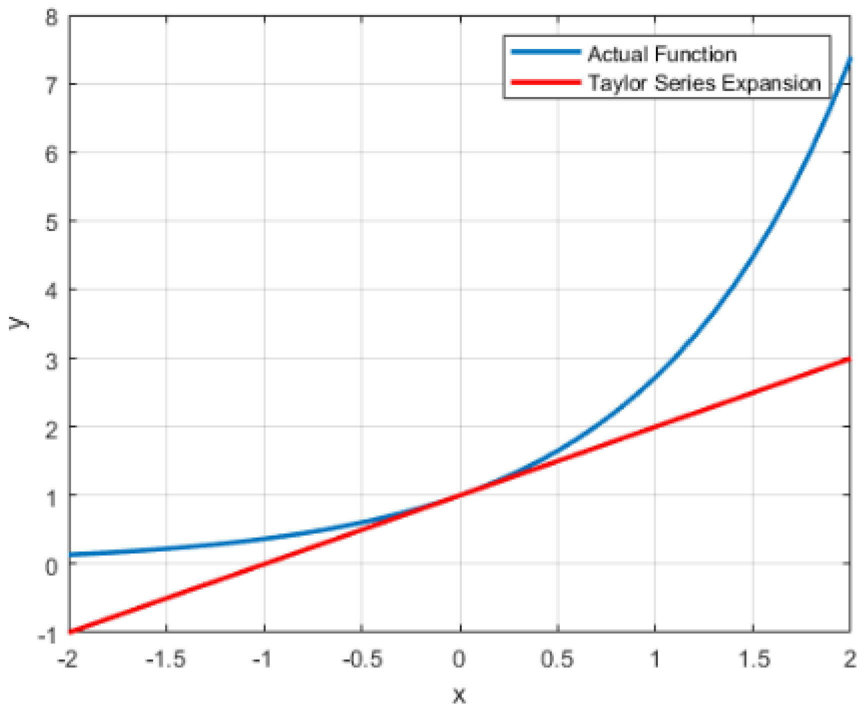


3.1.4 Plotting the Taylor series expansion

(3.1.4a)

```
% Create an x vector

x = -2:0.1:2;

y = exp(x);

fig = figure();
set(fig, 'color', 'white')
plot(x, y, 'LineWidth', 2)
grid on
xlabel('x')
ylabel('y')

% Use x vector and create a y estimate using the Taylor series expansion
N = 1;
yest = 0 * y;

for n = 0:N
    yest = yest + (x.^n)./factorial(n);
end

hold on
plot(x, yest, 'r-', 'LineWidth', 2)
legend('Actual Function', 'Taylor Series Expansion')
```

(3.1.4b)



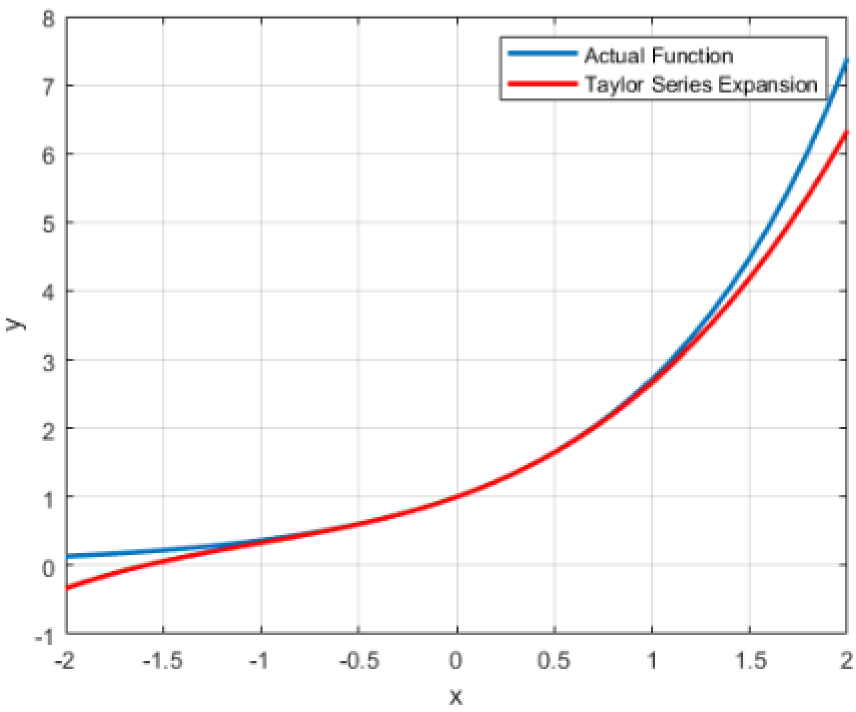
The diagram (3.1.4a) shows the code of plotting the Taylor series expansion. The graph shown in the diagram (3.1.4b) is the result of the Taylor series expansion when N = 1.

By changing the N, the "Taylor Series Expansion" line will be getting closer and closer to the "Actual Function" line, as shows in the diagrams below:

(3.1.4c)

```
N = 3;
yest = 0 * y;

for n = 0:N
    yest = yest + (x.^n)./factorial(n);
end

hold on
plot(x, yest, 'r-', 'LineWidth', 2)
legend('Actual Function', 'Taylor Series Expansion')
```

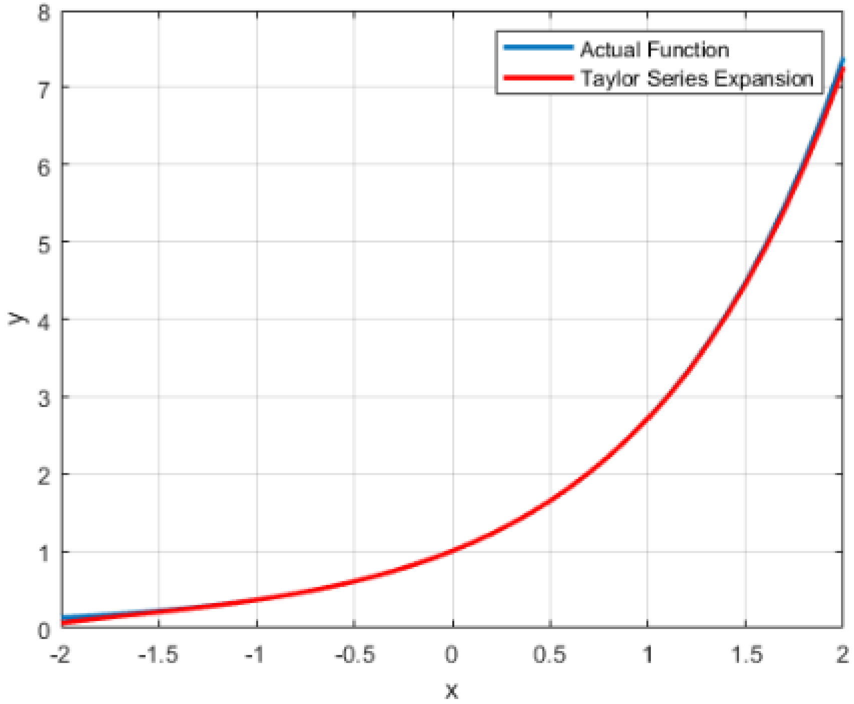
(3.1.4d)



(3.1.4e)

```
N = 5;
yest = 0 * y;

for n = 0:N
    yest = yest + (x.^n)./factorial(n);
end

hold on
plot(x, yest, 'r-', 'LineWidth', 2)
legend('Actual Function', 'Taylor Series Expansion')
```

(3.1.4f)



3.2 Classification/Discussion

A few examples are shown in the 3.1 section that indicate that Matlab is a significantly great tool for Taylor series calculation.

Matlab can be used to show the Taylor series and Maclaurin series expansion, as shown in the diagram (3.1.1b) and (3.1.3b).

The diagrams (3.1.3b), (3.1.4b), (3.1.4d), and (3.1.4f) show a clear example of how we are able to use Matlab to plot the graph for Taylor series expansion.

Matlat is without a doubt an excellent tool when it comes to calculate, expand, plot, and simplify Taylor series equations. It plays a heavy role by saving a lot of time and helping me to calculate the Taylor series equation efficiently and effectively, making my project become more completed.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Results

4.1.1 A new type of Taylor series expansion

(4.1.1a)

$$\int u\,dv = uv - \int v\,du.$$

(4.1.2a)

$$\begin{aligned}\int_a^b F(t)G(t)\,dt &= \left(F(t)G_1(t) - F'(t)G_2(t) + \cdots + (-1)^{n-1}F^{(n-1)}(t)G_n(t)\right)\Big|_a^b \\ &\quad + (-1)^n \int_a^b F^{(n)}(t)G_n(t)\,dt \\ &= \sum_{k=0}^{n-1}(-1)^k\left(F^{(k)}(t)G_{k+1}(t)\right)\Big|_a^b + (-1)^n\int_a^b F^{(n)}(t)G_n(t)\,dt,\end{aligned}$$

(4.1.3a)

Let $f \in C^{n+1}[a,b]$ *and* $x_0 \in [a,b]$. *Then, for all* $a \le x \le b$, *we have*

$$f(x) = \sum_{k=0}^{n} \frac{1}{k!}(x-x_0)^k f^{(k)}(x_0) + \frac{1}{n!}\int_{x_0}^{x} f^{(n+1)}(t)(x-t)^n\,dt.$$

(4.1.4a)

$$f(x) = \frac{1}{n!c_n}\left(\sum_{k=0}^{n} p_n^{(k)}(x-a)f^{(n-k)}(a) - \sum_{k=0}^{n-1} k!c_k f^{(n-k)}(x) + \int_a^x p_n(x-t)f^{(n+1)}(t)\,dt\right).$$

(4.1.5a)

$$f(x) = \sum_{k=0}^{n} \frac{(x-a)^{n-k}}{(n-k)!} f^{(n-k)}(a) + \frac{1}{n!}\int_a^x (x-t)^n f^{(n+1)}(t)\,dt,$$

(4.1.6a)

$$\begin{aligned}
&\sum_{k=0}^{n} k!C_k^{(\alpha,\beta,n)} f^{(n-k)}(x) \\
&\quad = \frac{1}{\Gamma(\alpha+\beta+n+1)}\sum_{k=0}^{n}(x-a)^k \sum_{j=0}^{n-k}\frac{1}{2^j} f^{(n-j)}(a)\Gamma(\alpha+\beta+n+1+j)C_k^{(\alpha+j,\beta+j,n-j)} \\
&\qquad + \sum_{k=0}^{n} C_k^{(\alpha,\beta,n)}\int_a^x (x-t)^k f^{(n+1)}(t)\,dt.
\end{aligned}$$

(4.1.7a)

$$\begin{aligned}\sum_{k=0}^{n} k!C_k^{(n)} f^{(n-k)}(x) \\ &= \sum_{k=0}^{n} \frac{\mathrm{d}^k}{\mathrm{d}x^k} \cos\big(n \arccos(x-a)\big) f^{(n-k)}(a) + \int_a^x \cos\big(n \arccos(x-t)\big) f^{(n+1)}(t)\, dt \\ &= \frac{2^{2n}}{(n-1)!} \sum_{k=0}^{n} (x-a)^k \sum_{j=0}^{n-k} 2^{-j}(n+j-1)! C_k^{(j-\frac{1}{2}, j-\frac{1}{2}, n-j)} f^{(n-j)}(a) \\ &\quad + \int_a^x \cos\big(n \arccos(x-t)\big) f^{(n+1)}(t)\, dt, \quad n \geq 1.\end{aligned}$$

From the formula (4.1.1a), we can recreate the missing steps before getting the formula (4.1.1a), as shown below:

Product Rule (of Differentiation)

(4.1.1b)

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\int \frac{d}{dx}(uv)dx = \int u\frac{dv}{dx}dx + \int v\frac{du}{dx}dx$$

$$\mathrm{uv} = \int udv + \int vdu$$

$$\int udv = \mathrm{uv} - \int vdu$$

(4.1.1b) shows the steps of how the formula (4.1.1a) is formed.

From the equation (4.1.2a), we can expand some steps in between the …, as shown below:

(4.1.2b)

$$\int_a^b F(t)G(t)dt$$

$$= [F(t)G_1(t) - F'(t)G_2(t) + \ldots + (-1)^{n-1}F^{(n-1)}(t)G_n(t)]|_a^b + (-1)^n \int_a^b F^{(n)}(t)G_n(t)\,dt$$

$$= \big(F(t)G_1(t) - F'(t)G_2(t) + F''(t)G_3(t) - F'''(t)G_4(t)$$

$$+ F''''(t)G_5(t) - \ldots + (-1)^{n-1}F^{(n-1)}(t)G_n(t)\big)|_a^b + (-1)^n \int_a^b F^{(n)}(t)G_n(t)\,dt$$

$$= \sum_{k=0}^{n-1}(-1)^k[F^{(k)}(t)G_{k+1}(t)]|_a^b + (-1)^n \int_a^b F^{(n)}(t)G_n(t)\,dt$$

The steps before getting the equation (4.1.3a) are shown below:

(4.1.3b)

$$
\begin{aligned}
f(x) &= \frac{1}{0!}(x-x_0)^0 f^{(0)}(x_0) + \frac{1}{1!}(x-x_0)^1 f^{(1)}(x_0) + \frac{1}{2!}(x-x_0)^2 f^{(2)}(x_0) \\
&\quad + \frac{1}{3!}(x-x_0)^3 f^{(3)}(x_0) + \ldots + \frac{1}{n!}(x-x_0)^n f^{(n)}(x_0) + \frac{1}{n!}\int_{x_0}^{x} f^{(n+1)}(t)(x-t)^n\,dt \\
&= f(x_0) + (x-x_0)f'(x_0) + \frac{1}{2}(x-x_0)^2 f''(x_0) \\
&\quad + \frac{1}{6}(x-x_0)^3 f'''(x_0) + \ldots + \frac{1}{n!}(x-x_0)^n f^{(n)}(x_0) + \frac{1}{n!}\int_{x_0}^{x} f^{(n+1)}(t)(x-t)^n\,dt \\
&= \sum_{k=0}^{n} \frac{1}{k!}(x-x_0)^k f^{(k)}(x_0) + \frac{1}{n!}\int_{x_0}^{x} f^{(n+1)}(t)(x-t)^n\,dt
\end{aligned}
$$

The equation (4.1.4a) is reduced to equation (4.1.5a) by letting the $P_n(x-t) = \frac{1}{n!}(x-t)^n$, $c_j = 0$ for every $j = 0,1,\ldots,n-1$ and $c_n = \frac{1}{n!}$

(4.1.4b)

$$
f(x) = \frac{1}{n!\,c_n}\left(\sum_{k=0}^{n} P_n{}^{(k)}(x-a) f^{(n-k)}(a) - \sum_{k=0}^{n-1} k!\,c_k f^{(n-k)}(x) + \int_a^x P_n(x-t) f^{(n+1)}(t)\,dt\right)
$$

$$
\begin{aligned}
f(x) &= \frac{1}{n!\left(\frac{1}{n!}\right)}\left(\sum_{k=0}^{n} \frac{1}{(n-k)!}(x-a)^{(n-k)} f^{(n-k)}(a) - \sum_{k=0}^{n-1} k!\,(0) f^{(n-k)}(x)\right. \\
&\quad \left. + \int_a^x \frac{1}{n!}(x-t)^n f^{(n+1)}(t)\,dt\right)
\end{aligned}
$$

$$
f(x) = 1\left(\sum_{k=0}^{n} \frac{(x-a)^{(n-k)}}{(n-k)!} f^{(n-k)}(a) - 0 + \frac{1}{n!}\int_a^x (x-t)^n f^{(n+1)}(t)\,dt\right)
$$

$$
f(x) = \sum_{k=0}^{n} \frac{(x-a)^{n-k}}{(n-k)!} f^{(n-k)}(a) + \frac{1}{n!}\int_a^x (x-t)^n f^{(n+1)}(t)\,dt
$$

(4.1.4b) shows the missing steps between (4.1.4a) and (4.1.5a).

(4.1.5b)

$$\sum_{k=0}^{n} k!\, C_k^{(\alpha,\beta,n)} f^{(n-k)}(x)$$

$$= \frac{1}{\gamma(\alpha+\beta+n+1)} \sum_{k=0}^{n} (x-a)^k \sum_{j=0}^{n-k} \frac{1}{2^j} \gamma(\alpha+\beta+n+1) C_k^{(\alpha+j,\beta+j,n-j)} f^{(n-j)}(a)$$

$$+ \sum_{k=0}^{n} C_k^{(\alpha,\beta,n)} \int_a^x (x-t)^k f^{(n+1)}(t)\, dt$$

$$= \frac{1}{(\alpha+\beta+n)!} \sum_{k=0}^{n} (x-a)^k \sum_{j=0}^{n-k} 2^{-j} (\alpha+\beta+n+j)!\, C_k^{(\alpha+j,\beta+j,n-j)} f^{(n-j)}(a)$$

$$+ \sum_{k=0}^{n} C_k^{(\alpha,\beta,n)} \int_a^x (x-t)^k f^{(n+1)}(t)\, dt$$

The equation (4.1.5b) expands one more step of the equation (4.1.6a).

(4.1.6b)

$$\sum_{k=0}^{n} k!\, C_k^{(n)} f^{(n-k)}(x)$$

$$= \sum_{k=0}^{n} \frac{d^k}{dx^k} \cos(n\, arccos(x-a))\, f^{(n-k)}(a) + \int_a^x \cos(n\, arccos(x-t))\, f^{(n+1)}(t)\, dt$$

$$= 2^{2n} \left(\frac{1}{\gamma(n)}\right) \sum_{k=0}^{n} (x-a)^k \sum_{j=0}^{n-k} \frac{1}{2^j} \gamma(n) C_k^{(\alpha+j,\beta+j,n-j)} f^{(n-j)}(a)$$

$$+ \int_a^x \cos(n\, arccos(x-t))\, f^{(n+1)}(t)\, dt$$

$$= \frac{2^{2n}}{(n-1)!} \sum_{k=0}^{n} (x-a)^k \sum_{j=0}^{n-k} 2^{-j} (n+j-1)!\, C_k^{(j-\frac{1}{2},j-\frac{1}{2},n-j)} f^{(n-j)}(a)$$

$$+ \int_a^x \cos(n\, arccos(x-t))\, f^{(n+1)}(t)\, dt$$

The equation (4.1.6b) recreates the missing step in the equation (4.1.7a).

4.1.2 Taylor series solution for Lane-Emden equation

(4.2.1a)

$$2u'' + xu''' - v^3(u^2+1) - 2xv^3uu' - x3v^2v'(u^2+1) = 0$$

$$2u''(0) - v^3(0)(u^2(0)+1) = 0$$

(4.2.2a)

$$4v'' + xv''' + v^5(u^2+3) + 2xv^5uu' + 5xv^4v'(u^2+3) = 0$$

$$4v''(0) + v^5(0)(u^2(0)+3) = 0$$

(4.2.1b)

$$2u'' + xu''' - v^3(u^2+1) - 2xv^3uu' - x3v^2v'(u^2+1) = 0$$

$$2u'' + (0)u''' - v^3(u^2+1) - 2(0)v^3uu' - (0)3v^2v'(u^2+1) = 0$$

$$2u'' + 0 - v^3(u^2+1) - 0 - 0 = 0$$

$$2u''(0) - v^3(0)(u^2(0)+1) = 0$$

The equation (4.2.1b) shows the missing steps in between the equation (4.2.1a).

(4.2.2b)

$$4v'' + xv''' + v^5(u^2+3) + 2xv^5uu' + 5xv^4v'(u^2+3) = 0$$

$$4v'' + (0)v''' + v^5(u^2+3) + 2(0)v^5uu' + 5(0)v^4v'(u^2+3) = 0$$

$$4v'' + 0 + v^5(u^2+3) + 0 + 0 = 0$$

$$4v''(0) + v^5(0)(u^2(0)+3) = 0$$

The equation (4.2.2b) shows the missing steps in between the equation (4.2.2a).

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

Throughout the development of this project, I have become skilled in the understanding of mathematical analysis in both theoretical and practical terms in the field of mathematics. .

As the title for my research is “Recent Development and Analysis of Taylor series”, I have reviewed a lot of materials from formal sources such as paper, websites, journals and so on that can be used in my research. The research materials are being used to show the recent development of Taylor series, by using a total of 11 recent research materials that are related to Taylor series, but involve many different areas.

Taylor series equations, formulas, methods, and expansions that are shown in the research materials are being used to produce my own result. For instance, I will calculate, simplify, expand, and recreate the missing steps of Taylor series equations shown in the research materials.

Due to the recommendation from my research supervisor, Mr. Chong Kam Yoon, Matlab is chosen by me as the choice of mathematical tool for my research. It best the online mathematics calculators such as Wolfram Alpha, Symbolab and so on when it comes to calculate, expand, and simplify Taylor series equations. It helps me to save a lot of time when calculating the Taylor series equations. It also allows me to produce clear and concise results, such as plotting graphs for Taylor series expansion.

My understanding of mathematical analysis has been expanded and enhanced throughout the duration of completing this project. I now have a better conceptual understanding about the Taylor series as I have built a strong foundation in the understanding of Taylor series. For example, the Taylor series allows us to be able to easily calculate the complicated functions and even the most complex functions. Whenever we want to approximate a complicated function numerically, all it takes is to take the first few terms of the Taylor series expansion of the function to get a nice polynomial approximation.

As I have found a lot of research materials that are related to the Taylor series, I now have a clearer vision about the development of Taylor series in recent years than I used to be. For example, there are researches such as a new type of Taylor series expansion, solving the Lane-Emden equation by using Taylor series, solving the nonlinear reaction-diffusion equation in the electroactive polymer film by using Taylor series method, and so on.

Last but not least, I would like to make a conclusion based on what I have learned by indicating the advantage and disadvantage of the Taylor series in my perspective. The biggest advantage of the Taylor series method is undoubtedly how easily we can calculate even the most complex and complicated functions. Take sin(x) for example, there are very few values of x which give a computable solution. However, we can use the Taylor series to estimate the value of any input. This is especially significant for computers, which can only compute using basic arithmetic. The Taylor series is the only way for computers to even solve the equation. The disadvantage will simply be how long it takes.

5.2 Problems and difficulties encountered

Throughout the work I have done for this project, I have encountered some problems. When I first started working on the report, one of the problems that I have faced is I am not able to find journals that are related to the Taylor series, and are in recent years.

After searching multiple times by using online browsers such as Google and Youtube, I have come to a solution of using Google Scholar. For example, when I need to find recent research materials related to the Taylor series, Google Scholar comes in handy to me because it has a convenient setting for the range of years. I can set the range of years and find recent materials related to the Taylor series with ease. Furthermore, Google Scholar brings me the convenience and efficiency of being able to cite the selected website easily, which saves me a great amount of time without needing to cite on the other specific websites.

Although Google Scholar has a lot of advantages, I have met a problem when using it. Google Scholar is very keyword sensitive. For instance, if I type in a long sentence, such as "Recent development and analysis of Taylor series", it may show me the recent development of other things that are not related to the Taylor series.

To solve this problem, my supervisor has given me a solution which is a better online browser, the TARUC libraries online browser. The TARUC browser best Google Scholar in every way, it has all the advantages that Google Scholar has, and without the disadvantage the Google Scholar has.

The weakness of the work done will be in Chapter 3, the Methodology section. This is because the methodology section mostly involves Matlab. As my coding skills are not good enough, I am not able to maximize the advantages that the Matlab can bring to me.

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APPENDICES

APPENDIX A: Taylor Series Example by Using Matlab

(6.1.1a)

```
%syms x
%f = 1/(5 + 4*cos(x));
%F = taylor(f, 'Order', 8)
%fprintf('===============================================')

h = exp(x*sin(x));
H = taylor(h, 'ExpansionPoint', 2, 'Order', 12);

size(char(H))
fprintf('===============================================')

H = simplify(H);
size(char(H))
fprintf('===============================================')

xd = 1:0.05:3;
yd = subs(h, x, xd);

fplot(H, [1, 3])

hold on
plot(xd, yd, 'r-.')

title('Taylor Approximation vs Actual Function')
legend('Taylor', 'Function')
```

The first few commands generate the first 12 nonzero terms of the Taylor series for g at the expansion point of x = 12. size(char(H)) is entered to find that H has about 100,000 characters in its printed form as shown in the diagram (6.1.1b). In order to proceed with using H, H = simplify(H) is used to simplify its presentation.

Next, a plot of these function together shows how well the Taylor approximation compares to the actual function h, as shown in the diagram below:

（6.1.1b)

```
ans = 1×2
          1       99791

==============================================
ans = 1×2
          1        6988

==============================================
```