Exercise 1

Introduction to Monte Carlo

Calculation of π

Consider a circle of diameter d surrounded by a square of length $l (l \geq d)$. Random coordinates within the square are generated. The value of π can be calculated from the fraction of points that fall within the circle.

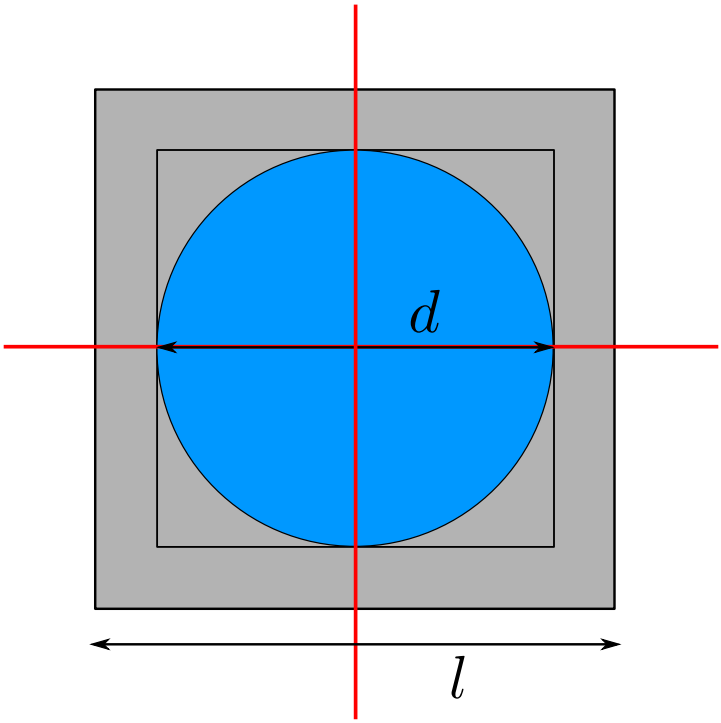

Figure 1: A circle of diameter d surrounded by a square of length l $(l \geq d)$

Questions:

1. How can π be calculated from the fraction of points that fall in the circle? Complete the small Monte Carlo program to calculate π using this method. Remark: the "exact" value of π can be computed numerically using $\pi = 4 \cdot \arctan(1)$.
2. How does the accuracy of the result depend on the ratio l/d and the number of generated coordinates? Derive a formula to calculate the relative standard deviation of the estimate of π
   Hint: The distribution of successful hits is binomial. Is there an optimal ratio?
3. Is it a good idea to calculate many decimals of π using this method?