The Puzzles of MrHeer

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1 Throwing eggs from a building

Question

Suppose that you have an N-story building and plenty of eggs. Suppose also that an egg is broken if it is thrown off floor F or higher, and unhurt otherwise. First, devise a strategy to determine the value of F such that the number of broken eggs is $\sim \lg N$ when using $\sim \lg N$ throws, then find a way to reduce the cost to $\sim 2 \lg F$.

Answer

$\sim \lg N$: start at the middle, always cut search space in half $\rightarrow$ binary search. $\sim 2 \lg F$: start at 1, next 2, 4, 8 (i.e., 2^i), once the egg breaks after ($\sim \lg F$ steps) do binary search in the smaller search space (range $< F$ and hence number of searches $< \sim \lg F$) $\rightarrow$ exponential search.

$$2^{\lceil \lg F \rceil - 1} < F \leqslant 2^{\lceil \lg F \rceil}$$

$$range = 2^{\lceil \lg F \rceil} - 2^{\lceil \lg F \rceil - 1} = 2^{\lceil \lg F \rceil - 1} < 2^{\lg F} = F$$

$$range < F$$

2 Throwing two eggs from a building

Question

Consider the previous question, but now suppose you only have two eggs, and your cost model is the number of throws. Devise a strategy to determine F such that the number of throws is at most $2\sqrt{N}$, then find a way to reduce the cost to $\sim c\sqrt{F}$. This is analogous to a situation where search hits (egg intact) are much cheaper than misses (egg broken).

Answer

Let us make our first attempt on k'th floor. If it breaks, we try remaining $(k-1)$ floors one by one. So in worst case, we make k trials. If it doesn't break, we jump $(k-1)$ floors (Because we have already made one attempt and we don't want to go beyond k attempts. Therefore $(k-1)$ attempts are available), Next floor we try is floor $k+(k-1)$ Similarly, if this drop does not break, next need to jump up to floor $k+(k-1)+(k-2)$, then $k+(k-1)+(k-2)+(k-3)$ and so on. Since the last floor to be tried is F'th floor, sum of series should be F for optimal value of k.

$$k+(k-1)+(k-2)+(k-3)+\cdots+1 \geqslant F$$

$$\frac{k(k+1)}{2} \geqslant F$$

$$k \geqslant \frac{\sqrt{8F+1}-1}{2}$$

$$k_{min} = \lceil \frac{\sqrt{8F+1}-1}{2} \rceil \sim \sqrt{2F}$$

Official solution

Solution to Part 1: To achieve $2\sqrt{N}$, drop eggs at floors $\sqrt{N}$, $2*\sqrt{N}$, $3*\sqrt{N}$, ..., $\sqrt{N}\sqrt{N}$. (For simplicity, we assume here that $\sqrt{N}$ is an integer.) Let assume that the egg broke at level $k\sqrt{N}$. With the second egg you should then perform a linear search in the interval $(k-1)\sqrt{N}$ to $k\sqrt{N}$. In total you will be able to find the floor F in at most $2\sqrt{N}$ trials.
Hint for Part 2: $1+2+3+\cdots+k \sim \frac{1}{2}k^2 \geqslant F$.

3 3-collinearity

Question

Suppose that you have an algorithm that takes as input N distinct points in the plane and can return the number of triples that fall on the same line. Show that you can use this algorithm to solve the 3-sum problem. Strong hint: Use algebra to show that (a, a^3), (b, b^3), and (c, c^3) are collinear if and only if $a + b + c = 0$.

Answer

Proof. (a, a^3), (b, b^3), and (c, c^3) are collinear if and only if $a + b + c = 0$:
We use a formulation of collinearity which equates gradients (assuming our points are distinct)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_1}{x_3 - x_1}$$

This becomes

$$\frac{b^3 - a^3}{b - a} = \frac{c^3 - a^3}{c - a}$$

which leaves us with

$$b^2 + ab + a^2 = c^2 + ac + a^2$$

so that

$$b^2 - c^2 = a(c - b)$$

$c \neq b$ so we have $a = -(b + c)$

$$a + b + c = 0$$

□

4 Queue with three stacks

Question

Implement a queue with three stacks so that each queue operation takes a constant (worst-case) number of stack operations.

Answer

5 Queue with two stacks

Question

Answer

Implement a queue with two stacks so that each queue operation takes a constant amortized number of stack operations. Hint: If you push elements onto a stack and then pop them all, they appear in reverse order. If you repeat this process, they're now back in order.