The Draft of MrHeer

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1 Throwing eggs from a building

Question

Suppose that you have an N-story building and plenty of eggs. Suppose also that an egg is broken if it is thrown off floor F or higher, and unhurt otherwise. First, devise a strategy to determine the value of F such that the number of broken eggs is $\sim\lg N$ when using $\sim\lg N$ throws, then find a way to reduce the cost to $\sim 2\lg F$.

Answer

$\sim\lg N$: start at the middle, always cut search space in half $\rightarrow$ binary search. $\sim 2\lg F$: start at 1, next 2, 4, 8 (i.e., 2^i), once the egg breaks after ($\sim\lg F$ steps) do binary search in the smaller search space (range $< F$ and hence number of searches $< \sim\lg F$) $\rightarrow$ exponential search.

$$2^{\lceil \lg F \rceil - 1} < F \leqslant 2^{\lceil \lg F \rceil}$$

$$range = 2^{\lceil \lg F \rceil} - 2^{\lceil \lg F \rceil - 1} = 2^{\lceil \lg F \rceil - 1} < 2^{\lg F} = F$$

$$range < F$$

2 Throwing two eggs from a building

Question

Consider the previous question, but now suppose you only have two eggs, and your cost model is the number of throws. Devise a strategy to determine F such that the number of throws is at most $2\sqrt{N}$, then find a way to reduce the cost to $\sim c\sqrt{F}$. This is analogous to a situation where search hits (egg intact) are much cheaper than misses (egg broken).

Answer

Let us make our first attempt on x'th floor. If it breaks, we try remaining $(x-1)$ floors one by one. So in worst case, we make x trials. If it doesn't break, we jump $(x-1)$ floors (Because we have already made one attempt and we don't want to go beyond x attempts. Therefore $(x-1)$ attempts are available), Next floor we try is floor $x+(x-1)$ Similarly, if this drop does not break, next need to jump up to floor $x+(x-1)+(x-2)$, then $x+(x-1)+(x-2)+(x-3)$ and so on. Since the last floor to be tried is N'th floor, sum of series should be N for optimal value of x.

$$x+(x-1)+(x-2)+(x-3)+\cdots+1 \geqslant N$$

$$\frac{x(x+1)}{2} \geqslant N$$

$$x \geqslant \frac{\sqrt{8N+1}-1}{2}$$

$$x_{min} = \lceil \frac{\sqrt{8N+1}-1}{2} \rceil \sim \sqrt{2N}$$

The egg is broken if it is thrown off floor F or higher

$$x+(x-1)+(x-2)+(x-3)+\cdots \geqslant F$$

Therefore, we start trying from 14'th floor. If Egg breaks we one by one try remaining 13 floors. If egg doesn't break we go to 27th floor. If egg breaks on 27'th floor, we try floors form 15 to 26. If egg doesn't break on 27'th floor, we go to 39'th floor. An so on...