

Datos aleatorios, Regre.Lineal

September 17, 2020

1 Modelo de Regresión Lineal

1.1 Modelo con datos simulados

- $y = a + b * x$
- $X = 100$ valores distribuidos según una $N(1.5, 2.5)$, media y desviación estandar
- $Y_e = 5 + 1 * X + e$
- e estará distribuida según una $N(0, 0.8)$, si media 0 no, ocurren desplazamientos en el modelo.

```
[103]: import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

```
[104]: x = 1.5 + 2.5 * np.random.randn(100)
```

```
[105]: res = 0 + 0.8 * np.random.randn(100)
```

```
[106]: y_pred = 5 + 1 * x
```

```
[107]: y_act = 5 + 1 * x + res
```

```
[108]: x_list = x.tolist()
y_pred_list = y_pred.tolist()
y_act_list = y_act.tolist()
```

```
[109]: data = pd.DataFrame(
{
    "x":x_list,
    "y_actual":y_act_list,
    "y_prediccion":y_pred_list
})
```

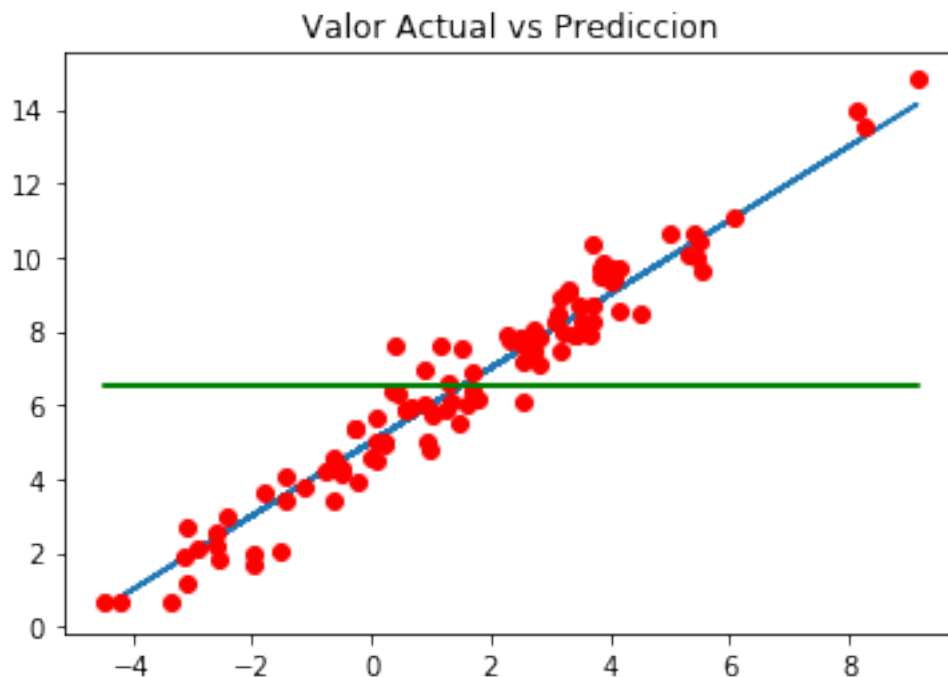
```
[110]: data.head()
```

```
[110]:      x    y_actual  y_prediccion
0  1.016354  5.701187    6.016354
1  2.514536  7.816246    7.514536
2  2.537257  6.124276    7.537257
3 -0.282380  5.335924    4.717620
4  8.254630 13.511166   13.254630
```

```
[111]: y_mean = [np.mean(y_act) for i in range(1, len(x_list) + 1)]
```

```
[112]: plt.plot(x, y_pred)
plt.plot(x, y_act, "ro")
plt.plot(x, y_mean, "g")
plt.title("Valor Actual vs Prediccion")
```

```
[112]: Text(0.5, 1.0, 'Valor Actual vs Prediccion')
```



```
[113]: data["SSR"] = (data["y_prediccion"] - np.mean(y_act))**2
data["SSD"] = (data["y_prediccion"] - data["y_actual"])**2
data["SST"] = (data["y_actual"] - np.mean(y_act))**2
```

```
[114]: data.head()
```

```
[114]:      x    y_actual  y_prediccion      SSR      SSD      SST
0  1.016354  5.701187    6.016354  0.285815  0.099330  0.722133
```

| | | | | | | |
|---|-----------|-----------|-----------|-----------|----------|-----------|
| 1 | 2.514536 | 7.816246 | 7.514536 | 0.928458 | 0.091029 | 1.600923 |
| 2 | 2.537257 | 6.124276 | 7.537257 | 0.972761 | 1.996517 | 0.182069 |
| 3 | -0.282380 | 5.335924 | 4.717620 | 3.361175 | 0.382300 | 1.476338 |
| 4 | 8.254630 | 13.511166 | 13.254630 | 44.939046 | 0.065811 | 48.444325 |

```
[115]: SSR = sum(data["SSR"])
      SSD = sum(data["SSD"])
      SST = sum(data["SST"])
```

```
[116]: SST
```

```
[116]: 854.4963653885451
```

```
[117]: SSR
```

```
[117]: 749.3628729511602
```

```
[118]: SSD
```

```
[118]: 44.79580780092574
```

```
[119]: SSD + SSR
```

```
[119]: 794.158680752086
```

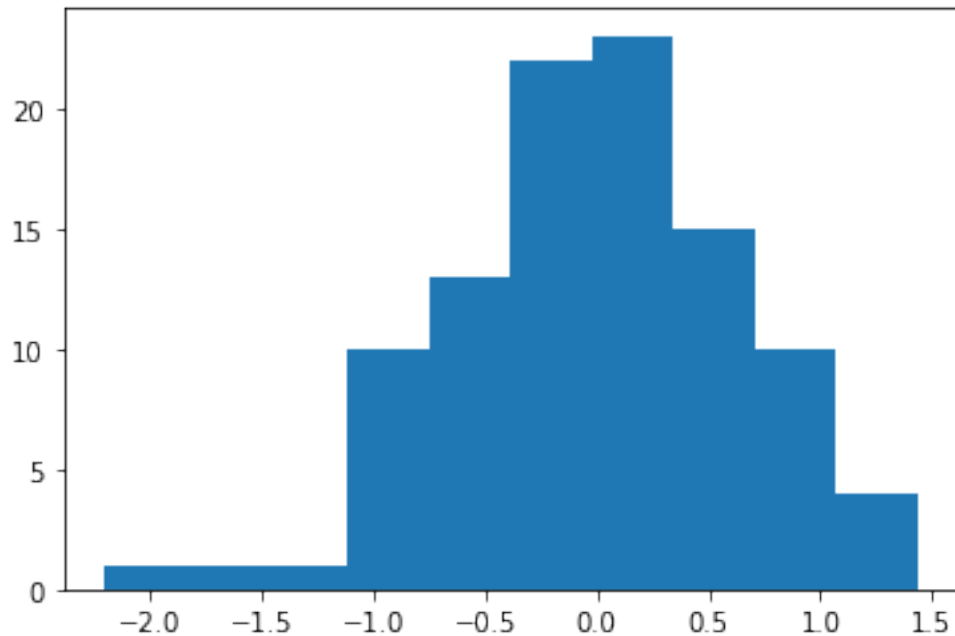
```
[120]: R2 = SSR/SST
```

```
[121]: R2
```

```
[121]: 0.8769643772684977
```

```
[123]: plt.hist((data["y_prediccion"]-data["y_actual"]))
```

```
[123]: (array([ 1.,  1.,  1., 10., 13., 22., 23., 15., 10.,  4.]),
      array([-2.20635247, -1.84242826, -1.47850405, -1.11457985, -0.75065564,
            -0.38673143, -0.02280723,  0.34111698,  0.70504119,  1.06896539,
             1.4328896 ]),
      <a list of 10 Patch objects>)
```



2 Obteniendo la recta de regresión

- $y = a + b \cdot x$
- $b = \frac{\sum((x_i - x_m)(y_i - y_m))}{\sum((x_i - x_m)^2)}$
- $a = y_m - b \cdot x_m$

```
[124]: x_m = np.mean(data["x"])
       y_m = np.mean(data["y_actual"])
       x_m, y_m
```

```
[124]: (1.538192936084344, 6.550970491703897)
```

```
[126]: data["beta_n"] = (data["x"] - x_m) * (data["y_actual"] - y_m)
       data["beta_d"] = (data["x"] - x_m) ** 2
```

```
[127]: beta = sum(data["beta_n"]) / sum(data["beta_d"])
```

```
[128]: alpha = y_m - beta * x_m
```

```
[129]: alpha, beta
```

```
[129]: (4.950816085564027, 1.040281988430695)
```

- El modelo lineal obtenido por regresion es: $y = 4.950816085564027 + 1.040281988430695 \cdot x$

```
[130]: data["y_model"] = alpha + beta * data["x"]
```

```
[131]: data.head()
```

```
[131]:
```

| | x | y_actual | y_prediccion | SSR | SSD | SST \ |
|---|-----------|-----------|--------------|-----------|----------|-----------|
| 0 | 1.016354 | 5.701187 | 6.016354 | 0.285815 | 0.099330 | 0.722133 |
| 1 | 2.514536 | 7.816246 | 7.514536 | 0.928458 | 0.091029 | 1.600923 |
| 2 | 2.537257 | 6.124276 | 7.537257 | 0.972761 | 1.996517 | 0.182069 |
| 3 | -0.282380 | 5.335924 | 4.717620 | 3.361175 | 0.382300 | 1.476338 |
| 4 | 8.254630 | 13.511166 | 13.254630 | 44.939046 | 0.065811 | 48.444325 |

| | beta_n | beta_d | y_model |
|---|-----------|-----------|-----------|
| 0 | 0.443450 | 0.272316 | 6.008111 |
| 1 | 1.235343 | 0.953246 | 7.566643 |
| 2 | -0.426296 | 0.998129 | 7.590279 |
| 3 | 2.212081 | 3.314487 | 4.657061 |
| 4 | 46.747715 | 45.110522 | 13.537959 |

```
[133]: SSR = sum((data["y_model"]-y_m)**2)
SSD = sum((data["y_model"]-data["y_actual"])**2)
SST = sum((data["y_actual"]-y_m)**2)
```

```
[134]: SSR,SSD,SST
```

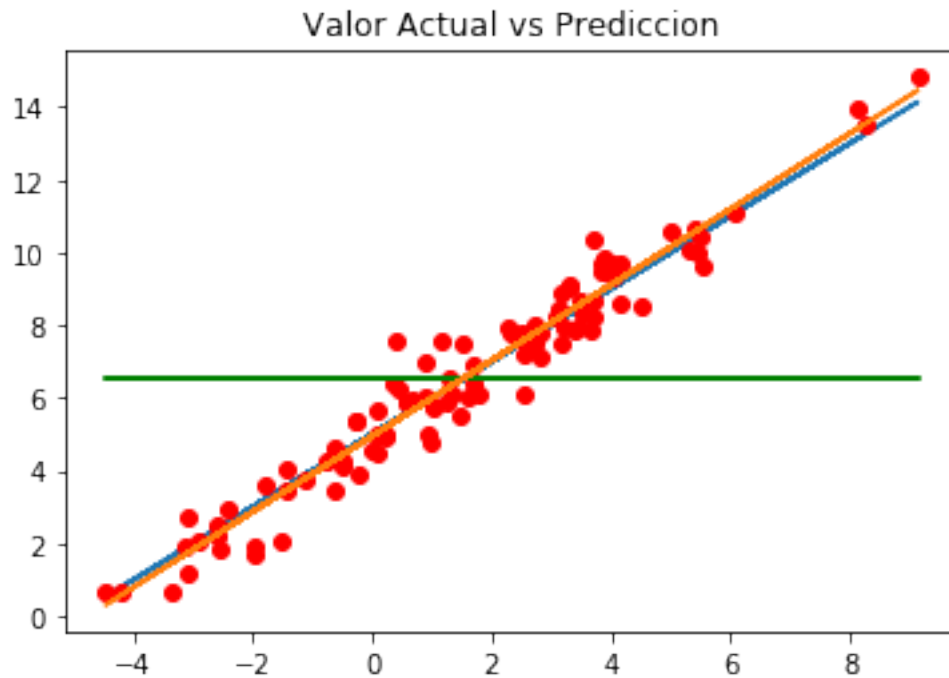
```
[134]: (810.9328028052313, 43.56356258331354, 854.4963653885453)
```

```
[135]: R2 = SSR/SST
R2
```

```
[135]: 0.949018434310712
```

```
[141]: y_mean =[np.mean(y_act) for i in range(1,len(x_list) + 1)]
plt.plot(x,y_pred)
plt.plot(x,y_act,"ro")
plt.plot(x,y_mean,"g")
plt.plot(data["x"],data["y_model"])
plt.title("Valor Actual vs Prediccion")
```

```
[141]: Text(0.5, 1.0, 'Valor Actual vs Prediccion')
```



2.1 Error estandar de los residuos (RSE)

```
[142]: RSE = np.sqrt(SSD)/(len(data)-2)
```

```
[143]: RSE
```

```
[143]: 0.06734969272609344
```

```
[144]: np.mean(data["y_actual"])
```

```
[144]: 6.550970491703897
```

```
[145]: RSE / np.mean(data["y_actual"])
```

```
[145]: 0.010280872553369706
```