EC330 HW2 Solution

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October 4, 2016

1. (a) Θ, Ω, O

   Both A and B are $\Theta(n \log n)$

   (b) ω, Ω

   To compare $2^{\log n}$ and $(\log n)^2$ we can use L'Hôpital's rule

$$\lim_{n\to\infty} \frac{2^{\log n}}{(\log n)^2} = \lim_{n\to\infty} \frac{2^{\log n} \frac{1}{\cancel{n} \ln 10}}{2(\log n) \frac{1}{\cancel{n} \ln 10}} = \lim_{n\to\infty} \frac{2^{\log n} \frac{1}{n \ln 10}}{2 \frac{1}{n \ln 10}}$$

   Since the result is infinity, $2^{\log n}$ rises faster than $(\log n)^2$

   (c) ω, Ω

   Via the Stirling's approximation, $n! = O\left(\sqrt{2\pi n}\left(\frac{n}{e}\right)^n\right)$.

$$\lim_{n\to\infty} \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{n^n} = \lim_{n\to\infty} \frac{\sqrt{2\pi n}}{e^n} = 0$$

   Therefore, $n!$ rises slower than n^n.

   (d) ω, Ω

   $\sum_{i=0}^{\infty} \frac{50}{7^i}$ is a constant, therefore n rises faster.

   (e) o, O

   e^n rises faster than any polynomial.

2.

$$2^{10} < \ln n < \sqrt{2}^{\lg n} < n < n \log n < 4^{\lg n} < n^5 + 7n < (\log n)^{\log n} < 2^n < (n+1)!$$

- $\sqrt{2}^{\log_2 n} = 2^{1/2 \log_2 n} = 2^{\log_2 \sqrt{n}} = \sqrt{n}$

  Similarly, $4^{\log_2 n} = 2^{\log_2 n^2} = n^2$

- For $n^5 + 7n < \sqrt{2}^{\lg n}$, if we take the derivatives, the polynomial will eventually reach 0 while the other one never will; therefore it rises faster.

- To compare $(\log n)^{\log n}$ and 2^n, let $x = \log n$

$$\begin{aligned} n &= 10^x \\ (\log n)^{\log n} &\stackrel{?}{=} 2^n \\ x^x &\stackrel{?}{=} 2^{10^x} \end{aligned}$$

  Taking the logarithm of both sides wouldn't change the equality/inequality

$$\begin{gathered} \log(x^x) \stackrel{?}{=} \log(2^{10^x}) \\ x \log x \stackrel{?}{=} 10^x \log 2 \\ O(x^2) < O(10^x) \end{gathered}$$

  Therefore, $(\log n)^{\log n} \equiv O(2^n)$.

- For $(n+1)!$, we can use Stirling's approximation, and take n^n. It is clear that $n^n \equiv \Omega(2^n)$.