EC330 Homework 1 Solution

Emre Ateş

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1. (a) Uses the normal exponential sum formula.

$$\sum_{n=0}^{N-1} r^n = \frac{1 - r^N}{1 - r}$$

$$\begin{aligned}
\sum_{i=3}^{30} \left(\frac{1}{8}\right)^i &= \sum_{i=0}^{27} \left(\frac{1}{8}\right)^{i+3} \\
&= \left(\frac{1}{8}\right)^3 \sum_{i=0}^{28-1} \left(\frac{1}{8}\right)^i \\
&= \frac{1}{8^3} \frac{1 - \left(\frac{1}{8}\right)^{28}}{1 - \frac{1}{8}} \\
&= \frac{8^{28} - 1}{7 \times 8^{30}} \approx \frac{1}{448}
\end{aligned}$$

   (b) Uses the infinite exponential sum formula.

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1 - r}$$

$$\begin{aligned}
\sum_{i=0}^{\infty} \frac{9}{7^i} &= 9 \sum_{i=0}^{\infty} \left(\frac{1}{7}\right)^i \\
&= 9 \frac{1}{1 - \frac{1}{7}} \\
&= \frac{21}{2} = 10.5
\end{aligned}$$

(c) Uses arithmetic sum formula and the sum of cubes.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$
$$\begin{aligned}\sum_{i=1}^{N}(6i^3+3i-9) &= 6\sum_{i=1}^{N} i^3 + 3\sum_{i=1}^{N} i - 9N \\ &= 6\left(\frac{N(N+1)}{2}\right)^2 + 3\frac{N(N+1)}{2} - 9N \\ &= \frac{3N^4}{2} + 3N^3 + 3N^2 - \frac{15N}{2}\end{aligned}$$

(d) We can use the ln approximation of the sum of harmonic series.

$$\begin{aligned}\sum_{i=8}^{280}\frac{1}{i} &= \sum_{i=1}^{280}\frac{1}{i} - \sum_{i=1}^{7}\frac{1}{i} \\ &= \ln 280 - \ln 7 + const = \ln 40 + const\end{aligned}$$

(e)

$$\text{Let } S = \sum_{i=1}^{\infty}\frac{i}{32^i}$$

$$\begin{array}{rl} 32S = & \frac{1}{32^0} + \frac{2}{32^1} + \frac{3}{32^2} + \frac{4}{32^3} + ... \\ - \quad S = & \frac{1}{32^1} + \frac{2}{32^2} + \frac{3}{32^3} + \frac{4}{32^4} + ... \\ \hline 31S = & \frac{1}{32^0} + \frac{1}{32^1} + \frac{1}{32^2} + \frac{1}{32^3} + ... \end{array}$$

$$\begin{aligned}S &= \frac{1}{31}\sum_{i=0}^{\infty}\left(\frac{1}{32}\right)^i \\ &= \frac{1}{31}\frac{1}{1-\frac{1}{32}} \\ &= \frac{32}{961}\end{aligned}$$

Using the formula ($\sum_{n=1}^{\infty} nr^n = \frac{r}{(r-1)^2}$) is also acceptable, differentiating both sides of the geometric sum is another explanation for the formula.

2. (a)

$$\begin{aligned}x^{21}x^{22}x^{23}...x^{72} &= x^{\sum_{k=21}^{73} k} \\ &= x^{\frac{72(72+1)}{2} - \frac{20(20+1)}{2}} \\ &= x^{2418}\end{aligned}$$

(b)

$$\log_{24} 76^3 = 3 \log_{24} 76$$

(c)

$$\text{Let } A = 32^{\log_{32} 841}$$

$$\begin{aligned} \log_{32} A &= \log_{32} 841 \\ A &= 841 \\ 32^{\log_{32} 841} &= 841 \end{aligned}$$

(d)

$$\begin{aligned} \log_{49}(7x)^y &= y \log_{49} 7x \\ &= y(\log_{49} 49^{1/2} + \log_{49} x) \\ &= y(0.5 + \log_{49} x) \end{aligned}$$

(e)

$$\begin{aligned} \sum_{i=1}^{3^N} \log_{54} i &= \log_{54} \prod_{i=1}^{3^N} i \\ &= \log_{54}(3^N)! \end{aligned}$$

3. (a) We are going to pick groups of 8 students 3 times.

$$\binom{48}{8} \times \binom{40}{8} \times \binom{32}{8} = 305,240,072,216,678,400,087,000$$

(b) There are two ways which we can choose 3 numbers whose sum is even: three even numbers (i) or two odd numbers and one even number (ii). There are 49 even and 50 odd numbers in the given set. The ordering of the numbers doesn't matter, the numbers must be unique.

$$\text{(i)}\ \frac{49 \times 48 \times 47}{3!} = 18,424$$

$$\text{(ii)}\ \frac{50 \times 49 \times 49}{2!} = 60,025$$

Note: (ii) is divided by 2 because the ordering of the two odd numbers doesn't matter.

$$\text{(i)} + \text{(ii)} = 78,449$$

Note: Not dividing the result by 2! and 3! is only -1 points, since the question might be interpreted to assume that the ordering is important.