EC330 HW6 Solution

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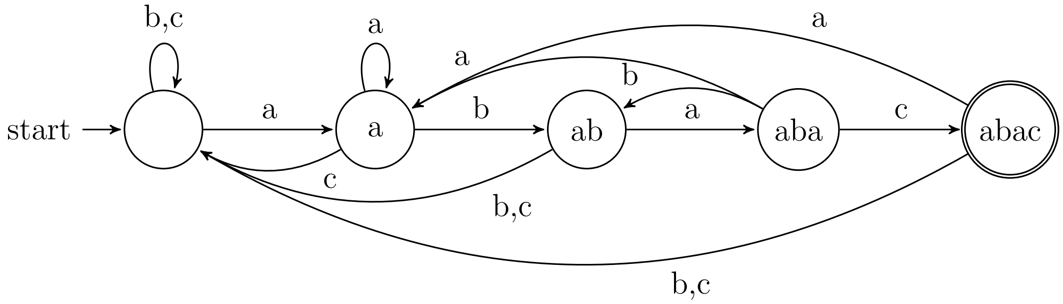
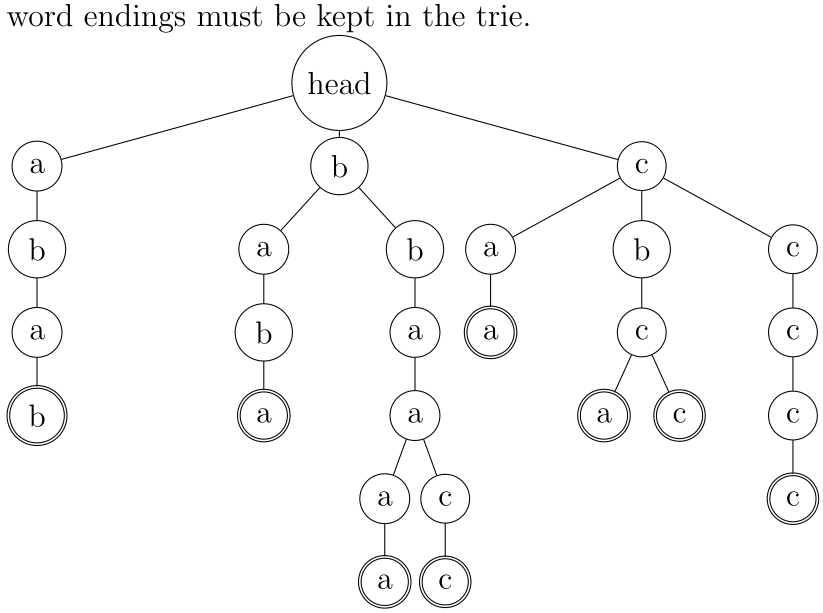
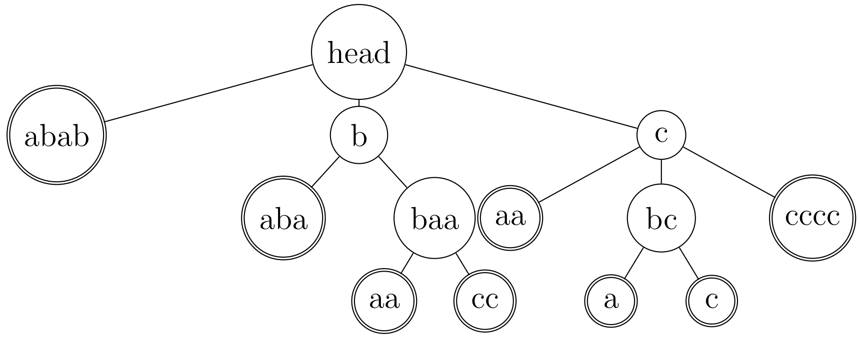
1. (a) The state machine is given below.



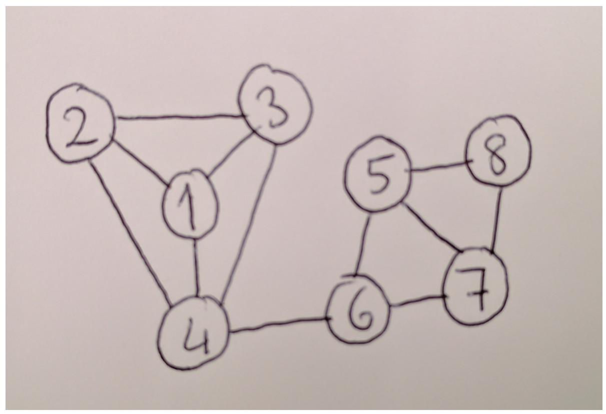
(b) The trie is given below. The word endings are indicated with double borders. Even though we only have word endings in leaves, it is not necessarily so, and word endings must be kept in the trie.



(c) The compressed trie is given below:



2. (a) The graph is given below



(b) For $O(V)$ edges, both an edge list and an adjacency list would be most space optimal. The space requirement would be $O(V)$.

For $O(V^2)$ edges, the representation doesn't matter. All representations (edge list, adjacency matrix and adjacency list) take $O(V^2)$ space.

If we care about adjacency time, we can use an adjacency matrix. It will allow us to determine whether two vertices are adjacent in $O(1)$ time.

(c) For a random vertex v, look at all the vertices u adjacent to it. For u, if there is a vertex in G that is not adjacent to u, disregard u. If we can find a u to which all vertices in G are adjacent, that is a universal sink.

```
Function Main(G = (E,V)):
    List L
    v ∈ V                                  /* v is a random element from V */
    for u | (v,u) ∈ E do
        if outdegree(u) = 0 then
            L.add(u)
        end
    end
    for t ∈ V do
        if len(L) = 0 then
            return False                   /* No sinks found */
        end
        for u ∈ L do
            if (u,t) ∉ E then
                L.remove(u)
            end
        end
    end
    return L.pop()    /* We are guaranteed to have one element left in L */
```

3. (a) For cycle graphs, we need to have an even number of vertices. For odd n, we will have the endpoints with the same color if we use two colors. The chromatic number is 2 for cycle graphs where n is even.

   (b) For star graphs, if we color the center vertex with one color, all of the other vertices can be the other color. The chromatic number is 2 for all star graphs.

   (c) For wheel graphs, we need to have an even number of outer vertices. The center can be one color, and the outer vertices can have alternating colors. Three colors are enough if the outer vertices are even numbered. The chromatic number is 3 for wheel graphs with an odd vertex count.

   (d) Proof by induction: For Q_1, we only need two colors since we only have two vertices. For any hypercube graph Q_n, we can construct it from two copies of Q_{n-1} by connecting every vertex to its counterpart. This requires only two colors. Therefore, any Q_n has a chromatic number of 2.

   This means that while the hypercubes are 3-colorable, the chromatic numbers are 2.

   (e) For every vertex in a complete bipartite graph, the only adjacent vertices are the edges in the opposite group. This means that opposing groups can be colored with two colors. Therefore, all $K_{n,n}$ has a chromatic number of 2.

   This means that all bipartite complete graphs are 3-colorable, but their chromatic numbers are 2.