


# Relational Operators

COURSE 8: Databases

# Relational model

# Relational model

- Codd rules 1985 → Is DBMS relational? If yes, to what degree?



Relational  
Integrity  
constraints

RELATIONS

OPERATORS

# Relational model


- Database = collection of RELATIONS
  - relation in relational model  $\neq$  relationship in ERD.
  - **relation** in relation model  $\leftrightarrow$  **table** with lines and columns
- **Relation Schema**: A relation schema represents the name of the relation with its attributes.
- Attribute **domain** – Each attribute has some pre-defined values.

Relational  
Integrity  
constraints

RELATIONS

OPERATORS

- Relational schema  $R(A_1, A_2, \dots, A_n)$
- $R \subset D_1 \times D_2 \times \dots \times D_n, D_i \text{ domain}$
- Example
  - Participant(participant\_id, last\_name, first\_name)
  - A1 - - participant\_id      D1 - - integer size 6
  - A2 - - last\_name          D2 - - string, length 20
  - A3 - - first\_name         D3 - - string, length 20




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constraints

RELATIONS

OPERATORS

- Domain constraints
  - “the value of each attribute must be unique”, specifies data types: integers, real numbers, characters, Booleans; variable length for strings, numbers etc.
- Key constraint
  - Unique + not null -- **PK**
- Referential integrity constraints
  - the value of a **FK** is null or it corresponds to the value of a PK.



Relational  
Integrity  
constraints

RELATIONS

OPERATORS

- UNION, INTERSECT, PRODUCT, DIFFERENCE
- PROJECT
- SELECT
- JOIN
- DIVISION

# Relational operators

Relational algebra



# Relational algebra

- Operands → relations (tables)
- Operators → operate on a relation/combine two relations  
and obtain as a result a new relation  
→ compositional

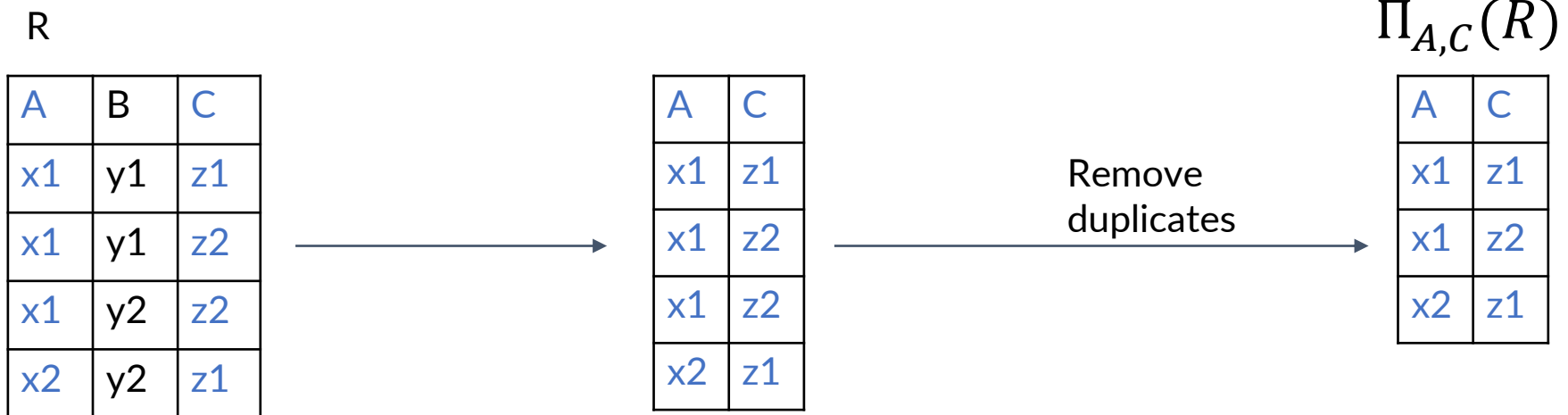
PROJECT, SELECT, DIFFERENCE, PRODUCT, UNION

Derived operators: JOIN, DIVISION, INTERSECT

# Project

- Unary operator
- Notations: **PROJECT(R, X)** or  **$\Pi_X (R)$** 
  - R is a relation; X is a set of attributes of R
- The result is a relation with a subset of the attributes X.
- Eliminating attributes from R may lead to duplicate rows. Hence after eliminating attributes, project also eliminates duplicate lines.

# PROJECT(R, X) $\Pi_X(R)$



$\text{PROJECT}(R, X) \quad \Pi_X (R)$

SQL

```
select distinct last_name, first_name  
from employees;
```

```
select last_name, first_name  
from employees  
group by last_name, first_name;
```

Usually needs a temporary table. Optimizations use indexes [1].

# Relational algebra properties $\Pi_x (R)$

## Rule 1: Project composition:

$$\Pi_{\{A_1, \dots, A_n\}} \left( \Pi_{\{B_1, \dots, B_m\}}(R) \right) = \Pi_{\{A_1, \dots, A_n\}}(R), \quad \{A_1, \dots, A_n\} \subseteq \{B_1, \dots, B_m\}$$

```
select last_name, first_name, salary
from
  (select last_name, first_name, salary, job_id
   from employees);
```

```
select last_name, first_name, salary
from employees;
```

# Select

- Unary operator
- Notations: **SELECT(R, C)** or  $\sigma_C(R)$
- R is a relation; C is a logical formula with attributes of R, constants and operators: AND, OR, NOT, <, =, >, <=, >=, !=
- The result is a relation with all the attributes of R but with only those lines satisfying **C**.

# SELECT(R, C)    $\sigma_C(R)$

R

A	B	C
x1	y1	z1
x1	y1	z2
x1	y2	z2
x2	y2	z1

$\sigma_{B='y2' \text{ or } C='z2'}(R)$

$\sigma_C(R)$

A	B	C
x1	y1	z2
x1	y2	z2
x2	y2	z1

$\text{SELECT}(R, C) \quad \sigma_C(R)$

SQL

```
select * from employees  
where last_name = 'King' or first_name = 'Steven';
```

Optimizations use indexes.



# Relational algebra properties $\sigma_C(R)$

Rule 2: Selection composition:

$$\sigma_{C_1}(\sigma_{C_2}(R)) = \sigma_{C_2}(\sigma_{C_1}(R)) = \sigma_{C_1 \wedge C_2}(R)$$

```
select job_id, job_title, min_salary, max_salary
from
  (select job_id, job_title, min_salary, max_salary
   from jobs
   where min_salary > 8000)
where min_salary < 10000;
```

```
select job_id, job_title, min_salary, max_salary
from jobs
where min_salary > 8000 and min_salary < 10000;
```

# Relational algebra properties $\Pi_X(R)$ and $\sigma_C(R)$

Rule 3a: Selection and projection commute:

$$\Pi_{\{A_1, \dots, A_n\}}(\sigma_C(R)) = \sigma_C(\Pi_{\{A_1, \dots, A_n\}}(R)), \quad C \text{ operands are in } \{A_1, \dots, A_n\}$$

```
select job_title, min_salary
from
  (select job_id, job_title, min_salary, max_salary
   from jobs
   where min_salary > 8000);
```

```
select job_title, min_salary
from
  (select job_title, min_salary
   from jobs)
where min_salary > 8000;
```

# Relational algebra properties $\Pi_X(R)$ and $\sigma_C(R)$

Rule 3b: Selection and projection commute:

$$\Pi_{\{A_1, \dots, A_n\}}(\sigma_C(R)) = \Pi_{\{A_1, \dots, A_n, B_1, \dots, B_m\}}(\sigma_C(\Pi_{\{A_1, \dots, A_n, B_1, \dots, B_m\}}(R))),$$

*C operands are in  $\{A_1, \dots, A_n, B_1, \dots, B_m\}$*

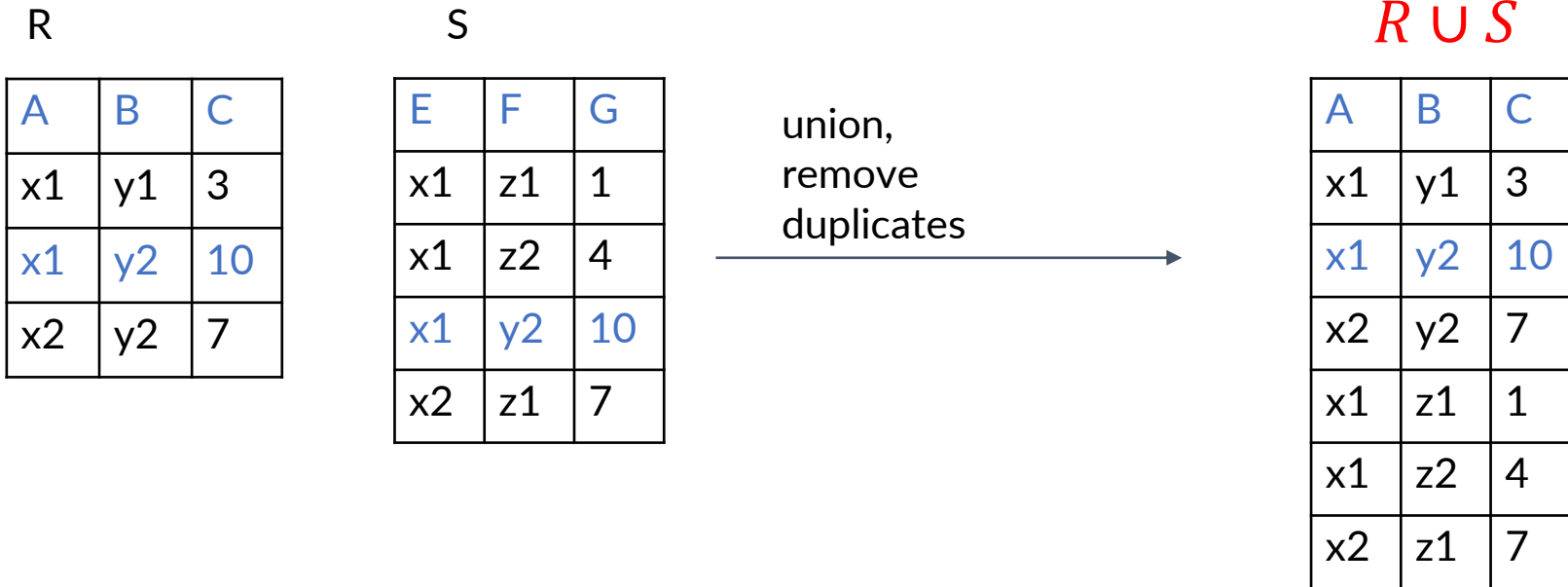
```
select job_title, min_salary
from
  (select job_id, job_title, min_salary, max_salary
   from jobs
   where min_salary > 8000
        and max_salary < 10000);
```

```
select job_title, min_salary
from
  (select job_title, min_salary, max_salary
   from jobs
   where min_salary > 8000
        and max_salary < 10000);
```

# UNION

- Binary operator
- Notations: **UNION(R, S)** or  **$R \cup S$**
- R and S relations; the result of union is the set of all tuples in R or S. --  
- - union on set of tuples.
- R and S must have compatibles relational schemas, i.e., the same number of attributes and the same attributes types.
- **$R \cup S = S \cup R$**

# UNION(R, S)    $R \cup S$



# UNION(R, S)    $R \cup S$

## SQL

```
select employee_id, start_date from job_history  
union /*sorts the result*/  
select employee_id, hire_date from employees;
```

```
select employee_id, start_date from job_history  
union all /* keeps duplicates */  
select employee_id, hire_date from employees;
```

Optimizations: AVOID union, use temporary tables, see  
QueryOptimization.sql

# UNION(R, S)    $R \cup S$

## SQL

```
select id, uuid, random_number  
from no_index  
union  
select id, uuid, random_number  
from no_index;
```

/\* ~210sec

```
select id, uuid, random_number  
from no_index  
union all  
select id, uuid, random_number  
from no_index;
```

/\* ~15sec

Optimizations: AVOID union, use temporary tables.

# Relational algebra properties **UNION** and $\sigma_C(R)$

Rule 4: Selection and union commute:

$$\sigma_C(R_1 \cup R_2) = \sigma_C(R_1) \cup \sigma_C(R_2)$$

```
select employee_id, start_date from job_history
where employee_id > 110
union all
select employee_id, hire_date from employees
where employee_id > 110;
```

```
select employee_id, start_date from (
select employee_id, start_date from job_history
union all
select employee_id, hire_date from employees
)
where employee_id > 110; /*see exec. plans*/
```



# Relational algebra properties **UNION** and $\Pi_x (R)$

Rule 5: Projection and union commute:

$$\Pi_{\{A_1, \dots, A_n\}}(R_1 \cup R_2) = \Pi_{\{A_1, \dots, A_n\}}(R_1) \cup \Pi_{\{A_1, \dots, A_n\}}(R_2)$$

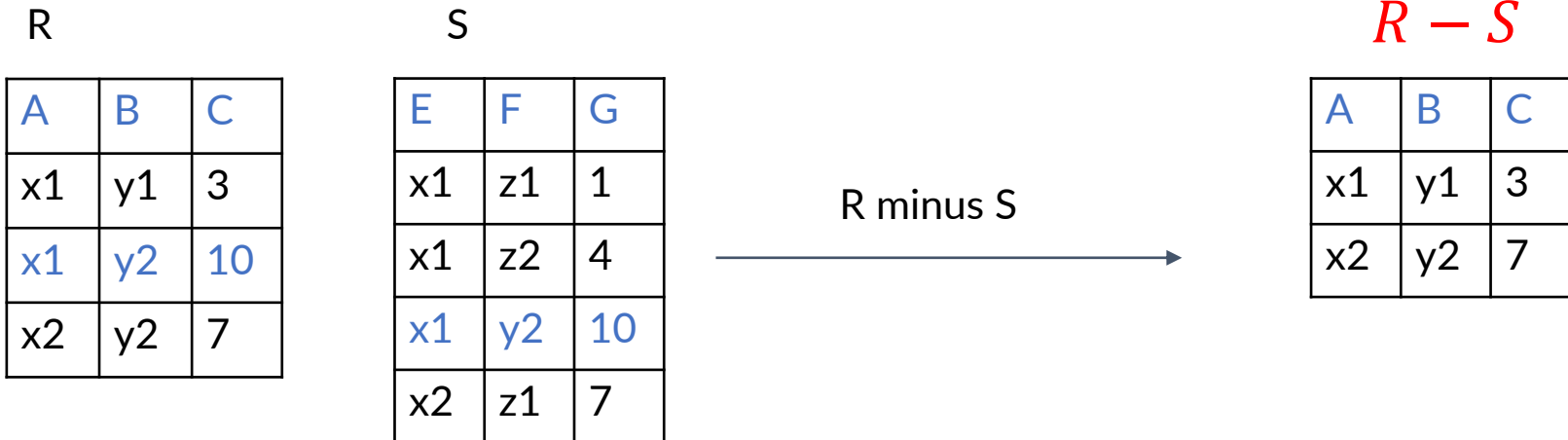
```
select start_date from (  
  select employee_id, start_date from job_history  
  union all  
  select employee_id, hire_date from employees  
);
```

```
select start_date from job_history  
union all  
select hire_date from employees
```

# DIFFERENCE

- Binary operator
- Notations: **DIFFERENCE(R, S)**, **MINUS(R,S)** or  $R - S$
- R and S relations; the result of difference is the set of all tuples in R that are not found in S.
  - difference on set of tuples.
- R and S must have compatibles relational schemas, i.e., the same number of attributes and the same attributes types.

# DIFFERENCE(R, S) $R - S$



# DIFFERENCE(R, S) $R - S$

## SQL

```
select department_id  
from departments  
minus  
select department_id  
from employees;
```

```
select department_id  
from departments d  
where d.department_id not in  
  (select department_id  
   from employees  
   where department_id is not null);
```

Optimizations: *not in* or *not exists*.

# Relational algebra properties **UNION** and **$R - S$**

Rule 6: Selection and difference commute:

$$\sigma_C(R_1 - R_2) = \sigma_C(R_1) - \sigma_C(R_2)$$

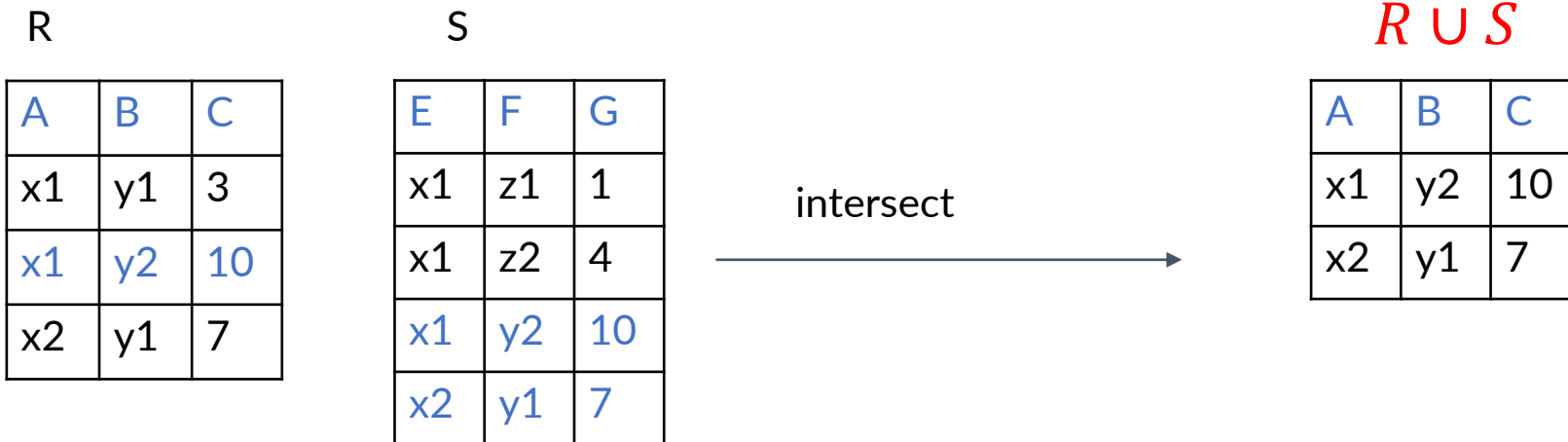
```
select department_id
from departments
where department_id > 120
minus
select department_id
from employees
where department_id > 120
```

```
select * from (
  select department_id
  from departments
  minus
  select department_id
  from employees
)
where department_id > 120; /*see exec. plans*/
```

# INTERSECT

- Binary operator
- Notations: **INTERSECT(R, S)** or  $R \cap S$
- R and S relations; the result of intersection is the set of all tuples that are both in R and in S. --- intersection on set of tuples.
- R and S must have compatibles relational schemas, i.e., the same number of attributes and the same attributes types.
- $R \cap S = S \cap R$
- $R \cap S = R - (R - S) = S - (S - R)$

# INTERSECT(R, S) $R \cap S$



# INTERSECT(R, S)    $R \cap S$

## SQL

```
select department_id  
from employees  
intersect  
select department_id  
from job_history;
```

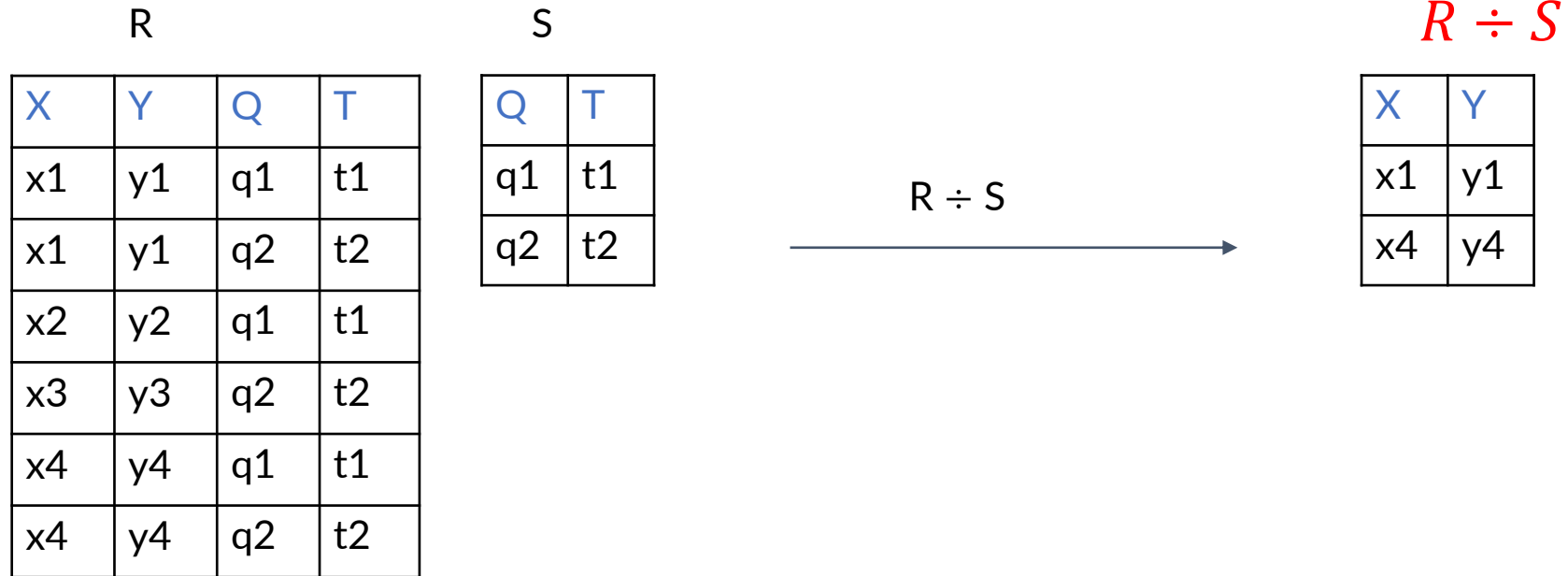
Optimizations: *in* or *exists*.



# DIVISION

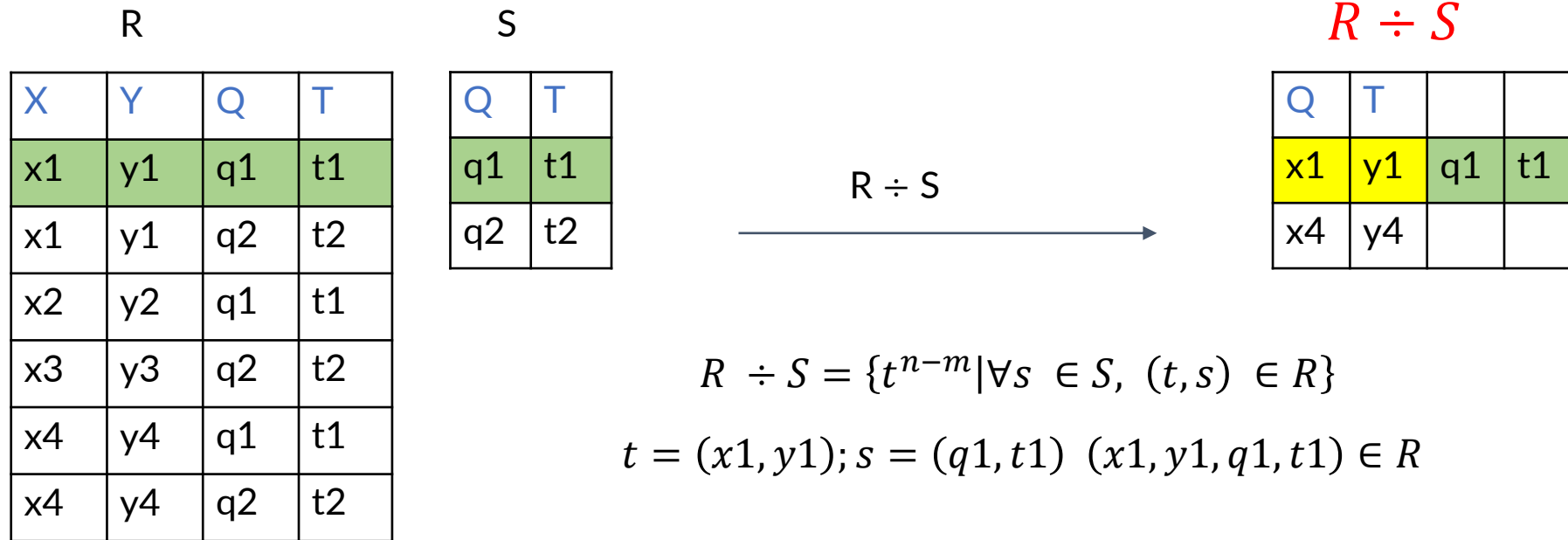
- Binary operator
- Notations: **DIVISION**(R, S) or  $R \div S$
- R and S relations; the result of difference is the set of all tuples to which any of the tuples in S can be added to obtain a tuple in R
- $R \div S = \{t^{n-m} \mid \forall s \in S, (t, s) \in R\}$
- If R has n attributes and S has m < n attribute the result of DIVISION has n – m attributes.

# DIVISION( $R, S$ ) $R \div S$



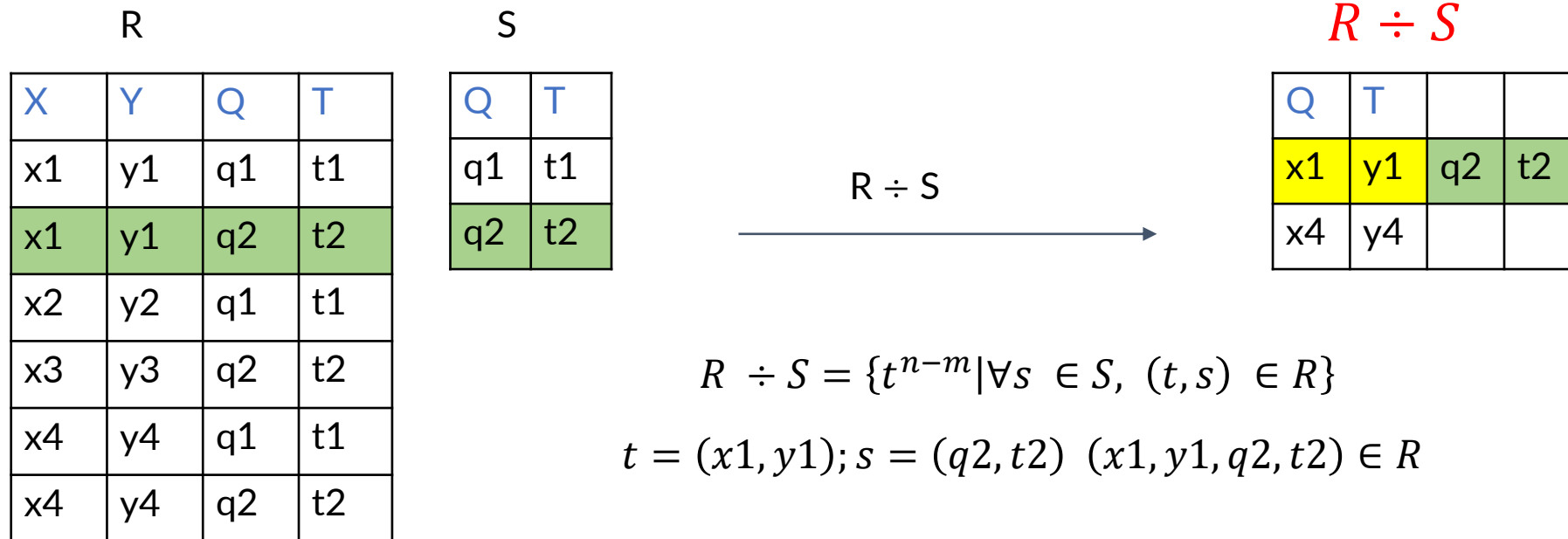
# DIVISION(R, S) $R \div S$

x1, y1 are “associated” with all tuples in S (R is the “associative” table):



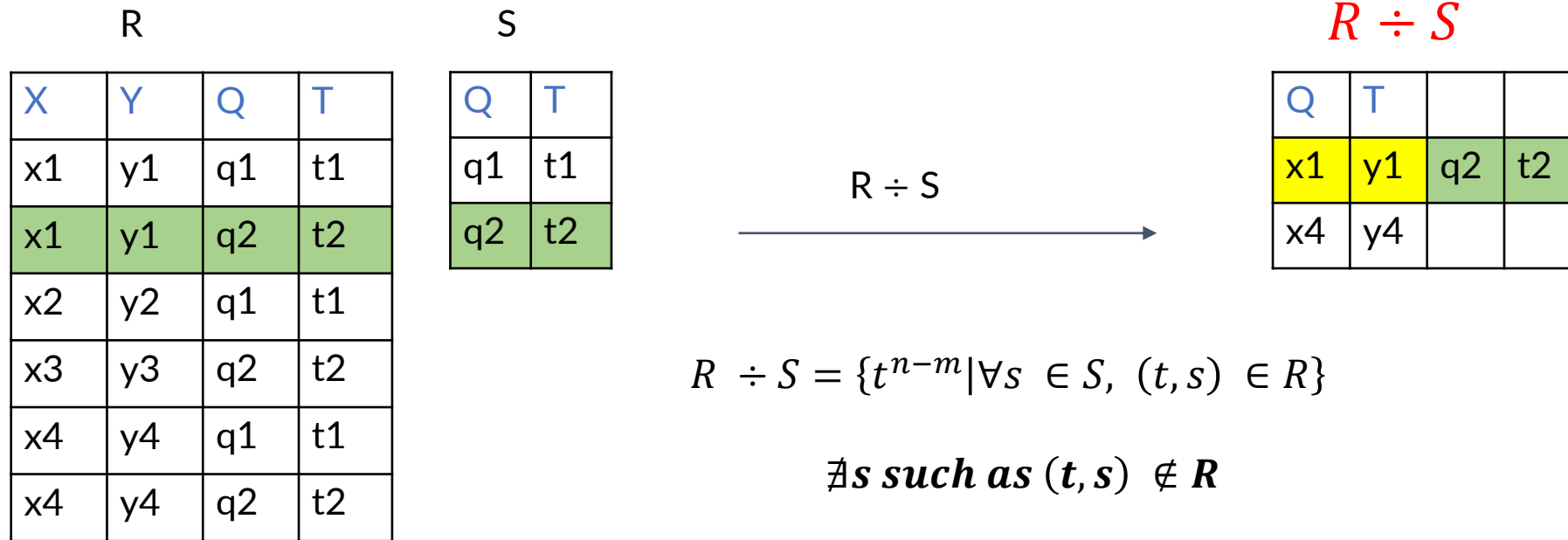
# DIVISION(R, S) $R \div S$

x1, y1 are associated with all tuples in S (R is the “associative” table):



# DIVISION( $R, S$ ) $R \div S$

$x_1, y_1$  are “associated” with all tuples in  $S$  ( $R$  is the “associative” table):



# DIVISION( $R, S$ ) $R \div S$

$(x2, y2)$  is not “associated” with all tuples in  $S$  :

R			
X	Y	Q	T
x1	y1	q1	t1
x1	y1	q2	t2
x2	y2	q1	t1
x3	y3	q2	t2
x4	y4	q1	t1
x4	y4	q2	t2

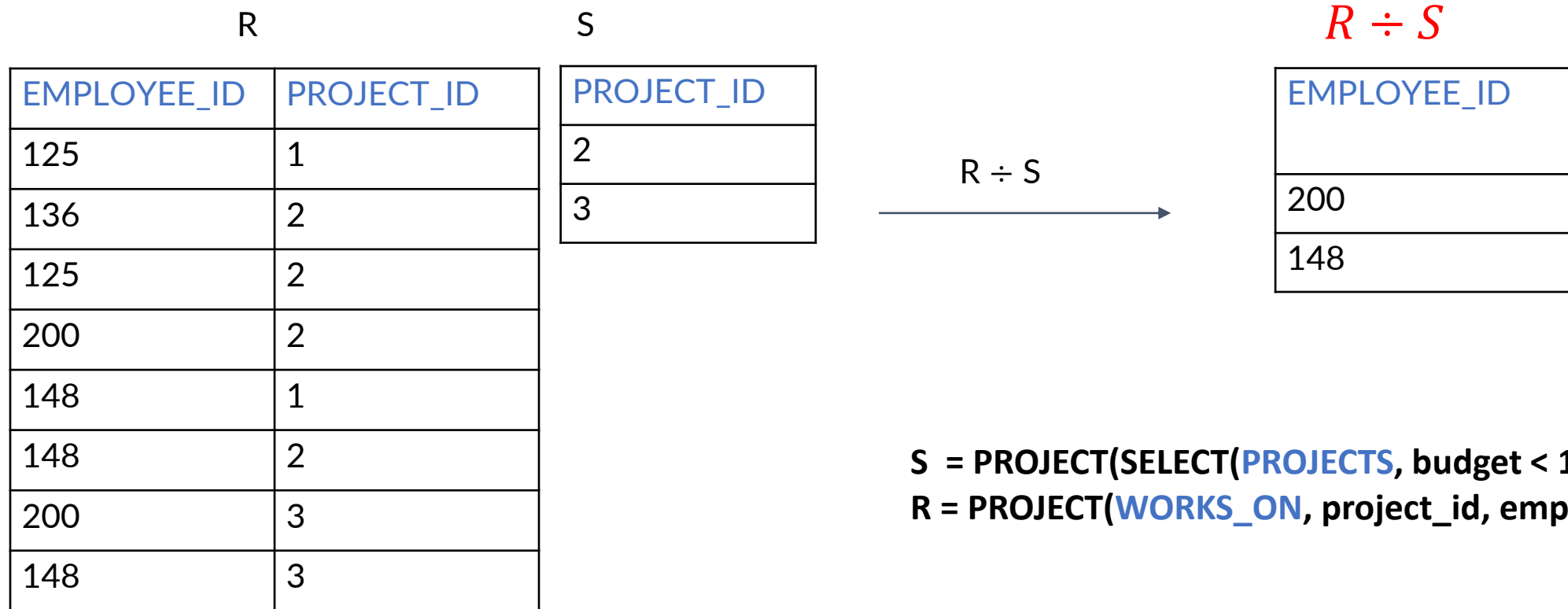
S	
Q	T
q1	t1
q2	t2

$R \div S$

$R \div S$			
Q	T		
x1	y1		
x4	y4		
x2	y2	q2	t2

# DIVISION(R, S) *SQL NOT EXISTS/COUNT*

Code for employees working on all projects with a budget < 10000

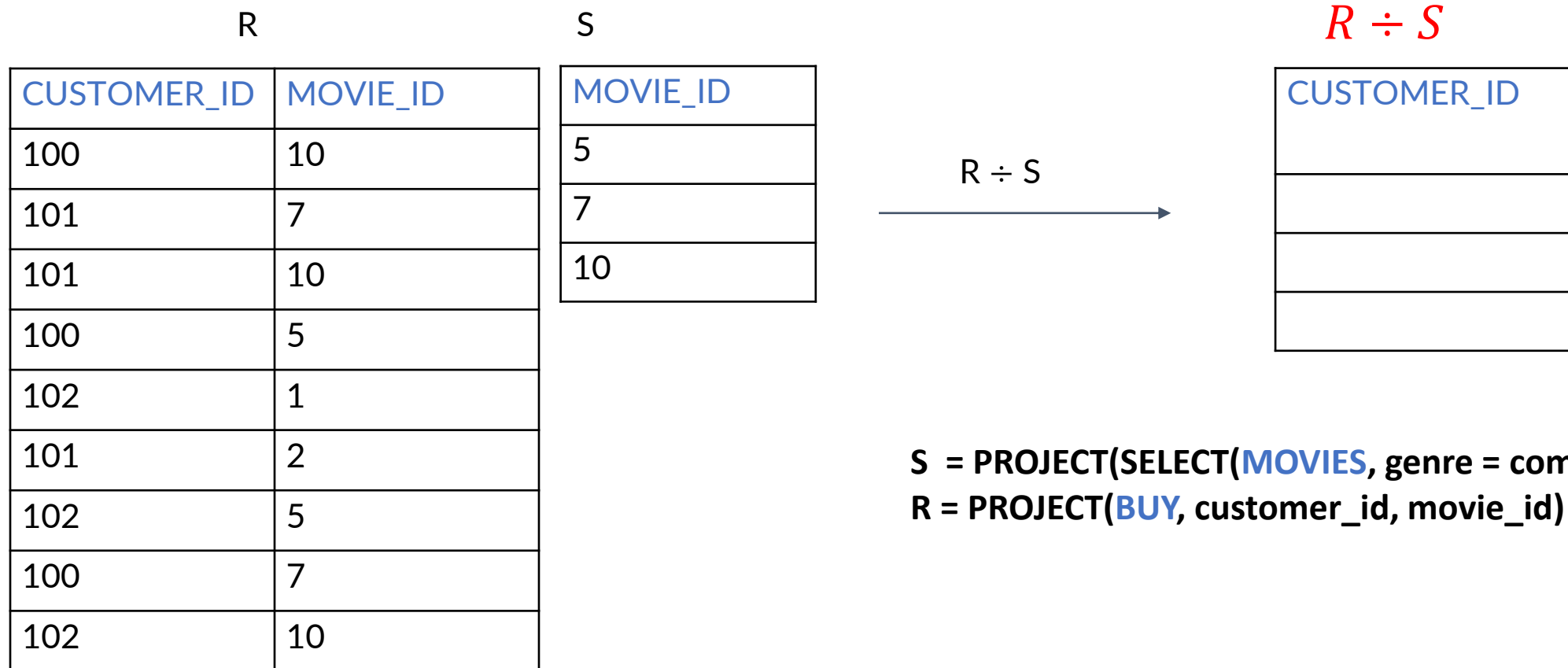


$S = \text{PROJECT}(\text{SELECT}(\text{PROJECTS}, \text{budget} < 10000), \text{project\_id})$

$R = \text{PROJECT}(\text{WORKS\_ON}, \text{project\_id}, \text{employee\_id})$

# DIVISION(R, S) *SQL NOT EXISTS/COUNT*

Code for customers buying tickets to all comedies < 10000



$S = \text{PROJECT}(\text{SELECT}(\text{MOVIES}, \text{genre} = \text{comedies}), \text{movie\_id})$

$R = \text{PROJECT}(\text{BUY}, \text{customer\_id}, \text{movie\_id})$



# PRODUCT

- Binary operator
- Notations: **PRODUCT**(R, S) or  $R \times S$
- R and S relations; the result of difference is the set of all tuples with  $m + n$  attributes, first  $m$  attributes form a tuple of R, last  $n$  attributes form a tuple of S
- $R \times S = \{(t, s) | t \in R, s \in S\}$

# PRODUCT(R, S) $R \times S$

R

X	Y	Q
x1	y1	q1
x2	y2	q2
x3	y3	q3
x4	y4	q4

S

U	V
u1	v1
u2	v2

$R \times S$

$R \times S$

X	Y	Q	U	V
x1	y1	q1	u1	v1
x2	y2	q2	u1	v1
x3	y3	q3	u1	v1
x4	y4	q4	u1	v1
x1	y1	q1	u2	v2
x2	y2	q2	u2	v2
x3	y3	q3	u2	v2
x4	y4	q4	u2	v2

PRODUCT(R, S)     $R \times S$

SQL

```
select e.*, d.*  
from employees e, departments d ;
```

```
select e.*, d.*  
from employees e cross join departments d ;
```

# JOIN

- Binary operator
- Notations: **JOIN(R, S)**

Natural Join:

$$\text{JOIN}(R, S) = \Pi_{\{i_1, \dots, i_m\}}(\sigma_{R.A1=S.A1 \wedge \dots R.An=S.An}(R \times S))$$

$A1, \dots, An$  are the intersection of attributes of  $R$  and  $S$ ,  $i1, \dots, im$  is the union of attributes of  $R$  and  $S$  minus  $A1, \dots, An$ .

# JOIN

- Binary operator
- Notations: **JOIN(R, S)**

Semi Join:

$$\text{SEMIJOIN}(R, S) = \Pi_{\{r_1, \dots, r_m\}}(\text{JOIN}(R, S))$$

$r_1, \dots, r_m$  is the union of attributes of R.

# JOIN

- Binary operator
- Notations: **JOIN(R, S)**

$\Theta$  join:

$$\Theta\text{-JOIN}(R, S, \sigma_{cond}) = \sigma_{cond}(R \times S)$$

# Relational algebra properties $\Pi_x(R)$ and $\sigma_c(R)$

Rule 6: JOIN/PRODUCT commute:

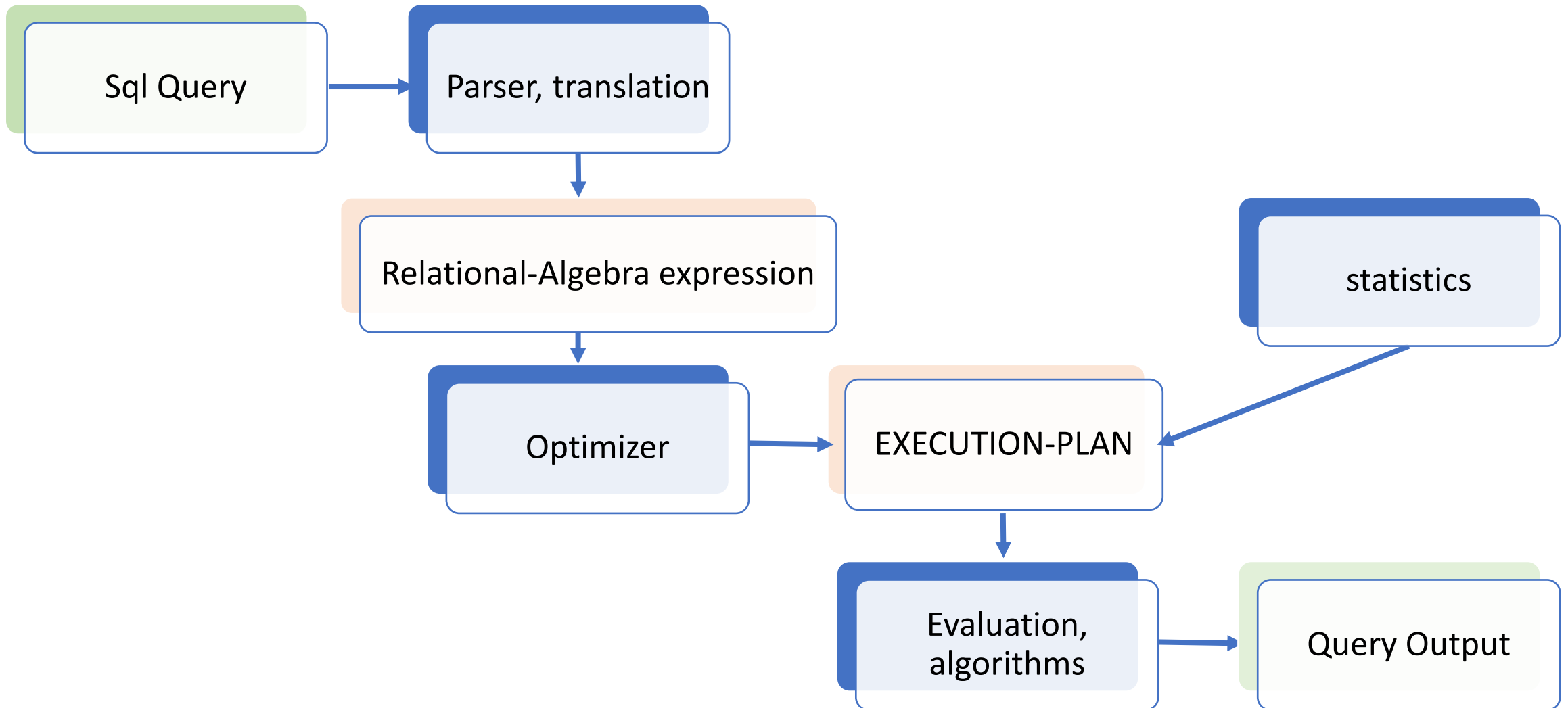
$$\begin{array}{ccc} JOIN(R, S) & = & JOIN(S, R), \\ R \times S & & = S \times R \end{array}$$

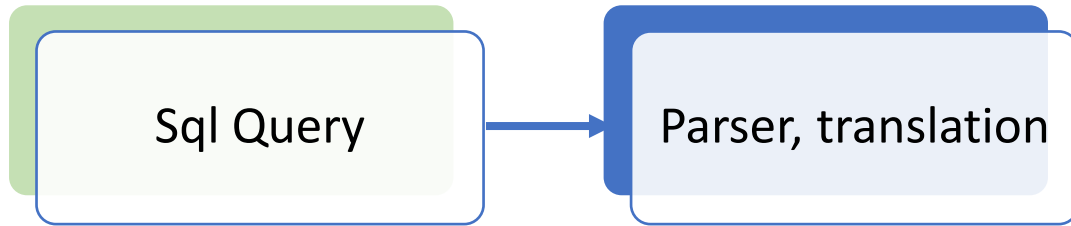
```
select last_name, department_name  
from employees e join departments d  
on (e.department_id = d.department_id);
```

```
select last_name, department_name  
from departments d join employees e  
on (e.department_id = d.department_id)
```

# Query execution

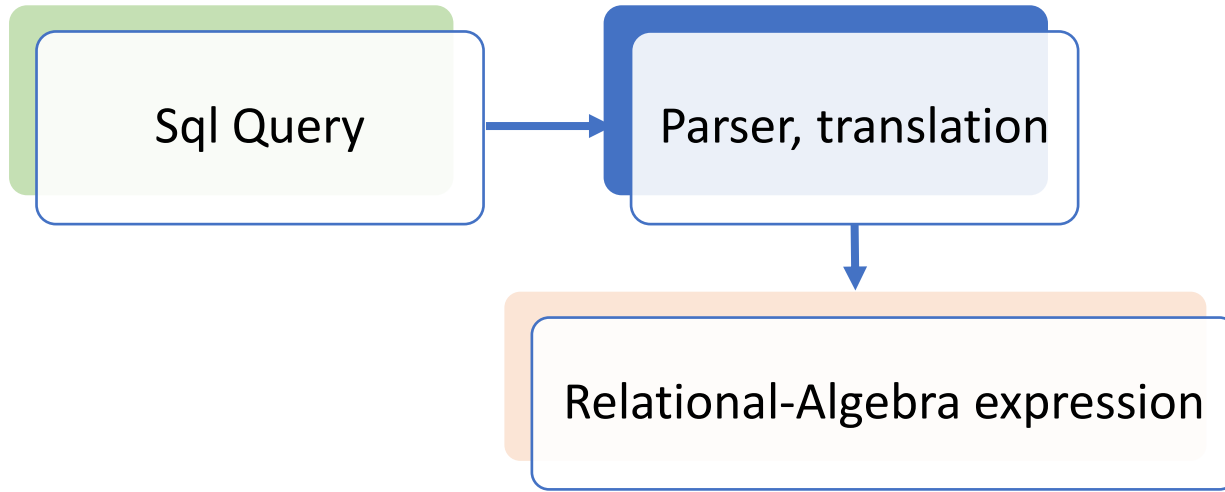






```
select p1.prod_name, p2.prod_name, p1.prod_min_price  
from products p1 join products p2  
on p1.prod_min_price = p2.prod_min_price
```

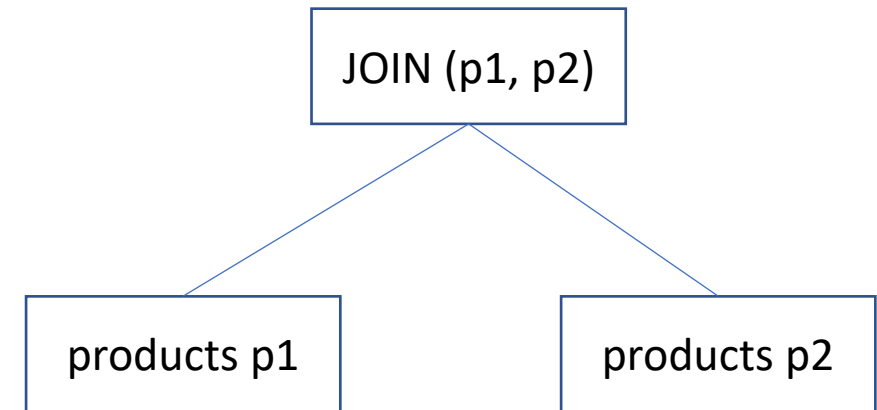
check syntax, table names, view names, column names

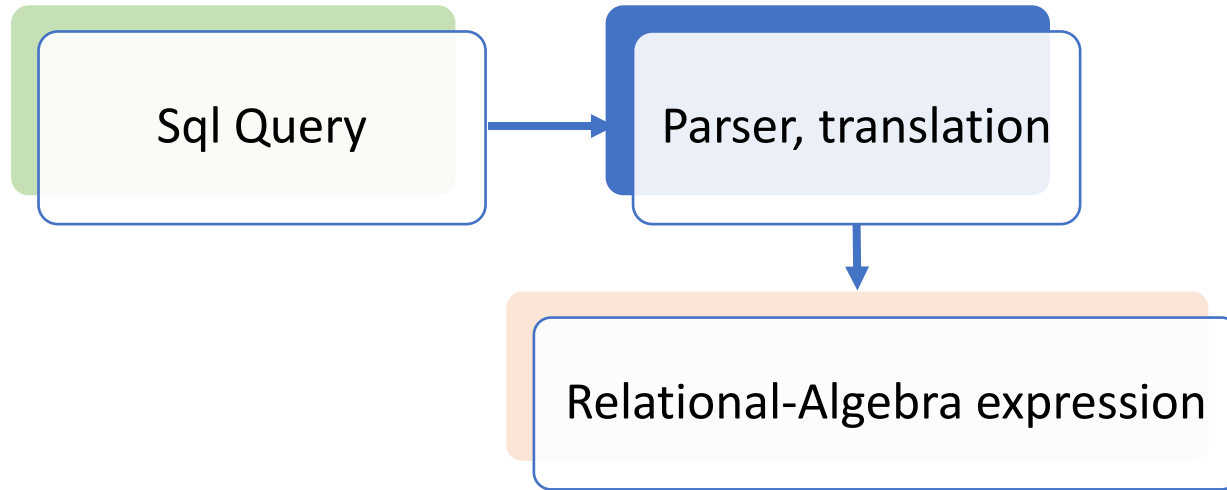


```
select p1.prod_name, p2.prod_name, p1.prod_min_price  
from products p1 join products p2  
on p1.prod_min_price = p2.prod_min_price
```

relations + operators

$JOIN(p1, p2)$

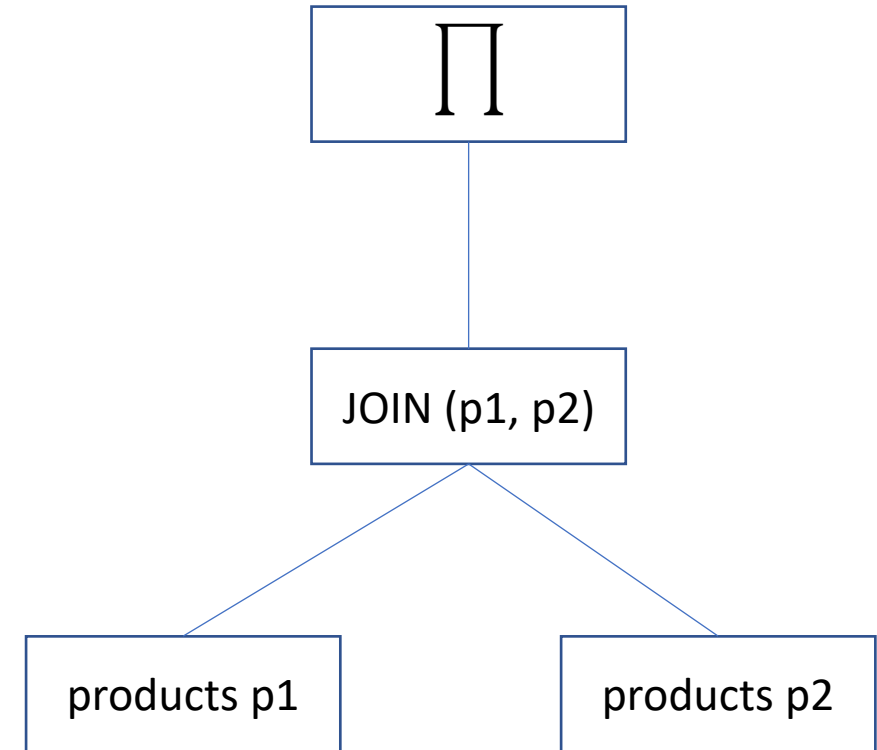


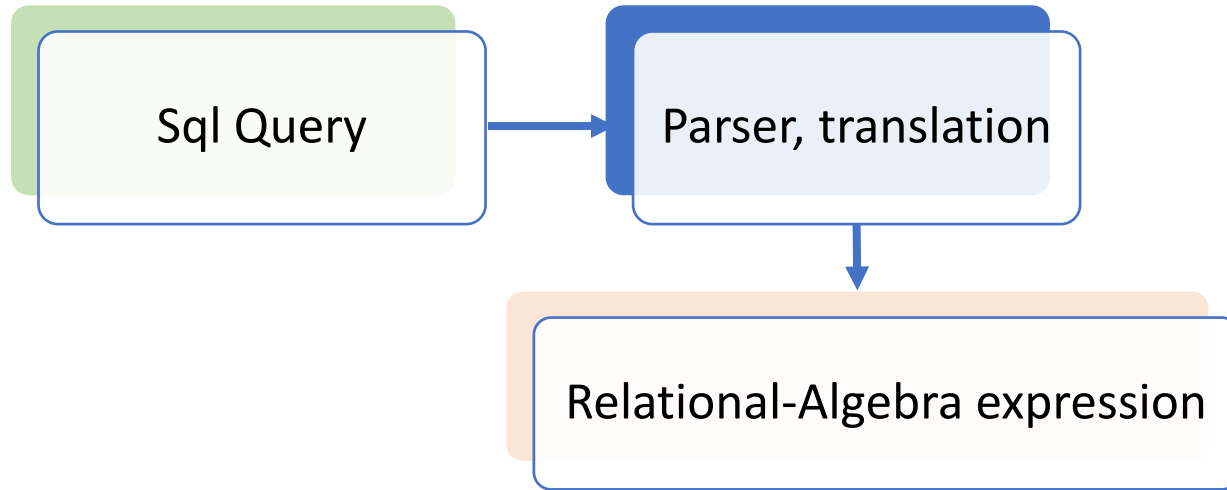


```

select p1.prod_name, p2.prod_name, p1.min_price
from products p1 join products p2
on p1.prod_min_price = p2.prod_min_price
  
```

$\Pi_{p1.prodname, p2.prodname, p1.minprice}$ 
 $JOIN(p1, p2)$

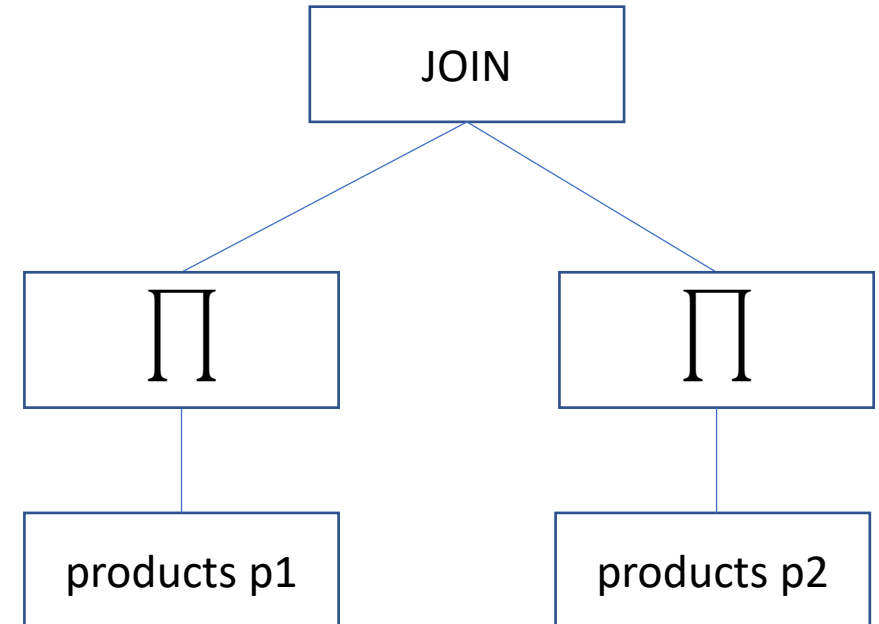


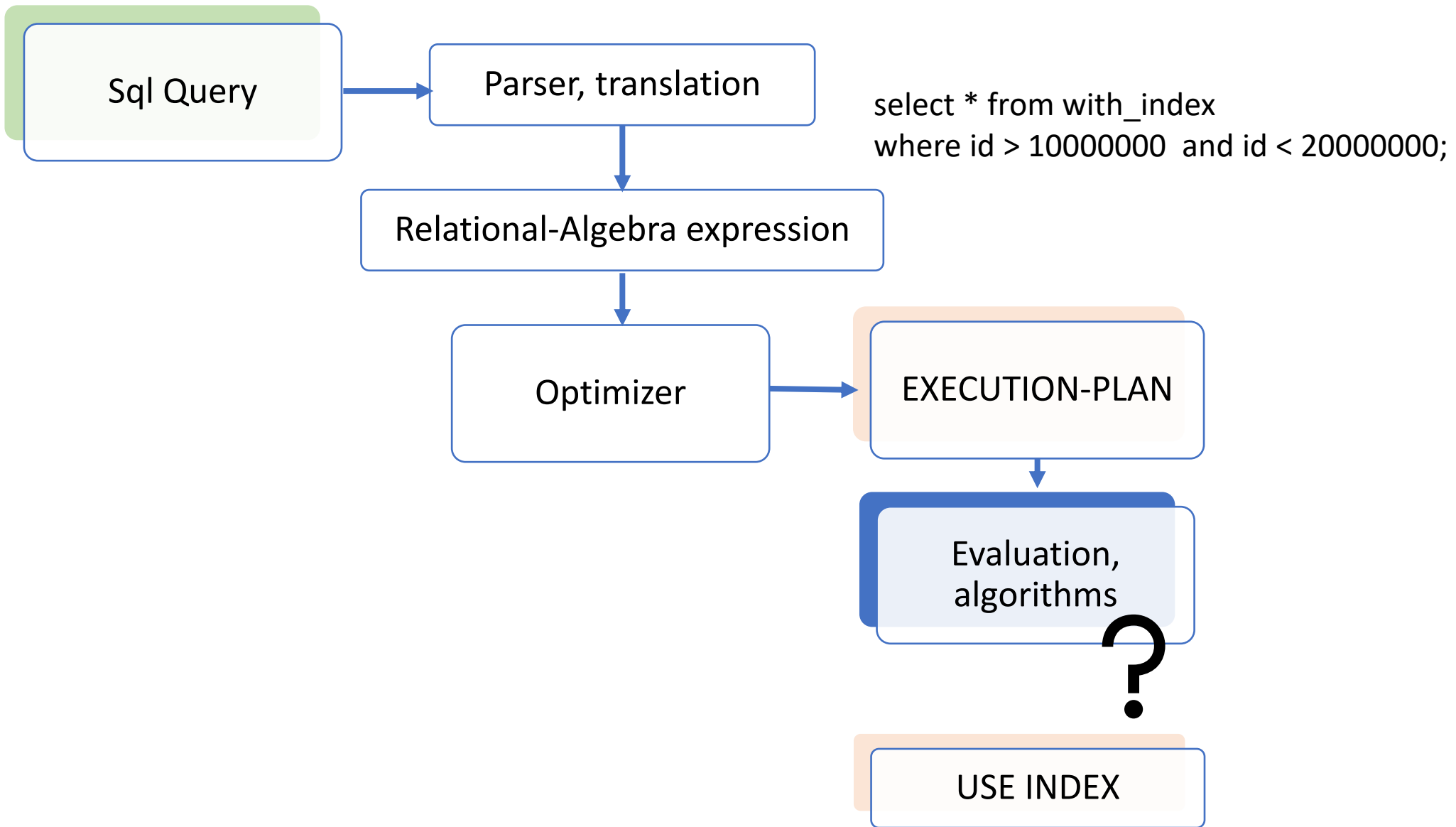


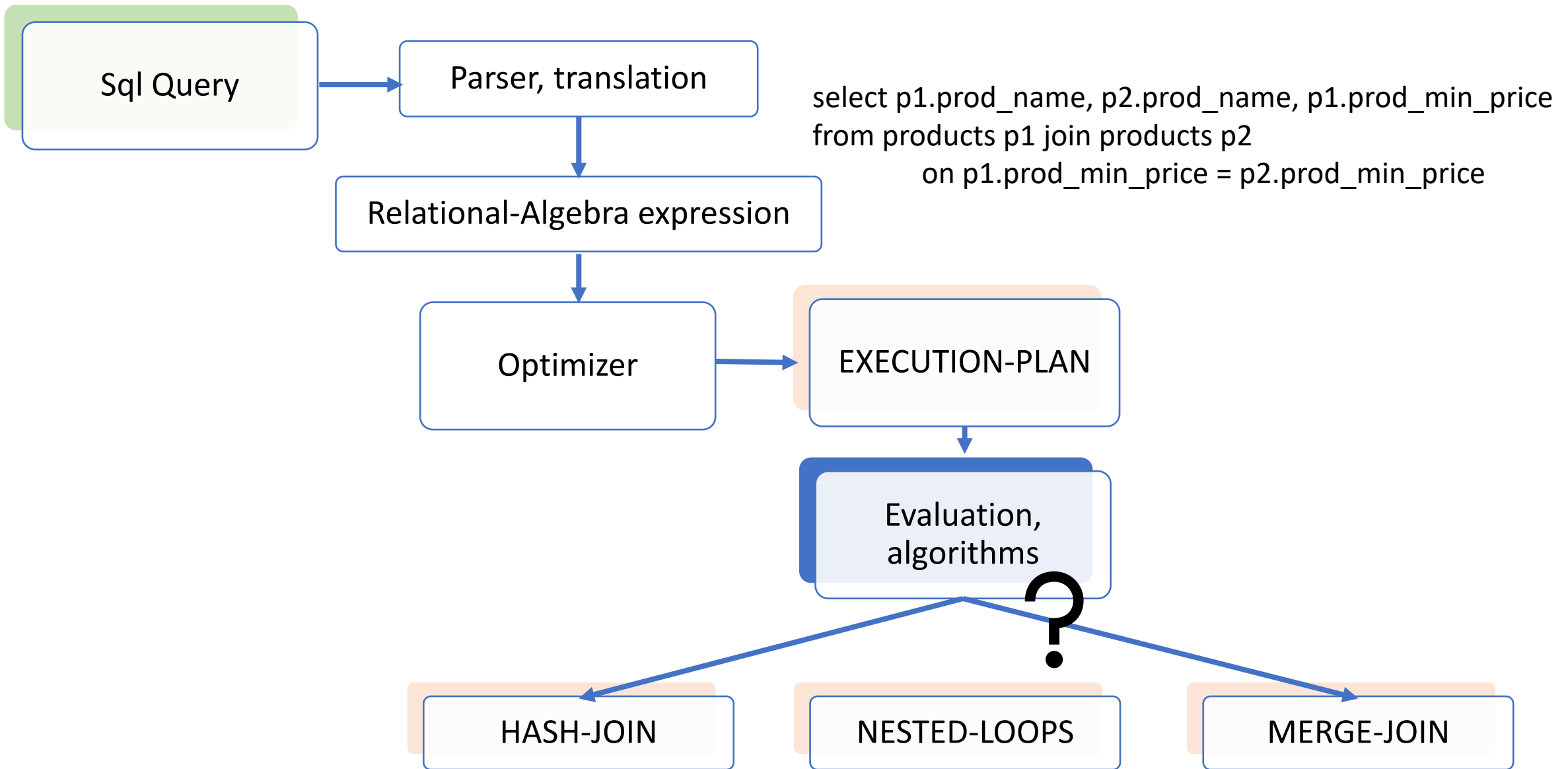
```
select p1.prod_name, p2.prod_name, p1.prod_min_price
from products p1 join products p2
on p1.prod_min_price = p2.prod_min_price
```

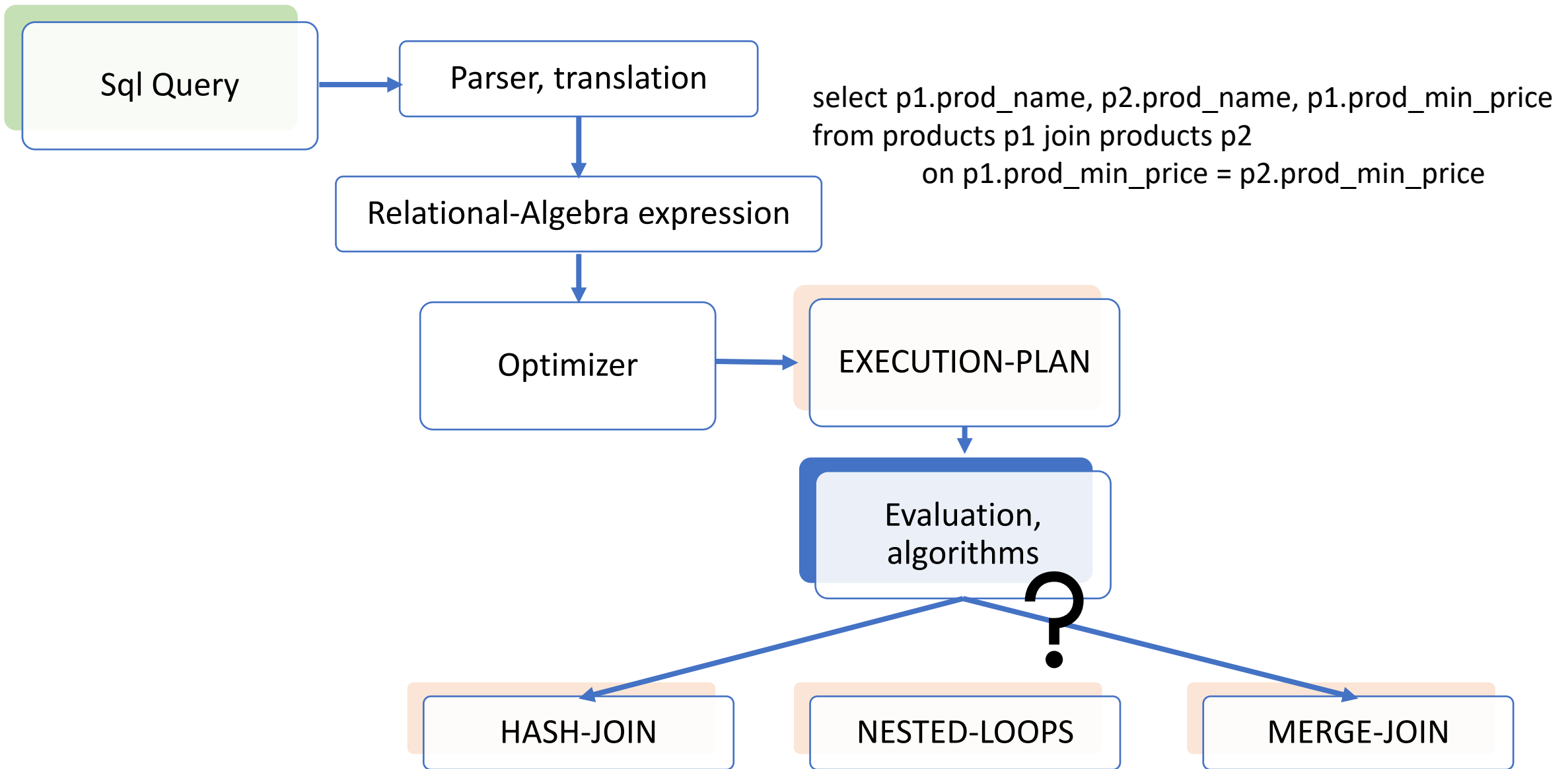
relations + operators

$JOIN(\prod_{name, minprice}^{p1}, \prod_{name, minprice}^{p2})$











## NESTED-LOOPS

```
for each tuple  $t_r$  in  $r$ 
    for each tuple  $t_s$  in  $s$ 
        if join condition  $\theta$  for pair  $(t_r, t_s) = \text{true}$ 
            result = result  $\cup (t_r, t_s)$ ;
        end
    end
end
```

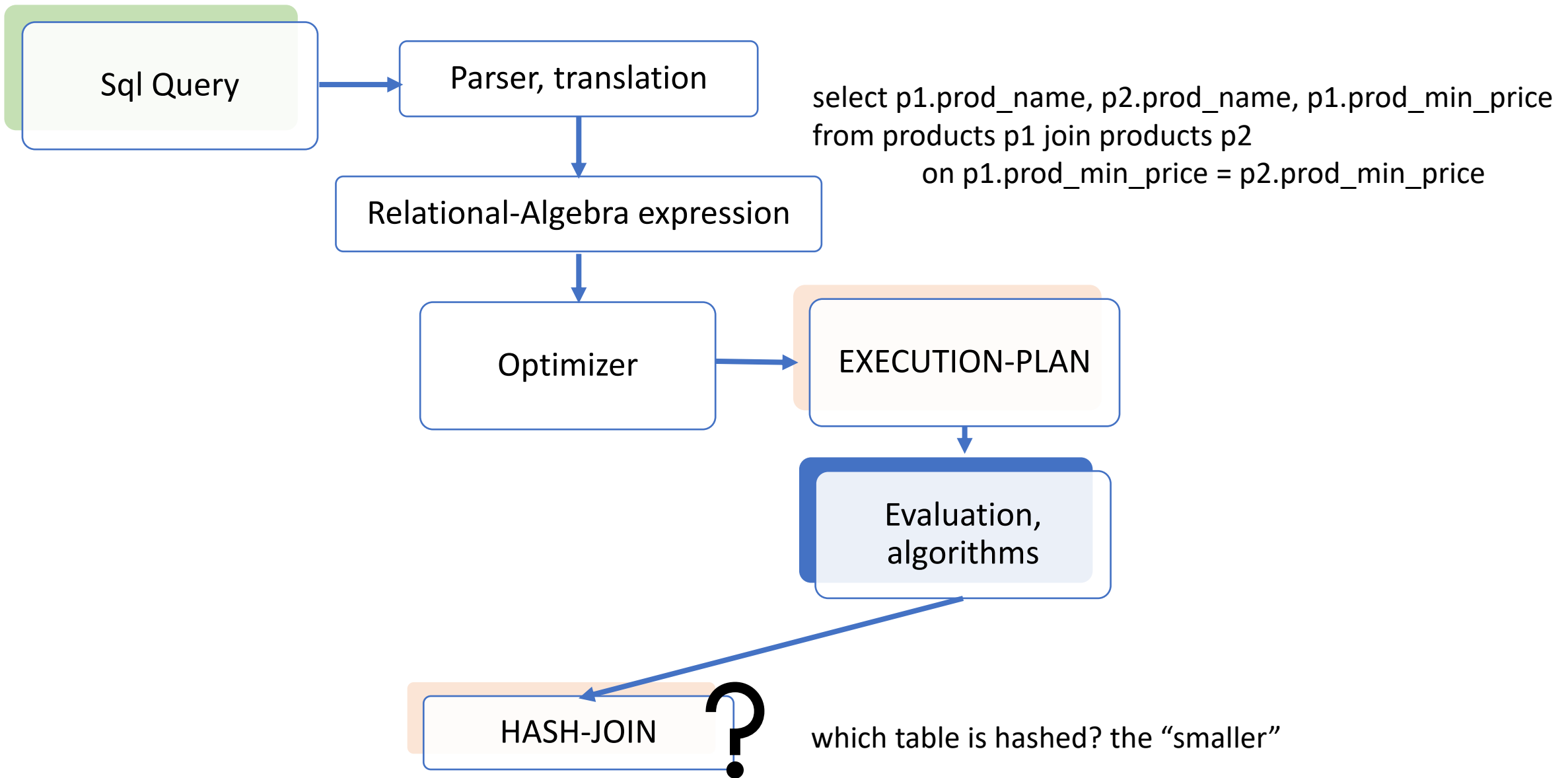
- Optimizations:
  - Block nested loops: each block in  $s$  is loaded in memory once for each block on  $r$ .
  - Index nested loops: if an index is present on  $s$ , it can be used to search for tuples satisfying join condition.

## MERGE-JOIN

```
sort R
sort S
r = R.first
s = S.first
while r <> null and s <> null
    if r.id > s.id
        s = next tuple in S
    else if r.id < s.id
        r = next tuple in R
    else
        result = result U (r,s)
        r' = next tuple in R
        while r' <> null and r'.id = s.id
            result = result U (r',s)
        s' = next tuple in S
        while s' <> null and r.id = s'.id
            result = result U (r,s')
        r = next tuple in R
        s = next tuple in S
```

## HASH-JOIN

```
partition R based on hash(key) -> Rn1,...,Rnp
partition S based on hash(key) -> Sn1,...,Snp
for each nkey
    build in-memory hash index
    for Rnkey-partition
    for each tuple ts in Snkey-partition
        probe ts , add matching tuples
```



# Bloom filters

# Bloom filters

- Probabilistic data structure, check membership for a value in a set.
- How it works:  $S$ , set of  $n$  values  $\rightarrow \text{const} * n$  bits  
calculate  $\text{hash}(v) \in [1, \text{const} * n]$   
set bit  $\text{hash}(v)$  to 1

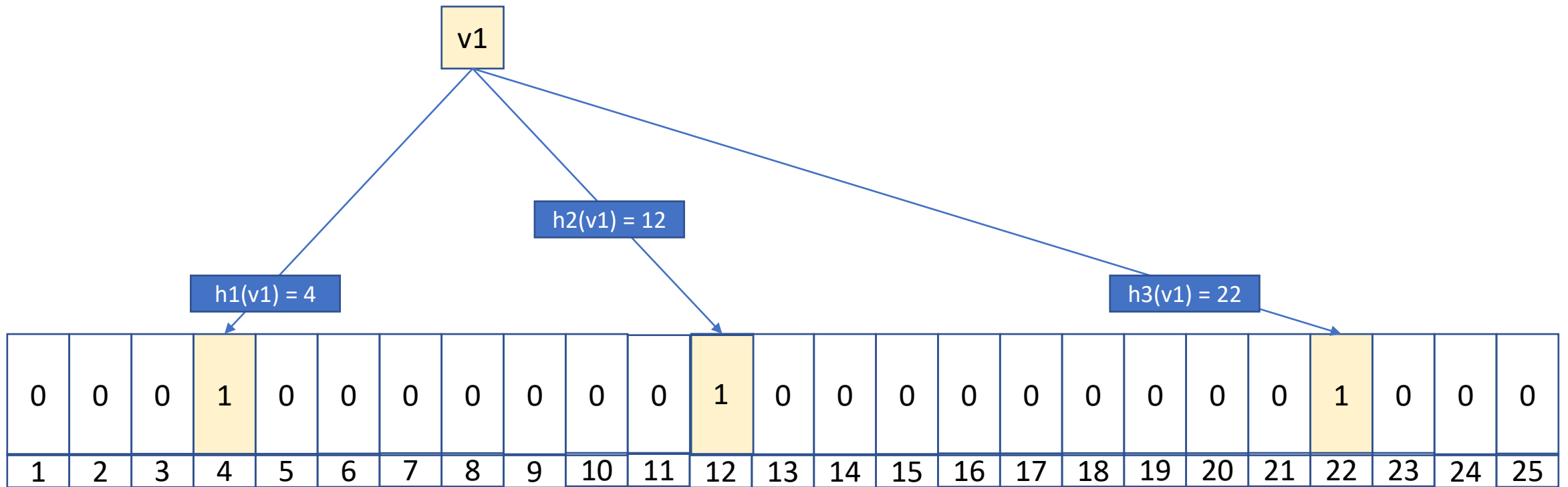
Test  $w \in S \rightarrow \text{hash}(w) = 1$  ?

- Small probability of **false positive**.  
 $w_1 \in S, w_2 \notin S \text{ hash}(w_1) = \text{hash}(w_2)$

# Bloom filters

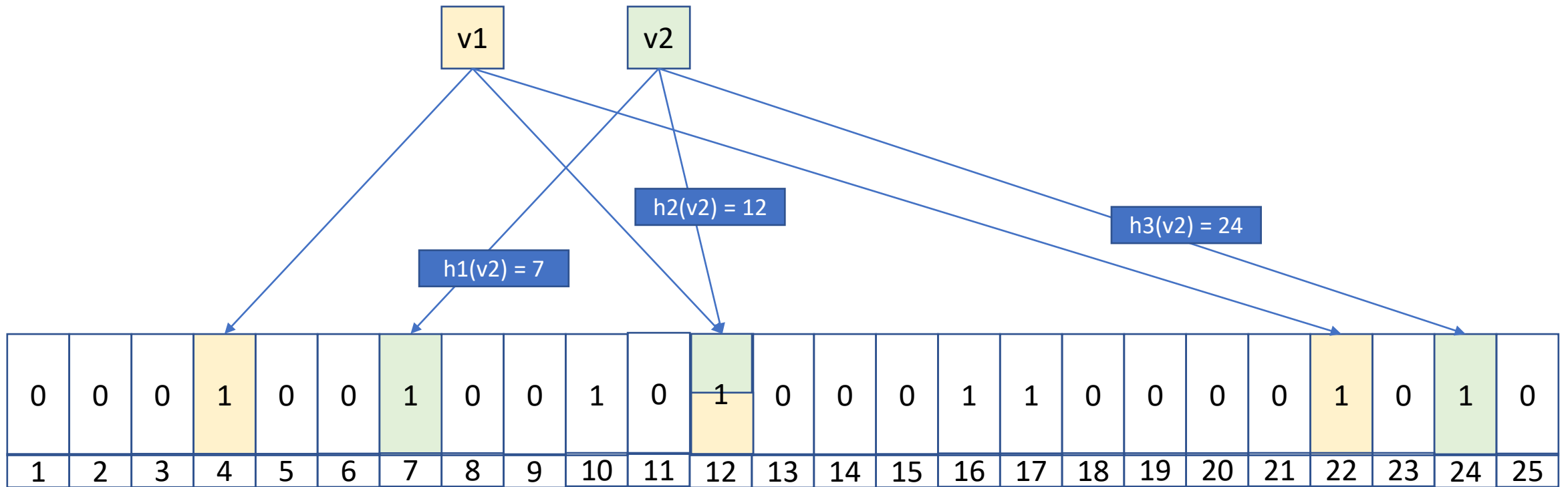
- To reduce the probability of false positives, use  $k > 1$  independent hash functions.
- How it works:  $S$ , set of  $n$  values  $\rightarrow \text{const} * n$  bits  
calculate  $h_1(v), h_2(v) \dots h_k(v) \in [1, \text{const} * n]$   
set bits  $h_1(v), h_2(v) \dots h_k(v)$  to 1

Test  $w \in S \rightarrow h_1(w) = 1$  and  $h_2(w) = 1 \dots$  and  $h_k(w) = 1$  ?



Small probability of **false positive**.

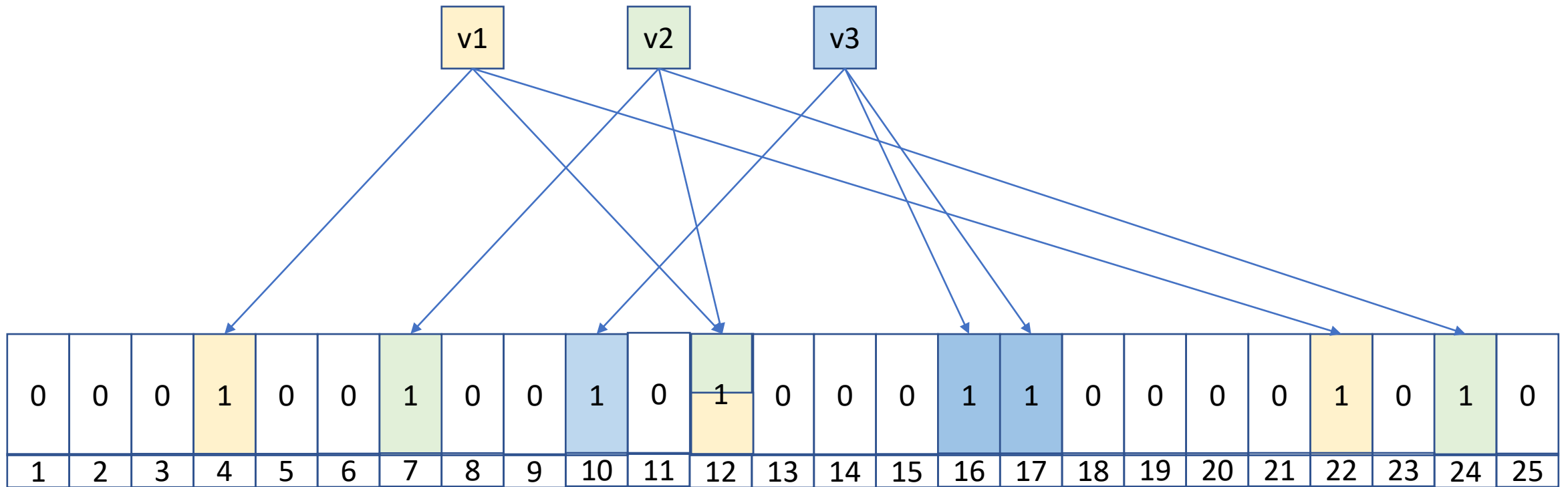
Probability of **false negative** = 0.



Small probability of **false positive**.

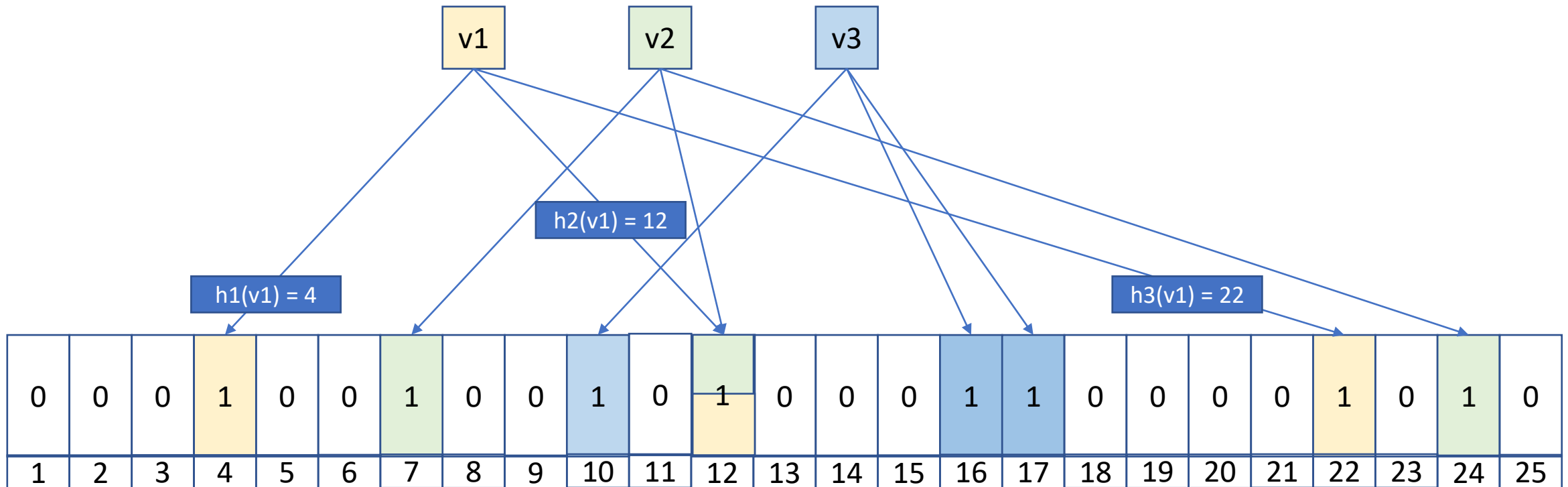
Probability of **false negative** = 0.





Small probability of **false positive**.

Probability of **false negative** = 0.

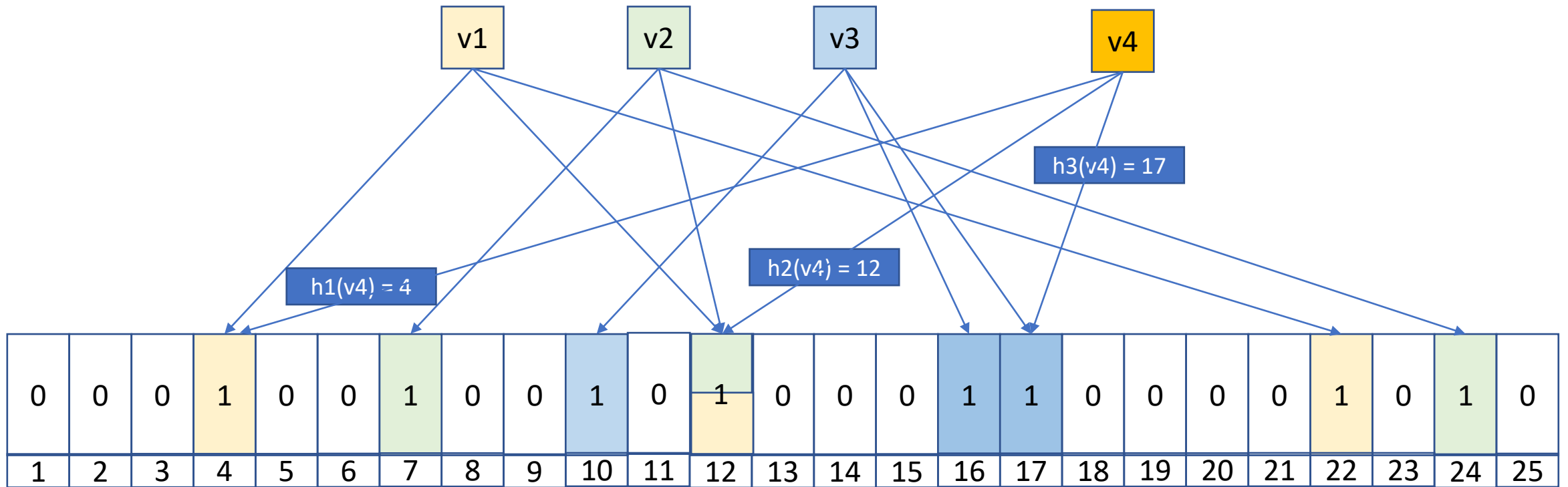


Small probability of **false positive**.

**false positive**. Value  $w$ :  $B[h1(w)] = 1 \ B[h2(w)] = 1 \ \dots \ B[hk(w)] = 1$

Probability of **false negative** = 0.

*Each hash of  $w$  equals a hash of an element in the set*



Small probability of **false positive**.

**false positive**. Value  $w$ :  $B[h_1(w)] = 1 \ B[h_2(w)] = 1 \ \dots \ B[h_k(w)] = 1$

Probability of **false negative** = 0.

*Each hash of  $w$  equals a hash of an element in the set*

# Bloom filters

- Used only to add elements or the test membership.
- Once an element is added to the filter it cannot be removed. Why?
- If all bits are set to 1, the probability of false positives increases.  
More space → more accuracy.
- More hash functions  
Latency → more accuracy.

# Bloom filters – independent hashing

- A family of hash functions  $H = \{h: U \rightarrow [1..m]\}$  is k-independent if  $\forall (x_1, x_2 \dots x_k) \in U^k$  and  $\forall (y_1, y_2 \dots y_k) \in [1..m]^k$  :
  - $Pr_{h \in H} [h(x_1) = y_1 \wedge h(x_2) = y_2 \dots \wedge h(x_k) = y_k] = \frac{1}{m^k}$
- $h(x_1)$  uniformly distributed.
- $h(x_1), h(x_2), \dots, h(x_k)$  independent random variables.

# Bloom filters – accuracy

- m size of array, n number of elements in S, k number of hash functions.
- Probability of false positive:

$$P = \left(1 - \left(1 - \frac{1}{m}\right)^{kn}\right)^k \text{ or}$$

$$P = \left(1 - e^{-\frac{kn}{m}}\right)^k$$

- $m = 10 * n$  and  $k = 7 \simeq 0,01$

# Bloom filters – accuracy

- m size of array, n number of elements in S, k number of hash functions.

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$h1(w) \neq h1(v1)$




- $m = 10 * n$  and  $k = 7 \simeq 0,01$

# Bloom filters – accuracy

- m size of array, n number of elements in S, k number of hash functions.

- Probability of false positive:

$$P = \left( 1 - \left( 1 - \frac{1}{m} \right)^{kn} \right)^k \quad \text{or}$$



$h_1(w) \neq h_1(v_1)$   
 $h_1(w) \neq h_2(v_1)$   
.....  
 $h_1(w) \neq h_k(v_1)$   
 $h_1(w) \neq h_1(v_2)$   
 $h_1(w) \neq h_2(v_2)$   
...  
 $h_1(w) \neq h_k(v_2)$   
...  
 $h_1(w) \neq h_k(v_n)$

- $m = 10 * n$  and  $k = 7 \simeq 0,01$



# Bloom filters – accuracy

- m size of array, n number of elements in S, k number of hash functions.

- Probability of false positive:

$$P = \left( 1 - \left( 1 - \frac{1}{m} \right)^{kn} \right)^k \quad \text{or}$$

$$h_1(w) = h_1(v1)$$

or

$$h_1(w) = h_2(v1)$$

or

.....

$$h_1(w) = h_k(v1)$$

or

$$h_1(w) = h_1(v2)$$

...

or

$$h_1(w) = h_k(vn)$$

- $m = 10 * n$  and  $k = 7 \simeq 0,01$

# General optimization rules

# General optimization rules

- Execute selections first
  - Reduce relation size (number of rows)
- Avoid cross-joins, use joins
- First join to be executed is the one obtaining the smaller relation
- Execute projections first

# Estimating Query Cost

rule-based execution  
plans/optimization

obsolete

cost-based execution  
plans

IO-cost

CPU-cost

IO cost

CPU time

number of blocks  
transferred from  
storage - b

number of random  
I/O accesses - s

$$b * t_T + s * t_S$$

cost for processing  
a tuple

cost for processing  
an index entry

cost for processing  
comparison  
operators

cost for processing  
a function .....

# Estimating cost

- Linear search

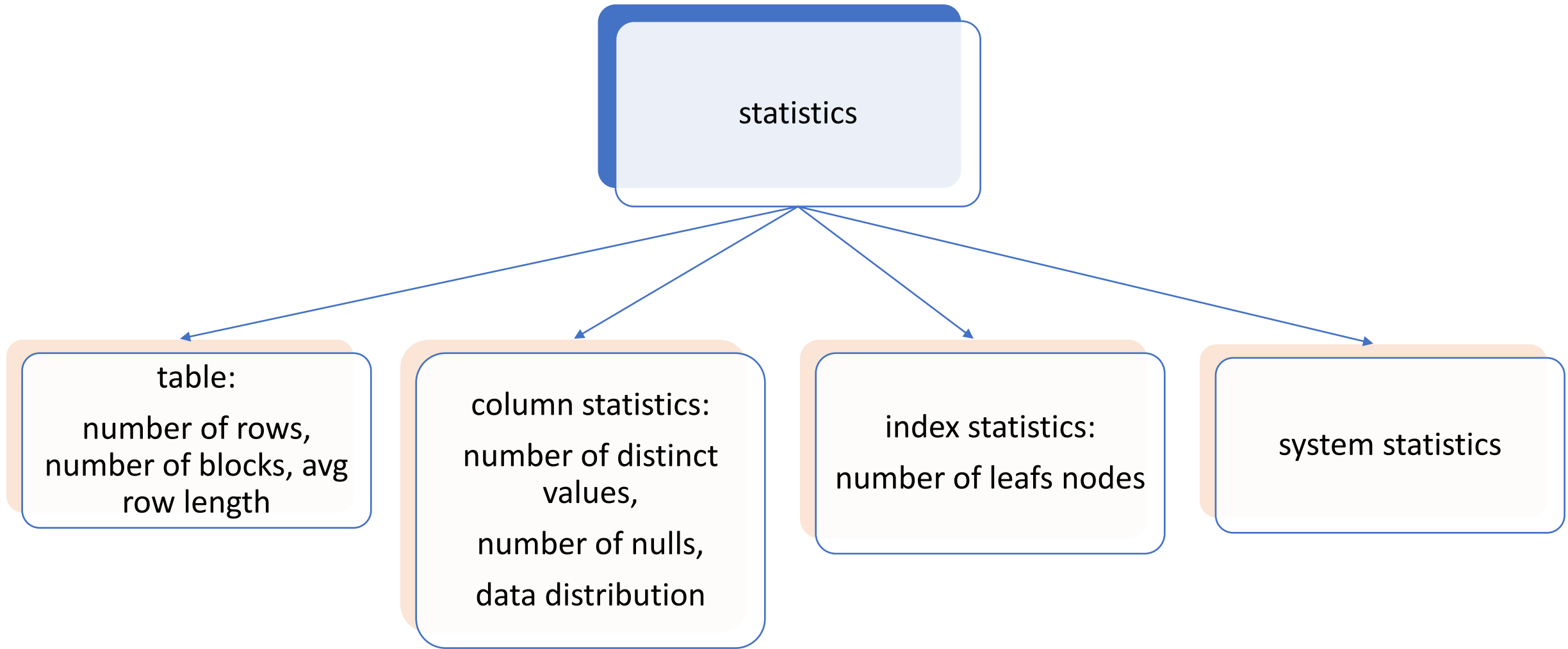
	COST	OBSERVATIONS
Linear search	$b * t_T + t_S$	Search for initial block Transfer r blocks
Linear search B-tree, Equality on key	$(h_i + 1) * (t_T + t_S)$	$h_i$ height of the index, each operation requires a seek and a block transfer.
Linear search clustering B-tree, Equality on non-key	$h_i * (t_T + t_S) + b * t_T + t_S$	b blocks storing the specified search key are stored sequentially
Linear search secondary B-tree, Equality on non-key	$(h_i + n) * (t_T + t_S)$	n number of records fetched; each record may be on a different block.

# Estimating cost - example

- Linear search

	COST	OBSERVATIONS
Linear search	$b * t_T + t_S$	Search for initial block Transfer $r$ blocks
Linear search B-tree, Equality on key	$(h_i + 1) * (t_T + t_S)$	$h_i$ height of the index, each operation requires a seek and a block transfer.
Linear search clustering B-tree, Equality on non-key	$h_i * (t_T + t_S) + b * t_T + t_S$	$b$ blocks storing the specified search key are stored sequentially
Linear search secondary B-tree, Equality on non-key	$(h_i + n) * (t_T + t_S)$	$n$ number of records fetched; each record may be on a different block.





- [1] <https://dev.mysql.com/doc/refman/8.0/en/group-by-optimization.html>
- [2] <https://computing.derby.ac.uk/c/codds-twelve-rules/>
- [3] <https://antognini.ch/papers/BloomFilters20080620.pdf>
- [4] <https://docs.oracle.com/en/database/oracle/oracle-database/19/tgsql/joins.html#GUID-5FCD34FE-ED04-4AB2-BC90-9752FED94F4F>