

Physics 3610H: Assignment VII

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Problem 1. In class we showed that the eigenfunctions of the harmonic oscillator Hamiltonian have the form $\psi(\xi) = H(\xi)e^{i\xi^2/2}$ where $\xi = \alpha x$. Considering just the even solutions, we expanded $H(\xi)$ in a power series

$$H(\xi) = \sum_{k=0}^{\infty} c_k \xi^{2k},$$

and we derived a recursion relation for the coefficients of this series

$$c_{k+1} = \frac{4k + 1 - \lambda}{2(k+1)(2k+1)} c_k.$$

Use this recursion relation to calculate $H_6(\xi)$. Be sure to scale your result, following convention, such that the coefficient of ξ^6 is 2^6 .

Solution 1.

Problem 2. Now consider the odd series

$$H(\xi) = \sum_{k=0}^{\infty} d_k \xi^{2k+1}.$$

Using the equation for $H(\xi)$ which we derived in class, derive the recursion relation for the coefficients in this series. Explain your steps. You should obtain

$$d_{k+1} = \frac{4k + 3 - \lambda}{2(k+1)(2k+3)} d_k.$$

Solution 2.

Problem 3. Using Dirac notation:

- (a) If \hat{A} and \hat{B} are two linear operators show that $(\hat{A}\hat{B})^\dagger = \hat{B}^\dagger \hat{A}^\dagger$.
- (b) If \hat{C} and \hat{D} are two Hermitian operators, is the product $\hat{C}\hat{D}$ always Hermitian? (You may use your result from (a)).

Solution 3.

- (a) We know that for states ψ and χ $\langle \psi | \hat{A} | \chi \rangle = \langle \hat{A}^\dagger \psi | \chi \rangle$. Applying this then to the product we obtain

$$\langle \psi | \hat{A}\hat{B} | \chi \rangle = \langle \psi | \hat{A} | \chi' \rangle$$

where $|\chi'\rangle = \hat{B} |\chi\rangle$. Then,

$$\langle \psi | \hat{A} | \chi' \rangle = \langle \hat{A}^\dagger \psi | \chi' \rangle = \langle \psi' | \hat{B} | \chi \rangle = \hat{B}^\dagger \langle \psi' | \chi \rangle.$$

Here we've written $\langle \hat{A}^\dagger \psi | = \langle \psi' |$ undoing this we obtain

$$\langle \psi | \hat{A}\hat{B} | \chi \rangle = \hat{B}^\dagger \langle \psi' | \chi \rangle = \hat{B}^\dagger \hat{A}^\dagger \langle \psi | \chi \rangle.$$

Because $|\psi\rangle$ and $|\chi\rangle$ are generic we can say then that $(\hat{A}\hat{B})^\dagger = \hat{B}^\dagger \hat{A}^\dagger$.

- (b) As we know from part (a) linear operators in general satisfy $(\hat{C}\hat{D})^\dagger = \hat{D}^\dagger \hat{C}^\dagger$. If this product were Hermitian we could say that $(\hat{C}\hat{D})^\dagger = \hat{C}\hat{D} = \hat{C}^\dagger \hat{D}^\dagger$ which we know is not the case again from part (a). Therefore the product of two Hermitian operators is not necessarily Hermitian unless the operators commute.