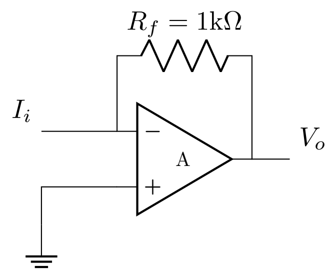
Physics 2250: Problem Set IV

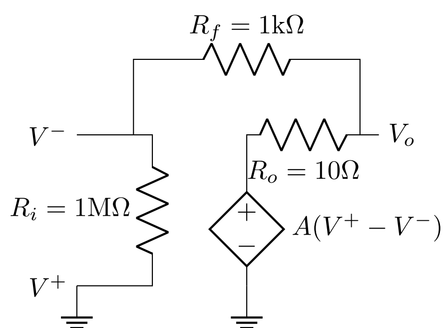
Jeremy Favro

October 7, 2024 - thank you for losing the key.

Problem 1. Consider a current-to-voltage converter that uses one op-amp and one resistor.



Solution 1.
a) Using simplified op-amp approximations here means that no current will flow inside the op amp between the terminals. Therefore all of I_i flows through R_f. Then, by the virtual ground approximation, $V^- \approxeq V^+ = 0 \implies I_i = \frac{V^- - V_o}{R_f} = -\frac{V_o}{R_f}$ which can be rearranged to obtain $V_o = -I_iR_f$.
b)

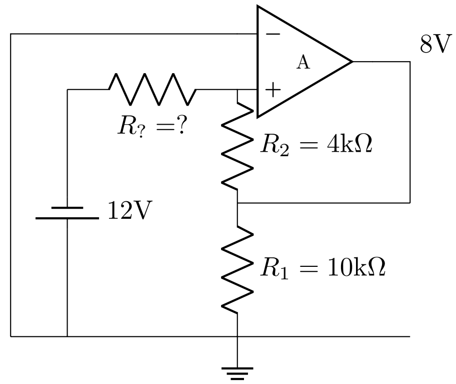


For some reason this question really messed with me, likely because I initially put it off until later and then got sick and am now attempting it while sick. Anyway:
The current coming in at the V^- node, I_i (yarrr), splits between the R_f and R_i branches. Knowing that V^+ will be zero because of the ground point V_o becomes $-V^-$ and $V^- = I_iR_i\frac{R_f + R_o}{R_f + R_o + R_i}$ which is the voltage drop across R_i due to the current through that branch. This means that $V_o = -I_iR_i\frac{R_f + R_o}{R_f + R_o + R_i}$. To compare this to part a) it's easiest to sub in the given numbers and partially evaluate

$$V_o = -I_iR_i\frac{R_f + R_o}{R_f + R_o + R_i}$$
$$V_o = -I_i\frac{10^6\left(10^3 + 10\right)}{10^3 + 10 + 10^6}$$
$$V_o \approxeq -1009 \cdot I_i$$

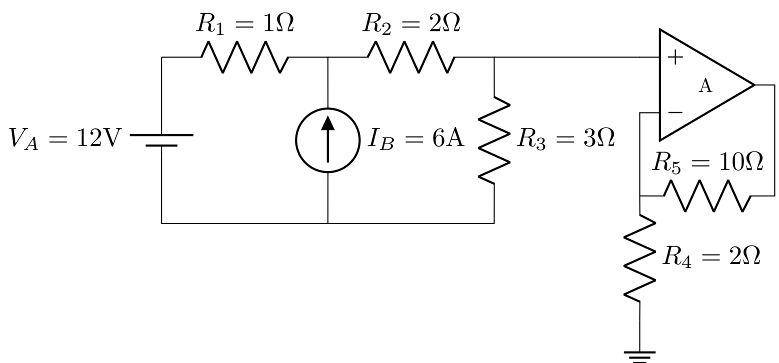
which is only slightly greater than the answer to part a) ($V_o = 1000 \cdot I_i$).

Problem 2. Determine the value of the leftmost resistor in the circuit below.



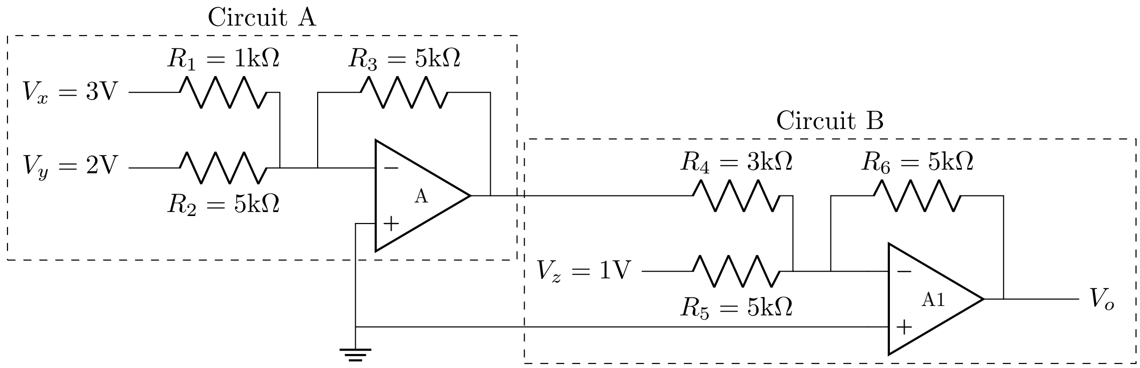
Solution 2. Defining the potential at the ground junction as 0V we get that the potential across R_1 is $8V$ and by virtual ground the potential at the positive terminal is 0V which means that the voltage drop across R_2 must be 8V which implies that $I_{R2} = \dfrac{-8\text{V}}{4\text{k}\Omega} = -2\text{mA}$ with the negative sign indicating that the current is flowing upwards because the potential drop across R_2 goes from the bottom up. In order to satisfy the loop law the potential after $R_?$ must be -12V so $V_? = -12\text{V} = I_{R2}R_? \implies R_? = \frac{-12\text{V}}{-12\text{mA}} = 6\text{k}\Omega$

Problem 3 (BONUS). Determine the output voltage, V_o in the circuit below.



Solution 3. The right side of the circuit can be solved a number of ways to determine that the voltage where it connects to the positive terminal of the op-amp is $V^+ = 3\text{V}$. Assuming the amplifier is standard (high gain), it will output the maximum voltage that it can as $V_o = A(V^+ - V^-)$ and though V^- will only be slightly less than V^+ (whatever the voltage drop across R_5 is), the gain factor will increase that slight difference significantly. I believe the standard maximum output for an op-amp is 15V, so $V_o = 15\text{V}$ provided this is a standard op-amp.

Problem 4. What is the output V_o of the following circuit

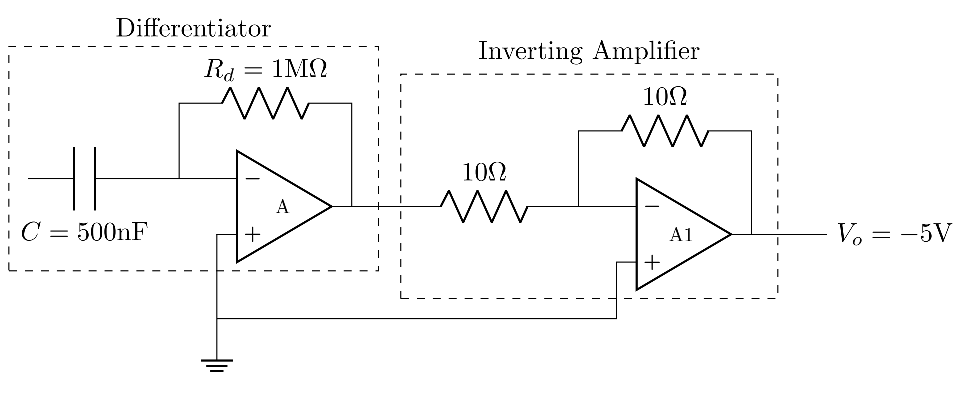


Solution 4. We can break this circuit down into two subcircuits (illustrated in the problem statement to avoid repetition here), circuit A and circuit B. To determine the output voltage of circuit A, V_{oA}, we start by analyzing current flow. The currents through R_1 and R_2 created by their respective voltages combine at the negative terminal junction into a single current, all of which travels through R_3 by the ideal amplifier approximations ($R_i \to \infty$). Mathematically this means that $I = I_1 + I_2$ and that $I_1 = \frac{V_x}{R_1}$, $I_2 = \frac{V_y}{R_2}$, and $I = -\frac{V_{oA}}{R_3}$ which means that $\frac{V_x}{R_1} + \frac{V_y}{R_2} = -\frac{V_{oA}}{R_3} \implies V_{oA} = -R_3\left(\frac{V_x}{R_1} + \frac{V_y}{R_2}\right)$.

Next for circuit B we basically perform the same steps except $I_4 = -\frac{R_3}{R_4}\left(\frac{V_x}{R_1} + \frac{V_y}{R_2}\right)$, $I_5 = \frac{V_z}{R_5}$, and $I = -\frac{V_{oB}}{R_6}$. Again we also have that $I = I_4 + I_5$ which means that $-\frac{R_3}{R_4}\left(\frac{V_x}{R_1} + \frac{V_y}{R_2}\right) + \frac{V_z}{R_5} = -\frac{V_{oB}}{R_6} \implies V_{oB} = -R_6\left[-\frac{R_3}{R_4}\left(\frac{V_x}{R_1} + \frac{V_y}{R_2}\right) + \frac{V_z}{R_5}\right]$. Finally, $V_{oB} = V_o = -5\text{k}\Omega\left[-\frac{5\text{k}\Omega}{3\text{k}\Omega}\left(\frac{3\text{V}}{1\text{k}\Omega} + \frac{2\text{V}}{5\text{k}\Omega}\right) + \frac{1\text{V}}{5\text{k}\Omega}\right] = \frac{82}{3}\text{V}$

Problem 5 (BONUS). You are to be a writer for the most boring episode of MacGyver. Our hero needs to briefly power a device with -5V, but only has a defective battery. In testing this battery, MacGyver finds that the voltage steadily (linearly) ramps down from 10V to 0V over precisely one minute. Conveniently, there is a box of op-amps, wires, resistors, and capacitors. Devise a circuit that accomplishes the desired task. BRIEFLY explain in words the basic idea behind your circuit design.

Solution 5.



The idea here is that the differentiator converts the linear ramp-down voltage of the battery to a constant signal with voltage given by $10R_dC$. The goal for this part of the circuit was to have an output of 5V that I could then invert with an inverting amplifier with a gain of 1. I fixed $C = 500\text{nF}$ and then solved for R_d after trying a few other capacitors which didn't have a nice equivalent resistor in the given set. The second part of the circuit was just pulled from my notes and the resistors chosen so that the amplifier had an effective gain of 1 meaning that $V_o = -V_i = -5\text{V}$.