Physics 2610H: Assignment I

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Problem 1. At what wavelength does the human body emit the maximum electromagnetic radiation? Use Wien's law from Exercise 14 and assume a skin temperature of 70°F.

Solution 1. By Wien's law, $\lambda_{max}T = \alpha$ where $\alpha = 2.898 \times 10^{-3}\,\mathrm{m\,K}$. Here $T = 294.261\,\mathrm{K}$ so $\lambda_{max} = \frac{\alpha}{T} \approx 10\mu\mathrm{m}$

Problem 2. With light of wavelength 520nm, photoelectrons are ejected from a metal surface with a maximum speed of $1.78 \times 10^5\,\mathrm{m\,s^{-1}}$.

(a) What wavelength would be needed to give a maximum speed of $4.81 \times 10^5\,\mathrm{m\,s^{-1}}$?

(b) Can you guess what metal it is?

Solution 2.

(a) Here we first need to determine the work function, ϕ, of the surface. Using equation 1.2 from the formula sheet,

$$E_{kmax} = \frac{1}{2}m_e v^2 = hf - \phi$$
$$E_{kmax} = \frac{hc}{\lambda} - \frac{1}{2}m_e v^2 = \phi$$

Then the required wavelength for $v' = 4.81 \times 10^5\,\mathrm{m\,s^{-1}}$ will be proportional to the base kinetic energy of those electrons and the work function energy

$$\frac{1}{2}m_e\left(v'\right)^2 + \phi = \frac{hc}{\lambda'}$$
$$\frac{hc}{\frac{1}{2}m_e\left(v'\right)^2 + \phi} = \lambda'$$
$$\frac{2hc}{m_e\left[\left(v'\right)^2 - v^2\right] + \frac{2hc}{\lambda}} = \lambda' = 420\mathrm{nm}$$

(b) Going by table 1 from the textbook with $\phi = \frac{hc}{\lambda} - \frac{1}{2}m_e v^2 \approx 2.3\mathrm{eV}$ the metal is probably sodium.

Problem 3. When a beam of monoenergetic electrons is directed at a tungsten target, X-rays are produced with wavelengths no shorter than 0.062nm. How fast are the electrons in the beam moving?

Solution 3. This is probably wrong. If it's moving that fast idk if $1/2mv^2$ holds. $\lambda_{min} = 0.062\mathrm{nm} \implies E = \frac{hc}{\lambda} = \frac{1}{2}m_e v^2 \implies \sqrt{\frac{2hc}{m_e\lambda}} = v = 84\,000\,000\,\mathrm{m\,s^{-1}}$

Problem 4. A 0.057nm X-ray photon "bounces off" an initially stationary electron and scatters with a wavelength of 0.061nm. Find the directions of scatter of

(a) The photon.

(b) The electron.

Solution 4.

(a) Using equation 1.3 from the formula sheet,

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos(\theta))$$
$$\theta = \arccos\left(1 - \frac{\Delta\lambda m_e c}{h}\right) \approx 2.3\text{rad}$$

(b) Using equations 4 and 5 from the textbook,

$$\frac{h(\lambda' - \lambda\cos\theta)}{\lambda\lambda'} = \gamma_u m_e u \cos\phi$$

and

$$\frac{h\sin\theta}{\lambda'} = \gamma_u m_e u \sin\phi$$

then,

$$\frac{\frac{h\sin\theta}{\lambda'}}{\frac{h(\lambda'-\lambda\cos\theta)}{\lambda\lambda'}} = \frac{\gamma_u m_e u \sin\phi}{\gamma_u m_e u \cos\phi}$$
$$\phi = \arctan\left(\frac{\frac{h\sin\theta}{\lambda'}}{\frac{h(\lambda'-\lambda\cos\theta)}{\lambda\lambda'}}\right) = \arctan\left(\frac{\lambda\sin\theta}{\lambda' - \lambda\cos\theta}\right) = 0.4\text{rad}$$

Problem 5. The setup depicted in Figure 6 is used in a diffraction experiment using X-rays of 0.26nm wavelength. Constructive interference is noticed at angles of 23° and 51.4°, but none between. What is the spacing d of atomic planes?

Solution 5. Using equation 1.4 from the formula sheet, $\frac{m\lambda}{2\sin\theta} = d$. Here, $m = 1$, $\lambda = 0.26$nm and $\theta = \frac{23\pi}{180}$rad so $d \approx 330$pm

Problem 6. The average kinetic energy of a particle at temperature T is $\frac{3}{2}k_B T$.

(a) What is the wavelength of a room-temperature (22°C) electron?

(b) Of a room-temperature proton?

(c) In what circumstances should each behave as a wave?

Solution 6. $E = \frac{3}{2}k_B T \approx 6 \times 10^{-4}$ J

(a) Knowing that $\lambda = \frac{h}{p}$,

$$E = \frac{1}{2}m_e v^2$$
$$\implies \sqrt{\frac{2E}{m_e}} = v$$
$$\implies \frac{h}{m_e\sqrt{\frac{2E}{m_e}}} = \frac{h}{\sqrt{2m_e E}} = \lambda \approx 6.3\text{nm}$$

(b) We can use the same derived equation as in part (a), $\dfrac{h}{\sqrt{2m_e E}} = \lambda \approx 15$nm

(c) In what circumstances should each behave as a wave?