Physics 2610H: Assignment I

Jeremy Favro

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Problem 1. At what wavelength does the human body emit the maximum electromagnetic radiation? Use Wien's law from Exercise 14 and assume a skin temperature of 70°F.

Solution 1. By Wien's law, $\lambda_{max}T = \alpha$ where $\alpha = 2.898 \times 10^{-3}\,\mathrm{m\,K}$. Here $T = 294.261\,\mathrm{K}$ so $\lambda_{max} = \frac{\alpha}{T} \approx 10\mathrm{\mu m}$

Problem 2. With light of wavelength 520nm, photoelectrons are ejected from a metal surface with a maximum speed of $1.78 \times 10^5\,\mathrm{m\,s^{-1}}$.

(a) What wavelength would be needed to give a maximum speed of $4.81 \times 10^5\,\mathrm{m\,s^{-1}}$?

(b) Can you guess what metal it is?

Solution 2.

(a) Here we first need to determine the work function, ϕ, of the surface. Using equation 1.2 from the formula sheet,

$$\begin{aligned} E_{kmax} &= \frac{1}{2}m_e v^2 = hf - \phi \\ E_{kmax} &= \frac{hc}{\lambda} - \frac{1}{2}m_e v^2 = \phi \end{aligned}$$

Then the required wavelength for $v' = 4.81 \times 10^5\,\mathrm{m\,s^{-1}}$ will be proportional to the base kinetic energy of those electrons and the work function energy

$$\begin{aligned} \frac{1}{2}m_e\left(v'\right)^2 + \phi &= \frac{hc}{\lambda'} \\ \frac{hc}{\frac{1}{2}m_e\left(v'\right)^2 + \phi} &= \lambda' \\ \frac{2hc}{m_e\left[\left(v'\right)^2 - v^2\right] + \frac{2hc}{\lambda}} &= \lambda' = 420\mathrm{nm} \end{aligned}$$

(b) Going by table 1 from the textbook with $\phi = \frac{hc}{\lambda} - \frac{1}{2}m_e v^2 \approx 2.3\mathrm{eV}$ the metal is probably sodium.

Problem 3. When a beam of monoenergetic electrons is directed at a tungsten target, X-rays are produced with wavelengths no shorter than 0.062nm. How fast are the electrons in the beam moving?

Solution 3. This is probably off by a bit because it doesn't account for relativistic effects. $\lambda_{min} = 0.062\mathrm{nm} \implies E = \frac{hc}{\lambda} = \frac{1}{2}m_e v^2 \implies \sqrt{\frac{2hc}{m_e\lambda}} = v \approx 8.4 \times 10^7\,\mathrm{m\,s^{-1}}$

Problem 4. A 0.057nm X-ray photon "bounces off" an initially stationary electron and scatters with a wavelength of 0.061nm. Find the directions of scatter of

(a) The photon.

(b) The electron.

Solution 4.

(a) Using equation 1.3 from the formula sheet,

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos(\theta))$$
$$\theta = \arccos\left(1 - \frac{\Delta\lambda m_e c}{h}\right) \approx 2.3\text{rad}$$

(b) Using equations 4 and 5 from the textbook,

$$\frac{h(\lambda' - \lambda\cos\theta)}{\lambda\lambda'} = \gamma_u m_e u \cos\phi$$

and

$$\frac{h\sin\theta}{\lambda'} = \gamma_u m_e u \sin\phi$$

then,

$$\frac{\frac{h\sin\theta}{\lambda'}}{\frac{h(\lambda'-\lambda\cos\theta)}{\lambda\lambda'}} = \frac{\gamma_u m_e u \sin\phi}{\gamma_u m_e u \cos\phi}$$
$$\phi = \arctan\left(\frac{\frac{h\sin\theta}{\lambda'}}{\frac{h(\lambda'-\lambda\cos\theta)}{\lambda\lambda'}}\right) = \arctan\left(\frac{\lambda\sin\theta}{\lambda' - \lambda\cos\theta}\right) = 0.4\text{rad}$$

Problem 5. The setup depicted in Figure 6 is used in a diffraction experiment using X-rays of 0.26nm wavelength. Constructive interference is noticed at angles of 23° and 51.4°, but none between. What is the spacing d of atomic planes?

Solution 5. Using equation 1.4 from the formula sheet, $\frac{m\lambda}{2\sin\theta} = d$. Here, $m = 1$, $\lambda = 0.26\text{nm}$ and $\theta = \frac{23\pi}{180}\text{rad}$ so $d \approx 330\text{pm}$

Problem 6. The average kinetic energy of a particle at temperature T is $\frac{3}{2}k_B T$.

(a) What is the wavelength of a room-temperature (22°C) electron?

(b) Of a room-temperature proton?

(c) In what circumstances should each behave as a wave?

Solution 6. $E = \frac{3}{2}k_B T \approx 6 \times 10^{-4}$ J

(a) Knowing that $\lambda = \frac{h}{p}$,

$$E = \frac{1}{2}m_e v^2$$
$$\implies \sqrt{\frac{2E}{m_e}} = v$$
$$\implies \frac{h}{m_e\sqrt{\frac{2E}{m_e}}} = \frac{h}{\sqrt{2m_e E}} = \lambda \approx 6.3\text{nm}$$

(b) We can use the same derived equation as in part (a), $\dfrac{h}{\sqrt{2m_e E}} = \lambda \approx 0.15\text{nm}$

(c) When interacting with surroundings of similar scale. E.g. the electron will behave like a wave when interacting with a slit of width on the order of 10nm whereas the proton would probably only act as a wave when diffracted with something on the order of an Angstrom.