q4

December 2, 2025

```
[23]: # a
clear_vars()
p = 9
q = 5
r = 22

x, n, t = var("x n t")
h = q
kappa = p
L = q*p
k = p/(q+p)
T1 = 0 # Assumed because this is in the sample code
T0 = 0
f = 10*x*(L-x)*sin(pi*x/L)
bn(s) = integral(f*sin(s*x), x, 0, L)/integral(sin(s*x)^2, x, 0, L)

w = 0
for n in [1..20]:
    wn=w
    EQ=tan(x*L)+kappa*x/h
    lambn=EQ.find_root((2*n-1)*pi/(2*L),n*pi/L)
    bnn=N(bn(lambn))
    w=wn+bnn*sin(lambn*x)*e^(-lambn^2*k*t)
    show(rf"lambda_{n}: {lambn}")
```

lambda_1: 0.06714051111739618

lambda_2: 0.13435344136504954

lambda_3: 0.2017003456182572

lambda_4: 0.26922477792503907

lambda_5: 0.33695044796079876

lambda_6: 0.4048837225601791

lambda_7: 0.473018293558356

lambda_8: 0.5413400467651973

lambda_9: 0.6098310313607336

lambda_10: 0.6784722169145844

lambda_11: 0.7472451706647032

lambda_12: 0.8161329398175129

lambda_13: 0.8851204159390972

lambda_14: 0.9541943936557794

lambda_15: 1.0233434667116108

lambda_16: 1.092557849800966

lambda_17: 1.161829177050288

lambda_18: 1.2311503042395802

lambda_19: 1.3005151276895273

lambda_20: 1.3699184247385792

```
[24]:  # b
       v=T0+x*h*(T1-T0)/(kappa+h*L)
       u(x,t)=w+v
```
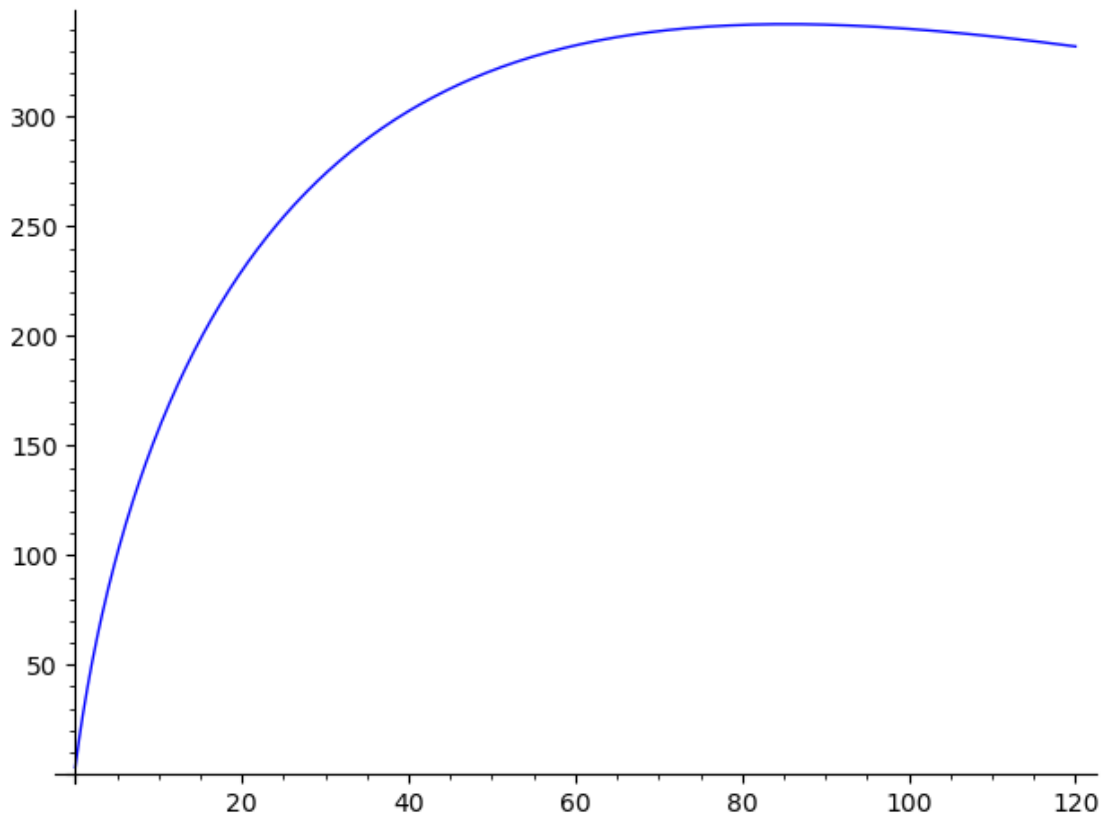
```
[25]:  # c
       EQ1 = u(L,t)
       plot(EQ1,t,0,120)
```

[25]:

```
[26]:  # d
       maxtime, maxtemp  = find_local_maximum(EQ1, 60, 120) # Guessed range based on␣
         ↪graph

       show(rf"max temp occurs at {maxtime}s and is {maxtemp} units")
```

max temp occurs at 342.2351971425617s and is 85.36838592140077 units

```
[27]:  # e
       fun = EQ1 - 25 # will cross at T=25 units
       plot(fun, t, 0, 120)
       tft = fun.find_root(0,10)
       show(rf"25 deg occurs at {tft}s")
```

25 deg occurs at 0.8255692025802429s

```
[28]:  # f
       plot(u(x, tft), x, 0, L)
```

[28]: