Physics 2250: Problem Set II

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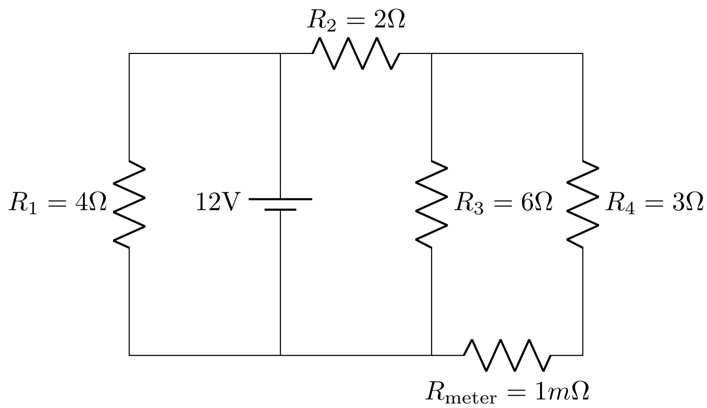
Problem 1. ...

Solution 1.

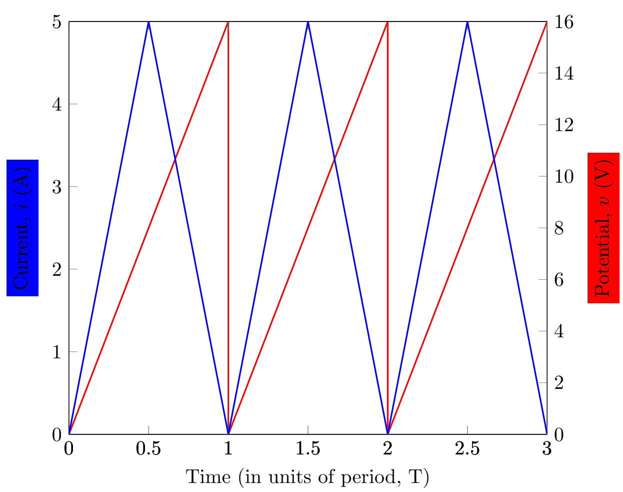


a) By simplifying to equivalent resistance and using Ohm's law in the loop containing ϵ_{a} and ϵ_{b} (without R_{meter}) $I_{\text{tot}} = \dfrac{V}{R} = \dfrac{12\text{V}}{4\Omega} = 3\text{A}$ so by the current divider rule and KCL the current through R_4 is the same as the current through points ϵ_{a} and ϵ_{b}, $I_{\text{ab}} = 3\text{A}\dfrac{6\Omega}{6\Omega + 3\Omega} = \boxed{2\text{A}}$

b) This is effectively the same until the "second" (really the first one calculated) current divider $\dfrac{6\Omega}{6\Omega + 3\Omega}$ because the branch we are calculating current for now contains the extra resistance of the meter and thus draws *very* slightly less current. The new current through this branch therefore becomes $3\text{A}\dfrac{6\Omega}{6\Omega + 3.001\Omega} = \boxed{1.99978\text{A}}$

Problem 2. The figure below displays a periodic current and potential delivered to some load (resistor). Note, the maximum values for v and i are 16V and 5A, respectively.

Solution 2.



a) $\bar{v}(t) = \frac{1}{T}\int_0^T v(t)\,dt = \frac{1}{\cancel{T}}\frac{1}{2}\cancel{T}\cdot 16\text{V} = 8\text{V}$

$\bar{i}(t) = \frac{1}{T}\int_0^T i(t)\,dt = \frac{1}{\cancel{T}}\frac{1}{2}\cancel{T}\cdot 5\text{A} = 2.5\text{A}$

b) Voltage here can be expressed as a linear function $v(t) = \frac{16}{T}t$ for one period so

$v_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = \sqrt{\frac{1}{T}\frac{16^2}{T^2}\cdot\frac{T^3}{3}} = \frac{16\sqrt{3}}{3} \approxeq 9.238\text{V}$ The same almost works for current $i(t)$ but it has to be made piecewise to account for the downward slope from $\frac{T}{2}$ to T, not that the sign really matters when doing the integration

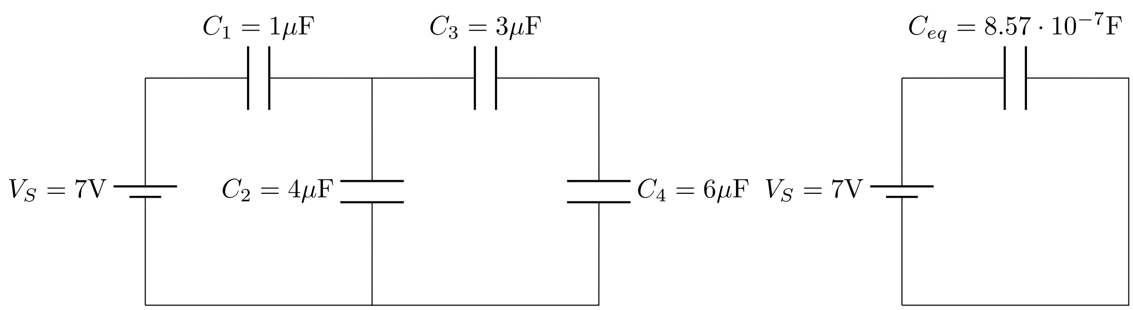
$$i(t) = \begin{cases} \frac{5}{T}t & 0 \le t \le \frac{T}{2} \\ -\frac{5}{T}t & \frac{T}{2} \le t \le T \end{cases}$$

which integrates as $i_{rms} = \sqrt{\frac{1}{T}\left[\int_0^{\frac{T}{2}} i^2(t)\,dt + \int_{\frac{T}{2}}^{T} i^2(t)\,dt\right]} = \sqrt{\frac{1}{T}\frac{25}{T^2}\left[\frac{T^3}{6} + \frac{T^3}{3} - \frac{T^3}{24}\right]} = \sqrt{\frac{25}{\cancel{T^3}}\frac{8\cancel{T^3}}{24}} = \frac{5\sqrt{3}}{3} \approxeq$ 2.887A

c) $P_{avg} = v_{rms}i_{rms} = 26.67\text{W}$

Problem 3. Consider the following network of capacitors

Solution 3.



a) Capacitor C_{eq} must have a voltage drop of 7V to satisfy KVL therefore $Q_{tot} = C_{eq}V = 6\mu\text{C}$. Re-complicating the circuit and using Q_{tot} we can find that the voltage drop across C_1 is $V_{C1} = \frac{Q_{tot}}{C_1} = 6\text{V}$. By KVL this means that the rest of the circuit must contribute a potential drop of 1V which means that C_2 will experience a charge of $Q_{C2} = 1\text{V}4\mu\text{F} = 4\mu\text{C}$ therefore because charge is conserved the C_3-C_4 branch experiences the rest of Q_{tot} or $Q_{C3C4} = Q_{tot} - Q_{C2} = 2\mu\text{C}$. Therefore the voltages across C_3 & C_4 are $0.\bar{6}\text{V}$ & $0.\bar{3}\text{V}$ respectively.

Capacitor	Charge	Voltage
C_1	$6\mu\text{C}$	6V
C_2	$4\mu\text{C}$	1V
C_3	$2\mu\text{C}$	$0.\bar{6}\text{V}$
C_4	$2\mu\text{C}$	$0.\bar{3}\text{V}$

b) Once at equilibrium we would see a voltage drop across C_1 of 7V (so $Q = C_1V = 7\mu\text{C}$) and no drop or charge across any other component because C_1, when fully charged, will act as a break in the circuit and not permit current to flow.