Math 3310H: Assignment I

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Problem 1. Define a relation $\mathbb{R} \times \mathbb{R}$ by $(a, b)\ \ (c, d)$ if $2(a - c) - 3(b - d) = 0$

(a) Show that   is an equivalence relation on $\mathbb{R}$.

(b) Give an example of two pairs $(a, b), (c, d) \in \mathbb{R} \times \mathbb{R}$, which lie in the same equivalence class, and two pairs that don't.

(c) This equivalence relation partitions the 2D plane $\mathbb{R} \times \mathbb{R}$ into subregions. What does the equivalence class (a, b) look like as a region of the plane?

Solution 1.

Problem 2. For each of the following sets S, determine whether S is closed under addition modulo n, or multiplication modulo n, or both or neither. (Addition and multiplication modulo n are defined in Exercise Set 2).

(a) $S = \{0, 4, 8, 12\}, n = 16$.

(b) $S = \{0, 3, 6, 9, 12\}, n = 15$.

(c) $S = \{1, 2, 3, 4\}, n = 5$.

(d) $S = \{0, 2, 3, 4, 6, 8, 9, 10\}, n = 12$.

(e) $S = \{1, 5, 7, 11\}, n = 12$.

Solution 2.

Problem 3. Determine whether the given binary operation $*$ is commutative, associative, both or neither. Justify your answers with proof.

(a) The operation $*$ on $\mathbb{Z}$ given by $a * b = a + b + ab$

(b) The operation $*$ on $\mathbb{R}$ given by $a * b = a + b - ab$

(c) The operation $*$ on $\mathbb{R}$ given by $a * b = a + 2ab$

(d) The operation $*$ on $\mathbb{Z} \times \mathbb{Z}$ given by $(a, b) * (c, d) = (ad + bc, bd)$

(e) The operation $*$ on $\mathbb{Z} \times \mathbb{Z}$ given by $(a, b) * (c, d) = (ad, bc)$

Solution 3.

Problem 4. Let S be a nonempty set. A binary algebraic structure $(S, *)$ is called a semigroup if $*$ is associative.

(a) Let S be the set of positive rational numbers. Show that $(S, *)$ is a commutative semigroup if

$$a * b = \frac{ab}{a + b}$$

(the usual operations on the right) for all $a, b \in S$

(b) Let S be a set containing more than one element. Define

$$a * b = b$$

for all $a, b \in S$. Show that $(S, *)$ is a noncommutative semigroup with no identity element.

Solution 4.