Physics 2250: Problem Set VIII

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November 22, 2024

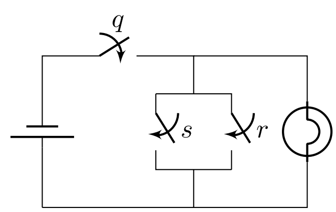
Problem 1.

(a) Design a simple circuit comprising only a battery, a bulb (load), and simple switches, where the bulb lights up only when $q \cdot \overline{(r+s)}$ is true

(b) Fill out a complete truth table for the above expression.

Solution 1.

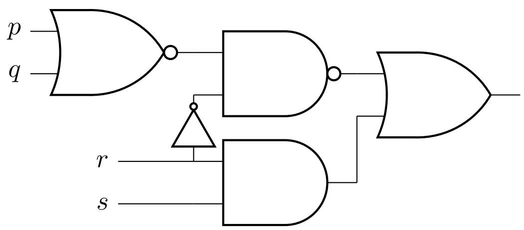
(a) Here if q is closed and either r or s is also closed, the bulb is bypassed. If q is open obviously no current reaches the bulb. If both r and s are open (ergo $\overline{(r+s)} = 1$ only true when $r, s = 0$) the lamp sees current and turns on.



(b)

q	r	s	$\overline{(r+s)}$	$q \cdot \overline{(r+s)}$
T	T	T	F	F
T	T	F	F	F
T	F	F	T	T
F	F	F	T	F
F	F	T	F	F
F	T	T	F	F
T	F	T	F	F
F	T	F	F	F

Problem 2. For the following logic circuit,

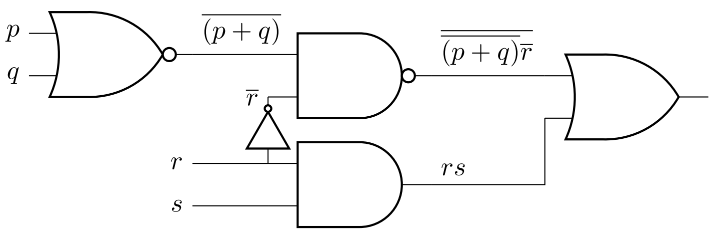


(a) Write a simplified logic expression for the output. (Top marks will be given for an expression that cannot be simplified further.)

(b) Write a complete truth table for this circuit.

Solution 2.

(a)

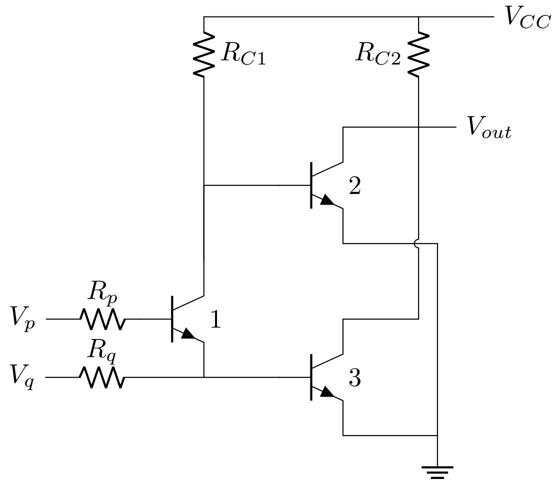


$rs + \overline{\overline{(p+q)}\,\overline{r}} = rs + p + q + r = p + q + r + rs$

(b)

p	q	r	s	rs	$p+q$	$(p+q)r$	$p+q+r+rs$
F	F	F	F	F	F	F	F
F	F	F	T	F	F	F	F
F	F	T	F	F	F	F	T
F	T	F	F	F	T	F	T
T	F	F	F	F	T	F	T
F	F	T	T	T	F	F	T
F	T	T	T	T	T	T	T
T	T	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	F	F	T	T	T
T	F	T	F	F	T	T	T
F	T	F	T	F	T	F	T
T	F	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	T	T	F	F	T	T	T
T	T	F	T	F	T	F	T

Problem 3 (**BONUS**)**.** Consider the circuit shown below with the two inputs p and q (of voltages V_p and V_q). Note that in this logic circuit we are only interested in knowing when the inputs and outputs voltages are high (HI) or low (LO), not theiractual value in volts. The convention is HI=1 and LO=0.



(a) Construct and complete a full truth table for q, p, and V_{out} (and any intermediate conditions you wish to include). Highlight the p, q conditions for which the output V_{out} =HI (or 1).

(b) Write a simplified logic expression in terms of p and q for the network.

(c) Reproduce an equivalent logic network using only NOR gates.

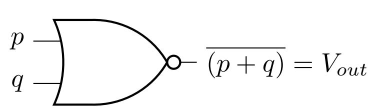
Solution 3.

(a)

p	q	1	2	3	V_{out}
1	1	1	1	1	0
1	0	1	1	1	0
0	1	0	1	1	0
0	0	0	1	0	1

(b) If I did the truth table correctly we're looking for a logic expression that is only true when both p and q are false, i.e., $\overline{p} \cdot \overline{q}$

(c) By DeMorgan's theorem, $\overline{p} \cdot \overline{q} = \overline{p + q}$ which is just a single NOR:



Problem 4. Consider the seven-segment (a-g) LED numeric display. To display a given "decimal" number (i.e. 0-9), various segments need to be powered. A 4-bit input (DCBA) is more than sufficient to create each of the 10 needed numerals. Let's presume that a 4-bit device is only being used to display the numerals 0-9 (and no other useful "patterns").

(a) Complete the (DCBA and a-g) portion of the truth table for the necessary inputs and outputs of the circuit that controls the LED display. An example for the numeral 6 has already been filled out. (for now, ignore the Decoder-4 column)

(b) How many inputs and how many outputs are needed to control the display?

(c) Consider the Decoder #4 circuit, used for controlling one of the seven segments. Note how the circuit only uses NAND/OR logic. Fill in the Decoder #4 column in the truth table and determine which segment (a-g) this circuit controls.

(d) Write an algebraic expression for the logic gate portion of Decoder #4, then simplify the expression using logic algebra. (Hint: You should be able to simplify the expression to include only OR and NOT operations.)

(e) [**BONUS**] Construct the analogous NAND/OR logic circuit (using only NAND/OR gates) to control segment **f**. (My solution uses four gates. You may be able to do it with fewer)

Solution 4.

(a)

Dec	$DCBA$	a	b	c	d	e	f	g
1	0001	0	0	1	1	0	0	0
2	0010	0	0	1	0	1	1	1
3	0011	0	0	1	1	1	1	1
4	0100	1	0	1	1	0	1	0
5	0101	1	0	0	1	1	1	1
6	0110	1	1	0	1	1	1	1
7	0111	0	0	1	1	1	0	0
8	1000	1	1	1	1	1	1	1
9	1001	1	0	1	1	1	1	1
0	0000	1	1	1	1	1	0	1

(b) We are trying to represent 10 distinct input states in binary, so we need

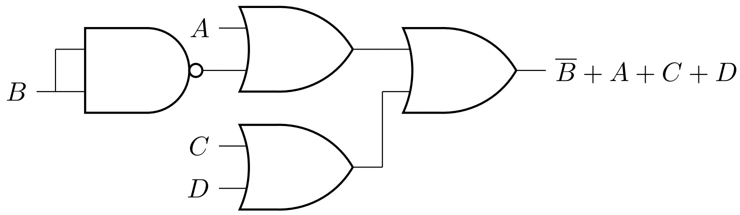
(c) When the decoder's "result" (signal at the base of the transistor) is HI, the transistor will be on and the diode will in turn be ON. The opposite occurs when the output is LO as the transistor is cut off and therefore no current flows to activate the diode.

Dec	$DCBA$	a	b	c	d	e	f	g	$Dec\,\#4$
1	0001	0	0	1	1	0	0	0	1
2	0010	0	0	1	0	1	1	1	0
3	0011	0	0	1	1	1	1	1	1
4	0100	1	0	1	1	0	1	0	1
5	0101	1	0	0	1	1	1	1	1
6	0110	1	1	0	1	1	1	1	1
7	0111	0	0	1	1	1	0	0	1
8	1000	1	1	1	1	1	1	1	1
9	1001	1	0	1	1	1	1	1	1
0	0000	1	1	1	1	1	0	1	1

As can be seen in the table, the d segment is the segment that is off when the "result" is LO, meanining the transistor is cut off and the lamp is OFF.

(d) Traversing the circuit we get $\overline{B\overline{(A+C+D)}}$ then, by DeMorgan's Theorem, $\overline{BX} = \overline{B}+\overline{X} = \overline{B}+(A+C+D)$. This result is consistent with the above truth table.

(e) [**BONUS**]



A
B
C
D
$\overline{B} + A + C + D$

Problem 5.

(a) $1100011_2 + 77_{10} = ?_2$

(b) $(101_2)^3 = ?_2$

(c) $111.11_2 = ?_{10}$

(d) $17.34_{10} = ?_2$

(e) $101.01_2 + 110.10_2 = ?_{10}$

Solution 5.

(a) First convert 77 to binary

$$\begin{aligned}
\frac{77}{2} &= 38R1\\
\frac{38}{2} &= 19R0\\
\frac{19}{2} &= 9R1\\
\frac{9}{2} &= 4R1\\
\frac{4}{2} &= 2R0\\
\frac{2}{2} &= 1R0\\
\frac{1}{2} &= 0R1 \implies 77_{10} = 1001101_2
\end{aligned}$$

Then add,
$$\begin{array}{cccccccc}
(1) & & & (1) & (1) & (1) & (1) & \\
 & 1 & 0 & 0 & 1 & 1 & 0 & 1\\
+ & 1 & 1 & 0 & 0 & 0 & 1 & 1\\
\hline
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0
\end{array}$$

(b)
$$\begin{array}{cccccc}
 & & & 1 & 0 & 1\\
 & & \times & 1 & 0 & 1\\
\hline
 & & & 1 & 0 & 1\\
 & & 0 & 0 & 0 & \\
+ & 1 & 0 & 1 & & \\
\hline
 & 1 & 1 & 0 & 0 & 1
\end{array}$$

Then,
$$\begin{array}{ccccccc}
 & & & & 1 & 0 & 1\\
 & \times & 1 & 1 & 0 & 0 & 1\\
\hline
 & & 1 & 1 & 0 & 0 & 1\\
 & 0 & 0 & 0 & 0 & 0 & \\
1 & 1 & 0 & 0 & 1 & & \\
\hline
1 & 1 & 1 & 1 & 1 & 0 & 1
\end{array}$$

(c) $111.11_2 = 1\left(2^2 + 2^1 + 2^0 + 2^{-1} + 2^{-2}\right)_{10} = 7.75_{10}$

(d) First convert 17 to base 2

$$
\begin{aligned}
\frac{17}{2} &= 8R1 \\
\frac{8}{2} &= 4R0 \\
\frac{4}{2} &= 2R0 \\
\frac{2}{2} &= 1R0 \\
\frac{1}{2} &= 0R1 \implies 17_{10} = 10001_2
\end{aligned}
$$

Then convert 0.34 to base 2. This process would repeat forever as $0.34 = \frac{17}{50}$ and 50 is not a power of 2 so I'm stopping one a reasonable approximation is reached.

$$
\begin{aligned}
0.34 \cdot 2 &= 0 + 0.68 \\
0.68 \cdot 2 &= 1 + 0.36 \\
0.36 \cdot 2 &= 0 + 0.72 \\
0.72 \cdot 2 &= 1 + 0.44 \\
0.44 \cdot 2 &= 0 + 0.88 \\
0.88 \cdot 2 &= 1 + 0.76 \\
0.76 \cdot 2 &= 1 + 0.52 \\
0.52 \cdot 2 &= 1 + 0.04 \implies 0.34_{10} \approx 0.01010111
\end{aligned}
$$

So $17.34_{10} \approx 10001.01010111_2$

(e)
$$
\begin{array}{ccccccc}
 & (1) & & & & & \\
 & 1 & 0 & 1 & . & 0 & 1 \\
+ & 1 & 1 & 0 & . & 1 & 0 \\
\hline
1 & 0 & 1 & 1 & . & 1 & 1
\end{array}
$$

Then $1011.11_2 = (2^3 + 2^1 + 2^0)_{10} + (2^{-1} + 2^{-2})_{10} = 11.75_{10}$