

# Physics 2700H: Assignment III

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March 4, 2025

**Problem 1.** Five kg of water at 25 °C is added to 10.0 kg of water at 85 °C. After the mixture has reached equilibrium, how much has entropy changed. (Assume no energy is exchanged between the water and its surroundings.)

**Solution 1.** The equilibrium temperature of the mixture can be determined using  $Q = mc\Delta T$ ,

$$\begin{aligned}m_C c \Delta T_C &= -m_H c \Delta T_H \\ \Rightarrow m_C (T_f - T_{iC}) &= -m_H (T_f - T_{iH}) \\ \Rightarrow T_f &= \frac{m_H T_{iH} + m_C T_{iC}}{m_C + m_H} = 338.15 \text{ K}\end{aligned}$$

Which means that  $\Delta Q_C = 836400 = -Q_H$ . To determine entropy change,

$$\begin{aligned}dS &= \frac{dQ}{T} \\ \Rightarrow \Delta S &= \int_{T_i}^{T_f} \frac{1}{T} dQ\end{aligned}$$

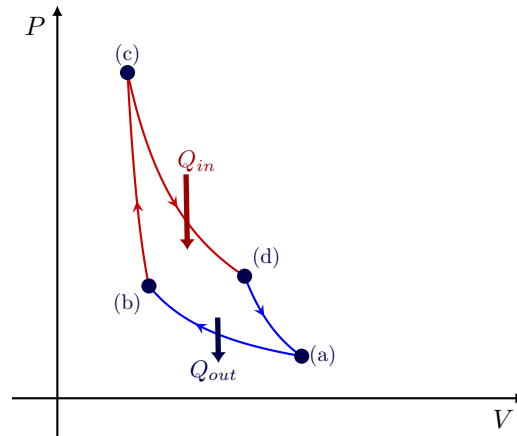
Where  $Q = mc\Delta T \Rightarrow dQ = mcdT$  so

$$\begin{aligned}\Delta S &= mc \int_{T_i}^{T_f} \frac{1}{T} dT \\ &= mc \ln \left( \frac{T_f}{T_i} \right)\end{aligned}$$

The entropy change of the system then is  $\Delta S_{sys} = \Delta S_H + \Delta S_C = c \left[ m_H \ln \left( \frac{T_f}{T_{iH}} \right) + m_C \ln \left( \frac{T_f}{T_{iC}} \right) \right] \approx 229.46 \text{ J K}^{-1}$  which is greater than zero as we would expect by the principle of increasing entropy.

**Problem 2.** The July 2023 Veritasium video about entropy, <https://www.youtube.com/watch?v=DxL2HoqLbyA>, introduces a Carnot engine within the first six minutes of the video. Draw on a  $PV$  diagram the cycle for this engine, with the bottom-right-most point labelled (a), and continue the cycle to points (b), (c) and (d). Identify which timestamps in the video correspond to the four points (a)...(d) and explain using a sentence per point why this is so.

**Solution 2.**



- (a) 5:03 the cold block is brought into contact and the piston begins to move downwards with heat moving into the cold block, pressure increasing, and volume decreasing.
- (b) 5:12 the cold block is removed and the gas continues to be compressed, now isothermally, pressure increases, and volume decreases.
- (c) 4:41 is when the hot block is brought into contact with the heat “aperture” and heat flows into the gas increasing temperature, pressure, and thereby driving an increase in volume.
- (d) 4:53 the hot block is removed but the piston continues to climb and so volume increases and pressure decreases along an isotherm.

**Problem 3.** One mole of helium gas is initially at  $P_0 = 1.0 \text{ atm}$  and  $T_0 = 273 \text{ K}$ .

- (a) Compute the entropy change if the gas is heated at constant pressure to temperature  $400 \text{ K}$ .
- (b) Starting again from the initial state  $(P_0, T_0)$ , what is the entropy change if the gas expands isothermally to twice its original volume?

**Solution 3.**

(a)

$$\begin{aligned}\Delta S &= \int_{T_i}^{T_f} \frac{1}{T} dQ \\ &= \int_{T_i}^{T_f} \frac{nC_P}{T} dT\end{aligned}$$

Where  $C_P = C_V + nR$  where  $C_V$  for an ideal monatomic gas is  $\frac{3}{2}R$  so for one mole we have,

$$\begin{aligned}\Delta S &= \int_{T_i}^{T_f} \frac{\frac{5}{2}R}{T} dT \\ &= \frac{5}{2}R \ln \left( \frac{T_f}{T_i} \right) = 7.94 \text{ J K}^{-1}\end{aligned}$$

(b) Again,

$$\Delta S = \int_{V_i}^{2V_i} \frac{1}{T} dQ$$

Here we use the first law,  $dU = dQ + dW$  where  $dU$  is zero as  $T$  is held constant and  $dW = -PdV \implies dQ = PdV$  so,

$$\begin{aligned}\Delta S &= \int_{V_i}^{2V_i} \frac{P}{T} dV \\ &= \int_{V_i}^{2V_i} \frac{nR\cancel{T}}{V\cancel{T}} dV \\ &= \int_{V_i}^{2V_i} \frac{R}{V} dV \\ &= R \ln \left( \frac{2V_i}{V_i} \right) \approx 5.76 \text{ J K}^{-1}\end{aligned}$$