Math 2120H: Assignment V

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Problem 1. Verify that the below vector field is conservative and find a potential function for $\mathbf{F}$

$$\mathbf{F} = e^{y+2z}\left(\mathbf{i} + x\mathbf{j} + 2x\mathbf{k}\right)$$

Solution 1. For conservative fields

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \frac{\partial P}{\partial x} = \frac{\partial M}{\partial z}.$$

Here $M = e^{y+2z}$, $N = xe^{y+2z}$, $P = 2xe^{y+2z}$ so,

$$\begin{aligned} \frac{\partial M}{\partial y} &= e^{y+2z}, & \frac{\partial M}{\partial z} &= 2e^{y+2z} \\ \frac{\partial N}{\partial x} &= e^{y+2z}, & \frac{\partial N}{\partial z} &= 2xe^{y+2z} \\ \frac{\partial P}{\partial y} &= 2xe^{y+2z}, & \frac{\partial P}{\partial x} &= 2e^{y+2z} \end{aligned}$$

which satisfies the previous set of equalities so the field is conservative. For the potential function we find $f(x, y, z) \int M\,dx = \int e^{y+2z}\,dx = xe^{y+2z} + G(y, z)$, then differentiate with respect to y and z to determine G. First with respect to y,

$$\frac{\partial f}{\partial y} = N = xe^{y+2z} = xe^{y+2z} + \frac{\partial G}{\partial y} \implies \frac{\partial G}{\partial y} = 0 \therefore G(y, z) = G(z)$$

Then with respect to z,

$$\frac{\partial f}{\partial z} = P = 2xe^{y+2z} = 2xe^{y+2z} + \frac{\partial G}{\partial z} \implies \frac{\partial G}{\partial z} = 0 \therefore G = C$$

so $f(x, y, z) = xe^{y+2z} + C$ where C is a constant of integration.

Problem 2. Find a potential function for the field below and evaluate the integral

$$\int_{(1,1,1)}^{(1,2,3)} 3x^2\,dx + \frac{z^2}{y}\,dy + 2z\ln y\,dz$$

Solution 2.

Problem 3. Find the outward flux of the field $\mathbf{F} = xy\mathbf{i} + y^2\mathbf{j}$ over the boundary of the region enclosed by the curve $y = x^2$ and the line $y = x$.

Solution 3.

Problem 4. Apply Green's Theorem to evaluate the integral below

$$\oint 3y\,dx + 2x\,dy, \qquad \text{Where } C \text{ is the boundary of } 0 \leq x \leq \pi,\ 0 \leq y \leq \sin x$$

Solution 4.