Physics 3130H: Assignment I

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Problem 1. A Skier weighing 90 kg starts from rest down a hill inclined at 17°. He skis 100 m down the hill and then coasts for 70 m along level snow until he stops. Find the coefficient of kinetic friction between the skis and the snow. What is his velocity when he reaches the bottom of the hill?

Solution 1. On the hill we can use conservation of energy to find the velocity the skier will have at the bottom of the slope. At the top of the hill the skier has potential energy

$$U = mgh = 100mg\sin(17)$$

and at the bottom all of this will have been converted into kinetic energy or lost to friction as work,

$$\frac{1}{2}mv^2 = mgh - 100\mu_k mg \implies v = \sqrt{2g(h - 100\mu_k)}$$

where we've taken the positive root implicitly because the skier should not have negative velocity. Now we have sufficient initial conditions to investigate the flat part of the problem, (by shifting our origin) $v(t=0) = v(x=0) = \sqrt{2g(h-100\mu_k)}$ and $x(t=0) = 0$. Our equation of motion in this region is

$$F = m\ddot{x} = \mu_k N = -mg\mu_k \implies \frac{d^2x}{dt^2} = -g\mu_k$$

which we rewrite using $\frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$ as

$$v\frac{dv}{dx} = -g\mu_k \implies v^2/2 = -g\mu_k x + C_1 \implies v = \sqrt{C_1 - 2g\mu_k x}.$$

applying $v(x=0) = \sqrt{2g(h-100\mu_k)}$ here,

$$v(0) = \sqrt{C_1} = \sqrt{2g(h-100\mu_k)} \implies C_1 = 2g(h - 100\mu_k) \implies v(x) = \sqrt{2g(h - 100\mu_k - \mu_k x)}.$$

Now we know that $v(70) = 0$ so

$$v(70) = \sqrt{2g(h - 100\mu_k - 70\mu_k)} = 0 \implies \mu_k = \frac{h}{170}$$

so

$$\frac{h}{170} = \frac{100\sin(17)}{170} \approx 0.17$$

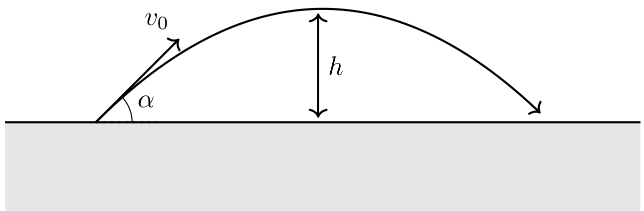
now we just substitute this into our initial condition to determine that the skiers velocity at the bottom of the hill is

$$v(x=0) = \sqrt{2g(h - 100\frac{100\sin(17)}{170})} = \sqrt{2g(100\sin(17) - \frac{10000\sin(17)}{170})} \approx 15.4\,\mathrm{m\,s^{-1}}$$

Problem 2.

(a) If a projectile is fired from the origin of the coordinate system with an initial velocity v_0 and in a direction making an angle α with the horizontal, calculate the time required for the projectile to cross a line passing through the origin and making an angle $\beta < \alpha$ with the horizontal.

(b) When a projectile is fired with an initial speed of v_0, it passes through the points P and Q which are at a distance h. Find the distance between P and Q in the figure



Solution 2.

(a) To do this we'll need to find the point of intersection between $y_p(x)$, the trajectory of the projectile, and $y_l(x) = \tan(\beta)x$, the given line. To do this we solve the equations of motion for both $y(t)$ and $x(t)$. Starting with x,

$$\begin{aligned} F_x &= m\ddot{x} = 0 \\ \implies \dot{x} &= C \\ \implies x(t) &= Ct + C_1 \end{aligned}$$

and because $x(0) = 0$ and $\dot{x}(0) = v_{0x}$, $C_1 = 0$ and $C = v_{0x}$ so $x(t) = v_{0x}t$. Now for $y(t)$,

$$\begin{aligned} F_y &= m\ddot{y} = -mg \\ \implies \dot{y} &= -gt + C \\ \implies y &= -\frac{gt^2}{2} + Ct + C_1. \end{aligned}$$

Applying initial conditions again we get that $y(t) = -\frac{gt^2}{2} + v_{0y}t$. We then solve for $t(x) = \frac{x}{v_{0x}}$ and substitute this into $y(t)$ to obtain

$$y(x) = -\frac{gx^2}{2v_{0x}^2} + \frac{v_{0y}x}{v_{0x}}.$$

Now we just solve

$$\tan(\beta)x = -\frac{g}{2v_{0x}^2}x^2 + \frac{v_{0y}}{v_{0x}}x \implies \frac{2v_{0x}}{g}\left[v_{0x}\tan(\beta) - v_{0y}\right] = x$$

(b) To do this we just shift down the trajectory by h so that P and Q are zeroes, so

$$\begin{aligned} 0 &= -\frac{g}{2v_{0x}^2}x^2 + \frac{v_{0y}}{v_{0x}}x - h \\ \implies x &= \frac{-\frac{v_{0y}}{v_{0x}} \pm \sqrt{\left(\frac{v_{0y}}{v_{0x}}\right)^2 - 4\left(-\frac{g}{2v_{0x}^2}\right)(h)}}{2\left(-\frac{g}{2v_{0x}^2}\right)} \\ &= \frac{v_{0y}v_{0x} \mp v_{0x}\sqrt{2gh + v_{0y}^2}}{g} \end{aligned}$$

then we can take the difference $Q - P$. With Q being the greater,

$$\frac{v_{0y}v_{0x} + v_{0x}\sqrt{2gh + v_{0y}^2}}{g} - \frac{v_{0y}v_{0x} - v_{0x}\sqrt{2gh + v_{0y}^2}}{g} = \frac{2v_{0x}\sqrt{2gh + v_{0y}^2}}{g}$$

Problem 3. A particle is under the influence of a force $F = -kx + \frac{k}{\alpha}x^3$, where k and α are constants and k is positive.

(a) Determine $U(x)$ and plot $U(x)$ as a function of x using computer graphing tools (if you use python, three cheers to you).

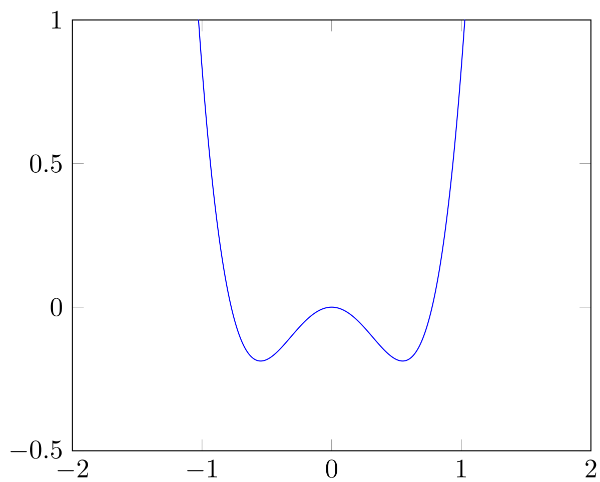
(b) Look at the figure carefully. Determine the energy E_1 in terms of k and α, based on the discussion we had in the class.

(c) If the particle has the energy E_2 discuss the motion of the particle.

Solution 3.

(a)

$$U(x) = \int F\,dx = \int -kx + \frac{k}{\alpha}x^3\,dx = -\frac{k}{2}x^2 + \frac{k}{4\alpha}x^4$$



(b) Look at the figure carefully. Determine the energy E_1 in terms of k and α, based on the discussion we had in the class.

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