Physics 2700H: Assignment II

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Problem 1. Consider a mix of N_2 and O_2, which we may treat as an ideal gas, inside a car engine's cylinder that follows the idealized Otto cycle. Assume points a, b, c, d in Fig. 4.15(b) correspond to (V, P) values of: $\{(7V_1, 1\,\text{atm}), (V_1, 15.25\,\text{atm}), (V_1, 30.50\,\text{atm}), (7V_1, 2\,\text{atm})\}$, respectively.

(a) Confirm that these (V, P) values are consistent with the gas experiencing one adiabatic compression and one adiabatic expansion over each cycle.

(b) If the car's engine size (the displacement of here, <u>four</u> cylinders) due to compression/expansion is 2.4 L, what is V_1?

(c) Find the net work done by the gas in all four cylinders of the engine over one cycle (you may use the relevant expression for work from either p. 44 or p. 77 of the textbook).

Solution 1.

(a) Here the adiabats are along $\overrightarrow{ab}$ and $\overrightarrow{cd}$. For an adiabat $P_iV_i^\gamma = P_fV_f^\gamma$ holds so

$$\begin{aligned}(7V_1)^\gamma \cdot 1\,\text{atm} &= V_1^\gamma \cdot 15.25\,\text{atm}\\ V_1^\gamma \cdot 1\,544\,734.579\,\text{Pa} &= V_1^\gamma \cdot 1\,545\,206.25\,\text{Pa}\end{aligned}$$

Which is approximately adiabatic. For $\overrightarrow{cd}$

$$\begin{aligned}V_1^\gamma \cdot 30.50\,\text{atm} &= (7V_1)^\gamma \cdot 2\,\text{atm}\\ V_1^\gamma \cdot 3\,090\,412.5\,\text{Pa} &= V_1^\gamma \cdot 3\,089\,469.158\,\text{Pa}\end{aligned}$$

Which is also approximately adiabatic.

(b) The volume difference between compression and expansion **for a single cylinder** is $7V_1 - V_1 = 6V_1$ so $4 \cdot 6V_1 = 2.4\,\text{L} \implies V_1 = 0.1\,\text{L} = 1 \times 10^{-4}\,\text{m}^3$

(c) This formula gives work done on the gas, so we must flip the sign to determine work done by the gas

$$\begin{aligned}W_{\overrightarrow{ab}} &= -\frac{P_iV_i}{\gamma - 1}\left[\left(\frac{V_i}{V_f}\right)^{\gamma-1} - 1\right]\\ &= -\frac{7\times 10^{-4}\,\text{m}^3 \cdot 101\,325\,\text{Pa}}{\frac{7}{5} - 1}\left[\left(\frac{7\times 10^{-4}\,\text{m}^3}{1\times 10^{-4}\,\text{m}^3}\right)^{\frac{7}{5}-1} - 1\right]\\ &\approx -208.9\,\text{J}\end{aligned}$$

$$\begin{aligned}W_{\overrightarrow{cd}} &= -\frac{1\times 10^{-4}\,\text{m}^3 \cdot 3\,090\,412.5\,\text{Pa}}{\frac{7}{5} - 1}\left[\left(\frac{1\times 10^{-4}\,\text{m}^3}{7\times 10^{-4}\,\text{m}^3}\right)^{\frac{7}{5}-1} - 1\right]\\ &\approx 417.9\,\text{J}\end{aligned}$$

Net work is $W_{net} = 417.9\,\text{J} - 208.9\,\text{J} = 209\,\text{J}$ **Does this need to be quadrupled??**

Problem 2. An inventor claims to have developed an engine that takes in 1.1×10^8 J at 400 K, rejects 5.07×10^7 J at 200 K, and delivers 16.7 kW hours of work. Would you advise investing money in this project?

Solution 2. A Carnot engine operating in this environment has $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{200\,\mathrm{K}}{400\,\mathrm{K}} = 0.5$ and this engine has efficiency $\eta = 1 - \frac{5.07\times10^7\,\mathrm{J}}{1.1\times10^8\,\mathrm{J}} \approx 0.54$. As the efficiency of a "real" engine cannot exceed that of a Carnot engine I would not reccomend investing in this project.

Problem 3. Suppose a house requires 4.3 GJ of heating in a winter month. The utility company charges \$0.14 per kW h.

(a) Find the cost savings of using a heat pump versus a 95%-efficient natural gas furnace. Assume a Carnot heat pump with average temperatures of 20 °C indoors and 0 °C outdoors.

(b) Repeat part (a) using a more realistic coefficient of performance of 4.0 for the heat pump.

Solution 3. (a) Find the cost savings of using a heat pump versus a 95%-efficient natural gas furnace. Assume a Carnot heat pump with average temperatures of 20 °C indoors and 0 °C outdoors.

(b) Repeat part (a) using a more realistic coefficient of performance of 4.0 for the heat pump.

Problem 4. A hypothetical engine, with an ideal gas as the working substance, operates in the cycle shown in Figure 4.17. Show that the efficiency of the engine is

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{1 - \frac{P_3}{P_1}}{1 - \frac{V_1}{V_3}}\right)$$

Solution 4.