Physics 3610H: Assignment IV

Jeremy Favro (0805980)
Trent University, Peterborough, ON, Canada

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Problem 1. In class we found the general form of the wavefunction in each region of a finite well to be

$$\Big\{x < -aDe^{+\kappa x} - a < x < aA\cos kx + B\sin kxx > aCe^{-\kappa x}$$

Using the continuity of the wavefunction and its first derivative at both $x = -a$ and $x = +a$, we arrived at the following four equations.

$$\psi(-a) = A\cos ka - B\sin ka = De^{-\kappa a} \tag{1}$$

$$\psi(+a) = A\cos ka + B\sin ka = Ce^{-\kappa a} \tag{2}$$

$$\left.\frac{d\psi}{dx}\right|_{-a} = kA\sin ka + kB\cos ka = D\kappa e^{-\kappa a} \tag{3}$$

$$\left.\frac{d\psi}{dx}\right|_{+a} = -kA\sin ka + kB\cos ka =_C \kappa e^{-\kappa a} \tag{4}$$

Together with normalization these determine A, B, C, D and E. In particular, by considering $(2) - (4)/\kappa$ and $(3) + (5)/\kappa$ we showed that

$$A\left(1 - \frac{k}{\kappa}\tan ka\right) = B\left(\tan ka + \frac{k}{\kappa}\right) = 0$$

In class we considered the even solutions by setting $B = 0$. Here, consider the odd solutions by setting $A = 0$.

(a) What two equations connect k and κ in this case?

(b) Let $x \equiv ka$ and $y \equiv \kappa a$ and plot both functions on a single plot for $2mV_o a^2/\hbar^2 = 25$.

(c) How many allowed values of energy are there in this case?

(d) Give an approximate value of κa which is allowed.

(e) Use $(2)+(4)/k$ and $(3)-(5)/\kappa$ to determine the values of C and D, and write the form of the odd wavefunctions in each region in terms of B, k and κ.

Solution 1.