

Physics 3200Y: Lab 1

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1 Introduction

This lab seeks to derive and verify an expression for the potential at every point inside a coaxial cylindrical capacitor. Herein such a capacitor is constructed and analyzed at a variety of frequencies to determine the validity of the derived equation. Significant deviation from expected results was observed in extreme regions of the capacitor and is determined to likely be caused by the invalidation of assumptions made during the derivation of the equation

This is inconvenient however as the charge Q_{enc} is unknown. $V_0 = V(a) - V(b)$ is however known as it is the voltage applied between the two cylinders which can also be determined using the previous equation,

$$V_0 = \frac{Q_{enc}}{2\epsilon_0\pi L} \ln\left(\frac{a'}{b}\right) \\ \Rightarrow Q_{enc} = \frac{2\epsilon_0\pi L V_0}{\ln(a/b)}$$

which, when substituted back into the equation for potential at arbitrary distance from b gives

$$V(r') = \frac{\ln(r'/b)}{\ln(a/b)} V_0$$

which is entirely in terms of known quantities.

2 Theory

2.1 Deriving the Potential

By the integral form of Gauss' law

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

in the setup depicted in Figure 1 a cylindrical Gaussian surface can be constructed, giving an area element

$$d\vec{a} = r\phi d\phi dz \hat{r}.$$

Because the electric field here is unknown it cannot be directly integrated. Instead an assumption must be made that the electric field solely in the radial direction. This is reasonable for a capacitor where we can ignore edge effects which would make it invalid. Using this the \vec{E} term can be extracted from the integral which reduces to the area of a cylinder. Additionally, as the field is entirely radial (again ignoring edge effects), the dot product of the vectors reduces to the product of their only components, both of which are in the radial direction. This yields

$$E = \frac{Q_{enc}}{2\epsilon_0\pi r L}.$$

From this the definition of the potential at some point r' (neglecting the vector due, again, to symmetries requiring only a value of radius) relative to an origin a can be applied:

$$V(r') = - \int_b^{r'} E dr = - \frac{Q_{enc}}{2\epsilon_0\pi L} \int_b^{r'} \frac{dr}{r} = \frac{Q_{enc}}{2\epsilon_0\pi L} \ln\left(\frac{r'}{b}\right).$$

3 Methods

3.1 Coaxial Capacitor

The first setup involved two coaxial aluminum cylinders held at known but different potentials (See Figure 1). The outer cylinder with inner radius $r_b \approx 21.25 \pm 0.01$ cm and the inner (solid) cylinder with radius $r_a \approx 5.77 \pm 0.01$ cm. The inner cylinder was connected to a variable frequency function generator set to a root-mean-square (RMS) voltage of $V_{gen} \approx 7.76$ V. The outer cylinder was held at ground. Tap water was used as a dielectric and filled the space between the cylinders at a constant depth. A Siglent SDS1202X-E oscilloscope was connected to a needle-tip probe. The probe was used to measure voltage at points on a printed template of ≈ 1 cm grid squares. Measurements were performed under an assumption of perfect radial symmetry which is a suitable approximation for such a configuration. Measurements were performed with a fixed input signal RMS voltage of V_{gen} on the intersection of every neighbouring grid square in a line outward from the inner cylinder. Measurements were performed at a variety of frequencies to determine if the potential inside the capacitor has any dependence on frequency.

3.2 Second Capacitor

A second capacitor consisting of parallel aluminum plates centered inside the outer cylinder in the original coaxial capacitor was also studied (see Figure 2). The setup was similar to the coaxial capacitor with the inner element(s) at a known potential V_{gen} and the outer element at ground. Tap water was again used as a dielectric. The configuration was quadrant-wise symmetric which allowed measurements to be made for only the upper-left quadrant using the same oscilloscope and needle tipped probe as before. The collected data was then mirrored into the other quadrants using Python.

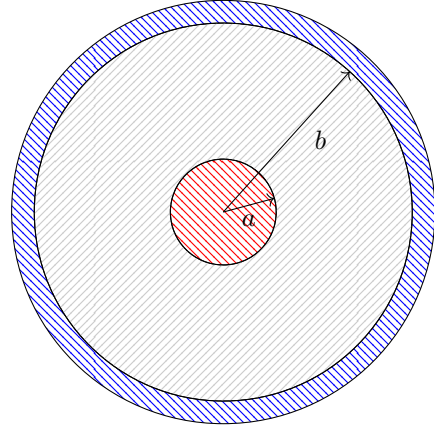


Figure 1: Coaxial Capacitor Structure

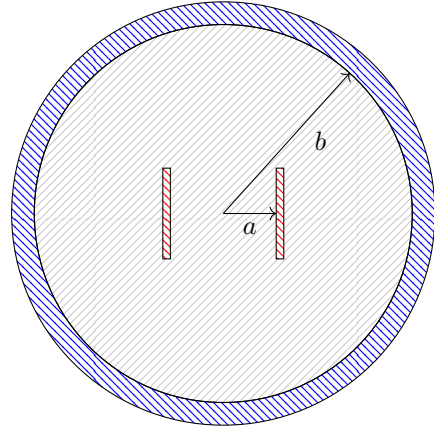


Figure 2: Second Capacitor Structure

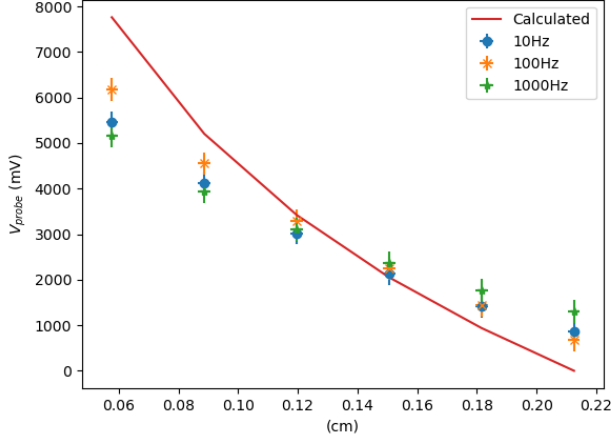


Figure 3: Coaxial Potential Plot

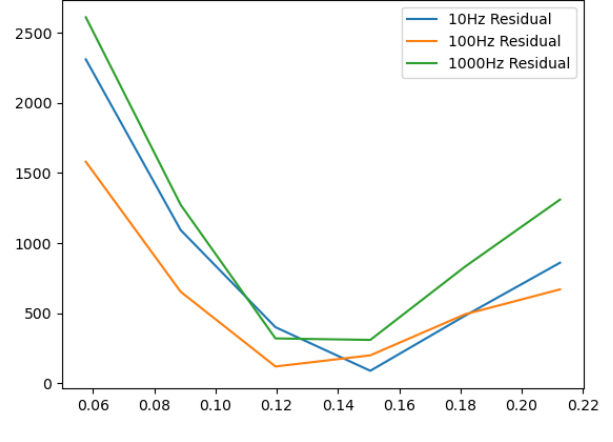


Figure 4: Coaxial Potential Residuals
Table 1: Coaxial Data

4 Results

4.1 Coaxial Capacitor

The general curve shape of the derived potential vs. position curve was successfully retrieved for the coaxial capacitor. There is significant error near the edges of the measured region (see Figure 4). These regions are where the assumptions made in deriving the ideal potential equation begin to fall apart as the electric field is no longer radial and begins to curve significantly. Uncertainty is also present in the position of the probe. No strategies were employed to accurately locate the probe beyond visual inspection. There does appear to be some dependence on frequency though if it does exist it is fairly subtle and washed out by semi-random error in the data.

Grid Position	V_{probe} (mV)	Frequency (Hz)
I16/J15	5150 ± 100	10
I15/H17	3930 ± 100	-
H17/G18	3100 ± 100	-
G18/F19	2360 ± 100	-
F19/E20	1770 ± 100	-
E20/D21	1310 ± 100	-
I16/J15	5450 ± 100	100
I15/H17	4110 ± 100	-
H17/G18	3220 ± 100	-
G18/F19	2140 ± 100	-
F19/E20	1420 ± 100	-
E20/D21	860 ± 100	-
I16/J15	6180 ± 100	1000
I15/H17	4560 ± 100	-
H17/G18	3300 ± 100	-
G18/F19	2250 ± 100	-
F19/E20	1430 ± 100	-
E20/D21	670 ± 100	-

4.2 Second Capacitor

A model was not developed for the second capacitor. It does behave somewhat as expected qualitatively. The capacitor overall exhibits similar behaviour to that seen in the purely cylindrical case but also behaves similarly to a parallel plate capacitor near the center. There is again a significant uncertainty in the measurements made due to uncertain probe position, height, and edge effects.

There are two significant anomalies (which are mirrored in all four quadrants) near the outer edge of the parallel plates. Both are likely due to the probe being further from the plates than in other measurements as both are in regions where, due to the mounting system of the probe, it was difficult to center the measuring tip.

5 Discussion

5.1 Sources of Uncertainty

Systematic uncertainty in the experiment is largely caused by (in order of severity)

1. Imprecise probe placement. The exact position of the probe was not precisely determined during each measurement. An estimated 2 mm position uncertainty is included in the horizontal error bars in Figure 4 to represent this, though the actual deviation from an ideal measurement is likely significantly more than this in some cases and makes determining subtle influences on the equation, such as a potential frequency dependence, difficult.
2. Imprecise potential measurement. This arises from several factors. Most notably is direct fluctuations in the potential displayed by the oscilloscope making it difficult to settle on an exact value.

Error propagation was not performed using any of these uncertainties due to time constraints. Uncertainties were instead estimated based on the smallest divisions of measurement devices and observed fluctuations.

5.2 Edge Effect

While measuring the potentials in lab it was observed that as the probe neared the edges of the cylindrical capacitor significant fluctuations in the potential displayed by the oscilloscope were observed. We initially came to the conclusion that this caused by the probe in some way altering the electric field when placed close enough to an edge. It was then suggested that the fluctuations observed were due to edge effects. Edge effects are a phenomenon where, in this specific case, the electric field gains some not insignificant non-radial component. As in our derivation of the potential we assumed that the electric field was perfectly radial this causes our predicted values to fall away from the observed values significantly.

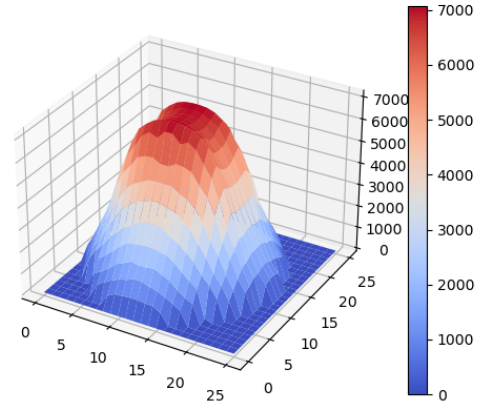


Figure 5: The mapped potentials of the second capacitor.

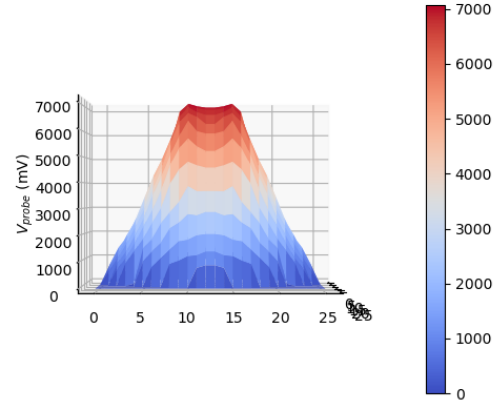


Figure 6: Side-on view of Figure 5.

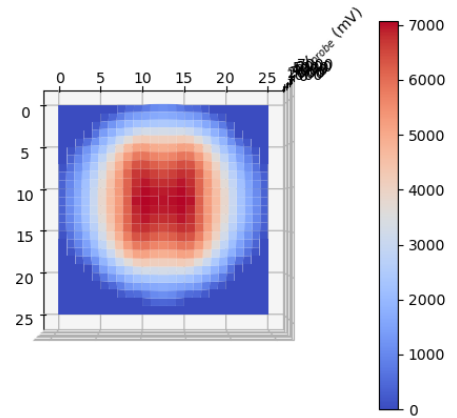


Figure 7: Top down view of Figure 5. Note the distinct behaviours per “region”.

6 Conclusion

General graph shapes for the modeled coaxial cylinder capacitor were successfully observed in real-world data. Uncertainties in measurements makes determining additional dependence on other factors such as frequency difficult to determine though data collected across a range of frequencies does suggest some dependence on frequency not present in the derived equation. Significant deviation from expected results is present in the regions near the edges, likely due to edge effects invalidating some assumptions made in the derivation of the potential equation.

The second capacitor's potentials were successfully mapped and qualitatively behave as expected.

7 Appendix

7.1 Images

7.2 Data

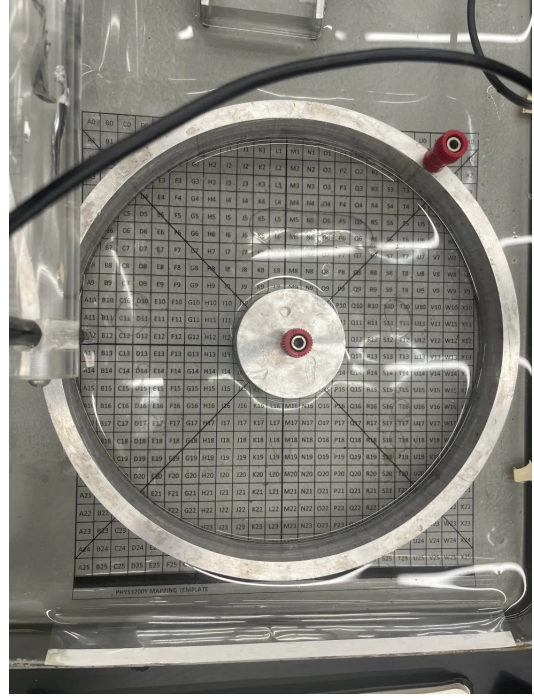


Figure 8: Coaxial cylinder capacitor. Probe visible at left. Mapping template visible beneath the aluminum elements.

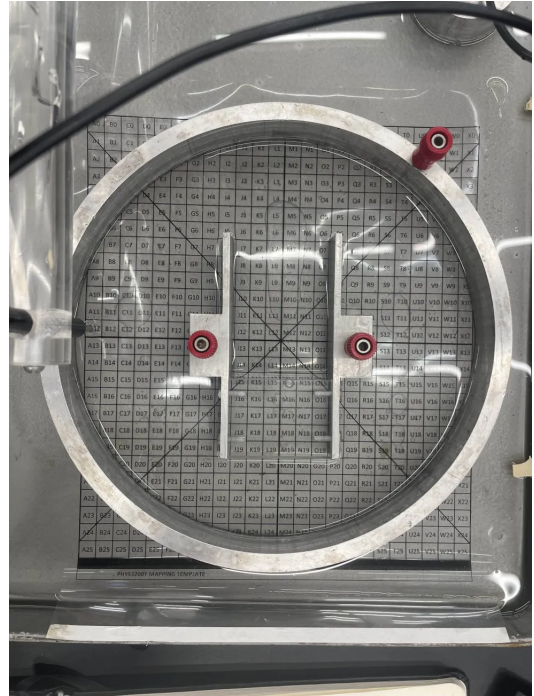


Figure 9: Second capacitor. Note that the plates of the inner element were electrically connected during the potential mapping.

Table 2: Second Capacitor Data

Grid Position	V_{probe} (mV)
L1	860 ± 10
L2	1360 ± 10
L3	2020 ± 10
L4	2860 ± 10
L5	3670 ± 10
L6	4580 ± 10
L7	5230 ± 10
L8	5920 ± 10
L9	6300 ± 10
L10	6670 ± 10
L11	6890 ± 10
L12	6980 ± 10
K1	760 ± 10
K2	1220 ± 10
K3	1970 ± 10
K4	2730 ± 10
K5	3660 ± 10
K6	4670 ± 10
K7	5410 ± 10
K8	6010 ± 10
K9	6580 ± 10
K10	6780 ± 10
K11	6950 ± 10
K12	7030 ± 10
J2	1180 ± 10
J3	1800 ± 10
J4	2780 ± 10
J5	3860 ± 10
J6	4810 ± 10
J7	5720 ± 10
J8	6420 ± 10
J9	6730 ± 10
J10	6970 ± 10
J11	7090 ± 10
J12	7130 ± 10
I2	1010 ± 10
I3	1740 ± 10
I4	2450 ± 10
I5	3250 ± 10
I6	4510 ± 10
I7	5340 ± 10
I8	5830 ± 10
I9	6380 ± 10
...	...

7.3 Code

```

1 import numpy as np
2 import matplotlib.pyplot as pyplot
3
4 V_0 = 7.76
5 a = 0.0577
6 b = 0.2125
7 dat_10 = [5450, 4110, 3020, 2140, 1420, 860]
8 dat_100 = [6180, 4550, 3300, 2250, 1430, 670]
9 dat_1000 = [5150, 3930, 3100, 2360, 1770, 1310]
10 yerrs = [250] * 6
11 xerrs = [0.002]*6
12 xax = np.linspace(a, b, 6)
13
14
15 def fun(x):
16     return 1000 * (V_0 * np.log(x / b) / np.log(a / b))
17
18
19 pyplot.errorbar(xax, dat_10, yerr=yerrs, xerr=xerrs,
20     ↪ label="10Hz", fmt="o")
21 pyplot.errorbar(xax, dat_100, yerr=yerrs, xerr=xerrs,
22     ↪ label="100Hz", fmt="x")
23 pyplot.errorbar(xax, dat_1000, yerr=yerrs, xerr=xerrs,
24     ↪ label="1000Hz", fmt="*")
25 pyplot.plot(xax, fun(xax),
26     label="Calculated")
27 pyplot.ylabel(r"$V_{\text{probe}}$ (mV)")
28 pyplot.xlabel(r"(cm)")
29 pyplot.legend()
30 pyplot.savefig("PHYS/PHYS 3200Y/Lab 1/Figure_1.png")
31
32 pyplot.figure(2)
33 pyplot.plot(xax, abs(dat_10 - fun(xax)), label="10Hz
34     ↪ Residual")
35 pyplot.plot(xax, abs(dat_100 - fun(xax)), label="100Hz
36     ↪ Residual")
37 pyplot.plot(xax, abs(dat_1000 - fun(xax)), label="1000Hz
38     ↪ Residual")
39
40 pyplot.legend()
41 pyplot.savefig("PHYS/PHYS 3200Y/Lab 1/Figure_2.png")
42 pyplot.show()

```

```

1 import numpy as np
2 import matplotlib.pyplot as pyplot
3 from matplotlib import cm
4
5 # Recall our template was
6 # A0 .. X0
7 # . . .
8 # . . .
9 # A25 .. X25
10 data = np.genfromtxt(
11     "PHYS/PHYS 3200Y/Lab 1/EM heatmap data.csv",
12     ↪ delimiter=",")

```

```

12 nums = data[:, 1]
13
14 x = np.linspace(0, 25, 26)
15 y = np.linspace(0, 25, 26)
16 xv, yv = np.meshgrid(x, y)
17
18 full_dat = np.zeros(shape=(26, 26))
19
20 # L
21 full_dat[12, 0:24] = np.concatenate(
22     (nums[0:12], np.flip(nums[0:12])))
23
24 # K
25 full_dat[11, 0:24] = np.concatenate(
26     (nums[12:24], np.flip(nums[12:24])))
27
28 # J
29 full_dat[10, 1:23] = np.concatenate(
30     (nums[24:35], np.flip(nums[24:35])))
31
32 # I
33 full_dat[9, 1:23] = np.concatenate(
34     (nums[35:46], np.flip(nums[35:46])))
35
36 # H
37 full_dat[8, 1:23] = np.concatenate(
38     (nums[46:57], np.flip(nums[46:57])))
39
40 # G
41 full_dat[7, 2:22] = np.concatenate(
42     (nums[57:67], np.flip(nums[57:67])))
43
44 # F
45 full_dat[6, 2:22] = np.concatenate(
46     (nums[67:77], np.flip(nums[67:77])))
47
48 # E
49 full_dat[5, 3:21] = np.concatenate(
50     (nums[77:86], np.flip(nums[77:86])))
51
52 # D
53 full_dat[4, 4:20] = np.concatenate(
54     (nums[86:94], np.flip(nums[86:94])))
55
56 # C
57 full_dat[3, 5:19] = np.concatenate(
58     (nums[94:101], np.flip(nums[94:101])))
59
60 # B
61 full_dat[2, 6:18] = np.concatenate(
62     (nums[101:107], np.flip(nums[101:107])))
63
64 # A
65 full_dat[1, 8:16] = np.concatenate(
66     (nums[107:111], np.flip(nums[107:111])))
67
68 full_dat = full_dat + np.flip(full_dat, axis=0)
69
70 ax = pyplot.figure().add_subplot(projection='3d')

```

```

71 ax.set_zlabel(r"$V_{probe}$ (mV)")
72
73 p = ax.plot_surface(xv, yv, full_dat, cmap=cm.coolwarm)
74 pyplot.colorbar(p)
75 pyplot.savefig("PHYS/PHYS 3200Y/Lab 1/hplot.png")
76 ax.view_init(elev=0, azim=0)
77 pyplot.savefig("PHYS/PHYS 3200Y/Lab 1/hplot-sideon.png")
78 ax.view_init(elev=90, azim=0)
79 pyplot.savefig("PHYS/PHYS 3200Y/Lab
↪ 1/hplot-topdown.png")
80
81 pyplot.show()

```