Math 3770H: Assignment I

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Problem 1. Verify that each of the two numbers $z = 1 \pm i$ satisfies the equation $z^2 - 2z + 2 = 0$.

Solution 1.

Problem 2. Prove that multiplication of complex numbers is commutative, as stated at the beginning of Sec. 2.

Solution 2.

Problem 3. Reduce each of these quantities to a real number:

$$\text{(a) } \frac{1+2i}{3-4i} + \frac{2-i}{5i}; \qquad \text{(b) } \frac{5i}{(1-i)(2-i)(3-i)}; \qquad \text{(c) } (1-i)^4.$$

Solution 3.

Problem 4. Verify that $\sqrt{2}|z| \geq |\text{Re}\{z\}| + |\text{Im}\{z\}|$

Solution 4.

Problem 5. Using the fact that $|z_1 - z_2|$ is the distance between two points z_1 and z_2, give a geometric argument that $|z-1| = |z+1|$ represents the line through the origin whose slope is -1.

Solution 5.

Problem 6. By factoring $z^4 - 4z^2 + 3$ into two quadratic factors and using inequality (2), Sec. 5, show that if z lies on the circle $|z| = 2$, then

$$\left| \frac{1}{z^4 - 4z^2 + 3} \right| \leq \frac{1}{3}$$

Solution 6.

Problem 7. Find the principal argument $\arg z$ when

Solution 7.