Math 3770H: Assignment III

Jeremy Favro (0805980)
Trent University, Peterborough, ON, Canada

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Problem 1. Show that $\left|\exp\left(z^2\right)\right| \leq \exp\left(|z|^2\right)$.

Solution 1.

Problem 2. Find the principal value of

$$\text{(a) } (-i)^i; \qquad \text{(b) } \left[\tfrac{e}{2}\left(-1-\sqrt{3}i\right)\right]^{3\pi i}; \qquad \text{(c) } (1-i)^{4i}.$$

Solution 2.

Problem 3. Derive expression (9), Sec. 40, for $\cosh^{-1}(z)$.

Solution 3. Expression (9) says

$$\cosh^{-1}(z) = \log\left[z + (z^2-1)^{1/2}\right].$$

Problem 4. Let C_R denote the upper half of the circle $|z| = R\,(R > 2)$, taken in the counterclockwise direction. Show that

$$\left|\int_{C_R} \frac{2z^2-1}{z^4+5z^2+4}\,dz\right| \leq \frac{\pi R(2R^2+1)}{(R^2-1)(R^2-4)}.$$

Then, by dividing the numerator and denominator on the right here by R^4, show that the value of the integral tends to zero as R tends to infinity. (Compare with Example 2 in Sec. 47.)

Solution 4.

Problem 5. Apply the Cauchy-Goursat theorem to show that

$$\int_C f(z)\,dz = 0$$

when the contour C is the unit circle $|z| = 1$, in either direction, and when

$$\text{(a) } f(z) = \frac{z^2}{z+3}; \qquad \text{(b) } f(z) = ze^{-z}; \qquad \text{(c) } f(z) = \frac{1}{z^2+2z+2};$$
$$\text{(d) } f(z) = \operatorname{sech}(z); \qquad \text{(e) } f(z) = \tan(z); \qquad \text{(f) } f(z) = \operatorname{Log}(z+2)$$

Solution 5.