Physics 3610H: Assignment III

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Problem 1. Consider a particle in the infinite square well from $0 < x < a$. The eigenstates and eigenvalues of the T.D.S.E for this system are

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$
$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$$

respectively. We also found that these states form an orthonormal set,

$$\int_{\mathbb{R}} \psi_n^*(x)\psi_m(x)dx = \delta_{mn}.$$

Suppose a particle in such a well is in the following state at $t = 0$:

$$\Psi(x, 0) = A(\psi_1(x) + 2\psi_2(x))$$

(a) Find A such that $\Psi(x, 0)$ is normalized.

(b) Draw $\Psi(x, 0)$ as a function of x.

(c) Where is the particle most likely to be found at $t = 0$? (Draw arrow(s) on your plot.)

(d) Where is the particle least likely to be found at $t = 0$? (Draw arrow(s) on your plot.)

(e) What is the probability of finding the particle in the left half of the well (i.e. $0 < x < a/2$) at $t = 0$?

(f) Find $\Psi(x, t)$.

(g) Show $\Psi(x, t)$ is normalized for all times t.

(h) What is the expectation value of x?

(i) If you measured the energy of this particle, what values might you get and what is the probability that you will get each of these values?

(j) What is the expectation value of the energy?

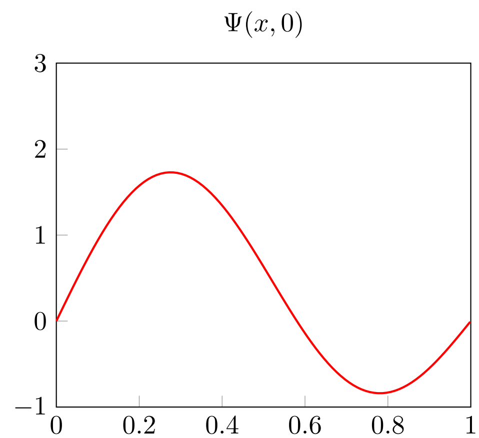
Solution 1.

(a) Here we force the integral over all space of $|\Psi|^2$ to be 1 and solve for an A which satisfies this. We can note that because we are dealing with an infinite well here the wavefunction is only nonzero within the well so we can trim down the bounds a bit,

$$
\begin{aligned}
1 &= \int_0^a \Psi^*(x,0)\Psi(x,0)\,dx \\
&= |A|^2 \int_0^a (\psi_1^*(x) + 2\psi_2^*(x))(\psi_1(x) + 2\psi_2(x))\,dx \\
&= |A|^2 \int_0^a \psi_1^*(x)\psi_1(x) + 2\psi_2^*(x)\psi_1(x) + 2\psi_2(x)\psi_1^*(x) + 4\psi_2(x)\psi_2^*(x)\,dx \\
&= |A|^2 \int_0^a \psi_1^*(x)\psi_1(x) + 2\delta_{12} + 2\delta_{12} + 4\psi_2(x)\psi_2^*(x)\,dx \\
&= |A|^2 \left[\delta_{11} + \cancelto{0}{\delta_{12} + \delta_{12}} + 4\delta_{22}\right] \\
&= |A|^2\,[1+4] \implies |A|^2 = 1/5 \implies A = \sqrt{1/5}
\end{aligned}
$$

Why doesn't this work for $0 < a \leq 3$????

(b)



(c) Here we take the derivative of the wavefunction $\Psi(x,0)$ and determine the turning points,

$$
\begin{aligned}
0 &= \frac{d}{dx}\Psi(x,0) \\
&= A\left[\frac{d}{dx}(\psi_1(x) + 2\psi_2(x))\right] \\
&= A\left[\frac{d}{dx}\psi_1(x) + 2\frac{d}{dx}\psi_2(x)\right] \\
&= A\left[\frac{d}{dx}\sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right) + 2\frac{d}{dx}\sqrt{\frac{2}{a}}\sin\left(\frac{2\pi x}{a}\right)\right] \\
&= A\sqrt{\frac{2}{a}}\left[\frac{\pi}{a}\cos\left(\frac{\pi x}{a}\right) + \frac{4\pi}{a}\cos\left(\frac{2\pi x}{a}\right)\right] \\
&= A\frac{\pi}{a}\sqrt{\frac{2}{a}}\left[\cos\left(\frac{\pi x}{a}\right) + 4\cos\left(\frac{2\pi x}{a}\right)\right]
\end{aligned}
$$

Which will be zero when

$$\cos\left(\frac{\pi x}{a}\right) = -4\cos\left(\frac{2\pi x}{a}\right)$$

and when

$$\cos\left(\frac{\pi x}{a}\right) = \cos\left(\frac{2\pi x}{a}\right) = 0.$$

Solving the second equation by taking acos of both sides we get

$$\frac{\pi x}{a} = \frac{2\pi x}{a} = 0 \implies x = 2x \implies x = 0$$

(d) See blue

(e) What is the probability of finding the particle in the left half of the well (i.e. $0 < x < a/2$) at $t = 0$?

(f) Find $\Psi(x, t)$.

(g) Show $\Psi(x, t)$ is normalized for all times t.

(h) What is the expectation value of x?

(i) If you measured the energy of this particle, what values might you get and what is the probability that you will get each of these values?

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