Physics 2610H: Assignment III

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Problem 1. A $50\,\mathrm{eV}$ electron is trapped between electrostatic walls $200\,\mathrm{eV}$ high. How far does its wave function extend beyond the walls?

Solution 1. The penetration depth, given in the text as equation 24, is

$$\delta = \frac{\hbar}{\sqrt{2m_e\left(U_0 - E\right)}} = \frac{\hbar}{\sqrt{2m_e\left(200\,\mathrm{eV} - 50\,\mathrm{eV}\right) \cdot 1.602 \times 10^{-19}\,\mathrm{J\,eV^{-1}}}} \approx 1.59 \times 10^{-11}\,\mathrm{m}$$

Problem 2. To a good approximation, the hydrogen chloride molecule, HCl, behaves vibrationally as a quantum harmonic oscillator of spring constant $480\,\mathrm{N\,m^{-1}}$ and with effective oscillating mass just that of the lighter atom, hydrogen. If it were in its ground vibrational state, what wavelength photon would be just right to bump this molecule up to its next-higher vibrational energy state?

Solution 2.

Problem 3. Calculate the reflection probability for a 5 eV electron encountering a step in which the potential drops by 2 eV.

Solution 3.

Problem 4. A beam of particles of energy E and incident upon a potential step of $U_0 = \frac{5}{4}E$ is described by the wave function

$$\psi_{inc}(x) = e^{ikx}$$

(a) Determine completely the reflected wave and the wave inside the step by enforcing the required continuity conditions to obtain their (possibly complex) amplitudes.

(b) Verify by explicit calculation that the ratio of reflected probability density to the incident probability density is 1.

Solution 4.

Problem 5. What fraction of a beam of $50\,\mathrm{eV}$ electrons would get through a $50\,\mathrm{volt}$, $1\,\mathrm{nm}$ wide electrostatic barrier?

Solution 5.