

Physics 2610H: Assignment II

Jeremy Favro

February 3, 2025

Problem 1. Write out the total wave function $\Psi(x, t)$ for an electron in the $n = 3$ state of a 10 nm wide infinite well. Other than the symbols x and t , the function should include only numerical values.

Solution 1. We can split $\Psi(x, t) = \psi(x)\phi(t) = \psi(x)\exp\left(\frac{-iEt}{\hbar}\right)$ then for a particle in an infinite potential well $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\sqrt{\frac{2mE}{\hbar^2}}x\right)$ and $E = \frac{n^2\pi^2\hbar^2}{2mL^2}$. So,

$$\begin{aligned}\Psi(x, t) &= \sqrt{\frac{2}{L}} \sin\left(\sqrt{\frac{2m \cdot \frac{n^2\pi^2\hbar^2}{2mL^2}}{\hbar^2}}x\right) \exp\left(\frac{-i \cdot \frac{n^2\pi^2\hbar^2}{2mL^2}t}{\hbar}\right) \\ &= \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \exp\left(-i \frac{n^2\pi^2\hbar t}{2mL^2}\right) \\ &= \sqrt{\frac{2}{10 \times 10^{-9}}} \sin\left(\frac{3\pi x}{10 \times 10^{-9}}\right) \exp\left(-i \frac{3^2\pi^2\hbar t}{2 \cdot 9.109 \times 10^{-31} \cdot (10 \times 10^{-9})^2}\right) \\ &\approx 1.414 \times 10^3 \text{ m}^{-0.5} \sin(9.425 \times 10^{17} \text{ m}^{-1}x) \exp(-i \cdot 5.142 \times 10^{13} \text{ s}^{-1}t)\end{aligned}$$

Problem 2. An electron in the $n = 4$ state of a 5 nm wide infinite well makes a transition to the ground state, giving off energy in the form of a photon. What is the photon's wavelength?

Solution 2. By equation 1.11 from the formula sheet the energy of a particle in state n in an infinite well is

$$E = \frac{n^2\hbar^2}{8mL^2}$$

Here we're looking at ΔE of an electron between $n = 4$ and $n = 1$ so,

$$\Delta E = \frac{n^2\hbar^2}{8mL^2} \Big|_4^1 = \frac{15\hbar^2}{8m_e L^2}$$

$$\text{Then, } E = \frac{hc}{\lambda} \implies \lambda = hc \frac{8m_e(5 \text{ nm})^2}{15\hbar^2} = \frac{8cm_e(5 \text{ nm})^2}{15h} \approx 5500 \text{ nm}$$

Problem 3. What is the probability that a particle in the first excited ($n = 2$) state of an infinite well would be found in the middle third of the well? How does this compare with the classical expectation? Why?

Solution 3.

$$\begin{aligned}
&= \int_{\frac{L}{3}}^{\frac{2L}{3}} \left[\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \right]^2 dx = \frac{2}{L} \int_{\frac{L}{3}}^{\frac{2L}{3}} \sin^2\left(\frac{n\pi x}{L}\right) dx \\
&= \frac{2}{L} \left[\frac{x}{2} - \frac{\sin\left(\frac{2n\pi x}{L}\right)}{\frac{4n\pi}{L}} \right]_{\frac{L}{3}}^{\frac{2L}{3}} \quad (\text{From formula sheet}) \\
&= \frac{2}{L} \left[\frac{x}{2} - \frac{L}{4n\pi} \sin\left(\frac{2n\pi x}{L}\right) \right]_{\frac{L}{3}}^{\frac{2L}{3}} \\
&= \frac{2}{L} \left[\frac{\frac{2L}{3}}{2} - \frac{L}{4n\pi} \sin\left(\frac{2n\pi \frac{2L}{3}}{L}\right) - \frac{\frac{L}{3}}{2} + \frac{L}{4n\pi} \sin\left(\frac{2n\pi \frac{L}{3}}{L}\right) \right] \\
&= \frac{2}{3} - \frac{2}{6} + \frac{2}{4n\pi} \sin\left(\frac{2n\pi}{3}\right) - \frac{2}{4n\pi} \sin\left(\frac{4n\pi}{3}\right) \\
&= \frac{1}{3} + \frac{1}{2n\pi} \left[\sin\left(\frac{2n\pi}{3}\right) - \sin\left(\frac{4n\pi}{3}\right) \right] \approx 0.196
\end{aligned}$$

Classically I think we'd expect that the particle has an even probability density over the entire well and therefore the probability that it is inside one third of the well is $\frac{1}{3}$. The difference between these two arises from the fact that in the $n = 2$ state we have an antinode in the middle third which means that the probability is lesser than the outer two thirds.