Math 2120H: Assignment IV

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Problem 1. Evaluate $\int_C (xy + y + z)\, ds$ along the curve $\mathbf{r}(t) = 2t\mathbf{i} + t\mathbf{j} + (2 - 2t)\mathbf{k}$, $0 \leq t \leq 1$.

Solution 1.

$$\begin{aligned}\int_C (xy + y + z)\, ds &= \int_0^1 [(2t)(t) + t + 2 - 2t]\, |2\mathbf{i} + 1\mathbf{j} + (2 - 2t)\mathbf{k}|\; dt \\ &= \int_0^1 [(2t)(t) + t + 2 - 2t]\, |2\mathbf{i} + 1\mathbf{j} - 2\mathbf{k}|\; dt \\ &= \int_0^1 \left[2t^2 - t + 2\right] \sqrt{9}\, dt = \frac{13}{2}\end{aligned}$$

Problem 2. Find the mass of a thin wire lying along the curve $\mathbf{r}(t) = (\sqrt{2})t\mathbf{i} + (\sqrt{2})t\mathbf{j} + (4 - t^2)\mathbf{k}$, $0 \leq t \leq 1$, if the density is $\delta = 3t$.

Solution 2. The mass of an object with a continuous density function is given by $\int_C \delta(x, y, z)\, ds$ so,

$$\begin{aligned}\int_C \delta(x, y, z)\, ds &= \int_0^1 3t \left|(\sqrt{2})\mathbf{i} + (\sqrt{2})\mathbf{j} - 2t\mathbf{k}\right|\; dt \\ &= \int_0^1 3t\sqrt{2 + 2 + 4t^2}\, dt \\ &= \int_1^2 3t\sqrt{4 + 4t^2}\, dt \\ &= 3\int_0^1 \sqrt{u}\, du = 2^{5/2} - 2\end{aligned}$$

Problem 3. Find the line integral of $F = 3y\mathbf{i} + 2x\mathbf{j} + 4z\mathbf{k}$ over the path $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^4\mathbf{k}$, $0 \leq t \leq 1$.

Solution 3.

$$\begin{aligned}&= \int_0^1 F(\mathbf{r}(t))\mathbf{r}'(t)\, dt \\ &= \int_0^1 \left[3t^2\mathbf{i} + 2t\mathbf{j} + 4t^4\mathbf{k}\right] \left[\mathbf{i} + 2t\mathbf{j} + 4t^3\mathbf{k}\right]\; dt \\ &= \int_0^1 3t^2 + 4t^2 + 16t^7\mathbf{k}\, dt = \frac{13}{3}\end{aligned}$$

Problem 4. Find the flux of the fields $F = 2x\mathbf{i} + (x - y)\mathbf{j}$ across the circle $\mathbf{r}(t) = (a\cos t)\mathbf{i} + (a\sin t)\mathbf{j}$, $0 \leq t \leq 2\pi$.

Solution 4. The outward normal vector for $\mathbf{r}(t)$ is $\nabla\mathbf{r}(t) = -a\sin t\mathbf{i} + a\cos t\mathbf{j}$, for $a \neq 1$ this must be divided by a to unitize(? is that a word) it so,

$$\begin{aligned}&= \int_0^{2\pi} [-2\sin t\mathbf{i} + (-\sin t - \cos t)\mathbf{j}]\, [-\sin t\mathbf{i} + \cos t\mathbf{j}]\; dt \\ &= \int_0^{2\pi} -2\sin^2 t - \sin(t)\cos(t) - \cos^2 t\, dt \\ &= \int_0^{2\pi} \sin(t)\cos(t) - 1\, dt = -2\pi\end{aligned}$$