Physics 2700H: Assignment III

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Problem 1. Five kg of water at 25 °C is added to 10.0 kg of water at 85 °C. After the mixture has reached equilibrium, how much has entropy changed. (Assume no energy is exchanged between the water and its surroundings.)

Solution 1. The equilibrium temperature of the mixture can be determined using $Q = mc\Delta T$,

$$\begin{aligned} m_C c\Delta T_C &= -m_H c\Delta T_H \\ \implies m_C\left(T_f - T_{iC}\right) &= -m_H\left(T_f - T_{iH}\right) \\ \implies T_f &= \frac{m_H T_{iH} + m_C T_{iC}}{m_C + m_H} = 338.15\,\mathrm{K} \end{aligned}$$

Which means that $\Delta Q_C = 836400 = -Q_H$. To determine entropy change,

$$\begin{aligned} dS &= \frac{dQ}{T} \\ \implies \Delta S &= \int_{T_i}^{T_f} \frac{1}{T}\,dQ \end{aligned}$$

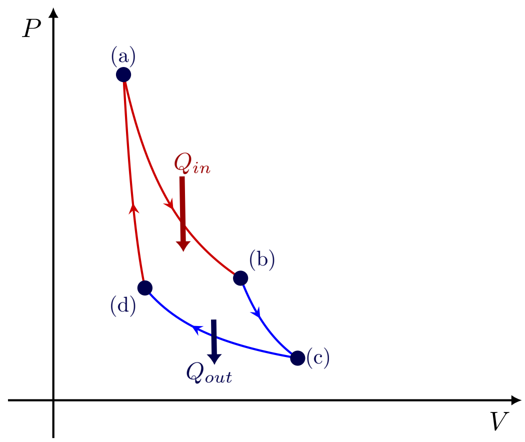
Where $Q = mc\Delta T \implies dQ = mcdT$ so

$$\begin{aligned} \Delta S &= mc\int_{T_i}^{T_f} \frac{1}{T}\,dT \\ &= mc\ln\left(\frac{T_f}{T_i}\right) \end{aligned}$$

The entropy change of the system then is $\Delta S_{sys} = \Delta S_H + \Delta S_C = c\left[m_H \ln\left(\frac{T_f}{T_{iH}}\right) + m_C \ln\left(\frac{T_f}{T_{iC}}\right)\right] \approx 229.46\,\mathrm{J\,K^{-1}}$ which is greater than zero as we would expect by the principle of increasing entropy.

Problem 2. The July 2023 Veritasium video about entropy, https://www.youtube.com/watch?v=DxL2HoqLbyA, introduces a Carnot engine within the first six minutes of the video. Draw on a PV diagram the cycle for this engine, with the bottom-right-most point labelled (a), and continue the cycle to points (b), (c) and (d). Identify which timestamps in the video correspond to the four points (a)... (d) and explain using a sentence per point why this is so.

Solution 2.



(a) 4:41 is when the hot block is brought into contact with the heat "aperture" and heat flows into the gas increasing temperature, pressure, and thereby driving an increase in volume.

(b) 4:53 the hot block is removed but the piston continues to climb and so volume increases and pressure decreases along an isotherm.

(c) 5:03 the cold block is brought into contact and the piston begins to move downwards with heat moving into the cold block, pressure increasing, and volume decreasing.

(d) 5:12 the cold block is removed and the has continues to be compressed, isothermally, meaning pressure increases and volume decreases.

Problem 3. One mole of helium gas is initially at $P_0 = 1.0\,\text{atm}$ and $T_0 = 273\,\text{K}$.

(a) Compute the entropy change if the gas is heated at constant pressure to temperaturFe $400\,\text{K}$.

(b) Starting again from the initial state (P_0, T_0), what is the entropy change if the gas expands isothermally to twice its original volume?

Solution 3.

(a)

$$\begin{aligned}\Delta S &= \int_{T_i}^{T_f} \frac{1}{T}\,dQ \\ &= \int_{T_i}^{T_f} \frac{nC_P}{T}\,dT\end{aligned}$$

Where $C_P = C_V + nR$ where C_V for an ideal monatomic gas is $\frac{3}{2}R$ so for one mole we have,

$$\begin{aligned}\Delta S &= \int_{T_i}^{T_f} \frac{\frac{5}{2}R}{T}\,dT \\ &= \frac{5}{2}R\ln\left(\frac{T_f}{T_i}\right) = 7.94\,\text{J}\,\text{K}^{-1}\end{aligned}$$

(b) Again,

$$\Delta S = \int_{V_i}^{2V_i} \frac{1}{T}\,dQ$$

Here we use the first law, $dU = dQ + dW$ where dU is zero as T is held constant and $dW = -PdV$ so $dQ = PdV$ so,

$$\begin{aligned}\Delta S &= \int_{V_i}^{2V_i} \frac{P}{T}\,dV \\ &= \int_{V_i}^{2V_i} \frac{nR\cancel{T}}{V\cancel{T}}\,dV \\ &= \int_{V_i}^{2V_i} \frac{R}{V}\,dV \\ &= R\ln\left(\frac{2V_i}{V_i}\right) \approx 5.76\,\text{J}\,\text{K}^{-1}\end{aligned}$$