

# Physics 2610H: Assignment II

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**Problem 1.** Write out the total wave function  $\Psi(x, t)$  for an electron in the  $n = 3$  state of a 10 nm wide infinite well. Other than the symbols  $x$  and  $t$ , the function should include only numerical values.

**Solution 1.** We can split  $\Psi(x, t) = \psi(x)\phi(t) = \psi(x)\exp\left(\frac{-iEt}{\hbar}\right)$  then for a particle in an infinite potential well  $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\sqrt{\frac{2mE}{\hbar^2}}x\right)$  and  $E = \frac{n^2\pi^2\hbar^2}{2mL^2}$  So,

$$\begin{aligned}\Psi(x, t) &= \sqrt{\frac{2}{L}} \sin\left(\sqrt{\frac{2m \cdot \frac{n^2\pi^2\hbar^2}{2mL^2}}{\hbar^2}}x\right) \exp\left(\frac{-i \cdot \frac{n^2\pi^2\hbar^2}{2mL^2}t}{\hbar}\right) \\ &= \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \exp\left(-i \frac{n^2\pi^2\hbar t}{2mL^2}\right) \\ &= \sqrt{\frac{2}{10 \times 10^{-9}}} \sin\left(\frac{3\pi x}{10 \times 10^{-9}}\right) \exp\left(-i \frac{3^2\pi^2\hbar t}{2 \cdot 9.109 \times 10^{-31} \cdot (10 \times 10^{-9})^2}\right) \\ &\approx 1.414 \times 10^3 \text{ m}^{-0.5} \sin(9.425 \times 10^{17} \text{ m}^{-1}x) \exp(-i \cdot 5.142 \times 10^{13} \text{ s}^{-1})\end{aligned}$$

**Problem 2.** An electron in the  $n = 4$  state of a 5 nm wide infinite well makes a transition to the ground state, giving off energy in the form of a photon. What is the photon's wavelength?

**Solution 2.** By equation 1.11 from the formula sheet the energy of a particle of mass  $m$  in state  $n$  in an infinite well of width  $L$  is

$$E = \frac{n^2\hbar^2}{8mL^2}$$

Here we're looking at  $\Delta E$  of an electron between  $n = 4$  and  $n = 1$  so,

$$\Delta E = \frac{n^2\hbar^2}{8mL^2} \Big|_4^1 = \frac{15\hbar^2}{8m_e L^2}$$

$$\text{Then, } E = \frac{hc}{\lambda} \implies \lambda = hc \frac{8m_e(5 \text{ nm})^2}{15\hbar^2} = \frac{8cm_e(5 \text{ nm})^2}{15h} \approx 5500 \text{ nm}$$

**Problem 3.** What is the probability that a particle in the first excited ( $n = 2$ ) state of an infinite well would be found in the middle third of the well? How does this compare with the classical expectation? Why?

**Solution 3.** Because this is a particle in an infinite well the probability is the integral between the two given points of  $\left| \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \right|^2$  so,

$$\begin{aligned}
 &= \int_{\frac{L}{3}}^{\frac{2L}{3}} \left[ \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \right]^2 dx = \frac{2}{L} \int_{\frac{L}{3}}^{\frac{2L}{3}} \sin^2\left(\frac{n\pi x}{L}\right) dx \\
 &= \frac{2}{L} \left[ \frac{x}{2} - \frac{\sin\left(\frac{2n\pi x}{L}\right)}{\frac{4n\pi}{L}} \right]_{\frac{L}{3}}^{\frac{2L}{3}} \quad (\text{From formula sheet}) \\
 &= \frac{2}{L} \left[ \frac{x}{2} - \frac{L}{4n\pi} \sin\left(\frac{2n\pi x}{L}\right) \right]_{\frac{L}{3}}^{\frac{2L}{3}} \\
 &= \frac{2}{L} \left[ \frac{\frac{2L}{3}}{2} - \frac{L}{4n\pi} \sin\left(\frac{2n\pi \frac{2L}{3}}{L}\right) - \frac{\frac{L}{3}}{2} + \frac{L}{4n\pi} \sin\left(\frac{2n\pi \frac{L}{3}}{L}\right) \right] \\
 &= \frac{2}{3} - \frac{2}{6} + \frac{2}{4n\pi} \sin\left(\frac{2n\pi}{3}\right) - \frac{2}{4n\pi} \sin\left(\frac{4n\pi}{3}\right) \\
 &= \frac{1}{3} + \frac{1}{2n\pi} \left[ \sin\left(\frac{2n\pi}{3}\right) - \sin\left(\frac{4n\pi}{3}\right) \right] \approx 0.196
 \end{aligned}$$

Classically I think we'd expect that the particle has an even probability density over the entire well and therefore the probability that it is inside one third of the well is  $\frac{1}{3}$ . The difference between these two arises from the fact that in the  $n = 2$  state we have an antinode in the middle third which means that the probability is lesser than the outer two thirds.