Physics 3200Y: Assignment I

Jeremy Favro (0805980)
Trent University, Peterborough, ON, Canada

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Problem 1. Let $\boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}'$ be the separation between $\mathbf{r}$ and $\mathbf{r}'$, where $\mathbf{r}' = (x', y', z')$ is a fixed point and $\mathbf{r} = (x, y, z)$. Let $\mathscr{r} = |\boldsymbol{\mathscr{r}}|$ be the magnitude of the separation.

(a) Show that $\nabla\left(\mathscr{r}^2\right) = 2\boldsymbol{\mathscr{r}}$.

(b) Show that $\nabla \exp\left(\vec{k} \cdot \vec{\boldsymbol{\mathscr{r}}}\right) = \vec{k} \exp\left(\vec{k} \cdot \vec{\boldsymbol{\mathscr{r}}}\right)$, where $\vec{k}$ is a vector constant.

(c) Show that $\nabla \exp\left(k\mathscr{r}\right) = k\hat{\boldsymbol{\mathscr{r}}} \exp\left(k\mathscr{r}\right)$.

(d) Show that $\nabla\left(\mathscr{r}^{-1}\right) = -\hat{\boldsymbol{\mathscr{r}}}/\mathscr{r}^2$.

Solution 1.

(a) *Proof.*

$$
\begin{aligned}
&= \nabla\left(\mathscr{r}^2\right) \\
&= \left[\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right] \sqrt{\left(x - x'\right)^2 + \left(y - y'\right)^2 + \left(z - z'\right)^2}^{\,2} \\
&= \left[\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right] \left[\left(x - x'\right)^2 + \left(y - y'\right)^2 + \left(z - z'\right)^2\right] \\
&= \frac{\partial}{\partial x}\left(x - x'\right)^2\hat{x} + \frac{\partial}{\partial y}\left(y - y'\right)^2\hat{y} + \frac{\partial}{\partial z}\left(z - z'\right)^2\hat{z} \quad \text{Note[1]} \\
&= 2\left(x - x'\right)\hat{x} + 2\left(y - y'\right)\hat{y} + 2\left(z - z'\right)\hat{z} \quad \text{By chain rule} \\
&= 2(\left(x - x'\right), \left(y - y'\right), \left(z - z'\right)) = 2\boldsymbol{\mathscr{r}}
\end{aligned}
$$

□

(b) *Proof.*

$$
\begin{aligned}
&= \nabla \exp\left(\vec{k} \cdot \vec{\boldsymbol{\mathscr{r}}}\right) \\
&= \nabla \exp\left(k_x(x - x') + k_y(y - y') + k_z(z - z')\right) \\
&= \nabla \exp\left(k_x(x - x')\right) \exp\left(k_y(y - y')\right) \exp\left(k_z(z - z')\right)
\end{aligned}
$$

□

(c) Show that $\nabla \exp\left(k\mathscr{r}\right) = k\hat{\boldsymbol{\mathscr{r}}} \exp\left(k\mathscr{r}\right)$.

(d) Show that $\nabla\left(\mathscr{r}^{-1}\right) = -\hat{\boldsymbol{\mathscr{r}}}/\mathscr{r}^2$.

[1] The partials kill the terms that don't contain their variable of differentiation, omitted for brevity