Physics 3200Y: Assignment I

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Problem 1. A hollow cone has a constant surface charge density σ. The cone's vertex is at $z = 0$ and its axis lies along the positive z-axis. It has a height h, which is also the radius of the cone at its widest end (the top). You are asked to find the potential difference $V(h) - V(0)$ between the top (on the cone's axis) and the vertex.

(a) Draw a picture that defines $\vec{r}$, $\vec{r}'$, $\imath$, etc.

(b) Discuss how you will parameterize your surface and find expressions for $\vec{r}'$ and da' in terms of your parameters.

(c) Write out and solve the integrals for $V(h)$ and $V(0)$ to find the final answer. The integral for $V(0)$ is easy to do, and I have some hints for the integrals for $V(h)$. First, I found it helpful to make the substitution $z' = h/2 + t$, and then with a little bit of work the integral simplifies to a constant times something of the form

$$\int_{-a}^{a} \frac{dt}{\sqrt{a^2 + t^2}} = 2\sinh^{-1}(\frac{1}{\sqrt{2}})$$

Show your work. Don't simply look up the integral!

Solution 1.

Problem 2. Consider the infamous triple-plate capacitor (I just made that up), consisting of three parallel plates. Let the middle plate lie in the xy plane, let the top plate lie a distance d_1 above it and the bottom plate lie a distance d_2 below it. Assume that the capacitor charges in such a way that the top plate has a 2D charge density σ, while the other two plates have charge densities$-\sigma/2$; treat the plates as infinite.

(a) Find the electric field everywhere along the z-axis, and the voltage differences between the different plates.

(b) Find the energy per unit area required to charge up the system by incrementally moving amounts of charge dQ from the lower two plates to the upper plate and calculating the work done for each increment. Describe carefully the sequence of steps in each case, since that is what you are being graded on.

(c) Show that you get the same answer for the work by integrating the electric field over all space

Solution 2.

Problem 3. A uniformly charged solid cylinder has length L, radius R, and charge density ρ.

(a) Find the potential on the axis of the cylinder as a function of z, the distance along the central axis from the centre of the cylinder.

(b) Does the solution to part (a) give you enough information to solve for the electric field along the z-axis? Why or why not? Hint: Look at the gradient operator in cylindrical coordinates. What assumptions would you have to make to calculate $\vec{E}$? Are they reasonable?

Solution 3.