Physics 2700H: Assignment I

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Problem 1. An ideal gas undergoes the following reversible cycle:

(i) an isobaric expansion from the state (P1,V1) to the state (P1,V2)

(ii) an isochoric reduction in pressure to the state (P2,V2)

(iii) an isobaric reduction in volume to the state (P2,V1)

(iv) an isochoric increase in pressure back to the original state (P1,V1)

(a) What work is done on the gas in this cycle?

(b) If $P_1 = 3.0\text{atm}$, $P_2 = 1.0\text{atm}$, $V_1 = 1.0\text{L}$ and $V_2 = 2.0\text{L}$, how much work is done on the gas in traversing the cycle 100 times?

Solution 1.

(a)

$$\begin{tikzcd}
(P_1,V_1) \arrow[r, "\text{Isobaric}", "W=-P(V_2-V_1)"'] & (P_1,V_2) \arrow[r, "W=0", "\text{Isochoric}"'] & (P_2,V_2) \arrow[r, "\text{Isobaric}", "W=-P(V_1-V_2)"'] & (P_2,V_1) \arrow[lll, bend right, "\text{Isochoric}", "W=0"']
\end{tikzcd}$$

One cycle of work is therefore $-P_1(V_2 - V_1) - P_2(V_1 - V_2)$

(b) One hundred cycles of work is $100\left[(P_2 - P_1)(V_2 - V_1)\right] = -2.03 \times 10^4$ J

Problem 2. A hypothetical substance has an isothermal compressibility $\kappa = \frac{a}{v}$ and volume expansion coefficient $\beta = \frac{2bT}{v}$, where a and b are constants and v is the molar volume. Show that the equation of state is

$$v - bT^2 + aP = \text{constant}$$

Solution 2.

$$\begin{aligned}
\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P &= \frac{2bT}{v} \\
\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P &= \frac{2bnT}{V} \\
\left(\frac{\partial V}{\partial T}\right)_P &= 2bnT
\end{aligned}$$

$$\begin{aligned}
-V\left(\frac{\partial P}{\partial V}\right)_T &= \frac{1}{\kappa} \\
-V\left(\frac{\partial P}{\partial V}\right)_T &= \frac{V}{na} \\
-\left(\frac{\partial P}{\partial V}\right)_T &= \frac{1}{na}
\end{aligned}$$

Problem 3. Researchers from the Universities of Maryland and Vermont found that adults' daily energy needs at rest (their resting metabolic rate, RMR) is closely related to the mass of their bones & muscles & organs. By selecting any person from this data set, find:

(a) Their total bone/muscle/organs mass and their RMR in $\mathrm{kcal\,d^{-1}}$.

(b) Their minimum daily energy needs written in J, in kJ, and in MJ.

(c) Their heat output at rest in watts (to high accuracy all energy we use at rest ends up emitted as heat).

Solution 3.

(a) There are a few individuals at around $1500\,\mathrm{kcal\,d^{-1}}$ and 60kg so I will work with an ideal individual at exactly these values.

(b) $1500\,\mathrm{kcal\,d^{-1}} = 6276000\mathrm{J} = 6276\mathrm{kJ} = 6.276\mathrm{MJ}$

(c) $\dfrac{6276000\mathrm{J}}{86400\mathrm{s}} = 72.64\mathrm{W}$

Problem 4. A gas is contained in a cylinder fitted with a frictionless piston and is taken from state a to state b along the path acb shown in Figure 3.8. 80J of heat flows into the system, and the system does 30J of work.

(a) If instead the work done by the gas system is only 10J along adb, how much heat flows into the system?

(b) When the system is returned from b to a along the curved path, the work done on the system is 20J. What is the heat transfer?

(c) If $U_a = 0$ and $U_d = 40\mathrm{J}$, find the heat absorbed in the processes ad and db.

Solution 4.

(a) Net system energy must be conserved so if work decreases by 20J heat must increase by 20J meaning that the new heat flow must be 110J

(b) By the same logic as above, the heat transfer is 100J

(c) If $U_a = 0$ and $U_d = 40\mathrm{J}$, find the heat absorbed in the processes ad and db.

Problem 5. You have a pure metal that is unknown except for the fact that it happens to be among those listed in Table 3.1. A 36.7g piece of this metal at 50.0°C is placed in a calorimeter containing 150g of water, initially at 10°C. The final equilibrium temperature in the calorimeter is 2.0°C. What is the metal?

Solution 5.

Problem 6. A gas with adiabatic exponent γ is compressed adiabatically from an initial state (P_i, V_i) to final state (P_f, V_f).

(a) Show that the work done in this process is $W = \dfrac{P_i V_i}{\gamma - 1}\left[\left(\dfrac{V_i}{V_f}\right)^{\gamma-1} - 1\right]$

(b) Evaluate the result numerically for one mole of helium gas initially at $P = 1.0\mathrm{atm}$ and $T = 300\mathrm{K}$ compressed to half its initial volume.

(c) Compute the work done in an isothermal compression from the same initial point to half the initial volume. Explain the difference between the numerical results for work done in adiabatic and isothermal compression.

Solution 6.