Physics 2700H: Assignment I

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Problem 1. An ideal gas undergoes the following reversible cycle:

(i) an isobaric expansion from the state (P1,V1) to the state (P1,V2)

(ii) an isochoric reduction in pressure to the state (P2,V2)

(iii) an isobaric reduction in volume to the state (P2,V1)

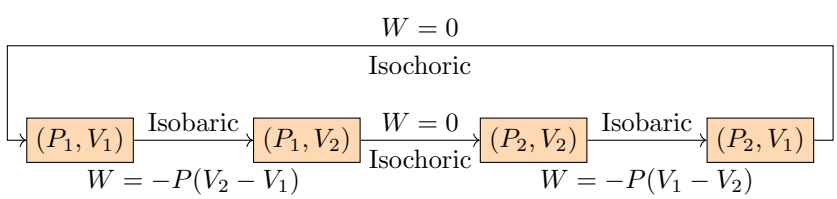
(iv) an isochoric increase in pressure back to the original state (P1,V1)

(a) What work is done on the gas in this cycle?

(b) If $P_1 = 3.0\text{atm}$, $P_2 = 1.0\text{atm}$, $V_1 = 1.0\text{L}$ and $V_2 = 2.0\text{L}$, how much work is done on the gas in traversing the cycle 100 times?

Solution 1.

(a)



One cycle of work is therefore $-P_1(V_2 - V_1) - P_2(V_1 - V_2)$

(b) One hundred cycles of work is $100\left[(P_2 - P_1)(V_2 - V_1)\right] = -2.03 \times 10^4\,\text{J}$

Problem 2. A hypothetical substance has an isothermal compressibility $\kappa = \frac{a}{v}$ and volume expansion coefficient $\beta = \frac{2bT}{v}$, where a and b are constants and v is the molar volume. Show that the equation of state is

$$v - bT^2 + aP = \text{constant}$$

Solution 2. From equations 2.1 and 2.2 on the forumla sheet, $\beta = \dfrac{1}{V}\dfrac{\partial V}{\partial T}_P = \dfrac{2bT}{v}$ and $\kappa = -\dfrac{1}{V}\dfrac{\partial V}{\partial P}_T = \dfrac{a}{v}$ then,

$$\begin{aligned}
dV &= \frac{\partial V}{\partial T}dT + \frac{\partial V}{\partial P}dP \rightsquigarrow \text{(chain rule)} \\
dV &= \frac{2bVT}{v}dT - \frac{Va}{v}dP = 2bnTdT - andP \\
V &= bnT^2 - anP + C \\
\frac{V}{n} &= v = bT^2 - aP + C \\
C &= aP - bT^2 + v = v - bT^2 + aP
\end{aligned}$$

Note that C changes a bit here sign wise and gets divided by n but it remains constant so I have left it as C.

Problem 3. Researchers from the Universities of Maryland and Vermont found that adults' daily energy needs at rest (their resting metabolic rate, RMR) is closely related to the mass of their bones & muscles & organs. By selecting any person from this data set, find:

(a) Their total bone/muscle/organs mass and their RMR in $\mathrm{kcal\,d^{-1}}$.

(b) Their minimum daily energy needs written in J, in kJ, and in MJ.

(c) Their heat output at rest in watts (to high accuracy all energy we use at rest ends up emitted as heat).

Solution 3.

(a) There are a few individuals at around $1500\,\mathrm{kcal\,d^{-1}}$ and 60kg so I will work with an ideal individual at exactly these values.

(b) $1500\,\mathrm{kcal\,d^{-1}} = 6276000\mathrm{J} = 6276\mathrm{kJ} = 6.276\mathrm{MJ}$

(c) $\dfrac{6276000\mathrm{J}}{86400\mathrm{s}} = 72.64\mathrm{W}$

Problem 4. A gas is contained in a cylinder fitted with a frictionless piston and is taken from state a to state b along the path acb shown in Figure 3.8. 80J of heat flows into the system, and the system does 30J of work.

(a) If instead the work done by the gas system is only 10J along adb, how much heat flows into the system?

(b) When the system is returned from b to a along the curved path, the work done on the system is 20J. What is the heat transfer?

(c) If $U_a = 0$ and $U_d = 40\mathrm{J}$, find the heat absorbed in the processes ad and db.

Solution 4.

(a) Net system energy change must be conserved so if work **done by the system** decreases by 20J heat **into the system** must decrease by 20J meaning that the new heat flow must be 60J

(b) We've kept the change in energy of the system as 50J by the first law of thermodynamics for the first two processes. Because work is path-independent, the change in energy along the curved path is the same as the change in energy along acb but with reversed sign as we are going backwards. So, $\Delta U = -50\mathrm{J} = Q + 20\mathrm{J} \implies Q = -70\mathrm{J}$.

(c) For ad, $\Delta U = 40\mathrm{J}$, then for db we know that the total change must be 50J in process ab, so $50\mathrm{J} = 40\mathrm{J} + \Delta U_{db} \implies \Delta U_{db} = 10\mathrm{J}$

Problem 5. You have a pure metal that is unknown except for the fact that it happens to be among those listed in Table 3.1. A 36.7g piece of this metal at 50.0°C is placed in a calorimeter containing 150g of water, initially at 10°C. The final equilibrium temperature in the calorimeter is 12.0°C. What is the metal?

Solution 5. Heat is the same between the water and the metal at equilibrium because they are in (presumably) perfect thermal contact so, with subscript m denoting metal and w denoting water,

$$m_m c_m \Delta T_m = m_w c_w \Delta T_w$$
$$c_m = \frac{m_w c_w \Delta T_w}{m_m \Delta T_m} \approx -900\mathrm{J}$$

So, according to the table this metal is likely aluminum. I am not sure about the sign but it is kind of irrelevant I think, mostly because logically if it takes −900J (J/kg°C, really) to cool something it will logically take 900J to heat it up.

Problem 6. A gas with adiabatic exponent γ is compressed adiabatically from an initial state (P_i, V_i) to final state (P_f, V_f).

(a) Show that the work done in this process is $W = \dfrac{P_i V_i}{\gamma - 1}\left[\left(\dfrac{V_i}{V_f}\right)^{\gamma-1} - 1\right]$

(b) Evaluate the result numerically for one mole of helium gas initially at $P = 1.0\text{atm}$ and $T = 300\text{K}$ compressed to half its initial volume.

(c) Compute the work done in an isothermal compression from the same initial point to half the initial volume. Explain the difference between the numerical results for work done in adiabatic and isothermal compression.

Solution 6.

(a) Starting with $dW = -PdV$ we can recognize that because this process is adiabatic $P = \dfrac{P_i V_i^\gamma}{V^\gamma} \left(= \dfrac{P_f V_f^\gamma}{V^\gamma} \right)$

$$
\begin{aligned}
dW &= -PdV \\
W &= -\int_{V_i}^{V_f} \frac{P_i V_i^\gamma}{V^\gamma}\, dV \rightsquigarrow V_n^\gamma \text{ is constant} \\
W &= -P_i V_i^\gamma \int_{V_i}^{V_f} V^{-\gamma}\, dV \\
W &= -\frac{P_i V_i^\gamma}{1-\gamma}\, V^{1-\gamma}\Big|_{V_i}^{V_f} \rightsquigarrow \text{ assume } \gamma \neq 1 \\
W &= -\frac{P_i V_i^\gamma}{1-\gamma} \left[V_f^{1-\gamma} - V_i^{1-\gamma} \right] \\
W &= \frac{P_i V_i^{\cancel{\gamma}} V_i^{1-\cancel{\gamma}}}{\gamma - 1} \left[\left(\frac{V_f}{V_i} \right)^{1-\gamma} - 1 \right] \\
W &= \frac{P_i V_i}{\gamma - 1} \left[\left(\frac{V_i}{V_f} \right)^{\gamma-1} - 1 \right]
\end{aligned}
$$

(b) $\gamma = 1.66$ for approximately room temperature Helium[1]

$$V = \frac{nRT}{P} \implies W = \frac{nRT}{\gamma - 1} \left[2^{\gamma-1} - 1 \right] = 2192\text{J}$$

(c) $dW = -PdV$ and $P = \dfrac{nRT}{V} = \dfrac{RT}{V}$ so

$$
\begin{aligned}
W &= -RT \int_{V_i}^{V_f} \frac{1}{V} dV \\
&= -RT \left[\ln V_f - \ln V_i \right] \\
&= -RT \ln \left(\frac{V_f}{V_i} \right) = -RT \ln \left(\frac{V_i}{2V_i} \right) = RT \ln 2 \approx 1729\text{J}
\end{aligned}
$$

Because ΔU is the same for an adiabat as it is for an isotherm, $\Delta U = Q + W$ holds for both. In the case of an adiabat, $Q = 0$ by definition so W increases (compared to an isotherm between the same two states) to "take up the slack".

[1] White, Frank M. (1998). Appendix A. Fluid Mechanics (4th ed.) (pp. 769-773). McGraw Hill.