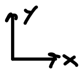
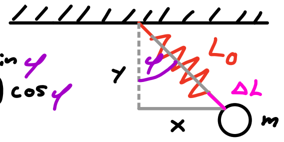


$x = (L_0 + \Delta L)\sin\varphi$
$y = -(L_0 + \Delta L)\cos\varphi$



L_0 = const baselength of spring
ΔL = length of springdisplacement

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (L_0+\Delta L)\sin\varphi \\ -(L_0+\Delta L)\cos\varphi \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \dot{\Delta L}\sin\varphi + (L_0+\Delta L)\cos(\varphi)\dot{\varphi} \\ -\dot{\Delta L}\cos\varphi + (L_0+\Delta L)\sin(\varphi)\dot{\varphi} \end{pmatrix}$$

$$T = \frac{m}{2}\dot{\underline{x}}^2 = \frac{m}{2}(\dot{x}^2 + \dot{y}^2)$$

$$\dot{x}^2 = (\dot{\Delta L}\sin\varphi + (L_0+\Delta L)\cos\varphi\,\dot{\varphi})^2$$
$$= \dot{\Delta L}^2\sin^2\varphi + (L_0+\Delta L)^2\cos^2\varphi\,\dot{\varphi}^2 + 2\dot{\Delta L}\dot{\varphi}(L_0+\Delta L)\sin\varphi\cos\varphi$$

$$\dot{y}^2 = \dot{\Delta L}^2\cos^2\varphi + (L_0+\Delta L)^2\sin^2\varphi\,\dot{\varphi}^2 - 2\dot{\Delta L}\dot{\varphi}(L_0+\Delta L)\sin\varphi\cos\varphi$$

$$\dot{x}^2 + \dot{y}^2 = \dot{\Delta L}^2\sin^2\varphi + (L_0+\Delta L)^2\cos^2\varphi\,\dot{\varphi}^2 + \dot{\Delta L}^2\cos^2\varphi + (L_0+\Delta L)^2\sin^2\varphi\,\dot{\varphi}^2$$
$$= \dot{\Delta L}^2(\sin^2\varphi + \cos^2\varphi) + \dot{\varphi}^2(L_0+\Delta L)^2[\cos^2\varphi + \sin^2\varphi]$$

$$\cos^2\varphi + \sin^2\varphi = 1$$

$$\Rightarrow \dot{\Delta L}^2 + \dot{\varphi}^2(L_0+\Delta L)^2$$

$$\Rightarrow T = \frac{m}{2}(\dot{\Delta L}^2 + \dot{\varphi}^2(L_0+\Delta L)^2)$$

$$V = mgy + \frac{1}{2}k\Delta L^2 = -mg(L_0+\Delta L)\cos\varphi + \frac{1}{2}k\Delta L^2$$

$\Rightarrow$ lagrangian:

$$L(\Delta L, \dot{\Delta L}, \varphi, \dot{\varphi}) = T - V = \frac{m}{2}(\dot{\Delta L}^2 + \dot{\varphi}^2(L_0+\Delta L)^2) + mg(L_0+\Delta L)\cos\varphi - \frac{1}{2}k\Delta L^2$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q} = 0$$

φ:
$$\frac{\partial L}{\partial \varphi} = -mg(L_0+\Delta L)\sin\varphi$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\varphi}} = \frac{d}{dt}\left(m\dot{\varphi}(L_0+\Delta L)^2\right) = m\ddot{\varphi}(L_0+\Delta L)^2 + 2m\dot{\varphi}(L_0+\Delta L)\dot{\Delta L}$$

$$\Rightarrow m\ddot{\varphi}(L_0+\Delta L)^2 + 2m\dot{\varphi}\dot{\Delta L}(L_0+\Delta L) + mg(L_0+\Delta L)\sin\varphi = 0$$

$$\Rightarrow \ddot{\varphi} = -\frac{2\dot{\varphi}\dot{\Delta L}}{(L_0+\Delta L)} - \frac{g}{(L_0+\Delta L)}\sin\varphi$$

ΔL: $\quad \frac{d}{dt}\frac{\partial L}{\partial \dot{\Delta L}} - \frac{\partial L}{\partial \Delta L} = 0$

$$\frac{\partial L}{\partial \Delta L} = m\dot{\varphi}^2 (L_0 + \Delta L) + mg\cos\varphi - K\Delta L$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\Delta L}} = \frac{d}{dt}\left(m\dot{\Delta L} \right) = m\ddot{\Delta L}$$

$$\Rightarrow \quad m\ddot{\Delta L} - m\dot{\varphi}^2 (L_0 + \Delta L) - mg\cos\varphi + K\Delta L = 0$$

$$\Rightarrow \quad \ddot{\Delta L} = \dot{\varphi}^2 (L_0 + \Delta L) + g\cos\varphi - \frac{k}{m}\Delta L$$

$$\boxed{\begin{aligned} \ddot{\varphi} &= -\frac{2\dot{\varphi}\dot{\Delta L}}{(L_0 + \Delta L)} - \frac{g}{(L_0 + \Delta L)}\sin\varphi \\ \ddot{\Delta L} &= \dot{\varphi}^2 (L_0 + \Delta L) + g\cos\varphi - \frac{k}{m}\Delta L \end{aligned}}$$