Stochastic Processes with Application to Finance

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Abstract

Knowledge about stochastic processes is the foundation needed in order to explore the market. These are used to model asset dynamics and their properties allow us to build model serving several purposes. One of these is pricing. In this thesis we will explore some types of stochastic processes, the theory on which they are based on and some application related to finance. More precisely, we will see diffusion and jump diffusion processes, what makes them palatable for us and how we can play with them to perform different tasks. The focus will be on the theory and the steps needed in order to obtain a valid system to price options. After some introductory, but necessary, topics we will see how pricing of barrier options can be carried out having the process at hand, and the legitimacy of this procedure.

In probability theory and related fields, a stochastic or **random process** is a mathematical object usually defined as a family of random variables in a probability space, where the index of the family often has the interpretation of time. Stochastic processes are widely used as mathematical models of systems and phenomena that appear to vary in a random manner. Examples include the growth of a bacterial population, an electrical current fluctuating due to thermal noise, or the movement of a gas molecule.

Asset prices are quantities which are governed by uncertainty. Models to try to predict and forecast prices have been developed with more or less success, but the bottom line is that there is no possibility to do such thing. A reasonable starting point, tough, is to try to study the behaviour of such *quantities in the most general form possible, backing up the treatment with solid mathematical ratiocination. This is the purpose of the theory of stochastic process in relation to finance. The need to identify and shape the quantities we are interested in, goes beyond the mere prediction aim, but rather an exploratory rationale. The idea of using stochastic processes to model asset price is an old one, even if the big leaps forward achieved on this matter are rather recent.*

The reason why is quite straightforward: stochastic processes describe the evolution, over time in our case, of a random quantity, which, in turn, give rise to aleatory dynamics. Once the theory for the treatment is established, we can start to operate on them, within the limits of it. We will mainly make use of the theory in order to arrive to a fair price for a derivative instrument, which is dependent from a stochastic process itself. The argument could be of different nature too. For example, how to perfectly hedge a position in an option or with an option might be the logical next step.

Financial institutions use stochastic models to assess the risk of their portfolios by simulating a wide range of possible future scenarios. This allows them to estimate Value at Risk (VaR) and Conditional Value at Risk (CVaR), which are crucial metrics for understanding potential losses under adverse market conditions. By employing stochastic processes, risk managers can better prepare for extreme events and make more informed decisions about capital allocation and risk mitigation strategies.

Another significant application is in the field of asset pricing. Stochastic models enable analysts to capture the dynamic behavior of asset prices, which is essential for valuing complex financial instruments. For instance, the Cox-Ross-Rubinstein binomial model uses a discrete-time stochastic process to price options by simulating different paths an asset’s price might take over time. This model provides a flexible and intuitive approach to option pricing, accommodating various market conditions and allowing for the incorporation of dividends and other factors. The ability to model the probabilistic nature of asset prices enhances the accuracy of pricing and helps investors identify mispriced securities, thereby uncovering arbitrage opportunities.