1.5

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- for something to be one-to-one, every value in the input should match a unique value in the range
  - ↳ never takes the same value twice
    $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$
  - ↳ use the horizontal line test

- The inverse function: has to be one-to-one
  ∴
  the inverse function

$$f^{-1}(x) = y \iff f(y) = x$$

Range ⟷ Domain
Domain ⟷ Range

they're switched

Cancellation equations:

$f^{-1}(f(x)) = x$ for every x in A

$f(f^{-1}(x)) = x$ for every x in B

- how to find the inverse function of a one-to-one function:

① write $y = f(x)$
② solve the equation for x in terms of y (if possible)
③ interchange x & y ∴
$y = f^{-1}(x)$

- The graph of f^{-1} is gotten by reflecting the graph of f about the line $\underline{\underline{y=x}}$

- Logarithmic functions

related to
$f^{-1}(x) = y \iff f(y) = x$

$$\log_b x = y \iff b^y = x$$

$\log_b(b^x) = x$ for every $x \in \mathbb{R}$

$b^{\log_b x} = x$ for every $x > 0$

- Laws of logs:

① $\log_b(xy) = \log_b x + \log_b y$

② $\log_b(x/y) = \log_b x - \log_b y$

③ $\log_b(x^r) = r\log_b x$ (when r is any real number)

- natural logs:

$\log_e x = \ln x$

$\ln x = y \iff e^y = x$

$\ln(e^x) = x$ $x \in \mathbb{R}$
$e^{\ln x} = x$ $x > 0$

$\ln e = 1$

↳ cancel out

- change of base formula:

$$\log_b x = \frac{\ln x}{\ln b}$$

- inverse trigonometric functions:

$\sin^{-1} x = y \iff \sin y = x$ & $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(cancellation holds)

$\cos^{-1} x = y \iff \cos y = x$ & $0 \leq y \leq \pi$

(cancellation holds)

$\tan^{-1}(x) = y \iff \tan(y) = x$ & $-\frac{\pi}{2} < y < \frac{\pi}{2}$

The rest:

$y = \csc^{-1} x\ (|x| \geq 1) \iff \csc y = x$ & $y \in (0, \pi/2] \cup (\pi, 3\pi/2]$

$y = \sec^{-1} x\ (|x| \geq 1) \iff \sec y = x$ & $y \in [0, \pi/2) \cup [\pi, 3\pi/2)$

$y = \cot^{-1} x\ (x \in \mathbb{R}) \iff \cot y = x$ & $y \in (0, \pi)$