2.4: The Precise Definition of a Limit

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· Motivation:

$$f(x) = \begin{cases} 2x - 1 & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$$

$\lim_{x \to 3} f(x) = 5$ (draw a graph)

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Q: how close to 3 does x have to be so that f(x) differs from 5 by less than 0.1?

distance ① $|x-3|$ ← a

distance ② $|f(x)-5|$ ← L

$|f(x)-5| < 0.1$ if $|x-3| < \delta$ but $x \neq 3$

$0.1 \to \varepsilon$

need to find δ

→ notice:

$|f(x)-5| = |(2x-1)-5|$

$= |2x-6|$

$= 2|x-3|$

$\therefore$

$2|x-3| < 0.1$

$|x-3| < \boxed{0.05}$

$0.05 \to \delta$

$0 < |x-3| < 0.05$

→ we can get a general formula for this & extend it

$|f(x)-5| < \varepsilon$ if $0 < |x-3| < \delta = \frac{\varepsilon}{2}$

· Precise definition of a limit:

$\lim_{x \to a} f(x) = L$

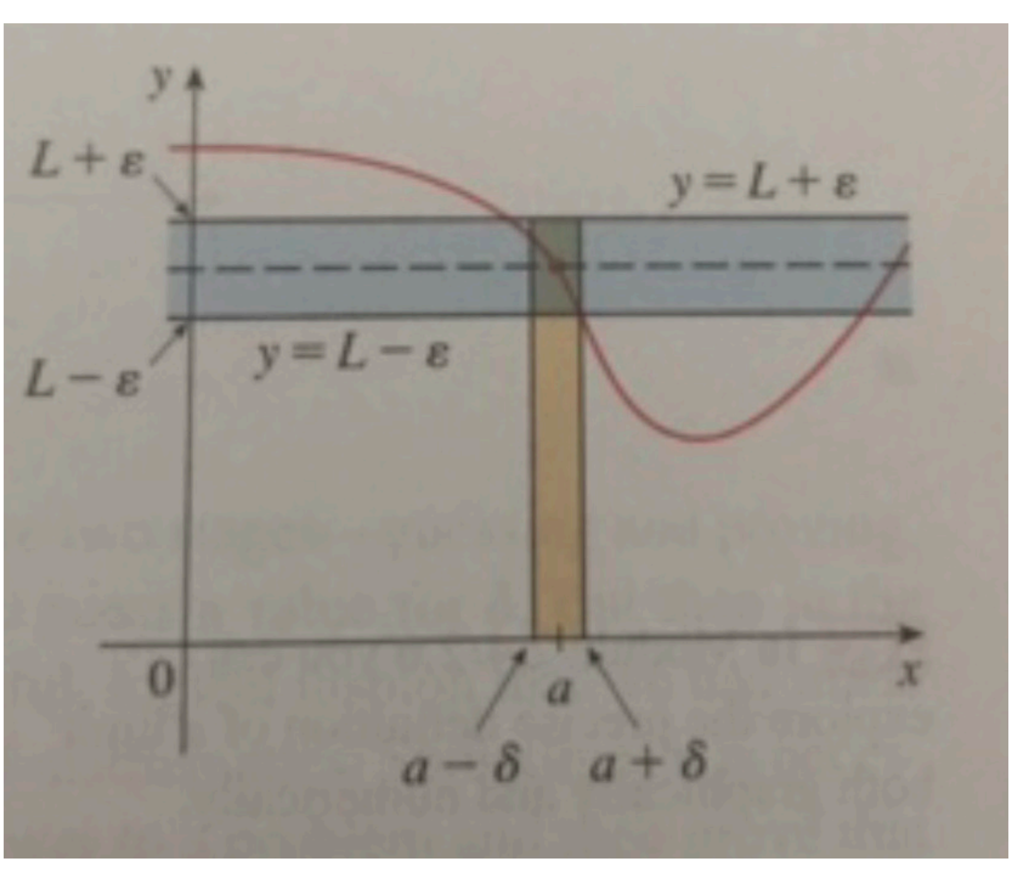
for every # $\varepsilon > 0$ there is a # $\delta > 0$ such that

if $0 < |x-a| < \delta$ then $|f(x)-L| < \varepsilon$

$a-\delta < x < a+\delta$ $\quad$ $L-\varepsilon < f(x) < L+\varepsilon$

$\therefore$ we can find the open interval $(a-\delta, a+\delta)$ & $(L-\varepsilon, L+\varepsilon)$

Example graph:



· Left & Right hand limit:

Left:

$\lim_{x \to a^-} f(x) = L$

if for every # $\varepsilon > 0$ there is a # $\delta > 0$ such that

if $a-\delta < x < a$ then $|f(x)-L| < \varepsilon$

Right:

$\lim_{x \to a^+} f(x) = L$

if for every # $\varepsilon > 0$ there is a # $\delta > 0$ such that

if $a < x < a+\delta$ then $|f(x)-L| < \varepsilon$

→ all of these help in proving limits ⊕ & ⊖

· Precise definition of an infinite limit:

$\lim_{x \to a} f(x) = \infty$

for every positive # M there is a positive # δ such that

if $0 < |x-a| < \delta$ then $f(x) > M$