CS 202

Section 1

Homework 1

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Question 1:

a) Show that $f(n) = 8n^4 + 5n^3 + 7$ is $O(n^5)$ by specifying appropriate c and n_0 values in Big-O definition

We need to find 2 positive constants c and n_0 such that:

$0 \leq 8n^4 + 5n^3 + 7 \leq cn^5 \; for \; all \; n \; \geq n_0$

$\frac{8}{n} + \frac{5}{n^2} + \frac{7}{n^5} \leq c \; for \; all \; n \; \geq n_0$

Choose c = 20 and n_0 = 1:

$8n^4 + 5n^3 + 7 \leq 20n^5 \; for \; all \; n \; \geq 1$

Therefore $f(n) = 8n^4 + 5n^3 + 7 = \; O(n^5)$

b) Trace the following sorting algorithms to sort the array [22, 8, 49, 25, 18, 30, 20, 15, 35, 27] in ascending order. Use the array implementation of the algorithms as described in the textbook and show all major steps.

1) Selection Sort

Initial Array: [22, 8, 49, 25, 18, 30, 20, 15, 35, 27]
Pass number 1:
Number of key comparisons: 9
Key comparison pairs: (8, 22), (49, 22), (25, 49), (18, 49), (30, 49), (20, 49), (15, 49), (35, 49), (27, 49)
Largest index and value: 2, 49
Last index and value: 9, 27
Swap between indexes 2 and 9: temp = 49, x = 27, y = 49
Total swap and moves so far: 1, 3
After iteration: [22, 8, 27, 25, 18, 30, 20, 15, 35, 49]

Pass number 2:
Number of key comparisons: 8
Key comparison pairs: (8, 22), (27, 22), (25, 27), (18, 27), (30, 27), (20, 30), (15, 30), (35, 30)
Largest index and value: 8, 35
Last index and value: 8, 35
Swap between indexes 8 and 8: temp = 35, x = 35, y = 35
Total swap and moves so far: 2, 6
After iteration: [22, 8, 27, 25, 18, 30, 20, 15, 35, 49]

Pass number 3:
Number of key comparisons: 7
Key comparison pairs: (8, 22), (27, 22), (25, 27), (18, 27), (30, 27), (20, 30), (15, 30)

Largest index and value: 5, 30
Last index and value: 7, 15
Swap between indexes 5 and 7: temp = 30, x = 15, y = 30
Total swap and moves so far: 3, 9
After iteration: [22, 8, 27, 25, 18, 15, 20, 30, 35, 49]

Pass number 4:
Number of key comparisons: 6
Key comparison pairs: (8, 22), (27, 22), (25, 27), (18, 27), (15, 27), (20, 27)
Largest index and value: 2, 27
Last index and value: 6, 20
Swap between indexes 2 and 6: temp = 27, x = 20, y = 27
Total swap and moves so far: 4, 12
After iteration: [22, 8, 20, 25, 18, 15, 27, 30, 35, 49]

Pass number 5:
Number of key comparisons: 5
Key comparison pairs: (8, 22), (20, 22), (25, 22), (18, 25), (15, 25)
Largest index and value: 3, 25
Last index and value: 5, 15
Swap between indexes 3 and 5: temp = 25, x = 15, y = 25
Total swap and moves so far: 5, 15
After iteration: [22, 8, 20, 15, 18, 25, 27, 30, 35, 49]

Pass number 6:
Number of key comparisons: 4
Key comparison pairs: (8, 22), (20, 22), (15, 22), (18, 22)
Largest index and value: 0, 22
Last index and value: 4, 18
Swap between indexes 0 and 4: temp = 22, x = 18, y = 22
Total swap and moves so far: 6, 18
After iteration: [18, 8, 20, 15, 22, 25, 27, 30, 35, 49]

Pass number 7:
Number of key comparisons: 3
Key comparison pairs: (8, 18), (20, 18), (15, 20)
Largest index and value: 2, 20
Last index and value: 3, 15
Swap between indexes 2 and 3: temp = 20, x = 15, y = 20
Total swap and moves so far: 7, 21
After iteration: [18, 8, 15, 20, 22, 25, 27, 30, 35, 49]

Pass number 8:
Number of key comparisons: 2
Key comparison pairs: (8, 18), (15, 18)
Largest index and value: 0, 18

Last index and value: 2, 15
Swap between indexes 0 and 2: temp = 18, x = 15, y = 18
Total swap and moves so far: 8, 24
After iteration: [15, 8, 18, 20, 22, 25, 27, 30, 35, 49]

Pass number 9:
Number of key comparisons: 1
Key comparison pairs: (8, 15)
Largest index and value: 0, 15
Last index and value: 1, 8
Swap between indexes 0 and 1: temp = 15, x = 8, y = 15
Total swap and moves so far: 9, 27
After iteration: [8, 15, 18, 20, 22, 25, 27, 30, 35, 49]
2) Bubble Sort
Initial Array: [22, 8, 49, 25, 18, 30, 20, 15, 35, 27]
Pass number 1:
Number of key comparisons: 9
Swap between indexes 0 and 1: temp = 22, x = 8, y = 22
Total swap and moves so far: 1, 3
After swap: [8, 22, 49, 25, 18, 30, 20, 15, 35, 27]

Swap between indexes 2 and 3: temp = 49, x = 25, y = 49
Total swap and moves so far: 2, 6
After swap: [8, 22, 25, 49, 18, 30, 20, 15, 35, 27]

Swap between indexes 3 and 4: temp = 49, x = 18, y = 49
Total swap and moves so far: 3, 9
After swap: [8, 22, 25, 18, 49, 30, 20, 15, 35, 27]

Swap between indexes 4 and 5: temp = 49, x = 30, y = 49
Total swap and moves so far: 4, 12
After swap: [8, 22, 25, 18, 30, 49, 20, 15, 35, 27]

Swap between indexes 5 and 6: temp = 49, x = 20, y = 49
Total swap and moves so far: 5, 15
After swap: [8, 22, 25, 18, 30, 20, 49, 15, 35, 27]

Swap between indexes 6 and 7: temp = 49, x = 15, y = 49
Total swap and moves so far: 6, 18
After swap: [8, 22, 25, 18, 30, 20, 15, 49, 35, 27]

Swap between indexes 7 and 8: temp = 49, x = 35, y = 49
Total swap and moves so far: 7, 21
After swap: [8, 22, 25, 18, 30, 20, 15, 35, 49, 27]

Swap between indexes 8 and 9: temp = 49, x = 27, y = 49

Total swap and moves so far: 8, 24
After swap: [8, 22, 25, 18, 30, 20, 15, 35, 27, 49]

After pass: [8, 22, 25, 18, 30, 20, 15, 35, 27, 49]

Pass number 2:
Number of key comparisons: 8
Swap between indexes 2 and 3: temp = 25, x = 18, y = 25
Total swap and moves so far: 9, 27
After swap: [8, 22, 18, 25, 30, 20, 15, 35, 27, 49]

Swap between indexes 4 and 5: temp = 30, x = 20, y = 30
Total swap and moves so far: 10, 30
After swap: [8, 22, 18, 25, 20, 30, 15, 35, 27, 49]

Swap between indexes 5 and 6: temp = 30, x = 15, y = 30
Total swap and moves so far: 11, 33
After swap: [8, 22, 18, 25, 20, 15, 30, 35, 27, 49]

Swap between indexes 7 and 8: temp = 35, x = 27, y = 35
Total swap and moves so far: 12, 36
After swap: [8, 22, 18, 25, 20, 15, 30, 27, 35, 49]

After pass: [8, 22, 18, 25, 20, 15, 30, 27, 35, 49]

Pass number 3:
Number of key comparisons: 7
Swap between indexes 1 and 2: temp = 22, x = 18, y = 22
Total swap and moves so far: 13, 39
After swap: [8, 18, 22, 25, 20, 15, 30, 27, 35, 49]

Swap between indexes 3 and 4: temp = 25, x = 20, y = 25
Total swap and moves so far: 14, 42
After swap: [8, 18, 22, 20, 25, 15, 30, 27, 35, 49]

Swap between indexes 4 and 5: temp = 25, x = 15, y = 25
Total swap and moves so far: 15, 45
After swap: [8, 18, 22, 20, 15, 25, 30, 27, 35, 49]

Swap between indexes 6 and 7: temp = 30, x = 27, y = 30
Total swap and moves so far: 16, 48
After swap: [8, 18, 22, 20, 15, 25, 27, 30, 35, 49]

After pass: [8, 18, 22, 20, 15, 25, 27, 30, 35, 49]

Pass number 4:

Number of key comparisons: 6
Swap between indexes 2 and 3: temp = 22, x = 20, y = 22
Total swap and moves so far: 17, 51
After swap: [8, 18, 20, 22, 15, 25, 27, 30, 35, 49]

Swap between indexes 3 and 4: temp = 22, x = 15, y = 22
Total swap and moves so far: 18, 54
After swap: [8, 18, 20, 15, 22, 25, 27, 30, 35, 49]

After pass: [8, 18, 20, 15, 22, 25, 27, 30, 35, 49]

Pass number 5:
Number of key comparisons: 5
Swap between indexes 2 and 3: temp = 20, x = 15, y = 20
Total swap and moves so far: 19, 57
After swap: [8, 18, 15, 20, 22, 25, 27, 30, 35, 49]

After pass: [8, 18, 15, 20, 22, 25, 27, 30, 35, 49]

Pass number 6:
Number of key comparisons: 4
Swap between indexes 1 and 2: temp = 18, x = 15, y = 18
Total swap and moves so far: 20, 60
After swap: [8, 15, 18, 20, 22, 25, 27, 30, 35, 49]

After pass: [8, 15, 18, 20, 22, 25, 27, 30, 35, 49]

Pass number 7:
Number of key comparisons: 3
After pass: [8, 15, 18, 20, 22, 25, 27, 30, 35, 49]

Question 2:

c)

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Analysis of Insertion Sort:
Unsorted Array: [ 9, 6, 7, 16, 18, 5, 2, 12, 20, 1, 16, 17, 4, 11, 13, 8 ]
Sorted Array: [ 1, 2, 4, 5, 6, 7, 8, 9, 11, 12, 13, 16, 16, 17, 18, 20 ]
Key Comparisons: 69
Number of Data Moves: 88
--------------------------------------------------
Analysis of Bubble Sort:
Unsorted Array: [ 9, 6, 7, 16, 18, 5, 2, 12, 20, 1, 16, 17, 4, 11, 13, 8 ]
Sorted Array: [ 1, 2, 4, 5, 6, 7, 8, 9, 11, 12, 13, 16, 16, 17, 18, 20 ]
Key Comparisons: 110
Number of Data Moves: 174
Sorted Array: [ 1, 2, 4, 5, 6, 7, 8, 9, 11, 12, 13, 16, 16, 17, 18, 20 ]
--------------------------------------------------
Analysis of Merge Sort:
Unsorted Array: [ 9, 6, 7, 16, 18, 5, 2, 12, 20, 1, 16, 17, 4, 11, 13, 8 ]
Sorted Array: [ 1, 2, 4, 5, 6, 7, 8, 9, 11, 12, 13, 16, 16, 17, 18, 20 ]
Key Comparisons: 47
Number of Data Moves: 128
--------------------------------------------------
Analysis of Quick Sort:
Unsorted Array: [ 9, 6, 7, 16, 18, 5, 2, 12, 20, 1, 16, 17, 4, 11, 13, 8 ]
Sorted Array: [ 1, 2, 4, 5, 6, 7, 8, 9, 11, 12, 13, 16, 16, 17, 18, 20 ]
Key Comparisons: 50
Number of Data Moves: 125
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d)

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Analysis using Random Arrays:
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Analysis of Insertion Sort
Array Size          Elapsed Time        compCount           moveCount
5000                15                  6180381             6185439
10000               59                  25003650            25013660
15000               161                 56545220            56560230
20000               265                 100107502           100127508
25000               382                 156927036           156952043
30000               528                 224312219           224342224
35000               706                 304313988           304348997
40000               933                 398672705           398712714

-----------------------------------------------------
Analysis of Bubble Sort
Array Size          Elapsed Time        compCount           moveCount
5000                49                  12494725            18526155
10000               214                 49978710            74980986
15000               627                 112487550           169590696
20000               979                 199970694           300262530
25000               1572                312473805           470706135
30000               2298                449969775           672846678
35000               3242                612480847           912836997
40000               4676                799950597           1195898148
-----------------------------------------------------
Analysis of Merge Sort
Array Size          Elapsed Time        compCount           moveCount
5000                1                   55250               123616
10000               2                   120451              267232
15000               3                   189396              417232
20000               3                   260890              574464
25000               3                   334037              734464
30000               4                   408493              894464
35000               5                   484377              1058928
40000               5                   561800              1228928

-----------------------------------------------------
Analysis of Quick Sort
Array Size          Elapsed Time        compCount           moveCount
5000                0                   84796               130738
10000               1                   150404              253443
15000               2                   247245              447996
20000               2                   314140              495275
25000               3                   437987              644850
30000               3                   517122              897860
35000               4                   630932              968971
40000               4                   718896              1064699
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Analysis using Almost Sorted Arrays:
-----------------------------------------------------
Analysis of Insertion Sort
Array Size          Elapsed Time        compCount           moveCount
5000                2                   771965              776964
10000               13                  3288505             3298504
15000               16                  7152715             7167714
20000               26                  11179787            11199786
25000               74                  19544639            19569642
30000               84                  29242049            29272048
35000               120                 35670985            35705984
40000               98                  38707459            38747460

-----------------------------------------------------
Analysis of Bubble Sort
Array Size          Elapsed Time        compCount           moveCount
5000                30                  12488589            2300898
10000               133                 49970910            9835518
15000               389                 112377540           21413148
20000               565                 199960839           33479364
25000               984                 312039569           58558932
30000               1382                449737544           87636150
35000               1612                608339619           106907958
40000               1982                765936374           116002386
-----------------------------------------------------
Analysis of Merge Sort
Array Size          Elapsed Time        compCount           moveCount
5000                0                   51125               123616
10000               1                   111083              267232
15000               2                   173694              417232
20000               2                   237876              574464
25000               2                   308858              734464
30000               2                   379022              894464
35000               3                   435514              1058928
40000               3                   483003              1228928

-----------------------------------------------------
Analysis of Quick Sort
Array Size          Elapsed Time        compCount           moveCount
5000                1                   349821              185452
10000               1                   645895              467113
15000               2                   853154              745382
20000               3                   1198840             1277869
25000               4                   1514562             1523612
30000               7                   1729288             2299925
35000               10                  5428614             1795492
40000               38                  27702093            1630652
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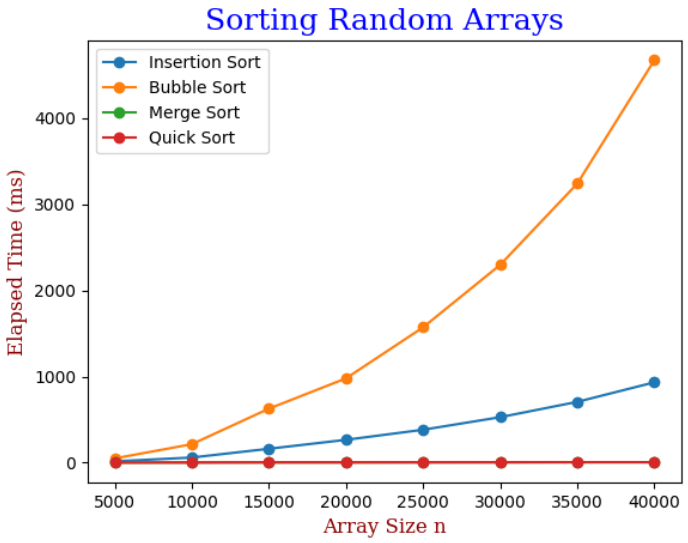
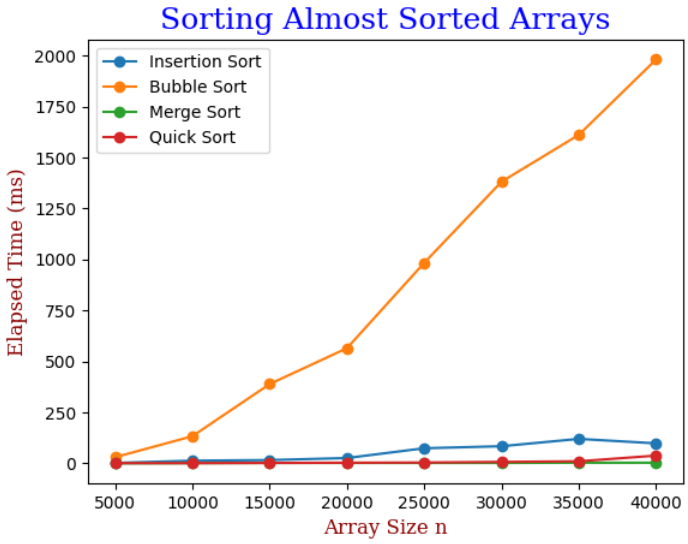
Analysis using Almost Unsorted Arrays:
-----------------------------------------------------
Analysis of Insertion Sort
Array Size          Elapsed Time        compCount           moveCount
5000                28                  11779819            11784840
10000               115                 46912641            46922672
15000               255                 105300999           105316026
20000               459                 188897569           188917580
25000               762                 293409738           293434760
30000               1133                421376972           421407008
35000               1609                577427615           577464882
40000               2277                760074594           760121838

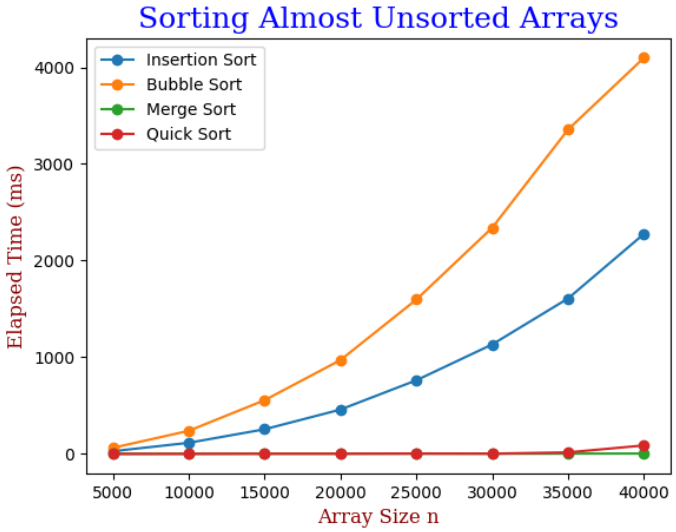
-----------------------------------------------------
Analysis of Bubble Sort
Array Size          Elapsed Time        compCount           moveCount
5000                63                  12497500            35324526
10000               239                 49995000            140708022
15000               556                 112492500           315858084
20000               971                 199989999           566632746
25000               1597                312487500           880154286
30000               2341                449985000           1264041030
35000               3357                612482500           1732184652
40000               4099                799980000           -2014841776
-----------------------------------------------------
Analysis of Merge Sort
Array Size          Elapsed Time        compCount           moveCount
5000                0                   49547               123616
10000               1                   108691              267232
15000               2                   172531              417232
20000               2                   237617              574464
25000               2                   306583              734464
30000               2                   376312              894464
35000               3                   435721              1058928
40000               4                   484490              1228928

-----------------------------------------------------
Analysis of Quick Sort
Array Size          Elapsed Time        compCount           moveCount
5000                1                   121483              191358
10000               1                   367813              585540
15000               2                   571278              892085
20000               2                   568573              914815
25000               4                   1055059             1727286
30000               3                   873315              1406946
35000               16                  4944236             7538032
40000               88                  27405350            41251057
```

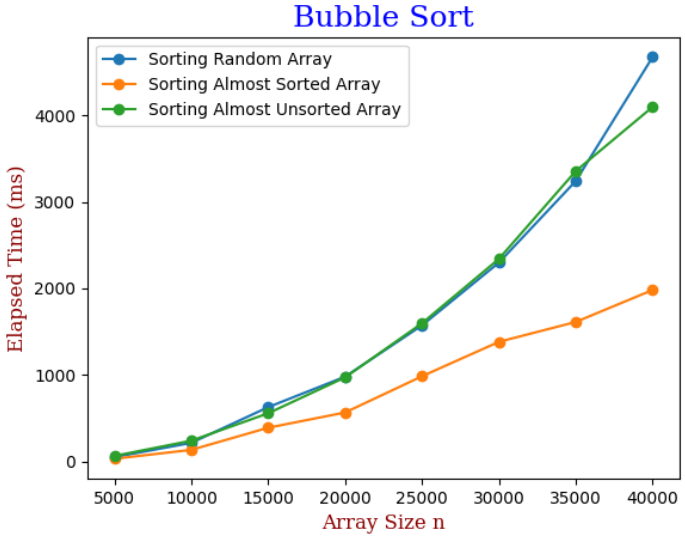
Question 3:

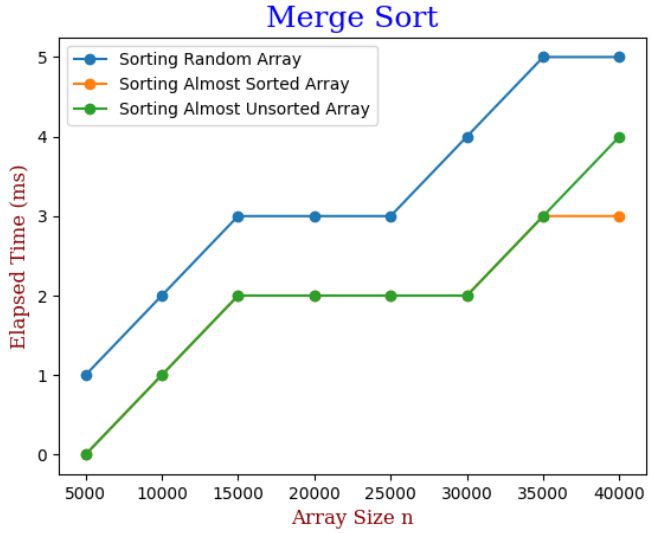

Sorting Random Arrays
Insertion Sort
Bubble Sort
Merge Sort
Quick Sort
Elapsed Time (ms)
0
1000
2000
3000
4000
5000
10000
15000
20000
25000
30000
35000
40000
Array Size n

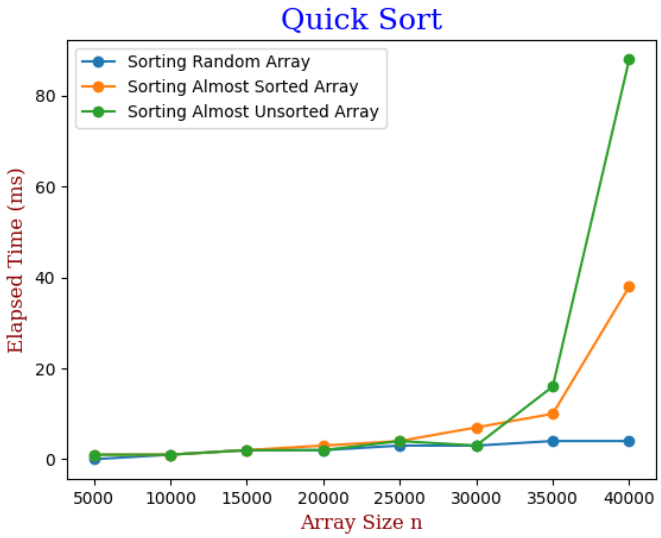

Sorting Almost Sorted Arrays
Insertion Sort
Bubble Sort
Merge Sort
Quick Sort
Elapsed Time (ms)
0
250
500
750
1000
1250
1500
1750
2000
5000
10000
15000
20000
25000
30000
35000
40000
Array Size n


Sorting Almost Unsorted Arrays
Insertion Sort
Bubble Sort
Merge Sort
Quick Sort
Elapsed Time (ms)
Array Size n
0
1000
2000
3000
4000
5000
10000
15000
20000
25000
30000
35000
40000


Insertion Sort
Sorting Random Array
Sorting Almost Sorted Array
Sorting Almost Unsorted Array
Elapsed Time (ms)
Array Size n
0
500
1000
1500
2000
5000
10000
15000
20000
25000
30000
35000
40000


Bubble Sort
Sorting Random Array
Sorting Almost Sorted Array
Sorting Almost Unsorted Array
Elapsed Time (ms)
Array Size n
0
1000
2000
3000
4000
5000
10000
15000
20000
25000
30000
35000
40000


Merge Sort
Sorting Random Array
Sorting Almost Sorted Array
Sorting Almost Unsorted Array
Elapsed Time (ms)
Array Size n
0
1
2
3
4
5
5000
10000
15000
20000
25000
30000
35000
40000


Quick Sort
Sorting Random Array
Sorting Almost Sorted Array
Sorting Almost Unsorted Array
Elapsed Time (ms)
Array Size n
0
20
40
60
80
5000
10000
15000
20000
25000
30000
35000
40000

Report

Theoretically, we expect that insertion sort is $O(n)$ for best case, $O(n^2)$ for average and worst cases. From experimental results, we see that for random array (average case) it looks like parabola, for almost sorted array (best case) it looks like line and for almost unsorted array (worst case) it even more parabola than average case. That is theoratical = experimental values.

Theoretically, we expect that bubble sort is $O(n)$ for best case, $O(n^2)$ for average and worst cases. From experimental results, we see that for random array (average case) it looks like parabola, for almost sorted array (best case) it looks like line but it's slope much larger than insertion sort's best case slope and for almost unsorted array (worst case) it very much resembles average case line. That is theoratical = experimental values.

Theoretically, merge sort is $O(n * \log n)$ for all possible cases. From experimental results, ladder-like lines suggest that it is not linear but some multiplication of n and for our case, factor should be $\log n$. Therefore, merge sort is also experimentally correct.

Theoretically, we expect that quick sort is $O(n * logn)$ for best and average case, $O(n^2)$ for worst case. From experimental results, almost sorted array (worst case) shows parabola and we expect that the others should be ladder-like lines. Our claim is true for random array, but not for almost unsorted array which has some outlier value at the end. Ignoring the last value of almost unsorted array we can see that quick sort is experimentally correct.

Between 4 sorting algorithms we see that merge and quick sort is much more efficient that insertion and bubble sorts which we expect from their time complexities. It seems among them the worst algorithm is bubble sort and the best is merge sort. This homework clearly shows that how $O(n * logn)$ and $O(n)$ much more efficient than $O(n^2)$ since even in very high values the first two ones not change much.