

PHYS 101 EXPERIMENT 3. CONSERVATION OF LINEAR MOMENTUM:COLLISIONS

Objective:

In this experiment collision types, their significances and the state of the momentum during the collisions will be examined.

Introduction:

Linear momentum of a particle with mass m moving with velocity \vec{v} is defined as $\vec{p} = m \vec{v}$. For a system of particles, the linear momentum of the system is equal to the sum of the linear momenta of the individual particles making up the system, i.e., $\vec{P} = \sum m \vec{v}$. If the net external force acting on the system is zero, the linear momentum of the system is constant in time. This, so called, principle of conservation of momentum has important applications in physics.

In many cases the laws of conservation of momentum and energy alone can be used to obtain important results concerning the properties of various mechanical processes. It should be noted that these properties are independent of the particular type of interaction between the particles involved. (One of the fundamental concepts in mechanics is that of a particle. By this we mean a body whose dimensions may be neglected in describing its motion. The possibility of doing so depends, of course, on the conditions of the problem concerned.)

The total momentum of a system of two bodies (or particles) has different values in different (inertial) frames of reference. If a frame S' moves with velocity \vec{V} relative to another frame S , then the velocities \vec{v}_n and \vec{v}'_n ($n = 1, 2, \dots$), of the particles relative to the two frames are such that

$$\vec{v}_1 = \vec{v}'_1 + \vec{V}, \quad \vec{v}_2 = \vec{v}'_2 + \vec{V} \quad (1)$$

Total momenta of the two particles \vec{P} and \vec{P}' in the two frames are therefore related by

$$\vec{P} = m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 (\vec{v}'_1 + \vec{V}) + m_2 (\vec{v}'_2 + \vec{V}) = (m_1 \vec{v}'_1 + m_2 \vec{v}'_2) + (m_1 + m_2) \vec{V} \quad (2)$$

or

$$\vec{P} = \vec{P}' + (m_1 + m_2) \vec{V} \quad (3)$$

In particular, there is always a frame of reference S' in which the total momentum is zero. Putting $\vec{P}' = 0$, in the above equation, we find the velocity of this frame as

$$\vec{V} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} \quad (4)$$

The right-hand side of the above equation can be written as the total time derivative of the expression

$$\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} \quad (5)$$

where \vec{r}_1 and \vec{r}_2 denote the positions of the first and second particles, respectively. We can thus say that the velocity of the system as a whole is the rate of motion whose position vector is \vec{R} . This point is called the

Center of Mass of the system. The law of "Conservation of Momentum" can therefore be formulated as stating that *The center of mass of the system moves uniformly in a straight line.*

Collisions: When two bodies collide, the equal and opposite forces are internal forces in what is called a two-body system and consequently have no effect on the total momentum of the system. This phenomenon means that the sum of the momenta of the bodies before impact is equal to the sum of the momenta after impact. The relation between the kinetic energies before and after impact depends on the elasticity of the bodies.

A collision between two particles is said to be *elastic* if it involves no change in their internal state. Accordingly, when the law of conservation of energy is applied to such a collision, the internal energy of the particles may be neglected. In the case the kinetic energy is not conserved, the collision is said to be *inelastic*. The case in which the particles stick and go together as a rigid entity is referred to as a *completely inelastic collision*.

In this experiment, you are going to study elastic and inelastic collisions of two pucks on a frictionless plane (on an air table) by assuming that they behave as point particles.

Formally, from the law of conservation of momentum, one can write down two independent equations relating the initial and final speeds of the particles and the angles at which they get scattered. Yet, if the collision is elastic, a third equation follows from the conservation of energy. It is possible to solve these equations for quantities that interest us. Therefore, we refrain ourselves from an elaborate algebraic analysis of these equations and be content only by the empirical verification of the law of conservation of linear momentum.

Questions to Think About:

1. If kinetic energy is not conserved during the collision where does it go?
2. A ball falls toward earth, its momentum increases. How would you reconcile this fact with the law of conservation of momentum?

Equipment:

Following equipment will be supplied.

- Air Table and Accessories

The following items must be brought by you and will not be supplied:

- Ruler
- A Scientific Calculator

Procedure:

1. Level the air table using the set screws beneath.
2. Activate the air pump and the spark timer and project two ordinary (nonmagnetic) pucks towards one another so that they collide at some place at the center of the air table, and thereafter follow different straight paths. Be careful not to mix the right and left hand sides when you turn over the paper to see the printed paths.
3. By pairing the black prints on the paper, locate the successive positions of the center of mass before and after the collision. Measure the distances before and after the collisions having the collision point set as the origin and tabulate your data in Table 1. Verify that the center of mass of the system moves uniformly in a straight line during the complete process, i.e. show that the center of mass velocity remains invariant during the collision (and hence, the momentum is conserved).
4. Attach velcro tapes around the pucks and repeat steps 2-3. The pucks will stick to each other and move together after the collision. (Be sure that the pucks do not contact each other at the sealing points of the velcro tapes and that they do not rotate after sticking.) Use Table 2 for your data.

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Data & Results: [45]

	Before the Collision			After the Collision		
	ΔR (cm)	Δt (ms)	V_{cm} (m/s)	ΔR (cm)	Δt (ms)	V_{cm} (m/s)
1	1.9	50	3.8×10^{-2}	1.8	50	3.6×10^{-2}
2	3.8	100	3.8×10^{-2}	3.6	100	3.6×10^{-2}
3	5.7	150	3.8×10^{-2}	5.4	150	3.6×10^{-2}
4	7.6	200	3.8×10^{-2}	7.2	200	3.6×10^{-2}

Table 1a: Collision with two steel disks (center of mass)

	Left Puck			Right Puck		
	ΔR (cm)	Δt (ms)	V (m/s)	ΔR (cm)	Δt (ms)	V (m/s)
1	1.8	50	3.6×10^{-2}	2.0	50	4×10^{-2}
2	3.6	100	3.6×10^{-2}	4.0	100	4×10^{-2}
3	5.4	150	3.6×10^{-2}	6.0	150	4×10^{-2}
4	7.2	200	3.6×10^{-2}	8.0	200	4×10^{-2}

Table 1b: Collision with two steel disks before the collision

$$E_{right} = \frac{1}{2} m v^2 = \frac{1}{2} m_{puck} V_{avg\ right}^2 = \frac{1}{2} \cdot 420 \text{ gr} \cdot (4 \times 10^{-2} \frac{\text{cm}}{\text{ms}})^2 = 0.336 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$$

$V_{avg\ left} = 3.6 \times 10^{-2} \text{ cm/ms}$	$E_{left} = 0.272 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$	$V_{avg\ right} = 4 \times 10^{-2} \text{ cm/ms}$	$E_{right} = 0.336 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$
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$$m_{puck} = 420 \text{ gr}$$

$$E_{left} = \frac{1}{2} m v^2 = \frac{1}{2} m_{puck} \cdot V_{avg\ left}^2 = \frac{1}{2} \cdot 420 \text{ gr} \cdot (3.6 \times 10^{-2} \frac{\text{cm}}{\text{ms}})^2 = 0.272 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$$

$$E_{initial} = E_{left} + E_{right} = (0.272 + 0.336) \text{ gr} \frac{\text{cm}^2}{\text{ms}^2} = 0.608 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$$

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Left Puck			Right Puck		
$\Delta R (cm)$	$\Delta t (ms)$	$V (\frac{cm}{ms})$	$\Delta R (cm)$	$\Delta t (ms)$	$V (\frac{cm}{ms})$
1	1.7	50	3.4×10^{-2}	1.9	50
2	3.5	100	3.5×10^{-2}	3.7	100
3	5.3	150	3.53×10^{-2}	5.5	150
4	7.1	200	3.55×10^{-2}	7.3	200

Table 1c: Collision with two steel disks after the collision

$$m_{puck} = 420 \text{ gr}$$

$$E_{left+} = \frac{1}{2} m v^2 = \frac{1}{2} m_{puck} V_{avg\ left+}^2 = \frac{1}{2} \cdot 420 \text{ gr} \cdot (3.5 \times 10^{-2} \text{ cm/ms})^2 = 0.257 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$$

$$V_{avg\ left+} = 3.5 \times 10^{-2} \frac{\text{cm}}{\text{ms}} \quad E_{left+} = 0.257 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2} \quad V_{avg\ right+} = 3.7 \times 10^{-2} \frac{\text{cm}}{\text{ms}} \quad E_{right+} = 0.287 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$$

$$E_{right+} = \frac{1}{2} m v^2 = \frac{1}{2} m_{puck} V_{avg\ right+}^2 = \frac{1}{2} \cdot 420 \text{ gr} \cdot (3.7 \times 10^{-2} \text{ cm/ms})^2 = 0.287 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$$

$$E_{final} = E_{left+} + E_{right+} = (0.257 + 0.287) \text{ gr} \frac{\text{cm}^2}{\text{ms}^2} = 0.544 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$$

$$\frac{E_{final}}{E_{initial}} = \frac{0.544}{0.608} = 0.895 \text{ Energy Loss \% = 10.5 \%}$$

Before the Collision			After the Collision		
$\Delta R (cm)$	$\Delta t (ms)$	$V_{cm} (\frac{cm}{ms})$	$\Delta R (cm)$	$\Delta t (ms)$	$V_{cm} (\frac{cm}{ms})$
1	1.5	3×10^{-2}	1.4	50	2.8×10^{-2}
2	2.9	2.9×10^{-2}	2.8	100	2.8×10^{-2}
3	4.4	2.93×10^{-2}	4.1	150	2.73×10^{-2}
4	5.9	2.95×10^{-2}	5.5	200	2.75×10^{-2}

Table 2a: Collision with two steel disks & Velcro (center of mass)

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	Left Puck			Right Puck		
	ΔR (cm)	Δt (ms)	$V \left(\frac{cm}{ms} \right)$	ΔR (cm)	Δt (ms)	$V \left(\frac{cm}{ms} \right)$
1	1.6	50	3.2×10^{-2}	1.4	50	2.8×10^{-2}
2	3.2	100	3.2×10^{-2}	2.8	100	2.8×10^{-2}
3	4.8	150	3.2×10^{-2}	4.2	150	2.8×10^{-2}
4	6.4	200	3.2×10^{-2}	5.6	200	2.8×10^{-2}

Table 2b: Collision with two steel disks & Velcro before the collision

$$m_{puck} = 420 \text{ gr}$$

$$E_{left+} = \frac{1}{2} mv^2 = \frac{1}{2} m_{puck} V_{avg\ left+}^2 = \frac{1}{2} \cdot 420 \text{ gr} \cdot (3.2 \times 10^{-2} \frac{\text{cm}}{\text{ms}})^2 = 0.215 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$$

$V_{avg\ left+} = 3.2 \times 10^{-2} \frac{\text{cm}}{\text{ms}}$	$E_{left+} = 0.215 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$	$V_{avg\ right+} = 2.8 \times 10^{-2} \frac{\text{cm}}{\text{ms}}$	$E_{right+} = 0.165 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$
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$$E_{right+} = \frac{1}{2} mv^2 = \frac{1}{2} m_{puck} V_{avg\ right+}^2 = \frac{1}{2} \cdot 420 \text{ gr} \cdot (2.8 \times 10^{-2} \frac{\text{cm}}{\text{ms}})^2 = 0.165 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$$

$$E_{initial} = E_{left+} + E_{right+} = (0.215 + 0.165) \text{ gr} \frac{\text{cm}^2}{\text{ms}^2} = 0.380 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$$

	Left Puck			Right Puck		
	ΔR (cm)	Δt (ms)	$V \left(\frac{cm}{ms} \right)$	ΔR (cm)	Δt (ms)	$V \left(\frac{cm}{ms} \right)$
1	1.6	50	3.2×10^{-2}	1.2	50	2.4×10^{-2}
2	3.2	100	3.2×10^{-2}	2.4	100	2.4×10^{-2}
3	4.8	150	3.2×10^{-2}	3.6	150	2.4×10^{-2}
4	6.4	200	3.2×10^{-2}	4.8	200	2.4×10^{-2}

Table 2c: Collision with two steel disks & Velcro after the collision

$$E_{left+} = \frac{1}{2} mv^2 = \frac{1}{2} m_{puck} V_{avg\ left+}^2 = \frac{1}{2} \cdot 420 \text{ gr} \cdot (2.8 \times 10^{-2} \frac{\text{cm}}{\text{ms}})^2 = 0.215 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$$

$V_{avg\ left+} = 3.2 \times 10^{-2} \frac{\text{cm}}{\text{ms}}$	$E_{left+} = 0.215 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$	$V_{avg\ right+} = 2.4 \times 10^{-2} \frac{\text{cm}}{\text{ms}}$	$E_{right+} = 0.120 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$
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$$E_{right+} = \frac{1}{2} mv^2 = \frac{1}{2} m_{puck} V_{avg\ right+}^2 = \frac{1}{2} \cdot 420 \text{ gr} \cdot (2.8 \times 10^{-2} \frac{\text{cm}}{\text{ms}})^2 = 0.120 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$$

$$E_{final} = E_{left+} + E_{right+} = (0.215 + 0.120) \text{ gr} \frac{\text{cm}^2}{\text{ms}^2} = 0.335 \text{ gr} \frac{\text{cm}^2}{\text{ms}^2}$$

$$E_{final} / E_{initial} = \frac{0.335}{0.380} = 0.88 \text{ Energy Loss \%} = 11.9 \%$$

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Questions:

1. [5] If you are given a data sheet with only spark timer prints on it, can you distinguish between

the initial and final states of the pucks? Yes we can. There can be 2 possibilities, it can be elastic or inelastic collision. Since in real life, no collisions can be totally elastic, we can compare two of the states by calculating kinetic energy of the particles. Lesser ones should be the final states and greater ones should be initial states. For completely inelastic collisions, the pucks will stick and move together. Therefore, we can measure where is where by measuring the distance between the dots, if it is nearly stable, this is final states and if it is decreasing, then that part is the initial state.

2. [5] A particle collides obliquely with an identical particle initially at rest. Assuming elastic

Collision, show that the two particles move at 90° from each other after the collision.

$$\text{Elastic collision} \Rightarrow P_x \text{ initial} = P_x \text{ finally} \Rightarrow Mv = Mv_1 \cos\theta_1 + Mv_2 \cos\theta_2$$

$$P_y \text{ initial} = P_y \text{ final} \Rightarrow 0 = v_1 \sin\theta_1 + v_2 \sin\theta_2 \quad (1)$$

$$0 + 0 = Mv_1 \sin\theta_1 - Mv_2 \sin\theta_2 \Rightarrow 0 = v_1 \sin\theta_1 - v_2 \sin\theta_2 \quad (2)$$

$$\Rightarrow v^2 = v_1^2 \cos^2\theta_1 + v_2^2 \cos^2\theta_2 + 2v_1 v_2 \cos\theta_1 \cos\theta_2 + v_1^2 \sin^2\theta_1 + v_2^2 \sin^2\theta_2 = v^2 = (v_1 \cos\theta_1 + v_2 \cos\theta_2)^2 + (v_1 \sin\theta_1 - v_2 \sin\theta_2)^2$$

$$\Rightarrow v^2 = v_1^2 (\cos^2\theta_1 + \sin^2\theta_1) + v_2^2 (\cos^2\theta_2 + \sin^2\theta_2) + 2v_1 v_2 (\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) = v^2 = v_1^2 + v_2^2 + 2v_1 v_2 \cos(\theta_1 + \theta_2)$$

$$\Rightarrow KE \text{ is conserved (Elastic)} \Rightarrow KE_i = KE_f \Rightarrow \frac{1}{2} Mv^2 = \frac{1}{2} Mv_1^2 + \frac{1}{2} Mv_2^2 \Rightarrow v^2 = v_1^2 + v_2^2 \quad (3)$$

Put (3) in (2): Conclusion: [15]

$$\Rightarrow 2v_1 v_2 \cos(\theta_1 + \theta_2) = 0 \Rightarrow \cos(\theta_1 + \theta_2) = 0 \Rightarrow \theta_1 + \theta_2 = 90^\circ, \text{two particles move at } 90^\circ \text{ from each other after the collision}$$

This experiment was aimed to show conservation of linear momentum in collisions. We expected to see the center of mass velocity remains invariant during the collisions. We draw Table 1a and Table 2a by tracking the path of the center of mass and finding the distance between the successive points in both collisions. I found less difference in elastic collision in terms of the velocity of center of mass before and after the collision than inelastic one. Even though velocities not equal before and after the collision, this result is expected since the error caused by friction, energy loss in collision, not being point object, some minor rotational force (in inelastic collision, not having same initial velocity results in faster one hits the slower one above its center cause some rotation) vb.. In Table 1b, 1c, 2b, 2c we look at the pucks individually and found that energy loss in the systems. For elastic one, I found 10.5% energy loss and 11.9% for inelastic one. They seem to have close errors but due to friction and energy loss at the collision of two not point particles elastic collision is not totally elastic. However, in inelastic collision, they start to trace diagonal path (caused by some minor rotation it is hard to give them some initial velocity and some angle and throwing at the same time to have perfect collision at the center). That's why, one puck has greater displacement than other. The energy loss while rotating and sticking together makes the error difference 11.9 - 10.5 = 1.4% reasonable. As a result, momentum is conserved with some losses in all collisions but energy is not always conserved.