PHYS 102 Lab Project

The Capacitance of Parallel Plate Capacitor with Dielectric Slab

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Section: 9

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Part A Calculating Dielectric Constants of Some Materials:

Objective:

This part of the experiment aims to find the dielectric constants of materials used in both Part B and C. This part will assume found dielectric constants will be correct since there is no reference value to be compared. Therefore, no error will be calculated for this part.

Theory:

1) $C = \epsilon_0 \frac{A}{d}$ [1]
2) $C = \kappa\, C_0$ [1]
3) $E = \frac{V}{d}$ [1]
4) $\kappa_{\text{air}} = 1.00059 \approx 1 = \kappa_{\text{vacuum}}$ [1]

(ϵ_0 is the permittivity of free space, A is the area of the single plate in meters, d is the distance between the plates in meters, C_0 is capacitance of the capacitor filled with vacuum, κ dielectric constant, E is the electric field, V is the potential difference between plates)

The capacitance of the parallel plate capacitor depends only on the area of one plate and the distance between the plates with constant ϵ_0 (formula 1). When the plates are bigger, they will store more charge, and when the plates are closer, the opposite charges will attract each other more. However, when d is very small, E may be high enough that the breakdown may occur between the plates, thus losing the capacitance feature (formula 3). One may solve the problem by putting an insulating material between the plates to increase the resistance of the capacitor to electric field changes [1]. Thus, the capacitance value will be increased with insulator material with dielectric constant κ (formula 2). Three homemade parallel plate capacitors will be prepared in advance, and the distance d between the plates will be kept stable while the area of plates A will be changed. That way, the effects of different materials will be directly observed at the capacitance value of the capacitor. Knowing C and C_0, κ can be easily extracted from formula 2.

Setup of Experiment:

Equipment List:

1) 3 pairs of plates with different areas
2) Aluminium Foil
3) Ruler
4) Multimeter
5) Scientific Calculator
6) Paper block
7) Wood block
8) Foam block

Procedure and Data:

1) Cover the plates with aluminum foil
2) Create four conductors by keeping the distance between the plates constant
3) Using the feature of the multimeter, measure the capacitance of every pair of plates while there is air, paper, wood, and foam between the plates, respectively. (Subtract the capacitance value of the multimeter when it is not measuring from found values)
4) $C_{Material}$ / C_{air} should give $\kappa_{material}$ from previous formulas.
5) Find each $\kappa_{material}$ and take their arithmetic mean to estimate them better.
6) Save dielectric constants to Table 1.2 for later usage.

Table 1.1:

A_n= Area of one plate of the nth conductor where:

A_1= 10 cm width * 12.5 cm height = 125 cm^2

A_2 = 15.5 cm width * 7.5 cm height = 116.25 cm^2

A_3 = 15.5 cm width * 9 cm height = 139.5 cm^2

C_n = Capacitance of nth conductor filled with given material

Material	$C_1(A_1)$(μF)	$C_2(A_2)$(μF)	$C_3(A_3)$(μF)
Air	0.010	0.008	0.012
Paper	0.015	0.012	0.017
Wood	0.016	0.013	0.019
Foam	0.012	0.009	0.014

C_{paper} / C_{air} = (0.015 / 0.010 + 0.012 / 0.008 + 0.017 / 0.012) / 3 = 1.47

C_{wood} / C_{air} = (0.016 / 0.010 + 0.013 / 0.008 + 0.019 / 0.012) / 3 = 1.60

C_{foam} / C_{air} = (0.012 / 0.010 + 0.009 / 0.008 + 0.014 / 0.012) / 3 = 1.17

Table 1.2:

Material	Dielectric Constant
Paper	1.47
Wood	1.60
Foam	1.17

Conclusion:

This part aims to calculate some Dielectric Constants to be able to use them in the next parts. Since there are no reference points present, the error rates for these measurements were not calculated. This part also suggests that there may be a correlation between the density and molecular structure of the matter and its dielectric constant. For example, the foam's low dielectric constant seems correlated with its thin, light, and porous structure; on the other hand, the wood's high dielectric constant may result from its dense, heavy, and non-porous structure.

Part B Calculating Capacitance of Half-Filled Capacitor:

Objective:

The purpose of this part of the experiment is to indicate the behavior of the capacitor when it is partially filled. The distance between the plates will be half-filled in the first vertical and then horizontal direction, respectively. The expected result proves that a single partially filled capacitor acts like multiple capacitors connected in series or parallel. The error will be found by comparing the experimental and theoretical results. The result will be the fundamental assumption of Part C.

Theory:

1) $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$ [2]
2) $C_p = C_1 + C_2 + C_3 + \cdots$ [2]
3) $C = \kappa \epsilon_0 \frac{A}{d}$ [1]
4) $V = V_1 + V_2 + V_3 + \cdots$ [2]
5) $V = \frac{Q}{C}$ [2]
6) $C_p V = C_1 V + C_2 V + C_3 V + \cdots$ [2]

(ϵ_0 is the permittivity of free space, A is the area of the single plate in meters, d is the distance between the plates in meters, C_0 is the capacitance of the capacitor filled with vacuum, Q charge, κ dielectric constant, C_s and C_p are total capacitances in series and parallel respectively)

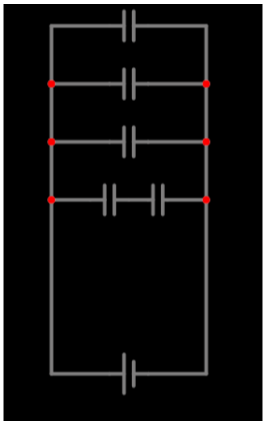

Figure 1

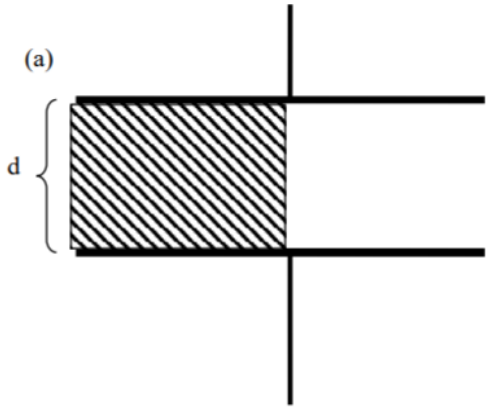

Figure 2

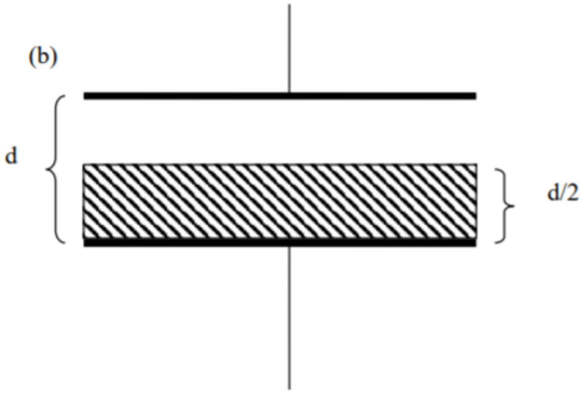

Figure 3

Since capacitances in series share total voltage between them, using formulas 4 and 5, one can get formula 1. On the other hand, capacitances in parallel have the same voltage values, writing formula 5 for Q; formula 6 is obtained. Eliminating V from formula 6, formula 2 is received.

For figure 2, since the capacitor plates are at the same potential, the capacitor is like two capacitors connected in parallel with half area each.

For figure 3, the inner boundary of the inductor can be thought of as a new plate. Thus, the capacitor is like two capacitors connected in series with half the distance between the plates each.

From experimental results, we should get formulas 1 and 2, respectively.

Setup of Experiment:

Equipment List:

1) 1 conductor from the previous part (A_1= 125 cm^2)
2) Ruler
3) Multimeter
4) Scientific Calculator
5) Paper block
6) Wood block
7) Foam block

Procedure and Data:

1) For each material:
    a. Insert material as in figure 2
    b. Measure capacitance of the conductor
    c. Calculate theoretical result
    d. Save both theoretical and experimental results in table 2.1
    e. Insert material as in figure 3
    f. Measure capacitance of the conductor
    g. Calculate theoretical result
    h. Save both theoretical and experimental results in table 2.2
2) For each table:
    a. Find errors in every row where Error = $\frac{C_{theo} - C_{exp}}{C_{theo}} * 100$

b. Take the average of the errors of three materials values

Table 2.1:

For Capacitances in Parallel

Material	C_{exp}(μF)	C_{theo}(μF)	Error (%)
Paper	0.010	0.012	16.67
Wood	0.012	0.013	7.69
Foam	0.010	0.010	0

C_{theo} = $C_p = C_1 + C_2 + C_3 + \cdots$ => $C_p = C_{material} + C_0$. Recall that $C_{material} = \kappa\, C_0$ and $C = \epsilon_0 \frac{A}{d}$. When A become half of its previous value, C is also become half. Therefore, we need to find $C_{material}$ from dielectric constants found in part A and divide it and C_0 by two.

$C_{wood} = 1.6 * C_0 * \frac{1}{2}, C_0 = C_0 * \frac{1}{2},$ => $C_p = C_{wood} + C_0$ and C_0 was 0.010 => $C_p = 0.013$

$C_{paper} = 1.47 * C_0 * \frac{1}{2}, C_0 = C_0 * \frac{1}{2},$ => $C_p = C_{paper} + C_0$ and C_0 was 0.010 => $C_p = 0.012$

$C_{foam} = 1.17 * C_0 * \frac{1}{2}, C_0 = C_0 * \frac{1}{2},$ => $C_p = C_{foam} + C_0$ and C_0 was 0.010 => $C_p = 0.010$

E_p = (16.67 + 7.69 + 0) / 3 = 8.12 %

Table 2.2:

For Capacitances in Series

Material	C_{exp}(μF)	C_{theo}(μF)	Error (%)
Paper	0.010	0.012	16.67
Wood	0.013	0.012	8.33
Foam	0.009	0.010	11.11

C_{theo} = $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots => \frac{1}{C_s} = \frac{1}{C_{material}} + \frac{1}{C_0}$. Recall that $C_{material} = \kappa\, C_0$ and $C = \epsilon_0 \frac{A}{d}$. When d become half of its previous value, C is doubled. Therefore, we need to find $C_{material}$ from dielectric constants found in part A and double it with C_0.

$C_{wood} = 1.6 * C_0 * 2,\, C_0 = \ C_0 * 2 => C_s = \frac{C_{material} * C_0}{C_0 + C_{material}}$ and C_0 $was\ 0.010 =>$ $C_s = \ 0.012$

$C_{paper} = 1.47 * C_0 * 2,\, C_0 = \ C_0 * 2 => C_s = \frac{C_{material} * C_0}{C_0 + C_{material}}$ and C_0 $was\ 0.010 =>$ $C_s = \ 0.012$

$C_{foam} = 1.17 * C_0 * 2,\, C_0 = \ C_0 * 2 => C_s = \frac{C_{material} * C_0}{C_0 + C_{material}}$ and C_0 $was\ 0.010 =>$ $C_s = \ 0.010$

E_s = (16.67 + 8.33 + 11.11) / 3 = 12.04 %

Conclusion:

This part aims to check whether the idea of partial fullness and especially thinking the system as multiple pieces of smaller capacitors is hold or not. From the theoratical and experimental results, error rates were calculated and for parallel capacitors it was 8.12%; whereas, for series it was 12.04%. It suggests us that the idea holds very likely and from that idea we can assume what we will done in Part C should be hold as well. Also, Paper's error rate suggest that there may be systemmatical error about multimeters, since it has the same error percentage and value, that may not be user error. Another point to remark is that, capacitance with dielectric materials always lose when they combined with air as in our sfystem as air has dielectric constant lower than all materials.

Part C Calculating Capacitance of Partially Filled Capacitor:

Objective:

This part of the experiment aims to find a general formula for calculating the capacitance of the partially filled capacitor. The experiment will be restricted to partial fullness as in figure 2 (there will be no air between plate and the edge of the insulator). The result of this part is to find linear dependence between partial fullness and capacitance of the capacitor.

Theory:

1) $C_p = C_1 + C_2 + C_3 + \cdots$ [2]
2) $C_1 = (1 - \mathrm{f})\,\mathrm{C}_0$ [3]
3) $C_2 = f\kappa\mathrm{C}_0$ [3]
4) $C(f) = C_0 * (1 - f + f\kappa)$ [3]

(C_0 is capacitance of the capacitor filled with vacuum, κ dielectric constant, C_s and C_p are total capacitances in series and parallel respectively, 0 <= f <= 1)

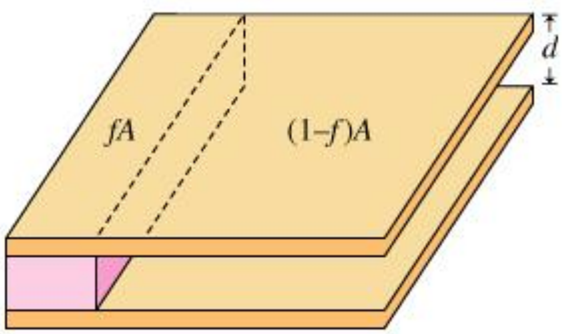

Figure 4

Part B proves that the configuration like Figure 4 divides the capacitor into two capacitors in parallel. For individual parts, the capacitance can be found by multiplying them with the division ratio f (formula 2 and 3). Summing the two leads us to formula 4. Using certain fractions, $C_{theo}(f)$ can be calculated and be compared with $C_{exp}(f)$. Th error will show the correctness of the general formula, formula 4.

Setup of Experiment:

Equipment List:

1) 1 conductor from previous parts (A_1= 125 cm^2)
2) Ruler
3) Multimeter
4) Scientific Calculator
5) Paper block
6) Wood block
7) Foam block

Procedure and Data:

1) For each material:
    a. Insert material as in figure 4 by f = next value in table 3.x where x in {1, 2, 3} respectively (Take the first and last values from previous parts, they are for completeness).
    b. Measure capacitance of the conductor
    c. Calculate theoretical result
    d. Save both theoretical and experimental results in table 3.x
2) For each table:
    a. Find errors in every row (Do not add (C_0) to the calculation. It will be zero)
    b. Take the average of the errors of three values
3) Take the average of the errors of three materials

Table 3.1:

Paper

F	C_{exp}(μF)	C_{theo}(μF)	Error (%)
12/12 (C_{paper})	0.015	0.015	0 (Not Used)
9/12	0.015	0.014	7.14
6/12	0.014	0.012	16.67
3/12	0.012	0.011	9.09
0/12 (C_0)	0.010	0.010	0 (Not Used)

(From Theorems Part) $C(f) = C_0 * (1 - f + f\kappa)$:

F = 3/12 =>$C(f) = C_0 * (1 - 0.25 + 0.25\kappa) => 0.010 * (1 - 0.25 + 0.25 * 1.47)$ = 0.011

F = 6/12 =>$C(f) = C_0 * (1 - 0.50 + 0.50\kappa) => 0.010 * (1 - 0.50 + 0.50 * 1.47)$ = 0.012

F = 9/12 =>$C(f) = C_0 * (1 - 0.75 + 0.75\kappa) => 0.010 * (1 - 0.75 + 0.75 * 1.47)$ = 0.014

E_{paper} = (16.67 + 7.14 + 9.09) / 3 = 10.97 %

Table 3.2:

Wood

f	C_{exp}(µF)	C_{theo}(µF)	Error (%)
12/12 (C_{wood})	0.016	0.016	0 (Not Used)
9/12	0.017	0.015	13.33
6/12	0.015	0.013	15.38
3/12	0.013	0.012	8.33
0/12 (C_0)	0.010	0.010	0 (Not Used)

(From Theorems Part) $C(f) = C_0 * (1 - f + f\kappa)$:

F = 3/12 =>$C(f) = C_0 * (1 - 0.25 + 0.25\kappa) => 0.010 * (1 - 0.25 + 0.25 * 1.60)$ = 0.012

F = 6/12 =>$C(f) = C_0 * (1 - 0.50 + 0.50\kappa) => 0.010 * (1 - 0.50 + 0.50 * 1.60)$ = 0.013

F = 9/12 =>$C(f) = C_0 * (1 - 0.75 + 0.75\kappa) => 0.010 * (1 - 0.75 + 0.75 * 1.60)$ = 0.015

E_{wood} = (15.38 + 13.33 + 8.33) / 3 = 12.35 %

Table 3.3:

Foam

f	C_{exp}(μF)	C_{theo}(μF)	Error (%)
12/12 (C_{foam})	0.012	0.012	0 (Not Used)
9/12	0.013	0.011	18.18
6/12	0.012	0.011	9.09
3/12	0.010	0.010	0
0/12 (C_0)	0.010	0.010	0 (Not Used)

(From Theorems Part) $C(f) = C_0 * (1 - f + f\kappa)$:

F = 3/12 =>$C(f) = C_0 * (1 - 0.25 + 0.25\kappa) => 0.010 * (1 - 0.25 + 0.25 * 1.17)$ = 0.010

F = 6/12 =>$C(f) = C_0 * (1 - 0.50 + 0.50\kappa) => 0.010 * (1 - 0.50 + 0.50 * 1.17)$ = 0.011

F = 9/12 =>$C(f) = C_0 * (1 - 0.75 + 0.75\kappa) => 0.010 * (1 - 0.75 + 0.75 * 1.17)$ = 0.011

E_{foam} = (18.18 + 9.09 + 0) / 3 = 9.09 %

E_{avg} = (10.97 + 12.35 + 9.09) / 3 = 10.80 %

Conclusion:

In this part, the effect of partial fullness is examined based on the assumptions that made on the results of part A and part B. Although, sometimes the values do not comply with the theoretical results, but overall trend is a linear as the more dielectric material inserted the more capacitance value is read from the multimeter. The error rate of wood insertion is a bit high from the others as I need to hold the wood block but others can be stay standalone which one can see in the video. Overall error rate is acceptable since the multimeter is very sensitive and although I drop out the waiting capacitance of the multimeter, it may not be so careful. To sum up, this project trying to go deep into understanding capacitors by testing theories and understanding how they actually work.

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YOUTUBE LINK: https://youtu.be/Eh5Wd0myX7U