I pledge on my honor that in the solutions of homework assignments, I will NOT present anyone else's work in whole or in part. I will answer the homework questions on my own, will NOT receive any unauthorized assistance

I also acknowledge that it is my responsibility to be aware of the university's regulations on plagiarism.

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Math-230 Section 5

Answers:

1-a) Let $A = \{$ all physics books placed next to each other $\}$

$$P(A) = \frac{n(A)}{n(S)}, \text{ Find } n(A) \text{ and } n(S)$$

By fundamental counting principle, $n(S) = \underline{9}\ \underline{8}\ \underline{7}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 9!$

To find $n(A)$, we can think the physics books together, then we have 6 items to arrange which can be find by ${}_6P_6 = 6!$.

Also we can change the places of the physics books among them $\Rightarrow$ ${}_4P_4 = 4! \Rightarrow n(A) = 6!.4! \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{6!.4!}{9!} = \frac{\cancel{4}\cancel{3}\cancel{2}}{3\cancel{9}\cancel{8}7} = \frac{1}{21}$ //

b) Let $A = \{$ no two math books placed next to each other $\}$

First, we can arranged other books in a ${}_6P_6 = 6!$ ways.

^ O ^ O ^ O ^ O ^ O ^ O ^ , where O = other books. There is 7 place math books can come $\Rightarrow \binom{7}{3}.\underbrace{3!}_{\text{in between them}}$. Recall that $P(A) = \frac{n(A)}{n(S)}$

where $n(S) = 9!$. $n(A) = 6!.\binom{7}{3}.3! \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{6!\,7\,6\,5}{9!}$

$$= \frac{\cancel{7}\cancel{6}5}{3\cancel{9}\cancel{8}\cancel{7}} = \frac{5}{12} \text{ //}$$

c) Let $A = \{$ some subyects placed next to each other $\}$
We can think of the subyects together and arrange them, also we need to arrange subyects among them.

$$n(A) = {}_3P_3 \cdot {}_3P_3 \cdot {}_4P_4 \cdot {}_2P_2 = 3! \cdot 3! \cdot 4! \cdot 2!$$

Recall that $P(A) = \frac{n(A)}{n(S)}$ and $n(S) = 9!$

$$\Rightarrow P(A) = \frac{3!\ 3!\ 4!\ 2!}{9!} = \frac{\cancel{6} \cdot \cancel{6} \cdot \cancel{2}}{3\cancel{9} \cdot {}^2\cancel{8} \cdot 7 \cdot \cancel{6} \cdot 5} = \frac{1}{210} \text{ //}$$

2-a) Let $A = \{$ all the balls drawn will be the same color $\}$

$$P(A) = \frac{n(A)}{n(S)}, \quad n(S) = \binom{12}{3}, \quad n(A) = \underbrace{\binom{3}{3}}_{\text{red}} + \underbrace{\binom{4}{3}}_{\text{blue}} + \underbrace{\binom{5}{3}}_{\text{green}} = 1 + 4 + 10 = 15$$

$$\Rightarrow P(A) = \frac{\cancel{15}\ 3}{\frac{2\ \cancel{12}\ 11\ \cancel{10}\ 2}{\cancel{3}\ \cancel{2}}} = \frac{3}{44} \text{ //}$$

b) Let $A = \{$ all the balls drawn will be different color $\}$

$P(A) = \frac{n(A)}{n(S)}$, $n(S) = \binom{12}{3}$, we need to select 1 ball from all colors $\Rightarrow n(A) = \binom{3}{1}\binom{4}{1}\binom{5}{1} = 3 \cdot 4 \cdot 5 \Rightarrow P(A) = \frac{3 \cdot \cancel{4} \cdot \cancel{5}}{\frac{2\ \cancel{12}\ 11\ \cancel{10}}{3\ 2\ 1}} = \frac{3}{11}$ //

3-a) Let $A = \{$ Box 1 has exactly k balls $\}$ and $P(A) = \frac{n(A)}{n(S)}$. Balls can be in any box in sample space in other words, all balls have N box choices

$$\Rightarrow n(S) = \underbrace{\underline{N}\ \underline{N}\ \underline{N}\ \underline{N}\ \underline{N} \ \cdots \ \underline{N}}_{n \text{ ball}} = N^n.$$

We can select k balls in $\binom{n}{k}$ ways to place in Box 1, other $(n-k)$ balls will be distributed into $N-1$ boxes $\Rightarrow$

$$\underbrace{\underline{N-1}\ \underline{N-1}\ \underline{N-1}\ \underline{N-1} \ \cdots \ \underline{N-1}}_{(n-k) \text{ balls}} \Rightarrow (N-1)^{(n-k)} \cdot \binom{n}{k} = n(A)$$

$$\Rightarrow P(A) = \frac{\binom{n}{k}(N-1)^{(n-k)}}{N^n} \text{ //}$$

b) Let $A = \{$ Box N is empty $\}$, Recall that $P(A) = \frac{n(A)}{n(S)}$ and $n(S) = N^n$. $n(A) \Rightarrow$ 1'st ball has $N-1$ box options, 2'nd ball has $N-1$ box options .... n'st ball has $N-1$ box options. By fundamental counting principles,

$$n(A) = (N-1)^n \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{(N-1)^n}{N^n} \quad //$$

4) Let $A_i = \{$ For the first time, some box has two balls with adding i-th ball $\}$

$P(A_1) = 0$, since there is no ball.

$P(A_2) = $ (1'st ball adding ok). (second adding into same box) $= \frac{r}{r} \cdot \frac{1}{r}$

$P(A_3) = $ (1'st ball ok) (2'nd ball ok) (3'rd ball wrong) $= \frac{r}{r} \cdot \frac{r-1}{r} \cdot \frac{2}{r}$

$P(A_4) = \frac{r}{r} \cdot \frac{r-1}{r} \cdot \frac{r-2}{r} \cdot \frac{3}{r}$

$\vdots$

$$P(A_n) = \frac{r}{r} \cdot \frac{r-1}{r} \cdot \frac{r-2}{r} \cdots \frac{r-(n-2)}{r} \cdot \frac{(n-1)}{r} \cdot \frac{(r-(n-1))!}{(r-(n-1))!}$$

$$= \frac{r! \cdot (n-1)}{r^n \cdot (r-n+1)!} = P(A_n) \quad //$$

5-a) $P(A|C) \cup P(A|C') = P(A)$ since $P(C)$ and $P(C')$ are complements of each other, in other words $P(C) + P(C') = 1$. In the same way, $P(B|C) \cup P(B|C') = P(B)$. Since $P(x) \geq 0$ for X be any event, we can take union of $P(A|C) \geq P(B|C)$ and $P(A|C') \geq P(B|C')$ to get $P(A|C) \cup P(A|C') \geq P(B|C) \cup P(B|C')$

$\Rightarrow$ $\underline{P(A) \geq P(B)}$ as we said (union do not change the direction of $\geq$ since $P(x) \geq 0$)

b) $1 \geq P(A) + P(B) \Rightarrow 1 \geq a + b \Rightarrow 0 \geq a + b - 1$. Since $b \geq 0$, dividing both sides with b don't change the direction of $\geq$ sign $\Rightarrow 0 \geq \frac{a+b-1}{b}$. By definition, $P(A|B) \geq 0$. Then

$$P(A|B) \geq \frac{a+b-1}{b} \quad //$$

6) If they are independent, the property $P(A\cap B) = P(A)\,P(B)$ must be satisfied.

$A = \{HHT, HTH, THH, HHH\}$, $B = \{THH, TTH, THT, HTH, HTT, HHT\}$, $A \cap B = \{HHT, HTH, THH\}$

$n(S) = \underline{2}\ \underline{2}\ \underline{2} = 8 \Rightarrow P(A) = \dfrac{n(A)}{n(S)}$, $P(B) = \dfrac{n(B)}{n(S)}$,

$P(A\cap B) = \dfrac{n(A\cap B)}{n(S)} \Rightarrow P(A) = \dfrac{4}{8} = \dfrac{1}{2}$, $P(B) = \dfrac{6}{8} = \dfrac{3}{4}$,

$P(A\cap B) = \dfrac{3}{8} \Rightarrow P(A).\,P(B) = \dfrac{1}{2}\cdot\dfrac{3}{4} = \dfrac{3}{8} = P(A\cap B)$ //

7) E and F be independent events means $P(E\cap F) = P(E).\,P(F)$.
Since $P(E) = 1 - P(E')$ and $P(F) = 1 - P(F')$, we can write

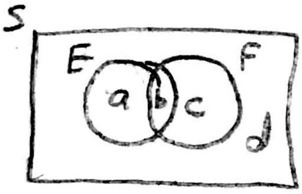
$P(E\cap F) = (1-P(E')).(1-P(F')) = 1 - P(F') - P(E') + P(E')\,P(F')$. ★

We can use De Morgan's rules to find complement of $(E\cap F)$.
$\Rightarrow (E\cap F)' = E' \cup F'$. Since $P(x) + P(x') = 1$ for any event x and its complement, $P(E\cap F) + P(E'\cup F') = 1 \Rightarrow$
$P(E\cap F) = 1 - P(E'\cup F')$. Putting that into the equation ★,

$$1 - P(E'\cup F') = 1 - P(F') - P(E') + P(E')\,P(F')$$

$$\Rightarrow P(E') + P(F') - P(E'\cup F') = P(E')\,P(F') \quad \Delta$$

Using Venn diagrams; $P(E') = \dfrac{c+d}{a+b+c+d}$, $P(F') = \dfrac{a+d}{a+b+c+d}$,



$P(E'\cup F') = \dfrac{c+d+a}{a+b+c+d} \Rightarrow P(E') + P(F') - P(E'\cup F') = \dfrac{c+d+a+d-c-d-a}{a+b+c+d} = \dfrac{d}{a+b+c+d}$

$= P(E'\cap F')$. Putting it into the equation Δ,

$P(E'\cap F') = P(E')\,P(F') \Rightarrow E'$ and F' are independent //