Objective:

The aim of this experiment is to investigate the behavior of circuits that consist of a resistor and a capacitor in series. For that, you will first study the behavior of the circuit with a constant applied voltage. And then study the response of the circuit to a rapidly varying square-wave voltage. You will also investigate the charging and discharging characteristics of capacitors.

Introduction:

A capacitor is a device that consist essentially of a sandwich of two plates of conducting material separated by an insulating material (dielectric). The primary function of a capacitor is to store charge and electrical energy when a potential difference is applied to its plates. The electricity stored by a capacitor when charged by a direct current (DC) potential is in the form of discrete electrical charges held on the surface of the plates. One plate has positive charges and the other an equal number of negative charges. When the capacitor is charged, the charge q accumulated on its plates gives rise to a potential difference V across its plates. The potential difference is proportional to the charge accumulated and the ratio of the charge to the potential difference is a constant. This constant is called the capacitance C of the capacitor and determines its ability to store charge per unit applied potential

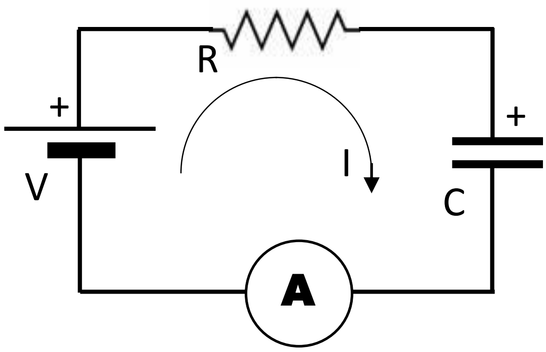
$$C = \frac{q}{V}. \qquad (1)$$

Charging a Capacitor

The amount of charge that can be stored per unit potential on a capacitor is defined as the capacitance. The capacitance of a parallel plate capacitor depends on the area of the plates, the spacing between them and the dielectric material (mica, paper, ceramic, etc) in the space. The amount of electrostatic energy U_E (in Joules) that a capacitor of C (Farads) can store when a charge of q (in Coulombs) is stored on it is given by

$$U_E = \frac{q^2}{2C}. \qquad (2)$$

A typical charging circuit is given in Fig. 1.



The course of current over time when the capacitor is charged through the resistor R at a fixed voltage V can be determined from Kirchhoff's loop theorem. The sum of the voltages gives

$$V = V_C(t) + V_R(t), \qquad (3)$$

which can be written as

$$V = \frac{q(t)}{C} + I(t)R. \qquad (4)$$

Rearranging, we get

$$I(t) = \frac{V}{R} - \frac{q(t)}{RC}. \qquad (5)$$

Differentiating both sides with respect to t and noting that we have,

$$I(t) = \frac{dq(t)}{dt}, \qquad (6)$$

we get the differential equation

$$\frac{dI(t)}{dt} = -\frac{I(t)}{RC}. \qquad (7)$$

Equation (7) shows that at any instant, the current decreases at a rate proportional to the current flowing in the circuit at that time. The only function whose rate of change (i.e., derivative) is proportional to the function itself is the exponential function. In particular, along with the initial condition that $I(t = 0) = I_0$ we write the solution of current as a function of time as

$$I(t) = I_0 e^{-t/(RC)}. \qquad (8)$$

At $t = 0$ there is no charge on the capacitor, i.e, $Q(0) = 0$, since the capacitor is fully discharged. But, at that moment, $I(0) = I_0 = V/R$, namely the current due to the resistor only. Hence, in an RC circuit when the capacitor is fully discharged at the moment of closing the switch the current will be determined by the resistor alone. At any later time, the current will decrease as the charge is accumulated on the plates. As time goes to infinity, the current will reach zero and the charge will reach a constant value VC. This relation can be obtained by integrating Eq. 8.

$$q(t) = VC\left(1 - e^{-t/(RC)}\right) \qquad (9)$$

The product RC is called the capacitive time constant, or the relaxation time of the circuit. We note that after a time t equal to RC the current drops to a fraction $\frac{I(RC)}{I_0} = e^{-1} = 0.368$, or 36.8% of its original value. Similar calculation for the charge shows that, after $t = RC$, charge accumulation on the plates of the capacitors will reach 63% of its value. These behaviors can be seen in Fig. 2.

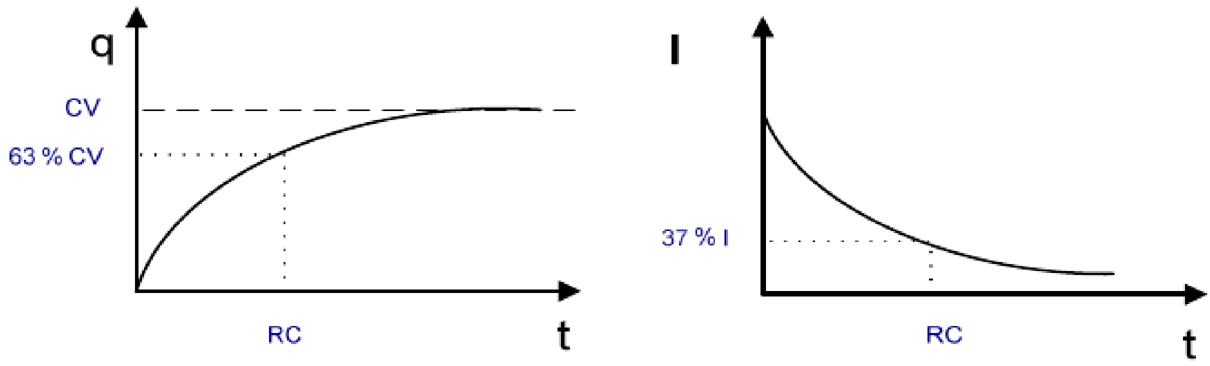


Figure 2: Charge and current behaviour of a charging capacitor

A related quantity, usually easier to measure experimentally, is the time required for the current I to drop to one-half of its original value. Denoting this time by $T_{1/2}$, we have

$$\frac{I}{I_0} = \frac{1}{2} = \exp\left(\frac{-T_{1/2}}{RC}\right). \qquad (9)$$

Taking natural logarithms of both sides and rearranging we find

$$T_{1/2} = RC \ln 2 = 0.693\, RC. \qquad (10)$$

This time is called the half-life time, a term which is also used in the description of radioactive decay processes.

Discharging a Capacitor

When the capacitor is disconnected from the power supply it starts to discharge; a current starts to flow from the positive plate, over the resistor R to the negative plate. This current causes the charge accumulated on the plates of the capacitor to decrease, which in turn decreases the potential and hence the current. Thus, the charge Q decreases rapidly at first, and then more slowly; correspondingly, the current has a relatively large initial value immediately after the switch is opened, but then falls off and approaches zero after the capacitor becomes nearly completely discharged. A typical discharge circuit is given in Fig. 3. A procedure similar to the analysis of charging can be followed for the discharging, starting with loop equation

$$V = V_C(t) - V_R(t) \qquad (11)$$

and repeating the same steps, we obtain

$$I(t) = I_0 e^{-t/(RC)}, \qquad I_0 = \frac{V}{R}, \qquad (12)$$

and

$$Q(t) = Q_0 e^{-t/(RC)}, \qquad Q_0 = VC. \qquad (13)$$

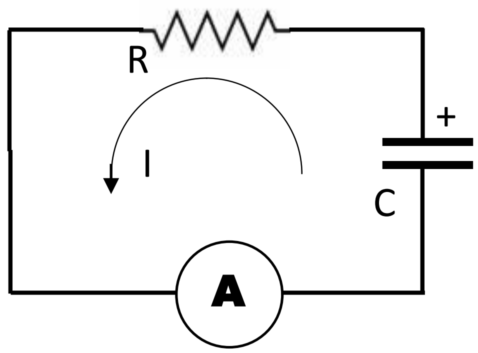


Figure 3: RC discharging circuit diagram

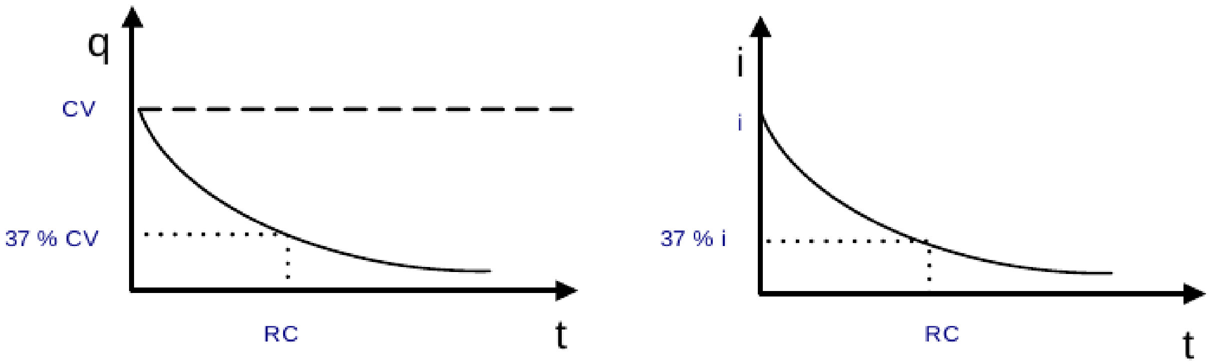


Figure 4: Charge and current behaviour of a discharging capacitor

Questions to Think About:

1. The flash light of a camera is produced by discharging a capacitor across the lamp. Why is the photoflash lamp not just connected directly to the power supply that charges the capacitor? Why is the capacitor used?
2. A battery, a resistor and a capacitor are connected in series. The resistor does not affect the amount of the charge that the capacitor stores. What purpose does the resistor serve?
3. A capacitor is found to leak charge over time. Model this leaking capacitor with the right combination of circuit elements. What does the leak time depend on?

Equipment:

The following equipment will be supplied:

- A multimeter;
- A 30 V DC power supply;
- Circuit elements (resistors, capacitors etc.);
- Oscilloscope;
- Cable sets;
- Stopwatch.

The following items must be brought by you and will not be supplied at the laboratory

- A ruler;
- A scientific calculator.

Procedure:

1. Construct the circuit using shown in Fig. 5 using the 100 μF electrolytic capacitor and one of the $k\Omega$ range resistances supplied.

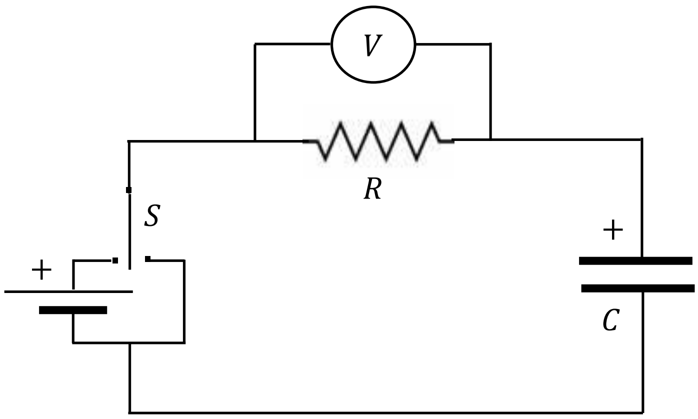


Figure 5: RC circuit

The electrolytic capacitor has polarity. If you connect it in wrong orientation, it can explode and harm you (or the other people around you). If you are not sure how to use it, consult your assistant

2. Make sure that the capacitor is initially uncharged. You can discharge a capacitor simply by short circuiting its terminals with a crocodile clamp.
3. Calculate the capacitive time constant (RC) and record it in Table 1. Use this value to estimate the time required to charge the capacitor to its half maximum and write it down in Table 1.
4. Adjust the voltage of the power supply to 20 V and set the circuit to the charging position and fully charge the capacitor.

5. Note the initial value of the current i_0 when the capacitor is fully charged. Measure the half life time $T_{1/2}$ using a stopwatch while discharging the capacitor. Record this value in Table 1.
6. Using this measurement, calculate the value of the capacitive time constant RC and print it into Table 1. Compare your empirical value with that you obtain from the values provided on the capacitor and the resistor.
7. Recharge the capacitor. This time, while discharging the capacitor, observe the decay of the voltage. Record the value of the current at every 2-3 seconds in Table 2 (a).
8. Plot i versus t on a semi-logarithmic graph paper with ***i*** recorded on the log scale using the data in Table 2(a). Note that the numbers along the side of the graph paper are just for guiding purposes. Neither the scaling nor the axis must be at that side.
9. From the slope of your plot, determine the time constant once again, and compare your result with its theoretical value. Record the results into Table 2(b).

Hint: Note that, if the current $I(t)$ is plotted against the time t on a semi-logarithmic scale, the curve is a straight line with negative slope $m = -(RC)^{-1}\log_{10} e$ and the intercept at the current axis is $\log_{10} I_0$.

10. Repeat Steps 7 through 9 for two more different values of R and C. Record the data in Tables 3(a)-(b) and 4(a)-(b). Plot the graphs on the same semi logarithmic graph paper you have used in step 8.
11. To observe rapid relaxation, set up the following circuit. Use a bipolar capacitor whose capacitance is 100 nF and a 100 Ω resistance.

Remark: In situations for which the RC time constant is too small (much shorter than a second), most multimeters of common use become incapable of responding to extremely rapid changes in voltage or current. Even if they would, the human eye cannot follow the rapidly varying meter movement (or the digital display). In this part of the experiment we shall select somewhat small values for the resistance and the capacitance. For example, if R = 10 $k\Omega$ and C = 0.1 µF, the corresponding time constant RC is 1 millisecond, which is a value far below to be detected by bare eye. To measure such a rapid relaxation of the current or a rapid change in the capacitor voltage, an oscilloscope may be used. To obtain a repetitive relaxation, we use a voltage source, sine or square wave generator, whose output can be selected as either a sinusoidal wave or a square wave. It is possible to vary the amplitude and the frequency ($f = T^{-1}$) of the output wave by playing with the wave generator control

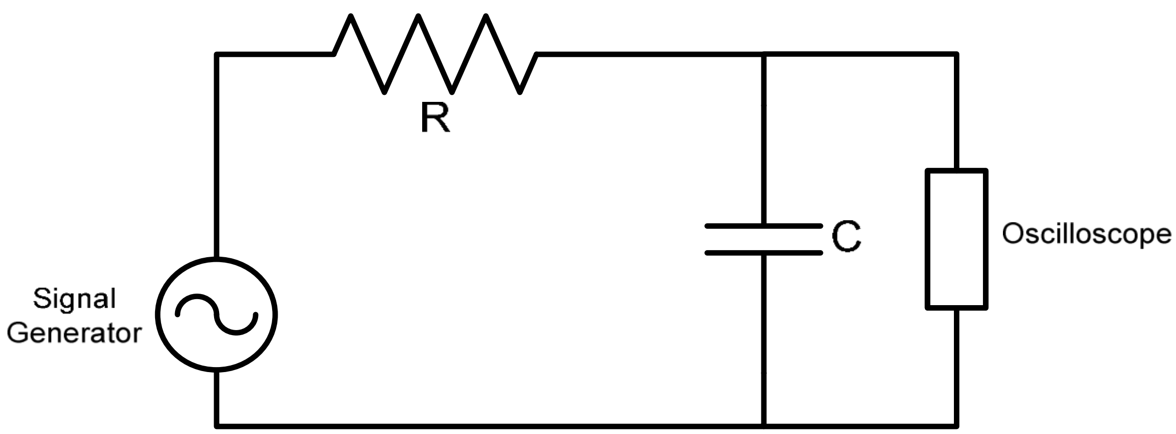


Figure 6: Set up for rapid relaxation

12. Apply a square wave at a known frequency using the signal generator.

13. Observe at least one period of the charging and discharging of the capacitor. Plot this waveform in the plot space provided in Data & Results Section.

14. Measure the half-life time and compute the RC time constant. Record this value in Table 5 and compare it with the theoretical value.

15. By changing the frequency of the signal source, alter the period of the square wave to be much higher and much smaller than the RC time constant observe the effect on the waveform, and comment on your observations.

PHYS 102 EXPERIMENT 3: RESISTANCE - CAPACITANCE CIRCUITS

Name & Surname :	**ID#:**	**Section:**

Data & Results: **[20]**

Theoretical		***Experimental***		**% Error**
RC ( )	**$T_{1/2}$ ( )**	**RC ( )**	**$T_{1/2}$ ( )**	

Table 1: Half life time and RC time constant of the circuit

R:………… C:…………..

Time (s)										
Voltage ( )										
Current ( )										

Table 2:a) Current change during discharge and b) RC time constant comparison

RC (*theoretical*)	**RC (from the slope)**	**% Error**

R:………… C:…………..

Time (s)										
Voltage ( )										
Current ( )										

Table 3:a) Current change during discharge and b) RC time constant comparison

RC (*theoretical*)	**RC (experimental)**	**% Error**

PHYS 102 EXPERIMENT 3: RESISTANCE - CAPACITANCE CIRCUITS

Name & Surname :	**ID#:**	**Section:**

R:………… C:…………..

Time (s)											
Voltage ( )											
Current ( )											

Table 4a: Current change during discharge and b) RC time constant comparison

RC ( ) **(*theoretical*)**	**RC ( )** **(experimental)**	**% Error**

[5] Plot the waveform for rapid relaxation here and show the parameters of the waveform (period, amplitude etc.):

Name & Surname :	**ID#:**	**Section:**

RC ( ) (*theoretical*)	**RC ( ) (calculated)**	**% Error**

Table 5: RC time constant comparison for rapid relaxation

Questions:

1) [5] Why do all the three lines have different slopes? Do they also have different intercepts on the current axis? Explain.

2) [5] Show that the capacitive time constant has units of time.

3) [5] What happens when RC is much higher or smaller than the period of the square wave?

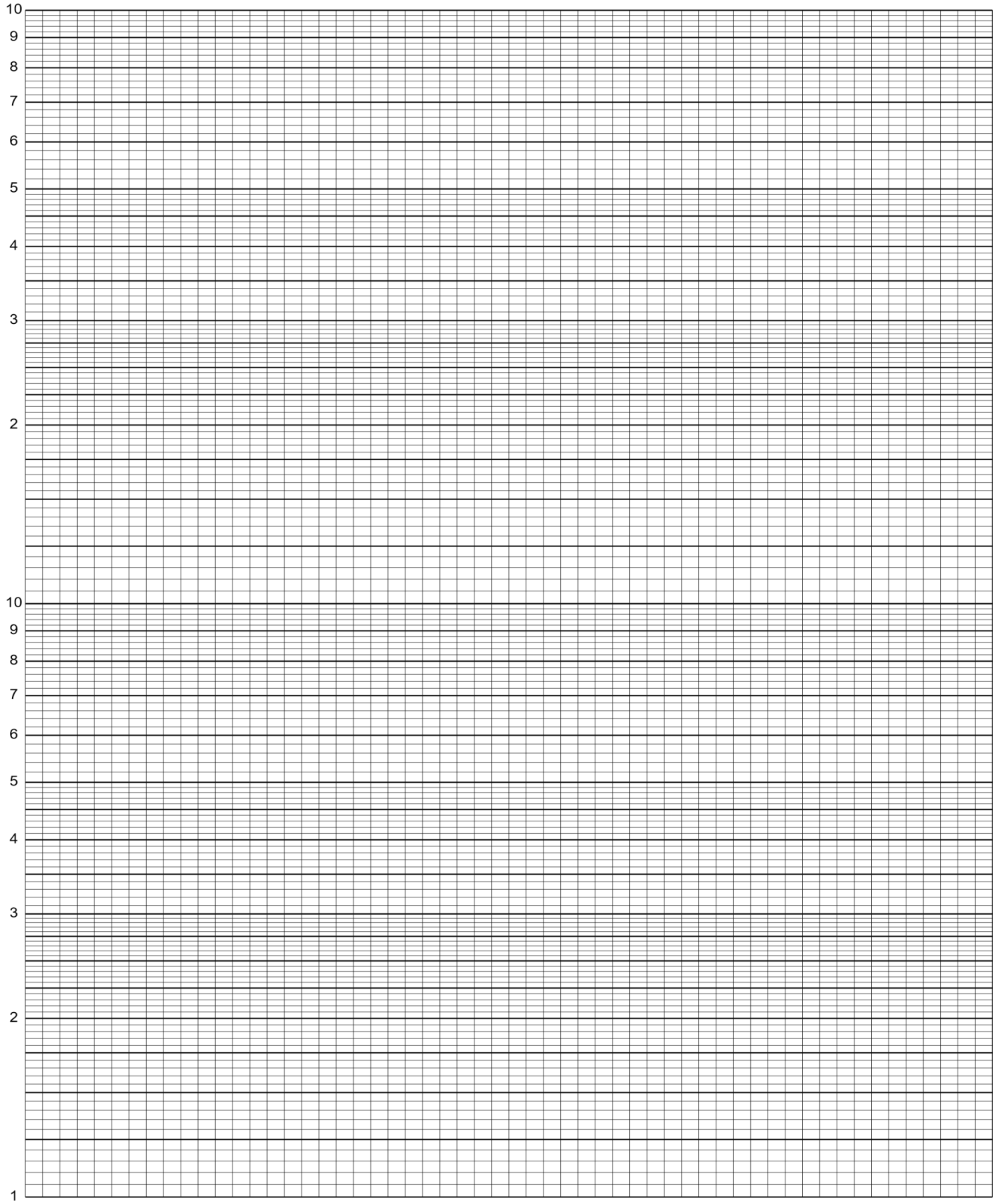
4) [5] Suppose that you want to construct a parallel plate capacitor whose capacitance is 1F. If the separation between the plates is 1mm what would be the area of the plates?

Conclusion: [10]

PHYS 102 EXPERIMENT 3: RESISTANCE - CAPACITANCE CIRCUITS

Name & Surname :	**ID#:**	**Section:**

Plot [15]



***RC* =**