EE-391 Section 2
Analytical Assignment 1
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1) Solve the following equation for $\theta$:

Find all possible answers. Make sure to give the final answer in radians.

$Re\left[(1-j)e^{j\theta}\right] = 1 - Re\left[(1-j)e^{-j\theta}\right]$

$$Re\left[\left(1-e^{j\frac{\pi}{2}}\right)e^{j\theta}\right] = 1 - Re\left[\left(1-e^{j\frac{\pi}{2}}\right)e^{-j\theta}\right]$$

$$Re\left[e^{j\theta} - e^{j\left(\frac{\pi}{2}+\theta\right)}\right] = 1 - Re\left[e^{-j\theta} - e^{j\left(\frac{\pi}{2}-\theta\right)}\right]$$

using Euler's formula:

$$Re\left[\cos\theta + j\sin\theta - \cos\left(\frac{\pi}{2}+\theta\right) - j\sin\left(\frac{\pi}{2}+\theta\right)\right]$$

$$= 1 - Re\left[\cos(-\theta) + j\sin(-\theta) - \cos\left(\frac{\pi}{2}-\theta\right) - j\sin\left(\frac{\pi}{2}-\theta\right)\right]$$

$\cos(-\theta) = \cos(\theta)$, $\sin(-\theta) = -\sin(\theta)$, $\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$, $\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$,

$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$, $\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$, $\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$

$$\Rightarrow Re\left[\cos\theta + j\sin\theta + \sin\theta - j\cos\theta\right] = 1 - Re\left[\cos\theta - j\sin\theta - \sin\theta - j\cos\theta\right]$$

$$\Rightarrow Re\left[2\cos\theta - 2j\cos\theta\right] = 1$$

$$\Rightarrow 2\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2}$$

$\nearrow$ $\theta = \arccos\left(\frac{1}{2}\right) \Rightarrow \theta = \frac{\pi}{3} + 2\pi k$ where $k \in \mathbb{Z}$

$\searrow$ $\theta = -\arccos\left(\frac{1}{2}\right) \Rightarrow \theta = -\frac{\pi}{3} + 2\pi k$ where $k \in \mathbb{Z}$

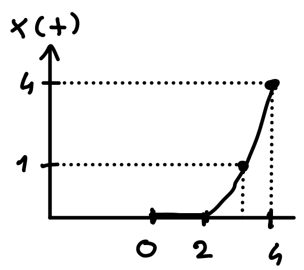
2) A periodic signal $x(t)$ with a period of $T = 4$ is described over one period $0 < t < 4$ by the equation:

$$x(t) = \begin{cases} (t-2)^2 & 2 \le t \le 4 \\ 0 & 0 \le t < 2 \end{cases}$$

This signal can be represented by the Fourier series which is valid for all time.

a) Sketch the periodic function $x(t)$
b) Find the $a_k$ coefficient.
c) Compare the area of curve in one period and DC coefficient $a_0$ values.
d) If $y(t) = \frac{d}{dt}x(t)$, and $b_k$ is the Fourier series of $y(t)$, find $b_2$

a)



b) $$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j\left(\frac{2\pi}{T_0}\right)kt}\,dt = \frac{1}{4}\int_0^4 x(t)\, e^{-j\frac{\pi}{2}kt}\,dt$$

$$= \frac{1}{4}\left[\int_0^2 0\,dt + \int_2^4 (t-2)^2 e^{-j\frac{\pi}{2}kt}\,dt\right] = \frac{1}{4}\int_2^4 (t-2)^2 e^{-j\frac{\pi}{2}kt}\,dt$$

Lets use Integration by Parts to solve that integral:

$$\left.\begin{aligned} u &= (t-2)^2 \Rightarrow du = 2(t-2)\,dt \\ dv &= e^{-j\frac{\pi}{2}kt}dt \Rightarrow v = \frac{e^{-j\frac{\pi}{2}kt}}{-j\frac{\pi}{2}k}\end{aligned}\right\} \Rightarrow a_k = \frac{1}{4}\left(\left[(t-2)^2 \cdot \frac{e^{-j\frac{\pi}{2}kt}}{-j\frac{\pi}{2}k}\right]\Bigg|_2^4 - \int_2^4 \frac{e^{-j\frac{\pi}{2}kt}}{-j\frac{\pi}{2}k}\cdot 2\cdot(t-2)\,dt\right)$$

$$\Rightarrow a_k = \frac{1}{4}\left(\frac{4\cdot e^{-2j\pi k}}{-j\frac{\pi}{2}k} - 0 + \frac{4}{j\pi k}\int_2^4 e^{-j\frac{\pi}{2}kt}\cdot(t-2)\,dt\right)$$

$$= \frac{1}{j\pi k}\int_2^4 e^{-j\frac{\pi}{2}kt}(t-2)\,dt - \frac{2e^{-2j\pi k}}{j\pi k}$$

use one more integration by parts;

$$\left.\begin{aligned} u &= t-2 \rightarrow du = dt \\ dv &= e^{-j\frac{\pi}{2}kt}dt \rightarrow v = \frac{e^{-j\frac{\pi}{2}kt}}{-j\frac{\pi}{2}k}\end{aligned}\right\} \Rightarrow a_k = \frac{1}{j\pi k}\left(\left[(t-2)\cdot\frac{e^{-j\frac{\pi}{2}kt}}{-j\frac{\pi}{2}k}\right]\Bigg|_2^4 - \int_2^4 \frac{e^{-j\frac{\pi}{2}kt}}{-j\frac{\pi}{2}k}\,dt\right) - \frac{2e^{-2j\pi k}}{j\pi k}$$

$$\Rightarrow a_k = \frac{1}{j\pi k}\left(\frac{2\,e^{-2j\pi k}}{-j\frac{\pi}{2}k} - 0 + \frac{2}{j\pi k}\int_2^4 e^{-j\frac{\pi}{2}kt}\,dt\right) - \frac{2e^{-2j\pi k}}{j\pi k}$$

$$\Rightarrow a_k = \frac{2}{(j\pi k)^2}\,\frac{e^{-j\frac{\pi}{2}kt}}{-j\frac{\pi}{2}k}\Bigg|_2^4 - \frac{4e^{-2j\pi k}}{(j\pi k)^2} - \frac{2e^{-2j\pi k}}{j\pi k}$$

$$\Rightarrow a_k = \frac{2}{(j\pi k)^2}\,\frac{e^{-2j\pi k} - e^{-j\pi k}}{-j\frac{\pi}{2}k} - \frac{4e^{-2j\pi k}}{(j\pi k)^2} - \frac{2e^{-2j\pi k}}{j\pi k}$$

3

$$\left.\begin{array}{l} e^{j\pi} = e^{-j\pi} = -1 \\ j^2 = -1 \end{array}\right\} \Rightarrow a_k = \left[\frac{4\left[(-1)^{2k} - (-1)^{k}\right]}{j(\pi k)^2} + \frac{4\,(-1)^{2k}}{(\pi k)^2} - \frac{2\,(-1)^{2k}}{j\pi k}\right]$$

$(k \neq 0)$

for $k = 0$:

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^0\, dt = \frac{1}{4}\int_0^2 0\, dt + \frac{1}{4}\int_2^4 (t-2)^2\, dt = \frac{1}{4}\left.\frac{(t-2)^3}{3}\right|_2^4$$

$$= \frac{1}{4}\left(\frac{8}{3} - 0\right) = \frac{2}{3}$$

$$a_k = \begin{cases} \dfrac{8}{j(\pi k)^2} + \dfrac{4}{(\pi k)^2} - \dfrac{2}{j\pi k} & \text{for } k \text{ odd} \\ \dfrac{2}{j\pi k} - \dfrac{4}{(\pi k)^2} & \text{for } k \text{ even} \\ \dfrac{2}{3} & \text{for } k = 0 \end{cases}$$

c) $\text{Area} = \int_0^4 x(t)\, dt = \int_0^4 x(t)\, e^{-j\frac{\pi}{2} t 0}\, dt = a_0 = \frac{2}{3}$ (calc. at b)

Therefore, they are equal.

d) $y(t) = \begin{cases} 2(t-2) & 2 \leq t \leq 4 \\ 0 & 0 \leq t < 2 \end{cases}$

$$b_k = \frac{1}{T_0}\int_0^{T_0} y(t)\, e^{-j\frac{2\pi}{T_0}kt}\, dt \Rightarrow b_2 = \frac{1}{4}\int_2^4 2(t-2)\, e^{-j\frac{\pi}{2}2t}\, dt$$

$$= \frac{1}{2}\int_2^4 (t-2)\, e^{-j\pi t}\, dt$$

using integration by parts:

$$\left.\begin{array}{l} u = t-2 \rightarrow du = dt \\ dv = e^{-j\pi t} dt \rightarrow v = \dfrac{e^{-j\pi t}}{-j\pi} \end{array}\right\} \Rightarrow b_2 = \frac{1}{2}\left.\left((t-2)\cdot\frac{e^{-j\pi t}}{-j\pi}\right)\right|_2^4 - \frac{1}{2}\int_2^4 \frac{e^{-j\pi t}}{-j\pi}\, dt$$

$$\Rightarrow b_2 = \frac{1}{2}\left[\frac{2\,(-1)^4 - 0}{-j\pi} - \frac{(-1)^4 - (-1)^2}{(j\pi)^2}\right] = -\frac{1}{j\pi} = b_2$$

3) Figure 1 shows an ideal C to D and D to C converter.
   a) Suppose that the discrete-time signal $x[n]$ is:

$$x[n] = 5\cos(0.26\pi n + 30^{\circ})$$

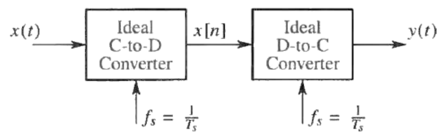



Figure 1

If the sampling rate is $f_s = 500$ samples per second, determine two different continuous-time signals that could have been inputs to the system.

$x[n] = 5\cos(0.26\pi n + 30^{\circ})$ $\quad f_s = 500$ hz $\quad \phi = 30^{\circ} = \frac{\pi}{6}$

Recall that $x(t)$ uses $\omega$, while $x[n]$ uses $\hat{\omega}$ (normalized radian frequency). Then,

$$\hat{\omega} = \frac{\omega}{f_s} = \frac{2\pi f_o}{f_s} = 0.26\pi \pm 2\pi \text{ (for alias frequencies)}$$

we need to find two $f_o$ to determine two different input to the system.

Let $\hat{\omega} = 0.26\pi$;

$$f_o = \frac{0.26\pi \cdot f_s}{2\pi} = 0.13 \times 500 = 65 \text{ Hz}$$

$x_1(t) = 5\cos(\omega_o t + \varphi) = 5\cos(2\pi f_o t + \varphi) = 5\cos(130\pi t + \pi/6)$ is the first signal.

To find the second signal, let $\hat{\omega} = 2.26\pi$;

$$f_o = \frac{2.26\pi \cdot f_s}{2\pi} = 1.13\pi \times 500 = 565 \text{ hz}$$

$\Rightarrow x_2(t) = 5\cos(2\pi(565)t + \frac{\pi}{6}) = 5\cos(1130\pi t + \frac{\pi}{6})$ is the second signal.

at could have been inputs to the system.

b) $x(t)$ is shown in Figure 2. Determine a formula for $y(t)$ when $f_s = 1000$ for both converters.

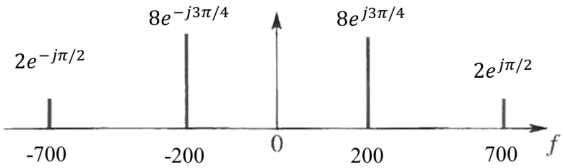



Figure 2

$$x(t) = 2e^{-j\pi/2}\, e^{-j2\pi(700)t} + 8e^{-j3\pi/4} \cdot e^{-j2\pi(200)t} + 0 + 2e^{j\pi/2}\, e^{j2\pi(700)t} + 8e^{j3\pi/4} \cdot e^{j2\pi(200)t}$$

using Inverse Euler Formulas $\Rightarrow$ $x(t) = 4\cos(2\pi 700t + \pi/2) + 16\cos(2\pi 200t + \frac{3\pi}{4})$

$$x[n] = x\left(\frac{n}{f_s}\right) = x\left(\frac{n}{1000}\right) = 4\cos\left(\frac{2\pi 700 n}{1000} + \pi/2\right) + 16\cos\left(\frac{2\pi 200 n}{1000} + 3\pi/4\right)$$

Let's apply reconstruction process to cosine components separately. We expect to get 200 Hertz signal right and 700 Hertz signal wrong since $1000 > 200 \times 2$ and $1000 < 700 \cdot 2$ (Nyquist rate). Reconstructor by convention takes spectral lines in $[-\pi, \pi]$. For first signal:

$$\left.\begin{array}{ll} \hat{\omega}_1 = 1.4\pi + 2\pi l & l = \pm 1, \pm 2, \pm 3, \ldots \\ \hat{\omega}_1 = -1.4\pi + 2\pi l & l = \pm 1, \pm 2, \pm 3, \ldots \end{array}\right\} \Rightarrow \begin{array}{l} \hat{\omega}_0 = \{-0.6\pi, 0.6\pi\} \\ \text{will be in } [-\pi, \pi] \end{array}$$

reconstruction:

$$\hat{\omega}_1 = \frac{\omega_1}{f_s} \rightarrow \omega_1 = \pm\hat{\omega}_1 f_s = \pm 0.6\pi \cdot 1000 = 600\pi \text{ rad/sec}$$

$$\rightarrow f_1 = \pm\frac{\omega_1}{2\pi} = \pm\frac{600\pi}{2\pi} = 300 \text{ Hz} \rightarrow \text{we get signal with 700 Hertz wrong}$$

For the second signal, same idea gives $\hat{\omega}_0 = \{-0.4\pi, 0.4\pi\} \in [-\pi, \pi]$

reconstruction:

$\omega_2 = \pm\hat{\omega}_2 f_s = \pm 0.4\pi \cdot 1000 = 400\pi$ rad/sec, $f_2 = 400\pi/2\pi = 200$Hz ✓

Then $y(t) = 4\cos(2\pi 300t + \pi/2) + 16\cos(2\pi 200t + 3\pi/4)$ //

4) A signal is given as below:

$$x(t) = [3 + \cos(3\pi(500)t)]\cos(3\pi(1000)t)$$

a) Is this signal periodic? If so, what is the period of the signal.
b) Plot the two-sided spectrum of this signal.
c) What relation should the sampling rate satisfy so that $y(t) = x(t)$ in figure 1?

$x(t) = 3\cos(2\pi(1500)t) + \cos(2\pi(750)t).\cos(2\pi(1500)t)$

Let $\cos(2\pi(750)t) = \cos x$, $\cos(2\pi(1500)t) = \cos 2x$

using trig. identity:

$\cos a.\cos b = \left(\frac{1}{2}\right)\cos(a-b) + \left(\frac{1}{2}\right)\cos(a+b) \Rightarrow$

$\cos x.\cos 2x = \left(\frac{1}{2}\right)\cos(2x-x) + \frac{1}{2}\cos(x+2x) = \frac{\cos x}{2} + \frac{\cos 3x}{2}$

$\Rightarrow x(t) = 3\cos(2\pi(1500)t) + \frac{\cos(2\pi(750)t)}{2} + \frac{\cos(2\pi(2250)t)}{2}$

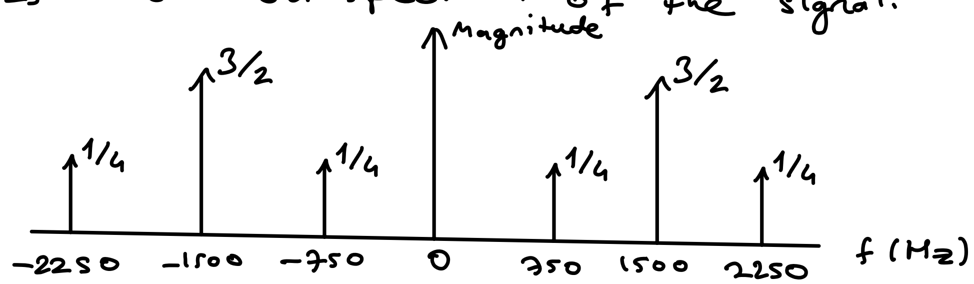
a) finding the periods of individual components using the structure of $A\cos(2\pi f_0 t)$,

period of the signal $= \gcd(1500, 750, 2250) = 750$ hz
signal is periodic.

b) writing $x(t)$ in exponential form:

$$x(t) = \frac{3}{2}\left[e^{j2\pi(1500)t} + e^{-j2\pi(1500)t}\right] +$$
$$\frac{1}{4}\left[e^{j2\pi(750)t} + e^{-j2\pi(750)t}\right] +$$
$$\frac{1}{4}\left[e^{j2\pi(2250)t} + e^{-j2\pi(2250)t}\right]$$

$\Rightarrow$ Two sided spectrum of the signal:




c) According to the rule of Nyquist rate which requires sampling rate to be the twice of the maximum frequency, the sample rate fs should be at least $f_s = 2 \cdot 2250 = 4500$ hz

5) Filter coefficients of an FIR system are $\{b_k\} = \{-2, 2, 4, 6\}$. Determine the $y[n]$ if $x[n]$ is:

$$x[n] = \begin{cases} -1 & n = 3k \\ 0 & n = 3k+1 \\ 1 & n = 3k+2 \end{cases} \qquad k \in Z$$

$$y[n] = \sum_{k=0}^{4} b_k\, x[n-k] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3] + b_4 x[n-4]$$

for some values of $n$, $y$ equals:

$$y[0] = -2\,x[0] + 2\,x[-1] + 4\,x[-2] + 6\,x[-3]$$
$$= -2.(-1) + 2.1 + 4.0 + 6.(-1) = -2$$

$$y[1] = -2\,x[1] + 2\,x[0] + 4\,x[-1] + 6\,x[-2]$$
$$= -2.0 + 2.(-1) + 4.1 + 6.0 = 2$$

$$y[2] = -2\,x[2] + 2\,x[1] + 4\,x[0] + 6\,x[-1]$$
$$= -2.1 + 2.0 + 4.(-1) + 6.1 = 0$$

$$y[3] = -2\,x[3] + 2\,x[2] + 4\,x[1] + 6\,x[0]$$
$$= -2.(-1) + 2.1 + 4.0 + 6(-1) = -2$$

$$y[4] = -2\,x[4] + 2\,x[3] + 4\,x[2] + 6\,x[1]$$
$$= -2.0 + 2.(-1) + 4.1 + 6.0 = 2$$

$\vdots$

$y$ output is repeated in every three unit increase in $n$. Therefore $y$ can be defined as,

$$y[n] = \begin{cases} -2 & n = 3k \\ 2 & n = 3k+1 \\ 0 & n = 3k+2 \end{cases} \qquad k \in Z$$

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6) For each of the systems, determine whether or not the system is (1) linear (2) time-invariant and (3) casual.

(a) $y[n] = 2x[n]\cos(\pi n)$

(b) $y[n] = x[n] - x[2n+1]$

(c) $y[n] = -x[n]^2$

(d) $y[n] = x[n] - \text{u}[n]$

(e) $y[n] = 2^{x[n]}$

(f) $y[n] = 5 + x[n]$

For linearity; consider $x_1[n] \rightarrow y_1[n]$

$x_2[n] \rightarrow y_2[n]$

form $\alpha x_1[n] + \beta x_2[n] = x_3[n] \rightarrow y_3[n]$

check where $y_3[n] \overset{?}{=} \alpha y_1[n] + \beta y_2[n]$ for all $\alpha, \beta, x_1[n], x_2[n], n$

$x[n] \rightarrow y[n]$, $x[n-n_0] = x_1[n] \rightarrow y_1[n]$,

$y_1[n] = y[n-n_0]$ for all $x[n], n, n_0 \Rightarrow$ time invariant

if $x[n+1], x[n+2], \ldots$ needed to compute $y[n] \Rightarrow$ Causal

a) <u>Linearity:</u>

$y_1[n] = 2x_1[n]\cos(\pi n)$, $y_2[n] = 2x_2[n]\cos(\pi n)$

$y_3[n] = 2x_3[n]\cos(\pi n) = (2\alpha x_1[n] + 2\beta x_2[n])\cos(\pi n)$

$= 2\alpha x_1[n]\cos(\pi n) + 2\beta x_2[n]\cos(\pi n)$

$= \alpha y_1[n] + \beta y_2[n] =$ linear ✓

<u>Time-Invariance:</u>

$y_1[n] = 2x_1[n]\cos(\pi n) = 2x[n-n_0]\cos(\pi n)$

but $y[n-n_0] = 2x[n-n_0]\cos(\pi(n-n_0))$ (replace $n$ with $n-n_0$)

since $y_1(n) \neq y[n-n_0] \Rightarrow$ Not time-invariant //

<u>Causality:</u>

Causal since we only need to know $x[n]$ at $n=n_0$ to compute $y[n_0]$ for any $n_0$.

b) Linearity

$y_1[n] = x_1[n] - x_1[2n+1]$ , $y_2[n] = x_2[n] - x_2[2n+1]$

$$y_3[n] = x_3[n] - x_3[2n+1] = \alpha x_1[n] + \beta x_2[n] - \alpha x_1[2n+1] - \beta x_2[2n+1]$$

$$= \alpha y_1[n] + \beta y_2[n] = \text{linear} \checkmark$$

Time-Invariance:

$$y_1[n] = x_1[n] - x_1[2n+1] = x_1[n-n_0] - x_1[2n-2n_0+1]$$

since $y[n] = x[n] - x[2n+1]$; we see that

$$y[n-n_0] = x[n-n_0] - x[2n-2n_0+1]$$

$\Rightarrow y_1[n] = y[n-n_0] \Rightarrow$ Time-Invariant $\checkmark$

Causality

To compute $y[n]$, we need to know $x[n]$ and $x[2n+1]$ since future value is used $\Longrightarrow$ not causal //

c) Linearity:

$y_1[n] = -x_1[n]^2$, $y_2[n] = -x_2[n]^2$,

$$y_3[n] = -x_3[n]^2 = -[\alpha x_1[n] + \beta x_2[n]]^2$$

$$= -\alpha^2 x_1^2[n] - \beta^2 x_2^2[n] - 2\alpha\beta x_1[n] x_2[n]$$

$$\neq -\alpha x_1^2[n] - \beta x_2^2[n] \Rightarrow \text{not linear} //$$

Time Invariance:

$$y_1[n] = -x_1[n]^2 = -x_1[n-n_0]^2 = y[n-n_0] \Rightarrow \text{Time Invariant} \checkmark$$

Causality:

Causal since $y[n_0]$ requires only $x[n_0]$ for any $n_0$. //

d) $y[n] = x[n] - u[n]$ (10)

Linearity:

$y_1[n] = x_1[n] - u[n]$ , $y_2[n] = x_2[n] - u[n]$

$y_3[n] = x_3[n] - u[n] = \alpha x_1[n] + \alpha x_2[n] - u[n]$

$\neq \alpha x_1[n] - \alpha u[n] + \beta x_2[n] - \beta u[n]$

$\neq \alpha y_1[n] + \beta y_2[n] \Rightarrow$ Not linear

Time Invariance:

$y_1[n] = x_1[n] - u[n] = x_1[n-n_0] - u[n]$

$\neq y[n-n_0] = x_1[n-n_0] - u[n-n_0]$

$\Rightarrow$ Not time invariant

Causality:

$n_0$ is enough to find $y[n_0]$ for any $n_0 \Rightarrow$ causal ✓

e) $y[n] = 2^{x[n]}$

Linearity:

$y_1[n] = 2^{x_1[n]}$ , $y_2[n] = 2^{x_2[n]}$

$y_3[n] = 2^{x_3[n]} = 2^{\alpha x_1[n] + \beta x_2[n]}$

$\neq \alpha 2^{x_1[n]} + \beta 2^{x_2[n]} = \alpha y_1[n] + \beta y_2[n]$

$\Rightarrow$ not linear

Time Invariance:

$y[n] = 2^{x[n]} \xRightarrow[n=n-n_0]{} 2^{x[n-n_0]} = y[n-n_0] \Rightarrow$ time-invariant ✓

Causality:

$n_0$ is enough to find $y[n_0]$ for any $n_0 \Rightarrow$ causal ✓

f) $y[n] = 5 + x[n]$

Linearity:

$y_1[n] = 5 + x_1[n]$ , $y_2[n] = 5 + x_2[n]$

$y_3[n] = 5 + x_3[n] = 5 + \alpha x_1[n] + \beta x_2[n]$

$\neq 5\alpha + \alpha x_1[n] + 5\beta + \beta x_2[n]$

$= \alpha y_1[n] + \beta y_2[n] \Rightarrow$ not linear //

Time-Invariance:

$y[n] = 5 + x[n] \xRightarrow[(n = n - n_0)]{} y[n - n_0] = 5 + x[n - n_0] \Rightarrow$ time-invariant ✓

Causality:

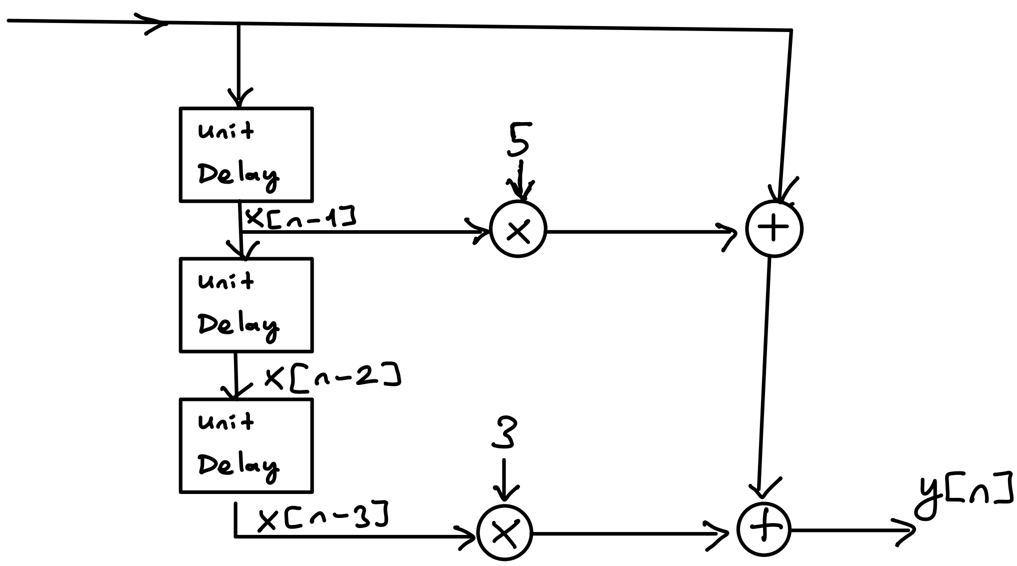
$n_0$ is enough to find $y[n_0]$ for any $n_0 \Rightarrow$ causal ✓

7) A linear time-invariant system is described by the difference equation.

$$y[n] = x[n] + 5x[n-1] + 3x[n-3]$$

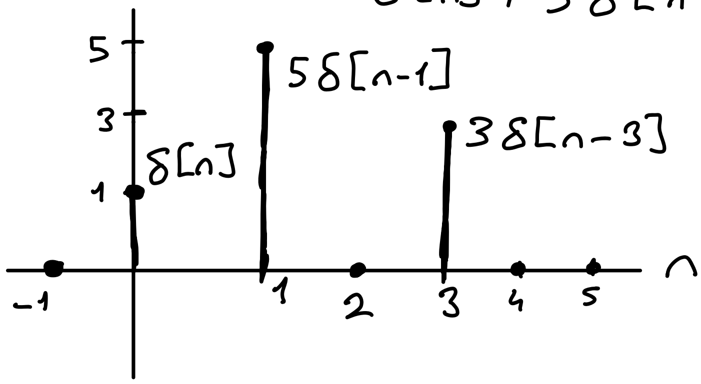
a) Draw the implementation of this system as a block diagram in direct form.
b) Write the impulse response for this system and plot it.

a) $x[n]$

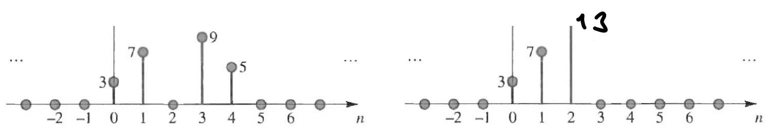


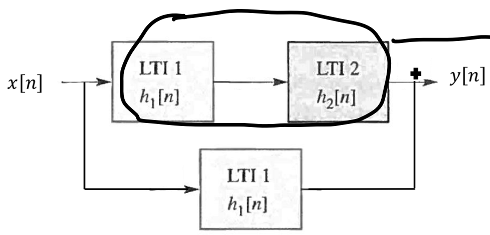
b) set $x[n] = \delta[n]$, then $y[n] = h[n]$

$\Rightarrow h[n] = \delta[n] + 5\delta[n-1] + 3\delta[n-3]$




8) For two linear time-invariant systems, $h_1[n], h_2[n]$ are given in the figures below. Find $y[n]$, when $x[n] = 2u[n]$




→ cascaded

$\rightarrow h_{12}[n] = h_1[n] * h_2[n]$

Needed to perform convolution to get $h_{12}[n]$

Write the system in impulse response form,

$h_1[n] = 3\delta[n] + 7\delta[n-1] + 9\delta[n-3] + 5\delta[n-4]$

$h_2[n] = 3\delta[n] + 7\delta[n-1] + 13\delta[n-2]$

$h_{12}[n] = h_1[n] * (3\delta[n] + 7\delta[n-1] + 13\delta[n-2])$

$= h_1[n] * 3\delta[n] + h_1[n] * 7\delta[n-1] + h_1[n] * 13\delta[n-2]$

Recall that $f[n] * \delta[n-n_0] = f[n-n_0]$ (1)

$h_{12}[n] = 3h_1[n] + 7h_1[n-1] + 13h_1[n-2]$

$$3h_1[n] = 9\delta[n] + 21\delta[n-1] + 27\delta[n-3] + 15\delta[n-4]$$

$$7h_1[n-1] = 21\delta[n-1] + 49\delta[n-2] + 63\delta[n-4] + 35\delta[n-5]$$

$$13h_1[n-2] = 39\delta[n-2] + 91\delta[n-3] + 117\delta[n-5] + 65\delta[n-6]$$

$$\Rightarrow h_{12}[n] = 9\delta[n] + 42\delta[n-1] + 88\delta[n-2] + 118\delta[n-3] + 78\delta[n-4] + 152\delta[n-5] + 65\delta[n-6]$$

In the next step, $h_{12}[n]$ is parallel to the $h_1[n]$

$$\Rightarrow h[n] = h_{12}[n] + h_1[n]$$
$$= 12\delta[n] + 49\delta[n-1] + 88\delta[n-2] + 127\delta[n-3] + 83\delta[n-4] + 152\delta[n-5] + 65\delta[n-6]$$

Next, $x[n]$ and $h[n]$ is cascaded

$$\Rightarrow y[n] = x[n] * h[n] \text{ and } x[n] = 2u[n]$$

$$\Rightarrow y[n] = x[n] * 12\delta[n] + x[n] * 49\delta[n-1] + x[n] * 88\delta[n-2] + x[n] * 127\delta[n-3] + x[n] * 83\delta[n-4] + x[n] * 152\delta[n-5] + x[n] * 65\delta[n-6]$$

using (1) again:

$$y[n] = 12x[n] + 49x[n-1] + 88x[n-2] + 127x[n-3] + 83x[n-4] + 152x[n-5] + 65x[n-6]$$

put $x[n] = 2u[n]$,

The output of the system, $y(n)$ is the following:

$$y[n] = 24u[n] + 98u[n-1] + 176u[n-2] + 254u[n-3] + 166u[n-4] + 304u[n-5] + 130u[n-6]$$