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Dynamic Programming in Orienteering: Route Choice and the Siting of Controls

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A simple functional equation of dynamic programming is used to generate optimal orienteering routes in mountainous country. Comparisons are made between computed and actual results, further refinements in the handling of data considered, and the benefits to competitors and organizers of orienteering-type events discussed.

INTRODUCTION

THERE ARE MANY applications, both actual and potential, of operational research in sport, and particularly of dynamic programming. It has been said that dynamic programming is a technique looking for applications, but it may well be that in sport there are at least as many applications of dynamic programming as of any other O.R. technique.[1] In this paper we aim to help planners of orienteering events site their 'controls', places that a competitor must visit. We do this by analyzing optimal routes between controls which have been determined from a simple functional equation.

THE NATURE OF THE PROBLEM

At the start of an orienteering event, competitors are given a map on which are pin-pointed, by small circles or six-figure map-references, certain locations, known as control sites, that they must visit. A control site may be a natural feature, such as a col or stream bed, or man-made, such as a cairn or sheep pen. It is usual to place a small labelled flag, about knee-high, at a control so that the competitor knows that he has found the right location. It is also usual for the competitor to be able to prove that he has visited the right location either by having a marshall placed at the control site, or by attaching to the small flag a clipper-type punch, which can be used by the competitor to stamp a card that he carries. Competitors receive a copy of the map with the controls only when they start, so that it is not possible for them to work out which route to take prior to the event.

We shall be considering some of the more arduous of these competitions, held in the Lake District and taking from four to eight hours to complete. A planner of such events needs to exercise considerable care in designing a course. He must take account of the needs of marshalls who may be manning the control sites, of rescue teams who might have to search for lost competitors, and of the varying abilities of participants. But within these constraints, he would like to design a course that is not only physically, but mentally challenging.

REPRESENTATION OF THE PROBLEM

Consider the map in Figure 1, where competitors had to find their way from the third control site at Knott Rigg to the fourth at High Spy. This was one of the seven legs in the 1974 Lake District Mountain Trial. How will the competitor interpret the map to help him make the best choice? In this case it will depend on his successful interpretation of the contours (which, in our illustrations, have been simplified to intervals of 50 metres). In particular, he will be wary of dangerous steep ground, where the contours are very close together, and he will want to avoid unnecessary climbing and descending when a series of ridges and valleys are crossed at right angles. In arriving at a decision, the competitor will be hindered by such factors as rain, wind, heat, fog or his own physical exhaustion, and it will require further skill for him to implement his choice. Our problem is much simpler: to arrive at an armchair solution using some (principally the contour) information available to him.

TRANSFORMING AND SIMPLIFYING THE INFORMATION ON THE MAP

Over the relevant area of Figure 1 we imposed a 19×19 grid, each grid line being at 250 metres intervals on the ground. The grid was used to identify a 19×19 table of spot heights, each height being recorded at the intersection of the grid lines. We also distinguished those intersections which occurred on rough ground (in areas of crags requiring rock-climbing skills to move about). This extracted information is given in the Appendix. We then generated optimal routes solely by referring to the data in this Appendix.

GENERATING TRAVEL TIMES

Travel times were calculated between each grid point and all its adjacent points. 'Adjacent' is taken to mean the nearest point (if there is one) in a N, S, E, W, NW, NE, SW or SE direction. By referring to the spot heights in the Appendix, travel times were calculated according to the following rules.

(1) *Flat or uphill.* Allow 10 minutes per mile + 10 minutes per 1000′ of ascent.
(2) *Gentle downhill* (under 500′ per mile). Allow eight minutes per mile.
(3) *Steep downhill* (over 500′ per mile). Allow 10 minutes per mile + two minutes per 1000′ of descent.
(4) *Rough ground on flat, uphill or gently downhill sections.* Allow 15 minutes per mile + 15 minutes per 1000′ of ascent.
(5) *Rough, steep downhill.* Allow 10 minutes per mile + 10 minutes per 1000′ of descent. Note that if either of two adjacent points was classed as 'rough', the connecting section between was classed likewise.

The calculations used the metric equivalent of these rules, though they are given here in their non-metric form as they use measurements which are more assessable. In particular, they are an extension of the well-known 'Naismith Rules'[2] used by generations of hikers: "Allow 20 minutes per mile + 30 minutes for every 1000′ climbed". This was extended to the five rules above by one of the authors (M.H.) when supervising and assisting fell runners to complete a classic, 24-hour Lakeland circuit (the Bob Graham Round). This round covers 42 summits, 70 miles and 27,000′ of ascent and descent. The extended Naismith rules were used to predict arrival times at various points to assist observers and pacemakers. The derivation and testing of the extended Naismith rules will be described in a separate paper.

APPLICATION OF DYNAMIC PROGRAMMING TO THE TRANSFORMED PROBLEM

We have explained how the grid network allows eight connections, at equal points of the compass, to be made from any point. This enables a programme to generate optimal routes with a varied choice of twists and turns. However, it also means that, even between points fairly close together, there is a large number of distinct routes which would have to be evaluated in the search for the optimum. For example, if travelling from one point to an adjacent point is regarded as a 'step', then from any central point in the grid:

there are 56 distinct ways to move 2 steps from this point;

there are 368 distinct ways to move 3 steps from this point.

In our grid, an optimal route might contain 20 or 30 steps. Even with powerful computing

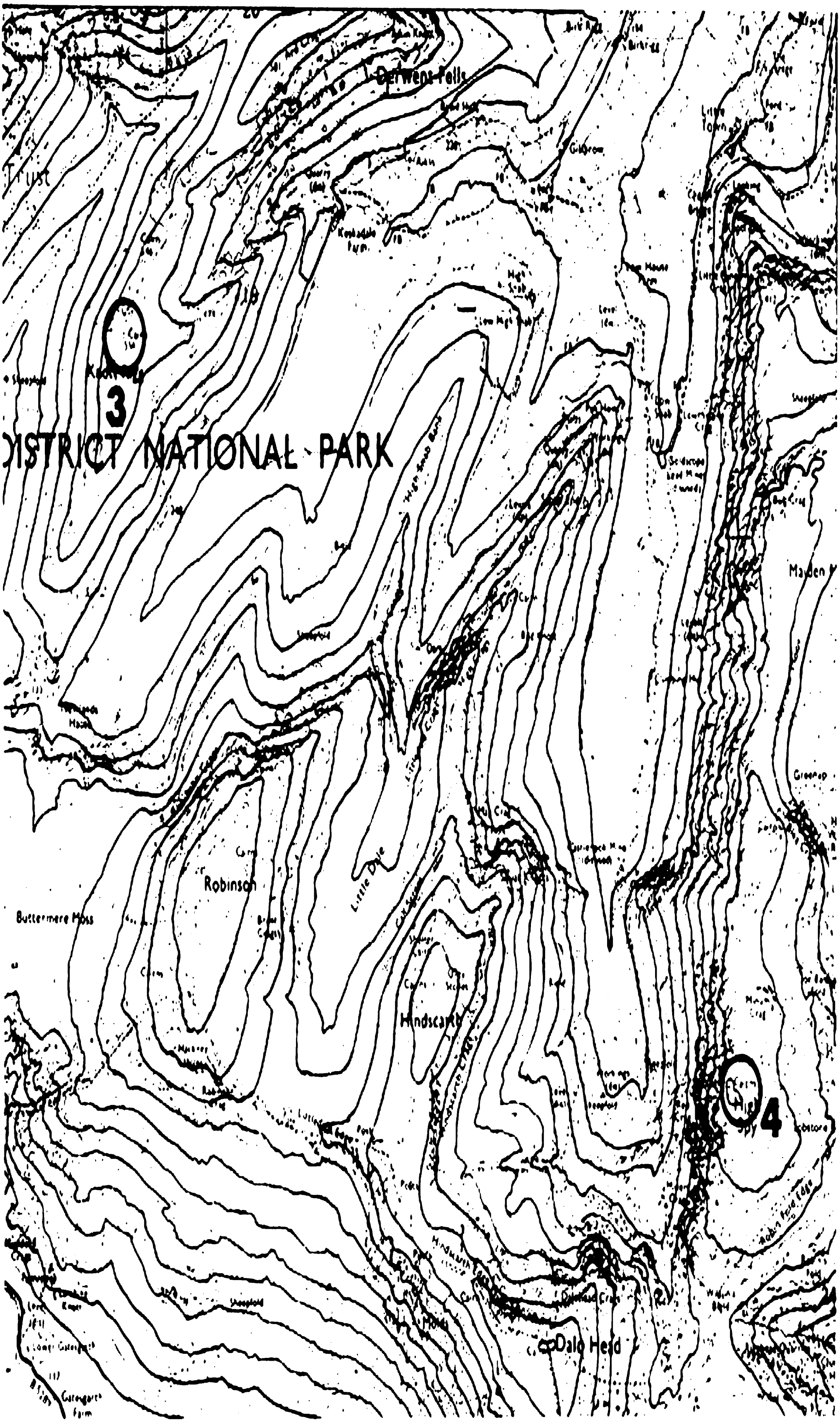

FIG. 1. *1974 Lake District Mountain Trial Course, Legs 3–4. Scale 1:25,000. 1000 m on ground = 40 mm on map. Major contour lines are at intervals of 50 metres.* (*Reproduced by kind permission of H.M.S.O. © Crown copyright reserved.*)

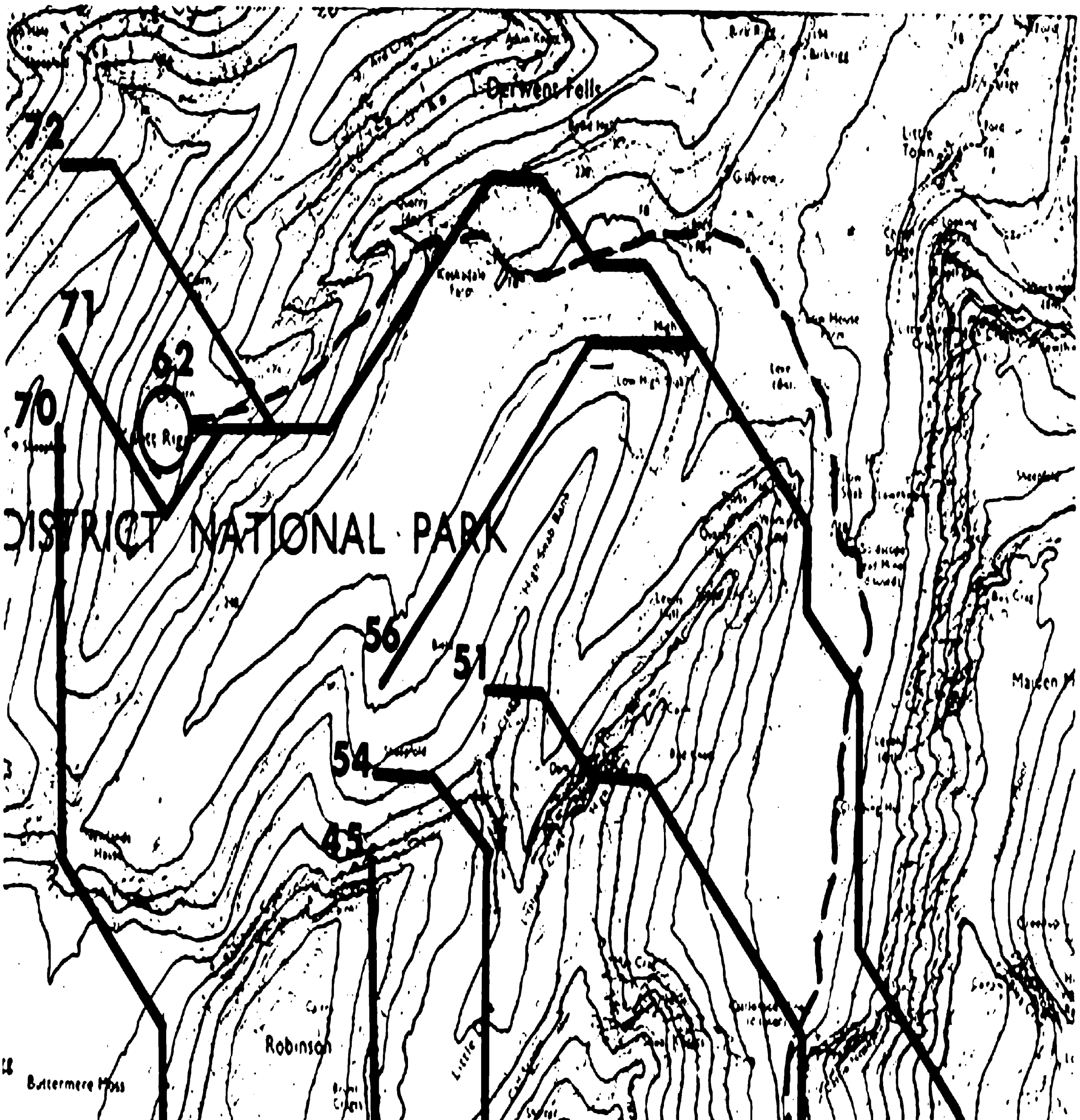

FIG. 2. *1974 Lake District Mountain Trial Course, Legs 3–4. Scale 1:25,000. 1000 m on ground = 40 mm on map. Major contour lines are at intervals of 50 metres.* (*Reproduced by kind permission of H.M.S.O.* © *Crown copyright reserved.*)

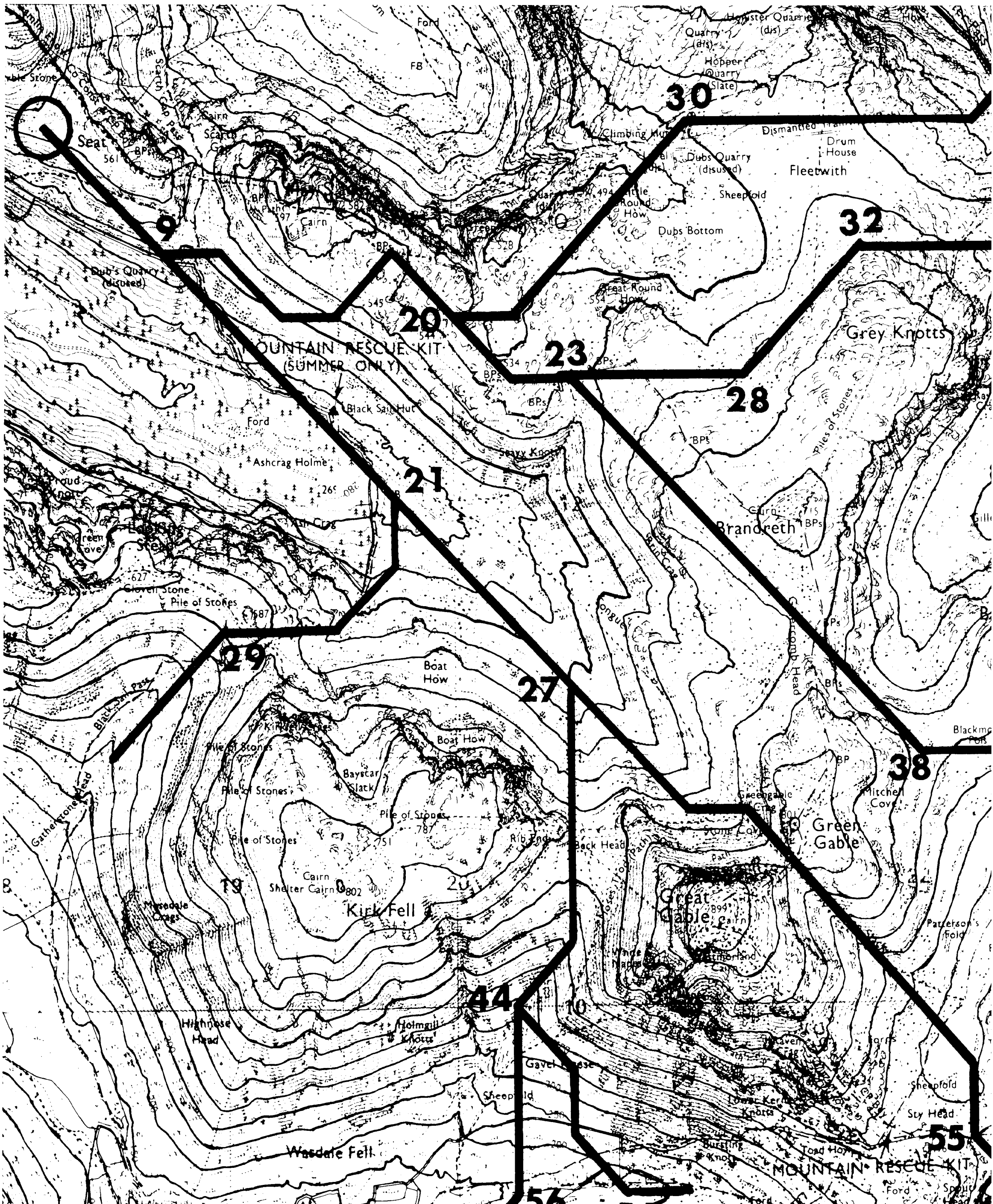

FIG. 3. *1981 "Karrimor" Mountain Marathon Leg*. From *stream junction east*
Scale 1:25,000. 1000 m on ground = 40 mm on map. Major contour lines are at
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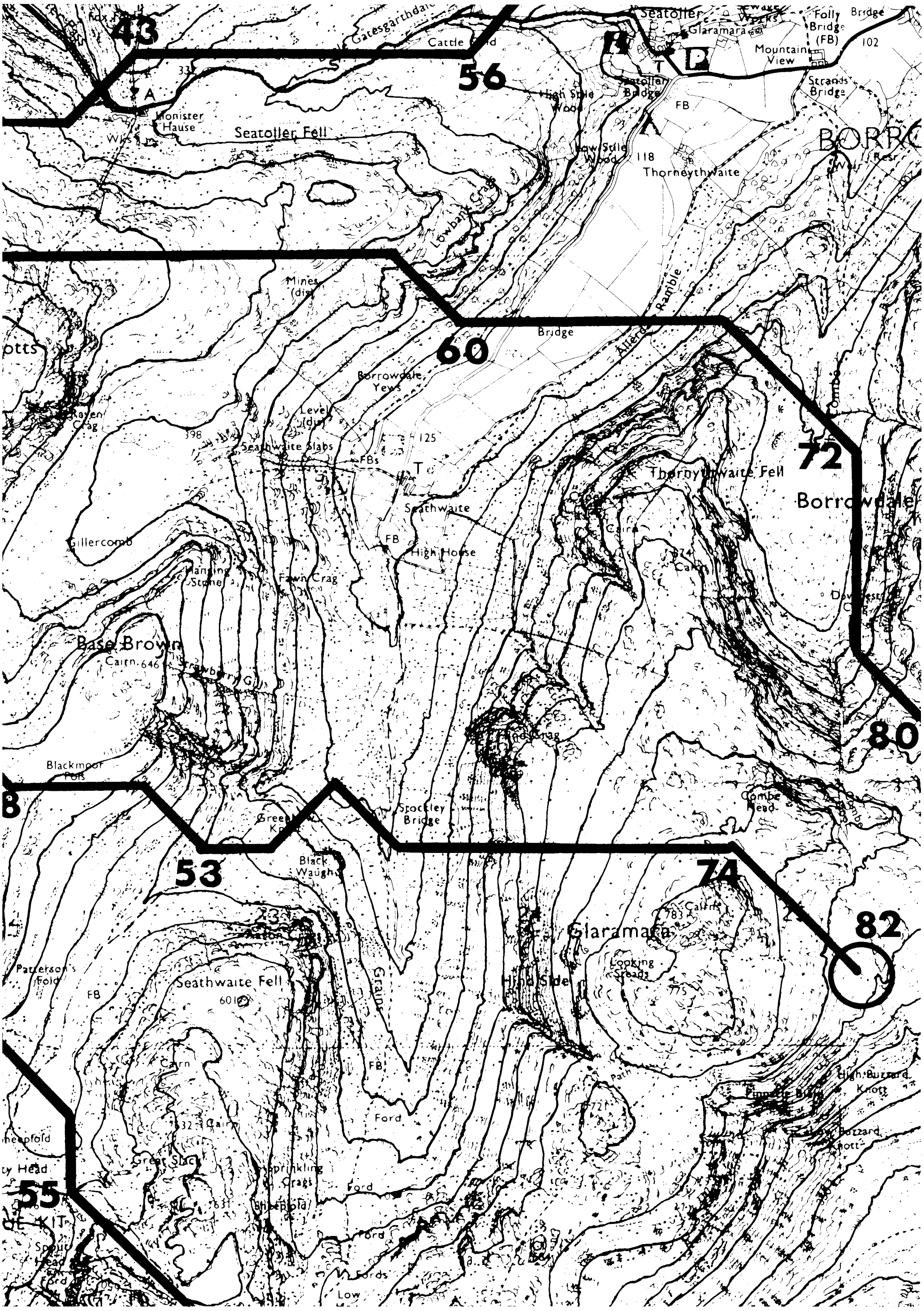

unction east of Glaramara to *tarn west of Seat.*
lines are at intervals of 50 metres. (*Reproduced*
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facilities, it is impracticable to evaluate all possible routes over such a range. Thus, this problem serves to demonstrate the efficiency of a familiar functional equation of dynamic programming used to find the shortest route in a network.[3]

For any point i in the network, let

$$f_n(i) = \min_j [t(i,j) + f_{n-1}(j)],$$

where

$f_n(i)$ = the optimal (in this case minimal) time to get from i to some terminal point (e.g. High Spy in Figure 1) in at most n steps;

$t(i,j)$ = the time taken to get from i to some adjacent point j;

$f_{n-1}(j)$ = the (previously found) minimum time to get from j to the terminal point in at most $n-1$ steps.

The computational procedure first finds all $f_1(i)$, then finds all $f_2(i)$, then all $f_3(i)$, etc.

AN ILLUSTRATION OF THE MODEL'S RESULTS

The thick, angular lines imposed on the map in Figure 2 are a convergent network of optimal routes heading for High Spy, generated by the computer model. This information includes the optimal route from Knott Rigg (the problem posed in Figure 1), and the "62" at Knott Rigg indicates the time it would take to reach High Spy. The model generates optimal routes from every point in the network, but for clarity, we have illustrated only the main relevant variants.

The thick, dashed line shows the actual route taken by the winner (Joss Naylor) in this particular race. The agreement between Naylor's and the computed optimal route is quite good, except towards the end of the leg. The divergence might have been due to the runner's need for water on a hot day, and Naylor's knowledge that there was a fresh spring at Miner's Crag south of High Spy (mentioned by Naylor in his written account after the race). Alternatively, Naylor's diversion round Miner's Crag might have been optimal for him, if not for the rest of the competitors. He was the only person able to *run* all the way to High Spy. Of those competitors who stuck closely to the computer's optimal route, most (including one of the authors) walked up the steep slope to High Spy on exactly the line suggested by the model and felt no inclination to do otherwise.

It is also interesting to study optimal routes to High Spy from points other than Knott Rigg. An organizer should pay special attention to those spots where major routes merge (e.g. Littledale Edge, South of Hindscarth, labelled 27). This would be a sensible location for an organizer to place a mountain rescue team. In passing, it is worth commenting that certain places in the mountains act as a focal point for travellers, regardless of their source and destination, and regardless of the fact that the place as such has no intrinsic value in being visited. In the Lake District, Esk Hause is a classic example.

Another point highlighted by Figure 2 is the care needed to be taken by an organizer in choosing an 'interesting' site for a control. Assume that a destination (say High Spy) has already been decided upon. Where should the preceding control be placed? A good place would be where several equally efficient routes branch out to reach the destination. For example, there is a combe NE of Robinson within which are the heads of four optimal routes to High Spy (labelled 45, 54, 56 and 51). Contrarily, a bad place to site a control prior to High Spy would be Buttermere Moss (West of Robinson, near the 43 label). From here it is obvious that there is only one possible route to take.

A FURTHER APPLICATION

Figure 3 illustrates one leg tackled by competitors (teams of two) in the Karrimor Mountain Marathon, October 1981. The leg goes from bottom right (a circle east of Glaramara, labelled 82) to top left (a circle just north of Seat). Superimposed on the map is a network of computer-generated optimal routes converging on Seat. The numbered labels indicate 'minutes to Seat' from any point. It is interesting to compare this convergent network with routes actually taken. The course planner identified eight such routes when questioning participants after the race. The most popular actual route closely followed the optimal computer route running through the centre of the map. But the fastest time was recorded by a team which ran north to Seatoller and then stuck closely to the computer route emanating from the 56 label. This team (the overall event winners) were better runners than their nearest rivals; they reasoned that on the direct route, they would have to walk up the steep slopes at the same speed as the other teams; but on the detour with its gentler slopes, they would be able to run whilst all others would have to walk.

A few competitors (including the third fastest) traversed the south slopes of Great Gable using a narrow path which threads its way through scree, crags and rockfalls. When designing the course, it did not occur to the planner that anyone would use this route.[4] Likewise, this path was not used by any optimal computer-route because we classified the whole south slope of Great Gable as 'rough ground'. To have allowed this way to become a feasible option, we would have needed a 'special way through' classification in our data bank. There are not many places in the Lake District where such a classification is required. On the other hand, they are attractive places for competitors to pass through (e.g. Lord's Rake on Scafell or Shamrock Traverse on Pillar).

Apart from the above exception, the computer's convergent network reflects very closely the routes taken by teams *after* they had left the Glaramara region. But why did teams take so many divergent routes immediately on leaving the Glaramara control? We suspect that many teams failed to determine an all-the-way strategy to Seat but instead adopted a temporary heuristic strategy; for example, "Follow a direct compass bearing", or "Don't lose height" or "Head for the nearest path". Such teams found that they had to reconsider their strategy because the planner had ensured that no simple heuristic would give a good result.

DISCUSSION

Having considered the results of the model with those achieved in practice by competitors, what refinements to the model would be worth considering?

Classifying terrain

Paths—We have already examined the value of having 'special way through' connections when paths run through rough ground. Otherwise, where paths run over normal ground, we feel that the extra speed on these paths is too insignificant to justify special classification.

Impassable barriers and excluded ground—Examples would be an unbroken line of sheer crags, a deep river, a large lake or extensive private land. On the particular maps which were analyzed, these were not restraining features.

Increasing or decreasing the grid size which records spot heights

Using a coarser, 500-metre grid eliminates major features of relief which have a significant impact on the optimal route taken. For example, using a 500-metre grid for the Knott Rigg to High Spy problem would lead to the computer choosing a direct, near-diagonal route, as such grid points take no account of the three major ridges which in fact block the direct route.

Using a finer, 125-metre grid gives a better rounding of spurs and ravines (for example in Figure 2, by giving a better approximation to Joss Naylor's route in the Low Snab area). However, whether a 125-metre or a 500-metre grid is used, there are few differences in the *pattern* of the convergent network. This fact might be exploited where it is regarded as too tedious to use a 125-metre grid but where routes need to be defined accurately. For example, consider the following 'path restriction method'.[5]

(a) Derive a complete network of routes to the terminus using the 250-metre grid.
(b) Examine (a)'s results and select those routes which need special attention.

(c) Introduce a narrow band of 125-metre grid points on each side of the selected routes and compute optimal routes within this band.

Alternative ways of generating travel times

It may be that some users would be quite happy with a cruder estimating system. We have already mentioned the original Naismith rules. Or even more simply, a hiking club might be quite happy to work from a constant 2 m.p.h. estimate—putting effort into walking uphill and stopping to admire the view on the way down.

Conversely, a more sophisticated estimating method is needed to reflect the times taken by the faster runners. We have hinted at why this is necessary when we described winners' behaviour in the example routes that have been analyzed. Currently, we are carrying out field trials to determine what appears to be a nonlinear effect of gradient on travel times for faster runners. The matter is further complicated in downhill running, where the nature of the terrain has a significant effect on travel times.

Refining the analytical method

At present we find an optimal route from every point in the grid. This could be extended to compute all second-best and third-best routes as well, but with a significant increase in computing time. However, we feel that our current system, representing all optimal routes as a convergent network, gives an event planner all the information he needs on alternative routes.

We also considered changing the state description by including an indication of the runner's physical condition at a point in time. In certain circumstances, a runner may rightly prefer one route between two points to another, even if the preferred route takes longer. This is because he may then be in a better condition to run the remainder of the course. We discounted this factor partly on the empirical evidence that it is wrong for a competitor to pace himself so that his performance deteriorates over time. True, this is a common occurrence, but a study of split times in long races, such as road marathons, showed that in nearly every case, winning performances were obtained from an even distribution of effort.[6] We surmise that what is best strategy for a winner is best strategy for all other competitors, at their own level of competence.

CONCLUSION

We have shown how a simple dynamic-programming model can help plan orienteering routes and site controls. Collection of data, in particular the spot heights for each point on the grid, is a little time-consuming, but once this work has been done, events in this area can be planned again and again, using the same data. In planning courses, it will still be necessary for the organizer to make a ground survey of the area to be used. But if he does some advance planning with our computer-based system, he can make a thorough study of the alternatives open to him.

ACKNOWLEDGEMENT

We are grateful to Peter Nelson, Course Planner of the 1981 Karrimor Mountain Marathon, for his investigations into routes taken by competitors and his comments on them.

APPENDIX

Spot Heights of the 250-Metre Grid

The numbers below, when multiplied by 10, give spot heights in metres above the sea. Rough ground points are circled. Knott Rigg (top left) and High Spy (bottom right) are starred.

43	36	32	43	52	47	35	34	25	23	20	19	21	18	16	14	15	19	20
33	29	40	52	45	36	34	25	21	20	22	22	20	17	15	14	18	33	23
36	41	54	49	45	40	28	22	22	24	26	25	22	17	16	15	17	33	40
30	45	53	47	40	30	23	22	25	30	35	27	20	22	23	15	17	36	41
31	46	49	35	27	23	23	24	29	36	30	22	26	37	21	15	20	36	50
37	48	37	28	24	27	28	28	33	37	24	26	39	33	18	16	21	44	57
40	39	29	25	28	36	29	31	38	33	29	39	39	26	17	17	25	47	57
34	30	26	29	40	43	38	42	45	31	45	42	33	21	17	18	32	48	56
36	33	31	37	51	54	57	56	48	44	53	39	28	20	17	19	34	54	51
44	47	48	50	68	71	63	52	45	46	54	37	30	22	18	22	41	59	55
47	49	53	65	73	70	57	48	45	51	57	54	40	24	19	30	45	62	59
48	51	57	68	72	68	56	47	51	66	68	58	41	27	22	33	49	62	57
48	51	59	68	70	64	56	49	59	70	68	53	40	27	21	30	49	64	59
40	51	54	59	62	61	57	56	62	70	57	46	41	31	25	30	53	64	60
35	40	42	48	51	52	56	58	66	67	53	52	46	34	30	34	54	63	61
24	30	31	39	42	43	45	48	56	65	65	54	51	45	44	38	54	56	55
15	20	24	29	31	32	34	38	42	58	64	63	65	67	58	47	51	50	41
11	14	17	21	22	23	25	29	30	40	52	67	73	70	57	50	51	51	41
10	11	12	13	14	15	17	19	21	29	41	58	68	67	57	52	55	55	51

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