Lista 4 - Estatística

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Fórmulas

$$\text{IC(1-}\alpha\text{)\%}= \hat{y} \pm \tau_{\frac{\alpha}{2}} \text{ ; } \times \text{ n-2 } \times S_e \times \sqrt{1 + \frac{1}{n} + \frac{n(X_0 - \bar{X})^2}{S_{xx}}}$$

$$\text{IC(1-}\alpha\text{)\%}= \hat{y} \pm \tau_{\frac{\alpha}{2}} \text{ ; } \times \text{ n-2 } \times S_e \times \sqrt{\frac{1}{n} + \frac{n(X_0 - \bar{X})^2}{S_{xx}}}$$

$$\text{Corr(x,y)} = \frac{S_{xy}}{\sqrt{S_{xx} \times S_{yy}}}$$

$$S_{xx} = n \times \sum x^2 - (\sum x)^2$$

$$S_{yy} = n \times \sum y^2 - (\sum y)^2$$

$$S_{xy} = n \times \sum xy - (\sum x)(\sum y)$$

$$\hat{\beta}1 = \frac{S_{xy}}{S_{xx}}$$

$$S_e = \sqrt{\frac{\sum y^2 - \hat{\beta}0 \sum y - \hat{\beta}1 \sum xy}{n-2}}$$

Questão *1)*

$S_x = 30$

$S_y = 11240$

${S_x}^2 = 128$

${S_y}^2 = 20353600$

$S_{xy} = 50480$

a) $S_{xx} = n \times \sum x^2 - (\sum x)^2$

$S_{xx} =$ 8 $\times$ 128 - (30)^2

$S_{xx} =$ 1024 - 900

$S_{xx} =$ 124

$S_{yy} = n \times \sum y^2 - (\sum y)^2$

$S_{yy} =$ 8 $\times$ 20353600 - (11240)^2

$S_{yy} =$ 162828800 - 126337600

$S_{yy} =$ 36491200

$S_{xy} = n \times \sum xy - (\sum x)(\sum y)$

$S_{xy} =$ 8 $\times$ 50480 - 337200

$S_{xy} =$ 403840 - 337200

$S_{xy} =$ 66640

Corr(x,y) $= \frac{S_{xy}}{\sqrt{S_{xx} \times S_{yy}}}$

Corr(x,y) $= \frac{66640}{\sqrt{124 \times 36491200}}$

Corr(x,y) $= \frac{66640}{67267}$

Corr(x,y) $=$ 0.9907

Interpretação do resultado: Há uma forte correlação positiva.

b) $\hat{\beta}1 = \frac{S_{xy}}{S_{xx}}$

$\hat{\beta}1 = \frac{66640}{124} = 537.42$

$\hat{\beta}0 = \hat{y}$ - $\hat{\beta}1 \times \bar{x}$

$\hat{\beta}0 =$ 1405 - (537.42) $\times$ 3.75

$\hat{\beta}0 =$ -610.325

Interpretação do resultado:

$\hat{\beta}0$: Não há interpretação prática. $\hat{\beta}1$: Para cada ano, existe um acréscimo ná média em 537.42 palavras no vocabulário de cada criança.

c) $R^2 = (0.9907)^2 = 0.9815$ ou 98.15%

Interpretação do resultado:

d) $S_e = \sqrt{\frac{\sum y^2 - \hat{\beta}0 \sum y - \hat{\beta}1 \sum xy}{n-2}}$

$S_e = \sqrt{\frac{20353600-(610.325 \times 11240)+(537.42 \times 50480)}{6}}$

$S_e = \sqrt{\frac{84691.4}{6}}$

$S_e = 118.81$

$\hat{y} = \hat{\beta}0 + \hat{\beta}1 \times$ n = -610.325 + 537.42 $\times$ 7 = 3151.615

IC(95%)= $\hat{y} \pm \tau_{\frac{\alpha}{2}}$; $\times$ n-2 $\times$ $S_e \times \sqrt{1 + \frac{1}{n} + \frac{n(X_0 - \bar{X})^2}{S_{xx}}}$

IC(95%) = 3151.615 $\pm$ 2.4469 $\times$ (118.81 $\times \sqrt{1 + \frac{1}{8} + \frac{8(7-3.75)^2}{124}}$)

IC(95)% = 3151.615 $\pm$ 390.73

IC(95)% = [2760.885 ; 3542.345]

e) IC(95%)= $\hat{y} \pm \tau_{\frac{\alpha}{2}}$; $\times$ n-2 $\times$ $S_e \times \sqrt{\frac{1}{n} + \frac{n(X_0 - \bar{X})^2}{S_{xx}}}$

IC(95%) = 3151.615 $\pm$ 2.4469 $\times$ (118.81 $\times \sqrt{\frac{1}{8} + \frac{8(7--3.75)^2}{124}}$)

IC(95)% = 3151.615 $\pm$ 261.07

IC(95)% = [2890.545 ; 3412.685]

Questão *2)*

$S_x = 60$

$S_y = 891$

${S_x}^2 = 346$

${S_y}^2 = 65451$

$S_{xy} = 4620$

a) $S_{xx} = n \times \sum x^2 - (\sum x)^2$

$S_{xx} = 13 \times 346 - (60)^2$

$S_{xx} = 4498 - 3600$

$S_{xx} = 898$

$S_{yy} = n \times \sum y^2 - (\sum y)^2$

$S_{yy} = 13 \times 65451 - (891)^2$

$S_{yy} = 850863 - 793881$

$S_{yy} = 56982$

$S_{xy} = n \times \sum xy - (\sum x)(\sum y)$

$S_{xy} = 13 \times 4620 - 53460$

$S_{xy} = 60060 - 53460$

$S_{xy} = 6600$

Corr(x,y) $= \frac{S_{xy}}{\sqrt{S_{xx} \times S_{yy}}}$

Corr(x,y) $= \frac{6600}{\sqrt{898 \times 56982}}$

Corr(x,y) $= \frac{6600}{7153.30}$

Corr(x,y) $= 0.9226$

b) $\hat{\beta}1 = \frac{S_{xy}}{S_{xx}}$

$\hat{\beta}1 = \frac{6600}{898} = 7.35$

$\hat{\beta}0 = \hat{y} - \hat{\beta}1 \times \bar{x}$

$\hat{\beta}0 = 68.54 - (7.35) \times 4.62$

$\hat{\beta}0 = 34.58$

c) $R^2 = (0.9226)^2 = 0.8512$ ou 85.12%

d) $S_e = \sqrt{\frac{\sum y^2 - \hat{\beta}0 \sum y - \hat{\beta}1 \sum xy}{n-2}}$

$S_e = \sqrt{\frac{65451-(34.58\times 891)+(7.35\times 4620)}{11}}$

$S_e = \sqrt{\frac{683.22}{11}}$

$S_e = 7.88$

$\hat{y} = \hat{\beta}0 + \hat{\beta}1 \times \text{n} = 34.58 + 7.35 \times 3 = 56.63$

IC(99%)$= \hat{y} \pm \tau_{\frac{\alpha}{2}}$; $\times$ n-2 $\times$ $S_e \times \sqrt{1 + \frac{1}{n} + \frac{n(X_0 - \bar{X})^2}{S_{xx}}}$

IC(99%) $= 56.63 \pm 3.1058 \times (7.88 \times \sqrt{1 + \frac{1}{13} + \frac{13(3-4.62)^2}{898}})$

IC(99)% $= 56.63 \pm 25.84$

IC(99)% $=$ [30.79 ; 82.47]

e) IC(99%)$= \hat{y} \pm \tau_{\frac{\alpha}{2}}$; $\times$ n-2 $\times$ $S_e \times \sqrt{\frac{1}{n} + \frac{n(X_0 - \bar{X})^2}{S_{xx}}}$

IC(99%) $= 56.63 \pm 3.1058 \times (7.88 \times \sqrt{\frac{1}{13} + \frac{13(3-4.62)^2}{898}})$

IC(99)% $= 56.63 \pm 8.29$

IC(99)% $=$ [48.34 ; 64.92]

Questão *3)*

$S_x = 54$

$S_y = 908$

${S_x}^2 = 332$

${S_y}^2 = 70836$

$S_{xy} = 3724$

a) $S_{xx} = n \times \sum x^2 - (\sum x)^2$

$S_{xx} = 12 \times 332 - (54)^2$

$S_{xx} = 3984 - 2916$

$S_{xx} = 1068$

$S_{yy} = n \times \sum y^2 - (\sum y)^2$

$S_{yy} = 12 \times 70836 - (908)^2$

$S_{yy} = 850032 - 824464$

$S_{yy} = 25568$

$S_{xy} = n \times \sum xy - (\sum x)(\sum y)$

$S_{xy} = 12 \times 3724 - 49032$

$S_{xy} = 44688 - 49032$

$S_{xy} = -4344$

Corr(x,y) $= \frac{S_{xy}}{\sqrt{S_{xx} \times S_{yy}}}$

Corr(x,y) $= \frac{-4344}{\sqrt{1068 \times 25568}}$

Corr(x,y) $= \frac{-4344}{5225.57}$

Corr(x,y) $= -0.8313$

b) $\hat{\beta}1 = \frac{S_{xy}}{S_{xx}}$

$\hat{\beta}1 = \frac{-4344}{1068} = -4.07$

$\hat{\beta}0 = \hat{y} - \hat{\beta}1 \times \bar{x}$

$\hat{\beta}0 = 75.66 - (-4.07) \times 4.5$

$\hat{\beta}0 = 93.97$

c) $R^2 = (0.8313)^2 = 0.691$ ou 69.10%

d) $S_e = \sqrt{\frac{\sum y^2 - \hat{\beta}0 \sum y - \hat{\beta}1 \sum xy}{n-2}}$

$S_e = \sqrt{\frac{70836-(93.97\times908)+(4.07\times3724)}{10}}$

$S_e = \sqrt{\frac{667.92}{10}}$

$S_e = 8.17$

$\hat{y} = \hat{\beta}0 + \hat{\beta}1 \times \text{n} = 93.97 + 4.07 \times 9 = 57.34$

IC(95%)$= \hat{y} \pm \tau_{\frac{\alpha}{2}}$; $\times$ n-2 $\times S_e \times \sqrt{1 + \frac{1}{n} + \frac{n(X_0-\bar{X})^2}{S_{xx}}}$

IC(95%) $= 57.34 \pm 2.2281 \times (8.17 \times \sqrt{1 + \frac{1}{12} + \frac{12(9-4.5)^2}{1068}})$

IC(95)% $= 57.34 \pm 20.84$

IC(95)% $=$ [36.5 ; 78.18]

e) IC(95%)$= \hat{y} \pm \tau_{\frac{\alpha}{2}}$; $\times$ n-2 $\times S_e \times \sqrt{\frac{1}{n} + \frac{n(X_0-\bar{X})^2}{S_{xx}}}$

IC(95%) $= 57.34 \pm 2.2281 \times (8.17 \times \sqrt{\frac{1}{12} + \frac{12(9-4.5)^2}{1068}})$

IC(95)% $= 57.34 \pm 10.15$

IC(95)% $=$ [47.19 ; 67.49]