CMPT 379
Compilers

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Parsing - Roadmap

- Parser:
  - decision procedure: builds a parse tree
- Top-down vs. bottom-up
- LL(1) – Deterministic Parsing
  - recursive-descent
  - table-driven
- LR(k) – Deterministic Parsing
  - LR(0), SLR(1), LR(1), LALR(1)
- Parsing arbitrary CFGs – Polynomial time parsing

Top-Down vs. Bottom Up

Grammar: $S \rightarrow A\ B$ Input String: ccbca

$A \rightarrow c \mid \varepsilon$

$B \rightarrow cbB \mid ca$

Top-Down/leftmost		Bottom-Up/rightmost	
$S \Rightarrow AB$	$S \rightarrow AB$	$ccbca \Leftarrow Acbca$	$A \rightarrow c$
$\Rightarrow cB$	$A \rightarrow c$	$\Leftarrow AcbB$	$B \rightarrow ca$
$\Rightarrow ccbB$	$B \rightarrow cbB$	$\Leftarrow AB$	$B \rightarrow cbB$
$\Rightarrow ccbca$	$B \rightarrow ca$	$\Leftarrow S$	$S \rightarrow AB$

Rightmost derivation for **id + id * id**

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow (E)$

$E \rightarrow - E$

$E \rightarrow \mathbf{id}$

$E \Rightarrow E * E$

$\Rightarrow E * \mathbf{id}$

$\Rightarrow E + E * \mathbf{id}$

$\Rightarrow E + \mathbf{id} * \mathbf{id}$ reduce with $E \rightarrow \mathbf{id}$

$\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}$ shift

$E \Rightarrow^*_{rm} E + E \backslash * \text{ id}$

Bottom-up parsing overview

- Start from terminal symbols, search for a path to the start symbol
- Apply shift and reduce actions: postpone decisions
- LR parsing:
  - L: left to right parsing
  - R: rightmost derivation (in reverse or bottom-up)
- LR(0) $\rightarrow$ SLR(1) $\rightarrow$ LR(1) $\rightarrow$ LALR(1)
  - 0 or 1 or *k* lookahead symbols

Actions in Shift-Reduce Parsing

- Shift
  - add terminal to parse stack, advance input
- Reduce
  - If αw is on the stack, $\alpha, w \in (N \cup T)^*$ and $A \rightarrow w$, and there is a $\beta \in T^*$ such that $S \Rightarrow^*_{rm} \alpha A \beta \Rightarrow_{rm} \alpha w \beta$ then we can reduce αw to αA on the stack (called *pruning the handle* w)
  - αw is a *viable prefix*
- Error
- Accept

Questions

- When to shift/reduce?
  - What are valid handles?
  - Ambiguity: Shift/reduce conflict
- If reducing, using which production?
  - Ambiguity: Reduce/reduce conflict

LR Parsing

- Table-based parser
  - Creates rightmost derivation (in reverse)
  - For “less massaged” grammars than LL(1)
- Data structures:
  - Stack of states/symbols {s}
  - Action table: **action**[s, a]; a $\in$ **T**
  - Goto table: **goto**[s, X]; X $\in$ **N**

Productions	
1	T → F
2	T → T*F
3	F → id
4	F → (T)

Action/Goto Table

	*	(	)	id	$	T	F
0		S5		S8		2	1
1	R1	R1	R1	R1	R1		
2	S3				Acc!		
3		S5		S8			4
4	R2	R2	R2	R2	R2		
5		S5		S8		6	1
6	S3		S7				
7	R4	R4	R4	R4	R4		
8	R3	R3	R3	R3	R3		

Trace “(id)*id”

Stack	Input	Action
0	**(id) * id $**	**Shift S5**
0 5	**id) * id $**	**Shift S8**
0 5 8	**) * id $**	**Reduce 3 F→id,**
		pop 8, goto [5,F]=1
0 5 1	**) * id $**	**Reduce 1 T→ F,**
		pop 1, goto [5,T]=6
0 5 6	**) * id $**	**Shift S7**
0 5 6 7	*** id $**	**Reduce 4 F→ (T),**
		pop 7 6 5, goto [0,F]=1
0 1	*** id $**	**Reduce 1 T → F**
		pop 1, goto [0,T]=2

Productions	
1	T → F
2	T → T*F
3	F → id
4	F → (T)

“(id)*id”

	*	(	)	id	$	T	F
0		S5		S8		2	1
1	R1	R1	R1	R1	R1		
2	S3				A		
3		S5		S8			4
4	R2	R2	R2	R2	R2		
5		S5		S8		6	1
6	S3		S7				
7	R4	R4	R4	R4	R4		
8	R3	R3	R3	R3	R3		

t	Input	Ac
0	(id) * id $	Sh
0 5	id) * id $	Sh
0 5 8	) * id $	Re po
0 5 1	) * id $	Reduce 1 T→ F, pop 1, goto [5,T]=6
0 5 6	) * id $	Shift S7
0 5 6 7	* id $	Reduce 4 F→ (T), pop 7 6 5, goto [0,F]=1
0 1	* id $	Reduce 1 T → F pop 1, goto [0,T]=2

Trace “(id)*id”

Stack	Input	Action
0 1	***** id $**	**Reduce 1 T→F, pop 1, goto [0,T]=2**
0 2	***** id $**	**Shift S3**
0 2 3	**id $**	**Shift S8**
0 2 3 8	**$**	**Reduce 3 F→id, pop 8, goto [3,F]=4**
0 2 3 4	**$**	**Reduce 2 T→T * F pop 4 3 2, goto [0,T]=2**
0 2	**$**	**Accept**

Productions	
1	T → F
2	T → T*F
3	F → id
4	F → (T)

“(id)*id”

	*	(	)	id	$	T	F
0		S5		S8		2	1
1	R1	R1	R1	R1	R1		
2	S3				A		
3		S5		S8			4
4	R2	R2	R2	R2	R2		
5		S5		S8		6	1
6	S3		S7				
7	R4	R4	R4	R4	R4		
8	R3	R3	R3	R3	R3		

Stack	Input	Actio
0 1	*** id $**	**Reduc pop 1,**
0 2	*** id $**	**Shift S**
0 2 3	**id $**	**Shift S8**
0 2 3 8	**$**	**Reduce 3 F→id, pop 8, goto [3,F]=4**
0 2 3 4	**$**	**Reduce 2 T→T * F pop 4 3 2, goto [0,T]=2**
0 2	**$**	**Accept**

Tracing LR: **action**[s, a]

- case **shift** *u*:
  - push state *u*
  - read new *a*
- case **reduce** *r*:
  - lookup production *r*: $X \rightarrow Y_1..Y_k$;
  - pop *k* states, find state *u*
  - push **goto**[u, X]
- case **accept**: done
- no entry in action table: **error**

Configuration set

- Each set is a parser state
- We use the notion of a dotted rule or item:

$$\mathbf{T \rightarrow T * \bullet F}$$

- The dot is before **F**, so we predict all rules with **F** as the left-hand side

$$\mathbf{T \rightarrow T * \bullet F}$$

$$\mathbf{F \rightarrow \bullet (T)}$$

$$\mathbf{F \rightarrow \bullet id}$$

- This creates a configuration set (or item set)
  - Like NFA-to-DFA conversion

Closure

Closure property:

- If $\mathbf{T \rightarrow X_1 \ldots X_i \bullet X_{i+1} \ldots X_n}$ is in set, and $\mathbf{X_{i+1}}$ is a nonterminal, then $\mathbf{X_{i+1} \rightarrow \bullet Y_1 \ldots Y_m}$ is in the set as well for all productions $\mathbf{X_{i+1} \rightarrow Y_1 \ldots Y_m}$
- Compute as fixed point
- The closure property creates a configuration set (item set) from a dotted rule (item).

Starting Configuration

- Augment Grammar with S’
- Add production S’ → S
- Initial configuration set is

closure(S’ → • S)

Example: I = closure(S' → • T)

S' → T
T → F | T * F
F → id | (T)

Example: I = closure(S' → • T)

S' → • T

T → • T * F

T → • F

F → • id

F → • (T)

S' → T
T → F | T * F
F → id | (T)

Successor(I, X)

Informally: "move by symbol X"

1. move dot to the right in all items where dot is before X
2. remove all other items

   (viable prefixes only!)
3. compute closure

Successor Example

$I = \{S' \rightarrow \bullet\ T,$

$T \rightarrow \bullet\ F,$

$T \rightarrow \bullet\ T * F,$

$F \rightarrow \bullet\ id,$

$F \rightarrow \bullet\ (\ T\ )\ \}$

$S' \rightarrow T$
$T \rightarrow F \mid T * F$
$F \rightarrow id \mid (\ T\ )$

Compute **Successor**(I, “(“)

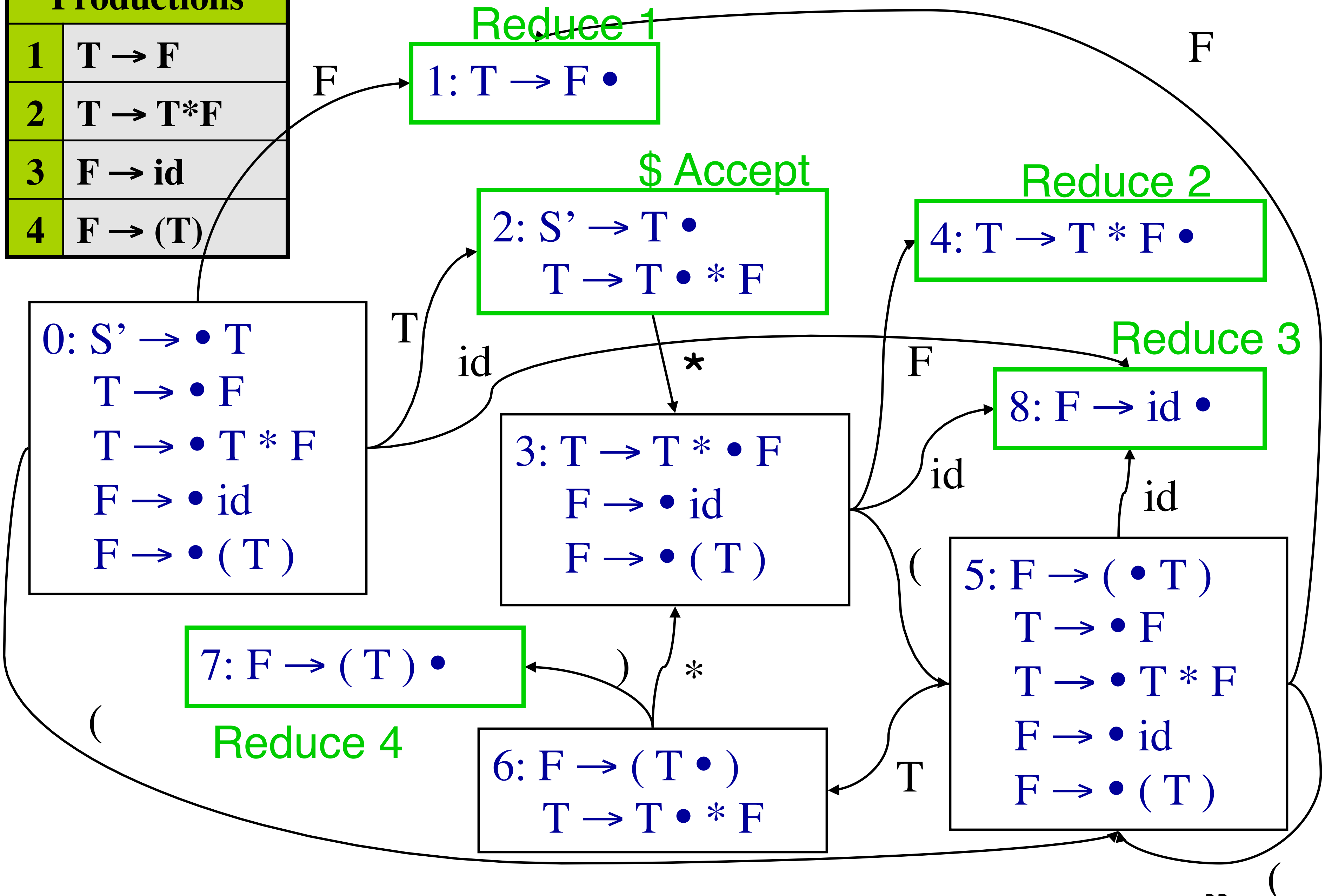
$\{\ F \rightarrow (\ \bullet\ T\ ),\ T \rightarrow \bullet\ F,\ T \rightarrow \bullet\ T * F,$
$F \rightarrow \bullet\ id,\ F \rightarrow \bullet\ (\ T\ )\ \}$

Sets-of-Items Construction

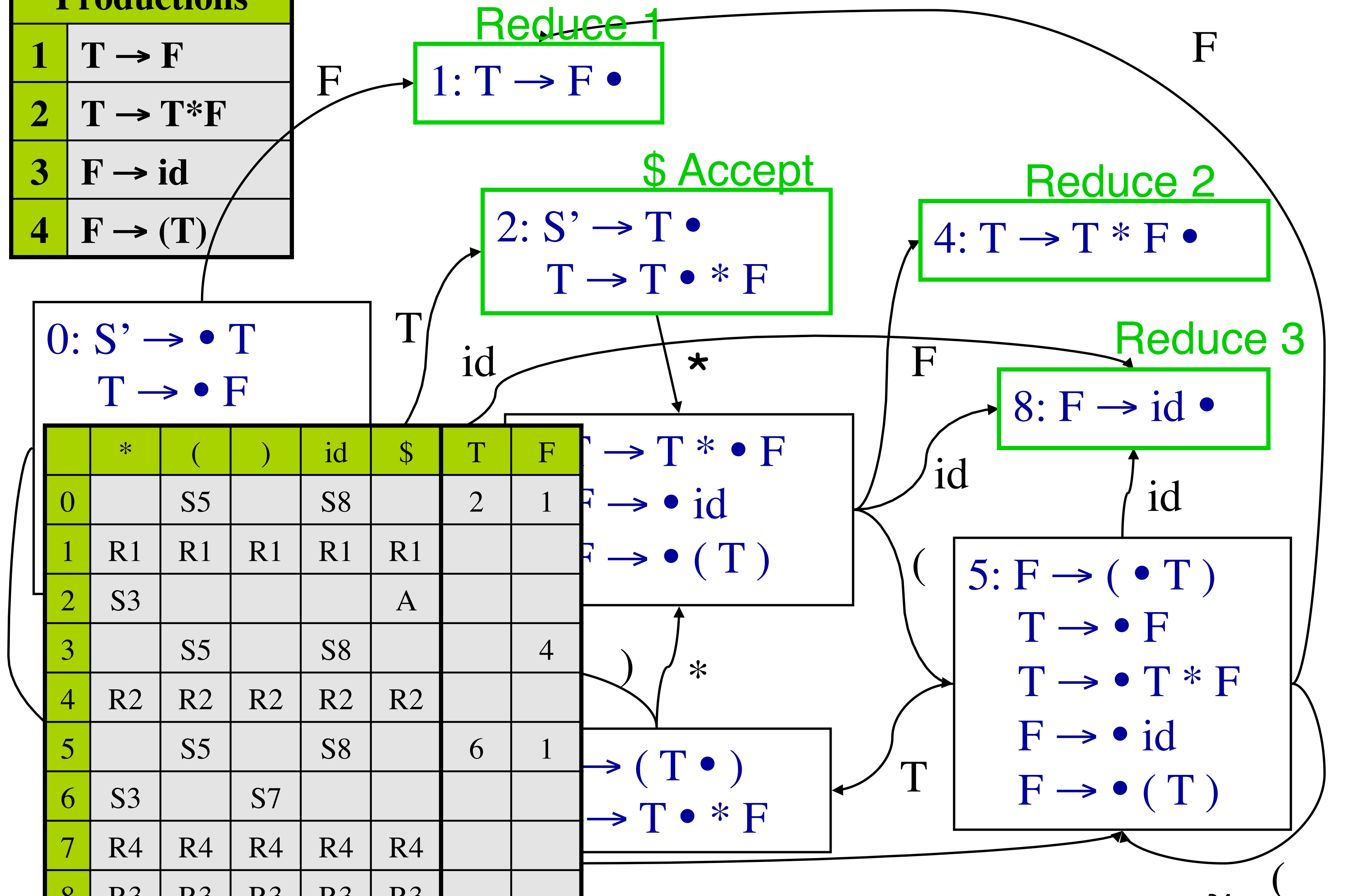
Family of configuration sets

function items(G')
    C = { closure({S' → • S}) };
    do foreach I ∈ C **do**
        foreach X ∈ **(N ∪ T) do**
            C = C ∪ { **Successor**(I, X) };
    while C changes;

Productions	
1	T → F
2	T → T*F
3	F → id
4	F → (T)



Productions	
1	**T → F**
2	**T → T*F**
3	**F → id**
4	**F → (T)**



	*	(	)	id	$	T	F
0		S5		S8		2	1
1	R1	R1	R1	R1	R1		
2	S3				A		
3		S5		S8			4
4	R2	R2	R2	R2	R2		
5		S5		S8		6	1
6	S3		S7				
7	R4	R4	R4	R4	R4		
8	R3	R3	R3	R3	R3		

LR(0) Construction

1. Construct $F = \{I_0, I_1, \ldots I_n\}$
2. a) if $\{A \rightarrow \alpha\bullet\} \in I_i$ and A != S'

   then action[i, _] := reduce $A \rightarrow \alpha$

   b) if $\{S' \rightarrow S\bullet\} \in I_i$

   then action[i,$] := accept

   c) if $\{A \rightarrow \alpha\bullet a\beta\} \in I_i$ and $\text{Successor}(I_i,a) = I_j$

   then action[i,a] := shift j
3. if $\text{Successor}(I_i,A) = I_j$ then goto[i,A] := j

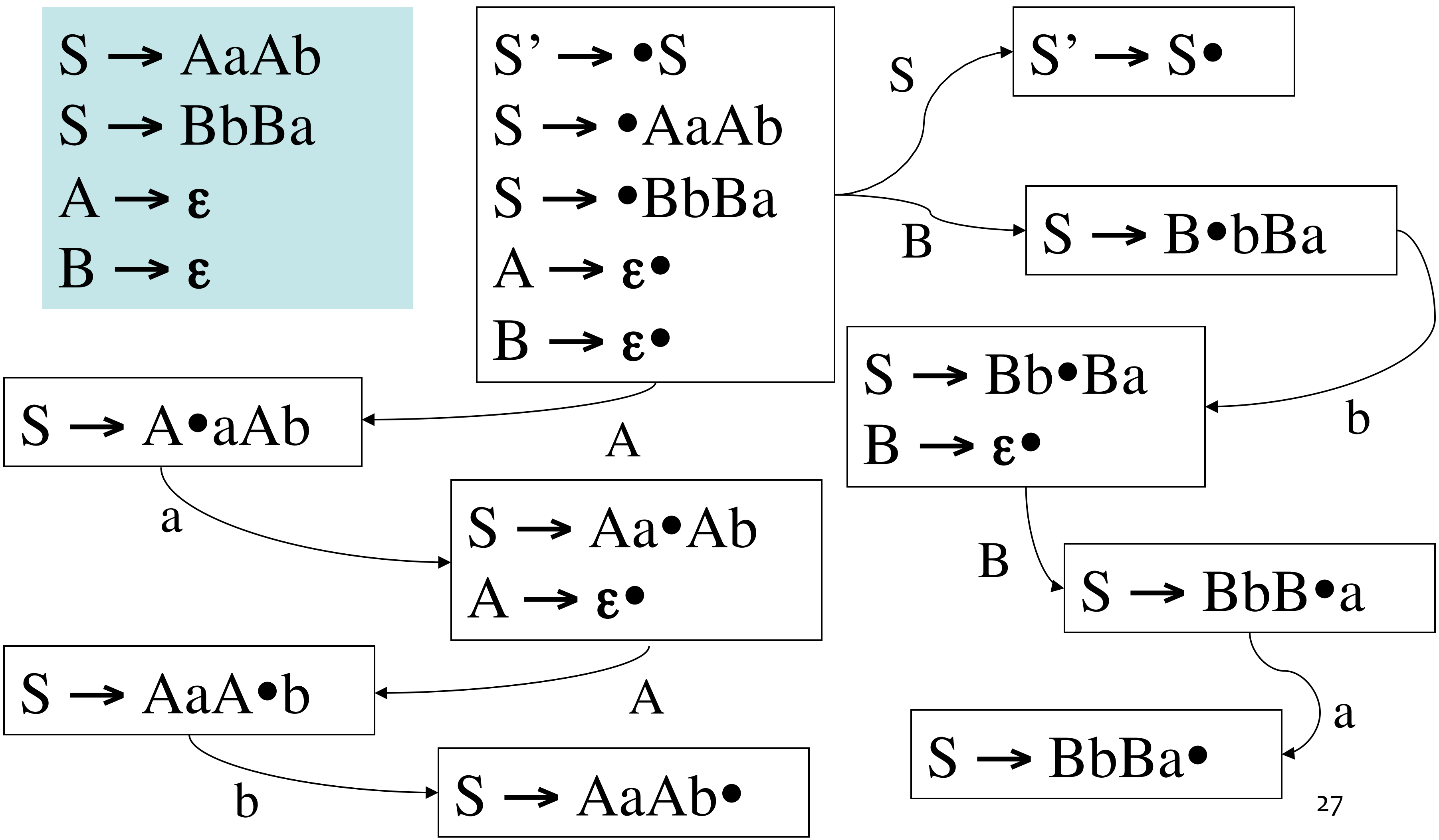
LR(0) Construction (cont'd)

4. All entries not defined are errors
5. Make sure I_0 is the initial state

- Note: LR(0) always reduces if $\{A \rightarrow \alpha\bullet\} \in I_i$, no lookahead
- Shift and reduce items can't be in the same configuration set
  - Accepting state doesn't count as reduce item
- At most one reduce item per set

Set-of-items with Epsilon rules

$S \rightarrow AaAb$
$S \rightarrow BbBa$
$A \rightarrow \varepsilon$
$B \rightarrow \varepsilon$



LR(0) conflicts:

$S' \rightarrow T$
$T \rightarrow F$
$T \rightarrow T * F$
$T \rightarrow id$
$F \rightarrow id \mid (T)$
$F \rightarrow id = T ;$

11: $F \rightarrow id \bullet$
$F \rightarrow id \bullet = T$
Shift/reduce conflict

1: $F \rightarrow id \bullet$
$T \rightarrow id \bullet$
Reduce/Reduce conflict

Need more lookahead: SLR(1)

LR(0) Grammars

- An LR(0) grammar is a CFG such that the LR(0) construction produces a table without conflicts (a deterministic pushdown automata)
- $S \Rightarrow^*_{rm} \alpha A \beta \Rightarrow_{rm} \alpha w \beta$ and $A \rightarrow w$ then we can *prune the handle* w
  - pruning the handle means we can reduce αw to αA on the stack
- Every viable prefix αw can recognized using the DFA built by the LR(0) construction

LR(0) Grammars

- Once we have a viable prefix on the stack, we can prune the handle and then restart the DFA to obtain another viable prefix, and so on ...
- In LR(0) pruning the handle can be done without any look-ahead
  - this means that in the rightmost derivation,
  - $S \Rightarrow^*_{rm} \alpha A \beta \Rightarrow_{rm} \alpha w \beta$ we reduce using a unique rule $A \rightarrow w$ without ambiguity, and without looking at β
- No ambiguous context-free grammar can be LR(0)

LR(0) Grammars $\subset$ Context-free Grammars

FIRST and FOLLOW

$a \in \mathrm{FIRST}(\alpha)$ if $\alpha \Rightarrow^* a\beta$

if $\alpha \Rightarrow^* \epsilon$ then $\epsilon \in \mathrm{FIRST}(\alpha)$

$a \in \mathrm{FOLLOW}(A)$ if $S \Rightarrow^* \alpha A a \beta$

$a \in \mathrm{FOLLOW}(A)$ if $S \Rightarrow^* \alpha A \gamma a \beta$

and $\gamma \Rightarrow^* \epsilon$

Example First/Follow

$S \rightarrow AB$

$A \rightarrow c \mid \varepsilon$

$B \rightarrow cbB \mid ca$

First(A) = {c, ε}

First(B) = {c}

First(cbB) =

First(ca) = {c}

First(S) = {c}

Follow(A) = {c}

Follow(A) ∩

First(c) = {c}

Follow(B) = {\$}

Follow(S) = {\$}

Example First/Follow

$S \rightarrow cAa$

$A \rightarrow cB \mid B$

$B \rightarrow bcB \mid \varepsilon$

$\text{First}(A) = \{b, c, \varepsilon\}$

$\text{First}(B) = \{b, \varepsilon\}$

$\text{First}(S) = \{c\}$

$\text{Follow}(A) = \{a\}$

$\text{Follow}(B) = \{a\}$

$\text{Follow}(S) = \{\$\}$

SLR(1) : Simple LR(1) Parsing

$S' \rightarrow T$
$T \rightarrow F \mid T * F \mid C (T)$
$F \rightarrow id \mid id ++ \mid (T)$
$C \rightarrow id$

What can the next symbol be when we reduce $F \rightarrow id$?

$S'\$ \Rightarrow T\$ \Rightarrow F\$ \Rightarrow id\underline{\$}$

$S'\$ \Rightarrow T\$ \Rightarrow T*F\$ \Rightarrow T*id\$ \Rightarrow F*id\$ \Rightarrow id\underline{*}id\$$

$S'\$ \Rightarrow T\$ \Rightarrow C(T)\$ \Rightarrow C(F)\$ \Rightarrow C(id\underline{)}\$$

The top of stack will be id and the next input symbol will be either \$, or * or)

Follow(F) = { *,), \$ }

SLR(1) : Simple LR(1) Parsing

$S' \rightarrow T$
$T \rightarrow F \mid T * F \mid C\ (\ T\ )$
$F \rightarrow id \mid id\ {+}{+} \mid (\ T\ )$
$C \rightarrow id$

What can the next symbol be when we reduce $C \rightarrow id$?

$$S'\$ \Rightarrow T\$ \Rightarrow C(T)\$ \Rightarrow C(F)\$ \Rightarrow C(id) \Rightarrow id\underline{(}id)\$$$

Follow(C) = { (}

SLR(1) : Simple LR(1) Parsing

0: $S' \to \bullet\, T$
$T \to \bullet\, F$
$T \to \bullet\, T * F$
$T \to \bullet\, C\ (T)$
$F \to \bullet\, id$
$F \to \bullet\, id\ ++$
$F \to \bullet\, (\ T\ )$
$C \to \bullet\, id$

id

1: $F \to id\, \bullet$
$F \to id\, \bullet\ ++$
$C \to id\, \bullet$

$S' \to T$
$T \to F \mid T * F \mid C\ (\ T\ )$
$F \to id \mid id\ ++ \mid (\ T\ )$
$C \to id$

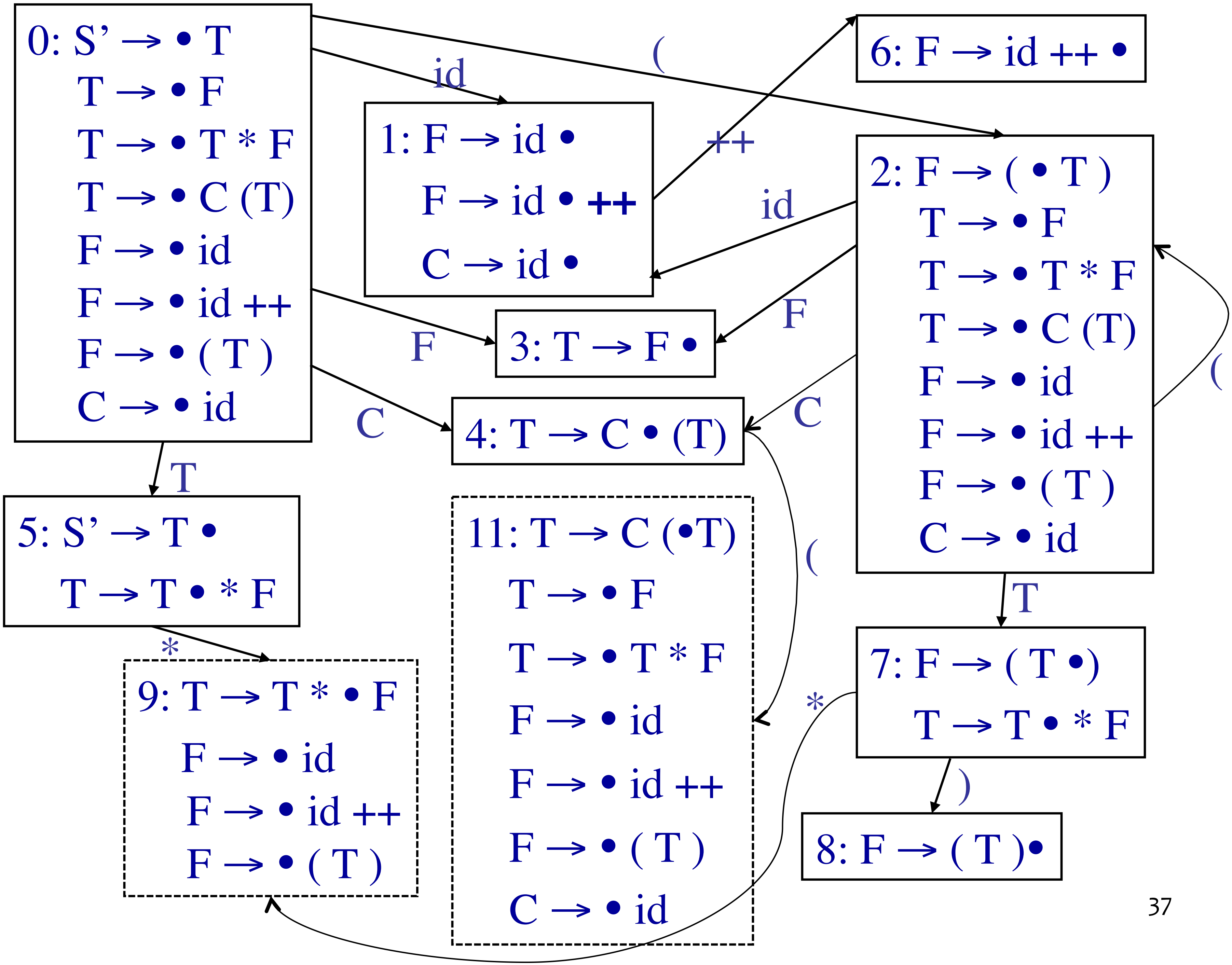
Follow(F) = { *,), $ }

Follow(C) = { (}

action[1,*]= action[1,)] = action[1,$] = Reduce $F \to id$

action[1,(] = Reduce $C \to id$

action[1,++] = Shift


0: S’ → • T
T → • F
T → • T * F
T → • C (T)
F → • id
F → • id ++
F → • (T)
C → • id
1: F → id •
F → id • ++
C → id •
2: F → (• T)
T → • F
T → • T * F
T → • C (T)
F → • id
F → • id ++
F → • (T)
C → • id
3: T → F •
4: T → C • (T)
5: S’ → T •
T → T • * F
6: F → id ++ •
7: F → (T •)
T → T • * F
8: F → (T)•
9: T → T * • F
F → • id
F → • id ++
F → • (T)
11: T → C (•T)
T → • F
T → • T * F
F → • id
F → • id ++
F → • (T)
C → • id
(
id
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id
F
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C
C
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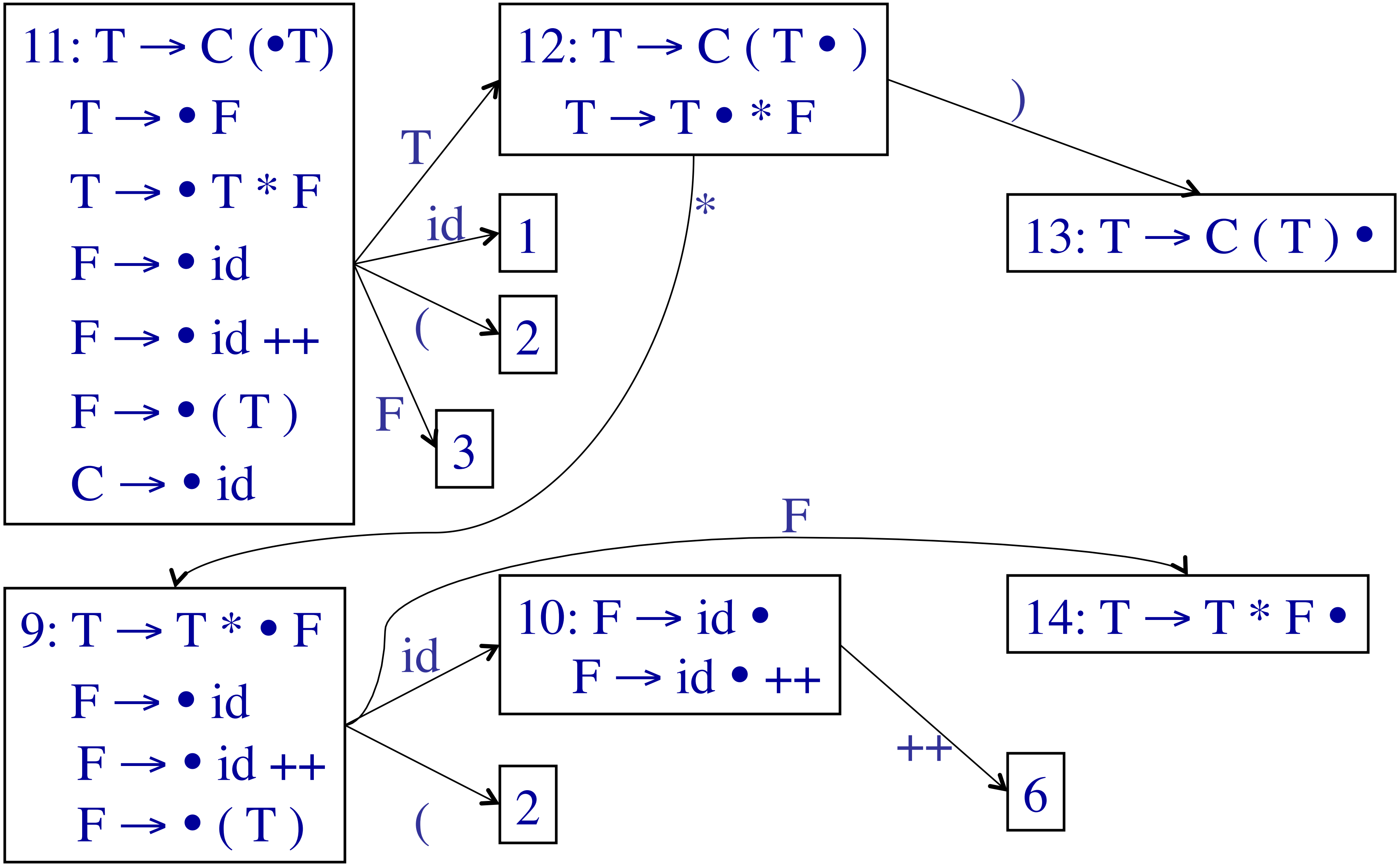
(
(
T
*
*
)



11: T → C (•T)
T → • F
T → • T * F
F → • id
F → • id ++
F → • (T)
C → • id
12: T → C (T •)
T → T • * F
13: T → C (T) •
T
id
(
F
1
2
3
)
*
F
9: T → T * • F
F → • id
F → • id ++
F → • (T)
10: F → id •
F → id • ++
14: T → T * F •
id
(
++
2
6

	Productions
1	**T → F**
2	**T → T*F**
3	**T → C(T)**
4	**F → id**
5	**F → id ++**
6	**F → (T)**
7	**C → id**

	*	(	)	id	++	$	T	F	C
0		S2		S1			5	3	4
1	R4	R7	R4		S2	R4			
2		S2		S1			7	3	4
3	R1		R1			R1			
4		S11							
5	S9					A			
6	R5		R5			R5			
7	S9		S8						
8	R6		R6			R6			
9		S2		S10				14	
10	R4		R4		S6	R4			
11		S2		S1			12	3	
12	S9		S13						
13	R3		R3			R3			
14	R2		R2			R2			

SLR(1) Construction

1. Construct $F = \{I_0, I_1, \dots I_n\}$
2. a) if $\{A \rightarrow \alpha\bullet\} \in I_i$ and $A \mathrel{!}= S'$
   then action[i, b] := reduce $A \rightarrow \alpha$
   for all $b \in \text{Follow}(A)$

   b) if $\{S' \rightarrow S\bullet\} \in I_i$
   then action[i, $] := accept

   c) if $\{A \rightarrow \alpha\bullet a\beta\} \in I_i$ and $\text{Successor}(I_i, a) = I_j$
   then action[i, a] := shift j
3. if $\text{Successor}(I_i, A) = I_j$ then goto[i, A] := j

SLR(1) Construction (cont'd)

4. All entries not defined are errors
5. Make sure I_0 is the initial state

- Note: SLR(1) only reduces $\{A \rightarrow \alpha\bullet\}$ if lookahead in Follow(A)
- Shift and reduce items or more than one reduce item can be in the same configuration set as long as lookaheads are disjoint

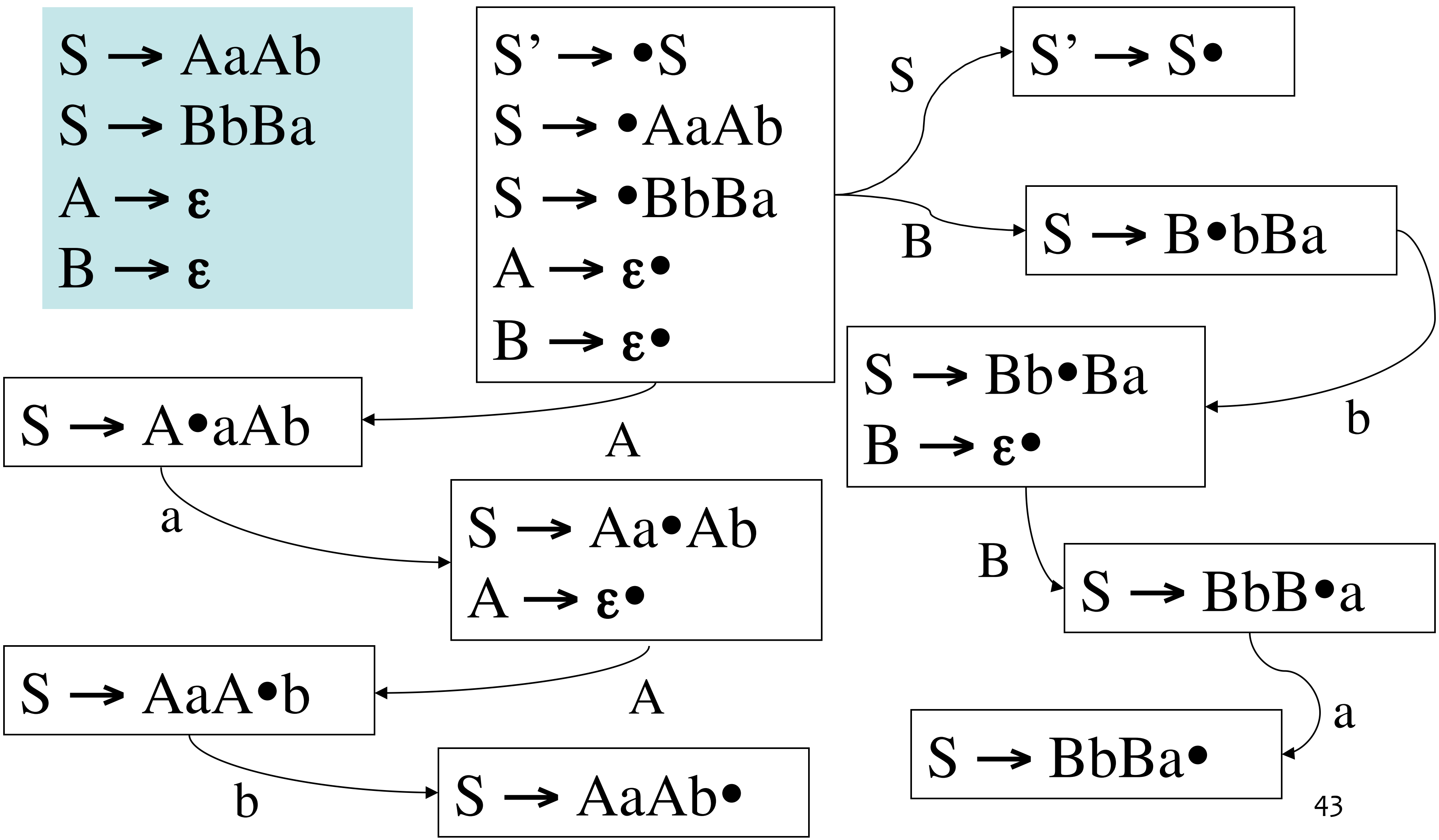
SLR(1) Conditions

- A grammar is SLR(1) if for each configuration set:
  - For any item $\{A \rightarrow \alpha \bullet x\beta : x \in T\}$ there is no $\{B \rightarrow \gamma \bullet : x \in \text{Follow}(B)\}$
  - For any two items $\{A \rightarrow \alpha \bullet\}$ and $\{B \rightarrow \beta \bullet\}$ $\text{Follow}(A) \cap \text{Follow}(B) = \emptyset$

LR(0) Grammars $\subset$ SLR(1) Grammars

Is this grammar SLR(1)?

$S \rightarrow AaAb$
$S \rightarrow BbBa$
$A \rightarrow \varepsilon$
$B \rightarrow \varepsilon$



SLR limitation: lack of context

$S' \rightarrow S$
$S \rightarrow L = R \mid R$
$L \rightarrow {*}R \mid id$
$R \rightarrow L$

Input: id = id

0: $S' \rightarrow \bullet\ S$
$S \rightarrow \bullet\ L = R$
$S \rightarrow \bullet\ R$
$L \rightarrow \bullet\ {*}\ R$
$L \rightarrow \bullet\ id$
$R \rightarrow \bullet\ L$

id

1: $L \rightarrow id\ \bullet$

L

2: $S \rightarrow L\ \bullet = R$
$R \rightarrow L\ \bullet$

=

3: $S \rightarrow L = \bullet\ R$
$R \rightarrow \bullet\ L$
$L \rightarrow \bullet\ {*}\ R$
$L \rightarrow \bullet\ id$

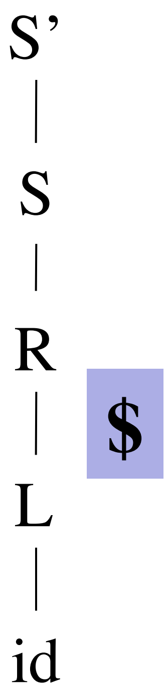
Follow(R) = { =, $ }

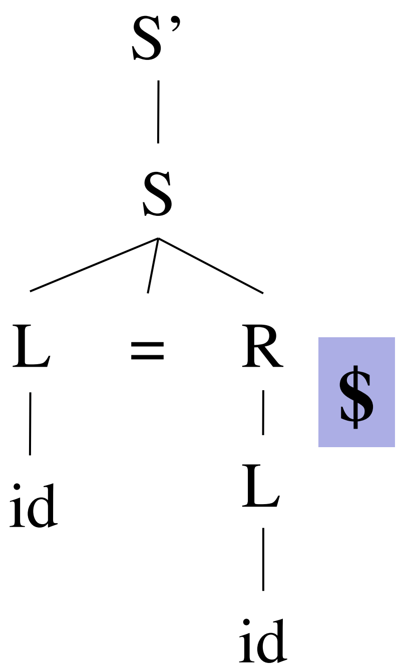
$S' \rightarrow S$
$S \rightarrow L = R \mid R$
$L \rightarrow *R \mid id$
$R \rightarrow L$

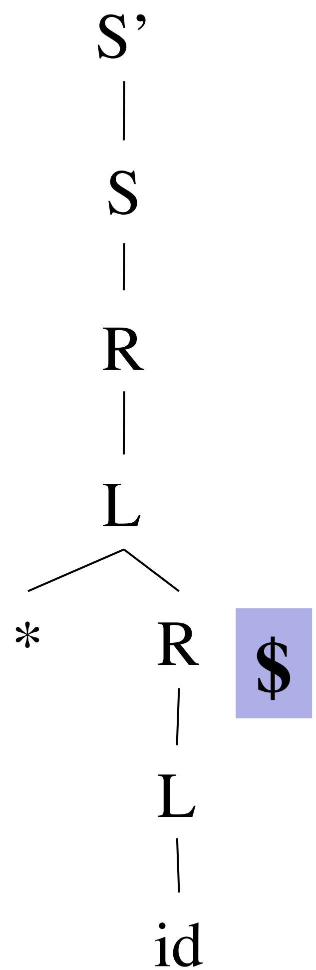
Follow(R) = { =, $ }

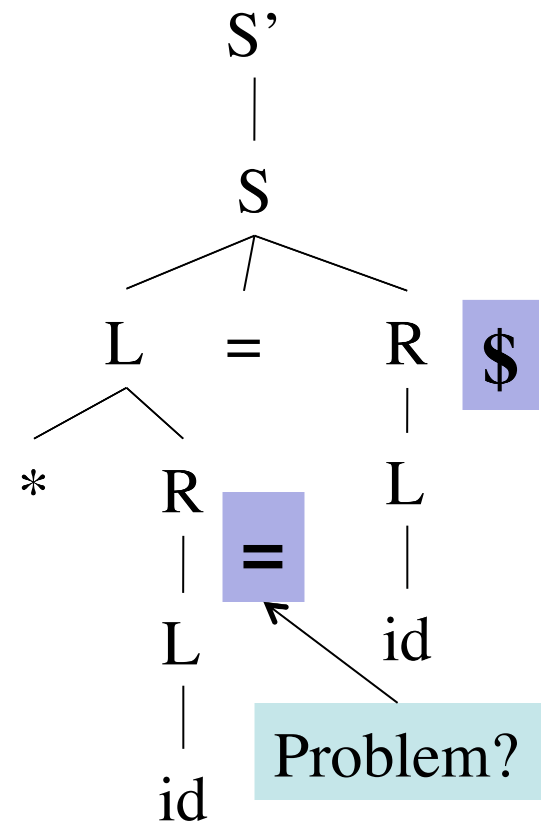
2: $S \rightarrow L \bullet = R$
$R \rightarrow L \bullet$

Find all lookaheads for reduce $R \rightarrow L \bullet$









No! $R \rightarrow L \bullet$ reduce and $S \rightarrow L \bullet = R$ do not co-occur due to the $L \rightarrow *R$ rule

Solution: Canonical LR(1)

- Extend definition of configuration
  - Remember lookahead
- New closure method
- Extend definition of Successor

LR(1) Configurations

- $[A \rightarrow \alpha \bullet \beta, a]$ for $a \in T$ is valid for a viable prefix $\delta\alpha$ if there is a rightmost derivation
  $S \Rightarrow^* \delta A \eta \Rightarrow^* \delta\alpha\beta\eta$ and
  $(\eta = a\gamma)$ or $(\eta = \varepsilon$ and $a = \$)$
- Notation: $[A \rightarrow \alpha \bullet \beta, a/b/c]$
  - if $[A \rightarrow \alpha \bullet \beta, a]$, $[A \rightarrow \alpha \bullet \beta, b]$, $[A \rightarrow \alpha \bullet \beta, c]$ are valid configurations

LR(1) Configurations

$$S \Rightarrow BB \Rightarrow BaB \Rightarrow Bab$$
$$\Rightarrow aBab \Rightarrow aaBab \Rightarrow aaaBab$$

$S \rightarrow B\ B$

$B \rightarrow a\ B \mid b$

- $S \Rightarrow^*_{rm} aaBab \Rightarrow_{rm} aaaBab$
- Item $[B \rightarrow a \bullet B, a]$ is valid for viable prefix *aaa*
- $S \Rightarrow^*_{rm} BaB \Rightarrow_{rm} BaaB$
- Also, item $[B \rightarrow a \bullet B, \$]$ is valid for viable prefix *Baa*

$$S \Rightarrow BB \Rightarrow BaB \Rightarrow BaaB$$

LR(1) Closure

Closure property:

- If $[A \rightarrow \alpha \bullet B\beta, a]$ is in set, then $[B \rightarrow \bullet \gamma, b]$ is in set if $b \in \mathrm{First}(\beta a)$
- Compute as fixed point
- Only include contextually valid lookaheads to guide reducing to B

Starting Configuration

- Augment Grammar with S' just like for LR(0), SLR(1)
- Initial configuration set is

$$I = \text{closure}([S' \rightarrow \bullet\, S,\ \$])$$

Example: closure([S’ → • S, $])

[S’ → • S, $]

[S → • L = R, $]

[S → • R, $]

[L → • * R, =]

[L → • id, =]

[R → • L, $]

[L → • *R, $]

[L → • id, $]

S’ → S
S → L = R | R
L → *R | id
R → L

concisely written as:

S’ → • S, $
S → • L = R, $
S → • R, $
L → • * R, =/$
L → • id, =/$
R → • L, $

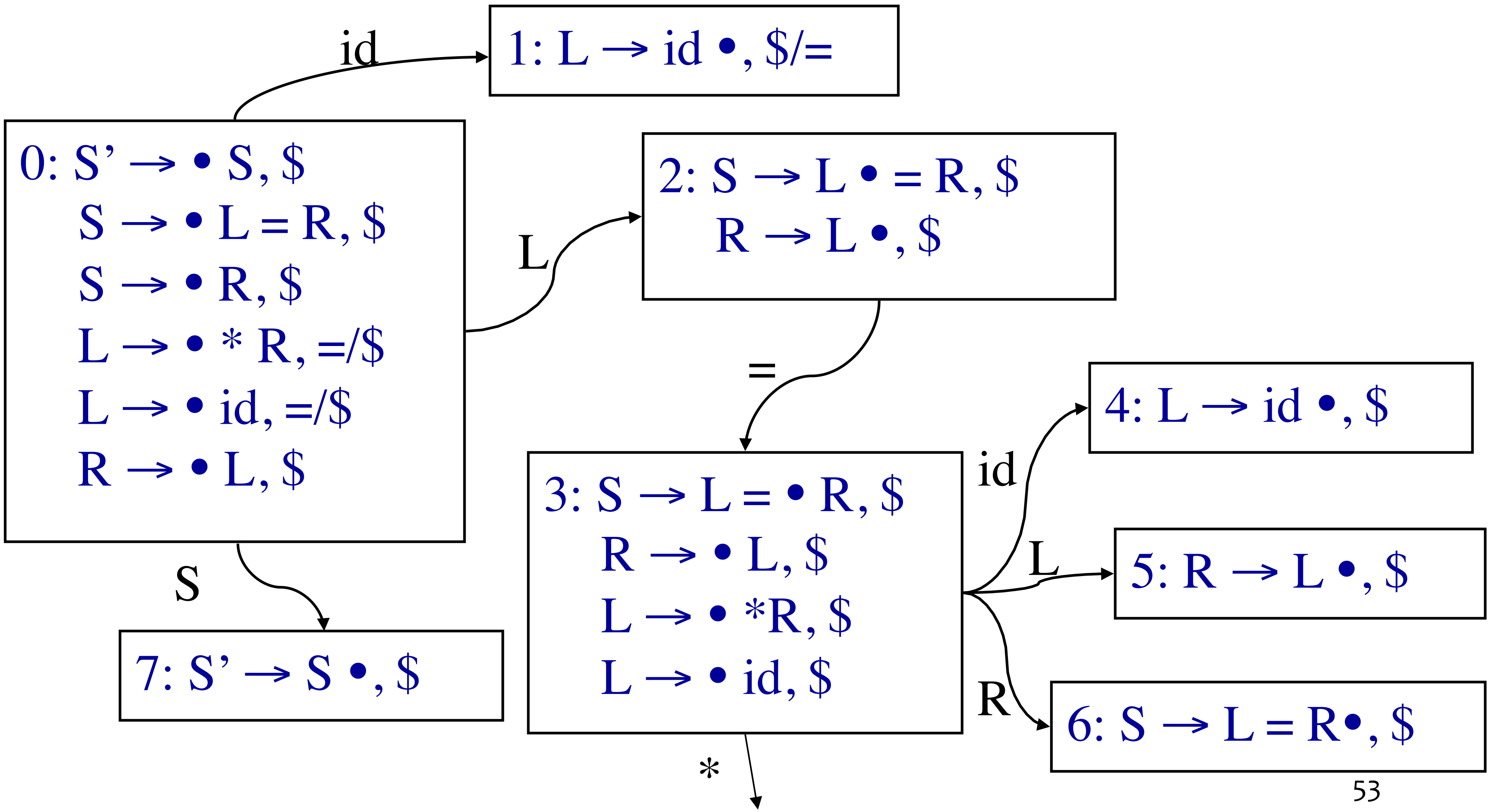
LR(1) Successor(C, X)

- Let I = $[A \rightarrow \alpha \bullet B\beta, a]$ **or** $[A \rightarrow \alpha \bullet b\beta, a]$
- Successor(I, B)

  = closure($[A \rightarrow \alpha B \bullet \beta, a]$)
- Successor(I, b)

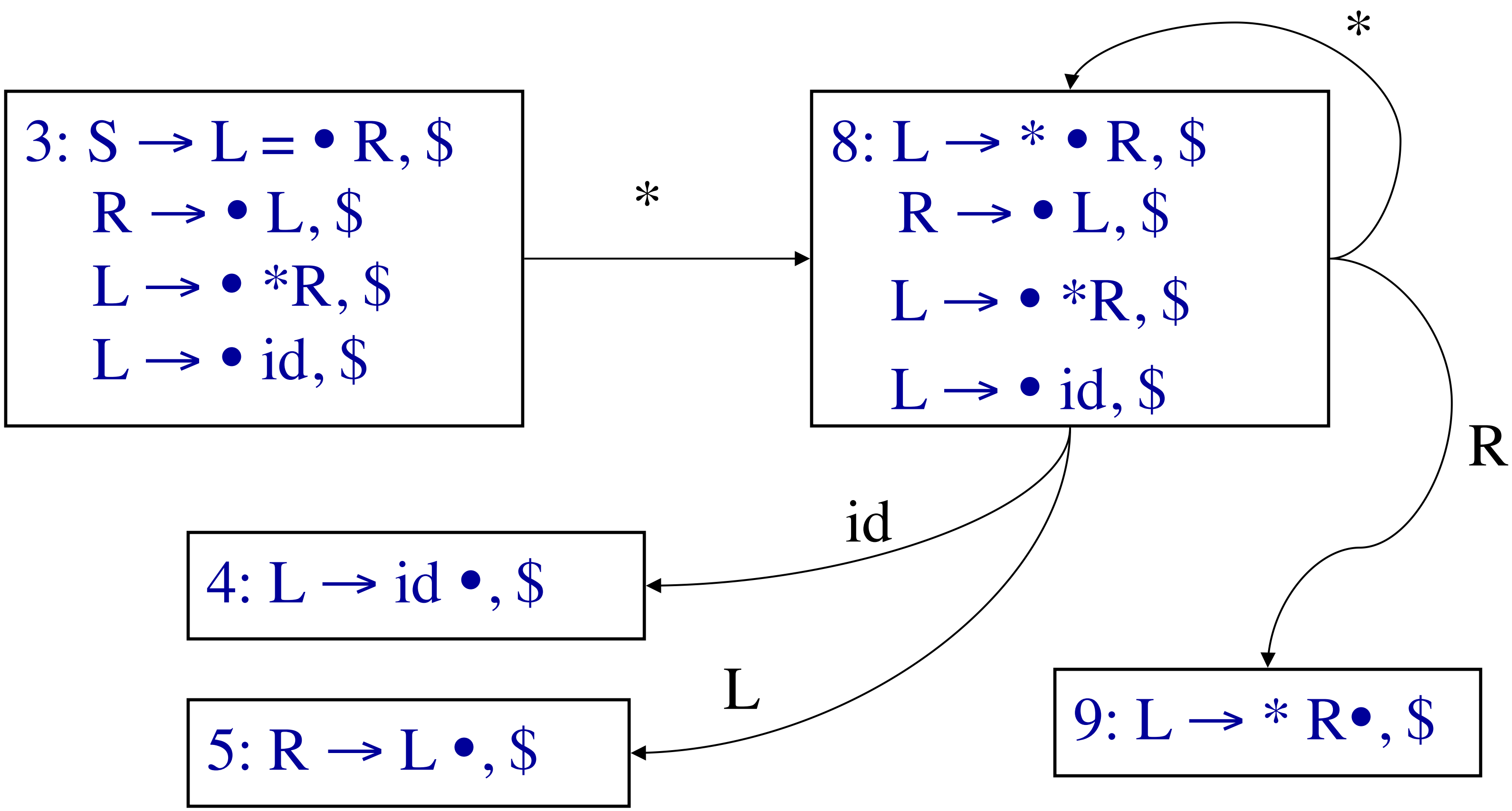
  = closure($[A \rightarrow \alpha b \bullet \beta, a]$)

LR(1) Example

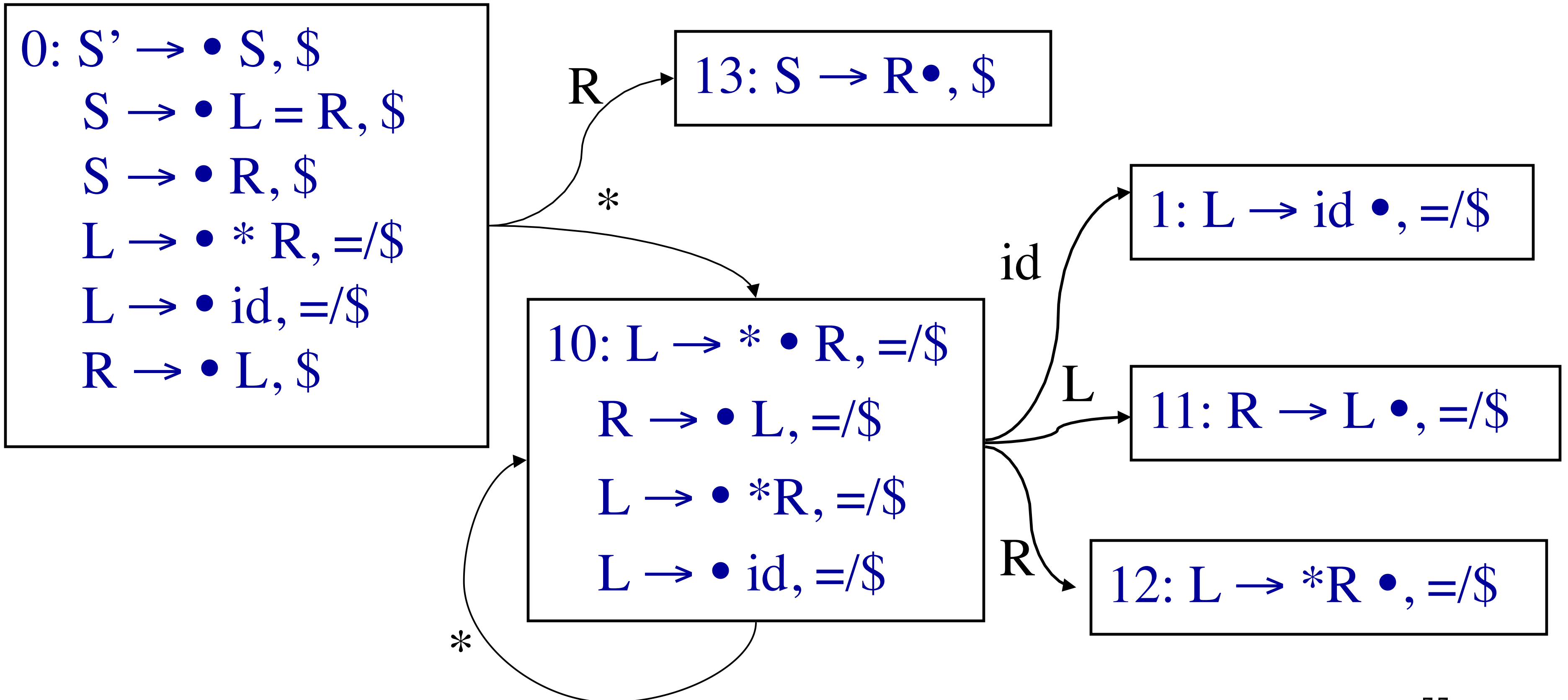


id
1: L → id •, $/=
0: S’ → • S, $
S → • L = R, $
S → • R, $
L → • * R, =/$
L → • id, =/$
R → • L, $
L
2: S → L • = R, $
R → L •, $
=
3: S → L = • R, $
R → • L, $
L → • *R, $
L → • id, $
id
4: L → id •, $
L
5: R → L •, $
R
6: S → L = R•, $
S
7: S’ → S •, $
*

LR(1) Example (contd)



LR(1) Example (contd)



Productions	
1	**S → L = R**
2	**S → R**
3	**L → * R**
4	**L → id**
5	**R → L**

	id	=	*	$	S	L	R
0	S1		S10		7	2	13
1		R4		R4			
2		S3		R5			
3	S4		S8			5	6
4				R4			
5				R5			
6				R1			
7				Acc			
8	S4					5	9
9				R3			
10	S1		S10			11	12
11		R5		R5			
12		R3		R3			
13				R2			

LR(1) Construction

1. Construct $F = \{I_0, I_1, \ldots I_n\}$
2. a) if $[A \rightarrow \alpha\bullet, a] \in I_i$ and $A \mathrel{!}= S'$

   then $action[i, a] := \text{reduce } A \rightarrow \alpha$

   b) if $[S' \rightarrow S\bullet, \$] \in I_i$

   then $action[i, \$] := \text{accept}$

   c) if $[A \rightarrow \alpha\bullet a\beta, b] \in I_i$ and $Successor(I_i, a) = I_j$

   then $action[i, a] := \text{shift } j$
3. if $Successor(I_i, A) = I_j$ then $goto[i, A] := j$

LR(1) Construction (cont'd)

4. All entries not defined are errors
5. Make sure I_0 is the initial state

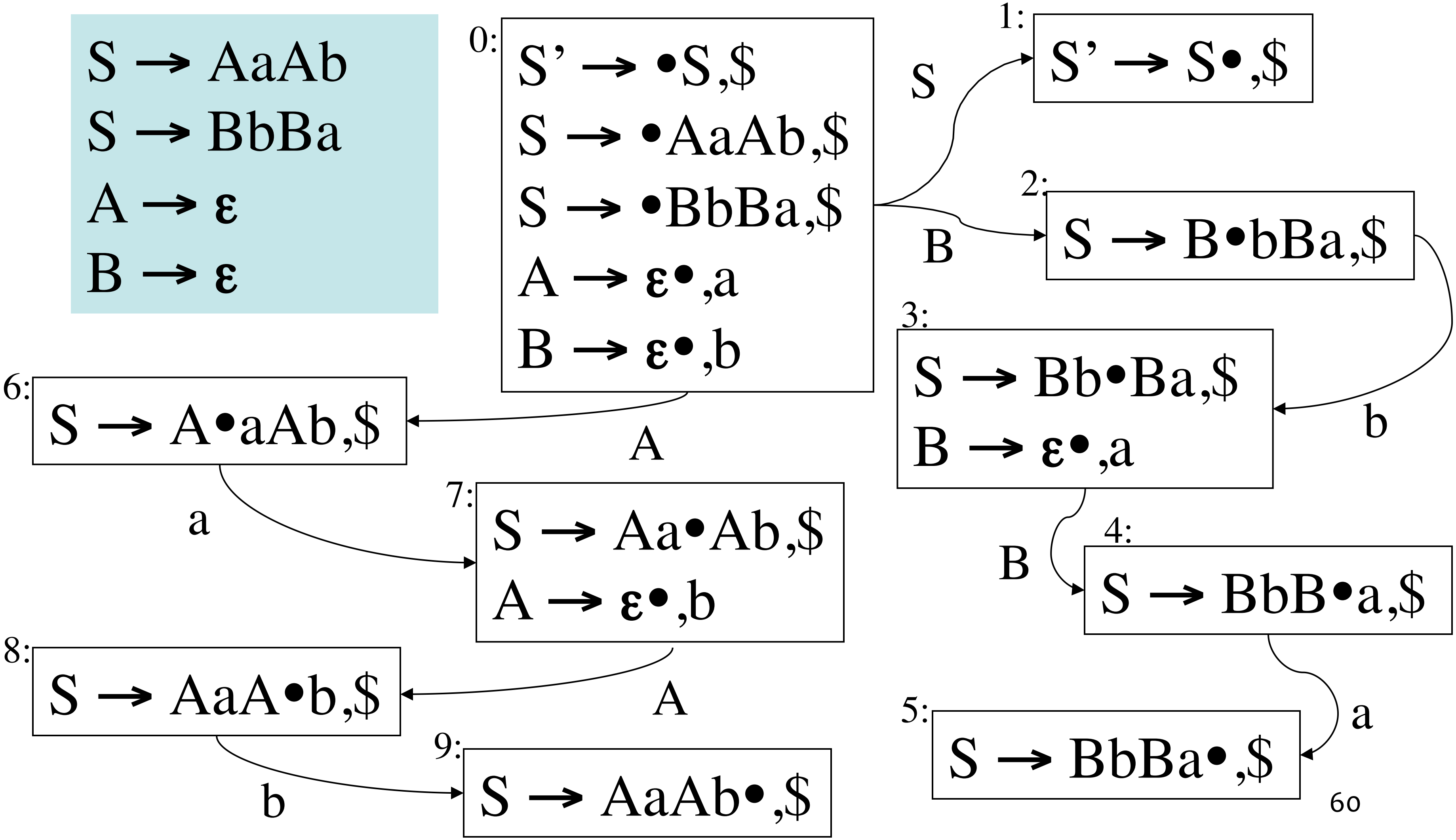
- Note: LR(1) only reduces using $A \rightarrow \alpha$ for $[A \rightarrow \alpha\bullet, a]$ if a follows
- LR(1) states remember context by virtue of lookahead
- Possibly many states!
  - LALR(1) combines some states

LR(1) Conditions

- A grammar is LR(1) if for each configuration set (itemset) the following holds:
  - For any item $[A \rightarrow \alpha \bullet x\beta, a]$ with $x \in T$ there is no $[B \rightarrow \gamma \bullet, x]$
  - For any two complete items $[A \rightarrow \gamma \bullet, a]$ and $[B \rightarrow \beta \bullet, b]$ then a != b.
- Grammars:
  - $LR(0) \subset SLR(1) \subset LR(1) \subset LR(k)$
- Languages expressible by grammars:
  - $LR(0) \subset SLR(1) \subset LR(1) = LR(k)$

Set-of-items with Epsilon rules

S → AaAb
S → BbBa
A → ε
B → ε



Canonical LR(1) Recap

- LR(1) uses left context, current handle and lookahead to decide when to reduce or shift
- Most powerful parser so far
- LALR(1) is practical simplification with fewer states

Merging States in LALR(1)

- $S' \rightarrow S$
  $S \rightarrow XX$
  $X \rightarrow aX$
  $X \rightarrow b$
- Same **Core Set**
- Different lookaheads

3: $X \rightarrow a \bullet X$, a/b
$X \rightarrow \bullet\, a\, X$, a/b
$X \rightarrow \bullet\, b$, a/b

6: $X \rightarrow a \bullet X$, \$
$X \rightarrow \bullet\, a\, X$, \$
$X \rightarrow \bullet\, b$, \$

36: $X \rightarrow a \bullet X$, a/b/\$
$X \rightarrow \bullet\, a\, X$, a/b/\$
$X \rightarrow \bullet\, b$, a/b/\$

R/R conflicts when merging

- $B \to d$
  $B \to f\ X\ g$
  $X \to \ldots$

2: $B \to d\ \bullet,\ c$
$B \to f\ X\ g\ \bullet,\ e$

4: $B \to d\ \bullet,\ g$
$B \to f\ X\ g\ \bullet,\ c$

24: $B \to d\ \bullet,\ c/g$
$B \to f\ X\ g\ \bullet,\ c/e$

- If R/R conflicts are introduced, grammar is not LALR(1)!

LALR(1)

- LALR(1) Condition:
  - Merging in this way does not introduce reduce/reduce conflicts
  - Shift/reduce can't be introduced
- Merging brute force or step-by-step
- More compact than canonical LR, like SLR(1)
- More powerful than SLR(1)
  - Not always merge to full Follow Set

S/R & ambiguous grammars

- Lx(k) Grammar vs. Language
  - Grammar is Lx(k) if it can be parsed by Lx(k) method – according to criteria that is specific to the method.
  - A Lx(k) grammar may or may not exist for a language.
- Even if a given grammar is not LR(k), shift/reduce parser can *sometimes* handle them by accounting for ambiguities
  - Example: ‘dangling’ else
    - Preferring shift to reduce means matching inner ‘if’

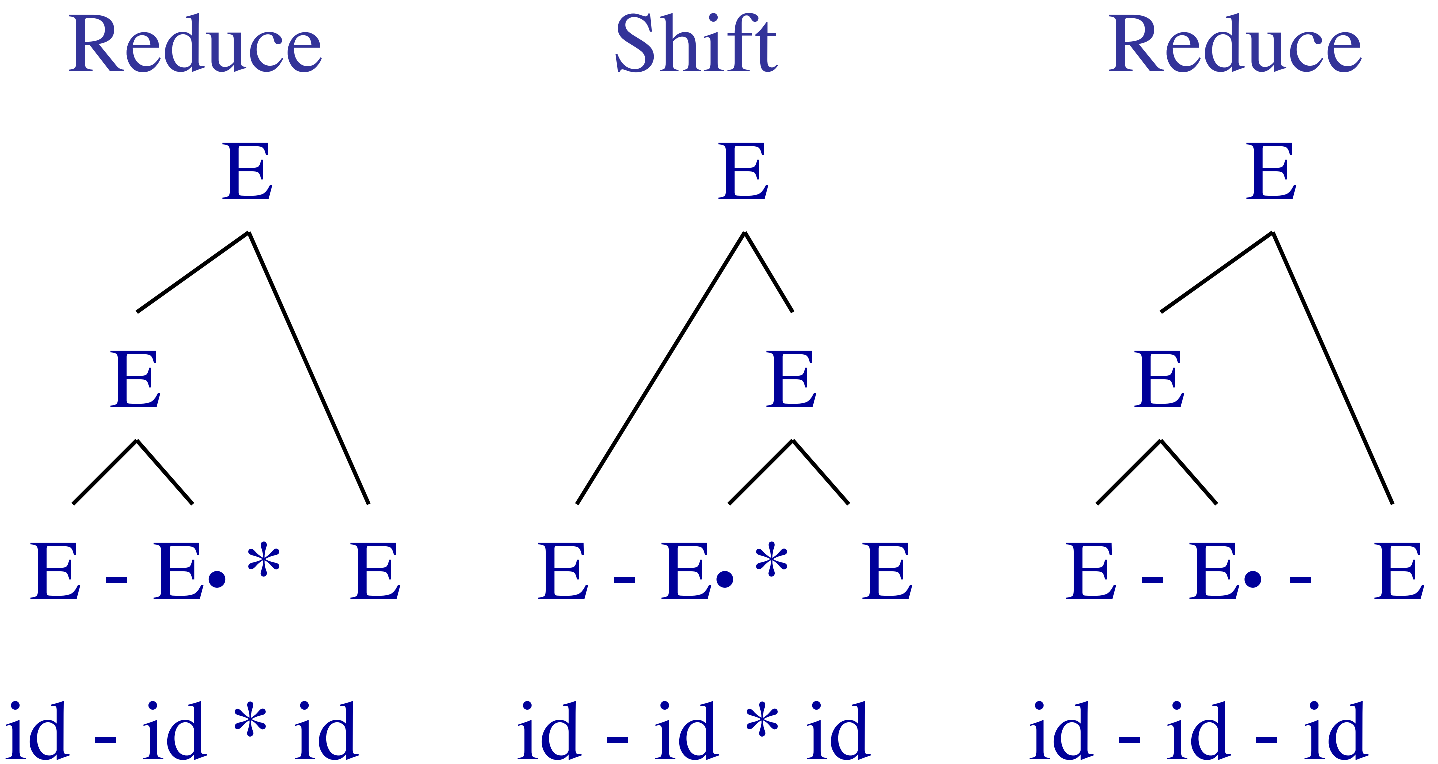
Dangling ‘else’

1. $S \rightarrow$ if E then S
2. $S \rightarrow$ if E then S else S

- Viable prefix “if E then if E then S”
  - Then read else
- Shift “else” (means go for 2)
- Reduce (reduce using production #1)
- NB: dangling else as written above is ambiguous
  - NB: Ambiguity can be resolved, but there’s still no LR(k) grammar

Precedence & Associativity

- Consider $E \rightarrow E - E \mid E * E \mid id$



Precedence Relations

- Let $A \rightarrow w$ be a rule in the grammar
- And b is a terminal
- In some state q of the LR(1) parser there is a shift-reduce conflict:
  - either reduce with $A \rightarrow w$ or shift on b
- Write down a rule, either:

  $A \rightarrow w, < b$ or $A \rightarrow w, > b$

Precedence Relations

- $A \rightarrow w, < b$ means rule has less precedence and so we shift if we see *b* in the lookahead
- $A \rightarrow w, > b$ means rule has higher precedence and so we reduce if we see *b* in the lookahead
- If there are multiple terminals with shift-reduce conflicts, then we list them all:

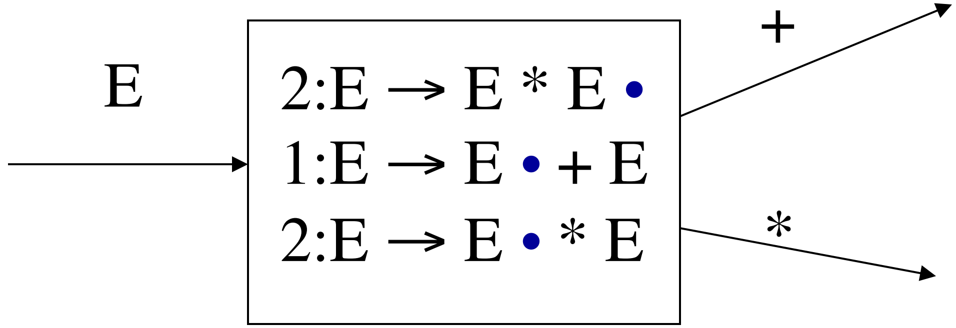
  $A \rightarrow w, > b, < c, > d$

Precedence Relations

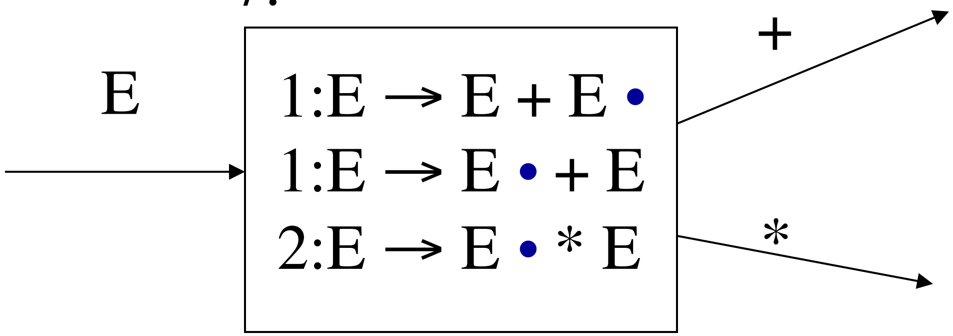
- Consider the grammar
  $E \rightarrow E + E \mid E * E \mid (E) \mid a$
- Assume left-association so that E+E+E is interpreted as (E+E)+E
- Assume multiplication has higher precedence than addition
- Then we can write precedence rules/relns:
  $E \rightarrow E + E, > +, < *$
  $E \rightarrow E * E, > +, > *$

Precedence & Associativity

10:



7:



$E \rightarrow E + E, > +, < *$

$E \rightarrow E * E, > +, > *$

	+	*
7	R1	Shift
10	R2	R2

Handling S/R & R/R Conflicts

- Have a conflict?
  - No? – Done, grammar is compliant.
- Already using most powerful parser available?
  - No? – Upgrade and goto 1
- Can the grammar be rearranged so that the conflict disappears?
  - While preserving the language!

Conflicts revisited (cont'd)

- Can the grammar be rearranged so that the conflict disappears?
  - Yes: Is it worth it?
    - Yes, resolve conflict.
    - No: live with default or specified conflict resolution (precedence, associativity)

Compiler (parser) compilers

- Rather than build a parser for a particular grammar (e.g. recursive descent), write down a grammar as a text file
- Run through a compiler compiler which produces a parser for that grammar
- The parser is a program that can be compiled and accepts input strings and produces user-defined output

Compiler (parser) compilers

- For LR parsing, all it needs to do is produce action/goto table
  - Yacc (yet another compiler compiler) was distributed with Unix, the most popular tool. Uses LALR(1).
  - Many variants of yacc exist for many languages
- As we will see later, translation of the parse tree into machine code (or anything else) can also be written down with the grammar
- Handling errors and interaction with the lexical analyzer have to be precisely defined

Parsing - Summary

- Top-down vs. bottom-up
- Lookahead: FIRST and FOLLOW sets
- LL(1) – Parsing: $O(n)$ time complexity
  - recursive-descent and table-driven predictive parsing
- LR(k) – Parsing : $O(n)$ time complexity
  - LR(0), SLR(1), LR(1), LALR(1)
- Resolving shift/reduce conflicts
  - using precedence, associativity