LEX6: NFA to DFA

Lexical Analysis

CMPT 379: Compilers

Instructor: Anoop Sarkar

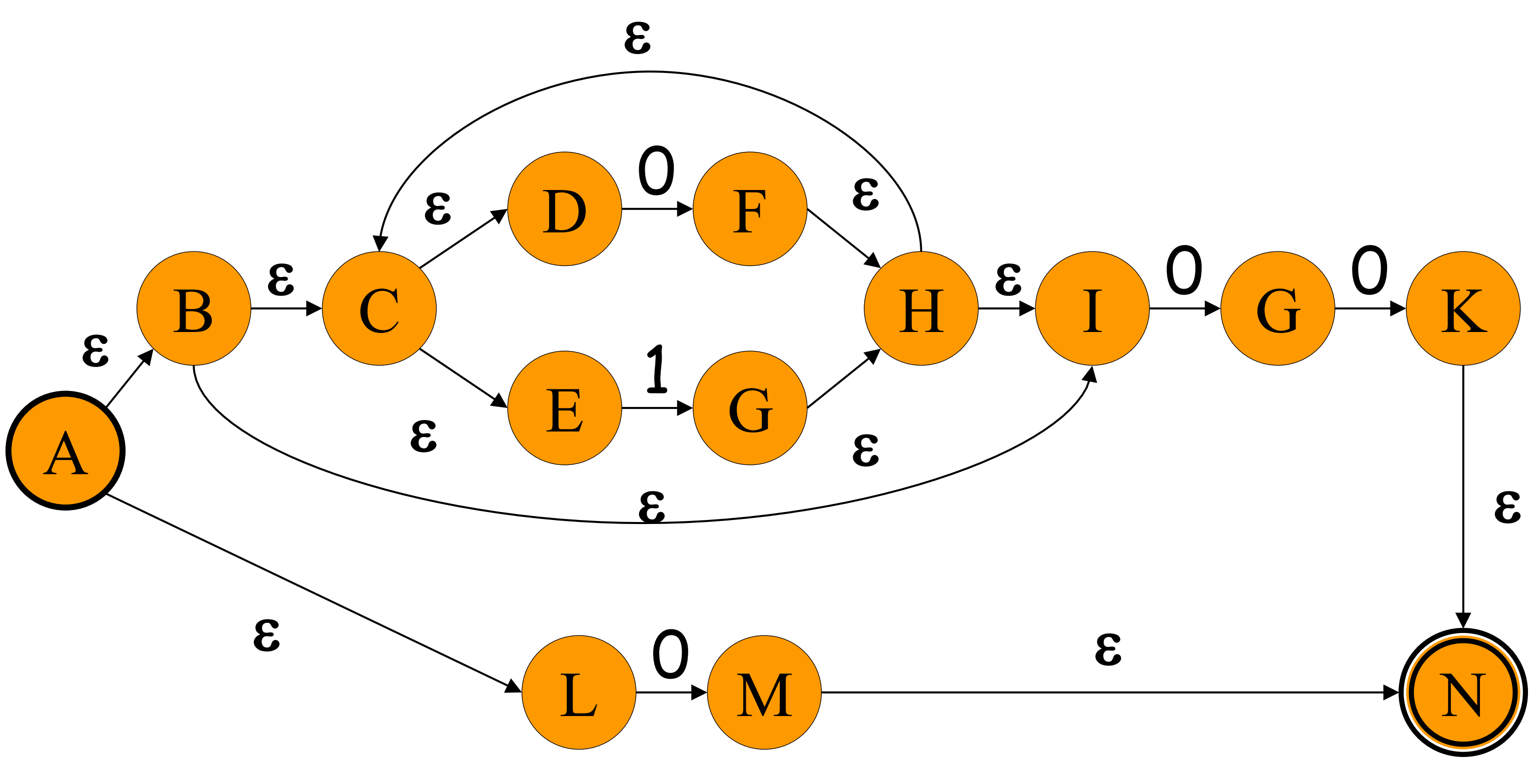
anoopsarkar.github.io/compilers-class

Building a Lexical Analyzer

- Token $\Rightarrow$ Pattern
- Pattern $\Rightarrow$ Regular Expression
- Regular Expression $\Rightarrow$ NFA
- NFA $\Rightarrow$ DFA
- DFA $\Rightarrow$ Table-driven implementation of DFA

ε-closure

ε-closure(s)= all states reached by following only ε-transitions



((0|1)*00)|0

ε-closure

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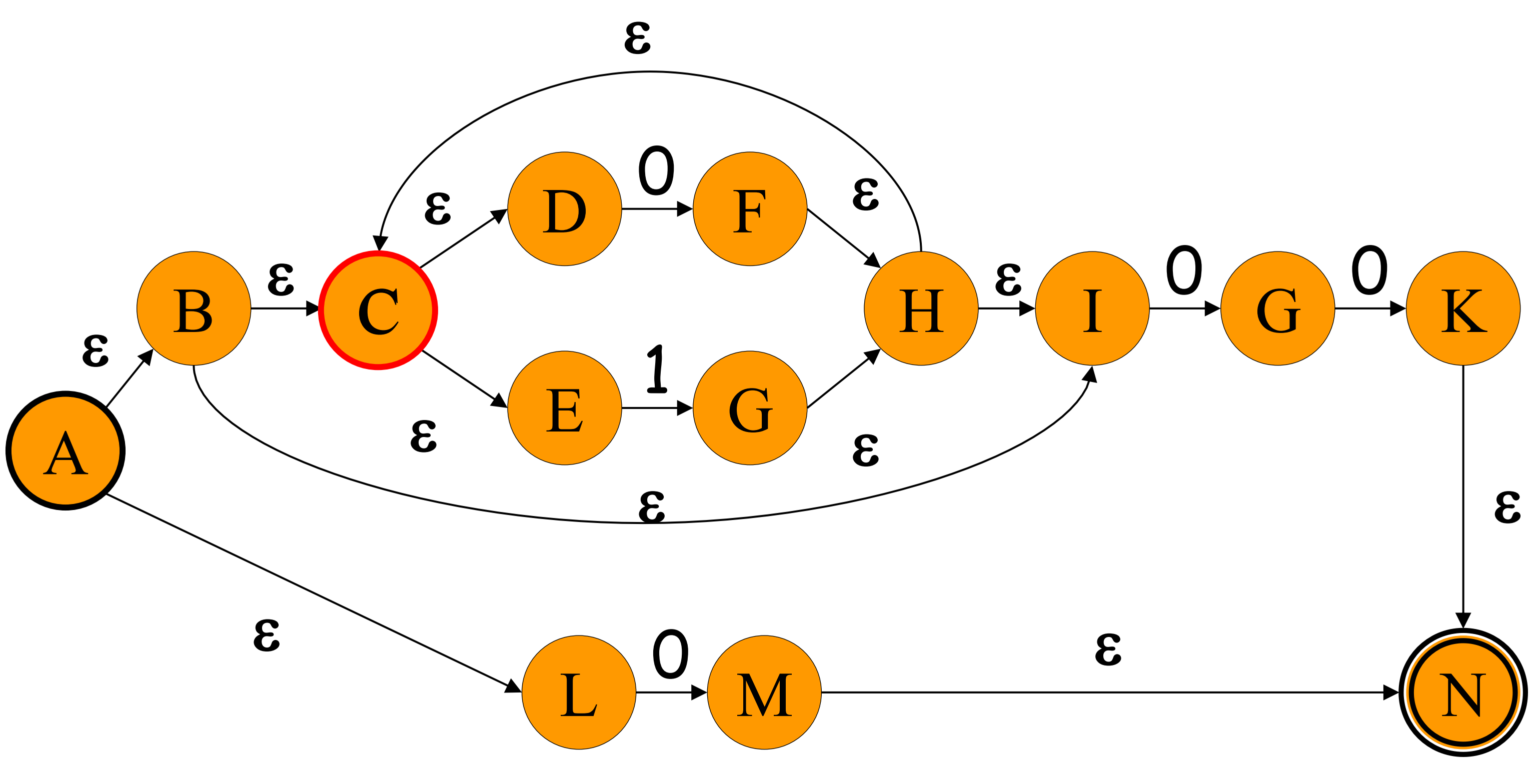
ε-closure(C) =



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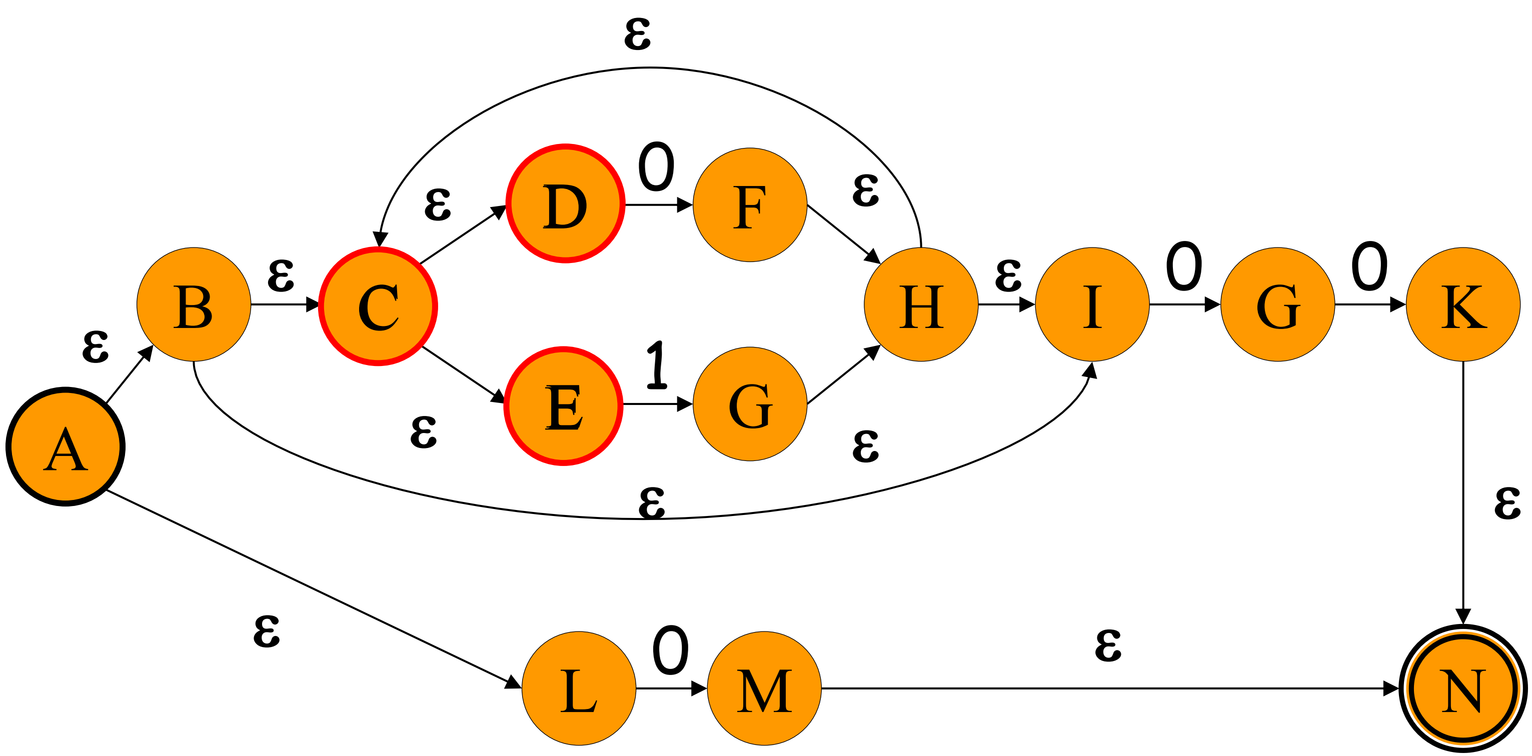

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ε-closure

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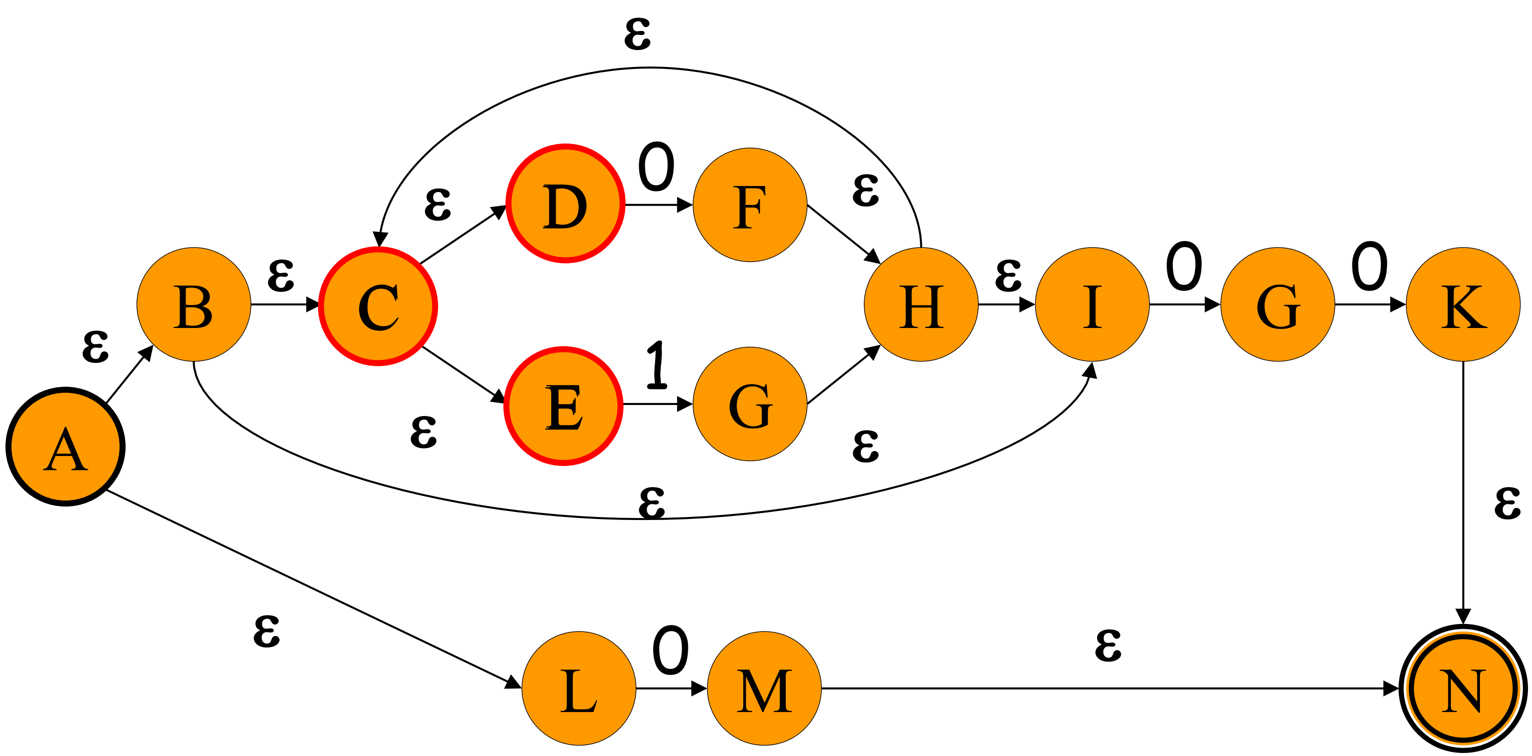
ε-closure(C) = {C, D, E}



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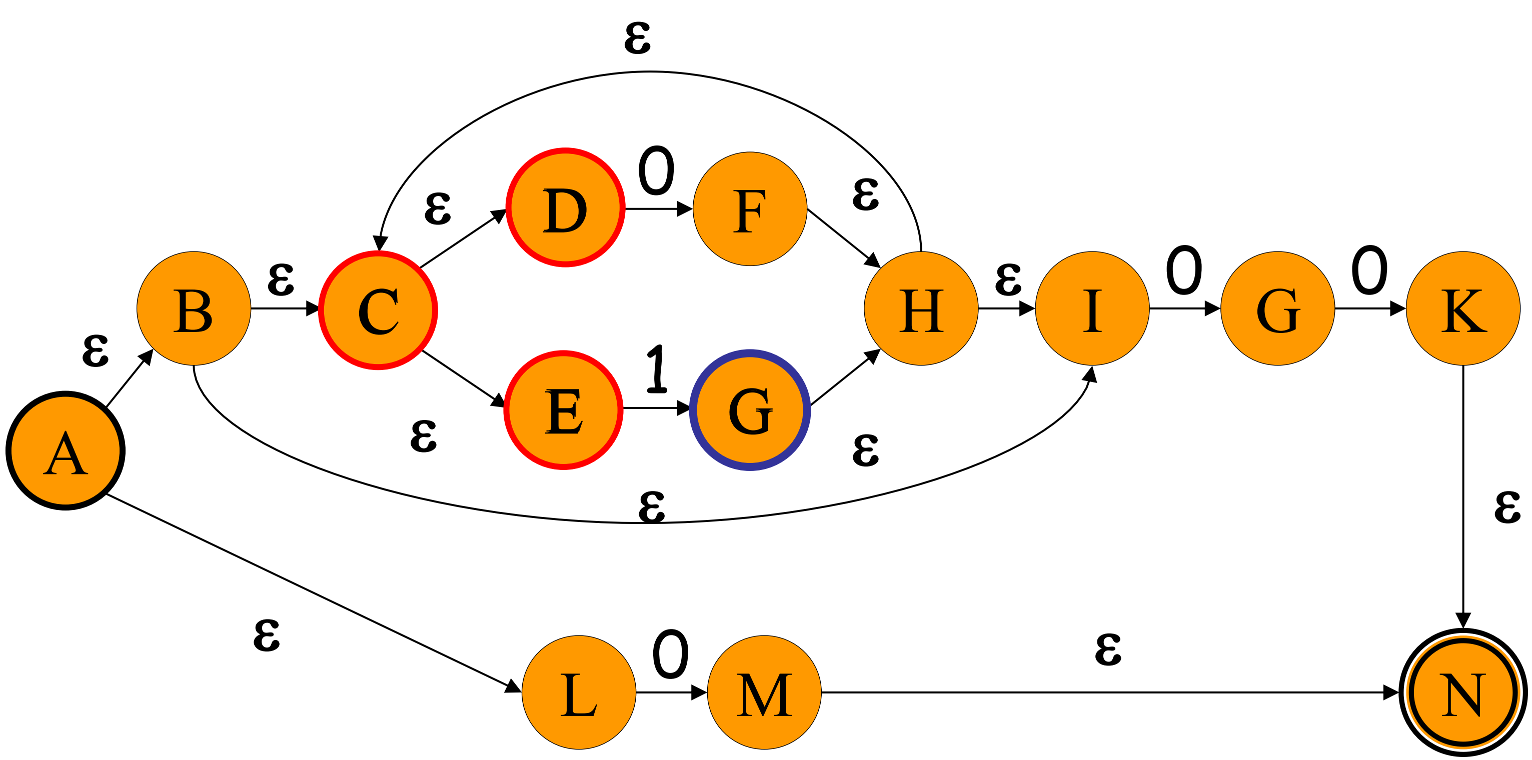

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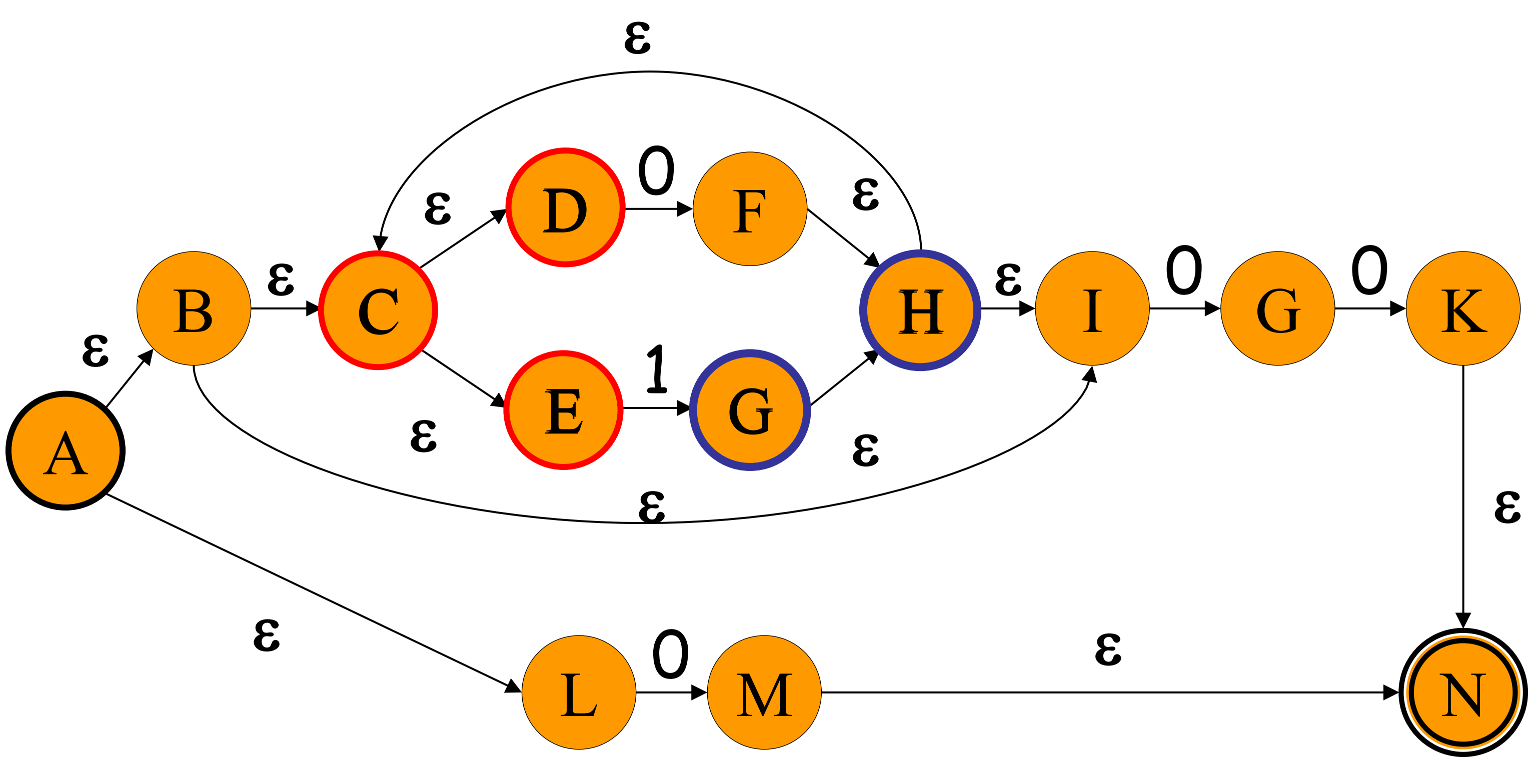

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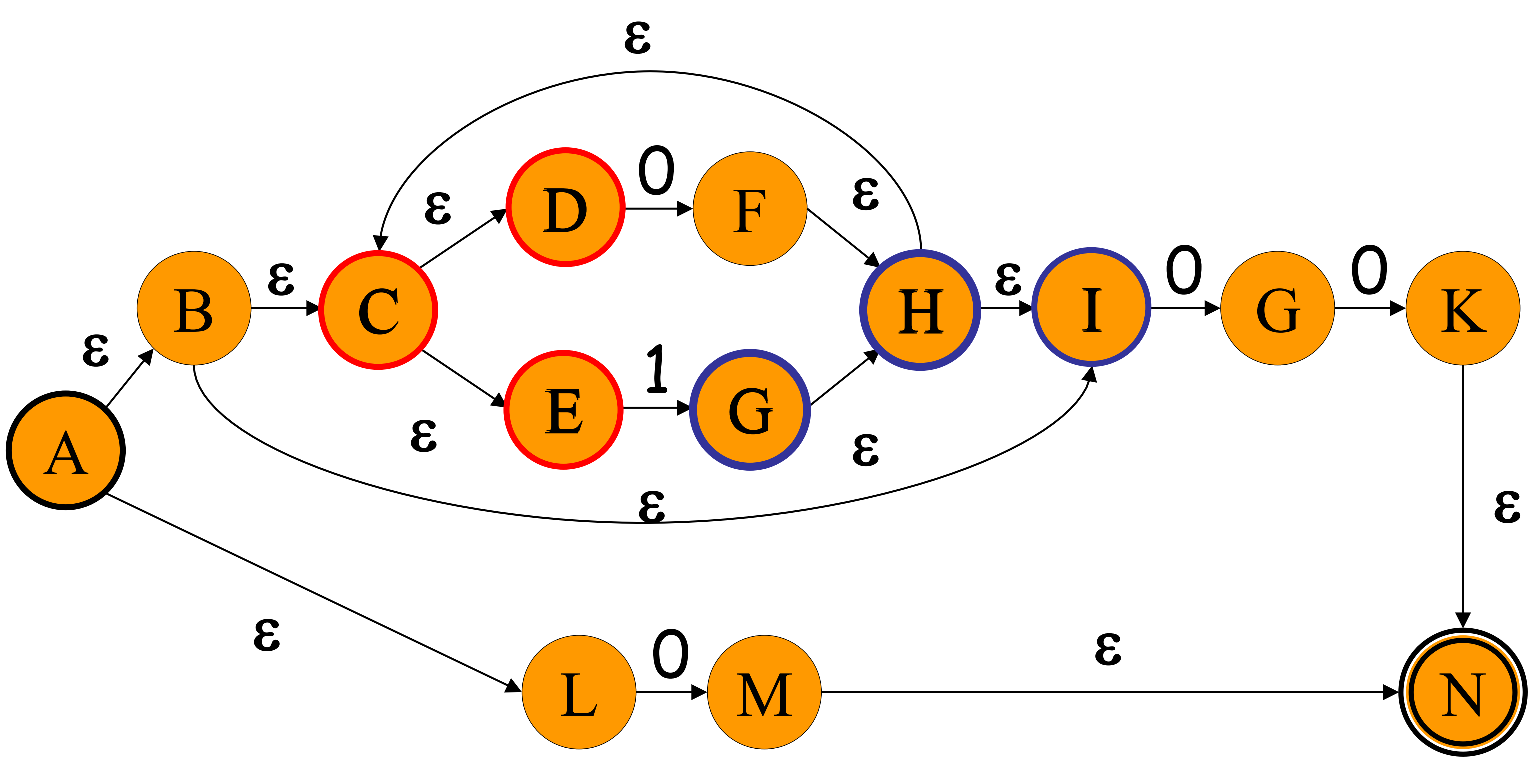

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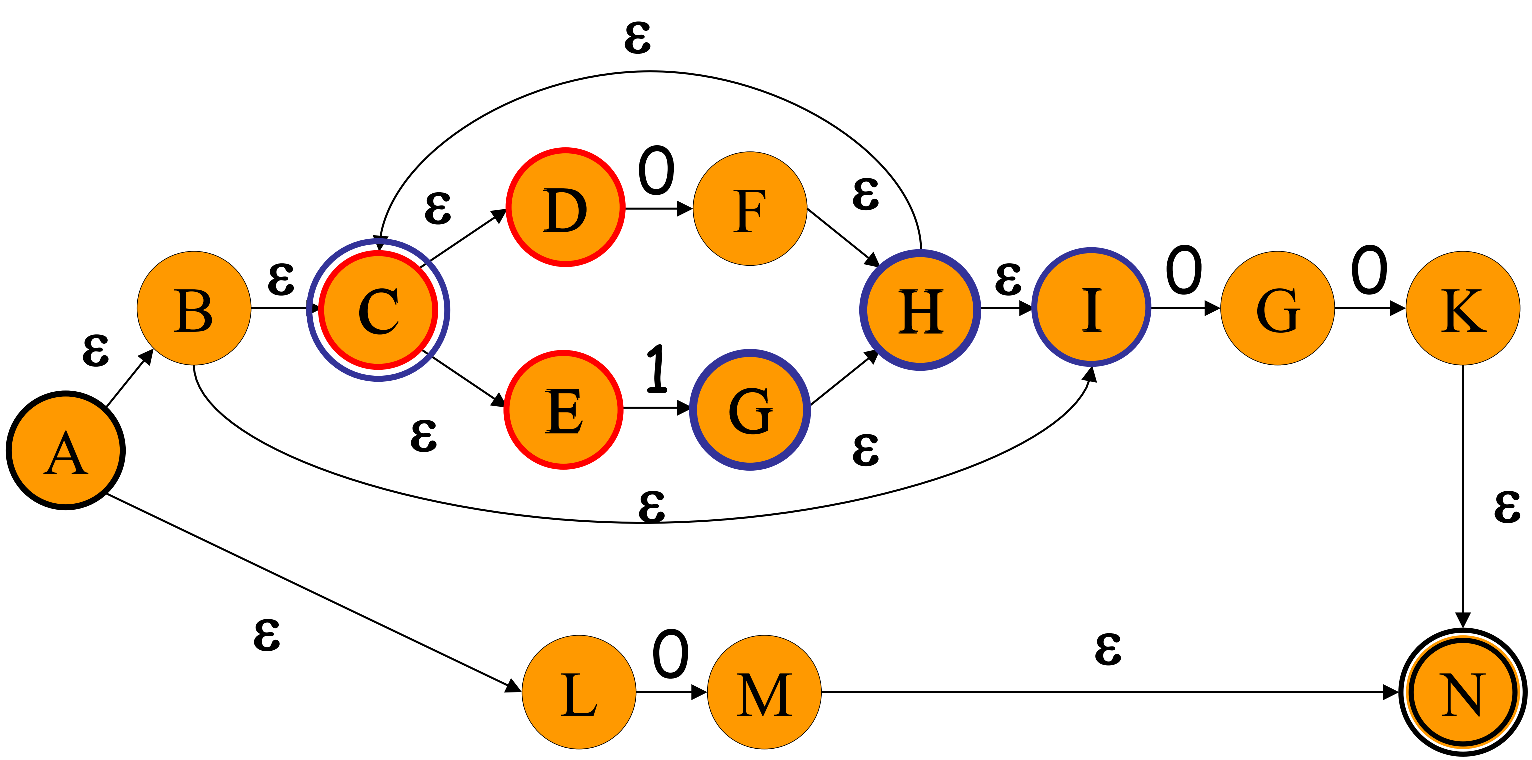

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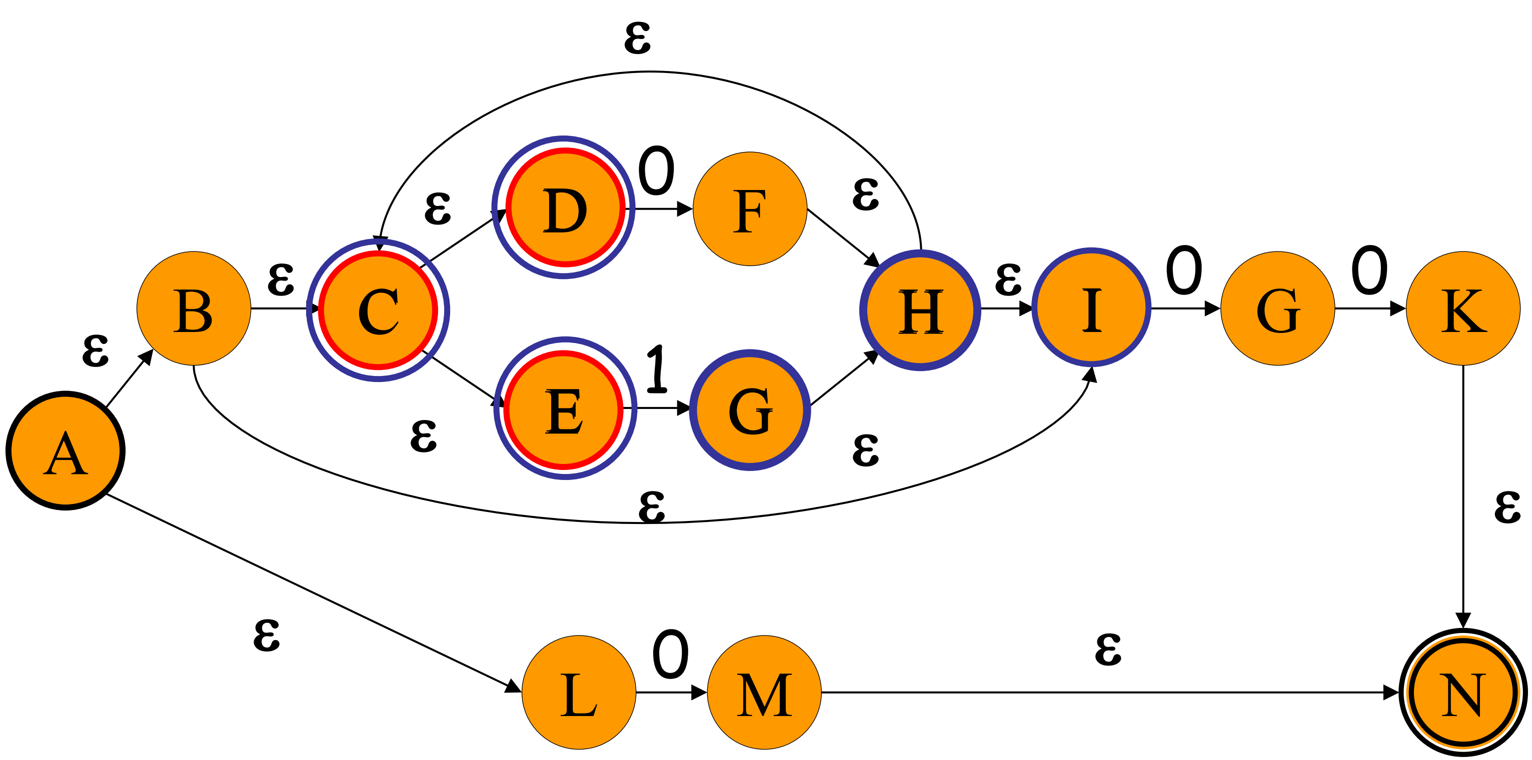


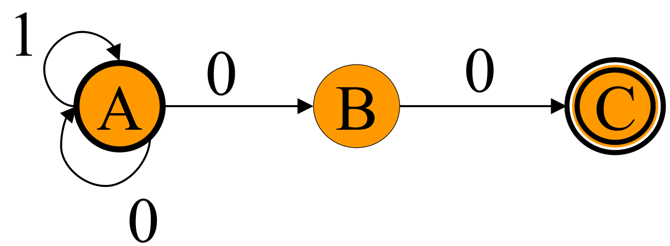
((0|1)*00)|0

ε-Closure (T: set of states)

push all states in T onto *stack*
initialize ε-*closure*(T) to T
while *stack* is not empty **do begin**
    pop t off *stack*
    for each state u with u ∈ **move**(t, ε) **do**
        if u ∉ ε-*closure*(T) **do begin**
            add u to ε-*closure*(T)
            push u onto *stack*
        end
end

Simulating NFAs

- An NFA may be in many states at any time

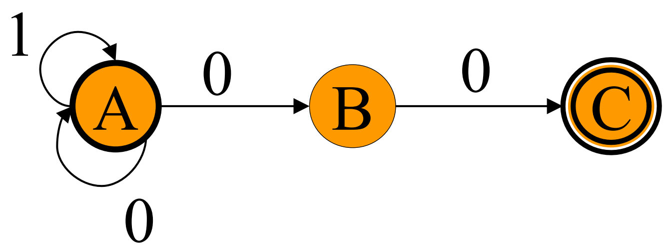


state	input
{A}	100
{A}	100
{A,B}	100
{A,B,C}	100

S

Simulating NFAs

- An NFA may be in many states at any time



state	input
{A}	100
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{A,B}	100
{A,B,C}	100

s

- How many different states?

Simulating NFAs

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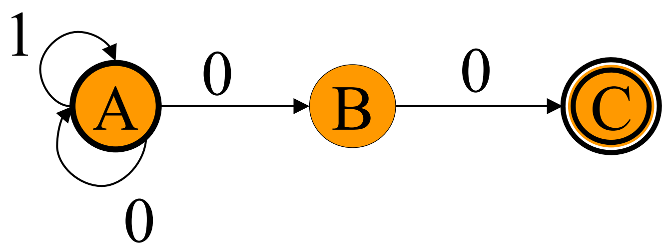

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- How many different states?

  |S|=N No. of states

Simulating NFAs

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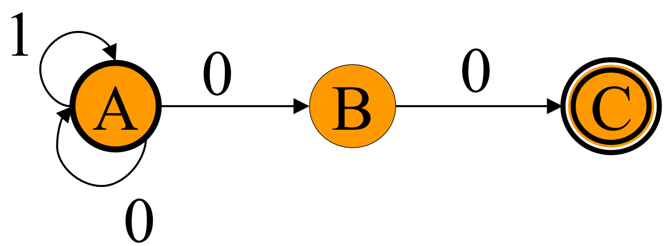

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s

- How many different states?

|S|=N No. of states

|**s**| possible states in each step

Simulating NFAs

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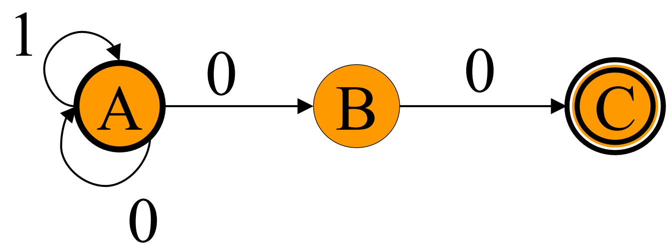

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s

- How many different states?

$|S|=N$ No. of states

$|\mathbf{s}| \leq N$ possible states in each step

Simulating NFAs

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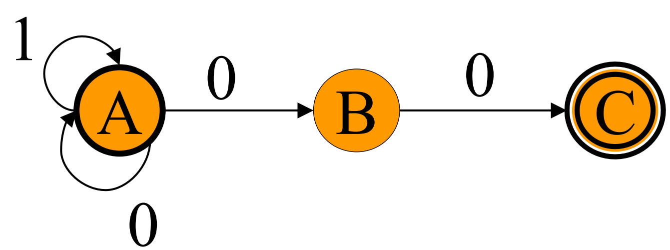

state	input
{A}	100
{A}	100
{A,B}	100
{A,B,C}	100

s

- How many different states?

|S|=N No. of states

|**s**| $\leq$ N possible states in each step

$\mathbf{2^N - 1}$

Non-empty subsets

NFA to DFA Conversion

NFA

- states
- start
- final
- transition

NFA to DFA Conversion

NFA

- states S
- start
- final
- transition

NFA to DFA Conversion

	NFA
• states	S
• start	q_0
• final	
• transition	

NFA to DFA Conversion

NFA

- states S
- start q_0
- final $F \subseteq S$
- transition

NFA to DFA Conversion

NFA

- states S
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- transition $\delta(x, a) = Y$

NFA to DFA Conversion

	NFA	DFA
• states	S	
• start	q_0	
• final	$F \subseteq S$	
• transition	$\delta(x, a) = Y$	

NFA to DFA Conversion

	NFA	DFA
• states	S	X ⊆ S
• start	q_0	
• final	F ⊆ S	
• transition	$\delta(x, a) = Y$	

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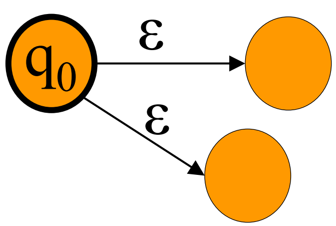

NFA to DFA Conversion

	NFA	DFA
• states	S	$X \subseteq S$
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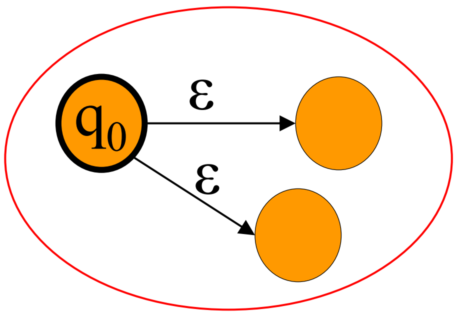
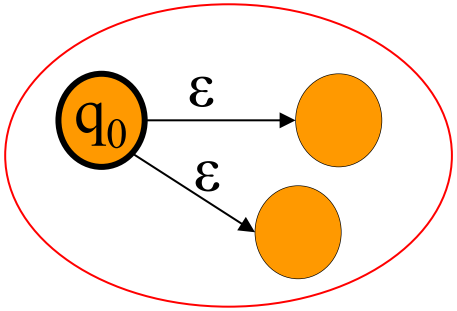


NFA to DFA Conversion

	NFA	**DFA**
• states	S	$X \subseteq S$
• start	q_0	ε-closure(q_0)
• final	$F \subseteq S$	
• transition	$\delta(x, a) = Y$	



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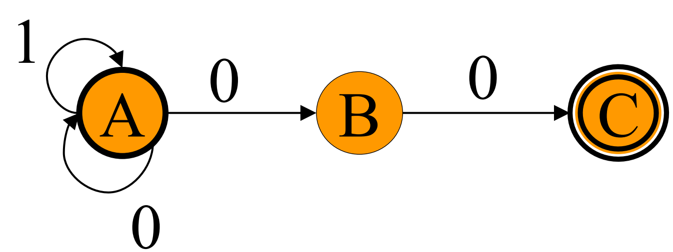

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{A}	100
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NFA to DFA Conversion

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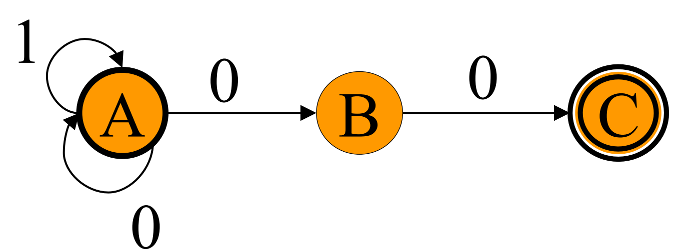

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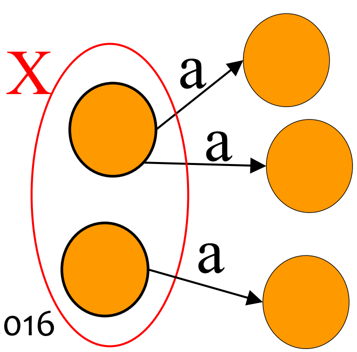

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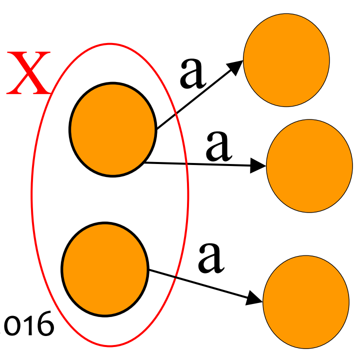

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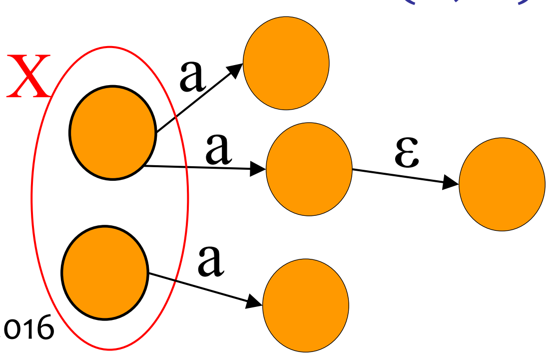
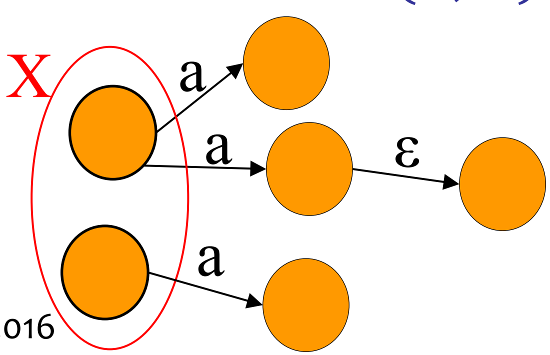


NFA to DFA Conversion

	NFA	DFA
• states	S	X ⊆ S
• start	q_0	ε-closure(q_0)
• final	F ⊆ S	{X | X∩F ≠ ∅}
• transition	$\boldsymbol{\delta(x,a)=Y}$	$\boldsymbol{\delta(X,a)} = \bigcup_{x \in X} \delta(x,a)$
		ε-closure($\boldsymbol{\delta(X,a)}$)



NFA to DFA Conversion

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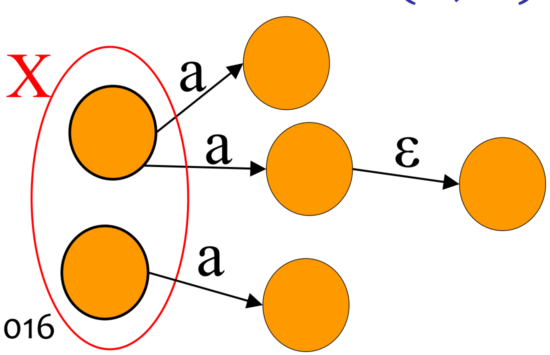

ε-closure($\boldsymbol{\delta(X, a)}$)

DFAedge(X,a)=ε−closure($\bigcup_{x \in X} \delta(x, a)$**)**

DFA construction

Dstates = {}, Dtrans = []

add ε-*closure*(q_0) to *Dstates* unmarked

```
while ∃ unmarked T ∈ Dstates do
      mark T;
      for each symbol c do
            U := DFAedge(T,c);
            if U ∉ Dstates then
                  add U to Dstates unmarked
            Dtrans[T, c] := U;
```

DFA construction

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add ε-*closure*(q_0) to *Dstates* unmarked

while ∃ unmarked T ∈ *Dstates* **do**

    mark T;

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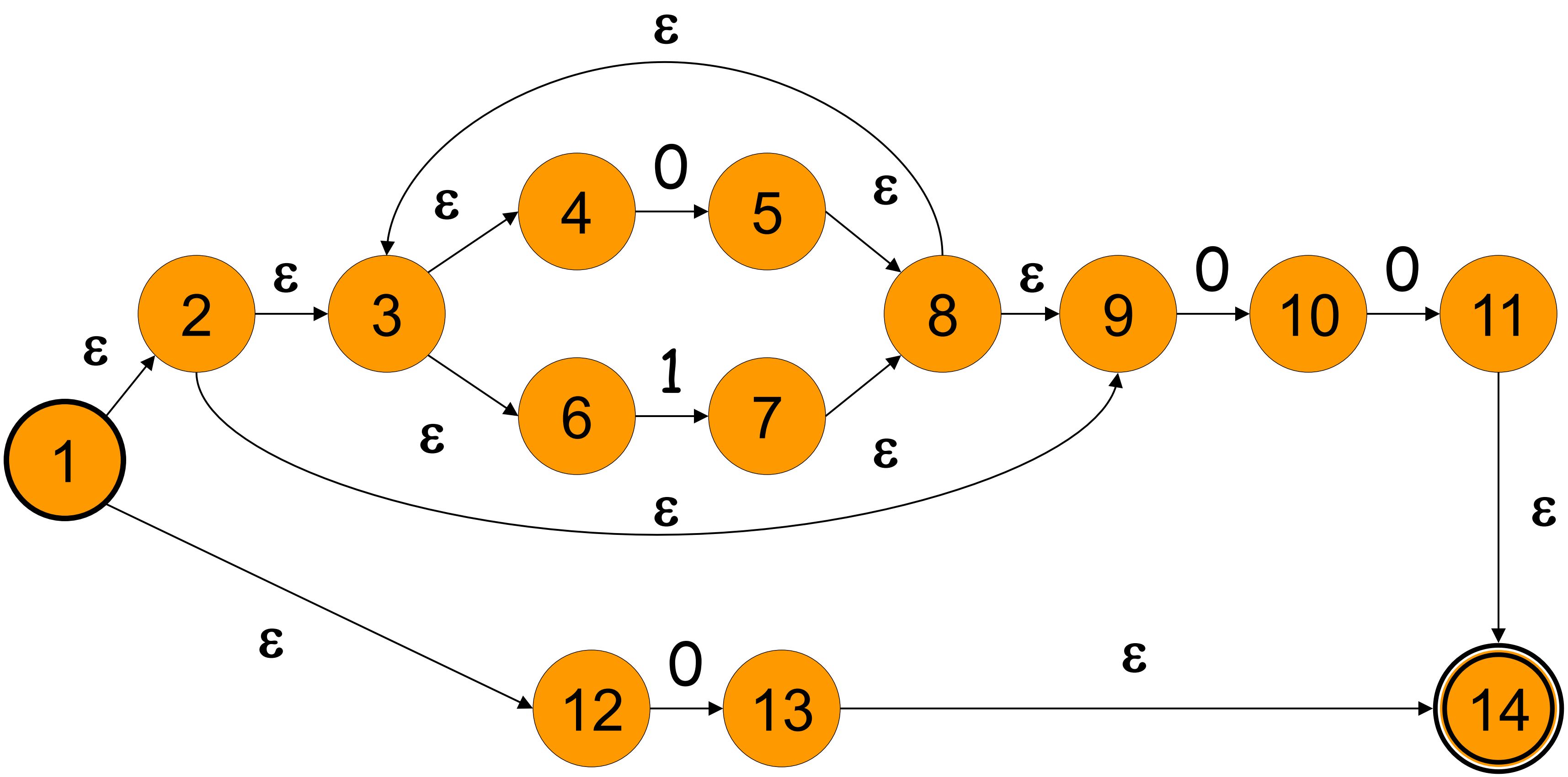
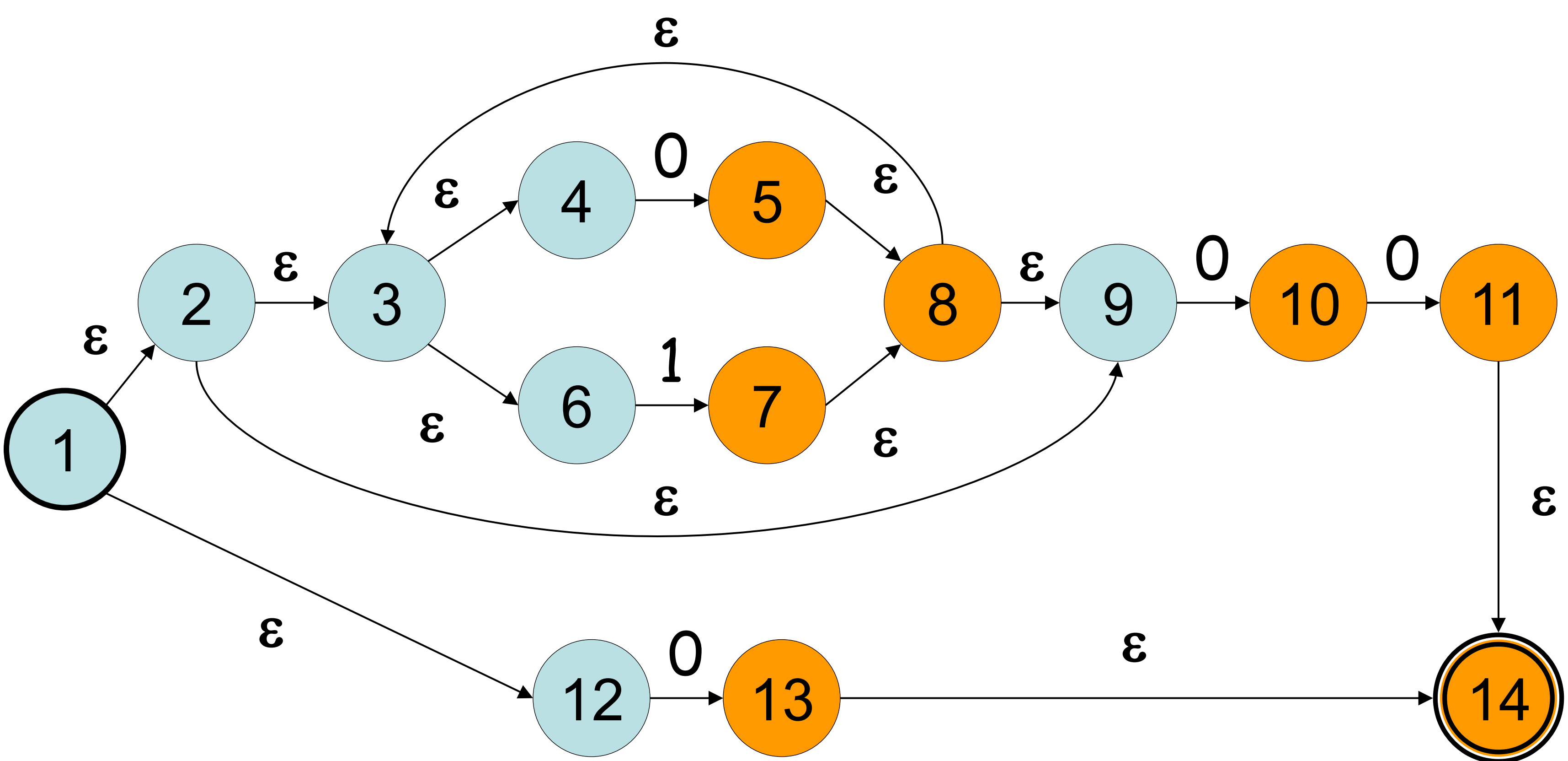
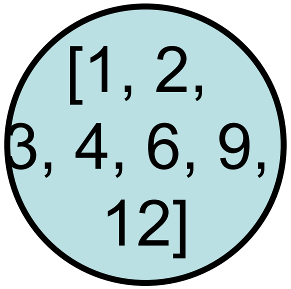
$\textbf{DFAedge}(\boldsymbol{T}, \boldsymbol{c}) =$
ε–closure($\bigcup_{t \in T} \delta(t, c)$)

NFA to DFA

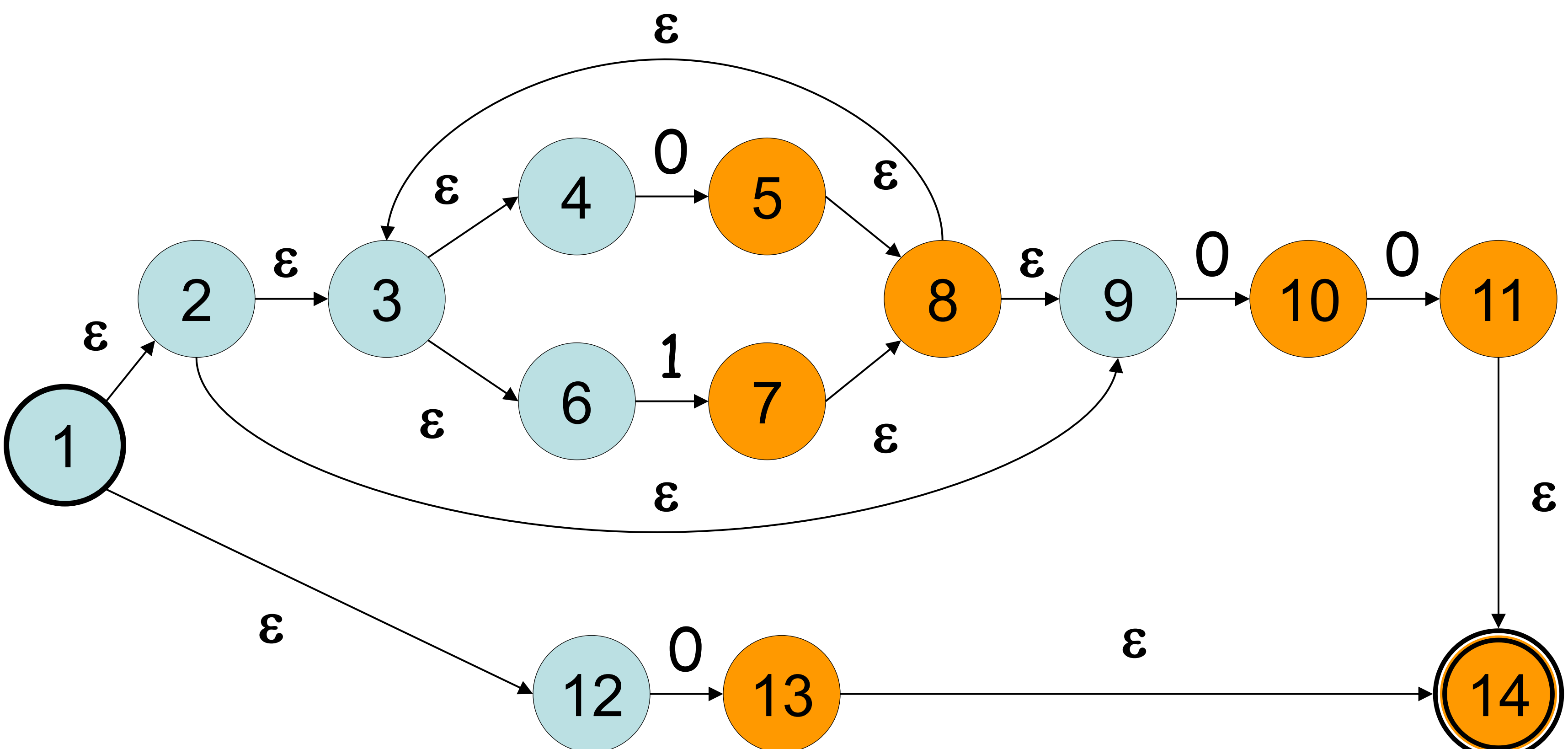


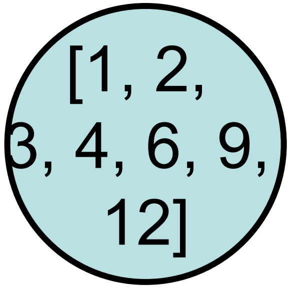
ε-*closure*(q_0)



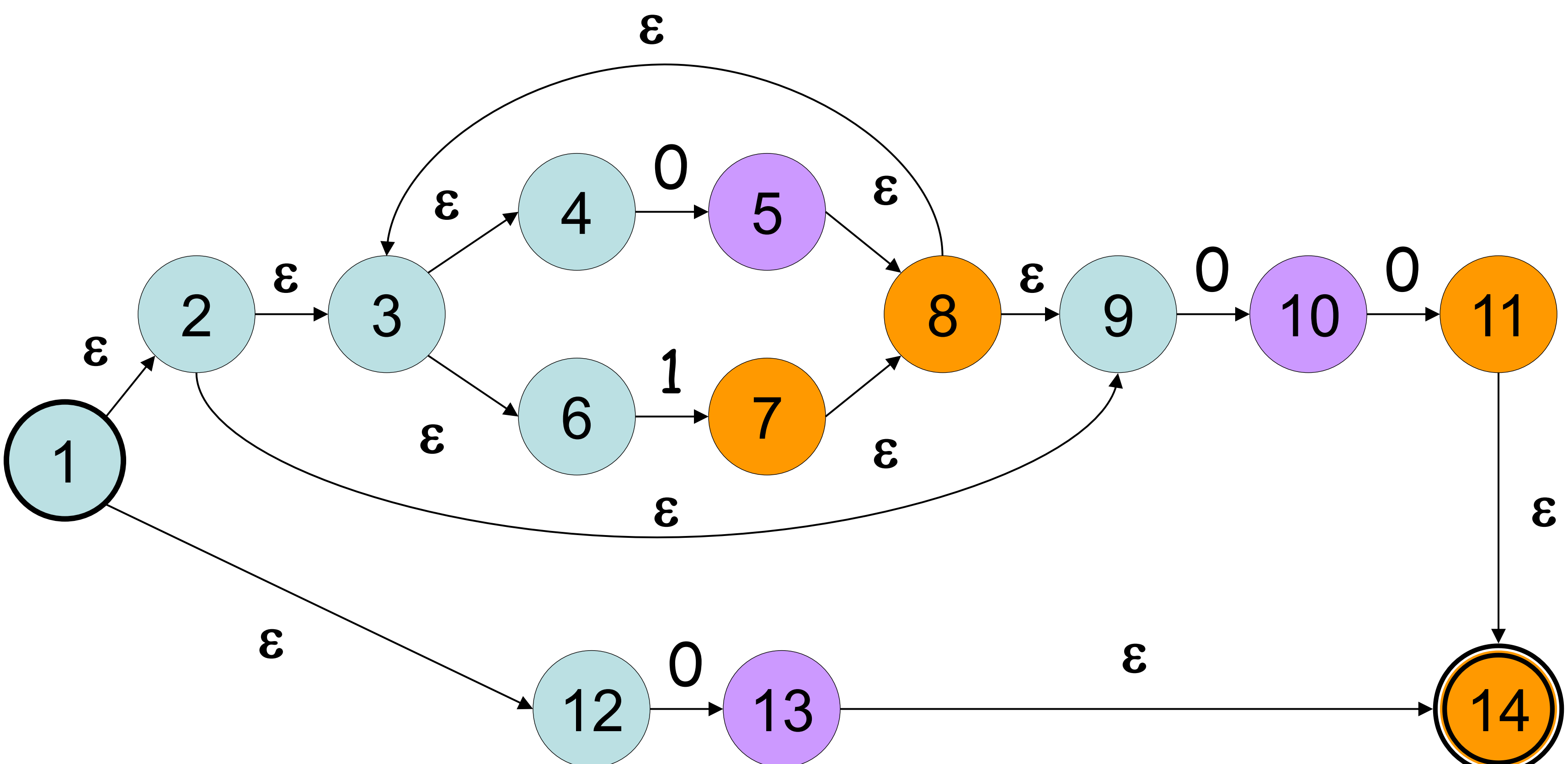


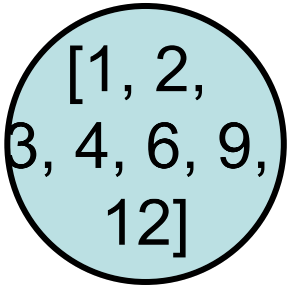
ε-*closure*(q_0)



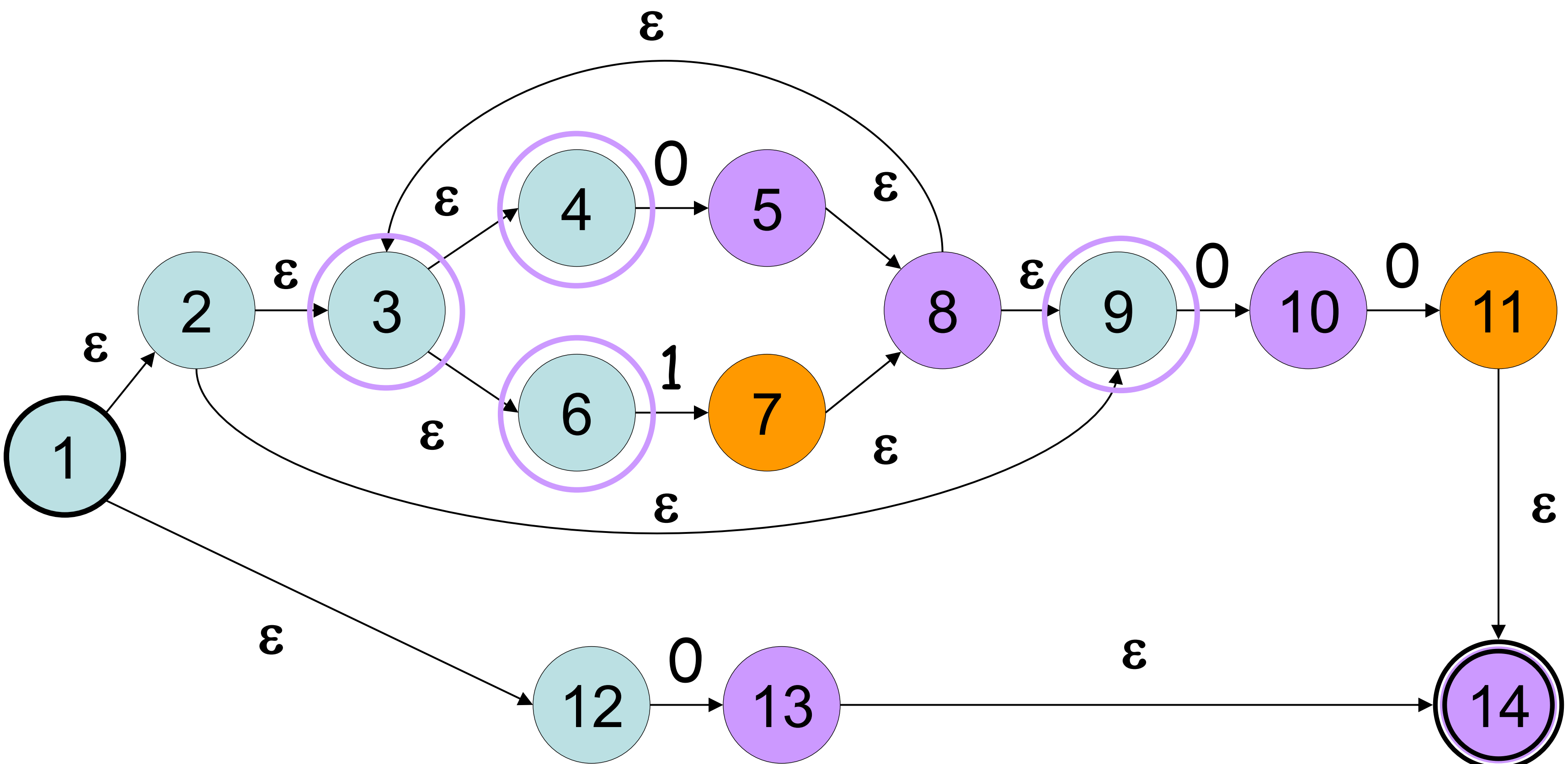


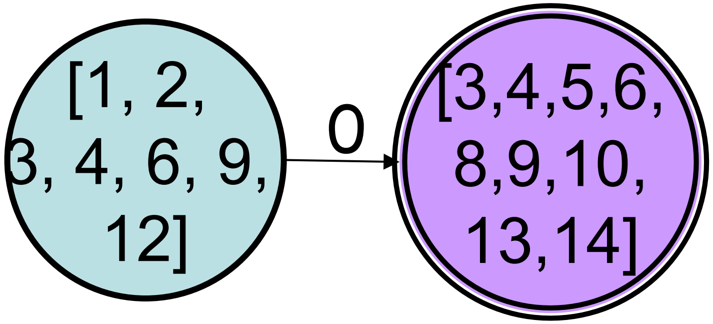
DFAedge(ε-*closure*(q_0), 0)



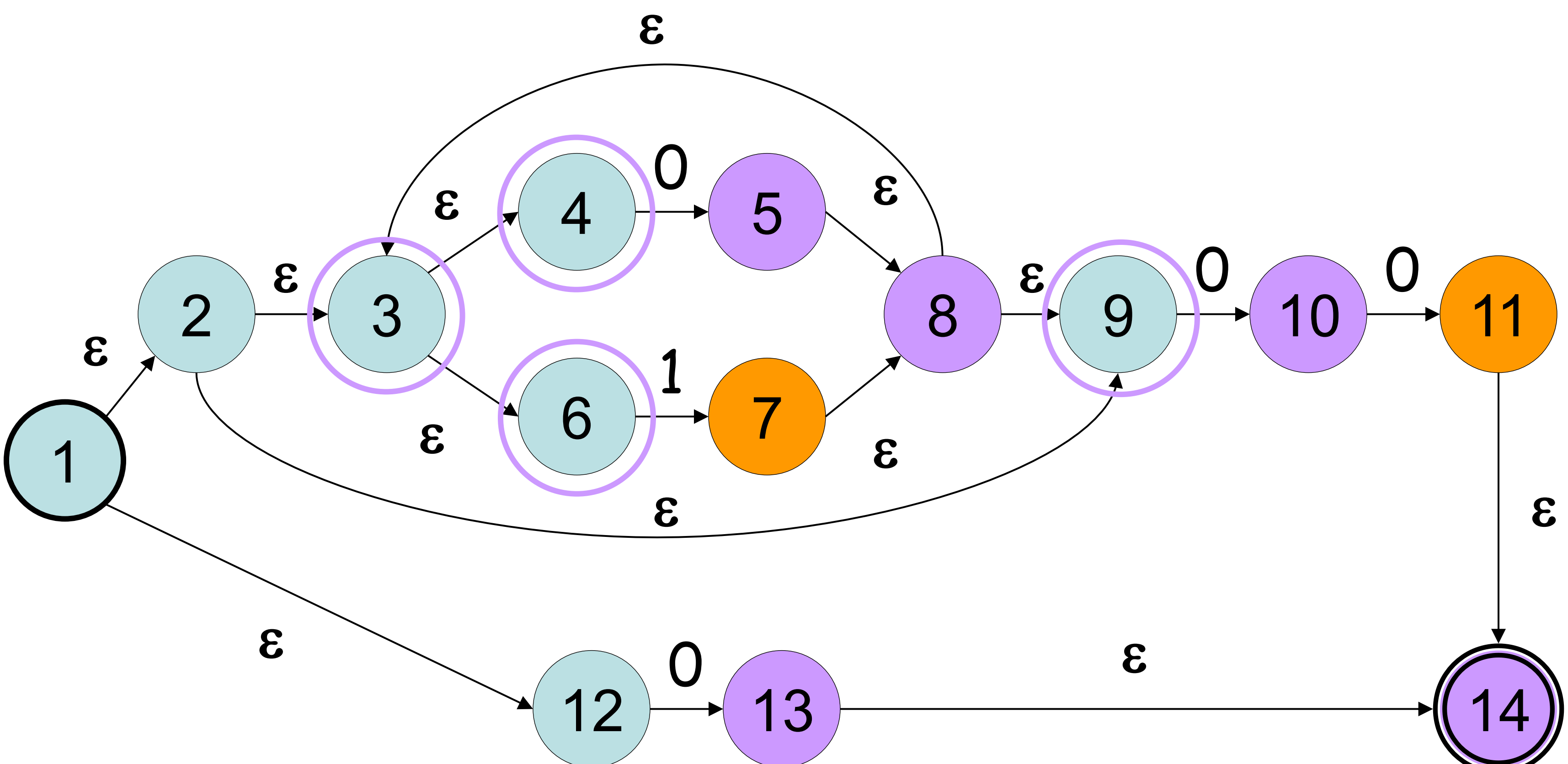


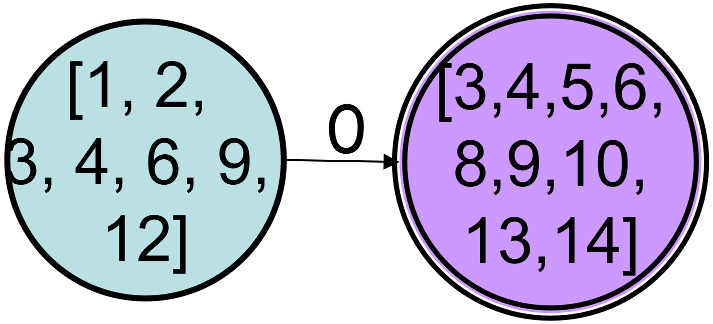
DFAedge(ε-*closure*(q_0), 0)



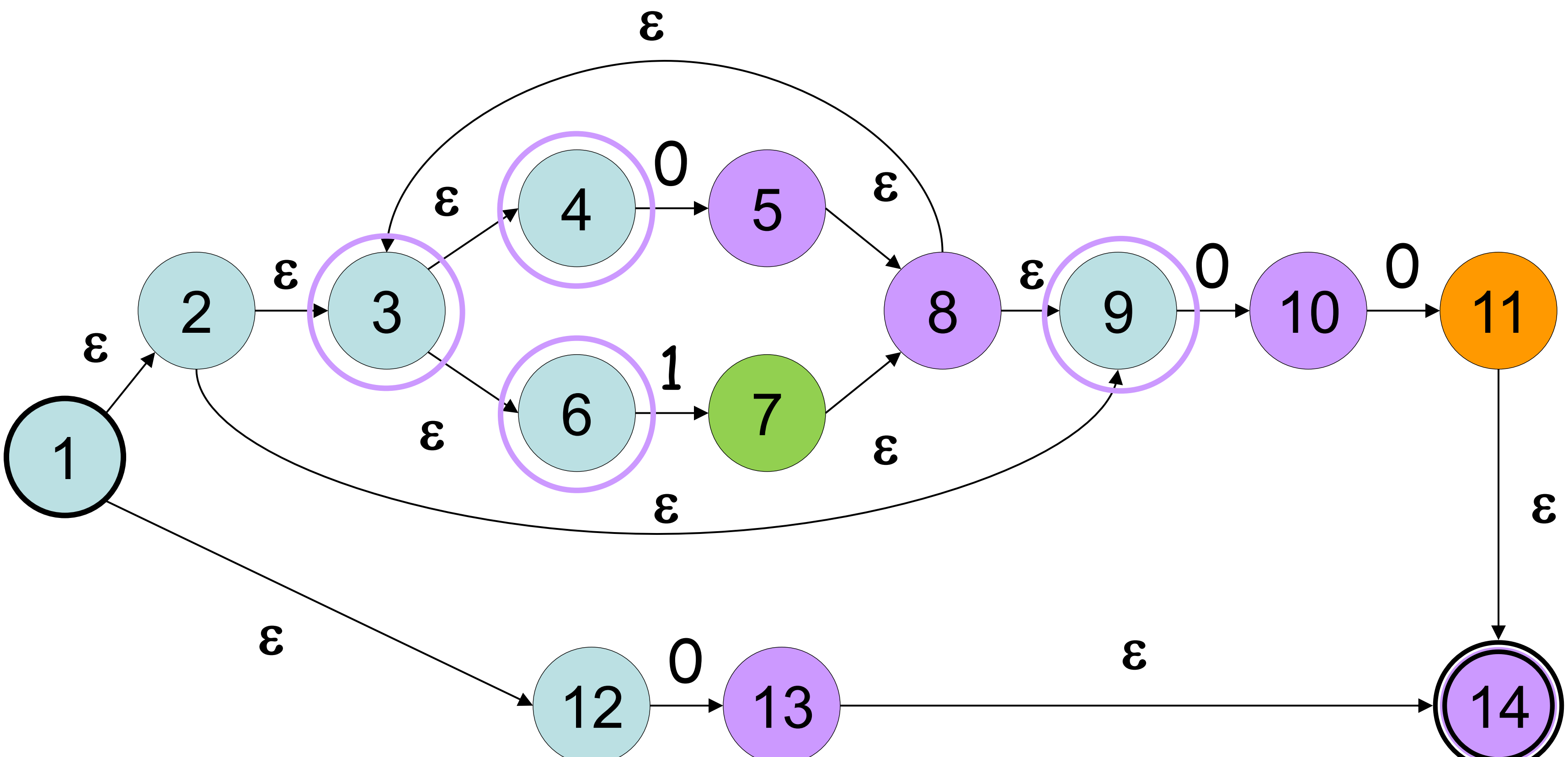


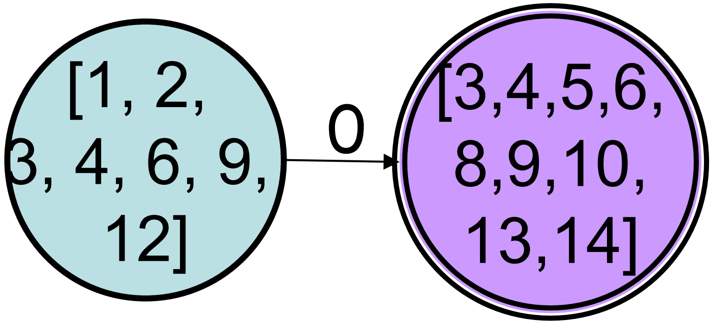
DFAedge(ε-*closure*(q_0), 0)



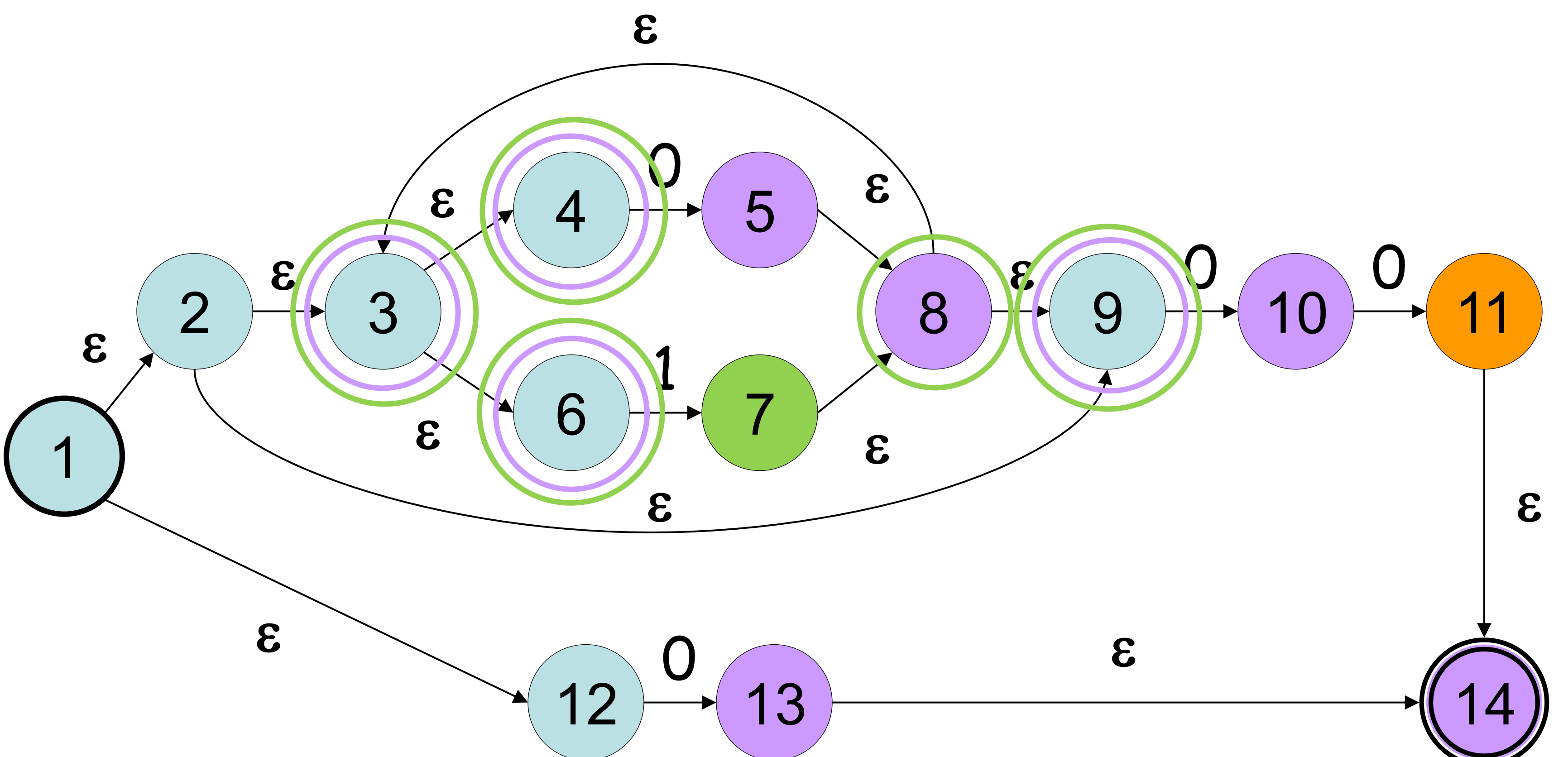


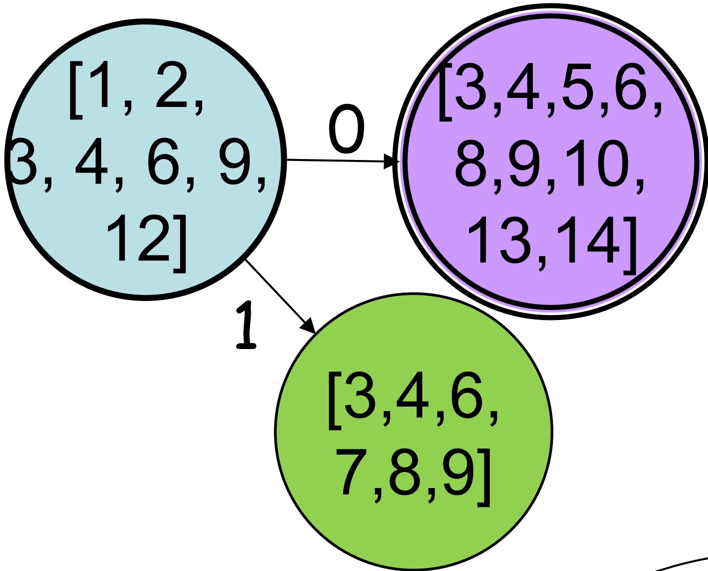
DFAedge(ε-*closure*(q_o), 1)



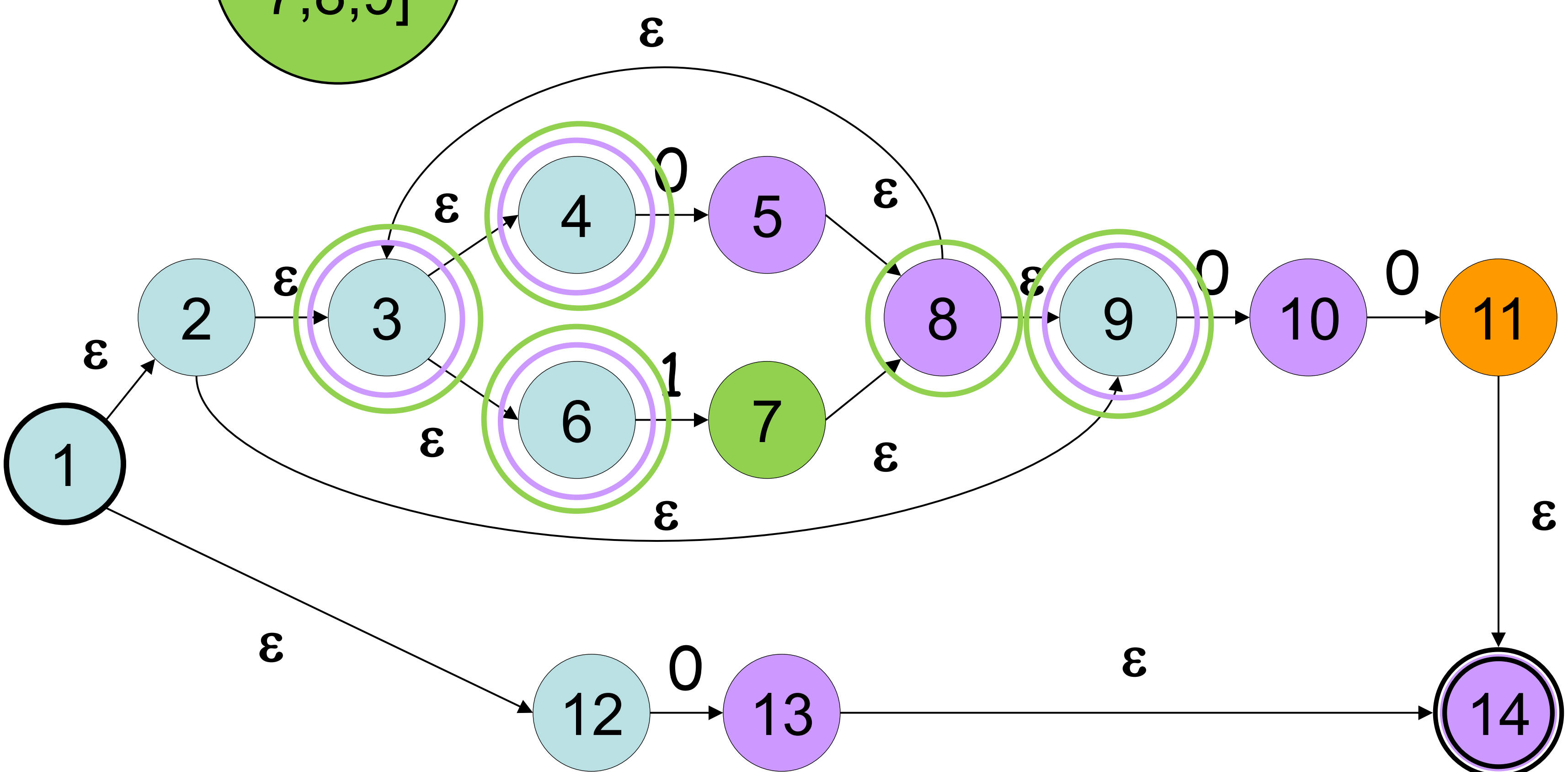


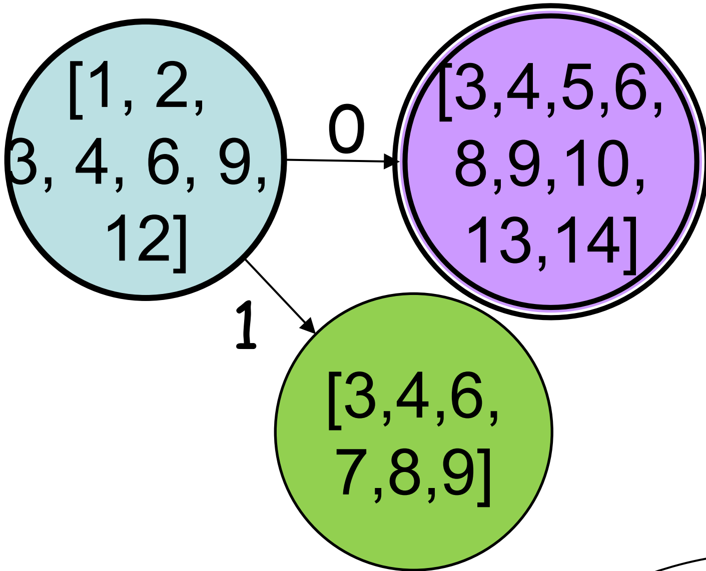
DFAedge(ε-*closure*(q_0), 1)



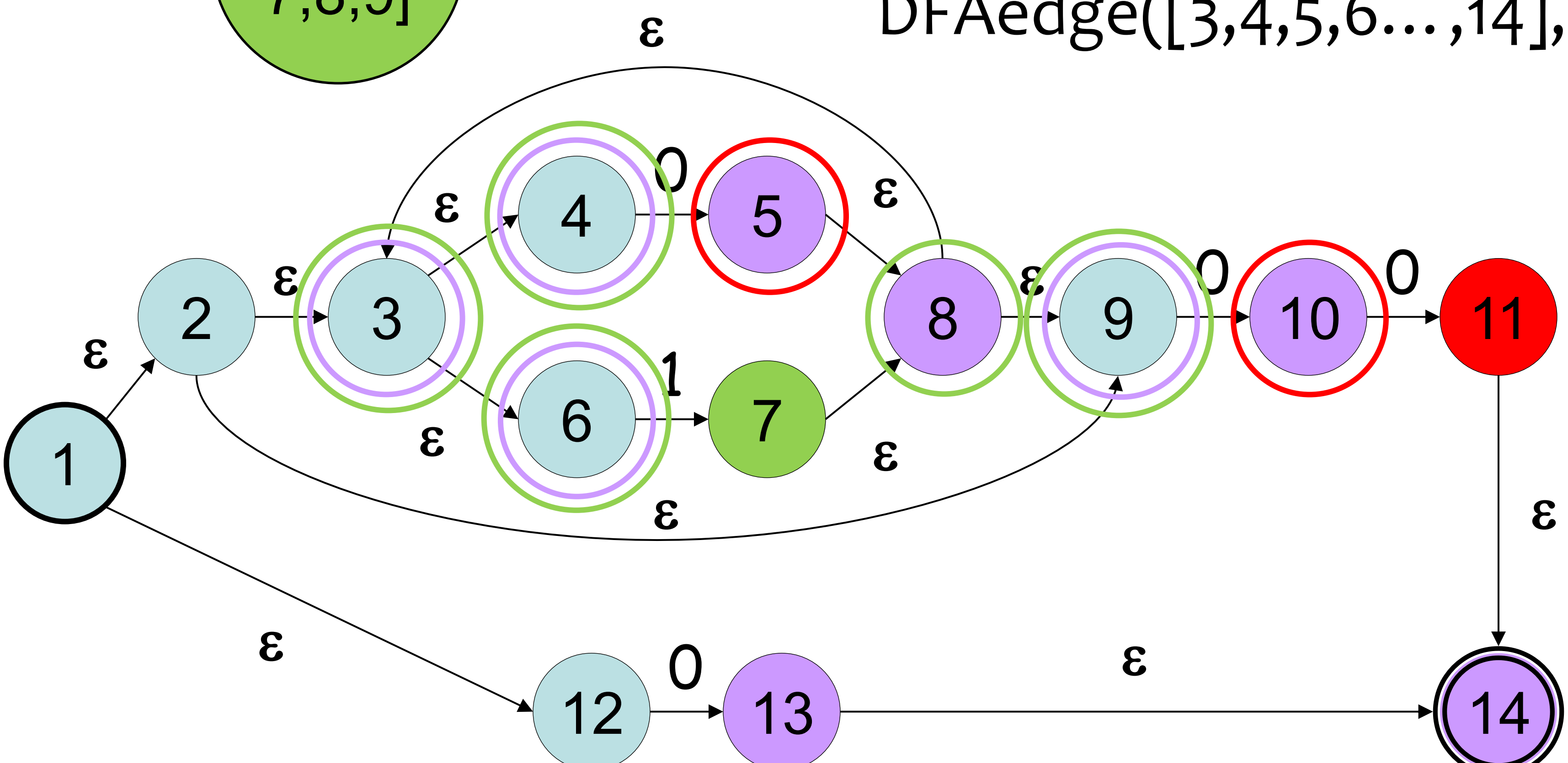


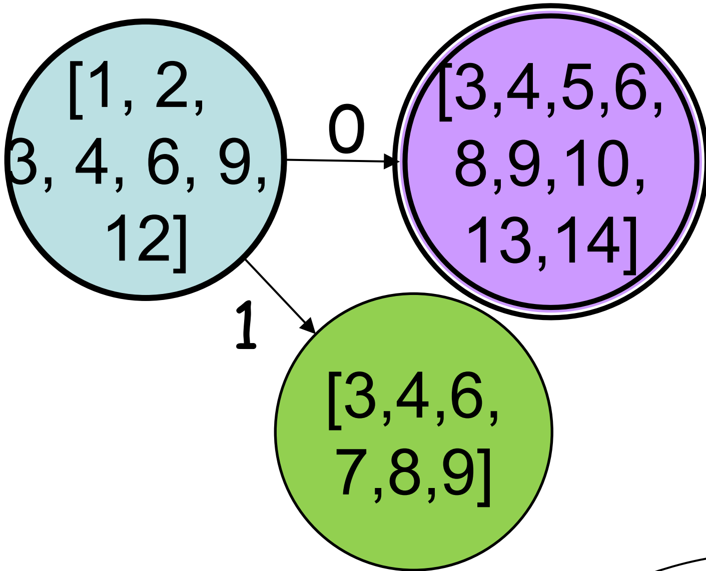
DFAedge(ε-*closure*(q_0), 1)





DFAedge([3,4,5,6…,14], 0)





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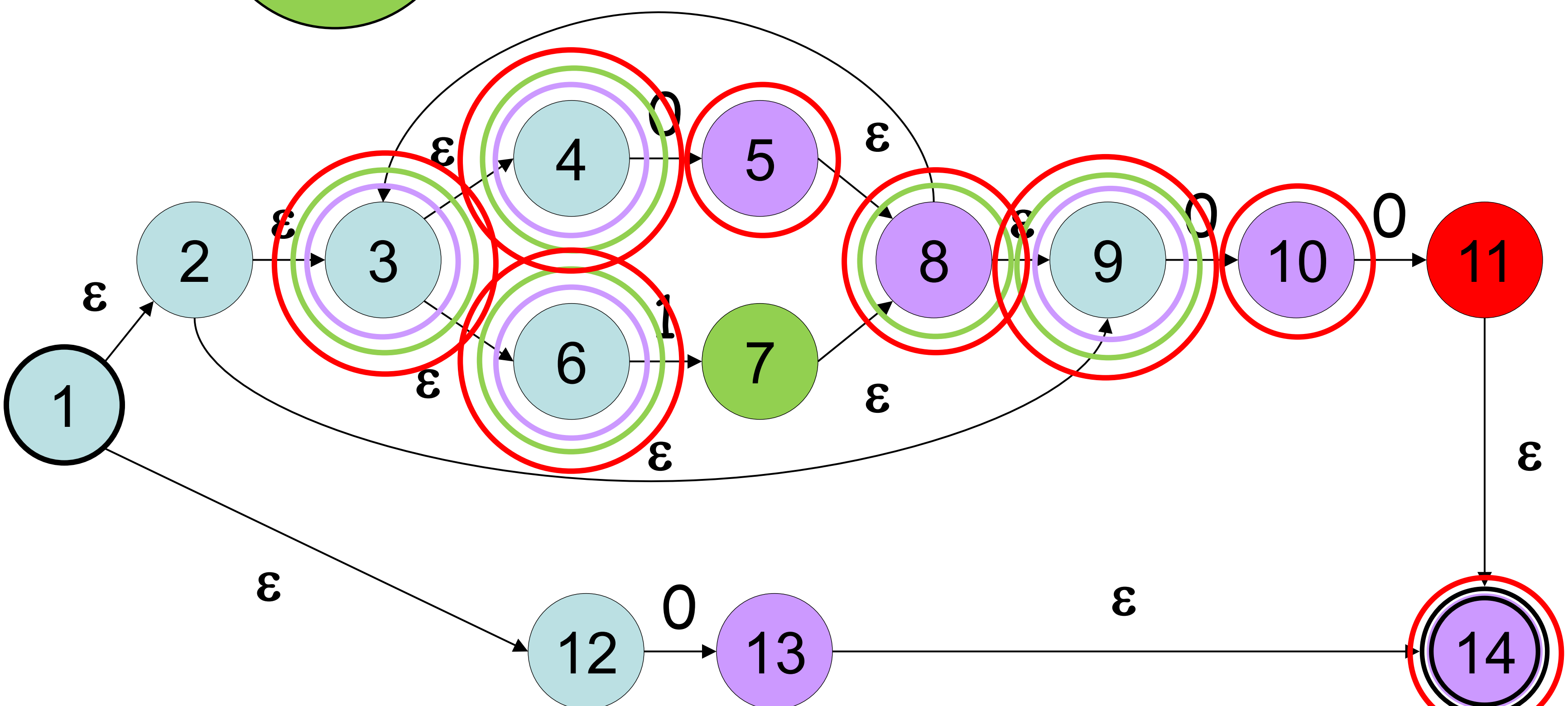
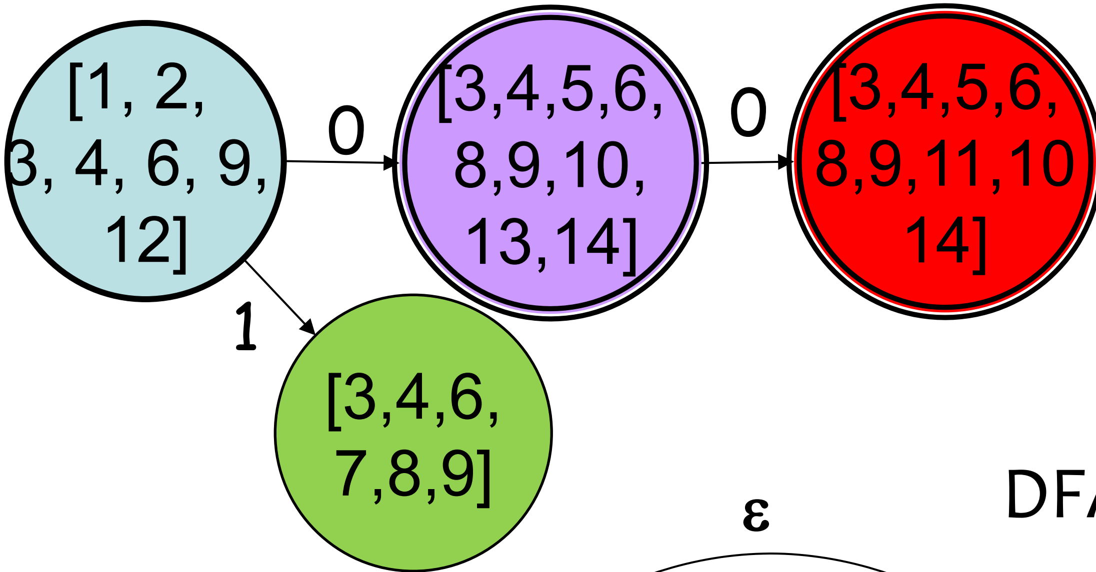
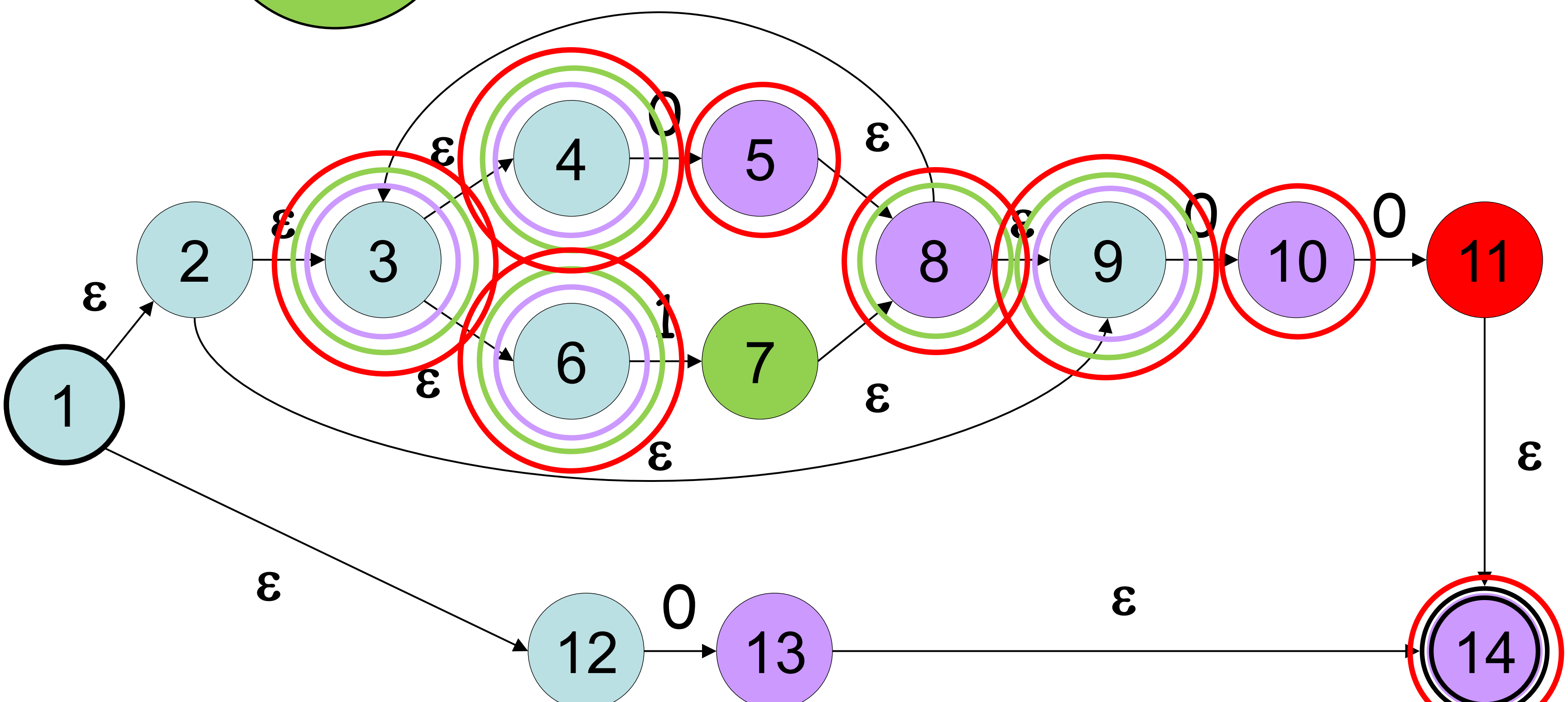
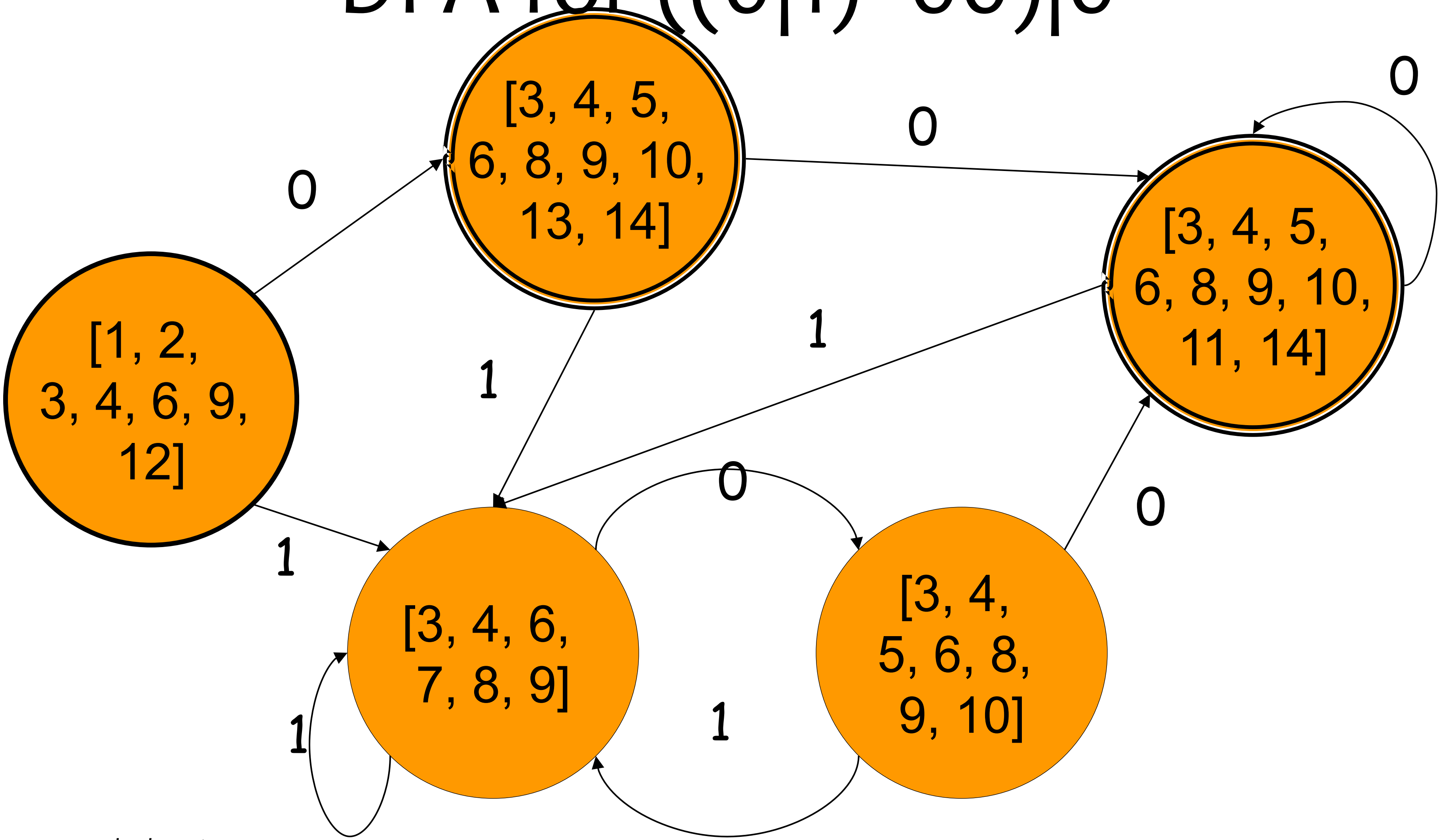




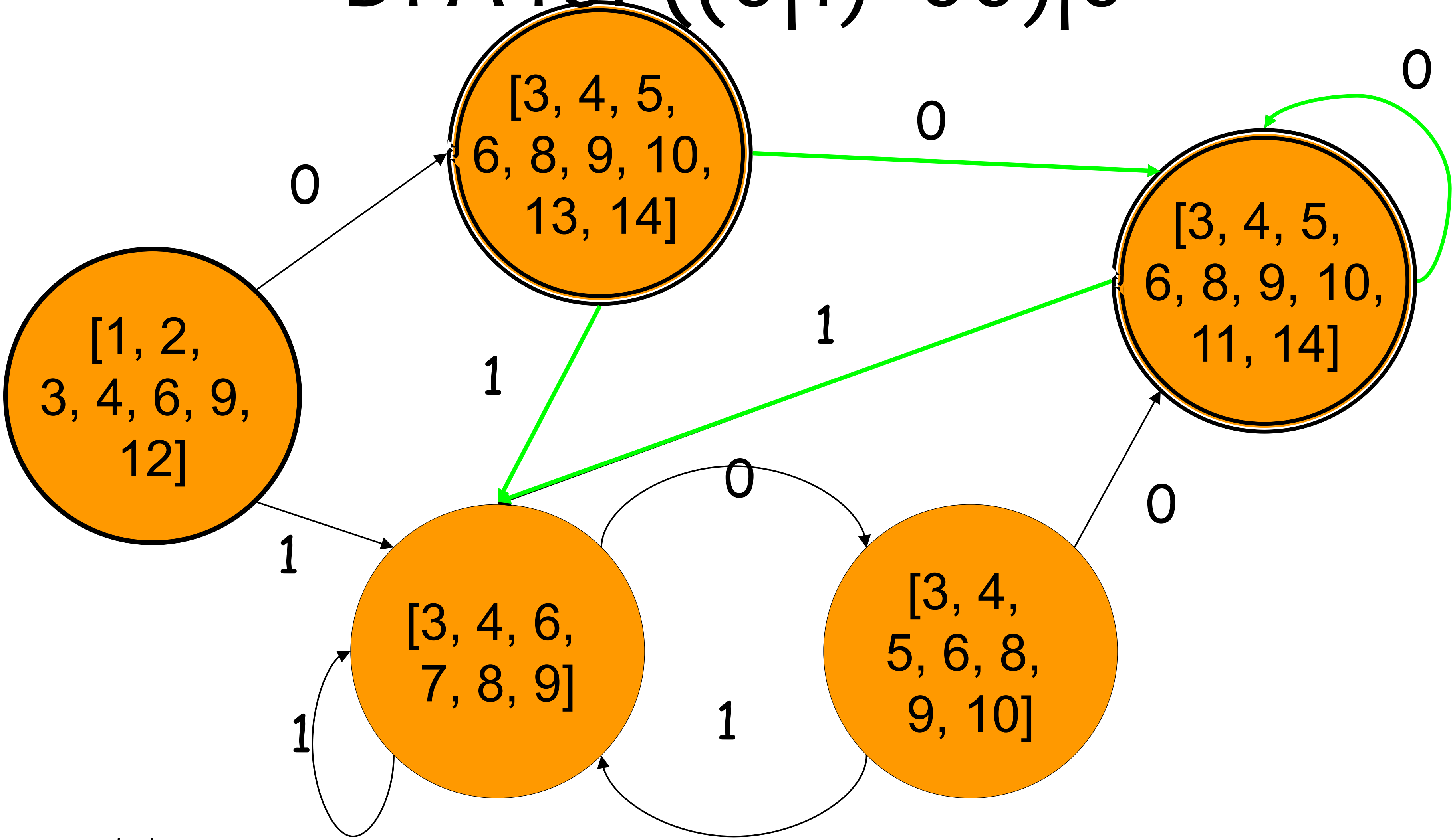
DFAedge([3,4,5,6…,14], 0)



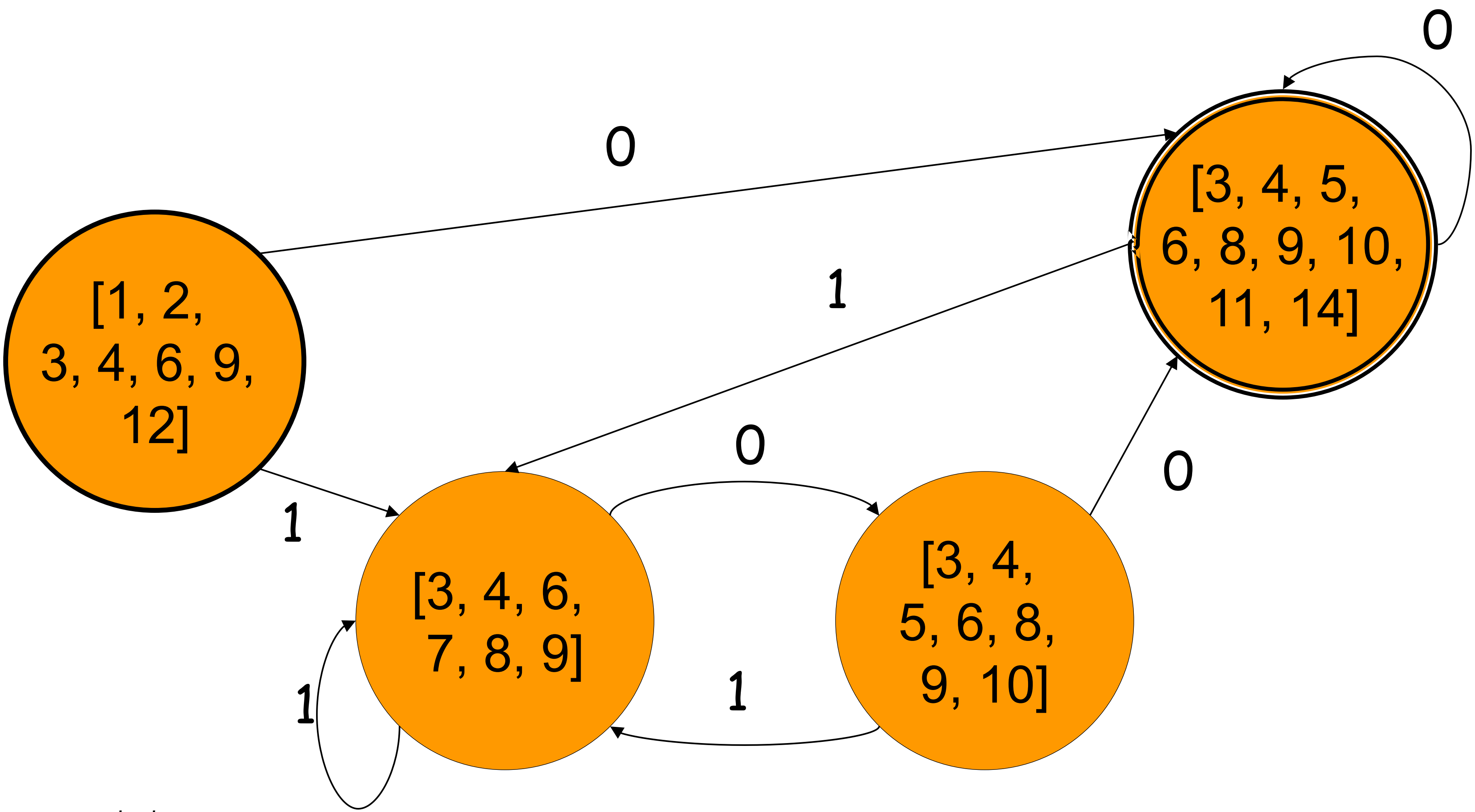
DFA for ((0|1)*00)|0



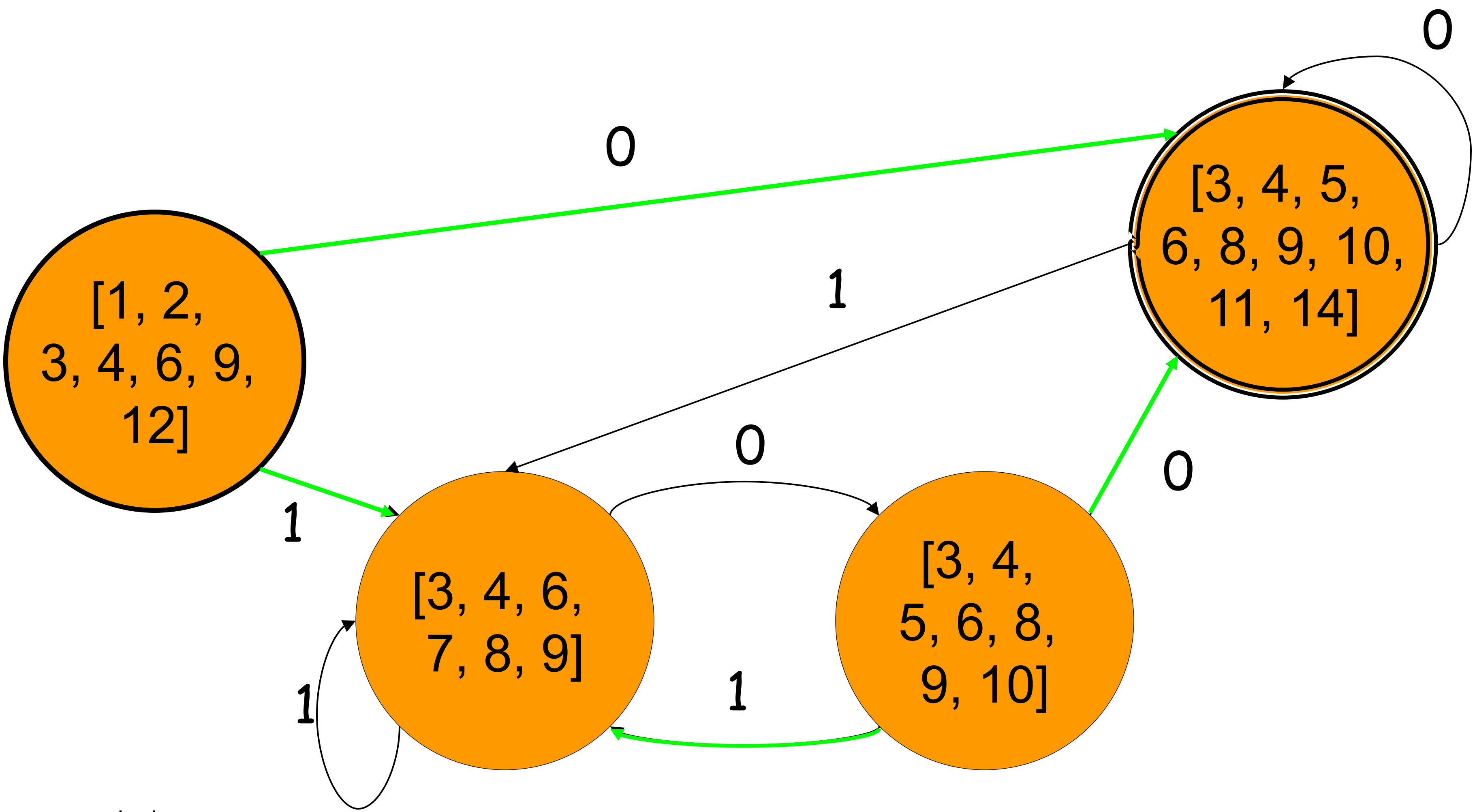
DFA for ((0|1)*00)|0



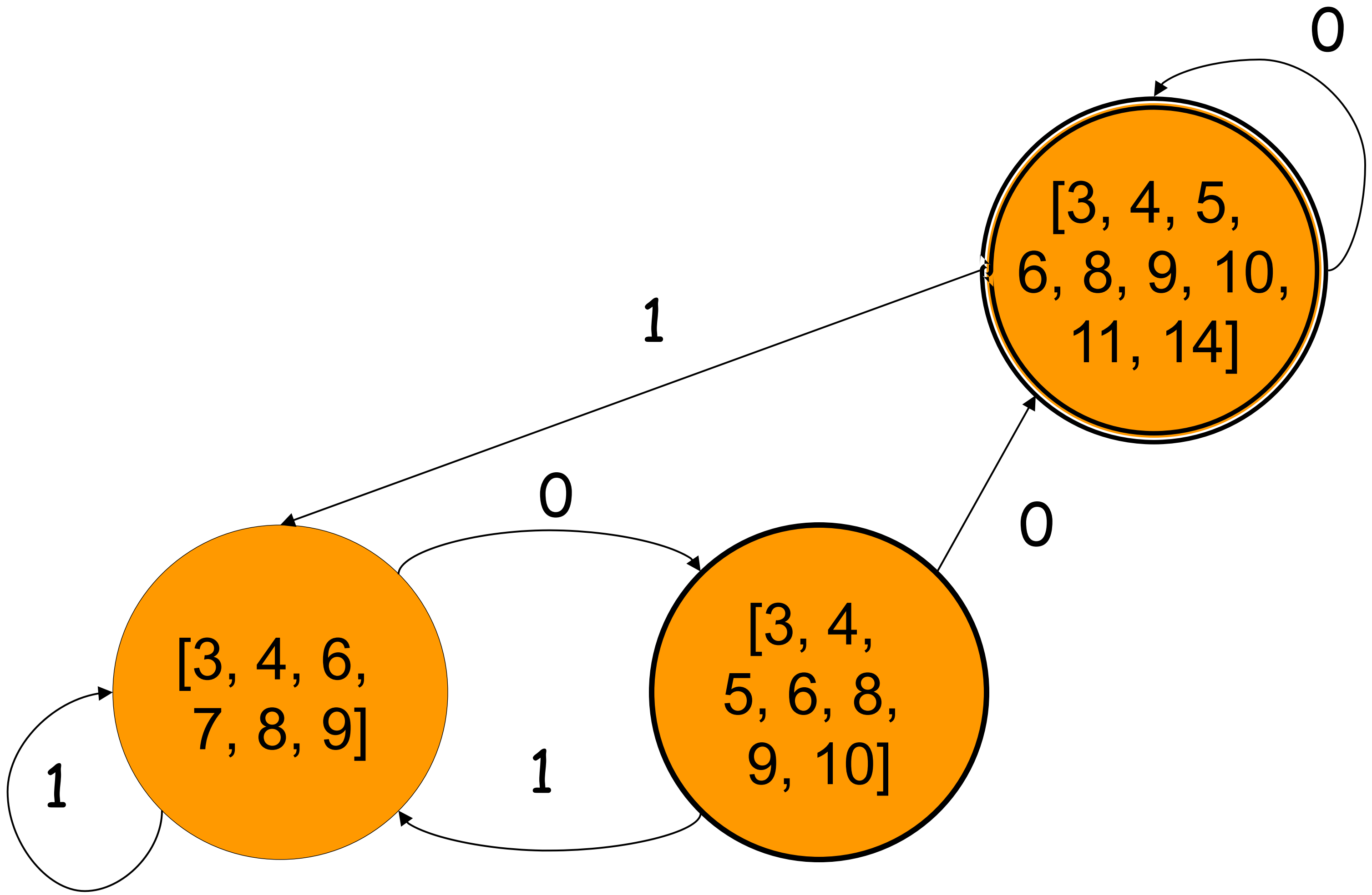
Minimization of DFAs



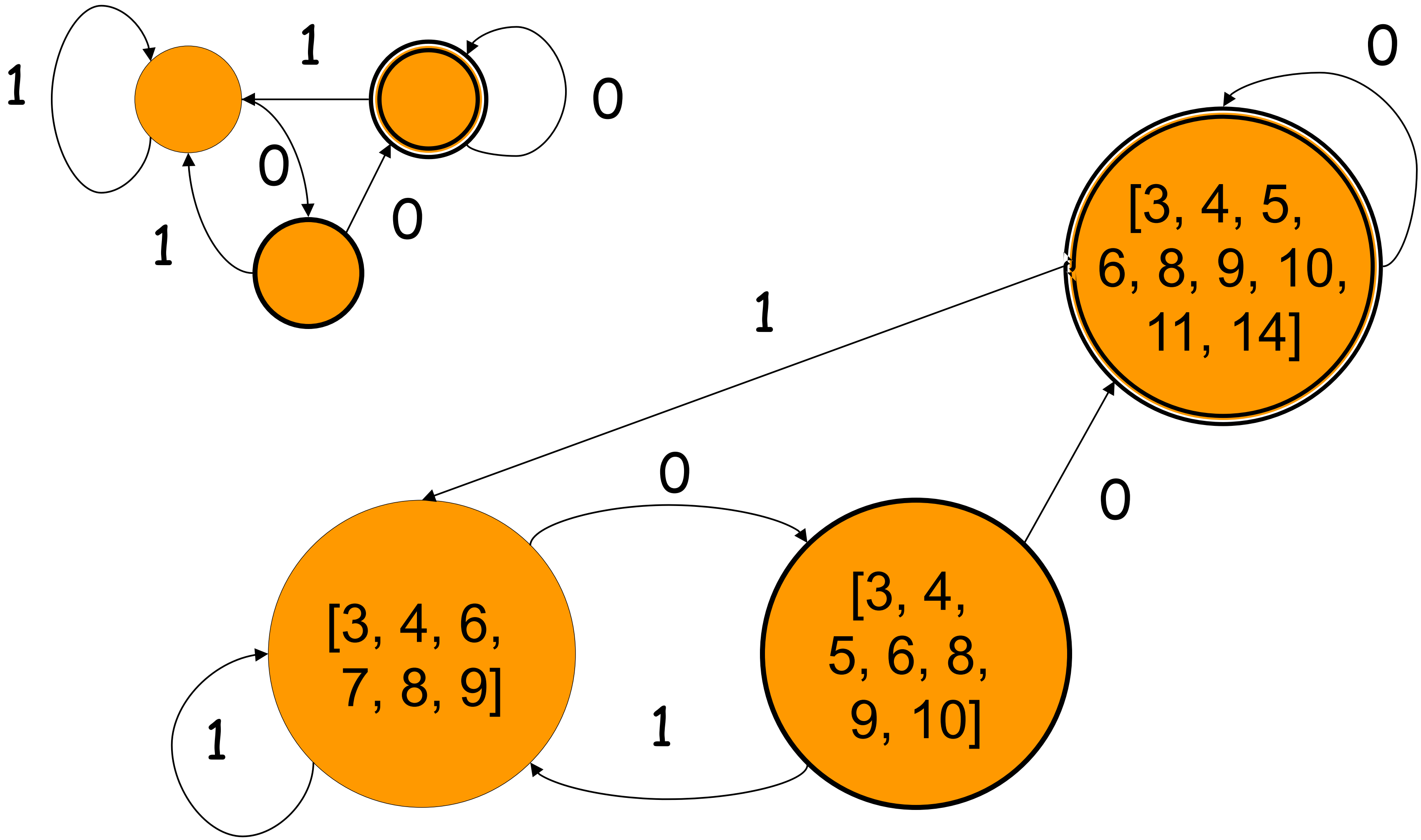
Minimization of DFAs



Minimization of DFAs



Minimization of DFAs



NFA to DFA Conversion

- Conversion method closely follows the NFA simulation algorithm
- Instead of simulating, we can collect those NFA states that behave identically on the same input
- Group this set of states to form one state in the DFA

NFA to DFA

```
states[0] = ε-closure({q0})
p = j = 0
while j ≤ p do
        for each symbol c ∈ ∑     do
                e = DFAedge(states[j], c)
                if e = states[i] for some i ≤ p
                then    Dtrans[j, c] = i
                else    p = p+1
                        states[p] = e
                        Dtrans[j, c] = p
        j = j + 1
```