CFG1: Intro to CFGs

Context-Free Grammars

CMPT 379: Compilers

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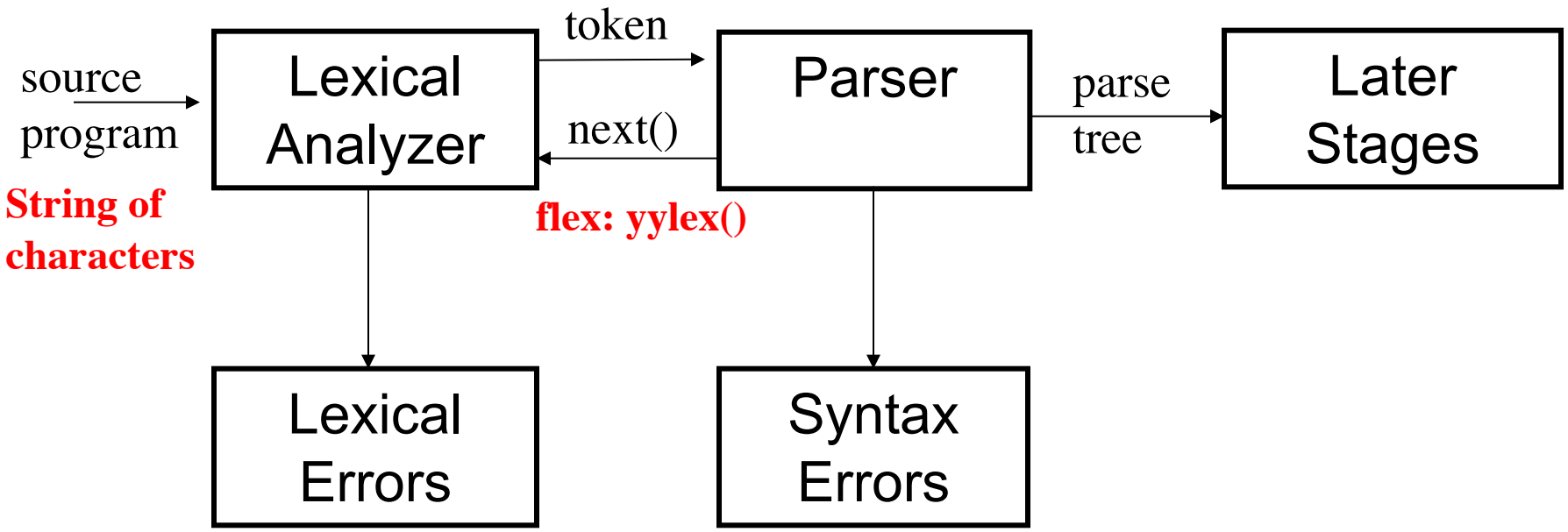
anoopsarkar.github.io/compilers-class

Parsing


source
program
Lexical
Analyzer
token
next()
Parser
parse
tree
Later
Stages
String of
characters
flex: yylex()
Lexical
Errors
Syntax
Errors

Parsing

- Not all string of tokens are valid programs
- Parser distinguishes between valid and invalid programs
- We need
  - A language for describing valid string of tokens
  - A method for distinguishing valid from invalid programs

Context-free Grammars (CFGs)

- Programming languages have recursive structure
- An EXP is …

```
if EXP then
    EXP
else
    EXP
```

```
while EXP do
    EXP
end
```

- Context Free Grammars are natural notation for the recursive structures

Context-free Grammars (CFGs)

- A CFG consists of
  - A set of terminals T
  - A set on non-terminals N
  - A start symbol $S \in N$
  - A set of productions $X \rightarrow Y_1 \ldots Y_n$

    $X \in N$

    $Y_i \in N \cup T \cup \{\varepsilon\}$

Context-free Grammars (CFGs)

- $\{(^i)^i \mid i \geq 0\}$

Productions:

$S \rightarrow (S)$

$S \rightarrow \varepsilon$

$N = \{S\}$

$T = \{ (,) \}$

Context-free Grammars (CFGs)

1. Begin with string that has only start symbol S
2. Replace any non-terminal X in the string by the right-hand side of some production $X \rightarrow Y_1 \ldots Y_n$
3. Repeat (2) until there is no non-terminals

r1: $S \rightarrow (S)$
r2: $S \rightarrow \varepsilon$

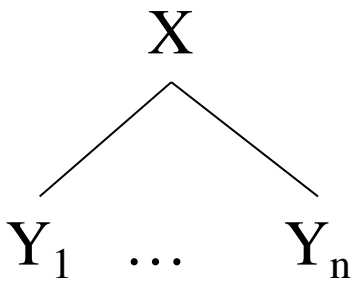
$S \Rightarrow^{r1} (S) \Rightarrow^{r1} ((S)) \Rightarrow^{r2} ((\ \ ))$

Derivation and Parse Tree

- A derivation is a sequence of productions

$$S \Rightarrow \ldots \Rightarrow \ldots \Rightarrow \ldots \Rightarrow \ldots$$

- A derivation can be drawn as a parse tree
  - Start symbol is the tree's root
  - For a production $X \rightarrow Y_1 \ldots Y_n$ add children $Y_1 \ldots Y_n$ to node X



Language of CFGs

- Let G be a context free grammar with start symbol S, and terminals T
  - The language L(G) of G is:

$$\{\alpha_1 \dots \alpha_n | \ \forall_i \ \alpha_i \in T \ and \ S \Rightarrow^* \alpha_1 \dots \alpha_n\}$$

{ε, (), (()), ((())), …}

Arithmetic Expressions

- $E \rightarrow E + E$
- $E \rightarrow E * E$
- $E \rightarrow (E)$
- $E \rightarrow - E$
- $E \rightarrow \mathbf{id}$

Derivation for id + id * id

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow (E)$

$E \rightarrow - E$

$E \rightarrow \mathbf{id}$

$E \Rightarrow E + E$

$\Rightarrow \mathbf{id} + E$

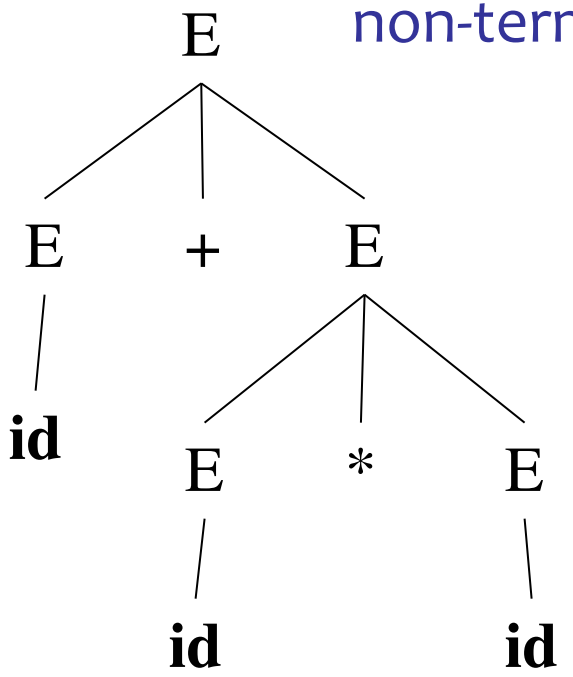
$\Rightarrow \mathbf{id} + E * E$

$\Rightarrow \mathbf{id} + \mathbf{id} * E$

$\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}$

Leaves nodes: terminals

Interior nodes: non-terminals



Leftmost derivation for id + id * id

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow (E)$

$E \rightarrow - E$

$E \rightarrow \mathbf{id}$

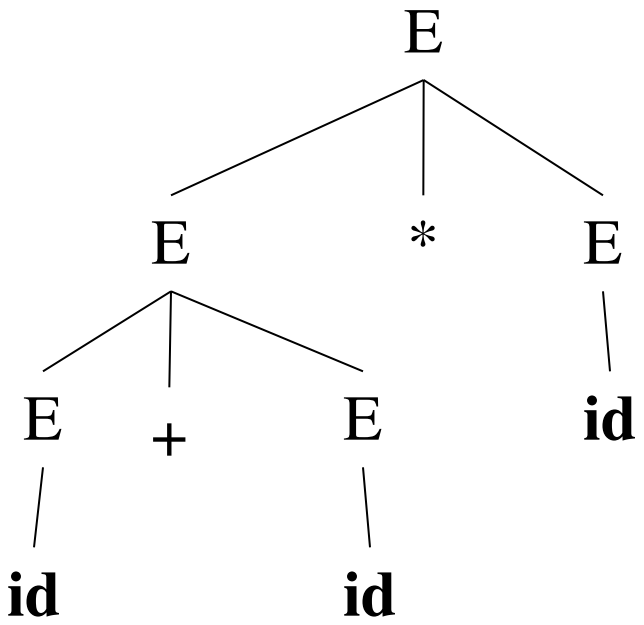
$E \Rightarrow E * E$

$\Rightarrow E + E * E$

$\Rightarrow \mathbf{id} + E * E$

$\Rightarrow \mathbf{id} + \mathbf{id} * E$

$\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}$



Parse tree gives a meaning to the string

(id+id)*id vs id+(id*id)

Rightmost derivation for id + id * id

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow (E)$

$E \rightarrow - E$

$E \rightarrow \mathbf{id}$

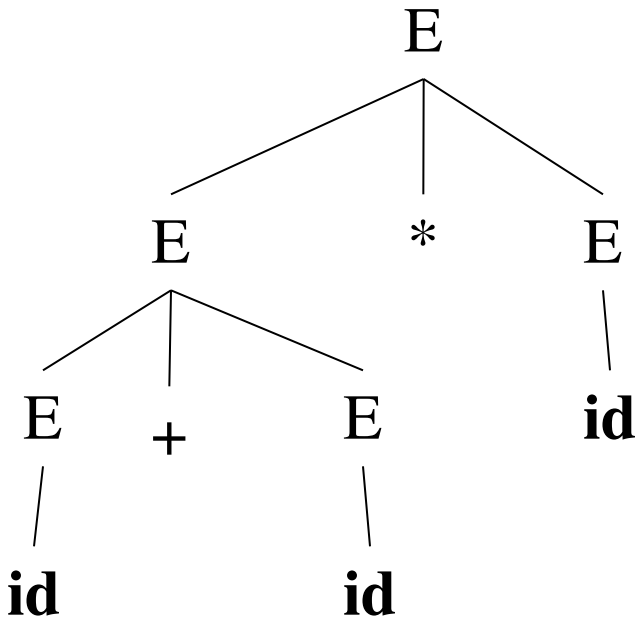
$E \Rightarrow E * E$

$\Rightarrow E * \mathbf{id}$

$\Rightarrow E + E * \mathbf{id}$

$\Rightarrow E + \mathbf{id} * \mathbf{id}$

$\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}$



Rightmost vs. Leftmost Derivation

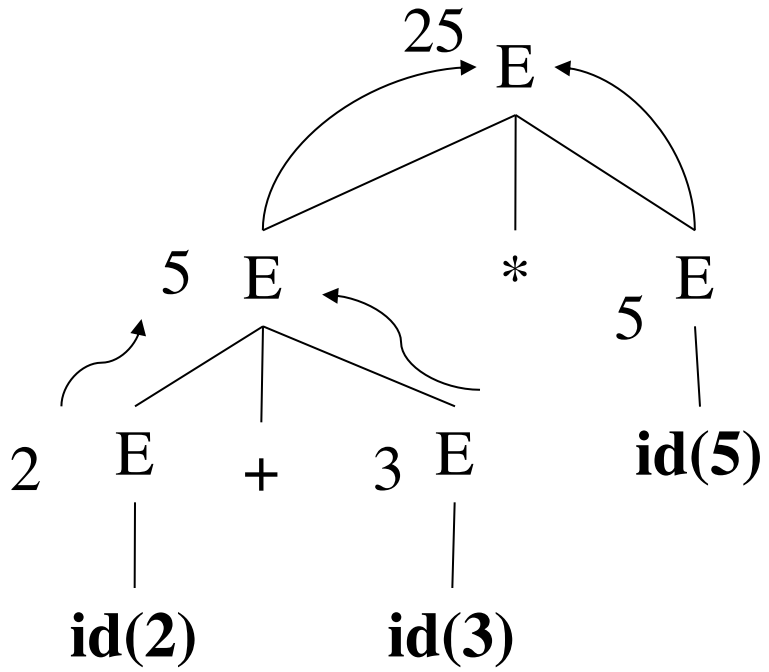
- Note that rightmost and leftmost derivations have the same parse tree
  - Every parse tree has a rightmost and a leftmost derivation
  - Important in resolving ambiguity

Writing a CFG for a PL

- First write (or read) a reference grammar of what you want to be valid programs
- For now, we only worry about the structure, so the reference grammar might choose to over-generate in certain cases (e.g. `bool x = 20;`)
- Convert the reference grammar to a CFG

Arithmetic Expressions

- E → E + E { $$ = $1 + $3 }
- E → E * E { $$ = $1 * $3 }
- E → (E) { $$ = $2 }
- E → - E { $$ = -1 * $2 }
- E → **id** { $$ = $1 }



CFG Notation

- Normal CFG notation

  $E \rightarrow E * E$

  $E \rightarrow E + E$

- Backus Naur notation

  E ::= E * E | E + E

  (an or-list of right hand sides)

  Also:

  E = E “*” E | E “+” E