Type Systems

Semantics

CMPT 379: Compilers

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Equality of types

- Main semantic tasks involve liveness analysis and checking equality
- Equality checking of types (basic types) is crucial in ensuring that code generation can target the correct instructions
- Coercions also rely on equality checking of types
- But what about those objects in PLs (records, functions, etc) that are not basic types?
- Can we perform any semantic checks on these as well?

Type Systems

- So far we have seen simple cases of type checking and coercion
- Basic types for data types: *boolean, char, integer, real*
- A basic type for lack of a type: *void*
- A basic type for a type error: *type_error*
- Based on these basic types we can build new types using type constructors

Type Constructors

- Arrays: int p[10];
  - type: *array(10, integer)*
  - multi-dim arrays: int p[3][2]: *array(3, array(2, integer))*
- Products/tuples: pair<int, char> p(10,'a');
  - type: $integer \times char$
- Records: struct { int p; char q; } data;
  - Type: $record((p \times integer) \times (q \times char))$
- Pointers: int *p;
  - Type: *pointer(integer)*

Type Constructors

- Functions: int foo (int p, char q) { return 2; }
  - Type: $integer \times char \rightarrow integer$
  - A function maps elements from the domain to the range
  - Function types map a domain type D to a range type R
  - A type for a function is denoted by $D \rightarrow R$
- In addition, type expressions can contain type variables
  - Example: $\alpha \times \beta \rightarrow \alpha$

Equivalence of Type Exprs

- Check equivalence of type exprs: *s* and *t*
- If *s* and *t* are basic types, then return true
- If $s = array(s_1, t_1)$ and $t = array(s_2, t_2)$ then return true if equal(s_1, s_2) and equal(t_1, t_2)
- If s = $s_1 \times t_1$ and $t = s_2 \times t_2$ then return true if equal(s_1, s_2) and equal(t_1, t_2)
- If $s = pointer(s_1)$ and $t = pointer(t_1)$ then return true if equal(s_1, t_1)

Polymorphic Functions

- Consider the following ML program:

```
fun null [] = true
   |  null (_::_) = false;
fun tl (_::xs) = xs;
fun length (alist) =
      if null(alist) then 0
      else length(tl(alist)) + 1;
```

- *null* tests if a list is empty
- *tl* removes first element and returns rest

Polymorphic Functions

- *length* is a polymorphic function (different from polymorphism in object inheritance)
- The function *length* accepts lists with elements of any basic type:

  length(['a', 'b', 'c'])

  length([1, 2, 3])

  length([[1,2,3], [4,5,6]])
- The type for *length* is *list*(α) $\rightarrow$ *integer*
- α can stand for any basic type: *integer* or *char*

Polymorphic Functions

- Consider the following ML program:

  fun *map f [] = []*

  | *map f (x::xs) = (f(x)) :: map f xs;*

- *map* takes two arguments: a function *f* and a list
- It applies *f* to each element of the list and creates a new list with the range of *f*
- Type of *map*: $(\alpha \rightarrow \beta) \rightarrow list(\alpha) \rightarrow list(\beta)$

Type Inference

- *Type inference* is the problem of determining the type of a statement from its body
- Similar to type checking and coercion
- But inference can be much more expressive when type variables can be used
- For example, the type of the *map* function on previous page uses type variables

Type Variable Substitution

- We can take a type variable in a type expression and substitute a value
- In *list*(α) we can substitute the type *integer* for the variable α to get *list(integer)*
- *list(integer)* < *list*(α) means *list(integer)* is an instance of *list*(α)
- *S(t)* is a substitution for type expr *t*
- Replacing *integer* for α is a substitution

Type Variable Substitution

- $s < t$ means s is an instance of t
- Or s is more specific than t
- Or t is more general than s
- Some more examples:
  - $integer \rightarrow integer < \alpha \rightarrow \alpha$
  - $(integer \rightarrow integer) \rightarrow (integer \rightarrow integer) < \alpha \rightarrow \alpha$
  - $list(\alpha) < \beta$
  - $\alpha < \beta$ and $\beta < \alpha$

Type Expr Unification

- Incorrect type variable substitutions:
  - *integer* < *boolean* (in some languages *boolean* < *integer* is true)
  - *integer* → *boolean* < $\alpha \to \alpha$
  - *integer* → $\alpha < \alpha \to \alpha$
- In general, there are many possible substitutions
- Type exprs *s* and *t* unify if there is a substitution *S* that is most general such that *S(s) = S(t)*
- Such a substitution *S* is the *most general unifier* which imposes the fewest constraints on variables

Example of Type Inference

- Example:

  fun *length (alist) =*

  if *null(alist)* ***then*** *0*

  else *length(tl(alist)) + 1;*

- *length* : α_1
- *null : list*(α_2) $\rightarrow$ *boolean*
- *alist : list*(α_2)
- *null(alist) : boolean*

Example (cont'd)

fun *length (alist) =*

if *null(alist)* ***then*** *0*

else *length(tl(alist)) + 1;*

- $0 : integer$
- $tl : list(\alpha_3) \rightarrow list(\alpha_3)$
- $tl(alist) : list(\alpha_2)$
- $length : list(\alpha_2) \rightarrow \alpha_4$
- $length(tl(alist)) : \alpha_4$
- $1 : integer$
- $+ : integer \times integer \rightarrow integer$
- $if : boolean \times \alpha_5 \times \alpha_5 \rightarrow \alpha_5$

$list(\alpha_2) \rightarrow \alpha_4 < \alpha_1$

$integer < \alpha_5$

$integer < \alpha_4$

$$length : list(\alpha_2) \rightarrow integer$$

Unification

- Algorithm for finding the ***most general substitution*** *S* such that *S(s) = S(t)*
- Also called the ***most general unifier***
- *unify(m, n)* unifies two type exprs *m* and *n* and returns true/false if they can be unified
- Side effect is to keep track of the *mgu* substitution for unification to succeed

Unification Algorithm

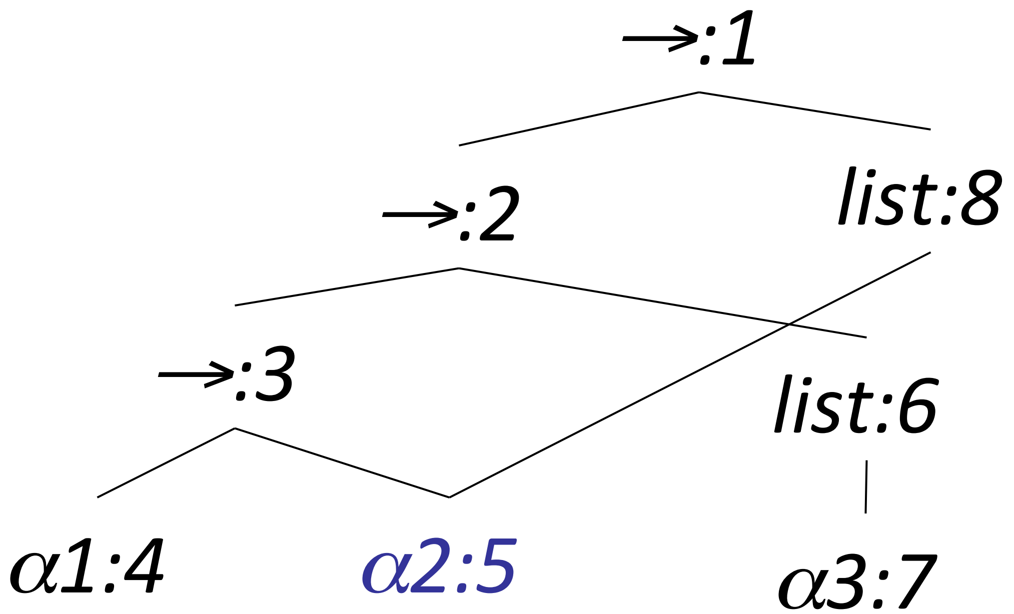
- We will explain the algorithm using an example:
  - E: $((\alpha 1 \rightarrow \alpha 2) \rightarrow list(\alpha 3)) \rightarrow list(\alpha 2)$
  - F: $((\alpha 3 \rightarrow \alpha 4) \rightarrow list(\alpha 3)) \rightarrow \alpha 5$
- What is the most general unifier?
  - $S_1(E) = S_1(F)$ $((\alpha 1 \rightarrow \alpha 1) \rightarrow list(\alpha 1)) \rightarrow list(\alpha 1)$

√ – $S_2(E) = S_2(F)$ $((\alpha 1 \rightarrow \alpha 2) \rightarrow list(\alpha 1)) \rightarrow list(\alpha 2)$

√ – $S_3(E) = S_3(F)$ $((\alpha 3 \rightarrow \alpha 2) \rightarrow list(\alpha 3)) \rightarrow list(\alpha 2)$

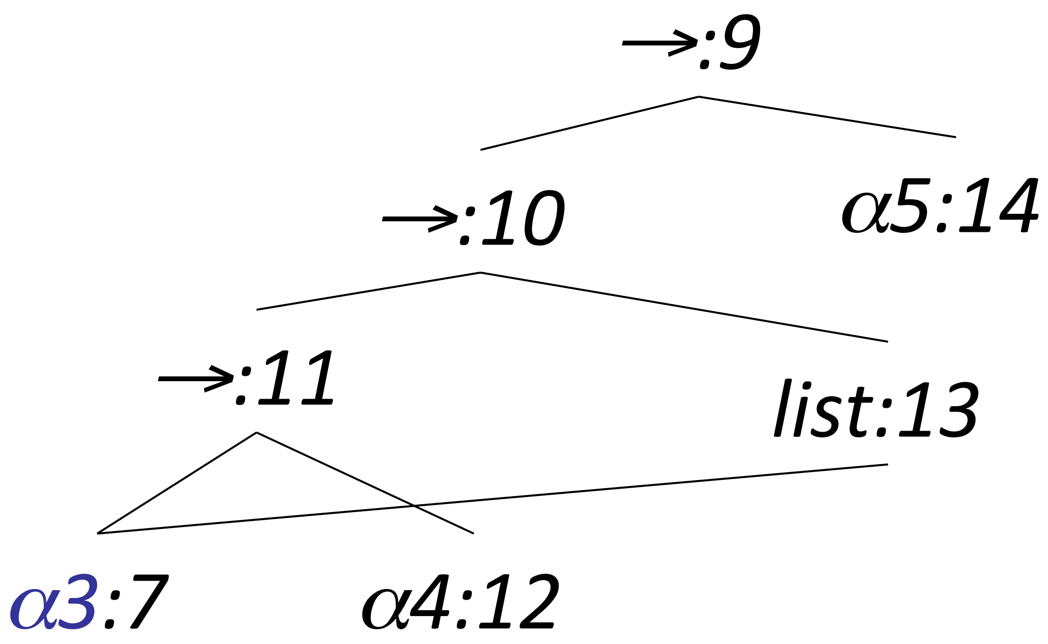
Unification Algorithm

E: $((\alpha 1 \rightarrow \alpha 2) \rightarrow list(\alpha 3)) \rightarrow list(\alpha 2)$

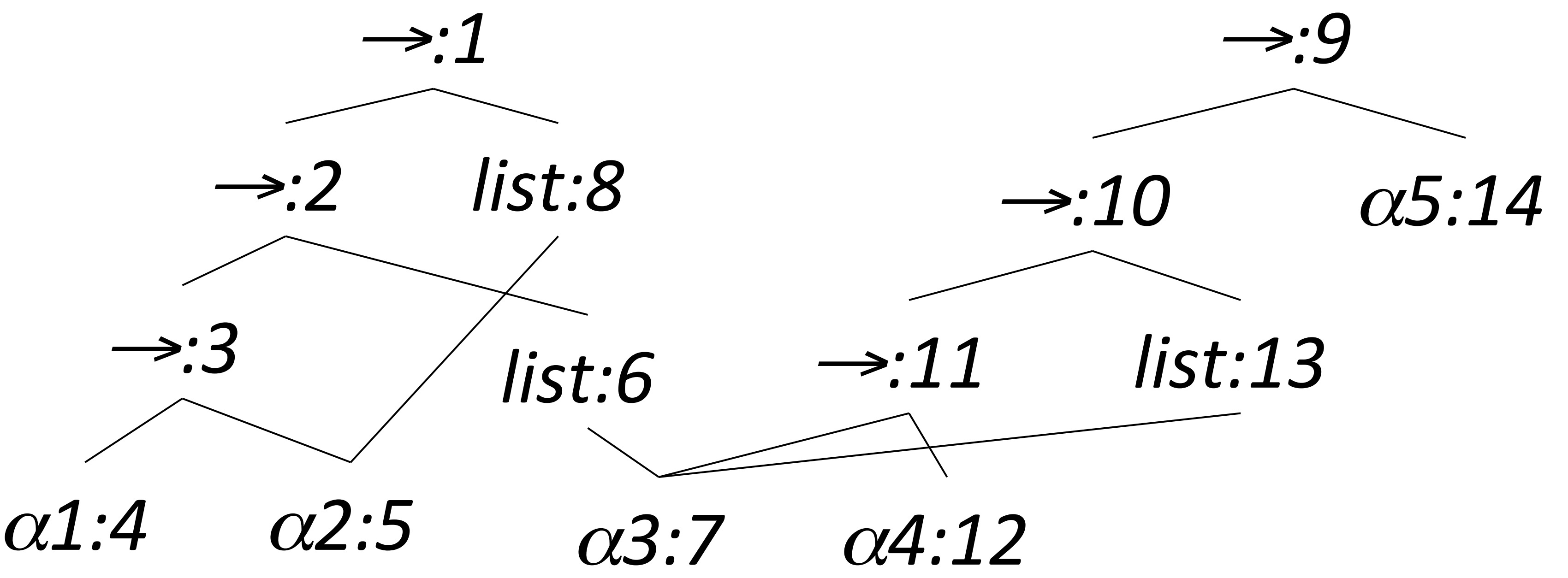


Unification Algorithm

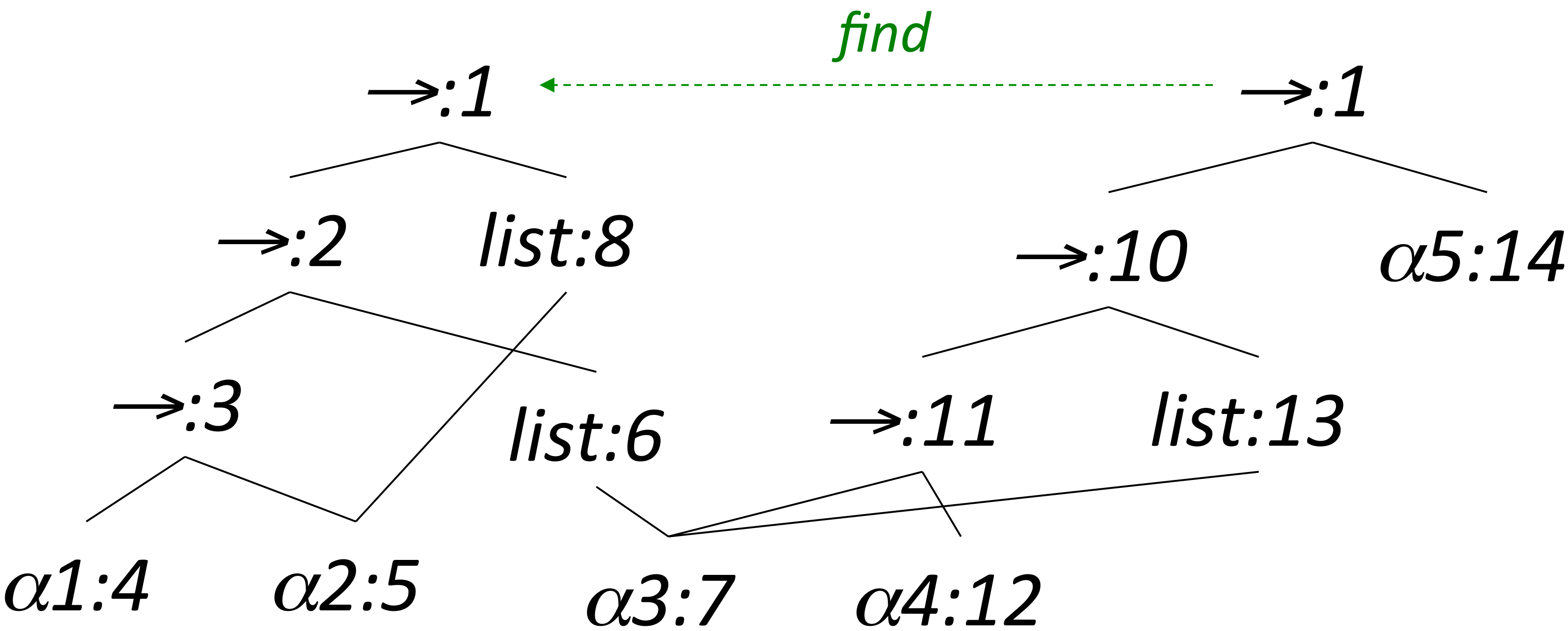
F: $((\alpha 3 \rightarrow \alpha 4) \rightarrow list(\alpha 3)) \rightarrow \alpha 5$



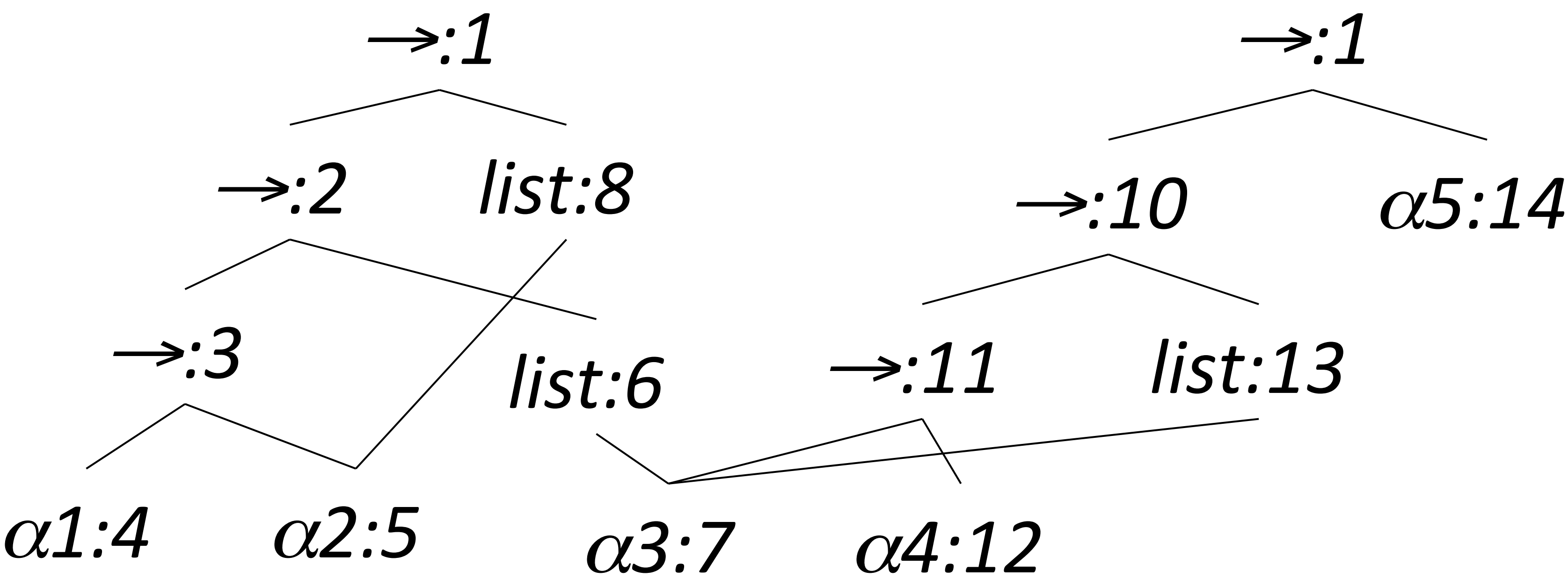
Unify(1,9)



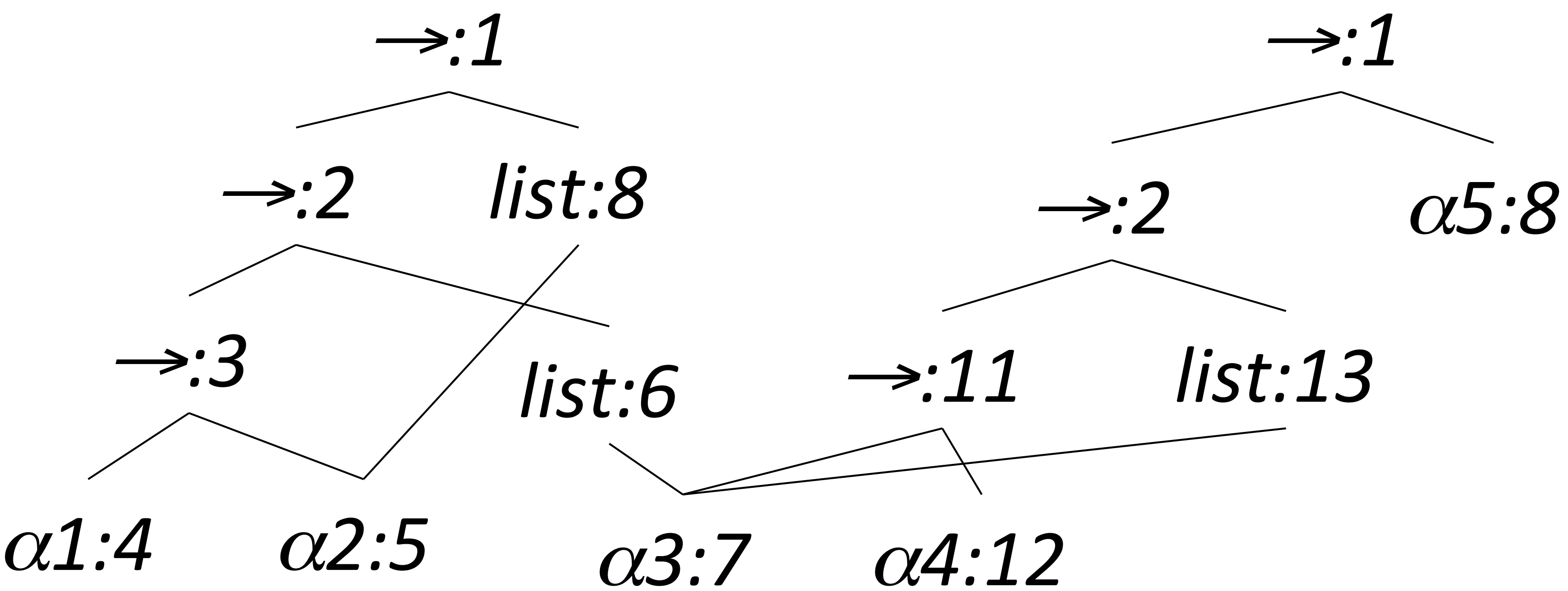
Unify(1,9)



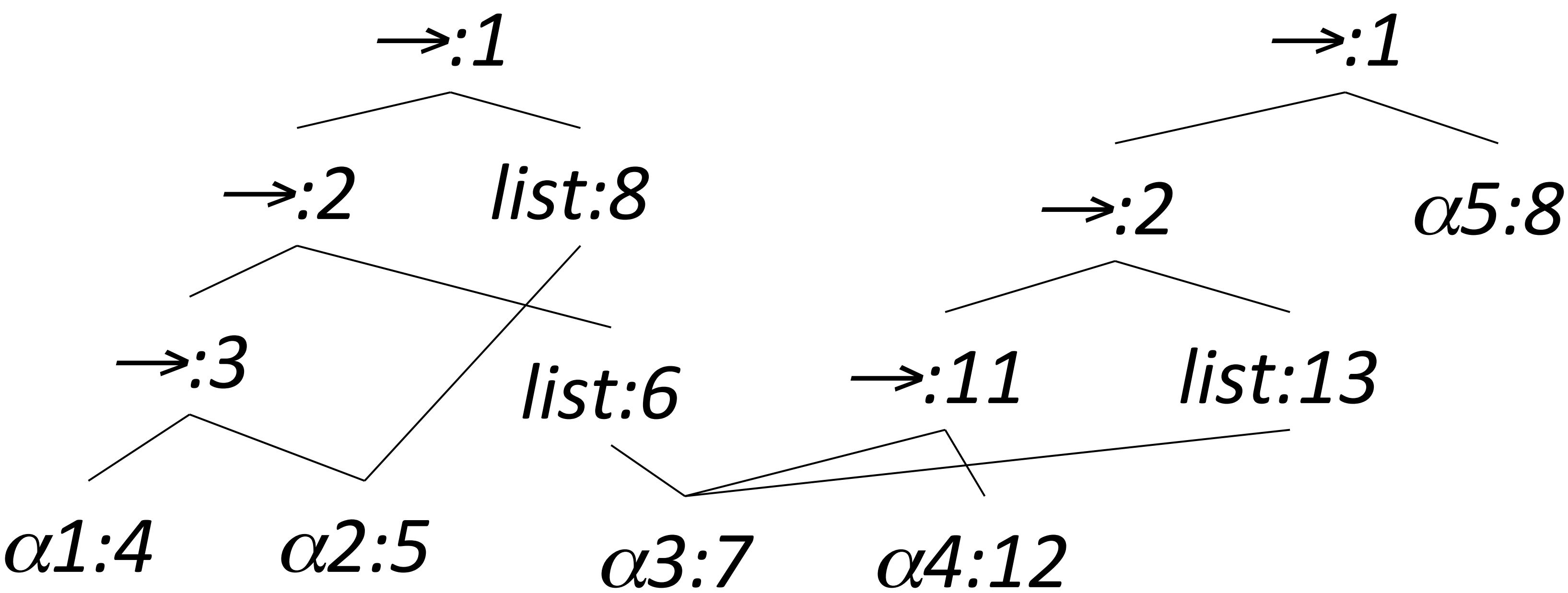
Unify(2,10) *and* Unify(8,14)



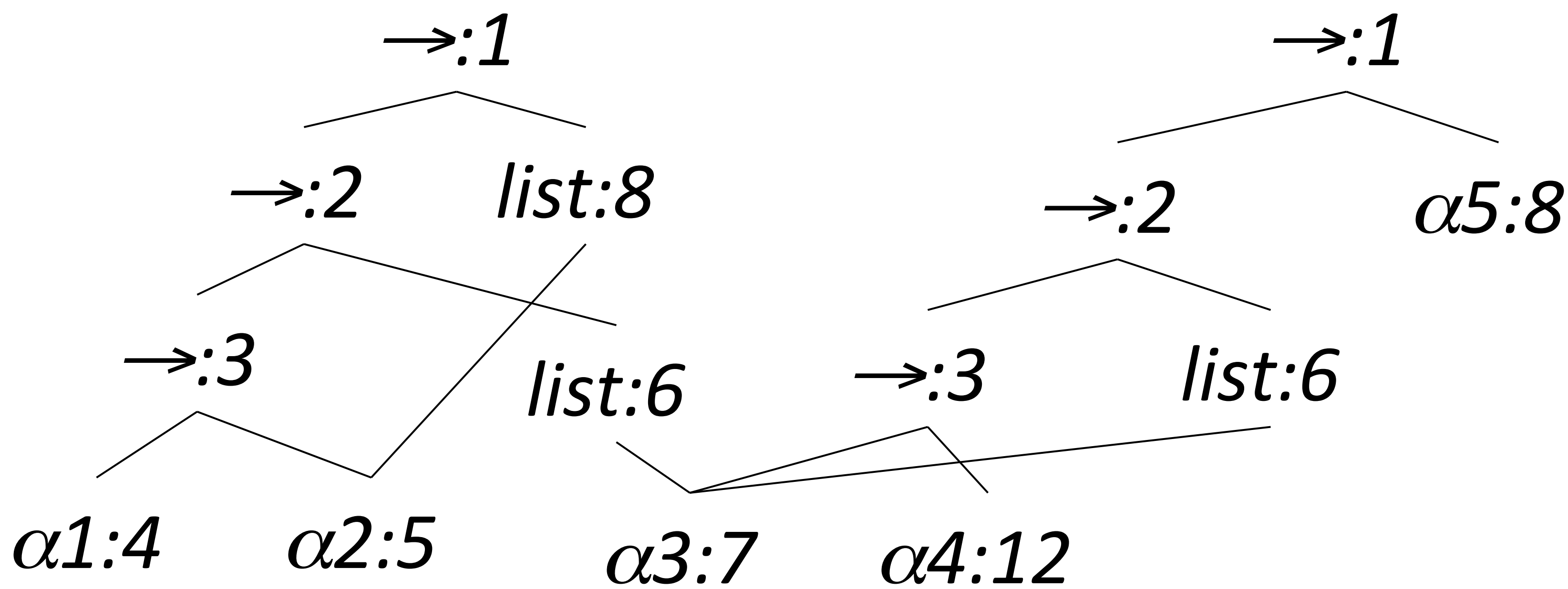
Unify(2,10) *and* Unify(8,14)



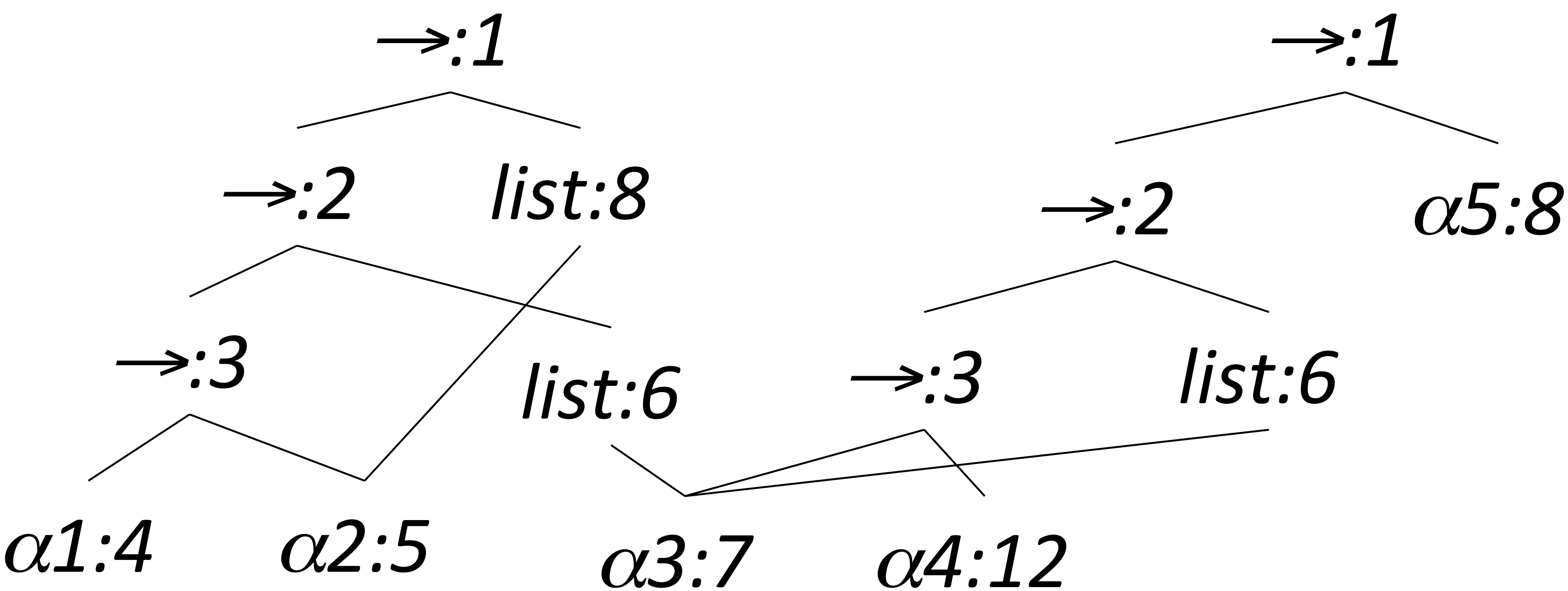
Unify(3,11) *and* Unify(6,13)



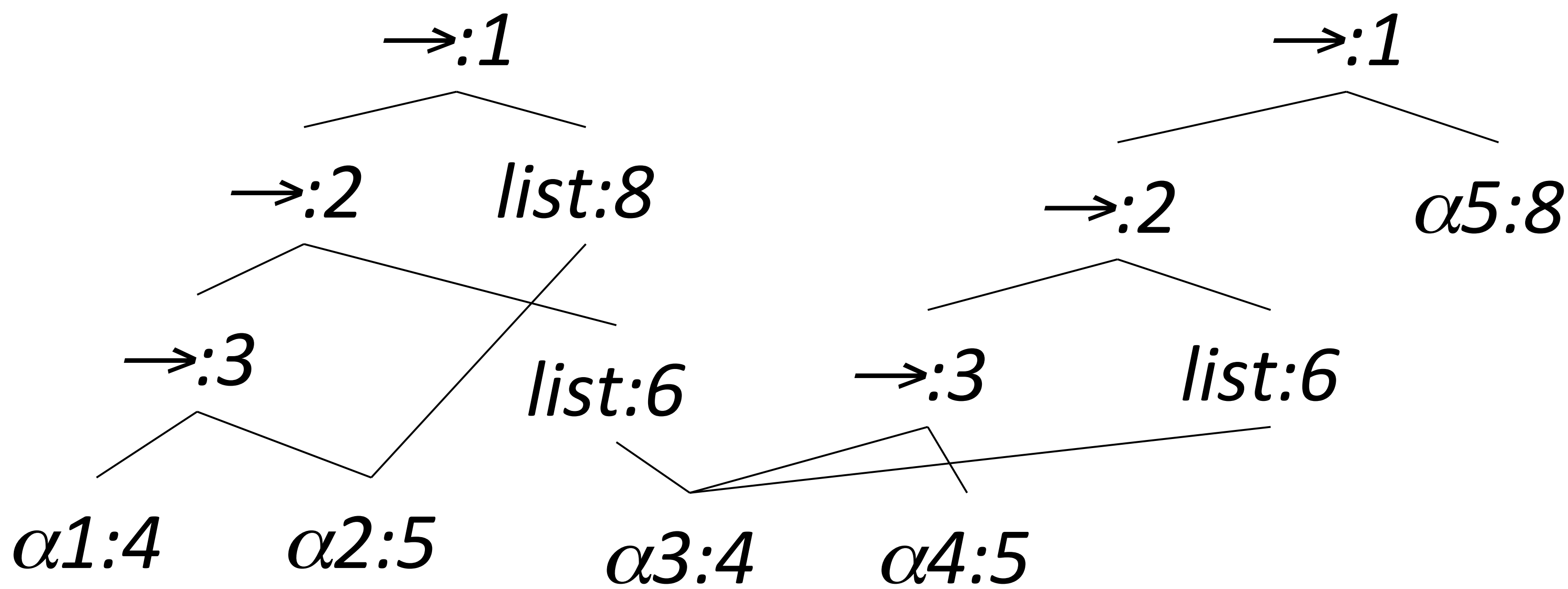
Unify(3,11) *and* Unify(6,13)



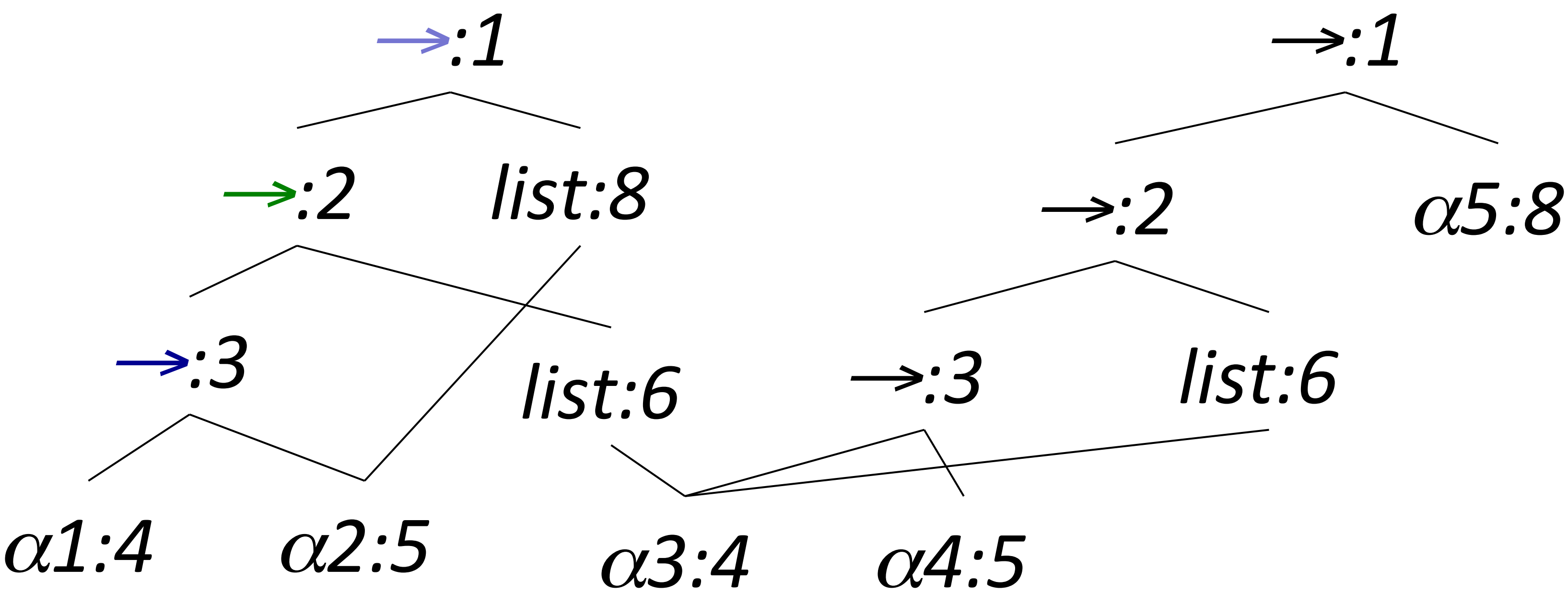
Unify(4,7) *and* Unify(5,12)



Unify(4,7) *and* Unify(5,12)



Unification success

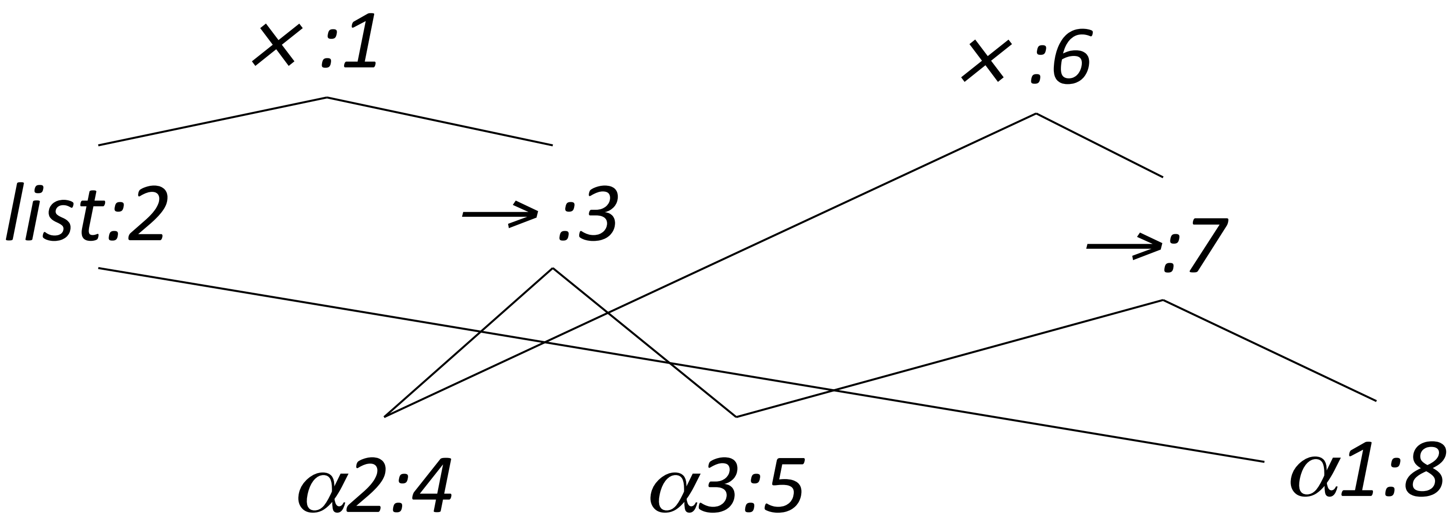


$((\alpha 1 \rightarrow \alpha 2) \rightarrow list(\alpha 1)) \rightarrow list(\alpha 2)$

Unification: Occur Check

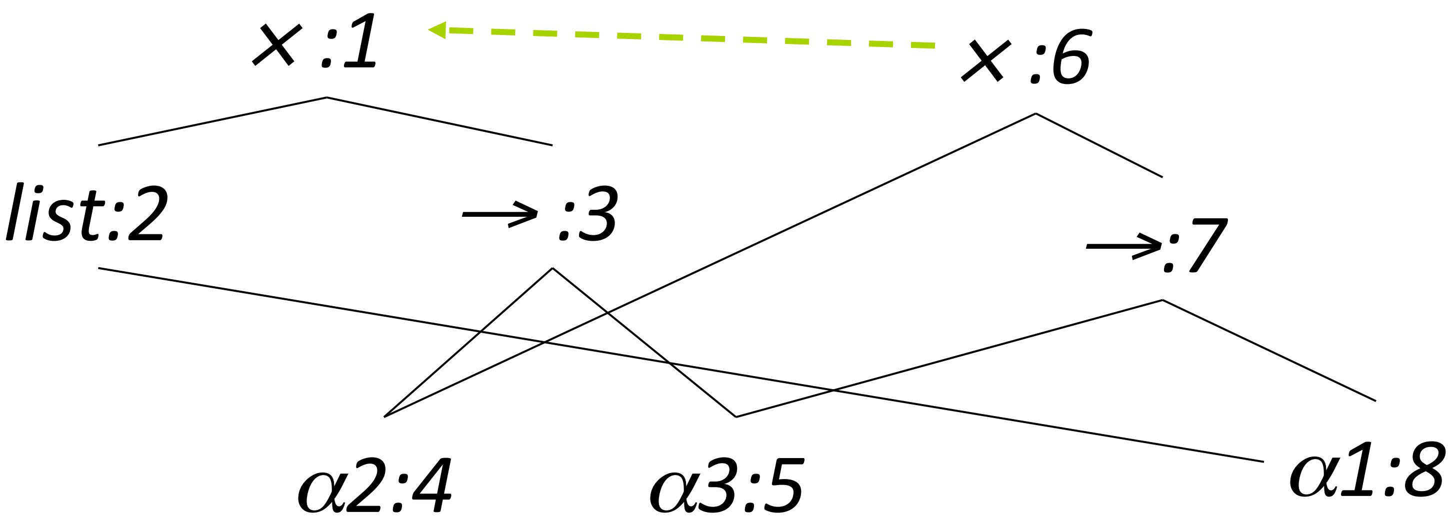
$$list(\alpha 1) \times (\alpha 2 \rightarrow \alpha 3)$$

$$\alpha 2 \times (\alpha 3 \rightarrow \alpha 1)$$



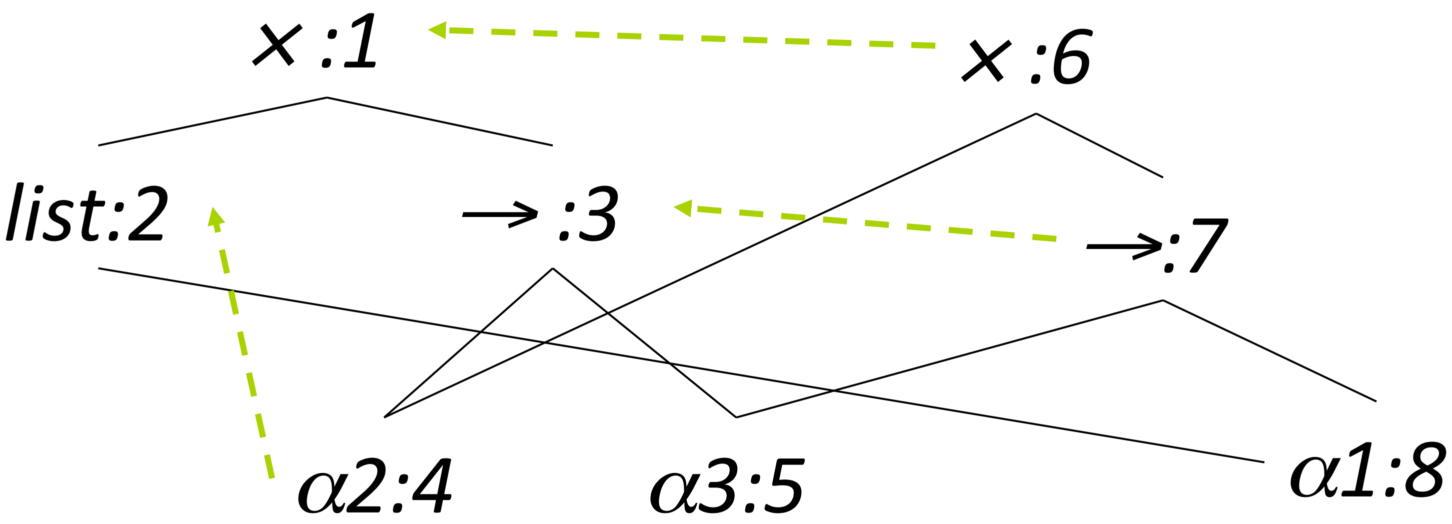
Unify(1,6)

6--1



Unify(2,4) and Unify(3,7)

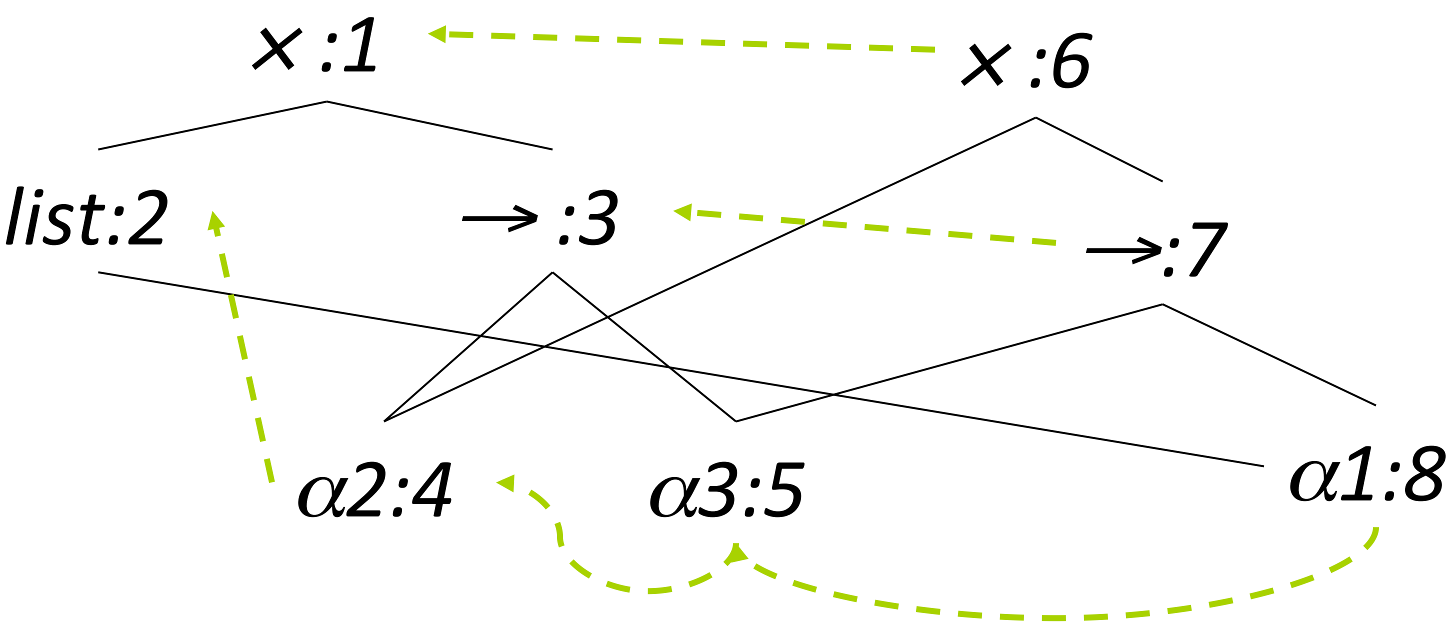
6--1, 4--2, 7--3



Unify(4,5) and Unify(5,8)

6--1, 4--2, 7--3, 5--4, 8--5

- *list(α1)*
- *= list(α2)*
- *= list(list(α1))*



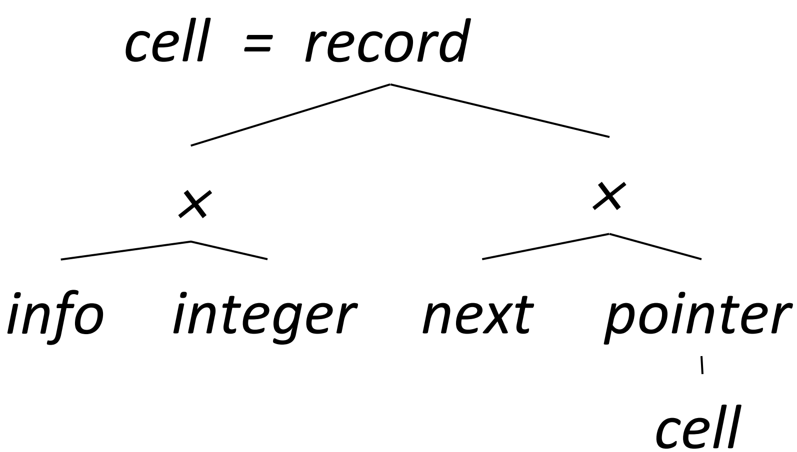
Occur Check

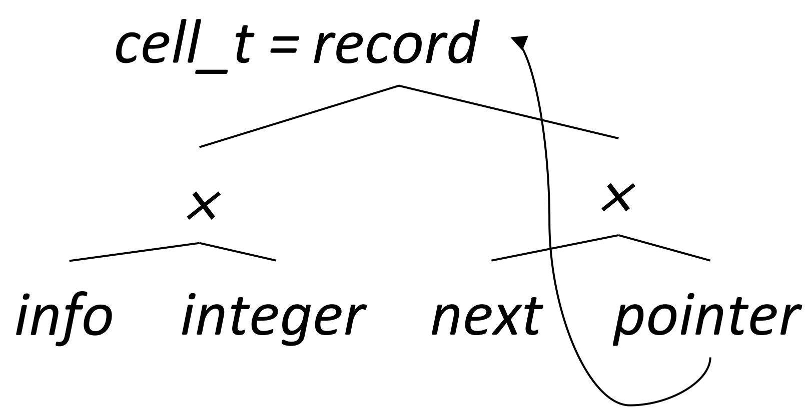
- Our unification algorithm creates a cycle in *find* for some inputs
- The cycle leads to an infinite loop. Note that Algorithm 6.32 in the Purple Dragon book has this bug
- A solution to this is to unify only if no cycles are created: the *occur check*
- Makes unification slower but correct

Recursive types

- Recursive types arise naturally in PLs
- For example, in pseudo-C:

*struct cell { int info; cell_t *next; } cell_t;*



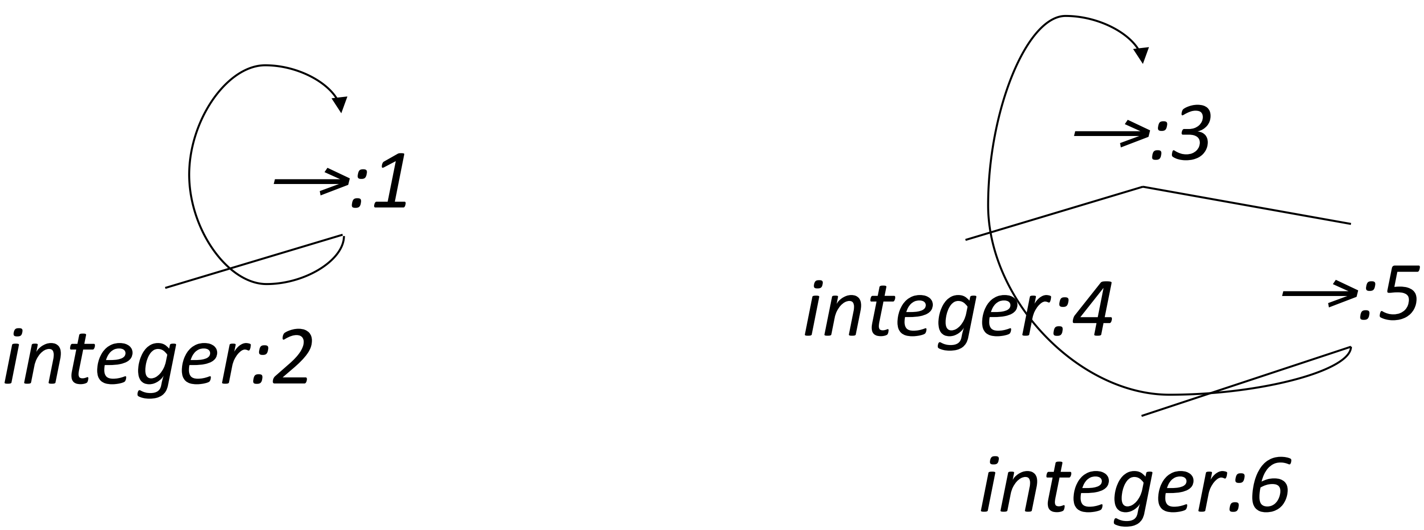


Recursive type equivalence

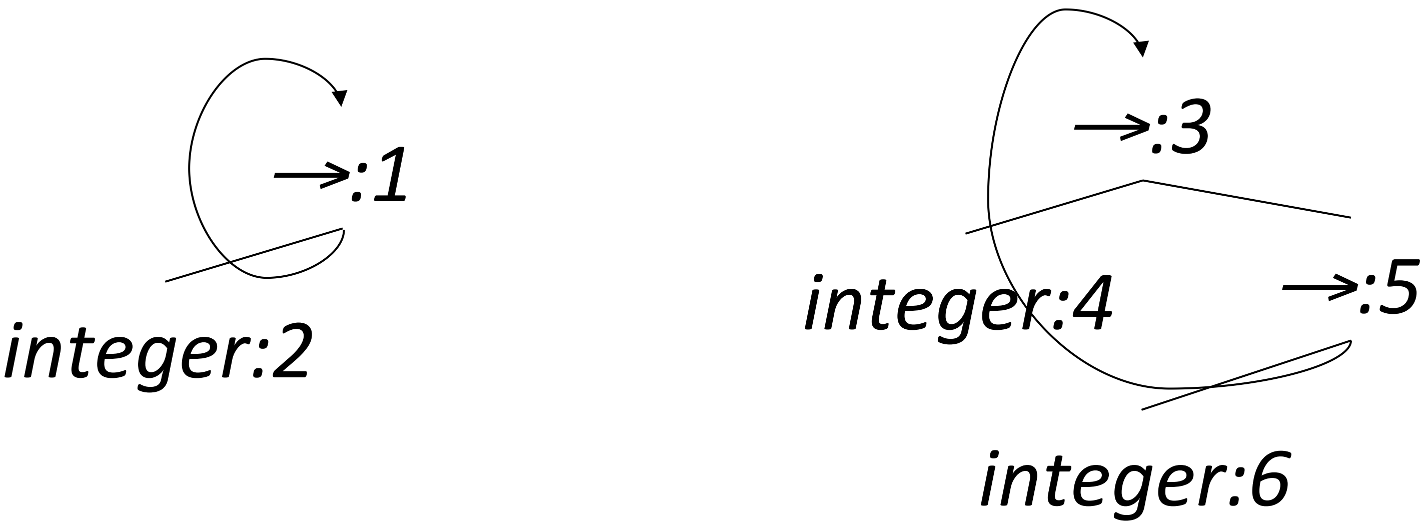
- Are these recursive type expressions equivalent:

  $\alpha 1$ = *integer* $\rightarrow \alpha 1$

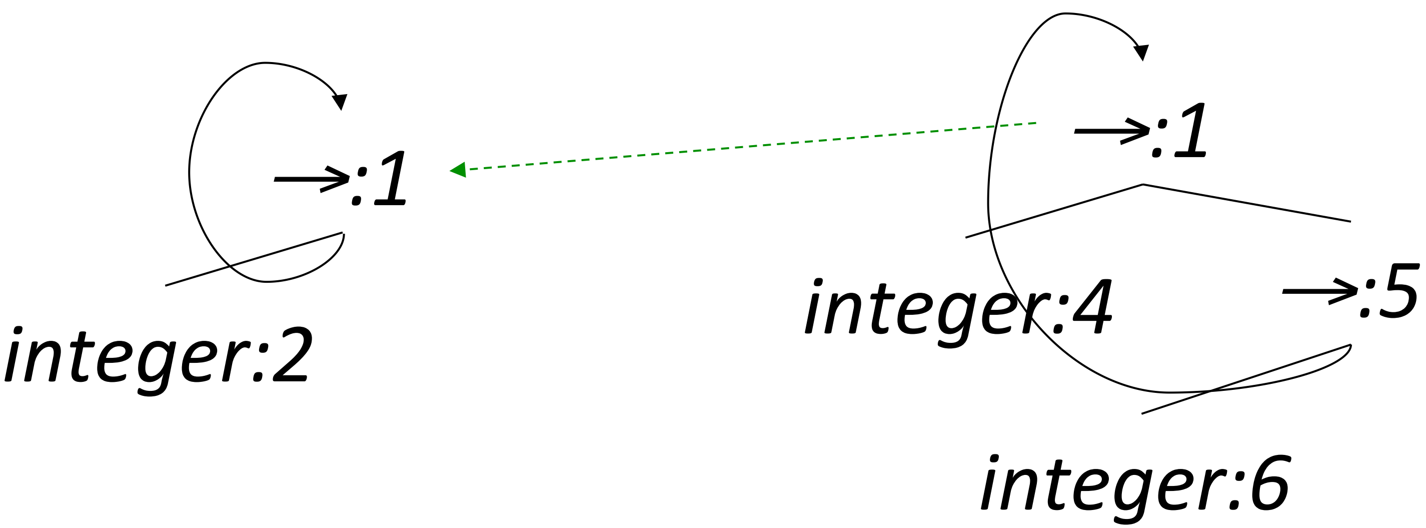
  $\alpha 2$ = *integer* $\rightarrow$ (*integer* $\rightarrow \alpha 2$)



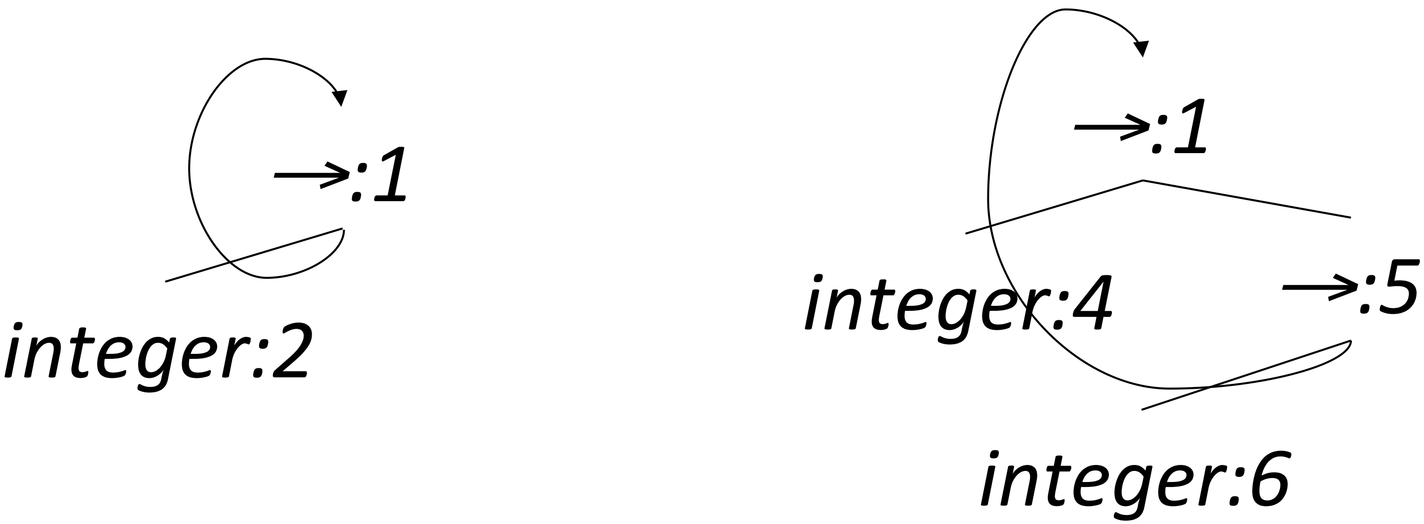
Unify(1,3)



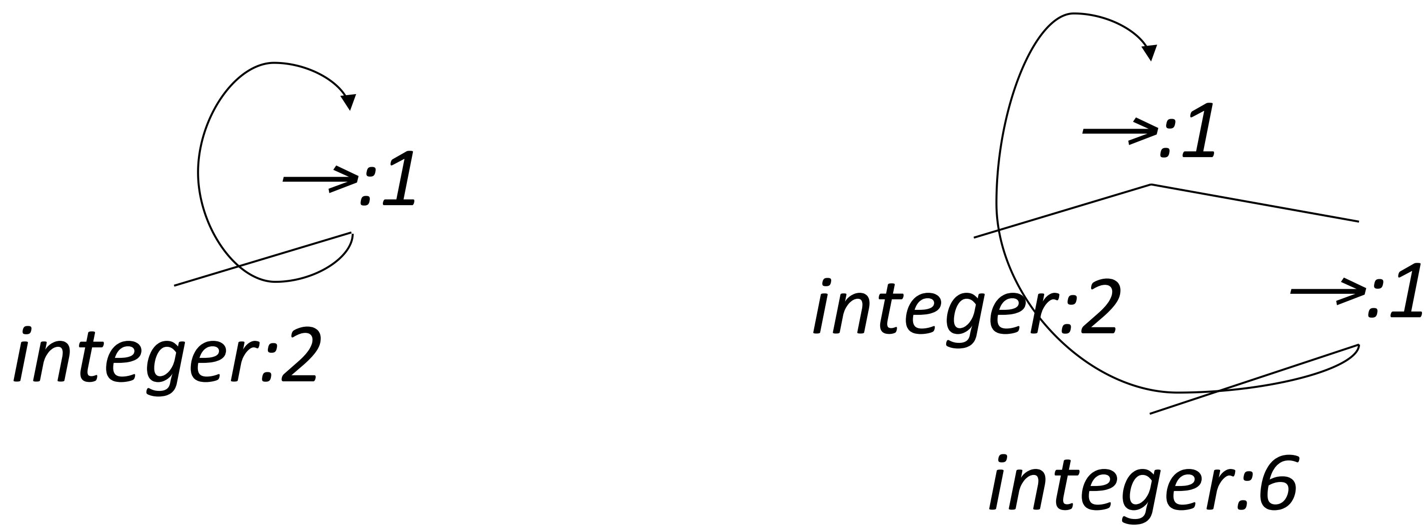
Unify(1,3)



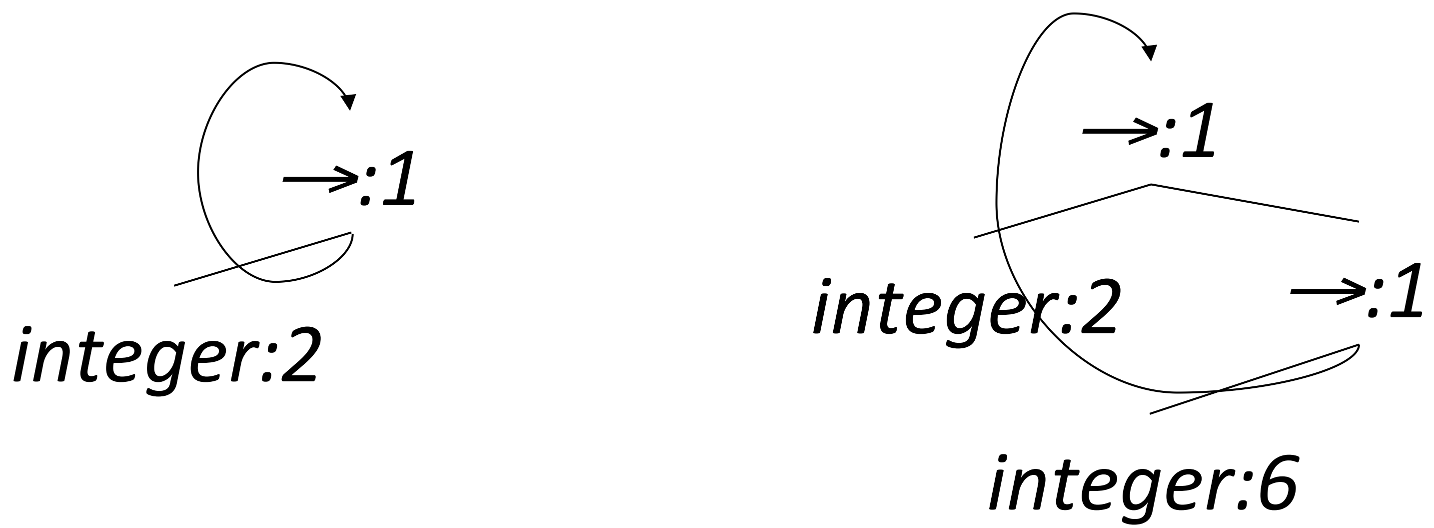
Unify(2,4) *and* Unify(1,5)



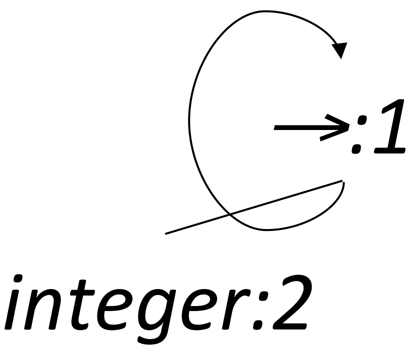
Unify(2,4) *and* Unify(1,5)

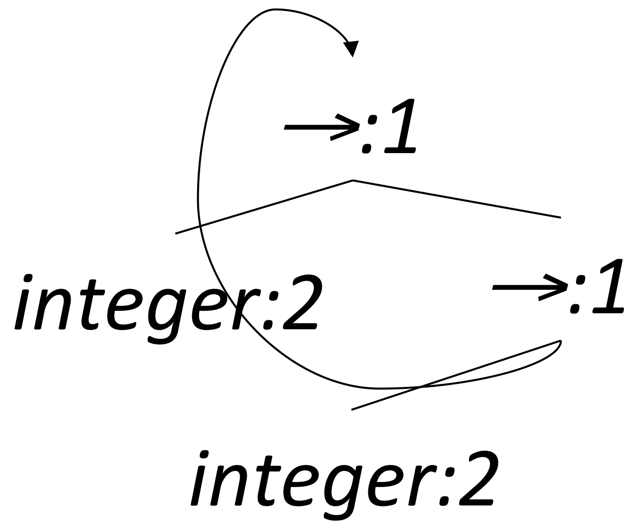


Unify(2,6) *and* Unify(1,1)



Unify(2,6) *and* Unify(1,1)





Unification is successful!

Summary

- Semantic analysis: checking various well-formedness conditions
- Most common semantic conditions involve types of variables
- Symbol tables
- Discovering types for variables and functions using inference (unification)