Register Allocation

CMPT 379: Compilers

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Register Allocation

- Intermediate code uses unlimited temporaries
  - Simplifying code generation and optimization
  - Complicates final translation to assembly

Register Allocation

- The problem:

  Rewrite the intermediate code to use no more temporaries than there are machine registers

- Method:
  - Assign multiple temporaries to each register
  - But without changing the program behavior

Example

- Consider the program

  $a = c + d$

  $e = a + b$

  $f = e - 1$

- Assume a & e dead after use
  - A dead temporary can be "reused"

- Can allocate a, e and f all to one register (r1)

  $r1 = r_2 + r_3$

  $r_1 = r_1 + r_4$

  $r_1 = r_1 - 1$

History

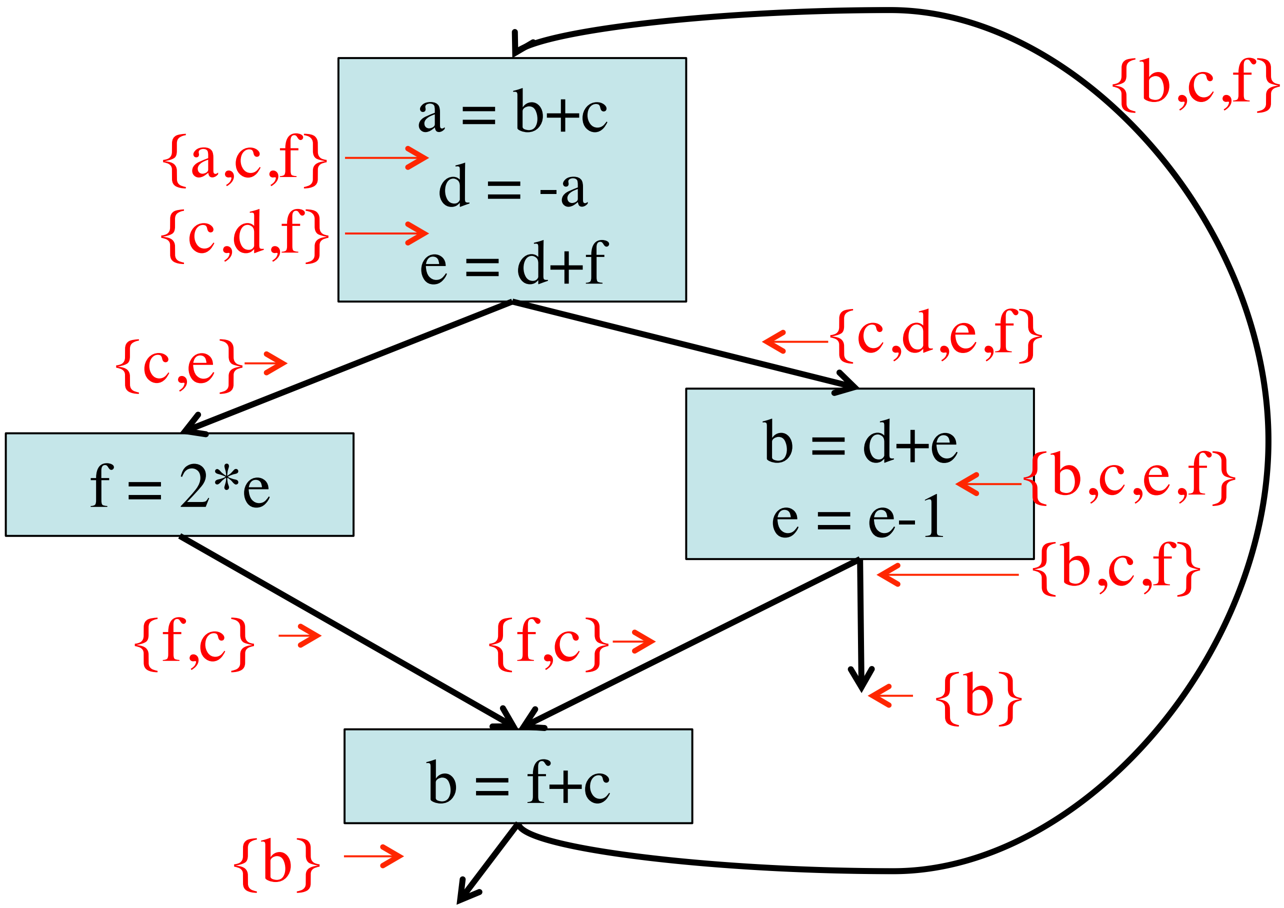
- Register allocation is as old as compilers
  - Register allocation was used in the original FORTRAN compiler in 1950's
  - Very crude algorithm
- A breakthrough came in 1980
  - Register allocation scheme based on graph coloring
  - Relatively simple, global and works well in practice

Principles of Register Allocation

- *Temporaries t_1 can t_2 can share the same register if at any point in the program at most one of t_1 or t_2 is live*
  - *If t_1 and t_2 are live at the same time, they cannot share a register*
- We need liveness analysis

Live Variables

- Compute live variables for each point

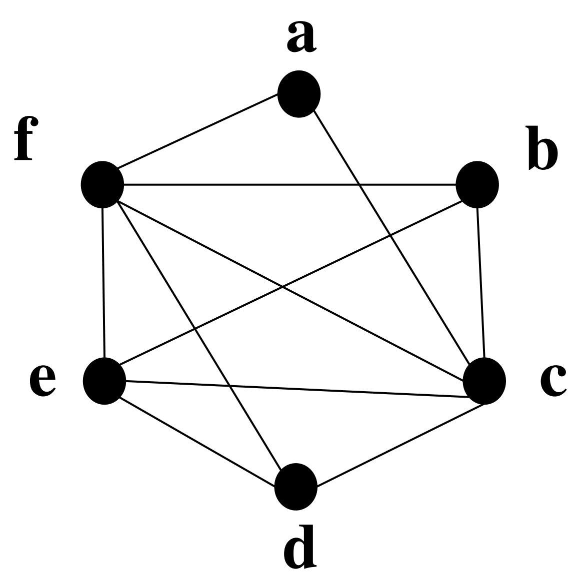


Register Interference Graph

- Construct an undirected graph
  - A node for each temporary
  - An edge between t_1 and t_2 if they are live simultaneously at some point in the program
- This is the *register interference graph* (RIG)
  - Two temporaries can be allocated to the same register if there is no edge connecting them

Register Interference Graph

- For our example



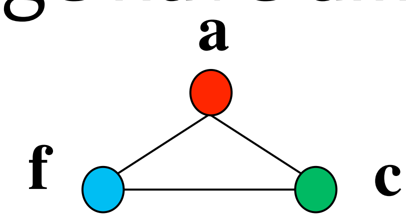
- a and c cannot be in the same register
- a and d could be in the same register

Register Interference Graph

- Extracts exactly the information we need to characterize legal register allocation
- Gives the global view (i.e., over the entire control flow graph) picture of the register requirements
- After RIG construction the register allocation algorithm is architecture independent

Graph Coloring

- A coloring of a graph is an assignment of colors to nodes, such that nodes connected by an edge have different colors



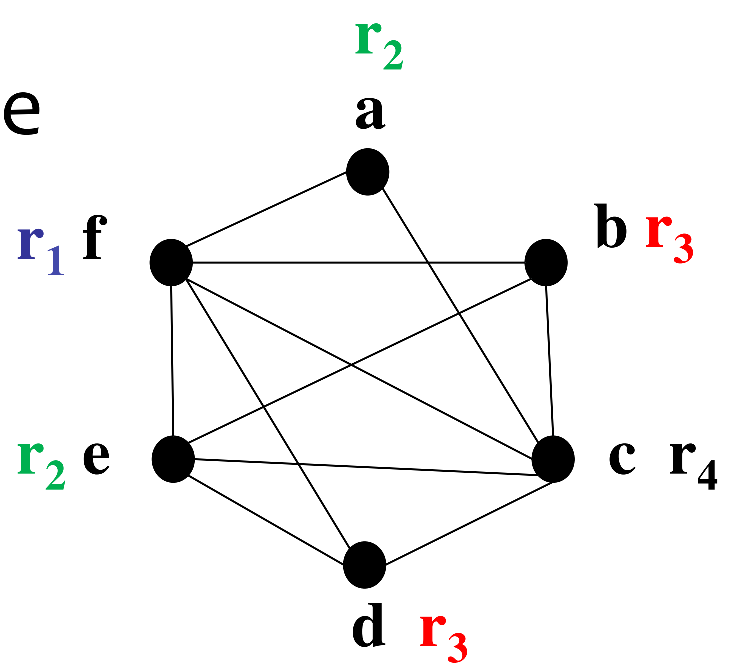
- A graph is k-colorable if it has a coloring with k colors

Register Allocation as Graph Coloring

- In our problem, colors = registers
- We need to assign colors (registers) to graph nodes (temporaries)
- Let k = number of machine registers
- If the RIG is k-colorable then there is a register assignment that uses no more than k registers

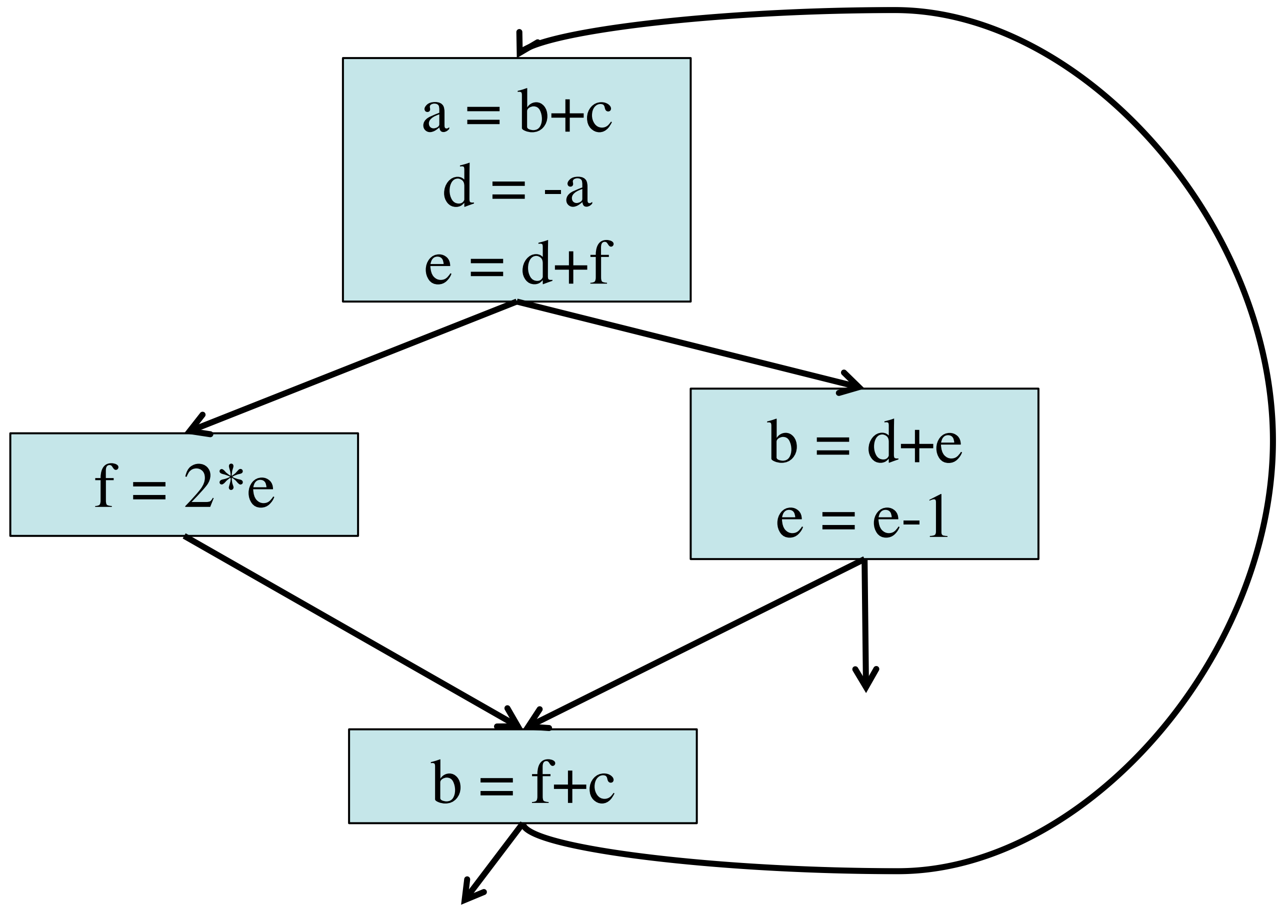
Example

- For our example

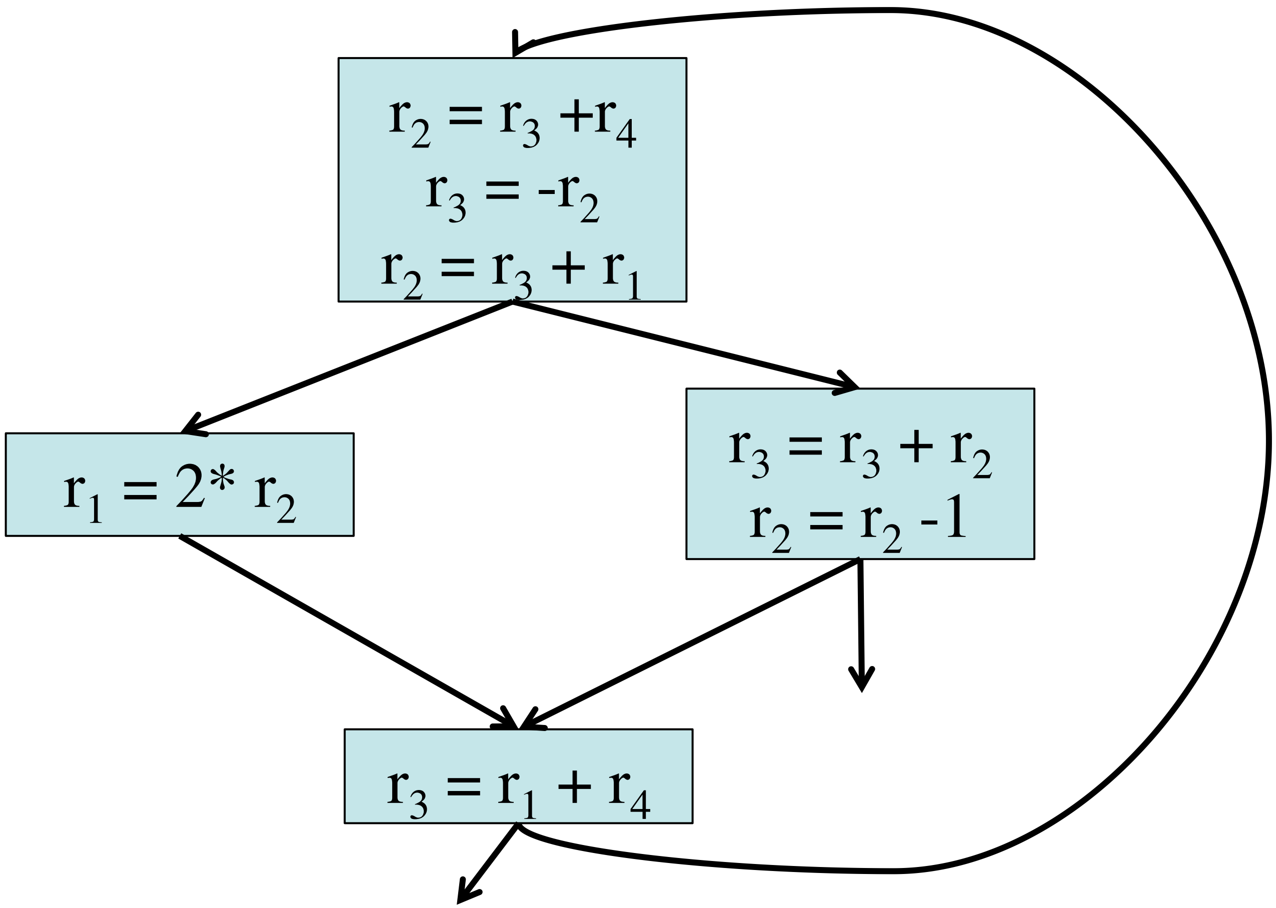


- There is no coloring with less than 4 colors
- There is a 4-coloring of this graph

Control Flow Graph



Register Allocation



Graph Coloring

- How do we compute graph coloring?
- It is not easy :
  - The problem is NP-hard. No efficient algorithms are known
    - Solution: use heuristics
  - A coloring might not exist for a given number of registers
    - Solution: register spilling

Register Allocation as Graph Coloring

- Main idea for solving whether a graph G is *k*-colorable:
- Pick any node *t* with fewer than *k* neighbors
- Remove *n* adjacent edges to create a new graph G'
- If G' is *k*-colorable, then so is G (the original graph)
- Let $c_1, \ldots, c_n$ be the colors assigned to the neighbors of t in G'
- Since n<k we can pick some color for t that is different from its neighbors

Register Allocation as Graph Coloring

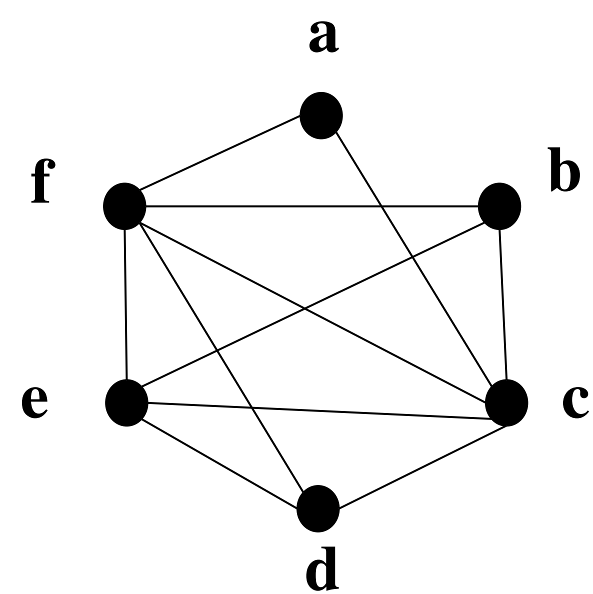
- Heuristic for graph coloring:
  - Ordering nodes (in an stack)
    1. Pick a node t with fewer than k neighbors
    2. Put t on a stack and remove it from the register interference graph (RIG)
    3. Repeat until the graph is empty
  - Assigning color to nodes on the stack:
    1. Start with the last node added
    2. At each step pick a color different from those assigned to already colored neighbors

Example

- Assume k=4

  Remove a

  stack={}

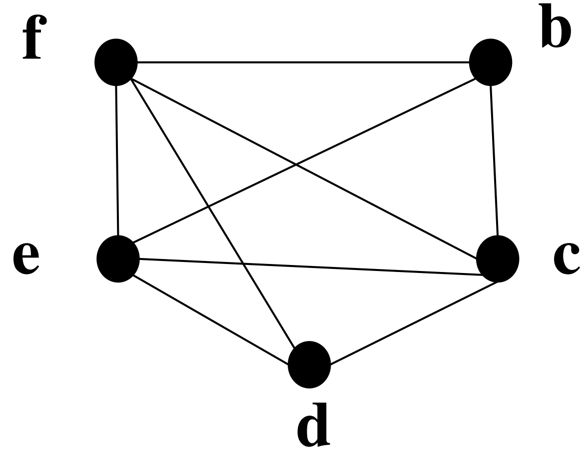


Example

- Assume k=4

Remove d

stack={a}

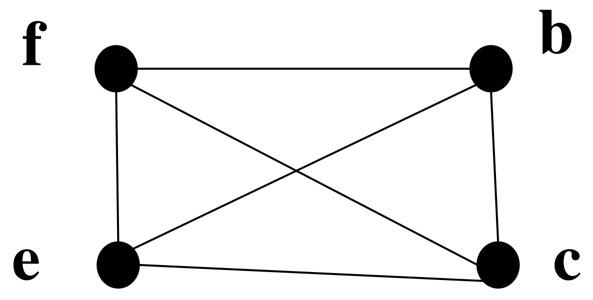


Example

- Assume k=4

  Note: All nodes now have fewer than 4 neighbors

  The graph coloring is guaranteed to succeed



  Remove c

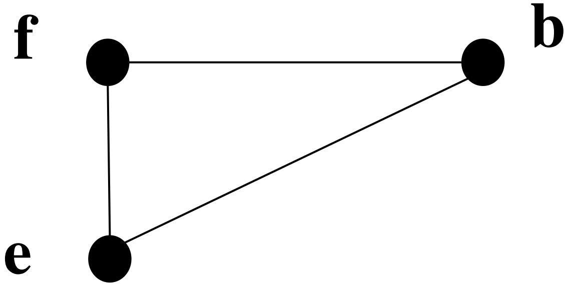
  stack={d,a}

Example

- Assume k=4

Remove b

stack={c,d,a}

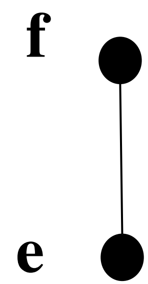


Example

- Assume k=4

Remove e

stack={b,c,d,a}



Example

- Assume k=4

Remove f

stack={e,b,c,d,a}

Example

- Assume k=4

  Empty graph – done with the first part

  Now we have the order for assigning colors to nodes, start coloring the nodes (from the top of the stack)

  stack={f,e,b,c,d,a}

Example

- Assume k=4

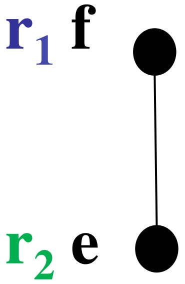
$\mathbf{r_1}$ **f**

stack={e,b,c,d,a}

Example

- Assume k=4
  e must be in a different register from f

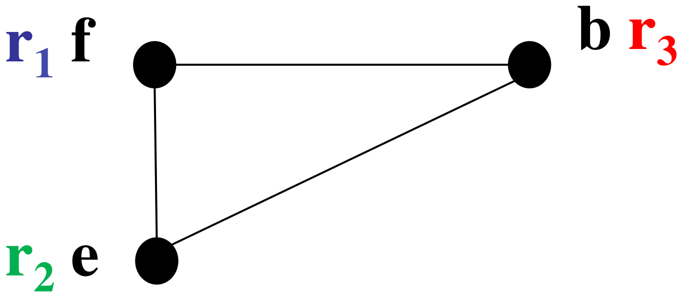
  stack={b,c,d,a}



Example

- Assume k=4

stack={c,d,a}

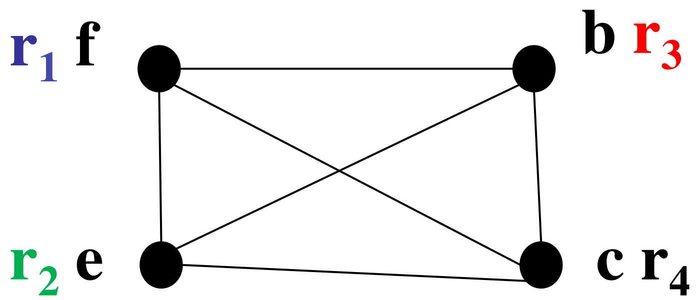


Example

- Assume k=4

  The ordering insures we can find a color for all nodes

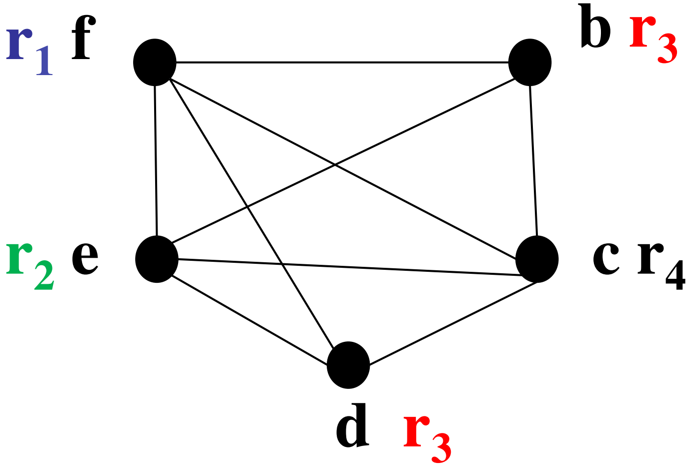
  stack={d,a}



Example

- Assume k=4
  - d can be in the same register as b

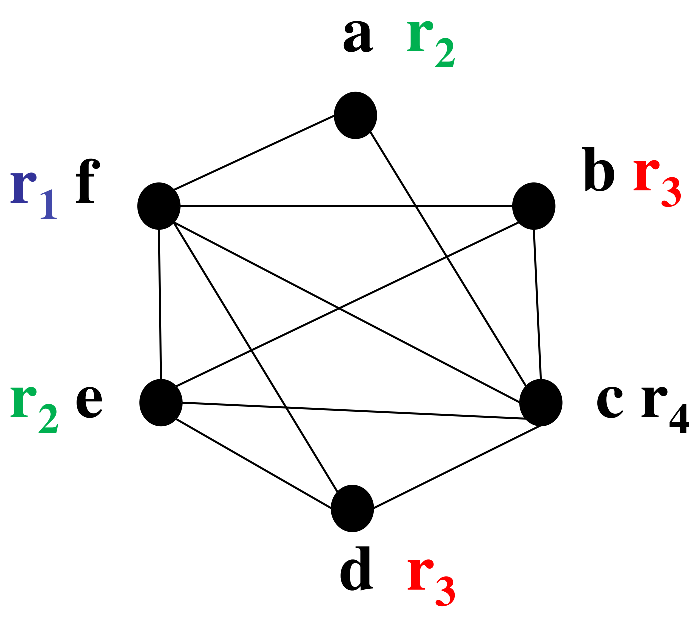
  stack={a}



Example

- Assume k=4

stack={}



Register Allocation as Graph Coloring

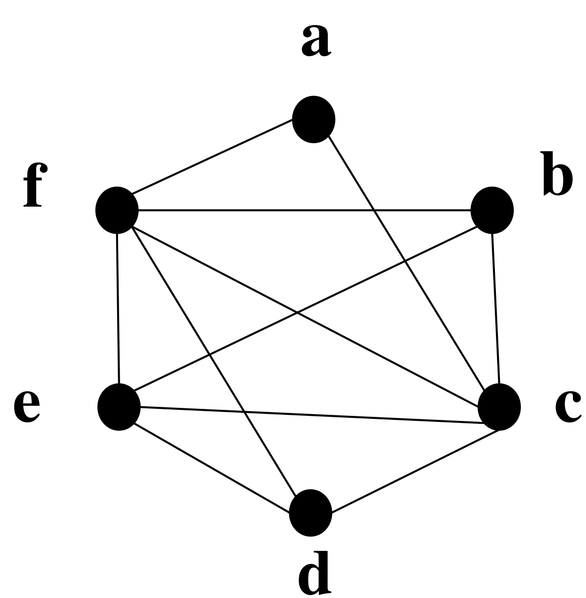
- What happens if the graph coloring heuristic fails to find a coloring?

- In this case we cannot hold all values in the registers
  - Some values should be *spilled* to memory

K-coloring fails

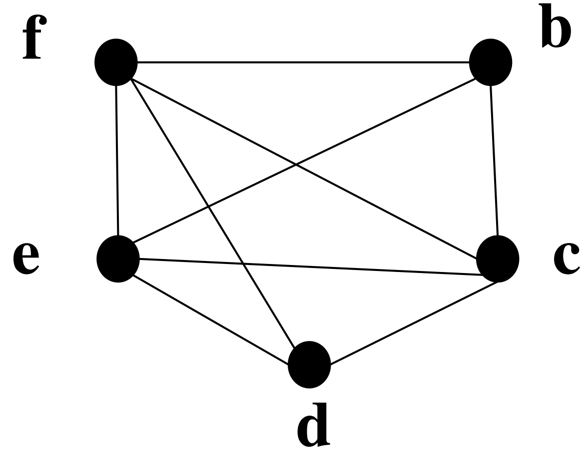
- What if all nodes have k or more neighbors?
- Try to find a 3 coloring of this graph

Remove a



Example of 3-coloring

- There is no node such that if we remove it then 3-coloring for
  the graph is available



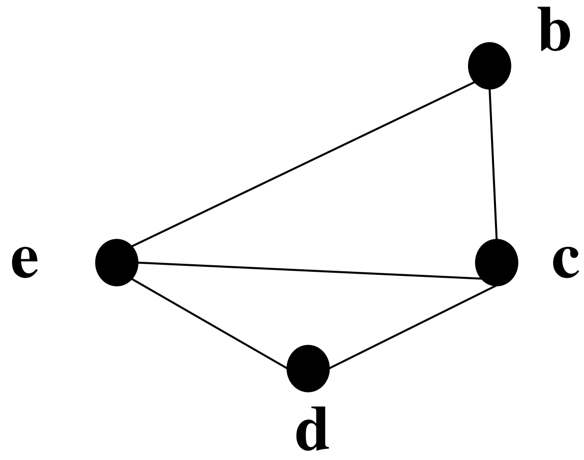
Optimistic Coloring

- If every node in G has more than *k* neighbors, *k*-coloring of G might not be possible

- Pick a node as candidate for spilling, remove it from the graph and continue *k*-coloring

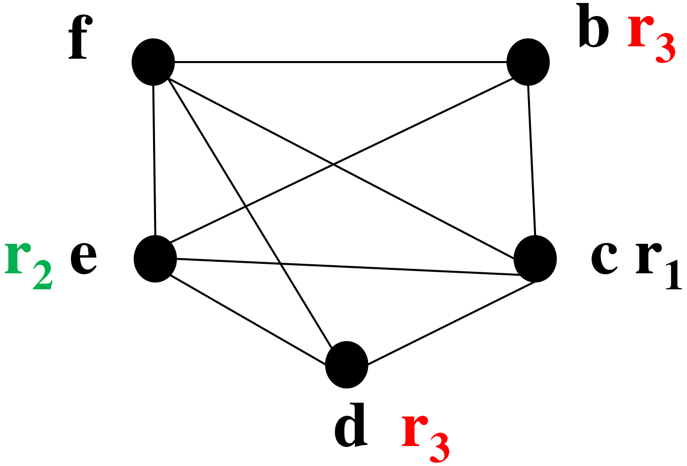
Optimistic Coloring

- Remove f and continue:
  - The ordering: {c,e,d,b,f,a}



Optimistic Coloring

- Color the nodes {c,e,d,b,f,a}
- Try to assign a color to f
- We hope that among 4 neighbors of f we use less than 3 colors (*optimistic coloring*)



Spilling

- If optimistic coloring fails, we spill f
  - Allocate a memory location for f
    - Typically in the current stack frame
    - Call this address fa
- Before each operation that reads f, insert

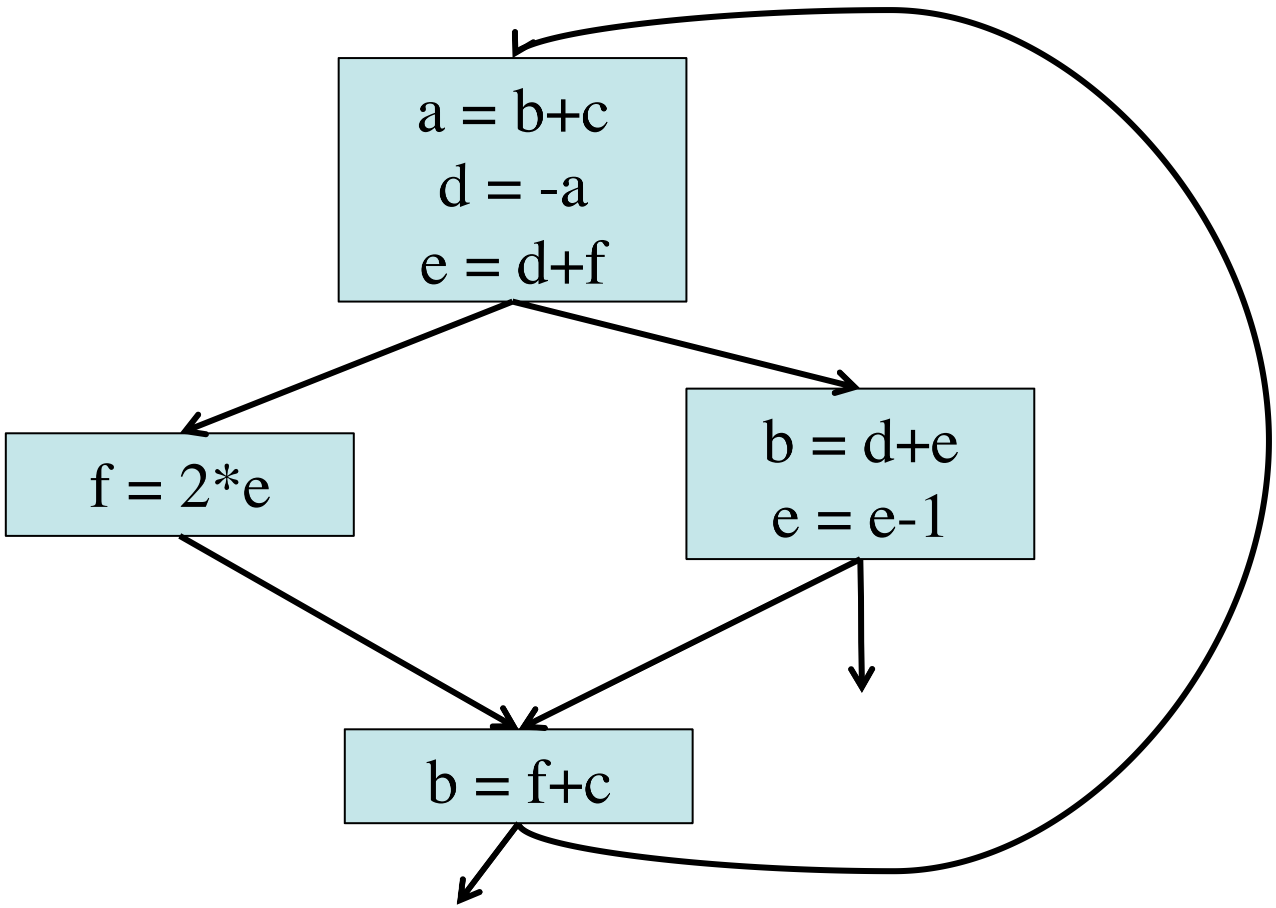
  f = load fa

- After each operation that writes f, insert

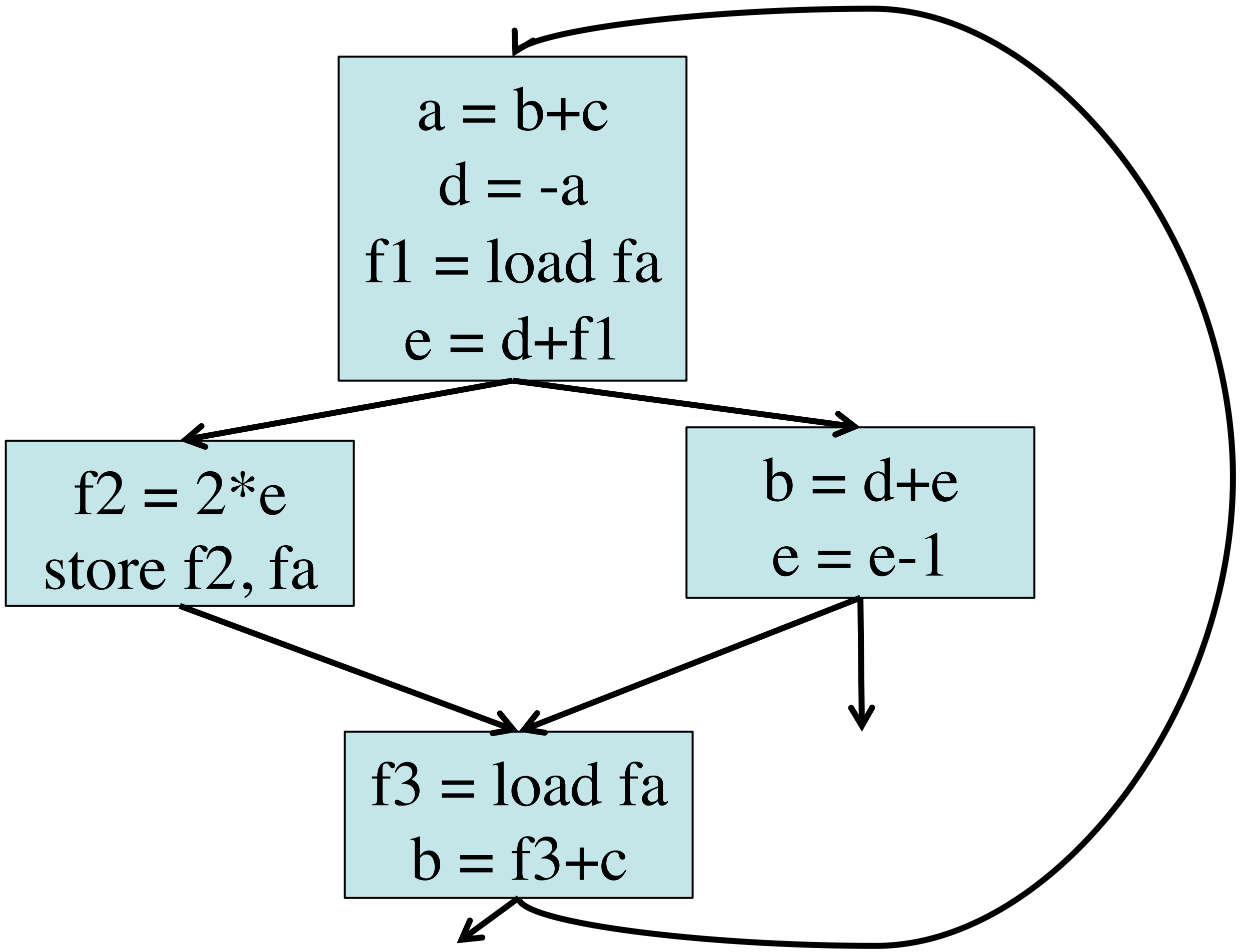
  store f, fa

- Spilling is slow but sometimes necessary.

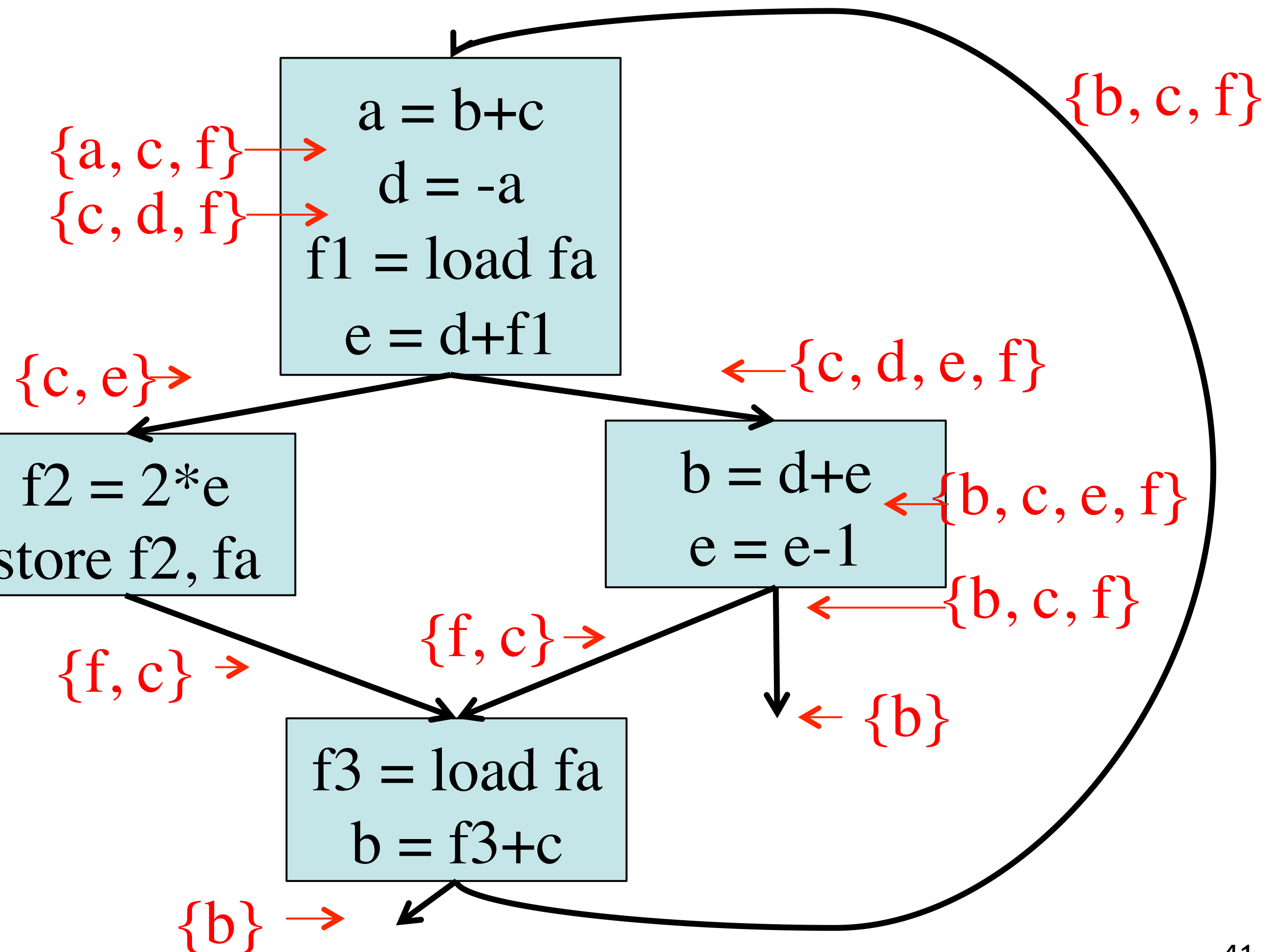
Original Code



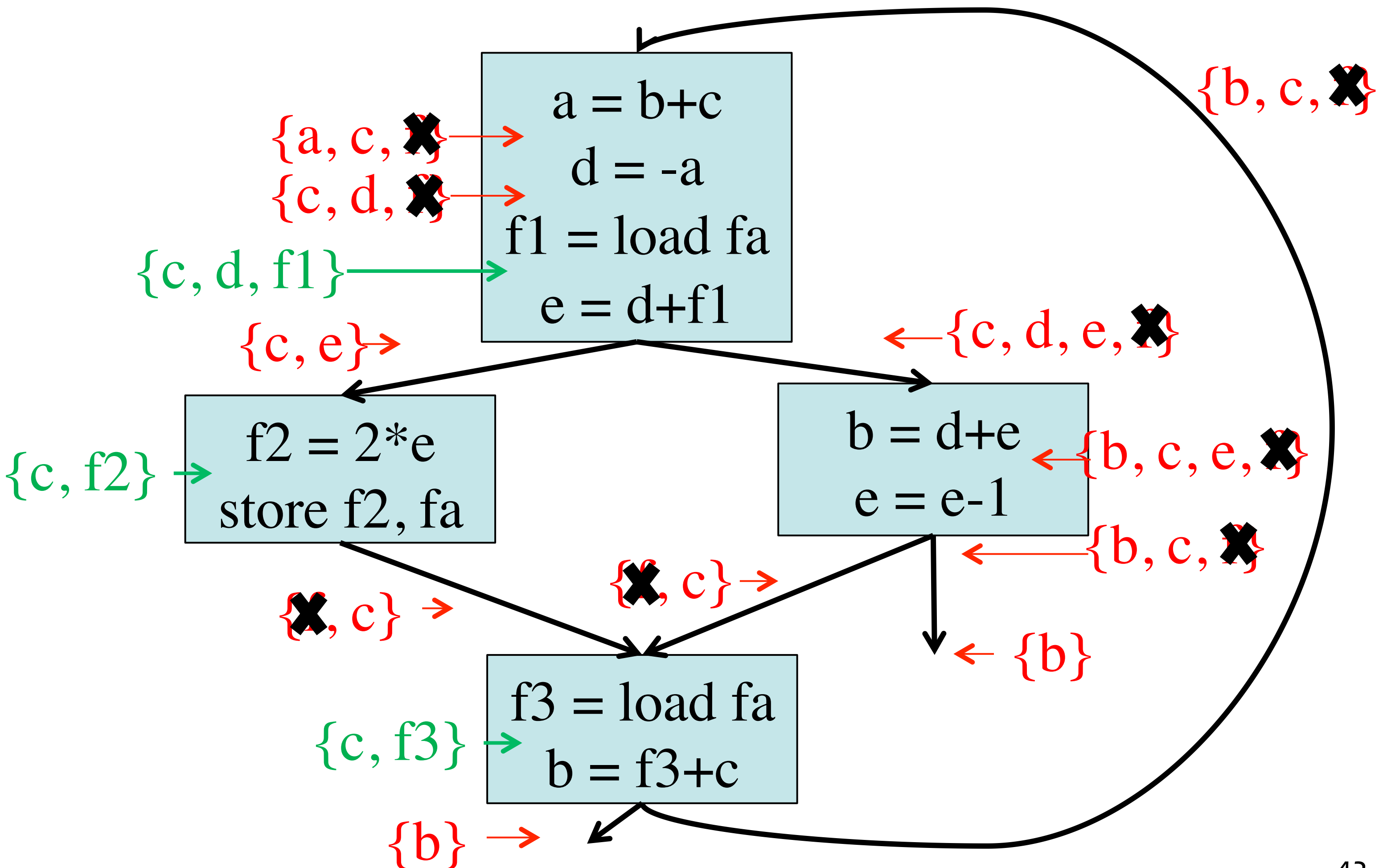
Code after Spilling f



Recompute the Liveness



Recompute the Liveness

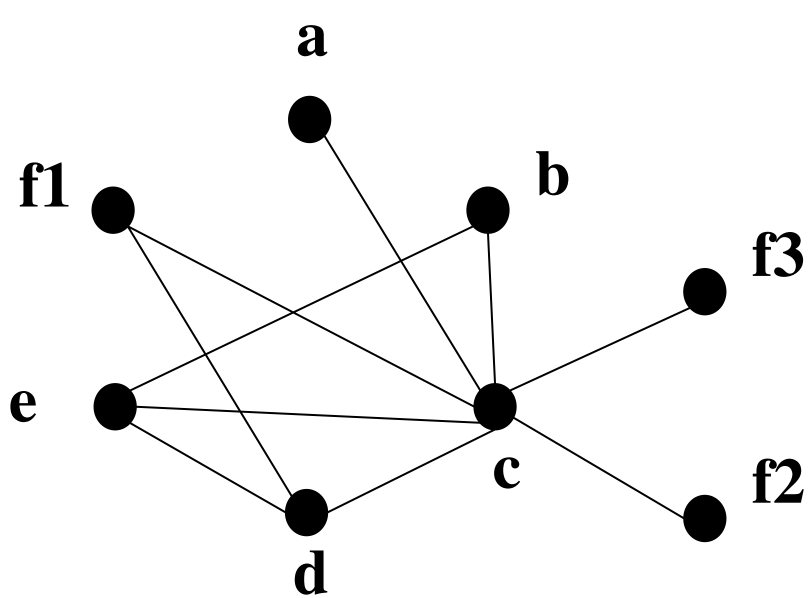


Rebuild the Interference Graph

- New liveness information is almost as before
  - Note f has been split into three temporaries
- fi is live only
  - Between a fi = load fa and the next instruction
  - Between a store fi, fa and the preceding instr.
- Spilling reduces the live range of f
  - And thus reduces its interferences
  - Which results in fewer RIG neighbors

Rebuild the Interference Graph

- Some edges of the spilled nodes are removed
- In our case f still interferes only with c and d
- And the new RIG is 3-colorable



Spilling

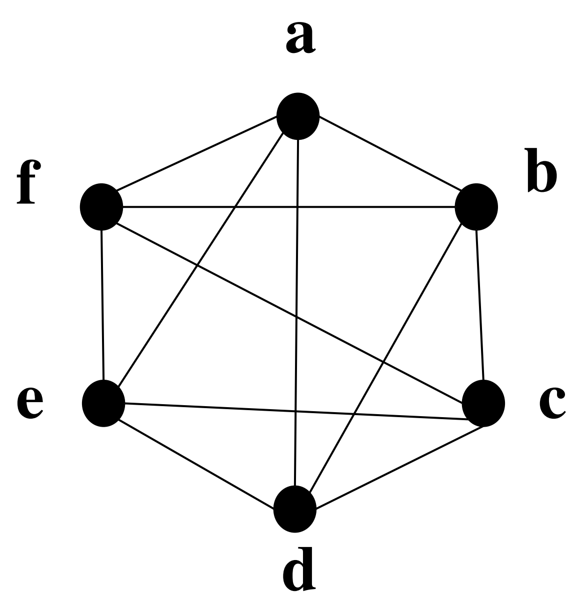
- Additional spilling might be required before a coloring is found

Example

K=3

remove a

Stack: {}

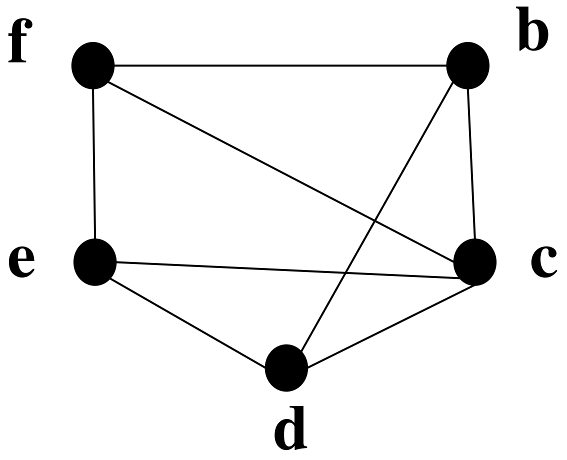


Example

K=3

remove c

Stack: {a}

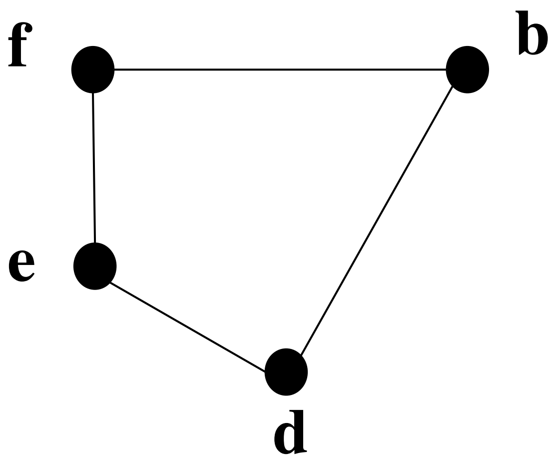


Example

K=3

remove b

Stack: {c,a}

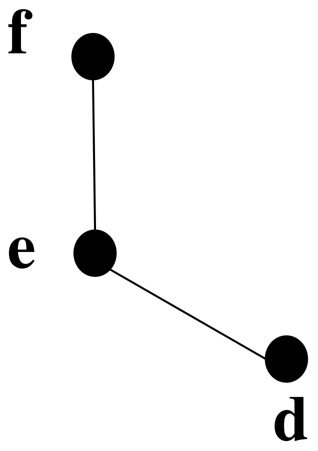


Example

K=3

remove e

Stack: {b,c,a}

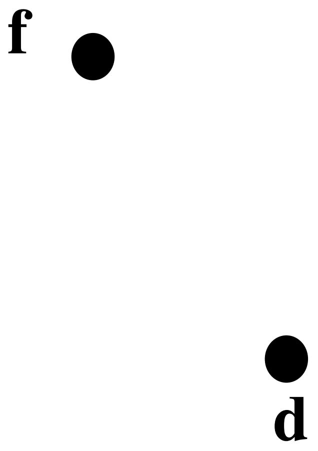


Example

K=3

remove f

Stack: {e,b,c,a}

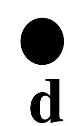


Example

K=3

remove d

Stack: {f,e,b,c,a}



Example

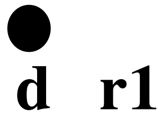
K=3

Stack: {d,f,e,b,c,a}

Example

K=3

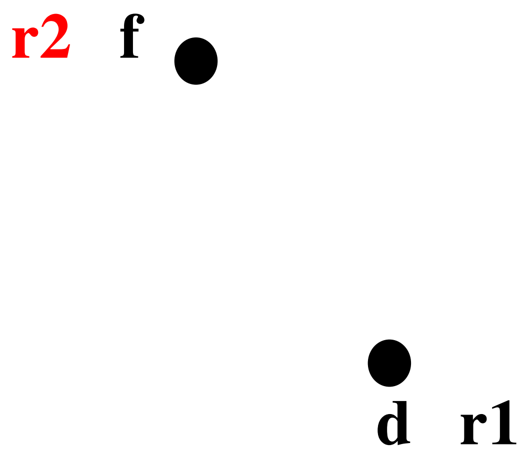
Stack: {f,e,b,c,a}



Example

K=3

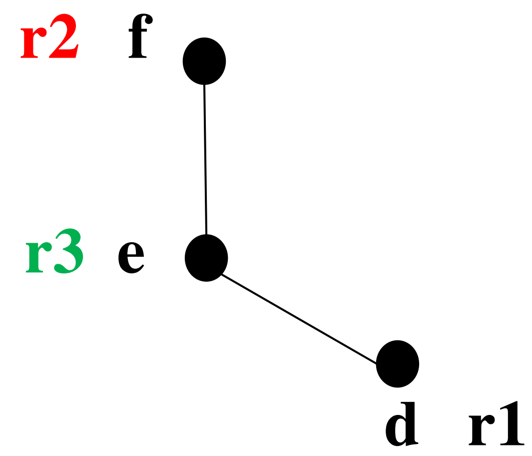
Stack: {e,b,c,a}



Example

K=3

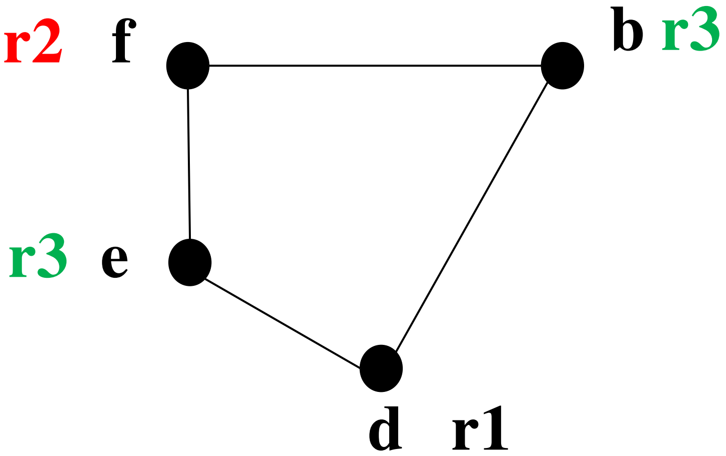
Stack: {b,c,a}



Example

K=3

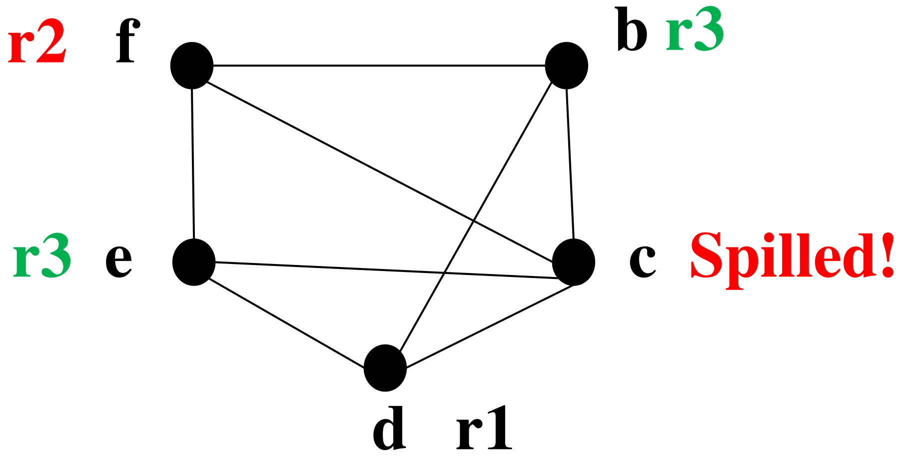
Stack: {c,a}



Example

K=3

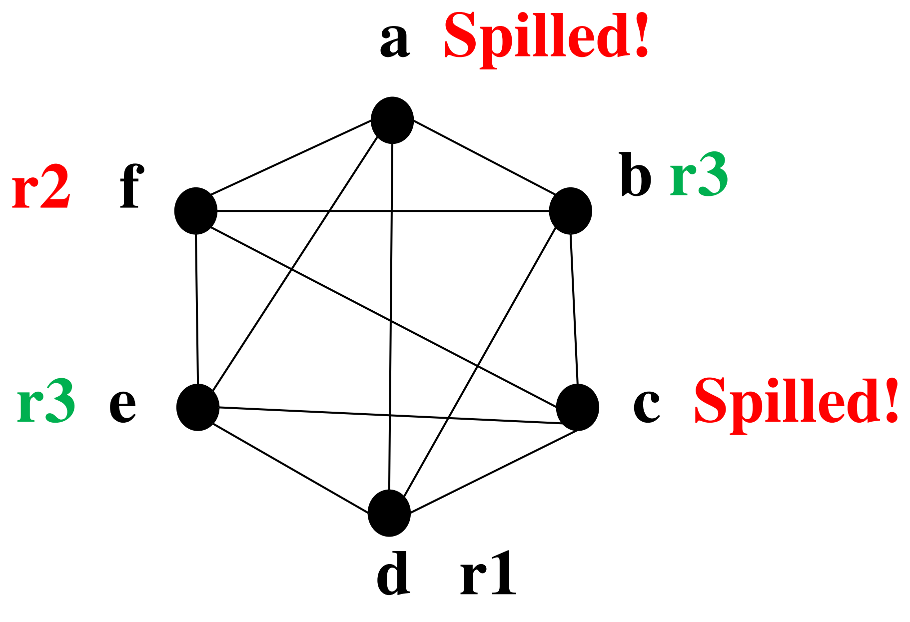
Stack: {a}



Example

K=3

Stack: {}



Example

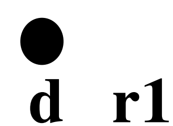
K=3

Stack: {d,f,e,b,c,a}

Example

K=3

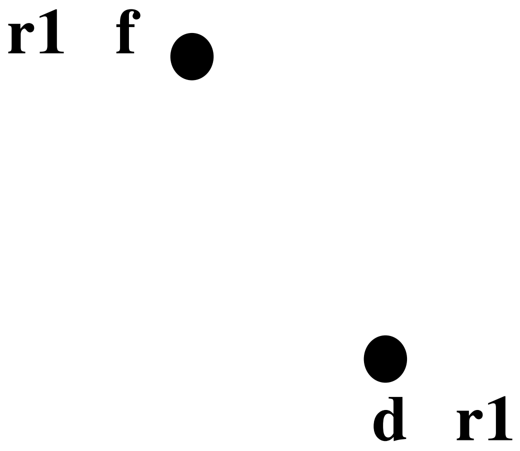
Stack: {f,e,b,c,a}



Example

K=3

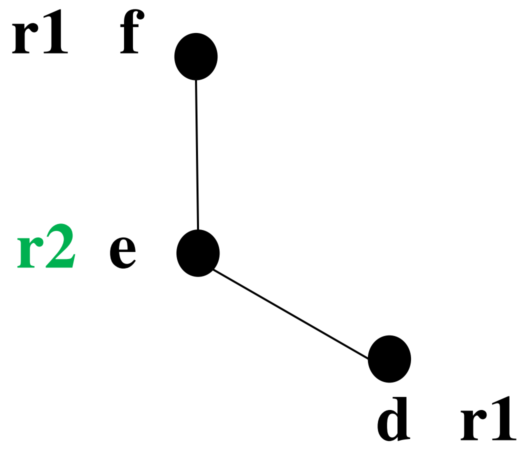
Stack: {e,b,c,a}



Example

K=3

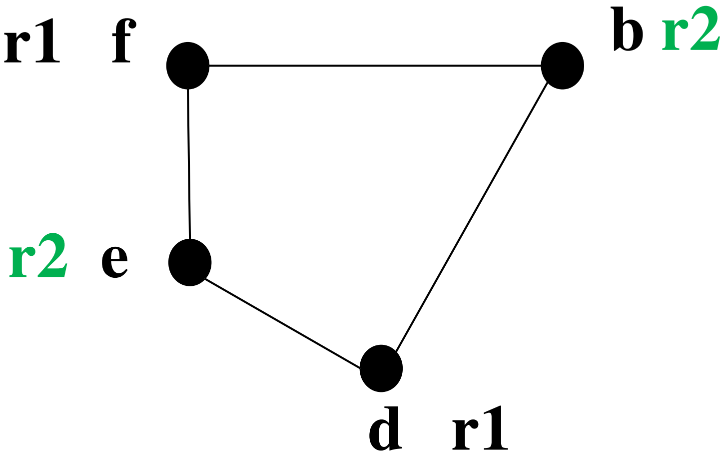
Stack: {b,c,a}



Example

K=3

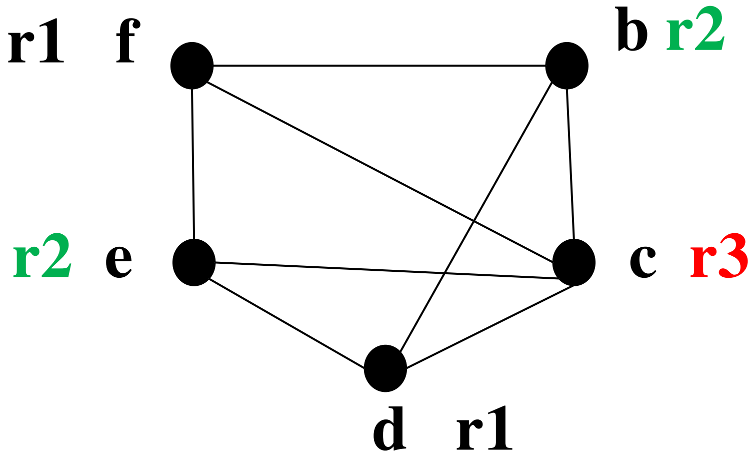
Stack: {c,a}



Example

K=3

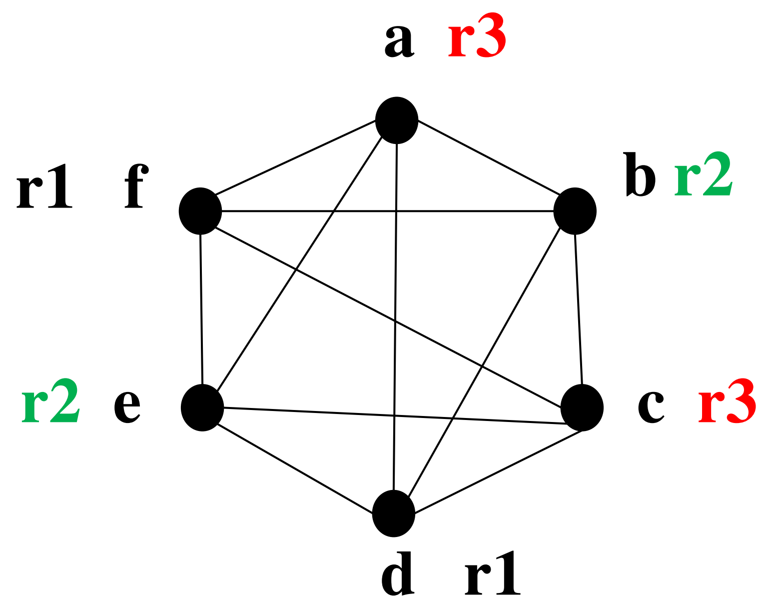
Stack: {a}



Example

K=3

Stack: {}



Spilling

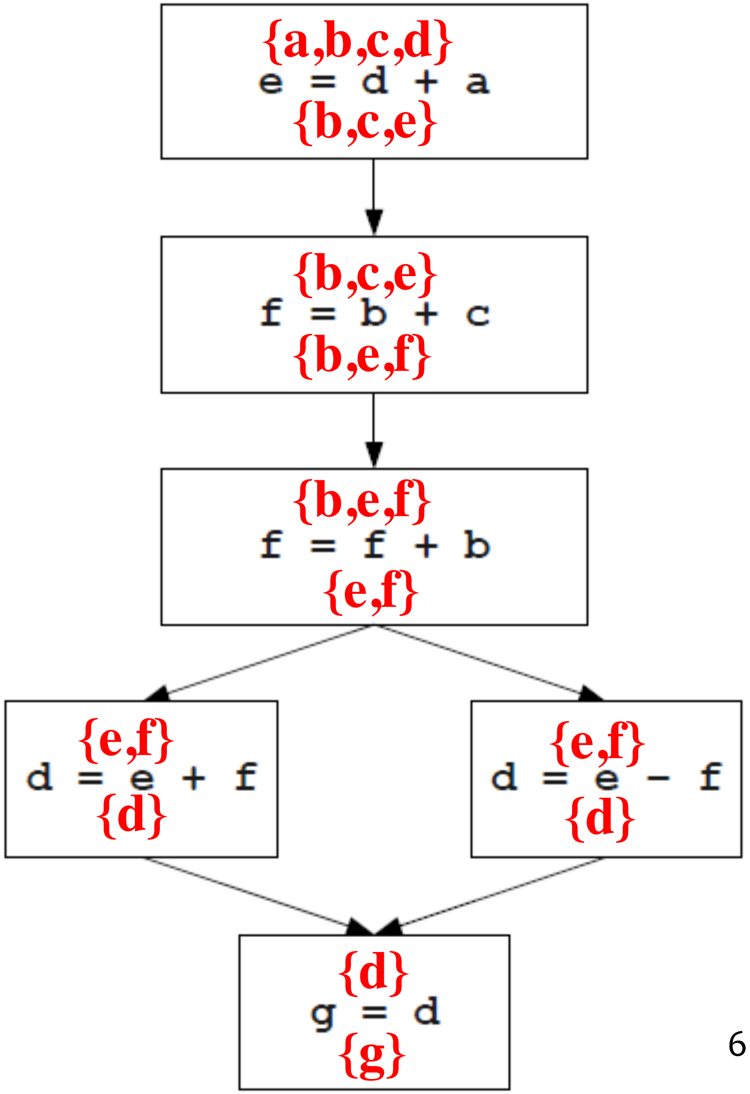
- Many different heuristics for picking a node to spill
  - Spill temporaries with most conflicts
  - Spill temporaries with few definitions and uses
  - Avoid spilling in inner loops (heavily visited regions of the code)
- C allows a *register* keyword to direct the compiler whether a variable contains a value that is heavily used.

Live Ranges and Live Intervals

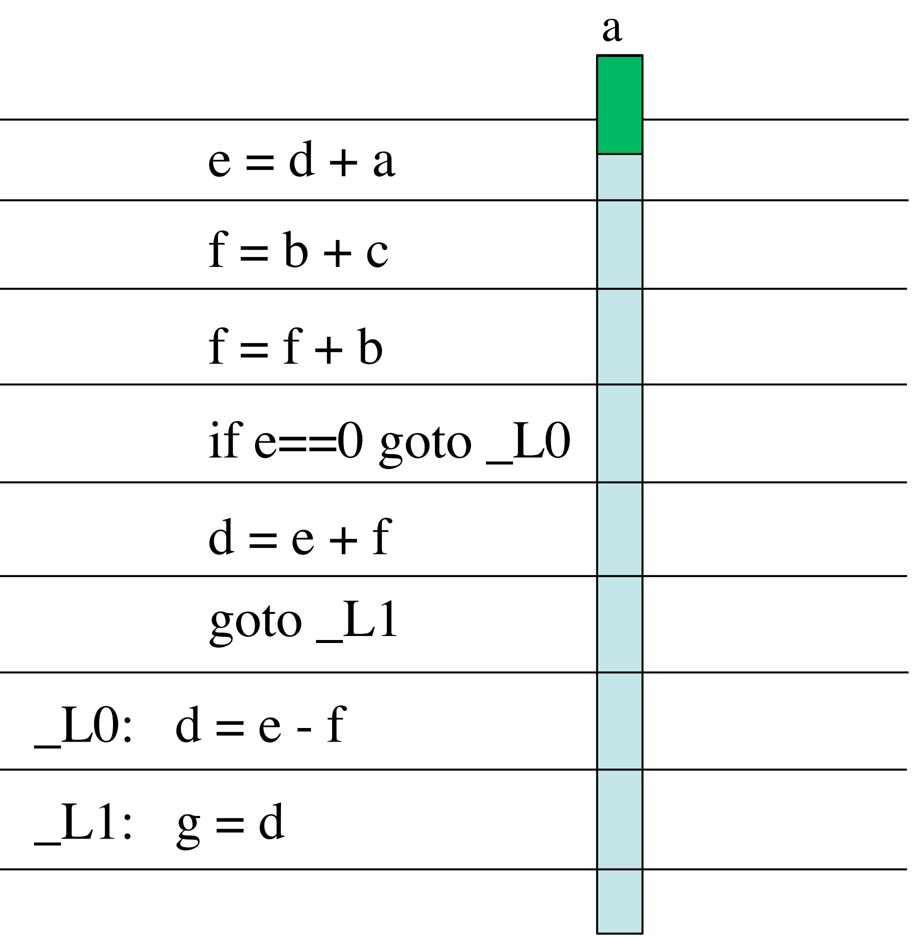
- The live range for a variable is the set of program points at which that variable is live.
- The live interval for a variable is the smallest subrange of the IR code containing all a variable's live ranges.
  - A property of the IR code, not CFG.
  - Less precise than live ranges, but simpler to work with

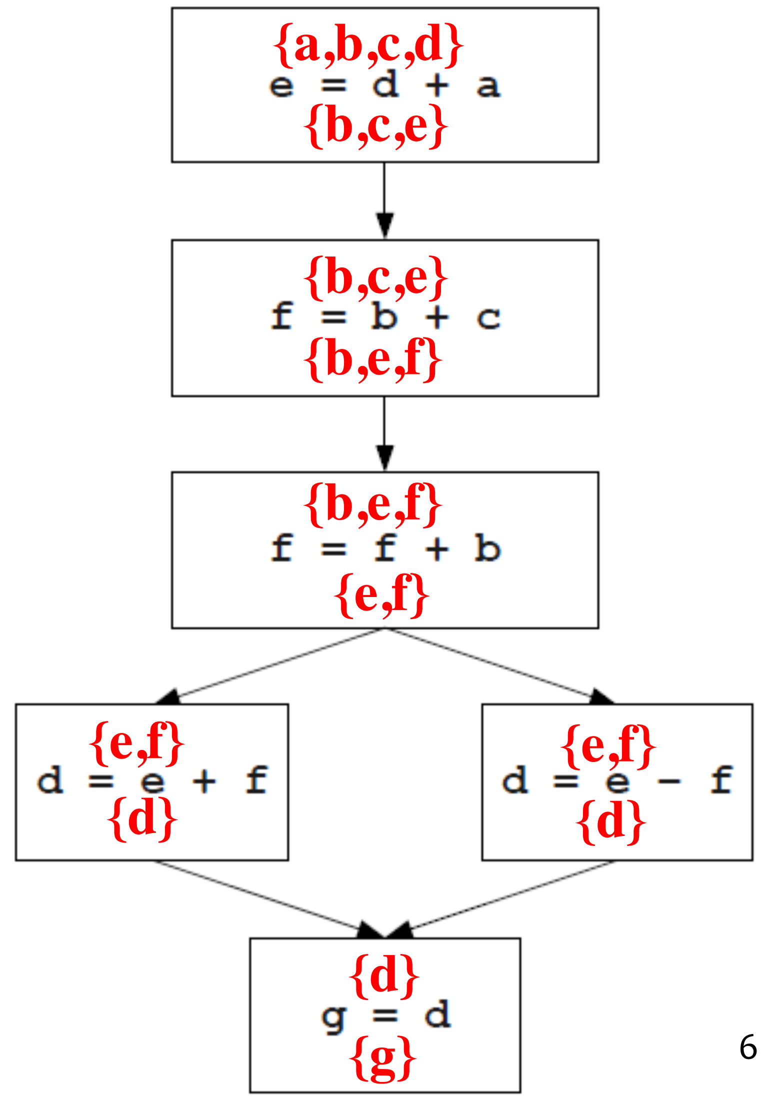
Live Intervals

```
        e = d + a
        f = b + c
        f = f + b
        if e==0 goto _L0
        d = e + f
        goto _L1
_L0:    d = e - f
_L1:    g = d
```

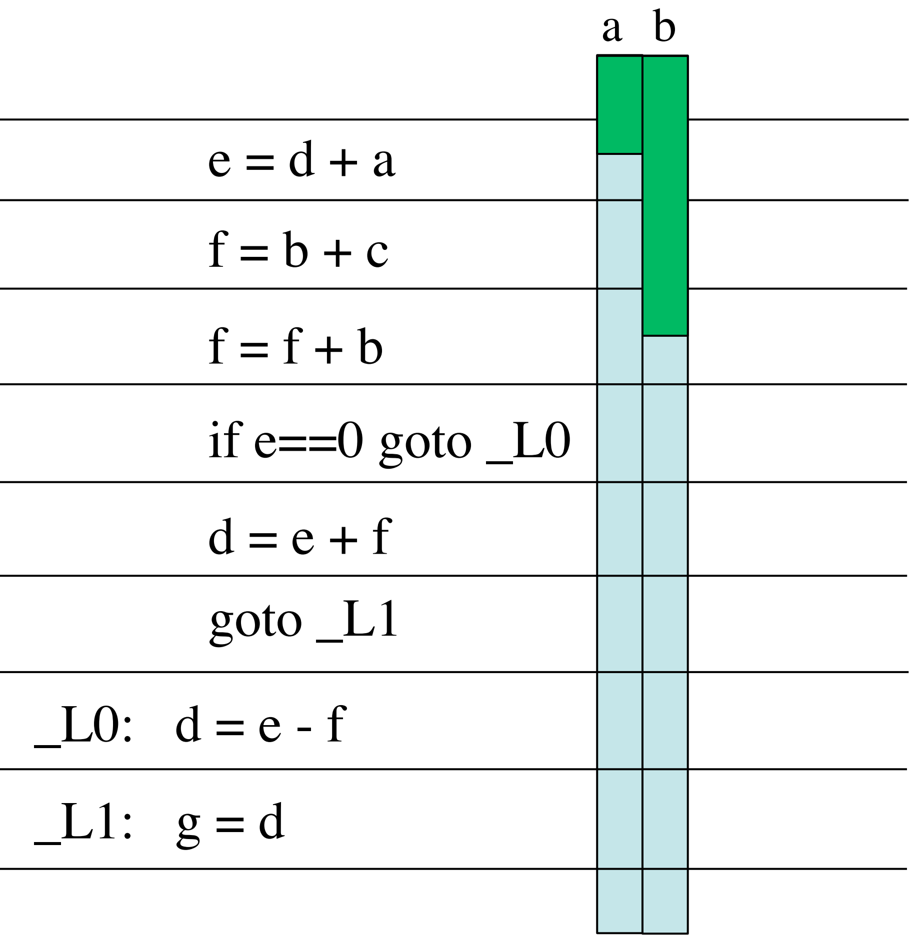


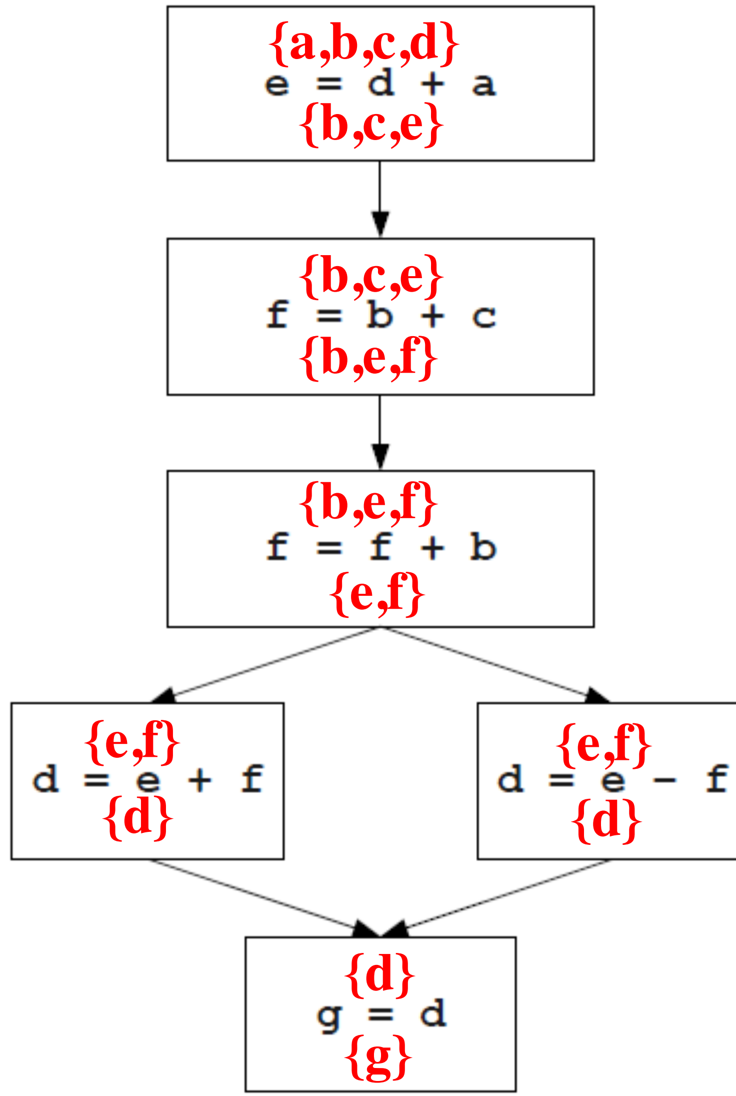
Live Intervals



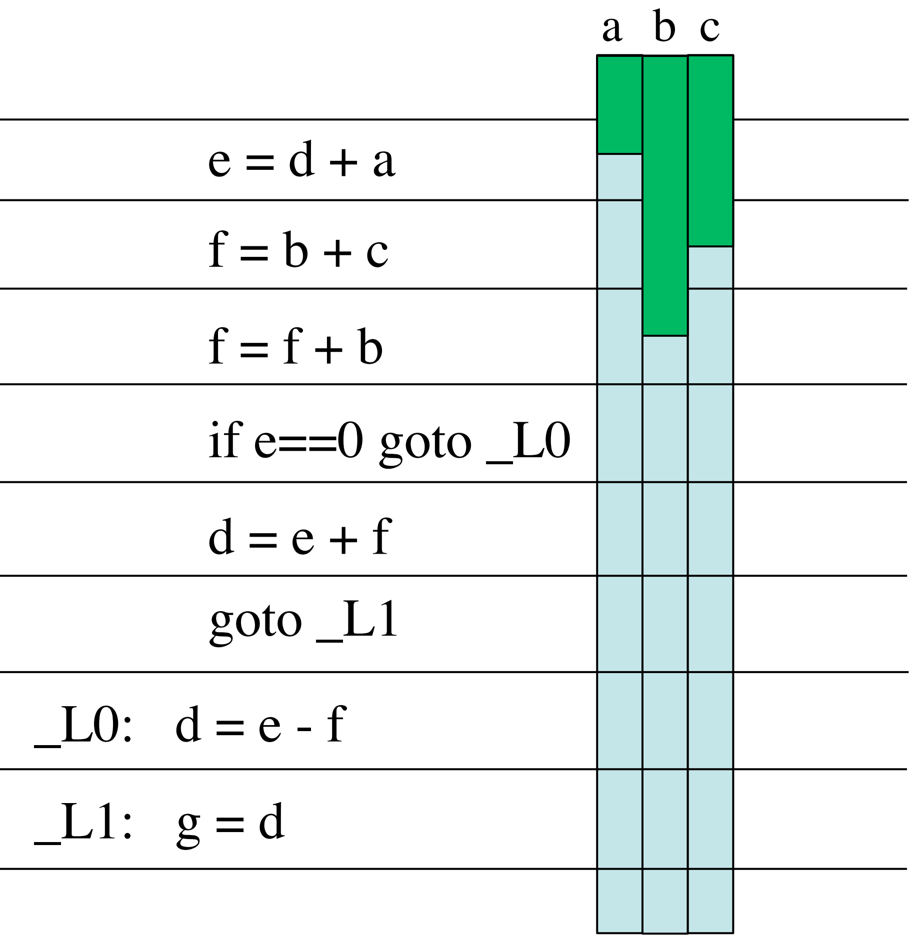


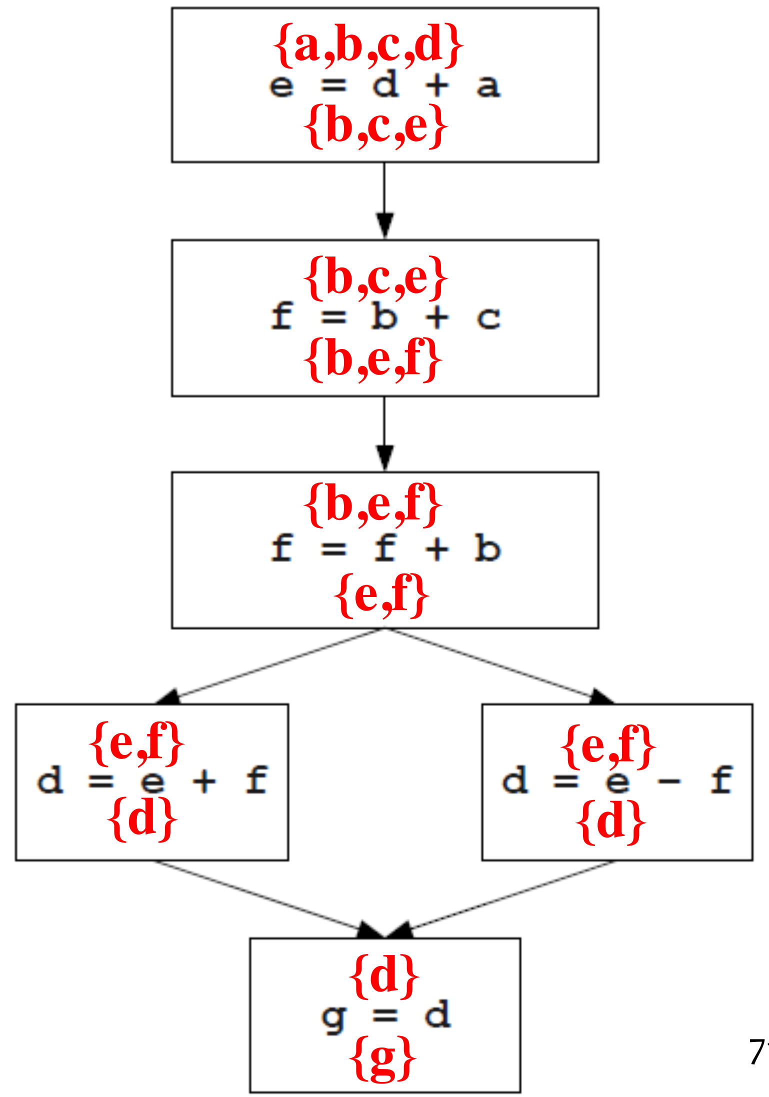
Live Intervals



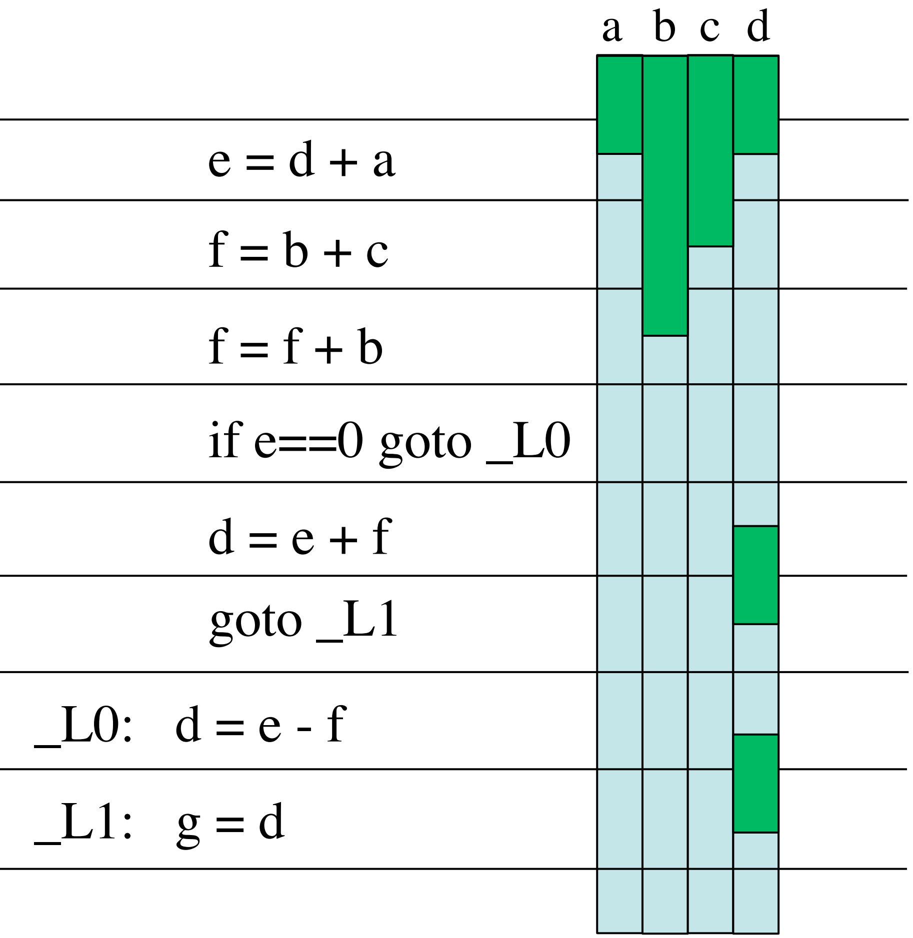


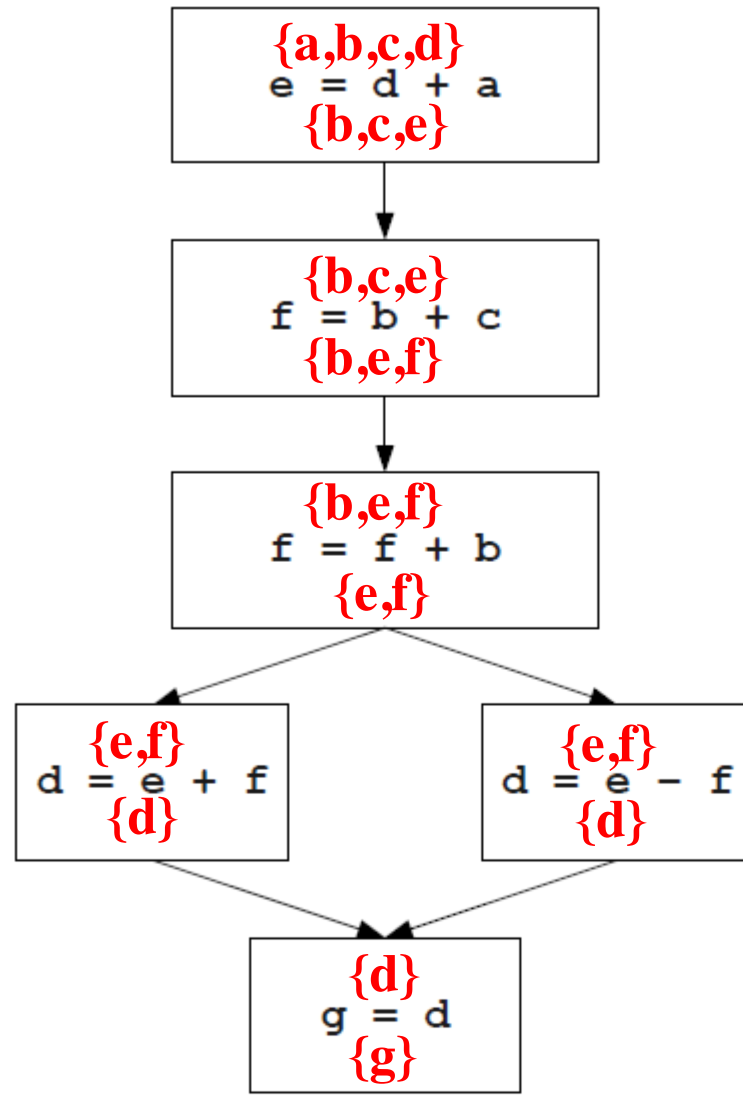
Live Intervals



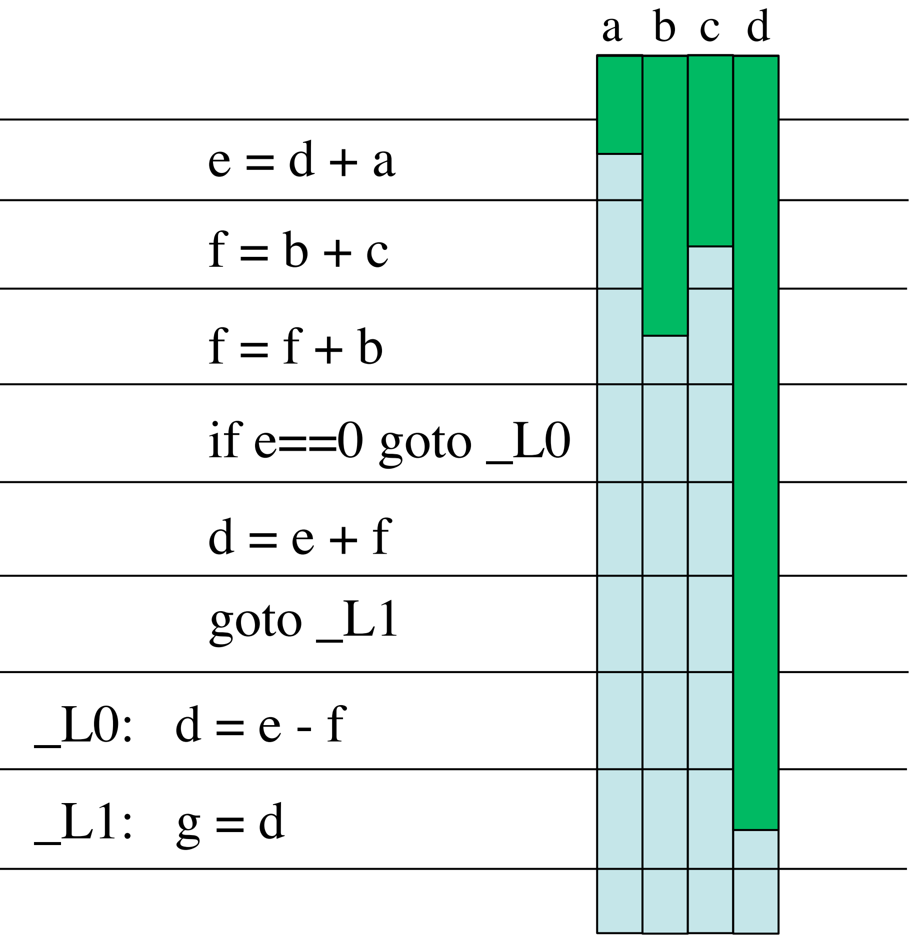


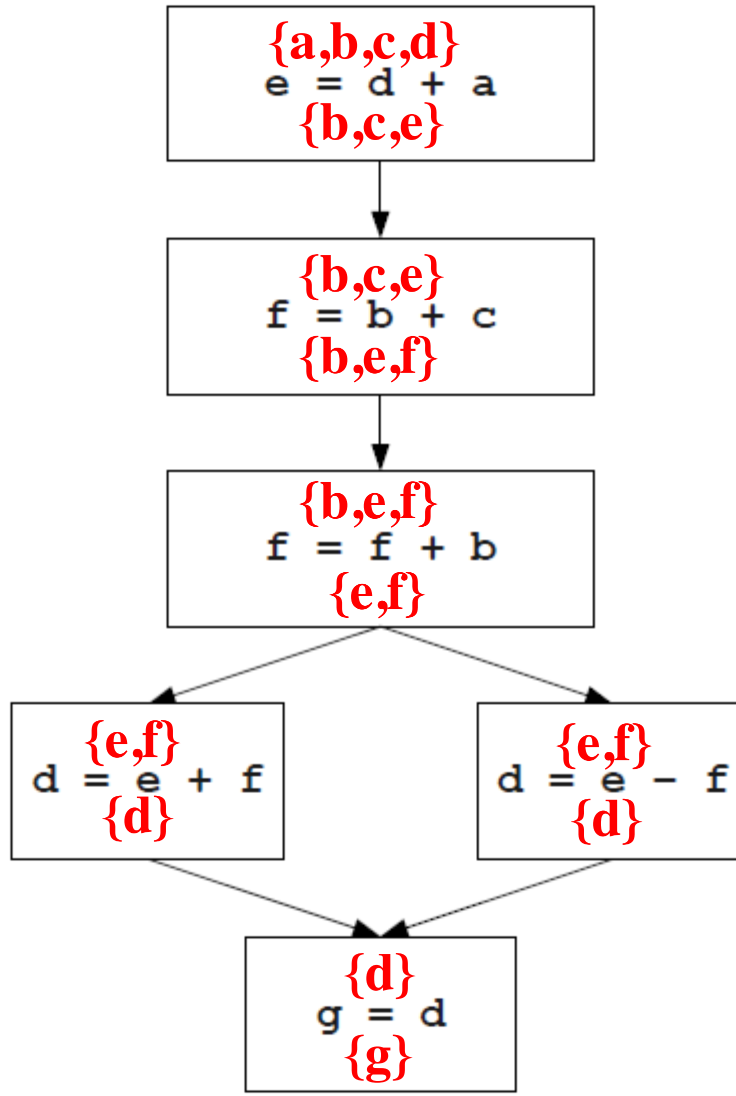
Live Intervals





Live Intervals





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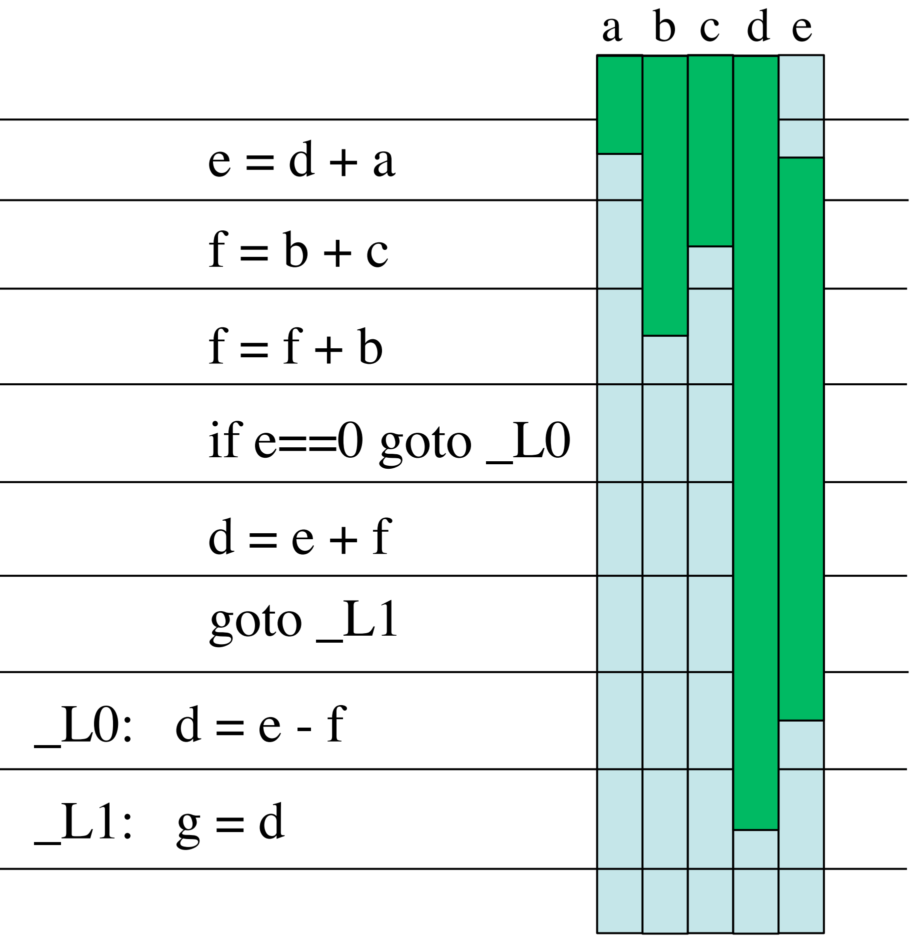
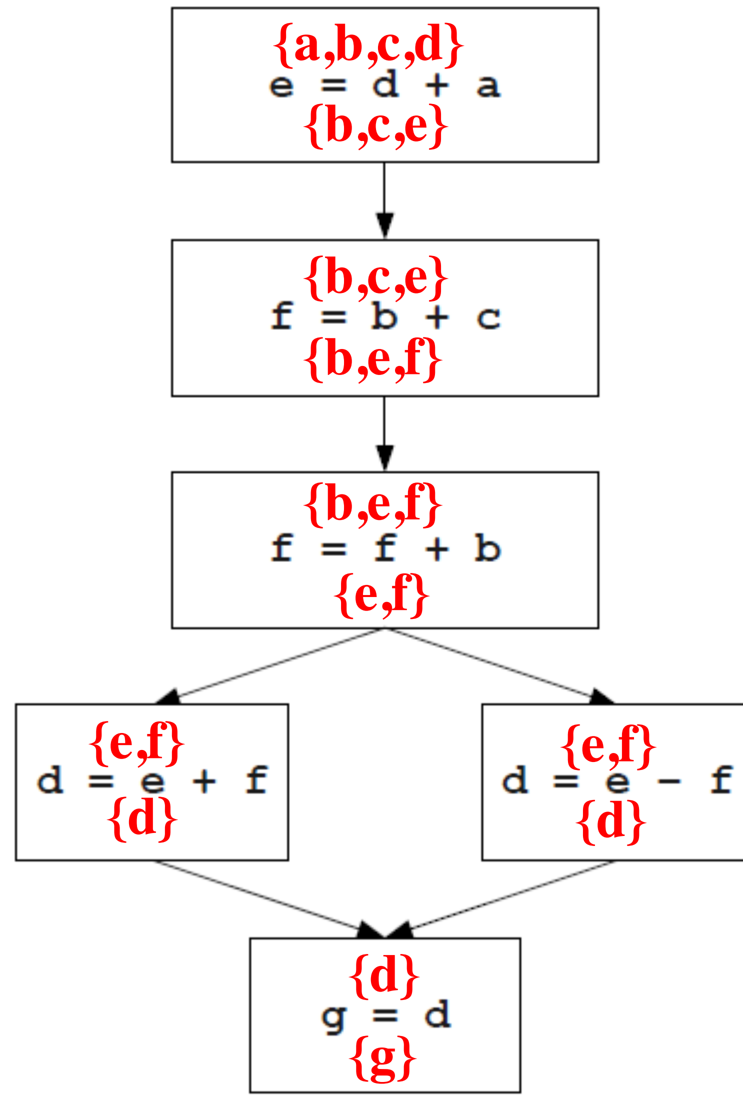
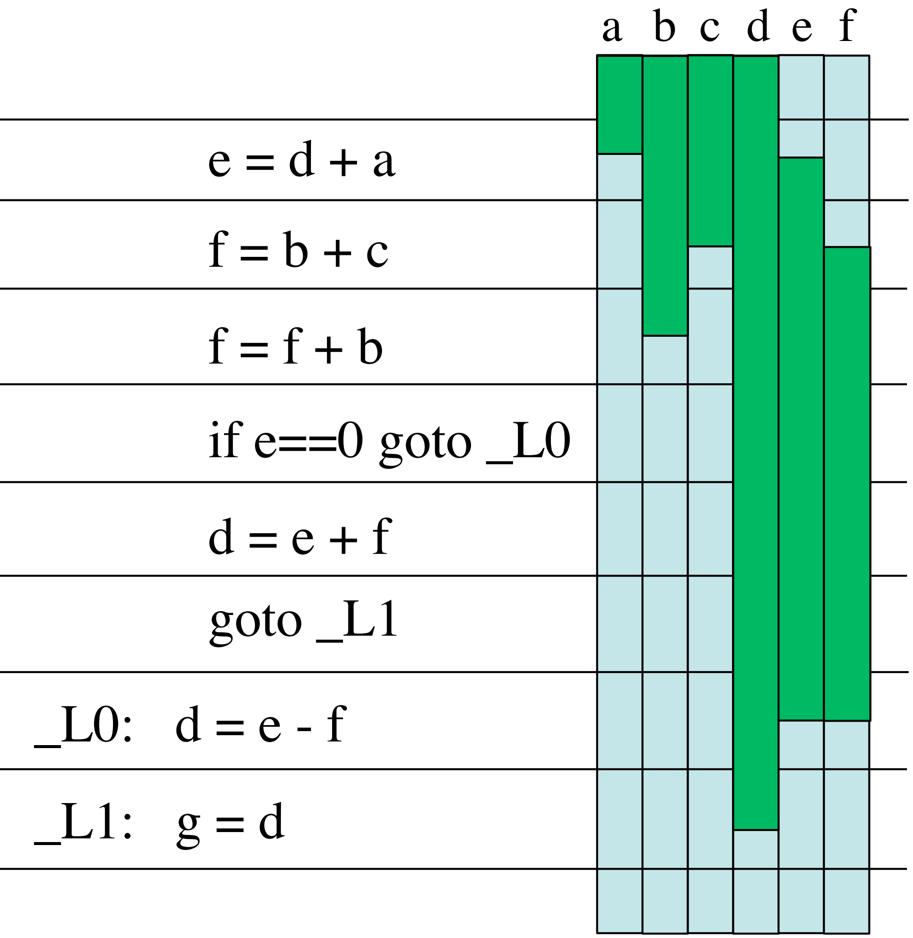
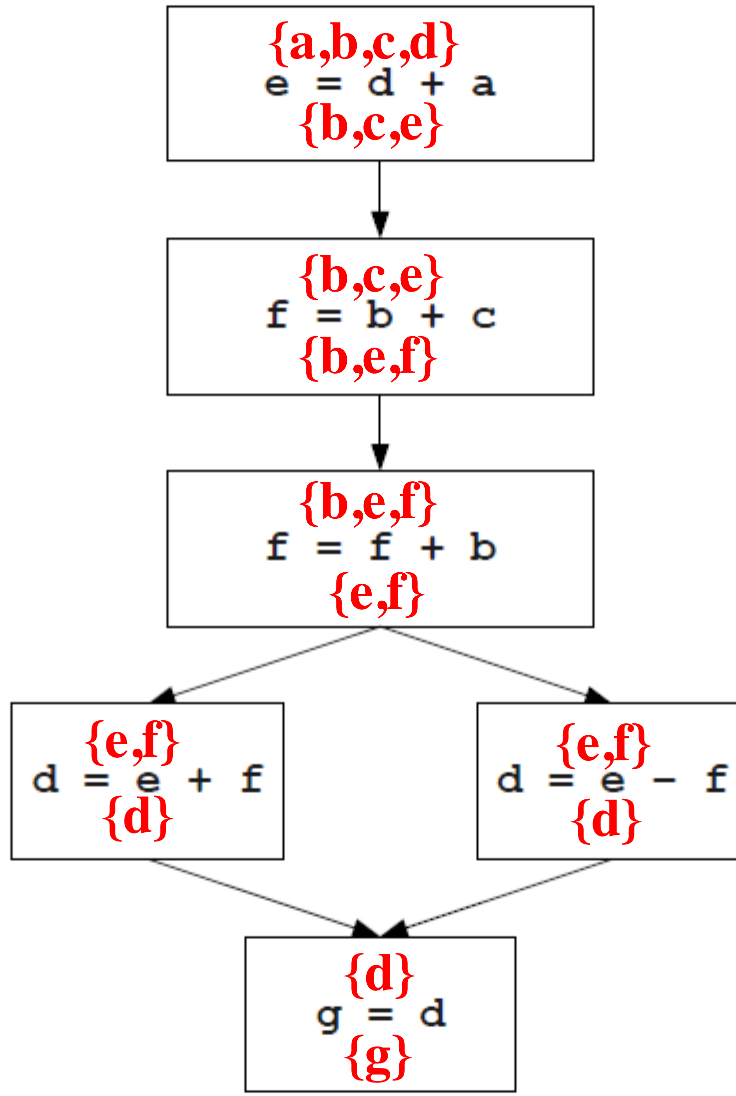




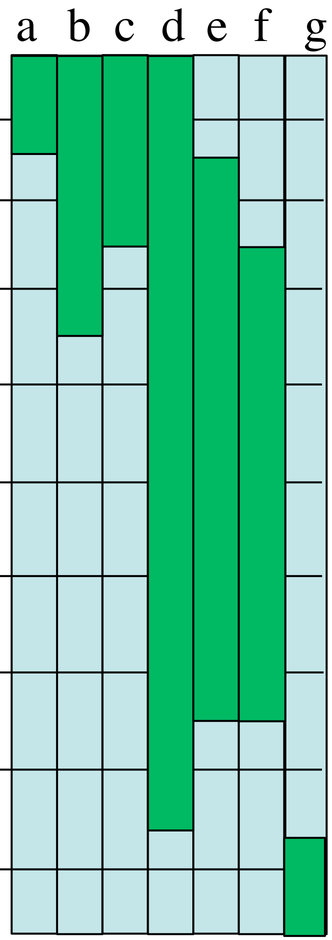
Live Intervals

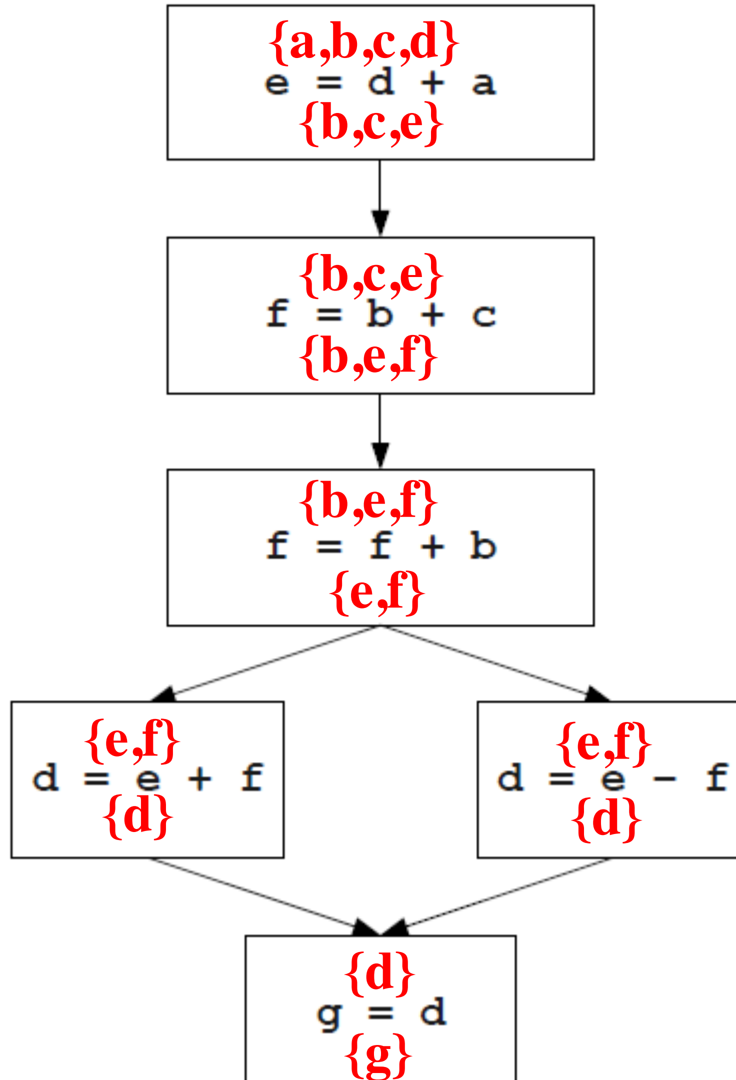




Live Intervals

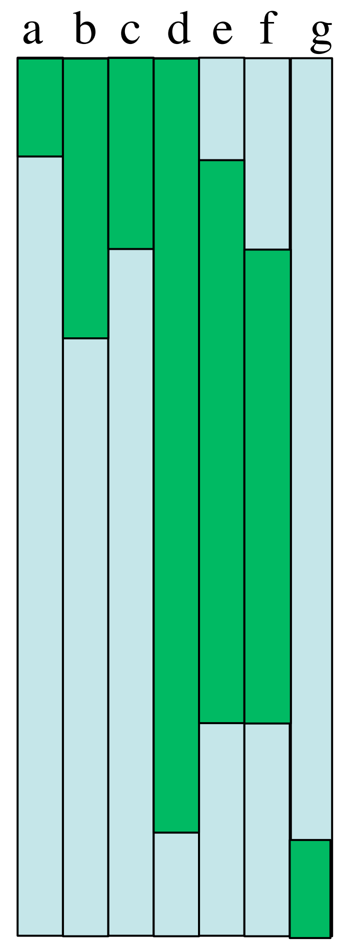
```
     e = d + a
     f = b + c
     f = f + b
     if e==0 goto _L0
     d = e + f
     goto _L1
_L0: d = e - f
_L1: g = d
```



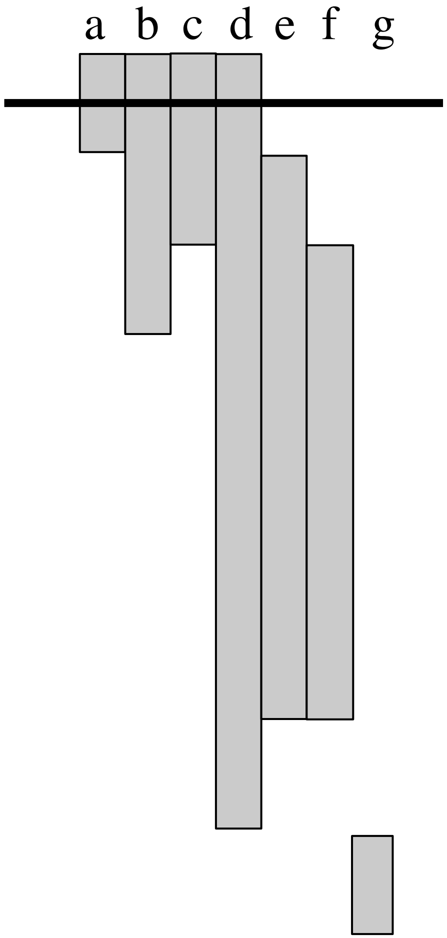


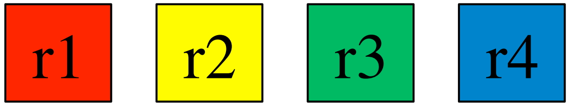
Register Allocation with Live Intervals

- Given the live intervals for all the variables in the program, we can allocate registers using a simple greedy algorithm.
- Idea: Track which registers are free at each point.
- When a live interval begins, give that variable a free register.
- When a live interval ends, the register is once again free.



Register Allocation with Live Intervals





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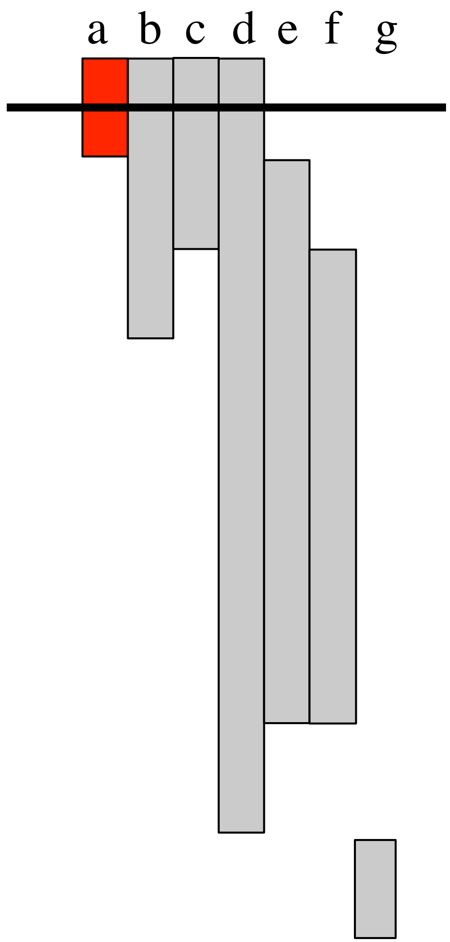

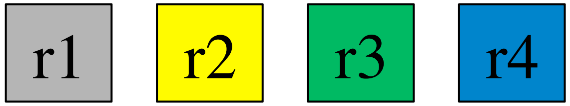

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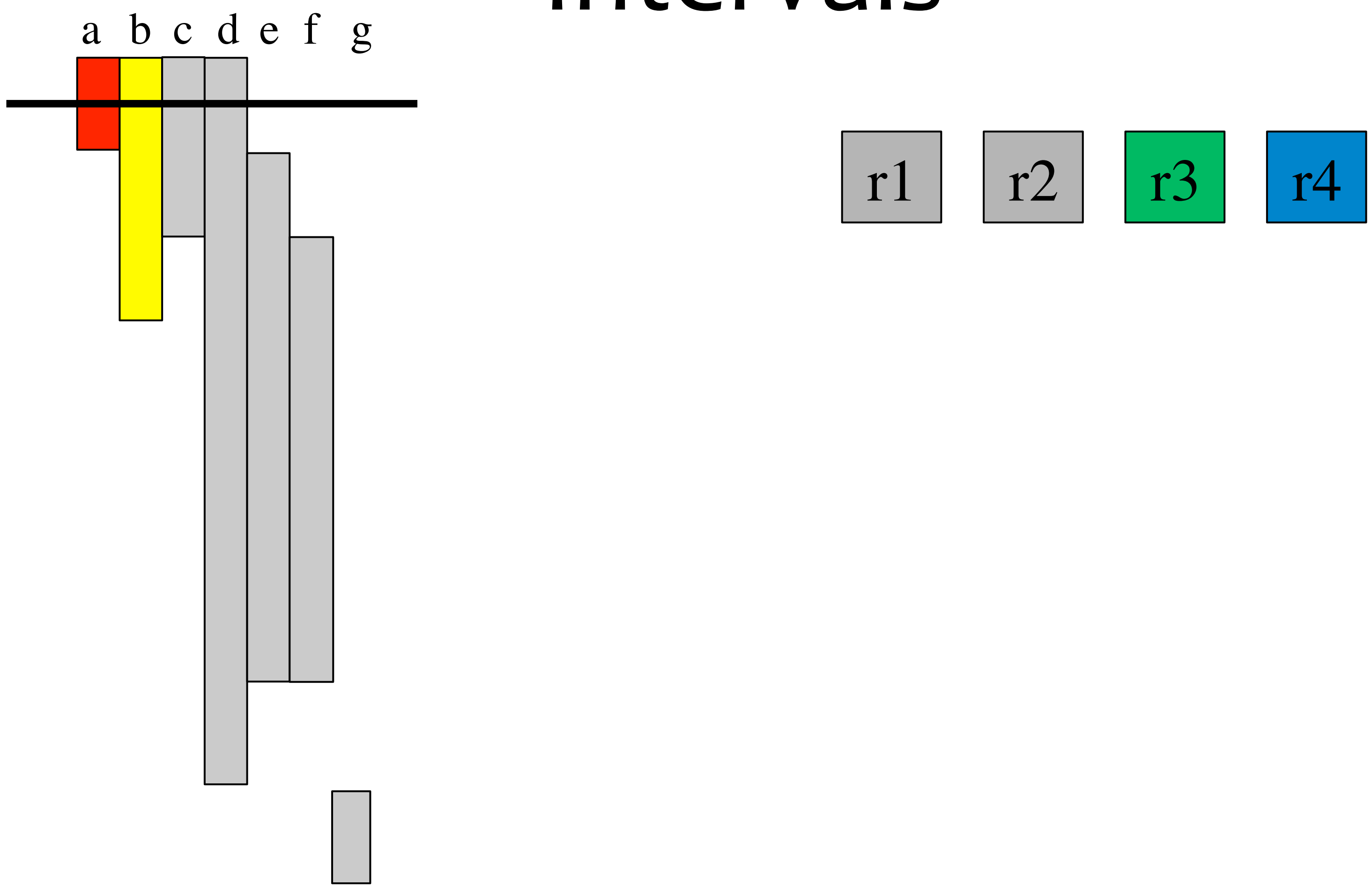

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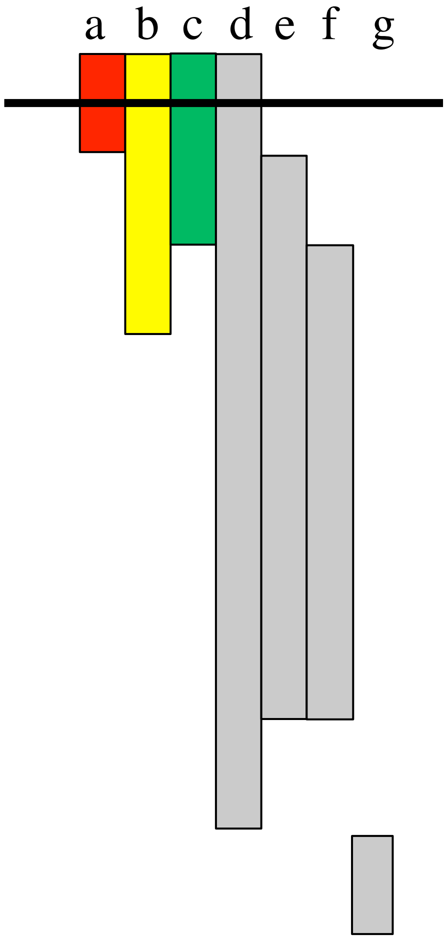

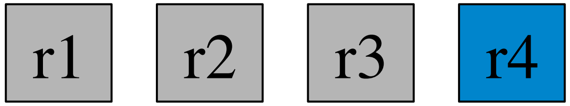

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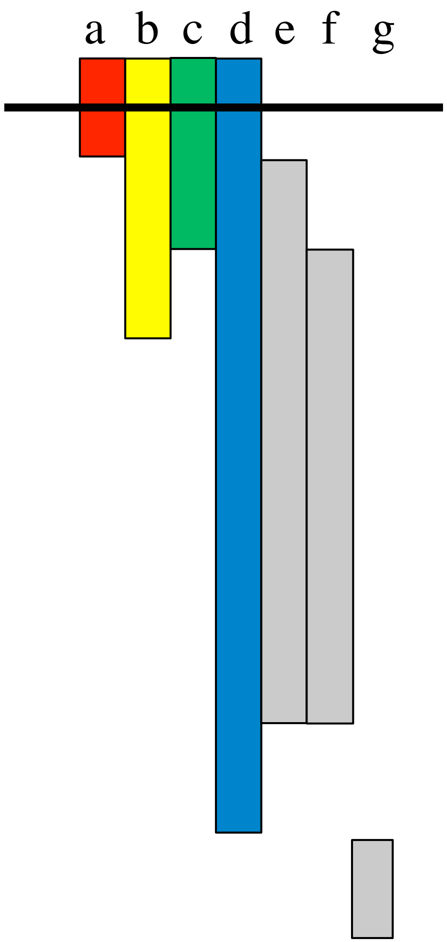

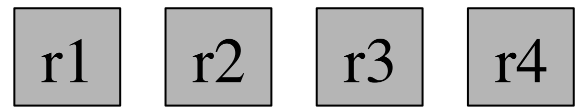

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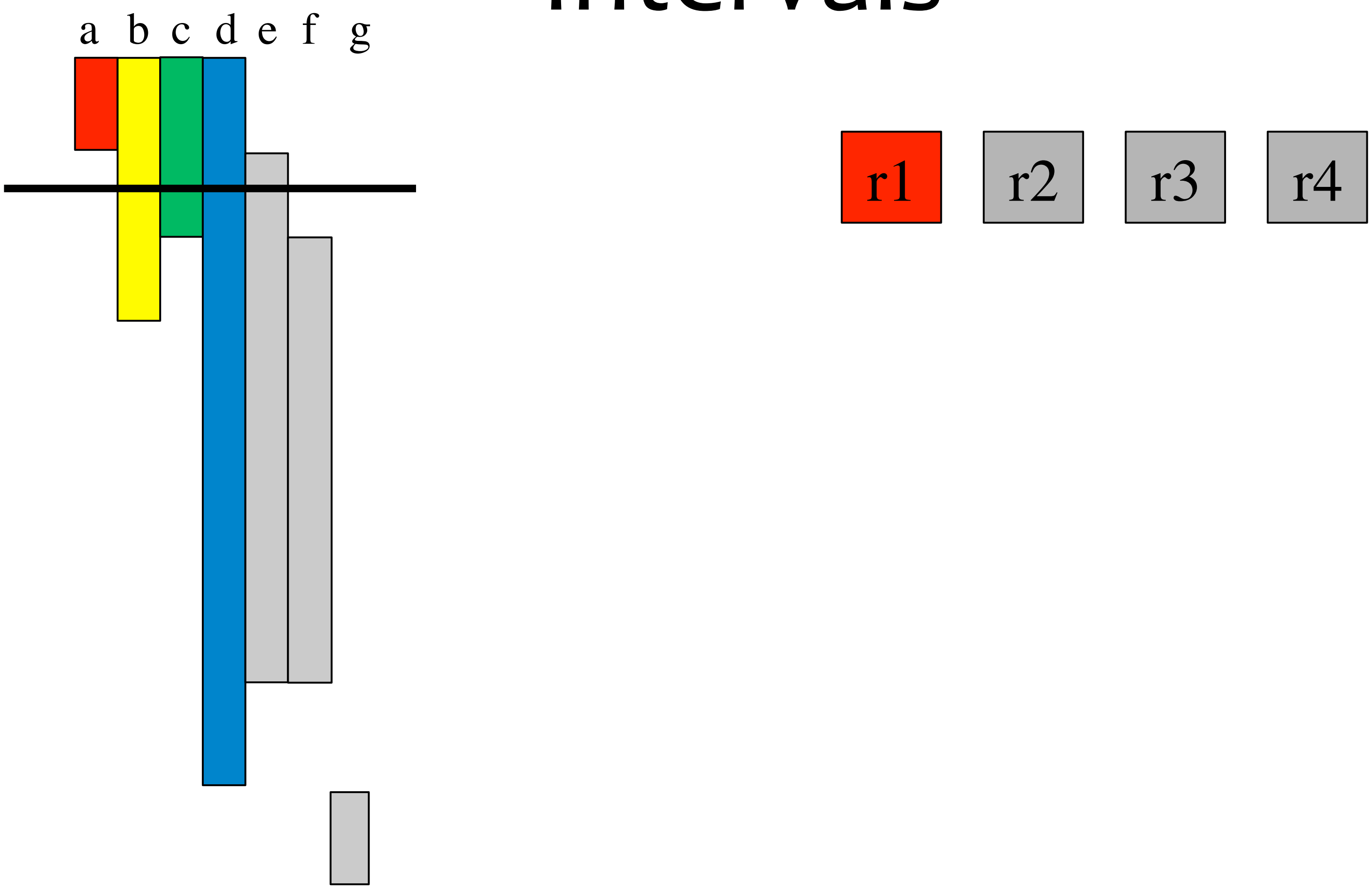

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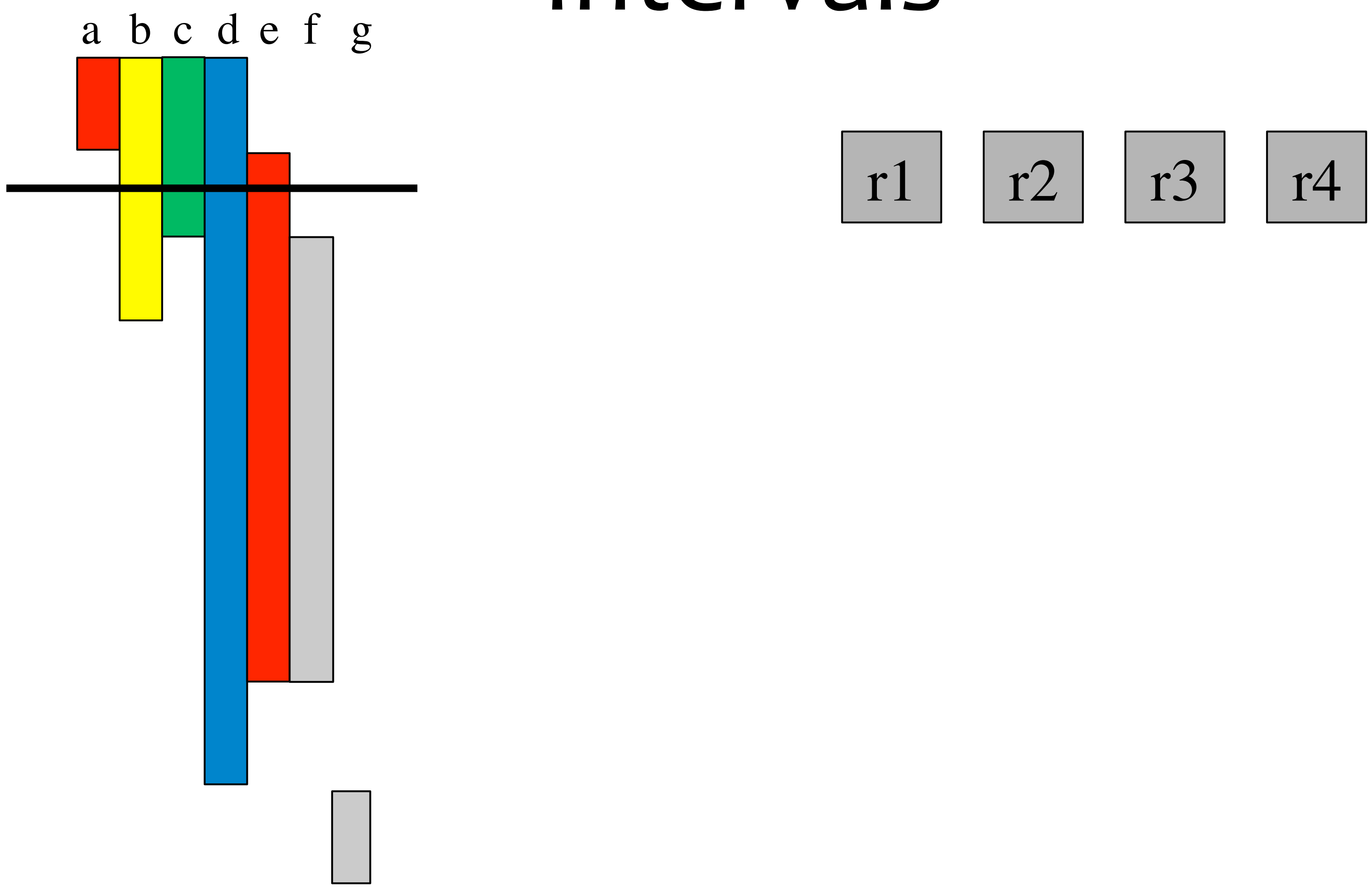

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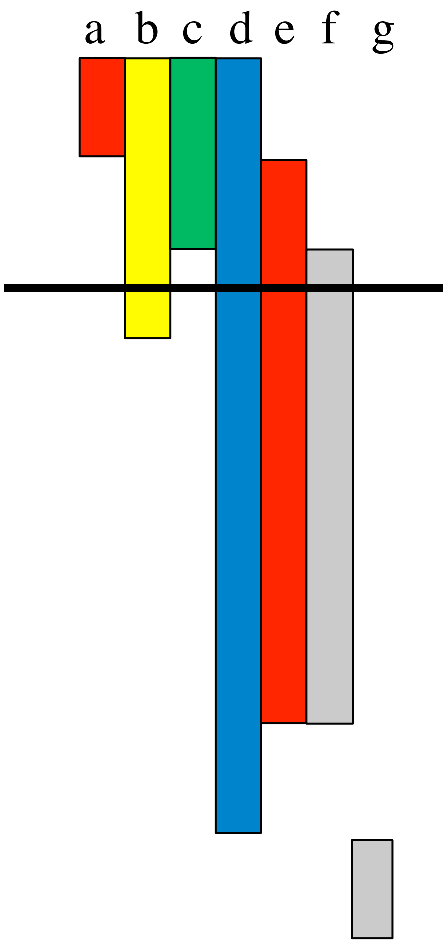

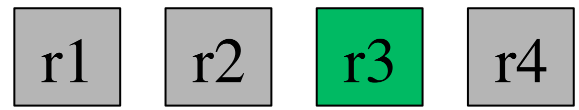

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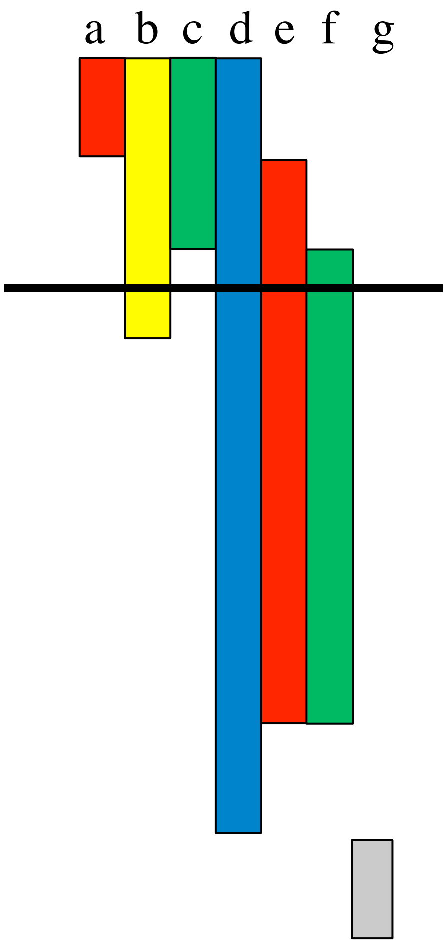

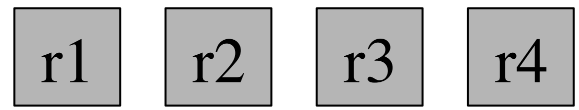

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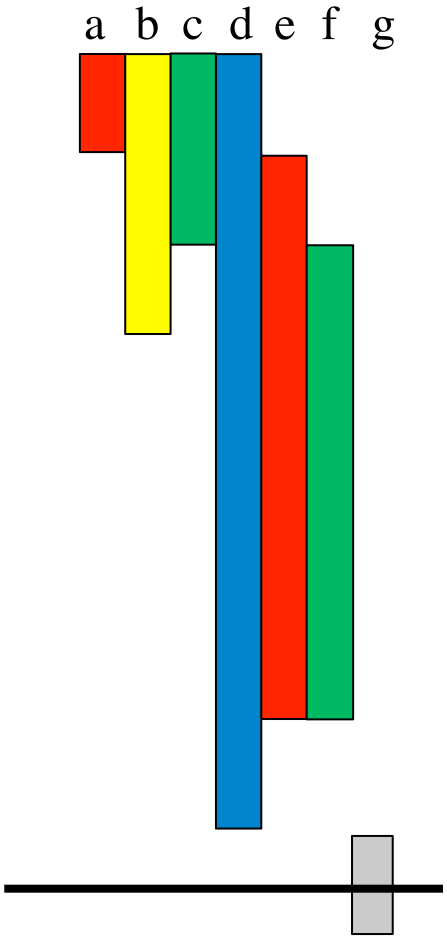

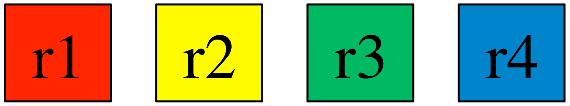

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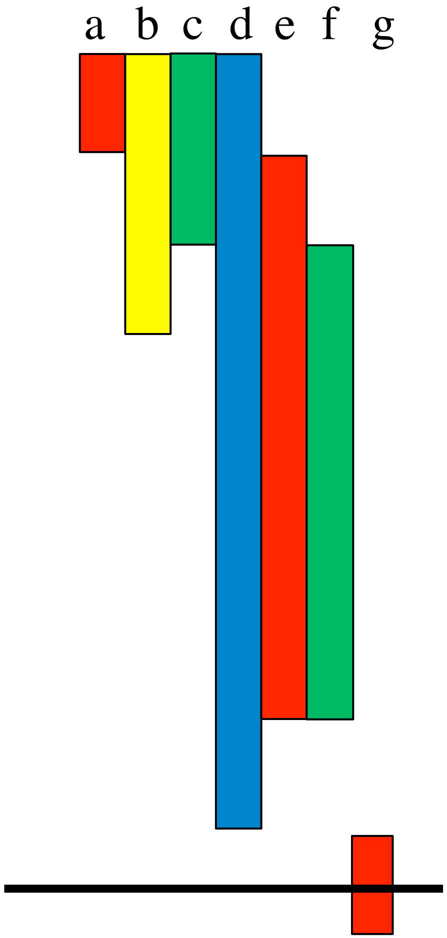

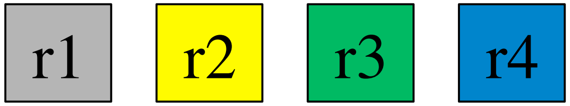

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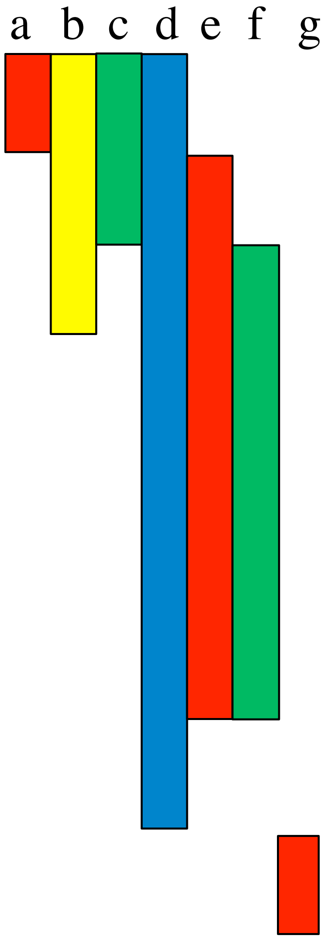

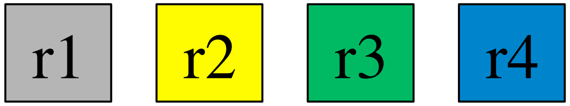

Linear Scan Register Allocation

- If a register cannot be found for a variable v, we may need to spill a variable.
- This algorithm is called linear scan register allocation and is a comparatively new algorithm.
- Pros:
  - Very efficient
  - Works well in many cases
  - Allocation needs one pass, the code can be generated simultaneously
  - Used in JIT compilers like Java HotSpot
- Cons:
  - Not as good as graph coloring approach

Summary

- Register allocation is a "must have" in compilers, because:
  - Intermediate code uses too many temporaries
  - It makes a big difference in performance
- The liveness at each location can be used for register allocation
- Register allocation as heuristic graph coloring uses live ranges
  - The basis for the technique used in GCC
- Linear scan register allocation uses live intervals
  - Often used in JIT compilers due to efficiency