

Syntax Directed Translation

CMPT 379: Compilers

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Syntax directed Translation

- Models for translation from parse trees into assembly/machine code
- Representation of translations
 - Attribute Grammars (semantic actions for CFGs)
 - Tree Matching Code Generators
 - Tree Parsing Code Generators

Attribute Grammars

- Syntax-directed translation uses a grammar to produce code (or any other “semantics”)
- Consider this technique to be a generalization of a CFG definition
- Each grammar symbol is associated with an attribute
- An attribute can be anything: a string, a number, a tree, any kind of record or object

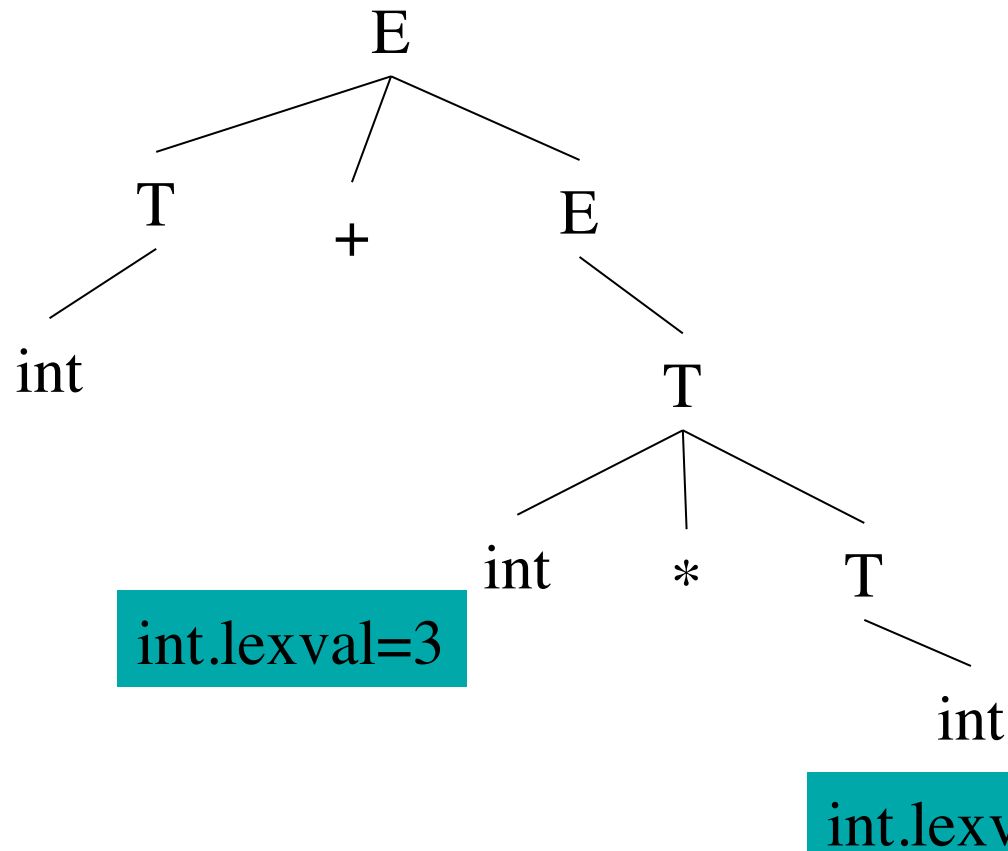
Attribute Grammars

- A CFG can be viewed as a (finite) representation of a function that relates strings to parse trees
- Similarly, an attribute grammar is a way of relating strings with “meanings”
- Since this relation is syntax-directed, we associate each CFG rule with a semantic (rules to build an abstract syntax tree)
- In other words, attribute grammars are a method to *decorate* or *annotate* the parse tree

Expr concrete syntax tree

Input:

4+3*5



int.lexval=4

int.lexval=3

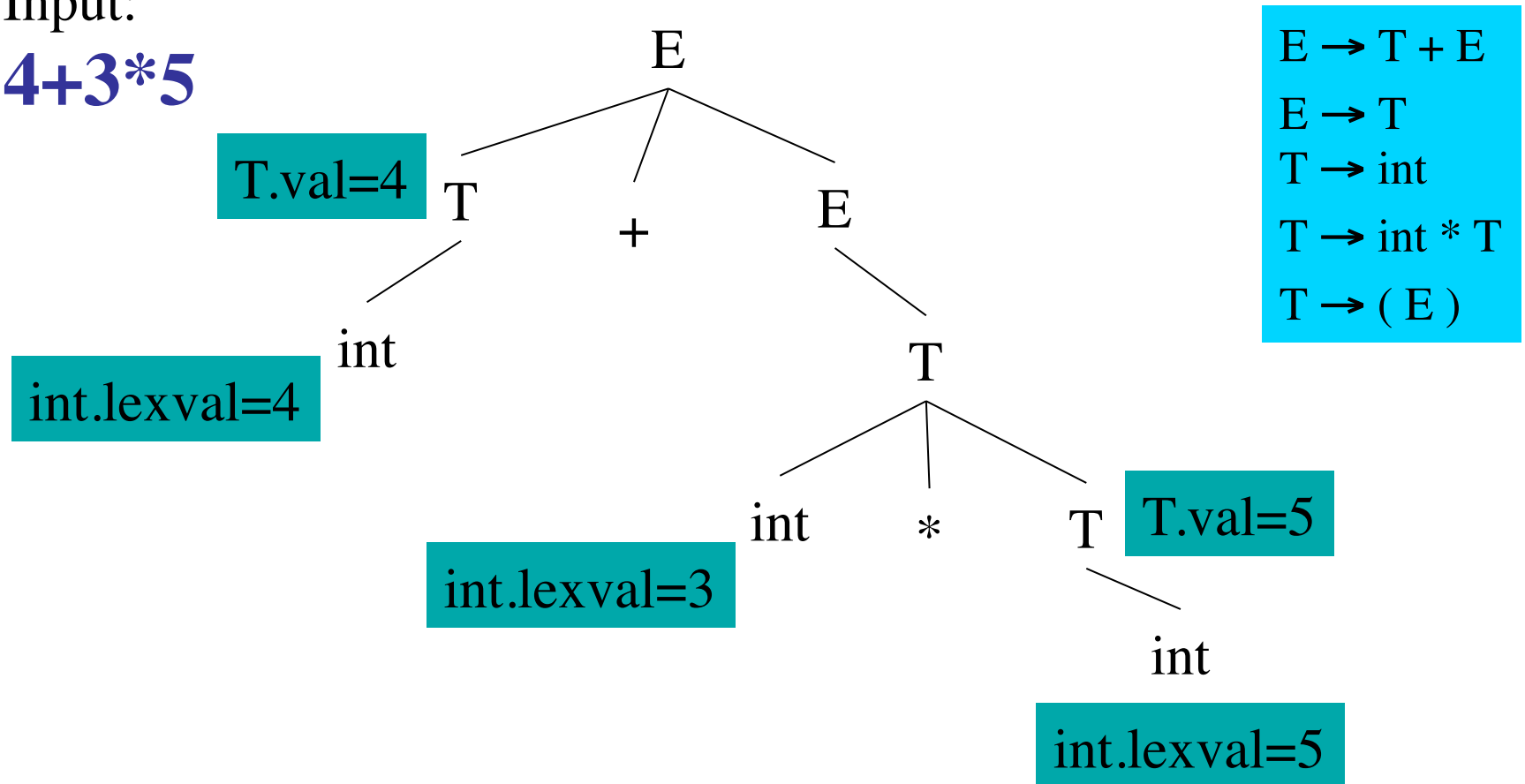
int.lexval=5

$E \rightarrow T + E$
 $E \rightarrow T$
 $T \rightarrow \text{int}$
 $T \rightarrow \text{int} * T$
 $T \rightarrow (E)$

Expr concrete syntax tree

Input:

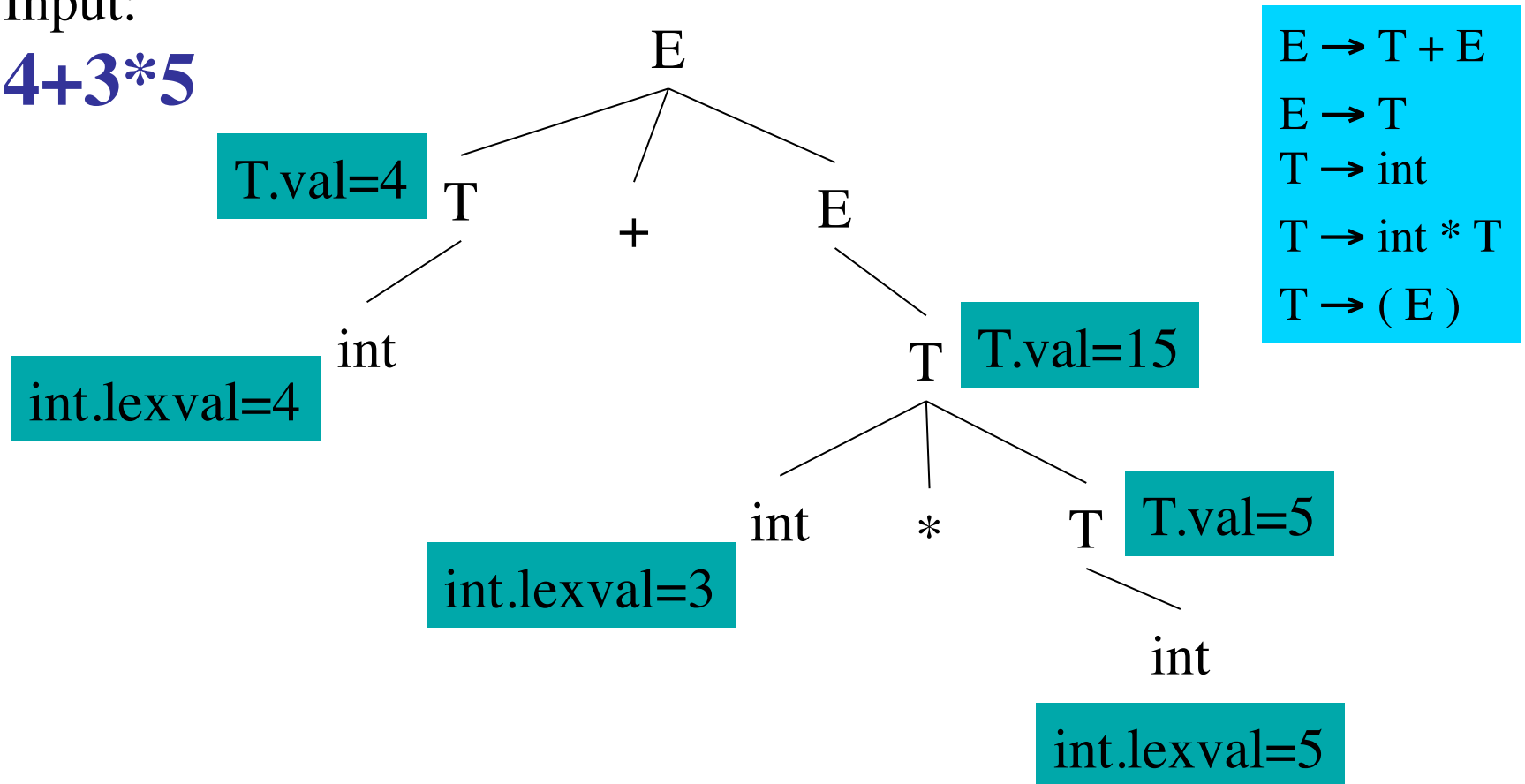
4+3*5



Expr concrete syntax tree

Input:

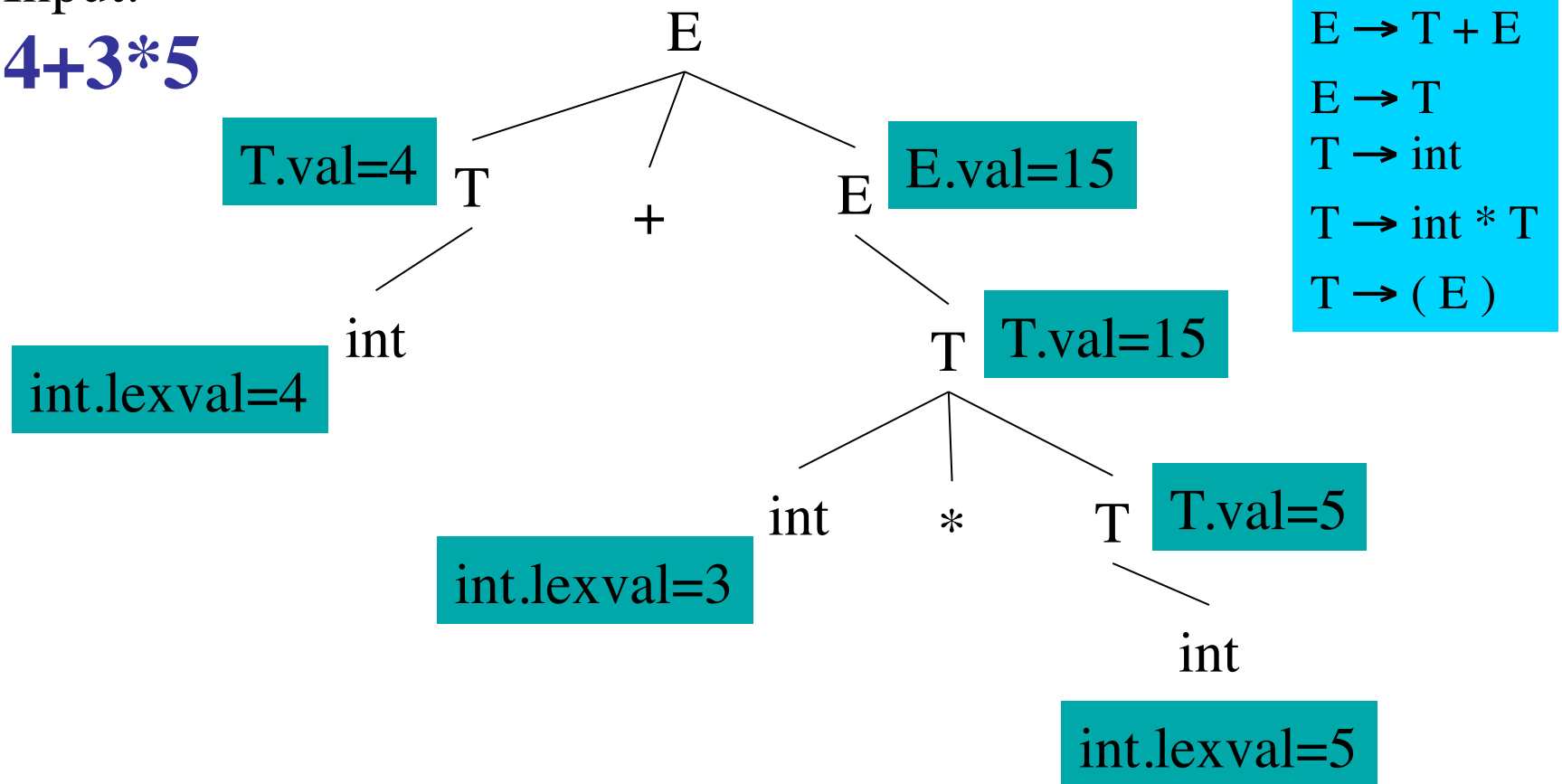
4+3*5



Expr concrete syntax tree

Input:

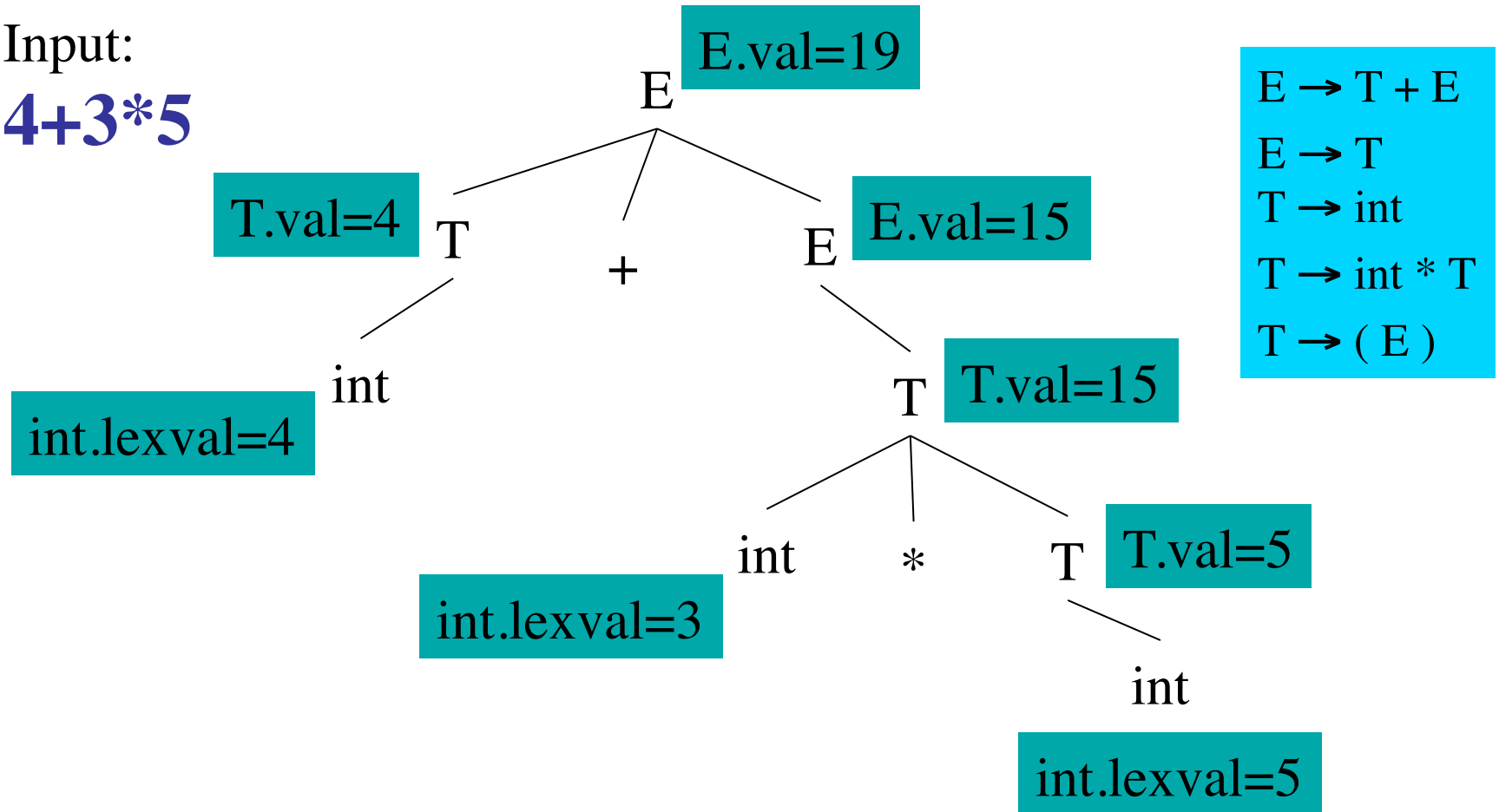
4+3*5



Expr concrete syntax tree

Input:

4+3*5



Syntax directed definition

$T \rightarrow \text{int}$

$\{ \$0.\text{val} = \$1.\text{lexval}; \}$ \dashrightarrow

In yacc: $\{ \$\$ = \$1 \}$

$T \rightarrow \text{int} * T$

$\{ \$0.\text{val} = \$1.\text{lexval} * \$3.\text{val}; \}$

$E \rightarrow T$

$\{ \$0.\text{val} = \$1.\text{val}; \}$

$E \rightarrow T + E$

$\{ \$0.\text{val} = \$1.\text{val} + \$3.\text{val}; \}$

$T \rightarrow (E)$

$\{ \$0.\text{val} = \$2.\text{val}; \}$

Flow of Attributes in *Expr*

- Consider the flow of the attributes in the *E* syntax-directed defn
 - The lhs attribute is computed using the rhs attributes
- Purely bottom-up:
 - compute attribute values of all children (rhs) in the parse tree
 - And then use them to compute the attribute value of the parent (lhs)

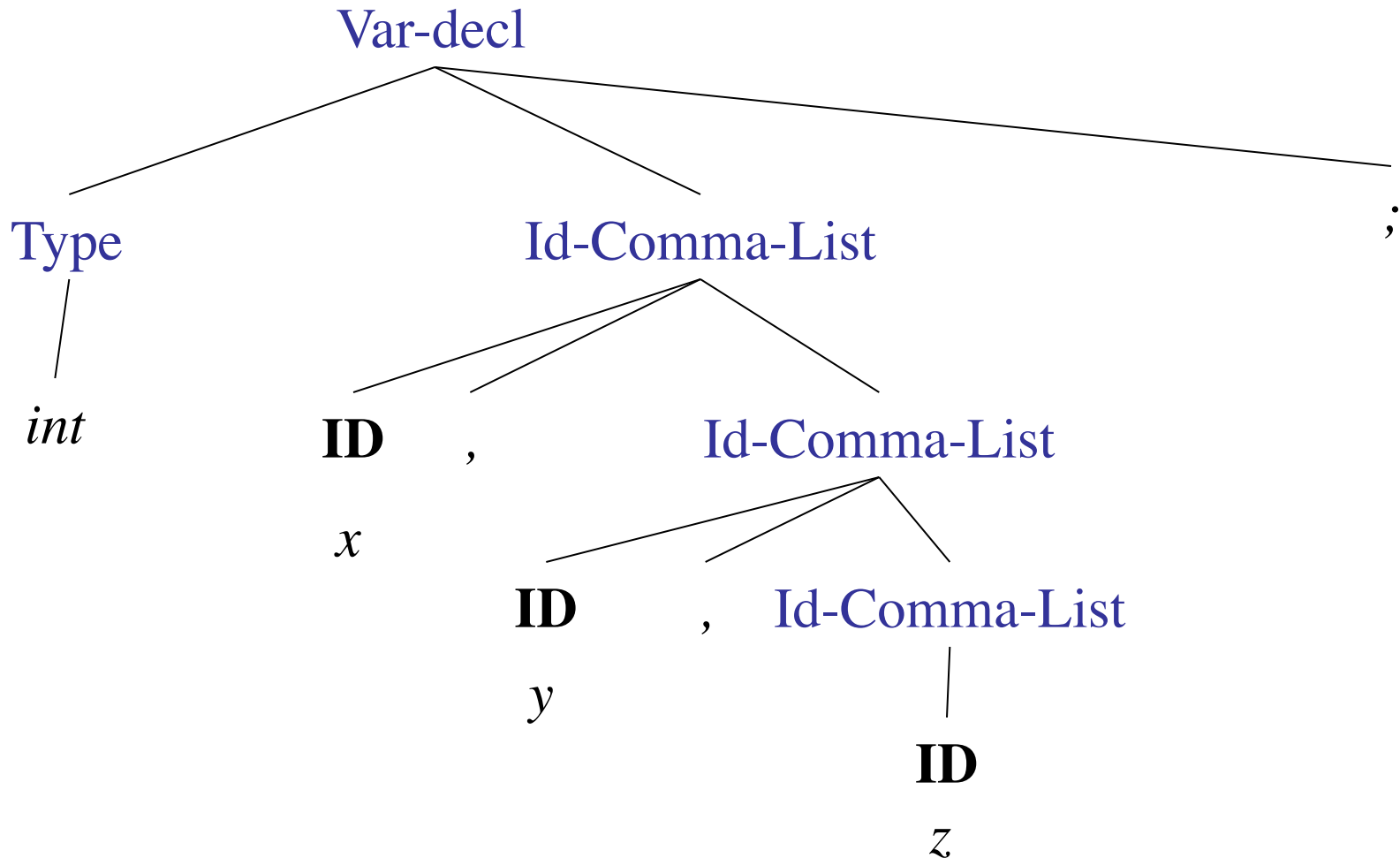
Synthesized Attributes

- **Synthesized attributes** are attributes that are computed purely bottom-up
- A grammar with semantic actions (or syntax-directed definition) can choose to use *only* synthesized attributes
- Such a grammar plus semantic actions is called an **S-attributed definition**

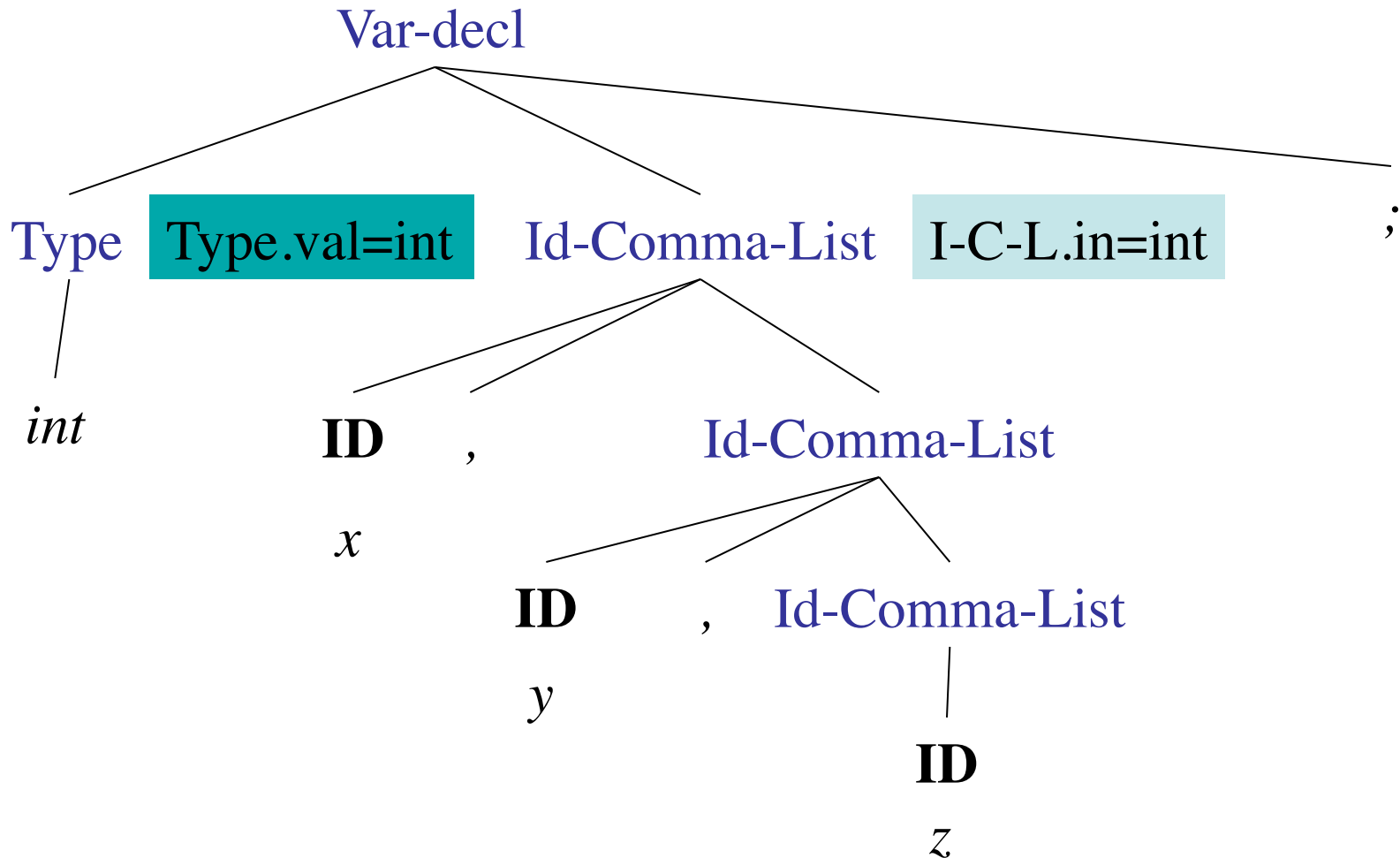
Inherited Attributes

- Synthesized attributes may not be sufficient for all cases that might arise for semantic checking and code generation
- Consider the (sub)grammar:
Var-decl \rightarrow Type Id-comma-list ;
Type \rightarrow **int** | **bool**
Id-comma-list \rightarrow **ID**
Id-comma-list \rightarrow **ID** , Id-comma-list

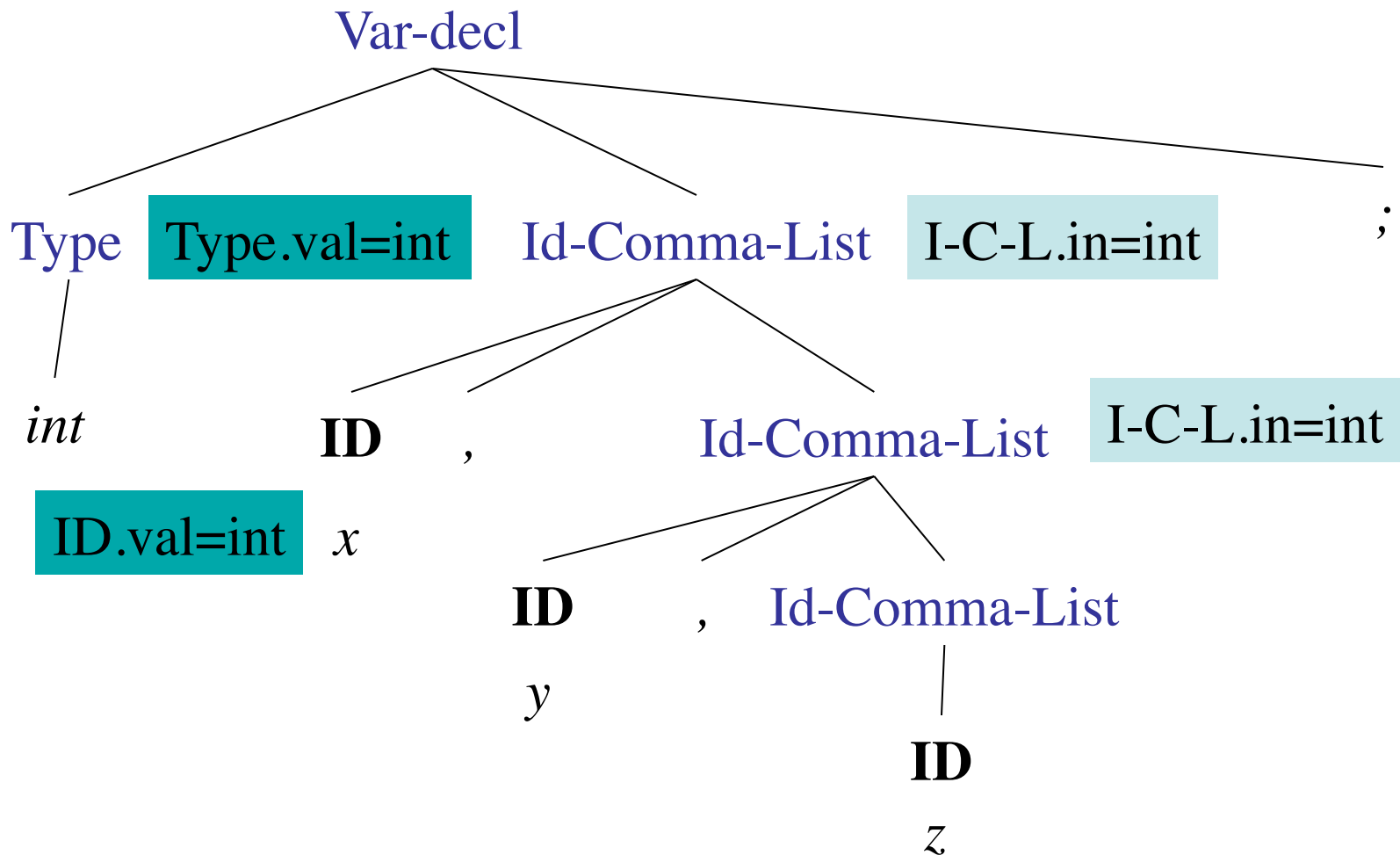
Example input: *int x, y, z ;*



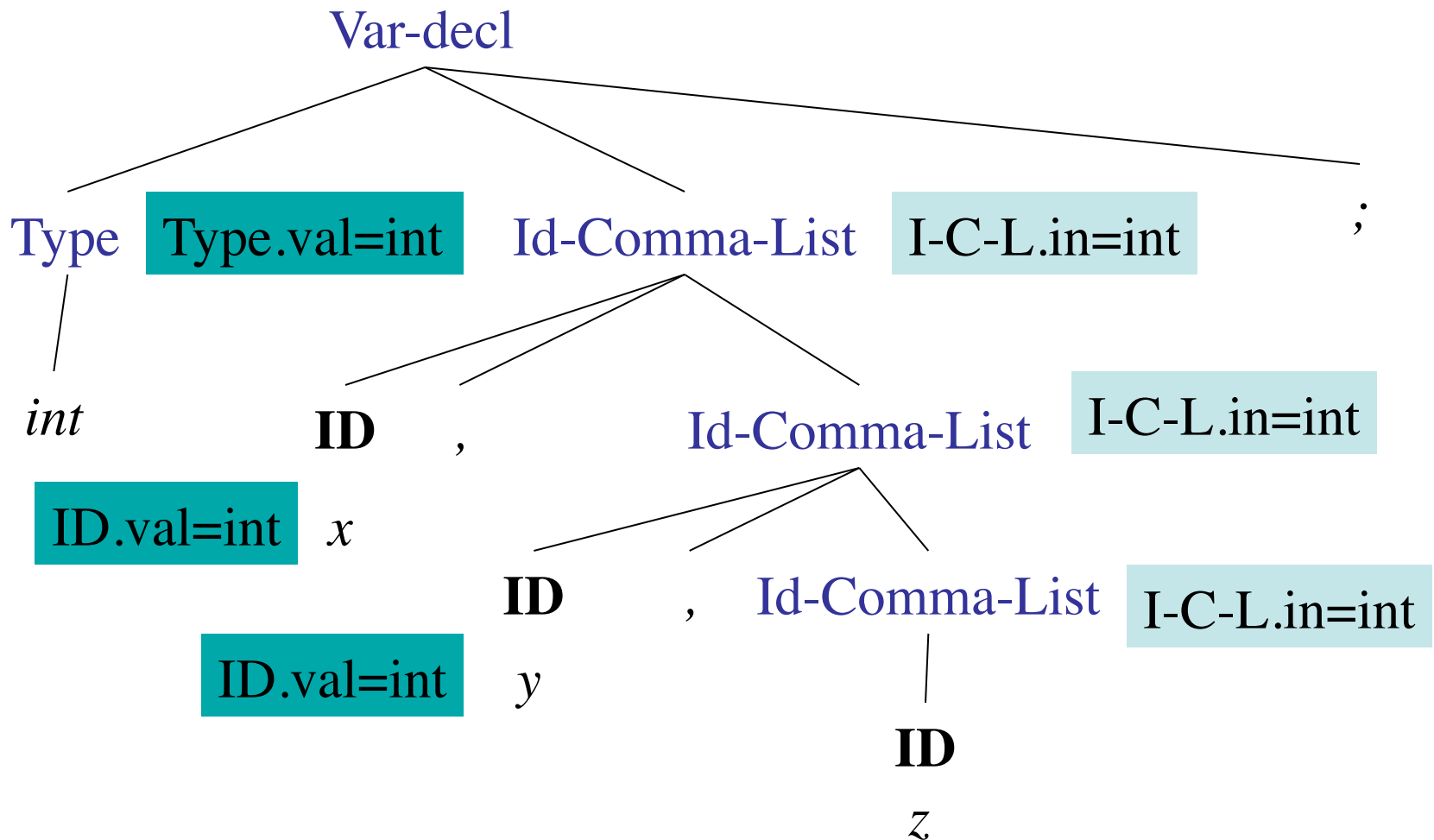
Example input: *int x, y, z ;*



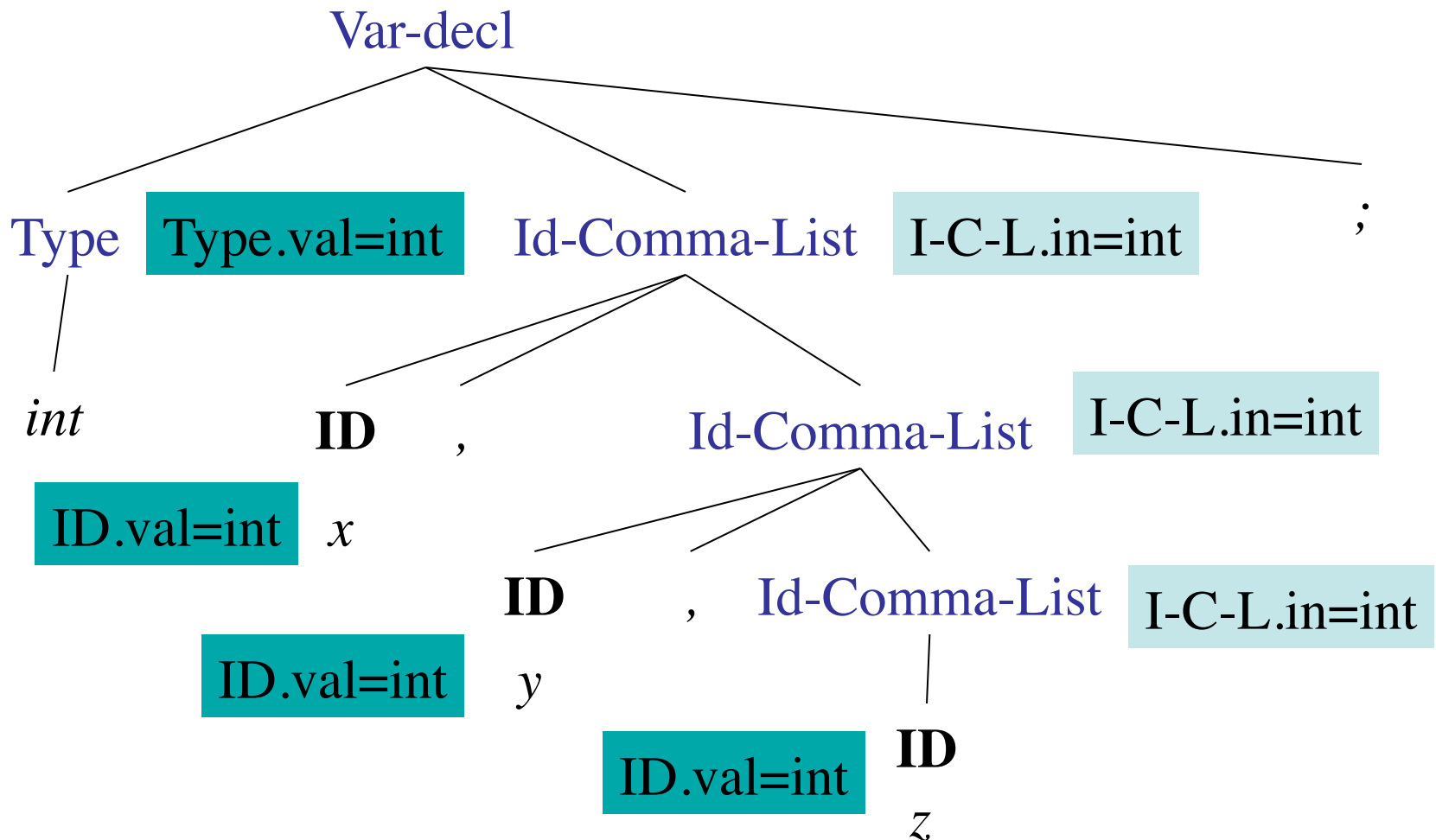
Example input: *int x, y, z ;*



Example input: *int x, y, z ;*



Example input: *int x, y, z ;*



Flow of Attributes in *Var-decl*

- How do the attributes flow in the *Var-decl* grammar?
- **ID** takes its attribute value from its parent node
- *Id-Comma-List* takes its attribute from its left sibling *Type* (or from its parent *Id-Comma-list*)

Syntax-directed definition

Var-decl \rightarrow Type Id-comma-list ;

{ \$2.in = \$1.val; }

Type \rightarrow **int**

{ \$0.val = int; }

| **bool**

{ \$0.val = bool; }

Id-comma-list \rightarrow **ID**

{ \$1.val = \$0.in; }

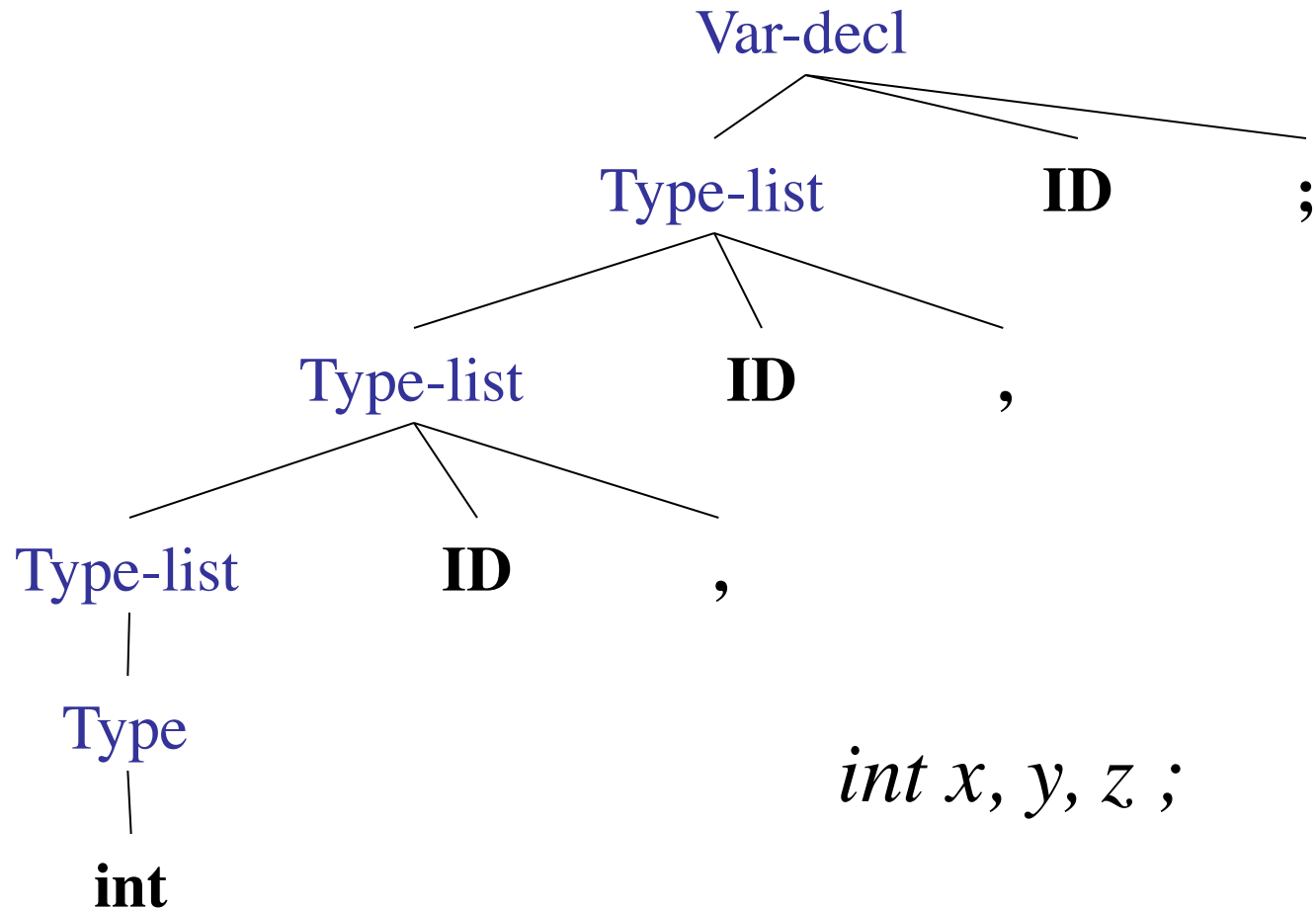
Id-comma-list \rightarrow **ID** , Id-comma-list

{ \$1.val = \$0.in; \$3.in = \$0.in; }

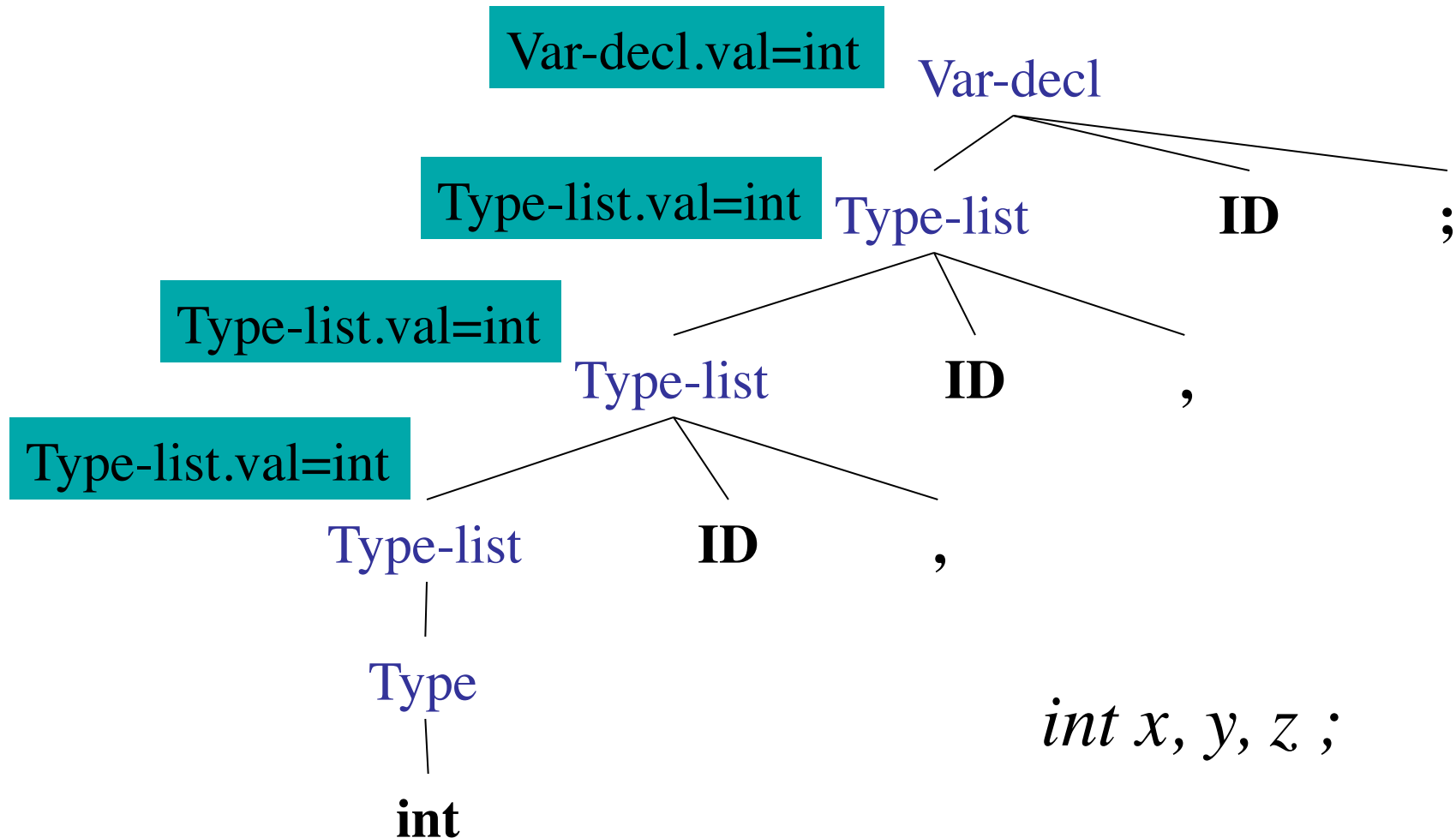
Inherited Attributes

- **Inherited attributes** are attributes that are computed at a node based on attributes from siblings or the parent
- Typically we combine synthesized attributes and inherited attributes
- It is possible to convert the grammar into a form that *only* uses synthesized attributes

Removing Inherited Attributes



Removing Inherited Attributes



Removing inherited attributes

Var-decl \rightarrow Type-List **ID** ;

{ \$0.val = \$1.val; }

Type-list \rightarrow Type-list **ID** ,

{ \$0.val = \$1.val; }

Type-list \rightarrow Type

{ \$0.val = \$1.val; }

Type \rightarrow **int**

{ \$0.val = int; }

| **bool**

{ \$0.val = bool; }

Direction of inherited attributes

- Consider the syntax directed defns:

$A \rightarrow L M$

$\{ \$1.in = \$0.in; \$2.in = \$1.val; \$0.val = \$2.val; \}$

$A \rightarrow Q R$

$\{ \$2.in = \$0.in; \$1.in = \$2.val; \$0.val = \$1.val; \}$

- Problematic definition: $\$1.in = \$2.val$
- Difference between incremental processing vs. using the completed parse tree

Incremental Processing

- Incremental processing: constructing output as we are parsing
- Bottom-up or top-down parsing
 - Both can be viewed as left-to-right and depth-first construction of the parse tree
- Some inherited attributes cannot be used in conjunction with incremental processing

L-attributed Definitions

- A syntax-directed definition is **L-attributed** if for each production $A \rightarrow X_1 \dots X_{j-1} X_j \dots X_n$, for each $j=1 \dots n$, each inherited attribute of X_j depends on:
 - The attributes of $X_1 \dots X_{j-1}$
 - The inherited attributes of A
- These two conditions ensure left to right and depth first parse tree construction
- Every **S-attributed** definition is **L-attributed**

Syntax-directed defns

- Different SDTs are defined based on the parser which is used.
- Two important classes of SDTs:
 1. LR parser, syntax directed definition is S-attributed
 2. LL parser, syntax directed definition is L-attributed

Syntax-directed defns

- LR parser, **S-attributed** definition
 - Implementing S-attributed definitions in LR parsing is easy: execute action on reduce, all necessary attributes have to be on the stack
- LL parser, **L-attributed** definition
 - Implementing L-attributed definitions in LL parsing: we need an additional action record for storing synthesized and inherited attributes on the parse stack

Top-down translation

- Assume that we have a top-down predictive parser
- Typical strategy: take the CFG and eliminate left-recursion
- Suppose that we start with an attribute grammar
- We should still eliminate left-recursion

Top-down translation example

$E \rightarrow E + T$

{ \$0.val = \$1.val + \$3.val; }

$E \rightarrow E - T$

{ \$0.val = \$1.val - \$3.val; }

$T \rightarrow \text{int}$

{ \$0.val = \$1.lexval; }

$E \rightarrow T$

{ \$0.val = \$1.val; }

$T \rightarrow (E)$

{ \$0.val = \$2.val; }

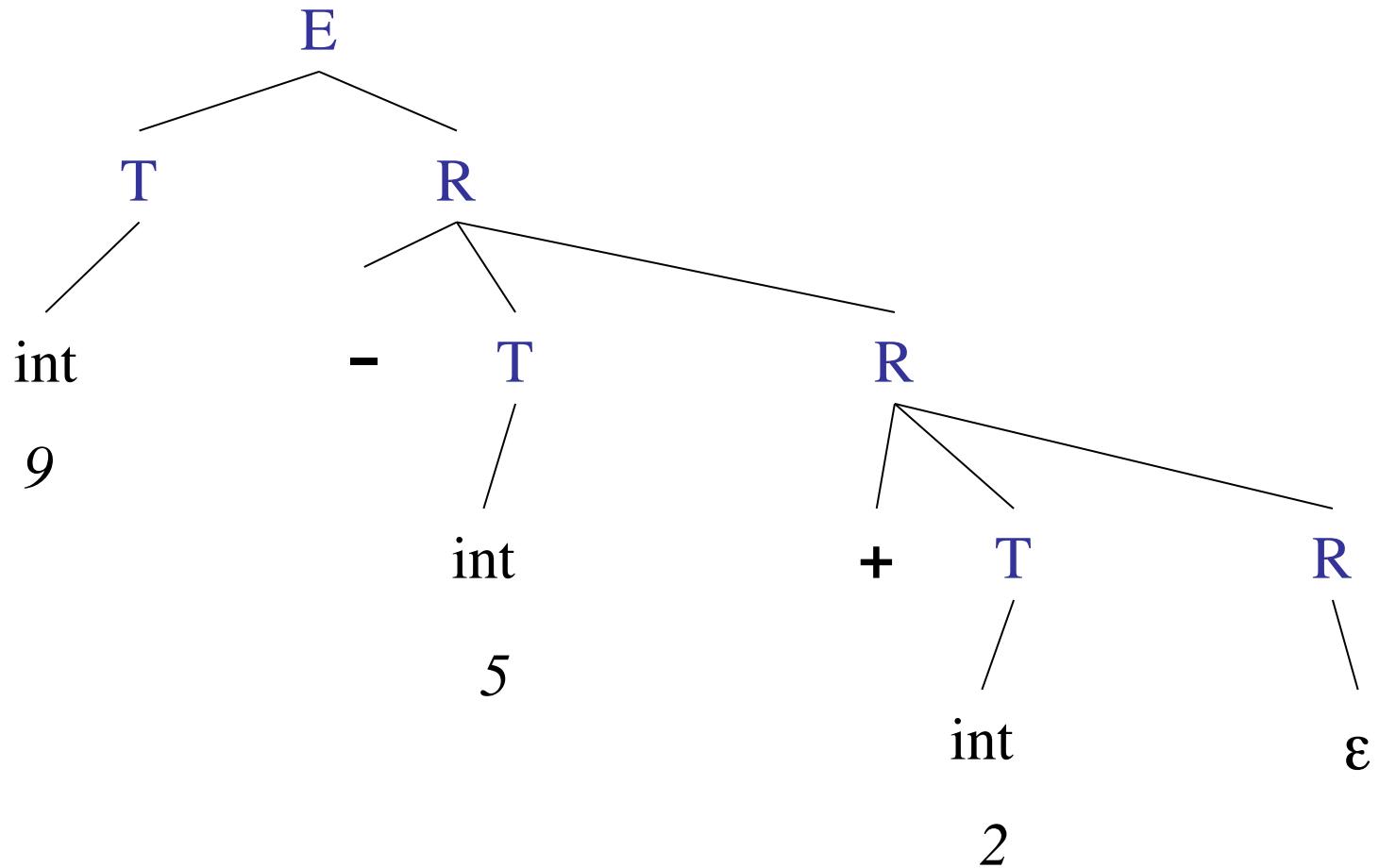
Top-down translation example

Remove left recursion

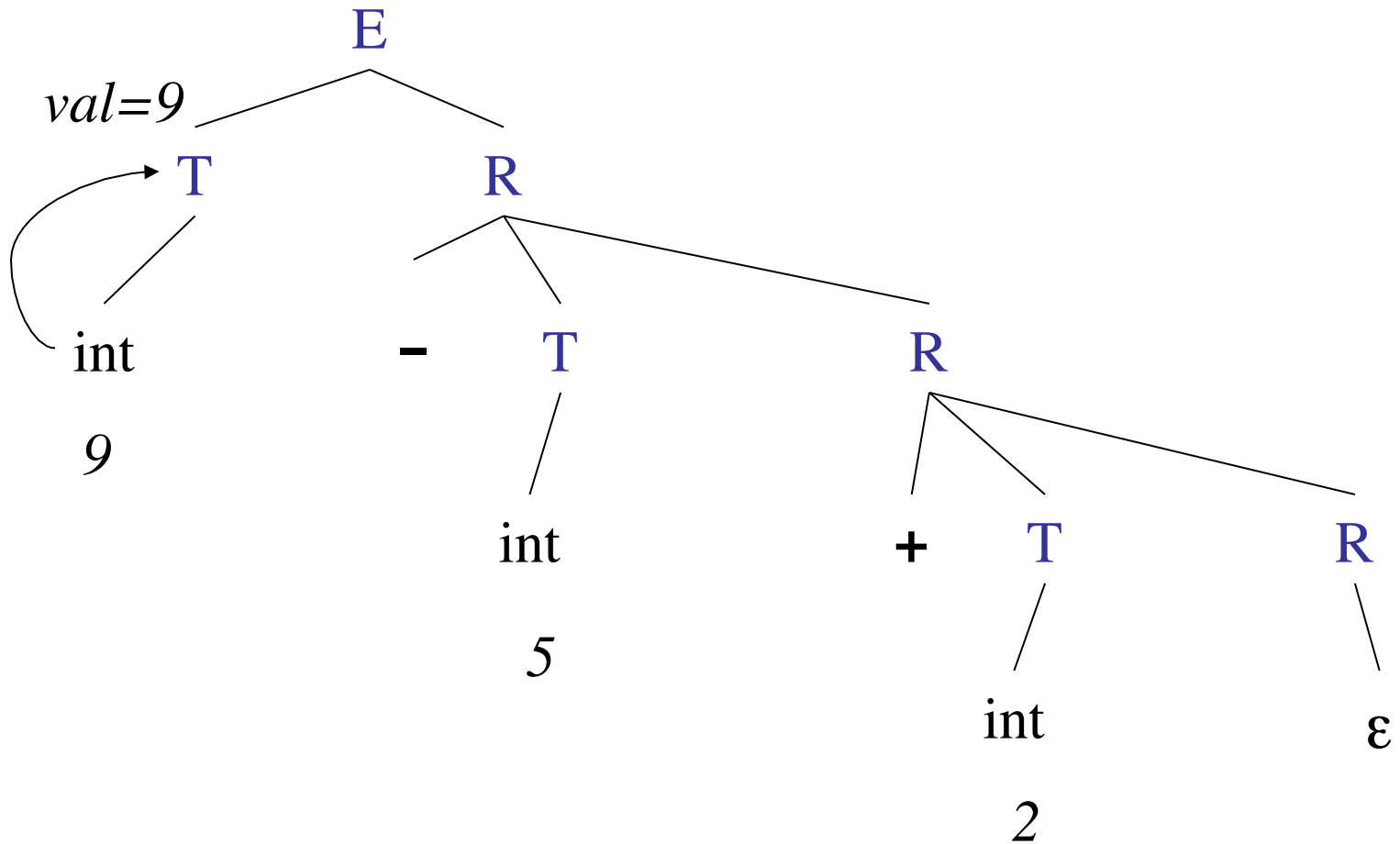
$$\begin{aligned} E &\rightarrow E + T \\ E &\rightarrow E - T \\ E &\rightarrow T \\ T &\rightarrow (E) \\ T &\rightarrow \mathbf{int} \end{aligned}$$

$$\begin{aligned} E &\rightarrow T R \\ R &\rightarrow + T R \\ R &\rightarrow - T R \\ R &\rightarrow \varepsilon \\ T &\rightarrow (E) \\ T &\rightarrow \mathbf{int} \end{aligned}$$

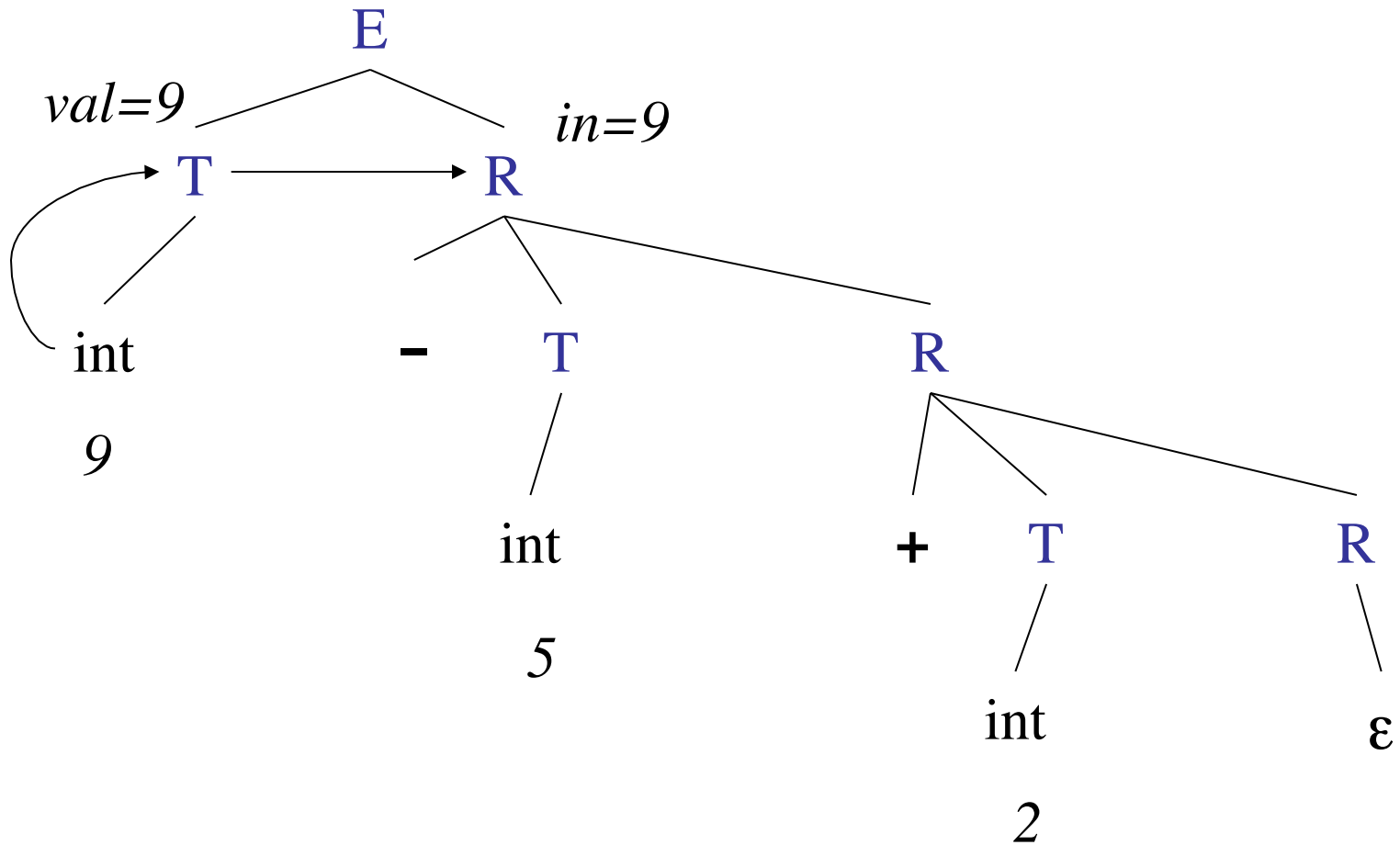
input: 9 - 5 + 2



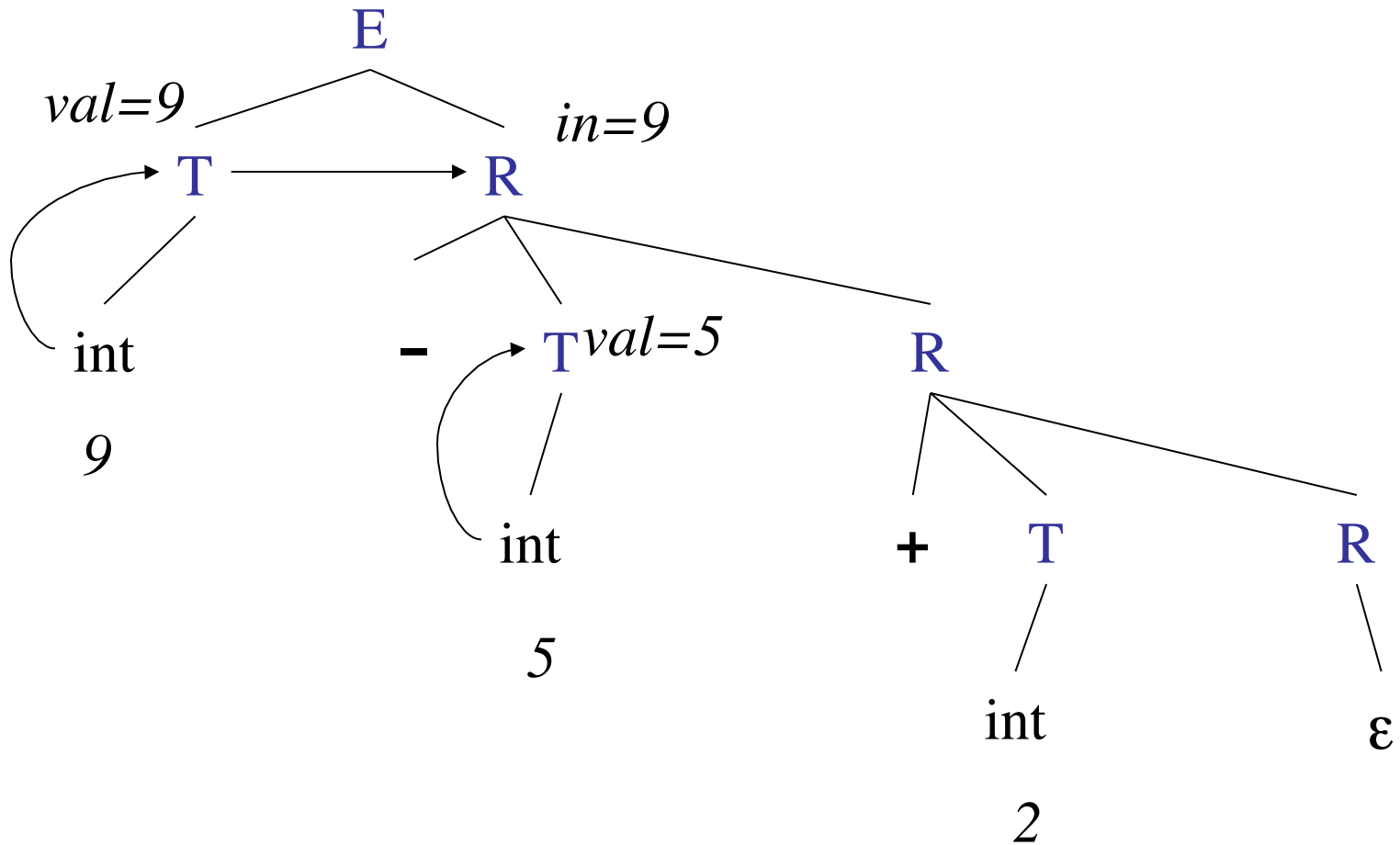
input: 9 - 5 + 2



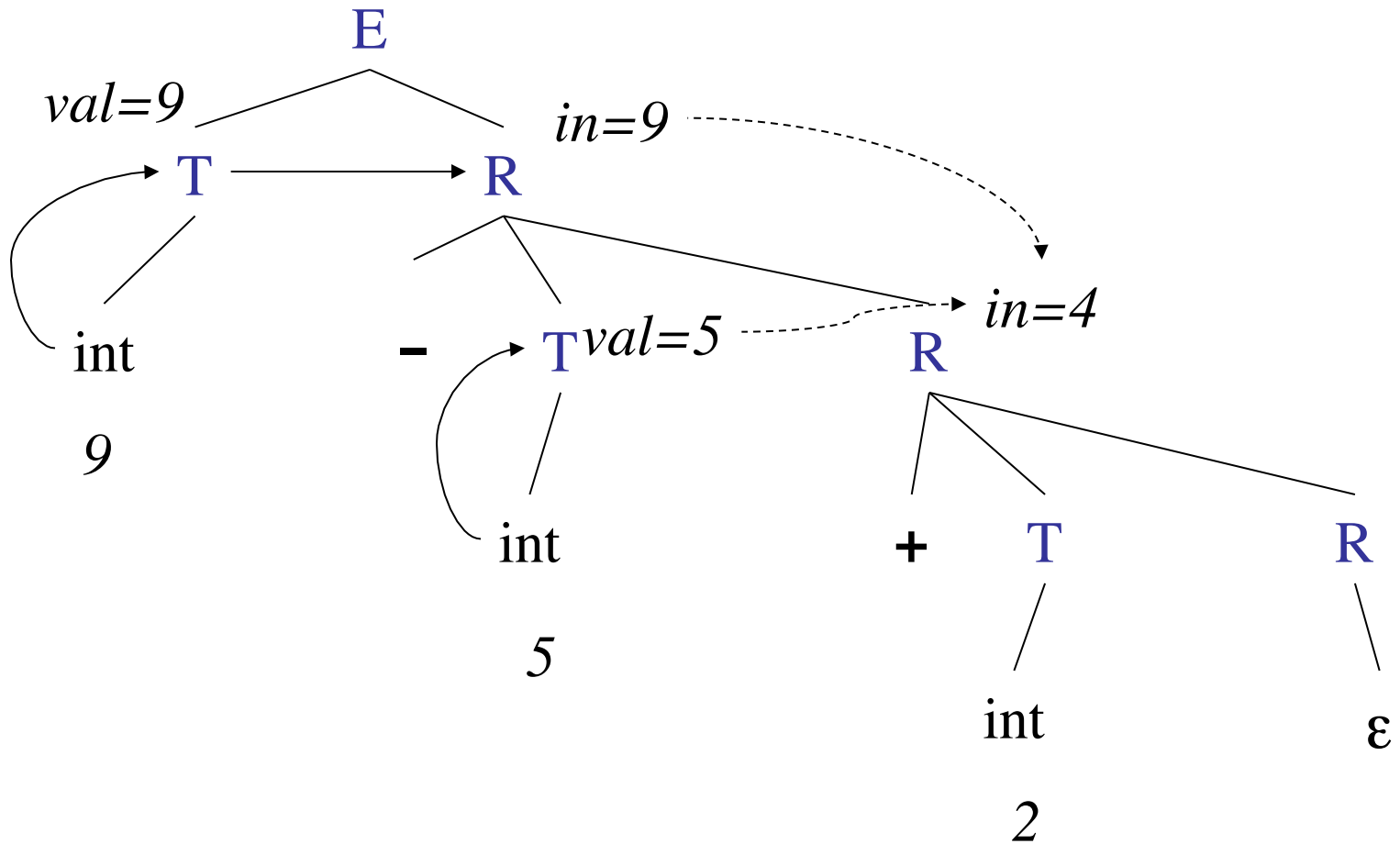
input: 9 - 5 + 2



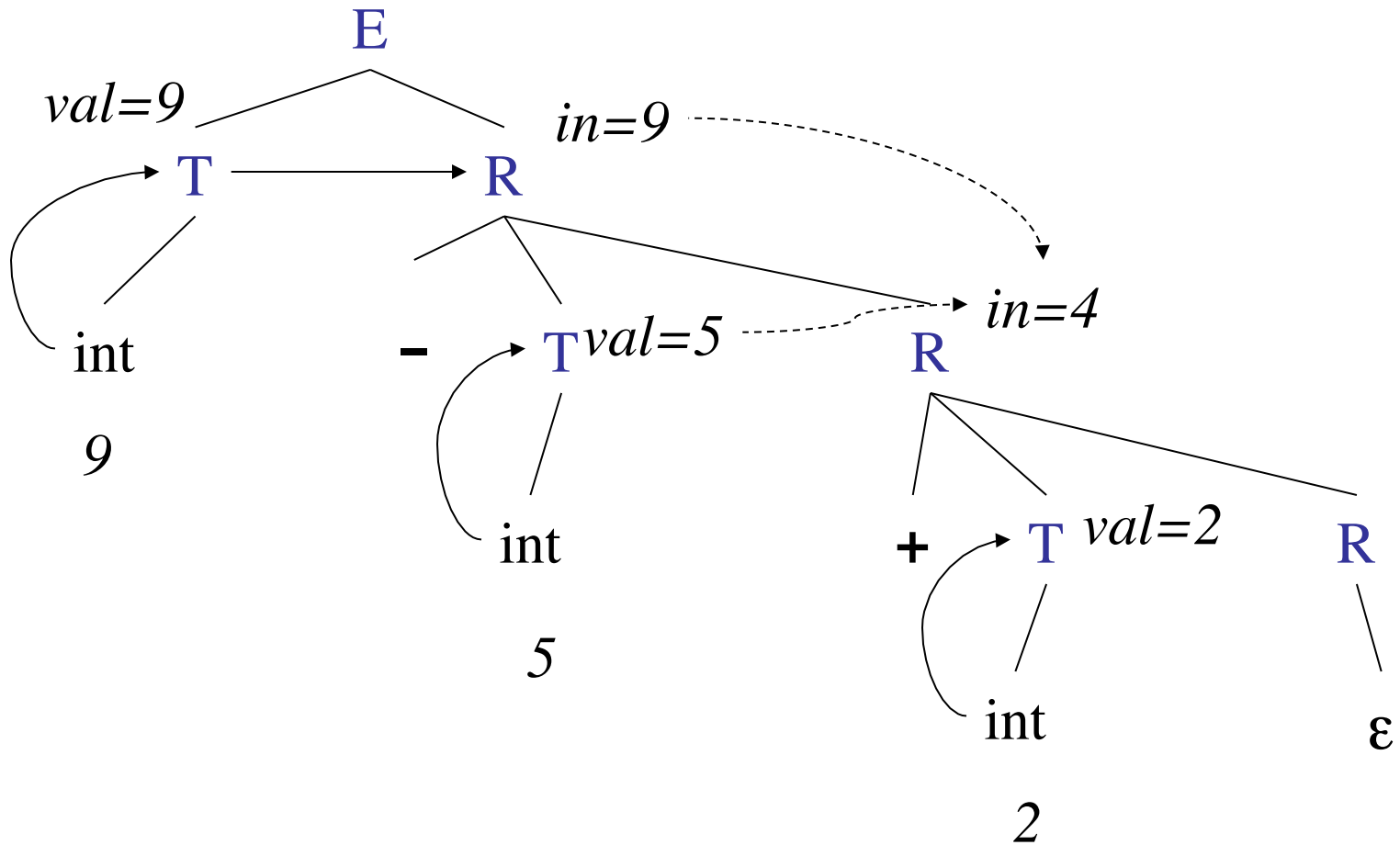
input: 9 - 5 + 2



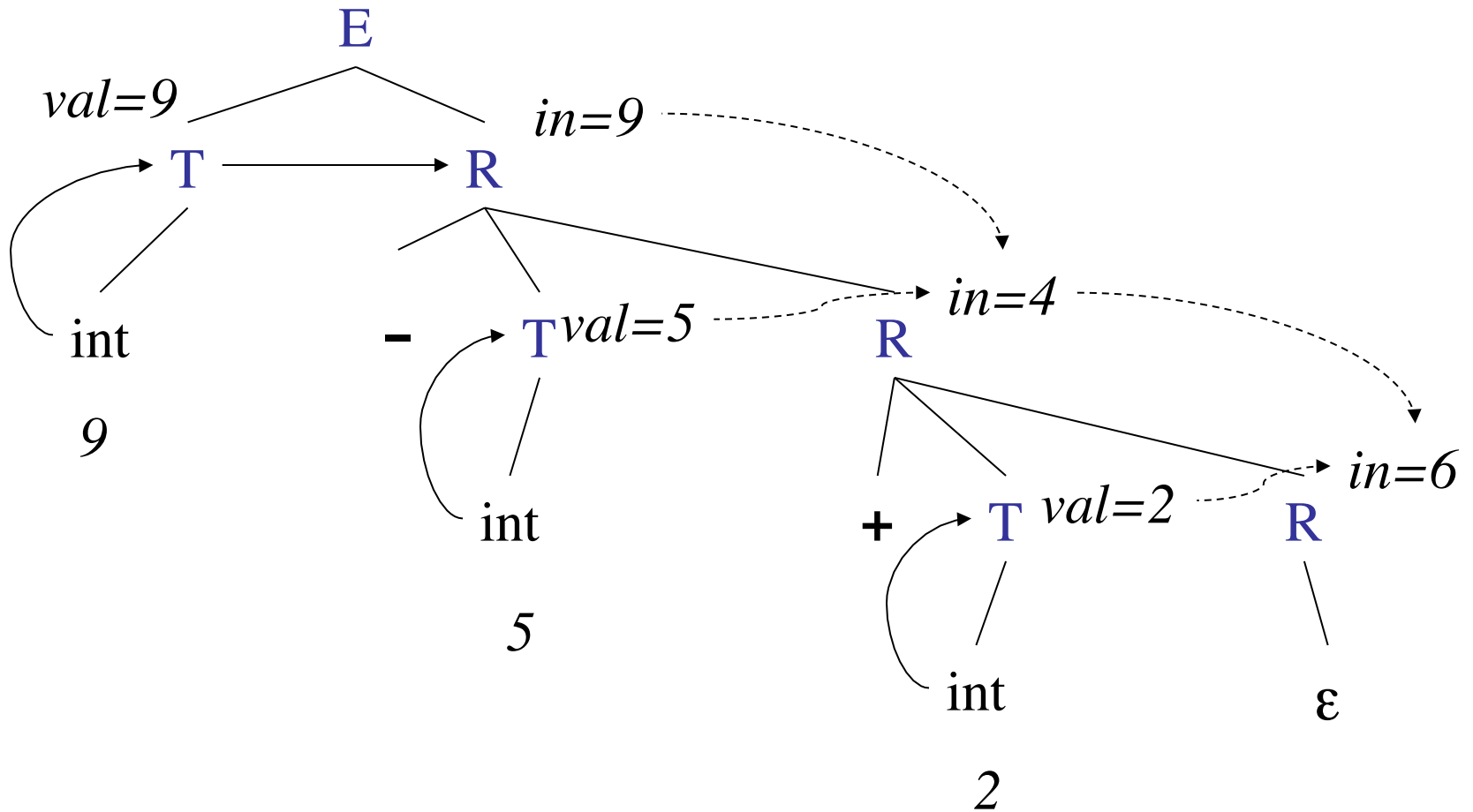
input: 9 - 5 + 2



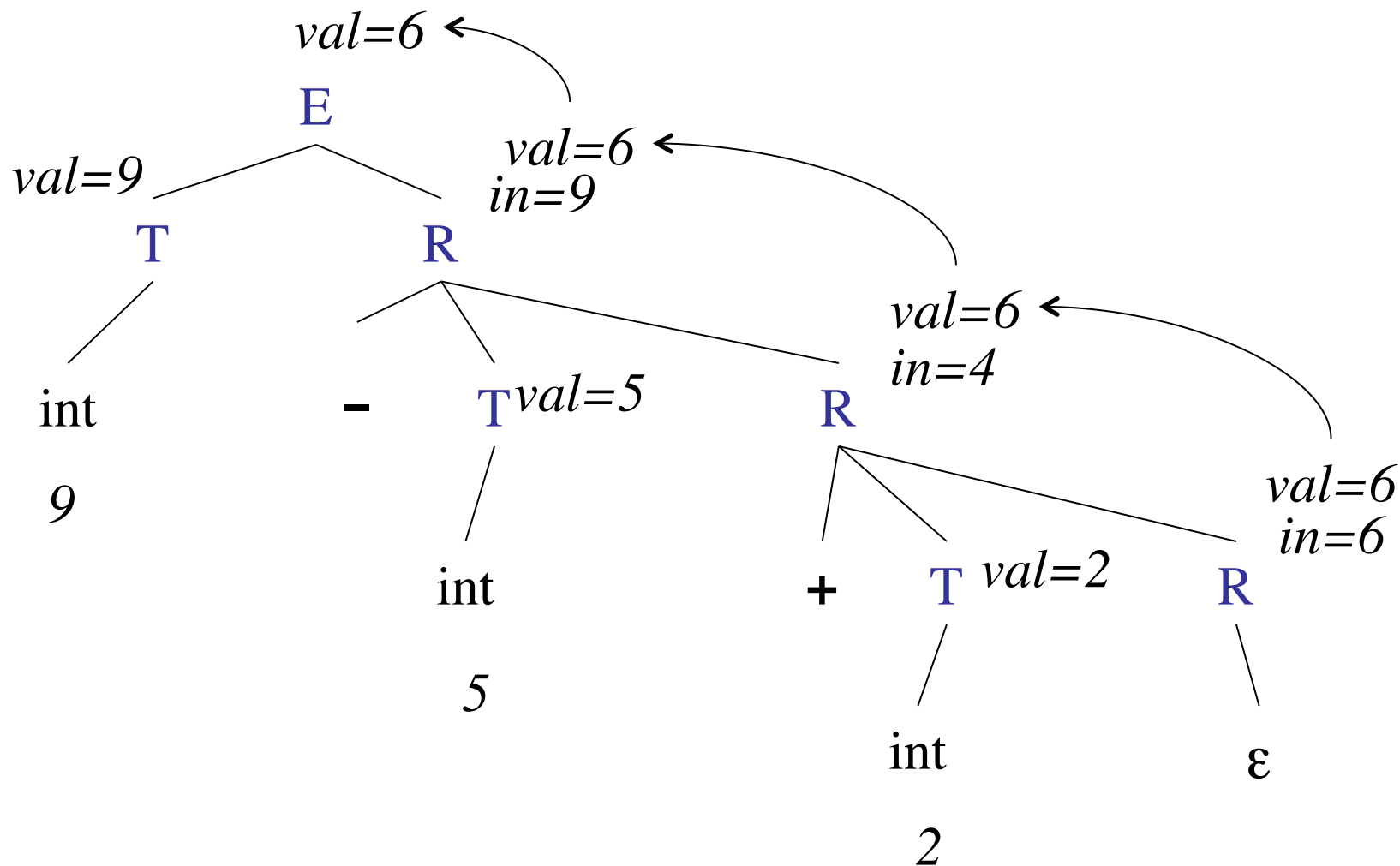
input: 9 - 5 + 2



input: 9 - 5 + 2



input: 9 - 5 + 2



Top-down translation example

SDT for the LL(1) grammar:

$E \rightarrow E + T$
 { $\$0.val = \$1.val + \$3.val$; }
 $E \rightarrow E - T$
 { $\$0.val = \$1.val - \$3.val$; }
 $E \rightarrow T$
 { $\$0.val = \$1.val$; }
 $T \rightarrow (E)$
 { $\$0.val = \$2.val$; }
 $T \rightarrow \text{int}$
 { $\$0.val = \$1.lexval$; }



$E \rightarrow T R$
 { $\$2.in = \$1.val$; $\$0.val = \$2.val$; }
 $R \rightarrow + T R$
 { $\$3.in = \$0.in + \$2.val$;
 $\$0.val = \$3.val$; }
 $R \rightarrow - T R$
 { $\$3.in = \$0.in - \$2.val$;
 $\$0.val = \$3.val$; }
 $R \rightarrow \epsilon$
 { $\$0.val = \$0.in$; }
 $T \rightarrow (E)$
 { $\$0.val = \$2.val$; }
 $T \rightarrow \text{int}$
 { $\$0.val = \$1.lexval$; }

Dependencies and SDTs

- There can be circular definitions:

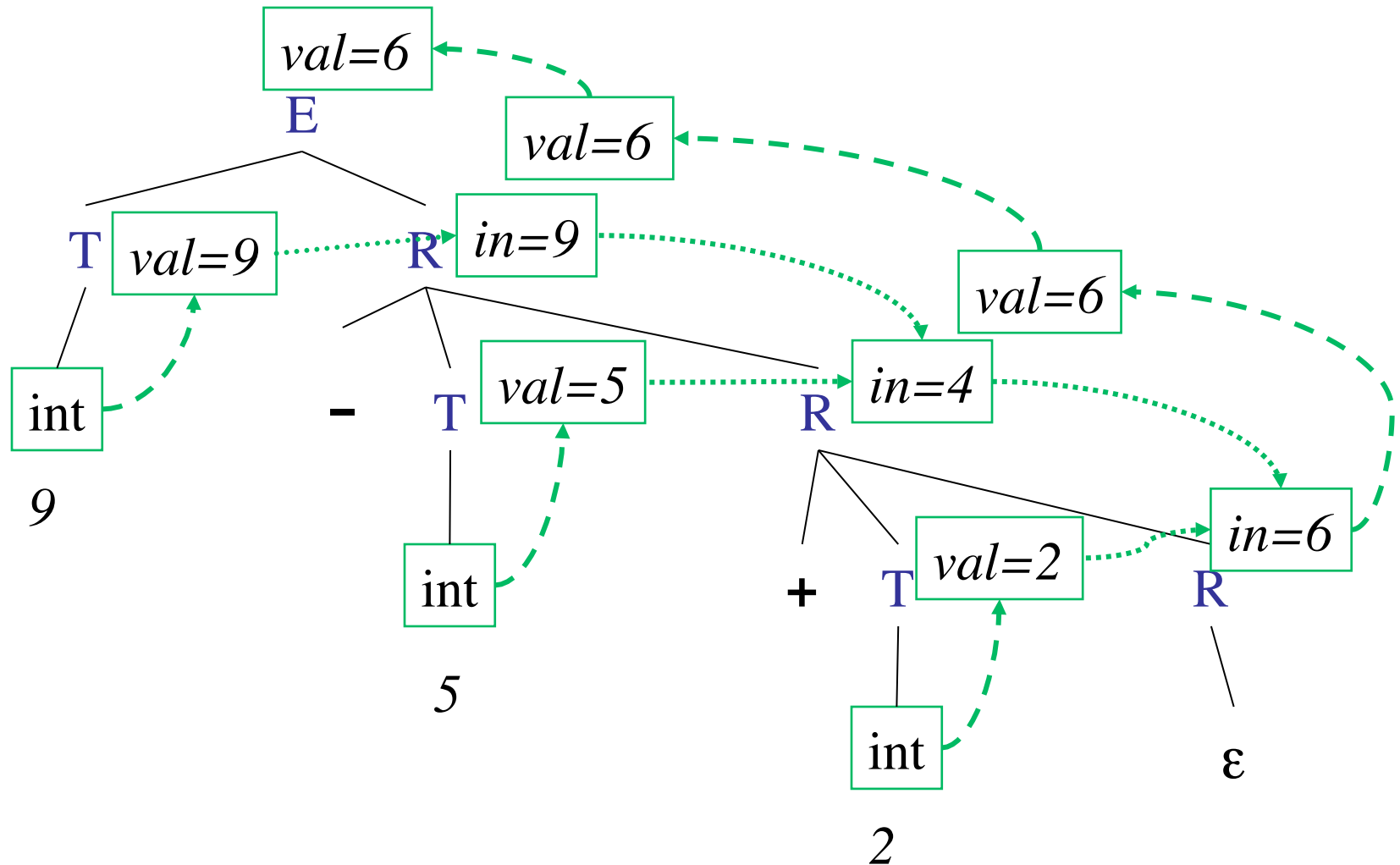
$A \rightarrow B \{ \$0.val = \$1.in; \$1.in = \$0.val + 1; \}$

- It is impossible to evaluate either $\$0.val$ or $\$1.in$ first (each value depends on the other)
- We want to avoid circular dependencies
- Detecting such cases in all parse trees takes exponential time!
- S -attributed or L -attributed definitions cannot have cycles

Dependency Graphs

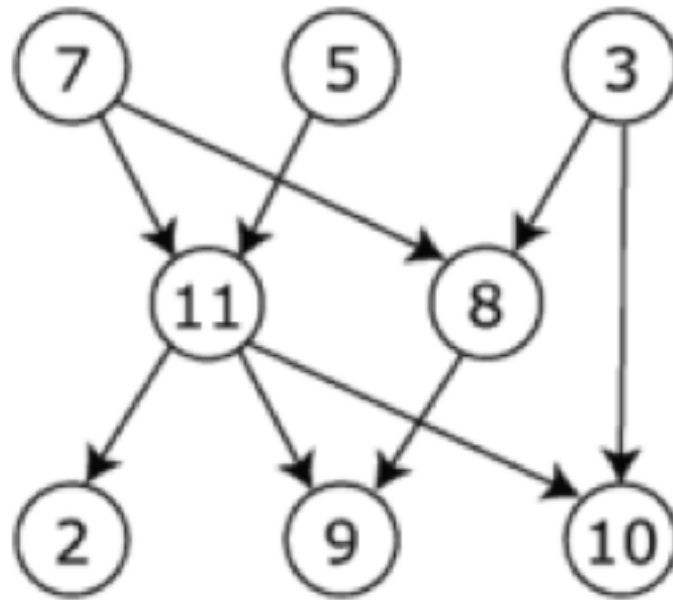
- Each dependency shows the flow of information in the parse tree
- All dependencies in each parse tree create a dependency graph

Dependency Graphs



Dependency Graphs

- Each dependency shows the flow of information in the parse tree
- All dependencies in each parse tree create a dependency graph
- Dependencies can be ordered and each ordering is called a **topological sort** of the dependency edges
- Each topological sort is a valid order of evaluation for semantic rules



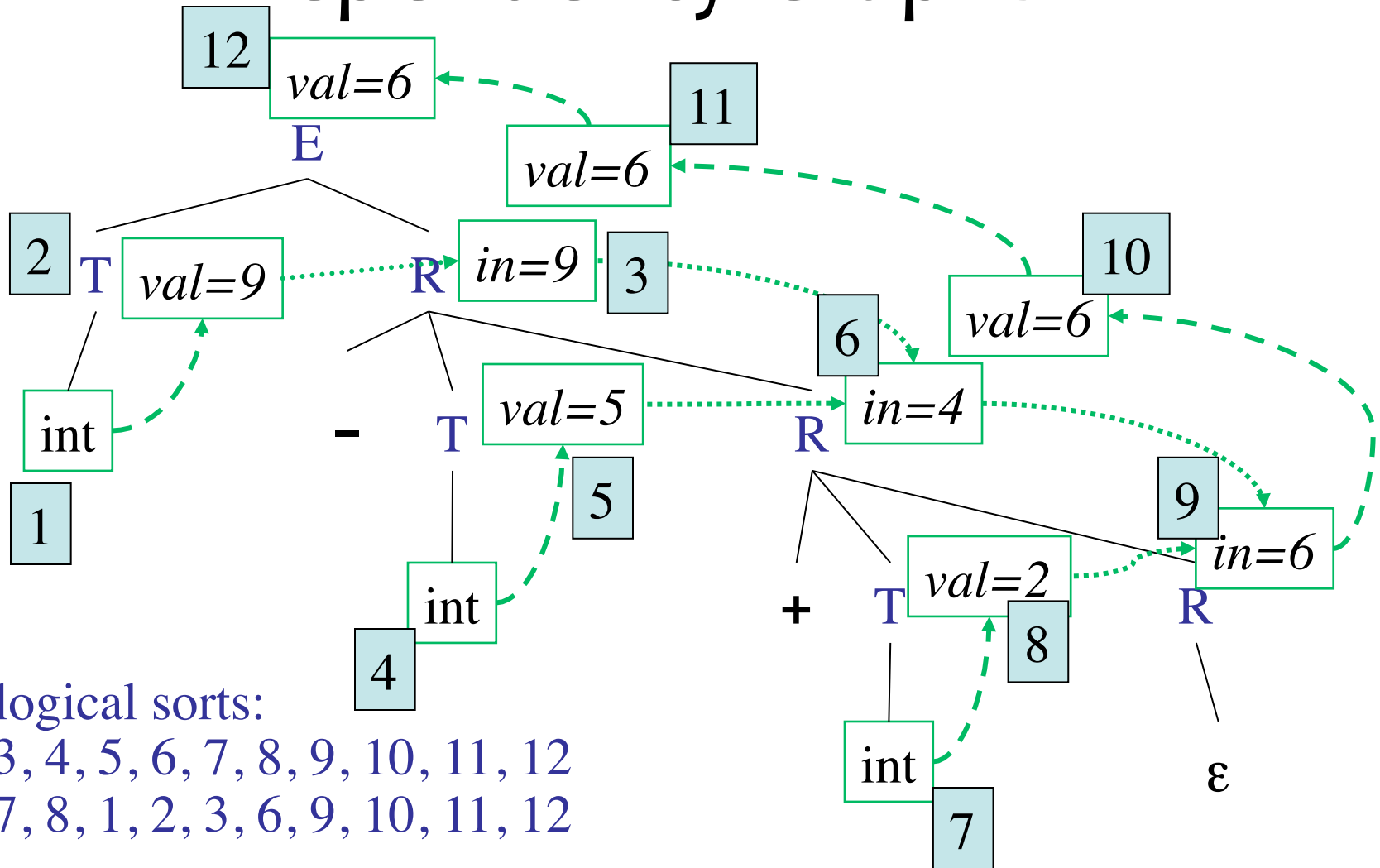
- A directed acyclic graph has many valid topological sort:
 - 7,5,3,11,8,2,9,10 (visual left-to-right top-to-bottom)
 - 3,5,7,8,11,2,9,10 (smallest-numbered available vertex first)
 - 3,7,8,5,11,10,2,9
 - 5,7,3,8,11,10,9,2 (least number of edges first)
 - 7,5,11,3,10,8,9,2 (largest-numbered available vertex first)
 - 7,5,11,2,3,8,9,10

Source: Wikipedia

Dependency Graphs

- Topological sort :
 - Order the nodes of the graph as N_1, \dots, N_k such that no edge in the graph goes from N_i to N_j ($j < i$)

Dependency Graphs



Topological sorts:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

4, 5, 7, 8, 1, 2, 3, 6, 9, 10, 11, 12

Syntax-directed definition with actions

- Some definitions can have side-effects:

$E \rightarrow T R$

```
{ $2.in = $1.val; $0.val = $2.val; printf("%s", $2.in); }
```

- When will these side-effects occur?
- The order of evaluating attributes is linked to the order of creating nodes in the parse tree

Syntax-directed definition with actions

- A definition with side-effects:
 - $B \rightarrow X \{a\} Y$
- Bottom-up parser:
 - Perform action a as soon as X appears on top of the stack (if X is a nonterminal, we can move action a to $X \rightarrow A_1 \dots A_n \{a\}$)
- Top-down parser:
 - Perform action a just before we attempt to expand Y (if Y is a non-terminal) or check for Y on the input (if Y is a terminal)

SDTs with Actions

- A syntax directed definition with actions:

$E \rightarrow T R$

$R \rightarrow + T \{ \text{print('+'); } \} R$

$R \rightarrow - T \{ \text{print('-'); } \} R$

$R \rightarrow \varepsilon$

$T \rightarrow \mathbf{int} \{ \text{print(int.lookup); } \}$

Stack	Input	Action
E \$	9-5+2\$	
T R \$	9-5+2\$	
int R \$	9-5+2\$	print(9)
R \$	-5+2\$	
- T R \$	-5+2\$	
T R \$	5+2\$	
int R \$	5+2\$	print(5)
R \$	+2\$	print(-)
+T R \$	+2\$	
T R \$	2\$	
int R \$	2\$	print(2)
R \$	\$	print(+)
\$	\$	terminate

Input: 9 - 5 + 2

$E \rightarrow T R$

$R \rightarrow + T \{ \text{print('+'); } \} R$

$R \rightarrow - T \{ \text{print('-'); } \} R$

$R \rightarrow \epsilon$

$T \rightarrow \text{int} \{ \text{print(int.lookup); } \}$

	+	-	int	\$
E			T R	
T			int	
R	+ T R	- T R		ϵ

output: 9 5 - 2 +

SDT maps infix expressions
to postfix

Actions in stack

- Action a for $B \rightarrow X \{a\} Y$ is pushed to the stack when the derivation step $B \rightarrow X \{a\} Y$ is made
- But the action is performed only after complete derivations for X has been carried out

Stack	Input	Action
E \$	9-5+2\$	
T R \$	9-5+2\$	
int R \$	9-5+2\$	print(9)
R \$	-5+2\$	
- T #A R \$	-5+2\$	
T #A R \$	5+2\$	
int #A R \$	5+2\$	print(5)
#A R \$	+2\$	print(-)
+T #B R \$	+2\$	
T #B R \$	2\$	
int #B R \$	2\$	print(2)
#B R \$	\$	print(+)
\$	\$	terminate

Input: 9 - 5 + 2

$E \rightarrow T R$

$R \rightarrow + T \{ \text{print('+'); } \} R$

$R \rightarrow - T \{ \text{print('-'); } \} R$

$R \rightarrow \epsilon$

$T \rightarrow \text{int} \{ \text{print(int.lookup); } \}$

	+	-	int	\$
E			T R	
T			int	
R	+ T R	- T R		ϵ

#A: print('-')

#B: print('+')

SDTs with Actions

- Syntax directed definition that tries to map infix expressions to prefix:

$E \rightarrow T R$

$R \rightarrow \{ \text{print('+'); } \} + T R$

$R \rightarrow \{ \text{print('-'); } \} - T R$

$R \rightarrow \epsilon$

$T \rightarrow \text{int} \{ \text{print(int.lookup); } \}$

Impossible to implement SDT during either top-down or bottom-up parsing, because the parser would have to perform printing actions long before it knows whether these symbols will appear in its input.

Marker non-terminals

- Bottom-up translation for L-attributed definitions:
 - Assumption: each symbol X has one synthesized (X_{val}) and one inherited (X_{in}) attribute (or actions)
1. Replace each $A \rightarrow X_1 \dots X_n$ by:

$$A \rightarrow M_1 X_1 \dots M_n X_n, M_i \rightarrow \varepsilon \text{ (new marker non-terminals)}$$
 2. When reducing by $M_i \rightarrow \varepsilon$

Compute $X_i.in$ ($M_i.val = X_i.in$)
 push it into stack

M_i	$X_i.in$
X_{i-1}	$X_{i-1}.val$
M_{i-1}	$X_{i-1}.in$
\vdots	\vdots
\vdots	\vdots
X_1	$X_1.val$
M_1	$X_1.in$
M_A	$A.in$
\vdots	\vdots

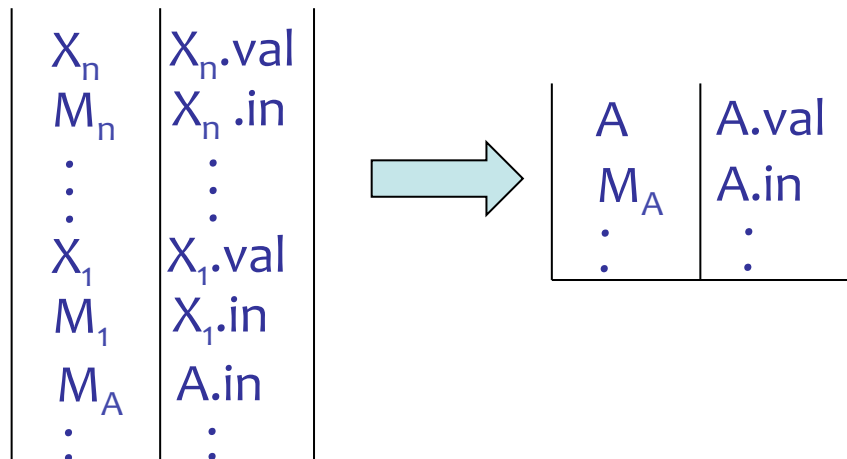
Marker non-terminals

3. When reducing by $A \rightarrow M_1 X_1 \dots M_n X_n$:

$$A.val = f(M_1.val, X_1.val \dots M_n.val, X_n.val)$$

Push A into stack

($M_i.val = X_i.in$)



4. Simplification: if X_j has no attributes or is computed by a copy rule $X_j.in = X_{j-1}.val$ discard M_j ; adjust indices suitably. If $X_1.in$ exist and $X_1.in = A.in$, omit M_1

Marker Non-terminals

$E \rightarrow T R$

$R \rightarrow + T \{ \text{print('+'); } \} R$

$R \rightarrow - T \{ \text{print('-'); } \} R$

$R \rightarrow \varepsilon$

$T \rightarrow \mathbf{int} \{ \text{print(int.lookup); } \}$

Marker Non-terminals

$E \rightarrow T R$

$R \rightarrow + T M R$

$R \rightarrow - T N R$

$R \rightarrow \varepsilon$

$T \rightarrow \mathbf{int} \{ \text{print}(\mathbf{int.lookup}); \}$

$M \rightarrow \varepsilon \{ \text{print}(' + '); \}$

$N \rightarrow \varepsilon \{ \text{print}(' - '); \}$

Equivalent SDT using
marker non-terminals

Impossible Syntax-directed Definition

$E \rightarrow \{ \text{print('+'); } \} E + T$
 $E \rightarrow \{ \text{print('-'); } \} E - T$
 $E \rightarrow T$
 $T \rightarrow (E)$
 $T \rightarrow \text{int}\{ \text{print(int.lookup); } \}$

Tries to convert
infix to prefix

$E \rightarrow M E + T$
 $E \rightarrow N E - T$
 $E \rightarrow T$
 $M \rightarrow \epsilon \{ \text{print('+'); } \}$
 $N \rightarrow \epsilon \{ \text{print('-'); } \}$
 $T \rightarrow (E)$
 $T \rightarrow \text{int}\{ \text{print(int.lookup); } \}$

Causes a reduce/reduce conflict
when marker non-terminals are
introduced.

Summary

- The parser produces concrete syntax trees
- Abstract syntax trees: define semantic checks or a syntax-directed translation to the desired output
- Attribute grammars: static definition of syntax-directed translation
 - Synthesized and Inherited attributes
 - S-attribute grammars
 - L-attributed grammars
- Complex inherited attributes can be defined if the full parse tree is available

Extra Slides

Syntax-directed defns

- LR parser, S-attributed definition
 - more details later ...
- LL parser, L-attributed definition

Stack	Input	Output
\$T')T'F	id)*id\$	$T \rightarrow F T' \{ \$2.in = \$1.val \}$
\$T')T'id	id)*id\$	$F \rightarrow id \{ \$0.val = \$1.val \}$

\$T')T'))*id\$

action record:
T'.in = F.val

The action record stays on the stack when T' is replaced with rhs of rule

LR parsing and inherited attributes

- As we just saw, inherited attributes are possible when doing top-down parsing
- How can we compute inherited attributes in a bottom-up shift-reduce parser
- Problem: doing it incrementally (while parsing)
- Note that LR parsing implies depth-first visit which matches L-attributed definitions

LR parsing and inherited attributes

- Attributes can be stored on the stack used by the shift-reduce parsing
- For synthesized attributes: when a reduce action is invoked, store the value on the stack based on value popped from stack
- For inherited attributes: transmit the attribute value when executing the **goto** function

Example: Synthesized Attributes

$T \rightarrow F \quad \{ \$0.val = \$1.val; \}$

$T \rightarrow T * F$

$\{ \$0.val = \$1.val * \$3.val; \}$

$F \rightarrow \mathbf{id}$

$\{ \text{val} := \mathbf{id}.lookup();$

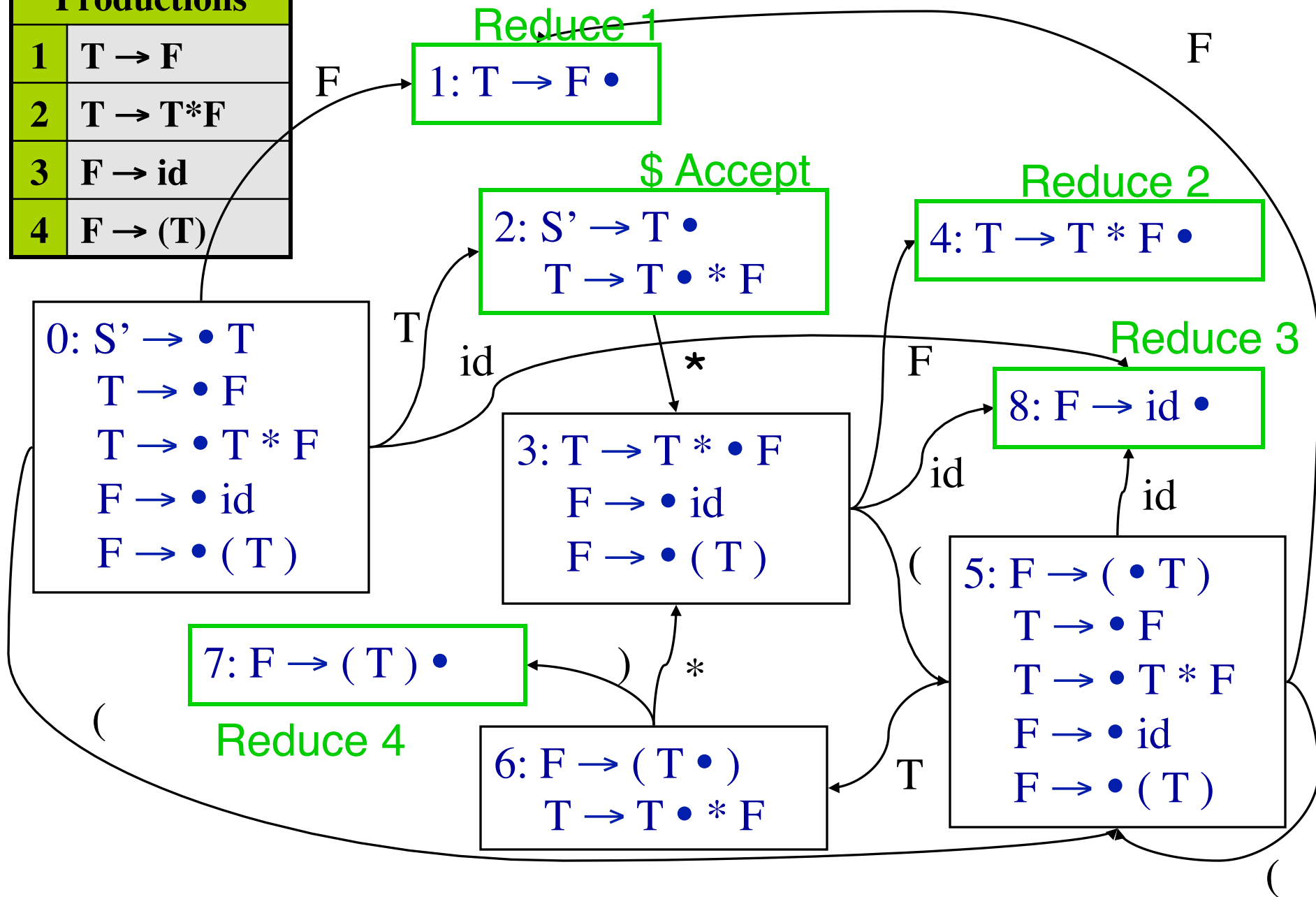
$\text{if (val)} \{ \$0.val = \$1.val; \}$

$\text{else } \{ \text{error}; \}$

$\}$

$F \rightarrow (T) \quad \{ \$0.val = \$2.val; \}$

Productions	
1	$T \rightarrow F$
2	$T \rightarrow T * F$
3	$F \rightarrow id$
4	$F \rightarrow (T)$



Trace “(id_{val=3})*id_{val=2}”

Stack	Input	Action	Attributes
0	(id) * id \$	Shift 5	
0 5	id) * id \$	Shift 8	a.Push id.val=3;
0 5 8) * id \$	Reduce 3 F→id, pop 8, goto [5,F]=1	{ \$0.val = \$1.val }
0 5 1) * id \$	Reduce 1 T→ F, pop 1, goto [5,T]=6	a.Pop; a.Push 3;
0 5 6) * id \$	Shift 7	{ \$0.val = \$1.val }
0 5 6 7	* id \$	Reduce 4 F→ (T), pop 7 6 5, goto [0,F]=1	a.Pop; a.Push 3;
			{ \$0.val = \$2.val }
			3 pops; a.Push 3

Trace “(id_{val=3})*id_{val=2}”

Stack	Input	Action	Attributes
0 1	* id \$	Reduce 1 T→F, pop 1, goto [0,T]=2	{ \$0.val = \$1.val } a.Pop; a.Push 3
0 2	* id \$	Shift 3	a.Push mul
0 2 3	id \$	Shift 8	a.Push id.val=2
0 2 3 8	\$	Reduce 3 F→id, pop 8, goto [3,F]=4	a.Pop a.Push 2
0 2 3 4	\$	Reduce 2 T→T * F pop 4 3 2, goto [0,T]=2	{ \$0.val = \$1.val * \$3.val; }
0 2	\$	Accept	3 pops; a.Push 3*2=6

Example: Inherited Attributes

$E \rightarrow T R$

$\{ \$2.in = \$1.val; \$0.val = \$2.val; \}$

$R \rightarrow + T R$

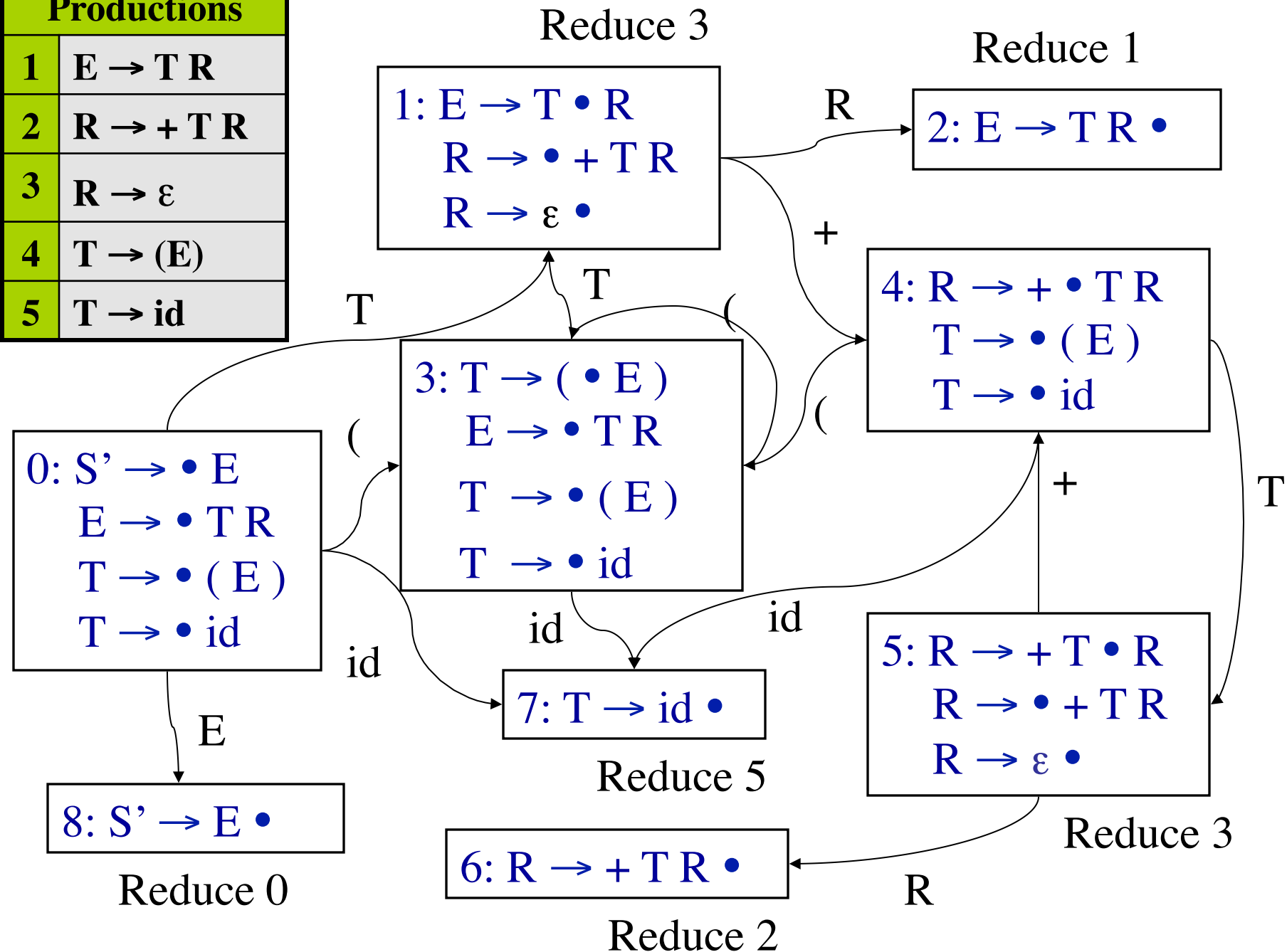
$\{ \$3.in = \$0.in + \$2.val; \$0.val = \$3.val; \}$

$R \rightarrow \epsilon \{ \$0.val = \$0.in; \}$

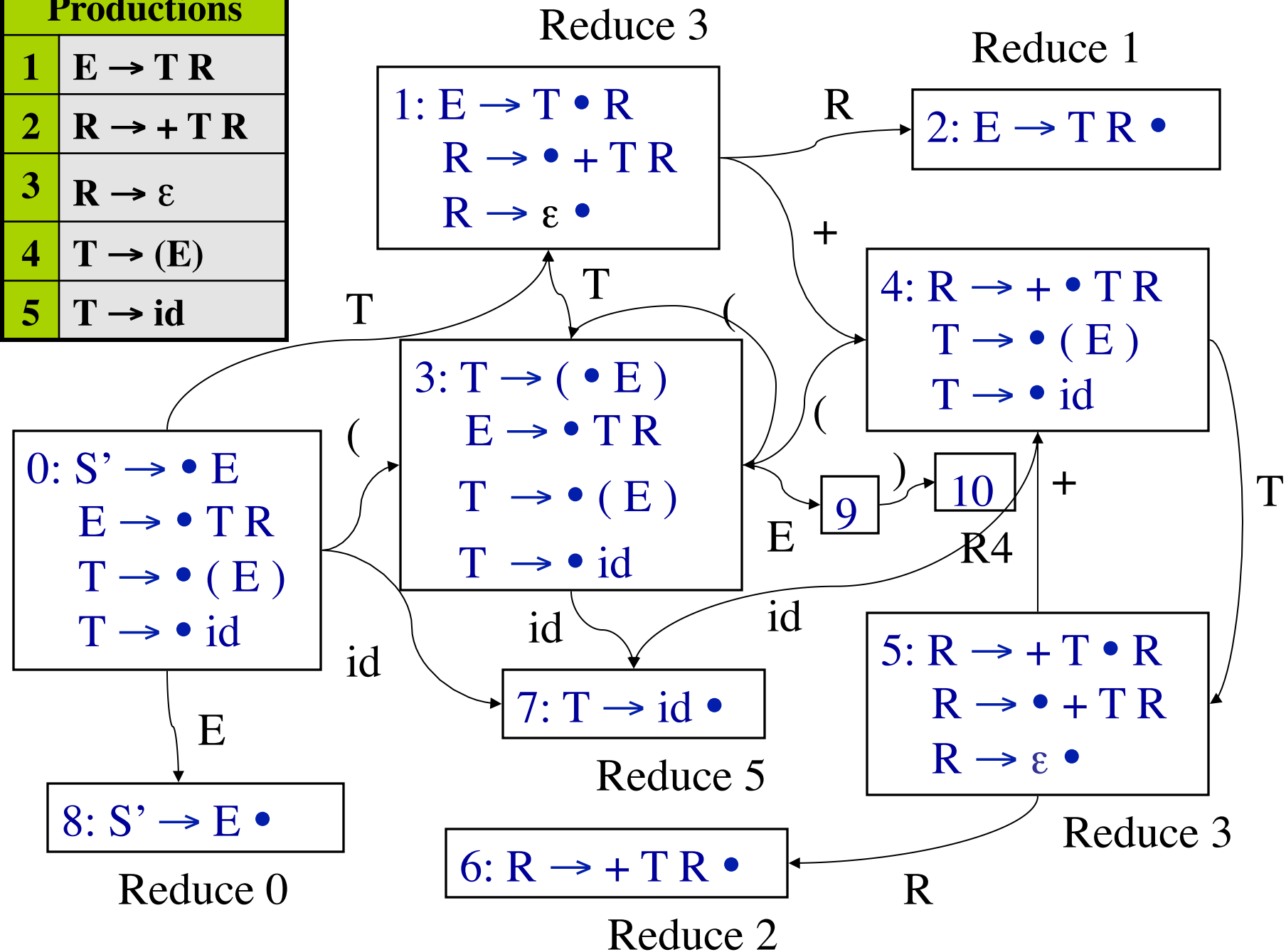
$T \rightarrow (E) \{ \$0.val = \$1.val; \}$

$T \rightarrow \mathbf{id} \{ \$0.val = \mathbf{id}.lookup; \}$

Productions	
1	$E \rightarrow T R$
2	$R \rightarrow + T R$
3	$R \rightarrow \varepsilon$
4	$T \rightarrow (E)$
5	$T \rightarrow id$



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1	$E \rightarrow T R$
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Productions	
1	$E \rightarrow T R \{ \$2.in = \$1.val; \$0.val = \$2.val; \}$
2	$R \rightarrow + T R \{ \$3.in = \$0.in + \$2.val; \$0.val = \$3.val; \}$
3	$R \rightarrow \epsilon \{ \$0.val = \$0.in; \}$
4	$T \rightarrow (E) \{ \$0.val = \$1.val; \}$
5	$T \rightarrow id \{ \$0.val = id.lookup; \}$

Attributes
$\{ \$0.val = id.lookup \}$ $\{ \text{pop}; attr.Push(3) \}$ $\$2.in = \$1.val$ $\$2.in := (1).attr \}$
$\{ \$0.val = id.lookup \}$ $\{ \text{pop}; attr.Push(2); \}$
$\{ \$3.in = \$0.in + \$1.val$ $(5).attr := (1).attr + 2$ $\$0.val = \$0.in$ $\$0.val = (5).attr = 5 \}$

0 7	T id \$	Reduce 5 T→id
0 1	+ id \$	pop 7, goto [0,T]=1
0 1 4	id \$	Shift 4
0 1 4 7	\$	Shift 7
0 1 4 5	\$	Reduce 5 T→id
		pop 7, goto [4,T]=5
		Reduce 3 R→ε
		goto [5,R]=6

Trace “ $\text{id}_{\text{val}=3} + \text{id}_{\text{val}=2}$ ”

Stack	Input	Action	Attributes
0	id + id \$	Shift 7	
0 7	+ id \$	Reduce 5 $T \rightarrow \text{id}$ pop 7, goto [0,T]=1	{ $\\$0.\text{val} = \text{id.lookup}$ } { pop; attr.Push(3)
0 1	+ id \$	Shift 4	$\\$2.\text{in} = \\$1.\text{val}$
0 1 4	id \$	Shift 7	$\\$2.\text{in} := (1).\text{attr}$ }
0 1 4 7	\$	Reduce 5 $T \rightarrow \text{id}$ pop 7, goto [4,T]=5	{ $\\$0.\text{val} = \text{id.lookup}$ } { pop; attr.Push(2); }
0 1 4 5	\$	Reduce 3 $R \rightarrow \epsilon$ goto [5,R]=6	{ $\\$3.\text{in} = \\$0.\text{in} + \\$1.\text{val}$ $(5).\text{attr} := (1).\text{attr} + 2$ $\\$0.\text{val} = \\$0.\text{in}$ $\\$0.\text{val} = (5).\text{attr} = 5$ }

Trace “ $\text{id}_{\text{val}=3} + \text{id}_{\text{val}=2}$ ”

Stack	Input	Action	Attributes
0 1 4 5 6	\$	Reduce 2 $R \rightarrow + T R$ Pop 4 5 6, goto [1,R]=2	{ $\\$0.\text{val} = \\$3.\text{val}$ pop; attr.Push(5); } <hr/>
0 1 2	\$	Reduce 1 $E \rightarrow T R$ Pop 1 2, goto [0,E]=8	{ $\\$0.\text{val} = \\$3.\text{val}$ pop; attr.Push(5); } <hr/>
0 8	\$	Accept	{ $\\$0.\text{val} = 5$ attr.top = 5; }

$A \rightarrow c \{ \$0.val = \$0.in \}$

LR parsing with inherited attributes

Bottom-Up/rightmost

ccbca \Leftarrow Acbca

$A \rightarrow c$

\Leftarrow AcbB

$B \rightarrow ca$

line 3

\Leftarrow AB

$B \rightarrow cbB$

\Leftarrow S

$S \rightarrow AB$

Parse stack at line 3:

['x'] A ['x'] c b B

$\$1.in = 'x'$

$\$2.in = \$1.val$

Consider:

$S \rightarrow AB$

$\{ \$1.in = 'x';$
 $\$2.in = \$1.val \}$

$B \rightarrow cbB$

$\{ \$0.val = \$0.in + 'y'; \}$

Parse stack at line 4:

['x'] A B

['xy']