

Register Allocation

CMPT 379: Compilers

Instructor: Anoop Sarkar

anoopsarkar.github.io/compilers-class

Register Allocation

- Intermediate code uses unlimited temporaries
 - Simplifying code generation and optimization
 - Complicates final translation to assembly
- Intermediate code uses too many temporaries

Register Allocation

- The problem:

Rewrite the intermediate code to use no more temporaries than there are machine registers

- Method:

- Assign multiple temporaries to each register
- But without changing the program behavior

Example

- Consider the program

$$a = c + d$$

$$e = a + b$$

$$f = e - 1$$

- Assume a & e dead after use
 - A dead temporary can be “reused”

- Can allocate a , e and f all to one register (r_1)

$$r_1 = r_2 + r_3$$

$$r_1 = r_1 + r_4$$

$$r_1 = r_1 - 1$$

History

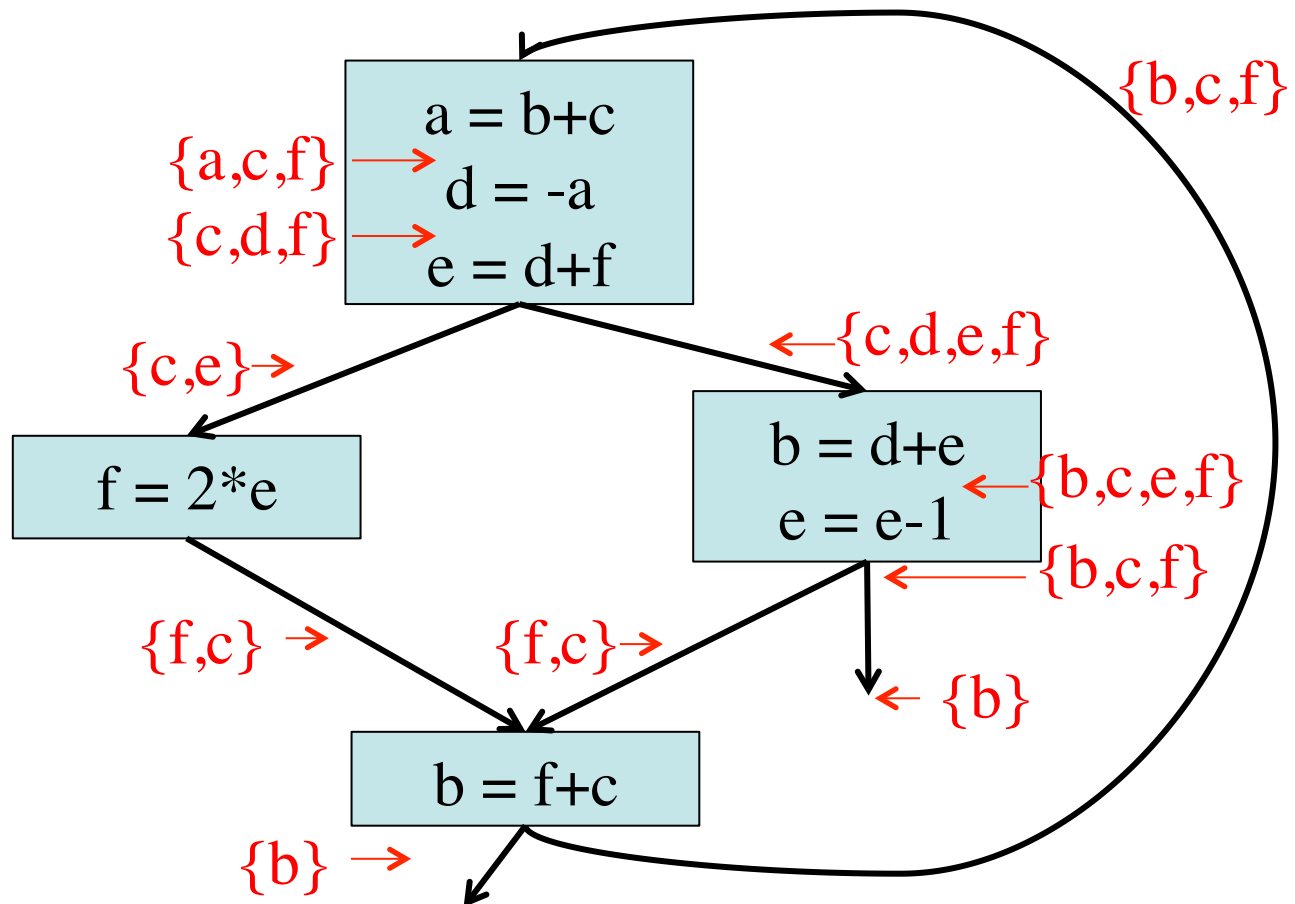
- Register allocation is as old as compilers
 - Register allocation was used in the original FORTRAN compiler in 1950's
 - Very crude algorithm
- A breakthrough came in 1980
 - Register allocation scheme based on graph coloring
 - Relatively simple, global and works well in practice

Principles of Register Allocation

- Temporaries t_1 and t_2 can share the same register if *at any point in the program at most one of t_1 or t_2 is live*
 - If t_1 and t_2 are live at the same time, they cannot share a register
- We need liveness analysis

Live Variables

- Compute live variables for each point

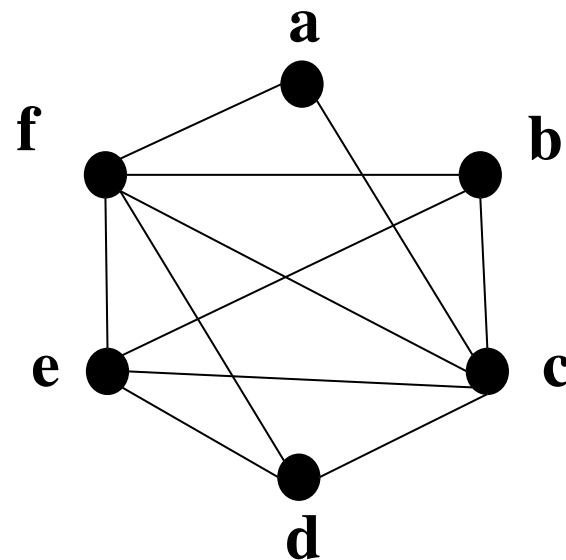


Register Interference Graph

- Construct an undirected graph
 - A node for each temporary
 - An edge between t_1 and t_2 if they are live simultaneously at some point in the program
- This is the *register interference graph* (RIG)
 - Two temporaries can be allocated to the same register if there is no edge connecting them

Register Interference Graph

- For our example



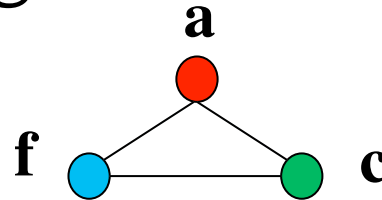
- **a** and **c** cannot be in the same register
- **a** and **d** could be in the same register

Register Interference Graph

- Extracts exactly the information we need to characterize legal register allocation
- Gives the global view (i.e., over the entire control flow graph) picture of the register requirements
- After RIG construction the register allocation algorithm is architecture independent

Graph Coloring

- A coloring of a graph is an assignment of colors to nodes, such that nodes connected by an edge have different colors



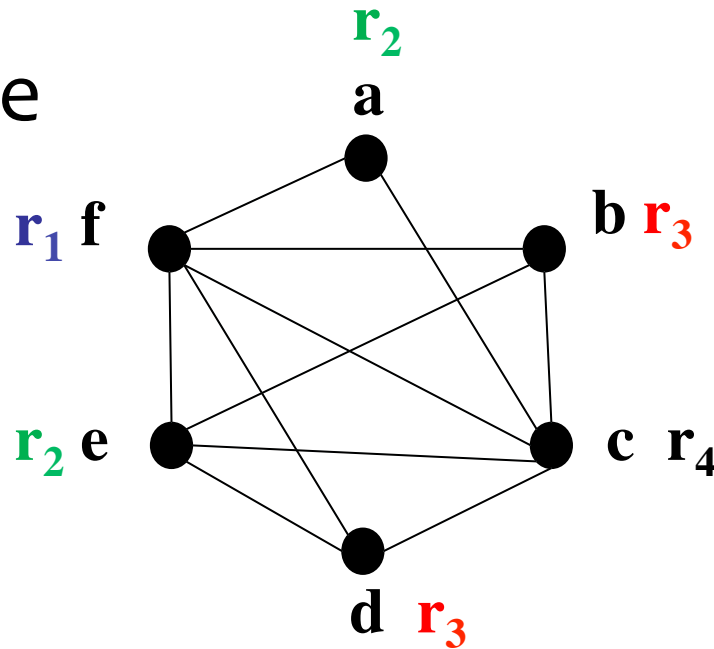
- A graph is k -colorable if it has a coloring with k colors

Register Allocation as Graph Coloring

- In our problem, colors = registers
- We need to assign colors (registers) to graph nodes (temporaries)
- Let k = number of machine registers
- If the RIG is k -colorable then there is a register assignment that uses no more than k registers

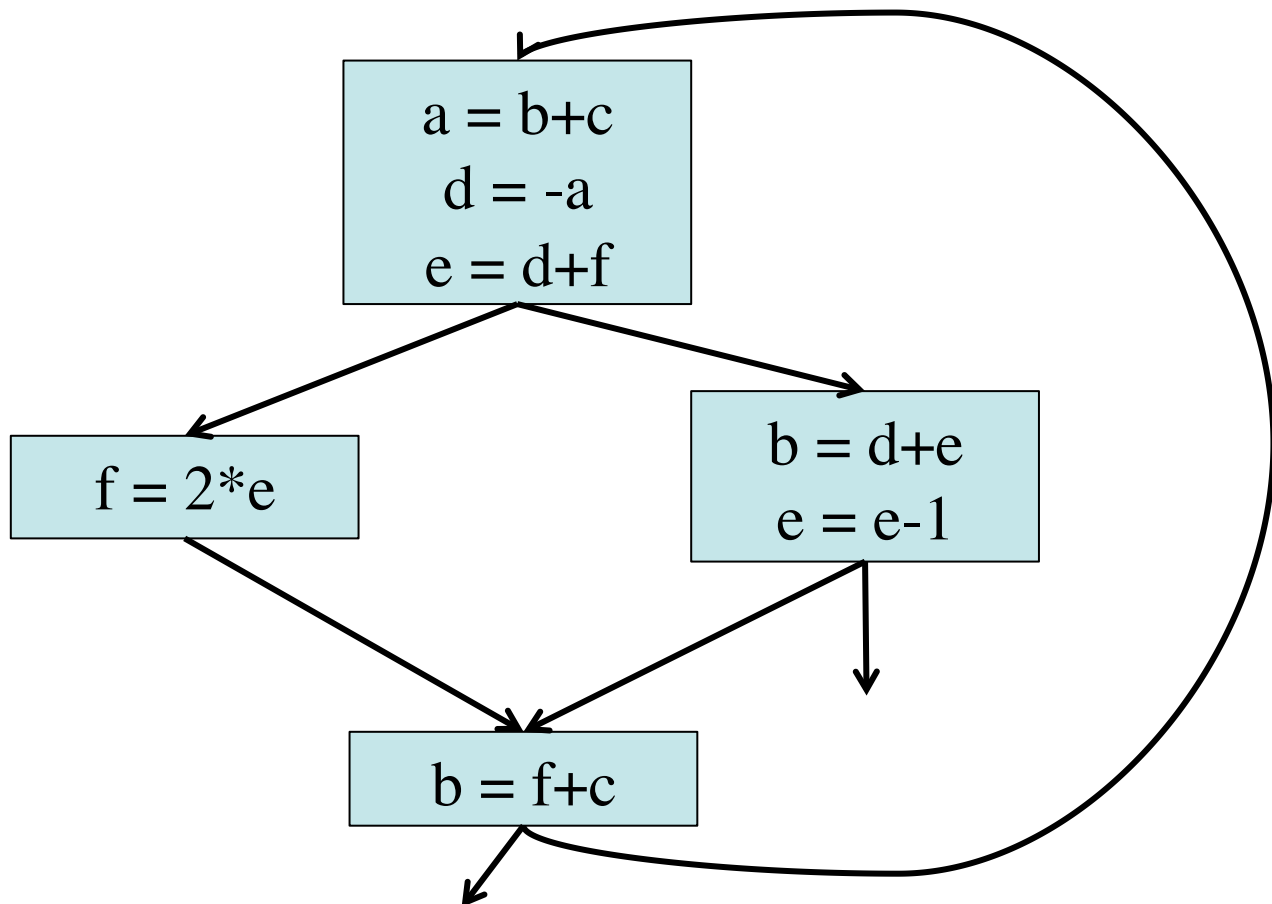
Example

- For our example

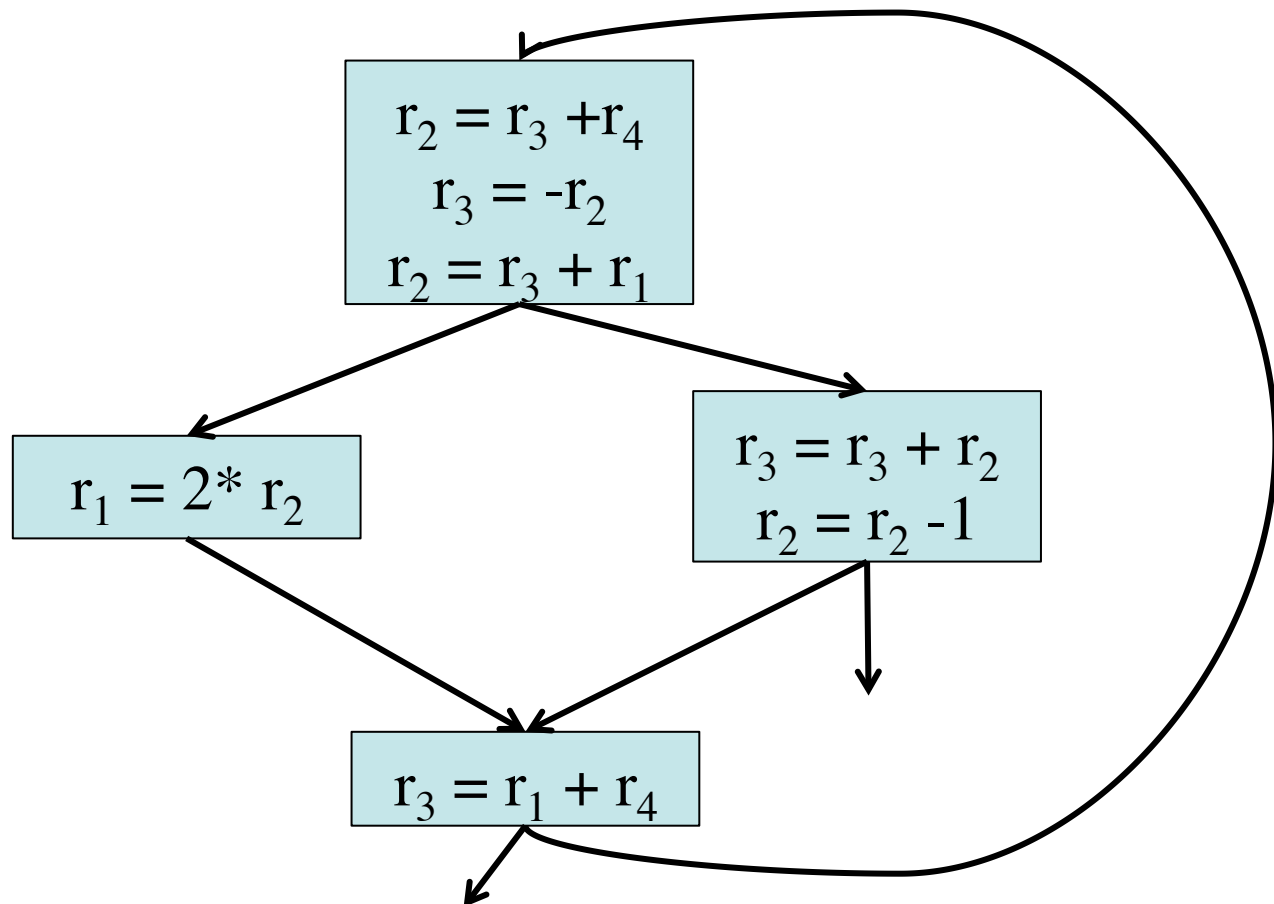


- There is no coloring with less than 4 colors
- There is a 4-coloring of this graph

Control Flow Graph



Register Allocation



Graph Coloring

- How do we compute graph coloring?
- It is not easy :
 - The problem is NP-hard. No efficient algorithms are known
 - Solution: use heuristics
 - A coloring might not exist for a given number of registers
 - Solution: register spilling

Register Allocation as Graph Coloring

- Main idea for solving whether a graph G is k -colorable:
- Pick any node t with fewer than k neighbor's
- Remove n and adjacent edges to create a new graph G'
- If G' is k -colorable, then so is G (the original graph)
- Let c_1, \dots, c_n be the colors assigned to the neighbors of t in G'
- Since $n < k$ we can pick some color for t that is different from its neighbors

Register Allocation as Graph Coloring

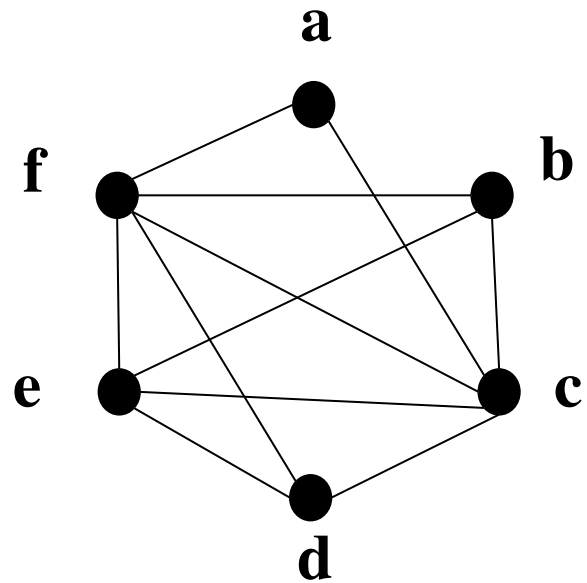
- Heuristic for graph coloring:
 - Ordering nodes (in an stack)
 1. Pick a node t with fewer than k neighbors
 2. Put t on a stack and remove it from the register interference graph (RIG)
 3. Repeat until the graph is empty
 - Assigning color to nodes on the stack:
 1. Start with the last node added
 2. At each step pick a color different from those assigned to already colored neighbors

Example

- Assume $k=4$

Remove **a**

stack={}

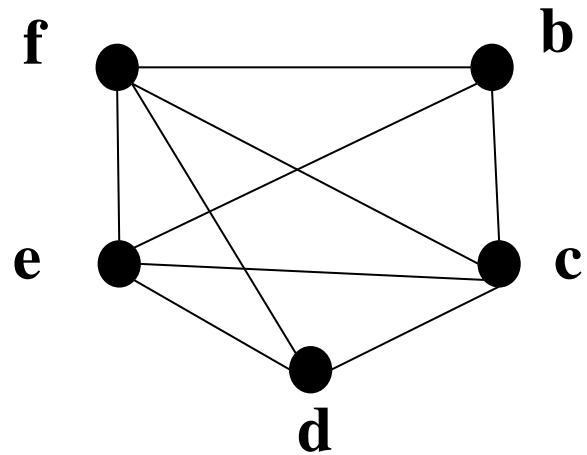


Example

- Assume $k=4$

Remove **d**

stack={a}

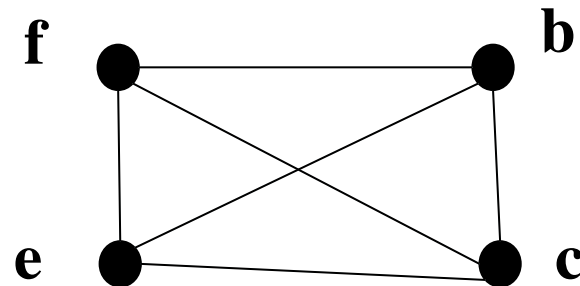


Example

- Assume $k=4$

Note: All nodes now have fewer than 4 neighbors

The graph coloring is
guaranteed to succeed



Remove **c**

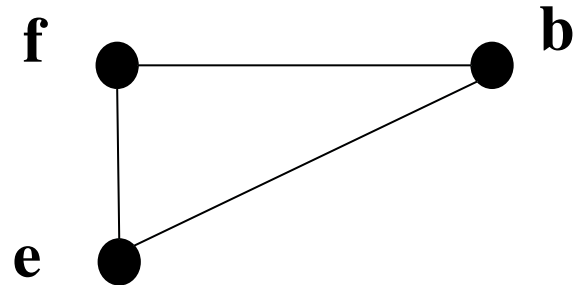
stack={d,a}

Example

- Assume $k=4$

Remove **b**

stack={c,d,a}

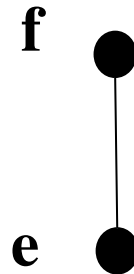


Example

- Assume $k=4$

Remove **e**

stack={b,c,d,a}



Example

- Assume $k=4$

f ●

Remove f

stack={e,b,c,d,a}

Example

- Assume $k=4$

Empty graph – done with the first part

Now we have the order for assigning colors to nodes, start coloring the nodes (from the top of the stack)

$\text{stack}=\{f,e,b,c,d,a\}$

Example

- Assume $k=4$

r_1 f ●

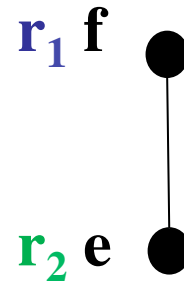
stack={e,b,c,d,a}

Example

- Assume $k=4$

e must be in a different register from f

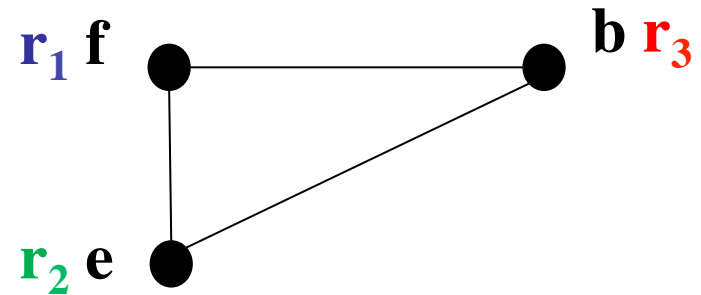
stack={b,c,d,a}



Example

- Assume $k=4$

stack={c,d,a}

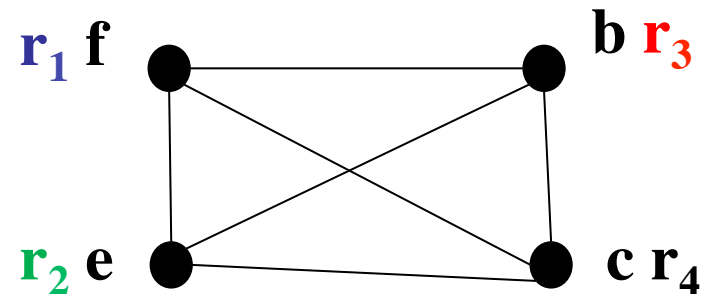


Example

- Assume $k=4$

The ordering insures we can find a color for all nodes

stack={d,a}

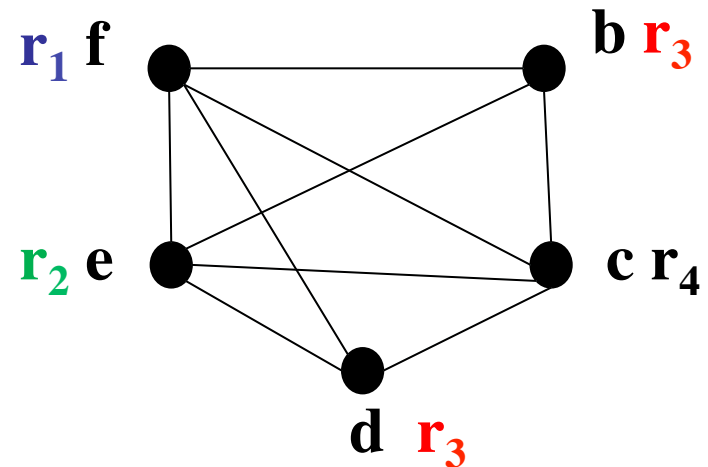


Example

- Assume $k=4$

d can be in the same register as b

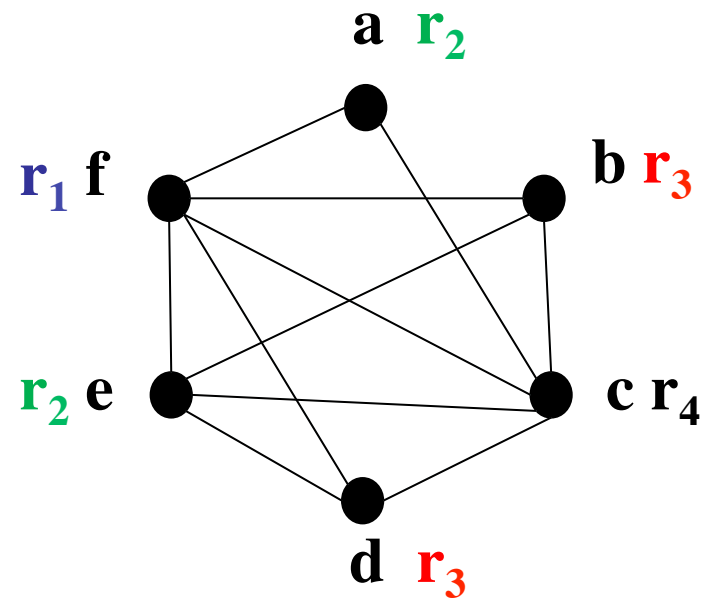
stack={a}



Example

- Assume $k=4$

stack={}



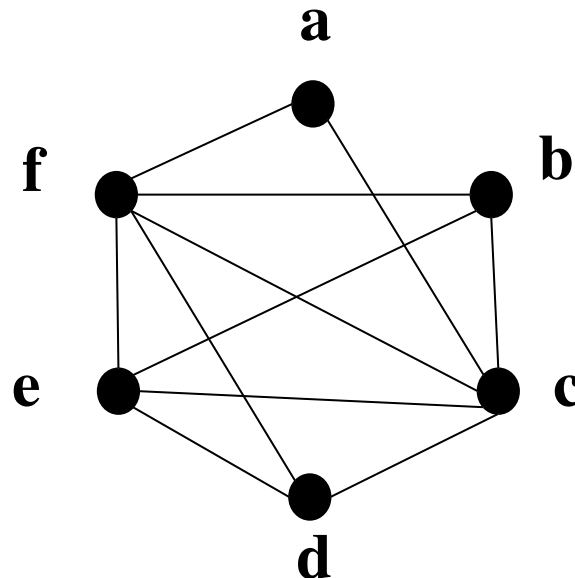
Register Allocation as Graph Coloring

- What happens if the graph coloring heuristic fails to find a coloring?
- In this case we cannot hold all values in the registers
 - Some values should be *spilled* to memory

K-coloring fails

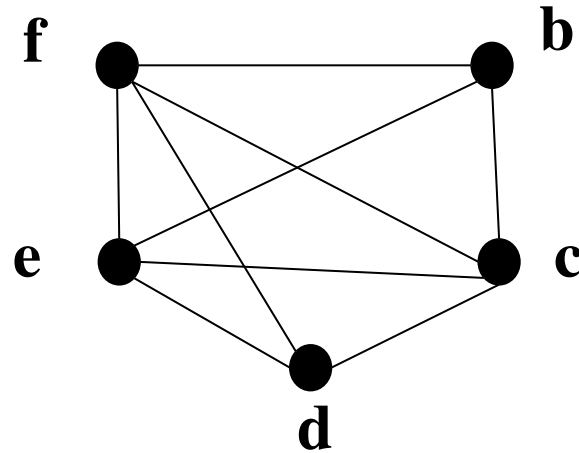
- What if all nodes have k or more neighbors?
- Try to find a 3 coloring of this graph

Remove **a**



Example of 3-coloring

- There is no node such that if we remove it then 3-coloring for the graph is available

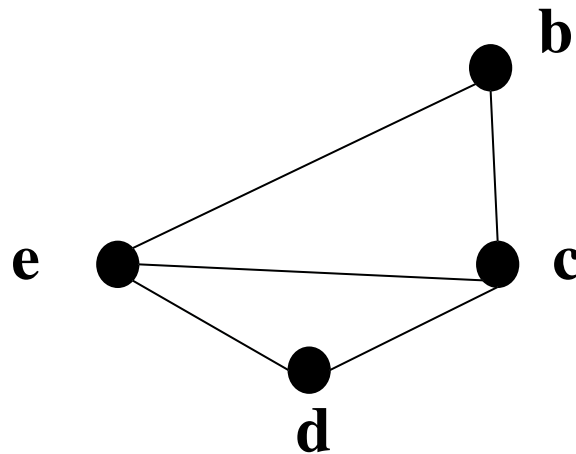


Optimistic Coloring

- If every node in G has more than k neighbors, k -coloring of G might not be possible
- Pick a node as candidate for spilling, remove it from the graph and continue k -coloring

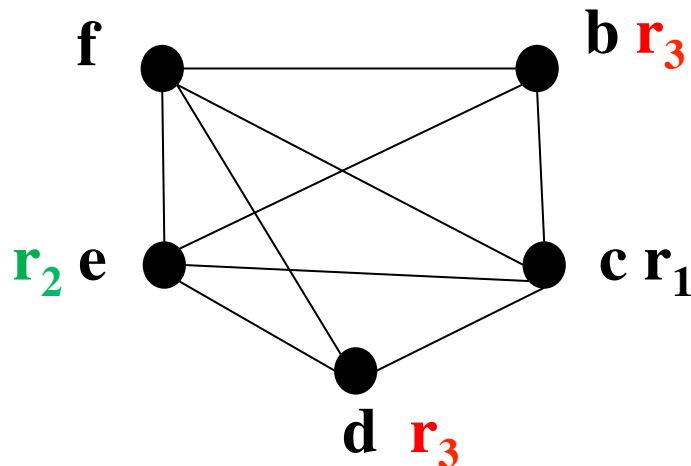
Optimistic Coloring

- Remove **f** and continue:
 - The ordering: {c,e,d,b,**f**,a}



Optimistic Coloring

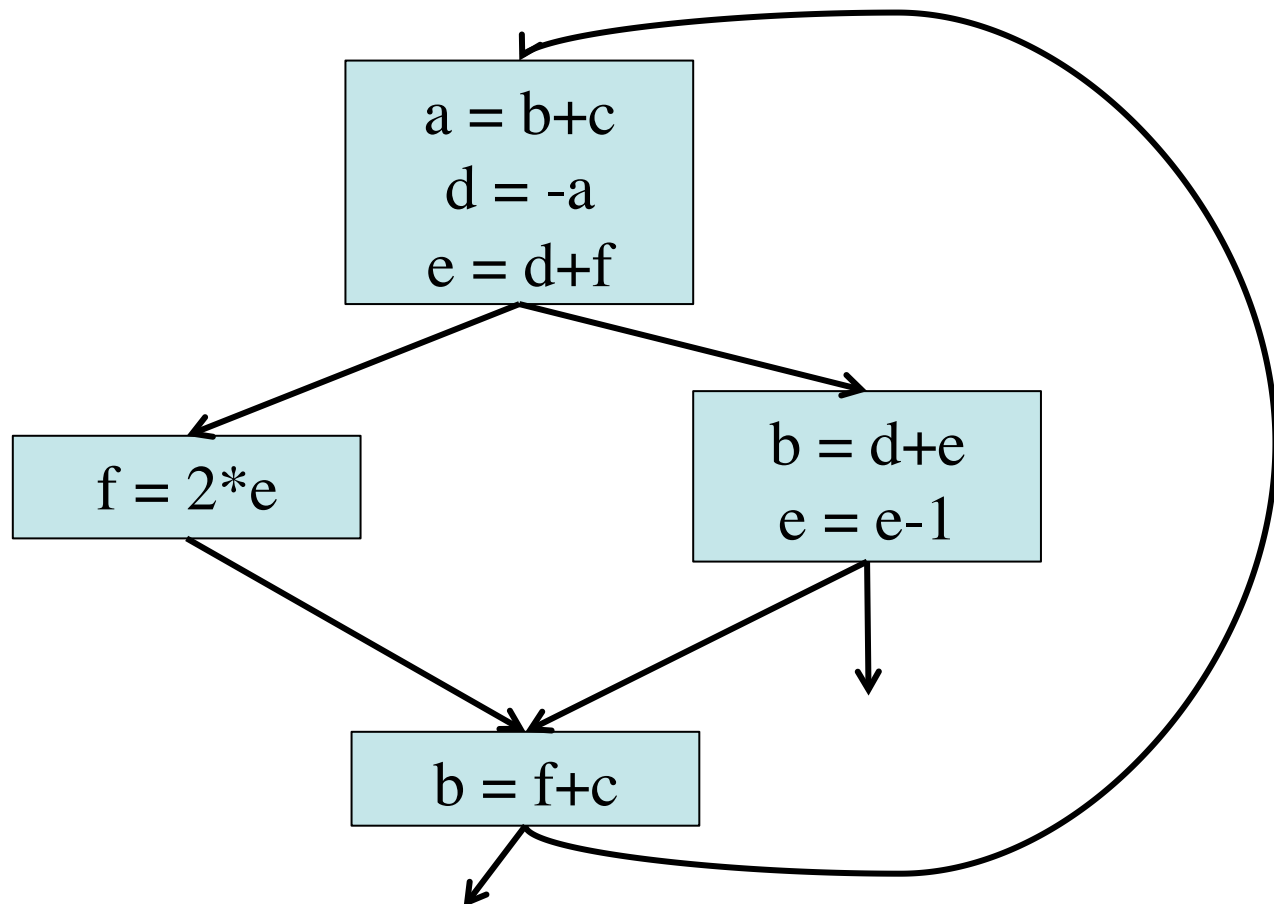
- Color the nodes $\{c, e, d, b, f, a\}$
- Try to assign a color to f
- We hope that among 4 neighbors of f we use less than 3 colors (*optimistic coloring*)



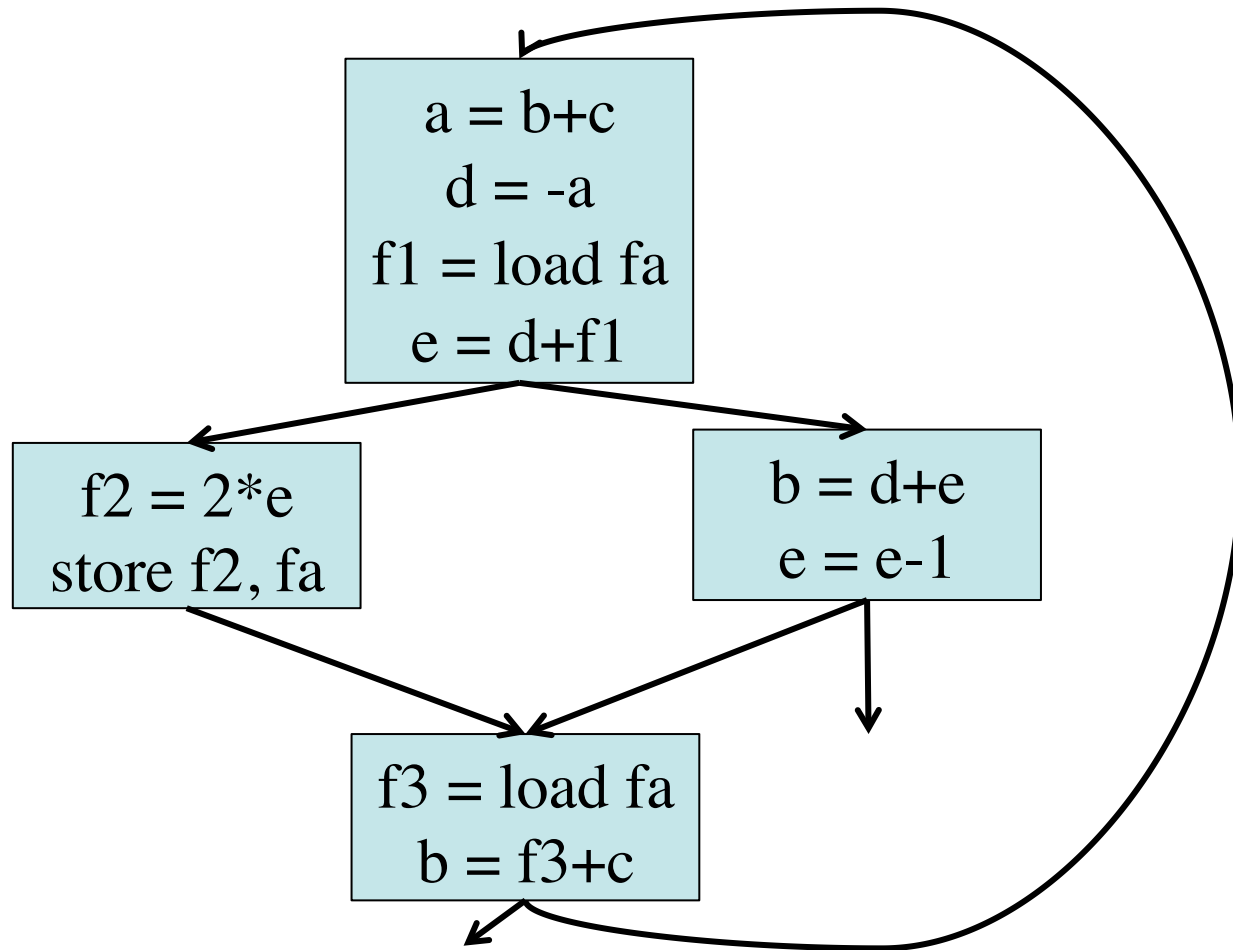
Spilling

- If optimistic coloring fails, we spill **f**
 - Allocate a memory location for **f**
 - Typically in the current stack frame
 - Call this address **fa**
- Before each operation that reads **f**, insert
f = load fa
- After each operation that writes **f**, insert
store f, fa
- Spilling is slow but sometimes necessary.

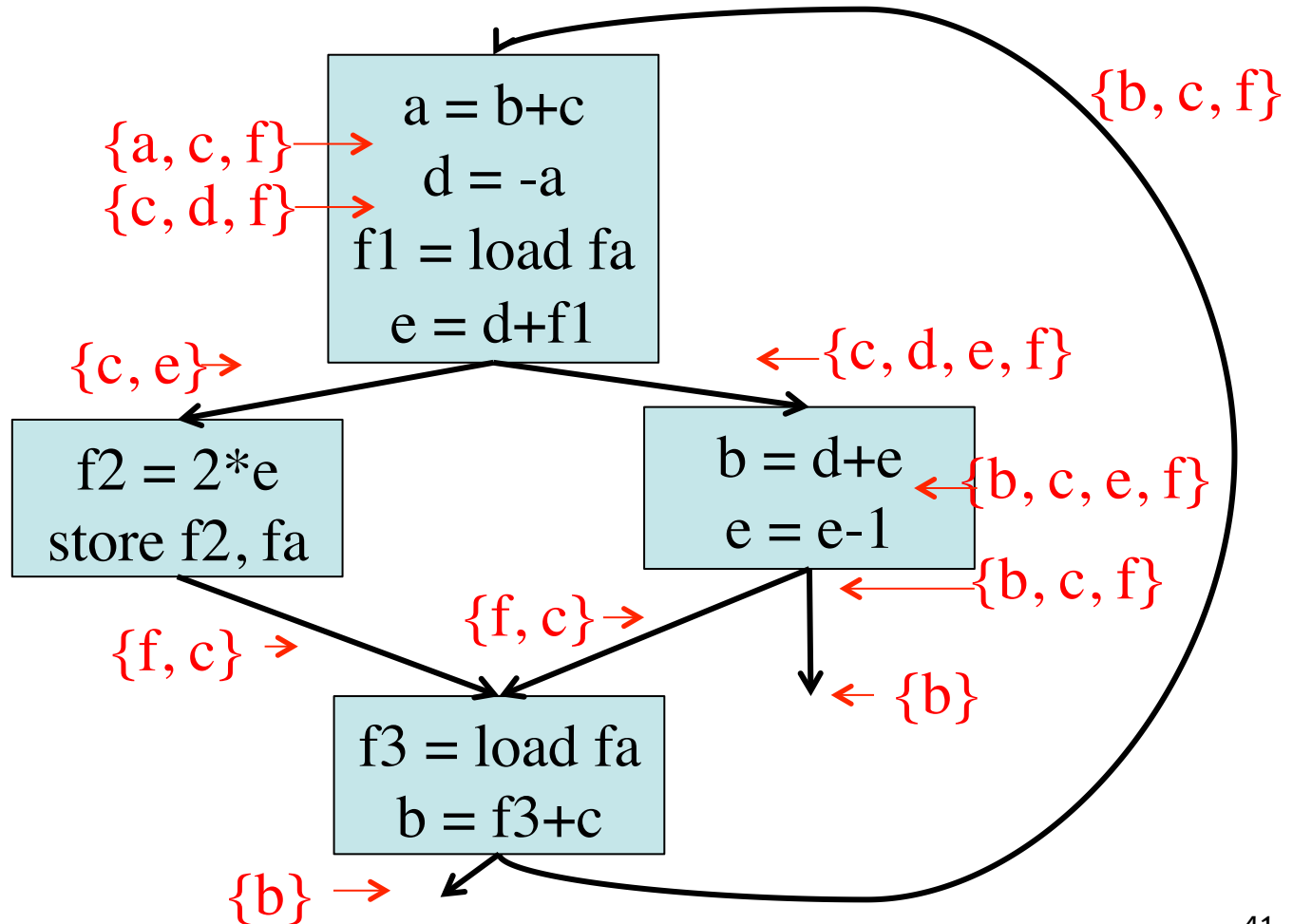
Original Code



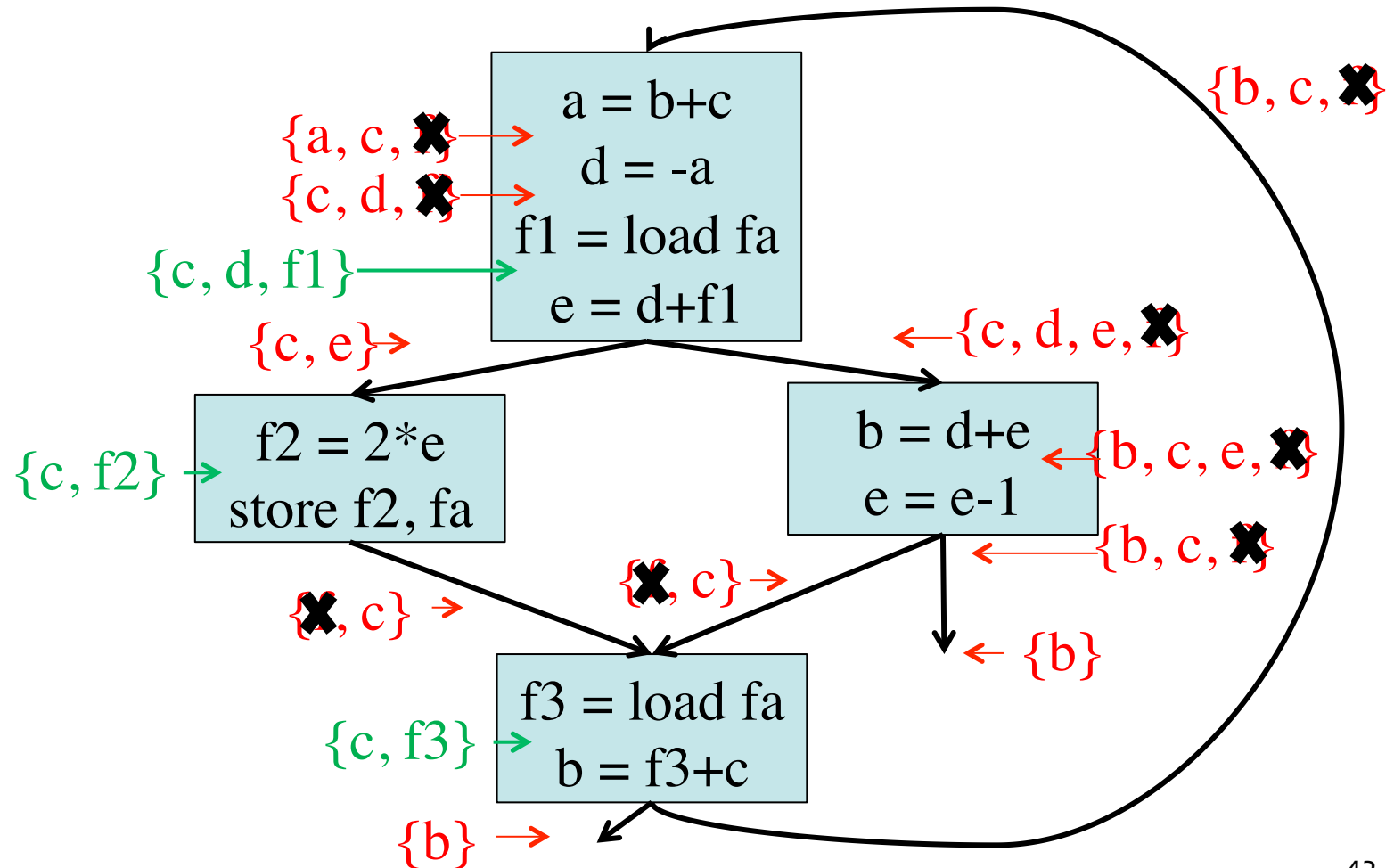
Code after Spilling f



Recompute the Liveness



Recompute the Liveness

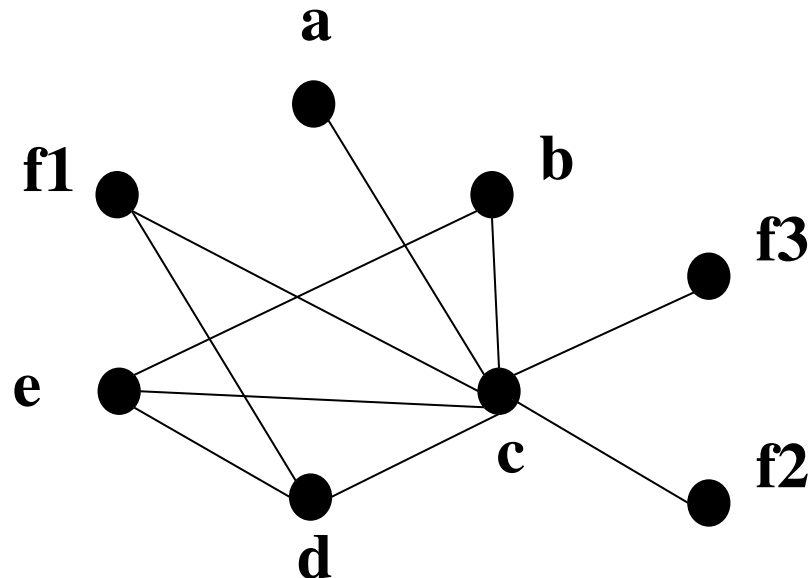


Rebuild the Interference Graph

- New liveness information is almost as before
 - Note **f** has been split into three temporaries
- **fi** is live only
 - Between a **fi = load fa** and the next instruction
 - Between a **store fi, fa** and the preceding instr.
- Spilling reduces the live range of **f**
 - And thus reduces its interferences
 - Which results in fewer RIG neighbors

Rebuild the Interference Graph

- Some edges of the spilled nodes are removed
- In our case **f** still interferes only with **c** and **d**
- And the new RIG is 3-colorable



Spilling

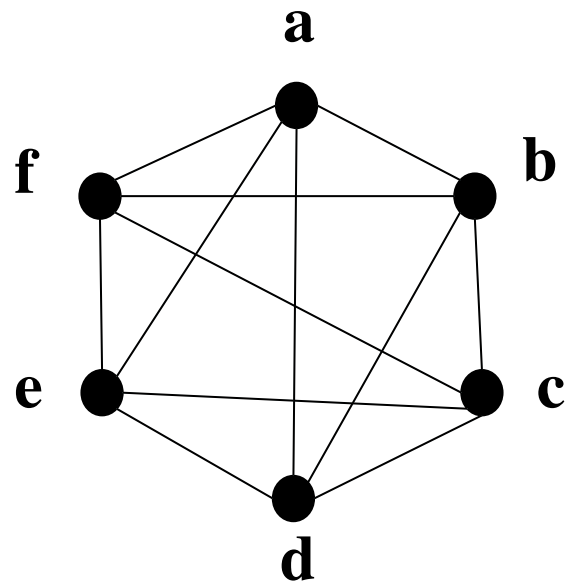
- Additional spilling might be required before a coloring is found

Example

$K=3$

remove a

Stack: {}

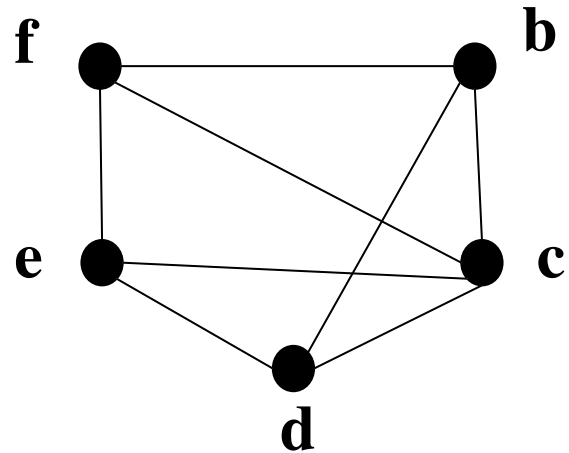


Example

$K=3$

remove c

Stack: {**a**}

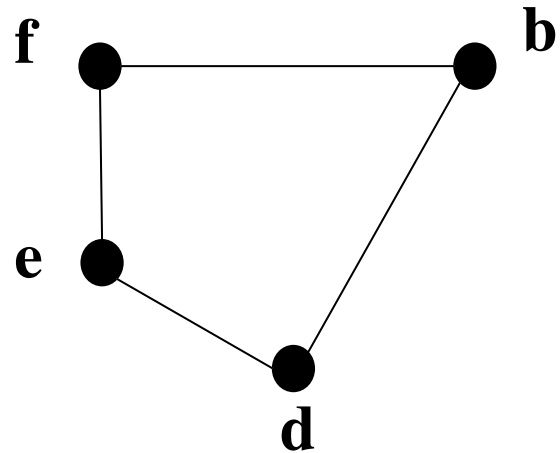


Example

$K=3$

remove b

Stack: {c,a}

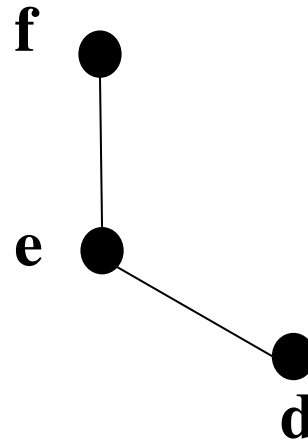


Example

$K=3$

remove e

Stack: {b, c, a}



Example

$K=3$

remove f

Stack: {e,b,c,a}

f ●

●
d

Example

$K=3$

remove d

Stack: {f,e,b,c,a}

●
d

Example

$K=3$

Stack: {d,f,e,b,c,a}

Example

$K=3$

Stack: {f,e,b,c,a}

●
d r1

Example

$K=3$

Stack: {e,b,c,a}

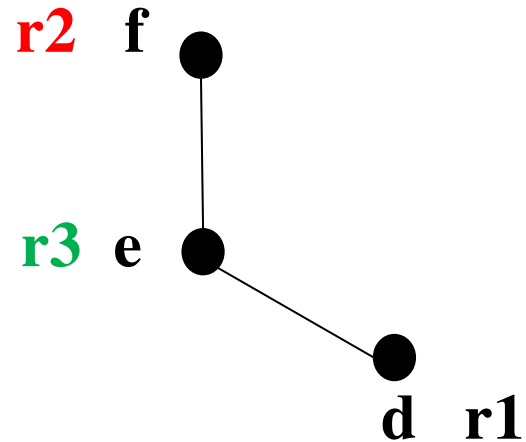
r2 f ●

●
d r1

Example

$K=3$

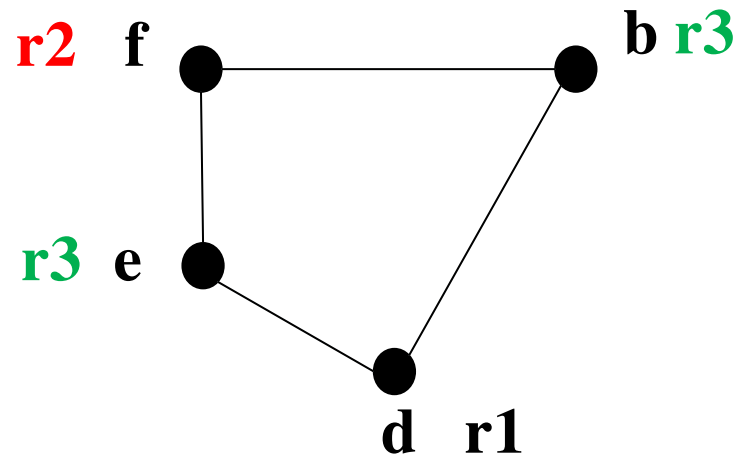
Stack: {b, **c**, **a**}



Example

$K=3$

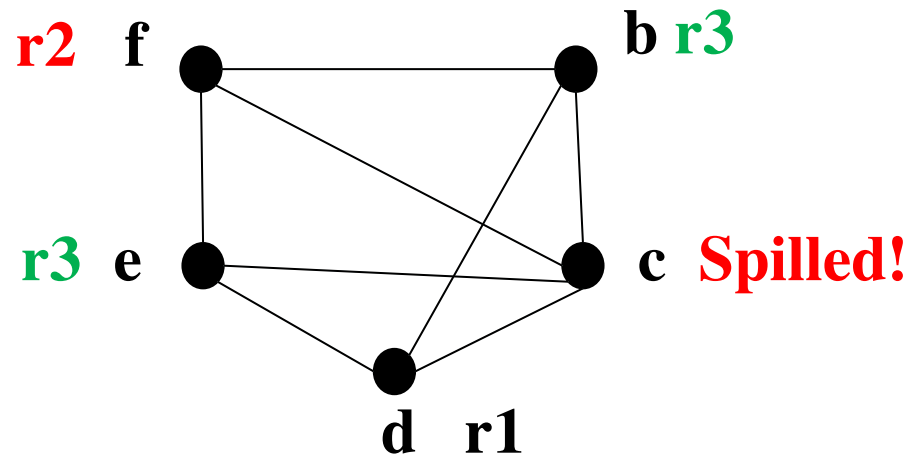
Stack: {**c**,a}



Example

$K=3$

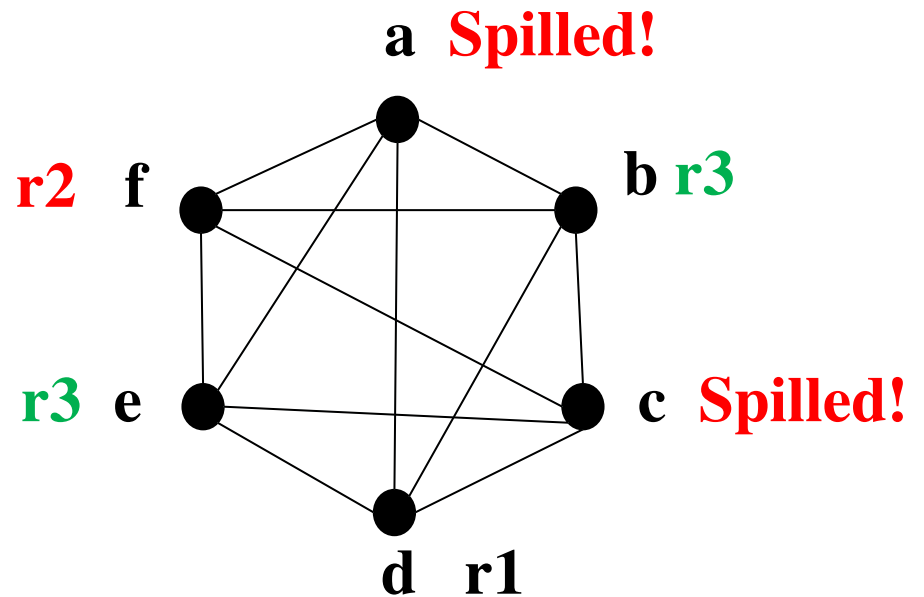
Stack: {**a**}



Example

$K=3$

Stack: {}



Example

$K=3$

Stack: {d,f,e,b,c,a}

Example

$K=3$

Stack: {f,e,b,c,a}

●
d r1

Example

$K=3$

Stack: {e,b,c,a}

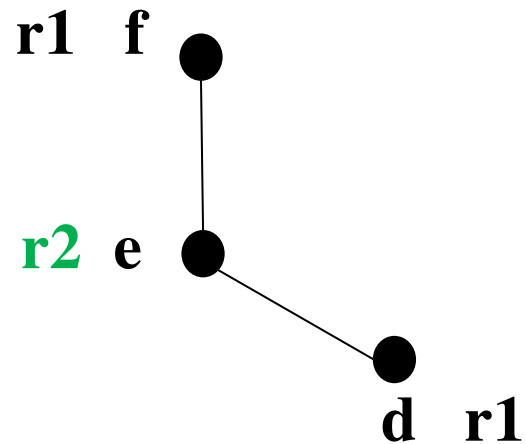
r1 f ●

●
d r1

Example

$K=3$

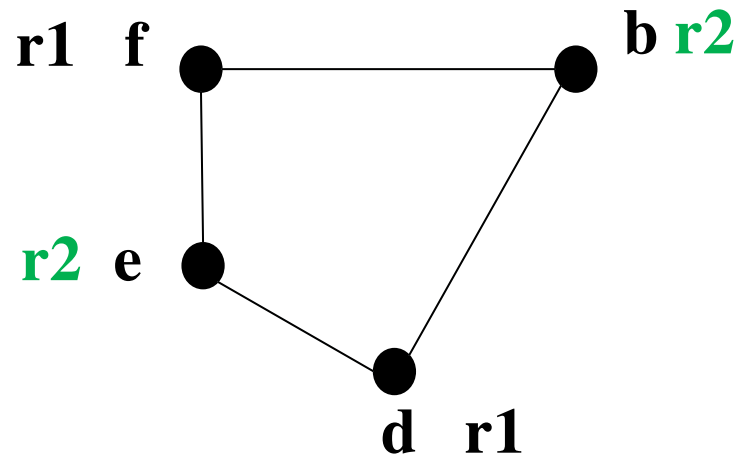
Stack: {b, c, a}



Example

$K=3$

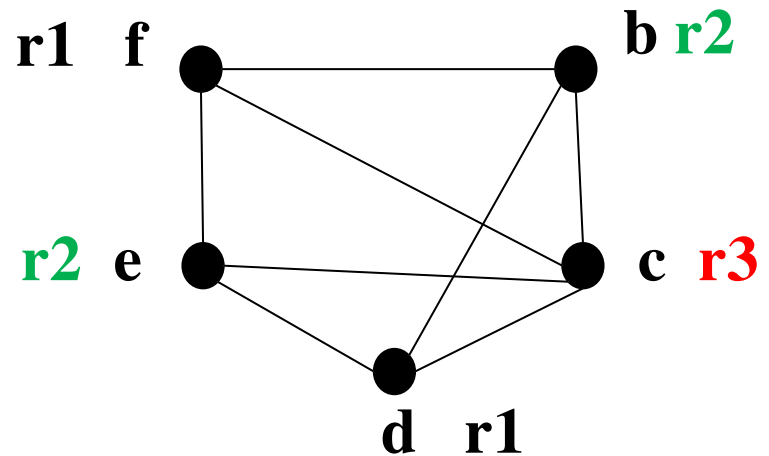
Stack: {**c**,**a**}



Example

$K=3$

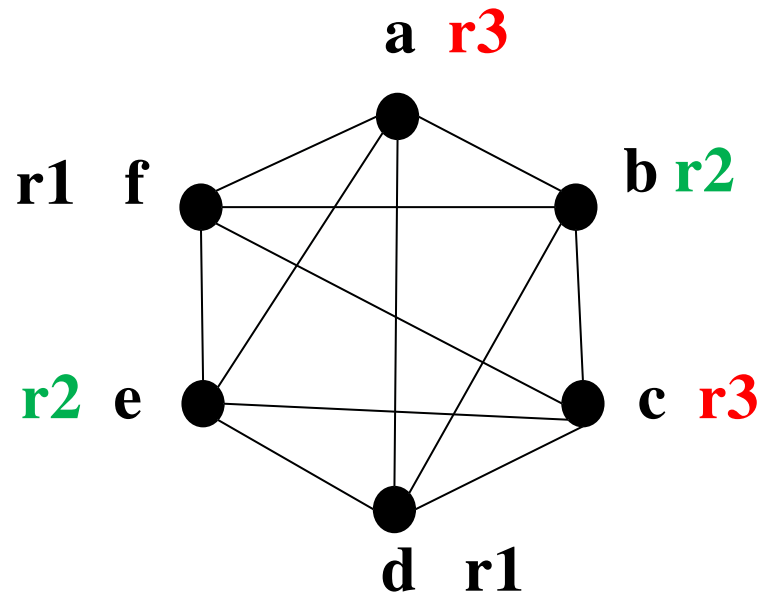
Stack: {**a**}



Example

$K=3$

Stack: {}



Spilling

- Many different heuristics for picking a node to spill
 - Spill temporaries with most conflicts
 - Spill temporaries with few definitions and uses
 - Avoid spilling in inner loops (heavily visited regions of the code)
- C allows a *register* keyword to direct the compiler whether a variable contains a value that is heavily used.

Live Ranges and Live Intervals

- The live range for a variable is the set of program points at which that variable is live.
- The live interval for a variable is the smallest subrange of the IR code containing all a variable's live ranges.
 - A property of the IR code, not CFG.
 - Less precise than live ranges, but simpler to work with

Live Intervals

$e = d + a$

$f = b + c$

$f = f + b$

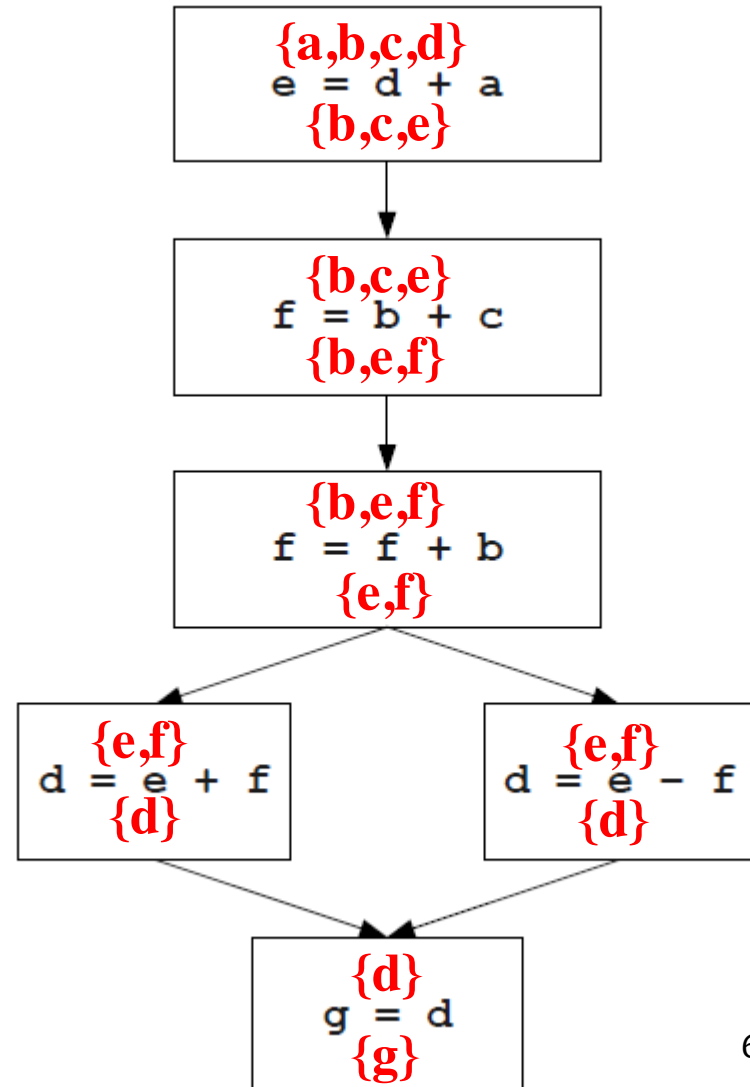
if $e == 0$ goto _L0

$d = e + f$

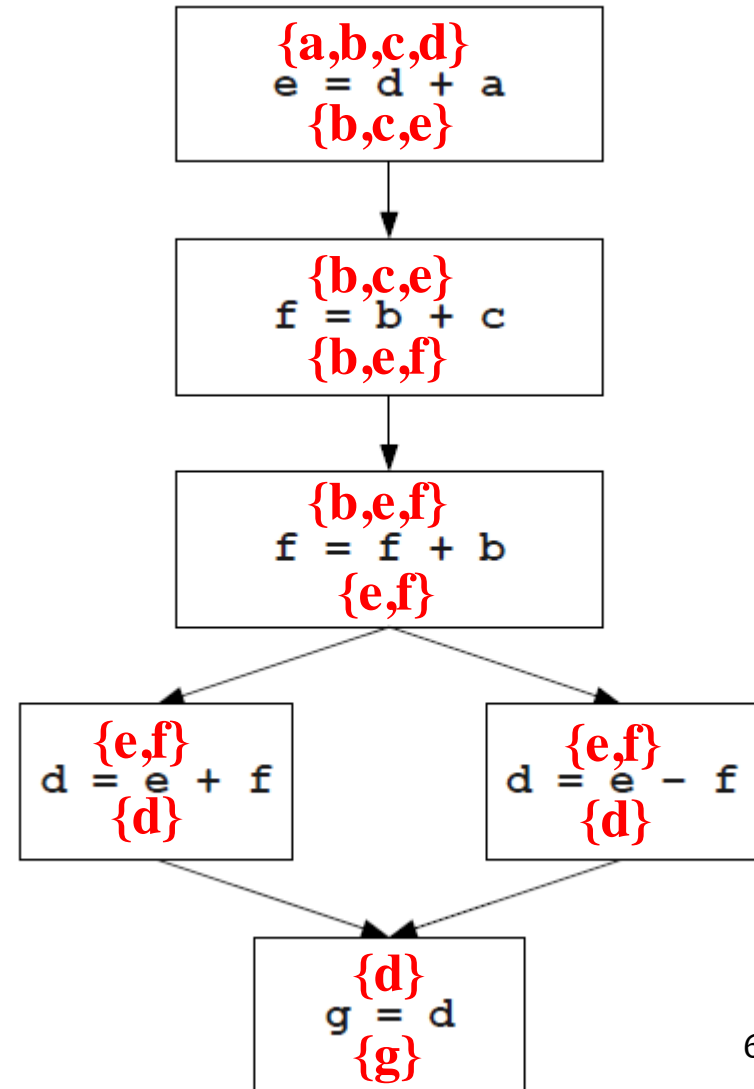
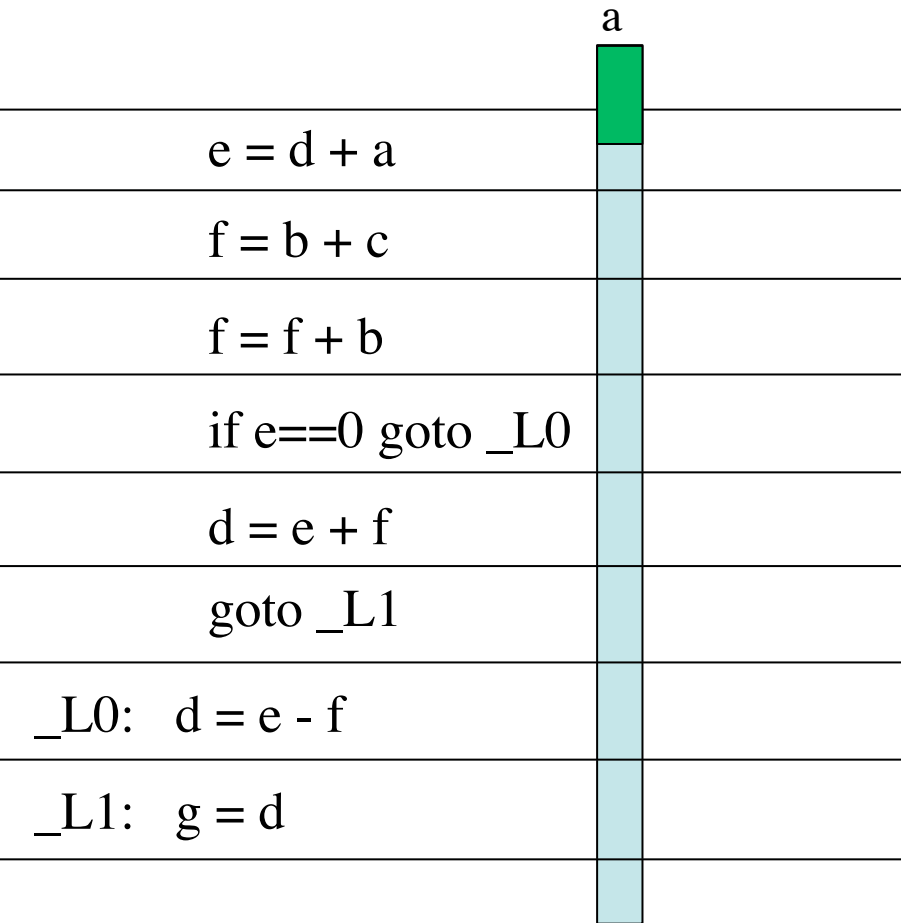
goto _L1

_L0: $d = e - f$

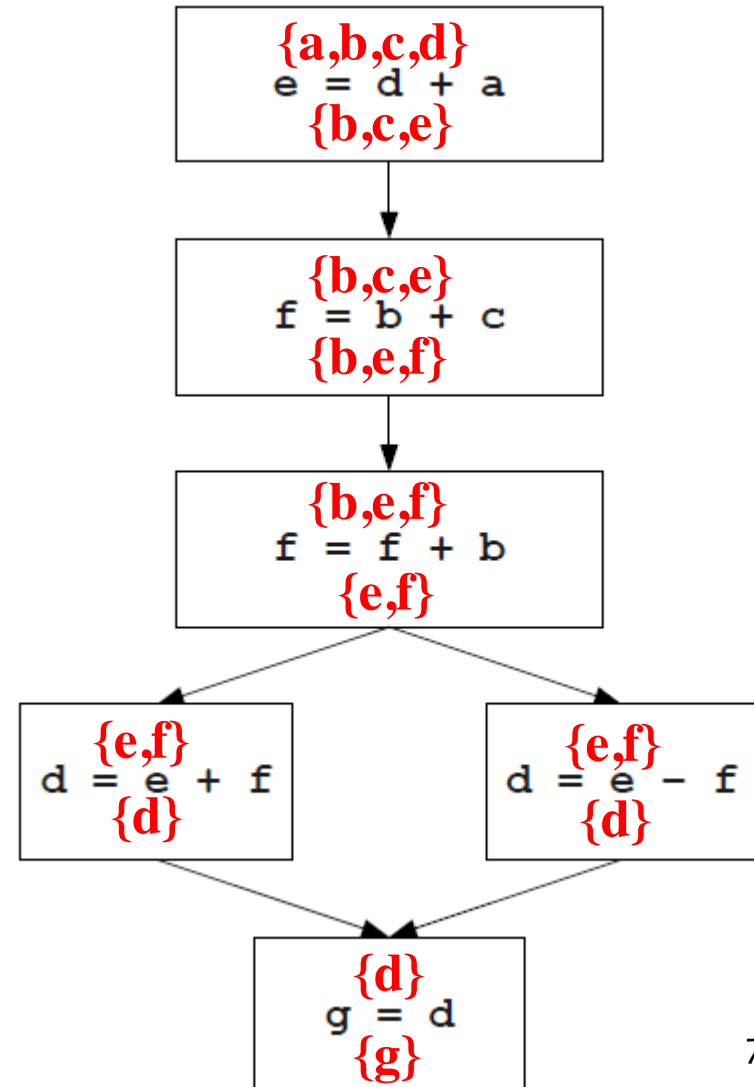
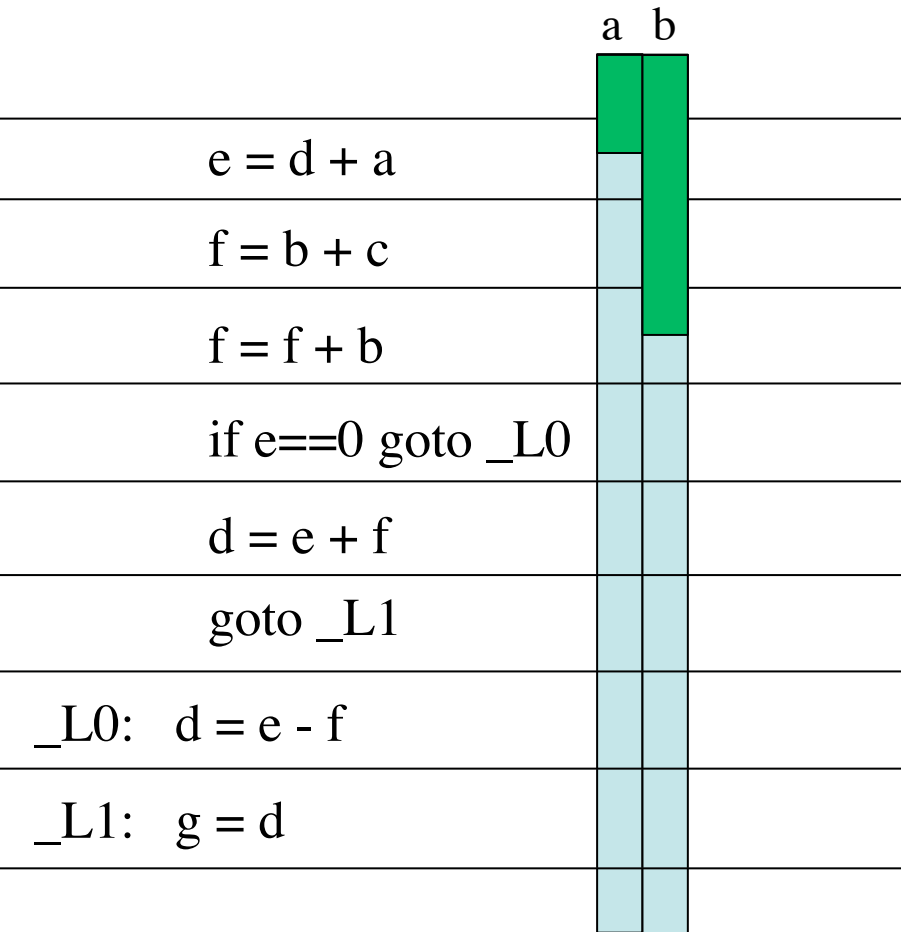
_L1: $g = d$



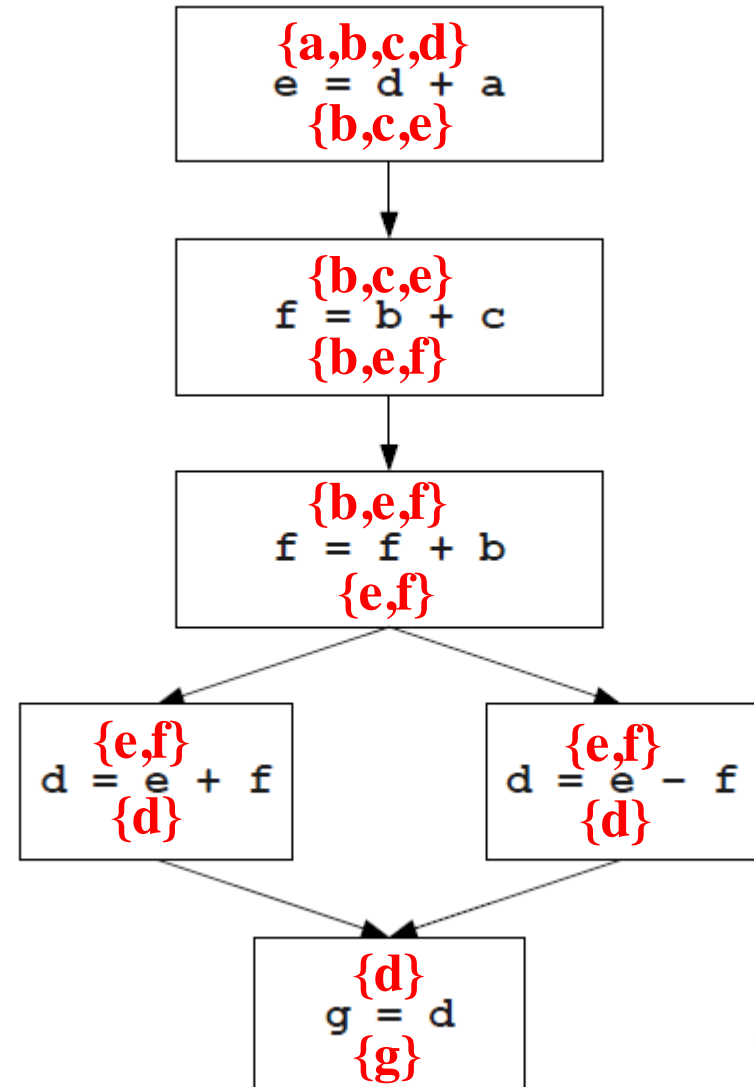
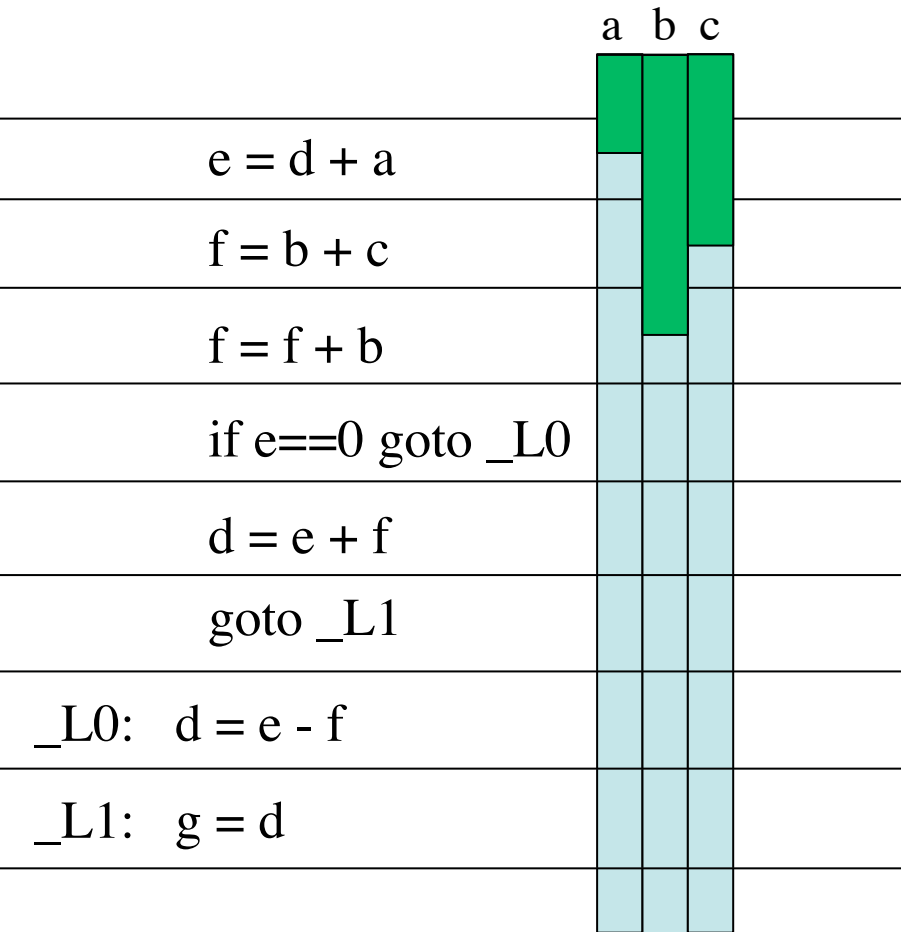
Live Intervals



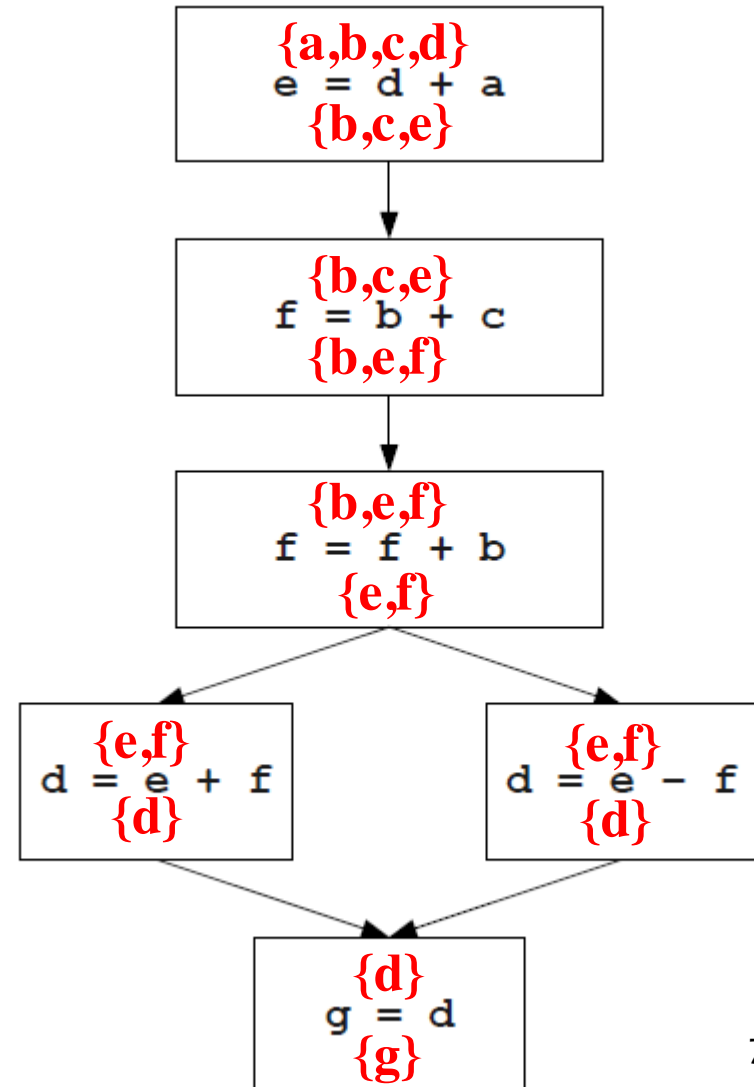
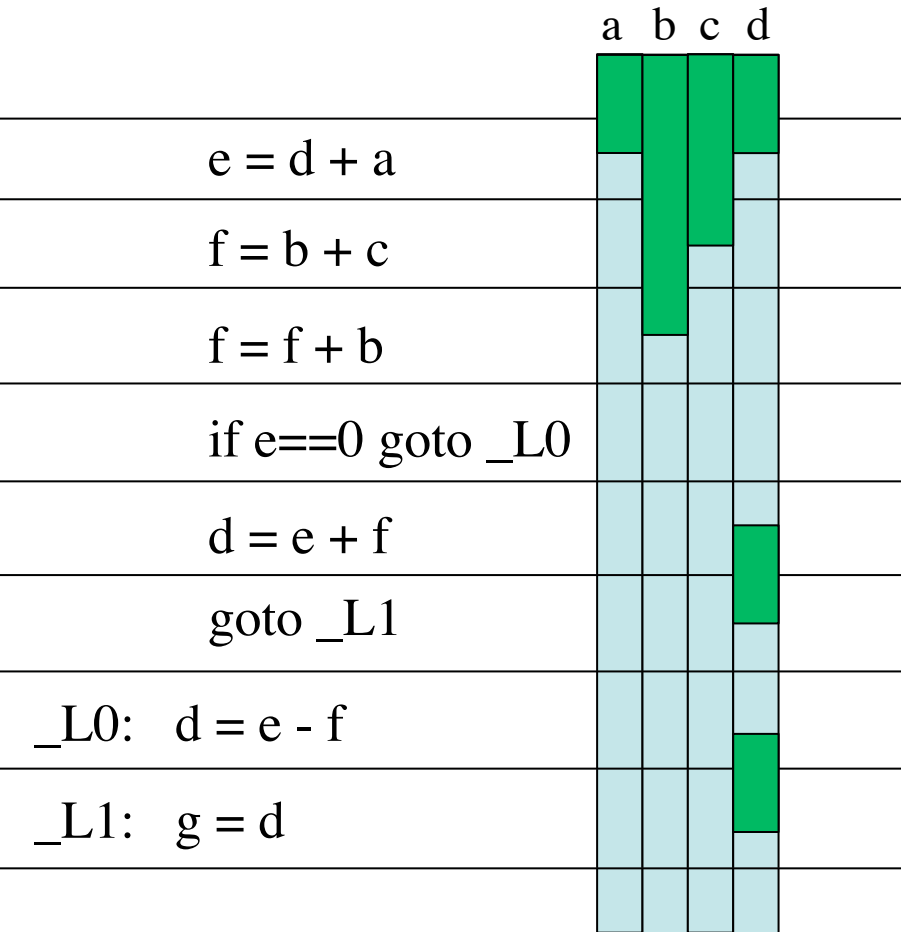
Live Intervals



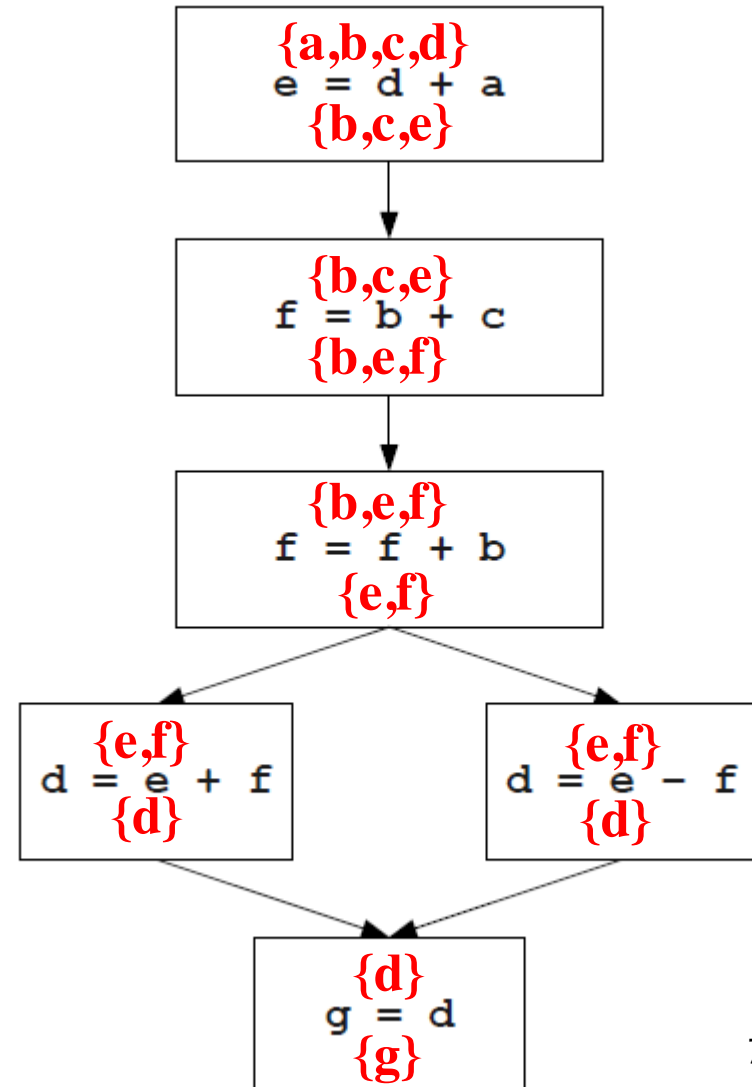
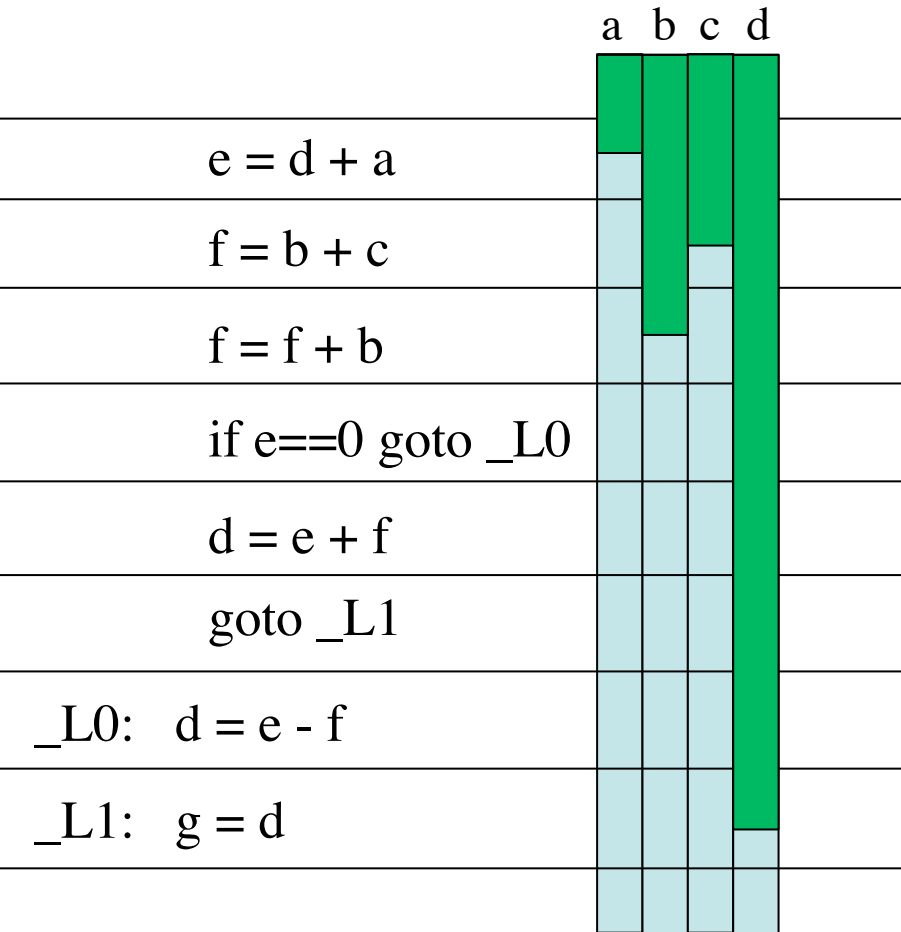
Live Intervals



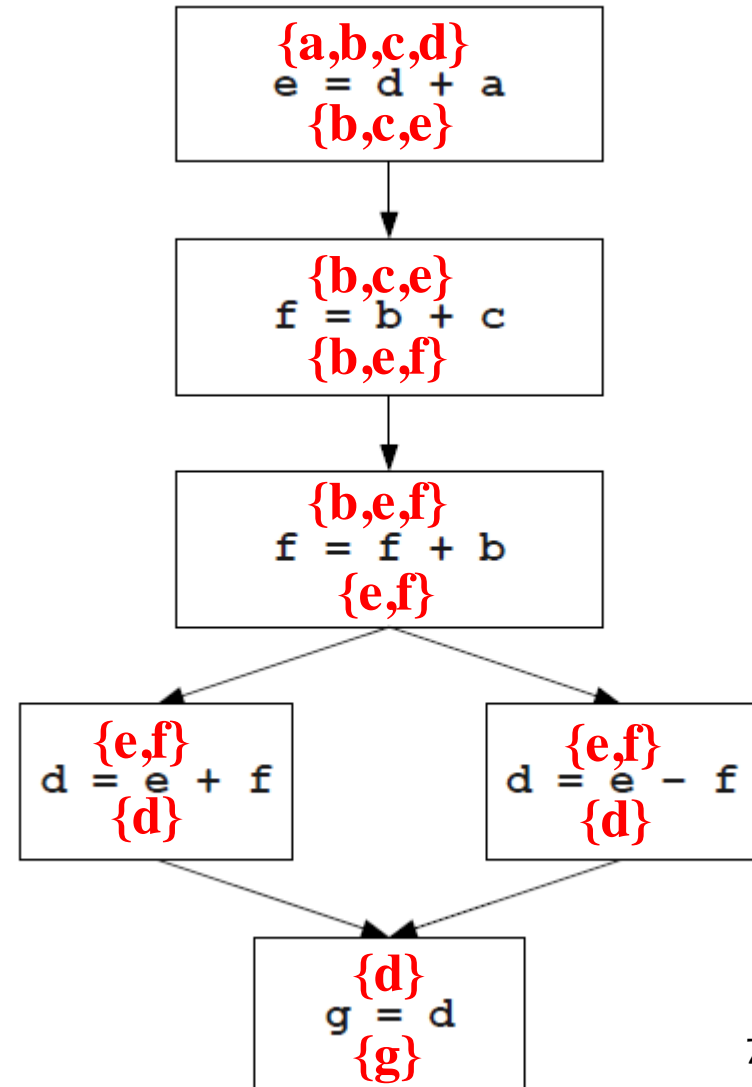
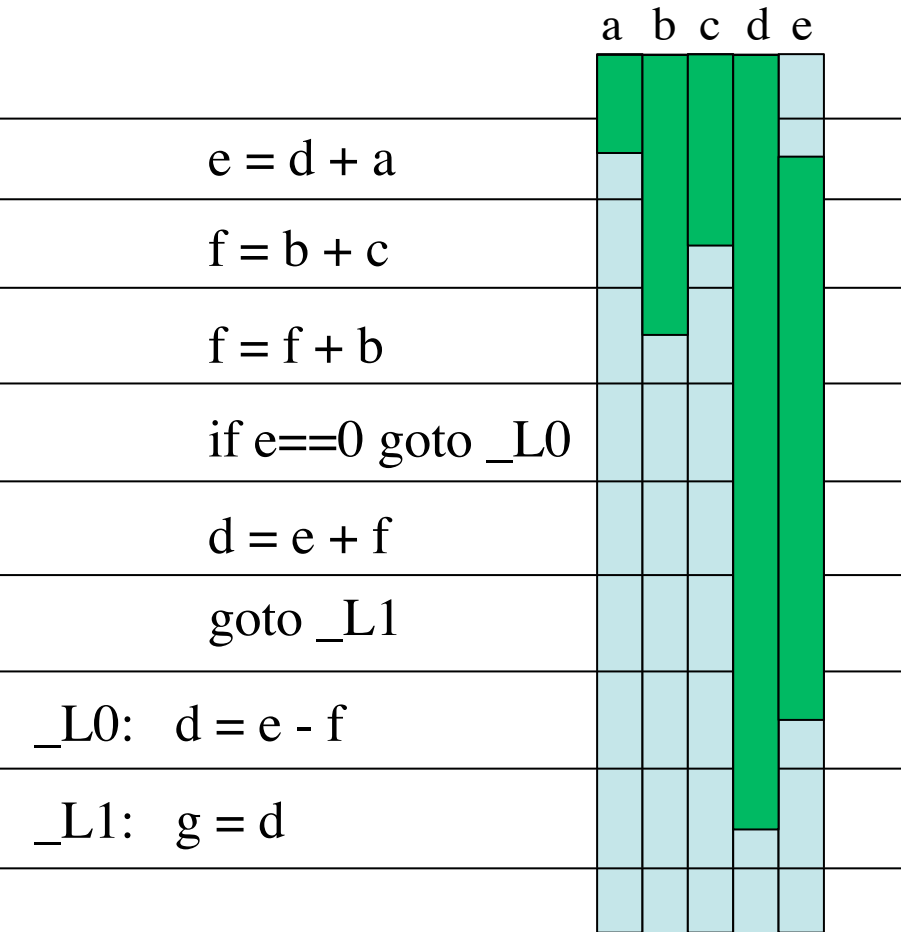
Live Intervals



Live Intervals

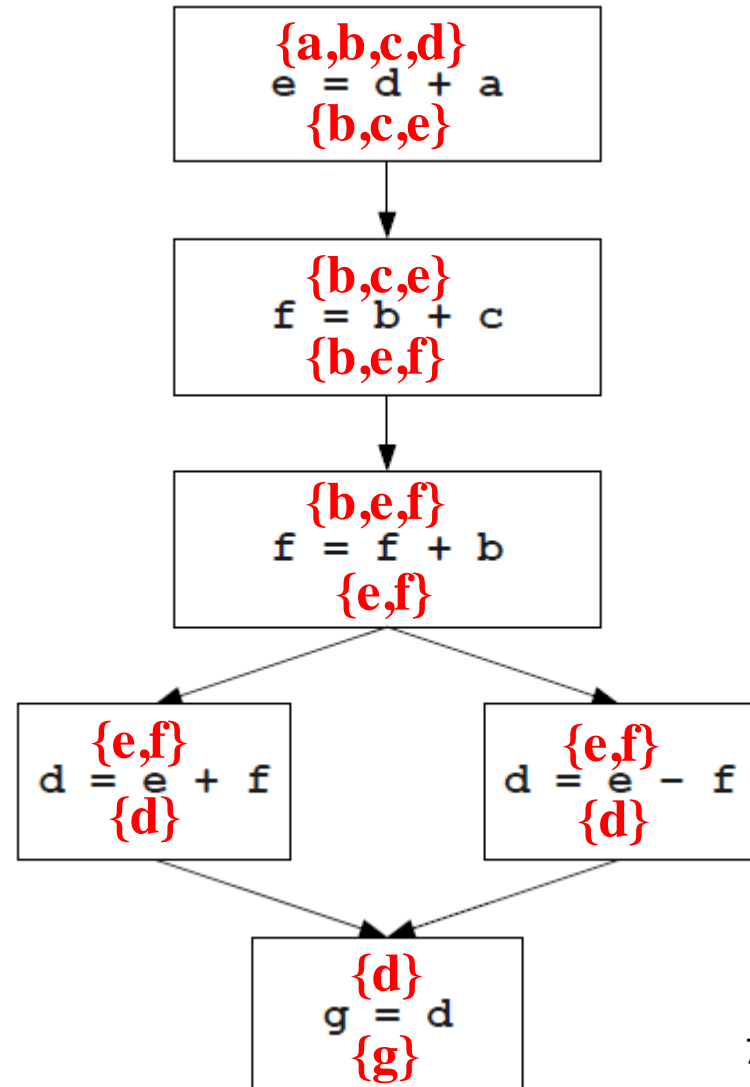


Live Intervals



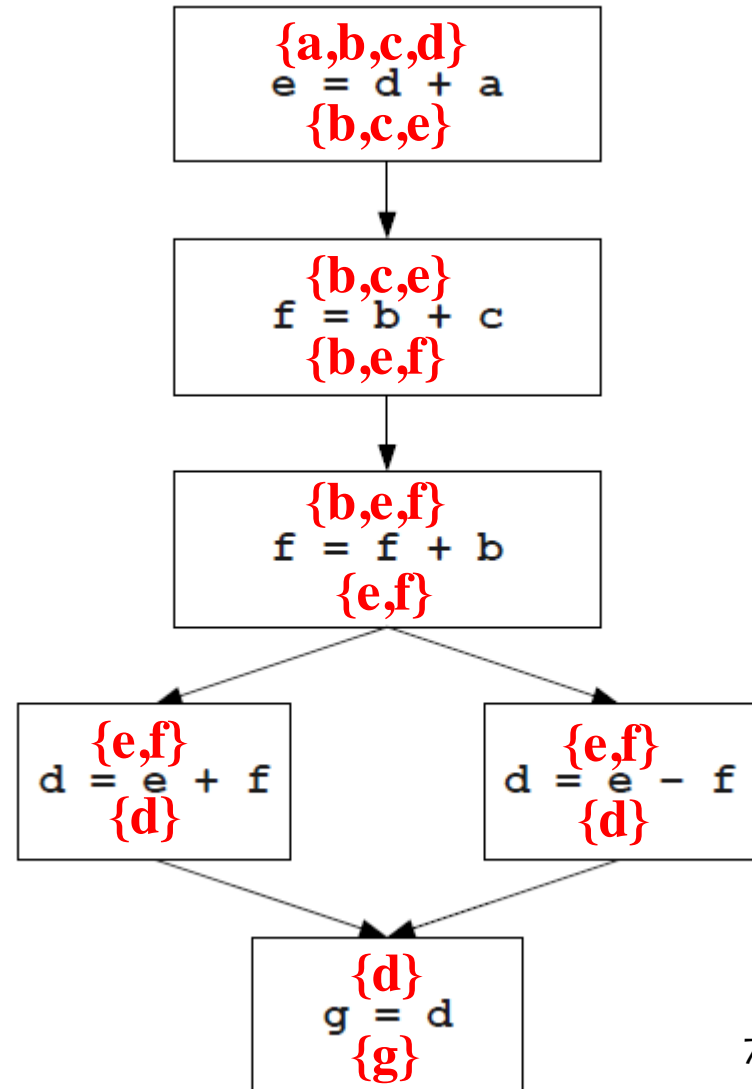
Live Intervals

	a	b	c	d	e	f
	█	█	█	█	█	█
$e = d + a$	█	█	█	█	█	█
$f = b + c$	█	█	█	█	█	█
$f = f + b$	█	█	█	█	█	█
if $e == 0$ goto _L0	█	█	█	█	█	█
$d = e + f$	█	█	█	█	█	█
goto _L1	█	█	█	█	█	█
_L0: $d = e - f$	█	█	█	█	█	█
_L1: $g = d$	█	█	█	█	█	█



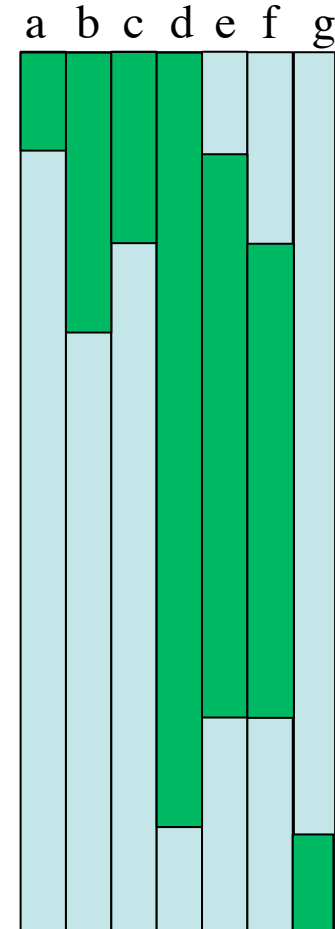
Live Intervals

	a	b	c	d	e	f	g
	█	█	█	█			
$e = d + a$		█	█	█	█		
$f = b + c$		█		█		█	
$f = f + b$		█		█		█	
if $e == 0$ goto _L0				█	█		
$d = e + f$				█	█	█	
goto _L1				█	█	█	
_L0: $d = e - f$							
_L1: $g = d$							█

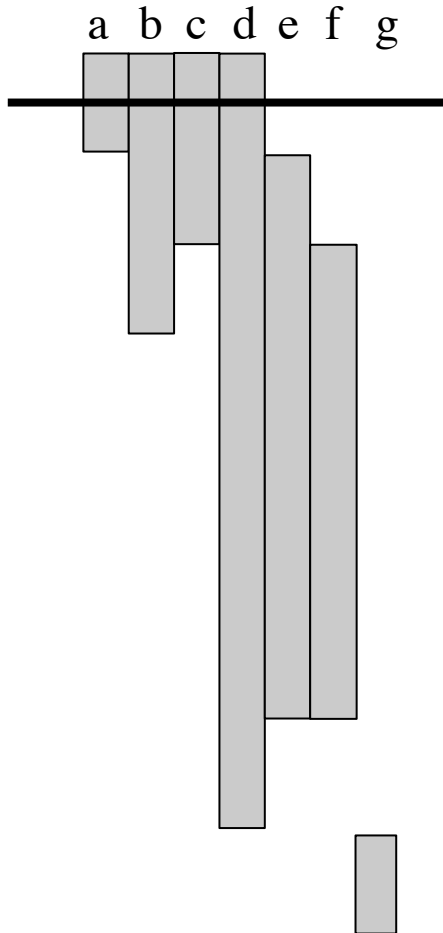


Register Allocation with Live Intervals

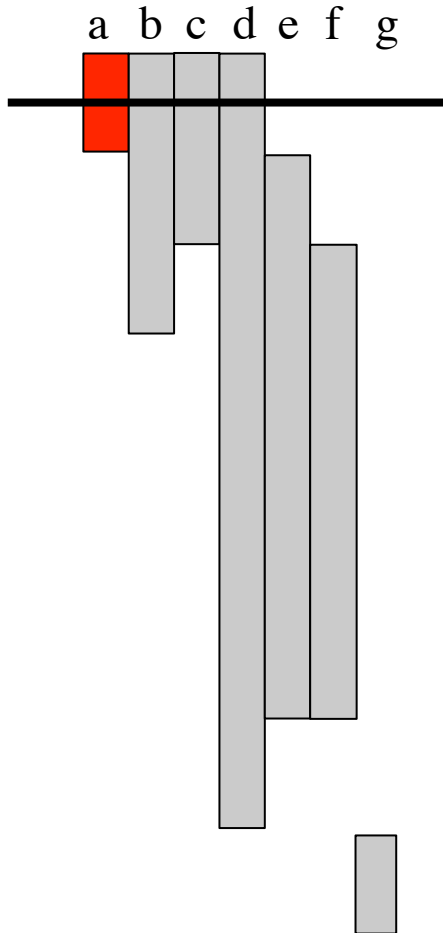
- Given the live intervals for all the variables in the program, we can allocate registers using a simple greedy algorithm.
- Idea: Track which registers are free at each point.
- When a live interval begins, give that variable a free register.
- When a live interval ends, the register is once again free.



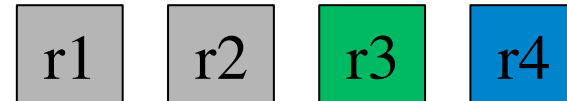
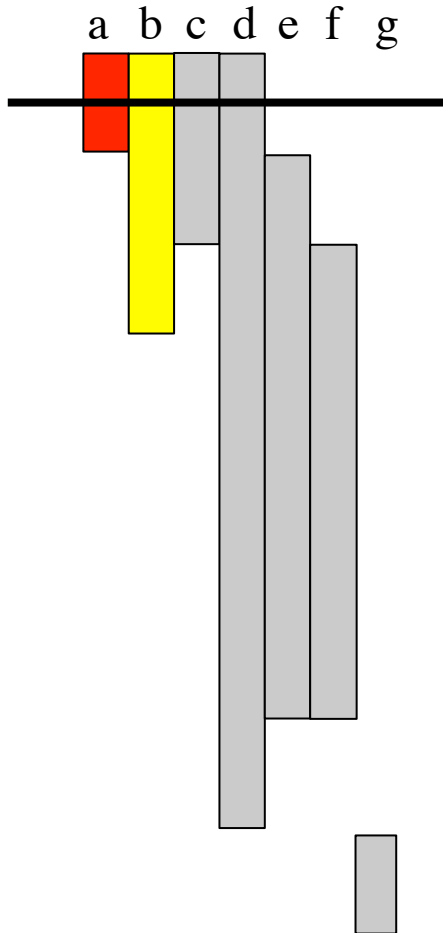
Register Allocation with Live Intervals



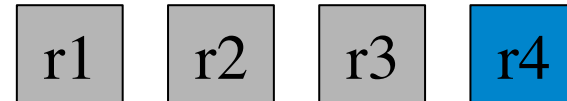
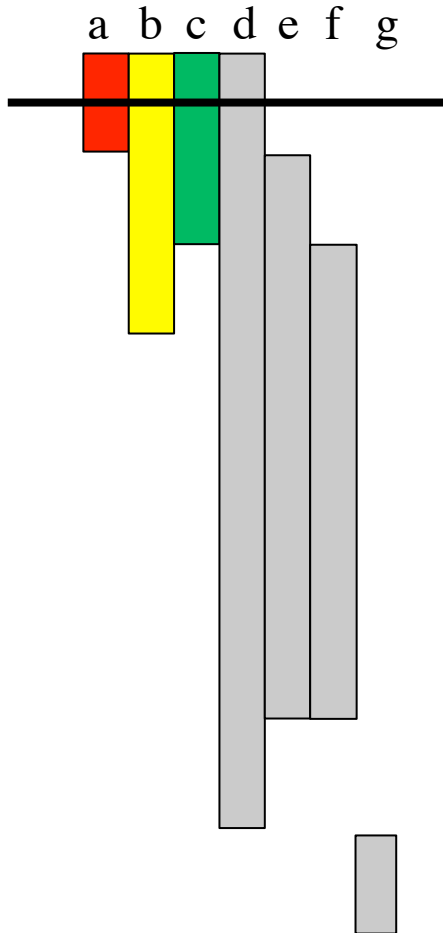
Register Allocation with Live Intervals



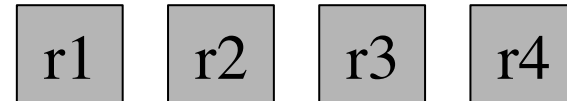
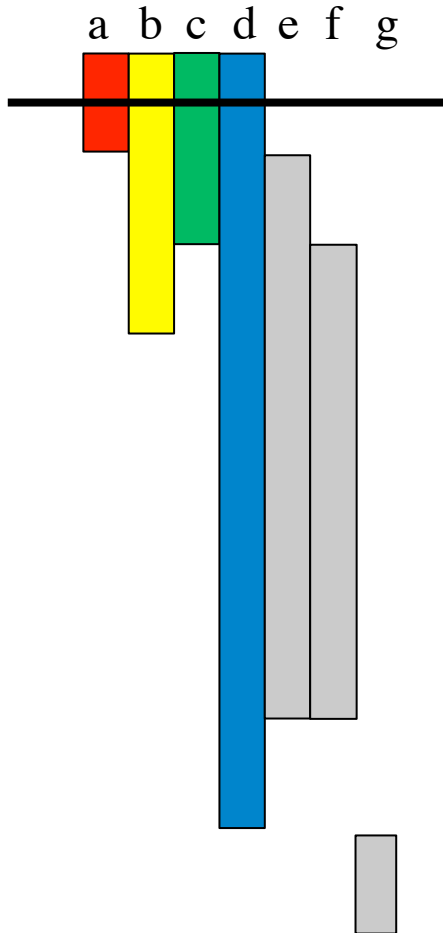
Register Allocation with Live Intervals



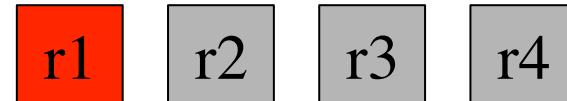
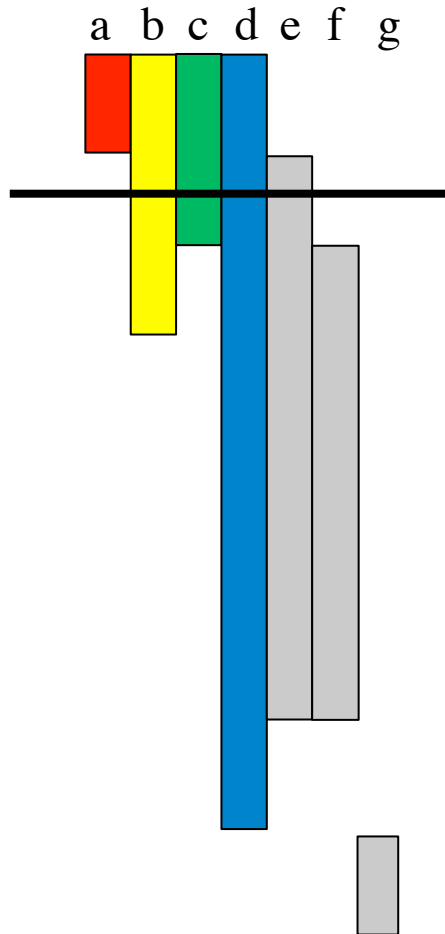
Register Allocation with Live Intervals



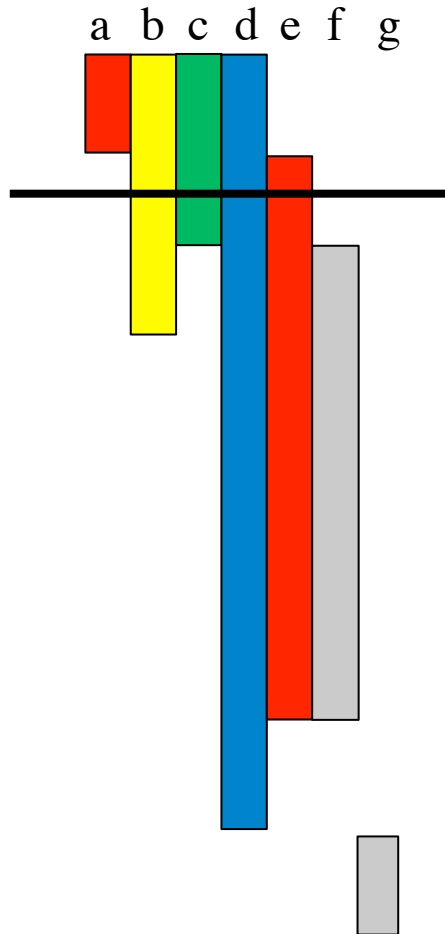
Register Allocation with Live Intervals



Register Allocation with Live Intervals

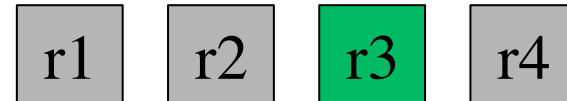
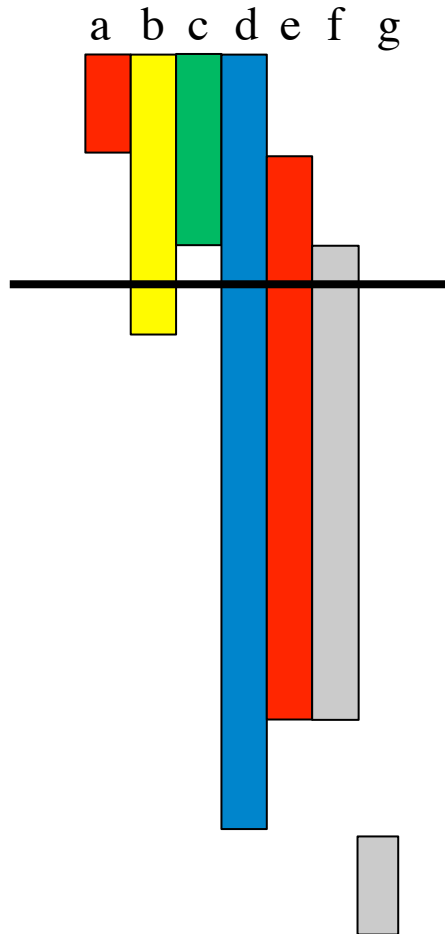


Register Allocation with Live Intervals

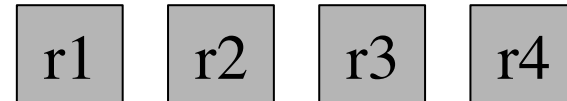
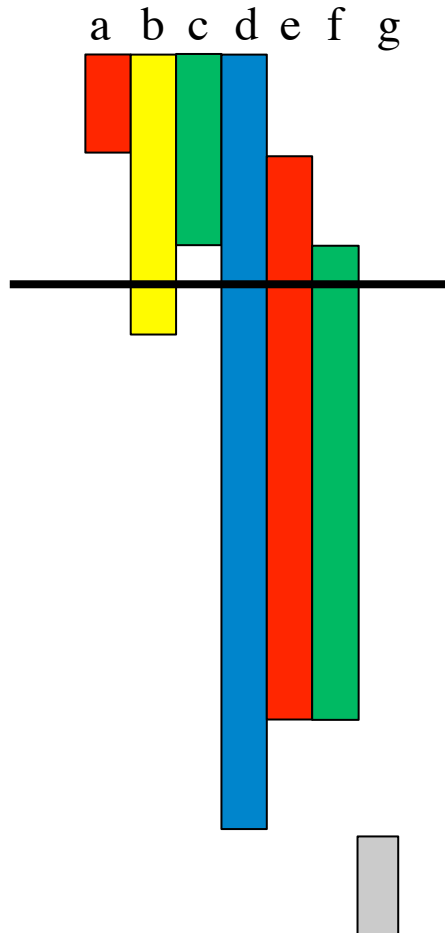


r1 r2 r3 r4

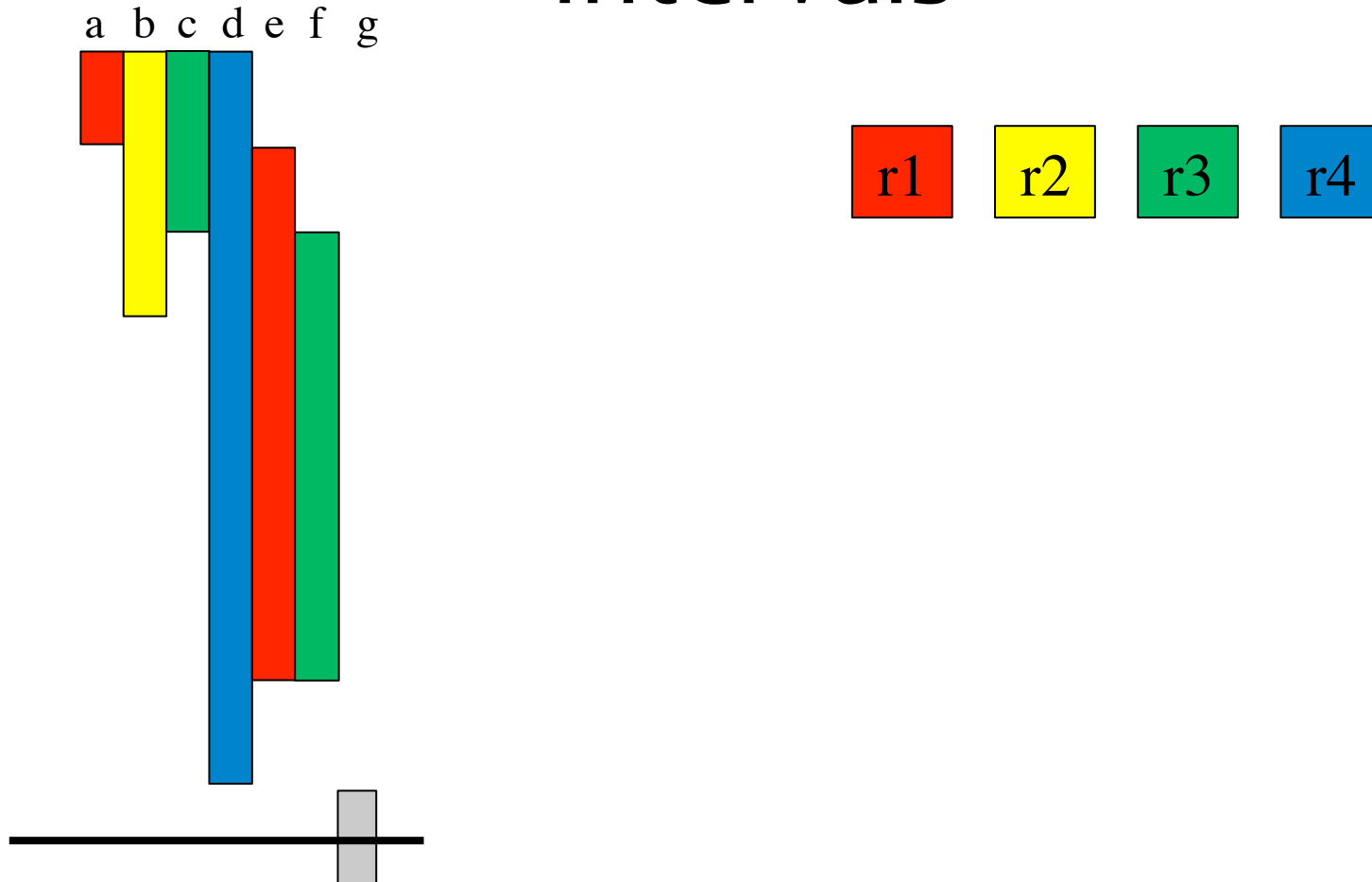
Register Allocation with Live Intervals



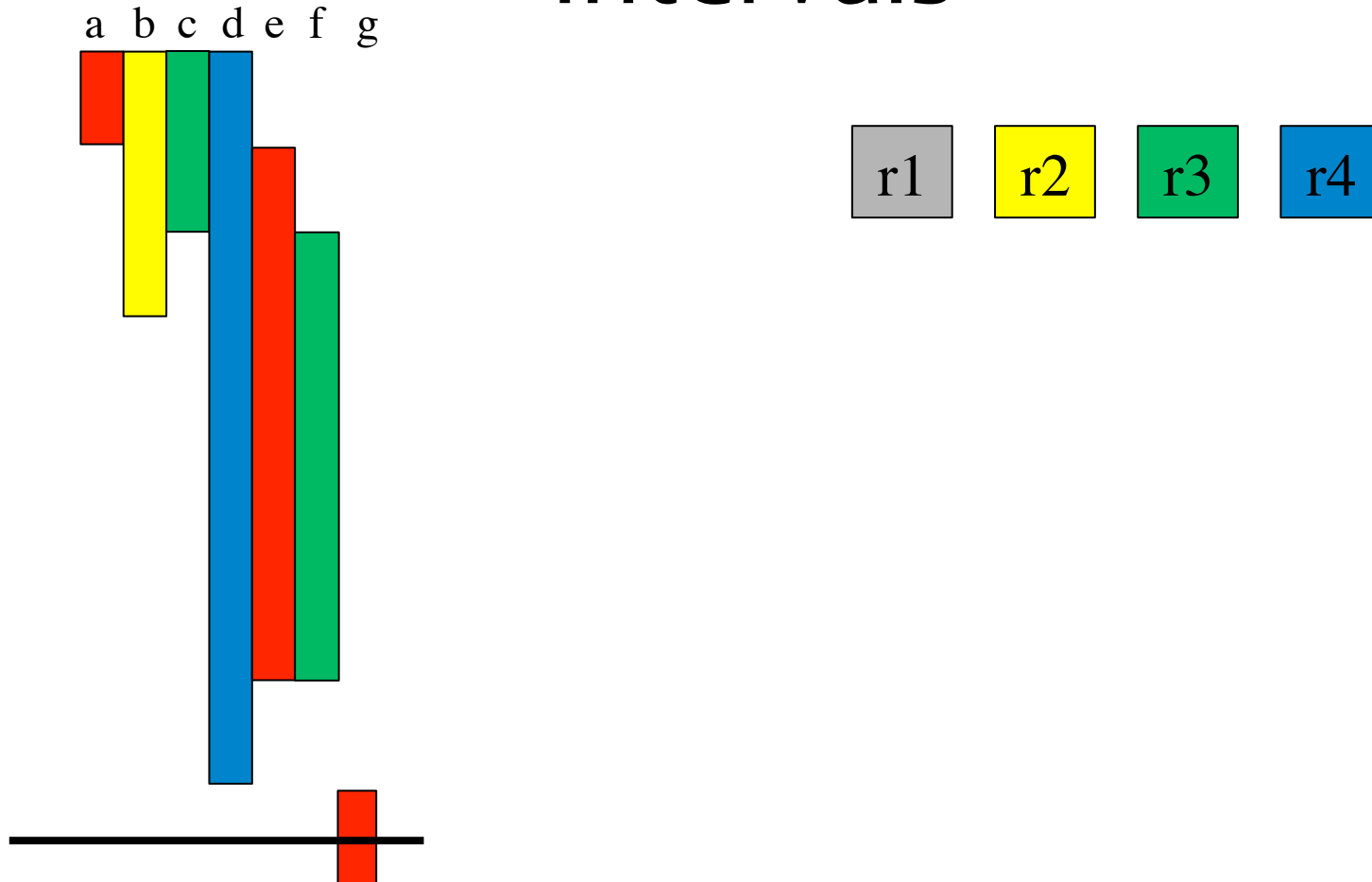
Register Allocation with Live Intervals



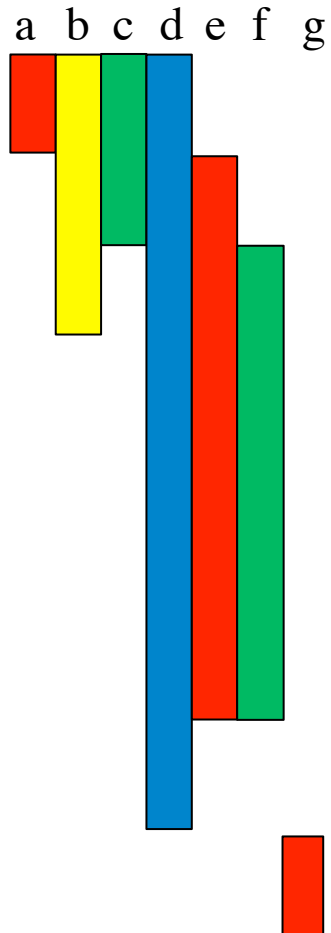
Register Allocation with Live Intervals



Register Allocation with Live Intervals



Register Allocation with Live Intervals



Linear Scan Register Allocation

- If a register cannot be found for a variable **v**, we may need to spill a variable.
- This algorithm is called linear scan register allocation and is a comparatively new algorithm.
- Pros:
 - Very efficient
 - Works well in many cases
 - Allocation needs one pass, the code can be generated simultaneously
 - Used in JIT compilers like Java HotSpot
- Cons:
 - Not as good as graph coloring approach

Summary

- Register allocation is a “must have” in compilers, because:
 - Intermediate code uses too many temporaries
 - It makes a big difference in performance
- The liveness at each location can be used for register allocation
- Register allocation as heuristic graph coloring uses live ranges
 - The basis for the technique used in GCC
- Linear scan register allocation uses live intervals
 - Often used in JIT compilers due to efficiency