LL: Top-down Parsing

Parsing

CMPT 379: Compilers

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Parsing - Roadmap

- Parser:
  - decision procedure: builds a parse tree
- Top-down vs. bottom-up
- LL(1) – Deterministic Parsing
  - recursive-descent
  - table-driven
- LR(k) – Deterministic Parsing
  - LR(0), SLR(1), LR(1), LALR(1)
- Parsing arbitrary CFGs – Polynomial time parsing

Top-Down vs. Bottom Up

Grammar: $S \rightarrow A\ B$ | Input String: ccbca

$A \rightarrow c \mid \varepsilon$

$B \rightarrow cbB \mid ca$

Top-Down/leftmost		Bottom-Up/rightmost	
$S \Rightarrow AB$	$S \rightarrow AB$	$ccbca \Leftarrow Acbca$	$A \rightarrow c$
$\Rightarrow cB$	$A \rightarrow c$	$\Leftarrow AcbB$	$B \rightarrow ca$
$\Rightarrow ccbB$	$B \rightarrow cbB$	$\Leftarrow AB$	$B \rightarrow cbB$
$\Rightarrow ccbca$	$B \rightarrow ca$	$\Leftarrow S$	$S \rightarrow AB$

Top-Down: Backtracking

$S \rightarrow A\ B$

$A \rightarrow c \mid \varepsilon$

$B \rightarrow cbB \mid ca$

True/False

$S \Rightarrow^* cbca?$

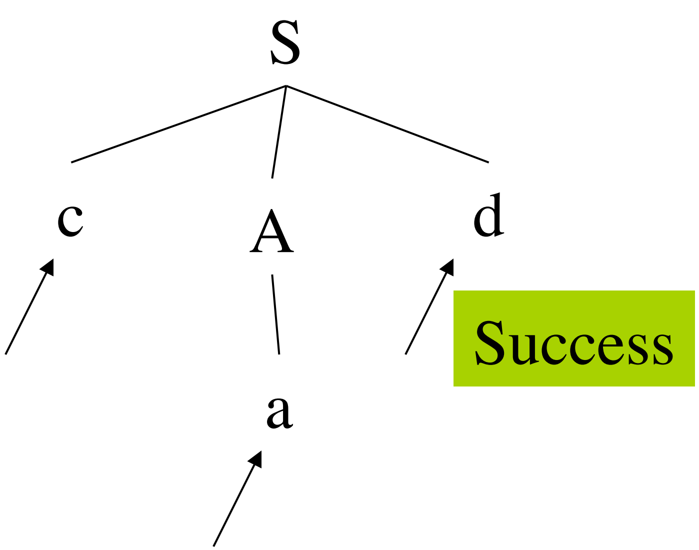
S	cbca	try $S \rightarrow AB$
AB	cbca	try $A \rightarrow c$
cB	cbca	match c
B	bca	dead-end, try $A \rightarrow \varepsilon$
εB	cbca	try $B \rightarrow cbB$
cbB	cbca	match c
bB	bca	match b
B	ca	try $B \rightarrow cbB$
cbB	ca	match c
bB	a	dead-end, try $B \rightarrow ca$
ca	ca	match c
a	a	match a, Done!

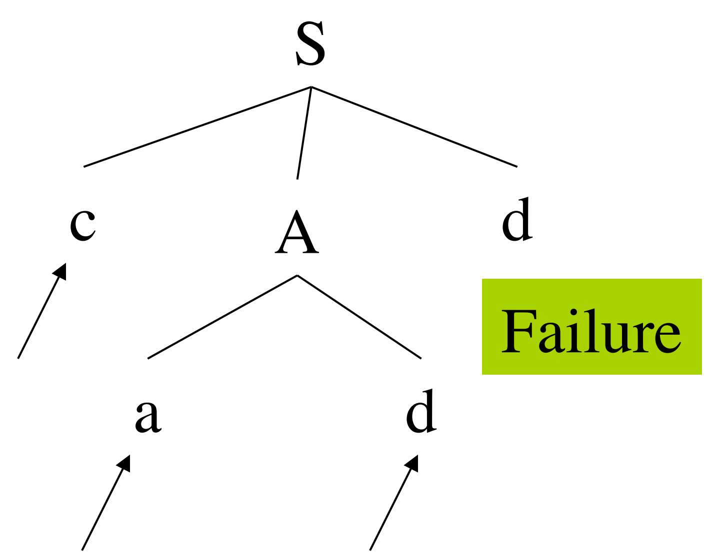
Backtracking

S → cAd | c
A → a | ad

Input: cad

S → cAd | c
A → ad | a



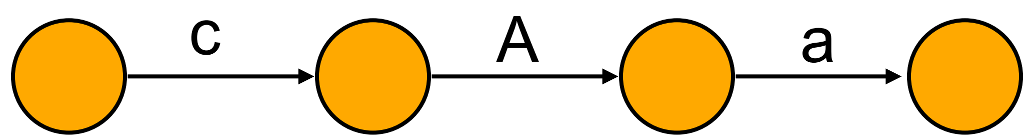


For some grammars, rule ordering is important for backtracking parsers, e.g S → aSa, S → aa

Transition Diagram

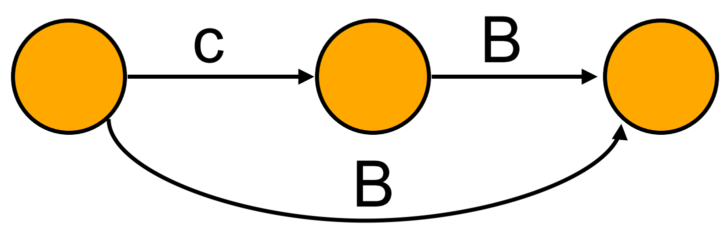
$S \rightarrow cAa$

S:



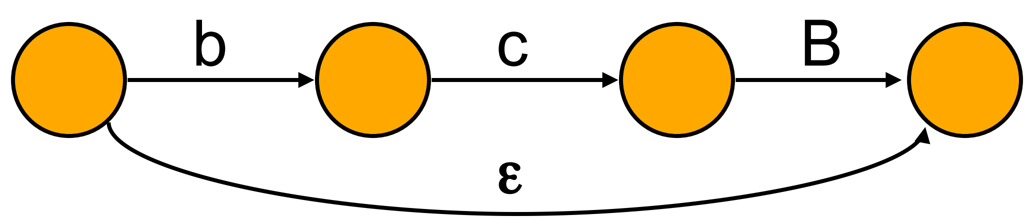
$A \rightarrow cB \mid B$

A:



$B \rightarrow bcB \mid \varepsilon$

B:



Predictive Top-Down Parser

- Knows which production to choose based on single lookahead symbol
- Need LL(1) grammars
  - First L: reads input Left to right
  - Second L: produce Leftmost derivation
  - 1 : one symbol of lookahead
- Can't have left-recursion
- Must be left-factored (no left-factors)
- Not all grammars can be made LL(1)

Leftmost derivation for **id + id * id**

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow (E)$

$E \rightarrow - E$

$E \rightarrow \mathbf{id}$

$E \Rightarrow E + E$

$\Rightarrow \mathbf{id} + E$

$\Rightarrow \mathbf{id} + E * E$

$\Rightarrow \mathbf{id} + \mathbf{id} * E$

$\Rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id}$

$E \Rightarrow^{*}_{lm} \mathrm{id} + E \backslash * E$

Predictive Parsing Table

Productions	
1	**T → F T'**
2	**T' → ε**
3	**T' → * F T'**
4	**F → id**
5	**F → (T)**

	*	(	)	id	$
T		**T → F T'**		**T → F T'**	
T'	**T' → * F T'**		**T' → ε**		**T' → ε**
F		**F → (T)**		**F → id**	

Trace "(id)*id"

	*	(	)	id	$
T		**T → FT'**		**T → FT'**	
T'	**T' → *FT'**		**T' → ε**		**T' → ε**
F		**F → (T)**		**F → id**	

Stack	Input	Output
$T	(id)*id$	
$T'F	(id)*id$	**T → F T'**
$T')T(	(id)*id$	**F → (T)**
$T')T	id)*id$	
$T')T'F	id)*id$	**T → F T'**
$T')T'id	id)*id$	**F → id**
$T')T'	)*id$	
$T')	)*id$	**T' → ε**

Trace “(id)*id”

	*	(	)	id	$
T		**T → FT’**		**T → FT’**	
T’	**T’ → *FT’**		**T’ → ε**		**T’ → ε**
F		**F → (T)**		**F → id**	

Stack	Input	Output
$T’	*id$	
$T’F*	*id$	**T’ → * F T’**
$T’F	id$	
$T’id	id$	**F → id**
$T’	$	
$	$	**T’ → ε**

Table-Driven Parsing

```
stack.push($); stack.push(S);
a = input.read();
forever do begin
   X = stack.peek();
   if X = a and a = $ then return SUCCESS;
   elsif X = a and a != $ then
      pop X; a = input.read();
   elsif X != a and X ∈ N and M[X,a] then
      pop X; push right-hand side of M[X,a];
   else ERROR!
end
```

Predictive Parsing table

- Given a grammar produce the predictive parsing table
- We need to to know for all rules $A \rightarrow \alpha \mid \beta$ the lookahead symbol
- Based on the lookahead symbol the table can be used to pick which rule to push onto the stack
- This can be done using two sets: FIRST and FOLLOW

FIRST and FOLLOW

$a \in \mathrm{FIRST}(\alpha)$ if $\alpha \Rightarrow^* a\beta$

if $\alpha \Rightarrow^* \epsilon$ then $\epsilon \in \mathrm{FIRST}(\alpha)$

$a \in \mathrm{FOLLOW}(A)$ if $S \Rightarrow^* \alpha A a \beta$

$a \in \mathrm{FOLLOW}(A)$ if $S \Rightarrow^* \alpha A \gamma a \beta$

and $\gamma \Rightarrow^* \epsilon$

Conditions for LL(1)

- Necessary conditions:
  - no ambiguity
  - no left recursion
  - Left factored grammar
- A grammar G is LL(1) if - whenever
  $A \rightarrow \alpha \mid \beta$

1. $\text{First}(\alpha) \cap \text{First}(\beta) = \emptyset$
2. $\alpha \Rightarrow^* \varepsilon$ implies **!**$(\beta \Rightarrow^* \varepsilon)$
3. $\alpha \Rightarrow^* \varepsilon$ implies $\text{First}(\beta) \cap \text{Follow}(A) = \emptyset$

ComputeFirst(α: string of symbols)

```
// assume α = X1 X2 X3 … Xn
if X1 ∈ T then First[α] := {X1}
else begin
  i:=1; First[α] := ComputeFirst(X1)\{ε};
  while Xi ⇒* ε do begin
    if i < n then
      First[α] := First[α] ∪ ComputeFirst(Xi+1)\{ε};
    else
      First[α] := First[α] ∪ {ε};
    i := i + 1;
  end
end
```

Recursion in computing FIRST causes problems when faced with recursive grammar rules

ComputeFirst; modified

foreach $X \in$ **T do** First[X] := {X};

foreach $p \in$ **P** : $X \rightarrow \varepsilon$ **do** First[X] := {ε};

repeat foreach $X \in$ **N**, $p : X \rightarrow Y_1\ Y_2\ Y_3 \ldots\ Y_n$ **do**

    begin i:=1;

      while $Y_i \Rightarrow^* \varepsilon$ and i <= n **do begin**

        First[X] := First[X] $\cup$ First[Y_i]\{ε};

        i := i+1;

      end

    if i = n+1 **then** First[X] := First[X] $\cup$ {ε};

until no change in First[X] for any X;

ComputeFirst; modified

foreach $X \in$ **T** **do** First[X] := X;

foreach $p \in$ **P** : $X \rightarrow \varepsilon$ **do** First[X] := {ε};

repeat foreach $X \in$ **N**, p : $X \rightarrow Y_1\, Y_2\, Y_3 \ldots\; Y_n$ **do**

begin i:=1;

while $Y_i \Rightarrow^* \varepsilon$

First[X] := F

i := i+1;

end

if i = n+1 **then** First[X] := First[X] ∪ {ε};

until no change in First[X] for any X;

Non-recursive FIRST computation works with left-recursive grammars. Computes a fixed point for FIRST[X] for all non-terminals X in the grammar. But this algorithm is very inefficient.

ComputeFollow

```
Follow(S) := {$};
repeat
 foreach p ∈ P do
    case p = A → αBβ begin
      Follow[B] := Follow[B] ∪ ComputeFirst(β)\{ε};
      if ε ∈ First(β) then
        Follow[B] := Follow[B] ∪ Follow[A];
    end
   case p = A → αB
      Follow[B] := Follow[B] ∪ Follow[A];
until no change in any Follow[N]
```

Example First/Follow

$S \rightarrow AB$

$A \rightarrow c \mid \varepsilon$

$B \rightarrow cbB \mid ca$

Not an LL(1) grammar

$\text{First}(A) = \{c, \varepsilon\}$

$\text{First}(B) = \{c\}$

$\text{First}(cbB) =$

$\text{First}(ca) = \{c\}$

$\text{First}(S) = \{c\}$

$\text{Follow}(A) = \{c\}$

$\text{Follow}(A) \cap$

$\text{First}(c) = \{c\}$

$\text{Follow}(B) = \{\$\}$

$\text{Follow}(S) = \{\$\}$

ComputeFirst on Left-recursive Grammars

- ComputeFirst as defined earlier loops on left-recursive grammars
- Here is an alternative algorithm for ComputeFirst
  1. Compute non left-recursive cases of FIRST
  2. Create a graph of recursive cases where FIRST of a non-terminal depends on another non-terminal
  3. Compute Strongly Connected Components (SCC)
  4. Compute FIRST starting from root of SCC to avoid cycles

ComputeFirst on Left-recursive Grammars

- Each Strongly Connected Component can have recursion
- But the connections between SCC means that (by defn) what we have now is a directed acyclic graph – hence without left recursion
- Unlike top-down LL parsing, bottom-up LR parsing allows left-recursive grammars, so this algorithm is useful for LR parsing

ComputeFirst on Left-recursive Grammars

- $S \rightarrow BD \mid D$
- $D \rightarrow d \mid Sd$
- $A \rightarrow CB \mid a$
- $C \rightarrow Bb \mid \varepsilon$
- $B \rightarrow Ab \mid b$

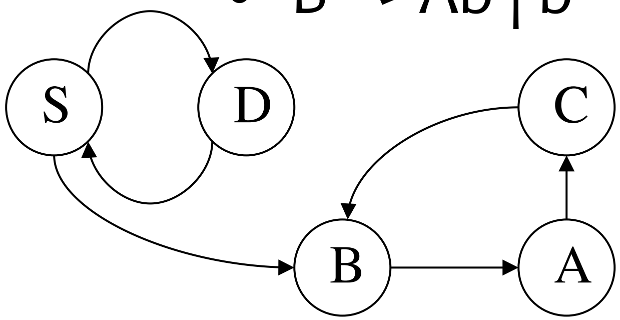
$FIRST_0[A] := \{a\}$

$FIRST_0[C] := \{\}$

$FIRST_0[B] := \{b\}$

$FIRST_0[S] := \{b, d\}$

$FIRST_0[D] := \{d\}$



Compute Strongly Connected Components

2 SCCs: e.g. consider B-A-C

$FIRST[B] := FIRST_0[B] + ComputeFirst(A)$

$FIRST[A] := FIRST_0[A] + ComputeFirst(C\ )$

$FIRST[A] := FIRST[A] + FIRST_0[B]$

$FIRST[C] := FIRST_0[C] + FIRST_0[B]$

$FIRST[C] := FIRST[C] + \{\varepsilon\}$

How to compute: Does $X \Rightarrow^* \varepsilon$?

- The question `Does $X \Rightarrow^* \varepsilon$?' can be written as the predicate: nullable(X)

```
Nullable = {} (set containing nullable non-terminals)
Changed = True
While (changed):
    changed = False
    if X is not in Nullable:
        if
            1. X → ε is in the grammar, or
            2. X → Y1 … Yn is in the grammar and Yi is in Nullable for all i
        then
            add X to Nullable; changed = True
```

Converting to LL(1)

$S \rightarrow AB$

$A \rightarrow c \mid \varepsilon$

$B \rightarrow cbB \mid ca$

Note that grammar is regular: c? (cb)* ca

c (c b c b … c b) c a (c b c b … c b) c a	c c (b c b … c b c) a c (b c b … c b c) a

same as:

c c? (bc)* a

$S \rightarrow cAa$

$A \rightarrow cB \mid B$

$B \rightarrow bcB \mid \varepsilon$

Verifying LL(1) using F/F sets

$S \rightarrow cAa$

$A \rightarrow cB \mid B$

$B \rightarrow bcB \mid \varepsilon$

$\text{First}(A) = \{b, c, \varepsilon\}$ $\quad$ $\text{Follow}(A) = \{a\}$

$\text{First}(B) = \{b, \varepsilon\}$ $\quad$ $\text{Follow}(B) = \{a\}$

$\text{First}(S) = \{c\}$ $\quad$ $\text{Follow}(S) = \{\$\}$

Building the Parse Table

- Compute First and Follow sets
- For each production $A \rightarrow \alpha$
  - foreach $a \in$ First(α) add $A \rightarrow \alpha$ to M[A,a]
  - If $\boldsymbol{\varepsilon} \in$ First(α) add $A \rightarrow \alpha$ to M[A,b] for each b in Follow(A)
  - If $\boldsymbol{\varepsilon} \in$ First(α) add $A \rightarrow \alpha$ to M[A,$] if $ $\in$ Follow(α)
  - All undefined entries are errors

Predictive Parsing Table

Productions	
1	**T → F T'**
2	**T' → ε**
3	**T' → * F T'**
4	**F → id**
5	**F → (T)**

FIRST(T) = {id, (}
FIRST(T') = {*, ε}
FIRST(F) = {id, (}

FOLLOW(T) = {$,)}
FOLLOW(T') = {$,)}
FOLLOW(F) = {*,$,)}

	*	(	)	id	$
T		**T → F T'**		**T → F T'**	
T'	**T' → * F T'**		**T' → ε**		**T' → ε**
F		**F → (T)**		**F → id**	

Revisit conditions for LL(1)

- A grammar G is LL(1) iff - whenever $A \rightarrow \alpha \mid \beta$
  1. $\text{First}(\alpha) \cap \text{First}(\beta) = \emptyset$
  2. $\alpha \Rightarrow^* \varepsilon$ implies $!(\beta \Rightarrow^* \varepsilon)$
  3. $\alpha \Rightarrow^* \varepsilon$ implies $\text{First}(\beta) \cap \text{Follow}(A) = \emptyset$
- No more than one entry per table field

Error Handling

- Reporting & Recovery
  - Report as soon as possible
  - Suitable error messages
  - Resume after error
  - Avoid cascading errors
- Phrase-level vs. Panic-mode recovery

Panic-Mode Recovery

- Skip tokens until *synchronizing set* is seen
  - Follow(A)
    - garbage or missing things after
  - Higher-level start symbols
  - First(A)
    - garbage before
  - Epsilon
    - if nullable
  - Pop/Insert terminal
    - “auto-insert”
- Add “synch” actions to table

Summary so far

- LL(1) grammars, necessary conditions
  - No left recursion
  - Left-factored
- Not all languages can be generated by LL(1) grammar
- LL(1) – Parsing: $O(n)$ time complexity
  - recursive-descent and table-driven predictive parsing
- LL(1) grammars can be parsed by simple predictive recursive-descent parser
  - Alternative: table-driven top-down parser

Examples

$S \rightarrow A\ B\ C$

$A \rightarrow a \mid \varepsilon$

$B \rightarrow b\ B \mid \varepsilon$

$C \rightarrow c \mid \varepsilon$

Is this LL(1)?

$S \rightarrow F$

$F \rightarrow A\ (\ B\ ) \mid B\ A$

$A \rightarrow x \mid y$

$B \rightarrow a\ B \mid b\ B \mid \varepsilon$

Is this LL(1)?