# LR Parsing

CMPT 379: Compilers
Instructor: Anoop Sarkar
anoopsarkar.github.io/compilers-class

# Top-Down vs. Bottom Up

Grammar: $S \rightarrow A\ B$

$A \rightarrow c \mid \varepsilon$

$B \rightarrow cbB \mid ca$

Input String: ccbca

| Top-Down/leftmost | | Bottom-Up/rightmost | |
|---|---|---|---|
| $S \Rightarrow AB$ | $S \rightarrow AB$ | $ccbca \Leftarrow Acbca$ | $A \rightarrow c$ |
| $\Rightarrow cB$ | $A \rightarrow c$ | $\Leftarrow AcbB$ | $B \rightarrow ca$ |
| $\Rightarrow ccbB$ | $B \rightarrow cbB$ | $\Leftarrow AB$ | $B \rightarrow cbB$ |
| $\Rightarrow ccbca$ | $B \rightarrow ca$ | $\Leftarrow S$ | $S \rightarrow AB$ |

# Rightmost derivation for **id + id * id**

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow ( E )$

$E \rightarrow - E$

$E \rightarrow id$

rightmost derivation step

$E \Rightarrow_{rm} E * E$

$\Rightarrow_{rm} E * \mathbf{id}$

$\Rightarrow_{rm} E + E * \mathbf{id}$

$\Rightarrow_{rm} E + \mathbf{id} * \mathbf{id}$

$\Rightarrow_{rm} \mathbf{id} + \mathbf{id} * \mathbf{id}$

reduce with $E \rightarrow id$

shift

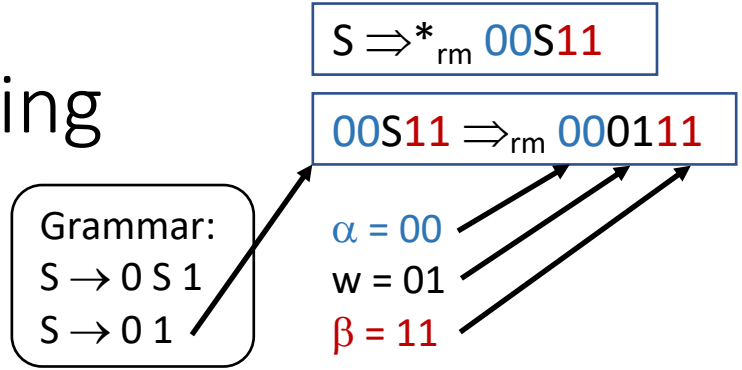
$E \Rightarrow^*_{rm} E \text{ ‘+’ } E \text{ ‘*’ } id$

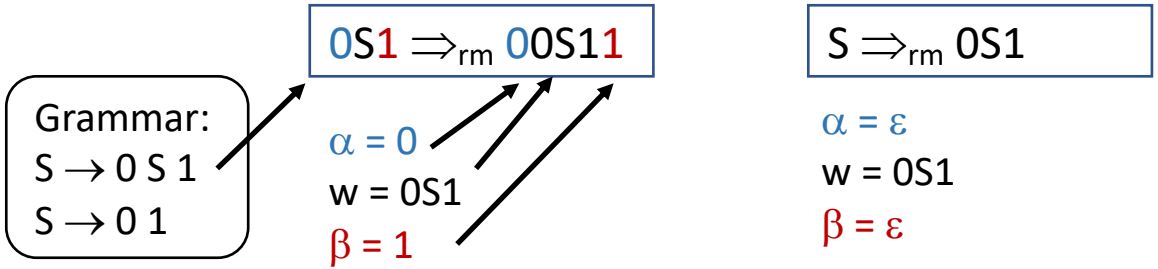
# LR parsing overview

- Start from terminal symbols, search for a path to the start symbol
- Apply shift and reduce actions (postpone decisions when possible)
- LR parsing:
  - L: left to right parsing
  - R: rightmost derivation (in reverse or bottom-up)
- LR(0) $\rightarrow$ SLR(1) $\rightarrow$ LR(1) $\rightarrow$ LALR(1)
  - 0 or 1 or *k* lookahead symbols

# Actions in Shift-Reduce Parsing

- Shift
  - add terminal to parse stack, advance input
- Reduce
  - If $\alpha w$ is on the stack, $\alpha, w \in (N \cup T)^*$ and $A \rightarrow w$, and there is a $\beta \in T^*$ such that $S \Rightarrow^*_{rm} \alpha A \beta \Rightarrow_{rm} \alpha w \beta$ then we can reduce $\alpha w$ to $\alpha A$ on the stack (called *pruning the handle* w)
  - $\alpha w$ is a *viable prefix*
- Error
- Accept



# Questions

- When to shift/reduce?
  - What are valid handles?
  - Ambiguity: Shift/reduce conflict
- If reducing, using which production?
  - Ambiguity: Reduce/reduce conflict

# Table-based Shift Reduce Parsing

# LR Parsing

- Table-based parser
  - Creates rightmost derivation (in reverse)
  - Works with left- and right- recursive context-free grammars
- Data structures:
  - Stack of states/symbols {s}
  - Action table: **action**[s, a]; a ∈ **T**
  - Goto table: **goto**[s, X]; X ∈ **N**

# Action/Goto Table

Action

S5: shift to state 5

Goto

| Productions | |
|---|---|
| 1 | T → F |
| 2 | T → T*F |
| 3 | F → id |
| 4 | F → (T) |

R1: reduce using rule 1

| | * | ( | ) | id | $ | T | F |
|---|---|---|---|---|---|---|---|
| 0 | | S5 | | S8 | | 2 | 1 |
| 1 | R1 | R1 | R1 | R1 | R1 | | |
| 2 | S3 | | | | Acc! | | |
| 3 | | S5 | | S8 | | | 4 |
| 4 | R2 | R2 | R2 | R2 | R2 | | |
| 5 | | S5 | | S8 | | 6 | 1 |
| 6 | S3 | | S7 | | | | |
| 7 | R4 | R4 | R4 | R4 | R4 | | |
| 8 | R3 | R3 | R3 | R3 | R3 | | |

# Trace "(id)*id"

| Productions | |
|---|---|
| 1 | T → F |
| 2 | T → T*F |
| 3 | F → id |
| 4 | F → (T) |

| | * | ( | ) | id | $ | T | F |
|---|---|---|---|---|---|---|---|
| 0 | | S5 | | S8 | | 2 | 1 |
| 1 | R1 | R1 | R1 | R1 | R1 | | |
| 2 | S3 | | | | A | | |
| 3 | | S5 | | S8 | | | 4 |
| 4 | R2 | R2 | R2 | R2 | R2 | | |
| 5 | | S5 | | S8 | | 6 | 1 |
| 6 | S3 | | S7 | | | | |
| 7 | R4 | R4 | R4 | R4 | R4 | | |
| 8 | R3 | R3 | R3 | R3 | R3 | | |

0 is the start state

| Stack | Input | Action/Goto |
|---|---|---|
| 0 | ( id ) * id $ | Shift S5 |
| 0 5 | id ) * id $ | Shift S8 |
| 0 5 8 | ) * id $ | Reduce 3 F→id, pop 8, goto [5,F]=1 |
| 0 5 1 | ) * id $ | Reduce 1 T→ F, pop 1, goto [5,T]=6 |
| 0 5 6 | ) * id $ | Shift S7 |
| 0 5 6 7 | * id $ | Reduce 4 F→ (T), pop 7 6 5, goto [0,F]=1 |
| 0 1 | * id $ | Reduce 1 T → F pop 1, goto [0,T]=2 |

# Trace “(id)*id”

| Productions | |
|---|---|
| 1 | T → F |
| 2 | T → T*F |
| 3 | F → id |
| 4 | F → (T) |

| | * | ( | ) | id | $ | T | F |
|---|---|---|---|---|---|---|---|
| 0 | | S5 | | S8 | | 2 | 1 |
| 1 | R1 | R1 | R1 | R1 | R1 | | |
| 2 | S3 | | | | A | | |
| 3 | | S5 | | S8 | | | 4 |
| 4 | R2 | R2 | R2 | R2 | R2 | | |
| 5 | | S5 | | S8 | | 6 | 1 |
| 6 | S3 | | S7 | | | | |
| 7 | R4 | R4 | R4 | R4 | R4 | | |
| 8 | R3 | R3 | R3 | R3 | R3 | | |

| Stack | Input | Action/Goto |
|---|---|---|
| 0 1 | * id $ | Reduce 1 T→F, |
| | | pop 1, goto [0,T]=2 |
| 0 2 | * id $ | Shift S3 |
| 0 2 3 | id $ | Shift S8 |
| 0 2 3 8 | $ | Reduce 3 F→id, |
| | | pop 8, goto [3,F]=4 |
| 0 2 3 4 | $ | Reduce 2 T→T * F |
| | | pop 4 3 2, goto [0,T]=2 |
| 0 2 | $ | Accept |

# Tracing LR: **action**[*s*, *a*]

- case **shift** *u*:
  - push state *u*
  - read new *a*
- case **reduce** *r*:
  - lookup production *r*: $X \rightarrow Y_1..Y_k$;
  - pop *k* states, find state *u*
  - push **goto**[*u*, *X*]
- case **accept:** done
- no entry in action table: **error**

# Algorithm to build the Action/Goto table

# Configuration set

- Each set is a parser state
- We use the notion of a dotted rule or item:

  **T → T * • F**

- The dot is before **F**, so we predict all rules with **F** as the left-hand side

  **T → T * • F**

  **F → • ( T )**

  **F → • id**

- This creates a configuration set (or item set)
  - Like NFA-to-DFA conversion

# Closure

Closure property:

- If $\mathbf{T \rightarrow X_1 \ldots X_i \bullet X_{i+1} \ldots X_n}$ is in set, and $\mathbf{X_{i+1}}$ is a nonterminal, then $\mathbf{X_{i+1} \rightarrow \bullet Y_1 \ldots Y_m}$ is in the set as well for all productions $\mathbf{X_{i+1} \rightarrow Y_1 \ldots Y_m}$
- Compute as fixed point
- The closure property creates a configuration set (item set) from a dotted rule (item).

# Starting Configuration

- Augment Grammar with S’
- Add production S’ $\rightarrow$ S
- Initial configuration set is

$$\text{closure}(S' \rightarrow \bullet\, S)$$

# Example: I = closure(S’ → • T)

S’ → T
T → F | T * F
F → id | ( T )

# Example: I = closure(S’ → • T)

S’ → • T

T → • T * F

T → • F

F → • id

F → • ( T )

S’ → T
T → F | T * F
F → id | ( T )

# Successor(I, X)

Informally: “move by symbol X”

1. move dot to the right in all items where dot is before X
2. remove all other items

   (viable prefixes only!)
3. compute closure

# Successor Example

$I = \{S' \rightarrow \bullet\ T,$

$T \rightarrow \bullet\ F,$

$T \rightarrow \bullet\ T * F,$

$F \rightarrow \bullet\ id,$

$F \rightarrow \bullet\ (\ T\ )\ \}$

$S' \rightarrow T$
$T \rightarrow F \mid T * F$
$F \rightarrow id \mid (\ T\ )$

Compute Successor(I, '(')

$\{ F \rightarrow (\ \bullet\ T\ ),\ T \rightarrow \bullet\ F,\ T \rightarrow \bullet\ T * F,$

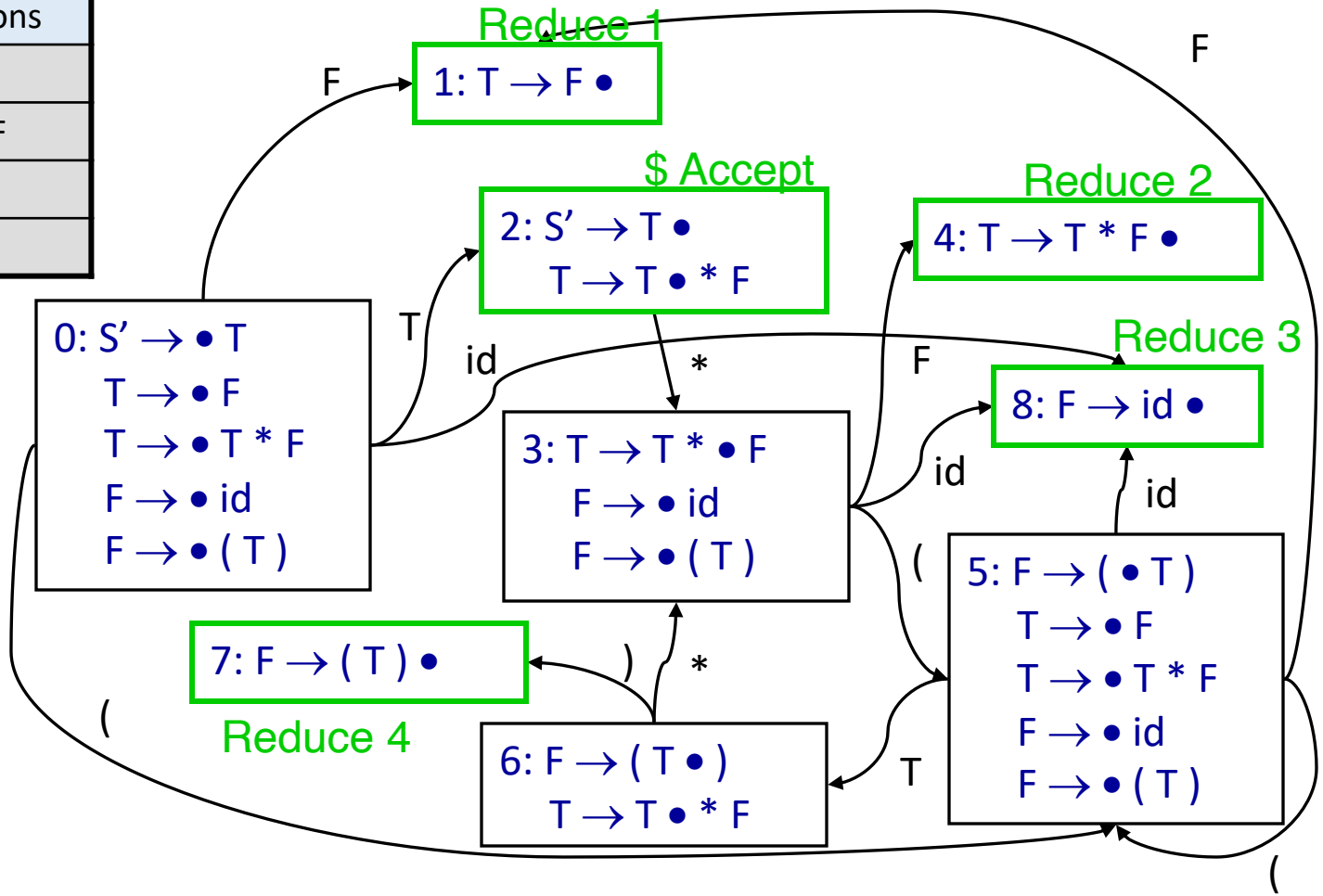
$F \rightarrow \bullet\ id,\ F \rightarrow \bullet\ (\ T\ )\ \}$

# Sets-of-Items Construction

Family of configuration sets

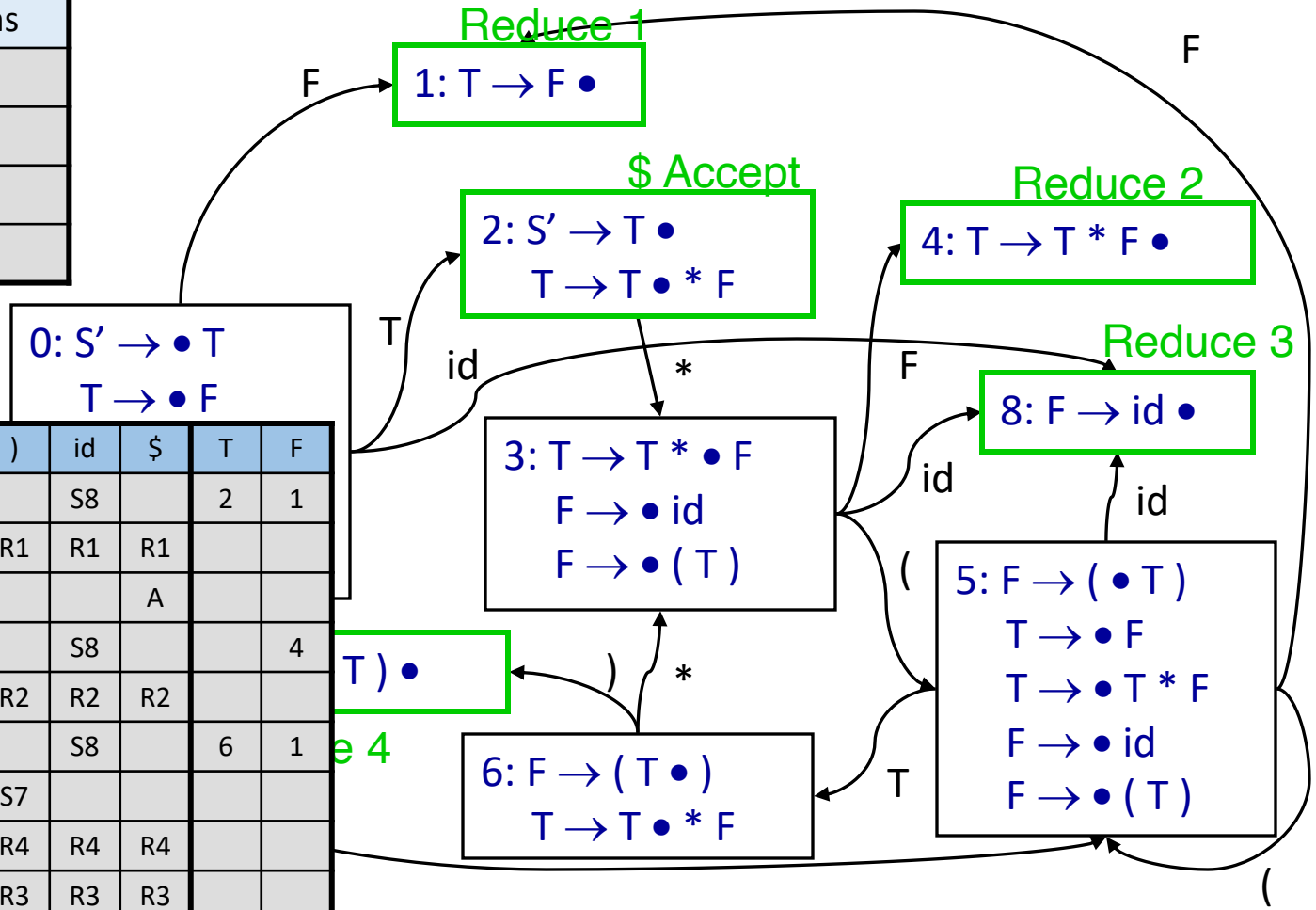
**function** items(G')
    C = { closure({S' $\rightarrow \bullet$ S}) };
    **do foreach** I $\in$ C **do**
        **foreach** X $\in$ **(N $\cup$ T) do**
            C = C $\cup$ { **Successor**(I, X) };
    **while** C changes;

| Productions | |
|---|---|
| 1 | T → F |
| 2 | T → T*F |
| 3 | F → id |
| 4 | F → (T) |



| Productions | |
|---|---|
| 1 | T → F |
| 2 | T → T*F |
| 3 | F → id |
| 4 | F → (T) |

| | * | ( | ) | id | $ | T | F |
|---|---|---|---|---|---|---|---|
| 0 | | S5 | | S8 | | 2 | 1 |
| 1 | R1 | R1 | R1 | R1 | R1 | | |
| 2 | S3 | | | | A | | |
| 3 | | S5 | | S8 | | | 4 |
| 4 | R2 | R2 | R2 | R2 | R2 | | |
| 5 | | S5 | | S8 | | 6 | 1 |
| 6 | S3 | | S7 | | | | |
| 7 | R4 | R4 | R4 | R4 | R4 | | |
| 8 | R3 | R3 | R3 | R3 | R3 | | |



# LR(0) Construction

1. Construct F = $\{I_0, I_1, \ldots I_n\}$
2. a) if $\{A \rightarrow \alpha\bullet\} \in I_i$ and A != S'

   then action[i, _] := reduce $A \rightarrow \alpha$

   b) if $\{S' \rightarrow S\bullet\} \in I_i$

   then action[i,$] := accept

   c) if $\{A \rightarrow \alpha\bullet a\beta\} \in I_i$ and Successor($I_i$,a) = $I_j$

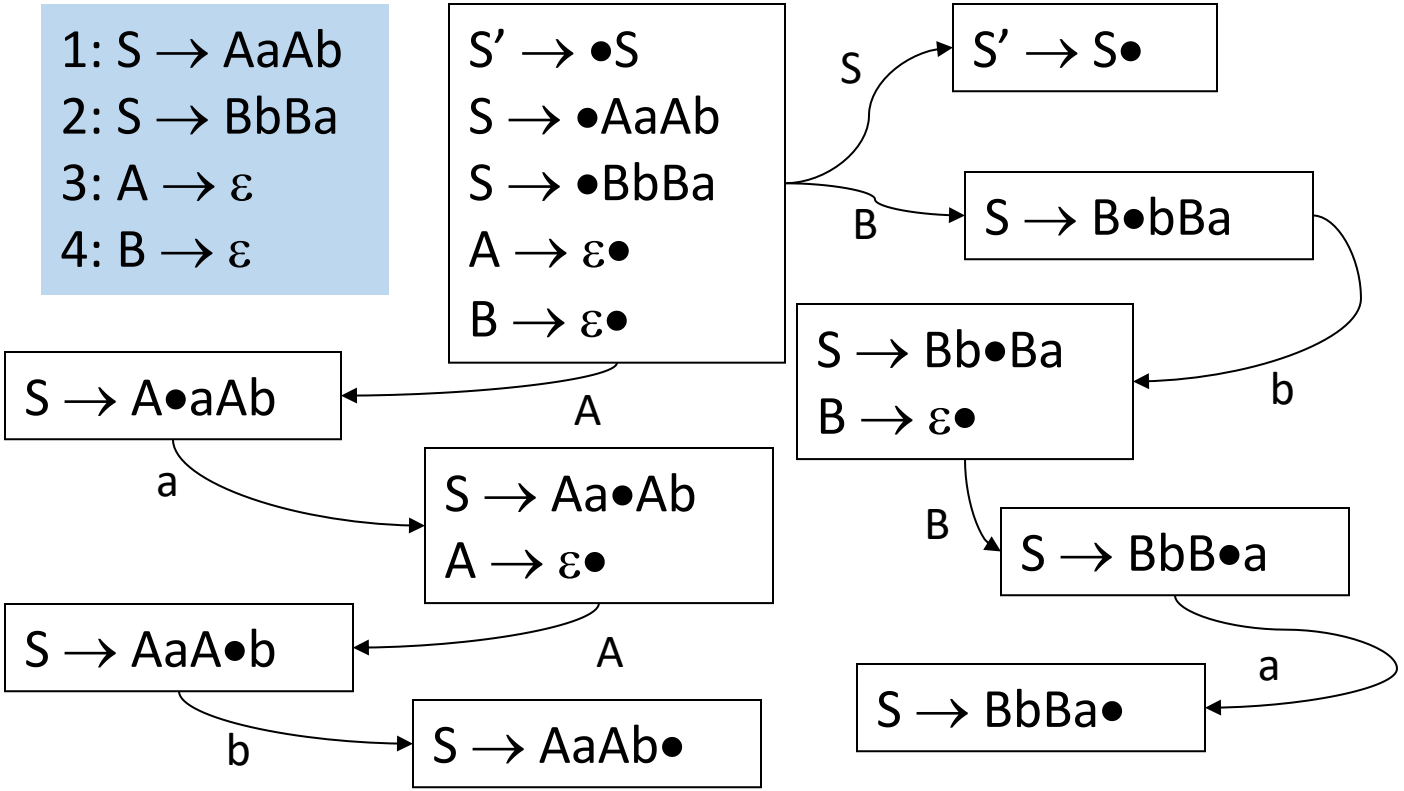
   then action[i,a] := shift j
3. if Successor($I_i$,A) = $I_j$ then goto[i,A] := j

# LR(0) Construction (cont'd)

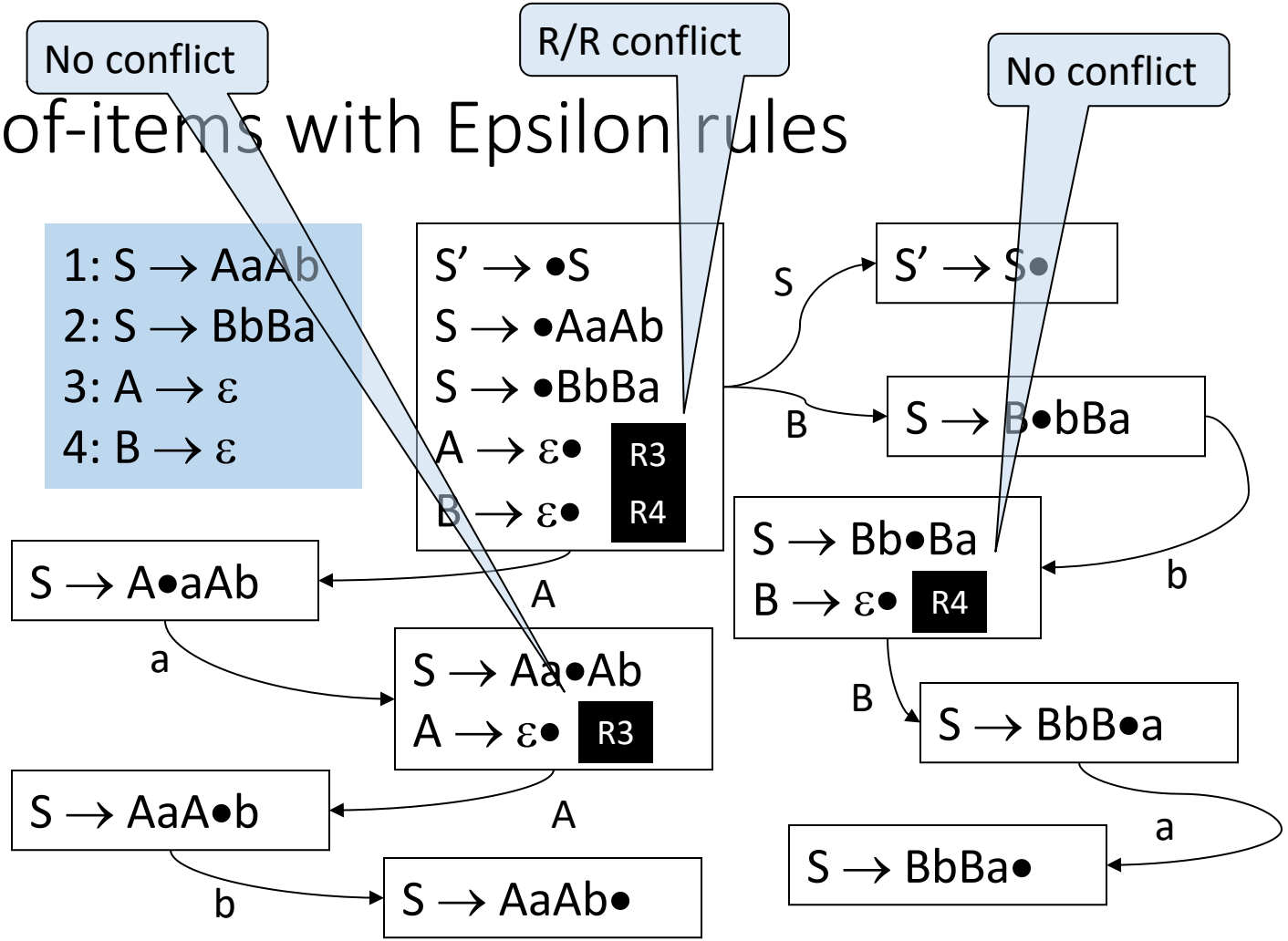
4. All entries not defined are errors
5. Make sure $I_0$ is the initial state

- Note: LR(0) always reduces if $\{A \rightarrow \alpha\bullet\} \in I_i$, no lookahead
- Shift and reduce items can't be in the same configuration set
  - Accepting state doesn't count as reduce item
- At most one reduce item per set

# Set-of-items with Epsilon rules



# Set-of-items with Epsilon rules



# LR(0) conflicts:

$S' \rightarrow T$

$T \rightarrow F$

$T \rightarrow T * F$

$T \rightarrow id$

$F \rightarrow id \mid ( T )$

$F \rightarrow id = T ;$

1: $F \rightarrow id \bullet$

$F \rightarrow id \bullet = T$

Shift/reduce conflict

1: $F \rightarrow id \bullet$

$T \rightarrow id \bullet$

Reduce/Reduce conflict

Need more lookahead: SLR(1)

# Viable Prefixes

- γ is a viable prefix if there is some ω such that γ|ω is a state of a shift-reduce parser


- Important fact: A viable prefix is a prefix of a handle
- An LR(0) item [X→ α • β] says that
  - α is on top of the stack (α is a suffix of γ)
  - The parser is looking for an X
  - Expects to find input string derived from β
- We can recognize viable prefixes via a NFA (DFA)
  - States of NFA are LR(0) items
  - States of DFA are sets of LR(0) items

# LR(0) Grammars

- An LR(0) grammar is a CFG such that the LR(0) construction produces a table without conflicts (a deterministic pushdown automata)
- $S \Rightarrow^*_{rm} \alpha A \beta \Rightarrow_{rm} \alpha w \beta$ and $A \rightarrow w$ then we can *prune the handle* w
  - pruning the handle means we can reduce $\alpha w$ to $\alpha A$ on the stack
- Every viable prefix $\alpha w$ can be recognized using the DFA built by the LR(0) construction

# LR(0) Grammars

- Once we have a viable prefix on the stack, we can prune the handle and then restart the DFA to obtain another viable prefix, and so on ...
- In LR(0) pruning the handle can be done without any look-ahead
  - this means that in the rightmost derivation,
  - $S \Rightarrow^*_{rm} \alpha A \beta \Rightarrow_{rm} \alpha w \beta$ we reduce using a unique rule $A \rightarrow w$ without ambiguity, and without looking at $\beta$
- No ambiguous context-free grammar can be LR(0)

LR(0) Grammars $\subset$ Context-free Grammars

# Parsing - Roadmap

- Parser:
  - decision procedure: builds a parse tree
- Top-down vs. bottom-up
- LL(1) – Deterministic Parsing
  - recursive-descent
  - table-driven
- LR(k) – Deterministic Parsing
  - LR(0), SLR(1), LR(1), LALR(1)
- Parsing arbitrary CFGs – Polynomial time parsing