

LR Parsing

CMPT 379: Compilers

Instructor: Anoop Sarkar

anoopsarkar.github.io/compiler-class

LR(0) conflicts:

$S' \rightarrow T$

$T \rightarrow F$

$T \rightarrow T * F$

$T \rightarrow id$

$F \rightarrow id \mid (T)$

$F \rightarrow id = T ;$

11: $F \rightarrow id \bullet$

$F \rightarrow id \bullet = T$

Shift/reduce conflict

1: $F \rightarrow id \bullet$

$T \rightarrow id \bullet$

Reduce/Reduce conflict

Need more lookahead: SLR(1)

FIRST and FOLLOW

$a \in \text{FIRST}(\alpha)$ if $\alpha \Rightarrow^* a\beta$

if $\alpha \Rightarrow^* \epsilon$ then $\epsilon \in \text{FIRST}(\alpha)$

$a \in \text{FOLLOW}(A)$ if $S \Rightarrow^* \alpha A a \beta$

$a \in \text{FOLLOW}(A)$ if $S \Rightarrow^* \alpha A \gamma a \beta$

and $\gamma \Rightarrow^* \epsilon$

Example First/Follow

$$S \rightarrow AB$$

$$A \rightarrow c \mid \varepsilon$$

$$B \rightarrow cbB \mid ca$$

$$\text{First}(A) = \{c, \varepsilon\}$$

$$\text{Follow}(A) = \{c\}$$

$$\text{First}(B) = \{c\}$$

$$\text{Follow}(A) \cap$$

$$\text{First}(cbB) =$$

$$\text{First}(c) = \{c\}$$

$$\text{First}(ca) = \{c\}$$

$$\text{Follow}(B) = \{\$ \}$$

$$\text{First}(S) = \{c\}$$

$$\text{Follow}(S) = \{\$ \}$$

Example First/Follow

$$S \rightarrow cAa$$

$$A \rightarrow cB \mid B$$

$$B \rightarrow bcB \mid \varepsilon$$

If $X \rightarrow \alpha A a$ and a is terminal then the set $\text{Follow}(A)$ includes a

If $X \rightarrow \alpha A$ then the set $\text{Follow}(A)$ includes $\text{Follow}(X)$

$$\text{First}(A) = \{b, c, \varepsilon\}$$

$$\text{First}(B) = \{b, \varepsilon\}$$

$$\text{First}(S) = \{c\}$$

$$\text{Follow}(A) = \{a\}$$

$$\text{Follow}(B) = \{a\}$$

$$\text{Follow}(S) = \{\$ \}$$

SLR(1) : Simple LR(1) Parsing

$$\begin{aligned} S' &\rightarrow T \\ T &\rightarrow F \mid T * F \mid C (T) \\ F &\rightarrow id \mid id ++ \mid (T) \\ C &\rightarrow id \end{aligned}$$

What can the next symbol be when we reduce $F \rightarrow id$?

$$S' \$ \Rightarrow T \$ \Rightarrow F \$ \Rightarrow id \underline{\$} \quad S' \$ \Rightarrow T \$ \Rightarrow T * F \$ \Rightarrow T * id \$ \Rightarrow F * id \$ \Rightarrow id \underline{*} id \$$$
$$S' \$ \Rightarrow T \$ \Rightarrow C(T) \$ \Rightarrow C(F) \$ \Rightarrow C(id) \underline{\$}$$

The top of stack will be id and the next input symbol will be either $\$$, or $*$ or $)$

$$\text{Follow}(F) = \{ *,), \$ \}$$

SLR(1) : Simple LR(1) Parsing

$$\begin{aligned} S' &\rightarrow T \\ T &\rightarrow F \mid T * F \mid C (T) \\ F &\rightarrow id \mid id ++ \mid (T) \\ C &\rightarrow id \end{aligned}$$

What can the next symbol be when we reduce $C \rightarrow id$?

$$S' \$ \Rightarrow T \$ \Rightarrow C(T) \$ \Rightarrow C(F) \$ \Rightarrow C(id) \Rightarrow id(id) \$$$

$\text{Follow}(C) = \{ (\}$

SLR(1) : Simple LR(1) Parsing

0: $S' \rightarrow \bullet T$
 $T \rightarrow \bullet F$
 $T \rightarrow \bullet T * F$
 $T \rightarrow \bullet C (T)$
 $F \rightarrow \bullet id$
 $F \rightarrow \bullet id ++$
 $F \rightarrow \bullet (T)$
 $C \rightarrow \bullet id$

id

$S' \rightarrow T$
 $T \rightarrow F \mid T * F \mid C (T)$
 $F \rightarrow id \mid id ++ \mid (T)$
 $C \rightarrow id$

1: $F \rightarrow id \bullet$
 $F \rightarrow id \bullet ++$
 $C \rightarrow id \bullet$

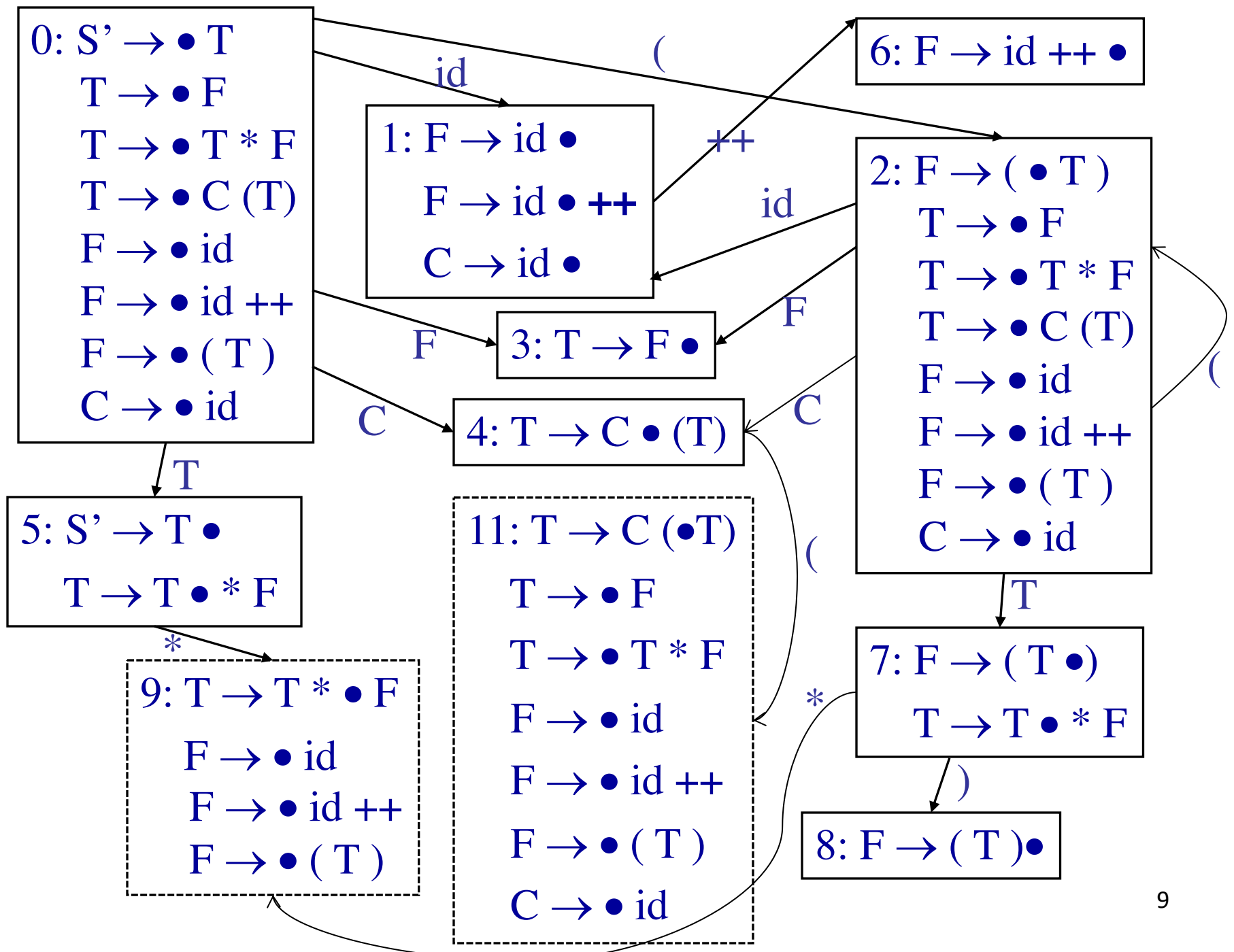
$\text{Follow}(F) = \{ *,), \$ \}$

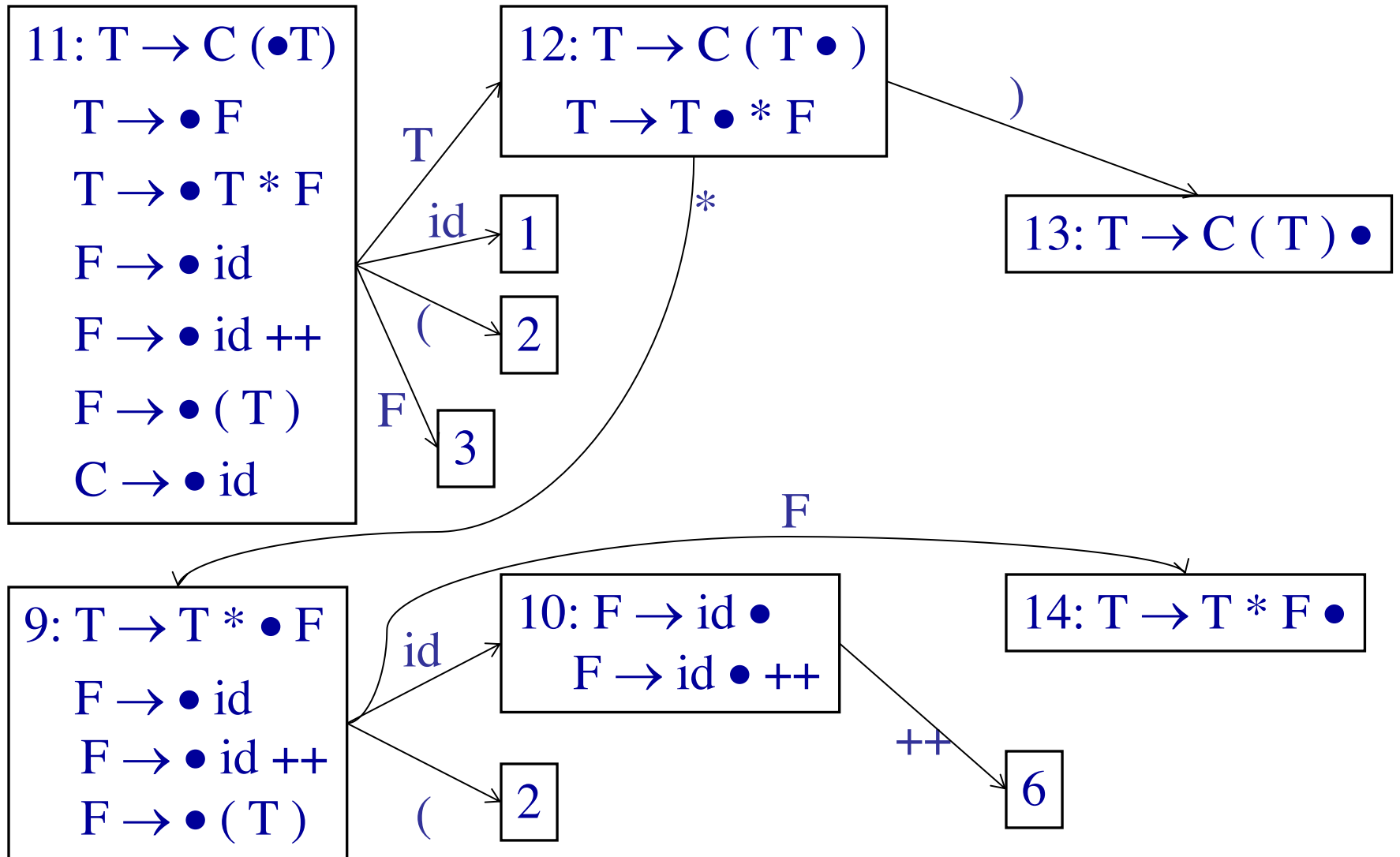
$\text{Follow}(C) = \{ (\}$

$\text{action}[1,*] = \text{action}[1,)] = \text{action}[1,\$] = \text{Reduce } F \rightarrow id$

$\text{action}[1,(] = \text{Reduce } C \rightarrow id$

$\text{action}[1,++] = \text{Shift}$





Productions	
1	$T \rightarrow F$
2	$T \rightarrow T * F$
3	$T \rightarrow C(T)$
4	$F \rightarrow id$
5	$F \rightarrow id ++$
6	$F \rightarrow (T)$
7	$C \rightarrow id$

	*	()	id	++	\$	T	F	C
0		S2		S1			5	3	4
1	R4	R7	R4		S2	R4			
2		S2		S1			7	3	4
3	R1		R1			R1			
4		S11							
5	S9					A			
6	R5		R5			R5			
7	S9		S8						
8	R6		R6			R6			
9		S2		S10				14	
10	R4		R4		S6	R4			
11		S2		S1			12	3	
12	S9		S13						
13	R3		R3			R3			
14	R2		R2			R2			

If there are still conflicts under these rules, grammar is not SLR

SLR Parsing

- Assume:
 - Stack contains α and next input is t
 - DFA on input α terminates in state s
- Reduce by $X \rightarrow \beta$ if
 - s contains item $X \rightarrow \beta \bullet$
 - $t \in \text{Follow}(X)$
- Shift if
 - s contains item $X \rightarrow \beta \bullet t w$
 - If $X \rightarrow \beta \bullet$ is in s then t cannot be in $\text{Follow}(X)$

$S' \rightarrow E$

$E \rightarrow T + E$

$E \rightarrow T$

$T \rightarrow id$

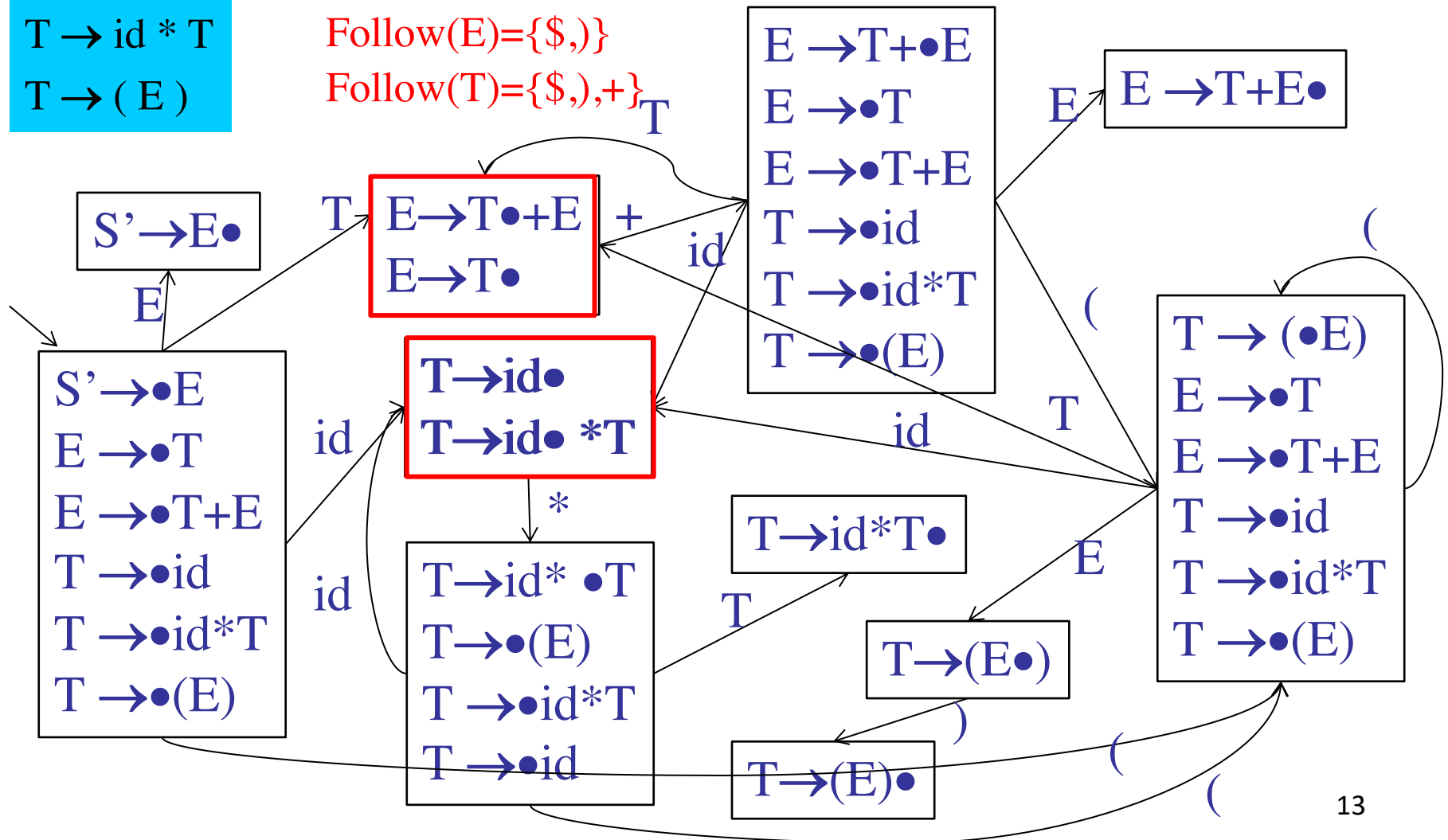
$T \rightarrow id * T$

$T \rightarrow (E)$

SLR Parsing

$Follow(E) = \{\$, \,)\}$

$Follow(T) = \{\$, \,)\, +\}$



SLR Parsing

- Let M be DFA for viable prefixes of G
- Let $|x_1 \dots x_n \$$ be initial configuration
- Repeat until configuration is $S | \$$
 - Let $\alpha | \omega$ be current configuration
 - Run M on current stack α
 - If M rejects α , report parsing error
 - Stack α is not a viable prefix
 - If M accepts α with items I , let a be the next input
 - Shift $[X \rightarrow \beta \bullet a \gamma] \in I$
 - Reduce if $[X \rightarrow \beta \bullet] \in I$ and $a \in \text{Follow}(X)$
 - Report parsing error if neither applies

If there is any conflict in the last step (more than two valid action), grammar is not SLR(k) in practice $k=1$

Trace 'id*id'

configuration (Stackinput)	DFA halt state	Action
id * id \$		

$S' \rightarrow E$

$E \rightarrow T + E$

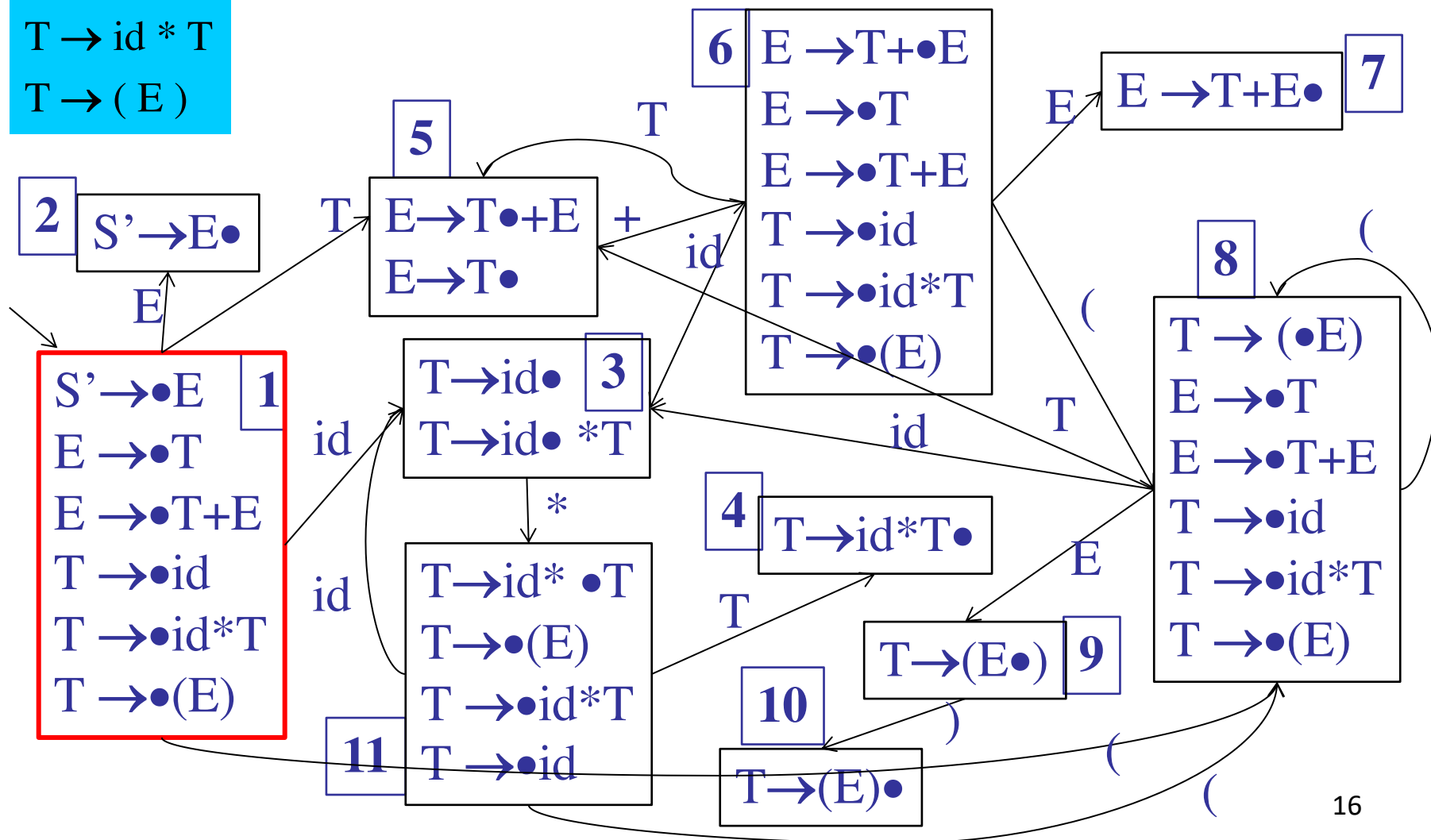
$E \rightarrow T$

$T \rightarrow id$

$T \rightarrow id * T$

$T \rightarrow (E)$

| id * id \$



Trace 'id*id'

configuration (Stackinput)	DFA halt state	Action
id * id \$ id * id \$	1	Shift

$S' \rightarrow E$

$E \rightarrow T + E$

$E \rightarrow T$

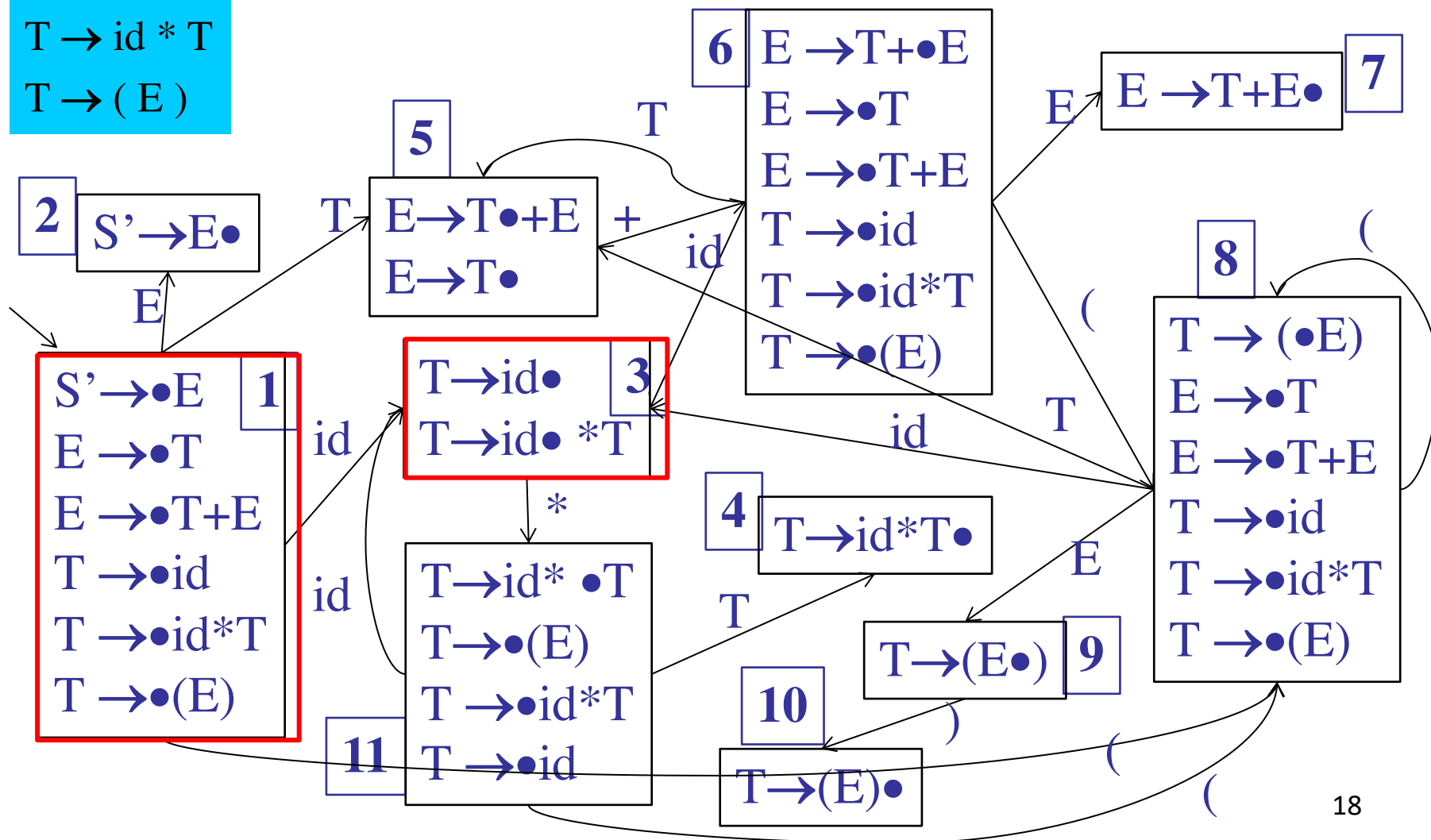
$T \rightarrow id$

$T \rightarrow id * T$

$T \rightarrow (E)$

$Follow(T) = \{ \$,), + \}$

id | * id \$



Trace 'id*id'

configuration (Stack input)	DFA halt state	Action
id * id \$	1	Shift
id * id \$	3 * $\notin \text{Follow}(T)$	Shift
id * id \$		

$S' \rightarrow E$

$E \rightarrow T + E$

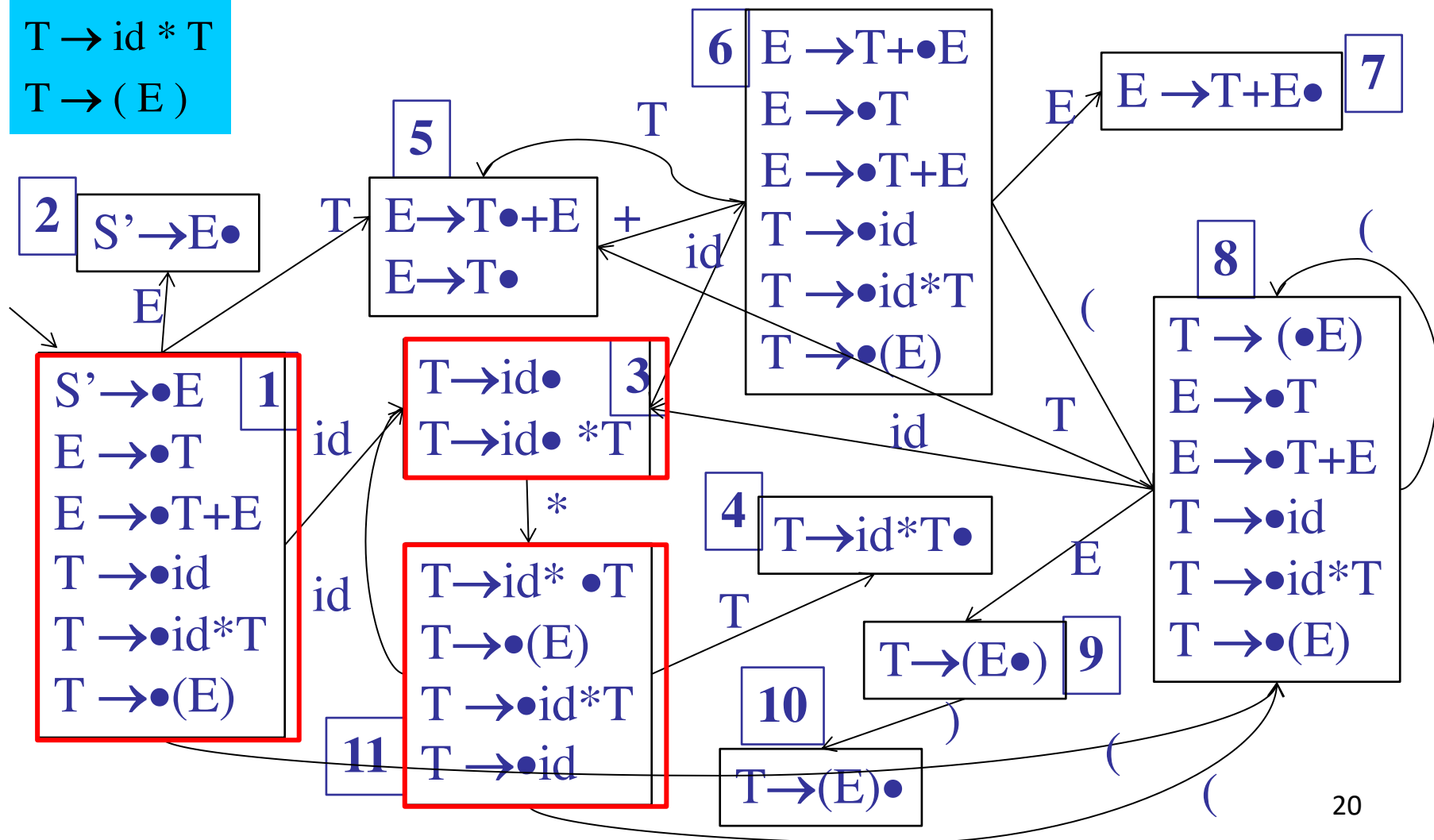
$E \rightarrow T$

$T \rightarrow id$

$T \rightarrow id * T$

$T \rightarrow (E)$

$id * \textcolor{red}{|} id \$$



Trace 'id*id'

configuration (Stackinput)	DFA halt state	Action
id * id \$	1	Shift
id * id \$	3 * $\notin \text{Follow}(T)$	Shift
id * id \$	11	Shift
id * id \$		

$S' \rightarrow E$

$E \rightarrow T + E$

$E \rightarrow T$

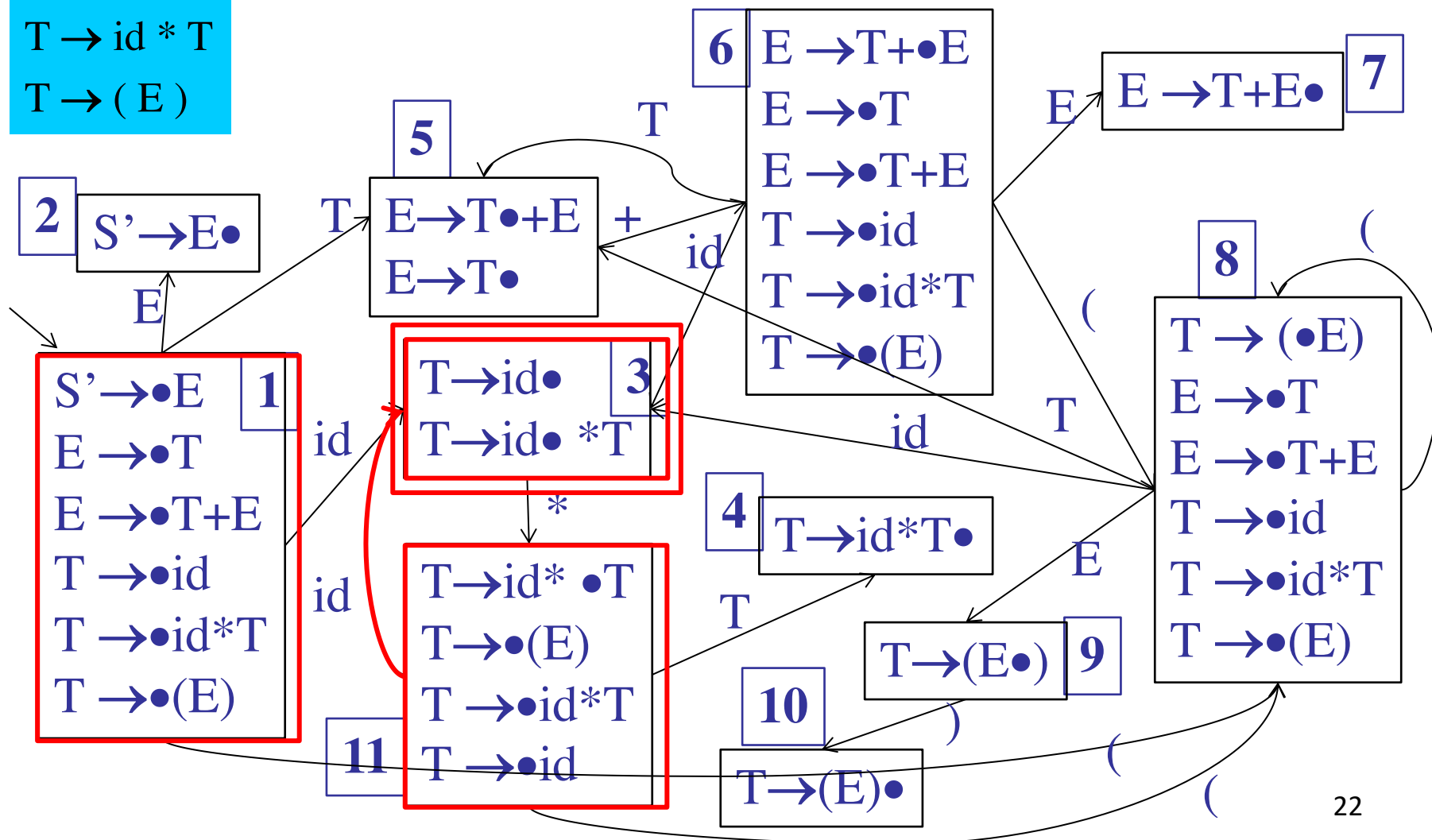
$T \rightarrow id$

$T \rightarrow id * T$

$T \rightarrow (E)$

$Follow(T) = \{\$,), +\}$

$id * id \mid \$$



Trace 'id*id'

configuration (Stack input)	DFA halt state	Action
id * id \$	1	Shift
id * id \$	3 * $\notin \text{Follow}(T)$	Shift
id * id \$	11	Shift
id * id \$	3 \$ $\in \text{Follow}(T)$	Reduce $T \rightarrow id$
id * T \$		

$S' \rightarrow E$

$E \rightarrow T + E$

$E \rightarrow T$

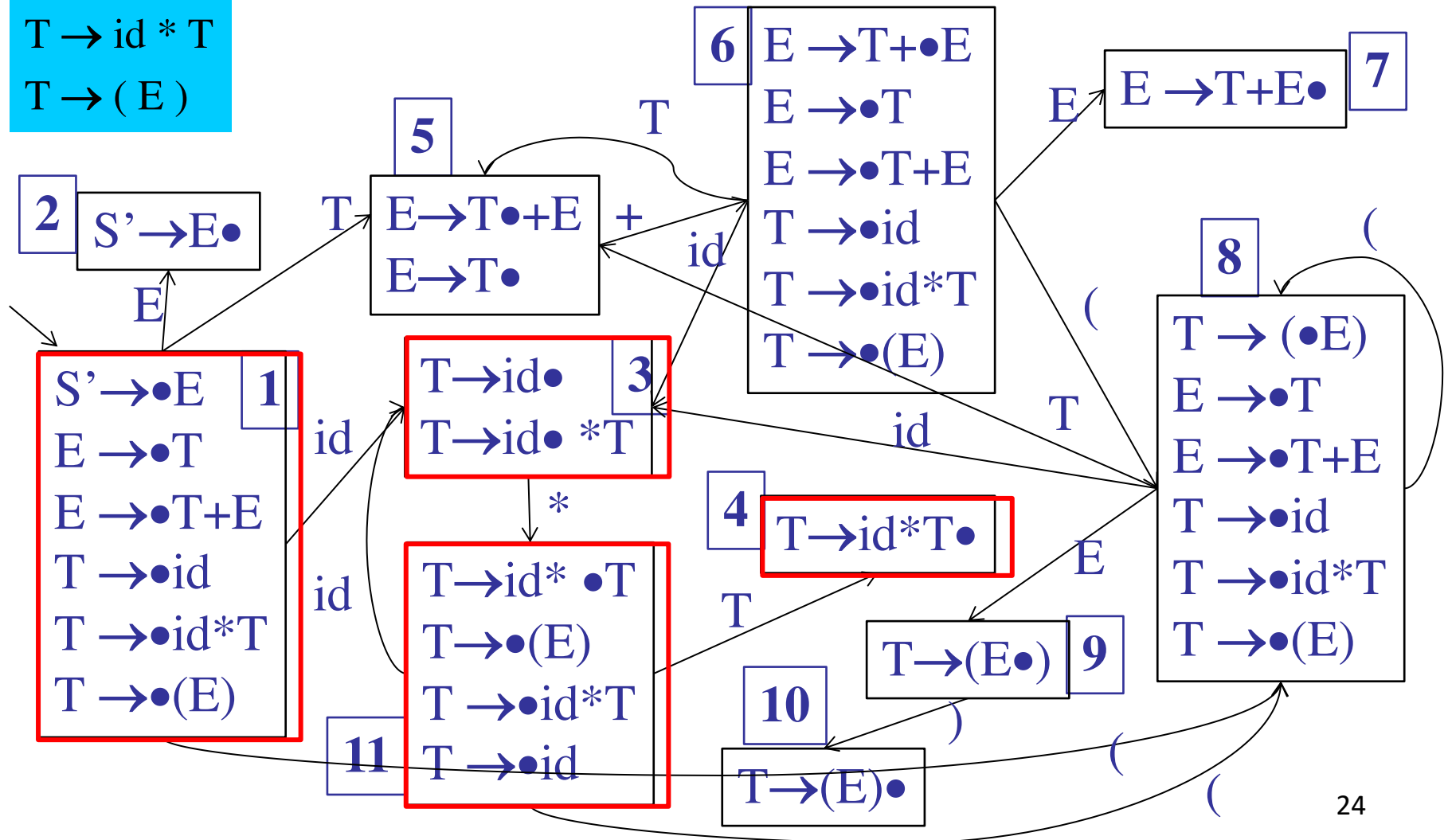
$T \rightarrow id$

$T \rightarrow id * T$

$T \rightarrow (E)$

$Follow(T) = \{ \$,), + \}$

$id * T \mid \$$



Trace 'id*id'

configuration (Stack input)	DFA halt state	Action
id * id \$	1	Shift
id * id \$	3 * $\notin \text{Follow}(T)$	Shift
id * id \$	11	Shift
id * id \$	3 \$ $\in \text{Follow}(T)$	Reduce $T \rightarrow \text{id}$
id * T \$	4 \$ $\in \text{Follow}(T)$	Reduce $T \rightarrow \text{id} * T$
T \$		

$S' \rightarrow E$

$E \rightarrow T + E$

$E \rightarrow T$

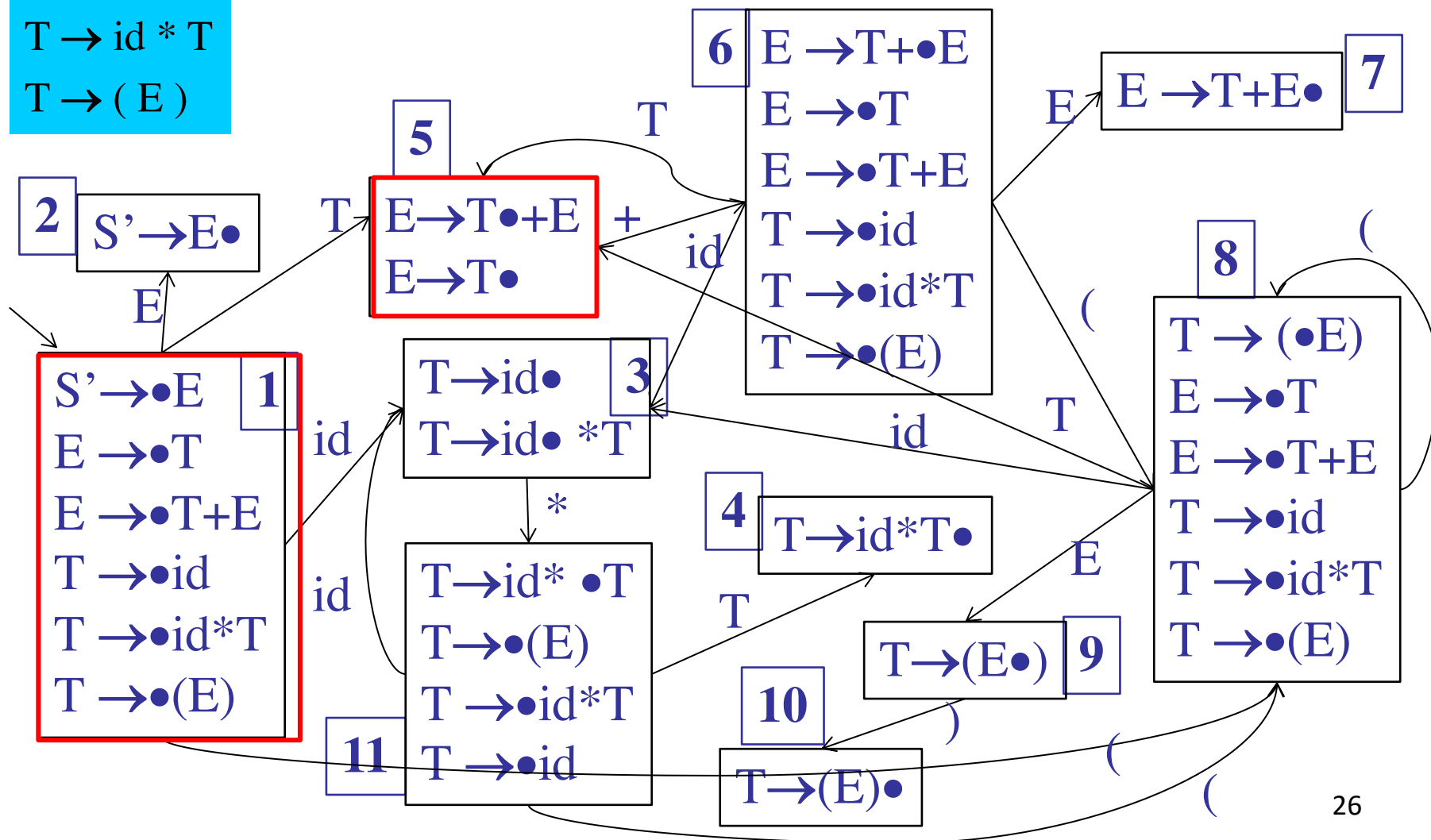
$T \rightarrow id$

$T \rightarrow id * T$

$T \rightarrow (E)$

$Follow(E) = \{\$,)\}$

$T \mid \$$



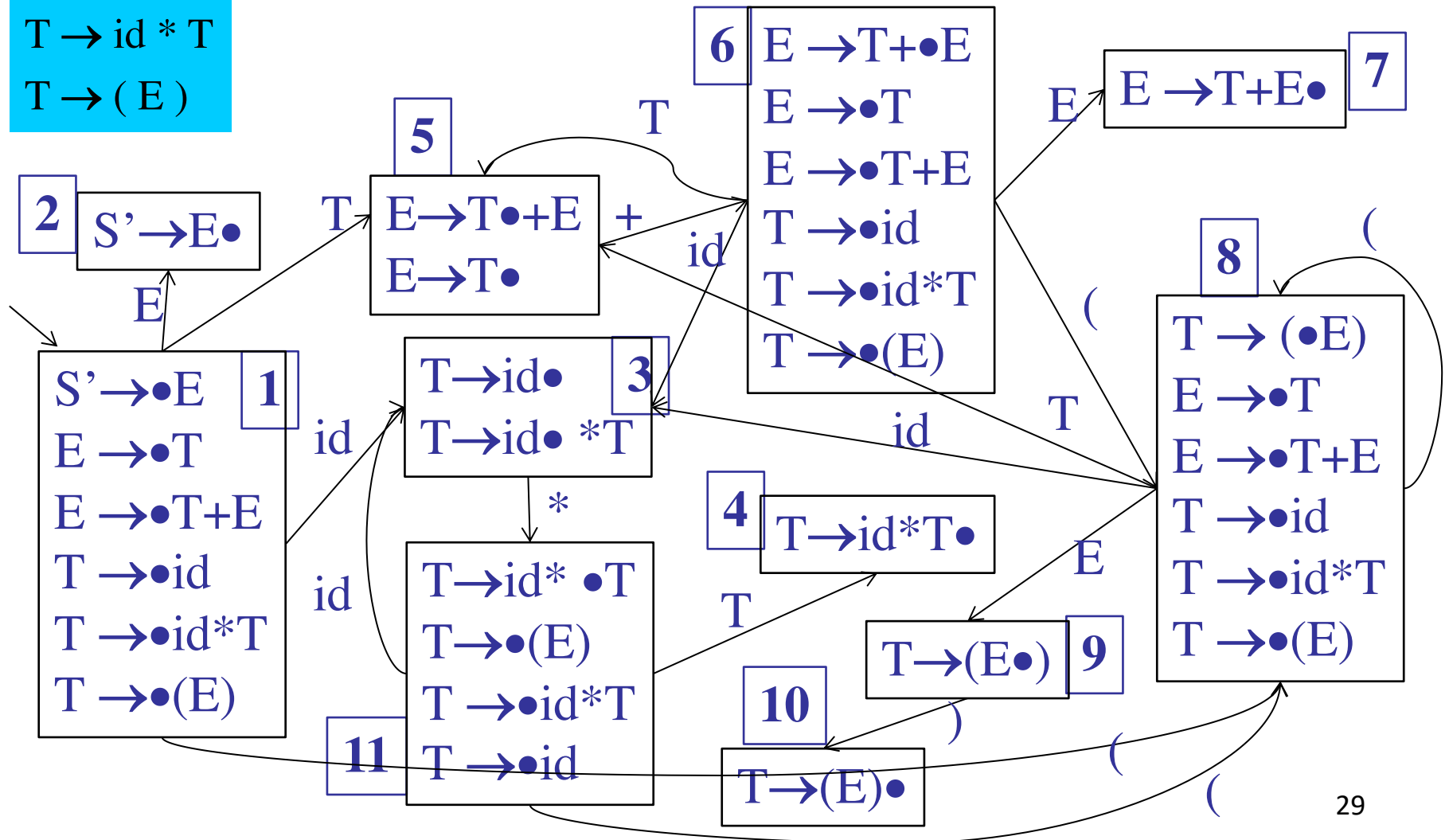
Trace 'id*id'

configuration (Stack input)	DFA halt state	Action
id * id \$	1	Shift
id * id \$	3 * \notin Follow(T)	Shift
id * id \$	11	Shift
id * id \$	3 \$ \in Follow(T)	Reduce $T \rightarrow id$
id * T \$	4 \$ \in Follow(T)	Reduce $T \rightarrow id * T$
T \$	5 \$ \in Follow(E)	Reduce $E \rightarrow T$
E \$		

Trace 'id*id'

configuration (Stack input)	DFA halt state	Action
id * id \$	1	Shift
id * id \$	3 * $\notin \text{Follow}(T)$	Shift
id * id \$	11	Shift
id * id \$	3 \$ $\in \text{Follow}(T)$	Reduce $T \rightarrow \text{id}$
id * T \$	4 \$ $\in \text{Follow}(T)$	Reduce $T \rightarrow \text{id} * T$
T \$	5 \$ $\in \text{Follow}(T)$	Reduce $E \rightarrow T$
E \$		Accept

$S' \rightarrow E$
 $E \rightarrow T + E$
 $E \rightarrow T$
 $T \rightarrow id$
 $T \rightarrow id * T$
 $T \rightarrow (E)$



Constructing SLR states

- Begin with item $S' \rightarrow \bullet S$, calculate related items (**closure**)
- Determine following states: what states can be reached on a single input token or non-terminal (**GOTO**)
- Construct closure of each resulting states

SLR(1) Construction

1. Construct $F = \{I_0, I_1, \dots, I_n\}$
2. a) if $\{A \rightarrow \alpha \bullet\} \in I_i$ and $A \neq S'$
then $\text{action}[i, b] := \text{reduce } A \rightarrow \alpha$
for all $b \in \text{Follow}(A)$
b) if $\{S' \rightarrow S \bullet\} \in I_i$
then $\text{action}[i, \$] := \text{accept}$
c) if $\{A \rightarrow \alpha \bullet a \beta\} \in I_i$ and $\text{Successor}(I_i, a) = I_j$
then $\text{action}[i, a] := \text{shift } j$
3. if $\text{Successor}(I_i, A) = I_j$ then $\text{goto}[i, A] := j$

SLR(1) Construction (cont'd)

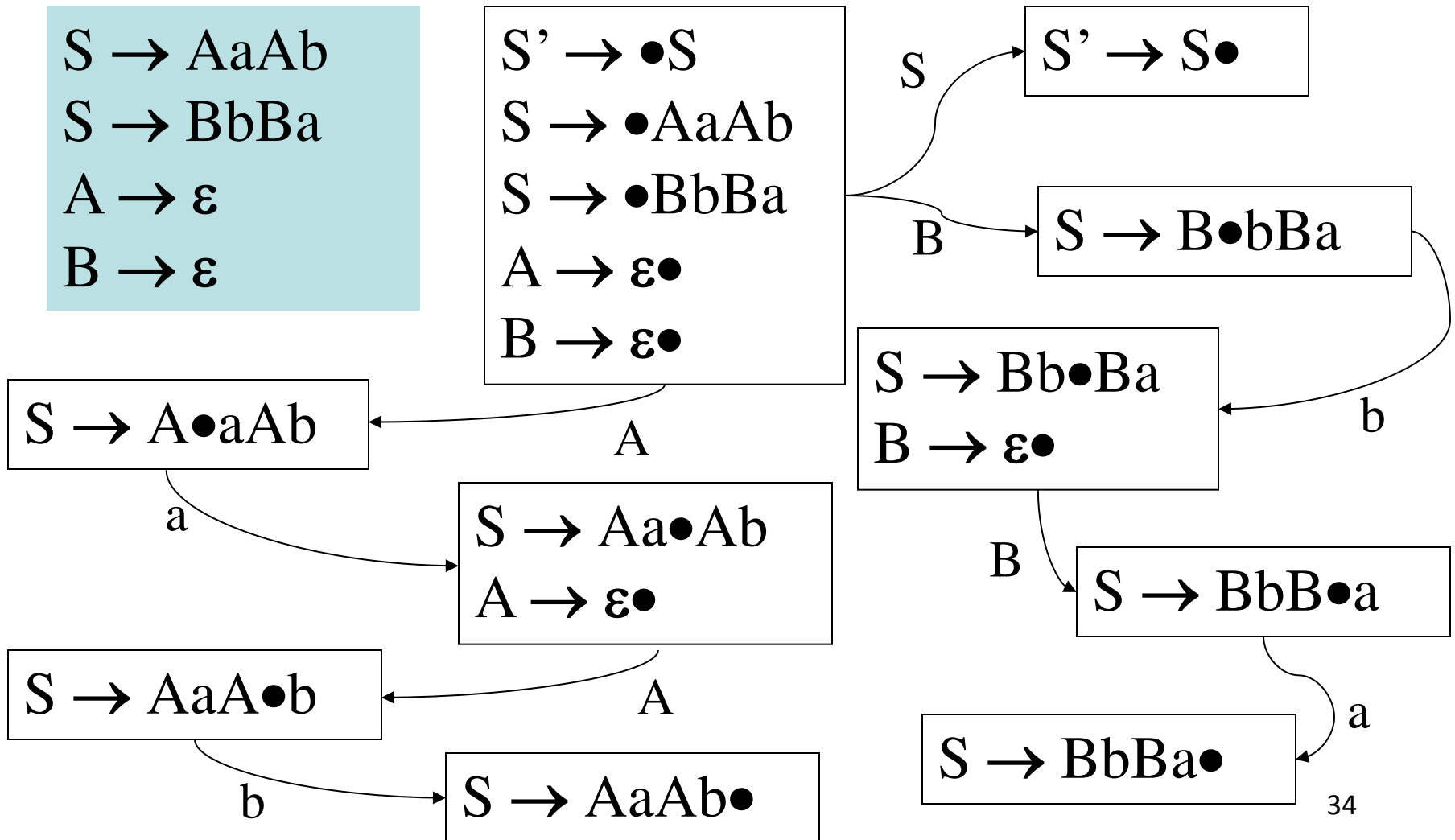
4. All entries not defined are errors
 5. Make sure I_0 is the initial state
- Note: SLR(1) only reduces $\{A \rightarrow \alpha \bullet\}$ if lookahead in $\text{Follow}(A)$
 - Shift and reduce items or more than one reduce item can be in the same configuration set as long as lookaheads are disjoint

SLR(1) Conditions

- A grammar is SLR(1) if for each configuration set:
 - For any item $\{A \rightarrow \alpha \bullet x \beta : x \in T\}$ there is no $\{B \rightarrow \gamma \bullet : x \in \text{Follow}(B)\}$
 - For any two items $\{A \rightarrow \alpha \bullet\}$ and $\{B \rightarrow \beta \bullet\}$ $\text{Follow}(A) \cap \text{Follow}(B) = \emptyset$

LR(0) Grammars \subset SLR(1) Grammars

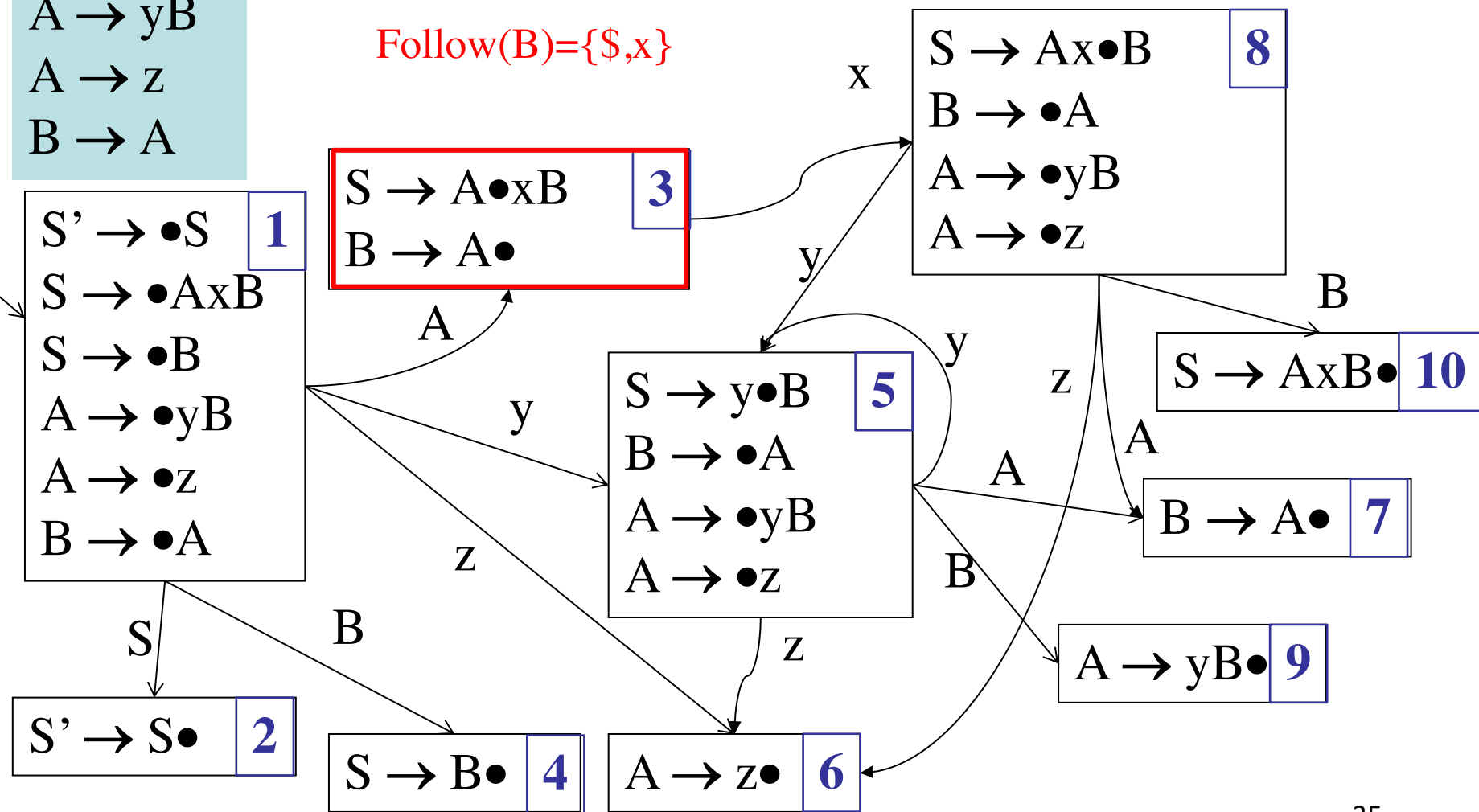
Is this grammar SLR(1)?



$S' \rightarrow S$
 $S \rightarrow Ax B$
 $S \rightarrow B$
 $A \rightarrow y B$
 $A \rightarrow z$
 $B \rightarrow A$

Is this grammar SLR(1)?

$\text{Follow}(B) = \{\$, x\}$



- 1) $S \rightarrow AxB$
- 2) $S \rightarrow B$
- 3) $A \rightarrow yB$
- 4) $A \rightarrow z$
- 5) $B \rightarrow A$

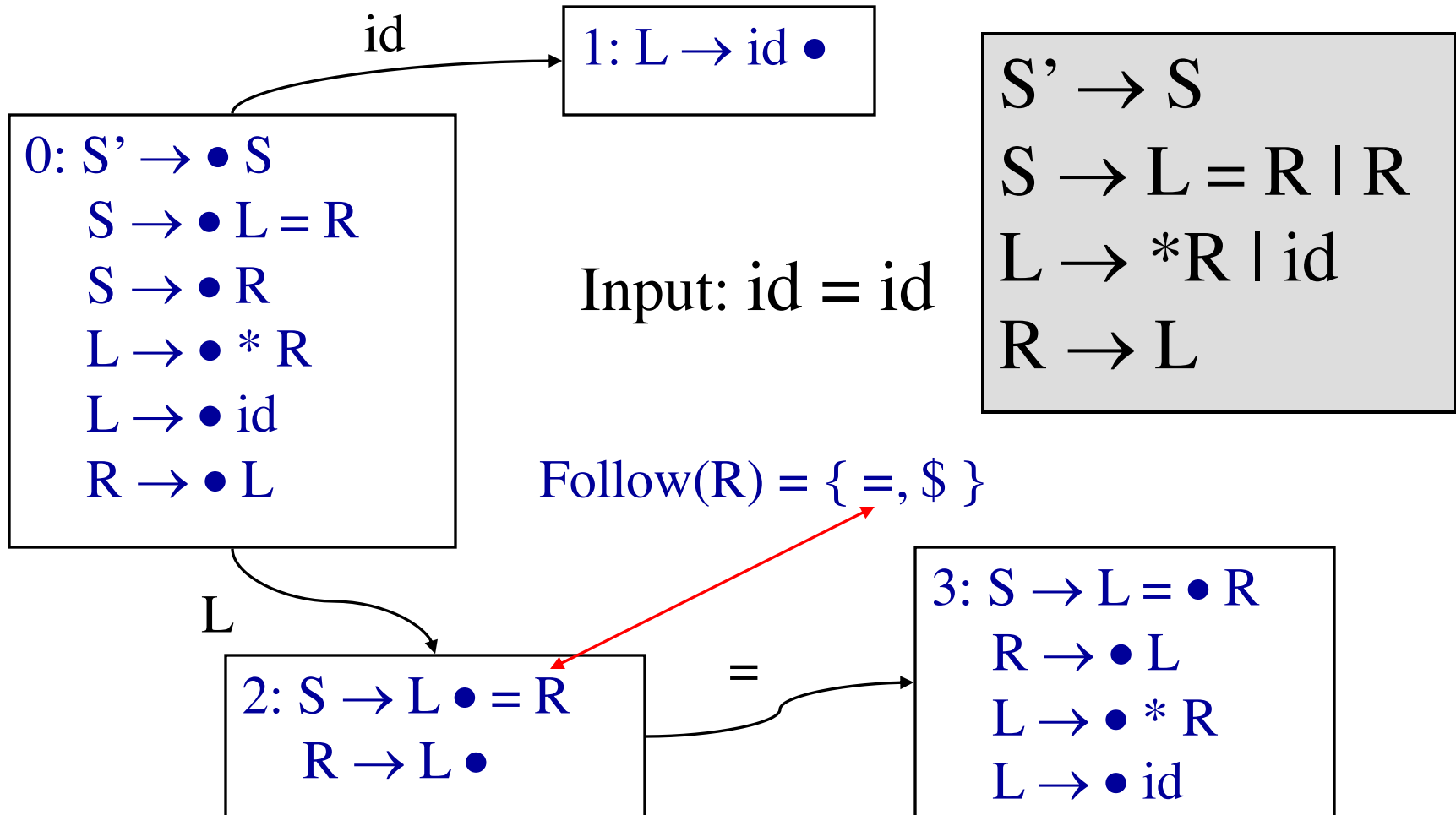
SLR Parsing Table

Grammar is not SLR

Reduce is a bad choice

	x	y	z	\$	S	A	B
1		S5	S6		2	3	4
2				ACC!			
3	S8,R5			R5			
4				R2			
5		S5	S6			7	9
6	R4			R4			
7	R5			R5			
8		S5	S6			7	10
9	R3			R3			
10				R1			

SLR limitation: lack of context



$$S' \rightarrow S$$

$$S \rightarrow L = R \mid R$$

$$L \rightarrow *R \mid \text{id}$$

$$R \rightarrow L$$

$$\text{Follow}(R) = \{ =, \$ \}$$

$$2: S \rightarrow L \bullet = R$$

$$R \rightarrow L \bullet$$

Find all lookaheads
for reduce $R \rightarrow L \bullet$

$$\begin{array}{c} S' \\ | \\ S \\ | \\ R \\ | \\ L \\ | \\ \text{id} \end{array} \quad \$$$

$$\begin{array}{c} S' \\ | \\ S \\ | \\ L \quad = \quad R \\ | \quad \quad | \\ \text{id} \quad L \\ \quad \quad | \\ \quad \quad \text{id} \end{array} \quad \$$$

$$\begin{array}{c} S' \\ | \\ S \\ | \\ R \\ | \\ L \\ | \\ * \quad R \\ \quad | \\ \quad L \\ \quad | \\ \quad \text{id} \end{array} \quad \$$$

$$\begin{array}{c} S' \\ | \\ S \\ | \quad | \quad | \\ L \quad = \quad R \quad \$ \\ | \quad \quad | \\ * \quad R \\ | \quad \quad | \\ L \quad \quad \text{id} \\ | \quad \quad | \\ \text{id} \quad \quad \text{id} \end{array} \quad \begin{array}{c} = \\ \text{Problem?} \end{array}$$

No! $R \rightarrow L \bullet$ reduce
and $S \rightarrow L \bullet = R$ do
not co-occur due to
the $L \rightarrow *R$ rule

Solution: Canonical LR(1)

- Extend definition of configuration
 - Remember lookahead
- New closure method
- Extend definition of Successor

LR(1) Parsing

- Limit introduced by SLR parsing in using Follow set to decide reductions
- Idea: augment LR items with 1 character lookahead $[B \rightarrow A\bullet, \$]$ making an LR(1) item
 - Reduce to B only if lookahead token is $\$$
- More accurate than just Follow set
- Similar to SLR parsing just use LR(1) items rather than LR(0) items