NØGLE TIL MCT

A:	Plimpton 322 (p. 38)
Aa:	YBC 6295 (p. 42)
B:	YBC 4675 (p. 44 - +1)
C:	YBC 9852 (p. 45)
Ca:	MLC 1950 (p. 48)
D:	YBC 4608 (p. 49 - +n)
E:	YBC 8633 (p. 53)
Ea:	NBC 7934 (p. 55)
Eb:	MLC 1354 (p. 56 - +1)
Ec:	YBC 8600 (p. 57)
F:	YBC 5037 (p. 59 - +1)
G:	YBC 4657 (p. 66 - +1)
H:	YBC 4663 (p. 69)
J:	YBC 4662 (p. 71 - +1)
Ja:	YBC 8588 (p. 75)
K:	YBC 4666 (p. 76 - +1)
L:	YBC 7164 (p. 81 - +1)
M:	YBC 9874 (p. 90)
N:	YBC 4186 (p. 91)
O:	YBC 4607 (p. 91 - +1)
P:	YBC 10772 (p. 98)
Pa:	YBC 7997 (p. 98)
Q:	YBC 9856 (p. 99)
R:	YBC 4652 (p. 100 - +1)
S:	YBC 4612 (p. 103 - +1)
Sa:	YBC 6492 (p. 105)
Sb:	MLC 1842 (p. 106)
T:	A 24194 (p. 107 - 112)
U:	A 24195 (p. 119 - 126)
Ua:	YBC 6967 (p. 129)
Ub:	YBC 7326 (p. 130)
Uc:	YBC 10522 (p. 131)
Y:	VAT 7848 (p. 141)

A 24194:	T
A 24195:	U
MLC 1354:	Eb
MLC 1842:	Sb
MLC 1950:	Ca
NBC 7934:	Ea
Plimpton 322:	A
VAT 7848:	Y
YBC 4186:	N
YBC 4607:	O
YBC 4608:	D
YBC 4612:	S
YBC 4652:	R
YBC 4657:	G
YBC 4662:	J
YBC 4663:	H
YBC 4666:	K
YBC 4675:	B
YBC 5037:	F
YBC 6295:	Aa
YBC 6492:	Sa
YBC 6967:	Ua
YBC 7164:	L
YBC 7326:	Ub
YBC 7997:	Pa
YBC 8588:	Ja
YBC 8600:	Ec
YBC 8633:	E
YBC 9852:	C
YBC 9856:	Q
YBC 9874:	M
YBC 10522:	Uc
YBC 10772:	P

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MATHEMATICAL CUNEIFORM TEXTS

EDITED BY

O. NEUGEBAUER AND A. SACHS

WITH A CHAPTER BY

A. GOETZE

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This Volume
is Gratefully Dedicated to
Professor Raymond Clare Archibald,
Indefatigable Student of the History of Mathematics,
for Many Acts of Kindness and
for Friendly Encouragement

PREFACE

This volume is devoted to the edition of hitherto unpublished mathematical texts chiefly from American collections. Not only does this new group of documents supplement the previously published material in many important respects, but is itself sufficiently extensive to offer a fair impression of the main types of Babylonian mathematical texts. New frontiers, the existence of which had been no more than suspected, are actually reached by a tablet (Plimpton 322) involving "Pythagorean" Number Theory.

In gathering this new material we have enjoyed the cooperation and generosity of many museums. For this help we herewith publicly extend our thanks to the staffs in charge of the collections of The Oriental Institute of The University of Chicago, The Free Public Museum of Liverpool (England), The Metropolitan Museum of Art in New York City, The Plimpton Library of Columbia University in New York City, and The University Museum of The University of Pennsylvania in Philadelphia. The Director of The Pierpont Morgan Library kindly granted permission to publish tablets belonging to the Morgan collection, which is on deposit at Yale University. The late Dr. H. Ehelolf kindly furnished the photographs of VAT 7848. Dr. F. W. Geers generously agreed to publish in this volume his preliminary copies of two tablets in the British Museum. Dr. F. R. Steele was patiently helpful in sending us descriptions and collations of a number of table-texts in The University Museum of The University of Pennsylvania. Several collations of texts in the same museum were made for us by Dr. S. N. Kramer. Special mention must be made of The Yale Babylonian Collection, the source of the largest part of the material presented here. More mathematical problem-texts have now been published from this collection than from any other museum in the world. Dr. F. J. Stephens, the curator, proved untiring in the baking and cleaning of the tablets, the collating of doubtful readings, and in putting excellent photographs of the texts at our disposal. In addition to numerous table-texts, most of the problem-texts published in this book were discovered by Dr. A. Goetze. We are also deeply indebted to Dr. Goetze for having contributed to this volume by his important chapter on the dialects of the Old-Babylonian Akkadian mathematical texts, frequently the only reliable means at our disposal for localizing (at least roughly) the tablets of unknown provenance.

While arrangements for printing were being worked out, eight new mathematical problem-texts were discovered by Drs. Goetze and Stephens. These documents (problem-texts Aa, Eb, Ec, Ja, Pa, Sa, Sb, Ue) have now been included in this volume. We call attention to this fact because of the possibility that some cross-references to these texts may have been overlooked in spite of our efforts to take account of all possible repercussions.

Our thanks are due to The American Oriental Society for the inclusion of this work in its Series, and to The Mathematical Association of America for having made a grant from the income of The A. Buffum Chace Memorial Fund toward the printing costs. We are also grateful to the American Council of Learned Societies and to the American Schools of Oriental Research for financial aid in the publication of this volume. We are, finally, indebted to The Rockefeller Foundation for having made possible our collaboration here at Brown University.

O. NEUGEBAUER
A. SACHS

January, 1945

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CHAPTER I. INTRODUCTION

§ 1. The Texts

The mathematical texts from ancient Babylonia fall into two main categories: the "table-texts" and the "problem-texts." The table texts are undoubtedly the older group and are in turn closely related to the metrological lists. The practical needs satisfied by the metrological texts are evident, since they enumerate in systematic arrangement certain measures (e.g., weights or lengths), their multiples, and fractions so that it is possible to read directly from the tablet how many units of a certain large measure correspond to a given number of units of a smaller. Analogously, a multiplication table immediately gives the result of the multiplication of a given number by a consecutive series of factors. All other table-texts are constructed according to the same principle. Problem-texts, however, sometimes exhibit only very remote connections, if any, with practical questions. They are school products intended to illustrate the rules for dealing with problems which are properly called "algebraic." This, of course, does not mean that the table-texts did not also belong in the curriculum of the scribal schools; indeed, many multiplication tables, especially those from Nippur, are typical school tablets written as exercises by pupils.[1] The difference in use between problem-texts and table-texts lies only in the fact that the ordinary table-texts belonged to a much lower level of scribal education and enjoyed wide use outside the schools, whereas the problem-texts are essentially a manifestation of higher education and were undoubtedly written and understood only by a very restricted group of scribes. It is therefore not at all surprising that many more table-texts than problem-texts have been preserved.[2]

We meet serious difficulties when we attempt to localize our material and to establish its exact date. A rather large number of the table-texts are of known provenance, particularly Nippur and Kiš,[3] but it is very difficult to give narrow time limits for these texts.[4] The majority of the problem-texts, on the other hand, are clearly Old-Babylonian (i.e., they are to be dated to the centuries around 1700 B.C.), but we are almost completely in the dark as to their provenance except for the information obtained from dialectic differences which make it possible to distinguish between northern and southern Old-Babylonian Akkadian.[5] Only a few texts from the later periods can be localized: two from Uruk[6] and two from Nineveh, but the latter may not even be mathematical.[7] It seems likely, nevertheless, that the extant Old-Babylonian problem-texts should be traced back to a small number of archives. This can be gathered from the following indications. (1) Many tablets of a fairly large group are arranged serially, as is explicitly indicated by numbering in the colophons.[8] (2) Problem-texts G, H, and J[9] form a clear unit. Text G contains a collection of problems, groups of which are solved in H and J in exactly the same order as they appear in G. (3) Two of the problem-texts published in this volume, M and N,[10] may have been written with the same stylus, which left characteristic faint parallel lines when a vertical wedge was written. (4) Text C is a copy of the latter half of B.[11] (5) Problem-text L is the direct continuation of K.[12]

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[1] CBS 13567+···, e.g., contains the same multiplication table three times (cf. below p. 22 No. 99,3).

[2] About 300 table-texts, but less than 100 problem-texts.

[3] This is illustrated by the following statistics for table-texts of known provenance: Adab 1, Aššur 1, Babylon 1, Kiš 48, Larsa 6, Nippur 117, Sippar 4, Susa 9, Lagaš 2, Uruk 1. It is worth emphasizing that not a single table-text is known to have come from Nineveh although more than 15000 tablets from this site have been catalogued.

[4] The reason for this difficulty is the fact that the signs for numbers do not furnish paleographical peculiarities which are sufficiently distinctive to determine their date within reasonable limits. We cannot say more than that most of these texts belong to the period between 1800 and 300 B.C. Only a few table-texts are clearly old (ca. 2000–1800 B.C.) or obviously late (the last three centuries B.C.). The latest group is easily recognizable by the fact that the relevant tables contain much more extensive numerical material, e.g., reciprocals with ten or more places; this is due to the influence of the development of a theoretical astronomy during the Seleucid period which made extensive computations necessary.

[5] We owe this information to Dr. Goetze's contribution (cf. below pp. 146ff.).

[6] MKT I p. 96 and possibly p. 141 below.

[7] Below pp. 139ff.

[8] Cf. MKT I Chapter VII and problem-texts T and U, pp. 107ff. below. An attempt made to localize these so-called "Series Texts" at Kiš (MKT I p. 387) can no longer be upheld. The basis for this was the alleged special meaning "volume" for sig_4 instead of the ordinary meaning "brick" (cf. problem-text O and the discussion thereof pp. 91ff. below).

[9] Pp. 66ff.

[10] Pp. 90f.

[11] Pp. 44ff.

[12] Pp. 76ff.

§ 2. The Sexagesimal Number System

The number system used in the mathematical texts has two characteristic properties: (a) it is based on sixty (hence the term "sexagesimal") and (b) it uses a place-value notation. The second property is especially remarkable and of great historical importance because it is very likely that from it originated the Hindu-Arabic place-notation system.[13]

The notation which we apply in the following is intended to reflect as closely as possible the characteristic features of the notation used in the texts themselves. This is essential for the understanding of the computing technique used in Babylonian mathematical texts. The replacement of the sexagesimal notation by the decimal would, to mention only one result, destroy many of the advantages of divisibility. This applies especially to calculation with fractions. The sign "20", e.g., represents not only 20 units but can also be used for $\frac{20}{60} = \frac{1}{3}$. The corresponding decimal fraction would be the infinite fraction 0.333..., which is certainly not an adequate reproduction of the original text. But also $\frac{1}{3}$ would not be a convenient transcription because this would mean only the fraction $\frac{1}{3}$, whereas "20" in a text can represent not only $\frac{1}{3}$ but also 20 or 20 times any power of 60, e.g., 1200 or $\frac{20}{3600} = \frac{1}{180}$, etc. Analogously, 1,20 not only stands for 60 + 20 = 80, but also for $1\frac{1}{3}$, or for 80·60 = 4800, or $\frac{80}{3600} = \frac{1}{45}$, etc. Only when we explicitly wish to separate integers from fractions do we introduce the symbol ";" as a mark of separation; thus, we write 1;20 for $1\frac{1}{3}$. The use of this sign always implies an *interpretation* of the text and therefore will be found only in our translations and commentaries, never in the transcriptions.

A somewhat similar situation prevails in the case of "zero". The number 3610 would be written 1,0,10 in the sexagesimal system. Only in the latest period, however, did there exist a notation which clearly corresponds to this writing. Seleucid texts, both mathematical and astronomical, use for "zero" the sign  which appears elsewhere as a mark of separation; we therefore reproduce it by the symbol ".". The number 3610 would accordingly be written 1,.,10. The earlier history of this symbol is very dark; the only thing which seems to be certain is that no sign for zero was in use in the Old-Babylonian period, except perhaps that a blank space was employed occasionally.[14] It is not known, however, when and in what form a special sign was introduced in the intervening period.[15] In our translations and commentaries the symbol "0" has frequently been inserted in order to indicate the correct place-value of the other numbers, but it must always be kept in mind that there is no corresponding sign in the text whenever we write "0". In all cases where the text has a special sign for zero we write ".".

It is perhaps not out of place to emphasize that the ambiguity in the Babylonian writing of numbers is by no means a disadvantage. Just as we multiply 0.0325 by 73.20, or 3.25 by 0.732, or 325 by 732 in exactly the same way, a sexagesimal computation can be carried out regardless of the place-value, which can be determined at the end in the same way as we determine the place of the decimal-point. A reciprocal table

2	30
3	20
4	15

etc. can be used for all possible orders of magnitude: the reciprocal of 2 is 0;30, but 0;2 is the reciprocal of 30, and 0;0,2 the reciprocal of 30,0, etc. There would be no point in assigning a definite order of magnitude to these numbers. The flexibility of the numerical notation is one of the most significant features of Babylonian mathematics and perhaps constituted the most important element for its further development.

§ 3. Translation and Transcription

With regard to our *translations*, some explanation is required for readers who are not familiar with the languages of the original texts. We have attempted, of course, to give translations which are as literal as possible. To such an attempt, however, very strict limits are set by the character of the texts themselves. The documents at our disposal employ an already well developed technical terminology and follow established patterns of arrangement. It is pointless to render a mathematical technical term by its ordinary meaning in non-mathematical texts. We therefore translate, e.g., (*x*) *a-na* (*y*) *i-ši* by "multiply (*x*) by (*y*)" although the non-technical meaning is "raise (*x*) to (*y*)" or "carry (*x*) to (*y*)". Another example is our translation of saḫar by "volume" instead of giving its literal equivalent "earth" or "dirt"; or gagar by "area" instead of "ground" or "surface". The neces-

[13] Cf. Neugebauer [2] pp. 267f. For the historical development of the sexagesimal system cf. Neugebauer, Vorlesungen pp. 93ff.

[14] See the instances in Plimpton 322 cited p. 39 below.

[15] The present material includes only one text from the intermediate period which uses "zero" (cf. below p. 34, No. 33). For other examples cf. MKT I pp. 15ff., 73, 77, 78 (note 24a), 98ff., 128 (obv. edge 2), 209, 355 (note 6); MKT II p. 48 (rev. I, 6); and Neugebauer [4].

sity of this procedure becomes clear if one tries to translate an example [16] where a saḫar is added to a gagar; it would be misleading to reproduce such a passage by "I added dirt to the ground."

The replacement of special technical expressions by the corresponding modern terms is not the only point at which we are forced to abandon a strictly literal translation. The phrasing of many tablets, especially when they employ Sumerian writings, is so condensed that the meaning becomes clear only if the structure of the whole text is taken into account. Many such texts are virtually untranslatable but could best be represented by mathematical formulas.[17] In order, however, to pave the way between the original text and the commentary, we have attempted to make the translation more intelligible by inserting explanatory words within parentheses. We cannot emphasize too strongly that such additions, always inside parentheses, are only a kind of modern commentary and are not a direct reflection of the wording of the text.

A special problem of translation is raised by Sumerian writings which are not compatible with the rules of Sumerian grammar. Many such forms are to be found scattered in Old-Babylonian mathematical texts which are otherwise written in Akkadian as well as in texts in which no specifically Akkadian writings occur. For example, zi with the meaning "subtract" (imperative) or "I subtracted"; daḫ "add" or "I added"; [18] the omission of the suffixes -šè, -ra, and -ta, etc. Our translation of these peculiar forms consciously conforms to the pattern imposed by the texts written unambiguously in Akkadian and by the fuller, correct Sumerian writings which sometimes occur. According to this pattern, the first person singular preterite is generally used in the description of the problem,[19] and the second person singular (imperative or present-future) for the operations carried out toward the solution of the problem.

The *transcriptions* strictly follow the original texts in the sense that we do not replace Sumerian writings by their Akkadian equivalents. Such correspondences are listed in the Vocabulary given in the last chapter (pp. 158ff.). In conformity with the modern convention, we use italics for Akkadian and roman for Sumerian. It should be mentioned for the benefit of those readers who are not Assyriologists that no conclusion can be drawn from the use of the Sumerian language as to the time or place of origin of the texts; the use of Sumerian is somewhat similar to the use of Latin in the Middle Ages. None of our tablets appears to be older than about 1800 B.C.

In the transcriptions, we of course follow the system of notation proposed by Thureau-Dangin.[20] A few values which we use, but which are not to be found or are not uniformly given in the standard lists of Thureau-Dangin or Deimel, are:

am_5 = AN (following Scholtz SV p. 174,1; [21] cf. for this reading Thureau-Dangin [3] p. 91 note 2 and Poebel GSG p. 25 § 60.)
bur^{2}u = GAŠAN (sign-form REC 510)
ganám = U_8 (following Thureau-Dangin; Deimel ŠL I gives $ganam_4$.)
ganba = KI-LAM (following Landsberger MSL p. 26, note.)
kára = KAR (following Thureau-Dangin; Deimel gives the value $kara_2$ in ŠL I.)
kid_9 = BU
sila = QA (following Thureau-Dangin; Deimel lists the sign as $sila_3$ in ŠL I.)
úṣ = AZ
zarà = BE-AŠ (following Thureau-Dangin; Deimel enters the sign as $zara_2$ in ŠL I.)

The principles followed in transcribing numbers are explained above, p. 2. The same holds for fractions expressed in the sexagesimal notation. There are, however, four special signs for the fractions $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$ which we shall reproduce as such in our transcriptions. It is sometimes useful to distinguish sharply between these special symbols and fractions expressed by igi-*n*-gál meaning $\frac{1}{n}$; wherever we feel it to be convenient, we shall write $\bar{n}$ when igi-*n*-gál is used. All such distinctions have been considered unnecessary in the commentaries.

In the transcription of *metrological units*, we distinguish between ordinary number-signs and special symbols used for certain measures in the original texts. Groups like 5 GAR or $\frac{1}{2}$ GAR present no problem because the numbers are written in the normal form. Multiples of iku are always written with

[16] Problem-text G (pp. 66ff. below) Nos. 15–18.

[17] The decipherment of texts of this type is almost a "calculation" of the translation. Cf. for this method Neugebauer [3].

[18] It is quite possible that such writings in mathematical texts, where the terminological advantages of a shortened notation are obvious and where tense, person and other grammatical refinements are never in doubt because of the rather rigorous literary pattern common to the mathematical texts, are the forerunners of the so-called "ideographic" writings in Akkadian texts of the later period, when the writing of the bare Sumerian root carried with it the choice of person and tense.

[19] Except in trivial cases like "the length exceeded the width by (*x*)."

[20] Thureau-Dangin SA and HS.

[21] Note, however, that Deimel ŠL I has assigned the value am_5 to the sign ÀMA.

horizontal number-signs (only 1 to 5 being possible) in the texts, and special symbols are accordingly unnecessary. Units like bùr and šár are not combined with numbers but are repeated to the required multiplicity. To avoid transcriptions like "bùr bùr bùr" or "ešè ešè," we write 3(bùr) and 2(ešè). To be consistent, we write 1(ešè), 1(bur^{3}u), etc., even when only one of these special units occurs. The sign "iku" which is written after units which are larger than iku we transcribe above the line as if it were a determinative, e.g., 2(bùr)^{iku}.

For symbols which we use to indicate such things as omissions or excesses in the text, see below p. 156.

§ 4. Metrology

Knowledge of the exact relations between the various units of length, surface, volume, weight, etc., is indispensable for the understanding of the mathematical texts. The teaching of metrological rules was undoubtedly the purpose of many examples in our texts which are very simple from the mathematical point of view but require the mastery of the ratios between the various units. In the following paragraphs we give a short outline of metrological relations so far as they occur in the texts treated in this volume. It must be emphasized, however, that these lists are neither complete nor of general value regardless of regional or other differences; although they undoubtedly represent an important standard for *Old-Babylonian* times, it is by no means excluded (indeed, it is certain in some cases) that different relations obtained in other groups of texts.[22]

a. Measures of Length

The most essential measures of length are the "cubit" (about 50 cm or 20 inches) and the GAR, which equals 12 cubits.[22a] The relation between the various units of length is given in the following list:

Name	Meaning	šu-si	kùš	GAR	UŠ[23]
šu-si	finger	1	0;2	0;0,10	
kùš	cubit	30	1	0;5	0;0,5
GAR	(?)	6,0	12	1	0;1
UŠ[23]	length[23]	6,0,0	12,0	1,0	1

The principle for the interpretation of this and the following lists will become clear from the following example: the third horizontal line means that

1 kùš (i.e., 1 cubit) = 30 šu-si = 1 kùš
= 0;5 GAR = 0;0,5 UŠ

whereas the fourth vertical column indicates that

0;2 kùš = 1 šu-si

1 kùš = 1 kùš

12 kùš = 1 GAR

12,0 kùš = 1 UŠ.

A blank field shows that the corresponding relation will scarcely be needed in practice; if, however, such a relation should be required, it can easily be derived from the neighboring fields. Thus, to supplement

0;0,10	
0;5	0;0,5

one must obviously write 0;0,0,10 in the free field in order to establish the same proportion in the second column as in the first.

In the *Neo-Babylonian* period, the relations between the various units deviate considerably from those given above. For example,

1 kùš = 24 šu-si

1 GAR = 14 kùš.

b. Measures of Area

The basic unit of area is the SAR, which equals the area of a square with sides measuring 1 GAR. An important larger unit is the iku of 1,40 SAR = (10 GAR)^{2}.

[22] Cf., e.g., note 24 on p. 5.

[22a] It is now possible to prove that the grounds for the assumption of an Old-Babylonian GAR of 24 kùš are false; cf. p. 96.

[23] Frequently read giš, i.e., "one," but for insufficient reasons; cf. Neugebauer [5] p. 274, note 126.

Name	še	gín	SAR	ubu	iku	ešè	bùr	bur^{3}u	šár	šar^{3}u
še[24]	1	0;0,20	0;0,0,20							
gín	3,0	1	0;1							
SAR	3,0,0	1,0	1	0;1,12	0;0,36	0;0,6	0;0,2			
ubu			50	1	0;30	0;5	0;1,40			
iku			1,40	2	1	0;10	0;3,20			
ešè			10,0	12	6	1	0;20	0;2		
bùr			30,0	36	18	3	1	0;6	0;1	
bur^{3}u			5,0,0	6,0	3,0	30	10	1	0;10	0;1
šár			30,0,0	36,0	18,0	3,0	1,0	6	1	0;6
šar^{3}u			5,0,0,0	6,0,0	3,0,0	30,0	10,0	1,0	10	1

c. Measures of Volume

The names of the measures of volume are identical with the names of the measures of area. The units of volume originate from the units of area by multiplication with a height of one cubit (= 1 kùš). The (volume-)SAR is thus not 1 GAR^3, but 1 GAR^2 · 1 kùš. In other words,

1 (volume-)SAR = 1 (surface-)SAR · 1 kùš
= 1 GAR^2 · 1 kùš = 2,24 kùš^3 = 0;5 GAR^3.

The table given in the previous section (b) for measures of area therefore also holds for measures of volume.

d. Bricks

One of the texts published below[25] shows that bricks were counted in units of SAR, one SAR being the equivalent of 12,0 bricks. The multiples and fractions of this SAR had the same names and the same ratios as the multiples and fractions of the surface- and the volume-SAR.[26]

We thus obtain the following scheme:

Name	Number of Bricks
gín	12
SAR	12,0
ubu	10,0,0
iku	20,0,0

In the Kassite and following periods, these units were no longer used; instead, the actual number of bricks in terms of thousands, hundreds, and sixties was indicated.

e. Weights

The units of weight are of great importance because the weights of silver correspond to what we today call money. The basic units are ma-na ("mina") and gín ("shekel").[27] The 180th part of the shekel is the še ("barley").

[24] From a text found at Tell Ṣifr, Thureau-Dangin [1] p. 99 concluded that 1 še = 0;20 gín = 0;0,20 SAR. This relation was considered standard (e.g., also MKT I p. 86) until Thureau-Dangin [2] and Waschow [1] independently discovered that 1 še = 0;0,20 gín is the relation assumed in other texts, e.g., the mathematical text YBC 4669 rev. II 1ff. (MKT III pp. 27f.). This relation, which corresponds exactly to that between še and gín as units of weight, is also used by the texts published here.

[25] Problem-text O, pp. 91ff.

[26] This will be shown below, pp. 94ff.

[27] The higher unit gú ("talent") = 1,0 ma-na is disregarded because it does not occur in the texts published in this volume. One ma-na corresponds to about 500 grams (approximately 1 pound).

Name	še	gín	ma-na
še	1	0;0,20	0;0,0,20
gín	3,0	1	0;1
ma-na	3,0,0	1,0	1

The fractions gín and še were used to determine $\frac{1}{60}$th and $\frac{1}{1800}$th of a 60th in general, as can be seen from the preceding lists.

f. Measures of Capacity

The volume of barley, oil, etc., was measured by special units. We need mention only the following relations:

Old-Babylonian Period

Name	še	gín	sìla	bán	PI	gur
še	1	0;0,20	0;0,0,20			
gín	3,0	1	0;1			
sìla	3,0,0	1,0	1	0;6	0;1	0;0,12
bán			10	1	0;10	0;2
PI			1,0	6	1	0;12
gur			5,0	30	5	1

Neo-Babylonian Period

Name	ninda	sìla	bán	PI
ninda	1	0;6	0;1	0;0,10
sìla	10	1	0;10	0;1,40
bán	1,0	6	1	0;10
PI	6,0	36	6	1

There is evidence for the acceptance of the following relation between sìla and the volume-SAR in the Old-Babylonian period:

1 sìla = 0;0,0,12 SAR or 1 SAR = 5,0,0 sìla.

For this relation see below, p. 96 and note 257.

For the sìla unit used in connection with "thickness," see p. 58.

§ 5. Examples of Metrological Calculations

The close relationship between mathematical and metrological texts is clearly illustrated by two tablets from Susa recently published by Scheil.[28] The two texts constitute a unit insofar as the second is the direct continuation of the first.[29] Together they contain almost 100 metrological examples systematically arranged in three distinct groups. The first group deals with rectangular areas, the second with squares, and the third with circular areas.

It is not our intention to republish these two tablets here, but merely to describe their contents by a simple table headed by examples given in full to show the terminology common to each group.

Group I. Rectangular Areas

Example of terminology (No. 1):

½ kùš uš	½ kùš is the length;
⅓ kùš sag	⅓ kùš is the width.
a-šà-bi 12½ še	Its[30] area is 12½ še.

The proof of the correctness of this and the following examples is facilitated by converting all lengths to units of GAR and the areas to GAR², i.e., SAR. We then obtain (cf. p. 4):

length (x) = 0;2,30 GAR, width (y) = 0;1,40 GAR; hence (pp. 4f.)

area (xy) = 0;0,4,10 SAR = 12;30 še.

We give below a complete list of the examples of this first group, all of which follow the above-given pattern.[31] For the transcription of fractions, see above p. 3.

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[28] Scheil [1]. These texts are "Old-Babylonian", as is obvious from the script. No details concerning their exact archeological provenance at Susa or their present location are given.

[29] Correspondingly, the first tablet ends with the remark "first tablet" while the second merely gives the common colophon without a tablet-number. Unfortunately, this colophon is too badly preserved to be intelligible.

[30] "It" in such phrases refers to the figure in question: here a rectangle, in the following sections a square or a circle, respectively.

[31] It can be observed that most of the examples follow the rule that the width in No. $n + 1$ equals the length in No. n. This principle frequently permits the restoration of damaged passages (indicated by [ ]).

	Length	Width	Area
Tablet I			
Obverse			
No. 1	½ kùš	⅓ kùš	12½ še
2	1 kùš	⅔ kùš	4 (gín) 5 še
3	1½ kùš	[1 kùš]	[½] gín 22½ še
4	2 kùš	1½ kùš	1 gín 4
5	2½ kùš	2 kùš	2 gín 15 še[32]
6	2⅔ kùš	2½ kùš	2⅔ gín 20 še
7	3 kùš	2⅔ kùš	3⅓ gín
8	3⅔ kùš	3 kùš	4½ gín 15 še
9	4 kùš	3⅔ kùš	6 gín 20 še
10	5½ kùš	4⅔ kùš	10⅔ gín 5 še
11	½ GAR	5⅓ kùš	13⅓ gín
12	½ GAR 1½ kùš	½ GAR	18⅔ gín 15 še
13	½ GAR 2 kùš	½ GAR 1½ kùš	⅓ SAR 5 gín
14	½ GAR 2⅔ kùš	½ GAR 1½ kùš	⅓ SAR 7 [gín 1]5 še[33]
15	½ GAR 3 kùš	½ GAR 2 kùš	½ SAR
Reverse			
16	½ GAR 4 kùš	½ GAR 3 kùš	½ SAR [7]½ gín
17	½ GAR 4⅔ kùš	½ GAR 4 kùš	⅔ SAR 4⅓ gín [2]0 še[34]
18	1 GAR	½ GAR [3] kùš	⅔ SAR 5 gín
19	1 GAR 3 kùš	1 GAR	1 SAR 15 gín
20	1 GAR 4 kùš	1 GAR 3 kùš	1⅔ SAR
21	[2] GAR	[1 GAR 4 kùš]	[2⅔] SAR
22	2½ GAR	2 GAR	5 [SAR]
23	3 GAR 4 kùš	2½ GAR	8⅓ SAR
24	3½ GAR 3 kùš	3 GAR 4 kùš	12½ SAR
25	4 GAR	3½ GAR 3 kùš	15 SAR
26	[5]½ GAR	[5 GAR]	[27]½ SAR
27	[6½ GAR 2] kùš	[5½ GAR]	[36]⅔ SAR
28	[7½ GAR]	6½ GAR 2 kùš	50 SAR
29	8 GAR 4 kùš	7½ GAR	1,2½ [SAR]
30	9 GAR	8 GAR [4 kùš]	[1,15 SAR]
31	10 GAR	9 GAR	1,30 [SAR]
32	12½ GAR	10 GAR	1 iku 25 SAR
33	13 GAR 4 kùš	12½ GAR	1 iku 1,6⅔ SAR

[32] Scheil's copy has 14 instead of 15.
[33] Scheil's copy gives 6 instead of 7.
[34] One would read ⅔ SAR 4 [gí]n igi-4-gál 10 še according to Scheil's copy of the damaged passage.

	Length	Width	Area
Tablet II			
Obv. I			
No. 34(1)[35]	[15] GAR	[1]3 GAR 4 kùš	2 iku
35(2)	16½ GAR 2 kùš	15 GAR[36]	[2 iku 50 SAR]
36(3)	[18 GAR]	[16½ GAR 2 kùš]	3 iku[37]
37(4)	[25 GAR]	[18 GAR]	4 iku 50 SAR
38(5)	30 GAR	25 GAR	1(ešè) 1 iku 50 SAR
39(6)	35½ GAR ⅔ kùš	30 GAR	1(ešè) 4 iku 1,6⅔ SAR
40(7)	40 GAR	37½ <GAR>	2(ešè) 3 <iku>[38]
41(8)	[45 GAR]	[40 GAR]	[1(bùr)]
42[39]	[50 GAR]	[45 GAR]	[1(bùr) 4] iku [5]0 [SAR]
Obv. II			
43(9)	1,[20 GAR]	50 GAR	[2(bùr) 4] iku
44(10)	[1,30 GAR]	[1,20 GAR]	4(bùr)
45(11)	1,52½ GAR	1,30 GAR	5(bùr) 1(ešè) 5 iku 25 SAR
46(12)	2,30 GAR	1,[5]2½ GAR[40]	9(bùr) 1(ešè) 1,15 SAR
47(13)	3,[45] GAR	2,30 GAR	1(bur³u) 8(bùr) 2(ešè) 1 iku 50 SAR
48(14)	4,30 GAR	3,45 GAR	3(bur³u) 3(bùr) 2(ešè) 1 iku 50 SAR
49(15)	7,30 GAR	6,40 GAR	[1(šár) 4(bur³u)]
50[41]	[10 GAR][41a]	[7,30 GAR]	[2(šár) 3(bur³u)]
Obv. III			
51(16)	[1]5 GAR	[10] GAR	5(šár)
52(17)	20 GAR	15 GAR	1(šar³u)
53(18)	25 GAR	20[GAR]	1(šar³u) 6(šár) 4(bur³u)

Group II. Squares

In direct continuation of the preceding examples, there now follows a new group dealing with squares. The first example in which the terminology is fully preserved is No. 57 (22):

4 šu-si íb-si$_8$	4 šu-si is the square-side.[42]
a-šà-bi 1 še igi-3-gál še	Its[43] area is 1 še (and) one-third še.

The best way to check these examples consists in first calculating the areas of the squares of the fundamental units. Thus we obtain (cf. above pp. 4f., sections a and b):

$x = 1$ GAR	$x^2 = 1$ GAR$^2 = 1$ SAR
$x = 1$ kùš $= 0;5$ GAR	$x^2 = 0;0,25$ SAR $= 0;25$ gín $= \frac{1}{3}$ gín 15 še
$x = 1$ šu-si $= 0;2$ kùš	$x^2 = 0;0,4$ kùš^2 $= 0;0,4 \cdot 1,15$ še $= 0;5$ še $= \frac{1}{12}$ še.

All further relations can be derived from these three by multiplying x^2 by the factor a^2 if the side of the given square contains ax units.

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[35] Numbers in parentheses correspond to Scheil's numbering.

[36] Scheil: 14 GAR

[37] Scheil: [. .] SAR

[38] In this example, not only are the signs enclosed within < > omitted but also sag and a-šà-bi.

[39] This restoration is based on the following considerations. According to the principle of arrangement mentioned above p. 6 note 31, sag of No. 41 should be 40. The sag of No. 42 is most likely a number ending in 5 in order to give a surface ending in [. .] iku and 50 SAR (cf. Nos. 37 and 38). It must, moreover, be a number between 40 and 50; hence 45. This brings No. 42 in direct connection with No. 41.

[40] Scheil: 2,[5]2½ GAR.

[41] This restoration is based on the assumption that sag in No. 50 has the numerical value of uš in No. 49 and that the corresponding relation holds between Nos. 50 and 51.

[41a] Here 10 GAR means 10,0 GAR, and analogously in the three examples which follow.

[42] For this technical term cf. the instances cited in the Vocabulary under si$_8$.

[43] Cf. above p. 6 note 30.

	Side of Square	Area
54 (19)	1 šu[-si]	[$\overline{12}$ še]
55 (20)	2 šu[-si]	[$\overline{3}$ še]
56 (21)	3 š[u-si]	[½ še] $\overline{4}$ še
57 (22)	4 šu[-si]	1 še $\overline{3}$ še
58 (23)	5 šu-si	2 še $\overline{12}$ še
59 (24)	6 šu-si	3 še
60 (25)	7 šu-si	4 še $\overline{12}$ še
61	[8 šu-si]	[5 še $\overline{3}$ še]
62	[9 šu-si]	[6½ še $\overline{4}$ še]
Rev. I[44]		
63	[⅓ kùš]	[8 še $\overline{3}$ še]
64	[½ kùš]	[18½ še $\overline{4}$ še]
65	[1 kùš]	[⅓ gín 15 še]
66 (26)	1½ kùš	⅚ gín 18½ še $\overline{4}$ še[45]
67 (27)	1⅓ kùš	⅔ gín 13 še $\overline{3}$ še[46]
68 (28)	<1⅔ kùš>[47]	1 gín 28 še $\overline{3}$ še
69 (29)	2 kùš	1⅔ gín
70 (30)	2½ kùš	2½ gín 18½ še $\overline{4}$ še[48]
71 (31)	2⅓ kùš	2 gín $\overline{4}$ 3 še [$\overline{3}$] še
72 (32)	[2⅔ kùš]	2⅚ gín [23 še] $\overline{3}$ še
73 (33)	[3 kùš]	3⅔ gín [1]5 še
74 (34)	[3½ kùš]	5 gín [18½ še $\overline{4}$ še]
Rev. II		
75	[4 kùš]	[6⅔ gín]
76	[½ GAR]	[15 gín]
77	[1 GAR]	[1 SAR]
78 (35)	[1½] GAR	[2 SAR] 15 gín
79 (36)	2 GAR	4 SAR
80 (37)	2½ GAR	6 SAR 15 gín
81 (38)	3 GAR	9 SAR
82 (39)	3½ GAR	(12 SAR 15 gín)[49]
83 (40)	4 GAR	16 SAR

Group III. Circles

The last group of problems is characterized by the use of the word KA-ŠÌR ≈ *kippatu*[50] "circumference." Let c designate the magnitude given in the text as KA-ŠÌR; then "its area" (a) corresponds to the value

$$a = 0;5\ c^2 = \frac{c^2}{12}.$$

The explanation of this formula is given by the fact that π is here approximated by 3. The area a of a circle with radius r and circumference c is then given by[51]

$$a = r^2\pi = \frac{(2r\pi)^2}{4\pi} = \frac{c^2}{4\pi} \approx \frac{c^2}{12}$$

as in the text.

The wording of this relation is illustrated by No. 84 (41):

½ GAR KA-ŠÌR	½ GAR is the circumference.
a-šà-bi 1 gín	Its[52] area is 1 gín (and)
igi-4-gál	one-fourth (gín).

The following examples complete the text.

	Circumference	Area
84 (41)	½ GAR	1 gín $\overline{4}$
85 (42)	1 GAR	5 gín
86 (43)	1½ GAR	11 gín $\overline{4}$
87 (44)	2 GAR	⅓ SAR
88 (45)	2½ GAR	½ SAR 1 gín $\overline{4}$
89 (46)	3 GAR	[⅔ SAR 5 gín]
Rev. III		
90	[5 GAR]	[2 SAR 5 gín]
91	[10 GAR]	[8⅓ SAR]
92 (47)	[2]0 GAR	33⅓ SAR
93 (48)	1 GAR[53]	3 iku
94 (49)	2 GAR	2(ešè)
95 (50)	3 GAR	1(bùr) 1(ešè) 3 iku
96 (51)	4 GAR	2(bùr) 2(ešè)
97 (52)	1 <GAR> (?)	1(š[ar^{2}u]) (?)[54]

It is perhaps useful to compare the measurements involved here with magnitudes familiar to us. The smallest unit, šu-si (Nos. 54ff.), is about 1⅔ cm (⅗ of an inch). The GAR is about 6 meters (or 6½ yards); the circumference of the largest circle in the last group is accordingly about 21.6 kilometers (more than 13 miles), its diameter about 7 kilometers.

It is plain that some of the fractions like the one-twelfth še which occurs in Nos. 54, 58, and 60 are nothing more than calculation units. It is therefore

[44] It should be noted that we are confronted here by the abnormal situation in which Column I is the left(!)-hand column of the reverse.

[45] Scheil's copy gives 16 instead of 18.

[46] Scheil's copy has 12 instead of 13.

[47] Omitted by the scribe.

[48] Scheil: 28 instead of 18.

[49] According to Scheil's copy, the text seems to bave 10⅓ SAR 6½ gín.

[50] The equivalence KA-ŠÌR = *kippatu* is attested by AO 6458 (last published TCL VI No. 51) where KA-ŠÌR ki-sur-bi corresponds to *kip-pat ki-sur-ri-šú-nu* (lines 35f.). This Akkadian value is missing in Deimel ŠL 15, 158.

[51] Discovered by W. Struve in 1929; cf. Neugebauer-Struve [1] §2. For an example from a later period, see No. 4 of problem-text Y (p. 142).

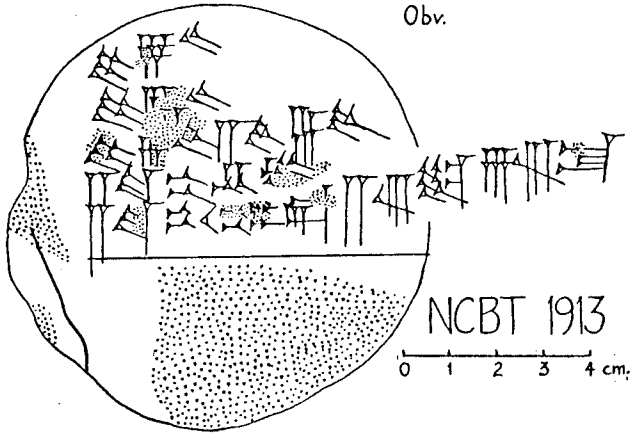
[52] Cf. above p. 6, note 30.

[53] 1 GAR here means 1,0 GAR (usually called 1 UŠ), and analogously in the following three examples.

[54] The reading of this last group is very uncertain. The only clear sign is the 1, written as an especially large wedge, indicating that 1,0,0 GAR (= 1,0 UŠ) is meant. The following signs are damaged; GAR is certainly missing, and KA-ŠÌR is perhaps the most plausible interpretation of the signs given in Scheil's copy. Then follows a-šà[-bi], and the next line bears traces which conform to the expected 1(šar^{2}u).

not surprising that the only fraction of the še written with a special fraction-sign is ½ še, in contrast to igi-3-gál še for ⅓ (Nos. 57, 67, 68, and 72). It is furthermore interesting to note that only the special sign for ½ is used in connection with GAR, whereas ⅓ GAR is always expressed by 4 kùš (cf. Nos. 9 and 24).[55] By way of contrast, kùš, gín and SAR are combined with the fractional signs[56] ½, ⅓ and ⅔. All of these rules are without exception in the Old-Babylonian mathematical problem-texts and metrological texts.[57]

One simple metrological example is given in **NCBT 1913** (cf. the accompanying copy), which reads as follows:[57a]



1 58,20
2 58,20
3 56,42,46,40
4 a-šà-bi en-nam
5 a-šà-bi 1(bùr) 2(ešè) 4 iku 2⅔ SAR 6⅔ gín

This is to be interpreted as

1 58;20 (GAR and)
2 58;20 (GAR are the sides of a square).
3 56,42;46,40 (SAR is the area).
4 What is its area (,transformed to standard units)?
5 Its area is 1 bùr, 2 ešè, 4 iku, 2⅔ SAR, (and) 6⅔ gín.

The sexagesimal places and the metrological units involved in the first three lines are uniquely determined by the final answer given in line 5. The identities in question are

$$(58;20\ \text{GAR})^2 = 56,42;46,40\ \text{SAR} = 1\ \text{bùr}\ 2\ \text{ešè}\ 4\ \text{iku}\ 2\tfrac{2}{3}\ \text{SAR}\ 6\tfrac{2}{3}\ \text{gín}.$$

A single metrological problem is to be found on the obverse of **NBC 8082**, a tablet which was written in the Old-Babylonian period—perhaps in the early part of this period if one chooses to be impressed by the four vertical wedges in the first half of the sign BI. The reverse is destroyed. The text reads:

20(?)	1,20
	1,20
	1,46,40
4	16
1,20	1 GAR 4 kùš íb-si$_8$
	a-šà-bi en-n[am]
	a-šà-bi 1⅔ [SAR 6⅔ gín]

The problem, which is presented in the lower right corner, is:

1 GAR (and) 4 kùš is the side of the square.
What is its area?
Its area is 1⅔ [SAR (and) 6⅔ gín].

The numbers 1,20 1,20 and 1,46,40 at the top of the same column mean $1;20^2 = 1;46,40$. The remaining numbers perhaps indicate that 1;20 was treated in the form $4 \cdot 0;20$ and that $1;20^2$ was found through $4^2 \cdot 0;20^2$.

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[55] The same certainly holds for ⅔ GAR.
[56] Also ⅚ gín occurs in Nos. 66 and 72.
[57] A preliminary investigation has shown us that the exceptions to these rules in contemporary economic documents are so rare that one can expect to find the reasons for these special cases by a study of the whole material.
[57a] The reverse is uninscribed.

CHAPTER II. TABLE-TEXTS

§ 1. Introduction

The mathematical tables, by means of which all numerical calculations were carried out, played a basic rôle in the high development of Old-Babylonian mathematics and of Babylonian astronomy in the Seleucid period. They can be separated into two main groups: (1) an older group, mainly from the "Old-Babylonian" period and most fully represented by the "combined" multiplication tables, and (2) from the very latest period, a group of tables of great numerical extent sufficient for astronomical computation on a large scale, but no longer linked together by a single clear system.

The tables of the second group are self-explanatory, but the common elements in the older tablets are not so obvious. Formally, the older group falls into two subdivisions: (1) small tablets containing a single table of reciprocals, squares, or products; and (2) larger "combined" texts containing a group of consecutive tables constituting a collection on one tablet of the contents of numerous single tables. From the combined tables as well as from the collected material of single tablets, it follows that all these tables are arranged according to the following well planned system: a table of reciprocals is followed by a series of about 40 different multiplication tables, after which come tables for squares or square-roots. The connection between the table of reciprocals and the multiplication tables lies in the fact that the latter concern the multiples of only[58] those numbers which are reciprocals of simple numbers, for the most part contained in the standard table of reciprocals. This shows that the multiplication tables were used not only for the products $a \cdot b$ of integers but simultaneously for general fractions $\frac{a}{c}$ if b is the reciprocal $\bar{c}$ of c.[59]

In publishing the table texts, it is most convenient to arrange them according to the following groups:

It must be kept in mind, however, that by this arrangement it sometimes happens that tables which are actually inscribed on the same tablet are listed separately. Tables of reciprocals, e.g., which come at the beginning of combined tables, are listed under section (a), separated from the multiplication tables (d) which follow. The same holds for so-called "school texts" which have, e.g., a combined multiplication table (d) on the obverse, but one or more copies of a single multiplication table (c) written by the pupil as an exercise on the reverse. In all such cases, cross-references have been given.

§ 2. Reciprocals

Let n be a number written in the sexagesimal notation. The sexagesimal representation of the reciprocal $\bar{n} = \frac{1}{n}$ can then be obtained by dividing 1 by n. Two results are possible: either this division ends after a finite number of steps[60] or it continues indefinitely.[61] The necessary and sufficient condition for the first case is that n contain no prime number which is not also contained in the base 60, i.e., n must be of the form

$$n = 2^{\alpha}3^{\beta}5^{\gamma}$$

where α, β, γ are integers or zero. We shall call such numbers "*regular numbers.*" With the exception of the table given in section b*, tables of reciprocals contain *only* reciprocals of regular numbers.

a. Tables of Reciprocals of Standard Type

Reciprocal tables of the "standard type" contain the following pairs of reciprocals:

2	30	16	3,45	45	1,20
3	20	18	3,20	48	1,15
4	15	20	3	50	1,12
5	12	24	2,30	54	1,6,40
6	10	25	2,24	1	1
8	7,30	27	2,13,20	1,4	56,15
9	6,40	30	2	1,12	50
10	6	32	1,52,30	1,15	48
12	5	36	1,40	1,20	45
15	4	40	1,30	1,21	44,26,40

[58] The single exception is 7.

[59] For details and historical significance, see MKT I pp. 4ff. or Neugebauer, Vorlesungen pp. 18ff.

[60] Example: $\bar{9} = 0;6,40$.

[61] For example, $\bar{7} = 0;8,34,17,8,34,17,\ldots$ with 8,34,17 being repeated indefinitely.

Many texts omit some pairs between 1 and 1,21.[62] Such cases will always be mentioned in the remarks to the texts. All of the following texts are published here for the first time unless otherwise noted.[63]

No.[64]	Museum No.[65]	For Remainder of Text cf.	Terminology		End	Remarks
			Beginning	Main Part		
32	NBC 6344	—	1-da ⅔-bi 40-àm šu-ri-bi 30-àm	igi-*n*-gál-bi *n̄*	1,12 1,15 and 1,20 omitted	Bought from Gejou in 1925; probably from Larsa. For the form šu-ri(!)-bi, cf. MKT I p. 10 No. 4.
33	CBS 29.15.482	—	[..]⅔-da-bi 40-[àm] [š]u-ri-a-bi 30-à[m]	igi-*n*-gál-bi *n̄*	1,12 1,15 and 1,20 omitted	Traces of catch-line at the end: [50] a-rá 1 [5]0.
34	CBS 29.15.489	—	[..]-da-bi [4]0-àm šu-ri-a-bi 30-àm igi-2-gál-bi 30-àm	igi-*n*-gál-bi *n̄*	1,12 1,15 and 1,20 omitted	Catch-line: 50 a-rá 1 50.
35	YBC 7303	§ 3d No. 143	1-da ⅔-bi 40-àm šu-ri-a-bi 30-àm	igi-*n*-gál-bi *n̄*	1,12 1,15 and 1,20 omitted	
36	YBC 6953	—	Destroyed	igi-*n*-gál-bi *n̄*	1,12 1,15 and 1,20 omitted	
37	CBS 8210	§ 3d No. 148	Destroyed	[igi-*n*-]gál-bi *n̄*	Destroyed	
37a	NBC 8061	—	1-da ⅔-bi 40-àm šu-ri-a-bi 30-àm	*n* *n̄*		Colophon: im<-gíd>-da *Ḳarad-...-...*.[65a]
38	CBS 8153	§ 3d No. 145	1 40-àm 2 30-àm	*n* *n̄*	Destroyed	
39	CBS 29.15.497	§ 3d No. 147	[..] 40-àm [..] 30-àm	[*n*] *n̄*	Destroyed	
40	A 7897	§ 3d No. 140 and pp. 24f.	Destroyed; space for at least 3 lines before 2 [30][66]	*n* *n̄*		Old-Babylonian cylinder from the Diyālā region.
41	CBS 8309	§ 3d No. 142	Destroyed	*n* *n̄*	1,12 1,15 and 1,20 omitted	
42	CBS 8215	§ 3d No. 141	Destroyed	*n* *n̄*	Destroyed	
43	CBS 29.16.90	§ 3d No. 146	Destroyed	*n* [*n̄*]	Destroyed	
44	CBS 7364	—	Destroyed	[*n*] *n̄*	Destroyed	

[62] The complete list is given in Nos. 37a and 40.

[63] All texts listed below are of "Type A" (cf. MKT I p. 9). They are arranged according to their terminology in the order of the probable age of the forms.

[64] For Nos. 1 to 28 see MKT I pp. 10–12; for No. 29 see MKT II p. 36; and for Nos. 30 and 31 see MKT III p. 49 (MMAP 27, 297 and 296).

[65] All CBS texts are presumably from Nippur.

[65a] "Tablet of *Ḳarad-...-...*."

[66] According to traces, preceding line perhaps 2[30-à]m.

b. Tables of Reciprocals of Larger Extent

The following tables are single tablets, all of unknown provenance except CBS 29.13.21, which is from Nippur. The date of the first tablet is undetermined, the second and the last two are Old-Babylonian, and the rest belong to the Seleucid (or a slightly earlier) period.

MM 86.11.409

No edge is preserved; the reverse is destroyed. On the obverse only the right-hand column is preserved; there is, consequently, no way of determining the terminology employed.

Obverse

1)	[1,.....]	57,[.....]
2)	[1,3,.....]	56,50[.....]
3)	[1,4]	56,15
4)	[1,4,48]	55,33,[20]
5)	[1,5,6,15]	55,17,[45,36]
6)	[1,5,36,36]	54,52,[10,51,51,6,40]
7)	[1,5,50,37,2,13,20]	54,40,3[0]
8)	[1,5,55,4,41,15]	54,36,[48]
9)	[1,6,21,18,43,12]	54,15,1[2,30]
10)	[1,7,30]	53,20
11)	[1,8,16]	52,44,[3,45]
12)	[1,8,20,37,30]	52,40,[29,37,46,40]
13)	[1,9,26,40]	51,50,[24]
14)	[1,.....]	51,[.....]

In line 1 the reading 57 is not absolutely certain; 56 would also be possible. The following restoration of the first two lines seems plausible:

[1,2,30]	57,[36]
[1,3,16,52,30]	56,5[3,20]

CBS 29.13.21

(Photograph: Plate 24)

We judge from the writing that this tablet, conservatively speaking, belongs to the earlier part of the Old-Babylonian period. It was found at Nippur. Only about a third of the original tablet is preserved; cf. the restoration given below. The frequent occurrence of igi-bi shows at once that the text deals with reciprocals and follows the well known pattern *n* igi-bi $\bar{n}$, here written in two lines. Before commenting on and restoring the whole text, we give a transcription of the preserved portion.

Obverse Col. I | **Col. II**

2,2[2,13,20]	[.....]2]5,18,45
igi-bi 25,18,[45]	1,25,20,58,9,11,6,40[68]
4,44,26,40	igi-bi 41,42,49,22,21,12,
igi-bi 12,39,22,30	39,22,30
9,28,53,20	
igi-bi 6,18,41,15[67]	*a-ra-kúra ša* 1,25,20
18,57,46,40	4,38,5,29,9,1,24,22,30
[igi-b]i 3,9,[50],37,30	*a-ra-kúra a-ra-ka-re-e ša* 1,20
[37,55,33],20	18,32,21,56,30[69]
[igi-bi 1,34,55,18,45]	6,5,[37],30

Reverse Col. III | **Col. II** | **Col. I**

[32],24	igi-bi 8,26,15	2,40
igi-bi 1,51,6,40	14,13,20	igi-bi 22,30
1,4,48	igi-bi 4,13,7,30	5,20
igi-bi 55,33,20	———	igi-bi 11,15
2,9,36	1,4	10,40
igi-bi 27,46,40	igi-bi 56,15	igi-bi 5,37,30
4,19,12	2,8	21,20
igi-bi 13,53,20	i[gi-b]i [28,7,30]	igi-bi 2,48,[4]5
8,38,24	[.....]	42,[40]
igi-bi 6,56,40	[.....]	[igi-b]i [1],24,2[2,3]0

In order to restore the original extent of this table of reciprocals, we need only observe that consecutive lines are derived from one another by simple duplication or halving of *n* and $\bar{n}$. If we continue this process beyond the end of Col. I of the obverse, we reach the beginning of the preserved portion of Col. II after 12 steps. The original length of the first section of Col. II accordingly amounted to about 15 pairs of reciprocals. If we take into account the fact that the last numbers are very long and each therefore requires two lines, we can assume that the first column contained 17 pairs. Under this assumption, we obtain as the original starting-point the relation

$$\overline{2,5} = 28,48$$

from which all the following pairs are derived by duplication and halving. Omitting for a moment the short section at the end of Col. II, to which we shall return later, we restore the numerical contents of Cols. I and II of the obverse as follows:

[67] *Sic*, instead of 6,19,41,15. This error does not affect the following pair of reciprocals.

[68] Reading of 1,25,20 uncertain except for the 5 in 25. Actually, one should expect 1,26,18,9,11,6,40. A possible explanation of the error would be that in computing, the scribe wrote two terms (20 and 58) side by side instead of giving their sum. Thus, his result was ...,25,20,58,... instead of ...,26,18,....

[69] The 30 at the end of this line and the 6 at the beginning of the next should be read together as 36. The same type of split writing occurs twice in YBC 4716 (p. 31 No. 154a); cf. also CBS 8309 (p. 27 No. 142), CBS 29.15.497 (p. 29 No. 147), and YBC 4704 (p. 16).

Obverse Col. I		Col. II	
[2, 5	28,48]	[1,15,51, 6,40	47,27,39,22,30]
[4,10	14,24]	[2,31,42,13,20	23,43,49,41,15]
[8,20	7,12]	[5, 3,24,26,40	11,51,54,50,37,30]
[16,40	3,36]	[10, 6,48,53,20	5,55,57,25,18,45]
[33,20	1,48]	[20,13,37,46,40	2,57,58,42,39,22,30]
[1, 6,40	54]	[40,27,15,33,20	1,28,59,21,19,41,15]
[2,13,20	27]	[1,20,54,31, 6,40	44,29,40,39,50,37,30]
[4,26,40	13,30]	[2,41,49, 2,13,20	22,14,50,19,55,18,45]
[8,53,20	6,45]	[5,23,38, 4,26,40	11, 7,25, 9,57,39,22,30]
[17,46,40	3,22,30]	[10,47,16, 8,53,20	5,33,42,34,58,49,41,15]
[35,33,20	1,41,15]	[21,34,32,17,46,40	2,46,51,17,29,24,50,37,30]
[1,11, 6,40	50,37,30]	[43, 9, 4,35,33,20	1,23,25,38,44,42,2]5,18,45
2,2[2,13,20]	25,18,[45]	1,26,18, 9,11, 6,40	41,42,49,22,21,12,39,22,30
4,44,26,40	12,39,22,30		
9,28,53,20	6,19,41,15		
18,57,46,40	3,9,[50],37,30		
[37,55,33],20	[1,34,55,18,45]		

The left-hand number of the last pair seems to be given erroneously in the text as 1,*25*,*20*,58,9,11,6,40, but the right-hand side is correct; for the error cf. p. 13 note 68.

We can now proceed also to restore, within certain limits, the reverse. That we cannot do so with complete certainty is due to the fact that we are now dealing with four different groups, the ends of which are not quite determined. The following gives the most plausible restoration of the three columns of the reverse rearranged for modern convenience in an order opposite to that of the text.

Reverse Col. I		Col. II		Col. III	
2,40	22,30	[7, 6,40]	8,26,15	[32],24	1,51, 6,40
5,20	11,15	14,13,20	4,13, 7,30	1, 4,48	55,33,20
10,40	5,37,30			2, 9,36	27,46,40
21,20	2,48,45	1, 4	56,15	4,19,12	13,53,20
42,[40	1,]24,2[2,3]0	2, 8	[28, 7,30]	8,38,24	6,56,40
[1,25,20	42,11,15]	[4,16	14, 3,45]	[17,16,48	3,28,20]
		[8,32	7, 1,52,30]	(Following destroyed)	
[1,40	36]	[17, 4	3,30,56,15]		
[3,20	18]	[34, 8	1,45,28, 7,30]		
[6,40	9]	[1, 8,16	52,44, 3,45]		
[13,20	4,30]	[2,16,32	26,22, 1,52,30]		
[26,40	2,15]				
[53,20	1, 7,30]	[4, 3	14,48,53,20]		
[1,46,40	33,45]	[8, 6	7,24,26,40]		
[3,33,20	16,52,30]	[16,12	3,42,13,20]		

This shows that all pairs of reciprocals given in the text can be obtained from the following pairs by continuous multiplication and division by 2:

2,5	28,48	(obv.)
2,40	22,30	(rev. I)
1,40	36	(rev. I/II)
1,4	56,15	(rev. II)
4,3	14,48,53,20	(rev. II/III)

These numbers show clearly how the whole text was computed. Two of the above-given pairs (involving 36 and 1,4) are already contained in the "standard" table of reciprocals. The three remaining pairs can be derived by a single step from such a "standard" table:

multipl. by 2		multipl. by 3		multipl. by 5	
1,20	45	1,21	44,26,40	25	2,24
2,40	22,30	4,3	14,48,53,20	2,5	28,48

All succeeding pairs are then obtainable by continuous duplication and halving.[70]

We now turn to the last two pairs of numbers given in Col. II of the obverse. Before doing so, however, it is convenient to introduce a simple notation for "regular numbers," i.e., numbers which can be decomposed into a product of powers of 2, 3 and 5 only.[71] We shall write

$$n = 2^{\alpha}3^{\beta}5^{\gamma} = (\alpha, \beta, \gamma)$$

for such a number.[72] We shall now apply this notation to the passage in question:

The *arakarūm* of 1,25,20 is 4,38,5,29,9,1,24,22,30.

The *arakarūm* of the *arakarūm*[73] of 1,20 is 18,32,21, 56,36,5,37,30.

All four of these numbers have a very simple structure because

$$\begin{aligned} 1,25,20 &= (10,0,1) \\ 4,38,5,29,9,1,24,22,30 &= (25,0,1) \\ 1,20 &= (4,0,1) \\ 18,32,21,56,36,5,37,30 &= (23,0,1). \end{aligned}$$

These numbers can thus be obtained by multiplying 5 by a power of 2. The same is true of the numbers in the following column (rev. I), which begins with 2,40 = (5,0,1). Our passage is therefore an introduction to the following section. Why just these numbers were selected is not clear.

There remains the question as to the meaning of the term *arakarūm*. As we have seen, the operation consists in multiplying the given number by a high power of 2 and taking the reciprocal. This process, however, is so simple that we are by its very simplicity deprived of the means to determine the specific meaning intended here. We shall again meet the term *arakarūm* in a quite different context.[74] The only common element seems to be the multiplication process. The analysis of Sumerian a-rá-kára, the source of the Akkadian loan-word *arakarūm*,[75] to a certain extent confirms this, since a-rá is the technical term used for multiplication, although none of the attested meanings[76] of kára seems to give a clue to the more specific meaning here. "Factor" or "coefficient" seems to cover fairly well both the literal and the mathematical meanings.

[70] The details of this process of calculating larger tables of reciprocals are explained in Neugebauer, Vorlesungen pp. 9ff. Two texts of a similar type (both starting with 2,5) are published in MKT I pp. 23f.; cf. MKT III, top of p. 52.

[71] Cf. above p. 11.

[72] For example, 30 = (1,1,1); and therefore 30^2 = 15,0 = (2,2,2).

[73] Or *arakarū* (plural)?

[74] Below p. 48.

[75] The word a-rá-kára has hitherto occurred only once: in a vocabulary published by Poebel HGT No. 148, line 20, which reads a-rá-kára = *a-ra-ka-ru-u₄-um*.

[76] Deimel ŠL 105II.

MM 86.11.410

No edges are preserved; the reverse is destroyed. Lines 2, 3 and 8 contain two pairs of reciprocals each.

1)	[1,22,56,38,2]4	[43,24,10]
2)	[1,23,20 . 4]3,12 .	1,[24,22,30 . 42,40]
3)	[1,25,20 .] 42,11,15 .	1,26,24 . [41,40]
4)	]1,2]6,48,20	4[1,28,19,12]
5)	[1],27,28,48	4[1,9,8,8,53,20]
6)	[1],27,47,29,22,57,46,40	[41, .,22,30]
7)	[1],27,53,26,15	40,[57,36]
8)	[1],27,53,20 . 40,30 .	1,30 [. 40]
9)	[1,3]1,7,30	3[9,30,22,13,20]
10)	[1,3]2,9,36	3[9,3,45]
11)	[1,32,3]5,33,20	3[8,52,48]

The sign transcribed here by "." is written in lines 2 and 8, in line 3. No real difference can be detected between 20 . in line 8, and 29 in line 6. No zero is preserved.

In line eight, [1],27,53,20 is an error for [1],28,53,20. The first preserved sign in line 11 could also be read 4, but [3]5 is required by the most plausible restoration. Cf. the following fragment.

Liverpool 29.11.77.34

This is a small fragment, of which only the left-hand part is preserved. It is not impossible that it forms a join with the preceding fragment MM 86.11. 410.

Obv. 1)	1,[32,35,33,20	38,52,48]
2)	1,33,1[8,43,12	38,34,48,53,20]
3)	1,33,45	[38,24]
4)	1,34,48,53,[20	37,58,7,30]
5)	1,34,55,18,4[5	37,55,33,20]
6)	1,36	[37,30]
7)	1,36,27,2,13,20	[37,19,29,16,48]
8)	1,37,12	[37,2,13,20]
9)	1,37,39,22,[30	36,51,50,24]
10)	1,38,[18,14,24	36,37,15,56,15]
11)	1,38,24,[54	36,34,47,14,34,4,26,40]
12)	[1,3]8,[45,55,33,20	36,27]

An alternative restoration of line 2 is

1,33,1[2,24,19,12 38,37,25,42,55,11,43,7,30].

Only traces of 4 lines are visible on the reverse. The first number is either 2 or 3; assuming the latter reading, the following restoration seems possible:

Rev. 1)	[. . .]20	[. . . . .]
2)	[3],12	[18,45]
3)	3,22,30	17,[46,40]
4)	3,24,[4]8	[17,34,41,15]
5)	3,[54,22],30	[15,21,36]

In line 3, both 30 and 17 are doubtful. In line 4, there are only traces of 24, and [4]8 does not agree with the visible remains, which indicate rather clearly [..]4, which must therefore be assumed to be an error. This can of course be considered a serious objection against our restoration, which, however, accounts very well for the spacing of the numbers in the text.

YBC 4704

A transcription of this Old-Babylonian text was given MKT I, p. 81[76a] among the fragments. It is now clear that reciprocals are involved. The following is a corrected transcription which has been checked with a photograph:

Obverse	Reverse
3,41,26,1,[30]	6,3,12,[2]4,[9,]
igi-bi 16,15,27,	1,21
39,48,28,38,	[igi-bi] 9,54,42,
31,6,40	9,[3]7,6,48,50(?),[76c]
33,12,54,13,30	50,42,57(?)[76d],25,
igi-bi 1,48,23,	48,58,16,17,
4,25,23,10,50	46,40
6[76b],47,24,26,40	

The following three pairs of reciprocals are given by the text:

$$\overline{3,41,26,1,30} = 16,15,27,39,48,28,38,31,6,40$$
$$\overline{33,12,54,13,30} = 1,48,23,4,25,23,10,56,47,24,26,40$$
$$\overline{6,3,12,24,9,1,21} = 9,54,42,9,37,9,15,32,10,54,17,25,48,58,16,17,46,40.$$

The decomposition[76c] of each of the three key-numbers into a product of powers of 2, 3 and 5 is interesting because of the high powers of 3:

$$3,41,26,1,30 = (1,14,1)$$
$$33,12,54,13,30 = (1,16,1)$$
$$6,3,12,24,9,1,21 = (0,24,0).$$

VAT 5457

This tablet was also given in transcription MKT I, p. 81 among the fragments. By a different combination of the signs, namely,

9,6,8	7,30
6,35,30,28,7,30	2 ,

the pair of reciprocals

$$\overline{9,6,8} = 6,35,30,28,7,30$$

results. The analysis of 9,6,8 as 2^{15} may be significant for the remaining numbers 7,30 and 2, since $7,30 \cdot 2 = 15$.

b*. Reciprocals of Regular and Irregular Numbers
YBC 10529

All edges except the top of this Old-Babylonian text are preserved; not much more than one-third of the text is missing at the top unless the original tablet was abnormally long. Each line follows the type igi-(n)-gál-bi ($\bar{n}$). The unique character of this table of reciprocals of successive integers lies in the fact that it gives approximative values for the reciprocals of the irregular numbers which occur. The degree of the approximations is described by the following table:

	n	$\bar{n}$ Text	$\bar{n}$ Calculated to one place beyond text
Obverse	56	[.....]	
	57	[.....]	
	58	1,2,4,[....]	1,2,4,8
	59	1,1,1	1,1,1,1
	1	1	1
	1,1	59,59[76f]	59,0,59,0,59
	1,2	58,3,52	58,3,52,15
	1,3	57,8,24[sic]	57,8,34,17
	1,[4]	56,15	56,15
	1,[5]	55,23,4,30[sic]	55,23,4,36,55
	1,[6]	54,32,43,30[sic]	54,32,43,38,11
	1,[7]	53,43,52	53,43,52,50
	1,[8]	52,56,53[sic],14	52,56,28,14,7
	1,9	52,10,28[sic]	52,10,26,5
	1,10	51,25,42	51,25,42,51
	1,11	50,42,15	50,42,15,12
Reverse	1,12	50	50
	1,13	[4]9,18,55[sic]	49,18,54,14
	1,14	48,38,55	48,38,55,8
	1,15	48	48
	1,1[6]	47,22,6	47,22,6,20
	1,[17]	46,45,11	46,45,11,41
	1,[18]	46,9,13,50	46,9,13,50,46
	1,[19]	45,34,9[sic],54[sic]	45,34,10,37,58
	[1,20]	45	45

[76a] The side of the tablet which was called "obverse(?)" is actually the reverse.

[76b] This sign must be read together with the last sign of the previous line as 56; for references to other split writings of this type, see p. 13, note 69.

[76c] This and the following line should have read: 9,37,9,15,32,10, 54,17,25.

[76d] Or 50(?),17(?) instead of 57(?).

[76e] Cf. p. 11.

[76f] As is usual in Old-Babylonian texts, "zero" is not indicated paleographically.

The remainder of the preserved part of the reverse is uninscribed.

b**. Operations with Reciprocals

The following texts from the Yale Babylonian Collection, all presumably Old-Babylonian, are constructed according to a common pattern which is best described by an example:

YBC 11127[76g]

[1]	1	2	2	4,54
2	30	1	2	
3	20	40	2	
4	15	30	2	
5	12	24	2	
6	10	20	2	

The numbers in the second column are the reciprocals of those in the first; the values given in the third column are double those of the second, and the constant factor $c = 2$ is listed in the fourth column. The number 4,54 in the upper right corner is the sum of the numbers given in the third column.

The following four texts show the same structure:

YBC 7354[76g]

			1,13,3[0]
1	1	30	30
2	30	15	30
3	20	10	30
4	15	7,30	30
5	12	6	30
6	10	5	[30]

YBC 7355[76h]

			1,38
1	1	40	40
[2]	30	20	40
[3]	[2]0	13,20	40
[4]	[15]	10	40
[5]	[12]	8	40
6	[10]	6,40	40

YBC 7358[76g]

			1,42,45
1	1	45	45
2	30	22,30	45
3	20	15	45
4	15	11,15	45
[5]	[12]	9	45

YBC 7234[76g]

			2,51,[30]
1	1	1,10	1,10
2	30	35	1,10
3	20	23,20	1,[10]
4	15	17,30	1,10
5	12	14	1,10
6	10	11,40	1,10

The only deviation from the scheme given above consists in the fact that YBC 7358 contains only five steps. The constants c are 30, 40, 45 and 1,10, respectively.

[76g] Length 9 cm.; width 8 cm.; reverse uninscribed.
[76h] Length 10 cm.; width 9 cm.; reverse uninscribed.

The next text follows a slightly different pattern:

YBC 7235[76i]

				1,3,20
1,40	1	1	40	1,6,40
5	3	20	13,20	1,6,40
6,40	4	15	10	1,6,40
				40

If we call the numbers in the first column $n \cdot 1,40$ (i.e., multiples of 100), then the second column gives n, the third $\bar{n}$, the fourth column $40\bar{n}$, and the fifth $40 \cdot 1,40 = 1,6,40$. The factor $c = 40$ appears at the bottom of the fifth column; the number 1,3,20 in the upper right corner is the sum of the numbers $c\bar{n}$ of the fourth column. The reverse of this tablet contains the partly erased numbers

				35
				3,30
10	6		7	15 4
			28	4
			..	30

which apparently have no connection with the obverse.

The final two tablets of this group are the following:

YBC 11125[76j]

(Obverse)				4,15
	7	2,23	1,35,20	11,7,20
	11	1,31	1,40	11,7,20
	13	1,17	51,20	11,7,20
	14	1,11,30	47,40	11,7,20

The reverse gives the first and last lines of the following tablet (YBC 7353) separated by a blank space.

YBC 7353[76k]

			3,11,15
7	2,23	1,11,30	8,20,30
11	1,31	45,30	8,20,30
[13]	1,17	38,30	8,20,30
14	1,11,30	35,45	8,20,30

The first two columns of both texts yield the constant product $a = 16,41$; thus, $7 \cdot 2,23 = 16,41$ or $11 \cdot 1,31 = 16,41$. The number 16,41 itself is the product of the prime numbers 7, 11 and 13, i.e., the least common multiple needed to produce finite sexagesimal expressions for $\frac{a}{n} = n'$ if n is one of the numbers of the

[76i] Length 8 cm.; width 8.5 cm.
[76j] Length 9.5 cm.; width 9 cm.
[76k] Length 9.5 cm.; width 9 cm.; reverse uninscribed.

first column. In other words, the previous examples are based on the ordinary concept of reciprocals for "regular" numbers n, namely, pairs of numbers n and $\bar{n}$ whose product $n \cdot \bar{n}$ equals a power of 60; the last two examples, however, deal with the case of pairs of numbers where $n \cdot n' = a$, where a is a number which contains all the prime factors of n not contained in 60.

Except for this modification, the remainder of the two tablets again follows the previous scheme. The third column is in the one case (YBC 11125) 40 times the second column, in the other case (YBC 7353) 30 times the second column. The sum of the numbers in the third column is given by the number in the upper right corner. The fourth column is $40 \cdot 16{,}41 = 11{,}7{,}20$ and $30 \cdot 16{,}41 = 8{,}20{,}30$, respectively.

The pattern common to all these texts can accordingly be described as giving n and $\bar{n}$ or n and n' (where $n \cdot \bar{n} = 60^k$ and $n \cdot n' = a = 7 \cdot 11 \cdot 13$); then $c\bar{n}$ or cn'; and finally c or ca or $c \cdot 1{,}40$ (where c is 2 or 30 or 40 or 50 or 1,10). In the upper right corner of these lists appears the number which is the sum of all numbers $c\bar{n}$ or cn'. A hint as to the possible use of tables of this sort in the preparation or solution of certain problems is offered in the Appendix which follows.

Closely related to the preceding texts, but involving squares instead of irregular numbers, is **PTS 247**, an Old-Babylonian text which was discovered by Dr. A. Goetze:

Upper Edge 4 *li-im* 3 *me-at ù* 20
Obverse 3,45

[10]	1,40	2	1,12
7,30	56,15	1,7,30	1,12
5	25	30	1,12
2,30	6,15	7,30	1,12

The first column gives the reciprocals $\bar{n}$ of 6, 8, 12 and 24, respectively; the second column is $\bar{n}^2$, and the third gives $c\bar{n}^2$, where $c = 1{,}12$, which is repeated in the last column. The total 3,45 of the third column is written at the top of that column. The words on the upper edge mean: "4 thousand, 3 hundred and 20"; since the sexagesimal equivalent of this number is 1,12,0, it is clear that the number c in the last column should be interpreted as 1,12,0.

Appendix: VAT 7530

VAT 7530 (published MKT I pp. 287–289) contains a number of more or less parallel examples dealing with prices (*maḫīrum*[761]); unfortunately, no calculations or answers are given. The only attempt to explain the nature of the problems in detail was made by Thureau-Dangin.[76m] As a sample of the type of problem involved, we give obv. 7–10:

On the left margin

2,23 [7]7 ma-na-ta-àm *ù* 11 ma-na-t[a-àm]
1,31 [8]13 ma-na-ta-àm *ù* 14 ma-na-t[a-àm]
1,17 [9]1 gín 11 še *ši-za-a-at* še kù-b[abbar]
1,11,30 [10]kù-babbar *li-li ù li-ri-id ma-ḫi-r[u] li-im-ta-ḫar*

For the sake of simplicity of presentation, the translation which follows is based on Thureau-Dangin's:

[7]Each 7 ma-na and each 11 ma-na,
[8]each 13 ma-na and each 14 ma-na:
[9]1 gín, 11 še, (and) $\frac{1}{3}$ še of silver.
[10]Let the silver rise or descend (such that) the *maḫīrum*[761] be equal.

According to Thureau-Dangin, whose notation we shall employ with some additions, the problem deals with four kinds of goods: A, B, C, and D. The price of $\alpha = 7$ ma-na of A or $\beta = 11$ ma-na of B or $\gamma = 13$ ma-na of C or $\delta = 2\alpha = 14$ ma-na of D is the same, namely, $p = 1$ gín $11\frac{1}{3}$ še $= 1{;}3{,}46{,}40$ gín of silver. The problem for him consists in determining the respective prices of the same number x of ma-na of A, B, C, and D. These prices will be $\frac{xp}{\alpha}$, $\frac{xp}{\beta}$, $\frac{xp}{\gamma}$, and $\frac{xp}{\delta}$. Since p is not exactly divisible by the product of α, β, γ, and δ, it is necessary that x be exactly divisible by 7, 11, and 13 in order to get a finite sexagesimal number as the answer. Thureau-Dangin correctly analyzed the numbers appearing on the left margin as

$$2{,}23 = \frac{7 \cdot 11 \cdot 13}{7} = \frac{\alpha\beta\gamma}{\alpha}$$

$$1{,}31 = \frac{7 \cdot 11 \cdot 13}{11} = \frac{\alpha\beta\gamma}{\beta}$$

$$1{,}17 = \frac{7 \cdot 11 \cdot 13}{13} = \frac{\alpha\beta\gamma}{\gamma}$$

$$1{,}11{;}30 = \frac{7 \cdot 11 \cdot 13}{14} = \frac{\alpha\beta\gamma}{2\alpha}.$$

Thus, according to Thureau-Dangin, 16 ma-na and 41 gín[76n] of each of the goods A, B, C, and D would cost $2{;}23p$, $1{;}31p$, $1{;}17p$, and $1{;}11{,}30p$, respectively.

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[761] For this term, cf. below, p. 106.
[76m] Thureau-Dangin [6], pp. 162f.
[76n] $16{,}41 = 7 \cdot 11 \cdot 13$.

There would be little point in questioning this interpretation were it not for the fact that we are now in a position to make it virtually certain that there is a purely numerical relation between α, β, γ, δ on the one hand and p on the other.

We have already seen[76o] in the case of YBC 11125 and YBC 7353 that, using the same terminology as above, there were tables of the type

			y
$\alpha = 7$	$\frac{\alpha\beta\gamma}{\alpha}$	$k \cdot \frac{\alpha\beta\gamma}{\alpha} = k \cdot \beta\gamma$	$k \cdot \alpha\beta\gamma$
$\beta = 11$	$\frac{\alpha\beta\gamma}{\beta}$	$k \cdot \frac{\alpha\beta\gamma}{\beta} = k \cdot \alpha\gamma$	$k \cdot \alpha\beta\gamma$
$\gamma = 13$	$\frac{\alpha\beta\gamma}{\gamma}$	$k \cdot \frac{\alpha\beta\gamma}{\gamma} = k \cdot \alpha\beta$	$k \cdot \alpha\beta\gamma$
$\delta = 2\alpha = 14$	$\frac{\alpha\beta\gamma}{2\alpha}$	$k \cdot \frac{\alpha\beta\gamma}{2\alpha} = k \cdot \frac{\beta\gamma}{2}$	$k \cdot \alpha\beta\gamma$

where k, in the two extant texts, is 40 or 30, and y is the sum of the numbers given in the third column. Using $k = 10$ instead of $k = 40$ or $k = 30$ would yield for y the value 1;3,45, a value which differs from $p = 1;3,46,40$ gín in VAT 7530 by 0;0,1,40 gín, which is only $\frac{1}{12}$ še. A glance at the other examples in VAT 7530 shows that similar tables can be set up. We would thus obtain the following relations:

We thus find a constant difference of 0;0,1,40 gín between the p given in the text and the value calculated; in all likelihood, this small difference is a magnitude below the lowest fraction of a še considered as a reasonable metrological unit, and was therefore disregarded by the text.

We are completely in the dark as to the exact application of this set of coincidences to the problems given in VAT 7530, but it seems obvious that there is a significant relation between VAT 7530 and the tables given above.

§ 3. Multiplication Tables

We shall call the number whose multiples are listed in a multiplication table the "principal number" of that table. Let c denote the principal number; then the following numbers are given in a table with principal number c:

1	c
2	$2c$
3	$3c$
.	
.	
.	
19	$19c$
20	$20c$
30	$30c$
40	$40c$
50	$50c$

VAT 7530	Quantities of goods	p		
		text	calculation	
obv. 7–10	$\alpha = 7$ $\beta = 11$ $\gamma = 13$ $\delta = 14$	1;3,46,40 gín	1;3,45 gín	$k = 10$
obv. 11–16	$\alpha = 7$ $\beta = 11$ $\gamma = 13$ $\delta = 14$ $\epsilon = 19$	1,8;54,16,40[76p] gín	1,8;54,15 gín	$k = 30$
obv. 17–21	$\alpha = 1$ $\beta = 2$ $\gamma = 3$ $\delta = 4$ $\epsilon = 5$ $\zeta = 6$ $\eta = 7$ $\theta = 8$ $\iota = 9$ $\kappa = 10$	10;15,6,40 gín	10;15,5 gín	$k = 30$
rev. 1–7	$\alpha = 1$ $\beta = 1;10$ $\gamma = 2$ $\delta = 2;20$ $\epsilon = 3$ $\zeta = 3;30$ $\eta = 4$ $\theta = 4;40$ $\iota = 5$ $\kappa = 5;50$	2;28,26,40 gín	2;28,25 gín	$k = 5$

[76o] P. 17.

[76p] The text mistakenly gives $11\frac{1}{2}$ še instead of $12\frac{1}{2}$ še.

There sometimes follows the value of c^2 or a catch-line indicating the table which follows in the canonical arrangement.[77]

The texts vary in regard to terminology. The characteristic term is a-rá, "times". The main types are

Type A		Type B		Type C	
c a-rá 1	*c*	*c* a-rá 1	*c*	1	*c*
a-rá 2	2*c*	2	2*c*	2	2*c*
.		.		.	
.		.		.	
.		.		.	
a-rá 50	50*c*	50	50*c*	50	50*c*

These three are the most common types, A being especially frequent in single tables, C in combined tables.[78] There are also some intermediate types which are only rarely represented:[79]

Type A′			Type B′		
	c a-rá 1	*c*		1 a-rá	*c*
	c a-rá 2	2*c*		2	2*c*
	.			.	
	.			.	
	.			.	
	c a-rá 50	50*c*		50	50*c*

Type B″			Type C′			
	c	1	*c*		*c*	1	*c*
	2		2*c*		*c*	2	2*c*
	.				.		
	.				.		
	.				.		
	50		50*c*		*c*	50	50*c*.

c. Single Multiplication Tables

No.[80]	Museum No.[81]	Principal Number	Type	End[82]	Remarks[83]
65	A 1555	50	A′	Destroyed.	Published by Holt [1] p. 212 as RCT 1; overlooked in MKT. Upper edge, remains of [..].. SI; left-hand edge, ..[..].. [...]1 21 ..... (ignored by Holt).
66	CBS 7375	50	A	Destroyed.	
67	NBC 6343	45	A		19 written 20-lá-1.
68	NBC 7338	44,26,40	B	[a-rá] 44,26,40 [3]2,55,18,31,6,40	Traces of [a] visible.
69	CBS 29.15.477	44,26,40	A	44,26,40 a-rá 44,26,40 [32],55,18,[31],6,40 40 a-rá 1 40	19 written 20-lá-1.
70	NBC 7336	36	A		19 written 20-lá-1.
71	A 678	36	A	Traces of number signs.	Excavated at Adab; 19 written 20-lá-1.
72	CBS 29.15.485	36	A	36 a-rá 6 30(!) 21,36 30 a-rá 1 30	19 written 20-lá-1. Notice the false position of the double line.
72a	YBC 8617	36	A		Colophon: im-gíd-da In-bi-ì-lí-šu/itu-kin-^{d}inanna ki-2 u_4-5-kam[83a].
73	CBS 29.15.503	30	C		19 written 20-lá-1.

[77] Cf. above p. 11.

[78] Cf. the graph given in MKT I p. 64 fig. 1.

[79] Only B′ and B″ were listed in MKT I p. 32, A′ in MKT III p. 50 (No. 61).

[80] Continuation of MKT I pp. 36–42, MKT II p. 36 and MKT III pp. 49f. At the latter place the following numbers should be added: 18d = MMAP 27, 295; 42b = MMAP 27, 292; 55d = MMAP 27, 61.

[81] All CBS texts are presumably from Nippur.

[82] Only deviations from the ordinary end (giving 50*c*) are indicated.

[83] Unless otherwise stated, the number 9 is written as three superimposed rows of three vertical wedges.

[83a] "Tablet of Inbi-ilišu. Month VI$_2$, day 5."

No.[80]	Museum No.[81]	Principal Number	Type	End [82]	Remarks [83]
74	Ni —	25	C′		Published in transcription by Hilprecht in BE 20,1 p. 16 (repeated in abbreviated form EAB p. 532); overlooked in MKT. From Nippur, lost in Istanbul (cf. Hilprecht BE 20,1 p. 19 and note 3).
75	NBC 6349	25	A′		Bought from Gejou in 1925; probably from Larsa. 19 written 20-lá-1, but 9 is written 5 wedges over 4. Colophon: 1 E(?) 1 E(?) ŠU si DI(?).
76	MLC 1611	25	A		Traces of 2 lines of a colophon; the first line can be restored: [itu-gi]š-apin-du$_8$-a u$_4$[-. . .-kam], "Month VIII, day . . . ."
77	CBS 29.15.480	25	A	Destroyed.	Fragment of a larger tablet with the table occupying only Col. I, leaving the remaining part of the obverse empty. Reverse blank.
78	Cornell Univ. Library 31	25	B		19 written 20-lá-1.
79	CBS 29.15.488	25	Not A, A′ or C′.	Traces of signs.	Almost completely destroyed; written on the reverse of combined multiplication table No. 160.
80	MLC 2231	24	Not A, A′ or C′.	Destroyed.	
81	NBC 7332	24	A		19 written 20-lá-1.
82	CBS 29.15.647	24	A	[24 a-]rá 24 9,36	19 written 20-lá-1.
83	CBS 29.15.493	24	C	9,36 1 22,30	19 written 20-lá-1. 9,36 = 24^2.
83a	YBC 8456	24	A	Destroyed.	
83b	PTS 252	24	A		
84	NBC 6778	20	A		19 written 20-lá-1.
85	CBS 7373	18	A	Destroyed.	19 written 20-lá-1.
86	NBC 7343	16,40	A		
87	CBS 7378	16,40	A	Destroyed.	
88	YBC 6949	16,40	A		19 written 20-lá-1.
89	CBS 29.15.712	16,40	A		19 written 20-lá-1
90	CBS 29.15.502	16,40	Not A, A′ or C′.	4,37,46,40 [1] 16	19 written 20-lá-1. 4,37,46,40 = 16,40^2.
91	CBS 7355	16,40	Not A	[16 a-r]á 1 16	
92	YBC 6951	16	A		19 written 20-lá-1.
92,1	YBC 11138	16	Not A, A′ or C′.		Colophon: [im-]gíd-da S[i(?)-n]a(?)-tum *mār* Ṣíl-lí-^{d}Nin-. . . . After a blank space, there follows: [itu-. . . . . -] DU 2 (or A?) u$_4$-1-kam/[mu Sa-am-s]u-i-lu-[n]a [. . . . .] [83b].

[83b] "Tablet of Sin(?)atum, son of Ṣilli-^{d}NIN. . . ." Then follows a date, presumably the first regnal year of Samsu-iluna.

No.[80]	Museum No.[81]	Principal Number	Type	End [82]	Remarks [83]
93	CBS 7376	16	A		19 written 20-lá-1.
94	CBS 29.15.501	16	C		19 written 20-lá-1.
95	CBS 29.15.496	16	C	1 15	19 written 20-lá-1.
96	NBC 7347	15	C		Edge: u_4-9-kam, "9th day".
97	CBS 29.15.487	15	A		19 written 20-lá-1.
98	CBS 29.15.478	15	Not A, A′ or C′.	1 12,30	This table is written in two copies on the tablet; on the reverse is the combined multiplication table No. 168.
98a	YBC 8306	10	C	Destroyed.	
99	YBC 6950	9	A		19 written 20-lá-1.
99′	YBC 11094	9	C		Remains of a very broken colophon at end.
99″	YBC 6705	9	A		Error in line 1: 9 a-rá (!) 9. Colophon: im-gíd-da U-bar-rum/itu-kin-^{d}inanna u_4-25-kam.[83c]
99,1	YBC 6952	8,20	A		
99,2	CBS 7890	8,20	C	8,20 a-rá 8,20 1,9,26,40-e	19 written 20-lá-1. The text contains two errors: 26 instead of 25 in line 3, and 2,6 instead of 2,5 in line 15.
99,2a	YBC 8595	8,20	A	[8,20 a-rá 8],20 [1,9,26],40-e	
99,3	CBS 13567+ 13574+13579 +13582+ 13591+13595 +13599+ 13606+13613 +13616	8	C	Destroyed.	This table is copied three times on a very badly preserved tablet; reverse destroyed.
99,3a	YBC 6739	8	A		Colophon: im-gíd-da U-bar-rum/itu-kin-^{d}inanna u_4-20-kam ba-zal.[83d]
99,4	Cornell Univ. Library No. 58	7,30	A		19 written 20-lá-1. Error in line 1: 7,30 a-rá (!) 7,30.
99,5	CBS 29.15.592	7,12	A		19 written 20-lá-1.
99,6	CBS 13324	7	C		
99,7	CBS 7365	7	A	6,40 a-rá 1 [6,40]	
99,7a	NBC 8138	7	A		
99,8	CBS 29.15.495	6,40	A	Destroyed.	
99,9	MLC 1593	6	A	Destroyed.	
99,10	NBC 7344	5	A		
99,11	CBS 7370	5	A	Destroyed.	

[83c] "Tablet of Ubar(r)um. Month VI, day 25." For another table owned by the same man (or another person bearing the same name), see No. 99,3a.

[83d] "Tablet of Ubar(r)um. Month VI, day 20." For another table owned by the same person (or another man with the same name), see No. 99″.

No.[80]	Museum No.[81]	Principal Number	Type	End [82]	Remarks [83]
99,12	M 406	5	C		Colophon: u_4-18-kam, "18th day". Published Moore [1] pl. 74, No. 94.
99,13	YBC 4690	4,30	B	Destroyed.	Traces at the beginning of lines 2 and 3 indicate that the scribe started to write a-rá (type A instead of B) but left the space free in all following lines.
99,13a	YBC 6048	4,30	Not A, A′ or C′.		
99,13b	YBC 11924	4	A		Colophon: im-gíd-da ^{d}Sin(EN-ZU)-a-pil-urim^{ki}/itu-kin-^{d}inanna u_4-11-kam ba-zal/mu é ^{d}en-ki šà urim^{ki}-ma[83e].
99,14	CBS 6095	3,45	C	Destroyed.	First column of the obverse of a large tablet, the remaining part blank. Reverse: combined multiplication table No. 144.
99,14a	NBC 7701	3,45	A	3,45 a-rá [3],45 14,3,45	Colophon: itu-ab-è u_4-20-kam/im-gíd-da E-še-i-pa-ni-AN-[. . (?)].[83f]
99,15	CBS 29.15.504	3,20	A	Destroyed.	
99,16	CBS 29.15.605	3,20	A	Destroyed.	
99,17	CBS 8149	3,20	A	Destroyed.	19 written 20-lá-1.
99,18	CBS 7366	3,20	A	Destroyed.	
99,18a	YBC 6403	3,20	A		
99,19	YBC 4706	2,30	A		
99,20	NBC 7345	2,30	A	Destroyed.	
99,20a	PTS 259	2,30	A		Illegible colophon.[83g]
99,20b	Univ. of Illinois —	2,24	Not B′, B″, C or C′.	Uncleaned.	
99,21	CBS 8226	2	Not A, A′ or C′.	Destroyed.	Written with large signs; table probably repeated in the second column, of which only a few numbers of the left-hand side are preserved. Reverse: combined multiplication table No. 156. 19 written 20-lá-1.
99,22	NBC 7346	1,40	A		Colophon: im-gíd-da/níg(?) ì-lí-ip-pa-al-sà-am/itu-giš-apin-du_8-a/u_4-25-kam ba-zal.[84]
99,23	NBC 7334	1,40	B		Error in line 1: 1,40 a-rá (!) 1,40.
99,24	YBC 9883	1,30	B		19 written 20-lá-1.
99,25	CBS 8136	1,30	Not A, A′ or C′.	Destroyed.	
99,26	CBS 29.15.498	1,15	?		Left-hand part completely destroyed. Error in line 5: 16,5 instead of 6,15.

[83e] "Tablet of Sīn-apil-urim. Month VI, day 11, the year (the king built) the temple of Enki in Ur." The year-name dates the tablet to the 8th year of Rīm-sīn.

[83f] "Month X, day 20. Tablet of Eše"ī-pāni-ili(?)."

[83g] Now that the tablet has been baked and cleaned, Dr. A. Goetze informs us that the "colophon" actually consists of six lines, each of which begins with the vertical personal wedge and is followed by a personal name. The names are badly preserved, but the first three clearly begin with *ì-lí-*. This uniformity of the first element of the names points to a school practice list of names, not real persons.

[84] "Tablet of Ilī-ippalsam. Month VIII, day 25."

The preceding list adds 77 single multiplication tables to the 85 published in MKT. The complete system of multiplication tables contains 40 individual tables.[85] With the exception of the tables for 48[86] 2,15 and 1,20 all these tables are now also represented by single tables.

d. Combined Multiplication Tables

Before giving the schematic list of 30 combined multiplication tables to be added to the 44 published in MKT, we shall discuss briefly the most complete specimen of the new group, the cylinder A 7897 (No. 140 below), as an example.

A 7897 is an Old-Babylonian cylinder from the Diyālā region[87] containing the remains of 12 columns in varying states of preservation. Fortunately, only one of the original 13 columns is now completely gone. The diameter of the cylinder is about 10.6 cm ($4\frac{1}{4}$ inches), its original height approximately 22.5 cm (9 inches). A hole of about 1.3 cm ($\frac{1}{2}$ in.) diameter passes through the cylinder along its axis.[88] It may be noted in passing that A 7897 is the first known mathematical table written on a cylinder.[89]

The distribution of the individual tables over the 13 columns is shown by the schematic drawing given below, fig. 1. At the beginning is the table of reciprocals already mentioned above, p. 12 No. 40. The succeeding tables for 50, 48 and 45 represent the most complete sequence of initial tables; many combined tables omit the table for 48. The following tables correspond exactly to the standard arrangement with only one exception, between 8 and 6,40 (Cols. VIII–IX). The available space only permits two tables for the three principal numbers 7,30 7,12 and 7 of the standard list. It seems plausible that 7 was omitted, since it is the only "irregular" number of the whole list. On the other hand, it is the practice of the text to give the square of the principal number at the end of all individual tables for two-place numbers, and it is clear from the traces that the last line preceding the table for 6,40 does not contain a-rá, a word required in the terminology for the square. It therefore remains undecided which table is missing. The restoration of a table for 1,30 at the end is of course not necessary, but very plausible.

The terminology of all individual tables is of Type B (at least so far as can be determined from the preserved portions) whereas the usual terminology in combined tables is of Type C (about 75% of all tables[90]), B′ or A.[91]

We now give the list of all combined tables and a description of the individual tables of which they consist. The remark "reverse type" means that the first column is the right-hand column (which is the normal case for the reverse). This has no deep significance; it is merely the consequence of the fact that school tablets frequently contain not only a combined multiplication table but also other texts, e.g., metrological lists, vocabularies, or single multiplication tables. Under these circumstances, it is purely accidental which text was first inscribed on the tablet.

Two of the tablets listed below (Nos. 163 and 165) are not combined tables of the usual form but are small tablets containing a single table on each side. These texts are accordingly nothing more than a pair of coordinated individual tables. Another tablet of this kind was already published in MKT II p. 37 (No. 122a).

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[85] Cf. the lists given in MKT I pp. 34f. and the following section.

[86] This table is also omitted in most of the combined tables; cf. MKT I p. 35.

[87] The cylinder belongs to a group of more than 150 sculptures and cuneiform tablets purchased for The Oriental Institute by H. Frankfort from two Baghdad dealers, Messaye and Samhiry, in 1930. More than 60 of the tablets are Old-Akkadian (ca. 2200 B.C.), the rest Old-Babylonian (ca. 1700 B.C.). The dealers' statement giving the Diyālā region as the provenance of the collection is amply corroborated by many coincidences (e.g., in personal names) between these tablets and others actually excavated in the Diyālā area by the Iraq Expedition of The Oriental Institute. There is accordingly not the slightest reason to doubt that also the cylinder in question comes from the same district.—We are indebted to Dr. Thorkild Jacobsen for having placed the dealer's photographs at our disposal and for data concerning the collection.

[88] When A 7897 was still in the dealer's possession, someone had already gone to great lengths to make the cylinder appear to be in an excellent state of preservation. This was done by filling the gaps with smaller fragments from the same cylinder as well as pieces belonging to other cylinders (A 7895 and A 7896, which belong to a type now generally called "forerunners to the series HAR-ra = *ḫubullu*"). When a fragment exceeded the dimensions of a lacuna, an instrument which left comb-like marks was used to shave the back and edges to form a well-tailored fit; somewhat lavishly, two holes inside the cylinder were even filled with parts of two economic tablets. Accordingly, before baking and the concomitant detachment of the various fragments were accomplished in the spring of 1941, A 7897 was a cylinder with writing visible on every part of its lateral surface, on which a particularly strong native paste had been liberally applied. Almost anticlimactically, all the fragments which had been prepared and joined with so much care were pasted in tilted or upside-down positions. At present, A 7897 consists of one large piece which forms the bulk of the cylinder and five smaller fragments which cannot be directly joined partly because of the deterioration or disappearance of the points of contact and partly because of the effects of the above-mentioned comb-like instrument.

[89] Tables for squares and cubes are inscribed on a six-sided *prism* (AO 8865; cf. MKT I p. 69 No. 24) and a problem-text on a four-sided *prism* (AO 8862; cf. MKT I pp. 108ff.). Both texts are Old-Babylonian.

[90] Cf. MKT I pp. 62ff. Among the tables published below, at least 16 use terminology C and 8 terminology A.

[91] Type B occurs only three times in the previously published material and then always in the individual table for the three-place number 44,26,40 (cf. MKT I p. 63, I and note 3).

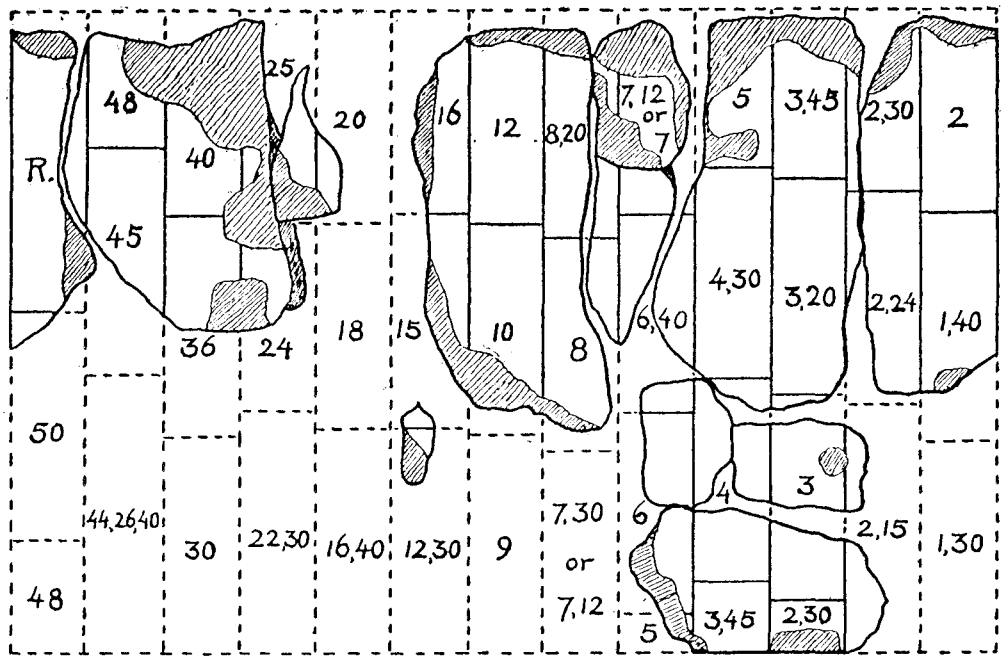


Fig. 1.

Another tablet (No. 154a) is peculiar in two ways: the tables begin with 2*c*; and the table for 44,26,40 is followed by the table for 3,45 against the canonical order.

The arrangement of the texts in the following list is quite arbitrary; the most extensive table beginning with the highest number precedes a table of smaller size or beginning with a lower number. The serial numbers continue those used in MKT.[92]

No.	Museum No.	Individual Table			Remarks
		Principal Number	Type	End	
140	A 7897	50	B	Destroyed.	For the general description of this cylinder, provenance, etc., see above, pp. 24f. For the table of reciprocals cf. p. 12 No. 40.
		48	Not A.	50 40 48 [a-rá] 48 38,24	
		45	B	Destroyed.	
		[44,26,40]	Destr.	Destroyed.	
		40	Not A.	50 3[3,20]	
		36	B	Destroyed.	
		[30]	Destr.	Destroyed.	
		25	Not A.	Destroyed.	
		24	Not A.	Destroyed.	

[92] Cf. MKT I pp. 44–59, MKT II p. 37, MKT III p. 49. The following numbers should be added in MKT III p. 49: 120a = MMAP 27, 296; 128a = MMAP 27, 294; 132a = MMAP 27, 293.

No.	Museum No.	Individual Table			Remarks
		Principal Number	Type	End	
		[22,30]	Destr.	Destroyed.	
		[20]	Not A.	Destroyed.	
		[18]	Destr.	Destroyed.	
		[16,40]	Destr.	Destroyed.	
		16	Destr.	[50] 13,20	
		15	B	50 [12,30]	
		12,30	B	Destroyed.	
		12	Not A.	50 10	
		10	B	Destroyed.	
		[9]	Destr.	Destroyed.	
		8,20	Not A.	50 6,56,40 8,20 a-rá 8,[2]0 1,9,26,40	
		8	B	Destroyed.	
		[7,30] or [7,12]	Destr.	Destroyed.	Cf. above, p. 24.
		[7,12] or [7]	Destr.	[50] [···]	
		6,40	B	[50] 5,33,20 [6,]40 a-rá 6,40 44,26,40	
		6	B	[50] 5	
		5	Not A.	50 4,10	
		4,30	B	50 3,45 4,30 a-rá 4,30 20,15	
		4	B	50 3,20	
		3,45	B	50 3,7,30 3,45 a-rá 3,45 14,3,45	
		3,20	B	50 2,46,40 3,20 a-rá 3,20 1[1,6,40]	
		3	B	50 2,30	
		2,30	B	50 2,5 2,30 a-rá 2,30 6,15	
		2,24	B	[50] 2 [2,24 a-r]á [2,24 5,45,36]	
		2,15	B	50 1,[52,30] 2,15 a-[rá 2,15 5,3,45]	

No.	Museum No.	Individual Table			Remarks
		Principal Number	Type	End	
		2	B	50 1,40	
		1,40	B	Destroyed.	
		[1,30]	Destr.	Destroyed.	See above, p. 24.
141	CBS 8215	50	C	Destroyed.	For the table of reciprocals see p. 12 No. 42. 19 written 20-lá-1.
		45	Destr.	50 37,30	
		44,26,40	C	Destroyed.	
		[40]	Destr.	50 [33,20]	
		[36]	C	Destroyed.	
		Following tables destroyed.			
		6,40	Destr.	50 5,33,20	
		6	C	Destroyed.	
		5	Destr.	[50 4],10	Error: 20 2(!),40.
		4,30	C	Destroyed.	Error: 2 9,30(!).
142	CBS 8309	50	Destr.	50 41,40	For the table of reciprocals see p. 12 No. 41.
		45	C	50 [37,30]	
		44,26,40	Destr.	50 [37,2],13,20 32,55,18,30 1,6,40	Note the splitting of 31 into 30 (line 2) and 1 (line 3); cf. No. 147 and p. 13 note 69.
		40	C	Destroyed.	A graphical restoration based on the preserved parts shows clearly that one individual table between 40 and 7,12 was omitted. 19 written 20-lá-1.
		Following tables destroyed.			
		7,12	Destr.	50 6 7,12 a-rá [7,12] 51,50,[24]	
		7	C	Destroyed.	
143	YBC 7303	50	A	a-rá 50 41,40	For the table of reciprocals see p. 12 No. 35. 19 written 20-lá-1. On the reverse Col. IV seems to have been empty (now destroyed). On the other hand, there is no room in rev. Col. VI for a table for 9 between 10 and 8,20.
		45	A	a-rá 50 37,30	
		44,26,40	A	a-rá 50 37,2,13,20	
		40	A	a-rá 50 33,20	
		36	A	a-rá 50 30	
		30	A	a-rá 50 25	
		25	A	a-rá [50 20,50]	

No.	Museum No.	Individual Table			Remarks
		Principal Number	Type	End	
		24	A	a-rá 50 20	
		22,30	A	Destroyed.	
		20	A	a-rá 50 [16,40]	
		18	A	a-rá 50 15	Error: a-rá 4 1,3.
		16,40	A	a-rá 50 13,53,20	
		16	A	Destroyed.	
		15	A	[a-rá 5]0 12,30	
		[12,30]	Destr.	Destroyed.	
		[12]	Destr.	Destroyed.	
		10	A	Desrtoyed.	From 6 to 9, the results are one line too low.
		8,20	A	a-rá 50 6,56,40	Error: a-rá 19 2,25,20 instead of 2,38,20.
144	CBS 6095	50	Not A.	Destroyed.	Reverse type. Obverse: single multiplication table for 3,45. Cf. above, p. 23 No. 99,14.
		[45]	Destr.	Destroyed.	
		44,26,40	Not A.	50 37,2,13,20	
		40	C	Destroyed.	
		36	Not A.	50 30	
		30	C	Destroyed.	
		25	Not A.	50 20,50	
		[24]	Destr.	Destroyed.	
		22,30	Not A.	Destroyed.	
		[20]	Destr.	Destroyed.	
		18	C	Destroyed.	
		[16,40]	Destr.	Destroyed.	
		16	Not A.	Destroyed.	
		[15]	Destr.	Destroyed.	
		12,30	Not A.	Destroyed.	
145	CBS 8153	[50]	Destr.	Destroyed.	For the table of reciprocals see p. 12 No. 38. Reverse: left-hand side of a single multipl. table and remains of a vocabulary. 19 written 20-lá-1.
		45	C	Destroyed.	
		[44,26,40]	Destr.	Destroyed.	
		40	Not A.	Destroyed.	

No.	Museum No.	Individual Table			Remarks
		Principal Number	Type	End	
		[36]	Destr.	Destroyed.	
		30	Not A.	Destroyed.	
		[25]	Destr.	Destroyed.	
		24	Not A.	24 a-rá [24 9,36]	
		[22,30]	Destr.	Destroyed.	
		20	Not A.	Destroyed.	
		[18]	Destr.	Destroyed.	
146	CBS 29.16.90	50	C	50 [41,40]	For the table of reciprocals see p. 12 No. 43. Note omission of tables for 48 *and* 45. The end of the last column is blank. Reverse type. Obverse: remains of vocabulary; right half blank. 19 written 20-lá-1.
		44,26,40	Not A.	50 37,[2,13,20]	
		40	C	Destroyed.	
		36	Not A.	50 21,36	
		30	C	50 25	
		25	C	50 20,50	Error: 5 2,30(!).
		24	C	50 20	
		22,30	C	Destroyed.	
		20	Not A.	Destroyed.	
147	CBS 29.15.497	50	Not A.	50 41,40	For the table of reciprocals see p. 12 No. 39. Three columns left blank on the reverse. 19 written 20-lá-1.
		48	Destr.	Destroyed.	
		45	Not A.	50 37,30 45 a-rá 45 33,45	
		44,26,40	C	50 [37,2,13,20] 44,2[6,40. . . . .] 32,5[5,18,30] 1,[6,40]	Line 2: hardly space for a-rá 44,26,40. Line 3/4: for the assumption of 30/1 cf. p. 13, note 69.
		40	C	Destroyed.	
		[36]	Destr.	Destroyed.	
		[30]	Destr.	Destroyed.	
		[25]	Destr.	Destroyed.	
		[24]	Destr.	Destroyed.	
		22,30	Not A.	Destroyed.	
148	CBS 8210	50	A	a-rá 50 41,40	For the table of reciprocals see p. 12 No. 37. 19 written 20-lá-1.

No.	Museum No.	Individual Table			Remarks
		Principal Number	Type	End	
		45	A	Destroyed.	
		44,26,40	A	Destroyed.	
149	CBS 8156	50	Not A.	50 41,40	19 written 20-lá-1. Reverse: metrological table (measures of length).
		[45]	Destr.	Destroyed.	
		44,26,40	Not A.	50 [37,2,13,20]	
150	CBS 29.15.492	50	C	Destroyed.	Reverse type. Obverse empty.
		45	Not A.	50 37,[3]0 45 a-rá 45 [33,45]	
151	YBC 6632	50	A	Destroyed.	
		[48]	Destr.	Destroyed.	
		[45]	Destr.	Destroyed.	
		44,26,40	A	Destroyed.	
152	CBS 7369	45	A	Destroyed.	A whole column is missing between 7,30 and 4. At the end follows a table of squares (cf. § 4e, No. 28) and a table of measur s (kùš).
		36	A	Destroyed.	
		30	A	Destroyed.	
		22,30	A	Destroyed.	
		16,40	A	Destroyed.	
		12,30	A	Destroyed.	
		7,30	A	Destroyed.	
		4	A	a-rá 50 [3,20]	
		2,30	A	a-rá <50> 2,5	40 is also omitted.
153	CBS 8274	[45]	Not A.	50 [37,30]	Reverse type. Obverse destroyed. Join with No. 144 impossible.
		44,26,40	C	Destroyed.	
		40	Not A.	[50] 33,20	
		36	Destr.	Destroyed.	
154	CBS 8308	44,26,40	Destr.	Destroyed.	19 written 20-lá-1.
		40	Not A.	Destroyed.	
		[36]	Destr.	Destroyed.	
		30	Not A.	50 25	
		25	C	Destroyed.	

No.	Museum No.	Individual Table			Remarks
		Principal Number	Type	End	
		24	Not A.	Destroyed.	
154a	YBC 4716	44,26,40	A	a-rá 50 37,2,13,20 íb-si$_8$ 32,[5]5,[1]8, 31,6,40	Two-column tablet. Col. II (except for first line) of obverse and whole of reverse uninscribed. First line broken away but must be restored: [44,26,40 a-rá 2 1,28,53,20]. Note the split writings 5,55,30/3,20 and 37,2,10/3,20; cf. p. 13, note 69.
		3,45	A	a-rá 5 [3,42,13,20]	Begins: 3,45 a-rá 2 7,30.
155	CBS 8281	40	Destr.	Destroyed.	Reverse empty.
		[36]	Destr.	Destroyed.	
		30	Not A.	Destroyed.	
		[25]	Destr.	Destroyed.	
		24	Not A.	Destroyed.	
		[22,30]	Destr.	Destroyed.	
		20	Not A.	Destroyed.	
		[18]	Destr.	Destroyed.	
		16,40	C	Destroyed.	
		[16]	Destr.	Destroyed.	
156	CBS 8226	40	Not A.	Destroyed.	Reverse type. Obverse: two single multiplication tables; cf. above, p. 23 No. 99,21.
		[36]	Not A.	50 [30]	
		30	C	Destroyed.	
		25	Not A.	[5]0 [20,50]	
		24	C	Destroyed.	
157	CBS 29.16.589	25	Not A.	Destroyed.	19 written 20-lá-1.
		[24]	Destr.	Destroyed.	
		22,30	Not A.	[50 18],45 [22,30] 22,30 8,[2]6,15	
		20	C	Destroyed.	
		[18]	Destr.	Destroyed.	
		16,40	Not A.	Destroyed.	
		[16]	Destr.	Destroyed.	
		15	Not A.	50 [12,30]	
		12,30	C	Destroyed.	

No.	Museum No.	Individual Table			Remarks
		Principal Number	Type	End	
		12	Not A.	Destroyed.	
		[10] or [9]	Destr.	Destroyed.	Space for only *one* table.
		8,20	Not A.	[50] 6,56,40 8,20 8,20 1,9,26,40	Remaining space blank.
158	CBS 8151	25	Destr.	Destroyed.	19 written 20-lá-1. Reverse empty. Catch-line: 1 16,40.
		[24]	Destr.	Destroyed.	
		22,30	Not A.	Destroyed.	
		[20]	Destr.	Destroyed.	
		18	Not A.	50 15	
159	CBS 29.15.476	24	Not A.	[5]0 20 9,30 6	*Sic* instead of 9,36.
		22,30	C	Destroyed.	19 written 20-lá-1.
		20	Not A.	50 16,40	
		18	C	Destroyed.	
		16,40	Not A.	50 13,53,20 4,36,46,40	
		16	C	Destroyed.	
		15	Destr.	[50 12],30	
		12,30	Destr.	Destroyed.	
160	CBS 29.15.488	24	Not A.	Destroyed.	Reverse: single multiplication table for 25 (cf. p. 21 No. 79).
		[22,30]	Destr.	Destroyed.	
		20	C	Destroyed.	
		[18]	Destr.	Destroyed.	
		16,40	C	Destroyed.	
161	CBS 29.15.484	20	Not A.	Destroyed.	Reverse type. Obverse destroyed.
		[18]	Destr.	Destroyed.	
		16,40	Not A.	Destroyed.	
162	CBS 29.15.474	7,30	Not A.	Destroyed.	19 written 20-lá-1. Badly preserved fragment. Obverse one large col. containing the beginning of the table for 7,30, smaller right-hand part empty, perhaps erased. Reverse from left to right: table for 8, table for 9, and a narrow third column with only 20, 30, 40, 50, but no results.
		9	Not A.	[5]0 7,30	
		8	Not A.	Destroyed.	

No.	Museum No.	Individual Table			Remarks
		Principal Number	Type	End	
163	YBC 9854	8,20	A	a-rá 50 6,56,40	19 written 20-lá-1. Cf. p. 24.
		8	A	Destroyed.	
164	CBS 29.15.76	6,40	Destr.	[a-rá 50 5,33],20	19 written 20-lá-1.
		6	A	Destroyed.	
		5	A	a-rá 50 4,10	
		4	A	Destroyed.	
		3,45	Destr.	[a-rá 50] 3,7,30	
		3,20	Destr.	Destroyed.	
165	CBS 8379	3,45	Not A.	50 3,7,30 [3,45] a-rá 3,45 1[4,3,45]	19 written 20-lá-1. Cf. p. 24.
		3,20	Not A.	Destroyed.	
166	CBS 8247	3,20	Not A.	[50] 2,46,40	Error: [20] 1,6,20(!). 19 written 20-lá-1. Small fragment; reverse(?) destroyed.
		3	C	Destroyed.	
167	CBS 29.13.174	2	A	Destroyed.	19 written 20-lá-1. Reverse: metrological table (measures of length) and blank space.
		1,40	A	Destroyed.	
168	CBS 29.15.478	1,40	C	Destroyed.	Very badly preserved; readings very doubtful. Reverse type. Obverse: two single multipl. tables for 15 (cf. p. 22 No. 98).
		1,30	Not A.	50 [1,15]	
		1,20	C	Destroyed.	

§ 4. Squares, Cubes, and Varia

e. Squares and Square Roots; Cube Roots

The canonical system apparently includes a table of squares. This is indicated by the fact that at least some of the combined tables contain a table of squares at the end.[93] Cubes are not preserved on any combined multiplication table but are so closely related to the tables of squares that there is no need to treat them separately. There is only one recognizable difference in the material available today: although we have tables of squares (16 tables) and square roots (17 tables), only tables for cube *roots* (6 tables) are hitherto attested.

The following numbers continue those listed in MKT I Chapter I § 4.

[93] E.g., Nos. 101 and 102 (MKT I p. 35).

No. 28. CBS 7369 rev. IV. Table of squares at the end of a combined multiplication table (cf. p. 30 No. 152) followed by a table of measures of length (kùš and GAR).

1 a-rá 1 1
2 a-rá 2 4
etc.
[11] a-rá 11 2,1

The following is destroyed, but it is plausible that the table extended to 1,0.

No. 29. CBS 8266. Single table of square roots.

From 15-e 30 íb-si$_8$ to [24,4]-e 38 íb-si$_8$
16,1-e 31 íb-si$_8$ 25,21-e 39 íb-si$_8$
17,4-e 32 íb-si$_8$ 26,40-e 40 íb-si$_8$
etc.

The last three lines are on the reverse, the remainder of which is blank.

No. 30. Plimpton 318. Single table of square roots.

From	15	30	to	54,9	57
	16,1	31		56,4	5[8]
	17,4	32		5[8],1	59
	etc.			1,2,3,2,1	1,2,1

The numbers in the last line are the squares of 1,1,1 and 1,1.[94]

No. 31. CBS 8270. Fragment containing five lines of a table of square roots. Reverse empty. Terminology destroyed.

Only the left-hand side of the fragment is preserved:

45,[4	corresponding to	52
46,4[9		53
48,3[6		54
50,25		55
52,[1]6		56

No. 31a. W 1923–366, published by Van der Meer, Syllabaries, No. 156. Six-sided prism, of which most of Col. V and all of Col. VI are destroyed. The first two columns and almost all of Col. III contain two metrological tables of measures of length.

The last four lines of Col. III, all of Col. IV, and what is left of Col. V constitute a table of square roots. The table begins

1-e 1 íb-si$_8$
4-e 2 íb-si$_8$

and continues in this fashion through Col. IV, which ends with

41,40-e 50 íb-si$_8$.

The first seven lines of Col. V are missing, but can easily be restored as having contained the squares of the integers between 51 and 57 inclusive. The few lines which are preserved in Col. V read as follows:

56,[4-e 58 íb-si$_8$]
58,1[-e 59 íb-si$_8$]

1[-e 1 íb-si$_8$]

1,2,1[-e 1,1 íb-si$_8$]
1,4,4[-e 1,2 íb-si$_8$]
1,6,9[-e 1,3 íb-si$_8$]
1,8,16[-e 1,4 íb-si$_8$]
1,10,15^{sic}[-e 1,5 íb-si$_8$]
1,12,36[-e 1,6 íb-si$_8$]
1,14,49[-e 1,7 íb-si$_8$].

The rest of the text is broken.

[94] Cf. MKT III p. 52.

The following errors are to be noted:

Col. IV 25:	10,41-e 29 íb-si$_8$ should read	14,1-e etc.
28:	17-e 32 íb-si$_8$	17,4-e etc.
38:	29,14-e 42 íb-si$_8$	29,24-e etc.
Col. V 8′:	1,10,15[-e 1,5 íb-si$_8$]	1,10,25-e etc.

No. 32. CBS 8165. Fragment of a single table. Obverse cube roots beginning with

1-e 1 ba-si$_8$
8-e 2 ba-si$_8$
27-e 3 ba-si$_8$.

Only parts of seven lines are preserved. The reverse is inscribed with scattered, half-erased numbers which are obviously connected with the calculation of cubes, e.g., 2,13,20 (= 20^3).

The six preceding tables all belong to the standard type of reciprocal and multiplication tables. They give the squares for all integers from 1 to 60 or the corresponding square or cube roots, and are frequently distributed on two tablets, as illustrated by the examples given above. The following text belongs to a more extensive type, and is closely related to a tablet found at Kiš and tentatively dated to the Persian period.[95]

No. 33. CBS 1535. Not from Nippur. Table of squares. Three badly damaged columns on the obverse, one on the reverse. Column I undoubtedly began with 1 a-rá 1 1; the first preserved lines are:

[5,30 a-rá 5,30	30,1]5
[6 a-rá 6	3]6
[6,30 a-rá] 6,30	42,15
[7 a-rá] 7	49
[7,30 a-r]á 7,30	56,15

etc.

The second column contains the squares from [18 a-rá 18 5,24] to [33,30 a-rá 3]3,30 [18,42],15, continued in Column III from [34 a-rá 34 19,16] to 46,30 a-rá 46,30 36,2,15. The concluding part from 47 [a-rá 47] 36,[49] to [1 a-rá 1 1] only fills the major part of one column of the reverse (end destroyed); the remainder of the reverse is blank.

Among the squares of this type, seven cases occur where a "zero" is called for. Unfortunately, only two of these numbers are not totally destroyed, namely,

24,30 a-rá 24,30 10,.,15
44,30 a-rá 44,30 33,0,15.

The sign which we transcribe here by "." looks

[95] Cf. MKT I p. 72 No. 27 and p. 73 note 21. A more precise dating to the end of the eighth century B.C. would be possible if, as seems likely, the person mentioned in the colophon turns out to be the same as the owner of the tablet published by Langdon VT Plate IV (p. 25).

exactly like the ordinary sign for 10, but no sign at all is given for "0" in the second case. The square of 42,30 is given as 30,.6,15 where "." again stands for a sign written like the sign for 10; but in this case the function of this special sign is not to indicate a "zero", but simply to separate 30 from 6 to prevent a mistaken reading 36.[96] The only other case where a unit should be separated from a ten-group of a higher order is destroyed. Hence this text sheds no new light on the problem of the use of a sign for zero before the Seleucid period.

The square of 51,30 is erroneously given as 43,12,15 instead of 44,12,15.

The following two texts in all probability belong to the Old-Babylonian period.

The obverse of **YBC 7294** ($6\frac{1}{2}$ by $5\frac{1}{2}$ cm.) has a line drawn parallel to the longer side and slightly left of the center, and contains three numbers written in a large hand:

2,30	6,15
2,30	

The explanation of these numbers is simple: $2{,}30^2 = 6{,}15{,}0$. The reverse is uninscribed.

Another tablet of exactly the same type is **YBC 10801**, which measures $7\frac{1}{2}$ by 6 cm. The line on the obverse is drawn to the left of the center, but this time parallel to the shorter side. The numbers, which are written large, read:

4,35	21,25
4,35	

As before, the numbers are explained by the fact that $4{,}35^2 = 21{,}0{,}25$. It is interesting to note that, as is usual in Old-Babylonian texts, the presence of the internal "zero" is not indicated in any way in the text. The reverse is again uninscribed.

For a more extensive text of a similar type, cf. MKT III p. 51.

f. Logarithms

Tablets which contain tables of exponents a^n, where n is an integer between 2 and 10, and a is one of the numbers 9, 16, 1,40, 3,45 (note that all of these are squares), are known.[96a] We now have an Old-Babylonian tablet which answers the question: to what power must a certain number a be raised in order to yield a given number? This problem is identical with finding the *logarithm* to the base a of a given number.

[96] Cf. Neugebauer [4].

[96a] Cf. MKT I pp. 77ff., Neugebauer [6], and Neugebauer, Vorlesungen, pp. 199–202.

One side of the text in question (**MLC 2078**) is destroyed except for slight traces; all edges are preserved. On the other side and on the left margin appears the following:

1 [1]15-e 2 íb-si$_8$
[2]30-e 4 íb-si$_8$
[3]45-e 8 íb-si$_8$
[4]1-e 16 íb-si$_8$
[5]*ga-mi-ru-um* níg (or: 4) *i du*(?) *uk* PI(?) . . . *ma*(?) . . . *i du*(?) *uk*(?)

2 [6]2-e 1 íb-si$_8$
[7]4-e 2 íb-si$_8$
[8]8-e 3 íb-si$_8$
[9]16-e 4 íb-si$_8$
[10]32-e 5 íb-si$_8$
[11]1,4-e 6 íb-si$_8$

Left Edge: 1,16[96b]-e 32 íb-si$_8$
1,30-e 1,4 íb-si$_8$

The meaning of the numbers in No. 1 is clearly

$$16^{0;15} = 2$$
$$16^{0;30} = 4$$
$$16^{0;45} = 8$$
$$16^{1} = 16,$$

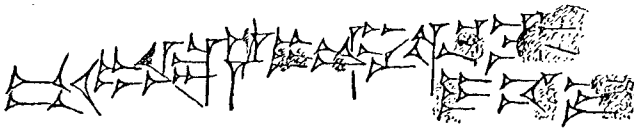
or, in other words,

$$0;15 = \log_{16} 2$$
$$0;30 = \log_{16} 4$$
$$0;45 = \log_{16} 8$$
$$1 = \log_{16} 16,$$

It is also evident that the two lines on the left edge are the direct continuation of this group, namely,

$16^{1;15} = 32$ $\quad$ or $\quad$ $1;15 = \log_{16} 32$
$16^{1;30} = 1,4$ $\quad$ $\quad$ $1;30 = \log_{16} 1,4.$

Line 5 (see the drawing below) may give some indica-



tion of the fact that the lines on the left edge are to be read after line 4, but we are unable to grasp what is said. It is, however, equally possible that line 5 may have served as an introductory heading to No. 2.

The second group of numbers, No. 2, means

$2 = 2^1$ $\quad$ $\quad$ $1 = \log_2 2$
$4 = 2^2$ $\quad$ $\quad$ $2 = \log_2 4$
$8 = 2^3$ $\quad$ or $\quad$ $3 = \log_2 8$
etc. $\quad$ $\quad$ etc.
$1,4 = 2^6$ $\quad$ $\quad$ $6 = \log_2 1,4.$

[96b] Sic, instead of 1,15.

The fact that the term íb-si$_8$ is used in both groups confirms the conclusion reached on the basis of the material published in MKT that íb-si$_8$ (or ba-si$_{(8)}$) not only means square root (or cube root) but is also used in connection with arithmetical operations in general where numbers are to be found which satisfy certain conditions[96c], e.g., in our case, $a^x = b$. The general character of the term is underlined in the present text by its use in two opposite directions: in No. 1 to indicate the number b, in No. 2 the logarithm x.

The new "logarithmic" tables are the logical supplement to the exponential tables mentioned at the beginning. Both exhibit a knowledge of the basic laws of operating with exponents. In a comparison with our concept of logarithm, the only missing element is the selection of a common base and the tabulation for constant intervals, which would be needed if the tables were to be used for practical computations in general. It is accordingly clear that the Old-Babylonian mathematicians were very close to an important discovery but failed to take the final, essential step. The starting-point for the whole problem is in all probability to be found in the computation of interest, which is dealt with in several previously published texts.[96d]

g. Varia, Fragments

Various tablets containing isolated numbers, obviously written down in the course of some calculation,[97] are preserved. One such note is made on CBS 3551:

7,36
7,36 íb(?)-si$_8$(?)
57,47,36

The last line should read 57,45,36, the square of 7,36. Many isolated numbers are written on CBS 7356. Numbers written in a disorderly manner within square fields are to be found on CBS 1215. CBS 7360 and CBS 7362 are very small fragments with a few number signs.

Another example, CBS 29.15.481, deals with areas, as is shown by the occurrence of uš "length" and the surface measure SAR. The following is a tentative transcription of the obverse:

40	2	20	uš	SI(?)
40	30			
12	SAR	9(?)	erim(?)	... 15,42
50		...	uš	40(?)
42	55		SI(?)	1,20
½	SAR	6⅓	...	26½ ....
30		7	....	
30		6	....	
⅓		....	....	

The last section of the badly preserved reverse reads:

15	50	1	50	7	20	24	10
	54	2	40	8	45	...	
		1	24	4	23	20	

The following are fragments of multiplication tables: CBS 7363 (50 or 48 or 45), CBS 7359 (45 or 44,26,40), CBS 7358 (40?), CBS 29.15.499 (9?).

Three small fragments in the Metropolitan Museum are from the Seleucid (or a slightly earlier) period. The first fragment seems to be part of a larger table of reciprocals although we were able to find satisfactory restorations for no more than two lines:

MM 86.11.408. No edge preserved.

1)	traces	8) [1,33,18,43,12	38,3]4,48,53,2[0]
2)	blank	9) [...(?)...	]9,34,11(?)
3)		10) [1,40	36]
4)	10	11) [1,49,13,36	32,57,32],20,37,30
5)	5,24	12) [...(?)...	]2,9,30
6)	37,3,13		
7)	6,4,41,14		

The following two fragments may even belong to astronomical texts:

MM 86.11.406. No edge preserved.

1) traces
2) ... 46(?) 20 ...
3) ... 3 10 30 9 20 ....
4) ... 3 55 .....
5) .. 32 54(?) ...
6) .... 32 ....

MM 86.11.407. No edge preserved.

1) ... 10(?) 50 ...
2) ... 10 28 ....
3) ... 10 1 6 ....
4) ..... 1 11(?) ...

[96c] Cf. Neugebauer, Vorlesungen, pp. 199ff. and Neugebauer [1] pp. 201ff., especially p. 203b.

[96d] See MKT III p. 83 under "Zinseszins."—It is worth noting that the words "30-e 4 íb-si$_8$", which appear in line 2 of our present text, also occur in Strassburg 366 obv. 4 (Neugebauer MKT I p. 257; Thureau-Dangin TMB p. 205). Unfortunately, although the problem in Strassburg 366 is perfectly preserved, the precise character of the question which is asked and the reasons for the calculations which are carried out remain obscure; it is quite possible that the subject is interest on money.

[97] Examples are given MKT I pp. 80ff.

CHAPTER III. PROBLEM-TEXTS

§ 1. Introduction

As was stated in Chapter I, our knowledge with regard to the exact date and place of origin of our texts is so inadequate that we cannot expect to be able to arrange our material according to such viewpoints. What we can do is merely to separate the large group of Old-Babylonian tablets (ca. 1900 to 1600 B.C.) from the Neo-Assyrian (ca. 700 B.C.) and Neo-Babylonian and Seleucid (last six centuries B.C.) texts. Within the Old-Babylonian group, which consists of problem-texts A to Ue, we have arranged the tablets, so far as possible, according to subject matter. Problem-texts V to Y, of a later date, neither allow nor require special internal classification.

The Old-Babylonian texts vary greatly as to type and content. Some tablets contain only one example, which gives all the details of the solution of the problem stated at the beginning of the text. We even have tablets which contain only a part of the working out of the solution of a problem (C and Uc). At the other extreme are tablets which state hundreds of problems in a very condensed form but give no answers (e.g., T and U). Between these two extremes lie all sorts of intermediate types: texts with two or more examples which are worked out in detail and which are well arranged according to the degree of mathematical complication, texts with many examples of quite diverse character arranged very carelessly (this type is represented by the texts BM 85194 and BM 85210, both published in MKT I), and texts which represent a smaller collection of coordinated problems.

Several of the texts which present large numbers of problems without giving answers bear colophons giving the tablet a serial number. This gave rise to the name "Series Texts" used in MKT for this whole group of tablets. We think it wise, however, to abandon this name because the new material makes it difficult to define the borders of this group. It is worth stressing that there is no evidence for canonical mathematical "series" of the type of the astrological series "*Enūma Anu Enlil*" or the lexicographical "ḪAR-ra/*ḫubullu*." The same tablet number occurs two or more times on tablets with different contents; problem-text T, for instance, is called "Tablet 10" although the same number is given to another tablet.[98] On the other hand, T is clearly related to U, but U is not given a serial number. Thus, it is fairly clear that the numbering of these texts implies nothing more than an arrangement of tablets of various groups by a scribe to keep them in order.

The relationship between texts which state problems and merely give the answers and texts which give the details of the solution is nicely illustrated by problem-texts G, H and J. Problem-text G states 31 problems in a well planned order; the writing down of the details of the solution of all these problems required three tablets, the first and last of which are preserved in H and J. It is clear that the same might be assumed for many tablets with single problems and corresponding collections of examples.

The first two texts published below stand somewhat between the table-texts and the problem-texts. The first (§ 2) tabulates the answers to a problem containing Pythagorean numbers (or Pythagorean triangles). It is the oldest preserved document in ancient number theory. The second text (§ 3) deals with the finding of a cube root in a special case.

The next group of texts (§ 4) deals with geometrical problems. Most of the texts have to do with triangles and trapezoids and their subdivision according to given conditions. Although these problems are sometimes accompanied by figures (usually very much out of scale) and although their terminology is geometrical, the whole treatment is strongly algebraic. Geometry is involved only insofar as it leads to the necessary relations between the given quantities; the solution, however, proceeds in a purely algebraic manner, and we find nowhere any certain traces of the type of geometrical argument which is so familiar to us from the Greek treatment of problems of this sort.

Problems involving solid geometry (prism and truncated pyramid) are dealt with in texts characterized by the word ki-lá, which probably refers to some sort of excavation (§ 5). The texts which we collect under the heading "irrigation" (§ 6) are of a similarly practical character. Most of these examples have to do with the volume of earth excavated in the digging or enlarging of canals. The mathematical aspect is usually very simple, but we learn from these texts a great deal about wages, the number of workers employed, and metrological relations.

The greatest amount of new information about practical questions is yielded by texts dealing with bricks (§ 7). Here we learn for the first time the dimensions of several standard types of brick and the metrological system used in counting bricks. It is clear that such information is not only of use for the

[98] Cf. the list given MKT I p. 385.

better understanding of similar mathematical texts but will also influence our interpretation of economic documents and archeological data.

In § 8 we reach the most typical part of Babylonian mathematics, namely, problems of a predominantly algebraic character. The first three texts in this group contain short examples of various types: a simple inheritance problem (Q No. 2), linear equations resulting from the determination of the weight of a stone from certain given conditions (R), and quadratic equations for the sides of a rectangle (S). The last text is placed here because its geometrical element consists of nothing more than the fact that the area of a rectangle is given by the product xy of its sides. But this text leads to the problem of determining x and y from a given product xy and a sum $x + y$ or a given difference $x - y$. This is the normal form for quadratic equations, to which the more involved forms must be reduced. The great weight attributed to the solution of quadratic equations in Babylonian mathematics is evident from all our material. This is also shown by texts T and U, which contain 247 and 177 problems, respectively, all leading to quadratic equations. These texts are systematic compendia of exercises arranged according to a consistent scheme. The common principle which must be used in order to solve all these problems is similar to our use of an algebraic formula in which we are allowed to substitute special values for the letters a, b, c etc. which occur as coefficients.

An entirely new type of "mathematical" text is represented by Ud and Ue (§ 9). The tablets contain lists of numbers (to which are added short explanations) which occur in mathematical texts. We find here, e.g., coefficients referring to bricks, work assignment, etc.—in short, just those parameters which must be known by anyone dealing with various types of mathematical texts. For the first time we have texts which have the character of pages from a general handbook.

The later periods of Mesopotamian history are very meagrely represented in our material (§ 10). Only two problem-texts from the Seleucid period were published in MKT; we now add another text from the same period (Y) and two somewhat doubtful Late-Assyrian fragments (V and W). Between these late texts and the Old-Babylonian material there still remains a gap of about one thousand years, to which not a single problem-text can be assigned with certainty. It is worth noticing, however, that there is no basic deviation in character between the latest material and the Old-Babylonian.

§ 2. Pythagorean Numbers

A. Plimpton 322

(Photograph: Plate 25)

Obverse	I	II	III	IV
1	*[ta-k]i-il-ti ṣi-li-ip-tim*	íb-si$_8$ sag	íb-si$_8$ *ṣi-li-ip-tim*	mu-bi-im
2	*[ša in-]na-as-sà-ḫu-ú-[m]a* sag *i-. . . .-ú*			
3	[1,59],15	1,59	2,49	ki-1
4	[1,56,56],58,14,50,6[105],15	56,7	3,12,1[108]	ki-2
5	[1,55,7],41,15,33,45	1,16,41	1,50,49	ki-3
6	[1],5[3,1]0,29,32,52,16	3,31,49	5,9,1	ki-4
7	[1],48,54,1,40	1,5	1,37	ki[-5]
8	[1],47,6,41,40	5,19	8,1	[ki-6]
9	[1],43,11,56,28,26,40	38,11	59,1	ki-7
10	[1],41,33,59,3,45	13,19	20,49	ki-8
11	[1],38,33,36,36	9,1[106]	12,49	ki-9
12	[1],35,10,2,28,27,24,26,40	1,22,41	2,16,1	ki-10
13	[1],33,45	45	1,15	ki-11
14	[1],29,21,54,2,15	27,59	48,49	ki-12
15	[1],27,3,45	7,12,1[107]	4,49	ki-13
16	[1],25,48,51,35,6,40	29,31	53,49	ki-14
17	[1],23,13,46,4[0]	56	53[109]	ki[-15]

[105] 50,6 written like 56.
[106] 9,1 error for 8,1.
[107] 7,12,1 (the square of 2,41) error for 2,41.
[108] 3,12,1 error for 1,20,25.
[109] 53 error for 1,46 (i.e., 2·53); cf. below pp. 40 and 41.

Commentary

a. Description of the Tablet

In its present state, the tablet represents the right-hand part of a larger text. The presence of modern glue, until the recent baking of the tablet, on the left (broken) edge shows that the missing part must have been lost after the excavation of the tablet. The size of the preserved part, $4\frac{7}{8}$ by $3\frac{3}{8}$ in. (12.7 by 8.8 cm), would make it unlikely that much more than half (or even less) of the existing part is missing. The reverse is uninscribed.

The script is clearly Old-Babylonian, i.e., it falls in the period between 1900 and 1600 B.C. The sign for 9 consists of three superimposed rows of three vertical wedges each. Zeros are not indicated by a special sign, but a blank space occurs in lines 3 and 15 where zero is called for; on the other hand, lines 7, 8, 10, etc., show that a blank space does not necessarily indicate zero.

Nothing is known concerning the provenance of the tablet. [The sixth plate in Mendelsohn, Catalogue, which was published months after this section was written, contains a photograph of the tablet before it was baked. The photo which we publish on Plate 22 is of the baked and cleaned tablet. The content of the tablet is characterized as "commercial account" by Mendelsohn.]

b. Content

The text deals with "Pythagorean triangles": right triangles whose sides are integers. Let l denote the longer, b the shorter leg of a right triangle, d its hypotenuse; then l, b and d are integers which fulfill the relation

$$l^2 + b^2 = d^2. \tag{1}$$

The values of d and b for 15 such Pythagorean triangles are given in the second and third preserved columns of our text. One might assume that the missing part contained the corresponding values of l. The first preserved column gives the ratios of d^2 to l^2. Although the values of d and b vary in a very irregular manner, the ratios $\frac{d^2}{l^2}$ decrease almost linearly (cf. fig. 2). Because the difference from line to line is very small (average 0;2,34), this is virtually equivalent to saying that $\frac{d}{l}$ decreases almost linearly (average difference 0;0,59,17,$\cdots$ i.e., almost 0;1), and we shall see that this is indeed the proper formulation of the problem. Formulating the problem with respect to the triangles, we can say that we start out with almost half a square (because the value of $b:l$ which corresponds to the first line [line 3] is 0;59,30) and gradually diminish the angle between l and d step by step, the lowest value being almost exactly 31°. It must, however, be kept in mind that the actual size of these triangles varies considerably owing to the fact that all sides are integral solutions of (1) and not approximations.

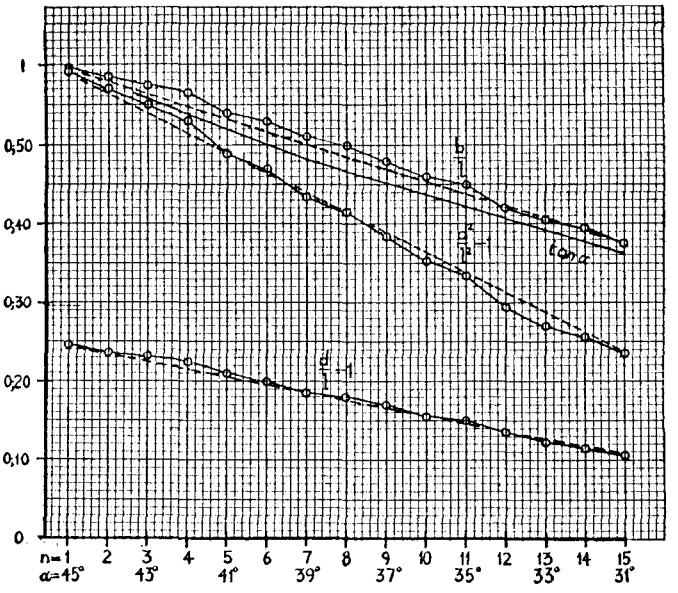


FIG. 2.

The numbers 1, 2, $\cdots$, 15 in the last column have no exact relationship to the preceding numbers but merely indicate the number of the steps, like the units on the abscissa in our diagram. The preceding ki gives them the character of ordinal numbers.[110]

c. The Headings

Each column contains a heading describing the contents. The fourth column has mu-bi-im, "its name," a translation which goes well with the interpretation of the numbers 1 to 15 made above.

The columns for b and d are headed íb-si$_8$ sag, "íb-si$_8$ of the width," and íb-si$_8$ *ṣi-li-ip-tim*, "íb-si$_8$ of the diagonal," respectively. The íb-si$_8$ is difficult to translate because of its unprecise character, or, more accurately stated, because of the lack of a corresponding expression in our terminology. The most common meaning is "side of a square" or "square root."[111] More generally, however, íb-si$_8$ seems to indicate the number which is the result of some operation or the number solving a certain problem.[112] In the present case, íb-si$_8$ sag (or *ṣi-li-ip-tim*) means something like "solving number of the width (or the diagonal)."

Two lines are given at the top of the first column but both are unfortunately damaged at the beginning.

[110] This use of ki is also known from other mathematical texts; cf., e.g., MKT I p. 248 and the Vocabulary below, s.v.

[111] See the instances cited in the Vocabulary under si$_8$.

[112] Cf. above p. 36 and note 96c.

The restoration given below seems probable although we are unable to read the last word of the second line.

[ta-k]i-il-ti ṣi-li-ip-tim
[ša in-]na-as-sà-ḫu-ú-[m]a sag *i-· · · ·-ú*

The translation causes serious difficulties. The most plausible rendering seems to be: "The *takiltum* of the diagonal which has been subtracted such that the width. . . ." The reading *[ta-k]i-il-ti* is, however, not certain; in addition, we do not know how it is to be translated here.[113] The mention of a subtraction and of the width in the second line could indicate that l^2 in $\frac{d^2}{l^2}$ is replaced by $d^2 - b^2$. This would be natural if l were given in the missing first column and if d and b were considered the unknown quantities to be determined which satisfy (1). We are, however, unable to give a sensible translation of this passage leading to $\frac{d^2}{d^2 - b^2}$.

d. Method of Solution

We turn now to the question of great historical interest: How were the mathematicians of the Old-Babylonian period able not only to solve the Pythagorean equation (1) in integers but to adapt the solutions to the further condition that the proportion $\frac{d}{l}$ decrease from step to step by a number deviating very little from one-sixtieth?

In order to answer this question, we must first compare the solutions for l, b, and d given in the tablet. These are listed in the following table with the correction of the four errors in lines 4, 11, 15, and 17 mentioned in the foot-notes to the transcription:

Line	l	b	d
3	2, 0	1,59	2,49
4	57,36	56, 7	1,20,25
5	1,20, 0	1,16,41	1,50,49
6	3,45, 0	3,31,49	5, 9, 1
7	1,12	1, 5	1,37
8	6, 0	5,19	8, 1
9	45, 0	38,11	59, 1
10	16, 0	13,19	20,49
11	10, 0	8, 1	12,49
12	1,48, 0	1,22,41	2,16, 1
13	1, 0	45	1,15
14	40, 0	27,59	48,49
15	4, 0	2,41	4,49
16	45, 0	29,31	53,49
17	1,30	56	1,46

[113] See the discussion p. 130.

Before proceeding, it is necessary to note in the first place that line 13 gives the 15-fold values of the triangle with the sides $l = 4$, $b = 3$, $d = 5$; and secondly, that the last line contains the common factor 2. All the other solutions are relatively prime.

The most important feature in this table is the obvious difference in character of the numbers l on the one hand and the b's and d's on the other: the b's and d's are "complicated" numbers, but the l's are very "simple." This offhand impression can be translated into precise terms by using the well known theorem[114] that all relatively prime Pythagorean numbers are contained exactly once in the set of numbers

$$(2) \qquad l = 2pq \qquad b = p^2 - q^2 \qquad d = p^2 + q^2$$

where p and q are relatively prime integers, both not being simultaneously odd and $p > q$. If one calculates the numbers p and q which bring our numbers l, b, and d to the form (2), one will find that the p's as well as the q's are "regular numbers." In other words, the p's and q's can be characterized by three numbers α, β, and γ, the exponents of 2, 3, and 5, respectively. If we write[115]

$$2^\alpha 3^\beta 5^\gamma = (\alpha, \beta, \gamma),$$

we then obtain the following list of numbers p and q which satisfy (2):[116]

Line	p	q
3	12 = (2,1,0)	5 = (0,0,1)
4	1, 4 = (6,0,0)	27 = (0,3,0)
5	1,15 = (0,1,2)	32 = (5,0,0)
6	2, 5 = (0,0,3)	54 = (1,3,0)
7	9 = (0,2,0)	4 = (2,0,0)
8	20 = (2,0,1)	9 = (0,2,0)
9	54 = (1,3,0)	25 = (0,0,2)
10	32 = (5,0,0)	15 = (0,1,1)
11	25 = (0,0,2)	12 = (2,1,0)
12	1,21 = (0,4,0)	40 = (3,0,1)
13	2 = (1,0,0)	1 = (0,0,0)
14	48 = (4,1,0)	25 = (0,0,2)
15	15 = (0,1,1)	8 = (3,0,0)
16	50 = (1,0,2)	27 = (0,3,0)
17	9 = (0,2,0)	5 = (0,0,1)

The sense in which the numbers $l = 2pq$ are "simple" is now made clear by this list: they are numbers of the form (α, β, γ), i.e., so-called "regular

[114] Kronecker, Zahlentheorie p. 31.

[115] As above, p. 15.

[116] We include here the values from lines 13 and 17 although a common factor 15 of l, b, and d occurs in line 13, and p and q are both odd in line 17.

numbers" or numbers whose reciprocals are *finite* sexagesimal fractions.[117]

This latter quality of the p's and q's also yields the answer to the question how the numbers of our list were found which not only solve equation (1) but also furnish within narrow limits given proportions $\frac{d}{l}$. Using (2), we find for this proportion

$$\frac{d}{l} = \tfrac{1}{2}(p\cdot\bar{q} + q\cdot\bar{p}) \tag{3}$$

where $\bar{p}$ and $\bar{q}$ represent the reciprocals of p and q, respectively. In other words, our problem *requires* that p and q be regular numbers in order to obtain expressions (3) with finite sexagesimal fractions.

We can say more. With the single exception of $2,5 = 5^3$, all the numbers p and q of our list are contained in the group of regular numbers which constitute the "reciprocal tables."[118] As we have already mentioned,[119] the complete system of "multiplication tables" (represented in the "combined multiplication tables"[120]) is not a system of all products $a\cdot b$ ($1 \leqq a < 60$, $1 \leqq b < 60$) but only of products $a\bar{b}$ where $\bar{b}$ is the reciprocal of a regular number included in the reciprocal table.

Our final result, then, is that our tablet was calculated by selecting numbers $p\bar{q}$ and $q\bar{p}$ from combined multiplication tables such that (3) has a value as near as possible to the required values of $\frac{d}{l}$; Pythagorean numbers were then formed with these values of p and q according to (2).

Several remarks remain to be made. The exception of 2,5 mentioned above is not to be considered serious because we know that the usual reciprocal tables were occasionally enlarged in this very direction.[121] Secondly, instead of using (3), one can also produce Pythagorean numbers by using one parameter α and its reciprocal $\bar{\alpha}$ where $\alpha = \frac{p}{q}$.[122] But a comparison of the following four lines shows immediately that neither α nor $\bar{\alpha}$ can have been the point of departure, but only the simple numbers p and q; hence formula (3) must necessarily have been used. Finally, the values $d = 53$ and $b = 56$ in the last line are obviously incorrect because this would imply $d < b$. The correction adopted in the preceding discussion assumes the triplet $d = 1,46$ (twice the value given in the text), $b = 56$, and $l = 1,30$. If, however, we keep $d = 53$ but assume $b = 28$ (half the value given in the text), we would also get Pythagorean numbers ($l = 45$). In this case, however, b, and not l, should be represented as $2pq$ and $p = 2$, $q = 7$ would be the corresponding numbers. Because $\frac{1,46}{1,30} = \frac{53}{45}$, the value of $\frac{d}{l}$ (and thus of $\frac{d^2}{l^2}$) is the same in both cases and in agreement with the value given in Col. I. Only the first triplet, however, fits in with the method followed in the preceding lines.

p	q	α	$\bar{\alpha}$
12	5	2;24	0;25
1,4	27	2;22,13,20	0;25,18,45
1,15	32	2;20,37,30	0;25,36
2,5	54	2;18,53,20	0;25,55,12

[117] Cf., e.g., Neugebauer, Vorlesungen pp. 6 and 12ff.; and above p. 15.
[118] Cf. p. 11.
[119] P. 11.
[120] Pp. 24ff.
[121] Cf. pp. 13ff. above and MKT I pp. 23f. (Nos. 3 and 4).
[122] Cf. Dickson, History II, pp. 163ff.

e. Historical Consequences

The final remarks about the character of Babylonian mathematics in MKT III contain the sentence:[123] "Man wird also erwarten können, dass noch eine Art elementarer Zahlentheorie erkennbar wird—etwa so, dass das "pythagoreisch" der älteren historischen Schule besser "babylonisch" wird heissen dürfen." This is fully confirmed by the text discussed here. We now have a text of purely number theoretical character, treating a problem organically developed from other problems already well known and solved by using exactly those tools the development of which is so characteristic for Babylonian numerical methods.

We now see that Babylonian number theory was acquainted with rules like (2) to produce Pythagorean numbers, i.e., a theorem like Euclid X 29 lemma 1.

There can be little doubt that the Pythagorean numbers did not remain the only problem treated by this part of Babylonian mathematics. We have an explicit hint in this direction from the extant material itself: tables giving the powers c^n for exponents $n = 1, 2, \ldots, 10$ for the bases $c = 9$, 16, 1,40 and 3,45.[124] All these are regular numbers, and it would be only natural to extend both problems and methods as described above to combinations of other numbers and different exponents. The study of sequences Σn, Σn^2 etc.[125] points in the same direction. Details can be disclosed only by the discovery of new texts, but their general direction seems evident.

In summary, our text gives the final link which connects the different parts of Old-Babylonian mathematics by the investigation of the fundamental laws of the numbers themselves.

[123] MKT III p. 80.
[124] MKT I pp. 77ff. and p. 35 above.
[125] MKT I pp. 102f., 497f., and MKT III pp. 13f.

§ 3. Cube Root

Aa. YBC 6295

(Photograph: Plate 46; copy: Plate 22)

Transcription

Obverse

1[*ma-*]*ak-ṣa-ru-um ša* ba-si
2ba-si 3,22,30 en-nam
3*aš-šum* 3,22,30 ba-si *la id-di-nu-kum*
47,30 *ša* ba-si *i-na-di-nu-kum*
5*ša-pa-al* 3,22,30 gar-ra-ma
63,22,30 7,30
7ba-si 7,30 en-nam 30
8igi 7,30 *pu-ṭur-ma* 8
98 *a-na* <3,>22,30 ÍL 27
10ba-si 27 en-nam 3
113 ba-si *a-na* 30 ba-si *ša-ni-im*
12ÍL 1,30
13ba-si 3,22,30-e 1,30

Reverse

(Top and most of left edge broken; remainder uninscribed except for *a-na* ^{d}*sin*(EN-ZU) written about two-thirds of the way down, starting at about halfway across the width.)

Translation

Obverse

1[*M*]*akṣarum*[125a] of the cube root.
2What is the cube root of 3,22,30?
3Since, (as regards) 3,22,30, they did not give you the cube root,
4–5put (down) 7,30,0, the cube root of which they do give you, below 3,22,30:
63,22,30 7,30,0[125b].
7What is the cube root of 7,30,0? (The answer is) 30.
8Take the reciprocal of 7,30,0, and (the result is) 0;0,0,8.
9Multiply 0;0,0,8 by 3,22,30, (and the result is) 27.
10What is the cube root of 27? (The answer is) 3.
11–12Multiply 3, the cube root, by 30, the other cube root,(and the result is) 1,30.
13The cube root of 3,22,30 is 1,30.

[125a] See p. 55, note 152 for an inconclusive discussion of this word. The meaning here is possibly something like "model example."

[125b] The use of *šapal*, "below," in line 5 to indicate the relative position of the numbers in line 6 is apparently further proof that Old-Babylonian tablets were written and meant to be read turned 90° clockwise with respect to the position conventionally given by Assyriologists. For evidence from the figures in mathematical texts, cf. below, p. 49, note 135d and Neugebauer, Vorlesungen, pp. 34 and 176.

Reverse

(See transcription.)

Commentary

The cube root of a given number b = 3,22,30 is to be found. The solution of the problem presented in the text can be followed with ease. The text proceeds toward the solution by introducing an auxiliary number a = 7,30,0, of which the cube root $\sqrt[3]{7,30,0}$ = 30 is said to be "given" (presumably in a table-text). The cube root of b is then calculated by

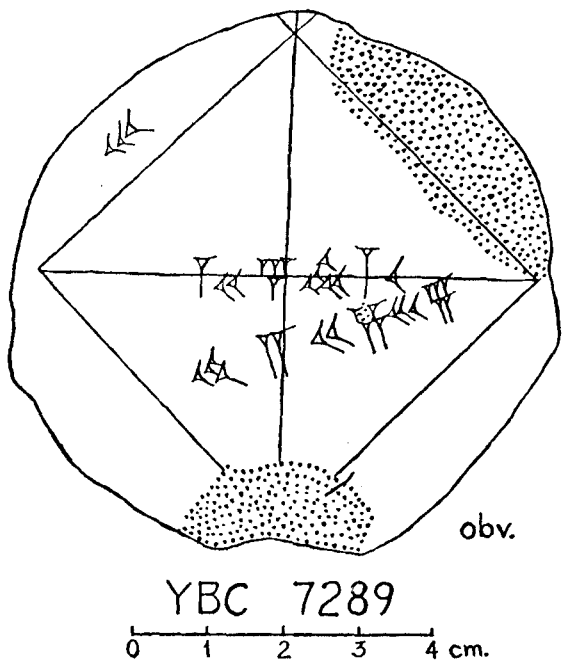
$$\sqrt[3]{b} = \sqrt[3]{a} \cdot \sqrt[3]{\frac{b}{a}} = 1,30.$$

For this method of solution, it is necessary that the auxiliary number a satisfy the following three conditions: (1) that a be the cube of a rational number; (2) that a be a regular number; (3) that $\sqrt[3]{\frac{b}{a}}$ can be found.

§ 4. Geometrical Problems

Simple Problems

a. Diagonal of a Square



YBC 7289, like the four texts which follow, is apparently Old-Babylonian. The obverse gives the fol-

lowing figure:[125c]

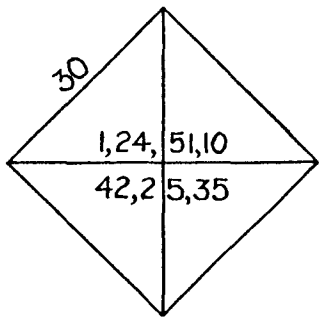


FIG. 3.

The number 30 indicates the side a of the square, and 1,24,51,10 means

(1) $$1;24,51,10 \approx \sqrt{2};$$

we therefore find

$$d = a\sqrt{2} = 42;25,35$$

for the diagonal. The value (1) for $\sqrt{2}$ is very good, as can be seen from

$$1;24,51,10^2 = 1;59,59,59,38,1,40.$$

Only the approximation $\sqrt{2} \approx 1;25$ occurred in previously published material (from the Seleucid period).[125d] Our new value, however, now also occurs in an Old-Babylonian list of coefficients, published p. 136 below, where we find in line 10 the entry

1,24,51,10 Diagonal, square root.

The question naturally arises how the value (1) for $\sqrt{2}$ was obtained. The following might give the answer, but it is impossible to furnish direct proof that precisely this way was followed. Two factors speak in favor of this explanation: first, that the process indicated leads to the very number found in (1); and, secondly, that the same procedure is attested in another text in finding the approximate value of $\sqrt{28,20}$.[125e]

The procedure in question consists in the alternating approximation of $\sqrt{a}$ by arithmetic and harmonic means of previously found approximations. Let α_1 be any approximation of $\sqrt{a}$ such that $\alpha_1 > \sqrt{a}$. Then $\beta_1 = \frac{a}{\alpha_1}$ is also an approximation of $\sqrt{a}$ but deviates from the true value in the opposite direction because it follows from $\alpha_1 > \sqrt{a}$ that $\beta_1 = \frac{a}{\alpha_1} < \sqrt{a}$. We now derive a new pair of approximations, α_2 and β_2, by

$$\alpha_2 = \frac{\alpha_1 + \beta_1}{2} \qquad \beta_2 = \frac{a}{\alpha_2}$$

and continue this process by computing

$$\alpha_3 = \frac{\alpha_2 + \beta_2}{2} \qquad \beta_3 = \frac{a}{\alpha_3}$$

etc.[125f] It is evident that all the α's are greater than $\sqrt{a}$, all β's less than $\sqrt{a}$, but that each step diminishes the difference between corresponding approximations. We apply this method to $\sqrt{2}$ by starting with $\alpha_1 = \frac{3}{2}$ as the first rough approximation (α_1^2 would be 2;15). Then we obtain for the first pair

$$\alpha_1 = 1;30 \qquad \beta_1 = \frac{2}{1;30} = 1;20.$$

The next step leads to

$$\alpha_2 = \tfrac{1}{2}(1;30 + 1;20) = 1;25$$

$$\beta_2 = \frac{2}{1;25} = 1;24,42,21,\ldots.$$

Here we have already reached the above mentioned value $\sqrt{2} \approx 1;25$. The very next step leads to

$$\alpha_3 = \tfrac{1}{2}(1;25 + 1;24,42,21,\ldots.) = 1;24,51,10,\ldots.$$

the value given in (1).[125g] The fact that both values for $\sqrt{2}$ found in our texts are links of the same chain seems to be a rather strong argument in support of our explanation.[125h]

[125c] The reverse contains the figure of a rectangle with an inscribed diagonal, but the numbers are too badly preserved to warrant a restoration of the dimensions.

[125d] MKT I p. 104; cf. also MKT I p. 140.

[125e] VAT 6598; cf. MKT I pp. 279ff. and Neugebauer, Vorlesungen pp. 33ff.

[125f] From $\alpha_2 = \frac{1}{2}(\alpha_1 + \beta_1)$ and $\beta_2 = \frac{a}{\alpha_2}$ it follows that $\beta_2 = \frac{2a}{\alpha_1 + \beta_1} = \frac{2\alpha_1\beta_1}{\alpha_1 + \beta_1}$. This expression is known as the "harmonic mean" of α_1 and β_1.

[125g] More accurately: $\alpha_3 = 1;24,51,10,35,\ldots.$

[125h] It should be remarked that the expansion of $\sqrt{2}$ into a continued fraction also leads to (1), but not until the seventh step.

b. Trapezoids

The figure given on the obverse of **YBC 7290**[125i]

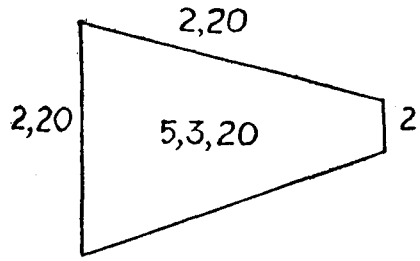


FIG. 4.

indicates that the area is obtained by

$$5;3,20 = 2;20 \cdot \frac{2;20 + 2}{2}.$$

On the reverse is given a trapezoid without inscribed numbers.

YBC 11126 is uninscribed on the reverse. The obverse gives the figure of a trapezoid with numbers.

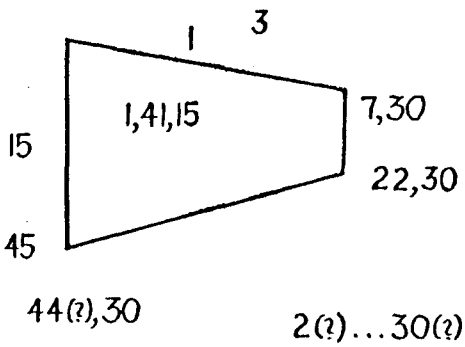


FIG. 5.

One set of these numbers satisfies the following relation

$$1,41,15 = 3,0 \cdot \frac{45 + 22;30}{2}$$

as expected for the area and sides. As for the remaining numbers, it is obvious that

$$45 = 3 \cdot 15 \qquad 22;30 = 3 \cdot 7;30 \qquad 3,0 = 3 \cdot 1,0$$

but the meaning of the coefficient 3 as well as the number 44(?),30 is not clear.

c. Circle

YBC 7302[125j] gives the figure of a circle whose circumference $c = 3$. We therefore have $c^2 = 9$ and for the

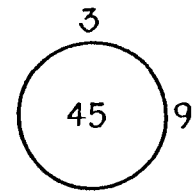


FIG. 6.

area

$$a = \frac{c^2}{4\pi} \approx \frac{c^2}{12} = 0;5 \cdot 9 = 0;45$$

using the value $\pi \approx 3$.[125k]

Analogously, we find in **YBC 11120**[125l]

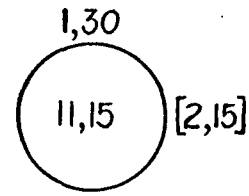


FIG. 7.

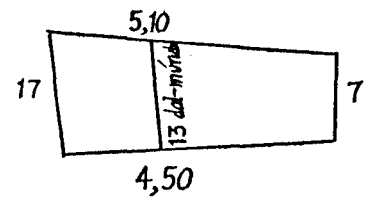
$$c = 1;30 \qquad c^2 = 2;15 \qquad a = 0;5 \cdot 2;15 = 0;11,15.$$

B and C. YBC 4675 and YBC 9852

Transcription of B (YBC 4675)

(Photograph: Plate 26; copy: Plate 1)

Obverse



1 *šum-ma* a(?)-šà(?)[126] uš uš *i* kú uš-1-e 5,10 uš-2-e 4,50
2 sag-an-ta 17 sag-ki-ta 7 a-šà-bi 2(bùr)^{iku}
3 1(bùr)^{iku}-ta-àm a-šà-*lam a-na ši-na zu-ú-uz*[127] *ta-al-li qá-ab-lu-ú ki ma-ṣi*

[125i] The tablet measures 7 by 8 cm.

[125j] The shape of the tablet is roughly circular; diameter 8 cm.; the reverse is uninscribed.

[125k] Cf. above p. 9.

[125l] Width 8 cm.; length 6.5 cm. The figure appears on the reverse; the obverse is uninscribed.

[126] The reading of "a" in a(?)-šà(?) is doubtful because of a horizontal wedge and the low position of the lower wedge at the end; šà is suspicious because of the uneven arrangement of the diagonal wedges (contrast, e.g., line 20) and perhaps a fourth diagonal wedge above. The reading of the signs which follow, uš uš *i*, is based on a final cleaning of this line by Dr. A. Goetze after our copy of the text was prepared.

[127] Or should one emend to <*a->zu-ú-uz* and translate "I divided"?

4uš-gíd-da *ù* uš-lugúd-da *ki ma-ṣi lu-uš-ku-un-ma*
51(bùr)iku *lu-ú sà-ni-iq ù a-na* 1(bùr)iku *ša-ni-i-im*
6*ki ma-ṣi* uš-gíd-da *ù ki ma-ṣi* uš-lugúd-da *lu-uš-ku-un-ma*
71(bùr)iku *lu-ú sà-ni-iq* uš-ḫá *ga-me-ru-ú-tim*
8*ki-la-a-al-le-e-en ta-ka-mar-ma ba-a-[ši-n]a te-ḫe-pe-e-ma*
95 *i-il-li-a-kum* igi 5 *ša i-li-a-ku[m t]a-pa-ṭa-ar-ma*
10*a-na* sag-an-na *ša* 10 *e-li* sag-ki-ta *i-te-ru*
11*a-na* 10 *ṷa-at-ri-im ta-na-aš-ši-ma* 2 *i-na-an-di-kum*
12*ta-as-sà-ḫa-ar* 17 sag-an-na *tu-uš-ta-ak-ka-al-ma*
1334,49 *i-il-li-a-kum i-na li-ib-bi* 4,49
1422 *ta-ḫa-ar-ra-aṣ-ma* 2,49 *a-ḫe-er-[t]um*
15íb-si8-*šu te-le-qé-e-ma*
161313 *ta-al-lum qá-ab-lu-ú-um i-il-li-a-kum*
171313 *ta-al-lam qá-ab-li-a-am ša i-li-a-[kum]*
18*ù* 17 sag-an-na *ta-ka-mar-ma ba-a-[ši-na te-ḫe-pe]-e-ma*
191915 *i-il-li-a-ak-kum* igi 15 *ta-p[a-ṭa-ar-ma]*
20*a-na* 1(bùr)iku a-šà-*im ta-na-[aš-ši-m]a*

Reverse

12 *i-na-an-di-kum* 2 *ša i-li-a-k[um]*
2*a-na* 2 *a-ra-ka-re-e-em ta-na-aš-ši-m[a]*
34 *i-il-li-a-kum* 4 *ša i-li-a-k[um]*
4*a-na* 2 UŠ *tu-uṣ-ṣa-ab-ma* 2,4 uš-gíd-d[a]
54 *i-na* 2 UŠ ki-2 *ta-ḫa-ar-ra-aṣ-ma*
61,56 uš-lugúd-da *te-ep-pe-eš-ma* 1(bùr)iku *sà-ni-iq*
7*ta-as-sà-ḫa-ar* 13 *ta-al-lam qá-ab-li-a-am*
8*ša i-li-a-kum ù* 7 sag-ki-ta <*ta-*>*ka-mar*
9*ba-a-ši-na te-ḫe-pe-e-ma* 10 *i-il-li-a-ak-kum*
10igi 10 *ta-pa-ṭa-ar-ma a-na* 1(bùr)iku a-šà-*im ta-na-aš-ši-ma*
113 UŠ *i-il-li-a-kum* 3 UŠ *ša i-li-a-ak-kum*
12*a-na* 2 *a-ra-ka-re-e-em ta-na-aš-ši-ma*
136 *i-il-li-a-ak-kum* 6 *a-na* 3 UŠ *tu-uṣ-ṣa-ab-ma*
143,6 uš-gíd-da 6 *i-na* 3 UŠ *ta-ḫa-ar-ra-aṣ-ma*
152,54 uš-lugúd-da *tu-uš-ta-ak-ka-al-ma*
161(bùr)iku *sà-ni-iq*

C (YBC 9852) is a duplicate of B (YBC 4675) reverse 7–16. Except for the last two lines, the distribution of the wording in the two texts is the same, line for line.

Transcription of C (YBC 9852)

(Photograph: Plate 29; copy: Plate 1)

Obverse

1*ta-as-sà-ḫa-ar-ma* 13 *ta-[al-lam qá-ab-li-a-am]*
2*ša i-li-a-kum ù* 7 sa[g-ki-ta *ta-ka-mar]*
3*ba-a-ṣi-na te-ḫe-pe-e-ma* 10 *[i-il-li-a-ak-kum]*
4igi 10 *ta-pa-ṭa-ar-ma a-na* 1(bùr)[iku a-šà-*im ta-na-aš-ši-ma]*
53 *i-il-li-a-ak-kum* 3 *ša i-[li-a-ak-kum]*
6*a-na* 2 *a-ra-ka-re-e-em ta-na-aš-[ši-ma]*
76 *i-il-li-a-kum* 6 *a-na* 3 *t[u-uṣ-ṣa-ab-ma]*
83,6 uš-gíd-da 6 *i-na* 3 *ta-ḫa-a[r-ra-aṣ-ma]*
92,54 uš-lugúd-da

Reverse

1*tu-uš-ta-ak-ka-* <*al-*> *ma* 1(bùr)iku *sà-ni-i[q]*

(Remainder of reverse uninscribed.)

Translation of B (YBC 4675)

Obverse

(For the figure see the transcription. Dal-múrub means "middle dividing-line.")

1If ..... The first length is 5,10, the second length 4,50,
2the upper width 17, the lower width 7, its area 2 bùr;
3divide[127] the area in two, each (part) 1 bùr. How large is my middle dividing-line?
4How much should I set the longer length and the shorter length so that
51 bùr should border (on one side of the dividing-line), and for the second 1 bùr
6how much should I set the longer length and how much the shorter length so that
7–81 bùr should border (on the other side)? Both the complete lengths you shall add together, then you shall halve [the]m, and
95,0 will result for you. You shall take the reciprocal of 5,0 which resulted for you, and then
10–11you shall multiply (the result) with the excess 10 by which the upper width exceeded the lower width, and you will get 2,0.[128]
12You shall turn about; you shall square 17, the upper width, and
134,49 will result for you. From 4,49
14you shall subtract 2,0, and (as for) the remaining 2,49,
15you shall take its square root, and
1613, the middle dividing line, will result for you.
17–18You shall add 13, the middle dividing-line which resulted for you, and 17, the upper width, then you shall hal[ve them], and
191915 will result for you. You shall [take] the reciprocal of 15, [and]
20you shall mult[iply (it)] by 1 bùr, the area, and

[128] Our translation of lines 10–11 deliberately departs from the awkward wording of the text: "to (= as for?) the upper width which exceeded the lower width by 10, you shall multiply (the result) with the excess 10, and you will get 2,0."

Reverse

[1]you will get 2,0. The 2,0 which resulted for you
[2]you shall multiply by 0;2, the *arakarūm*[129], and
[3]4 will result for you. The 4 which resulted for you
[4]you shall add to 2 UŠ, and the (resulting) 2,4 is the longer length.
[5]You shall subtract 4 from the second 2 UŠ, and
[6]the (resulting) 1,56 is the shorter length. You shall perform the (required) procedure, and the (resulting) 1 bùr borders (on the one side).
[7]You shall turn about; 13, the middle dividing-line
[8]which resulted for you, and 7, the lower width, you shall add together.
[9]You shall halve them, and 10 will result for you;
[10]you shall take the reciprocal of 10, and you shall multiply (it) by 1 bùr, the area, and
[11]3 UŠ will result for you. The 3 UŠ which resulted for you
[12]you shall multiply by 0;2, the *arakarūm*[129], and
[13]6 will result for you. You shall add 6 to 3 UŠ, and
[14]the (resulting) 3,6 is the longer length. You shall subtract 6 from 3 UŠ, and
[15]the (resulting) 2,54 is the shorter length. You shall make the (required) multiplication, and
[16]the (resulting) 1 bùr borders on the other side.

Commentary

a. Mathematical Commentary

The problem involves a trapezoid of given dimensions b_1, b_2, l_1, l_2 and area A to be divided into two parts with equal area (cf. the figure given in the transcription and fig. 8). The length d of the bisecting

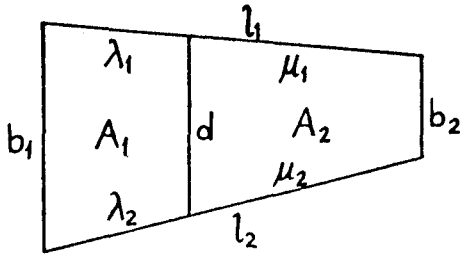


FIG. 8.

line and the resulting partial lengths of l_1 and l_2 are to be calculated. From the procedure followed by the text, it follows that b_1, b_2 and d are assumed to be parallel.

The given numerical values are

$$(1)\quad \begin{matrix} b_1 = 17\,\text{GAR} & l_1 = 5{,}10\,\text{GAR} \\ b_2 = 7\,\text{GAR} & l_2 = 4{,}50\,\text{GAR} \end{matrix} \qquad A_1 = A_2 = 30{,}0\,\text{GAR}^2.$$

[129] Cf. below p. 48.

In the text, A_1 and A_2 are expressed as 1 bùr each, which is equivalent[130] to 30,0 GAR²; in the case of the other four values, GAR is not given by the text but must be supplied. Because $l_1 = 5{,}10$ is about 18 times the width $b_1 = 17$, the trapezoid is a long strip. The assumption that one should read $l_1 = 5;10$ instead of 5,10 and correspondingly $l_2 = 4;50$ is excluded by the fact that we would have

$$l_1 + b_2 + l_2 = 5;10 + 7 + 4;50 = 17 = b_1,$$

an equation which would indicate the degeneration of the trapezoid into a single line of length b_1 (fig. 9).

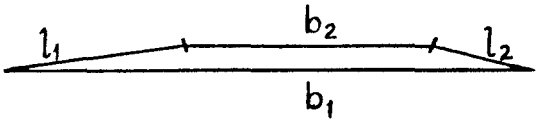


FIG. 9.

We must therefore assume the stretched form of the field as expressed by the numbers (1). This also follows from the units which are expressly indicated for the areas[131] and the terminology.[132] The scribe scarcely realized, however, that the numbers given are incompatible with the basic assumption of his calculations that b_1, b_2 and d are parallel lines. From the parallelism of b_1 and b_2, it would follow that one could form a triangle of sides $b_1 - b_2$, l_1 and l_2 (cf. fig. 10). But we known from (1) that

$$5{,}10 = l_1 \qquad l_2 + (b_1 - b_2) = 4{,}50 + 10 = 5{,}0$$

which shows the impossibility of such a triangle. In

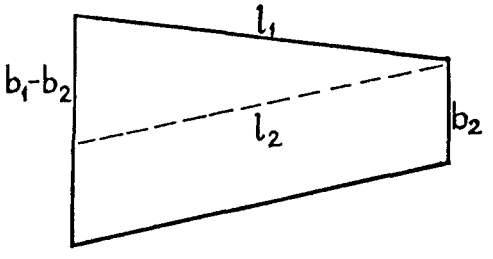


FIG. 10.

other words, the given dimensions exclude an arrangement of the sort indicated in fig. 8 and assumed by the calculation.[133] The formulas used are accordingly only of an *approximative* character.

[130] Cf. above p. 5.
[131] 1 bùr = 30,0 GAR², not 30 GAR².
[132] The "length" is never shorter than the "width".
[133] It can be shown by similar arguments that no angle can be 90°, but that l_1 can be assumed parallel to l_2; this, however, would not make the calculations of the text exact.

It follows from (1) that the area A of the figure is computed according to the approximative formula

$$(2) \qquad A = \frac{b_1 + b_2}{2} \cdot \frac{l_1 + l_2}{2},$$

and the calculations which follow are based on the corresponding formulas for the partial areas

$$(3) \quad A_1 = \frac{b_1 + d}{2} \cdot \frac{\lambda_1 + \lambda_2}{2} \qquad A_2 = \frac{d + b_2}{2} \cdot \frac{\mu_1 + \mu_2}{2}.$$

Before turning to the single steps of the calculation, we must derive still another relation used in the text.

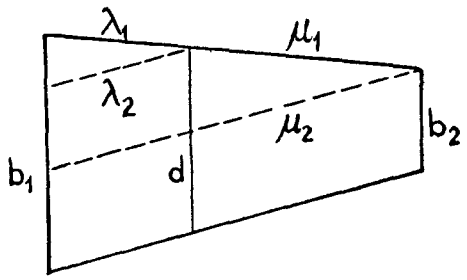


FIG. 11.

Assuming an arrangement like that indicated in fig. 8, we obtain (cf. fig. 11)

$$\frac{\lambda_1}{b_1 - d} = \frac{l_1}{b_1 - b_2} \qquad \frac{\lambda_2}{b_1 - d} = \frac{l_2}{b_1 - b_2}$$

and therefore

$$\lambda_1 + \lambda_2 = (b_1 - d)\frac{l_1 + l_2}{b_1 - b_2}.$$

Analogously,

$$\mu_1 + \mu_2 = (d - b_2)\frac{l_1 + l_2}{b_1 - b_2}.$$

Now, using the given equality $A_1 = A_2$ and (3), we obtain

$$(b_1 + d)(\lambda_1 + \lambda_2) = (d + b_2)(\mu_1 + \mu_2)$$

and from the two preceding equations by multiplying the first by $b_1 + d$, the second by $d + b_2$, we find that

$$(b_1 + d)(b_1 - d) = (d + b_2)(d - b_2)$$

or

$$b_1^2 - d^2 = d^2 - b_2^2$$

or finally

$$(4) \qquad d^2 = \tfrac{1}{2}(b_1^2 + b_2^2).$$

To repeat, this formula is derived under the assumption of the approximative formulas (3) and of the parallelism of b_1, b_2 and d.

We are now in a position to describe the procedure followed in the text, which is actually based on (4), albeit in a rather awkward form. Instead of calculating d directly from (4) and (1), the text employs a method which starts out as if only $b_1 - b_2$ were known but which thereafter makes use of the numerical value of b_2. The first step is

$$\tfrac{1}{2} \cdot 10{,}0 = 5{,}0 = \tfrac{1}{2}(l_1 + l_2).$$

The text now forms the reciprocal of this 5,0 but fails to mention a multiplication by 1,0,0 which, in view of the fact that "zeros" cannot be expressed, would have no effect on the numbers which are written in the text:

$$\frac{1{,}0{,}0}{5{,}0} \cdot 10 = 2{,}0 = \frac{2A}{l_1 + l_2}(b_1 - b_2) = \tfrac{1}{2}(b_1^2 - b_2^2).$$

The necessity of the multiplication by $A = 1{,}0{,}0$ follows from the next step:

$$17^2 = 4{,}49 = b_1^2$$

$$4{,}49 - 2{,}0 = 2{,}49 = b_1^2 - \tfrac{1}{2}(b_1^2 - b_2^2) = \tfrac{1}{2}(b_1^2 + b_2^2)$$

which assumes the correct order of magnitude for the "2". In accordance with (4), we have

$$d = \sqrt{\tfrac{1}{2}(b_1^2 + b_2^2)} = \sqrt{2{,}49} = 13,$$

which is the goal of the first part of the calculation.

The remaining part of the text is devoted to the determination of the sections λ_1, μ_1 of l_1 and λ_2, μ_2 of l_2. The main idea followed consists in computing both the sum and the difference of the λ's and μ's from which their single values follow immediately. To this end the text computes

$$\frac{13 + 17}{2} = 15 = \frac{d + b_1}{2}$$

and, using (3),

$$\overline{15} \cdot 30{,}0 = 2{,}0 = \frac{2A_1}{d + b_1} = \frac{\lambda_1 + \lambda_2}{2}$$

In order also to find $\frac{\lambda_1 - \lambda_2}{2}$, the text uses the proportion

$$\frac{\lambda_1}{\lambda_2} = \frac{l_1}{l_2}$$

which follows from fig. 8, assuming that b_1, d and b_2 are parallel. We therefore have

$$2{,}0 \cdot 0{;}2 = \frac{\lambda_1 + \lambda_2}{2} \cdot \frac{l_1 - l_2}{l_1 + l_2} = \frac{\lambda_1 - \lambda_2}{2}$$

and finally

$$(5) \quad 2{,}0 \pm 4 = \begin{Bmatrix} 2{,}4 \\ 1{,}56 \end{Bmatrix} = \frac{\lambda_1 + \lambda_2}{2} \pm \frac{\lambda_1 - \lambda_2}{2} = \begin{Bmatrix} \lambda_1 \\ \lambda_2 \end{Bmatrix}$$

which yields the points of intersection of the dividing line with the two longer sides.

We would now calculate the μ's by

$$\mu_1 = l_1 - \lambda_1 \qquad \mu_2 = l_2 - \lambda_2,$$

but the text prefers to repeat the same process for the second area as well. The single steps are

$$\frac{13 + 7}{2} = 10 = \frac{d + b_2}{2}$$

$$\overline{10} \cdot 30{,}0 = 3{,}0 = \frac{2A_2}{d + b_2} = \frac{\mu_1 + \mu_2}{2}$$

$$3{,}0 \cdot 0;2 = 6 = \frac{\mu_1 + \mu_2}{2} \cdot \frac{l_1 - l_2}{l_1 + l_2} = \frac{\mu_1 - \mu_2}{2}$$

hence

$$(6) \quad 3{,}0 \pm 6 = \begin{Bmatrix} 3{,}6 \\ 2{,}54 \end{Bmatrix} = \frac{\mu_1 + \mu_2}{2} \pm \frac{\mu_1 - \mu_2}{2} = \begin{Bmatrix} \mu_1 \\ \mu_2 \end{Bmatrix}$$

which furnishes the remaining parts of the longer sides.

b. Terminology

1. *arakarūm*

The coefficient $\frac{l_1 - l_2}{l_1 + l_2}$ which is used to transform $\frac{\lambda_1 + \lambda_2}{2}$ and $\frac{\mu_1 + \mu_2}{2}$ into $\frac{\lambda_1 - \lambda_2}{2}$ and $\frac{\mu_1 - \mu_2}{2}$, respectively, is called *arakarūm*. For a discussion of the term, see p. 15.

2. *tallu* ≈ dal

The Akkadian word which is spelled *ta-al-li* (obv. 3), *ta-al-lum* (obv. 16) and *ta-al-lam* (obv. 17, rev. 7) proves definitely that the Sumerian word from which it was borrowed is to be read dal, not ri.[134] It also proves that *pirkum*[135] is not the unique Akkadian equivalent of Sumerian dal with the meaning indicated in the mathematical contexts. The translation "dividing-line" is intended to cover the technical meaning of dal ≈ *tallum*, *pirkum*, namely, the line which cuts through any area (e.g., circle, trapezoid, triangle).

[134] For previous occurrences of dal in mathematical texts, see MKT II p. 31a under RI. For the use of dal as the altitude in a trapezoid, cf. below p. 50, No. 1.

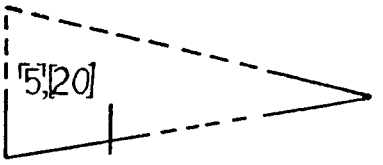
[135] Cf. Neugebauer [1] p. 199. Outside the mathematical texts, *pirkum* has meanings such as "bar (of a door)" or "border (of a country)"; the meaning "width" is at present restricted to the Nuzi texts (cf. Lewy [1] p. 33).

Ca. MLC 1950

(Photograph: Plate 27; copy: Plate 2)

Transcription

Obverse



[1]sag-dù 20 GAR uš
[2]5,20 a-šà-bi
[3]30 GAR ... LU(?)

[4]sag-an-ta *ù* sag-ki-ta *mi-nu-um* za-e kì-ta[-zu-dè]
[5]igi 20 du$_8$ 3 *ta-mar* 3 *a-na* 5,20 *i-ši-ma* 1[6]
[6]16 *a-na* sag-an-ta ù ...-...-LU-*bu*
[7]...-...-*ma* 30 uš *a-na* 2 *e-ṣí*(?)-*ip*[135a] 1
[8]*ù* 20 *mu-ta-ri-tam* an-ta UL-gar 1,20
[9]igi 1,20 du$_8$ 45 *a-na* 5,20 a-šà [*i-ši-ma* 4]
[10][4 *a-na*] 16 daḫ *i-na* 16 zi 2[0 sag-an-ta]
[11][12 sag-ki-ta]

Reverse destroyed except for traces at end.

Translation

Obverse

[1]A triangle. 20 GAR is the length,
[2]5,20 its area,
[3]30 GAR the .....
[4]What are the upper width and the lower width? [When you] perform (the operations),
[5]take the reciprocal of 20, (and) you will see 0;3. Multiply 0;3 by 5,20, and (the result is) 1[6].
[6]16 to(?) the upper width and .....
[7–8]..... 30, the length, multiply by 2, add (the resulting) 1,0 and 20, the upper perpendicular, (and the result is) 1,20.
[9]Take the reciprocal of 1,20, [multiply] (the resulting) 0;0,45 by 5,20, the area, [and (the result is) 4].
[10]Add [4 to] 16, subtract from 16. [The upper width is] 2[0];
[11][the lower width is 12.]

Reverse destroyed except for traces at the end.

Commentary

The problem presented here belongs to a class which is well attested:[135b] a triangle is subdivided by a line parallel to the base into a trapezoid and a smaller triangle. One group of dimensions is given; find the others from the given relations. In our particular case, we are given a triangle, the altitude 50, sub-

[135a] This reading, if correct, presupposes that the scribe inverted the order of the two components of the sign ZI.

[135b] Cf. the numerous examples cited in MKT III p. 82 under "Dreieck".

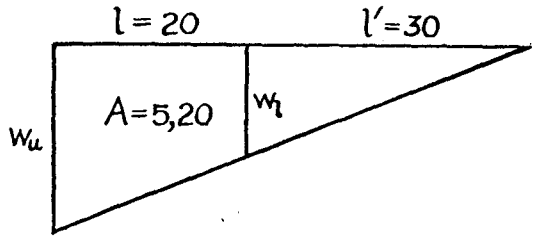


FIG. 12.

divided into two parts $l = 20$ and $l' = 30$.[135c] In addition, the area $A = 5{,}20$ of the trapezoid is given. Find the lengths w_u and w_l of the parallel sides of this trapezoid.[135d]

The method of solving the problem in question consists in finding both $\frac{1}{2}(w_u + w_l)$ and $\frac{1}{2}(w_u - w_l)$ such that the sum and difference of these expressions give the answer. The first expression is easily found. From the formula

(1) $$A = \tfrac{1}{2}(w_u + w_l)l$$

for the area of the trapezoid, it follows that we have

(2) $$\frac{A}{l} = \frac{5{,}20}{20} = 5{,}20 \cdot 0;3 = 16 = \tfrac{1}{2}(w_u + w_l),$$

as given in the text.

The next step requires explanation. We introduce

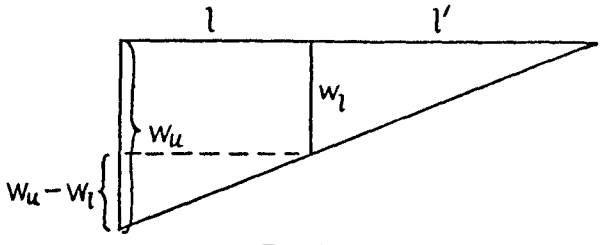


FIG. 13.

the dotted line as given in fig. 13 and then find, from similar triangles,

$$\frac{w_u}{l + l'} = \frac{w_l}{l'} = \frac{w_u - w_l}{l}$$

or

$$w_u = \frac{l + l'}{l}(w_u - w_l) \qquad w_l = \frac{l'}{l}(w_u - w_l).$$

By adding, we obtain

$$w_u + w_l = \frac{2l' + l}{l}(w_u - w_l)$$

and consequently

$$\tfrac{1}{2}(w_u - w_l) = \frac{(w_u + w_l)l}{2(2l' + l)}.$$

Using (1), we can therefore write

(3) $$\tfrac{1}{2}(w_u - w_l) = \frac{A}{2l' + l}$$

and this is precisely the relation used by the scribe. He computes:

$$2l' = 2 \cdot 30 = 1{,}0 \qquad 2l' + l = 1{,}0 + 20 = 1{,}20$$

$$\frac{1}{2l' + l} = 0;0{,}45$$

and therefore

(4) $$\frac{A}{2l' + l} = 0;0{,}45 \cdot 5{,}20 = 4 = \tfrac{1}{2}(w_u - w_l).$$

The relations (2) and (4) now provide the final answer

$$\tfrac{1}{2}(w_u + w_l) + \tfrac{1}{2}(w_u - w_l) = 16 + 4 = 20 = w_u$$

and

$$\tfrac{1}{2}(w_u + w_l) - \tfrac{1}{2}(w_u - w_l) = 16 - 4 = 12 = w_l.$$

Only parts of two lines of another problem are preserved on the reverse.

D. YBC 4608

(Photograph: Plate 28; copy: Plate 3)

Transcription

Obverse

1 [1]sag-ki-gu$_4$ [. . . . . .[136] a-]šà *a*[-*na ši-na zu-ú-uz*[137]]
[2]42,11,15 a-šà íd-ki-t[a 14,3,45 a-šà íd-an-na]
[3]igi-5-gál íd-ki-ta íd-a[n-na 52,30 dal-bi]
[4]sa[g-an-n]a *ù* sag-ki-ta en[-nam] za-e k[ìd-da-zu-dè]
[5]5 [*i-n*]*a* sag-ki-ta[139] ḫé-gar 1 [*i-*]*na* sag-an-na[138] ḫ[é-gar]
[6][1]4,3,45 *ù* 42,11,15 [gar-]gar-ma 56,15 in-sì
[7]5 *ù* 1 gar-gar-ma 6 igi-6-gál-bi du$_8$-ma 10 in-sì
[8][10] *a-na* 56,15 nim-ma 9,22,30 *a-na ši-na* e-tab 18,45
[9][1]8,45 *re-eš-ka li-ki-il* igi-1-gál-bi *ša* sag-an-na[138] du$_8$-ma 1
[10]1 *a-na* 14,3,45 nim-ma 14,3,45 in-sì *a-na ši-na* e-tab

[135c] For the sake of convenience, a right triangle is assumed in our commentary. It is equally possible to assume an isosceles triangle, each side 50. One would then also have to assume the approximative formula for the area of a trapezoid, which is found, e.g., in problem-text B (see p. 47, formula (2)).

[135d] The left side is, as usual, called the "upper width" because we must turn the figure 90° in clockwise direction (cf. p. 42, note 125b and Neugebauer, Vorlesungen, pp. 34 and 176).

[136] Probably blank.

[137] Restoration after problem-text B, obv. 3, q.v.

[138] Error for uš-an-na, "the upper length."

[139] Error for uš-ki-ta, "the lower length."

11 28,7,30 in-sì 18,4[5 *i-*]*n*[*a* 28],7,30 zi-ma
12 9,22,30 *re-eš-ka li*[*-ki-il* igi-]5-gál-bi íd-ki-ta du_8-ma 12
13 12 *a-na* 42,11,15 nim-ma [8],26,15 in-sì
14 *ba-ma-at* 9,22,30 *ša re-eš-ka ú-ka-al-lu* gaz-ma
15 4,41,15 in-sì 4,41,15 *a-na* 8,26,15 daḫ-ma
16 13,7,30 igi 13,7,30 *ú-ul ip-pa-aṭ-ṭa-ar*
17 *mi-nam a*[*-na*] 13,7,30 *lu-uš-ku-un ša* 52,30 dal-bi *i-na-di-nam*
18 4 [*šu-ku-un* igi-]4-gál-bi 15 in-sì 15 *a-na* 1 *ša i-na* sag-an-na[138]
19 *aš-ku-nu ta-na-aš-ši-i-ma* 15 uš íd-an-na
20 15 *a-na* 5 *ša i-na* sag-ki-ta[139] *aš-ku-nu ta-na-aš-ši-i-ma*
21 1,15 uš íd-ki-ta in-sì *aš-šu-um* sag-an-na *ù* sag-k[i-ta]
22 *a-ma-ri-i-ka* a-šà *a-na ši-na* e-tab 28,7,30 in-sì
23 igi-15-gál-bi du_8-ma 4 in-sì 4 *a-na* 28,7,30 nim-ma
24 1,52,30 in-sì *i-na* 1,52,30 52,30 dal-bi *ta-b*[*a-al*]
25 1 sag-an-na a-šà *ka-la-šu a-na ši-na* e-tab 1,52,3[0 in-sì]
26 igi 1,30 du_8-ma 40 in-sì 40 *a-na* 1,52,30 nim-ma
27 1,15 in-sì *i-na* 1,15 1 sag-an-na *ta-ba-al*
28 15 sag-ki-ta *aš-šu* a-šà [*a-*]*ma-ri-i-ka*
29 [1] *ù* 15 gar-gar-ma 1,15 *ba-ma-at* [1],15 <gaz-ma> 37[,30 in-sì]
30 [3]7,30 *a-na* 1,30 nim-ma 56,15 a-šà [in-sì]

2 31 [.....] *ú-ul i*[*-de*] *ù* sag-an-n[a .....]
32 [.....] 1,30 *ṣi-l*[*i-ip-tum* ...]

Reverse

(About 6 lines missing at beginning)

3 1 [.....]....[.....]
2 a-šà *ab-ni* gar-ga[r.....]
3 uš *ù* sag en[-nam.....]
4 ....-bi igi-5-gál .....[.....]
5 igi-5-gál-bi du_8-ma [.....]
6 6 uš 1,12 sag [.....]

4 7 [.....]
8 gar-gar ..[.....]
9 1,12 sa[g.....] *ip*(?)[....]
10 [.....] *li*(?) [..... igi-]12-gál-b[i du_8-ma]
11 5 in-sì 1 [*i-*]*na* sag ..[.....] 1 *a-*[*n*]*a* 5 da[ḫ]-ma
12 6 uš 1,12 sag 7,12 a-šà gar-gar u[š *ù*] sag 7,12

5 13 sag-dù 6,30 uš [11,22],30 a-šà sag[-bi(?)] *ú-ul i-de*
14 6 šeš-meš *i-zu-zu-uš* šeš ugu <šeš> *ú-te-l*[*e-el-*]*le-ma*
15 *ma-li ú-te-le-el-lu-ú ú-ul i-de*
16 šeš ugu šeš *ki-ṭa-a*[140] *ú-te-le-el-le*
17 za-e kìd-da<-zu>-dè a-šà *a-na ši-na* e-tab 22,45
18 igi 6,30 *ú-ul ip-pa-aṭ-ṭa-ar mi-nam a-na* 6,30
19 *lu-uš-ku-un ša* 22,45 *i-na-di-nam* 3,30 *šu-ku-un-ma*
20 sag-an-na igi 6 šeš-meš du_8-ma 10 *a-na* 6,30 [nim-m]a
21 1,5-ta-àm uš *il-qú*[*-ú* .....]
22 35 GAR dagal 35 [*i-na* 3,30 .....]
23 35 *i-na* 2,5[5 .....]
24 35 *i-na* 2,2[0 .....]
25 35 *i-na* 1,4[5 .....]
26 35 *i-na* 1,10 z[i-ma .....]
27 35 zi-ma sag(?) [.....]

Two loose fragments contain:

...]-ma uš [...
...] *ù* sag [...

and

...]-ma a-šà [...
...] a-šà igi [...
.....] DA(?) [...

Translation

Obverse

1 [1]A trapezoid .....[136]. [Divide the ar]ea in [two];[141]
[2]42,11;15 is the area of the low[er] strip, [14,3;45 the area of the upper strip;
[3]one-fifth of the (length of the) lower strip is the (length of the) up[per] strip; [52;30 is its dividing-line].
[4]Wh[at] are (the upper length, the lower length,) the [upp]er wid[th] and the lower width? When you pe[rform (the operations)],
[5]let 5 be put (aside) in the lower width,[139] (and) let 1 be [put (aside)] in the upper width.[138]
[6]Add [1]4,3;45 and 42,11;15, and you will get 56,15.
[7]Add 5 and 1, and (the result is) 6; take the reciprocal of 6, and you will get 0;10;
[8]multiply [0;10] by 56,15, and multiply the (resulting) 9,22;30 by two: (the result is) 18,45;
[9]keep [1]8,45 in your head.[142] Take the reciprocal of 1 of the upper width,[138] and (the result is) 1;
[10]multiply 1 by 14,3;45, and you will get 14,3;45; multiply by two, (and)
[11]you will get 28,7;30. Subtract 18,4[5 fr]o[m 28],7;30, and
[12]k[eep] the (resulting) 9,22;30 in your head.[142] Take the [reciprocal] of 5 (of the length) of the lower strip, and (the result is) 0;12;
[13]multiply 0;12 by 42,11;15, and you will get [8],26;15.

---

[140] Dr. A. Goetze, p. 149 below, note 358, reads *ki-i a-a* and analyzes *a-a* as the absolute state of *aḫḫum*. The following additional occurrences should be noted: *ki-ṭa-a il-qú-ú*, VAT 8522 obv. II 7 (falsely read KI-*ia* 2 *il-lu-ú* MKT I p. 368); and [*k*]*i-ṭa il-qú-ú*, VAT 6597 obv. I 5 (which was read ...*-ia il-qú-ú* MKT I p. 274). See now also Sb (p. 106) obv. 6.

[141] Or: "[I divided (*a-zu-ú-uz*) the ar]ea in [two]"? Or: "[the ar]ea [is divided] in [two]"?

[142] Literally, "let your head hold."

[14]Halve 9,22;30 which you are keeping in your head,[143] and
[15]you will get 4,41;15; add 4,41;15 to 8,26;15, and (the result is)
[16]13,7;30. The reciprocal of 13,7;30 is not obtainable.
[17]What should I put to 13,7;30 which will give me 52;30, its dividing-line?
[18–19][Put] 0;4. The [rec]iprocal of 0;4 will give you 15; you shall multiply 15 by 1, which I put (aside) in the upper width,[138] and the (resulting) 15 is the length of the upper strip.
[20]You shall multiply 15 by the 5 which I put (aside) in the lower width,[139] and
[21–22]you will get 1,15, the length of the lower strip. In order for you to see the upper width and the lo[wer] width, multiply the area by two, (and) you will get 28,7;30.
[23]Take the reciprocal of 15, and you will get 0;4; multiply 0;4 by 28,7;30, and
[24]you will get 1,52;30; tak[e away] 52;30, its dividing-line, from 1,52;30, (and)
[25]the (resulting) 1,0 is the upper width. Multiply the whole area by two, (and) [you will get] 1,52,30.
[26]Take the reciprocal of 1,30, and you will get 0;0,40; multiply 0;0,40 by 1,52,30, and
[27]you will get 1,15; take away 1,0, the upper width, from 1,15, (and)
[28]the (resulting) 15 is the lower width. In order for you to see the area,
[29]add [1,0] and 15, and (the result is) 1,15; halve [1],15, [and you will get] 37;[30];
[30]multiply [3]7;30 by 1,30, and [you will get] 56,15, the area.

2 [31][.....] I did not know, and the upper width [.....]
[32][.....] 1,30 the dia[gonal .....]

Reverse

(About 6 lines missing at the beginning.)

3 [1].....(traces).....
[2]I formed an area. The su[m of .....]
[3]What are the length and the width? [.....]
[4]..... one-fifth ...[.....]
[5]Take the reciprocal of 5, and [.....]
[6]6,0 if the length; 1,12 is the width [.....[144]]

4 [7][.....]
[8]The sum (or: I added) ..[.....]
[9]1,12 the width [.....]
[10][..... take the rec]iprocal of 12, [and]
[11]you will get 0;5. 1,0(?) in(?) the width .....; add 1,0 to 5,0, and (the resulting)
[12]6,0 is the length. 1,12 is the width. 7,12,0 is the area. The sum of the length [and] the width is 7,12.

5 [13]A triangle. 6,30 is the length, [11,22],30 the area; I did not know [its(?)] width.
[14]6 brothers divided it. One brother('s share) exceeded the other('s), but
[15]how much he exceeded I did not know.
[16]How much did one brother exceed the other?
[17]When you perform (the operations), multiply the area by two, (and the result is) 22,45,0.
[18–19]The reciprocal of 6,30 is not obtainable. What should I put to 6,30 which will give me 22,45,0? Put 3,30, (which is)
[20]the upper width. Take the reciprocal of 6, the brothers, (and) [multiply] the (resulting) 0;10 by 6,30, and (the resulting)
[21]1,5 (is) the length which each too[k .....]
[22]35 GAR is the breadth. 35 [from 3,30 .....]
[23]35 from 2,5[5 .....]
[24]35 from 2,2[0 .....]
[25]35 from 1,4[5 .....]
[26]subtr[act] 35 from 1,10 [.....]
[27]subtract 35, and the width(?) [.....]

Commentary

No. 1

No figure is given in the text itself, but it follows from the context that a trapezoid of area A is to be divided into two parts, A_1 and A_2, such that the partial lengths have the proportion 1:5. The partial areas and the length of their common side are given. All calculations of the text are correct if we assume that l_1 and l_2 are at right angle with b_1, d and b_2 (cf.

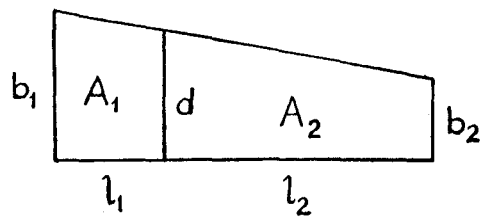


FIG. 14.

fig. 14). The numerical values given are:

$$A_1 = 14,3;45 \qquad d = 52;30$$
$$A_2 = 42,11;15 \qquad \tfrac{1}{5}l_2 = l_1.$$

[143] Literally, "your head is holding."

[144] The rest of this line is probably to be restored, at least in part, according to line 12.

The process by which the text finds the unknown quantities b_1, b_2, l_1, l_2 is the following:

$$14,3;45 + 42,11;15 = 56,15 = A_1 + A_2 = A$$
$$1 + 5 = 6 = \lambda\,(l_1 + l_2),$$

where λ is an unknown factor by which the numbers 1 and 5 are to be divided in order to obtain the actual values of l_1 and l_2. The first goal is the determination of λ. This is done by determining the corresponding coefficient of d. To this end, it calculates

$$\overline{6} \cdot 56,15 \cdot 2 = 9,22;30 \cdot 2 = 18,45$$
$$= \frac{2A}{\lambda(l_1 + l_2)} = \frac{1}{\lambda}\,(b_1 + b_2).$$

Now the same operation is applied to the partial areas. First we obtain

$$\overline{1} \cdot 14,3;45 \cdot 2 = 28,7;30 = \frac{2A_1}{\lambda l_1} = \frac{1}{\lambda}\,(b_1 + d)$$

and hence

$$28,7;30 - 18,45 = 9,22;30 = \frac{1}{\lambda}\,(b_1 + d) - \frac{1}{\lambda}\,(b_1 + b_2)$$
$$= \frac{1}{\lambda}\,(d - b_2).$$

For the second area we have

$$\overline{5} \cdot 42,11;15 = 8,26;15 = \frac{A_2}{\lambda l_2} = \frac{1}{\lambda} \cdot \frac{d + b_2}{2}.$$

From the preceding relation it follows that

$$\tfrac{1}{2} \cdot 9,22;30 = 4,41;15 = \frac{1}{\lambda} \cdot \frac{d - b_2}{2};$$

hence

$$4,41;15 + 8,26;15 = 13,7;30$$
$$= \frac{1}{\lambda}\left(\frac{d + b_2}{2} + \frac{d - b_2}{2}\right) = \frac{d}{\lambda}.$$

Now d is given as 52;30, hence

$$\frac{52;30}{13,7;30} = 0;4 = \lambda.$$

We therefore get for the partial lengths:

$$\overline{0;4} \cdot 1 = 15 = \frac{1}{\lambda} \cdot \lambda l_1 = l_1$$

$$\overline{0;4} \cdot 5 = 1,15 = \frac{1}{\lambda} \cdot \lambda l_2 = l_2.$$

Now it is easy to find b_1 and b_2 by

$$14,3;45 \cdot 2 \cdot \overline{15} = 28,7;30 \cdot 0;4$$
$$= 1,52;30 = \frac{2A_1}{l_1} = b_1 + d$$
$$1,52;30 - 52;30 = 1,0 = (b_1 + d) - d = b_1$$

and

$$56,15 \cdot 2 \cdot \overline{1,30} = 1,52,30 \cdot 0;0,40 = 1,15$$
$$= \frac{2A}{l_1 + l_2} = b_1 + b_2$$
$$1,15 - 1,0 = 15 = (b_1 + b_2) - b_1 = b_2.$$

Thus the problem is completely solved.

Finally, as a check, the text calculates

$$1,0 + 15 = 1,15 = b_1 + b_2$$
$$\tfrac{1}{2} \cdot 1,15 \cdot 1,30 = 37;30 \cdot 1,30 = 56,15$$
$$= \tfrac{1}{2}(b_1 + b_2)(l_1 + l_2) = A$$

which agrees with the value of $A_1 + A_2$.

Nos. 2, 3 and 4

No. 2 is almost completely destroyed with the exception of small parts of two lines, which are sufficient to show the geometrical character of the problem. The mention of the "upper width" implies a trapezoid. If our partial restoration of "diagonal" is correct, we would have here the first example of the use of *şiliptum* in the sense of "diagonal (of a trapezoid)" since the otherwise attested meaning is "diagonal (of a rectangle) or hypotenuse (of a right triangle)."

Nos. 3 and 4 are badly damaged, but the numbers which are preserved indicate that both deal with a rectangle of length $l = 6,0$, width $b = 1,12$, and area $lb = 7,12,0$. The most plausible restoration of No. 3 seems to be that $l + b = 7,12$ and $b = \frac{1}{5}l$ were the given relations, but it is not clear how this problem would lead to calculations compatible with the remains still visible in lines 4 and 5. No. 4 is too damaged to permit a restoration of the problem.

No. 5

We have here one of the "inheritance" problems which require the partition of property to be distributed among a given number of brothers.[145] The field in question is of triangular shape (cf. fig. 15)

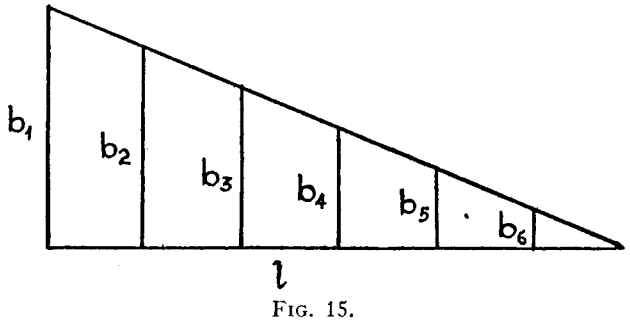


FIG. 15.

[145] A number of such problems are attested; cf., e.g., MKT I pp. 126ff., 274ff., 368ff. See also problem-text Q No. 2, p. 100 below. For the general background cf. Gandz [2].

with length l and area A:

$$l = 6{,}30 \qquad A = 11{,}22{,}30.$$

This area is divided among 6 brothers by equidistant lines parallel to the base of the triangle. The question asked by the text concerns the difference between the allotments of the brothers.

The first step consists in computing the base b_1 of the triangle (cf. fig. 15) by

$$2 \cdot 11{,}22{,}30 = 22{,}45{,}0 = 2A$$

$$\frac{22{,}45{,}0}{6{,}30} = 3{,}30 = \frac{2A}{l} = b_1.$$

We now divide l by 6, which gives

$$\tfrac{1}{6} \cdot 6{,}30 = 1{,}5 = \lambda$$

for the length of each partial field. Then the corresponding b's are found by dividing b_1 into 6 parts

$$\tfrac{1}{6} \cdot 3{,}30 = 35 = \frac{b_1}{6} = \delta$$

and subtracting the result step by step[146]

$$\begin{aligned}
&[3{,}30 - 35 = 2{,}55 = b_1 - \delta = b_2]\\
&2{,}55 - 35\ [= 2{,}20 = b_2 - \delta = b_3]\\
&2{,}20 - 35\ [= 1{,}45 = b_3 - \delta = b_4]\\
&1{,}45 - 35\ [= 1{,}10 = b_4 - \delta = b_5]\\
&1{,}10 - 35\ [= \ \ 35 = b_5 - \delta = b_6].
\end{aligned}$$

One would expect the last line to state that

$$b_6\lambda = 35 \cdot 1{,}5 = 37{,}55$$

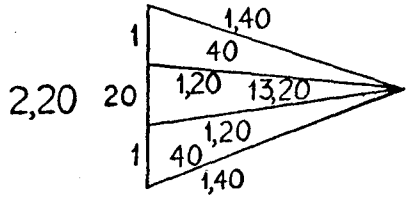
is the difference between the individual fields, but we find it difficult to see how this could be reconciled with the preserved words.

E. YBC 8633

(Photograph: Plate 29; copy: Plate 4)

Transcription

Obverse[147]



1 [1]sag-dù 1,40 uš-ta[148] *ki-l[a-l]a-an* 2,20 sag-ki a-šà *mi-n[u-u]m*
[2]*at-ta i-na* 2,20 sag *ša* ..[...]...[...]...
[3]20 *ú-sú-uḫ-ma a-na* sag sag-dù ....[....]
[4]*ù* 2 *ša te-zi-bu a-na ši-na ḫe-pé-ma* 1
[5]1 sag-ki sag-dù *iš-te-en* 1 sag sag-dù *ša-ni-im*
[6]uš *ša-nu-um mi-nu-um*
[7]20 *ma-ak-ṣa-ra-am a-na* 4 *i-ši-ma* 1,20
[8]1,20 uš *ki-nu-um*[150] *ù* 1 sag-ki sa[g-dù]
[9][*a-n*]*a ši-na ḫe-pe-ma* 30 *a-na* 1,20 u[š *ša-nim*]
[10][*i-*]*ši-ma* 40 a-šà sag[-dù .....]
[11]20 sag-ki s[ag]-d[ù *a-*]*na ši-na* [*ḫe-pe-ma*]
[12]10 *a-na* 1,2[0 uš *ša-nim i-ši-ma*]
[13]13,20 a-šà sag-d[ù .....]

Reverse

[1]1 sag-ki sag-dù *a-na š*[*i-na ḫe-pe-ma*]
[2]30 *a-na* 1,20 uš *ki*(?)-*nim*[150] [*i-ši-ma*]
[3][4]0 a-šà sag-dù *ša-a*[*l-ši-im*]
[4]1,33,20 a-šà *ki-nu-um* [.....]

2 [5]*ma-ak-ṣa-ru-um ša* sa[g-ki-gu$_4$ .....]
[6]20 *a-na* 5 *ša*(?) *ṣí-li-i*[*p-*]*ti*[*m i-š*]*i*[*-ma*]
[7]1,40 *ṣí-l*[*i-i*]*p*[*-ta*]*m i*[*-na-*]*di-kum-ma*
[8]20 *a-na* 4 uš *i-ši-ma* 1,20 uš *ki-nu-um*[150]
[9]20 *a-na* 3 *i-ši-ma* 1 sag sag-dù *i-na-di-ku*

3 [10]*ma-ak-ṣa-ru-um ša* sag-ki-gu$_4$ *ṣí-li-ip-tim*
[11]20 *a-na* 5 *i-ši-ma* 1,40 uš
[12]20 *a-na* 4 *i-ši-ma* 1,20 uš *ša-nu-um*
[13]20 *a-na* 3 *i-ši-ma* 1 sag-ki sag-d[ù]

Translation

Obverse[147]

(For the figure, see the transcription.)

1 [1]A triangle. 1,40 is each of the two lengths, 2,20 the width. What is the area?
[2]As for you—from 2,20 the width which ..[...] ...[...]...
[3]subtract 20 ..... width of the triangle ...[.....].
[4]And then halve the 2,0 which you left, and (the result is) 1,0.
[5]1,0 is the width of the one triangle (and) 1,0 is the width of the second triangle.
[6]What is the second length?
[7]Multiply 20, the *makṣarum*, by 4, and (the result is) 1,20;
[8–10]1,20 is the second[151] length. And then halve 1,0, the width of the tr[iangle], and [mu]ltiply the (resulting) 30 by 1,20, the [second] len[gth]; the (resulting) 40,0 is the area of the [first(?)] tri[angle].
[11]Ha[lve] 20, the width of the triangle, [and]

[146] The parts in [ ] are restored.
[147] The first six lines show so many traces of erasure that it is plausible to assume that our text was written over the beginning of another.
[148] For -ta-àm?
[150] Sic instead of *ša-nu-um* (or, in rev. 2, instead of *ša-nim*).

[12]multiply the (resulting) 10 by 1,2[0, the second length;]
[13]the (resulting) 13,20 is the area of the [second(?)] triang[le].

Reverse

[1]Ha[lve] 1,0, the width of the triangle, [and]
[2][multiply] the (resulting) 30 by 1,20, the second[151] length;
[3]the (resulting) [4]0,0 is the area of the third triangle.
[4]1,33,20 is the true area [. . . . .]

2 [5]As to the *makṣarum* of the tr[apezoid of the diagonal(?) —]
[6][multip]ly 20 by 5 of(?) the diagonal, [and]
[7]you will get 1,40, the dia[go]nal.
[8]Multiply 20 by 4, the length, and the (resulting) 1,20 is the second[151] length.
[9]Multiply 20 by 3, and [you will] ge[t] 1,0, the width of the triangle.

3 [10]As to the *makṣarum* of the trapezoid of the diagonal—
[11]multiply 20 by 5, and the (resulting) 1,40 is the length.
[12]Multiply 20 by 4, and the (resulting) 1,20 is the second length.
[13]Multiply 20 by 3, and the (resulting) 1,0 is the width of the triang[le].

Commentary

The three problems contained in this text are very closely related, as is shown by the identity of corresponding parameters. The first example concerns a "triangle" (sag-dù) subdivided into three triangles as illustrated by the figure given at the beginning of the text. The last two problems mention a "trapezoid" (sag-ki-gu$_4$), but we shall see that this refers to an auxiliary figure.

No. 1

The first line of the text indicates that the determination of the area is the problem to be solved. If we use the notation indicated in fig. 16, the parameters given are $d = 1,40$ and $B = 2a + b = 2,20$. As will be shown below, the text derives the values $a = 1,0$ $b = 20$ and $c = 1,20$ from these parameters. Once these values are obtained, the areas in question are computed (obv. 8 to rev. 4) as follows:

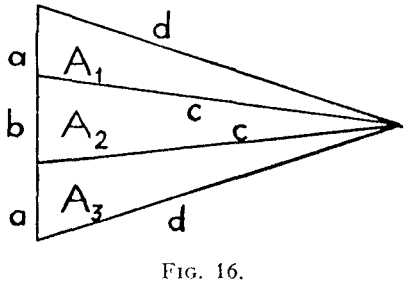


FIG. 16.

[151] Text: "true."

$$\tfrac{1}{2}ac = \tfrac{1}{2} \cdot 1,0 \cdot 1,20 = 40,0 = A_1$$

$$\tfrac{1}{2}bc = \tfrac{1}{2} \cdot 20 \cdot 1,20 = 13,20 = A_2$$

$$\tfrac{1}{2}ac = \tfrac{1}{2} \cdot 1,0 \cdot 1,20 = 40,0 = A_3 = A_1$$
$$A_1 + A_2 + A_3 = 1,33,20.$$

Before analyzing the steps preceding the computation of these areas, we must discuss the shape of the figure in question. We have already seen that

$$a = 1,0 \qquad c = 1,20 \qquad d = 1,40$$

are the sides of the two outer triangles. The numbers are all multiples of 20:

$$(1) \qquad a = 3 \cdot 20 \qquad c = 4 \cdot 20 \qquad d = 5 \cdot 20$$

and satisfy the relation

$$(2) \qquad a^2 + c^2 = d^2.$$

The two outer triangles are therefore right triangles with the right angle at the common base and adjacent to the interior triangle. The formula $A_1 = \frac{1}{2}ac$, used above, therefore gives the exact area of these triangles. The innermost triangle, however, has an area slightly less than $A_2 = \frac{1}{2}bc$ because c is the side, not the altitude, of this triangle. And, finally, it follows from (2) that the three bases cannot form a single straight line as assumed in the figure of the text (and in fig. 16) but that the exact figure should look like fig. 17 when drawn to correct scale.

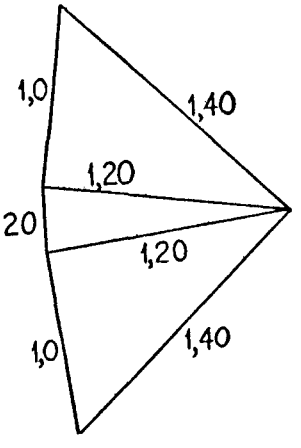


FIG. 17.

We must now turn back to the beginning of the text. The only parameters explicitly given are $d = 1,40$ and $B = 2a + b = 2,20$. Nothing, however, is said

about the value of b, although the first step of the solution (lines 2 to 5) consists in computing

$$a = \tfrac{1}{2}(B - b) = \tfrac{1}{2}(2{,}20 - 20) + 1{,}0.$$

One explanation for this curious situation would be the assumption that the value $b = 20$ is taken from the figure; but the figure contains all the numbers involved, those given as well as those required. Another possibility would consist in decomposing the total triangle into three parts such that the outer parts are pythagorean triangles; but it is hard to see how such a problem could have been solved. We are therefore forced to suppose that the problem is incompletely stated in its present form.

There now follows the computation of c, the final quantity needed for the determination of the areas mentioned above. This c is called "second length" in contrast to the "length" d of the outside contour. The calculation consists in multiplying a magnitude $\alpha = 20$, called *makṣarum*, by 4, which gives

$$\alpha \cdot 4 = 1{,}20 = c.$$

The explanation of this step is simple if one looks at equations (1) and (2) given above. From the fact that a, c and d should form a right triangle and from $a = 3 \cdot 20$ $d = 5 \cdot 20$, it follows that the third side must be $4 \cdot 20$ because $3^2 + 4^2 = 5^2$. The *makṣarum* is therefore the common factor[152] by which 3, 4 and 5 must be multiplied in order to give a, c and d.

Nos. 2 and 3

The following two examples confirm the preceding interpretation of *makṣarum*. Here the two outer triangles are treated separately; in each case, a, c and d are found according to formula (1), c being also called "diagonal" (*ṣiliptum*). A serious difficulty lies in the expression at the beginning of No. 3: "*makṣarum* of the trapezoid of the diagonal". As we have seen above, the *makṣarum* is the factor 20 needed to enlarge the pythagorean triangle with sides measuring 3, 4 and 5 to the actual dimensions 1,0, 1,20 and 1,40. One could assume that the *makṣarum* for the diagonal

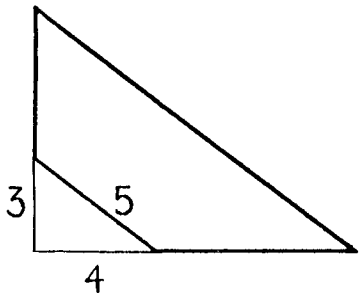


FIG. 18.

was represented by two similar triangles with a common right angle in the manner indicated in fig. 18. The interior triangle is the triangle with sides measuring 3, 4 and 5; the larger triangle corresponds to the actual triangle. The space between the two diagonals could therefore have been called "the trapezoid of the diagonal". It must be emphasized, however, that we can point to no other instance of this sort of geometrical argument in Babylonian mathematics.

[152] The philological analysis of *makṣarum* offers no difficulties: it is a form of the *maqtalum* type from the root *kṣr* "to bind, tie." The word *makṣarum* occurs outside mathematical texts with the meanings "bale (of straw)" (cf., e.g., Ungnad [1] p. 25b, note 2), "band" (in the sense of apparel) (restoration of Deimel ŠL 597, 176a by Landsberger); cf. also *makṣaru* "of the mouth of a horse" which is given as an explanation of *napsamu* (for the latter see Landsberger, Fauna p. 112, note 1 against Delitzsch, HWB p. 532b). The linguistic transparency of *makṣarum* does not seem to contribute to the elucidation of its mathematical meaning. [Since the above was written, *makṣarum* has turned up in problem-text Aa (p. 42), obv. 1, q.v.; the meaning is also none too clear there.]

Ea. NBC 7934

(Copy: Plate 2)

Transcription

Obverse

1 [1] [6½ GAR 5] kùš uš
[2] 3 kùš dagal-an-na
[3] 3½ kùš GAM-2-kam
[4] saḫar-bi en-nam
[5] 5⅚ SAR 1⅚ gín 7½ še

2 [6] 6½ GAR 5 kùš uš
[7] 7½ GAR dagal-ki-ta
[8] 8½ kùš GAM
[9] saḫar-bi en-nam

Reverse

[1] 1⅔ SAR 3⅔ gín 15 še

[2] ŠU-NIGÍN[152a] 2½ SAR 5½ gín 22½ še saḫar

3 [3] ù 15 gín saḫar
[4] ½ GAR-ta-àm *mi-[i]t-[ḫ]a[-ar-tum]*
[5] 1 kùš [GAM]
[6] [ŠU-NIGI]N 2⅚ [SAR ½ gín 22½ še saḫar]

Edge

[1] [. . . . .] . . UD(?) bi [. . . . .]

[152a] Ligature.

Translation

Obverse

1 [6½ GAR (and) 5] kùš is the length,
3 kùš the upper width,
3½ kùš the second depth.
What is its volume?
5⅚ SAR, 1⅚ gín (and) 7½ še (is the volume).

2 6½ GAR (and) 5 kùš is the length,
7½ GAR the lower width,
8½ kùš the depth.
What is its volume?

Reverse

1 1⅔ SAR, 3⅔ gín (and) 15 še (is the volume).

2 Total: 2½ SAR, 5½ gín (and) 22½ še volume.

3 And 15 gín volume.
4½ GAR is the sid[e of the square],
1 kùs [the depth].
[Grand tota]l: 2⅚ [SAR, ½ gín (and) 22½ še volume].

The volumes, obtained by multiplying length, width and depth, are consequently

No. 1:	0;51,52,30 SAR	= ⅚ SAR	1⅚ gín 7½ še
No. 2:	1;43,45 SAR	= 1⅔ SAR	3⅔ gín 15 še
No. 3:	0;15 SAR	=	15 gín,

giving a total of 2;50,37,30 SAR = 2⅚ SAR ½ gín 22½ še.

The above relations are too simple in themselves to determine the shape and purpose of the figure as a whole. A clue in this direction may have been contained in the almost completely destroyed line which we have called line 1 of the edge, although it is not completely excluded that it was in reality the first line of the obverse. As a mere conjecture, we propose an arrangement of the three volumes as indicated in fig. 19. The whole figure might be interpreted as the foundation for a wall and a pillar; this would perhaps explain the peculiar terminology of the depths.

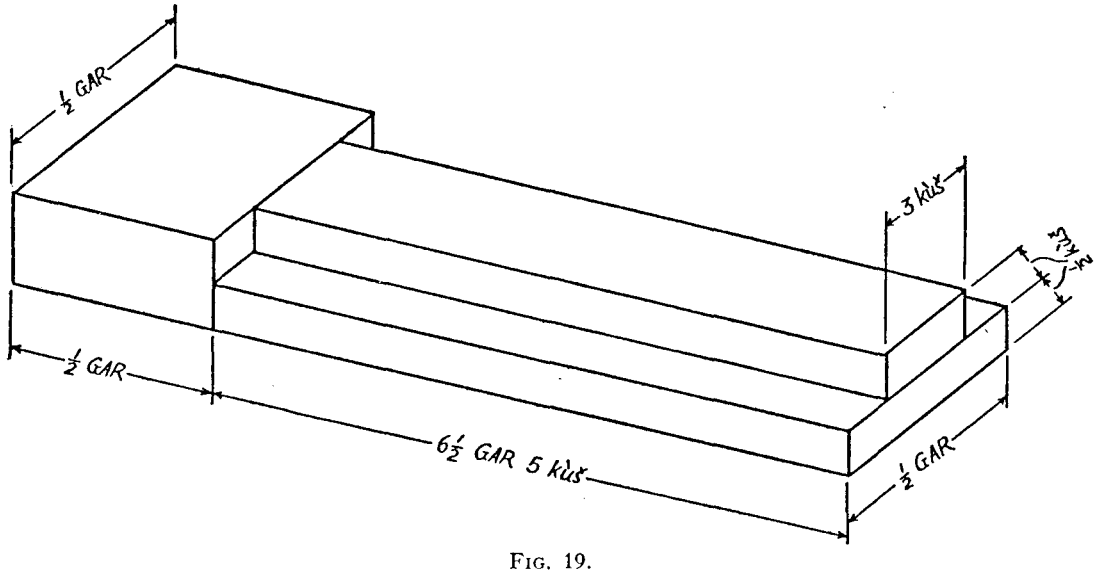


FIG. 19.

Edge

1 [. . . . .] . . . . . [. . . . .]

Commentary

The text deals with three rectangular prisms with the following dimensions:

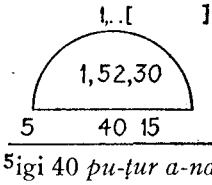
	Length	Width	Depth
No. 1:	6½ GAR 5 kùš	3 kùš	½ kùš
No. 2:	6½ GAR 5 kùš	½ GAR	½ kùš
No. 3:	½ GAR	½ GAR	1 kùš.

Eb. MLC 1354

(Photograph: Plate 47; copy: Plate 21)

Transcription

Obverse



1 GÂN-UD-SAR ⅔ [. . . . .]
2 *i-na* sag 5 GAR *a*-ZU(?)-. . .[162b]
3 1 iku 12½ SAR a-šà . . . . . .
4 uš sag *mi-nu* za-e kì-ta-zu-dè
5 igi 40 *pu-ṭur a-na* 1,52,30 a-šà *i-ši-ma* 2,48,45 *t*[*a-mar*

[162b] One would expect *azūz* here in view of *zi-zi* in line 8.

6igi 15 igi-gub-ba *pu-ṭur-ma* 4 *ta-mar* 4 *a-na* 2,4[8,45]]
7*i-ši-ma* 11,15 *ta-mar* ½-bi gar-ra
8igi 40 *pu-ṭur a-na* 5 *ša zi-zi i-ši-ma*
97,30 *ta-mar ba-ma-at* 7,30 *ḫe-pé šu-tam-ḫír-ma*

Reverse

114,3,45 *ta-mar* 14,3,45
2*a-na* šà 11,[1]5 [gar-gar-ma] 11,29,3,45

(Remainder destroyed.)

Translation

Obverse

(For the figure here, see the transcription.)

1Segment of a circle. Two-thirds [.....]
2from (or: in) the width 5 GAR I divided(?).
31 iku 12½ SAR the area .....
4What are the length and the width? When you perform (the operations),
5Take the reciprocal of 0;40, multiply (it) by 1,52;30, the area, and you will see 2,48;45.
6–7Take the reciprocal of 0;15[152c], the fixed coefficient, and you will see 4. Multiply 4 by 2,4[8;45], and you will see 11,15. Put (down) half of it.[152d]
8Take the reciprocal of 0;40, multiply by 5, of the divided(?) (part(?)), and
9you will see 7;30. Halve 7;30, square (the result), and

Reverse

1–2you will see 14;3,45. [Add] 14;3,45 to 11,[1]5, [and you will see] 11,29;3,45.

(Remainder of reverse destroyed.)

Commentary

We unfortunately know only part of the problem solved in this text. The subject is a segment of a circle, the area of which is given as 1 iku 12½ SAR = 1,52;30 SAR. The role played by the 5 GAR mentioned in line 2 is not clear; the meaning of "length" and "width" is even more obscure.

The general trend of the calculations which are carried out in the text seems obvious. Using the notation

$$A = 1{,}52{;}30 \text{ SAR} \qquad b = \tfrac{2}{3}$$
$$a = 5 \text{ GAR} \qquad c = 0{;}15,$$

we find that the calculations, at the point where the text is destroyed, have led to

$$\frac{A}{bc} + \left(\frac{a}{2b}\right)^2.$$

There is little doubt that this was followed by

$$\sqrt{\frac{A}{bc} + \left(\frac{a}{2b}\right)^2}$$

and the final step for one of the unknown quantities:

$$x = \frac{a}{2b} + \sqrt{\frac{A}{bc} + \left(\frac{a}{2b}\right)^2} = 30.$$

The original expression for x must accordingly have been reducible to

$$x(bx - a) = \frac{A}{c}.$$

We do not know how to interpret this in the geometrical context of the problem.

[152c] Our choice of sexagesimal order here is arbitrary.

[152d] We consider it probable that this sentence is out of place; it should come after "you will see 7;30" in line 9.

Ec. YBC 8600

(Photograph: Plate 47; copy: Plate 21)

Transcription

Obverse

11 kùš *ki-ip-pa-at i-ṣí-*[*i*]*m*
2*i-na ki ma-ṣi ik-bi-ir*
3*at-ta* 5 *it-ti* 5 *šu-ta-ki-il-ma*
425 *a-na* 4,48 igi-gub-ba
5*ta-na-aš-ši-ma* 2 2 sìla *ku-bu-ur* giš
62 s[ìla *ku-*]*bu-ur* giš
7[*ki-ip-pa-a*]*t* giš *mi-nu-ú*

Edge

1[*a*]*t*(?) <*-ta*> igi 4,48
212,30

Reverse

1*a-na* 2 sìla *i-ši-ma*
225 íb-si$_8$ 25
35 1 kùš *ki-pa-at* giš

(Remainder of reverse blank; there are some traces of erasure.)

Translation

Obverse

11 kùš is the circumference of a log.[152e]
2How thick was it?
3As for you—multiply together 0;5 with 0;5, and (the resulting)

[152e] Literally, "wood", "tree."

4–50;0,25 you shall multiply by 4,48, the fixed coefficient, and (the result is) 2. 2 sìla is the thickness of the log.
62 s[ìla is the th]ickness of a log.
7What is [the circumferen]ce of the log?

Edge

1(As for) you(?)—the reciprocal of 4,48, (namely,)
20;0,12,30,

Reverse

1multiply by 2 sìla, and (the result is)
20;0,25. The square root of 0;0,25 is
30;5. 1 kùš is the circumference of the log.

Commentary

The interest of this text lies in the light which it sheds on the details of a hitherto obscure metrological transformation.

Evidence for the curious use of the unit sìla, otherwise known only as a measure of capacity,[152f] to indicate the "thickness" of logs has been available for years.[152g]

The present text gives by example the technique for making the metrological transformation from the circumference measured in units of GAR to the "thickness" measured in units of sìla.

In lines 1–5, the circumference is given as c = 1 kùš = 0;5 GAR, and the "thickness" t is then calculated by

$$\alpha c^2 = t = 2 \text{ sìla},$$

where α, called the "fixed coefficient" (i.e., constant), has the value 4,48. The remainder of the text solves the inverse problem, i.e., the circumference is calculated by

$$\sqrt{\frac{t}{\alpha}} = c = 0;5 \text{ GAR} = 1 \text{ kùš}.$$

The constant α = 4,48 is also mentioned in line 58 of problem-text Ud[152h], which is actually an Old-Babylonian list of constants, as "4,48 of the thickness of a log". Although the value and use of α are thus firmly established, the analysis of the various elements which combined to yield the value 4,48—not to mention an explanation of the use of sìla units in connection with "thickness"—is not easy. The hypothesis, which at first sight would seem to be the most plausible, that the "thickness" merely refers to the *diameter* of the log is definitely ruled out by the fact that the text operates with the *square* of the circumference. The operation points unmistakably toward an area or volume. The hypothesis that an *area* is involved, although not impossible, is not very likely because it would leave us with a new, mysterious sìla unit. Accordingly, in view of the fact that all indications seem to point in the direction of a *volume* relation, we offer the following tentative explanation for α = 4,48. Since the "thickness" is given in units of sìla, it is only natural to start with the now well-attested Old-Babylonian relation between the capacity-sìla and the volume-SAR:[152i]

(1) 1 capacity-sìla = 0;0,0,12 volume-SAR.

Let us, in addition, consider as standard the shape and dimensions (a cube with sides measuring 6 šu-si) given for a sìla container in a mathematical text.[152j] The number of sìla units of the "thickness" of a given log would then correspond to the number of capacity-sìla contained in a cylindrical slice 6 šu-si thick. In the example given by our text, where c = 1 kùš = 0;5 GAR, the volume V of such a slice with a thickness h = 6 šu-si = 0;12 kùš would be

$$V = \frac{c^2}{4\pi}h = (0;5)^2 \cdot 0;5 \cdot 0;12 = 0;0,0,25 \text{ volume-SAR};$$

since the text gives 2 sìla as the "thickness", this would mean that

(2) 1 sìla = 0;0,0,12,30 volume-SAR

and that the value 4,48 for α could be stated as $\frac{0;5 \cdot 0;12}{0;0,0,12,30}$. Since relations (1) and (2) are not reconcilable, let us investigate the effects of a constant α' based on the firmly established relation (1). The value of α' would be $\frac{0;5 \cdot 0;12}{0;0,0,12} = 5,0$. The following table shows a possible explanation for the substitution of α = 4,48 for the more "correct" α' = 5,0:

c	"thickness" calculated on the basis of	
	α = 4,48	α' = 5,0
1 kùš	2 sìla	2;5 sìla
2	8	8;20
3	18	18;45
4	32	33;20
5	50	52;5

[152f] See the table of capacity measures on p. 6.
[152g] Cf., e.g., the Old-Babylonian letter treated by Ungnad BB No. 52,10 and the Neo-Assyrian letter (approximately 7th century B.C.) published by Harper ABL VI No. 566, 12–15 and last translated by Waterman RCAE I pp. 400f.; see also the Neo-Babylonian letter published by Clay NBLE No. 200, 29f. and translated by Ebeling NBU pp. 158f.
[152h] P. 135; this line is parallel to line 35 of Uc (p. 137).
[152i] Cf. p. 96.
[152j] YBC 4669, No. 4 (MKT I, p. 514).

The use of $\alpha' = 5{,}0$ leads to inconvenient fractions of sìla for the "thickness" when the circumference is an integer number of kùš. The deliberate substitution of $\alpha = 4{,}48$ for $\alpha' = 5{,}0$ would be motivated by the desire to establish a relation between circumferences of an integer number of kùš and corresponding "thicknesses" of an integer number of sìla.[152k]

The only other mathematical text which involves the "thickness" of logs is still obscure despite all efforts to penetrate the calculations made by the text.[152l]

§ 5. Excavations (ki-lá)

F. YBC 5037

(Photograph: Plate 30; copy: Plate 5)

Transcription

Obverse

1 [1]ki-lá 3½ GAR 3 kùš-ta-àm íb-si$_8$ 2½ kùš bùr-bi 7½ gín éš-kàr
[2]6 še á-bi a-šà saḫar erim-ḫá ù kù-babbar en-nam

2 [3]kù-babbar ki-lá 9⅓ gín 7½ še 2½ bùr-bi <7½ gín éš-kàr> 6 še á-bi en-ta-à<m> íb-si$_8$ 3½ GAR 3 kùš

3 [4]kù ki-lá 9⅓ gín 7½ <še> 3½ GAR 3 kùš-ta-àm 7½ gín éš-kàr
[5]6 še á-bi bùr-bi en-nam 2½ kùš {kùš}

4 [6]ki-lá 3 GAR 4 kùš uš 2½ GAR sag 3⅓ kùš bùr-bi 10 gín éš-kàr 6 še á-bi a-šà saḫar erim kù <en-nam>

5 [7]kù ki-lá 5½ gín 10 še 2½ GAR sag 3⅓ kùš bùr-bi 10 gín <éš->kàr 6 še á-bi uš en<-nam> 3 GAR 4 kùš

6 [8]kù ki-lá 5½ gín 10 še 3 GAR 4 kùš uš 3⅓ kùš bùr-bi 10 gín éš-kàr
[9]6 še á-bi sag-bi en-nam 2½ GAR sag

7 [10]kù ki-lá 5½ gín 10 še 3 GAR 4 kùš uš 2½ GAR sag 10 gín éš-kàr
[11]6 še á-bi lú-ḫun-gá bùr-bi en-nam 3⅓ kùš bùr-bi

8 [12]kù ki-lá 5½ gín 10 še 3⅓ kùš bùr-bi 10 gín éš-kàr 6 še á-bi
[13]uš sag gar-gar-ma 5½ GAR 4 kùš uš sag en-nam 3 GAR 4 kùš uš 2½ GAR sag-bi

9 [14]kù ki-lá 5½ gín 10 še 3⅓ kùš bùr-bi 10 gín éš-kàr 6 še á-bi uš ugu sag ½ GAR 4 kùš dirig
[15]uš-bi sag-bi en-nam 3 GAR 4 kùš uš-bi 2½ GAR sag-bi

10 [16]ki-lá 5 GAR uš 1½ GAR sag ½ GAR bùr-bi 10 gín éš-kàr 6 še á-bi a-šà saḫar erim kù-babbar en-nam

11 [17]kù ki-lá 9 gín ½ GAR bùr-bi 10 gín éš-kàr 6 še á-bi uš sag gar-gar-ma
[18]6½ GAR uš sag en-nam 5 GAR uš 1½ GAR sag-bi

12 [19]kù ki-lá 9 gín ½ GAR bùr-bi 10 gín éš-kàr 6 še á-bi
[20]uš ugu sag 3½ GAR dirig uš sag en-nam 5 GAR uš 1½ sag-bi

13 [21]kù ki-lá 9 gín 2½ GAR[153] sag 10 gín éš-kàr 6 še á-bi uš bùr-bi gar-gar 5½ GAR uš bùr en-nam

14 [22]kù ki-lá 9 gín 2½ GAR[153] sag 10 gín <éš->kàr 6 á-bi uš ugu bùr 4½ GAR dirig uš bùr en-nam

15 [23]kù ki-lá 9 gín 5 GAR uš 10 gín <éš->kàr 6 á-bi sag bùr gar-gar 2 kùš[154] uš[155] bùr en-nam

16 [24]kù ki-lá 9 gín 5 GAR uš 10 gín <éš->kàr 6 á-bi sag ugu bùr 1 GAR dirig sag bùr en-nam

17 [25]saḫar ki-lá 45 [SA]R ½ GAR bùr-bi šu-ri-a uš sag 1 GAR sag ba-zi uš sag en-nam

18 [26]saḫar ki-lá 45 SAR ½ GAR bùr igi-3-gál uš sag 2 kùš sag ba-zi uš sag en-nam

19 [27]saḫar ki-lá 45 SAR ½ GAR bùr igi-4-gál uš sag 3 kùš sag daḫ uš sag en-nam

20 [28]saḫar ki-lá 45 SAR ½ GAR bùr igi-3-gál uš sag 2 GAR uš daḫ ½ GAR sag daḫ uš [sag en-]nam

21 [29]saḫar ki-lá 45 SAR ½ GAR bùr igi-4-gál uš sag 1 GAR uš daḫ ½ GAR sag daḫ u[š sag en-nam]

22 [30]saḫar ki-lá 45 SAR ½ GAR bùr igi-6-gál uš sag 2 GAR uš daḫ 1 GAR sag daḫ u[š sag en-nam]

23 [31]saḫar ki-lá 45 SAR ½ GAR bùr igi-3-gál uš sag ½ GAR uš daḫ uš sag en[-nam]

[152k] The assumption of the value $\pi \approx 3;7,30$ (decimally, 3.125) would lead to $V = 0;0,0,24$ volume-SAR and thus directly to $\alpha = \frac{4;48 \cdot 0;12}{0;0,0,12} = 4{,}48$. This assumption, however, seems to us unwarranted because this approximation of π is not attested elsewhere in Babylonian mathematics.

[152l] VAT 8522 (MKT I, p. 368), obverse I. Cf. MKT III, p. 61, Waschow [1], p. 247, von Soden [1], p. 200, and von Soden [2], p. 151. The only new contribution to the reading of this text which we can offer is the reading 4 ba-si 2 ba-si for the line given directly below the figure in the transcription in MKT.

[153] Mistake for 1½ GAR.

[154] Error for GAR.

[155] Mistake for sag, "the width."

24 [32]saḫar ki-lá 45 SAR 1½ GAR sag igi-3 <-gál> uš bùr 3½ GAR uš daḫ uš bùr-bi [en-nam]

25 [33]saḫar ki <-lá> 45 SAR 1½ GAR sag igi-3-gál uš bùr 4 GAR uš daḫ 2 kùš [bùr daḫ uš bùr en-nam]

[33a]5 GAR uš ½ GA[R bùr]

Reverse

26 [1]saḫar ki-lá 45 SAR 1½ GAR sag igi-4-gál uš bùr 4 GAR uš daḫ 3 kùš bùr daḫ uš bùr [en-nam]

27 [2]saḫar ki-lá 4[5 SAR] 1½ sag igi-5-gál uš bùr 2½ uš daḫ uš bùr en-nam 5 uš [½ bùr]

28 [3]saḫar ki-lá 45 SAR 1½ sag igi-6-gál uš bùr 2 GAR uš daḫ uš bùr-bi en-nam

29 [4]saḫar ki-lá 45 SAR 1½ sag igi-11-gál uš bùr ½ GAR uš ba-zi uš bùr-bi en-nam

30 [5]saḫar ki-lá 45 SAR 1½ GAR sag igi-3-gál uš bùr 3 GAR uš daḫ 2 kùš bùr ba-zi uš bùr en[-nam]

31 [6]saḫar ki-lá 4[5 SAR] 5 GAR uš ig[i-3-gál] sag bùr sag bùr en-nam 1½ sag ½[155a] bùr[-bi]

32 [7]saḫar ki-lá 45 SAR 5 uš š[u-ri-a] sag bùr 3 kùš bùr b[a-z]i sag bùr en-nam

33 [8]saḫar ki-lá 45 SAR 5 uš i[gi-6-]gál sag bùr ½ GAR <sag> b[a-z]i 2 kùš bùr daḫ sag bùr en-nam

34 [9]saḫar ki-lá 45 SAR 5 [uš ig]i-4-gál sag bùr 1½ kùš bùr daḫ sag bùr en-nam 1½ sag ½ bùr

35 [10]ki-lá ½ GA[R-ta-à]m an-ta íb-si$_8$ 4 kùš-ta-àm ki <-ta> íb-si$_8$ ½ GAR bùr

[11]saḫar-bi en-nam {1} 1 SAR 5 gín saḫar-bi

36 [12]saḫar ki-lá 1 SAR 5 gín <½ GAR bùr> 4 kùš ki-ta íb-si$_8$ an-ta en-ta-àm íb-si$_8$ ½ GAR an-ta

37 [13]saḫar ki-lá 1 SAR 5 gín <½ GAR bùr> ½ GAR-ta-àm an-na íb-si$_8$ en-ta-à <m> ki-ta 4 kùš

38 [14]saḫar ki-lá 1 SAR 5 gín ½ GAR-ta-àm an-ta íb-si$_8$ 4 kùš-ta-àm ki-ta íb-si$_8$

[15]bùr-bi [en-n]am ½ GAR bùr-bi

39 [16]saḫar ki[-lá 1 SAR 5] gín <½ GAR bùr> íb-si$_8$ an-ta ù íb[-si$_8$] ki-ta gar-gar ½ GAR 4 kùš

[17]íb-s[i$_8$ an-]ta íb-si$_8$ ki-ta <en-nam> ½ GAR íb-si$_8$ an-ta 4 kùš íb-si$_8$ ki-ta

40 [18]saḫar ki-lá 1 SAR 5 gín ½ GAR bùr íb-si$_8$ an-na ugu íb-si$_8$ ki-ta 2 kùš dirig

[19]íb-si$_8$ a[n-t]a íb-si$_8$ <ki->ta <en-nam> ½ GAR íb-si$_8$ an-na 4 kùš íb-si$_8$ ki-ta

41 [20]saḫar ki[-lá 1 SAR 5] gín ½ GAR bùr ⅔ íb-si$_8$ an-ta íb-si$_8$ ki-ta

[21]íb-si$_8$ [an-ta í]b-si$_8$ ki-ta en-nam ½ GAR íb-si$_8$ an-ta 4 kùš ki-ta

42 [22]saḫar ki-lá 1 SAR 5 gín ½ GAR bùr šu-ri-a íb-si$_8$ an-ta ù 1 kùš íb-si$_8$ ki-ta

[23]íb-si$_8$ an-ta íb-si$_8$ <ki->ta en-nam ½ GAR an-na 4 kùš ki-ta

43 [24]saḫar ki-lá 1 SAR 5 gín ½ GAR bùr igi-3-gál íb-si$_8$ an-ta ù 2 kùš ki-ta

[25]íb-si$_8$ an-na ù íb-si$_8$ ki-ta en-nam ½ GAR an-na 4 kùš ki-ta

44 [26]saḫar ki-lá 1 SAR 5 gín ½ GAR <bùr> igi-6-gál íb-si$_8$ an-ta ù 3 kùš ki-ta

[27]íb-si$_8$ an-na ù íb-si$_8$ ta-ta[156] en-nam ½ GAR an-na 4 kùš ki-ta

[28]44 im-šu

Translation

Obverse

1 [1]A ki-lá. 3½ GAR (and) 3 kùš is each square-side, 2½ kùš its depth, 7½ gín (volume) the assignment,

[2]6 še (silver) the wages. What are the area, the volume, the (number of) workers, and the (total expenses in) silver?

2 [3]The (total expenses in) silver of a ki-lá are 9⅓ gín (and) 7½ še, 2½ (kùš) its depth, <7½ gín (volume) the assignment>, 6 še (silver) the wages. What is each square-side? 3½ GAR (and) 3 kùš.

3 [4]The (total expenses in) silver of a ki-lá are 9⅓ gín (and) 7½ <še>, 3½ GAR (and) 3 kùš each (square-side), 7½ gín (volume) the assignment,

[5]6 še (silver) the wages. What is its depth? 2½ kùš {kùš}.

4 [6]A ki-lá. 3 GAR (and) 4 kùš is the length, 2½ GAR the width, 3⅓ kùš its depth, 10 gín (volume) the assignment, 6 še (silver) the wages. <What are> the area, the volume, the (number of) workers, (and) the (total expenses in) silver?

5 [7]The (total expenses in) silver of a ki-lá are 5½ gín (and) 10 še, 2½ GAR the width, 3⅓ kùš its depth, 10 gín (volume) the assignment, 6 še (silver) the wages. What is the length? 3 GAR (and) 4 kùš.

[155a] The sign for ½ was left out by mistake in our copy on Plate 5.

[156] Error for ki-ta.

6 [8]The (total expenses in) silver of a ki-lá are 5½ gín (and) 10 še, 3 GAR (and) 4 kùš the length, 3⅓ kùš its depth, 10 gín (volume) the assignment,
[9]6 še (silver) the wages. What is its width? 2½ GAR is the width.

7 [10]The (total expenses in) silver of a ki-lá are 5½ gín (and) 10 še, 3 GAR (and) 4 kùš the length, 2½ GAR the width, 10 gín (volume) the assignment,
[11]6 še (silver) the wages of a hired man. What is its depth? 3⅓ kùš is its depth.

8 [12]The (total expenses in) silver of a ki-lá are 5½ gín (and) 10 še, 3⅓ kùš its depth, 10 gín (volume) the assignment, 6 še (silver) the wages;
[13]I added the length and the width, and (the result is) 5½ GAR (and) 4 kùš. What are the length (and) the width? 3 GAR (and) 4 kùš is the length; 2½ GAR is its width.

9 [14]The (total expenses in) silver of a ki-lá are 5½ gín (and) 10 še, 3⅓ kùš its depth, 10 gín (volume) the assignment, 6 še (silver) the wages; the length exceeded the width by ½ GAR (and) 4 kùš.
[15]What are its length (and) its width? 3 GAR (and) 4 kùš is its length; 2½ GAR is its width.

10 [16]A ki-lá. 5 GAR is the length, 1½ GAR the width, ½ GAR its depth, 10 gín (volume) the assignment, 6 še (silver) the wages. What are the area, the volume, the (number of) workers, (and) the (total expenses in) silver?

11 [17]The (total expenses in) silver of a ki-lá are 9 gín, ½ GAR its depth, 10 gín (volume) the assignment, 6 še (silver) the wages; I added the length and the width, and (the result is)
[18]6½ GAR. What are the length and the width? 5 GAR is the length; 1½ GAR is its width.

12 [19]The (total expenses in) silver of a ki-lá are 9 gín, ½ GAR its depth, 10 gín (volume) the assignment, 6 še (silver) the wages;
[20]the length exceeded the width by 3½ GAR. What are the length (and) the width? 5 GAR is the length; 1½ (GAR) is its width.

13 [21]The (total expenses in) silver of a ki-lá are 9 gín, 2½ GAR[153] the width, 10 gín (volume) the assignment, 6 še (silver) the wages; I added the length (and) its depth, (and the result is) 5½ GAR. What are the length and the depth?

14 [22]The (total expenses in) silver of a ki-lá are 9 gín, 2½ GAR[153] the width, 10 gín (volume) the assignment, 6 (še silver) the wages; the length exceeded the depth by 4½ GAR. What are the length and the depth?

15 [23]The (total expenses in) silver of a ki-lá are 9 gín, 5 GAR the length, 10 gín (volume) the assignment, 6 (še silver) the wages; I added the width and the depth, (and the result is) 2 kùš[154]. What are the length[155] and the depth?

16 [24]The (total expenses in) silver of a ki-lá are 9 gín, 5 GAR the length, 10 gín (volume) the assignment, 6 (še silver) the wages; the width exceeded the depth by 1 GAR. What are the width and the depth?

17 [25]The volume of a ki-lá is 45 [SA]R, ½ GAR its depth; one-half of the length is the width (after) I added[157] 1 GAR (to) the width. What are the length and the width?

18 [26]The volume of a ki-lá is 45 SAR, ½ GAR the depth; one-third of the length is the width (after) I added[157] 2 kùš (to) the width. What are the length and the width?

19 [27]The volume of a ki-lá is 45 SAR, ½ GAR the depth; one-fourth of the length is the width (after) I subtracted[158] 3 kùš (from) the width. What are the length and the width?

20 [28]The volume of a ki-lá is 45 SAR, ½ GAR the depth; one-third of the length is the width (after) I subtracted[158] 2 GAR (from) the length (and) I subtracted[158] ½ GAR (from) the width. What are the length and [the width]?

21 [29]The volume of a ki-lá is 45 SAR, ½ GAR the depth; one-fourth of the length is the width (after) I subtracted[158] 1 GAR (from) the length (and) I subtracted[158] ½ GAR (from) the width. [What are the] len[gth and the width]?

22 [30]The volume of a ki-lá is 45 SAR, ½ GAR the depth; one-sixth of the length is the width (after) I subtracted[158] 2 GAR (from) the length (and) I subtracted[158] 1 GAR (from) the width. [What are the] len[gth and the width]?

23 [31]The volume of a ki-lá is 45 SAR, ½ GAR the depth; one-third of the length is the width (after) I subtracted[158] ½ GAR (from) the length. What are the length and the width?

24 [32]The volume of a ki-lá is 45 SAR, 1½ GAR the width; one-third of the length is the depth

[157] Text: "subtracted (from)"; cf. commentary to Nos. 17–34 on p. pp. 64f.
[158] Text: "added (to)"; cf. commentary to Nos. 17–34 on pp. 64f.

(after) I subtracted[158] $3\frac{1}{2}$ GAR (from) the length. [What] are the length and its depth?

25 [33]The volume of a ki<-lá> is 45 SAR, $1\frac{1}{2}$ GAR the width; one-third of the length is the depth (after) I subtracted[158] 4 GAR (from) the length (and) [I subtracted[158]] 2 kùš [(from) the depth. What are the length and the depth?]

[33a]5 GAR is the length; $\frac{1}{2}$ GA[R is the depth].

Reverse

26 [1]The volume of a ki-lá is 45 SAR, $1\frac{1}{2}$ GAR the width; one-fourth of the length is the depth (after) I subtracted[158] 4 GAR (from) the length (and) I subtracted[158] 3 kùš (from) the depth. [What] are the length and the depth?

27 [2]The volume of a ki-lá is 4[5 SAR], $1\frac{1}{2}$ (GAR) the width; one-fifth of the length is the depth (after) I subtracted[158] $2\frac{1}{2}$ (GAR from) the length. What are the length and the depth? 5 (GAR) is the length; [$\frac{1}{2}$ (GAR) is the depth].

28 [3]The volume of a ki-lá is 45 SAR, $1\frac{1}{2}$ (GAR) the width; one-sixth of the length is the depth (after) I subtracted[158] 2 GAR (from) the length. What are the length and its depth?

29 [4]The volume of a ki-lá is 45 SAR, $1\frac{1}{2}$ (GAR) the width; one-eleventh of the length is the depth (after) I added[157] $\frac{1}{2}$ GAR (to) the length. What are the length and its depth?

30 [5]The volume of a ki-lá is 45 SAR, $1\frac{1}{2}$ GAR the width; one-third of the length is the depth (after) I subtracted[158] 3 GAR (from) the length (and) I added[157] 2 kùš (to) the depth. What are the length (and) the depth?

31 [6]The volume of a ki-lá is 4[5 SAR], 5 GAR the length; one[-third] of the width is the depth. What are the width and the depth? $1\frac{1}{2}$ (GAR) is the width; $\frac{1}{2}$ (GAR) is [its] depth.

32 [7]The volume of a ki-lá is 45 SAR, 5 (GAR) the length; [one-hal]f of the width is the depth (after) I a[dd]ed[157] 3 kùš (to) the depth. What are the width (and) the depth?

33 [8]The volume of a ki-lá is 45 SAR, 5 (GAR) the length; o[ne-six]th of the width is the depth (after) I a[dd]ed[157] $\frac{1}{2}$ GAR (to) <the width> (and) I subtracted[158] 2 kùš (from) the depth. What are the width (and) the depth?

34 [9]The volume of a ki-lá is 45 SAR, 5 (GAR) [the length; on]e-fourth of the width is the depth (after) I subtracted[158] $1\frac{1}{2}$ kùš (from) the depth. What are the width (and) the depth? $1\frac{1}{2}$ (GAR) is the width; $\frac{1}{2}$ (GAR) is the depth.

35 [10]A ki-lá. [Ea]ch upper square-side is $\frac{1}{2}$ GAR, each lower square-side 4 kùš, $\frac{1}{2}$ GAR the depth.

[11]What is its volume? {1} 1 SAR (and) 5 gín is its volume.

36 [12]The volume of a ki-lá is 1 SAR (and) 5 gín, <$\frac{1}{2}$ GAR the depth>, (each) lower square-side 4 kùš. What is each upper square-side? $\frac{1}{2}$ GAR is (each) upper (square-side).

37 [13]The volume of a ki-lá is 1 SAR (and) 5 gín, <$\frac{1}{2}$ GAR the depth>, each upper square-side $\frac{1}{2}$ GAR. What is each lower (square-side)? 4 kùš.

38 [14]The volume of a ki-lá is 1 SAR (and) 5 gín, each upper square-side $\frac{1}{2}$ GAR, each lower square-side 4 kùš.

[15]What is its depth? $\frac{1}{2}$ GAR is its depth.

39 [16]The volume of a ki[-lá is 1 SAR (and) 5] gín, <$\frac{1}{2}$ GAR the depth>; I added the upper square-side and the lower squ[are]-side, (and the result is) $\frac{1}{2}$ GAR (and) 4 kùš.

[17]<What are> the [up]per squa[re]-side (and) the lower square-side? $\frac{1}{2}$ GAR is the upper square-side; 4 kùš is the lower square-side.

40 [18]The volume of a ki-lá is 1 SAR (and) 5 gín, $\frac{1}{2}$ GAR the depth; the upper square-side exceeded the lower square-side by 2 kùš.

[19]<What are> the upper square-side and the lower square-side? $\frac{1}{2}$ GAR is the upper square-side; 4 kùš is the lower square-side.

41 [20]The volume of a ki[-lá is 1 SAR (and) 5] gín, $\frac{1}{2}$ GAR the depth; $\frac{2}{3}$ of the upper square-side is the lower square-side.

[21]What are the [upper] square-side and the lower square-side? $\frac{1}{2}$ GAR is the upper square-side; 4 kùš is the lower (square-side).

42 [22]The volume of a ki-lá is 1 SAR (and) 5 gín, $\frac{1}{2}$ GAR the depth; one-half of the upper square-side and 1 kùš is the lower square-side.

[23]What are the upper square-side (and) the lower square-side? $\frac{1}{2}$ GAR is the upper (square-side); 4 kùš is the lower (square-side).

43 [24]The volume of a ki-lá is 1 SAR (and) 5 gín, $\frac{1}{2}$ GAR the depth; one-third of the upper square-side and 2 kùš is the lower (square-side).

[25]What are the upper square-side and the lower square-side? $\frac{1}{2}$ GAR is the upper (square-side); 4 kùš is the lower (square side).

44 [26]The volume of a ki-lá is 1 SAR (and) 5 gín, ½ GAR <the depth>; one-sixth of the upper square-side and 3 kùš is the lower (square-side).

[27]What are the upper square-side and the lower square-side? ½ GAR is the upper (square-side); 4 kùš is the lower (square-side).

[28]44 sections.

Commentary

a. Mathematical Commentary

This tablet contains a systematically arranged list of 44 problems dealing with the volumes of prisms (square and rectangular) and a truncated square pyramid. The shapes in question, however, cannot be concluded from the terminology alone but only by the simultaneous analysis of the numerical relations between the given quantities and the indicated solutions. Each group of problems appears to suppose the same basic relation for all problems of the group. Designating the volume by V, we have

$$(1)\quad \begin{array}{ll} \text{Nos. 1 to 3:} & V = x^2z = 35;9,22,30 \text{ SAR} \\ \text{Nos. 4 to 9:} & V = xyz = 27;46,40 \text{ SAR} \\ \text{Nos. 10 to 34:} & V = xyz = 45 \text{ SAR} \\ \text{Nos. 35 to 44:} & V = \tfrac{1}{2}(x_l^2 + x_u^2)z = 1;5 \text{ SAR} \end{array}$$

where x is the length (uš), y the width (sag), z the depth (bùr); and x_l the lower (íb-si$_8$ ki-ta), x_u the upper square-side (íb-si$_8$ an-ta) of the truncated square pyramid. These volumes, however, only occur indirectly in many examples through a specific volume λ which indicates one man's output of work (éš-kàr) per day. In addition, daily wages per man w (á) and the total expenditure for labor E (kù = "silver") are given. This means that the relation

$$(2)\quad E = \frac{V}{\lambda} w$$

holds. The wages w are the same in all examples, namely, 6 še = 0;2 gín silver, while the given output λ fluctuates between 7;30 gín (Nos. 1 to 3) and 10 gín (Nos. 4 to 34) volume. In three examples (Nos. 1, 4 and 10), the number m of workers (erim-ḫá) is given as a problem but never answered. The answer would be given by

$$(3)\quad m = \frac{E}{w}$$

and need not be an integer because the number of men of course means nothing more than the number of single man-days of work.

The following list shows the arrangement of the problems and gives the values of the magnitudes involved which satisfy the basic relations (1) and (2) for V and E. The solutions frequently call for quadratic equations (Nos. 8 and 9; 11 to 34; 39 to 44).

Nos. 1 to 3.

Square prism:

x = 3½ GAR 3 kùš = 3;45 GAR
z = 2;30 kùš
λ = 7;30 gín = 0;7,30 SAR
w = 6 še = 0;2 gín
E = 9⅓ gín 7½ še = 9;22,30 gín.

The corresponding volume would be

$$V = x^2z = 35;9,22,30 \text{ SAR.}$$

The following tabulation describes the problems:

No.	given	find	result indicated by the text
1	x, z, λ, w	x^2, V, m, E	—
2	$E, z, \langle\lambda\rangle, w$	x	$x = 3\frac{1}{2}$ GAR 3 kùš
3	E, x, λ, w	z	$z = 2\frac{1}{2}$ kùš

Nos. 4 to 9.

Rectangular prism:

x = 3 GAR 4 kùš = 3;20 GAR
y = 2;30 GAR
z = 3;20 kùš
λ = 10 gín = 0;10 SAR
w = 6 še = 0;2 gín
E = 5½ gín 10 še = 5;33,20 gín.

The volume is given by

$$V = xyz = 27;46,40 \text{ SAR.}$$

The problems are:

No.	given	find	result indicated by the text
4	x, y, z, λ, w	xy, V, m, E	—
5	E, y, z, λ, w	x	$x = 3$ GAR 4 kùš
6	E, x, z, λ, w	y	$y = 2\frac{1}{2}$ GAR
7	E, x, y, λ, w	z	$z = 3\frac{1}{3}$ kùš
8	$E, z, \lambda, w, x + y$	x, y	$x = 3$ GAR 4 kùš
9	$E, z, \lambda, w, x - y$	x, y	$y = 2\frac{1}{2}$ GAR

The last two examples lead to the fundamental types of quadratic equations, which require the calculation of two unknown quantities if their product and their sum or difference are given.

Nos. 10 to 34.

This new group also deals with a rectangular prism but assumes the following new set of values:

x = 5 GAR λ = 10 gín = 0;10 SAR
y = 1;30 GAR w = 6 še = 0;2 gín
z = ½ GAR = 6 kùš E = 9 gín.
$V = xyz$ = 45 SAR

The first example, like the first four examples in the preceding group, requires nothing more than the knowledge of the basic relation (1). The next six examples lead to the basic types of quadratic equations for all combinations of x, y and z, as the following list shows:

No.	given	find	result indicated by the text
10	x, y, z, λ, w	xy, V, m, E	—
11	$E, z, \lambda, w, x + y$	x, y	x = 5 GAR, $y = 1\frac{1}{2}$ GAR
12	$E, z, \lambda, w, x - y$	x, y	x = 5 GAR, $y = 1\frac{1}{2}$ (GAR)
13	$E, y, \lambda, w, x + z$	x, z	—
14	$E, y, \lambda, w, x - z$	x, z	—
15	$E, x, \lambda, w, y + z$	(y)[159], z	—
16	$E, x, \lambda, w, y - z$	y, z	—

In the remaining examples of this group involving a rectangular prism, the linear relation between two of the unknowns is of the general type $ax + by = c$ (and analogously for x, z and y, z):

No.	given	find	result indicated by the text
17	$V, z, \frac{x}{2} = y + 1$	x, y	—
18	$V, z, \frac{x}{3} = y + \frac{2}{12}$	x, y	—
19	$V, z, \frac{x}{4} = y - \frac{3}{12}$	x, y	—
20	$V, z, \frac{x-2}{3} = y - \frac{1}{2}$	x, y	—
21	$V, z, \frac{x-1}{4} = y - \frac{1}{2}$	x, y	—
22	$V, z, \frac{x-2}{6} = y - 1$	x, y	—
23	$V, z, \frac{x-\frac{1}{2}}{3} = y$	x, y	—
24	$V, y, \frac{x-3\frac{1}{2}}{3} = z$	x, z	—
25	$V, y, \frac{x-4}{3} = z - \frac{2}{12}$	x, z	x = 5 GAR, $z = \frac{1}{2}$ GAR
26	$V, y, \frac{x-4}{4} = z - \frac{3}{12}$	x, z	—
27	$V, y, \frac{x-2\frac{1}{2}}{5} = z$	x, z	x = 5, $[z = \frac{1}{2}]$
28	$V, y, \frac{x-2}{6} = z$	x, z	—
29	$V, y, \frac{x+\frac{1}{2}}{11} = z$	x, z	—
30	$V, y, \frac{x-3}{3} = z + \frac{2}{12}$	x, z	—
31	$V, x, \frac{y}{3} = z$	y, z	$y = 1\frac{1}{2}$, $z = \frac{1}{2}$
32	$V, x, \frac{y}{2} = z + \frac{3}{12}$	y, z	—
33	$V, x, \frac{y+\frac{1}{2}}{6} = z - \frac{2}{12}$	y, z	—
34	$V, x, \frac{y}{4} = z - \frac{1\frac{1}{2}}{12}$	y, z	$y = 1\frac{1}{2}$, $z = \frac{1}{2}$

Except for No. 31, all these problems require the solution of quadratic equations.

The scribe of this text makes consistent use of a peculiar notation for addition and subtraction in Nos. 17 to 34. Thus, although the literal translation of a part of No. 17 is "one-half of the length is the width; I *subtracted* 1 GAR (from) the width", this signifies in modern notation

$$\frac{x}{2} = y + 1. \tag{4}$$

Conversely, "one-fourth of the length is the width; I *added* 3 kùš (to) the width" in No. 19 corresponds to

$$\frac{x}{4} = y - \frac{3}{12}. \tag{5}$$

The simplest explanation, namely, the supposition of an erroneous interchange of daḫ and zi (which otherwise mean "add" and "subtract", respectively), is

[159] Erroneously replaced by x in the text.

almost certainly excluded by the same strange-looking use of daḫ and zi in analogous contexts in Strassburg 363 (MKT I pp. 243ff.) obv. 1f., 13f., rev. 8f. Thus Strassburg 363 obv. 13 f., literally, "(one) square-side is $\frac{2}{3}$ of the (other) square-side; I *added* 10 to the large(r) square-side, I *added* 5 to the small(er) square-side", corresponds to

(6) $\quad \frac{2}{3}(x - 10) = y - 5 \qquad x > y.$

The difficulty in interpreting daḫ and zi in a literal manner in all these passages is in all probability our own fault, and we would tentatively offer the following alternative interpretation, which at least has the advantage of preserving the normal literal meanings of daḫ and zi. If we interpret "length", "width" and "square-side" of the original text as unprecise terms for "(tentative) length", etc., and if we then denote the *real* length by x, the *real* width by y, and the *preliminary*, or *tentative*, length and width by x_1 and y_1, respectively, we would get instead of equations (4) and (5),

(4′) $\quad \frac{x_1}{2} = y_1 \qquad y = y_1 - 1$

and

(5′) $\quad \frac{x_1}{4} = y_1 \qquad y = y_1 + \frac{3}{12}$

and, correspondingly, instead of (6),

(6′) $\quad \frac{2}{3}x_1 = y_1 \qquad x = x_1 + 10 \qquad y = y_1 + 5.$

It is immediately obvious that these equations are equivalent with (4), (5) and (6). In this way, no tampering with the usual meanings of daḫ and zi is necessary. For the sake of convenience, our translations reflect equations of the type (4), (5) and (6) instead of (4′), (5′) and (6′).

Nos. 35 to 44.

Square truncated pyramid:

$x_l = 0;20$ GAR $\qquad z = 0;30$ GAR $= 6$ kùš
$x_u = 0;30$ GAR $\qquad V = \frac{1}{2}(x_l^2 + x_u^2)z = 1;5$ SAR,

where x_l and x_u denote the lower and upper square-sides, respectively, and V the volume calculated according to the approximative formula indicated. Expenses and wages are not mentioned in this group.

A published tablet[160] of the same type also contains examples dealing with a truncated pyramid, the volume of which is calculated according to the same approximative formula as here.[161] The only difference lies in the fact that YBC 4708 assumes the truncated pyramid as the form of a pile of bricks and therefore makes the lower square-side greater than the upper, whereas in the present text, the upper square-side is the greater—which indicates that a hole in the ground, like the foundation of a building or the like, is assumed.[162]

No.	given	find	result indicated by the text
35	x_l, x_u, z	V	V = 1 SAR 5 gín
36	V, $<z>$, x_l	x_u	$x_u = \frac{1}{2}$ GAR
37	V, $<z>$, x_u	x_l	x_l = 4 kùš
38	V, x_l, x_u	z	$z = \frac{1}{2}$ GAR
39	V, $<z>$, $x_l + x_u$	x_l, x_u	$x_u = \frac{1}{2}$ GAR
40	V, z, $x_u - x_l$	x_l, x_u	x_l = 4 kùš
41	V, z, $\frac{2}{3}x_u = x_l$	x_l, x_u	
42	V, z, $\frac{x_u}{2} + \frac{1}{12} = x_l$	x_l, x_u	
43	V, z, $\frac{x_u}{3} + \frac{2}{12} = x_l$	x_l, x_u	$x_u = \frac{1}{2}$ GAR, x_l = 4 kùš
44	V, z, $\frac{x_u}{6} + \frac{3}{12} = x_l$	x_l, x_u	

b. Terminology

ki-lá

Although the mathematical meaning of ki-lá is not subject to doubts (cf. the mathematical commentary), the term itself is very obscure. The attested Akkadian equivalents (cf. Deimel, ŠL 461, 238) consist of the verb *napalsuḫu*, various shades of meaning connected with the verb *sanāqu*, the noun *šuqultu* ("weight"), and some specific use of the term *šikittu*. None of these seems, at least superficially, to fit our context, for although *šikittu* is indeed used in an architectural context (cf. Delitzsch, HWB 660a = Streck, Assurb. p. 272, l. 13 and p. 276, l. 13), the meaning seems to be more general ("shape", "plan", "form", or the like) and is apparently not directly pertinent to the use of ki-lá in our texts. The Akkadian correspondences for giš-ki-lá (cf. Deimel, ŠL 461, 240) also do not seem to be helpful, since, to judge from the contexts of the vocabularies in which they occur, *mekū* is a type of instrument used to breach a wall and *maštaktu* some kind of instrument for measuring time.

In view of all that we know about ki-lá from the extant mathematical texts, it is perhaps worth suggesting the possibility that ki-lá is the Sumerian

[160] YBC 4708 (MKT I pp. 389ff.).

[161] Nos. 53 to 60. The arrangement of these examples follows exactly the present scheme.

[162] Cf. MKT I pp. 187 and 229.

origin of the Akkadian loan-word *kalakkum*, "cellar", "silo", or the like: ki-lá-k> *kalá-k> *kalákkum*.[162a] Cf., however, the signs NIGÍN containing AL, BUR, or KI—all with the value gala and corresponding to Akkadian *kalakkum* (Deimel, ŠL 498, 2; 511, 23; and 514, 2, respectively).

éš-kàr

For the sake of brevity and for want of a better term, we have translated éš-kàr throughout this book by "assignment". In general, éš-kàr seems to refer to the amount of material or work turned over or assigned to a worker for a specific task. In our texts, the term refers uniquely to the work which one worker was expected to perform in one day.[162b]

G, H and J. YBC 4657, YBC 4663, YBC 4662

Transcription of G (YBC 4657)

(Photograph: Plate 31; copy: Plate 6)

Obverse

1 [1][k]i-lá [5 GAR uš 1½] GAR sag ½ GAR bùr-bi 10 gín saḫar éš-kàr 6 še á-[b]i [lú-ḫ]un-g[á]
[2]gagar saḫa[r-ḫá erim-ḫá] ù kù-bab[bar] <en->
n[am] 7½ gagar 45 saḫar-ḫá
[3]4,30 [erim-ḫá] 9 gín kù-babbar

2 [4]kù ki[-lá 9 gín] 1½ GAR sag ½ GAR <bùr-bi>
10 gín éš-kàr 6 še á-bi lú-ḫun-gá
[5]uš-bi [en-nam] 5 GAR {GAR} uš

3 [6]kù [k]i-l[á 9 gí]n 5 GAR uš ½ GAR bùr-bi 10 gín éš-kàr 6 še á-bi lú-ḫun-gá
[7]sag-bi [en-na]m 1½ GAR sag

4 [8]kù ki[-lá 9 gín 5] GAR uš-bi 1½ GAR sag 10 gín éš-kàr
[9]6 še á-bi lú-ḫun-gá bùr-bi en-nam [½ GAR] bùr-bi

5 [10]kù ki-lá 9 gín 5 GAR uš 1½ G[AR s]ag [½ GAR bùr-b]i 6 še á-bi lú-ḫun-gá
[11]saḫar éš-kàr en-[na]m [10 gín saḫar] éš-kàr

6 [12]kù ki-lá [9 gín] 5 [GAR uš 1½ GAR sag ½ G]AR bùr-bi 10 gín éš-kàr
[13]á-bi lú-ḫ[un-g]á e[n-nam 6 še] á-bi lú-ḫun-gá

7 [14]kù ki-lá [9 gín ½] bùr[-bi 10 gín éš-kàr 6 š]e á-bi lú-ḫun-gá
[15]uš [ù sag] gar-g[ar-ma 6,30 uš ù] sag en-nam

8 [16]kù [ki-lá 9 gín ½ GAR bùr-bi 10 gín] éš-kàr 6 še á-bi lú-ḫun-gá
[17][uš ugu sag 3,30 dirig] uš ù sag en-nam 5 GAR uš 1½ sag

9 [18]k[i-lá 5 GAR uš 1½ GAR sag ½ GAR bùr-b]i gagar-ḫá saḫar-ḫá en-nam 7½ SAR gagar 45 saḫar-ḫá

10 [19]saḫar ki[-lá 45 SAR 1½ GAR sag] ½ GAR bùr-bi uš-bi en-nam 5 GAR uš-bi

11 [20]saḫar ki-lá 4[5 SAR 5 GA]R uš ½ GAR bùr-bi sag-bi en-nam 1½ GAR sag-bi

12 [21]saḫar ki-lá 4[5 SAR 5] GAR uš 1½ GAR sag bùr-bi en-nam ½ GAR bùr-bi

13 [22]saḫar ki-lá 4[5 SAR] ½ GAR bùr-bi uš sag gar-gar-ma 6½ GAR uš ù sag en-nam
[23]5 GAR uš 1½ GAR uš[163]

14 [24]saḫar ki-lá 45 SAR ½ GAR bùr-bi uš ugu sag 3½ GAR dirig
[25]uš sag en-nam 5 GAR uš 1½ GAR sag

15 [26]ki-lá gagar saḫar-ḫá gar-gar-ma 52,30 1½ GAR sag ½ GAR bùr-bi uš-bi en-nam 5 GAR uš

16 [27][ki-]lá gagar saḫar-ḫá gar-gar-ma 52,30 5 GAR uš ½ GAR bùr-bi
[28][sag-bi] en-nam 1½ GAR [sag]

17 [29][ki-lá gagar saḫar-ḫá] gar-gar-ma <52,30> ½ GAR bùr-bi uš ù s[ag gar-gar-ma] 6[½ GAR]
[30][uš sag] e[n-na]m

18 [31][ki-lá gagar saḫar-ḫá] gar-gar-ma 52,30 ½ GAR bùr-bi uš ugu sag 3[½ GAR dirig]
[32][uš sa]g en-nam

Reverse

19 [1]ki-lá *i-na* 7½ SAR gagar 45 saḫar-ḫá uš ù sag gar-gar-ma 6½ GAR
[2]uš sag bùr-bi en-nam

20 [3]ki-lá *i-na* 7½ SAR gagar 45 saḫar-ḫá uš ugu sag 3½ GAR dirig
[4]uš sag ù bùr-bi en-nam 5 GAR uš 1½ GAR sag ½ GAR bùr-bi

21 [5]ki-lá *i-na* 7½ SAR gagar 45 saḫar-ḫá igi-7 <-gál> uš ugu sag dirig bùr-bi
[6]uš ù sag en-nam 5 GAR uš 1½ GAR sag

[162a] Problem-text Ja (p. 75), which turned up after the above was written, mentions both ki-lá and *kalakkum*; unfortunately, it is not yet clear whether this constitutes brilliant proof or emphatic refutation of the proposed relation between ki-lá and *kalakkum*.

[162b] Cf. Thureau-Dangin TMB, p. XVIII.

[163] Mistake for sag, "width".

22 [7]ki-lá 5 GAR uš 1½ GAR sag ½ bùr-bi 10 gín saḫar éš-kàr lú-1-e uš en-nam al-díb
[8]6⅔ šu-si uš al-díb

23 [9]ki-lá 5 GAR uš 1½ GAR sag ½ bùr-bi 10 gín saḫar éš-kàr 30 erim-ḫá
[10]uš en-nam al-díb ½ GAR ⅔ kùš uš ì-díb-bé-eš

24 [11]ki-lá 5 GAR uš 1½ GAR sag ½ GAR bùr-bi 10 gín saḫar éš-kàr 30 erim-ḫá u_4 en-nam ì-til-eš
[12]u_4-9-kam in-til-eš

25 [13]ki-lá 1½ GAR sag ½ GAR bùr-bi 10 gín éš-kàr 30 erim-ḫá u_4-9-kam in-til-eš
[14]uš-bi en-nam 5 GAR uš-bi

26 [15]ki-lá 5 GAR uš ½ GAR bùr-bi 10 gín éš-kàr 30 erim-ḫá u_4-9-kam ì-til-eš sag-bi en-nam 1½ GAR sag

27 [16]ki-lá 5 GAR uš 1½ GAR sag 10 gín éš-kàr 30 erim-ḫá *i-na* u_4-9-kam ì-til-eš
[17]bùr-bi en-nam ½ GAR bùr-bi

28 [18]ki-lá 5 GAR uš 1½ GAR sag ½ GAR bùr-bi 30 erim-ḫá *i-na* u_4-9-kam in-til-eš
[19]saḫar éš-kàr en-nam 10 gín éš-kàr

29 [20]ki-lá 30 erim-ḫá *i-na* u_4-9-kam ì-til-eš ½[164] GAR bùr-bi 10 gín éš-kàr uš ù sag gar-gar-ma
[21]6½ GAR uš ù sag en-nam 5 GAR uš 1½ GAR sag-bi

30 [22]ki-lá 30 erim-ḫá *i-na* u_4-9-kam ì-til-eš ½[164] GAR bùr-bi 10 gín éš-kàr uš ugu sag 3½ GAR dirig
[23]uš sag en-nam 5 GAR uš 1½ GAR sag-bi

31 [24]ki-lá 2½ GAR íb-si_8 3⅓ kùš bùr-bi 10 gín éš-kàr 1(bán) še-ta-àm á-bi lú-ḫu[n-gá]
[25]gagar saḫar-ḫá erim-ḫá ù še en-nam 6 SAR igi-4-gál gagar
[26]20⅚ SAR saḫar-ḫá 2,5 erim-ḫá 4(gur) 5(bán) gur še

Bottom Edge

31 im-šu ki-lá

Translation of G (YBC 4657)

Obverse

1 [1]A ki-lá. [5 GAR is the length, 1½] GAR the width, ½ GAR its depth, 10 gín volume the assignment, 6 še (silver) the wages of a hire[d man].
[2]What are the area, the volume, [the (number of) workers], and the (total expenses in) silver? 7½ (SAR) is the area; 45 (SAR) is the volume;
[3]4,30 is [the (number of) workers]; 9 gín is the (total expenses in) silver.

2 [4]The (total expenses in) silver of a ki-lá [are 9 gín], 1½ GAR the width, ½ GAR <its depth>, 10 gín (volume) the assignment, 6 še (silver) the wages of a hired man.
[5]What is] its length? 5 GAR {GAR} is the length.

3 [6]The (total expenses in) silver of a ki-l[á are 9 gí]n, 5 GAR the length, ½ GAR its depth, 10 gín (volume) the assignment, 6 še (silver) the wages of a hired man.
[7][What] is its width? 1½ GAR is the width.

4 [8]The (total expenses in) silver of a ki-lá [are 9 gín, 5] GAR its length, 1½ GAR the width, 10 gín (volume) the assignment,
[9]6 še (silver) the wages of a hired man. What is its depth? [½ GAR] is its depth.

5 [10]The (total expenses in) silver of a ki-lá are 9 gín, 5 GAR the length, 1½ GAR the width, [½ GAR] its [depth], 6 še (silver) the wages of a hired man.
[11]What is the volume of the assignment? [10 gín is the volume] of the assignment.

6 [12]The (total expenses in) silver of a ki-lá are [9 gín], 5 [GAR the length, 1½ GAR the width, ½] GAR its depth, 10 gín (volume) the assignment.
[13]What are the wages of a hired man? [6 še (silver)] is the wages of a hired man.

7 [14]The (total expenses in) silver of a ki-lá [are 9 gín, ½] (GAR) [its] depth, [10 gín (volume) the assignment, 6] še (silver) the wages of a hired man;
[15]I added the length [and the width, and (the result is) 6;30 (GAR)]. What are [the length and] the width?

8 [16]The (total expenses in) silver [of a ki-lá are 9 gín, ½ GAR its depth, 10 gín (volume)] the assignment, 6 še (silver) the wages of a hired man;
[17][the length exceeded the width by 3;30 (GAR)]. What are the length and the width? 5 GAR is the length; 1½ (GAR) is the width.

9 [18]A ki-lá. [5 GAR is the length, 1½ GAR the width, ½ GAR] its [depth]. What are the area (and) the volume? 7½ SAR is the area; 45 (SAR) is the volume.

10 [19]The volume of a ki-lá [is 45 SAR, 1½ GAR the width], ½ GAR its depth. What is its length? 5 GAR is its length.

[164] Faint traces of 1 before ½.

11 [20]The volume of a ki-lá is 45 [SAR, 5] GAR the length, ½ GAR its depth. What is its width? 1½ GAR is its width.

12 [21]The volume of a ki-lá is 45 [SAR, 5] GAR the length, 1½ GAR the width. What is its depth? ½ GAR is its depth.

13 [22]The volume of a ki-lá is 45 [SAR], ½ GAR its depth; I added the length (and) the width, and (the result is) 6½ GAR. What are the length and the width?
[23]5 GAR is the length; 1½ GAR is the length.[163]

14 [24]The volume of a ki-lá is 45 SAR, ½ GAR its depth; the length exceeded the width by 3½ GAR.
[25]What are the length (and) the width? 5 GAR is the length; 1½ GAR is the width.

15 [26]A ki-lá. I added the area (and) the volume, and (the result is) 52;30; 1½ GAR is the width, ½ GAR its depth. What is its length? 5 GAR is the length.

16 [27]A [ki-]lá. I added the area (and) the volume, and (the result is) 52;30; 5 GAR is the length, ½ GAR its depth.
[28]What is [its width]? 1½ GAR is [the width].

17 [29][A ki-lá.] I added [the area (and) the volume], and (the result is) <52;30>; ½ GAR is its depth; [I added] the length and the width, [and (the result is)] 6½ [GAR].
[30]What [are the length (and) the width]?

18 [31][A ki-lá.] I added [the area (and) the volume], and (the result is) 52;30; ½ GAR is its depth; the length exceeded the width by 3½ [GAR].
[32]What are the [length (and) the wid]th?

Reverse

19 [1]A ki-lá .....[165] 7½ SAR the area, 45 (SAR) the volume; I added the length and the width, and (the result is) 6½ GAR.
[2]What are the length, the width, (and) its depth?

20 [3]A ki-lá .....[165] 7½ SAR the area, 45 (SAR) the volume; the length exceeded the width by 3½ GAR.
[4]What are the length, the width, and its depth? 5 GAR is the length; 1½ GAR is the width; ½ GAR is its depth.

21 [5]A ki-lá .....[165] 7½ SAR the area, 45 (SAR) the volume; one-seventh of that by which the length exceeded the width is its depth.
[6]What are the length and the width? 5 GAR is the length; 1½ GAR is the width.

22 [7]A ki-lá. 5 GAR is the length, 1½ GAR the width, ½ (GAR) its depth, 10 gín volume the assignment. What length did one man take?
[8]He took 6⅔ šu-si length.

23 [9–10]A ki-lá. 5 GAR is the length, 1½ GAR the width, ½ (GAR) its depth, 10 gín volume the assignment. What length did 30 workers take? They took ½ GAR (and) ⅔ kùš length.

24 [11]A ki-lá. 5 GAR is the length, 1½ GAR the width, ½ GAR its depth, 10 gín volume the assignment. On what day did 30 workers finish?
[12]They finished on the 9th day.

25 [13]A ki-lá. 1½ GAR is the width, ½ GAR its depth, 10 gín (volume) the assignment; 30 workers finished on the 9th day.
[14]What is its length? 5 GAR is its length.

26 [15]A ki-lá. 5 GAR is the length, ½ GAR its depth, 10 gín (volume) the assignment; 30 workers finished on the 9th day. What is its width? 1½ GAR is the width.

27 [16]A ki-lá. 5 GAR is the length, 1½ GAR the width, 10 gín (volume) the assignment; 30 workers finished on the 9th day.
[17]What is its depth? ½ GAR is its depth.

28 [18]A ki-lá. 5 GAR is the length, 1½ GAR the width, ½ GAR its depth; 30 workers finished on the 9th day.
[19]What is the volume of the assignment? 10 gín is the assignment.

29 [20]A ki-lá. 30 workers finished on the 9th day; ½[164] GAR is its depth, 10 gín (volume) the assignment; I added the length and the width, and (the result is)
[21]6½ GAR. What are the length and the width? 5 GAR is the length; 1½ GAR is its width.

30 [22]A ki-lá. 30 workers finished on the 9th day; ½[164] GAR is its depth, 10 gín (volume) the assignment; the length exceeded the width by 3½ GAR.
[23]What are the length (and) the width? 5 GAR is the length; 1½ GAR is its width.

31 [24]A ki-lá. 2½ GAR is the square-side, 3⅓ kùš its depth, 10 gín (volume) the assignment, 1 bán barley the wages of each hired man.

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[165] For the dots in the translation of Nos. 19, 20 and 21, the text has the word *i-na*, literally, "in". The syntactic function of *i-na* here and in the corresponding Nos. (with the exception of No. 21) in J (YBC 4662) escapes us.

[25]What are the area, the volume, the (number of) workers, and the (total expenses in) barley? 6 SAR and one-fourth (SAR) is the area; [26]20⅚ SAR is the volume; 2,5 is the (number of) workers; 4 gur (and) 5 bán is the (total expenses in) barley.

Bottom Edge

31 sections: ki-lá.

Transcription of H (YBC 4663)

(Photograph: Plate 32; copy: Plate 7)

Obverse

1 [1]ki-lá 5 GAR uš 1½ GAR <sag> ½ GAR bùr-bi 10 saḫar éš-kàr 6 [še á-bi]
[2]gagar saḫar-ḫá erim-ḫá ù kù-babbar en-nam za-e kid$_9$-da-zu-dè
[3]uš sag UR-UR-ta 7,30 *i-na-ad-di-ik-ku*
[4]7,30 *a-na* bùr-bi *i-ši* 45 *i-na-ad-di-ik-ku*
[5]igi éš-kàr du$_8$ 6 *i-na-ad-di-ku a-na* 45 *i-ši* 4,30 *i-na-di-ku*
[6]4,30 *a-na i-di i-ši* 9 *i-na-di-ku ki-a-am né-p*[*e-š*]*u*

2 [7]9 gín kù-babbar ki-lá 1½ GAR <sag> ½ GAR bùr-bi 10 saḫar éš-kàr 6 še [á-b]i
[8]uš-bi en-nam za-e kid$_9$-da-zu<-dè> sag *ù* bùr-bi UR-UR-ta
[9]9 *i-na-ad-di-ku-um* igi éš-kàr *pu-ṭù-ur*
[10]*a-na* 9 *i-ši* 54 *i-na-ad-di-ik-ku-um*
[11]54 *a-na i-di i-ši* 1,48 *i-na-ad-di-ku-um*
[12]i[gi] 1,48 <du$_8$> 33,20 *i-na-ad-di-ik-ku* 33,20 *a-na* 9 kù *i-ši*
[13]5 *i-na-ad-di-ku-um* 5 GAR uš-bi *ki-a-am né-pe-šu*[166]

3 [14]9 kù-babbar ki-lá 5 GAR uš ½ GAR bùr-bi 10 éš-kàr 6 še á-bi
[15]sag-bi en-nam za-e kid$_9$-da-zu-dè
[16]uš *ù* bùr-bi UR-U[R-ta] 30 *i-na-ad-di-ku*
[17]igi éš-kàr du$_8$ *a-na* 30 *i-ši* 3 *i-na-ad-di-ku*
[18]3 *a-na i-di i-ši* 6 *i-na-di-ku* igi 6 du$_8$ *a-na* 9 kù *i-ši*
[19]sag-bi *i-na-ad-di-ku* 1½ GAR sag *ki-a-am né-pe-šu*

4 [20]9 gín kù ki-lá 5 GAR uš 1½ GAR sag 10 saḫar éš-kàr 6 še á-am$_5$
[21]bùr-bi en-nam za-e kid$_9$-da-zu-dè
[22]uš sag UR-UR-ta 7,30 *i-na-di-ku* igi éš<-kàr> *pu-ṭù-ur*
[23]*a-na* 7,30 *i-ši* 45 *i-na-di-ku* 45 *a-na i-di i-ši*
[24]1,30 *i-na-di-ku* igi 1,30 *pu-ṭù-ur* 40 *i-na-di-ku*
[25]40 *a-na* 9 kù *i-ši* 6 bùr-bi *i-na-di-ku* ½ GAR bùr-bi

5 [26]9 kù-babbar ki-lá 5 GAR uš 1½ GAR sag ½ GAR bùr-bi 6 še á-am$_5$
[27]éš-kàr en-nam za-e kid$_9$-da-zu-dè uš *ù* sag
[28]*šu-ta-ki-il* 7,30 *i-na-ad-di-ku* 7,30 *a-na* bùr-bi *i-ši*
[29]45 *i-na-ad-di-ik-ku* 45 *a-na i-di i-ši*
[30]1,30 *i-na-di-ik-ku* igi 9 kù-babbar *pu-ṭù-ur*
[31]6,40 *i-na-di-ik* <*-ku*> 6,40 a-rá 1,30 *i-ši*[167]
[32]éš-kàr *i-na-di-ku* 10 gín éš-kàr

6 [33]9 kù ki-lá 5 GAR uš 1½ GAR sag ½ GAR bùr-bi 10 gín éš-kàr
[34]á-bi en-nam za-e kid$_9$-da-zu-dè uš *ù* sag
[35]*šu-ta-ki-il* 7,30 *i-na-di-ik-ku* 7,30 *a-na* bùr-bi *i-ši*
[36]45 *i-na-di-ik-ku* igi éš-kàr du$_8$ *a-na* 45 *i-ši*
[37]4,30 *i-na-ad-di-kum* igi 4,30 du$_8$
[38]13,20 *i-na-ad-di-kum* 13,20 *a-na* 9 kù-babbar *i-ši*
[39]á-bi [*i-*]*na-ad-di-kum* 6 še á-bi-am$_5$

7 [40]{9 gín kù-babbar ki-lá}[168]

Reverse

[1]9 kù ki-lá uš *ù* sag gar-gar-ma 6,30 ½ GAR [bùr-bi]
[2]10 gín éš-kàr 6 še á-bi uš sag-bi en-nam
[3]za-e kid$_9$-da-zu-dè igi á-bi *pu-ṭù-ur*
[4]*a-na* 9 gín kù-babbar *i-ši* 4,30 *i-na-di-ku-um*
[5]4,30 *a-na* éš-kàr *i-ši* 45 *i-na-di-ik-ku*
[6]igi bùr-bi du$_8$ *a-na* 45 *i-ši* 7,30 *i-na-di-ku*
[7]½ uš *ù* sag *ša* gar-gar-*ru ḫé-pe* 3,15 *i-na-di-ku*
[8]3,15 UR-UR-ta 10,33,45 *i-na-di-ku*
[9]7,30 *i-na li-bi* 10,33,45 *ú-sú-uḫ*
[10]3,3,45 *i-na-ad-di-ik-ku* íb-si$_8$-*šu le-qé*
[11]1,45 *i-na-di-ku a-na* 1 *ṣí-ib a-na* 1 *ḫu-ru-úṣ*
[12]uš sag *i-na-di-ku* 5 uš 1½ GAR sag

8 [13]9 kù-babbar ki-lá uš ugu sag 3,30 *i-te-er*
[14]½ GAR bùr-bi 10 gín éš-kàr 6 še á-am$_5$
[15]uš sag en-nam za-e kid$_9$-da-zu-dè
[16]igi *i-di* du$_8$ *a-na* 9 kù-babbar *i-ši* 4,30 *i-na-di-ku*[169]
[17]4,30 *a-na* éš-kàr *i-ši* 45 *i-na-di-ik-ku*
[18]igi ½ GAR du$_8$ *a-na* 45 *i-ši* 7,30 *i-na-di-ku*
[19]½ *ša* uš ugu sag *i-te-ru ḫe-pe* 1,45 *i-na-di-ku*
[20]1,45 *šu-ta-ki-il* 3,3,45 *i-na-di-ku*
[21]*a-na* 3,3,45 7,30 *ṣí-ib* 10,33,45 *i-na-di* <*-ku*>
[22]íb-si$_8$-*šu le-qé* 3,15 *i-na-di-ku*
[23]3,15 *a-na* 2 *lu-pu-ut-ma* 1,45 a-rá[170] 1 *ṣí-ib*
[24]1,45 *a-na* 1 *ḫu-ru-úṣ* uš *ù* sag *i-na-di-ku*
[25]5 GAR uš 1½ GAR sag *ki*[171]*-a-am né-pe-šu*[172]

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[166] The text seems to have *né-pu-šu*.

[167] Erasure at the end of the line shows traces of éš-kàr *i-na-*; cf. the beginning of the next line.

[168] The whole line presumably represents a false start on the scribe's part.

[169] The interior wedges of *-di-* are missing.

[170] A-rá, "times", is presumably an error for *a-na*, "to".

[171] Text: *di-*.

[172] On the dividing-line, below "5" in line 25, appears the erasure of what seems to have been the sign for "9", probably intended to begin a new example.

Translation of H (YBC 4663)

Obverse

1 [1]A ki-lá. 5 GAR is the length, 1½ GAR <the width>, ½ GAR its depth, 10 (gín) volume the assignment, 6 [še (silver) the wages].
[2]What are the area, the volume, the (number of) workers, and the (total expenses in) silver? When you perform (the operations),
[3]multiply together the length (and) the width, (and) you will get 7;30 (SAR, the area).
[4]Multiply 7;30 by its depth, (and) you will get 45 (SAR, the volume).
[5]Take the reciprocal of the assignment, (and) you will get 6; multiply by 45, (and) you will get 4,30 (, the number of workers).
[6]Multiply 4,30 by the wages, (and) you will get 9 (gín, the total expenses in silver). Such is the procedure.

---

2 [7]9 gín is the (total expenses in) silver of a ki-lá, 1½ GAR <the width>, ½ GAR its depth, 10 (gín) volume the assignment, 6 še (silver) the [wage]s.
[8]What is its length? When you perform (the operations), multiply together the width and its depth, (and)
[9]you will get 9; take the reciprocal of the assignment,
[10]multiply by 9, (and) you will get 54;
[11]multiply 54 by the wages, (and) you will get 1;48;
[12]<take> the reciprocal of 1;48, (and) you will get 0;33,20; multiply 0;33,20 by 9, the (total expenses in) silver, (and)
[13]you will get 5. 5 GAR is its length. Such is the procedure.

---

3 [14]9 (gín) is the (total expenses in) silver of a ki-lá, 5 GAR the length, ½ GAR its depth, 10 (gín volume) the assignment, 6 še (silver) the wages.
[15]What is its width? When you perform (the operations),
[16]multiply [together] the length and its depth, (and) you will get 30;
[17]take the reciprocal of the assignment, multiply by 30, (and) you will get 3,0;
[18]multiply 3,0 by the wages, (and) you will get 6; take the reciprocal of 6, multiply by 9, the (total expenses in) silver, (and)
[19]you will get its width. 1½ GAR is the width. Such is the procedure.

---

4 [20]9 gín is the (total expenses in) silver of a ki-lá, 5 GAR the length, 1½ GAR the width, 10 (gín) volume the assignment, 6 še (silver) the wages.
[21]What is its depth? When you perform (the operations),
[22]multiply together the length (and) the width, (and) you will get 7;30; take the reciprocal of the assignment,
[23]multiply by 7;30, (and) you will get 45; multiply 45 by the wages,
[24](and) you will get 1;30; take the reciprocal of 1;30, (and) you will get 0;40;
[25]multiply 0;40 by 9, the (total expenses in) silver, (and) you will get 6 (kùš), its depth. ½ GAR is its depth.

---

5 [26]9 (gín) is the (total expenses in) silver of a ki-lá, 5 GAR the length, 1½ GAR the width, ½ GAR its depth, 6 še (silver) the wages.
[27–28]What is the assignment? When you perform (the operations), multiply together the length and the width, (and) you will get 7;30; multiply 7;30 by its depth,
[29](and) you will get 45; multiply 45 by the wages,
[30](and) you will get 1;30; take the reciprocal of 9, the (total expenses in) silver, (and)
[31]you will get 0;6,40; multiply 0;6,40 by 1;30, (and)
[32]you will get the assignment; 10 gín (volume) is the assignment.

---

6 [33]9 (gín) is the (total expenses in) silver of a ki-lá, 5 GAR the length, 1½ GAR the width, ½ GAR its depth, 10 gín (volume) the assignment.
[34–35]What are the wages? When you perform (the operations), multiply together the length and the width, (and) you will get 7;30; multiply 7;30 by its depth, (and)
[36]you will get 45; take the reciprocal of the assignment, multiply by 45, (and)
[37]you will get 4,30; take the reciprocal of 4,30, (and)
[38]you will get 0;0,13,20; multiply 0;0,13,20 by 9, the (total expenses in) silver, (and)
[39]you will get the wages. 6 še (silver) is the wages.

---

7 [40]9 gín is the (total expenses in) silver of a ki-lá.[168]

Reverse

[1]9 (gín) is the (total expenses in) silver of a ki-lá; I added the length and the width, and (the result is) 6;30 (GAR); ½ GAR is [its depth],
[2]10 gín (volume) the assignment, 6 še (silver) the wages. What are the length (and) its width?
[3]When you perform (the operations), take the reciprocal of the wages,
[4]multiply by 9 gín, the (total expenses in) silver, (and) you will get 4,30;
[5]multiply 4,30 by the assignment, (and) you will get 45;
[6]take the reciprocal of its depth, multiply by 45, (and) you will get 7;30;
[7]halve the length and the width which I added together, (and) you will get 3;15;

[8]square 3;15, (and) you will get 10;33,45;
[9]subtract 7;30 from 10;33,45, (and)
[10]you will get 3;3,45; take its square root, (and)
[11]you will get 1;45; add it to the one, subtract it from[173] the other, (and)
[12]you will get the length (and) the width. 5 (GAR) is the length; 1½ GAR is the width.

8 [13]9 (gín) is the (total expenses in) silver of a ki-lá; the length exceeded the width by 3;30 (GAR);
[14]½ GAR is its depth, 10 gín (volume) the assignment, 6 še (silver) the wages.
[15]What are the length (and) the width? When you perform (the operations),
[16]take the reciprocal of the wages, multiply by 9, the (total expenses in) silver, (and) you will get 4,30;
[17]multiply 4,30 by the assignment, (and) you will get 45;
[18]take the reciprocal of ½ GAR (, the depth), multiply by 45, (and) you will get 7;30;
[19]halve that by which the length exceeded the width, (and) you will get 1;45;
[20]square 1;45, (and) you will get 3;3,45;
[21]to 3;3,45 add 7;30, (and) you will get 10;33,45;
[22]take its square root, (and) you will get 3;15;
[23]operate with 3;15 twice: add 1;45 to[170] the one,
[24]subtract 1;45 from[173] the other, (and) you will get the length and the width.
[25]5 GAR is the length; 1½ GAR is the width. Such is the procedure.

Transcription of J (YBC 4662)

(Photograph: Plate 33; copy: Plate 8)

Obverse

19 [1]ki-l[á *i-n]a* 7½ SAR gagar 45 SAR saḫar-ḫ[á]
[2]uš *ù* sag gar-gar-ma 6,30 uš sag *ù* [bùr-bi en-nam]
[3]za-e kìd-da-zu-dè igi 7½ gagar [du$_8$-]m[a]
[4]*a-na* 45 SAR saḫar-ḫá *i-ši* 6 bùr-bi *i-na-d[i-ku]*
[5]igi bùr-bi du$_8$ 10 *i-na-di-ku* 10 *a-na* 45 saḫar-ḫá *[i-ši]*
[6]7,30 *i-na-ad-di-kum* ½ uš *ù* sag *ša* gar-gar[*-ru ḫé-pe*]
[7]3,15 *i-na-ad-di-kum* 3,15 a-rá 3,15 U[R-UR-a]
[8]10,33,45 *i-na-di-kum* 7,30 *i-na li-ib[-bi* 10,33,45]
[9]*ta-ba-al* 3,3,45 *i[-na-]di-k[u* íb-si$_8$-*šu le-qé]*
[10]1,45 *i-na-di-kum a-na* 1 [1,4]5 daḫ *i-n[a* 1 1,45 *ḫu-ru-úṣ]*
[11]uš *ù* sag *i-n[a]-ad-di-kum* 5 GAR [uš 1½ sag]

20 [12]ki-lá *i-na* 7½ gagar 45 SAR saḫar-ḫá uš ugu sag
[13]3,30 <*i-te-er*> uš sag *ù* bùr-bi en-nam za-e kìd-da-zu-dè

[14]igi 7,30 gagar du$_8$ 8 *i-na-di-ku* 8 a-rá 45 saḫar-ḫá *i-ši*
[15]6 bùr-bi *i-na-di-ku* igi 6 bùr-bi du$_8$ 10 *i-na-di-ku*
[16]10 *a-na* 45 saḫar-ḫá *i-ši* 7,30 *i-na-ad-d[i-k]u*
[17][½ *ša* uš] ugu sag *i[-te-r]u* 3,30 *ḫé-p[e]*
[18][1,45 *i-*]*na-di-ku* 1,[45] a-rá 1,45 UR-UR[-a]
[19][3,3,4]5 *i-na-di-ku* 7,30 a-rá[170] *li-ib-b[i* 3,3,45 daḫ]
[20]1[0],33,45 *i-na-di-ku* 10,33,45 íb-si$_8$-*šu le-qé*
[21]3,15 *i-na-di-ku* 3,15 *a-na* 2 *lu-pu-ut-ma*
[22]1,[4]5 *a-[n]a* 1 daḫ 1,45 *a-na* 1 *ḫu-ru-úṣ*
[23]u[š *ù* sa]g *i-na-di-ku* 5 GAR uš 1½ sag

21 [24]ki-lá <*i-na*> 7½ SAR gagar 45 SAR saḫar-ḫá igi-7-gál
[25]uš ugu sag *i-te-ru* bùr-bi uš sag *ù* bùr-bi en-nam
[26]za-e kìd-da-zu-dè igi 7½ SAR gagar du$_8$ *a-na* 45 [saḫar *i-ši]*
[27]bùr-bi *i-na-di-ku* ½ 7-*bi-tim ša la-ap-tu-ma ḫé-pe*
[28]3,30 *i-na-di-ku* igi bùr-bi du$_8$ 10 *i-na-di-ku*
[29]10 *a-na* 45 saḫar-ḫá *i-ši* 7,30 *i-na-ad-di-ku*
[30]½ 3,30 *ḫé-pe* 1,45 *i-na-di-ku* 1,45 a-rá 1,45
[31]UR-UR-a 3,3,45 *i-na-di-ku* 7,30 a-rá[170] *li[-ib-bi]* 3,3,45 daḫ
[32]10,33,45 *i-na-di-ku* 10,33,45 íb-si$_8$-š[*u le-qé]*
[33]3,15 *i-na-di-ku* 3,15 *a-na* <2> *lu-pu-ut-m[a]*
[34]1,45 a-rá 1 daḫ 1,45 *a-na* 1 *ḫu-ru-ú[ṣ]*
[35]uš *ù* sag *i-na-di-ku* 5 GAR uš [1½ GAR sag]

22 [36]ki[-lá 5 GAR uš 1½ GAR sag] ½ GAR bùr-bi 10 [gín éš-kàr]
[37][lú-1-e uš en-nam al-díb][174] za-e kìd[-da-zu-dè]
[38][sag *ù* bùr-bi UR-UR-a] 9 *i-na-di[-ku]*
[39][igi 9 du$_8$ 6,40 *i-na-d]i-ku* 6,40 a-rá éš-kàr
[40][*i-ši* 1,6,40 *i-na-d]i-ku* 1,6,40 díb lú-1

23 [41][ki-lá 5 GAR uš 1½ GAR sag ½ GAR] bùr-bi 10 gín éš-kàr
[42][30 erim-ḫá uš en-nam al-díb za-e kìd-]da-z[u-d]è

(three or four

Reverse

lines missing)

24 [3][ki-lá 5 GAR uš 1½ GAR sag ½ GAR bùr-b]i 10 gín [éš-kàr]
[4][30 erim-ḫá] *i-na ki ma-ṣí* [u$_4$] ì-til-le
[5][za-e] kìd-da-zu-dè uš *ù* s[ag UR-]UR-ta
[6][7],30 *i-na-di-ku* 7,30 *a-na* bùr-bi *i-*[š]*i* 45 *i-na-di-ku*
[7]igi éš-kàr du$_8$ 6 *i-na-di-ku* 45 *a-na* 6 *i-ši*
[8]4,30 *i-na-di-ku* igi 30 erim-ḫá du$_8$ 2 *i-na-di-ku*
[9]*a-na* 4,30 *i-ši* 9 *i-na-di-ku*
[10]30 erim-ḫá *i-na* u$_4$-9-kam ì-til-le

[173] Text: "to".

[174] Traces at the end of the gap do not seem to conform to the end of our restoration, which follows the phrasing of the corresponding No. in G (YBC 4657).

25 11ki-lá 1½ GAR sag ½ GAR bùr-bi 10 gín éš-kàr
12 30 erim-ḫá u4-9-kam in-til-eš
13 uš-bi en-nam za-e kìd-da-zu-dè
14 sag *ù* bùr-bi UR-UR-ta 9 *i-na-di-ku* igi éš-kàr d[u8 6 *i-na-di-ku*]
15 6 *a-na* 9 *i-ši* 54 *i-na-di-ku* igi 54 du8 1,6,40 *i-na-di-ku*
16 30 *ù* 9 UR-UR-a 4,30 *i-na-di-ku* 4,30
17 *a-na* 1,6,40 *i-ši* uš *i-na-di-ku* 5 GAR uš

26 18ki-lá 5 GAR uš ½ GAR bùr-bi 10 gín éš-kàr 30 erim-ḫá
19 *i-na* u4-9-kam in-til-eš sag-bi en-nam
20 za-e kìd-da-zu-dè uš *ù* bùr-bi UR-UR-ta
21 [3]0 *i-na-di-ku* igi éš-kàr du8 6 *i-na-di-ku*
22 [30] *a-na* 6 *i-ši* <3 *i-na-di-ku* igi> 3 du8-a 20 *ta-mar* 30 erim-ḫá *ù* 9
23 [UR-UR-]ta 4,30 *i-na-di-ku* 4,30 *a-na* 20 *ta-na-ši-ma*
24 sa[g *i-n*]*a-di-ku* 1½ GAR sag

27 25ki-l[á 5 GAR uš 1½ GAR sa]g 10 gín éš-kàr <30 erim-ḫá *i-na* u4-9-kam in-til-eš>
26 bùr-bi e[n-nam za-e kìd-]da-zu-dè
27 uš *ù* sag UR-UR-a [7,30 *i-n*]*a-di-ku* igi éš-kàr du8 <*a-na* 7,30 *i-ši*>
28 45 *i-na-di-ku* igi 45 du8 1,20 *i-na-di-ku*
29 30 erim-ḫá u4-9-kam UR-U[R-a 4,3]0 *i-na-di-ku*
30 4,30 *a-na* 1,20 *i-š*[*i* 6 *i-na-di-ku* ½ GAR bùr-bi]

28 31ki-lá 5 GAR uš [1½ GAR sag] ½ GAR bùr-bi <30 erim-ḫá *i-na* u4-9-kam in-til-eš> éš-kàr en-nam
32 za-e kìd-da-zu-dè uš *ù* sag UR-UR-a
33 7,30 *ta-mar* 7,30 *a-na* bùr-bi *i-ši* 45 *ta-mar*
34 30 erim-ḫá *ù* u4-9-kam UR-UR-a 4,30 *ta-mar*
35 igi 4,30 du8 13,20 *ta-mar* 13,20 *a-na* [45 *i-ši*]
36 éš-kàr *i-na-di-ku* 10 gín é[š-kàr]

Translation of J (YBC 4662)

Obverse

19 1A ki-lá[. . . . .][175] 7½ SAR the area, 45 SAR the volume;
2I added the length and the width, and (the result is) 6;30 (GAR). [What are] the length, the width, and [its depth]?
3–4When you perform (the operations), take the reciprocal of 7½, the area, multiply by 45 SAR, the volume, (and) [you] will get 6 (kùš), its depth.
5Take the reciprocal of its depth, (and) you will get 0;10; [multiply] 0;10 by 45, the volume, (and)
6you will get 7;30. Hal[ve] the length and the width which I added together, (and)
7you will get 3;15; multiply [together] 3;15 times 3;15, (and)
8–9you will get 10;33,45; take away 7;30 fr[om 10;33,45], (and) you will get 3;3,45; [take its square root, (and)]
10you will get 1;45; add [1;4]5 to the one, [subtract 1;45] from [the other, (and)]
11you will get the length and the width. 5 GAR [is the length; 1½ GAR is the width.]

20 12–13A ki-lá. . . . . [175] 7½ (SAR) the area, 45 SAR the volume; the length <exceeded> the width by 3;30 (GAR). What are the length, the width, and its depth? When you perform (the operations),
14take the reciprocal of 7;30, the area, (and) you will get 0;8; multiply 0;8 times 45, the volume, (and)
15you will get 6 (kùš), its depth. Take the reciprocal of 6, its depth, (and) you will get 0;10;
16multiply 0;10 by 45, the volume, (and) you will get 7;30.
17Halve 3;30, by which the length exceeded the width, (and)
18you will get [1;45]; multiply together 1;[45] times 1;45, (and)
19you will get [3;3,4]5; [add] 7;30 to [3;3,45], (and)
20you will get 1[0];33,45; as for 10;33,45, take its square root, (and)
21you will get 3;15; operate with 3;15 twice:
22add 1;[4]5 to the one, subtract 1;45 from[176] the other, (and)
23you will get the length [and] the width. 5 GAR is the length; 1½ (GAR) is the width.

21 24A ki-lá. 7½ SAR is the area, 45 SAR the volume; one-seventh
25of that by which the length exceeded the width is its depth. What are the length, the width, and its depth?
26When you perform (the operations), take the reciprocal of 7½ SAR, the area, [multiply by] 45, [the volume, (and)]
27you will get its depth. Halve the one-seventh which has been assumed, (and)
28you will get 3;30. Take the reciprocal of its depth, (and) you will get 0;10;
29multiply 0;10 by 45 (SAR), the volume, (and) you will get 7;30.
30–31Halve 3;30, (and) you will get 1;45; multiply together 1;45 times 1;45, (and) you will get 3;3,45; add 7;30 to 3;3,45, (and)
32you will get 10;33,45; as for 10;33,45, [take] its square root, (and)
33you will get 3;15; operate with 3;15 <twice>:
34add 1;45 to[177] one, subtract 1;45 from[176] the other, (and)

[175] See p. 68, note 165.

[176] Text: "to".

[177] Text: "times".

35you will get the length and the width. 5 GAR is the length; [1½ GAR is the width].

22 36A ki[-lá. 5 GAR is the length, 1½ GAR the width], ½ GAR its depth, 10 [gín (volume) the assignment].
37[How much length did one man take? When] you perform (the operations),
38[multiply together the width and its depth, (and) you] will get 9;
39–40[take the reciprocal of 9, (and)] you will get [0;6,40; multiply] 0;6,40 times the assignment, (and) you will get [0;1,6,40]. 0;1,6,40 (GAR) is the taking of one man.

23 41[A ki-lá. 5 GAR is the length, 1½ GAR the width, ½ GAR] its depth, 10 gín (volume) the assignment.
42[How much length did 30 workers take?] When you [perform (the operations)],
..... (three or four

Reverse

lines missing) .....

24 3[A ki-lá. 5 GAR is the length, 1½ GAR the width, ½ GAR] its [depth], 10 gín (volume) [the assignment].
4In how many [days] did [30 workers] finish?
5When you perform (the operations), multiply together the length and the width, (and)
6you will get [7;]30; multiply 7;30 by its depth, (and) you will get 45.
7Take the reciprocal of the assignment, (and) you will get 6; multiply 45 by 6, (and)
8you will get 4,30. Take the reciprocal of 30 workers, (and) you will get 0;2;
9multiply by 4,30, (and) you will get 9.
1030 workers finished on the 9th day.

25 11A ki-lá. 1½ GAR is the width, ½ GAR its depth, 10 gín (volume) the assignment;
1230 workers finished on the 9th day.
13What is its length? When you perform (the operations),
14multiply together the width and its depth, (and) you will get 9. Take the reciprocal of the assignment, [(and) you will get 6];
15multiply 6 by 9, (and) you will get 54; take the reciprocal of 54, (and) you will get 0;1,6,40.
16–17Multiply together 30 and 9, (and) you will get 4,30; multiply 4,30 by 0;1,6,40, (and) you will get the length. 5 GAR is the length.

26 18A ki-lá. 5 GAR is the length, ½ GAR its depth, 10 gín (volume) the assignment; 30 workers
19finished on the 9th day. What is its width?
20When you perform (the operations), multiply together the length and its depth, (and)
21you will get [3]0. Take the reciprocal of the assignment, (and) you will get 6;
22multiply [30] by 6, (and) <you will get 3,0>; take <the reciprocal> of 3,0, (and) you will see 0;0,20. 30 workers and 9
23[multiply] together, (and) you will get 4,30; multiply 4,30 by 0;0,20, and
24you will get the width. 1½ GAR is the width.

27 25A ki[-lá. 5 GAR is the length, 1½ GAR] the width, 10 gín (volume) the assignment; <30 workers finished on the 9th day>.
26What is its depth? When [you] perform (the operations),
27multiply together the length and the width, (and) you will get [7;30]. Take the reciprocal of the assignment, <multiply by 7;30>, (and)
28you will get 45; take the reciprocal of 45, (and) you will get 0;1,20.
29Multiply [together] 30 workers (and) the 9th day, (and) you will get [4,3]0;
30multiply 4,30 by 0;1,20, [(and) you will get 6. ½ GAR is its depth].

28 31A ki-lá. 5 GAR is the length, [1½ GAR the width], ½ GAR its depth; <30 workers finished on the 9th day.> What is the assignment?
32When you perform (the operations), multiply together the length and the width, (and)
33you will see 7;30; multiply 7;30 by its depth, (and) you will see 45.
34Multiply together 30 workers and the 9th day, (and) you will see 4,30;
35take the reciprocal of 4,30, (and) you will see 0;0,13,20; [multiply] 0;0,13,20 by [45], (and)
36you will get the assignment. 10 gín (volume) is the as[signment].

Commentary

These three texts form a closely knit group. G (YBC 4657) contains the statement of 31 problems which were worked out on three other tablets, two of which are preserved: H (YBC 4663), which deals with the problems Nos. 1 to 8; and J (YBC 4662), which treats Nos. 19 to 28, omitting the last three examples of the main text.

The subject of these problems is again a volume of earth, called ki-lá. The system of parameters is the same for all problems with the sole exception of the last (No. 31), which is perhaps the first example of a tablet which followed (cf. below p. 74). The volume of the ki-lá is determined by the following magnitudes:

l = 5 GAR length (uš)
b = 1;30 GAR width (sag)
(1a) $g = lb$ = 7;30 SAR area (gagar) of the base
h = 6 kùš (= 0;30 GAR) depth (bùr)
$V = lbh$ = 45 SAR volume (saḫar).

From the last relation it follows that the ki-lá is considered to be a prism. As for the work on the ki-lá, the following assumptions are made:

λ = 0;10 SAR work output to be expected daily for each worker (éš-kàr, translated "assignment")
w = 6 še = 0;2 gín wages per man per day (á) paid in silver
(1b) t = 9 duration of the work in days (u_4)
m = 30 number of workers (erim-ḫá)
M = 4,30 number of man-days (erim-ḫá)
E = 9 gín total expenses in silver (kù-babbar).

It is clear that the following relations must hold:

(2a) $$E = \frac{V}{\lambda} w$$

(2b) $$\lambda = \frac{V}{mt}$$

where $V = lbh$. In one of the examples (No. 22), the specific output λ_l is asked for; this is defined as the work which one worker could be expected to complete in one day expressed in terms of the length which he digs (translation, "the length which he took"). This amount is given by the formula

(2c) $$\lambda_l = \frac{\lambda}{bh} = \frac{l}{mt}.$$

The next example calculates $m\lambda_l$, i.e., the sum of the λ_l's of the 30 men.

The last example (No. 31) assumes a ki-lá with a square base $a = 2\frac{1}{2}$ GAR and a depth $h = 3\frac{1}{3}$ kùš. The volume of output per man each day, λ, is again 0;10 SAR, as in the other examples. The wages, however, are now expressed in barley: 1 bán = 10 sìla.[179] The quantities to be found are $g = a^2$, V, m and E. The text also indicates the answers: $g = 6\frac{1}{4}$ SAR, $V = 20\frac{5}{6}$ SAR, m = 2,5 men, E = 20,50 sìla = 4 gur 5 bán.

[178] The main text G (YBC 4657) does not ask for h, but J (YBC 4662) asks for and computes its value.

[179] Assuming the wages are equivalent in all the examples of our text, we would obtain the relation that 6 še silver corresponded to 10 sìla of barley, or 1 še of silver to $\frac{5}{3}$ sìla of barley, This equivalence, which also occurs in problem-text K (cf. p. 79), is actually attested for the period of the Third Dynasty of Ur (Schwenzner [1] p. 102).

The following is a tabulation of the first 30 problems in G (YBC 4657) with indications on the right margin to show which of the problems are treated in H (YBC 4663) and J (YBC 4662).

No.	given	find	
1	l, b, h, λ, w	g, V, M, E	H (YBC 4663)
2	E, b, h, λ, w	l	
3	E, l, h, λ, w	b	
4	E, l, b, λ, w	h	
5	E, l, b, h, w	λ	
6	E, l, b, h, λ	w	
7	$E, h, \lambda, w, l + b$	l, b	
8	$E, h, \lambda, w, l - b$	l, b	
9	l, b, h	g, V	
10	V, b, h	l	
11	V, l, h	b	
12	V, l, b	h	
13	$V, h, l + b$	l, b	
14	$V, h, l - b$	l, b	
15	$g + V, b, h$	l	
16	$g + V, l, h$	b	
17	$g + V, h, l + b$	l, b	
18	$g + V, h, l - b$	l, b	
19	$g, V, l + b$	l, b, h	
20	$g, V, l - b$	l, b, h	
21	$g, V, h = \frac{1}{7}(l - b)$	l, b, h[178]	
22	l, b, h, λ	λ_l	J (YBC 4662)
23	l, b, h, λ, m	$m\lambda_l$	
24	l, b, h, λ, m	t	
25	b, h, λ, m, t	l	
26	l, h, λ, m, t	b	
27	l, b, λ, m, t	h	
28	l, b, h, m, t	λ	
29	$m, t, h, \lambda, l + b$	l, b	
30	$m, t, h, \lambda, l - b$	l, b	

We now turn to the calculations carried out by H (YBC 4663) and J (YBC 4662).

No. 1 computes

$$g = lb = 7;30 \text{ SAR} \qquad \frac{V}{\lambda} = 4,30 \text{ man-days}$$

$$V = gh = 45 \text{ SAR} \qquad E = \frac{V}{\lambda} w = 9 \text{ gín}.$$

It should be noticed that none of these magnitudes is explicitly called in the text what it actually is; in this first example, the scribe restricts himself to the bare numbers. The following five examples need no special commentary because they strictly follow the formulas (2a) and (1a). Nos. 7 and 8 lead to quadratic equations of the standard form in which $l \pm b$ and lb are given. They are solved by

$$\frac{E}{w} \cdot \lambda = 45 \text{ SAR} = V$$

$$\frac{V}{h} = 7;30 \text{ SAR} = lb,$$

followed in No. 7 by

$$\frac{l+b}{2} \pm \sqrt{\left(\frac{l+b}{2}\right)^2 - lb} = \begin{cases} 5 \text{ GAR} = l \\ 1;30 \text{ GAR} = b \end{cases}$$

and in No. 8 by

$$\sqrt{\left(\frac{l-b}{2}\right)^2 + lb} \pm \frac{l-b}{2} = \begin{cases} 5 \text{ GAR} = l \\ 1;30 \text{ GAR} = b. \end{cases}$$

No solutions are preserved for Nos. 9 to 18, but it is clear that they can be solved in exactly the same way. Nos. 9 to 12 and Nos. 15 and 16 lead to linear equations, Nos. 13, 14 and 17, 18 to quadratic equations of the standard type.

From No. 19 on, the solutions are again preserved, in J (YBC 4662). Nos. 19 and 20 solve the same quadratic equations as Nos. 13 and 14.

In No. 21 the problem consists in finding l, b and h from $lb = 7;30$ SAR, $V = 45$ SAR and $h = \frac{1}{7}(l - b)$. The text computes

$$\frac{V}{lb} = h = 6 \text{ kùš} = 0;30 \text{ GAR} \qquad \text{and} \qquad \tfrac{1}{2}\, 7 = 3;30.$$

The next step $\frac{V}{h} = lb = 7;30$ is superfluous because lb is given. Then follows

$$0;30 \cdot 3;30 = h \cdot \tfrac{7}{2} = \tfrac{1}{2}(l - b) = 1;45$$

and therefore

$$\sqrt{\left(\frac{l-b}{2}\right)^2 + lb} = 3;15$$

and finally

$$\sqrt{\left(\frac{l-b}{2}\right)^2 + lb} \pm \frac{l-b}{2} = \begin{cases} 5 \text{ GAR} = l \\ 1;30 \text{ GAR} = b. \end{cases}$$

Nos. 22 and 23 involve the specific length λ_l corresponding to one man's assignment for a single day. The calculation in No. 22 follows the formula (2c)

$$\lambda_l = \frac{\lambda}{bh}$$

whereas No. 23, which is destroyed, is to be restored as having computed $m \cdot \lambda_l$ as the length assigned to $m = 30$ men.

The last five examples (Nos. 24–28) are based on the relation (2b)

$$\lambda = \frac{lbh}{mt}$$

assuming $m = 30$. Since four of the remaining five parameters are given, the last is easily computed.

Ja. YBC 8588

(Photograph: Plate 46; copy: Plate 21)

Transcription

Obverse

1 ki-lá 1,30 uš 30 sag
2 *i-na iš-te-en ka-la-ak-ki-im*
3 9 *ka-la-ak-ku*
4 *šu-up-lum* en-nam
5 1,30 uš gar-ra 30 sag gar-ra
6 *i-na iš-te-en ka-la-ak-ki-im*
7 9 *ka-la-ak-ku ša iq-bu-ú*
8 *iš-te-en ka-la-ak-kum ša iq-bu* éš-kàr
9 15 éš-kàr gar-ra
10 9 *ka-la-ak-ku ša iq-bu* erim-ḫá
11 9 erim-ḫá gar-ra-ma
12 15 éš-kàr *a-na* 9 erim-ḫá
13 ÍL 2,15 saḫar-ḫá
14 1,30 uš *a-na* 30 sag ÍL 45 a-šà
15 igi 45 a-šà *pu-ṭur-ma* 1,20
16 1,20 *a-na* 2,15 saḫar-ḫá
17 ÍL 3 *šu-up-l*[*um*]

Edge

1 *šum-ma* 1,30 uš 30 sag
2 [. . . . .] 15 éš-kàr

Reverse

1 9 erim-ḫá . . . . . .
2 1,30 uš *a-na* 30 sag ÍL 45
3 45 a-šà *a-na* 3 *šu-up-lim*
4 ÍL 2,15 saḫar
5 igi 15 éš-kàr *pu-ṭur-ma* 4
6 4 *a-na* 2,15 saḫar ÍL 9
7 9 erim-ḫá *it-ta-di-kum*

Translation

Obverse

1 A ki-lá. 1;30 is the length, 0;30 the width;
2 in one *kalakkum*

3(there are (?)) 9 *kalakkū*.[179a]
4What is the depth?
5Put (down) 1;30, the length. Put (down) 0;30, the width.
6(As for(?)) "in one *kalakkum*
7(there are(?)) 9 *kalakkū*", which he (*or*: they) mentioned,
8"one *kalakkum*", which he (*or*: they) mentioned—(as for) the assignment,
9put (down) 0;15, the assignment.
10(As for(?)) "9 *kalakkū*", which he (*or*: they) mentioned—(as for) the workers,
11put (down) 9 workers.
12Multiply 0;15, the assignment, by 9, the (number of) workers, (and the resulting)
132;15 is the volume.
14Multiply 1;30, the length, by 0;30, the width, (and the resulting) 0;45 is the area.
15Take the reciprocal of 0;45, and (the result is) 1;20.
16–17Multiply 1;20 by 2;15, the volume, (and the resulting) 3 is the depth.

Edge

1If 1;30 is the length, 0;30 the width,
2[. . . . .] 0;15 the assignment,

Reverse

19 workers . . . . . .
2Multiply 1;30, the length, by 0;30, the width, (and the result is) 0;45.
3–4Multiply 0;45, the area, by 3, the depth, (and the resulting) 2;15 is the volume.
5Take the reciprocal of 0;15, the assignment, and (the result is) 4.
6Multiply 4 by 2;15, the volume, (and the result is) 9.
7It has given you 9, the (number of) workers.

Commentary

The chief difficulty in understanding this text lies in the fact that we do not quite see the role played by the information given in lines 2–3: "in one *kalakkum* (there are(?)) 9 *kalakkū*."[179a] Even if the suggestion[179b] that *kalakkum* is the Akkadian equivalent of Sumerian ki-lá is correct, the consequences are not clear. In other respects, the problem (lines 1–4) is simple. The length l = 1;30 GAR and the width w = 0;30 GAR are given; find the depth h. In lines 5–11, the numerical values to be used in the actual calculation are set forth, and we are told that the number of workers (here to be understood in the sense of work-days) m is 9[179c] and the daily assignment (éš-kàr) of each worker λ is 0;15 SAR. The calculation (lines 12–17) proceeds smoothly, and the depth h is found by

$$h = \frac{\lambda m}{lw} = 3 \text{ kùš}.$$

The remainder of the text apparently serves as a check for the result obtained in the first part of the tablet. The number of workers is found by

$$m = \frac{lwh}{\lambda} = 9.$$

§ 6. Irrigation (Canals, Cistern)

K. YBC 4666

(Photograph: Plate 34; copy: Plate 9)

Transcription

Obverse

1 [1][pa$_5$-sig] 5 GAR UŠ uš-bi 2 kùš dagal 1 kùš bùr-b[i]
[2][⅓ gín saḫar] éš-kàr 1(bán) še á-bi lú-ḫun-gá
[3]gagar saḫar-ḫá erim-ḫá ù še en-nam 1(ubu)^{iku} gagar 1(ubu)^{iku} saḫar-ḫá
[4]2,30 erim-ḫá 5 gur še-e

2 [5]še pa$_5$-sig 5 gur 2 kùš dagal 1 kùš bùr-bi ⅓ gín éš-kàr 1(bán) á lú-ḫun-gá
[6]uš-bi en-nam 5 GAR UŠ uš[-bi]

3 [7]še pa$_5$-sig 5 gur 5 GAR UŠ uš 1 kùš bùr-b[i ⅓] gín éš-kàr
[8]1(bán) á-bi lú-ḫun-gá dagal-bi en-nam 2 k[ùš dagal-]bi

4 [9]še pa$_5$-sig 5 gur 5 GAR UŠ uš 2 kù[š dagal ⅓] gín éš-kàr
[10]1(bán) á-bi lú-ḫun-gá bùr-bi en-n[am 1 kùš b]ùr-bi

5 [11]pa$_5$-sig 5 GAR[179d] UŠ uš-bi 2 kùš daga[l 1 kùš bùr-bi ⅓ gín é]š-kàr
[12]lú-1-e uš en-n[am] al-díb 2 G[AR uš al-d]íb

6 [13]pa$_5$-sig 5 UŠ uš 2 kùš dagal 1 kùš bùr-bi ⅓ gín éš-kàr
[14]15 erim-ḫá uš en-nam al-díb-bé-eš 30 GAR uš al-díb-bé

[179a] Dr. Goetze suggests the translation, "nine *kalakkū* according to one *kalakkum*", i.e., 9 *kalakkū* of equal size. He compares *x* (*ammatu*) *ina* 1 *ammati*, "*x* cubits according to one (standard) cubit."
[179b] Made pp. 65f. *Kalakkū* is the plural of *kalakkum*.
[179c] The number of workers seems somehow to be connected with the number of *kalakkū*.
[179d] 5 GAR appears to be written as a ligature.

7 [15]pa$_5$-sig 5 UŠ uš 2 kùš dagal 1 kùš bùr-bi ⅓ gín éš-kàr
[16]15 erim-ḫá u$_4$ en-nam in-til-eš u$_4$-10-kam in-til-eš

8 [17]pa$_5$-sig 2 kùš dagal 1 kùš bùr-bi ⅓ gín saḫar éš-kàr
[18]15 erim-ḫá u$_4$-10-kam in-til-eš uš en-n[am] 5 UŠ uš-bi

9 [19]pa$_5$-sig 1 kùš dagal 1 kùš bùr-bi ⅓ gí[n saḫ]ar éš-kàr
[20]lú-1-e uš en-nam al-díb 4 G[AR u]š al-díb

10 [21]pa$_5$-sig 1 kùš dagal ½ kùš bùr-bi ⅓ [gí]n saḫar éš-kàr
[22]lú-1-e uš en-nam al-díb 8 GAR uš al-díb

11 [23]pa$_5$-sig 1,40 GAR uš 1 kùš dagal ½ kùš bùr-bi ⅓ gín éš-kàr
[24]lú-1-e u$_4$ en-nam ì-til u$_4$-12½-kam ì-til

Reverse

12 [1][p]a$_5$-sig 1 k[ùš] dagal 1 k[ùš] bùr-bi ⅓ gín éš-kàr 1[180] lú-ḫun-gá á-b[i 1(bán)] 5 [sì]la
[2]ì-ne-ḪUN 1 kùš dagal ⅔ kùš bùr-bi *še-a-am* en-nam ḫu-mu-ra-a[n-s]ì
[3]2(bán) še mu-na-an-sì

13 [4]pa$_5$-sig 1 kùš dagal 1 kùš bùr-bi ⅓ gín éš-kàr 1 lú-ḫun-gá á-bi 1(bán) 5 sìla
[5]ì-ne-ḪUN 1 kùš dagal 1⅓ kùš bùr-bi *še-a-am* en-nam ḫu-mu-ra-an-sì
[6]2(bán) še mu-na-an-sì

14 [7]pa$_5$-sig 5 UŠ uš 3 kùš dagal-an-ta 2 kùš dagal-ki-ta 2 kùš bùr-bi
[8]10 gín éš-kàr 6 še á lú-ḫun-gá gagar saḫar-[ḫ]á erim-ḫá ù kù-babbar en-nam
[9][1,2,30 gagar 2,]5 saḫar 12,30 erim-ḫá ⅓ [m]a-na 5 gín kù-babbar

15 [10]kù pa$_5$-sig ⅓ m[a-n]a 5 gín 3 kùš dagal-an-[t]a 2 kùš dagal-ki-ta 2 kùš bùr-bi
[11]10 gín saḫar éš-kàr 6 še á lú-ḫun-g[á] uš en-nam 5 UŠ uš

16 [12]kù pa$_5$-sig ⅓ ma-na 5 gín 5 UŠ uš 2 kùš dagal-ki-ta 2 kùš bùr-bi
[13]10 gín éš-kàr 6 še á lú-ḫun-gá d[agal]-an-ta {dagal-ki-ta ù bùr-bi} en-nam

17 [14]pa$_5$-sig 5 UŠ uš 3 kùs dagal-a[n-ta] 2 kùš dagal-ki-ta 2 kùš bùr-bi
[15]10 gín éš-kàr lú-1-e uš en-nam al-díb 4⅔ kùš 4 šu-si <al->díb

18 [16]pa$_5$-sig 5 UŠ uš 3 kùš dagal-an-ta 2 kùš dagal-ki-ta 2 kùš bùr-bi
[17]10 gín éš-kàr 30 erim-ḫá uš en-nam al-díb 12 GAR ì-díb-bé-eš

19 [18]pa$_5$-sig 5 UŠ uš 3 kùš dagal-an-ta 2 kùš dagal-ki-ta 2 kùš bùr-bi
[19]10 gín éš-kàr 30 erim-ḫá u$_4$ en-nam ì-til-eš u$_4$-25-kam ì-til

20 [20]pa$_5$-sig 3 kùš dagal-an-ta 2 kùš dagal-ki-ta 2 kùš bùr-bi
[21]10 gín éš-kàr 30 erim-ḫá u$_4$-25-kam ì-til-eš uš en-nam 5 UŠ uš

21 [22]pa$_5$-sig 5 UŠ <uš> 3 kùš dagal-an-ta 2 kùš dagal-ki-ta 2 kùš bùr-bi
[23]10 gín éš-kàr *i-na* 1 kùš bùr en-nam kú ì-kú ½ kùš ì-kú

22 [24]pa$_5$-sig 5 UŠ <uš> 2 kùš dagal-ki-ta 2 kùš bùr-bi [10 g]ín éš-kàr
[25][*i-n*]*a* 1 kùš bùr-bi ½ kùš kú ì-kú dagal-an-t[a en-nam]

23 [26][pa$_5$-]sig 5 UŠ uš 3 kùš dagal 3 kùš bùr-bi 10 g[ín éš-kàr]
[27][6] še á-bi lú-ḫun-gá

Left Edge

26[sic] im-šu pa$_5$-sig

Translation

Obverse

1 [1][A little canal.] 5 UŠ is its length, 2 kùš the width, 1 kùš its depth,
[2][⅓ gín volume] the assignment, 1 bán barley the wages of a hired man.
[3]What are the area, the volume, the (number of) workers, and the (total expenses in) barley? 1 ubu is the area; 1 ubu is the volume;
[4]2,30 is the (number of) workers; 5 gur is the (total expenses in) barley.

2 [5]The (total expenses in) barley of a little canal are 5 gur, 2 kùš the width, 1 kùš its depth, ⅓ gín (volume) the assignment, 1 bán (barley) the wages of a hired man.
[6]What is its length? 5 UŠ is [its] length.

3 [7]The (total expenses in) barley of a little canal are 5 gur, 5 UŠ the length, 1 kùš its depth, [⅓] gín (volume) the assignment,
[8]1 bán (barley) the wages of a hired man. What is its width? 2 kùš is its [width].

4 [9]The (total expenses in) barley of a little canal are 5 gur, 5 UŠ the length, 2 kùš [the width, ⅓] gín (volume) the assignment.

[180] Or 1 {1}?

[10]1 bán (barley) the wages of a hired man. What is its depth? [1 kùš] is its depth.

5 [11]A little canal. 5 UŠ is its length, 2 kùš the width, [1 kùš its depth, ⅓ gín (volume) the as]signment.
[12]What length did 1 man take? He took 2 GAR [length].

6 [13]A little canal. 5 UŠ is the length, 2 kùš the width, 1 kùš its depth, ⅓ gín (volume) the assignment.
[14]What length did 15 workers take? They took 30 GAR length.

7 [15]A little canal. 5 UŠ is the length, 2 kùš the width, 1 kùš its depth, ⅓ gín (volume) the assignment.
[16](In) how many days did 15 workers finish? They finished the 10th day.

8 [17]A little canal. 2 kùš is the width, 1 kùš its depth, ⅓ gín volume the assignment;
[18]15 workers finished the 10th day. What is the length? 5 UŠ is its length.

9 [19]A little canal. 1 kùš is the width, 1 kùš its depth, ⅓ gí[n vol]ume the assignment.
[29]What length did 1 man take? He took 4 GAR length.

10 [21]A little canal. 1 kùš is the width, ½ kùš its depth, ⅓ gín volume the assignment.
[22]What length did 1 man take? He took 8 GAR length.

11 [23]A little canal. 1,40 GAR is the length, 1 kùš the width, ½ kùš its depth, ⅓ gín (volume) the assignment.
[24](In) how many days did one man finish? He finished the 12½th day.

Reverse

12 [1]A little canal. 1 kùš is the width, 1 kùš its depth, ⅓ gín (volume) the assignment; 1[180] hired man his wages [1 bán] (and) 5 sìla
[2]..... 1 kùš the width, ⅔ kùš its depth. How much barley should I give you?
[3]I gave him 2 bán barley.

13 [4]A little canal. 1 kùš is the width, 1 kùš its depth, ⅓ gín (volume) the assignment; 1 hired man his wages 1 bán (and) 5 sìla
[5]..... 1 kùš the width, 1⅓ kùš its depth. How much barley should I give you?
[6]I gave him 2 bán barley.

14 [7]A little canal. 5 UŠ is the length, 3 kùš the upper width, 2 kùš the lower width, 2 kùš its depth,
[8]10 gín (volume) the assignment, 6 še (silver) the wages of a hired man. What are the area, the volume, the (number of) workers, and the (total expenses in) silver?
[9][1,2;30 (SAR) is the area; 2],5 (SAR) is the volume; 12,30 is the (number of) workers; ⅓ ma-na (and) 5 gín is the (total expenses in) silver.

15 [10]The (total expenses in) silver of a little canal are ⅓ ma-na (and) 5 gín, 3 kùš the upper width, 2 kùš the lower width, 2 kùš its depth,
[11]10 gín volume the assignment, 6 še (silver) the wages of a hired man. What is the length? 5 UŠ is the length.

16 [12]The (total expenses in) silver of a little canal are ⅓ ma-na (and) 5 gín, 5 UŠ the length, 2 kùš the lower width, 2 kùš its depth,
[13]10 gín (volume) the assignment, 6 še (silver) the wages of a hired man. What is the upper width {, the lower width, and its depth}?

17 [14]A little canal. 5 UŠ is the length, 3 kùš the upper width, 2 kùš the lower width, 2 kùš its depth,
[15]10 gín (volume) the assignment. How much length did 1 man take? He took 4⅔ kùš (and) 4 šu-si.

18 [16]A little canal. 5 UŠ is the length, 3 kùš the upper width, 2 kùš the lower width, 2 kùš its depth,
[17]10 gín (volume) the assignment. How much length did 30 workers take? They took 12 GAR.

19 [18]A little canal. 5 UŠ is the length, 3 kùš the upper width, 2 kùš the lower width, 2 kùš its depth,
[19]10 gín (volume) the assignment. (In) how many days did 30 workers finish? They finished the 25th day.

20 [20]A little canal. 3 kùš is the upper width, 2 kùš the lower width, 2 kùš its depth,
[21]10 gín (volume) the assignment; 30 workers finished the 25th day. What is the length? 5 UŠ is the length.

21 [22]A little canal. 5 UŠ is <the length>, 3 kùš the upper width, 2 kùš the lower width, 2 kùš its depth,
[23]10 gín (volume) the assignment. What is the inclination[182] per 1 kùš depth? ½ kùš is the inclination.[182]

[182] For the literal meaning, cf. p. 81 of the commentary.

22 [24]A little canal. 5 UŠ is <the length>, 2 kùš the lower width, 2 kùš its depth, [10] gín (volume) the assignment;
[25]the inclination[182] per 1 kùš depth is ½ kùš. [What is] the upper width?

23 [26]A little [canal]. 5 UŠ is the length, 3 kùš the width, 3 kùš its depth, 10 g[ín (volume) the assignment],
[27][6] še (silver) the wages of a hired man.

Left Edge

26[sic] sections: little canal.

Commentary

a. Mathematical Commentary

The 23 problems collected in this text exhibit a great deal of similarity with the ki-lá problems of the preceding texts. The object to be worked on is now supposed to be a little canal (pa_5-sig) of a certain length and with a rectangular cross-section.[183] The number of workers, time, expenses, etc. are again combined in various ways to form a large number of examples.

Nos. 1–8.

The common parameters in these eight examples are the following:

l = 5,0 GAR,[184] length (uš) of the canal
b = 2 kùš = 0;10 GAR, width (dagal) of the canal
h = 1 kùš, depth (bùr) of the canal.

The area of the canal is therefore

$$A = lb = 50 \text{ GAR}^2 = 1 \text{ ubu}$$

and the volume to be removed

$$V = lbh = 50 \text{ GAR}^2 \text{ kùš} = 1 \text{ ubu}.$$

If λ denotes the volume of earth excavated by one man in one day, m the number of men required to finish the whole canal in one day, w the wages of one man for one day's work, W the total wages expended, then the following relations must hold:

$$(1) \qquad \frac{V}{\lambda} = \frac{lbh}{\lambda} = m \qquad m \cdot w = W.$$

The value of λ (éš-kàr) given in the text is ⅓ gín, written with the special sign for ⅓; but since m = 2,30 and V = 50 SAR, λ should be ⅓ SAR = 20 gín, not ⅓ gín.[185] This discrepancy[186] continues through the first 13 examples.

If we denote the length of the canal dug by one man in one day by l_1, we obtain

$$l_1 = \frac{\lambda}{bh} = 2 \text{ GAR}.$$

If l_{15} stands for the length dug by 15 men in one day, then

$$l_{15} = 15\, l_1 = 30 \text{ GAR}.$$

For 15 men to finish the whole work, a time interval of

$$t_{15} = 10 \text{ days}$$

would be required. We furthermore let m_{10} denote the number of men required to finish the work in 10 days.

Daily wages per man and total expenses are here expressed in barley (še). The former amounts to

$$w = 1 \text{ bán} = 10 \text{ sìla}$$

and the total expenses for m = 2,30 men are

$$W = 25{,}0 \text{ sìla} = 5 \text{ gur}.$$

Later (Nos. 14ff.), w is assumed to be 6 še silver. The resulting equivalence between ⅗ še silver and 1 sìla barley is the same which we have already observed above, p. 74 note 179.

The arrangement of the problems is as follows:

No.	given	find	result indicated by the text
1	l, b, h, λ, w	A, V, m, W	A = 1 ubu V = 1 ubu m = 2,30 W = 5 gur
2	W, b, h, λ, w	l	l = 5,0 GAR
3	W, l, h, λ, w	b	b = 2 kùš
4	W, l, b, λ, w	h	h = [1 kùš]
5	l, b, h, λ	l_1	l_1 = 2 G[AR]
6	l, b, h, λ	l_{15}	l_{15} = 30 GAR
7	l, b, h, λ, m_{10}	t_{15}	t_{15} = 10 days
8	b, h, λ, m_{10}, t_{15}	l	l = 5,0 GAR

Nos. 9–13.

These five examples assume new parameters. Nos. 9 and 10 ask for the length l_1 which one man digs in one day if his daily output λ is 0;20 SAR,[187]

[183] Analogous problems are also given in VAT 7528 (MKT I pp. 508–513).

[184] Written 5 GAR UŠ or 5 UŠ; we have translated uniformly "5 UŠ".

[185] The correct value ⅓ SAR occurs in the analogous examples of VAT 7528 (MKT I pp. 508f., Nos. 1–4; p. 511, No. 18). The values given in YBC 5037 (problem-text F above) for the daily output per man vary from 7;30 gín to 10 gín. In the second part of our text, λ = 10 gín.

[186] Cf. also Nos. 9ff. of the following text. For similar discrepancies in economic texts of the same period, cf. Thureau-Dangin [2], p. 157.

[187] Written ⅓ gín as in Nos. 1–8.

and b and h of the cross-section are given as follows:

No. 9 b = 1 kùš = 0;5 GAR
h = 1 kùš
No. 10 b = 1 kùš = 0;5 GAR
h = ½ kùš.

It obviously follows that

$$l_1 = \frac{\lambda}{bh}$$

which yields the values given in the text: l_1 = 4 GAR in No. 9, and 8 GAR in No. 10.

No. 11 requires the determination of the number t_1 of days required for one man to dig a canal with the dimensions

l = 1,40 GAR h = ½ kùš
b = 1 kùš = 0;5 GAR

if λ = 0;20 SAR[187]. The result is

$$t_1 = \frac{lbh}{\lambda} = 12\tfrac{1}{2} \text{ days.}$$

Nos. 12 and 13 both give

b = 1 kùš λ = 0;20 SAR[187]
h = 1 kùš

but then are mentioned additional magnitudes which we denote by

No. 12 b' = 1 kùš No. 13 b' = 1 kùš
h' = ⅔ kùš h' = 1⅓ kùš
w' = [1(bán)] 5 sìla w' = 1(bán) 5 sìla

and payments are asked, for which 2 bán seems to be given as the answer in both cases. The relationship assumed between all these magnitudes is not clear to us.

Nos. 14–22.

This group is uniform with respect to the values of the parameters; the cross-section of the canal is no longer a rectangle, but is now a trapezoid with the upper width (b_u) greater than the lower width (b_l). The dimensions are

l = 5,0 GAR[184], length (uš)
b_u = 3 kùš = 0;15 GAR, upper width (dagal-an-ta)
b_l = 2 kùš = 0;10 GAR, lower width (dagal-ki-ta)
h = 2 kùš, depth (bùr).

The corresponding volume (saḫar) is given by

$$V = l\frac{b_u + b_l}{2}h = 2{,}5 \text{ SAR.}$$

In No. 14 the "area" is also asked for (as in No. 1 in the first group), but the answer is unfortunately not preserved, leaving us in doubt as to which area is meant.[188] The output λ per man per day is

λ = 10 gín = 0;10 SAR

i.e., half of the output assumed in the previous examples. The wages are now expressed in silver instead of barley. The wages for one man's daily work is

w = 6 še = 0;2 gín,

and the total expenses corresponding to the number of men

m = 12,30

working (one day each), are

W = 25 gín = ⅓ ma-na 5 gín.

As in the previous examples, we denote the length which n men are able to dig in one day by l_n, the number of men required to finish the whole work in 10 days by m_{10}, and the time interval for the completion of the work by 30 men by t_{30}. The first seven problems are:

No.	given	find	result given in the text
14	l, b_u, b_l, h, λ, w	A, V, m, W	[A = 1,2;30] V = 2,5 m = 12,30 W = ⅓ ma-na 5 gín
15	W, b_u, b_l, h, λ, w	l	l = 5,0 GAR
16	W, l, b_l, h, λ, w	b_u, {b_l, h}[189]	—
17	l, b_u, b_l, h, λ	l_1	l_1 = 4⅔ kùš 4 šu-si
18	l, b_u, b_l, h, λ	l_{30}	l_{30} = 12 GAR
19	l, b_u, b_l, h, λ, m_{25}	t_{30}	t_{30} = 25 days
20	b_u, b_l, h, λ, m_{25}, t_{30}	l	l = 5,0 GAR

A new concept, the slope of the sides of the canal,[190] is introduced in the last two examples (Nos. 21 and

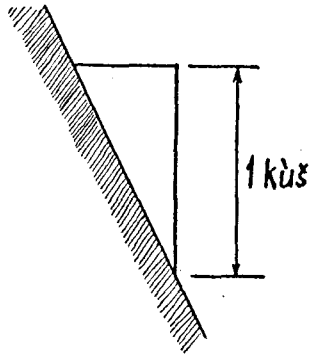

FIG. 20.

[188] One could think of lb_u or lb_l or of the mean value $l\frac{b_u + b_l}{2}$. Our restoration assumes the third possibility.

[189] Added by mistake, since these magnitudes are already given. The source of the error is apparently the telescoping of *three* examples asking for b_u, b_l, h, respectively.

[190] Cf. the following section for the terminology.

22) of this group. The slope is measured by the deviation d corresponding to a vertical height of 1 kùš

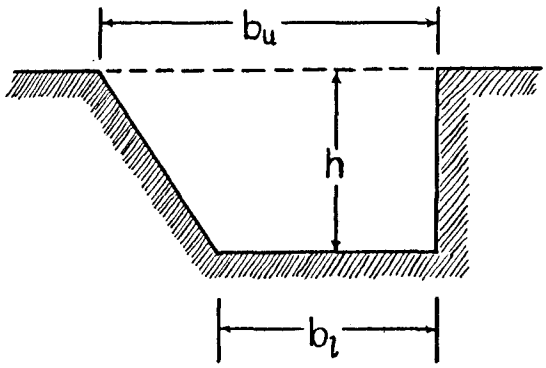

FIG. 21.

(cf. fig. 20). For a profile like the one assumed here, d will be (cf. fig. 21)

$$d = \frac{b_u - b_l}{h}.$$

No. 21 requires the calculation of d if b_u, b_l and h are given; this leads to $d = \frac{1}{2}$ kùš. No. 22 assumes d, b_l and h as known and asks for the value of b_u. Both examples superfluously give also l and λ.

As to the profile itself, the unsymmetrical arrangement in fig. 21 was chosen in order to give the simplest possible explanation for the parameter d which characterizes the slope. One could, however, assume that d means the *total* decrease in width for one kùš depth and that the actual slope corresponds to only $\frac{1}{2}d$ on each side. This would, of course, make for a technically better construction for a canal, but the fact that the previous group of examples assumes rectangular profiles, i.e., vertical(!) sides, shows how cautious one must be in estimating the validity of practical considerations of this type.

No. 23.

This final example consists of two lines which merely give the parameters of a problem. That these two lines constitute a kind of "catch-line" giving the first words of a tablet which was supposed to follow can fortunately be shown by the fact that the first example of the next problem-text, L, begins with the identical words. This proves that problem-text L is the direct continuation of problem-text K.

According to the colophon, this text was supposed to consist of 26 examples, although there are only 23.

b. Terminology

kú ì-kú

Our text gives a fuller form of the expression for the slope than any of the previously published texts, which offer the following terminology:[191]

(1) *i-na* (*n*) (kùš) (*m*) kùš šà-gal
(2) (*n*) šà-gal
(3) *i-na* (*n*) kùš (*m*) kùš (kú) ì-kú(-e)
(4) *i-na* (*n*) kùš en-nam *i-ku-ul*
(5) *i-na* (*n*) kùš *ú-ku* <*-la*>[192] en-nam *i-ku-ul*
(6) *i-na* (*n*) kùš (*m*) šu-si é-gar$_8$ *ú-ku-la*[193] *i-ku-ul*
(7) [š]*a i-na* (*n*) kùš *i-ku-lu*.

The present text asks (reverse, 23) *i-na* 1 kùš bùr en-nam kú ì-kú, and answers $\frac{1}{2}$ kùš ì-kú; in reverse, 25, the parameter for the slope is given as *i-na* 1 kùš bùr-bi $\frac{1}{2}$ kùš kú ì-kú.

The literal meaning of the key terms is quite clear, since šà-gal, *ukullū*, and the noun kú all mean "fodder, food," and the verbs kú and *akālu* mean "eat, consume." The literal meaning of the passages in our text is, accordingly, "In 1 kùš of the depth how much fodder did it (i.e., the canal[194]) eat," the answers being, "It ate $\frac{1}{2}$ kùš" and "In 1 kùš of its depth it ate $\frac{1}{2}$ kùš fodder." The practical meaning in all cases is simply that for a given unit of height (or depth), the profile of the figure in question recedes by a certain number of units.

L. YBC 7164

(Photograph: Plate 35; copy: Plate 10)

Transcription

Obverse

1 [1]pa$_5$-sig 5 UŠ uš 3 kùš dagal 3 kùš bùr-bi [10 gín é]š-kàr
[2]6 še á lú-ḫun-gá [gag]ar saḫar erim-ḫá kù-babb[ar en-na]m
[3]1,15 SAR gagar <2> [ik]u 25 SAR saḫar 2(geš^{2}u) 2,30 erim[-ḫá ù $\frac{2}{3}$ m]a-na 5 gín kù-babbar

2 [4]pa$_5$-sig 5 UŠ uš 3 kùš dagal 3 kùš [bùr-bi 1 kùš] *šu*[195] $\frac{1}{3}$ ma-na[196] *ši-lu-tum*
[5]2 kùš *šu*[195] saḫar 10 gín dusu lú-1-e [uš en-nam al-]díb 3 kùš 6 šu-si

3 [6]pa$_5$-sig 5 UŠ uš 3 kùš dagal 3 kù[š bùr-bi 1 kùš š]*u*[195] $\frac{1}{3}$ ma-na[196] saḫar *ši-lu-tum*

[191] For the texts cf. MKT II p. 32 under ŠÀ-GAL, ibid. p. 12 under *akâlu*, and MKT III, p. 71 under kú; for discussions of the mathematical meaning cf. the references given MKT III p. 82 under Böschungen.

[192] Read so according to Thureau-Dangin, TMB p. 129, **231**, 13. Note, however, that von Soden [1] p. 198 reads úr instead of *ú-ku* <*-la*>.

[193] So according to Thureau-Dangin, TMB p. 129, **231**, 18.

[194] That the subject of ì-kú is the figure in question follows from (6) above.

[195] Abbreviation for *šuplum* (cf. below p. 88).

[7]2 kùš *šu*[195] saḫar 10 gín dusu 30 erim-ḫá uš e[n-nam al-]díb-eš 8 GAR uš <al->díb

4 [8]pa$_5$-sig 5 UŠ uš 3 kùš dagal 3 kùš bùr-bi 1 kùš *šu*[195] ⅓ ma-na[196] saḫar *ši-lu-tum*
[9]2 kùš *šu*[195] saḫar 10 gín dusu 30 erim-ḫá u$_4$ en-nam in-til i[t]u-1 7½-kam in-til

5 [10]pa$_5$-sig 3 kùš dagal 3 kùš bùr-bi 1 kùš *šu*[195] <⅓ ma-na> saḫar *ši-lu-tum* 2 kùš *šu*[195] <saḫar 10 gín> dusu
[11]30 erim-ḫá itu-1 7½-kam in-til uš en-nam 5 UŠ uš-bi

6 [12]pa$_5$-sig 5 UŠ uš 3 kùš dagal 4½ kùš bùr-bi 1 kùš *šu*[195] ⅓ ma-na[196] *ši-lu-tum*
[13]2 kùš *šu*[195] saḫar 10 gín dusu 1½ kùš *šu*[195] 8½[197] gín dusu
[14]lú-1-e uš en-na[sic] al-díb 5⅔[198] kùš 3,20 šu-si uš al-díb

7 [15]pa$_5$-sig 5 UŠ uš 3 kùš dagal 3 kùš bùr-bi 1 kùš *šu*[195] ⅓ ma-na[196] *ši-lu-tum*
[16]2 kùš *šu*[195] 10 gín dusu lú-1-e igi-TE-en u$_4$ *ši-lu-ta-am iḫ-re*
[17]igi-TE-en u$_4$ dusu *iḫ-r[e]* *ù* saḫ[ar] en-nam igi-5-gál u$_4$ *ši-lu-ta-am*
[18]4 gín saḫar *is-sú-uḫ* ⅔ u$_4$ *ù* igi-5-gál ⅔ u$_4$ dusu *iḫ-re* 8 gín saḫar

8 [19]pa$_5$-sig-[li]bir-ra 5 UŠ uš 1 kùš dagal 1 kùš bùr-bi
[20]*te-er-di-is-sà* ½ kùš *ta-ra-ḫi-ša* *ú-ša-mi-iṭ* saḫar-bi en-nam ⅓ SAR 5 gín[199]

9 [21]pa$_5$-sig-libir-ra 5 UŠ uš 1 kùš dagal 1 kùš bùr-bi *de*[sic]-*e[r-di-sà]*
[22]½ kùš *ta*[200] *ú-ša-mi-iṭ* ⅓ gín[196] éš-kàr lú-1-e uš e[n-nam al-díb 4 GAR] uš

10 [23]pa$_5$-sig-libir-ra 5 UŠ uš 1 kùš dagal 1 kùš bùr-bi *[te-er-di-sà]*
[24]½ kùš *ta*[200] *ú-ša-mi-iṭ* ⅓ gín[196] éš-kàr 10 erim-ḫá u[š en-nam al-díb]
[25]40 GAR u[š] in-díb-bé[-eš]

11 [26][pa$_5$-s]ig[-libir-]ra 5 UŠ uš 1 kùš dagal 1 k[ùš bùr-bi]
[27]*te-e[r-di-i]s-sà* ½ kùš *ta*[200] *ú-ša-mi-i[ṭ* ⅓ gín éš-kàr]
[28]10 erim-ḫá u$_4$ en-nam in-til u$_4$-7½-kam [in-til]

Reverse

12 [1][p]a$_5$-sig-libir-ra 1 kùš dagal 1 kùš bùr-bi *de*[sic]-*er*<-*di*>-*sà* ½ kùš *ta*[200] <*ú-ša-mi-iṭ* ⅓ gín éš-kàr>
[21][0 er]im-ḫá u$_4$-7½ ì-til-eš uš-bi en-nam 5 UŠ [uš-bi]

13 [3]pa$_5$-sig-libir-ra 5 UŠ uš-bi 2 kùš dagal 1 kùš bùr-bi
[4]*te-er-di-sà* ½ kùš *ta*[200] *ú-ša-am-mi-iṭ*
[5]1 kùš *a-na* bùr-bi bí-daḫ saḫar-bi en-nam 1 iku saḫar

14 [6]pa$_5$-sig-libir-ra 5 UŠ <uš->bi 2 kùš dagal 1 kùš bùr-bi
[7]*te-e[r-d]i-sà* ½ kùš *ta*[200] *ú-ša-am-mi-iṭ* 1 kùš *a-na* bùr-bi bí-d[aḫ]
[8]⅓ gín[196] saḫ[ar] éš-kàr lù-1-e uš en-nam al-díb
[9]1 GAR uš al-díb

15 [10]pa$_5$-sig-libir-ra 5 UŠ uš-bi 3 kùš dagal 1 kùš bùr-bi
[11]*te-er-di-sà* ½ kùš *ta*[200] *ú-ša-am-mi-iṭ* 1½ kùš
[12]*a-na* bùr-bi bí-daḫ saḫar-bi en-nam [. . . . . .]. .[201]

16 [13]pa$_5$-sig-libir-ra 5 UŠ <uš->bi 2 kùš dagal 1 k[ùš bù]r-b[i]
[14]*te-er-di-sà* ½ kùš *ta*[200] *ú-ša-am-mi-iṭ* 1½ kùš *a-na* bùr-bi bí-daḫ
[15]1 kùš bùr-bi ⅓ gín[196] saḫar éš-kàr 1½ kùš bùr-bi 10 gín saḫar . . .[202]
[16]lú-1-e uš en-nam al-díb 4⅔ kùš 4 šu-si uš al-díb

17 [17]pa$_5$-sig 1 kùš dagal 1 kùš búr-bi *me-e pa-ta-a-tim*
[18]en-nam A NIGIN nim 4 RA nim

18 [19]pa$_5$-sig 2 kùš dagal 1½ kùš bùr-bi *me-e pa-ta-a-tim*
[20]en-nam A NIGIN ⅓ A . . .

Translation

Obverse

1 [1]A little canal. 5 UŠ is the length, 3 kùš the width, 3 kùš its depth, [10 gín] (volume) the assignment,
[2]6 še (silver) the wages of a hired man. What are the area, the volume, the (number of) workers, the (total expenses in) silver?
[3]1,15 SAR is the area; <2> iku (and) 25 SAR is the volume; 2 geš'u (and) 2,30 is the (number of) workers; [and ⅔] ma-na (and) 5 gín is the (total expenses in) silver.

2 [4]A little canal. 5 UŠ is the length, 3 kùš the width, 3 kùš [its depth; (for the first) 1 kùš] depth,[203] ⅓ ma-na[196] (volume) is the *šīlūtum*;
[5](for the next) 2 kùš depth,[203] a volume of[204]

[196] For: SAR.
[197] Mistake for 7½.
[198] Mistake for 1⅔.
[199] Mistake for 25 SAR.
[200] Abbreviation for *ta-ra-ḫi-ša*.

[201] See commentary, pp. 87f.
[202] We feel unsure about the identification of this sign; one would expect éš-kàr; the sign is perhaps intended to be dusu.
[203] The text abbreviates the word in question by writing the first syllable only.
[204] The syntax is peculiar here; the translation is probably not literal.

10 gín is the dusu. [How much length did] one man take? 3 kùš (and) 6 šu-si.

3 [6]A little canal. 5 UŠ is the length, 3 kùš the width, 3 kùš [its depth; (for the first) 1 kùš] depth,[203] ⅓ ma-na[196] volume is the *šīlūtum*; [7](for the next) 2 kùš depth,[203] a volume of[204] 10 gín is the dusu. How much length did 30 workers take? They took 8 GAR length.

4 [8]A little canal. 5 UŠ is the length, 3 kùš the width, 3 kùš its depth; (for the first) 1 kùš depth,[203] ⅓ ma-na[196] volume is the *šīlūtum*; [9](for the next) 2 kùš depth,[203] a volume of[204] 10 gín is the dusu. (In) how many days did 30 workers finish? They finished (after) 1 month (and) 7½ (days).[205]

5 [10]A little canal. 3 kùš is the width, 3 kùš its depth; (for the first) 1 kùš depth,[203] <⅓ ma-na[196]> volume is the *šīlūtum*; (for the next) 2 kùš depth,[203] <a volume of 10 gín> is the dusu; [11]30 workers finished (after) 1 month (and) 7½ (days).[205] What is the length? 5 UŠ is its length.

6 [12]A little canal. 5 UŠ is the length, 3 kùš the width, 4½ kùš its depth; (for the first) 1 kùš depth,[203] ⅓ ma-na[196] (volume) is the *šīlūtum*; [13](for the next) 2 kùš depth,[203] a volume of[204] 10 gín is the dusu; (for the last) 1½ kùš depth,[203] (a volume of) 8½[197] gín is the dusu. [14]How much length did one man take? He took 5⅔[198] kùš (and) 3;20 šu-si length.

7 [15]A little canal. 5 UŠ is the length, 3 kùš the width, 3 kùš its depth; (for the first) 1 kùš depth,[203] ⅓ ma-na[196] (volume) is the *šīlūtum*; [16–18](for the next) 2 kùš depth,[203] (a volume of) 10 gín is the dusu. What fraction of a day did one man dig the *šīlūtum*? What fraction of a day did he dig the dusu? And what is the volume? One-fifth of a day he extracted the *šīlūtum*, (amounting to) 4 gín volume; ⅔ of a day and one-fifth of ⅔ of a day he dug the dusu, (amounting to) 8 gín volume.

8 [19]An old, little canal. 5 UŠ is the length, 1 kùš the width, 1 kùš its depth; [20]its increase cut off ½ kùš of its *taraḫḫum*. What is its volume? ⅓ SAR (and) 5 gín.[199]

9 [21]An old, little canal. 5 UŠ is the length, 1 kùš the width, 1 kùš its depth; its increase [22]cut off ½ kùš of its *taraḫḫum*[203]; ⅓ gín[196] (volume) is the assignment. How much length [did] one man [take? 4 GAR] length.

10 [23]An old, little canal. 5 UŠ is the length, 1 kùš the width, 1 kùš its depth; [its increase] [24]cut off ½ kùš of its *taraḫḫum*[203]; ⅓ gín[196] (volume) is the assignment. [How much] length [did] 10 workers [take]? [25]They took 40 GAR length.

11 [26]An [old], little [canal]. 5 UŠ is the length, 1 kùš the width, 1 kùš [its depth]; [27]its increase cut off ½ kùš of its *taraḫḫum*[203]; [⅓ gín (volume) is the assignment]. [28](In) how many days did 10 workers finish? [They finished] (after) 7½ days.[205]

Reverse

12 [1]An old, little canal. 1 kùš is the width, 1 kùš its depth; its increase <cut off> ½ kùš of its *taraḫḫum*[203]; <⅓ gín is the assignment>. [2]10 workers finished (in) 7½ days. What is its length? 5 UŠ is [its length].

13 [3]An old, little canal. 5 UŠ is its length, 2 kùš the width, 1 kùš its depth; [4]its increase cut off ½ kùš of its *taraḫḫum*[203] (and) [5]added 1 kùš to its depth. What is its volume? 1 iku is the volume.

14 [6]An old, little canal. 5 UŠ is its <length>, 2 kùš the width, 1 kùš its depth; [7]its increase cut off ½ kùš of its *taraḫḫum*[203] (and) [add]ed 1 kùš to its depth; [8]⅓ gín[196] volume is the assignment. How much length did one man take? [9]He took 1 GAR length.

15 [10]An old, little canal. 5 UŠ is its length, 3 kùš the width, 1 kùš its depth; [11–12]its increase cut off ½ kùš of its *taraḫḫum*[203] (and) added 1½ kùš to its depth. What is its volume? [. . . . .].[201]

16 [13]An old, little canal. 5 UŠ is its <length>, 2 kùš the width, 1 kùš its depth; [14]its increase cut off ½ kùš of its *taraḫḫum*[203] (and) added 1½ kùš to its depth; [15](for the first) 1 kùš of its depth, ⅓ gín[196] volume is the assignment; (for the next) 1½ kùš of its depth, 10 gín volume is the . . . . .[206] [16]How much length did one man take? He took 4⅔ kùš (and) 4 šu-si length.

[205] The text uses the ordinal for the length of time; to avoid awkwardness, this has been ignored in the translation.

[206] Unidentified sign, perhaps intended for dusu; one would expect the word for "assignment."

17 [17–18]Λ little canal. 1 kùš is the width, 1 kùš its depth. What ..... waters of the water-courses .....

18[19–20]Λ little canal. 2 kùš is the width, 1½ kùš its depth. What ..... waters of the water-courses .....

Commentary

a. Mathematical Commentary

As was noted above, p. 81, commentary to No. 23, problem-text L is the direct continuation of problem-text K.

All the examples of this text deal with a little canal (pa$_5$-sig) of rectangular cross-section. The first seven assume a width of 3 kùš and a depth of 3 kùš or more. Nos. 8–16 have to do with an old canal (pa$_5$-sig-libir-ra) of similar dimensions which must be enlarged because of deposited silt or because a flow of water greater than that permitted by the original canal is required. The work involved therefore consists in cleaning out and/or expanding the existing canal. The last two examples apparently deal with the water supplied by the canal.

No. 1.

The dimensions given for the length (uš), width (dagal), and depth (bùr) are

l = 5,0 GAR b = 3 kùš = 0;15 GAR h = 3 kùš.

The work λ assigned to one man per day amounts to

$$\lambda = 0;10 \text{ SAR}$$

volume. The wages are

$$w = 6 \text{ še} = 0;2 \text{ gín silver}$$

per man per day.

The text requires the answers for: the area (gagar) $g = lb$ covered by the canal; its volume (saḫar) V; the number m of workers (erim-ḫá) necessary to complete the whole work in one day, i.e., the total number of man-days; and, finally, the total expenses E for the work expressed in silver.

The answers given satisfy the following relations:

$$g = lb = 1{,}15 \text{ SAR}$$
$$V = lbh = 3{,}45 \text{ SAR} = [2]^{207} \text{ iku } 25 \text{ SAR}$$
$$m = \frac{V}{\lambda} = 22{,}30 = 2 \text{ geš}^{2}\text{u } 2{,}30$$
$$E = mw = 45 \text{ gín} = [\tfrac{2}{3}]^{208} \text{ ma-na } 5 \text{ gín}.$$

The geš^{2}u in the third line literally means ten-sixties, i.e., 600.

[207] Omitted by the scribe.

[208] Restored.

Nos. 2–5.

The dimensions given for the canal in this group of four examples are the same as in No. 1, namely,

l = 5,0 GAR b = 3 kùš = 0;15 GAR h = 3 kùš.

The volume assigned to one man each day, however, is no longer 0;10 SAR but is raised to

$$\lambda = 0;12 \text{ SAR}.$$

This statement is not made explicitly, but the discussion of No. 7 will show that it is implicitly contained in the wording

1 kùš *šu* ⅓ ma-na (saḫar) *ši-lu-tum*
2 kùš *šu* saḫar 10 gín dusu.

Anticipating this conclusion, we obtain the following results:

No. 2. The length l_λ which corresponds to one man's work per day is therefore given by

$$l_\lambda = \frac{\lambda}{bh} = \frac{0;12}{0;15 \cdot 3} = \frac{4}{15} = 0;16 \text{ GAR} = 3 \text{ kùš } 6 \text{ šu-si}.$$

No. 3. The length of the canal worked by 30 men during one day is therefore

$$30 \cdot l_\lambda = 0;16 \cdot 30 = 8 \text{ GAR}.$$

No. 4. The time required for 30 men to complete the total work on l = 5,0 GAR length is therefore

$$t = \frac{5{,}0}{8} = 37\tfrac{1}{2} \text{ days}.$$

This number is expressed as "1 month (and) 7½ (days)". The assumption of 30 days as the length of one month for purposes of computation is well attested by other texts.[209]

No. 5. This problem is the inverse of No. 4: 30 men finish a total length of l = 5,0 GAR in 37½ days.

Nos. 6 and 7.

It is more convenient to begin the discussion of these two examples with No. 7 because it assumes simpler conditions. The canal considered in No. 7 has the following dimensions:

l = 5,0 GAR b = 3 kùš = 0;15 GAR h = 3 kùš.

The rate of work involved, however, is not considered to be the same for different levels of depth. The depth h of 3 kùš is therefore subdivided into two parts

$$h' = 1 \text{ kùš} \qquad h'' = 2 \text{ kùš}$$

such that the rate of output λ′ for the upper part is

[209] Cf., e.g., MKT III p. 63.

twice the rate λ'' for the deeper level:

$$\lambda' = 0;20 \text{ SAR} \qquad \lambda'' = 0;10 \text{ SAR}.$$

Each of these different rates of output is expressed in the text by indicating the volume produced by one man in one day if he spent the full day in working at the relevant level. This is clearly the meaning of

1 kùš *šu* ⅓ ma-na (saḫar) *ši-lu-tum*
= "(for the first) 1 kùš depth, ⅓ ma-na[210] (volume) is the *šīlūtum*",

2 kùš *šu* 10 gín dusu
= "(for the next) 2 kùš depth, (a volume) of 10 gín is the dusu,"

even if we for the moment leave undetermined the literal meanings of *šīlūtum* and dusu in this context.[211]

The text now asks what parts of a man's work-day are expended on these two types of labor, and the answer is given as $\tau_1 = \frac{1}{5}$, $\tau_2 = \frac{4}{5}$ of a day. This result can be obtained in the following way. The required time is certainly proportional to the depth to be dug, but in inverse proportion to the rate of output at the relevant level. We therefore have on the one hand

$$\tau_1 : \tau_2 = \frac{h'}{\lambda'} : \frac{h''}{\lambda''}$$

and on the other hand, of course,

$$\tau_1 + \tau_2 = 1.$$

Inserting the values given for h', h'' and λ', λ'', we obtain $\tau_1 : \tau_2 = 1 : 4$, and combining this with the second equation, we finally obtain $\tau_1 = 0;12$ $\tau_2 = 0;48$, the answer given in the text.

We are now able to compute the volume actually dug during the times τ_1 and τ_2. The rate of output per man per day for the upper stratum is λ', and $\tau_1\lambda' = \lambda_1$ is therefore the actual output during this period; correspondingly, $\tau_2\lambda'' = \lambda_2$ gives the actual output for the rest of the day (τ_2) for the deeper stratum. Hence the total assignment during one day is

$$\lambda = \lambda_1 + \lambda_2 = \tau_1\lambda' + \tau_2\lambda'' = 0;4 + 0;8 = 0;12.$$

This is the value for λ which was needed above in the explanation of the relations in Nos. 2–5; the partial values $\lambda_1 = 0;4$ and $\lambda_2 = 0;8$ are called saḫar, i.e., dirt or volume, corresponding to τ_1 and τ_2.

Having now discovered the principles behind No. 7, we are prepared to turn to No. 6, which distinguishes three (instead of two) levels of work on a canal. The dimensions given are

(1) $l = 5,0$ GAR $\quad b = 3$ kùš $= 0;15$ GAR
$\quad h = 4;30$ kùš.

The length and the width are the same as in No. 7, but the increase in depth beyond 3 kùš produces a third level with an even smaller rate of output, as is clearly indicated by the following list:

	$h' = 1$ kùš	$\lambda' = 0;20$ SAR	*šīlūtum*
(2)	$h'' = 2$ kùš	$\lambda'' = 0;10$ SAR	dusu
	$h''' = 1\frac{1}{2}$ kùš	$\lambda''' = 0;8,30$ SAR	dusu.

The text asks for the length l_λ dug by one man during one day and gives the answer

(3) $l_\lambda = 5\frac{2}{3}$ kùš (and) 3;20 šu-si $= 0;28,53,20$ GAR.

In order to check this answer, we must know the total assignment λ of one man each day which results from the different rate of output given above. We therefore compute the partial times τ_1, τ_2 and τ_3 devoted to each type of work and shall determine λ by

(4) $$\lambda = \tau_1\lambda' + \tau_2\lambda'' + \tau_3\lambda'''$$

as in No. 7. In order to find the τ's, we have the conditions

(5a) $$\tau_1 + \tau_2 + \tau_3 = 1$$

(5b) $$\tau_1 : \tau_2 : \tau_3 = \frac{h'}{\lambda'} : \frac{h''}{\lambda''} : \frac{h'''}{\lambda'''}.$$

Using the values given in (2), we obtain for (5b)

$$\tau_1 : \tau_2 : \tau_3 = 3 : 12 : \frac{1,30}{8;30} = 1 : 4 : \frac{1,0}{17}.$$

The last term $\frac{1,0}{17}$ introduces a fraction which cannot be expressed as a finite sexagesimal fraction and would therefore only lead to approximate results. But even so, the final result would show no similarity with that given in the text, as one can easily discover by carrying out all the necessary computations. We are therefore forced to assume that $\lambda''' = 0;8,30$ is due to a scribal error[212] and should be replaced by

$$\lambda''' = 0;7,30 \text{ SAR}.$$

Adopting this correction, we obtain

$$\tau_1 : \tau_2 : \tau_3 = 3 : 12 : 12 = 1 : 4 : 4$$

and therefore, because of (5a),

$$\tau_1 = \tfrac{1}{9} \qquad \tau_2 = \tau_3 = \tfrac{4}{9}.$$

[210] The expression ⅓ ma-na here means 20 gín = ⅓ SAR volume; this contamination is perhaps due to the occurrence of ma-na in No. 1, where ma-na and gín indicate as usual the weight in silver for the total expenses.

In problem-text M, the *šīlūtum* again has the value "20" (i.e., 0;20 SAR). In line 21 of Ue, an Old-Babylonian list of coefficients, appears the entry "20, the *šīlūtum*."

[211] Cf. below p. 88.

[212] It should be remarked that the reading 8 is certain, although the spacing of the wedges, here as elsewhere in this text, is very awkward. A meaningless duplication of one wedge occurs in the 5 of 25 in line 3 of the obverse.

This gives for the partial assignment per man per day at the different levels

$$\lambda_1 = \tau_1\lambda' = \tfrac{1}{9} \cdot 0;20 \qquad \lambda_2 = \tau_2\lambda'' = \tfrac{1}{9} \cdot 0;40$$
$$\lambda_3 = \tau_3\lambda''' = \tfrac{1}{9} \cdot 0;30$$

and therefore for the total assignment per man per day

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3 = \tfrac{1}{9} \cdot \tfrac{3}{2} = 0;10 \text{ SAR.}$$

If we divide this amount by the cross-section bh of the canal, we obtain the daily assignment of length to be dug by one man[213]

$$(6a) \quad l_\lambda = \frac{\lambda}{bh} = \frac{0;10}{0;15 \cdot 4;30} = 0;8,53,20 \text{ GAR.}$$

To express this result in terms of kùš and fractions thereof, we must use the relation 1 kùš = 0;5 GAR. At this point the scribe committed a new error. Instead of writing

(6b) 0;8,53,20 GAR = $1\frac{2}{3}$ kùš $3\frac{1}{3}$ šu-si,

he apparently failed to banish from his mind the 5 in the relation 1 kùš = 0;5 GAR and wrote $5\frac{2}{3}$ kùš $3\frac{1}{3}$ šu-si. The incorrectness of this answer, if taken literally, is also evident from the fact that $l_\lambda = 5\frac{2}{3}$ kùš $3\frac{1}{3}$ šu-si could only result from λ = 0;32,30 SAR, i.e., an output which is considerably greater than the largest assignment given in the whole text. It is therefore clear that (6b) is the answer which should have been given in the text.

Nos. 8–12.

Here begins a new type of problem characterized by the term pa$_5$-sig-libir-ra, "an old, little canal". The work no longer consists of digging out the entire profile of the canal but simply cleaning out the silted parts or perhaps even enlarging the existing profile beyond that of the canal when it was first dug. All five examples of this group assume dimensions for the existing profile as indicated in fig. 22, where the

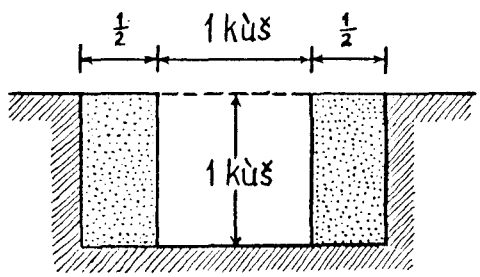

FIG. 22

dotted area represents the part which is to be removed (called *tarahḫu*):

[213] For another way to obtain the same result cf. below p. 87.

No. 8 gives the dimensions of the free profile of the canal

l = 5,0 GAR $\quad b$ = 1 kùš = 0;5 GAR $\quad h$ = 1 kùš

and asks for the volume V^* to be dug if the width should be increased by

$$c = \tfrac{1}{2} \text{ kùš} = 0;2,30 \text{ GAR}$$

on each side. The answer should be

$$V^* = l \cdot 2c \cdot h = 5,0 \cdot 0;5 \cdot 1 = 25 \text{ SAR,}$$

but by misinterpreting the order of magnitude, the scribe gives $\frac{1}{3}$ SAR 5 gín, i.e., 0;25 SAR.

No. 9 introduces the assignment of work per man per day

$$\lambda = 0;20 \text{ SAR,}$$

written, however, $\frac{1}{3}$ gín here and in the following examples where this value occurs.[214] The values given for l, b, h and c are the same as above. The problem consists in finding the length dug by one man given by

$$\frac{\lambda}{2ch} = \frac{0;20}{0;5} = 4 \text{ GAR.}$$

Nos. 10 and 11 make the same assumptions but ask for the length l_{10} for 10 men (hence 40 GAR is the result) and the time needed to complete the whole work by 10 men, which is

$$\frac{l}{l_{10}} = \frac{5,0}{40} = 7\tfrac{1}{2} \text{ days.}$$

No. 12 is the inverse problem to No. 11, computing l from the number of men and the time.

Nos. 13–16.

This new group not only assumes an increase of the "old canal" in width but in depth as well (cf. fig. 23).

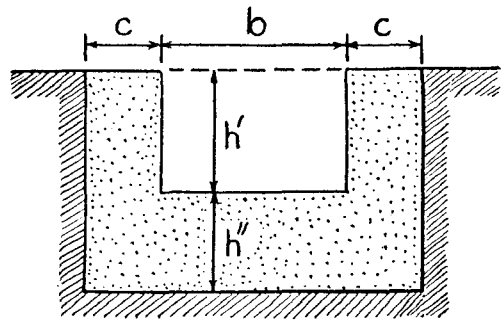

FIG. 23

[214] For a discussion of this phenomenon, see p. 79 and notes 185 and 186.

Nos. 13 and 14 assume

$$l = 5{,}0 \text{ GAR} \qquad b = 2 \text{ kùš} = 0;10 \text{ GAR}$$

$$c = \tfrac{1}{2} \text{ kùš} = 0;2{,}30 \text{ GAR} \quad h' = 1 \text{ kùš} \quad h'' = 1 \text{ kùš}.$$

No. 13 inquires about the total volume V^* which is given by

$$l(2ch' + (b + 2c)h'') = 5{,}0\ (0;5 + 0;15)$$
$$= 1{,}40 \text{ SAR} = 1 \text{ iku}.$$

No. 14 asks for the length dug by one man. For this, one must divide the daily work assigned to one man $\lambda = 0;20$ by the cross-section of the volume to be removed. Hence

$$\frac{\lambda}{2ch' + (b + 2c)h''} = \frac{0;20}{0;5 + 0;15} = 1 \text{ GAR}$$

is the solution to this problem.

No. 15 is perfectly parallel to No. 13; they differ only in some of the numerical values given. No. 15 assumes:

$$l = 5{,}0 \text{ GAR} \qquad b = 3 \text{ kùš} = 0;15 \text{ GAR}$$

$$c = \tfrac{1}{2} \text{ kùš} = 0;2{,}30 \text{ GAR} \quad h' = 1 \text{ kùš} \quad h'' = 1\tfrac{1}{2} \text{ kùš}.$$

This would lead to a total volume $V^* = 2{,}55$ SAR $= 1$ iku 1 ubu 25 SAR. Unfortunately, the answer given by the text is destroyed except for wedges at the end of line 12 which do not fit 25, SAR, or saḫar ("volume"), but only a sign like 30. This might be explained by comparing No. 15 with the following examples, in which all parameters are the same as in No. 15 with the sole exception of $b = 2$ kùš. Assuming $b = 3$ kùš to be an error[215] for 2 kùš, we would obtain $V^* = 2{,}17;30$ SAR. It is therefore possible that the scribe merely gave the number 2,17,30 without converting it to the standard 1 iku $37\frac{1}{2}$ SAR.

No. 16 again introduces a difference with regard to the expected output at different levels according to the pattern already encountered in Nos. 2–7. The work consists in enlarging an old canal under circumstances represented by fig. 23 and

$$l = 5{,}0 \text{ GAR} \qquad b = 2 \text{ kùš} = 0;10 \text{ GAR}$$

$$c = \tfrac{1}{2} \text{ kùš} = 0;2{,}30 \text{ GAR} \quad h' = 1 \text{ kùš} \quad h'' = 1\tfrac{1}{2} \text{ kùš}.$$

If λ' denotes the volume assigned to a man if he worked a full day in the upper level (i.e., above a depth h') and λ'' the corresponding assignment in the lower level, we are given

$$\lambda' = 0;20 \text{ SAR} \qquad \lambda'' = 0;10 \text{ SAR}.$$

We now proceed along exactly the same lines as in Nos. 6 and 7 by computing the complementary fractions of a day τ_1 and τ_2 to be spent in working at the upper and lower levels, respectively. First we have by definition

$$\tau_1 + \tau_2 = 1.$$

Let A' denote the area of the upper part of the cross-section, A'' the area of the lower part, i.e.,

$$A' = 2ch' = 0;5 \text{ GAR kùš}$$
$$A'' = (2c + b)h'' = 0;22{,}30 \text{ GAR kùš};$$

then

$$\tau_1 : \tau_2 = \frac{A'}{\lambda'} : \frac{A''}{\lambda''} = \frac{0;5}{0;20} : \frac{0;22{,}30}{0;10} = 1 : 9$$

because the greater the cross-section, the longer the time needed for each part; but the greater the assignment, the shorter the time. Hence we find

$$\tau_1 = 0;6 \qquad \tau_2 = 0;54$$

and for the volume to be dug out by one man in one day

$$\lambda = \tau_1\lambda' + \tau_2\lambda'' = 0;6 \cdot 0;20 + 0;54 \cdot 0;10 = 0;11.$$

We are now able to compute the length l_λ which indicates the actual progress of the total work as a result of one man's digging for one day:

$$l_\lambda = \frac{\lambda}{A' + A''} = \frac{0;11}{0;5 + 0;22{,}30} = \frac{2}{5} = 0;24 \text{ GAR}.$$

The text reaches the same result by giving

$$l_\lambda = 4\tfrac{2}{3} \text{ kùš } 4 \text{ šu-si}$$

since this equals 4;48 kùš $= 0;24$ GAR.

It is perhaps worth remarking that, without computing the partial times, the same result can be obtained by

$$l_\lambda = \frac{\lambda'}{A' + A''\dfrac{\lambda'}{\lambda''}} = \frac{1}{\dfrac{A'}{\lambda'} + \dfrac{A''}{\lambda''}}.$$

Correspondingly, No. 6 could be solved by

$$l_\lambda = \frac{1}{\dfrac{A'}{\lambda'} + \dfrac{A''}{\lambda''} + \dfrac{A'''}{\lambda'''}}$$

where $A' = bh'$, $A'' = bh''$, $A''' = bh'''$.

Nos. 17 and 18.

Here begins a new type of problem, the meaning of which escapes us. The cross-section of the canal is rectangular, $b = 1$ kùš, $h = 1$ kùš in No. 17, and $b = 2$ kùš, $h = 1\frac{1}{2}$ kùš in No. 18; no length is given. From this it follows that the area of the profile in No. 18 is three times the area in No. 17. There is, however, no indication that this ratio plays a rôle in the answers which are given: 4 and $\frac{1}{3}$ of ?.

[215] Cf. the parallelism between Nos. 13 and 14.

The text discussed here shows so many peculiarities and errors on the part of the scribe that we think it might be useful to enumerate them in order to substantiate this general impression.

Omission of words or syllables (indicated by < >) in obv. 3, 10 (twice), rev. 1 (twice), 6, 13
Omission of u_4 in itu-1 <*u*$_4$>-7½-kam in obv. 9 and 11
Expressing ⅓ SAR by ⅓ ma-na in obv. 4, 6, 8, 12 and 15, by ⅓ gín in rev. 8 and 15
Error: 8½ gín instead of 7½ gín in obv. 13
Error: 5⅔ kùš 3,20 šu-si instead of 1⅔ kùš 3,20 šu-si in obv. 14
Error: ⅓ SAR 5 gín instead of 25 SAR in obv. 20
Error(?): 3 kùš instead of 2 kùš in rev. 10 and perhaps writing 2,17,30 instead of 1 iku 37½ SAR in rev. 12.

For abbreviations used by the scribe, cf. the following paragraph. It is clear from all this that the present text can be used for general conclusions only with the greatest of caution.

b. Terminology

Two abbreviations occur frequently. In obv. 20, we read ½ kùš *ta-ra-ḫi-ša*, but in all subsequent examples (Nos. 9ff.) only ½ kùš *ta* is written in the identical context.

The second instance is the use of *šu* in Nos. 2–7, where the mathematical context requires an expression for depth. It is clear that *šuplum* is the most plausible assumption for the term in question, although we do not have a single full writing in our text. Because we find bùr-bi used twice in rev. 15 in the identical context as *šu* in Nos. 2–7, we should perhaps assume *šupulša* as the proper form.

dusu and *šīlūtum*

As shown in the mathematical commentary, the work of digging the canal is broken up into different categories: the easier work in the upper level and the slower work in the lower levels (cf. especially No. 6). Since the latter is always connected with the word dusu, which means "basket" (particularly in connection with construction projects),[216] this is perhaps an indication that digging at the lower levels required shovelling the excavated dirt into baskets which had to be raised to the top and emptied, whereas the work at the upper level merely involved digging out the earth and casting it from the shovel directly to the ground-level. The work in the upper level is characterized by the abstract word *šīlūtum*, which is perhaps to be connected with the verb *šalū* "to cast, throw" via *šil*$^{?}$*u* > *šīlu*.

taraḫḫum

The context of Nos. 8–16 makes it probable that *taraḫḫu* means the sides of the canal in contrast to its bottom. For a discussion of the meaning of the word and its previous occurrences, see Thureau-Dangin [8] p. 19, note 6. Dr. A. Goetze has kindly called our attention to another occurrence of the word in an Old-Babylonian omen text which he will publish, YBC 4638 IV 51ff.: ^{51}*šum-ma mar-tum* ^{52}*li-pi-a-am* ^{53}*ta-ra-ḫa-a-ša ú-ka-al-la*, which he translates, "Supposing the gall-bladder, its two *taraḫḫu* 'hold' fat." To this can be added the occurrence of *ta-ra-ḫu* in a broken passage of a medical text published by Lutz.[217]

terdītu

The translation "increase" for *terdītu* is based on the assumption that it is the archaic II,1 infinitive parallel to *ruddū*, "to add". We have no explanation for the variant initial syllable *de-* instead of *te-*.

Equally possible translations of the clause in which the word occurs, *t/de-er-di-(is-)sà* ½ kùš *ta(-ra-ḫi-ša) ú-ša-(am-)mi-iṭ* ((*x*) kùš *a-na* bùr-bi bí-daḫ), would be "as for its increase, I cut off ½ kùš of its *taraḫḫum* (, I added (*x*) kùš to its depth)," or "as for its increase, I cut off ½ kùš as its two *taraḫḫu*, etc.", instead of "its increase cut off ½ kùš of its *taraḫḫum* ((and) added (*x*) kùš to its depth)."

BM 85196, No. 16

The new information derived from the text just discussed, namely, that the output at different levels in the excavation of a canal are classified according to a sliding scale, now makes it possible to give an intelligible analysis of at least the main outlines of a problem which was previously discussed without success by Thureau-Dangin and Neugebauer.[218]

The text reads as follows:

16 ^{36}[id]*nam-ka-rum* 15 sag *i-na* 1 kùš *li*-LUM *i*[*-na* 2 kùš . . . .]
^{37}*i-na* 3 kùš *ḫa-da-lum iḫ-di-il a-na* 1 l[ú uš *pu-lu-uk*]
^{38}*ù* bal éš-kàr *qí-bi* za-e igi éš-kàr du$_8$-a 1[8 *ta-mar*]
^{39}igi 18 du$_8$-a 3,20 *ta-mar* uš *a-na* 1 lú *ta-pa*[*-la-ak*]

Reverse II

1[igi 2]0 du$_8$-a 3 *ta-mar* 3,20 *a-na* 3 *i-ši* 10 *ta*[*-mar*]
2igi 10 du$_8$-a 6 *ta-mar* 3,20 *a-na* 6 *i-ši* 20 *ta-mar*
3igi 6,40 du$_8$-a 9 *ta-mar* 3,20 *a-na* 9 *i-ši* 30 *ta-mar*
4[10 *a-na* 20 *i-ši* 3,20 *ta-mar* 3,20 *i-na* igi-6-[g]ál *u*$_4$*-mi*

[216] Cf. Deimel ŠL 85, 350 and Meissner [1] p. 48.

[217] Lutz [2] No. 111 rev. 2.

[218] Thureau-Dangin [8] pp. 22f. and TMB pp. 44f.; Neugebauer MKT II pp. 45f., 49, 56f.

5saḫar-ḫá *i-sú-uḫ* 20 *a-na* 10 *i-ši* 3,20 *ta-mar i-na* $\frac{1}{3}$ u_4-*mi*
6[saḫ]ar-ḫá *i-sú-uḫ* 30 *a-na* 6,40 *i-ši* 3,20 *ta-mar* 3,20
7*i-na* u_4-$\frac{1}{2}$-kam *is-sú-uḫ ki-a-am ne-pé-šum*

Translation:

16 36An irrigation-canal. 0;15 (GAR) is the width; in 1 kùš, the *li*-LUM; i[n 2 kùš, ...;]
37in 3 kùš, ...[219] [Mark off the length] for 1 man,
38and speak (i.e., give) the fraction of the assignment. As for you—take the reciprocal of the assignment, (and) [you will see] 1[8];
39take the reciprocal of 18, (and) you will see 0;3,20. You shall mark off the length for 1 man.

Reverse II

1Take [the reciprocal] of [0;2]0, (and) you will see 3; multiply 0;3,20 by 3, (and) you will see 0;10.
2Take the reciprocal of 0;10, (and) you will see 6; multiply 0;3,20 by 6, (and) you will see 0;20.
3Take the reciprocal of 0;6,40, (and) you will see 9; multiply 0;3,20 by 9, (and) you will see 0;30.
4Multiply 0;10 by 0;20, (and) you will see 0;3,20; in one-sixth of a day
5he extracted the volume. Multiply 0;20 by 0;10, (and) you will see 0;3,20; in $\frac{1}{3}$ of a day
6he extracted the volume. Multiply 0;30 by 0;6,40, (and) you will see 0;3,20; 0;3,20
7he extracted in $\frac{1}{2}$ of a day. Thus is the procedure.

The problem consists in computing the fractions τ_1, τ_2, τ_3 of a day spent in digging three levels of a canal. The profile is given in fig. 24 with

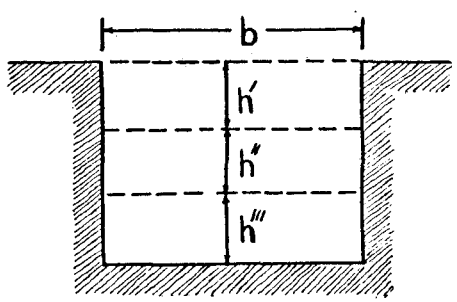

FIG. 24

$$b = 0;15 \text{ GAR} \qquad h' = h'' = h''' = 1 \text{ kùš}^{220}$$

and with assignments per man as if he worked a full day at each level

$$\lambda' = 0;20 \text{ SAR} \quad \lambda'' = 0;10 \text{ SAR} \quad \lambda''' = 0;6,40 \text{ SAR}$$

for the first, second and third level. From the same considerations as those used in No. 6 of problem-text L,[221] we obtain

$$\tau_1 + \tau_2 + \tau_3 = 1$$

$$\tau_1 : \tau_2 : \tau_3 = \frac{h'}{\lambda'} : \frac{h''}{\lambda''} : \frac{h'''}{\lambda'''}$$

and thus

$$\tau_1 = \tfrac{1}{6} \qquad \tau_2 = \tfrac{1}{3} \qquad \tau_3 = \tfrac{1}{2}$$

as the fractions of a day. Multiplying these τ's by the corresponding assignments per man expressed as if he worked a full day at each level, we find (cf. lines 4 to 7)

$$\lambda_1 = \tau_1\lambda' = \frac{0;20}{6} = 0;3,20 \text{ SAR}$$

$$\lambda_2 = \tau_2\lambda'' = \frac{0;10}{3} = 0;3,20 \text{ SAR}$$

$$\lambda_3 = \tau_3\lambda''' = \frac{0;6,40}{2} = 0;3,20 \text{ SAR}$$

as the corresponding actual volume of work per man per day at each level, called bal éš-kàr, "fraction of the assignment," in line 38.

The details of the text are almost certainly corrupt, as are many other examples in the same tablet. The first step consists in the inverse statements (lines 38f.)

$$\frac{1}{\lambda} = 18 \qquad \frac{1}{18} = 0;3,20.$$

Thereafter (line 39), the length l_λ is mentioned, which would be one man's daily assignment, without giving its value. From $\lambda = \lambda_1 + \lambda_2 + \lambda_3 = 0;10$ it would follow that

$$l_\lambda = \frac{\lambda}{b(h' + h'' + h''')} = \frac{0;10}{0;15 \cdot 3} = 0;13,20 \text{ GAR} = 2\tfrac{2}{3} \text{ kùš}$$

which, however, occurs nowhere in the text. Then follow (lines 1–3)

$$\frac{\lambda_1}{\lambda'} = \tau_1 = 0;10 \qquad \frac{\lambda_2}{\lambda''} = \tau_2 = 0;20 \qquad \frac{\lambda_3}{\lambda'''} = \tau_3 = 0;30$$

[219] Here the text has an expression consisting of the infinitive followed by the third person preterite of the obscure verb *ḫa-d/ṭālum*. Because the infinitive is in the nominative case, we consider it better to interpret the next word as *iḫ-d/ṭi-il* instead of *aḫ-d/ṭi-il* (so read by Thureau-Dangin and, following him, Neugebauer).

[220] In the text, this is presented in the form $h' = 1$, $h' + h'' = 2$, $h' + h'' + h''' = 3$.

[221] Cf. pp. 84ff.

and again an inverse statement (lines 4–7)

$$\tau_1\lambda' = \lambda_1 \qquad \tau_2\lambda'' = \lambda_2 \qquad \tau_3\lambda''' = \lambda_3.$$

It almost seems as if two inverse examples were thrown together by the scribe.

In problem-text L, the first level is called *šīlūtum*, the second and third dusu.[222] In the present text, we find *li*-LUM for the first, *ḫad/ṭālum iḫd/ṭil* for the third, the term for the second being destroyed. We do not know the literal meaning of these words.

M. YBC 9874

(Photograph: Plate 36; copy: Plate 11)

Transcription

Obverse

1,40	5	1,40	3,20	3,36
	1		12	
	40		20	

Reverse

1 [a-]*ta-a*[*p*] *ta-ra-aḫ-ḫi-im-mi*
2 . .[. . . .]. . *a-ma-at pu-ta-am ù a-ma-at šu-up-lam*
3 *e*[*ḫ*(?)-*r*]*e*(?)-*a*-[*m*]*a ṷa-ar-ka-a-nu-um ú da a*(?) *e na ta ra nu*
4 . . . .-*ša*[223]-*mi ú-ṣi-ib ú* UD *še*(?) *da ak ka*(?) *ta mi*
5 *eḫ*(?)-*re-e* 5 *a-na* 1,30 *ta*[-*a*]*t-ta-na-aš-ši-i-ma*
6 7,30 *i-na-ad-di-ik-ku* 8 *a-na* 20 *ši-i-lu-ú-tim*
7 *ta-at-ta-na-aš-ši-i-ma* 2,40 *iš-ka-ar a-ṷa-ti*
8 *iš-te-en i-na-ad-di-ik-ku*
9 13,20 *a-na* 1,30 *ta-at-ta-na-aš-ši-i-ma*
10 20 *ši-i-lu-ta-am*
11 *i-na-di-ik-ku*

Translation

Obverse

(See transcription.)

Reverse

1 A canal of *taraḫḫu*[224].
2 . .[. . . . .]. . a cubit(?) width and a cubit(?) depth
3 I(?) dug(?), but later . . . . . .
4 . . . . . I added . . . . . .
5 I(?) dug. You shall multiply 5 by 1,30, and
6–8 you will get 7,30. You shall multiply 8 by 20, the *šīlūtum*, and you will get 2,40, the "assignment" of one[225] thing.
9 You shall multiply 13,20 by 1,30, and
10–11 you will get 20, the *šīlūtum*.

Commentary

We are unable to give an interpretation for this text because we cannot establish a relation between the numbers on the obverse and the text of the reverse. If, as seems quite possible, the obverse and reverse belong to independent, although similar, problems, our failure is not surprising.

There is little doubt that the lines drawn on the obverse are in some sense to be interpreted as a figure. The critical problem is whether it should be read as the map or as the cross-section of a canal. In the first case, 5 could represent the length l = 5,0 GAR of the canal, 1,40 and 1,40 the width c = 0;1,40 GAR of two silted sides of the canal, leaving an unsilted interior width b = 1 kùš; the depth would be h = 0;40 kùš (cf. fig. 22 on p. 86). The second interpretation, supported by an analogous drawing in BM 85196[226], would lead to the following cross-section of a canal: upper width b = 0;5 GAR enlarged on both sides by c = 0;1,40 GAR[227]; depth h' = 1 kùš enlarged by an additional h'' = 0;40 kùš (cf. fig. 23 on p. 86).

The numbers at the side of the figure fulfill the relation

$$3{,}20 \cdot 3{,}36 = 12$$

in which 3,20 could be interpreted as $2c$ = 0;3,20 GAR. The real meaning of 3,36 is unclear to us. The last entry 20 could be the assignment λ' = 0;20 SAR per man per day.

The text of the reverse contains two terms which are known from the preceding texts: *taraḫḫu*, referring to the sides of a canal to be enlarged,[228] and *šīlūtu* for the work in the upper level of the excavation.[229] The value 20 corresponds to the usual assignment λ' = 0;20 SAR for the *šīlūtu*. The computation

$$5 \cdot 1;30 = 7;30 \qquad [\overline{7;30} = 0;8] \qquad 0;8 \cdot 0;20 = 0;2,40$$

could give the output (*iš-ka-ar*) $\lambda_1 = 0;8 \cdot 0;20 = 0;2,40$ in the *šīlūtu*. It is, however, unclear to what the numbers 5 and 1,30 refer.

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[222] Cf. p. 88.
[223] Or *ta*?
[224] At this point and possibly at the end of line 4 occurs *-mi*, which indicates direct discourse.
[225] Or "the one."
[226] Cf. MKT II pp. 45 and 56. This drawing precedes the problem discussed above, pp. 88f..
[227] Needless to say, GAR could be replaced by kùš and the sexagesimal place values changed.
[228] Cf. above p. 88.
[229] Cf. above p. 88.

The remaining part of the calculation seems to be the inverse operation designed to find $\lambda' = 0;20$:

$$0;13,20 \cdot 1;30 = 0;20,$$

where $0;13,20 = 5 \cdot 0;2,40 = 5\lambda_1$.

N. YBC 4186[229a]

(Photograph: Plate 36; copy: Plate 11)

Transcription

Obverse

[1][t]úl 10 GAR *im-ta-ḫar* 10 GAR *iš-pi-i[l]*
[2]*me-e-ša az/ṣ-z/ṣu-ul-ma i-na me-e-ša*
[3]*a-na* 1 šu-si *šu-up-lim* a-šà *ki ma-ṣi am-qú-ur*
[4]10 *ù* 10 *ša im-t[a]ḫ-ru* gar-ra
[5]10 *šu-pu-ul* túl gar-ra
[6]*ù* 10 *šu-pu-ul me-e ša* a-šà *im-qú-r[u]*
[6a]gar-ra-ma
[7]igi 10 *šu-pu-ul me-e ša* a-šà *[i]m-q[ú]-ru*
[8]*pu-ṭur-ma* 6 *a-na* 10 *šu-pu-u[l* túl Í]L 1
[9]1 *re-eš-ka li-k[i-il]*
[10]10 *ša im-taḫ-ru* DA . . .[. .]. .[. .] 1,40
[11]1,40 *a-na* 1 *ša re-eš-ka ú-ka-al-l[u]*
[12]ÍL 1,40 a-šà *am-qú-ur*

Reverse blank.

Translation

[1]A cistern was 10 GAR square, 10 GAR deep.
[2–3]I emptied out(?)[230] its water; with its water how much field did I irrigate to a depth of 1 šu-si?
[4]Put (aside) 10 and 10 which formed the square.
[5]Put (aside) 10, the depth of the cistern.
[6–6a]And put (aside) 0;0,10, the depth of the water which irrigated the field.
[7–8]Take the reciprocal of 0;0,10, the depth of the water which [irri]gated the field, and (the resulting) 6,0 [mul]tiply by 10, the depth of the cistern, (and the result is) 1,0,0.
[9]1,0,0 ke[ep] in your head.[231]
[10][Square(?)] 10, which formed the square, [and (the result is)] 1,40.
[11–12]Multiply 1,40 by 1,0,0, which you are ke[eping] in your head.[232] I irrigated 1,40,0,0 (SAR) field.

Commentary

The text assumes a cistern (túl) in the shape of a cube, such that its length l, width b, and depth h are 10 GAR each. The problem which is posed requires the calculation of the area A of a field irrigated to a depth h_A of 1 šu-si by the water contained in the cistern. After the transformation of $h_A = 1$ šu-si to 0;0,10 GAR, which is necessary because h is expressed in units of GAR, is made, the actual computation is carried out according to the formula

$$\frac{h}{h_A} \cdot l \cdot b = A.$$

The transformation of the final answer 1,40,0,0 (SAR) to the standard 3 šár 2 bur³u is not made in the text.

The situation described in the text is strongly idealized in that the water is required to be spread to a uniform depth of one finger's breadth over a field which is approximately $3\frac{1}{2}$ kilometers square.

§ 7. Bricks

O. YBC 4607

(Photograph: Plate 37; copy: Plate 12)

Transcription

Obverse

1 [1]sig$_4$ $\frac{1}{2}$ kùš uš-bi
[2]$\frac{1}{3}$ kùš sag-bi 5 šu-si sukud-bi
[3]gagar saḫar-bi ù i šám saḫar-bi en-nam
[4]12 še šu-ri-a še gagar-bi 2 še ù <igi->12-gál še saḫar-bi
[5]3$\frac{1}{3}$ sìla 8$\frac{1}{3}$ gín i šám saḫar-bi

2 [6]sig$_4$ 18 šu-si uš-bi
[7]12 šu-si sag-bi 5 šu-si sukud-bi
[8]gagar saḫar-bi ù i šám saḫar-bi en-nam
[9]18 še gagar 3 še saḫar-bi [5 sìl]a i šám saḫar-bi

[229a] By an oversight, the museum number on Plate 11 is given as YBC 4185; the correct number is YBC 4186.

[230] This translation is based on the requirements of the context and (only incidentally) on a comparison with Hebrew *nzl*, for which cf. especially the use in connection with a cistern cited by Jastrow DTTB p. 892a. Still better in all probability is the suggestion made to us by Dr. W. F. Albright: to compare Syriac *nṣl*, "to pour, discharge, distill, of liquid," and especially *naṣlâ* or *niṣlâ*, "dipper" (= Arabic *ṭarğârah*, "dipper for pouring, skimming"); for our passage he suggests the meaning "bail out." The actual meaning is of course the same in either case.

Additional occurrences of *nz/ṣl* in the so-called chemical texts have come to our attention since the above was written. The following forms are attested (the page numbers refer to Zimmern [1], which is the source of our transcriptions): (1) *ta-na-az-za-la-šum-ma* (p. 184, I § 1,20); (2) *[ta-na-az-za-]la-aš-šum-ma* (p. 184, I § 2,31); (3) *ta-na-za-al* (p. 188, I § 7,24; § 8,28); (4) *ta-na-az-za-lam*(variant: *la*)*-ma* (p. 192, II § 1,6; § 2,14; p. 194, II § 3,3). Forms (1), (2) and (4) are preceded by *ana muḫḫi agurri*, "upon burnt-brick"; (3) by *ina libbi mê*, "into water"; and (4) by *ana ṭâbti*, "upon salt". In all cases, the object is a molten mass, and Zimmern therefore translated the verb by "du lässt *fliessen*" (Zimmern's italics). For observations on the technical significance of pouring the liquified mass over or into the various materials mentioned, see Zimmern [2].

[231] Literally, "let your head keep."

[232] Literally, "your head keeps."

3 10sig$_4$-áb ⅔ kùš uš-bi
11⅓ kùš sag-bi 5 šu-si sukud-bi
12gagar saḫar-bi ù ì šám saḫar-bi en-nam
1316 še šu-ri-a še ù igi-6-gál še še-b[i][233]
142 še šu-ri-a še igi-4-gá[l še ù igi-9-gál igi-]4-gál saḫar
1541/2 sìla 6⅔[235] <gín> 20 še ì š[ám saḫar-bi]

4 16sig$_4$-al-ùr[-ra]
17⅔ kùš-ta-àm íb-si$_8$ 5 šu-si su[kud-bi]
18gagar saḫar-bi ù ì šám saḫar-bi en-nam
19igi-6-gál 3 še ù igi-3-gál še gagar
205 še šu-ri-a še ù igi-9-gál šu-ri-a še saḫar

Reverse

5 1sig$_4$-al-ùr-ra 1 kùš-ta-àm íb-si$_8$
25 šu-si sukud-bi gagar saḫar-bi ù ì šám saḫar-bi en-nam
3⅓ gín ù 15 še gagar 12 še šu-ri-a še saḫar-bi

6 4sig$_4$ ½ kùš uš-bi ⅓ kùš sag-bi
55 šu-si sukud-du 1 SAR gagar *ta-ad-di-ta-a*[*m* en-nam ì-díb]
61 SAR 2,24 ì-díb

7 7sig$_4$ 18 šu-si uš-bi 12 šu-si sag-bi 5 šu-si sukud-bi
81 SAR gagar *ta-ad-di-ta-am* en-nam ì-díb
91(geš^{2}u) sig$_4$ ì-díb

8 10sig$_4$-áb ⅔ kùš uš-bi ⅓ kùš sag-bi 5 šu-si[236] sukud-bi
111 SAR gagar *ta-ad-di-ta-am* en-nam ì-díb
121(geš^{2}u) 48 sig$_4$-áb ì-díb

9 13sig$_4$-al-ùr-ra ⅔ kùš-ta-àm íb-si$_8$
145 šu-si sukud-bi 1 SAR gagar *ta-ad-di-ta-am*
14aen-nam ì-díb
155,24 sig$_4$-al-ùr-ra ì-díb

10 16sig$_4$-al-ùr-ra 1 kùš-ta-àm <íb-si$_8$> 5 šu[-si] sukud-bi
171 SAR gagar *ta-ad-di-ta* e[n]-n[am] ì-díb
182,24 sig$_4$-al-ù[r-ra] ì-díb

Left Edge

10 im-šu-meš

Translation

Obverse

1 1A brick. ½ kùš is its length,
2⅓ kùš its width, 5 šu-si its height.
3What are the area, its volume, and oil, the equivalent[237] of its volume?
412 še (and) one-half še is its area; 2 še and one-twelfth še is its volume;
53⅓ sìla (and) 8⅓ gín oil is the equivalent[237] of its volume.

2 6A brick. 18 šu-si is its length,
712 šu-si its width, 5 šu-si its height.
8What are the area, its volume, and oil, the equivalent[237] of its volume?
918 še is the area; 3 še is its volume; [5] sìla oil is the equivalent[237] of its volume.

3 10A half-brick. ⅔ kùš is its length,
11⅓ kùš its width, 5 šu-si its height.
12What are the area, its volume, and oil, the equivalent[237] of its volume?
1316 še (and) one-half še and one-sixth še is its še;[233]
142 še (and) one-half še (and) one-fourth [še and one-ninth of one-fourth [še is the volume];
1541/2 sìla (and) 6⅔[235] <gín> (and) 20 še oil is the equ[ivalent[237] of its volume].

4 16A kiln-burnt brick.
17⅔ kùš is each square-side, 5 šu-si its height.
18What are the area, its volume, and oil, the equivalent[237] of its volume?
19One-sixth (gín and) 3 še and one-third še is the area;
205 še (and) one-half še and one-ninth of one-half še is the volume.

Reverse

5 1A kiln-burnt brick. 1 kùš is each square-side,
25 šu-si its height. What are the area, its volume, and oil, the equivalent[237] of its volume?
3⅓ gín and 15 še is the area; 12 še (and) one-half še is its volume.

6 4A brick. ½ kùš is its length, ⅓ kùš its width,
55 šu-si the height. [How] much laying (of bricks) [did] 1 SAR area [take]?
6It took 1 SAR (and) 2,24 (bricks).

7 7A brick. 18 šu-si its length, 12 šu-si its width, 5 šu-si its height.
8How much laying (of bricks) did 1 SAR area take?
9It took 1 geš^{2}u bricks.

8 10A half-brick. ⅔ kùš is its length, ⅓ kùš its width, 5 šu-si its height.
11How much laying (of bricks) did 1 SAR area take?
12It took 1 geš^{2}u (and) 48 half-bricks.

9 13A kiln-burnt brick. ⅔ kùš is each square-side,
14–14a5 šu-si its height. How much laying (of bricks) did 1 SAR area take?
15It took 5,24 kiln-burnt bricks.

[233] Mistake for gagar-bi, "its area."
[235] Mistake for 7⅓.
[236] The signs šu- and -si are run together in such a way that two wedges of -si have been eliminated.
[237] Cf. below p. 97.

10 [16]A kiln-burnt brick. 1 kùš is each <square-side>, 5 šu-si its height.
[17]How much laying (of bricks) did 1 SAR area take?
[18]It took 2,24 kiln-burnt bricks.

Left Edge

10 sections.

Commentary

a. Contents of O (YBC 4607)

The ten examples of this tablet deal with bricks of various dimensions. Five types are distinguished, all of which have the same height (sukud) of 5 fingers (šu-si) in common but differ as to length (uš) and width (sag). This is shown by the following list:

No.	"Name" of Brick		Length	Width	Type
1 and 6	sig$_4$	= brick	½ kùš	⅓ kùš	1
2 and 7	sig$_4$	= brick	18 šu-si	12 šu-si	2
3 and 8	sig$_4$-áb	= half-brick	⅔ kùš	⅓ kùš	3
4 and 9	sig$_4$-al-ùr-ra	= kiln-burnt brick	⅔ kùš		4
5 and 10	sig$_4$-al-ùr-ra	= kiln-burnt brick	1 kùš		5

The numbers assigned to the types of bricks in the last vertical column of this list are merely intended to simplify our terminology.[238] These types, however, are not the only ones known from texts; an additional type is inferred below p. 96 and four others pp. 137f.

[238] Bricks of type 1 also occur in YBC 4673 (MKT III p. 29) No. 1; the fourth line of the same section should be read gagar saḫar en-nam, following Thureau-Dangin [11] p. 83.

If a_k denotes the area of a brick of type k, then we have the simple relations

(1) $$a_5 : a_4 : a_3 : a_1 = 18 : 8 : 4 : 3$$

and

(2) $$a_5 : a_2 = 25 : 6.$$

The problem in the first five examples consists in finding the area (gagar), the volume (saḫar), and (in the first three examples only) the equivalent of this volume in terms of measures of capacity. All examples follow exactly the same pattern, which we now analyze in the case of the first problem. Here we are given the following dimensions of a brick:

$$l = \tfrac{1}{2} \text{ kùš} = 0;2,30 \text{ GAR}$$
$$b = \tfrac{1}{3} \text{ kùš} = 0;1,40 \text{ GAR}$$
$$h = 5 \text{ šu-si} = 0;10 \text{ kùš}.$$

From this we obtain for the area and volume of one brick

$$lb = 0;0,4,10 \text{ SAR} = 12;30 \text{ še}$$
$$lbh = 0;0,0,41,40 \text{ SAR} = 2;5 \text{ še}$$

which are the answers given in the text. The transformation to measures of capacity can be obtained in all cases by multiplying the number of SAR which constitute the volume lbh by 5,0,0 and calling the resulting units sìla: thus, in No. 1:

$$0;0,0,41,40 \cdot 5,0,0 = 3;28,20 \text{ sìla}.$$

This formula agrees in all cases with the results given in the text. A discussion of other details connected with this transformation will be found below, pp. 96f.

The following table describes the numerical relations of the first five examples:

	No. 1 Type 1	No. 2 Type 2	No. 3 Type 3	No. 4 Type 4	No. 5 Type 5
l	½ kùš = 0;2,30 GAR	18 šu-si = 0;3 GAR	⅔ kùš =0;3,20 GAR	⅔ kùš = 0;3,20 GAR	1 kùš = 0;5 GAR
b	⅓ kùš = 0;1,40 GAR	12 šu-si = 0;2 GAR	⅓ kùš =0;1,40 GAR		
h	5 šu-si = 0;10 kùš	5 šu-si = 0;10 kùš	5 šu-si = 0;10 kùš	5 šu-si = 0;10 kùš	5 šu-si =0;10 kùš
lb	0;0,4,10 SAR = 12;30 še	0;0,6 SAR = 18 še	0;0,5,33,20 SAR = 16;40 še	0;0,11,6,40 SAR = 33;20 še = ⅙ gín 3⅓ še	0;0,25 SAR = 1,15 še = ⅓ gín 15 še
lbh	0;0,0,41,40 SAR = 2;5 še	0;0,1 SAR = 3 še	0;0,0,55,33,20 SAR =2;46,40 še = 2½ še ¼ še $_{3}^{1}{}_{8}$ <še>	0;0,1,51,6,40 SAR = 5;33,20 še = 5½ še $_{1}^{1}{}_{8}$ še	0;0,4,10 SAR = 12;30 še
Equivalent of Volume	0;0,0,41,40 · 5,0,0 = 3;28,20 sìla = 3⅓ sìla 8⅓ gín	0;0,1 · 5,0,0 = [5 sìla]	0;0,0,55,33,20 · 5,0,0 = 4;37,46,40 = 4½ sìla 7⅔ <gín> 20 še	—	—

The second part of the text (Nos. 6–10) refers to the identical types of brick but now asks for the number n of bricks necessary to cover a surface of 1 SAR. This question is expressed in the text by the words 1 SAR gagar *ta-ad-di-ta-am* en-nam ì-dí̀b, which we translate, "How much laying (of bricks) did 1 SAR area take?". Our translation of *taddītu* by "laying (of bricks)" is based on the use of the II1 conjugation of *ndi* in the sense of "lay" in connection with bricks.[239] The weak link of this interpretation is the fact that we can find no trace of the use of the II1 conjugation of *ndi*, to which *taddītu* would correspond as an archaic infinitive.

It is evident that n must result from the division of 1 SAR by *lb*. This leads to the following list, in which n has the value of the reciprocal of *lb* in the preceding scheme.

	No. 6 Type 1	No. 7 Type 2	No. 8 Type 3	No. 9 Type 4	No. 10 Type 5
l	½ kùš	18 šu-si	⅔ kùš	⅔ kùš	1 kùš
b	⅓ kùš	12 šu-si	⅓ kùš		
n	14,24	10,0	10,48	5,24	2,24

For another occurrence of n = 14,24 of No. 6, cf. below p. 134.

b. Counting of Bricks

The sixth problem of this text yields interesting new information about the counting of bricks. This example expresses 14,24, the number of bricks, by "1 SAR 2,24", from which it follows that 1 SAR is the equivalent of 12,0 bricks.[240] The use of the unit SAR for the counting of bricks in the pre-Kassite periods is well attested outside of the mathematical texts,[241] but the specific relation

(1) 1 SAR = 12,0 bricks,

regardless of the particular type of brick, is here definitively proved for the first time and explains various passages which had not been fully understood.[242] Before discussing these passages, we must say at once that the origin of this terminology is still unknown. The most plausible assumption would of course be that at one time 12,0 bricks of a certain type either covered a surface of 1 area-SAR or had a volume of 1 volume-SAR. None of the types of brick mentioned in our texts fits such a theory. The essential point, however, is that the relation (1) not only holds for a specific type of brick but is a generally valid notation in counting bricks of any type in the texts now at our disposal.

The first text which we are able to explain by means of the relation (1) is a mathematical text, **YBC 4708,** published MKT I pp. 389–402. We can limit our discussion to the first example of this text because all those which follow are constructed on exactly the same scheme. The object in question is a "pile of bricks" (sig_4-anše[243]) of 5 GAR length (uš), 1½ GAR width (sag) and ½ GAR height (sukud). The number of bricks (sig_4) contained in such a pile is the problem posed by the text. The volume of the brick-heap (not given in the text) is easily computed as

$$V = lbh = 45 \text{ SAR}.$$

Assuming that the bricks (sig_4) are of type 1, we know from our new text that the volume of a single brick is

$$v = 0;0,0,41,40 \text{ SAR},$$

and we therefore find the number N of bricks to be

$$N = \frac{V}{v} = \frac{45}{0;0,0,41,40} = 1,4,48,0.$$

The text expresses this result as

$$N = 3 \text{ iku } 24 \text{ SAR}.$$

According to (1), we have

$$24 \text{ SAR} = 24 \cdot 12,0 = 4,48,0;$$

hence

$$3 \text{ iku} = 1,0,0,0 \text{ bricks}$$

or

(2) 1 iku = 20,0,0 bricks = 1,40 SAR.

This shows that the relation 1 iku = 1,40 SAR which is known from the measures of area and volume also holds for the "SAR" which represents 12,0 bricks. Analogously, as one would expect, the same text reveals that

(3) 1 gín = 0;1 SAR = 12 bricks.

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[239] Cf., e.g., Muss-Arnolt HWB, p. 646b.

[240] Scheil [2] p. 162 and Genouillac ITT 5 p. 6 proposed the relation 1 SAR = 722 (instead of 720) bricks, but this apparent discrepancy is due to the fact that the texts on which the figure 722 is based deliberately ignored fractions of units which were of no interest for their purposes; cf. below p. 95 and note 248. Thureau-Dangin[10] p. 192, note 1 and [11] p. 83 proposed the correct relation on the basis of BM 85194 rev. III 7–12, for a new interpretation of which cf. below pp. 96f.

[241] Cf., e.g., RTC No. 137 (pre-Old-Akkadian), Scheil [2] p. 164 and p. 166 (Ur III), Legrain [1] pp. 127f.(Ur III), Schorr AR No. 106 (time of Ḫammurapi) and the texts discussed below.

[242] This by no means implies that *all* texts dealing with bricks can now be satisfactorily explained. All previous attempts, however, must be subjected to a critical examination which starts from the new point of view of the relation (1).

[243] Transcribed SIG_4-GÍR in MKT I pp. 389ff. The correct reading was proposed by Thureau-Dangin [6] p. 165.

This shows that the assumption made in MKT[244] that SIG_4 means "volume" must be abandoned and is to be replaced by the ordinary translation "bricks". The volume of the pile of bricks will of course be obtained by multiplying the number of bricks by the volume of a single brick of the special type considered in the text.[245]

YBC 4673 (MKT III pp. 29f.) No. 4 is a problem which shows that the relation 1 ubu = 50 SAR, which is well known from the measures of volume and area, also holds for the brick-SAR which has the value 12,0 bricks. The problem reads:

5*i-na* 30 G[AR uš ....]
61(ubu)^{iku} 4 SAR sig_4 ..[....][245a]
7erim-ḫá en-nam gar-ra
8*i-na* u_4-1-k[am] ì-til[245b]
91,12 erim-ḫá

The translation follows:

5In [a distance of] 30 G[AR ....]
61 ubu (and) 4 SAR bricks [were transported(?)].
7How many workers were put (to the task so that)
8they were finished in the first day?
91,12 workers.

Whether the condition was actually given in this problem or merely assumed, one is safe in supplying the fact that one worker transported 9 "sixties" bricks the given distance in one day; this is actually stated in the preceding problems, Nos. 2 and 3, of the same text as well as problem-text P and AO 8862.[245c] Working back from the answer given, we find that the number of bricks involved are therefore 1,12 · 9,0 = 10,48,0. Since 4 SAR = 4 · 12,0 = 48,0 bricks, it follows that

1 ubu = 10,0,0 bricks;

since 50 · 12,0 = 10,0,0, we see that the brick-SAR stands in the same relation to the brick-ubu as the volume- or area-SAR to the volume- or area-ubu.

Another case where the new relations explain the calculations is **AO 7667**, an Ur III tablet published by Scheil.[246] This text contains nine entries concerning piles of bricks, four of which we do not discuss here because of the occurrence of a term[247] the exact significance of which we are unable to grasp. The remaining five examples can be arranged as follows:

The dimensions of the piles are given by length (l), width (b), and height (h). The second numbers given in the last column are computed according to (1) and (3); the text, of course, gives only the number of SAR and gín in each case. We now show that we are right in applying (1) and (3).

We must first remark that the lengths form an arithmetic progression with a difference of ½ GAR whereas all other dimensions are the same. The numbers of bricks should therefore also form such a progression. Starting with No. I, we would obtain for these numbers

No. I	2,34,48		
No. II	1,56, 6	instead of	1,56,0
Nos. III and IV	1,17,24		
No. VI	38,42	instead of	38,36.

This shows that Nos. II and VI omit 6 bricks = ½ gín each, apparently in order to avoid fractions of the gín.[248]

We now compute the volume of the two piles in No. I. The product of l, b and h gives a volume of 2;6,40 SAR for the first pile and 2;40 SAR for the

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[244] MKT I pp. 126 and 387f.

[245] In MKT I p. 399, the number N of bricks was called V^*, which was obtained from the volume V by multiplying V by $\frac{36}{5} = 7;12$. This coefficient is now explained as the combination of *two* coefficients: the volume v of a single brick and the number 12,0 of bricks in one SAR. Indeed, in order to obtain N, the total volume V must be divided by 12,0 v = 0;8,20, i.e., multiplied by its reciprocal 7;12.

[245a] The reading 1(ubu)^{iku}, instead of nu-zu, is based on a recent examination of the text.

[245b] The text has ì-til, not si[g_4 n]u<-zu>.

[245c] Cf. p. 98 below.

[246] Scheil [2].

[247] 1 kùš ba-an-gi_4. In No. VII one must restore (rev. I 8) 2[½ kùš dagal], which follows from a comparison with No. VIII. The piles of bricks in No. VIII are ⅕ as long as the piles in No. VII, and the number of bricks in No. VIII is ⅕ the number in No. VII. All other dimensions are the same.

[248] The same type of approximation is also found in another Ur III tablet published by Legrain [2] pl. II (p. 63) No. 15 and later transcribed by Scheil [2] p. 162 (where it is mistakenly called an unpublished text). Computing the total of the five entries, one finds 19 SAR and 40,6 bricks, whereas the text gives 22 SAR and 4,0 bricks; both are the same if we use (1) and disregard the 6 bricks = ½ gín.

No.	First Pile			Second Pile		Number of Bricks
	l	b	h	l	$b = h$	
I	2 GAR	½ GAR ⅓ kùš	2 kùš	2 GAR	4 kùš	12⅚ SAR 4 gín = 2,34,48
II	1½ GAR	½ GAR ⅓ kùš	2 kùš	1½ GAR	4 kùš	9⅔ SAR = 1,56,0
III and IV	1 GAR	½ GAR ⅓ kùš	2 kùš	1 GAR	4 kùš	6⅓ SAR 7 gín = 1,17,24
VI	½ GAR	½ GAR ⅓ kùš	2 kùš	½ GAR	4 kùš	3 SAR 13 gín = 38,36

second. The total volume is therefore 4;46,40 SAR. Assuming that according to (1) and (3) the number $12\frac{5}{6}$ SAR 4 gín given in the text means 2,34,48 bricks, we obtain for the volume of a single brick

$$v = \frac{4;46,40}{2,34,48} = 0;0,1,51,6,40 \text{ SAR}.$$

This is precisely the volume of the "type 4" brick (called sig_4-al-ùr-ra,[248a] "kiln-burnt brick") of our main text (p. 93), i.e., a brick which is $\frac{2}{3}$ kùš square and 5 šu-si high. This shows that the same relations hold for texts of the Ur III as well as the Old-Babylonian period.

The explanation given here proves that there is no need to assume the existence of a measure of length called GAR which was the equivalent of 24 kùš (instead of the ordinary 1 GAR = 12 kùš) in order to explain the relations in AO 7667.[249]

Another type of brick, also called sig_4-al-ur_5-ra "kiln-burnt brick", seems to be assumed in VAT 6598,[250] in which the volume V = 4 SAR of a wall[251] or prismatic shape contains 9 SAR bricks, i.e., according to our new relation, N = 1,48,0 bricks. The volume of a single brick is therefore

$$\frac{V}{N} = 0;0,2,13,20 \text{ SAR}.$$

Assuming that the height of the brick is the same as in all the examples of YBC 4607, namely, h = 5 šu-si = 0;10 kùš, we obtain for the area covered by our brick

$$lb = 0;0,13,20 \text{ SAR}.$$

Plausible solutions would be either l = 24 šu-si and $b = \frac{2}{3}$ kùš or l = 1 kùš and b = 16 šu-si.

c. The Capacity Equivalent of a Volume-SAR

A fresh interpretation of **BM 85194** rev. III 7–12, from which Thureau-Dangin[252] has already extracted corroborative evidence for the volume occupied by a sìla and for the relation 1 SAR = 12,0 bricks, yields still another detail, which is of importance for the interpretation of our present tablet, problem-text O. BM 85194 rev. III 7–12 reads as follows:[253]

7[šu]*m-ma* gi[š-má] 1 SAR sig_4 *i-na-aš-ši-i*
8[š]*e-a-am* en[-nam] *i-na-aš-ši* za-e 41,40 saḫar-ḫá
9[1] sig_4(text: sig_4-ḫá) [41],40 *a-na* 5 GÁN *i-ši* 3,28,20
10saḫar-ḫá [$3\frac{1}{3}$] sìla $8\frac{1}{3}$ gín še(text: sig_4) 1 sig_4
113,2[8,20] *a-na* 12 šu-ši(text: šu-si) *i-ši* 41,40 *ta-mar*
128(gur) [1],40 še-gur *ta-mar ne-pé-šum*

Taking into account the various emendations, we translate:[254]

7[I]f a [ship] carries 1 SAR bricks,
8how [much ba]rley will it carry? As for you— 0;0,0,41,40 is the volume of
9[1] brick. Multiply [0;0,0,41],40 by 5,0,0, (and the resulting) 3;28,20 is
10the volume; [$3\frac{1}{3}$] sìla (and) $8\frac{1}{3}$ gín is the barley of 1 brick.
11Multiply 3;2[8,20] by 12 sixties, (and) you will see 41,40, (i.e.,)
12you will see 8 gur (and) [1],40 (sìla) barley. (This is) the procedure.

This translation is based on the following interpretation of the single steps taken. The essential problem, as Thureau-Dangin saw, is the conversion of the volume occupied by 1 SAR = 12,0 bricks to units of capacity, namely, sìla and gur. At the beginning of the actual calculation, the text states as a known fact that v = 0;0,0,41,40 volume-SAR is the volume of one brick. This, as we have already seen,[255] is precisely the volume of the brick which we have called "type 1".[256] To convert this volume of a single brick expressed in units of the volume-SAR to capacity units of the sìla-class, the text multiplies v = 0;0,0,41,40 volume-SAR by a coefficient 5,0,0 (called 5 GÁN, a notation for which we have no explanation), and the resulting 3;28,20 sìla is decomposed into $3\frac{1}{3}$ sìla and $8\frac{1}{3}$ gín, which is called the barley of 1 brick. The coefficient 5,0,0 is easily explicable on the basis of the accepted relation[257] for the Old-Babylonian period

$$1 \text{ sìla} = 0;0,0,12 \text{ volume-SAR}$$

or, in other words,

$$1 \text{ volume-SAR} = 5,0,0 \text{ sìla};$$

[248a] In the summation after VI in AO 7667, the bricks are more specifically called sig_4-ù-ku-ru-um; this Sumerian word we take to be a (back?) loanword from Akkadian *agurrum* as follows: *agúrrum* > akúrrum > ukúrrum.

[249] This assumption was made by Scheil [2] p. 169 and adopted by Neugebauer MKT I pp. 87f.

[250] Published MKT I pp. 278f.; for these examples, cf. also von Soden [1] pp. 196ff.

[251] Erroneously transcribed $^{giš}SIG_4$ in MKT. The correct reading é-gar_8, was given by Thureau-Dangin [7] p. 58.

[252] Thureau-Dangin [10] p. 192, note 1; Thureau-Dangin [11] pp. 82f.

[253] We accept here all the restorations and emendations proposed by Thureau-Dangin [10] p. 192, note 1 and Thureau-Dangin [11] p. 82. In addition, we emend sig_4-ḫá of line 9 to sig_4.

[254] Contrast the translation and interpretation given Thureau-Dangin [10] p. 192, note 1 and Thureau-Dangin [11] p. 82.

[255] P. 93.

[256] Thureau-Dangin [11] p. 83 discovered that this type of brick was involved here.

[257] Thureau-Dangin [11] is the latest discussion of this relation.

accordingly, in order to transform a given number of volume-SAR into sìla units, it is necessary to multiply the number by 5,0,0. The text then multiplies the sìla equivalent of a single brick, 3;28,20, by 12,0, the number of bricks involved, and the resulting 41,40 sìla is in turn decomposed to the standard 8 gur and 1,40 sìla.[258]

The coefficient 5,0,0 which was explained in the previous paragraph is also used implicitly in our main text (O)[259] in the conversion of the SAR-volume of a single brick to sìla units. The philological interpretation involved, however, is not without difficulties. The question is expressed by the words ì šám saḫar-bi en-nam, and the answer is given as a number of sìla followed by ì šám saḫar-bi. Before we had connected the coefficient 5,0,0 of BM 85194 rev. III 7–12 with the analysis of our text, we translated "oil, the price of its volume", taking the implicit coefficient 5,0,0 as a price coefficient. Aside from the fact that we were forced to suggest 3600 bricks as the price unit involved in order to reach the correct order of magnitude of prices known more or less directly from other texts,[260] we were made uncomfortable by the resulting inference that prices of bricks merely took into account the volume of the bricks but disregarded the size or quality (e.g., kiln-burnt or sun-dried) of the individual brick. The discovery of the same coefficient 5,0,0 in BM 85194 rev. III 7–12 in connection with the conversion of the SAR-volume of a single brick to units of sìla moved us to abandon our original translation and interpretation of ì šám saḫar-bi, which we now interpret as "oil, the (capacity-)equivalent of its (i.e., the brick's) volume" in the question, and so-and-so-much "oil is the (capacity-)equivalent of its (i.e., the brick's) volume" in the answer. In connection with economic documents, the verb šám is translated "to buy" and the noun šám "price", but it is by no means excluded that the basic meanings are "to be equivalent" and "equivalent", respectively.

Forms like íb-si$_8$ (in the sense of *miṯḫartu*) and íb-tag$_4$ (≈*šapiltu*), which are Sumerian verbal forms which have taken on the meaning of a noun, suggest the possibility of the alternative reading ì-šám instead of ì šám throughout our present text. This would eliminate the "oil" from the problems and would lead to the translation "the (capacity-)equivalent" instead of "oil, the (capacity-)equivalent".

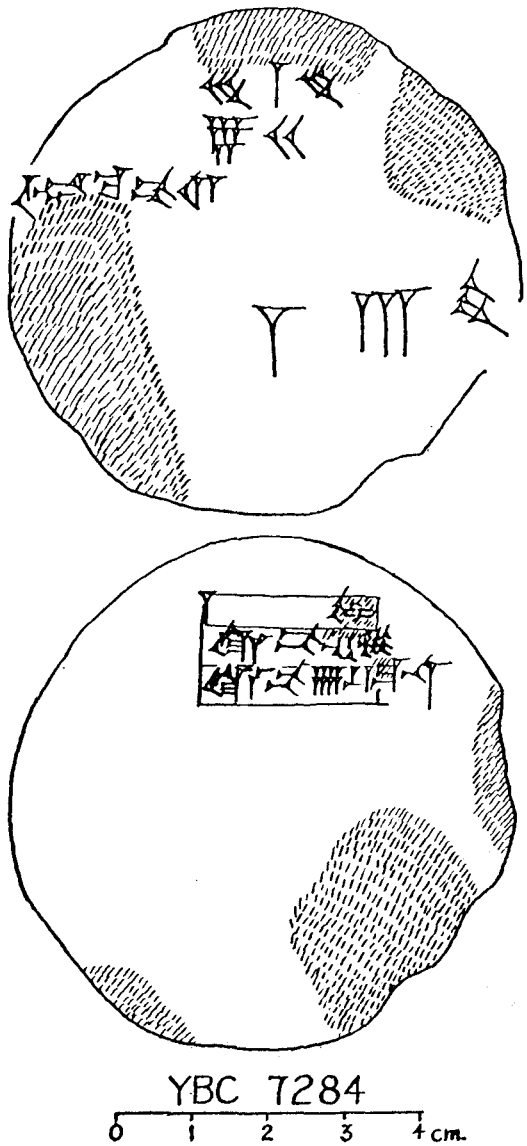
[258] It is not clear why 1,40 sìla was not subjected to the conversion to 1 PI 4 bán.

[259] Cf. p. 93 and the last horizontal row in the table at the bottom of the same page.

[260] From Pohl RVU No. 310 and No. 165, it follows that 1 gur of barley was the price of 8,24 kiln-burnt bricks (Ur III). Schorr UAZP No. 106 seems to indicate that 16 SAR bricks (i.e., 3,12,0 bricks) cost 1 gín silver (time of Ḫammurapi).

Oa. YBC 7284

YBC 7284 is a small, round tablet which reveals the existence of a coefficient accepted in the Old-Babylonian period for the transformation of the volume of bricks to units of weight.



On the reverse (convex side), we read:

1 sig$_4$	One brick.
ki-lá-bi en-nam	What is its weight?
ki-lá-bi 8⅓ ma-na	Its weight is 8⅓ ma-na.

The following numbers are inscribed on the obverse (flat side):

41,40
8,20
igi-gub-ba-bi 12
1,3,40

We know that v = 0;0,0,41,40 volume-SAR is the volume of one brick of "type 1" (called sig_4).[260a] If we multiply this value with the accepted constant, called "its igi-gub-ba",[260b]

$$c = 12,0,0 \text{ ma-na/SAR},$$

we obtain for the weight of one brick 8;20 ma-na, as indicated in the second line of the reverse.

The connection between this problem and the number 1,3,40 at the end of the obverse is not apparent.

P. YBC 10722

(Photograph: Plate 38; copy: Plate 4)

Transcription

Reverse

1[. . . . .] *ag-ra-am a-gu-ur* [. . . .[261]]
2[*i-na* 30[262]] GAR uš 9 šu-ši sig_4 *iz-bi-l*[*am*]
3[. . . (bán)] 5 sìla *še-a-am i-di-šu il-qé-ma*
4*i-na-an-na* 4 lú-ḫu[n-g]á *a-gu-ur* . . .
5 1 7
6 1 11
6a*an-nu-um-ma*(-)*aš*(?)-*šu*(?) . . . . . .
7 1 13
8 1 14

Obverse blank, except for the number 9 written on the left edge at right angles to the normal direction of writing.

Translation

1[. . . . .] I hired a hired-man [. . . .[261]]
2He transported 9 sixties of bricks a distance of [30(?)[262]] GAR;
3he took as wages [. . . bán] (and) 5 sìla barley.
4Now I hired 4 hired-men. . . .
5 1 7
6 1 11
6a. . . . . .
7 1 13
8 1 14

Commentary

Although much erased and incomplete, the problem set forth in this text clearly belongs to a group involving the transportation of 9 "sixties" (i.e., 540) of bricks: AO 8862 III 27–IV (MKT I pp. 111ff.) and YBC 4673 Nos. 2, 3, 4 (MKT III pp. 29ff.). The incomplete nature of the present text makes it impossible to state the exact details of the problem. Our tentative interpretation of the numbers in lines 5–8 is that the four 1's at the beginning of these lines refer to the four hired-men mentioned in line 4; the numbers which then follow perhaps refer to the fractions[263] $\frac{1}{7}$, $\frac{1}{11}$, $\frac{1}{13}$, and $\frac{1}{14}$ which, although unmentioned, may somehow be involved. Since, however, we are ignorant of the exact nature of the problem, we do not feel it wise to rule out the possibility that the numbers are to be interpreted as 7, 11, 13, and 14 or even as 1,7 1,11 1,13 and 1,14; in either case, the sum of these numbers, 45 or 4,45, could then perhaps be connected with the amount of barley [. . . bán] and 5 sìla mentioned in line 3.

Pa. YBC 7997

(Photograph: Plate 49; copy: Plate 23)

Transcription

Obverse

1*ú-tu-num* 1,30 GAR *ki-ip-pa-tum*
2*ba-ma-at ta-al-li-im*
3*a-na er-bé-et ta-za-az-ma*
4 15 *ra-bi-a-tim tu-uš-ta-kal-ma*
5 3,45 *i-li-a-ma*
6*a-na* 12 *šu-up-li-im*
7*tu-ub-ba-al-ma*
8 45 *šu-up-lu-um i-li-a-am*
9. . . -. . . -*tar*[263a] 1,30 *ki-i*[*p-pa-*]*tam*

Reverse

1[*tu-*]*uš-ta-kal*{*-al*}
2*a-n*[*a*] 5 *tu-ub-ba-al-ma*
3[1]1,15 *i-li-a-ma*
4 45 *ù* 11,15
5*tu-uš-ta-kal-ma*
6[*m*]*a-la i-li-a-am*
7 7,12 *ta-ta-na-aš-ši-ma*
8sig_4 *i-na-an-di-in*

Translation

Obverse

1An oven. 1;30 GAR is the circumference.
2Half of the diameter
3you shall divide in four.

[260a] Cf. p. 93.
[260b] For this term see below p. 132.
[261] Perhaps nothing missing.
[262] This restoration is based on the traces and on YBC 4673 (MKT III p. 29) No. 2, line 5; No. 3, line 15; No. 4, line 5.
[263] For this possibility cf. the figure in VAT 6597 No. 4 (MKT I p. 274).
[263a] Dr. Goetze suggests the reading *tu-ṷa-tar* here.

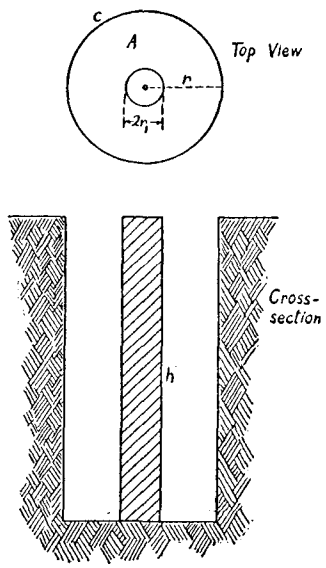
[4]You shall square 0;15, of the quarter,[263b] and
[5]0;3,45 (, which) will come up,
[6-7]you shall multiply by 12, the depth, and
[8]0;45, the depth[sic], will come up.
[9-Rev. 1]....., you shall square 1;30, the circumference,

Reverse

[2]multiply by 0;5, and
[3]0;11,15 will come up.
[4-5]Multiply together 0;45 and 0;11,15, and
[6]whatever comes up
[7]you shall multiply by 7;12,
[8]and it will give the brick(s).

Commentary

Although the text does not specifically state the problem to be solved, it is possible to reconstruct it with some degree of plausibility from the given data and the calculations which are carried out. The accompanying figure gives the basis for our interpretation of the text. The only parameter given in conventional form in the text is the circumference c = 1;30 GAR of the oven. The problem which we assume is the following: calculate the number of bricks of what we call "type 1"[263c] contained in a cylindrical core with a radius r_1 one-fourth of the radius r of the oven and with a height h of 12 kùš. The calculation proceeds as follows:



$$\alpha = \frac{r_1}{r} = \frac{1}{4} = 0;15$$
$$\alpha^2 = 0;3,45$$
$$\alpha^2 h = 0;3,45 \cdot 12 = 0;45$$
$$c^2 = 2;15$$
$$A = \frac{c^2}{4\pi} = 2;15 \cdot 0;5 = 0;11,15,$$

after which the volume V of the cylindrical pile is calculated by

$$V = \alpha^2 A h = 0;45 \cdot 0;11,15$$
$$(= 0;8,26,15 \text{ volume-SAR}).$$

The text ends by recommending the calculation of the number of bricks in the cylindrical core by the multiplication of V by 7;12.[263d] The background of the coefficient 7;12 has been explained p. 95, note 245; it is the number by which a volume measured in units of the volume-SAR must be multiplied to obtain the number (measured in units of the brick-SAR, which equals 12,0 bricks) of bricks of "type 1" occupying that volume.

The failure to mention the problem involved, the omission of the value of h until it is used in the actual calculation, and the omission of the results of the calculations in the last three lines seem to indicate that our text was never meant to be anything more than a set of sketchy notes.

The problem itself contains elements which are strongly idealized: the height of the cylindrical core seems excessive, and the bricks in the *cylindrical* core are in the shape of *rectangular prisms*.

§ 8. Equations

Q. YBC 9856

(Photograph: Plate 38; copy: Plate 4)

Transcription

Obverse

1 [1]*iš-ka-ra-am i-na ku-ZI-im id-di-nu-nim-ma*
[2]*ma-li pi-ḫa* 1 *ši-in pi-ḫa* 2
[3]*ša-lu-uš-ti pi-ḫa* 20 *ma-li pi-ḫa*
[4]*ši-in pi-ḫa ù ša-lu-uš-ti pi-ḫa*
[5]*ki ma-ṣi*
[6]*ma-li pi-ḫa* 18[263e] *ši-in* [*pi-*]*ḫa* 36
[7]*ša-lu-uš-ti pi-ḫa* 6

2 [8]1 ma-na kù-babbar 5 šeš-*a-ni ma-li*
[9]*ši-it-ti-in a-ḫi ṣe-eḫ-ri-im*
[10]*a-ḫu-um e-li a-ḫi-im li-te-le-le*
[11]1 4 2 8 3 12 4 1[6 5 2]0

Reverse uninscribed.

[263b] "Of the quarter" is to be understood as an indication of the source of 0;15, namely, the division of the radius into quarters as described in the previous line.
[263c] Cf. p. 93.
[263d] The answer would be 1;0,45 brick-SAR, i.e., 12,9 bricks.
[263e] Read so against our copy on plate 4.

Translation

1 [1]They gave me an assignment in ..... such that
[2]one of[264] my *pū* is 1, two of my *pū* are 2,
[3–5]one-third of my *pū* is 0;20. How much are one of[264] my *pū*, two of my *pū*, and one-third of my *pū*?
[6]One of[264] my *pū* is 18, two of my [*pū*] are 36,
[7]one-third of my *pū* is 6.

2 [8]1 ma-na silver of 5 brothers. As much
[9](as the difference between the shares) of two brothers is (that) of the smallest one; (and)
[10]let one brother exceed the (next) brother (by this amount).
[11]1 4, 2 8, 3 12, 4 1[6, 5 2]0.

Commentary

No. 1.

The mathematical content of this problem can easily be deduced from the numbers which occur. Three magnitudes λ_1, λ_2 and λ_3 shall be found such that

$$\lambda_1 + \lambda_2 + \lambda_3 = 1{,}0$$

and

$$\lambda_1 : \lambda_2 : \lambda_3 = 1 : 2 : \tfrac{1}{3}$$

The result is

$$\lambda_1 = 18 \qquad \lambda_2 = 36 \qquad \lambda_3 = 6.$$

We must, however, confess that the wording of the text, or at least our translation, does not clearly reveal the details of the problem. The total 1,0 is not mentioned, unless it is somehow included in the word *iškarum* "assignment". We do not know whether *ku-ZI-im* should be translated by "chair" (*kussēm*) or "cold", "winter" (*kuṣṣim* or *kūṣim*); even these alternatives perhaps do not exhaust the possibilities. The meaning of *pū* is also unclear to us in this context.[265] To our ignorance of the background of the problem is added the awkwardness of expression for the given ratios between the λ's.

In spite of these difficulties, it is perhaps worth suggesting that the problem has to do with the parts (λ_1, λ_2, λ_3) of a work-day devoted to three separate kinds of work performed at different speeds. The answers would then have to be considered as 0;18 and 0;36 and 0;6 of a day, making a total of 1. For this general type of problem, cf., e.g., the commentary to Nos. 6 and 7 of problem-text L, pp. 84ff.

No. 2.

Problem: 1 ma-na silver (= 1,0 gín) shall be distributed among five brothers, it being understood that the shares a_1, a_2, a_3, a_4, a_5 form an arithmetic progression ($a_1 < a_2 < \ldots < a_5$) with difference $d = a_1$. The result is

$$a_1 = 4 \qquad a_2 = 8 \qquad a_3 = 12 \qquad a_4 = 16 \qquad a_5 = 20.$$

"Inheritance problems" of this or a similar character are not new. One problem was discussed above pp. 52f. No. 5, and several others have previously been published.[265a] Problems dealing with progressions of various types are also well attested.[266]

R. YBC 4652

(Photograph: Plate 39; copy: Plate 13)

Transcription

Obverse

. . . . . .

3 [1][na$_4$ ì-pa k]i-lá n[u-na-tag . . . . . .
[2][ì-lá 1 ma-]na sag n[a$_4$ en-nam sag na$_4$ . . . . . .
. . . . . .

4 [3][na$_4$ ì-p]à ki-l[á nu-]na-t[ag . . . . . .
. . . . . .
[4][ig]i-11-gál ù [. . . . . .
. . . . . ì-lá 1 ma-na]
[5]sag na$_4$ e[n-nam sag na$_4$ . . . . . .
. . . . . .

5 [6]na$_4$ ì-pà ki[-lá nu-na-tag . . . . . .
. . . . . .
[7]ì-lá 1 ma-na s[ag na$_4$ en-nam sag na$_4$ . . . . . .
. . . . . .]

6 [8]na$_4$ ì-pà ki[-lá nu-na-tag . . . . . .
. . . . . .
[9]ì-lá 1 ma-na [sag na$_4$ en-na]m sag n[a$_4$ . . . . . .
. . . . . .

7 [10]na$_4$ ì-pà ki-lá [nu-na-ta]g igi-7-gál bí-daḫ igi-11-gá[l b]í-[da]ḫ
[11]ì-lá 1 ma-na sag na$_4$ en-nam sag na$_4$ $\frac{2}{3}$[266a] ma-na 8 gín 22½ še

8 [12]na$_4$ ì-pà ki-lá nu-na-tag igi-7-gál ba-zi igi-13-gál ba-zi-ma
[13]ì-lá 1 ma-na sag na$_4$ en-nam sag na$_4$ 1 ma-na 15$\frac{5}{6}$ gín

9 [14]na$_4$ ì-pà ki-lá nu-na-tag igi-7-gál ba-zi igi-11-gál bí-daḫ
[15][igi-1]3-gál ba-zi ì-lá 1 ma-na sag na$_4$ en-nam
[16][sag] na$_4$ 1 ma-na 9½ gìn 2½ še

---

[264] Literally, "as much as."

[265] Cf. perhaps Speiser [1] pp. 18ff.

[265a] For references see MKT III p. 83 under "Verteilungsaufgaben."

[266] Cf. MKT III p. 83 under "Reihen."

[266a] Our copy on plate 13 has ⅓ by mistake.

10 [17][na$_4$ ì-p]à ki-lá nu-na-tag igi-7-gál ba-zi igi-11-gál bí-daḫ
[18][. . . . . b]í-d[aḫ] ì-lá 1 ma-na sa[g na$_4$ en-nam]
. . . . . .

Reverse

. . . . . .

19 [1]na$_4$ ì-pá ki-lá nu-na-tag 6-bi ì-lá 2 gín [bí-daḫ-ma]
[2]igi-3-gál igi-7-gál a-rá-24-kam tab[267] bí-daḫ-ma
[3]ì-lá 1 ma-na sag na$_4$ en-nam sag na$_4$ 4⅓ gín

20 [4]na$_4$ ì-pà ki-lá nu-na-tag 8-bi ì-lá 3 gín bí-daḫ-ma
[5]igi-3-gál igi-13-gál a-rá 21 e-tab bí-daḫ-ma
[6]ì-lá 1 ma-na sag na$_4$ en-nam sag na$_4$ 4½ gín

21 [7]na$_4$ ì-pà ki-lá nu-na-tag igi-6-gál ba-zi
[8]igi-3-gál igi-8-gál bí-daḫ-ma ì-lá 1 ma-na
[9]sag na$_4$ en-nam sag na$_4$ 1 ma-na 9 gín 21½ še
[10]ù <igi-> 10-gál še kam

22 [11]na$_4$ ì-pà ki-lá nu-na-tag ⅔ igi-6[-gál . . . .]
[12]igi-3-gál igi-8-gál bí-daḫ-ma ì-l[á 1 ma-na]
[13]sag na$_4$ en-nam sag na$_4$ 1 m[a-na . . . . . .]

[14]22 [im-šu-meš]

Translation

Obverse

. . . . . .

3 [1][I found a stone], (but) [did n]ot we[igh it; . . . . . .]
[2][I weighed (it): 1 ma-]na. [What was] the origin(al weight) of the sto[ne? . . . . . .]

4 [3][I fou]nd [a stone], (but) did [not] we[igh it; . . . . . .]
[4]one-eleventh and [. . . . . , I weighed (it): 1 ma-na].
[5][What] was the origin(al weight) of the stone? [. . . . . .]

5 [6]I found a stone, [(but) did not] we[igh it; . . . . . .],
[7]I weighed (it): 1 ma-na. [What was the] origin(al weight) of the stone? [. . . . . .]

6 [8]I found a stone, [(but) did not] we[igh it; (after) . . . . . .],
[9]I weighed (it): 1 ma-na. What [was the origin(al weight) of the stone]? The origin(al weight) of the sto[ne was . . . . . .]

[7][10]I found a stone, [(but) did not] weig[h it]; (after) I a[dd]ed one-seventh (and) a[dd]ed one-eleventh,
[11]I weighed (it): 1 ma-na. What was the origin(al weight) of the stone? The origin(al weight) of the stone was ⅔ ma-na, 8 gín, (and) 22½ še.

8 [12]I found a stone, (but) did not weigh it; (after) I subtracted one-seventh (and) subtracted one-thirteenth,
[13]I weighed (it): 1 ma-na. What was the origin(al weight) of the stone? The origin(al weight) of the stone was 1 ma-na (and) 15⅚ gín.

9 [14]I found a stone, (but) did not weigh it; (after) I subtracted one-seventh, added one-eleventh, (and)
[15]subtracted one-thir[teenth], I weighed (it): 1 ma-na. What was the origin(al weight) of the stone?
[16][The origin(al weight)] of the stone was 1 ma-na, 9½ gín, (and) 2½ še.

10 [17][I fou]nd [a stone], (but) did not weigh (it); (after) I subtracted one-seventh, added one-eleventh,
[18][. . . . .] added [. . . . .], I weighed (it): 1 ma-na. What was the origin(al weight) of the stone?
. . . . . .

Reverse

. . . . . .

19 [1]I found a stone, (but) did not weigh it; (after) I weighed (out) 6 times (its weight), [added] 2 gín, (and)
[2]added one-third of one-seventh multiplied by 24,
[3]I weighed (it): 1 ma-na. What was the origin(al weight) of the stone? The origin(al weight) of the stone was 4⅓ gín.

20 [4]I found a stone, (but) did not weigh it; (after) I weighed (out) 8 times (its weight), added 3 gín,
[5]one-third of one-thirteenth I multiplied by 21, added (it), and then
[6]I weighed (it): 1 ma-na. What was the origin(al weight) of the stone? The origin(al weight) of the stone was 4½ gín.

21 [7]I found a stone, (but) did not weigh it; (after) I subtracted one-sixth (and)
[8]added one-third of one-eighth, I weighed (it): 1 ma-na.
[9]What was the origin(al weight) of the stone? The origin(al weight) of the stone was 1 ma-na, 9 gín, 21½ še,
[10]and one-tenth of a še.

[267] Reverse 5 would lead us to expect a-rá 24 e-tab here.

22 [11]I found a stone, (but) did not weigh it; (after) I [. . . .] ⅔ of one-sixth [. . . .],
[12]added one-third of one-eighth, I weighed (it): [1 ma-na].
[13]What was the origin(al weight) of the stone? The origin(al weight) of the stone was 1 m[a-na . . . . .].

[14]22 [sections].

Commentary

a. Mathematical Commentary

According to the colophon, the tablet contained 22 examples when complete, but only 11 are even partly preserved; of these, 6 can be fully restored. All the problems of the tablet are obviously of the same type, resulting in a linear equation for one unknown quantity, the "original" weight of a stone.

The preserved problems are the following:

No. 7

$$\left(x + \frac{x}{7}\right) + \frac{1}{11}\left(x + \frac{x}{7}\right) = 1{,}0$$

Solution: $x = \frac{2}{3}$ ma-na + 8 gín + 22½ še = 48;7,30 gín.

Indeed, $\frac{x}{7} = 6;52{,}30$ $x + \frac{x}{7} = 55$ $\frac{1}{11}\left(x + \frac{x}{7}\right) = 5.$

No. 8

$$\left(x - \frac{x}{7}\right) - \frac{1}{13}\left(x - \frac{x}{7}\right) = 1{,}0$$

Solution: $x = 1$ ma-na + $15\frac{5}{6}$ gín = 1,15;50 gín.

Indeed, $\frac{x}{7} = 10;50$ $x - \frac{x}{7} = 1{,}5$ $\frac{1}{13}\left(x - \frac{x}{7}\right) = 5.$

No. 9

$$\left(x - \frac{x}{7}\right) + \frac{1}{11}\left(x - \frac{x}{7}\right) - \frac{1}{13}\left[\left(x - \frac{x}{7}\right) + \frac{1}{11}\left(x - \frac{x}{7}\right)\right] = 1{,}0$$

Solution: $x = 1$ ma-na + $9\frac{1}{2}$ gín + $2\frac{1}{2}$ še = 1,9;30,50 gín.

Indeed, $\frac{x}{7} = 9;55;50$ $x - \frac{x}{7} = 59;35$

$\frac{1}{11}\left(x - \frac{x}{7}\right) = 5;25$

$\frac{1}{13}\left[\left(x - \frac{x}{7}\right) + \frac{1}{11}\left(x - \frac{x}{7}\right)\right] = 5.$

No. 19

$$(6x + 2) + \frac{1}{3} \cdot \frac{1}{7} \cdot 24(6x + 2) = 1{,}0$$

Solution: $x = 4\frac{1}{3}$ gín.

Indeed, $6x + 2 = 28$ $\frac{24}{21}(6x + 2) = 32.$

No. 20

$$(8x + 3) + \frac{1}{3} \cdot \frac{1}{13} \cdot 21(8x + 3) = 1{,}0$$

Solution: $x = 4\frac{1}{2}$ gín.

Indeed, $8x + 3 = 39$ $\frac{21}{39}(8x + 3) = 21.$

No. 21

$$\left(x - \frac{x}{6}\right) + \frac{1}{3} \cdot \frac{1}{8}\left(x - \frac{x}{6}\right) = 1{,}0$$

Solution: $x = 1$ ma-na + 9 gín + $21\frac{1}{2}$ še + $\frac{1}{10}$ še = 1,9;7,12 gín.

Indeed, $\frac{x}{6} = 11;31{,}12$ $x - \frac{x}{6} = 57;36$ $\frac{1}{24}\left(x - \frac{x}{6}\right) = 2;24.$

The remaining problems are completely broken away or too badly preserved to be restored with certainty.

The main difficulty encountered in interpreting the text of the problem consists in placing the parentheses correctly. The terminology alone is in itself inadequate; only experience with analogous problems,[268] when combined with the given solution, indicates the correct interpretation. The ancient scribes of course had the oral interpretation of their teachers at their disposal.

b. Terminology

ki-lá nu-na-tag

The expression ki-lá-tag(-ga) or the negative ki-lá-nu-tag(-ga) occurs frequently in Ur III economic texts

[268] Cf. e.g., MKT I p. 483.

dealing with cloth of various kinds.[269] Insofar as we have been able to check the references, the negative appears when the weights of the bolts of cloth have not been mentioned, the affirmative expression when weights have been given. This in itself would constitute strong evidence in favor of a general meaning like "weighed/unweighed". The following passages from the series *ana ittišu*[270] are also relevant:

Tablet 2, IV 35′ f.: [kù-pad-d]u-ni ki-lá-nu-tag-ga | *ši-bír-ta-šu la-a ša-qí-il-ta*, "sein Metallblock, nicht abgewogen".
Tablet 2, IV 50′ f.: kù-pad-du-ni ki-lá-[nu-tag-ga] | [š]*i-bír-ta-šu* [*la-a ša-q*]*íl-ta*, "seinen Metallblock, den nicht gewogenen".
Tablet 3, II 12: kù-pad-du-ki-lá-tag-ga | "(= *ši-be-ir-tum*) *ša-qil-tu*, "mit Gewichtsstempel versehener Metallblock".[271]
Tablet 3, II 13: kù-pad-du-ki-lá-nu-tag-ga | "(= *ši-be-ir-tum*) *la-a* "(= *ša-qil-tu*), "nicht mit Gewichtsstempel versehener Metallblock".[272]

In all these passages from *ana ittišu*, the Akkadian version means "weighed", whereas the Sumerian phrase according to Landsberger signifies "mit Gewichtsstempel versehen". The latter meaning would not be inappropriate in the Ur III texts mentioning cloth if we are to assume an "ink" stamp or a stamp applied to a lump of clay attached to the cloth; but it seems to be completely out of the question in connection with the stone mentioned in the present mathematical text. We have therefore translated simply, "I did not weigh it".

YBC 4669, rev. I 16–20

Thanks to a recent collation, we are now able to understand YBC 4669 rev. I 16–20 (published MKT III p. 27), a problem which has not hitherto found a satisfactory analysis.[273] The text reads:

16 *a-na* ⅔ ⅔-*ia*
17 1(bán)[274] daḫ-ma
18 *še-e ma-ši-il*
19 sag še en-nam[275]
20 3(PI)[276] še sag

The translation follows:

16 To ⅔ of my ⅔
17 I added 1 bán, and (the result was that)
18 the barley was halved.
19 What is the origin(al quantity) of the barley?
20 3 PI barley is the origin(al quantity).

The mathematical interpretation is now simple. Let x represent the original quantity of barley. The text gives

$$\frac{2}{3} \cdot \frac{2}{3} \cdot x + 10 \text{ sìla} = \frac{x}{2};$$

hence

$$x = 3{,}0 \text{ sìla} = 3 \text{ PI},$$

which is the answer given in the text.

S. YBC 4612

(Photograph: Plate 40; copy: Plate 14)

Transcription

Obverse

1 1 3,45 GAR uš 1,20 GAR sag a-šà-bi en-n[am]
2 a-šà-bi 1(bur³u)[iku]

2 3 1(bur³u)[iku] a-šà 3,45 GAR uš
4 sag-bi en-nam 1,20 GAR sag

3 5 1(bur³u)[iku] a-šà 1,20 GAR sag
6 uš-bi en-nam 3,45 GAR uš

4 7 1(bur³u)[iku] a-šà uš ù sag gar-gar-ma
8 5,5 uš ù sag en-nam
9 3,45 GAR uš 1,20 GAR sag

5 10 1(bur³u)[iku] a-šà uš ugu sag 2,25 dir<ig>
11 uš ù sag en-nam
12 3,45 uš 1,20 GAR sag

6 13 2,30 GAR uš 24 GAR sag
14 a-šà-bi en-nam 2(bùr)[iku] a-šà

---

[269] To the references given by Deimel, ŠL 126,53d; 461,239; and 536,260 may be added the following: Reisner TT No. 265 rev. 6; Genouillac ITT 3[II] pl. 31 (5597) obv. 3; Genouillac TCL V pl. XL (AO 6054) rev. IV 2; Lutz [1] p. 231 No. 9, 17; Schneider, Montserrat, No. 152, 9; No. 280, 5.

[270] Quoted according to the edition of the series in Landsberger MSL.

[271] Landsberger's footnote: Übersetzung nach dem Sum.; akk. nur "gewogen."

[272] Cf. also Tablet 3, I 34ff., which, if Landsberger's suggested restoration is correct, would be most illuminating because Sumerian tag (≈Akkadian *mḫṣ*) would then be used in connection with the "stamping" of various symbols on measures of capacity.

[273] Cf. Thureau-Dangin [12] p. 89; Thureau-Dangin TMB p. 209, **611**.

[274] In MKT, "½" was read instead of the paleographically equivalent "1(bán)"; the same mistake was made by Thureau-Dangin.

[275] After en-nam appears an erased form of the first sign of the next line.

[276] The sign form is a reversed "A," i.e., one vertical wedge superposed on another, followed by a single vertical wedge.

7 [15]2(bùr)[iku] a-šà 2,30 uš
[16]sag-bi en-nam 24 sag

8 [17]2(bùr)[iku] a-šà 24[276aa] sag
[18]uš en-nam 2,30 G[AR u]š

9 [19]2(bùr)[iku] a-šà uš ù sag gar-gar-ma
[20]2,54 uš ù sag en-nam

Reverse

[1]2,30 GAR uš 24 GAR sag

10 [2]2(bùr)[iku] a-šà uš u <gu> sag 2,6 dirig
[3]uš ù sag en-nam
[4]2,30 GAR uš 24 GAR sag

11 [5]8,53 GAR 4 kùš uš
[6]6½ GAR 3 kùš sag
[7][a-šà-b]i en-nam 2(bùr)[iku] a-šà

12 [8][1,52]½ GAR uš
[9][32] GAR sag
[10][a-šà-b]i 2(bùr)[iku]

13 [11]17½ <GAR> 3⅓ kùš uš
[12]3 GAR 4½ kùš sag
[13]a-šà-bi 2(bùr)[iku]

14 [14]2,13 GAR 4 kùš uš
[15]27 GAR sag
[16]a-šà-bi 2(bùr)[iku]

15 [17]1,6½ GAR 2 kùš uš
[18]54 GAR sag
[19]a-šà-bi 2(bùr)[iku]

Translation

Obverse

1 [1]3,45 GAR is the length, 1,20 GAR the width. What is its area?
[2]Its area is 1 bur[2]u.

2 [3]1 bur[2]u is the area, 3,45 GAR the length.
[4]What is its width? 1,20 GAR is the width.

3 [5]1 bur[2]u is the area, 1,20 GAR the width.
[6]What is its length? 3,45 GAR is the length.

4 [7]1 bur[2]u is the area; I added the length and the width, and (the result is)
[8]5,5. What are the length and the width?
[9]3,45 GAR is the length; 1,20 GAR is the width.

5 [10]1 bur[2]u is the area; the length exceeded the width by 2,25.
[11]What are the length and the width?
[12]3,45 (GAR) is the length; 1,20 GAR is the width.

6 [13]2,30 GAR is the length, 24 GAR the width.
[14]What is its area? 2 bùr is the area.

7 [15]2 bùr is the area, 2,30 (GAR) the length.
[16]What is its width? 24 (GAR) is the width.

8 [17]2 bùr is the area, 24 (GAR) the width.
[18]What is the length? 2,30 GAR is the length.

9 [19]2 bùr is the area; I added the length and the width, and (the result is)
[20]2,54 (GAR). What are the length and the width?

Reverse

[1]2,30 GAR is the length; 24 GAR is the width.

10 [2]2 bùr is the area; the length exceeded the width by 2,6 (GAR).
[3]What are the length and the width?
[4]2,30 GAR is the length; 24 GAR is the width.

11 [5]8,53 GAR (and) 4 kùš is the length,
[6]6½ GAR (and) 3 kùš the width.
[7]What is its [area]? 2 bùr is the area.

12 [8][1,52]½ GAR is the length,
[9][32] GAR the width. (What is its area?)
[10]Its [area] is 2 bùr.

13 [11]17½ <GAR> (and) 3⅓ kùš is the length,
[12]3 GAR (and) 4½ kùš the width. (What is its area?)
[13]Its area is 2 bùr.

14 [14]2,13 GAR (and) 4 kùš is the length.
[15]27 GAR the width. (What is its area?)
[16]Its area is 2 bùr.

15 [17]1,6½ GAR (and) 2 kùš is the length,
[18]54 GAR the width. (What is its area?)
[19]Its area is 2 bùr.

Commentary

The mathematical aspect of this group of 15 problems is extremely simple. The only relation used is the fact that the area of a rectangle of sides x and y ($x > y$) is xy. The numerical values of the magnitudes used are

Nos. 1 to 5 x = 3,45 GAR y = 1,20 GAR
xy = 5,0,0 GAR² = 1 bur[2]u

Nos. 6 to 10 x = 2,30 GAR y = 24 GAR
xy = 1,0,0 GAR² = 2 bùr

and the problems are:

[276aa] Two corner wedges have fallen out in our copy on plate 14.

Nos.	given	find
1 and 6	x, y	xy
2 and 7	xy, x	y
3 and 8	xy, y	x
4 and 9	$xy, x + y$	x, y
5 and 10	$xy, x - y$	x, y

The solution of the last two pairs requires quadratic equations of the standard type.

The last group (Nos. 11 to 15) is even simpler: find xy if x and y are given. The emphasis of these problems obviously lies in the metrological aspect. If we express the area in GAR², we find the value 1,0 GAR² for the area in No. 13, but 1,0,0 GAR² in the remaining four examples. The text, however, gives all five areas as 1,0,0 GAR².

Because the area is supposed to be 1,0,0 GAR² (i.e., 2 bùr) in all five cases, x and y (expressed in GAR) are reciprocal numbers. The values are:

No.	x	y
11	8,53;20 GAR	6;45 GAR
12	[1,52;]30 GAR	[32] GAR
13	17;46,40 GAR	3;22,30 GAR
14	2,13;20 GAR	27 GAR
15	1,6;40 GAR	54 GAR

The restoration in No. 12 is uniquely determined by the fact that there is no other pair of numbers x and y such that $x \cdot y = 1,0,0$ and $x = a + 0;30$ $y = b$, where a and b are integers and where $x > y$.

As indicated above, the solution given in No. 13 is wrong because the product xy is only 1,0, not 1,0,0. This shows that the scribe's starting-point was a table of reciprocals but that he committed an error in determining the sexagesimal place. The correct value would have been 17,46;40 GAR = 17,46½ GAR 2 kùš.

Sa. YBC 6492

(Photograph: Plate 48; copy: Plate 22)

Transcription

Obverse

1 1⅔ uš sag a-šà 10 uš [sag en-n]am
2 2⅔ uš sag a-šà 2,30 [uš sag en-na]m
3 3⅔ uš sag a-šà 17,46,[40 uš sag en-na]m
4 4⅔ uš sag a-šà ...25(?),[..... uš sag en-]nam
5–7 5–7⅔ uš sag a-šà at the beginning of each line and [en]-nam at the end is the maximum which can be read.
8–14 8–14Maximum reading: *ba-ma-at* at the beginning of each line and en-nam at the end. The type was obviously: *ba-ma-at* uš sag a-šà (*c*) uš sag en-nam.
15–18 15–18Maximum visible: igi-3-gál at the beginning of each line and en-nam at the end. The type involved is: igi-3-gál uš sag a-šà (*c*) uš sag en-nam.
19 19[igi]-15-g[ál uš sag a-šà ..... uš sag en-na]m
20–21 20–21Broken away. Restore to correspond to the type of line 19.

Reverse

22 1[igi-1]5-gál uš sag a-šà 56,15 uš sag [en-nam]
23 2igi-15-gál uš sag a-šà 10,25 uš sag en-na[m]
24 3⅚(?) sag a-na uš ..... a-šà 13,20 uš sag en-na[m]

(The rest of the reverse is uninscribed.)

Commentary

With the possible exception of the last one, the problems stated by this text consist in finding x and y from

$$y = \alpha x \qquad xy = c,$$

where α and c are given. No answers are indicated. The text calls x uš ("length"), y sag ("width"), and xy a-šà ("area"). Each line, except perhaps the last, reads:

"α of the length is the width; the area is c. What are the length and the width?".

In **Nos. 1–7**, α is ⅔; c is given as 10, 2,30 and 17,46[,40], respectively, in Nos. 1–3, but the values for c are destroyed in Nos. 4–7. The answers can easily be found to be:

No. 1: $x = 30$, $y = 20$ — No. 2: $x = 15$, $y = 10$

No. 3: $x = 40$, $y = 26;40$.

In the second group, **Nos. 8–14**, α is given as ½, but c is destroyed in all cases.

In **Nos. 15–18**, α is ⅓, but c is broken away.

Nos. 19–23 give $\alpha = \frac{1}{15}$, and Nos. 22 and 23 give $c = 56,15$ and $c = 10,25$, respectively. This leads to $x = 3,45$ and $y = 15$ in No. 22, and $x = 12,30$ and $y = 50$ in No. 23.

Our inability to read with certainty some of the signs in the last problem, **No. 24**, raises difficulties for the interpretation. It is perhaps worth pointing out that

$$y = \tfrac{5}{6}x \qquad xy = 13;20$$

would lead to $x = 4$ and $y = 3;20$.

Sb. MLC 1842

(Photograph: Plate 48; copy: Plate 22)

Transcription[276a]

Obverse

1 ganba-e *i-li-i-ma* 30 š[e g]ur *a-ša-a*[*m*]
2 [g]anba *iš-pí-il-ma* 30 še gur *a-ša-am*
3 *ma-ḫi-ri-ja ak-mu-ur-ma* 9
4 kù-babbar *ma-ḫi-ri-ja ak-mu-ur-ma*
5 1 ma-na 7½ gín
6 ganba *a-ša-am ù ki-ja*[276b]
7 *ap-šu-ur at-ta i-na e-pé-š*[*i-ka*]
8 1,7,30 kù-babbar *lu-pu-ut-ma*
9 igi 1,7,30 *pu-ṭur-ma*
10 53,20 *i-li* 53,20 š[*a i-lu*(?)-*ú*(?)]
11 *a-na* 9 *ma-ḫi-ri-ka*

Reverse

1 *i-ši-ma* 8[276c] *i-li*
2 8 *ša i-li-a-kum*
3 *a-na* 2,30 *ma-ḫ*[*i-ri-ka*(?)] . . . (?)
4 *i-ši-*[*m*]*a* 20 *i-li* 20 *ša*(?) *i*(?)-[*lu*(?)-*ú*(?)]
5 . . . . . .
6 [*tu*(?)]-*úr*(?)-*m*[*a* . . . . .] . . . . . .
7 [. . .] . . *ma* [. . .] . . . . . .
8 4,3[0 . . .] . . . . . [. . . .] . . . . . .
9 *šu-ta-ki-il-ma* 20,15 *i-li-a-kum*(?)
10 *i-na* šà 20,15 *ša i-lu-ú*
11 [. . .] . . . . . [. . . .] . . . . -*ka* . . . -*lu-ú*
12 [. . . . .]

Lower Edge

1 [. . . . .] . . . . *li* . . . . . .
2 . . . . . .

Left Edge

1 [. . . . .] . . . . . -*im-ma i-na ša am*(?)-*ḫu-ru*

[276a] After a copy of this tablet was prepared on Plate 22, Dr. A. Goetze succeeded in removing several strips of foreign matter from the tablet. The photograph on Plate 48 shows the present state of the tablet.

[276b] Probably nothing, except possibly [-*a*], is missing at the end of this line. For this word, cf. p. 50, note 140.

[276c] Written over an erasure.

Translation

Obverse

1 (When) the equivalent rose, I bought 30 gur of barley;
2 (when) the equivalent became low, I bought 30 gur of barley.
3 I added my equivalents, and (the result is) 9.
4 I added the silver of my equivalents, and (the result is)
5 1 ma-na (and) 7½ gín.
6 (At what(?)) equivalent(s) did I buy, and how much
7 did I sell(?) (for(?))? (As for) you—when you perform (the operations),
8 assume 1;7,30, the silver.
9 Take the reciprocal of 1;7,30, and
10—Rev. 10 10;53,20 will come up. 0;53,20, wh[ich came up], multiply by 9, of your equivalents, and 8 will come up.

Reverse

2–4 8, which came up for you, multiply by 2;30, of [your] equ[ivalents . . .], and 20 will come up. 20, which(?) c[ame up(?)],
5 . . . . . .
6–7 [Tu]rn about(?), [. . . . .] . . . . [. . .] . . . . . .
8–9 square 4;3[0, . . .] . . . . . [. . .] . . . , and 20;15 will come up for you(?).
10 From 20;15, which came up,
11–12 [. . . .] . . . . . [. . . . .] . . . . . .

Lower Edge

1–2 [. . . . .] . . . . . [. . . . .] . . . .

Left Edge

1 [. . . . .] . . . . . in that which I(?) received(?).

Commentary

Unfortunately, the published mathematical texts concerning prices are badly preserved or obscure for other reasons.[276cc] The characteristic term is Akkadian *maḫīrum*, which corresponds to Sumerian ganba. Landsberger[276d] defines it as "die für die Einheit des Geldes eingetauschte Warenmenge", a definition which can be shown to be only partly true.[276e] Thu-

[276cc] For one of them, VAT 7530, see above, p. 18.

[276d] MSL, p. 124.

[276e] In the case of HE 113 (published by Scheil, RA 15 (1918), pp. 184f.; republished by Boyer CHJ, Plate VI and pp. 33ff.), line 6, the phrase ganba *a-na* 1 gín heads a column containing entries which give the quantity of fish of various sorts corresponding to 1 gín of silver. On the other hand, lines 1–3 of HE 111 (published by Scheil, RA 15 (1918), pp. 190f.; republished

reau-Dangin[276f] writes, "Ce terme désigne le rapport inverse de celui que nous appelons le 'cours';" he translates *maḫīrum* by "marché". The few unequivocal passages from Old-Babylonian texts which we have sampled indicate that *maḫīrum* refers to the quantity of goods which could be bought for a unit weight of silver or to the weight of silver for which a unit quantity of goods could be bought. The inherent concept of equivalence is reflected by the non-commercial meaning of *maḫīrum*, "an equal", and *mḫr* in the sense of "to be equal". For the sake of conciseness, we have translated the term by "the equivalent".

In the present text, the problem involves the purchase of 30 gur of barley when the *maḫīrum* went up (which presumably means that the price fell) and another 30 gur when the *maḫīrum* dropped (i.e., when the price rose). The sum of the two *maḫīrū*,[276g] which we take to mean the quantities q_1 and q_2 of barley purchasable for 1 ma-na of silver on each occasion, is given as

$$q_1 + q_2 = 9.$$

In addition, the sum of the silver of the *maḫīrū*, which we assume refers to the weights of silver p_1 and p_2 required to purchase 1 gur of barley on each occasion, is given by

$$p_1 + p_2 = 1;7,30 \text{ ma-na.}$$

A double question is posed: the first apparently calls for the finding of the *maḫīrū*, and the second seems to have something to do with selling.[276h]

We offer the following interpretation with no greater pretension than the feeling that it points in the correct general direction. Let us assume that the average quantity q_0 of goods purchasable for 1 ma-na of silver,

$$(1) \qquad q_0 = \frac{q_1 + q_2}{2} = \frac{9}{2} = 4;30,$$

differs from q_1 and q_2 by a constant quantity ϵ; in other words,

$$q_1 = q_0 + \epsilon \qquad q_2 = q_0 - \epsilon.$$

We can then set up the relation

$$1;7,30 = p_1 + p_2 = \frac{1}{q_0 + \epsilon} + \frac{1}{q_0 - \epsilon} = \frac{q_1 + q_2}{q_0^2 - \epsilon^2}$$

whence

$$(2) \qquad q_0^2 - \epsilon^2 = \frac{q_1 + q_2}{p_1 + p_2} = \frac{9}{1;7,30} = 8,$$

which leads to

$$(3) \qquad \epsilon^2 = q_0^2 - \frac{q_1 + q_2}{p_1 + p_2} = 20;15 - 8 = 12;15$$

and

$$\epsilon = 3;30.$$

Accordingly,

$$q_1 = q_0 + \epsilon = 4;30 + 3;30 = 8$$

$$q_2 = q_0 - \epsilon = 4;30 - 3;30 = 1$$

and

$$p_1 = \frac{1}{q_1} = 0;7,30$$

$$p_2 = \frac{1}{q_2} = 1.$$

Thus, the first 30 gur of barley would have cost $30p_1 = 30 \cdot 0;7,30 = 3;45$ ma-na of silver; and the second 30 gur of barley, $30p_2 = 30 \cdot 1 = 30$ ma-na of silver.

The first calculation ordered by the text itself (obv. 8 to rev. 1) is the computation of (2). Then (rev. 2 to 4) comes the multiplication of the result by 2;30, a step which we do not understand. In the next stage (rev. 6 to 8), we reach (1). The last part of the text which we can follow (rev. 9f.) brings us to the beginning of (3).

by Boyer CHJ, Plate V and pp. 27f.) read: 8 gú sig ..... / ganba 1 gú-e 7½ gín / kù-bi 1 ma-na; in other words, the price of one talent (ganba 1 gú-e) of wool is given as 7½ gín of silver.

Landsberger contrasts *maḫīrum* with the term kar, which he says is "der auf die Einheit der Ware berechnete Preis." This definition, too, is only true in part, as can readily be seen by contrasting two passages in Jean TCL X, No. 17: lines 14f. (6 gú urudu kar-bi 6 ma-na / kù-bi 1 ma-na) and lines 18f. (4 gín guškin kar-bi 9 gín-ta-àm / kù-bi ⅔ ma-na 6 gín). In the former passage, the kar is given as 6 ma-na of copper for 1 gín of silver—but in the latter, the kar is 1 gín of gold for 9 gín of silver. In fact, to judge from the numerous contexts in which kar occurs as an indicator of value, kar almost always indicates the quantity of goods purchasable for a unit weight of silver.

[276f] TMB p. 221 under *maḫīru*.

[276g] Plural of *maḫīrum*.

[276h] As pointed out by Waschow [2], p. 246a, the verb *pšr* occurs in two badly preserved mathematical texts (VAT 6469 and VAT 6546, both published MKT I p. 269) dealing with purchases. Waschow translates *pšr* by "einlösen, für Geld weggeben."

T. A 24194

(Photograph: Plate 41; copy: Plate 15)

Transcription

Obverse I

1 [1][a-šà 1(eše)[iku]]
[2]a-n[a(?) .....]
[3]igi-5(?)[-gál-bi ....]
[4]20 daḫ [.....]
[5]igi-7[-gál igi-11-gál]
[6]uš-še daḫ-ma [32]

2 [7]a-rá 2 e-tab daḫ 34

3 [8]ba-zi-ma 28

4 [9]a-rá 2 e-tab zi 26

5 [10]a-rá 15 e-tab-ma uš

6 [11]a-rá 20 e-tab 10 dirig

7 [12]a-rá 10 e-tab-ma 10 ba-lá

8 [13]sag-šè daḫ 22

9 [14]a-rá 2 e<-tab> daḫ 24

10 [15]zi-ma 18

11 [16]a-rá 2 e<-tab> zi-m[a 16]

12 [17]a-rá 10 e-tab sag

13 [18]a-rá [1]5 e<-tab> 10 dirig

14 [19]uš-sag-šè daḫ 52

15 [20]a-rá 2 e<-tab> daḫ 54

16 [21]zi 48

17 [22]a-rá 2 e<-tab> zi 46

18 [23]a-rá 25 e<-tab> uš sag

19 [24]a-rá 30 e-tab 10 dirig

20 [25]a-rá 20 e<-tab> 10 ba-lá

21 [26]a-rá 3 uš a-rá 2 sag
[27]daḫ 2,12

22 [28]a-rá 2 e-tab daḫ 2,14

23 [29]zi-ma 2,8

24 [30]a-rá 2 e<-tab> zi 2,6

25 [31][a-rá] 1,5 e<-tab> uš sag

26 [32]a-rá 1 e<-tab> 10 ba-lá

27 [33][a-]rá 1,10 e<-tab> 10 dirig

Obverse II

28 [1][uš-šè ù a-na uš]
[2][ugu sag dirig daḫ 4]2

29 [3][a-rá] 2 e<-tab> [daḫ 4]4

30 [4][zi 38]

31 [5][a-rá 2 e-tab zi 3]6

32 [6][a-rá 20 e-tab uš s]ag

33 [7][a-rá 25 e-tab 10 di]rig

34 [8][a-rá] 1[5 e-tab 10 b]a-lá

35 [9]u[š sag ù a-rá 2]
[10]a-n[a uš ugu sag]
[11][dirig daḫ-ma 1,12]

36 [12][a-rá 2 e-tab daḫ 1,14]

37 [13]zi-ma 1[,8]

38 [14]a-rá 2 e<-tab> zi 1,6

39 [15]a[-rá 3]5 e<-tab> uš [sag]

40 [16]a[-rá 4]0 e<-tab> 10 dir[ig]

41 [17]a-rá 30 e-tab 10 b[a-lá]

42 [18]a-šà 1(eše)[iku]
[19]igi-14-gál u[š sa]g
[20]ù a-rá 2 [a-na uš ug]u
[21]sag dirig [2,29 daḫ]
[22]igi-7-gál igi-11-gál
[23]uš-šè daḫ 32

43 [24]a-rá 2 e<-tab> daḫ 34

44 [25]zi 28

45 [26]a-rá 2 e<-tab> zi [26]

46 [27]a-rá [1]5 [e<-tab> uš]

47 [28]a-rá 20 e<-tab> 10 dirig

48 [29]a-rá 10 e<-tab> 10 ba-lá

49 [30]sag-šè daḫ 22

50 [31]a-rá 2 e<-tab> daḫ 24

51 [32]zi 18

52 [33]a-rá 2 e<-tab> zi 16

53 [34]a-rá 10 e-tab sag

54 [35]a-šà 1(eše)[iku]

Obverse III

[1][. . . . .]
[2][. . . . .]
[3][. . . . .]
[4][. . . . .]
[5][. . . . .]
[6][uš-šè daḫ] 33

55 [7]a[-rá 2 e-tab daḫ 3]6

56 [8]zi 27

57 [9][a-rá] 2 e<-tab> zi 24

58 [10]a-rá 10 e<-tab> uš

59 [11]a-rá 15 e<-tab> 15 dirig

60 [12]sag daḫ 23

61 [13][a-rá 2] e<-tab> daḫ 2[6]

62 [14]zi 17

63 [15]a-rá 2 e<-tab> zi 14

64 [16]a-rá 1[0 e<-tab>] 10 dirig

65 [17]u[š-sag-šè daḫ] 53

66 [18][a-r]á 2 e<-tab> daḫ 56

67 [19][zi] 47

68 [20]a-rá 2 e<-tab> z[i 4]4

69 [21]a-rá 20 e<-tab> 10 dirig

70 [22]a-rá 15 e<-tab> 5 ba-lá

71 [23]a-rá 3 uš a-rá 2 sag
[24]daḫ-ma 2,13

72 [25]a-rá 2 e<-tab> daḫ 2,16

73 [26]zi 2,7

74 [27]a-rá 2 e<-tab> zi 2,4

75 [28]a-rá 50 e<-tab> 20 dirig

76 [29]a-rá 40 e<-tab> 10 ba-lá

77 [30]uš-šè ù a-na uš ugu
[31]sag dirig daḫ 43

78 [32]a-rá 2 e<-tab> daḫ 4[6]

79 [33]zi 37

80 [34]a-rá 2 e<-tab> zi 34

Obverse IV

81 [1][a-šà 1(ešè)[iku]]
[2][. . . . . .]
[3][. . . . . .]
[4][uš-šè daḫ 32]

82 [5]a-rá [2 e-tab daḫ 34]

83 [6][zi] 2[8]

84 [7][a-rá 2 e-tab] zi 26

85 [8]a-r[á 15] e-tab uš

86 [9]a[-rá 2]0 e-tab 10 dirig

87 [10]a-rá 10 e-tab 10 ba-lá

88 [11][sa]g daḫ 22

89 [12]a-rá [2 e<-tab> da]ḫ 24

90 [13]zi 18

91 [14]a-rá 2 e<-tab> zi 16

92 [15]a-rá 10 e-tab sag

93 [16]a-rá 15 e<-tab> 10 di[rig]

94 [17]a-rá 5 [e-tab 10 ba-lá]

95 [18]uš-sag[-šè daḫ 52]

96 [19]a-rá [2 e-tab daḫ 54]

97 [20]zi [4]8

98 [21]a-rá [2 e-tab zi 4]6

99 [22]a-rá 2[5 e-tab u]š sag

100 [23]a-rá 30 e<-tab> 10 dirig

101 [24]a-rá 20 e<-tab> 10 ba-lá

102 [25]uš-šè ù [a-]na uš ugu sag <dirig>
[26]daḫ[-ma] 42

103 [27][a-rá 2 e-tab daḫ] 44

104 [28][zi 3]8

105 [29]a[-rá 2 e-tab zi] 36

106 [30][a-rá 20] e<-tab> uš

107 [31]a[-rá 25] e<-tab> 10 [di]rig

108 [32]a-rá 15 [e-tab 10 ba-]lá

Obverse V

109 [1]a-rá 3 uš a[-rá 2 s]ag
[2]daḫ 2,12

110 [3][a-]rá 2 e<-tab> daḫ 2,14

111 [4]zi 2,8

112 [5]a-rá 2 e<-tab> zi 2,6

113 [6]i[gi-3-gá]l uš igi-4-gál sag
[7][daḫ-]ma 17

114 [8]a-rá 2 e<-tab> daḫ 2,19[277]

115 [9]zi 2,13[277]

116 [10]a-rá 2 e<-tab> zi 2,11[277]

117 [11]a-rá 10 e-tab 5 dirig

118 [12]a-rá [5 e-tab] 5 ba-lá

119 [13]a-šà 1(ešè)[iku]
[14]šu-ri-a [uš . . . . . .]
[15]. . . . . [. . . . . igi-]6-gál
[16]ig[i . . . . .]. . . 8 . . . zi
[17]ig[i . . . . . u]š-šè daḫ 32

[277] The 2 should have been omitted. This error seems to be due to contamination by the numbers in Nos. 109–112. In No. 113 there are traces of a correction before 17.

120 [18][a-rá 2 e<-tab> daḫ 3]4

121 [19]z[i] 28

122 [20][a-rá 2 e-tab z]i 26

123 [21][a-rá 15 e-ta]b uš

124 [22]a-rá 20 e<-tab> 10 dirig

125 [23][a-]rá 10 e<-tab> 10 [ba-]lá

126 [24]sag-šè daḫ 22

127 [25]a-rá 2 [e<-tab> daḫ] 24

128 [26]z[i 18]

129 [27]a-r[á 2 e<-tab> zi 16]

130 [28]a-rá 10 e<-tab> [sa]g

131 [29]a-rá [15 e<-tab>] 10 [dirig]

132 [30][uš] sag [daḫ 5]2

133 [31]a-rá [2 e-tab] daḫ 54

134 [32]zi 48

135 [33]a-rá 2 e<-tab> zi 46

136 [34][a-rá 25 e-tab] uš sag

Reverse I

137 [1]a-rá 20 e<-tab> 10 ba-lá

138 [2]a-rá 30 e-tab 10 dirig

139 [3]a-šà 1(ešè)[iku]
[4]šu-ri-a uš igi-3-gál zi
[5]25 daḫ igi-7-gál ù .....[278]
[6]igi-11-gál-bi uš daḫ 32 ...[278]

140 [7]a-rá 2 e<-tab> daḫ 34

141 [8]zi 28

142 [9]a-rá 2 e<-tab> zi 25[279]

143 [10]a-rá 15 e-tab uš

144 [11]a-rá 20 e<-tab> 10 dirig

145 [12]a-rá 10 e<-tab> 10 ba-lá

146 [13]sag daḫ 22

147 [14]a-rá 2 e<-tab> daḫ 24

148 [15]zi 18

149 [16]a-rá 2 e<-tab> zi 16

150 [17]a-rá 10 e<-tab> sag

151 [18]a-rá 15 e<-tab> 10 dirig

152 [19]uš sag daḫ 52

153 [20]a-rá 2 e<-tab> daḫ 54

154 [21]zi 48

155 [22]a-rá 2 e<-tab> zi 46

156 [23]a-rá 25 e-tab uš sag

157 [24]a-rá 30 e-tab 10 dirig

158 [25]a-rá 20 e<-tab> 10 ba-lá

159 [26]a-rá 3 uš a-rá 2 sag
[27]daḫ-ma 2,12

160 [28]a-rá 2 e-tab daḫ 2,14

161 [29]zi 2,8

162 [30]a-rá 2 e<-tab> zi 2,6

163 [31]a-rá 1,5 e<-tab> uš sag

164 [32]a-rá 1,10 e<-tab> 10 dirig

165 [33]a-rá 1 e-tab 10 ba-lá

166 [34]uš-šè ù a-na uš [u]gu sag dirig
[35]daḫ-ma 42

167 [36]a-rá 2 e<-tab> daḫ 44

Reverse II

168 [1]zi [38]

169 [2]a-rá 2 e<-tab> zi 36

170 [3]a-rá 20 e<-tab> uš

171 [4]a-rá 25 e<-tab> 10 dirig

172 [5]a-rá 15 e<-tab> 10 ba-lá

173 [6]..... ù a-rá 2 .....[280]
[7]a-na uš ugu sag dirig
[8]daḫ 1,12

174 [9]a-rá 2 e<-tab> daḫ 1,14

175 [10]zi 1,8

176 [11]a-rá 2 e<-tab> zi 1,6

177 [12]a-rá 35 e<-tab> uš sag

178 [13]a-rá 40 e<-tab> 10 dirig

179 [14]a[-rá 3]0 e<-tab> 10 ba-lá

[278] The signs at the end of line 6 must be the continuation of the end of line 5 because the problem ends with 32.
[279] Error for 26.
[280] The traces do not conform to the expected uš sag ù a-rá 2.

180 [15]igi-3-gál uš [igi-]4-gál sag[281]
[16][a-na uš ugu] sag dirig
[17]daḫ 27

181 [18]a-rá 2 e<-tab> daḫ 29

182 [19]zi 23

183 [20]a-rá 2 e<-tab> zi 21

184 [21]a-rá 15 e<-tab> 5 dirig

185 [22][a-rá] 10 e-tab 5 ba-lá

186 [23]uš sag igi-3-gál [uš]
[24]igi-4-gál sag daḫ 1,7

187 [25]a-rá 2 e<-tab> daḫ 1,9

188 [26]zi 1,3

189 [27]a-rá 2 e<-tab> zi 1,1

190 [28]a-rá [3]5 e<-tab> 5 dirig

191 [29][a-na uš ugu] sag dirig
[30][daḫ-ma] 12

192 [31][a-r]á 2 e<-tab> daḫ 14

193 [32][zi] 8

194 [33]a-rá 2 e<-tab> zi 6

195 [34]a-rá 5 e-tab
[35]a-na uš ugu sag dirig

Reverse III

196 [1]a-rá 8 e<-tab> 6 dirig

197 [2]a[-šà] 1(ešè)[iku]
[3]uš ù a-na uš ugu
[4]sag d[irig] ù
[5]igi-4-gál ù 6 d[aḫ]
[6]igi-8-gál-bi 15 daḫ
[7]igi-11-gál-bi uš daḫ
[8]i[gi-]8-gál-bi
[9][uš-šè] da[ḫ 3]4

198 [10]a-rá 2 e<-tab> daḫ 38

199 [11]zi 26

200 [12]a-rá 2 e<-tab> zi 22

201 [13]a-rá 7,30 e<-tab> uš

202 [14]a-rá 10 e<-tab> 10 dirig

203 [15]a-rá 5 e<-tab> 10 ba-lá

204 [16]sag daḫ [24]

205 [17]a-rá 2 e<-tab> d[aḫ 28]

206 [18]zi 16

207 [19]a-r[á 2 e<-tab> z]i 12

208 [20]a[-rá 5 e<-tab>] sag

209 [21]a[-rá] 6 e<-tab> 4 dirig

210 [22][uš sag da]ḫ 54

211 [23]a-rá 2 e<-tab> daḫ [58]

212 [24]zi 46

213 [25]a-rá 2 e<-tab> zi 42

214 [26][a-rá 12],30 e<-tab> uš sag

215 [27][a-rá] 15 e<-tab> 10 dirig

216 [28][a-rá] 10 e<-tab> 10 ba-lá

217 [29][uš-šè ù a-na uš]
[30][ugu sag dirig daḫ] 44

218 [31]a[-rá 2 e<-tab> da]ḫ 48

219 [32]zi 36

220 [33]a-rá 2 e<-tab> zi 32

221 [34]a[-rá 10 e-tab] uš

222 [35]a-rá 12 e<-tab> 8 dirig

Reverse IV

223 [1]a-rá 8 e-tab
[2][8] ba-lá

224 [3]uš-šè ù igi-3[-gál u]š
[4]sag[-šè ù] igi-4-gál sag
[5]daḫ-ma 1,9

225 [6][a-rá] 2 e<-tab> da[ḫ] 1,13

226 [7]zi-ma 1,1

227 [8]a-rá 2 e-tab zi
[9]57

228 [10]a-rá 20 e<-tab> 1[5 di]rig

229 [11]a-rá 15 e<-tab> 5 ba-lá

230 [12][igi-3-g]ál u[š igi-4-g]ál sag
[13][ù a-rá] 3 a-na [u]š
[14]ugu sag dirig
[15]daḫ-ma 49

231 [16]a-rá 3[282] e<-tab> daḫ 53

232 [17]zi-ma 41

[281] The sign sag is written below the line.

[282] Mistake for 2.

233 [18]a-rá 2 e<-tab> zi [37]

234 [19]a-rá 10 e<-tab> 5 b[a-lá]

235 [20]a-rá 15 e<-tab> 1[5 dirig]

236 [21]a-rá 3 uš a-rá 2 [sag]
[22]daḫ-ma 2,14

237 [23]a-rá 2 e<-tab> d[aḫ] 2,18

238 [24]zi 2,6

239 [25]a-rá 2 e<-tab> zi 2,2

240 [26]a-rá 30 e<-tab> 10 ba-lá

241 [27]a-rá 40 e<-tab> 30 dirig

242 [28]a-.....
[29]a-rá(?) .....
[30]..... daḫ [2,54]

243 [31]a-rá 2 e-tab
[32]da[ḫ 2,58]

Reverse V

244 [1][zi-ma] 2,46

245 [2][a-r]á 2 e-tab
[3]zi 2,42

246 [4]a-rá 40 e<-tab> 10 ba-lá

247 [5]a-rá 45 e<-tab> 10 dirig

[6]4 šu-ši
[7]im-šu
[8]dub-10-kam-ma

Translation

Obverse I

1 [1][Area, 1 ešè.]
[2]That by wh[ich(?) .....];
[3]one-fif[th(?) thereof .....]
[4]I added 20 [.....]
[5]one-seven[th of one-eleventh]
[6]I added to the length: [32].

2 [7]I multiplied by 2, I added: 34.

3 [8]I subtracted: 28.

4 [9]I multiplied by 2, I subtracted: 26.

5 [10]I multiplied by 15: the length.

6 [11]I multiplied by 20: it exceeded by 10.

7 [12]I multiplied by 10: it was less by 10.

8 [13]I added to the width: 22.

9 [14]I multiplied by 2, I added: 24.

10 [15]I subtracted: 18.

11 [16]I multiplied by 2, I subtracted: [16].

12 [17]I multiplied by 10: the width.

13 [18]I multiplied by [1]5: it exceeded by 10.

14 [19]I added to the length (and) the width: 52.

15 [20]I multiplied by 2, I added: 54.

16 [21]I subtracted: 48.

17 [22]I multiplied by 2, I subtracted: 46.

18 [23]I multiplied by 25: the length (and) the width.

19 [24]I multiplied by 30: it exceeded by 10.

20 [25]I multiplied by 20: it was less by 10.

21 [26–27]I added to 3 times the length (and) 2 times the width: 2,12.

22 [28]I multiplied by 2, I added: 2,14.

23 [29]I subtracted: 2,8.

24 [30]I multiplied by 2, I subtracted: 2,6.

25 [31]I multiplied [by] 1,5: (thrice) the length (and twice) the width.

26 [32]I multiplied by 1,0: it was less by 10.

27 [33]I multiplied by 1,10: it exceeded by 10.

Obverse II

28 [1–2][I added to the length and that by which the length exceeded the width: 4]2.

29 [3]I multiplied [by] 2, [I added: 4]4.

30 [4][I subtracted: 38].

31 [5][I multiplied by 2, I subtracted: 3]6.

32 [6][I multiplied by 20: the length (and that by which the length exceeded) the wi]dth.

33 [7][I multiplied by 25: it ex]ceeded [by 10].

34 [8][I multiplied by] 1[5: it] was less [by 10].

35 [9–11][I added to the] le[ngth, the width, and 2 times] that by whi[ch the length exceeded the width: 1,12].

36 [12][I multiplied by 2, I added: 1,14].

37 [13]I subtracted: 1,[8].

38 [14]I multiplied by 2, I subtracted: 1,6.

39 [15]I multiplied [by 3]5: the length, [the width, (and 2 times that by which the length exceeded the width)].

40 [16]I multiplied [by 4]0: it exc[eed]ed by 10.

41 [17]I multiplied by 30: it w[as less] by 10.

42 [18]Area, 1 ešè.
[19]One-fourteenth of the len[gth and the wid]th,
[20–21]and 2 times [that by which the length] exceeded the width; [I added 2,29];
[22]one-seventh of one-eleventh (thereof)
[23]I added to the length: 32.

43 [24]I multiplied by 2, I added: 34.

44 [25]I subtracted: 28.

45 [26]I multiplied by 2, I subtracted: [26].

46 [27][I multiplied] by [1]5: [the length].

47 [28]I multiplied by 20: it exceeded by 10.

48 [29]I multiplied by 10: it was less by 10.

49 [30]I added to the width: 22.

50 [31]I multiplied by 2, I added: 24.

51 [32]I subtracted: 18.

52 [33]I multiplied by 2, I subtracted: 16.

53 [34]I multiplied by 10: the width.

54 [35]Area, 1 ešè.

Obverse III

[1–5] . . . . . .
[6][I added to the length]: 33.

55 [7][I multiplied] b[y 2, I added: 3]6.

56 [8]I subtracted: 27.

57 [9]I multiplied [by] 2, I subtracted: 24.

58 [10]I multiplied by 10: the length.

59 [11]I multiplied by 15: it exceeded by 15.

60 [12]I added to the width: 23.

61 [13]I multiplied [by 2], I added: 2[6].

62 [14]I subtracted: 17.

63 [15]I multiplied by 2, I subtracted: 14.

64 [16][I multiplied] by 1[0]: it exceeded by 10.

65 [17][I added to the len]gth [(and) the width]: 53.

66 [18]I multiplied [by] 2, I added: 56.

67 [19][I subtracted]: 47.

68 [20]I multiplied by 2, I sub[tracted: 4]4.

69 [21]I multiplied by 20: it exceeded by 10.

70 [22]I multiplied by 15: it was less by 5.

71 [23–24]I added to 3 times the length (and) 2 times the width: 2,13.

72 [25]I multiplied by 2, I added: 2,16.

73 [26]I subtracted: 2,7.

74 [27]I multiplied by 2, I subtracted: 2,4.

75 [28]I multiplied by 50: it exceeded by 20.

76 [29]I multiplied by 40: it was less by 10.

77 [30–31]I added to the length and that by which the length exceeded the width: 43.

78 [32]I multiplied by 2, I added: 4[6].

79 [33]I subtracted: 37.

80 [34]I multiplied by 2, I subtracted: 34.

Obverse IV

81 [1][Area, 1 ešè]
[2–3][. . . . . .]
[4][I added to the length: 32].

82 [5][I multiplied] by [2, I added: 34].

83 [6][I subtracted]: 2[8].

84 [7][I multiplied by 2], I subtracted: 26.

85 [8]I multiplied b[y 15]: the length.

86 [9]I multiplied b[y 2]0: it exceeded by 10.

87 [10]I multiplied by 10: it was less by 10.

88 [11]I added to the [wid]th: 22.

89 [12][I multiplied] by [2, I add]ed: 24.

90 [13]I subtracted: 18.

91 [14]I multiplied by 2, I subtracted: 16.

92 [15]I multiplied by 10: the width.

93 [16]I multiplied by 15: it ex[ceeded] by 10.

94 [17][I multiplied] by 5: [it was less by 10].

95 [18][I added to] the length (and) the width: [52].

96 [19][I multiplied] by [2, I added: 54].

97 [20]I subtracted: [4]8.

98 [21][I multiplied] by [2, I subtracted: 4]6.

99 [22]I multiplied by 2[5: the len]gth (and) the width.

100 [23]I multiplied by 30: it exceeded by 10.

101 [24]I multiplied by 20: it was less by 10.

102 [25–26]I added to the length and that by which the length <exceeded> the width: 42.

103 [27][I multiplied by 2, I added]: 44.

104 [28][I subtracted: 3]8.

105 [29][I multiplied by 2, I subtracted]: 36.

106 [30]I multiplied b[y 20]: the length.

107 [31]I multiplied b[y 25]: [it exc]eeded by 10.

108 [32][I multiplied] by 15: [it was] less [by 10].

Obverse V

109 [1–2]I added to 3 times the length (and) [2] times the width: 2,12.

110 [3]I multiplied by 2, I added: 2,14.

111 [4]I subtracted: 2,8.

112 [5]I multiplied by 2, I subtracted: 2,6.

113 [6–7][I added to] one[-thi]rd of the length (and) one-fourth of the width: 17.

114 [8]I multiplied by 2, I added: 2,19.[277]

115 [9]I subtracted: 2,13.[277]

116 [10]I multiplied by 2, I subtracted: 2,11.[277]

117 [11]I multiplied by 10: it exceeded by 5.

118 [12][I multiplied] by [5]: it was less by 5.

119 [13]Area, 1 ešè.
[14]One-half of [the length .....]
[15][..... one-]sixth
[16]one[-.....]...8... I subtracted
[17]one[-.....] I added to the [len]gth: 32.

120 [18][I multiplied by 2, I added: 3]4.

121 [19]I sub[tracted]: 28.

122 [20][I multiplied by 2, I sub]tracted: 26.

123 [21][I multipl]ied [by 15]: the length.

124 [22]I multiplied by 20: it exceeded by 10.

125 [23]I multiplied by 10: it was [less] by 10.

126 [24]I added to the width: 22.

127 [25][I multiplied] by 2, [I added]: 24.

128 [26]I sub[tracted: 18].

129 [27][I multiplied b]y [2, I subtracted: 16].

130 [28]I multiplied by 10: the [wid]th.

131 [29][I multiplied] by [15: it exceeded by] 10.

132 [30][I added to the length (and)] the width: [5]2.

133 [31][I multiplied] by [2], I added: 54.

134 [32]I subtracted: 48.

135 [33]I multiplied by 2, I subtracted: 46.

136 [34][I multiplied by 25]: the length (and) the width.

Reverse I

137 [1]I multiplied by 20: it was less by 10.

138 [2]I multiplied by 30: it exceeded by 10.

139 [3]Area, 1 ešè.
[4]I subtracted one-third from one-half the length,
[5]I added 25, one-seventh (thereof) and .....;
[6]I added one-eleventh thereof to the length: 32.

140 [7]I multiplied by 2, I added: 34.

141 [8]I subtracted: 28.

142 [9]I multiplied by 2, I subtracted: 25.[279]

143 [10]I multiplied by 15: the length.

144 [11]I multiplied by 20: it exceeded by 10.

145 [12]I multiplied by 10: it was less by 10.

146 [13]I added to the width: 22.

147 [14]I multiplied by 2, I added: 24.

148 [15]I subtracted: 18.

149 [16]I multiplied by 2, I subtracted: 16.

150 [17]I multiplied by 10: the width.

151 [18]I multiplied by 15: it exceeded by 10.

152 [19]I added to the length and the width: 52.

153 [20]I multiplied by 2, I added: 54.

154 [21]I subtracted: 48.

155 [22]I multiplied by 2, I subtracted: 46.

156 [23]I multiplied by 25: the length (and) the width.

157 [24]I multiplied by 30: it exceeded by 10.

158 [25]I multiplied by 20: it was less by 10.

159 [26–27]I added to 3 times the length (and) 2 times the width: 2,12.

160 [28]I multiplied by 2, [added: 2,14.

161 [29]I subtracted: 2,8.

162 [30]I multiplied by 2, I subtracted: 2,6.

163 [31]I multiplied by 1,5: (thrice) the length (and twice) the width.

164 [32]I multiplied by 1,10: it exceeded by 10.

165 [33]I multiplied by 1,0: it was less by 10.

166 [34–35]I added to the length and that by which the length [ex]ceeded the width: 42.

167 [36]I multiplied by 2, I added: 44.

Reverse II

168 [1]I subtracted: [38].

169 [2]I multiplied by 2, I subtracted: 36.

170 [3]I multiplied by 20: the length.

171 [4]I multiplied by 25: it exceeded by 10.

172 [5]I multiplied by 15: it was less by 10.

173 [6–8]I added to ..... and 2 times that by which the length exceeded the width: 1,12.

174 [9]I multiplied by 2, I added: 1,14.

175 [10]I subtracted: 1,8.

176 [11]I multiplied by 2, I subtracted: 1,6.

177 [12]I multiplied by 35: the length, the width, (and twice that by which the length exceeded the width).

178 [13]I multiplied by 40: it exceeded by 10.

179 [14]I multiplied b[y 3]0: it was less by 10.

180 [15–17]I added to one-third of the length, [one-] fourth of the width, (and) [that by which the length] exceeded the width: 27.

181 [18]I multiplied by 2, I added: 29.

182 [19]I subtracted: 23.

183 [20]I multiplied by 2, I subtracted: 21.

184 [21]I multiplied by 15: it exceeded by 5.

185 [22]I multiplied [by] 10: it was less by 5.

186 [23–24]I added to the length, the width, one-third of the [length], (and) one-fourth of the width: 1,7.

187 [25]I multiplied by 2, I added: 1,9.

188 [26]I subtracted: 1,3.

189 [27]I multiplied by 2, I subtracted: 1,1.

190 [28]I multiplied by [3]5: it exceeded by 5.

191 [29–30][I added to that by which the length] exceeded the width: 12.

192 [31]I multiplied [b]y 2, I added: 14.

193 [32][I subtracted]: 8.

194 [33]I multiplied by 2, I subtracted: 6.

195 [34]I multiplied by 5:
[35]that by which the length exceeded the width.

Reverse III

196 [1]I multiplied by 8: it exceeded by 6.

197 [2]Ar[ea], 1 ešè.
[3]The length and that by which the length
[4][exce]eded the width and
[5]one-fourth (thereof) and 6 I added together;
[6]I added 15 to one-eighth thereof;
[7]to one-eleventh thereof I added the length;
[8]one-eighth thereof
[9]I ad[ded to the length: 3]4.

198 [10]I multiplied by 2, I added: 38.

199 [11]I subtracted: 26.

200 [12]I multiplied by 2, I subtracted: 22.

201 [13]I multiplied by 7;30: the length.

202 [14]I multiplied by 10: it exceeded by 10.

203 [15]I multiplied by 5: it was less by 10.

204 [16]I added to the width: [24].

205 [17]I multiplied by 2, I a[dded: 28].

206 [18]I subtracted: 16.

207 [19][I multiplied] b[y 2, I subtra]cted: 12.

208 [20][I multiplied] b[y 5]: the width.

209 [21]I multiplied b[y] 6: it exceeded by 4.

210 [22][I ad]ded [to the length (and) the width]: 54.

211 [23]I multiplied by 2, I added: [58].

212 [24]I subtracted: 46.

213 [25]I multiplied by 2, I subtracted: 42.

214 [26]I multiplied [by 12;]30: the length (and) the width.

215 [27]I multiplied [by] 15: it exceeded by 10.

216 [28]I multiplied [by] 10: it was less by 10.

217 [29–30][I added to the length and that by which the length exceeded the width]: 44.

218 [31][I multiplied] b[y 2, I ad]ded: 48.

219 [32]I subtracted: 36.

220 [33]I multiplied by 2, I subtracted: 32.

221 [34][I multiplied] b[y 10]: the length (and that by which the length exceeded the width).

222 [35]I multiplied by 12: it exceeded by 8.

Reverse IV

223 [1]I multiplied by 8: it was less by [8].

224 [3–5]I added to the length and one-th[ird of the len]gth, (and) [to] the width [and] one-fourth of the width: 1,9.

225 [6]I multiplied [by] 2, I added: 1,13.

226 [7]I subtracted: 1,1.

227 [8]I multiplied by 2, I subtracted: [9]57.

228 [10]I multiplied by 20: it [exc]eeded by 1[5].

229 [11]I multiplied by 15: it was less by 5.

230 [12–15]I added to [one-third] of the len[gth, one-four]th of the width, [and] 3 [times] that by which the length exceeded the width: 49.

231 [16]I multiplied by 3,[282] I added: 53.

232 [17]I subtracted: 41.

233 [18]I multiplied by 2, I subtracted: [37].

234 [19]I multiplied by 10: it was [less] by 5.

235 [20]I multiplied by 15: [it exceeded by] 1[5].

236 [21–22]I added to 3 times the length (and) 2 times [the width]: 2,14.

237 [23]I multiplied by 2, I ad[ded]: 2,18.

238 [24]I subtracted: 2,6.

239 [25]I multiplied by 2, I subtracted: 2,2.

240 [26]I multiplied by 30: it was less by 10.

241 [27]I multiplied by 40: it exceeded by 30.

242 [28–30]I added to ..... times(?) ....: [2,54].

243 [31]I multiplied by 2,
[32]I ad[ded: 2,58].

Reverse V

244 [1][I subtracted]: 2,46.

245 [2]I multiplied [b]y 2,
[3]I subtracted: 2,42.

246 [4]I multiplied by 40: it was less by 10.

247 [5]I multiplied by 45: it exceeded by 10.

[6]4 sixties (i.e., 4,0)
[7]sections;
[8]10th tablet.

Commentary

a. General Commentary

This text, like the following one, can best be compared to an extensive collection of problems from a chapter of a textbook. It is obvious that a collection of this sort was used in teaching mathematical methods. They constitute a large reservoir of problems from which individual problems of any required type (say, speaking from a modern point of view, of a certain category of quadratic equations) could be selected.[283] This explains the schematic arrangement of the examples and made possible the employment of a terse style which, if isolated, would be extremely ambiguous.

The tablets are written with very small characters; this permitted the scribe to include 200 or more examples on a single tablet measuring about 3 by 4 inches. The difficulty in reading such small writing is considerably increased by the state of preservation, which is far from good. As a matter of fact, it is frequently only the systematic structure of the contents which makes it possible to interpret faint traces of signs.

We now turn to the explanation of the contents and the arrangement of the examples in our text, all of which deal with quadratic equations for two unknown quantities: "length" (uš, henceforth called x) and "width" (sag, denoted by y). It is necessary to distinguish between two types of examples: "main examples" and "variants." A characteristic case of a main example is the following:[284] the first condition for the unknown quantities is that the "area" (a-šà) is 1 ešè = 10,0 GAR², i.e.,

$$xy = 10,0. \tag{1}$$

[283] For a classification of Babylonian quadratic equations, see Gandz [1].
[284] YBC 4711 (MKT III pp. 45ff.).

Then a function $f(x,y)$ is defined by the expression

(2a) $f(x,y) = \frac{1}{11}\{\frac{1}{13}[(x + 4y) + 2(x - y)] + 45\}$

and it is furthermore stated that

(2b) $x + f(x,y) = 35.$

It is evident that (1) and (2) lead to a quadratic equation for x and y.

The "variants" assume that (1) and (2a) are kept unchanged while (2b) is replaced by one of the following equations:

$$
\begin{aligned}
x + 2f(x,y) &= 40\\
x - f(x,y) &= 25\\
x - 2f(x,y) &= 20\\
x &= 6f(x,y)\\
x &= 8f(x,y) - 10\\
y + f(x,y) &= 25\\
y + 2f(x,y) &= 30\\
y - f(x,y) &= 15\\
&\text{etc.}\\
(x + y) + f(x,y) &= 55\\
(x + y) + 2f(x,y) &= 1{,}0\\
&\text{etc.}\\
(x - y) + f(x,y) &= 15\\
&\text{etc.}
\end{aligned}
\qquad \text{(2c)}
$$

This list of variants shows the principle of arrangement: after a certain $f(x,y)$ is defined, it is added or subtracted with a factor 1 or 2 to or from x, y, $x + y$, $x - y$, etc.

A further observation can easily be made. If we compute x and y from (1), (2a) and (2b), or any of the equations (2c), we will always obtain

(3) $x = 30 \qquad y = 20.$

This statement can be immediately confirmed from our equations: it is evident that (1) is satisfied by (3); it follows from (2a) that $f(x,y)$ has the value 5 if we use (3); finally, it is again trivial that $x = 30 \quad y = 20$ and $f(x,y) = 5$ satisfy all equations (2b) or (2c).

The conditions (1) and (2) mentioned above are of course only typical examples of a main problem and some of its variants. The fact remains, however, that the same set of values for x and y which is given by (3) satisfies all the problems of the two tablets discussed here as well as many tablets published previously. This clearly illustrates the procedure followed in composing these collections of problems. In order to guarantee real and positive solutions of the quadratic equations, one started from simple solutions like (3) and constructed the equations accordingly. All variants follow the same principle and their purpose lies exclusively in the teaching of methods to reduce problems of types (1) and (2) to the "normal" form of quadratic equations which gives the product of the unknown quantities and their sum or difference. The factor ± 2 in the variants is of course chosen only to indicate any coefficient other than 1 in the previous variants, since a coefficient 1 can be overlooked in the numerical computation. The whole system of a main example with all its variants serves the purpose of giving the general rule of solution; this corresponds in a certain sense to an algebraic formula in which the letters can be replaced by special numbers in each individual case.

b. Mathematical Commentary

A 24194 contains 7 main examples and 240 variants. In 5 of the main examples (Nos. 1, 42, 81, 119, 139), the function $f(x,y)$ has the value 2 if we give x and y the values 30 and 20, respectively. The value 3 occurs in No. 54, the value 4 in No. 197. Unfortunately, the state of preservation of the text is such that only three of the main examples can be read or restored with some degree of probability. These are:

No. 42. $xy = 10{,}0 \qquad x + f(x,y) = 32,$

where $f(x,y)$ is an abbreviation for

$$\tfrac{1}{7} \cdot \tfrac{1}{11}\{\tfrac{1}{11}(x + y) + 2(x - y) + 2{,}29\} = f(x,y).$$

The number 2,29 is, to be sure, restored; but this is necessary if the other readings are correct.

No. 139. The only restoration of this example which seems to conform to the preserved portion of the text is the following:

$$xy = 10{,}0 \qquad x + f(x,y) = 32,$$

where

$$f(x,y) = \frac{1}{11}\left\{\frac{1}{7}\left(\frac{x}{2} - \frac{1}{3}\frac{x}{2} + 25\right) + 17\right\}.$$

The number 17 which is added at the end is necessary to yield the correct result, but we do not see how this element of the formula was expressed in the text because we are unable to read the traces at the end of line 5.

No. 197. This is the only main example which is almost completely preserved in the text. We have

$$xy = 10{,}0 \qquad x + f(x,y) = 34,$$

where

$$f(x,y) = \tfrac{1}{8}\{\tfrac{1}{11}[\tfrac{1}{8}[x + (x - y) + \tfrac{1}{4}(x + (x - y)) + 6] + 15] + x\}$$

with the value $f(30,20) = 4$.

The Variants. The restoration of the complete system of variants involves no serious difficulty because of their schematic arrangement. In order to reproduce the contents in roughly the same form as

in the text, we shall adopt an abbreviated notation by omitting such parts of an equation which remain unchanged from the previous example. Thus, we write

$$\begin{array}{l} x + f(x,y) = 32 \\ \quad + 2f(x,y) = 34 \end{array} \quad \text{for} \quad \begin{array}{l} x + f(x,y) = 32 \\ x + 2f(x,y) = 34 \end{array}$$

etc. Analogously, we write

$$\begin{array}{l} 15f(x,y) = x \\ 20f(x,y) = \quad + 20 \end{array} \quad \text{for} \quad \begin{array}{l} 15f(x,y) = x \\ 20f(x,y) = x + 20. \end{array}$$

This corresponds closely to the expression of the text. We shall, moreover, frequently write f for $f(x,y)$ without distinguishing different f's belonging to different main examples. The f in each group of variants following a main example refers, of course, to the same function.

Adopting this notation, we can now give a list of all variants arranged according to groups, each of which belongs to the same main example.

No.		No.	
1	$xy=10,0 \quad f(x,y)=\ldots.$	21	$3x+2y+f=2,12$
	$x+f(x,y)=32$	22	$+2f=2,14$
2	$+2f=34$	23	$-f=2,8$
3	$-f=28$	24	$-2f=2,6$
4	$-2f=26$	25	$1,5f=3x+2y$
5	$15f=x$	26	$1,0f=\quad -10$
6	$20f=\quad +10$	27	$1,10f=\quad +10$
7	$10f=\quad -10$		
		28	$x+(x-y)+f=42$
8	$y+f=22$	29	$+2f=44$
9	$+2f=24$	30	$-f=38$
10	$-f=18$	31	$-2f=36$
11	$-2f=16$	32	$20f=x+(x-y)$
12	$10f=y$	33	$25f=\quad +10$
13	$15f=\quad +10$	34	$15f=\quad -10$
14	$x+y+f=52$	35	$x+y+2(x-y)+f=1,12$
15	$+2f=54$	36	$+2f=1,14$
16	$-f=48$	37	$-f=1,8$
17	$-2f=46$	38	$-2f=1,6$
18	$25f=x+y$	39	$35f=x+y+2(x-y)$
19	$30f=\quad +10$	40	$40f=\quad +10$
20	$20f=\quad -10$	41	$30f=\quad -10$

This list shows the principle of arrangement within each group: in the first example, $f(x,y)$ is defined; then follow subgroups in which this $f(x,y)$ is added to a simple function $g(x,y)$ which is x in the first subgroup, y in the second, $x + y$ in the next, etc. The examples of each subgroup are finally:

$$(4) \qquad \begin{array}{ll} g + f = a_1 & af = g \\ g + 2f = a_2 & bf = g + 10 \\ g - f = a_3 & cf = g - 10 \\ g - 2f = a_4 & \end{array}$$

where $a_1, a_2, a_3, a_4, a, b, c$ are given numbers. Occasionally, the arrangement in the text is not exactly as given above, e.g., $g + 10$ and $g - 10$ can be interchanged (e.g., Nos. 26 and 27), or one or even both can be omitted[284n] (cf. Nos. 13 and 53). In No. 196 occurs another value instead of $g + 10$, namely, $8f = x - y + 6$. In general, however, the scheme (4) is followed well enough so that it is sufficient to indicate which function g was used in each subgroup. For all groups where $f(30,20) = 2$, we thus obtain the following list:

$g(x,y)$	a_1	Nos.				
x	32	1	42	81	119	139
y	22	8	49	88	126	146
$x+y$	52	14		95	132	152
$3x+2y$	2,12	21		109		159
$x+(x-y)$	42	28		102		166
$x+y+2(x-y)$	1,12	35				173
$\frac{x}{3}+\frac{y}{4}$	17			113		
$\frac{x}{3}+\frac{y}{4}+(x-y)$	27					180
$\frac{x}{3}+\frac{y}{4}+(x+y)$	1,7					186
$x-y$	12					191

Essentially the same device is followed in the problems where $f(30,20) = 3$. Here we have

No.		No.	
54	$xy=10,0 \quad f(x,y)=\ldots.$	65	$x+y+f=53$
	$x+f=33$	66	$+2f=56$
55	$+2f=36$	67	$-f=47$
56	$-f=27$	68	$-2f=44$
57	$-2f=24$	69	$20f=x+y+10$
58	$10f=x$	70	$15f=\quad -5$
59	$15f=\quad +15$		
		71	$3x+2y+f=2,13$
60	$y+f=23$	72	$+2f=2,16$
61	$+2f=26$	73	$-f=2,7$
62	$-f=17$	74	$-2f=2,4$
63	$-2f=14$	75	$50f=3x+2y+20$
64	$10f=y+10$	76	$40f=\quad -10$
		77	$x+(x-y)+f=43$
		78	$+2f=46$
		79	$-f=37$
		80	$-2f=34$

The reason why the second part of the scheme (4) is here less complete lies in the fact that some of these relations are not expressible in integers if $f = 3$; e.g., $10f = x$ but $x \pm 10$ is not a multiple of f.

[284n] The reason for the omission is sometimes clear; e.g., in Nos. 195ff., $5f = x - y$ would lead to $0 \cdot f = g - 10$ as the last equation in (4).

In the last group (Nos. 197–247), we have $f(30,20) = 4$ and the following subgroups:

$g(x,y)$	a_1	Nos.
x	34	197
y	24	204
$x + y$	54	210
$3x + 2y$	2,14	236
$x + (x - y)$	44	217
$\frac{x}{3} + \frac{y}{4} + (x + y)$	1,9	224
$\frac{x}{3} + \frac{y}{4} + 3(x - y)$	49	230
?	2,54	242

As for the variants, it is sufficient to present here only those which correspond to the second part in (4).

No.		No.	
201	$7;30f = x$	221	$10f = x + (x - y)$
202	$10f = +10$	222	$12f = +8$
203	$5f = -10$	223	$8f = -8$
208	$5f = y$		
209	$6f = +4$	228	$20f = \frac{x}{3} + \frac{y}{4} + (x+y) + 15$
		229	$15f = -5$
214	$12;30f = x + y$		
215	$15f = +10$		
216	$10f = -10$	234	$10f = \frac{x}{3} + \frac{y}{4} + 3(x-y) - 5$
		235	$15f = +15$
		240	$30f = 3x + 2y - 10$
		241	$40f = +30$
		246	$40f = \ldots\ldots - 10$
		247	$45f = +10$

In Nos. 201 and 214, fractional coefficients are introduced in order to obtain the complete scheme of (4).

c. Terminology

For reasons discussed above, p. 3, we have consistently based our translation on the practice of the mathematical texts written in Akkadian, which generally employ the first person in stating the problem, but the second person for the execution of the computations leading to the solution. Since texts like T and U only describe the problems, the first person is used here throughout.

In translating -ma, which indicates the statement of an equivalence, we have used ":" for the sake of brevity and convenience, in contrast to our usual translation of -ma by "and (the result is)" or the like.

uš sag

A tendency to abbreviate long expressions is visible in the replacement of whole phrases containing the words uš (length) and sag (width) by uš sag alone. Thus, uš sag in No. 25 stands for a-rá 3 uš a-rá 2 sag in No. 21. This is the same as if we would abbreviate $3x + 2y$ by x,y and is therefore analogous to our practice of writing $f(x,y)$, $g(x,y)$ ... for expressions previously defined in terms of x and y. Another example of the same abbreviation occurs in No. 163. In Nos. 39 and 177, uš sag means $(x + y) + 2(x - y)$, whereas Nos. 106, 170 and 221 give uš alone for $x + (x - y)$. This sort of abbreviation is of course only possible in texts with a strongly schematic arrangement where that which is actually meant follows from the scheme alone; it is never found outside texts like T and U.[285]

U. A 24195

(Photograph: Plate 42; copy: Plate 16)

Transcription

Obverse I

1 [1][a-rá 5] sag
[2][a-šà u]š igi-7-gál uš
[3][da]ḫ uš ù sag gim(?)-nam(?)
[4]i[gi]-4-gál sag a-šà daḫ 15

2 [5]a-rá 2 e-tab daḫ-ma [20]

3 [6]zi-ma 5

4 [7][a-r]á 2 e-tab a-šà gim-nam

5 [8]a-rá 3 e-tab 5 dirig

6 [9]a-rá 2 a-šà daḫ-ma 25

7 [10]a-rá 2 e-tab daḫ [30]

8 [11]a-rá 2 a-šà zi 15

9 [12]a-rá 2 e-tab zi-ma 10

10 [13]a-rá 4 e-tab a-rá 2 a-šà

11 [14]a-rá 6 e-tab a-rá 3 a-šà

12 [15][. . . . .] ig[i-. . . .]gál uš sag
[16][gim-nam] igi-4-gál sag a-šà daḫ 15

13 [17]a-rá 2 e-tab <daḫ->ma 20

14 [18]zi 5

15 [19]a-rá 2 e-tab a-šà gi[m-na]m

16 [20]a-rá 3 e-tab-ma 5 dirig

17 [21]a-rá 3 a-šà daḫ 35

[285] For analogous cases, cf. the following text and MKT I p. 455 d).

18 [22]a-rá 2 e-tab daḫ 40

19 [23][a-rá 3 a-šà] zi 25

20 [24][a-rá 2 e-tab] zi 20

21 [25][a-rá 6 e-]tab a-rá [3 a-]šà

22 [26][a-rá 8 e-]tab-ma 10 dirig

23 [27][a-rá 4] e-tab-ma 10 ba-lá

24 [28]...[.....]... a-na uš ugu sag
[29][diri]g ....[.....].... gim-nam
[30]igi-4-gál [sag a-šà da]ḫ 15

25 [31][a-rá 2 e-tab daḫ-]ma 20

26 [32]zi-ma 5

27 [33][a-rá 2 e-tab a-šà gim-nam]

Obverse II

28 [1]a-rá 3 <e-tab> 5 dirig

29 [2]uš ù ..... 5(?)
[3]a-na(?) uš ugu sag(?) [dir]ig(?)
[4]igi-6-gál ..... 5 zi
[5]igi-11(?)-gál ..... gim-nam
[6]igi-6-gál <uš> a-šà daḫ-ma 15

30 [7]a-rá 2 e<-tab> daḫ-ma 20

31 [8]zi-ma 5

32 [9]a-rá 2 e-tab a-šà gim-nam

33 [10]a-rá 3 e-tab-ma 5 dirig

34 [11]..... a-rá 3 e<-tab> 20 daḫ
[12]igi-..... ù 5 gim-nam
[13]igi-6-gál uš a-šà daḫ 15

35 [14]a-rá 2 e-tab daḫ 20

36 [15]zi-ma 5

37 [16]a-rá 2 e-tab a-šà gim-nam

38 [17]a-rá 3 e-tab-ma 5 dirig

39 [18]..... a-rá 2 e-tab sag
[19]igi-..... ù 5 [gim-na]m
[20]igi-4[-gál s]ag [a-šà daḫ-ma 15]

40 [21]a-rá 2 e-tab daḫ 20

41 [22]zi-ma 5

42 [23]a-rá 2 e-tab-ma a-šà gim-nam

43 [24]a-rá 20(?) e-tab-ma 6(?) dir[ig]

44 [25]..... a-rá 2 uš(?)
[26].....

[27]uš(?) sag a-na [uš ug]u sag dirig
[28]igi-1[2(?)-]gál-b[i a-šà] daḫ 15

45 [29]a-rá 2 e-tab daḫ-ma 20

46 [30]zi-ma 5

47 [31]a-rá 2 e-tab a-šà gim-nam

48 [32]a-rá 3 e-tab-ma 5 dirig

49 [33]...[.....]...
[34]...[.....]
[35]igi-11-gál ...[.....]....
[36]..... igi-6-gál-bi(?)
[37]<a-šà daḫ-ma 15>

Obverse III

50 [1]a-rá 2 e-tab daḫ-ma [2]0

51 [22]zi 5

52 [3]a-rá 2 e-tab-ma a-š[à gim]

53 [4][a-rá] 3 e-tab 5 dirig

54 [5]a-rá 4 a-šà daḫ 45

55 [6]a-rá 2 e-tab daḫ-ma [5]0

56 [7][z]i-ma [3]5

57 [8]a-rá 2 e-tab zi 30

58 [9]a-rá 8 e-tab-ma a-rá 4 a-šà

59 [10]a-rá 10 e-tab-ma 10 dirig

60 [11]a-rá 6 e-tab-ma 10 ba-lá

61 [12]a-šà(?) ù a-na uš [ugu s]ag dirig
[13]15 daḫ igi-7-gál-bi 5 [gim-]nam
[14]igi[-6(?)-]gál uš(?) a-šà daḫ-ma 15

62 [15]a-rá 2 e-tab daḫ-ma 20

63 [16][z]i-ma 5

64 [17]a-rá 2 e-tab-ma a-šà gim

65 [18][a-rá 3] e-tab-ma 5 dirig

66 [19][a-rá 1 e-]tab-ma 5 b[a-l]á

67 [20][igi-4-gá]l sag 5 uš-šè(?) da[ḫ]
[21][igi-7-]gál ù 10 sag gim-n[am]
[22]igi-4-g[ál sa]g a-šà ù u[š da]ḫ 45

68 [23]a-rá 2 [e-tab daḫ-]ma 50

69 [24]zi-ma 35

70 [25]a-rá 2 e-tab zi 30

71 [26]a-rá 8 e-tab-ma a-šà uš gim

72 [27]a-rá 10 e-tab-ma 10 dirig

73 [28]a-rá 6 e-tab-ma 10 ba-lá

74 [29]a-šà sag [daḫ-ma 3]5

75 [30][a-rá 2 e-]tab daḫ-ma [4]0

76 [31][zi-ma 25]

77 [32]a-rá [2 e-tab zi 20]

78 [33]a-rá [6 e-tab a-šà sag gim]

79 [34]a-rá [8 e-tab 10 dirig]

Obverse IV

80 [1]a-rá 4 e-tab-ma 10 ba-lá

81 [2]a-šà uš sag daḫ-ma 1,5

82 [3]a-rá 2 e-tab daḫ-ma 1,10

83 [4]a-šà uš [sag zi 5]5

84 [5]a-rá 2 e-tab z[i 50]

85 [6]a-rá 12 e-tab-ma [a-šà uš sag]

86 [7]a-rá 14 e-tab-ma 1[0 dirig]

87 [8]a-rá 10 e-tab-ma 10 b[a-lá]

88 [9]a-šà ù a-rá [2 e-tab uš daḫ-ma 1],15

89 [10]a-rá 2 e-tab [daḫ 1,20]

90 [11]zi-ma [1,5]

91 [12]a-rá 2 e-t[ab zi 1]

92 [13]a-rá 14 e-tab [a-šà ù a-rá 2 uš]

93 [14]a-rá [1]6 e-ta[b 10 dirig]

94 [15][a-r]á 12 e-t[ab 10 ba-lá]

95 [16]a-rá(?) 4(?) e<-tab>(?) uš(?) a-na [uš ugu s]ag dirig
[17]igi-13-gál a-šà uš [daḫ-]ma 50

96 [18]a-rá 2 e-tab daḫ-ma 1

97 [19]a-šà uš zi 30

98 [20]a-rá 2 e-tab zi 20

99 [21]a-rá 4 e-tab-ma a-šà uš

100 [22]a-rá 5 e-tab-ma 10 dirig

101 [23]a-rá 3 e-tab-ma 10 ba-lá

102 [24]a-šà ù sag(?) daḫ[-ma] 40

103 [25]a-rá 2 e-tab daḫ 50

104 [26]a-šà sag zi-ma 20

105 [27]a-rá 2 e-tab zi[-ma 10]

106 [28]a-rá 3 e-tab [a-šà sag]

107 [29]a-rá 4 [e-tab 10 dirig]

108 [30]a[-rá 2 e-tab 10 ba-lá]

About 4 lines destroyed
(Nos. **109–112**).

Reverse I

About 4 lines destroyed
(Nos. **113–115** and beginning of No. **116**).

[5]a[-. . . . . .]
[6]sag a[-. . . . . .]
[7]25 . . .[. . . . . a-šà daḫ-ma 15]

117 [8]a-rá 2 e<-tab> daḫ [20]

118 [9]zi-ma [5]

119 [10]a-rá 2 e-tab a[-š]à g[im]

120 [11]a-rá 3 e-tab-ma 5 [dirig]

121 [12]a-šà uš daḫ-ma 45

122 [13]a-rá 2 e<-tab> daḫ-ma 50

123 [14]zi-ma 35

124 [15]a-rá 2 e-tab zi 30

125 [16]a-rá 8 e-tab-ma a-šà ù uš

126 [17]a-rá 6 e-tab-ma 10 ba-lá

127 [18]a-rá 10 e-tab-ma 10 dirig

128 [19]a-šà sag daḫ-ma 35

129 [20]a-rá 2 e-tab daḫ 40

130 [21]zi 25

131 [22]a-rá 2 e<-tab> zi 20

132 [23]a-rá 6 e-tab a-šà sag

133 [24]a-šà uš sag daḫ-ma 1,5

134 [25]a-rá 2 e-tab daḫ-ma 1,10

135 [26]zi-ma 55

136 [27]a-rá 2 e-tab zi 50

137 [28]a-rá 12 e-tab a-šà uš sag

138 [29]a-rá 15 e-tab 15 dirig

139 [30]a-rá 10 e-tab-ma 10 ba-lá

140 [31]a-šà ù a-rá 3 uš daḫ 1,45

141 [32]a-rá 2 e-tab daḫ 1,50

142 [33]zi-ma 1,35

143 [34]a-rá 2 e-tab zi 1,3[0]

144 [35]a-rá 20 e-tab a-šà ù
[36]a-rá 3 uš

Reverse II

145 [1]a-šà a-rá 3 uš a-rá 2 [sag]
[2]daḫ-ma 2,25

146 [3]a-rá 2 e-tab daḫ 2,30

147 [4]zi-ma 2,15

148 [5]a-rá 2 e-tab zi 2,10

149 [6]a-rá 28 e-tab-ma
[7]a-šà a-rá 3 uš ù a-rá 2 sag

150 [8]a-rá 30 e-tab 10 dirig

151 [9]a-šà ù a-na uš ugu sag dirig
[10]daḫ-ma 25

152 [11]a-rá 2 e-tab daḫ 30

153 [12]zi-ma 15

154 [13]a-rá 2 e-tab zi 10

155 [14]a-rá 4 e-tab a-šà gim-nam

156 [15]a-rá 6 e-tab-ma 10 dirig

157 [16]a-šà uš
[17]ù a-na uš ugu sag dirig
[18]daḫ-ma 55

158 [19]a-rá 2 e-tab daḫ 1

159 [20]zi 50

160 [21]a-rá 2 e-tab zi [45]

161 [22]a-rá 10 e-tab-ma
[23]a-šà ù uš ù a-na u[š]
[24]ugu sag dirig

162 [25]a-rá 12 e-tab 10 dirig

163 [26]a-rá 8 e-tab 10 ba-lá

164 [27]3 uš sag uš a-rá 3 e-tab
[28]sag a-rá 2 e-tab
[29]igi-3-gál ù uš
[30]a-šà bí-daḫ
[31]aká-bi EŠ-àm[286]
[32]níg uš daḫ igi-6-gál
[33]uš ù níg daḫ
[34]ù 10 sag daḫ igi-7-gál sag
[35]a-šà ù(?) níg(?) daḫ(?) [..... 5]5

Reverse III

165 [1]a-rá 2 e-tab daḫ 1

166 [2]zi-ma 45

167 [3]a-rá 2 e-tab zi 40

168 [4]a-rá 10 e-tab-ma a-šà uš

169 [5]a-rá 15 e-tab 25 dirig

170 [6]a-rá 6 e-tab 20 ba-lá

171 [7]uš a-rá 2 e-tab
[8]a-šà ù 10 GAR daḫ-ma 1,25

172 [9]a-rá 2 e<-tab> daḫ-ma 1,30

173 [10]zi-ma 1,15

174 [11]a-rá 2 e-tab zi 1,10

175 [12]a-rá 16 e-tab a-šà gim

176 [13]a-rá 12 e-tab 20 ba-lá

177 [14]a-rá 20 e-tab 20 dirig

Translation

Obverse

1 [1][5 times] the width,
[2–3][the "area", the len]gth; one-seventh thereof I [ad]ded to the length: it is equal(?) to the length and the width.
[4]I added one-fourth of the width to the "area": 15.

2 [5]I multiplied by 2, I added: [20].

3 [6]I subtracted: 5.

4 [7]I multiplied [b]y 2: it is equal to the "area".

5 [8]I multiplied by 3: it exceeded by 5.

6 [9]I added 2 times the "area": 25.

7 [10]I multiplied by 2, I added: [30].

8 [11]I subtracted from 2 times the "area": 15.

9 [12]I multiplied by 2, I subtracted: 10.

10 [13]I multiplied by 4: 2 times the "area".

11 [14]I multiplied by 6: 3 times the "area".

12 [15–16][.....] one-[...]th (thereof) [is equal to] the length and the width. I added one-fourth of the width to the "area": 15.

[286] We follow here the suggestion made by Stephens (Thureau-Dangin TMB p. 164, note 3) to read aká for the sign which was read NE(?) in MKT I p. 423 and elsewhere.

13 [17]I multiplied by 2, <I added>: 20.

14 [18]I subtracted: 5.

15 [19]I multiplied by 2: it is e[qu]al to the "area".

16 [20]I multiplied by 3: it exceeded by 5.

17 [21]I added to 3 times the "area": 35.

18 [22]I multiplied by 2, I added: 40.

19 [23]I subtracted from [3 times the "area"]: 25.

20 [24][I multiplied by 2], I subtracted: 20.

21 [25]I multi[plied by 6]: [3] times the ["ar]ea".

22 [26]I multi[plied by 8]: it exceeded by 10.

23 [27]I multiplied [by 4]: it was less by 10.

24 [28–29]...[.....]... that by which the length [excee]ded the width ....[....]....: it is equal to .....
[30][I ad]ded one-fourth of [the width] to [the "area"]: 15.

25 [31][I multiplied by 2, I added]: 20.

26 [32]I subtracted: 5.

27 [33][I multiplied by 2: it is equal to the "area".]

Obverse II

28 [1]<I multiplied> by 3: it exceeded by 5.

29 [2]The length and ..... 5(?)
[3]that by which(?) the length exceeded(?) the width(?)
[4]one-sixth ..... 5 I subtracted(?);
[5]one-eleventh (thereof) is equal to .....
[6]I added one-sixth of <the length> to the "area": 15.

30 [7]I multiplied by 2, I added: 20.

31 [8]I subtracted: 5.

32 [9]I multiplied by 2: it is equal to the "area".

33 [10]I multiplied by 3: it exceeded by 5.

34 [11]..... I multiplied by 3, I added 20;
[12]one-...th (thereof) is equal to ..... and 5.
[13]I added one-sixth of the length to the "area": 15.

35 [14]I multiplied by 2, I added: 20.

36 [15]I subtracted: 5.

37 [16]I multiplied by 2: it is equal to the "area".

38 [17]I multiplied by 3: it exceeded by 5.

39 [18]..... 2 times the width;
[19]one-...th (thereof) is equal to ..... and 5.
[20][I added] one-four[th of the w]idth [to the "area": 15].

40 [21]I multiplied by 2, I added: 20.

41 [22]I subtracted: 5.

42 [23]I multiplied by 2: it is equal to the "area".

43 [24]I multiplied by 20(?): it ex[ceeded by] 6(?).

44 [25]..... 2 times the length(?)
[26].....
[27]the length(?), the width (and) that by which [the length ex]ceeded the width;
[28]I added one-t[welfth(?)] there[of to the "area"]: 15.

45 [29]I multiplied by 2, I added: 20.

46 [30]I subtracted: 5.

47 [31]I multiplied by 2: it is equal to the "area".

48 [32]I multiplied by 3: it exceeded by 5.

49 [33–34].....
[35]one-eleventh .....
[36]..... <I added> one-sixth thereof(?) <to the "area": 15>

Obverse III

50 [1]I multiplied by 2, I added: [2]0.

51 [2]I subtracted: 5.

52 [3]I multiplied by 2: [it is equal to] the "area".

53 [4]I multiplied [by] 3: it exceeded by 5.

54 [5]I added to 4 times the "area": 45.

55 [6]I multiplied by 2, I added: [5]0.

56 [7][I subt]racted: [3]5.

57 [8]I multiplied by 2, I subtracted: 30.

58 [9]I multiplied by 8: 4 times the "area".

59 [10]I multiplied by 10: it exceeded by 10.

60 [11]I multiplied by 6: it was less by 10.

61 [12–13]I added 15 to the "area"(?) and that by which the length [ex]ceeded the [wi]dth; one-seventh thereof is [equal] to 5.
[14]I added one-[six]th of the length(?) to the "area": 15.

62 [15]I multiplied by 2, I added: 20.

63 [16][I subt]racted: 5.

64 [17]I multiplied by 2: it is equal to the "area".

65 [18]I multiplied [by 3]: it exceeded by 5.

66 [19]I mult[iplied by 1]: it was [less] by 5.

67 [20][One-fourth] of the width, 5 I added to(?) the length,
[21][one-seven]th (thereof) and 10 are equal to the width.
[22][I ad]ded one-fourth of the width to the "area" and the len[gth]: 45.

68 [23][I multiplied] by 2, [I added]: 50.

69 [24]I subtracted: 35.

70 [25]I multiplied by 2, I subtracted: 30.

71 [26]I multiplied by 8: it is equal to the "area" (and) the length.

72 [27]I multiplied by 10: it exceeded by 10.

73 [28]I multiplied by 6: it was less by 10.

74 [29][I added] to the "area" (and) the width: [3]5.

75 [30]I multip[lied by 2], I added: [4]0.

76 [31][I subtracted: 25.]

77 [32][I multiplied] by 2, [I subtracted: 20].

78 [33][I multiplied] by [6: it is equal to the "area" (and) the width].

79 [34][I multiplied] by 8: [it exceeded by 10].

Obverse IV

80 [1]I multiplied by 4: it was less by 10.

81 [2]I added to the "area", the length, (and) the width: 1,5.

82 [3]I multiplied by 2, I added: 1,10.

83 [4][I subtracted] from the "area", the length, (and) [the width]: [5]5.

84 [5]I multiplied by 2, I subtr[acted: 50].

85 [6]I multiplied by 12: [the "area", the length, (and) the width].

86 [7]I multiplied by 14: [it exceeded by] 1[0].

87 [8]I multiplied by 10: it was [less] by 10.

88 [9][I added] to the "area" (and) [2 times the length: 1],15.

89 [10]I multiplied by 2, [I added: 1,20].

90 [11]I subtracted: [1,5].

91 [12]I multiplied by 2, [I subtracted: 1,0].

92 [13]I multiplied by 14: [the "area" and 2 times the length].

93 [14]I multiplied by [1]6: [it exceeded by 10].

94 [15]I multiplied by 12: [it was less by 10].

95 [16]4(?) times the length(?) (and) that by which [the length ex]ceeded the width;
[17][I added] one-thirteenth (thereof) to the "area" (and) the length: 50.

96 [18]I multiplied by 2, I added: 1,0.

97 [19]I subtracted from the "area" (and) the length: 30.

98 [20]I multiplied by 2, I subtracted: 20.

99 [21]I multiplied by 4: the "area" (and) the length.

100 [22]I multiplied by 5: it exceeded by 10.

101 [23]I multiplied by 3: it was less by 10.

102 [24]I added to the "area" and the width(?): 40.

103 [25]I multiplied by 2, I added: 50.

104 [26]I subtracted from the "area" (and) the width: 20.

105 [27]I multiplied by 2, I subtracted: [10].

106 [28]I multiplied by 3: [the "area" and the width].

107 [29][I multiplied] by 4: [it exceeded by 10].

108 [30][I multiplied b]y [2: it was less by 10].

109–112 (About 4 lines destroyed.)

Reverse I

113–116 (About 4 lines destroyed, including beginning of 116.)
[5]..[. . . . . .]
[6]the width [. . . . . .]
[7]25 . . .[. . . . . I added to the "area": 15].

117 [8]I multiplied by 2, I added: [20].

118 [9]I subtracted: [5].

119 [10]I multiplied by 2: [it is] eq[ual] to the "a[re]a".

120 [11]I multiplied by 3: [it exceeded] by 5.

121 [12]I added to the "area" and the length: 45.

122 [13]I multiplied by 2, I added: 50.

123 [14]I subtracted: 35.

124 [15]I multiplied by 2, I subtracted: 30.

125 [16]I multiplied by 8: the "area" and the length.

126 [17]I multiplied by 6: it was less by 10.

127 [18]I multiplied by 10: it exceeded by 10.

128 [19]I added to the "area" (and) the width: 35.

129 [20]I multiplied by 2, I added: 40.

130 [21]I subtracted: 25.

131 [22]I multiplied by 2, I subtracted: 20.

132 [23]I multiplied by 6: the "area" (and) the width.

133 [24]I added to the "area", the length, (and) the width: 1,5.

134 [25]I multiplied by 2, I added: 1,10.

135 [26]I subtracted: 55.

136 [27]I multiplied by 2, I subtracted: 50.

137 [28]I multiplied by 12: the "area", the length, (and) the width.

138 [29]I multiplied by 15: it exceeded by 15.

139 [30]I multiplied by 10: it was less by 10.

140 [31]I added to the "area" and 3 times the length: 1,45.

141 [32]I multiplied by 2, I added: 1,50.

142 [33]I subtracted: 1,35.

143 [34]I multiplied by 2, I subtracted: 1,3[0].

144 [35]I multiplied by 20: the "area" and
[36]3 times the length.

Reverse II

145 [1–3]I added to the "area", 3 times the length, and 2 times [the width]: 2,25.

146 [3]I multiplied by 2, I added: 2,30.

147 [4]I subtracted: 2,15.

148 [5]I multiplied by 2, I subtracted: 2,10.

149 [6]I multiplied by 28:
[7]the "area", 3 times the length, and 2 times the width.

150 [8]I multiplied by 30: it exceeded by 10.

151 [9–10]I added to the "area" and that by which the length exceeded the width: 25.

152 [11]I multiplied by 2, I added: 30.

153 [12]I subtracted: 15.

154 [13]I multiplied by 2, I subtracted: 10.

155 [14]I multiplied by 4: it is equal to the "area" (and that by which the length exceeded the width).

156 [15]I multiplied by 6: it exceeded by 10.

157 [16–18]I added to the "area", the length, and that by which the length exceeded the width: 55.

158 [19]I multiplied by 2, I added: 1,0.

159 [20]I subtracted: 50.

160 [21]I multiplied by 2, I subtracted: [45].

161 [22]I multiplied by 10:
[23]the "area" and the length and that by which the len[gth]
[24]exceeded the width.

162 [25]I multiplied by 12: it exceeded by 10.

163 [26]I multiplied by 8: it was less by 10.

164 [27]3 (times) the length and the width, 3 times the length,
[28]2 times the width;
[29]one-third (thereof) and the length
[30]I added to the area
[31] . . . . . .
[32]that which I added to the length, one-sixth
[33]of the length and that which I added,
[34]and 10 I added to the width; one-seventh (thereof): the width.
[35]The "area" and(?) that(?) which I added(?) [. . . . . : 5]5.

Reverse III

165 [1]I multiplied by 2, I added: 1,0.

166 [2]I subtracted: 45.

167 [3]I multiplied by 2, I subtracted: 40.

168 [4]I multiplied by 10: the "area" and the length.

169 [5]I multiplied by 15: it exceeded by 25.

170 [6]I multiplied by 6: it was less by 20.

171 [7–8]I added to 2 times the length, the "area", and 10 GAR: 1,25.

172 [9]I multiplied by 2, I added: 1,30.

173 [10]I subtracted: 1,15.

174 [11]I multiplied by 2, I subtracted: 1,10.

175 [12]I multiplied by 16: it is equal to the "area", (2 times the length, and 10 GAR).

176 [13]I multiplied by 12: it was less by 20.

177 [14]I multiplied by 20: it exceeded by 20.

Commentary

a. General Commentary

The general arrangement of the examples of this text follows exactly the same pattern as in the preceding text. Everything written in the general commentary to problem-text T concerning "main problems" and "variants" therefore also holds here. A glance at the numbers occurring in the text shows that here too

(1) $x = 30 \quad y = 20$

is the pair of solving numbers.

We must, however, mention a new element, namely, the fact that the term "area" (a-šà) is here to be interpreted as *one-sixtieth* of the product xy. This follows immediately from any group of variants, e.g., the first (using the symbol A for "a-šà"):

$$A + f = 15 \qquad 2f = A$$
$$A + 2f = 20 \qquad 3f = A + 5$$
$$A - f = 5$$

It follows from these equations that $f = 5$ and $A = 10$, but not $A = xy = 10{,}0$, as one would at first sight interpret the term "area" according to (1).

The present text is not the first to use this peculiar terminology; exactly the same usage also occurs in YBC 4695.[287] A comparison of the two texts shows, however, that the ambiguity in terminology is not as great as one would suppose. Whereas texts in which "area" (a-šà) denotes the product $xy = 10{,}0$ always start with the words, "The area is 1 ešè", no such sentence occurs in the present text or in YBC 4695. The reason is clear: an area measuring 1 ešè *must* be interpreted as an area of 10,0 GAR² and can never mean only 10 GAR². In omitting this metrological statement, the text paves the way for the looser interpretation of "area" as the numerical value of xy disregarding the empty sexagesimal place, and the "area" of $x = 30 \quad y = 20$ is then simply 10, not 10,0. In order to keep our formulas correct, however, we must maintain a clear distinction between the exact value of a product and its sixtieth. We therefore introduce the notation

(2) $(xy) = 0;1 \cdot xy$

wherever the word "area" in the text means only $0;1\,xy$.

[287] Published MKT III pp. 34ff. YBC 4695 is called "5th tablet," but our present text bears no number.

As a consequence of the fact that $xy = 10{,}0$ is no longer to be considered as one of the given equations, each main example now contains *two* independent equations, each of which is subject to variations. The same holds for the above-mentioned YBC 4695, except that a large group of problems in YBC 4695 contains only two *linear* equations whereas our present text calls for quadratic equations.

b. The Main Examples

Most of the main examples consist of two very simple equations. The second equation, which is subject to variations, is usually of the form $(xy) + f = a$, where f is a simple function like $\frac{x}{6}$ or $\frac{y}{4}$. Only No. 164 seems to be of a more elaborate type.

No. 1. Although partially restored, the following interpretation seems fairly certain:

$$\tfrac{1}{7}(5y + (xy) + x) + x = x + y$$
$$\tfrac{y}{4} + (xy) = 15.$$

The variants which follow refer to the second equation; $f(x,y)$ is simply $\frac{y}{4}$.

Nos. 12 and 24. The first equations are destroyed, the second are again

$$\tfrac{y}{4} + (xy) = 15.$$

Nos. 29 and 34. The second equations are now

$$\tfrac{x}{6} + (xy) = 15.$$

The first equations are in a hopeless condition.

No. 39. This example again uses $\frac{y}{4} + (xy) = 15$ as the second equation. The first equation seems to be similar to the first equation in No. 34, but no complete restoration can be given.

No. 44. The first equation is destroyed. The second seems to be

$$\tfrac{1}{12}\{(x + y) + (x - y)\} + (xy) = 15.$$

No. 49. This is almost completely destroyed. It is difficult to see how the traces at the end of the last line can refer to ..... $+ (xy) = 15$.

No. 61. This apparently gives

$$\tfrac{1}{7}\{(xy) + (x - y) + 15\} = 5$$
$$\tfrac{x}{6} + (xy) = 15.$$

No. 67.

$$\frac{y}{4} + \frac{1}{7}(x + 5) + 10 = y$$

$$\frac{y}{4} + (xy) + x = 45.$$

For the variants, $f(x,y)$ is here only $\frac{y}{4}$ in the second equation.

No. 95. Only one equation seems to be given:

$$\tfrac{1}{13}\{4x + (x - y)\} + (xy) + x = 50$$

leading to $f(x,y) = \frac{1}{13}\{4x + (x - y)\}$ for the variants, and $f(30,20) = 10$. We must therefore assume that the first equation of the preceding group (No. 67) still holds for this group too.

No. 116: destroyed.

No. 164. It seems clear that the sentence in lines 29f.:

igi-3-gál ù uš a-šà bí-daḫ

"I added one-third (thereof) and the length to the area"

must be interpreted as

$$\tfrac{1}{3}\{\qquad\} + x + (xy).$$

The preceding lines must therefore contain the part to be inserted between the braces. One part is certainly given by lines 27f.:

uš a-rá 3 e-tab sag a-rá 2 e-tab

"3 times the length (and) 2 times the width", i.e., $3x + 2y$. This term, however, is not divisible by 3 if $x = 30$ $y = 20$; hence the preceding term (line 27) 3 uš sag must also be included within the braces. Nothing is gained, however, by interpreting 3 uš sag as $3(x + y)$ because $3(x + y) + 3x + 2y$ is also not divisible by 3 if $y = 20$. We must therefore interpret 3 uš sag to mean $3x + y$ and obtain for the contents of lines 27–30:

(1) $\quad \frac{1}{3}\{(3x + y) + (3x + 2y)\} + x + (xy).$

The value of this term for $x = 30$ $y = 20$ is 2,0.

The next line (31)

aká-bi EŠ-àm

is unintelligible to us, although it is attested several times in already published material. We shall therefore disregard it for the moment and try to interpret the remaining part. This second part begins with níg uš daḫ, "that which I added to the length", followed by igi-6-gál uš ù níg daḫ, "one-sixth of the length and that which I added". If we denote "that which I added" by α, we have up to this point the expression for $\alpha + \frac{1}{6}(x + \alpha)$. Finally, we have ù 10 sag daḫ igi-7-gál sag, "and 10 I added to the width; one-seventh (thereof): the width". This must be understood as $\{(\quad) + 10 + y\}\frac{1}{7} = y$. The part under discussion therefore means

(2) $\quad \frac{1}{7}\{\alpha + \frac{1}{6}(x + \alpha) + 10 + y\} = y.$

We must now look for the meaning of α. If the above relation should hold for $x = 30$ $y = 20$, we must obtain for α the value 1,30. We know, moreover, that α is "that which I added to the length". Looking at the expression (1) and taking into consideration that its value is 2,0 if $x = 30$ $y = 20$ and that it contains the term x, we would have

(3) $\quad \alpha = \frac{1}{3}\{(3x + y) + (3x + 2y)\} + (xy).$

In other words, the first equation in our problem consists of equation (2) with α defined by means of (1), namely, as the part of (1) which is added to x.

Two objections can be raised against this interpretation. First, it is not clear why in (1) the term $\alpha = \frac{1}{3}\{\qquad\} + (xy)$ was added to x although later only α, never $\alpha + x$, is used. The second and main objection, however, lies in disregarding the sentence aká-bi EŠ-àm which must have some sense but which plays no role in our interpretation. The only way out seems to be to assume that this expression merely indicates some sort of pause between the construction of (1) and the formula (2), and means "this is the first part" or the like. To test this assumption, one should be able to explain in the same way all examples in which this expression occurs. Unfortunately, none of these examples can at present be completely explained either with or without the sentence in question.[289]

The second equation of this problem must be contained in the last line (35). From a-šà at the beginning and from the following variants, it follows that this equation must be of the type $(xy) + g(x,y) = 55$. The same value 55 occurs on the right-hand side of No. 157, which is the first example of the preceding group:

(4) $\quad (xy) + x + (x - y) + f(x,y) = 55.$

It is possible that the last line merely contains a reference to this equation (4) of the preceding group to the effect that No. 164 has its second equation in com-

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[289] The previous examples are: YBC 4668 + YBC 4713 Nos. 13–16 (MKT I p. 423) and YBC 4697 No. 18 (MKT III p. 40). The latter example is especially analogous to our case. It starts with $xy = 10,0$, which proves that the whole remainder constitutes only *one* equation. The first part contains the construction of a certain expression which is easy to understand, namely, $x + 1^{5}[\frac{1}{3}(x - y) + 0;20]$. Then follows the sentence with aká-bi, but the rest is unintelligible.

mon with No. 157, whereas the first equation is new, namely, given by (2) and (3).[290]

c. The Variants

For the variants we need know nothing more than the fact that the value of $f(x,y)$ for $x = 30$ $y = 20$ is 5 in all examples except Nos. 95–115, where $f(30,20) = 10$. Altogether, there are five major groups of variants to be distinguished.

Nos. 1–66. The main body of variants of this first group repeats the same pattern nine times:

	Nos.								
$(xy) + f = 15$	1	12	24	29	34	39	44	49	61
$+ 2f = 20$	2	13	25	30	35	40	45	50	62
$- f = 5$	3	14	26	31	36	41	46	51	63
$2f = (xy)$	4	15	27	32	37	42	47	52	64
$3f = \quad + 5$	5	16	28	33	38	43[291]	48	53	65
$1f = \quad - 5$									66

The restored occurrence of a coefficient 1 (a-rá 1 e-tab-ma) in No. 66 is merely due to the consistent carrying out of the scheme. In an independent example, this coefficient would not have been mentioned.[292] It should furthermore be noted that $(xy) - 2f$ would be zero, which explains why this variant is missing. It reappears, however, in the continuation of this scheme where (xy) is replaced by $2(xy)$ or $3(xy)$ or $4(xy)$:

	No.		No.		No.
$2(xy)+f=25$	6	$3(xy)+f=35$	17	$4(xy)+f=45$	54
$+2f=30$	7	$+2f=40$	18	$+2f=50$	55
$2(xy)-f=15$	8	$3(xy)-f=25$	19	$-f=35$	56
$-2f=10$	9	$-2f=20$	20	$-2f=30$	57
$4f=2(xy)$	10	$6f=3(xy)$	21	$8f=4(xy)$	58
$6f=3(xy)$	11	$8f= \quad +10$	22	$10f= \quad +10$	59
		$4f= \quad -10$	23	$6f= \quad -10$	60

The deviation from the regular scheme at the end of the first group is apparently due to the fact that $2(xy) + 10$ and $3(xy)$ are both equal to 30 if $x = 30$ $y = 20$, so that the scribe could replace $2(xy) + 10$ by $3(xy)$, which actually belongs to the next group.

Nos. 67–94. This and the following two groups add to (xy) not only f but also simple functions, namely, x, y, $x + y$ and $2x$, followed by the usual system of variants.

	No.		No.
$x+(xy)+f=45$	67	$x+y+(xy)+f=1,5$	81
$+2f=50$	68	$+2f=1,10$	82
$-f=35$	69	$-f=55$	83
$-2f=30$	70	$-2f=50$	84
$8f=x+(xy)$	71	$12f=x+y+(xy)$	85
$10f= \quad +10$	72	$14f= \quad +10$	86
$6f= \quad -10$	73	$10f= \quad -10$	87
$y+(xy)+f=35$	74	$2x+(xy)+f=1,15$	88
$+2f=40$	75	$+2f=1,20$	89
$-f=25$	76	$-f=1,5$	90
$-2f=20$	77	$-2f=1,0$	91
$6f=y+(xy)$	78	$14f=2x+(xy)$	92
$8f= \quad +10$	79	$16f= \quad +10$	93
$4f= \quad -10$	80	$12f= \quad -10$	94

Nos. 95–115. Only two of the three sections are preserved; they correspond to the first two sections of the preceding group except that now $f(30,20) = 10$.

	No.		No.
$x + (xy) + f = 50$	95	$y + (xy) + f = 40$	102
$+ 2f = 1,0$	96	$+ 2f = 50$	103
$- f = 30$	97	$- f = 20$	104
$- 2f = 20$	98	$- 2f = 10$	105
$4f = x + (xy)$	99	$3f = y + (xy)$	106
$5f = \quad + 10$	100	$4f = \quad + 10$	107
$3f = \quad - 10$	101	$2f = \quad - 10$	108

The last section (Nos. 109–115) can be restored, e.g., as $x + y + (xy) + f = 1,10$, followed by the corresponding variants.

Nos. 116–163. This large group of variants is composed of eight sections headed by the following equations:[293]

	No.		No.
$[(xy)+f=15]$	116	$(xy)+3x+f=1,45$	140
$(xy)+x+f=45$	121	$(xy)+3x+2y+f=2,25$	145
$(xy)+y+f=35$	128	$(xy)+(x-y)+f=25$	151
$(xy)+x+y+f=1,5$	133	$(xy)+x+(x-y)+f=55$	157

Each of these examples is followed by the usual set of three problems replacing f by $2f$, $-f$, $-2f$ and changing the right-hand side accordingly, $f(20,30)$ being 5.

If we now call the function to which f is added in the above list $g(x,y)$, or g for short, we can describe the remaining variants as follows:

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[290] This interpretation is supported by No. 168 (cf. below, p. 129) but difficult to bring into agreement with the traces following a-šà in line 35.

[291] We mention No. 43 at this place because of the general scheme, but we do not see how the traces can be reconciled with this interpretation.

[292] For the occurrence of the coefficient 1 in the expression $1x + 4y$, see MKT III p. 47 No. 1.

[293] The first example is destroyed, but the restoration is obvious.

	No.		No.
$2f = g$	119	$20f = g$	144
$3f = g + 5$	120		
		$28f = g$	149
$8f = g$	125	$30f = g + 10$	150
$6f = g - 10$	126		
$10f = g + 10$	127	$4f = g$	155
		$6f = g + 10$	156
$6f = g$	132		
		$10f = g$	161
$12f = g$	137	$12f = g + 10$	162
$15f = g + 15$	138	$8f = g - 10$	163
$10f = g - 10$	139		

The scribe seems to have tired somewhat of the strict adherence to the scheme ± 10 toward the end.

Nos. 164–177. The last two sections contain the following examples:

	No.
$(xy) + \ldots + f = 55$	164
$+ 2f = 1{,}0$	165
$- f = 45$	166
$- 2f = 40$	167
$10f = (xy) + x + (x - y)$	168
$15f = + 25$	169
$6f = - 20$	170
$(xy) + 2x + 10 + f = 1{,}25$	171
$+ 2f = 1{,}30$	172
$- f = 1{,}15$	173
$- 2f = 1{,}10$	174
$16f = (xy) + 2x + 10$	175
$12f = - 20$	176
$20f = + 20$	177

As already mentioned in the commentary to No. 164,[294] this problem might be restored, like No. 157, as $(xy) + x + (x - y) + f = 55$. This is supported by No. 168, in which we find $(xy) + x$ on the right-hand side, x being understood as an abbreviation for a longer expression beginning with x.

d. Terminology

The tendency of this text to abbreviate is by no means as pronounced as in the preceding text. In our present text, however, we do find examples of the same practice, e.g., e instead of e-tab in Nos. 117, 122, 131, 172. The expression a-šà ù a-na uš ugu sag dirig of No. 151 (i.e., $(xy) + x - y$) becomes simply a-šà in No. 155. In No. 175, however, a-šà stands for $(xy) + 2x + 10$ of No. 171, while a-šà uš in No. 168 is an abbreviation for $(xy) + x + (x - y)$ introduced in No. 157.

[294] Pp. 127f. above.

Ua. YBC 6967

(Photograph: Plate 27; copy: Plate 17)

Transcription

Obverse

[1][igi-b]i *e-li* igi 7 *i-ter*
[2][igi] *ù* igi-bi *mi-nu-um*
[3]*a[t-t]a* 7 *ša* igi-bi
[4]ugu igi *i-te-ru*
[5]*a-na ši-na ḫe-pé-ma* 3,30
[6]3,30 *it-ti* 3,30
[7]*šu-ta-ki-il-ma* 12,15
[8]*a-na* 12,15 *ša i-li<-a>-kum*
[9][1 *eq(?)-l]a-am ṣí-ib-ma* 1,12,15
[10][íb-si$_8$ 1],12,15 *mi-nu-um* 8,30
[11][8,30 *ù*] 8,30 *me-ḫe-er-šu i-di-ma*

Reverse

[1]3,30 *ta-ki-il-tam*
[2]*i-na iš-te-en ú-su-uḫ*
[3]*a-na iš-te-en ṣí-ib*
[4]*iš-te-en* 12 *ša-nu-um* 5
[5]12 igi-bi 5 *i-gu-um*

Translation

Obverse

[1][The *igib*]*ūm* exceeded the *igūm* by 7.
[2]What are [the *igūm* and] the *igibūm?*
[3–5]As for you—halve 7, by which the *igibūm* exceeded the *igūm*, and (the result is) 3;30.
[6–7]Multiply together 3;30 with 3;30, and (the result is) 12;15.
[8]To 12;15, which resulted for you,
[9]add [1,0, the produ]ct, and (the result is) 1,12;15.
[10]What is [the square root of 1],12;15? (Answer:) 8;30.
[11]Lay down [8;30 and] 8;30, its equal, and then

Reverse

1–2subtract 3;30, the *takīltum*, from the one,
3add (it) to the other.
4One is 12, the other 5.
512 is the *igibūm*, 5 the *igūm*.

Commentary

The problem treated here belongs to a well known class of quadratic equations[295] characterized by the terms igi and igi-bi (in Akkadian, *igūm* and *igibūm*, respectively). These terms refer to a pair of numbers which stand in the relation to one another of a number and its reciprocal, to be understood in the most general sense as numbers whose product is a power of 60. We must here assume the product

$$xy = 1{,}0 \tag{1}$$

as the first condition to which the unknowns x and y are subject. The second condition is explicitly given as

$$x - y = 7. \tag{2}$$

From these two equations it follows that x and y can be found from

$$\left.\begin{matrix} x \\ y \end{matrix}\right\} = \sqrt{\left(\frac{7}{2}\right)^2 + 1{,}0} \pm \frac{7}{2},$$

a formula which is followed exactly by the text, leading to

$$\left.\begin{matrix} x \\ y \end{matrix}\right\} = \sqrt{1{,}12;15} \pm 3;30 = 8;30 \pm 3;30 = \left\{\begin{matrix} 12 \\ 5 \end{matrix}\right.$$

We have refrained from translating *takīltu*[295a] because the mathematical contexts in which this word appears do not lead to a common meaning. Here and in VAT 8520 rev. 21,[295b] *takīltu* refers to the term which is to be added to or subtracted from the square root in the solution of a quadratic equation. Also in VAT 8512 obv. 19,[295c] the word is applied to a term to be subtracted from the square root. In VAT 8389 obv. II 4,[295d] however, the ratio of the yield of a field to its area is called *takīltu*. For another, but uncertain occurrence in connection with Pythagorean numbers, cf. p. 40. The suggestion made by Thureau-Dangin[295e] that *takīltu* was used to designate "un nombre qui a été croisé avec lui-même, porté au carré" suits only the first three passages cited above.

[295] Cf. especially VAT 8520 (MKT I pp. 346ff.).
[295a] The reading *ša-ki-il-tum* given in MKT was corrected by Thureau-Dangin [5] p. 23 note 1.
[295b] MKT I p. 347.
[295c] MKT I p. 341.
[295d] MKT I p. 318.
[295e] Thureau-Dangin [5] p. 23 note 1.

Ub. YBC 7326

(Photograph: Plate 43; copy: Plate 17)

Transcription

Obverse

1*šum-ma ši-na ta-ar-ba-ṣú-ú-a-mi*
2*ta-ar-ba-ṣú-ú e-li ta<-ar>-ba-ṣí*
38,20 *i-te-er li-li-du e-li li[-li-]di*
410,50 *i-te-er*

Reverse

[1,4]0	50	8,20
1,20	40	
1,40	41,40	10,50
1,10	29,10	

Translation

Obverse

1If, (with regard to) two of my sheep-folds,[295f]
2–4(one) sheep-fold[295g] exceeded the (other) sheep-fold[295g] by 8,20; the l[am]bs[295g] (of the one) exceeded the lambs[295g] (of the other) by 10,50.

Reverse

See transcription.

Commentary

It is obvious that the obverse does not state the complete problem. Let n_1 and n_2 denote the number of sheep in the first and second folds respectively, x_1 the number of lambs[295h] bred in the first fold, x_2 the corresponding number in the second; then the given conditions

$$n_1 - n_2 = 8{,}20 \tag{1}$$

$$x_1 - x_2 = 10{,}50 \tag{2}$$

would not be sufficient to determine four unknown quantities n_1, n_2, x_1, x_2.

The problem, however, is in a sense completed by the numbers given on the reverse, although these numbers in turn become intelligible only when our

[295f] Followed by *-mi*, which indicates direct discourse. One can also translate with no essential change in meaning, "If my sheep-folds are two (in number)".
[295g] The text uses the plural of the noun, but the singular for the verb.
[295h] The meaning "young ram" for *lillidu* was established by Landsberger [2] p. 156; for the mathematical texts, we make the assumption that *lillidu* and silá-DU, its Sumerian synonym, simply refer to the age, not the sex, of the lambs.

text is compared with another, published example[295i] which has been understood only in part. This latter problem also deals with two sheep-folds and gives two conditions parallel to (1) and (2):

$$n_1 - n_2 = a \qquad x_1 - x_2 = b,$$

where a and b are given numbers.[295j] At the beginning, however, four additional parameters are given in the form of a statement that in the first fold 1,40 bred 1,20 (text: 1 tùr 1,40 1,20 *ú-ṭa-li-id*) whereas in the second 1,40 bred 1,15. It is evident that these numbers are analogous to the numbers 1,40 1,20 and 1,40 1,10 on the left-hand side of the list given on the reverse of our present text. These numbers acquire significance if we interpret them as percentages, the original sheep of the fold being taken as 100%; in our text, the lambs numbered 80% and 70% of the original sheep. In other words, if β_1 lambs are produced per α = 1,40 sheep in the first fold, and if β_2 lambs are bred per α = 1,40 sheep in the second fold, then we are given two additional conditions, namely,

$$(3) \quad \frac{n_1}{x_1} = \frac{\alpha}{\beta_1} \qquad (4) \quad \frac{n_2}{x_2} = \frac{\alpha}{\beta_2}.$$

The equations (1) to (4) are sufficient to determine the unknown quantities, and one finds[295k] exactly the numbers given in the list on the reverse. This list can then be interpreted as follows:

α = 1,40	n_1 = 50,0	$n_1 - n_2$ = 8,20
β_1 = 1,20	x_1 = 40,0	
α = 1,40	n_2 = 41,40	$x_1 - x_2$ = 10,50
β_2 = 1,10	x_2 = 29,10	

Two other problems concerning sheep in previously published material are too fragmentary to permit full understanding.[295l] Finally, one should mention the obverse of **YBC 7273**:[295m]

1,40	.....[295n]	26	8,20
1,20	48[295o]	7(?),40(?)	8
1,40	30(?)[295p]	8	4,10(?)
1,10	42		
		.....	.....

The percentages are again 80% and 70%, respectively.

It should be noted that it follows from these examples that *tarbaṣu* (Sumerian tùr) not only means sheep-fold but also the number of sheep in the fold.

Uc. YBC 10522

(Photograph: Plate 44; copy: Plate 18)

Transcription

Obverse

[1]*tu-úr-ma ni-in(?)-da(?)-nam(?) ṣa-[b]a-at*
[2]1,20 GAR *da(?)-an-nu-um*
[3]*a-na(?)* 6 uš *ša* SAL+KU-a-ni *iṣ-ba-[t]u*
[3a]*bi-il-ma*
[4]8 *i-il-li*<*-a*>*-ku* <½->*šu ḫe-pé-ma* 4 *[i-]i[l-li-a-ku(?)]*
[5][4] *ša i-li-a-kum šu-ta[-ki-il-ma* 16 .....]

Reverse uninscribed.

Translation

[1]Turn around, and take .....
[2]1,20 GAR .....[296]
[3–3a]multiply by 6, the length which his sister(?) took, and
[4]8 will come up for you. <Hal>ve it, and 4 will [come up for you].
[5]Squ[are 4], which came up for you, [and 16 ....].

Commentary

This text is clearly only part of the calculation of a problem which is lost. For a similar case of a partial calculation, see problem-text C (p. 45), which, fortunately for us, is the duplicate of the last half of problem-text B.

The numerical calculations involved are:

$$1{,}20 \cdot 6 = 8 \qquad \tfrac{1}{2} \cdot 8 = 4 \qquad 4^2 = 16.$$

[295i] YBC 4669, No. 8, published MKT III p. 28. Von Soden [1] p. 203, note 1 corrected the readings but retained the erroneous interpretation of the numbers.

[295j] Unfortunately, only a = 1,0 is preserved; one can show, however, that b must be an integer greater than 48 to admit a solution.

[295k] By $x_1 = \frac{\alpha b - a\beta_2}{\beta_1 - \beta_2} \cdot \frac{\beta_1}{\alpha}$, $x_2 = \frac{\alpha b - a\beta_1}{\beta_1 - \beta_2} \cdot \frac{\beta_2}{\alpha}$.

[295l] YBC 4669, No. 7 (MKT III pp. 27f.) and VAT 8522 rev. 1ff. (MKT I p. 369); cf. von Soden [1] pp. 200 and 203, note 1. In VAT 8522, α = 1,40 β = 1,20 again occur, and correspondingly we have $\frac{1{,}20}{1{,}40} \cdot 16{,}40 = 13{,}20$. YBC 4669 rev. II 13 reads 9 (instead of 6) according to a recent collation.

[295m] The tablet measures 7 by 7 cm. The reverse has been erased except for two lines at the bottom of the right-hand column which give the personal names Ib-qú-ša and Ì-lí-...-....

[295n] Traces of an erased(?) number-sign written vertical to the remaining signs of the obverse.

[295o] Much pressed.

[295p] Probably erased.

[296] This translation assumes an erroneous nominative for accusative at this point.

§ 9. Old-Babylonian Lists of Coefficients

Introduction

The two lists published here, Ud and Ue, are the first of their kind to be discovered.[297] They contain the values of various coefficients which it was necessary to know in order to solve certain types of mathematical problems. The structure of both lists is identical: on the left is given the value of the coefficient, on the right the object or figure to which it applies. Only Ud has a heading, in line 1, describing the nature of the entries:

"Its igi-gub-ba. The thing in connection with which the operation is to be performed."[298]

Igi-gub-ba means literally something like "fixed (or established) fraction."[299] A good illustration of the application of this concept in Old-Babylonian mathematics is to be found in problem-text Oa (p. 97), where the volume v of a certain type of brick is implicitly given as 0;0,0,41,40 volume-SAR, and the weight w (in units of ma-na) is found by multiplying v by 12,0,0, which is expressly called "its igi-gub-ba". Here the igi-gub-ba has the value which was considered standard in the Old-Babylonian period for the relation between a weight reckoned in ma-na units and the volume of bricks computed in units of the volume-SAR. In another mathematical text,[300] the term igi-gub-ba refers to the value of $\frac{1}{4\pi}$ which is used in connection with the formula for the area of a circle,[301] namely, $A = \frac{c^2}{4\pi}$. Note also the instructive use of *igigubbūm* (Akkadian loanword from Sumerian igi-gub-ba) mentioned below in the analysis of Ud, line 18. It seems to follow from these examples that the literal meaning of igi-gub-ba, "fixed (or established) fraction", is to be understood in the more general sense of "coefficient"—whether an absolute constant which follows from definitions, as in the case of $\frac{1}{4\pi}$ in the formula for the area of a circle, or an "empirical" constant, as when it expresses the relation between the weight and volume of bricks or the relation between a quantity of asphalt and the area which it will cover.

In the following pages, we present the contents of Ud and Ue by combining transcription, translation and commentary line by line or, where convenient, in groups of lines. In the translations, the values of the coefficients are left out.

These two lists make it only too clear how small a fraction of the Old-Babylonian mathematical repertoire we now have at our disposal.

Ud. YBC 5022

(Photograph: Plate 44; copy: Plate 18)

Obverse

1igi-gub-ba- *ša né-pi-iš-tum*

Translated and discussed above.

2 4,30 *na-az-ba-al* sig$_4$
3 7,12 *na-al-ba-an* saḫar
4 5,24 *na-al-ba-an* sig$_4$-áb
5 3,22,30 *na-az-ba-al-ša*
6 2,42 *na-al-ba-an* sig$_4$-al-ùr-ra
7 1,41,15 *na-az-ba-al-ša*
8 1,40 *na-az-ba-al* saḫar
9 1,12 *na-al-ba-an* 1 kùš sig$_4$
10 45 *na-az-ba-al-ša*
11 4,48 *na-al-ba-an* ½ kùš sig$_4$
12 4,23,36 *na-az-ba-al-ša*
13 9 *na-al-ba-an* ⅓ kùš sig$_4$

2"*Nazbalum* of brick.
3*Nalbanum* of soil.
4*Nalbanum* of half-brick.
5Its (i.e., the half-brick's) *nazbalum*.
6*Nalbanum* of kiln-burnt brick.
7Its (i.e., the kiln-burnt brick's) *nazbalum*.
8*Nazbalum* of soil.
9*Nalbanum* of 1-kùš brick.
10Its (i.e., the 1-kùš brick's) *nazbalum*.
11*Nalbanum* of ½-kùš brick.
12Its (i.e., the ½-kùš brick's) *nazbalum*.
13*Nalbanum* of ⅓-kùš brick."

We cannot decide at present whether *nazbalum* (which also occurs in lines 15 and 44 of Ud and lines 32, 41 and 49 of Ue) refers to a receptacle for transporting bricks and similar materials or, perhaps less likely,

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[297] Both were discovered by Dr. A. Goetze, M.M.E., to whom are due a number of readings, particularly in Ue.

[298] The syntax here is based on the reading igi-gub-ba-*ša né-pi-iš-tum* in spite of the position of the *ša* at the beginning of the right-hand side of the tablet; the only other possible reading, igi-gub-ba *ša né-pi-iš-tum*, would be ungrammatical. "Its" of course refers to the second half of the line.

The rather long-winded translation of *nēpištum* is based on the context as well as the use of *nēpešum* in the sense of "(mathematical) procedure, solution" and $^{\circ}$*ṣṣpš* with the meaning "to perform (mathematical operations)".

[299] For discussions of igi-gub(-ba) and references to its occurrences in previously published mathematical texts, see Thureau-Dangin [13] pp. 80–82 and TMB pp. XII, 218 and 240; Neugebauer MKT II pp. 17a, 28a and 51.

[300] Cf. the discussion by Thureau-Dangin [13] p. 80 or Neugebauer MKT II p. 21.

[301] For details, see above, p. 9.

the transportation itself. Similarly, there seems to be room for doubt whether *nalbanum*, which is also attested in line 14 of Ud, refers to the "mold", the process of "molding", or even a 'structure' mentioned in mathematical texts under the name giš-ù-šub.[302] Cf. also the latest discussion of *nalbanum* by von Soden [4] p. 64, note 3, who reaches the conclusion that *nalbanum* is a "Ziegelbehälter", which, depending on the size, can be a small "Ziegelform" or a large "Ziegelgrube". For a discussion of the relations between the values of the coefficients and the dimensions of the bricks which may be deduced therefrom, see the Excursus on the Bricks, p. 137. The item in line 8 also occurs in line 32 of Ue, but the coefficient given there is 6,40.

148,20 *na-al-ba-an* im-. . .
"*Nalbanum* of . . . . . -clay."

For *nalbanum*, see the discussion of lines 2–13. Read perhaps im-dugud (cf. Deimel ŠL 399, 209b and 209c), for which we are given the choice between the meanings "clod" and "'heavy' clay"; or read perhaps im-ge$_6$, "'black' clay".

157,38 *na-az-ba-al-ša*
"Its (i.e., the . . . . . -clay's) *nazbalum*."

For *nazbalum*, see the discussion of lines 2–13.

167,12 *ša*[303] sig$_4$-anše
"Of[303] the brick pile."

The value 7,12 is to be identified with the coefficient 7;12 which is implicitly involved in a series of problems[304] concerning a pile of bricks (sig$_4$-anše). As we have seen, this is the number by which one must multiply a volume V expressed in units of the volume-SAR in order to get the number N (expressed in units of the brick-SAR = 12,0) of bricks of "type 1". The number 7;12 thus represents the proportion $\frac{N}{V}$. Since the size and shape of the "type 1" brick are fixed, it follows that the value 7;12 for $\frac{N}{V}$ is a constant. Line 40 of Ue is parallel to this line.

173,45 *ša* im-dù-a
"Of the im-dù-a."

Im-dù-a apparently means "a wall of rammed earth".[305] It is quite possible that 3,45, the coefficient, is to be associated with 3,45 (probably to be interpreted as 0;3,45 SAR[306]) which is given as the volume of the daily work assignment per laborer in three problems[307] dealing with an im-dù-a. Unfortunately, no satisfactory mathematical interpretation of these problems has yet been offered. Line 13 of Ue is a parallel entry.

1815 *ša* esir(?)-è-a
"Of asphalt(?)."

The reading esir is not without difficulty since the second element is written with an enclosed A instead of NUMUN. A similar difficulty is encountered in the case of esír in the apparently parallel line 31 of Ue.

In AO 6770,[308] the number 15 is expressly called *igi-gub-bu-um*[309] in connection with asphalt (esir-è-a). According to the interpretation proposed by Thureau-Dangin, 0;15 is the coefficient by which one must multiply an area expressed in units of square-kùš in order to find the number of sìla of asphalt required to cover the area in question.

1930 ša sag-dù
"Of the triangle."

The coefficient is perhaps to be interpreted as 0;30, the constant by which the product of the length and width of a right

[302] Cf. MKT II p. 34b for references.

[303] The comparison of lines 16 and 38 of Ud permits us to make a conjecture as to why the numbers in some lines are immediately followed by a noun (or nominal phrase) while other lines show the pattern number-*ša*-noun. In line 38, where the noun directly follows the number, we can observe that the number is actually the value for the following noun, namely, the number of bricks of a certain type covering a standard area. In line 16, however, where the number is followed by *ša* and then a noun, the number is not the actual value of the noun in question but simply a fixed metrological proportion between disparate units—a proportion which must be known before making the transformation from one type of metrological unit (the volume-SAR) to another (SAR = 12,0 bricks). If this analysis of the difference in meaning dependent on the presence or absence of *ša* is correct, one should understand *ša* ("of") in the sense of "in connection with" or the like. The above remarks would apply at best only to Ud, since *ša* is rarely used in Ue; contrast, e.g., Ud, line 17 with the parallel line 13 in Ue.

[304] Discussed p. 94 and note 245.

[305] According to Thureau-Dangin TMB p. 239.

[306] The small assignment involved (cf. the further references cited MKT II p. 57) is probably due to a combination of two or more different operations which had to be performed by the worker. For the combination of operations, cf. e.g., pp. 85f.

[307] YBC 4673, Nos. 9–11; published MKT III p. 30.

[308] Thureau-Dangin [13] pp. 79–81; Neugebauer MKT II p. 65 No. 4.

[309] For this term, cf. above, p. 132.

triangle must be multiplied to get the area.

[20]6(?) ša GÁN-MAN
"Of GÁN-MAN."

Dr. Goetze suggests a comparison with GÁN-TAB-*am* in Strassburg 364 rev. 6 (MKT I p. 249), for which, however, von Soden [2] p. 147 has suggested the emendation *bit-qá-am*.

[21]45(?) *ù* 10 *ša* GÁN-UD-SAR
"Of the segment of a circle."

For references to GÁN-UD-SAR in mathematical texts, see MKT II p. 33b and the Vocabulary below.

[22]1,12 *ra-ṭù-um ša* urudu
[23]1,30 *ra-ṭù-um ša* kù-babbar
[24]1,48 *ra-ṭù-um ša* guškin
[22]"*Rāṭum* of copper.
[23]*Rāṭum* of silver.
[24]*Rāṭum* of gold."

To judge from the available contexts in which it appears, the meaning of *rāṭum* seems to lie in the sphere of ditches or pipes and tubes. Line 22 is parallel to Ue, line 36. For line 24, cf. Ue, line 34.

[25]2,15 *ru-uq-qú ša* guškin
[26]4 *ru-uq-qú ša* kù-babbar
[27]2,24 *ru-uq-qú ša* an-na
[28]2,8 *ru-uq-qú* ša sud-...
[25]"*Ruqqū* of gold.
[26]*Ruqqū* of silver.
[27]*Ruqqū* of lead.
[28]*Ruqqū* of ....."

For the term *ruqqum* (plural: *ruqqū*[309a]), see the Excursus on *ruqqum*, p. 138, where an attempt is made to explain the coefficient of line 26. The last word in line 28 is probably sud-á[g](?), for which cf. sù-ud-ág(-gá), which Thompson DACG p. 76 proposes to translate by "brass."

[29]6 *ša* giš-má-lá
"Of the cargo boat."

For giš-má-lá, cf. Thureau-Dangin [8] pp. 9f. and Salonen WB p. 36.

[30]6,40 *ša* GÁ-DUB še
[31]3,36 *ša* GÁ-DUB in[309b]-nu-da
[32]3,45 *ša* GÁ-DUB ...
[33]15 *ša* GÁ-DUB a
[30]"Of GÁ-DUB of barley.
[31]Of GÁ-BUD of straw.
[32]Of GÁ-DUB of .....
[33]Of GÁ-DUB of water."

The meaning of GÁ-DUB (read é-dub?) here as well as in line 61 is not clear. Perhaps the meaning is something like "storage receptacle" or "storage house".

In connection with line 31, it should be mentioned that Ue, line 22, lists the coefficient 3,36 for in-nu-da, "straw". In Deimel ŠL p. 1134b, N 626, the only references given for in-nu-da are those supplied by Goetze from Hittite texts. Two other occurrences are concealed in gi-gur-in-nu-da (Deimel ŠL 85, 216) and gur-in-nu-da-mušen (Deimel ŠL 111, 21). The following are references to some occurrences of in-nu-da in Old-Babylonian texts: Thureau-Dangin TCL I No. 142, 16; Frank SKT No. 38, rev. 12; Faust YBT VIII No. 173, 14; and probably Grice YBT V No. 188, 3.

Reverse

[34]3,20 *ša* sig$_4$-al-ùr-ra
[35]1,52,30 *ša* sig$_4$-al-ùr-ra ki-2
[34]"Of kiln-burnt brick.
[35]Of the second kiln-burnt brick."

For a hypothesis about the values of the coefficients, see p. 138. "Second" in line 35 refers to the fact that kiln-burnt brick is already mentioned in the previous line.

[36]4,24 *ma-aš-qá-al-tum ša* sig$_4$
"*Mašqaltum* of brick."

The exact meaning of *mašqaltum* is unknown to us, although it seems fairly certain that the root is *šql*, "to weigh". Cf. *ma-aš-qa-la-tu* (plural), Scheil MMAP XXII No. 151, 3.

[37]1,12 *ša-ḫa-al-lu-ú-um*

Meaning unknown. Dr. Goetze calls attention to *šaḫillu*, Meissner SAW p. 93a.

[38]14,24[309b] *ta-ad-di-tum ša* sig$_4$
"Layer of brick(s)."

The number 14,24 is the same as "1 SAR (= 12,0) 2,24", which is given as the answer in problem-text O rev. 6 (pp. 92 and 94) to the question regarding the number of bricks of "type 1" necessary to cover an area of 1 area-SAR. The same term, *taddītum*, appears in the question asked there: 1 SAR gagar *ta-ad-di-ta-am* en-nam ì-díb.

[39]45 *mu-ut-ta-al-li-a-tum*

Meaning unknown; apparently the plural of *muttallītu*, feminine participle of *utlellū*.

[309a] It is possible that we are wrong in taking *ru-uq-qú* as a plural since the plural of *ruqqum* elsewhere ends in *-ātum*, e.g., CT 2, 1, lines 45 and 47 and Pfeiffer-Lacheman HSS XIII No. 435, line 22 (*ru-uq-qé-tu*$_4$); contrast, on the other hand, the late plurals *ruq-qe* entered in Delitzsch HWB 627b. Our reason for considering *ru-uq-qú* a plural is the otherwise strict use of *-um* for the singular in this text.

[309b] Read so against our copy on plate 18.

40 18 *i-im-lu-ú[-um]*
In all probability, this corresponds to line 14 of Ue, which gives the coefficient 18 for im-lá. Akkadian *ʾimlūm* would accordingly be a loanword from Sumerian im-lá. The coefficient 18 is mentioned in mathematical texts dealing with im-lá (MKT I pp. 145f. obv. III 6, 11, 15 and 20).

41 6 *ma-aš-šu-ú-u[m ša* si]g$_4$(?)

42 1,40 *ma-aš-šu-ú-um ša* saḫar
[41] "*Maššū[m* of bri]ck(?).
[42] *Maššūm* of soil."
For *maššūm*, cf. Deimel ŠL 371, 48p and 481, 53f; since the root is almost beyond question *nšį*, we have here some instrument or receptacle for lifting or raising.

43 10 gín éš-kàr lú-1
"10 gín (volume) the assignment of 1 man."
This volume as the amount of daily work per laborer is frequently attested in mathematical texts; cf., e.g., p. 162 under éš-kàr. Cf. also Ue, line 20.

44 1,20 *ù* 15 *na-az-ba-al* a
"*Nazbalum* of water."
For *nazbalum*, see the discussion of lines 2–13.

45 2,13,20[310] *ù* 1,15 *ša* GIŠ-DUB-ÍL
"Of GIŠ-DUB-ÍL."

46 2[. . .],30 *ša* im
"Of clay."

47 [12,3]0 *ša* GÁN-má-ru$_4$
"Of GÁN-má-ru$_4$."
The reading -ru$_4$ was proposed by Dr. Goetze, to whom the possibility occurred that we may have here a variant spelling of the word for "arrow-quiver", for which cf. Ehelolf [1]. Read possibly -kur$_9$? The restoration of the value of the coefficient is based on line 6 of Ue.

48 53(?),20 *ša* GÁN-zà-mí
"Of GÁN-zà-mí".
For the stringed musical instrument called zà-mí, which also occurs in line 50, cf. Landsberger [1] p. 155. Another coefficient is given for GÁN-zà-mí in Ue, line 4.

49 22,30 *ù* 12,30 *ša* GÁN-zarà
"Of GÁN-zarà."
Another coefficient for the same object is given in line 51 of Ud. The coefficient 12,30 is given for GÁN-zarà in Ue, line 5.

50 26,40 *ša* geštú zà-mí
"Of the 'ear' of zà-mí."
For zà-mí, see the discussion of line 48.

51 3,30 *ša* GÁN-zarà ki-2
"Of the second GÁN-zarà."
"Second" refers to the fact that the same object is already mentioned once before, in line 49.

52 4,20 *ša* GÁN-UD-SAR ki-2
"Of the second segment of a circle."
"Second" refers to the previous entry of the same object in line 21, q.v.

53 4,36,10 *ša* GÁN-MAN ki-2
"Of the second GÁN-MAN."
The previous mention of the same object in line 20, q.v., explains "second" here.

54 45 *ša* GÁN-UD-SAR ki-3
"Of the third segment of a circle."
Lines 21 (q.v.) and 52 explain the use of "third" here.

55 [2]0,48,6 *ša* gir$_4$-ad-KID
"Of the wicker-worker's (asphalt) oven."
For gir$_4$-ad-KID, see Deimel ŠL 430, 2.

56 26(?) im-babbar 54 *ka-lu-ú-um*
"Gypsum." "Yellow ochre(?)."
For im-babbar, "gypsum", cf. Thompson DACG pp. 148f. See also Ue, line 15. The meaning "yellow ochre" is proposed for *kalūm* by Thompson DACG pp. 31f.

57 30 *ša* pisan(?) 20 *ša* á-kur-ra
"Of the pisan(?)-vessel." "Of á-kur-ra."

58 4,48 *ša ku-bu-ur-re-e* giš
"Of the thickness of a log."
In problem-text Ec (p. 57), the square of the circumference of a log, measured in units of GAR, is multiplied by the coefficient 4,48 to give the thickness in units of the thickness-sìla. For details, other references to the use of sìla in connection with the thickness of logs, and an attempt to explain the value of this coefficient, see the commentary to Ec. Line 35 of Ue is parallel to this line

59 40 *ša*-KAK(-)UD-*šu-rum*

60 12 *ma-al-ta-ak-tum ša mu-ši-tim*
"*Maltaktum* of the night."
For *maltaktum*, see von Soden [3] p. 43, note 2. For a parallel line, see Ue, line 50.

61 4,48 GÁ-DUB še
"GÁ-DUB of barley."
See the discussion of line 30 for GÁ-DUB.

62 24 PI-*lu-ú-tum ša* G[I . . .]

63 1 *ši-li-ip-tum* 1 *zi-it-tum*
"Diagonal." "Inheritance."
Line 11 of Ue corresponds to the first part of this line.

64 1 gi-kud-a
"Cut reed."

---

[310] The first number may start with 1, and the 13 may be 15 or more.

For references to the occurrence of "cut reed" in the mathematical texts, see MKT II p. 30a under kud.

6 5 57,36 *ša-...-lu(?)-ú-um*
6 6 ^{d}nidaba
"The deity Nidaba."

This line implies that the whole text belonged to the so-called "wisdom" literature. For mathematical texts which mention Nidaba in their colophons, see MKT II p. 35b.

Ue. YBC 7243

(Photograph: Plate 49; copy: Plate 23)

Obverse

1 5 KA-[.....]
2 15 GÁN-[.....]
3 10 GÁN-...[-.....]
4 26,40 GÁN-zà-mí

Another value is given for the coefficient "of GÁN-zà-mí" in Ud, line 48, q.v. It is perhaps significant, however, that the same coefficient is given for the "ear" of zà-mí in line 50 of Ud.

5 12,30 GÁN-zarà

The same value for the coefficient of GÁN-zarà is given in Ud, line 49.

6 12,30 GÁN-má-ru$_4$

This corresponds to Ud, line 47, q.v.

7 5,24(?) GÁN-UD-A(?)-SAR
8 14,36,10 GÁN-UD-SAR(?) ki(?) in-ŠUB
9 45(?) GÁN-...
10 1,24,51,10 *ṣi-li-ip-tum* íb-si$_8$
"Diagonal, square root."

This line is discussed on p. 43.

11 1 *ṣi-li-ip-tum*
"Diagonal."

This line corresponds to the first part of line 63 of Ud.

12 45(?) giš-a-bal
"Water-dipper(?)."
13 3,45 im-dù-a

This line corresponds to Ud, line 17, q.v.

14 18 im-lá

This probably corresponds to Ud, line 40, q.v.

15...8(?) im-PAR-PAR

For im-PAR-PAR, which (like im-babbar in line 56 of Ud) is supposed to mean "gypsum", see Deimel ŠL 399, 194 and Thompson DACG p. 150.

16 44 im-KI(?)-a
17 6,40 im-dub(?)
18 4,16 .....
19 5 giš-má
"Ship."
20 10 éš-kàr <ki(?)->lá
"Assignment (for) a <ki(?)>-lá."

For references to a daily work assignment of 10 volume-gín per worker in connection with a ki-lá, see the Vocabulary, p. 162, under éš-kàr. Cf. also Ud, line 43.

21 20 ^{pa_5}*ši-lu-tum*

As we have shown in connection with problem-texts L and M (cf. especially pp. 85, 88 and 90), 0;20 volume-SAR (in apposition to *šīlūtum*) is the work assignment per man per day at the upper level in the digging of a canal.

22 3,36 in-nu-da
"Straw."

For what is probably a parallel entry and for a discussion of in-nu-da, see Ud, line 31.

23 1,16 áraḫ in-nu-da
"Storehouse(?) (for) straw."

Áraḫ is written É-UŠ-GÍD-DA; for the reading and meaning, cf. Meissner [2], col. 242.

24 10,45,6 túg-ḫá *ša i-na ki(?)-ri-im*
"Bolts of cloth (or garments) which are in ....."
25 15 *ni-d/ki-ir-tum*
26 48 *me-e* a-šà
"Water of the field."

Reverse

27 2,13,20 *ki-it-mu-um*

For the latest discussion of *kitmum*, see Meissner [3] pp. 25f.; according to him, the meaning is "black dye".

28 20 ...
29 20 làl(?)
"Honey(?)."
30 15 a
"Water."
31 15 esír(?)-ḫá(?)
"Asphalt(?)."

For the difficulty in reading esír and for an attempt to explain the use of the coefficient, see Ud, line 18, which seems to be parallel to this line. In Ud, line 18, the material is called esir(?)-è-a, which, according to Deimel ŠL 579, 458, corresponds to Akkadian *kupru*. Esír(?)-ḫá(?), if the reading is correct, should be compared with esirḫa (written with the sign ESÍR containing ḪÁ), which also corresponds to Akkadian *kupru*; cf. Deimel ŠL 488.

326,40 *na-az-ba-al* saḫar
"*Nazbalum* of soil."
For *nazbalum*, see the discussion of Ud, lines 2–13. The same item has the coefficient 1,40 in Ud, line 8.

339 ki-lá(?)-bi(?) *ša* urudu(?)
Weight(??) of copper(?)."

341,20 *ra-ṭù-um* <*ša*> guškin
"*Rāṭum* <of> gold."
The reading of the first part of this line is due to Dr. Goetze. A priori, we would prefer to restore *ra-ṭù-um* at the beginning of line 37, which would make it parallel to Ud, line 24.

354,48 *ku-bu-ur i-ṣí-im*
"Thickness of a log."
This line is parallel to Ud, line 58, q.v.

361,12 *ra*(?)-*ṭù*(?)-*um ša* urudu(?)
"*Rāṭum*(?) of copper(?)."
If the reading is correct, this is parallel to line 22 of Ud.

371,48 ...-...-... *ša* [guš]kin
"..... of gold."
See the discussion of line 34.

382,24 *a*[-*gu-ru-*]*um*
"Kiln-burnt brick."
If the restoration is correct, the coefficient would correspond to the number of bricks of "type 5", called sig_4-al-ùr-ra, required to cover an area of 1 area-SAR; cf. p. 94.

391,12 .....

407,12 sig_4-[an]še
"Brick pile."
For an explanation, see the parallel entry, line 16 of Ud.

414,30 *na-az*[-*ba-lu-um*] ...-*lu-um*
"..... *naz*[*balum*]."
For *nazbalum*, see the discussion of Ud, lines 2–13.

423,35 .....-*tum*(?)

4348 1,40 1,20 *ú-k*/*lu-*...-...-*um*

442(?),42 ^{sig_4}*a-gu-ru-um*
"Kiln-burnt brick."
If the restoration is correct, the coefficient corresponds to that given for *nalbanum* of sig_4-al-ùr-ra in Ud, line 6.

451,41,15 *ša-*.....-*tum*(?)

4645 *ta-la-ak-tum*
For the various meanings of *tallaktum* ("going", "step", "way", "street", "avenue", etc.), see Baumgartner [1] pp. 123ff.

473 *ša* ½ kùš
"Of ½-kùš <brick(?)>."
See below, note 311.

485,20 sa-tuk
The last sign is possibly KIN.

491,15 *na-az-ba-lu-um*
For *nazbalum*, see Ud, lines 2–13.

5012 *ma-al-ta-ak-tum*
This is parallel to Ud, line 60 which gives the coefficient 12 for "*maltaktum* of the night".

5126,40 *ru-qú-um ša* kù-babbar
"*Ruqqum* of silver."
Ud, line 26, gives 4 as the coefficient for "*ruqqū* of silver", for which cf. pp. 138f.

5220 NI LU

Excursus on the Bricks

The list given below tabulates the information about various types of brick contained in lines 2 and 4–13 of Ud. The numbers assigned to the types of brick in the last column are arbitrary, except that 1, 3, 4 and 5 were deliberately chosen to coincide with the same numbers in the table given on p. 93.

Object	*Nalbanum* (α)	*Nazbalum* (β)	Type
sig_4		4,30,0 (line 2)	1
sig_4-áb	5,24 (line 4)	3,22,30 (line 5)	3
sig_4-al-ùr-ra	2,42 (line 6)	1,41,15 (line 7)	4
1 kùš sig_4	1,12 (line 9)	45,0 (line 10)	5
½ kùš sig_4	4,48 (line 11)	4,23,36 (line 12)	6
⅓ kùš sig_4	9,0 (line 13)		7

Let $\alpha_1, \alpha_2, \ldots, \alpha_7$ represent the numbers given in the text for the *nalbanum* of bricks of the types 1, 2, ..., 7; and $\beta_1, \beta_2, \ldots, \beta_7$ the numbers given for the *nazbalum*. It is evident that the proportion between the *nalbanum* and *nazbalum* has the same value for types 3, 4 and 5;[311] in other words,

$$\frac{\alpha_3}{\beta_3} = \frac{\alpha_4}{\beta_4} = \frac{\alpha_5}{\beta_5} = \frac{8}{5}.$$

We may further note that

$$\alpha_3 : \alpha_4 : \alpha_5 = \beta_3 : \beta_4 : \beta_5 = 18 : 9 : 4,$$

which can easily be extended to

$$\beta_1 : \beta_3 : \beta_4 : \beta_5 = 24 : 18 : 9 : 4.^{312}$$

The meaning of this proportion is clear, since we have already seen[313] that the volumes (or areas) of single bricks of types 1, 3, 4 and 5 in problem-text O stand

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[311] In the case of type 6, line 12 gives $\beta_6 = 4,23,36$ instead of the expected 3. It is perhaps significant that Ue, line 47 gives 3 as the coefficient "of ½-kùš <brick(?)>".

[312] One is tempted to emend saḫar in line 3 to sig_4, since the number 7,12 given there is precisely what one would expect for the *nalbanum* (α_1) of the brick of type 1 (sig_4).

[313] P. 93.

in the relation:

$$v_1 : v_3 : v_4 : v_5 = 3 : 4 : 8 : 18.$$

and therefore

$$\bar{v}_1 : \bar{v}_3 : \bar{v}_4 : \bar{v}_5 = 24 : 18 : 9 : 4$$

which is precisely the relation of the α's and β's. The names given for these types of brick are the same in both texts, except that type 5 is called sig_4-al-ùr-ra in problem-text O but 1 kùš sig_4 in the present text.[314] It should also be pointed out that the numbers given for the *nalbanum* of these types of brick correspond to the absolute value of $\bar{v} \cdot \overline{12,0}$, which, as we have seen,[315] represents the ratio between the number of bricks expressed in units of SAR = 12,0 and the volume of a pile of bricks expressed in terms of volume-SAR. Because of this, one is tempted to view the values of the *nalbanum* as the number of SAR (= 12,0) of bricks of the different types in 1 volume-SAR, which may have been a unit of special significance in the molding of bricks. For our present purposes, however, it is sufficient to say that the numerical relations can be stated in the form

$$\alpha_3 = \frac{0;5}{v_3} \qquad \alpha_4 = \frac{0;5}{v_4} \qquad \alpha_5 = \frac{0;5}{v_5}.$$

If we extend this relation to $\alpha_6 = 4{,}48$, we find the volume of a single brick of type 6 to be

$$v_6 = 0;0,1,2,30 \text{ volume-SAR};$$

assuming that the height is the same as in the case of types 1 to 5, namely, 5 šu-si = 0;10 kùš, we can compute the area of a brick of type 6:

$$a_6 = \frac{0;0,1,2,30}{0;10} = 0;0,6,15 \text{ area-SAR}.$$

From this value and the fact that the type in question is called the $\frac{1}{2}$-kùš brick, it follows that the length and width are

$$l_6 = b_6 = \tfrac{1}{2} \text{ kùš}.$$

Repeating the same process for $\alpha_7 = 9{,}0$, we find

$$v_7 = 0;0,0,33,20 \text{ volume-SAR};$$

if we again assume a height of 5 šu-si as before, we get

$$a_7 = \frac{0;0,0,33,20}{0;10} = 0;0,3,20 \text{ area-SAR},$$

which would lead to the following dimensions for the "$\frac{1}{3}$-kùš brick":

$$l_7 = 12 \text{ šu-si} \qquad b_7 = \tfrac{1}{3} \text{ kùš}.$$

[314] The name 1 kùš sig_4 is explained by the fact that the brick is 1 kùš square.

[315] P. 95, note 245.

If we, perhaps foolhardily, set up the hypothesis that the numbers given in connection with sig_4-al-ùr-ra in lines 34 and 35 of Ud stand in the same relation to the volume of a single brick, namely, $\frac{0;5}{v}$, as the numbers given for the *nalbanum* in the case of types 1, 3, 4, 5, 6 and 7, we would get

$v_8 = 0;0,1,30$ volume-SAR $\quad v_9 = 0;0,2,40$ volume-SAR.

If we go still further and assume, as before, that the height is still 5 šu-si, then

$a_8 = 0;0,9$ area-SAR $\quad a_9 = 0;0,16$ area-SAR,

which would lead to the rather good dimensions

$$l_8 = b_8 = 18 \text{ šu-si} \qquad l_9 = b_9 = 24 \text{ šu-si}.$$

With all the necessary reservations, we summarize our present results in the following table which continues the one for bricks of types 1 to 5 on p. 93.

	Type 6	Type 7	Type 8	Type 9	Type 10[316]
l	½ kùš	12 šu-si	18 šu-si	24 šu-si	24 šu-si or 1 kùš
b		⅓ kùš			⅔ kùš or 16 šu-si
n[317]	9,36	18,0	6,40	3,45	4,30

Excursus on *ruqqum*

Some slight progress in the reading, if not the definitive understanding, of the single extant mathematical problem dealing with *ruqqum* has made it possible to present a hypothesis about one of the coefficients given in Ud for *ruqqum* made of various metals (lines 25–28). YBC 4669 (MKT III p. 27) rev. II, No. 6 reads as follows:

1 3 kùš 1 šu-si uš
2 2 kùš 6 šu-si sag
3 gagar[318]-bi en-nam
4 $2\frac{2}{3}$ gín $20\frac{1}{2}$ še
5 1(?) gín kù-babbar sì-ma
6 3 šu-si-ta[318] íb-si_8
7 *ru-uq-qá-am*[319] *im*(?)-*ḫa-ṣú*(?)[320]
8 *i-na-an-na*
9 kù-babbar en-nam sì-ma
10 $2\frac{2}{3}$ gín $20\frac{1}{2}$ še gagar[318]
11 ḫé-gar-ra

[316] Inferred p. 96 above from VAT 6598.

[317] Number of bricks covering an area-SAR.

[318] An examination of the original established this reading beyond doubt.

[319] This interpretation of the signs is due to von Soden [1] p. 203, note 1.

[320] Collation shows this to be the most probable reading of the signs.

The meaning of the problem is:

1 kùš (and) 1 šu-si is the length,
2 kùš (and) 6 šu-si the width.
3 What is its area?
4 2⅔ gín (and) 20½ še (is the area).
5 (When) 1(?) gín silver was given,
6–7 they beat(?) a *ruqqum* 3 šu-si square.
8 Now
9 how much silver was given
10–11 (so that) 2⅔ gín (and) 20½ še area .... put(?)..?

The full details, as can be seen, are still obscure, but we would seem to be justified in setting up a relation between 1 gín silver and an area 3 šu-si square. The area expressed in units of the area-SAR is

$$(0;0,30 \text{ GAR})^2 = 0;0,0,15 \text{ SAR}.$$

The reciprocal of 0;0,0,15, namely, 4,0,0, may be the source of the 4 which is given in Ud, line 26, for "*ruqqū* (plural) of silver".[321]

The mention of area, not volume, in connection with *ruqqum* seems to point to a flat object.[322] This, however, is hard to bring into agreement with previous interpretations. The literature as well as additional occurrences of *ruqqum* are cited by Kraus AB p. 9, note to line 16, where the generally accepted meaning "Metalltopf" is endorsed. E. R. Lacheman in Starr, Nuzi, pp. 539f., subsequently cited passages from Nuzi texts (of which SMN 174 has in the meantime been published in Pfeiffer-Lacheman HSS XIII) which suggested to him the meaning "feeding-trough". It is worth adding that Pfeiffer-Lacheman HSS XIII, No. 70, lines 1 and 4 mention *ruqqum*[322a] because the weight of the first *ruqqum* is given as 1 *bilat* 41 *ma-na*, i.e., about 100 pounds. CT 8, 34b, line 11 and Langdon [1], Plate XXIX (after p. 188), line 1 mention a *ruqqum* of 2 bán, while CT 2, 1, line 9, CT 2, 6, line 13, Ranke BE 6,1, No. 84, line 12 and Meissner BAP No. 7, line 6 mention one of 3 bán; all six of these texts are Old-Babylonian.

§ 10. Miscellaneous Texts from Later Periods

It is merely for the sake of completeness that we include the three following small fragments of mathematical texts. Experience shows that parts of the same tablet frequently turn up in different museums. In addition, since the overwhelming majority of the known mathematical texts are of Old-Babylonian origin, it is of no little interest to get as much information as possible about later periods with respect to such things as the development of the terminology.

For the first two texts of this group, problem-texts V and W, we present the preliminary copies which Dr. F. W. Geers has kindly agreed to publish in this volume. Each had been identified as a "Rechentext" by Dr. E. F. Weidner. Dr. Geers himself, however, suggests the possibility that V belongs with the liver omina, although close parallels are lacking. If the readings *niṣirti barūt*[*i*] (V 1) and *ba-ru-ti* (W 2) are correct, it seems quite possible that both V and W are instructions concerning the secrets of the *barū*-priests, among whose functions was the inspection of sheep livers;[323] if this is so, the term šu-si in V (and perhaps in W) refers to a part of the sheep's liver, not the measure of length of the same name. It ought to be remarked, in this connection, that Weidner [1] has called attention to examples of a type of number speculation in actual liver omen texts. We feel so unsure about all these matters at present that we have decided to include these two fragmentary texts in this book in order to remain on the safe side. Both V and W were found at Nineveh and are to be dated (at least in their present form) to the end of the eighth or the beginning of the seventh century B.C. Problem-text V is written in the Assyrian script, W in the Babylonian.

V. K 11097[324]

(Copy: Plate 19)

1 .....] 3(?) *zittu*(ḫa-la) *niṣirtu*(SAL-urì) *baru*(lú-ḫal)-*t*[*i*(?).....
2 ....] *taš-nin-tum ummāni*(um-me-a) *ša barī*(?)(lú-ḫa[l(?)) .....
3 ...] *abu*(ad) *ana māri-šú ša i-ram-mu ina*(-)*az/ṣ-z/ṣa* ..[....

[321] Ue, line 51, however, gives 26,40 for "*ruqqum* of silver."

[322] If one calculates the dimensions of the *ruqqum* mentioned in YBC 4669 using the generally accepted metric values for gín and šu-si, they turn out to be approximately 5 cm. (= 3 šu-si) by 5 cm. by 3.5 mm.

[322a] Falsely transcribed URUDU*pisannu*(PISÀN) instead of urudušen in spite of parallel passages; cf. the copy of the sign given op. cit., p. 99.

[323] Bezold, Cat. III p. 1137 expresses a similar opinion concerning our problem-text V.

[324] Dr. Geers made two independent copies of this text. The divergences are indicated in the margin on Plate 19. Bezold, Cat. III p. 1137 gives the measurements of the fragment as 2½ by 2 7/16 inches.

4 ...] KAM AM 15(?) *zittu*(ḫa-la) *iš-ta-at*(?) šu-si 7 ...[.....
5 ...].. níg-ŠID *ina*(?) *ši-kin*(?) šu-si *ina*(?) 5(?) šu-si *ina*(?) AN .. B/NA ..[...
6 ...].. *ma mi*(?) itu(?) mu-1-kám *e-si-ḫa aḫu*(šeš) *ana aḫi*(šeš) UD-DA [....
7 ....] *barū*(?)(lú-ḫal(?)) *apkallu*(?)(NUN(?)-ME(?)-...) *et-pe-šú ḫa-as-si mu-du-ú* [.....
8 .....].. B/NA ..... EN .... *qa-ba-a i-šak-ka-nu ši-ki*[n(?) .....
9 ... *ši-k*]*in*(?) šu-[s]i *ki-a-am ul*(nu) *idu*(zu)-*ú* ..... *ik kil la* ..[...
10 .....] níg-ŠID-*šú ina ṣa-a*(?)-*tim ina a-re-e ina* níg(?)-ŠID ...[...
11 .....]DI-*bi re-bu-*ᵓ*u-ú-ma it-tal-pi-it šu u* ...[.....
12 .....]... igi-10-gál-bi *ana* igi-6-gál-bi *i-ši-ma* 1 ..[.....
13 .....-]*ma*(?) *lu-mur* GÁN *ina*(?) *taš*(?)-*nin*(?)-*ti*(?) EŠ KI [....
14 .....]..... *ši-kin* šu-si [.. .....
15 .....:.]..DU ..... *mu* AD ...[.....
16 .....]..... níg(?)-ŠID ....[...
17 .....]....[..... .....

(Following destroyed.)

If the reading *zittu* "inheritance (portion)" in lines 1 and 4 is correct, this text might possibly refer to the division of property. Confirmation of a sort is possibly furnished by line 3: "the father to his son whom he loves ...",[325] and the end of line 6: "(one) brother to the (other) brother...". The continuation is too mutilated to permit an analysis of the details. The only completely preserved operation is contained in line 12: "multiply one-tenth by one-sixth, and (the result is) 1".

W. K 8705[326]

(Copy: Plate 19)

Beginning destroyed.

1 .....]...[.....
2 .....] *ba-ru-ti* ....[.....
3 .....-]*ti* šu-si ..[.....
4 .....] 7 7 *la* ....[.....
5 ..... k]ar(?) *šu*(-)*ši* ... EŠ SIG$_7$ AD ..[...
6 .....]. EŠ *ù* 1-*en ši-kin* šu-s[i.....
7 .....]. TAR *ti* SIG$_5$ *ul šá-ri-ir* 1 *šá* ...[...
8 .....] *ú šá* udu-nitá *šá* dug$_4$-ga-*ma ma ut-ta*[.. .....
9 ....] LUM(?) udu-nitá 6 1 kùš SUK-*lim* igi-6-gál ...[...
10 .. udu-]nitá *kīma ú-kal-li-mu-ka* 4 šu-si ...[.....
11 .....]... *kīma* níg-lal-*ma* 55 *tam-mar šá* dug$_4$-ga-*ma* [....
12 .....]-*ti mi-na-a-te ú-ter-ma iq-bi šá* [.....
13 ....-*šá*(?)-*r*]*a-ar-ma* 9 9 *a-na* 6 šu-si [.....
14 .....]..9 a-rá 6 54 *a-na-ši-*[*i* .....
15 .....] 5 *a-na-ši-i* udu-nitá [.....
16 .....]..10 9 [.....

Note the occurrence of "rams" (*immeru*, written udu-nitá) in lines 8, 9, 10 and 15. At the end (line 14) we find the statement 9 · 6 = 54.

X. MM 86.11.404

(Photograph: Plate 43; copy: Plate 20)

Obverse

(Beginning destroyed.)

1[.....] KU [.....
2[.....] ká(?) [.....
3[.....].. *ina* 1 ma[-na .....
4[..... m]a-na 4½ gín *annaku*(an-na) ..[...

5[*i*-]*na ka-a-ri ra-a-zu* 10 *sa-pa-ni*(?) [.....
6 4,26,40 LÁL KUR 10 *al-lu-ú* ME *ru*(?) TAR *ru*(?) [.....
7 sag al-*ja* en *ina* 1 ma-na 4,26,40[.....
8 nim-*ma* 55,33,20 igi 55,33,20 [.....
9 1,4,48 1,6,40 a-rá 1,4,48 [.....
10 [ni]m 1,12 al mi su si(?) bal lu [.....
11 [.....] nu-zu *a-n*[*a* ...]..... DU ...[.....
12 [.....]...[.....

(Following destroyed.)

Reverse

1[.....]...[.....
2[.....] NE *ja* [.....
3[.....]..... 5 *šú* ..[.....
4[.....].. a-rá 5,40 [.....
5[.....] 40 a-rá 2,30 [.....
6[.....].. 1,20,40 a-rá ..[.....
7[.....].. 7,4,40 a-rá 3 [.....
8[.....].. 1,15,14 uš [.....

[325] But cf. the similar phrase *a-pil-šu šá i-ram-mu*, "his heir whom he loves", in a *barū*-text, Zimmern, Ritualtafeln Nr. 24, line 20.

[326] This tablet measures 2¼ by 2 in. according to Bezold, Cat. III p. 954.

9[.....]... TU TAR LUM ...

10[.....]. 2 GAR [.....

11[.....]..[.....

Left Edge: ...]... KU ...[...

The text contains at least four examples, of which the second is the only one in which some relations can be established. Lines 5–7, which state the problem, end with the question "what is the initial (number?, weight?) of my hoe(s)[327]?". In line 5 we read "in the market(-place or -price?) a *rāzu* ..."; *rāzu* is perhaps to be identified with *ra'izu*, "pitch-fork".[328] In line 6 are mentioned "10 hoes" and the number 4,26,40. The computation begins in line 7 with the subtraction of 0;4,26,40 from 1 ma-na, the result being 0;55,33,20 (line 8). Then follows $\frac{1}{0;55,33,20} = 1;4,48$. Then 1;4,48 is multiplied by 1;6,40 (which is perhaps to be explained as the reciprocal of $0;54 = 1 - \frac{1}{10}$, where 10 is the given number of hoes). The result 1;12 is given in line 10.

Y. VAT 7848

(Photograph: Plate 45; copy: Plate 20)

Obverse

1 1[.....]..... 30 GAM 30 15 .....

2[.....]..... 51,45 . MA 45,4(?) [*mi* G]AM

3[*mi lu*-DU-*ma lu* 45,4 52] GAM 52 45,4 52(?) dal LA(?) UŠ(?) sìla(?) še(?)

4[.....].. GAR(?) . 26 a-š[à] . 2[6 GAM 21,3]6 DU-*ma* 9,21,36 5 sìla 4 ninda 5(?) še(?)

2 5[..... 1 uš 45 sag 1 GA]M 1 1 45 GAM 45 33,45 gar-gar-*ma* 1,33,45 *mi* GAM *mi lu*-DU-*ma*

6[*lu* 1,33,45 1,]15 GAM 1,15 BAR-NUN

3 7[.....].. 30 uš 30 uš 2-*ú* 50 sag-ki an-ta 14 sag ki-ta 30 GAM 30 15

8[14 t]a 50 DUL-DU-*ma ri-ḫi* 36 ½-*šú* 18 . 18 GAM[328a] 18 5,24 5,25[329] ta 15 DUL-DU-*ma*

9[*ri-*]*ḫi* 9,36 *mi* GAM *mi lu*-DU-*ma lu* 9,36 24 GAM 24 9,36 24 dal .....[330]

10[50] *ù* 14 sag-meš gar-gar-*ma* 1,4 ½-*šú* 32 24 dal GAM 32 DU-*ma* 12,48

11[12,4]8 GAM 21,36 DU-*ma* 4,36,28,48 2½ sìla 2 ninda ½ ninda 10-*u'* ninda .....

4 12[.....].. 1 *kip-pat* 1 GAM 1 1 . 1 GAM 5 5 . 5 a-šà . 5 GAM 21,36 DU-*ma* 1,48 . 1 *me* [8]

5 13[... *k*]*a-a-du* 10 kùš *a-na* 10 kùš *ù* 10 kùš sukud ...-meš *šá* 1 Ú .... *a-na* ugu QAR(?)

14[.....]-'A ta *iš-di ka-a-du lu di* ...[331] *a-m/ba* ... *šá ina ka-a-du lu* AD

15[.... igi-1-gá]l-bi 1 . 1 GAM 10 10 . 10 GAM 10 1,40 1 *me* kùš

6 16[.....*d*]*i*(?) ...[331] 10 kùš *a-na* 10 kùš *ù* 1 ma-na ki(?)-lá-*šú ina qabli*(múrub)-*šú* šu(?)-si *ana* šu-si sig-*ta*

17[.....].. [k]i-lá *šá* sig-*ta* 10 GAM 30 . 5 3 *me*[332] šu-si 5 GAM 5 25 . 25 1,30 *lim*

18[igi-25-]gál-bi 2,24 2,24 GAM 20 DU-*ma* 48 4 *ḫa-an-za* ninda še

7 19[.....]... -*ma* EŠ(?) *itti*(ki) *a-ḫa-meš di* BU(?) Ú Ú *ma ru* ... *ḫi* ... gíd-da-meš 1-*en iṣ*(?)-*ṣi*(?)

20[.....]..... DU *ki*(?) *ma* 'A *la*(?) *ši ma* 52,4 *it-ti-šú zaq-pi*

21[.....]..... [DU]L-D[U-*m*]*a ri-ḫi* 9 . 9 GAM 9 1,21 [.....]

The **reverse** is destroyed except for scattered traces of signs.

Translation of Nos. 2, 3 and 4

2 5[A triangle. 1,0 is the length, 45 the width. 1,0 tim]es 1,0 is 1,0,0. 45 times 45 is 33,45. Add together, and (the result is) 1,33,45. What should I multiply by what so that (the result)

6[would be 1,33,45? 1,]15 times 1,15. (It is) the hypotenuse.

3 7[A trapezoid.] 30 is the length, 30 the second length, 50 the upper width, 14 the lower width. 30 times 30 is 15,0.

8Subtract [14 fr]om 50, and the remainder is 36. Half of it is 18. 18 times 18 is 5,24. Subtract 5,25,[329] from 15,0 and

9the [rem]ainder is 9,36. What should I multiply by what so that (the result) would be 9,36?

---

[327] For *allu* cf. Thureau-Dangin [14] p. 145.

[328] For *ra'izu*, "pitch-fork", see Landsberger MSL 1 p. 142.

[328a] GAM is here written with three wedges. An astronomical "procedure text" of the Seleucid period, BM 35399 (to be published by Neugebauer in "Astronomical Cuneiform Texts"), frequently writes three inclined wedges (instead of two) in a row for GAM in the expression (*x*) GAM (*y*) DU.

[329] Error for 5,24.

[330] Traces of signs. These presumably are nothing more than some description of dal, since they are without influence on the mathematical interpretation.

[331] Lines 14 and 16 contain the same signs at this point.

[332] Written over erasure.

24 times 24 is 9,36. 24 is the dividing-line
.....

10 Add [50] and 14, the widths, and (the result is) 1,4. Half of it is 32. Multiply 24, the dividing-line, by 32, and (the result is) 12,48.

11 Multiply [12,4]8 by 0;0,0,21,36, and (the result is) 0;4,36,28,48. $2\frac{1}{2}$ sìla, 2 ninda, $\frac{1}{2}$ ninda, $\frac{1}{10}$ ninda,
.....

4 12 [A circle.] 1,0,0 is the circumference. 1,0,0 times 1,0,0 is 1,0,0,0,0. 1,0,0,0,0 times 0;5 is 5,0,0,0. 5,0,0,0 is the area. Multiply 5,0,0,0 by 0;0,0,21,36, and (the result is) 1,48. One hundred [and 8].

Commentary

The style of this text is reminiscent of AO 6484, a Seleucid tablet from Warka.[333] It is quite possible that the present text also came from Warka; it was written in the Seleucid period, i.e., some time during the last three centuries B.C. The ductus is very cursive and has a pronounced slant.

a. Mathematical Commentary

No. 1. More than half of this example is destroyed or damaged beyond hope of restoration. The only small sections which we are able to read with any confidence contain the following operations:

$$0;30 \cdot 0;30 = 0;15 \qquad .... = 51;45.$$

We have selected the relative order of magnitude of these numbers in such a way that the number 52 which then follows could perhaps be explained as 0;15 + 51;45. Following this, we read 52 · 52 = 45,4, where 52 seems to be explained as dal, i.e., altitude or diameter, etc. After a lacuna, the result 26 is given as the area. The subsequent multiplication by 21,36 is known to be an operation applied to areas in the transformation to seed-barley measures. We shall discuss this metrological question below, where it will be shown[334] that 26 must be interpreted as 26,0 kùš².

No. 2. In a right triangle with sides $a = 45$ $b = 1,0$, the hypotenuse c is computed according to

$$c = \sqrt{a^2 + b^2} = \sqrt{1,33,45} = 1,15.$$

The use of the "Pythagorean theorem" is well attested for all periods of Babylonian mathematics.

No. 3. A field in the shape of a trapezoid is given by (cf. fig. 25):

$$l_1 = l_2 = 30 \qquad b_u = 50 \qquad b_l = 14.$$

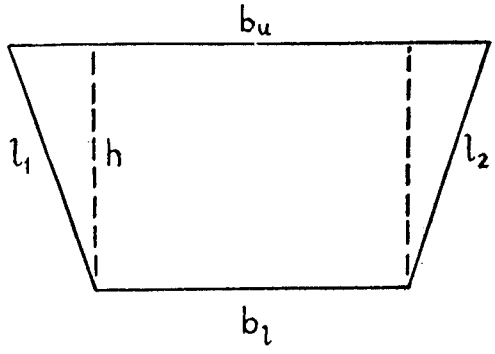

FIG. 25.

The altitude h of the trapezoid is computed by

$$h = \sqrt{l^2 - \left(\frac{b_u - b_l}{2}\right)^2}$$
$$= \sqrt{30^2 - \left(\frac{36}{2}\right)^2} = \sqrt{9,36} = 24.$$

Then follows the determination of the area by

$$A = \frac{b_u + b_l}{2} \cdot h = 32 \cdot 24 = 12,48.$$

The multiplication by 21,36 which follows is of a purely metrological character and will be discussed separately below. We will see[335] that all lengths are supposed to be measured in kùš and therefore $A = 12,48$ kùš².

No. 4. The area a of a circle is computed from its circumference $c = 1,0,0$ by

$$a = 0;5 \cdot c^2 = 5,0,0,0,$$

where $0;5 = \frac{1}{12}$ is an approximation for $\frac{1}{4\pi}$. This is exactly the procedure which we have already encountered in the Old-Babylonian tablets discussed pp. 9 and 44 above. For the multiplication by 21,36 see below, p. 143. The interpretation of c as 1,0,0 instead of any other power of 60 is based on metrological considerations.[336]

No. 5. This example seems at first glance to deal with a cube whose side measures 10 kùš. The whole computation seems to consist of the two steps $1 \cdot 10 = 10$ $10 \cdot 10 = 1,40$. The last number 1,40 is then called 100 kùš, which does not necessarily contradict the assumption that a volume is in question, since we know that the term kùš also occurs as a unit of area in the Neo-Babylonian period.[336a]

[333] TU No. 33 (pls. 61f.), MKT I pp. 96ff. and Thureau-Dangin TMB pp. 76ff.
[334] Cf. p. 144 below.
[335] Cf. p. 143 below.
[336] Cf. p. 145 below.
[336a] Cf., e.g., Oppert [1] p. 97.

No. 6. A volume seems to be involved again, with at least two dimensions equal to 10 kùš. The following calculations are carried out:

$$10 \cdot 30 = 5{,}0 = 300 \qquad 5{,}0 \cdot 5 = 25{,}0 = \tfrac{3}{2} \cdot 1000$$
$$\frac{1}{25{,}0} = 0;0,2,24 \qquad 0;0,2,24 \cdot 20 = 0;0,48.$$

By a different choice of the sexagesimal places, we could of course have obtained 0;48 as the result. The text gives 4 *ḫa-an-za*[337] ninda še, "four-fifths of a ninda of barley" as the answer. Accordingly, 0;0,48 would imply the use of bán as the unit, while 0;48 would assume ninda.

b. Metrological Problems

Nos. 1, 3 and 4 involve the computation of areas. Let A be the final value obtained for the area from geometrical relations; in all three examples the value of A is multiplied by a factor 21,36 which we denote by c. The very same parameter

$$c = 21{,}36$$

also occurs in AO 6484[333] No. 5 (obv. 10f.) and No. 7 (obv. 13–18). Example No. 5 in AO 6484 concerns a square whose side is $1\frac{2}{3}$ kùš, i.e., $A = 2;46,40$ kùš^2. No. 7 in the same text computes the area of a triangle but does not indicate the units assumed for the measurements of the sides. The units of the area obtained, $A = 12$, are accordingly likewise undefined.

The multiplication by $c = 21{,}36$, however, is not the only modification to which the value of A is subjected. Nos. 1 and 3 of our present text and Nos. 5 and 7 of AO 6484 transform the value cA into so-called še-numun, "seed-barley". This furnishes the following relations:

		A	cA	seed-barley
AO 6484	No. 5	2;46,40 kùš^2	1	1,48 = 108 še-meš
	No. 7	12	4,19,12	7,46;33,36 ≈ 466½ še-meš
VAT 7848	No. 1	26	9,21,36	5 sìla 4 ninda 5(?) še
	No. 3	12,48	4,36,28,48	2½ sìla 2 ninda ½ ninda $\frac{1}{10}$ ninda ....
	No. 4	5	1,48 = 108	——

A comparison of the values of A in Nos. 5 and 7 of AO 6484 with the seed-barley values in the last column makes it extremely probable that the units in No. 6 are also kùš^2, i.e., $A = 12$ kùš^2. The corresponding units of VAT 7848 will be discussed later. Our first step will be to explain the coefficient $c = 21{,}36$, after which we shall investigate the relation between cA and the seed-barley measures.

An explanation of the multiplication of the area A by the coefficient $c = 21{,}36$ can be derived from a passage in a Neo-Babylonian metrological text from Nippur published by Hilprecht:[338]

an-ni-ti šu-si *šá* 24 šu-si-meš
1 kùš *am-mat* še-numun *u* gi-meš
šá 1 *me* kùš uš 1 *me* kùš sag
5(bán) 3 sìla 3⅓ ninda *šá* še-numun .....
"This is the šu-si, 24 of which (equal)
1 kùš (i.e., cubit), the cubit of seed-barley and the square-gi[339]
such that (a field of) 100 kùš length and 100 kùš width
(corresponds to) 5 bán, 3 sìla (and) 3⅓ ninda of seed-barley .....

Only the last three lines have a bearing on our problem. They contain the statement that an area of 100 kùš length and 100 kùš width corresponds to 5 bán 3 sìla 3⅓ ninda = 33;20 sìla or

(1) $(1{,}40 \text{ kùš})^2 = 2{,}46{,}40 \text{ kùš}^2 = 33;20$ sìla seed-barley.

For the moment, we ignore the relation between area and seed-barley and concentrate our attention on the fact that this relation is based on the use of (100 kùš)^2 as the unit of area. If an area A is measured in kùš^2 and if the relation between area and seed-barley re-

[337] Our initial impulse to discredit the clearly written *-za* of *ḫa-an-za* by assuming a simple paleographical excess of a single vertical wedge which would lead to the emended reading *ḫa-an-šá* was well meant but false. Strassmaier AV p. 412, No. 3198 quotes the forms *ḫa-an-zu*, *ḫa-an-za* and *ḫa-an-zi-šú-nu* from two Seleucid economic texts—thus establishing beyond doubt that the basic form in the Seleucid period (at least at Warka) was *ḫanz-*, not the expected *ḫanš-*. To these Strassmaier references can be added *ḫa-an-zu*, Clay BRM II No. 36, 5 and No. 47, 5, in both of which places the reading *ḫa-an-zu* is established by collations kindly made for us by Dr. A. Goetze, against Clay's copy which gives *ḫa-an-su*. The same phonetic pattern *nš* > *nz* can be observed in Seleucid *ši-in-zi-ru-ú* (Clay BRM II No. 4, 2; No. 15, 2, 4, 8, 10, 13; Contenau TCL XIII No. 237, 2, 3, 7, 13, 15, 19, 21; Speleers MRC No. 294, 2, 21) for the older *šinširū*, "one-twelfth". Contrast, on the other hand, *sa-ma-šu-ru-ú*, "one-eighteenth", quoted by Strassmaier AV, p. 793, No. 6536 from a Seleucid text found at Warka (BM 93002) which was first published in transcription by Oppert-Ménant DJ, pp. 301ff.; a photograph shows that the reading is certain. Contrast also *ḫa-an-šú*, Contenau TCL XIII No. 234, 12, 14, 19, 21, 23, 25.

[338] Hilprecht BE 20,1 Pl. 20 (CBS 8539) rev. III 15–18; discussed by Thureau-Dangin [1] p. 85.

[339] 1 gi = 7 kùš.

quires new units of (100 kùš)² = 2,46,40 kùš², it is obvious that we must divide A by 2,46,40 in order to find the number of new units contained in A. The division by 2,46,40 can be carried out by a multiplication with its reciprocal, i.e., by 0;0,0,21,36. This shows that the coefficient 21,36 must be interpreted as

(2) $$c = 0;0,0,21,36,$$

the clear function of which is to convert areas measured in kùš² into areas measured in units of (100 kùš)².

We now turn to the second problem, the relation between areas and seed-barley. The original, underlying idea is clear: a field requires a certain amount of seed-barley; fields can therefore be measured by this quantity of seed-barley. The question is to find the relation assumed for the equivalence between an area of (100 kùš)² and barley-seed. The answer seems to be given directly by (1), which states that one unit of (100 kùš)² corresponds to 33;20 sìla seed-barley. We should therefore be able to obtain the results of VAT 7848 Nos. 1 and 3 by multilpying the area cA by 33;20. The results given in the text, however, are inconsistent with each other because from

No. 1 cA = 9,21,36,0 ≈ 5 sìla 4 ninda ... = 5;24 sìla ...
No. 3 cA = 4,36,28,48 ≈ 2½ sìla 2½ ninda $\frac{1}{10}$ ninda ... = 2;45,36 sìla ...,

it follows that the area in No. 1 is more than twice the area in No. 3, while 5;24 sìla is less than twice 2;45,36. We are accordingly confronted with the unpleasant fact that at least one of the results given in VAT 7848 must be incorrect. We must therefore investigate both cases independently. From (1), we obtain:

No. 1 0;9,21,36 · 33;20 = 5;12 sìla = 5 sìla 2 ninda
No. 3 0;4,36,28,48 · 33;20 = 2;33,36 sìla = 2½ sìla 0;36 ninda.

Both of these results deviate from the answers given in the text, but not very considerably. This leads to two conclusions: (a) the relation (1) only gives the order of magnitude of the correspondence assumed in VAT 7848 and (b) the areas in Nos. 1 and 3 must be

No. 1 A = 26,0 kùš²
No. 3 A = 12,48 kùš²

in order to yield for cA the order of magnitude 0;9,... and 0;4,..., respectively, necessary for our above results. The units in No. 3 by which l_1, l_2, b_u and b_l are measured must therefore be kùš; No. 1 is too damaged to draw any conclusions from it.

We can now try to apply the same method to AO 6484. These examples give the following values:

	A	cA	(seed-barley)
No. 5	2;46,40 kùš²	0;0,1 (100 kùš)²	1,48
No. 7	12 kùš²	0;0,4,19,36 (100 kùš)²	7,46;33,36

Both these examples follow the rule that the amount of seed-barley is obtained from cA by multiplying it with a factor

(3) $$d = 1,48,0,0.$$

The numbers thus obtained are called še. We must first demonstrate that "še" must have been the name of a definite unit here, not simply "barley". This follows from the fact that if še merely means "barley" here, the units meant should be either sìla or ninda. From (1), however, it follows that only about 0;0,33,20 sìla (≈ $\frac{1}{11}$ ninda) corresponds to cA = 0;0,1 (100 kùš)². If, therefore, the result is called 1,48 = 108 še, then both sìla and ninda are excluded as units. Hence še must be the name of the unit involved and we therewith get rid of the unpleasant assumption that the name of the units has been omitted. Moreover, a unit called še is well known from Old-Babylonian metrology, equivalent to 0;0,0,20 sìla.[340] Assuming for a moment not only (1) but also that 1 še = 0;0,0,20 sìla in our text too, we would obtain the result that cA = 0;0,1 (100 kùš)² corresponds to 0;0,33,20 sìla = 1,40 še, which is at least the same order of magnitude as the 1,48 še given in the text. In other words, we again reach a very close, if not total, agreement with the text.

This opens a new road by which to attack the examples in VAT 7848. We multiply cA by d = 1,48,0,0 in order to obtain the corresponding number of še units and then convert these še into sìla by assuming that 1 še = 0;0,0,20 sìla. We obtain:

VAT 7848	cA	dcA	text
No. 1	0;9,21,36	16,50,52;48 še = 5;36 sìla 2,52;48 še	5;24 sìla ... še
No. 3	0;4,36,28,48	8,17,39;50,24 še = 2;45,36 sìla 51;50,24 še	2;45,36 sìla ... še

The application of the rule of AO 6484, combined with the relation

(4) $$1 \text{ še} = 0;0,0,20 \text{ sìla},$$

leads to exactly the same number of sìla as given in No. 3. We cannot expect agreement with No. 1 as well because No. 1 and No. 3, as stated above, are contradictory. We must, however, admit that if the number of sìla appears correctly in No. 3, we have no

[340] Cf. p. 6 above.

way of knowing the form in which the remaining 51;50,24 še are mentioned in the final traces of the line. All that we can say, therefore, is that if we replace the relation (1) of Hilprecht's metrological text by

(5) $(100\ \text{kùš})^2 = 36\ \text{sìla} = 1\ \text{PI}$

and if we further assume

(4) $1\ \text{še} = 0;0,0,20\ \text{sìla},$

we obtain the following rule: an area A measured in kùš^2 corresponds to cdA še, where

(2) $c = 0;0,0,21,36$

and

(3) $d = 1,48,0,0,$

This rule yields exactly the values which we find in the two examples of AO 6484 and very closely the value of VAT 7848 No. 3. Though it must be admitted that Hilprecht's text gives (1), not (5), it is also true that at least some Neo-Babylonian economic texts calculate areas according to (5), not (1).[341]

We can now summarize our results, such as they are. Both AO 6484 and VAT 7848 presuppose lengths measured in kùš and areas in kùš^2. The multiplication by $c = 0;0,0,21,36$ means that the areas are transformed into units of (100 kùš)^2. VAT 7848 No. 4 erroneously considers this step as final. All the other examples then give the equivalents in seed-barley. Two ways are open: either cA is multiplied by $d = 1,48,0,0$ in order to obtain the corresponding barley measured in še units; or cA is multiplied by 36, the result being the number of sìla of seed-barley. These two processes are equivalent if 1 še = 0;0,0,20 sìla. The first way is followed by AO 6484, the second by VAT 7848. Because of an error in the text, the coefficients 36 and 0;0,0,20 cannot be considered more than plausible.

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[341] For instance, Pohl NRBSM I No. 23 and II No. 19.

CHAPTER IV. THE AKKADIAN DIALECTS OF THE OLD-BABYLONIAN MATHEMATICAL TEXTS

By A. Goetze

The bulk of the extant Old Babylonian texts other than mathematical of content can be divided in two large groups: a northern group and a southern group. Each shows certain characteristics in spelling, grammar, and lexicon. The northern group includes, in addition to the text of the Hammurapi Code and the royal letters, the texts from Dilbat and Sippar; the southern group comprises chiefly the texts from Larsa. It goes without saying that texts from other places will probably necessitate the positing of additional "dialects". As far as Mari is concerned, this necessity is already apparent from the few samples which have been published. It would be interesting, e.g., to learn about the peculiarities of Ur or Uruk texts. At present, the available material is not sufficiently extensive for more specific conclusions.

It seems promising to ask whether the classification derived from business documents and letters is also applicable to the mathematical texts, and whether perhaps a study of them can furnish criteria for positing additional sub-classes.

The characteristics of the two main Old Babylonian "dialects", insofar as they have a bearing on the mathematical tablets, are as follows:

(1) Old Babylonian does not possess special signs for the emphatic stops. For *ṭa*, *ṭe*, *ṭi*, *ṭu*, then, the following substitutes are used:

North:	TA, TE, TI, TU	i.e.	*ṭá*, *ṭe*$_4$, *ṭì*, *ṭú*
South:	DA	DI, DU	i.e.	*ṭa*,	*ṭi*, *ṭù*

(2) Syllables beginning with *samekh* are represented by special signs only in the North (the Code excepted). The South (and the Code) continue the Old Akkadian practice of using one and the same sign for *za*, *sa*, and *ṣa*, etc.[342] The correct choice of transliteration can be made only on the basis of later Akkadian (which created and continued to evolve better means of differentiation) or the Semitic etymology. *Samekh* with following vowel is spelled in the

North:	SA, SI, SU	i.e.	*sa*, *si*, *su*
South:	ZA, ZI, ZU	i.e.	*sà*, *sí*, *sú*

Only in exceptional cases SA, SI, SU occur also in southern texts, and then quite consistently. Such cases are *sāmum* "red", *sebe*, *sebet* "seven", *samnum* "eight", *salāmum* "maintain friendly relations", *šasūm* "shout" and their cognates and the combination resulting from *š* and the initial sibilant of the suffixed pronouns of the third person (e.g. *rēšum* "head" + *-šu*, *-ša*, *-šunu*, *-šina*). Furthermore SI-*it-tum* "remainder" in the South corresponds to *ši-it-tum* in the North. The interpretation of these facts remains to be found.[343]

(3) Syllables ending with *samekh* are in part distinguishable from syllables ending in *z* or *ṣ*. We find in the

North:	AŠ, IZ, UŠ	i.e.	*ás*, *is*, *ús*
South:	AZ, IZ, UZ	i.e.	*as*, *is*, *us*

Certain mathematical tablets employ furthermore AZ with the value *úṣ* (once *úz*).[344]

(4) With labials the distinction between the voiced and the voiceless variety is rather imperfect in all Akkadian dialects. In northern Old Babylonian only *ba* and *pa* are kept apart by distinct signs.[345] In this respect southern Old Babylonian is better equipped, since it also distinguishes *bi* and *pi*; for *pi* the sign is used which also denotes (both in the North and the South) *wa/wi/wu*.[346] Thus, we have in the

North:	BA, BI, BU[347]	i.e.	*ba*, *bi*, *bu*
	PA, BI, BU		*pa*, *pí*, *pu*
South:	BA, BI, BU	i.e.	*ba*, *bi*, *bu*
	PA, PI, BU		*pa*, *pi*, *pu*

[342] See Thureau-Dangin, RA 30, 93ff. and compare Goetze, Language 14, 136.

[343] It seems fairly certain that the pronunciation of the sibilant in the mentioned words must have varied in southern Old Babylonian from both *š* (etymologically *šin*, *śin*, or the voiceless interdental) and *s* (*samekh*). The possibility must be seriously considered whether we might be dealing here with another Primitive Semitic sibilant which is no longer recognizable in any other individual language.

[344] This transliteration is adopted here for the sake of uniformity; see p. 3.

[345] In Old Akkadian not even this was the case. A remnant of the use of BA for *pá* seems to be preserved in *ki-ip-pá-tum* BM 15285 (MKT I 137ff.; TMB 53ff.) obv. I 3.

[346] One may ask whether this indicates a change in pronunciation. It may very well be that in southern Old Babylonian *p* had shifted to *f* (as in South Semitic). In other words *pi* may have been pronounced *fi* for which *wi* was the closest approximation.

[347] The BE sign is of very limited currency and may therefore be neglected here.

In one group of the mathematical tablets (see p. 150) still another system becomes for the first time observable. It distinguishes four varieties:

BA, ?, BI, BU	i.e.	*ba*,	*bi*, *bu*
PA, BI, PI, BU		*pa*, *pé*,[348] *pi*, *pu*	

It is probably at home in Uruk.

(5) The sequence *aya* is expressed[349] in the

North by *a-ia*
South by *a-a*

(6) Babylonian possessed three sets of vowels: short vowels, long vowels (both inherited from Primitive Semitic), and "Schleifton vowels" (originating either from contraction or from a change in intonation). The Code and good northern texts mark Schleifton (but not simple length) by insertion of the vowel signs *a*, *e*, *i*, *ú* after syllables ending with the same vowels. Southern texts exhibit in varying degrees repeated vowels where neither contraction nor grammatical change of intonation can be made responsible for their presence.[350]

(7) Phonetic complements are virtually absent from the Code (*eqlum* "field" is almost the only exception). Northern texts prefer as complements syllables consisting of vowel + consonant (i.e. spelled syllables), southern texts, however, syllables consisting of consonant + vowel + consonant (i.e. spoken syllables) whenever such syllables are available. Hence, one finds, e.g., in the

North: *eqlum*^{um}, *eqlam*^{am}, *eqlim*^{im}
South: *eqlum*^{lum}, *eqlam*^{lam}, *eqlim*^{lim}

(8) In southern Old Babylonian a development begins which becomes more and more significant in Middle and Neo-Babylonian: the nasalization of double voiced stops. One says in the

North: *inaddin*
South: *inandin*[351]

(9) Constructions like *tuppī anniam ina amārim*, literally "on seeing this my tablet", are frequent in both the North and the South. The addition of the possessive suffix to the infinitive construction is peculiar, however, to the South. Thus we read in the

North: *kīma* (or *ūm*) *tuppī anniam tammaru*
South: *tuppī anniam ina amāri-ka*[352]

Before an individual text is assigned to either group, one has to point out the presence therein of several of the enumerated characteristics. As long as only one of these is found, the assignment is more or less uncertain.

The mathematical texts, even when of considerable length, are apt to be stereotyped in their style. This state of affairs is unfavorable to any linguistic investigation; the linguist may find himself without the clues he needs. For this reason it is advantageous, if not imperative, first to establish groups of texts which for reasons other than linguistic belong together, thus creating a broader basis for a search for dialectic peculiarities within these groups.

1st group.

AO 6770, 8862.
YBC 4675 (= B), 5022 (= Ud), 6504, 7243 (= Ue), 7997 (= Pa), 9852 (= C), 9856 (= Q)?, 9874 (= M).
Plimpton 322 (= A)?[353]

AO 6770 (MKT II 37ff.; TMB 71ff.).

1: *a-pa-ṭa-ar* (rev. 18); *ta-pa-aṭ-ṭa-ar* (obv. 5); *im-ṭi* (obv. 12).
4: *e-pe-ši-i-ka* (obv. 2, 11); *te-pe-iš* (obv. 11); *e-ḫe-pe* (rev. 20); *ša-pi-il-tam* (rev. 7).
6: *a-re-e-im* (obv. 6); *an-nu-ú-um* (obv. 12); but *a-ra-am* (obv. 3).

[348] It should be noted that the value *bé* of the sign BI is that of classical Sumerian as established by S. N. Kramer; see his The Sumerian Prefix Forms BE- and BI-, 3.

[349] For the problem compare F. Delitzsch, Assyrische Grammatik (2nd ed.), 55ff.

[350] This probably means that in the South simple length and Schleifton length had become indistinguishable. In consequence repeated vowels could now be used for both varieties indiscriminately.

[351] Spelled later on *i-nam-din* or the like. One should rather transliterate *i-nan-din;* the nasal is certainly not an *m*.

[352] Or with a typical southern word, borrowed from Sumerian, *unnedukkī* "my letter".

[353] For AO 8862 Larsa is well attested as place of provenience; there is no reason to doubt the dealer's explicit statement. AO 6770, though marked "Warka" in MKT (but not in Thureau-Dangin TMB), was published in TCL XVIII together with texts which for the most part come from Larsa. The tablet is connected with AO 8862 by the employment of the formulae *at-ta i-na e-pe-ši-i-ka* and *i-na-ad-di-kum*. YBC 4675 shares only the second of these formulae, but its phonetic manner of writing is very reminiscent of AO 8862 with which it has, moreover, the term *ba-a (x) ḫepūm* in common. If YBC 4675 belongs to the first group, then so does YBC 9852, a duplicate of the last part of the former. YBC 6504 has been added, despite divergences, because it uses at least the formula *i-na e-pe-ši-i-ka*, and furthermore the phonetic spellings *tu-uš-ta-kal* and *ta-pa-ṭar* in the manner of AO 8862, AO 6770, and YBC 4675. The phonetic spellings *tu-uš-ta-kal* and *tu-ub-ba-al* as well as the occurrence of *ta-ta-na-aš-ši* and (though lacking the suffix) *i-na-an-di-in* link YBC 7997 to the group. The appurtenance of YBC 9856 to the group is conjectural. YBC 9874 has been assigned here because of the recurrence therein of *i-na-ad-di-ku;* the use of *našūm* Btn has its only parallels in AO 6770 and YBC 7997. The Plimpton tablet may or may not belong here. YBC 5022 and YBC 7243, written in different hands, have the appearance of Larsa tablets of the time of Rīm-Sîn.

Note *ù-te-er-ši-im-ma* (rev. 2); *a-ga-na* (obv. 9).

AO 8862 (MKT I 108ff.; TMB 64ff.)

2: *a-si-i-ma* (III 34, IV 9).
3: *tu-úṣ-bu* (I 21); *ḫu-ru-úṣ-ma* (II 11, III 19).
4: *e-pe-ši-i-ka* (I 8, II 2, III 8); *te-ḫe-ep-pe-e-ma* (I 12), *te-ḫe-pe-e-ma* (III 14); *e-ḫe-pe-e-ma* (II 19); *uš-te-pi-ra-an-ni-i-ma* (III 33), *uš-te-pi-ra-an-ni* (IV 8), *e-pi-ri-ia* (IV 22, 26).
6: *ab-ni-i* (I 2), but *ab-ni* (I 31); *aš-li-i* (IV 4); *u_4-mu-ú* (IV 21); *ki-im-ra-tu-ú* (I 7, II 32, III 7); *iš-ti-šu-ú* (IV 1), *ḫa-am-ši-šu-ú* (III 39), *ša-la-ši-ri-šu-ú* (IV 12), *ir-bi-ši-ri-šu-ú* (IV 13), but also *ši-ni-šu* (III 36), *ša-la-ši-šu* (III 37), *ir-bi-e-šu* (III 38), *iš-ti-ši-ri-šu* (IV 11).
te-ḫe-ep-pe-e-ma (I 12), *te-ḫe-pe-e-ma* (III 14), *e-ḫe-pe-e-ma* (II 19); *e-pe-ši-i-ka* (I 8 etc.); *a-si-i-ma* (III 34, IV 9); *uš-te-pi-ra-an-ni-i-ma* (III 33); *ki-im-ra-ti-i-ka* (II 16); *u_4-mu-ú-a* (IV 20).
ša-ni-i (I 19), but *ša-nu-um* (III 36, IV 11), *ša-ni-im* (II 31).
7: *eqlam^{lam}* (I 2, II 35); *eqlim^{lim}* (I 5); but also *eqlam^{am}* (I 31).

YBC 4675 (= B).

1: *ta-pa-ṭa-ar-ma* (obv. 9, rev. 10).
2: *sà-ni-iq* (obv. 5, 7, rev. 6, 16).
4: *te-ḫe-pe-e-ma* (obv. 8, rev. 9); *te-ep-pe-eš-ma* (rev. 6).
6: *a-ra-ka-re-e-em* (rev. 2, 12); *ki-la-a-al-le-e-en* (obv. 8); *ša-ni-i-im* (obv. 5); *qá-ab-lu-ú-um* (obv. 16); *ga-me-ru-ú-tim* (obv. 7); *zu-ú-uz* (obv. 3).
te-ḫe-pe-e-ma (obv. 8, rev. 9); *te-le-qé-e-ma* (obv. 15).
7: *eqlam^{lam}* (obv. 3), but *eqlim^{im}* (obv. 20, rev. 10).
8: *i-na-an-di-kum* (obv. 11, rev. 1).

YBC 5022 (= Ud).

1: *ra-ṭù-um* (22ff.).
4: *né-pi-iš-tum* (1).
6: *ša-ḫa-al-lu-ú-um* (37); *i-im-lu-ú-um* (40); *ma-aš-šu-ú-um* (41f.); *ka-lu-ú-um* (56); *ku-bu-ur-re-e* (58); PI-*lu-ú-tum* (62); *ša*-x-x-*ú-um* (65).

YBC 6504 (MKT III 22ff.; TMB 134ff.).

4: *te-ḫe-ep-pe-ma* (obv. 5, 17, rev. 5); *e-pe-ši-i-ka* (obv. 3, 12).
6: *e-pe-ši-i-ka* (obv. 3, 12).

YBC 7243 (= Ue).

1: *ra-ṭù-um* (34, 36).

YBC 7997 (= Pa).

6: *ta-ta-na-aš-ši-ma* (rev. 7).
8: *i-na-an-di-in* (rev. 8).

YBC 9852 (= C).

1: *ta-pa-ṭa-ar-ma* (obv. 4).
2: *sà-ni-iq* (rev. 1).
4: *te-ḫe-pe-e-ma* (obv. 3).
6: *a-ra-ka-re-e-em* (obv. 6); *te-ḫe-pe-e-ma* (obv. 3).

YBC 9856 (= Q).

4: *pi-ia* (obv. 2, 3, 4, 6, 7).

YBC 9874 (= M).

6: *eḫ-re-e* (rev. 5); *ṷa-ar-ka-a-nu-um* (rev. 3); *ši-i-lu-ta-am* (rev. 10); *ši-i-lu-ú-tim* (rev. 6).
ta-at-ta-na-aš-ši-i-ma (rev. 5, 7, 9).

Plimpton 322 (= A).

6: [*in-*]*na-as-sà-ḫu-ú-ma* (I 2).

This group is certainly to be localized in the South, in all probability Larsa. It employs PI for both *pi* and *pe*, and shows numerous repeated vowels.

2nd group.

BM 13901.
YBC 4662 (= J), 4663 (= H), and probably YBC 7164 (= L).[354]

BM 13901 (MKT III 1ff.; TMB 1ff.).

1: *ip-pa-ṭa-ar* (obv. I 40, II 15, 23 etc.); *im-ṭi* (obv. II 12).
2: *se-bi-a-tim* (obv. II 12, 20).
3: *tu-*1Z-*bu* (rev. II 15).
4: *te-ḫe-pe*(-*ma*) (obv. I 2, 3, 13 etc.).
6: *ša-ni-im* (obv. II 10); *ši-ni-šu* (obv. II 8); *ta-na-ši-ma* (obv. I 22, II 25, 26 etc.).
ra-ma-ni-ša-a-ma (rev. I 52).
7: *eqlam^{lam}* (obv. I 1, rev. I 50, II 11); *eqlim^{lim}* (obv. I 10).

YBC 4662 (= J).

3: *ḫu-ru-úṣ* (obv. 22, 34).
4: *ḫé-pe* (obv. 17, 27, 30).
6: *ta-na-ši-ma* (rev. 23).

YBC 4663 (= H).

1: *pu-ṭù-ur* (obv. 9, 22, 24, 30, rev. 3).
2: *ú-sú-uḫ* (rev. 9).
3: *ḫu-ru-úṣ* (rev. 11, 24).
4: *ḫe-pe* (rev. 19), *ḫé-pe* (rev. 7); *né-pe-šu* (obv. 6, 19, rev. 25).

YBC 7164 (= L).

6: *pa-ta-a-tim* (rev. 17, 19); *ši-lu-tum* (obv. 4, 6, 8 etc.), *ši-lu-ta-am* (obv. 16).

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[354] The close connection of YBC 4662 and 4663 needs hardly any comment; both external appearance and content establish it firmly, not to speak of their being acquired in the same lot. Of their terminology only za-e kìd-da-zu-dè, *i-na-ad-di-ik-kum*, and ½ (*x*) *ḫepûm* may be mentioned here. YBC 7164 is hard to place. This is in part the consequence of predominantly ideographic spelling. The relationship of the tablet to the "series texts" in the Yale collection (MKT chapter VII) is obvious. But it very briefly states the solution of the problem without going into the details of the mathematical computations involved. The few Akkadian elements which the tablet presents may just as well indicate appurtenance to one of the following groups. I think it should not be separated, however, from BM 13901, whose relationship to the "series texts" Neugebauer has duly emphasized (MKT III 10), and which gives us the benefit of phonetic spellings. For linguistic reasons BM 13901 can be assigned neither to any of the following groups nor to the first group. I should have preferred other than linguistic reasons if there were any; as it is, the argument presented may be regarded as circular.

This is likewise a southern group. It employs PI for both *pi*[365] and *pe*, but exhibits repeated vowels only sparingly.

3rd group.

Strassburg 362, (363,)(364,) 366, (367,) 368.
VAT 7530, 7531, (7532,) 7535, 7620, (7621).
YBC 4608 (= D).
(MLC 1950 (= Ca)).[366]

Strassburg 362 (MKT I 239ff.; TMB 82ff.).
2: *sa-am-ni-im* (obv. 9).
6: *ta-na-aš-ši-i-ma* (obv. 8), but *ú-te-le-le-ma* (obv. 2).

Strassburg 366 (MKT I 257ff.; TMB 205).
1: *pa-ṭa-ra-am* (obv. 6), *ta-pa-ṭa?-ar-ma* (obv. 10).[367]
2: *sa-am-da-ka-am* (obv. 6).
6: *te-le-qé-e-ma* (obv. 8).
Note: *ur-ra-am* (obv. 1).

Strassburg 368 (MKT I 311ff.; TMB 91f.).
6: *ta-na-aš-ši-i-ma* (obv. 9); *el-qé-e-ma* (obv. 1); *aḫ-ṣú-bu-ú-šu* (obv. 3), *aḫ-ṣú-ub-šu-ú-ma* (obv. 2).
8: *mi-in-da-az-zu* (obv. 1).

VAT 7530 (MKT I 287ff.; TMB 100f.).
2: *ši-sà-a-at* (obv. 9, 13, 19, rev. 5).

VAT 7531 (MKT I 289ff.; TMB 98f.).
2: *si-ta-at* (obv. 11).

VAT 7535 (MKT I 303ff.; TMB 93ff.).
6: *el-qé-ma* (obv. 1, rev. 25); *iḫ-ḫa-aṣ-ba-an-ni-ma* (obv. 3, 5, rev. 3 etc.).
8: *mi-in-da-zu* (rev. 25).

VAT 7620 (MKT I 314ff.; TMB 208).
4: *ḫe-pé-ma* (rev. 2).

YBC 4608 (= D).
1: *ip-pa-aṭ-ṭa-ar* (obv. 16, rev. 18).
5: *ki-i a-a* (rev. 16).[368]
6: *ta-na-aš-ši-i-ma* (obv. 19, 20); *a-ma-ri-i-ka* (obv. 22, 28).
ú-te-l[e-el-]le-ma (rev. 14).
9: *aš-šum* sag.an-na *ù* sag.ki-ta *a-ma-ri-i-ka* (obv. 21f.).

This group, likewise southern, is localized in Uruk. It employs BI for *pé*.

4th group.

VAT 8389, 8390, 8391, 8512, 8520, (8521,) 8522, 8523, 8528.
YBC 4186 (= N), 6295 (= Aa), 8588 (= Ja), 8600 (= Ec), 8633 (= E).[369]

VAT 8389 (MKT I 317ff.; TMB 103ff.).
2: *im-ku-sú* (obv. 6, 8, 15, 19 etc.); *ú-sú-uḫ(-ma)* (obv. II 1, 11, rev. I 14).
3: *am-ku-us* (obv. I 1, 2).
4: *ḫe-pé-ma* (obv. I 12).
6: *ša-nu-um* (obv. II 13); *ša-ni-im* (obv. I 2, 18, rev. I 16 etc.); *ši-ni-šu* (obv. I 16, 20).

VAT 8390 (MKT I 335ff.; TMB 112ff.).
2: *ú-s[ú-uḫ-m]a* (obv. I 17).

VAT 8391 (MKT I 319ff.; TMB 107ff.).
2: *im-ku-sú* (obv. I 6, 7, 11 etc.); *ú-sú-uḫ(-ma)* (obv. II 11, 18, rev. I 16 etc.).
3: *am-ku-us* (obv. I 1, 2, 21 etc.).
4: *ḫe-pé-ma* (obv. I 30; rev. I 18).
6: *ša-nu-um* (obv. II 20), *ša-ni-im* (obv. I 13, 22, II 5 etc.).

VAT 8512 (MKT I 340ff.; TMB 101ff.).
2: *ú-sú-uḫ-ma* (obv. 19, edge 3).
4: *ḫe-pé-ma* (obv. 17, 25, rev. 11, 18).
pi-ir-kum (obv. 20, 21), *pi-ir-kam* (edge 1, rev. 11, 18), *pi-ir-ki* (rev. 6).

VAT 8520 (MKT I 346ff.; TMB 115ff.).
2: *ú-sú-uḫ(-ma)* (rev. 3, 6, 11, 21, 33).
4: *ḫe-pé-ma* (obv. 16, rev. 16).
6: *i-gu-um* (obv. 3, 29, rev. 24); *i-gi-im* (obv. 1, 4); *i-gi-bu-um* (rev. 27, 28); *i-gi-bi-im* (rev. 5, 29); *ša-nu-um* (rev. 22).

VAT 8522 (MKT I 368ff., TMB 208).
2: *i-na-ki-sú-nim* (obv. I 8a, 8); *li-ik-ki-sú-niṃ* (obv. I 6).
se-bi-at (rev. 6).

VAT 8523 (MKT I 373ff.; TMB 124ff.).
2: *sé-ke-ri-im* (obv. I 1, rev. 15); *ka-sí-ri-im* (obv. I 7, 19, 23 etc.); *ú-sú-uḫ-ma* (obv. I 15); *sà-a[n]-qum* (rev. 10), *sà-a[n]-qí* (rev. 13).
4: *ḫi-pi-tum* (rev. 8).

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[365] It must be admitted that no actual case of PI = *pi* has been observed in this group. But the two facts are complementary, and it is certain that PI serves for *pe*. This latter value distinguishes the 2nd group from the 3rd and 4th, not the former.

[366] The numbers in parentheses refer to tablets which contain no clues as to their linguistic position. They are included here, because they are connected by external appearance and content with other tablets of the group, with which they were purchased. The Strassburg tablets are said to come from Warka; the same statement was independently made with regard to the Berlin lot. The two strains of evidence corroborate one another. Further confirmation comes from the content and their terminology. Characteristic for the group is the combination of the formulae za-e kì-da-zu-de, in-sì, and ḫé-gar—in-sì—ḫé-gar. Strassburg 363 presents the latter formula phonetically to varying degrees. In this respect it recalls YBC 4608 which gives it phonetically in its entirety. YBC 4608 is furthermore linked to Strassburg 366 by the use it makes of the phrase *ba-ma-at* (x) gaz. It is for these reasons that the Yale tablet has been included in the group.

[367] Frank's autograph seems to present TA. The passage should be collated. For the time being this is impossible.

[368] *ki-i a-a* is obviously synonymous with *ki ma-ṣi* "how much?". I believe *a-a* to be absolute state like *maṣi*. Its use in such phrases is paralleled in Aramaic (as are almost all details in the use of the absolute state); see Th. Nöldeke, Syrische Grammatik p. 147 sub E. The respective determinatus is *ayyum*, the adjectival interrogative.

VAT 8528 (MKT I 353ff.; TMB 118ff.).
2: *sú-tim* (rev. I 25, II 1, 2).
4: *ḫe-pé-ma* (rev. II 4).
YBC 4186 (= N).
4: *iš-pi-il* (obv. 1).
YBC 6295 (= Aa).
6: *ša-ni-im* (11).
YBC 8588 (= Ja).
6: *iq-bu-ú* (obv. 7); *iq-bu* (obv. 8, 10).
YBC 8600 (= Ec).
6: *mi-nu-ú* (obv. 7); *ta-na-aš-ši-ma* (obv. 5); *i-ši-ma* (rev. 1).
YBC 8633 (= E).
2: *ú-sú-uḫ-ma* (obv. 3).
4: *ḫe-pé-ma* (obv. 4), but *ḫe-pe-ma* (obv. 9).
6: *ša-nu-um* (rev. 12); *ḫe-pé-ma* (obv. 4), *ḫe-pe-ma* (obv. 9).

As far as linguistics is concerned, this 4th group cannot be distinguished from the 3rd. The enumerated occurrences (particulatly those from VAT 8512) make it quite clear, however, that here PI is *pi* and BI *pé*. The provenience may likewise be Uruk.[360]

5th group.

MLC 1842 (= Sb).
YBC 6967 (= Ua), 10522 (= Uc).[360a]
MLC 1842 (= Sb).
4: *iš-pí-il-ma* (obv. 2), *e-pé-š*[*i-ka*] (obv. 7).
6: *i-li-i-ma* (obv. 1); *i-li* (rev. 1, 4, 9, 13).
Note also *ki-ia* (obv. 7), cf. VAT 6597 (obv. 5), and *i-na* šà (rev. 10), cf. *i-na lìb-ba* VAT 6599 (rev. 8).
YBC 6967 (= Ua).
2: *ú-su-uḫ* (rev. 2).
3: *ḫe-pé-ma* (obv. 5).
4: *ša-nu-um* (rev. 4); *i-gu-um* (rev. 5); *ḫe-pé-ma* (obv. 5); *i-di-ma* (obv. 11).
YBC 10522 (= Uc).
4: *ḫe-pé-ma* (4).

The employment of BI for *pi* and the occurrence of SU make this a northern group.

6th group.

BM 85194, 85196, 85200, 85210.
VAT 6597, 6598, 6599.

[359] This group employs for the most part *it-ta-di-ik-kum* (and variants), and the phrases (*x*) *ana šina ḫepūm* and *luškun—inaddinam*—gar-ra. YBC 8600 and 8633 are in their terminology particularly close to VAT 8512. YBC 4186, 6295 and 8588 are linked to the group by the use made of the phrases igi (*x*) *pu-ṭur* and (*x*) gar-ra.

[360] For VAT 8512 the dealer gave Larsa as provenience. Too much reliance cannot be placed on such information.

[360a] The group is best characterized by *illiakkum*. Unfortunately, the texts are all short.

MLC 1354.[361]
BM 85194 (MKT I 142ff.; TMB 21ff.).
2: *pí-sa-nu-um* (rev. III 35); *i-s*[*a*]*-an-ni*<*-iq*> (obv. II 29); *i-si-iq-tim* (obv. II 28, 35, 39, 43).
i-sà-an-ni-iq (obv. II 39, 43), *sà-na-qám* (rev. II 8).
3: *lu-uk-bu-ús* (rev. II 24).
ta-ka-ba-as (rev. II 33); *zu-úz* (rev. II 3); *lu-úṣ-ba-at* (rev. II 10, 25).
4: *ḫe-pé* (obv. I 5, 6, 8, etc.); *ne-pé-šum* (obv. I 12, 22, 36 etc.); *pí-sa-nu-um* (rev. III 35); *še-pí-šu* (rev. III 22), *še-pí-ia* (rev. III 24).
6: *ab-ni* (obv. I 39, 42); *i-na-aš-ši* (rev. III 8); *ša-ni-im* (obv. I 26, 27).
i-na-aš-ši-i (rev. III 7); *i-ba-ši-i* (rev. II 40); *ep-te-e*(*-ma*) (obv. II 27, 34, 41).
7: *eqlim*^{im} (rev. II 35, 38).
BM 85196 (MKT II 43ff.; TMB 39ff.).
1: *ta-aš-lu-ṭú* (rev. I 27, 28); *uṭ-ṭe*$_4$*-tim* (obv. I 32).
2: *ki-bi-su* (colophon).
ú-sà-an-ni-iq (obv. II 31); [*s*]*é-ru-uḫ-ma* (rev. II 19); ^{iti}*sú-sú-lum* (obv. II 17).
3: *aḫ-ru*-IZ (rev. II 23, 24).
4: *ḫe-pé* (obv. I 4, 11, 15 etc.); *ne-pé-šum* (obv. I 7, 12, 16 etc.).
6: *ša-ni-im* (rev. II 18); *la-bi-ru-tum* (rev. I 22, 26).
mi-it-ḫa-ru-ú (rev. II 28); *ep-te-e-ma* (obv. II 29).
Note *ša-at* (obv. I 31).
BM 85200 + VAT 6599 (MKT I 193ff.; TMB 11ff.).
2: *ki-ib-su* (colophon).
4: *ḫe-pé* (obv. Ia 2, 28 etc.); *ne-pé-šum* (obv. Ia 8, 14, 20 etc.).
Note *i-na lìb-ba* (rev. Ib 8).
BM 85210 (MKT I 219ff.; TMB 46ff.).
3: *ta-ka-ba-ás* (obv. I 6); *lu-uk-bu-ús* (obv. II 16); *lu-uk-bu-ús-ma* (obv. II 6).
lu-úṣ-ba-at (obv. I 3, II 6).
4: *ḫe-pé* (obv. II 10, 19, 25); *ne-pé-šum* (obv. I 7, 12, 16 etc.); *zi-iq-pí* (obv. II 9, 18).
VAT 6597 (MKT I 274ff.; TMB 205).
4: *ne-pé-šum* (obv. I 3, II 9, rev. 10).
6: *re-bu-um* (obv. II 9); *re-bi-im* (obv. II 8).
VAT 6598 (MKT I 277ff.; TMB 127).
4: *ḫe-pé* (obv. 9, 19; rev. I 9); *ne-pé-šum* (obv. 2, 12; rev. 1 24, II 4).
MLC 1354 (= Eb).
4: *ḫe-pé* (obv. 9).
6: *mi-nu* (obv. 4); *i-ši-ma* (obv. 5, 7, 8).

[361] The connection between the pieces in the British Museum and those in Berlin is so intimate that VAT 6599 and BM 85200 are actually parts of the same tablet, as Neugebauer has shown. MLC 1354 exhibits the characteristic *ta-mar* of the group; note, furthermore, *šu-tam-ḫír*. Otherwise it employs more phonetic spellings (particularly *ba-ma-at* for ½). The piece was purchased in Abu Haba (ancient Sippar).

This group combines northern and southern characteristics. It is slightly younger in date[362] than the other groups. Since by now it seems clear that Akkadian mathematics (as the other varieties of Akkadian writings) originated in the South, the situation is satisfactorily explained when it is assumed that the 5th group comprises tablets based on southern originals, but written and modernized in the North. The southern originals were close to the 1st or 2nd group.

To sum up, then, it can be stated that linguistically the six groups[363] treated above represent four varieties of Akkadian. The 1st and 2nd groups are witnesses of a southern Old Babylonian at home in Larsa; the 3rd and 4th groups represent another southern variety which is probably to be localized at Uruk. The 5th group shows northern characteristics. The 6th group comprises northern modernizations of southern (Larsa) originals.

[362] See F. Thureau-Dangin, TMB IX. Neugebauer dates BM 85200 late in the Old Babylonian period, the other BM tablets to the Kassite period.

[363] A number of pieces had to be neglected here. They either fall outside the scope of the present study because of their more recent date or they contain so little Akkadian that they are worthless for its purpose.

CHAPTER V. INDICES

§ 1. Bibliography and Abbreviations

A Plimpton 322 (cf. pp. 38ff. above)

A (24194 or 24195) A = Asiatic collection of The Oriental Institute of The University of Chicago

Aa YBC 6295 (cf. p. 42 above)

AfO Archiv für Orientforschung

AJSL The American Journal of Semitic Languages and Literatures

AO Antiquités Orientales (Louvre, Paris)

B YBC 4675 (cf. pp. 44f. above)

Baumgartner [1] W. Baumgartner, Untersuchungen zu den akkadischen Bauausdrücken. ZA 36 (1925), pp. 123–138

Bezold, Cat. III C. Bezold, Catalogue of the Cuneiform Tablets in the Kouyunjik Collection of the British Museum, Vol. III. London, British Museum, 1893

BM British Museum, London

Boyer CHJ G. Boyer, Contribution à l'histoire juridique de la 1re dynastie babylonienne. Paris, Geuthner, 1928

C YBC 9852 (cf. pp. 44ff. above)

Ca MLC 1950 (cf. pp. 48f. above)

CBM identical with CBS

CBS Catalogue of the Babylonian Section, University Museum, University of Pennsylvania, Philadelphia

Clay BRM II A. T. Clay, Legal Documents from Erech Dated in the Seleucid Era. Babylonian Records in the Library of J. Pierpont Morgan, Vol. II. New York, 1913

Clay NBLE A. T. Clay, Neo-Babylonian Letters from Erech. Yale Oriental Series, Babylonian Texts, Vol. III. New Haven, Yale University Press, 1919

Contenau TCL XIII G. Contenau, Contrats néo-babyloniens, II. Musée du Louvre, Département des antiquités orientales. Textes cunéiformes, Vol. XIII. Paris, Geuthner, 1929

CT Cuneiform Texts from Babylonian Tablets, &c., in the British Museum

D YBC 4608 (cf. pp. 49ff. above)

Deimel ŠL A. Deimel, Šumerisches Lexikon, Vol. I (2nd ed.) 1930; Vol. II 1928–33; Vol. III 1934–37. Scripta Pontificii Instituti Biblici, Rome

Delitzsch HWB F. Delitzsch, Assyrisches Handwörterbuch. Leipzig, Hinrichs, 1896

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E YBC 8633 (cf. pp. 53ff. above)

Ea NBC 7934 (cf. pp. 55f. above)

Eb MLC 1354 (cf. pp. 56f. above)

Ebeling NBU E. Ebeling, Neubabylonische Briefe aus Uruk. Berlin, 1930–1934

Ec YBC 8600 (cf. pp. 57ff. above)

Ehelolf [1] H. Ehelolf, Zur Etymologie und den Ideogrammen von *išpatu* "Köcher". ZA 35 (1924) pp. 46f.

F YBC 5037 (cf. pp. 59ff. above)

Faust YBT VIII D. E. Faust, Contracts from Larsa Dated in the Reign of Rîm-Sin. Yale Oriental Series, Babylonian Texts, Vol. VIII. New Haven, Yale University Press, 1941

Fossey MA II C. Fossey, Manuel d'assyriologie, Vol. II. Évolution des cunéiformes. Paris, Conard, 1926

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G YBC 4657 (cf. pp. 66ff. above)

Gandz [1] S. Gandz, The Origin and Development of the Quadratic Equations in Babylonian, Greek and Early Arabic Algebra. Osiris 3 (1937) pp. 405–557

Gandz [2] S. Gandz, The Algebra of Inheritance. Osiris 5 (1938) pp. 319–391

Genouillac ITT 3II H. de Genouillac, Inventaire des tablettes de Tello conservées au Musée Impérial Ottoman, Vol. 3, Part II. Paris, Leroux, 1912

Genouillac ITT 5 H. de Genouillac, Inventaire des tablettes de Tello conservées au Musée Impérial Ottoman, Vol. 5. Paris, Leroux, 1921

Genouillac TCL V H. de Genouillac, Textes économiques d'Oumma de l'époque d'Our. Musée du Louvre, Département des antiquités orientales. Textes cunéiformes, Vol. V. Paris, Geuthner, 1922.

Grice YBT V E. Grice, Records from Ur and Larsa Dated in the Larsa Dynasty. Yale Oriental Series, Babylonian Texts, Vol. V. New Haven, Yale University Press, 1919

H YBC 4663 (cf. pp. 66ff. above)

Harper ABL VI R. F. Harper, Assyrian and Babylonian Letters belonging to the Kouyunjik Collections of the British Museum, Part VI. Chicago, University of Chicago Press, 1902

Hilprecht BE 20,1 H. V. Hilprecht, Mathematical, Metrological and Chronological Tablets from the Temple Library of Nippur. The Babylonian Expedition of the University of Pennsylvania. Series A: Cuneiform Texts, Vol. 20,1. Philadelphia, 1906

Hilprecht EAB H. V. Hilprecht, The Excavations in Assyria and Babylonia. The Babylonian Expedition of the University of Pennsylvania. Series D: Researches and Treatises, Vol. 1. Philadelphia, 1904

Holt [1] I. L. Holt, Tablets from the R. Campbell Thompson Collection in Haskell Oriental Museum, The University of Chicago. AJSL 27 (1911) pp. 193–232

ITT see Genouillac ITT

J YBC 4662 (cf. pp. 66ff. above)

Ja YBC 8588 (cf. pp. 75f. above)

JA Journal asiatique

JAOS Journal of the American Oriental Society

Jastrow DTTB M. Jastrow, A Dictionary of the Targumim, the Talmud Babli and Yerushalmi, and the Midrashic Literature. New York, Verlag Choreb, 1926

Jean TCL X C.-F. Jean, Contrats de Larsa, I. Musée du Louvre, Département des antiquités orientales. Textes cunéiformes, Vol. X. Paris, Geuthner, 1926

K YBC 4666 (cf. pp. 76ff. above)

K (8705 or 11097) K = Ḳuyunjiḳ Collection, British Museum, London

Kraus AB P. Kraus, Altbabylonische Briefe, II. Teil. MVAG 36,1 (1932)

Kronecker, Zahlentheorie L. Kronecker, Vorlesungen über Zahlentheorie. Leipzig, Teubner, 1901

L YBC 7164 (cf. pp. 81ff. above)

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Legrain [2] L. Legrain, Collection Louis Cugnin. Textes cunéiformes. Catalogue, transcription et traduction. RA 10 (1913) pp. 41–68

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Liverpool The Free Public Museum of Liverpool, England

Lutz [1] H. F. Lutz, Sumerian Temple Records of the Late Ur Dynasty. University of California Publications in Semitic Philology, Vol. 9,2 (1928) pp. 117–268

Lutz [2] H. F. Lutz, Selected Sumerian and Babylonian Texts. UP 1,2 (1919)

M YBC 9874 (cf. pp. 90f. above)

M (406) M = University of Michigan Collection, Ann Arbor, Mich.

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Meissner SAW B. Meissner, Supplement zu den assyrischen Wörterbüchern. Leiden, Brill, 1898

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MKT see Neugebauer MKT

MLC Morgan Library Collection (deposited at Yale University, New Haven)

MM Metropolitan Museum of Art, New York

Moore [1] E. W. Moore, Neo-Babylonian Documents in the University of Michigan Collection. Ann Arbor, University of Michigan Press, 1939

Muss-Arnolt HWB W. Muss-Arnolt, Assyrisch-Englisch-Deutsches Handwörterbuch. Berlin, Reuther & Reichard, 1905

MVAG Mitteilungen der Vorderasiatisch-Aegyptischen Gesellschaft

N YBC 4186 (cf. p. 91 above)

NBC Nies Babylonian Collection, Yale University, New Haven

NCBT Newell Collection of Babylonian Tablets, Yale University, New Haven

Neugebauer MKT O. Neugebauer, Mathematische Keilschrifttexte. QS A 3. 3 vols. Berlin, Springer, 1935–37

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Neugebauer-Struve [1] O. Neugebauer and W. Struve, Über die Geometrie des Kreises in Babylonien. QS B 1 (1929) pp. 81–92

Ni Nippur Collection, Istanbul Museum

O YBC 4607 (cf. pp. 91ff. above)

Oa YBC 7284 (cf. pp. 97f. above)

OLZ Orientalistische Literaturzeitung

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Oppert-Ménant DJ J. Oppert & J. Ménant, Documents juridiques de l'Assyrie et de la Chaldée. Paris, Maisonneuve, 1877

P YBC 10722 (cf. p. 98 above)

Pa YBC 7997 (cf. pp. 98f. above)

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Poebel HGT A. Poebel, Historical and Grammatical Texts. UP 5 (1914)

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Pohl RVU A. Pohl, Rechts- und Verwaltungsurkunden der III. Dynastie von Ur. TMH N.F. 1/2. Leipzig, Hinrichs, 1937

Plimpton Plimpton Library, Columbia University, New York

PSBA Proceedings of the Society of Biblical Archaeology

PTS Princeton Theological Seminary

Q YBC 9856 (cf. pp. 99f. above)

QS Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik. Abteilung A: Quellen; Abteilung B: Studien

R YBC 4652 (cf. pp. 100ff. above)

RA Revue d'Assyriologie

Ranke BE 6,1 H. Ranke, Babylonian Legal and Business Documents from the Time of the First Dynasty of Babylon, Chiefly from Sippar. The Babylonian Expedition of the University of Pennsylvania. Series A: Cuneiform Texts, Vol. 6,1. Philadelphia, 1906

RCT R. Campbell Thompson Collection, The University of Chicago

REC see Thureau-Dangin REC

Reisner TT G. Reisner, Tempelurkunden aus Telloh. Königliche Museen zu Berlin. Mittheilungen aus den orientalischen Sammlungen, Heft XVI. Berlin, Spemann, 1901

RTC see Thureau-Dangin RTC

S YBC 4612 (cf. pp. 103ff. above)

Sa YBC 6492 (cf. pp. 105f. above)

Sachs [1] A. J. Sachs, Some Metrological Problems in Old-Babylonian Mathematical Texts. BASOR 96 (1944) pp. 29–39

Sb MLC 1842 (cf. pp. 106f. above)

Salonen WB A. Salonen, Die Wasserfahrzeuge in Babylonien. Studia Orientalia edidit Societas Orientalis Fennica, Vol. 8,4. Helsingfors, 1939

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T A 24194 (cf. pp. 107ff. above)
TCL I see Thureau-Dangin TCL I
TCL V see Genouillac TCL V
TCL VI see Thureau-Dangin TU
TCL X see Jean TCL X
TCL XIII see Contenau TCL XIII
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Thureau-Dangin REC F. Thureau-Dangin, Recherches sur l'origin de l'écriture cunéiforme. Paris, Leroux, 1898
Thureau-Dangin RTC F. Thureau-Dangin, Recueil de tablettes chaldéennes. Paris, Leroux, 1903
Thureau-Dangin SA F. Thureau-Dangin, Le syllabaire accadien. Paris, Geuthner, 1926
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Thureau-Dangin [14] F. Thureau-Dangin, Notes assyriologiques XLIV. Fragment de vocabulaire. RA 21 (1924) pp. 139–146
TMB see Thureau-Dangin TMB
TMH Texte und Materialen der Frau Professor Hilprecht Collection of Babylonian Antiquities im Eigentum der Universität Jena
TU see Thureau-Dangin TU
U A 24195 (cf. pp. 119ff. above)
Ua YBC 6967 (cf. pp. 129f. above)
Ub YBC 7326 (cf. pp. 130f. above)
Uc YBC 10522 (cf. p. 131 above)
Ud YBC 5022 (cf. pp. 132ff. above)
Ue YBC 7243 (cf. pp. 136ff.)
Ungnad BB A. Ungnad, Babylonische Briefe aus der Zeit der Ḫammurapi-Dynastie. VAB 6 (1914)
Ungnad [1] A. Ungnad, Aus den neubabylonischen Privaturkunden. Beihefte zur Orientalistischen Litteratur-Zeitung, 2 (1908) pp. 19–28
UP University of Pennsylvania. The Museum, Publications of the Babylonian Section
V K 11097 (cf. pp. 139f. above)

VAB Vorderasiatische Bibliothek
Van der Meer, Syllabaries P. E. Van der Meer, Syllabaries A, B¹ and B with Miscellaneous Lexicographical Texts from the Herbert Weld Collection. Oxford Editions of Cuneiform Texts, IV. Oxford, 1938
VAT Vorderasiatische Abteilung, Tontafeln, Staatliche Museen, Berlin
von Soden see under Soden
W K 8705 (cf. p. 140 above)
W 1923–366 W = Herbert Weld Collection, Ashmolean Museum, Oxford
Waschow [1] H. Waschow, review of Neugebauer MKT III. AfO 12 (1939) p. 277
Waschow [2] H. Waschow, review of Neugebauer MKT I–II. AfO 11 (1936/1937) pp. 245–247
Waterman RCAE I L. Waterman, Royal Correspondence of the Assyrian Empire. Part I. Ann Arbor, University of Michigan Press, 1930
Weidner [1] E. F. Weidner, Zahlenspielereien in akkadischen Leberschautexten. OLZ 20 (1917) pp. 257–266
X MM 86.11.404 (cf. pp. 140f. above)
Y VAT 7848 (cf. pp. 141ff. above)
YBC Yale Babylonian Collection, New Haven
ZA Zeitschrift für Assyriologie und verwandte Gebiete
ZDMG Zeitschrift der Deutschen Morgenländischen Gesellschaft
Zimmern, Ritualtafeln H. Zimmern, Beiträge zur Kenntnis der babylonischen Religion. Assyriologische Bibliothek, 12. Leipzig, Hinrichs, 1901
Zimmern [1] H. Zimmern, Assyrische chemisch-technische Rezepte, insbesondere für Herstellung farbiger glasierter Ziegel, in Umschrift und Übersetzung. ZA 36 (1925) pp. 177–208
Zimmern [2] H. Zimmern, Vorläufiger Nachtrag zu den assyrischen chemisch-technischen Rezepten. ZA 37 (1927) pp. 213f.

Symbols Used in Transcriptions and Translations

[ ]	restored	< >	omitted in the text
( )	not in the text (when used in translations); Sumerian writing (in transcriptions of late texts)	[. . . .]	destroyed signs which we cannot supply
		. . . . .	signs or traces of signs which we are unable to read
{ }	erroneous excess in the text		

§ 2. Concordance of Museum Numbers

A 678	p. 20, No. 71
A 1555	p. 20, No. 65
A 7897	p. 12, No. 40; p. 25, No. 140
A 24194	p. 107; plates 15 and 41
A 24195	p. 119; plates 16 and 42
AO 6484	p. 143
BM 85194 rev. III 7–12	p. 96
BM 85196 No. 16	p. 88
CBS 1215	p. 36
CBS 1535	p. 34, No. 33
CBS 3551	p. 36
CBS 6095	p. 23, No. 99,14; p. 28, No. 144
CBS 7355	p. 21, No. 91
CBS 7356	p. 36
CBS 7358	p. 36
CBS 7359	p. 36
CBS 7360	p. 36
CBS 7362	p. 36
CBS 7363	p. 36
CBS 7364	p. 12, No. 44
CBS 7365	p. 22, No. 99,7
CBS 7366	p. 23, No. 99,18
CBS 7369	p. 30, No. 152; p. 33, No. 28
CBS 7370	p. 22, No. 99,11
CBS 7373	p. 21, No. 85
CBS 7375	p. 20, No. 66
CBS 7376	p. 22, No. 93
CBS 7378	p. 21, No. 87
CBS 7890	p. 22, No. 99,2
CBS 8136	p. 23, No. 99,25
CBS 8149	p. 23, No. 99,17
CBS 8151	p. 32, No. 158
CBS 8153	p. 12, No. 38; p. 28, No. 145
CBS 8156	p. 30, No. 149
CBS 8165	p. 34, No. 32
CBS 8210	p. 12, No. 37; p. 29, No. 148
CBS 8215	p. 12, No. 42; p. 27, No. 141
CBS 8226	p. 23, No. 99,21; p. 31, No. 156
CBS 8247	p. 33, No. 166
CBS 8266	p. 33, No. 29
CBS 8270	p. 34, No. 31
CBS 8274	p. 30, No. 153

YBC 6295	p. 42; plates 22 and 46
YBC 6403	p. 23, No. 99,18a
YBC 6492	p. 105; plates 22 and 48
YBC 6632	p. 30, No. 151
YBC 6705	p. 22, No. 99″
YBC 6739	p. 22, No. 99,3a
YBC 6949	p. 21, No. 88
YBC 6950	p. 22, No. 99
YBC 6951	p. 21, No. 92
YBC 6952	p. 22, No. 99,1
YBC 6953	p. 12, No. 36
YBC 6967	p. 129; plates 17 and 27
YBC 7164	p. 81; plates 10 and 35
YBC 7234	p. 17
YBC 7235	p. 17
YBC 7243	p. 136; plates 23 and 49
YBC 7273	p. 131
YBC 7284	p. 97
YBC 7289	p. 42
YBC 7290	p. 44
YBC 7294	p. 35
YBC 7302	p. 44
YBC 7303	p. 12, No. 35; p. 27, No. 143
YBC 7326	p. 130; plates 17 and 43
YBC 7353	p. 17
YBC 7354	p. 17
YBC 7355	p. 17
YBC 7358	p. 17
YBC 7997	p. 98; plates 23 and 49
YBC 8306	p. 22, No. 98a
YBC 8456	p. 21, No. 83a
YBC 8588	p. 75; plates 21 and 46
YBC 8595	p. 22, No. 99,2a
YBC 8600	p. 57; plates 21 and 47
YBC 8617	p. 20, No. 72a
YBC 8633	p. 53; plates 4 and 29
YBC 9852	p. 44; plates 1 and 29
YBC 9854	p. 33, No. 163
YBC 9856	p. 99; plates 4 and 38
YBC 9874	p. 90; plates 11 and 36
YBC 9883	p. 23, No. 99,24
YBC 10522	p. 131; plates 18 and 44
YBC 10529	p. 16
YBC 10722	p. 98; plates 4 and 38
YBC 10801	p. 35
YBC 11094	p. 22, No. 99′
YBC 11120	p. 44
YBC 11125	p. 17
YBC 11126	p. 44
YBC 11127	p. 17
YBC 11138	p. 21, No. 92,1
YBC 11924	p. 23, No. 99,13b

§ 3. Vocabulary

The arrangement of the words in this vocabulary follows the order: ʾ/ʾa/aʾ/a; b; d; e; g; ḫ; i/i; k; l; m; n; p; q; r; s/ṣ/š; t/ṭ; u/u; z. *Akkadian verbal forms* are entered under the consonantal skeleton (thus, *i-du-ú* under *idʾ*$_4$, *te-le-qé-e* under *lqʾ*$_3$), *Sumerian verbal forms* under the syllabic root (e.g., bí-daḫ under daḫ, ba-zi under zi). *All other words* will be found listed under the full, normalized form (nominative singular for nouns), e.g., a-šà, *ina*, *libbu*, *makṣaru*, *terdītu*.

Reference is made to the problem-texts by capital letters, A, Aa, B, ..., X, Y, for which cf. the list of abbreviations (p. 152). The single bracket [followed by a line-number indicates that the first part of the word in question is restored, while] following a line-number indicates the restoration of the last part. In the rare cases where it was thought desirable to list the occurrence of a word which is completely destroyed, this fact is indicated by the enclosure of the line-number within [ ]. In addition to the use of parentheses with special measures (cf. p. 4) and in the transcriptions of late texts to indicate Sumerian writings (cf. p. 156), we also use parentheses in the vocabulary to indicate that the enclosed syllables or words are not present in some of the passages which are listed. Thus, uš(-bi) means that the occurrences listed read uš *or* uš-bi. The symbols (*x*) and (*y*) represent numbers, or nouns (like "length", "area") or phrases (like "that by which the length exceeded the width") which involve a numerical magnitude.

ʾ/ʾA/Aʾ/A

a (≈*mū*, q.v.) "water"
  a: Ue 30
  GÂ-DUB a: Ud 33
  na-az-ba-al a: Ud 44

á (≈*idu*, q.v.) "wages" (per man per day)
  silver
    6 še á lú-ḫun-gá: K rev. 8,11,13; L obv. 2
    6 (še) á-bi (lú-ḫun-gá): F obv. 2,3,5,6,7, and passim; G obv. 1,4,6,9,10,13,14,16; H obv. [7,14, rev. 2; K rev. 27
    6 še á-bi-am$_5$: H obv. 39
    6 še á-am$_5$; H obv. 20,26, rev. 14
    á-bi (lú-ḫun-gá) en-nam: G obv. 13; H obv. 34
    á-bi [*i-*]*na-ad-di-kum:* H obv. 39
    igi á-bi *pu-ṭù-ur:* H rev. 3
  barley
    1(bán) á(-bi) lú-ḫun-gá: K obv. 5,8,10

1(bán) še(-ta-àm) á-bi lú-ḫun-gá: G rev. 24; K obv. 2
á-bi 1(bán) 5 sìla: K rev. 1], 4

a-bal see giš-a-bal

abu "father"
abu(ad) *ana māri-šú:* V 3

$^{\flat}{}_{1}gr$ "to hire"
ag-ra-am a-gu-ur: P 1
4 lú-ḫu[n-g]á *a-gu-ur:* P 4

agru (≈lú-ḫun-gá, q.v.) "hired man"
ag-ra-am a-gu-ur: P 1

agurru (≈sig$_4$-al-ùr-ra, q.v.) "kiln-burnt brick"
^{sig_4}*a-gu-ru-um:* Ue 44
a[*-gu-ru-*]*um:* Ue 38

aḫameš "one another"
itti(ki) *a-ḫa-meš:* Y obv. 19

aḫru "remaining behind"
i-na li-ib-bi (*x*) (*y*) *ta-ḫa-ar-ra-aṣ-ma* (*x* − *y*) *a-ḫe-er-*[*t*]*um* íb-si$_8$-*šu te-le-qé-e:* B obv. 14

aḫu(≈šeš, q.v.) "brother"
5 šeš-*a-ni:* Q 8
ma-li ši-it-ti-in a-ḫi ṣe-eḫ-ri-im: Q 9
a-ḫu-um e-li a-ḫi-im li-te-le-le: Q 10
aḫu(šeš) *ana aḫi*(šeš): V 6

$^{\flat}$*imlū* see *imlū*

aká
aká-bi EŠ-àm: U rev. II 31

$^{\flat}{}_{1}kl$ (≈kú, q.v.) "to eat, consume"
to square
(*x*) *šu-ta-ki-il*(*-ma*) (*x*^{2}): H rev. 20*; Sb rev. 9; Uc [5
(*x*) *it-ti* (*x*) *šu-ta-ki-il-ma* (*x*^{2}): Ec obv. 3; Ua obv. 7
(*x*) *tu-uš-ta-ak-ka-al-ma* (*x*^{2}): B obv. 12
(*x*) *tu-uš-ta-kal*(*-ma*) (*x*^{2}): Pa obv. 4, rev. 1 ([*tu-*] *uš-ta-kal*{*-al*})
to multiply
(*x*) *ù* (*y*) *šu-ta-ki-il* (*xy*): H obv. 28, 35*
tu-uš-ta-ak-ka-al-ma: B rev. 15; C rev. 1 (<*-al-*>)
(*x*) *ù* (*y*) *tu-uš-ta-kal-ma* (*xy*): Pa rev. 5

á-kur-ra
Ud 57

al≈*allu*, q.v.

$^{\flat}{}_{4}lḫ$ "to go up" (see also nim and DUL-DU)
ganba-e *i-li-i-ma:* Sb obv. 1
statement of result
(*x*) *i-li*(*-a-am*/*ma*): Pa obv. 5,8, rev. 3,6; Sb obv. 10, rev. 1,4
(*x*) *ša i-li-a*(*-ak*)*-kum:* B obv. 9],17], rev. 1], 3],8,11; C obv. 2,5]; Sb rev. 2; Ua obv. 8(<*-a-*>); Uc 5
(*x*) *ša i-lu-ú:* Sb obv. [10(?)], rev. 4(?)],10
(*x*) *i-il-li-a*(*-ak*)*-kum*/*ku:* B obv. 9,13,16,19, rev. 3,9,11,13; C obv. 5,7; Sb rev. 9(?); Uc 4 (<*-a-*>)

$^{\flat}{}_{4}llḫ$ see *utlellū*

allu "hoe" (cf. p. 141, note 327)
al: X obv. 10(?)
al-*ḫa:* X obv. 7
10 *al-lu-ú:* X obv. 6

ammatu (≈kùš, q.v.) "cubit"
a-ma-at pu-ta-am ù a-ma-at šu-up-lam: M rev. 2 (*a-ma-at* = *ammat*?)

$^{\flat}{}_{1}mr$ "to see, find"
aš-šu-um (*x*) *ù* (*y*) *a-ma-ri-i-ka:* D obv. 22
aš-šu (*x*) [*a-*]*ma-ri-i-ka:* D obv. 28
lu-mur: V 13
statement of result
ta-mar: Ca Obv. 5; Eb, obv. 5],6,7,9, rev. 1; J rev. 22,33,34,35
tam-mar: W 11

a-na (≈*mala*/*i*, q.v.) "as much as"
difference
a-na (*x*) ugu (*y*) dirig: T obv. I 2(?), II 10], III 30, IV [25, rev. I 34, II 7,35, III 3, IV 13; U obv. I 28, II 3(?),27, III 12, IV 16, rev. II 9,17,23

ana "to, for", etc.
a-na 1(bùr)^{iku} *ša-ni-i-im ki ma-ṣi* uš-gíd-da *lu-uš-ku-un:* B obv. 5
a-na: Ca obv. 6; E obv. 3(?); X obv. 11
(*x*) *a-na* 2 *lu-pu-ut-ma:* H rev. 23; J obv. 21,33
i-na me-e-ša a-na 1 šu-si *šu-up-lim* a-šà *ki ma-ṣi am-qú-ur:* N 3
abu ana māri-šú: V 3
aḫu ana aḫi: V 6
9 *a-na* 6 šu-si . . . . : W 13
10 kùš *a-na* 10 kùš: Y obv. 13,16
. . . . *a-na* ugu QAR: Y obv. 13
ina qabli-šú šu(?)-si *a-na* šu-si sig-*ta:* Y obv. 16
addition
(*x*) *a-na* (*y*) *tu-uṣ-ṣa-ab-ma* (*x* + *y*): B rev. 4,13; C obv. 7
(*x*) *a-na* (*y*) daḫ-ma (*x* + *y*): D obv. 15, rev. 11]
(*x*) *a-na* šà (*y*) [gar-gar-ma] (*x* + *y*): Eb rev. 2
(*x*) *a-na* 1 *ṣí-ib:* H rev. 11
a-na (*x*) (*y*) *ṣí-ib* (*x* + *y*): H rev. 21
a-na 1 (*x*) daḫ: J obv. 10
(*x*) *a-na* 1 daḫ: J obv. 22
(*x*) kùš *a-na* bùr-bi bí-daḫ: L rev. 5,7,12,14
(*x*) *a-na* (*y*) *ṣí-ib*(*-ma*) (*x* + *y*): Ua obv. 8, rev. 3
halving, quartering
(*x*) *a-na ši-na zu-ú-uz:* B obv. 3; D obv. 1]
(*x*) *a-na ši-na ḫe-pé*$^{(-)}$*-ma* $\left(\frac{x}{2}\right)$: E obv. 4,[9,[11, rev. 1; Ua obv. 5
(*x*) *a-na er-bé-et ta-za-az-ma:* Pa obv. 3

* This text elsewhere uses UR-UR-ta for the same operation; see UR-UR.

multiplication
a-na sag-an-na *ša* 10 *e-li* sag-ki-ta *i-te-ru a-na* 10 *ṷa-at-ri-im ta-na-aš-ši-ma:* B obv. 10–11
(*x*) *a-na* (*y*) *ta-na*(*-aš*)*-ši*(*-i*)*-ma* (*xy*): B obv. 20, rev. 2,10,12; C obv. 4,6; D obv. 18,20; Ec obv. 4; J rev. 23
(*x*) *a-na* (*y*) *e-ṣi*(?)*-ip* (*xy*): Ca obv. 7
(*x*) *a-na* (*y*) *i-ši*(*-ma*) (*xy*): Ca obv. 5,9; E obv. 7,9,12, rev. 2,6,8,9,11,12,13; Eb obv. 5,6,8; Ec rev. 1; H obv. 4,5,6,10,11, and passim; J obv. 4,5,16,29, rev. 6, and passim; Sb obv. 11, rev. 3; V 12
(*x*) *a-na* (*y*) nim-ma (*xy*): D obv. 8,10,13,23, 26,30, rev. 20
(*x*) *a-na ši-na* c-tab (2*x*): D obv. 8,10,22,25, rev. 17
mi-nam a-na (*x*) *lu-uš-ku-un ša* (*y*) *i-na-di-nam:* D obv. 17, rev. 18
(*x*) *a-na* (*y*) *ta-at-ta-na-aš-ši-i-ma* (*xy*)*:* M rev. 5,6,9
(*x*) *a-na* (*y*) ÍL (*xy*): Aa 9,11; Ja obv. 12,14,16, rev. 2,3,6; N 8,11
(*x*) *a-na*(?) (*y*) *bi-il-ma* (*xy*): Uc 3
(*x*) *a-na* (*y*) *tu-ub-ba-al-ma* (*xy*): Pa obv. 6, rev. 2]
subtraction (probably error for *i-na*)
(*x*) *a-na* 1 *ḫu-ru-úṣ:* H rev. 11,24; J obv. 22,34

an-na (≈*elū*) "upper" (see also an-ta)
sag-an-na: B obv. 10,12,18; D obv. 4,5,9,18,21, 25,27,31], rev. 20
íd-an-na: D obv. 3],19
dagal-an-na: Ea obv. 2
½ GAR-ta-àm an-na íb-si$_8$: F rev. 13
íb-si$_8$ an-na: F rev. 18,19,25,27
½ GAR an-na: F rev. 23,25,27

an-na (≈*annaku*) "lead"
ru-uq-qú ša an-n[a]: Ud 27
4½ gín an-na: X obv. 4

annumma
an-nu-um-ma(*-*)*aš*(?)*-šu*(?)...: P 6a

an-ta (≈*elū*) "upper" (see also an-na)
sag-an-ta: B obv. 2; Ca obv. 4,6
mu-ta-ri-tam an-ta: Ca obv. 8
½ GAR-ta-àm an-ta íb-si$_8$: F rev. 10,14
½ GAR an-ta: F rev. 12
an-ta en-ta-àm íb-si$_8$: F rev. 12
íb-si$_8$ an-ta: F rev. 16,17,19],20,21,22,23,24,26
dagal-an-ta: K rev. 7,10,13,14],16,18,20,22,25
sag-ki an-ta: Y obv. 7

$^{\flat}{}_{37}pš$ (≈kìd and kid$_9$, q.v.) "to make, perform"
te-ep-pe-eš-ma: B rev. 6
at-ta i-na e-pé-š[*i-ka*]*:* Sb obv. 7

a-rá "times"
multiplication
(*x*) a-rá (*y*) or merely a-rá (*x*): Multipl. Tables, passim; p. 33, No. 28; p. 34, No. 33; X obv. 9, rev. 4,5,6,7
(*x*) a-rá (*y*) *i-ši* (*xy*): H obv. 31; J obv. 14,39
(*x*) a-rá (*x*) UR-UR-a (x^2): J obv. 7,18,30
a-rá-24-kam tab: R rev. 2
a-rá (*x*) e(-tab): R rev. 5; T obv. I 7,9,10,11,12, and passim; U obv. I 5,[7,8,10,12, and passim
a-rá (*x*) a-šà (or sag or uš or a-na uš ugu sag dirig): T obv. I 26, II 20, III 23, V 1, rev. I 26, II 6, IV 21; U obv. I 9,11,13,14,21,25, II 25, III 5,9, rev. I 31,36, II 1,7
uš (or sag) a-rá (*x*) e-tab: U rev. II 27,28, III 7
(*x*) a-rá (*y*) (*xy*) *a-na-ši*[*-i*...]: W 14
addition
(*x*) a-rá 1 *ṣí-ib:* H rev. 23 (probably error for *a-na*)
(*x*) a-rá *li-ib-bi* (*y*) daḫ (*x* + *y*): J obv. 19,31 (probably error for *a-na*)
(*x*) a-rá 1 daḫ: J obv. 34 (probably error for *a-na*)

áraḫ "store-house"
áraḫ in-nu-da: Ue 23

arakarū (≈a-rá-kára) "coefficient" (cf. pp. 15, 48)
a-ra-kára: p. 13, obv. II 5
a-ra-kára a-ra-ka-re-e: p. 13, obv. II 7
a-ra-ka-re-e-em: B rev. 2,12; C obv. 6

arū (≈a-rá, q.v.) "product"
ina a-re-e: V 10

a-šà (≈*eqlu*, q.v.) "field"
a-šà: N 3,6,7,12; Ue 26
area
a-šà: B obv. 1(?); Ca obv. 20; D obv. [1,2,22, 25,28,30, rev. 2,12,13,17; E obv. 1,10,13, rev. 3,4; Eb obv. 3,5; F obv. 2,6,16; Ja obv. 14,15, rev. 3; S obv. 3,5,7,10,14,15,17,19, rev. 2,7; Sa obv. 1,2,3,4,5, and passim; T obv. I [1], II 18,35, III 2, IV [1], V 13, rev. I 3; Y obv. 4],12
a-šà-bi: Scheil [1], passim; NCBT 1913 (p. 10) 4,5; NBC 8082 (p. 10) 6,7; B obv. 2; Ca obv. 2; S obv. 1,2,14, rev. 7,[10,13,16,19
a-šà-*im:* B obv. 20, rev. 10
a-šà-*lam:* B obv. 3
$\frac{1}{60}$ of the area (cf. p. 126)
a-šà: U obv. I 4,7,9,11,13, and passim

$^{\flat}{}_{47}sḫ$
e-si-ḫa: V 6

$^{\flat}{}_{47}šp$ (≈tab, q.v.) "to multiply"
(*x*) *a-na* (*y*) *e-ṣí*(?)*-ip* (*xy*): Ca obv. 7

aššu(*m*) "in order, because"
aš-šu-um (*x*) *ù* (*y*) *a-ma-ri-i-ka:* D obv. 21
aš-šum (x^3) ba-si *la id-di-nu-kum:* Aa 3
aš-šu (*x*) [*a-*]*ma-ri-i-ka:* D obv. 28
aš(?)*-šu*(?): P 6a

atappu (≈pa$_5$-sig, q.v.) "canal"
[*a-*]*ta-a*[*p*] *ta-ra-aḫ-ḫi-im-mi:* M rev. 1

attā (≈za-e, q.v.) "thou"
at-ta: E obv. 2; Ec obv. 3, edge 1(?); Sb obv. 7; Ua obv. 3]
aṷatu "word, thing, matter"
(*x*) *iš-ka-ar a-ṷa-ti iš-te-en:* M rev. 7
$^{\flat}{}_{4}zb$ "to leave behind"
result of subtraction
(*x*) *ša te-zi-bu:* E obv. 4

B

bamtu "one-half"
ba-ma-at (*x*): Pa obv. 2; Sa obv. 8–14
ba-ma-at (*x*) gaz-ma $\left(\frac{x}{2}\right)$: D obv. 14,29
ba-ma-at (*x*) *ḫe-pé:* Eb obv. 9
bán (≈*sūtu*) a measure of capacity (cf. p. 6)
1(bán) (še): K obv. 2,5,8,10, rev. 4
1(bán) še-ta-àm: G rev. 24
2(bán) še: K rev. 3,6
4(gur) 5(bán) gur še: G rev. 26
BAR-NUN (≈*ṣiliptu*, q.v.) "diagonal, hypotenuse"
BAR-NUN: Y obv. 6
barū a type of priest
barū(lú-ḫal): V 2(?),7(?)
barūtu "*barū*-priesthood"
barū(lú-ḫal)-*t*[*i*(?)]: V 1
ba-ru-ti: W 2
ba-si$_{(8)}$ see si$_{(8)}$
-bi "its, their"
passim with a-šà, sag, uš, etc.
½-bi: Eb obv. 7
multiplicative
(*x*)-bi ì-lá: R rev. 1,4
bū "one-half"
ba-a-ši-na te-ḫe-pe-e: B obv. 8],18], rev. 9; C obv. 3
bnį "to create, form, build"
a-šà *ab-ni:* D rev. 2
bùr a unit of area (cf. p. 5)
1(bùr): Scheil [1], No. 95; NCBT 1913 (p. 10) 5
1(bùr)^{iku}: B obv. 3,5,7,20, rev. 6,10,16; C obv. 4, rev. 1
2(bùr): Scheil [1], No. 96
2(bùr)^{iku}: B obv. 2; S obv. 14,15,17,19, rev. 2,7,10,13,16,19
3(bùr): Scheil [1], No. 48
4(bùr): Scheil [1], No. 44
5(bùr): Scheil [1], No. 45
8(bùr): Scheil [1], No. 47
9(bùr): Scheil [1], No. 46
bùr (≈*šuplu*, q.v.) "depth"
bùr(-bi): F obv. 1,3,5,6,7, and passim; G obv. 1,6,9,[10,12, and passim; H obv. 1,4,7,8,14, and passim; J obv. 4,5,13,15,25, and passim; K obv. 1],5,7],10,13, and passim; L obv. 1,8,10,12,15, and passim
bur$^{\flat}$u a unit of area (cf. p. 5)
1(bur$^{\flat}$u): Scheil [1], No. 47
1(bur$^{\flat}$u)^{iku}: S obv. 2,3,5,7,10
3(bur$^{\flat}$u): Scheil [1], No. 48
4(bur$^{\flat}$u): Scheil [1], No. 53

D

dagal (≈*rupšu*) "width"
dagal: D rev. 22; K obv. 1,5,11],13,15, and passim; L obv. 1,4,6,8,10, and passim
dagal-an-na: Ea obv. 2
dagal-an-ta: K rev. 7,10,[13,14],16,18,20,22,25]
dagal-bi: K obv. 8
dagal-ki-ta: Ea obv. 7; K rev. 7,10,12,13,14,16, 18,20,22,24
daḫ (≈*ṷṣb*, q.v.) "to add"
daḫ(-ma): T obv. I 4,6,7,13,14, and passim; U obv. I [3,4,5,9,10, and passim
a-na 1 (*x*) daḫ: J obv. 10
(*x*) a-rá *li-ib-bi* (*y*) daḫ (*x* + *y*): J obv. [19],31
(*x*) *a-na* 1 daḫ: J obv. 22
(*x*) a-rá 1 daḫ: J obv. 34
(*x*) *a-na* (*y*) daḫ(-ma) (*x* + *y*): Ca obv. 10; D obv. 15, rev. 11
(*x*) kùš *a-na* bùr-bi bí-daḫ: L rev. 5,7],12,14
bí-daḫ(-ma): R obv. 10,14,17,18], rev. 2,4,5,8, 12; U rev. II 30
peculiar use in subtraction (cf. pp. 64f.)
(*x*) (*y*) daḫ: F obv. 27,28,29,30,31,32,33, rev. 1,2,3,5,8,9
dal (≈*tallu*, q.v.) "dividing line" (cf. p. 48)
(*x*) dal múrub: B figure
dal: Y obv. 3,9,10
dal-bi: D obv. 17,24
derdītu see *terdītu*
díb (≈*ṣbt*, q.v.) "to take"
lú-1-e uš en-nam al-díb: G rev. 7; J obv. [37]; K obv. 12,20,22, rev. 15; L obv. [5,14,[22], rev. 8,16
(*x*) GAR (or kùš or šu-si) (uš) al-díb (singular): G rev. 8; K obv. [12,20,22, rev. 15(<al->); L obv. 14, rev. 9,16
(*x*) GAR uš <al->díb (plural): L obv. 7
(*x*) erim-ḫá uš en-nam al-díb: G rev. 10; K rev. 17
(*x*) GAR uš al-díb-bé (plural): K obv. 14
(*x*) erim-ḫá uš en-nam al-díb-bé-eš: K obv. 14
(*x*) erim-ḫá uš en-nam al-díb-eš: L obv. [7
(*x*) díb lú-1: J obv. 40
1 SAR gagar *ta-ad-di-ta-am* en-nam ì-díb: O rev. [5],8,11,14a,17
(*x*) sig$_{4}$ ì-díb: O rev. 6,9,12,15,18
(*x*) GAR uš ì-díb-bé-eš: G rev. 10; K rev. 17
(*x*) GAR uš in-díb-bé[-eš]: L obv. 25

dirig (≈*ṷtr*, q.v.) "to exceed"
difference
(*x*) ugu (*y*) (*x* − *y*) dirig: F obv. 14,20,22,24, rev. 18; G obv. 24, rev. 3,5,22; S obv. 10, rev. 2
(*x*) dirig: T obv. I 11,18,24,33, II [7, and passim; U obv. I 8,20,26, II 1,10, and passim
a-na uš ugu sag dirig: T obv. II 21], III 31, IV <25>, rev. I 34, II 7,16,29,35, III 4], IV 14; U obv. I [29, II [3(?), 27, III 12, IV 16, rev. II 9,17,24
DU "to multiply"
(*x*) GAM (*y*) DU-ma (*xy*): Y obv. 4,10,11,12,18
mi GAM *mi lu*-DU-*ma lu* (*x*): Y obv. 5,9
du_8 (≈*ṗṭr*, q.v.) "to undo, loosen, free"
reciprocal
<igi> (*x*) du_8-a (*x̄*): J rev. 22
igi (*x*) du_8(-ma) (*x̄*): Ca obv. 5,9; D obv. 26, rev. 20; H obv. 5,<12>,17,18,36,37, rev. 6,16,18; J obv. 5,14,15,26,28,[39], rev. 7,8, 14],15,21,27,28,35
igi-(*x*)-gál-bi du_8-ma (*x̄*): D obv. 7,9,12,23, rev. 5,[10]
dub (≈*tuppu*) "tablet"
[du]b-1-kam: p. 6, note 29
dub-10-kam-ma: T rev. V 8
DUB-ÍL see GIŠ-DUB-ÍL
dug_4 (≈*qbi*, q.v.) "to speak"
šá dug_4-ga-*ma:* W 8,11
DUL-DU "to subtract" (cf. also nim)
(*x*) ta (*y*) DUL-DU-*ma ri-ḫi* (*y* − *x*): Y obv. 8,[21
dusu (≈*tupšikku*) "basket" (cf. p. 88)
work at lower levels in the excavation of a canal
10 gín dusu: L obv. 5,7,9,10,13,16
$8\frac{1}{2}$(error for $7\frac{1}{2}$) gín dusu: L obv. 13
igi-TE-en u_4 dusu *iḫ-r*[*e*]: L obv. 17
$\frac{2}{3}$ u_4 *ù* igi-5-gál $\frac{2}{3}$ u_4 dusu *iḫ-re:* L obv. 18

E

-e
(*x*)-e ($\sqrt{x}$) íb-si_8: p. 33, No. 29, passim
(*x*) a-rá (*x*) (x^2)-e: p. 22, No. 99,2
(x^3)-e (*x*) ba-si_8: p. 34, No. 32, passim
ba-si (x^3)-e (*x*): Aa 13
lú-1-e: see lú
uš-(*x*)-e: see uš
see also MLC 2078 (p. 35)
eli (≈ugu, q.v.) "over, above"
(*x*) *ša* (*x* − *y*) *e-li* (*y*) *i-te-ru:* B obv. 10
a-ḫu-um e-li a-ḫi-im li-te-le-le: Q 10
(*x*) *e-li* (*y*) (*x* − *y*) *i-ter*/*-te-er:* Ua obv. 1; Ub 2,3
en (≈*minū*, q.v.) "what?"
en: X obv. 7
en-na: L obv. 14
en-nam: NCBT 1913 (p. 10) 4; NBC 8082 (p. 10) 6; Aa 2,7,10; D obv. 4], rev. 3]; Ea obv. 4,9; F obv. 2,5<6>,7(<-nam>),9,11, and passim; G obv. 2(<en->),[7,9,11,13],15, and passim; H obv. 2,8,15,21,27,34, rev. 2,15; J obv. 13,25, rev. 13,19,26],31; Ja obv. 4; K obv. 3,6,8,10],12], and passim; L obv. [2,7],9,11,17,20,22,28, rev. 2,5,8,12,16,18,20; O obv. 3,8,12,18, rev. 2,8,11,14a,17]; Oa rev. 2; R obv. 5],[9,11,13,15, rev. 3,6,9,13; S obv. 1,4,6,8,11,14,16,18,20, rev. 3,7; Sa obv. 1,2,3, 4,5 and passim
en-ta-àm (íb-si_8): F obv. 3, rev. 12,13
en-na(m) see en
en-ta-àm see en
eqlu (≈a-šà, q.v.) "field"
product
[*eq*(?)-*l*]*a-am:* Ua obv. 9
erbētu "four"
(*x*) *a-na er-bé-et ta-za-az-ma:* Pa obv. 3
erim (≈*ṣābu*) "men, workers"
erim: F obv. 6,16
erim-ḫá: F obv. 2; G rev. 9,11,13,15,16,18, 20, 22,25,26; H obv. 2; J rev. 8,10,12,18,22,29,34; Ja obv. 10,11,12, rev. 1,7; K obv. 3,4,14,16,18, rev. 8,9,17,19,21; L obv. 2,3],7,9,11,24,28, rev. [2
EŠ
aká-bi EŠ-àm: U rev. II 31
ešè (≈*eblu*) a unit of area (cf. p. 5)
1(ešè): Scheil [1], Nos. 38, 39, 45, 46, 95
1(ešè)^{iku}: T obv. I [1], II 18],35, IV [1], V 13, rev. I 3, III 12
2(ešè): Scheil [1], Nos. 40,47,48,94,96; NCBT 1913 (p. 10) 5
E-še-i-pa-ni-AN . . . (?)
p. 23, No. 99,14a, colophon
esír (≈*ittū*) "bitumen"
esír(?)-ḫá: Ue 31
esir-è-a (≈*kupru*) "asphalt"
esir(?)-è-a: Ud 18
éš-kàr (≈*iškaru*, q.v.) "assignment of work" (per man per day) (cf. p. 66)
$7\frac{1}{2}$ gín éš-kàr: F obv. 1,<3>,4
10 (gín) (saḫar) éš-kàr: F obv. 6,7(<éš->),8, 10,12,14,16,17,19,21,22(<éš->),23(<éš->), 24(<éš->); G obv. 1,4,6,8,11, and passim; H obv. 1,7,14,20,32,33, rev. 2,14; J obv. 41, rev. 11,18,25,36]; K rev. 8,11,13,15,17,19, 21, 23,24; L obv. [1; Ud 43; Ue 20
(15) éš-kàr: Ja obv. 8,9,12, edge 2, rev. 5
$\frac{1}{3}$ gín (meaning $\frac{1}{3}$ SAR) (saḫar) éš-kàr: K obv.

2,5,7,9,[11,13,15,17,19,21,23, rev. 1,4; L obv. 22,24,[27], rev. <1>,8,15
(saḫar) éš-kàr en-nam: G obv. 11, rev. 19; H obv. 27; J rev. 31
igi éš-kàr du_8(or *pu-ṭù-ur*): H obv. 5,9,17,22 (<-kàr>),36; J rev. 7,14,21,27
éš-kàr *i-na-di-ku:* H obv. 32; J rev. 36
(*x*) *a-na*(or a-rá) éš-kàr *i-ši:* H rev. 5,17; J obv. 39

ešrū "one-tenth"
10-*u*$_3$: Y obv. 11

etpešu "wise, clever"
et-pe-šú ḫa-as-si mu-du-ú: V 7

G

GÁ-DUB
GÁ-DUB še(or in-nu-da or ... or a): Ud 30,31,32,33,61

gagar (≈*qaqqaru*) "ground, surface"
area
gagar: G obv. 2,18,26,27, rev. 1,3,5,25; H obv. 2; J obv. 1,3,12,14,24,26; K obv. 3, rev. 8; L obv. [2,3; O obv. 3,8,9,12,18,19, rev. 2,3,5, 8,11,14,17
gagar-bi: O obv. 4,13(written še-b[i])
gagar-ḫá: G obv. 18

GAM "depth"
GAM(-2-kam): Ea obv. 3,8

GAM "times"
(*x*) GAM (*y*) (*xy*): Y obv. 1,3,5,7,8,12,15,17,21
(*x*) GAM (*y*) DU-*ma* (*xy*): Y obv. [4],10,11,12,18
mi GAM *mi lu*-DU-*ma lu* (*x*) ($\sqrt{x}$) GAM ($\sqrt{x}$): Y obv. [2,5f.,9

gameru "complete(?)"
uš-ḫá *ga-me-ru-ú-tim ki-la-a-al-le-e-en:* B obv. 7

ga-mi-ru-um
MLC 2078 (p. 35), 5

GÁN-...
Ue 9

ganba (≈*maḫīru*, q.v.) "price equivalent"
ganba(-e): Sb obv. 1,2,6

GÁN-MAN
Ud 20,53

GÁN-má-ru_4(?)
Ud 47; Ue 6

GÁN-UD-A(?)-SAR
Ue 7

GÁN-UD-SAR (≈*askaru*) "segment of a circle"
Eb obv. 1; Ud 21,52,54; Ue 8(?)

GÁN-zà-mí (see also zà-mí)
Ud 48; Ue 4

GÁN-zarà
Ud 49,51; Ue 5

GAR a unit of length (cf. p. 4)
(*x*) GAR: Scheil [1], Nos. 11–53,78–89,92–96; NBC 8082 (p. 10) 5; Ca obv. 1,3; D rev. 22; Ea obv. 6,7, rev. 4; Eb obv. 2; F obv. 1,3, 4,6,7, and passim; G obv. 1,4,5,6,7, and passim; H obv. 1,7,13,14,19, and passim; J obv. 11,23,35,36, rev. 11, and passim; K obv. 12],14,20],22,23, rev. 17; L obv. 7,25, rev. 9; N 1; P 2; Pa obv. 1; S obv. 1,3,4,5,6, and passim; U rev. III 8; Uc 2
1,40 GAR: K obv. 23
5 GAR UŠ uš(-bi): K obv. 1,6,7,9,11

gar (≈*šḳn*, q.v.) "to set, place"
(*x*) *ša-pa-al* (*y*) gar-ra-ma (*y*) (*x*): Aa 5
(*x*) gar-ra(-ma): Eb obv. 7; Ja obv. 5,9,11; N 4,5,6a
(*x*) [*i*-]*na* sag-ki-ta(or sag-an-na) ḫé-gar: D obv. 5

gar-gar (≈*kmr*, q.v.) (cf. also UL-gar) "to add"
(*x*) (*ù*) (*y*) gar-gar(-ma) (*x* + *y*): D obv. [6,7,29; F obv. 13,17,21,23, rev. 16; G obv. 15],22,26, 27,29,31, rev. 1,20; H rev. 1; J obv. 2; S obv. 7,19; Y obv. 5,10
½ (*x*) *ù* (*y*) *ša* gar-gar-*ru ḫé-pe:* H rev. 7; J obv. 6]

gar-gar (≈*kumurrū*) "sum"
gar-gar (*x*) *ù* (*y*) (*x* + *y*): D rev. 2],8,12

gaz (≈*ḫṭ*$'_{3-5}$, q.v.) "to break"
halving
ba-ma-at (*x*) gaz-ma $\left(\frac{x}{2}\right)$: D obv. 14,<29>

geš$'$u "ten sixties", i.e., "600"
1(geš$'$u) sig_4: O rev. 9,12
2(geš$'$u) 2,30 erim[-ḫá]: L obv. 3

geštú "ear"
geštú zà-mí: Ud 50

gíd (≈*arkū*) "long"
uš-gíd-da: B obv. 4,6, rev. 4,14; C obv. 8
gíd-da-meš: Y obv. 19
im-gíd-da, q.v.

gi-kud-a "cut reed"
Ud 64

gim (≈*kīma*, q.v.) "like, equal"
gim: U obv. III 17,26, rev. I 10], III 12·
gim-nam: U obv. I 3(?),7,19],29, II 5,9,12,16, [19,23,31, III [13,21], rev. II 14

gín (≈*šiqlu*) a unit of weight, area, volume, capacity
area (cf. p. 5)
Scheil [1], Nos. 2–13,16–19,66–74,78,80,82,84–86,88; NCBT 1913 (p. 10) 5; O rev. 3
capacity (cf. p. 6)
O obv. 5
volume (cf. p. 5)
Ea obv. 5, rev. 1,2,3; F obv. 1,<3>,4,6,7,8, and passim; G obv. 1,4,6,8,12, and passim; H obv. 32,33, rev. 2,14; J obv. 41, rev. 3,11, 18,25,36; K rev. 8,11,13,15,17, and passim; L obv. 5,7,9,<10>,13,16,18, rev. 15; Ud 43
⅓ gín (meaning ⅓ SAR): K obv. 5,7,9,13,15,17,

19,[21,23, rev. 1,4; L obv. 22,24, rev. <1>, 8,15
⅓ SAR 5 gín (error for 25 SAR): L obv. 20
weight (cf. p. 6)
F obv. 3,4,7,8,10, and passim; G obv. 3,[6,10; H obv. 7,20,40, rev. 4; K rev. 9,10,12; L obv. 3; R obv. 11,13,16, rev. 1,3,4,6,9; Sb obv. 5; X obv. 4
gir$_4$-ad-KID (≈*kīr ad*-DU-*pu*) "wicker-worker's (asphalt) oven"
Ud 55
giš (≈*išu*, q.v.) "wood, tree"
Ec obv. 5,6,7, rev. 3; Ud 58
giš-a-bal "dipper(?)"
Ue 12
GIŠ-DUB-ÍL
Ud 45
giš-má (≈*eleppu*) "boat"
Ue 19
giš-má-lá (≈*malallū*) "cargo boat" (cf. p. 134 ad line 29)
Ud 29
gur (≈*kurru*) a measure of capacity (cf. p. 6)
4(gur) 5(bán) gur še: G rev. 26
5 gur še-e: K obv. 4
še pa$_5$-sig 5 gur: K obv. 5,7,9
30 še gur: Sb obv. [1,2
guškin (≈*ḫurāṣu*) "gold"
Ud 24,25; Ue 34,[37

Ḫ

ḫanzu "one-fifth" (cf. p. 143, note 337)
4 *ḫa-an-za* ninda še: Y obv. 18
ḫassu "wise"
et-pe-šú ḫa-as-si mu-du-ú: V 7
ḫp$'_{3-5}$ (≈gaz, q.v.) "to break"
halving
½ (*x*) *ḫe-pe* $\left(\frac{x}{2}\right)$: H rev. 7,19; J obv. [6],17],27,30
(*x*) *a-na ši-na ḫe-pé-ma* $\left(\frac{x}{2}\right)$: E obv. 4,9,[11], rev. [1]; Ua obv. 5
(*x*) <½->*šu ḫe-pé-ma* $\left(\frac{x}{2}\right)$: Uc 4
ba-a-ši-na te-ḫe-pe-e-ma: B obv. 8,[18, rev. 9; C obv. 3
ba-ma-at (*x*) *ḫe-pé:* Eb obv. 9
ḫr$'_{3-5}$ "to dig"
eḫ(?)*-re-e:* M rev. 5
e[*ḫ*(?)*-r*]*e*(?)*-a-*[*m*]*a:* M rev. 3
iḫ-re: L obv. 16,17],18
ḫrṣ "to subtract"
(*x*) *a-na* 1 *ḫu-ru-úṣ:* H rev. 11,24; J obv. 22,34
i-n[*a* 1 (*x*) *ḫu-ru-úṣ*]: J obv. 10
i-na li-ib-bi (*x*) (*y*) *ta-ḫa-ar-ra-aṣ-ma* (*x* − *y*): B obv. 14
(*x*) *i-na* (*y*) *ta-ḫa-ar-ra-aṣ-ma* (*y* − *x*): B rev. 5,14; C obv. 8]
ḪUN
1 lú-ḫun-gá á-bi 1(bán) 5 sìla ì-ne-ḪUN: K rev. 2,5
ḫun-gá see lú-ḫun-gá

I/Į

ì (≈*šamnu*) "oil"
ì šám saḫar-bi: O rev. 3,5,8,9,12,15,18, rev. 2
íb-si$_8$ see si$_8$
íd (≈*nāru*) "river"
strip
íd-an-na: D obv. 3,19
íd-ki-ta: D obv. 2,3,12,21
įd$'_4$ "to know"
ú-ul i-de: D obv. 31], rev. 13,15
ul idu(zu)*-ú:* V 9
idu (≈á, q.v.) "wages" (per man per day)
(*x*) *a-na i-di i-ši:* H obv. 6,11,18,23,29
igi *i-di* du$_8$: H rev. 16
[. . . (bán)] 5 sìla *še-a-am i-di-šu il-qé:* P 3
igi (≈*igūm*, q.v.) "reciprocal"
igi (*x*) *ta-pa-ṭa-ar-ma:* B obv. 9,19, rev. 10; C obv. 4
igi (*x*) *ú-ul ip-pa-aṭ-ṭa-ar:* D obv. 16, rev. 18
igi (*x*) du$_8$(-ma) (*x̄*): Ca obv. 5,9; D obv. 26, rev. 20; H obv. 5,12],17,18,36,37, rev. 6,16,18; J obv. 3,5,14,15,26,28, rev. 7,8,14,15,21,27, 28,35
igi (*x*) (*x̄*): Ec edge 1
igi (*x*) *pu-ṭur*(or *-ṭù-ur*)(*-ma*) (*x̄*): Aa 8; Eb obv. 5,6,8; H obv. 9,22,24,30, rev. 3; Ja obv. 15, rev. 5; N 7; Sb obv. 9
igi: Ua obv. 1,4
(*x*) igi-bi (*x̄*): p. 13, passim
igi (*x*) . . . . . (*x̄*): X obv. 8
igi-bi (≈*igibū*) (cf. p. 130)
Ua obv. [1,2,3, rev. 5
igi-(*x*)-gál "one-(*x*)th"
Scheil [1], Nos. 2,4,56–58,60,66–68,70–72,84,86, 88; D obv. 3, rev. 4; F obv. 26,27,28,29,30,31, 32(<-gál>),33, rev. 1,2,3,4,5,6],[8,[9,24,26; G rev. 5(<-gál>),25; J obv. 24; L obv. 17,18; O obv. 4(<igi->),13,14,19,20; R obv. [4,10,12,14,15,17, rev. 2,5,7,8,10(<igi->), 11],12; Sa obv. 15–19, rev. [1,2; T obv. I 5, II 19,22, V 6,[15,16],17], rev. I 4,5, II 15,23, 24, III 5, IV 3], 4,[12; U obv. I 2,[4,[15,16,30, II 4,5,6,12,13,19],20],35, III 14,[20,[21,22], IV 17, rev. II 29,32,34; W 9
igi-(*x*)-gál-bi: T obv. I 3], rev. I 6, III 6,7,[8; U obv. II 28],36(?), III 13

iš-te-en ka-la-ak-kum/-ki-im: Ja obv. 2,6,8
(*x*) *iš-ka-ar a-ṷa-ti iš-te-en:* M rev. 8
(*x*) *i-na iš-te-en ú-su-uḫ:* Ua rev. 2
(*x*) *a-na iš-te-en ṣí-ib:* Ua rev. 3
1-*en:* W 6; Y 19

iṣu (≈giš, q.v.) "wood, log"
i-ṣí-im: Ec obv. 1; Ue 35

itti "with"
(*x*) *it-ti* (*x*) *šu-ta-ki-il-ma* (x^2): Ec obv. 3; Ua obv. 6
itti(ki) *a-ḫa-meš:* Y obv. 19
it-ti-šú: Y obv. 20

itu (≈*ṷarḫu*) "month"
itu: V 6(?)
itu-1 7½-kam (meaning 37½ days): L obv. 9,11
itu-ab-è: p. 21, No. 76, colophon; p. 23, No. 99,14a, colophon
itu-giš-apin-du$_8$-a: p. 23, No. 99,22, colophon
itu-kin-^{d}inanna (ki-2): p. 20, No. 72a, colophon; pp. 22f., No. 99″, colophon; No. 99,3a, colophon; No. 99,13b, colophon

K

kādu
ka-a-du: Y obv. [13,14

kalakku (?≈ki-lá, q.v.) (cf. pp. 65f.)
i-na iš-te-en ka-la-ak-ki-im 9 *ka-la-ak-ku:* Ja obv. 2f., 6f.
iš-te-en ka-la-ak-kum: Ja obv. 8
9 *ka-la-ak-ku:* Ja obv. 10

kalu "all"
a-šà *ka-la-šu a-na ši-na* e-tab: D obv. 25

kalū (cf. p. 135 ad line 56)
ka-lu-ú-um: Ud 56

-kam
a-rá-24-kam tab: R rev. 2
1 ma-na 9 gín 21½ še ù <igi->10-gál še kam: R rev. 10

ordinal
see dub, GAM, itu, mu, u$_4$

kāru "quay, market"
[*i*-]*na ka-a-ri:* X obv. 5

KA-ŠÌR (≈*kippatu*, q.v.) "circumference" (cf. p. 9)
Scheil [1], Nos. 84ff.

kbr "to be thick"
i-na ki ma-ṣi ik-bi-ir: Ec obv. 2

kī "how"
ki ma-ṣi$^{(?)}$: B obv. 3,4,6; Ec obv. 2; J rev. 4; N 3; Q 5

ki see *itti*

ki see gagar

kì see kìd

ki-(*x*) "(*x*)th"
p. 20, No. 72a, colophon; A IV 3ff.; B rev. 5; Ud 35,51,52,53,54

kiʾam "thus"
ki-a-am né-pe-šu: H obv. 6,13,19, rev. 25 (text: *di*^{sic}-)
ki-a-am ul idu-ú: V 9

kìd, kid$_9$, kìt (≈^{ʾ}$_{39}$*pš*, q.v.) "to make, perform"
za-e kìd-da-zu-dè: D obv. 4], rev. 17(<-zu->); J obv. 3,13,26,37],[42, rev. 5,13,20,[26,32
za-e kid$_9$-da-zu-dè: H obv. 2,8,15,21,27,34, rev. 3,15
za-e kì-ta-zu-dè: Ca obv. 4]; Eb obv. 4

kiṷā "how much" (cf. p. 50, note 140)
šeš ugu šeš *ki-ṷa-a ú-te-le-el-le:* D rev. 16
ki-ṷa ap-šu-ur: Sb obv. 6

ki-lá
(?≈*kalakku*, q.v.) an excavation of some sort (cf. pp. 65f.)
F obv. 1,3,4,6,7, and passim; G obv. [1,4],[6,8], 10, and passim; H obv. 1,7,14,20,26,33,40, rev. 1,13; J obv. 1],12,24,36], rev. 11,18,25], 31; Ja obv. 1; Ue 20(<ki(?)>-lá)
(≈*šuqultu*) "weight"
ki-lá nu-na-tag: R obv. [1,3],6],8],10,12,14,17, rev. 1,4,7,11
ki(?)-lá: Y obv. 17
ki-lá-bi: Oa rev. 2,3; Ue 33(?)
ki(?)-lá-*šú:* Y obv. 16

kilallān "both"
ki-l[*a-l*]*a-an:* E obv. 1
ki-la-a-al-le-e-en: B obv. 8

kīma (≈gim, q.v.) "as, like"
kīma ú-kal-li-mu-ka: W 10
kīma níg-lal-*ma:* W 11

kīnu "true, real"
uš *ki-nu-um* or *ki*(?)-*nim* (one expects *ša-nu-um* or *ša-nim*): E obv. 8, rev. 2,8
a-šà *ki-nu-um:* E rev. 4

kippatu (≈KA-ŠÌR, q.v.) "circumference"
ki-ip-pa-at giš (or *i-ṣí-*[*i*]*m*): Ec obv. 1,[7
ki-pa-at giš: Ec rev. 3
ki-ip-pa-tam/-tum: Pa obv. 1,9
(*x*) *kip-pat:* Y obv. 12

k(?)*iru*
túg-ḫá *ša i-na ki*(?)-*ri-im:* Ue 24

kìt see kìd

ki-ta (≈*šaplū*) "lower"
ki-ta íb-si$_8$: F rev. 10(<-ta->), 12, 14
(íb-si$_8$) ki-ta: F rev. 13,16,17,18,19,20,21,22,23, 24,25,26,27(ta^{sic}-ta)
dagal-ki-ta: Ea rev. 7; K rev. 7,10,12,13,14,16, 18,20,22,24
íd-ki-ta: D obv. 2,3,12,21
sag-ki-ta: B obv. 2,10, rev. 8; C obv. 2]; D obv. 4,5,20,21],28; Y obv. 7

kilmu
ki-it-mu-um: Ue 27
klm "to show"
kīma ú-kal-li-mu-ka: W 10
kmr (≈gar-gar, q.v.) "to add"
ak-mu-ur-ma: Sb obv. 3,4
(*x*) (*u*) (*y*) *ta-ka-mar(-ma)* (*x* + *y*): B. obv. 8,18, rev. 8(<*ta-*>)
½ (*x*) *u* (*y*) *ša* gar-gar-*ru ḫé-pe:* H rev. 7; J obv. 6]
kú (≈$^{\circ}$1*kl*, q.v.; ≈*ukullū*) "to eat, consume"; "fodder, food"
kú: B obv. 1(?)
inclination (cf. p. 81)
i-na 1 kùš bùr en-nam kú ì-kú: K rev. 23
[*i-n*]*a* 1 kùš bùr-bi ½ kùš kú ì-kú: K rev. 25
½ kùš ì-kú: K rev. 23
kù (≈*kaspu*) "silver" (see also kù-babbar)
F obv. 4,6,7,8,10,12,14,17,19,21,22,23,24; G obv. 4,6,8,10,12,14,16; H obv. 12,18,20,25,33, rev. 1; K rev. 10,12
kù-babbar (≈*kaspu*) "silver" (see also kù)
F obv. 2,3,16; G obv. 2],3; H obv. 2,7,14,26,30, 38,40, rev. 4,13,16; K rev. 8,9; L obv. 2],3; Q 8; Sb obv. 4,8; Ud 23,26; Ue 51
kubru "thickness"
ku-bu-ur giš(or *i-ṣí-im*): Ec obv. 5,[6; Ue 35
kuburrū "thickness"
ša ku-bu-ur-re-e giš: Ud 58
kud see gi-kud-a
kṷl "to hold"
(*x*) *re-eš-ka li-ki-il:* D obv. 9,12]; N 9]
(*x*) *ša re-eš-ka ú-ka-al-lu:* D obv. 14; N 11]
kùš (≈*ammatu*, q.v.) a unit of length (cf. p. 4)
Scheil [1], Nos. 1–21,23–25,27–29,33–35,39,66, 67,69–71; NBC 8082 (p. 10)5; Ea obv. 1,2,3,6,8, rev. 5; Ec obv. 1, rev. 3; F obv. 1,3,4,5,6, and passim; G rev. 10,24; K obv. 1,5,7,8],9], and passim; L obv. 1,4,5,6,7, and passim; O obv. 1,2,10,11,17, rev. 1,4,10,13,16; S rev. 5,6,11,12,14,17; Ud 9,11,13; Ue 47; Y obv. 13,15(volume?; cf. p. 142),16
1 kùš SUK-*lim:* W 9
ku-ZI
iš-ka-ra-am i-na ku-ZI-im id-di-nu-nim: Q 1

L

lā "not"
aš-šum (*x*^{3}) ba-si *la id-di-nu-kum:* Aa 3
lá
(≈*mṭṭ*) "to be less"
(*x*) ba-lá: T obv. I 12,25,32, II [8,17], and passim; U obv. I 27, III 11,19],28, IV 1, and passim
(≈*šql*) "to weigh"
ì-lá: R obv. 7,9,11,13,15,18, rev. 1,3,4,6,8,12]
làl (≈*dišpu*) "honey"
làl(?): Ue 29
libir-ra (≈*labiru*) "old"
pa$_{5}$-sig-libir-ra: L obv. [19,21,23,[26, rev. 1,3,6, 10,13
libbu (≈šà, q.v.) "heart, interior"
subtraction
i-na li-ib-bi (*x*) (*y*) *ta-ḫa-ar-ra-aṣ-ma* (*x* − *y*): B obv. 13
(*x*) *i-na li-bi* (*y*) *ú-sú-uḫ* (*y* − *x*): H rev. 9
(*x*) *i-na li-i*[*b-bi* (*y*)] *ta-ba-al* (*y* − *x*): J obv. 8
addition
(*x*) a-rá *li-ib-bi* (*y*) daḫ (*x* + *y*): J obv. 19],31]
lillidu "young ram"
li-li-du e-li li[*-li-*]*di:* Ub 3
lim "thousand"
Y 17
li-im: PTS 247 (p. 18), upper edge
lpt
it-tal-pi-it: V 11
"to operate with" or the like
(*x*) *a-na* 2 *lu-pu-ut-ma* (*y*) a-rá(or *a-na*) 1 daḫ(or *ṣí-ib*) (*y*) a-rá(or *a-na*) 1 *ḫu-ru-úṣ:* H rev. 23; J obv. 21,33
½ 7-*bi-tim ša la-ap-tu-ma ḫé-pe:* J obv. 27
(*x*) *lu-pu-ut-ma:* Sb obv. 8
lq$^{\circ}$$_{3}$ "to take"
(*x*)-ta-àm uš *il-qú*[*-ú*]: D rev. 21
(*x*) sìla *še-a-am i-di-šu il-qé-ma:* P 3
extraction of square-root
(*x*) íb-si$_{8}$-*šu le-qé:* H rev. 10,22; J obv. [9],20,[32]
(*x*) íb-si$_{8}$-*šu te-le-qé-e-ma:* B obv. 15
lū "so that"
1(bùr)^{iku} *lu-ú sà-ni-iq:* B obv. 5,7
mi GAM *mi lu*-DU-*ma lu* (*x*): Y obv. 9
lú (≈*aṷīlu*) "man"
lú-1-e uš en-na(m) al-díb: G rev. 7; K obv. 12,20,22, rev. 15; L obv. 5,14,22, rev. 8,16
(*x*) díb lú-1: J obv. 40
lú-1-e u$_{4}$ en-nam ì-til: K obv. 24
lú-1-e igi-TE-en u$_{4}$ *ši-lu-ta-am iḫ-re:* L obv. 16
éš-kàr lú-1: Ud 43
lú-ḫun-gá (≈*agru*, q.v.) "hired man"
F obv. 11; G obv. [1,4,6,9,10,13,14,16, rev. 24]; K obv. 2,5,8,10, rev. 1,4,8,11],13,27; L obv. 2; P 4]
lugúd (≈*kurū*) "short"
uš-lugúd-da: B obv. 4,6, rev. 6,15; C obv. 9

M

má see giš-má
maḫīru (≈ganba, q.v.) "price equivalent"
ma-ḫi-ri-ṷa[*-ka:* Sb obv. 3,4,11, rev. 3](?)
makṣaru (cf. p. 55, note 152)
ma-ak-ṣa-ra-am: E obv. 7
ma-ak-ṣa-ru-um: Aa [1; E rev. 5,10

mala (≈a-na, q.v.; see also *mali*) "as much as"
[*m*]*a-la i-li-a-am:* Pa rev. 6
má-lá see giš-má-lá
mali (≈a-na, q.v.; see also *mala*) "as much as"
ma-li ú-te-le-el-lu-ú ú-ul i-de: D rev. 15
ma-li pi-ia: Q 2,3,6
ma-li ši-it-ti-in a-ḫi še-eḫ-ri-im: Q 8
maltaktu (cf. p. 135 ad line 60)
ma-al-ta-ak-tum: Uc 50
ma-al-ta-ak-tum ša mu-ši-tim: Ud 60
ma-na (≈*manū*) a unit of weight
weight (cf. p. 6)
K rev. [9,[10,12; L obv. [3; Oa rev. 3; Q 8; R obv. [2,7,9,11,13,15,16,18, rev. 3,6,8,9,13]; Sb obv. 5; X obv. 3],[4,7; Y obv. 16(?)
consistent error for SAR (volume)
⅓ ma-na: L obv. 4,6,8,<10>,12,15
māru "son"
abu ana māri(dumu)-*šú:* V 3
maṣi see *mṣ*$_{17}$
mašqaltu
ma-aš-qá-al-tum ša sig$_4$: Ud 36
maššū (cf. p. 135 ad line 41)
ma-aš-šu-ú-u[*m ša* si]g$_4$(?): Ud 41
ma-aš-šu-ú-um ša saḫar: Ud 42
me (abbreviation for *me*$^{?}$*atu*?) "one-hundred"
(*x*) *me:* Y obv. 12,15,17
me$^{?}$*atu* "one-hundred"
me-at: PTS 247 (p. 18), upper edge
meḫru "equivalent"
(*x*) *ù* (*x*) *me-ḫe-er-šu i-di:* Ua obv. 11
mḫr (≈si$_8$, q.v.) "to be equal, be square, form a square"
[t]úl 10 GAR *im-ta-ḫar* 10 GAR *iš-pi-i*[*l*]: N 1
10 *ù* 10 *ša im-t*[*a*]*ḫ-ru* gar-ra: N 4
10 *ša im-taḫ-ru:* N 10
(*x*) *šu-tam-ḫír-ma* (*x*^{2}): Eb obv. 9
mi (abbreviation for *minū*, q.v.) "what?"
mi GAM *mi lu*-DU-*ma lu* (*x*): Y obv. 5,9
minītu
mi-na-a-te: W 12
minū (≈en, q.v.; see also *mi*) "what?"
mi-nam a-na (*x*) *lu-uš-ku-un ša* (*y*) *i-na-di-nam:* D obv. 17, rev. 18
(*x*) *mi-nu*(-*ú*): Eb obv. 4; Ec obv. 7
(*x*) *mi-nu-um:* Ca obv. 4; E. obv. 1],6; Ua obv. 2,10
miṯḫartu (≈íb-si$_8$, q.v. under si$_8$) "side of a square"
½ GAR-ta-àm *mi-*[*i*]*t-*[*ḫ*]*a*[*-ar-tum*]: Ea rev. 4
mqr "to irrigate"
a-šà (*ki ma-ṣi*) *am-qú-ur:* N 3,12
10 *šu-pu-ul me-e ša* a-šà *im-qú-ru:* N 6],[7
mṣ$_{17}$ "to suffice, extend"
question: how much/many
ki ma-ṣi$^{(?)}$: B obv. 3,4,6; Ec obv. 2; J rev. 4; N 3; Q 5
mu (≈*šattu*) "year"
mu-1-kam: V 6
mu é ^{d}en-ki šà urim^{ki}-ma: p. 23, No. 99,13b
mu (≈*šumu*) "name"
mu-bi-im: A IV 1
mū (≈a, q.v.) "water"
me-e a-šà: Uc 26
me-e pa-ta-a-tim: L rev. 17,19
me-e-ša az/ṣ-z/ṣu-ul-ma i-na me-e-ša a-na 1 šu-si *šu-up-lim* a-šà *ki ma-ṣi am-qú-ur:* N 2
10 *šu-pu-ul me-e:* N 6,7
mūdū "wise, informed"
et-pe-šú ḫa-as-si mu-du-ú: V 7
múrub (≈*qablū*, q.v.) "middle" (adjective)
(x) dal múrub: B figure
mušītu "night"
ma-al-ta-ak-tum ša mu-ši-tim: Ud 60
muttalli(')*ātu*
mu-ut-ta-al-li-a-tum: Ud 39
muttarittu "perpendicular"
mu-ta-ri-tam an-ta: Ca obv. 8

N

na$_4$ (≈*abnu*) "stone"
R obv. 2],5,6,8,9], and passim
nalbanu (cf. p. 132 ad line 3)
na-al-ba-an im-. . . .: Ud 14
na-al-ba-an 1 kùš sig$_4$: Ud 9
na-al-ba-an ½ kùš sig$_4$: Ud 11
na-al-ba-an ⅓ kùš sig$_4$: Ud 13
na-al-ba-an saḫar: Ud 3
na-al-ba-an sig$_4$-áb: Ud 4
na-al-ba-an sig$_4$-al-ùr-ra: Ud 6
nazbalu (cf. p. 132 ad line 2)
na-az-ba-al a: Ud 44
na-az-ba-al saḫar: Ud 8; Ue 32
na-az-ba-al sig$_4$: Ud 2
na-az-ba-al-ša: Ud 5,7,10,12,15
na-az-ba-lu-um: Ue 41],49
ndi "to throw, cast, lay"
(*x*) *ù* (*x*) *me-ḫe-er-šu i-di-ma:* Ua obv. 11
ndn (≈sì, q.v.) "to give"
aš-šum (*x*^{3}) ba-si *la id-di-nu-kum:* Aa 3
(*x*^{3}) *ša* ba-si *i-na-di-nu-kum:* Aa 4
iš-ka-ra-am i-na ku-ZI-*im id-di-nu-nim-ma:* Q 1
result
(*x*) *i-na*(-*ad*/-*an*)-*di*(-*ik*)-*kum*/-*ku*/-*ku-um:* B obv. 11, rev. 1; E rev. [7,9; H obv. 3,4,5,6,9, and passim; J. obv. 4],5,6,7,8, and passim; M rev. 6,8,11
(*x*) *i-na-an-di-in:* Pa rev. 8
mi-nam a-na (*x*) *lu-uš-ku-un ša* (*y*) *i-na-di-nam:* D obv. 17, rev. 19
it-ta-di-kum: Ja rev. 7

nēpešu "procedure, thing to do"
ki-a-am né-pe-šu: H obv. 6,13(*-pu-*),19, rev. 25
nēpištu (cf. p. 132)
igi-gub-ba-*ša né-pi-iš-tum:* Ud 1
^{d}nidaba
Ud 66
ni-d/ki-ir-tum
Ue 25
níg (≈*ša*, q.v.) "that which"
níg uš daḫ: U rev. II 32
níg daḫ: U rev. II 33,35(?)
NIGIN
L rev. 18,20
níg-lal
kīma níg-lal-*ma:* W 11
níg-ŠID "account(?)"
V 5,10
nim "to be high"
L rev. 18(??)
to multiply
(*x*) *a-na* (*y*) nim-ma (*xy*): D obv. 8,10,13,23,26, 30, rev. [20
(*x*) a-rá (*y*) [. . . . . ni]m (*xy*): X obv. 10
to subtract (cf. also DUL-DU)
ina (*x*) (*y*) [. . . . .] nim-*ma* (*x* − *y*): X obv. 8
ninda (≈*akālu*) a measure of capacity (cf. p. 6)
Y obv. 4,11,18
ni-in(?)-*da*(?)-*nam*(?)
Uc 1
niṣirtu "secret"
niṣirtu(SAL-urì): V 1
nsḫ (≈zi, q.v.) "to extract, withdraw"
igi-5-gál u$_4$ *ši-lu-ta-am* 4 gín saḫar *is-sú-uḫ:* L obv. 18
to subtract
[*ša in-*]*na-as-sà-ḫu-ú-*[*m*]*a:* A I 2
(*x*) *i-na li-bi* (*y*) *ú-sú-uḫ* (*y* − *x*): H rev. 9
(*x*) *i-na* (*y*) *ú-sú-uḫ* (*y* − *x*): Ua rev. 2
i-na (*x*) (*y*) *ú-sú-uḫ-ma* (*x* − *y*): E obv. 3
nšḫ (≈ÍL, q.v.) "to raise, carry"
to multiply
(*x*) a-rá (*y*) (*xy*) *a-na-ši-i:* W 14],15(?)
(*x*) *a-na* (*y*) *i-ši*(*-ma*) (*xy*): Ca obv. 5; E obv. 7,[10, rev. [6,8,9,11,12,13; Eb obv. 5,7,8; Ec rev. 1; H obv. 4,5,6,10,11, and passim; J obv. 4,16,29, rev. 6,7,9,15,17,22,30],33,35; Sb rev. 1,4; V 12
(*x*) a-rá (*y*) *i-ši* (*xy*): H obv. 31; J obv. 14,[40
(*x*) *a-na* (*y*) *ta-na*(*-aš*)*-ši*(*-i*)*-ma* (*xy*): B obv. 11,20], rev. 2],10,12; C obv. 6]; D obv. 19,20; Ec obv. 5; J rev. 23
(*x*) (*a-na*) (*y*) *ta*(*-at*)*-ta-na-aš-ši*(*-i*)*-ma* (*xy*): M rev. 5,7,9; Pa rev. 7
nṣ/zl "to empty(?), bail out(?)" (cf. p. 91, note 230)
me-e-ša az/ṣ-z/ṣu-ul-ma: N 2

P

pà (≈*ṷt*$^{\flat}$$_{\gamma}$) "to find"
na$_4$ì-pà: R obv. [3,6,8,10,12,14,[17, rev. 1,4,7,11
pa$_5$ see *šīlūtu*
pa$_5$-sig (≈*atappu*, q.v.) "small canal"
pa$_5$-sig: K obv. 5,7,9,11,13, and passim; L obv. 1,4,6,8,10,12,15, rev. 17,19
pa$_5$-sig-libir-ra: L obv. 19,21,23,[26, rev. [1,3,6, 10,13
pattu "water-course" (or the like)
me-e pa-ta-a-tim: L rev. 17,19
PI-*lu-ú-tum*
PI-*lu-ú-tum ša* G[I . . .]: Ud 62
pisan (≈*pisannu*) "container"
Ud 57(?)
pšr (cf. p. 107, note 276h)
ki-ṭa ap-šu-ur: Sb obv. 7
ptr (≈du$_8$, q.v.) "to loosen, undo, free"
reciprocal
igi (*x*) *ú-ul ip-pa-aṭ-ṭa-ar:* D obv. 16, rev. 18
igi (*x*) *pu-ṭur*/*-ṭù-ur*(*-ma*) (*x̄*): Aa 8; Eb obv. 5,6,8; H obv. 9,22,24,30, rev. 3; Ja obv. 15, rev. 5; N 8; Sb obv. 9
igi (*x*) *ta-pa-ṭa-ar-ma* (*x̄*): B obv. [9,19], rev. 10; C obv. 4
pū
ma-li pi-ṭa: Q 2,3,[6
ši-in pi-ṭa: Q 2,4,6
ša-lu-uš-ti pi-ṭa: Q 3,4,7
pūtu (≈sag, q.v.) "width"
pu-ta-am: M rev. 2

Q

qablu "middle" (noun)
ina qabli(múrub)*-šú:* Y obv. 16
qablū (≈múrub, q.v.) "middle" (adjective)
ta-al-lam qá-ab-li-a-am: B obv. 17, rev. 7
ta-al-li qá-ab-lu-ú: B obv. 3
ta-al-lum qá-ab-lu-ú-um: B obv. 16
qbṭ (≈dug$_4$, q.v.) "to speak"
ú-ter-ma iq-bi: W 12
ša iq-bu(*-ú*): Ja obv. 7,8,10
qa-ba-a i-šak-ka-nu: V 8

R

rabi'ātum "quarter"
15 *ra-bi-a-tim tu-uš-ta-kal-ma:* Pa obv. 4
r$^{\flat}$$_{1}$*m* "to love"
abu ana māri-šú ša i-ram-mu: V 3
rāṭu (cf. p. 134 ad line 22)
ra-ṭù-um ša urudu(or kù-babbar or guškin): Ud 22,23,24; Ue 34,36(?)
rāzu "pitch-fork(?)" (cf. p. 141)
ra-a-zu: X obv. 5

rebuʾū "fourth"
re-bu-ʾu-ú-ma: V 11
rēšu (≈sag, q.v.) "head"
(*x*) *re-eš-ka li-ki-il:* D obv. 9,12; N 9
(*x*) *ša re-eš-ka ú-ka-al-lu:* D obv. 14; N 11
riḫ "to be left over, remain"
(*x*) ta (*y*) DUL-DU-*ma ri-ḫi* (*y* − *x*): Y obv. 8,[9,21
ruqqu (cf. p. 139)
ru-uq-qú ša guškin(or kù-babbar or an-na or sud-á[g](?)): Ud 25,26,27,28
ru-qú-um ša kù-babbar: Ue 51

S/Ṣ/Š

ša (≈níg, q.v.) "which", "of"
"of, in connection with"
ša: p. 13, obv. II 5,7; Aa 1; D obv. 9; E obv. 2, rev. 5,6(?),10; Eb obv. 8; Ud 16,17,18,19,20, and passim; Ue 33,36,37,47,51; V 2(?); W 8
šá: Y obv. 17
"which"
ša: Aa 4; B obv. 9,10,17, rev. 1,3,8,11; C obv. 2,5; D obv. 14,17,18,20, rev. 19; E obv. 4; H rev. 7,19; J obv. 6,17,27; Ja obv. 7,8,10; N 4,6,7,10,11; Sb obv. 10], rev. 2,4(?),10; Ua obv. 3,8; Uc 3,5; Ue 24; V3
šá: W 7,8,11,12; Y obv. 13,14
šà (≈*libbu*, q.v.) "heart, interior"
subtraction
i-na šà(?) (*x*): Sb rev. 10
addition
(*x*) *a-na* šà (*y*) [gar-gar-ma] (*x* + *y*): Eb rev. 2
sag "head"
(≈*pūtu*, q.v.) "width"
sag(-bi): Scheil [1], Nos. 1–53; A I 2, II 1; D rev. 3,6,9],11,12,13; E obv. 2,3,5, rev. 9; Eb obv. 2,4; F obv. 6,7,9,10,13, and passim; G obv. 1,4,7,8,[10, and passim; H obv. 3,8, 15,19,20, and passim; J obv. 2,6,11,12,13, and passim; Ja obv. 1,5,14, edge 1, rev. 2; O obv. 2,7,11, rev. 4,7,10; S obv. 1,4,5,7,8, and passim; Sa obv. 1,2,3,4,5, and passim; T obv. I 17,23,26,31, II [6, and passim; U obv. I 1,3,4,15,16, and passim
sag-an-na/-ta: B obv. 2,10,12,18; Ca obv. 4,6; D obv. 4],5(error for uš-an-na),9(error for uš-an-na),18(error for uš-an-na),21,25,27,31], rev. 20
sag-ki: E obv. 1,5,8,11, rev. 1,13
sag-ki an-ta: Y obv. 7
sag-ki-ta: B obv. 2,10, rev. 8; C obv. 2]; Ca obv. 4; D obv. 4,5(error for uš-ki-ta),20 (error for uš-ki-ta),21],28; Y obv. 7
sag-meš: Y obv. 10
sag-šè daḫ: T obv. I 13,19(uš-sag-šè), II 30, IV [11,18(uš-sag[-šè]), V 24, rev. IV 4
(≈*rēšu*, q.v.) origin(al weight, number)
sag na_4: R obv. 2,5,7],9,11,13,15,18], rev. 3,6,9, 13
sag al-*ṣa:* X obv. 7
sag-dù (≈*santakku*) "triangle"
Ca obv. 1; D rev. 13; E obv. 1,3,5,8,10],11],13], rev. 1,3,9,13]; Ud 19
sag-ki see sag
sag-ki-gu_4 "trapezoid"
D obv. 1; E rev. 5],10
šaḫallū
ša-ḫa-al-lu-ú-um: Ud 37
saḫar (≈*epi/eru*) "earth, soil, dirt"
"earth"
Ud 3,8,42; Ue 32
volume
saḫar(-bi): Ea obv. 4,9, rev. 2,3; F obv. 2,6,16, 25,26, and passim; G obv. 1,11,19,20,21, and passim; H obv. 1,7,20; Ja rev. 4,6; K obv. 17,19,21, rev. 9,11; L obv. 2,3,5,6,7, and passim; O obv. 3,4,5,8,9, and passim
saḫar-ḫá: G obv. 2,18,26,27, rev. 1,3,5,25,26; H obv. 2; J obv. 1],4,5,12,14,16,24,29; Ja obv. 13,16; K obv. 3, rev. 8
ša-KAK(-)UD-*šu-rum*
Ud 59
SAL+KU (≈*aḫatu*) "sister"
6 uš *ša* SAL+KU-a-ni *iṣ-ba-*[*t*]*u:* Uc 3
šalšu "third"
sag-dù *ša-a*[*l-ši-im*]: E rev. 3
šalaštu "one-third"
ša-lu-uš-ti pi-ṣa: Q 3,4,7
šám "price"
capacity equivalent (cf. p. 97)
ì šám saḫar-bi: O obv. 3,5,8,9,12,15],18, rev. 2
$š^{\flat}{}_{1}m$ "to buy"
a-ša-am: Sb obv. 1],2,6
samašurū "one-eighteenth"
see p. 143, note 337
[Sa-am-s]u-i-lu-[n]a
p. 21, No. 92,1, colophon
šanū "second, other"
a-na 1(bùr)^{iku} *ša-ni-i-im:* B obv. 5
a-na (*x*) ba-si *ša-ni-im:* Aa 11
sag-dù *ša-ni-im:* E obv. 5
uš *ša-nim:* E obv. [9],[12]
uš *ša-nu-um:* E obv. 6, rev. 12
ša-nu-um (*x*): Ua rev. 4
uš 2-*ú:* Y obv. 7
(cf. also under *kīnu*)
šapal "beneath, below"
(*x*) *ša-pa-al* (*y*) gar-ra-ma: Aa 5
sa-pa-ni(?)
X obv. 5

SAR (≈*musarū*) a unit of area, volume, or bricks
area (cf. p. 5)
Scheil [1], Nos. 13–33,37–39,45–48,79–83,87,88, 92; NCBT 1913 (p. 10) 5; Eb obv. 3; G obv. 18, rev. 1,3,5,25; J obv. 1,24,26; L obv. 3; O rev. 5,8,11,14,17
volume (cf. p. 5)
Ea obv. 5, rev. 1,2; F obv. [25,26,27,28,29, and passim; G obv. 24, rev. 26; J obv. 1,4,12,24; L obv. 3,20
bricks (cf. p. 5)
O rev. 6
šár a unit of area (cf. p. 5)
Scheil [1], Nos. 51,53
šarʾu a unit of area (cf. p. 5)
Scheil [1], Nos. 52,53,97(?)
sa-tuk
Ue 48
ṣbt (≈díb, q.v.) "to take"
6 uš *ša* SAL+KU-a-ni *iṣ-ba-[t]u:* Uc 3
ni-in(?)*-da*(?)*-nam*(?) *ṣa-[b]a-at:* Uc 1
še, *še*ʾ*u* "barley"; a unit of area, volume, weight, or capacity
še-b[i]: O obv. 13 (mistake for gagar-bi)
area (cf. p. 5)
Scheil [1], Nos. 1–3,5,6,8–10,12,14,17,56–60, 66–68,70–73; O obv. 4,9,13,19, rev. 3
"barley"
še: G rev. 24,25,26; K obv. 2,3,5,7,9, rev. 3,6; Sb obv. 1],2; Ud 30,61; Y obv. 18
še-a-am: K rev. 2,5; P 3
še-e: K obv. 4
capacity (cf. p. 6)
O obv. 15; Y obv. 4(?)
volume (cf. p. 5)
Ea obv. 5, rev. 1,2; O obv. 4,9,14,20, rev. 3
weight (cf. p. 6)
F obv. 2,3,<4>,5,6,7, and passim; G obv. 1,4,6,9,10,[14,16; H obv. 7,14,20,26,39, rev. 2,14; K rev. 8,11,13,27; L obv. 2; R obv. 11,16, rev. 9,10
sebītu "one-seventh"
7-*bi-tim:* J obv. 27
ṣeḫru "small(est)"
ma-li ši-it-ti-in a-ḫi ṣe-eḫ-ri-im: Q 9
šeš (≈*aḫu*, q.v.) "brother"
šeš: D rev. 14,16
šeš-*a-ni:* Q 8
šeš-meš: D rev. 14,20
*še*ʾ*u* see še
sḫr "to turn around"
ta-as-sà-ḫa-ar(*-ma*)*:* B obv. 12, rev. 7; C obv. 1
sì (≈*ndn*, q.v.) "to give"
še-a-am en-nam ḫu-mu-ra-an-sì: K rev. 2],5
(*x*)(bán) še mu-na-an-sì: K rev. 3,6
result
(*x*) in-sì: D obv. 6,7,10,11,13, and passim
$si_{(8)}$
cube root
(x^3)-e (*x*) ba-si_8: p. 34, No. 32, passim
ba-si: Aa 1,2,3,4,7,10,11,13
(≈*miṯḫartu*, q.v., and *mḫr*, q.v.) side of a square
íb-si_8: Scheil [1], Nos. 54ff.; NBC 8082 (p. 10) 5; F obv. 1,3; G rev. 24; O obv. 17, rev. 1,13,<16>
íb-si_8 an-na/-ta: F rev. 16,17,18,19,20, and passim
an-na/-ta íb-si_8: F rev. 10,12,13,14
íb-si_8 ki-ta: F rev. 16],17,18,19,20, and passim
ki-ta íb-si_8: F rev. 10,12,14
solving number (or the like) (cf. p. 39)
íb-si_8: p. 35, MLC 2078, passim; A II 1, III 1
square root
íb-si_8: p. 31, No. 154a; Ue 10
íb-si_8 (x^2) (*x*): Ec rev. 2
(*x*)-e ($\sqrt{x}$) íb-si_8: p. 33, No. 29; p. 34, No. 31a
(*x*) íb-si_8-*šu te-le-qé-e-ma* ($\sqrt{x}$): B obv. 15
(*x*) íb-si_8-*šu le-qé:* H rev. 10,22; J obv. [9],20,32]
sig "small, low"
see pa_5-sig
sig-*ta:* Y obv. 16,17
sig_4 (≈*libittu*) "brick"
O obv. 1,6, rev. 4,7,9; Oa rev. 1; P 2; Pa rev. 8; Ud 2,9,11,13,36,38,[41
^{sig_4}*a-gu-ru-um:* Ue 44
sig_4-áb (≈*arḫu*) "half-brick"
O obv. 10, rev. 10,12; Ud 4
sig_4-al-ùr-ra (≈*agurru*, q.v.) "kiln-burnt brick"
O obv. 16], rev. 1,13,15,16,18]; Ud 6,34,35
sig_4-anše (≈*amaru*) "heap of bricks"
Ud 16; Ue 40]
šiknu "form" or the like
ši-kin: V 5,8,9,14; W 6
sìla (≈*qū*) a measure of capacity (cf. p. 6)
K rev. [1,4; O obv. 5,[9,15; P 3; Y obv. 4,11
a measure of thickness
2 sìla: Ec obv. 5,6], rev. 1
ṣiliptu (≈BAR-NUN, q.v.) "diagonal, hypotenuse"
ṣi$^{(?)}$*-li-ip-tam/-tim/-tum:* A I 1, III 1; D obv. 32]; E rev. 6],7],10; Ud 63; Ue 10,11
Ṣil-[*lí-*]^{d}NIN-... see *S*[*i*(?)*-n*]*a-tum*
šīlūtu (cf. p. 88)
igi-TE-en u_4 *ši-lu-ta-am iḫ-re:* L obv. 16
igi-5-gál u_4 *ši-lu-ta-am* 4 gín saḫar *is-sú-uḫ:* L obv. 17
20 *ši-i-lu-ta-am:* M rev. 11
20 *ši-i-lu-ú-tim:* M rev. 6
⅓ ma-na(error for SAR) (saḫar) *ši-lu-tum:* L obv. 4,6,8,10,12,15
^{pa_5}*ši-lu-tum:* Ue 21

šinā "two"
a-šà-*lam a-na ši-na zu-ú-uz:* B obv. 3
(*x*) *a-na ši-na* e-tab (2*x*): D obv. 8,10,22,25, rev. 17
(*x*) *a-na ši-na ḫe-pe*$^{(?)}$*-ma* $\left(\frac{x}{2}\right)$: E obv. 4,9,11, rev. 1]; Ua obv. 5
ši-in pi-ia: Q 2,4,6
ši-na ta-ar-ba-ṣú-ú-a-mi: Ub 1
S[*i*(?)-*n*]*a*(?)-*tum mār Ṣíl-*[*lá*-]^{d}NIN-. . .
p. 21, No. 92,1 colophon
^{d}*Sīn*(EN-ZU)-*a-pil-urim*^{ki}
p. 23, No. 99,13b, colophon
šinzirū "one-twelfth"
see p. 143, note 337
šittān "two"
ma-li ši-it-ti-in a-ḫi ṣe-eḫ-ri-im: Q 9
škn (≈gar, q.v.) "to place, set"
(*x*) *ša i-na* sag-an-na(or sag-ki-ta) *aš-ku-nu:* D obv. 19,20
qa-ba-a i-šak-ka-nu: V 8
mi-nam a-na (*x*) *lu-uš-ku-un ša* (*y*) *i-na-di-nam:* D obv. 17, rev. 19
lu-uš-ku-un-ma: B obv. 4,6
(*x*) *šu-ku-un-ma:* D obv. [18], rev. 19
šmṭ "to cut, smooth(?)"
ú-ša(*-am*)*-mi-iṭ:* L obv. 20,22,24,27], rev. <1>, 4,7,11,14
snq "to border"
sà-ni-iq: B obv. 5,7, rev. 6,16; C rev. 1]
špl "to be deep, low"
10 GAR iš-*pi-i*[*l*]: N 1
[g]anba *iš-pí-il-ma:* Sb obv. 2
šrr
ul šá-ri-ir: W 7
[. . .-*ša*(?)-*r*]*a-ar-ma:* W 13
šu abbreviation for *šuplu*, q.v.
sud-ág (≈*elmešu*) (cf. p. 134 ad line 28)
ru-uq-qú ša sud-á[g(?)]: Ud 28
SUK-*lim* a type of kùš
1 kùš SUK-*lim:* W 9
sukud (≈*mēlū*) "height"
sukud: Y obv. 13(?)
sukud-bi: O obv. 2,7,11,17], rev. 2,7,10,14,16
sukud-du: O rev. 5
sum see sì
šumma "if"
šum-ma: B obv. 1; Ja edge 1; Ub 1
ŠU+NIGÍN "total"
Ea rev. 2
ŠU+NIGIN "grand total"
Ea rev. [6
šuplu (≈bùr, q.v.) "depth"
šu (abbreviation; cf. p. 88): L obv. 4,5,[6,7,8, 9,10,12,13,15,16
šu-up-lam: M rev. 2
šu-up-lim/li-im: Ja rev. 3; N 3; Pa obv. 6
šu-up-lum/lu-um: Ja obv. 4,17]; Pa obv. 8
šu-pu-ul túl(or *me-e*): N 5,6,7,8]
šu-ri-a "one-half"
šu-ri-a: F obv. 25, rev. 7],22; O obv. 4,13,14,20, rev. 3; T obv. V 14, rev. I 4
šu-ri-a-bi: p. 12, Nos. 33–35,37a
šu-ri-bi: p. 12, No. 32
šu-si (≈*ubānu*) a unit of length (cf. p. 4)
Scheil [1], Nos. 54–60; G rev. 8; K rev. 15; L obv. 5,14, rev. 16; N 3; O obv. 2,6,7,11,17, rev. 2,5,7,10,14,16]; Y obv. 16,17
part of the liver(?)
V 4,5,9; W 3,5,10,13
šu-ši "sixty"
P 2; T rev. V 6

T/Ṭ

ta abbreviation for *taraḫḫu*, q.v.
ta (≈*ultu*) "from"
(*x*) ta (*y*) DUL-DU-*ma ri-ḫi* (*y* − *x*): Y obv. 8
tab (≈$^{?}$$_{4?}$*šp*, q.v.) "to multiply"
a-rá-24-kam tab: R rev. 2
a-rá (*x*) e<-tab>: T obv. I 14,16,18,20,22,23, 25,30,31,32,33, II 3,14,15,16,24,26,28,29, and passim; U obv. II 7,11, IV 16(?), rev. I 8,13, 22, II 27,28, III 9
(*x*) *a-na ši-na* e-tab (2*x*): D obv. 8,10,22,25, rev. 17
a-rá (*x*) e-tab(-ma): R rev. 5; T obv. I 7,9,10, 11,12,17,24,28, II 17,34, IV 8,9,10,15, V 11, 12, rev. I 2,10,23,24,28,33, II 22,34, IV 1,8,31, V 2; U obv. I 5,7,8,10,12, and passim
taddītu "laying" (of bricks) (cf. p. 94)
1 SAR gagar *ta-ad-di-ta-am* en-nam ì-díb: O rev. 5],8,11,14,17(<-*am*>)
ta-ad-di-tum ša sig$_4$: Ud 38
tag (cf. p. 102)
ki-lá nu-na-tag: R obv. [1,3],[6],[8],[10,12,14,17, rev. 1,4,7,11
$^{(?)}$*takīltu* (cf. p. 130)
(*x*) *ta-ki-il-tam:* Ua rev. 1
[*ta*(?)-*k*]*i*(?)-*il-ti ṣi-li-ip-tim:* A I 1
tallaktu
ta-la-ak-tum: Ue 46
tallu (≈dal, q.v.) "dividing line" (cf. p. 48)
ta-al-lam qá-ab-li-a-am: B obv. 17, rev. 7; C obv. 1
ta-al-li qá-ab-lu-ú: B obv. 3
ta-al-lum qá-ab-lu-ú-um: B obv. 16
diameter
ba-ma-at ta-al-li-im: Pa obv. 2

-ta-àm "each"
1,5-ta-àm uš *il-qú*[-*ú*]: D rev. 21
1(bùr)^{iku}-ta-àm: B obv. 3
en-ta-àm (íb-si$_8$): F obv. 3, rev. 12,13(-ta-à<m>)
(x) GAR(or kùš)-ta-àm (íb-si$_8$ or *mi-*[*i*]*t-*[*ḫ*]*a*[-*ar-tum*]): Ea rev. 4; F obv. 1,4, rev. 10,13,14; O obv. 17, rev. 1,13,16
1(bán) še-ta-àm á-bi lú-ḫu[n-gá]: G rev. 24
1,40 uš-ta<-àm(?)> *ki-l*[*a-l*]*a-an:* E obv. 1

taraḫḫu (cf. p. 88)
te-er-di-is-sà ½ kùš *ta-ra-ḫi-ša ú-ša-mi-iṭ:* L obv. 20
te(or *de*)*-er-di*(*-is*)*-sà* ½ kùš *ta*(abbreviation) *ú-ša*(*-am*)*-mi-iṭ:* L obv. 22,24,27, rev. 1,4,7, 11,14 (cf. p. 88)
[*a-*]*ta-a*[*p*] *ta-ra-aḫ-ḫi-im-mi:* M rev. 1

tarbaṣu "sheep-fold"
ta-ar-ba-ṣú-ú e-li ta<-ar>-ba-ṣí: Ub, 2
ši-na ta-ar-ba-ṣú-ú-a-mi: Ub, 1

tašnintu "rivalry, fight" or the like
taš(?)*-nin*(?)*-ti*(?): V 13
taš-nin-tum: V 2

tbl "to take away"
subtract
i-na (*x*) (*y*) *ta-ba-al* (*x* − *y*): D obv. 24],27
(*x*) *i-na li-i*[*b-bi* (*y*)] *ta-ba-al* (*y* − *x*)*:* J obv. 9

terdītu "increase" or the like (cf. p. 88)
de-er-di-sà: L obv. 21], rev. 1(<*-di-*>)
te-er-di-sà: L rev. 4,7,11,14
te-er-di-is-sà: L obv. 20,27

til (≈*gmr*) "to be finished"
((*x*) erim-ḫá) u$_4$-(*y*)-kam in-til: L obv. 9,11
(*x*) erim-ḫá u$_4$ en-nam in-til: L obv. 9,28
(*x*) erim-ḫá (*i-na*) u$_4$-(*y*)-kam in-til-eš: G rev. 12,13,18; J rev. 12,19; K obv. 16, 18
(*x*) erim-ḫá u$_4$ en-nam in-til-eš: K obv. 16
lú-1-e u$_4$ en-nam ì-til: K obv. 24
<lú-1-e> u$_4$-(*x*)-kam ì-til: K obv. 24
<(*x*) erim-ḫá> u$_4$-(*y*)-kam ì-til: K rev. 19
(*x*) erim-ḫá u$_4$ en-nam ì-til-eš: G rev. 11; K rev. 19
((*x*) erim-ḫá) u$_4$-(*y*)(-kam) ì-til-eš: G rev. 15,16, 20,22; K rev. 21; L rev. 2
[(*x*) erim-ḫá] *i-na ki ma-ṣí* [u$_4$] ì-til-le: J rev. 4
(*x*) erim-ḫá *i-na* u$_4$-(*y*)-kam ì-til-le: J rev. 10

túg (≈*ṣubātu*) "cloth"
túg-ḫá *ša i-na ki*(?)*-ri-im:* Ue 24

túl (≈*būr*(*t*)*u*) "well, cistern"
N [1,5,[8]

ṭur "to (re)turn, repeat"
tu-úr-ma: Sb rev. [6(?); Uc 1
ú-ter-ma iq-bi: W 12

U/Ṷ

ū, ù "and"
between nouns (including additive use)
ù(or ù): PTS 247 (p. 18), upper edge; B obv. 4,18, rev. 8; C obv. 2; Ca obv. 4,8; D obv. 4,6,7,21,29, rev. 3; Ea rev. 3; F obv. 2, rev. 16,22,24,25,26,27; G obv. 2,17,22,29, rev. 1,4,6,20,21,25; H obv. 2,8,16,27,34; rev. 1,7, 24; J obv. 2,6,11,13,25, and passim; K obv. 3, rev. 8,13; L obv. 18; M rev. 2; N 4; O obv. 3,4,8,12,13, and passim; Pa rev. 4; Q 4; R obv. 4, rev. 10; S obv. 7,8,11,19,20, rev. 3; T obv. II 20, III 30, IV 25, rev. I 5,34, II 6, III 3,4,5, IV 3; U obv. I 3, II 2,12,19, III 12, and passim; Ua obv. 2; Ud 44,45,49; Y obv. 10,13,16
between clauses
ù(or ù): B obv. 5,6; Ca obv. 6; D obv. 31; E obv. 4,8; L obv. 17; N 6; Sb obv. 6; W 6

u$_4$ (≈*ūmu*) "day"
u$_4$: G rev. 11; K obv. 16,24, rev. 19; L obv. 9,28
igi-TE-en u$_4$: L obv. 16,17
igi-5-gál u$_4$: L obv. 17
⅔ u$_4$ *ù* igi-5-gál ⅔ u$_4$: L obv. 18
u$_4$-(*n*)-kam (*n* being an integer): pp. 20f., No. 72a, colophon; No. 76, colophon; No. 92,1, colophon; p. 22, No. 96, colophon; No. 99″, colophon; No. 99,3a, colophon; p. 23, No. 99,12, colophon; No. 99,13b, colophon; No. 99,14a, colophon; p. 23, No. 99,22, colophon; G rev. 12,13,15,16,18,20,22; J rev. 10,12,19, 29,34; K obv. 16,18, rev. 19,21
u$_4$-12½-kam: K obv. 24
u$_4$-7½(-kam): L obv. 28, rev. 2
(Note itu-1 7½-kam: L obv. 9,11 meaning "37½ days")

ṷarkānu "later"
ṷa-ar-ka-a-nu-um: M rev. 3

ṷatru "excess" (or "exceeding")
a-na 10 *ṷa-at-ri-im:* B obv. 11

U-bar-rum
p. 22, No. 99″, colophon; No. 99,3a, colophon

ṷbl "to bring" (cf. also *tbl*)
multiply
(*x*) *a-na*(?) (*y*) *bi-il-ma* (*xy*): Uc 3a
(*x*) *a-na* (*y*) *tu-ub-ba-al-ma* (*xy*): Pa obv. 7, rev. 2

ubu a unit of area or volume or bricks
area (cf. p. 5)
1(ubu)^{iku}: K obv. 3
bricks (cf. p. 5)
volume (cf. p. 5)
1(ubu)^{iku}: K obv. 3

UD-SAR see GÁN-UD-SAR

udu-nitá (≈*immeru*) "ram"
W 8,9,10,15

ugu (≈*eli*, q.v.) "over, above"
šeš ugu šeš *ú-te-le-el-le:* D rev. 14],16
(*x*) ugu (*y*) (*x* − *y*) dirig: F obv. 14,20,22,24, rev. 18; G obv. 24,31, rev. 3,5,22; S obv. 10, rev. 2 (u<gu>)
(*x*) ugu (*y*) (*x* − *y*) *i-te-er:* H rev. 13; J obv. 12
½ *ša* uš ugu sag *i-te-ru:* H rev. 19; J obv. 17]
igi-7-gál uš ugu sag *i-te-ru:* J obv. 25
a-na uš ugu sag dirig: T obv. II [20, III 30, IV 25, rev. I 34, II 7,35, III 3, IV 14; U obv. I 28, II 3,[27, rev. II 9,17,24

(?)*ul* "not"
igi (*x*) *ú-ul ip-pa-aṭ-ṭa-ar:* D obv. 16, rev. 18
(*x*) *ú-ul i-de:* D obv. 31], rev. 13,15
ki-a-am ul(nu) *idu-ú:* V 9
ul: W 7

UL-gar (cf. also gar-gar) "to add"
(*x*) *ù* (*y*) UL-gar (*x* + *y*): Ca obv. 8

ultu see ta

ummānu "capitalist"; "artisan"
ummānu(um-me-a): V 2

urudu (≈*erū*) "copper"
Ue 33(?)
ra-ṭù-um ša urudu: Ud 22; Ue 36(?)

UR-UR (see ²₁*kl*)
to square
(*x*) a-rá (*x*) UR-UR-a (x^2): J obv. 7],18],31
(*x*) UR-UR-ta (x^2): H rev. 8
to multiply
(*x*) (*ù*) (*y*) UR-UR-a (*xy*): J rev. 16,27,29],32,34
(*x*) (*y*) UR-UR-ta (*xy*): H obv. 3,22
(*x*) *ù* (*y*) UR-UR-ta (*xy*): H obv. 8,16]; J rev. [5,14,20,22

uš (≈*šiddu*) "length"
uš *ša-nim*(or *ša-nu-um*): E obv. 6,9], rev. 12
uš *ki-nu-um* (or *ki-nim*): E obv. 8, rev. 2(?),8
uš 2-*ú:* Y obv. 7
uš(-bi): Scheil [1], Nos. 1–53; B obv. 1; Ca obv. 1,7; D obv. 19,21, rev. 3,6,12,13,21; E rev. 8,11; Ea obv. 1,6; Eb obv. 4; F obv. 6,7,8,10, 13, and passim; G obv. 5,6,7,8,10, and passim; H obv. 1,3,14,16,20, and passim; J obv. 2,6,11,12,13, and passim; Ja obv. 1,5,14, edge 1, rev. 2; K obv. 1,6,7,9,11, and passim; L obv. 1,4,6,7,8, and passim; O obv. 1,6,10, rev. 4,7,10; P 2; S obv. 1,3,6,7,8, and passim; Sa obv. 1,2,3,4,5, and passim; T obv. I 10,23,26,31, II 15, and passim; U obv. I 2,3,15,28, II 2, and passim; Uc 3; Y obv. 7
uš-1-e: B obv. 1
uš-2-e: B obv. 1
uš-gíd-da: B obv. 4,6, rev. 4,14; C obv. 8
uš-ḫá: B obv. 7
uš-lugúd-da: B obv. 4,6, rev. 6,15; C obv. 9
uš-šè: T obv. I 6,19, II 23, III 17],30, IV 3,25, V [17, rev. I 34; U obv. III 20(?)
uš-ta<-àm(?)>: E obv. 1

UŠ a unit of length (cf. p. 4)
2 UŠ: B rev. 4,5
3 UŠ: B rev. 11,13,14 (UŠ consistently omitted in duplicate C obv. 5,7,8)
5 UŠ: K obv. 13,15,18, rev. 7,11,12,14,16,18, 21,22,24,26; L obv. 1,4,6,8,11,12,15,19,21,23, 26, rev. 2,3,6,10,13
5 GAR UŠ uš(-bi): K obv. 1,6,7,9,11

uṣb (≈daḫ, q.v.) "to add"
(*x*) *a-na* 1 *ṣí-ib:* H rev. 11
a-na (*x*) (*y*) *ṣí-ib* (*x* + *y*): H rev. 21
(*x*) a-rá 1 *ṣí-ib:* H rev. 23
(*x*) *a-na* (*y*) *ṣí-ib* (*x* + *y*): Ua rev. 3
a-na (*x*) (*y*) *ṣí-ib-ma* (*x* + *y*): Ua obv. 9
(*x*) *a-na* (*y*) *tu-uṣ-ṣa-ab-ma* (*x* + *y*): B rev. 4,13; C obv. 7]
ú-ṣí-ib: M rev. 4

utlellū "to exceed"
a-ḫu-um e-li a-ḫi-im li-te-le-le: Q 10
šeš ugu šeš *ú-te-le-el-le*(*-ma*): D rev. 14,16
ma-li ú-te-le-el-lu-ú ú-ul i-de: D rev. 15

uṭr (≈dirig, q.v.) "to exceed"
(*x*) ugu(or *e-li*) (*y*) (*x* − *y*) *i-te-er*(or *i-ter*): H rev. 13; J obv. <13>; Ua obv. 1; Ub 3,4
(*x* − *y*) *ša* (*x*) *e-li* (*y*) *i-te-ru:* B obv. 10; Ua obv. 4
½ *ša* (*x*) ugu (*y*) *i-te-ru* (*x* − *y*) *ḫé-pe* $\left(\frac{x-y}{2}\right)$: H rev. 19; J obv. 17
igi-7-gál uš ugu sag *i-te-ru:* J obv. 25

utūnu "oven"
ú-tu-num: Pa obv. 1

Z

za-e (≈*attā*, q.v.) "thou"
Ca obv. 4; D obv. 4, rev. 17; Eb obv. 4; H obv. 2,8,15,21,27,34, rev. 3,15; J obv. 3,13, 26,37,[42], rev. [5],13,20,[26],32

zal (≈*nsḫ*) "to pass, elapse"
ba-zal: pp. 22f., No. 99,3a, colophon; No. 99, 22, colophon; No. 99,13b, colophon

zà-mí (≈*sammū*) (cf. p. 135 ad line 48) (cf. also GÁN-zà-mí)
geštú zà-mí: Ud 50

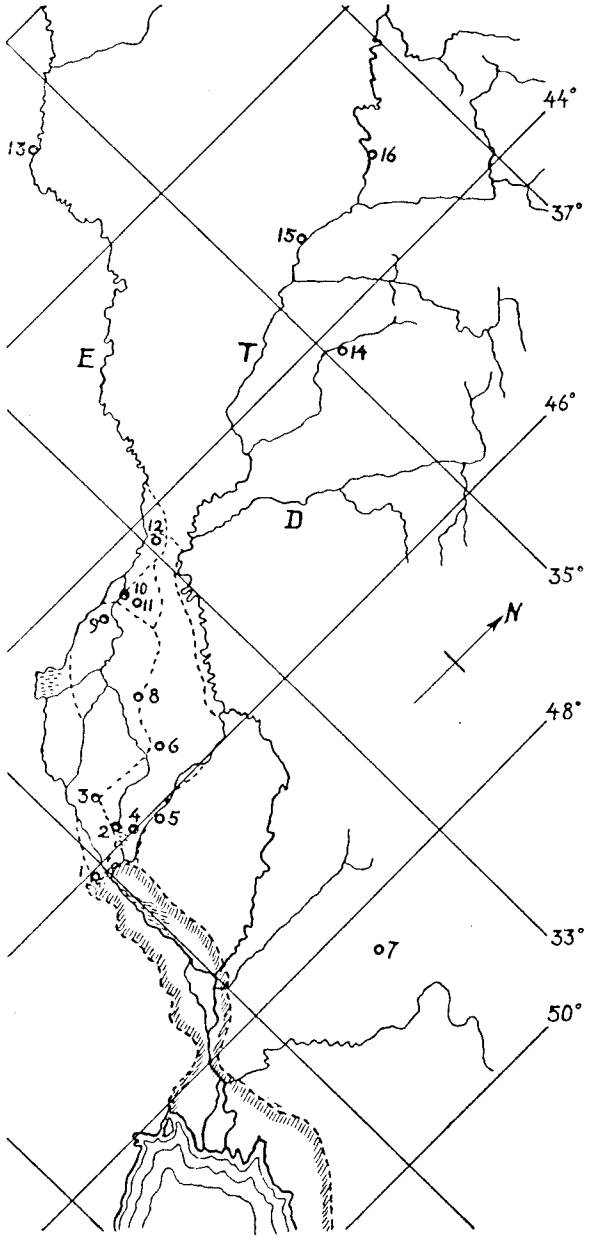
zaqpu
zaq-pi: Y obv. 20

zbl "to transport"
sig₄ *iz-bi-l*[*am*]: P 2

SUBJECT INDEX

MAP SHOWING ANCIENT SITES WHICH ARE MENTIONED

Broken lines indicate coast and rivers in antiquity. The numbers indicating ancient sites run from south to north.



Adab 6

Aššur 15

Babylon 10

Dilbat 9

Diyālā River D

Euphrates River E

Kiš 11

Kutalla, see Tell Ṣifr

Lagaš 5

Larsa 2

Mari 13

Nineveh 16

Nippur 8

Nuzi 14

Susa 7

Sippar 12

Tell Ṣifr (ancient Kutalla) 4

Tigris River T

Ur 1

Uruk (modern Warka) 3

Warka, see Uruk

PLATES

Problem-text	Copy	Photo-graph
(CBS 29.13.21)		24
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Eb	21	47
Ec	21	47
F	5	30
G	6	31
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Uc	18	44
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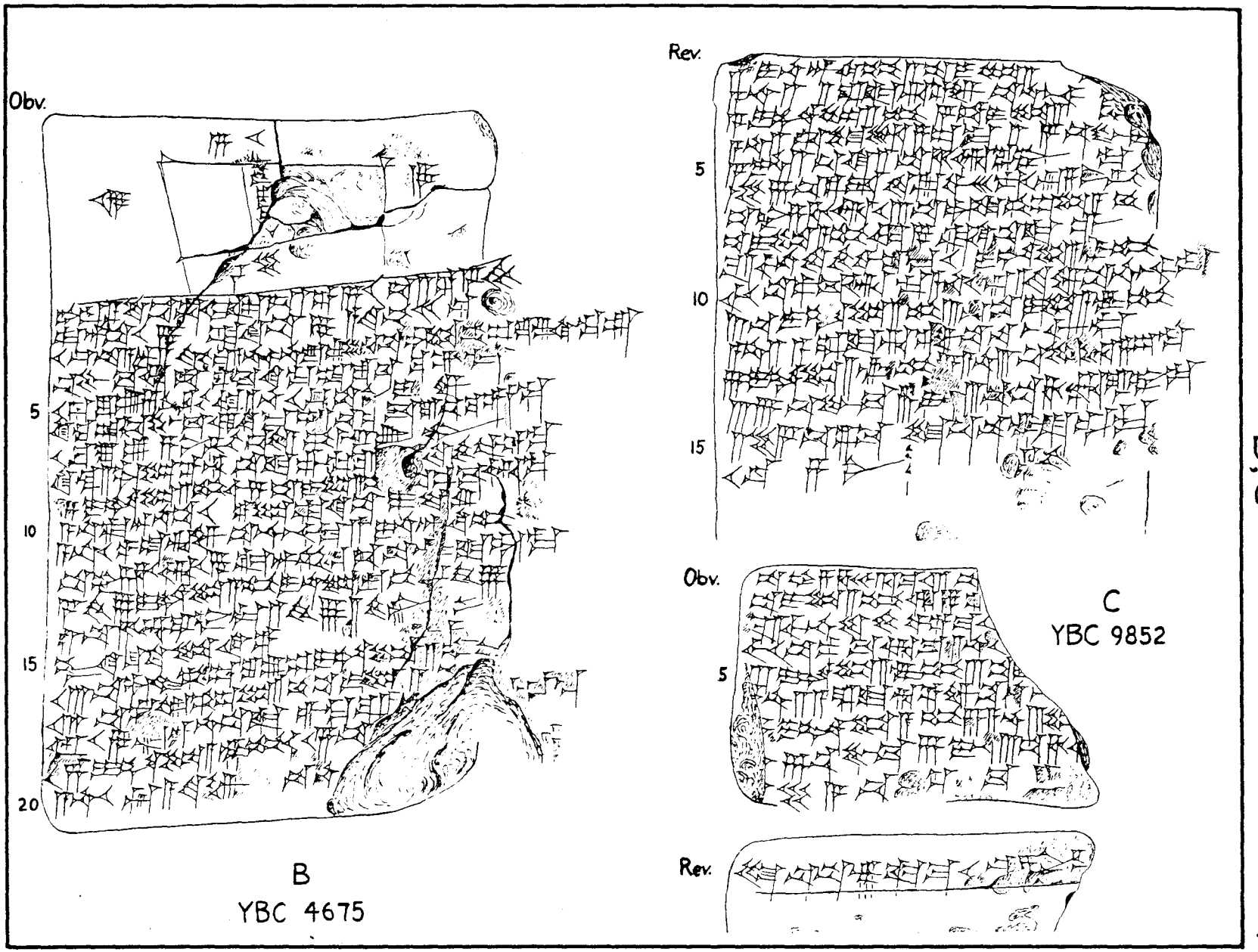

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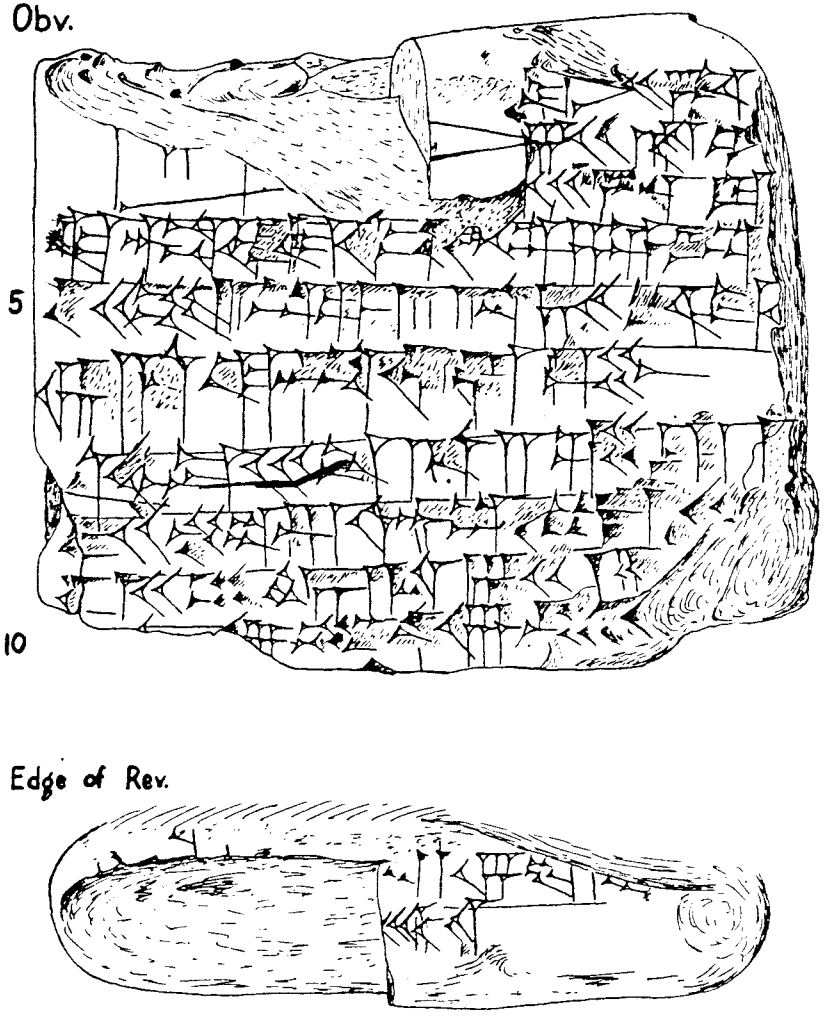

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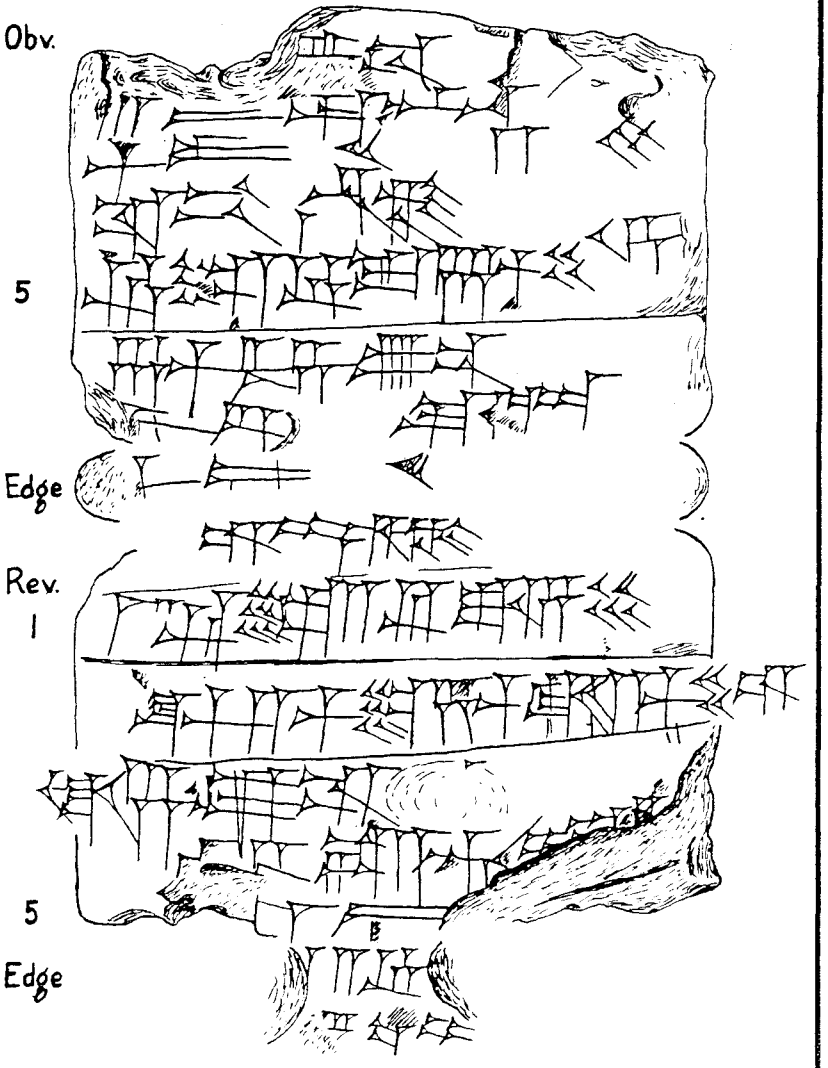

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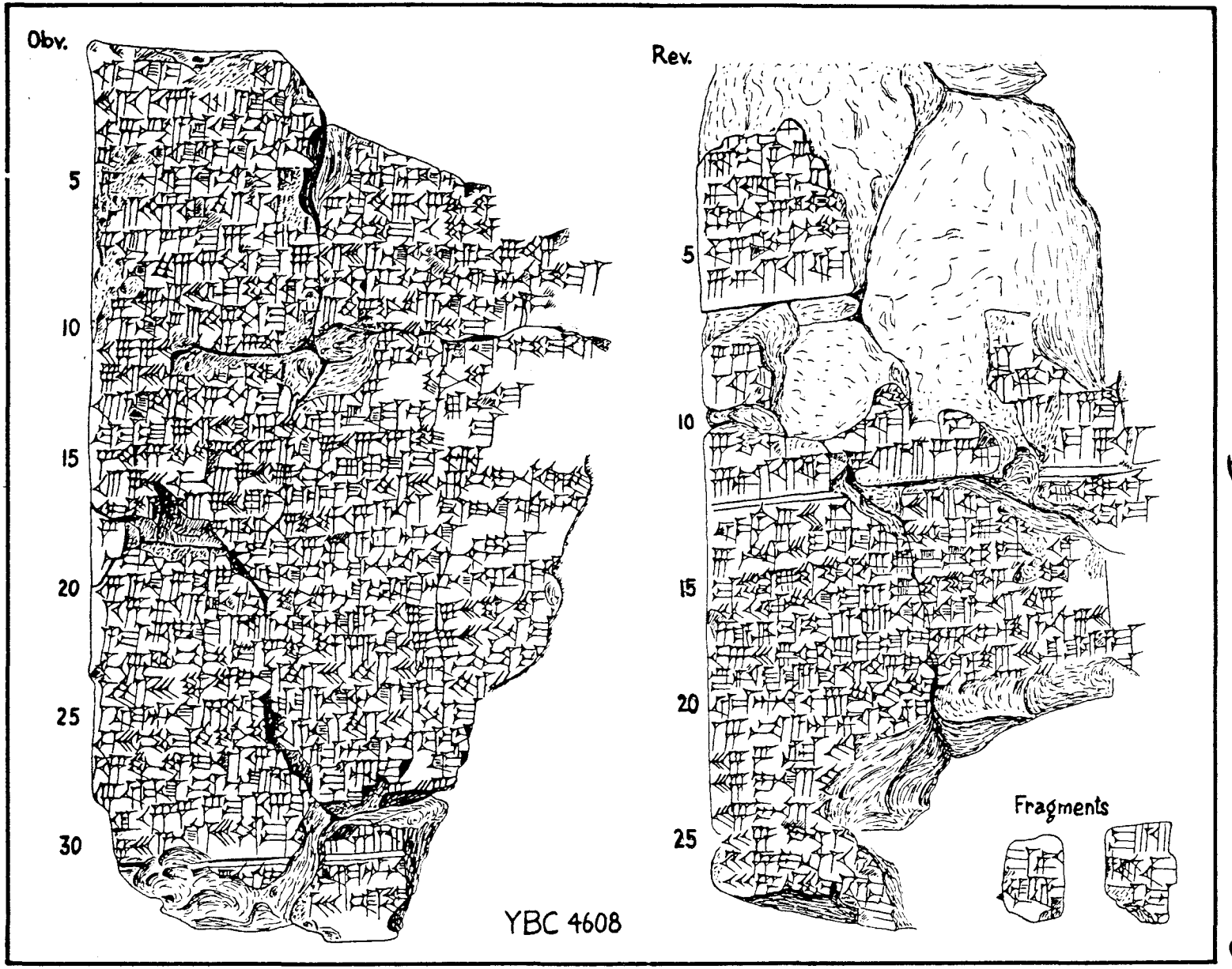

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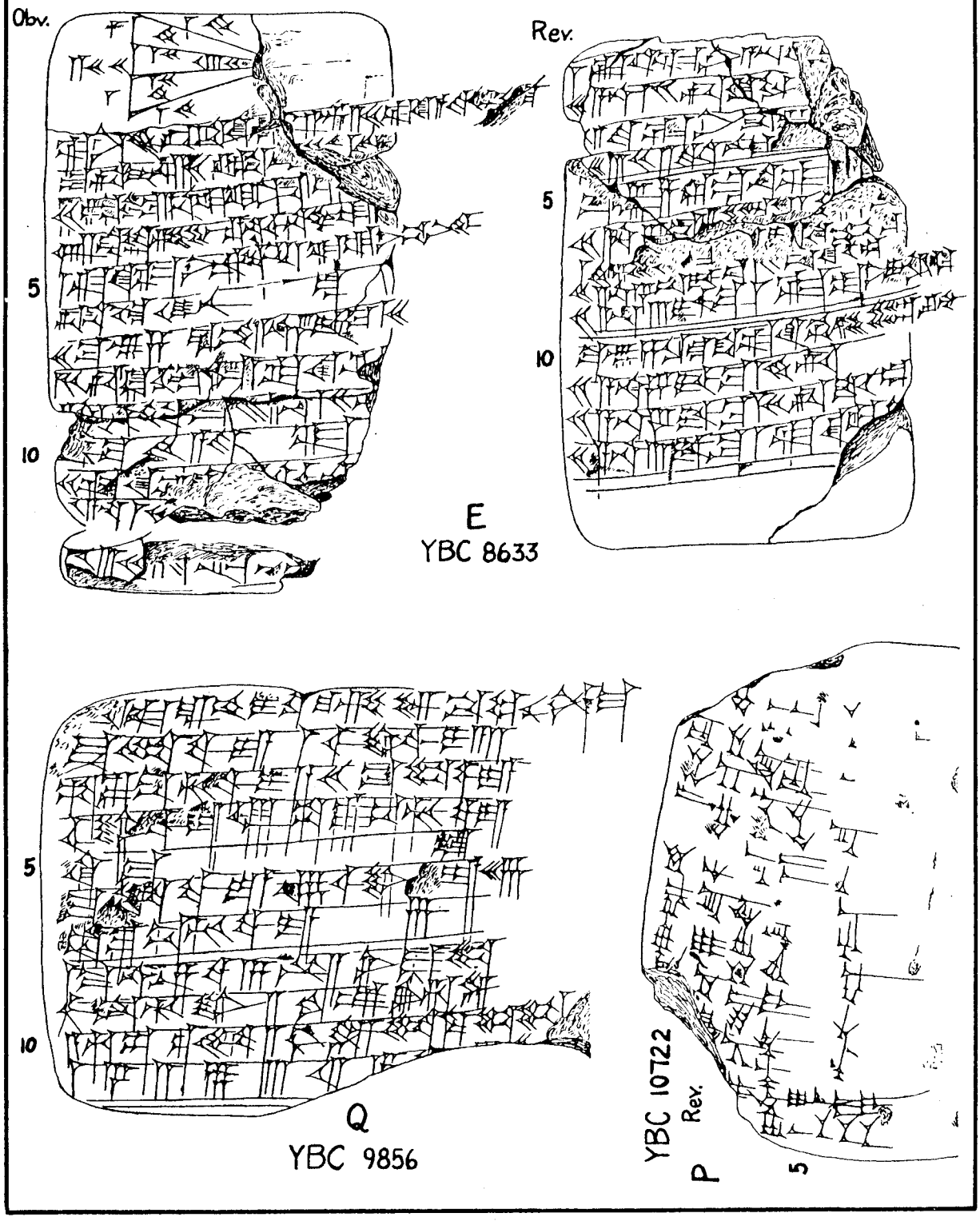
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YBC 10722
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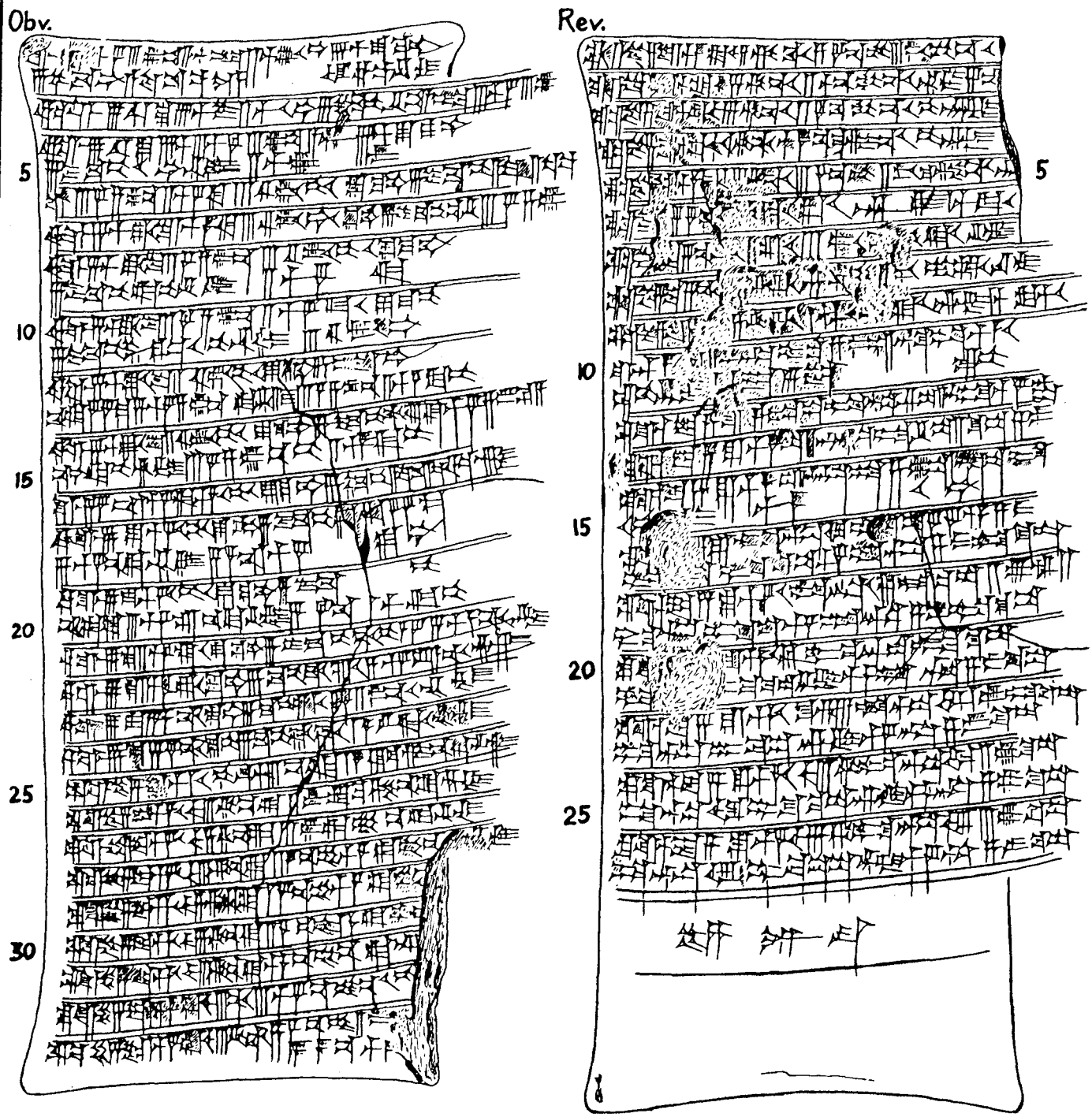

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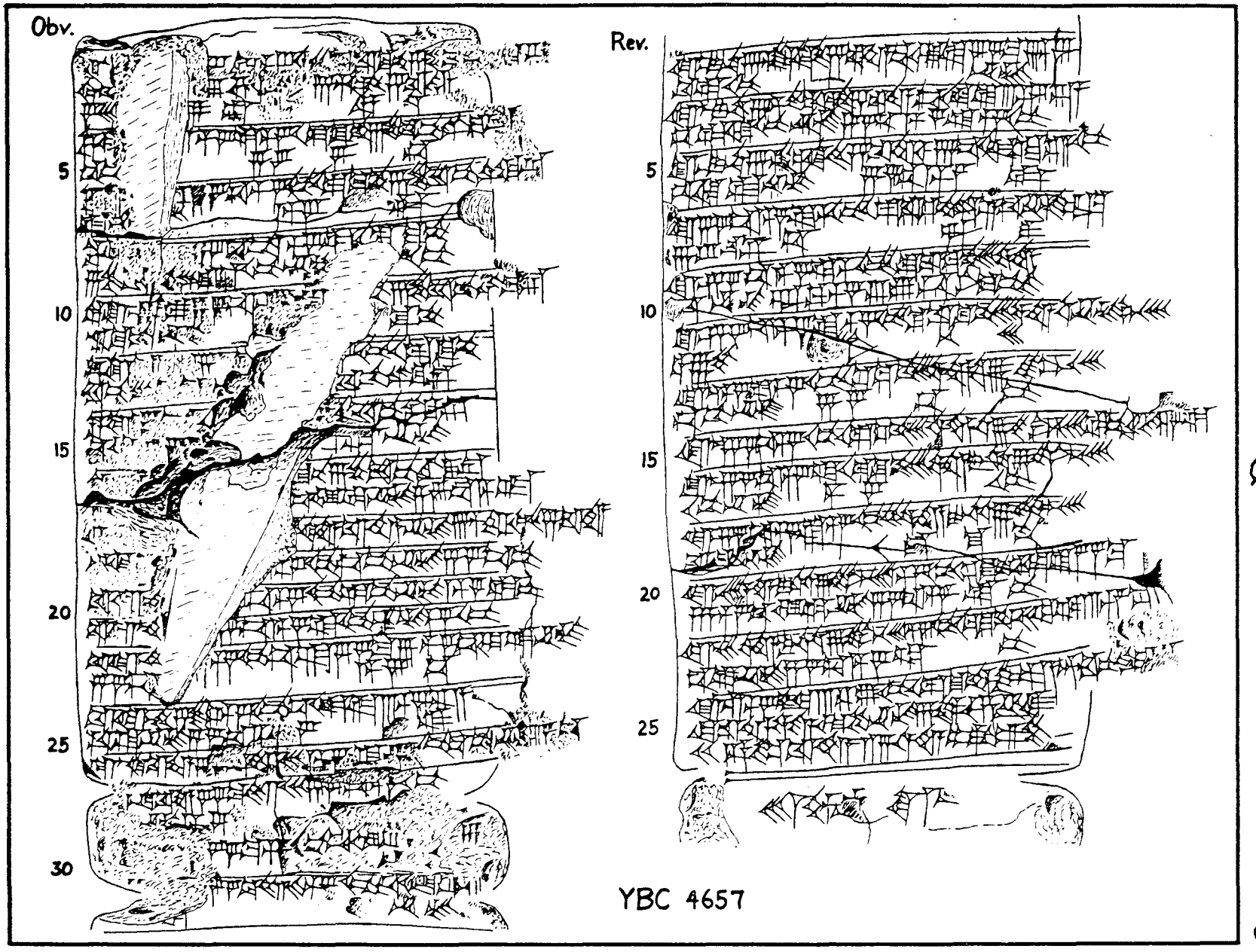
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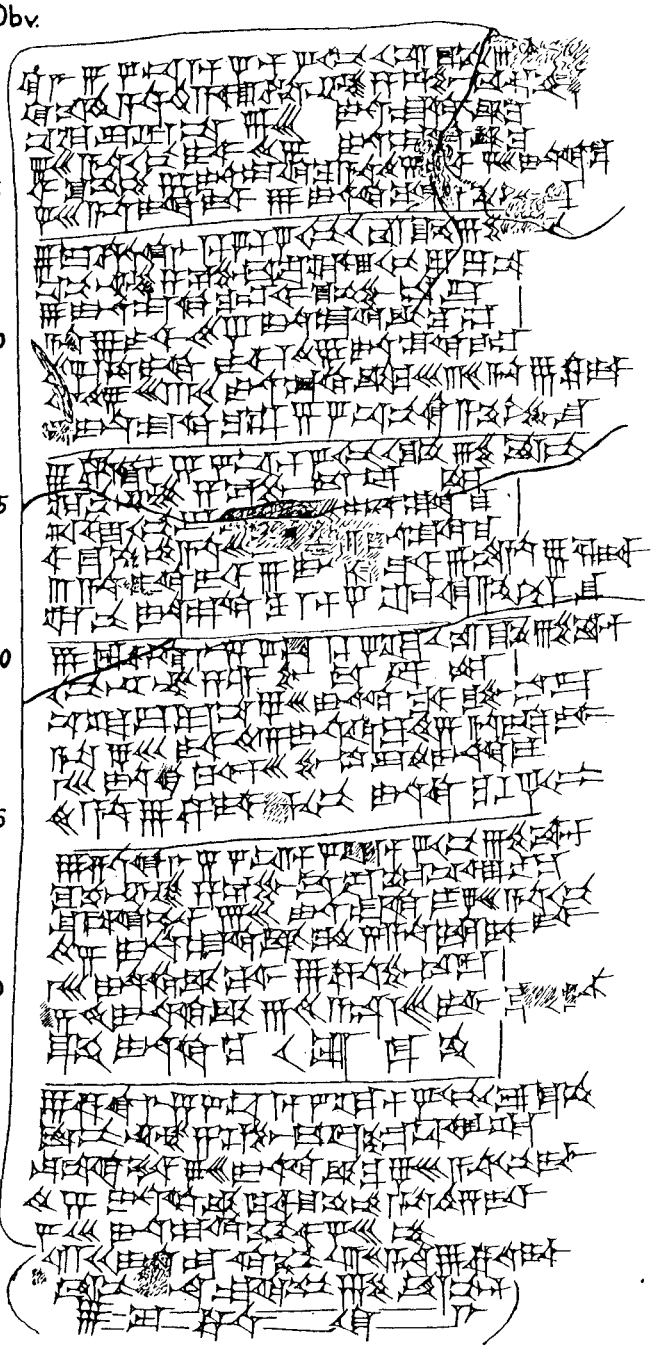
Obv.



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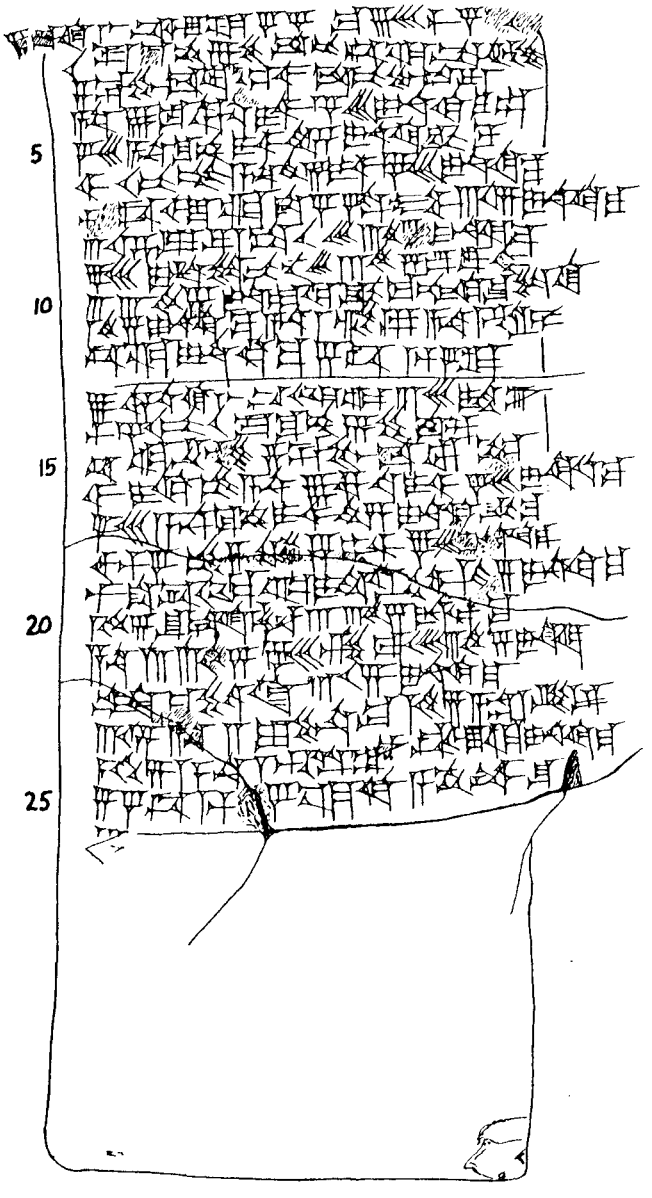

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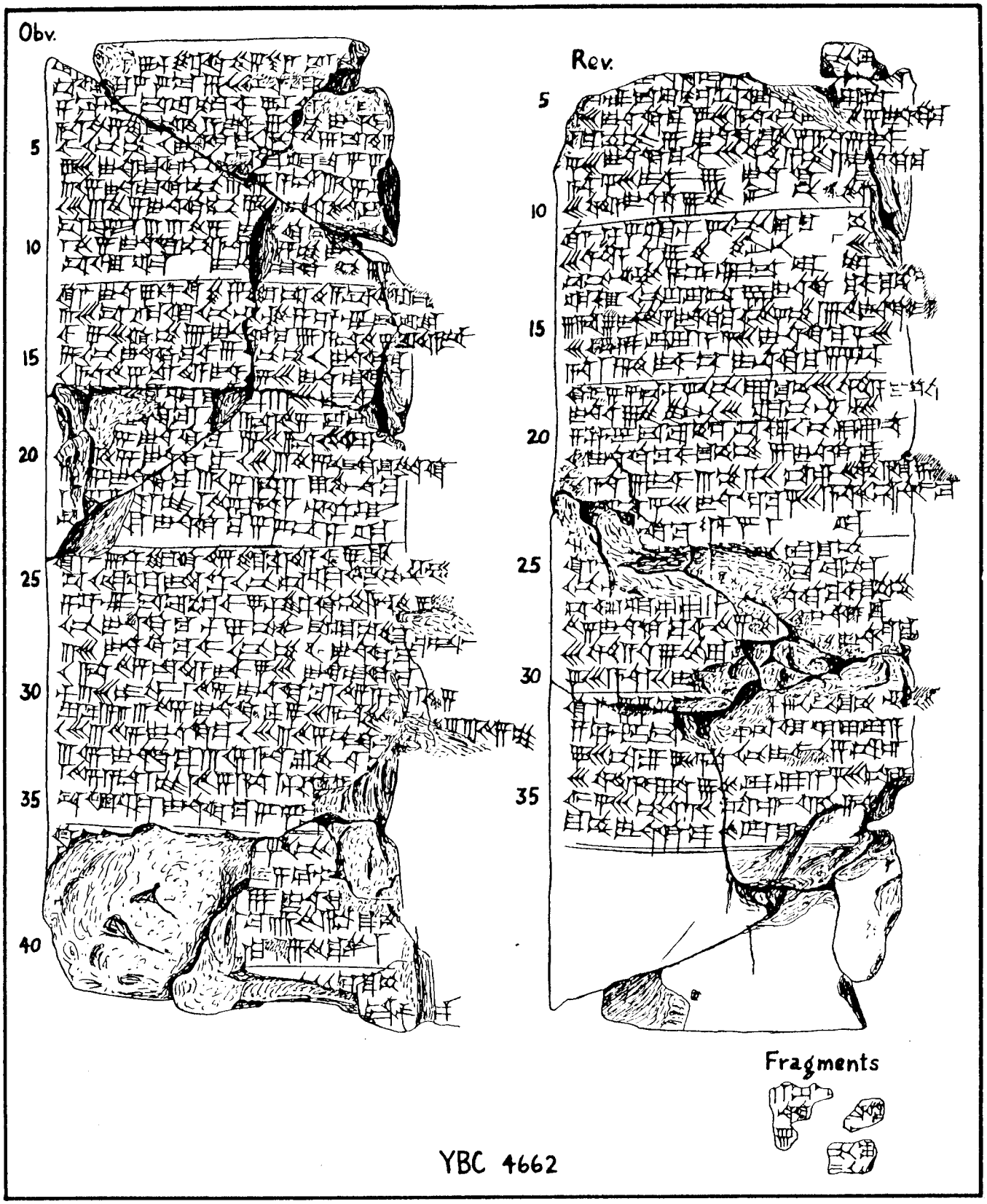

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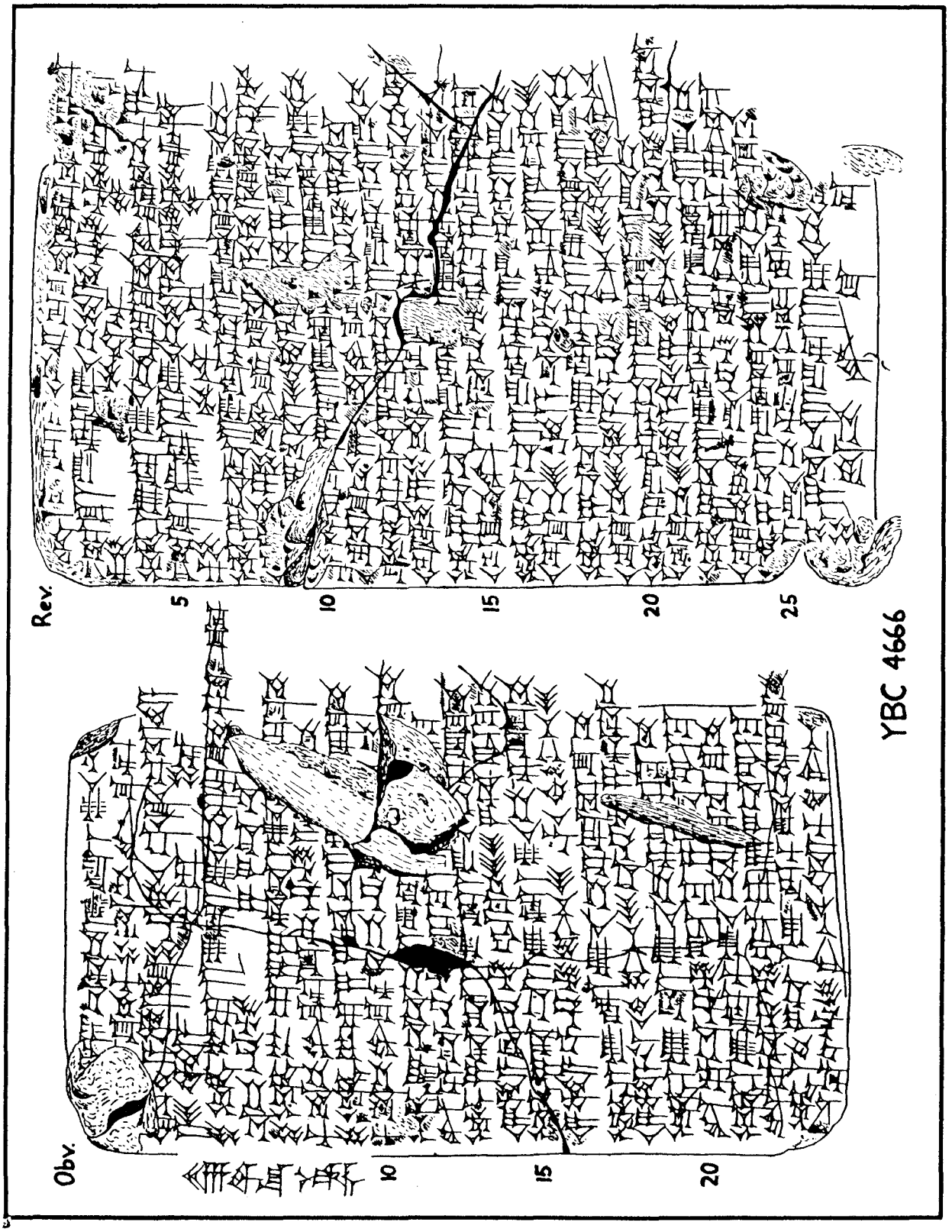
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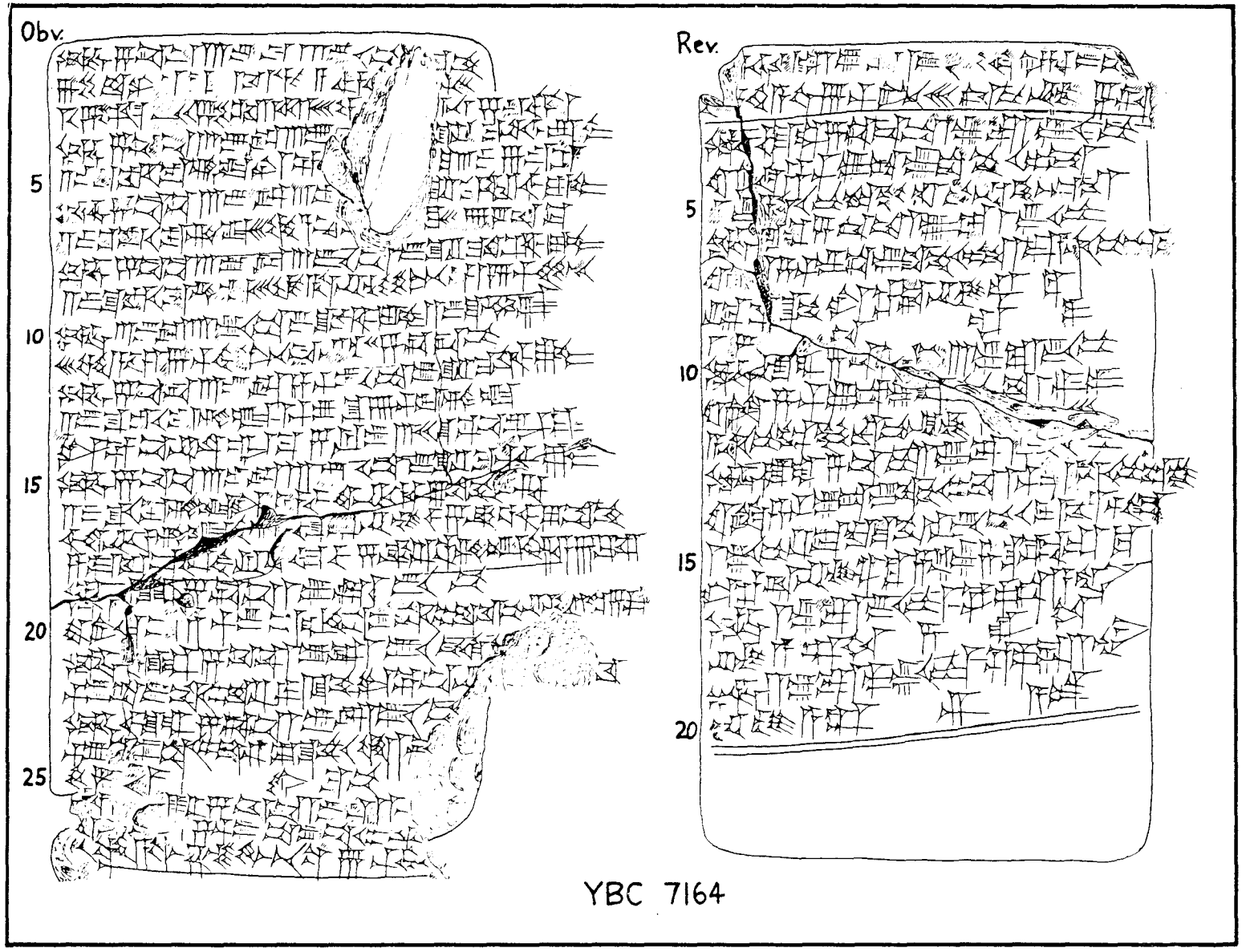

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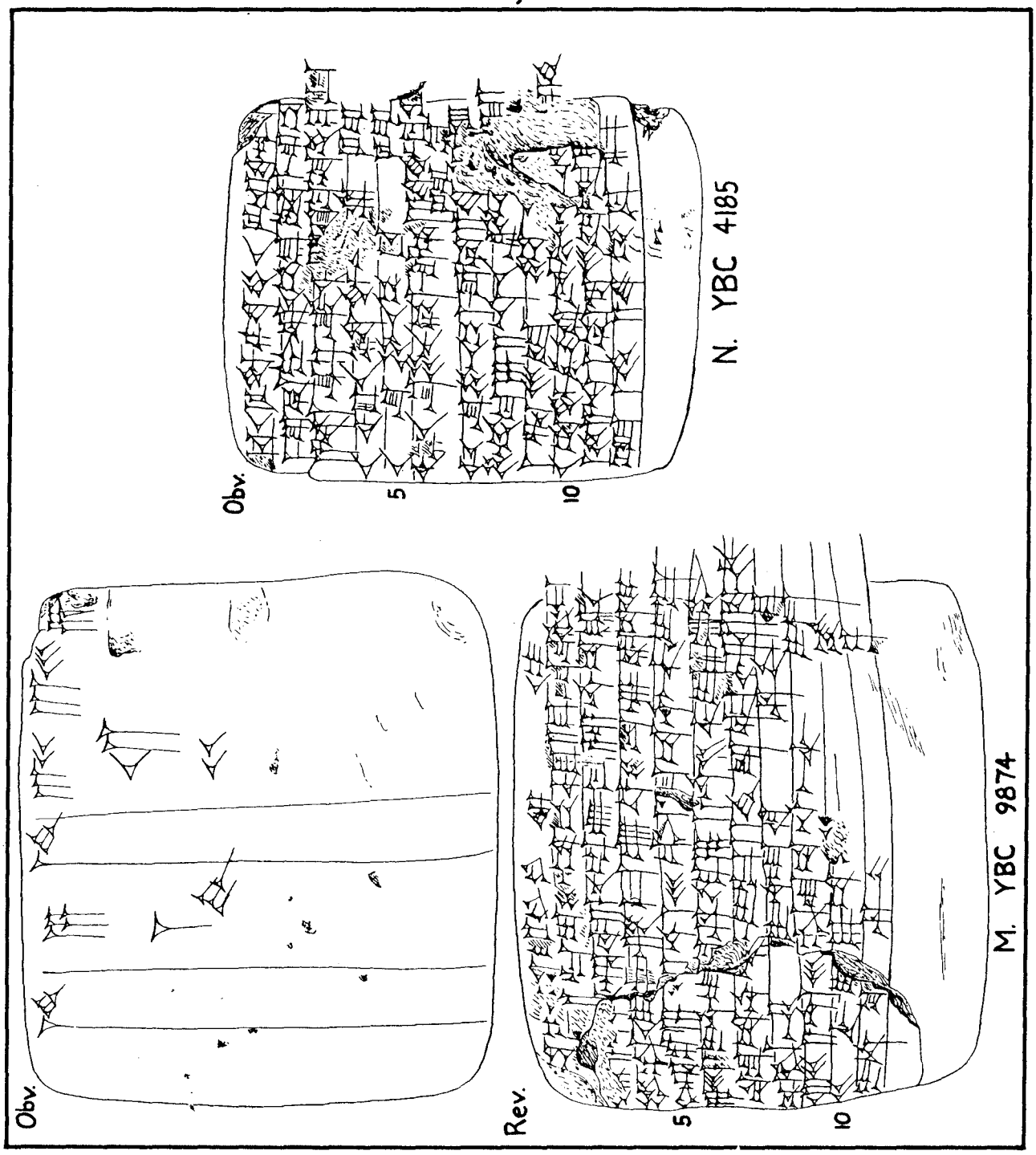
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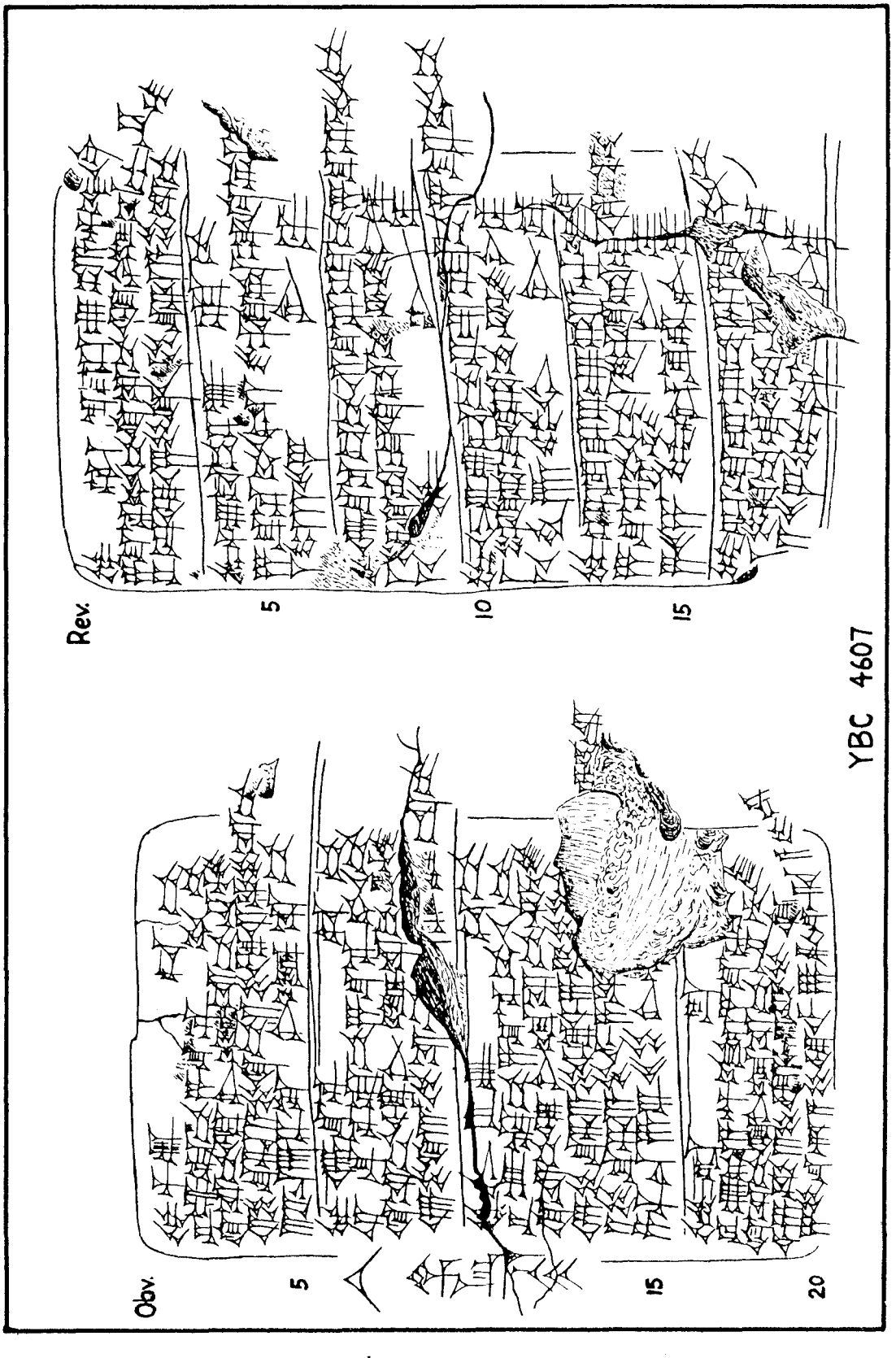
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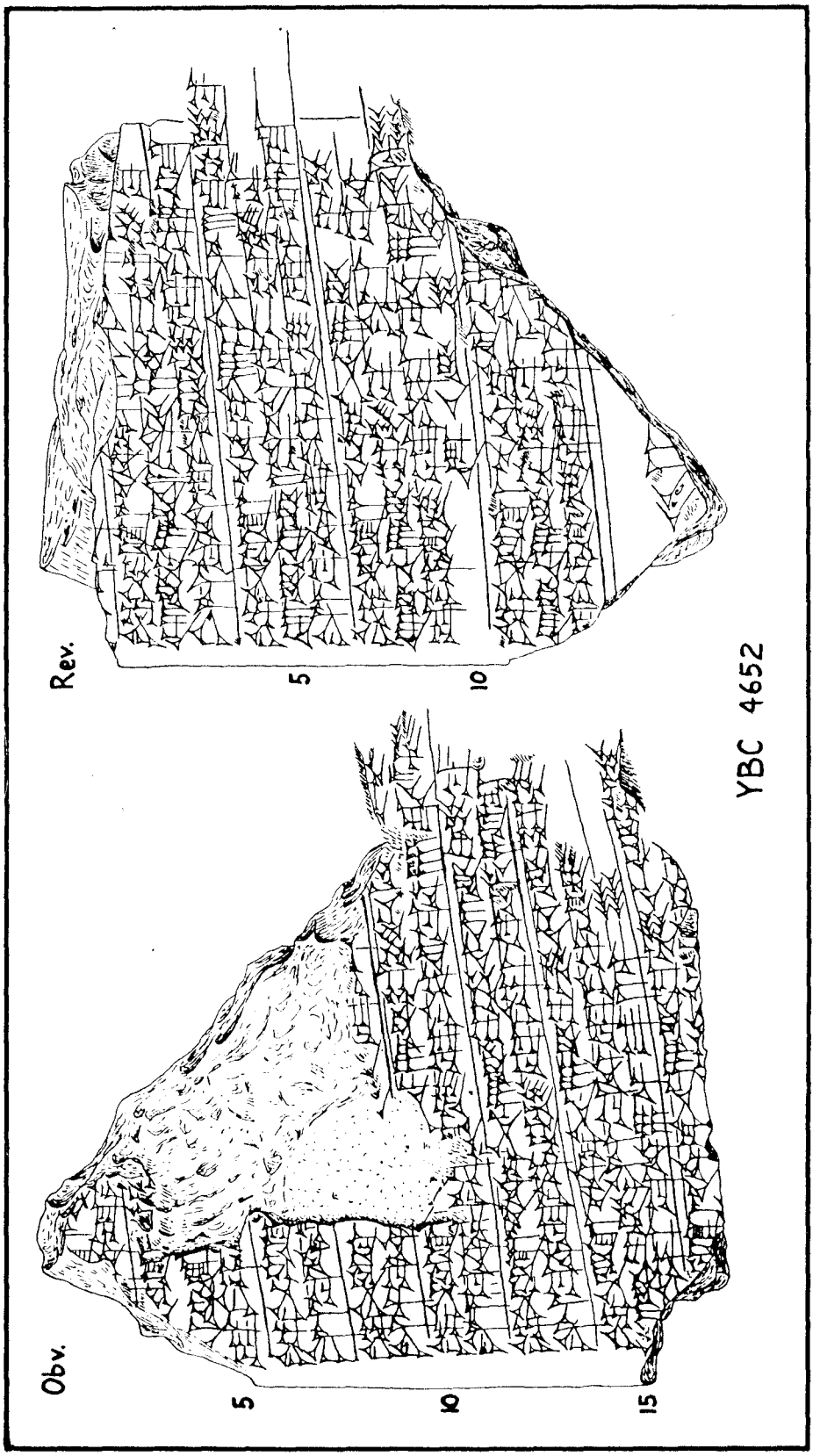
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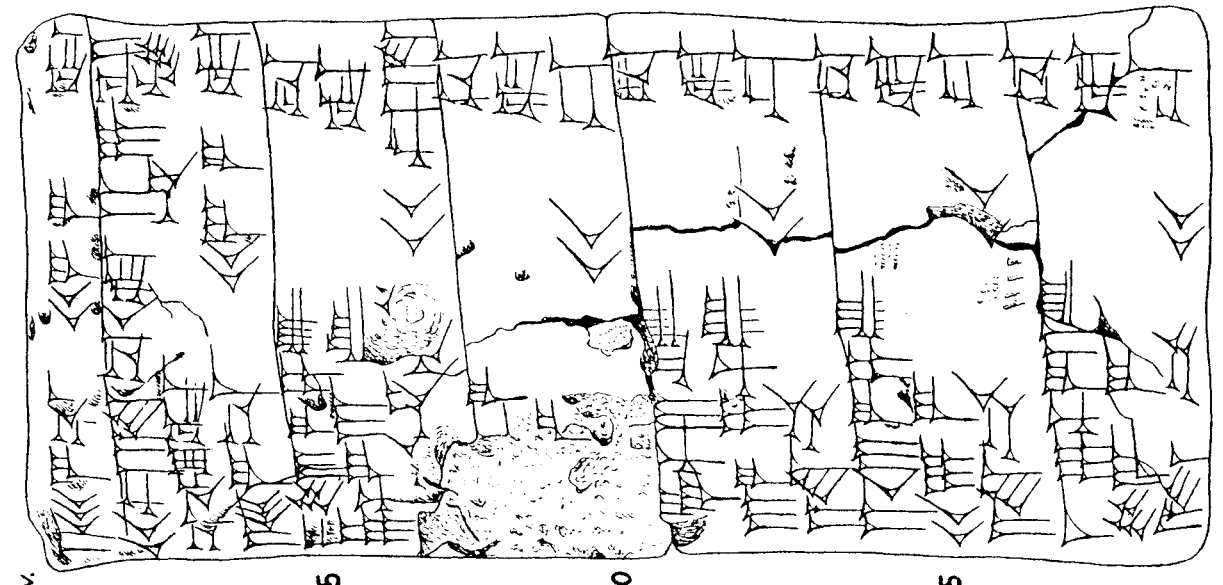

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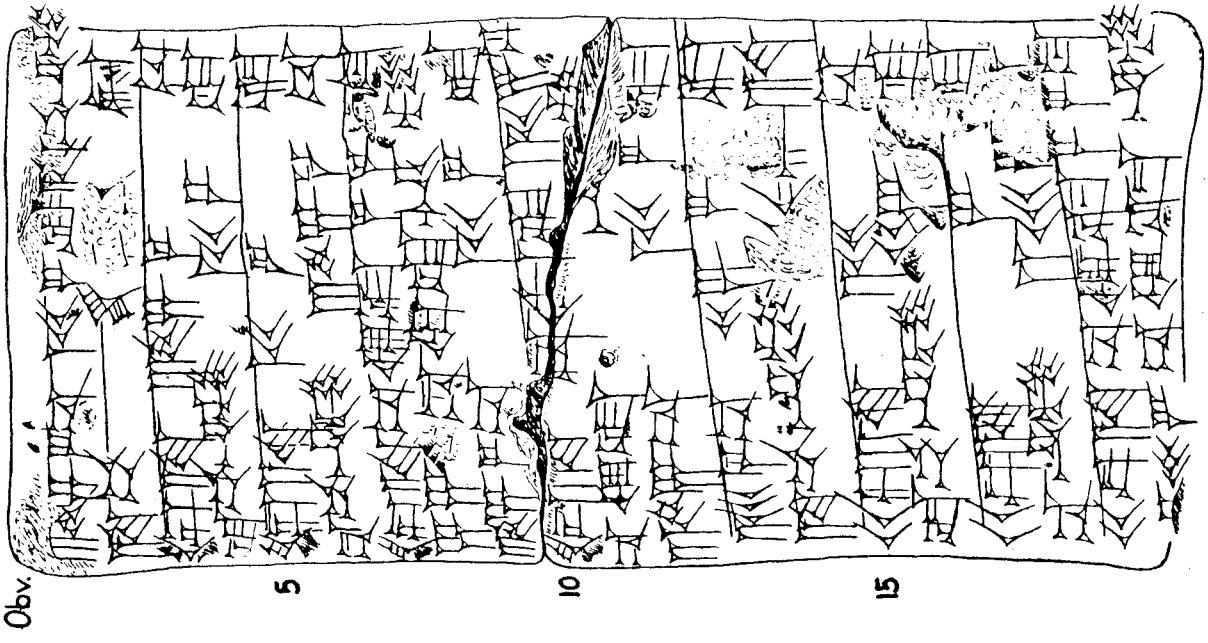

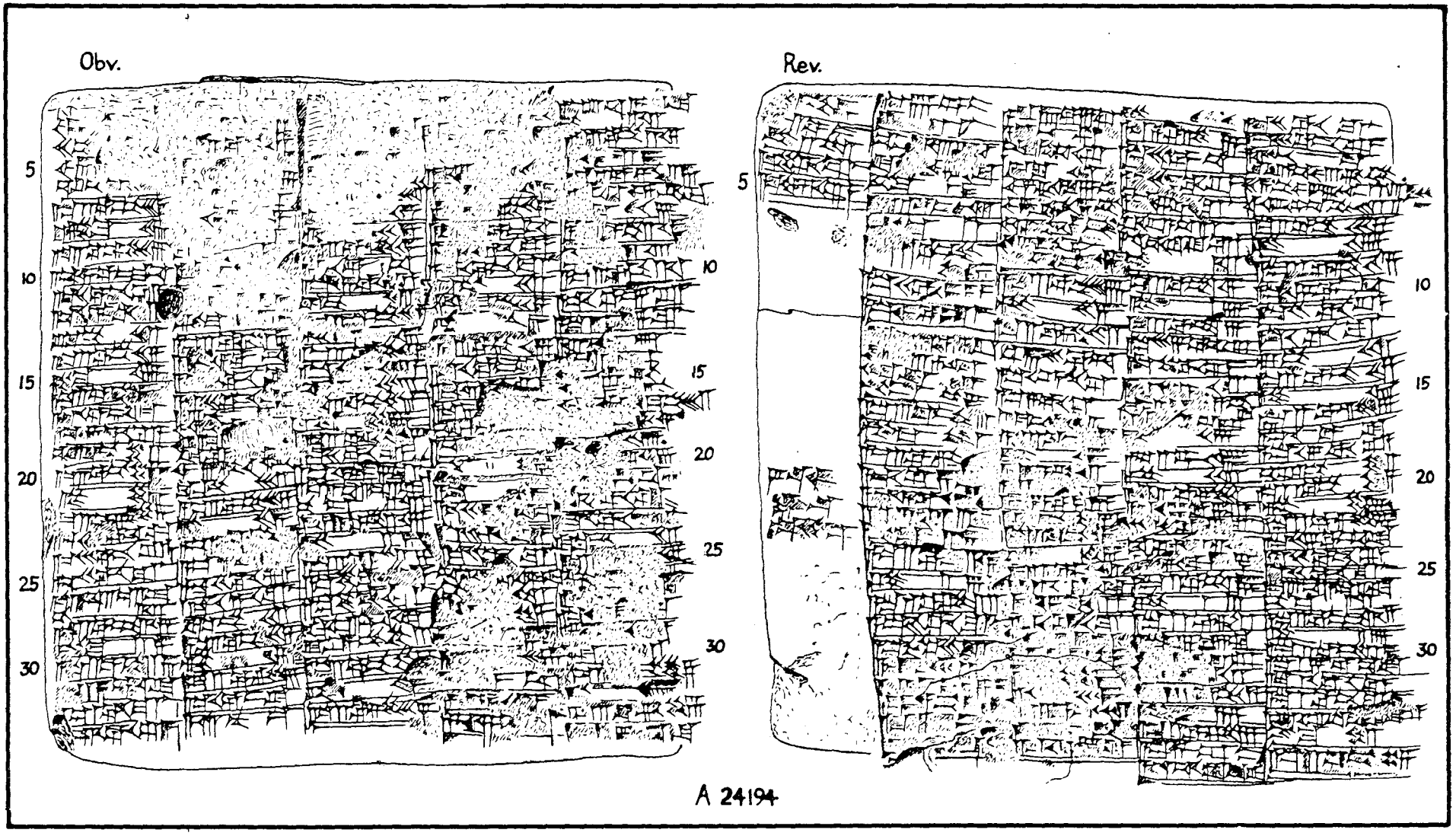

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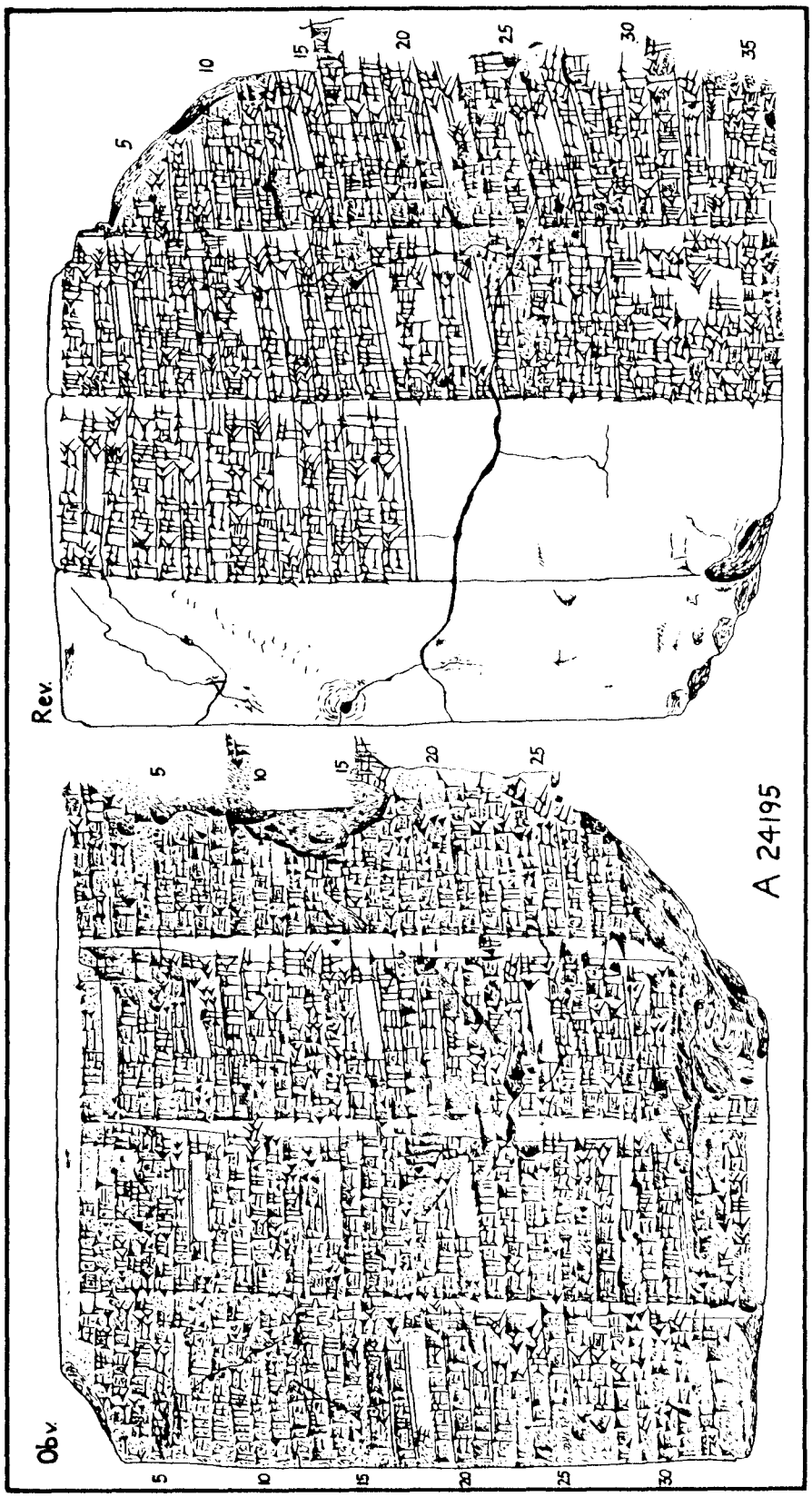
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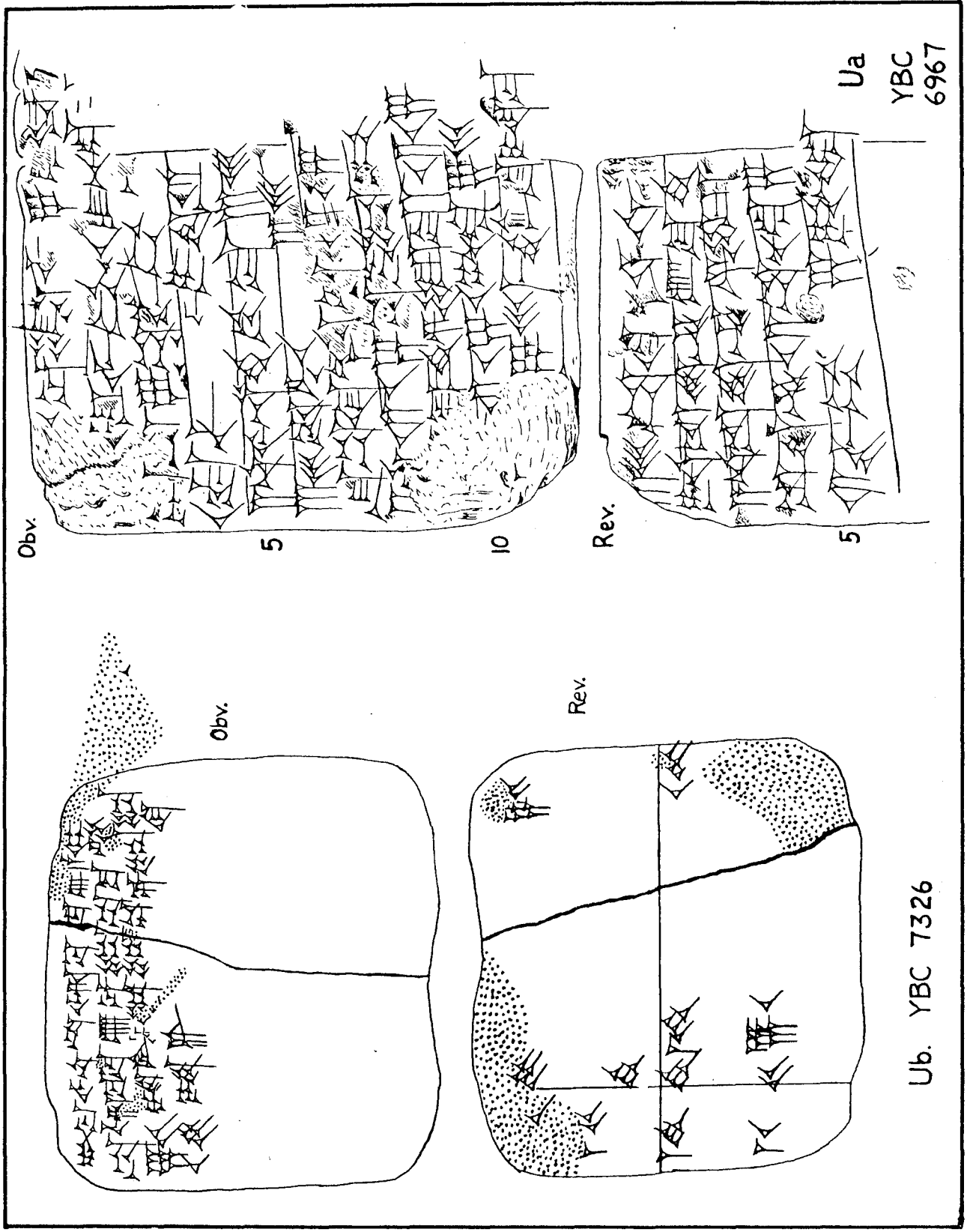
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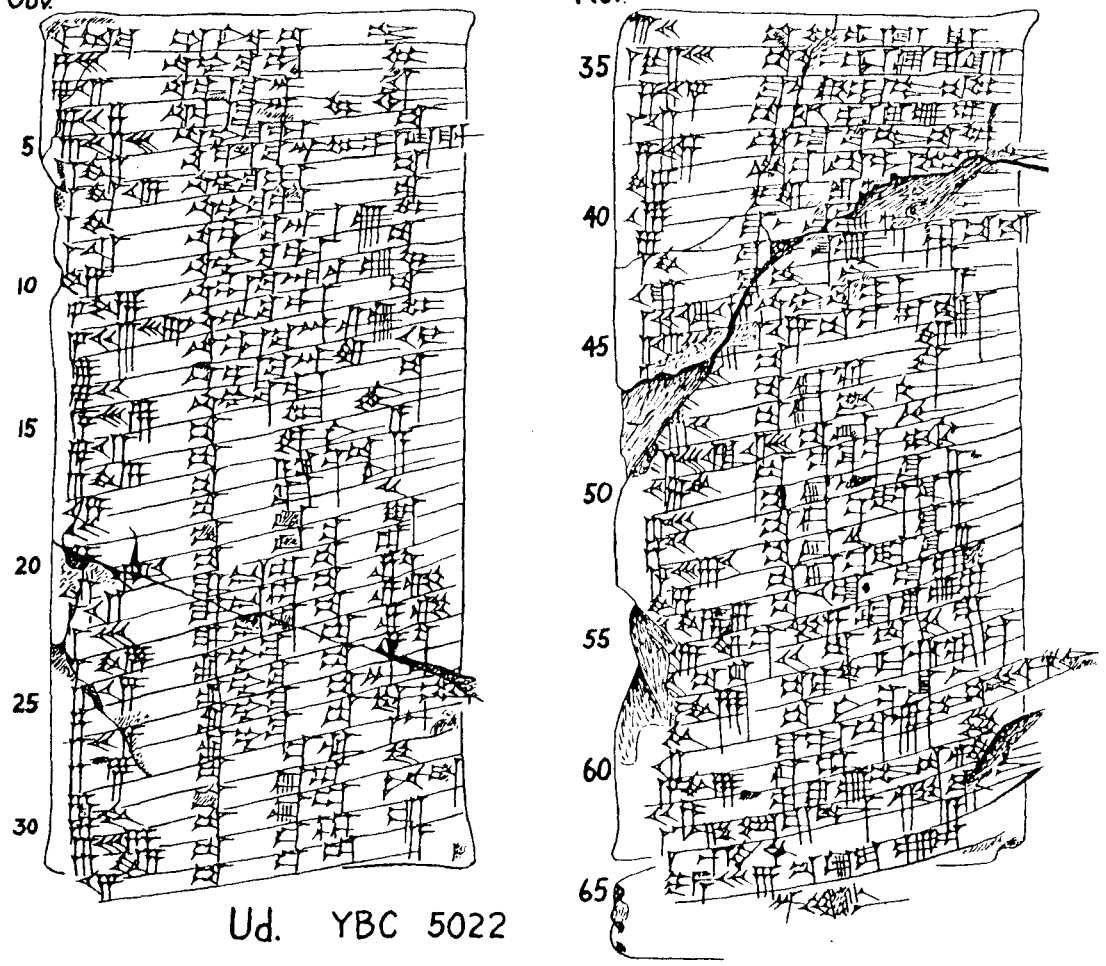

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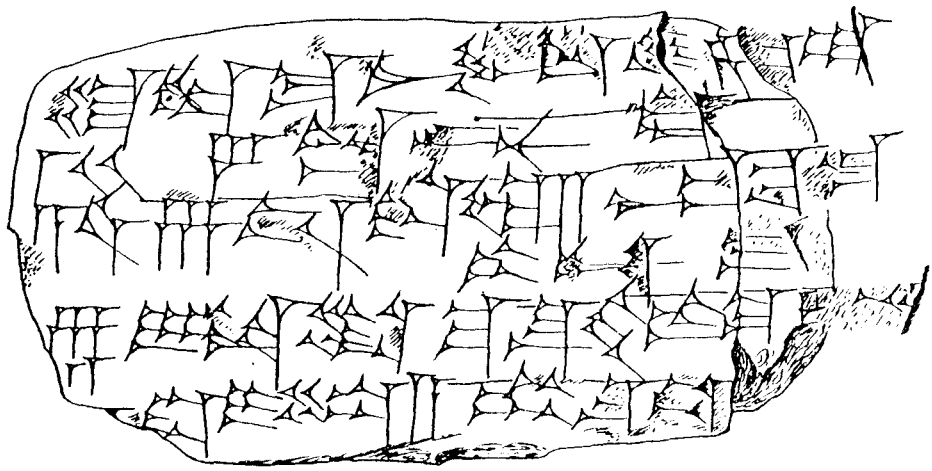

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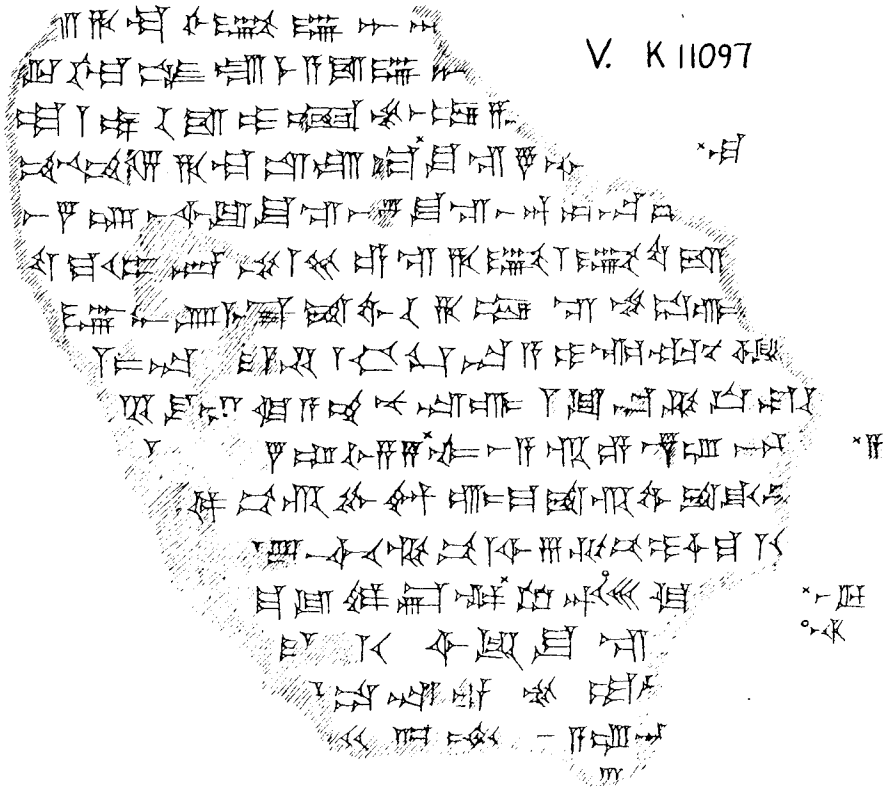

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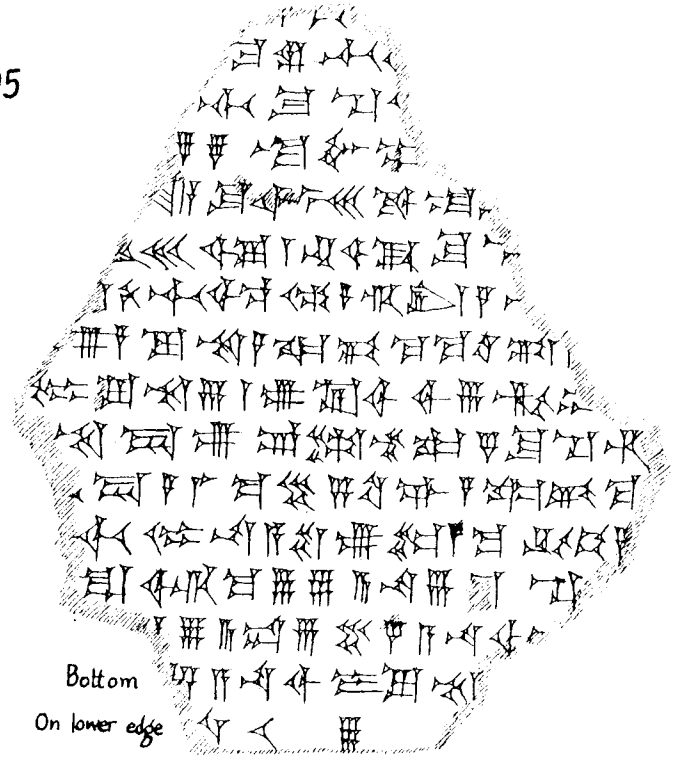
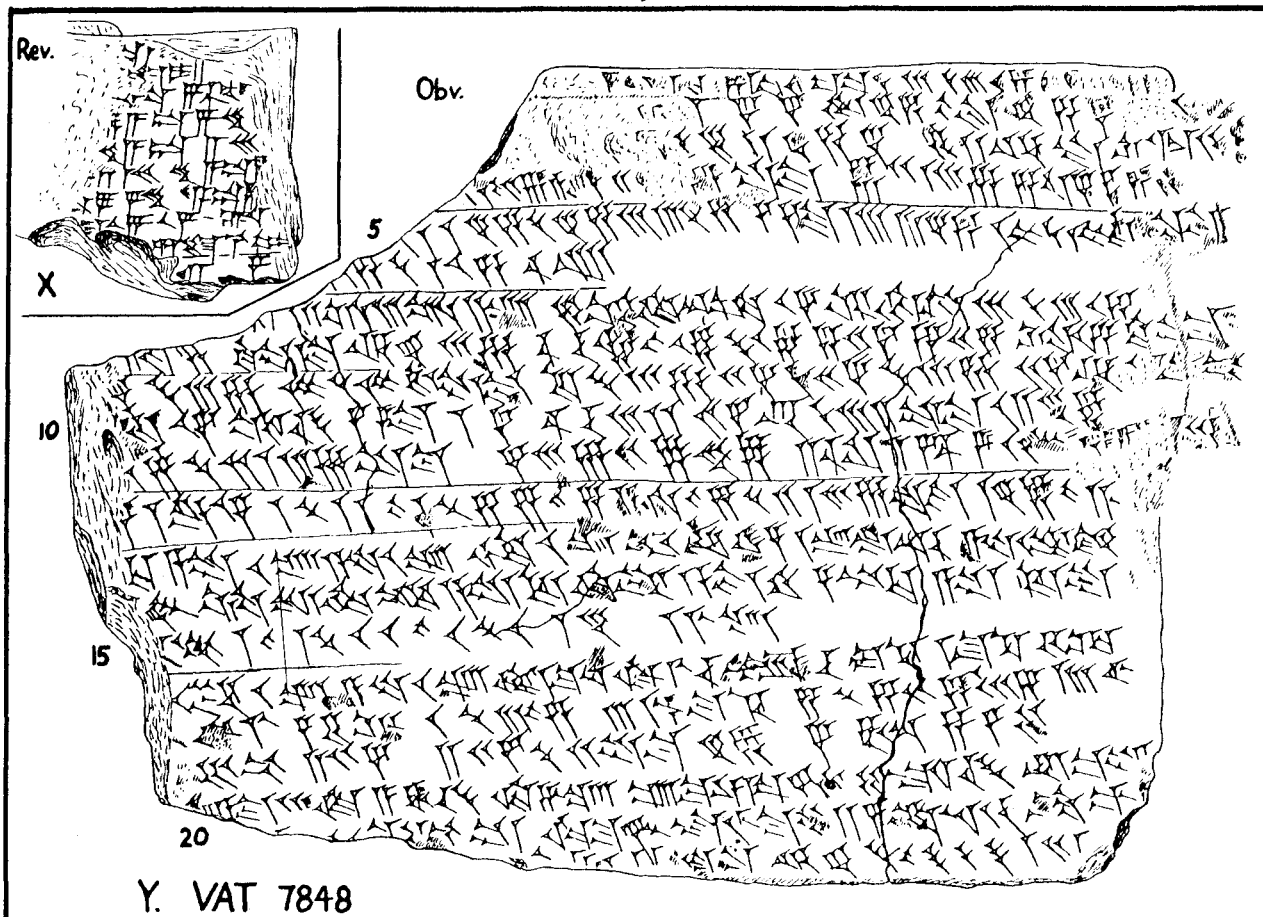
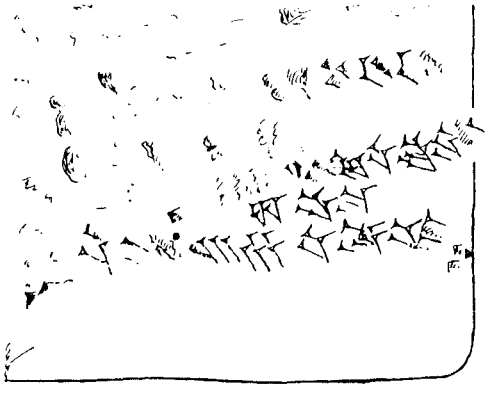


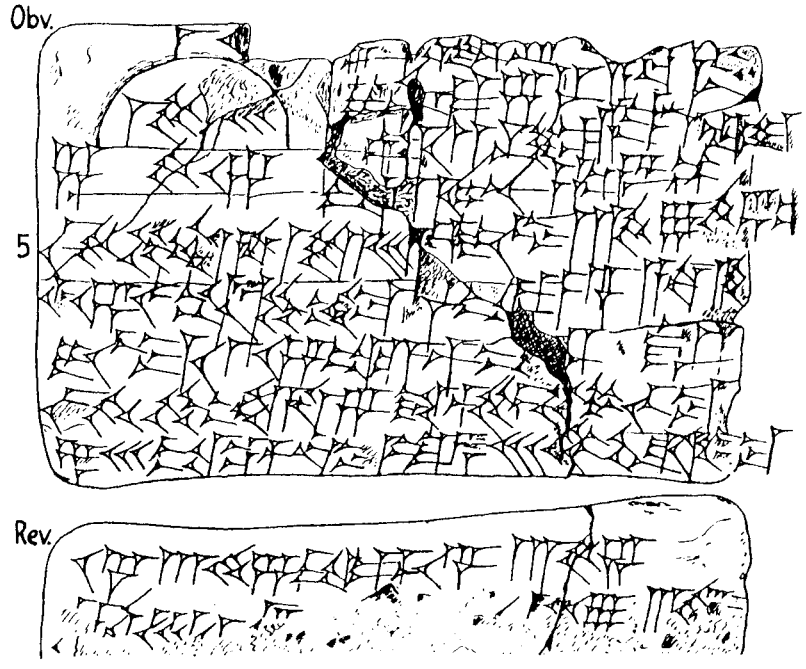


Y. VAT 7848

Part of reverse



MM 86.11.404



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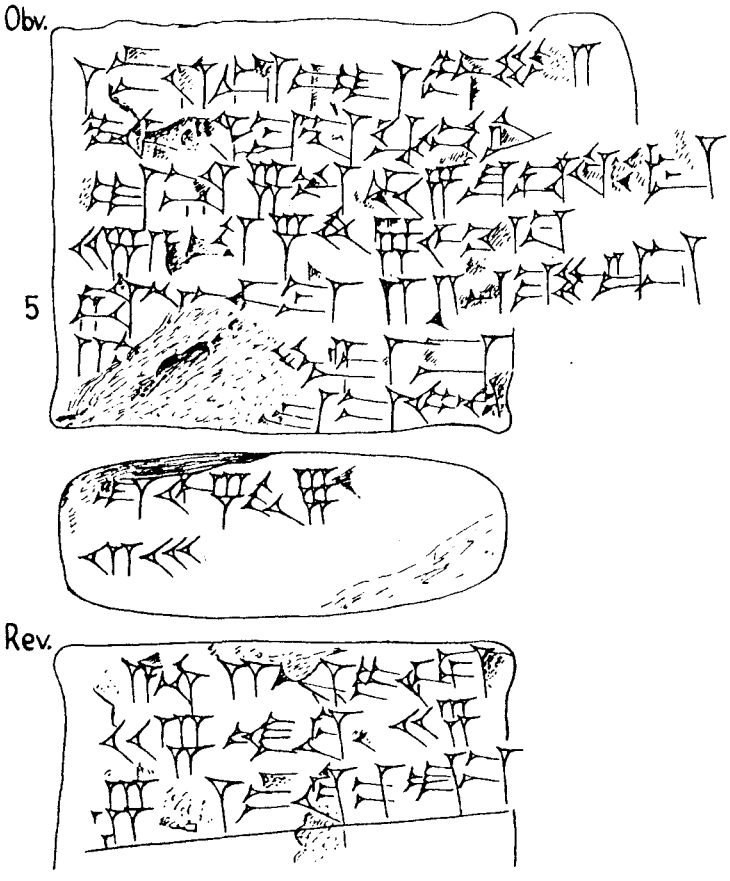

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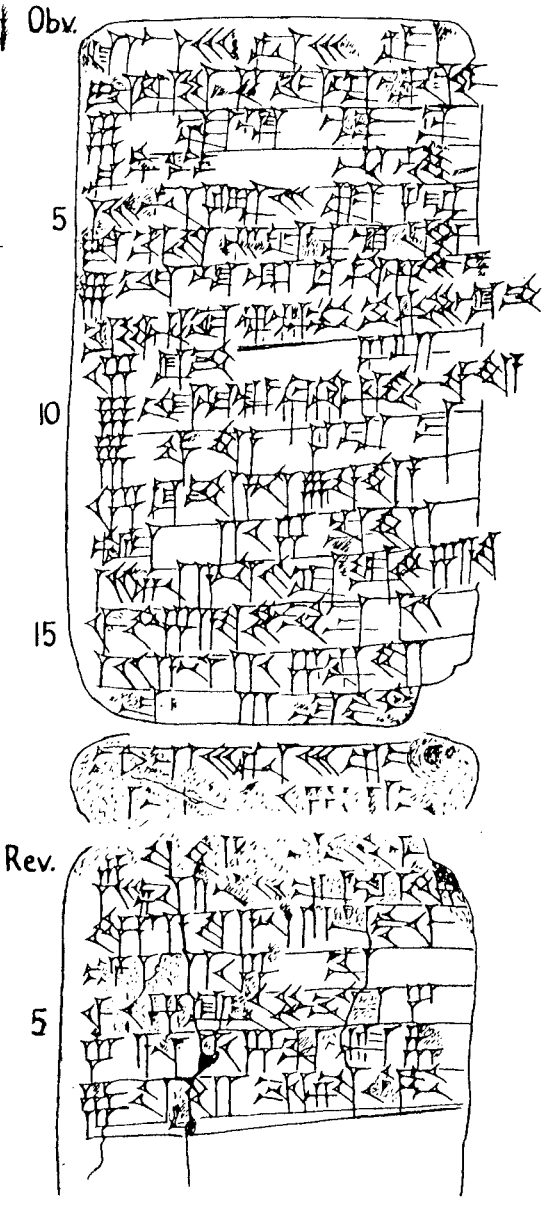

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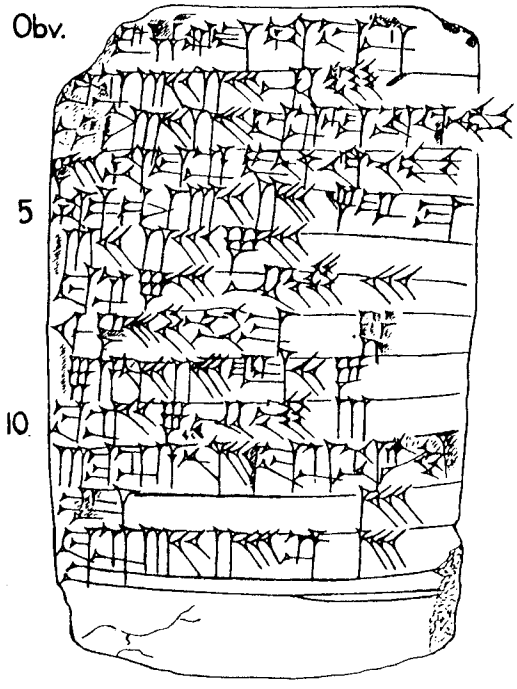

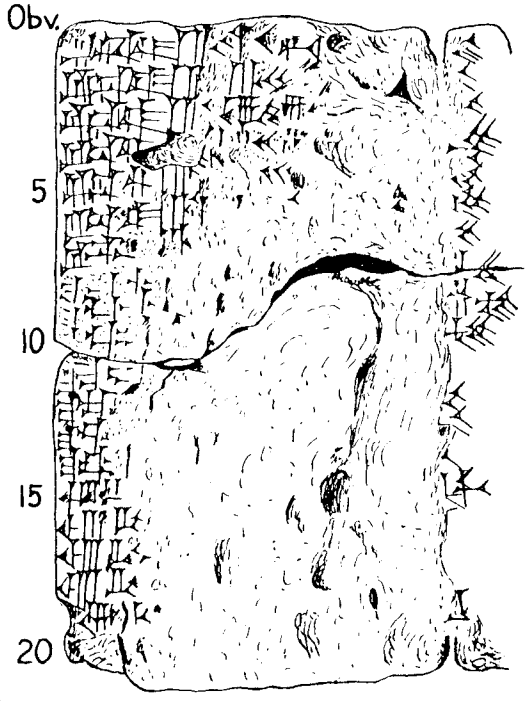

Rev.

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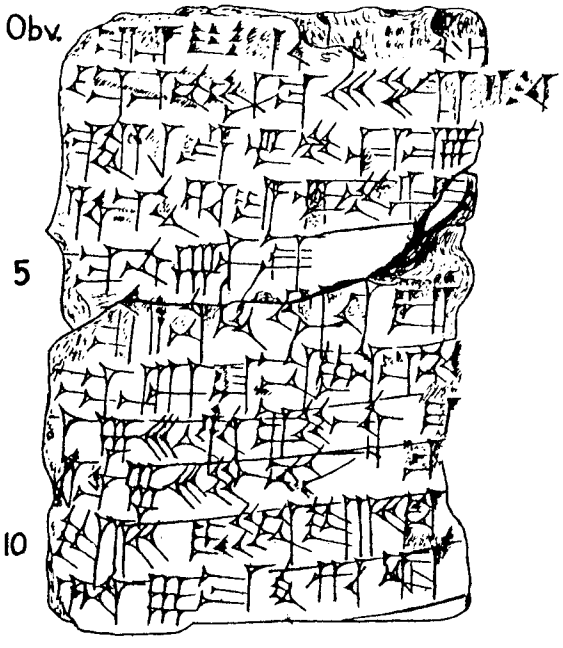
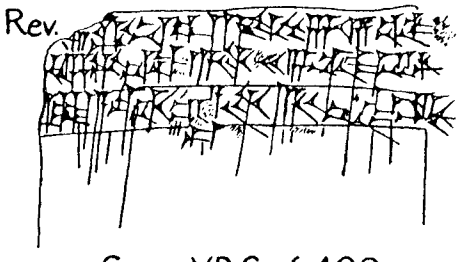
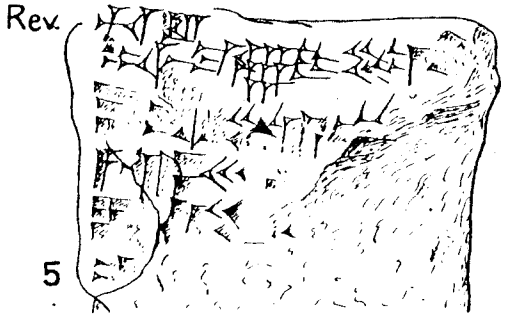




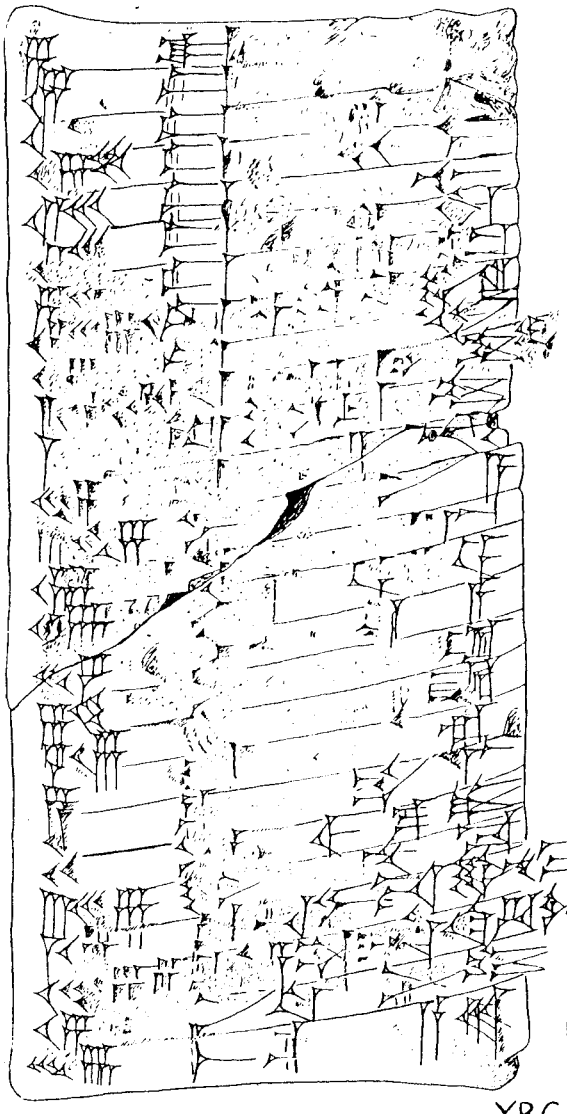
Sa. YBC 6492



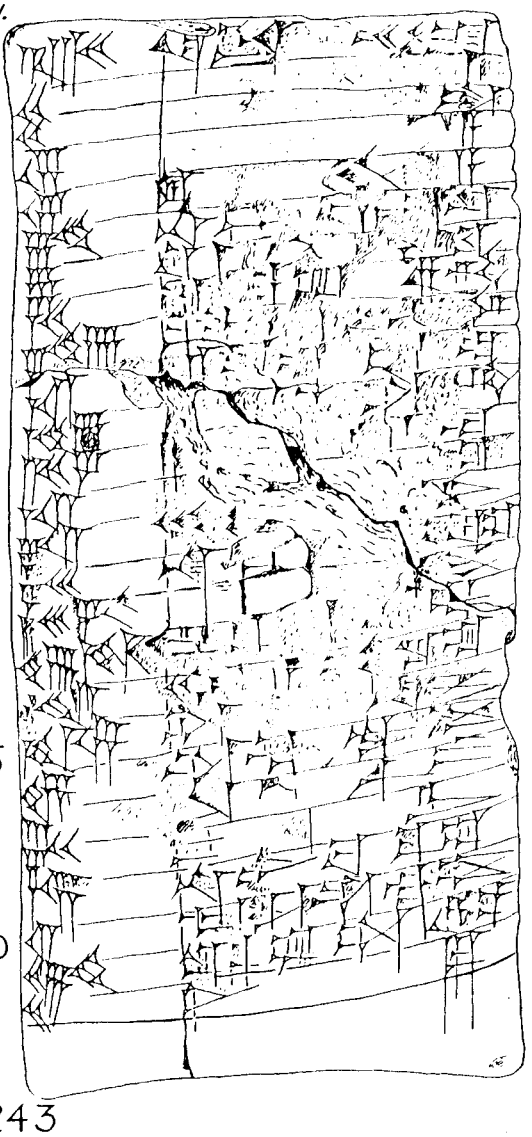
Sb

MLC 1842

Obv.



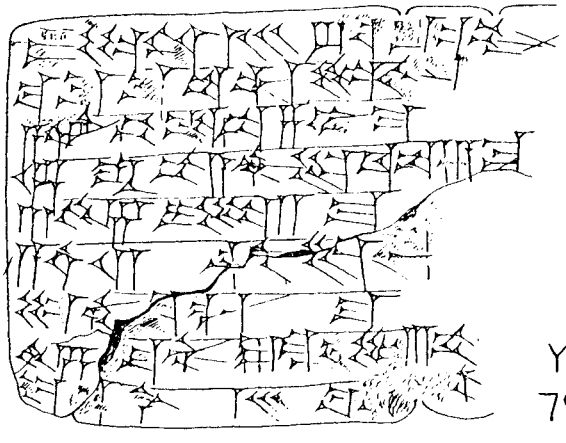
Rev.



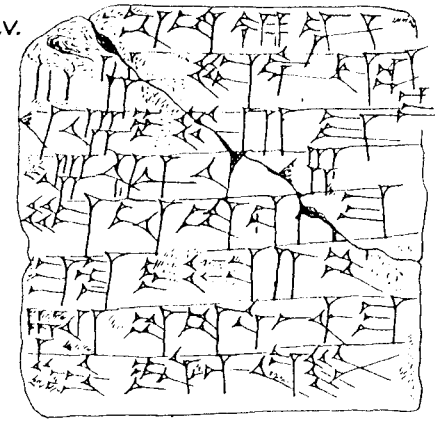
Ue

YBC 7243

Obv.



Rev.

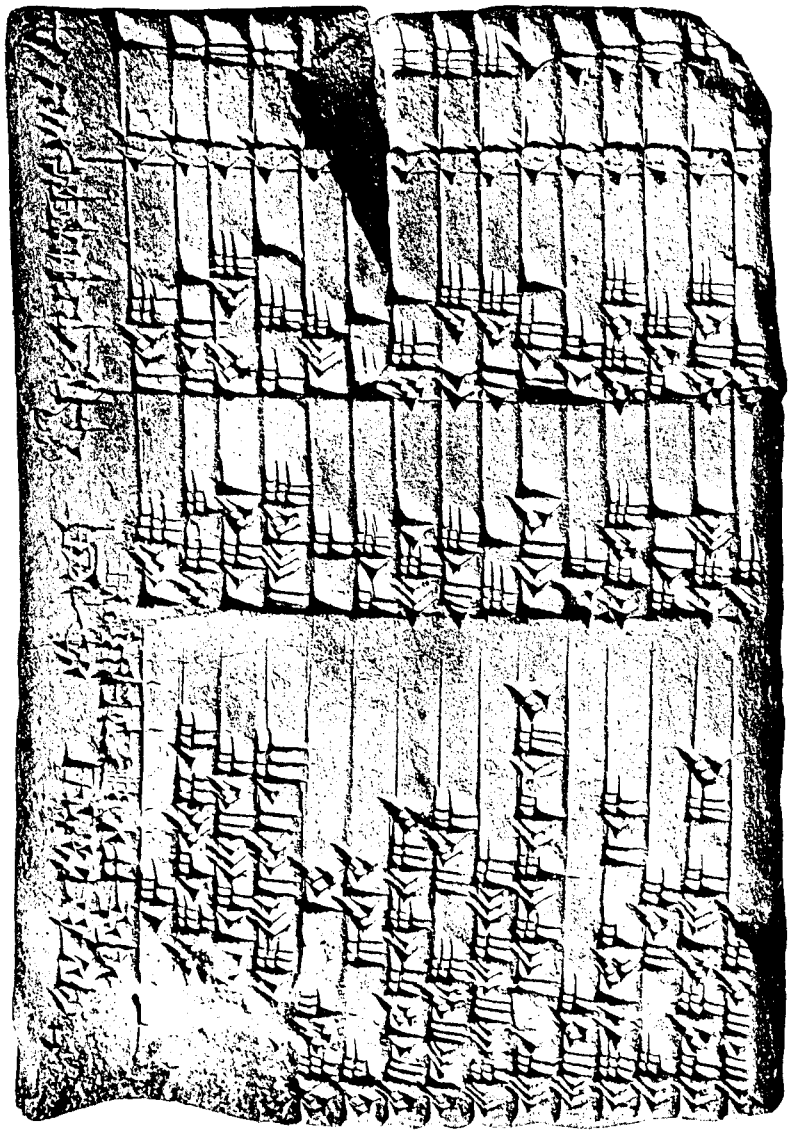


Pa

YBC 7997

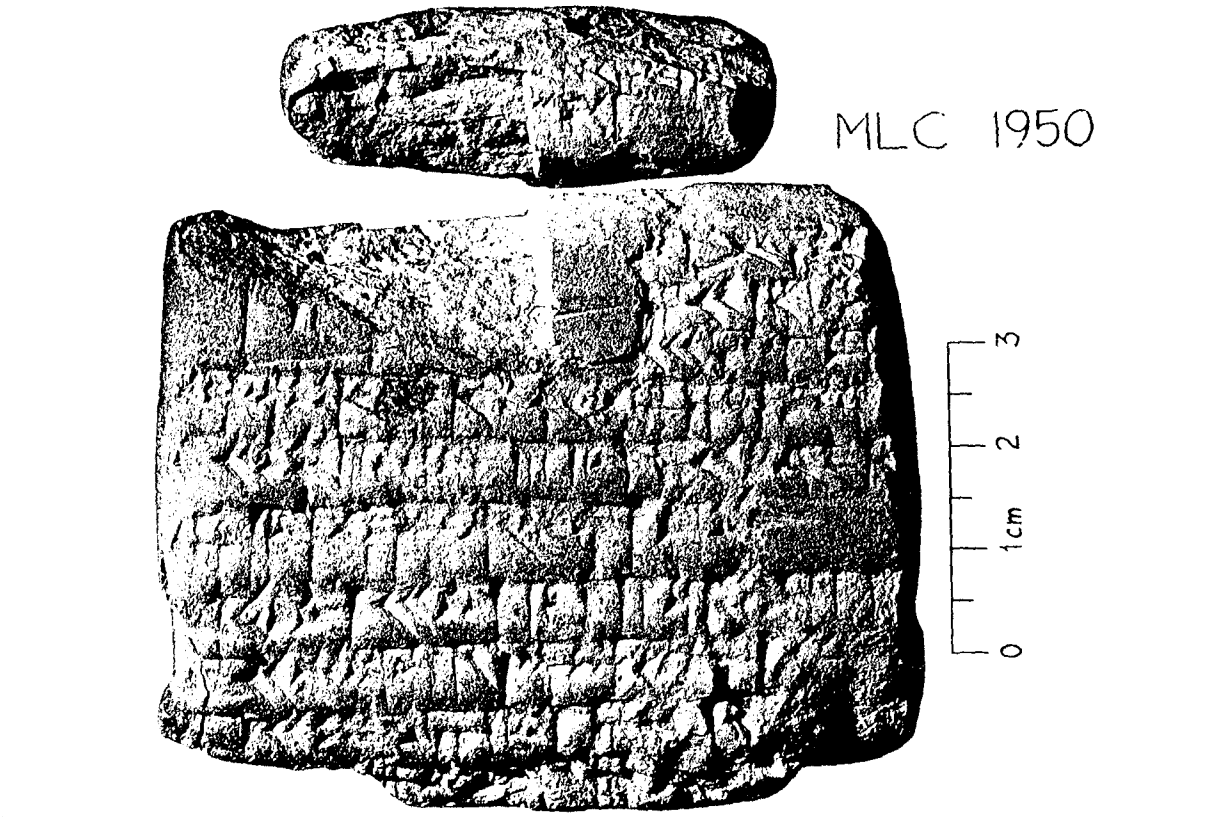
CBS 29.13.21

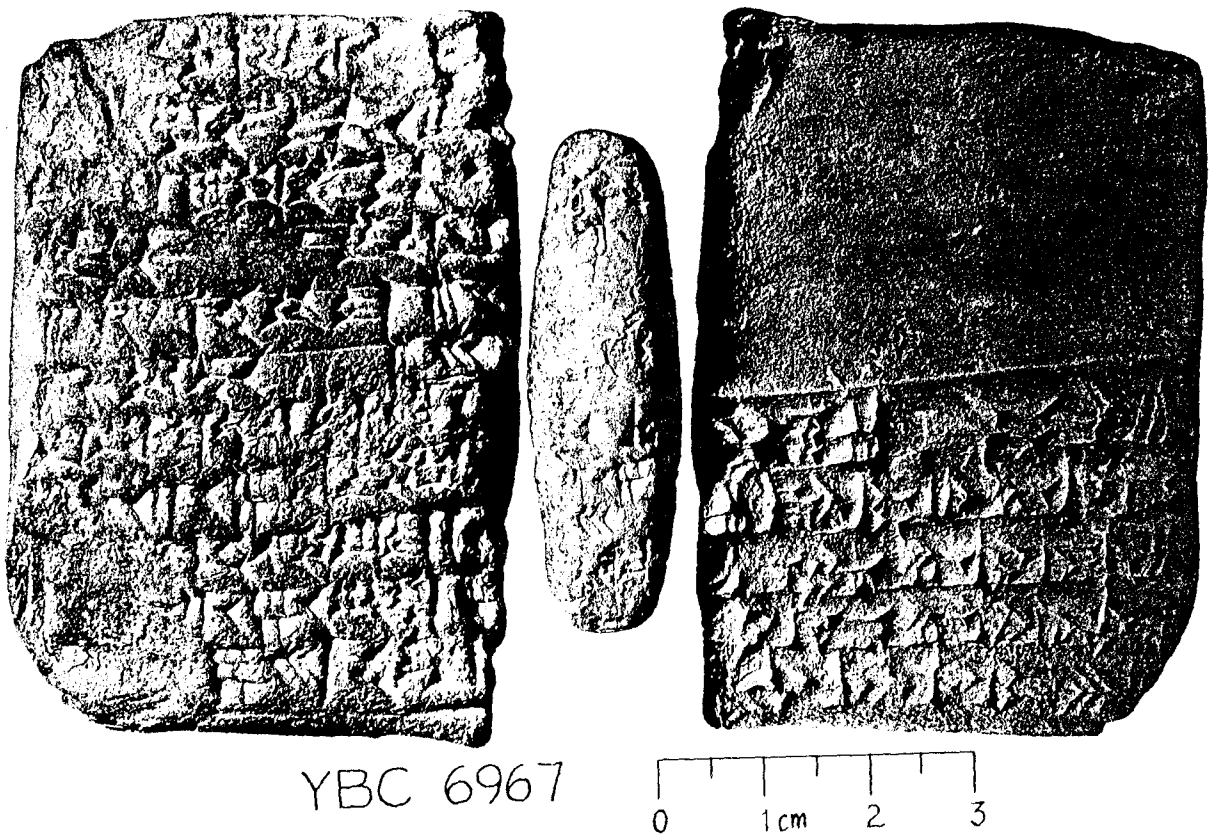
A



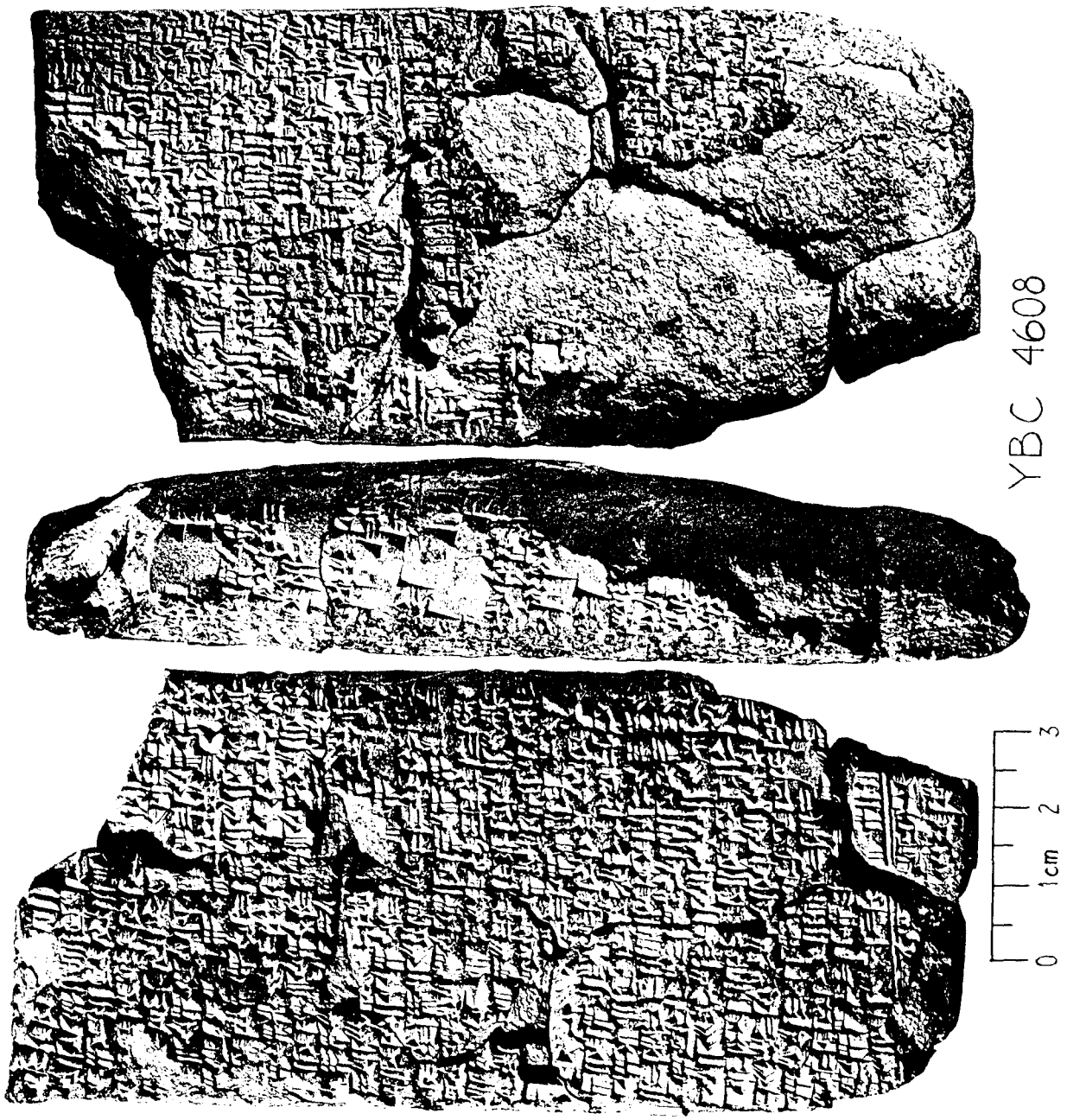
Plimpton 322

B

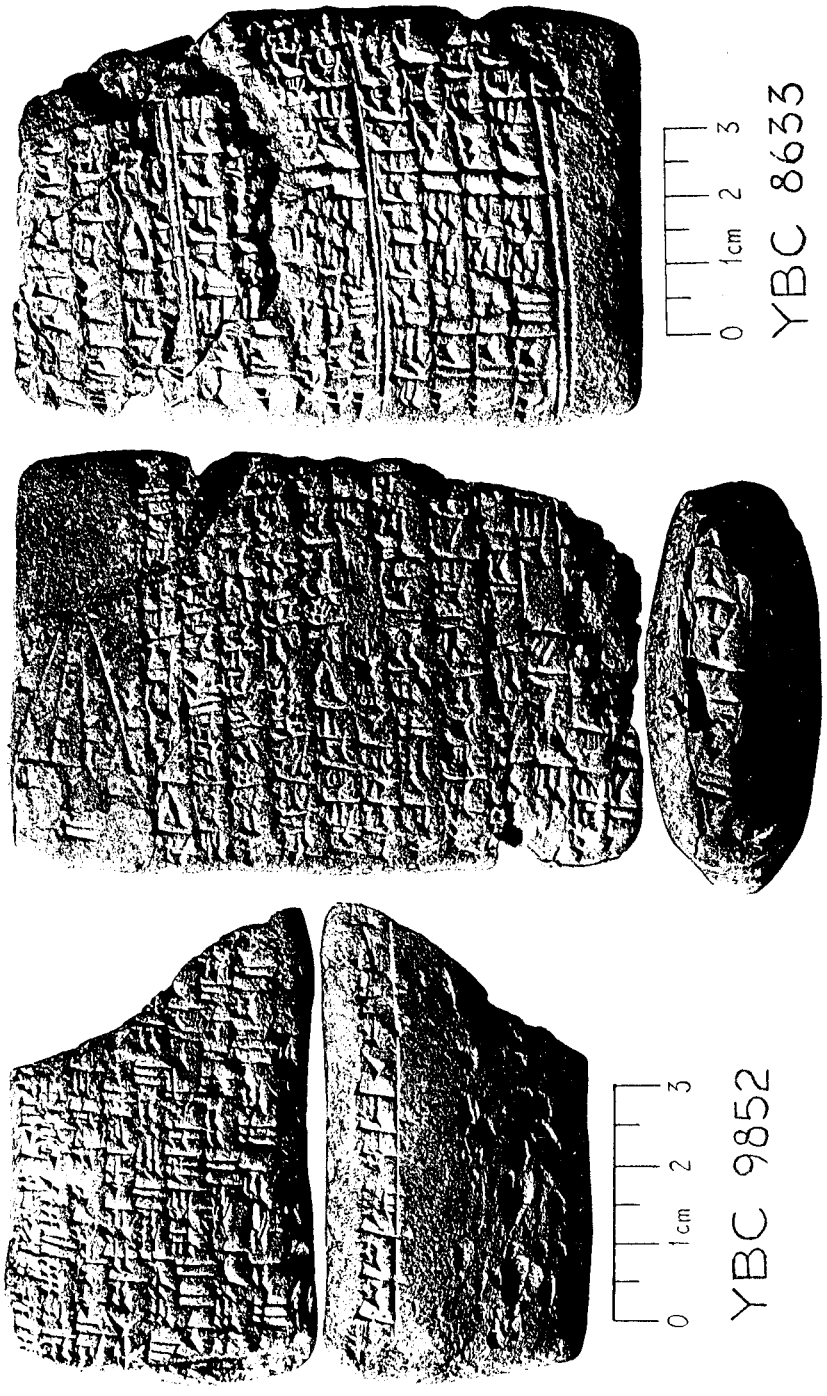

MLC 1950
0
1cm
2
3


YBC 6967
0
1cm
2
3

D

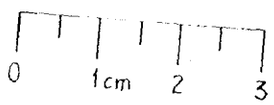


C, E

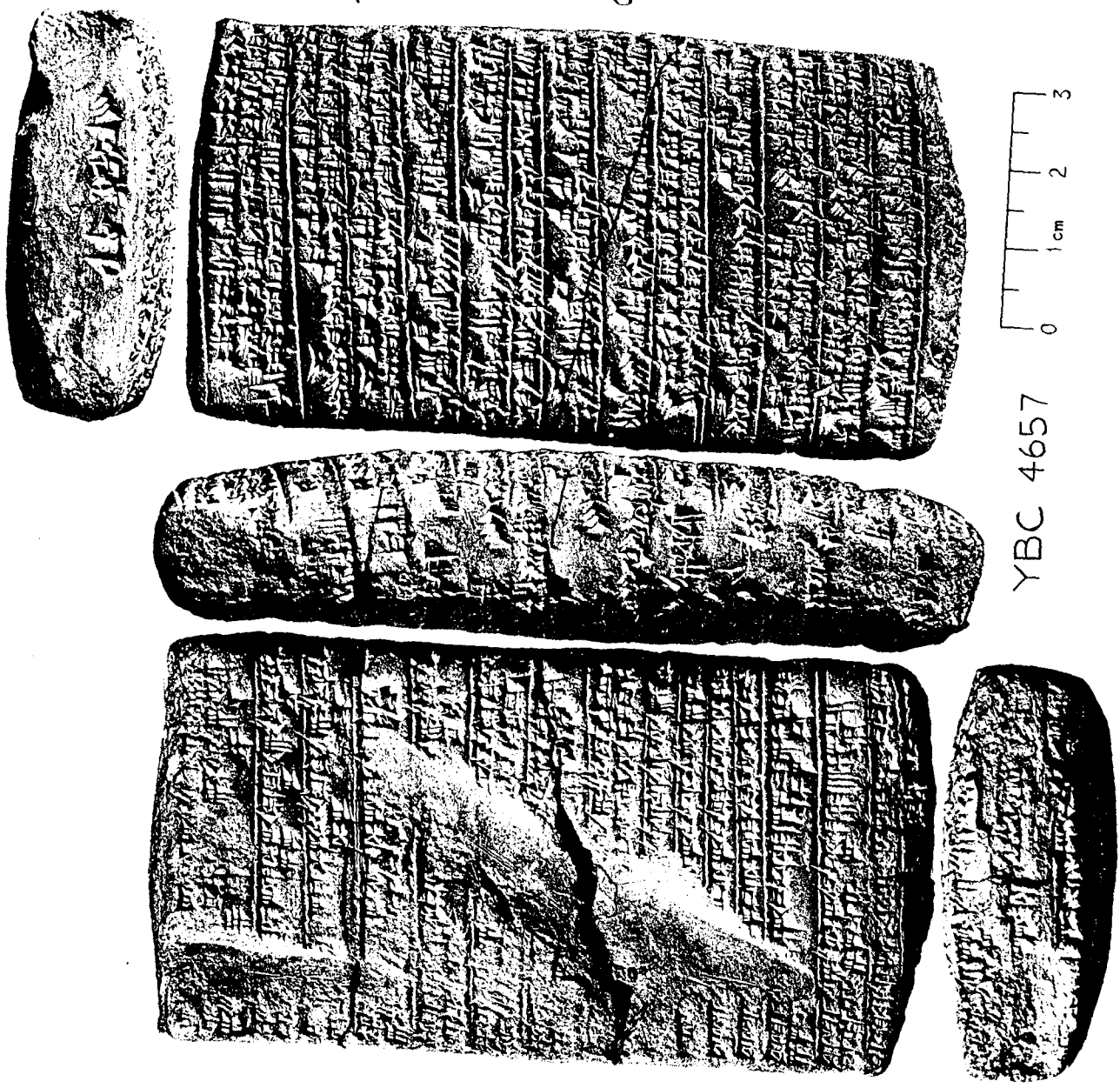


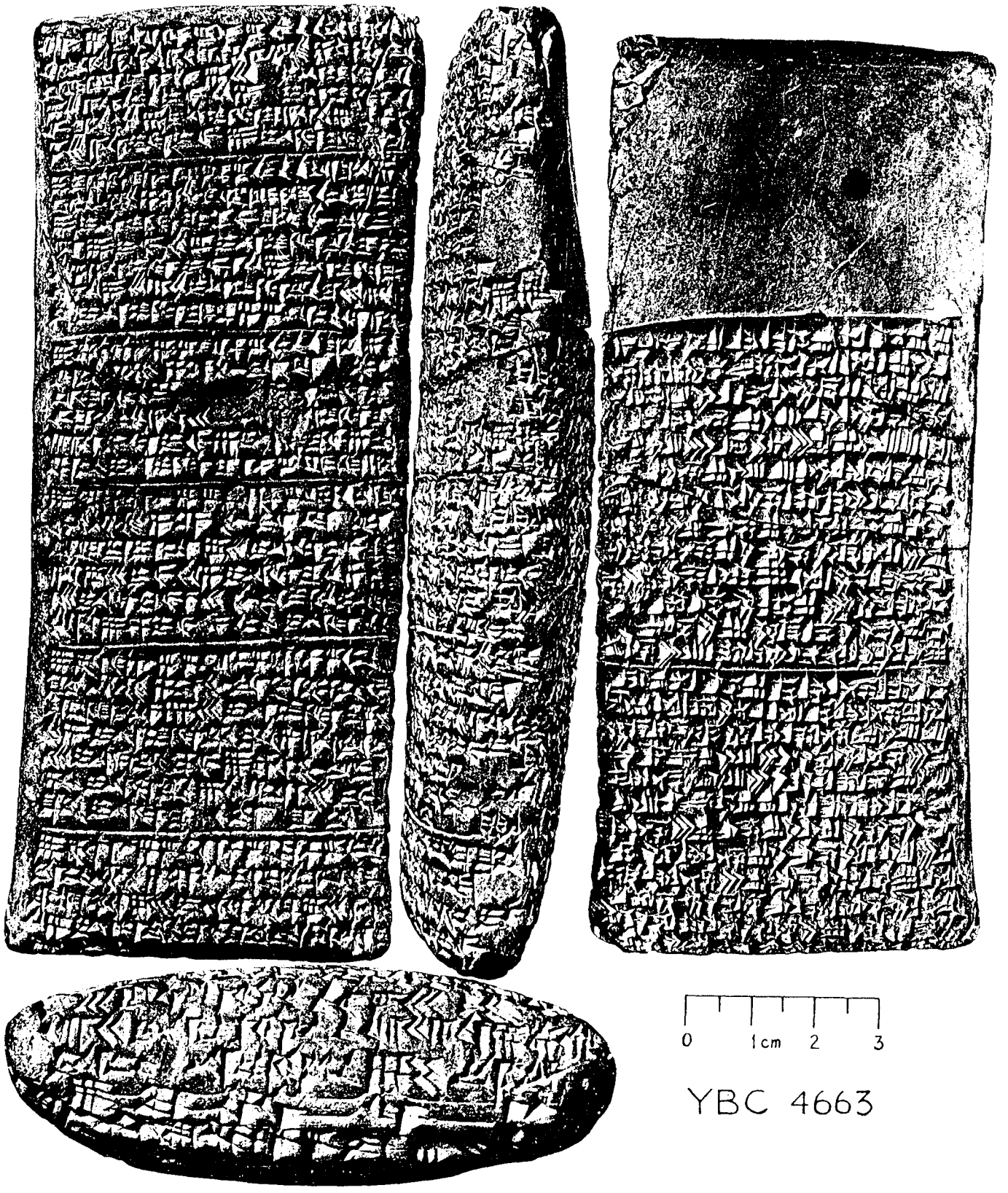
F

YBC 5037



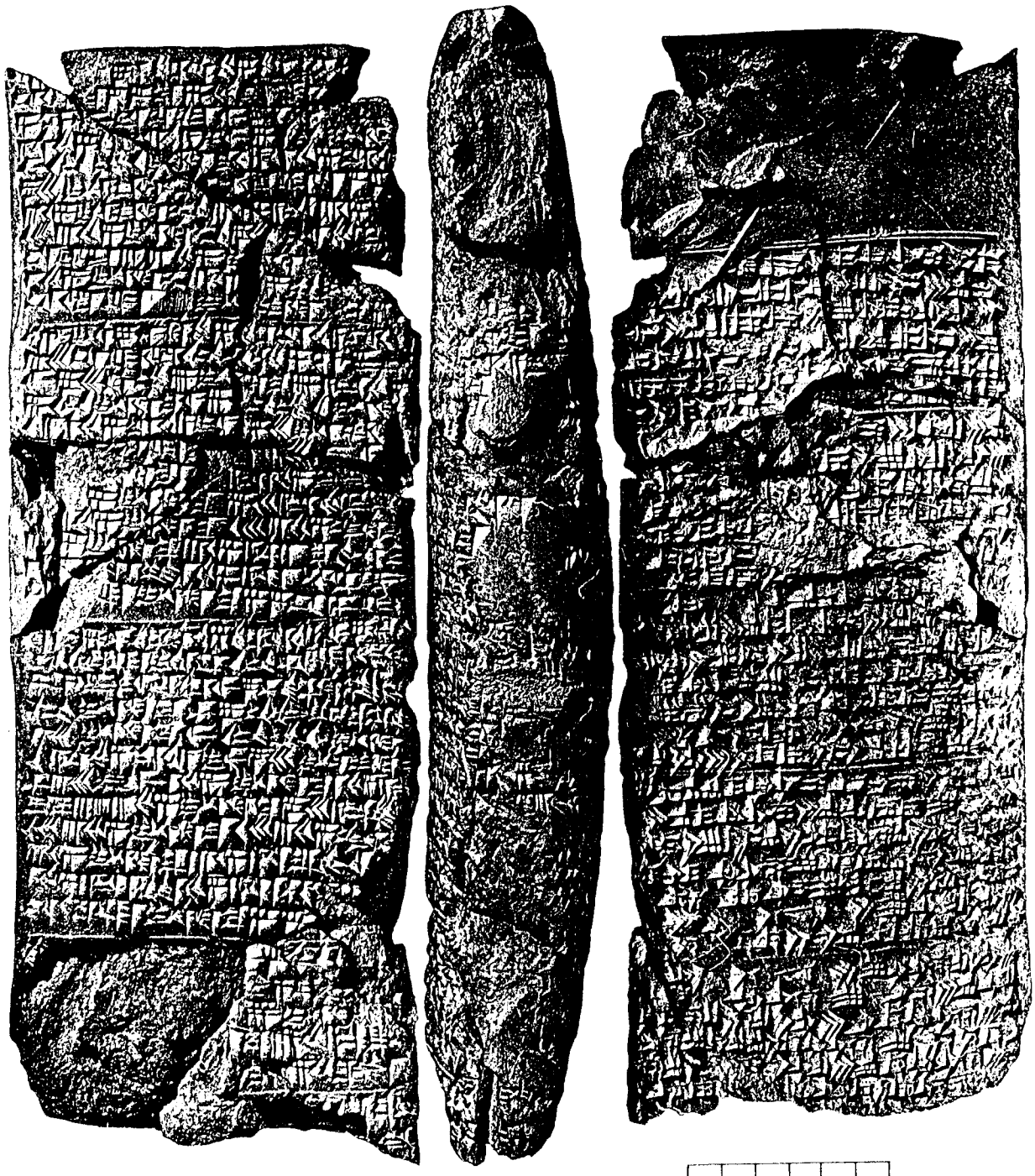
G





YBC 4663

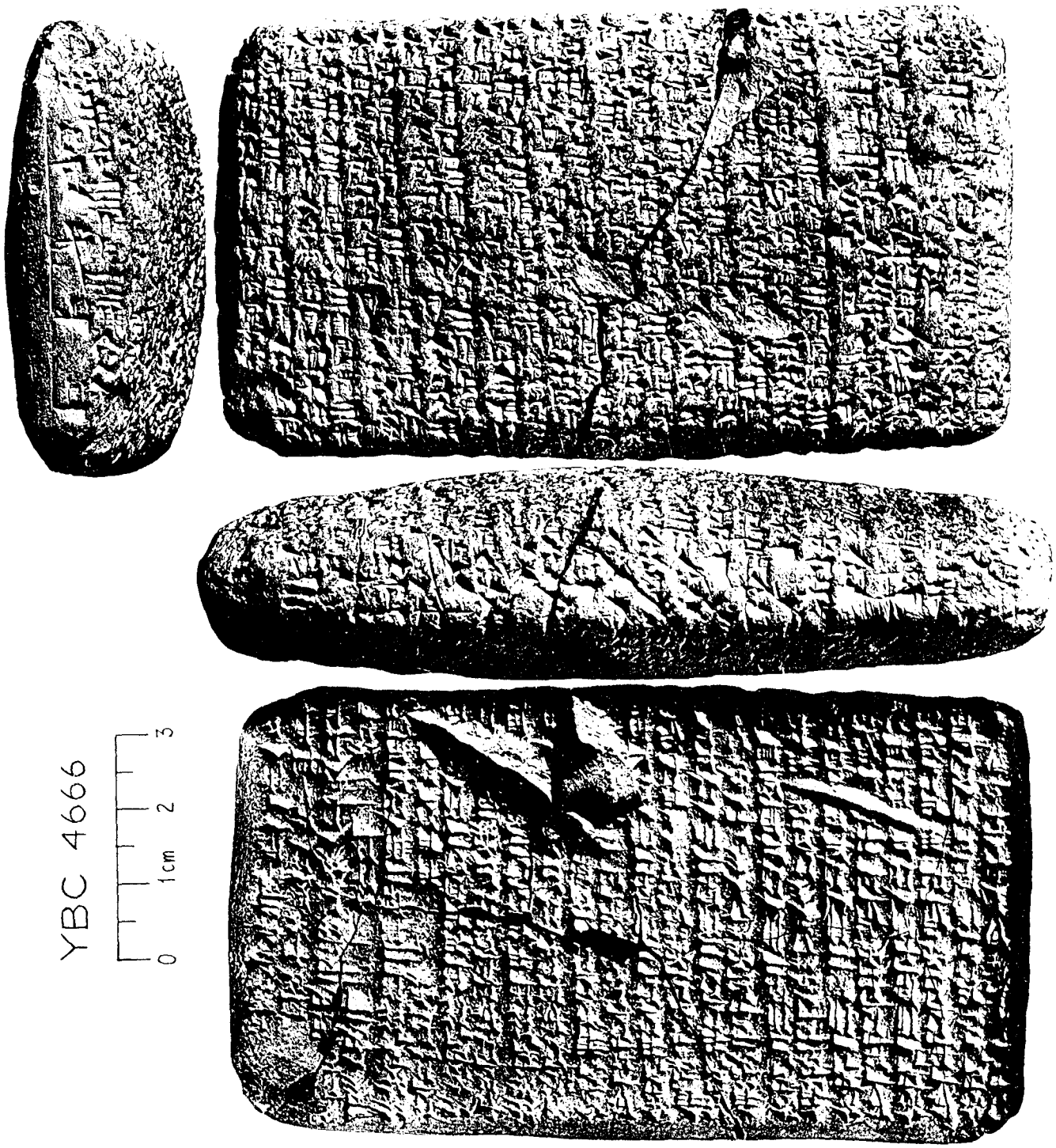
J



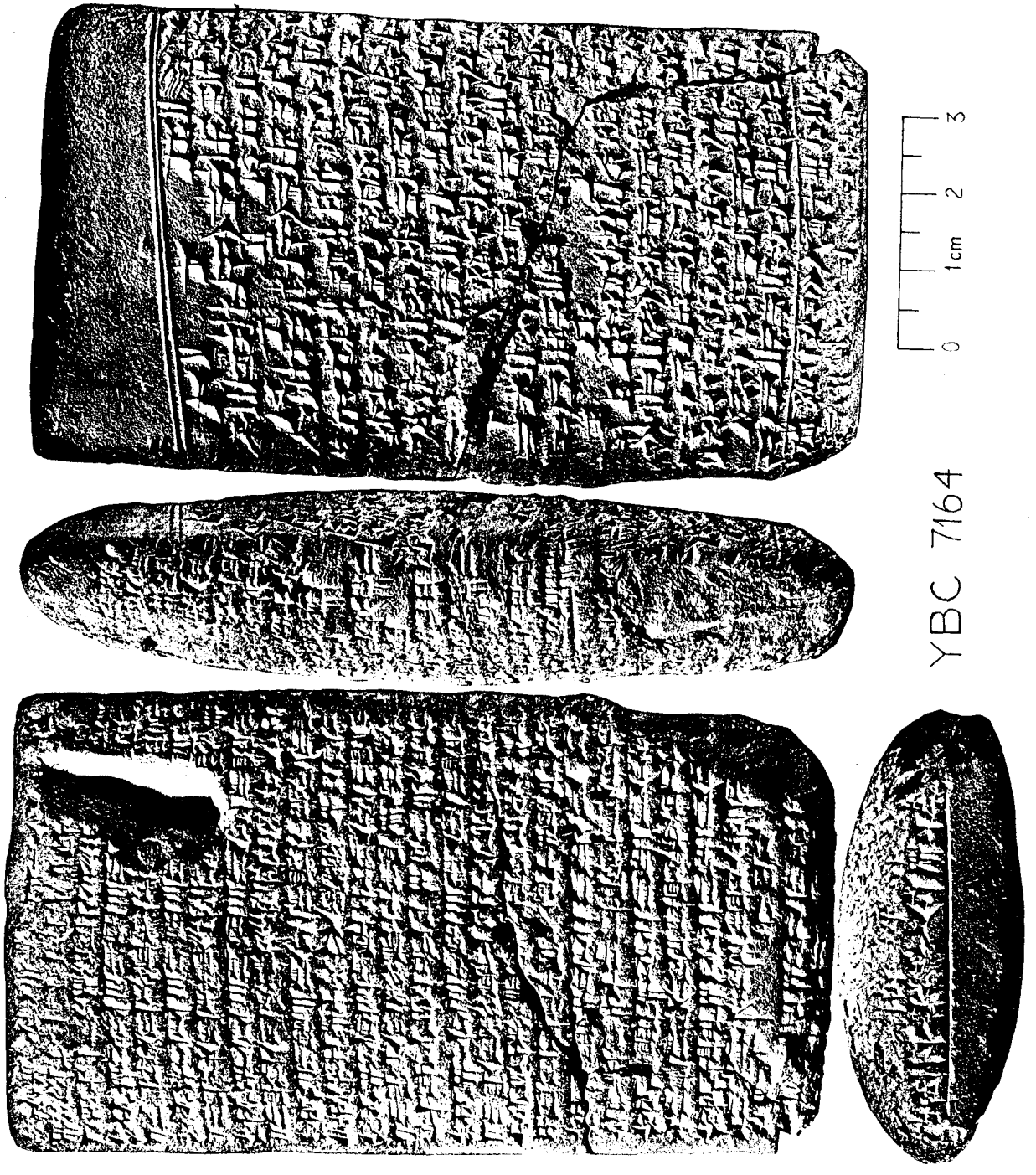
YBC 4662

0 1cm 2 3

K



L



M, N

YBC 9874

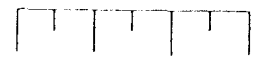
0 1cm 2 3

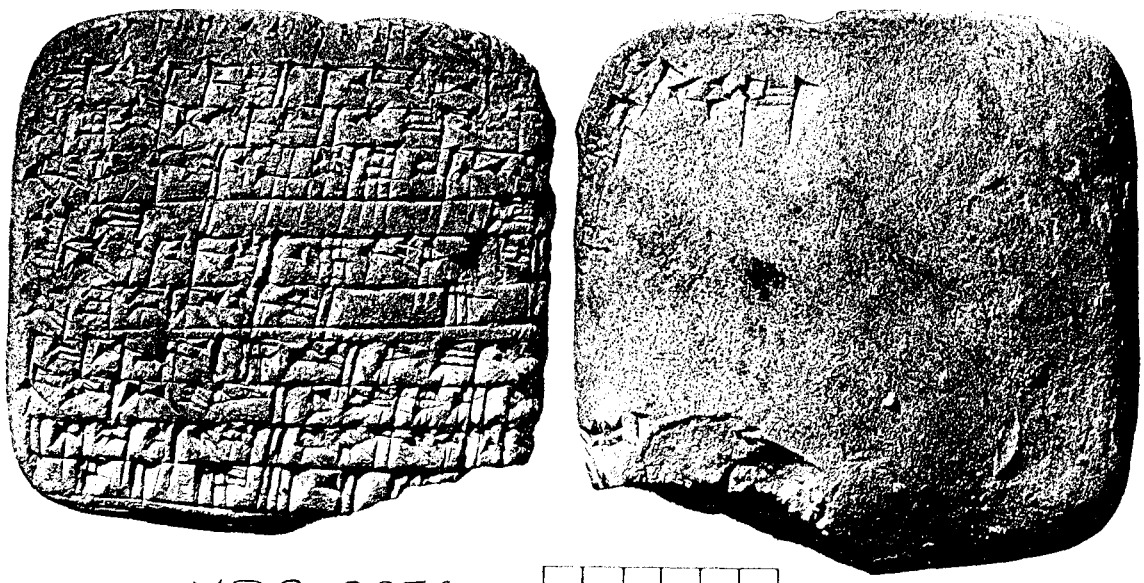
YBC 4186

O

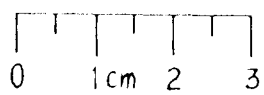
P, Q

YBC 10722

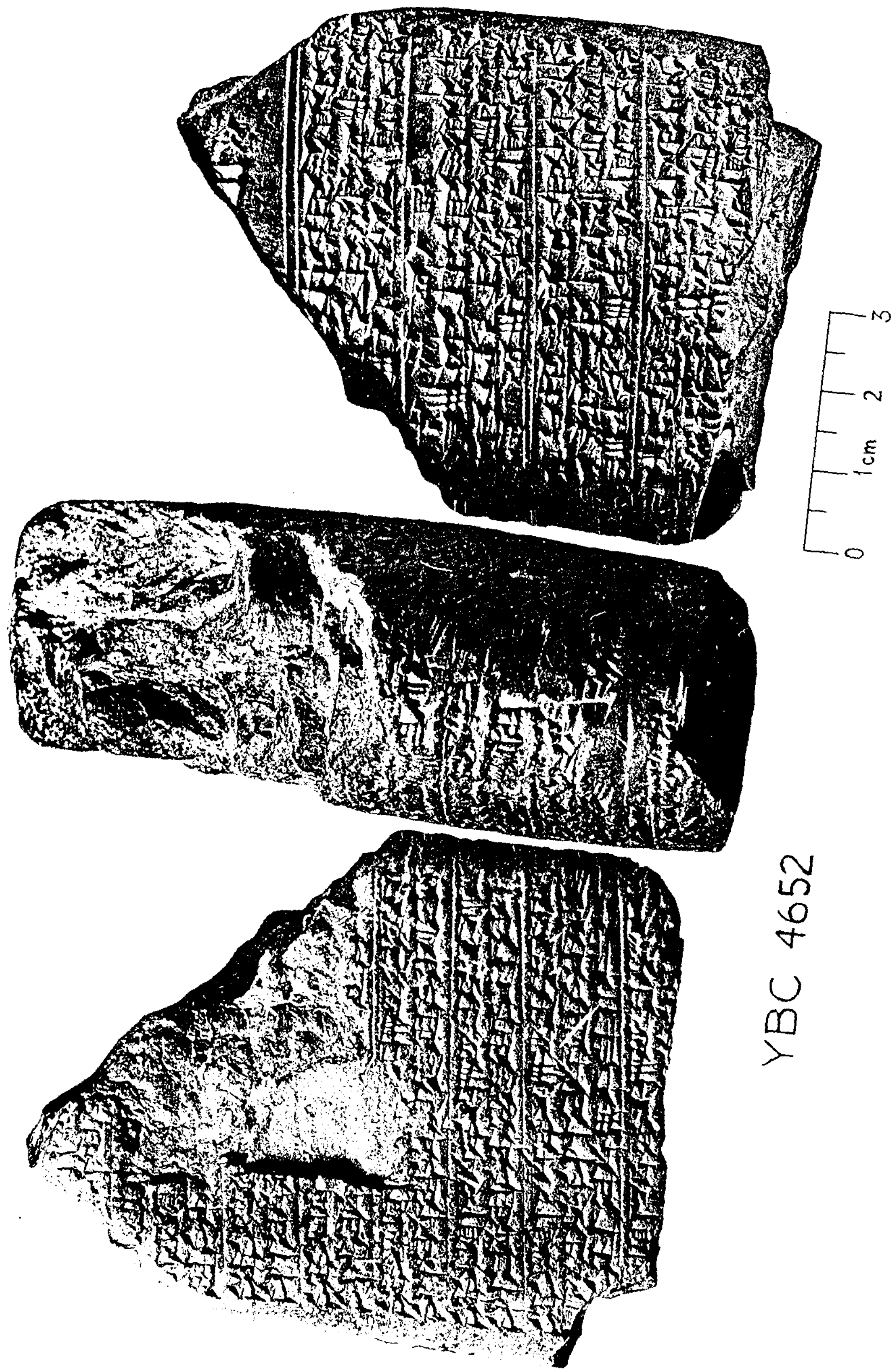




YBC 9856

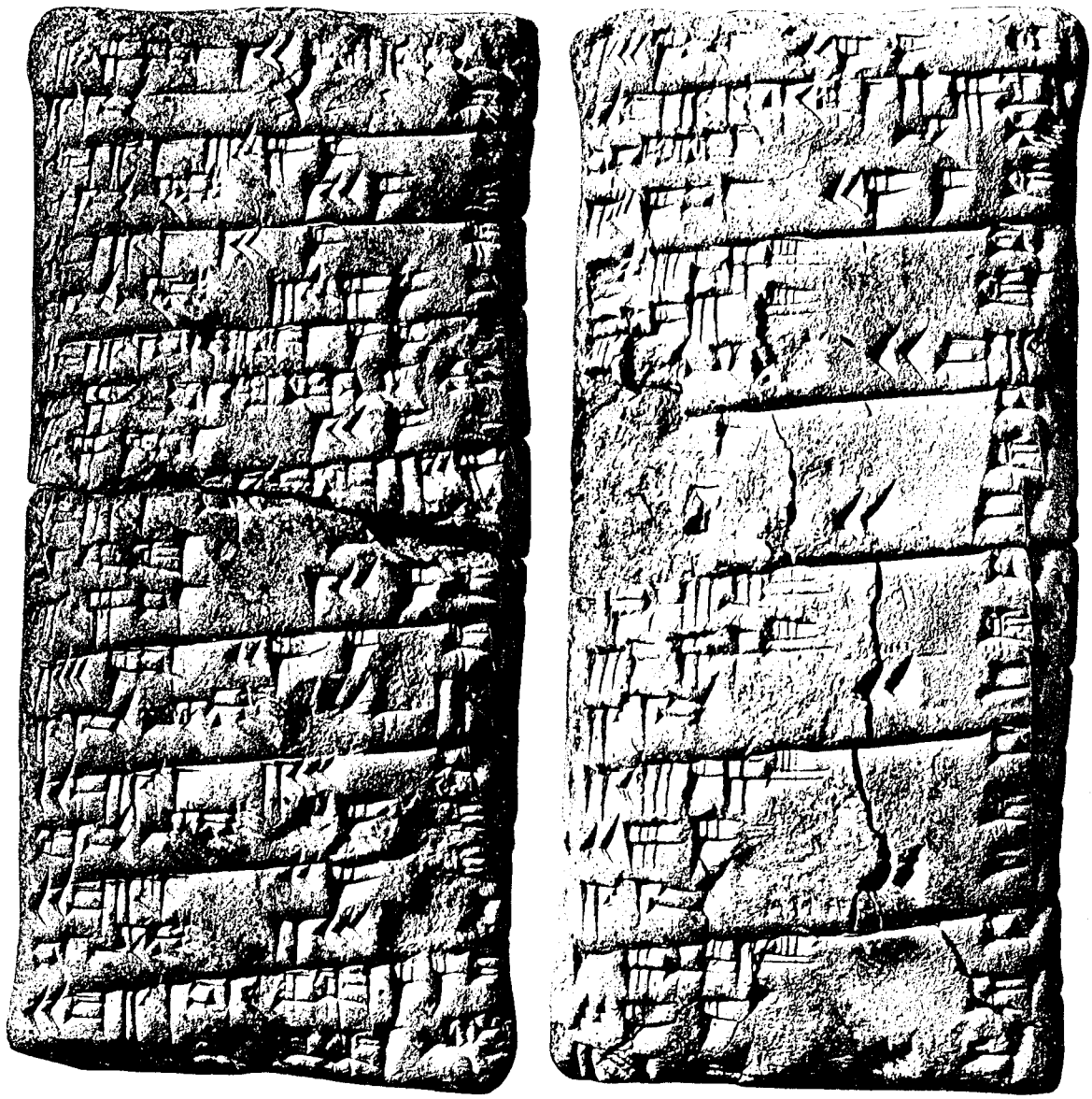


R

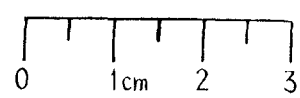


YBC 4652

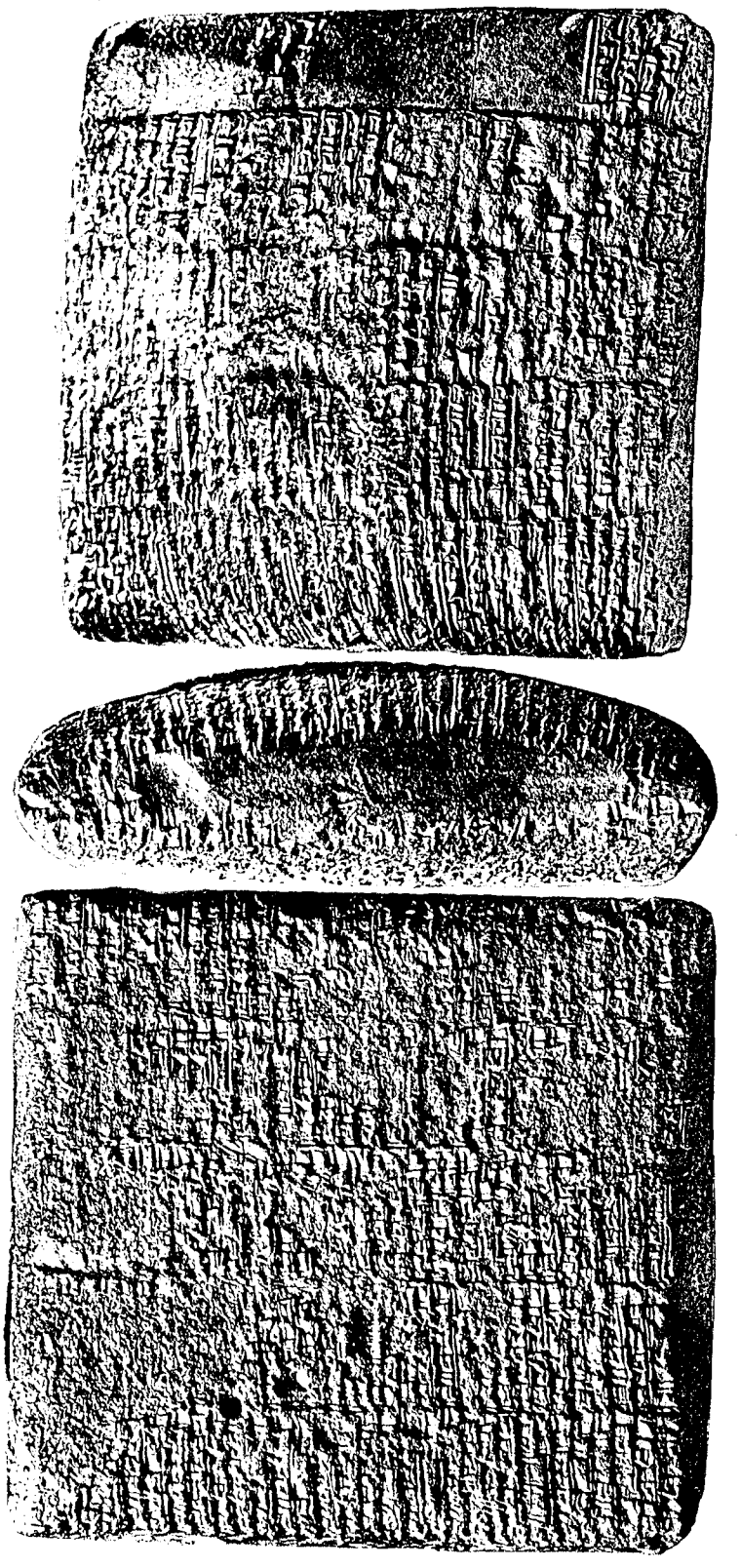
S



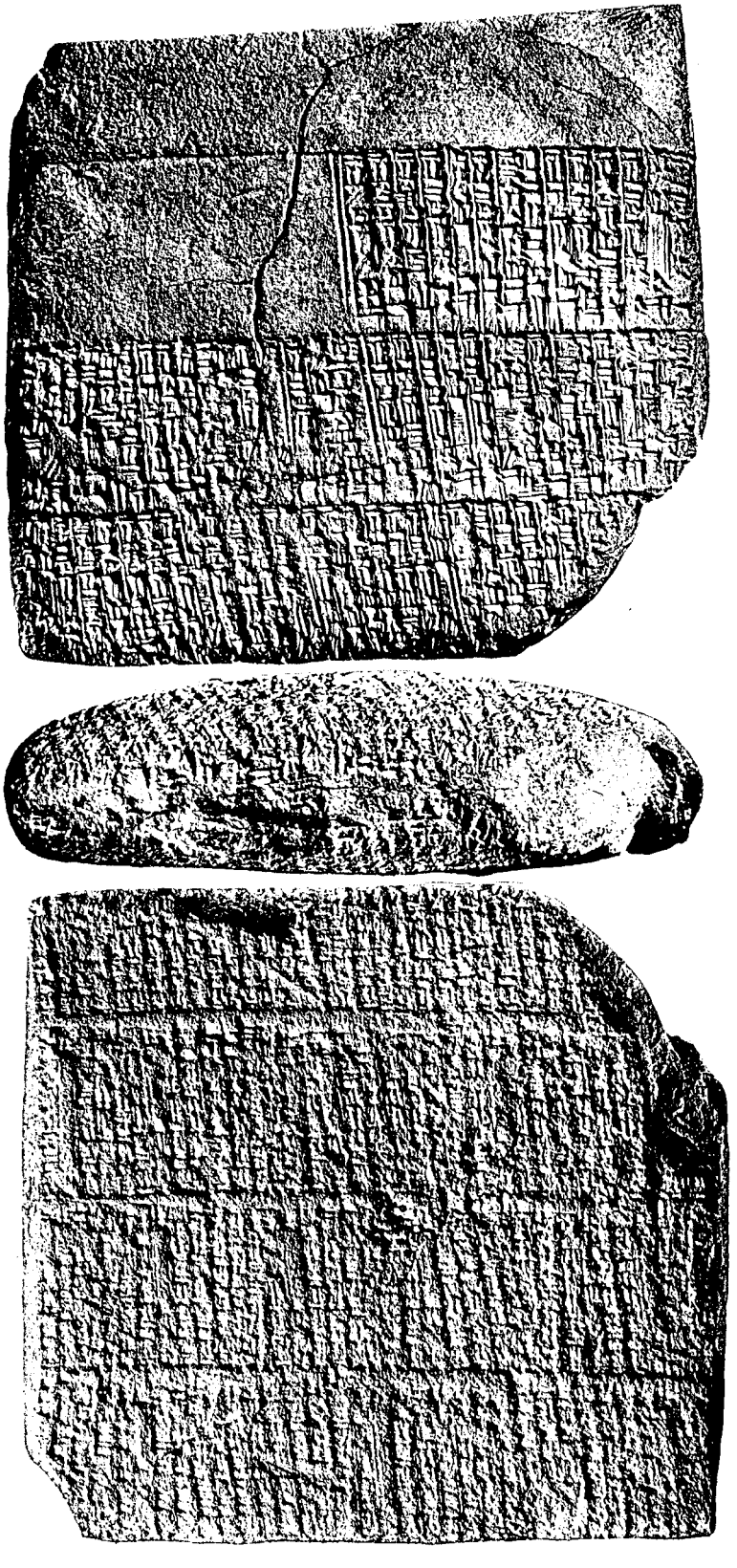
YBC 4612



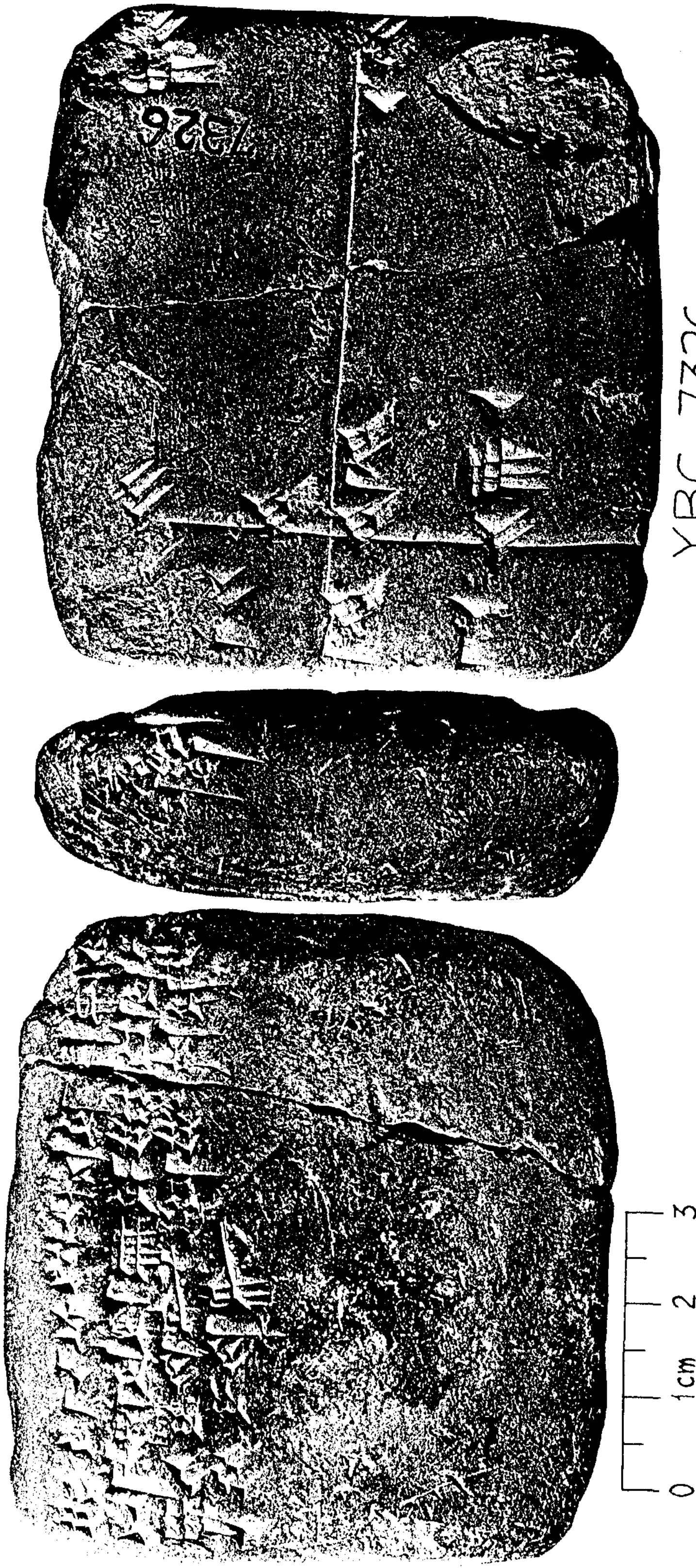
T



A 24194

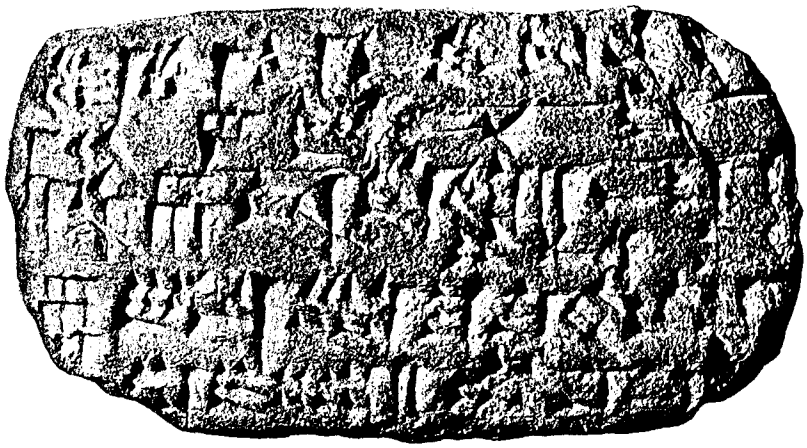


A 24195



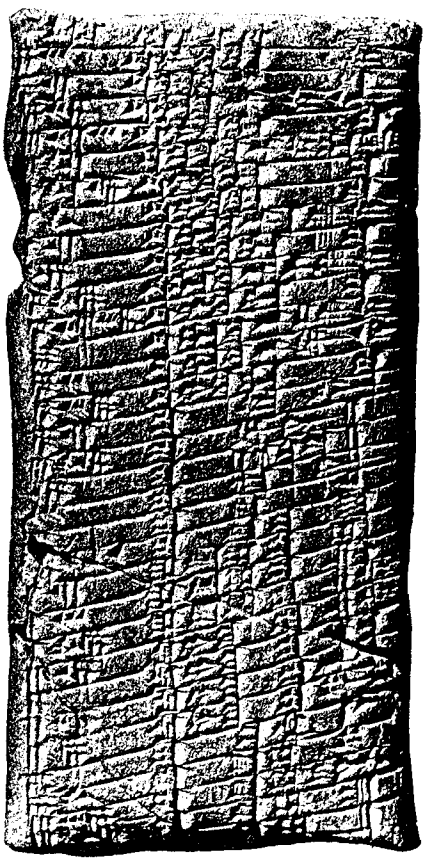
YBC 7326

MM 86-11-404



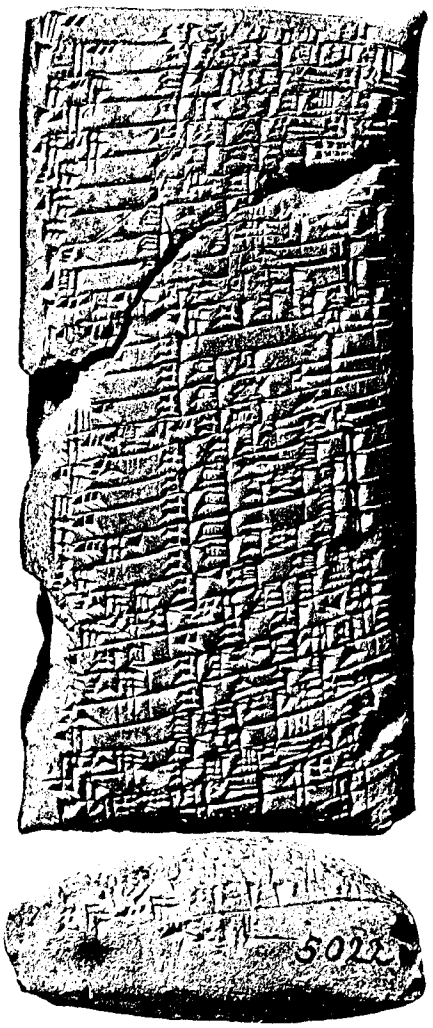
YBC 10522

0 1cm 2 3

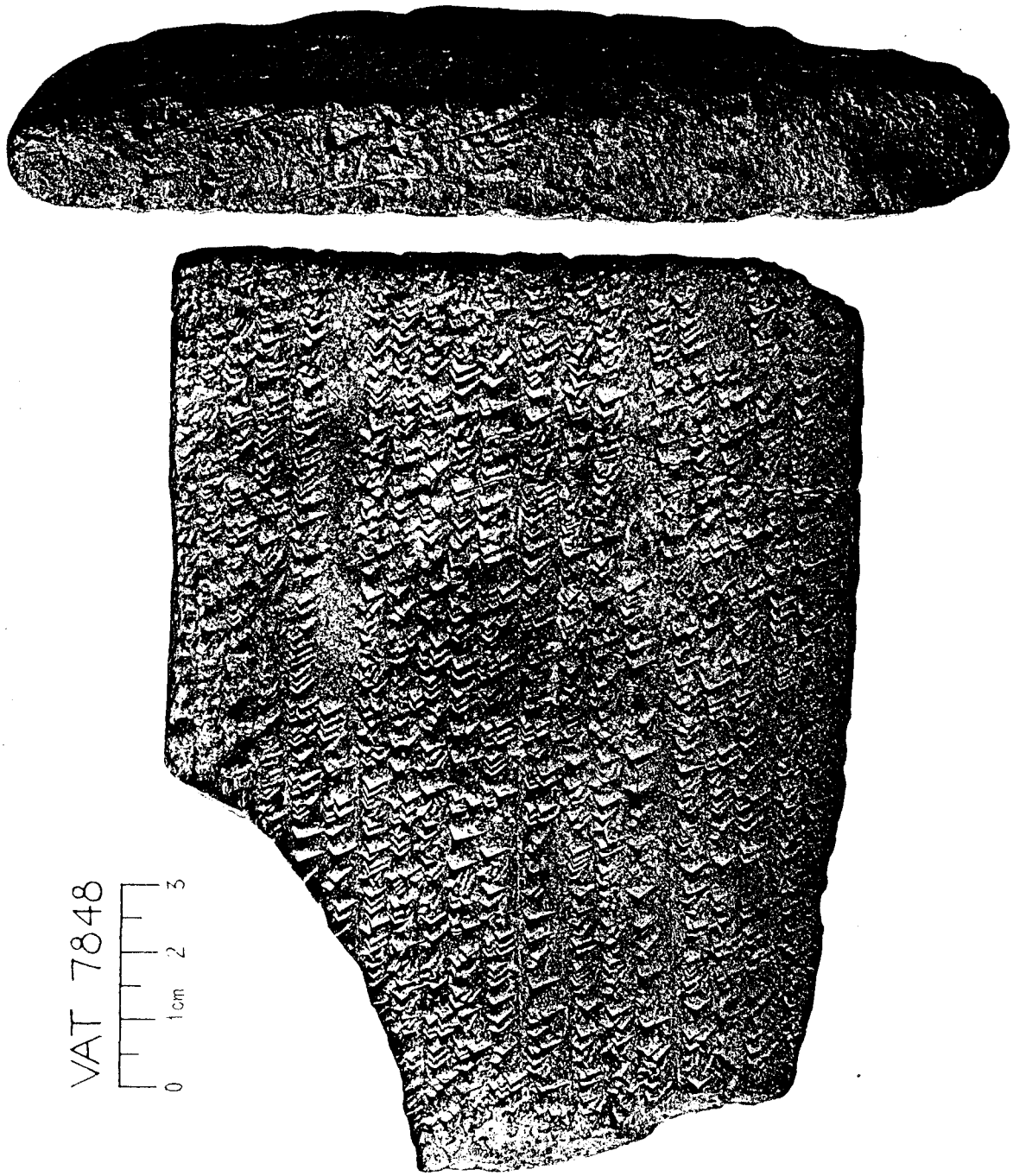


0 1cm 2 3

YBC 5022

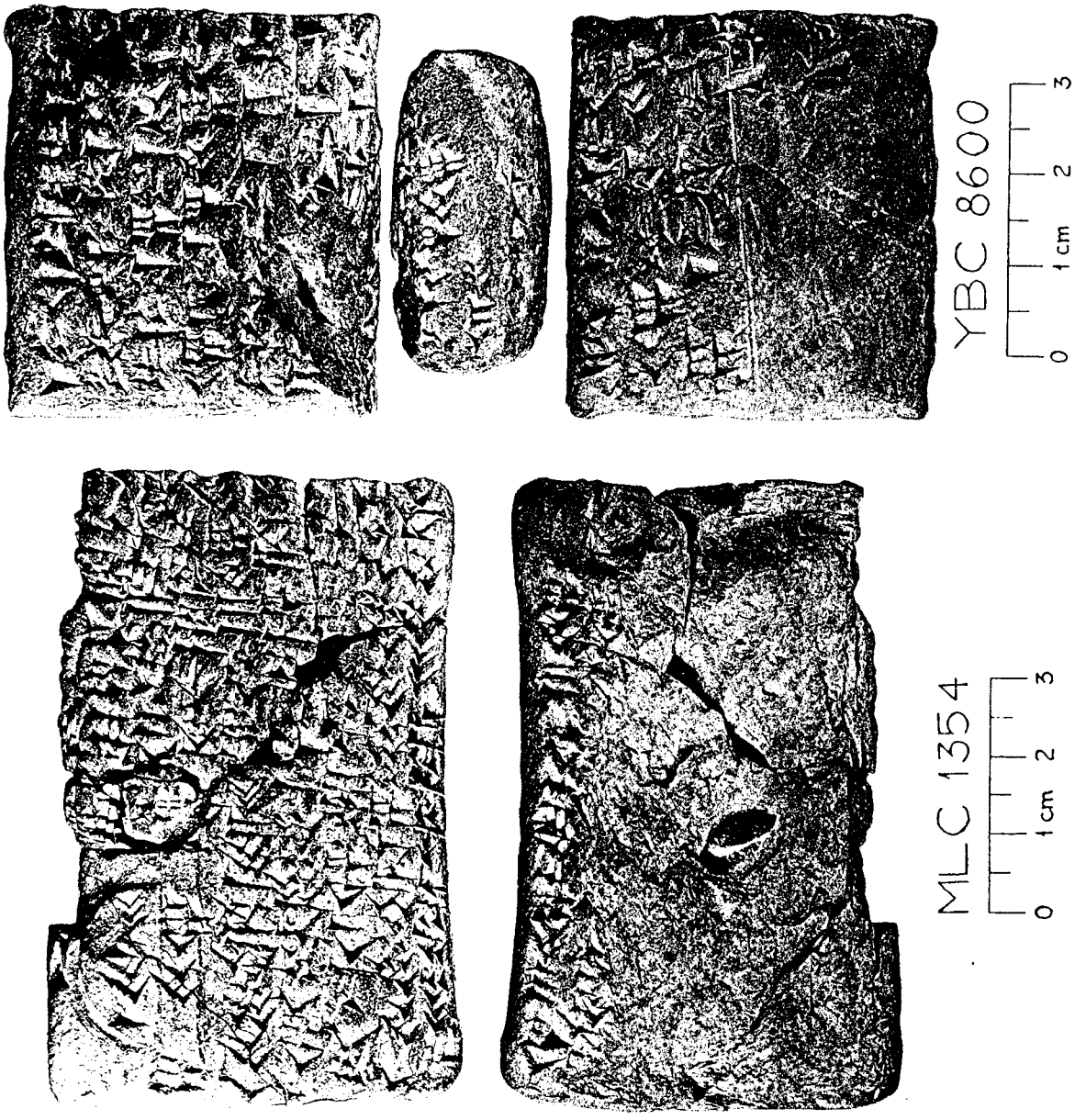


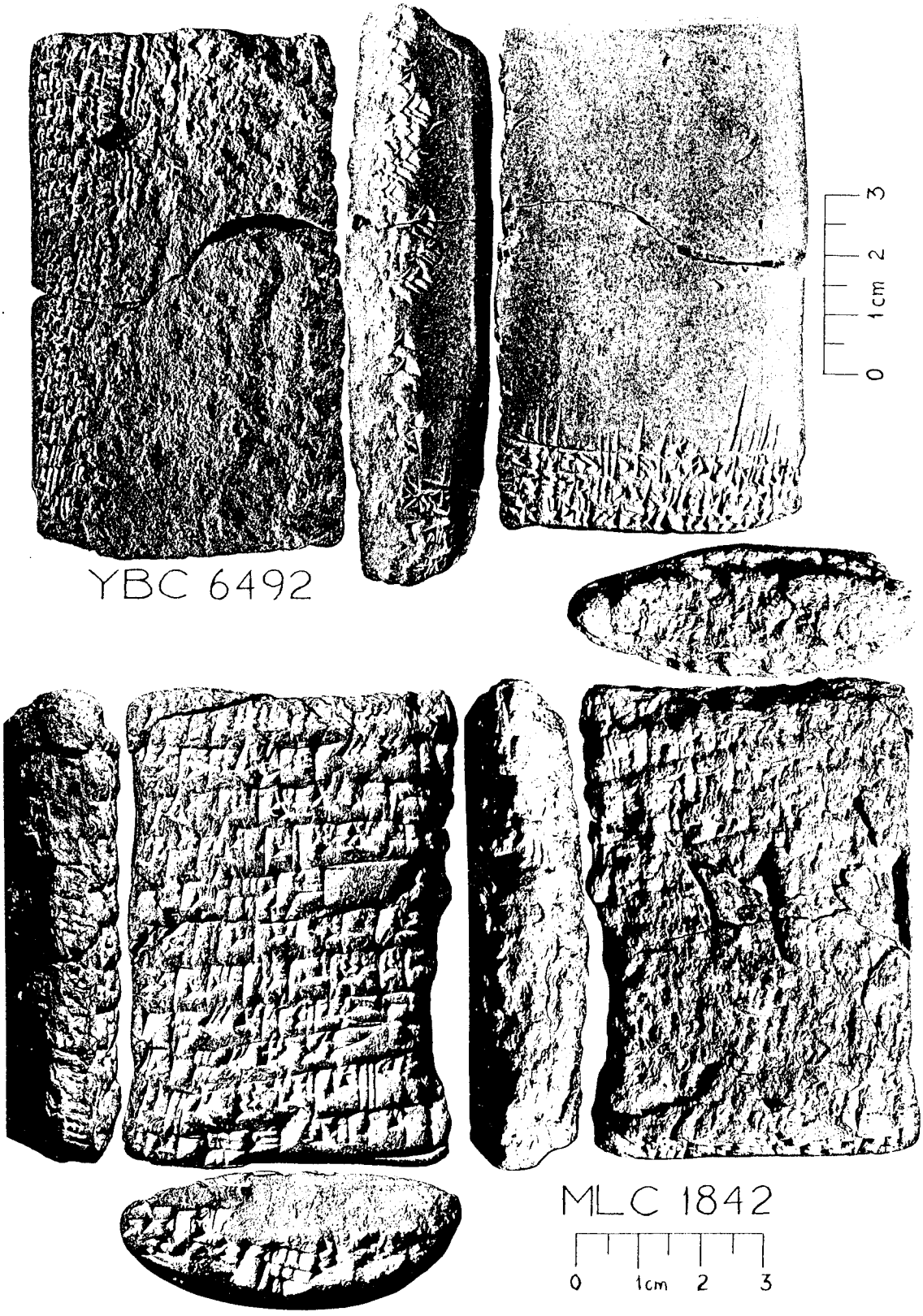
Y

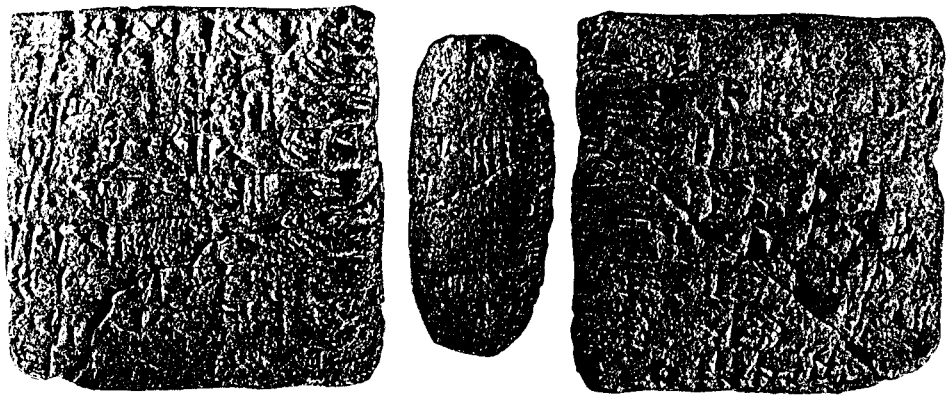



YBC 6295
0 1cm 2 3
YBC 8588
0 1cm 2 3

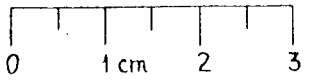
Eb, Ec




YBC 6492
0 1cm 2 3
MLC 1842
0 1cm 2 3



YBC 7997



YBC 7243