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import java.util.*;
import java.lang.*;

public class BinLinkedTree<T extends Comparable<T>,D>
implements Iterable<T>
{
    // ATTRIBUTES
    private TreeNode root = null;
    private int size = 0;

    class TreeNode {
        // ATTRIBUTES
        D data;
        T key;
        TreeNode left;
        TreeNode right;
        TreeNode parent;

        // CONSTRUCTOR
        TreeNode(T key, D data, TreeNode left, TreeNode right, TreeNode parent) {
            this.data = data;
            this.key = key;
            this.left = left;
            this.right = right;
            this.parent = parent;
        }
        TreeNode(T key, D data, TreeNode parent) {
            this(key, data, null, null, parent);
        }

        // METHODS
        public String toString(){
            return "" + this.key;
        }
    }

    // INSERT, DELTE METHODS
    public boolean empty() {
        /* Return true if tree is empty. */
        // O{1}
        return this.size == 0;
    }
    public void insert(T key, D data) {
        /* Insert new node with given key and data. */
        // O{log(n)}
        TreeNode node = this.root;
        if( this.empty() ){
            this.root = new TreeNode(key, data, null);
        } else {
            while( true ){
                if( key.compareTo(node.key) > 0 ){
                    if( node.right != null ){
                        node = node.right;
                    } else {
                        node.right = new TreeNode(key, data, node);
                        break;
                    }
                } else if( key.compareTo(node.key) < 0 ) {
                    if( node.left != null ){
                        node = node.left;
                    } else {

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        node.left = new TreeNode(key, data, node);
        break;
    }
    } else {
        throw new DuplicateKeyException();
    }
    }
    }
    size++;
}
public static class DuplicateKeyException extends RuntimeException {
    public DuplicateKeyException() { super(); }
}
public boolean delete(T key) {
    /* Delete the node with the given key and return true, if it does not exists, return false. */
    // O{(log(n))^2}
    try {
        TreeNode node = this.getNode(key);
        boolean isroot = node.key.equals(this.root.key);
        boolean rightchild = !isroot && node.key.compareTo(node.parent.key) > 0;
        if( node.right == null && node.left == null ){
            if( isroot ){
                this.root = null;
            } else if( rightchild ){
                node.parent.right = null;
            } else {
                node.parent.left = null;
            }
        } else if( node.right != null && node.left == null ){
            if( isroot ){
                this.root = node.right;
            } else if( rightchild ){
                node.parent.right = node.right;
            } else {
                node.parent.left = node.right;
            }
        } else if( node.left != null && node.right == null ){
            if( isroot ){
                this.root = node.left;
            } else if( rightchild ){
                node.parent.right = node.left;
            } else {
                node.parent.left = node.left;
            }
        } else {
            TreeNode successor = this.succ(node);
            this.delete(successor.key);
            this.size++;
            successor.left = node.left;
            successor.right = node.right;
            if( isroot ){
                this.root = successor;
            } else if( rightchild ){
                node.parent.right = successor;
            } else {
                node.parent.left = successor;
            }
        }
        this.size--;
        return true;
    } catch( NoSuchElementException e ){
        return false;
    }
}

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// GET, MIN, MAX, SUCC, PRED METHODS
public TreeNode getNode(T key) {
    /* Return the node with the given key */
    // O{log(n)}
    TreeNode node = this.root;
    while( node != null ){
        if( key.compareTo(node.key) > 0 ){
            node = node.right;
        } else if( key.compareTo(node.key) < 0 ) {
            node = node.left;
        } else {
            return node;
        }
    }
    throw new NoSuchElementException();
}
public D getData(T key){
    /* Return the data with the given key */
    // O{log(n)}
    return this.getNode(key).data;
}
public TreeNode minNode(TreeNode node) {
    /* Return the node with the smallest key in the subtree of the given node. */
    // O{log(n)}
    while( node.left != null ){
        node = node.left;
    }
    return node;
}
public TreeNode minNode() {
    /* Return the node with the smallest key. */
    // O{log(n)}
    return this.minNode(this.root);
}
public TreeNode maxNode(TreeNode node) {
    /* Return the node with the largest key in the subtree of the given node. */
    // O{log(n)}
    while( node.right != null ){
        node = node.right;
    }
    return node;
}
public TreeNode maxNode() {
    /* Return the node with the largest key. */
    // O{log(n)}
    return this.maxNode(this.root);
}
public TreeNode pred(TreeNode node) {
    /* Return the node with the next smaller key of the given node. */
    // O{log(n)}
    if( node.left != null ){
        return maxNode(node.left);
    } else {
        TreeNode elder = node.parent;
        while( elder != null && node.key.compareTo(elder.key) < 0 ){
            elder = elder.parent;
        }
        return elder;
    }
}
public TreeNode succ(TreeNode node) {
    /* Return the node with the next bigger key of the given node. */
    // O{log(n)}
    if( node.right != null ){
        return minNode(node.right);
    } else {

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        TreeNode elder = node.parent;
        while( elder != null && node.key.compareTo(elder.key) > 0 ){
            elder = elder.parent;
        }
        return elder;
    }
}

// DEPTH METHOD
public int deep() {
    /* Return depth of tree. */
    // O{n*log(n)}
    Stack<TreeNode> stack = new Stack<TreeNode>();
    stack.push(this.root);
    int max = 0;
    while( !stack.isEmpty() ){
        TreeNode node = stack.pop();
        if( node.left != null ) {
            stack.push(node.left);
            max = StrictMath.max(max, depth(node.left.key));
        }
        if( node.right != null ){
            stack.push(node.right);
            max = StrictMath.max(max, depth(node.right.key));
        }
    }
    return max;
}

public int depth(T key){
    /* Return depth of tree from given node to the root. */
    // O{log(n)}
    int depth = 0;
    TreeNode node = this.getNode(key);
    while( node.parent != null ){
        node = node.parent;
        depth++;
    }
    return depth;
}

// BALANCED METHOD
public boolean perfectBalanced(TreeNode node) {
    /* Return true, if subtree of given node is perfectly balanced. */
    // O{log(n)}
    int leftSize = getSize(node.left.key);
    int rightSize = getSize(node.right.key);
    if( leftSize > rightSize ){
        return leftSize == rightSize+1;
    } else if( leftSize < rightSize ){
        return leftSize+1 == rightSize;
    } else {
        return true;
    }
}

public boolean perfectBalanced(T key) {
    TreeNode node = this.getNode(key);
    return perfectBalanced(node);
}

public boolean perfectBalanced() {
    return perfectBalanced(this.root);
}

public int getSize(T key) {
    /* Return number of nodes in subtree of given node */
    // O{log(n)}
    Stack<TreeNode> stack = new Stack<TreeNode>();
    stack.push(this.getNode(key));

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    int size = 0;
    while( !stack.isEmpty() ){
        TreeNode node = stack.pop();
        size++;
        if( node.left != null ){
            stack.push(node.left);
        }
        if( node.right != null ){
            stack.push(node.right);
        }
    }
    return size;
}
public int getSize() {
    return this.size;
}

// STRING, ARRAY METHODS
public D[] toArray() {
    /* Return array with the data from all elements of the tree, sorted by the keys. */
    // O{n*log(n)}
    D[] array = (D[]) new Object[this.size];
    int i = 0;
    for( T key : this ){
        array[i] = this.getNode(key).data;
        i++;
    }
    return array;
}
public String toStringOrdered() {
    // O{n*log(n)}
    String out = "";
    for( T key : this ){
        out += key + " ";
    }
    return out;
}
public String toString() {
    // O{n*log(n)}
    String out = "";
    int depth = this.deep();
    for( int row=0; row<=depth; row++ ){
        String space = getSpaces( 2 * ((int)Math.pow(2,depth-row)-1) );
        out += space;
        for( int i=(int)Math.pow(2,row); i<(int)Math.pow(2,row+1); i++ ){
            try {
                out += this.getNode(i).key + space + " ";
            } catch( NoSuchElementException e ){
                out += " ." + space + space + " ";
            }
        }
        out += "\n";
    }
    return out;
}
public static String getSpaces(int n){
    String out = "";
    for( int i=0; i<n; i++ ){
        out += " ";
    }
    return out;
}
public static char[] getNodePath(int n) {
    String binString = Integer.toBinaryString(n);
    char[] binChars = binString.substring(1, binString.length()).toCharArray();
    return binChars;
}

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}
public TreeNode getNode(int n) {
    /** Returns node at given position. */
    // O{log(n)}
    TreeNode node = this.root;
    for( char c : getNodePath(n) ){
        if( c == '0' && node.left != null ){
            node = node.left;
        } else if( c == '1' && node.right != null ){
            node = node.right;
        } else {
            throw new NoSuchElementException();
        }
    }
    return node;
}

// ITERATOR METHOD
public Iterator<T> iterator() {
    return iteratorIO();
}

// IN ORDER ITERABLE METHODS
public Iterator<T> iteratorIO() {
    return new InOrderIterator();
}
private class InOrderIterator implements Iterator<T> {
    // ATTRIBUTES
    private Stack<TreeNode> stack = new Stack<TreeNode>();

    // CONSTRUCTOR
    InOrderIterator() {
        pushLeftTree(root);
    }

    // METHODS
    public boolean hasNext() {
        // O{1}
        return !stack.isEmpty();
    }
    public T next() {
        // O{log(n)}
        if( !hasNext() ){
            throw new NoSuchElementException();
        }
        TreeNode node = stack.pop();
        pushLeftTree(node.right);
        return node.key;
    }
    private void pushLeftTree(TreeNode node) {
        // O{log(n)}
        while (node != null) {
            stack.push(node);
            node = node.left;
        }
    }
    public void remove() {
        throw new UnsupportedOperationException();
    }
}

// POST ORDER ITERABLE METHODS
public Iterator<T> iteratorPO() {
    return new PostOrderIterator();
}
private class PostOrderIterator implements Iterator<T> {

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// ATTRIBUTES
Stack<TreeNode> stack = new Stack<TreeNode>();
Stack<Boolean> rightChild = new Stack<Boolean>();

// CONSTRUCTOR
PostOrderIterator() {
    pushLeftTree(root);
}

// METHODS
public boolean hasNext() {
    // O{1}
    return !stack.isEmpty();
}
public T next() {
    // O{log(n)}
    if( stack.peek().right == null || rightChild.peek() ){
        rightChild.pop();
        return stack.pop().key;
    } else {
        rightChild.pop();
        rightChild.push(true);
        pushLeftTree(stack.peek().right);
        return next();
    }
}
public void remove() {
    throw new UnsupportedOperationException();
}
private void pushLeftTree(TreeNode node) {
    // O{log(n)}
    if (node != null) {
        stack.push(node);
        rightChild.push(false);
        pushLeftTree(node.left);
    }
}
}
}

```

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public class TestBinLinkedTree {
    public static class Num implements Comparable<Num> {
        int num;
        public Num(int num) {
            if( num < 100 ){
                this.num = num;
            } else {
                throw new RuntimeException("Num just excepts Elements < 100");
            }
        }
        public int compareTo(Num other) {
            if( this.num < other.num ){
                return -1;
            } else if( this.num > other.num ){
                return 1;
            } else {
                return 0;
            }
        }
    }
    public String toString() {
        if( this.num < 10 ){
            return "0" + this.num;
        } else {
            return "" + this.num;
        }
    }
}

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    }
}
}

public static void print(Object o) {
    System.out.println(o);
}

public static BinLinkedTree<Num,String> newTree(int[] ints) {
    BinLinkedTree<Num,String> tree = new BinLinkedTree<Num,String>();
    for( int i : ints ) {
        tree.insert(new Num(i), "a");
    }
    return tree;
}

public static void main(String[] args) {

    // GET, MIN, MAX, SUCC, PRED
    int[] ints1 = {6,4,10,2,5,8,9,7,1,3,12,11};
    BinLinkedTree<Num,String> tree1 = newTree(ints1);
    print(tree1);
    print("depth: " + tree1.deep());
    print("node 06: " + tree1.getNode(new Num(6)));
    print("node 10: " + tree1.getNode(new Num(10)));
    print("");
    print("min: " + tree1.minNode());
    print("min 10: " + tree1.minNode(tree1.getNode(new Num(10))));
    print("max: " + tree1.maxNode());
    print("max 04: " + tree1.maxNode(tree1.getNode(new Num(4))));
    print("");
    print("pred 06: " + tree1.pred(tree1.getNode(new Num(6))));
    print("succ 06: " + tree1.succ(tree1.getNode(new Num(6))));
    print("pred 08: " + tree1.pred(tree1.getNode(new Num(8))));
    print("succ 09: " + tree1.succ(tree1.getNode(new Num(9))));
    print("pred 07: " + tree1.pred(tree1.getNode(new Num(7))));
    print("succ 05: " + tree1.succ(tree1.getNode(new Num(5))));
    print("pred 01: " + tree1.pred (tree1.getNode(new Num(01))));
    print("succ 12: " + tree1.succ(tree1.getNode(new Num(12))));

    // DELETING
    int[] ints2 = {53,27,69,13,34,63,95,8,17,30,46,66,5,9,15,18,32,
        50,29,68,71,64,98,99,97};
    BinLinkedTree<Num,String> tree2 = newTree(ints2);
    print(tree2);
    print("delete 15 =>");
    tree2.delete(new Num(15));
    print(tree2);
    print("delete 46 =>");
    tree2.delete(new Num(46));
    print(tree2);
    print("delete 27 =>");
    tree2.delete(new Num(27));
    print(tree2);
    print("delete 53 =>");
    tree2.delete(new Num(53));
    print(tree2);
    print("");

    // KOMMUTATIVE DELETING
    print("Ist komutativ, da der Knoten durch seinen Nachfolger \nersetzt wird
        und die Suche nach Nachfolgern Kommutativ \nist.\n");
    BinLinkedTree<Num,String> tree2a = newTree(ints2);
    BinLinkedTree<Num,String> tree2b = newTree(ints2);
    print(tree2a);
    print("delete 53 & 63 =>");
    tree2a.delete(new Num(53));

```



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tree2a.delete(new Num(63));
print(tree2a);
print("delete 63 & 53 =>");
tree2b.delete(new Num(63));
tree2b.delete(new Num(53));
print(tree2b);
print("");

// BALANCED DELETING
print("Tree size:");
print(tree2b.getSize());
print(tree2b.getSize(new Num(64)));
print("13 size:");
print(tree2b.getSize(new Num(13)));
print("");
print("Tree perfectly balanced: " + tree2b.perfectBalanced());
print("perfectly balanced at 13: " + tree2b.perfectBalanced(new Num(13)));
print("perfectly balanced at 27: " + tree2b.perfectBalanced(new Num(27)));
print("perfectly balanced at 95: " + tree2b.perfectBalanced(new Num(95)));
print("");

// ARRAY AND POST ORDER ITERATION
print(tree1);
print("Element in order:");
String elementsI0 = "";
for( Iterator<Num> iter = tree1.iteratorI0(); iter.hasNext(); ){
    elementsI0 += iter.next() + " ";
}
print(elementsI0);
print("");
// print("Data in order:");
// String dataString = "";
// for( String data : tree1.toArray() ){
//     dataString += data + " ";
// }
// print(dataString);
print("Java can not cast `(String[]) Object[n]`, so no tests for `toArray` here.");
print("");
print("Element post order:");
String elementsP0 = "";
for( Iterator<Num> iter = tree1.iteratorP0(); iter.hasNext(); ){
    elementsP0 += iter.next() + " ";
}
print(elementsP0);
}
}

```