Quiz

Supervised Learning-Logistic Regression

$$\hat{y} = w \cdot x + b$$

where:

- $\hat{y}$ is the predicted value.
- w is the weight (slope of the line).
- b is the bias (y-intercept).

Our goal is to minimize the **Mean Squared Error (MSE)** between the predicted values and the actual values y. The loss function for MSE is:

$$L(w,b) = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2 = \frac{1}{n}\sum_{i=1}^{n}(w \cdot x_i + b - y_i)^2$$

where n is the number of data points.

Step-by-Step Solution Using Gradient Descent

Let's assume:

- α is the learning rate (how big each step is).
- The parameters w and b are initialized to 0.

We'll go through several iterations of gradient descent to find optimal values for w and b.

Example Data

Suppose we have a small dataset:

x	y
1	2
2	2.8
3	3.6
4	4.5

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Step 1: Initialize Parameters

Set:

- Initial $w = 0$
- Initial $b = 0$
- Learning rate $\alpha = 0.01$

Step 2: Compute the Gradient of the Loss Function

For each iteration, we need to compute the gradients with respect to w and b:

1. **Gradient with respect to w:**

$$\frac{\partial L}{\partial w} = \frac{2}{n} \sum_{i=1}^{n} (w \cdot x_i + b - y_i) \cdot x_i$$

2. **Gradient with respect to b:**

$$\frac{\partial L}{\partial b} = \frac{2}{n} \sum_{i=1}^{n} (w \cdot x_i + b - y_i)$$

Step 3: Update Parameters

Update w and b using the gradients:

$$w = w - \alpha \cdot \frac{\partial L}{\partial w}$$

$$b = b - \alpha \cdot \frac{\partial L}{\partial b}$$

Iterative Computation Example

First Iteration

Let's go through the first iteration with the initial values $w = 0$ and $b = 0$.

1. **Calculate Predictions:**
   - For $x = 1$: $\hat{y} = 0 \cdot 1 + 0 = 0$
   - For $x = 2$: $\hat{y} = 0 \cdot 2 + 0 = 0$
   - For $x = 3$: $\hat{y} = 0 \cdot 3 + 0 = 0$
   - For $x = 4$: $\hat{y} = 0 \cdot 4 + 0 = 0$

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2. **Calculate Gradients:**

- Gradient with respect to w:

$$\frac{\partial L}{\partial w} = \frac{2}{n}\sum_{i=1}^{n}(w \cdot x_i + b - y_i) \cdot x_i$$

$$\frac{\partial L}{\partial w} = \frac{2}{4}\left[(0-2)\cdot 1 + (0-2.8)\cdot 2 + (0-3.6)\cdot 3 + (0-4.5)\cdot 4\right]$$

$$= \frac{2}{4}\left[-2\cdot 1 - 2.8\cdot 2 - 3.6\cdot 3 - 4.5\cdot 4\right]$$

$$= \frac{2}{4}(-2 - 5.6 - 10.8 - 18)$$

$$= \frac{2}{4}\cdot(-36.4) = -18.2$$

Example Data

Suppose we have a small dataset:

x	y
1	2
2	2.8
3	3.6
4	4.5

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- Gradient with respect to b:

$$\frac{\partial L}{\partial b} = \frac{2}{4}\left[(0-2)+(0-2.8)+(0-3.6)+(0-4.5)\right]$$

$$= \frac{2}{4}(-2-2.8-3.6-4.5)$$

$$= \frac{2}{4}\cdot(-12.9) = -6.45$$

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3. **Update Parameters:**

   Using $\alpha = 0.01$:

$$w = w - \alpha \cdot \frac{\partial L}{\partial w} = 0 - 0.01 \cdot (-18.2) = 0.182$$

$$b = b - \alpha \cdot \frac{\partial L}{\partial b} = 0 - 0.01 \cdot (-6.45) = 0.0645$$

Second Iteration

Repeat the process with updated values $w = 0.182$ and $b = 0.0645$.