AIML 425 | Assignment 4
Variational Autoencoders

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1 Introduction

In this study, we examine variational autoencoders, question 2 of the given assignment. The additional term, "variational", represents the relationship between the regularization and variational inference in statistics. Its aptitude for generation of basic shapes (rectangles, circles or triangles), and the information rate of the latent layer are subjects of interest in this paper. Additionally, we compare this standard variational autoencoder, to one that has its latent layer controlled to be Gaussian instead of a sampling layer, using the Maximum Mean Discrepancy(MMD) objective function.

2 Theory

2.1 Variational Autoencoders

Autoencoders are neural networks that attempt to reconstruct the input data given, a type of feature extraction algorithm. This is first done by first attempting to encode the data into a latent vector of lower dimensions then the given input data, via convolutional layers. A visual example would be attempting to decompose a 3 dimensional spiral into 1 dimension, θ, the phase of the spiral. The latent vector is then decoded to form the original input as closely as possible, the difference being quantified as the **reconstruction loss**. It is important to note that the autoencoder is only trained to encode and decode with as low a reconstruction loss as possible, regardless of how the latent space is organised.

A **variational** autoencoder(VAE) has its encodings' distributions regularized to ensure that its latent vector space is continuous and complete, enabling generative process. Instead of encoding each data point as a single point in latent space, it is encoded as a distribution. In the decoder, a point is sampled from that distribution, to generate the new datapoint or set, by introducing a second loss or regularization term, that tends to regularise the organisation of the latent space - the Kulback-Leibler divergence. The Kullback-Leibler divergence between two Gaussian distributions has a closed form that can be directly expressed in terms of the means and the covariance matrices of the the two distributions.

2.2 Probabilistic Interpretation

Let x denote our data, and that is in reality perfectly represented by a lower dimensional variable z, not directly observed. This means that: a latent distribution z is sampled from the prior distribution p(z), and that x is sampled from the conditional likelihood distribution p(x|z).
This way, the encoder is an analog of p(z|x), describing the distribution of the latent variable given the unencoded datapoint x, and the decoder is an analog of p(x|z), describing the distribution of the decoded variable given the encoded input.
Let p(z) be of a standard Gaussian distribution, and p(x|z) to be

$$z \sim N(0, \mathbf{I})$$

$$x|z \sim N(f(z), c\mathbf{I})$$

where f is unspecified for now.
The best Gaussian distribution, $q_x(z) \cong N(g(x), h(x))$, that describes the p(z|x) is the one which minimizes our given error measure- the Kulback-Leibler divergence-found via gradient descent over the parameters that describe the family of distribution(Rocca J, 2019). Hence, we need to find the optimal functions (g^*, h^*)=

$$\underset{(g,h)\in GxH}{\arg\min}\ KL(q_x(z), p(z|x))$$

$$= \underset{(g,h)\in GxH}{\arg\min}\,(\mathbb{E}_{z\sim q_x}(ln q_x(z)) - \mathbb{E}_{z\sim q_x}(ln\frac{p(x|z)p(z)}{p(x)}))$$

$$= \underset{(g,h)\in GxH}{\arg\max}\,(\mathbb{E}_{z\sim q_x}(ln p(x|z)) - KL(q_x(z), p(z)))$$

$$= \underset{(g,h)\in GxH}{\arg\max}\,(\mathbb{E}_{z\sim q_x}(-\frac{||x-f(z)||^2}{2c}) - KL(q_x(z), p(z)))$$

Given the **best** approximation of p(z|x), denoted by $q^*_x(z)$, we need to choose an f^* that maximizes the likelihood of x given z sampled from $q^*_x(z)$

$$f^* = \underset{f\in F}{\arg\max}\,\mathbb{E}_{z\sim q^*_x}(ln p(x|z))$$

$$= \underset{f\in F}{\arg\max}\,\mathbb{E}_{z\sim q^*_x}(-\frac{||x-f(z)||^2}{2c})$$

3 Results & Conclusion

Both models were trained on 10000 image arrays, for 20 epochs, with a 2D latent space. Adjusted model was trained with Gaussian noise of variance 5 for best results.

3.1 Image Generation

3.1.1 VAE Model

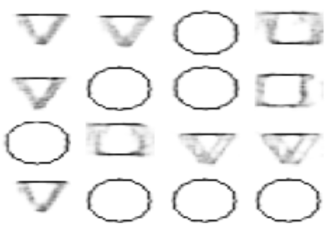


Figure 1: 16 test dataset Images generated with VAE (20 epochs)

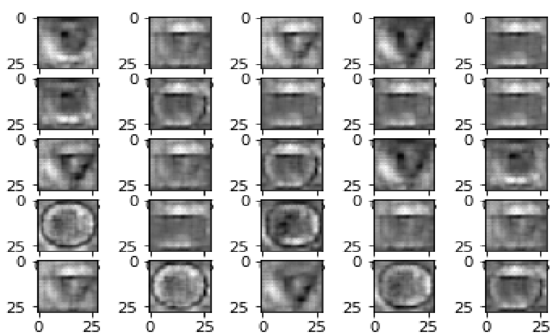


Figure 2: 25 Randomly generated images with VAE

3.1.2 Similar Model with controlled latent space

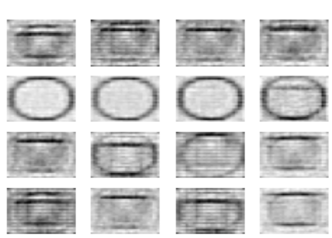


Figure 3: 16 test dataset Images generated with adjusted model (20 epochs)

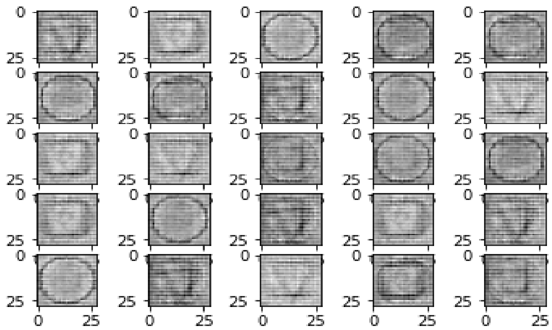
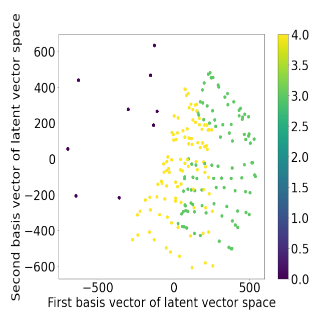


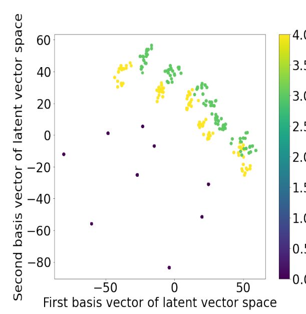
Figure 4: 25 Randomly generated images(adjusted model) $\sigma^2 = 5$

3.2 Performance Comparison

Performance Metrics				
Model	Info loss (MSE Metric)	Test Image Generation Quality	Random Generation Quality	Info Rate $I(\text{bits}) = dlog_2\frac{1}{\sigma_\epsilon^2}$
VAE Model	12.022	93.75%	72.00%	3980
Adjusted VAE Model	0.048484	68.75%	80.00%	4.640 (σ_ϵ^2=5)



(a) Latent Space plot for standard VAE model



(b) Latent Space plot for modified VAE model, variance 0.01

Quality is determined by number of distinguishable shapes in generation set. Purple dots represent circles, green ones are triangles and yellow ones are rectangles.

3.3 Conclusion

The information rate is a measure of the shared information across the bottleneck, indirectly indicating the proportion of information that is shared between the input and reconstructed output.

The 2D cluster plots indicate that both models generally separate shapes well enough in the latent space. The modified model can be adjusted have a centered and smaller latent space as shown in the cluster plot, at the cost of reconstruction.

MMD model appears to have smoother transitions between 2 shapes.

4 Code

Workspace on Colab `https://colab.research.google.com/drive/1iZ1BNwyySu08mg8Xjiaj0FLv2m5qmTdY?usp=sharing`. Additional higher epoch renderings are attached in the Appendix.

5 References

Diederik P. Kingma and Max Welling (2019), "An Introduction toVariational Autoencoders", Foundations and TrendsR©in Machine Learning: Vol. xx, No.xx, pp 1–18. DOI: 10.1561/XXXXXXXXX.

Keydana, S. (2021). RStudio AI Blog: Representation learning with MMD-VAE. RStudio AI Blog. Retrieved 8 September 2021, from `https://blogs.rstudio.com/ai/posts/2018-10-22-mmd-vae/`.

Lamberta, B., Daoust, M., Katariya, Y., Chen, W., Babu, S., Gardener, T. (2021). Convolutional Variational Autoencoder [Ebook]. Tensorflow. Retrieved 8 September 2021, from `https://colab.research.google.com/github/tensorflow/docs/blob/master/site/en/tutorials/generative/cvae.ipynb`

Rocca, J. (2019). Understanding Variational Autoencoders (VAEs). Medium. Retrieved 8 September 2021, from `https://towardsdatascience.com/understanding-variational-autoencoders-vaes-f70510919f73` .

6 Appendix

6.1 Random Image Generation for standard VAE model trained to 10 epochs

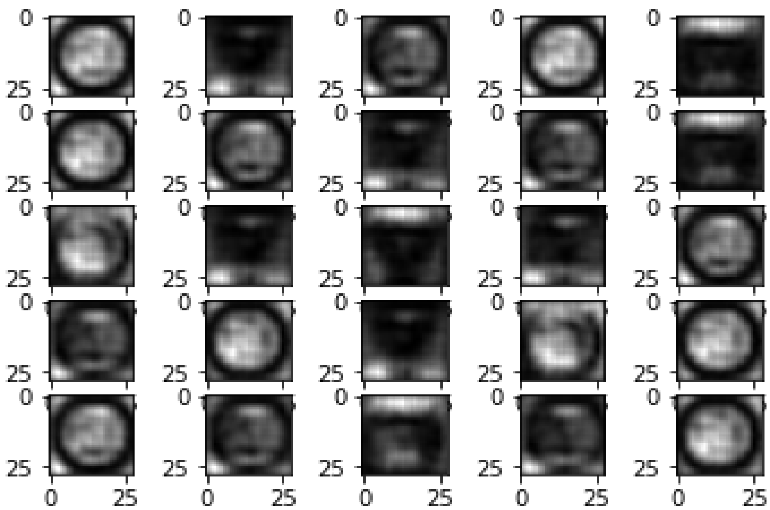


Figure 5: 25 Randomly generated images with VAE at 10 epochs only

6.2 Random Image Generation for modified model with Gaussian noise variance of 15

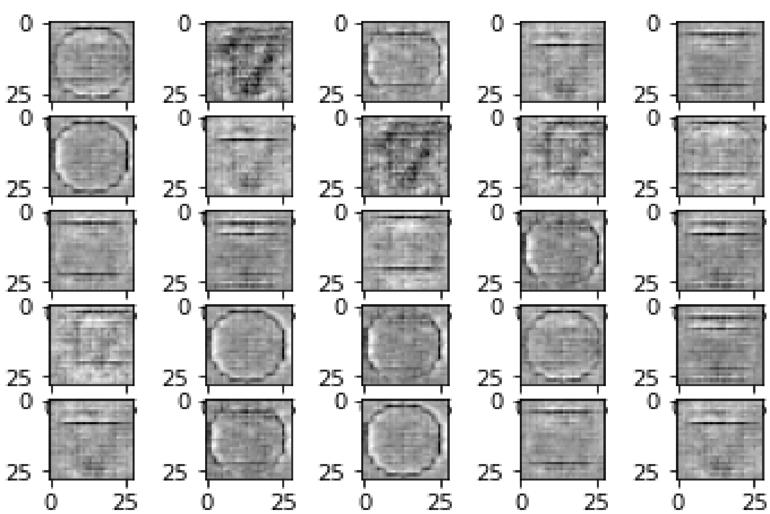


Figure 6: 25 Randomly generated images with VAE_ MMD

6.3 Test Image Generation for boths models

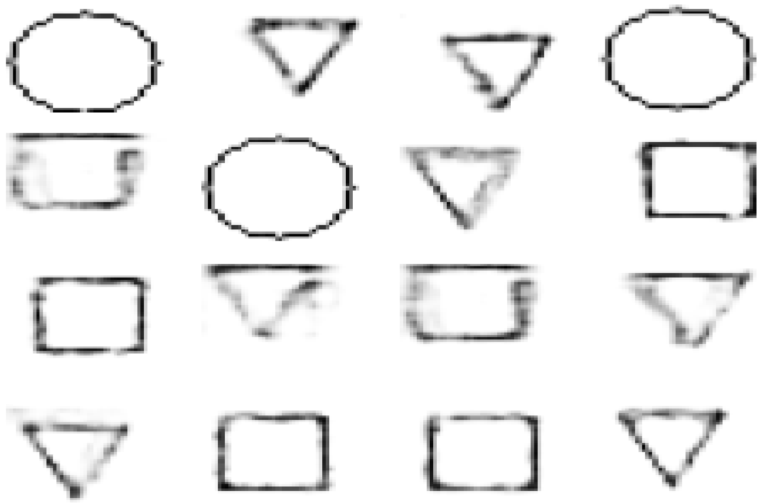


Figure 7: Test image generation at 50 epochs for standard VAE model

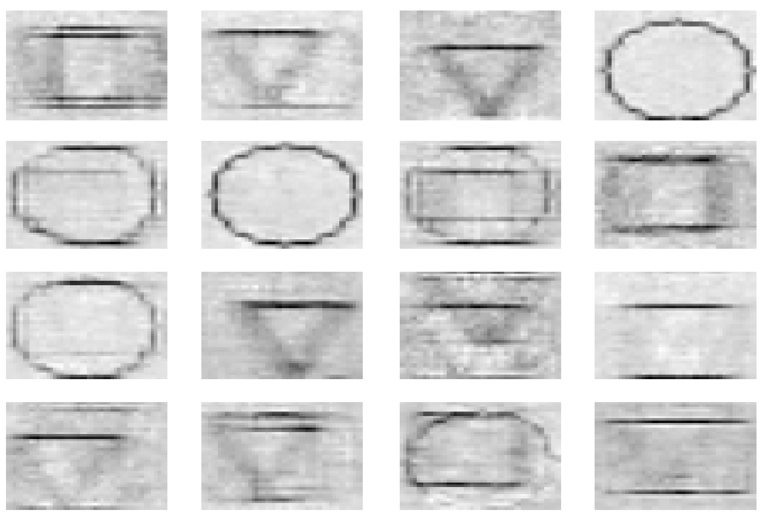


Figure 8: Test image generation at 50 epochs for modified model, with variance 2 for noise

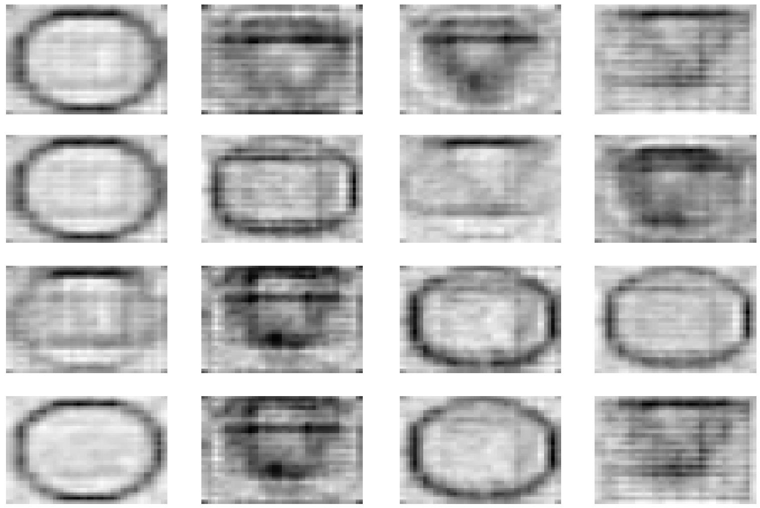


Figure 9: Different set of test image generation at 20 epochs for modified model, with variance 2 for noise