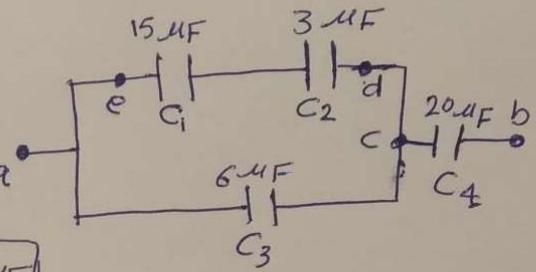
$\Delta V_{ab} = \Delta V_{ac} + \Delta V_{cb}$ $\quad Q_1 = Q_2$

ΔV_{ab} given $= 15$ V

Problem 23/ch.26

(a) $C_{ab} = ?$, first we find C_{ed}



$\Rightarrow \frac{1}{C_{ed}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{15} + \frac{1}{3} \Rightarrow \boxed{C_{ed} = 2.5\,\mu F}$

Now $C_{ac} = C_{ed} + C_3 = 2.5 + 6 \Rightarrow \boxed{C_{ac} = 8.5\,\mu F}$

$\Rightarrow \frac{1}{C_{ab}} = \frac{1}{C_{ac}} + \frac{1}{C_4} = \frac{1}{8.5} + \frac{1}{20} \Rightarrow \boxed{C_{eq} = C_{ab} = 5.96\,\mu F}$ ✓

(b) $Q_{tot} = Q_{ab} = ?$, Note that $\Delta V_{ab} = 15\,V \Rightarrow Q_{ab} = C_{ab}\Delta V_{ab} = (5.96\,\mu F)(5\,V) \Rightarrow \boxed{Q_{ab} = 89.4\,\mu C}$

We need to find $Q_1 = ?$, $Q_2 = ?$, $Q_3 = ?$, $Q_4 = ?$ (the charge on each capacitor).

- $Q_4 = Q_{ab} \Rightarrow \boxed{Q_4 = 89.4\,\mu C}$ / Now to find $Q_1, Q_2, Q_3 \Rightarrow$ we find $\Delta V_{ac}, \Delta V_{cb}$

$\Rightarrow \Delta V_{ac} = ?$: $\Delta V_{ab} = \Delta V_{ac} + \Delta V_{cb}$, but $\Delta V_{cb} = \frac{Q_4}{C_4} = \frac{89.4\,\mu C}{20\,\mu F} \Rightarrow \boxed{\Delta V_{cb} = 4.47\,V}$

$\Rightarrow$ Also $\Delta V_{ab} = 15\,V$ (given!) $\Rightarrow 15 = \Delta V_{ac} + 4.47 \Rightarrow \boxed{\Delta V_{ac} = 10.53\,V}$

Note that $\Delta V_3 = \Delta V_{ac} \Rightarrow Q_3 = C_3\,\Delta V_{ac} = (6\,\mu F)(10.53) \Rightarrow \boxed{Q_3 = 63.2\,\mu C}$

Note that $Q_1 = Q_2$ (series!) $= Q_4 - Q_3 \Rightarrow Q_1 = Q_2 = 89.4 - 63.2 = \boxed{26.3\,\mu C}$ $= Q_1 = Q_2$

OR to find $Q_1, Q_2 \Rightarrow Q_1 = Q_2 = C_{ed}\,\Delta V_{ed}$ [But $\Delta V_{ed} = \Delta V_{ac}$!!]

$= (2.5\,\mu F)\cdot(10.53\,V) = \boxed{26.3\,\mu C = Q_1 = Q_2}$

same

$C = \frac{Q}{\Delta V} \Rightarrow Q = C\Delta V$

Four capacitors are connected as shown in Figure P26.23. (a) Find the equivalent capacitance between points *a* and *b*. (b) Calculate the charge on each capacitor, taking $\Delta V_{ab} = 15.0$ V.

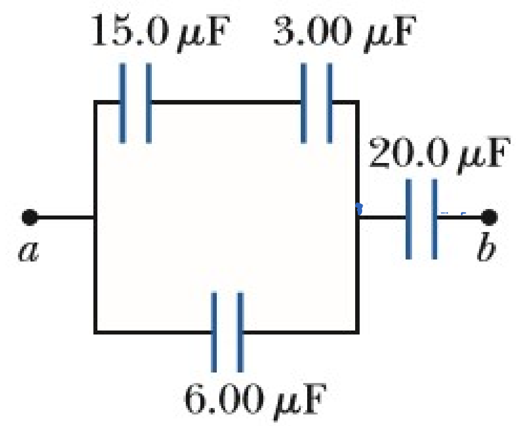


Figure P26.23

26.4 Energy Stored in a Charged Capacitor

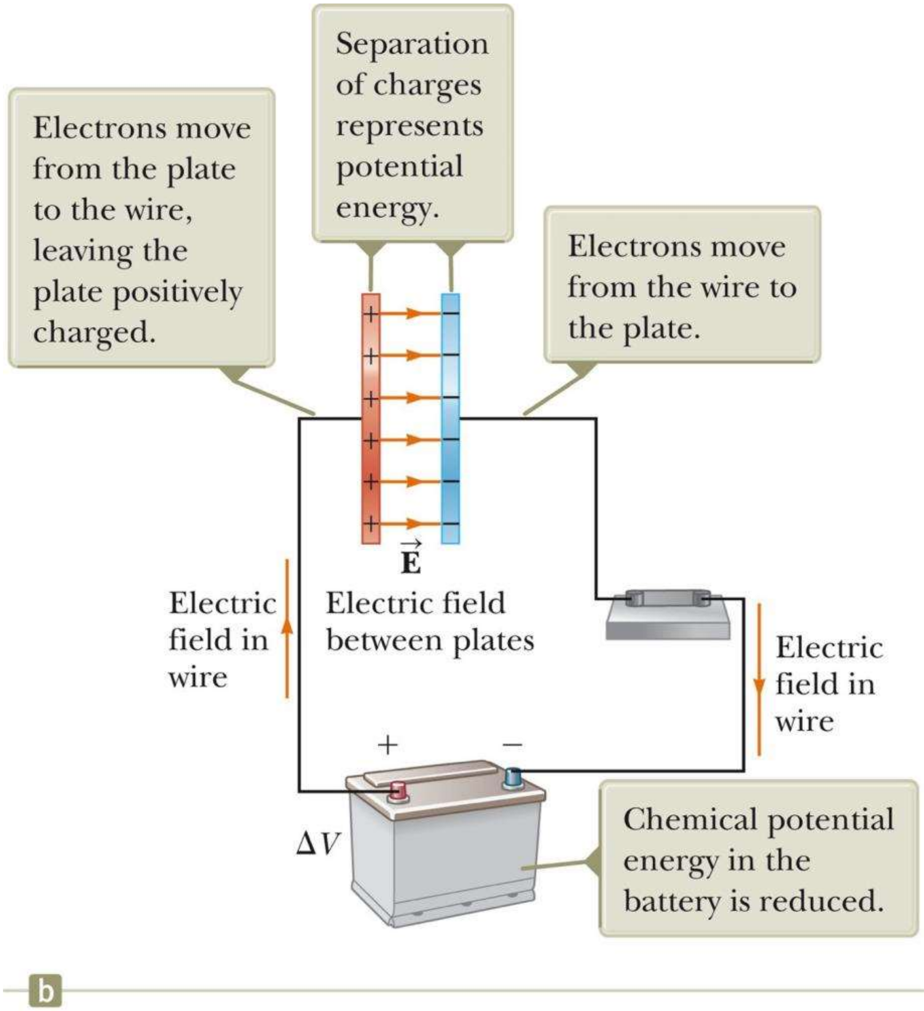
Consider the circuit to be a system.

Before the switch is closed, the energy is stored as chemical energy in the battery.

When the switch is closed, the energy is transformed from chemical potential energy to electric potential energy.

The electric potential energy is related to the separation of the positive and negative charges on the plates.

A capacitor can be described as a device that stores energy as well as charge.



Energy Stored in a Capacitor

Assume the capacitor is being charged and, at some point, has a charge q on it.

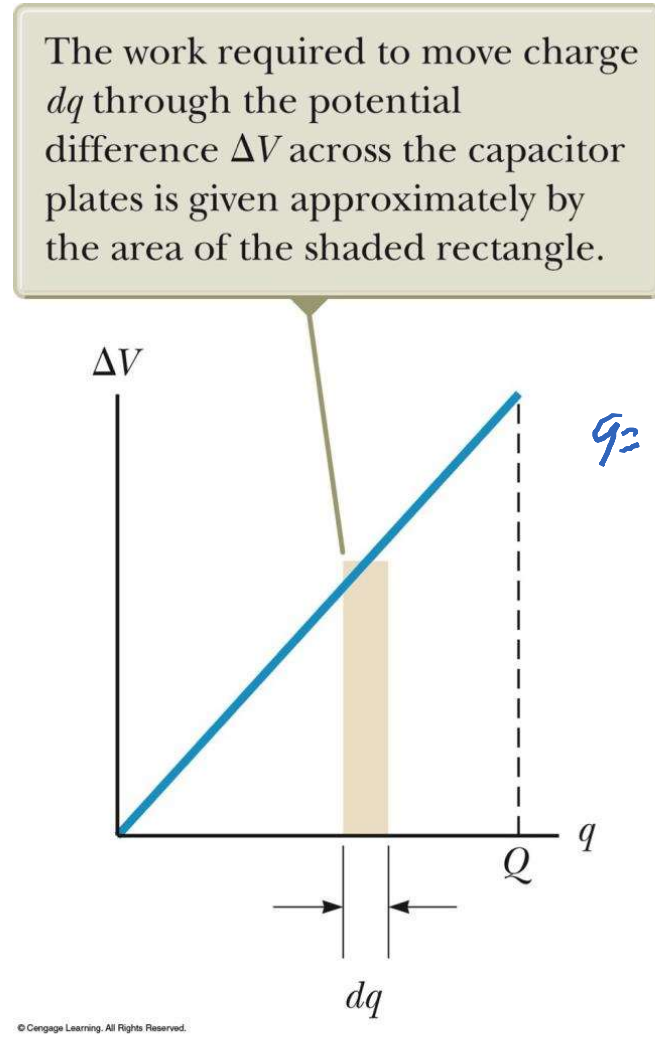
The work needed to transfer a charge from one plate to the other is

$$\int dW = \Delta V dq = \int \overbrace{\frac{q}{C}}^{=\Delta V} dq$$

The work required is the area of the tan rectangle.

The total work required is

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$



$W = q\Delta V$

$dW = dq\,\Delta V$

$q = 0 \rightarrow q = Q$

Energy, cont

$C = \frac{Q}{\Delta V} \Rightarrow U = \frac{Q^2}{2Q/\Delta V} = \frac{1}{2} Q \Delta V$ ✓ $= \frac{1}{2}(C\Delta V)\Delta V = \frac{1}{2} C (\Delta V)^2$

The work done in charging the capacitor appears as electric potential energy U:

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$$

$C = \frac{\varepsilon_0 A}{d}$, $\Delta V = Ed$

$U = \frac{1}{2} C \Delta V^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(E^2 d^2)$

$= \frac{1}{2}\varepsilon_0 E^2 (\underbrace{Ad}_{\text{volume}}) \Rightarrow \frac{U}{\text{volume}} = \boxed{u_E = \frac{1}{2}\varepsilon_0 E^2}$

This applies to a capacitor of any geometry.

The energy stored increases as the charge increases and as the potential difference increases.

Q $\quad$ ΔV

For a parallel-plate capacitor, the energy can be expressed in terms of the field as
$U = ½ (\varepsilon_o Ad)E^2$. ***Note that:*** $\Delta V = Ed$ and $C = \varepsilon_o A/d$

It can also be expressed in terms of the energy density (energy per unit volume)
$u_E = ½ \varepsilon_o E^2$. ***Note that:*** *volume = Ad*

Question: *Can you find u_E for cylindrical and spherical capacitors?*