Chapter 4 (Discrete)

Distributions for Discrete random Variables

- Bernoulli
- Binomial
- Geometric
- Poisson

Bernoulli Distribution

* Any experiment of only 2 outcomes $|S|=2$ is called a Bernoulli experiment.

__Ex__ Toss a coin once $S=\{H,T\}$

* From an experiment, we can form a new Bernoulli experiment to do so

* Focus on some event A (we call it the event of interest) or the event of success

Let $P(A) = p$ Notice that $\bar{A} \equiv$ not A

$$p(\bar{A}) = 1 - p = q$$

* The new experiment is the old experiment focusing on the occurance of A. The new experiment has

two outcomes $\{A, \bar{A}\}$

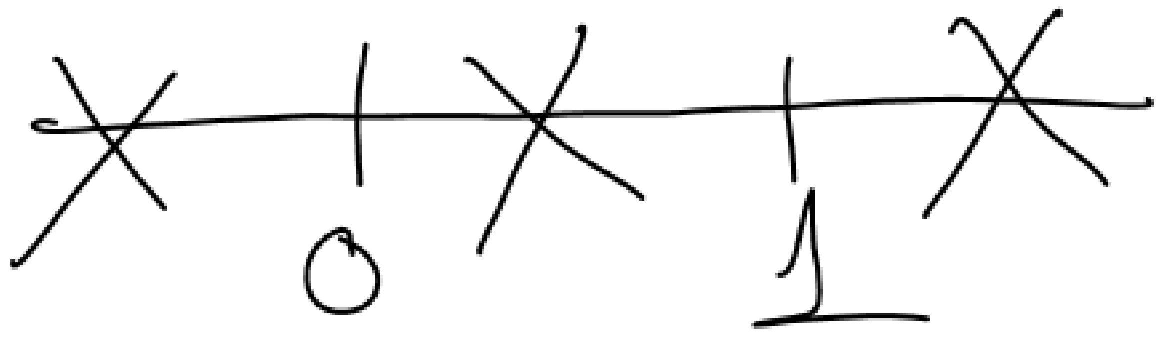
$X(A) = 1$

$X(\bar{A}) = 0$

X	0	1
$p(X=x)$	$q = 1-p$	p

OR

$$P(X=x) = (1-p)^{1-x} p^{x} \quad x = 0, 1$$



prob.

$$F_X(x) = P(X \le x) = \begin{cases} 0 & X < 0 \\ 1-p & 0 \le X < 1 \\ 1 & X \ge 1 \end{cases}$$

intervals

$\underline{\underline{X<0}}$: $P(\underline{\underline{X<0}}) = 0$

$0 \le X < 1$: $P(X \le 0) = P(X=0) = 1-p$

$X \ge 1$: $P(X \le 1) = P(X=0) + P(X=1) = 1-p+p = 1$

$$E(X) = \sum_{\substack{\forall x \\ =0,1}} x\, P(X=x) = 0\, P(X=0) + 1\, P(X=1)$$

$$= p$$

$$Var(X) = \sum_{\forall x=0,1} (x - E(X))^2 P(X=x)$$

$$= (0-p)^2 P(X=0) + (1-p)^2 P(X=1)$$

$$= p^2(1-p) + (1-p)^2 p = p(1-p)\left[\cancel{p} + 1 - \cancel{p}\right] = pq$$

Ex Toss a dice, Focus on the appearance of the outcome 3

Sol $A = \{3\}$ $\qquad \bar{A} = \{1, 2, 4, 5, 6\}$

Bernoulli experiment

Bernoulli distribution

X	0	1
$P(X=x)$	$\frac{5}{6}$	$\frac{1}{6}$

① Find the CDF of X?

$$P(X \le x) = F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{5}{6} & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

② What is the Expected value of X?

Sol $E(X) = p = \frac{1}{6}$

(3) Find the variance of X

Sol $Var(X) = pq = \frac{1}{6}\frac{5}{6} = \frac{5}{36}$

(4) Find $F_X(\frac{1}{2}) = 5/6$

OR $F_X(\frac{1}{2}) = P(X \leq \frac{1}{2}) = P(X = 0) = \frac{5}{6}$

Binomial Distribution

* Consider an initial experiment.
* Focus on an event (event of interest) whose $P(A) = p$
* Repeat this initial experiment (in Bernoulli form) n-times independently

X: Count the number of times the event A occured among n times

$$X = \{0, 1, 2, \ldots, n\}$$

$${}_nC_x = \frac{n!}{(n-x)!\,x!}$$

$$P(X=x) = \binom{n}{x} p^{x} (1-p)^{n-x} \qquad X = 0, 1, 2, \ldots, n$$

$$P(X \le x) = F_X(x) = \sum_{\forall X \le x} P(X=x) = \sum_{\forall X \le x} \binom{n}{x} p^{x} (1-p)^{n-x},$$

$E(X) = np$

$Var(X) = npq$

Remark:- The probability of a number of items that satisfy some property, taken from a large sociaty is computed by binomial Distribution

n: The number of society members

X: The number of items that satisfy the property

Event of interest: is the property with p

$\equiv$ probability that one event satisfies this property without repetition

Ex Four coins are tossed together. Let X be the number of heads.

① Find the pmf of X?

$X \sim Bin(\underset{n}{4}, \underset{p}{\tfrac{1}{2}})$

$P(X=x) = \binom{4}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{4-x}$

$= \binom{4}{x}\left(\frac{1}{2}\right)^{4}$

$x = 0, 1, 2, 3, 4$

(2) Find the prob. that we get 2 heads

Sol $P(X=2) = \binom{4}{2}\left(\frac{1}{2}\right)^4$