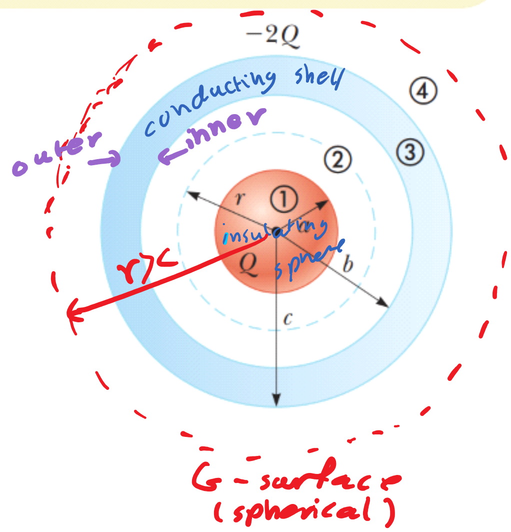
Lecture 15 (Saturday 27-3)

Saturday, March 27, 2021 3:15 AM

Example 24.7 A Sphere Inside a Spherical Shell

A solid insulating sphere of radius a carries a net positive charge Q uniformly distributed throughout its volume. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and carries a net charge $-2Q$. Using Gauss's law, find the electric field in the regions labeled ①, ②, ③, and ④ in Figure 24.19 and the charge distribution on the shell when the entire system is in electrostatic equilibrium.



For regions ① and ②

↳ Same as in Ex 24.3.

- Region ① $(r < a)$

$$E = k_e \frac{Q}{a^3} r$$

- Region ② $(a < r < b)$

$$E = k_e \frac{Q}{r^2}$$

- Region ③ $(b < r < c)$

↳ $E = 0$ (conductor in electrostatic equilibrium).

- Region ④ $(r > c)$

↳ $\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} \Rightarrow E(4\pi r^2) = \frac{q_{in}}{\varepsilon_0}$

$q_{in} = ? \Rightarrow q_{in} = q_{\text{insulating sphere}} + q_{\text{conducting shell}} = Q + (-2Q) = -Q$

$\Rightarrow q_{in} = -Q$

$\Rightarrow E(4\pi r^2) = \frac{-Q}{\varepsilon_0}$

$$\Rightarrow E = -k_e \frac{Q}{r^2} \quad (\text{for } r > c, \text{ region } (4))$$

$q_{inner} = ?$, $q_{outer} = ?$ of the conducting shell

Given: $q_{shell} = -2Q \Rightarrow q_{shell} = q_{inner} + q_{outer}$

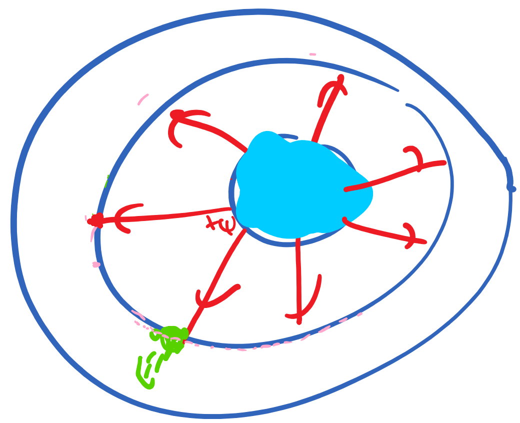
$$\Rightarrow -2Q = q_{inner} + q_{outer}$$

$\Phi_E = 0 \Rightarrow q_{in} = 0$

$q_{inner} + q_{sphere} = 0$

$q_{inner} + Q = 0$

$$\Rightarrow \boxed{q_{inner} = -Q}$$



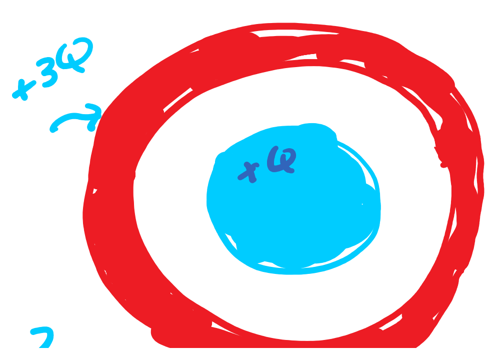
Note that $q_{inner} = -Q$ to cancel $q = +Q$ on the solid sphere to give $E = 0$ $(q_{in} = 0)$.

$$\Rightarrow -2Q = -Q + q_{outer} \Rightarrow \boxed{q_{outer} = -Q}$$

<u>Question</u>

$q_{shell} = +3Q$

$q_{sphere} = +Q$



$q_{sphere} = \ldots$ 

$\Rightarrow q_{inner} = ?\ ,\ q_{outer} = ?$

$q_{inner} = -q_{sphere} = \boxed{-Q}$

But $q_{shell} = q_{inner} + q_{outer} \Rightarrow +3Q = -Q + q_{outer}$

$\Rightarrow \boxed{q_{outer} = +4Q}$

[What If the sphere is conducting
NOT insulating?

The only difference will be
for region ① $\Rightarrow$ ($E = 0$ for $r<a$)

Other regions $\rightarrow$ same !!