Chapter 26: Capacitance and Dielectrics

Capacitors are devices that store electric charge.

Examples of where capacitors are used include:

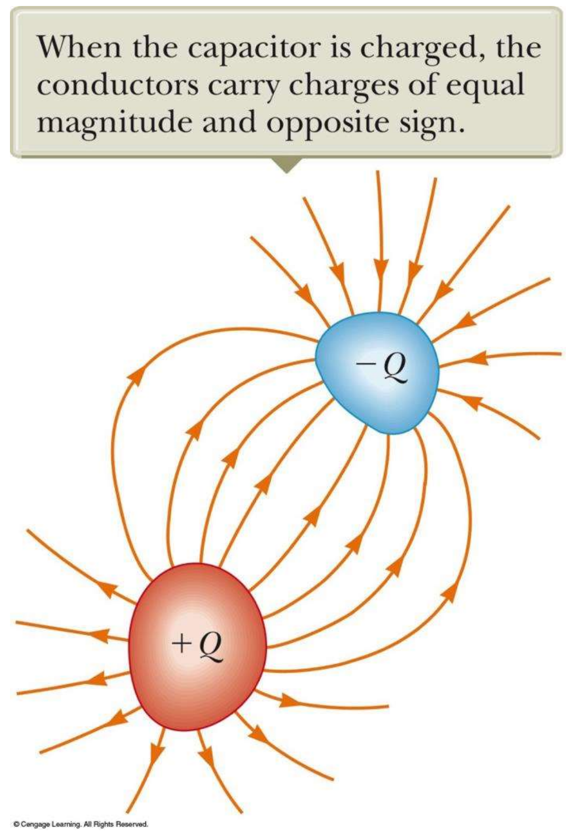
- radio receivers
- filters in power supplies
- energy-storing devices in electronic flashes

26.1 Definition of Capacitance

A capacitor consists of two conductors.

- These conductors are called plates.
- When the conductor is charged, the plates carry charges of equal magnitude and opposite signs.

A potential difference exists between the plates due to the charge.



Definition of Capacitance

The **capacitance**, *C*, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the potential difference between the conductors.

$$C \equiv \frac{Q}{\Delta V}$$ → units → 1 C/V = 1 F

The SI unit of capacitance is the **farad** (F).

The farad is a large unit, typically you will see microfarads (μF) and picofarads (pF).

? ×10⁻⁶ 10⁻¹²

Capacitance will always be a positive quantity

The capacitance of a given capacitor is constant.

The capacitance is a measure of the capacitor's ability to store charge .

- The capacitance of a capacitor is the amount of charge the capacitor can store per unit of potential difference (You may need to remember *heat capacity*: the amount of energy an object can store)

Parallel Plate Capacitor → PPC

Each plate is connected to a terminal of the battery.

- The battery is a source of potential difference.

If the capacitor is initially uncharged, the battery establishes an electric field in the connecting wires.

This field applies a force on electrons in the wire just outside of the plates.

The force causes the electrons to move onto the negative plate.

This continues until equilibrium is achieved.

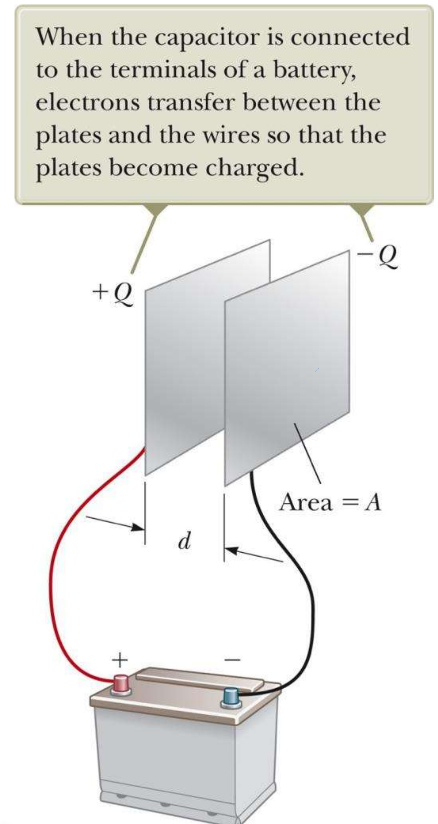
- The plate, the wire and the terminal are all at the same potential.

At this point, there is no field present in the wire and the movement of the electrons ceases.

The plate is now negatively charged.

A similar process occurs at the other plate, electrons moving away from the plate and leaving it positively charged.

In its final configuration, the potential difference across the capacitor plates is the same as that between the terminals of the battery.



26.2 Calculating Capacitance

$C = \frac{Q}{\Delta V}$, $\Delta V = Ed$, $E = \frac{\sigma}{\varepsilon_0}$, $\sigma = \frac{Q}{A}$

$\Rightarrow \Delta V = \frac{\sigma}{\varepsilon_0} d = \frac{Qd}{\varepsilon_0 A}$

$\Rightarrow C = \frac{\cancel{Q}}{\frac{\cancel{Q} d}{\varepsilon_0 A}} = \frac{\varepsilon_0 A}{d}$

Parallel-Plate Capacitors

The charge density on the plates is $\sigma = Q/A$.

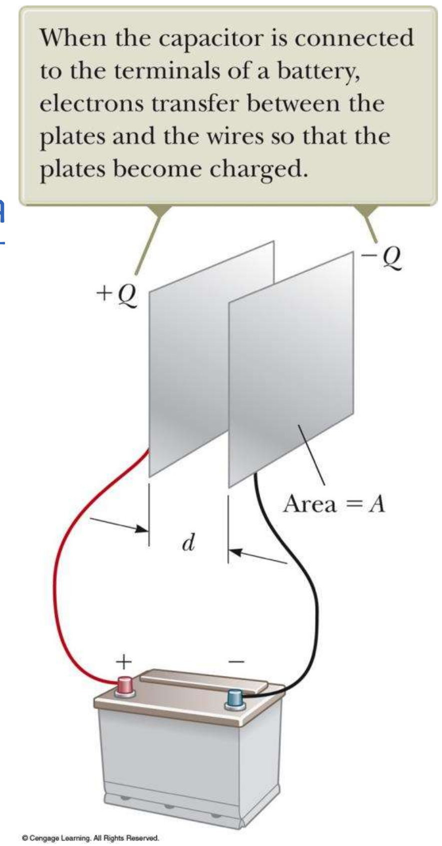
- A is the area of each plate, the area of each plate is equal
- Q is the charge on each plate, equal with opposite signs

The electric field is uniform between the plates ($E=\sigma/\varepsilon_0$) and zero elsewhere.

The capacitance is proportional to the area of its plates and inversely proportional to the distance between the plates.

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{Q}{Qd/\varepsilon_o A} = \frac{\varepsilon_o A}{d}$$

Note that C is proportional to A and $1/d$



The Cylindrical Capacitor

$\Delta V = -\int_a^b \vec{E} \cdot d\vec{s}$

For more details, see example 26.1

$\Delta V = -2k_e\lambda \ln(b/a)$

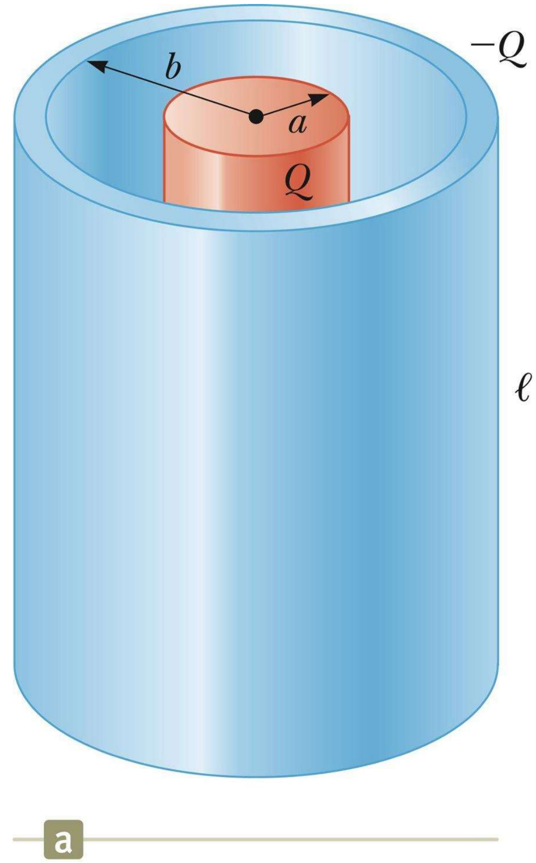
$\lambda = Q/\ell$

The capacitance is:

$$C = \frac{Q}{\Delta V} = \frac{\ell}{2k_e \ln(b/a)} \Rightarrow \frac{C}{\ell} = \frac{1}{2k_e \ln\left(\frac{b}{a}\right)}$$

Capacitance per unit length=?

Note that if you increase ***a*** or decrease ***b***, this will bring the conductors closer together ➔ capacitance increases. *How would this compare to the parallel-plate capacitor?*



The Spherical Capacitor

$\Delta V = -\int_A^B \vec{E}\cdot d\vec{s}$

For more details, see example 26.2

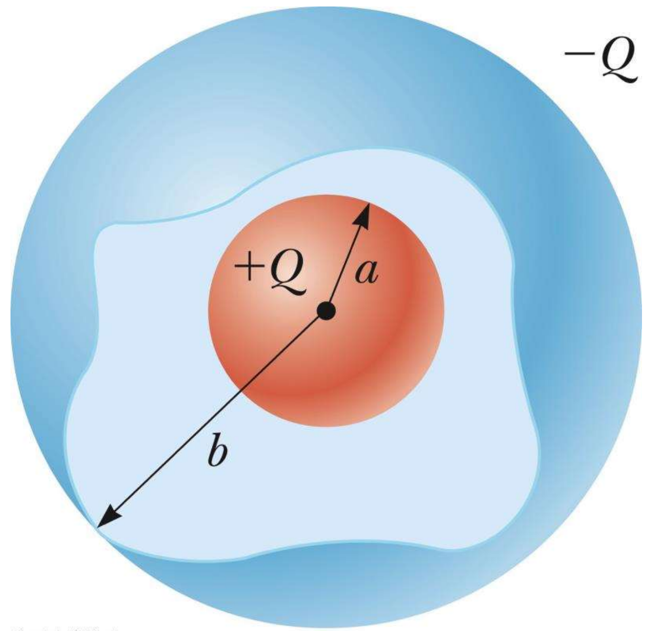
The potential difference will be

$$\Delta V = k_e Q\left(\frac{1}{b} - \frac{1}{a}\right)$$

The capacitance will be:

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e(b-a)}$$

What will happen to the capacitance if we increase ***a*** and decrease ***b***?



$b \to \infty$

WHAT IF? If the radius b of the outer sphere approaches infinity, what does the capacitance become?

Answer In Equation 26.6, we let $b \to \infty$:

$$C = \lim_{b\to\infty} \frac{ab}{k_e(b-a)} = \frac{a\cancel{b}}{k_e(\cancel{b})} = \frac{a}{k_e} = 4\pi\epsilon_0 a$$

Notice that this expression is the same as Equation 26.2, the capacitance of an isolated spherical conductor.

Capacitance – Isolated Sphere

Assume a spherical charged conductor with radius *a.*

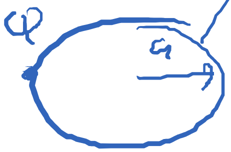
The sphere will have the same capacitance as it would if there were a conducting sphere of infinite radius, concentric with the original sphere.

Assume $V = 0$ for the infinitely large shell

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / a} = \frac{a}{k_e} = 4\pi\varepsilon_o a$$

Note, this is independent of the charge on the sphere and its potential.

$V = \frac{k_e Q}{a}$



$V = 0$ $\Delta V = \frac{k_e Q}{a}$

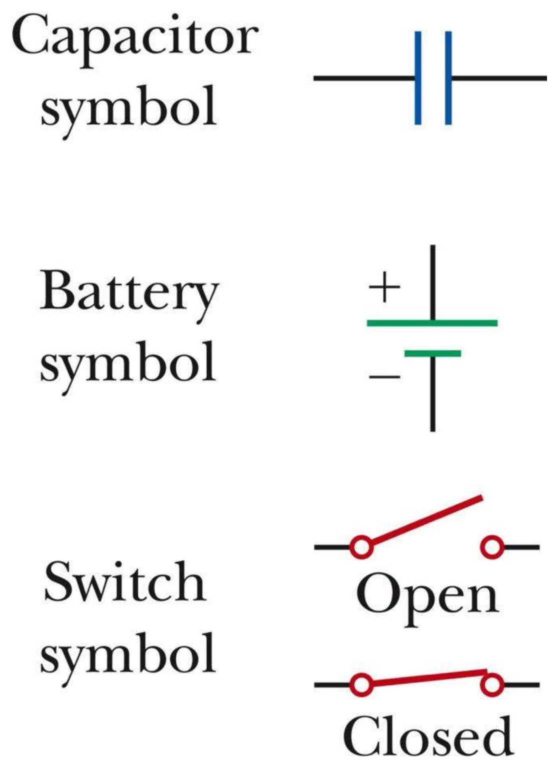
Circuit Symbols

A circuit diagram is a simplified representation of an actual circuit.

Circuit symbols are used to represent the various elements.

Lines are used to represent wires.

The battery's positive terminal is indicated by the longer line.



26.3 Combinations of Capacitors

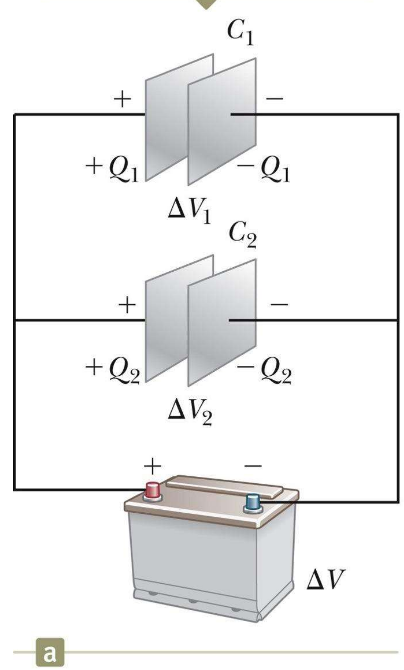
Capacitors in Parallel

When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged.

$C = \frac{Q}{\Delta V}$

$$\boxed{\begin{aligned} \Delta V_1 &= \Delta V_2 = \Delta V \\ Q &= Q_1 + Q_2 \end{aligned}}$$

A pictorial representation of two capacitors connected in parallel to a battery



Capacitors in Parallel, 2

The flow of charges ceases when the voltage across the capacitors equals that of the battery.

The potential difference across the capacitors is the same.

- And each is equal to the voltage of the battery
- $\Delta V_1 = \Delta V_2 = \Delta V$
  - ΔV is the battery terminal voltage

The capacitors reach their maximum charge when the flow of charge ceases.

The total charge is equal to the sum of the charges on the capacitors.

- $Q_{tot} = Q_1 + Q_2$

$$C = \frac{Q}{\Delta V} \Rightarrow Q = C\,\Delta V$$

$$\Rightarrow Q = Q_1 + Q_2 \Rightarrow C_{eq}\cancel{\Delta V} = C_1\cancel{\Delta V_1} + C_2\cancel{\Delta V_2} \quad \{\Delta V_1 = \Delta V_2 = \Delta V\}$$

$$\Rightarrow \boxed{C_{eq} = C_1 + C_2 + \ldots}$$

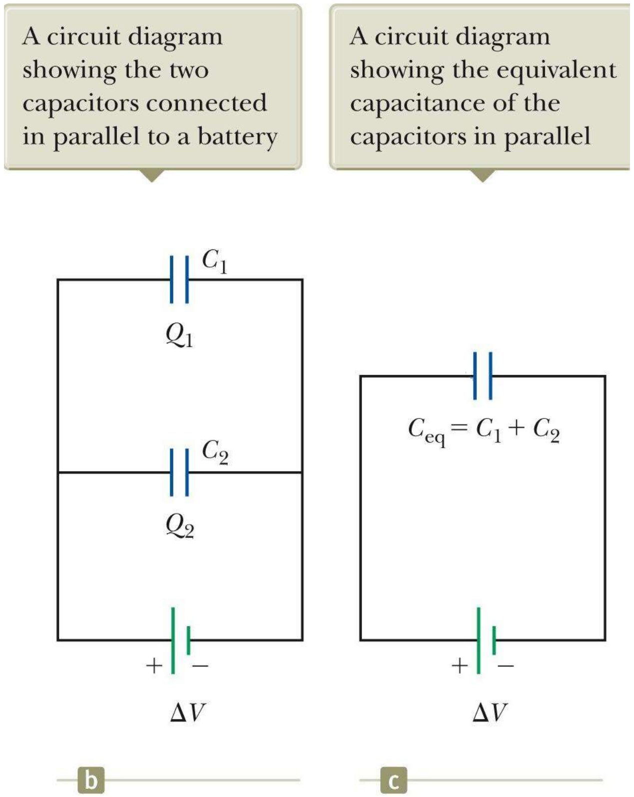
Capacitors in Parallel, 3

The capacitors can be replaced with one capacitor with a capacitance of C_{eq}.

- The *equivalent capacitor* must have exactly the same external effect on the circuit as the original capacitors.

$$\mathbf{C_{eq} = C_1 + C_2 + C_3 + \dots}$$

The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors.

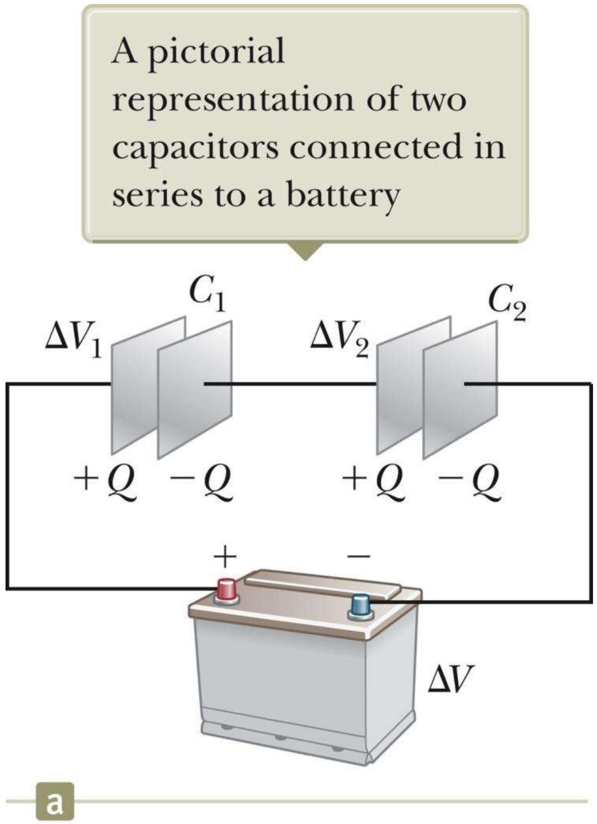


Capacitors in Series

When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_2 through the battery.

As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is removed from the left plate of C_2, leaving it with an excess positive charge.

All of the right plates gain charges of –Q and all the left plates have charges of $+Q$.



Capacitors in Series, cont.

An equivalent capacitor can be found that performs the same function as the series combination.

The charges are all the same.

$$Q_1 = Q_2 = Q$$

The potential differences add up to the battery voltage.

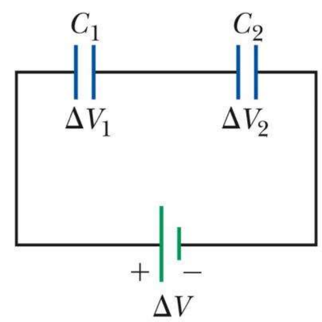
$$\Delta V_{tot} = \Delta V_1 + \Delta V_2 + \ldots$$

Therefore:

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots}$$

The equivalent capacitance (C_{eq}) of a series combination is always less than any individual capacitor in the combination.

A circuit diagram showing the two capacitors connected in series to a battery



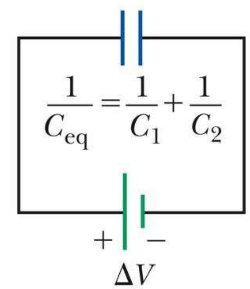
b

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$\Rightarrow$

A circuit diagram showing the equivalent capacitance of the capacitors in series



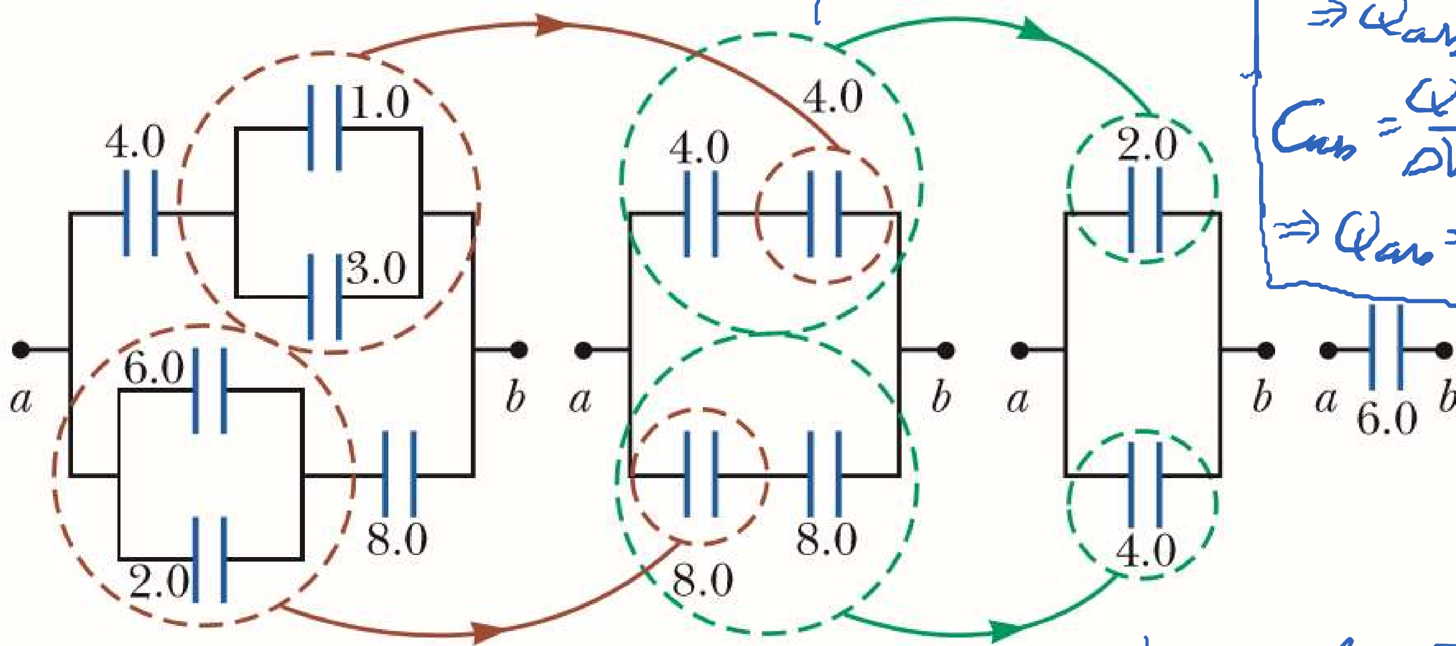
c

$$C = \frac{Q}{\Delta V} \Rightarrow \Delta V = \frac{Q}{C}$$

$$\{Q_1 = Q_2 = Q\}$$

Equivalent Capacitance, Example 26.3

$C = 2\,\mu C$



$C_{eq} = 6\,\mu F = C_{ab}$

If $\Delta V_{ab} = 15\,V$

$\Rightarrow Q_{ab} = ?$

$C_{ab} = \frac{Q_{ab}}{\Delta V_{ab}}$

$\Rightarrow Q_{ab} = (6\,\mu F)(15\,V) = 90\,\mu C$

$\hookrightarrow \frac{1}{C} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \Rightarrow C = 4\,\mu F$