1. Since $F_{XY}(x, y)$ is a probability $0 \leq F_{XY}(x, y) \leq 1$ for every $-\infty < x < \infty$ and $-\infty < y < \infty$.

2. If $x_1 \leq x_2$ and $y_1 \leq y_2$ then $F_{XY}(x_1, y_1) \leq F_{XY}(x_2, y_1) \leq F_{XY}(x_2, y_2)$, $F_{XY}(x_1, y_1) \leq F_{XY}(x_1, y_2) \leq F_{XY}(x_2, y_2)$ because $F_{XY}(x, y)$ is a nondecreasing function of X and Y.

3. $F_{XY}(-\infty, -\infty) = 0$

4. $F_{XY}(-\infty, y) = 0$

5. $F_{XY}(x, -\infty) = 0$

6. $F_{XY}(\infty, \infty) = 1$

7. $F_{XY}(x, \infty) = \lim_{y \to \infty} F_{XY}(x, y) = F_X(x)$

8. $F_{XY}(\infty, y) = \lim_{x \to \infty} F_{XY}(x, y) = F_Y(y)$

9. $P(x_1 < X \leq x_2, y_1 < Y \leq y_2) = F_{XY}(x_1, y_1) + F_{XY}(x_2, y_2) - F_{XY}(x_1, y_2) - F_{XY}(x_2, y_1)$

10. $P(x_1 < X \leq x_2, Y = y) = F_{XY}(x_2, y) + F_{XY}(x_1, y)$

11. $P(X = x, y_1 < Y \leq y_2) = F_{XY}(x, y_2) - F_{XY}(x, y_1)$

12. $P(X > a, Y > b) = 1 - F_X(a) - F_Y(b) + F_{XY}(a, b)$

13. $f_{XY}(x, y) = \dfrac{\partial^2 F_{XY}(x, y)}{\partial x \partial y}$

14. $f_{XY}(x, y) \geq 0 \ \forall x, \forall y.$

15. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(x, y) \ dx \ dy = 1$

16. $F_{XY}(a, b) = \int_{-\infty}^{a} \int_{-\infty}^{b} f_{XY}(x, y) \ dy \ dx$

17. $F_X(x) = \int_{-\infty}^{x} \int_{-\infty}^{\infty} f_{XY}(\xi_1, \xi_2) \ d\xi_2 \ d\xi_1$

18. $F_Y(y) = \int_{-\infty}^{y} \int_{-\infty}^{\infty} f_{XY}(\xi_1, \xi_2) \ d\xi_1 \ d\xi_2$

19. $P(x_1 < X \leq x_2, y_1 < Y \leq y_2) = \int_{y_1}^{y_2} \int_{x_1}^{x_2} f_{XY}(x, y) \ dx \ dy$

20. $f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x, y) \ dy$, for $\forall x$.

21. $f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x, y) \ dx$, for $\forall y$.

22. $F_X(x|Y = y) = \dfrac{\int_{-\infty}^{x} f_{XY}(\xi, y) d\xi}{f_Y(y)}$

23. $f_X(x|Y=y) = \dfrac{f_{XY}(x,y)}{f_Y(y)}$

24. $F_X(x|y_1 < Y \le y_2) = \dfrac{F_{XY}(x,y_2) - F_{XY}(x,y_1)}{F_Y(y_2) - F_Y(y_1)} = \dfrac{\int_{y_1}^{y_2}\int_{-\infty}^{x} f_{XY}(\xi,y)d\xi dy}{\int_{y_1}^{y_2}\int_{-\infty}^{\infty} f_{XY}(x,y)dxdy}$

25. $f_X(x|y_1 < Y \le y_2) = \dfrac{\int_{y_1}^{y_2} f_{XY}(x,y)dx}{\int_{y_1}^{y_2}\int_{-\infty}^{\infty} f_{XY}(x,y)dxdy}$

26. Let X and Y be discrete random variables.
The conditional pmf of Y, given $X=x$ has occured, is given by $p_{Y|X}(y|x) = \dfrac{P(X=x, Y=y)}{P(X=x)} = \dfrac{p_{XY}(x,y)}{p_X(x)}$.
The conditional pmf of X, given $Y=y$ has occured, is given by $p_{X|Y}(x|y) = \dfrac{P(X=x, Y=y)}{P(Y=y)} = \dfrac{p_{XY}(x,y)}{p_y(y)}$.

27. If X and Y are independent then $p_{X|Y}(x|y) = p_X(x)$ and $p_{Y|X}(y|x) = p_Y(y)$.

28. The conditional CDF are defined by:
$F_{X|Y}(x_0|y) = P(X \le x_0, Y=y) = \sum_{x \le x_0} p_{X|Y}(x|y)$.
$F_{Y|X}(y_0|x) = P(Y \le y_0, X=x) = \sum_{y \le y_0} p_{Y|X}(y|x)$.

29. The conditional expected value of X, given $Y=y$ is defined by: $\mu_{X|Y} = E(X|Y) = \sum_x x p_{X|Y}(x|y)$.
The conditional expected value of Y, given $X=x$ is defined by $\mu_{Y|X} = E(Y|X) = \sum_y y p_{Y|X}(y|x)$.
The conditional variance value of X, given $Y=y$ is defined by:
$\sigma^2_{X|Y} = E((X-\mu_{X|Y})^2|Y) = \sum_x (x-\mu_{X|Y})^2 p_{X|Y}(x|y)$
$=E(X^2|Y=y) - E^2(X|Y=y)$.
The conditional variance value of Y, given $X=x$ is defined by:
$\sigma^2_{Y|X} = E((Y-\mu_{Y|X})^2|X) = \sum_y (y-\mu_{Y|X})^2 p_{Y|X}(y|x)$
$=E(Y^2|X=x) - E^2(Y|X=x)$.

30. Let X and Y be continuous random variables.
The conditional pdf of X, given $Y=y$ has occured, is given by $f_{X|Y}(x|y) = \dfrac{f_{X,Y}(x,y)}{f_Y(y)}$.
The conditional pdf of Y, given $X=x$ has occured, is given by $f_{Y|X}(y|x) = \dfrac{f_{X,Y}(x,y)}{f_X(x)}$.

31. If X and Y are independent then $f_{X|Y}(x|y) = f_X(x)$ and $f_{Y|X}(y|x) = f_Y(y)$.

32. The conditional CDF are defined by:
$F_{X|Y}(x_0|y) = P(X \le x_0, Y=y) = \int_{-\infty}^{x_0} f_{X|Y}(x|y)\ dx$.
$F_{Y|X}(y_0|x) = P(Y \le y_0, X=x) = \int_{-\infty}^{y_0} f_{Y|X}(y|x)\ dy$.

33. The conditional expected value of X, given $Y = y$ is defined by: $\mu_{X|Y} = E(X|Y) = \int_{-\infty}^{\infty} x f_{X|Y}(x|y)\ dx$.

The conditional expected value of Y, given $X = x$ is defined by $\mu_{Y|X} = E(Y|X) = \int_{-\infty}^{\infty} y f_{Y|X}(y|x)\ dy$.

The conditional variance value of X, given $Y = y$ is defined by:

$\sigma^2_{X|Y} = E((X - \mu_{X|Y})^2|Y) = \int_{-\infty}^{\infty} (x - \mu_{X|Y})^2 f_{X|Y}(x|y)\ dx$

$\sigma^2_{X|Y} = E(X^2|Y = y) - E^2(X|Y = y)$.

$\sigma^2_{X|Y} = \int_{-\infty}^{\infty} x^2 f_{X|Y}(x|y)\ dx - \left(\int_{-\infty}^{\infty} x f_{X|Y}(x|y)\ dx \right)^2$

The conditional variance value of Y, given $X = x$ is defined by:

$\sigma^2_{Y|X} = E((Y - \mu_{Y|X})^2|X) = \int_{-\infty}^{\infty} (y - \mu_{Y|X})^2 f_{Y|X}(y|x)\ dy$

$= E(Y^2|X = x) - E^2(Y|X = x)$.

$\sigma^2_{Y|X} = \int_{-\infty}^{\infty} y^2 f_{Y|X}(y|x)\ dy - \left(\int_{-\infty}^{\infty} y f_{Y|X}(y|x)\ dy \right)^2$