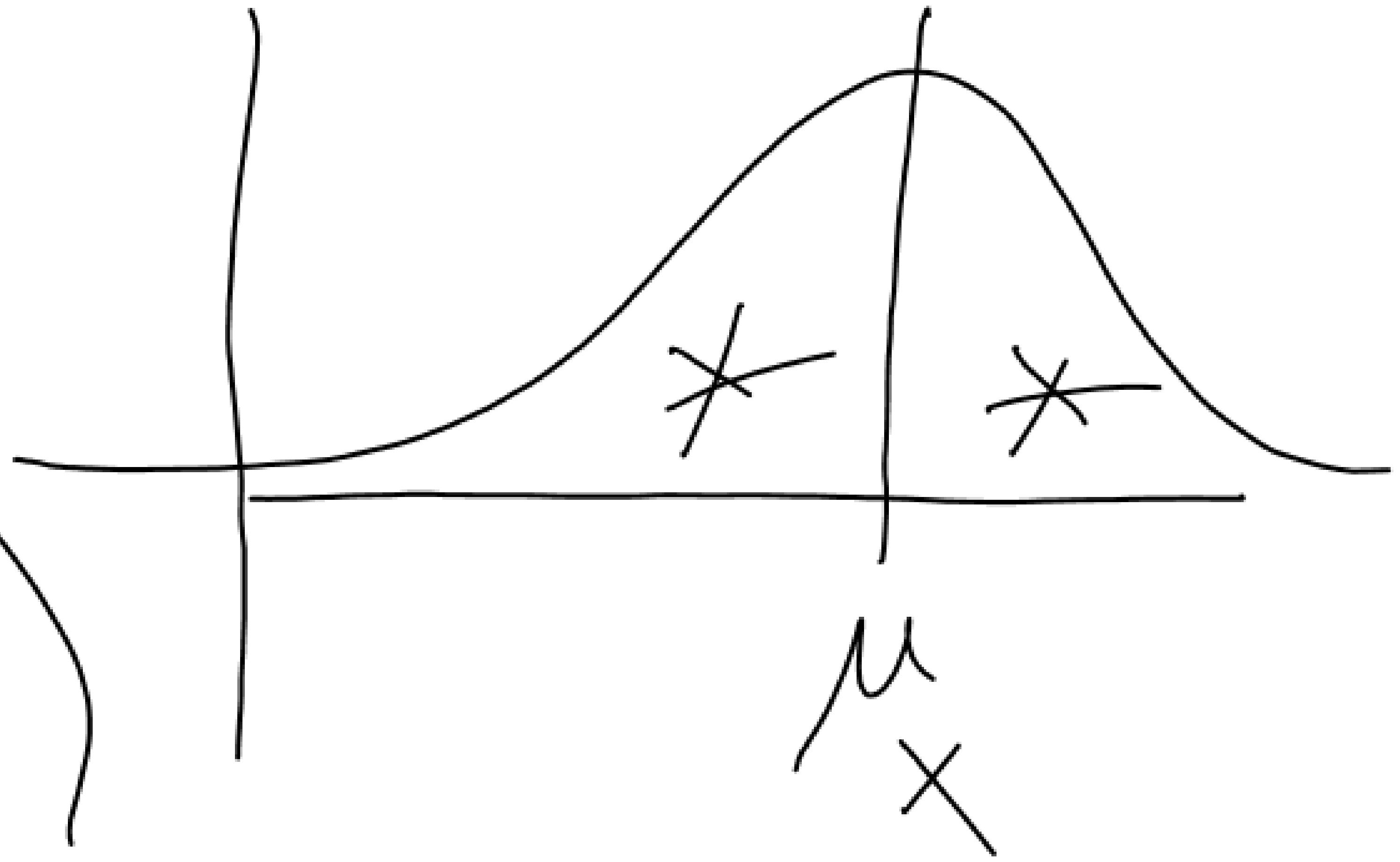
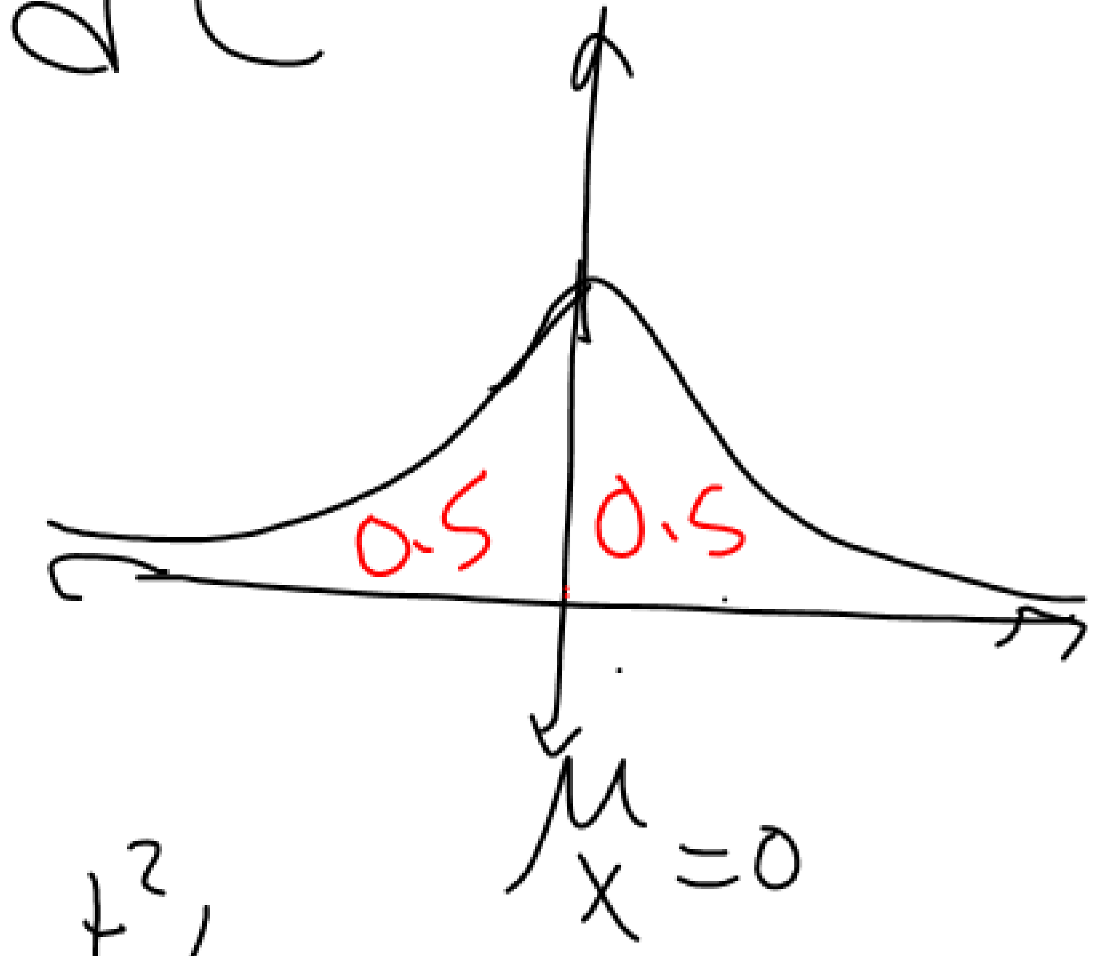
$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma_X} e^{-\frac{1}{2}\left(\frac{x-\mu_X}{\sigma_X}\right)^2}$$

$$N(\mu_X, \sigma_X)$$



$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\,\sigma_X} e^{-\frac{1}{2}\left(\frac{t-\mu_X}{\sigma_X}\right)^2} dt$$

$$N(\mu_X, \sigma_X) \longrightarrow N(0,1)$$ Standard Normal



$$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

$$F_Z(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

$F_X(x) = P(\underbrace{X}_{\text{standard Normal}} \leq x)$

$$X \longrightarrow Z \qquad Z = \frac{X - \mu_X}{\sigma_X}$$

$$Z \longrightarrow X \qquad X = \sigma_X Z + \mu_X$$

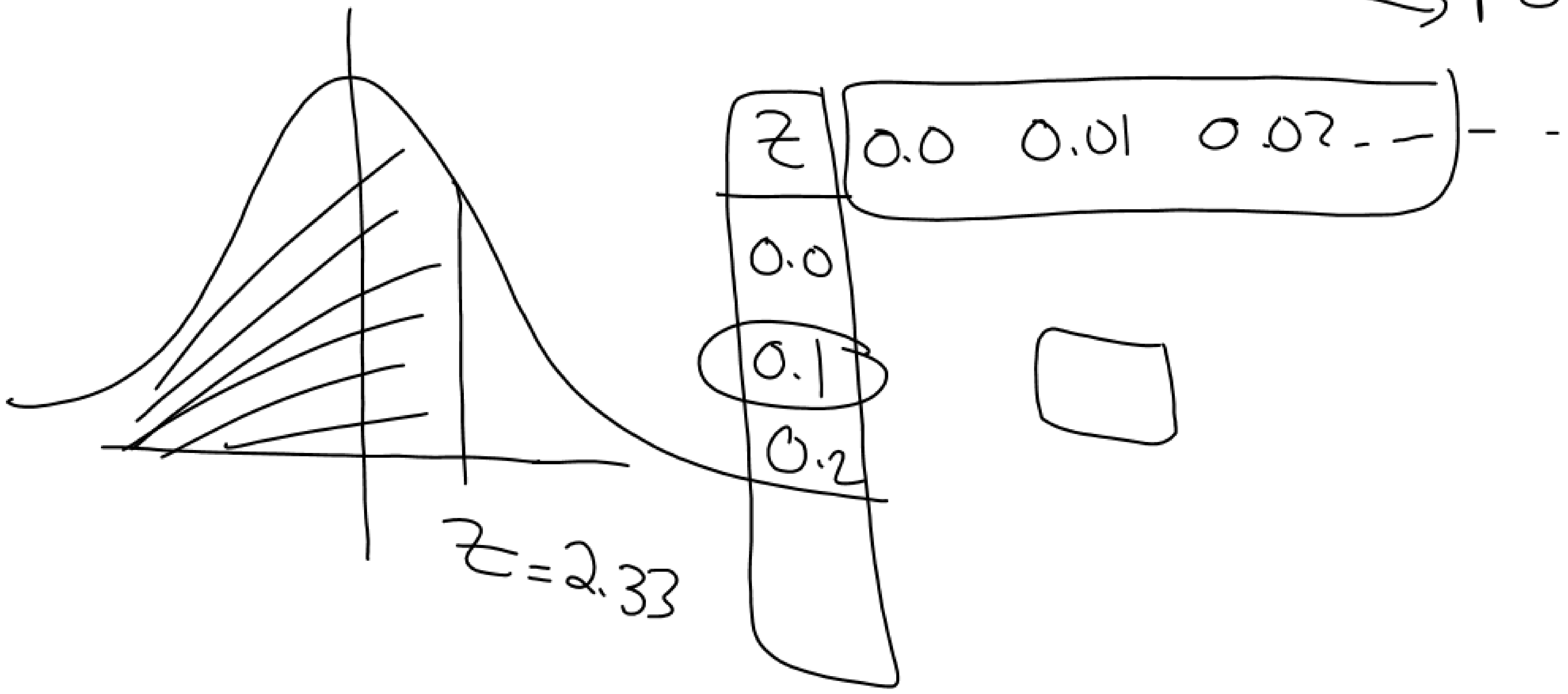
Ex

Find

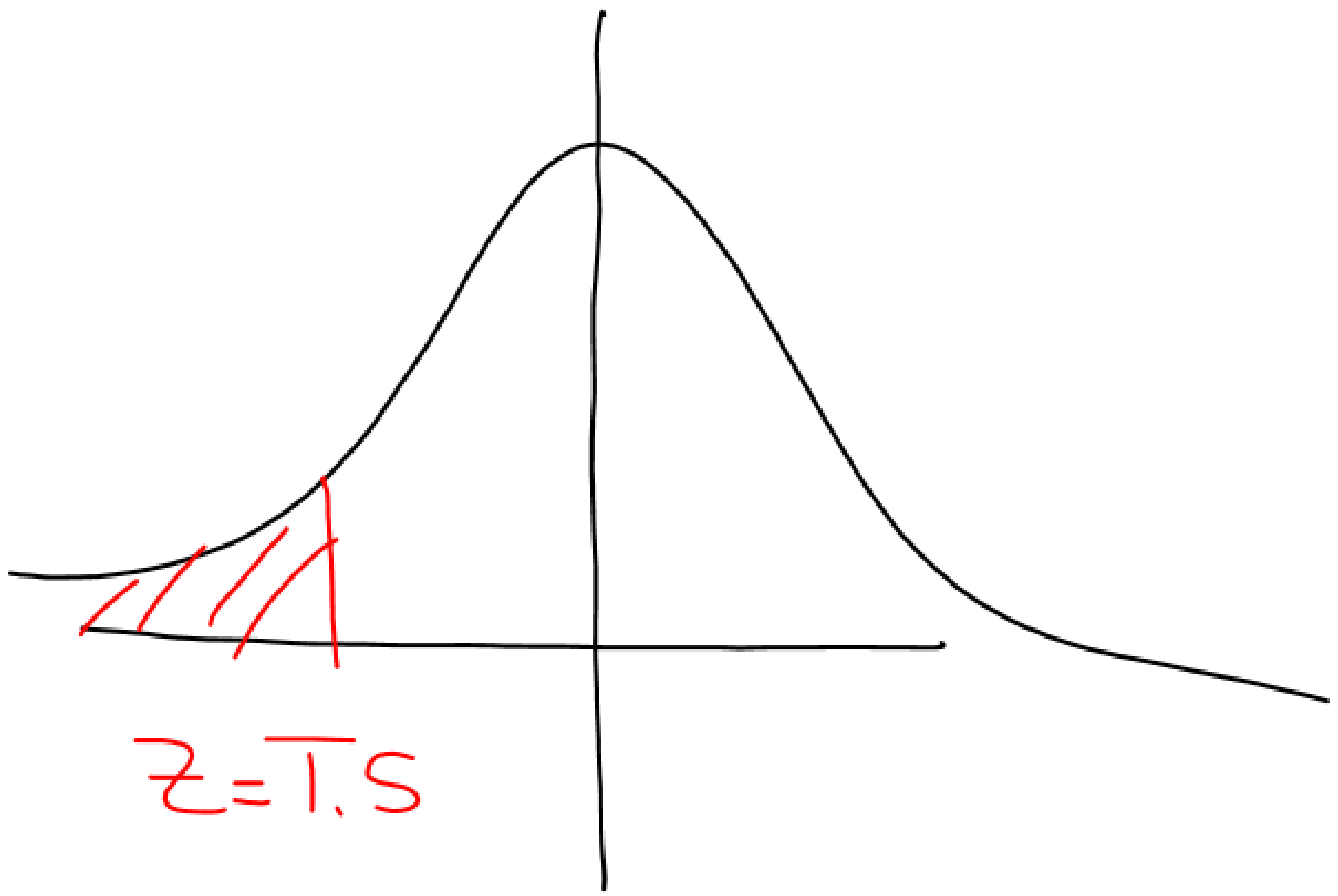
① $F_Z(2.3254) = P(Z \le 2.3254) \cong P(Z \le 2.33)$

rounding

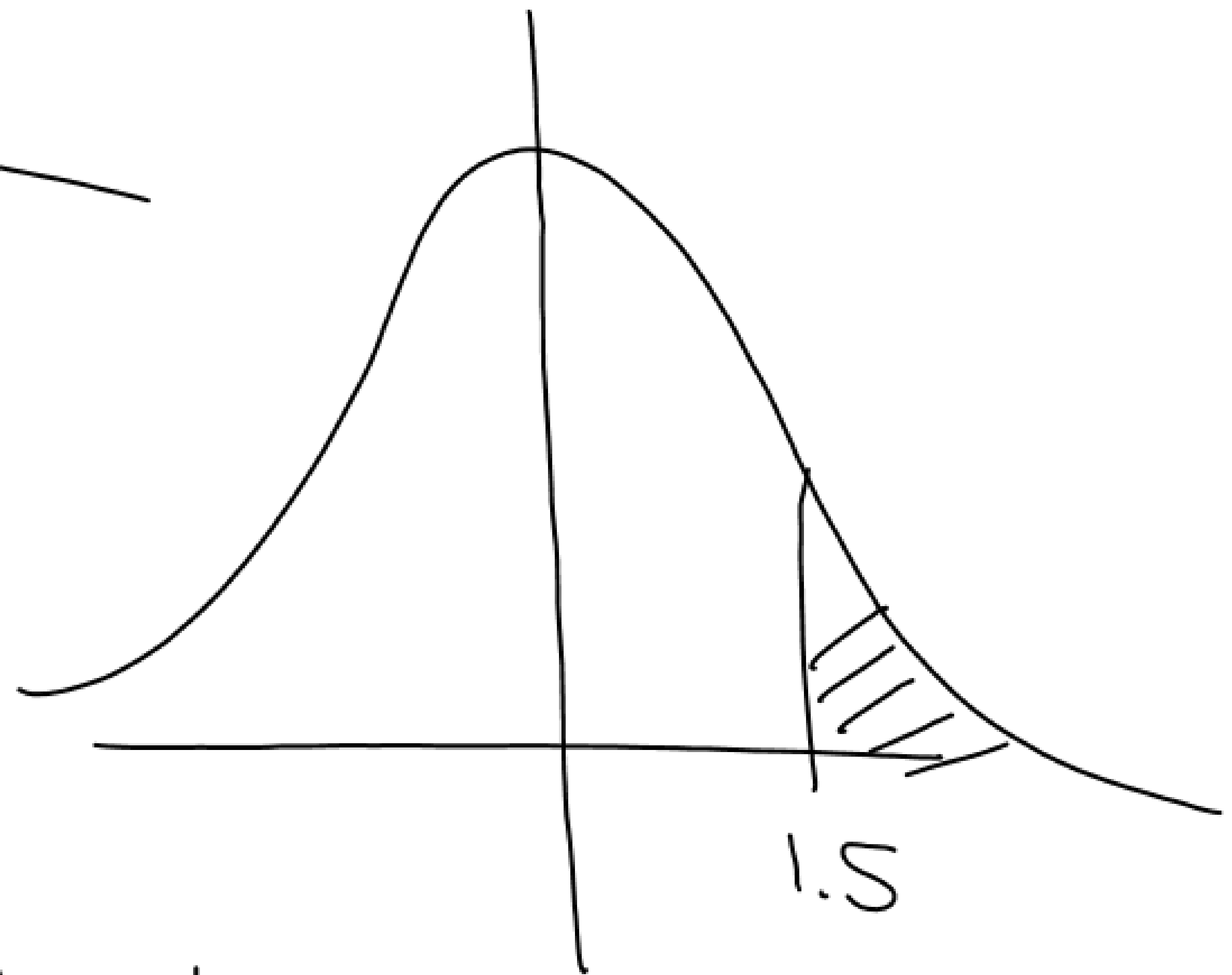
$= 0.9901$



② $P(Z \le -1.5) = 0.0668$

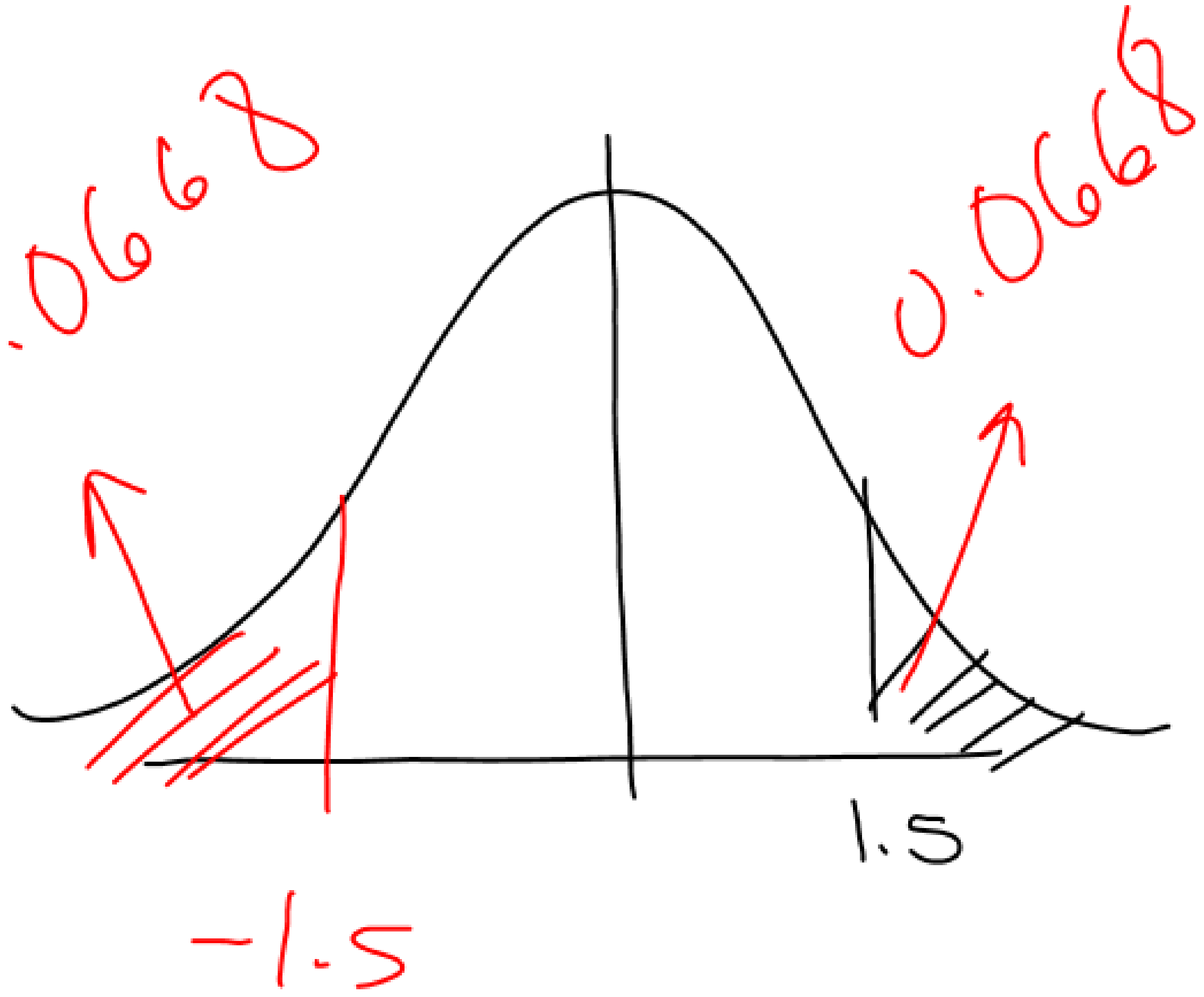


③ $P(Z \geq 1.5)$



$= 1 - P(Z \leq 1.5) = 1 - 0.9332 = 0.0668$

OR



Remark

① $P(-\infty < X < \infty) = 1$ where X is any cont. random variable

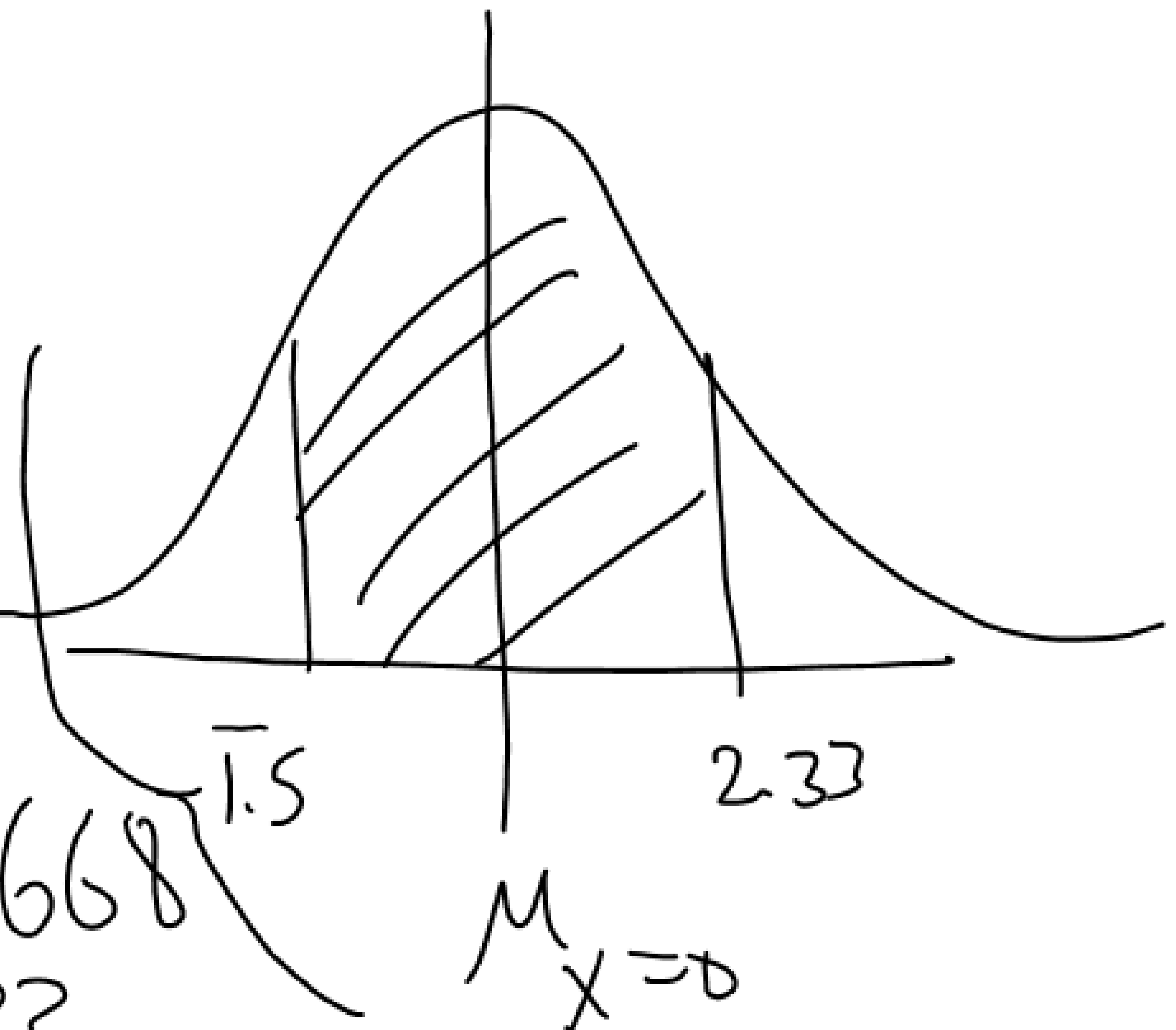
② $P(X \leq \mu_X) = P(X \geq \mu_X) = 1/2$ for any symmetric cont. random variable

Ex Find ① $P(-1.5 \leq Z \leq 2.33)$

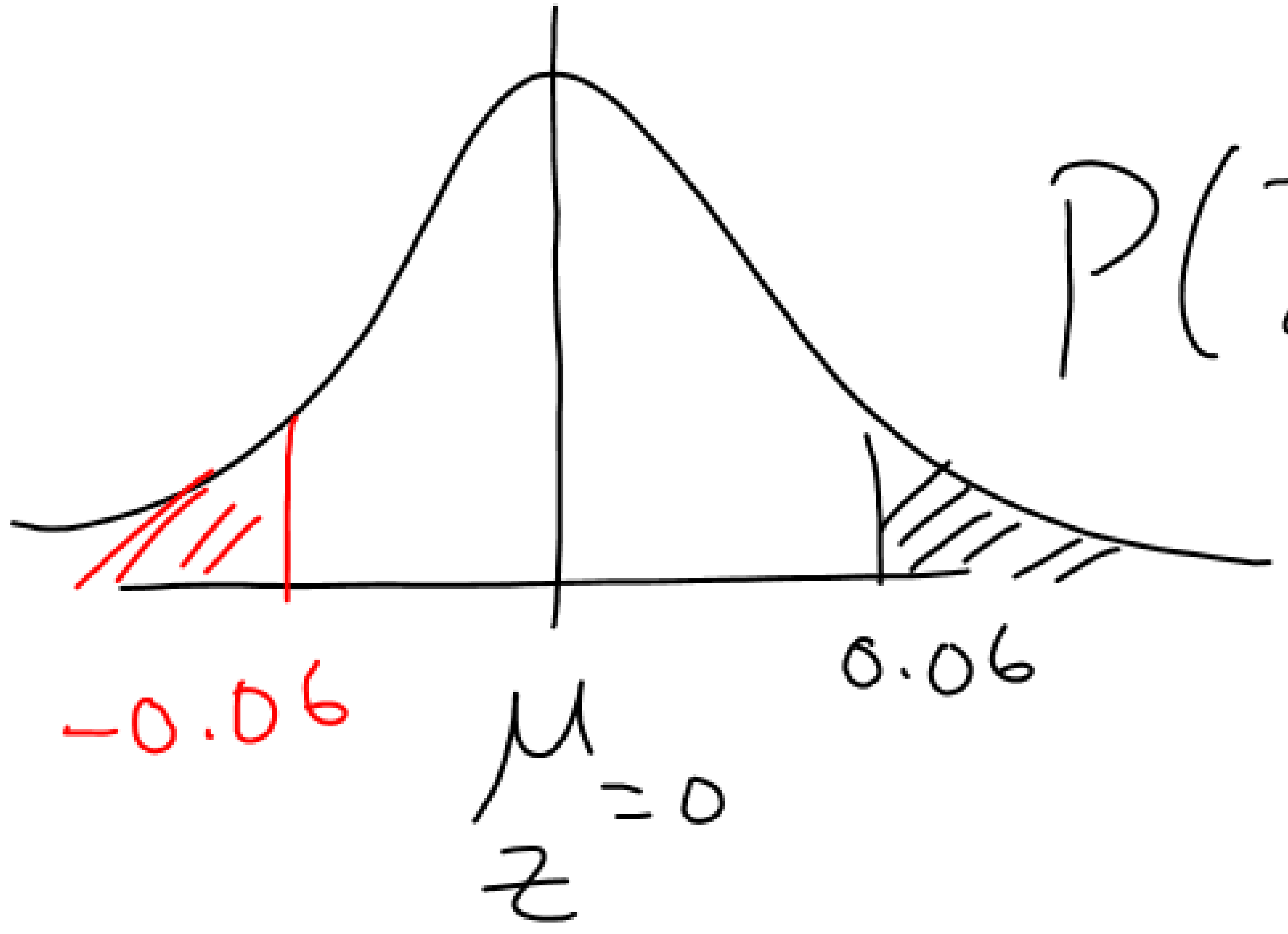
$= P(Z \leq 2.33) - P(Z \leq -1.5)$

$= 0.9901 - 0.0668$

$= 0.9233$



② $P(Z>0.06)$



$$P(Z>0.06) \equiv P(Z<-0.06) = 0.4761$$

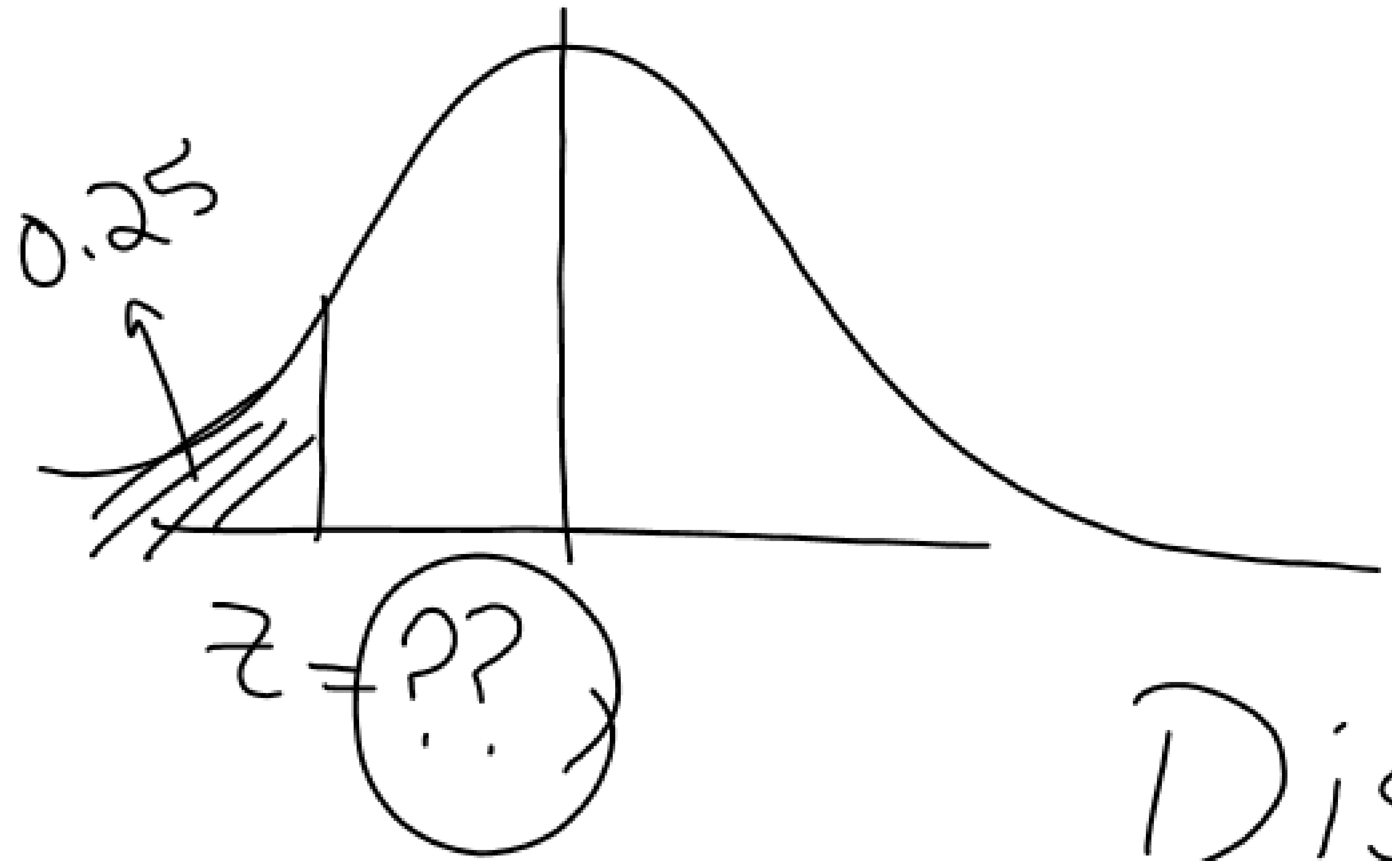
$$1-P(Z\leq 0.06) = 1-0.5239 = 0.4761$$

Note: $P(Z \geq a) \equiv P(Z \leq -a)$

1^{st} quartile	Q1	25% percentile
2^{nd} quartile	Q2	50% percentile
3^{rd} quartile	Q3	75% percentile

Ex Q1 is the z value such that $F_Z(z) = 0.25$

Sol $P(Z \le z) = 0.25$



-0.67

	0.07	0.08
-0.6	0.2514	.2483
	1.4×10^{-3}	1.7×10^{-3}

Distance between the closest found values and the given value