Ex Find the probability that a day selected randomly from a 365 days lies in January $\cup$ (OR) May?

Sol Let A: be the event of choosing a day in January

B: " " " " " " " " May

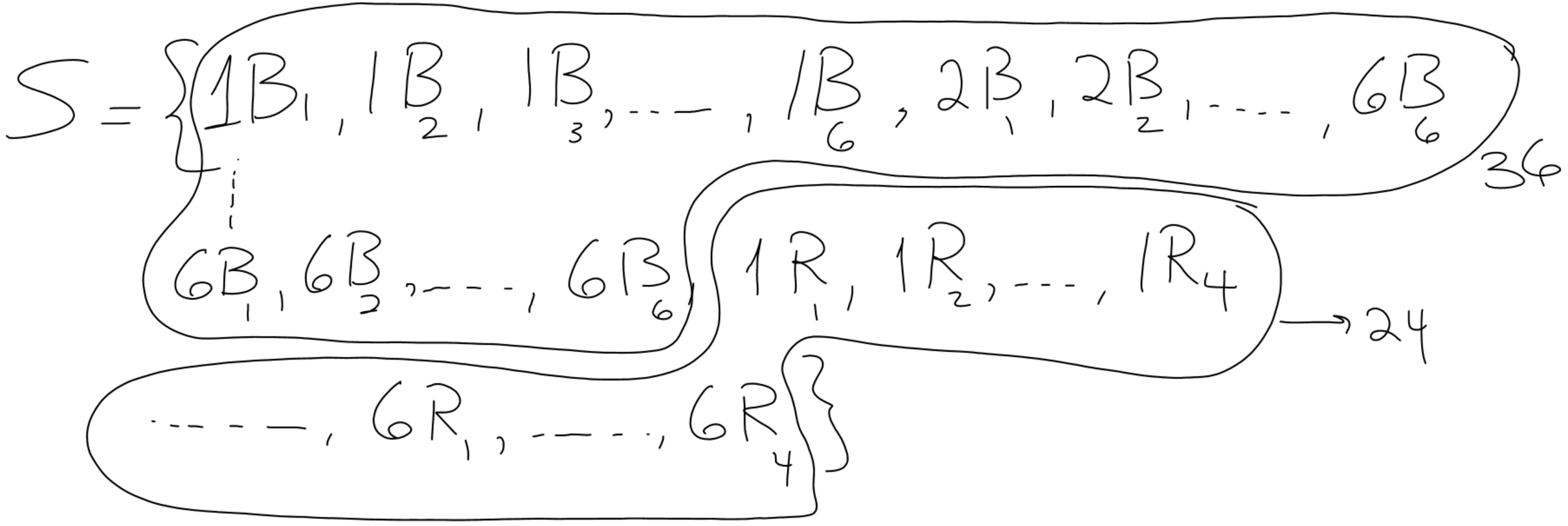
$$P(A \cup B) = P(A) + P(B) - \cancel{P(A \cap B)}, \quad A \cap B = \phi \text{ m.e}$$

$$= \frac{31}{365} + \frac{31}{365} = \frac{62}{365}$$

Ex Two boxes have cards enumerated from 1 → 6 in the 1st box and 10 balls (6 blue, 4 red) in the 2nd box. We first draw a card from the 1st box and then one ball from the 2nd box

(1) Find the probability that the selected ball is red?

Sol $|S| = 6 \cdot 10 = 60$



Let A be the event of getting a red ball

$P(A) = \frac{|A|}{|S|}$ $|A|$ $6 \; 4 = 24$ $P(A) = \frac{24}{60}$

(2) What is the probability that the selected ball is blue or the card number is < 3

Sol Let A be the event that the selected ball is blue

B be " " " " " card is < 3

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$|A| \quad 6 \quad 6 = 36 \qquad |A \cap B| \quad 2 \quad 6 = 12$

$|B| \quad 2 \quad 10 = 20 \qquad P(A \cup B) = \frac{36}{60} + \frac{20}{60} - \frac{12}{60} = \frac{44}{60}$

Ex In a car parking 40% ($P(A)$) of the cars are manufactured in the USA, 30% ($P(B)$) of the cars are black and 10% of the cars are black and made in the USA. ($P(A \cap B)$)

(1) What is the probability that the car is made in the USA or black

Sol Let A be the event that the car is made in USA

B " " " " " " " black

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.3 - 0.1 = 0.6$$

(2) What is the prob. that the car is not black?

Sol $P(B^c) = 1 - P(B)$

$= 1 - 0.3$

$= 0.7$

(3) Find the probability that the car is neither from the USA nor black?

neither nor $\equiv$ and not

$$P(\bar{A} \cap \bar{B}) = P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cup \bar{B})$$

Easier $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

DeMorgan's Law

$= 1 - 0.6$

$= 0.4$

Contingency Table

Ex A company consists of 145 offices connected by a local network through the PCs in the officies. The PCs have

different levels of Security (Low, Medium, High) based on the security softwares installed, activated in the PCs (Antiviruses, firewalls, ---) There are two shifts for the offices (Day shift, Night shift)

The officies are discribed by the contingency table

(1) Find the probability that a randomly selected office in the day shift has a medium security level?

$$P(D \cap M) = \frac{|D \cap M|}{|S|} = \frac{40}{145}$$

(2) What is the probability that a randomly selected office is in the night shift

<u>Sol</u> $P(D^c) = \frac{|D^c|}{|S|} = \frac{50}{145}$

(3) Find the probability that a randomly selected office

is open in the night shift or has a high security level?

Sol $P(D^c \cup H) = P(D^c) + P(H) - P(D^c \cap H)$

$$= \frac{50}{145} + \frac{50}{145} - \frac{30}{145}$$

$$= \frac{70}{145}$$

(4) Find the probability that a randomly selected office is medium level of security, if you know that the office opens at night shift?

$$P(M|D^c) = \frac{P(M \cap D^c)}{P(D^c)} = \frac{15/145}{50/145} = \frac{15}{50}$$

(5) Test whether the security level and shift time are independent?

Sol $P(M|D^c) = P(M)$

$$\frac{15}{50} \neq \frac{55}{145}$$

The security level and the shift time are dependent

Remark Given that two events A, B, if any pair of events

A and B

A^c and B

A and B^c

A^c and B^c

is independent then the other pairs are the same.

Ex In a car parking, 60% of the cars are made in the USA, and 90% of the USA cars are black. What is the probability that a randomly selected car is made in the USA and Black?

P(A)

P(B|A)

Sol Let A be the event that the selected car is made in the USA

B be the event that the selected car is black

$$P(A \cap B) = P(B|A)\,P(A) = (0.6)(0.9) = 0.54$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A|B)\,P(B)$$

$$= P(B|A)\,P(A)$$