Equipotentials

$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s} = \dots - \vec{E} \cdot \vec{s}$

take points B, C $\rightarrow \theta = 90° \rightarrow \cos 90° = 0$

$\vec{E} \perp \vec{s}$

$\Rightarrow \Delta V = V_B - V_C = 0$

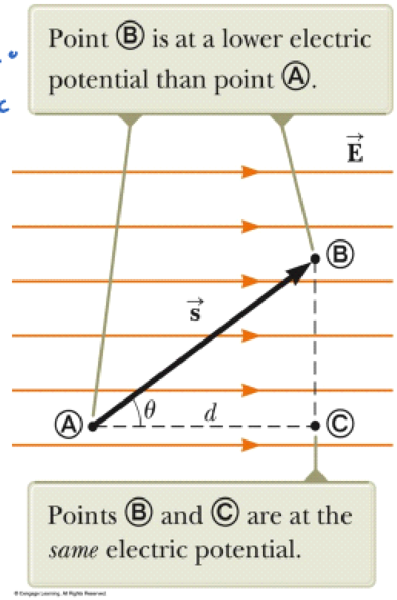
$\Rightarrow V_B = V_C$

Point *B* is at a lower potential than point *A*.

Points *B* and *C* are at the same potential ($V_B = V_C$).

- All points in a plane perpendicular to a uniform electric field are at the same electric potential (note that in this case **E·s** = 0 and thus $\Delta V = 0$)

The name **equipotential surface** is given to any surface consisting of a continuous distribution of points having the same electric potential.



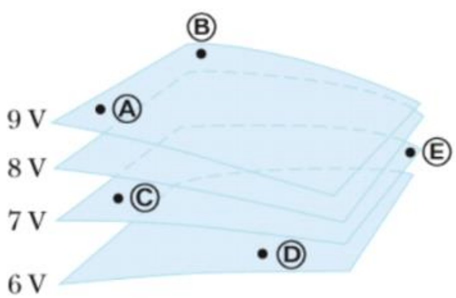
Quick Quiz 25.2 The labeled points in Figure 25.4 are on a series of equipotential surfaces associated with an electric field. Rank (from greatest to least) the work done by the electric field on a positively charged particle that moves from Ⓐ to Ⓑ, from Ⓑ to Ⓒ, from Ⓒ to Ⓓ, and from Ⓓ to Ⓔ.

$W_{A \to B} =$ zero

$W_{B \to C} = q(V_B - V_C) = q(9-7) = 2q$

$W_{C \to D} = q(7-6) = q$

$W_{D \to E} = q(6-7) = -q$



ANSWER

$W = -\Delta U = -q\Delta V = q(V_i - V_f)$

Therefore:

B➔C > C➔D > A➔B > D➔E

Example 25.1 The Electric Field Between Two Parallel Plates of Opposite Charge

A battery has a specified potential difference ΔV between its terminals and establishes that potential difference between conductors attached to the terminals. A 12-V battery is connected between two parallel plates as shown in Figure 25.5. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and we do not consider locations near the plate edges.) Find the magnitude of the electric field between the plates.

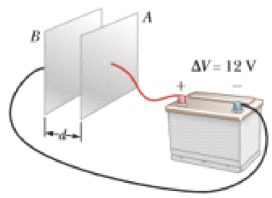


Figure 25.5 (Example 25.1) A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference ΔV divided by the plate separation d.

$$\Delta V = -Ed$$
$$\Rightarrow |\Delta V| = Ed$$
$$\Rightarrow E = \frac{|\Delta V|}{d}$$

SOLUTION

Conceptualize In Example 24.5, we illustrated the uniform electric field between parallel plates. The new feature to this problem is that the electric field is related to the new concept of electric potential.

Categorize The electric field is evaluated from a relationship between field and potential given in this section, so we categorize this example as a substitution problem.

Use Equation 25.6 to evaluate the magnitude of the electric field between the plates:

$$E = \frac{|V_B - V_A|}{d} = \frac{12 \text{ V}}{0.30 \times 10^{-2} \text{ m}} = \boxed{4.0 \times 10^3 \text{ V/m}}$$

The configuration of plates in Figure 25.5 is called a *parallel-plate capacitor* and is examined in greater detail in Chapter 26.

Example 25.2 Motion of a Proton in a Uniform Electric Field

A proton is released from rest at point Ⓐ in a uniform electric field that has a magnitude of 8.0×10^4 V/m (Fig. 25.6). The proton undergoes a displacement of magnitude $d = 0.50$ m to point Ⓑ in the direction of $\vec{\mathbf{E}}$. Find the speed of the proton after completing the displacement.

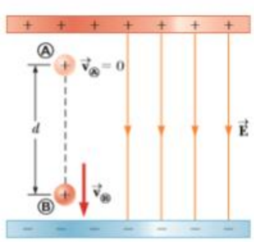


Figure 25.6 (Example 25.2) A proton accelerates from Ⓐ to Ⓑ in the direction of the electric field.

$$\Delta K + \Delta U = 0 \qquad (E_f = E_i)$$

$$\Rightarrow \Delta V = -Ed$$

$$(\tfrac{1}{2}mv^2 - 0) + e\Delta V = 0$$

$$v = \sqrt{\frac{-2e\Delta V}{m}} = \sqrt{\frac{-2e(-Ed)}{m}} = \sqrt{\frac{2eEd}{m}}$$

$$v = \sqrt{\frac{2(1.6 \times 10^{-19} \text{ C})(8.0 \times 10^4 \text{ V})(0.50 \text{ m})}{1.67 \times 10^{-27} \text{ kg}}}$$

$$= \boxed{2.8 \times 10^6 \text{ m/s}}$$

Note that since ΔV is negative ➔ ΔU is negative ➔ U decreases as the proton moves in the direction of **E**

THUS: As the proton accelerates in the direction of **E**, it gains K. E. while the electric P. E. of the system decreases.

Remember Ex 23.10

$$K = \frac{1}{2}mv^2$$

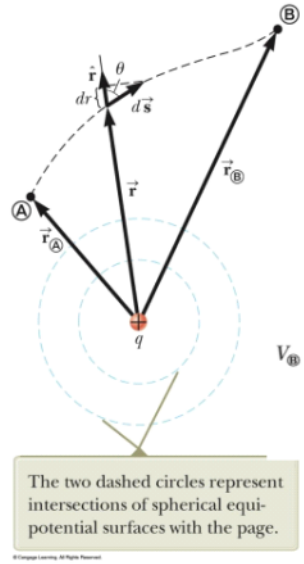
Electric Potential Due to Point Charges

An isolated positive point charge produces a field directed radially outward.

The potential difference between points *A* and *B* will be

$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$-\int_A^B \frac{dr}{r^2} = -\left[\frac{-1}{r}\right] = \left[\frac{1}{r}\right]_A^B$$



$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s}$$

$$\vec{E}\cdot d\vec{s} = k_e \frac{q}{r^2}\overbrace{\hat{r}\cdot d\vec{s}}^{dr}$$

$$\hat{r}\cdot d\vec{s} = ds\cos\theta$$

$$ds\cos\theta = dr.$$

$$\vec{E}\cdot d\vec{s} = (k_e q/r^2)\,dr;$$

$$V_B - V_A = -k_e q\int_{r_A}^{r_B}\frac{dr}{r^2} = k_e \frac{q}{r}\bigg|_{r_A}^{r_B}$$

$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

Potential and Point Charges, cont.

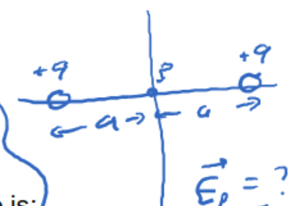
The electric potential is independent of the path between points *A* and *B*.

Choose a reference potential of $V = 0$ at $r_A = \infty$.

Substitute in

$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

E P



Then the electric potential due to a point charge at any distance r from the charge is:

$$V = k_e \frac{q}{r}$$

$\vec{E}_P = ? = \text{zero}$

$V_P = ? \neq 0$
$= k_e\left(\frac{q}{a} + \frac{q}{a}\right)$
$= 2k_e\frac{q}{a}$

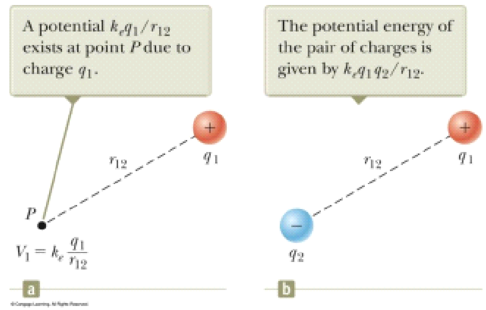
For a group of point charges, the total electric potential at some point is:

$$\boxed{V = k_e \sum_i \frac{q_i}{r_i}} = k_e\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \cdots\right]$$

Potential Energy of Two Charges

Assume q_1 is fixed as shown
Now, to bring q_2 from ∞ to point P,
The work done needed is:
$W=\Delta U=q_2\Delta V \Rightarrow U-\cancel{U(\infty)}=q_2[V_P-\cancel{V(\infty)}]$

Given that $U(\infty)=V(\infty)=0$
and $V_P = k_e q_1/r_{12}$



➔The potential energy of the system is

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

If the two charges are the same sign, U is positive and work must be done to bring the charges together.

If the two charges have opposite signs, U is negative and work is done to keep the charges apart.