Ibrahim Albluwi

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 $median(): O(n \log n)$

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Solution 2. Maintain an unordered array:

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Add to the end of the list. Note that the array might resize, so the running time is amortized.

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Use Quickselect to find the median. Note that this is the expected case if the array is shuffled.

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Solution 3. Maintain a sorted array:

insert(): *O*(*n*)

Search for the right position and then shift any elements that come after.

median(): $\Theta(1)$

The median is always at index $\frac{n}{2}$.

Solution 4. Use a max-heap to store the lower half of the elements (\leq median) and a min-heap to store the upper half of the elements (> median).

Assume that the max-heap is named **left** and the min-heap is named **right**. Ensure that:

• Any element in **left** is smaller than or equal to all the elements in **right**.

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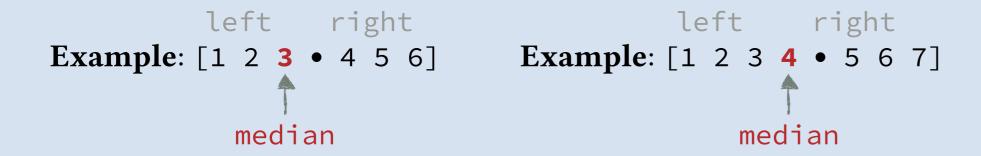
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Example: [1 2 3 • 4 5 6] **Example**: [1 2 3 4 • 5 6 7]

General Idea:

- **Insert** the new element into the left heap if it is less than or equal to the current median and to the right if it is greater than the current median.
- **Rebalance** the heaps by moving an element from the larger heap to the smaller heap if the size invariant is violated.

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```
insert into
the correct heap
```

rebalance the heaps if necessary

```
If left.size() > right.size()+1: right.insert(left.delMax()).
If right.size() > left.size(): left.insert(right.delMin()).
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Running Time:

```
insert(): O(\log n)
```

Inserting into the left or the right heaps is $O(\log n)$ and rebalancing is $O(\log n)$.

```
\textbf{median()} \colon \Theta(1)
```

The median is always left.max().