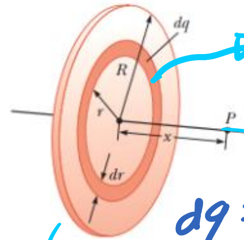
# Lecture 9 (12-1)

Saturday, March 13, 2021 11:33 PM

**Example 23.9** **The Electric Field of a Uniformly Charged Disk**

A disk of radius $R$ has a uniform surface charge density $\sigma$. Calculate the electric field at a point $P$ that lies along the central perpendicular axis of the disk and a distance $x$ from the center of the disk (Fig. 23.17).




$dA = 2\pi r\,dr$

$\vec{E}$

$dq = \sigma\,dA$
$= 2\pi\sigma r\,dr$

$$E = 2\pi k_e \sigma \left[1 - \frac{x}{(R^2 + x^2)^{1/2}}\right]$$

• If $x \ll R$ (or if $R \to \infty$)

$\hookrightarrow E = 2\pi k_e \sigma = \cancel{2}\pi\left(\frac{1}{2\cancel{4}\pi\varepsilon_0}\right)\sigma$

$\Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$ → The electric field is independent of the distance from the disc when $x \ll R$ or when $R \to \infty$.

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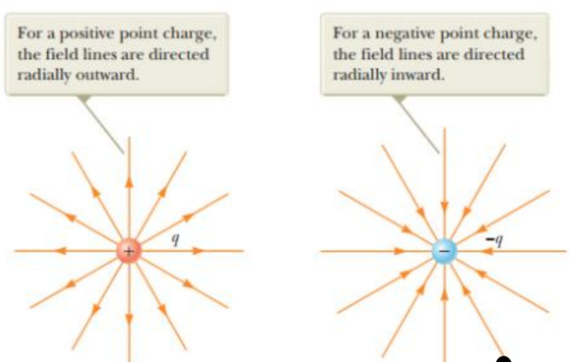
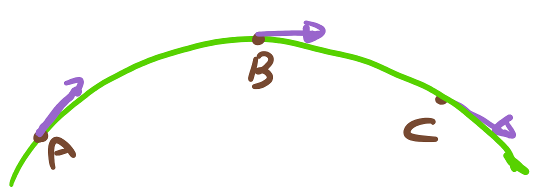
## 23.6 Electric Field Lines




• The direction of the $\vec{E}$-field at a given point is tangent to the electric field line at that point.




• The number of electric field lines per unit area through a surface perpendicular to the lines is proportional to $|\vec{E}|$ in that region.

↳ Means that lines are closer together where the electric field is strong, and far apart where the field is weak.

- Electric field lines do NOT cross.
- Electric field lines leave the positive charge and enter the negative charge, and the number of lines is proportional to the magnitude of the charge.

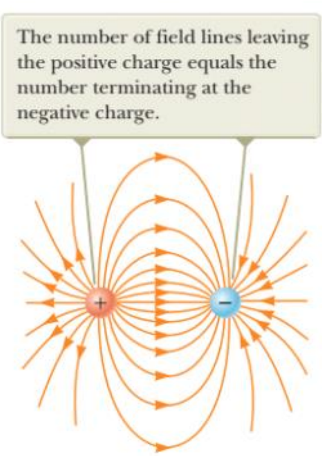



**Figure 23.20** The electric field lines for two point charges of equal magnitude and opposite sign (an electric dipole).

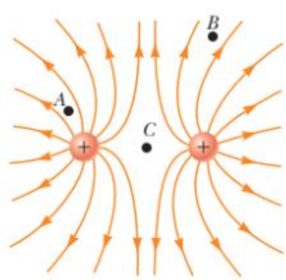
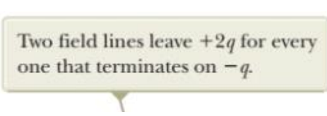



**Figure 23.21** The electric field lines for two positive point charges. (The locations *A*, *B*, and *C* are discussed in Quick Quiz 23.5.)

$E_A > E_B > E_C$

where $E_C = 0$




16 lines leaving $+2q$

entering $-q$

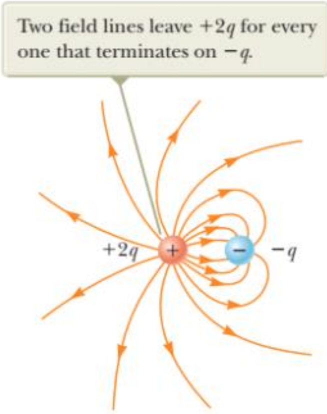



**Figure 23.22** The electric field lines for a point charge $+2q$ and a second point charge $-q$.

16 lines leaving

8 lines entering $-q$

The rest $\Rightarrow$ infinity

**Question:** 12 lines leave $Q_1 = 60\ nC$, and 4 lines enter $Q_2$. $Q_2 = ?$

$$Q_2 = \frac{-4}{12} Q_1 \Rightarrow Q_2 = -20\ nC$$

## 23.7: Motion of a Charged Particle in a Uniform Electric Field

$$\sum \vec{F} = m\vec{a}, \quad \sum \vec{F} = \vec{F}_e = q\vec{E}$$

$$\Rightarrow q\vec{E} = m\vec{a} \Rightarrow \boxed{\vec{a} = \frac{q\vec{E}}{m}}$$

$\vec{E}$ is uniform $\Rightarrow \vec{a}$ is constant $\Rightarrow$ we use eqns of motion with constant acceleration.

**In 1-dimension:**

$$v_{xf} = v_{xi} + a_x t$$

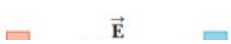
$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

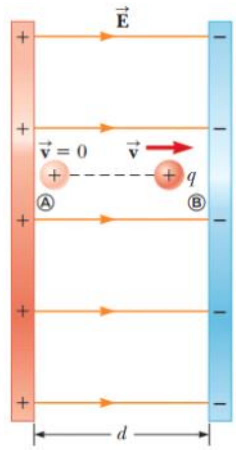
$$v_{xf}^2 = v_{xi}^2 + 2a_x (x_f - x_i)$$

$$\Delta x = x_f - x_i$$

**Ex 23.10** ($v_B = ?$)

(A) $v_B^2 = \cancel{v_A^2} + 2\underset{=}{a}\,(\underbrace{x_B - x_A}_{= d})$






(A) $v_B^2 = \cancel{v_A^2} + 2\underset{=}{a}\underbrace{(x_B - x_A)}_{= d}$

$a = \dfrac{qE}{m}$

$\Rightarrow v_B^2 = 2\left(\dfrac{qE}{m}\right) d$

$\Rightarrow \boxed{v_B = \sqrt{\dfrac{2qEd}{m}}}$

(B) OR $W = \Delta K \Rightarrow F \Delta x = K_B - K_A$

$\Rightarrow \underset{= qE}{F_e} \underbrace{\Delta x}_{d} = \frac{1}{2} m v_B^2 - \cancel{\frac{1}{2} m v_A^2}$

$\Rightarrow qEd = \frac{1}{2} m v_B^2 \Rightarrow \boxed{v_B = \sqrt{\dfrac{2qEd}{m}}}$

## Ex 23.11:

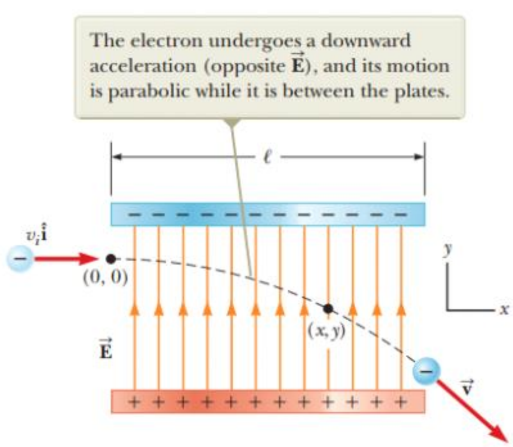



**Figure 23.24** (Example 23.11) An electron is projected horizontally into a uniform electric field produced by two charged plates.

$\rightarrow a_y = -\dfrac{eE}{m_e}, \quad a_x = 0$

## Ex 23.6:

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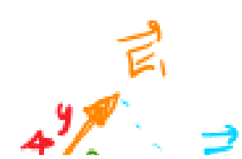
Electric dipole



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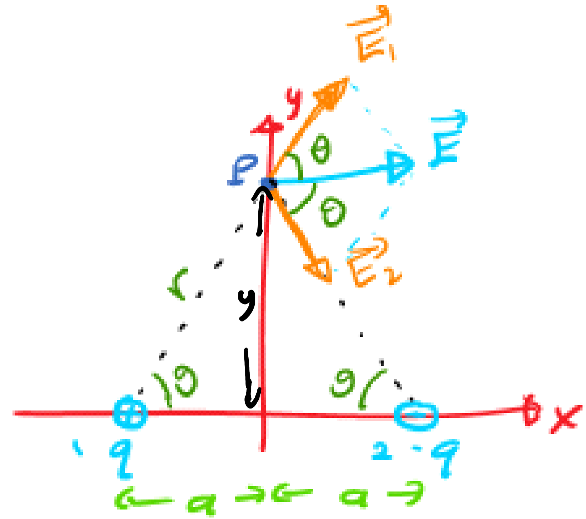
## Electric dipole

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$|\vec{E}_1| = |\vec{E}_2|$$

$$E_y = 0, \quad E_x = E_{1x} + E_{2x}$$




$$E_x = k_e \frac{q}{a^2+y^2}\cos\theta + k_e \frac{q}{a^2+y^2}\cos\theta$$

$$E_x = 2k_e \frac{q}{a^2+y^2}\cos\theta$$

$$\cos\theta = \frac{a}{r} = \frac{a}{(a^2+y^2)^{\frac{1}{2}}}$$

$$\Rightarrow \boxed{E_x = k_e \frac{2aq}{(a^2+y^2)^{3/2}}}$$

- Whe P is a distance $y \gg a$ from the origin
  ⇒ The electric field due to the electric dipole is:

$$\boxed{E \approx k_e \frac{2aq}{y^3}}$$