Chapter 23/Suggested Problems

11. M Three point charges are arranged as shown in Figure P23.11. Find (a) the magnitude and (b) the direction of the electric force on the particle at the origin.

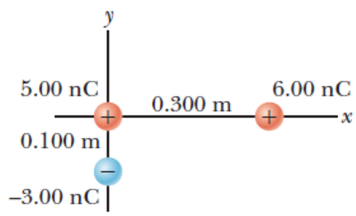
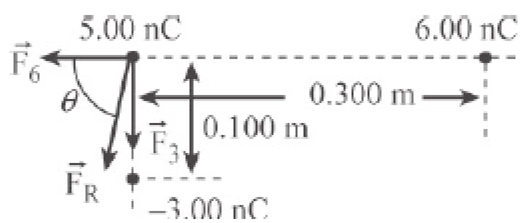


Figure P23.11 Problems 11 and 35.

P23.11 The particle at the origin carries a positive charge of 5.00 nC. The electric force between this particle and the –3.00-nC particle located on the –y axis will be attractive and point toward the –y direction and is shown with $\vec{F}_3$ in the diagram, while the electric force between this particle and the 6.00-nC particle located on the x axis will be repulsive and point toward the –x direction, shown with $\vec{F}_6$ in the diagram. The resultant force should point toward the third quadrant, as shown in the diagram with $\vec{F}_R$. Although the charge on the x axis is greater in magnitude, its distance from the origin is three times larger than the –3.00-nC charge. We expect the resultant force to make a small angle with the –y axis and be approximately equal in magnitude with F_3.



ANS. FIG. P23.11

From the diagram in ANS. FIG. P23.11, the two forces are perpendicular, and the components of the resultant force are

$$F_x = -F_6 = -\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6.00\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(0.300\ \text{m})^2}$$

$$= -3.00\times10^{-6}\ \text{N} \quad (\text{to the left})$$

$$F_y = -F_3 = -\left(8.99\times10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{(3.00\times10^{-9}\ \mathrm{C})(5.00\times10^{-9}\ \mathrm{C})}{(0.100\ \mathrm{m})^2}$$

$$= -1.35\times10^{-5}\ \mathrm{N}\quad\text{(downward)}$$

(a) The forces are perpendicular, so the magnitude of the resultant is

$$F_R = \sqrt{(F_6)^2 + (F_3)^2} = \boxed{1.38\times10^{-5}\ \mathrm{N}}$$

(b) The magnitude of the angle of the resultant is

$$\theta = \tan^{-1}\left(\frac{F_3}{F_6}\right) = 77.5°$$

The resultant force is in the third quadrant, so the direction is

$$\boxed{77.5°\ \text{below}\ -x\ \text{axis}}$$

16. Two small metallic spheres, each of mass m = 0.200 g, are suspended as pendulums by light strings of length L as shown in Figure P23.16. The spheres are given the same electric charge of 7.2 nC, and they come to equilibrium when each string is at an angle of θ = 5.00° with the vertical. How long are the strings?

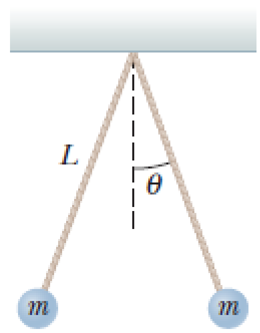


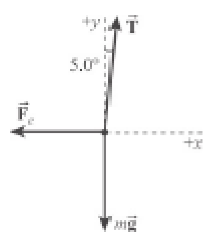
Figure P23.16

P23.16 Consider the free-body diagram of one of the spheres shown in ANS. FIG. P23.16. Here, T is the tension in the string and F_e is the repulsive electrical force exerted by the other sphere.

$$\sum F_y = 0 \Rightarrow T\cos 5.0° = mg$$

or
$$T = \frac{mg}{\cos 5.0°}$$

$$\sum F_x = 0 \Rightarrow F_e = T\sin 5.0° = mg\tan 5.0°$$



ANS. FIG. P23.16

At equilibrium, the distance separating the two spheres is $r = 2L\sin 5.0°$.

Thus, $F_e = mg\tan 5.0°$ becomes $\frac{k_e q^2}{(2L\sin 5.0°)^2} = mg\tan 5.0°$, which yields

$$L = \sqrt{\frac{k_e q^2}{mg\tan 5.0°(2\sin 5.0°)^2}}$$

$$= \sqrt{\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.20\times10^{-9}\ \mathrm{C})^2}{(0.200\times10^{-3}\ \mathrm{kg})(9.80\ \mathrm{m/s^2})\tan 5.0°(2\sin 5.0°)^2}} = \boxed{0.299\ \mathrm{m}}$$

25. Four charged particles are at the corners of a square of side a as shown in Figure P23.25. Determine (a) the electric field at the location of charge q and (b) the total electric force exerted on q.

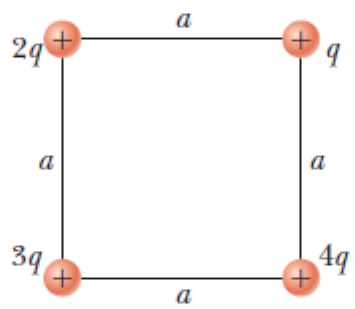


Figure P23.25

P23.25 We sum the electric fields from each of the other charges using Equation 23.7 for the definition of the electric field.

The field at charge q is given by

$$\vec{\mathbf{E}} = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3$$

(a) Substituting for each of the charges gives

$$\vec{\mathbf{E}} = \frac{k_e(2q)}{a^2}\hat{\mathbf{i}} + \frac{k_e(3q)}{2a^2}\left(\hat{\mathbf{i}}\cos 45.0° + \hat{\mathbf{j}}\sin 45.0°\right) + \frac{k_e(4q)}{a^2}\hat{\mathbf{j}}$$

$$= \frac{k_e q}{a^2}\left[\left(2 + \frac{3}{2}\cos 45.0°\right)\hat{\mathbf{i}} + \left(\frac{3}{2}\sin 45.0° + 4\right)\hat{\mathbf{j}}\right]$$

$$= \boxed{\frac{k_e q}{a^2}\left(3.06\hat{\mathbf{i}} + 5.06\hat{\mathbf{j}}\right)}$$

(b) The electric force on charge q is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \boxed{\frac{k_e q^2}{a^2}\left(3.06\hat{\mathbf{i}} + 5.06\hat{\mathbf{j}}\right)}$$

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29. M In Figure P23.29, determine the point (other than infinity) at which the electric field is zero.

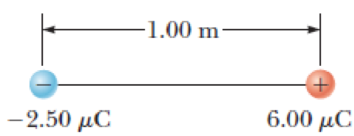
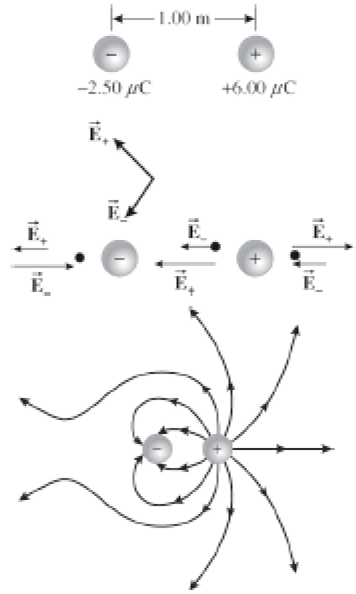


Figure P23.29

P23.29 The field of the positively-charged object is everywhere pointing radially away from its location. The object with negative charge creates everywhere a field pointing toward its different location. These two fields are directed along different lines at any point in the plane except for points along the extended line joining the particles; so the two fields cannot be oppositely-directed to add to zero except at some location along this line, which we take as the x axis. Observing the middle panel of ANS. FIG. P23.29, we see that at points to the left of the negatively-charged object, this particle creates field pointing to the right and the positive object creates field to the left. At some point along this segment the fields will add to zero. At locations in between the objects, both create fields pointing toward the left, so the total field is not zero. At points to the right of the positive 6-μC object, its field is directed to the right and is stronger than the leftward field of the –2.5-μC object, so the two fields cannot be equal in magnitude to add to zero. We have argued that only at a certain point straight to the left of both charges can the fields they separately produce be opposite in direction and equal in strength to add to zero.



ANS. FIG. P23.29

Let x represent the distance from the negatively-charged particle (charge q_-) to the zero-field point to its left. Then 1.00 m + x is the distance from the positive particle (of charge q_+) to this point. Each field is separately described by

$$\vec{\mathbf{E}} = k_e q\hat{\mathbf{r}}/x^2$$

so the equality in magnitude required for the two oppositely-directed vector fields to add to zero is described by

$$\frac{k_e|q_-|}{x^2} = \frac{k_e|q_+|}{(1\text{ m}+x)^2}$$

It is convenient to solve by taking the square root of both sides and cross-multiplying to clear of fractions:

$$|q_-|^{1/2}(1\text{ m}+x) = q_+^{1/2}x$$

$$1\text{ m}+x = \left(\frac{6.00}{2.50}\right)^{1/2} x = 1.55x$$

$$1\text{ m} = 0.549x$$

and $x = \boxed{1.82\text{ m}}$ to the left of the negatively-charged object.

31. Three point charges are located on a circular arc as shown in Figure P23.31. (a) What is the total electric field at P, the center of the arc? (b) Find the electric force that would be exerted on a -5.00-nC point charge placed at P.

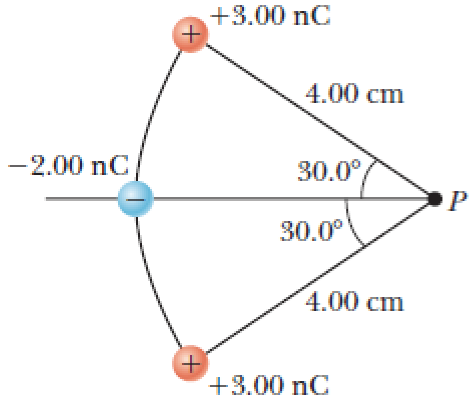
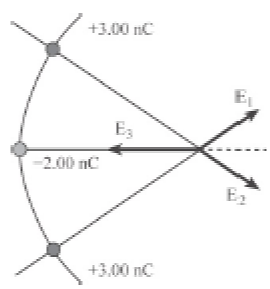


Figure P23.31

P23.31 Call $Q = 3.00$ nC and $q = |-2.00\text{ nC}| = 2.00$ nC, and $r = 4.00$ cm $= 0.040\,0$ m. Then,

$$E_1 = E_2 = \frac{k_e Q}{r^2} \quad \text{and} \quad E_3 = \frac{k_e q}{r^2}$$



ANS. FIG. P23.31

Then,

$$E_y = 0$$

$$E_x = E_{total} = 2\frac{k_e Q}{r^2}\cos 30.0° - \frac{k_e q}{r^2}$$

$$E_x = \frac{k_e}{r^2}(2Q\cos 30.0° - q)$$

$$E_x = \left[\frac{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{(0.040\,0\ \text{m})^2}\right] \times\left[2\left(3.00\times10^{-9}\ \text{C}\right)\cos 30.0° - 2.00\times10^{-9}\ \text{C}\right]$$

$$= 1.80\times10^4\ \text{N/C}$$

(a) $\boxed{1.80\times10^4\ \text{N/C to the right}}$

(b) The electric force on a point charge placed at point P is

$$F = qE = \left(-5.00\times10^{-9}\ \text{C}\right)E = \boxed{-8.98\times10^{-5}\ \text{N (to the left)}}$$

33. AMT A small, 2.00-g plastic ball is suspended by a 20.0-cm-long string in a uniform electric field as shown in Figure P23.33. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical, what is the net charge on the ball?

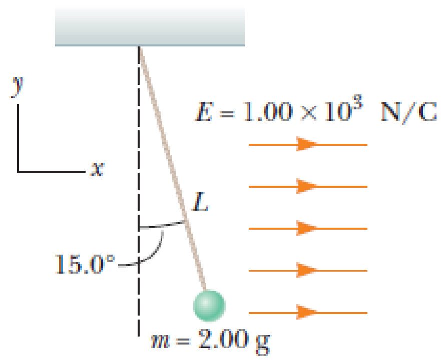


Figure P23.33

***23.33** From the free-body diagram shown in ANS. FIG. P23.33,

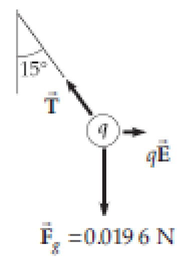
$$\sum F_y = 0: \qquad T\cos 15.0° = 1.96 \times 10^{-2}\text{ N}$$

So $\quad T = 2.03 \times 10^{-2}$ N.

From $\sum F_x = 0$, we have $qE = T\sin 15.0°$,

or

$$q = \frac{T\sin 15.0°}{E} = \frac{(2.03 \times 10^{-2}\text{ N})\sin 15.0°}{1.00 \times 10^3\text{ N/C}}$$
$$= 5.25 \times 10^{-6}\text{ C} = \boxed{5.25\ \mu\text{C}}$$



ANS. FIG. P23.33

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43. W A continuous line of charge lies along the x axis, extending from $x = +x_0$ to positive infinity. The line carries positive charge with a uniform linear charge density λ_0. What are (a) the magnitude and (b) the direction of the electric field at the origin?

P23.43 (a) Magnitude $|E| = \int \frac{k_e dq}{x^2}$, where $dq = \lambda_0 dx$

$$E = k_e\lambda_0 \int_{x_0}^{\infty} \frac{dx}{x^2} = k_e\lambda_0\left(-\frac{1}{x}\right)\Bigg|_{x_0}^{\infty} = \boxed{\frac{k_e\lambda_0}{x_0}}$$

(b) The charge is positive, so the electric field points away from its source, $\boxed{\text{to the left}}$.

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44. A thin rod of length ℓ and uniform charge per unit length λ lies along the x axis as shown in Figure P23.44. (a) Show that the electric field at P, a distance d from the rod along its perpendicular bisector, has no x component and is given by $E = 2k_e\lambda \sin\theta_0/d$. (b) **What If?** Using your result to part (a), show that the field of a rod of infinite length is $E = 2k_e\lambda/d$.

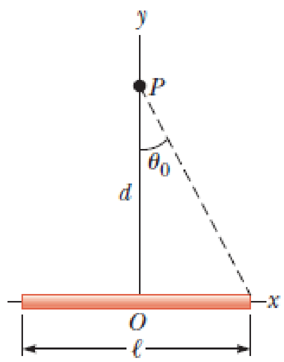


Figure P23.44

P23.44 (a) The electric field at point P, due to each element of length dx, is $dE = \dfrac{k_e dq}{x^2 + d^2}$ and is directed along the line joining the element to point P. By symmetry,

$$E_x = \int dE_x = 0$$

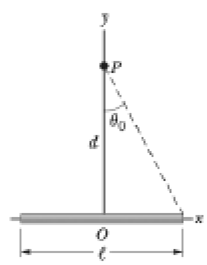
and since $dq = \lambda dx$,

$$E = E_y = \int dE_y = \int dE \cos\theta$$

where $\cos\theta = \dfrac{y}{\sqrt{x^2 + d^2}}$.

Therefore, $E = 2k_e\lambda d \displaystyle\int_0^{\ell/2} \frac{dx}{\left(x^2 + d^2\right)^{3/2}} = \boxed{\dfrac{2k_e\lambda \sin\theta_0}{d}}$

with $\sin\theta_0 = \dfrac{\ell/2}{\sqrt{(\ell/2)^2 + d^2}}$.



ANS. FIG. P23.44

(b) For a bar of infinite length, $\theta_0 = 90°$ and $E_y = \boxed{\dfrac{2k_e\lambda}{d}}$.

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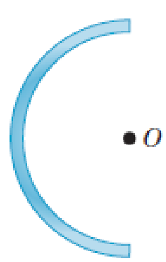
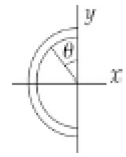
45. **M** A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown in Figure P23.45. The rod has a total charge of $-7.50\ \mu$C. Find (a) the magnitude and (b) the direction of the electric field at O, the center of the semicircle.



P23.45 Due to symmetry, $E_y = \int dE_y = 0$, and

$E_x = -\int dE \sin\theta = -k_e \int \frac{dq \sin\theta}{r^2}$ where $dq = \lambda ds = \lambda r d\theta$; the component E_x is negative because charge $q = -7.50\ \mu\text{C}$, causing the net electric field to be directed to the left.



ANS. FIG. P23.45

$$E_x = -\frac{k_e\lambda}{r}\int_0^\pi \sin\theta d\theta = -\frac{k_e\lambda}{r}(-\cos\theta)\Big|_0^\pi = -\frac{2k_e\lambda}{r}$$

where $\lambda = \frac{|q|}{L}$ and $r = \frac{L}{\pi}$. Thus,

$$E_x = -\frac{2k_e|q|\pi}{L^2} = -\frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(7.50\times10^{-6}\ \text{C})\pi}{(0.140\ \text{m})^2}$$

$$E_x = -2.16\times10^7\ \text{N/C}$$

(a) magnitude $E = \boxed{2.16\times10^7\ \text{N/C}}$

(b) $\boxed{\text{to the left}}$

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67. **M** A charged cork ball of mass 1.00 g is suspended on a light string in the presence of a uniform electric field as shown in Figure P23.67. When $\vec{\mathbf{E}} = (3.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}) \times 10^5$ N/C, the ball is in equilibrium at $\theta = 37.0°$. Find (a) the charge on the ball and (b) the tension in the string.

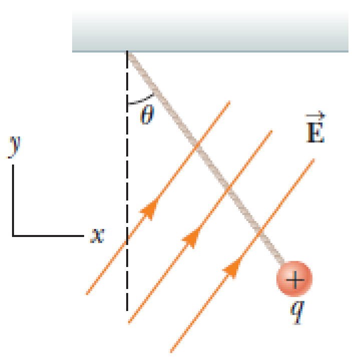


Figure P23.67
Problems 67 and 68.

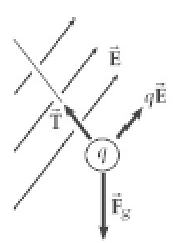
P23.67 ANS. FIG. P23.67 shows the free-body diagram for Newton's second law gives

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{T}} + q\vec{\mathbf{E}} + \vec{\mathbf{F}}_g = 0$$

We are given

$$E_x = 3.00 \times 10^5\ \text{N/C}$$

and $$E_y = 5.00 \times 10^5\ \text{N/C}$$



ANS. FIG. P23.67

Applying Newton's second law or the first condition for equilibrium in the x and y directions,

$$\sum F_x = qE_x - T\sin 37.0° = 0 \qquad [1]$$

$$\sum F_y = qE_y + T\cos 37.0° - mg = 0 \qquad [2]$$

(a) We solve for T from equation [1]:

$$T = \frac{qE_x}{\sin 37.0°}$$

and substitute into equation [2] to obtain

$$q = \frac{mg}{E_y + \dfrac{E_x}{\tan 37.0^\circ}}$$

$$= \frac{(1.00 \times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{5.00 \times 10^5\ \text{N/C} + \left(\dfrac{3.00 \times 10^5\ \text{N/C}}{\tan 37.0^\circ}\right)}$$

$$q = \boxed{1.09 \times 10^{-8}\ \text{C}}$$

(b) Using the above result for q in equation [1], we find that the tension is

$$T = \frac{qE_x}{\sin 37.0^\circ} = \frac{(1.09 \times 10^{-8}\ \text{C})(3.00 \times 10^5\ \text{N/C})}{\sin 37.0^\circ}$$

$$= \boxed{5.44 \times 10^{-3}\ \text{N}}$$

82. **Review.** A negatively charged particle $-q$ is placed at the center of a uniformly charged ring, where the ring has a total positive charge Q as shown in Figure P23.82. The particle, confined to move along the x axis, is moved a small distance x along the axis (where $x << a$) and released. Show that the particle oscillates in simple harmonic motion with a frequency given by

$$f = \frac{1}{2\pi}\left(\frac{k_e qQ}{ma^3}\right)^{1/2}$$

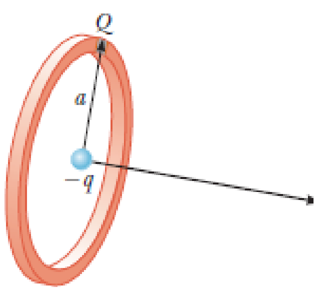


Figure P23.82

P23.82 The field on the axis of the ring is calculated in Example 19.6 in the chapter text as

$$E = E_x = \frac{k_e xQ}{(x^2 + a^2)^{3/2}}$$

The force experienced by a charge $-q$ placed along the axis of the ring is

$$F = -k_e Qq\left[\frac{x}{(x^2 + a^2)^{3/2}}\right]$$

and when $x << a$, this becomes

$$F = -\left(\frac{k_e Qq}{a^3}\right)x$$

This expression for the force is in the form of Hooke's law, with an effective spring constant of

$$k = \frac{k_e Qq}{a^3}$$

Since $\omega = 2\pi f = \sqrt{\dfrac{k}{m}}$, we have

$$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{k_e Qq}{ma^3}}}$$