



King Abdullah School of Engineering — Department of Basic Sciences
Spring Semester 2022-2023 — Principles of Probability 20336 — First Exam — March 29
Duration: 60 minutes — Instructor: Dr. Maisa Khader

Name (Arabic) and ID: _____

First Part

1. (10 points) Fill in the blanks with the **CORRECT ANSWER ONLY**. Approximate to five decimal places as needed.

(1) A little girl has six building blocks and is required to select four of them at a time to build a model. If the order of the blocks in each model is not important, how many models can she build $15 = {}^6C_4$

(2) Two events A and B have the following probabilities: $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$. The $P(A^c \cup B^c) =$ 0.7

(3) A bag contains 8 red balls, 4 green, and 8 yellow balls. A ball is drawn at random from the bag and it is found not to be one of the red balls. What is the probability that it is a green ball?

Let G denote the event that the selected ball is green ball and R denote the event that the selected ball is red $P(\text{ } \underline{G | R^c} \text{ }) =$ $\frac{4/20}{12/20} = \frac{4}{12} = \frac{1}{3}$

(4) Let the random variable X denote the number of heads in four flips of a fair coin. The $P(X \geq 3) =$ $1 - P(X \leq 2) = 1 - [({}^4C_0)(\frac{1}{2})^0(\frac{1}{2})^4 + ({}^4C_1)(\frac{1}{2})^1(\frac{1}{2})^3 + ({}^4C_2)(\frac{1}{2})^2(\frac{1}{2})^2]$

(5) Twenty percent of the population of a particular city wear glasses. If you randomly stop people from the city, determine the probability that it takes exactly 10 tries to get a person who wears glasses. $P(X = 10) =$ $(0.2)(1-0.2)^{10-1} = (0.2)(0.8)^9 = 0.0268435$

(6) The number of traffic tickets that a certain traffic officer gives out on any day has been shown to have a Poisson distribution with mean of 7. What is the probability that on one particular day the officer gave out no ticket?

$P(X = 0) =$ $\frac{e^{-7} 7^0}{0!} = e^{-7} = 9.1188 \times 10^{-4}$

(7) The random variable X is discrete random variable if and only if CDF is discontinuous function.

2. (4 points) Identify whether the following statements are TRUE or FALSE

- (1) If A and B are independent events, then so are the events A and B^c True
- (2) Let A , B and C be three events. The expression for the event A occurs but neither B nor C occurs in set notation is $A \cap (B^c \cup C^c)$ False
- (3) The variance σ_X^2 of the random variable X can be any real number. False
- (4) Let A and B be two events. The $P(A - B) = P(A \cap B^c)$ True

Second Part

Show all your work! If insufficient work is shown, full credit may not be awarded.

3. The probability of rain on a day of the year selected at random is 0.25 in a certain city. The local weather forecast is correct 60% of the time when the forecast is rain and 80% of the time for other forecasts.

Please identify the events before you start solving the problem. Also, write all the rules you are applying to solve the problem.

- (1) (2 points) What is the probability that the forecast on a day selected at random is correct?

$$\begin{array}{l}
 0.25 \text{ } R \begin{cases} 0.6 \text{ } CF \\ 0.4 \text{ } \overline{CF} \end{cases} \\
 0.75 \text{ } \overline{R} \begin{cases} 0.8 \text{ } CF \\ 0.2 \text{ } \overline{CF} \end{cases}
 \end{array}$$
 { Let R be the event for rain forecast
 CF " " " for correct forecast

$$\begin{aligned}
 P(CF) &= P(R) P(CF|R) + P(\overline{R}) P(CF|\overline{R}) \quad \textcircled{1} \\
 &= (0.25)(0.6) + (0.75)(0.8) \quad \textcircled{0.5} \\
 &= \frac{3}{4} = 0.75
 \end{aligned}$$

- (2) (2 points) Given that the forecast is correct. What is the probability that it rains?

$$\begin{aligned}
 P(R|CF) &= \frac{P(R \cap CF)}{P(CF)} = \frac{P(R) P(CF|R)}{P(CF)} \quad \textcircled{1} \\
 &= \frac{(0.25)(0.6)}{(0.25)(0.6) + (0.75)(0.8)} = \frac{0.15}{0.75} = \frac{1}{5} = 0.2
 \end{aligned}$$

$$\frac{{}^6C_3 {}^4C_2}{{}^{10}C_5} = \frac{10}{21} = 0.4761904762$$

4. (2 points) A certain library has a collection of 10 books on probability theory. Six of these books were written by American authors and four were written by foreign authors. If I select five of these books at random, what is the probability that two of them were written by American authors and three of them were written by foreign authors?

$$p = \frac{{}^6C_2 {}^4C_3}{{}^{10}C_5} = \frac{60}{252}$$

$$\frac{5}{21} = 0.2380952381$$

5. The discrete random variable X has the following pmf:

X	0	1	2
$P(X=x)$	b $\frac{1}{6}$	$2b$ $\frac{2}{6}$	$3b$ $\frac{3}{6}$

$$\frac{3}{6}$$

$$\frac{1}{6}$$

$$\frac{2}{6}$$

- (1) (1 point) What is the value of b ?

$$b + 2b + 3b = 1$$

$$6b = 1 \Rightarrow b = \frac{1}{6}$$

- (2) (3 points) Determine the CDF of X ?

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{6} & 0 \leq x < 1 \\ \frac{3}{6} & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

$$\begin{aligned} x &< 0 \\ 0 &\leq x < 1 \\ 1 &\leq x < 2 \\ x &\geq 2 \end{aligned}$$

$$\left\{ \begin{array}{l} 0 \\ \frac{3}{6} \\ \frac{4}{6} \\ 1 \end{array} \right.$$

$$\begin{aligned} x &< 0 \\ 0 &\leq x < 1 \\ 1 &\leq x < 2 \\ x &\geq 2 \end{aligned}$$

(3) (1 point) Determine the value of $P(0 < X \leq 2)$?

$$P(0 < X \leq 2) = F(2) - F(0) \\ = 1 - \frac{1}{6} = \frac{5}{6}$$

$$1 - \frac{3}{6} = \frac{3}{6}$$

or

$$P(X=1) + P(X=2) = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$

$$\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$$

(4) (1 point) Find $E(X)$

$$E(X) = \sum_{x=0,1,2} x P(X=x)$$

$$= 0P(X=0) + 1P(X=1) + 2P(X=2)$$

$$= \frac{2}{6} + 2 \frac{3}{6}$$

$$= \frac{2}{6} + 1 = \frac{8}{6} = \frac{4}{3}$$

$$0 \frac{3}{6} + 1 \frac{1}{6} \\ + 2 \frac{2}{6} \\ = \frac{1}{6} + \frac{4}{6} \\ \frac{5}{6}$$