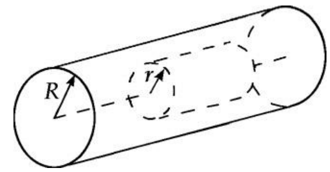
P24.33 If ρ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length L and radius r, contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho\pi r^2 L$. Because the charge distribution is long, no electric flux passes through the circular end caps; $\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = EdA\cos 90.0° = 0$. The curved surface has $\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = EdA\cos 0°$, and E must be the same strength everywhere over the curved surface.



ANS. FIG. P24.33

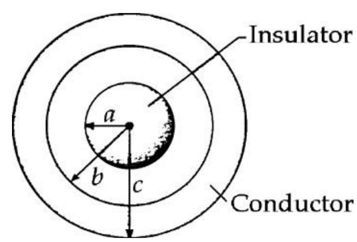
Gauss's law, $\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \dfrac{q}{\epsilon_0}$, becomes $E \displaystyle\int_{\substack{\text{Curved}\\\text{Surface}}} dA = \dfrac{\rho\pi r^2 L}{\epsilon_0}$.

Now the lateral surface area of the cylinder is $2\pi rL$:

$$E(2\pi r)L = \frac{\rho\pi r^2 L}{\epsilon_0}$$

Thus, $\vec{\mathbf{E}} = \boxed{\dfrac{\rho r}{2\epsilon_0} \text{ radially away from the cylinder axis}}$.

P24.54 Choose as each gaussian surface a concentric sphere of radius r. The electric field will be perpendicular to its surface, and will be uniform in strength over its surface. The density of charge in the insulating sphere is



ANS. FIG. P24.54

$$\rho = Q / \left(\tfrac{4}{3}\pi a^3\right)$$

(a) The sphere of radius $r < a$ encloses charge

$$q_{\text{in}} = \rho\left(\frac{4}{3}\pi r^3\right) = \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = \boxed{Q\left(\frac{r}{R}\right)^3}$$

(b) Applying Gauss's law to this sphere reveals the magnitude of the field at its surface.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}\left(\frac{r}{a}\right)^3 \to E = \frac{1}{4\pi \epsilon_0}\frac{Qr}{a^3} = \boxed{k_e \frac{Qr}{a^3}}$$

(c) For a sphere of radius r with $a < r < b$, the whole insulating sphere is enclosed, so the charge within is Q: $q_{\text{in}} = \boxed{Q}$.

(d) Gauss's law for this sphere becomes:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0} \to E = \frac{1}{4\pi \epsilon_0}\frac{Q}{r^2} = \boxed{k_e \frac{Q}{r^2}}$$

(e) For $b \le r \le c$, $\boxed{E = 0}$ because there is no electric field inside a conductor.

(f) For $b \le r \le c$, we know $E = 0$. Assume the inner surface of the hollow sphere holds charge Q_{inner}. By Gauss's law,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$0 = \frac{Q + Q_{\text{inner}}}{\epsilon_0} \to Q_{\text{inner}} = \boxed{-Q}$$

(g) The total charge on the hollow sphere is zero; therefore, charge on the outer surface is opposite to that on the inner surface:

$$Q_{\text{outer}} = -Q_{\text{inner}} = \boxed{+Q}$$

(h) A surface of area A holding charge Q has surface charge $\sigma = q/A$. The solid, insulating sphere has small surface charge because its total charge Q is uniformly distributed throughout its volume. The $\boxed{\text{inner surface of radius } b}$ has smaller surface area, and therefore larger surface charge, than the outer surface of radius c.