$f_X(x)$ to determine whether a pdf

① $f_X(x) \geq 0$ ② $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(t)\,dt$$

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx \qquad Var(X) = E(X - E(X))^2$$

$$= E(X^2) - E^2(X)$$

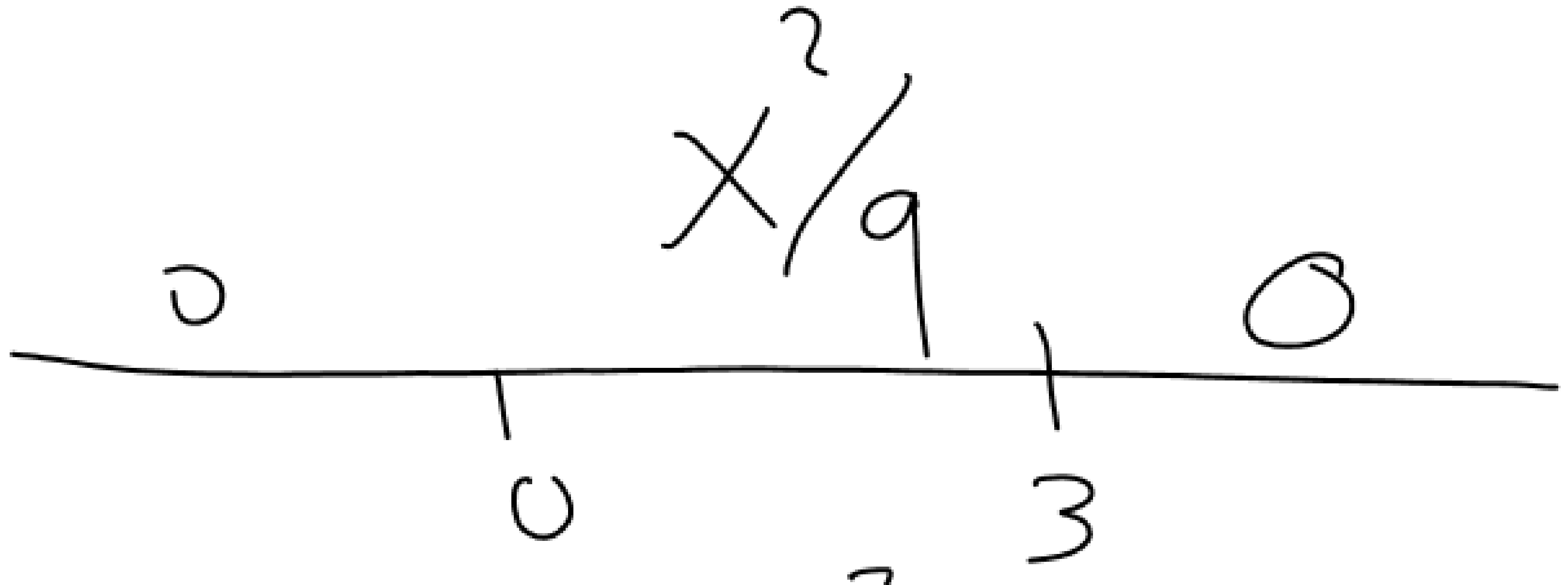
$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

$$E(g(X)) = \begin{cases} \sum_{\forall x} g(x)\, P(X = x) & X \text{ is discrete random variable} \\ \int_{-\infty}^{\infty} g(x)\, f_X(x)\,dx & X \text{ is cont. random variables.} \end{cases}$$

$$f_X(x) = \boxed{\quad} \text{ on } \boxed{a \le x \le b}\ ;\ f_X(x) \text{ is zero otherwise}$$

$\underline{Ex}$ Back to our example

$$f_X(x) = \begin{cases} \dfrac{x^2}{9} & 0 < x < 3 \\ 0 & \text{o.w} \end{cases}$$



⑥ Find $E(X)$

$\underline{\underline{Sol}}$ $E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_0^3 x \frac{x^2}{9}\,dx = \int_0^3 \frac{x^3}{9}\,dx = \frac{x^4}{36}\Big|_0^3$

$$= \frac{1}{36}\left[3^4 - 0\right] = \frac{3^4}{36} = \frac{9}{4}$$

⑦ Find $Var(X)$

$$Var(X) = E(X^2) - E^2(X)$$

$$E(X^2) = \int_0^3 \frac{x^4}{9}\,dx = \frac{x^5}{45}\Big|_0^3 = \frac{1}{45}\left[3^5 - 0\right] = \frac{3^5}{45} = \frac{27}{5}$$

$$Var(X) = \frac{27}{5} - \left(\frac{9}{4}\right)^2 = \frac{27}{80}$$

Ex A cont. random variable X has a pdf

$$f_X(x) = \begin{cases} \frac{x}{50} & 0 \le x < \cancel{b}^{\,10} \\ 0 & \text{o.w} \end{cases}$$

① Find the value of b?

Sol $\int_0^b \frac{x}{50}\,dx = 1$

$$\left.\frac{x^2}{100}\right|_0^b = 1$$

$$\frac{1}{100}\left[b^2 - 0\right] = 1$$

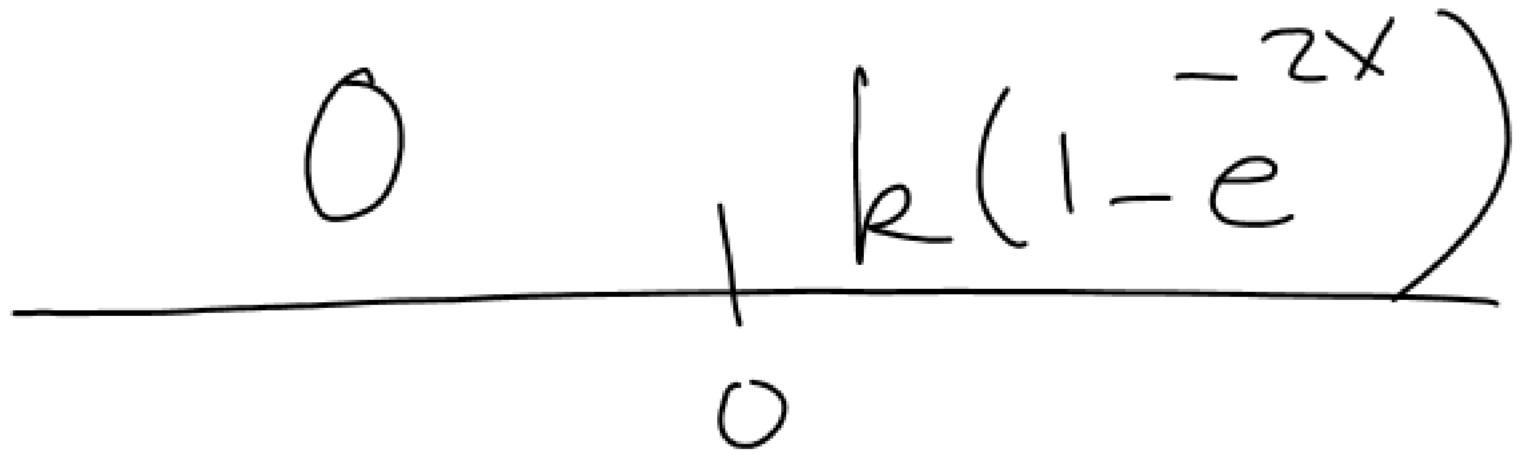
$$\frac{b^2}{100} = 1 \Rightarrow b^2 = 100$$

$$b = \mp 10$$

b is +ve

Ex The CDF of a cont. random variable is

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \overset{1}{\cancel{k}}(1-e^{-2x}) & x \geq 0 \end{cases}$$

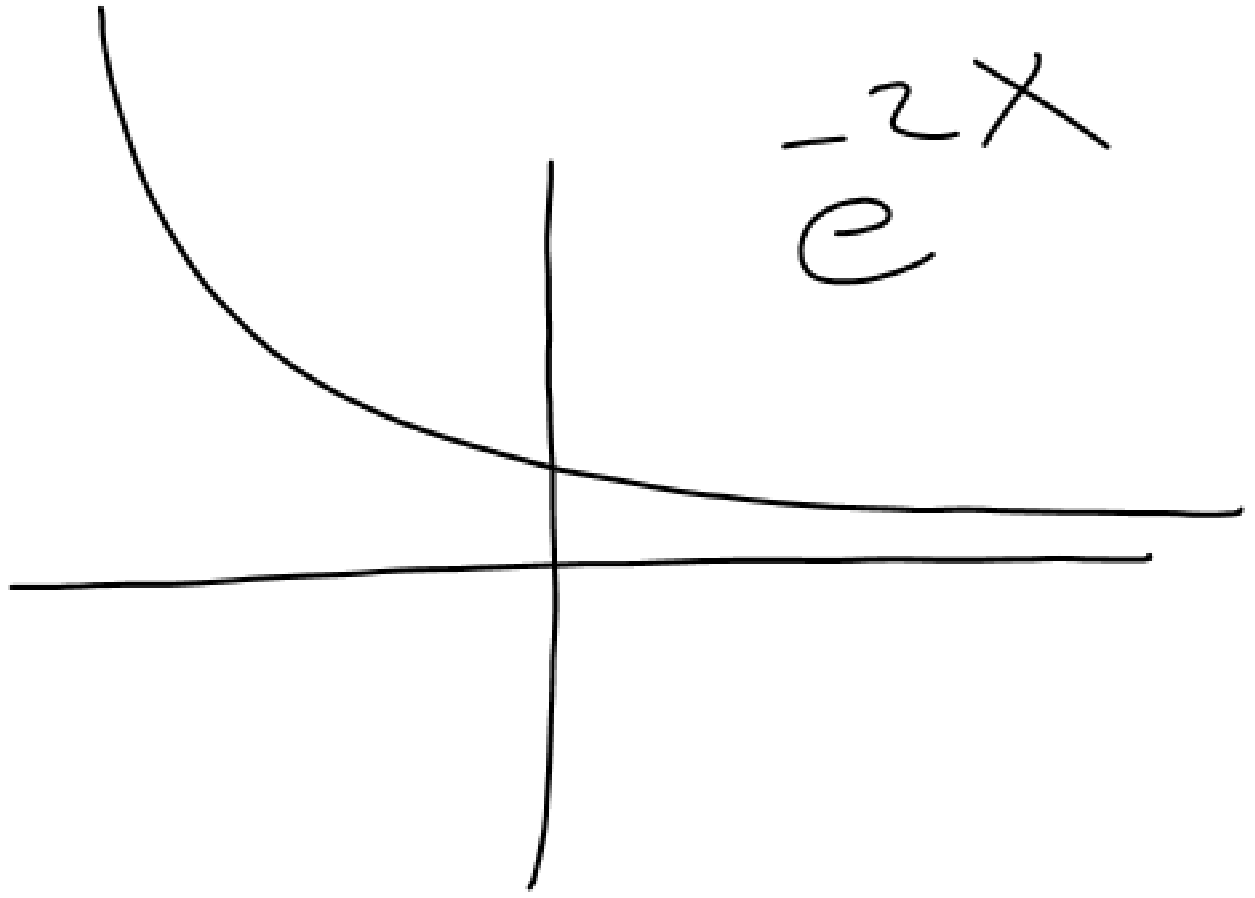


Find the value of k?

Sol

$$\lim_{x \to 0^+} F_X(x) \overset{?}{=} \lim_{x \to 0^-} F_X(x)$$ Since X is cont. random variable variable. $F_X(x)$ has to be cont. fcn.

$$\lim_{x\to 0^+} F_X(x) = \lim_{x\to 0} k(1-e^{-2x}) = k(1-e^{0}) = 0$$



$$\lim_{x\to 0^-} F_X(x) = \lim_{x\to 0} 0 = 0$$

$$|||$$

We know $\lim_{x\to\infty} F_X(x) = 1$

$$\lim_{x\to\infty} k(1-e^{-2x}) = 1 \Rightarrow k(1-0) = 1 \quad \therefore k = 1$$

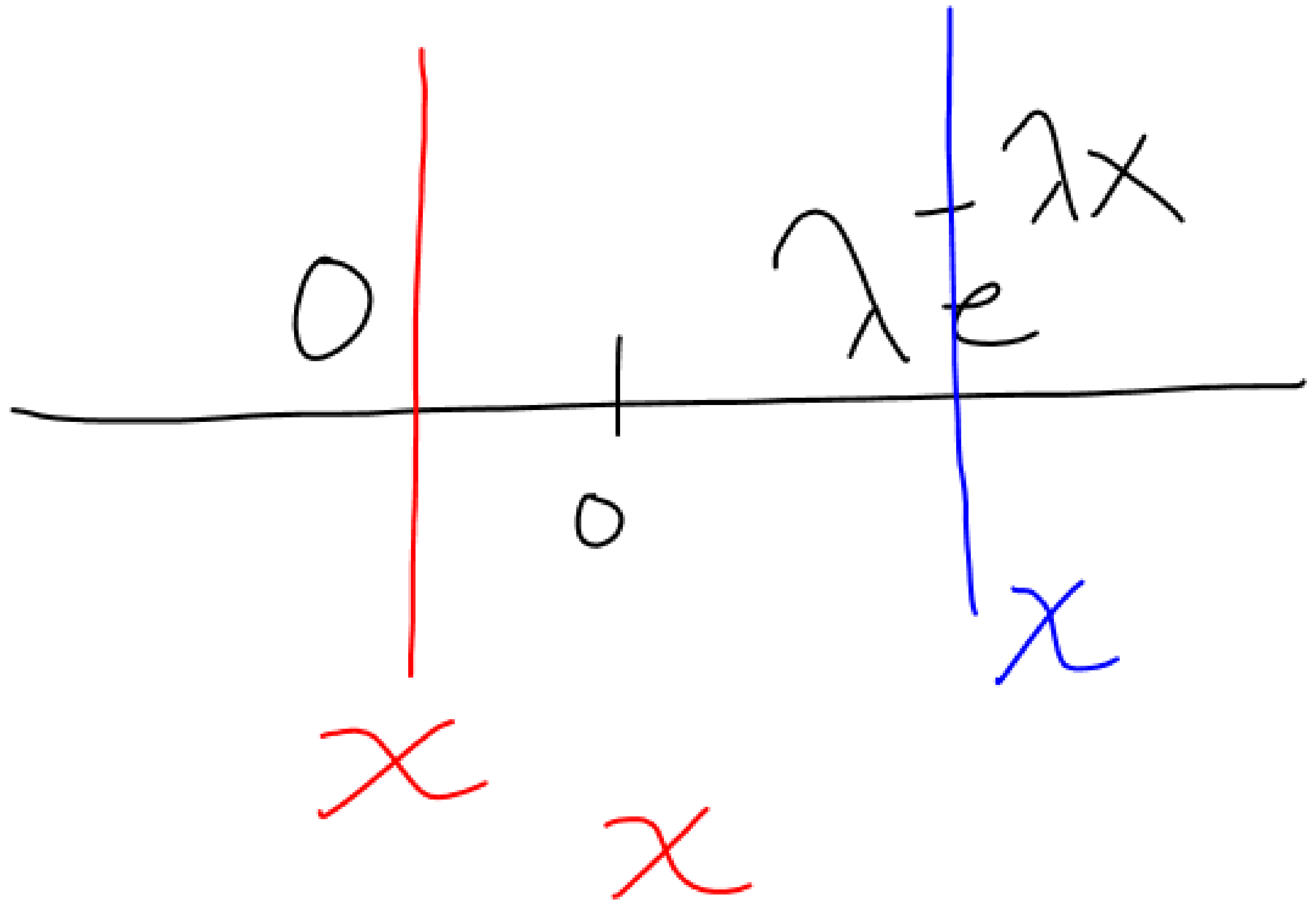
Chapter 4

Famous Continuous Probability Distributions

(1) Exponential Distribution

A cont. random variable X is called exponential random variable (X has exponential probability Distribution) if for some $\lambda > 0$ the pdf of X is

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$



Where $$F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\lambda x} & x \geq 0 \end{cases}$$

$$\lim_{x \to 0^+} F_X(x) = \lim_{x \to 0^-} F_X(x)$$

$$\lim_{x \to 0} \left(1 - e^{-\lambda x}\right) = 1 - 1 = 0$$

$$\lim_{x \to 0} 0 = 0$$

$x < 0$: $$\int_{-\infty}^{x} 0 \, dt = 0$$

$x \geq 0$ $$\int_{-\infty}^{x} f_X(t)\,dt = \int_{-\infty}^{0} 0 \, dt + \int_{0}^{x} \lambda e^{-\lambda t} \, dt$$

Note $\int a^{bx+c}\,dx = \dfrac{a^{bx+c}}{b\ln a} + C$

$$\int_0^x \lambda e^{-\lambda t}\,dt = \frac{\lambda e^{-\lambda t}}{-\lambda}\Bigg|_0^x = -\left[e^{-\lambda x} - e^{0}\right] = 1 - e^{-\lambda x}$$

$$E(X) = \int_{-\infty}^{\infty} x\, f_X(x)\,dx = \int_0^{\infty} x\lambda e^{-\lambda x}\,dx$$ Improper integral

$$= \lambda \lim_{b\to\infty} \int_0^b x e^{-\lambda x}\,dx$$

$$\int_0^b x e^{-\lambda x}\, dx$$

LIATE

u		dv
x	$+$	$e^{-\lambda x}$
1	$-$	$\dfrac{e^{-\lambda x}}{-\lambda}$
0	$+\int$	$\dfrac{e^{-\lambda x}}{\lambda^2}$