Experiment

↓

Sample space

↓

Random variable

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Cont. — prob. density fcn

pdf $f_X(x)$ fcn

CDF $F_X(x) = P(X \leq x)$

Discrete — pmf: $P(X = x)$ Prob. [prob. mass fcn]

CDF: $P(X \leq x)$ Prob. [commulative density fcn]

Ex Toss a dice twice, Let X be the sum of the tosses.

① What is the sample space?

Sol $S = \{(1,1),(1,2),\cdots(1,6),(2,1),(2,2),\cdots,(2,6)$
$(3,1),(3,2),\cdots(3,6),(4,1)\cdots\cdots,(4,6)$
$(5,1),\cdots\cdots(5,6),(6,1),\cdots\cdots,(6,6)\}$

② What is the random variable?

$X = \{2,3,4,5,6,7,8,9,10,11,12\}$

(3) Find the probability that X is less than or equal to 4

Sol $P(X \leq 4) = P(X=2) + P(X=3) + P(X=4)$

$P(X=2)$: (1,1) — $P(X=3)$: (1,2), (2,1) — $P(X=4)$: (1,3), (2,2), (3,1)

$$= \frac{1}{36} + \frac{2}{36} + \frac{3}{36}$$

$$= \frac{6}{36} = \frac{1}{6}$$

(4) Find the pmf of X $\left[P(X=x)\right]$

X	2	3	4	5	6	7	8	9	10	11	12	
$P(X=x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{36}{36}=1$

(1) $0 \leq P(X=x_i) \leq 1 \quad \forall i=1,2,3,\ldots,n$

(2) $\sum_{i=1}^{n} P(X=x_i) = 1$

Example Toss a dice twice. Let X be the maximum of the tosses

① What is the random variable X

Sol $X = \{1, 2, 3, 4, 5, 6\}$

② What is the pmf $\left(P(X = x)\right)$

Sol

X	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$	$\frac{36}{36}$

OR

$$P(X = x) = \frac{2x-1}{36}$$

$x = 1, 2, 3, 4, 5, 6$

OR

$$P(X=x)=\begin{cases}\frac{1}{36} & X=1\\ 3/36 & X=2\\ 5/36 & X=3\\ 7/36 & X=4\\ 9/36 & X=5\\ 11/36 & X=6\end{cases}$$

$P(X=7)=$

Properties of pmf. $P(X=x)$

1. $0 \le P(X=x_i) \le 1 \quad \forall i=1,2,\dots,n$
2. $\sum_{i=1}^{n} P(X=x_i)=1$
3. $P(X=x)=0; \forall x \notin X$
4. Let A be an event interms of X
   $P(A)=\sum_{\forall x \in A} P(X=x)$

Ex Let X be a discrete random variable whose pmf is given by

X	-1	1	2	3	5
$P(X=x)$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{4}{8}$	0

(1) Determine whether the given table is truely a pmf?

(1) $0 \leq P(X=x_i) \leq 1$ ✓

(2) $\sum_{\forall x} P(X=x_i) = P(X=-1) + P(X=1) + P(X=2) + P(X=3) = \frac{2}{8} + \frac{1}{8} + \frac{1}{8} + \frac{4}{8} = 1$ ✓

$\forall x = -1, 1, 2, 3$

(2) Find the probability that $X=2$

Sol $P(X=2)=\frac{1}{8}$

(3) Find the probability that X is equal to 0

Sol $P(X=0)=0$

(4) Find the probability that X is less than or equal to 2.6

Sol $P(X\leq 2.6)=\sum_{\forall X\leq 2.6} P(X=x)=\sum_{X=-1,1,2} P(X=x)=P(X=-1)+P(X=1)+P(X=2)$

$$= \frac{2}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

OR $P(X \leq 2.6) = 1 - P(\overline{X \leq 2.6}) = 1 - P(X > 2.6)$

$$= 1 - P(X = 3)$$

$$= 1 - \frac{4}{8} = \frac{4}{8} = 1/2$$

(5) Find the probability that $|X| = 1$

Sol $P(|X| = 1) = \sum_{|X|=1} P(X = x) = \sum_{X=-1,1} P(X = x) = P(X=-1) + P(X=1)$

$$= \frac{2}{8} + \frac{1}{8} = 3/8$$

(6) Find the probability that X is greater than 0

Sol $P(X>0) = P(X=1) + P(X=2) + P(X=3)$

$= \frac{1}{8} + \frac{1}{8} + \frac{4}{8}$

$= \frac{6}{8} = \frac{3}{4}$

OR $P(X>0) = 1 - P(\overline{X>0}) = 1 - P(X \leq 0)$

$= 1 - P(X=-1) - P(X=0) = 1 - \frac{2}{8} = \frac{6}{8} = \frac{3}{4}$

The Commulative Density fcn for Discrete Random Variables

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pmf: $P(X=x)$ prob. CDF $P(X \leq x) = F_X(x)$

$= p_X(x)$

- $0 \leq P(X \leq x) \leq 1$
- As x increases $F_X(x)$ does Not decrease.

$x_1 \leq x_2 \qquad F_X(x_1) \leq F_X(x_2)$

- $P(X \leq x_0) = \sum_{\forall X \leq x_0} P(X = x)$
- $x < x_{min} \qquad P(X \leq x) = F_X(x) = 0$

  $x \geq x_{max} \qquad P(X \leq x) = F_X(x) = 1$

} Discrete

$$\lim_{x \to -\infty} F_X(x) = 0 = F_X(-\infty)$$

$$\lim_{x \to \infty} F_X(x) = 1 = F_X(\infty)$$

} Cont R.V