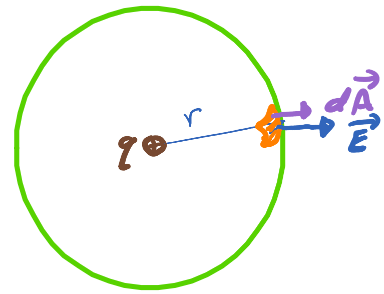
Lecture 11 (12-1)

Thursday, March 18, 2021 12:01 PM

24.2 Gauss's Law

Assume a point charge +q surrounded by a spherical surface of radius r.

+q is at the center



$\Rightarrow \Phi_E = ?$

$\Phi_E = \oint \vec{E}\cdot d\vec{A}$

But $d\vec{A} \parallel \vec{E} \Rightarrow \theta = 0^\circ$

$\Rightarrow \vec{E}\cdot d\vec{A} = E\,dA\cos 0^\circ$

$= E\,dA$

$\Rightarrow \Phi_E = \oint E\,dA = \underbrace{E}_{E = k_e\frac{q}{r^2}} \underbrace{\oint dA}_{\text{surface area of a sphere} = 4\pi r^2}$

$\Rightarrow \Phi_E = \left(k_e \frac{q}{\cancel{r^2}}\right)(4\pi \cancel{r^2})$

$= (k_e q)(4\pi) = \left(\frac{q}{\cancel{4\pi}\varepsilon_0}\right)(\cancel{4\pi})$

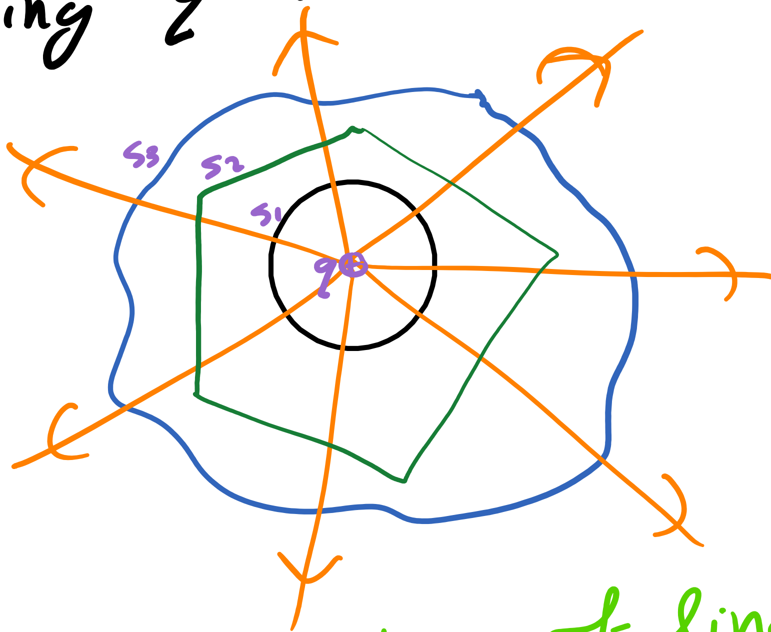
$\Rightarrow \boxed{\Phi_E = \frac{q}{\varepsilon_0}}$ $\rightarrow \Phi_E \propto q$ inside the closed surface

$\rightarrow \Phi_E$ is indep. of r !!

• Now consider several colsed surfaces

shown:

surrounding q as shown.



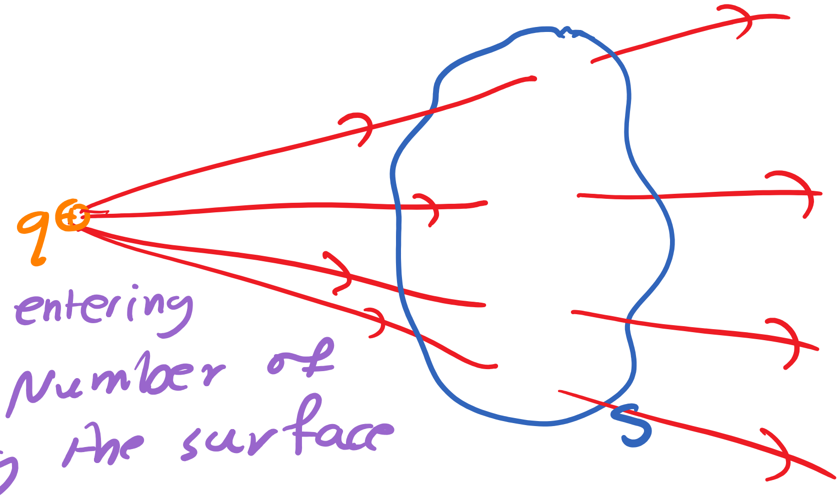
$\Phi_{S_1} = \frac{q}{\varepsilon_0}$

Note that $\Phi_E \propto$ number of lines passing through the surface, where the number of lines passing through S_1 = through S_2 = through S_3

$$\Rightarrow \Phi_{S_1} = \Phi_{S_2} = \Phi_{S_3} = \frac{q}{\varepsilon_0}$$

$\Rightarrow$ The net flux through any closed surface surrounding a point charge q is given by $\frac{q}{\varepsilon_0}$ and is independent of the shape of that surface.

• Consider q outside a closed surface



We note that:

Number of lines entering the surface = Number of lines leaving the surface

$\Rightarrow$ The net Φ_E through any closed surface surrounding NO charge is ZERO.

Back to Ex 24.1 (cube)
$\Rightarrow \Phi_{net} = 0$ since no charge inside the surface

Gauss's Law: $\boxed{\Phi_E = \oint \vec{E} \cdot d\vec{A} = \dfrac{q_{in}}{\varepsilon_0}}$

q_{in} = Net charge inside the surface

Conceptual Example 24.2 **Flux Due to a Point Charge**

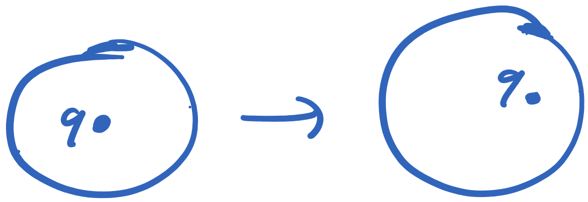
A spherical gaussian surface surrounds a point charge q. Describe what happens to the total flux through the surface if **(A)** the charge is tripled, **(B)** the radius of the sphere is doubled, **(C)** the surface is changed to a cube, and **(D)** the charge is moved to another location inside the surface.

$\Phi_E = \frac{q}{\varepsilon_0}$

(A) If $q \to 3q \Rightarrow \Phi_E \to \frac{3q}{\varepsilon_0}$

(B) $r \to 2r \Rightarrow \Phi_E$ same

(C) If sphere $\to$ cube $\Rightarrow \Phi_E$ same

(D)  $\Rightarrow \Phi_E$ same

Q: Find Φ_E through the closed surfaces S_1, S_2, S_3, and $S4$ shown below:

$\Phi_E = \frac{q_{in}}{\varepsilon_0}$

$\Phi_1 = \frac{-q+2q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$

$\Phi_2 = \frac{+q+2q-3q}{\varepsilon_0} = \text{zero}$

$\Phi_3 = \frac{+q+q+2q}{\varepsilon_0} = \frac{4q}{\varepsilon_0}$

$\Phi_4 = \text{zero}$