1st Quiz @ 7:30 on Monday Not 8:00

Experiment

$\downarrow$

Sample Space

$\downarrow$

Event $\longrightarrow$ Prob.

1.3 Definition of Probability

Defn:- A collection of events $A_1, A_2, \ldots, A_n$ are called mutually exclusive (mutually disjoint) (m.e) if $A_i \cap A_j = \phi \quad \forall\, i \neq j$

Defn:- A collection of events $A_1, A_2, \ldots, A_n$ are called collectively exchuastive (c.e) if $A_1 \cup A_2 \cup \cdots \cup A_n = S$

Defn:- A collection of events which are m.e and c.e are called a partition of S

Ex Toss a dice once. Determine whether the following events forming a partition of S or not by giving the reason(s)

① $A = \{2,4,6\}$ $B = \{1\}$ $C = \{3,5\}$

الحل $S = \{1, 2, 3, 4, 5, 6\}$

$$\left.\begin{array}{l} A \cap B = \phi \\ A \cap C = \phi \\ B \cap C = \phi \end{array}\right\} m.e$$

$$A \cup B \cup C = \{1, 2, 3, 4, 5, 6\}$$
$$= S$$
$$(c.e)$$

A, B, C are patitions of S

② $A = \{2, 4, 6\}$ $B = \{3, 5\}$

Sol

$A \cap B = \phi$ (m.e)

$A \cup B = \{2, 3, 4, 5, 6\} \neq S$ Not (c.e)

A, B are not partition

③ $A = \{2, 4, 6\}$ $B = \{3, 5\}$ $C = \{1, 5\}$

Sol $A \cap B = \phi$ $A \cap C = \phi$ $B \cap C = \{5\}$ Not (m.e)

$A \cup B \cup C = \{1, 2, 3, 4, 5, 6\} = S$ (c.e)

A, B, C Not partition

4 $A = \{2, 4, 6\}$ $B = \{5\}$ $C = \{2\}$

Sol $A \cap B = \emptyset$ $A \cap C = \{2\}$ $B \cap C = \emptyset$

Not m.e

$A \cup B \cup C = \{2, 4, 5, 6\} \neq S$ Not c.e

A, B, C are not partition

Consider a random experiment whose sample space is S. Let A be an event the probability of A occurs denoted by

$P(A)$ is a real number

① $0 \leq P(A) \leq 1$ ② $P(S) = 1$

③ If $A_1, A_2, \ldots, A_n$ are collection of m.e events

Then

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$$

OR $$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

The classical Defn of probability

Consider a random experiment whose sample space is S is finite, and all outcomes have the same chance of occurance (equally likely events). Then

$$P(A) = \frac{|A|}{|S|} = \frac{\text{\# of elements of } A}{\text{\# of elements of } S}$$

proof $0 \leq P(A) \leq 1$??

$$\frac{0}{|S|} \leq \frac{|A|}{|S|} \leq \frac{|S|}{|S|} \Rightarrow 0 \leq P(A) \leq 1 \checkmark$$

② $P(S) = 1$??

$$P(S) = \frac{|S|}{|S|} = 1 \checkmark$$

③ $$P\left(\bigcup_{i=1}^{n} A_i\right) \stackrel{?}{=} \sum_{i=1}^{n} P(A_i) \quad , \quad A_i \cap A_j = \phi \quad \forall i \neq j$$

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = \frac{|A_1 \cup A_2 \cup \cdots \cup A_n|}{|S|}$$

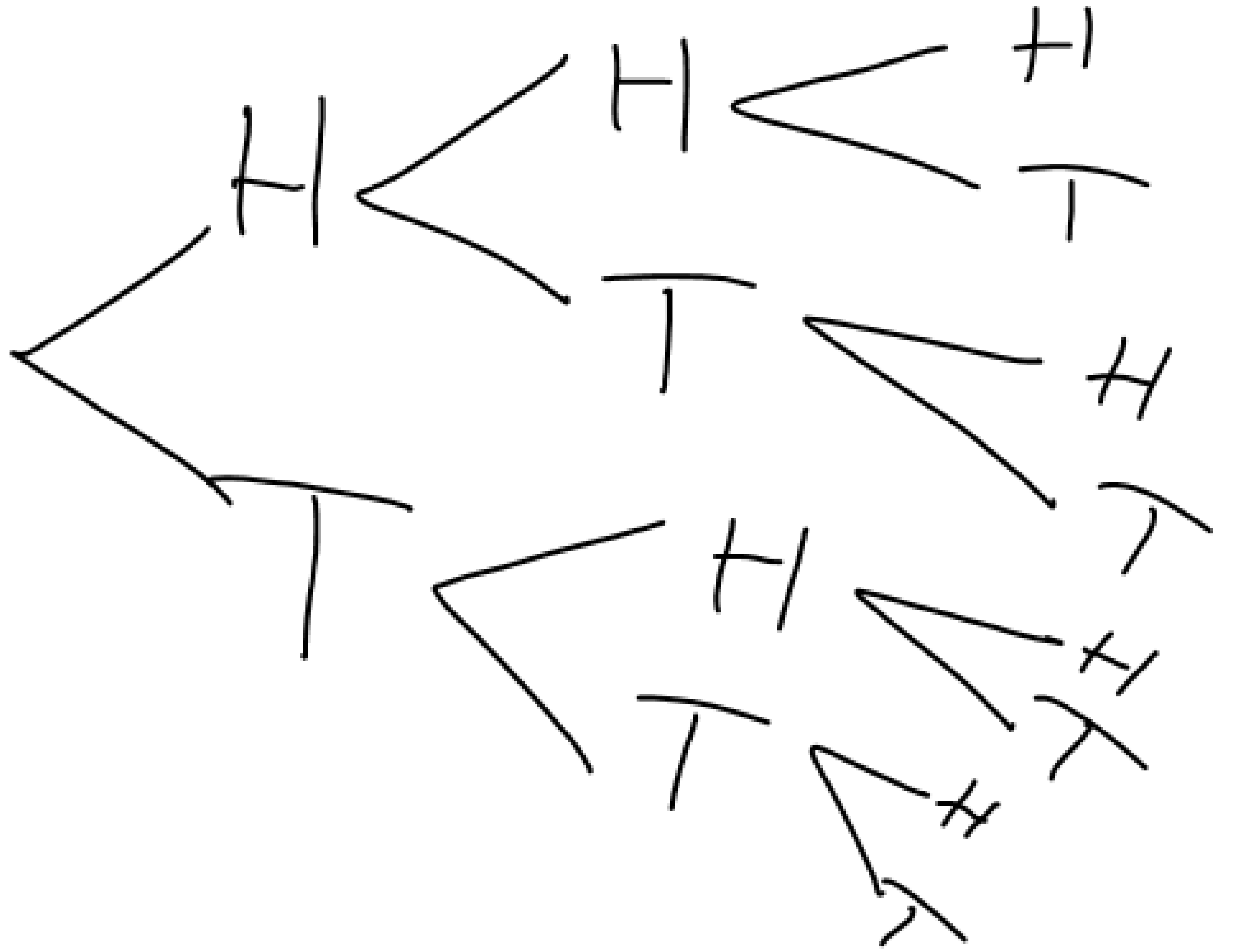
$$= \frac{|A_1| + |A_2| + \cdots + |A_n|}{|S|} = P(A_1) + P(A_2) + \cdots + P(A_n)$$

<u>Ex</u> The experiment is Toss a coin 3 times

① Find the number of elements in the sample space

<u>Sol</u> $\underline{2}\ \underline{2}\ \underline{2} = 2^3 = 8$

② What is the sample space



$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

(3) Find the prob. that the outcome has 3 tails

Sol

Let A be the event of getting 3 tails

$$A = \{TTT\}$$

$$P(A) = \frac{1}{8}$$

(4) What is the prob. that the 2^{nd} toss is head

Sol Let B be the event that the 2^{nd} toss is head

$B = \{HHH, HHT, THH, THT\}$

$$P(B) = \frac{4}{8} = \frac{1}{2}$$

5 Find the prob. that the number of heads in the outcomes is 1 OR 2

one

Sol Let C_1 be the event of getting head

C_2 be the event of getting two heads

$P(C_1 \cup C_2)$

$C_1 = \{HTT, THT, TTH\}$

$C_2 = \{HHT, HTH, THH\}$

$C_1 \cap C_2 = \phi$

m, e

$C_1 \cup C_2 = \{HTT, THT, TTH, HHT, HTH, THH\}$

$$P(C_1 \cup C_2) = 6/8 = \frac{3}{4}$$

OR $P(C_1 \cup C_2) = P(C_1) + P(C_2)$

$$= \frac{3}{8} + \frac{3}{8} = \frac{6}{8}$$

1.5 Elementary Set Theory

1.6 Probability Laws

Let A, B, C be events from a random experiment

1. $A \cup B = B \cup A$
2. $A \cap B = B \cap A$
3. $(A \cup B) \cup C = A \cup (B \cup C)$

   $(A \cap B) \cap C = A \cap (B \cap C)$
4. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
5. $A \cup \overline{A} = S$ $\quad$ $A \cap \overline{A} = \emptyset$

$A \cup S = S$

$A \cap S = A$

$A \cup \phi = A$

$A \cap \phi = \phi$

$\overline{\phi} = S$

$\overline{S} = \phi$

Probability Laws:-

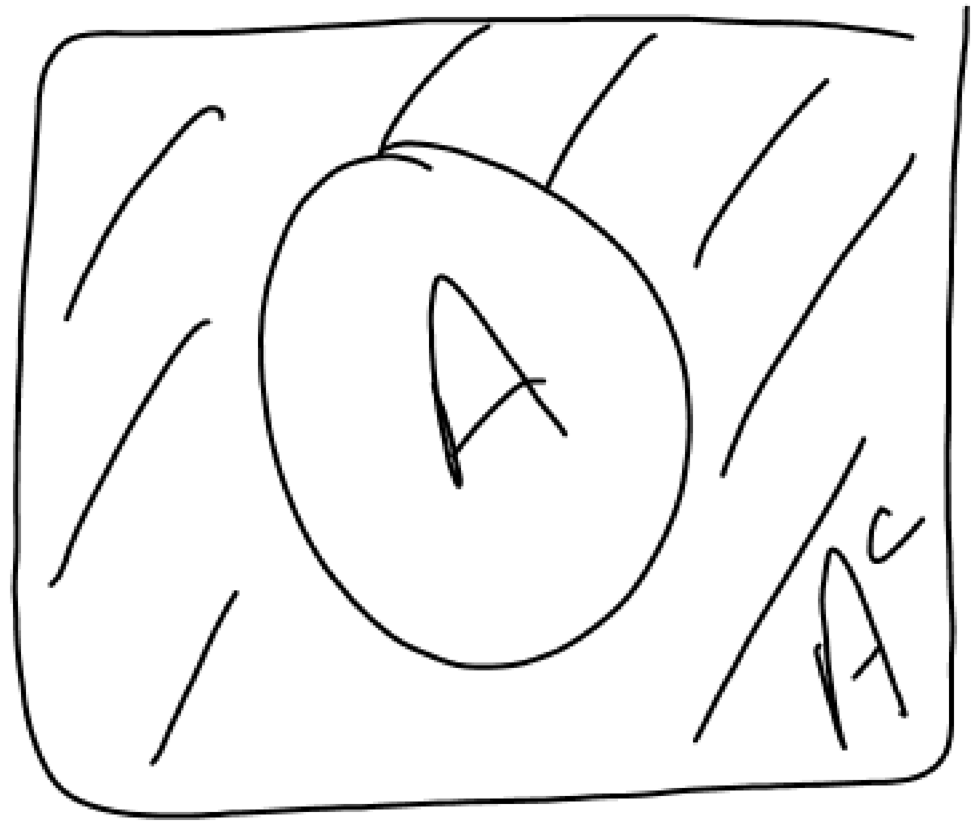
1. $P(\phi) = 0$
2. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

③ $P(A) + P(\bar{A}) = 1$

④ Total probability Law

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

<u>proof</u> ③ $P(A) + P(\bar{A}) = 1$

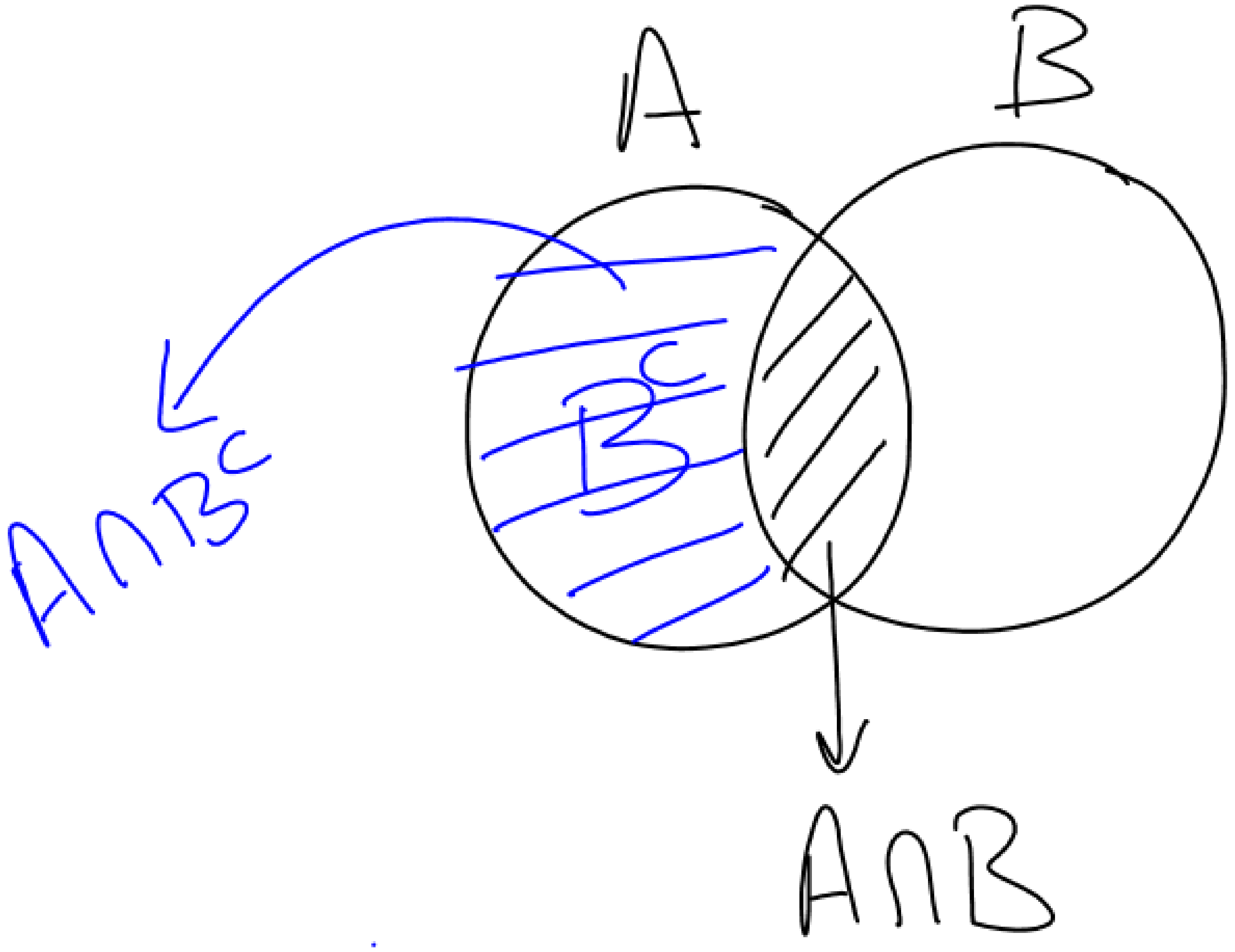


$$A \cup \bar{A} = S$$

$$P(A \cup \bar{A}) = P(S)$$

$P(A) + P(\bar{A}) = 1$; $A, \bar{A}$ are m.e

(4) $P(A) = P(A \cap B^c) + P(A \cap B)$



$$A = \underbrace{(A \cap B) \cup (A \cap B^c)}_{m.e}$$

$P(A) = P(A \cap B) + P(A \cap B^c)$