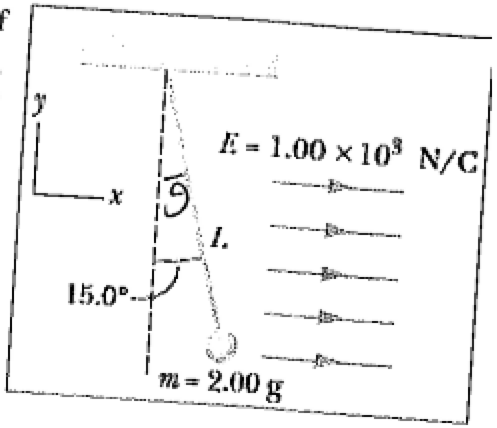
Problem 1 (28 points)

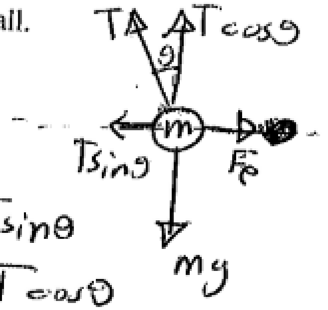
A) A small plastic ball of mass $m = 2.0$ g is suspended by a string of length $L = 20$ cm in a uniform electric field $\vec{E}$ as shown in the figure. If the ball is in equilibrium when the string makes an angle $\theta = 15°$ with the vertical. ***(20 points)***



1) Draw the **Free-Body-Diagram** of the ball.

2) Find the net **charge** (q) on the ball.

①



② $\Sigma F_x = 0 \Rightarrow F_e = T\sin\theta$

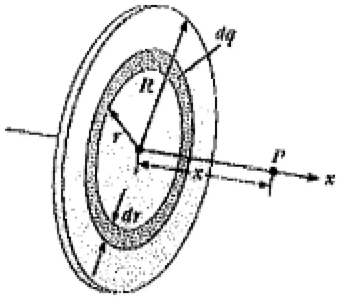
$\Sigma F_y = 0 \Rightarrow mg = T\cos\theta$

$$\Rightarrow \frac{F_e}{mg} = \tan\theta, \quad \text{but } F_e = qE \Rightarrow \frac{qE}{mg} = \tan\theta$$

$$\Rightarrow q = \frac{mg\tan\theta}{E} = \frac{(2\times10^{-3})(10)\tan 15°}{1\times10^{3}}$$

$$\boxed{q = 5.36\ \mu C}$$

B) A disk of radius R has a uniform surface charge density σ. The electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk is given by: $E = 2\pi k_e \sigma \left[1 - \frac{x}{(R^2 + x^2)^{\frac{1}{2}}}\right]$, calculate the **electric field** at a point: ***(8 points)***



1) Close to the disk ($x << R$).

$$\Rightarrow E \approx 2\pi k_e \sigma [1 - 0] = 2\pi k_e \sigma$$

$$\Rightarrow \boxed{E \approx \frac{\sigma}{2\varepsilon_0}}$$

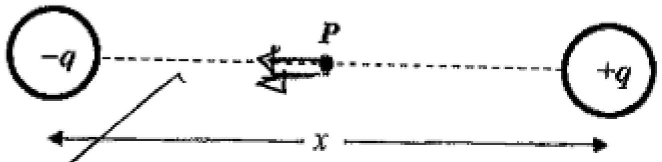
2) Far away from the disk ($x >> R$).

↳ It look like a point charge!!

You can use binomial expansion to show this if you want!!

Problem 2 (22 points)

A) Two point charges q and $-q$ are separated by a distance x as shown below. **Draw** the electric field lines between the two charges and determine the magnitude **and** the direction of the **net electric** field at point ***P***, midway between the charges. ***(12 points)***



$$E = \frac{k_e q}{r^2} = \frac{k_e q}{\left(\frac{x}{2}\right)^2} = \frac{4 k_e q}{x^2} \quad \text{for each charge.}$$

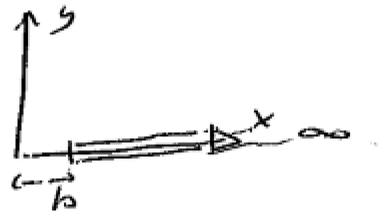
$$\Rightarrow \vec{E}_{net} = 2E(-\hat{i}) = \boxed{\frac{8 k_e q}{x^2}(-\hat{i}) = \vec{E}_{net}}$$

OR $E_{net} = \dfrac{8 k_e q}{x^2}$ to the left

B) A line of charge starts at $x = +b$ and extends to positive infinity. The linear charge density is $\lambda = c/x$, where c is a constant. Determine the **electric field** at the origin. ***(10 points)***

$$E = k_e \int \frac{dq}{r^2}, \qquad dq = \lambda\, dx = \frac{c}{x}\, dx$$

$$r^2 \longrightarrow x^2$$



$$\Rightarrow E = k_e c \int_b^{\infty} \frac{dx}{x^3}$$

$$= \frac{k_e c}{2}\left[\frac{-1}{x^2}\right]_b^{\infty} = \frac{k_e c}{2}\left[0 - \left(\frac{-1}{b^2}\right)\right]$$

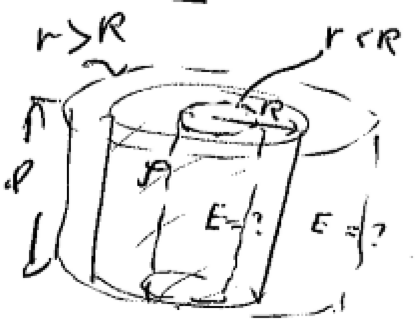
$$\Longrightarrow \boxed{E = \frac{k_e c}{2b^2}}$$

Note that the closed surface integral over **E**.d**A** equals:
The surface integral through the flat surfaces + the surface integral through the side surface, but through the flat surfaces is equal to ZERO since **E** is perpendicular to d**A**
➔ the flux through the closed surface will be that from the side surface only, which is equal to E(2πr*l*)

Problem 3 (27 points)

A) Consider a long, cylindrical charge distribution of radius R with a uniform charge density ρ. Use Gauss's law to find the **electric field** at distance r from the axis for **(1)** $r < R$ and **(2)** $r > R$. ***(20 points)***

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$



$$\oint E\,dA = \frac{q_{in}}{\varepsilon_0} \Rightarrow E\oint dA = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow E(2\pi r l) = \frac{q_{in}}{\varepsilon_0}$$

(1) $r<R \Rightarrow q_{in} = \rho\pi r^2 l$

$$\Rightarrow E(2\cancel{\pi} \cancel{r} \cancel{l}) = \frac{\rho \cancel{\pi} r^{\cancel{2}} \cancel{l}}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\rho}{2\varepsilon_0} r}$$

(2) $r>R \Rightarrow q_{in} = \rho\pi R^2 l$

$$\Rightarrow E(2\cancel{\pi} r \cancel{l}) = \frac{\rho \cancel{\pi} R^2 \cancel{l}}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\rho R^2}{2\varepsilon_0 r}}$$

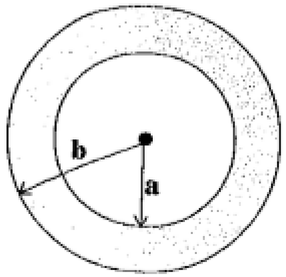
B) The total electric flux through a closed surface of area $A = 20\ m^2$ is $\Phi = 1.81$ N. m²/C. Find the **net charge** enclosed by this surface. ***(7 points)***

$$\Phi = \frac{q_{in}}{\varepsilon_0} \Rightarrow q_{in} = \Phi\,\varepsilon_0 = (1.81)(8.85\times 10^{-12})$$

$$\Rightarrow \boxed{q_{in} = 16\times 10^{-12}\,C}$$

Problem 4 (23 points)

A) A spherical conducting shell, of inner radius **a** and outer radius **b**, carries net charge of $+3q$. A point charge $-7q$ is placed in the center of the shell as shown. ***(15 points)***



1) Find the **electric field** for $r < a$ and $a < r < b$.

$r<a \Rightarrow E = -7\frac{k_e q}{r^2}$

$a<r<b \Rightarrow E = 0$, conductor in electrostatic equilibrium.

2) Find the **surface charge density** on the **outer** surface of the conducting shell.

$\sigma_{out} = \frac{q_{outer}}{A_{outer}}$, $A_{outer} = 4\pi b^2$, $q_{outer} = ?$

$q_{inner} = +7q \Rightarrow q_{shell} = q_{inner} + q_{outer} \Rightarrow +3q = +7q + q_{outer}$

$\Rightarrow q_{outer} = -4q$

$\Rightarrow \sigma_{out} = \frac{-4q}{4\pi b^2} = \frac{-q}{\pi b^2}$

B) Two parallel and infinite non-conducting sheets, one carries a positive surface charge density $\sigma_1 = 5\,\sigma$ and the other carries a ~~negative~~ positive surface charge density $\sigma_2 = 2\,\sigma$. Calculate the **electric field** at a point **outside** the sheets. (*Note that the magnitude of the electric field due to an infinite non-conducting sheet carries a surface charge density* σ *is* $E = \frac{\sigma}{2\varepsilon_0}$). **(8 points)**

If σ_1 & σ_2 are both positive

$$E = \left|\frac{\sigma_1}{2\varepsilon_0}\right| + \left|\frac{\sigma_2}{2\varepsilon_0}\right| = \frac{5\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} = 3.5\frac{\sigma}{\varepsilon_0}$$

If σ_1 is (+) & σ_2 is (−)

$$\Rightarrow E = \left|\left|\frac{\sigma_1}{2\varepsilon_0}\right| - \left|\frac{\sigma_2}{2\varepsilon_0}\right|\right| = \left|\frac{5\sigma}{2\varepsilon_0} - \frac{2\sigma}{2\varepsilon_0}\right| = 1.5\frac{\sigma}{\varepsilon_0}$$