U with Multiple Charges

If there are more than two charges, then find U for each pair of charges and add them.

For three charges:

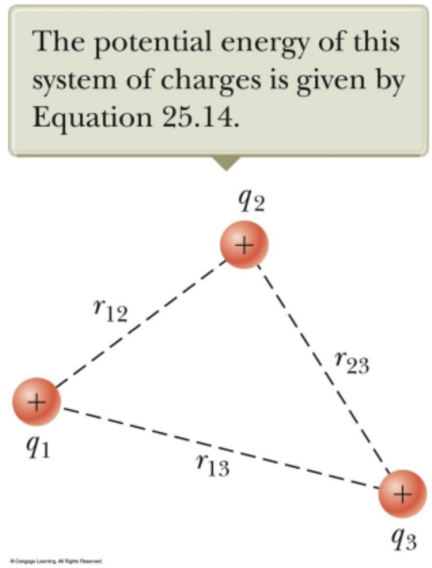
$$U = k_e\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right) \longrightarrow \text{Eqn. 25.14}$$

- The result is independent of the order of the charges.

Suppose q_1 is fixed, with q_2 and q_3 are at ∞

➔ W needed to bring q_2 from ∞ to its position near q_1 is given by the **first** term of Eqn. 25.14.

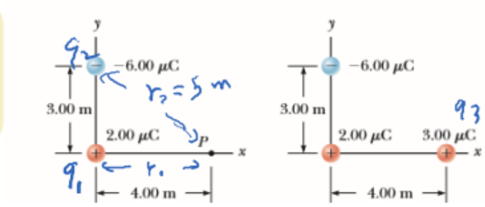
➔ W needed to bring q_3 from ∞ to its near q_1 and q_2 is given by the **second and third** terms Eqn. 25.14.



Example 25.3 The Electric Potential Due to Two Point Charges

As shown in Figure 25.10a, a charge $q_1 = 2.00\ \mu C$ is located at the origin and a charge $q_2 = -6.00\ \mu C$ is located at (0, 3.00) m.

(A) Find the total electric potential due to these charges at the point P, whose coordinates are (4.00, 0) m.



For part (B), note that we Did not use Eqn. 25.14. Why?

Since we are bringing only one Charge from infinity!

SOLUTION

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (8.988 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{2.00 \times 10^{-6}\ \text{C}}{4.00\ \text{m}} + \frac{-6.00 \times 10^{-6}\ \text{C}}{5.00\ \text{m}}\right)$$

$$= \boxed{-6.29 \times 10^3\ \text{V}}$$

(B) Find the change in potential energy of the system of two charges plus a third charge $q_3 = 3.00\ \mu C$ as the latter charge moves from infinity to point P (Fig. 25.10b).

$\Delta U = U_P - \cancel{U_\infty}$

$U_f = q_3V_P$

$\{\ \Delta U = q_3\,\Delta V$

$\ \ U_f - \cancel{U_i} = q_3(V_P - \cancel{V_\infty})\ \Rightarrow U_f = q_3 V_P$

Note that $U_i = 0$

$\Delta U = U_f - U_i = q_3V_P - 0 = (3.00 \times 10^{-6}\ \text{C})(-6.29 \times 10^3\ \text{V})$

$= \boxed{-1.89 \times 10^{-2}\ \text{J}}$

$= q_3\left[\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right]$

$= \left(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$

Obtaining the Value of the Electric Field from the Electric Potential (25.4)

$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

$\vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

Remember $\boxed{\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}}$ ➔ $dV = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$

Assume, to start, that the field has only an *x* component ➔ $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E_x\,dx$. ➔ $E_x = -\dfrac{dV}{dx}$, $V \equiv V(x)$

Similar statements would apply to the *y* and *z* components. $dV = -E_x dx$

In general, the electric potential is a function of all three dimensions.

Given $V(x, y, z)$ you can find E_x, E_y and E_z as partial derivatives: $E_x = -\dfrac{\partial V}{\partial x}$ $E_y = -\dfrac{\partial V}{\partial y}$ $E_z = -\dfrac{\partial V}{\partial z}$

Where: $\vec{\mathbf{E}} = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}}$ and $E = \sqrt{E_x^2 + E_y^2 + E_z^2}$

$dV = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ and for equipotential surface $dV = 0$ and thus $dV = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$, which requires $\vec{\mathbf{E}} \perp d\vec{\mathbf{s}}$ $\theta = 90^\circ$

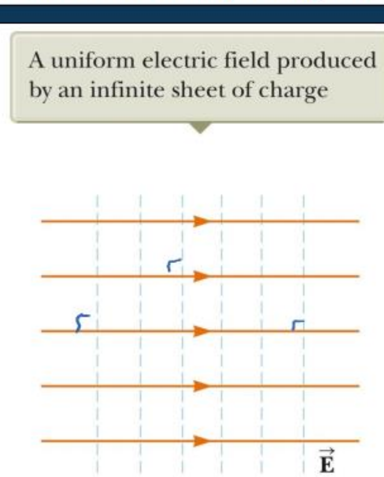
➔ Equipotential surfaces must always be perpendicular to the electric field lines passing through them.

E and *V* for an Infinite Sheet of Charge

The equipotential lines are the dashed blue lines.

The electric field lines are the brown lines.

The equipotential lines are everywhere perpendicular to the field lines.



E and *V* for a Point Charge

The equipotential lines are the dashed blue lines.

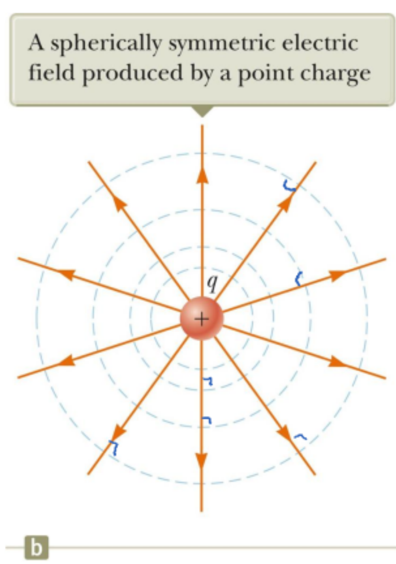
The electric field lines are the brown lines.

The electric field is radial.

$E_r = -dV/dr$

The equipotential lines are everywhere perpendicular to the field lines.

QUESTION: Given that the electric potential due to a point charge *q* is $V = k_e q/r$, find the electric field due to this charge using $E_r = -dV/dr$. (Easy!!)



$V = k_e \dfrac{q}{r} \Rightarrow E = -\dfrac{dV}{dr} = -\dfrac{d}{dr}\left(k_e \dfrac{q}{r}\right)$

$= -k_e q \dfrac{d}{dr}\left(\dfrac{1}{r}\right) = -k_e q\left(\dfrac{-1}{r^2}\right)$

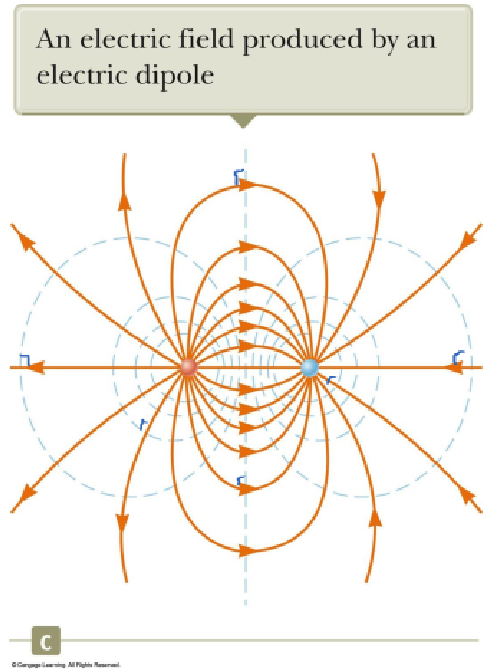
$= \dfrac{k_e q}{r^2}$

E and V for a Dipole

The equipotential lines are the dashed blue lines.

The electric field lines are the brown lines.

The equipotential lines are everywhere perpendicular to the field lines.



Electric Field from Potential/Questions

$V \equiv V(x, y, z)$

39. **W** Over a certain region of space, the electric potential is $V = 5x - 3x^2y + 2yz^2$. (a) Find the expressions for the x, y, and z components of the electric field over this region. (b) What is the magnitude of the field at the point P that has coordinates (1.00, 0, −2.00) m?

SAMPLE from a previous exam

$\Rightarrow V \equiv V(x, y)$

C) Over a certain region of space, the electric potential is $V = 6x^2y - 4xy$. Where x and y are in meters and V is in volts. *(10 points)*

$\vec{E} = E_x\hat{i} + E_y\hat{j}$, $E_x = -\frac{\partial V}{\partial x} = -(12xy - 4y)$

$E_y = -\frac{\partial V}{\partial y} = -(6x^2 - 4x)$

1) Determine the electric field vector $\vec{\mathbf{E}}$.

$\Rightarrow \vec{E} = (-12xy + 4y)\hat{i} + (-6x^2 + 4x)\hat{j}$

2) Find the **magnitude** of the electric when $x = y = 1.0$ m.

$\Rightarrow \vec{E} = (-8\hat{i} - 2\hat{j})$ V/m

$\Rightarrow |\vec{E}| = E = \sqrt{(-8)^2 + (-2)^2}$

$\Rightarrow E = \sqrt{68}$ V/m

Electric Potential for a Continuous Charge Distribution

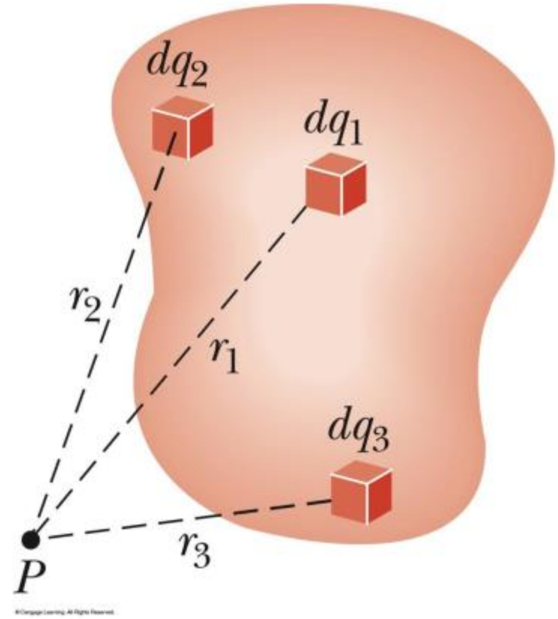
Method 1: The charge distribution is known.

Consider a small charge element *dq*

- Treat it as a point charge.

The potential at some point due to this charge element is

$$\int dV = \int k_e \frac{dq}{r}$$



V for a Continuous Charge Distribution, cont.

To find the total potential, you need to integrate to include the contributions from all the elements.

$$V = k_e \int \frac{dq}{r} \quad \rightarrow \quad dq \begin{cases} \rho\, dV \\ \sigma\, dA \\ \lambda\, dl \end{cases}$$

- This value for *V* uses the reference of *V* = 0 when *P* is infinitely far away from the charge distributions.

V for a Continuous Charge Distribution, final

If the electric field is already known from other considerations, the potential can be calculated using the original approach:

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

- If the charge distribution has sufficient symmetry, first find the field from Gauss' Law and then find the potential difference between any two points,
  - Choose V = 0 at some convenient point

$$V \begin{cases} k_e \int \frac{dq}{r} \\ \Delta V = -\int_A^B \vec{E} \cdot d\vec{s} \end{cases}$$

Example 25.4 The Electric Potential Due to a Dipole

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$ as shown in Figure 25.13. The dipole is along the x axis and is centered at the origin.

(A) Calculate the electric potential at point P on the y axis.

$$V_P = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{q}{\sqrt{a^2+y^2}} + \frac{-q}{\sqrt{a^2+y^2}}\right) = \boxed{0}$$

(B) Calculate the electric potential at point R on the positive x axis.

$$V_R = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{-q}{x-a} + \frac{q}{x+a}\right) = \boxed{-\frac{2k_e qa}{x^2-a^2}}$$

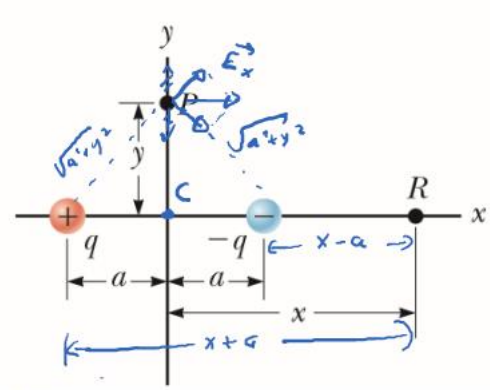
(C) Calculate V and E_x at a point on the x axis far from the dipole.

$$V_R = \lim_{x>>a}\left(-\frac{2k_e qa}{x^2-a^2}\right) \approx \boxed{-\frac{2k_e qa}{x^2} \quad (x>>a)}$$

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_e qa}{x^2}\right)$$

$$= 2k_e qa \frac{d}{dx}\left(\frac{1}{x^2}\right) = \boxed{-\frac{4k_e qa}{x^3} \quad (x>>a)}$$

$\frac{d}{dx}(x^{-2}) = -2x^{-3} = \frac{-2}{x^3}$



Question: Is the electric field zero at point P in part (A)?!

No. The answer in (A) tells us that the y-comp of the electric field is zero. However, the electric field has an x-comp at P, which we can't calculate from V since we don't have V(x).