24.3 Applications of Gauss's Law to Various Charge Distributions

$$E = k_e \int \frac{dq}{r^2}, \quad dq \begin{cases} \lambda \, dl \\ \sigma \, dA \\ \rho \, dV \end{cases}$$

$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}} \rightarrow \text{Gauss's Law}$$

→ for any closed surface, BUT when you use it to find E it is a gaussian surface.

The goal in this type of calculation is to determine a surface for which each portion of the surface satisfies one or more of the following conditions:

1. The value of the electric field can be argued by symmetry to be constant over the portion of the surface.
2. The dot product in Equation 24.6 can be expressed as a simple algebraic product $E\,dA$ because $\vec{\mathbf{E}}$ and $d\vec{\mathbf{A}}$ are parallel. $\vec{E} \cdot d\vec{A} = E\,dA$ $(\vec{E} \parallel d\vec{A})$
3. The dot product in Equation 24.6 is zero because $\vec{\mathbf{E}}$ and $d\vec{\mathbf{A}}$ are perpendicular. $\vec{E} \cdot d\vec{A} = 0$ $(\vec{E} \perp d\vec{A})$
4. The electric field is zero over the portion of the surface. $\vec{E} = 0$

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when you use G-law to find E, Remember:

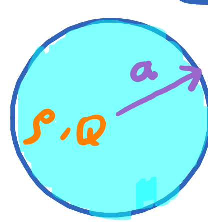
$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}}$$

→ The net electric flux through a G-surface is equal to the charge enclosed by G-surface divided by ε_0

Ex 24.3: $E = ?$ for:

(A) $r > a$ (outside)

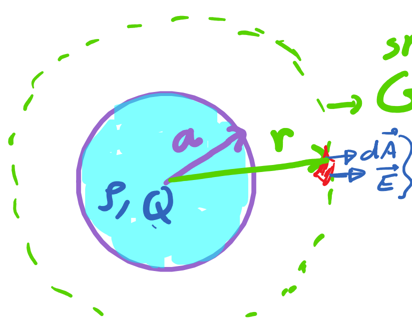
(B) $r < a$ (inside)



(A) $r > a$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

→ G-surface



spherical G-surface $(r > a)$

$\vec{E} \parallel d\vec{A}$

$\Rightarrow \vec{E} \cdot d\vec{A} = E\,dA$

→ $q_{in} = ?$ → charge inside G-surface

$\oint \vec{E}\cdot d\vec{A} \Rightarrow$ flux through G-surface

$\boxed{q_{in} = Q}$ But $\oint \vec{E}\cdot d\vec{A} = ?$

$$\oint \vec{E}\cdot d\vec{A} = \oint E\, dA = E \oint dA$$

{since by symmetry E has the same value everywhere on the surface}

$\Rightarrow$ area of the G-surface $= 4\pi r^2$

$$\Rightarrow \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2} \Rightarrow \boxed{E = k_e\frac{Q}{r^2} \quad (r>a)}$$

(B) for $r<a$

Note that $q_{in} < Q$

Note that the charge is uniformly distributed $\Rightarrow$ volume charge density is the same for $r = a$ and for $r<a$

$$\Rightarrow \rho = \rho'$$

$$\Rightarrow \frac{Q}{V} = \frac{q_{in}}{V'} \quad \left\{ V = \frac{4}{3}\pi a^3, \quad V' = \frac{4}{3}\pi r^3 \right.$$

Q $\quad$ q_{in} $\quad \Rightarrow \boxed{q = Q\frac{r^3}{a^3}}$

$$\Rightarrow \frac{Q}{\cancel{\frac{4}{3}}\cancel{\pi} a^3} = \frac{q_{in}}{\cancel{\frac{4}{3}}\cancel{\pi} r^3} \Rightarrow \boxed{q_{in} = Q\frac{r^3}{a^3}}$$

↳ indeed $q_{in} < Q$ since $r < a$

$$\Rightarrow \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$E(4\pi \cancel{r^2}) = \frac{Q\, r^{\cancel{3}}}{\varepsilon_0\, a^3}$$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0}\frac{r}{a^3}$$

$$\Rightarrow \boxed{E = k_e \frac{Q}{a^3} r \quad (\text{for } r < a)}$$