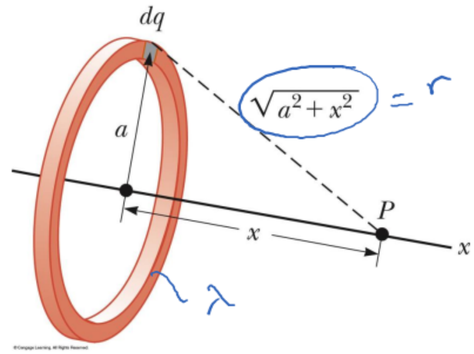
Example 25.5: *V* for a Uniformly Charged Ring, *V*=? at *P*

P is located on the perpendicular central axis of the uniformly charged ring .

The symmetry of the situation means that all the charges on the ring are the same distance from point P.

The ring has a radius a and a total charge *Q*.



Potential at P

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{a^2+x^2}}$$

$$V = \frac{k_e}{\sqrt{a^2+x^2}} \underbrace{\int dq}_{=Q} = \boxed{\frac{k_e Q}{\sqrt{a^2+x^2}}}$$

Magnitude of the electric field at P

$$E_x = -\frac{dV}{dx} = -k_e Q \frac{d}{dx}(a^2+x^2)^{-1/2}$$

$$= -k_e Q(-\tfrac{1}{2})(a^2+x^2)^{-3/2}(2x)$$

$$E_x = \boxed{\frac{k_e x}{(a^2+x^2)^{3/2}} Q}$$

Same result we obtained in Ch-23!

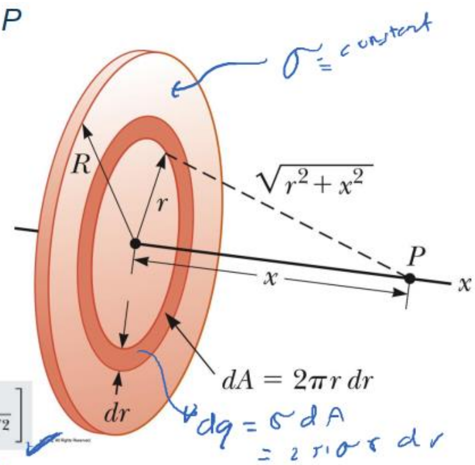
Example 25.6: *V* for a Uniformly Charged Disk, *V*=? at *P*

The ring has a radius *R* and surface charge density of *σ*.

P is along the perpendicular central axis of the disk.

P is on the central axis of the disk, symmetry indicates that all points in a given ring are the same distance from *P*.

The potential and the field are given by



Potential at *P*

$$V = k_e \int \frac{dq}{r} \quad \text{between dq and P}$$

$$dq = \sigma\, dA = \sigma(2\pi r\, dr) = 2\pi\sigma r\, dr$$

$$V = \pi k_e \sigma \int_0^R \frac{2r\, dr}{\sqrt{r^2+x^2}}$$

$$= \pi k_e \sigma \int_0^R (r^2+x^2)^{-1/2}\, 2r\, dr$$

$$V = \boxed{2\pi k_e \sigma[(R^2+x^2)^{1/2} - x]}$$

Magnitude of the electric field at *P*

$$E_x = -\frac{dV}{dx} = \boxed{2\pi k_e \sigma\left[1 - \frac{x}{(R^2+x^2)^{1/2}}\right]}$$

Same result we obtained in Ch-23!

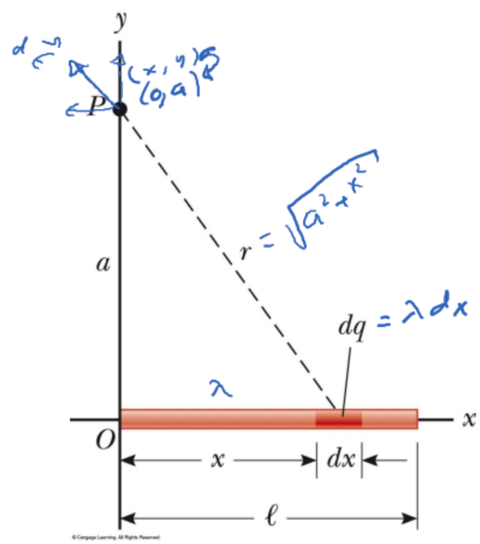
Example 25.7: V for a Finite Line of Charge, V=? at P

A rod of line ℓ has a total charge of Q and a linear charge density of λ.

$$V = k_e \int \frac{dq}{r} = \int_0^{\ell} k_e \frac{\lambda\, dx}{\sqrt{a^2 + x^2}} \checkmark$$

$$V = k_e \lambda \int_0^{\ell} \frac{dx}{\sqrt{a^2 + x^2}} = k_e \frac{Q}{\ell} \ln\left(x + \sqrt{a^2 + x^2}\right)\Big|_0^{\ell}$$

$$V = k_e \frac{Q}{\ell}\left[\ln\left(\ell + \sqrt{a^2 + \ell^2}\right) - \ln a\right] = k_e \frac{Q}{\ell} \ln\left(\frac{\ell + \sqrt{a^2 + \ell^2}}{a}\right) \checkmark$$



Question: Can you calculate the electric field at point P?!
We can find E_y but not E_x in this case .. How?

You can find E_y by having ***a*** ➔ y in V above and then $E_y = -dV/dy$

There is E_x, but you can't find it from V above since V was calculated at a specific value (x=0) rather than a general value!

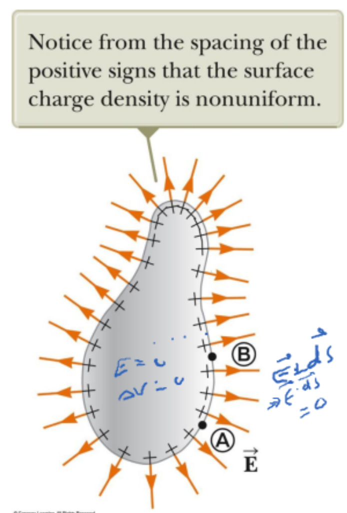
V Due to a Charged Conductor (25.6)

Consider two points on the surface of the charged conductor as shown.

$\vec{\mathbf{E}}$ is always perpendicular to the displacement $d\vec{\mathbf{s}}$. Therefore, $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$

Remember: $\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ $\Rightarrow \Delta V = 0$

➔ the potential difference between A and B is also zero. $\rightarrow V_A = V_B$



V is constant everywhere on the surface of a charged conductor in equilibrium
➔ ΔV = 0 between any two points on the surface

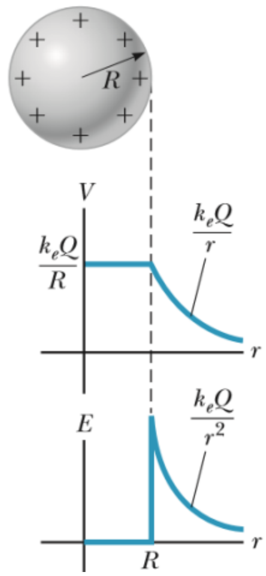
The surface of any charged conductor in electrostatic equilibrium is an equipotential surface.

Every point on the surface of a charge conductor in equilibrium is at the same electric potential.

Because the electric field is zero inside the conductor, we conclude that the electric potential is constant everywhere inside the conductor and equal to the value at the surface.

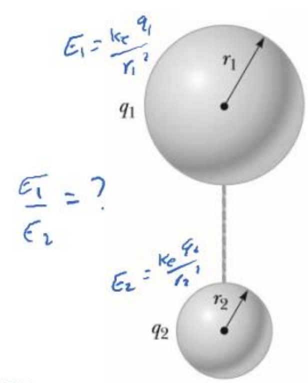
V Due to a Charged Conductor, cont.

Charged conducting sphere
V* vs *r* and *E* vs *r



Example 25.8 **Two Connected Charged Spheres**

Two spherical conductors of radii r_1 and r_2 are separated by a distance much greater than the radius of either sphere. The spheres are connected by a conducting wire as shown in Figure 25.19. The charges on the spheres in equilibrium are q_1 and q_2, respectively, and they are uniformly charged. Find the ratio of the magnitudes of the electric fields at the surfaces of the spheres.



- The spheres are so far apart ➔ the field of one does not affect the charge distribution on the other.
- The conducting wire between them ensures that both spheres have the same electric potential ($V_1 = V_2$).
- We can model the field and potential outside the spheres to be that due to point charges.

$$V = \cancel{k_e} \frac{q_1}{r_1} = \cancel{k_e} \frac{q_2}{r_2} \longrightarrow \frac{q_1}{q_2} = \frac{r_1}{r_2}$$

The magnitudes of the electric fields at the surfaces of the spheres: $E_1 = k_e \frac{q_1}{r_1^2}$ and $E_2 = k_e \frac{q_2}{r_2^2}$

$$\frac{E_1}{E_2} = \frac{q_1}{q_2}\frac{r_2^2}{r_1^2} \longrightarrow \frac{E_1}{E_2} = \frac{\cancel{r_1}}{\cancel{r_2}}\frac{r_2^2}{r_1^2} = \boxed{\frac{r_2}{r_1}}$$