$$F(x)=\begin{cases} 0 & x<1 \\ \frac{2}{5} & 1\le x<2 \\ 1 & x\geq 2 \end{cases}$$

X

$$P(X=x)=\begin{cases} \frac{x+1}{5} & x=1,2 \\ 0 & o.w \end{cases}$$

X	1	2	
$P(X=x)$	$\frac{2}{5}$	$\frac{3}{5}$	1

$$P(Y=y)=\begin{cases} \frac{\ }{15} & \\ 0 & o.w \end{cases}$$

Y	1	2	3	
$P(Y=y)$	$\frac{4}{15}$	$\frac{5}{15}$	$\frac{6}{15}$	$\frac{15}{15}=1$

(c) Are X and Y independent?

$$P(X=x, Y=y) \stackrel{?}{=} P(X=x)P(Y=y)$$

$$\frac{2x+y}{30} \stackrel{?}{\neq} \left(\frac{x+1}{5}\right)\left(\frac{3+y}{15}\right)$$

X, Y are dependent.

(d) Find $P(X \leq 1.5, Y \leq 2.3)$?

$$= \sum_{\substack{\forall y \leq 2.3 \\ \boxed{y=1,2}}} \sum_{\substack{\forall x \leq 1.5 \\ \boxed{x=1}}} \frac{1}{30}(2x+y) = \frac{1}{30}\sum_{y=1,2}(2+y) = \frac{1}{30}\left[(2+1)+(2+2)\right]$$

$$= 7/30$$

(e) Find $P(X \leq 1.5, Y > 1)$?

$$\sum_{\substack{y>1 \\ \boxed{y=2,3}}} \sum_{\substack{x \leq 1.5 \\ \boxed{x=1}}} \frac{1}{30}(2x+y) = \frac{1}{30}\sum_{y=2,3}(2+y) = \frac{1}{30}\left[(2+2)+(2+3)\right]$$

$$= 9/30$$

(f) Find $P(X > 1, Y > 1)$?

$$= \frac{1}{30}\sum_{\substack{y>1 \\ \boxed{y=2,3}}} \sum_{\substack{x>1 \\ \boxed{x=2}}} (2x+y) = \frac{1}{30}\sum_{y=2,3}(4+y) = \frac{1}{30}(4+2+4+3) = 13/30$$

$$\cancel{= \frac{1}{30}(4+2+3)}$$

2. Five students are going to take two exams. Let X be the number of students that pass Exam #1, and Y the number of students that pass Exam #2. The following table is the joint pmf of X and Y

$Y \backslash X$	0	1	2	3	4	5	$P(Y=y)$
0	0.02	0.02	0.01	0.01	0.03	0.01	0.1
1	0.05	0.05	0.04	0.03	0.01	0.02	0.2
2	0.01	0.01	0.01	0.01	0.06	0.1	0.2
3	0.03	0.03	0.04	0.03	0.03	0.04	0.2
4	0	0.05	0.05	0.05	0	0.05	0.2
5	0.02	0.02	0.01	0.01	0.02	0.02	0.1
$\cdot$	0.13	0.18	0.16	0.14	0.15	0.24	1.00

$P(X<4, Y<2)$

(c)

(d)

(f)

$$\sum_y P(X=x$$

$P(X\le 1, Y\ge 4)$

0.05

0.06

$P(Y\ge 4)$

$P(X=x)$ ~~$f_X(x)$~~

(a) Is the table indeed a joint pmf of X and Y?

$$0 \le P(X=x, Y=y) \le 1 \quad \checkmark$$

$$\sum_{\forall y}\sum_{\forall x} P(X=x, Y=y) = 1 \quad \checkmark$$

X	0	1	2	3	4	5
$P(X=x)$	0.13	0.18	0.16	0.14	0.15	0.24

(b) What is the probability that 3 students pass Exam # 1 and 5 students pass Exam # 2?

$$P(X=3, Y=5) = 0.01$$

(c) What is the probability that at least one student passes Exam # 1 and no students pass Exam # 2?

$$P(X \geq 1, Y=0) = p(1,0) + p(2,0) + p(3,0) + p(4,0) + p(5,0)$$

$$= 0.08$$

OR $= 0.1 - P(0,0) = 0.1 - 0.02$

(d) What is the probability that more than 2 students pass each exam?

$$P(X>2, Y>2) = P(X \geq 3, Y \geq 3) = 0.25$$

(e) Compute the probability that more than 2 students pass Exam # 1?

$$P(X>2, Y=y) = P(X>2) = 0.14 + 0.15 + 0.24$$

$$0.53$$

(f) Compute the probability that at most 4 students pass Exam # 2?

$$P(Y \leq 4) = 1 - P(Y \geq 5) = 1 - P(Y = 5) = 1 - 0.1 = 0.9$$

(g) Compute the probability that less than 4 students pass Exam # 1 or less than 2 students pass Exam # 2?

$$P(X < 4 \text{ OR } Y < 2) = P(X < 4) + P(Y < 2) - P(X < 4, Y < 2)$$

$$\cup = 1 - P(X \geq 4) + P(Y = 0) + P(Y = 1) - P(X < 4, Y < 2)$$

$$= 1 - 0.39 + 0.1 + 0.2 - 0.23$$

(h) Find the marginal pmfs of X and Y?

On the Table

(i) Are X and Y independent random variables?

$$P(X=1, Y=1) \stackrel{?}{=} P(X=1)\,P(Y=1)$$

$$0.05 \stackrel{?}{\neq} (0.18)(0.2)$$ They are dependent.

(j) Compute the probability that $X \leq 1$ given that $Y \geq 4$?

$$P(X \leq 1 \mid Y \geq 4) = \frac{P(X \leq 1, Y \geq 4)}{P(Y \geq 4)} = \frac{0.09}{0.3}$$

Section 5.4 Continuous Bivariate Random Variables

1. Let X and Y are two continuous random variables whose joint pdf is given by

$$f_{XY}(x, y) = \begin{cases} e^{-(x+y)} & 0 \leq x < \infty, 0 \leq y < \infty \\ 0 & \text{otherwise} \end{cases}$$

   (a) Are X and Y independent?

   (b) Find $F_{XY}(5, \ln 4)$?

(c) Find $P(X > 1, Y > 9.9)$?

(d) Find $P(X < 0.5 \text{ or } Y > 9.9)$?

(e) Given that $X < 0.5$, find the probability that $Y > 9.9$?

2. Let X and Y are two continuous random variables whose joint pdf is given by

$$f_{XY}(x,y) = \begin{cases} 2e^{-(x+y)} & 0 \leq x \leq y, 0 \leq y < \infty \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $f_{XY}(x,y)$ is indeed a joint pdf of X and Y ?

(b) Are X and Y independent variables?

(c) Find the probability that $X < 3$ and $Y < 5$?

3. The joint pdf of continuous variables X and Y is given by $f_{XY}(x, y) = k\sqrt{x^2 + y^2}$, where $(x, y) \in R$ and R is shown in the figure.

   (a) Find the value of k in order for $f_{XY}(x, y)$ to be a legitimate pdf?

   (b) Find the marginal pdfs $f_X(0)$ and $f_Y(3)$?

(c) Compute $F_{XY}(0,0) = P(X \le 0, Y \le 0)$?

(d) What is the probability that (x, y) is in the second quadrant?

(e) What is the probability that $X > 0$ and $Y < 1$?

(f) What is the probability that the point (x, y) is inside the square $-4 \leq x \leq 4$, $-3 \leq x \leq 5$?

(g) What is the probability that the point (x, y) lies on the upper half plane?

(h) What is the probability that the point lies on the x–axis?

4. Let The joint pdf of continuous variables X and Y is given by $f_{XY}(x, y) = \frac{1}{2}x^3y$, where $0 \leq x \leq 2$ and $0 \leq y \leq 1$

(a) Determine whether X and Y are independent?

(b) Find the marginal pdfs?

5. The joint CDF of two discrete random variables is given by

$$F_{XY}(x, y) = \begin{cases} \frac{1}{8} & \text{if } x = 1,\ y = 1 \\ \frac{1}{8} & \text{if } x = 1,\ y = 2 \\ \frac{1}{8} & \text{if } x = 2,\ y = 1 \\ \frac{1}{8} & \text{if } x = 2,\ y = 2 \end{cases}$$

(a) Find the joint pmf of X and Y?

(b) Find the marginal pmf of X and Y

6. Let X be a uniform random variable over the interval $[9, 100]$ and Y be an exponential random variable with $\lambda = 3$. X and Y are independent random variables?

   (a) Find the joint pdf $f_{XY}(x, y)$?

   (b) Find $P(X < 50, Y < 3.2)$?

(c) Find $P(X > 49.6, Y > 6)$?

(d) Find $P(X < 20, Y = 8.99)$

7. The joint pdf of two continuous random variables X and Y is $f_{XY}(x, y) = 2x + Ay$ for $x \in \left[0, \frac{1}{2}\right]$ and $y \in [1, 2]$.

(a) Find the value of A?

(b) Are X and Y independent?

(c) Given that $X < \frac{1}{4}$, compute the probability that $Y < \frac{3}{2}$?

Section 5.6 Conditional Distributions

1. The joint pmf of the discrete random variables X and Y is given by

$$p_{XY}(x,y) = \begin{cases} \frac{1}{30}(2x+y) & x = 1,\ 2;\ y = 1,\ 2,\ 3 \\ 0 & \text{Otherwise} \end{cases}$$

(a) Find the conditional pmf of X given Y?

(b) Find the conditional pmf of Y given X?

(c) Find the conditional pmf of $Y = 2$ given $X = 1$?

2. The joint pdf of continuous random variables X and Y is given by

$$f_{XY}(x,y) = \begin{cases} xe^{-x(y+1)} & 0 \leq x < \infty;\ 0 \leq y < \infty \\ 0 & \text{Otherwise} \end{cases}$$

(a) Find the conditional pdf of X given Y?

(b) Find the conditional pdf of Y given X?

3. Let X and Y be discrete variables whose joint pmf is given by the table

	0	1	2
1	0.2	0.1	0.1
2	0.3	0.2	0.1

(a) Find the conditional $E(X|Y) = \mu_{X|Y}$ when $Y = 2$

(b) Find the conditional $\sigma^2_{X|Y}$ when $Y = 2$

4. Compute the conditional mean $E(X|Y = y)$ if the joint pdf of X and Y is given by

$$f_{XY}(x, y) = \begin{cases} \frac{e^{-x/y}e^{-y}}{y} & 0 \leq x < \infty;\ 0 < y < \infty \\ 0 & \text{Otherwise} \end{cases}$$

Section 5.7 Covariance and Correlation Coefficient

1. The joint pdf of the random variables X and Y is defined as follows:

$$f_{XY}(x,y) = \begin{cases} 25e^{-5y} & 0 \leq x < 0.2;\ y \geq 0 \\ 0 & \text{Otherwise} \end{cases}$$

   (a) Find the marginal pdfs of X and Y.

   (b) What is the covariance of X and Y?

2. Two discrete random variables X and Y have the joint pmf given by

$$P(X = x, Y = y) = \begin{cases} 0 & \text{if } x = -1,\ y = 0 \\ \frac{1}{3} & \text{if } x = -1,\ y = 1 \\ \frac{1}{3} & \text{if } x = 0,\ y = 0 \\ 0 & \text{if } x = 0,\ y = 1 \\ 0 & \text{if } x = 1,\ y = 0 \\ \frac{1}{3} & \text{if } x = 1,\ y = 1 \end{cases}$$

(a) Are X and Y independent?

(b) What is the covariance of X and Y?