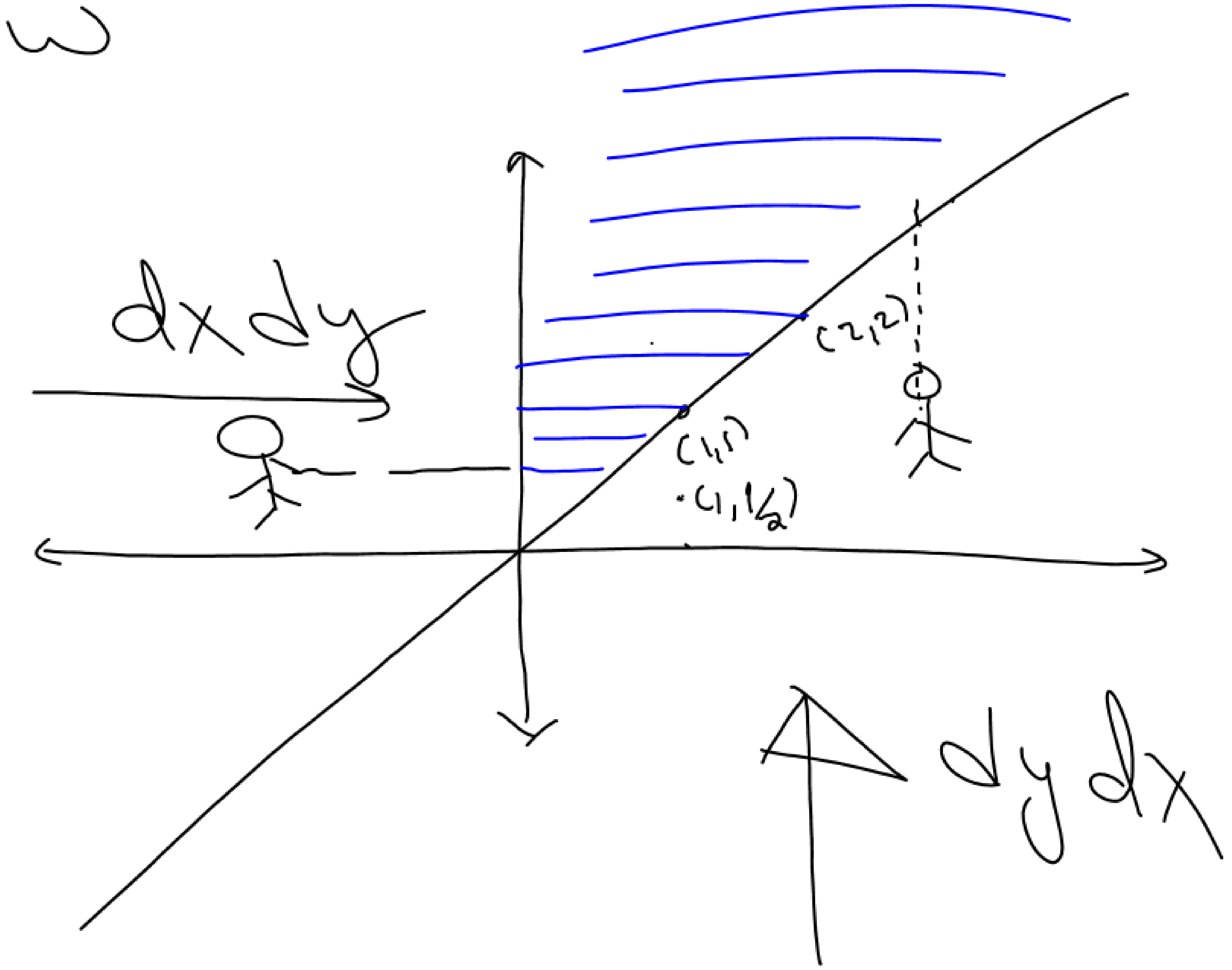
Ex

$$f_{X,Y}(x,y) = \begin{cases} k e^{-(x+y)} & 0 \le x \le y \quad 0 \le y < \infty \\ 0 & o.w \end{cases}$$

$$\int_{x=0}^{x=\infty} \int_{y=x}^{y=\infty} k e^{-(x+y)} \, dy \, dx$$

$$\int_{y=0}^{y=\infty} \int_{x=0}^{x=y} k e^{-(x+y)} \, dx \, dy$$



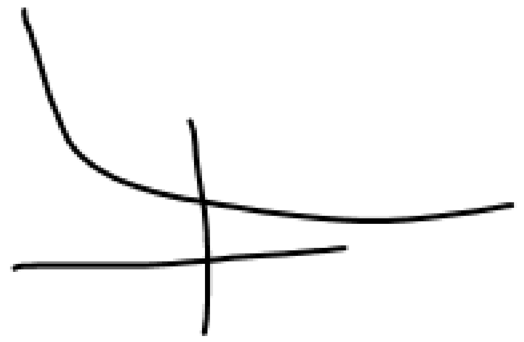
$$\int\int k e^{-x} e^{-y}\, dx\, dy$$

$$\int_{x=1}^{x=2}\int_{y=x}^{y=7}\left(x^2+2y\right)dy\,dx = \int_{x=1}^{x=2}\left(x^2y+\frac{2y^2}{2}\Bigg|_{y=x}^{y=7}\right)dx$$

$$\cancel{\int_{y=x}^{y=7} 2y\,dy + \int_{x=1}^{x=2} x^2\,dx}$$

$$f_{X,Y}(x,y) = \begin{cases} 2e^{-(x+y)} & 0 \le x \le y \qquad 0 \le y < \infty \\ 0 & \text{o.w} \end{cases}$$

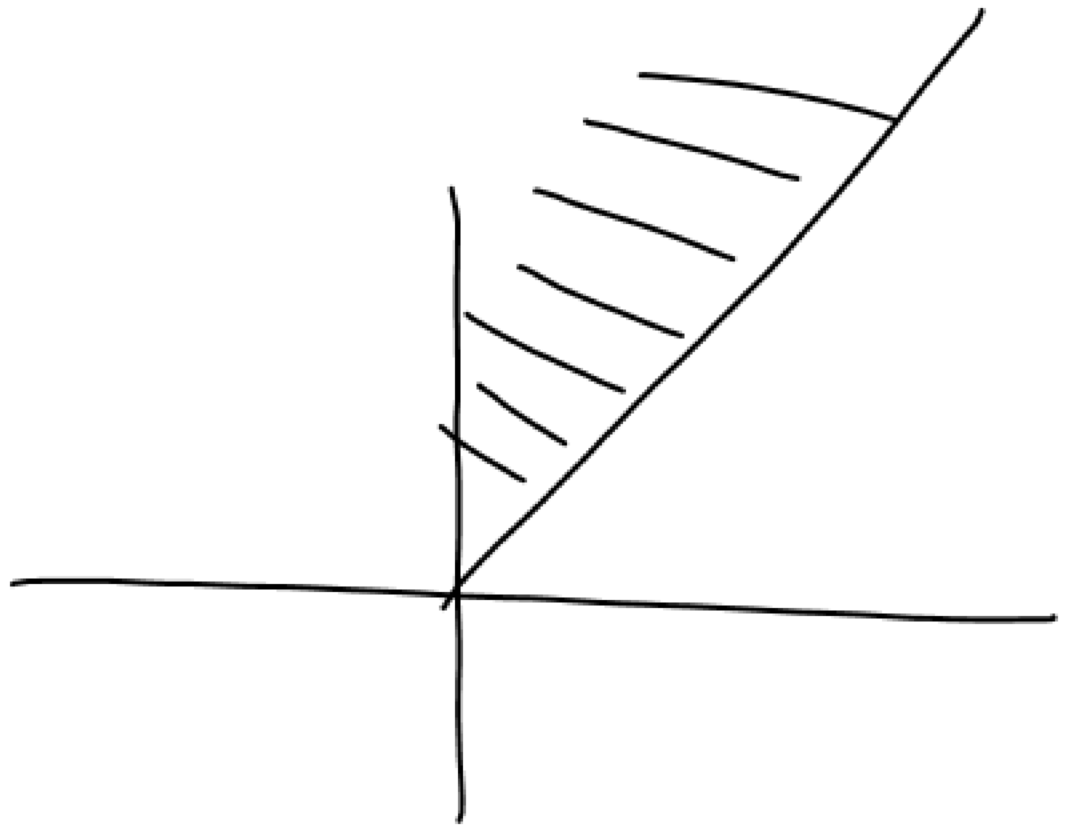
$$\int e^{ax+b}\,dx = \frac{e^{ax+b}}{a} + c$$



④ Find the marginal pdf of X and Y?

Sol

$$f_X(x) = \int_{y=x}^{y=\infty} 2e^{-(\overset{\text{const}}{x}+y)} = \lim_{b\to\infty} \left(-2e^{-(x+y)}\right)\Big|_{y=x}^{y=b}$$



$$= -2\lim_{b\to\infty}\left[e^{-(x+b)} - e^{-(x+x)}\right] = -2\lim_{b\to\infty}\left[e^{-x}e^{-b} - e^{-2x}\right] = 2e^{-2x}$$

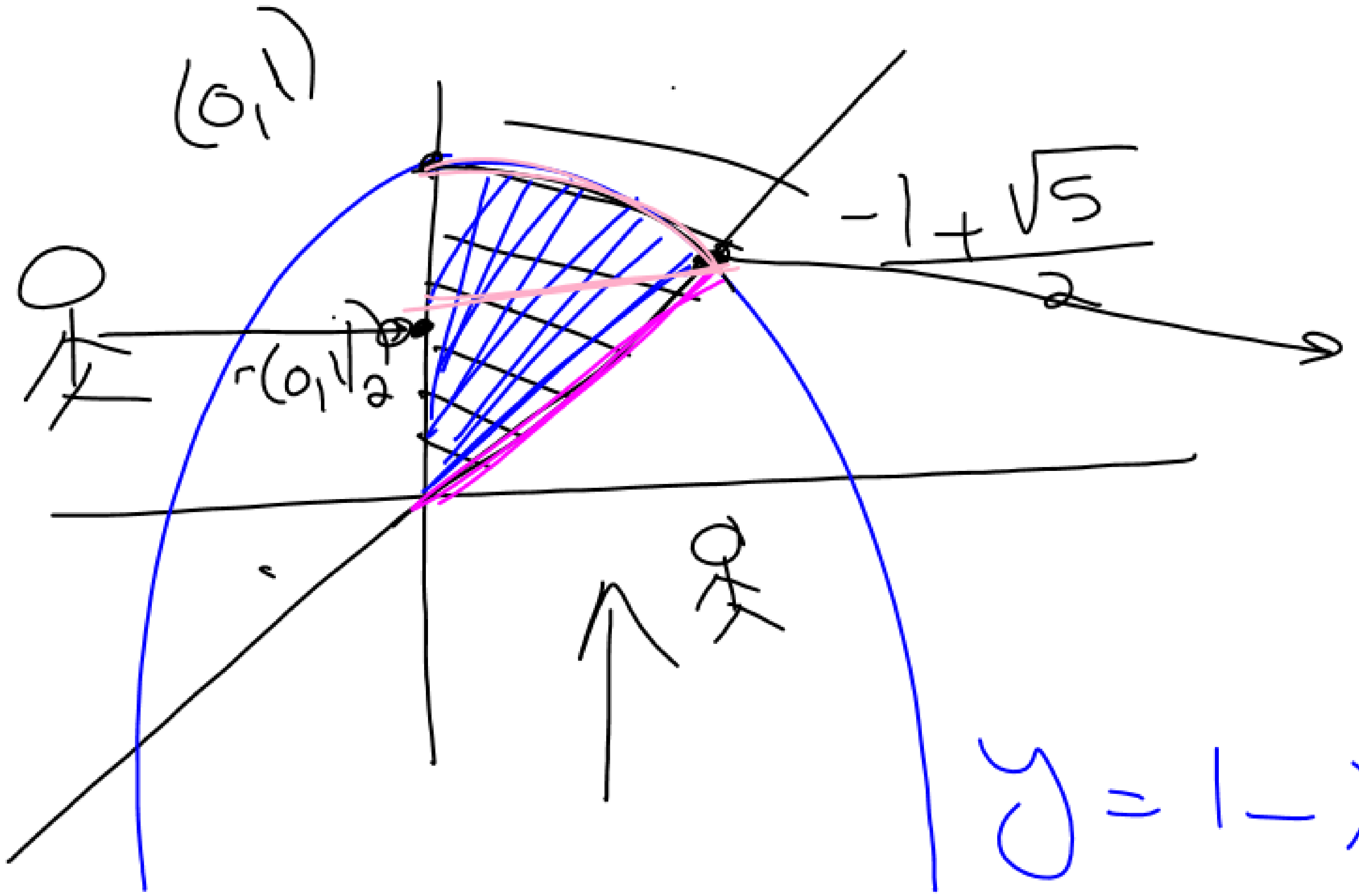
$$f_X(x) = \begin{cases} 2e^{-x} & 0 \le x < \infty \\ 0 & \text{o.w} \end{cases}$$

$$f_y(y) = \int_{x=0}^{x=y} 2e^{-(x+y)}\,dx = -2e^{-(x+y)}\Big|_{x=0}^{x=y}$$

$$= -2\left[e^{-2y} - e^{-y}\right]$$

$$f_Y(y) = \begin{cases} -2\left[e^{-2y} - e^{-y}\right] & y \ge 0 \\ 0 & \text{o.w} \end{cases}$$

6) Find $P(Y<1-X^2) = \displaystyle\int_{x=0}^{x=\frac{-1+\sqrt{5}}{2}}\int_{y=x}^{y=1-x^2} 2e^{-(x+y)}\,dy\,dx$



$y=1-x^2$

$$x = 1-x^2$$

$$x^2+x-1=0$$

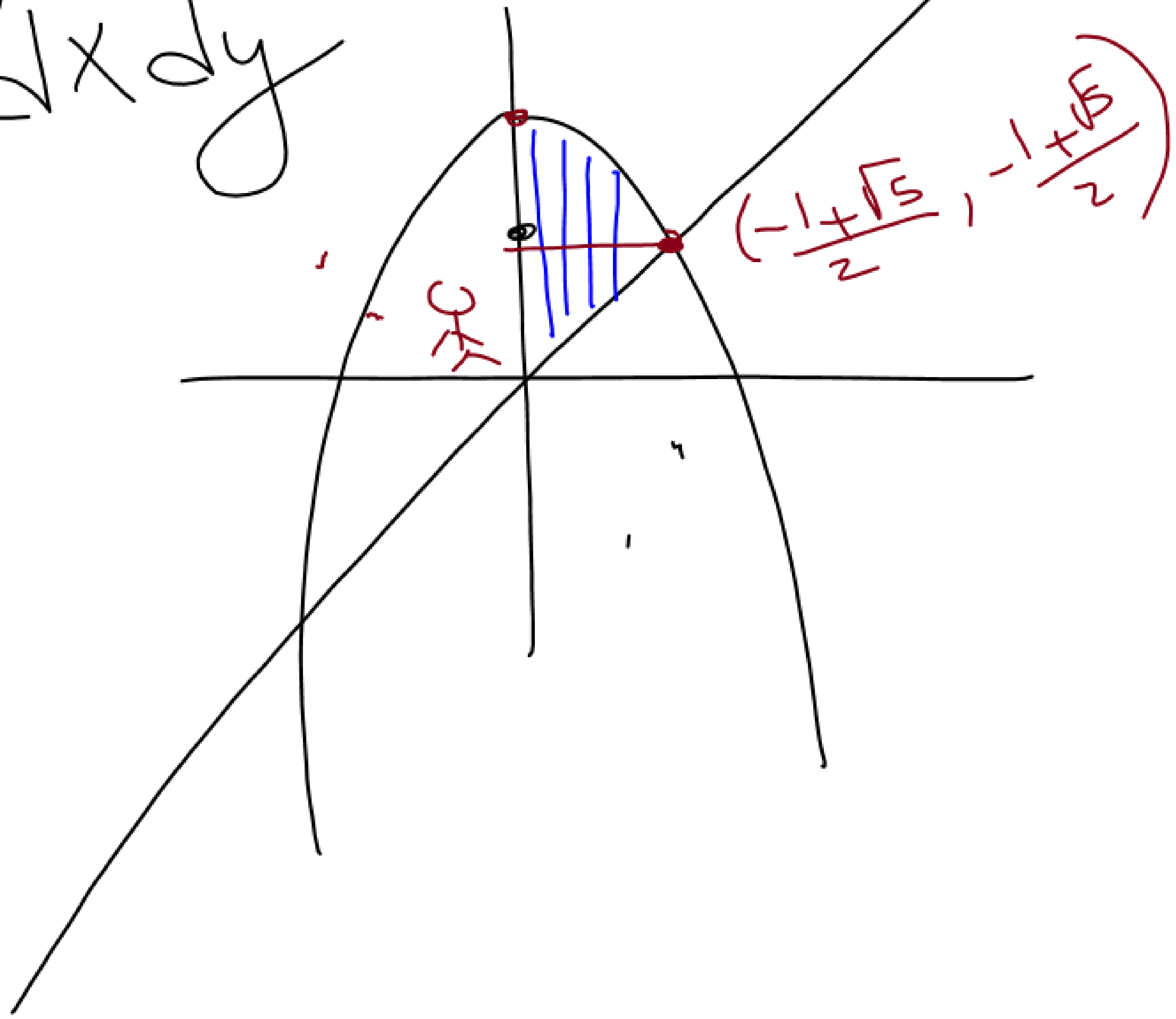
$$x = \frac{-b\mp\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-1\mp\sqrt{1-4(1)(-1)}}{2} = \frac{-1\mp\sqrt{5}}{2}$$

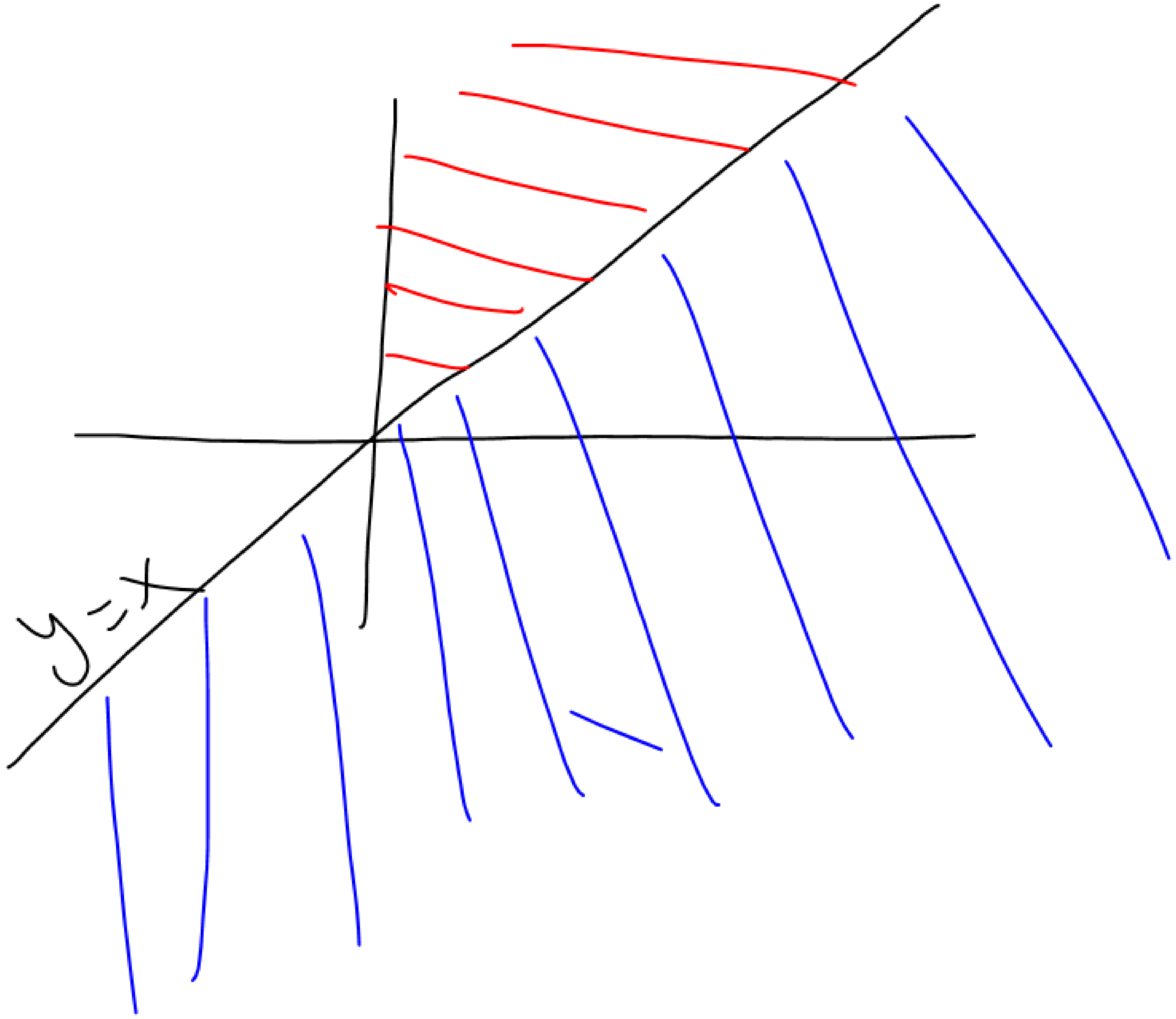
dx dy

We have 2 regions.

$$\int_{y=0}^{y=\frac{-1+\sqrt{5}}{2}}\int_{x=0}^{x=y} 2e^{-(x+y)}\,dx\,dy+\int_{y=\frac{-1+\sqrt{5}}{2}}^{y=1}\int_{x=0}^{y=1-x^2} 2e^{-(x+y)}\,dx\,dy$$



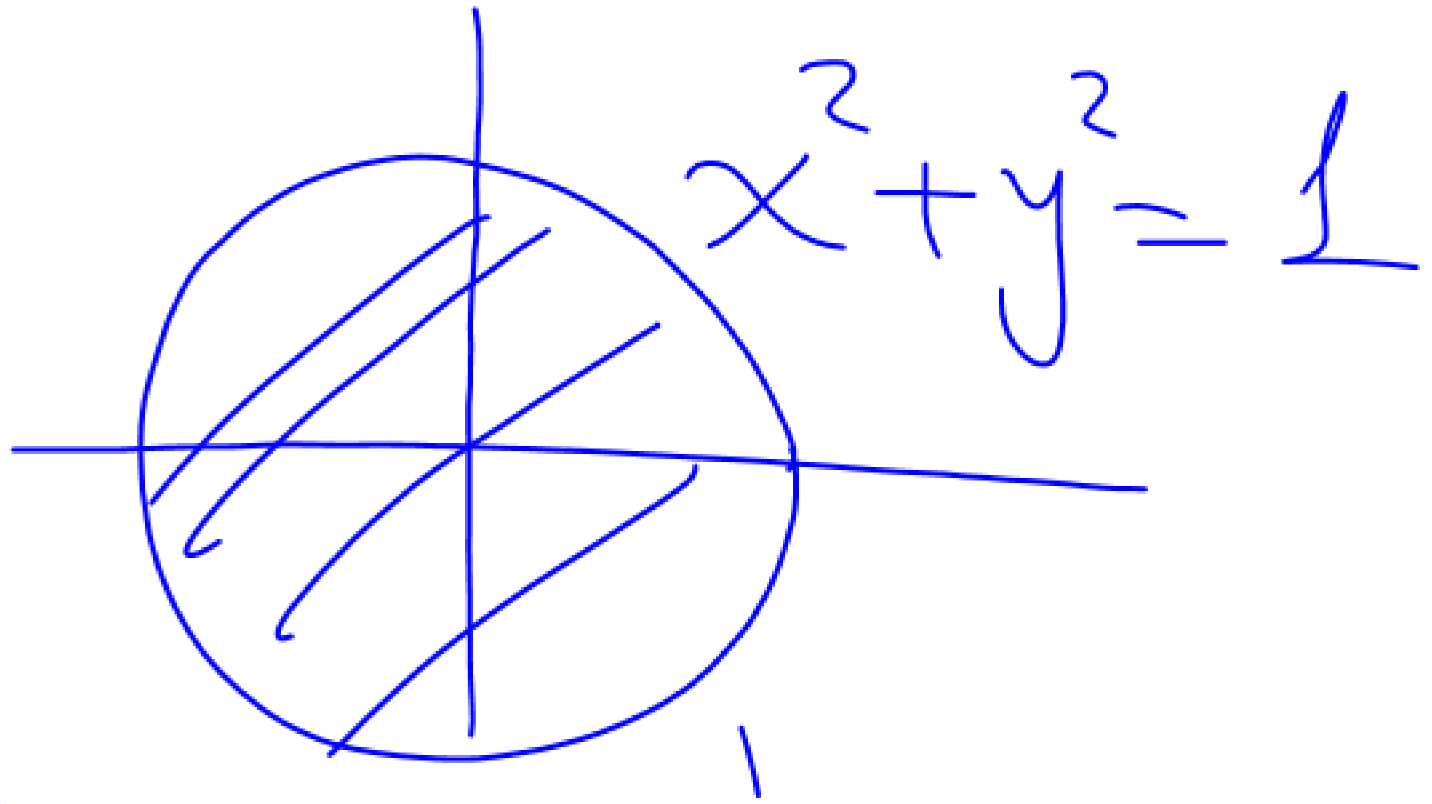
7 Find $P(Y < X) = 0$

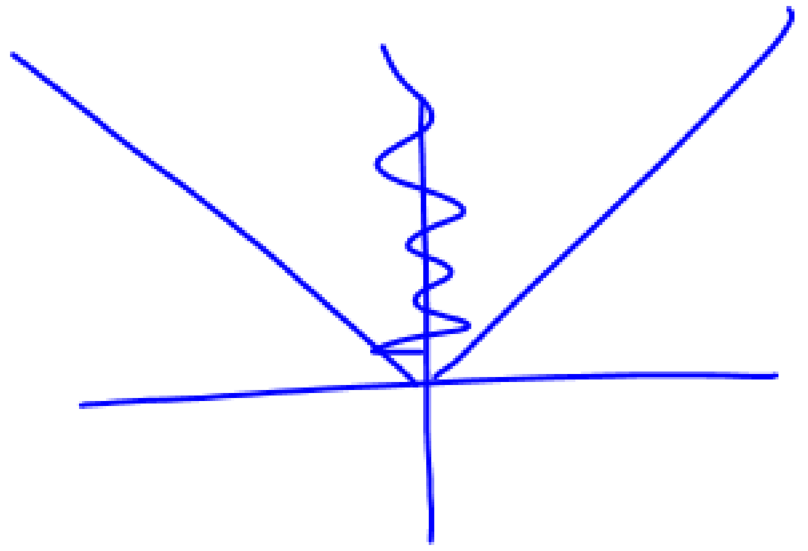


Imposible event

Ex The joint pdf of X, Y is given by

$f_{X,Y}(x,y) = \frac{3}{2\pi}\sqrt{x^2+y^2}$, $(x,y) \in R$ is the shaded region





① Find $f_X(0) = \frac{3}{2\pi}\int_{y=-1}^{1}\sqrt{0+y^2}\,dy = \frac{3}{2\pi}\int_{y=-1}^{y=1}|y|\,dy$

Note

$$\int_{-a}^{a} f(x)\,dx \begin{cases} \xrightarrow{\text{even fcn}} 2\int_{0}^{a} f(x)\,dx \\ \xrightarrow{\text{odd fcn}} 0 \end{cases}$$

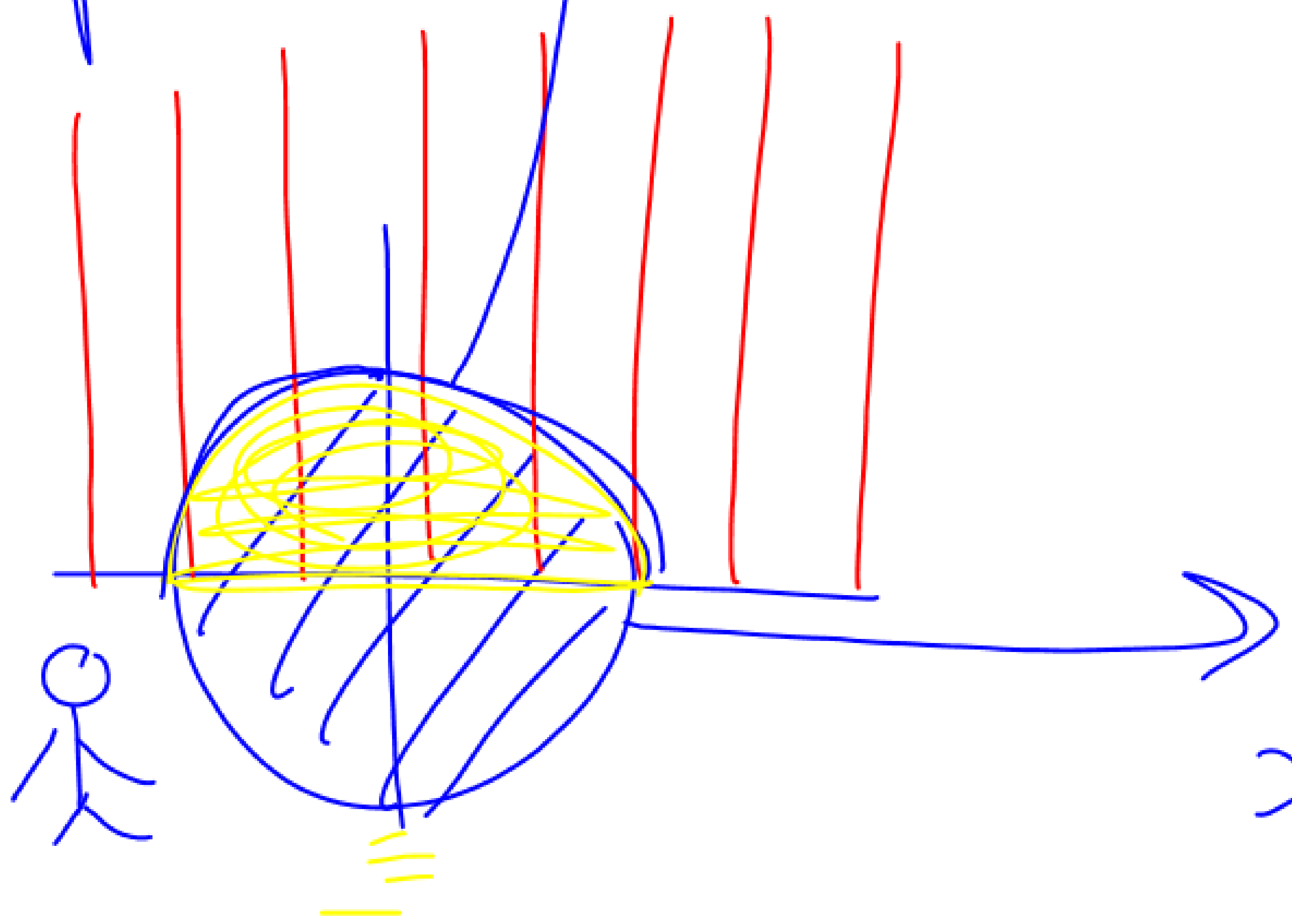
$$=\frac{3}{\cancel{2\pi}}\cancel{2}\int_0^1 y\,dy = \frac{\cancel{2}y^2 3}{\cancel{2}\pi}\Big|_0^1 = \frac{3}{\pi}$$

② $$f_y(0) = \frac{3}{2\pi}\int_{x=-1}^{x=1}\sqrt{x^2+0}\,dx = \frac{3}{\pi}$$

③ Compute the prob. that (X,Y) lies in the upper half plane

$y=\sqrt{1-x^2}$

Sol



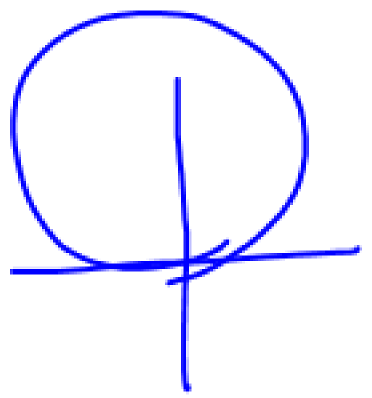
$x^2+y^2=1$

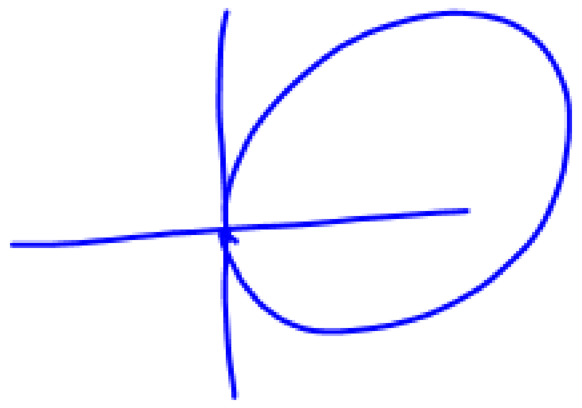
$x=\mp\sqrt{1-y^2}$

$$P\big((X,Y) \text{ lies in the upper half plane}\big) = P(Y\geq 0) = \int_{x=-1}^{x=1}\int_{y=0}^{y=\sqrt{1-x^2}} \frac{3}{2\pi}\sqrt{x^2+y^2}\,dy\,dx$$

No easy

Note



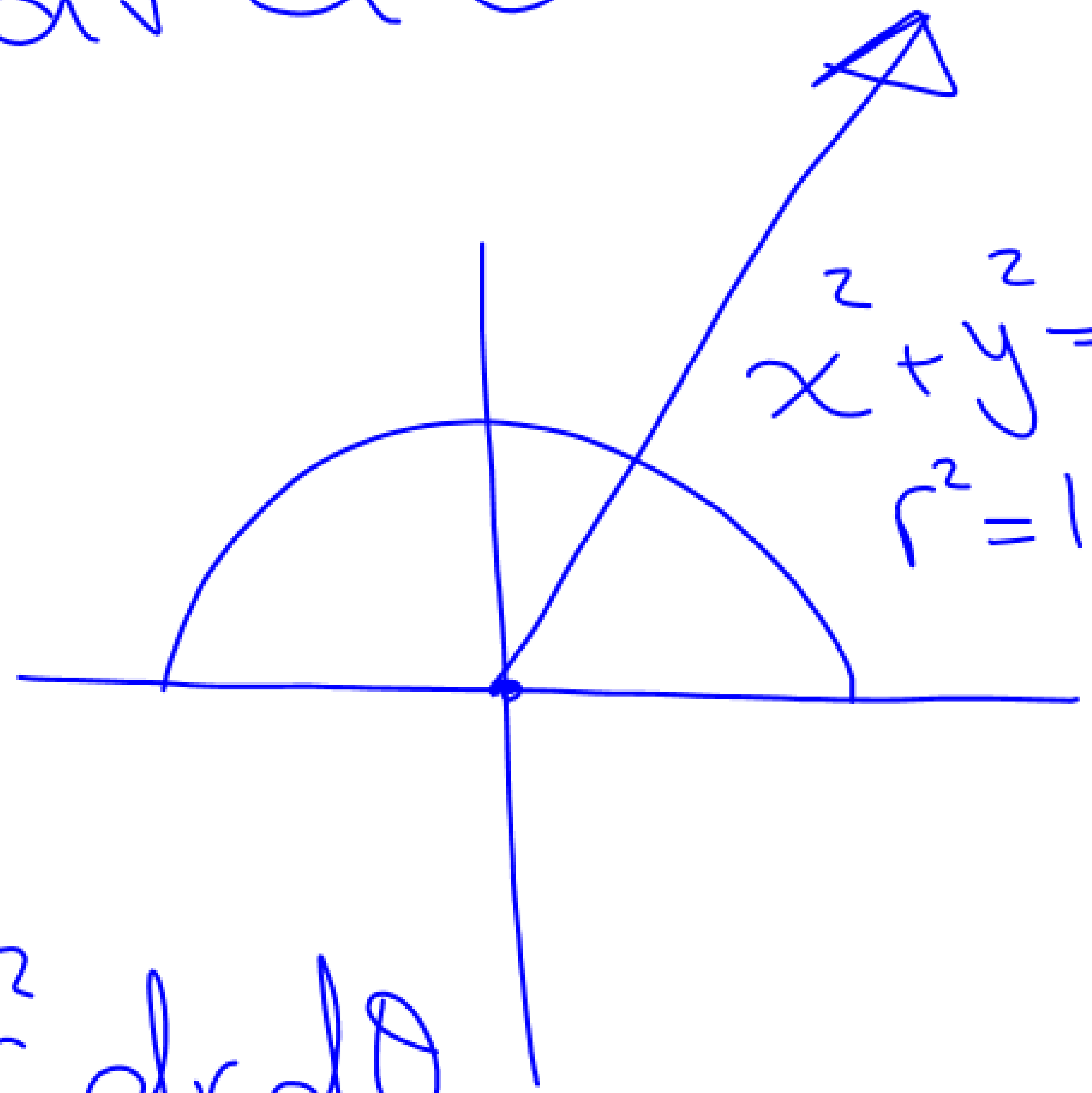


$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$x^2 + y^2 = r^2$$

$$dA = r\,dr\,d\theta$$



$x^2+y^2=1$

$r^2 = 1 \Rightarrow r = 1$

$|r|$

$$\frac{3}{2\pi}\int_{\theta=0}^{\theta=\pi}\int_{r=0}^{r=1}\sqrt{r^2}\; r\,dr\,d\theta = \int_{\theta=0}^{\theta=\pi}\int_{r=0}^{r=1} r^2\,dr\,d\theta$$

$x^2+y^2=a^2$

$\boxed{r=a}$