Lecture 1 (28/2/22)
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Chapter One

Counting Rules

① The Fundamental Counting Rules: FCR

If a process consists of m steps (m positive integer)

m - slots, step i

Each step i can be preformed in n_i ways (n_i positive integer)

Then the whole process can be per formed in

$n_1 n_2 n_3 \cdots n_m$ way.

Ex A password consists of 6 characters. If each character can be only a digit 0 to 9. How many possible password do we have

sol $\underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = 10^6$ possible password.

② In the same password example, how many passwords of 6 characters (0→9) such that the last character is even number and the 3rd character is > 7

sol $\underline{10}\ \underline{10}\ \underline{2}\ \underline{10}\ \underline{10}\ \underline{5}$ (3rd: 8, 9; last: even 0,2,4,6,8)

$10^4(5)(2) = 10^5$

2 Permutation

The number of ways we can select arrange in order r objects from n objects (Without replacement)

$0 \leq r \leq n$

. $P(n,r) \equiv {}_nP_r = \frac{n!}{(n-r)!}$

$\frac{n(n-1)(n-2)\cdots(3)(2)(1)}{(n-r)(n-r-1)\cdots(3)(2)(1)}$

$P(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$

5 Shift ☒ 2 $P(5,2) = \frac{5!}{3!} = 5(4) = 20$

Ex Let $A = \{1,2,3,4,5,6\}$

① In how many ways we can select in order 4 elements from A without replacement.

sol ${}_6P_4 = \frac{6!}{2!} = \frac{(6)(5)(4)(3)\cancel{2!}}{\cancel{2!}}$

$= 360$

$\underline{6}\ \underline{5}\ \underline{4}\ \underline{3} = 360$

② In how many ways we can arrange the elements of A

sol $\underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 6!$

or ${}_6P_6 = P(6,6)$

③ In how many ways we can select in order without replacement, three elements of A such that the last number is even?

sol $(\underline{5}\ \underline{4})\ \underline{3} = 60$ ↑ even number 2, 4, 6

${}_5P_2$

③ Combinations :-

Is the number of ways we select without order at once r objects from n objects $(0 \leq r \leq n)$

. $\binom{n}{r} \equiv {}_nC_r \equiv C(n,r) \equiv C^r_n$

$= \frac{n!}{(n-r)!\,r!} = \frac{P(n,r)}{r!}$

. $P(n,r) \geq C(n,r)$

. $C(n,r) \equiv C(n, n-r)$

$\frac{n!}{(n-r)!\,r!} \equiv \frac{n!}{(n-r)!\,(n-n+r)!}$

$C(n,0) \equiv C(n,n) = 1$

Ex We have 13 students (8 males 5 females) are candidates to join a competetion

① In how many ways the director can select 4 students without order.

sol ${}_{13}C_4 \equiv \frac{13!}{(13-4)!\,4!} = 715$ ways

$\frac{13}{4}\ \frac{12}{3}\ \frac{11}{2}\ \frac{10}{1}$ We divide

② I how many ways we can select 4 students that all of them female at once.

sol $\frac{5}{4}\ \frac{4}{3}\ \frac{3}{2}\ \frac{2}{1}$ or ${}_5C_4 = \frac{5!}{4!\,1!}$

$= 5$

③ In how many ways we can select 4 students such that 3 of them are males.

sol $\frac{8}{3}\ \frac{7}{2}\ \frac{6}{1}\ \frac{5}{1} = 280$

${}_8C_3\ \ {}_5C_1$