1-a $\frac{1}{5} \times \left(\frac{75}{100} + \frac{95}{100} + \frac{80}{100} + \frac{65}{100} + \frac{85}{100} \right) = 0.8$

1-b Yes, he should choose label 4, and the incorrect rate decreases to : $1 - 0.35 = 0.65$

2. Orange.

$$g_{blue}(\vec{X}) = -\frac{1}{2}\log\left|\begin{bmatrix} 50.56 & 57.49 \\ 57.49 & 65.54 \end{bmatrix}\right| - \frac{1}{2}[X1-29.42,\ X2-33.98]\begin{bmatrix} 50.56 & 57.49 \\ 57.49 & 65.54 \end{bmatrix}^{-1}\begin{bmatrix} X1-29.42 \\ X2-33.98 \end{bmatrix}$$

$$= -1.076014805 - \frac{1}{2}[X1-29.42,\ X2-33.98]\begin{bmatrix} 7.618892622 & -6.683096381 \\ -6.683096381 & 5.877497878 \end{bmatrix}\begin{bmatrix} X_1-29.42 \\ X_2-33.98 \end{bmatrix}$$

$$g_{blue}\left(\begin{bmatrix} 11.1 \\ 27.8 \end{bmatrix}\right) = -635.2052648$$

$g_{blue}\left(\begin{bmatrix} 11.1 \\ 27.8 \end{bmatrix}\right) < g_{orange}\left(\begin{bmatrix} 11.1 \\ 27.8 \end{bmatrix}\right)$ therefore, the type is more likely to be orange.

$$g_{orange}(\vec{X}) = -\frac{1}{2}\log\left|\begin{bmatrix} 46.79 & 52.19 \\ 52.19 & 58.59 \end{bmatrix}\right| - \frac{1}{2}[X1-33.88,\ X2-37.80]\begin{bmatrix} 46.79 & 52.19 \\ 52.19 & 58.59 \end{bmatrix}^{-1}\begin{bmatrix} X1-33.88 \\ X2-37.80 \end{bmatrix}$$

$$= -1.434800998 - \frac{1}{2}[X1-33.88,\ X2-37.80]\begin{bmatrix} 3.323312535 & -2.960294952 \\ -2.960294952 & 2.653998866 \end{bmatrix}\begin{bmatrix} X1-33.88 \\ X_2-37.80 \end{bmatrix}$$

$$g_{orange}\left(\begin{bmatrix} 11.1 \\ 27.8 \end{bmatrix}\right) = -322.0601825$$

a. $\theta \cdot \theta \cdot (1-\theta) \cdot \theta \cdot \theta = \theta^4(1-\theta)$

b. $\log[\theta^4 \cdot (1-\theta)^1] = \log(\theta^4) + \log(1-\theta)$

$= 4\log\theta + \log(1-\theta)$

c.

$L(\theta) = 4\log\theta + \log(1-\theta)$

$\frac{dL}{d\theta} = 4 \cdot \frac{1}{\theta} + \frac{1}{1-\theta} \cdot (-1)$

$= \frac{4}{\theta} - \frac{1}{1-\theta}$

$= \frac{4(1-\theta)-\theta}{\theta(1-\theta)}$

$= \frac{4-5\theta}{\theta(1-\theta)} = 0$

$\theta = 0.8$

When $\theta = 0.8$, P(HHTHH) is maximum.