1-a x_1: cancer volume

x_2: patient's age

x_3: cancer type

Model 1: $\hat{y}_1 = w_1x_1 + w_0 = [w_0\ w_1]\begin{bmatrix}1\\ x_1\end{bmatrix}$

Model 2: $\hat{y}_2 = w_2x_2 + w_1x_1 + w_0 = [w_0\ w_1\ w_2]\begin{bmatrix}1\\ x_1\\ x_2\end{bmatrix}$

1-b $\hat{y}_3 = w_2x_2 + w_1x_1 \cdot x_3 + w_1'x_1(1-x_3) + w_0$

$= w_2x_2 + w_1(x_1x_3) + w_1'x_1 - w_1'(x_1x_3) + w_0$

$\cancel{= w_2x_2 + (w_1 - w_1')x_1x_3 + w_1'x_1 + w_0}$

$= w_2x_2 + w_1(x_1x_3) + w_1'(x_1 - x_1x_3) + w_0$

$= [w_0\ w_1'\ w_1\ w_2]\begin{bmatrix}1\\ x_1 - x_1x_3\\ x_1x_3\\ x_2\end{bmatrix}$

1-c model 1 has 2 parameters

model 2 has 3 parameters

model 3 has 4 parameters → the most complex model

If we consider model 3, then it would be most complex model.

If we only consider model 1 and 2, then model 2 is more complex than model 1.

1-d: model 1: $\begin{bmatrix}1 & 0.7\\ 1 & 1.3\\ 1 & 1.6\end{bmatrix}$ model 2: $\begin{bmatrix}1 & 0.7 & 55\\ 1 & 1.3 & 65\\ 1 & 1.6 & 70\end{bmatrix}$ model 3: $\begin{bmatrix}1 & 0 & 0.7 & 55\\ 1 & 1.3 & 0 & 65\\ 1 & 1.6 & 0 & 70\end{bmatrix}$

model 3 has 3 features: x1-x1*x3, x1*x3 and x2

1-e: model 2, for 2 reasons:

(1) model 2 has less MSE than model 1

(2) model 3's ~~has~~ training error is much lower than its validation error, which may implies overfitting.

2. feature selection,

x_1 — rainfall

x_2 — fertilizer

x_3 — average temperature

x_4 — number of sunny days

result

y — crop yields

Linear model: $y = w_0 + w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4$

3-a

$$X = \begin{bmatrix} 1 & 28540 \\ 1 & 40133 \\ 1 & 39900 \\ 1 & 0 \\ 1 & 0 \\ 1 & 42050 \\ 1 & 43220 \\ 1 & 39565 \\ 1 & 40400 \\ 1 & 54506 \\ 1 & \end{bmatrix}$$

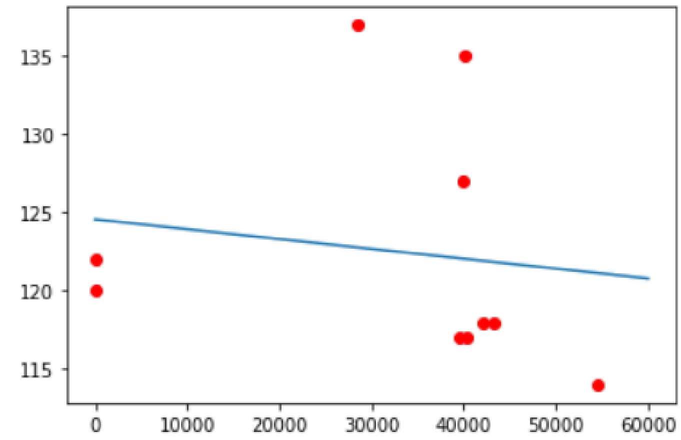
3-b:

$y = [w_0 \; w_1] \cdot X$

$w_0 \approx 124$

$w_1 \approx -6.3 \times 10^{-5}$

y = (-6.3*10^(-5))*x + 124



3-c before removing the noise

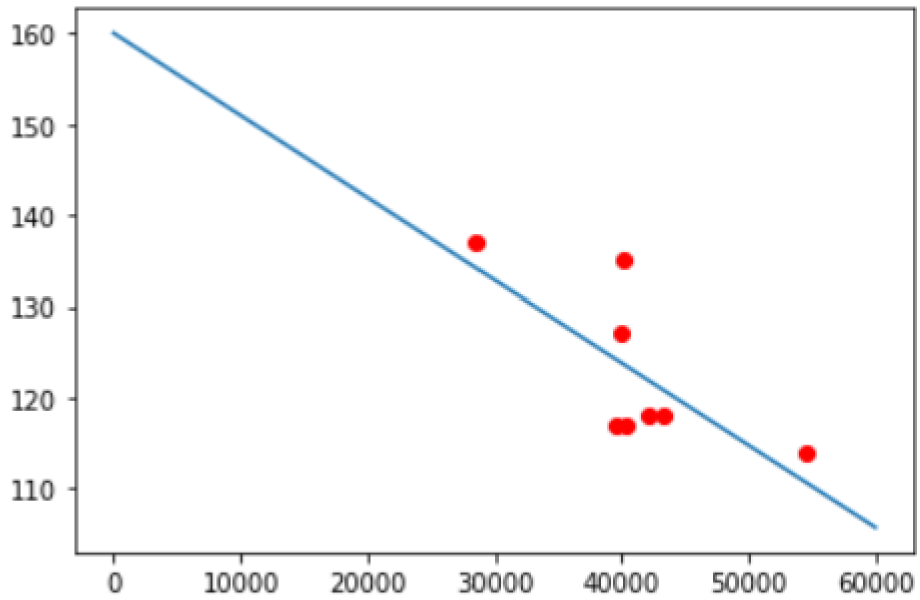
3-d $R^2 = \frac{ESS}{TSS} = \frac{12.087}{566.5} = 0.02$

3-e $X = 40000$

$y = w_0 + w_1 X = 122.05$

3-f-b: $\begin{bmatrix} w_0 \\ w_1 \end{bmatrix} = \begin{bmatrix} 160 \\ -9 \times 10^{-4} \end{bmatrix}$

$y = 160 - (9 \times 10^{-4}) X$



3-f-c after removing the noise

3-f-d: $R^2 = \cancel{0.51}\ \dfrac{284.89}{558.875} = 0.51$

3-f-e: 123.815

4-a
$$RSS(w) = \sum_{i=1}^{N} (\hat{y_i} - y_i)^2$$
$$= \sum_{i=1}^{N} (wx_i - y_i)^2$$
$$= \|Xw - y\|_2^2$$
$$= (Xw-y)^T (Xw-y)$$
$$= \boxed{w^T X^T X w - 2wX^T y + y^T y}$$

4-b $\nabla_w = 2X^TXw - 2X^Ty = 0$

$X^TXw = X^Ty$ $\quad \boxed{w = \frac{X^Ty}{X^TX}}$

w is scalar, so I write in this way.

5-a $p(y|w,x) = p(\varepsilon)$

$\varepsilon^{(i)} = y^{(i)} - w^Tx^{(i)}$

$W_{ML} = \arg\max_{w} \prod_{i=1}^{N} p(\varepsilon^{(i)})$

$L(w) = \prod_{i=1}^{N} p(\varepsilon^{(i)})$

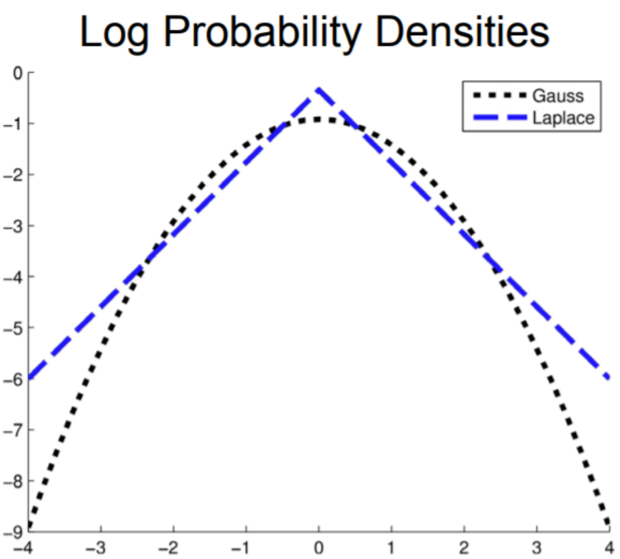
$\log(L(w)) = \sum_{i=1}^{N} \log \frac{1}{2b} e^{\frac{-|\varepsilon^{(i)}|}{b}}$

$= \underbrace{N \log \frac{1}{2b}}_{\text{same for any } w} + \sum_{i=1}^{N} \frac{1}{-b} |\varepsilon^{(i)}|$

$\log(L(w)) \Rightarrow g(w) = \frac{1}{-b} \sum_{i=1}^{N} |\varepsilon^{(i)}|$

$= -\frac{1}{b} \sum_{i=1}^{N} |y^{(i)} - w^T x^{(i)}|$ #

5-b



The probability of noise data appearing in Laplace Distribution is higher than Gaussian Distribution, therefore the Laplace model would be more robust than the Gaussian.

6 $g(w) = (y - Xw)^T \Omega (y - Xw)$

$= (y^T - w^T X^T)\Omega(y - Xw)$ $\quad x^T\Omega y = y^T\Omega x$

$= (y^T\Omega - w^T X^T \Omega)(y - Xw)$

$= y^T\Omega y \boxed{- w^T X^T \Omega y - y^T \Omega X w} + w^T X^T \Omega X w$

$= w^T(X^T\Omega X)w - 2(y^T\Omega^T X)w + y^T\Omega y$

$\frac{\partial g(w)}{\partial w} = 2X^T\Omega X w - 2y^T\Omega^T X$

$X^T\Omega X w = y^T\Omega^T X$

$w = (X^T\Omega X)^{-1} y^T\Omega^T X$