Laporan Praktikum 5 AMP

Antonius Aditya Rizky Wijaya

G5402221003

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Cross-Validation and the Bootstrap

The Validation Set Approach

```
library(ISLR2)

## Warning: package 'ISLR2' was built under R version 4.3.3

set.seed(1)
train <- sample(392, 196)
```

Penjelasan: Training set dipilih secara acak sebanyak 196 observasi dari total 392 data.

```
lm.fit <- lm(mpg ~ horsepower, data = Auto, subset = train)
```

Penjelasan: Model regresi linear sederhana dibuat dengan `horsepower` sebagai prediktor untuk memprediksi `mpg`.

```
attach(Auto)
mean((mpg - predict(lm.fit, Auto))[-train]^2)

## [1] 23.26601
```

Penjelasan: MSE dihitung untuk test set, yang mengukur seberapa baik model memprediksi `mpg` untuk data yang belum terlihat.

```
lm.fit2 <- lm(mpg ~ poly(horsepower, 2), data = Auto,
    subset = train)
mean((mpg - predict(lm.fit2, Auto))[-train]^2)

## [1] 18.71646

lm.fit3 <- lm(mpg ~ poly(horsepower, 3), data = Auto,
    subset = train)
mean((mpg - predict(lm.fit3, Auto))[-train]^2)

## [1] 18.79401
```

Penjelasan: Model polinomial mungkin memberikan MSE lebih kecil dibanding model linear, artinya model lebih cocok menangkap pola data. Perbandingan MSE antar model bisa membantu memilih model terbaik.

```r
set.seed(2)
train <- sample(392, 196)
lm.fit <- lm(mpg ~ horsepower, subset = train)
mean((mpg - predict(lm.fit, Auto))[-train]^2)
```

```
## [1] 25.72651
```

```r
lm.fit2 <- lm(mpg ~ poly(horsepower, 2), data = Auto,
    subset = train)
mean((mpg - predict(lm.fit2, Auto))[-train]^2)
```

```
## [1] 20.43036
```

```r
lm.fit3 <- lm(mpg ~ poly(horsepower, 3), data = Auto,
    subset = train)
mean((mpg - predict(lm.fit3, Auto))[-train]^2)
```

```
## [1] 20.38533
```

Penjelasan:

> Hasilnya bisa berbeda karena pemilihan training set yang berbeda. Jika MSE polinomial tetap lebih rendah dari linear, ini memperkuat bahwa regresi polinomial lebih baik dalam menangkap pola hubungan antara horsepower dan mpg. Variasi MSE antar pemilihan training set menunjukkan bahwa model bisa dipengaruhi oleh pemilihan data training secara acak.

Leave-One-Out Cross-Validation

```r
glm.fit <- glm(mpg ~ horsepower, data = Auto)
coef(glm.fit)
```

```
## (Intercept)  horsepower
##  39.9358610  -0.1578447
```

Penjelasan:

> Model regresi linear dibuat untuk memprediksi mpg berdasarkan horsepower, dan hasilnya sama dengan model dari `lm()`.

```r
lm.fit <- lm(mpg ~ horsepower, data = Auto)
coef(lm.fit)
```

```
## (Intercept)  horsepower
##  39.9358610  -0.1578447
```

Penjelasan:

> Menggunakan `glm()` atau `lm()` untuk regresi linear tidak membuat perbedaan signifikan dalam konteks ini.

```r
library(boot)
glm.fit <- glm(mpg ~ horsepower, data = Auto)
```

```r
cv.err <- cv.glm(Auto, glm.fit)
cv.err$delta

## [1] 24.23151 24.23114
```

Penjelasan:

> LOOCV digunakan untuk mengukur akurasi model regresi. Hasilnya adalah MSE estimasi model pada data baru.

```r
cv.error <- rep(0, 10)
for (i in 1:10) {
  glm.fit <- glm(mpg ~ poly(horsepower, i), data = Auto)
  cv.error[i] <- cv.glm(Auto, glm.fit)$delta[1]
}
cv.error

##  [1] 24.23151 19.24821 19.33498 19.42443 19.03321 18.97864 18.83305
18.96115
##  [9] 19.06863 19.49093
```

Penjelasan:

> LOOCV digunakan untuk membandingkan model polinomial dari derajat 1 hingga 10. Jika MSE semakin kecil seiring bertambahnya derajat, model polinomial lebih baik dari model linear.

k-Fold Cross-Validation

```r
set.seed(17)
cv.error.10 <- rep(0, 10)
for (i in 1:10) {
  glm.fit <- glm(mpg ~ poly(horsepower, i), data = Auto)
  cv.error.10[i] <- cv.glm(Auto, glm.fit, K = 10)$delta[1]
}
cv.error.10

##  [1] 24.27207 19.26909 19.34805 19.29496 19.03198 18.89781 19.12061
19.14666
##  [9] 18.87013 20.95520
```

Penjelasan:

> 10-fold CV lebih efisien daripada LOOCV karena tidak perlu membangun model sebanyak jumlah data. Hasil MSE dibandingkan untuk menemukan model polinomial terbaik. Jika tren mirip dengan LOOCV, berarti model polinomial lebih cocok untuk data ini.

The Bootstrap

Estimating the Accuracy of a Statistic of Interest

```
alpha.fn <- function(data, index) {
  X <- data$X[index]
  Y <- data$Y[index]
  (var(Y) - cov(X, Y)) / (var(X) + var(Y) - 2 * cov(X, Y))
}
```

Penjelasan:

Fungsi ini akan digunakan untuk melakukan bootstrap estimasi alpha dengan berbagai sampel dari dataset `Portfolio`.

```
alpha.fn(Portfolio, 1:100)

## [1] 0.5758321
```

Penjelasan:

Nilai alpha dihitung tanpa resampling, sehingga ini menjadi dasar perbandingan dengan metode bootstrap nanti.

```
set.seed(7)
alpha.fn(Portfolio, sample(100, 100, replace = T))

## [1] 0.5385326
```

Penjelasan:

Bootstrap memungkinkan kita memperkirakan variabilitas alpha berdasarkan sampel yang berbeda-beda.

```
boot(Portfolio, alpha.fn, R = 1000)

##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
##
## Call:
## boot(data = Portfolio, statistic = alpha.fn, R = 1000)
##
##
## Bootstrap Statistics :
##      original       bias    std. error
## t1* 0.5758321 0.0007959475  0.08969074
```

Penjelasan:

Bootstrap digunakan untuk mengestimasi distribusi alpha, termasuk standar errornya.

Estimating the Accuracy of a Linear Regression Model

```
boot.fn <- function(data, index)
  coef(lm(mpg ~ horsepower, data = data, subset = index))
boot.fn(Auto, 1:392)

## (Intercept)  horsepower
##  39.9358610  -0.1578447
```

Penjelasan:

Fungsi ini akan digunakan untuk melakukan bootstrap estimasi koefisien regresi linear.

```
set.seed(1)
boot.fn(Auto, sample(392, 392, replace = T))

## (Intercept)  horsepower
##  40.3404517  -0.1634868

boot.fn(Auto, sample(392, 392, replace = T))

## (Intercept)  horsepower
##  40.1186906  -0.1577063
```

Penjelasan:

Bootstrap digunakan untuk memahami variabilitas koefisien regresi. Hasil koefisien bisa berbeda tiap iterasi, menunjukkan distribusi kemungkinan nilai koefisien.

```
boot(Auto, boot.fn, 1000)

##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
##
## Call:
## boot(data = Auto, statistic = boot.fn, R = 1000)
##
##
## Bootstrap Statistics :
##       original        bias    std. error
## t1* 39.9358610  0.0544513229 0.841289790
## t2* -0.1578447 -0.0006170901 0.007343073
```

Penjelasan:

Bootstrap memberikan distribusi yang lebih baik untuk koefisien regresi dibandingkan metode konvensional.

```
summary(lm(mpg ~ horsepower, data = Auto))$coef
```

```
##                Estimate  Std. Error   t value      Pr(>|t|)
## (Intercept) 39.9358610 0.717498656  55.65984 1.220362e-187
## horsepower  -0.1578447 0.006445501 -24.48914  7.031989e-81
```

Penjelasan:

Bootstrap memungkinkan perbandingan antara estimasi standar regresi dan distribusi bootstrap untuk mengukur ketidakpastian estimasi.

```
boot.fn <- function(data, index)
  coef(
      lm(mpg ~ horsepower + I(horsepower^2),
        data = data, subset = index)
    )
set.seed(1)
boot(Auto, boot.fn, 1000)

##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
##
## Call:
## boot(data = Auto, statistic = boot.fn, R = 1000)
##
##
## Bootstrap Statistics :
##         original        bias     std. error
## t1* 56.900099702  3.511640e-02 2.0300222526
## t2* -0.466189630 -7.080834e-04 0.0324241984
## t3*  0.001230536  2.840324e-06 0.0001172164

summary(
    lm(mpg ~ horsepower + I(horsepower^2), data = Auto)
  )$coef

##                     Estimate   Std. Error   t value      Pr(>|t|)
## (Intercept)     56.900099702 1.8004268063  31.60367 1.740911e-109
## horsepower      -0.466189630 0.0311246171 -14.97816  2.289429e-40
## I(horsepower^2)  0.001230536 0.0001220759  10.08009  2.196340e-21
```

Penjelasan:

Bootstrap memungkinkan estimasi lebih akurat untuk regresi polinomial dengan memperhitungkan ketidakpastian lebih baik dibandingkan metode konvensional. Jika hasil bootstrap mirip dengan regresi standar, berarti model stabil. Jika berbeda, berarti ada ketidakpastian tinggi dalam estimasi koefisien.

Exercise

Nomor 5

In Chapter 4, we used logistic regression to predict the probability of `default` using `income` and `balance` on the `Default` data set. We will now estimate the test error of this logistic regression model using the validation set approach. Do not forget to set a random seed before beginning your analysis.

a. Fit a logistic regression model that uses `income` and `balance` to predict `default`.

```
library(ISLR2)
defult_model <- glm(default ~ income + balance, data = Default, family =
"binomial")
```

b. Using the validation set approach, estimate the test error of this model. In order to do this, you must perform the following steps:
   i. Split the sample set into a training set and a validation set.
   ii. Fit a multiple logistic regression model using only the training observations.
   iii. Obtain a prediction of default status for each individual in the validation set by computing the posterior probability of default for that individual, and classifying the individual to the `default` category if the posterior probability is greater than 0.5.
   iv. Compute the validation set error, which is the fraction of the observations in the validation set that are misclassified.

```
set.seed(2)
# i. Split the sample
train_index <- sample(nrow(Default), nrow(Default)/2)
train_data <- Default[train_index, ]
valid_data <- Default[-train_index, ]

# ii. Fit a multiple logistic regression model
model_train <- glm(default ~ income + balance, data = train_data, family =
"binomial")

# iii. Obtain a prediction of default status and classifying for each
individual in the validation set
prob_predict <- predict(model_train, valid_data, type = "response")
class_predict <- ifelse(prob_predict > 0.5, "Yes", "No")

# iv. Compute the validation set error
(table(class_predict, valid_data$default))

##
## class_predict   No  Yes
##           No  4819  101
##           Yes   18   62
```

```
(mean(class_predict != valid_data$default))

## [1] 0.0238
```

c. Repeat the process in (b) three times, using three different splits of the observations into a training set and a validation set. Comment on the results obtained.

```
# Pengulangan pertama
set.seed(2)
train_index <- sample(nrow(Default), nrow(Default) *0.8)
train_data <- Default[train_index, ]
valid_data <- Default[-train_index, ]
model_train_d <- glm(default ~ income + balance, data = train_data, family =
"binomial")
prob_predict <- predict(model_train_d, valid_data, type = "response")
class_predict <- ifelse(prob_predict > 0.5, "Yes", "No")
(table(class_predict, valid_data$default))

##
## class_predict   No  Yes
##           No  1929   33
##           Yes    9   29

(mean(class_predict != valid_data$default))

## [1] 0.021

# Pengulangan kedua
set.seed(2)
train_index <- sample(nrow(Default), nrow(Default) *0.7)
train_data <- Default[train_index, ]
valid_data <- Default[-train_index, ]
model_train_d <- glm(default ~ income + balance, data = train_data, family =
"binomial")
prob_predict <- predict(model_train_d, valid_data, type = "response")
class_predict <- ifelse(prob_predict > 0.5, "Yes", "No")
(table(class_predict, valid_data$default))

##
## class_predict   No  Yes
##           No  2894   53
##           Yes   11   42

(mean(class_predict != valid_data$default))

## [1] 0.02133333

# Pengulangan ketiga
set.seed(2)
train_index <- sample(nrow(Default), nrow(Default) *0.4)
train_data <- Default[train_index, ]
valid_data <- Default[-train_index, ]
model_train_d <- glm(default ~ income + balance, data = train_data, family =
```

```
"binomial")
prob_predict <- predict(model_train_d, valid_data, type = "response")
class_predict <- ifelse(prob_predict > 0.5, "Yes", "No")
(table(class_predict, valid_data$default))

##
## class_predict   No  Yes
##           No  5782  130
##           Yes   19   69

(mean(class_predict != valid_data$default))

## [1] 0.02483333
```

Dengan menggunakan `set.seed(2)`, proporsi 80: 20 menghasilkan validation error yang lebih kecil (2,1%) dibanding proporsi 70: 30 (2,13%) dan proporsi 40: 60 (2,48%), bahkan proporsi 50: 50 pada soal (b) (2,38%).

Dari sini saya bisa menyimpulkan:

1. Jika proporsi `training set` lebih besar, akan memungkinkan model lebih akurat karena lebih banyak data untuk di `train`. Namun, `validation set` yang terlalu kecil dapat membuat `validation error` menjadi kurang stabil akibat kurangnya data untuk divalidasi.
2. Jika proporsi `training set` lebih kecil, akan membuat model kurang stabil (`underfitting`) karena data untuk di-`train`-nya terbatas. Namun, `validation set` yang lebih besar akan membuat `validation error` menjadi lebih representatif terhadap populasi (distribusi datanya lebih mendekati distribusi populasi).

Bagaimana menentukan proporsi data yang tepat?

1. `Training data` > `validation data` (umum digunakan), dapat digunakan jika `dataset` kecil atau model yang kompleks. Karena butuh lebih banyak data untuk di `train` agar model lebih akurat, selain itu sedikit persentase `validation data` untuk `dataset` yang kecil sudah cukup untuk merepresentasikan populasi.
2. `Training data` < `validation data`, dapat digunakan jika `dataset` besar (misal 1 juta observasi) atau variasi data yang kompleks. Karena butuh lebih banyak data untuk validasi agar hasil evaluasi model lebih akurat, selain itu sedikit persentase `training data` untuk `dataset` yang besar sudah cukup besar untuk di `train`.
3. Bisa menggunakan tujuan pemodelan sebagai alasan membagi data. Jika tujuannya untuk generalisasi model, gunakan training data lebih besar. Jika tujuannya untuk evaluasi model (gunakan validation data lebih besar).

d. Now consider a logistic regression model that predicts the probability of `default` using `income`, `balance`, and a dummy variable for `student`. Estimate the test error for this model using the validation set approach. Comment on whether or not including a dummy variable for `student` leads to a reduction in the test error rate.

```
# Pengulangan pertama
set.seed(2)
train_index <- sample(nrow(Default), nrow(Default) *0.8)
train_data <- Default[train_index, ]
valid_data <- Default[-train_index, ]
model_train_stu <- glm(default ~ income + balance + student, data =
train_data, family = "binomial")
prob_predict <- predict(model_train_stu, valid_data, type = "response")
class_predict <- ifelse(prob_predict > 0.5, "Yes", "No")
(table(class_predict, valid_data$default))

##
## class_predict   No  Yes
##           No  1928   35
##           Yes   10   27

(mean(class_predict != valid_data$default))

## [1] 0.0225

# Pengulangan kedua
set.seed(2)
train_index <- sample(nrow(Default), nrow(Default) *0.7)
train_data <- Default[train_index, ]
valid_data <- Default[-train_index, ]
model_train_stu <- glm(default ~ income + balance + student, data =
train_data, family = "binomial")
prob_predict <- predict(model_train_stu, valid_data, type = "response")
class_predict <- ifelse(prob_predict > 0.5, "Yes", "No")
(table(class_predict, valid_data$default))

##
## class_predict   No  Yes
##           No  2894   57
##           Yes   11   38

(mean(class_predict != valid_data$default))

## [1] 0.02266667

# Pengulangan ketiga
set.seed(2)
train_index <- sample(nrow(Default), nrow(Default) *0.4)
train_data <- Default[train_index, ]
valid_data <- Default[-train_index, ]
model_train_stu <- glm(default ~ income + balance + student, data =
train_data, family = "binomial")
```

```
prob_predict <- predict(model_train_stu, valid_data, type = "response")
class_predict <- ifelse(prob_predict > 0.5, "Yes", "No")
(table(class_predict, valid_data$default))
```

```
##
## class_predict   No  Yes
##           No  5784  139
##           Yes   17   60
```

```
(mean(class_predict != valid_data$default))
```

```
## [1] 0.026
```

Bisa dilihat untuk semua proporsi data, `validation error`-nya meningkat dibanding yang tanpa variabel dummy `student`, hal ini mengindikasikan bahwa `student` tidak relevan karena model menjadi lebih kompleks namun tidak memberikan manfaat tambahan (`overfitting`).

Nomor 6

> We continue to consider the use of a logistic regression model to predict the probability of `default` using `income` and `balance` on the `Default` data set. In particular, we will now compute estimates for the standard errors of the `income` and `balance` logistic regression coefficients in two different ways: (1) using the bootstrap, and (2) using the standard formula for computing the standard errors in the `glm()` function. Do not forget to set a random seed before beginning your analysis.

a. Using the `summary()` and `glm()` functions, determine the estimated standard errors for the coefficients associated with `income` and `balance` in a multiple logistic regression model that uses both predictors.

```
library(ISLR2)
set.seed(3)
model_6 <- glm(default ~ income + balance, data = Default, family = binomial)
summary(model_6)
```

```
##
## Call:
## glm(formula = default ~ income + balance, family = binomial,
##     data = Default)
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.154e+01  4.348e-01 -26.545  < 2e-16 ***
## income       2.081e-05  4.985e-06   4.174 2.99e-05 ***
## balance      5.647e-03  2.274e-04  24.836  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 2920.6  on 9999  degrees of freedom
## Residual deviance: 1579.0  on 9997  degrees of freedom
## AIC: 1585
##
## Number of Fisher Scoring iterations: 8
```

Didapat $\beta_1 = 2.081 * 10^{-05}$ dengan `standard error`-nya $= 4.985 * 10^{-06}$ dan $\beta_2 = 5.647 * 10^{-03}$ dengan `standard error`-nya $= 2.274 * 10^{-04}$

b. Write a function, `boot.fn()`, that takes as input the `Default` data set as well as an index of the observations, and that outputs the coefficient estimates for `income` and `balance` in the multiple logistic regression model.

```
boot.fn <- function(data, index) {
  model <- glm(default ~ income + balance, data = data[index, ], family =
'binomial')
  return(coef(model)[2:3])
}
```

c. Use the `boot()` function together with your `boot.fn()` function to estimate the standard errors of the logistic regression coefficients for income and balance.

```
library(boot)
set.seed(3)
(hasil_bootstrap <- boot(data = Default, statistic = boot.fn, R = 1000))

##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
##
## Call:
## boot(data = Default, statistic = boot.fn, R = 1000)
##
##
## Bootstrap Statistics :
##         original       bias     std. error
## t1* 2.080898e-05 2.058964e-07 4.857844e-06
## t2* 5.647103e-03 1.968308e-05 2.310337e-04
```

d. Comment on the estimated standard errors obtained using the `glm()` function and using your bootstrap function.

Output dari `summary(model_6)` pada bagian (a) memberikan `standard error` berdasarkan metode formula bawaan dari regresi logistik (`glm()`). Sementara output dari `boot()` pada bagian (c) memberikan `standard error` berdasarkan metode bootstrap, yang menghitung variasi.

Dengan metode bootstrap didapat $\beta_1 = 2.080898 * 10^{-05}$ dengan `standard error`-nya $= 4.857844 * 10^{-06}$ dan $\beta_2 = 5.647103 * 10^{-03}$ dengan `standard error`-nya $= 2.310337 * 10^{-04}$.

`Standard error` berdasarkan metode bootstrap sedikit lebih besar dari `standard error` berdasarkan metode formula bawaan, hal ini karena metode bootstrap lebih sensitif terhadap variasi data.

Nomor 7

In Sections 5.3.2 and 5.3.3, we saw that the `cv.glm()` function can be used in order to compute the LOOCV test error estimate. Alternatively, one could compute those quantities using just the `glm()` and `predict.glm()` functions, and a for loop. You will now take this approach in order to compute the LOOCV error for a simple logistic regression model on the `Weekly` data set. Recall that in the context of classification problems, the LOOCV error is given in (5.4).

a. Fit a logistic regression model that predicts `Direction` using `Lag1` and `Lag2`.

```
library(ISLR2)
data(Weekly)
model_7a <- glm(Direction ~ Lag1 + Lag2, data = Weekly, family = 'binomial')
summary(model_7a)

##
## Call:
## glm(formula = Direction ~ Lag1 + Lag2, family = "binomial", data = Weekly)
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.22122    0.06147   3.599 0.000319 ***
## Lag1        -0.03872    0.02622  -1.477 0.139672
## Lag2         0.06025    0.02655   2.270 0.023232 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1496.2  on 1088  degrees of freedom
## Residual deviance: 1488.2  on 1086  degrees of freedom
## AIC: 1494.2
##
## Number of Fisher Scoring iterations: 4
```

b. Fit a logistic regression model that predicts `Direction` using `Lag1` and `Lag2` *using all but the first observation*.

```
model_7b <- glm(Direction ~ Lag1 + Lag2, data = Weekly[-1, ], family =
'binomial')
summary(model_7b)

##
## Call:
## glm(formula = Direction ~ Lag1 + Lag2, family = "binomial", data =
Weekly[-1,
```

```
##     ])
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.22324    0.06150   3.630 0.000283 ***
## Lag1        -0.03843    0.02622  -1.466 0.142683
## Lag2         0.06085    0.02656   2.291 0.021971 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1494.6  on 1087  degrees of freedom
## Residual deviance: 1486.5  on 1085  degrees of freedom
## AIC: 1492.5
##
## Number of Fisher Scoring iterations: 4
```

c. Use the model from (b) to predict the direction of the first observation. You can do this by predicting that the first observation will go up if $P(\texttt{Direction="Up"} \mid \texttt{Lag1}, \texttt{Lag2}) > 0.5$. Was this observation correctly classified?

```
prob_predict <- predict(model_7b, Weekly[1,], type = "response")
class_predict <- ifelse(prob_predict > 0.5, "Up", "Down")

cat("Prediksi:\n")

## Prediksi:

class_predict

##    1
## "Up"

cat("\nYang benar:\n")

##
## Yang benar:

Weekly$Direction[1]

## [1] Down
## Levels: Down Up
```

Prediksinya salah, data `Direction` pada observasi pertama seharusnya “Down”, namun hasil prediksinya “Up”.

d. Write a for loop from $i = 1$ to $i = n$, where n is the number of observations in the data set, that performs each of the following steps:
   i. Fit a logistic regression model using all but the ith observation to predict `Direction` using `Lag1` and `Lag2` .

ii. Compute the posterior probability of the market moving up for the ith observation.
iii. Use the posterior probability for the ith observation in order to predict whether or not the market moves up.
iv. Determine whether or not an error was made in predicting the direction for the ith observation. If an error was made, then indicate this as a 1, and otherwise indicate it as a 0.

```
error <- numeric(nrow(Weekly))
for (i in 1:nrow(Weekly)) {
  model <- glm(Direction ~ Lag1 + Lag2, data = Weekly[-i, ], family =
"binomial")
  pred <- predict(model, Weekly[i, ], type = "response")
  class <- ifelse(pred > 0.5, "Up", "Down")
  error[i] <- ifelse(class != Weekly$Direction[i], 1, 0)
}
```

e. Take the average of the n numbers obtained in (d) in order to obtain the LOOCV estimate for the test error. Comment on the results.

```
loocv_error <- mean(error)
cat("LOOCV Test Error:", loocv_error, "\n")

## LOOCV Test Error: 0.4499541
```

LOOCV Test Error sekitar 45% yang mengindikasikan bahwa prediksi mungkin akan sedikit lebih sering benar daripada salah, tapi memang tidak menutup fakta bahwa kemungkinan salahnya pun besar.

Nomor 8

We will now perform cross-validation on a simulated data set.

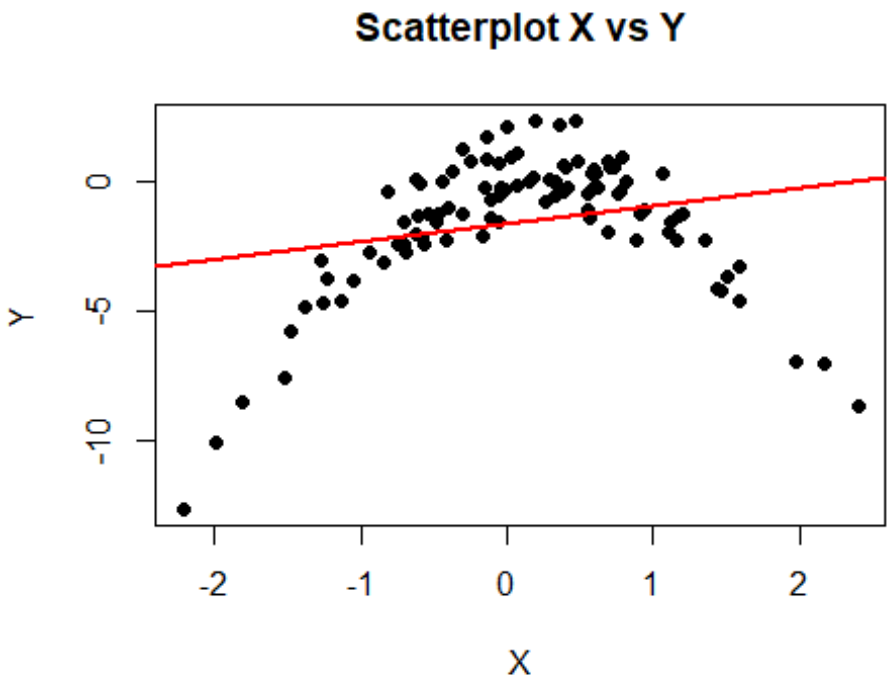
a. Generate a simulated data set. In this data set, what is n and what is p? Write out the model used to generate the data in equation form.

```
set.seed(1)
x <- rnorm(100)
y <- x - 2 * x^2 + rnorm(100)
n <- length(x)  # n = 100
p <- 1
cat("Jumlah observasi (n):", n, "\n")

## Jumlah observasi (n): 100

cat("Jumlah prediktor (p):", p, "\n")

## Jumlah prediktor (p): 1
```

Model matematis: $y = -2x^2 + x + \epsilon$, di mana $\epsilon \sim \mathrm{N}(0, 1)$

b. Create a scatterplot of X against Y. Comment on what you find.

```
plot(x, y, main = "Scatterplot X vs Y", xlab = "X", ylab = "Y", pch = 16, col
= "black")
abline(lm(y ~ x), col = "red", lwd = 2)
```



Scatterplot menunjukkan hubungan non-linear antara x dan y. Secara visual, terlihat bahwa y memiliki pola kuadratik terhadap x.

c. Set a random seed, and then compute the Leave-One-Out Cross-Validation (LOOCV) errors that result from fitting the following four models using least squares:

i. $Y = \beta_0 + \beta_1 X + \epsilon$
ii. $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \epsilon$
iii. $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + \epsilon$
iv. $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + \beta_4 X^4 + \epsilon.$

Note you may find it helpful to use the `data.frame()` function to create a single data set containing both X and Y.

```
data <- data.frame(x = x, y = y)

loocv_error <- function(degree) {
  errors <- rep(0, n)
  for (i in 1:n) {
    train_data <- data[-i, ]
    test_data <- data[i, ]
    model <- lm(y ~ poly(x, degree), data = train_data)
```

```
    pred <- predict(model, newdata = test_data)
    errors[i] <- (pred - test_data$y)^2
  }
  return(mean(errors))
}

set.seed(1)
loocv_1 <- loocv_error(1)
loocv_2 <- loocv_error(2)
loocv_3 <- loocv_error(3)
loocv_4 <- loocv_error(4)

cat("LOOCV Error Linear:", loocv_1, "\n")

## LOOCV Error Linear: 7.288162

cat("LOOCV Error Kuadratik:", loocv_2, "\n")

## LOOCV Error Kuadratik: 0.9374236

cat("LOOCV Error Kubi:", loocv_3, "\n")

## LOOCV Error Kubi: 0.9566218

cat("LOOCV Error Kuartik:", loocv_4, "\n")

## LOOCV Error Kuartik: 0.9539049
```

d. Repeat (c) using another random seed, and report your results. Are your results the same as what you got in (c)? Why?

```
set.seed(2)
loocv_1_d <- loocv_error(1)
loocv_2_d <- loocv_error(2)
loocv_3_d <- loocv_error(3)
loocv_4_d <- loocv_error(4)

cat("Hasil LOOCV dengan seed berbeda:\n")

## Hasil LOOCV dengan seed berbeda:

cat("LOOCV Error Linear:", loocv_1, "\n")

## LOOCV Error Linear: 7.288162

cat("LOOCV Error Kuadratik:", loocv_2, "\n")

## LOOCV Error Kuadratik: 0.9374236

cat("LOOCV Error Kubik:", loocv_3, "\n")

## LOOCV Error Kubik: 0.9566218

cat("LOOCV Error Kuartik:", loocv_4, "\n")
```

```
## LOOCV Error Kuartik: 0.9539049
```

Hasil LOOCV error kemungkinan sama, karena LOOCV tidak bergantung pada pengacakan subset seperti metode `k-fold`.

e. Which of the models in (c) had the smallest LOOCV error? Is this what you expected? Explain your answer.

Model linear (derajat 1) memiliki error lebih tinggi karena tidak mampu menggambarkan hubungan non-linear yang jelas dalam data.

Model kubik (derajat 3) dan kuartik (derajat 4) dapat memiliki error yang lebih rendah dibandingkan model linear, tetapi mungkin tidak jauh lebih baik dari model kuadratik. Sehingga model kuadratik (derajat 2) memiliki LOOCV error terkecil.

Ini sesuai ekspektasi, karena model yang sebenarnya digunakan untuk mensimulasikan data adalah kuadratik.

f. Comment on the statistical significance of the coefficient estimates that results from fitting each of the models in (c) using least squares. Do these results agree with the conclusions drawn based on the cross-validation results?

```r
model_8f_1 <- lm(y ~ poly(x, 1), data = data)
model_8f_2 <- lm(y ~ poly(x, 2), data = data)
model_8f_3 <- lm(y ~ poly(x, 3), data = data)
model_8f_4 <- lm(y ~ poly(x, 4), data = data)

summary(model_8f_1)
```

```
##
## Call:
## lm(formula = y ~ poly(x, 1), data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.5161 -0.6800  0.6812  1.5491  3.8183
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -1.550      0.260  -5.961 3.95e-08 ***
## poly(x, 1)     6.189      2.600   2.380   0.0192 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.6 on 98 degrees of freedom
## Multiple R-squared:  0.05465,    Adjusted R-squared:  0.045
## F-statistic: 5.665 on 1 and 98 DF,  p-value: 0.01924
```

```r
summary(model_8f_2)
```

```
##
## Call:
```

```
## lm(formula = y ~ poly(x, 2), data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9650 -0.6254 -0.1288  0.5803  2.2700
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -1.5500     0.0958  -16.18  < 2e-16 ***
## poly(x, 2)1   6.1888     0.9580    6.46 4.18e-09 ***
## poly(x, 2)2 -23.9483     0.9580  -25.00  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.958 on 97 degrees of freedom
## Multiple R-squared:  0.873,  Adjusted R-squared:  0.8704
## F-statistic: 333.3 on 2 and 97 DF,  p-value: < 2.2e-16

summary(model_8f_3)

##
## Call:
## lm(formula = y ~ poly(x, 3), data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9765 -0.6302 -0.1227  0.5545  2.2843
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -1.55002    0.09626 -16.102  < 2e-16 ***
## poly(x, 3)1   6.18883    0.96263   6.429 4.97e-09 ***
## poly(x, 3)2 -23.94830    0.96263 -24.878  < 2e-16 ***
## poly(x, 3)3   0.26411    0.96263   0.274    0.784
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9626 on 96 degrees of freedom
## Multiple R-squared:  0.8731, Adjusted R-squared:  0.8691
## F-statistic: 220.1 on 3 and 96 DF,  p-value: < 2.2e-16

summary(model_8f_4)

##
## Call:
## lm(formula = y ~ poly(x, 4), data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.0550 -0.6212 -0.1567  0.5952  2.2267
##
```

```
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -1.55002    0.09591 -16.162  < 2e-16 ***
## poly(x, 4)1   6.18883    0.95905   6.453 4.59e-09 ***
## poly(x, 4)2 -23.94830    0.95905 -24.971  < 2e-16 ***
## poly(x, 4)3   0.26411    0.95905   0.275    0.784
## poly(x, 4)4   1.25710    0.95905   1.311    0.193
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9591 on 95 degrees of freedom
## Multiple R-squared:  0.8753, Adjusted R-squared:  0.8701
## F-statistic: 166.7 on 4 and 95 DF,  p-value: < 2.2e-16
```

Koefisien Signifikan (p-value < 0.05): Model Linear: signifikan namun kurang kuat Model Kuadratik: semua koefisien sangat signifikan Model Kubik: ada koefisien yang tidak signifikan Model Kuartik: ada koefisien yang tidak signifikan

Jika digabungkan dengan hasil LOOCV Error setiap model, dapat disimpulkan bahwa model kuadratik kemungkinan besar optimal, dengan koefisien yang signifikan dan LOOCV error yang rendah. Meski model kubik dan kuartik memiliki LOOCV error yang cukup rendah juga, namun dua model ini memiliki koefisien yang tidak signifikan, sehingga menunjukkan bahwa terjadi overfitting.

Nomor 9

We will now consider the `Boston` housing data set, from the `ISLR2` library.

a. Based on this data set, provide an estimate for the population mean of `medv`. Call this estimate $\hat{\mu}$.

```
library(ISLR2)
data("Boston")
mean_medv <- mean(Boston$medv)
cat("Rata-rata populasi (μ):", mean_medv, "\n")

## Rata-rata populasi (μ): 22.53281
```

b. Provide an estimate of the standard error of $\hat{\mu}$. Interpret this result.

Hint: We can compute the standard error of the sample mean by dividing the sample standard deviation by the square root of the number of observations.

```
se_mean <- sd(Boston$medv) / sqrt(length(Boston$medv))
cat("SE mean:", se_mean, "\n")

## SE mean: 0.4088611
```

c. Now estimate the standard error of $\hat{\mu}$ using the bootstrap. How does this compare to your answer from (b)?

```
set.seed(4)
bootstrap_means <- replicate(1000, mean(sample(Boston$medv,
```

```
length(Boston$medv), replace = TRUE)))
se_mean_bootstrap <- sd(bootstrap_means)
cat("SE mean (Bootstrap):", se_mean_bootstrap, "\n")

## SE mean (Bootstrap): 0.4069406
```

Standard error menggunakan bootstrap adalah `0.4069406`. Nilai ini sangat dekat dengan standard error yang didapat dari formula soal (b) yaitu `0.4088611`. Standard error dari bootstrap biasanya lebih akurat karena memperhitungkan variasi distribusi data secara empiris.

d. Based on your bootstrap estimate from (c), provide a 95% confidence interval for the mean of `medv`. Compare it to the results obtained using `t.test(Boston$medv)`.

Hint: You can approximate a 95% confidence interval using the formula $[\hat{\mu} - 2SE(\hat{\mu}), \hat{\mu} + 2SE(\hat{\mu})]$.

```
ci_bootstrap <- c(mean_medv - 2 * se_mean_bootstrap, mean_medv + 2 *
se_mean_bootstrap)
cat("95% CI (Bootstrap):", ci_bootstrap, "\n")

## 95% CI (Bootstrap): 21.71893 23.34669

hasil_t_test <- t.test(Boston$medv)
cat("95% CI (t-test):", hasil_t_test$conf.int, "\n")

## 95% CI (t-test): 21.72953 23.33608
```

e. Based on this data set, provide an estimate, $\hat{\mu}_{med}$, for the median value of `medv` in the population.

```
median_medv <- median(Boston$medv)
cat("Median populasi (μ̂med):", median_medv, "\n")

## Median populasi (μ̂med): 21.2
```

f. We now would like to estimate the standard error of $\hat{\mu}_{med}$. Unfortunately, there is no simple formula for computing the standard error of the median. Instead, estimate the standard error of the median using the bootstrap. Comment on your findings.

```
set.seed(4)
bootstrap_medians <- replicate(1000, median(sample(Boston$medv,
length(Boston$medv), replace = TRUE)))
se_median_bootstrap <- sd(bootstrap_medians)
cat("SE Median (Bootstrap):", se_median_bootstrap, "\n")

## SE Median (Bootstrap): 0.3749919
```

Estimasi standard error median adalah `0.3749919`, ini lebih kecil dibanding standard error mean (`0.4069406`).

g. Based on this data set, provide an estimate for the tenth percentile of `medv` in Boston census tracts. Call this quantity $\hat{\mu}_{0.1}$. (You can use the `quantile()` function.)

```
persentil_10 <- quantile(Boston$medv, 0.1)
cat("Persentil ke-10 (μ0.1):", persentil_10, "\n")

## Persentil ke-10 (μ0.1): 12.75
```

h. Use the bootstrap to estimate the standard error of $\hat{\mu}_{0.1}$. Comment on your findings.

```
set.seed(4)
bootstrap_percentile_10 <- replicate(1000, quantile(sample(Boston$medv,
length(Boston$medv), replace = TRUE), 0.1))
se_percentile_10 <- sd(bootstrap_percentile_10)
cat("SE Percentile ke-10 (Bootstrap):", se_percentile_10, "\n")

## SE Percentile ke-10 (Bootstrap): 0.4900443
```

Kita dapatkan standard error = `0.4900443`, ini lebih besar dibanding standard error median (`0.3749919`).