

# Praktikum 4 SMA

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## NOMOR 1

Kita tahu kalau:

$$X \sim \Gamma\left(\text{shape} = \frac{r}{2}, \text{scale} = 2\right) \sim \chi_r^2$$

Bukti bisa dilihat dari fkp sebaran  $\chi_r^2$ :

$$f(x) = \frac{1}{2^{\frac{r}{2}} \cdot \Gamma\left(\frac{r}{2}\right)} \cdot x^{\frac{r}{2}-1} \cdot e^{-\frac{x}{2}}$$

Dimana fkp ini sama dengan fkp sebaran  $\Gamma\left(\frac{r}{2}, 2\right)$

Kita juga tahu kalau

$$Z \sim N(0,1)$$

maka

$$Z^2 \sim \chi_1^2$$

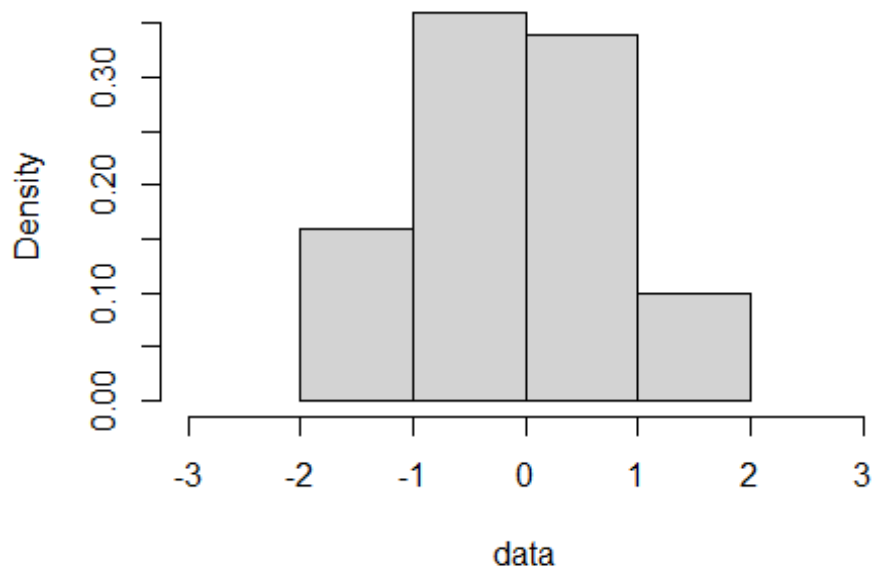
Sehingga jika  $W \sim \Gamma\left(\frac{1}{2}, 2\right)$ , maka  $W = Z^2 \sim \chi_1^2$ , yang artinya

$$\sqrt{W} = Z \sim N(0,1)$$

```
set.seed(123)
n <- 100
W <- rgamma(n, shape=0.5, scale = 2) #chi_kuadrat, df = 1
tanda <- ifelse(runif(n) > 0.5, 1, -1)
data <- tanda * sqrt(W)

batas <- c(-Inf, -2, -1, 0, 1, 2, Inf)
hist(data, breaks=batas, xlim = c(-3,3))
```

## Histogram of data



```
oi <- hist(data, breaks=batas, plot=FALSE)$counts
names(oi) <- c("(-inf,-2)", "[-2,-1)", "[-1,0)", "[0,1)", "[1,2)", "[2,inf)")
print("Tabel Frekuensi Observasi:")

## [1] "Tabel Frekuensi Observasi:"

print(oi)

## (-inf,-2)  [-2,-1)  [-1,0)  [0,1)  [1,2)  [2,inf)
##          4       16      36     34     10      0

peluang <- c(pnorm(-2), pnorm(-1)-pnorm(-2), pnorm(0)-pnorm(-1), pnorm(1)-
pnorm(0), pnorm(2)-pnorm(1), 1-pnorm(2))
ei <- n*peluang

# Manual
diskrepansi <- function(oi, ei) {
  d <- sum((oi - ei)^2 / ei)
  return(d)
}
hasil_d <- diskrepansi(oi,ei)
p_value <- 1 - pchisq(hasil_d, df = (length(oi)-1))

# Dengan function
hasil_tes <- chisq.test(oi, p=peluang)
```

```

## Warning in chisq.test(oi, p = peluang): Chi-squared approximation may be
## incorrect

# Tampilkan hasil
cat("Peluang teoritis pada selang ke i (pi_i):\n", round(peluang, 4), "\n")

## Peluang teoritis pada selang ke i (pi_i):
## 0.0228 0.1359 0.3413 0.3413 0.1359 0.0228

cat("\nEkspektasi banyak data pada selang ke i (e_i):\n", round(ei, 2), "\n")

##
## Ekspektasi banyak data pada selang ke i (e_i):
## 2.28 13.59 34.13 34.13 13.59 2.28

cat("\np_value manual:", round(p_value, 4), "\n")

##
## p_value manual: 0.4085

print(hasil_tes)

##
## Chi-squared test for given probabilities
##
## data: oi
## X-squared = 5.0612, df = 5, p-value = 0.4085

```

$p\text{-value} = 0.4085 > 0.1$ , sehingga tidak ada bukti untuk menolak  $H_0$ . Artinya dengan uji c-squared pada tingkat kepercayaan 95%, 100 nilai data tersebut menyebar normal baku.

```

library(fitdistrplus)

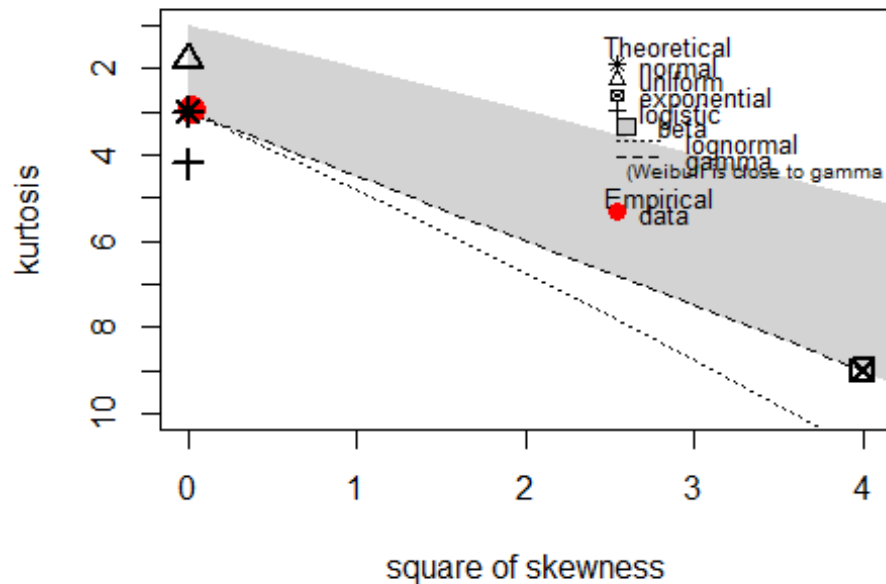
## Loading required package: MASS

## Loading required package: survival

descdist(data, discrete = FALSE)

```

## Cullen and Frey graph



```
## summary statistics
## -----
## min:  -2.515241   max:  1.953266
## median: -0.1955804
## mean:  -0.1956884
## estimated sd:  0.9326193
## estimated skewness:  -0.1777538
## estimated kurtosis:  2.988486
```

## NOMOR 2

### 2a

```
orang <- 0:6
jam <- c(0, 6, 5, 4, 4, 3, 2)
data_pengunjung <- rep(orang, jam)
cat("Data:\n", data_pengunjung)

## Data:
## 1 1 1 1 1 1 2 2 2 2 2 3 3 3 3 4 4 4 4 5 5 5 6 6

total_orang <- sum(jam * orang)
cat("\nTotal orang:", total_orang, "\n")

##
## Total orang: 71
```

```

n <- length(data_pengunjung)
cat("Total data:", n, "\n")

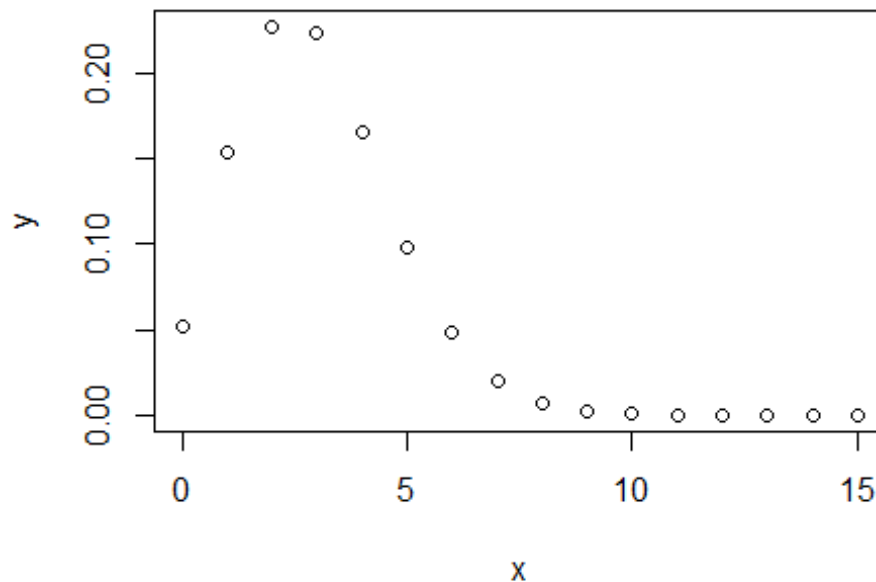
## Total data: 24

lambda <- total_orang / n
cat("Estimasi lambda:", round(lambda, 4), "\n")

## Estimasi lambda: 2.9583

set.seed(123)
x <- c(0:15)
y <- dpois(x, lambda)
plot(x, y)

```



```

data_fmp <- data.frame(x, round(y, 4))
colnames(data_fmp) <- c("x", "f(x)")
print(data_fmp)

```

```

##      x  f(x)
## 1  0 0.0519
## 2  1 0.1536
## 3  2 0.2271
## 4  3 0.2240
## 5  4 0.1656
## 6  5 0.0980
## 7  6 0.0483
## 8  7 0.0204

```

```

## 9 8 0.0076
## 10 9 0.0025
## 11 10 0.0007
## 12 11 0.0002
## 13 12 0.0000
## 14 13 0.0000
## 15 14 0.0000
## 16 15 0.0000

prob <- dpois(2, lambda) + dpois(3, lambda) + dpois(6, lambda);prob

## [1] 0.4994313

o1 <- NULL
o2 <- NULL
for(i in data_pengunjung){
  if(i == 2 | i == 3 | i == 6){
    o1 <- c(o1, i)
  }
  else{
    o2 <- c(o2, i)
  }
}
oi <- c(length(o1), length(o2))

e1 <- n*prob
e2 <- n*(1-prob)
ei <- c(e1, e2)

d <- sum((oi - ei)^2/ei)
p_value <- 1 - pchisq(d, df = length(oi) - 1)
cat("Diskrepansi =", d, "\n")

## Diskrepansi = 0.1621483

cat("P-value =", p_value)

## P-value = 0.6871863

# Dengan function
uji_chi <- chisq.test(oi, p=c(prob, 1-prob))
cat("\nHasil:\n")

##
## Hasil:

print(uji_chi)

##
## Chi-squared test for given probabilities
##

```

```
## data: oi
## X-squared = 0.16215, df = 1, p-value = 0.6872
```

$p\text{-value} = 0.6872 > 0.1$ , sehingga tidak ada bukti untuk menolak  $H_0$ . Artinya dengan uji chi-squared dengan kepercayaan 95%, banyaknya orang yang datang menyebar Poisson.

## 2b

```
# Uji Kolmogorov-Smirnov
ks.test(data_pengunjung, "ppois", lambda = lambda, alternative = "two.sided",
simulate.p.value = TRUE)

## Warning in ks.test.default(data_pengunjung, "ppois", lambda = lambda,
## alternative = "two.sided", : ties should not be present for the one-sample
## Kolmogorov-Smirnov test

##
## Asymptotic one-sample Kolmogorov-Smirnov test
##
## data: data_pengunjung
## D = 0.20546, p-value = 0.2631
## alternative hypothesis: two-sided
```

$p\text{-value} = 0.2631 > 0.1$ , sehingga tidak ada bukti untuk menolak  $H_0$ . Artinya dengan uji K-S dengan kepercayaan 95%, banyaknya orang yang datang menyebar Poisson.

## NOMOR 3

### 3a

```
data <- c(6.5, 4.6, 5.6, 8.5, 6.0, 6.9, 4.4, 6.3, 6.2, 7.0, 5.7, 6.1)
norm <- fitdist(data, "norm", method = "mle")
print(norm)

## Fitting of the distribution ' norm ' by maximum likelihood
## Parameters:
##      estimate Std. Error
## mean 6.150000  0.2998842
## sd   1.038829  0.2120493

logLik(norm)

## [1] -17.4844

lnorm <- fitdist(data, "lnorm", method = "mle")
print(lnorm)

## Fitting of the distribution ' lnorm ' by maximum likelihood
## Parameters:
##      estimate Std. Error
```

```
## meanlog 1.8021117 0.04913476
## sdlog    0.1702078 0.03473812

logLik(lnorm)

## [1] -17.40378

gamma <- fitdist(data, "gamma", method = "mle")
print(gamma)

## Fitting of the distribution ' gamma ' by maximum likelihood
## Parameters:
##      estimate Std. Error
## shape 35.034661   14.23526
## rate   5.696836    2.33135

logLik(gamma)

## [1] -17.37203

cat("Loglikelihood normal: ", logLik(norm), "\n")

## Loglikelihood normal: -17.4844

cat("Loglikelihood normal: ", logLik(lnorm), "\n")

## Loglikelihood normal: -17.40378

cat("Loglikelihood normal: ", logLik(gamma), "\n")

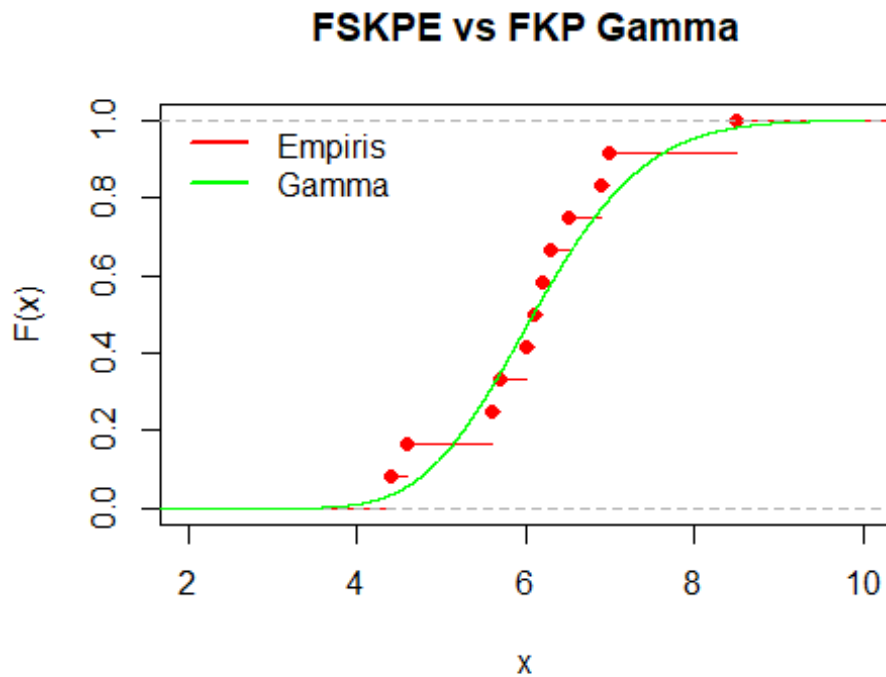
## Loglikelihood normal: -17.37203
```

Gamma fit terbaik dengan nilai loglikelihood terbesar.

### 3b

```
set.seed(123)
x <- c(0:10000)/1000
y <- pgamma(x, shape = gamma$estimate["shape"], rate =
gamma$estimate["rate"])
plot(ecdf(data), xlim = c(2,10), col = "red", ylab = "F(x)", main = "FSKPE vs
FKP Gamma")
lines(x, y, col = "green", lwd = 1)
legend("topleft", legend = c("Empiris", "Gamma"), col = c("red", "green"),
bty = "n", lwd = 2)
```





Plot FSK empiris dengan FSK gamma terlihat hampir fit yang artinya jaraknya hampir nol, sehingga nantinya akan membuat diskrepansi D menjadi sangat kecil, yang menyebabkan  $p\_value$  yang besar.

### 3c

Karena jumlah data 12 dan ingin nilai  $e_i \geq 5$ , maka selang yang mungkin adalah 2.

```
sel <- 2
ei <- 6

q <- qgamma(0.5, shape=gamma$estimate["shape"], rate=gamma$estimate["rate"])
o1 <- sum(data <= q)
o2 <- sum(data > q)
oi <- c(o1, o2)

d <- sum((oi-ei)^2 / ei)
p_value <- 1-pchisq(d, df = sel-1)

cat("Diskrepansi =", d, "\n")
## Diskrepansi = 0.3333333
cat("P-value =", p_value)
## P-value = 0.5637029
```

$p\text{-value} = 0.5637 > 0.1$ , sehingga tidak ada bukti untuk menolak  $H_0$ . Artinya dengan uji chi-squared dengan kepercayaan 95%, data mengikuti sebaran gamma.

### 3d

```
ks.test(data, "pgamma", shape=gamma$estimate["shape"],
rate=gamma$estimate["rate"], alternative = "two.sided", simulate.p.value =
TRUE)
```

```
##
## Exact one-sample Kolmogorov-Smirnov test
##
## data: data
## D = 0.14573, p-value = 0.9291
## alternative hypothesis: two-sided
```

$p\text{-value} = 0.9291 > 0.1$ , sehingga tidak ada bukti untuk menolak  $H_0$ . Artinya dengan uji K-S dengan kepercayaan 95%, data mengikuti sebaran gamma.

```
data <- sort(data)
fske <- c(1:length(data))/length(data)

fgam <- pgamma(data, shape = gamma$estimate["shape"], rate =
gamma$estimate["rate"])

du <- NULL
dl <- c(0-fgam[1])
for(i in 1:length(fske)){
  du[i] <- fske[i] - fgam[i]
}
for(i in 2:length(fske)){
  dl[i] <- fske[i-1] - fgam[i]
}

d <- max(abs(du), abs(dl));d
## [1] 0.1457286

KSgamma <- function(KSd, m, size){
  KS <- NULL

  upper <- c(1:size)/size
  lower <- upper - 1/size
  for(i in 1:m){
    ndata <- sort(rgamma(size, shape = gamma$estimate["shape"], rate =
gamma$estimate["rate"]))
    fit <- fitdist(ndata, "gamma", method = "mle")
    shape <- fit$estimate["shape"]
    rate <- fit$estimate["rate"]
    U <- abs(upper - pgamma(ndata, shape, rate))
  }
}
```

```

    L <- abs(lower - pgamma(ndata, shape, rate))
    ddot <- max(U, L)
    KS <- c(KS, ddot)
  }

  p <- length(KS[KS > KSd]) / m
  return(p)
}

p <- KSGamma(d, 1000, length(data));p
## [1] 0.701

```

## NOMOR 4

```

nilai_uas <- c(30, 47, 84, 54, 56, 70, 45, 10, 21, 61, 52, 61)
rata_rata <- mean(nilai_uas)
standar_deviasi <- sd(nilai_uas)
cat("Rata-rata:", round(rata_rata, 2), "\n")

## Rata-rata: 49.25

cat("Standar deviasi:", round(standar_deviasi, 2), "\n")

## Standar deviasi: 20.72

ks.test(nilai_uas, "pnorm", mean=rata_rata, sd=standar_deviasi, alternative =
"two.sided", simulate.p.value = TRUE)

## Warning in ks.test.default(nilai_uas, "pnorm", mean = rata_rata, sd =
## standar_deviasi, : ties should not be present for the one-sample
## Kolmogorov-Smirnov test

##
## Asymptotic one-sample Kolmogorov-Smirnov test
##
## data:  nilai_uas
## D = 0.16874, p-value = 0.8841
## alternative hypothesis: two-sided

```

Diskrepansi  $D = 0.16874$  sangat kecil, menunjukkan jarak antara FSK empiris data dan FSK normal( $49.25, 20.72^2$ ) hampir nol, dan data sangat dekat dengan distribusi normal.

$p\text{-value} = 0.8841 > 0.1$ , sehingga tidak ada bukti untuk menolak  $H_0$ . Artinya dengan uji K-S dengan kepercayaan 95%, data nilai UAS ini berasal dari distribusi normal  $N(49.25, 20.72^2)$

```

uas <- sort(nilai_uas)
n <- length(uas)
mean <- mean(uas)
sd <- sd(uas)

```

```

up <- c(1:n)/n
low <- up - 1/n

du <- NULL
dl <- NULL
for(i in 1:n){
  du[i] <- abs(up[i] - pnorm(uas[i], mean, sd))
  dl[i] <- abs(low[i] - pnorm(uas[i], mean, sd))
}

d <- max(du, dl);d
## [1] 0.1687387

KSnorm <- function(KSd, m, n){
  KS <- NULL
  upper <- c(1:n)/n
  lower <- upper - 1/n

  for(i in 1:m){
    baru <- sort(rnorm(n, mean, sd))
    xbar <- mean(baru)
    stdev <- sd(baru)

    U <- abs(upper - pnorm(baru, xbar, stdev))
    L <- abs(lower - pnorm(baru, xbar, stdev))
    ddot <- max(U, L)
    KS <- c(KS, ddot)
  }

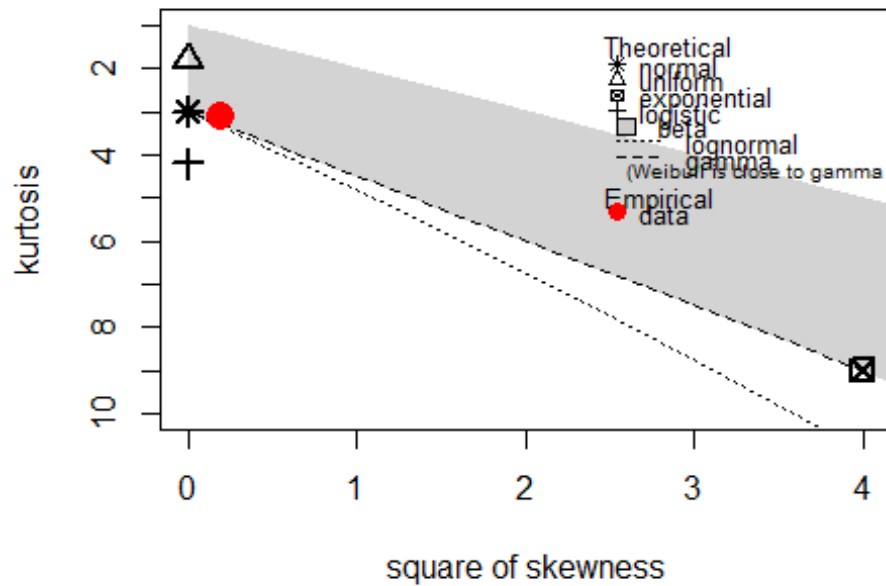
  pval <- length(KS[KS > KSd]) / m
  return(pval)
}

p <- KSnorm(d, 1000, n);p
## [1] 0.451

descdist(nilai_uas, discrete = FALSE)

```

## Cullen and Frey graph



```
## summary statistics
## -----
## min: 10   max: 84
## median: 53
## mean: 49.25
## estimated sd: 20.71945
## estimated skewness: -0.4373181
## estimated kurtosis: 3.098352
```