Assignment 1:
Source Localization from EEG Signals

Noah Steidle

March 29, 2022

Abstract

Neuroimaging techniques like electroencephalography (EEG) and magnetoencephalography (MEG) are important tools to evaluate a variety of brain functions as they are easily realizable and non-invasive. In order to identify and interpret the data taken from an EEG, Source Localization can be applied which is a mathematical problem describing the solution of a forward and an inverse problem. In the following paper, neuroimaging procedures are briefly introduced with a focus on electroencephalography. Moreover, some information about the mathematical background is given, leading to the consideration of a problem with simulated data. Finally, results of this problem are presented and a conclusion is drawn.

1 Introduction

Modern medicine increasingly relies on technology to measure and visualize all parts of the human body. A variety of electrobiological measurement techniques, which has been developed in the past decades, enables doctors to identify all types of diseases early and to choose a well-adjusted treatment of the patient which overall results in a higher number of recoveries. Exemplary to be listed are electrocardiography (ECG), electromyography (EMG), electroencephalography (EEG), magnetoencephalography (MEG), electrogastrography (EGG), electrooptigraphy (EOG) to obtain measurements of the heart, muscular contractions, the brain, the stomach and the eyes. [2]

In the proceeding of this report the focus will lay on electroencephalography (EEG); a method that works by reading scalp electrical activity generated by brain structures measured from the head surface. One outstanding property of this procedure is the non-invasive nature, enabling a broadly and repetitive usage on patients of all ages.

The general idea of an EEG is to measure the local current flows that are produced when brain cells (neurons) are activated. Hereby, only large clusters of active neurons can be measured due to the low amount of electrical energy recordable on the head surface and skin, skull, as well as other layers impeding the connection of electrode and neuronal layers. Due to the position of the electrodes on the surface, EEG measurements work best for impulses coming from the cerebral cortex describing activities as "movement initiation, conscious awareness of sensation, complex analysis, and expression of emotions and behaviour" as stated by [2].

2 Methods

In order to identify sources of the brain from data taken from EEG recordings the method of Source Localization is implemented. This process exists of solving a forward and an inverse problem.
For the forward problem a head model is constructed, in which the connection between the scalp recordings and the discretized head model volume can be described with the linear matrix equation

$$V = Gx + \epsilon, \qquad V \in \mathbb{R}^{M\times 1},\ G \in \mathbb{R}^{M\times N},\ x \in \mathbb{R}^{N\times 1},\ \epsilon \in \mathbb{R}^{M\times 1}$$

where V contains the recording of M channels, G is the lead field matrix with information about head geometry and conductivities, x is unknown containing the intensities of N sources, and ϵ describes the noise in the measurements.
The inverse problem is ill-defined as there are possible infinite combinations of positions and intensities. Therefore, the problem is expressed as a linear optimization problem with regularization

$$\hat{x} = \min_x \left\{ ||V - Gx||_2^2 + \sum_{i=1}^{k} \alpha_i ||W_i x||_p \right\}.$$

Hereby, k is the number of regularization constraints and W_i is a weighted matrix related to the imposed constraints.

3 Problem and Results

A network with $M = 5$ recording channels and $N = 20$ sources was considered at $t = \{0, \ldots, 100\}$ time steps. In order to facilitate the model the brain was approximated by a sphere. The situation can be seen in Figure 1 in two dimensions and in Figure 2 in three dimensions, respectively.

After simulating the EEG recordings (with initial conditions $Y_0 = (0, 0, 0, 0, 0)$), constructing the lead field matrix G (with constant $c = 2$) and solving the

regularization problem, the final numerical consideration of the problem presented in the previous section 2 resulted in the following four images Figure 3, Figure 4, Figure 5, Figure 6.
The resulting time series of the signals of the electrodes by position can be seen in Figure 7.

4 Conclusions

Electroencephalography is a powerful technology to detect a variety of specifications of the human brain. For instance, R. Bickford states amongst others the following applications of EEGs in [1]

- monitor alertness, coma and brain death;
- locate areas of damage following head injury, stroke, tumour, etc.;
- monitor cognitive engagement (alpha rhythm);
- control anaesthesia depth;
- investigate epilepsy and locate seizure origin;
- test epilepsy drug effects;
- monitor human and animal brain development;
- test drugs for convulsive effects;
- investigate sleep disorder and physiology.

Considering this excerpt of applications and the general accessibility of EEG technology, electroencephalography provides a prime example for modern technology helping a great number of people.

References

[1] R.D. Bickford. "Electroencephalography". In: *Adelman G. ed. Encyclopedia of Neuroscience* (1987), pp. 371–373.

[2] M. Teplan. "FUNDAMENTALS OF EEG MEASUREMENT". In: *Measurement Science Review* (2002).

A Code

```
%% (a) Construct a 3D sphere of appropriate size

[X,Y,Z] = sphere; % Unit sphere presented with 20 tiles

%surf(X,Y,Z)
%axis equal
```

```
%% (b) Distribute 20 sources uniformly within the sphere

N = 20; % Number of sources
eps = 0.1; % Distance to the boundary

% Calculate points
rvals = 2*rand(N,1)-1;
el = asin(rvals);
az = 2*pi*rand(N,1);
ra = (1-eps)*(rand(N,1).^(1/3));

[px,py,pz] = sph2cart(az,el,ra);

% Visualize

% In 3D
figure('Name',sprintf('3D-plot with eps = %0.2f',eps))
mesh(X,Y,Z,'FaceAlpha',0.5)
hold on
plot3(px,py,pz,'o')
hold off
axis('equal')
legend('Sphere','Sources')

% In 2D
figure('Name',sprintf('2D-plot with eps = %0.2f',eps))
% x-y
subplot(1,3,1)
plotcircle(1,0,0)
hold on
plot(px,py,'x')
hold off
axis('equal')
title('x-y-plane')
legend('Sphere','Sources','Location','South')
% x-z
subplot(1,3,2)
plotcircle(1,0,0)
hold on
plot(px,pz,'x')
hold off
```

```
axis('equal')
title('x-z-plane')
legend('Sphere','Sources','Location','South')
% y-z
subplot(1,3,3)
plotcircle(1,0,0)
hold on
plot(py,pz,'x')
hold off
axis('equal')
title('y-z-plane')
legend('Sphere','Sources','Location','South')
```

```
%% (c) Simulate EEG recordings

tN = 101;

% Initiate variables
Y1 = zeros(1,tN);
Y2 = zeros(1,tN);
Y3 = zeros(1,tN);
Y4 = zeros(1,tN);
Y5 = zeros(1,tN);

% Set Initial Conditions = 0

% Generate noise
w1 = randn(1,tN);
w2 = randn(1,tN);
w3 = randn(1,tN);
w4 = randn(1,tN);
w5 = randn(1,tN);

% Simulate data
for t = 4:tN

    Y1(t) = 0.95*sqrt(2)*Y1(t-1) - 0.9025*Y1(t-2) + w1(t);
    Y2(t) = 0.5*Y1(t-2) + w2(t);
    Y3(t) = -0.4*Y1(t-3) + w3(t);
    Y4(t) = -0.5*Y1(t-2) + 0.25*sqrt(2)*Y4(t-1) + w4(t);
    Y5(t) = 0.25*sqrt(2)*Y4(t-1) + 0.25*sqrt(2)*Y5(t-1) + ...
        w5(t);

end
```

```
%% (d) Distribute 5 electrodes at the surface of the sphere

M = 5; % Number of electrodes

% Calculate points
TH = 2*pi*rand(1,M);
PH = asin(-1+2*rand(1,M));
[pex,pey,pez] = sph2cart(TH,PH,1);

% Assign electrodes 1-5 to signals Y1-Y5
E1 = {pex(1), pey(1), pez(1), Y1};
E2 = {pex(2), pey(2), pez(2), Y2};
E3 = {pex(3), pey(3), pez(3), Y3};
E4 = {pex(4), pey(4), pez(4), Y4};
E5 = {pex(5), pey(5), pez(5), Y5};

% Visualize

% In 3D

figure('Name',sprintf('3D-plot with eps = %0.2f',eps))
mesh(X,Y,Z,'FaceAlpha',0.5)
hold on
plot3(px,py,pz,'o')
plot3(pex,pey,pez,'s','Color','r')
hold off
axis('equal')
legend('Sphere','Sources','Electrodes')

% In 2D
figure('Name',sprintf('2D-plot with eps = %0.2f',eps))
% x-y
subplot(1,3,1)
plotcircle(1,0,0)
hold on
plot(px,py,'x')
plot(pex,pey,'s','Color','r')
hold off
axis('equal')
title('x-y-plane')
legend('Sphere','Sources','Electrodes','Location','South')
% x-z
subplot(1,3,2)
plotcircle(1,0,0)
hold on
plot(px,pz,'x')
plot(pex,pez,'s','Color','r')
hold off
axis('equal')
title('x-z-plane')
legend('Sphere','Sources','Electrodes','Location','South')
% y-z
subplot(1,3,3)
```

```
plotcircle(1,0,0)
hold on
plot(py,pz,'x')
plot(pey,pez,'s','Color','r')
hold off
axis('equal')
title('y-z-plane')
legend('Sphere','Sources','Electrodes','Location','South')
```

```
%% (e) Construct the lead field (gain matrix)

c = 2; % Constant c
G = zeros(M,N); % Initiate empty matrix

% Fill matrix
for m = 1:M

    for n = 1:N

        R = ...
            sqrt((pex(m)-px(n))^2+(pey(m)-py(n))^2+(pez(m)-pz(n))^2);
        G(m,n) = c / (R^2);

    end

end
```

```
%% (f) Solve the regularization problem for determining the ...
    intensities

x_star = zeros(tN,N); % Initiate empty x_star

% Fill x_star
for t = 1:tN

    V = [ Y1(t); Y2(t); Y3(t); Y4(t); Y5(t) ];
    x_star(t,:) = G.' * inv(G*G.') * V;

end
```

```
%% (g) Visualize the results

% Visualize solution in time of each source
% Distribute results to four different figures
for j = 1:4

    figure('Name',sprintf('Results: Part %d',j))
    for k = ((5*j)-4):(5*j)

```

```
        z = mod(k+4,5)+1;

        subplot(5,1,z)
        plot(0:tN-1,x_star(:,k))
        axis([0 tN-1 -inf inf])
        title(sprintf('Source: x=%0.4f, y=%0.4f, z=%0.4f', ...
            px(k),py(k),pz(k)))

    end

end

% Visualize signals of the electrodes
figure('Name','Signals of Electrodes')
subplot(5,1,1)
plot(0:tN-1,E1{4})
[phii,thetaa] = cart2pol(E1{1},E1{2},E1{3});
set(gca,'xticklabel',[])
title(sprintf('Position: \\theta =%0.4f, \\phi =%0.4f', ...
    phii, thetaa))
subplot(5,1,2)
plot(0:tN-1,E2{4})
[phii,thetaa] = cart2pol(E2{1},E2{2},E2{3});
set(gca,'xticklabel',[])
title(sprintf('Position: \\theta=%0.4f, \\phi=%0.4f', phii, ...
    thetaa))
subplot(5,1,3)
plot(0:tN-1,E3{4})
[phii,thetaa] = cart2pol(E3{1},E3{2},E3{3});
set(gca,'xticklabel',[])
title(sprintf('Position: \\theta=%0.4f, \\phi=%0.4f', phii, ...
    thetaa))
subplot(5,1,4)
plot(0:tN-1,E4{4})
[phii,thetaa] = cart2pol(E4{1},E4{2},E4{3});
set(gca,'xticklabel',[])
title(sprintf('Position: \\theta=%0.4f, \\phi=%0.4f', phii, ...
    thetaa))
subplot(5,1,5)
plot(0:tN-1,E5{4})
[phii,thetaa] = cart2pol(E5{1},E5{2},E5{3});
title(sprintf('Position: \\theta=%0.4f, \\phi=%0.4f', phii, ...
    thetaa))
```

The complete Matlab-file can be found here.

B Pictures

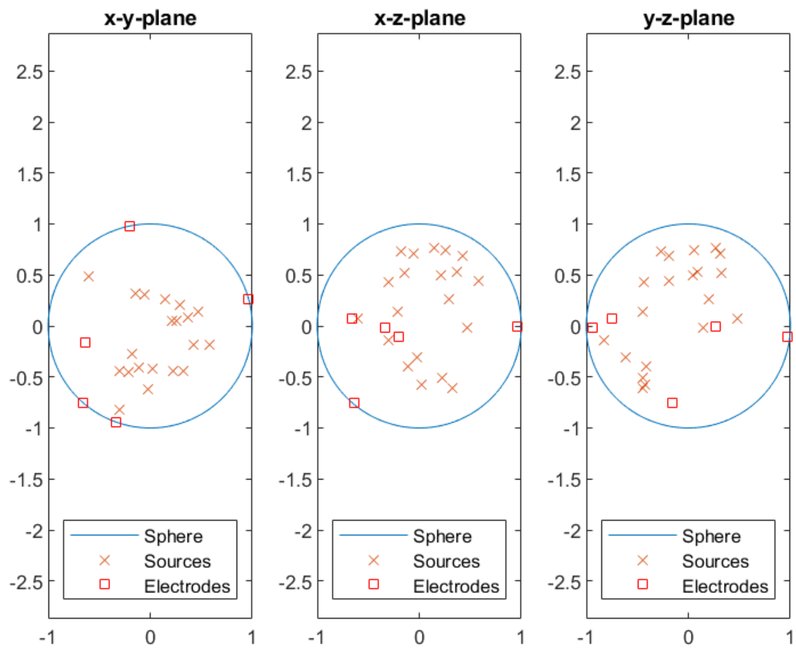

Figure 1: Overview of Electrodes and Sources in all two-dimensional planes.

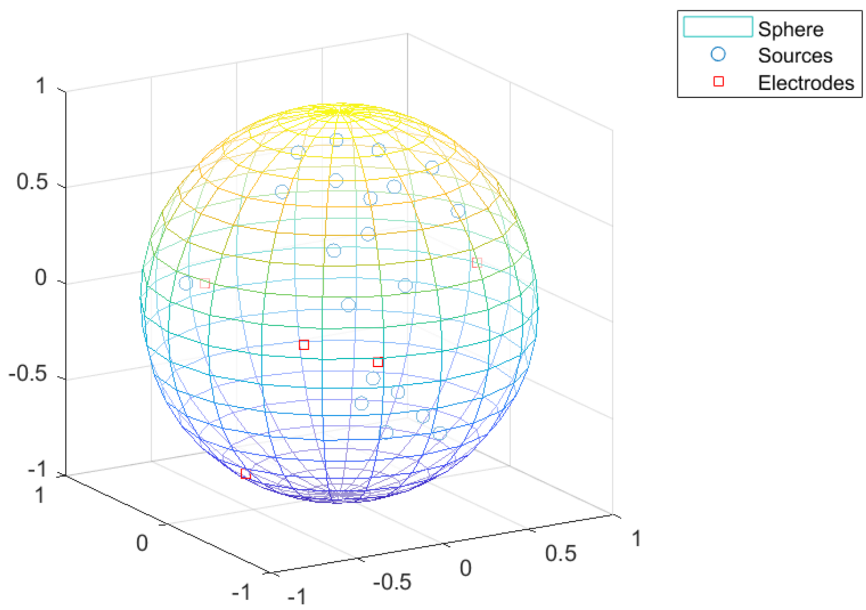

Figure 2: Overview of Electrodes and Sources in three dimensions.

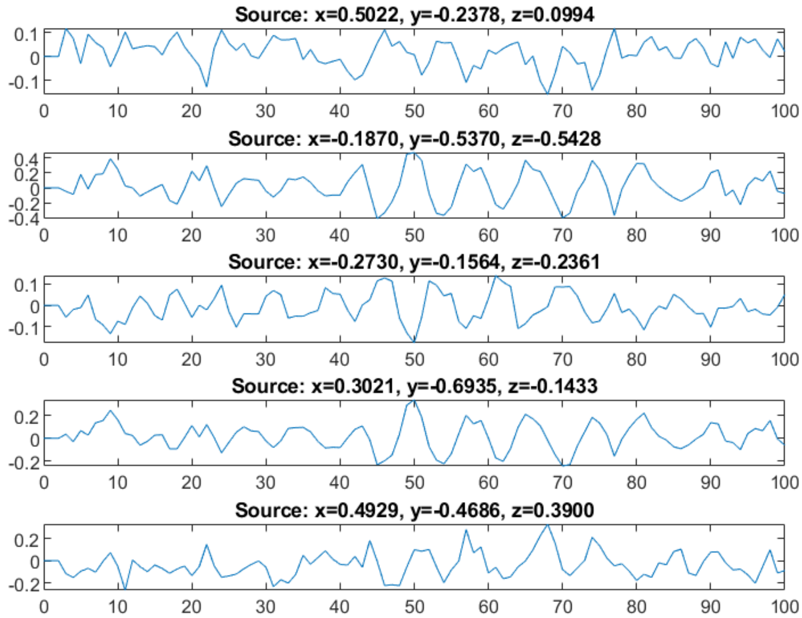

Figure 3: Solution in team of sources 1-5.

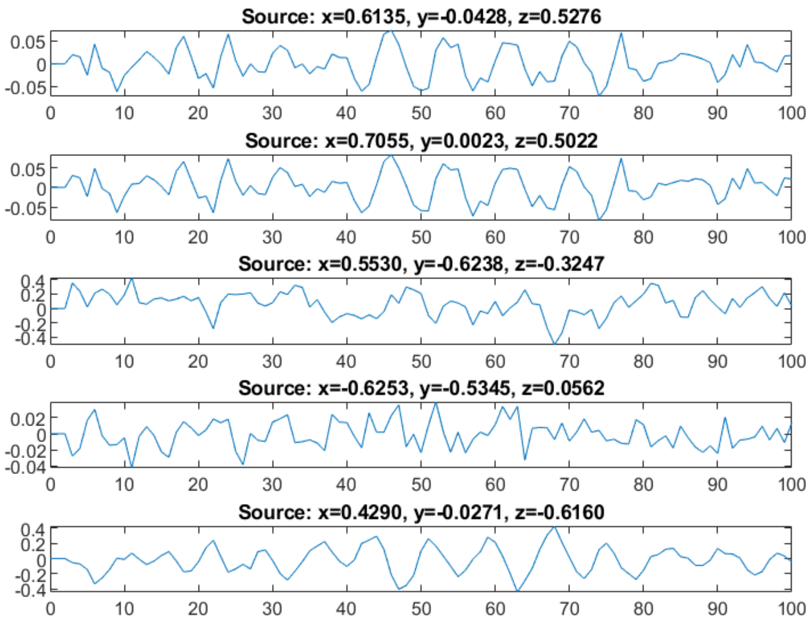

Figure 4: Solution in team of sources 6-10.

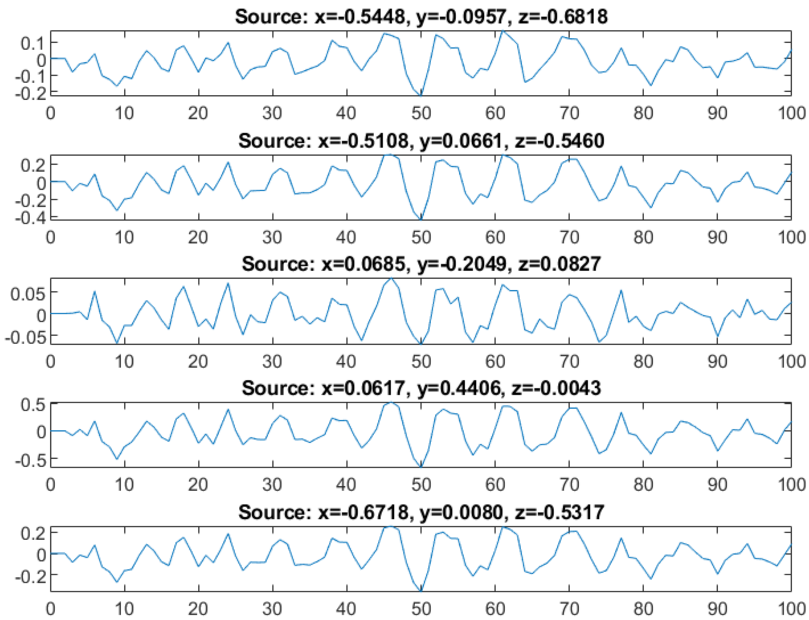

Figure 5: Solution in team of sources 11-15.

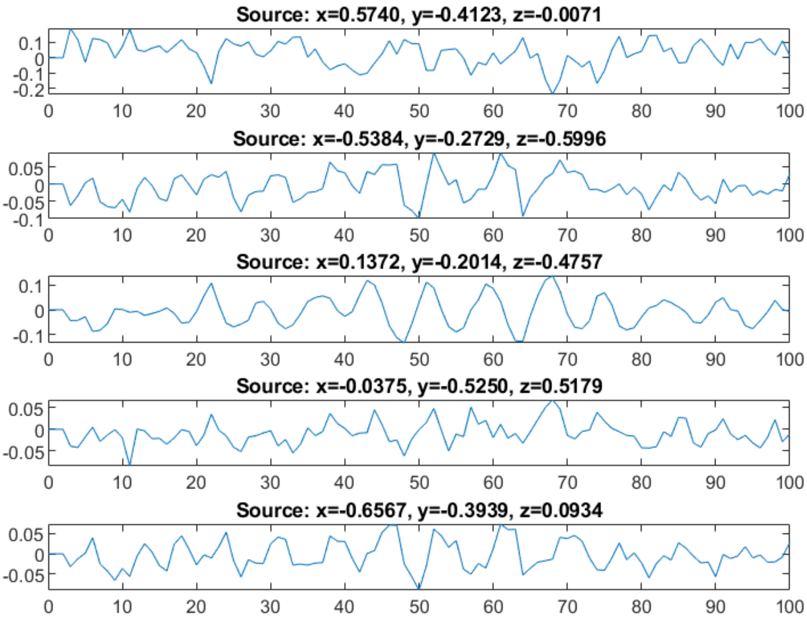

Figure 6: Solution in team of sources 16-20.

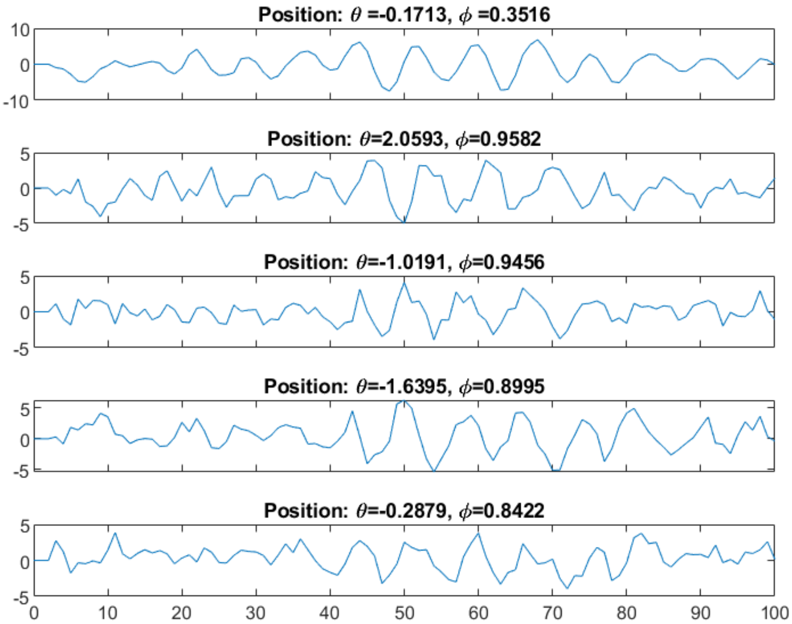

Figure 7: Simulated signals of the electrodes.