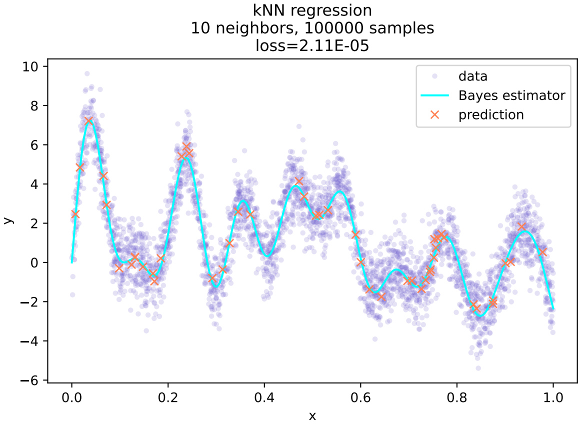
# Machine learning I, supervised learning: metrics



# Metrics

Let $D = \{x_1, \dots, x_n\} \subset \mathcal{X}$ be a dataset of $n$ samples, with labels $\{y_1, \dots, y_n\} \subset \mathcal{Y}$.
There is a metric in the input space $\mathcal{X}$ and in the output space $\mathcal{Y}$.

- The **metric** in $\mathcal{X}$ determines to what extent two samples $x_i$ and $x_j$ should be considered similar or dissimilar.
- The **metric** in $\mathcal{Y}$ determines to what extent two labels $y_i$ and $y_j$ should be considered similar or dissimilar.

This is very important during the complete processing of the data.

# Metrics in output space

A **loss function** $l$ is a map that measures the discrepancy between to elements of a set (for instance of a linear space).

$$l : \begin{cases} \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}_+ \\ (y, z) \mapsto l(y, z) \end{cases}$$

Typically, $z$ can represent our prediction for a given input $x$, $z = \tilde{f}(x)$, and $y$ the correct label.

# Most common losses

**"0-1" loss for binary classification.**
$\mathcal{Y} = \{0, 1\}$ or $\mathcal{Y} = \{-1, 1\}$.

$$l(y, z) = 1_{y \neq z} \tag{1}$$

**Squared loss for regression**
$\mathcal{Y} = \mathbb{R}$.

$$l(y, z) = (y - z)^2 \tag{2}$$

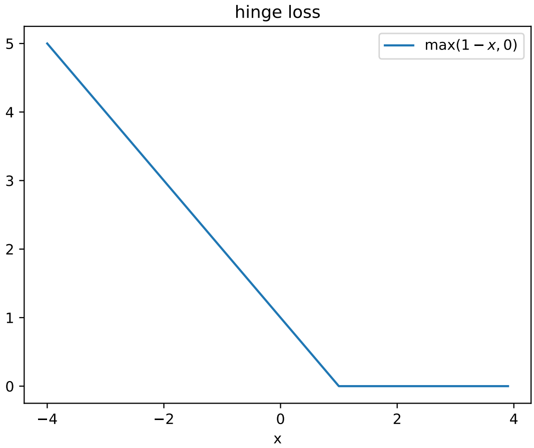
**absolute loss for regression.**
$\mathcal{Y} = \mathbb{R}$.

$$l(y, z) = |y - z| \tag{3}$$

# Cross entropy loss (more advanced)

- $\mathcal{Y} = \{0, 1\}$
- $$l(z, y) = y \log(1 + e^{-z}) + (1 - y) \log(1 + e^{z}) \tag{4}$$
- typically used for logistic regression or neural networks (note that sometimes $\mathcal{Y} = \{-1, 1\}$, and then the writing is different).

Other losses exist and are relevant in some contexts, such as the hinge loss (used for support vector machines).



## Geometric metrics

Often, the input space $\mathcal{X}$ is $\mathbb{R}^p$. We compare $p$-dimensional **vectors**, $x$ and $y$ that write $x = (x_1, ..., x_p)$ and $y = (y_1, ..., y_p)$ with each $x_i$ and each $y_i$ a real number.
In this case, **geometric** metrics are used.

## Geometric distances

$x = (x_1, ..., x_p)$ and $y = (y_1, ..., y_p)$ are $p$-dimensional **vectors**.

- $L_2 : ||x - y||_2 = \sqrt{\sum_{k=1}^{p} (x_k - y_k)^2}$ (Euclidian distance, 2-norm distance)

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- $L_1 : ||x - y||_1 = \sum_{k=1}^{p} |x_k - y_k|$ (Manhattan distance, 1-norm distance)
- weighted $L_1 : \sum_{k=1}^{p} w_k |x_k - y_k|$, with each $w_k > 0$.

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- L1 : $||x - y||_1 = \sum_{k=1}^{p}|x_k - y_k|$ (Manhattan distance, 1-norm distance)
- weighted $L_1$ : $\sum_{k=1}^{p} w_k|x_k - y_k|$, with each $w_k > 0$.
- $||x - y||_\infty$ : $\max(|x_i - y_i|, i \in [1, n])$ (infinity norm distance, Chebyshev distance)

These definitions are also in the memo :
`https://github.com/nlehir/MLI_SupervisedLearning/blob/master/documents/Math_memo.pdf`

# Non-geometric data

Not all data are geometric !

# Hamming distance

- $\#\{x_i \neq y_i\}$ (Hamming distance)
- Levenshtein distance for strings (allows deletions and additions)

## General definition of a distance

A **distance** on a set $E$ is an map $d : E \times E \to \mathbb{R}_+$ that must :

- be **symetric** : $\forall x, y, d(x, y) = d(y, x)$
- **separate the values** : $\forall x, y, d(x, y) = 0 \Leftrightarrow x = y$
- respect the **triangular inequality** $\forall x, y, z, d(x, y) \leq d(x, z) + d(y, z)$

We could verify that the distances defined in the previous slides are all proper distances.

# Similarities

Sometimes, it is not possible to define a proper **distance** in the input space $\mathcal{X}$ ! This may happen for instance is $\mathcal{X}$ is a dataset of texts.

- When distances are unavailable, we can use **Similarities** or **Dissimilarity** to compare points.
- Dissimilarites are more general and don't always abide by the distance axioms.
- Other examples : Adjacency in an oriented graph, Custom agregated score to compare data.

# Example : cosine similarity

The **cosine similarity** may be used to compare texts.
If $u$ and $v$ are vectors,

$$S_C(u, v) = \frac{(u|v)}{||u||||v||} \tag{5}$$

- the **bag of words representation** allows us to build a vector from a text (one hot encoding).
- **cosine_similarity/scraper.py**
- **cosine_similarity/similarity.py**

# Hybrid data

Sometimes each sample contains both numerical data and non-numerical data (text, categorical data.)
See **code/metrics/hybrid_data/**
This is often the case in machine learning applications ! (database of customers, database of cars, etc.)