Supervised learning: metrics

12 octobre 2022

Metrics

Let $D = \{x_1, \ldots, x_n\} \subset \mathcal{X}$ be a dataset of n samples, with labels $\{y_1, \ldots, y_n\} \subset \mathcal{Y}$.
There is a metric in the input space $\mathcal{X}$ and in the output space $\mathcal{Y}$.

- The **metric** in $\mathcal{X}$ determines to what extent two samples x_i and x_j should be considered similar or dissimilar.
- The **metric** in $\mathcal{Y}$ determines to what extent two labels y_i and y_j should be considered similar or dissimilar.

This is very important during the complete processing of the data.

Metrics in output space

A **loss function** l is a map that measures the discrepancy between to elements of a set (for instance of a linear space).

$$l : \begin{cases} \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}_+ \\ (y, z) \mapsto l(y, z) \end{cases}$$

Typically, z can represent our prediction for a given input x, $z = \tilde{f}(x)$, and y the correct label.

"0-1" loss for **binary classification.**

$\mathcal{Y} = \{0, 1\}$ or $\mathcal{Y} = \{-1, 1\}$.

$$l(y, z) = 1_{y \neq z} \tag{1}$$

square loss for **regression**.

$\mathcal{Y} = \mathbb{R}$.

$$l(y, z) = (y - z)^2 \tag{2}$$

absolute loss for **regression**.

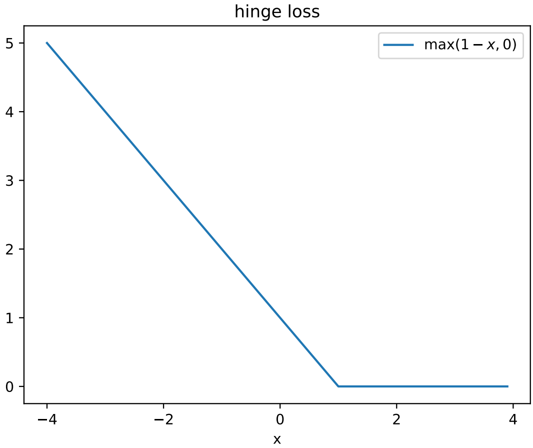
$\mathcal{Y} = \mathbb{R}$.

$$l(y, z) = |y - z| \tag{3}$$

Cross entropy loss (more advanced)

- $\mathcal{Y} = \{0, 1\}$
-
$$l(z, y) = y \log(1 + e^{-z}) + (1 - y) \log(1 + e^{z}) \tag{4}$$
- typically used for logistic regression or neural networks (note that sometimes $\mathcal{Y} = \{-1, 1\}$, and then the writing is different).

Other losses exist and are relevant in some contexts, such as the hinge loss.



Metrics in input space

Often, $\mathcal{X} = \mathbb{R}^p$ (input space). In this case, **geometric** metrics are used.

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- weighted L_1 : $\sum_{k=1}^{p} w_k|x_k - y_k|$
- L_∞ : $\max(x_1, \ldots, x_n)$ (infinity norm distance, Chebyshev distance)

https://www.geogebra.org/geometry?lang=fr

Non-geometric data

Not all data are geometric !

Hamming distance

- $\#\{x_i \neq y_i\}$ (Hamming distance)
- Levenshtein distance for strings (allows deletions and additions)

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- respect the **triangular inequality**
  $\forall x, y, z, d(x, y) \leq d(x, z) + d(y, z)$

General definition of a distance

We could verify that :

- L2 is a distance
- Hamming is a distance

Similarities

Sometimes, it is not possible to define a proper **distance** in the input space $\mathcal{X}$! This may happen for instance is $\mathcal{X}$ is a dataset of texts.

- When distances are unavailable, we can use **Similarities** or **Dissimilarity** to compare points.
- Dissimilarites are more general and don't always abide by the distance axioms.
- Other examples : Adjacency in an oriented graph, Custom agregated score to compare data.

Example : cosine similarity

The **cosine similarity** may be used to compare texts.
If u and v are vectors,

$$S_C(u, v) = \frac{(u|v)}{||u||||v||} \tag{5}$$

- the **bag of words representation** allows us to build a vector from a text (one hot encoding).
- **cosine_similarity/scraper.py**
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Hybrid data

Sometimes each sample contains both numerical data and non-numerical data (text, categorical data.)
See **hybrid_data/**
This is often the case in machine learning applications ! (database of customers, database of cars, etc.)

Exercice

Manual distances computations.